Introducing Make-to-stock to Make-to-order Platforms

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Abstract

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Many popular apparel print-on-demand (POD) services, such as Merch by Amazon, use the POD approach to meet scattered demand for a large variety of products on their platform. The POD approach is a form of the make-to-order (MTO) production mode; its flexibility in handling variety provides a quick response to uncertain customer demands, but its unit production cost is higher than the batch-producing make-to-stock (MTS) production mode.

The business model for POD platforms is as follows. The platform licenses product designs from content creators and sells them on the website. These designs all use the same basic blanks (t-shirts, hoodies, tank tops, etc.). Once an order is received, the platform prints the requested design on site and ships the item(s) to the customer. The platform also handles all logistical issues, such as payment. The content creator receives a certain portion of each sale. The content creator is responsible only for making product decisions: managing portfolio size, marketing, pricing and so on.

Compared to MTO, the traditional MTS production mode has advantages from a cost perspective. MTS production is cheaper per unit but is less flexible, and there is a fixed fee associated with each order. Its effective use requires demand to be sufficiently high and less variable. In this dissertation, we study whether MTS can be integrated into POD

platforms and, if so, how to do so most effectively. In our analysis, we find that in the real-life decentralized system where the platform does not control what the content creators do, content creators will take actions that are optimal for themselves, but the result is that product demands are not “good” enough for the platform to use MTS. To address this issue, we take actions in two areas: product assortment and effort allocation. They are presented in Chapter 3 and Chapter 4, respectively.

In Chapter 3, we study how the platform can limit the number of designs by a content creator so that demand can be more concentrated in each product, making MTS effective and improving profit at the same time. We use a Salop’s circular model to reflect the underlying competition among products and to derive the relationship between the number of products and individual product demand. We propose a hybrid production mode and find conditions under which the hybrid mode can offer a lower overall cost than the MTO production mode. To implement this approach in a decentralized setting where the platform can induce but not control the actions of the content creators, however, we find that the platform must overcome an incentive misalignment with independent content creators. This misalignment occurs because the content creator’s profit is determined only by sales, as is the prevalent practice on such platforms nowadays; consequently, content creators will offer as many designs as possible on the platform to maximize sales. However, the platform prefers to have a limited number of designs so that each design will have enough demand to employ hybrid production. To overcome this misalignment, we first analyze a centralized system to set a target for improvement and propose that the platform incorporate a listing fee for each item. We show that this simple addition can effectively align the content creator with the platform and improve profitability for both. Numerical studies demonstrate the potential profit improvement and identify conditions under which the approach using hybrid production and a listing fee can yield significant profit improvement. In particular, we find when the profit margin on each product is lower or the unit production cost saving is significant, the hybrid production mode can have the most positive impact on system profit.

Inspired by the success in profit improvement achieved by the hybrid production mode, we study in Chapter 4 another operational aspect that can impact the addition of MTS to MTO; namely, how content creators allocate effort when adopting the hybrid production mode. Content creators can exert effort in various ways, including polishing their designs and marketing. Content creators promise to invest a fixed amount of effort for their products to be listed on the platform. With diminishing returns of the effort's effect on demand, we find that content creators prefer to spread their efforts across all products in their portfolio. When a content creator's amount of effort is low, this can cause the demand for an individual product to be too low for the platform to use the hybrid production mode. To address this inefficiency, we propose that the platform adopt a coordinating strategy for the hybrid mode that rewards the products with higher demand with a higher portion of the profits. We show that this strategy can effectively encourage content creators to concentrate their effort on fewer products so that individual demands for these products are high enough to justify hybrid production. Compared to the approach of limiting the number of designs in Chapter 3, here in Chapter 4 we argue that the coordinating strategy is less restrictive on total demand because non-promoted products remain in the portfolio, generating demand that can be satisfied using MTO production mode.

In Chapter 4, we explore another strategy to enhance the implementation of the hybrid production mode by proposing that the platform exert additional effort to increase demand. These efforts from the platform can take the form of enhancing customer service, promotions for the platform, sharing data with content creators on trending designs, and so on. We first find the optimal amount of platform's effort; we then show that, with a small enough unit effort cost for the platform, a cooperative effort strategy can enable the hybrid production mode to achieve higher system profit improvement than the coordinated hybrid mode and can eliminate the system's incentive misalignment regarding allocation of effort. Numerical studies further quantify the magnitude of potential profit improvement achieved by both the coordinated hybrid production mode and the cooperative effort strategy and demonstrate that even when the platform exerts less than the optimal amount of effort, system profit can still benefit from the additional effort that the platform does exert.

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priest. I can always count on his help when faced with difficulties.

DEDICATION

To my loving parents, Shiming Ma and Geng Li,
and my dear husband, Ryan Lopopolo.

Chapter 1

INTRODUCTION

Make-to-order (MTO) refers to the production mode in which products are manufactured after a customer has placed an order. The MTO mode produces small batches that are specific to demand, and those batches can contain different products. By contrast, in the traditional make-to-stock (MTS) production mode, products are manufactured in advance of demand and then stockpiled as inventory for customers to purchase. The MTS mode produces larger batches of one design. Comparatively, MTO allows customers to purchase products customized to their precise requirements and saves manufacturers the cost of holding finished goods in inventory. It usually also has a shorter production lead time and a higher unit cost.

T-shirt and apparel print-on-demand (POD) platforms, which have become increasingly popular in recent years, use the MTO production approach. A report by Berzina-Pudule (2021) indicates that the industry has grown by 12% cumulatively over the previous four years. The t-shirt printing business alone was valued at $3.64 billion in 2020 and is projected to reach $7.57 billion by 2028. Another report, Dukurs (2021), found that 49% of Americans are interested in buying customized items, which suggests the vast potential of this niche market.

There are many POD platforms in the market; notable examples are Printify, Printful, Print Aura, Merch by Amazon and Redbubble. These platforms partner with content creators and print customized graphics on blank products. While Printify, Printful, and Print Aura provide integration with e-commerce platforms such as Shopify, Etsy, and Wix to help content creators to sell products, Merch by Amazon and Redbubble sell POD products on their own marketplaces. Our research questions are motivated by Merch by Amazon, and we focus on its business processes in our modeling approach while noting that our analysis is generalizable to other platforms.

The Merch by Amazon business model involves three parties: the POD platform (Merch), content creators, and customers. The POD platform allows individuals to register as content creators and upload their unique 2-D graphic designs, which the platform will list on its website as final products to be printed on t-shirts, hoodies, tank tops, and other items to attract customers. The designs are screened to meet the Merch content policy's requirements, which forbid intellectual property (IP) infringement and offensive or controversial content[1]. The Merch platform carries inventory of blank t-shirts, hoodies, and the like in different colors and sizes in their warehouses, all sourced from a single supplier. Once an order for a specific product is received, the Merch platform collects payment, prints the design on-site on the blank material, and ships the final product directly to the customer placing the order. The Merch platform bears the costs for blank product purchases, printing, and shipping. Revenues are shared between the Merch platform and the content creator, according to a publicly available formula.

Content creators set the product price and invest the time and effort into creating the designs to be listed on the Merch platform. They also have the freedom to choose which product options (type, size, color, etc.) to make available to customers. The profit the content creators receive on each unit sold is a portion of the price of the product using the published formula.

Customers browse through products with different designs on the platform's website, check their prices, and then decide whether to make a purchase and, if so, which design to choose. Customer tastes can be heterogeneous, and the closer the design is to what a given customer is looking for, the more likely a product with that design will be purchased.

Merch's POD production process can be categorized as an MTO and boasts lower inventory risk and shorter time to respond to out-of-stock situations. It also affords the platform

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[1] See `https://merch.amazon.com/resource/201858630`.

the opportunity to monetize the creativity of content creators and to invest in unique customized products for customers. Because the Merch platform prints the final product in small batches (often one at a time), the unit production cost is higher than when using traditional MTS production. In the operations context, MTO extends and combines the concepts of lean manufacturing, push-pull demand, and delayed differentiation.

Currently, all POD platforms use MTO to fulfill orders because, even though the platforms could go to an outside vendor that could offer a lower unit production cost through MTS, the outside vendor would also require much longer lead times; with MTO, there is effectively no lead time, as long as the blank materials are on hand. In order to fulfill customers' orders quickly (a crucial competitive factor), platforms would have to stock up on inventory in anticipation of customers' orders. Given the high uncertainty of the order quantity for each design (especially when the number of designs is high), this could result in both high leftovers and high stock-out probabilities. Hence, using the MTO mode is a rational choice that the platforms make when deciding between the two options.

We see an opportunity to improve on the MTO production strategy if we can properly combine it with an MTS production mode. We propose such a hybrid MTO–MTS strategy in which the platform procures (or produces) different products in predetermined MTS quantities and uses them to fulfill the orders first. If that inventory runs out, the platform then switches to MTO to fulfill the rest of the orders. Essentially, to effectively combine the advantages of both modes, the lower-unit cost MTS mode is used to provide a base level of supply, and the faster, more flexible MTO mode is used to absorb the remaining, variable portion of demand. However, the hybrid production mode inherits from MTS a non-zero fixed fee associated with each product to reflect the setup costs incurred in both production and inventory management. Thus, for hybrid production to be effective in reducing operational costs, individual product demand needs to be high enough and stable enough to justify the fixed cost. The description of the POD platform business mode above shows that the platform does not control content creators' choices. In Chapter 3, the content creators choose the number of products to list on the platform, and in Chapter 4, the content creators choose the number of products to promote. In both these essays, we find that content creators will take actions that are optimal for themselves, but the result is that they generate product demand patterns that are not amenable to the platform's use of the hybrid mode. To address this issue, we take actions from two perspectives in this dissertation: product assortment and effort allocation.

In the first essay, we study how to limit the number of designs by a given content creator so that demand can be more concentrated in each product, making MTS effective and improving profit at the same time. Using Salop's circular demand model, we derive the relationship between the number of products and individual product demand. We first examine the current decentralized setting and find that the hybrid production mode cannot be effectively used due to an incentive misalignment with content creators that occurs because their profit on such platforms is determined only by sales; thus, they can offer a theoretically infinite number of designs to meet all possible customer preferences and maximize sales. However, the platform prefers to carry a limited number of designs so that demand for individual products is high enough to justify using the hybrid production mode. To address this misalignment, we analyze a centralized system to set a target for improvement and propose a coordinating contract where the platform charges a listing fee for each design. We prove that this simple addition allows both the platform and content creators to capture the maximum possible profit.

In the second essay, we focus on how the content creators' allocation of effort affects the adoption of the hybrid production mode. The content creators' effort includes polishing their designs, marketing, and so on. Content creators promise to invest fixed amount of effort for their product portfolio to be listed on the platform, and the platform routinely terminates content creator's accounts if they do not show evidence of the required level of effort. Assuming the effort's effect on product demand has diminishing returns, we see that the content creators prefer to spread their effort across all products in their portfolio.

When the amount of content creator effort is low, this could cause demand for individual products to be too low for the platform to use the hybrid production mode. To overcome this inefficiency, we propose that the platform adopt a coordinating strategy for the hybrid mode, which rewards products with higher demand with a higher portion of the profits. Furthermore, we explore an alternative strategy of cooperative effort, which enhances the profitability of the hybrid production mode. The platform can exert effort in addition to the effort exerted by content creators to increase individual product demand, effectively use the hybrid mode, and make the overall pie bigger. We analyze the condition under which the cooperative effort strategy can further improve the profitability of the hybrid production mode and eliminate the incentive misalignment on the allocation of effort in the system.

To the best of our knowledge, this is the first study to examine the novel problem of operations improvement with a combined MTS and MTO production mode for POD platforms. We propose three creative yet practical strategies that have non-trivial improvement on profitability of the system. The models that we formulate in this dissertation are both tractable and representative of reality.

Chapter 2

LITERATURE REVIEW

The two essays in this dissertation study the same POD platform operations problem from two different perspectives. This problem relates to the streams of literature on MTS and MTO comparison and dual and reactive sourcing, which are reviewed in Section 2.1, together with the supply chain contracting literature and product assortment literature, which are also relevant for the first essay. In Section 2.2, we review the literature related to effort-dependent supply chain contracting and cooperative advertising.

2.1 Managing product portfolio when introducing MTS into MTO platforms

There are many studies comparing the MTS and MTO production modes. Popp (1965) is one of the earliest papers that examines the stochastic inventory problem of a product with zero replenishment lead time. He compares the effects of using MTS and MTO strategies on the inventory cost of such a product. Li (1992) considers customers with delivery time sensibility and the competition between firms for early delivery, builds a stochastic model for inventory, and derives conditions under which a firm chooses MTS vs. MTO. Rajagopalan (2002) adopts a non-linear, integer programming formula and derives a heuristic for a company with multiple products to decide which products will use MTS and which products will use MTO. All these papers treat the MTS–MTO issue as choosing one over the other for a single product. Our approach is more flexible because we use a newsvendor model for each product to determine the portion of demand to be produced with MTS and MTO so that we take advantage of both modes.

The MTO production mode studied in this dissertation, where the platform keeps blank products in stock and later prints different designs on them based on customer orders, has been studied in previous literature as delayed differentiation. Swaminathan and Tayur (1998) developed a model to solve for the optimal configurations and inventory levels of semi-finished products using a two-stage integer program with recourse. In contrast with their model, this dissertation assumes that the blank-product inventory level is ample and that the configuration level of the semi-finished products is fixed. Building on this foundation, this dissertation seeks ways to improve the manufacturer's profitability by incorporating the MTS production mode, in which fully finished products are also kept in stock and used to fulfill orders.

The MTO production mode has a shorter lead times and a higher variable cost than the MTS mode, which makes MTO comparable to the expedited supply channel and MTS comparable to the regular supply channel in a dual and reactive sourcing problem. Moinzadeh and Nahmias (1988) built an approximation of (Q,R) policy that allowed for two sets of lot size Q and reorder level R for the dual sourcing problem, with the assumption that there can only be a single outstanding order of each supply mode. Moinzadeh and Schmidt (1991), meanwhile, studied an (S-1, S) approximation model in dual sourcing mode. The proposed policy takes into consideration the age of the outstanding orders. Moinzadeh and Aggarwal (1997) extended this approach to a multi-echelon case in which each stocking location has access to two different sourcing modes and follows the (S-1, S) inventory policy, based on the remaining lead times for the outstanding orders. Veeraraghavan and Scheller-Wolf (2008) analyzed a dual sourcing problem with back orders under general lead times. They developed a dual order-up-to policy that uses regular and expedited inventory positions; this policy is proved to be nearly optimal. Allon and Van Mieghem (2010) analyzed a tailored base-surge sourcing policy where there is a constant rate of replenishment from the supplier with longer lead times and lower unit costs; emergency orders are only placed with the pricier alternative supplier when the inventory falls below a preset level. Of all these studies, this dissertation is closest to the inventory approach of Allon and Van Mieghem (2010). We both treat the supplier with a longer lead time and a lower unit price as the primary or default sourcing options and use the other supplier to prevent a loss of sales. However, this dissertation studies a single-period newsvendor model that fits the nature of apparel products on the Merch platform, considers multiple products, and incorporates an additional player in the form of content creators. The first essay adds the layer of a consumer choice model that we use to change the demand pattern to cater to the MTO–MTS hybrid production mode.

The first essay is also related to the literature on supply chain contracting. The survey paper by Lariviere (2016) and Cachon (2003)'s book chapter provide thorough reviews of the literature in this field. The earliest related work is Pasternack (1985), which studies pricing and partial return policies for a supplier of a product with short shelf life in a newsvendor setting. Various forms of contract have also been examined in the literature; for example, Cachon (2004) examined the advanced-purchase discount contract to explore the balance of inventory risk allocation between supplier and retailer compared to traditional push or pull contracts. Cachon and Lariviere (2005) compared revenue sharing contracts with buy-back, price-discount, quantity-flexible, and other contracts in a newsvendor model setting between supplier and retailer. Aydın and Hausman (2009) investigated how slotting fees paid by a manufacturer to a retailer can be used to coordinate the assortment decisions in a supply chain based on a multinomial logit consumer choice model and periodic review inventory model. The first essay is structured similarly to these studies in comparing centralized and decentralized scenarios and searching for the contract form that will optimize profit for the supply chain. The proposed approach of using a listing fee to control the number of products listed on the platform is most similar to Aydın and Hausman (2009). However, the first essay has an added layer of evaluating the novel hybrid production mode against the currently adopted MTO-only production mode and further finding the right contract form for the most efficient production mode. The design licensing nature of the situation studied in the first essay means that the platform is both manufacturer and retailer, with content creators supplying only designs. This setup differs from the traditional supplier–

manufacturer and retailer arrangement.

Furthermore, the coordinating mechanism of limiting the number of products offered on the platform places the first essay in the literature of assortment planning. Mahajan and Ryzin (1999)'s book chapter provides an in-depth review of literature in this field. Smith and Agrawal (2000), Mahajan and Van Ryzin (2001), Noonan (1993), and Ryzin and Mahajan (1999) are all representative works in this area and study joint assortment planning and inventory management under consumer choice and stock-out-based substitution. Studies like Gaur and Honhon (2006) and Honhon et al. (2010) build on previous literature by proposing search algorithms and heuristics to determine the optimal assortment and inventory levels for these models. Similar to the previously reviewed Aydιn and Hausman (2009), our first essay lies at the intersection of assortment planning and supply chain contracting and considers the decentralized nature of the supply chain channel.

2.2 Managing demand when introducing MTS to MTO platforms

There is supply chain contracting research that studies effort-dependent demand model coordination; again, Cachon (2003)'s book chapter provides an extensive review of that literature, pointing out that when a retailer's effort and ad spending can be observed by the supplier, verified by the courts, and directly benefit the supplier, a cost-sharing contract can achieve effort coordination in the system. When these conditions are not met, a quantity-discount contract can coordinate the system.

The hybrid mode coordinating strategy of imposing royalty parameter discrimination in the second essay is similar to the quantity-discount contract. Jeuland and Shugan (1983) is among the early work on coordinating the manufacturer–retailer–consumer channel with a quantity-discount contract; they also consider a model extension to include promotional effort. Desai and Srinivasan (1995) study the coordination between a franchiser and a franchisee with a two-part pricing scheme. They model effort as the choice between adopting high or low service levels. Krishnan et al. (2004) discussed the category of non-linear, quantity-dependent pricing schemes, including quantity discounts and two-part tariffs to coordinate the supply chain between the manufacturer and retailer for both ordering quantity and promotional effort. Compared to these studies, our contract encourages content creators to induce more individual product demand instead of focusing on coordinating ordering quantity. With their limited effort budgets, content creators need to reconfigure their effort allocation schemes and bear the total demand loss caused by the diminishing returns of effort in order to take advantage of the contract.

The alternative hybrid mode-enhancing strategy of cooperative effort in the second essay is related to the cost-sharing contract, which aims to address the issue that arises when effort increases the profits for both parties in the supply chain, while the costs are borne by only one party. Sharing the costs of effort in forms of cooperation by adding to the effort invested, subsidizing by offering to pay for a portion of the effort, and endorsing by offering the platform's or supplier's existing brand recognition influence to the end products are some of the approaches that can be taken to coordinate the systems. Netessine and Rudi (2000) study drop shipping models between a wholesaler and a retailer with different relative decision power levels and propose that the wholesaler subsidize part of the retailer's marketing cost to coordinate the system. Chu and Desai (1995) compare a contract in which the manufacturer partially covers the retailer's customer satisfaction cost against a contract that pays the retailer a bonus based on a customer satisfaction index. They use a two-period model and find the cost-sharing contract to be more efficient at coordinating the channel when the retailer values the long term benefit of customer satisfaction. Lal (1990) models the supplier or franchiser's effort in promoting the brand name to boost product demand and points out that the supplier or franchiser's commitment to effort supplement is nonenforceable under the contract. Ren and Zhou (2008) investigate the outsourcing supply chain between a company and a call center the company uses for customer service. For the situation where the call center's effort can be observed and verified, they propose a cost-sharing contract with pay per call resolved to coordinate both staffing and effort levels for the call center. Although most of the cited papers adopt a contract form in which the supplier subsidizes part of the retailer's advertising cost, there are also papers such as Lal (1990) and Xie and Wei (2009) that cover strategies similar to the cooperative effort approach in our study, in which the supplier invests in effort or advertising expenditures that do not depend on how much the retailer or franchisee spends on advertising. Unlike Lal (1990), we do not face the nonenforceable contract problem for the platform, because the platform's additional effort increases the platform's profit. We also differ from Xie and Wei (2009) because in addition to the supplier committing to additional advertising costs to boost product demand, they also have the supplier subsidizing part of the retailer's advertising expenses.

The literature cited above all studies channel coordination between a supplier, manufacturer, or franchiser and a retailer or franchisee. Our second essay makes a novel advance on previous research because we apply the demand model and hybrid mode-coordinating strategy along with the cooperative effort strategy to the dynamic between the POD platform and the content creators with operational cost considerations. In this dynamic, the POD platform is responsible for the production and shipping of goods and connecting to customers, while the content creators are responsible for creating the IP and investing effort into promoting their products.

The cooperative effort strategy in our study is also related to the cooperative advertising research stream. Jørgensen and Zaccour (2014)'s survey paper provides an in-depth review of this field. Berger (1972) is among the early work that studies the cooperative advertising between a supplier and a retailer from an optimization approach. Demand is modeled as a deterministic function of advertising amount by the retailer, while the supplier optimizes a fixed discount per unit to give to the retailer as an advertising allowance. Huang and Li (2001) discuss three advertising models in which the manufacturer is the leader and the retailer the follower in a non-cooperative game, a simultaneous non-cooperative game, and a cooperative game to reflect on the power shifts from manufacturer to retailer in driving channel coordination. The product demand function depends on both the retailer's advertising effort and the manufacturer's brand investment; both have diminishing returns on demand. Karray and Zaccour (2006) study the effect of a retailer's private label product (store brand) on manufacturer profit and suggest that cooperative advertising can be used by the manufacturer to counter the private label when strong competition exists. The demand model used in their paper considers the prices of the private label and national brand products, as well as the retailer's advertising expenditure on the national brand. The advertising expenditure's diminishing return effect on the demand is additive to the effect of prices on demand. Xie and Wei (2009) also look at a cooperative advertising strategy as a way to coordinate the manufacturer–retailer channel but consider product price and advertising expenditure from both the retailer and the manufacturer in their demand model. The manufacturer compensates the retailer for a portion of its advertising expenditure, on top of its own advertising investment. The advertising expenditure's diminishing returns effect on demand is multiplicative to the effect of prices on demand. We build our demand model on top of all these studies and choose an additive relationship between the diminishing effect of effort and the effect of price on product demand. We generalize advertising expenditure into effort, which includes money, time, and influence spent on marketing, and product design quality efforts that improve demand for the product. All these previous models ignore competition between retailers, suppliers, and channels; in our model, we ignore the competition between products obtained by effort investment and assume instead that competition is constant and exogenous because of the apparel market outside the platform.

Furthermore, the second essay contributes to the literature by adding the MTS production mode to the MTO production mode that is widely used by POD platforms. Other well-established topics in the literature, such as dual and reactive sourcing, also study the trade-offs between faster delivery lead time and higher unit cost, similar to the MTO–MTS debate. The related literature is reviewed in Section 2.1.

Chapter 3

MANAGING PRODUCT PORTFOLIO WHEN INTRODUCING MAKE-TO-STOCK TO MAKE-TO-ORDER PLATFORMS

3.1 Introduction

In Chapter 1, we describe in detail the business model for POD platforms and the advantages and disadvantages for both the currently employed MTO mode and the MTS production mode, which could decrease the cost of operations. We also propose a hybrid production strategy that takes advantage of the low unit cost offered by MTS and the flexibility of MTO.

Because of a non-zero fixed setup cost for each MTS inventory order, for the hybrid strategy to be effective, the demand for each design must be high enough and not too variable. In this chapter, we first determine the best hybrid strategy for the platform to lower overall production and inventory costs; we then determine the conditions under which the hybrid strategy achieves a higher profit than the MTO-only strategy.

Platforms like Merch are examples of decentralized systems that do not control their content creators' behavior. Anyone with good design ideas can join the platform and become a content creator; the cost to the content creator of listing a design is very low. Moreover, the content creator's profit is determined solely on the basis of the sales generated by their products, regardless of production costs – a prevalent practice on POD platforms like Merch nowadays. As a result of this low barrier to entry and low risk, and to maximize total sales, content creators will offer a theoretically infinite number of slightly differentiated products on the platform. Although all the designs, when considered in the aggregate, generate considerable revenue for the platform, the sales for each design are rare and scattered. In this case, even when the hybrid strategy can theoretically reduce the platform's production cost, the content creator's product assortment decision can make it unfeasible for the platform to implement.

In a centralized system where the platform employs content creators to design products or otherwise controls the product assortment decision, this misalignment does not occur because the platform can limit the number of designs on its platform to concentrate demand in a small enough number of products to use the hybrid production mode to reduce costs.

As the notion of centralizing platforms such as Merch is not presently the focus, our main research question in this chapter is how to resolve this incentive misalignment so that the MTS mode can be effectively integrated into the existing MTO mode to improve profit. To do that, we first quantify the conflicting incentives and then show that by charging an appropriate per-design listing fee, the platform can incentivize content creators to generate a demand pattern that supports the hybrid MTS–MTO production mode and increases system profitability. The listing fee is not only an effective measure that the platform can adopt but is also practical.

The rest of the chapter is organized as follows: Section 3.2 defines the product demand based on the consumer choice model and illustrates the currently adopted benchmark case with an MTO-only production mode. In Section 3.3, we characterize the optimal setup of the hybrid production mode and discuss what profit improvement the hybrid production mode can bring to a centralized system and how the platform could achieve the improved profits in a decentralized system. Following that, Section 3.4 presents a numerical study that demonstrates the magnitude of potential profit improvement resulting from our recommendations and the boundaries of parameters for when to adopt the hybrid production mode. Lastly, in Section 3.5, we conclude with a summary, managerial insights, limitations of our study, and future research directions.

3.2 Base model setup and analysis

In this section, we describe and analyze the current setup on platforms like Merch, which we denote as the base model. It serves as the benchmark for improvements that we propose in Section 3.3.

To develop sharper insights and for clarity of exposition, we assume that there is one content creator who sells n total products on the platform. The case of multiple content creators is discussed at the end of this chapter.

The most important features of the base model are that 1) the system is decentralized, meaning that the platform and the content creator make their individual decisions to maximize their own profit, and 2) the platform uses only the MTO production mode to satisfy all demand.

We use a single-period demand model to study the production decisions. There are two reasons for this choice. First, the inspiration for our study is primarily a t-shirt selling business, in which designs may have short lifespans. Second, when we introduce the MTS production mode, the inventory replenishment lead time is typically very long—most suppliers are located in Asia, while the platform sells to the North American market. Practically, the platform has one opportunity to place an MTS order for each design. The single-period model is widely used in papers studying similar situations, such as Aydιn and Porteus (2008) and Pasternack (1985).

At the beginning of the period, both the platform and the content creator know the distributions of potential demand for all products but not their exact values. Based on this distribution information, the content creator designs and lists their products on the platform. In particular, since all designs compete with one another for customers, the content creator must decide how many designs to sell (which we denote by n) and the attributes of each design. During the period, demand is realized: customers arrive with heterogeneous attribute preferences and will choose the item among the available n products that provides the maximum utility to purchase. If that maximum utility is below zero, then the customer will leave without making a purchase (we assume they are lost to outside options). When the platform receives an order, it prints the product design on a prestocked generic blank and then ships the finished product to the customer. Revenues are split between the platform and the content creator according to a formula that is agreed upon beforehand.

The rest of this section provides details of the model and the players' decisions. We describe the Salop's circular demand model in Section 3.2.1, derive the content creator's product placement decisions on the circle in Section 3.2.2, and then derive the resulting product demand distribution in Section 3.2.3. Finally, in Section 3.2.4, we derive the optimal decisions for both the content creator and the platform.

3.2.1 Salop's circular model

As all the designs on the platform compete with one another and with customers' outside options, the demand for each design depends on all other designs. To model the underlying demand function for each product, we use the circular city model from Salop (1979), which is a variant of Hotelling's model (Hotelling, 1929). We make this modeling choice because the circular city model is a spatial differentiation model, and we can use the locations on the circular market as representations of the key differentiating attributes for the product designs. The circular city model also allows for customer heterogeneity. It assumes that a customer's preference for a key attribute is located evenly on a circular market. The distance between a customer's location and a product's location represents how close the product is to the customer's ideal product and can be used to calculate the customer's utility loss, with a unit cost of c, if the customer were to purchase that product. We assume that each customer's reservation price for a product that perfectly matches their preference is represented by a random variable W. To avoid trivial cases, we assume that any possible realization of W is higher than the product price p.

As is the case on Merch, the platform may make suggestions, but it is the content creator who ultimately makes the pricing decision. We assume that the content creator prices all n products identically at p because all the printed-product types on Merch use the same generic blanks. Moreover, p is fixed because of pricing pressures from the large, competitive apparel market outside the platform. A further study on heterogeneous and possibly dynamic pricing is worth considering in the future.

To derive the demand for all products, we start with $n = 1$. Denote the location of this lone product on the circle as l. For all customers also located at l, the total potential demand for that product is linearly decreasing in the price p, which we call the *market potential* and denote by $M = W - p$. Since W is a random variable that is always greater than p, M is a positive random variable. For customers located away from l at l^*, there is an additional utility loss proportional to that distance: $c\|l - l^*\|$. Therefore, the total potential market demand at any location l^* is calculated as

$$M - c\|l - l^*\|. \tag{3.1}$$

As the customer location l^* moves farther away from the product location l, the product becomes less attractive, decreasing demand. When that distance goes beyond a certain level, there is no demand for that product. Denote that maximum distance by Δ_0; then, from Equation (3.1),

$$\Delta_0 = \frac{M}{c}. \tag{3.2}$$

Thus, the interval $[l - \Delta_0, l + \Delta_0]$ represents the range of customers for product l.

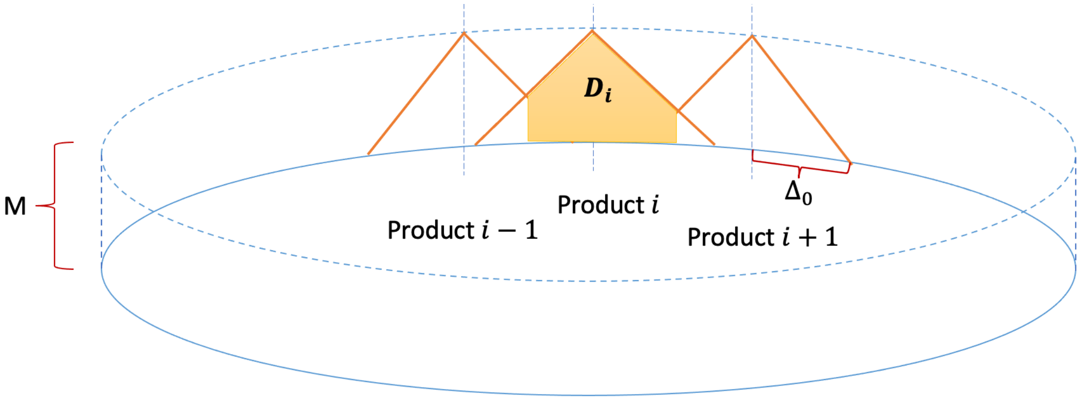

Figure 3.1: Salop's circular city model: product demand

Now consider $n > 1$. When the products are located far away from one another (i.e., at least $2\Delta_0$ away), they do not compete for the same customers. Demand for each product is represented by the area of a triangle with a base of $2\Delta_0$ centered at l and a height of M, which equals $M\Delta_0 = M^2/c$. When two products' demand ranges do overlap, however, they compete for customers in the overlapping area. Customers will choose between the two products and purchase the one that is closer. It is clear that the break-even point for customers to choose between two products lies in the middle of two products, and the product demand is calculated as the area between the break-even points of the two products (the shaded area in Figure 3.1). Thus, for a full listing of products l_i, $i \in \{1, 2, \ldots, n\}$, we can write the demand function for product i as follows:

$$D_i = \frac{M^2}{c} - \max\left(0, \frac{M}{c} - \frac{l_{i+1} - l_i}{2}\right)^2 \frac{c}{2} - \max\left(0, \frac{M}{c} - \frac{l_i - l_{i-1}}{2}\right)^2 \frac{c}{2}. \tag{3.3}$$

Note that this and the demand expressions in Section 3.2.2 are derived for any given realization of the random variable M.

All notations are summarized in Table 3.1.

Symbol	**Description**
Indices	
$j \in \{1, 2, ..., m\}$	index for possible values of market potential
$i \in \{1, 2, ..., n\}$	index for each product on the market
Parameters	
c	traveling cost on the Salop's circular market
W	random variable of customer' effective reservation price
M, M_j	random variable of market potential for the Salop's model, possible values of the random variable M
q_j	probabilities associated with D_js for the discrete distribution
l_i	location for product i on the Salop's circular market

D, D_i	random variable of demand for a product and demand for product i
n	number of products in the market
p, p_i	price of a product, price of product i
Δ_0	range from the product location within which customers are willing to buy
r	royalty that content creators receive per item sold
a	profit split portion for content creators per item sold, less minimum revenue for the platform
b	minimum revenue for the platform
c_{MTS}	unit cost of MTS production mode
c_{fix}	fixed cost of MTS production mode
c_{MTO}	unit cost of MTO production mode
c_{list}	listing fee for every product
Profit functions and products assortment decisions	
$\pi_{sys}^{D\text{-}MTO}$	decentralized total profit for the system with MTO-only production mode
$\pi_{sys}^{C\text{-}H}$	centralized total profit for the system with hybrid production mode
$\pi_{sys}^{D\text{-}H}$	decentralized total profit for the system with hybrid production mode
$\pi_{sys}^{D\text{-}H\text{-}L}$	decentralized total profit for the system with hybrid production mode and incentive-aligning contract
$\pi_{cc}^{D\text{-}MTO}$	content creators' total profit in the decentralized system with MTO-only production mode
$\pi_{cc}^{D\text{-}H}$	content creators' total profit in the decentralized system with hybrid production mode
$\pi_{cc}^{D\text{-}H\text{-}L}$	content creators' total profit in the decentralized system with hybrid production mode and incentive-aligning contract

$\pi_{pl}^{D\text{-}MTO}$	platform's profit in decentralized system with MTO-only production mode
$\pi_{pl}^{D\text{-}H}$	platform's profit in decentralized system with hybrid production mode
$n^{D\text{-}MTO}$	optimal number of products for decentralized system with MTO-only production mode
$n^{C\text{-}H}$	optimal number of products for centralized system with hybrid production mode
$n^{D\text{-}H}$	optimal number of products for decentralized system with hybrid production mode
$n^{D\text{-}H\text{-}L}$	optimal number of products for decentralized system with hybrid production mode and incentive-aligning contract

Table 3.1: Table of Notations in Chapter 3

3.2.2 Distribution of products on the circle

As is evident from Equation (3.3), the demand for a product depends on its location in relation to its adjacent products, so the total demand for all products depends on the locations of all products on the Salop's circle. If there are n products in the market, then we show in the following lemma where the products should be located on the circle to maximize demand:

Lemma 3.1. *When competing with adjacent products on both sides, the demand for a single product is maximized when this product is centered between the two adjacent products. The total demand for all* n *products is maximized when all products are evenly distributed on the circular market.*

Proof. Proofs of all the analytical results in Chapter 3 are presented in Appendix A. □

When we increase the number of products, n, the demand for each product decreases, while the total demand for all products increases. With optimal distribution of products along the circle from Lemma 3.1, the demand function for each product becomes

$$D_i(n) = \begin{cases} \frac{M^2}{c} & \text{if } n \leq \lfloor \frac{c}{2M} \rfloor, \\ \frac{M}{n} - \frac{c}{4n^2} & \text{if } n > \lfloor \frac{c}{2M} \rfloor. \end{cases} \tag{3.4}$$

The interpretation of this result is quite intuitive: we know from Lemma 3.1 that all n products are evenly located on the unit circle. We also know from Equation (3.2) that each product's maximum demand region has a length of $2\Delta_0$. So when n is small enough such that $n{*}2\Delta_0 \leq 1$, the products do not compete with one another, and each has a monopolistic demand of M^2/c. When n is large, an overlapping of customer ranges between products exists, and the single product demand function changes accordingly. The total product demand on the Salop's circle is as follows:

$$\sum_i D_i(n) = \begin{cases} n\frac{M^2}{c} & \text{if } n \leq \lfloor \frac{c}{2M} \rfloor \\ M - \frac{c}{4n} & \text{if } n > \lfloor \frac{c}{2M} \rfloor. \end{cases} \tag{3.5}$$

3.2.3 Distribution of total demand

Everything derived so far is for a fixed value of M, but as noted above, M is a positive random variable. In this section, we incorporate M's randomness to derive the distribution of total demand. Let M follow a general discrete distribution described as:

$$M = M_j \text{ with probability } q_j, (j = 0, ..., m), \tag{3.6}$$

where $M_0 < M_1 < ... < M_m$ denote all possible values that M can take, and $\sum_{j=0}^{m} q_j = 1$. We denote the mean and standard deviation of this distribution as $E(M)$ and $\sigma(M)$, respectively. The graph in Figure 3.2 shows one possible distribution.

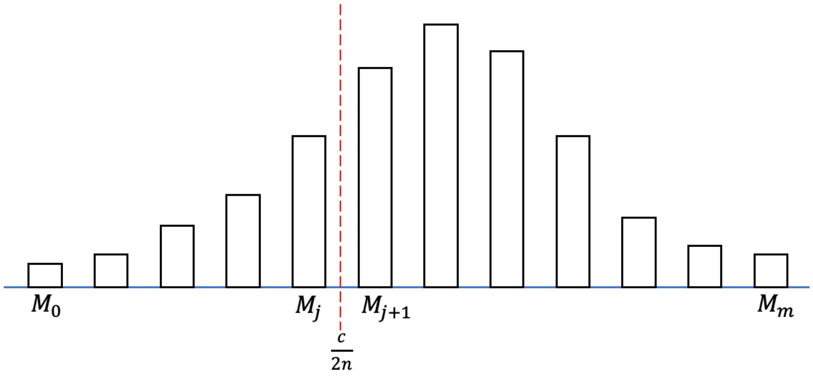

Figure 3.2: Discrete distribution for market potential M

The demand for each product clearly depends on the materialized market potential M and the number of products n on the platform. Using Equation (3.4) and Figure 3.1, we see that:

- When $n > \frac{c}{2M_0}$, all products are competing with one another, regardless of the realized value of M.

- When $\frac{c}{2M_{j+1}} < n \leq \frac{c}{2M_j}$ for some $0 \leq j < m$, products are competing with one another if and only if the realized $M > M_j$.

- When $n \leq \frac{c}{2M_m}$, none of the products is competing with one another, regardless of the realized value of M.

Now that we know how the demand for each product can be determined from Equation (3.4), we in effect know the whole distribution of the demand for each product and can calculate all its moments. Below, we show the expressions of its first two moments, the expected value $E(D)$, and the standard deviation $\sigma(D)$. Since all products are identical, we drop the i subscript notation for ease of exposition whenever there is no confusion.

$$E(D) = \begin{cases} \frac{E(M)}{n} - \frac{c}{4n^2} & \text{when } n > \frac{c}{2M_0}, \\ \sum_{k=0}^{j} \frac{M_k^2}{c} q_k + \sum_{k=j+1}^{m} \left(\frac{M_k}{n} - \frac{c}{4n^2} \right) q_k & \text{when } \frac{c}{2M_{j+1}} < n \leq \frac{c}{2M_j}, \\ \sum_{k=0}^{m} \frac{M_k^2}{c} q_k & \text{when } n \leq \frac{c}{2M_m}. \end{cases} \tag{3.7}$$

$$\sigma(D) = \begin{cases} \frac{\sigma(M)}{n} & \text{when } n > \frac{c}{2M_0}, \\ \sqrt{\sum_{k=0}^{j} \left(\frac{M_k^2}{c} - E(D) \right)^2 q_k + \sum_{k=j+1}^{m} \left(\frac{M_k}{n} - \frac{c}{4n^2} - E(D) \right)^2 q_k} & \text{when } \frac{c}{2M_{j+1}} < n \leq \frac{c}{2M_j}, \\ \sqrt{\sum_{k=0}^{m} \left(\frac{M_k^2}{c} - E(D) \right)^2 q_k} & \text{when } n \leq \frac{c}{2M_m}. \end{cases} \tag{3.8}$$

3.2.4 Decisions and system performance

At the beginning of the period, the content creator decides to offer n products on the platform and designs their attributes—namely their locations, l_i, $i \in \{1, 2, \ldots, n\}$, on the circular market. Since we know from Lemma 3.1 that all n products will be evenly distributed on the circular market, their locations naturally follow the selection of n. Therefore, the content creator only needs to determine n.

Then, the market potential M is realized according to Equation (3.6), and sales for each product take place during the period according to Equations (3.7) and (3.8). For each sale, the revenue p is shared between the content creator and the platform based on a publicly disclosed formula. We adopt a linear formula in which the content creator's share of each sale is

$$r = a(p - b). \tag{3.9}$$

The platform receives the remaining $p - r$ from each sale. Here, b serves as a floor for the content creator's pricing, as the platform needs to cover all production costs. Parameter a determines the revenue split for the content creator. This type of revenue split is easy to understand and implement and is commonly used in the industry[1].

Recall that in this base case the system is decentralized and the platform uses only MTO production. Hence, we call this the **Case D-MTO** and denote all case-specific expressions by a D-MTO superscript. The subscripts "cc," "pl," and "sys" refer to the content creator, the platform, and the system, respectively.

Since the platform uses MTO production to satisfy all demand and there are plenty of blank products, all customer demand will be captured. The overall expected profit function that the content creator will optimize is thus expressed as

$$\begin{aligned}\pi_{cc}^{D\text{-}MTO}(n) &= a(p-b)nE(D)\\ &= \begin{cases} a(p-b)\left(E(M)-\frac{c}{4n}\right) & \text{when } n > \frac{c}{2M_0}\\ a(p-b)\left(\sum\limits_{k=0}^{j}\frac{nM_k^2}{c}q_k+\sum\limits_{k=j+1}^{m}\left(M_k-\frac{c}{4n}\right)q_k\right) & \text{when } \frac{c}{2M_{j+1}} < n \le \frac{c}{2M_j}\\ a(p-b)\left(\sum\limits_{k=0}^{m}\frac{nM_k^2}{c}q_k\right) & \text{when } n \le \frac{c}{2M_m}\end{cases}.\end{aligned} \tag{3.10}$$

We note that $\pi_{cc}^{D\text{-}MTO}(n) = a(p-b)nE(D)$ is continuous across all three intervals in Equation (3.10). More importantly, it is also *increasing* on all three intervals. Therefore, it is optimal for the content creator to sell an infinite number of designs. The following lemma shows the content creator's optimal number of products and the corresponding profits.

Lemma 3.2. *Let $n^{D\text{-}MTO}$ denote the optimal number of products selected by the content creator in Case D-MTO. Then, we have $n^{D\text{-}MTO} = \infty$. Moreover,*

$$\pi_{cc}^{D\text{-}MTO}(n^{D\text{-}MTO}) = \lim_{n\to\infty}\pi_{cc}^{D\text{-}MTO}(n) = a(p-b)E(M). \tag{3.11}$$

Lemma 3.2 is quite intuitive. Because the content creator receives a fixed per-item profit

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[1] At present, Merch uses $r = 0.75*(p-13)$ for t-shirts on its platform (`https://merch.amazon.com/resource/201858580`). This means that for every item sold, 75% of the price, after deducting \$13, goes to the content creator.

of r, their incentive is to maximize total expected demand. From Figure 3.1, we see that the more products are evenly distributed on the circular market, the more total demand is captured. The maximum profit, which is represented by the rectangular band with height M (the realized value), is achieved with an infinite number of products. Of course, this is a theoretical limit. In reality, other practical considerations and/or limitations will result in a large but finite number of products. Nonetheless, the result of Lemma 3.2 suggests that the content creator will design and list as many products as they can. For consistency of exposition, we will continue to say that the number of designs on the platform is infinite.

The corresponding profit functions are given:

$$\pi_{cc}^{D\text{-}MTO}(n^{D\text{-}MTO}) = \lim_{n\to\infty} \pi_{cc}^{D\text{-}MTO}(n) = a(p-b)E(M), \tag{3.12}$$

$$\pi_{pl}^{D\text{-}MTO}(n^{D\text{-}MTO}) = \left[p - a(p-b) - c_{MTO}\right] E(M), \tag{3.13}$$

$$\pi_{sys}^{D\text{-}MTO}(n^{D\text{-}MTO}) = (p - c_{MTO})E(M). \tag{3.14}$$

Using a similar logic, we see that because the platform and the system receive a fixed per-item profit, they also prefer to list infinite products on the platform (though they do not make this decision). Thus, in Case D-MTO, the incentives of content creator and platform are aligned. Both parties prefer to list infinite products ($n^{D\text{-}MTO} \to \infty$) to saturate the market and maximize total demand. This preference also maximizes system profit.

3.3 Introduction of the MTS production mode

Although the MTO production mode is flexible when dealing with uncertain demand, its high unit cost presents an opportunity for the platform to achieve higher profitability. In this section, we explore the potential profit improvements by optimally supplementing the MTO production mode with MTS, which has a lower unit cost. This combination is referred to as the hybrid production mode.

Assuming that the hybrid production mode is to be used, we first characterize the optimal MTS–MTO combination in terms of inventory and production quantities in Section

3.3.1. On one hand, it is clear that having the MTS option will reduce the platform's costs. On the other, adding an MTS mode entails extra overhead cost. If this extra setup cost is higher than the cost savings, then the platform should not deploy MTS even if it has the option to do so. We investigate this issue in Section 3.3.2.

If it is optimal to use only MTO production for a product, that is often because the demand for that product is too low or its variability too high—both of which are deterrents for the MTS mode. In such cases, the platform will prefer to have fewer products so that the resultant demand for each product is more concentrated. They can do that if the system is centralized. In Section 3.3.3, we will analyze the centralized case to see what the highest possible system profit is and then try to devise a mechanism that works in the decentralized setting for the content creator to optimally reduce the number of products to the system-optimal level. This coordination strategy can lead to profit improvements for both platform and content creator.

3.3.1 Optimal MTS quantity for the hybrid strategy

The MTO production mode is straightforward: each customer order is printed based on the design and shipped to the customer. The platform can quickly capture and respond to each order since there are plenty of generic blanks. This also means that the product is printed in a location close to the customers, which has a higher cost. With the MTS production mode, the platform can order *pre-printed* products ahead of demand realization and often do so from a low-cost supplier that is farther away. While MTS offers lower production cost, it is less flexible. When used alone and demand is variable, the MTS inventory quantity is either too high (leftover) or too low (lost demand). Combining the MTO and MTS production mode has the potential to lower costs (like MTS) and still capture all customer demand (like MTO). A natural way to combine MTO and MTS is to use the following hybrid policy: forecast demand and order a quantity h (at a unit cost of $c_{MTS} < c_{MTO}$) before the sales period starts. Once the sales period begins, if all h units of inventory are depleted, then use MTO mode (at a unit cost of c_{MTO}) to satisfy the remaining demand during the rest of the period. At the end of the period, any remaining inventory is salvaged with zero value.

If the hybrid production mode is used for a given product, the only decision for the platform to make is h: how much to preprint using the MTS mode in order to maximize overall profit:

$$g(h) = p\int_0^\infty xf(x)dx - c_{MTS}h - c_{MTO}\int_h^\infty (x-h)f(x)dx, \tag{3.15}$$

where $f(x)$ is the pdf of demand D during this time period. This single-period model admits the newsvendor-type solution given in lemma 3.3. Define $\rho = \frac{c_{MTS}}{c_{MTO}}$, and $\bar{F}(d) = 1 - \int_{-\infty}^{d} f(x)\,dx$:

Lemma 3.3. *The optimal stocking level h^* that maximizes* (3.15) *is given by*

$$\bar{F}(h^*) = \frac{c_{MTS}}{c_{MTO}} = \rho. \tag{3.16}$$

In Equation (3.16), demand can have any general distribution. To derive additional analytical results, we assume for the rest of the chapter that the demand has a normal distribution, which is a good approximation of many practical distributions and is commonly used in the single-period inventory management setting. We provide additional justifications for and with numerical validation of this assumption in Appendix B. Specifically, we let the demand for a single product have a normal distribution whose first two moments match those of the general demand distribution given in Equations (3.7) and (3.8): $D \sim Normal(E(D), \sigma(D))$. To minimize possible negative demand values due to the normal distribution, we adopt the rule of thumb in the newsvendor inventory literature that $3\sigma(D) \leq E(D)$ (e.g., Agrawal and Nahmias, 1997; Lau, 1997).

With normally distributed demand, $f(x) = \frac{1}{\sigma(D)\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-E(D)}{\sigma(D)}\right)^2}$ in Equation (3.15). It is then solved to give the following:

Corollary 3.1. *The optimal profit for Equation* (3.15) *is*

$$g(h^*) = \begin{cases} (p - c_{MTS})\left(\frac{E(M)}{n} - \frac{c}{4n^2}\right) - c_{MTO}\frac{\sigma(M)}{n}\phi(\Phi^{-1}(\rho)) - c_{fix} & \textit{when } n > \frac{c}{2M_0}, \\ \begin{array}{l} (p - c_{MTS})\left(\sum\limits_{k=0}^{j} \frac{M_k^2}{c} q_k + \sum\limits_{k=j+1}^{m} (\frac{M_k}{n} - \frac{c}{4n^2}) q_k\right) - c_{fix} \\ -c_{MTO}\phi(\Phi^{-1}(\rho)) \\ \sqrt{\sum\limits_{k=0}^{j}\left(\frac{M_k^2}{c} - E(D)\right)^2 q_k + \sum\limits_{k=j+1}^{m}\left(\frac{M_k}{n} - \frac{c}{4n^2} - E(D)\right)^2 q_k} \end{array} & \textit{when } \frac{c}{2M_{j+1}} < n \leq \frac{c}{2M_j}, \\ \begin{array}{l} (p - c_{MTS})\sum\limits_{k=0}^{m} \frac{M_k^2}{c} q_k \\ -c_{MTO}\sqrt{\sum\limits_{k=0}^{m}\left(\frac{M_k^2}{c} - E(D)\right)^2 q_k}\,\phi(\Phi^{-1}(\rho)) - c_{fix} \end{array} & \textit{when } n \leq \frac{c}{2M_m}. \end{cases} \tag{3.17}$$

3.3.2 Introducing the hybrid production mode to the decentralized system

In this section, we study what will change when the platform adopts the hybrid production mode by adding MTS to MTO, as discussed in the last section. Since the platform and the content creator are still decentralized, we denote this as **Case D-H**.

Denote the system's profit by $\pi_{sys}^{D\text{-}H}(n)$, the content creator's profit by $\pi_{cc}^{D\text{-}H}(n)$, and the optimal number of products by $n^{D\text{-}H}$. The royalty contract defined by Equation (3.9) remains the same, and the content creator's profit is as follows:

$$\pi_{cc}^{D\text{-}H} = a(p - b)nE(D). \tag{3.18}$$

We see that $\pi_{cc}^{D\text{-}H}$ in (3.18) is identical to $\pi_{cc}^{D\text{-}MTO}$ in (3.10). That is, the content creator's profit function has not changed with the introduction of the hybrid production mode. This makes sense because the change in production mode affects only production cost, which is entirely borne by the platform, and thus has no impact on the content creator's profitability function. The content creator will continue to prefer to list infinite designs:

Proposition 3.1. *In a decentralized system with the hybrid production mode available,* $n^{D\text{-}H} = \infty$.

As for the platform, because of Lemma 3.1, we know that all products have identical demand, which is a function of n. Whenever MTS is used, the outsourced manufacturer can incur setup costs, and the warehouse needs to set aside or add shelf space for each product using the MTS production mode. We use a fixed cost c_{fix} to represent this fee associated with each product. Therefore, in the hybrid mode, MTS will be used for a product if and only if its profit minus c_{fix} is higher than that in the MTO-only production mode. There could be two outcomes in Case D-H:

1. adopting the MTO-only production mode for all products,

2. adopting the hybrid production mode for all products,

and the platform takes the option with higher profit. Thus, the profit function for the platform in Case D-H is as follows:

$$\pi_{pl}^{D\text{-}H}(n) = \max\begin{pmatrix} (p - a(p-b) - c_{MTS})\sum E(D) - c_{MTO}\phi(\Phi^{-1}(\rho))\sum\sigma(D) - nc_{fix}, \\ (p - a(p-b) - c_{MTO})\sum E(D) \end{pmatrix}. \tag{3.19}$$

Recall that in the decentralized system, it is the content creator who makes the product decision, and from Proposition 3.1, $n^{D\text{-}H} = \infty$. Plugging this into the platform's decision (3.19), we see that for the hybrid option, the non-zero MTS fixed cost and the infinite number of products the content creator lists drive the platform's profit to $-\infty$, which is always worse than the profit the platform receives under the MTO option. In other words, although the platform would like to make the hybrid production decision based on its own parameters, the content creator's decision to list infinite products means each will have an infinitesimally small demand and MTS would be cost prohibitive due to the fixed cost. As a result, we have

Proposition 3.2. *In a decentralized system with the hybrid production mode available, the platform will use only the MTO production mode. Moreover,*

$$\lim_{n\to\infty} \pi_{sys}^{D\text{-}H}(n) = \lim_{n\to\infty} \pi_{sys}^{D\text{-}MTO}(n) = (p - c_{MTO})E(M).$$

Proposition 3.2 indicates that simply making the hybrid production mode available makes no difference to the status quo. The content creator is not affected by the production cost, so they will continue to list infinite products on the platform. This forces the platform to stick with the MTO-only production mode. Without changing the incentive relationship between the platform and content creators, the hybrid production mode, no matter how economically attractive it could be to the platform, is not viable. All the decisions and profits in Case D-H are exactly the same as those in Case D-MTO.

3.3.3 Achieving system alignment

In this section, we propose a measure for the platform to address the incentive misalignment detailed in the previous section. To evaluate the effectiveness of the hybrid production mode, we first study a centralized system to establish the maximum possible profit improvement.

In a centralized system, decisions are taken to maximize system profit. One can think of this arrangement as having a platform that is so powerful that it makes the design decisions, in addition to the production mode decisions. We call this **Case C-H**.

We denote total system profit by $\pi_{sys}^{C\text{-}H}(n)$ when a total of n products are listed on the platform. Equation (3.17) gives the profit function for a single product under the hybrid production mode. Extending it to n identical products on the platform gives us

$$\pi_{sys}^{C\text{-}H}(n) = (p - c_{MTS})\sum E(D) - c_{MTO}\phi(\Phi^{-1}(\rho))\sum \sigma(D) - nc_{fix}$$

$$= \begin{cases} (p - c_{MTS})\left(E(M) - \frac{c}{4n}\right) - c_{MTO}\phi(\Phi^{-1}(\rho))\sigma(M) - nc_{fix} & \text{when } n > \frac{c}{2M_0}, \\ (p - c_{MTS})\left(\sum\limits_{k=0}^{j} \frac{nM_k^2}{c} q_k + \sum\limits_{k=j+1}^{m} \left(M_k - \frac{c}{4n}\right) q_k\right) - nc_{fix} & \\ -c_{MTO}\phi(\Phi^{-1}(\rho)) & \\ \sqrt{\sum\limits_{k=0}^{j} \left(\frac{nM_k^2}{c} - nE(D)\right)^2 q_k + \sum\limits_{k=j+1}^{m} \left(M_k - \frac{c}{4n} - nE(D)\right)^2 q_k} & \text{when } \frac{c}{2M_{j+1}} < n \leq \frac{c}{2M_j}, \\ (p - c_{MTS}) \sum\limits_{k=0}^{m} \frac{nM_k^2}{c} q_k & \\ -c_{MTO}\phi(\Phi^{-1}(\rho))n\sqrt{\sum\limits_{k=0}^{m} \left(\frac{M_k^2}{c} - E(D)\right)^2 q_k} - nc_{fix} & \text{when } n \leq \frac{c}{2M_m}. \end{cases} \tag{3.20}$$

Denote the optimal number of products to carry by $n^{C\text{-}H}$.

Proposition 3.3. *In Case C-H,* $n^{C\text{-}H} \leq \max(\frac{1}{2}\sqrt{\frac{(p-c_{MTS})c}{c_{fix}}}, \frac{c}{2D_0})$. *Moreover, under the following conditions,* $\pi_{sys}^{C\text{-}H}(n^{C\text{-}H}) > \pi_{sys}^{D\text{-}MTO}(n)$*:*

1. *when* $n^{C\text{-}H} = \frac{1}{2}\sqrt{\frac{(p-c_{MTS})c}{c_{fix}}}$ *and*

$$(c_{MTO} - c_{MTS})E(M) > c_{MTO}\phi(\Phi^{-1}(\rho))\sigma(M) + \sqrt{(p - c_{MTS})c_{fix}c}; \tag{3.21}$$

2. *when* $n^{C\text{-}H} = \frac{c}{2M_0}$ *and*

$$(c_{MTO} - c_{MTS})E(M) > c_{MTO}\phi(\Phi^{-1}(\rho))\sigma(M) + \frac{M_0(p - c_{MTS})}{2} + \frac{c_{fix}c}{2M_0}. \tag{3.22}$$

From Proposition 3.3, we see that when the system is centralized, the optimal number of products is finite and bounded. The system will sacrifice some demand in order to achieve a lower production cost by using the MTS mode and by focusing on a smaller number of products to avoid excessive fixed fees. A smaller number of products can also lead to higher demand for each product, which can take advantage of the lower unit cost that is a feature of the hybrid production mode. This is in contrast to the situation in the decentralized systems D-MTO and D-H, where the content creator always chooses to list an infinite number of products.

There are sufficient conditions where Case C-H yields a higher system profit than Case D-MTO, even though we list only two explicit expressions here. Conversely, when Case C-H yields a lower system profit than Case D-MTO, the platform should use the MTO-only production mode even when the hybrid production mode is available. Recall from Lemma 3.2 that both the platform and the content creators prefer to have an infinite number of products when this happens.

The system improvement in Case C-H is only feasible because centralization allows a proper trade-off between demand (experienced by both the content creator and the platform) and cost (experienced by only the platform). While it provides a goal, it is not practically feasible to centralize the system. In the many real-world platform examples that motivated our study, content creators work independently and are given substantial creative and operational flexibility. Below, we propose a measure that the platform can use to incentivize but not force the content creator to choose a system-optimal number of products. The key is to motivate the content creators to limit their number of products n.

The platform can find various ways to impose a cost on the number of products to give content creators an incentive to reduce n. Here, we choose to study the simple measure of a listing fee that is easy to understand and implement in practice. Specifically, we propose that the platform charge a listing fee c_{list} on each product design sold on the platform. We call this system **Case D-H-L**. The platform chooses the hybrid production mode if it achieves a higher profit than under the MTO-only production mode. Thus, the system profit, $\pi_{sys}^{D\text{-}H\text{-}L}(n)$, can be calculated as

$$\pi_{sys}^{D\text{-}H\text{-}L}(n) = \max \begin{pmatrix} \left((p - c_{MTS}) \sum E(D) - c_{MTO}\phi(\Phi^{-1}(\rho)) \sum \sigma(D) - nc_{fix}\right), \\ (p - c_{MTO})E(M) \end{pmatrix}. \tag{3.23}$$

The profit function for content creators, denoted as $\pi_{cc}^{D\text{-}H\text{-}L}(n)$, is as follows:

$$\begin{aligned} \pi_{cc}^{D\text{-}H\text{-}L}(n) &= a(p-b)nE(D) - nc_{List} \\ &= \begin{cases} a(p-b)\left(E(M) - \frac{c}{4n}\right) - nc_{List} & \text{when } n > \frac{c}{2M_0}, \\ a(p-b)\left(\sum\limits_{k=0}^{j} \frac{nM_k^2}{c} q_k + \sum\limits_{k=j+1}^{m} (M_k - \frac{c}{4n}) q_k\right) - nc_{List} & \text{when } \frac{c}{2M_{j+1}} < n \le \frac{c}{2M_j}, \\ a(p-b)\left(\sum\limits_{k=0}^{m} \frac{nM_k^2}{c} q_k\right) - nc_{List} & \text{when } n \le \frac{c}{2M_m}. \end{cases} \end{aligned} \tag{3.24}$$

In situations where Case D-MTO yields a higher profit than Case C-H, we know the system will choose to use the MTO-only production mode, and the platform's preferred number of products $n^{D\text{-}H\text{-}L} = \infty$. Therefore, the platform should charge no listing fee ($c_{List} = 0$), as Equation (3.24) shows. In this situation, Case D-H-L is identical to Case D-H, which itself is identical to the status quo Case D-MTO. These are situations where the MTS production mode is not appropriate; it is in fact better to adopt a strategy to list infinite products to capture all potential market demand and to use MTO to produce one product at a time to satisfy all demand.

In the other situations, MTS does have a role to play. When Case C-H yields a higher profit than Case D-MTO, we know the system will choose to use the hybrid production mode. The listing fee is paid by the content creator to the platform, so it does not affect the system's total profit function. By comparing (3.23) and (3.20), we also see that $\pi_{sys}^{D\text{-}H\text{-}L}(n) = \pi_{sys}^{C\text{-}H}(n)$ in such situations. This immediately implies that the system's optimal number of products under our proposed listing fee scheme is the same as that for the centralized hybrid system ($n^{D\text{-}H\text{-}L} = n^{C\text{-}H}$). In Case C-H, the platform can simply choose this number of products. By contrast, in Case D-H-L, the platform needs to choose an appropriate listing fee c_{List} to motivate content creators to choose the platform's preferred number of products.

We do not have a closed-form expression for the optimal number of products in Case C-H, $n^{C\text{-}H}$, but in Proposition 3.3, we show that it does have an upper bound. In Proposition 3.4, we show that by changing the listing fee c_{list}, the content creator's optimal number of products in Case D-H-L can be equal to any value between 0 and the upper bound for $n^{C\text{-}H}$. Therefore, there must exist in Case D-H-L an appropriate listing fee that can induce the content creator to choose the $n^{C\text{-}H}$ and achieve the highest possible overall system profit.

Proposition 3.4. *There exists a listing fee c_{list} value such that when the content creator is charged c_{list} for every product they list on the platform and compensated with royalty r for every unit sold, $n^{D\text{-}H\text{-}L} = n^{C\text{-}H}$ and $\pi_{sys}^{D\text{-}H\text{-}L} = \pi_{sys}^{C\text{-}H}$.*

For the listing fee scheme to be incentive-compatible so that the content creator is willing to participate, we also need to ensure that the content creator's profit in Case D-H-L is no worse than that in the current Case D-MTO. This can be done by adjusting the royalty parameter a and guaranteeing that both the platform and content creators can share the profit improvement brought about by the new contract. The simultaneous adjustments of c_{list} and a offer the platform a range of profit-sharing outcomes with the content creator. The platform can set a so that the content creator's profit in Case D-H-L is exactly the same as in Case D-MTO if the platform wants to capture the maximum amount of profit improvement, but it does not have to do so. It may be better to let the content creator make a higher profit in Case D-H-L to make its implementation easier, and our proposal is flexible enough to permit that because it lets the platform choose from a range of possible outcomes based on multiple practical considerations.

3.4 Numerical studies

In Section 3.3, we show that adding MTS to the existing MTO production mode could lead to improved profitability for the platform. Moreover, a listing fee strategy can help the platform remove incentive misalignment and induce the content creator to design the "right" number of products to achieve maximum profitability. In this section, we use comprehensive numerical studies to illustrate the conditions for and magnitude of potential profit improvement. Moreover, we conduct sensitivity analyses to understand the role of various system parameters. Finally, we investigate the optimal listing fee and how it depends on the system parameters.

3.4.1 Parameters

The parameters used in the numerical study come from both observing the Merch platform and reasonable assumptions and estimations. First, we set product price $p = 23$, which is a common price for Merch products on the Amazon website. The unit cost for MTS $c_{MTS} = \$1$ is chosen based on the unit price of one to two dollars for bulk-produced t-shirts on TaoBao.com in China. The unit cost of MTO $c_{MTO} = \$11$ is our estimate based on the royalty example provided by the Merch platform[2]. From the same example, we know that $a = 0.75$, and $b = \$13$. The traveling cost related to the Salop's circular city is normalized to $c = 1$, and the fixed cost for MTS $c_{fix} = \$0.02$ is set considering the normalization of market potential example distribution.

In Section 3.2.3, we assume that the market potential M follows a general discrete distribution. In the numerical studies, we choose 10-point discrete distribution for M. The point values and the corresponding probabilities for this distribution are shown in Table 3.2. All possible values of M are set below 0.2 so that in the Salop's circular model with a normalized market length of 1 and a traveling cost of $c = 1$, all the conditions of the demand function in Equation (3.7) can occur.

point values	$\{0.01, 0.03, 0.05, ..., 0.19\}$
probabilities	$\{0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1\}$

Table 3.2: Market potential $\boldsymbol{M}$ distribution for numerical study

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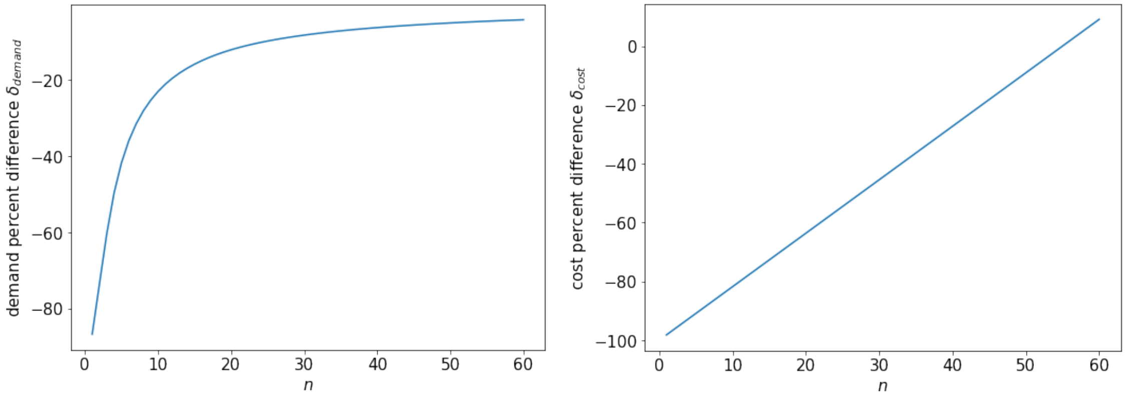
[2] See `https://merch.amazon.com/resource/201858580`.

3.4.2 Optimal number of products n

When we switch from the MTO-only mode to the hybrid production mode, we can reduce the total cost in certain situations by limiting number of products on the platform so that demand for each product is higher and can justify the fixed cost in the hybrid production mode. A downside of limiting the number of products is demand loss. The optimal $n^{C\text{-}H}$ balances the trade-off between cost savings and demand loss. We investigate how $n^{C\text{-}H}$ is identified and how it balances the trade-off by demonstrating the changes in total demand, total cost, and system profit in Figure 3.3. These value changes, denoted by δ_{demand}, δ_{cost}, and δ_{π}, respectively, are measured in percentage terms with Case D-MTO as the base case, where the content creator lists an infinite number of products:

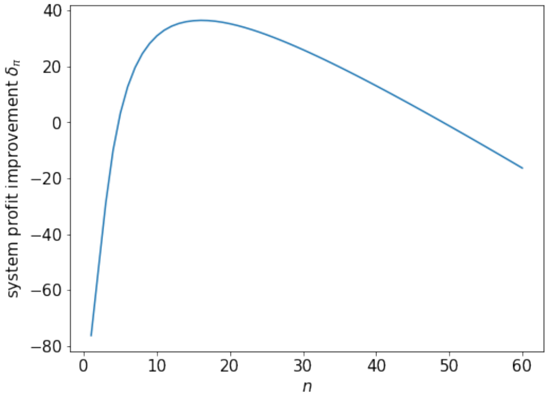
$$\begin{cases} \delta_{demand} & = \left(\frac{nE(D)(n)}{E(M)} - 1\right) \times 100\% \\ \\ \delta_{cost} & = \left(\frac{c_{MTS}nE(D)(n)+c_{MTO}\phi(\Phi^{-1}(\rho))n\sigma(D)(n)+nc_{fix}}{c_{MTO}E(M)} - 1\right) \times 100\% \\ \\ \delta_{\pi} & = \left(\frac{\pi_{sys}^{C\text{-}H}(n)}{\pi_{sys}^{D\text{-}MTO}(n^{D\text{-}MTO})} - 1\right) \times 100\%. \end{cases} \tag{3.25}$$

Figure 3.3a shows that in Case C-H, when n is relatively small, total demand increases dramatically as n increases; when n is relatively large, the rate of total demand increase is slower as n increases. But the total demand by Case C-H is always lower than what is achieved under Case D-MTO. Referring back to Figure 3.1, we see that as more designs are added to the circular market, additional demand between the peaks of the existing triangles are captured and as n gets larger, the additional spaces between them that are captured become smaller and smaller, hence the diminishing total demand increase brought about by n. This suggests that limiting the number of products from infinity to a large number causes only a small demand loss in percentage terms but can enable the platform to use the hybrid production mode, which can be more profitable. Figure 3.3b shows that the



(a) total demand

(b) total cost



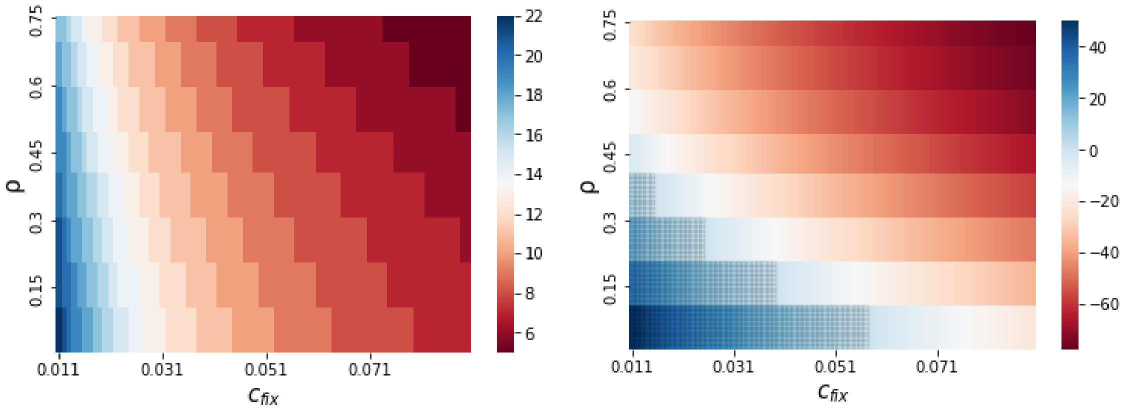
(c) system profit improvement

Figure 3.3: Total demand, total cost, and system profit improvement in *Case C-H* as % change compared to *Case D-MTO* in percentage terms using the parameters in Section 3.4.1

total cost incurred in Case C-H increases almost linearly with the number of products n and exceeds the cost in Case D-MTO when n is large. Considering these two figures, it is intuitive that the optimal number of products for Case C-H, $n^{C\text{-}H}$, should be in the range where the increase rate for total demand decreases dramatically so that the majority of potential market demand is captured while the total cost is not too high. This intuition is reflected in Figure 3.3c.

Figure 3.3 shows how the optimal number of products, $n^{C\text{-}H}$, is determined for one parameter combination. Next, we study how $n^{C\text{-}H}$ is affected by the trade-off relationship between the unit cost saving vs. the fixed cost in the hybrid production mode. Figure 3.4a is a heat map for $n^{C\text{-}H}$ over the unit cost saving parameter ρ and the fixed cost c_{fix}. Here, we vary c_{fix} around the value given in the example parameter of \$0.02, and vary ρ by changing the value of c_{MTS} while fixing the value of c_{MTO} at \$11. The graph indicates that when the fixed cost is low and the difference between c_{MTO} and c_{MTS} is significant, the optimal $n^{C\text{-}H}$ can be high. This means that the more ideal MTS costs are, the more products the system can optimally make with the hybrid production mode. Doubling the fixed cost has a larger impact on the value of $n^{C\text{-}H}$ than doubling the unit cost saving parameter ρ; thus, if the costs are comparable, reducing c_{fix} is much more effective than reducing ρ. We also see that the highest value of the optimal number of products in Case C-H is around 22 for the set of parameters chosen, which is significantly different from that of Case D-MTO, in which $n^{D\text{-}MTO} = \infty$.

The trade-off relationships between the unit cost saving and the fixed cost also affects the decision to switch from MTO-only to hybrid production. In Figure 3.4b, we demonstrate how the value of δ_π changes over the combination of unit cost saving parameter ρ and fixed cost c_{fix}. We see that when the fixed cost is low and the unit cost saving is significant, switching from MTO-only to hybrid production can achieve a very large profit improvement that in some cases can be over 40% for the set of parameters we choose. The shaded area in Figure 3.4b shows the parameter boundary of when adopting hybrid production is beneficial



(a) optimal number of products $n^{C\text{-}H}$

(b) System profit improvement δ_π

Figure 3.4: $n^{C\text{-}H}$ and δ_π over the trade-off of unit cost saving vs. fixed cost

to the system. When the value of ρ and c_{fix} are in the dark blue area, the platform should adopt the hybrid production mode; otherwise, the platform should adopt the MTO-only production mode. Within the area for the hybrid production mode, as the value of ρ decreases further, the unit cost saving from the hybrid production mode is more significant and can make up for a higher fixed cost and still achieve a higher system profit than the MTO-only production mode.

3.4.3 Sensitivity analysis

In the previous section, we show how both the optimal number of products in Case C-H $n^{C\text{-}H}$ and the system profit improvement δ_π change over the trade-off between unit cost saving and fixed cost parameters; all other parameters are fixed. In this section, we conduct a more extensive sensitivity analysis of $n^{C\text{-}H}$ and δ_π over other parameters. Specifically, we change the selling price p and the combination of MTS unit cost c_{MTS} and MTO unit cost c_{MTO} and study the resulting changes in $n^{C\text{-}H}$ and δ_π. Because we do not have a closed-form solution for $n^{C\text{-}H}$, its value is found via a grid search. The range of parameters being searched is shown in Table 3.3 and is set around the practical parameter values used in the previous example.

parameter name	range
p	$\{13, 14, 15, ..., 27\}$
c	1
c_{fix}	0.02
c_{MTO}	$\{8, 9, 10, 11\}$
c_{MTS}	$\{1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5\}$

Table 3.3: Grid search parameter ranges

We summarize the results of the grid search in Table 3.4, showing the trend of how key values of the system, such as the percentage increase in system profit δ_π and the optimal number of products $n^{C\text{-}H}$ change, along with parameters like p and c_{MTS}, for different combinations of other parameters. The slopes for each parameter combination are calculated using the beginning and end values for each line with the corresponding lowest and highest values of p or c_{MTS}.

Table 3.4 shows that as p increases, the trends of δ_π vary and are sometimes negative and sometimes positive, based on other parameter values. At the same time, the trends of $n^{C\text{-}H}$ are all positive. This situation is examined more closely in Figures 3.5 and 3.6. We see that as p increases, the profit margin for each product increases. The platform is more interested in capturing demand than saving production costs. Thus, in this situation, the optimal number of products is usually larger, making the centralized system more similar to the decentralized system. This is also reflected in the fact that the percentage difference

Dependent var.	Independent var.	Min (slope)	Max (slope)	Median (slope)	Mean (slope)	Std (slope)
δ_π	p	-29.30	1.14	-3.47	-5.32	6.17
$n^{C\text{-}H}$	p	0.43	0.5	0.43	0.45	0.03
δ_π	c_{MTS}	-112.49	-6.21	-13.58	-18.55	15.26
$n^{C\text{-}H}$	c_{MTS}	-0.67	-0.33	-0.67	-0.53	0.165

Table 3.4: δ_π and $n^{C\text{-}H}$ trend summary when varying different parameters

in system profit δ_π is close to zero when p is large in all cases. However, when the margin is relatively small, the situation is different. When ρ is small, and the introduction of hybrid production mode can bring very significant unit cost saving, δ_π is positive; otherwise, δ_π is negative. Therefore, the important managerial insight to take away from Figures 3.5 and 3.6 is that the introduction of a hybrid production mode is most effective for improving the system's profit when the margin of the product is small and the unit cost saving is significant.

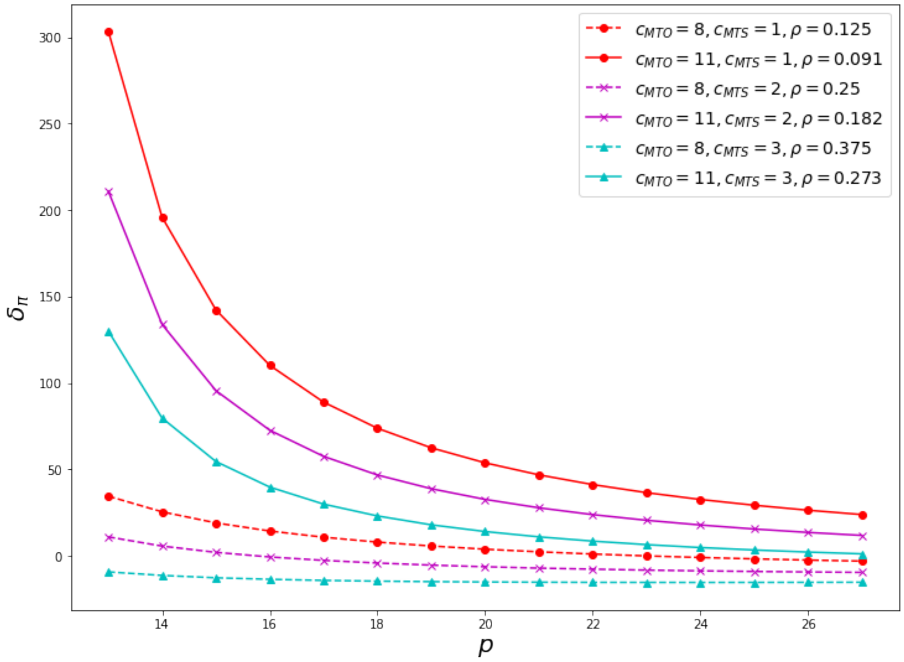


Figure 3.5: Changes in system profit improvement δ_π as price p changes

Table 3.4 also demonstrates that increasing unit cost c_{MTS} will cause a decrease in both δ_π and $n^{C\text{-}H}$, because increasing c_{MTS} makes the hybrid production mode more expensive. As a result, the hybrid production mode becomes less desirable than the MTO-only production mode. To make up for the fixed cost that comes with the hybrid mode, each product should have an even higher mean demand, which requires $n^{C\text{-}H}$ to be smaller. But in the meantime, the concentration of demand will cause more total demand loss, which deepens

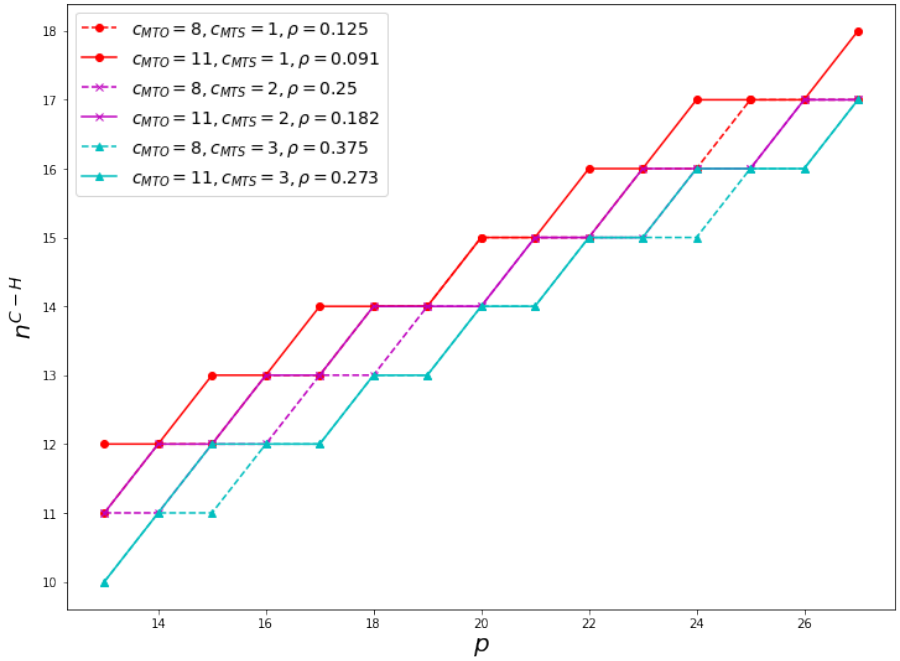


Figure 3.6: Changes in optimal number of products in *Case C-H* $n^{C\text{-}H}$ as price p changes

the loss of advantage of using hybrid production over MTO-only production. Figure 3.7 demonstrates this situation in greater detail. For the purpose of readability, we demonstrate only parameter combinations when $c_{MTO} = \$9$ in the figures, consistent with our example. Figure 3.7 shows that as the product margin decreases, lowering the MTS unit cost has a greater impact on system profit improvement δ_{π}.

3.4.4 Setting the optimal listing fee

So far, we have analyzed the misalignment in the system and demonstrated the situations in which it is most pronounced. In this section, we focus on the cases where misalignment happens and investigate how to set a listing price c_{list} in Case D-H-L to maximize system profit $\pi_{sys}^{D\text{-}H\text{-}L}$. In cases where the hybrid production mode fails to improve system profit, the platform can simply set the listing price $c_{list} = 0$ to encourage the number of products and adopt the MTO-only production mode. Figure 3.8 shows the relationship between the

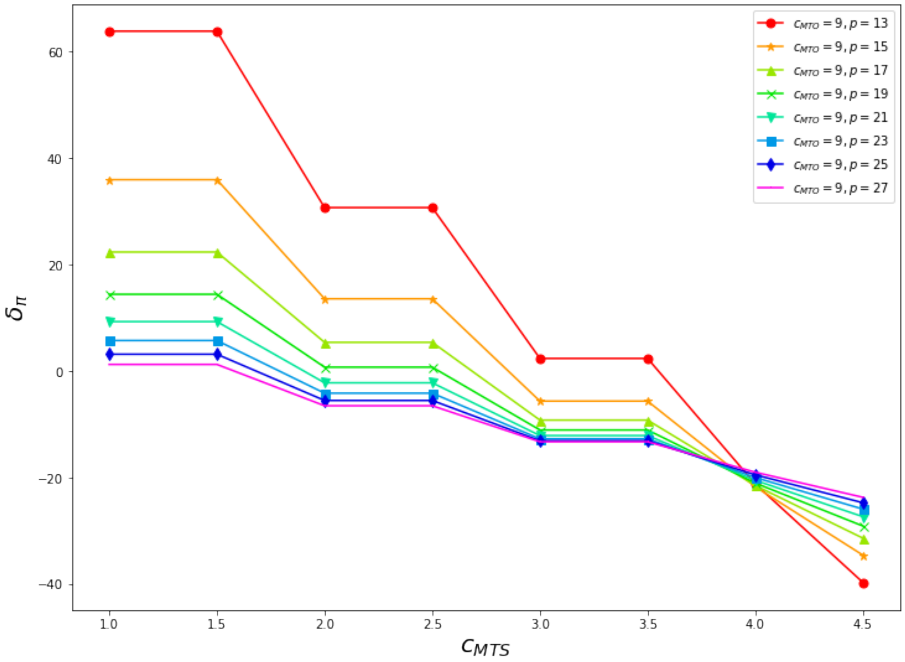


Figure 3.7: Changes in system profit improvement δ_π as MTS unit cost c_{MTS} changes

value of c_{list} set by the platform and how the content creators are going to choose the optimal number of products $n^{D\text{-}H\text{-}L}$. We see that the number of products chosen by the content creators decreases rapidly as the listing fee increases when the listing fee is small. This indicates that charging even a small amount as a listing fee goes a long way toward reducing the number of products listed on the platform.

Using this relationship, combined with the boundary for when $\delta_\pi > 0$ shown in Figure 3.4b and the value for $n^{C\text{-}H}$ shown in Figure 3.4a, we can calculate the value of c_{list} that can achieve optimal system profit for Case D-H-L, as shown in Figure 3.9. For areas where $\delta_\pi > 0$, c_{list} is set such that $n^{D\text{-}H\text{-}L} = n^{C\text{-}H}$. The value for c_{list} in this area increases with both ρ and c_{fix}. Intuitively, when both the unit cost and fixed cost increase, the platform needs to increase c_{list} to share more production costs with content creators. From what we observe in the numerical study, the listing fee is around 30% to 40% of the fixed cost and increases slightly with the unit cost saving parameter ρ. We see that the non-zero area for

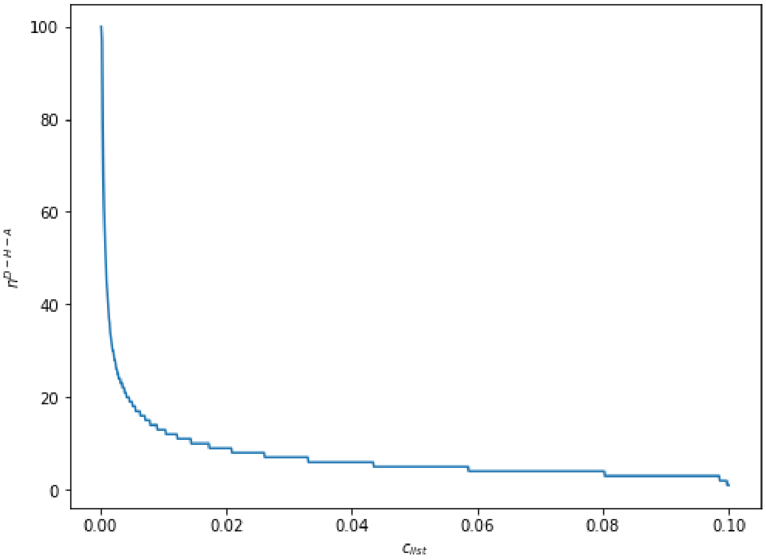


Figure 3.8: Relationship between listing fee c_{list} and number of products in *Case D-H-L* $n^{D\text{-}H\text{-}L}$ using the parameters in Section 5.1

the optimal listing fee has the same shape as the shaded area in Figure 3.4b, where the hybrid production mode is more profitable. Inside the non-zero area for the optimal listing fee, the values for the listing fee follows the same pattern as the optimal number of products $n^{C\text{-}H}$ in Figure 3.4a, as we adjust the listing fee to match $n^{D\text{-}H\text{-}L}$ to $n^{C\text{-}H}$.

Referring back to the example in Figure 3.3, in which we use the most realistic parameter estimation, we calculate that the optimal number of products to be 16 and the listing fee that can achieve coordination to be \$0.00701, which is 35.05% of the MTS fixed cost.

To summarize, we use a combination of practical parameters to demonstrate how the optimal number of products is identified and quantify the magnitude of the system profit improvement. We find that when c_{fix} is low and unit cost saving is significant, the hybrid production mode is the most beneficial. Both the optimal number of products and system profit improvement are high. Furthermore, switching to the hybrid production mode is most profitable when the product margin is low and the unit cost saving is high. The MTS unit cost has a bigger impact on the system profit improvement δ_π when the product margin is

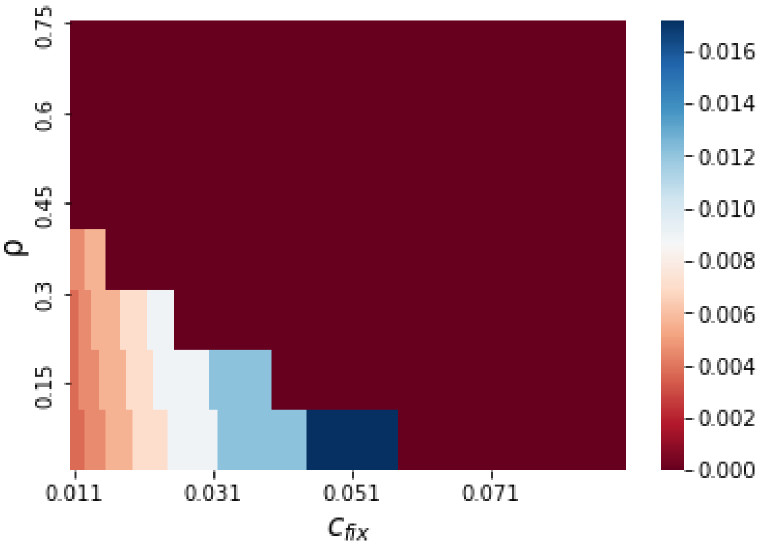


Figure 3.9: Optimal listing fee c_{list} over the trade-off of unit cost saving vs. fixed cost

low. We also demonstrate the pattern for the optimal listing fee in Case D-H-L and observe that the listing fee is around 30% to 40% of the fixed cost, increasing slightly with ρ.

3.5 Conclusion

In this chapter, we have examined platforms that license designs from content creators and print products on demand. For these platforms, supplementing the currently popular MTO production mode with the traditional MTS inventory system can achieve higher profits. However, these systems are often decentralized, meaning that content creators have the power to determine the number of products listed on the platform and are paid a royalty for each unit of product sold, while the platform makes production decisions and incurs all associated costs. When MTO is the only production mode, both the content creator and the platform desire to list infinite product designs. When adopting the theoretically more profitable MTS–MTO hybrid production mode, however, an incentive misalignment occurs. The platform is motivated to sell fewer products so that the more concentrated demand for each product allows them to benefit from the economy of scale enabled by MTS. Under the current royalty agreement, however, the content creator is motivated to list infinite products to saturate the market and capture as much customer demand as possible.

In this chapter, we show that an appropriate listing fee assessed on each product for sale, when accompanied by an appropriate adjustment of the royalty parameter, will align the incentives between content creators and the platform and lead to potentially higher profitability for both parties. We also find conditions on the parameters that make this approach more appealing and important.

There are two broad managerial insights. First, for the practical costs of both MTS and MTO, it is more profitable for the platform to adopt a hybrid production mode that uses MTS for its cheaper unit cost and MTO to capture variations in demand. The introduction of the hybrid production mode can have the most positive impact on system profit when the product margin is low and the unit cost saving is significant, but when the product margin is low, the system profit improvement is highly sensitive to changes in MTS unit cost c_{MTS}, thus requiring the platform to invest in acquiring an accurate parameter estimation.

Second, the platform should start charging content creators a listing fee for each design to offset the fixed cost introduced by MTS and help limit the number of products carried on the platform. The listing fee is lower than the fixed cost to ensure that the incentives for both content creators and platform are aligned to take advantage of the hybrid production mode and achieve higher profits.

Like all research, our study has certain limitations. For model tractability, we assume that only one content creator designs all products on the platform. While that allows us to focus on the main trade-off between content creators and platform, it is clearly a simplification. When multiple content creators exist on the same platform, their decisions will affect one another. This additional layer of game between content creators could cause them to be more resistant to limiting the number of products they list. This is because each content creator can benefit from other content creators' decisions to reduce the number of products that they list. Thus, they might prefer rather to wait for others to delist some of their products before delisting their own products. We conjecture that the use of a listing fee to limit the total number of products will still hold in this context. To achieve the same limiting effect on the optimal number of products, $n_{D\text{-}H\text{-}L}$, we may need to impose a higher listing fee.

Moreover, the use of Salop's demand model means that we have simplified heterogeneity among product features and customer preferences into only one attribute. This allows both the product designs and customer preferences to be modeled in a one-dimensional space in a circular market. When additional heterogeneity is incorporated into the analysis, we should start to see equilibria where different types of content creators co-exist—for example, those with a large customer base and those who are just starting out with a small customer base. This topic is worth considering in the future.

We also assume that each product is listed with the same price, whereas in reality, content creators have the freedom to choose their own prices and adjust them throughout the product life cycle, especially when the content creators are heterogeneous.

To better reflect realistic and important business models, many other factors are worth considering in future research, such as marketing efforts to attract more demand from outside the platform and the effort that content creators put into designing their products, which clearly affects demand and pricing. Aside from short-lifespan designs, there are also designs that generate sales throughout the year, which are known as "evergreen" designs. For these designs, a multi-period inventory model could better describe the costs than the single-period model used in our study.

Chapter 4

MANAGING DEMAND WHEN INTRODUCING MAKE-TO-STOCK TO MAKE-TO-ORDER PLATFORMS

4.1 Introduction

In this chapter, we continue our study of profit improvement for POD platforms by using a combination of MTS and MTO production modes. The POD platforms' business model and the hybrid MTS–MTO production strategy are detailed in Chapter 1.

In Chapter 3, we show that while adopting the MTO–MTS hybrid production mode can benefit the platform by combining lower MTS unit costs with MTO flexibility, its practical implementation is hindered by the incentive dynamic between the platform and content creators: the platform is interested in concentrating demand across fewer products to take advantage of MTS, while content creators are only interested in generating maximum total demand across all products, which can be achieved by offering as many products as possible. We address this problem by introducing a contract with a listing fee for each product, which creates an incentive for content creators to limit the number of products they list. While this approach is profitable to the system due to the cost savings of using the MTS production mode, it may decrease total demand. In this chapter, we explore alternative approaches that are focused on increasing demand per product without limiting the number of offerings. Specifically, we study how the content creators and/or the platform could optimally exert effort to increase product demand.

Content creators and the platform can exert effort in various forms. For content creators, effort includes the time and expertise invested in perfecting the designs and increasing product awareness and reputation through channels such as YouTube and TikTok. It can also include money spent on buying external ads[1]. We refer to the products into which content creators invest effort as *promoted products*. Effort for the platform can include enhancing customer service, promotions for the platform as a whole, building better design tools for content creators, sharing data on popular design trends, and so on. Since all these forms of platform effort indirectly affect product demand, we recognize that to achieve the same magnitude of influence on demand, the cost of effort for the platform is higher than for the content creators. The platform chooses to exert effort that affects all products fairly. While all effort activities can increase product demand, the effect of effort on demand has diminishing returns (Basuroy and Nguyen, 1998). Content creators promise to invest e_{cc} amount of effort so their product portfolio can be listed on the platform, and the platform routinely terminates content creator accounts that do not show evidence of enough effort, such as design updates or innovations or sales activities. Thus, we assume that content creators have a fixed effort budget that they use in full. However, the platform is free to choose how much effort, if any, to exert.

Diminishing returns on effort can cause content creators to spread their effort across all products in their portfolios. Combined with the content creators' limited effort budgets, spreading effort across all products can lead to the demand for each product being too low to allow the platform to take advantage of MTS production. To ensure that the hybrid production mode is adopted more effectively, we look for strategies to encourage content creators to concentrate their efforts on fewer products with the aim of making the demand for those products high enough to justify MTS production. We note that we cannot contract directly on effort, because the content creators' effort is neither fully observable nor fully verifiable by the platform. Instead, we propose the platform give products with higher demand a larger portion of the profits and products with lower demand a smaller portion of the profits. This kind of royalty discrimination contract can be easily justified in practice: products with higher demand can be made with the hybrid production mode

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[1] Here we specify purchasing ads off the platform for the simplicity of the model.

and its lower unit production costs, so they merit higher royalty payments. By contrast, products with lower demand can only use the MTO production mode, which is more costly and thus justifies a lower royalty payment. In other words, the platform *shares* the benefits of MTS with the content creators in the form of a higher royalty percentage. We show that this approach can align the content creators' incentive with the platform's objective and thus lead to the optimal system profit under a hybrid production mode. Alternatively, we propose a cooperative effort strategy in which the platform exerts additional effort to increase demand. We analyze when this strategy could further enhance the profitability of the hybrid production mode and eliminate the incentive conflict between content creators and the platform.

The rest of the chapter is arranged as follows. In Section 4.2, we set up and analyze the benchmark model using the MTO production mode and the royalty contract currently used in practice. In Section 4.3, we add the hybrid production mode to the model and propose a coordinating mechanism to achieve the highest system profit. In Section 4.4, we allow the platform to also exert effort, describe the optimal cooperative effort strategy, and quantify additional profit improvement attributable to the platform's additional effort. In Section 4.5, we demonstrate the sensitivity of our solutions to important system parameters using numerical studies. We summarize our insights and discuss limitations and future research directions in Section 4.6.

4.2 Benchmark model

In this section, we set up the benchmark model that adopts the MTO production mode and the existing royalty contract. First, we describe the demand function and the role of effort, which apply to all models in this chapter. We then describe the optimal decisions for the content creators and the system and demonstrate that the incentives of the involved parties are aligned in the benchmark scenario.

We list all the notations used in our study in Table 4.1.

Symbol	Description
Abbreviations	
MTO	make-to-order production mode
MTS	make-to-stock production mode
H	hybrid production mode
All-H	Case All-Hybrid
n-H	Case n-Hybrid
Mix	Case Mix
cc	content creator
pl	platform
sys	system consisting of content creator and platform
sys-prod	system evaluated for a single product
CH	coordinated contract for the hybrid production mode
CE	cooperative effort strategy
Variables	
e_0	effort amount allocated to one product
n	number of promoted products
e_{pl}	platform's additional effort amount for one content creator
a_1	content creator's revenue split portion for products with demand lower than $\overline{\mu}$ in effort-coordinating contract
a_2	content creator's revenue split portion for products with demand greater than or equal to $\overline{\mu}$ in effort-coordinating contract
a_3	content creator's profit split portion in cooperative effort strategy
Parameters	
D	demand for each product

e_{cc}	total amount of effort each content creator has to invest into their product portfolio
p	product selling price
v	reservation price for customers
α	parameter for modulating the influence of price p and reservation price v in the demand calculation
β	negative effect of competition on demand that is modulated by effort
γ	parameter defining the linear ratio of the standard deviation to the mean demand
r	royalty content creators receive per item sold
a	revenue split portion for content creators per item sold, less minimum revenue per product for the platform
b	minimum revenue for the platform
N	total number of products in a content creator's portfolio
c_{MTS}	unit cost for MTS production mode
c_{fix}	fixed cost for MTS production mode
c_{MTO}	unit cost for MTO production mode
k	unit effort cost for the platform
Placeholders	
$\hat{\mu}$	minimum required demand for products to be made with hybrid production mode
$\hat{n}$	maximum number of promoted products for product to be made with hybrid production mode
$\widehat{e_{pl}}$	amount of platform effort to qualify for hybrid production mode
n_{max}	local optimal number of promoted products for system profit function in Case n-Hybrid

n_i^*	optimal number of promoted products for Case $i, i \in \{n\text{-}H, Mix\}$
$\overline{\mu}$	lower threshold on μ for higher royalty percentages in hybrid mode coordinating contract
$\overline{n}$	upper threshold on n for higher royalty percentages in hybrid mode coordinating contract
$\overline{e_{pl}}$	amount of platform effort to eliminate system misalignment
e_{pl}^i	optimal amount of platform effort with production mode $i, i \in \{H, MTO\}$
e_{pl}^*	optimal amount of platform effort
Functions	
$f(x)$	density function of demand for each product, D
$\mu(e_0)$	mean demand for a promoted product with effort e_0
$\sigma(e_0)$	standard deviation of demand for a promoted product with effort e_0
π_{cc}	content creator's profit under current royalty contract
π_i^j	profit for i when in j, $i \in \{pl, sys, sys\text{-}prod\}$, $j \in \{MTO, H, All\text{-}H, n\text{-}H, Mix, CH, CE\}$
$\Delta\pi_{sys}^j$	system profit improvement in j compared to the benchmark scenario in percentage terms, $j \in \{CH, CE\}$

Table 4.1: Table of notations in Chapter 4

4.2.1 Demand model

Assume demand D for each product is i.i.d. and follows a normal distribution with the density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}. \tag{4.1}$$

The mean demand μ increases with the effort e_0 put in by the content creator sub-linearly to capture the diminishing return effect (Basuroy and Nguyen, 1998). The standard deviation σ is assumed to grow linearly with the mean demand (Khouja and Robbins, 2003). We assume that there is constant competition β that has a negative effect on product demand. This competition β originates from the abundant alternatives in the apparel market outside the platform; its effect on product demand can be modulated by the amount of effort e_0 invested in the product. We interpret that the effort invested in the product wins demand from apparel options outside the platform and do not model competition among products on the platform. Denote the reservation price for the customers by v and price for every product by p; the functions for the mean and standard deviation of demand are as follows:

$$\begin{cases} \mu(e_0) = \alpha(v-p) - \frac{\beta}{1+e_0}, \\ \sigma(e_0) = \gamma\mu(e_0). \end{cases} \tag{4.2}$$

In this function, α is the effect of price on the mean demand, and γ defines the coefficient of variation. We assume that $\gamma \leq \frac{1}{3}$ to ensure that the majority of possible values of demand lying within three standard deviations from the mean are positive (a common assumption in the newsvendor literature; see, e.g., Agrawal and Nahmias (1997) and Lau (1997)).

4.2.2 Content creator's decision

We assume that all content creators are identical and thus study only one content creator with a product portfolio of N designs. The content creators commit to investing effort e_{cc} to have their product portfolio listed on the platform, and the platform routinely terminates content creator accounts that do not show evidence of effort, such as design updates or additions or sales promotions. We assume that each content creator selects a subset of n products ($1 \leq n \leq N$) to promote and spreads the effort e_{cc} equally amongst those products, such that each product in the selected subset receives an effort of $e_0 = \frac{e_{cc}}{n}$. According to these assumptions, the mean demand function for each product in the selected subset is

$$\mu(\frac{e_{cc}}{n}) = \alpha(v-p) - \frac{\beta}{1+\frac{e_{cc}}{n}} = \alpha(v-p) - \frac{\beta n}{n+e_{cc}}. \tag{4.3}$$

Similar to Chapter 3, the platform offers the content creators a royalty contract that splits the revenue generated between content creator and the platform. This form of contract is widely adopted by the POD platforms in the industry, including Merch by Amazon, Printify, Printful, and so on. For every unit of product sold, the content creator receives the royalty, which is calculated as a percentage a of the selling price, less a base cost b determined by the platform:

$$r = a(p-b). \tag{4.4}$$

Without loss of generality, we assume that for the content creators, each unit of effort has a cost of \$1. Combining the mean demand function in Equation (4.3) and the royalty function in Equation (3.9), we derive the profit function for a generic content creator who has N products in their portfolio and e_{cc} amount of effort to invest as follows:

$$\pi_{cc}(n) = a(p-b)\left\{(N-n)\left[\alpha(v-p)-\beta\right] + n\left[\alpha(v-p) - \frac{\beta n}{n+e_{cc}}\right]\right\} - e_{cc}. \tag{4.5}$$

Maximizing this profit function, we obtain the optimal decision for the content creators in Proposition 4.1:

Proposition 4.1. *The optimal effort allocation decision for the content creator is to split the fixed amount of effort e_{cc} equally among all N products; that is, $n^* = N$. The resulting optimal profit is then*

$$\pi_{cc}^{MTO}(n^*) = a(p-b)N\left[\alpha(v-p) - \frac{\beta N}{N+e_{cc}}\right] - e_{cc}. \tag{4.6}$$

Proofs of this and all further lemmas, propositions, and corollaries in Chapter 4 are presented in Appendix C. We can see that due to the effort's diminishing return effect on individual product demand, the content creator's optimal effort allocation strategy is to evenly spread their effort across all their products.

4.2.3 System-optimal solution

In this subsection, we derive the first-best solution for the benchmark scenario that uses only MTO production. The profit function for the platform can be calculated as

$$\pi_{pl}^{MTO}(n) = [p - a(p-b) - c_{MTO}] \left\{ (N-n)\left[\alpha(v-p) - \beta\right] + n \left[\alpha(v-p) - \frac{\beta n}{n + e_{cc}}\right] \right\}, \tag{4.7}$$

and the profit function for the system that consists of the content creator and the platform can be calculated as

$$\pi_{sys}^{MTO}(n) = (p - c_{MTO}) \left\{ (N-n)\left[\alpha(v-p) - \beta\right] + n \left[\alpha(v-p) - \frac{\beta n}{n + e_{cc}}\right] \right\} - e_{cc}. \tag{4.8}$$

Maximizing the profit in Equation (4.8) with respect to n, we obtain the following result:

Proposition 4.2. *Under the MTO production mode, the optimal solution for the system is to spread effort evenly among* ***all*** *products:* $n^* = N$*. The resulting optimal profit is then*

$$\pi_{sys}^{MTO}(n^*) = (p - c_{MTO}) N \left[\alpha(v-p) - \frac{\beta N}{N + e_{cc}}\right] - e_{cc}. \tag{4.9}$$

A comparison of Propositions 4.1 and 4.2 shows that when the platform uses the MTO production mode, the content creator's decision coincides with the solution that is optimal for the system, indicating that the royalty contract currently in practice is coordinating. This is to be expected, as under the MTO system all products contribute equally to both content creator and the platform.

4.3 Hybrid production mode

As proposed in Chapter 3, the platform and the system may benefit from adding MTS production to the MTO mode, and we refer to such production mode as *the hybrid production mode*. In Section 4.3.1, we first add the hybrid production mode to the benchmark model introduced in Section 4.2.3 and derive when the hybrid production mode is profitable for the system. In Section 4.3.2, we find that there exists an incentive conflict between the two parties that leads to a profit reduction relative to the centralized system. We then propose and analyze a coordinating contract that achieves system profit improvement under the hybrid production mode in Section 4.3.3.

4.3.1 When to adopt hybrid production mode

Under the hybrid production mode, the platform makes a fixed number of units of certain products at the beginning of the time period to keep in stock at a unit cost c_{MTS} and incurs a fixed cost c_{fix} for each product selected for MTS mode. To satisfy uncertain demand beyond this MTS stock, the platform then fulfills any excess demand using the MTO mode at a unit cost c_{MTO}. Any leftover inventory is salvaged at the end of the period. We find the optimal quantity to produce with MTS using a single-period newsvendor model because the apparel industry that is in focus has a short product life cycle compared to the long replenishment cycle of the MTS production mode. The order needs to be made substantially earlier than the demand is observed, and subsequent replenishment is not feasible from the MTS source. The optimal quantity to be made with MTS is given by Lemma 3.3 in Chapter 3, and the system profit function with the optimal MTS quantity is given by Corollary 3.1 in Chapter 3. We also note that since the MTS production incurs a fixed cost c_{fix} per product, the demand for each product has to be high enough to justify that expense.

To determine when the hybrid production mode is profitable, we compare the system's profit function for each promoted product with $\frac{e_{cc}}{n}$ amount of effort under the MTO production mode $\pi^{MTO}_{sys\text{-}prod}$ to the system's profit under the hybrid production mode $\pi^{H}_{sys\text{-}prod}$.

The profit functions are given as follows:

$$\begin{cases} \pi_{sys\text{-}prod}^{MTO} = (p - c_{MTO})\mu(\frac{e_{cc}}{n}) - \frac{e_{cc}}{n}, \\ \pi_{sys\text{-}prod}^{H} = \left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right] \mu(\frac{e_{cc}}{n}) - c_{fix} - \frac{e_{cc}}{n}, \end{cases} \tag{4.10}$$

where $\rho = \frac{c_{MTS}}{c_{MTO}}$ indicates the relative unit cost saving for the MTS production with respect to the MTO production mode.

Comparing these two profit functions, we find the condition on demand that makes the hybrid production mode profitable in the next lemma:

Lemma 4.1. *A product qualifies for hybrid production mode if and only if its mean demand* $\mu(\frac{e_{cc}}{n})$ *is greater than* $\hat{\mu} = \frac{c_{fix}}{c_{MTO}[1-\rho-\phi(\Phi^{-1}(\rho))\gamma]}$.

Lemma 4.1 suggests that in order to use the hybrid production mode to improve system profit, the mean demand of a product should be higher than a certain volume, and $\hat{\mu}$ represents the number of units per product at which the fixed cost of hybrid production mode is covered by the savings from the difference between the hybrid and MTO production modes. In other words, given a limited amount of effort e_{cc}, content creators should promote a small enough number of products to make the demand for the promoted products high enough to qualify for the hybrid production mode. The lower the fixed cost for hybrid production mode and the more profitable the hybrid production is when compared to MTO production, the lower the requirement for mean product demand.

Depending on the comparison of the threshold $\hat{\mu}$ to the mean demand of products in a content creator's portfolio, the optimal production strategy can be in one of four cases, which are summarized in Table 4.2.

Case All-Hybrid. All products qualify for the hybrid production mode. This occurs when, for each product, the mean demand of a non-promoted product is high enough to qualify for hybrid production:

$$\mu(0) = \alpha(v - p) - \beta > \hat{\mu}. \tag{4.11}$$

	Production mode for	
Case name	promoted products	non-promoted products
All-Hybrid	Hybrid	Hybrid
n-Hybrid	Hybrid	MTO
Mix	when $\mu(\frac{e_{cc}}{n}) > \hat{\mu}$ – Hybrid ; when $\mu(\frac{e_{cc}}{n}) \leq \hat{\mu}$ – MTO	MTO
All-MTO	MTO	MTO

Table 4.2: Summary of the four possible production strategies

Case n-Hybrid. Promoted products qualify for the hybrid production mode, and non-promoted products use the MTO production mode. This case occurs when the mean demand of the non-promoted product $\mu(0)$ is not high enough to meet the requirement for the hybrid production mode, but the lowest amount of non-zero effort $\frac{e_{cc}}{N}$ expended for a product qualifies it for hybrid production. The boundary conditions for this case are as follows:

$$\begin{cases} \mu(0) = \alpha(v-p) - \beta \leq \hat{\mu}, \\ \mu(\frac{e_{cc}}{N}) > \hat{\mu}. \end{cases} \tag{4.12}$$

Case Mix. Promoted products can only qualify for the hybrid production mode when the number of such products n is small enough for the mean demand to be higher than $\hat{\mu}$; otherwise, they are made using the MTO mode, just like the non-promoted products. This case occurs when the lowest amount of non-zero effort $\frac{e_{cc}}{N}$ does not qualify the product for hybrid production. However, the product with the highest possible demand in the portfolio, which is achieved by concentrating all effort into one product, $\mu(e_{cc})$, does qualify for hybrid production. The boundary conditions for this case are as follows:

$$\begin{cases} \mu(\frac{e_{cc}}{N}) \leq \hat{\mu}, \\ \mu(e_{cc}) = \alpha(v-p) - \frac{\beta}{1+e_{cc}} > \hat{\mu}. \end{cases} \tag{4.13}$$

Case All-MTO. All products are made using MTO production. This case occurs when even the highest concentration of effort e_{cc} in one product does not qualify that product for hybrid production. The boundary condition for this situation is as follows:

$$\mu(e_{cc}) \leq \hat{\mu}. \tag{4.14}$$

To illustrate the relationships among these four cases, we plot their areas in Figure 4.1 with the fixed cost for hybrid production mode on the x-axis and the content creator's total effort e_{cc} on the y-axis. The boundary conditions are calculated using a representative case with the parameter values shown in Table 4.3[2].

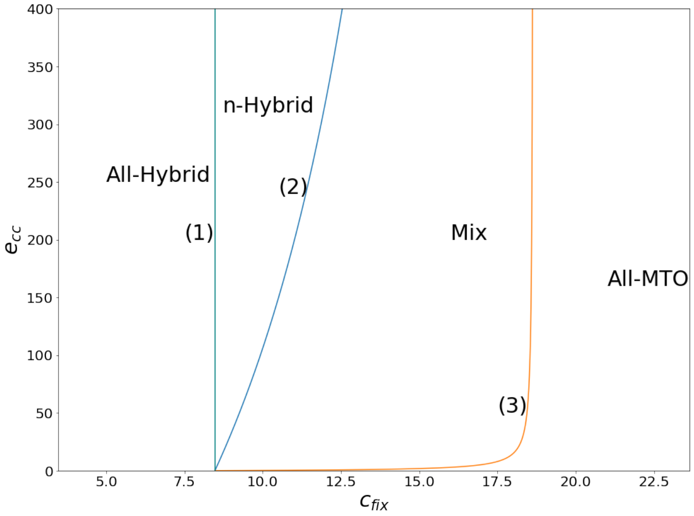

Figure 4.1: Regions for each case

Parameter	c_{MTO}	c_{MTS}	γ	v	p	α	β	N
Value	10	1	0.3	30	25	0.44	1.2	600

Table 4.3: Parameter values for a representative case

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[2] The justification for choosing these parameters is presented in Section 6.1.

We note that boundaries (1), (2), and (3) correspond to the conditions of $\mu(0) = \hat{\mu}$, $\mu(\frac{e_{cc}}{N}) = \hat{\mu}$, and $\mu(e_{cc}) = \hat{\mu}$, respectively. We observe that when c_{fix} is low enough, every product qualifies for hybrid production (*Case All-Hybrid*). When c_{fix} is in the middle range, if the content creator's exertion of effort e_{cc} is high enough, all promoted products qualify for the hybrid mode (*Case n-Hybrid*); if the content creator's exertion of effort e_{cc} is low, that effort needs to be concentrated so that the promoted products to qualify for hybrid production (*Case Mix*). When c_{fix} is too high, all products must use MTO production (*Case All-MTO*). In *Case All-MTO*, the hybrid production mode is no longer effective in improving system profit. Thus, this case is equivalent to the benchmark model, and we do not explicitly model it in the analysis below.

4.3.2 Optimal effort allocation strategy

In this section, we derive the optimal effort allocation strategy from the system's perspective and compare it to the content creators' strategy derived in Proposition 4.1.

4.3.2.1 Case All-Hybrid

In *Case All-Hybrid*, all products qualify for the hybrid production mode. The system profit is as follows:

$$\begin{aligned}\pi_{sys}^{\textit{All-H}}(n) = \left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right] \{(N-n)\left[\alpha(v-p) - \beta\right] \\ +n\left[\alpha(v-p) - \frac{\beta n}{n + e_{cc}}\right]\Big\} - Nc_{fix} - e_{cc}.\end{aligned} \tag{4.15}$$

We see that changing the number of promoted products n affects only total product demand and does not influence the number of products being made under the hybrid production mode. Thus the system prefers to maximize total demand and chooses to promote all N products in a content creator's portfolio. The content creator's incentive is aligned with the system's incentive in *Case All-Hybrid*.

4.3.2.2 Case n-Hybrid

Recall that n represents the number of promoted products. In this case, n products qualify for the hybrid production mode, while $N - n$ products do not. The system profit function is thus calculated as follows:

$$\begin{aligned}\pi_{sys}^{n\text{-}H}(n) =&(N-n)(p-c_{MTO})\left[\alpha(v-p)-\beta\right]+ \\ &n\left\{\left[p-c_{MTS}-c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]\left[\alpha(v-p)-\frac{\beta n}{n+e_{cc}}\right]-c_{fix}\right\}-e_{cc}.\end{aligned} \tag{4.16}$$

We denote the unconstrained maximizer for Equation (4.16) with

$$n_{max} = e_{cc}\left(\frac{\sqrt{W}}{\sqrt{Z}}-1\right), \tag{4.17}$$

where

$$\begin{cases} W = \left[p-c_{MTS}-c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]\beta, \\ Z = \left[c_{MTS}+c_{MTO}\phi(\Phi^{-1}(\rho))\gamma-c_{MTO}\right]\left[\alpha(v-p)-\beta\right]+c_{fix}, \end{cases} \tag{4.18}$$

and derive the optimal number of products to promote in the following lemma:

Lemma 4.2. *In Case n-Hybrid, the optimal number of promoted products for the system is*

$$n^*_{n\text{-}H} = \max\left(\min\left(N, n_{max}\right), 1\right). \tag{4.19}$$

We can interpret W as the product of the hybrid production margin and the sensitivity to effort and Z as the difference between MTO mode profit relative to hybrid mode profit for a single non-promoted product. To gain intuition on how W and Z influence the value of n_{max}, we re-write Equation (4.16) as

$$\pi_{sys}^{n\text{-}H}(n) = \underbrace{N(p - c_{MTO})\left[\alpha(v-p) - \beta\right]}_{\substack{\text{profit earned on } N \text{ non-promoted} \\ \text{products with MTO mode}}} - \underbrace{nZ}_{\substack{\text{profit loss due to switching} \\ \text{from MTO to hybrid mode} \\ \text{for } n \text{ non-promoted products}}} + \underbrace{nW\frac{e_{cc}}{n + e_{cc}}}_{\substack{\text{profit improvement due to} \\ \text{effort with hybrid mode} \\ \text{for } n \text{ products}}} - e_{cc}, \tag{4.20}$$

and further summarize that n_{max} balances the trade-off between the profit lost due to switching from MTO to hybrid production for non-promoted products and the profit improvement due to the demand increase caused by exerting effort on these products with hybrid mode. When Z is low and W is high, the value of n_{max} is high.

Lemma 4.2 shows that there are scenarios in *Case n-Hybrid* when the content creator's decision is not aligned with the system's decision, because the content creator's preferred number of products to promote remains N, while in some scenarios the system prefers otherwise. We summarize such misalignment scenarios as follows:

Scenario n-Hybrid-A (Interior solution) When Conditions (4.12) are satisfied *and* $1 \leq n_{max} \leq N$, the system prefers that content creators promote n_{max} products. The promoted products are produced using the hybrid production mode, while non-promoted products are produced using MTO.

Scenario n-Hybrid-B (Boundary solution) When Conditions (4.12) are satisfied *and* $n_{max} \leq 1 \leq N$, the system prefers that content creators promote one product. The one promoted product is produced with the hybrid production mode, while non-promoted products are produced with MTO production mode.

4.3.2.3 Case Mix

When the non-promoted products do not qualify for the hybrid production mode, and the promoted products can only qualify for the hybrid production mode when the effort is more concentrated, we can write the system's profit function as follows:

$$\pi_{sys}^{Mix}(n) = \begin{cases} \pi_{sys}^{n\text{-}H}(n) & \text{when } \mu(\frac{e_{cc}}{n}) > \hat{\mu}, \\ \pi_{sys}^{MTO}(n) & \text{when } \mu(\frac{e_{cc}}{n}) \leq \hat{\mu}. \end{cases} \tag{4.21}$$

We optimize Equation (4.21), and find the optimal number of promoted products in Lemma 4.3. Denote the solution to $\mu(\frac{e_{cc}}{n}) = \hat{\mu}$ as $\hat{n}$, and $\tilde{n}_1 = \max(\min(\hat{n}, n_{max}), 1)$. We know $\hat{n}$ is the upper bound on n for the product to qualify for the hybrid production mode, and $\hat{n} \leq N$ is equivalent to $\mu(\frac{e_{cc}}{N}) \leq \hat{\mu}$.

Lemma 4.3. *In Case Mix, the optimal number of promoted products for the system is:*

$$n_{Mix}^{*} = \arg\max_{n}(\pi_{sys}^{n\text{-}H}(\tilde{n}_1), \pi_{sys}^{MTO}(N)). \tag{4.22}$$

Lemma 4.3 shows two scenarios in *Case Mix* where the content creator's incentive is not aligned with the platform's incentive. In these scenarios, the content creator prefers to promote all products, while the system prefers otherwise. We summarize such scenarios where misalignment happens as follows:

Scenario Mix-A (Interior solution) When Conditions (4.13) are satisfied, $1 \leq n_{max} \leq \hat{n}$, and $\pi_{sys}^{n\text{-}H}(n_{max}) > \pi_{sys}^{MTO}(N)$, the system prefers the content creators to promote n_{max} products. The promoted products are produced with the hybrid production mode, while non-promoted products are produced with MTO production mode.

Scenario Mix-B (Boundary condition) When Conditions (4.13) are satisfied, $n_{max} \leq 1$, and $\pi_{sys}^{n\text{-}H}(1) > \pi_{sys}^{MTO}(N)$, the system prefers the content creators to promote one product. The one promoted product is produced with the hybrid production mode, while non-promoted products are produced with MTO production mode.

For Scenario Mix-A to exist, the system profit achieved at n_{max} should be higher than investing effort in all N products and using the MTO production mode. To gain intuition on when this condition is satisfied, we follow the same logic as in Equation (4.20). Because $\pi_{sys}^{MTO}(N)$ is used in these conditions, we first rewrite this profit function as follows:

$$\pi_{sys}^{MTO}(N) = \underbrace{N(p - c_{MTO})\left[\alpha(v-p) - \beta\right]}_{\substack{\text{profit earned on } N \text{ non-promoted} \\ \text{products with MTO mode}}} + \underbrace{N(p - c_{MTO})\frac{\beta e_{cc}}{N + e_{cc}}}_{\substack{\text{profit improvement due to} \\ \text{effort with MTO mode} \\ \text{for } N \text{ products}}} - e_{cc}. \tag{4.23}$$

Comparing $\pi_{sys}^{n\text{-}H}(n_{max})$ in Scenario Mix-A to the profit in Equation (4.23), we obtain the condition for when Scenario Mix-A exists:

$$\begin{aligned}
\pi_{sys}^{n\text{-}H}(n_{max}) - \pi_{sys}^{MTO}(N) = {} & \underbrace{n_{max}W\frac{e_{cc}}{n_{max} + e_{cc}}}_{\substack{\text{profit improvement due to} \\ \text{effort with hybrid mode} \\ \text{for } n_{max} \text{ products}}} - \underbrace{n_{max}Z}_{\substack{\text{profit loss due to switching} \\ \text{from MTO to hybrid mode} \\ \text{for } n_{max} \text{ non-promoted products}}} \\
& - \underbrace{N(p - c_{MTO})\frac{\beta e_{cc}}{N + e_{cc}}}_{\substack{\text{profit improvement due to} \\ \text{effort with MTO mode} \\ \text{for } N \text{ products}}} \\
= {} & e_{cc}(\sqrt{W} - \sqrt{Z})^2 - \underbrace{N(p - c_{MTO})\frac{\beta e_{cc}}{N + e_{cc}}}_{\substack{\text{profit improvement due to} \\ \text{effort with MTO mode} \\ \text{for } N \text{ products}}} > 0.
\end{aligned} \tag{4.24}$$

When the profit loss from non-promoted products when switching from MTO to hybrid production mode Z is low and the hybrid production margin W is high, there are more instances for Scenario Mix-A to exist. We also note that n_{max} is high under the same situation. The condition on profit for Scenario Mix-B to exist can be similarly interpreted by replacing n_{max} with 1 in the condition.

Based on the regions for each case in Figure 4.1, we also use the conditions associated with each scenario to outline their corresponding regions on the map for effort and fixed cost in Figure 4.2. We note that the regions for Scenarios n-Hybrid-B and Mix-B are very small, indicating that these two scenarios rarely occur. In fact, to find these regions for demonstration purposes, we had to substantially adjust the parameters used in the calculations to the values shown in Table 4.4. Neither the parameter values shown in Table 4.4 nor the other parameter sets that we identified in a numerical search to demonstrate the existence of these scenarios are representative of reality: either the product price for all products has to be set just above the minimum revenue for the platform b, or every content creator has a very small number of products listed on the platform. Thus, we conclude that Scenarios n-Hybrid-B and Mix-B are not going to be frequently observed in practice, and we focus on Scenarios n-Hybrid-A and Mix-A in all further analysis. The regions for these scenarios are plotted using the parameter values in Table 4.3.

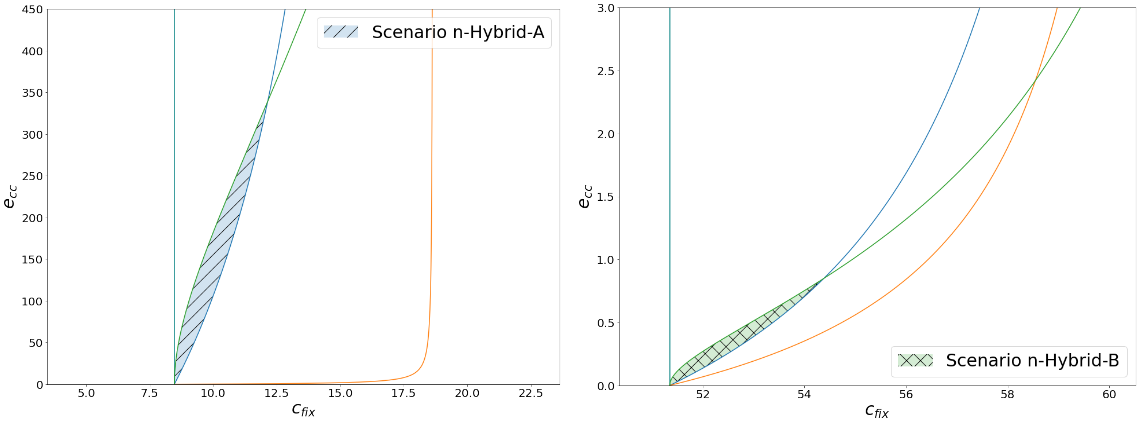
Scenario/Parameter	c_{MTO}	c_{MTS}	γ	v	p	α	β	N
Scenario n-Hybrid-B	10	1	0.3	30	13.5	0.44	1.2	2
Scenario Mix-B	10	1	0.3	30	13.5	0.44	1.2	600

Table 4.4: Parameter values for Scenarios n-Hybrid-B and Mix-B

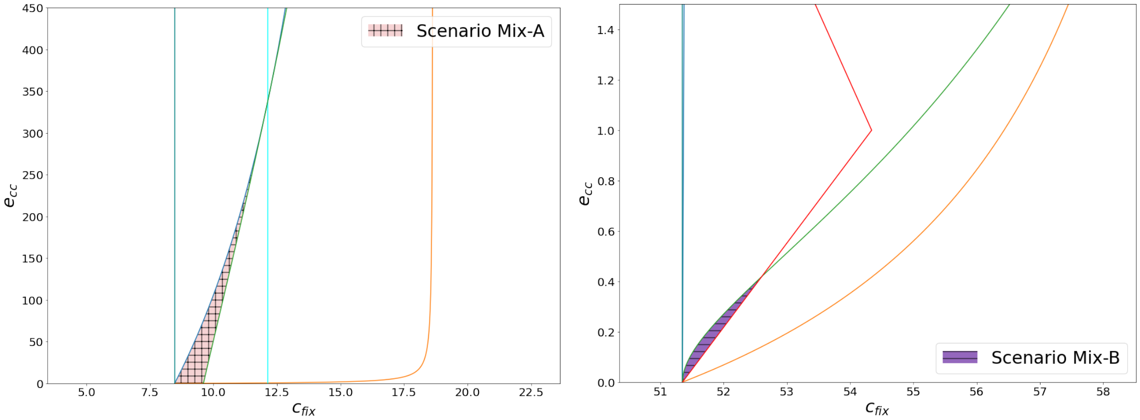
4.3.3 Coordinating contract for effective implementation of hybrid production mode

In Section 4.3.2, we discovered scenarios in which the content creator's choice of the number of products to promote does not optimize system profit when the hybrid production mode is available. The content creator prefers to spread their investment of effort equally across all products in their portfolio because of the diminishing returns of that investment, while the system prefers to concentrate demand by limiting number of promoted products and make better use of the hybrid production mode.

We note that this incentive gap exists because content creators do not benefit from the hybrid production mode and are thus not motivated to concentrate the effort allocation



(a) Scenario n-Hybrid-A (b) Scenario n-Hybrid-B



(c) Scenario Mix-A (d) Scenario Mix-B

Figure 4.2: Regions in which the optimal choice for content creators is not aligned with the system's optimal choice

to take advantage of the lower MTS unit cost, as the platform prefers. To eliminate this incentive gap, we propose that the platform give products with higher demand a larger portion of the revenue for each unit sold. This strategy incentivizes content creators to choose a subset of their products to promote so that they can take advantage of the higher royalty percentage. The new royalty is structured as follows:

$$r = \begin{cases} a_1(p-b) & \text{when } \mu(\frac{e_{cc}}{n}) < \overline{\mu}, \\ a_2(p-b) & \text{when } \mu(\frac{e_{cc}}{n}) \geq \overline{\mu}. \end{cases} \tag{4.25}$$

Here, $\overline{\mu} \in (\mu(0), \mu(e_{cc}))$ is a threshold for determining whether a product's mean demand is high enough to be rewarded with a higher royalty percentage, and we naturally assume $a_2 > a_1$. Substituting (4.3), we see that for mean demand to reach at least the threshold $\overline{\mu}$, the requirement for n should satisfy the following condition:

$$n \leq \overline{n} = \frac{e_{cc}}{\frac{\beta}{\alpha(v-p)-\overline{\mu}} - 1}. \tag{4.26}$$

According to the new contract, we can write the content creator's profit function as[3]

$$\pi_{cc}^{CH}(n) = \begin{cases} a_1(p-b)\left\{(N-n)\left[\alpha(v-p)-\beta\right] + n\left[\alpha(v-p) - \frac{\beta n}{n+e_{cc}}\right]\right\} - e_{cc} & \text{when } n > \overline{n}, \\ a_1(p-b)(N-n)\left[\alpha(v-p)-\beta\right] + a_2(p-b)n\left[\alpha(v-p) - \frac{\beta n}{n+e_{cc}}\right] - e_{cc} & \text{when } n \leq \overline{n}. \end{cases} \tag{4.27}$$

To achieve the first-best system profit in Scenarios n-Hybrid-A and Mix-A, the contract parameters need to be set such that the content creators choose the n that matches the optimal number of promoted products in these scenarios, n_{max}. In Proposition 4.3, we

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[3] The superscript *CH* refers to a coordinated contract for the hybrid production mode. It differs from *C-H* in Chapter 3, which stands for *Case C-H*.

characterize the contract parameters that achieve system coordination. Denote

$$\begin{cases} \underline{a_2} = \frac{1}{n_{max}\overline{\mu}}\left[aN\mu(\frac{e_{cc}}{N}) - a_1(N-n_{max})\mu(0)\right], \\ \overline{a_2} = \dfrac{1}{(p-b)n_{max}\overline{\mu}}\{\left[p-c_{MTO}-a_1(p-b)\right](N-n_{max})\mu(0) - \left[p-c_{MTO}-a(p-b)\right]N\mu(\dfrac{e_{cc}}{N}) \\ \qquad\qquad + \left[p-c_{MTS}-c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]n_{max}\overline{\mu} - n_{max}c_{fix}\}. \end{cases} \tag{4.28}$$

We then find

Proposition 4.3. *The system is coordinated with the new royalty contract described in Equation* (4.25) *when* $\overline{\mu} = \alpha(v-p) - \frac{\beta n_{max}}{n_{max}+e_{cc}}$ *and* $a_2 > a_1\frac{n_{max}\mu(0)+\frac{\beta N e_{cc}}{N+e_{cc}}}{n_{max}\overline{\mu}}$.

The contract is incentive-compatible if and only if $a_2 \in \left[\underline{a_2}, \overline{a_2}\right]$*;* a_2 *exists when*

$$a_1 \leq \frac{1}{(p-b)N\mu(\frac{e_{cc}}{N})}\{(p-c_{MTO})(N-n_{max})\mu(0) + \left[p-c_{MTS}-c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]n_{max}\overline{\mu} \\ - n_{max}c_{fix} - \left[p-c_{MTO}-a(p-b)\right]N\mu(\frac{e_{cc}}{N})\}, \tag{4.29}$$

and a_1 *always exists.*

Because of the diminishing returns of effort, the tendency among content creators to spread out their effort remains the same on both intervals corresponding to the high and the low royalty percentage payoffs. In order to coordinate the system, the platform needs to set the threshold $\overline{\mu}$ to correspond to the system profit-maximizing level of concentration of effort n_{max} and adjust the royalty percentages a_1 and a_2 such that the content creator's profit at n_{max} is higher than when the content creator promotes all products. The conditions in Equation (4.28) guarantee that the contract is incentive-compatible by ensuring that both content creators and the platform are not worse off than under the benchmark case. The values of (a_1, a_2) in the hybrid mode coordinating contract can have multiple solutions as long as they satisfy the conditions in Proposition 4.3. This gives the platform the flexibility to choose how much they want to penalize effort spread beyond n_{max}, even though changing the (a_1, a_2) values does not change the preferred level of concentration of effort among content creators after the contract change. The lower a_1 is, the higher the penalty to promote more than n_{max} products.

4.4 Cooperative effort

We recognize that there are alternative ways to enhance the performance of the hybrid production mode. Motivated by the practice of cooperative advertising, we propose a *cooperative effort* strategy, in which the platform can exert effort (in addition to the effort exerted by content creators) to impact demand. This strategy can increase demand for all products on the platform. In Section 4.4.1, we study the optimal cooperative effort strategy to enhance the hybrid production mode. Then, in Section 4.4.2, we compare the optimal system profit achieved by the platform exerting additional effort against that achieved by adoption of coordinated hybrid production and derive the conditions under which the cooperative effort strategy achieves higher system profit.

4.4.1 Optimal cooperative effort strategy

Recall that the content creators promise to invest e_{cc} amount of effort to be listed on the platform, and the platform routinely terminates content creator accounts that do not show evidence of effort. Thus, when the platform is adding to the content creator's effort, the content creator continues to investing all e_{cc} of the effort into all N products, while the platform adds on an extra e_{pl} to the content creator's product portfolio. The extra e_{pl} of effort can take the form of enhanced customer service, promotion of the platform as a whole, building better design tools, sharing sales trend data with content creators, and so on. Note that these activities affect all products simultaneously, so e_{pl} is evenly distributed across all products. We also note that compared to the efforts of content creators, these platform efforts can only indirectly affect product demand. Thus, to achieve the same magnitude of influence on demand, the cost of effort for the platform is higher than the effort cost for content creators. Here, we assume that for the platform each unit of effort has a cost of $\$k$, and $k \geq 1$.

The platform's effort increases the demand for all products, which not only increases the total sales of the product portfolio but also allows for better use of the hybrid production mode and reduces total operational costs.

In *Case All-Hybrid*, all products qualify for the hybrid production mode regardless of the effort invested in individual products. In *Case n-Hybrid*, the content creators' effort $\frac{e_{cc}}{N}$ qualifies the product for the hybrid production mode. Thus, in these two cases, the added platform effort of $\frac{e_{pl}}{N}$ does not change the production mode but does increase total profits. The system profit function in these two cases is as follows:

$$\begin{aligned}\pi_{sys-H}^{CE}(N, e_{cc}+e_{pl}) =& N\left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]\left[\alpha(v-p) - \frac{\beta N}{N+e_{cc}+e_{pl}}\right] \\ & - Nc_{fix} - e_{cc} - ke_{pl}.\end{aligned} \tag{4.30}$$

Maximizing Equation (4.30) with respect to e_{pl}, we find the optimal platform's effort amount:

$$e_{pl}^{H} = \max(0, N\left(\sqrt{\frac{W}{k}} - 1\right) - e_{cc}). \tag{4.31}$$

However, in *Case Mix* and *Case All-MTO*, since the content creators' effort investment of $\frac{e_{cc}}{N}$ does not qualify the products for the hybrid production mode, the platform's effort of $\frac{e_{pl}}{N}$ can potentially change the production mode for these products from MTO to hybrid. We denote the threshold for e_{pl} to change the production mode for the products in these cases as $\widehat{e_{pl}} = N(\frac{\beta}{\alpha(v-p)-\hat{\mu}} - 1) - e_{cc}$. When $e_{pl} \geq \widehat{e_{pl}}$, the hybrid production mode is adopted, and the system profit function is given by Equation (4.30); when $e_{pl} < \widehat{e_{pl}}$, all products are made using the MTO mode, and the system profit function is as follows:

$$\pi_{sys-MTO}^{CE}(N, e_{cc}+e_{pl}) = N\left(p - c_{MTO}\right)\left[\alpha(v-p) - \frac{\beta N}{N+e_{cc}+e_{pl}}\right] - e_{cc} - ke_{pl}. \tag{4.32}$$

Maximizing Equation (4.32) with respect to e_{pl}, we obtain

$$e_{pl}^{MTO} = \max(0, N\left(\sqrt{\frac{(p - c_{MTO})\,\beta}{k}} - 1\right) - e_{cc}). \tag{4.33}$$

We can now summarize the optimal cooperative effort strategy for the system in Proposition 4.4.

Proposition 4.4. *The optimal amount of platform effort for the system depends on the production mode strategy and the platform's unit effort cost:*

- *In Cases All-Hybrid and n-Hybrid,*

$$e_{pl}^{*} = e_{pl}^{H}; \tag{4.34}$$

- *In Cases Mix and All-MTO,*

$$e_{pl}^{*} = \begin{cases} e_{pl}^{H} \quad \textit{when } k \leq \frac{p-c_{MTO}}{\beta}\left[\alpha(v-p)-\hat{\mu}\right]^2, \\ \\ \arg\max\left(\pi_{sys-MTO}^{CE}(N, e_{cc}+e_{pl}^{MTO}), \pi_{sys-H}^{CE}(N, e_{cc}+e_{pl}^{H})\right) \\ \qquad \textit{when } \frac{p-c_{MTO}}{\beta}\left[\alpha(v-p)-\hat{\mu}\right]^2 < k \leq \frac{W}{\beta^2}\left[\alpha(v-p)-\hat{\mu}\right]^2, \\ \\ e_{pl}^{MTO} \quad \textit{when } k > \frac{W}{\beta^2}\left[\alpha(v-p)-\hat{\mu}\right]^2. \end{cases} \tag{4.35}$$

From Equations (4.31) and (4.33), we see that the value for optimal e_{pl} decreases in each case when the platform's unit cost of effort k increases. This is intuitive, as a higher value of k makes it more expensive for the platform to exert effort. In *Case Mix* and *Case All-MTO*, the boundaries on platform unit effort cost k decrease with MTS fixed cost c_{fix}, because the higher the fixed cost, the harder it is to qualify for the hybrid production mode, which makes the unit effort cost requirement more stringent.

With the platform's effort, the content creator still gets paid a royalty per unit of product sold. The royalty calculation is in the same form as the original contract defined in Equation (3.9). To ensure that both platform and content creator can benefit from the cooperative effort strategy, the royalty percentage a needs to be reduced accordingly. We denote the updated royalty percentage for the cooperative effort strategy as a_3. In Corollary 4.1, we characterize the range for a_3 to ensure that the contract is incentive-compatible for both parties after the platform effort has been exerted. Denote

$$\begin{cases} \underline{a_{3-H}} = a\frac{\mu(\frac{e_{cc}}{N})}{\mu(\frac{e_{cc}+e_{pl}^{H}}{N})}, \\ \overline{a_{3-H}} = \frac{1}{N(p-b)\mu(\frac{e_{cc}+e_{pl}^{H}}{N})}\{N\left[p-c_{MTS}-c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]\mu(\frac{e_{cc}+e_{pl}^{H}}{N}) \\ \quad -Nc_{fix}-ke_{pl}^{H}-N\left[p-c_{MTO}-a(p-b)\right]\mu(\frac{e_{cc}}{N})\}, \end{cases} \tag{4.36}$$

and

$$\begin{cases} \underline{a_{3-MTO}} = a\frac{\mu(\frac{e_{cc}}{N})}{\mu(\frac{e_{cc}+e_{pl}^{MTO}}{N})}, \\ \overline{a_{3-MTO}} = \frac{1}{N(p-b)\mu(\frac{e_{cc}+e_{pl}^{MTO}}{N})}\{N\left(p-c_{MTO}\right)\mu(\frac{e_{cc}+e_{pl}^{MTO}}{N}) \\ \qquad -ke_{pl}^{MTO}-N\left[p-c_{MTO}-a(p-b)\right]\mu(\frac{e_{cc}}{N})\}. \end{cases} \tag{4.37}$$

We then find

Corollary 4.1. *When the optimal amount of platform effort is* $e_{pl}^{*}=e_{pl}^{H}$*, the contract is incentive-compatible for both parties if and only if* a_3 *is in* $\left[\underline{a_{3-H}},\overline{a_{3-H}}\right]$*. Moreover, when* $\pi_{sys-H}^{CE}(N,e_{cc}+e_{pl}^{H})>\pi_{sys}^{MTO}(N)$*, the range for* a_3 *is non-empty.*

When the optimal platform's effort amount is $e_{pl}^{*}=e_{pl}^{MTO}$*, the contract is incentive compatible for both parties if and only if* a_3 *is in* $\left[\underline{a_{3-MTO}},\overline{a_{3-MTO}}\right]$*. Moreover, when* $\pi_{sys-MTO}^{CE}(N,e_{cc}+e_{pl}^{MTO})>\pi_{sys}^{MTO}(N)$*, the range for* a_3 *is non-empty.*

4.4.2 When to adopt the cooperative effort strategy

In this section, we compare system profits under the hybrid production mode achieved by cooperative effort strategy to the profits achieved by the coordinating contract and analyze the conditions under which the cooperative effort strategy should be adopted.

In all cases without misalignment (*Case All-Hybrid* and *Case n-Hybrid* excluding Scenario n-Hybrid-A), as long as e_{pl}^* from Proposition 4.4 is greater than zero, any non-zero platform effort is optimal. Substituting the expression for the optimal platform effort in these cases in Equation(4.34), we derive that when $k < W\frac{N^2}{(N+e_{cc})^2}$, the platform should adopt the cooperative effort strategy with the hybrid production mode.

For Scenarios n-Hybrid-A and Mix-A, where misalignment exists between platform and content creators, the coordinated system profit function when adopting the hybrid production mode without platform effort is

$$\begin{aligned}\pi_{sys}^{CH}(n_{max}) = (N - n_{max})\left(p - c_{MTO}\right)\left[\alpha(v-p) - \beta\right] + \\ n_{max}\left\{\left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]\left[\alpha(v-p) - \frac{\beta n_{max}}{n_{max}+e_{cc}}\right] - c_{fix}\right\} - e_{cc}.\end{aligned} \tag{4.38}$$

We show that in Scenario Mix-A, as long as

$$\begin{aligned}\sqrt{k} <& \frac{\sqrt{\beta}}{2\left(\sqrt{p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma} - \sqrt{p - c_{MTO}}\right)} \\ &\times \frac{\left[c_{MTO} - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right](p - c_{MTO})}{p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma},\end{aligned} \tag{4.39}$$

the optimal platform effort enables all products to be made with the hybrid production mode and that $e_{pl}^* = e_{pl}^H$. The condition in Equation (4.39) is easily satisfied with practical values of the unit effort cost for the platform k[4]; hence, for further analysis we focus on situations where the optimal platform effort enables products to be made with the hybrid

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[4] When calculated using parameter values in Table 4.3, the upper bound on k is 9.31.

production mode ($e^*_{pl} = e^H_{pl}$)[5]. When Condition (4.39) holds, the system profit function of optimal platform effort under the hybrid production mode is

$$\begin{aligned}\pi^{CE}_{sys-H}(N, e_{cc} + e^H_{pl}) = N\left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right] \\ \times\left[\alpha(v-p) - \frac{\beta N}{N + e_{cc} + e^H_{pl}}\right] - Nc_{fix} - e_{cc} - ke^H_{pl}.\end{aligned} \tag{4.40}$$

Comparing the profit functions in Equations (4.40) and (4.38), we derive Corollary 4.2:

Corollary 4.2. *Under the hybrid production mode, when* $k < (\sqrt{W} - \sqrt{Z})^2$*:*

$$\pi^{CE}_{sys-H}(N, e_{cc} + e^H_{pl}) > \pi^{CH}_{sys}(n_{max}).$$

To explain Corollary 4.2, we rewrite the profit functions for the two contracts as follows:

$$\begin{cases}\pi^{CH}_{sys}(n_{max}) = & N(p - c_{MTO})\left[\alpha(v-p) - \beta\right] - n_{max}Z + n_{max}W\frac{e_{cc}}{n_{max}+e_{cc}} - e_{cc}, \\ \pi^{CE}_{sys-H}(N, e_{cc} + e^H_{pl}) = & N\left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]\left[\alpha(v-p) - \beta\right] - Nc_{fix} \\ & +NW\frac{e_{cc}+e^H_{pl}}{N+e_{cc}+e^H_{pl}} - e_{cc} - ke^H_{pl}.\end{cases} \tag{4.41}$$

Then, $\pi^{CE}_{sys-H}(N, e_{cc} + e^H_{pl}) > \pi^{CH}_{sys}(n_{max})$ is equivalent to

$$NW\frac{e_{cc} + e^H_{pl}}{N + e_{cc} + e^H_{pl}} - NZ - ke^H_{pl} > n_{max}W\frac{e_{cc}}{n_{max} + e_{cc}} - n_{max}Z. \tag{4.42}$$

Solving Equation (4.42), we get the boundary on k for the cooperative effort strategy to be more profitable than the coordinated hybrid production mode. Intuitively, when $e^H_{pl} > 0$, as the unit effort cost for the platform increases, the optimal platform's effort amount e^H_{pl} decreases, resulting in a decrease in the system profit improvement brought about by the cooperative effort strategy. When the effort is too costly for the platform, the platform chooses the coordinating contract for the hybrid production mode.

We also note that platform exerting effort enables each product to qualify for the hybrid

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[5] When k exceeds the bound in Equation (4.39), for some (e_{cc}, c_{fix}) combinations with higher c_{fix} values in Scenario Mix-A, $e^*_{pl} = e^{MTO}_{pl}$, the products are made using MTO with optimal platform effort.

production mode. This way, the system's preference for the number of promoted products will increase to N, which aligns with the content creator's preference.

In Proposition 4.5, we characterize the minimum amount of platform effort in each scenario for the incentive misalignment to go away. Substituting e_{cc} with $e_{cc} + e_{pl}$ in the expression for n_{max} in Equation (4.17), we denote the solution to $(e_{cc} + e_{pl})(\frac{\sqrt{W}}{\sqrt{Z}} - 1) = N$ as $\overline{e_{pl}}$, so that

$$\overline{e_{pl}} = -e_{cc} + N\frac{\sqrt{Z}}{\sqrt{W} - \sqrt{Z}} \tag{4.43}$$

Proposition 4.5. *When* $e_{pl} > \overline{e_{pl}}$*, the system is aligned in all scenarios, and every product is produced using the hybrid production mode.*

Through calculation, we see that $\frac{e_{cc}}{n_{max}} = \frac{e_{cc}+\overline{e_{pl}}}{N} = \frac{\sqrt{Z}}{\sqrt{W}-\sqrt{Z}}$, which indicates that when the platform exerts effort $\overline{e_{pl}}$, each product has the same effort investment as the promoted products under the coordinated contract.

Combining Proposition 4.5 and Corollary 4.2, we derive Corollary 4.3:

Corollary 4.3. *When* $k < (\sqrt{W} - \sqrt{Z})^2$, $e_{pl}^H > \overline{e_{pl}}$.

This suggests that when unit effort cost k is small enough for the cooperative effort strategy to achieve higher system profit than the effort coordinating strategy, the misalignment is eliminated by the additional platform effort.

4.5 Numerical study

In Section 4.3, we characterize the boundaries for cases when the hybrid production mode is profitable and proposed a coordinating contract that allows for the hybrid production mode to be implemented in the most profitable way. In Section 4.4, we propose an additional cooperative effort strategy inspired by a commonly used cooperative advertising approach to enhance the hybrid production mode and calculated the boundary condition for when this strategy should be adopted. In this section, we use a parameter set that is grounded in practical examples to demonstrate how the scenario boundaries change with different parameters in Section 4.5.2 and quantify the optimal profit improvement brought about by the hybrid production mode when it is coordinated in Section 4.5.3. We then quantify the system profit improvement resulting from implementing hybrid production mode with additional platform effort in Section 4.5.4 and explore how system profit improvement changes when the platform does not exert all of the optimal amount of effort under the hybrid production mode e_{pl}^{H} in Section 4.5.5.

4.5.1 Choices for parameter values

As noted above, there exist many POD platforms with business models similar to the one analyzed in our study. We use information provided by the Merch by Amazon website and reports posted by content creators to inform the practical parameters for our numerical study. Based on our understanding of the business model for similar POD platforms, these values are representative of general POD platforms. Because effort in our study is difficult to quantify, we use advertising dollars to proxy the cost of effort.

Merch by Amazon suggests that cost-per-click bids should start at \$0.50[6]. The conversion rate is reported to be around 10%[7]. Thus, for each unit sold, the ad spending is around \$5. We assume that the ad campaign has a relatively low advertising cost of sales of around 20%[8]; we can then derive the price for the product as $p = \frac{\$5}{20\%} = \$25$, which is close to what we observe on the Merch platform. Assume that one unit of sales is achieved by increasing the effort investment from \$0 to \$5; then, using the equation

$$\alpha(v-p) - \frac{\beta}{1+5} - \left[\alpha(v-p) - \frac{\beta}{1+0}\right] = 1, \tag{4.44}$$

we can derive the negative competition effect on demand as $\beta = 1.2$.

The Merch platform only allows content creators with more than 100 products in their

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[6] See `https://merch.amazon.com/resource/PXS2VF2PLDDQZAE`.

[7] See `https://www.smart-minded.com/en/business/merch-by-amazon/`.

[8] See `https://www.adbadger.com/blog/amazon-ppc-education/acos-amazon/`.

portfolios to have access to the platform's advertising tools. Here, we assume $N = 600$ to be representative of typical content creators in our study who consider creating designs on POD platforms as a significant source of income and are rational when making effort investment decisions. This assumption is comparable to another example, where a portfolio of 400 products generates monthly revenue of $\$3,324$ without advertising[9]. In our newsvendor model, we assume that the period over which we evaluate product demand and profit is three months and can thus derive from the portfolio example that every product generates one unit of demand in the period without advertising. Assuming that the customer reservation price is \$5 above the selling price and substituting the value for β using

$$\alpha(v - p) - \frac{\beta}{1 + 0} = 1, \tag{4.45}$$

we obtain $\alpha = 0.44$. We also assume $\gamma = 0.3$, which is within the range of $[0, \frac{1}{3}]$.

The royalty parameters of $a = 0.75$ and $b = 13$ are calculated using the examples given on the Merch website[10]. We assume the unit cost for the MTO production mode to be $c_{MTO} = \$10$, based on the platform's minimum revenue per unit of sales b. We assume the unit cost for MTS $c_{MTS} = \$1$, based on the unit price for ordering t-shirts in bulk on TaoBao.com in China.

The set of parameter values is summarized in Table 4.3.

4.5.2 When should the platform offer a coordinating contract?

Figure 4.2 reveals the regions in which the platform needs to offer a coordinating contract to obtain the maximum benefit from the hybrid production mode. Here, we show the sensitivity of these regions to the changes in system parameters. Recall that we focus only on Scenarios n-Hybrid-A and Mix-A, as Scenarios n-Hybrid-B and Mix-B are not practical.

Figure 4.3 shows how changes in the MTO unit cost c_{MTO} affect the regions in which

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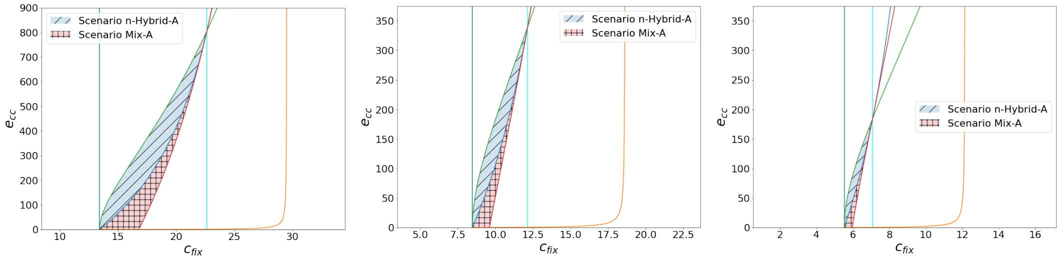
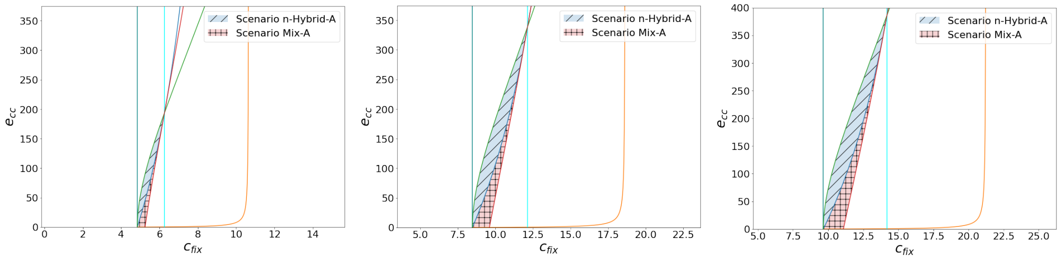
[9] See `https://empireflippers.com/sell-merch-by-amazon-business/`

[10] See `https://merch.amazon.com/resource/201858580`.



(a) $c_{MTO} = 15$ (b) $c_{MTO} = 10$ (c) $c_{MTO} = 7$

Figure 4.3: Regions in which the platform needs to provide a coordinating contract as MTO unit cost c_{MTO} changes

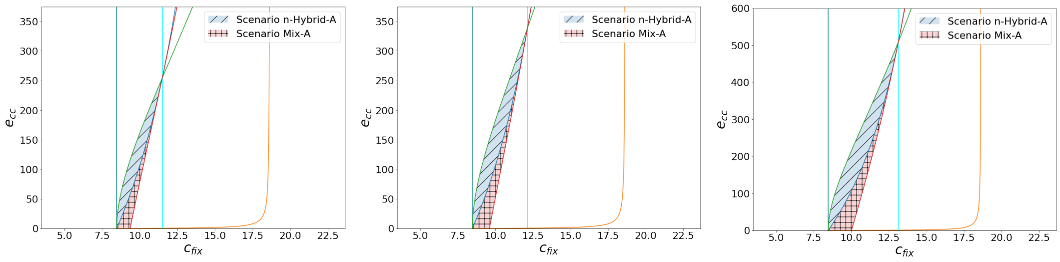


(a) $c_{MTS} = 4$ (b) $c_{MTS} = 1$ (c) $c_{MTS} = 0.2$

Figure 4.4: Regions in which the platform needs to provide a coordinating contract as MTS unit cost c_{MTS} changes

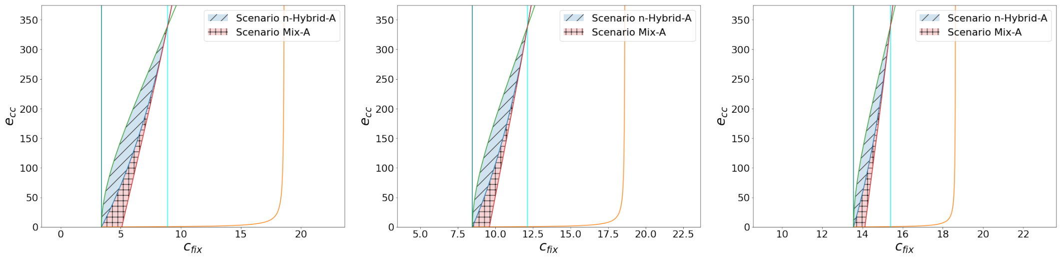
the platform needs to offer a coordinating contract. As c_{MTO} decreases from \$15 to \$10 to \$7, the advantage of the hybrid production mode decreases. Consequently, the hybrid production mode becomes profitable at lower values of c_{fix}, and we observe that the area in which the coordinating contract is needed shrinks. We observe the opposite dynamic in Figure 4.4, where we change the value of c_{MTS}. This is intuitive, because the benefit of the hybrid production mode decreases as c_{MTS} increases.

Changes in product margin can also change the misalignment regions, as indicated by Figure 4.5, where we analyze the sensitivity of the regions in which a coordinating contract is needed to the changes in the margin. We keep $v - p$ constant to ensure that product demand remains constant and that we can focus on the effect of the margin on the mis-



(a) $v = 35, p = 30$ (b) $v = 30, p = 25$ (c) $v = 25, p = 20$

Figure 4.5: Regions in which the platform needs to provide a coordinating contract as product margin changes



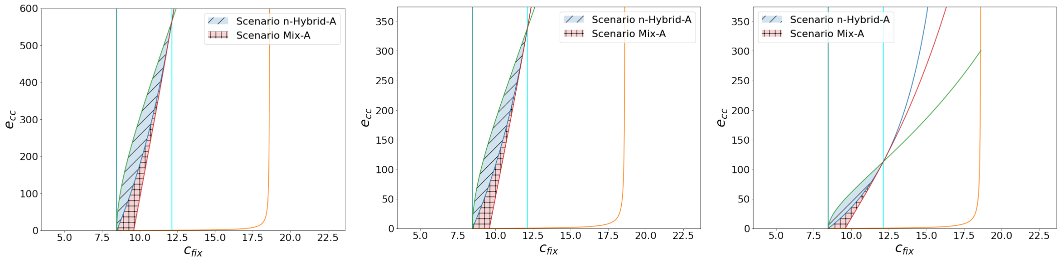
(a) $\beta = 1.8$ (b) $\beta = 1.2$ (c) $\beta = 0.6$

Figure 4.6: Regions in which the platform needs to provide a coordinating contract as the negative effect of competition β changes

alignment regions. As the product margin increases, the hybrid production mode becomes less attractive to the platform. As a result, the amount of content creator effort e_{cc} required to adopt the hybrid production mode and achieve optimal profit decreases, and the area needing a coordinating contract is smaller.

Figure 4.6 demonstrates the effect of the competition parameter β on the misalignment regions. As β increases, the competition from outside market becomes fiercer, which leads to a decrease in the hybrid production mode's ability to hedge against fixed cost c_{fix}. We observe that the area that requires a coordinating contract shifts to the left and becomes wider.

Lastly, we evaluate the influence of total number of products in a portfolio N on the



(a) $N = 1000$ (b) $N = 600$ (c) $N = 200$

Figure 4.7: Regions in which the platform needs to provide a coordinating contract as the total number of products in a portfolio N changes

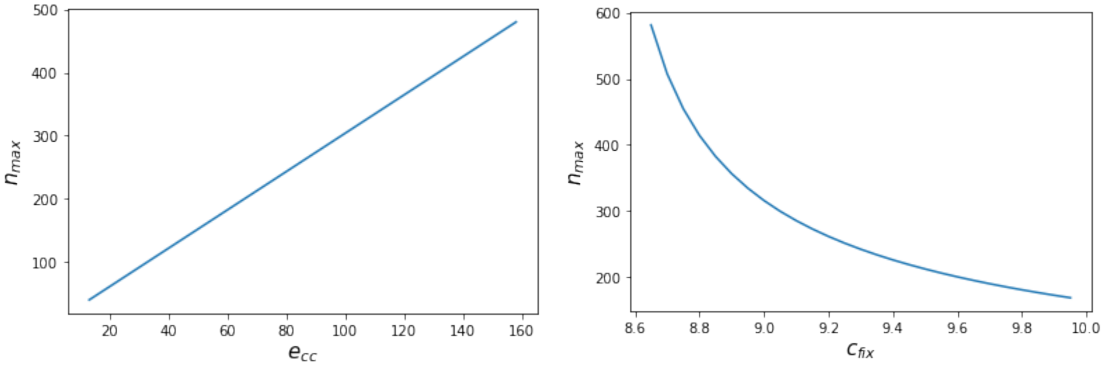
misalignment regions (see Figure 4.7). Our assumption in Section 4.5.1 is compared to when N increases to $1,000$ and when N decreases to 200. As the portfolio size N increases, even though the optimal effort for each product to adopt hybrid production mode does not change, the overall amount of content creator effort e_{cc} required increases, and the area in which a coordinating contract is needed is larger.

4.5.3 Coordinated hybrid production mode adoption

In Section 4.3.3, we show that the coordinating contract can achieve the first-best scenario system profit by encouraging content creators to promote n_{max} products. Here, in Figure 4.8, we quantify the value of n_{max} and explore how the amount of content creator effort e_{cc} and the fixed cost for hybrid production mode c_{fix} affect n_{max}. We focus on these two parameters because they play a crucial role in defining the regions in which the hybrid production mode is optimal and where the system needs to implement a new coordinating contract.

Figure 4.8a shows that n_{max} increases linearly with e_{cc}. The intuition is that the more resource a content creator has, the more products they can promote, so the promoted products can optimally be made with the hybrid production mode.

Figure 4.8b demonstrates that n_{max} decreases with c_{fix}. This is because as the fixed cost



(a) Change of n_{max} with e_{cc}, $c_{fix} = 10.2$ (b) Change of n_{max} with c_{fix}, $e_{cc} = 50$

Figure 4.8: Changes in local optimal number of promoted products n_{max} over content creator effort e_{cc} and MTS fixed cost c_{fix}

rises, the demand requirement for products to optimally adopt hybrid production mode also rises to justify the higher fixed cost. Thus, with the same level of limited e_{cc}, the content creator has to concentrate their effort into fewer products.

Based on the optimal effort concentration level n_{max}, we calculate the optimal system profit improvement that the coordinating contract can achieve. Here, the system profit improvement is calculated by comparing it to the system profit in the benchmark scenario:

$$\Delta\pi_{sys}^{CH} = \frac{\pi_{sys}^{CH}(n_{max})}{\pi_{sys}^{MTO}(N)} - 1. \tag{4.46}$$

When we set $n_{max} = N$, we can also calculate the system profit improvement achieved by adopting the hybrid production mode in *Case All-Hybrid* and *Case n-Hybrid*, excluding Scenario n-Hybrid-A, where the system is aligned.

Using the assumption values for parameters in Section 4.5.1, we generated a heat map (see Figure 4.9) that demonstrates the system profit improvement brought about by the coordinated hybrid production mode $\Delta\pi_{sys}^{CH}$ for all combinations of (e_{cc}, c_{fix}) in *Case All-Hybrid*, *Case n-Hybrid*, and Scenario Mix-A.

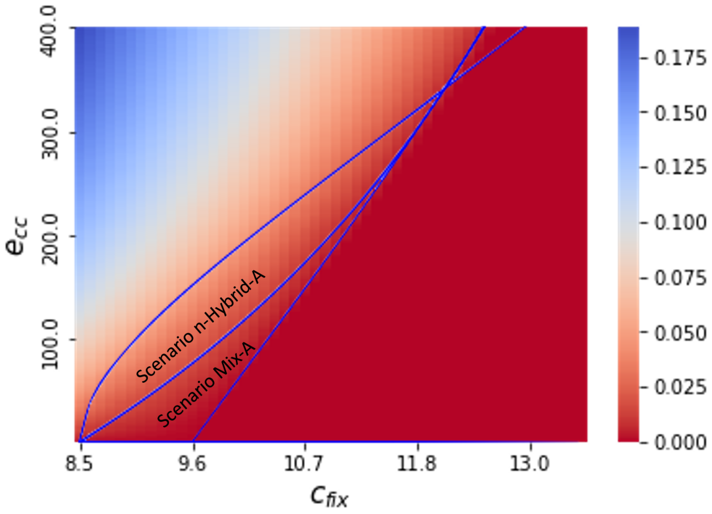


Figure 4.9: System profit improvement $\Delta\pi_{sys}^{CH}$ for coordinated hybrid production mode in *Case All-Hybrid*, *Case n-Hybrid*, and Scenario Mix-A

We see that, within the misalignment scenarios, the higher content creator effort e_{cc} and lower fixed cost in the hybrid production mode c_{fix} lead to greater system profit improvement achieved by coordinating effort. This occurs because higher e_{cc} means that the system has more effort resources to optimize over and that a greater number of products can optimally be made with hybrid production mode, leading to greater system profit improvement. Similarly, a lower fixed cost for the hybrid production mode means that more products can be made with that mode, leading to greater system profit improvement.

These observations also hold in *Case All-Hybrid* and *Case n-Hybrid*, excluding Scenario n-Hybrid-A, where the misalignment does not exist. The higher e_{cc} means more demand is now satisfied with the hybrid mode, indicating a larger profit increase. The lower c_{fix} means that the hybrid production mode is more profitable, thus justifying larger profit increase when switching from MTO production.

4.5.4 Optimal cooperative effort strategy

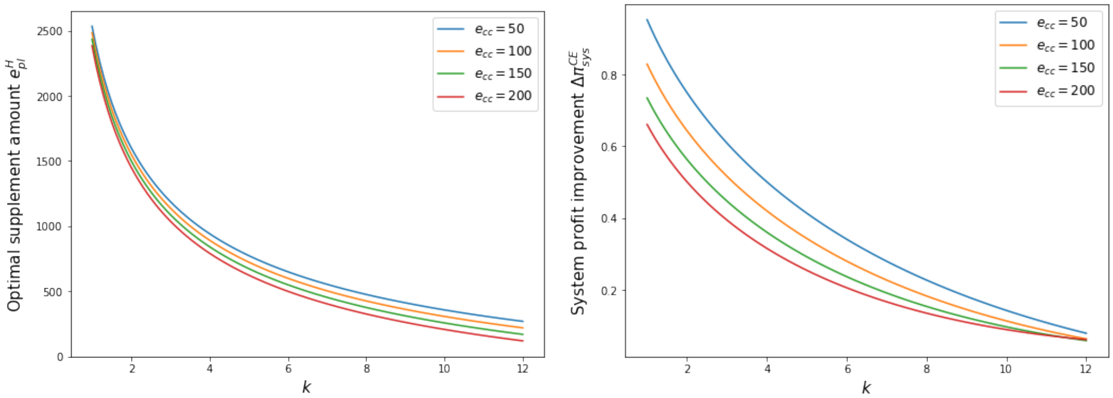
In Section 4.4, we analyze the system profit achieved by implementing the hybrid production mode with additional platform effort. Since with a reasonable unit effort cost to the platform, the hybrid production mode leads to a larger system profit than the MTO production mode, we limit our numerical study here to the platform's exertion of effort e_{pl}^{H} under the hybrid production mode.

In Figure 4.10, we explore how the value of e_{pl}^{H} and the corresponding system profit improvement with platform effort and the hybrid production mode are affected by the change in unit effort cost for the platform k. In line with Section 4.5.3, the system profit improvement for cooperative effort strategy is calculated by comparing it to the system profit in the benchmark scenario:

$$\Delta\pi_{sys}^{CE} = \frac{\pi_{sys-H}^{CE}(N, e_{cc} + e_{pl}^{H})}{\pi_{sys}^{MTO}(N)} - 1. \tag{4.47}$$

We choose to demonstrate these relationships for different levels of content creator effort. When the fixed cost for the hybrid production mode c_{fix} is \$10.2: $e_{cc} = 50$ is in *Case Mix*, where no incentive misalignment exists; $e_{cc} = 100$ represents Scenario Mix-A; $e_{cc} = 150$ represents Scenario n-Hybrid-A; and $e_{cc} = 200$ is in the scenario where no incentive misalignment exists for *Case n-Hybrid*.

We see that even in scenarios where the content creator effort e_{cc} is high enough that an incentive gap does not exist, exerting platform effort can still further boost system profit. As the unit effort cost k increases, exerting platform effort becomes more expensive, so the optimal amount of platform effort e_{pl}^{H} decreases. As a result, the system profit improvement compared to the benchmark scenario also decreases. We also observe that as the content creator effort level e_{cc} increases, the less effort the platform needs to put into the product portfolio. This, together with the increasing benchmark system profit due to the higher e_{cc}, causes the system profit improvement brought about by the cooperative effort strategy to



(a) Optimal platform effort amount e_{pl}^{H}

(b) System profit improvement from platform effort $\Delta\pi_{sys}^{CE}$

Figure 4.10: Changes in optimal platform effort e_{pl}^{H} and system profit improvement $\Delta\pi_{sys}^{CE}$ with platform unit effort cost k when value of content creator effort e_{cc} varies

decrease.

To compare the magnitude of system profit improvement brought about by the cooperative effort strategy to the effort coordinating contract, we generate a heatmap similar to Figure 4.9 in Figure 4.11. Here, we choose $k = 6$ in the calculations.

Comparing Figures 4.11 and 4.9 shows that even when the effort is six times more costly for the platform than for the content creators, exerting the optimal amount of platform effort e_{pl}^{*} can achieve higher profit improvement than simply coordinating the system. This occurs because the cooperative effort strategy brings more resources into the system to make all products more attractive when compared to the outside market, while the coordinated hybrid production mode only optimizes over the amount of resources that are already in the system. We also observe that a lower content creator effort level e_{cc} causes a larger amount of platform effort e_{pl}^{H}, leading to greater system profit improvement $\Delta\pi_{sys}^{CE}$. Higher fixed cost for the hybrid production mode c_{fix} will reduce system profit improvement when compared to the benchmark scenario in which only the MTO mode is adopted, because the hybrid production mode becomes more expensive.

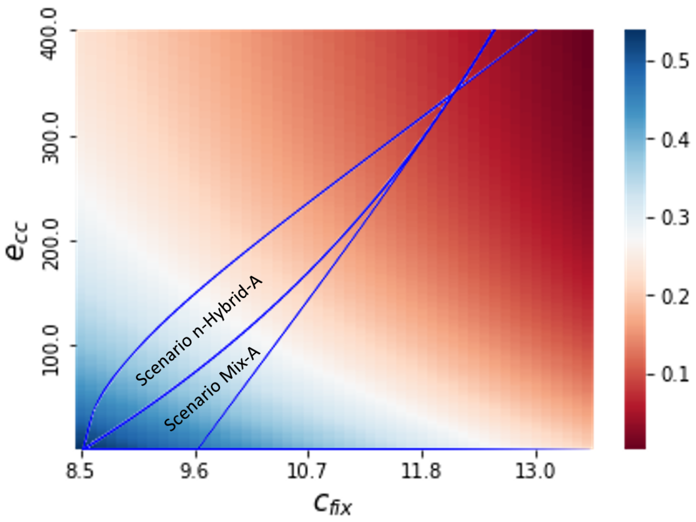


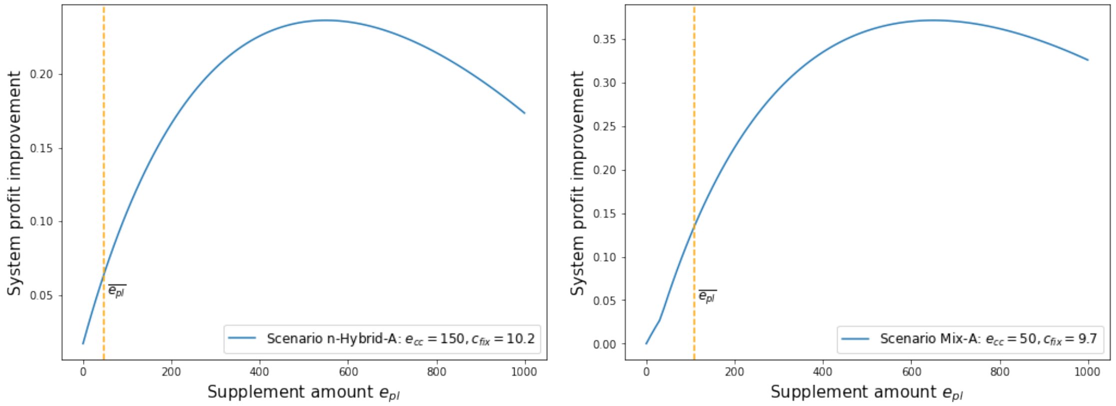
Figure 4.11: System profit improvement $\Delta\pi_{sys}^{CE}$ due to adopting the hybrid production mode and cooperative effort strategy

4.5.5 Exerting less than the optimal platform effort under the hybrid production mode

Even though we have demonstrated that the cooperative effort strategy can lead to tremendous system profit improvement, the large number of existing content creators may mean that the platform does not have enough resources to exert the full amount of e_{pl}^{H} for each content creator. In this section, we explore how the system profit improvement changes when the platform does not exert the full optimal amount of effort e_{pl}^{H}.

For a (e_{cc}, c_{fix}) combination in Scenario n-Hybrid-A, as the platform's effort amount e_{pl} increases from zero to beyond $\overline{e_{pl}}$, the system's preferred production mode remains the hybrid production mode. For a (e_{cc}, c_{fix}) combination in Scenario Mix-A, as the platform's effort amount increases from zero, the products are made using the MTO mode first. When the amount of platform effort continues to increase beyond $\widehat{e_{pl}}$, the platform shifts to the hybrid production mode. We demonstrate how the system profit improvement changes with platform effort amount e_{pl} for both examples in Figure 4.12.

We see from Figure 4.12 that even when the platform does not exert the full optimal



(a) Scenario n-Hybrid-A example with $e_{cc} = 150$, $c_{fix} = 10.2$

(b) Scenario Mix-A example with $e_{cc} = 50$, $c_{fix} = 9.7$

Figure 4.12: System profit improvement in Scenarios n-Hybrid-A and Mix-A

amount of effort, the system profit still improves with platform effort e_{pl}. Using practical parameters in our example, the boundary condition for when the cooperative effort strategy is more profitable in cases without misalignment is \$18.00, and the boundary condition derived in Corollary 4.2 is \$15.90, which are well above the reasonable range for k. Since the effort cost for the platform $k = 6$ is less than this boundary, we show the optimal platform effort amount e_{pl}^H is more than the amount needed to eliminate the incentive misalignment $\overline{e_{pl}}$. We test for multiple (e_{cc}, c_{fix}) combinations for all scenarios and find that the insights we derive from Figure 4.11 still hold.

4.6 Conclusion

In this chapter, we focus on effort allocation across products while continuing to explore the potential profit gains for POD platforms brought about by the hybrid production mode. The platform can take advantage of both the lower unit cost for MTS production and the flexibility of MTO production, but it needs to boost the demand for products to make up for the fixed cost that comes with the MTS production mode. In a decentralized setting, because of the diminishing returns of effort on product demand, content creators prefer to spread their effort evenly across all their products. When the effort of content creators is too low, their preference can cause product demand to be too low to efficiently use the hybrid production mode. We propose a coordinating contract that encourages content creators to aggregate their effort allocation by offering a higher royalty percentage to products that enjoy higher demand. The coordinated hybrid production mode achieves the optimal system profit. We also investigate the cooperative effort strategy in which the platform exerts additional effort, at its own expense, to promote the content creators' products. This strategy enhances the hybrid production mode and achieves system profit improvement by increasing individual product demand, thus making the pie bigger.

Compared to the strategy of limiting the number of listed products on the platform in Chapter 3, the coordinated hybrid mode strategy and cooperative effort strategy in Chapter 4 impose fewer restrictions on total demand while achieving the same goal of optimal use of the hybrid production mode.

We note the following managerial insights that have been revealed in the work presented above: the hybrid production mode can be used to lower production costs and improve system profits, but it requires the demand for a single product to be higher than a certain threshold to justify the fixed cost that comes with adding MTS to MTO. When content creator effort levels are low or the fixed cost is high, the content creators' tendency to spread their effort investment across all products in their portfolios will lead to inefficiency in using the hybrid production mode and cause an incentive misalignment. The platform can impose a contract that rewards products with higher demand with a higher royalty percentage to encourage content creators to aggregate their effort allocation and coordinate the system. The platform can also exert additional effort into the product portfolios at its own expense to improve the profitability of the hybrid production mode. With reasonable unit effort cost for the platform, the system profit improvement brought about by the cooperative effort strategy is much higher than that of simply coordinating the system by using a hybrid production mode. Even when the platform exerts less than the optimal amount of effort, the system profit can still benefit from the additional platform effort.

Our study has certain limitations. To make the model tractable, we make the assumption that there is no competition among products on the platform and that the only competition comes from outside the platform. When accounting for competition among products on the platform, the profit improvement brought about by the cooperative effort strategy will be reduced due to cannibalization among products on the platform.

We study the practical setting in which content creators cannot choose the amount of effort e_{cc} to exert because of the platform's requirement but do choose how to spread that effort over their product portfolio. In other settings where content creators are free to choose how much effort to exert, we see that there exists an optimal amount of content creator effort. The cooperative effort strategy can only be effective when the content creators' effort budget is lower than the content creators' optimal effort amount. Otherwise, the content creators will reduce the amount of effort they exert in response to the platform's effort, rendering the cooperative effort strategy ineffective. In this case, the platform can choose the royalty discrimination strategy.

The assumption of homogeneous content creators we made for the simplicity of modeling is also different from reality, where there are content creators who are popular YouTubers with high-demand products and generic graphic designers with low-demand products. Future research can investigate heterogeneity and model the demand while considering competition from both products on the platform and the outside market.

Chapter 5

CONCLUDING REMARKS

This dissertation explores potential profit improvements of the rapidly growing t-shirt and apparel print-on-demand platforms that license designs from content creators. The research was inspired by an internship the author completed at Merch by Amazon. During that experience, the author observed that the managers had hopes that adopting the MTS production mode would lead to a reduction in operational costs, but MTS adoption was halted because the low, scattered demand for individual products was hard to forecast and could easily lead to overstock or loss of sales. Furthermore, the high fixed cost incurred by trying to use MTS with a large number of products could not be overcome by the individual product demand pattern.

To solve this problem, which many POD platforms face, this dissertation first proposes an important operational innovation: combining the MTS adoption with the original MTO production mode so that the platform can enjoy both low MTS unit cost and the flexibility of MTO production. In this combined MTS–MTO hybrid production mode, the platform orders pre-determined quantities of different products prior to the sales period and uses them to fulfill the customer orders first. If the inventory runs out, the platform then switches to MTO production mode to fulfill the rest of the orders.

The hybrid production mode inherits the non-zero fixed setup cost associated with each product from MTS. In a decentralized setting (which is the case for most POD platforms) where the content creators optimize their own profits by promoting total demand, the individual demand pattern ends up being low and scattered, which prevents the effective adoption of the hybrid production mode. This dissertation further addresses this problem from two operations aspects, product assortment and effort allocation, which are presented in Chapter 3 and Chapter 4, respectively.

In Chapter 3, we propose that the platform charge a listing fee for each design to offset the fixed cost introduced by the hybrid production mode. This strategy, when combined with the existing royalty contract, encourages content creators to limit the number of products they list on the platform so that individual demand will be boosted enough to enable the adoption of the hybrid production mode.

In Chapter 4, we first propose that instead of rewarding the demand generated by all products equally, the platform impose a contract that rewards products with higher demand with a higher royalty percentage. This strategy shares the benefits of hybrid mode with content creators to encourage them to aggregate their effort allocation on a subset of their products and enables this subset of products to optimally use the hybrid production mode. We then propose that the platform exert additional effort—on top of content creator efforts—to increase the demand for all products on the platform. This strategy will boost individual product demand and make the pie bigger, thus further enhancing the profitability of the hybrid production mode.

Combining these insights from the two chapters, we offer the following suggestions to POD platform managers:

- When the platform contains a large number of products that rarely generate sales, the listing fee tool can be used to clean out inactive designs.
- When the demand for all products is too low and reducing the number of listings could hurt the customer loyalty to the platform as a whole, a royalty discrimination contract can be adopted to encourage content creators to pick their favorite designs and focus on promoting them.
- Whenever effort is not too costly, the platform should, so far as it is able, always exert cooperative effort to help content creators to improve the quality of their products and to amplify the content creators' marketing endeavors.

In conclusion, this dissertation demonstrates that adopting the MTS production mode to reduce operational costs in POD platforms not only is viable but also has great potential for profit improvement. The key to success in this regard is to combine the MTS production mode with the current MTO production mode optimally and take measures to induce demand patterns with higher demand for individual products.

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Appendix A

PROOFS FOR CHAPTER 3

Proof. **Lemma 3.1** For product i, which competes with adjacent products on both sides, we denote $\frac{l_i - l_{i-1}}{2}$ as x_1, and $\frac{l_{i+1} - l_i}{2}$ as x_2. Then, the total length that product i occupies on the market would be $length_i = x_1 + x_2$. If we fix the locations of product $i-1$ and $i+1$, we can optimize the demand for product i by changing l_i:

$$\begin{aligned} D_i(l_i) &= \frac{M^2}{c} - \left(\frac{M}{c} - \frac{l_{i+1} - l_i}{2}\right)^2 \frac{c}{2} - \left(\frac{M}{c} - \frac{l_i - l_{i-1}}{2}\right)^2 \frac{c}{2} \\ &= \frac{M^2}{c} - \left(\frac{M}{c} - x_2\right)^2 \frac{c}{2} - \left(\frac{M}{c} - x_1\right)^2 \frac{c}{2} \\ &= M\left(x_1 + x_2\right) - \frac{c}{2}\left(x_1^2 + x_2^2\right) \\ &\leq M\left(x_1 + x_2\right) - c x_1 x_2. \end{aligned} \tag{A.1}$$

The "=" sign is satisfied when $x_1 = x_2 = \frac{l_{i+1} - l_{i-1}}{4}$. This means that when the position of product i is in the middle of l_{i+1} and l_{i-1}, the demand for product i is maximized.

The overall demand between l_{i+1} and l_{i-1} is two times the demand for product i, as indicated in Figure 3.1, so when we are maximizing the demand for product i, we are also maximizing the total demand between l_{i+1} and l_{i-1}. From a centralized point of view, when the total number of products is fixed, the products need to be located evenly on the Salop's circle to maximize total demand. ☐

Proof. **Lemma 3.2** For all the content creators selling the collection of all n items, their total profit $\pi_{cc}^{D\text{-}MTO}(n)$ is calculated as shown in Equation (3.10).

We see that Equation (3.10) is continuous on the boundaries for n, and for each interval of the n, $\pi_{cc}^{D\text{-}MTO}(n)$ increases as n increases, so we can draw the conclusion that

$$\begin{aligned} n^{D\text{-}MTO} &\to \infty \\ \lim_{n\to\infty} \pi_{cc}^{D\text{-}MTO}(n) &= a(p-b)E(M) \end{aligned} \tag{A.2}$$

Similarly, we can write the profit function for the platform as

$$\begin{aligned} \pi_{pl}^{D\text{-}MTO}(n) &= (p-a(p-b)-c_{MTO})\, nE(M) \\ &= \begin{cases} (p-a(p-b)-c_{MTO})\left(E(M)-\frac{c}{4n}\right) & \text{when } n > \frac{c}{2M_0}, \\ (p-a(p-b)-c_{MTO})\left(\sum_{k=0}^{j}\frac{nM_k^2}{c}q_k + \sum_{k=j+1}^{m}\left(M_k-\frac{c}{4n}\right)q_k\right) & \text{when } \frac{c}{2M_{j+1}} < n \le \frac{c}{2M_j}, \\ (p-a(p-b)-c_{MTO})\left(\sum_{k=0}^{m}\frac{nM_k^2}{c}q_k\right) & \text{when } n \le \frac{c}{2M_m}, \end{cases} \end{aligned} \tag{A.3}$$

and the profit function for the system as

$$\begin{aligned} \pi_{sys}^{D\text{-}MTO}(n) &= (p-c_{MTO})\, nE(M) \\ &= \begin{cases} (p-c_{MTO})\left(E(M)-\frac{c}{4n}\right) & \text{when } n > \frac{c}{2M_0}, \\ (p-c_{MTO})\left(\sum_{k=0}^{j}\frac{nM_k^2}{c}q_k + \sum_{k=j+1}^{m}\left(M_k-\frac{c}{4n}\right)q_k\right) & \text{when } \frac{c}{2M_{j+1}} < n \le \frac{c}{2M_j}, \\ (p-c_{MTO})\left(\sum_{k=0}^{m}\frac{nM_k^2}{c}q_k\right) & \text{when } n \le \frac{c}{2M_m}. \end{cases} \end{aligned} \tag{A.4}$$

Both these functions are continuous on the boundaries for n and increase for each interval of n; thus $n^{D\text{-}MTO} \to \infty$ also maximizes $\pi_{pl}^{D\text{-}MTO}(n)$ and $\pi_{sys}^{D\text{-}MTO}(n)$, and their optimal profits are

$$\pi_{pl}^{D\text{-}MTO}(n^{D\text{-}MTO}) = (p-a(p-b)-c_{MTO})\, E(M), \tag{A.5}$$

$$\pi_{sys}^{D\text{-}MTO}(n^{D\text{-}MTO}) = (p-c_{MTO})E(M). \tag{A.6}$$

□

Proof. **Lemma 3.3** The first order condition of Equation (3.15) is as follows:

$$\frac{dg(h)}{dh} = 0 \Leftrightarrow -c_{MTS} - c_{MTO}\frac{\partial \int_h^\infty (x-h)f(x)dx}{\partial h} = 0. \tag{A.7}$$

Using the Leibniz rule, we have $\frac{\partial \int_h^\infty (x-h)f(x)dx}{\partial h} = -\bar{F}(h)$, where $\bar{F}(\cdot)$ is the complementary cumulative distribution function of demand. It is easy to see that the second order condition is positive. Thus, we find the h that maximizes $g(h)$ to be

$$\bar{F}(h^*) = \frac{c_{MTS}}{c_{MTO}} = \rho. \tag{A.8}$$

□

Proof. **Corollary 3.1** Using the normal approximation and optimal pre-printing quantity h^*, we have $h^* = \sigma(D)\Phi^{-1}(\rho) + E(D)$. We further simplify Equation (3.15) as follows:

$$g(h) = pE(D) - c_{MTS}h - c_{MTO}\left(\int_h^\infty xf(x)dx - h\int_h^\infty f(x)dx\right) - c_{fix}. \tag{A.9}$$

From Hadley and Whitin (1963), in Appendix 4 Equation 24, we have

$$\begin{aligned}
\int_h^\infty xf(x)dx &= \int_h^\infty x\frac{1}{\sigma(D)}\phi\left(\frac{x-E(D)}{\sigma(D)}\right)dx \\
&= \frac{1}{\sigma(D)}\left[\sigma(D)^2\phi\left(\frac{h-E(D)}{\sigma(D)}\right) + E(D)\sigma(D)\Phi\left(\frac{h-E(D)}{\sigma(D)}\right)\right] \\
&= \sigma(D)\phi\left(\frac{h-E(D)}{\sigma(D)}\right) + E(D)\Phi\left(\frac{h-E(D)}{\sigma(D)}\right),
\end{aligned} \tag{A.10}$$

and we know that

$$\begin{aligned}
\int_h^\infty f(x)dx &= \bar{F}(h) \\
&= \int_h^\infty \frac{1}{\sigma(D)}\phi\left(\frac{h-E(D)}{\sigma(D)}\right)dx \\
&= \int_h^\infty \phi\left(\frac{h-E(D)}{\sigma(D)}\right)D\frac{h-E(D)}{\sigma(D)} \\
&= \Phi\left(\frac{h-E(D)}{\sigma(D)}\right).
\end{aligned} \tag{A.11}$$

Thus, we can write Equation (A.9) as

$$\begin{aligned} g(h) =& pE(D) - c_{MTS}h \\ & - c_{MTO}\sigma(D)\phi\left(\frac{h-E(D)}{\sigma(D)}\right) + c_{MTO}\Phi\left(\frac{h-E(D)}{\sigma(D)}\right)(h-E(D)) - c_{fix}. \end{aligned} \tag{A.12}$$

Substituting the expression for h^*, we obtain

$$\begin{aligned} g(h^*) =& pE(D) - c_{MTS}h^* \\ & - c_{MTO}\sigma(D)\phi\left(\frac{h^*-E(D)}{\sigma(D)}\right) + c_{MTO}\Phi\left(\frac{h^*-E(D)}{\sigma(D)}\right)(h^*-E(D)) - c_{fix} \\ =& pE(D) - c_{MTS}h^* - c_{MTO}\sigma(D)\phi(\Phi^{-1}(\rho)) + c_{MTO}\Phi(\Phi^{-1}(\rho))(h^*-E(D)) - c_{fix} \\ =& pE(D) - c_{MTS}h^* - c_{MTO}\sigma(D)\phi(\Phi^{-1}(\rho)) + c_{MTO}\rho(h^*-E(D)) - c_{fix} \\ =& pE(D) - c_{MTS}E(D) - c_{MTO}\sigma(D)\phi(\Phi^{-1}(\rho)) - c_{fix}. \end{aligned} \tag{A.13}$$

We substitute the expression of the expected and the standard deviation of the product demand in Equation (3.7) and Equation (3.8) and write the profit function for a single product when using the optimal h^* as Equation (3.17). □

Proof. **Propositions 3.1 and 3.2** To calculate the optimal number of products in a decentralized system, we need to look at the content creator's profit function, as shown in Equation (3.18), which has the exact same form as the profit for content creators in Equation (3.10). Referring back to Equation (A.2), we know that

$$n^{D\text{-}H} \to \infty. \tag{A.14}$$

Because the platform has a profit function in the form of Equation (3.19), we see that the term nc_{fix} would approach ∞ because of Equation (A.14). This would drive the profit for the hybrid function to $-\infty$, so the platform would abandon the hybrid production mode and shift back to MTO-only production, which would mean the overall profit for the decentralized case with hybrid mode available, denoted as $\pi_{sys}^{D\text{-}H}(n^{D\text{-}H})$, the same as $\pi_{sys}^{D\text{-}MTO}(n^{D\text{-}MTO})$, as seen in Equation (A.2). □

Proof. **Proposition 3.3** We first look at the expression for $\pi_{sys}^{C\text{-}H}(n)$. For the interval of when $n > \frac{c}{2M_0}$,

$$\begin{aligned}\pi_{sys}^{C\text{-}H}(n) &= (p - c_{MTS})\left(E(M) - \frac{c}{4n}\right) - c_{MTO}\phi(\Phi^{-1}(\rho))\sigma(M) - nc_{fix} \\ &\leq (p - c_{MTS})E(M) - c_{MTO}\phi(\Phi^{-1}(\rho))\sigma(M) - \sqrt{(p - c_{MTS})c_{fix}c},\end{aligned} \tag{A.15}$$

and at this interval, the local optimal point is

$$n^{C\text{-}H} = \begin{cases} \frac{1}{2}\sqrt{\frac{(p-c_{MTS})c}{c_{fix}}} & \text{when } M_0^2(p - c_{MTS}) \geq c_{fix}c, \\ \frac{c}{2M_0} & \text{when } M_0^2(p - c_{MTS}) < c_{fix}c. \end{cases} \tag{A.16}$$

When $n < \frac{c}{2M_0}$, the local optima would fall into the region of $[0, \frac{c}{2M_0})$. The optimal number of products to carry for the centralized case $n^{C\text{-}H}$ is chosen from among these local optima, so it is bounded by $\max(\frac{1}{2}\sqrt{\frac{(p-c_{MTS})c}{c_{fix}}}, \frac{c}{2M_0})$.

We now compare $\pi_{sys}^{C\text{-}H}(n)$ with the upper bound of $\pi_{sys}^{D\text{-}MTO}(n)$ and again look at the interval for when $n^{C\text{-}H} \geq \frac{c}{2M_0}$.

If $n^{C\text{-}H} = \frac{1}{2}\sqrt{\frac{(p-c_{MTS})c}{c_{fix}}}$, then $\pi_{sys}^{C\text{-}H}(n^{C\text{-}H}) > \lim_{n\to\infty} \pi_{sys}^{D\text{-}MTO}(n)$, as long as the condition in Equation (3.21) is satisfied. If $n^{C\text{-}H} = \frac{c}{2M_0}$, then $\pi_{sys}^{C\text{-}H}(n^{C\text{-}H}) > \lim_{n\to\infty} \pi_{sys}^{D\text{-}MTO}(n)$, as long as the condition in Equation (3.22) is satisfied. □

Proof. **Proposition 3.4** For the case when $n^{C\text{-}H} > \frac{c}{2M_0}$, if $c_{list}c \leq a(p - b)M_0^2$,

$$\pi_{cc}^{D\text{-}H\text{-}L}(n) \leq a(p - b)E(M) - \sqrt{a(p - b)c_{list}c}. \tag{A.17}$$

Meanwhile, $\pi_{cc}^{D\text{-}H\text{-}L}(n)$ is maximized at $n^{D\text{-}H\text{-}L} = \frac{1}{2}\sqrt{\frac{a(p-b)c}{c_{list}}}$. In this scenario, $n^{D\text{-}H\text{-}L} \geq \frac{c}{2M_0}$.

If $c_{list}c > a(p - b)M_0^2$, $n^{D\text{-}H\text{-}L} = \frac{c}{2M_0}$, and the local optima is $\pi_{cc}^{D\text{-}H\text{-}L}(\frac{c}{2M_0})$.

For the case when $\frac{c}{2M_{j+1}} \leq n^{D\text{-}H\text{-}L} < \frac{c}{2M_j}$ where $j \in \{1, 2, ..., m-1\}$, we write the first order condition of $\pi_{cc}^{D\text{-}H\text{-}L}(n)$ w.r.t n:

$$\frac{\partial \pi_{cc}^{D\text{-}H\text{-}L}(n)}{\partial n} = a(p-b)\left(\sum_{k=0}^{j} \frac{M_k^2}{c} q_k + \sum_{k=j+1}^{m} \frac{c}{4n^2} q_k\right) - c_{list}. \tag{A.18}$$

We can show the curve is concave by deriving the second order condition:

$$\frac{\partial^2 \pi_{cc}^{D\text{-}H\text{-}L}(n)}{\partial n^2} = -2a(p-b) \sum_{k=j+1}^{m} \frac{c}{4n^3} q_k < 0. \tag{A.19}$$

If we set Equation (A.18) to zero, then we have $n^{D\text{-}H\text{-}L} = \frac{1}{2}\sqrt{\frac{c\sum_{k=j+1}^{m} q_k}{\frac{c_{list}}{a(p-b)} - \sum_{k=0}^{j} \frac{M_k^2}{c} q_k}}$. Combining this with the boundary of the interval, we know that if $\frac{1}{2}\sqrt{\frac{c\sum_{k=j+1}^{m} q_k}{\frac{c_{list}}{a(p-b)} - \sum_{k=0}^{j} \frac{M_k^2}{c} q_k}} < \frac{c}{2M_{j+1}}$, then $n^{D\text{-}H\text{-}L} = \frac{c}{2M_{j+1}}$; if $\frac{1}{2}\sqrt{\frac{c\sum_{k=j+1}^{m} q_k}{\frac{c_{list}}{a(p-b)} - \sum_{k=0}^{j} \frac{M_k^2}{c} q_k}} \geq \frac{c}{2M_j}$, then $n^{D\text{-}H\text{-}L} = \frac{c}{2M_j}$. To translate these into the conditions for c_{list}, we do the following:

$$n^{D\text{-}H\text{-}L} = \begin{cases} \frac{c}{2M_{j+1}} & \text{when } \frac{c_{list}c}{a(p-b)} > M_{j+1}^2 \sum_{k=j+1}^{m} q_k + \sum_{k=0}^{j} \frac{M_k^2}{c} q_k, \\ \frac{c}{2M_j} & \text{when } \frac{c_{list}c}{a(p-b)} \leq M_j^2 \sum_{k=j+1}^{m} q_k + \sum_{k=0}^{j} \frac{M_k^2}{c} q_k, \\ \frac{1}{2}\sqrt{\frac{c\sum_{k=j+1}^{m} q_k}{\frac{c_{list}}{a(p-b)} - \sum_{k=0}^{j} \frac{M_k^2}{c} q_k}} & \text{otherwise.} \end{cases} \tag{A.20}$$

We can further prove that the profit function for $\pi_{cc}^{D\text{-}H\text{-}L}$ is continuous on the edge of each interval by proving $\lim_{n \to (\frac{c}{2M_j})^-} \pi_{cc}^{D\text{-}H\text{-}L}(n) = \pi_{cc}^{D\text{-}H\text{-}L}(\frac{c}{2M_j})$. This indicates that the global optimal of $\pi_{cc}^{D\text{-}H\text{-}L}$ is located in the interval of $(\frac{c}{2M_{j+1}}, \frac{c}{2M_j})$ when the value of $\frac{c_{list}c}{a(p-b)}$ is in the interval of $\left(M_{j+1}^2 \sum_{k=j+1}^{m} q_k + \sum_{k=0}^{j} \frac{M_k^2}{c} q_k, M_j^2 \sum_{k=j+1}^{m} q_k + \sum_{k=0}^{j} \frac{M_k^2}{c} q_k\right)$.

As to the case when $n_{cent}^{MTO/MTS} < \frac{c}{2M_m}$, the first order condition shows it is linear:

$$\frac{\partial \pi_{cc}^{D\text{-}H\text{-}L}(n)}{\partial n} = a(p-b)\sum_{k=0}^{m} \frac{M_k^2}{c} q_k - c_{list}. \tag{A.21}$$

So, if $c_{list} < a(p-b)\sum_{k=0}^{m} \frac{M_k^2}{c} q_k$, $n^{D\text{-}H\text{-}L} = \frac{c}{2M_m}$; otherwise, $n^{D\text{-}H\text{-}L} = 0$. Even though $n^{D\text{-}H\text{-}L}$ cannot be in the interval of $(0, \frac{c}{2M_m})$, we argue that $n^{C\text{-}H}$ also does not exist in this interval, because in this interval, each product does not compete with adjacent products. Assume $n^{C\text{-}H} \in (0, \frac{c}{2M_m})$; if the profit for each product under the hybrid production mode $g(h^*) > 0$, then the number of products n should increase until competition among products emerges. In this case, $n^{C\text{-}H} = \frac{c}{2M_m}$ is out of this interval, and the value of $\pi_{sys}^{C\text{-}H}(n)$ will be evaluated on the interval of $[\frac{c}{2M_m}, \max(\frac{1}{2}\sqrt{\frac{(p-c_{MTS})c}{c_{fix}}}, \frac{c}{2M_0})]$. If the profit for each product for the hybrid production mode $g(h^*) \leq 0$, then the number of products n should decrease until zero, which is also outside the interval of $(0, \frac{c}{2M_m})$.

$\square$

Appendix B

VALIDATING THE NORMAL APPROXIMATION FOR THE PRODUCT DEMAND IN CHAPTER 3

In Section 3.3.1, we choose to use the newsvendor model and solve it for the optimal MTS level for a general distribution of product demand D. Since product demand is motivated by the Salop's model with discretely distributed market potential M, we know that product demand D also follows a discrete distribution. To simplify the analysis, we approximate D with a normal distribution of $Normal(E(D), \sigma(D))$. To validate our approximation decision, we choose 10 discrete distributions for the market potential M and calculate the total profit for Case C-H using both the original distribution and the normal approximation. The distributions are listed in Table B.1. We choose three two-point distributions, three five-point distributions and four ten-point distributions with different distribution shapes, including bell shape, skewed shape to either side, bimodal shape with peaks on the lower and higher values, and evenly distributed shape. The value of points for M is set to be below 0.2 for the same reason when setting the example distribution in the numerical study in Section 3.4.

We plot the value for total profit of Case C-H calculated with both the original distribution for product demand D and the normal approximation for D on the y axis and the number of products n on the x axis. This comparison is done for every example distribution. Other parameters remain the same as those set in the numerical study, where $p = 23$, $c_{MTS} = 1$, $c_{MTO} = 11$, $c = 1$, and $c_{fix} = 0.02$. We summarize our results in Figure B.1 and Table B.2. For all distributions tested, the total profit values calculated using the original distribution and using the normal approximation are very close to one another, and the optimal values for n are almost identical for both approaches, with a maximum difference of one. Thus, our approach of using normal approximation for the distribution of product

index	point values	probabilities
1	$\{0.01, 0.1\}$	$\{0.5, 0.5\}$
2	$\{0.01, 0.1\}$	$\{0.05, 0.95\}$
3	$\{0.01, 0.1\}$	$\{0.95, 0.05\}$
4	$\{0.01, 0.05, 0.09, 0.13, 0.17\}$	$\{0.2, 0.2, 0.2, 0.2, 0.2\}$
5	$\{0.01, 0.05, 0.09, 0.13, 0.17\}$	$\{0.1, 0.2, 0.4, 0.2, 0.1\}$
6	$\{0.01, 0.05, 0.09, 0.13, 0.17\}$	$\{0.3, 0.15, 0.1, 0.15, 0.3\}$
7	$\{0.01, 0.03, 0.05, ..., 0.19\}$	$\{0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1\}$
8	$\{0.01, 0.03, 0.05, ..., 0.19\}$	$\{0.1, 0.3, 0.15, 0.1, 0.1, 0.05, 0.05, 0.05, 0.05, 0.05\}$
9	$\{0.01, 0.03, 0.05, ..., 0.19\}$	$\{0.05, 0.05, 0.05, 0.05, 0.05, 0.1, 0.1, 0.15, 0.3, 0.1\}$
10	$\{0.01, 0.03, 0.05, ..., 0.19\}$	$\{0.05, 0.2, 0.15, 0.05, 0.05, 0.05, 0.05, 0.15, 0.2, 0.05\}$

Table B.1: Example distributions for market potential M

demand D is validated.

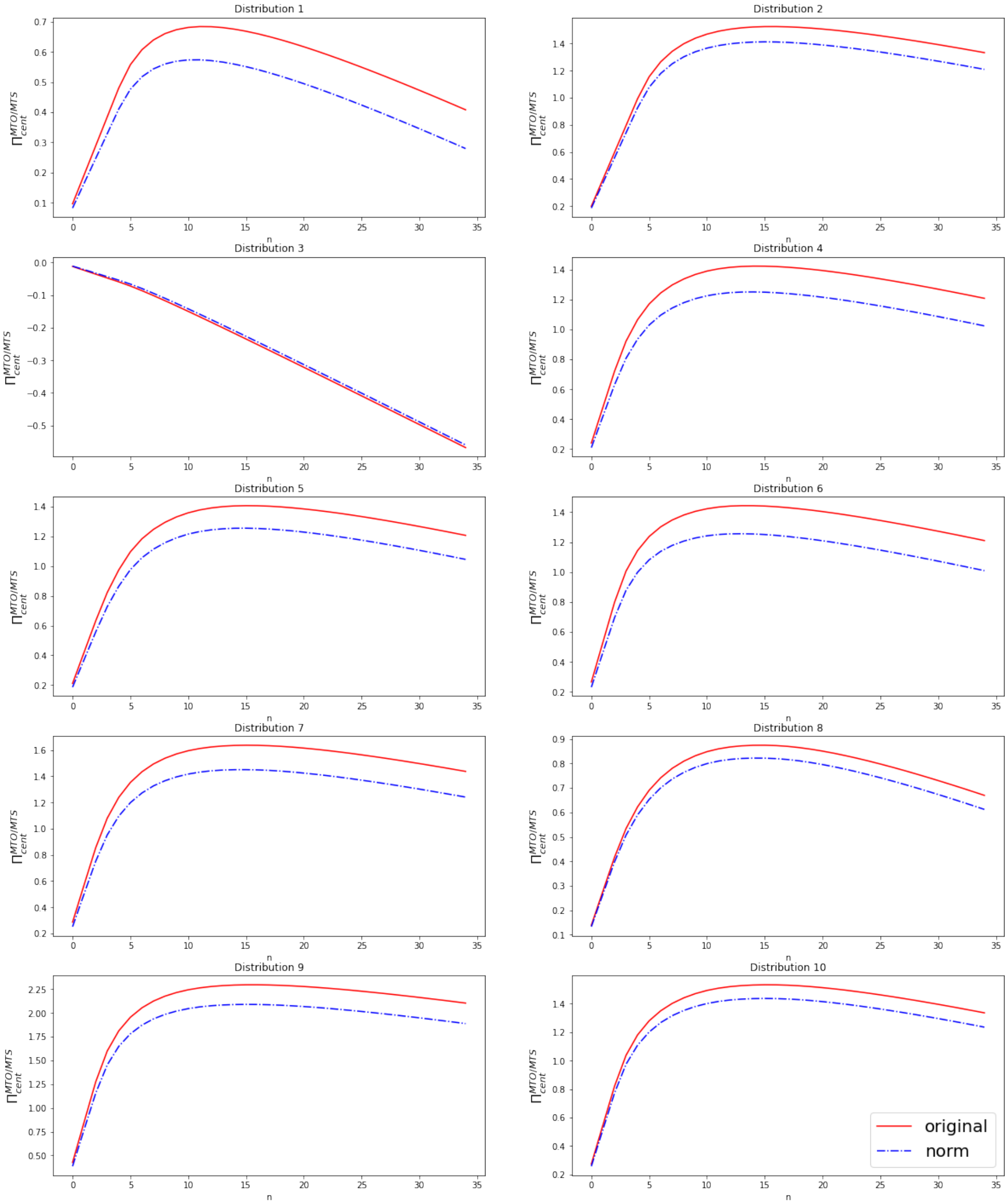

Figure B.1: Validation of the normal approximation for the newsvendor model

distribution index	distribution used in calculation	$n^{C\text{-}H}$	optimal $\pi_{sys}^{C\text{-}H}$
1	original distribution	12	0.684
	normal approximation	12	0.574
2	original distribution	17	1.526
	normal approximation	16	1.413
3	original distribution	1	−0.012
	normal approximation	1	−0.011
4	original distribution	15	1.424
	normal approximation	15	1.252
5	original distribution	16	1.407
	normal approximation	16	1.256
6	original distribution	14	1.443
	normal approximation	14	1.256
7	original distribution	16	1.637
	normal approximation	16	1.451
8	original distribution	16	0.875
	normal approximation	15	0.822
9	original distribution	17	2.296
	normal approximation	16	2.089
10	original distribution	16	1.533
	normal approximation	16	1.437

Table B.2: Results comparison: original distribution vs. normal approximation

Appendix C

PROOFS FOR CHAPTER 4

Proof. **Proposition 4.1** We calculate the F.O.C. of Equation (4.5) as follows:

$$\begin{aligned}\frac{d\pi_{cc}(n)}{dn} &= -a(p-b)\left[\alpha(v-p)-\beta\right]+a(p-b)\alpha(v-p)-a(p-b)\frac{2\beta n(n+e_{cc})-\beta n^2}{(n+e_{cc})^2}\\ &= a(p-b)\beta\frac{e_{cc}^2}{(n+e_{cc})^2}>0.\end{aligned} \tag{C.1}$$

Thus, we know that $\pi_{cc}(n)$ increases with n, and because $n \in [1, N]$, then $n^* = N$ optimizes Equation (4.5). This indicates that content creators prefer to use the fixed amount of effort to promote all N products in their portfolios.

By substituting $n = N$ in Equation (4.5), we obtain the optimal content creator profit in the benchmark scenario in Equation (4.6). ☐

Proof. **Proposition 4.2** We calculate the F.O.C. of Equation (4.8) as follows:

$$\frac{d\pi_{sys}^{MTO}(n)}{dn} = (p-c_{MTO})\beta\frac{e_{cc}^2}{(n+e_{cc})^2}>0. \tag{C.2}$$

Based on Equation (C.2), we know that $\pi_{sys}^{MTO}(n)$ increases with n, and because $n \in [1, N]$, then $n^* = N$ optimizes $\pi_{sys}^{MTO}(n)$. Thus, the centralized system is aligned with the content creators to promote all N products in a content creator's portfolio.

By substituting $n = N$ in Equation (4.8), we obtain the optimal content creator profit in the benchmark scenario in Equation (4.9). ☐

Proof. **Lemma 4.1** We compare the two equations in (4.10) and calculate conditions when the hybrid production mode yields a higher profit than the MTO production mode:

$$\begin{aligned}\pi^{H}_{sys\text{-}prod} - \pi^{MTO}_{sys\text{-}prod} &= (c_{MTO} - c_{MTS})\mu(\frac{e_{cc}}{n}) - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\mu(\frac{e_{cc}}{n}) - c_{fix} \\ &= c_{MTO}\left[1 - \rho - \phi(\Phi^{-1}(\rho))\gamma\right]\mu(\frac{e_{cc}}{n}) - c_{fix}.\end{aligned} \tag{C.3}$$

When $\pi^{H}_{sys\text{-}prod} > \pi^{MTO}_{sys\text{-}prod}$, then $\mu(\frac{e_{cc}}{n}) > \hat{\mu} = \frac{c_{fix}}{c_{MTO}[1-\rho-\phi(\Phi^{-1}(\rho))\gamma]}$.

□

Proof. **Lemma 4.2** We calculate the F.O.C. for Equation (4.16) as

$$\begin{aligned}\frac{d\pi^{n\text{-}H}_{sys}(n)}{dn} = &- (p - c_{MTO})\left[\alpha(v-p) - \beta\right] \\ &+ \left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]\left[\alpha(v-p) - \beta\frac{n^2 + 2km}{(n+e_{cc})^2}\right] - c_{fix},\end{aligned} \tag{C.4}$$

and the S.O.C. for Equation (4.16) as

$$\frac{d^2\pi^{n\text{-}H}_{sys}(n)}{dn^2} = -\left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]\beta e_{cc}^2\frac{2}{(n+e_{cc})^3} < 0. \tag{C.5}$$

Thus, we know the local maximizer for Equation (4.16) is achieved when $\frac{d\pi^{n\text{-}H}_{sys}(n)}{dn} = 0$. Solving this, we obtain

$$\frac{n^2 + 2km}{(n+e_{cc})^2} = \frac{\left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]\alpha(v-p) - c_{fix} - (p - c_{MTO})\left[\alpha(v-p) - \beta\right]}{\left[p - c_{MTS} - cMTO\phi(\Phi^{-1}(\rho))\gamma\right]d}. \tag{C.6}$$

Using the notations defined in Equation (4.18), we solve for the unconstrained maximizer as follows:

$$n_{max} = e_{cc}\left(\frac{\sqrt{W}}{\sqrt{Z}} - 1\right). \tag{C.7}$$

Considering the boundary conditions for Case Mix and the practical range of n, there are three situations for the optimal solution of Equation (4.16):

$$n^*_{n\text{-}H} = \begin{cases} N & \text{when } \alpha(v-p)-\beta < \hat{\mu},\ 1 \leq N \leq n_{max}, \text{ and } \mu(\frac{e_{cc}}{N}) > \hat{\mu}, \\ n_{max} & \text{when } \alpha(v-p)-\beta < \hat{\mu},\ 1 \leq n_{max} \leq N \text{ and } \mu(\frac{e_{cc}}{N}) > \hat{\mu}, \\ 1 & \text{when } \alpha(v-p)-\beta < \hat{\mu},\ n_{max} \leq 1 \leq N \text{ and } \mu(\frac{e_{cc}}{N}) > \hat{\mu}. \end{cases} \tag{C.8}$$

To simplify the results given in Equation (C.8), we have

$$n^*_{n\text{-}H} = \max\left(\min\left(N, n_{max}\right), 1\right). \tag{C.9}$$

□

Proof. **Lemma 4.3** Inside the interval of $[1, \hat{n}]$, $\pi^{Mix}_{sys}(n) = \pi^{n\text{-}H}_{sys}(n)$; thus, similar to Lemma 4.2, we deduct the maximizer for $\pi^{Mix}_{sys}(n)$ in this interval as follows:

$$\tilde{n}_1 = \begin{cases} \hat{n} & \text{when } \alpha(v-p)-\beta < \hat{\mu},\ 1 \leq \hat{n} \leq n_{max}, \text{ and } \hat{n} \leq N, \\ n_{max} & \text{when } \alpha(v-p)-\beta < \hat{\mu}, \text{ and } 1 \leq n_{max} \leq \hat{n} \leq N, \\ 1 & \text{when } \alpha(v-p)-\beta < \hat{\mu}, \text{ and } n_{max} \leq 1 \leq \hat{n} \leq N. \end{cases} \tag{C.10}$$

This can be simplified as $\tilde{n}_1 = \max\left(\min\left(\hat{n}, n_{max}\right), 1\right)$.

Inside the interval of $(\hat{n}, N]$, $\pi^{Mix}_{sys}(n) = \pi^{MTO}_{sys}(n)$; thus, from Proposition 4.2, we know the maximizer for $\pi^{Mix}_{sys}(n)$ in this interval is $\tilde{n}_2 = N$.

Thus, the optimal solution for $\pi^{Mix}_{sys}(n)$ in the interval of $[1, N]$ is chosen from the set of $\{\tilde{n}_1, \tilde{n}_2\}$, depending on which brings higher value to the profit function, as shown in Equation (4.22). □

Proof. **Proposition 4.3** We calculate the F.O.C. for the content creator's profit under the new contract in Equation (4.27) as follows:

$$\frac{d\pi_{cc}^{CH}(n)}{dn} = \begin{cases} a_1(p-b)\beta\frac{e_{cc}^2}{(n+e_{cc})^2} > 0 & \text{when } n > \overline{n}, \\ (a_2 - a_1)\left[\alpha(v-p) - \beta\right] + a_2\beta\frac{e_{cc}^2}{(n+e_{cc})^2} > 0 & \text{when } n \leq \overline{n}. \end{cases} \tag{C.11}$$

Because the F.O.C. is positive on both intervals, the content creator will choose n from the local maxima of $\overline{n}$ and N. To coordinate the system, the platform needs to set the parameters of $\overline{\mu}, a_1, a_2$ such that

$$\begin{cases} \overline{n} = n_{max}, \\ \pi_{cc}^{CH}(\overline{n}) > \pi_{cc}^{CH}(N). \end{cases} \tag{C.12}$$

Substituting the expressions of $\overline{n}$ in Equation (4.26) and the expression of $\pi_{cc}^{CH}(n)$ in Equation (4.27), we can solve the conditions for the coordinating contract as follows:

$$\begin{cases} \overline{\mu} = \alpha(v-p) - \frac{\beta n_{max}}{n_{max}+e_{cc}}, \\ a_2 > a_1\frac{n_{max}\mu(e_{cc}=0) + \frac{\beta N e_{cc}}{N+e_{cc}}}{n_{max}\overline{\mu}}. \end{cases} \tag{C.13}$$

To ensure the coordinating contract is incentive-compatible, we also need to guarantee that both the content creator and the platform are not worse off than under the benchmark case:

$$\begin{cases} \pi_{cc}^{CH}(n_{max}) \geq \pi_{cc}(N), \\ \pi_{pl}^{CH}(n_{max}) \geq \pi_{pl}^{MTO}(N). \end{cases} \tag{C.14}$$

We can expand these conditions as follows:

$$\begin{cases} \{a_1(N-n_{max})\left[\alpha(v-p)-\beta\right]+a_2 n_{max}[\alpha(v-p)-\frac{\beta n_{max}}{n_{max}+e_{cc}}]\}(p-b) \geq \\ \qquad\qquad a(p-b)N\left[\alpha(v-p)-\frac{\beta N}{N+e_{cc}}\right], \\ (N-n_{max})\left[p-c_{MTO}-a_1(p-b)\right]\left[\alpha(v-p)-\beta\right]+ \\ \quad n_{max}\left\{\left[p-c_{MTS}-c_{MTO}\phi(\Phi^{-1}(\rho))\gamma-a_2(p-b)\right]\left[\alpha(v-p)-\frac{\beta n_{max}}{n_{max}+e_{cc}}\right]-c_{fix}\right\}\geq \\ \quad \left[p-c_{MTO}-a(p-b)\right]N\left[\alpha(v-p)-\frac{\beta N}{N+e_{cc}}\right]. \end{cases} \tag{C.15}$$

Solving these conditions, we obtain

$$\begin{cases} a_2 \geq \underline{a_2}^{contract}, \\ a_2 \geq \underline{a_2}, \\ a_2 \leq \overline{a_2}, \end{cases} \tag{C.16}$$

where

$$\begin{cases} \underline{a_2}^{contract} = a_1 \frac{n_{max}\mu(e_{cc}=0)+\frac{\beta N e_{cc}}{N+e_{cc}}}{n_{max}\overline{\mu}}, \\ \underline{a_2} = \frac{1}{n_{max}\overline{\mu}}\left[aN\mu(\frac{e_{cc}}{N})-a_1(N-n_{max})\mu(0)\right], \\ \overline{a_2} = \frac{1}{(p-b)n_{max}\overline{\mu}}\{\left[p-c_{MTO}-a_1(p-b)\right](N-n_{max})\mu(0) \\ \quad -\left[p-c_{MTO}-a(p-b)\right]N\mu(\frac{e_{cc}}{N}) \\ \quad +\left[p-c_{MTS}-c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right]n_{max}\overline{\mu}-n_{max}c_{fix}\}. \end{cases} \tag{C.17}$$

For a_2 to exist, we need

$$\begin{cases} \underline{a_2}^{contract} \leq \overline{a_2}, and \\ \underline{a_2} \leq \overline{a_2}. \end{cases} \tag{C.18}$$

We see that $\underline{a_2} \leq \overline{a_2}$ is satisfied for all values of a_1 because

$$\overline{a_2}-\underline{a_2} = \frac{1}{(p-b)n_{max}\overline{\mu}}(\pi_{sys}^{n\text{-}H}(n_{max})-\pi_{sys}^{MTO}(N)), \tag{C.19}$$

and in Scenarios n-Hybrid-A and Mix-A, $\pi_{sys}^{n\text{-}H}(n_{max}) > \pi_{sys}^{MTO}(N)$. Solving $\underline{a_2}^{contract} \leq \overline{a_2}$, we obtain the condition in Equation (4.29).

We also note that the right-hand side in Equation (4.29) is greater than a in the misalignment scenarios, so a_1 always exists. We prove that the right-hand side in Equation (4.29) is greater than a as follows:

$$\begin{aligned}
&\frac{1}{(p-b)N\mu(\frac{e_{cc}}{N})}\{(p-c_{MTO})\,(N-n_{max})\mu(0) + \left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right] n_{max}\overline{\mu} \\
&- n_{max}c_{fix} - \left[p - c_{MTO} - a(p-b)\right] N\mu(\frac{e_{cc}}{N})\} - a \\
&= \frac{\pi_{sys}^{n\text{-}H}(n_{max}) - \pi_{sys}^{MTO}(N)}{(p-b)N\mu(\frac{e_{cc}}{N})} > 0
\end{aligned} \tag{C.20}$$

□

Proof. **Proposition 4.4** We calculate the F.O.C. for the system's profit function in Equation (4.30) as

$$\frac{\partial \pi_{sys-H}^{CE}(N, e_{cc}+e_{pl})}{\partial e_{pl}} = NWN\frac{1}{(N+e_{cc}+e_{pl})^2} - k, \tag{C.21}$$

and the S.O.C. as

$$\frac{\partial^2 \pi_{sys-H}^{CE}(N, m+e_{pl})}{\partial e_{pl}^2} = NWN\frac{-2}{(N+e_{cc}+e_{pl})^3}. \tag{C.22}$$

We see that $\frac{\partial^2 \pi_{sys-H}^{CE}(N,e_{cc}+e_{pl})}{\partial e_{pl}^2} < 0$; thus, Equation (4.30) is maximized when $\frac{\partial \pi_{sys-H}^{CE}(N,e_{cc}+e_{pl})}{\partial e_{pl}} = 0$. We solve for the e_{pl} that maximizes the system profit as

$$e_{pl}^H = N\left(\sqrt{\frac{W}{k}} - 1\right) - e_{cc}. \tag{C.23}$$

We note that because the platform adds to the content creator effort, e_{pl}^H is not negative. Thus the optimal amount of platform effort is derived as in Equation (4.31).

We calculate the F.O.C. of Equation (4.32) as

$$\frac{\partial \pi_{sys-MTO}^{CE}(N, e_{cc}+e_{pl})}{\partial e_{pl}} = N\left(p-c_{MTO}\right)\beta N \frac{1}{(N+e_{cc}+e_{pl})^2} - k, \tag{C.24}$$

and the S.O.C. as

$$\frac{\partial^2 \pi_{sys-MTO}^{CE}(N, e_{cc}+e_{pl})}{\partial e_{pl}^2} = N\left(p-c_{MTO}\right)\beta N \frac{-2}{(N+e_{cc}+e_{pl})^3}. \tag{C.25}$$

We see that $\frac{\partial^2 \pi_{sys-MTO}^{CE}(N, e_{cc}+e_{pl})}{\partial e_{pl}^2} < 0$; thus, Equation (4.32) is maximized when $\frac{\partial \pi_{sys-MTO}^{CE}(N, e_{cc}+e_{pl})}{\partial e_{pl}} = 0$. We solve for the e_{pl} that maximizes the system profit as

$$e_{pl}^{MTO} = N\left(\sqrt{\frac{\beta\left(p-c_{MTO}\right)}{k}} - 1\right) - e_{cc}. \tag{C.26}$$

We note that because the platform adds to the content creator effort, e_{pl}^{MTO} is not negative. Thus the optimal amount of platform effort is derived as in Equation (4.33).

For Case All-Hybrid and Case n-Hybrid, the system profit function with platform effort and hybrid production mode is the same as in Equation (4.30); thus, the optimal amount of platform effort is given in Equation (4.34).

For Case Mix and Case All-MTO, the profit function is as follows:

$$\pi_{sys-Mix-All-MTO}^{CE}(N, e_{cc}+e_{pl}) = \begin{cases} \pi_{sys-H}^{CE}(N, e_{cc}+e_{pl}) & \text{when } e_{pl} \geq \widehat{e_{pl}}, \\ \pi_{sys-MTO}^{CE}(N, e_{cc}+e_{pl}) & \text{when } e_{pl} < \widehat{e_{pl}}. \end{cases} \tag{C.27}$$

We first prove that this profit function is continuous at $e_{pl} = \widehat{e_{pl}}$:

$$\lim_{e_{pl}\to\widehat{e_{pl}}^-} \pi_{sys-Mix-All-MTO}^{CE}(N, e_{cc}+e_{pl}) = N\left(p-c_{MTO}\right)\mu\left(\frac{e_{cc}+\widehat{e_{pl}}}{N}\right) - e_{cc} - k\widehat{e_{pl}}, \tag{C.28}$$

and

$$\begin{aligned}\lim_{e_{pl}\to\widehat{e_{pl}}^{+}} \pi^{CE}_{sys-Mix-All-MTO}(N, e_{cc}+e_{pl}) =& N\left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right] \\ & \times \mu\left(\frac{e_{cc}+\widehat{e_{pl}}}{N}\right) - Nc_{fix} - e_{cc} - k\widehat{e_{pl}}.\end{aligned} \tag{C.29}$$

By substituting $\widehat{e_{pl}} = N(\frac{\beta}{\alpha(v-p)-\hat{\mu}} - 1) - e_{cc}$, we calculate that

$$\lim_{e_{pl}\to\widehat{e_{pl}}^{-}} \pi^{CE}_{sys-Mix-All-MTO}(N, e_{cc}+e_{pl}) = \lim_{e_{pl}\to\widehat{e_{pl}}^{+}} \pi^{CE}_{sys-Mix-All-MTO}(N, e_{cc}+e_{pl}).$$

Next, we discuss the optimal solutions. From previous analysis, we know that $\pi^{CE}_{sys-H}(N, e_{cc}+e_{pl})$ is concave and has an optimal solution shown in Equation (4.31). Similarly, we know that $\pi^{CE}_{sys-MTO}(N, e_{cc}+e_{pl})$ is concave and has an optimal solution shown in Equation (4.33). Thus, the optimal solution for $\pi^{CE}_{sys-Mix-All-MTO}(N, e_{cc}+e_{pl})$ is as follows:

$$\begin{cases} e^{H}_{pl} \quad \text{when } e^{H}_{pl} \geq \widehat{e_{pl}}, \text{ and } e^{MTO}_{pl} \geq \widehat{e_{pl}}, \\ \arg\max\left(\pi^{CE}_{sys-MTO}(N, e_{cc}+e^{MTO}_{pl}), \pi^{CE}_{sys-H}(N, e_{cc}+e^{H}_{pl})\right) \\ \qquad \text{when } e^{H}_{pl} \geq \widehat{e_{pl}}, \text{ and } e^{MTO}_{pl} < \widehat{e_{pl}}, \\ e^{MTO}_{pl} \quad \text{when } e^{H}_{pl} < \widehat{e_{pl}}, \text{ and } e^{MTO}_{pl} < \widehat{e_{pl}}. \end{cases} \tag{C.30}$$

By substituting $\widehat{e_{pl}} = N(\frac{\beta}{\alpha(v-p)-\hat{\mu}} - 1) - e_{cc}$, we derive the optimal solutions as in Equation (4.35).

□

Proof. **Corollary 4.1** To ensure incentive compatibility for both the platform and content creators with the optimal amount of platform effort, we need to guarantee that both the platform and the content creator are not worse off after the platform exerts additional effort with the hybrid production mode available, compared to the benchmark scenario.

When the optimal platform's effort amount is e_{pl}^{H}, for the content creators, we need to satisfy the condition of $\pi_{cc}^{CE}(N, e_{cc} + e_{pl}^{H}) \geq \pi_{cc}(N)$, where

$$\pi_{cc}^{CE}(N, e_{cc} + e_{pl}^{H}) = Na_3(p-b)\left[\alpha(v-p) - \frac{\beta N}{N + e_{cc} + e_{pl}^{H}}\right] - e_{cc}, \tag{C.31}$$

and

$$\pi_{cc}(N) = Na(p-b)\left[\alpha(v-p) - \frac{\beta N}{N + e_{cc}}\right] - e_{cc}. \tag{C.32}$$

Similarly, for the platform, we need to satisfy the condition of $\pi_{pl-H}^{CE}(N, e_{cc} + e_{pl}^{H}) \geq \pi_{pl}^{MTO}(N)$, where

$$\begin{aligned}\pi_{pl-H}^{CE}(N, e_{cc} + e_{pl}^{H}) =& N\left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma - a_3(p-b)\right] \\ & \times \left[\alpha(v-p) - \frac{\beta N}{N + e_{cc} + e_{pl}^{H}}\right] - Nc_{fix} - ke_{pl}^{H},\end{aligned} \tag{C.33}$$

and

$$\pi_{pl}^{MTO}(N) = N\left[p - c_{MTO} - a(p-b)\right]\left[\alpha(v-p) - \frac{\beta N}{N + e_{cc}}\right]. \tag{C.34}$$

Solving these, we obtain $a_3 \in \left[\underline{a_{3-H}}, \overline{a_{3-H}}\right]$.

For a_3 to exist in this case, we need

$$\underline{a_{3-H}} \leq \overline{a_{3-H}}, \tag{C.35}$$

and because

$$\overline{a_{3-H}} - \underline{a_{3-H}} = \frac{\pi_{sys-H}^{CE}(N, e_{cc} + e_{pl}^{H}) - \pi_{sys}^{MTO}(N)}{N(p-b)\mu(\frac{e_{cc}+e_{pl}^{H}}{N})}, \tag{C.36}$$

we know that a_3 exists when $\pi_{sys-H}^{CE}(N, e_{cc} + e_{pl}^{H}) > \pi_{sys}^{MTO}(N)$.

When the optimal platform's effort amount is e_{pl}^{MTO}, for the content creators, we need to satisfy the condition of $\pi_{cc}^{CE}(N, e_{cc} + e_{pl}^{MTO}) \geq \pi_{cc}(N)$, where

$$\pi_{cc}^{CE}(N, e_{cc} + e_{pl}^{MTO}) = Na_3(p-b)\left[\alpha(v-p) - \frac{\beta N}{N + e_{cc} + e_{pl}^{MTO}}\right] - e_{cc}. \tag{C.37}$$

Similarly, for the platform, we need to satisfy the condition of $\pi_{pl-MTO}^{CE}(N, e_{cc}+e_{pl}^{MTO}) \geq \pi_{pl}^{MTO}(N)$, where

$$\begin{aligned}\pi_{pl\text{-}MTO}^{CE}(N, e_{cc} + e_{pl}^{MTO}) =& N\left(p - c_{MTO} - a_3(p-b)\right)\left[\alpha(v-p) - \frac{\beta N}{N + e_{cc} + e_{pl}^{MTO}}\right] \\ & - ke_{pl}^{MTO}.\end{aligned} \tag{C.38}$$

Solving these, we obtain $a_3 \in \left[\underline{a_{3-MTO}}, \overline{a_{3-MTO}}\right]$.

Following a similar logic as that used in the proof of a_3's existence, we know that a_3 exists when $\pi_{sys-MTO}^{CE}(N, e_{cc} + e_{pl}^{MTO}) > \pi_{sys}^{MTO}(N)$. ☐

Proof. **Corollary 4.2** To compare the system profits achieved by the cooperative effort strategy against the effort coordinating contract, we calculate the difference between Equation (4.40) and Equation (4.38) as follows:

$$\begin{aligned}\pi_{sys}^{CE}(N, e_{cc} + e_{pl}^{H}) - \pi_{sys}^{CH}(n_{max}) = & NW\frac{e_{cc} + e_{pl}^{H}}{N + e_{cc} + e_{pl}^{H}} - NZ - ke_{pl}^{H} \\ & - n_{max}W\frac{e_{cc}}{n_{max} + e_{cc}} + n_{max}Z.\end{aligned} \tag{C.39}$$

By substituting Equation (4.31) and Equation (4.17), when $\pi_{sys}^{CE}(N, e_{cc} + e_{pl}^{H}) > \pi_{sys}^{CH}(n_{max})$, we obtain the following equation:

$$k(N + e_{cc}) - 2N\sqrt{W}\sqrt{E} + N(W - Z) - e_{cc}(\sqrt{W} - \sqrt{Z})^2 > 0. \tag{C.40}$$

Solving this inequation for k, we obtain

$$\begin{cases} \sqrt{k} < \frac{N\sqrt{W}-\sqrt{N^2W-(N+e_{cc})\left[N(W-Z)-e_{cc}(\sqrt{W}-\sqrt{Z})^2\right]}}{N+e_{cc}}, or \\ \sqrt{k} > \frac{N\sqrt{W}+\sqrt{N^2W-(N+e_{cc})\left[N(W-Z)-e_{cc}(\sqrt{W}-\sqrt{Z})^2\right]}}{N+e_{cc}}. \end{cases} \tag{C.41}$$

Because $e_{pl}^H \geq 0$, by substituting Equation (4.31), we know that $\sqrt{k} \leq \frac{N\sqrt{W}}{N+e_{cc}}$. Thus, we can eliminate the higher-value range for $\sqrt{k}$, and simplify the range as follows:

$$\begin{aligned} \sqrt{k} <& \frac{N\sqrt{W} - \sqrt{N^2W - (N+e_{cc})\left[N(W-Z) - e_{cc}(\sqrt{W}-\sqrt{Z})^2\right]}}{N+e_{cc}} \\ =& \frac{N\sqrt{W} - \sqrt{\left[N\sqrt{Z} - e_{cc}(\sqrt{W}-\sqrt{Z})\right]^2}}{N+e_{cc}}. \end{aligned} \tag{C.42}$$

Because we know for Scenarios n-Hybrid-A and Mix-A, $n_{max} \leq N$, by substituting Equation (4.17), we obtain $N\sqrt{Z} \geq e_{cc}(\sqrt{W} - \sqrt{Z})$. Thus,

$$\begin{aligned} \sqrt{k} <& \frac{N\sqrt{W} - \left[N\sqrt{Z} - e_{cc}(\sqrt{W}-\sqrt{Z})\right]}{N+e_{cc}} \\ =& \sqrt{W} - \sqrt{Z}. \end{aligned} \tag{C.43}$$

So, we know that when $k < (\sqrt{W} - \sqrt{Z})^2$, the cooperative effort strategy achieves a higher system profit than the effort coordinating contract.

□

Proof. **Proposition 4.5** From Equation (4.17), we know the expression of the local maximizer for the system profit function with hybrid production mode applied to promoted products is

$$n_{max} = e_{cc}\left(\frac{\sqrt{W}}{\sqrt{Z}} - 1\right). \tag{C.44}$$

Thus, for the system with platform effort, the updated n_{max} can be written as

$$n_{max} = (e_{cc} + e_{pl}) \left(\frac{\sqrt{W}}{\sqrt{Z}} - 1 \right). \tag{C.45}$$

The system achieves alignment when the platform's preferred number of promoted products $n^*_{n\text{-}H} = \max(\min(N, n_{max}), 1)$ is the same as the content creator's preferred number of promoted products N. Thus, to coordinate the system, the platform needs to exert additional effort $\overline{e_{pl}}$ such that

$$(e_{cc} + \overline{e_{pl}}) \left(\frac{\sqrt{W}}{\sqrt{Z}} - 1 \right) = N. \tag{C.46}$$

We solve Equation (C.46) and obtain (4.43).

Next, we prove $\overline{e_{pl}}$ is the minimum amount of effort for the platform to exert to align the system. We calculate the F.O.C. of Equation (C.45) w.r.t. e_{pl} as follows:

$$\frac{\partial n_{max}}{\partial e_{pl}} = \frac{\sqrt{W}}{\sqrt{Z}} - 1. \tag{C.47}$$

We need to prove that $W > Z$ to show that n_{max} increases with e_{pl} and thus any value of $e_{pl} \geq \overline{e_{pl}}$ will guarantee that $n_{max} \geq N$, such that the system is aligned.

Because the cases related to n_{max} exclude Case MTO, we know that $\mu(e_{cc}) > \hat{\mu}$ in these cases. Since $\alpha(v-p) > \mu(e_{cc})$, we know that $\alpha(v-p) > \hat{\mu}$. By substituting the expression for $\hat{\mu}$, we obtain

$$\alpha(v-p) > \frac{c_{fix}}{c_{MTO} - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma}, \tag{C.48}$$

which can be re-written as

$$\alpha(v-p)\left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right] - c_{fix} > \alpha(v-p)(p - c_{MTO}). \tag{C.49}$$

Thus, we know

$$\alpha(v-p)\left[p - c_{MTS} - c_{MTO}\phi(\Phi^{-1}(\rho))\gamma\right] - c_{fix} > \left[\alpha(v-p) - \beta\right](p - c_{MTO}), \tag{C.50}$$

which is equivalent to $W > Z$. ☐

Proof. **Corollary 4.3** $k < (\sqrt{W} - \sqrt{Z})^2$ is equivalent to $\sqrt{k} < \sqrt{W} - \sqrt{Z}$ because we have previously proven in Proposition 4.5 that $W > Z$.

By substituting $\sqrt{k} < \sqrt{W} - \sqrt{Z}$ into Equation (4.31), we have

$$\begin{aligned} e_{pl}^{H} &= N\left(\sqrt{\frac{W}{k}} - 1\right) - e_{cc} \\ &> N\left(\frac{\sqrt{W}}{\sqrt{W} - \sqrt{Z}} - 1\right) - e_{cc} \\ &= N\left(\frac{\sqrt{Z}}{\sqrt{W} - \sqrt{Z}}\right) - e_{cc} \\ &= \overline{e_{pl}}. \end{aligned} \tag{C.51}$$

☐