

Essays on Contract Theory

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Abstract

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This dissertation is primarily on the contractual design to account for various source of information asymmetry in a principal-agent(s) relationship. In the first chapter, I study the optimal provision of team incentives with the feasibility for the agents to coordinate private actions through repeated interaction with imperfect public monitoring. As the agents' imperfect monitoring of private actions is inferred from the stochastically correlated measurements, correlation of measurement noise, besides its risk sharing role in the conventional multiple-agent moral hazard problem, is crucial to the accuracy of each agent's inference on the other's private action. The principal's choice of performance pay to provide incentive via inducing competition or coordination among the agents thus exhibits the tradeoff between risk sharing and mutual

inference between the agents. I characterize the optimal form of performance pay with respect to the correlation of measurement noise and find that it is not monotonic as suggested by the literature.

In the second chapter, I study the optimal incentive provision in a principal-agent relationship with costly information acquisition by the agent. When it is feasible for the principal to induce or to deter perfect information acquisition, adverse selection or moral hazard arises in response to the principal's decision, as if she is able to design a contract not only to cope with an existing incentive problem, but also to implement the existence of an incentive problem. The optimal contract to implement adverse selection by inducing information acquisition, comparing to the second best menu, exhibits a larger rent difference between an agent in an efficient state and whom in an inefficient state. The optimal contract to implement moral hazard by deterring information acquisition, comparing to the second best debt contract, prescribes a lower debt and an equity share of output residual. With imperfect information acquisition or private knowledge of information acquiring cost, the contract offered to an uninformed agent is qualitatively robust, and that to the informed exhibits countervailing incentives.

I relax the assumption of complete contracting and study truthful information revelation in an incomplete contracting environment in the third chapter. Truthful revelation of asymmetric information through shared ownership (partnership) is incorporated into the Property Right Theory of the firms. Shared ownership is optimal as an information transmission device, when it is incentive compatible within the relationship as well as when the relationship breaks, at the expense of the ex-ante incentive to invest in the relationship-specific asset as the hold-up concern is not efficiently mitigated. Higher (lower) level of integration is optimal with a lower marginal value of asset if the information rent effect is stronger (weaker) than the hold-up effect.

JEL: D81, D82, D86

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Chapter 1

Team Incentives with Imperfect Mutual Inference

1.1 Introduction

In the studies of team incentive with agents who may coordinate hidden actions through side contracting or repeated interaction¹, each agent is assumed to observe an exogenous signal of each other's action, with the signal independent of the measurement noise. I provide an alternative remedy on how the agents observe each other's hidden action: they infer it through the publicly observable and contractible stochastic measurement of each agent's private action. Each agent, knowing his own private action, knows to what extent the measurement is attributed to noise. If the measurement of each agent's action is stochastically correlated to one another's, by observing another agent's measurement of action, he is able to update his belief on that agent's action. The principal does not have this information as she cannot distinguish private action from measurement noise of any agent. Such inference is more accurate as the measurement noise of each agent's action is more correlated to that of another. Higher correlation improves the agents' mutual inference if they were induced to make it.

Specifically, I borrow the model from Holmström and Milgrom (1990), and incorporate infinitely repeated interaction among the agents while assuming that the principal's equilibrium response is to offer a stationary contract in every period, provided that she cannot commit to a long-term contract, as Che and Yoo (2001) assumed. The key feature of this paper that distinguishes itself from the related literature is that the repeated interaction is enforced by the imperfect inference the agents made

¹Please refer to Holmström (1982), Mookherjee (1984), and Ramakrishnan and Thakor (1991) for pioneer research on team incentive with non-cooperative agents, and Holmström and Milgrom (1990), Itoh (1991, 1993), Che and Yoo (2001), and Rayo (2007) for that with agents coordinating actions with either a side contract or repeated interaction with perfect monitoring. Fleckinger and Roux (2012) surveyed this literature.

on each other's private action. More correlated the agents are in measurement noise, more accurate inference they are able to make, and the collusion/coordination between the agents is more firmly enforced. Thus, higher correlation of measurement noise potentially has the following effects: i) it allows for more efficient risk sharing under a contract with relative performance evaluation (RPE), the risk sharing effect highlighted by Holmström and Milgrom (1990); ii) it improves accuracy of inference among the agents, which better enforces the agents' coordination under a contract with joint performance evaluation (JPE), the monitoring effect emphasized by Rayo (2007). The principal's choice of contract and her decision on whether to induce the agents to coordinate then exhibit a tradeoff between these two effects: risk sharing under RPE is distorted to deter collusion among the agents, and inducing agents to coordinate actions is at the expense of higher risk premium.

I find that, for some parametric space of the common discount factor and intrinsic trust between the agents, the optimal contract is not monotonic in correlation parameter. The optimal contract has collusion-proof RPE for extreme correlations, and coordination-inducing JPE for intermediate correlations.

For sufficiently low correlation of measurement noise, the risk sharing effect with respect to an increase in the correlation is relatively weaker than the effect of improved mutual inference. The optimal contract has collusion-proof RPE for sufficiently low correlations, coordination-inducing JPE otherwise. This contradicts to Holmström and Milgrom (1990)'s claim that RPE is optimal for more correlated measurement noises, yet it is consistent to Rayo (2007) who suggested that a more accurate signal is in favor of JPE that induces implicit incentive.

For sufficiently correlated noise of measurement, the risk sharing effect with respect to an increase in the correlation is relatively prominent compared to the effect of improved mutual inference. The optimal contract has collusion-proof RPE for sufficiently high correlations, coordination-inducing JPE otherwise, even though a higher correlation implies a more accurate inference. This is consistent to Holmström and Milgrom (1990), yet it contradicts to Rayo (2007).

The chapter is organized as the following. The model and the agents' mutual inference and repeated game with grim trigger strategy is laid out in Section 1.2. Benchmark contracts are given in Section 1.3. Section 1.4 is devoted to the optimal contract to deter the agents from collusion and that to induce the agent to coordinate. Optimality of coordination-inducing and collusion-proofness with respect to the correlation of measurement noise and its underlying trade-off is discussed in Section 1.5. I examine a couple of other repeated game strategies in Section 1.6, and Section 1.7

concludes the paper. All proofs are in Appendix A.1.

1.1.1 Related Literature

The paper builds on a vast literature devoted to the study of optimal team incentive, with the option that agents are able to coordinate private actions through an explicit side contract (Holmström and Milgrom (1990), Itoh (1991, 1993)), relational side payment (Rayo (2007)), or enforced by repeated interaction (Che and Yoo (2001)). They, however, assume the private action to be either contractible among the agents or observable through an exogenous signal. I take one step further to argue that each agent's observation is an inference drawn from the stochastically correlated outputs, which attributes the accuracy of observation to the correlation of noise, and yields new conclusions to the literature.

Repeated game with imperfect public monitoring has been developed in the industrial organizational literature (Green and Porter (1984) and Abreu, Pearce, and Stacchetti (1986)) and made general by Abreu, Pearce, and Stacchetti (1990) and Fudenberg, Levine, and Maskin (1994). This paper, to the best of my knowledge, is the first attempt to apply it into agency theory to study the optimal provision of team incentive.

Fleckinger (2012) has similar findings that RPE is favored with lower (equilibrium) correlation of noise, a contradiction to Holmström and Milgrom (1990). However, he assumes that correlation of measurement noise depends on effort level, so that equilibrium correlation not only provides the principal with information on stochastic environment (informative correlation) but also signals all agents' private actions (technological correlation). I show in this paper that even if correlation of measurement noise is independent of the agents' actions (purely informative) as in Holmström and Milgrom's model, the principal can still benefit from inducing the agents to coordinate and make inference on each other's private action updated from the realization of outputs. Meyer and Vickers (1997) discuss incentive gain or loss of RPE with respect to different level of cross-sectional and inter-temporal correlations with the presence of career concern. Their focus is on the interaction between ratchet effect and insurance, whether they reinforce, or oppose each other. Collusion and coordination between the agents are absent in both papers.

Ishiguro (2004) and Kim (2011) characterize the optimal collusion-proof contract in a tournament, assuming that the risk neutral agents with limited liability can write side contract contingent on verifiable outcome before exerting private actions. There

is no loss of generality to consider only collusion-proof contracts in their models as the agents do not possess superior information to that of the principal. They conclude that shutting down one agent non-anonymously is essential to deter side contracting due to the tournament and limited liability setup, where performance pay contingent on actual realization of outcome is impossible.

1.2 Model

1.2.1 Model Specification

A principal contracts with two agents $i = 1, 2$ to execute a project at each period $t = 1, 2, \dots$, in which each agent exerts productive independent private actions $a_{it} \in \{a_L, a_H\}$, $a_L < a_H$, at a cost $c_{it}(a_{it}) \in \{c_L, c_H\}$, $c_L < c_H$, that generates a publicly observable and contractible output x_{it} , which is an imperfect measurement of the private action, $x_{it} = a_{it} + \varepsilon_{it}$, where ε_{it} is the stochastic nature agent i faced at period t that is time-independently distributed.² Let (x_{1t}, x_{2t}) jointly follows normal distribution with unit variance and covariance $0 \leq \sigma \leq 1$ across agents,

$$\begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} \sim N \left(\begin{pmatrix} a_{1t} \\ a_{2t} \end{pmatrix}, \begin{pmatrix} 1 & \sigma \\ \sigma & 1 \end{pmatrix} \right).$$

I assume throughout that implementing $a_i = a_H$ for $i = 1, 2$ is optimal, and focus on the characterization of the optimal contract to induce the agents to or to deter them from coordinating private actions, given the agents' ability to make inference on each other's action through correlated imperfect measurement, specified in the next section.

The principal is risk neutral with time separable utility $u_{pt} = \sum_i (x_{it} - w_{it}(x_{1t}, x_{2t}))$, where $w_{it}(x_{1t}, x_{2t})$ is the transfer paid to agent i contingent on realization of outputs at each period. To emphasize the agents' repeated interaction and how it enforces coordination of actions, suppose for simplicity that the principal is unable to commit to a long-term contract, so that he offers the stationary contract $w_i(x_{1t}, x_{2t})$ as the static equilibrium repeated infinitely³. Each agent is risk averse with time-separable expo-

²An alternative way to model is to assume that the agents exert private actions to produce a non-contractible noisy output, and that each agent's private action is measured separately and imperfectly by a contractible indicator, which is stochastically correlated among the agents. Modeling in this way, conclusions would be qualitatively the same even with synergy in production.

³Lack of commitment to long-term contracts can also be justified as the existence of a sequentially optimal short-term contract. Fudenberg, Holmström, and Milgrom (1990) constructed sufficient conditions for such existence. In brief, the principal and the agents are symmetrically informed of future history-independent contingent outcomes and technologies, and that preferences are time

nential utility $u_{it} = -\exp(-rw_{it}(x_{1t}, x_{2t}) - c_{it}(a_{it}))$, where $r > 0$ notes the agent's constant absolute risk aversion.

Assumption of exponential utility allows us to take advantage of its convenience without losing too much insight. In each agent's optimization problem given accepted contract, maximization of the expected utility with an exponential form is equivalent to maximization of its certainty equivalence, denoted as $\bar{u}_{it} = E(w_i(x_{1t}, x_{2t})) - c_{it}(a_{it}) - \frac{r}{2}Var(w_i(x_{1t}, x_{2t}))$, which allows us to directly model the risk sharing concern, captured by $R(w_i(x_{1t}, x_{2t})) \equiv \frac{r}{2}Var(w_i(x_{1t}, x_{2t}))$. In addition, with exponential utilities and time-separable production, Holmström and Milgrom (1987) have shown the optimality of linear contract, $w_i(x_{1t}, x_{2t}) = \alpha_i x_{it} + \beta_i x_{jt} + \gamma_i$, in which β_i as the rate contingent on the other agent's realization of output determines the contractual form I intend to study in this paper.

The optimal contract may take the form of relative performance evaluation, joint performance evaluation, or independent piece rate, defined respectively as the following. The contract has a relative performance evaluation (RPE) if a better realization of one agent's output lowers the other's payoff, i.e. if $\beta_i < 0$; it has a joint performance evaluation (JPE) if a better realization of one agent's output increases the other's payoff, i.e. if $\beta_i > 0$; it consists of an independent piece rate (IPR) if each agent's payoff is independent of the other's realization of output, i.e. if $\beta_i = 0$.

1.2.2 Imperfect Inference via Correlated Noise

After signing the contract and before production, the agents can coordinate with each other and agree on a non-verifiable action profile (a_1^0, a_2^0) , when the contract implements static Nash equilibrium (a_1', a_2') . The actions are not directly observable to each other, but given a production function with separable action and measurement noise, each agent is aware of to what extent his own realization of output is attributed to measurement noise, which is correlated to that of the other agent. Hence, each agent has partial information on the other's output realization that is attributed to the measurement noise instead of to the productive action. This information is not available to the principal, who does not observe any agent's action, thus cannot distinguish realization of output due to productive action or measurement noise of any agent. Specifically, I model this ability to make inference through correlated measurement noise as what follows.

separable, in which case a long-term contract has no extra value than a sequence of its short-term counterpart.

Agent $j \neq i$, knowing his own private action, forms a belief on agent i 's production, as a conditional density of x_{it} on x_{jt} ,

$$x_{it}(a_{it})|x_{jt}(a_j^0) \sim N(a_{it} + \sigma(x_{jt} - a_j^0), 1 - \sigma^2).$$

When the realization of agent i 's output falls far from the mean, at an extreme tail of the above conditional density, agent j believes that agent i has deviated from their agreement (a_1^0, a_2^0) . This is precisely defined in the following assumption.

Assumption 1.1. *Agent j believes that agent i has unilaterally deviated downward from their non-verifiable agreement (a_{1t}^0, a_{2t}^0) if the realization of x_{it} conditional on x_{jt} is $s > 0$ standard deviations lower than the conditional mean, i.e. if $x_{it}(a_{it})|x_{jt}(a_j^0) < a_i^0 + \sigma(x_{jt} - a_j^0) - s\sqrt{1 - \sigma^2}$, and has unilaterally deviated upward from their agreement if the realization of x_{it} conditional on x_{jt} is $s > 0$ standard deviations higher than the conditional mean, i.e. if $x_{it}(a_{it})|x_{jt}(a_j^0) > a_i^0 + \sigma(x_{jt} - a_j^0) + s\sqrt{1 - \sigma^2}$.*

s is assumed to be exogenous except in Section 1.6. It can be regarded as an intrinsic trust between the agents: if the agents place more trust on each other, each believes that the other has cheated when the realization of output is farther from the conditional mean, represented by a larger s .

Let q_i^d be the probability of agent j detecting agent i 's deviation correctly, and q_i^n be the probability of false detection in the case of no deviation. That is,

$$q_i^d = \begin{cases} \Pr(x_{it}(a'_i)|x_{jt}(a_j^0) < a_i^0 + \sigma(x_{jt} - a_j^0) - s\sqrt{1 - \sigma^2}) & \text{for } a'_i < a_i^0 \\ \Pr(x_{it}(a'_i)|x_{jt}(a_j^0) > a_i^0 + \sigma(x_{jt} - a_j^0) + s\sqrt{1 - \sigma^2}) & \text{for } a'_i > a_i^0 \end{cases},$$

$$q_i^n = \begin{cases} \Pr(x_{it}(a_i^0)|x_{jt}(a_j^0) < a_i^0 + \sigma(x_{jt} - a_j^0) - s\sqrt{1 - \sigma^2}) & \text{for } a'_i < a_i^0 \\ \Pr(x_{it}(a_i^0)|x_{jt}(a_j^0) > a_i^0 + \sigma(x_{jt} - a_j^0) + s\sqrt{1 - \sigma^2}) & \text{for } a'_i > a_i^0 \end{cases},$$

where a'_i is the optimal action to deviate given the contract.

Lemma 1.1. q_i^d is increasing in σ and q_i^n is independent of σ .

Proof. Appendix A.1.1. □

Higher correlation between the measurement noise of each agent's action results in a higher probability of a unilateral deviation to be detected, whereas probability of false detection is unaffected. At the extremes, with perfectly correlated measurement noise, agents infer each other's action perfectly, and for independently distributed noise, the agents know as much about each other's action as the principal does.

To see how this inference is incorporated into the agents' repeated interaction, suppose that the stationary contract (α, β, γ) is such that the static game between the agents is in the form of a prisoner's dilemma: the non-cooperative Nash equilibrium is (a'_1, a'_2) , yet the expected utility maximizing action profile is (a_1^0, a_2^0) .

Assumption 1.2. *The agents play the grim trigger strategy with bang-bang property: they continue to exert the coordinated actions (a_1^0, a_2^0) if a unilateral deviation is not detected; otherwise, the coalition breaks and the agents go back to the non-cooperative Nash equilibrium (a'_1, a'_2) infinitely.*

The stationary contract (α, β, γ) enforce (a_1^0, a_2^0) as a perfect public equilibrium (PPE)⁴ if

$$\begin{aligned} (1 - \delta)\bar{u}_i(a_i^0, a_j^0) + \delta \left((1 - q_i^n)\bar{u}_i(a_i^0, a_j^0) + q_i^n\bar{u}_i(a'_i, a'_j) \right) \\ \geq (1 - \delta)\bar{u}_i(a'_i, a'_j) + \delta \left((1 - q_i^d)\bar{u}_i(a_i^0, a_j^0) + q_i^d\bar{u}_i(a'_i, a'_j) \right), \end{aligned}$$

where δ is the common discount factor. Note that with imperfect monitoring and a symmetric setting as in this model, punishment for both agents must occur on the equilibrium path to enforce (a_1^0, a_2^0) , making the most punishing grim trigger strategy inefficient.⁵ Nevertheless, it is the optimal form of contract that admits (a_1^0, a_2^0) instead of the punishment strategy in the agents' coalition that is in the interest of this paper; I thereby adopt the simplest and briefly discuss trigger strategy with a finite period of punishment in Section 1.6.

1.3 Benchmarks

As the agents are different only in realization of measurement noise, implementing (a_H, a_H) , we can focus on symmetric equilibrium where the contract has $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2) = (\alpha, \beta, \gamma)$. In this section, we look at two benchmark contracts: the non-

⁴On general repeated games with imperfect public monitoring, please refer to Abreu, Pearce, and Stacchetti (1990) and Fudenberg, Levin, and Maskin (1994). Threshold grim trigger strategies were traditionally adopted in the study of repeated Cournot games to enforce collusion, e.g. Porter (1983) and Green and Porter (1984) as pioneer works and Aoyagi and Fr chet (2009) for a recent theoretical and experimental work.

⁵Fudenberg, Levine, and Maskin (1994) derived conditions under which a grim trigger strategy can yield efficient payoff. In brief, not only a unilateral deviation of a given player is detected stochastically as in this paper (individual full rank), but also the player who deviate can be detected stochastically (pairwise full rank).

cooperative benchmark in which collusion is impossible, and the coordinated benchmark in which agents can write side contract on actions.

In the non-cooperative benchmark where coordination between agents are assumed to be impossible, (a_H, a_H) is implemented as a Nash equilibrium among the agents if, for each agent $i = \{1, 2\}$, a_H is incentive compatible given the other agent exerting a_H , and each agent earns at least his reservation payoff (normalized to zero).

$$\bar{u}_i(a_H, a_j) \geq \bar{u}_i(a_L, a_j) \quad a_j \in \{a_H, a_L\} \quad (IC_n)$$

$$\bar{u}_i(a_H, a_H) \geq 0 \quad (IR)$$

The fixed payment in the contract, γ , is adjusted to satisfy constraint (IR) without affecting incentive compatibility, thus, along with implementation of (a_H, a_H) , the principal's optimization problem in the non-cooperative benchmark is reduced to minimize the risk premium subject to constraint (IC_n) ,

$$\mathcal{P}_n : \quad \min_{\alpha, \beta} R(\alpha, \beta) = \frac{r}{2}(\alpha^2 + \beta^2 + 2\sigma\alpha\beta)$$

$$s.t. (IC_n)$$

In the coordinated benchmark in which the agents can write a verifiable side contract on actions, (a_H, a_H) is implemented if it is incentive compatible for the team and each agent earns at least his reservation payoff.

$$\bar{u}_1(a_H, a_H) + \bar{u}_2(a_H, a_H) \geq \bar{u}_1(a_1, a_2) + \bar{u}_2(a_1, a_2) \quad (IC_c)$$

$$(a_1, a_2) \in \{(a_H, a_L), (a_L, a_H), (a_L, a_L)\}$$

$$\bar{u}_i(a_H, a_H) \geq 0 \quad (IR)$$

Given binding (IR) , the principal's optimization problem given the coordinated benchmark is to minimize the risk premium subject to constraint (IC_c) ,

$$\mathcal{P}_c : \quad \min_{\alpha, \beta} R(\alpha, \beta) = \frac{r}{2}(\alpha^2 + \beta^2 + 2\sigma\alpha\beta)$$

$$s.t. (IC_c)$$

The two benchmarks are merely a replication of Holmström and Milgrom (1990), the non-cooperative one as the case with no side-trade among agents and the coordinated one as the case with unrestricted side-trade. The benchmark optimal contracts

are summarized in the proposition below.

Proposition 1.0. *The Optimal contract in the non-cooperative benchmark $\mathbb{C}_n = (\alpha_n, \beta_n, \gamma_n)$ exhibits RPE, $\beta_n = -\frac{\sigma(c_H - c_L)}{a_H - a_L} < 0$. The optimal contract in the coordinated benchmark $\mathbb{C}_c = (\alpha_c, \beta_c, \gamma_c)$ exhibits JPE, $\beta_c = \frac{c_H - c_L}{2(a_H - a_L)} > 0$.*

Proof. Appendix A.1.2. □

Under the non-cooperative benchmark, the optimal contract is in the form of RPE for its better risk sharing across the agents, and under the coordinated benchmark, the optimal contract has JPE, for it motivates the team as one agent exerting multiple productively independent actions.⁶

1.4 Coordination via Repeated Interaction with Imperfect Monitoring

It is straightforward that under the benchmark RPE, $\bar{u}_i(a_H, a_H) \leq \bar{u}_i(a_L, a_L)$ for each agent, with equality holds at $\sigma = 0$. Thus, for all $\sigma > 0$, $(\alpha_n, \beta_n, \gamma_n)$ is vulnerable to collusion among the agents who may agree on (a_L, a_L) instead. The benchmark JPE, on the other hand, is vulnerable to free-riding if actions are not contractible, as $\bar{u}_1(a_H, a_H) < \bar{u}_1(a_L, a_H)$ and $\bar{u}_2(a_H, a_H) < \bar{u}_2(a_H, a_L)$. In this section, the collusion-proof contract and the coordination-inducing contract are characterized respectively, postponing the discussion on the optimal team incentive to the next section.

1.4.1 Optimal Collusion-proof Contract

A contract (α, β, γ) implementing (a_H, a_H) is collusion-proof if (a_H, a_H) is the Nash equilibrium among the non-cooperative agents, and neither (a_L, a_L) , (a_H, a_L) , or (a_L, a_H) can be enforced as a perfect public equilibrium of the agents' repeated game described in Assumptions 1.1 and 1.2. That is, in addition to (IC_n) , a collusion-proof (α, β, γ)

⁶As raised by Ishiguro (2002), under the non-cooperative benchmark, the implemented action profile (a_H, a_H) is a Nash equilibrium among the agents given the contract offered, but with binding (IC_n) as the solution is worked out, it is not a unique Nash equilibrium. However, the fact that optimal contract takes the form of RPE would still hold, as the principal can increase α_n by an infinitesimal amount to implement (a_H, a_H) as the unique Nash equilibrium within the team, which does not change the fact that the contract exhibits RPE.

satisfies the following constraints,

$$\begin{aligned} & (1 - \delta)\bar{u}_i(a_L, a_L) + \delta((1 - q_i^n)\bar{u}_i(a_L, a_L) + q_i^n\bar{u}_i(a_H, a_H)) \\ & \leq (1 - \delta)\bar{u}_i(a_i = a_H, a_j = a_L) + \delta((1 - q_i^d)\bar{u}_i(a_L, a_L) + q_i^d\bar{u}_i(a_H, a_H)) \end{aligned} \quad (1.1)$$

$$\begin{aligned} & (1 - \delta)\bar{u}_i(a_i = a_L, a_j = a_H) + \delta((1 - q_i^n)\bar{u}_i(a_i = a_L, a_j = a_H) + q_i^n\bar{u}_i(a_H, a_H)) \\ & \leq (1 - \delta)\bar{u}_i(a_H, a_H) + \delta((1 - q_i^d)\bar{u}_i(a_i = a_L, a_j = a_H) + q_i^d\bar{u}_i(a_H, a_H)) \end{aligned} \quad (1.2)$$

(1.2) holds when (IC_n) is satisfied, so only (1.1) is the relevant collusion-proof condition, which, given (α, β, γ) , can be written as

$$\alpha + h(\delta, \Delta)\beta \geq \frac{c_H - c_L}{a_H - a_L} \quad (CP)$$

where $\Delta \equiv q_i^d - q_i^n$ and $h(\delta, \Delta) \equiv \frac{\delta\Delta}{1 - \delta + \delta\Delta}$. The principal's problem to find the optimal collusion-proof contract is to minimize the risk premium, subject to (IC_n) and (CP) ,

$$\begin{aligned} \mathcal{P}_{CP} : \quad & \min_{\alpha, \beta} R(\alpha, \beta) = \frac{r}{2}(\alpha^2 + \beta^2 + 2\sigma\alpha\beta) \\ & s.t. (IC_n), (CP) \end{aligned}$$

Proposition 1.1. *i) JPE ($\beta > 0$) is never optimal to deter collusion. ii) The optimal collusion-proof contract $\mathbb{C}_{CP} = (\alpha_{CP}, \beta_{CP}, \gamma_{CP})$ exhibits RPE, where $\beta_{CP} = \frac{h(\delta, \Delta) - \sigma}{1 + h(\delta, \Delta)^2 - 2\sigma h(\delta, \Delta)} \frac{c_H - c_L}{a_H - a_L} < 0$, for $\delta < \delta_{CP} \equiv \frac{\sigma}{(1 - \sigma)\Delta + \sigma}$, and has IPR, where $\beta_{CP} = 0$, otherwise.*

Proof. Appendix A.1.3. □

Detering collusion, any $\beta > 0$ raises risk premium without providing additional incentive. For sufficiently impatient agents, their coalition is so loose that the principal, even when deterring collusion, is able to provide incentive and account for risk sharing through RPE. For sufficiently patient agents, however, to deter the agents from colluding with each other, the principal trades off the risk sharing advantage of RPE completely and relies on IPR to provide incentive.

1.4.2 Optimal Coordination-inducing Contract

A contract (α, β, γ) implementing (a_H, a_H) induces coordination between the agents if it is incentive compatible for the coalition, and (a_H, a_H) is enforced as a perfect

public equilibrium of the agents' repeated game described in Assumptions 1.1 and 1.2, when (a_L, a_L) is the non-cooperative Nash equilibrium. That is, in addition to (IC_c) , a coordination-inducing contract also satisfies

$$\begin{aligned} & (1 - \delta)\bar{u}_i(a_H, a_H) + \delta((1 - q_i^n)\bar{u}_i(a_H, a_H) + q_i^n\bar{u}_i(a_L, a_L)) \\ & \geq (1 - \delta)\bar{u}_i(a_i = a_L, a_j = a_H) + \delta((1 - q_i^d)\bar{u}_i(a_H, a_H) + q_i^d\bar{u}_i(a_L, a_L)) \end{aligned} \quad (1.3)$$

Given the model specification, (1.3) can be written as

$$\alpha + h(\delta, \Delta)\beta \geq \frac{c_H - c_L}{a_H - a_L} \quad (CI)$$

where $\Delta \equiv q_i^d - q_i^n$ and $h(\delta, \Delta) \equiv \frac{\delta\Delta}{1 - \delta + \delta\Delta}$.

Notice that if (IR) were binding as in the benchmark, at the punishment phase of the repeated game, the agents would prefer to reject the contract and seek outside options from which each of them earn exactly the reservation utility. That is, there would be no punishment at all. Thus, the binding participation constraint to induce coordination is the one in the punishment phase instead of that in the continuation phase,

$$\bar{u}_i(a_L, a_L) \geq 0 \quad (IR_p)$$

Given binding (IR_p) , the principal's problem to find the optimal coordination-inducing contract is to minimize the sum of expected difference in payment and risk premium, subject to (IC_c) and (CI) ,

$$\begin{aligned} \mathcal{P}_{CI} : \quad & \min_{\alpha, \beta} R(\alpha, \beta) = (\alpha + \beta)(a_H - a_L) + \frac{r}{2}(\alpha^2 + \beta^2 + 2\sigma\alpha\beta) \\ & s.t. \ (IC_c), (CI) \end{aligned}$$

Proposition 1.2. *i) RPE ($\beta < 0$) is never optimal to induce coordination. ii) The optimal coordination-inducing contract $\mathbb{C}_{CI} = (\alpha_{CI}, \beta_{CI}, \gamma_{CI})$ exhibits JPE, where $\beta_{CI} = \frac{h(\delta, \Delta) - \sigma}{1 + h(\delta, \Delta)^2 - 2\sigma h(\delta, \Delta)} \frac{c_H - c_L}{a_H - a_L} - \frac{(1 - h(\delta, \Delta))(a_H - a_L)}{r(1 + h(\delta, \Delta)^2 - 2\sigma h(\delta, \Delta))} > 0$, for $\delta > \delta_{CI} \equiv \frac{r\sigma \frac{c_H - c_L}{a_H - a_L} + (a_H - a_L)}{r((1 - \sigma)\Delta + \sigma) \frac{c_H - c_L}{a_H - a_L} + (a_H - a_L)}$, and has IPR, where $\beta_{CI} = 0$, otherwise.*

Proof. Appendix A.1.4. □

For sufficiently patient agents, the principal finds it less costly to offer a contract with JPE to provide incentive implicitly through repeated relationship between the agents, using the agents' ability to make inference on each other's action. For impatient

agents, on the other hand, inducing coordination by a contract with JPE, the principal is at the expense of a high risk premium; she thus trades off the agents' mutual inference and relies on IPR to provide incentive.

Proposition 1.2, however, is not flawless. The justification for the principal offering a series of short-term stationary contract relies on history-independent future outcome, which holds if collusion is deterred, but fails if coordination is induced. This is because of the imperfect nature of the agents' mutual inference: the agents know whether they are in a continuation or in a punishment phase in the repeated relationship, whereas the principal does not; the outcome of imperfect monitoring is public among the agents, but private to the coalition of agents. The principal thus has incentive to screen the continuation phase from the punishment phase, by offering a menu of contract $\{\mathbb{C}_{co}, \mathbb{C}_p\} = \{(\alpha_{co}, \beta_{co}, \gamma_{co}), (\alpha_p, \beta_p, \gamma_p)\}$ such that the coalition of agents in the continuation phase voluntarily act according to \mathbb{C}_{co} , and the agents in the punishment phase voluntarily follow \mathbb{C}_p , are motivated to exert a_H , and earn at least the reservation payoffs.

$$\bar{u}_i(\mathbb{C}_{co}, a_H) \geq \bar{u}_i(\mathbb{C}_p, a_L) \quad (TT_{co})$$

$$\bar{u}_i(\mathbb{C}_p, a_H) \geq \bar{u}_i(\mathbb{C}_{co}, a_L) \quad (TT_p)$$

$$\bar{u}_i(\mathbb{C}_p, a_H) \geq \bar{u}_i(\mathbb{C}_p, a_L) \quad (IC_P)$$

$$\bar{u}_i(\mathbb{C}_p, a_H) = 0 \quad (IR_P)$$

The fact that each agent's payoff in the punishment stage is reduced to his reservation payoff is unchanged, so constraint (CI) is unaffected with the presence of screening. If the principal can observe the phases of the repeated game, \mathbb{C}_{CI} as in Proposition 1.2 is chosen to induce coordination in the continuation phase, and \mathbb{C}_n as in the non-cooperative benchmark is offered to minimize risk premium in the punishment phase. These two contracts are not truth revealing with asymmetric information. As $\bar{u}_i(\mathbb{C}_{CI}, a_L) = \bar{u}_i(\mathbb{C}_n, a_H) = 0$, (TT_p) is binding and (TT_{co}) is violated at $\{\mathbb{C}_{CI}, \mathbb{C}_n\}$ if $(1 - h(\delta, \Delta))\beta_{CI} < -\beta_n$. Hence, $\beta_{co} \geq \beta_{CI} > 0$ and $0 > \beta_p \geq \beta_n$. Proposition 1.2 is qualitatively robust to screening.

1.5 Optimal Team Incentive

By offering a contract with collusion-proof RPE, the principal benefits from providing incentive through risk sharing, at a cost of deterring collusion; by offering a contract with coordination-inducing JPE, the principal benefits from the implicit incentive

built on the information inferred by the agents that is otherwise private, at a cost of higher risk premium. Straight forward from Propositions 1.1 and 1.2, the following proposition characterizes the optimal contract by how patient the agents are, other things remaining equal.

Proposition 1.3. *It is optimal to deter collusion with a RPE if $\delta < \delta_{CP}$, to induce coordination with a JPE if $\delta > \delta_{CI}$, and to provide incentive by IPR if $\delta_{CP} < \delta < \delta_{CI}$.*

Proof. Appendix A.1.5. □

Proposition 1.3 is a natural first thought with repeated interaction available. What's more to my interest is, given a fixed discount factor, how the tradeoff behind the principal's decision depends on the correlation of measurement noise.

1.5.1 Mutual Inference vs. Risk Sharing

To see the tradeoff behind the principal's decision on the optimal contractual form, revisit the optimality condition with respect to the linear contracting parameters (α, β) , which, in the scenario of deterring collusion, is

$$\underbrace{\frac{R_\beta(\alpha, \beta)}{R_\alpha(\alpha, \beta)}}_{\text{MRS in Risk Sharing}} = \underbrace{h(\delta, \Delta)}_{\text{Firmness of Coalition}} \quad (1.4)$$

In the scenario of inducing coordination, on the other hand, (α, β) is optimally decided where

$$\underbrace{\frac{(a_H - a_L) + R_\beta(\alpha, \beta)}{(a_H - a_L) + R_\alpha(\alpha, \beta)}}_{\text{Modified MRS in Risk Sharing}} = \underbrace{h(\delta, \Delta)}_{\text{Firmness of Coalition}} \quad (1.5)$$

Subscripts stand for partial derivative to avoid cluster.

The left-hand-side of (1.4) is straightforward and exploited by the literature, namely, the marginal rate of substitution in risk sharing when collusion is deterred. The left-hand-side of (1.5) is the modified marginal rate of substitution in risk sharing when coordination is induced, due to the two sources of risk: stochastic realization of output on which the transfer is contingent, and a break of coalition from false detection of unilateral deviation. These imply one consideration of the choice of β to provide incentive: through improvement of risk sharing.

$h(\delta, \Delta) \equiv \frac{\delta\Delta}{1-\delta+\delta\Delta}$ in the right-hand-sides is increasing in $\delta\Delta$, which is an interaction of agents' patience and their accuracy of inference on unilateral deviation from the

coalition, and in the agents' repeated game, it measures the discounted difference in likelihood to enter a punishment phase. Thus, $h(\delta, \Delta)$ is interpreted as an indicator for the firmness of the coalition independent of contract offered: higher $h(\delta, \Delta)$, stronger is the expected punishment for unilateral deviation, and the coordinated actions are enforced more firmly. This implies another consideration of the choice of β : to induce or to deter the agents' mutual inference to provide incentive.

At $\beta = 0$, if $h(\delta, \Delta) < \frac{R_\beta(\alpha, 0)}{R_\alpha(\alpha, 0)} = \sigma$, the coalition is sufficiently loose; offering a contract with $\beta < 0$ to deter collusion is valuable to the principal as it improves risk sharing more significantly than its cost on deterring collusion. Collusion-proof RPE is optimal if $h(\delta, \Delta) < \sigma$. If $h(\delta, \Delta) > \frac{(a_H - a_L) + R_\beta(\alpha, 0)}{(a_H - a_L) + R_\alpha(\alpha, 0)} = \frac{(a_H - a_L) + r\alpha\sigma}{(a_H - a_L) + r\alpha}$, the coalition is sufficiently firm; offering a contract with $\beta > 0$ to induce coordination is valuable to the principal as it induces implicit incentive through the agents' mutual inference, which is sufficiently accurate, at a relatively small cost from inefficient risk sharing. JPE that induces coordination is optimal if $h(\delta, \Delta) > \frac{(a_H - a_L) + r\alpha\sigma}{(a_H - a_L) + r\alpha} \equiv \varphi(\sigma)$. Otherwise, if $\sigma < h(\delta, \Delta) < \varphi(\sigma)$, the coalition is not only insufficiently loose for the cost of deterring collusion under $\beta < 0$ to be relatively small comparing to the benefit from risk sharing, but also insufficiently firm for the benefit of implicit incentive through mutual inference to be relatively large to outweigh the cost of risk under $\beta > 0$. IPR is optimal if $\sigma < h(\delta, \Delta) < \varphi(\sigma)$.

Comparing with the conventional literature, where the agents' monitoring ability is assumed to be independent of σ ($h(\delta, \Delta)$ does not depend on σ), it is optimal to offer coordination-inducing JPE for sufficiently small correlation, collusion-proof RPE for sufficiently large correlation, and IPR otherwise. However, with the agents' monitoring inferred from correlated stochastic output (i.e. $h(\delta, \Delta)$ depends on σ through Δ , the accuracy of inference), the optimal form of contract may not be monotonic in σ , as the following proposition suggests.

Proposition 1.4. *There exist some $a_H - a_L$, s , and δ such that $h(\delta, \Delta) > \varphi(\sigma)$ for $\sigma \in (\underline{\sigma}, \bar{\sigma})$, $0 < \underline{\sigma} < \bar{\sigma} < 1$, and $h(\delta, \Delta) < \varphi(\sigma)$ otherwise. That is, for $\sigma \in (\underline{\sigma}, \bar{\sigma})$, coordination-inducing JPE is optimal.*

Proof. Appendix A.1.6. □

The proposition essentially suggests that there exists some parametric values of the difference in the level of private action ($a_H - a_L$), those of the agents' exogenous trust (s), and those of the discount factor (δ), given which the optimal contract is not monotonic in correlation of measurement noise. Specifically, for sufficiently patient

agents, if the difference in correct and false detection of deviation (Δ) ascends significantly fast for some intermediate level of correlations, for instance, if the deviation is not too obvious even with low correlation ($a_H - a_L$ not being too large), or if the agents place sufficient trust on each other (s not being too small), it is optimal for the principal to induce the agents to coordinate with a JPE for intermediate level of correlation.

Figure 1.1 illustrates one of the examples that Proposition 1.4 holds. The agents' patience at level $\hat{\delta}$, for sufficiently small correlation of measurement noise, neither the effect of risk sharing and that of mutual inference is significant, $\sigma < h(\delta, \Delta) < \varphi(\sigma)$, IPR is optimal. For sufficiently small correlation, an increase in correlation improves risk sharing under RPE more significantly than its improvement on accuracy of mutual inference (as illustrated in the bottom of Figure 1.1), to an extent where $\sigma > h(\delta, \Delta)$, the benefit from risk sharing exceeds the cost from deterring collusion, collusion-proof RPE is optimal. For intermediate level of correlation, relatively to the risk sharing effect, improvement of accuracy of mutual inference with respect to a higher correlation is significantly fast, to an extent where $h(\delta, \Delta) > \varphi(\sigma)$, the benefit from implicit incentive through mutual inference exceeds the cost from inefficient risk sharing, coordination-inducing JPE is optimal. For sufficiently large correlation, the agents' mutual inference is so accurate that it has little room for improvement, risk sharing effect of a higher correlation exceeds the effect from improvement of inference accuracy, collusion-proof RPE is then optimal.

In comparison with the literature, for $\sigma \in (0, \bar{\sigma})$, $\bar{\sigma} < 1$, collusion-proof RPE is optimal for relatively small correlations of measurement noise, IPR is optimal for intermediate levels of correlation, and for sufficiently large correlations, the principal finds it optimal to induce coordination among the agents with a JPE to take advantage of their mutual inference, although at the expense of risk sharing. This contradicts to the conventional proposition introduced by Holmström and Milgrom (1990), yet is consistent to Rayo (2007)'s claim that a signal of higher accuracy is in favor of JPE that induces implicit incentive. For $\sigma \in (\underline{\sigma}, 1)$, $\underline{\sigma} > 0$, our proposition is almost⁷ consistent to that of Holmström and Milgrom (1990): coordination-inducing JPE is optimal for sufficiently small correlation of measurement, IPR is optimal for intermediate levels of correlation, and for sufficiently large correlations, the principal finds it optimal to deter collusion among the agents with a RPE for better risk sharing. This, however, contradicts to that of Rayo (2007) as more accurate signal does not favor JPE that induce implicit incentive, with the presence of risk sharing concern.

⁷The exception is that in their model, IPR is never optimal.

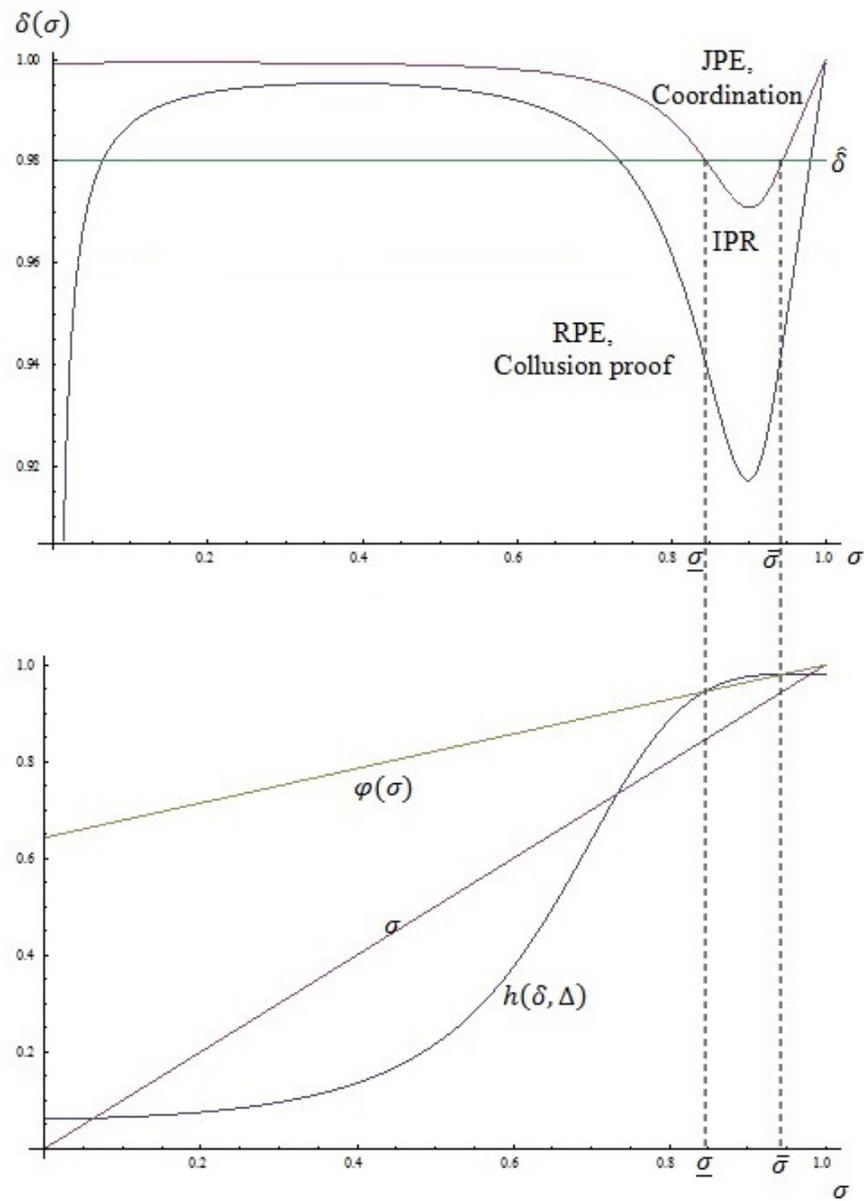


Figure 1.1: Optimal Team Contract
 Drawn with $s = 6$, $a_H - a_L = 3$, $c_H - c_L = 5$.

1.6 Discussion

Endogenous Belief. The agents' belief on each other's unilateral deviation may not be exogenous as assumed. Knowing that their actions are measured with a highly correlated noise, the agents might adjust their beliefs accordingly, that is, it is possible that s is also a choice variable of the coalition. The optimal s^* when the agents cooperate/collude will be such that the expected payoff under non-deviation is maximized subject to enforcement,

$$s^* \in \arg \max_s \sum_{j=1,2} \left((1 - \delta) \bar{u}_j(a_i^0, a_j^0) + \delta \left((1 - q_j^n) \bar{u}_j(a_i^0, a_j^0) + q_j^n \bar{u}_j(a_i', a_j') \right) \right)$$

subject to

$$\begin{aligned} (1 - \delta) \bar{u}_j(a_i^0, a_j^0) + \delta \left((1 - q_j^n) \bar{u}_j(a_i^0, a_j^0) + q_j^n \bar{u}_j(a_i', a_j') \right) \\ \geq (1 - \delta) \bar{u}_j(a_i^0, a_j') + \delta \left((1 - q_j^d) \bar{u}_j(a_i^0, a_j^0) + q_j^d \bar{u}_j(a_i', a_j') \right). \end{aligned}$$

Given the contract offered, the problem is reduced to

$$s^* \in \arg \min_s q_j^n$$

subject to

$$\Delta \geq \frac{1 - \delta}{\delta} \left(\frac{\bar{u}_j(a_i^0, a_j') - \bar{u}_j(a_i^0, a_j^0)}{\bar{u}_j(a_i^0, a_j^0) - \bar{u}_j(a_i', a_j')} \right) \quad (EF)$$

The optimal belief of unilateral deviation is such that the probability of false detection is minimized, subject to that the difference in correct and false detection is sufficiently large to ensure the enforcement of coordinated actions, with the lower bound depending on the contract offered at an earlier date. This is as if the principal can not only induce coordination, but also implement the level of trust the agents place on each other, which results in binding enforcement constraint at the optimal s^* for the agents.

Under a contract with RPE, if the belief on deviation is endogenous, the agents can adjust s to collude for any $\Delta < \max_s \Delta$. A RPE is collusion-proof only if (EF) is binding at $\max_s \Delta$. To induce coordination with JPE, the principal is better off with a higher Δ by construction. If it is optimal for her to induce coordination, it is optimal for her to implement the level of trust such that Δ is maximized. The optimal coordination-inducing JPE is where (EF) is binding at $\max_s \Delta$. Thus, (EF) is reduced

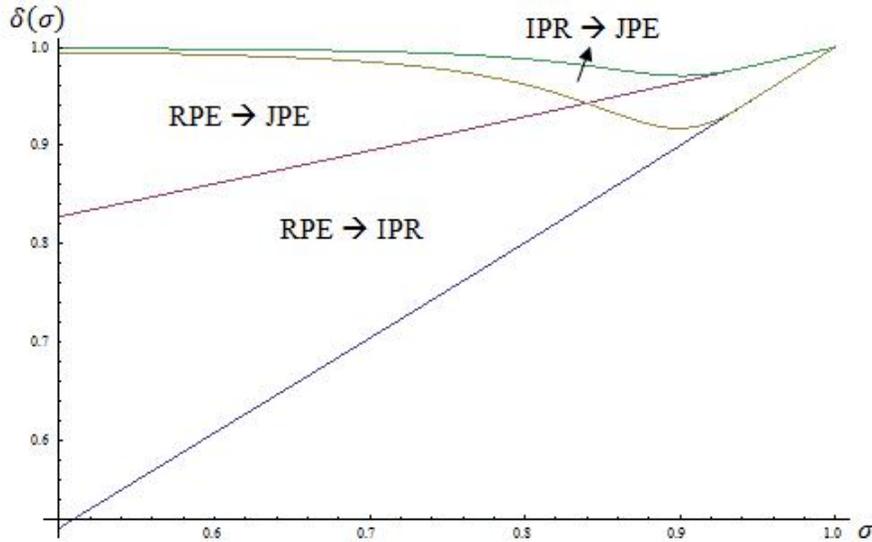


Figure 1.2: Endogenous Belief of Deviation
 Drawn with $a_H - a_L = 3$.

to $\max_s \Delta = \frac{1-\delta}{\delta} \left(\frac{\bar{u}_j(a_i^0, a_j') - \bar{u}_j(a_i^0, a_j^0)}{\bar{u}_j(a_i^0, a_j^0) - \bar{u}_j(a_i', a_j^0)} \right)$. Given standard normal density of the noise, $\max_s \Delta$ is where $s^* \sqrt{1 - \sigma^2} = \frac{a_H - a_L}{2}$, i.e. s^* standard deviation from the conditional mean is increasing in correlation. Intuitively, more correlated is the measurement noise, the agents are able to make more accurate inference, and trust each other more when observing an output away from the conditional mean.

An endogenously chosen s^* yields a larger region in the σ - δ space for coordination-inducing JPE to be optimal (Figure 1.2), as this threshold of entering a punishment stage is adjusted so that a unilateral deviation is detected with a higher probability than that under an exogenous threshold. For s^* is increasing in σ , difference in the probability to detect unilateral deviation in comparison to that with an exogenous belief is more significant at lower level of correlation. This results in a prediction consistent to that of Holmström and Milgrom (1990): given δ , collusion-proof RPE is optimal for sufficiently large correlation, coordination-inducing JPE is optimal otherwise, and contradict to that of Rayo (2007): coordination is optimal for sufficiently accurate signals of actions between the agents.

Fixed-Term Punishment. Considering a less punishing trigger strategy with a finite period T of punishment (as in Green and Porter (1984) and Porter (1983)), $h(\delta, \Delta)$ in (4) is replaced by $h_T(\delta, \Delta) \equiv \frac{\delta \Delta (1 - \delta^T)}{1 - \delta + \delta \Delta (1 - \delta^T)} < h(\delta, \Delta)$, and for $\delta < \delta_T$, collusion-proof RPE is optimal, where $\delta_T > \delta_{CP}$ solves $h_T(\delta, \Delta) = \sigma$. The shape of δ_T depends on the

length of punishment T , which may suggest an even more complicated pattern of optimal contractual form with respect to noise correlation than the previously discussed.

1.7 Conclusion

Explicitly modeling how the agents make inference on each other through correlated measurement of private action, I argue that the agents' monitoring accuracy is increasing in correlation of measurement noise, in contrast to the independency assumed in the conventional studies of optimal team incentive subject to the agents forming a coalition. More correlated measurement of action, incentive provision through filtering out common noise is less costly, so is the enforcement of the agents' coalition. Thus, the following two propositions in the literature do not co-exist: i) collusion-proof RPE is optimal for sufficiently large correlation of measurement noise due to its risk sharing advantage, emphasized Holmström and Milgrom (1990), and ii) coordination-inducing JPE is optimal for sufficiently accurate monitoring among the agents, emphasized by Rayo (2007). I show consistency to Holmström and Milgrom (1990) for higher correlation of measurement noise and consistency to Rayo (2007) for smaller correlation.

This paper is the first attempt to incorporate repeated interaction with imperfect public monitoring into the study of optimal provision of team incentive, and it is expecting a generalization in the future. Specifically, a general condition on the functional forms and the density of measurement noise for which the proposition of this paper holds is of interest; with a continuum of actions, the problem becomes more challenging as the magnitude of unilateral deviation is also related to the accuracy of mutual inference, and hence to the correlation of measurement noise.

Chapter 2

Information Acquisition and the Equilibrium

Incentive Problem

2.1 Introduction

Standard agency theories regard the sources of the incentive problem in the contractual relationship, either asymmetric information on the state of nature, privately observed action, or both, to be exogenous. This is attributed to the assumption that information structure on productivity is exogenous.¹ I depart from the exogenous availability of productive information by assuming that it is an investment decision of the agent at a sunk cost (non-monetary) upon being offered a contract yet before committing to the contractual relationship, in the spirit of Arrow,

A key characteristic of information costs is that...they typically represent an irreversible investment...I am thinking of the need for having made an adequate investment of time and effort to be able to distinguish one signal from another. (Arrow, 1974: 39)

With this remedy, the source of the incentive problem is an endogenous choice of the principal, which is decided jointly with whether to induce the agent to or to deter him from acquiring productive information. The conventional theories of contract

¹Consider a principal contracting with an agent to execute a project that yields output to the principal, which depends on the agent's private productive effort and stochastic productive state of nature. Agency theories are diverged into two branches given different exogenous information structure. The theory of adverse selection focused on the incentive problem resulted from an agent having private information on the realization of productivity, in which output is manipulated through private effort; the theory of moral hazard, on the other hand, studied the incentive problem due to imperfectly measured hidden action, where the parties have symmetric information on the density of the stochastic productivity.

discussed the optimal contract to cope with an existing incentive problem; in addition to such, I study how it is designed to implement the incentive problem to face.

Consider a principal contracting with an agent protected by limited liability, and both players are risk neutral. The principal's revenue is generated by the agent's privately observed productive effort, whose productivity depends on the stochastic state of nature. The agent can acquire information on the realized state of nature at a sunk cost. I examine two information acquiring technologies. Perfect information acquisition is defined in the sense that the correct signal is available to the agent as long as he acquires it. Imperfect information acquisition refers to an information acquiring effort that improves the probability for the agent to observe the correct signal on the realized state of nature, and no signal otherwise.

With perfect information acquisition, the principal is able to implement adverse selection in the production stage by inducing the agent to acquire information, so that asymmetric information is present without productive uncertainty, and the productive effort is an instrument to report the state of nature. The principal is also able to implement moral hazard in equilibrium by deterring the agent from information acquisition, in which scenario the principal and the agent are symmetrically informed, and the publicly observed output is an imperfect measurement of the agent's private effort. With imperfect information acquisition, the equilibrium incentive problem is stochastic, whose density is implemented by the principal through inducing information acquisition to a specific precision.

The emergence of the incentive problem and its interaction with information management has only received attention at one end: endogenous information acquisition which generates adverse selection.² Moral hazard as a consequence of deterrence of information acquisition has been, to the best of my knowledge, absent from contract theory. I fill this gap by investigating how information management interacts with the equilibrium incentive problem, and how the optimal contract is modified from the second best in response to such interaction.

Given perfect information acquisition, to implement adverse selection through inducing information acquisition, the principal creates a larger rent difference than the second best between an agent in a relatively efficient state and whom in a relatively inefficient state, so that the agent is motivated to distinguish the efficient states of nature from the inefficient. This is implemented via a menu contract in which higher (lower) output than the second best is specified for a sufficiently efficient (inefficient)

²Please refer to Lewis and Sappington (1997), Crémer, Khalil, and Rochet (1998a), and Terstiege (2012) in the literature review for more details.

state of nature.

To implement moral hazard through deterring information acquisition, the principal must deter the agent's opportunistic motives to acquire information off the equilibrium path. Given the second best debt contract³, the agent attempts to acquire information in order to distinguish a sufficiently inefficient state of nature to avoid the debt by rejecting the contract, and to discover a relatively efficient state of nature to extract maximal rent. The optimal contract to deter information acquisition and implement moral hazard is thus characterized by a downward distortion of debt from its second best and a lower equity share of output residual to the agent to restrict his ability to extract rent by acquiring information. The former implies a larger output residual, which motivates productive effort in equilibrium, whereas the latter discourages it. This results in an upward distortion of productive effort from the second best with sufficiently large cost of information acquisition, and downward distortion otherwise. That is, deterrence of information acquisition is complementary to higher-powered incentive for sufficiently large cost of information acquisition, which is different from Crémer, Khalil, and Rochet (1998a), who did not introduce moral hazard in production when information is deterred.

The key tradeoff behind the decision to induce or to deter information, to implement adverse selection or moral hazard in the production stage, involves rent and efficiency. Agent's acquisition of information benefits the principal as it allows for more efficient production, yet rent is given to induce hidden information acquiring effort and truthful revelation. For sufficiently small cost of information acquisition, it is optimal to induce information acquisition and implement adverse selection as the improvement in efficiency exceeds the net rent; for sufficiently costly information acquisition, it is optimal to deter information acquisition and implement moral hazard as the improvement in efficiency falls short of the net rent.

Consider in a firm-employee relationship, inducing information acquisition to implement adverse selection is optimal if the agent is an "expert" in the field, who is able to acquire productive information at a lower cost. Deterring information acquisition to implement moral hazard is optimal if the agent is a "mediocre," who acquires productive information at a higher cost. Applying to investment banking, the investment bank (principal) finds it optimal to induce the funds-seeking firm (agent) to conduct costly market investigation and reveal its finding through a menu of funding options

³Contracting with a risk neutral agent protected by limited liability, the second best contract with the presence of moral hazard is a debt contract, which prescribes a debt paid to the principal, leaving the agent the claimant of output residual. Please refer to Innes (1990) for the pioneer treatment, and Poblete and Spulber (2012) with a weaker assumption.

if the market is well-established and sufficiently transparent, and finds it optimal to deter the private firm from conducting costly market investigation with a single debt-with-equity-share contract if the firm participates in a newly-established market in which past data is limited.

The optimal debt-with-equity-share contract to deter information acquisition is qualitatively robust to imperfect information acquisition, as well as to private knowledge of the information acquiring cost. The main difference to perfect information acquisition with common knowledge of information acquiring cost is that the principal does not know perfectly upon offering the contract whether the agent is informed of the state of nature or not, which itself is an information advantage of the agent. The optimal contract under these two remedies is thus designed such that the informed or uninformed agent truthfully reveals being informed or uninformed. The additional incentive compatibility constraints distort the contract designed to the uninformed agent towards the same direction as does the constraint to deter perfect information acquisition with common knowledge of information acquiring cost. This is intuitive as the agent's opportunistic motive to acquire information still remains.

The optimal menu contract designed to the informed agent, however, exhibits pooled output menu over some intermediate states of nature, which is absent in the optimal menu contract to induce perfect information acquisition with common knowledge of information acquiring cost. This is due to the technical resemblance of the truth telling constraints for the informed agent to the type-dependent participation constraints that generate countervailing incentives⁴, although the reservation utility is assumed to be identical across states of nature.

The chapter is organized as the following. The model is outlined in Section 2.2. Given perfect information acquisition, I derive in Section 2.3 the optimal menu contract when information acquisition is induced, merely as a replication of Lewis and Sappington (1997) and Crémer et al. (1998a), and Section 2.4 is devoted to the optimal contract to deter information acquisition. Optimal information management and equilibrium incentive problem in the contractual relationship is discussed in Section 2.5. I examine the robustness of the optimal contract with imperfect information acquisition in Section 2.6, and that with private knowledge of the cost of information acquisition in Section 2.7. The paper is concluded in Section 2.8.

⁴Please refer to Lewis and Sappington (1989) for the pioneer work, and Jullien (2000) for a more general treatment.

2.1.1 Related Literature

Information acquisition in the environment with adverse selection has gained much attention in agency theory, and is roughly categorized into two forms of information acquisition: strategic and productive information gathering. The former refers to circumstances where information is realized at no cost at the production stage, but can be acquired at a cost *ex ante* to facilitate the agent's decision on accepting the contract, which affects the form of the agent's individual rationality but not truthful revelation of information. Crémer and Khalil (1992), Crémer, Khalil, and Rochet (1998b), and Szalay (2009) study this sort of information acquisition. As information would realize at the stage of production, information acquisition is only for strategic purpose and the only incentive problem at the production stage is due to asymmetric information.

I build my propositions on the latter form of information acquisition, which corresponds to situations where information is realized only if it is acquired at a cost. Thus, information acquisition affects both participation and incentive compatibility of the agent. Lewis and Sappington (1997), Crémer, Khalil, and Rochet (1998a), Kessler (1998), Krämer and Strausz (2011), Zermeno (2011), Terstiege (2012), and Hoppe and Schmitz (2013) fall into this category. They, however, either do not consider deterrence of information acquisition, or assume a deterministic output as a perfect measurement of the agent's productive effort when information acquisition is deterred. Adverse selection endogenously arises as a consequence of inducing information acquisition, whereas moral hazard is assumed away. The interaction between deterring information acquisition and moral hazard is absent from the principal's optimization program in these papers.

A considerable literature is also devoted to asymmetric information on productive noise in an environment with moral hazard and risk averse agent, in explanation of the empirical puzzle that a higher-powered incentive is given under a riskier environment.⁵ In this literature, information on productive noise is assumed to be a mean-preserving imperfect signal that is unable to be communicated through a contract; truthful revelation is absent and moral hazard is present from the outset. Thus, regardless of the level of information acquisition, the fundamental incentive problem is moral hazard, and their focus is on inducing information acquisition to facilitate productive decision that generates a risky return. Additionally, this literature attributed a higher-powered incentive to inducing information acquisition, implicitly implying a

⁵Refer to Demski and Sappington (1987), Malcomson (2009, 2011), Raith (2008), and Zábajník (1996) for theory, and Prendergast (2002) and Shi (2011) for the empiric.

lower-powered incentive if information acquisition is deterred. I argue in the appendix that deterring a risk-averse agent from information acquisition does not necessarily rely on a lower-powered incentive; it depends on the density of the state of nature.

The idea that information availability on state of nature distinguishes adverse selection from moral hazard is also emphasized by Sobel (1993) and Chu and Sappington (2009a). The former compares the principal's payoff given various timing that information becomes available: pre-contract, post-contract prior to production, or after production. The latter develops a dynamic model in which information becomes available at an interim stage, before which, the incentive problem is due to hidden action, and asymmetric information thereafter. In both papers, however, information is not acquired by the agent, and the timing of information availability is exogenous, so does the underlying incentive problem. What the present paper contributes to the literature is the endogenous choice of the incentive problem through inducing/deterring information acquisition, which provides a refutable modification to the standard contracts.

Endogeneity of incentive problems due to productive information acquisition is also noticed by an independent work of Iossa and Martimort (2013). They study imperfect information acquisition in a similar fashion to mine in Section 2.6. Their propositions rely on the assumptions that i) the agent is pessimistic about his own information acquisition; ii) the transfer scheme is linear in output regardless of whether the agent is informed; iii) information is acquired after the contract is signed; iv) pooling is defined over the slope of transfer instead of output schedule. Thus, the main conclusions are drawn differently from mine. First, costly information acquisition itself does not distort the optimal contract and equilibrium productive effort, it is the joint effect with the agent's pessimistic attitude that makes a difference. In contrast, I show that even with symmetric belief on the agent's information acquisition in equilibrium, information management distorts the optimal contract from the second best provided that the cost of information acquisition is not too extreme. Second, they find pooling to be optimal in sufficiently inefficient states of nature instead of in the intermediate states of nature, due to the assumed linear transfer and a different definition of pooling.

2.2 Model

A principal hires an agent to execute a project that yields the publicly observable and contractible output $q(e, \theta)$, depending on the agent's privately observed productive

effort (e) and the state of nature (θ). Let $q_e(e, \theta) > 0$,⁶ $q_\theta(e, \theta) > 0$, $q_{ee}(e, \theta) < 0$, $q_{\theta\theta}(e, \theta) < 0$, and $q_{e\theta}(e, \theta) > 0$ for $(e, \theta) > (0, 0)$, i.e. the output function is concave in both effort and state of nature, and higher θ indicates a relatively efficient state of nature with higher total and marginal product. θ follows prior distribution $F(\theta)$ defined over $[0, \bar{\theta}]$. The agent's cost of effort is given by a convex cost function $c(e)$, where $c(0) = 0$, $c_e(e) > 0$ and $c_{ee}(e) \geq 0$ for $e > 0$.

The principal and the agent are both risk neutral, with the principal's payoff defined as the output net of contingent transfer specified in the contract, $u^P = q(e, \theta) - t(q(e, \theta))$, and the agent's payoff defined as the contingent transfer net of cost of effort, $u^A = t(q(e, \theta)) - c(e)$. The agent is protected by limited liability.⁷

Upon being offered a contract, the agent can invest effort a in information acquisition, which allows him to observe the correct signal of the state of nature with probability $a \in [0, 1]$, or no signal otherwise, at a (sunk) non-monetary cost $d(\kappa, a)$, before accepting the contract, κ being the cost parameter of information acquisition. The acquired information is private to the agent, as well as his information acquiring action, but the cost of information acquisition is common knowledge. The non-monetary sunk cost of information acquisition captures the characteristic of information acquisition as "an irreversible investment of the agent's effort to distinguish one signal from another," which is unconstrained by his limited liability.

I proceed with the case of perfect information acquisition, i.e. $a \in \{0, 1\}$, once information gathering effort is exerted, the agent knows the realized state of nature perfectly. This is equivalent to $d(\kappa, a) = \kappa a$, the agent being risk neutral in information acquisition. I then discuss in Section 2.6 the implementation of imperfect information acquisition, when the optimal information acquiring effort is interior, given $d_a(\kappa, a) \geq 0$, with equality at $a = 0$, $d_{aa}(\kappa, a) > 0$, $d_a(\kappa, 1) \rightarrow \infty$.

The cost parameter of information acquisition, κ , can be interpreted accordingly in different applications of the model. For instance, in a firm-employee relationship, it represents the agent's expertise in his field, with a lower cost corresponding to a higher level of expertise as the agent is able to distinguish between productive signals at a lower cost. In investment banking, it captures the cost of market investigation (or more broadly, due diligence), which depends on market transparency or availability of data and past experiences, where a lower cost may be due to a well-established market with high level of information transparency. In insurance market, such cost of information acquisition may reflect the cost of conducting genetic test or other health

⁶Subscripts denote partial derivatives.

⁷Contracting with a risk averse agent without limited liability is discussed in Appendix B.

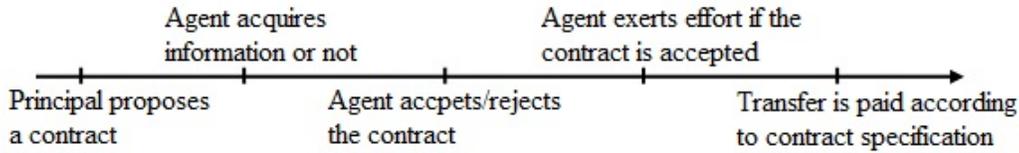


Figure 2.1: Time-line

examination, which, comparing with identifying accident, is less costly to reach the same accuracy.

If the principal induces perfect information acquisition with the contract, the agent has private information on the state of nature before accepting the contract and there is no productive uncertainty. The incentive problem is one due to ex-ante asymmetric information, the adverse selection. If the principal deters information acquisition with the contract, the principal and the agent have symmetric information, and the publicly observed output is an imperfect measurement of the agent's hidden action; the incentive problem is then moral hazard. The time-line of the game is given by Figure 2.1.

To capture my main arguments, the following assumptions are made so that the moral hazard problem when information acquisition is deterred is relevant, and such assumptions do not affect the adverse selection problem when information acquisition is induced.

Assumption 2.1. $q(e, 0)$ is a constant, normalized to zero, e.g. $q(e, \theta) = (m(\theta) - m(0))n(e)$, where $n(0) = 0$.

Assumption 2.2. $\rho_1(e, \theta) \equiv \frac{f(\theta)}{1-F(\theta)} \frac{q_e(e, \theta)}{q_\theta(e, \theta)}$ ⁸ is increasing in θ and $\rho_2(e, \theta) \equiv \frac{f(\theta)}{F(\theta)} \frac{q_e(e, \theta)}{q_\theta(e, \theta)}$ is decreasing in θ for $(e, \theta) > (0, 0)$.

Assumption 2.1 is imposed to assume out moving support in lower realization of output when θ is a stochastic variable in the production stage. Assumption 2.2 assumes a monotone hazard rate weighted by marginal rate of substitution in production to guarantee a second best separating equilibrium in an adverse selection environment and the optimality of a feasible debt contract with risk neutrality and limited liability if moral hazard is the underlying incentive problem.

⁸This is phrased as “critical ratio” in Poblete and Spulber (2012).

2.3 Inducing Information Acquisition

If it is optimal for the principal to induce information acquisition perfectly, she proposes a feasible contract that consists of a menu of options, $\{t(q(\theta)), q(\theta)\}$, in which the agent in state θ accepts the contract, has no incentive to produce $q(\theta')$ for any $\theta' \neq \theta$, and acquires information on the state of nature at a cost κ before acceptance. Applying the revelation principle, and let $h(q(\theta'), \theta) \equiv e$, $\hat{c}(q(\theta'), \theta) \equiv c(h(q(\theta'), \theta))$, where $q(\theta') = q(e, \theta)$, the set of feasible contract is restricted to $\{t(\theta), q(\theta)\}$ such that individual rationality, incentive compatibility, and induced information acquisition constraints are satisfied. Precisely,

$$t(\theta) - \hat{c}(q(\theta), \theta) \geq 0, \quad \forall \theta \in [0, \bar{\theta}] \quad (IR_\theta)$$

$$\theta \in \arg \max_{\theta'} t(\theta') - \hat{c}(q(\theta'), \theta), \quad \forall \theta \in [0, \bar{\theta}] \quad (IC_\theta)$$

$$\int_0^{\bar{\theta}} t(\theta) - \hat{c}(q(\theta), \theta) dF(\theta) - \kappa \geq \max_e \int_0^{\bar{\theta}} t(q(e, \theta)) dF(\theta) - c(e) \quad (II)$$

As the agent's utility $u^A(t(\theta'), q(\theta'), \theta)$ satisfies single crossing property⁹, (IC_θ) can be replaced by local incentive compatibility and monotonicity constraints:

$$t_\theta(\theta) - \hat{c}_q(q(\theta), \theta) q_\theta(\theta) = 0 \quad (LIC_\theta),$$

$$q_\theta(\theta) \geq 0 \quad (M).$$

The principal's optimization program to induce information acquisition is thus

$$\begin{aligned} \mathcal{P}_{II} : \quad & \max_{t(\theta), q(\theta)} \int_0^{\bar{\theta}} q(\theta) - t(\theta) dF(\theta) \\ & s.t. (IR_\theta), (LIC_\theta), (M), (II) \end{aligned}$$

The optimal contract to induce information $\mathbb{C}^{II} = \{t^{II}(\theta), q^{II}(\theta)\}$ in comparison to the second best menu $\mathbb{C}^{SM} = \{t^{SM}(\theta), q^{SM}(\theta)\}$ is characterized in Proposition 2.1.

Proposition 2.1. *For $\kappa > \kappa^a$, to induce information acquisition, higher-powered (lower-powered) incentive than the second best menu is given to the agent in the more*

⁹ $\frac{d(-\frac{u_q}{u_t})}{d\theta} = \frac{d(c_h h_q)}{d\theta} = \frac{1}{q_e^2} (-c_{hh} q_\theta - c_h q_{e\theta}) < 0$, i.e. marginal cost of output, relative to marginal utility of transfer, decreases in θ .

(less) efficient states, i.e. $q^{II}(\theta) > q^{SM}(\theta)$ for $\theta > \hat{\theta}$ and $q^{II}(\theta) \leq q^{SM}(\theta)$ for $\theta \leq \hat{\theta}$, where $\hat{\theta}$ is the state of nature that the agent would have expected to reveal ex ante if he did not acquire information.

Proof. Appendix A.2.1, or Crémer, Khalil, and Rochet (1998a) and Lewis and Sappington (1997) for the case where the stochastic state of nature is a cost parameter. \square

When information acquisition is induced, I am able to replicate the proposition of Crémer, Khalil, and Rochet (1998a) and Lewis and Sappington (1997). Additional (ex ante) rent is given when (II) is violated at the second best, as if the agent is rewarded rent to utilize his expertise. This is in the form of higher rent in the more efficient states via raising $q^{II}(\theta)$ above the second best, and lower rent for the less efficient states through lowering $q^{II}(\theta)$ below the second best, so that the agent is motivated to distinguish the relatively efficient states of nature from the less efficient.

2.4 Detering Information Acquisition

If it is optimal for the principal to deter information acquisition, she proposes a feasible contingent transfer $t(q)$ to implement productive effort e , which satisfies limited liability of the agent, prescribes both players with payoffs that are non-decreasing in output,¹⁰ and such that the agent does not acquire information on the state of nature before acceptance. The principal solves the following program subject to limited liability, non-decreasing payoff, incentive compatibility, and deterred information acquisition constraints.

$$\mathcal{P}_{DI} : \max_{t(q(e,\theta)), e} \int_0^{\bar{\theta}} q(e, \theta) - t(q(e, \theta)) dF(\theta)$$

subject to

$$t(q) \geq 0 \quad (LL)$$

$$0 \leq t_q(q) \leq 1 \quad (NDP)$$

$$e \in \arg \max_y \int_0^{\bar{\theta}} t(q(y, \theta)) - c(y) dF(\theta) \quad (IC)$$

¹⁰Defining feasible contract as satisfying limited liability and non-decreasing payoffs follows Innes (1990) and Pobleto and Spulber (2012).

$$\int_0^{\bar{\theta}} t(q(e, \theta)) - c(e) dF(\theta) \geq \int_0^{\bar{\theta}} \mathbf{1}_{\theta \geq \tilde{\theta}} t(q(e(\theta), \theta)) - c(e(\theta)) dF(\theta) - \kappa \quad (DI),$$

where $e(\theta) \in \arg \max_y t(q(y, \theta)) - c(y)$ and $\tilde{\theta}$ is such that $t(q(e(\tilde{\theta}), \tilde{\theta})) - c(e(\tilde{\theta})) = 0$, i.e. for $\theta < \tilde{\theta}$, the agent who acquired information off the equilibrium path finds it optimal to reject the contract.¹¹ Simply by the right-hand-side of (DI) one can have a glimpse of the agent's opportunistic motives to acquire information off the equilibrium path: to distinguish a sufficiently inefficient state of nature to avoid exerting effort at a loss, and to discover a relatively efficient state of nature to extract maximal rent.

This section is devoted to the discussion on how deterring information acquisition interacts with the moral hazard problem in the contractual relationship, by looking at the distortion of the optimal contract from the second best. In Section 2.4.1, I focus on a tractable example assuming a specific form of contingent transfer consisting of a debt and a share of output residual, and turn to the general case in Section 2.4.2. I assume in both sections risk neutrality, leaving the case with a risk averse agent to Appendix B.

2.4.1 Example

Poblete and Spulber (2012) have shown the optimality of debt contract $t(q) = \max\{q(e, \theta) - \underline{q}, 0\}$, $\underline{q} \geq 0$, in a moral hazard problem with continuous effort and state of nature, given risk neutrality, limited liability, and non-decreasing payoff, when $\rho_1(e, \theta) \equiv \frac{f(\theta)}{1-F(\theta)} \frac{q_e(e, \theta)}{q_\theta(e, \theta)}$ is increasing in θ . In this section I focus on a simplified example in which the contract to deter information acquisition, \mathbb{C}^{DI} , has a contingent transfer in the form $t^{DI}(q) = T^{DI} + \max\{s^{DI}(q(e^{DI}, \theta) - \underline{q}^{DI}), 0\}$, and discuss how deterring information acquisition modifies this contract from the second best \mathbb{C}^{SD} , in which $t^{SD}(q) = \max\{q(e^{SD}, \theta) - \underline{q}^{SD}, 0\}$, leaving a general contractual form to the next section.

Lemma 2.1. $T^{DI} = 0$.

Proof. If (DI) is violated at the second best, $T > 0$ does not bind (DI), as off the equilibrium path, the agent who acquires information can always accept the contract and exert any $e \geq 0$ to earn T , i.e. regardless of whether acquiring information or not, the agent's expected utility increases by T . $T < 0$ violates (LL) for $q(e, \theta) < \underline{q}$. \square

For the convenience of interpretation, I would phrase the simplified contract as a duo of debt (\underline{q}) and equity share of output residual (s).

¹¹This is by the envelope theorem of the informed agent's optimization problem off the equilibrium path.

In Assumption 2.3, I give a sufficient condition for the first order approach to be valid given the simplified contract and a class of commonly seen production and cost function that satisfy such condition, given which (IC) can be replaced by local incentive compatibility without loss of generality,

$$\int_{\underline{\theta}}^{\bar{\theta}} sq_e(e, \theta) dF(\theta) - c_e(e) = 0 \quad (LIC'),$$

where $\underline{\theta}$ is such that $q(e, \underline{\theta}) \equiv \underline{q}$.

Assumption 2.3. *The production function is such that the first order approach is valid, e.g. production and cost functions in the forms $q(e, \theta) = \theta^\alpha e^\beta$ and $c(e) = e$, where $0 < \alpha, \beta \leq 1$ and $\alpha \geq \frac{\beta}{1-\beta} \bar{\theta}$.*

Lemma 2.2. $\tilde{q} \equiv q(e(\tilde{\theta}), \tilde{\theta}) > \underline{q}$ and $\tilde{\theta} > \underline{\theta}$.

Proof. If $\tilde{q} \leq \underline{q}$, $t(\tilde{q}) - c(e(\tilde{\theta})) = -c(e(\tilde{\theta})) < 0$, contradicting to the definition of $\tilde{\theta}$. Thus, $\tilde{q} > \underline{q}$. Off the equilibrium path, an informed agent exerts effort $e(\theta)$ where $sq_e(e(\theta), \theta) = c(e(\theta))$. An uninformed agent exerts effort e^* such that $\int_{\underline{\theta}}^{\bar{\theta}} sq_e(e^*, \theta) dF(\theta) = c_e(e^*)$. Let θ^0 be such that $\int_{\underline{\theta}}^{\bar{\theta}} sq_e(e^*, \theta) dF(\theta) = sq_e(e^*, \theta^0)$, i.e. $e^* = e(\theta^0)$. If $\theta^0 \leq \underline{\theta}$, $q(e(\theta^0), \theta^0) \leq q(e(\theta^0), \underline{\theta}) = \underline{q}$. Thus, by definition of $\tilde{\theta}$, $\theta^0 < \tilde{\theta}$ for all $\theta^0 \leq \underline{\theta}$, implying that $\underline{\theta} < \tilde{\theta}$. If $\theta^0 > \underline{\theta}$, $e(\theta^0) > e(\underline{\theta})$; hence, $q(e(\theta^0), \underline{\theta}) = \underline{q} > q(e(\underline{\theta}), \underline{\theta})$, implying that $\underline{\theta} < \tilde{\theta}$. \square

Lemma 2.2 gave a preliminary hint on one of the agent's motives to acquire information off the equilibrium path: to distinguish a sufficiently inefficient state of nature to avoid the debt. It, along with Lemma 2.1, is applied to rewrite constraint (DI) into

$$\int_{\underline{\theta}}^{\bar{\theta}} s(q(e, \theta) - \underline{q}) dF(\theta) - c(e) \geq \int_{\tilde{\theta}}^{\bar{\theta}} s(q(e(\theta), \theta) - \underline{q}) - c(e(\theta)) dF(\theta) - \kappa \quad (DI')$$

The principal's optimization program to deter a risk neutral agent from acquiring information with the simplified contract is thus reduced to

$$\begin{aligned} \mathcal{P}'_{DI} : \quad & \max_{s, \underline{q}, e} \int_0^{\bar{\theta}} q(e, \theta) dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} s(q(e, \theta) - \underline{q}) dF(\theta) \\ & s.t. (NDP), (LIC'), (DI') \end{aligned}$$

Characterization of the optimal simplified contract with binding constraint (DI) is given in Proposition 2.2 below.

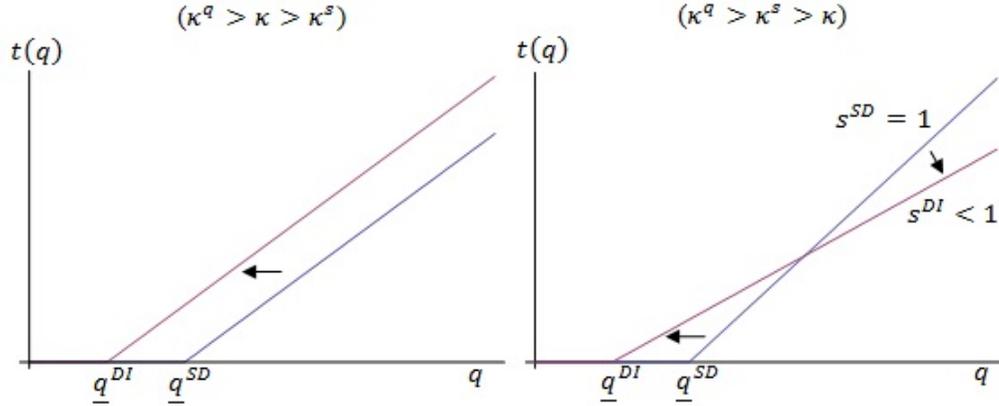


Figure 2.2: Debt-with-Equity-Share Contract to Deter Information Acquisition

Proposition 2.2. *There exists κ^s and κ^q , $\kappa^s \leq \kappa^q$, such that for $\kappa \in [\kappa^s, \kappa^q)$, it is optimal to deter information acquisition with a debt contract which has a lower debt than the second best, $\underline{q}^{DI} < \underline{q}^{SD}$ and $s^{DI} = s^{SD} = 1$, and for $\kappa < \kappa^s$, it is optimal to deter information acquisition with a debt-with-equity-share contract, in which $\underline{q}^{DI} < \underline{q}^{SD}$ and $s^{DI} < s^{SD} = 1$, if $\int_{\underline{\theta}^{SD}}^{\bar{\theta}} q(e^{SD}, \theta) - \underline{q}^{SD} dF(\theta) < \int_{\underline{\theta}^{SD}}^{\bar{\theta}} q(e(\theta), \theta) - \underline{q}^{SD} dF(\theta)$ ¹². (Illustrated in Figure 2.2)*

Proof. Given Lemma 2.2, lowering the debt $\underline{q} < \underline{q}^{SD}$ increases expected output residual more significantly on the equilibrium path than it does off the equilibrium path. Suppose that $s = 1$, for sufficiently small κ such that \underline{q} is arbitrarily close to zero to deter information acquisition, the principal earns arbitrarily close to nothing. Lowering s gives the principal a positive share of a smaller expected output. The complete proof is in Appendix A.2.2. \square

When the second best contract violates constraint (DI), one of the opportunistic motives of the agent to acquire information is to discover a sufficiently inefficient state of nature to avoid the debt. For $\kappa^s < \kappa < \kappa^q$, the debt is lowered to discourage the agent from attempting to discover a sufficiently inefficient state of nature. The optimal contract to deter information acquisition is as if the principal trades off a larger output residual to the agent as his rent, in order to incentivize productive effort and discourage him from acquiring information for opportunistic purpose. As a claimant for a larger output residual, the agent unambiguously exerts higher effort than that under the second best.

¹²The agent's expected full residual when uninformed is smaller than that when informed, given the second best debt contract, i.e. information is valuable to the agent. For instance, a Cobb-Douglas production function with uniformly distributed state of nature would satisfy this.

A larger output residual to the agent, however, creates another opportunistic motive to acquire information: to discover a relatively efficient state of nature and extract maximal rent by exerting $e(\theta)$. This motive is significant for an agent with sufficiently small cost of information acquisition, $\kappa < \kappa^s < \kappa^q$, and whose expected output residual when being uninformed is smaller than that when being informed under the second best debt contract, $\int_{\underline{\theta}^{SD}}^{\bar{\theta}} q(e^{SD}, \theta) - \underline{q}^{SD} dF(\theta) < \int_{\underline{\theta}^{SD}}^{\bar{\theta}} q(e(\theta), \theta) - \underline{q}^{SD} dF(\theta)$. The principal, instead of granting the entire output residual to the agent, finds it optimal to reduce the agent's share of output residual so that his ability to extract rent by acquiring information is restricted, which distort the implemented productive effort downwards.

For a lower debt incentivizes productive effort yet a reduced share of output residual discourages it, there exists $\kappa^e < \kappa^s$ such that for $\kappa < \kappa^e$, deterring information acquisition trades off productive effort, whereas for $\kappa > \kappa^e$, deterring information acquisition is accompanied by a higher-powered incentive than the second best, summarized as the following corollary.

Corollary 2.1. *There exists $\kappa^e < \kappa^s$ such that for $\kappa \geq \kappa^e$, $e^{DI} \geq e^{SD}$, and $e^{DI} < e^{SD}$ otherwise.*

The prediction is different from what Crémer, Khalil and Rochet (1998) suggests. This is because of the key departure from Crémer et al.: whether a moral hazard problem is present when information acquisition is deterred. In Crémer et al., productive uncertainty is absent even when information acquisition is deterred; contractible output is a perfect measurement for the agent's effort, as if the effort itself can also be contracted upon. In the current model, effort level is implemented with a transfer contingent on realization of contractible output. That is, in Crémer et al., the principal has two instruments to motivate productive effort and to deter information acquisition, whereas in the model here, the principal has only one instrument, the transfer.

In terms of agent's motive to acquire information, in Crémer et al., the second best transfer is a fixed payment, so the agent's motive to acquire information on the cost parameter is to use the information to reduce the cost of effort. Thus, to deter information acquisition, the principal increases the fixed payment, along with a lower contracted output (hence, a lower effort) if it is not too costly to acquire information. Here, the second best contract is a debt contract, given which the agent's motive to acquire information is to use the information to avoid the debt or to extract maximal rent. Therefore, the principal lowers the debt to deter information acquisition, accom-

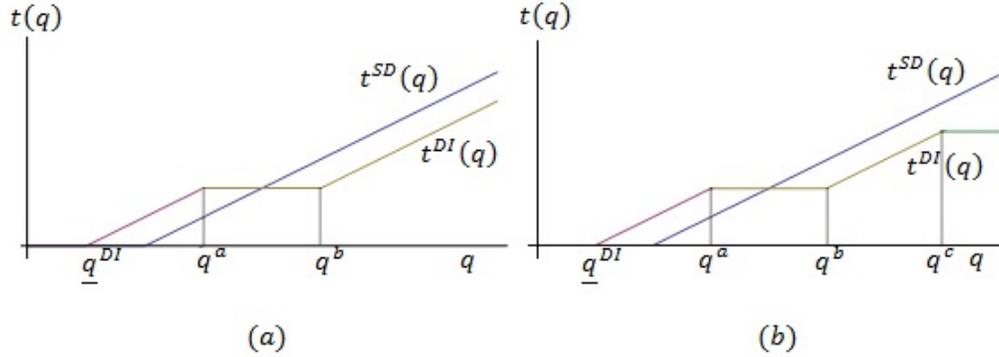


Figure 2.3: General Contract to Deter Information Acquisition

panied by a reduced share of output residual for insufficiently large cost of information acquisition. The former incentivize effort whereas the latter discourages it.

2.4.2 General Contract

The readers at this point may question the optimality of the proposed debt-with-equity-share contract with the presence of binding constraint to deter information acquisition. I respond by showing that the result of a lower debt than its second best counterpart is indeed optimal, and a reduced share of output residual is qualitatively robust, yet in a different form of transfer, in which $s \in \{0, 1\}$ for different output intervals beyond the debt, as the principal's objective function is linear in the slope of transfer.

Proposition 2.3. *The optimal contract to deter a risk neutral agent protected by limited liability from acquiring information has a lower debt and a reduced share of output residual, $t^U(q) = 0$ for $q \leq \underline{q}^{DI} < \underline{q}^{SD}$ and $t^{DI}(q) \leq q - \underline{q}^{DI}$ for $q > \underline{q}^{DI}$.*

Proof. Appendix A.2.3. Precise form of $t^{DI}(q)$ is derived in the same appendix with the proof and is illustrated in Figure 2.3. \square

The intuition discussed in the previous example prevails. Recall that in the second best environment, the agent's opportunistic motive to acquire information is to distinguish the inefficient states of nature to avoid the debt, and to discover the efficient states to extract maximal rent. A lower debt, $\underline{q}^{DI} < \underline{q}^{SD}$, is implemented to account for the former motive, and the transfer for sufficiently high realization of output is distorted downward from the second best to demotivate the latter, which violates monotonicity for some intermediate states of nature. Thus, a low-powered incentive ($t_q^{DI}(q) = 0$) is optimal for intermediate outputs, as illustrated in Figure 2.3(a). If cost

of information acquisition is sufficiently low, the rent extraction motive of the agent is so significant that the optimal contract to deter information acquisition is capped at a threshold level, i.e. $t_q^{DI}(q) = 0$ for $q > q^c$, as illustrated in Figure 2.3(b).¹³

The contract in Proposition 2.3 can also be regarded as a second degree *contract* discrimination, in a similar fashion to the second degree price discrimination. For sufficiently low output level, $q < q^a$, the agent is paid according to the transfer schedule $t^a(q) = \max\{q - \underline{q}^{DI}, 0\}$, a simple debt contract with the level of debt smaller than the second best. Intuitively, the principal rewards the agent in the form of a lower debt for sufficiently small output realization, which could have been avoided if he acquired information. For intermediate level of output, $q^a \leq q < q^c$, the agent receives $t^b(q) = T + \max\{q - q^b, 0\}$, $T = q^a - \underline{q}^{DI}$, i.e. a debt-with-fixed-payment contract. For sufficiently high level of output, $q \geq q^c$, he receives a fixed payment \bar{T} .

The agent's motive to acquire information off the equilibrium path contributes to the explanation of why deterrence of information acquisition generates this contract discrimination by output level. If the state of nature is sufficiently inefficient, the agent's benefit of acquiring information mainly comes from avoiding the debt. If, instead, the state of nature is relatively efficient, the agent's benefit of acquiring information is mostly attributed to rent extraction. Thus, from the principal's perspective, it is optimal to deter information acquisition with different transfer schemes associated to different sets of output.

2.5 The Equilibrium Incentive Problem – the Rent-Efficiency Tradeoff

The following lemma indicates that some information management, either to induce information acquisition or to deter it, is preferred to null information management for the principal.

Lemma 2.3. *Null information management is suboptimal.*

Proof. Consider $\kappa > \kappa^a$ such that (II) is strictly violated under \mathbb{C}^{SM} . Without inducing information acquisition, \mathbb{C}^{SM} implements the same outcome as $\mathbb{C}^0 = \{t^0(q(e^0, \theta))\}$,

¹³Alternatively, if we model the problem in the context of procurement or regulation as in Lewis and Sappington (1997), the optimal form of procurement contract to deter information acquisition would coincide qualitatively to what Chu and Sappington (2009b) characterizes in a model with adverse selection. Their driving force for the optimality is the shape of the density function of the state of nature, whereas in this paper, it is due to deterrence of information acquisition.

where $e^0 \in \arg \max_e \int_0^{\bar{\theta}} t^{SM}(q(e, \theta)) dF(\theta) - c(e)$ and $t^0(q) = t^{SM}(q)$ for all q . \mathbb{C}^0 satisfies (LL) , (IC) , and (DI) by construction, which must not be preferred to \mathbb{C}^{DI} for the principal. Consider $\kappa < \kappa^q$ such that (DI) is strictly violated under \mathbb{C}^{SD} . Without deterring information acquisition, \mathbb{C}^{SD} implements the same outcome as $\mathbb{C}^1 = \{t^1(q^1(\theta)), q^1(\theta)\}$, where $q^1(\theta) = q(e(\theta), \theta)$ for all $\theta \geq \tilde{\theta}$, zero otherwise, and $t^1(q^1(\theta)) = t^{SD}(q(e(\theta), \theta))$ for $\theta \geq \tilde{\theta}$, zero otherwise. \mathbb{C}^1 by construction satisfies (IR_θ) , (IC_θ) , and (II) , which the principal does not prefer to \mathbb{C}^{II} . \square

Lemma 2.3 allows us to restrict our attention to the comparison between inducing information acquisition and deterring it when studying the endogenous implementation of incentive problem. Define the principal's net value of information, $V(\kappa)$, as the difference between her ex ante expected utility when information acquisition is induced and that when it is deterred,

$$V(\kappa) \equiv E(u^P(\mathbb{C}^{II}, \kappa)) - E(u^P(\mathbb{C}^{DI}, \kappa)),$$

which is equivalent to the expected improvement in efficiency minus the expected net information rent given to the agent to incentivize information acquisition,

$$V(\kappa) = \underbrace{\int_0^{\bar{\theta}} q^{II}(\theta) - c(h(q^{II}(\theta), \theta)) dF(\theta) - \int_0^{\bar{\theta}} q(e^{DI}, \theta) - c(e^{DI}) dF(\theta)}_{\text{Expected Improvement in Efficiency}} - \underbrace{\left[\int_0^{\bar{\theta}} u^A(t^{II}(q^{II}(\theta)), q^{II}(\theta), \kappa) dF(\theta) - \int_0^{\bar{\theta}} u^A(t^{DI}(q(e^{DI}, \theta)), \kappa) dF(\theta) \right]}_{\text{Expected Net Information Rent}}.$$

For an agent with the cost of information acquisition κ , if information is crucial in the sense that the principal benefits more from an improvement in efficiency relative to the net information rent to motivate the agent to acquire and use the information, the principal finds it optimal to induce information acquisition and implement adverse selection in the production stage. Otherwise, it is optimal for her to deter information acquisition to avoid a high net information rent, at the expense of efficiency, and implement moral hazard in the production stage. The principal's information management and endogenous implementation of incentive problem exhibits a rent-efficiency trade-off. In a standard adverse selection problem, efficient production from the inefficient types of agent is traded off to save on rent given to the efficient types of agent, and in the scope of information management, efficient use of information is traded off to

save on rent given to the agent obtaining such information.

Straightforward from the optimization problem of the principal, $V_\kappa(\kappa) < 0$. For $\kappa \rightarrow 0$, the principal earns second best payoff if she induces the agent to acquire information, and she can only deter information acquisition by an extremely low-powered transfer scheme, i.e. effort is distorted far away from the efficient level. For $\kappa \rightarrow \infty$, the principal earns second best payoff if she deters the agent from acquiring information, and if she intends to induce information acquisition, the information rent goes to infinite. Hence,

Proposition 2.4. *There exists $0 < \kappa^I < \infty$ such that for $\kappa < \kappa^I$, improvement in efficiency exceeds the net information rent, and it is optimal to induce information acquisition and implement adverse selection in the production stage; for $\kappa > \kappa^I$, improvement in efficiency falls short of the net information rent, and it is optimal to deter information acquisition and implement moral hazard in the production stage.*

Application: Expert and Mediocre in a Production Relationship. Interpreting the cost of information acquisition as the agent’s expertise in this field, the principal finds it optimal to induce an “expert” (who has sufficiently small cost of information acquisition) to acquire productive information and to implement adverse selection in the production stage, as by acquiring this information, improvement in efficiency is more significant than the net information rent. If the agent is a “mediocre” (who has sufficiently large cost of information acquisition), it is then optimal to deter him from acquiring information and to implement moral hazard in the production stage, to avoid a significantly large information rent at the expense of efficiency.

In terms of the contractual form, if $\kappa^I \in (\kappa^a, \kappa^q)$, it is optimal i) to induce information acquisition with the second best menu contract, \mathbb{C}^{SM} , for $\kappa \leq \kappa^a$ (an agent with extremely high expertise); ii) to induce information acquisition with a modified menu contract, \mathbb{C}^{II} , in which $q^{II}(\theta) > q^{SM}(\theta)$ for $\theta \geq \hat{\theta}$ and $q^{II}(\theta) < q^{SM}(\theta)$ for $\theta < \hat{\theta}$, for $\kappa^a < \kappa \leq \kappa^I$ (an agent with high expertise); iii) to deter information acquisition with a debt-with-equity-share contract, \mathbb{C}^{DI} , in which $\underline{q}^{DI} < \underline{q}^{SD}$ and $s^{DI} \leq 1$, for $\kappa^I < \kappa < \kappa^q$ (an agent with mild expertise); to deter information acquisition with a second best debt contract, \mathbb{C}^{SD} , for $\kappa^q \leq \kappa$ (an agent with poor expertise).

However, level of κ^I depends on the exact functional form and the distribution of the state of nature, and is not guaranteed to be within the above-mentioned interval. If $k^I \leq k^a$, interval ii) does not exist, and if $k^I \geq k^q$, interval iii) does not exist. For example, given production function $q(e, \theta) = \sqrt{\theta}e$, cost of effort $c(e) = \frac{e^2}{2}$, and $\theta \sim Unif(0, \bar{\theta})$, a modified menu contract is never optimal if $\bar{\theta}$ is sufficiently low, i.e.

if information on the state of nature does not improve efficiency significantly relative to the net information rent, and a debt-with-equity-share contract is never optimal if $\bar{\theta}$ is sufficiently high, where information is crucial in production.

Application: Investment Banking. The model I present here can also be applied to address the agency problems in investment banking, where an investment bank (the principal) makes decision on funding a project executed by a private firm (the agent), the profitability of which depends on the firm's investment (human and physical capital), i.e. the hidden action in the model, and non-contractible stochastic market condition, i.e. the state of nature in the model. The firm can, before accepting the contract, conduct market investigation (information acquisition)¹⁴ at a sunk cost.

The cost of market investigation may be related to the characteristics of the market where the firm participates, such as market transparency, or whether the market is a newly formed or a well-established one. If the investment bank is contracting with a firm in a well-established market with high level of transparency, the firm is able to collect data and past experience at a lower cost. It is optimal for the investment bank to offer a menu of funding options that induce the firm to conduct market investigation prior to acceptance. On the other hand, contracting with a firm in a newly formed market or in one with low level of transparency, data and past experience is limited or relatively costly for the firm to acquire. It is optimal for the investment bank to propose a state-independent debt-with-equity-share contract, so that the firm is deterred from conducting market investigation.

I am aware of the complexity of the real investment banking industry than in this model, e.g. there involves more competition among investment banks and firms instead of a simple principal-agent relationship, the investment bank itself may acquire information as well, and there may also be a regulator involved, but this model serves as a benchmark for more sophisticated studies in which endogeneity of incentive problem is optimally chosen with information management.

2.6 Imperfect Information Acquisition

In the previous sections, information acquisition is perfect as long as acquiring effort is exerted, or equivalently, it is taken as a special case with linear cost of information

¹⁴I restrict information acquisition to only market investigation for explanatory convenience. Information acquisition by the private firm may also include interior investigation such as management and production audit.

acquisition. To emphasize the difference between perfect and imperfect information acquisition, in this section I examine the case with an interior solution of information management, assuming that $d(\kappa, a)$ has $d_a(\kappa, a) \geq 0$, with equality at $a = 0$, $d_{aa}(\kappa, a) > 0$, $d_a(\kappa, 1) \rightarrow \infty$ for all κ . That is, in equilibrium, the incentive problem in the production stage is stochastic, whose density is implemented by the contracts offered. Denote the contracts $\mathbb{C}^I = \{q^I(\theta), t^I(\theta)\}$ to an informed agent and $\mathbb{C}^U = \{t^U(q)\}$ to an uninformed agent.

Given a , if the agent observes productivity signal and is induced to reveal it truthfully, he has information advantage in the production stage and earns information rent $u^I(\theta) = t^I(\theta) - \hat{c}(q^I(\theta), \theta)$. If he does not observe any signal, output is an imperfect measurement of his productive effort and he earns $u^U(\theta) \equiv t^U(q(e^U, \theta)) - c(e^U)$, where e^U is the implemented effort by \mathbb{C}^U . The optimal investment in information acquisition is thus $a \in \arg \max_{a'} a' \int_0^{\bar{\theta}} u^I(\theta) dF(\theta) + (1 - a') \int_0^{\bar{\theta}} u^U(\theta) dF(\theta) - d(\kappa, a')$, or by the first order condition,

$$\int_0^{\bar{\theta}} u^I(\theta) dF(\theta) - \int_0^{\bar{\theta}} u^U(\theta) dF(\theta) = d_a(\kappa, a) \quad (A)$$

Information on productivity is not the only information advantage the agent has, however. Whether the agent observes a correct signal or nothing is also his private information. The feasible contracts $\{\mathbb{C}^I, \mathbb{C}^U\}$ are designed such that an informed agent prefers \mathbb{C}^I and the uninformed finds \mathbb{C}^U more attractive. Respectively,

$$u^I(\theta) \geq \max_e t^U(q(e, \theta)) - c(e) \quad \forall \theta \in [0, \bar{\theta}] \quad (TT_I)$$

and

$$\int_0^{\bar{\theta}} u^U(\theta) dF(\theta) \geq \max_e \int_0^{\bar{\theta}} t^I(q^I(e, \theta)) dF(\theta) - c(e) \quad (TT_U)$$

Adjusting notation accordingly for (LIC_θ) , (M) , (IC) , (LL) , and (NDP) , $\{\mathbb{C}^I, \mathbb{C}^U\}$ solves the following program to implement imperfect information acquisition,

$$\begin{aligned} \mathcal{P}_M : \quad & \max_{q^I(\theta), t^I(\theta), e^U, t^U(q), a} a \int_0^{\bar{\theta}} q^I(\theta) - t^I(q^I(\theta)) dF(\theta) \\ & + (1 - a) \int_0^{\bar{\theta}} q(e^U, \theta) - t^U(q(e^U, \theta)) dF(\theta) \\ & s.t. (LIC_\theta), (M), (IC), (LL), (NDP), (A), (TT_I), (TT_U) \end{aligned}$$

Lemma 2.4. *If (TT_I) is binding for some states of nature, it is binding at θ^T , $\tilde{\theta} <$*

$\theta^T \leq \bar{\theta}$, where $\tilde{\theta}$ is such that $e(\theta) \in \max_e t^U(q(e, \theta)) - c(e) = 0$ for $\theta < \tilde{\theta}$.

Proof. Appendix A.2.4. □

Proposition 2.5. *Optimal contract $\{\mathbb{C}^I, \mathbb{C}^U\}$ with imperfect information acquisition has the following properties*

1. $q^I(\theta) \geq q^{SM}(\theta)$ for $\theta > \hat{\theta}$, $q^I(\theta) \leq q^{SM}(\theta)$ for $\theta \leq \hat{\theta}$, and $q^I(\theta)$ exhibits a gap at $\hat{\theta}$, where $\hat{\theta}$ is the state that an uninformed agent expected to reveal ex ante if he pretends to be informed.
2. If $\theta^T < \bar{\theta}$, there exists interval (θ^a, θ^b) containing θ^T such that $q_\theta^I(\theta) = 0$ for $\theta \in (\theta^a, \theta^b)$.
3. $t^U(q)$ takes the form of debt-with-equity-share, with a lower debt than its second best counterpart, $t^U(q) = 0$ for $q \leq \underline{q}^U < \underline{q}^{SD}$ and $t^U(q) \leq q - \underline{q}^U$ for $q > \underline{q}^U$.

Proof. Appendix A.2.4. □

As the agent has private information in whether a correct signal or a null signal is observed, the optimal contract in comparison to the second best¹⁵ incorporates this dimension of truthful revelation. To induce truthful revelation of receiving no signal, $q^I(\theta)$ is lowered for $\theta < \hat{\theta}$ to restrict the ex ante expected rent given to an uninformed agent claiming to be informed of state $\hat{\theta}$, and the debt in $t^U(q)$, \underline{q}^U , is lowered to give a higher rent to an uninformed agent who truthfully report receiving no signal. In addition, an informed agent in $\theta < \tilde{\theta}$ has no attempt to pretend to be uninformed and give up his rent. Thus, to induce truthful revelation of receiving a correct signal, an equity share of output residual in $t^U(q)$ is offered in equilibrium to limit an informed agent's ability to extract rent by claiming to be uninformed, and $q^I(\theta)$ for $\theta < \theta^T$ is raised to give an informed agent a higher rent so that it is more costly for him to pretend uninformed, which violates monotonicity near θ^T ; pooled output schedule is then optimal for some intermediate states of nature containing θ^T . Whether $q^I(\theta)$ for $\theta < \min\{\hat{\theta}, \theta^T\}$ is above or below the second best then depends on the relative magnitude of the effects from inducing truthful revelation of an informed agent and that of an uninformed agent.

¹⁵The second best here is referred to the one with symmetric information on whether information is realized imperfectly. I find it more persuasive to compare the optimal contract to this second best instead of the one with perfect signal, as the latter includes the effect of information management and that of a possible null signal.

I thus conclude the qualitative robustness of the debt-with-equity-share contract in \mathbb{C}^U , with a lower debt than the second best debt contract. The intuition, however, is different from that under perfect information acquisition. With perfect information acquisition, a debt-with-equity-share contract is offered to deter different use of information off the equilibrium path. With imperfect information acquisition, a lower debt is offered to induce the uninformed agent to truthfully report his informativeness, and an equity share of output residual is offered to deter the informed agent from claiming to be uninformed. The pooled output schedule for intermediate states of nature in \mathbb{C}^I is attributed to the joint effect of truthful revelation of being informed of states $\theta \in (\theta^a, \theta^b)$ and the monotonicity constraint. The former technically resembles the type-dependent participation constraints that generate countervailing incentives. In fact, the contract designed for an uninformed agent is itself a type-dependent alternative for an informed agent.¹⁶

Iossa and Martimort (2013) provides a similar idea to that in this section, yet with several differences in the model. The most important difference is on the assumption that the agent, although observing the true level of information acquiring action, is pessimistic about the realization of information in the sense that he believes the information is realized at a lower probability than a , whereas I have ex ante symmetric belief on the equilibrium realization of information acquisition upon seeing the contractual specification. They assume linear transfer scheme for both the informed and the uninformed agent, and I assume neither. Information is acquired after signing the contract, instead of before acceptance. They define pooling over the slope of transfer, and I define pooling over the output schedule.

Due to these differences in assumptions and definitions, we predict some results differently. In Iossa and Martimort, costly information acquisition itself does not distort the optimal contract, it is the joint effect with the agent's pessimistic attitude that makes a difference, whereas I show that even with symmetric belief on the agent's information acquisition in equilibrium, information management distorts the optimal contract from the second best provided that the cost of information acquisition is not too extreme. They find pooling optimal in inefficient states of natures instead of in the intermediate states of nature. This difference relies on the assumed linear transfer, so

¹⁶Please refer to Lewis and Sappington (1989) for a pioneer work and to Jullien (2000) for a general discussion of countervailing incentives. Lemma 2.4 and Proposition 2.5 here can be regarded as a justification for the presence of countervailing incentive even with type-independent reservation payoff. However, it does not perfectly coincide with Lewis and Sappington (1989) and Jullien (2002), as the "type-dependent reservation payoff" for an informed agent here depends on the principal's endogenous choice of contract to an uninformed agent.

that (TT_I) is violated in sufficiently inefficient states of nature under the linear second best and can be made satisfied only by pooling the slope of transfer to an informed agent and that to an uninformed agent. Pooling in Iossa and Martimort is resulted from truthful revelation of observing sufficiently inefficient states of nature, whereas pooling in this paper is due to the joint forces of truthful revelation of observing intermediate states of nature and monotonicity of output schedule.

In the case of perfect information acquisition, information management exhibits a rent-efficiency tradeoff. With imperfect information acquisition, an additional consideration is included: the risk for having no signal, which is either absent in the environment with perfect information acquisition, or negligible when the agent is risk neutral in acquiring information. The principal implement a^* in equilibrium such that $\int_0^{\bar{\theta}} q^I(\theta) - c(h(q^I(\theta), \theta)) - u^I(\theta) dF(\theta) - \int_0^{\bar{\theta}} q(e^U, \theta) - c(e^U) - u^U dF(\theta) = \phi d_{aa}(a^*)$, or equivalently,

$$\underbrace{\int_0^{\bar{\theta}} q^I(\theta) - c(h(q^I(\theta), \theta)) - q(e^U, \theta) + c(e^U) dF(\theta)}_{\text{Expected Improvement in Efficiency}} - \underbrace{\phi d_{aa}(\kappa, a^*)}_{\text{Risk Premium}} = \underbrace{\int_0^{\bar{\theta}} u^I(\theta) - u^U dF(\theta)}_{\text{Expected Net Information Rent}} .$$

Information management thus involves a rent-risk-efficiency tradeoff.

Implementing a high information acquiring effort benefits the principal in the sense that, with a high probability of the agent being informed, productive effort is exerted more efficiently, and the rent given to the agent regarding whether he is informed or not is lower; it costs the principal a higher information rent to the informed agent to motivate such high information acquiring effort and reveal the truth. Implementing a low information acquiring effort benefits the principal as she pays an information rent to the informed agent with a small probability, a lower rent to the agent regarding whether he is informed or not for it is likely that the agent is uninformed, and a small transfer to motivate a low information acquiring effort, at a larger expense of efficiency. Implementing an intermediate information acquiring effort balance the probability that production is made more efficiently and information rent is paid to the informed agent with the transfer paid to motivate information acquiring effort, at the expense of higher rent given to the agent regarding whether information is realized.

2.7 Private Cost of Information Acquisition

I have adopted the assumption of common knowledge in the cost of information acquisition. It is not surprising that this cost, interpreted as the agent's expertise, may also be the agent's private information. Consider perfect information acquisition as assumed throughout the paper except in Section 2.6. For ease of illustration, let $\kappa \in \{\kappa^L, \kappa^H\}$, $\kappa^L < \kappa^I < \kappa^H$, $\kappa = \kappa^L$ with probability k . Under common knowledge of κ , the principal finds it optimal to implement adverse selection by inducing the agent of κ^L to acquire information, and to implement moral hazard by deterring the agent of κ^H from acquiring information.

If κ is private knowledge of the agent, the principal design a pair of contract $\{\mathbb{C}^I, \mathbb{C}^U\}$, where $\mathbb{C}^I = \{q^I(\theta), t^I(\theta)\}$ is designed to induce the agent of κ^L to acquire and reveal information truthfully, $\mathbb{C}^U = \{t^U(q)\}$ is designed to keep the agent of κ^H uninformed and motivated to exert effort, and that the agent voluntarily reveal his cost of information acquisition. In addition to the incentive compatibility, individual rationality, inducing information acquisition, and deterring information acquisition constraints in Sections 2.3 and 2.4, the pair of contracts satisfies

$$u^I(\theta) \geq \max_e t^U(q(e, \theta)) - c(e) \quad \forall \theta \in [0, \bar{\theta}] \quad (TT_I)$$

and

$$\int_0^{\bar{\theta}} u^U(\theta) dF(\theta) \geq \max_e \int_0^{\bar{\theta}} t^I(q^I(e, \theta)) dF(\theta) - c(e) \quad (TT_U)$$

as in Section 2.6. The agent who has acquired and learned information does not lie to be uninformed by accepting \mathbb{C}^U , vice versa. The principal's optimization program is then

$$\begin{aligned} \mathcal{P}_p : \quad & \max_{q^I(\theta), t^I(\theta), e^U, t^U(q)} k \int_0^{\bar{\theta}} q^I(\theta) - t^I(q^I(\theta)) dF(\theta) \\ & + (1 - k) \int_0^{\bar{\theta}} q(e^U, \theta) - t^U(q(e^U, \theta)) dF(\theta) \end{aligned}$$

$$s.t. (LIC_\theta), (M), (IC), (LL), (NDP), (II), (DI), (TT_I), (TT_U)$$

Proposition 2.6. *The debt-with-equity-share contract to deter information acquisition derived in Proposition 2.3 is qualitatively robust to private knowledge of κ , and the optimal menu contract to induce information acquisition has $q_\theta^I(\theta) = 0$ for $\theta \in (\theta^c, \theta^d)$, when κ is the agent's private information.*

Proof. As shown in Appendix A.2.4, (TT_I) and (TT_U) distort the contract to the uninformed agent \mathbb{C}^U from the second best \mathbb{C}^{SD} towards the same direction as does (DI) . The result predicted in Proposition 2.3 is re-enforced with asymmetric information on the cost of information acquisition. (TT_U) distort the contract to the informed agent \mathbb{C}^I from the second best \mathbb{C}^{SM} towards the same direction as does (II) , and as pointed out in Section 2.6, (TT_I) technically resembles a θ -dependent reservation payoff that generates countervailing incentives. The presence of (TT_U) re-enforce the result predicted in Proposition 2.1, and (TT_I) , along with monotonicity constraint, results in pooled output schedule in intermediate states of nature for the informed agent. \square

2.8 Conclusion

The main insight of this paper is the treatment of the two polar incentive problems as equilibrium responses via information management, and the optimal contract to implement the equilibrium incentive problem. Model-wise, this brings the two polar incentive problems under a unified framework. What's more, this fills the gap in the literature, in which abundant analysis is focused on how existing incentive problem affects equilibrium outcome, but little is said about how such incentive problem arise, and how the optimal contract responds respectively to its emergence.

The model presented in this paper is ready to be extended towards several directions that are left off. One drawback of the present model is that, given the assumed information acquiring technology, the two incentive problems are substitutes in equilibrium, which fails to explain the possible co-existence of the two incentive problems. Information acquiring effort that generates a noisy signal, which is communicated from the agent to the principal through a menu of contingent transfers, may be a more sophisticated way to model the interaction between information management and implementation of the incentive problems, yet at the expense of model complexity, as output options in the menu contract cannot be made singletons.

In addition, I only consider the agent to acquire information, implicitly assuming that it is impossible or infinitely costly for the principal to acquire information. Relaxing this assumption, one can incorporate into the model the principal's decision on whether to delegate information acquisition to the agent, or to acquire information by herself and communicate such information to the agent. This expands the support of endogenous incentive problem within the contractual relationship to include the possibility of an informed principal.

A static contractual relationship was assumed throughout the paper, and the tim-

ing of information acquisition is exogenously given. It would be interesting to extend the model to a dynamic contracting relationship, in which the timing of information acquisition is endogenously implemented, and the cost of information acquisition diminishes in time as partial information may be freely observed by the agent throughout the production process.

From an empirical standpoint, I suggested the importance of identifying the cost of information acquisition as well as the essential incentive problem(s) in empirical tests on agency theory. The incentive problem within the contractual relationship is an equilibrium response, and empirical research in which it is assumed exogenously may in some occasions fail to identify the true underlying incentive problem and thus generate bias conclusions. Specifically, in a scenario where the contractible variable depends on a stochastic and a choice variable, information on the former is acquirable at a cost, e.g. production, employment relationships, and investment banking, identification of the cost of information acquisition is more likely to play an important role in the analysis as it sheds light on the equilibrium incentive problem and the form of contract. For scenarios where it is impossible or very costly to manipulate the contractible variable, e.g. a trade contract after the object is produced, or situations where information on the stochastic state of nature is extremely costly or almost impossible to acquire, e.g. accident insurance, assuming the source of incentive problem from the outset may benefit the researcher for its simplicity.

Chapter 3

Information Revelation in the Property Right

Theory of the Firms

3.1 Introduction

I incorporate truthful revelation of asymmetric information through the allocation of ownership structure into the Property Right Theory of the firms, in which investment in a relationship-specific asset as well as trade between the buyer and the investor are not ex-ante contractible. The optimal ownership structure, in addition to its conventional role to mitigate the hold-up problem, also serve as an information transmission device.

Consider a seller investing in an asset that can be purchased and used by the buyer to produce a higher valued output. Neither the ex-ante investment and the ex-post surplus is contractible ex-ante. The players can only contract on the allocation of asset ownership and an upfront fixed payment, before investment and negotiation on terms of trade occur. If information is symmetric, traditional theories on ownership allocation of the asset or firm boundary are developed from two roots of the theories of the firms: the Property Right Theory and the Transaction Cost Economics¹. The former emphasize on how non-integration (investor-ownership) mitigates the holdup concern and stimulates ex-ante incentive to invest in the relationship-specific asset, assuming efficient ex-post bargaining of trade, whereas the latter focus on how integration

¹For the Property Right Theory, please refer to the pioneer works of Grossman and Hart (1986) and Hart and Moore (1990), or a survey by Segal and Whinston (2013) for more information. For the Transaction Cost Economics, please refer to the seminal works of Williamson (1975, 2002), or a survey by Tadelis and Williamson (2013) for more information. Also refer to Holmstrom and Roberts (1998) for a revisit and Gibbons (2005) for a neat comparison of the two and their extended theories.

(buyer-ownership) governs costly ex-post opportunism with various asset specificity.

Suppose that the marginal value of asset is the buyer's private information as she has better access to the output market. I recognize the allocation of asset ownership as an information transmission device in the incomplete contracting relationship with asymmetric information. To facilitate ex-post negotiation on the terms of trade, the allocation of asset ownership is designed as a menu of shared ownership allocation and upfront transfer such that the buyer truthfully reveals her information on the marginal value of asset. Without the ability to contract on the investment or the terms of trade, this truthful revelation is induced if the buyer is deterred from misreporting when trade occurs as well as when she turns down the trade. That is, there is an additional set of incentive compatibility constraints off the equilibrium path.

Non-integration, seller-investor being the sole owner of the asset predicted in the conventional Property Right Theory, is suboptimal for any marginal value of asset due to this incentive compatibility off the equilibrium path. Shared ownership (partnership) is optimal, with the share reflecting the tradeoff of mitigating hold-up and inducing information revelation. If the information rent effect is stronger than the hold-up effect in the sense that the marginal information rent (inverse hazard rate) relative to the marginal value of asset is diminishing in the marginal value of asset, it is optimal to have a lower level of integration (seller having a larger share) with a buyer who has a higher marginal value of asset, vice versa.

Optimality of shared ownership captures the share-holding behavior of the modern firm mergers and acquisitions. For instance, Facebook in 2014 bought Oculus VR, a virtual reality gaming company, for \$400 million in cash and 23.1 million shares of Facebook stock; MediaTek Inc., a semi-conductor company, acquired 69% stake in NuCORE Technology Inc. via share swapping in 2007. In this paper, I study one possible explanation for the optimality of shared ownership: the joint effect of information transmission and hold-up.²

The paper is organized as the following. The model is delivered in Section 3.2, and a benchmark with symmetric information at the trade negotiation stage is discussed in Section 3.3. I characterize the incentive compatibility constraints and the optimal truth revealing ownership allocation (level of integration) as a result of the joint effects of information rent and hold-up, with both binary and continuous marginal value of asset, in Section 3.4.1 and 3.4.2 respectively. Conclusion is made in Section 3.5.

²Aghion and Tirole (1994), Cao (2003), Dasgupta and Tao (2000), and Schmitz (2008, 2013) also recognize the optimality of joint/split asset ownership, with various explanations to that analyzed here, none of them on information revelation.

3.1.1 Related Literature

The current paper is built upon two lenses of literature. First, it contributes to the recent works in the theories of the firms with incomplete contracting and asymmetric information, e.g. Baldenius (2006), Malcomson (1997), Matouschek (2004), and Schmitz (2006, 2008). The main departure of the current paper to this lens of literature is on the role of allocating asset ownership to induce truthful revelation. They assume independency between ownership structure and information at the ex-ante investment stage as well as at the ex-post negotiation stage. The informed party never reveals his information that facilitates ex-post negotiation, i.e. ex-post bargaining is inefficient. Their focus is thus on the optimal ownership structure to motivate ex-ante investment and to reduce the probability of ex-post disagreement.

Truth revealing ownership or control is also studied in the recent literature of complete contract (Kuribko and Lewis (2010)), that of foreign direct investment (Raff, Ryan, and Stähler (2009), and Stähler (2005)), and that of financial contracting (Dessein (2005)). The difference between information revelation under an incomplete contracting framework as in the current paper and that under the above literature is on the contractibility of ex-post return or that of ex-ante action. With such contractibility in this lens of literature, participation can be induced by the contract, as well as incentive to take a certain action. Thus, it is sufficient to induce truthful information revelation within the relationship, and there is no concern of hold-up.

In comparison, I study the information transmitting role of asset ownership in an incomplete contracting environment. Efficient mitigation of hold-up is traded off to induce truthful information revelation. This tradeoff is attributed to incentive compatibility both on and off the equilibrium path.

The optimal ownership structure as an information transmission device analyzed in this paper can be regarded as a positive response to the argument by Holmström and Roberts (1998, p.91) that the theory of the firm has been “too narrowly focused on the hold-up problem and the role of asset specificity.” It can also be regarded as an endogenous solution to the observation by Riordan (1990) and Chou (2007) that the boundary of the firm is the boundary of information, as the boundary of the firm itself is an equilibrium result of truthful information revelation. Goldlücke and Schmitz (2014) also studies information transmission in an incomplete contracting environment. They focus on, however, informed seller’s signaling incentive through the observable investment, which mitigates the hold-up problem. Allocation of asset ownership is assumed exogenous. I characterize truth revealing allocation of shared ownership prior

to investment and ex-post negotiation, which eliminates information asymmetry in the bargaining stage at the expense of ex-ante investment due to inefficient mitigation of hold-up.

3.2 Model

A seller invest I in an asset (an interim output), which generates non-contractible but observable quality of asset $q(I) = I$, at a cost $c(I) = \frac{1}{2}I^2$. The asset can be used to produce a final output by the buyer's human capital, along with the seller's human capital to maintain the quality of the asset during production. Production of final output with the asset of quality I yields value $\mu(\beta, I) = \beta I$ within the relationship, or value $\nu(\theta, \beta, I) = \theta\beta I$ outside of the relationship. The marginal value of asset, $\beta \in \mathcal{B}$ and $\beta > 0$, follows the prior probability distribution $F(\beta)$. $F(\beta)$ is common knowledge. The buyer observes the realization of β , and there is a delay for the seller to observe β , specified at the end of this section. The buyer has information advantage due to his access to the final output market. The seller has full bargaining power. Both players are risk neutral.

Assumption 3.1. *The asset is relationship-specific, in the sense that $\theta \leq 1$.*

For instance, as an initial investor, the seller's human capital is more efficient than others in maintaining the quality of the asset, and the buyer's human capital is more efficient than others in turning the asset into the final output. θ is common knowledge, with a lower θ indicative of a more relationship-specific asset. Relationship-specificity implies that trade is efficient under perfect information.

At the first stage of the game, before investment and trade, the buyer has private information in the realization of marginal value of asset. The seller and the buyer can contract on the allocation of ownership and the upfront transfer payment. Instead of polar ownership structures, I assume that the ownership can be shared between the players, with the seller owning $s \in [0, 1]$ of the asset, and the buyer owning $1 - s$ of the asset, or that it can belong to no one, $s = \phi$. Smaller s implies a higher level of integration, with $s = 0$ corresponding to the case where the seller-investor is employed by the buyer, and $s = 1$ corresponding to the case where the seller-investor and the buyer are non-integrated players in the asset market. The share of ownership not only gives the share holder the right to claim a share of residual payoff when the relationship breaks, but also the power to veto usage of the asset by the other owner, with higher share of asset indicating a larger veto power. This shared ownership is assumed based

on the observation that modern firm acquisition is in the form of increasing the holding of the corporate stocks, and a larger share holder tends to have a larger power in the board of directors.³ The seller offers a menu of contract on ownership share and upfront transfer payment, $\{s(\beta), t(\beta)\}$, and the buyer accepts one of the options in the menu or reject all. If the buyer rejects the ownership contract, the asset ownership is undefined, in which case the reservation payoff of each player is normalized to zero. This is the only contract they can write ex-ante.

At the third stage, upon observing the quality of the asset, the players negotiate the share of the final output value. If negotiation is successful, trade occurs and the seller receives p while the buyer receives $\mu(\beta, I) - p$. In the case where trade negotiation fails and the relationship breaks, the seller collects $s(\beta)$ of the outside value, i.e. $s(\beta)\nu(\theta, \beta, I)$, and the buyer collects $1 - s(\beta)$ of the outside value, i.e. $(1 - s(\beta))\nu(\theta, \beta, I)$.

Some of the literature on joint ownership have a different assumption on the payoff the players earn when the relationship breaks: they earn zero payoff as each of the joint owners of the asset can veto the other owners from using the asset outside of the relationship. I do not follow this line of literature for two reasons. First, as a partial residual payoff claimant (e.g. a share holder), even if the asset is used outside of the relationship, the player still collects a share of the outside value. She then be at least weakly better off not to veto the use of the asset. Second, each player's veto power is increasing in her share of ownership. One can thus regard this share of outside value as the probability that the other player fails to veto the use of asset outside of the relationship. In this sense, the payoff each player receives outside of the relationship is her expected payoff before the use of asset is vetoed or is fail to be vetoed. Another interpretation for the share of ownership as share of outside value is in the spirit of Cao (2003). Suppose that there is an active asset market. On break of relationship, the players sell the asset in the asset market and share the revenue $\nu(\theta, \beta, I)$ according to the share of ownership.

I will investigate two scenarios regarding the delay of information to the seller. If the information on the realization of β is delayed until investment is made and before negotiation starts, e.g. the seller is able to learn the profitability of the asset from the investment activity, inducing information revelation at the stage of contracting is purely for investment. If the information on the realization of β is delayed until ex-post negotiation and trade are completed, e.g. the seller is able to learn the profitability of

³Other applications include and are not limited to partnerships in law and consulting firms, and joint child custody between the parents.

the asset from negotiating with the buyer, inducing information revelation at the stage of contracting is for both investment and negotiation. I characterize how and under what conditions can truthful information revelation be induced by shared ownership under each scenario.

3.3 Symmetric Information at Negotiation

As a benchmark, suppose that at the trade negotiation stage, β becomes common knowledge, yet the buyer's message $\hat{\beta}$ is already announced and the ownership structure $s(\hat{\beta})$ is contracted. Given ownership structure $s(\hat{\beta})$, and the level of investment I made in the previous stage, the seller makes a take-it-or-leave-it offer $p(\beta, s(\hat{\beta}), I)$ to be paid from the buyer. The asset being relationship-specific, it is ex-post efficient to trade if

$$p(\beta, s(\hat{\beta}), I) \in \arg \max_p p$$

subject to

$$\beta I - p \geq (1 - s(\hat{\beta}))\theta\beta I$$

That is, $p(\beta, s(\hat{\beta}), I) = (1 - \theta + s(\hat{\beta})\theta)\beta I$.

At the investment stage, anticipating trade negotiation, if the seller believes that the message $\hat{\beta}$ sent by the buyer is truthful, given the ownership structure $s(\hat{\beta})$, the seller invests

$$I(\hat{\beta}, s(\hat{\beta})) \in \arg \max_I p(\hat{\beta}, s(\hat{\beta}), I) - c(I)$$

$p(\hat{\beta}, s(\hat{\beta}), I)$ being linear in I and $c(I)$ satisfying Inada condition, $I(\hat{\beta}, s(\hat{\beta}))$ solves the first order condition $(1 - \theta + s(\hat{\beta})\theta)\hat{\beta} = c'(I)$. With quadratic cost of investment, $I(\hat{\beta}, s(\hat{\beta})) = (1 - \theta + s(\hat{\beta})\theta)\hat{\beta}$.

At the stage of contracting on ownership, the seller proposes a take-it-or-leave-it menu of ownership structures and upfront transfer payments, $\{s(\beta), t(\beta)\}$, such that the buyer is willing to accept the ownership allocation and truthfully reveal his information. $\{s(\beta), t(\beta)\}$ satisfies the following individual rationality constraint given truthful revelation.

$$\beta I(\beta, s(\beta)) - p(\beta, s(\beta), I(\beta, s(\beta))) - t(\beta) \geq 0 \quad \forall \beta \quad (IR^S)$$

For information is symmetric in the trade negotiation stage, the buyer anticipates that trade would occur regardless of his report. $\{s(\beta), t(\beta)\}$ is designed such that the

buyer does not reveal $\hat{\beta} \neq \beta$, anticipating trade ex-post.

$$\beta \in \arg \max_{\hat{\beta}} \beta I(\hat{\beta}, s(\hat{\beta})) - p(\beta, s(\hat{\beta}), I(\hat{\beta}, r(\hat{\beta}))) - t(\hat{\beta}) \quad \forall \hat{\beta} \neq \beta \quad (IC^S)$$

Proposition 3.1. *If information is symmetric at negotiation, the optimal allocation of ownership $s^S(\beta)$ has $s^S(\beta) = 1$ for all β , with the buyer voluntarily revealing information.*

Proof. Appendix A.3.1. □

In equilibrium, the parties share the surplus after investment is made. They thus have congruent interest with regards to the level of investment. Knowing that information would be symmetric in the trade negotiation stage, the buyer voluntarily reveal the true information to facilitate the seller's investment decision. Therefore, inducing information revelation does not distort the efficient allocation of ownership, non-integration. Conflict of interests arises only when information is asymmetric in the trade negotiation stage, where the buyer has incentive to opportunistically reveal incorrect information to extract a higher share of surplus.

3.4 Information Revelation to Facilitate Negotiation

If information is asymmetric at the trade negotiation stage, given the buyer's reported message $\hat{\beta}$, ownership structure $s(\hat{\beta})$, and level of investment I , the seller makes a take-it-or-leave-it offer $p(\hat{\beta}, s(\hat{\beta}), I)$ to be paid from the buyer. The asset being relationship-specific, it is ex-post efficient to trade if the buyer has truthfully revealed his private information. $p(\hat{\beta}, I)$ is then offered such that the buyer is willing to accept.

$$p(\hat{\beta}, s(\hat{\beta}), I) \in \arg \max_p$$

subject to

$$\hat{\beta}I - p \geq (1 - s(\hat{\beta}))\theta\hat{\beta}I$$

That is, $p(\hat{\beta}, s(\hat{\beta}), I) = (1 - \theta + s(\hat{\beta})\theta)\hat{\beta}I$.

At the investment stage, anticipating trade negotiation, given the buyer's reported message $\hat{\beta}$ and ownership structure $s(\hat{\beta})$, the seller invests

$$I(\hat{\beta}, s(\hat{\beta})) \in \arg \max_I p(\hat{\beta}, s(\hat{\beta}), I) - c(I)$$

$p(\hat{\beta}, s(\hat{\beta}), I)$ being linear in I and $c(I)$ satisfying Inada condition, $I(\hat{\beta}, s(\hat{\beta}))$ solves the first order condition $(1 - \theta + s(\hat{\beta})\theta)\hat{\beta} = c'(I)$. With quadratic cost of investment $c(I) = \frac{1}{2}I^2$, $I(\hat{\beta}, s(\hat{\beta})) = (1 - \theta + s(\hat{\beta})\theta)\hat{\beta}$.

At the stage of contracting on ownership, the seller proposes a take-it-or-leave-it menu of ownership structures and upfront transfer payments, $\{s(\beta), t(\beta)\}$, such that the buyer is willing to accept the ownership allocation and truthfully reveal his information. That is, $\{s(\beta), t(\beta)\}$ satisfies the following individual rationality constraint given truthful revelation.

$$\beta I(\beta, s(\beta)) - p(\beta, s(\beta), I(\beta, s(\beta))) - t(\beta) \geq 0 \quad \forall \beta \quad (IR^N)$$

The difference between contracting on shared ownership here and complete contracting on profit share is the commitment to trade. In the complete contracting framework, the contracting parties commit ex-ante to a sharing rule as well as to trade ex-post; thus, it is sufficient to induce truthful information revelation within the relationship. Under the current framework, asset ownership and its corresponding upfront payment are the only elements that can be contracted on ex-ante, and therefore, truthful information revelation must be induced within as well as outside of the relationship. That is, $\{s(\beta), t(\beta)\}$ is designed to deter the buyer from revealing $\hat{\beta} \neq \beta$ and accepting the seller's trade offer, as well as from revealing $\hat{\beta} \neq \beta$ and seeking an outside trading partner.⁴

$$\begin{aligned} & \beta I(\beta, s(\beta)) - p(\beta, s(\beta), I(\beta, s(\beta))) - t(\beta) \\ & \geq \beta I(\hat{\beta}, s(\hat{\beta})) - p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) - t(\hat{\beta}) \quad \forall \hat{\beta} \neq \beta \quad (IC'_I) \end{aligned}$$

⁴Deterring this sort of “off-schedule deviation” has drawn attention recently in the mechanism design theory. Athey and Segal (2013, 2475-2477) considers a general dynamic mechanism design problem when the decision rule is non-enforceable in the sense that each agent can freely choose a non-participation decision, resulting in a reservation payoff independent of the decision rule; efficiency can be achieved with this modification to the dynamic mechanism design problem. Compte and Jehiel (2009), on the other hand, draw an inefficiency result in a static trade mechanism with correlated types in which each player's outside payoff is his private information. In the former paper, the role of the optimal ownership structure to mitigate the hold-up problem is absent, and in the latter paper, the hold-up problem is not studied. That is, in both papers, information revelation is the only incentive problem that the contract is designed to solve. My focus here is to see the tradeoff of investment incentive due to inefficient mitigation of hold-up when inducing truthful information revelation.

and

$$\begin{aligned} \beta I(\beta, s(\beta)) - p(\beta, s(\beta), I(\beta, s(\beta))) - t(\beta) \\ \geq (1 - s(\hat{\beta}))\theta\beta I(\hat{\beta}, s(\hat{\beta})) - t(\hat{\beta}) \quad \forall \hat{\beta} \neq \beta \quad (IC'_O) \end{aligned}$$

With interim individually rational terms of trade, (IC'_I) and (IC'_O) can be expressed by

$$\beta \in \arg \max_{\hat{\beta}} \beta I(\hat{\beta}, s(\hat{\beta})) - p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) - t(\hat{\beta}) \quad (IC_I)$$

and

$$\beta \in \arg \max_{\hat{\beta}} (1 - s(\hat{\beta}))\theta\beta I(\hat{\beta}, s(\hat{\beta})) - t(\hat{\beta}) \quad (IC_O)$$

3.4.1 Binary Marginal Value of Asset

Starting with binary marginal value of asset, $\mathcal{B} = \{\underline{\beta}, \bar{\beta}\}$, $\underline{\beta} < \bar{\beta}$, $(f(\bar{\beta}), f(\underline{\beta})) = (\sigma, 1 - \sigma)$, and $\mathcal{C} = \{(\bar{s}, \bar{t}), (\underline{s}, \underline{t})\}$, the incentive compatibility constraints are expressed as

$$\bar{u} = \bar{\beta} I(\bar{\beta}, \bar{s}) - p(\bar{\beta}, \bar{s}, I(\bar{\beta}, \bar{s})) - \bar{t} \geq \bar{\beta} I(\underline{\beta}, \underline{s}) - p(\underline{\beta}, \underline{s}, I(\underline{\beta}, \underline{s})) - \underline{t} \quad (IC_I^H)$$

$$\underline{u} = \underline{\beta} I(\underline{\beta}, \underline{s}) - p(\underline{\beta}, \underline{s}, I(\underline{\beta}, \underline{s})) - \underline{t} \geq \underline{\beta} I(\bar{\beta}, \bar{s}) - p(\bar{\beta}, \bar{s}, I(\bar{\beta}, \bar{s})) - \bar{t} \quad (IC_I^L)$$

$$\bar{u} = \bar{\beta} I(\bar{\beta}, \bar{s}) - p(\bar{\beta}, \bar{s}, I(\bar{\beta}, \bar{s})) - \bar{t} \geq \theta \bar{\beta} (1 - \underline{s}) I(\underline{\beta}, \underline{s}) - \underline{t} \quad (IC_O^H)$$

$$\underline{u} = \underline{\beta} I(\underline{\beta}, \underline{s}) - p(\underline{\beta}, \underline{s}, I(\underline{\beta}, \underline{s})) - \underline{t} \geq \theta \underline{\beta} (1 - \bar{s}) I(\bar{\beta}, \bar{s}) - \bar{t} \quad (IC_O^L)$$

Or equivalently,

$$\bar{u} - \underline{u} \geq (\bar{\beta} - \underline{\beta}) I(\underline{\beta}, \underline{s}) \quad (IC_I^H)$$

$$\bar{u} - \underline{u} \leq (\bar{\beta} - \underline{\beta}) I(\bar{\beta}, \bar{s}) \quad (IC_I^L)$$

$$\bar{u} - \underline{u} \geq (\bar{\beta} - \underline{\beta}) \theta (1 - \underline{s}) I(\underline{\beta}, \underline{s}) \quad (IC_O^H)$$

$$\bar{u} - \underline{u} \leq (\bar{\beta} - \underline{\beta}) \theta (1 - \bar{s}) I(\bar{\beta}, \bar{s}) \quad (IC_O^L)$$

As shown by Lemma 3.1 below, (IC_I^H) and (IC_O^L) are the relevant binding incentive compatibility constraints, and it is without loss of generality to neglect the sufficient monotonicity constraints for incentive compatibility with binary marginal value of asset.

Lemma 3.1. *Truthful information revelation is induced if*

$$(\bar{\beta} - \underline{\beta})I(\underline{\beta}, \underline{s}) \leq \bar{u} - \underline{u} \leq (\bar{\beta} - \underline{\beta})\theta(1 - \bar{s})I(\bar{\beta}, \bar{s}) \quad (IC^B)$$

The sufficient monotonicity constraints for incentive compatibility

$$I(\underline{\beta}, \underline{s}) \leq I(\bar{\beta}, \bar{s}) \quad (M_I^B)$$

and

$$(1 - \underline{s})I(\underline{\beta}, \underline{s}) \leq (1 - \bar{s})I(\bar{\beta}, \bar{s}) \quad (M_O^B)$$

automatically holds given (IC^B) .

Proof. Appendix A.3.2. □

Non-integration with the efficient buyer is not incentive compatible as the inefficient buyer would have incentive to report upward and deviate from trade ex-post, to extract the rent and hold up the seller, i.e. $\bar{s} = 1$ violates (IC_O^L) , summarized and proven in Lemma 3.2. Incentive compatibility off the equilibrium path results in this distortion at the top, which differs from the prediction in the traditional complete contracting literature.

Lemma 3.2. $\bar{s} = 1$ *is not incentive compatible.*

Proof. If $\bar{s} = 1$, for any $0 \leq \underline{s} \leq 1$, $(\bar{\beta} - \underline{\beta})I(\underline{\beta}, \underline{s}) > (\bar{\beta} - \underline{\beta})\theta(1 - \bar{s})I(\bar{\beta}, \bar{s}) = 0$. The set of $\bar{u} - \underline{u}$ satisfying (IC^B) is empty. □

In addition, there is no incentive compatible allocation of ownership if the following assumption is violated such that the set of $\bar{u} - \underline{u}$ satisfying (IC^B) is empty.

Assumption 3.2. $\sigma\bar{\beta} < \underline{\beta} \leq \theta\bar{\beta}$.

Assumption 3.2 places condition on the value of inducing truthful information revelation relative to asset specificity such that an incentive compatible allocation of ownership exists. By rearrangement, it is equivalent to $\frac{\sigma}{1-\sigma} < \frac{\underline{\beta}}{\bar{\beta}-\underline{\beta}} \leq \frac{\theta}{1-\theta}$. That is, an incentive compatible ownership structure exists if the marginal information rent is relatively small (the first inequality) and the asset is not too relationship-specific (the second inequality).

With binding (IC^B) and $\underline{u} = 0$, the seller's reduced optimization problem at the ownership allocation stage is then

$$\max_{\bar{s}, \underline{s}} \sigma (\bar{\beta}I(\bar{\beta}, \bar{s}) - (\bar{\beta} - \underline{\beta})I(\underline{\beta}, \underline{s}) - c(I(\bar{\beta}, \bar{s}))) + (1 - \sigma) (\underline{\beta}I(\underline{\beta}, \underline{s}) - c(I(\underline{\beta}, \underline{s})))$$

subject to

$$I(\underline{\beta}, \underline{s}) = \theta(1 - \bar{s})I(\bar{\beta}, \bar{s})$$

Proposition 3.2. *If Assumption 3.2 holds, there exist a truth revealing contract on ownership structure such that $1 > \bar{s}, \underline{s} \geq 0$, and thus $I(\beta, s(\beta)) < I(\beta, s^S(\beta))$. $\bar{s} \geq \underline{s}$ if $\frac{\sigma}{1-\sigma} \geq \frac{\beta}{\bar{\beta}}$.*

Proof. Appendix A.3.3. □

With the concern of truthful information revelation to facilitate ex-post bargaining efficiency, non-integration predicted in the conventional Property Right Theory of the firms no longer holds. Equilibrium level of investment is traded off as the hold-up concern is not efficiently mitigated to induce truthful information revelation. If the marginal information rent ($\frac{\sigma}{1-\sigma}$) is larger than the relative marginal value of asset ($\frac{\beta}{\bar{\beta}}$), the optimal contract has $\underline{s} \leq \bar{s}$ to restrict the information rent given to the buyer with a higher marginal value of asset, at the expense of investment efficiency with the buyer having a lower marginal value of asset. That is, if the information rent effect is stronger than the hold-up effect, it is optimal to have a higher level of integration with the less efficient buyer than that with the more efficient buyer. This provides an explanation and characterization for joint ownership and partnerships to be optimal under the framework of Property Right Theory of the firm with asymmetric information.

3.4.2 A Continuum of Marginal Values of Asset

Turning to the general case with a continuum of marginal values of asset, $\mathcal{B} = [\underline{\beta}, \bar{\beta}]$, it is necessary that the seller has an additional instrument to induce truthful revelation off the equilibrium path, suggested by the following proposition.

Proposition 3.3. *With a continuum of marginal values of asset, truthful revelation requires the parties' ability to commit to a menu of penalty $\Delta(\beta)$ imposed on the buyer when the seller makes an individually rational offer given $\hat{\beta}$ yet the buyer deviates from trade. Otherwise, there is no truth revealing contract on ownership structure.*

Proof. Appendix A.3.4. □

With truthful revelation, an interim individually rational take-it-or-leave-it offer would be accepted. Thus, an individually rational offer based on the buyer's reported message is rejected only when the buyer lied about the asset value. If the parties are

able to commit to a menu of penalty⁵ in this scenario, information revelation can be induced by the menu contract $\mathbb{C} = \{s(\beta), t(\beta), \Delta(\beta)\}$ on allocation of asset ownership. Otherwise, any contract inducing truthful revelation within the relationship must leave the buyer with incentive to report upward and turn down the ex-post trade offer, and any contract inducing truthful revelation when the relationship breaks must leave the buyer with incentive to report downward and accept the ex-post trade offer. This is due to the fact that the buyer's revelation strategy to turn down the trade offer allows him to extract a larger share of the "general" value of asset, whereas his revelation strategy to accept the trade offer allows him to extract a larger share of not only the general value but also the "relationship-specific" value of asset.⁶

Thus, with the buyer having an additional mean to report untruthful message, it is necessary for the seller to have an additional instrument to induce truthful revelation. Truthful revelation can be achieved up to a certain level of incomplete contracting environment, which is less complete than when the terms of trade can be contracted upon ex-ante, and more complete than the environment analyzed in the Property Right Theory of the firms. In addition, in the truth-revealing equilibrium, $\Delta(\beta)$ is only realized off the equilibrium path, and thus does not affect the equilibrium level of investment.

We now turn to the characterization of the optimal truth revealing ownership structure, provided that $\Delta(\beta)$ can be committed. Truth revealing $\mathbb{C} = \{s(\beta), t(\beta), \Delta(\beta)\}$ satisfies incentive compatibility within the relationship and that when the relationship ends, i.e.

$$\beta \in \arg \max_{\hat{\beta}} \beta I(\hat{\beta}, r(\hat{\beta})) - p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) - t(\hat{\beta}) \quad \forall \hat{\beta} \neq \beta \quad (IC_I^N)$$

$$\beta \in \arg \max_{\hat{\beta}} (1 - s(\hat{\beta}))\theta \beta I(\hat{\beta}, s(\hat{\beta})) - t(\hat{\beta}) - \Delta(\hat{\beta}) \quad \forall \hat{\beta} \neq \beta \quad (IC_O^N)$$

⁵This penalty is verifiable ex-post if whether the terms of trade is individually rational given reported message, and whether the trade takes place are both verifiable ex-post. In an employment and partnership discussion, this penalty is in the form of golden parachute, a compensation to the employee (seller) when employment is terminated.

⁶Commitment to $\Delta(\beta)$ is not required for incentive compatibility with binary types as, for each type, there is by construction at most one direction to lie.

Lemma 3.3. (IC_I^N) can be replaced by local incentive compatibility constraint

$$\beta \frac{dI(\hat{\beta}, s(\hat{\beta}))}{d\hat{\beta}} - \frac{dp(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta})))}{d\hat{\beta}} - \frac{dt(\hat{\beta})}{d\hat{\beta}} \Big|_{\hat{\beta}=\beta} = 0 \quad \forall \beta \quad (LIC_I)$$

and monotonicity constraint

$$\frac{dI(\hat{\beta}, s(\hat{\beta}))}{d\hat{\beta}} \Big|_{\hat{\beta}=\beta} > 0 \quad \forall \beta \quad (M_I)$$

Proof. Appendix A.3.5. □

Lemma 3.4. (IC_O^N) can be replaced by local incentive compatibility constraint

$$\theta \beta \frac{d \left[(1 - s(\hat{\beta})) I(\hat{\beta}, s(\hat{\beta})) \right]}{d\hat{\beta}} - \frac{dt(\hat{\beta})}{d\hat{\beta}} - \frac{d\Delta(\hat{\beta})}{d\hat{\beta}} \Big|_{\hat{\beta}=\beta} = 0 \quad \forall \beta \quad (LIC_O)$$

and monotonicity constraint

$$\frac{d \left[(1 - s(\hat{\beta})) I(\hat{\beta}, s(\hat{\beta})) \right]}{d\hat{\beta}} \Big|_{\hat{\beta}=\beta} > 0 \quad \forall \beta \quad (M_O)$$

Proof. Appendix A.3.5. □

The seller's optimization problem in the ownership allocation stage is then

$$\max_{s(\beta), t(\beta), \Delta(\beta)} \int_{\beta} (p(\beta, s(\beta), I(\beta, s(\beta))) - c(I(\beta, s(\beta))) + t(\beta)) dF(\beta)$$

subject to (IR^N) , (LIC_I) , (LIC_O) , (M_I) , and (M_O) .

Condition 1. $\frac{d\left(\frac{1-F(\beta)}{f(\beta)}\right)}{d\beta} \leq 1$.

Condition 2. $\frac{1-F(\beta)}{f(\beta)} \left(1 - \frac{1-F(\beta)}{f(\beta)} \frac{1}{\beta}\right)$ is non-decreasing in β .

The above conditions characterize the marginal information rent $\left(\frac{1-F(\beta)}{f(\beta)}\right)$, the inverse of the hazard rate) and the marginal information rent relative to the marginal value of asset $\left(\frac{1-F(\beta)}{f(\beta)} \frac{1}{\beta}\right)$ such that both monotonicity constraints are strictly satisfied in equilibrium, as summarized in the following proposition.

Proposition 3.4. *If $\Delta(\beta)$ can be committed, in equilibrium,*

1. If Condition 1 and 2 hold, (M_I) and (M_O) are strictly satisfied
2. If Condition 1 holds and Condition 2 is violated, (M_I) is strictly satisfied, and $(1 - s(\beta))I(\beta, s(\beta)) = \gamma_1$, where $\gamma_1 > 0$ is constant of β .
3. If only Condition 2 holds for some $\beta \in [\beta_1, \beta_2]$, and $\theta \geq \frac{\beta_2 - \beta_1}{\beta_2}$, (M_O) is strictly satisfied, and $I(\beta, s(\beta)) = \beta_1$ for $\beta \in [\beta_1, \beta_2]$.

Proof. Appendix A.3.6. □

If the marginal information rent is not increasing too fast, i.e. the buyer's rent is not too convex in the equilibrium level of investment, the truth revealing ownership structure implements an equilibrium investment $(I(\beta, s(\beta)))$ that is increasing in the marginal value of asset. Otherwise, to induce truthful revelation within the relationship, the ownership structure is designed such that the level of equilibrium investment is independent of the marginal value of asset. If the marginal information rent relative to the marginal value of asset diminishes sufficiently faster or increases sufficiently slower than the marginal information rent, the truth revealing ownership structure implements an equilibrium share of investment to the buyer $((1 - s(\beta))I(\beta, s(\beta)))$ that is increasing in the marginal value of asset. Otherwise, to induce truthful revelation in case of break of relationship, the ownership structure is designed such that the equilibrium share of investment to the buyer is independent of the marginal value of asset.

Condition 3. $\frac{1-F(\beta)}{f(\beta)} \frac{1}{\beta}$ is non-increasing in β .

Condition 3 characterizes the relative marginal information rent to the marginal value of asset such that the optimal truth revealing ownership structure is weakly increasing in the marginal value of asset, as summarized in the following proposition.

Proposition 3.5. *If $\Delta(\beta)$ can be committed, the equilibrium ownership structure $\{s(\beta), t(\beta), \Delta(\beta)\}$ and corresponding level of investment $I(\beta, s(\beta))$ has the following property*

1. If Condition 1 and 2 hold, $s(\beta) < 1$ and $I(\beta, s(\beta)) < I(\beta, s^S(\beta))$. $s(\beta)$ is (weakly) increasing in β if Condition 3 holds, diminishing otherwise.
2. If Condition 1 holds and Condition 2 is violated, $s(\beta) < 1$ is increasing in β , and $I(\beta, s(\beta)) < I(\beta, s^S(\beta))$.

3. If only Condition 2 holds for some $\beta \in [\beta_1, \beta_2]$, and $\theta \geq \frac{\beta_2 - \beta_1}{\beta_2}$, $s(\beta) \leq 1$ and is decreasing in β , with equality at $\beta = \beta_1$; $I(\beta, s(\beta)) = \beta_1 \leq I(\beta, s^S(\beta))$ with equality at β_1 .⁷

Proof. Appendix A.3.6. □

Given Condition 1, there are several observations we are able to make from Proposition 3.5.

The optimal ownership structure and the level of integration reflects the tradeoff of efficient mitigation of hold-up for truthful information revelation. If the marginal information rent relative to the marginal value of asset is decreasing (Condition 3 holds), the incentive cost of not efficiently mitigate hold-up increases relatively fast to the effect of information rent when the asset generates a larger marginal value. The optimal level of integration is thus lower when the marginal value of asset is higher. Otherwise, the effect of information rent increases relatively fast to the hold-up effect when the asset generates a larger marginal value, and the optimal level of integration is higher when the marginal value of asset is higher.

Nevertheless, this does not imply non-integration under the highest marginal value of asset, near which Condition 2 is violated. If the ownership structure had non-integration only at the highest marginal value of asset, the buyer with a slightly lower marginal value of asset than the maximum would have had incentive to report upward at the stage of ownership allocation and reject the take-it-or-leave-it offer at the stage of trade to walk off with a higher rent. Distortion at the top is a result of incentive compatibility off the equilibrium path. On the other hand, if the ownership structure had non-integration for at least some realization of the marginal value of asset as with perfect information, the information rent given to the buyer to induce truthful revelation would be too high to be optimal.

Thus, non-integration conventionally predicted in the Property Right Theory of the firm no longer holds with asymmetric information, even with the presence of ex-post efficient negotiation.⁸ The optimal ownership structure to induce truthful revelation within as well as outside of the relationship has at least some degree of integration/partnership, $s(\beta) < 1$. This is at the expense of ex-ante investment incentive,

⁷If $\frac{d(\frac{1-F(\beta)}{f(\beta)})}{d\beta} > 1$ and $\theta < \frac{\beta_2 - \beta_1}{\beta_2}$, there is no truth revealing ownership structure.

⁸Matouschek (2004) and Schmitz (2008) explained optimal joint asset ownership with asymmetric information by the presence of ex-post inefficient bargaining, without truthful information revelation ex-ante.

$I(\beta, s(\beta)) < I(\beta, s^S(\beta))$, as the hold-up concern is not efficiently mitigated to induce truthful information revelation.⁹

3.4.2.1 Asset Specificity and Ownership

Considering the information transmission role of the optimal shared ownership does not reject the prediction of the Transaction Cost Economics that more relationship-specific is the asset, higher level of integration is optimal, as summarized in Corollary 1. More relationship-specific the asset is (lower θ), for any realization of the marginal value of asset, less valuable is mitigating the hold-up problem with a lower level of integration, relative to restricting the information rent with a higher level of integration.

Corollary 3.1. *The optimal ownership structure $s(\beta)$ is increasing in θ .*

Proof. Appendix A.3.7. □

3.4.2.2 Application: Length of Project and Firm Boundary

The marginal value of asset, β , can also be interpreted as the discount factor in a dynamic production process of the final output. Suppose that it takes a time sequence of efforts for the buyer to finalize the output with the asset and yield value I . The marginal value of asset in the previous section is denoted as $\beta(t)$ instead, with t being the length of time sequence of efforts that is the buyer's private information. $\beta(t) \in (0, 1)$ then discount the value of final output back to its present value prior to the negotiation stage.

$\beta(t)$ measuring the discounted value of final product which takes t sequence of efforts for the buyer to produce, $\beta'(t) < 0$. A higher marginal value of asset corresponds to a shorter sequence of efforts required for production of final output. The model then predicts a testable implication that it is optimal to have a lower level of integration with a buyer who takes a shorter time sequence of efforts to finalize the output, if the marginal information rent relative to the discount rate is increasing in t . In other words, a longer-term project is optimally accompanied with a higher level of integration.

Corollary 3.2. *$s(\beta(t))$ is diminishing in t if $\frac{1-F(\beta(t))}{f(\beta(t))} \frac{1}{\beta(t)}$ is increasing in t .*

⁹Goldlücke and Schmitz (2014) predicts an opposite trade-off. When the seller, who has private information on his outside option, has incentive to signal a higher outside option through higher investment, the equilibrium would exhibit higher level of investment, at the expense of ex-post inefficiencies when the buyer mistaken a true signal as a bluff.

On the contrary, if the final output is more valuable in the future ($\beta'(t) > 0$), the longer time sequence it takes for the buyer to finalize the output, the lower level of integration is optimal. A longer-term project is accompanied with a lower level of integration. The buyer extracts a higher information rent and the hold-up problem is more efficiently mitigated from having a greater total value of time or a smaller total cost of time.

3.5 Conclusion

Allocation of asset ownership or the level of integration as an instrument to mitigate hold-up and incentivize non-contractible investment is well recognized in the Property Right Theory of the firm with incomplete contract. This paper is complementary to the literature by studying the information transmission role of the asset ownership. The equilibrium level of integration itself is a result of truthful information revelation and mitigation of hold-up. Non-integration is not incentive compatible under the incomplete contracting environment, for the privately informed buyer would have incentive to lie and seek outside opportunities. Some level of integration is optimal, and whether it is decreasing or increasing in the marginal value of asset depends on whether the the information rent effect relative to the hold-up effect is decreasing or increasing.

In the literature of the Property Right Theory of the firms with asymmetric information, ex-post negotiation is inefficient due to the presence of asymmetric information at the negotiation stage. In this paper, ex-post negotiation is efficient as information is truthfully revealed ex-ante through the allocation of ownership. The next research question is, under what circumstances would it be optimal to truthfully reveal information via ownership allocation, under what conditions would it be optimal to leave information asymmetric, and in what situation would it be optimal to reveal some certain information through shared ownership, leaving the rest asymmetric.

Appendix A

Proof of Propositions

A.1 Proof of Propositions in Chapter 1

A.1.1 Proof of Lemma 2.1

$$q_i^d = \begin{cases} Pr(x_{it}(a'_i)|x_{jt}(a_j^0) < a_{it}^0 + \sigma(x_{jt} - a_{jt}^0) - s\sqrt{1-\sigma^2}) = \Phi\left(\frac{a_i^0 - a'_i}{\sqrt{1-\sigma^2}} - s\right) & \text{for } a'_i < a_i^0 \\ Pr(x_{it}(a'_i)|x_{jt}(a_j^0) > a_{it}^0 + \sigma(x_{jt} - a_{jt}^0) + s\sqrt{1-\sigma^2}) = 1 - \Phi\left(\frac{a_i^0 - a'_i}{\sqrt{1-\sigma^2}} + s\right) & \text{for } a'_i > a_i^0 \end{cases},$$

where $\Phi(\bullet)$ is the standard normal CDF. $\frac{\partial q_i^d}{\partial \sigma} = \phi(\bullet) \frac{|a_i^0 - a'_i|}{\sqrt{(1-\sigma^2)^3} \sigma} > 0$, where $\phi(\bullet)$ is the standard normal PDF.

$$q_i^n = \begin{cases} Pr(x_{it}(a_i^0)|x_{jt}(a_j^0) < a_{it}^0 + \sigma(x_{jt} - a_{jt}^0) - s\sqrt{1-\sigma^2}) = \Phi(-s) & \text{for } a'_i < a_i^0 \\ Pr(x_{it}(a_i^0)|x_{jt}(a_j^0) > a_{it}^0 + \sigma(x_{jt} - a_{jt}^0) + s\sqrt{1-\sigma^2}) = 1 - \Phi(s) & \text{for } a'_i > a_i^0 \end{cases}$$

$$\frac{\partial q_i^n}{\partial \sigma} = 0.$$

□

A.1.2 Proof of Proposition 1.0

On non-cooperative benchmark:

$$\mathcal{P}_n : \quad \min_{\alpha, \beta} R(\alpha, \beta) = \frac{r}{2}(\alpha^2 + \beta^2 + 2\sigma\alpha\beta)$$

$$s.t. (IC_n)$$

(IC_n) can be reduced to $\alpha \geq \frac{c_H - c_L}{a_H - a_L}$, i.e. α is decided to provide incentive, and β is decided to minimize the risk premium. The principal's problem is then reduced to

$\min_{\beta} \beta^2 + 2\sigma \frac{c_H - c_L}{a_H - a_L} \beta$, by the first order condition of this reduced problem, $\beta_n = \sigma \frac{c_H - c_L}{a_H - a_L}$.

On coordinated benchmark

$$\mathcal{P}_c : \quad \min_{\alpha, \beta} R(\alpha, \beta) = \frac{r}{2}(\alpha^2 + \beta^2 + 2\sigma\alpha\beta)$$

$$s.t.(IC_c)$$

(IC_c) can be reduced to $\alpha + \beta \geq \frac{c_H - c_L}{a_H - a_L}$, i.e. both α and β play a role in incentive provision. The principal's problem is then reduced to $\min_{\beta} \left(\frac{c_H - c_L}{a_H - a_L} \right)^2 - 2(1 - \sigma) \left(\frac{c_H - c_L}{a_H - a_L} - \beta \right) \beta$, by the first order condition of this reduced problem, $\beta_c = \frac{1}{2} \frac{c_H - c_L}{a_H - a_L}$.

□

A.1.3 Proof of Proposition 1.1

$$\mathcal{P}_{CP} : \quad \min_{\alpha, \beta} R(\alpha, \beta) = \frac{r}{2}(\alpha^2 + \beta^2 + 2\sigma\alpha\beta)$$

$$s.t.(IC_n), (CP)$$

$0 \leq h(\delta, \Delta) \equiv \frac{\delta\Delta}{1-\delta+\delta\Delta} \leq 1$. If $\beta \geq 0$, $\alpha + h(\delta, \Delta)\beta \geq \alpha \geq \frac{c_H - c_L}{a_H - a_L}$, (IC_n) is binding while (CP) slacks except at $\beta = 0$. With binding (IC_n) and ignoring (CP_n) and $\beta \geq 0$, the problem is that of the non-cooperative benchmark, in which the optimal $\beta_n < 0$. Thus, among all $\beta \geq 0$, $\beta_{CP} = 0$ is optimal to deter collusion, hence, part i). If $\beta < 0$, $\alpha + h(\delta, \Delta)\beta \geq \frac{c_H - c_L}{a_H - a_L}$, (CP) is the only binding constraint: $\alpha + h(\delta, \Delta)\beta = \frac{c_H - c_L}{a_H - a_L}$. Solving the first order condition, $\left(h(\delta, \Delta)\beta - \frac{c_H - c_L}{a_H - a_L} \right) h(\delta, \Delta) + \beta + \sigma \left(\frac{c_H - c_L}{a_H - a_L} - 2h(\delta, \Delta)\beta \right) = 0$, $\beta_{CP} = \frac{h(\delta, \Delta) - \sigma}{1 + h(\delta, \Delta)^2 - 2\sigma h(\delta, \Delta)} \frac{c_H - c_L}{a_H - a_L}$ if $h(\delta, \Delta) < \sigma$, and $\beta_{CP} = 0$ if $h(\delta, \Delta) \geq \sigma$. Rearranging $h(\delta, \Delta) < \sigma$ yields $\delta < \delta_{CP} \equiv \frac{\sigma}{(1-\sigma)\Delta + \sigma}$.

□

A.1.4 Proof of Proposition 1.2

$$\mathcal{P}_{CI} : \quad \min_{\alpha, \beta} R(\alpha, \beta) = (\alpha + \beta)(a_H - a_L) + \frac{r}{2}(\alpha^2 + \beta^2 + 2\sigma\alpha\beta)$$

$$s.t.(IC_c), (CI)$$

$0 \leq h(\delta, \Delta) \equiv \frac{\delta\Delta}{1-\delta+\delta\Delta} \leq 1$. If $\beta \leq 0$, $\alpha + h(\delta, \Delta)\beta \geq \alpha + \beta \geq \frac{c_H - c_L}{a_H - a_L}$, (IC_c) is binding while (CI) slacks except at $\beta = 0$. With binding (IC_c) and ignoring (CI)

and $\beta \leq 0$, the problem is that of the coordinated benchmark, in which the optimal $\beta_c > 0$. Thus, among all $\beta \leq 0$, $\beta_{CI} = 0$ is optimal to induce coordination, hence, part i). If $\beta > 0$, $\alpha + \beta > \alpha + h(\delta, \Delta)\beta \geq \frac{c_H - c_L}{a_H - a_L}$, (CI) is the only binding constraint: $\alpha + h(\delta, \Delta)\beta = \frac{c_H - c_L}{a_H - a_L}$. Solving the first order condition, $(a_H - a_L)(1 - h(\delta, \Delta)) + r \left(\left(h(\delta, \Delta)\beta - \frac{c_H - c_L}{a_H - a_L} \right) h(\delta, \Delta) + \beta + \sigma \left(\frac{c_H - c_L}{a_H - a_L} - 2h(\delta, \Delta)\beta \right) \right) = 0$, $\beta_{CI} = \frac{h(\delta, \Delta) - \sigma}{1 + h(\delta, \Delta)^2 - 2\sigma h(\delta, \Delta)} \times \frac{c_H - c_L}{a_H - a_L} - \frac{(1 - h(\delta, \Delta))(a_H - a_L)}{r(1 + h(\delta, \Delta)^2 - 2\sigma h(\delta, \Delta))}$ if $h(\delta, \Delta) - \sigma > \frac{(1 - h(\delta, \Delta))(a_H - a_L)^2}{r(c_H - c_L)}$, and $\beta_{CP} = 0$ otherwise. Rearranging the inequality yields $\delta > \delta_{CI} \equiv \frac{r\sigma \times \frac{c_H - c_L}{a_H - a_L} + (a_H - a_L)}{r((1 - \sigma)\Delta + \sigma) \times \frac{c_H - c_L}{a_H - a_L} + (a_H - a_L)}$. As $\frac{\sigma}{(1 - \sigma)\Delta + \sigma} < 1$, $\delta_{CI} > \delta_{CP}$.

□

A.1.5 Proof of Proposition 1.3

From Proposition 1.1 and 1.2, $\delta_{CP} \equiv \frac{\sigma}{(1 - \sigma)\Delta + \sigma}$ and $\delta_{CI} \equiv \frac{r\sigma \times \frac{c_H - c_L}{a_H - a_L} + (a_H - a_L)}{r((1 - \sigma)\Delta + \sigma) \times \frac{c_H - c_L}{a_H - a_L} + (a_H - a_L)}$. As $\frac{\sigma}{(1 - \sigma)\Delta + \sigma} < 1$, $\delta_{CI} > \delta_{CP}$. For $\delta < \delta_{CP}$, the optimal collusion-proof RPE is preferred to an IPR and the optimal coordination-inducing contract has IPR. Collusion-proof RPE is thus preferred to coordination-inducing JPE for $\delta < \delta_{CP}$. For $\delta > \delta_{CI}$, the optimal coordination inducing JPE is preferred to an IPR and the optimal collusion-proof contract has IPR. Coordination-inducing JPE is thus preferred to collusion-proof RPE for $\delta > \delta_{CI}$. For $\delta_{CI} \geq \delta \geq \delta_{CP}$, coordination is induced with IPR and collusion is deterred with IPR; IPR is optimal.

□

A.1.6 Proof of Proposition 1.4

It is easy to verify that $\delta \gtrless \delta_{CI}$ is merely a rearrangement of $h(\delta, \Delta) \gtrless \varphi(\sigma)$. Thus, Proposition 1.4 is equivalent to saying that there exists parameters s , and $a_H - a_L$ such that δ_{CI} is diminishing in σ for $\sigma_0 < \sigma < \sigma^*$, and is increasing in σ for $\sigma_1 > \sigma > \sigma^*$, where $(\sigma_0, \sigma_1) \subset [0, 1]$. $\frac{d\delta_{CI}}{d\sigma} \gtrless 0$ if $\Omega(\sigma) \equiv \frac{c_H - c_L}{a_H - a_L} (1 - \delta_{CI} (1 - \Delta + (1 - \sigma)\Delta_\sigma)) \gtrless 0$. For $\sigma \rightarrow 1$, $\Delta \rightarrow 1$ if s is sufficiently large, so $\Omega(\sigma) > 0$ for sufficiently correlated measurement noise. $\Omega(\sigma) < 0$ if $1 - \Delta + (1 - \sigma)\Delta_\sigma$ is sufficiently large. Thus, if parameters $a_H - a_L$ and s is such that Δ increases sufficiently fast in σ for some $\sigma \in (\sigma_0, \sigma^*)$, $h(\delta, \Delta) > \varphi(\sigma)$ for $\delta \in (\delta_{CI}(\sigma_0), \delta_{CI}(\sigma^*))$. Examples of such parameters are easily found, Figure 1.1 as one of which.

□

A.2 Proof of Propositions in Chapter 2

A.2.1 Proof of Proposition 2.1

$$\mathcal{P}_{II} : \max_{t(\theta), q(\theta)} E(u^P(\theta)) = \int_0^{\bar{\theta}} q(\theta) - t(\theta) dF(\theta)$$

subject to

$$t(0) - \hat{c}(q(0), 0) \geq 0 \quad (IR_0)$$

$$t_\theta(\theta) - \hat{c}_q(q(\theta), \theta)q_\theta(\theta) = 0 \quad \forall \theta \in [0, \bar{\theta}] \quad (LIC_\theta)$$

$$q_\theta(\theta) \geq 0 \quad (M)$$

$$\int_0^{\bar{\theta}} t(\theta) - \hat{c}(q(\theta), \theta) dF(\theta) - \kappa \geq \max_e \int_0^{\bar{\theta}} t(q(e, \theta)) - c(e) dF(\theta) \quad (II)$$

Subscripts stand for partial derivatives. Let $u^A(\theta) = \max_y t(y) - \hat{c}(q(y), \theta) = t(\theta) - \hat{c}(q(\theta), \theta)$. $u_\theta^A(\theta) = -\hat{c}_\theta(q(\theta), \theta) > 0$ by envelop theorem. Taking integral and by binding (IR_0) , $u^A(\theta) = \int_0^\theta -\hat{c}_\theta(q(x), x) dx$. Plug $t(\theta) = u^A(\theta) + \hat{c}(q(\theta), \theta)$ into $E(u^P(\theta))$ and rearrange by integration by parts,

$$E(u^P(\theta)) = \int_0^{\bar{\theta}} q(\theta) - \hat{c}(q(\theta), \theta) dF(\theta) - \int_0^{\bar{\theta}} \frac{1 - F(\theta)}{f(\theta)} (-\hat{c}_\theta(q(\theta), \theta)) dF(\theta).$$

Let $u^A(\hat{\theta}) = t(\hat{\theta}) - \hat{c}(q(\hat{\theta}), \hat{\theta}) = \max_e \int_0^{\bar{\theta}} t(q(e, \theta)) - c(e) dF(\theta)$, the certainty equivalence of the right hand side of (II) , then (II) can be rewritten as

$$\int_0^{\bar{\theta}} (\mathbf{1}_{\theta > \hat{\theta}} - F(\theta)) (-\hat{c}_\theta(q(\theta), \theta)) d\theta \geq \kappa.$$

The principal's reduced program to induce information acquisition is thus

$$\mathcal{P}_{II} : \max_{q(\theta)} \int_0^{\bar{\theta}} q(\theta) - \hat{c}(q(\theta), \theta) - \frac{1 - F(\theta)}{f(\theta)} (-\hat{c}_\theta(q(\theta), \theta)) dF(\theta)$$

subject to

$$\int_0^{\bar{\theta}} (\mathbf{1}_{\theta \geq \hat{\theta}} - F(\theta)) (-\hat{c}_\theta(q(\theta), \theta)) d\theta \geq \kappa \quad (II')$$

Let λ be the Lagrange multiplier for (II') , $q^{II}(\theta)$ solves

$$\left((1 - \hat{c}_q(q(\theta), \theta)) - \frac{1 - F(\theta)}{f(\theta)} (-\hat{c}_{\theta q}(q(\theta), \theta)) \right) + \lambda \frac{\mathbf{1}_{\theta > \hat{\theta}} - F(\theta)}{f(\theta)} (-\hat{c}_{\theta q}(q(\theta), \theta)) = 0$$

For $\kappa < \kappa^a$, where $\kappa^a = \lim_{q(\theta) \rightarrow q^{SM}(\theta)} \int_0^{\bar{\theta}} (\mathbf{1}_{\theta > \hat{\theta}} - F(\theta)) (-\hat{c}_{\theta}(q(\theta), \theta)) d\theta$, (II') slacks and the principal is able to induce information acquisition with the second best menu contract $\mathbb{C}^{SM} = \{t^{SM}(\theta), q^{SM}(\theta)\}$, $\lambda = 0$. Note that $\frac{1-F(\theta)}{f(\theta)} (-\hat{c}_{\theta q}(q(\theta), \theta)) = \frac{\partial}{\partial q(\theta)} \left(c_e(h(\theta, q(\theta))) \frac{1-F(\theta)}{f(\theta)} \frac{q_{\theta}(h(\theta, q(\theta)), \theta)}{q_e(h(\theta, q(\theta)), \theta)} \right) > 0$, which is decreasing in θ , so monotonicity constraint is strictly satisfied. For $\kappa > \kappa^a$, (II') is violated given the second best menu contract. $\lambda > 0$ in equilibrium. First claim that $\lambda \leq 1$. Suppose that the cost of information acquisition increased by δ , with the optimal contract provided, the principal's equilibrium payoff dropped by $\lambda\delta$. A weakly dominated response to this increment for the principal is to increase the transfer to the agent in any state of nature by δ , which does not violate any constraint. This results in a drop of the principal's payoff by δ . $\lambda\delta \leq \delta$, and thus $\lambda \leq 1$. With $0 < \lambda \leq 1$ and $\frac{1-F(\theta)}{f(\theta)} (-\hat{c}_{\theta q}(q(\theta), \theta)) > 0$, the optimal contract to induce information acquisition $\mathbb{C}^{II} = \{t^{II}(\theta), q^{II}(\theta)\}$ is such that $q^{II}(\theta) > q^{SM}(\theta)$ for $\theta > \hat{\theta}$, and $q^{II}(\theta) < q^{SM}(\theta)$ for $\theta \leq \hat{\theta}$. □

A.2.2 Proof of Proposition 2.2

$$\mathcal{P}'_{DI} : \max_{s, \underline{q}, e} \int_0^{\bar{\theta}} q(e, \theta) dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} s(q(e, \theta) - \underline{q}) dF(\theta)$$

subject to

$$0 \leq s \leq 1 \quad (NDP)$$

$$\int_{\underline{\theta}}^{\bar{\theta}} s q_e(e, \theta) dF(\theta) - c_e(e) = 0 \quad (LIC')$$

$$\begin{aligned} & \int_{\underline{\theta}}^{\bar{\theta}} s(q(e, \theta) - \underline{q}) dF(\theta) - c(e) \\ & \geq \int_{\hat{\theta}}^{\bar{\theta}} s(q(e(\theta), \theta) - \underline{q}) - c(e(\theta)) dF(\theta) - \kappa \quad (DI') \end{aligned}$$

Subscripts stand for partial derivatives. Let μ' , and ϕ' be the Lagrange multipliers associated to (LIC') and (DI') , respectively. The Lagrange function, ignoring (NDP) ,

$$\begin{aligned} \mathcal{L}' &= \int_0^{\bar{\theta}} q(e, \theta) dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} s(q(e, \theta) - \underline{q}) dF(\theta) + \mu' \left(\int_{\underline{\theta}}^{\bar{\theta}} sq_e(e, \theta) dF(\theta) - c_e(e) \right) \\ &+ \phi' \left(\int_{\underline{\theta}}^{\bar{\theta}} s(q(e, \theta) - \underline{q}) dF(\theta) - c(e) - \int_{\underline{\theta}}^{\bar{\theta}} s(q(e(\theta), \theta) - \underline{q}) - c(e(\theta)) dF(\theta) + \kappa \right) \end{aligned}$$

$\mathbb{C}^{DI} = \{q^{SD}, s^{SD}\}$ solves $\frac{\partial \mathcal{L}'}{\partial \underline{q}} q = 0$ and $\frac{\partial \mathcal{L}'}{\partial s} (1 - s) s = 0$.

$$\frac{\partial \mathcal{L}'}{\partial \underline{q}} q = \left(1 - \mu' \left(\frac{q_e(e, \underline{\theta})}{q_{\theta}(e, \underline{\theta})} \frac{f(\underline{\theta})}{1 - F(\underline{\theta})} \right) + \phi' \left(\frac{F(\underline{\theta}) - F(\tilde{\theta})}{1 - F(\underline{\theta})} \right) \right) q = 0$$

By Lemma 2.2, $\tilde{\theta} > \underline{\theta}$, so $F(\underline{\theta}) - F(\tilde{\theta}) < 0$. If $\phi_n = 0$, constraint (DI) slacks and the principal is able to deter information acquisition with the second best debt contract $\mathbb{C}^{SD} = \{s^{SD} = 1, \underline{q}^{SD}\}$. Define $\kappa^q = \lim_{\underline{q} \rightarrow \underline{q}^{SD}} \int_{\tilde{\theta}}^{\bar{\theta}} (q(e(\theta), \theta) - \underline{q}) - c(e(\theta)) dF(\theta) - \int_{\underline{\theta}}^{\tilde{\theta}} (q(e^{SD}, \theta) - \underline{q}) dF(\theta) + c(e^{SD})$. For $\kappa < \kappa^q$, $\phi' > 0$, and as $F(\underline{\theta}) - F(\tilde{\theta}) < 0$, the optimal debt contract to deter information acquisition has $0 \leq \underline{q}^{DI} < \underline{q}^{SD}$, where equality holds only if $\phi' \rightarrow \infty$.

$$\begin{aligned} \frac{\partial \mathcal{L}'}{\partial s} &= - \int_{\underline{\theta}}^{\bar{\theta}} (q(e, \theta) - \underline{q}) dF(\theta) + \mu^n \int_{\underline{\theta}}^{\bar{\theta}} q_e(e, \theta) dF(\theta) \\ &+ \phi' \left(\int_{\underline{\theta}}^{\bar{\theta}} (q(e, \theta) - \underline{q}) dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} (q(e(\theta), \theta) - \underline{q}) dF(\theta) \right) \end{aligned}$$

If $\phi' < \bar{\phi}' \equiv \frac{\mu'_{SD} \int_{\underline{\theta}}^{\bar{\theta}} q_e(e^{SD}, \theta) dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} (q(e^{SD}, \theta) - \underline{q}^{SD}) dF(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} (q(e(\theta), \theta) - \underline{q}^{SD}) dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} (q(e^{SD}, \theta) - \underline{q}^{SD}) dF(\theta)}$, the principal is able to deter information acquisition with $s = 1$. Let $\kappa^s = \lim_{s \rightarrow 1} \int_{\tilde{\theta}}^{\bar{\theta}} s(q(e(\theta), \theta) - \underline{q}^s) - c(e(\theta)) dF(\theta) - \int_{\underline{\theta}}^{\tilde{\theta}} s(q(e^s, \theta) - \underline{q}^s) dF(\theta) + c(e^s)$, superscript s being the level of equilibrium choice variables when $\phi' = \bar{\phi}'$. If $\int_{\underline{\theta}}^{\bar{\theta}} (q(e^{SD}, \theta) - \underline{q}^{SD}) dF(\theta) < \int_{\underline{\theta}}^{\bar{\theta}} (q(e(\theta), \theta) - \underline{q}^{SD}) dF(\theta)$, for $\kappa < \kappa^s$ such that $\phi' > \bar{\phi}'$, $s^{DI} < s^{SD}$. For $\kappa < \kappa^s$ such that $\phi' > \bar{\phi}'$, $s^{DI} < s^{SD} = 1$ and solves $-\int_{\underline{\theta}}^{\bar{\theta}} (q(e, \theta) - \underline{q}) dF(\theta) + \mu' \int_{\underline{\theta}}^{\bar{\theta}} q_e(e, \theta) dF(\theta) + \phi' \left(\int_{\underline{\theta}}^{\bar{\theta}} (q(e, \theta) - \underline{q}) dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} (q(e(\theta), \theta) - \underline{q}) dF(\theta) \right) = 0$. The solution is optimal as $\frac{\partial^2 \mathcal{L}'}{\partial s^2} = - \int_{\underline{\theta}}^{\bar{\theta}} q_e(e(\theta), \theta) e_s(\theta) dF(\theta) + (q(e(\tilde{\theta}), \tilde{\theta}) - \underline{q}) f(\tilde{\theta}) \tilde{\theta}_s < 0$.

□

A.2.3 Proof of Proposition 2.3

$$\mathcal{P}_{DI} : \max_{t(q(e,\theta)), e} \int_0^{\bar{\theta}} q(e, \theta) - t(q(e, \theta)) dF(\theta)$$

subject to

$$t(q(e, \theta)) \geq 0 \quad (LL)$$

$$0 \leq t_q(q(e, \theta)) \leq 1 \quad (NDP)$$

$$\int_0^{\bar{\theta}} t_q(q(e, \theta)) q_e(e, \theta) dF(\theta) = c_e(e) \quad (LIC)$$

$$\int_0^{\bar{\theta}} t(q(e, \theta)) dF(\theta) \geq \int_0^{\bar{\theta}} \mathbf{1}_{\theta \geq \tilde{\theta}} t(q(e(\theta), \theta)) - c(e(\theta)) dF(\theta) - \kappa \quad (DI)$$

Subscripts stand for partial derivatives. Let μ , and ϕ be the Lagrange multipliers associated to (LIC), and (DI), respectively.

With limited liability, $t(q(e, \theta)) = \int_0^{\theta} t_q(q(e, x)) q_{\theta}(e, x) dx$, and by integration by parts, $\int_0^{\bar{\theta}} t(q(e, \theta)) dF(\theta) = \int_0^{\bar{\theta}} (1 - F(\theta)) t_q(q(e, \theta)) q_{\theta}(e, \theta) d\theta$. By the envelope theorem of an informed agent off the equilibrium path and integration by parts, $t(q(e(\theta), \theta)) - c(e(\theta)) = \int_{\tilde{\theta}}^{\theta} t_q(q(e(x), x)) q_{\theta}(e(x), x) dx$, and $\int_{\tilde{\theta}}^{\bar{\theta}} t(q(e(\theta), \theta)) - c(e(\theta)) dF(\theta) = \int_{\tilde{\theta}}^{\bar{\theta}} (1 - F(\theta)) t_q(q(e(\theta), \theta)) q_{\theta}(e(\theta), \theta) d\theta$. The (point-wise) Lagrange function of the principal's problem to deter an agent from acquiring information, excluding (LL) and (NDP), is then written as

$$\begin{aligned} \mathcal{L} = & t_q(q(e, \theta)) \left(-(1 - F(\theta)) q_{\theta}(e, \theta) + \mu q_e(e, \theta) f(\theta) + \phi((1 - F(\theta)) q_{\theta}(e, \theta)) \right. \\ & \left. - \phi \mathbf{1}_{\theta' \geq \tilde{\theta}} (1 - F(\theta')) q_{\theta}(e(\theta'), \theta') \right) + q(e, \theta) f(\theta) - \mu c_e(e) - \phi(c(e) + \kappa), \end{aligned}$$

where θ' is such that $q(e, \theta) \equiv q(e(\theta'), \theta')$. If κ is sufficiently large that $\phi = 0$, $t_q(q(e, \theta)) = 1$ if $\mu \geq \frac{1-F(\theta)}{f(\theta)} \frac{q_{\theta}(e, \theta)}{q_e(e, \theta)} \equiv \frac{1}{\rho_1(e, \theta)}$, $t_q(q(e, \theta)) = 0$ otherwise. As $\frac{1}{\rho_1(e, \theta)}$ by Assumption 2.2 is decreasing in θ , the second best contract is in the form of debt, where $t^{SD}(q) = 0$ for $q \leq \underline{q}^{SD}$, and $t^{SD}(q) = q - \underline{q}^{SD}$ otherwise.

Let the solution to the above problem, ignoring monotonicity for now, be $\hat{t}(q)$. If κ is sufficiently small that $\phi > 0$, for $q < \tilde{q} \equiv q(e(\tilde{\theta}), \tilde{\theta})$, i.e. where $\theta' < \tilde{\theta}$, $\hat{t}_q(q) = 1$ if $\mu > \frac{1-\phi}{\rho_1(e, \theta)}$, $\hat{t}_q(q) = 0$ otherwise. For $q < \tilde{q}$, $\hat{t}(q) = \max\{q - \underline{q}^{DI}, 0\}$, where $\underline{q}^{DI} < \underline{q}^{SD}$. For $q \geq \tilde{q}$, i.e. where $\theta' \geq \tilde{\theta}$, $\hat{t}_q(q) = 1$ if $\mu > \frac{1-\phi}{\rho_1(e, \theta)} + \phi \frac{1-F(\theta')}{f(\theta)} \frac{q_{\theta}(e(\theta'), \theta')}{q_e(e, \theta)}$, $\hat{t}_q(q) = 0$ otherwise. $\frac{1-\phi}{\rho_1(e, \theta)} + \phi \frac{1-F(\theta')}{f(\theta)} \frac{q_{\theta}(e(\theta'), \theta')}{q_e(e, \theta)} = \frac{1}{\rho_1(e, \theta)} - \frac{\phi}{\rho_1(e, \theta)} \left(1 - \frac{(1-F(\theta')) q_{\theta}(e(\theta'), \theta')}{(1-F(\theta)) q_{\theta}(e, \theta)} \right)$. Let θ_0 be such that $q(e, \theta_0) \equiv q(e(\theta_0), \theta_0)$. For $\theta < \theta_0$, $\theta < \theta'$ and for $\theta > \theta_0$, $\theta > \theta'$, so $\frac{\partial \theta'}{\partial \theta} < 1$. $\frac{(1-F(\theta')) q_{\theta}(e(\theta'), \theta')}{(1-F(\theta)) q_{\theta}(e, \theta)}$ is then increasing in θ . If $\phi = 1$, $-\frac{\partial}{\partial \theta} \left(\frac{1}{\rho_1(e, \theta)} \right) <$

$\frac{\partial}{\partial \theta} \left(\frac{-1}{\rho_1(e, \theta)} \left(1 - \frac{(1-F(\theta'))q_{\theta}(e(\theta'), \theta')}{(1-F(\theta))q_{\theta}(e, \theta)} \right) \right)$. Thus, for sufficiently small κ such that $\phi > \hat{\phi}$, $\hat{\phi} < 1$, $\frac{1}{\rho_1(e, \theta)} - \frac{\phi}{\rho_1(e, \theta)} \left(1 - \frac{(1-F(\theta'))q_{\theta}(e(\theta'), \theta')}{(1-F(\theta))q_{\theta}(e, \theta)} \right)$ is increasing in θ . If $\phi < \hat{\phi}$, for $q \geq \tilde{q}$, $\hat{t}(q) = \max\{q - \hat{q}, 0\}$, where $\hat{q} > \underline{q}^{DI}$. As $\hat{q} > \underline{q}^{DI}$, $\hat{t}(q)$ violates non-decreasing transfer near \tilde{q} . Thus, there exists an interval $[q^a, q^b]$ containing \tilde{q} in which low-powered incentive ($t_q(q) = 0$) occurs. Therefore, $t^{DI}(q) \leq q - \underline{q}^{DI}$ for $q > \underline{q}^{DI}$. If $1 > \phi > \hat{\phi}$, there exists q^c such that $\hat{t}_q(q) = 0$ for $q > q^c$. The optimal contract is then as the following:

$$t^{DI}(q(e, \theta)) = \begin{cases} 0 & \text{for } q \in [q(e, 0), \underline{q}^{DI}] \\ q(e, \theta) - \underline{q}^{DI} & \text{for } q \in (\underline{q}^{DI}, q^a) \\ q^a - \underline{q}^{DI} & \text{for } q \in [q^a, q^b] \\ q(e, \theta) - \hat{q} & \text{for } q \in (q^b, q^c] \\ \bar{T} & \text{for } q \in (q^c, q(e, \bar{\theta})) \end{cases},$$

where interval $(q^c, q(e, \bar{\theta}))$ is empty if κ is sufficiently large. As $t_{qq}^{DI} = 0$ wherever second-order differentiable, $\int_0^{\bar{\theta}} t_{qq}^{DI}(q(e, \theta))q_e^2(e, \theta) + t_q^{DI}(q(e, \theta))q_{ee}(e, \theta)dF(\theta) < 0$, the first order approach is valid. □

A.2.4 Proof of Lemma 2.4 and Proposition 2.5

$u^I(\theta) = \int_0^{\bar{\theta}} -\hat{c}_{\theta}(q^I(x), x)dx + u^I(0)$. The principal's optimization program is then

$$\mathcal{P}_M : \max_{q^I(\theta), e^U, t^U(q), a} a \left(\int_0^{\bar{\theta}} q^I(\theta) - \hat{c}(q^I(\theta), \theta) - \frac{1-F(\theta)}{f(\theta)} (-\hat{c}_{\theta}(q^I(\theta), \theta))dF(\theta) - u^I(0) \right) \\ + (1-a) \left(\int_0^{\bar{\theta}} q(e^U, \theta) - t^U(q(e^U, \theta))dF(\theta) \right)$$

subject to

$$t^U(q(e^U, \theta)) \geq 0 \quad (LL)$$

$$0 \leq t_q^U(q) \leq 1 \quad (NDP)$$

$$q_{\theta}^I(\theta) \geq 0 \quad (M)$$

$$\int_0^{\bar{\theta}} t_q^U(q(e^U, \theta))q_e(e^U, \theta)dF(\theta) = c_e(e^U) \quad (LIC_U)$$

$$\int_0^{\bar{\theta}} u^I(\theta) dF(\theta) - \int_0^{\bar{\theta}} u^U(\theta) dF(\theta) = d_a(\kappa, a) \quad (A)$$

$$u^I(\theta) \geq \max_e t^U(q(e, \theta)) - c(e) \quad \forall \theta \in [0, \bar{\theta}] \quad (TT_I)$$

$$\int_0^{\bar{\theta}} u^U(\theta) dF(\theta) \geq \max_e \int_0^{\bar{\theta}} t^I(q^I(e, \theta)) dF(\theta) - c(e) \quad (TT_U)$$

Subscripts in the functions stand for derivatives. Let μ , ϕ , $\lambda^I(\theta)$, λ^U be the Lagrange multipliers for (LIC_U) , (A) , (TT_I) , (TT_U) , respectively. Denote $\hat{\theta}$ as such that $u^I(\hat{\theta}) = \max_e \int_0^{\bar{\theta}} t^I(q^I(e, \theta)) dF(\theta) - c(e)$.

$q^I(\theta)$ solves

$$\begin{aligned} & \left(a - a\hat{c}_q(q(\theta), \theta) - (a - \phi) \frac{1 - F(\theta)}{f(\theta)} (-\hat{c}_{\theta q}(q^I(\theta), \theta)) \right) \\ & + \frac{1}{f(\theta)} \int_{\theta}^{\bar{\theta}} \lambda^I(x) dx (-\hat{c}_{\theta q}(q^I(\theta), \theta)) - \frac{\lambda^U}{f(\theta)} (-\hat{c}_{\theta q}(q^I(\theta), \theta)) \mathbf{1}_{\theta \leq \hat{\theta}} = 0 \end{aligned}$$

and, by similar method as Appendix A.2.3, $t_q^U(q) = 1$ if

$$\mu > \frac{1 - a + \phi}{\rho_1(e, \theta)} - \frac{\lambda^U}{\rho_1(e, \theta)} + \frac{\int_{\theta'}^{\bar{\theta}} \lambda^I(\theta) q_{\theta}(e(\theta), \theta) d\theta}{q_e(e, \theta) f(\theta)}$$

where $\rho_1(e, \theta) \equiv \frac{f(\theta)}{1 - F(\theta)} \frac{q_e(e, \theta)}{q_{\theta}(e, \theta)}$ and θ' is such that $q(e, \theta) = q(e(\theta'), \theta')$.

Show Lemma 2.4. Let $\{\hat{C}^I, \hat{C}^U\}$ be the optimal contract excluding (TT_I) , in which $t_q^U(q) = 1$ if

$$\mu > \frac{1 - a + \phi - \lambda^U}{\rho_1(e, \theta)}.$$

Claim that $1 - a + \phi - \lambda^U \geq 0$. Suppose that in addition to the optimal contracts, that the principal increases the transfer to the uninformed agent by δ and adjust a downward by η to bind (A) , which does not violate any constraint excluding (TT_I) . Downward adjustment of a has a second order effect yet increment of transfer has a first order effect. The principal's indirect objective function is then changed by $(-1 + a - \phi + \lambda^U)\delta \leq 0$ as she is moving from the optimal solution to the suboptimal. As $1 - a + \phi - \lambda^U \geq 0$, the optimal contingent transfer excluding (TT_I) to an uninformed agent is a debt contract. Thus, given implemented productive effort, there exist $\tilde{\theta}$ such that $e(\theta) \in \max_e t^U(q(e, \theta)) - c(e) = 0$ for $\theta < \tilde{\theta}$. Along with individual rationality of the informed agent, (TT_I) is strictly satisfied for $\theta < \tilde{\theta}$. Hence, if $\hat{u}^I(\theta)$ is sufficiently convex such that (TT_I) is violated for some $\theta \in [\theta_1, \theta_2]$, it is where $\theta_1 > \tilde{\theta}$ and $\theta_2 \leq \bar{\theta}$.

To deter an informed agent in states $\theta \in (\theta_1, \theta_2)$ from lying to be uninformed, there exists θ^T such that $q^I(\theta)$ for $\theta < \theta^T$ are raised to increase the rent in these states. As (TT_I) is strictly satisfied in $\theta \leq \tilde{\theta}$, $\tilde{\theta} < \theta^T \leq \bar{\theta}$.

Given Lemma 2.4, $q^I(\theta)$ solves

$$\begin{aligned} & \left(a - a\hat{c}_q(q(\theta), \theta) - (a - \phi) \frac{1 - F(\theta)}{f(\theta)} (-\hat{c}_{\theta q}(q^I(\theta), \theta)) \right) \\ & + \frac{\lambda^I(\theta^T)}{f(\theta)} \mathbf{1}_{\theta \leq \theta^T} (-\hat{c}_{\theta q}(q^I(\theta), \theta)) - \frac{\lambda^U}{f(\theta)} (-\hat{c}_{\theta q}(q^I(\theta), \theta)) \mathbf{1}_{\theta \leq \hat{\theta}} = 0 \end{aligned}$$

and $t_q^U(q) = 1$ if

$$\mu > \frac{1 - a + \phi}{\rho_1(e, \theta)} - \frac{\lambda^U}{\rho_1(e, \theta)} + \lambda^I(\theta^T) \frac{q_{\theta}(e(\theta'), \theta')}{q_e(e, \theta) f(\theta)} \mathbf{1}_{\theta' \in [\tilde{\theta}, \theta^T]}$$

As $-\hat{c}_{\theta q}(q^I(\theta), \theta) = \frac{\partial}{\partial q(\theta)} \left(c_e(h(\theta, q(\theta))) \frac{q_{\theta}(h(\theta, q(\theta)), \theta)}{q_e(h(\theta, q(\theta)), \theta)} \right) > 0$, two binding constraints distort $q^I(\theta)$ from $q^{SM}(\theta)$: a) $q^I(\theta)$ for $\theta \leq \theta^T$ are raised to prevent an informed agent from pretending to be uninformed, implied by $\frac{\lambda^I(\theta^T)}{f(\theta)} \mathbf{1}_{\theta \leq \theta^T} (-\hat{c}_{\theta q}(q^I(\theta), \theta)) \geq 0$; b) $q^I(\theta)$ for $\theta \leq \hat{\theta}$ is lowered with a gap at $\hat{\theta}$ to prevent an uninformed agent from lying to be informed, captured by $-\frac{\lambda^U}{f(\theta)} (-\hat{c}_{\theta q}(q^I(\theta), \theta)) \mathbf{1}_{\theta \leq \hat{\theta}} < 0$. For $\theta > \hat{\theta}$, $q^I(\theta) \geq q^{SM}(\theta)$ as at most the first effect is present. For $\theta \leq \hat{\theta}$, depending on the countervailing effect between truth telling in the informed phase and that in the uninformed phase, $q^I(\theta) < q^{SM}(\theta)$ if the second effect is sufficiently significant to outweigh the first. Hence i) in Proposition 2.5. If $\theta^T < \bar{\theta}$ and $\theta^T \neq \hat{\theta}$, monotonicity must be violated near θ^T due to a). Optimal $q^I(\theta)$ thus have $q_{\theta}^I(\theta) = 0$ for $\theta \in (\theta^a, \theta^b)$, where $\theta^T \in (\theta^a, \theta^b)$, ii) in Proposition 2.5. The same constraints also distort $t^U(q)$ from $t^{SD}(q) = \max\{q(e^{SD}, \theta) - \underline{q}^{SD}, 0\}$: a) to prevent an uninformed agent from lying to be informed, the initial debt is lowered, $t^U(q) = 0$ for $q < \underline{q}^U < \underline{q}^{SD}$, as $-\frac{\lambda^U}{\rho(e, \theta)} < 0$; b) to deter an informed agent from lying to be uninformed, for $q \in [\tilde{q}, q^T]$, where $\tilde{q} \equiv q(e(\tilde{\theta}), \tilde{\theta})$ and $q^T \equiv q(e(\theta^T), \theta^T)$, $t^U(q)$ is lowered in the sense that $t_q^U(q) = 1$ for $q(e, \theta) > q_1 > \underline{q}^U$, as $\lambda^I(\theta^T) \frac{q_{\theta}(e(\theta'), \theta')}{q_e(e, \theta) f(\theta)} > 0$, which violates monotonicity near \tilde{q} . Hence, there exist an interval (q^a, q^b) containing \tilde{q} , such that $t_q^U(q) = 0$ for $q(e, \theta) \in (q^a, q^b)$. Thus, $t^U(q) \leq q - \underline{q}^U$ for $q > \underline{q}^U$, share of output residual to the agent is reduced.

□

A.3 Proof of Propositions in Chapter 3

A.3.1 Proof of Proposition 3.1

With $p(\beta, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) = (1 - \theta + s(\hat{\beta})\theta)\beta I(\hat{\beta}, s(\hat{\beta}))$, (IC^S) is equivalent to

$$\beta \in \arg \max_{\hat{\beta}} u^S(\beta, \hat{\beta}) \equiv (1 - s(\hat{\beta}))\theta\beta I(\hat{\beta}, s(\hat{\beta})) - t(\hat{\beta}) \quad \forall \beta \quad (IC^S)$$

(IC^S) is satisfied only if $\hat{\beta} = \beta$ solves the first order condition, $\frac{\partial u^S(\beta, \hat{\beta})}{\partial \hat{\beta}} = 0$, and if the second order condition is satisfied, $\frac{\partial^2 u^S(\beta, \hat{\beta})}{\partial \hat{\beta}^2} \leq 0$. Taking derivative of $\frac{\partial u^S(\beta, \hat{\beta})}{\partial \hat{\beta}} \Big|_{\hat{\beta}=\beta} \equiv 0$ with respect to β yields $\frac{\partial^2 u^S(\beta, \hat{\beta})}{\partial \hat{\beta}^2} \Big|_{\hat{\beta}=\beta} + \theta \frac{d[(1-s(\hat{\beta}))I(\hat{\beta}, r(\hat{\beta}))]}{d\hat{\beta}} \Big|_{\hat{\beta}=\beta} = 0$. If $\frac{d[(1-s(\hat{\beta}))I(\hat{\beta}, r(\hat{\beta}))]}{d\hat{\beta}} \Big|_{\hat{\beta}=\beta} \geq 0$, the second order condition holds. Thus, (IC^S) can be replaced by the local incentive compatibility constraint

$$\theta\beta \frac{d[(1-s(\hat{\beta}))I(\hat{\beta}, s(\hat{\beta}))]}{d\hat{\beta}} - \frac{dt(\hat{\beta})}{d\hat{\beta}} \Big|_{\hat{\beta}=\beta} = 0 \quad \forall \beta \quad (LIC^S)$$

and the monotonicity constraint

$$\frac{d[(1-s(\hat{\beta}))I(\hat{\beta}, s(\hat{\beta}))]}{d\hat{\beta}} \Big|_{\hat{\beta}=\beta} \geq 0 \quad \forall \beta \quad (M^S)$$

It is then straightforward that $s(\beta) = 1$ and $t(\beta) = 0$ for all β satisfy both (LIC^S) and (M^S) . With the producer having all bargaining power, $s(\beta) = 1$ implements first-best investment with the producer claiming the entire surplus from trade. $\{s(\beta) = 1, t(\beta) = 0\}$ is thus the optimal contract to induce information revelation.

□

A.3.2 Proof of Lemma 3.1

By construction, $(\bar{\beta} - \underline{\beta})I(\underline{\beta}, \underline{s}) \geq (\bar{\beta} - \underline{\beta})\theta(1 - \underline{s})I(\underline{\beta}, \underline{s})$ and $(\bar{\beta} - \underline{\beta})I(\bar{\beta}, \bar{s}) \geq (\bar{\beta} - \underline{\beta})\theta(1 - \bar{s})I(\bar{\beta}, \bar{s})$. Thus, $\mathbb{C} = \{(\bar{s}, \bar{t}), (\underline{s}, \underline{t})\}$ is incentive compatible if $(\bar{\beta} - \underline{\beta})I(\underline{\beta}, \underline{s}) \leq \bar{u} - \underline{u} \leq (\bar{\beta} - \underline{\beta})\theta(1 - \bar{s})I(\bar{\beta}, \bar{s})$. It satisfies both (IC_I^H) and (IC_I^L) if $(\bar{\beta} - \underline{\beta})I(\underline{\beta}, \underline{s}) \leq (\bar{\beta} - \underline{\beta})I(\bar{\beta}, \bar{s})$, which holds if (IC^B) holds as $(\bar{\beta} - \underline{\beta})I(\bar{\beta}, \bar{s}) \geq (\bar{\beta} - \underline{\beta})\theta(1 - \bar{s})I(\bar{\beta}, \bar{s})$. It satisfies both (IC_O^H) and (IC_O^L) if $(\bar{\beta} - \underline{\beta})\theta(1 - \underline{s})I(\underline{\beta}, \underline{s}) \leq (\bar{\beta} - \underline{\beta})\theta(1 - \bar{s})I(\bar{\beta}, \bar{s})$,

which holds if (IC^B) holds as $(\bar{\beta} - \underline{\beta})I(\underline{\beta}, \underline{s}) \geq (\bar{\beta} - \underline{\beta})\theta(1 - \underline{s})I(\underline{\beta}, \underline{s})$.

□

A.3.3 Proof of Proposition 3.2

The set of $\{\underline{s}, \bar{s}\}$ satisfying (IC^B) is empty if $\min_{\underline{s}} I(\underline{\beta}, \underline{s}) > \max_{\bar{s}} \theta(1 - \bar{s})I(\bar{\beta}, \bar{s})$ for any θ , i.e. if $\underline{\beta} > \theta\bar{\beta}$. With the second inequality of Assumption 3.2, the set of $\{\underline{s}, \bar{s}\}$ satisfying (IC^B) is not empty. The seller

$$\max_{\bar{s}, \underline{s}} \sigma (\bar{\beta}I(\bar{\beta}, \bar{s}) - (\bar{\beta} - \underline{\beta})I(\underline{\beta}, \underline{s}) - c(I(\bar{\beta}, \bar{s}))) + (1 - \sigma) (\underline{\beta}I(\underline{\beta}, \underline{s}) - c(I(\underline{\beta}, \underline{s})))$$

subject to

$$I(\underline{\beta}, \underline{s}) = \theta(1 - \bar{s})I(\bar{\beta}, \bar{s})$$

Let the Lagrange function be $\mathcal{L} = \sigma (\bar{\beta}I(\bar{\beta}, \bar{s}) - (\bar{\beta} - \underline{\beta})I(\underline{\beta}, \underline{s}) - c(I(\bar{\beta}, \bar{s}))) + (1 - \sigma) (\underline{\beta}I(\underline{\beta}, \underline{s}) - c(I(\underline{\beta}, \underline{s}))) + \lambda (\theta(1 - \bar{s})I(\bar{\beta}, \bar{s}) - I(\underline{\beta}, \underline{s}))$, with λ being the Lagrange multiplier of the constraint. The objective function is concave in $I(\bar{\beta}, \bar{s})$ and in $I(\underline{\beta}, \underline{s})$, which are linear in \bar{s} and \underline{s} respectively, and thus the objective function is concave in \bar{s} and in \underline{s} . The set of \bar{s} and that of \underline{s} satisfying (IC^B) are convex as the left-hand-side of (IC^B) is linear in \underline{s} and the right-hand-side is concave in \bar{s} . Hence, if \bar{s} and \underline{s} have interior solutions, $(\bar{s}, \underline{s}, \lambda)$ solve the following optimality conditions and the binding constraint.

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{s}} &= \sigma (\bar{\beta} - (1 - \theta(1 - \bar{s}))\bar{\beta}) + \lambda (2\theta(1 - \bar{s}) - 1) = 0 \\ \frac{\partial \mathcal{L}}{\partial \underline{s}} &= (1 - \sigma) (\underline{\beta} - (1 - \theta(1 - \underline{s}))\underline{\beta}) - \sigma (\bar{\beta} - \underline{\beta}) - \lambda = 0 \\ &(1 - \theta(1 - \underline{s}))\underline{\beta} = \theta(1 - \bar{s})(1 - \theta(1 - \bar{s}))\bar{\beta} \end{aligned}$$

Plugging the latter two equations into the first yields $\sigma\theta(1 - \bar{s})\bar{\beta} - (1 - 2\theta(1 - \bar{s}))(\underline{\beta} - \sigma\bar{\beta} - (1 - \sigma)\theta\bar{\beta}(1 - \bar{s})(1 - \theta(1 - \bar{s}))) = 0$. $\bar{s} < 1$ solves this equation if $\underline{\beta} - \sigma\bar{\beta} > 0$, i.e. if the first inequality of Assumption 3.2 holds. If $\underline{s} = 0$ is optimal, from the binding constraint, $(1 - \theta)\underline{\beta} = \theta(1 - \bar{s})(1 - \theta(1 - \bar{s}))\bar{\beta}$. At $\bar{s} = 1$, $(1 - \theta)\underline{\beta} > 0$, at $\bar{s} = 0$, $\underline{\beta} \leq \theta\bar{\beta}$, and the right-hand-side is strictly concave in $(1 - \bar{s})$; thus, there exist a unique $\bar{s} \in (0, 1)$ such that the constraint is binding at $\underline{s} = 0$. If \bar{s} and \underline{s} have interior solutions, from the optimality conditions, $\bar{s} = 1 - \frac{\lambda}{\sigma\theta\bar{\beta} + 2\theta\lambda}$, $\underline{s} = 1 - \frac{\sigma(\bar{\beta} - \underline{\beta}) + \lambda}{(1 - \sigma)\theta\bar{\beta}}$, and by construction $\lambda \geq 0$. $\bar{s} \geq \underline{s}$ if $\frac{\lambda}{\sigma\theta\bar{\beta} + 2\theta\lambda} \leq \frac{\sigma(\bar{\beta} - \underline{\beta}) + \lambda}{(1 - \sigma)\theta\bar{\beta}}$, which holds if $\sigma\theta\bar{\beta} \geq (1 - \sigma)\theta\underline{\beta}$, i.e. if $\frac{\sigma}{1 - \sigma} \geq \frac{\underline{\beta}}{\bar{\beta}}$.

□

A.3.4 Proof of Proposition 3.3

From (IC_I) , $\beta \in \arg \max_{\hat{\beta}} u(\beta, \hat{\beta}) \equiv \beta I(\hat{\beta}, s(\hat{\beta})) - p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) - t(\hat{\beta})$ only if $\hat{\beta} = \beta$ solves the first order condition, $\frac{\partial u(\beta, \hat{\beta})}{\partial \hat{\beta}} = \left(\beta - \frac{\partial p(\bullet)}{\partial t(\bullet)} \right) \frac{dI(\bullet)}{d\hat{\beta}} - \frac{\partial p(\bullet)}{\partial s(\hat{\beta})} \frac{ds(\hat{\beta})}{d\hat{\beta}} - \frac{\partial p(\bullet)}{\partial \hat{\beta}} - \frac{dt(\hat{\beta})}{d\hat{\beta}} = 0$. From (IC_O) , $\beta \in \arg \max_{\hat{\beta}} v(\beta, \hat{\beta}) \equiv (1 - s(\hat{\beta}))\theta\beta I(\hat{\beta}, s(\hat{\beta})) - t(\hat{\beta})$ only if $\hat{\beta} = \beta$ solves the first order condition, $\frac{\partial v(\beta, \hat{\beta})}{\partial \hat{\beta}} = \theta\beta(1 - s(\hat{\beta})) \frac{dI(\bullet)}{d\hat{\beta}} - \theta\beta I(\bullet) \frac{ds(\hat{\beta})}{d\hat{\beta}} - \frac{dt(\hat{\beta})}{d\hat{\beta}} = 0$. With $p(\hat{\beta}, s(\hat{\beta}), I) = (1 - \theta + s(\hat{\beta})\theta)\hat{\beta}I$, $\frac{\partial u(\beta, \hat{\beta})}{\partial \hat{\beta}} = \theta\beta(1 - s(\hat{\beta})) \frac{dI(\bullet)}{d\hat{\beta}} - \theta\beta I(\bullet) \frac{ds(\hat{\beta})}{d\hat{\beta}} - \frac{\partial p(\bullet)}{\partial \hat{\beta}} - \frac{dt(\hat{\beta})}{d\hat{\beta}} < \theta\beta(1 - s(\hat{\beta})) \frac{dI(\bullet)}{d\hat{\beta}} - \theta\beta I(\bullet) \frac{ds(\hat{\beta})}{d\hat{\beta}} - \frac{dt(\hat{\beta})}{d\hat{\beta}} = \frac{\partial v(\beta, \hat{\beta})}{\partial \hat{\beta}}$. Without commitment to a penalty $\Delta(\beta)$, any allocation of ownership that induces truthful revelation within the relationship would leave the buyer with incentive to report upward and deviate from trade, and any allocation of ownership that induces truthful revelation when the relationship ends would leave the buyer with incentive to report downward and trade. Only if the parties can commit to a penalty $\Delta(\beta)$ imposed on the buyer when the seller proposes an individually rational trade term $p \leq p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) = (1 - \theta + s(\hat{\beta})\theta)\hat{\beta}I(\hat{\beta}, s(\hat{\beta}))$ yet the buyer deviates from trade, would it be possible to induce truthful revelation with an allocation of ownership. $\Delta(\beta) = \int \frac{\partial p(\bullet)}{\partial \hat{\beta}} d\hat{\beta}$ such that $\frac{\partial v(\beta, \hat{\beta})}{\partial \hat{\beta}} = \theta\beta(1 - s(\hat{\beta})) \frac{dI(\bullet)}{d\hat{\beta}} - \theta\beta I(\bullet) \frac{ds(\hat{\beta})}{d\hat{\beta}} - \frac{dt(\hat{\beta})}{d\hat{\beta}} - \frac{\partial p(\bullet)}{\partial \hat{\beta}} = \frac{\partial u(\beta, \hat{\beta})}{\partial \hat{\beta}}$.

□

A.3.5 Proof of Lemma 3.3 and Lemma 3.4

From (IC_I^N) , $\beta \in \arg \max_{\hat{\beta}} u(\beta, \hat{\beta}) \equiv \beta I(\hat{\beta}, s(\hat{\beta})) - p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) - t(\hat{\beta})$ only if $\hat{\beta} = \beta$ solves the first order condition, $\frac{\partial u(\beta, \hat{\beta})}{\partial \hat{\beta}} = 0$, and if the second order condition is satisfied, $\frac{\partial^2 u(\beta, \hat{\beta})}{\partial \hat{\beta}^2} < 0$. If $\hat{\beta} = \beta$ solves (LIC_I) , $\left. \frac{\partial u(\beta, \hat{\beta})}{\partial \hat{\beta}} \right|_{\hat{\beta}=\beta} \equiv 0$. Taking derivative with respect to β yields $\left. \frac{\partial^2 u(\beta, \hat{\beta})}{\partial \hat{\beta}^2} \right|_{\hat{\beta}=\beta} + \left. \frac{dI(\hat{\beta}, s(\hat{\beta}))}{d\hat{\beta}} \right|_{\hat{\beta}=\beta} = 0$. If $\left. \frac{dI(\hat{\beta}, s(\hat{\beta}))}{d\hat{\beta}} \right|_{\hat{\beta}=\beta} > 0$, the second order condition holds. In addition, as investment and the terms of trade depend only on $\hat{\beta}$, let $I_1(\hat{\beta}) \equiv I(\hat{\beta}, s(\hat{\beta}))$ and $T_1(\hat{\beta}) \equiv p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) + t(\hat{\beta})$, and $V_1(\beta) \equiv \max_{\hat{\beta}} U_1(\beta, I_1(\hat{\beta})) - T_1(\hat{\beta})$. From (IC_I^N) , $V_1(\beta) - \beta I_1(\hat{\beta}) + T_1(\hat{\beta}) \geq 0$. $V_1(\beta) - \beta I_1(\hat{\beta}) + T_1(\hat{\beta}) = V_1(\beta) - V_1(\hat{\beta}) + (\hat{\beta} - \beta)I_1(\hat{\beta}) = \int_{\hat{\beta}}^{\beta} \frac{\partial U_1(x, I_1(x))}{\partial \beta} - \frac{\partial U_1(x, I_1(\hat{\beta}))}{\partial \beta} dx = \int_{\hat{\beta}}^{\beta} \int_{I_1(\hat{\beta})}^{I_1(x)} \frac{\partial^2 U_1(x, y)}{\partial \beta \partial I_1} dy dx$. As $\frac{\partial^2 U_1}{\partial \beta \partial I_1} = 1 > 0$, the Spence-Mirrlees property that allows us to replace (IC_I^N) with (LIC_I) and (M_I) without loss of generality is satisfied.

From (IC_O^N) , $\beta \in \arg \max_{\hat{\beta}} v(\beta, \hat{\beta}) \equiv (1 - s(\hat{\beta}))\theta\beta I(\hat{\beta}, s(\hat{\beta})) - t(\hat{\beta}) - \Delta(\hat{\beta})$ only if $\hat{\beta} = \beta$ solves the first order condition, $\frac{\partial v(\beta, \hat{\beta})}{\partial \hat{\beta}} = 0$, and if the second order condition is satisfied, $\frac{\partial^2 v(\beta, \hat{\beta})}{\partial \hat{\beta}^2} < 0$. If $\hat{\beta} = \beta$ solves $(LICO)$, $\frac{\partial v(\beta, \hat{\beta})}{\partial \hat{\beta}} \Big|_{\hat{\beta}=\beta} \equiv 0$. Taking derivative with respect to β yields $\frac{\partial^2 v(\beta, \hat{\beta})}{\partial \hat{\beta}^2} \Big|_{\hat{\beta}=\beta} + \theta \frac{d[(1-s(\hat{\beta}))I(\hat{\beta}, r(\hat{\beta}))]}{d\hat{\beta}} \Big|_{\hat{\beta}=\beta} = 0$. If $\frac{d[(1-s(\hat{\beta}))I(\hat{\beta}, r(\hat{\beta}))]}{d\hat{\beta}} \Big|_{\hat{\beta}=\beta} > 0$, the second order condition holds. In addition, as the buyer's share of investment and the transfers depend only on $\hat{\beta}$, let $I_2(\hat{\beta}) \equiv (1 - s(\hat{\beta}))I(\hat{\beta}, s(\hat{\beta}))$ and $T_2(\hat{\beta}) \equiv t(\hat{\beta}) + \Delta(\hat{\beta})$, and $V_2(\beta) \equiv \max_{\hat{\beta}} U_2(\beta, I_2(\hat{\beta})) - T_2(\hat{\beta})$. From (IC_O^N) , $V_2(\beta) - \theta\beta I_2(\hat{\beta}) + T_2(\hat{\beta}) \geq 0$. $V_2(\beta) - \theta\beta I_2(\hat{\beta}) + T_2(\hat{\beta}) = V_2(\beta) - V_2(\hat{\beta}) + \theta(\hat{\beta} - \beta)I_2(\hat{\beta}) = \int_{\hat{\beta}}^{\beta} \frac{\partial U_2(x, I_2(x))}{\partial \beta} dx = \int_{\hat{\beta}}^{\beta} \int_{I_2(\hat{\beta})}^{I_2(x)} \frac{\partial^2 U_2(x, y)}{\partial \beta \partial I_2} dy dx$. As $\frac{\partial^2 U_2}{\partial \beta \partial I_2} = \theta \geq 0$, the Spence-Mirrlees property that allows us to replace (IC_O^N) with $(LICO)$ and (M_O) without loss of generality is satisfied.

□

A.3.6 Proof of Proposition 3.4 and Proposition 3.5

First show (1). Let the buyer's equilibrium payoff when Lemma 3.3 and Lemma 3.4 holds be $u(\beta) \equiv \max_{\hat{\beta}} \left\{ u(\beta, \hat{\beta}) \equiv \beta I(\hat{\beta}, s(\hat{\beta})) - p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) - t(\hat{\beta}) \right\}$. By Envelope theorem, $\frac{du(\beta)}{d\beta} = I(\beta, s(\beta)) > 0$, $u(\beta) = \int_{\underline{\beta}}^{\beta} I(x, s(x)) dx$, and by integration by parts, $\int_{\underline{\beta}}^{\beta} u(\beta) dF(\beta) = \int_{\underline{\beta}}^{\beta} \frac{1-F(\beta)}{f(\beta)} I(\beta, s(\beta))$. As $p(\beta, s(\beta), I(\beta, s(\beta))) + t(\beta) = \beta I(\beta, s(\beta)) - u(\beta)$, the seller's take-it-or-leave-it offer on ownership structure solves

$$\max_{s(\beta)} \int_{\underline{\beta}}^{\beta} \left(\beta I(\beta, s(\beta)) - \frac{1-F(\beta)}{f(\beta)} I(\beta, s(\beta)) - c(I(\beta, s(\beta))) \right) dF(\beta)$$

The point-wise first order condition has

$$\beta - \frac{1-F(\beta)}{f(\beta)} - I(\beta, s(\beta)) = 0$$

Let $s^*(\beta)$ solves the above first order condition. The second order condition at $s^*(\beta)$ holds since $\frac{\partial I(\beta, s(\beta))}{\partial s(\beta)} > 0$. At $s^*(\beta)$, $\beta - \frac{1-F(\beta)}{f(\beta)} - I(\beta, s^*(\beta)) \equiv 0$. Taking derivative of the identity with respect to β , $1 - \frac{d(\frac{1-F(\beta)}{f(\beta)})}{d\beta} - \frac{dI(\beta, s(\beta))}{d\beta} = 0$. $\frac{dI(\beta, s(\beta))}{d\beta} \geq 0$ at $s^*(\beta)$ if $\frac{d(\frac{1-F(\beta)}{f(\beta)})}{d\beta} \leq 1$. Rearrange the first order condition yields $1 - s(\beta) = \frac{1-F(\beta)}{f(\beta)} \frac{1}{\theta\beta}$ and $I(\beta, s(\beta)) = \beta - \frac{1-F(\beta)}{f(\beta)}$. $\frac{d[(1-s(\beta))I(\beta, s(\beta))]}{d\beta} \geq 0$ at $s^*(\beta)$ if $\frac{1-F(\beta)}{f(\beta)} \left(1 - \frac{1-F(\beta)}{f(\beta)} \frac{1}{\beta} \right)$ is non-decreasing in β . Thus, (M_I) and (M_O) are both satisfied at $s^*(\beta)$ if $\frac{d(\frac{1-F(\beta)}{f(\beta)})}{d\beta} \leq 1$

(Condition 1) and $\frac{1-F(\beta)}{f(\beta)} \left(1 - \frac{1-F(\beta)}{f(\beta)} \frac{1}{\beta}\right)$ is non-decreasing in β (Condition 2). $s(\beta) = 1 - \frac{1-F(\beta)}{f(\beta)} \frac{1}{\theta\beta}$, which is non-decreasing in β if $\frac{1-F(\beta)}{f(\beta)} \frac{1}{\beta}$ is non-increasing in β (Condition 3).

Show (2). If Condition 1 holds and $\frac{1-F(\beta)}{f(\beta)} \left(1 - \frac{1-F(\beta)}{f(\beta)} \frac{1}{\beta}\right)$ is diminishing in β , (M_O) is violated at $s^*(\beta)$. The seller then

$$\max_{s(\beta), \gamma_1} \int_{\beta} \left(\beta I(\beta, s(\beta)) - \frac{1-F(\beta)}{f(\beta)} I(\beta, s(\beta)) - c(I(\beta, s(\beta))) \right) dF(\beta)$$

subject to

$$(1 - s(\beta))I(\beta, s(\beta)) = \gamma_1$$

$(1 - s(\beta))I(\beta, s(\beta)) = \gamma_1$ is attainable for all β if $\gamma_1 \in [\min \gamma_1, \max \gamma_1]$. It is straightforward that $\min \gamma_1 = 0$ at $s(\beta) = 1$ for any β . $\max \gamma_1 = \min_{\beta} \max_{s(\beta)} (1 - s(\beta))I(\beta, s(\beta))$. If $\theta \geq \frac{1}{2}$, $\max \gamma_1 = \frac{\beta}{4\theta}$, with $s(\underline{\beta}) = 1 - \frac{1}{2\theta}$; if $\theta < \frac{1}{2}$, $\max \gamma_1 = (1 - \theta)\underline{\beta}$, with $s(\underline{\beta}) = 0$. Thus, $\gamma_1 \in \left[0, \frac{\beta}{4\theta}\right]$ if $\theta \geq \frac{1}{2}$, and $\gamma_1 \in [0, (1 - \theta)\underline{\beta}]$ if $\theta < \frac{1}{2}$. Rearranging $(1 - s(\beta))I(\beta, s(\beta)) = \gamma_1$ yields $s(\beta) = 1 - \frac{1}{2\theta} \left(1 - \sqrt{1 - \frac{4\theta\gamma_1}{\beta}}\right)$. As $\gamma_1 \in \left[0, \frac{\beta}{4\theta}\right]$ if $\theta \geq \frac{1}{2}$, $0 \leq \sqrt{1 - \frac{\beta}{\beta}} \leq s(\beta) \leq 1$. The seller's optimization problem can be reduced to

$$\max_{\gamma_1} \int_{\beta} \left(\beta \hat{I}(\beta, \gamma_1) - \frac{1-F(\beta)}{f(\beta)} \hat{I}(\beta, \gamma_1) - c(\hat{I}(\beta, \gamma_1)) \right) dF(\beta)$$

subject to

$$\gamma_1 \in [0, \max \gamma_1]$$

where $\hat{I}(\beta, \gamma_1) = I(\beta, s(\beta))$ at $s(\beta) = 1 - \frac{1}{2\theta} \left(1 - \sqrt{1 - \frac{4\theta\gamma_1}{\beta}}\right)$. The objective function being concave in $s(\beta)$ and $s(\beta)$ being concave and monotonically decreasing in γ_1 , the objective function is concave in γ_1 . The equilibrium choice of γ_1 is then either 0, $\max \gamma_1$, or the solution of the first order condition

$$\int_{\beta} \left(\frac{\beta}{2} - \left(\frac{\beta}{2} - \frac{1-F(\beta)}{f(\beta)} \right) \frac{1}{\sqrt{1 - \frac{4\theta\gamma_1}{\beta}}} \right) dF(\beta) = 0$$

At $\gamma_1 = 0$, the left hand side becomes $\int_{\beta} \frac{1-F(\beta)}{f(\beta)} dF(\theta) > 0$. $\gamma_1 = 0$ with $s(\beta) = 1$ for all β is thus sub-optimal. Hence, it is optimal to have $\gamma_1 > 0$ and $s(\beta) = 1 - \frac{1}{2\theta} \left(1 - \sqrt{1 - \frac{4\theta\gamma_1}{\beta}}\right) < 1$, which is increasing in β . Note that for any $F(\beta)$, $\lim_{\beta \rightarrow \bar{\beta}} \frac{d\left[\frac{1-F(\beta)}{f(\beta)} \left(1 - \frac{1-F(\beta)}{f(\beta)} \frac{1}{\beta}\right)\right]}{d\beta} = \lim_{\beta \rightarrow \bar{\beta}} \frac{d\left(\frac{1-F(\beta)}{f(\beta)}\right)}{d\beta} = -1$, which strictly violates Condi-

tion 2. Thus, $s(\beta) < 1$ is optimal for $\beta = \bar{\beta}$; “no distortion at the top” predicted in the conventional screening model with complete contracting no longer holds.

Show (3). If Condition 2 holds and $\frac{d\left(\frac{1-F(\beta)}{f(\beta)}\right)}{d\beta} > 1$ for some $\beta \in [\beta_1, \beta_2]$, (M_I) is violated for these β . The seller then

$$\max_{s(\beta), \gamma_2} \int_{\beta} \left(\beta I(\beta, s(\beta)) - \frac{1-F(\beta)}{f(\beta)} I(\beta, s(\beta)) - c(I(\beta, s(\beta))) \right) dF(\beta)$$

subject to

$$I(\beta, s(\beta)) = \gamma_2$$

$I(\beta, s(\beta)) = \gamma_2$ if and only if $p(\beta, r(\beta), I(\beta, r(\beta))) = \hat{p}I(\beta, r(\beta)) = \gamma_2^2$ for all $\beta \in [\beta_1, \beta_2]$, where $s(\beta) = \frac{\gamma_2}{\beta\theta} - \frac{1}{\theta} + 1$. $0 \leq s(\beta) \leq 1$ if $\beta(1-\theta) \leq \gamma_2 \leq \beta$ for all $\beta \in [\beta_1, \beta_2]$, i.e. if $\beta_2(1-\theta) \leq \gamma_2 \leq \beta_1$. Thus, the set of γ_2 attainable for the seller is not empty if and only if $\beta_2(1-\theta) \leq \beta_1$, i.e. if and only if $\theta \geq \frac{\beta_2 - \beta_1}{\beta_2}$. Otherwise, there is no truth revealing $\{s(\beta), t(\beta), \Delta(\beta)\}$. γ_2 solves the seller’s optimization program in the investment stage,

$$\max_{\gamma_2} \frac{1}{2} \gamma_2^2$$

subject to

$$\beta_2(1-\theta) \leq \gamma_2 \leq \beta_1$$

It is straightforward that the above problem has a corner solution: $\gamma_2 = \beta_1$. Thus, $s(\beta) = \frac{\gamma_2}{\beta\theta} - \frac{1}{\theta} + 1$ is decreasing in β , and $s(\beta) \leq 1$, with equality at β_1 .

□

A.3.7 Proof of Corollary 3.1

If both Condition 1 and 2 hold, $s(\beta) = 1 - \frac{1-F(\beta)}{f(\beta)} \frac{1}{\theta\beta}$ as shown in Appendix A.3.6. With a higher θ , a higher $s(\beta)$ is optimal. If Condition 1 holds and Condition 2 is violated, and γ_1 solves $\int_{\beta} \left(\frac{\beta}{2} - \left(\frac{\beta}{2} - \frac{1-F(\beta)}{f(\beta)} \right) \frac{1}{\sqrt{1-\frac{4\theta\gamma_1}{\beta}}} \right) dF(\beta) = 0$ or $\gamma_1 = \frac{\beta}{4\theta}$ with $\theta \geq \frac{1}{2}$, $4\theta\gamma_1$ is constant in θ , $s(\beta) = 1 - \frac{1}{2\theta} \left(1 - \sqrt{1 - \frac{4\theta\gamma_1}{\beta}} \right)$ is thus increasing in θ . If Condition 2 holds, Condition 1 is violated, and $\theta \geq \frac{\beta_2 - \beta_1}{\beta_2}$, $s(\beta) = \frac{\gamma_2}{\beta\theta} - \frac{1}{\theta} + 1$ where $\gamma_2 = \beta_1 \leq \beta$. The optimal $s(\beta)$ is thus increasing in θ .

□

Appendix B

Deterrence of Information Acquisition with a Risk Averse Agent

Suppose that the agent is risk averse in the realization of transfer, in the sense that $u^A = v(t(q(e, \theta)) - c(e))$, where $v_t(t) > 0$ and $v_{tt}(t) < 0$. The *(IC)* constraint can be replaced by the local incentive compatibility constraint

$$\int_0^{\bar{\theta}} v_t(t) t_q(q) q_e(e, \theta) - c_e(e) dF(\theta) = 0 \quad (LIC_a)$$

if $v(t)$ is sufficiently concave and $c(e)$ is sufficiently convex, and transfer is non-decreasing, $t_q(q) \geq 0$.¹ We assume the former two hold, along with the following assumption for the second best contract to be monotonically non-decreasing.

Assumption B.1. $v(t)$ has non-increasing absolute risk aversion, i.e. $\frac{\partial \left(-\frac{v_{tt}(t)}{v_t(t)} \right)}{\partial t} \leq 0$.

Replacing *(IC)* by *(LIC_a)*, the principal's optimization program to deter a risk averse agent from information acquisition is

$$\begin{aligned} \mathcal{P}_a : \quad & \max_{t(q), e} \int_0^{\bar{\theta}} q(e, \theta) - t(q(e, \theta)) dF(\theta) \\ & s.t. (IR), (LIC_a), (DI) \end{aligned}$$

How the binding constraint *(DI)* distort the optimal (non-monotonic) contract, $t^{DI}(q)$, from the second best, $t^{SB}(q)$ is characterized in the following proposition.

Proposition B.1. *Implementing e^* ,*

¹This is straightforward from the second order derivative of the agent's optimization problem.

1. for $q(e^*, \theta) < q(e(\tilde{\theta}), \tilde{\theta})$, $t^{DI}(q) > t^{SB}(q)$;
2. for $q(e^*, \theta) \geq q(e(\tilde{\theta}), \tilde{\theta})$, $t^{DI}(q) > t^{SB}(q)$ if $f(\theta) > f(\theta')$, and $t^{DI}(q) \leq t^{SB}(q)$ if $f(\theta) \leq f(\theta')$ with equality holds at $f(\theta) = f(\theta')$, where θ' is such that $q(e(\theta'), \theta') \equiv q(e^*, \theta)$;
3. there is a downward gap of $t^{DI}(q)$ at $q(e(\tilde{\theta}), \tilde{\theta})$ from the left.

Proof. Let $\theta' > 0$ is such that $q(e^*, \theta) \equiv q(e(\theta'), \theta')$. For sufficiently small $\theta > 0$, $e(\theta) < e^*$ and $q(e(\theta), \theta) < q(e^*, \theta)$, for sufficiently large θ , $e(\theta) > e^*$ and $q(e(\theta), \theta) > q(e^*, \theta)$, and by $q_{e\theta} > 0$, $\theta' \in (0, \bar{\theta})$ exists. By the first order condition of the principal's point-wise optimization problem with respect to $t(q)$,

$$\frac{1}{v_t(t(q(e^*, \theta)))} = \lambda^{IR} + \mu^a \frac{v_{tt}(t(q(e^*, \theta)))}{v_t(t(q(e^*, \theta)))} t_q(q(e^*, \theta) q_e(e^*, \theta)) + \phi^a \left(1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)} \right),$$

where λ^{IR} , μ^a , and ϕ^a are the Lagrange multipliers associated to constraints (IR) , (LIC_a) , and (DI) , respectively. If κ is sufficiently small that $\phi^a > 0$, $1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)} = 1$ for $\theta' < \tilde{\theta}$, i.e. for $q(e^*, \theta) \equiv q(e(\theta'), \theta') < q(e(\tilde{\theta}), \tilde{\theta})$, hence part 1; for $\theta' \geq \tilde{\theta}$, i.e. for $q(e^*, \theta) \equiv q(e(\theta'), \theta') > q(e(\tilde{\theta}), \tilde{\theta})$, $0 < 1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)} < 1$ for $f(\theta) > f(\theta')$, and $1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)} \leq 0$ for $f(\theta) \leq f(\theta')$, with equality holds at $f(\theta) = f(\theta')$, hence part 2. As $1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)} = 1$ for $\theta' < \tilde{\theta}$ and $1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)} < 1$ for $\theta' \geq \tilde{\theta}$, part 3 is straightforward. \square

Part 1 in Proposition B.1 is intuitive: one motive for the agent to acquire information is to distinguish sufficiently inefficient state of nature to avoid exerting effort at a loss. Thus, to counter such opportunistic motive, the principal increases the transfer for sufficiently inefficient states of nature, reducing the loss subject to those states. It can also be seen as an reward for not acquiring information to avoid loss in the most inefficient states of nature, as $q(e^*, \theta) < q(e(\tilde{\theta}), \tilde{\theta})$ would have been avoided if the agent had acquired information.

Part 2 captures the other opportunistic motive for the agent to acquire information off the equilibrium path: to discover a relatively efficient state of nature to extract maximal rent. It would be clearer if we think of states of nature as discrete states, so that the density is the probability distribution. The principal is unable to judge directly whether a certain realization of output is produced by an uninformed or an informed agent. If an output level is more likely to be realized by an agent who opportunistically acquired information, $f(\theta') > f(\theta)$, it is optimal for the principal to

punish the agent for such realization relative to the second best contract, and if it is more likely to be realized by an agent who did not acquire information, $f(\theta) > f(\theta')$, it is then optimal for the principal to reward the agent for such realization more than the second best would have.

Proposition B.1 is derived without imposing monotonicity on the transfer scheme. Part 3 indicates that, even if the second best transfer is monotonically increasing, the binding constraint to deter information acquisition creates non-monotonicity to the optimal contract. Thus, imposing non-decreasing transfer, there are some non-contingency of transfer on outputs at least near $q(e(\tilde{\theta}), \tilde{\theta})$.²

Corollary B.1. *If $f_\theta(\theta) = 0 \forall \theta \in [0, \bar{\theta}]$, $t^{DI}(q) = t^{SB}(q)$ for $q(e^*, \theta) \geq q(e(\tilde{\theta}), \tilde{\theta})$; if $f_\theta(\theta) > 0 \forall \theta \in [0, \bar{\theta}]$, $t^{DI}(q) < t^{SB}(q)$ for $q(e(\theta^0), \theta^0) > q(e^*, \theta) \geq q(e(\tilde{\theta}), \tilde{\theta})$ and $t^{DI}(q) \geq t^{SB}(q)$ for $q(e^*, \theta) \geq q(e(\theta^0), \theta^0)$; if $f_\theta(\theta) < 0 \forall \theta \in [0, \bar{\theta}]$, $t^{DI}(q) > t^{SB}(q)$ for $q(e(\theta^0), \theta^0) > q(e^*, \theta) \geq q(e(\tilde{\theta}), \tilde{\theta})$ and $t^{DI}(q) \leq t^{SB}(q)$ for $q(e^*, \theta) \geq q(e(\theta^0), \theta^0)$, where θ^0 is such that $e^* = e(\theta^0)$.*

The corollary indicates that, in the case of accepting the contract, if the density of state of nature is increasing, upon observing a sufficiently low realization of output, the principal believes that it is more likely to be produced by an informed agent, and reward the agent less than what he would have been rewarded in the second best; otherwise, she rewards him more than what he would have been rewarded in the second best. The optimal contract to deter information acquisition offers a higher-powered (lower-powered) incentive than offered in the second best if the density of state of nature is increasing (decreasing). This serves as a complement to the literature on information acquisition with the presence of moral hazard mentioned in the literature review, which attributes a higher-powered incentive to inducing information acquisition that generates a mean preserving signal of the random noise. I argue that, deterring information acquisition does not necessarily rely on a lower-powered incentive. It depends on the density of states of nature.

²This non-monotonicity comes from utilization of information off the equilibrium path to reject the contract. If no rejection is possible, i.e. information can only be acquired after signing the contract, part 3 is absent.

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