

**Essays on Fiscal Policy, Monetary Policy and Currency Unions: Exploring the
Role of Informality and Labor Mobility**

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A dissertation
submitted in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy

University of Washington
2016

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Program Authorized to Offer Degree:
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Abstract

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In this dissertation I study how economic activity outside of government control –informality– impacts policy-making in a small open economy. I also study the impact of labor mobility in a currency union on the welfare of the union.

Chapter 1 is concerned with the impact of informality on the Ramsey optimal fiscal and monetary policy. In particular, I ask: how does economic activity outside of government control affect the conduct of fiscal and monetary policy? I study this question in a New Keynesian, small open economy model. The model is assumed to feature informality in both goods and labor markets. A non-traded sector produces a non-taxed informal good. The traded sector produces a formal good and is subject to taxation, but it can hire workers using both formal and informal contracts. I show that the presence of informality decreases the

optimal tax rate and increases macroeconomic volatility. Moreover, when the country cannot credibly precommit to the optimal policy, informality significantly increases the incentive to peg the currency. This result can help explain why many sub-Saharan African countries have plans to either expand existing currency unions or to form new ones.

In Chapter 1 I also investigate the impact of the informal sector on fiscal policy: the tax rate levied by the government in the formal sector and the amount of public debt. With the steady state of the theoretical model described above, I show that the presence of informality decreases the optimal tax rate and increases the level of public debt. Using a panel data of developing countries, I empirically document the negative relationship between the size of the informal sector on the tax rate and its positive relationship with public debt.

Chapter 2 is concerned with how migration within a currency union affects welfare across the union. In particular, I study this question in this paper with a New Keynesian currency union model. The union consists of two countries whose economies are characterized by labor market frictions. One country member has a higher job-finding rate and a lower unemployment rate compared to the other country, hence unemployed agents in the latter have an incentive to relocate to the former. I show that when firms have the ability to hire workers from abroad and when unemployed agents can relocate to a different country, the negative impact of asymmetric shocks is significantly reduced, improving welfare across the union on average.

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Acknowledgments

I first wish to thank my committee chairs Fabio Ghironi and Yu-chin Chen for their guidance, support and encouragement. I am deeply indebted to both for their advice and dedication to see this dissertation through. I have no doubt that without them this dissertation would either not have been completed or look very different.

I would also like to thank my committee member Philip Brock for taking interest in my work and making time to meet and discuss my progress. I also thank Danny Hoffman and Lynn Thomas for taking time out of their busy schedule to serve as the Graduate Student Representative on my committee.

I am grateful to all the participants in the International & Macroeconomics workshops for their questions and useful comments. I thank Emily Cacciatore for reading early drafts of my dissertation and providing very useful editing comments and suggestions.

Finally, I am forever indebted to my mother for all the sacrifices she made so that I can have the best education. Without her I'd not have gotten this far in life.

Dedication

To my mother.

Chapter 1

Fiscal and Monetary Policy in the Presence of Informality and the Incentive to Join a Currency Union

1.1 Introduction

This paper studies how informality affects optimal fiscal and monetary policy in a small open economy. Informality refers to economic activities outside of government regulation, such as firms not declared to the authorities and illegal employment. It is an empirically relevant phenomenon in both developing and industrial countries. Schneider [2005] estimates that the informal sector varies between 38% and 58% of GDP in a number of sub-Saharan African countries.¹ Several countries in the Euro area are similarly characterized by large informal sectors and those with the largest informality are at the center of the recent euro area crisis.² A common trait across these developing and industrial economies is that they have either given up monetary independence by joining the euro or they are considering tak-

¹Nigeria (58%), Benin (45%), Niger (42%), Mali (41%), Côte d'Ivoire (40%), Ghana (38%), Burkina-Faso (38%)

²Greece (29%), Italy (27%), Spain (23%), Belgium (23%), Germany (16%), France (15%)

ing the same action and joining a monetary union: the West African currency union of the Colonies Françaises Africaines (CFA) plans to expand from eight to fifteen member countries by 2020. All current members and those planning to join the new arrangement have large informal sectors. Since informality reduces the tax base and constrains tax revenues, it decreases the room to maneuver fiscal policy for counter-cyclical stabilization. With the loss of monetary policy independence that occurs when countries join a currency union, the constraint placed on fiscal policy by informality may have implications for the desirability of giving up monetary independence. At the same time, informality may exacerbate problems caused by time inconsistency in the conduct of policy. The purpose of this paper is to explore how the different aspects of informality affect fiscal and monetary policy incentives, including the desirability of giving up monetary independence by pegging the currency.

I set up a New Keynesian, dynamic stochastic general equilibrium (DSGE) that extends the benchmark model developed by Galí and Monacelli [2005] to study fiscal and monetary policy in small, industrial economies by introducing informality in goods and labor markets. As in Galí and Monacelli [2005], I assume a continuum of small open economies. Each country has two sectors of production: a non-traded sector and a traded sector. The Home country's non-traded sector is "informal": it is characterized by low productivity, and it is "invisible" to the government. Thus, it is unregulated and not taxed. Non-traded sectors in the rest of the world need not be unregulated. The Home country's traded sector is instead the formal one, regulated and taxed by the fiscal authority. The government collects taxes from firms operating in the formal sector and provides a productive public good that is used by firms in both sectors to produce their respective goods. I follow Loayza [1996] and assume that the informal firms only have partial access to the public good since they are not taxed and regulated by the government. The fiscal authorities have access only to distortionary taxes and can only see and tax the formal sector. When setting the optimal tax rate, the government must take into account the presence of the informal sector. On one hand, the tax decreases formal output since it increases the marginal cost of the formal firms. On the other

hand, tax proceeds are used to finance provision of a public good that decreases marginal costs also in the formal sector, implying a positive effect of taxation of output through this channel. On balance, taxes levied in the formal sector only increase informal output, since the public good decreases the marginal cost of the informal firms. Formal firms do not evade taxes, but they hire a mix of formal and informal workers in order to decrease their marginal cost of production. Formal firms pay the informal workers a lower wage compared to the formal workers.

I use the Ramsey approach of Schmitt-Gohé and Uribe [2004] to characterize optimal fiscal and monetary policy, and I compare optimal policy in the economy with no informality to the economy with informality. I find that the optimal tax rate in the presence of informality is significantly lower than the tax rate when there is no informality. This result is due to the fact that in the presence of informality, the government finds it optimal to decrease the tax rate to prevent the formal good from becoming relatively more expensive than the informal good. Importantly, if the only type of informality in the economy is the use of informal labor in the formal sector, then the government finds it optimal to increase the tax rate. This is because informal labor decreases the formal firms' marginal cost and allows them to increase output. In this case, an increase in the tax rate restores output to its constrained-efficient level. Long run inflation is zero under the optimal policy regardless of the presence of informality, consistent with much of the literature on optimal monetary policy in industrial countries. Informality does not alter the conclusion that zero inflation is optimal under commitment. However, in contrast to the current literature that prescribes price stability over the business cycle as optimal, I find that significant departure from price stability is optimal in response to shocks in the presence of informality. When there is no informality, the volatility of the tax rate and inflation are very low and similar across these policy instruments. Policy authorities should adopt a policy of price stability and smooth taxation over time, in line with the with standard results in the literature on optimal fiscal and monetary policy. In the presence of informality, tax smoothing remains

optimal, but price stability ceases to be so, as indicated by the drastic increase in optimal inflation volatility. When faced with significant informality, the Ramsey policy maker finds tax variation more costly than fluctuations in inflation. This result stands in contrast to those in Schmitt-Gohé and Uribe [2004], where changes in the tax rate were used more often than changes in inflation. If the country pegs its currency, then it can no longer use changes in inflation, and can only rely on fiscal policy. Historically, the CFA-zone countries adopted a common long-run tax rate of 18%. Comparing the welfare under the CFA countries' historical policy to the welfare under the optimal Ramsey policy under commitment and optimal policy under discretion, I find that informality decreases the incentive to join a currency union if the countries can credibly commit to their own optimal policy. However, if the countries do not have the technology or the credibility to commit, pegging the currency is better than pursuing discretionary policy: under discretion, the policy maker uses both changes in inflation and taxation significantly more often, with costs that are amplified in the presence of informality. Thus, informality strengthens the argument for currency pegs originally explored by Giavazzi and Pagano [1988].

The findings in this paper contribute to the literature on informality by focusing on the impact of informality on the conduct of fiscal and monetary policy. Very little is known about how informality affects these policies. The majority of studies on the informal sector, such as Áureo de Paula and Scheinkman [2007] and Marjit and Kar [2009], have been more concerned with its determinants and less with its impact on macroeconomic variables and policy making. Some recent papers have shown that informal labor markets have macroeconomic implications through their impact on aggregate variables, such as inflation (Castillo and Montoro [2012]), and their volatility (Shapiro [2015]). This paper contributes further by exploring the impact of informality on other macroeconomic variables. A small existing literature (Castillo and Montoro [2012] and Batini et al. [2011]) explores the impact of informality on inflation dynamics and the conduct of monetary policy. These papers consider informality in labor and credit markets, ignoring informality in the goods market, and they

conduct their analyses in the context of a closed economy. My framework differs from theirs in that it considers goods and labor informality in the context of a small open economy. Moreover, I address the question of optimal taxation, which many believe is the driver of informality, by studying its impact on the optimal tax rate.

This paper contributes also to the extensive literature on optimal fiscal and monetary policy for small economies. For instance, Galí and Monacelli [2005] and Benigno and Benigno [2003] explore the scenarios in which price stability is optimal for open economies. Schmitt-Gohé and Uribe [2004] find that in the presence of nominal rigidity, the Ramsey planner resorts to price stability and uses fiscal policy more often. The framework laid out in this paper follows the literature on optimal monetary policy by developing a New Keynesian DSGE (NK-DSGE) model with different features of informality. Informality in the labor market allows firms to decrease their marginal cost of production by hiring cheaper labor, and perfect competition in the informal goods market – which implies flexible prices – affects the overall (CPI) inflation dynamics in the economy. Informality thus has direct implications for the conduct of monetary policy and this paper contributes to the literature by laying out the conduct of optimal policy in the presence of informality.

Finally the paper also contributes to the literature on optimal exchange rate regimes for small open economies. Scholars have been giving opposing policy prescriptions on exchange rate regimes for small countries. On one hand, Aghion et al. [2009] conclude that developing countries with low financial development should adopt a less flexible exchange rate in order to enjoy faster economic growth. On the other hand, Levy-Yeyati and Sturzenegger [2003] and Galí and Monacelli [2005] suggest that countries should adopt a more flexible exchange rate regime. Except for Galí and Monacelli [2005], these studies are concerned with the impact of the exchange rate regime on economic growth. I re-evaluate the desirability of relinquishing monetary policy independence in the presence of informality, and I find that informality strengthens the classical argument for exchange rate pegs in Giavazzi and Pagano [1988]: if precommitment to optimal policy is not feasible, a peg (or a currency union) is

more desirable than discretion, the more so the larger the informal sector. This result can help explain why many sub-Saharan African countries have plans to either expand existing currency unions or the form new ones.

The rest of the paper is organized as follows. Section 2 discusses some characteristics of informality, which are the basis of the assumptions I make later in the model. Section 3 lays out the model. Section 4 studies the consequences of informality in a version of the model without nominal rigidity and solves numerically for the optimal tax rate in this environment. Section 5 solves for the optimal fiscal and monetary policies in the presence of price stickiness for the economy with and without informality. Section 6 explores alternative policies and performs a welfare ranking exercise. Section 7 concludes.

1.2 The Informal Sector

Informality can be defined in many ways, but in this paper I use informality to refer to economic activities out of the control of authorities. In the literature, it is also referred to as the underground economy and the black market. Batini et al. [2010] provides a very good summary of informality around the world. Informality can be defined in terms of goods, credit and lending, and labor. In this paper, I cast informality in terms of consumption good and labor. In particular, I use the term informal sector to refer to the non-traded, non-regulated (not taxed) and low productivity sector of the economy. I also introduce informality in the formal sector, to refer to the phenomenon of formal firms hiring informal labor, also known as “under the table” labor. This captures the informalization of the formal sector. Employees with “under the table” contracts receive lower wages than those with formal contracts. All countries have some degree of informality, but the phenomenon is more pervasive in developing countries, especially those in Sub-Saharan Africa. Schneider [2005] estimates the size of the underground economy between 1999 and 2004 at around 41% of GDP in developing economies and about 18% in the OECD countries. In recent years,

policy makers have shown a particular interest in the informal sector and its implication for different policies. Chen [2007] and Williams [2014] both discuss the shift in how the informal sector is viewed. Firstly, there is the change in how informality is defined, shifting from a narrower to a much broader definition. Secondly, there is increased recognition that informality is pervasive and here to stay as long as what is considered to be the formal sector thrives. Policy makers now talk about the accommodation, rather than the eradication, of the informal sector. Since theory predicted that as economies grow, the informal workforce moves to the formal sector and informality decreases, the informal sector was long viewed as a short term phenomenon. However, Kanbur [2015] and Williams [2014] document and discuss its pervasive and long term nature. Many reasons have been given to justify the existence of the informal sector ranging from the burden of regulation (taxation) to the shortage of employment in the formal sector. I do not explicitly model the creation of the informal sector: I take it as given and incorporate its key characteristics affecting the conduct of fiscal policy and monetary policy. I use several studies and surveys of the informal sector to support some of my assumptions. Below, I summarize some key assumptions and use a 2013 study from ANSD [2013] as reference.

Level of Education in the Informal Sector:

I take the informal sector as given and a low level of education as the main cause of its existence. Agents with high level of education are able to secure employment in the formal sector and those with less education find employment in the informal sector. This is in line with previous papers that have modeled the informal sector such as Azuma [2008], Rauch [1991] and Fiess et al. [2001]. The informal sector is documented to have very low productivity (McMillan and Rodrik [2012]) and this can be justified by the low level of education of its workforce, at least in Sub-Saharan Africa. A study from Senegal (ANSD [2013]) supports the claim that the education level of agents operating in the informal sector is low. Many agents did not attend school, some attended only a few years of elementary

education and many received traditional Islamic schooling (learning the Koran without any formal training). A very small fraction, less than 5%, graduated (at least) high school. The following summarizes the findings in the study.

Table 1.1: Level of Education in the Informal Sector in Senegal

Level of Education	Percentage
No School	28
Some Primary School	28.2
Some High School	18
Islamic School	16
Some High School and more	4.5

Competition:

Another important assumption that has implications for the results of the model is the assumption of a perfectly competitive informal sector. In this way, informal prices adjust every period and monetary policy has no direct impact on real (informal) variables. Hence, as the degree of informality increases, the economy becomes more and more competitive and overall price level (CPI) becomes more flexible. ANSD [2013] conducted a survey of firms in the informal sector and found that competition is the biggest problem encountered by those firms, which is due to the non-differentiation of their products. This finding constitutes the basis of my assumption of a perfectly competitive informal sector.

1.3 The Model

The world economy consists of a continuum of small open economies (of size zero) on the unit interval with identical preferences and technology, as in Galí and Monacelli [2005]. Each economy has two sectors of production, a traded sector and a non-traded sector. The representative household consists of a continuum of members with different levels of education. In the Home country, members of the household supply their labor to both the (traded)

formal sector and the (non-traded) informal sector. In the formal sector, some members of the household (those with the highest level of education) receive a formal wage contract and some (with a relatively lower level of education) receive an informal (“under the table”) wage contract, which pays a lower wage than the formal contract. Consumption consists of the informal good and a bundle of formal goods produced domestically and abroad. The government collects a unit tax on the production of formal goods and uses the proceeds to provide non-rival productive public services that are used by both formal and informal firms. Following Loayza [1996], formal firms are registered and known to the authorities, so they have full access to the public services provided by the government. The informal firms, on the other hand, are not declared to authorities and only have access to a fraction of the public services.

1.3.1 Household and Preferences

All households have identical preferences over a consumption bundle C_t and three types of labor N_t^I , N_t^F and L_t . With a discount rate β , the lifetime utility of each household is:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t; N_{\phi_i,t}^F; N_{\phi_i,t}^I; L_{\phi_i,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \int_{\phi} \frac{(N_{\phi_i,t}^F)^{1+\psi}}{1+\psi} d\phi_i - \dots \right\} \quad (1.1)$$

$$\left\{ \dots \int_{\phi} \frac{(N_{\phi_i,t}^I)^{1+\psi}}{1+\psi} d\phi_i - \int_{\phi} \frac{L_{\phi_i,t}^{1+\mu}}{1+\mu} d\phi_i \right\}$$

where ϕ_i represents individual i 's level of education. $N_{\phi_i,t}^F$; $N_{\phi_i,t}^I$ and $L_{\phi_i,t}$ represent the amount of hours individually spent by household members with talent level ϕ_i working. The individual hours are then aggregated over levels of education for each type of contract in order to get N_t^F and N_t^I , which are the aggregate hours devoted to working in the formal sector under formal contract and under informal contract, respectively. L_t represents the aggregate hours devoted to working in the informal sector. This specification of the household's utility function assumes risk sharing within the household; all members consume the

same consumption basket regardless of wage earnings. C_t is the consumption basket of the household and it is, in turn, a composite of consumption indexes defined as:

$$C_t = \left[\alpha^{\frac{1}{\eta}} C_{I,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (1.2)$$

where α is the proportion of the informal good in the consumption bundle. It can also act as a proxy for the size of the informal (goods) sector in the Home country. $C_{I,t}$ is the informal good and $C_{F,t}$ is a composite of consumption index of Home formal goods and Foreign formal goods. η is the elasticity of substitution between the informal good and the formal good. $C_{F,t}$ is defined as:

$$C_{F,t} = \left[(1-a)^{\frac{1}{\gamma}} C_{HF,t}^{\frac{\gamma-1}{\gamma}} + a^{\frac{1}{\gamma}} C_{FF,t}^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} \quad (1.3)$$

where a is the home country's degree of openness and also a measure of home bias ($a < 0.5$). γ is the elasticity of substitution between a domestically produced formal good and a foreign formal good. $C_{HF,t}$ and $C_{FF,t}$ are both indexes of formal goods:

$$C_{HF,t} = \left(\int_0^1 C_{HF,t}(j)^{\frac{\lambda-1}{\lambda}} dj \right)^{\frac{\lambda}{\lambda-1}} \quad C_{FF,t} = \left(\int_0^1 (C_{FF,t}^i)^{\frac{v-1}{v}} di \right)^{\frac{v}{v-1}}$$

where $C_{FF,t}^i$ is an index of all imported formal goods from country i and defined by:

$$C_{FF,t}^i = \left(\int_0^1 C_{FF,t}^i(j)^{\frac{\lambda-1}{\lambda}} dj \right)^{\frac{\lambda}{\lambda-1}}$$

$j \in [0, 1]$ represents the different varieties of formal goods in each country, $i \in [0, 1]$ represents the different countries in the world economy, λ is the elasticity of substitution between formal goods within the same country and v the elasticity between formal goods of two different countries. Each country is assumed to be in financial autarky, so each household maximizes

(1.1) subject to a sequence of budget constraints as follows:

$$P_t C_t \leq \int_{\phi_i} W_{\phi_i,t}^F N_{\phi_i,t}^F d\phi_i + \int_{\phi_i} W_{\phi_i,t}^I N_{\phi_i,t}^I d\phi_i + \int_{\phi_i} \Omega_{\phi_i,t} L_{\phi_i,t} d\phi_i \quad (1.4)$$

where $W_{\phi_i,t}^F$ is the wage earned by an individual with education level ϕ_i in the formal sector under a formal wage contract, $W_{\phi_i,t}^I$ is wage earned in the formal sector under informal wage contract and $\omega_{\phi_i,t}$ is wage earned in the informal sector. Given the above specification of preferences, the optimal allocation of spending yields the following demand functions. The Household's demand for home formal good j and the price index for home formal goods are :

$$C_{HF,t}(j) = \left(\frac{P_{HF,t}(j)}{P_{HF,t}} \right)^{-\lambda} C_{HF,t} \quad P_{HF,t} = \left(\int_0^1 P_{HF,t}(j)^{1-\lambda} dj \right)^{\frac{1}{1-\lambda}}$$

And the HH's demand for foreign formal goods is:

$$C_{FF,t}^i(j) = \left(\frac{P_{FF,t}^i(j)}{P_{FF,t}^i} \right)^{-\lambda} C_{FF,t}^i \quad C_{FF,t}^i = \left(\frac{P_{FF,t}^i}{P_{FF,t}} \right)^{-v} C_{FF,t}$$

$C_{FF,t}^i(j)$ is the HH's demand for foreign formal good j from country i and $C_{FF,t}^i$ is the total imports of formal goods from country i . The Price index for goods from country i , $P_{FF,t}^i$, and the Price index for total imports, $P_{FF,t}$, can be expressed as:

$$P_{FF,t}^i = \left(\int_0^1 P_{FF,t}^i(j)^{1-\lambda} dj \right)^{\frac{1}{1-\lambda}} \quad P_{FF,t} = \left(\int_0^1 (P_{FF,t}^i)^{1-v} di \right)^{\frac{1}{1-v}}$$

The demands for (index of) Home formal goods and Foreign formal good are respectively given by:

$$C_{HF,t} = (1-a) \left(\frac{P_{HF,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} \quad C_{FF,t} = a \left(\frac{P_{FF,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t}$$

The price index for formal goods is:

$$P_{F,t} = [(1 - a)P_{HF,t}^{1-\gamma} + aP_{FF,t}^{1-\gamma}]^{\frac{1}{1-\gamma}} \quad (1.5)$$

The demand functions for the informal good and formal goods are:

$$C_{I,t} = \alpha \left(\frac{P_{I,t}}{P_t} \right)^{-\eta} C_t \quad C_{F,t} = (1 - \alpha) \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t$$

The Consumer Price Index is:

$$P_t = [\alpha P_{I,t}^{1-\eta} + (1 - \alpha) P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}} \quad (1.6)$$

The households intratemporal optimality conditions are:

$$(N_{\phi_i,t}^F)^\psi C_t^\sigma = \frac{W_{\phi_i,t}^F}{P_t} \quad (N_{\phi_i,t}^I)^\psi C_t^\sigma = \frac{W_{\phi_i,t}^I}{P_t}$$

$$L_{\phi_i,t}^\mu C_t^\sigma = \frac{\Omega_{\phi_i,t}}{P_t} \quad (1.7)$$

1.3.2 Employment and Wage Setting

The labor market for each type of employment is perfectly competitive. Each household member takes the wage rate as given and decides how many hours to supply. The labor markets are segmented and labor is immobile. That is, members with no or very little schooling receive informal wages in the informal sector; members with high school education receive informal (“under the table”) wage contracts; and members with college or advanced degrees receive formal wage contracts. Given the competitiveness of the markets, the wage

is constant across each type of employment:

$$W_{\phi_{i,t}}^F = W_t^F \quad W_{\phi_{i,t}}^I = W_t^I \quad \Omega_{\phi_{i,t}} = \Omega_t$$

The household's optimality conditions can be then be written simply as:

$$(N_t^F)^\psi C_t^\sigma = \frac{W_t^F}{P_t} \quad (N_t^I)^\psi C_t^\sigma = \frac{W_t^I}{P_t} \quad L_t^\mu C_t^\sigma = \frac{\Omega_t}{P_t} \quad (1.8)$$

And the intertemporal optimality condition of the household can be written as:

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = Q_{t+1} \quad (1.9)$$

with Q_{t+1} being the discount factor.

1.3.3 Related Prices

I define the relative price of the informal good to the formal good as follows:

$$\Theta_t = \frac{P_{I,t}}{P_{F,t}} \quad (1.10)$$

I further define the bilateral terms of trade between the domestic country and a given country i as the price of country i 's goods in terms of the home goods: $S_{i,t} = \frac{P_{i,FF,t}}{P_{HF,t}}$. The effective terms of trade can then be written as:

$$S_t = \frac{P_{FF,t}}{P_{HF,t}} = \left(\int_0^1 S_{i,t}^{1-v} di \right)^{\frac{1}{1-v}}$$

The bilateral real exchange rate between the Home country and country i is defined as the price of goods from country i in terms of the price of the domestic country's goods, $Q_{i,t} = \frac{\xi_{i,t} P_{i,t}}{P_t}$, and the effective real exchange rate (the price of the goods from the rest of the world in terms of the price of the domestic goods) is $Q_t = \frac{\xi_t P_t^*}{P_t}$. I define the bilateral

real exchange rate for formal goods as $Q_{i,t}^F = \frac{\xi_{i,t} P_{iF,t}}{P_{F,t}}$ and the effective real exchange rate as $Q_t^F = \frac{\xi_t P_{F,t}^*}{P_{F,t}}$. Purchasing Power Parity (PPP) is violated due to the presence of the non-traded informal good.

1.3.4 International Risk Sharing

I assume financial autarky, which implies the value of all imports must equal the value of all exports. The Home country cannot borrow to finance its spending. This yields the following:

$$S_t \left(\frac{P_{FF,t}}{P_{F,t}} \right)^{-\gamma} C_t = \left(\frac{P_{HF,t}}{P_{F,t}} \right)^{-\gamma} \int_0^1 (S_t^i S_{i,t})^{v-\gamma} (Q_{i,t}^F)^{v-\eta} Q_{i,t}^\eta C_t^i di$$

$$S_t^{1-\gamma} C_t = \int_0^1 (S_t^i S_{i,t})^{v-\gamma} (Q_{i,t}^F)^{v-\eta} Q_{i,t}^\eta C_t^i di$$

Under the Cole and Obstfeld specification ($v = \gamma = \eta = 1$) this condition boils down to:

$$C_t = Q_t C_t^* \tag{1.11}$$

which is essentially equivalent to the market sharing condition under complete (market) risk sharing, where C_t^* is the rest of the world's consumption.

1.3.5 Firms and Production

Any given firm in the formal sector produces a differentiated traded good with two types of labor. Both types are perfectly substitutable and produce identical good j :

$$Y_{H,t}(j) = (A_t^F N_t^F(j) + A_t^I N_t^I(j)) G_t^X \tag{1.12}$$

where N_t^F and N_t^I are, respectively, the number of hours worked by the employees with formal wage contracts and employees with no formal wage contracts employed by firm j .

This specification is similar to the one used by Castillo and Montoro [2012]. A_t^F and A_t^I are their corresponding levels of productivity, for which I assume that A_t^I is a fraction κ of A_t^F . Following Loayza [1996] I introduce the flow of government spending as productivity-enhancing with an output elasticity of χ such that the formal firm production function is:

$$Y_{H,t}(j) = (N_t^F(j) + \kappa N_t^I(j)) A_t^F G_t^\chi \quad (1.13)$$

Aggregating over each type of formal good produced, the index of formal good is given by:

$$Y_{H,t} = \left(\int_0^1 Y_{H,t}(j)^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}} \quad (1.14)$$

This aggregate output can then be related to an aggregate level of employment and the price dispersion $\Delta_{p_h,t}$:

$$Y_{H,t} = (N_t^F + \kappa N_t^I) \Delta_{p_h,t} A_t^F G_t^\chi \quad (1.15)$$

The level of productivity follows an AR(1) process as $A_t^F = \rho_F A_{t-1}^F + \epsilon_t^F$.

The perfectly competitive informal firm combines labor and a fraction of the productive public good to produce the informal good. The production function is given by:

$$Y_{I,t} = A_{I,t} L_t (\delta G_t)^\chi \quad (1.16)$$

δ is the fraction of public services to which firms operating in the informal sector have access. Consequently, δ can also serve as an indicator of the effectiveness of the authorities' crackdown on informal firms. The harder they crack down the lower the fraction of public services informal firms have access to, hence the lower δ is. One could assume that public goods have different impacts on both sectors or that congestion might arise with rapid growth of the informal sector, but to simplify the analysis I assume that output elasticity of government spending is constant across both sectors. The level of productivity is an AR(1) process, $A_t^I = \rho_I A_{t-1}^I + \epsilon_t^I$.

1.3.6 The Government

The government or the fiscal authorities collect a unit tax τ_t on every formal good sold. It then uses the tax revenues to provide a non-rival public good G_t . The government runs a balanced budget in every period as follows:

$$G_t = \tau_t Y_{H,t} \quad (1.17)$$

From this relation, it is clear that the size of the public good directly depends on the size of the formal sector and the tax rate. Since the public good is productive, an expansion of the formal sector has a positive externality on the informal sector. The tax in itself is distortionary for the formal sector, because it decreases the amount of goods produced, but at the same time it is productivity-enhancing for both sectors through the provision of the public good.

1.3.7 Pricing

Since the informal sector is perfectly competitive, the price is equal to the marginal cost and is fully flexible.

$$P_{I,t} = MC_{I,t}$$
$$MC_{I,t} = \frac{\Omega_t}{A_{I,t} (\delta G_t)^\chi}$$

Using the labor consumption substitution rate from the household optimality, $\Omega_t/P_t = L_t^\mu C_t^\sigma$, the informal price setting relation becomes:

$$P_{I,t}^{1-\alpha} = \frac{L_t^\mu C_t^\sigma}{A_{I,t} (\delta G_t)^\chi} P_{F,t}^{1-\alpha} \quad (1.18)$$

And in terms of inflation rates:

$$\Pi_{I,t}^{1-\alpha} = \left(\frac{G_{t-1}}{G_t}\right)^\alpha \left(\frac{C_t}{C_{t-1}}\right) \left(\frac{A_{i,t-1}}{A_{i,t}}\right) \Pi_{F,t}^{1-\alpha} \quad (1.19)$$

Prices are sticky in the formal sector, à la Calvo (1983). In each period, a fraction θ of all formal firms get to reset their price. The firms set the optimal price by maximizing the discounted values of all expected profits. The optimal price must satisfy the following first order condition:

$$P_{H,t}^* = \left(\frac{\gamma}{\gamma-1}\right) \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t [Q_{t+k} P_{H,t+k}^{\gamma+1} Y_{H,t+k} MC_{t+k}^n]}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t [(1-\tau_{t+k}) Q_{t+k} P_{H,t+k}^\gamma Y_{H,t+k}]} \quad (1.20)$$

where $Q_{t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$. Expressed in terms of inflation rates and written recursively this equation becomes:

$$\begin{aligned} \Pi_{HF,t}^* &= \left(\frac{\gamma}{\gamma-1}\right) \frac{F_{1t}}{F_{2t}} \\ F_{1t} &= Y_{H,t} MC_{F,t} + \theta\beta \left(\frac{C_{t+1}}{C_t}\right)^{-1} \Pi_{t+1}^{-1} \Pi_{HF,t+1}^\gamma F_{1,t+1} \\ F_{2t} &= Y_{H,t} (1-\tau_t) + \theta\beta \left(\frac{C_{t+1}}{C_t}\right)^{-1} \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma-1} F_{2,t+1} \end{aligned}$$

The optimal price inflation and the overall formal sector inflation are linked by the following equation:

$$\Pi_{HF,t}^* = \left(\frac{1-\theta\Pi_{HF,t}^{\gamma-1}}{1-\theta}\right)^{\frac{1}{1-\gamma}} \quad (1.21)$$

The price dispersion $\Delta_{ph,t}$, due to price stickiness is expressed as:

$$\Delta_{ph,t} = \theta\Pi_{HF,t}^\gamma \Delta_{ph,t-1} + (1-\theta) \Pi_{HF,t}^{*(-\gamma)} \quad (1.22)$$

1.3.8 Equilibrium

The informal firms only have to meet domestic demand, so market clearing in the informal sector entails output being equal to domestic demand:

$$Y_{I,t} = \alpha \left(\frac{P_{I,t}}{P_t} \right)^{-\eta} C_t \quad (1.23)$$

The formal good is consumed both domestically and abroad, so market clearing in the formal sector entails output being equal to domestic demand plus foreign demand (exports):

$$Y_{H,t} = (1 - \alpha) \left(\frac{P_{HF,t}}{P_{F,t}} \right)^{-\gamma} \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} \left[(1 - a)C_t + a \int_0^1 (S_t^i S_{i,t})^{v-\gamma} (Q_{i,t}^F)^{v-\eta} Q_{i,t}^\eta C_t^i di \right] \quad (1.24)$$

where $Q_{i,t}^F$ is the formal good real exchange rate. And assuming the Cole and Obstfeld specification ($\sigma = \eta = v = \gamma = 1$) these two market clearing conditions reduce to:

$$Y_{I,t} = \alpha \Theta_t^{\alpha-1} C_t \quad (1.25)$$

$$Y_{H,t} = (1 - \alpha) S_t^a \Theta_t^\alpha [(1 - a)C_t + aQ_t C_t^*] \quad (1.26)$$

Using the risk sharing condition under financial autarky ($C_t = Q_t C_t^*$), the formal sector market-clearing equilibrium can be written as:

$$Y_{H,t} = (1 - \alpha) S_t^a \Theta_t^\alpha C_t \quad (1.27)$$

When there is no good informality $\alpha = 0$ and $Y_{I,t} = 0$. The formal sector market-clearing then becomes $Y_{H,t} = S_t^a C_t$. Combining the market-clearing in the informal and formal sectors and the international risk sharing, domestic consumption can be expressed as:

$$C_t = \left(\frac{Y_{H,t}}{1 - \alpha} \right)^{\frac{(1-\alpha)(1-a)}{1-2\alpha}} \left(\frac{Y_{I,t}}{\alpha} \right)^{\frac{-\alpha}{1-2\alpha}} (C_t^* \Theta_t^{\alpha})^{\frac{\alpha(1-\alpha)}{1-2\alpha}} \quad (1.28)$$

The terms of trade and the relative price of the informal good to the formal good can be expressed as:

$$\Theta_t = \left(\frac{Y_{I,t}}{\alpha C_t} \right)^{\frac{1}{\alpha-1}} \quad (1.29)$$

$$S_t = \left(\frac{Y_{I,t}}{\alpha} \right)^{\frac{\alpha}{(1-\alpha)(1-a)}} C_t^{\frac{1-2\alpha}{(1-\alpha)(1-a)}} \left(C_t^* \Theta_t^{*(-\alpha)} \right)^{\frac{1}{1-a}} \quad (1.30)$$

1.3.9 Marginal Costs and The Linkage Between Formal and Informal Sectors

The informal sector having access to public goods provided by the government through taxation of the formal sector makes the former dependent on the latter. The size of the public good is positively correlated with the size of the formal good sector for any given tax rate. By increasing informal sector productivity, public good effectively decreases the informal sector marginal cost. The formal firm's marginal cost chooses formal and informal labor until their respective marginal cost are equalized. This implies a constant proportion of formal labor to informal labor given by:

$$N_t^F = \kappa^{\frac{1}{\psi}} N_t^I \quad (1.31)$$

which implies that formal production in terms of formal labor can be expressed as:

$$Y_{H,t} = \bar{v} A_t^F N_t^F G_t^X \quad (1.32)$$

where $\bar{v} = \left(1 + \kappa^{1+\frac{1}{\psi}} \right)$. The marginal cost in the formal sector can then be expressed as:

$$MC_t = \frac{W_t^F}{\bar{v} A_t^F G_t^X (1 - \tau_t)} \quad (1.33)$$

The above marginal cost is in terms of formal wage contracts, but it makes use of the optimal labor proportion so that the marginal cost in terms of the informal wage contract is identical. The informal firm's marginal cost can be written as:

$$MC_{I,t} = \frac{\Omega_t}{A_{I,t} (\delta G_t)^\chi} \quad (1.34)$$

Since prices are fully flexible in the informal sector, we have that $MC_{I,t} = 1$ and the output level is given by:

$$Y_{I,t} = A_{I,t} (\delta G_t)^\chi \alpha^{\frac{1}{1+\mu}} \quad (1.35)$$

G_t is a function of $Y_{H,t}$, so the informal sector output increases with the formal sector output. The presence of informality in the formal sector decreases the elasticity of the marginal cost with respect to output. An increase in demand (positive demand shock) causes the marginal cost to increase less as informality (in the formal sector) increases; this also means that inflation increases less. The tax levied by fiscal authorities increases formal sector marginal cost and the provision of public good decreases the marginal cost. The informal sector, on the other hand, benefits from a decrease in marginal cost without the tax burden. The current set-up of the model is such that the informal sector has no impact on the productivity of the public good. that can be changed by making the public good less productive with an increase in the informal sector but not the formal sector, since that sector contributes to the financing of the public good. The public good is dependent on the size of the formal sector and the tax rate levied by the fiscal authorities. When $\chi = 0$, this equation boils down to the regular output level found in the New Keynesian literature; fully flexible price level of output is only a function of the productivity shocks.

1.3.10 The Rest of the World

The world economy is taken as given, and the home economy is considered to be too small to have any impact on its dynamics. The rest of the world is populated by identical countries

with two sectors of production, traded and non-traded (not necessarily informal). The traded sector, denoted by T, and the non-traded sector, denoted NT, are characterized by the following equations. The Marginal cost in the traded sector is:

$$MC_t^* = \frac{C_t^{*(\sigma+\psi)}}{A_t^*} \quad (1.36)$$

Firms in the traded sector that reset their prices choose the optimal price:

$$\Pi_{FF,t+k}^* = \left(\frac{\gamma}{\gamma-1} \right) \frac{F_{1t}^*}{F_{2t}^*} \quad (1.37)$$

where:

$$\begin{aligned} F_{1t}^* &= C_t^* MC_t^* + \theta \beta \Pi_{t+1}^{*(-1)} \Pi_{FF,t+1}^{*(\gamma+1)} F_{1,t+1}^* \\ F_{2t}^* &= C_t^* + \theta \beta \Pi_{t+1}^{*(-1)} \Pi_{FF,t+1}^{*\gamma} F_{2,t+1}^* \end{aligned}$$

The non-traded sector only produces a fraction of the traded sector output as follows:

$$Y_{NT,t}^* = \alpha C_t^* \quad (1.38)$$

The average marginal cost of a firm in the non-traded sector is given by:

$$MC_{NT,t}^* = \frac{Y_{NT,t}^{*(\sigma+\psi)}}{A_{NT}^*} \quad (1.39)$$

The respective inflation rates in the non-traded and traded sector are given by the expressions:

$$\begin{aligned} \Pi_{NT,t} &= \frac{MC_{NT,t}^*}{MC_{NT,t-1}^*} \\ \Pi_t^* &= \Pi_{NT,t}^\alpha \Pi_{FF,t}^{*(1-\alpha)} \end{aligned}$$

where, for simplicity, I assume the non-traded sector to have fully flexible prices, just like the non-traded informal sector in the Home country. I take France as the rest of the world in this set-up, because the CFA-Franc used in these countries was originally pegged to the French Franc and now to the Euro (I also used calibration parameters of the Euro and the results did not change).

1.3.11 The Distortions

In this section I look at the different distortions in the economy.

Informality: The presence of informality distorts the labor market by reducing the amount of formal labor and allowing firms to replace formal labor with informal labor. This affects the marginal cost of production in the formal sector. Informality in goods decreases formal labor ($1 - \alpha$) and labor informality increases formal labor ($v > 1$).

Distortionary Taxation: The provision of the public through a unit sales tax increases the marginal cost of production, thus decreasing the level of output and level of employment in the formal sector, since only the formal sector is taxed.

Nominal Rigidity: The presence of nominal rigidity, namely price stickiness, induces dispersion in price levels and thus impacts real variables like output.

Monopolistic Distortion: Firms with monopolistic power produce less than the optimal level of output and create a lower level of employment.

1.3.12 Calibration

I pull the parameter values used to obtain the simulated results from several sources. The countries in the CFA zone are similar in terms of economic structure and characteristics. In that sense the parameters used to simulate the model can be extended to all the countries

in the zone. I use the ratio of imports to GDP as a measure of openness and I obtain an estimate from the World Bank’s World Development Indicator. For the size of the non-traded informal sector, I use Schneider [2005] and Schneider et al. [2010] as a reference and the average of the estimates reported for the CFA zone countries. I set α , the fraction of household basket devoted to the informal good, so that the informal output as a percentage of total output match the estimates in Schneider [2005] and Schneider et al. [2010]. Different estimates of the elasticity of output with respect to government spending are used to simulate sensitivity analysis. Yasin [2009] reports an elasticity of government spending in SSA to be $\chi = 0.1$. However, I set $\chi = 0.01$ for the welfare analysis because the discretionary policy’s welfare loss is sensitive to the value of χ . I should note that different values of χ does not affect welfare ranking. The remaining parameters are standard in the literature. Table 2.3 in the Appendix summaries the parameters of the model and their sources.

1.4 Flexible Price Equilibrium and Optimal Fiscal Policy

Before I look at the conduct of monetary policy within the presence of nominal rigidity, I analyze the model under flexible prices. Under the assumption of flexible prices, the Ramsey optimal problem is static. I first consider the economy without informality and then I look at the implication of the presence of each type of informality separately.

1.4.1 Optimal Taxation and Informality

When there is no informality the optimal tax rate in the formal sector is well above zero at around 17.5%. This tax rate increases with χ , the elasticity of output with respect to the public good as reported in Table 1.5. The public good is financed by the tax proceed and enhances productivity, so as it becomes more productive, the authorities find it optimal to increase the tax rate since its positive effect is now higher. And the low volatility of the tax

rate under flexible prices indicates that the government spreads the tax burden over time and does not change often.

To isolate the effect of each type of informality in order to better understand the channels through which they impact the economy, I introduce each type separately and explore how each type impacts the optimal tax rate in the formal sector. In particular I first introduce labor market informality into the formal sector ($v > 1$ or $x > 0$ and $\alpha = 0$). With informal labor, formal firms are able to decrease their production cost. As informal labor in the formal sector increases, the optimal tax rate increases (Table 1.6). As the marginal cost decreases, taxation becomes less and less distortionary and the Ramsey planner finds it optimal to increase the tax rate. An increase in labor informality in the formal sector has an impact on the optimal tax rate similar to that of an increase in the output elasticity of the public good.

By setting $v = 1$ or $x = 0$ and $\alpha > 0$, I shut off labor informality in the formal sector and introduce goods informality into the economy. The less productive non-traded sector that produces the informal good is not taxed, but the formal sector is. The optimal tax rate decreases with the expansion of the informal good sector (Table 1.7). The presence of the informal sector increases the marginal cost of the formal good production and makes it relatively more expensive than the informal good. The Ramsey planner then finds it optimal to decrease the optimal tax rate to decrease the marginal cost of producing the formal good. This way, the formal good is not relatively more expensive compared to the informal good.

The two types of informality have opposite effect on the optimal tax rate in the formal sector. I introduce both into the model and I study the cumulative effect of both labor market informality and goods market informality on the optimal tax rate. In particular, I compare the economy with no informality ($\alpha = 0$ and $x = 0$) to the economy with both types of informality ($x = 0.7$ and $\alpha = 0.4$). Table 1.8 reports the simulated moments. The optimal tax rate is considerably lower in the presence of all types of informality, 0.72% compared to 17.5% when there is no informality. The impact of α , the size of the informal sector, on the

optimal tax rate is much larger than the combined effects of χ and x , the output elasticity of government spending and the measure of labor informality. In particular as the size of the informal sector increases, the optimal Ramsey tax decreases (Figure 1.1).

In the presence of informality, it is important to the Ramsey planner to a lower the tax rate in order to prevent the formal good from becoming relatively more expensive compared to the informal good. The policy implications suggest that fiscal authorities in countries with informal sectors should adopt lower tax rate so that goods from the (taxed) formal sector are relatively less expensive for consumers.

It is also worth noting that the extent to which the informal sector firms have access to the public good, as measured by the parameter δ , has no impact on the optimal tax rate. This suggests that the government should focus on the formal sector instead of spending resources to crack down on the informal sector.

Empirical Evidence

I empirically document the effect of the shadow economy on the tax rate and level in order to verify the steady results of the model. Figure 1.2 plots tax revenues (as a percentage of GDP) against the size of the informal sector; there is a negative relationship between these variables.

I collect data from the World Bank Development Indicators (WDI), the World Bank Government Indicators (WGI), and the IMF's World Economic Outlook. The variables used and their respective sources are listed in the appendix. I estimate the following two equations separately:

$$TRate_{i,t} = \alpha + \gamma IS_{i,t} + \psi X_{i,t} + \epsilon_{i,t} \quad (1.40)$$

where $TRate_t$ is the tax revenue collected as a percentage of total GDP. I use tax revenue as

a percentage of GDP as a proxy for the tax rate levied on firms in the formal sector. IS_t is the size of the informal sector as estimated by Schneider et al. [2010]. The panel data goes from 1999 to 2007 because estimates of the size of the informal sector is only available for those years. Due to missing data, the panel is unbalanced. Finally, X_t is a matrix of control variables known to have an impact on tax revenues.

Due to concerns over endogeneity, such as the tax rate inducing firms to relocate to the informal sector, I use general methods of moments (GMM) to estimate the model. This methodology also allows me to address other concerns such as measurement errors, especially about the size of the informal sector, and omitted variables. This identification method was also used by Ceyhun and Uras [2013].

Table 1.4 report the results of the estimations. As expected, the size of the informal sector negatively impacts tax revenues. The estimated coefficient on the informal sector is significant.

1.4.2 Public Debt and Informality

So far the government budget is balanced, but one could change this specification and allow the government to borrow and study the impact of the informal sector on public debt. Each period, the authorities are faced with a random level of public spending that must be financed with tax revenues and debt. In addition they must also repay existing debt. Now the government's budget constraint becomes:

$$G_t + R_{t-1}D_t = \tau_t Y_{H,t} + D_{t+1} \quad (1.41)$$

Here the provision of the public good follows a random walk: $G_t = \rho_g G_{t-1} + \epsilon_t^g$. Assuming a constant long run constant tax rate, public debt rises with the size of the informal sector. Figure 1.4 plots long run public debt against the size of the informal sector for select number of countries.

Empirical Evidence

I empirically investigate the findings of the theoretical model. Figure 1.3 plots government debt and the size of the informal sector and the fitted line exhibit a positive slope. Using GMM I estimate the following equation:

$$Pdebt_t = \delta + \lambda IS_t + \beta X_t + \epsilon_t \quad (1.42)$$

where $Pdebt_t$ is the country's public debt as a percentage of the total GDP. IS_t is again the size of the informal and the panel data goes from 1999 to 2007 because estimates of the size of the informal sector is only available for those years. Z_t is a matrix of control variables known to have an impact on public debt.

Table 1.3 report the results of the estimations and the size of the informal sector positively impacts public debt.

1.4.3 Dynamic Adjustment Under Flexible Prices

I now analyze the response of some select macroeconomic variables to a productivity shock in the formal sector. Figure 1.5 plots the responses of the variables. In both cases, there is an increase in formal and informal outputs. The presence of informality has no impact on the dynamic adjustment of the formal output. The difference is observed in the adjustment of consumption and government spending. Consumption increases significantly in the presence of informality and government spending increases considerably in the absence of informality. The increase in consumption can be attributed to the increase in both the informal good and the formal good. The tax rate remains constant over the business cycle in both cases. The increase in the government spending is then justified by the increase in the formal good. The informal output responds to the formal sector productivity shock due to the public good provided through taxation of formal firms. This replicates the empirical documentation in Kanbur [2015], which shows that the informal sector has kept up with the

expansion of the formal sector in emerging economies for the past three decades.

1.5 Sticky Price Equilibrium

This section introduces nominal rigidity in the form of price stickiness in the formal sector. I model price stickiness à la Calvo (1983). To close the model in this case, monetary and fiscal policy rules are needed. Here, I derive the optimal policies under commitment. Since a closed form solution is not attainable, I provide numerical simulations of the Ramsey allocations and optimal policies. The optimal monetary and fiscal policies derived in this section are used as benchmarks for comparing alternative monetary policy regimes, namely the optimal policy under discretion and an exchange rate peg policy. Nominal rigidity introduces forward-looking variables in the optimality conditions of the economy, so I first focus on the long run outcomes of the macro variables and then proceed to analyze their short run dynamic adjustments over the business cycle.

1.5.1 Optimal Fiscal and Monetary Policy in the Long Run

The Economy with No Informality

In the economy without informality, where $\alpha = \kappa = 0$, the Ramsey planner maximizes total welfare (lifetime utility) subject to market-clearing conditions and the FOCs of the households and firms:

$$\text{Max} \quad E_0 \left\{ \sum_{t=0}^{\infty} \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(N_t^F)^{1+\psi}}{1+\psi} \right\}$$

s.t. households' and firms' optimality condition are satisfied and all markets clear.

The allocations under sticky prices are identical to the flexible price allocations and the optimal inflation is equal to zero when I assume that the government levies the optimal tax rate from the flexible price allocations and the monetary authority takes that as given and set

monetary policy optimally (Ramsey commitment). The results are reported in Table 1.10. This is a standard result in the New Keynesian literature on monetary policy. When the tax is set at the flexible price optimal rate, the steady state is constrained efficient and the planner faces one distortion (nominal rigidity), so price stability is optimal and the flexible price allocation is replicated.

The long run outcomes become differ slightly when I assume that the Ramsey planner chooses the optimal tax rate and the optimal inflation rate simultaneously. The appendix contains the details of the derivation of the FOCs and the long run allocations are reported in Table 1.9. The long run optimal inflation is zero, which is consistent with most of the results in the New Keynesian literature on optimal monetary policy. One important observation is the sharp increase in the optimal tax rate with the introduction of nominal rigidity: the optimal tax rate under flexible prices stands at 17.52%, but in the presence of price stickiness it increases to 48%. The allocations of the remaining macroeconomic variables mirror quite well the allocations under flexible prices, but at the expense of higher taxation in the formal sector. Siu [2004] also report an increased optimal tax rate with increased price stickiness. In this situation, the steady state is neither efficient nor constrained efficient and the planner has only two instruments for more than two distortions. The authorities then set the tax rate higher than the optimal rate in the case of flexible prices to increase the size of the public good and increase output. This is necessary to replicate the flexible price allocations for the remaining macroeconomic variables.

The Economy with Informality

In the economy with informality, where α and x are both different than zero, the Ramsey planner solves the following problem:

$$\text{Max} \quad E_0 \left\{ \sum_{t=0}^{\infty} \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(N_t^F)^{1+\psi}}{1+\psi} - \frac{(N_t^I)^{1+\psi}}{1+\psi} - \frac{L_t^{1+\mu}}{1+\mu} \right\}$$

s.t. the households' and firms' optimality conditions are satisfied and all markets clear.

The details of the derivations are contained in the appendix. As in the case with no informality, if the tax rate is set at the optimal rate under flexible prices and the Ramsey planner solves for the optimal monetary policy, the long run allocations mirror the flexible prices allocations. Table 1.12 reports the simulated means of some select variables.

Table 1.11 shows the long run mean of some variables when the Ramsey planner chooses the optimal tax rate and inflation simultaneously. The sticky price allocations are different than the flexible price allocation except for consumption. All the remaining variables are higher in the presence of sticky prices. The tax rate increases with the presence of nominal rigidity, from 0.72% to 2.8%. The increase in the tax rate erases the inefficiency caused by nominal rigidity and even increases output due the productive nature of the public good provided by the fiscal authority.

1.5.2 Optimal Fiscal and Monetary Policy in the Short Run

Table 1.14 reports the standard deviation of the different variables. The introduction of informality induces higher volatility in all of the variables except for the optimal tax rate in the formal sector. When there is no informality, volatility of both the formal good inflation and the tax rate are low at 0.03 and 0.05, respectively. This indicates that the policy authorities do not change these two policy instruments very often over the business cycle and tend to resort to tax changes in the formal sector slightly more often than inflation changes. In the presence of informality, the volatility of formal good inflation increases to 0.10 and that of the tax rate decreases to about 0.003. It is now costly to resort to fiscal policy changes when the economy has both informal labor and goods market, so the policy authority makes more usage of inflation changes.

The economies with and without informality respond differently to a productivity shock in the formal sector. Looking at the impulse responses under a technology shock in the formal sector, for the economy with no informality and that with informality, the tax rate in the formal sector remains unchanged when there is informality but drops then increases

when there is informality. However, in both cases the optimal deviations from steady state replicate the deviations under flexible prices. Regardless of informality, the optimal policies call for deviations from price stability, which is in contrast with the results in the recent literature such as Galí and Monacelli [2005] and Benigno and Benigno [2003]. Furthermore, even though the optimal policy calls for departure from price stability, the directions of the departure are opposite for Home formal inflation on impact. With no informality, the optimal response of formal inflation is to decrease then increase before returning to its steady state level. However, in the presence of informality, the opposite happens: formal goods inflation responds by increasing then decreasing (remaining negative) before returning to its steady state. Another key difference concerns government spending, which increases significantly less in the presence of informality. That is due to the low optimal tax rate levied by the authority in the presence of informality. CPI inflation increases in both cases, but significantly more in the presence of informality due to the increases in the terms of trade, nominal exchange rate and informal price inflation. One can note that the magnitude of the responses tends to be larger in the presence of informality except for the responses of government spending and the tax rate. Informality induces higher macroeconomic volatility as reported in Table 1.14.

1.6 Alternative Policies/Incentive to Join a Currency Union

I explore the implications of alternative policies to the optimal Ramsey policy under commitment. Namely I examine the historical policy that has been adopted by the countries of the CFA zone and also the Ramsey problem under discretion. Soffritti and Zanetti [2008] perform a similar exercise and conclude that in the case in which the country cannot credibly commit to a policy, pegging its exchange rate is better than adopting a discretionary policy. What are the implications of the presence of informality in the choice of an exchange rate

regime?

1.6.1 Exchange Rate Peg and Ramsey Optimal Fiscal Policy

The first alternative policy I consider is a policy of exchange rate peg coupled with an optimal Ramsey taxation in the formal sector.

$$\text{Max} \quad E_0 \left\{ \sum_{t=0}^{\infty} \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(N_t^F)^{1+\psi}}{1+\psi} - \frac{(N_t^I)^{1+\psi}}{1+\psi} - \frac{L_t^{1+\mu}}{1+\mu} \right\}$$

s.t. the households' and firms' optimality conditions are satisfied, all markets clear and

$$\Delta E_t = 1.$$

In this case fiscal policy and very active and monetary policy becomes very passive, which is in line with the optimal volatility under the optimal Ramsey under commitment. This is confirmed in the welfare analysis. This policy of optimal Ramsey fiscal policy and an exchange rate performs as well as the optimal Ramsey fiscal and monetary policy. This result is due to flexible price in the informal sector. If there were nominal rigidity in the informal sector, then one active optimal policy would not be enough to achieve the optimal welfare.

1.6.2 Historical Policies: Exchange Rate Peg and Flat Tax Rate

The historical policies followed by these countries have been a flat tax rate and a fixed exchange rate. The common currency is currently pegged to the Euro; however it was pegged to the French Franc at the time of its creation. The parity was changed only once (January

1994) since its inception. The historical policy can be summarized by:

$$\tau_t = 0.18$$

$$\Delta E_t = 1$$

This flat tax is very close to the optimal tax rate under flexible price and no informality, which is 17.5%. This is just a mere coincidence and a different calibration would yield different values for the optimal tax rate under flexible prices, but would not change the reported results. The simulated moments of the variables under this historical regime, with and without informality, are reported in Table 1.15. One thing is very clear: in the long run, the allocations under the historical policy are almost identical to the allocations under the optimal Ramsey policy when there is no informality. Under the optimal Ramsey policy with commitment, the tax rate and formal inflation rate have very low volatility. Under the historical policy, the tax rate is constant so it mimics the optimal fiscal policy under commitment. However, the volatility of the formal good inflation is higher in the presence of informality than the volatility under the optimal commitment policy. Without the changes in the nominal exchange rate, the planner resorts to more changes in the formal inflation rate.

1.6.3 Discretionary Policy: Fiscal and Monetary

Under discretion, the monetary authorities choose the optimal policies in every period. I establish the optimal discretionary policy and calculate the welfare level under different degrees of informality. With $\alpha = \kappa = 0$, there is no informality in the economy and the

planner solves the following problem:

$$\text{Max} \quad \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(N_t^F)^{1+\psi}}{1+\psi} \right\}$$

s.t the HH and firms' optimality conditions are satisfied and all markets clear.

If instead the economy is now characterized by both labor and good informality, the planner solves the following problem:

$$\text{Max} \quad \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(N_t^F)^{1+\psi}}{1+\psi} - \frac{(N_t^I)^{1+\psi}}{1+\psi} - \frac{L_t^{1+\mu}}{1+\mu} \right\}$$

s.t the HH and firms' optimality conditions are satisfied and all markets clear.

The simulated moments of the variables under this discretionary regime for both economies with and without informality are reported in Table 1.16. The long run optimal inflation, as expected, is positive. Under discretionary policy the monetary authorities use inflation as a tool to increase output. With no informality, the tax rate and formal inflation have zero standard deviation. In the presence of informality, the volatility of the formal inflation is 0.04 and that of the tax rate is 0.3. Under discretion, the policy authority resorts to more changes in tax policy than inflation policy, which is contrary to the policy prescription under the optimal commitment policy. Recall that under the optimal commitment policy the standard deviation of the formal inflation is 0.1 while that of the optimal tax rate is 0.003.

1.6.4 Welfare Ranking

I calculate the long run welfare level under different policy regimes: the optimal Ramsey policy under commitment, the optimal Ramsey policy under discretion and an exchange rate peg. I use the long run level of welfare under the optimal Ramsey commitment policy and

then calculate the increase in consumption needed under the alternative regimes in order to close the gap in terms of welfare.

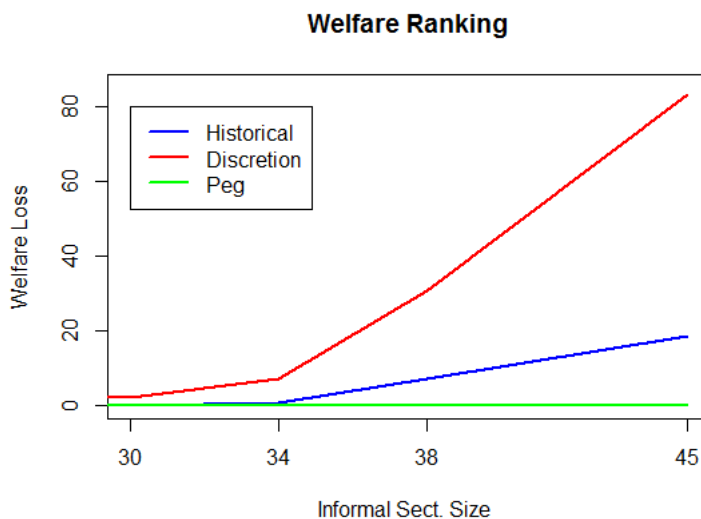
$$W^c = \frac{U \left[\left(1 + \frac{\Delta^p}{100}\right) C^p, N_F^p, N_I^d, L^p \right]}{1 - \beta} \quad (1.43)$$

where W^c is the long run welfare under the Ramsey commitment policy and p refers to the alternative policy being evaluated (i.e exchange rate peg). Figure 1.6.4 below plots the Δ^p for different sizes of the informal sector. The welfare loss increases with the size of the informal sector when the country pegs its currency.

The optimal Ramsey fiscal and monetary policy under commitment yield the highest welfare. This is a standard result in the literature on optimal monetary policy suggesting that policy authorities should follow a credible commitment policy over time. The policy of an exchange rate peg coupled with the optimal Ramsey fiscal policy performs as well as the policy of Ramsey fiscal and monetary policy. The welfare loss is greatest under the optimal Ramsey optimal fiscal and monetary policy under discretionary regardless of the presence of informality. The historic regime of an exchange rate peg and fixed tax rate, in turn outperforms the discretionary regime, and the gap is amplified in the presence of informality. As the size of the informal sector increases, the welfare loss under the historical policy and the discretionary policy rises. Hence the informal sector decreases a country's incentive to peg its currency and should make use of rule based fiscal and monetary policy.

However, If I assume that these countries' central banks do not have the independence and credibility for commitment and that the choice is then between the historical policy of pegging their exchange rate plus a constant tax rate and conducting discretionary policy every period, then I can conclude that the presence of informality *increases* the incentive of these countries to join a currency union. The ability to credibly and effectively of commit to an optimal fiscal and monetary policy hinges on the level of independence of the central bank and its strength. Dincer and Barry Eichengreen [2014] reports regional central transparency

and independence and the African as a whole had a score of 0.34 (out of 100) in 2010. Giavazzi and Pagano [1988] and Soffritti and Zanetti [2008] also arrive at a similar conclusions. These previous results are strengthened in the presence of informality.



Different calibrations of the model yield different allocations and welfare losses, but do not alter the welfare rankings. The welfare loss is sensitive to the elasticity of government spending; higher elasticity of public goods causes larger welfare loss under all sub-optimal policy regimes, particularly under the discretionary policy regime.

In conclusion, fiscal and monetary authorities should never adopt a discretionary policy regime, particularly if the economy is characterized by informality as the level of welfare worsens. Batini et al. [2011] report a similar result: the time inconsistency problem, observed under the discretionary regime, worsens with the introduction of informality. Under the discretionary regime, both the formal inflation and tax rate have higher volatility compared to the historical regime and the optimal commitment regime. Furthermore, the volatility of the tax rate is higher than that of the formal inflation, which is the opposite of the the prescription under the optimal regime. This can help explain why the discretionary regime performs worse than both the optimal commitment regime and the historical regime. The welfare cost of the historical regime comes from the too much volatility in the formal sector

inflation while the welfare cost of the discretionary regime results from high volatility of the formal tax rate and low volatility of the formal sector inflation. Figures 1.7 and 1.8 plot the impulse responses of the three different regimes under a productivity shock in the formal sector for the economy with no informality and the economy with informality, respectively. One can see that while the tax rate remains unchanged under the historical policy and optimal commitment policy, it increases considerably under the discretionary regime before returning to its steady state rate. Consumption increases in all the policy regimes, though it does so more under the optimal regime and less under discretion. This is one of the main sources of low level of welfare under the discretionary regime.

1.7 Conclusion

This paper developed a small open DSGE model characterized by informality in the goods market and labor market. The paper shows that informality significantly decreases the optimal tax rate levied on the formal sector. Policy makers in countries with significant informality should aim to keep taxes low and steady in the formal sector in order to avoid formal goods becoming relatively more expensive than the informal good. Even though it does not affect the long run optimal inflation, informality affects the long run levels of other macro variables and increases the volatility of all variables except the tax rate. If countries have neither the technology nor the credibility to commit to the optimal policy, it is desirable to peg the nominal exchange and adopt a flat tax rate.

The paper contribution to research on the implications of informality for policy making is two-fold. First, I shed light on the impact of informality on the optimal fiscal and monetary policy. Every economy has some degree of informality, but it is more pervasive in developing countries, and understanding its impact on policy making is of first order importance. Second, I re-evaluate policy prescription on optimal exchange rate volatility for small open economies and show that an exchange peg coupled with a flat tax rate outperforms a discre-

tionary regime.

The current setup of the model can be extended in several directions. An especially important one will be to relax the assumption of financial autarky, explore the consequences of capital flows, and allow the government to run budget deficits. This would give us insights into the dynamic of public debt under a currency peg with informality-constrained fiscal revenues. Another interesting extension would be to allow the productivity of the public good to decrease with the size of the informal sector. Existence of the informal sector would then decrease the positive effect of tax proceeds, which would have implications for optimal taxation. I intend to explore these extensions in future work.

Tables

Calibration

Table 1.2: Calibration

	Parameter	Value
Openness	a	0.3
Informality	α	0.4
Output elasticity of Gov. Sp.	χ	0.01
Formal Labor elasticity	ψ	3
Informal Labor elasticity	μ	3
Formal Sector Informalization	κ	0.7
Informal Sector Access to Public Good	δ	0.6
Price stickiness	θ	2/3
Coef. of autocor. Formal Output	ρ_F	0.9
Coef. of autocor. Informal Output	ρ_I	0.7

Table 1.3: GMM Regression with all countries

<i>Dependent variable: Government Debt</i>	
Size of Informal sector	6.237*** (0.823)
GDP per capita	0.001 (0.002)
GDP per capita growth	0.168 (0.388)
Inflation	0.203*** (0.059)
Current accounts	0.036 (0.203)
Openness (trade)	-0.419*** (0.086)
Corruption Control	-11.600* (6.361)
Voice & Accountability	-13.168** (5.279)
Observations	366
R ²	0.372
Adjusted R ²	0.303
F Statistic	19.612*** (df = 9; 298)
<i>Note:</i>	*p<0.1; **p<0.05; ***p<0.01

Table 1.4:

<i>Dependent variable: Tax Revenue (% of GDP)</i>	
Size of informal Sector	-0.157* (0.085)
GDP per Capita	0.002*** (0.0004)
Foreign aid	-0.052** (0.026)
GDP per capita growth	0.026 (0.037)
Inflation	0.016 (0.027)
Current accounts	0.007 (0.023)
Openness (Trade)	0.047*** (0.012)
Rule of Law	0.564 (0.860)
Corruption Control	-0.555 (0.632)
Governement Effectiveness	-2.170** (0.833)
Voice & accountability	-0.355 (0.580)
Observations	222
R ²	0.459
Adjusted R ²	0.348
F Statistic	12.973*** (df = 11; 168)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 1.5: Optimal Tax: Varying Elasticity χ

Elasticity χ	Optimal Tax Rate
0.1	17.5%
0.2	32.8%
0.3	45.2%
0.4	54.7%
0.5	62%

The optimal tax rate under flexible price, at varying degrees of output elasticity of government spending.

Table 1.6: Optimal Tax: Varying Degree of Labor Informality κ

Labor Informality κ	Optimal Tax Rate
0.1	17.8%
0.2	18.3%
0.3	18.8%
0.4	19.3%
0.5	19.8%

The optimal tax rate under flexible price, with varying degrees of labor market informality.

Table 1.7: Optimal Tax: Varying Size of the Informal Good α

Goods Informality α	Optimal Tax Rate
0.1	21%
0.2	13.5%
0.3	6.7%
0.4	0.7%

The optimal tax rate under flexible price, with varying degrees of goods informality.

Table 1.8: Flexible Price (Ramsey) Allocations

Variable	No Informality	Informality ($\alpha = 0.4$ and $\kappa = 0.7$)
Consumption	0.80	1.1
Tax	17.5%	0.72%
Gov. Spending	0.13	0.5%
Formal output	0.74	0.71
Formal Labor	0.91	0.74
Informal Output	-	0.44

Flexible price allocation with informality and without informality.

Sticky Prices

Table 1.9: Long Run (Ramsey) Allocations

Variable	Flexible Price	Sticky Price
CPI Inflation	–	1.0002
Formal Inflation	–	1.0002
Consumption	0.80	0.79
Tax	17.5%	48%
Gov. Spending	0.13	0.35
Formal output	0.74	0.73

Ramsey allocation (no informality) when choosing both monetary and fiscal policies simultaneously.

Table 1.10: Long Run (Ramsey) Allocations

Variable	Flexible Price	Sticky Price ($\tau = 17.5\%$)
CPI Inflation	–	1.00
Formal Inflation	–	1.00
Consumption	0.80	0.80
Tax	17.5%	17.5%
Gov. Spending	0.13	0.13
Formal output	0.74	0.74

Ramsey allocations (no informality) when holding the optimal tax rate at the rate under flexible price.

Table 1.11: Long Run (Ramsey) Allocations With Sticky Prices

Variable	Flexible Price	Sticky Price
CPI Inflation	–	1.0003
Formal Inflation	–	1.0003
Informal Inflation	–	1.0003
Consumption	1.1	1.1
Tax	0.72%	2.8%
Gov. Spending	0.0052	0.023
Formal output	0.71	0.82
Informal Output	0.44	0.52

Ramsey allocation (with informality) when choosing both monetary and fiscal policies simultaneously.

Table 1.12: Long Run (Ramsey) Allocations

Variable	Flexible Price	Sticky Price($\tau = 0.72\%$)
CPI Inflation	–	1.0003
Formal Inflation	–	1.0003
Informal Inflation	–	1.0003
Consumption	1.1	1.1
Tax	0.72%	0.72%
Gov. Spending	0.0052	0.0051
Formal output	0.71	0.71
Informal Output	0.44	0.44

Ramsey allocation (with informality) when holding tax rate fixed.

Table 1.13: Long Run (Ramsey) Allocations

Variable (Mean)	No Informality	Informality ($\alpha = 0.4$ and $x = 0.7$)
CPI Inflation	1.0002	1.0003
Formal Inflation	1.0002	1.0003
Tax	48%	2.80%
Gov. Spending	0.35	0.023
Formal output	0.73	0.82
Consumption	0.80	1.10
Informal Output	-	0.52

Ramsey allocation (mean): Informality vs No Informality

Table 1.14: Standard Deviation of Some Select Variables

Variable (Std Deviation)	No Informality	Informality ($\alpha = 0.4$ and $x = 0.7$)
CPI Inflation	0.3	2.30
Formal Inflation	0.03	0.10
Tax	0.05	0.0028
Gov. Spending	0.47	0.03
Formal output	1.03	1.25
Consumption	0.78	3.21
Informal Output	-	0.07

Ramsey allocation (std deviation): Informality vs No Informality

Table 1.15: Historic Policy

Variable	No Informality	Informality
CPI Inflation	1.00	1.00
Formal Inflation	1.00	1.00
Informal Inflation	-	1.00
Formal output	0.74	0.96
Informal output	-	0.63
Consumption	0.80	1.03
Gov. spending	0.13	0.17
Tax	0.18	0.18

Long run allocation under the historical policy: No Informality Vs Informality

Table 1.16: Discretionary Policy

Variable	No Informality	Informality
Π	1.07	1.07
Π_{HF}	1.07	1.07
Π_I	—	1.07
Y_H	0.28	0.26
Y_I	-	0.53
ΔE	1.07	1.07
C	0.41	0.10
G	0.18	0.03
τ	0.63	0.12

Ramsey long run allocations under discretionary policy: Non Informality Vs Informality

Figures

Figure 1.1: Optimal Ramsey Tax Rate and Informal Sector

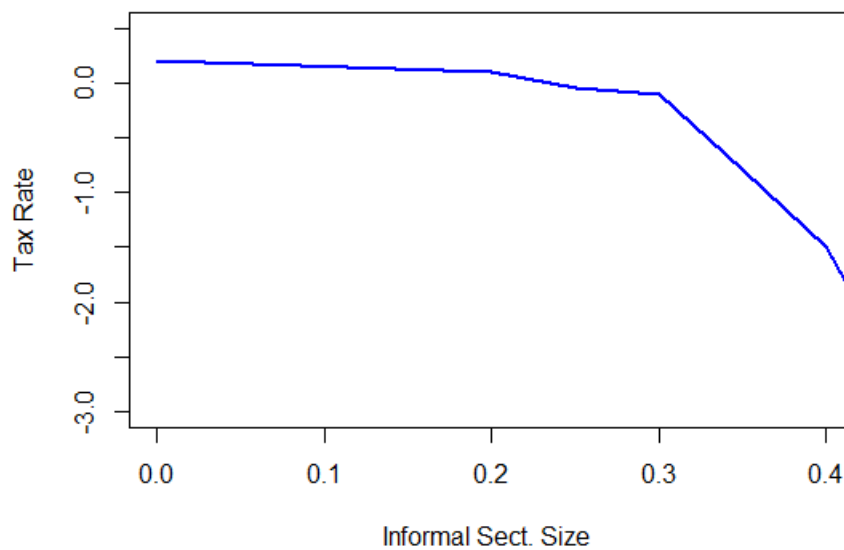


Figure 1.2: Tax Revenues and the Size of the Informal Sector

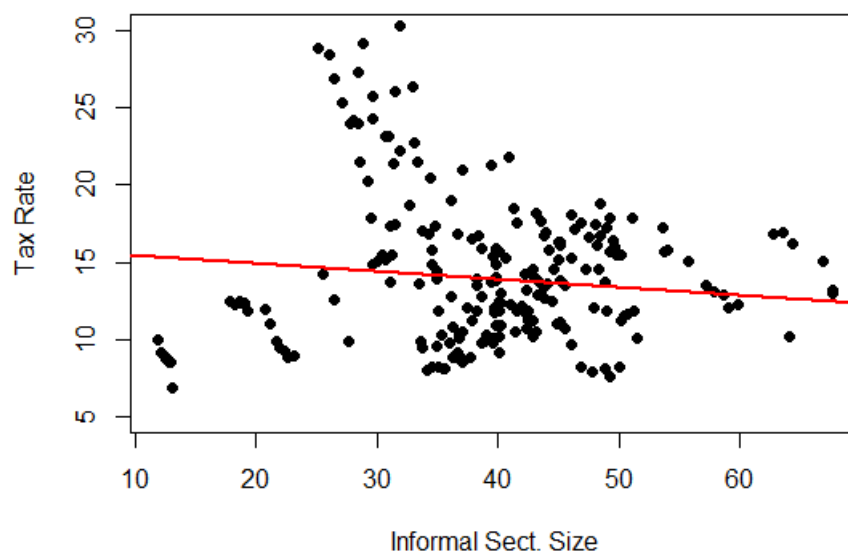


Figure 1.3: Government Debt and the Size of the Informal Sector

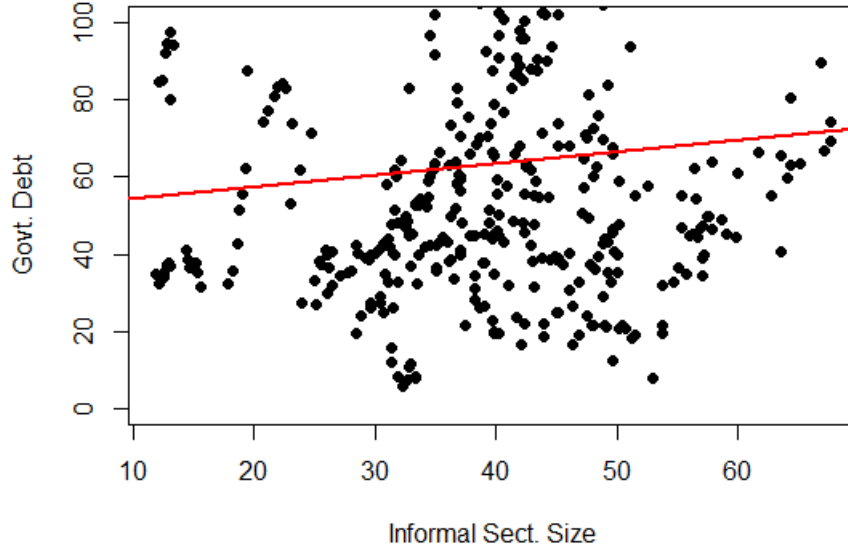


Figure 1.4: Optimal Ramsey Public Debt and Informal Sector

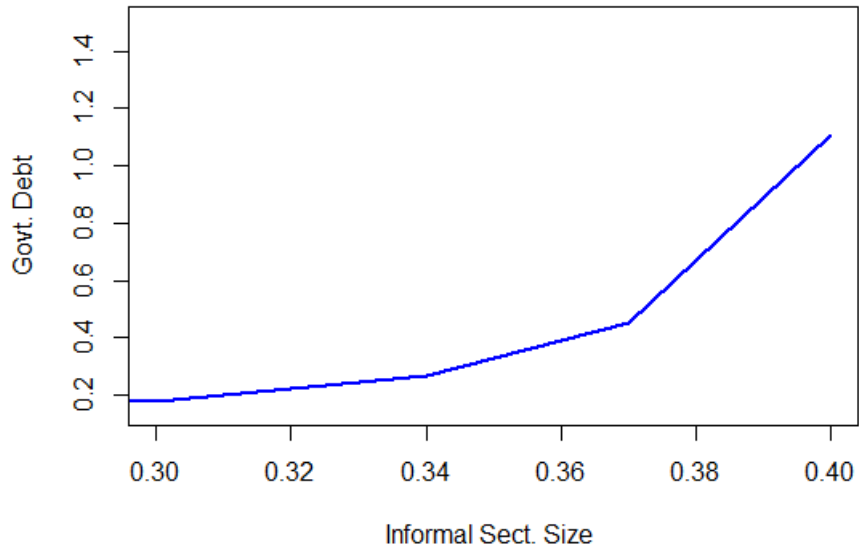


Figure 1.5: Impulse responses to a productivity shock in the formal sector under flexible prices

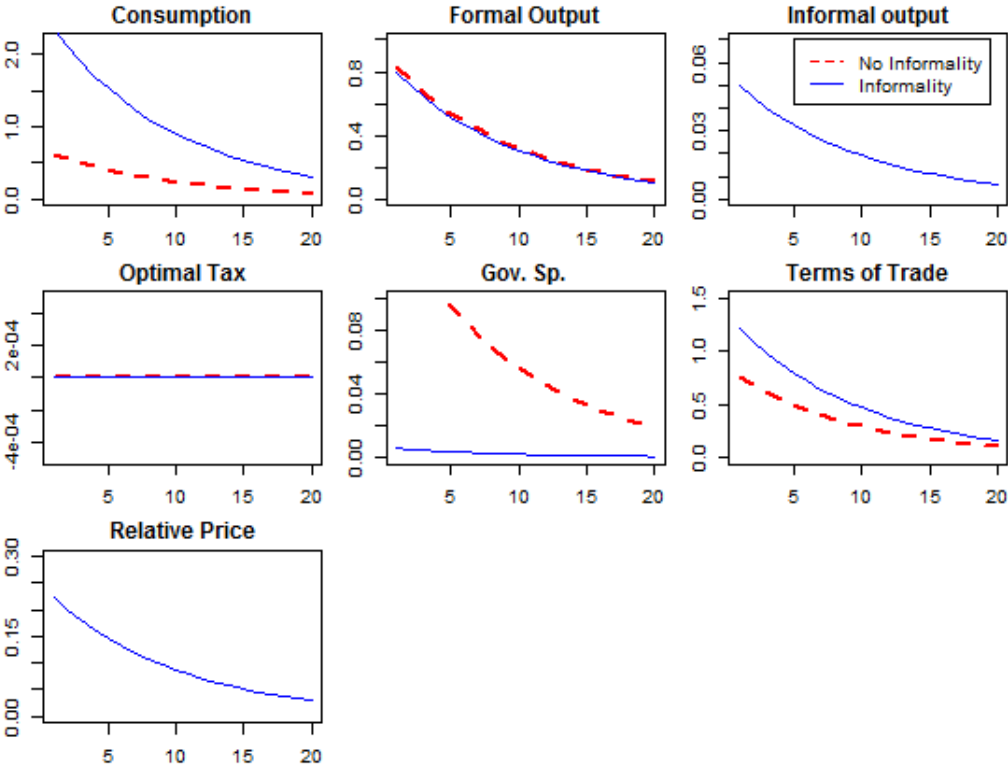


Figure 1.6: Impulse responses to a productivity shock in the formal sector under the optimal ramsey commitment

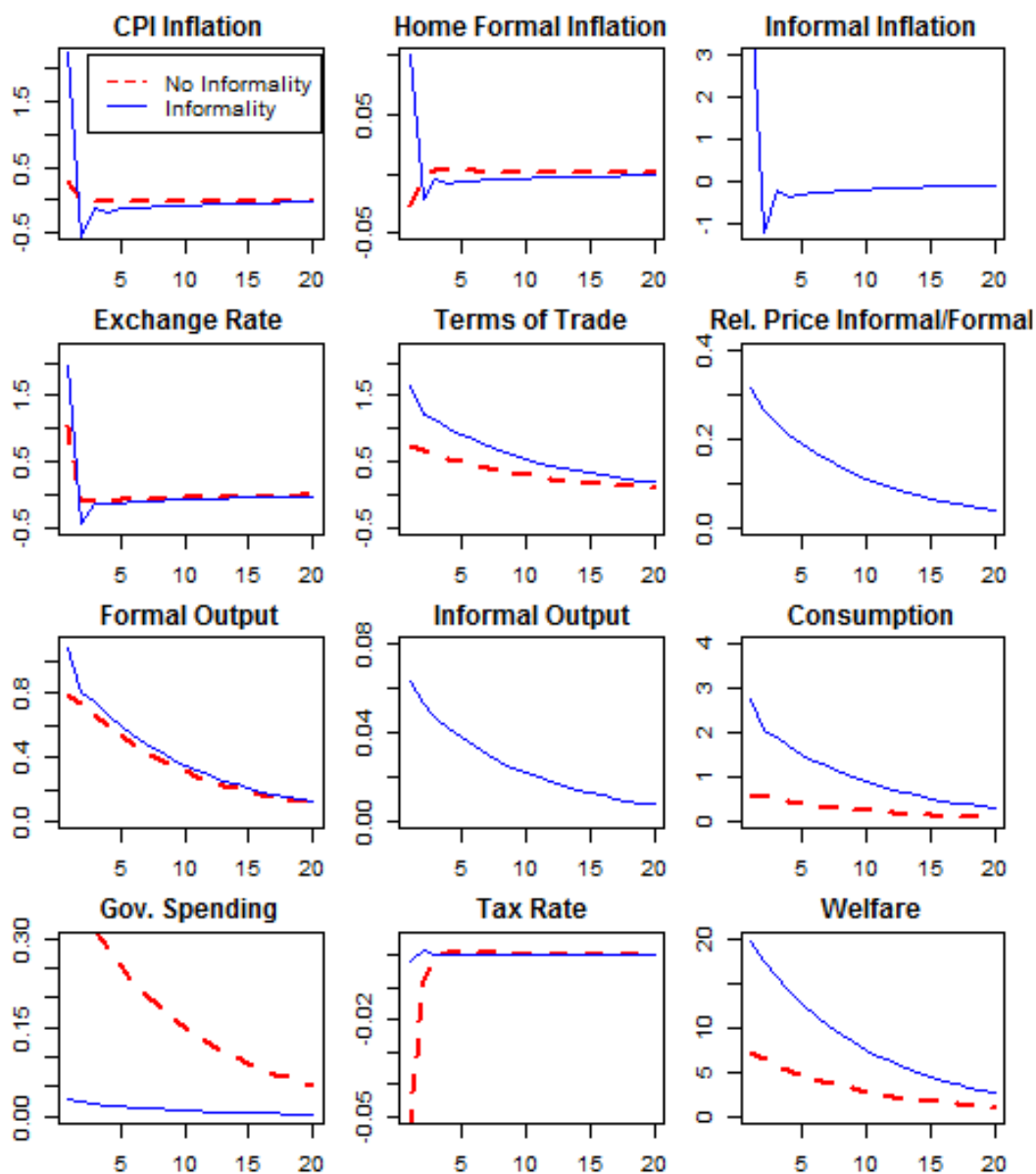


Figure 1.7: Impulse responses to a productivity under three different policy regimes with no informality

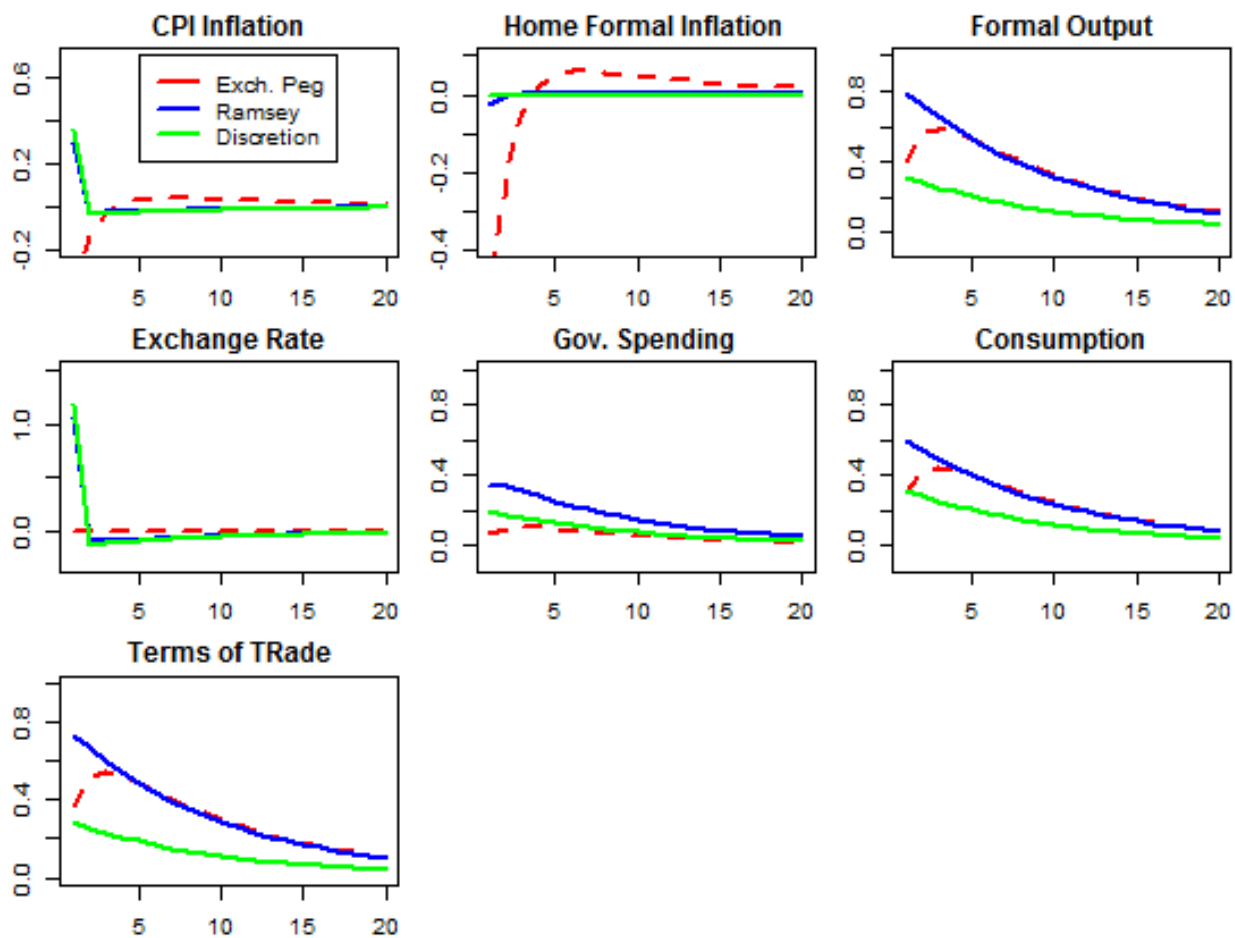
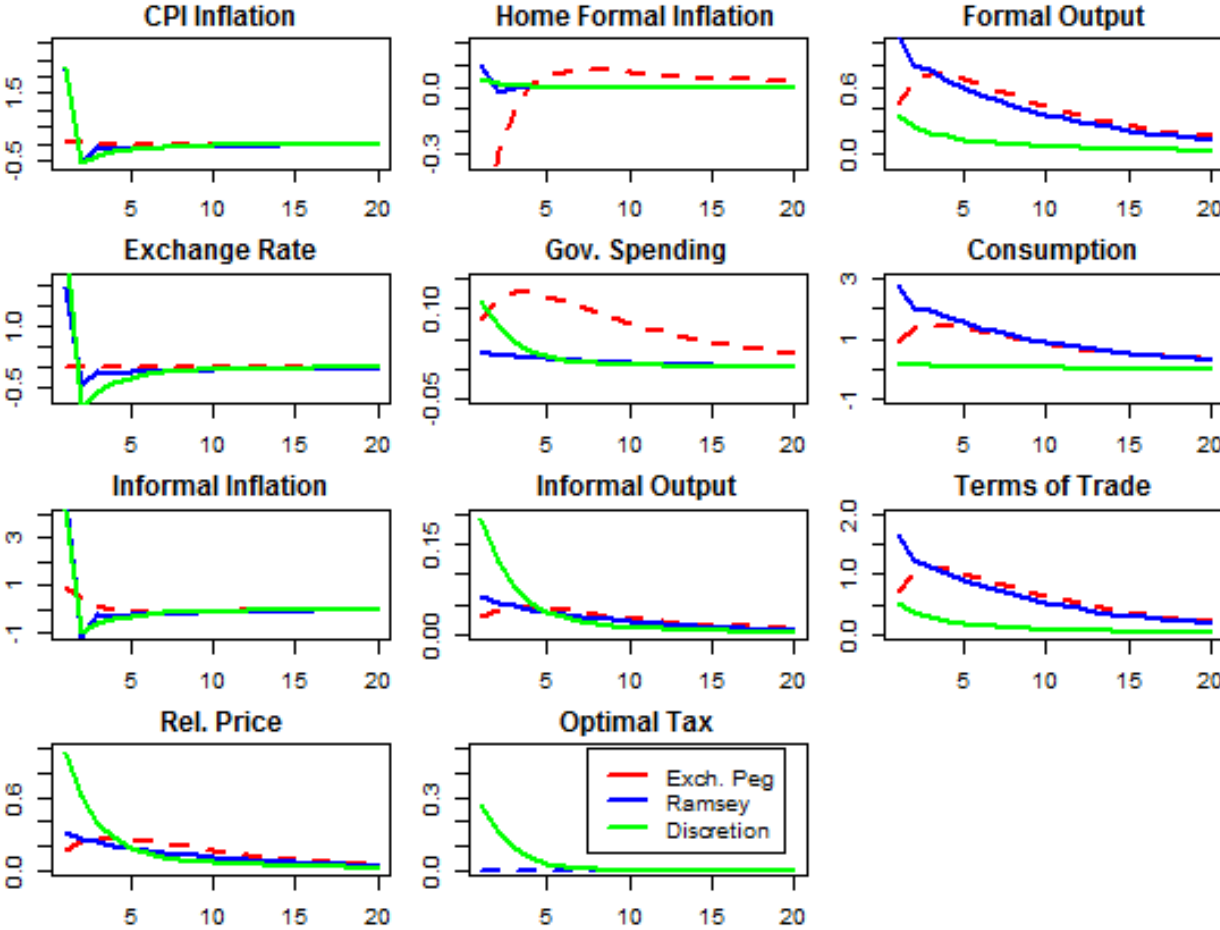


Figure 1.8: Impulse responses to a productivity shock in the formal sector with informality



Appendix

A- Flexible Price Equilibrium without Informality

In this section, I present the equilibrium conditions of the economy with no informality ($\alpha = x = 0$ and $v = 1$) under flexible prices.

Ramsey Problem with Lump Sum Tax:

When the public good is financed through lump sum taxation, the equilibrium conditions in the economic are:

$$Y_{H,t} = m^{\frac{1}{\psi+1}} A_t G_t^X$$

$$N_t^F = m^{\frac{1}{\psi+1}}$$

$$C_t = Y_{H,t}^{1-a} (C_t^*)^a$$

$$G_t = T_t$$

Ramsey Problem with Distortionary Tax:

The equilibrium conditions when the public good is financed by a unit tax on formal output are as follow:

$$Y_{H,t} = (1 - \tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} A_t G_t^X$$

$$N_t^F = m^{\frac{1}{\psi+1}} (1 - \tau_t)^{\frac{1}{\psi+1}}$$

$$C_t = Y_{H,t}^{1-a} (C_t^*)^a$$

$$G_t = \tau_t Y_{H,t}$$

The Ramsey planner chooses the unit tax rate by solving the following problem:

$$\begin{aligned}
 \text{Max} \quad & \sum_{t=0}^{\infty} \left\{ \log(C_t) - \frac{N_t^{F(1+\psi)}}{1+\psi} \right\} \\
 \text{s.t.} \quad & Y_{H,t} = (1 - \tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} A_t G_t^x \\
 & N_t^F = m^{\frac{1}{\psi+1}} (1 - \tau_t)^{\frac{1}{\psi+1}} \\
 & C_t = Y_{H,t}^{1-a} (C_t^*)^a \\
 & G_t = \tau_t Y_{H,t}
 \end{aligned}$$

B- Flexible Price Equilibrium with Informality

In this situation, prices are flexible in both the formal sector and the informal sector, so no nominal rigidity and no tax distortions. The existing distortions are: informality in the goods market with $(0 < \alpha < 1)$, informality in the labor market where $(0 < x < 1)$ and monopoly power with $(0 < m < 1)$.

Ramsey Problem with Lump Sum Tax:

If the Ramsey planner finances the public good through lump sum taxation, then the equilibrium conditions are:

$$\begin{aligned}
 Y_{H,t} &= (1 - \alpha)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{\psi}{\psi+1}} A_t G_t^\alpha \\
 Y_{I,t} &= \alpha^{\frac{1}{1+\mu}} A_{I,t} (\delta G_t)^\alpha \\
 N_t^F &= \left(\frac{m(1 - \alpha)}{v} \right)^{\frac{1}{\psi+1}} \\
 N_t^I &= x^{\frac{1}{\psi}} \left(\frac{m(1 - \alpha)}{v} \right)^{\frac{1}{\psi+1}} \\
 L_t &= \alpha^{\frac{1}{1+\mu}} \\
 C_t &= \left(\frac{Y_{H,t}}{1 - \alpha} \right)^{\frac{(1-\alpha)(1-a)}{1-2\alpha}} \left(\frac{Y_{I,t}}{\alpha} \right)^{\frac{-\alpha}{1-2\alpha}} (C_t^* \Theta_t^{*\alpha})^{\frac{a(1-\alpha)}{1-2\alpha}} \\
 G_t &= T_t
 \end{aligned}$$

Ramsey Problem with Distortionary Tax:

If the public good is financed through a unit tax on formal output then the equilibrium conditions in the economy are:

$$\begin{aligned}
 Y_{H,t} &= (1 - \alpha)^{\frac{1}{\psi+1}} (1 - \tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{\psi}{\psi+1}} A_t G_t^\alpha \\
 Y_{I,t} &= \alpha^{\frac{1}{1+\mu}} A_{I,t} (\delta G_t)^\alpha \\
 N_t^F &= (1 - \alpha)^{\frac{1}{\psi+1}} (1 - \tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{-1}{\psi+1}} \\
 N_t^I &= x^{\frac{1}{\psi}} (1 - \alpha)^{\frac{1}{\psi+1}} (1 - \tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{-1}{\psi+1}} \\
 L_t &= \alpha^{\frac{1}{1+\mu}} \\
 C_t &= \left(\frac{Y_{H,t}}{1 - \alpha} \right)^{\frac{(1-\alpha)(1-a)}{1-2\alpha}} \left(\frac{Y_{I,t}}{\alpha} \right)^{\frac{-\alpha}{1-2\alpha}} (C_t^* \Gamma_t^{*\alpha})^{\frac{a}{1-\alpha}} \\
 G_t &= \tau_t Y_{H,t}
 \end{aligned}$$

The Ramsey planner chooses the optimal unit tax by solving the following problem:

$$\begin{aligned}
& \text{Max} \quad \sum_{t=0}^{\infty} \left\{ \log(C_t) - \left(\frac{1+x^{\frac{1}{\psi}}}{1+\psi} \right) (N_t^F)^{1+\psi} - \frac{\alpha}{1+\mu} \right\} \\
& \text{s.t.} \quad Y_{H,t} = (1-\alpha)^{\frac{1}{\psi+1}} (1-\tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{\psi}{\psi+1}} A_t G_t^\chi \\
& \quad Y_{I,t} = \alpha^{\frac{1}{1+\mu}} A_{I,t} (\delta G_t)^\chi \\
& \quad N_t^F = (1-\alpha)^{\frac{1}{\psi+1}} (1-\tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{-1}{\psi+1}} \\
& \quad C_t = \left(\frac{Y_{H,t}}{1-\alpha} \right)^{\frac{(1-\alpha)(1-a)}{1-2\alpha}} \left(\frac{Y_{I,t}}{\alpha} \right)^{\frac{-\alpha}{1-2\alpha}} (C_t^* \Gamma_t^{*\alpha})^{\frac{a(1-\alpha)}{1-2\alpha}} \\
& \quad G_t = \tau_t Y_{H,t}
\end{aligned}$$

C- Sticky Price Equilibrium without Informality

With Lump Sum Tax:

When the public good is financed through a lump sum tax, the equilibrium conditions are:

$$\begin{aligned}
C_t &= Y_{H,t}^{1-a} (C_t^*)^a \\
\Delta_{p_h,t} &= \theta \Pi_{HF,t}^\gamma \Delta_{p_h,t-1} + (1-\theta) \Pi_{HF,t}^{*(-\gamma)}; \\
\Pi_{HF,t}^* &= \left(\frac{\gamma}{\gamma-1} \right) \frac{F_{1t}}{F_{2t}} \\
F_{1t} &= \frac{\Delta_{p_h,t}^\psi Y_{H,t}^{\psi+2}}{(1-\alpha)v^\psi A_t^{F(1+\psi)} G_t^{\chi(1+\psi)}} + \theta\beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^\gamma F_{1,t+1} \\
F_{2t} &= Y_{H,t} + \theta\beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma-1} F_{2,t+1} \\
1 &= (1-\theta) \Pi_{HF,t}^{*(1-\gamma)} + \theta \Pi_{HF,t}^{\gamma-1} \\
\Pi_t &= (\Pi_{HF,t})^{(1-a)} (\Pi_{FF,t}^*)^a (\Delta E_t)^a \\
\Delta E_t &= \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^*} \right) \left(\frac{C_{t-1}}{C_t} \right)^{\frac{1}{a}} \left(\frac{Y_{H,t}}{Y_{H,t-1}} \right)^{\frac{1}{a}}
\end{aligned}$$

With Distortionary Tax:

When the public good is financed through a unit tax on formal output, the equilibrium conditions are:

$$\begin{aligned}
C_t &= Y_{H,t}^{1-a} (C_t^*)^a \\
\Delta_{p_h,t} &= \theta \Pi_{HF,t}^\gamma \Delta_{p_h,t-1} + (1-\theta) \Pi_{HF,t}^{*(-\gamma)}; \\
\Pi_{HF,t}^* &= \left(\frac{\gamma}{\gamma-1} \right) \frac{F_{1t}}{F_{2t}} \\
F_{1t} &= \frac{\Delta_{p_h,t}^\psi Y_{H,t}^{\psi+2}}{(1-\alpha)v^\psi A_t^{F(1+\psi)} G_t^{\chi(1+\psi)}} + \theta\beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^\gamma F_{1,t+1} \\
F_{2t} &= Y_{H,t} - G_t + \theta\beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma-1} F_{2,t+1} \\
1 &= (1-\theta) \Pi_{HF,t}^{*(1-\gamma)} + \theta \Pi_{HF,t}^{\gamma-1} \\
\Pi_t &= (\Pi_{HF,t})^{(1-a)} (\Pi_{FF,t}^*)^a (\Delta E_t)^a \\
\Delta E_t &= \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^*} \right) \left(\frac{C_{t-1}}{C_t} \right)^{\frac{1}{a}} \left(\frac{Y_{H,t}}{Y_{H,t-1}} \right)^{\frac{1}{a}}
\end{aligned}$$

The Ramsey Planner chooses the optimal tax and inflation by solving the following problem:

$$\begin{aligned}
Max_{G_t, \Pi_{HF,t}} \sum_0^{\infty} \log(C_t) &- \left(\frac{1}{1+\psi} \right) \left(\frac{\Delta_{H,t} Y_{H,t}}{A_{F,t} G_t^\chi} \right)^{1+\psi} \\
&+ \lambda_{1t} \left[C_t - Y_{H,t}^{(1-a)} (C_t^*)^a \right] \\
&+ \lambda_{2t} \left[\Delta_{H,t} - \theta \Pi_{HF,t}^\gamma \Delta_{H,t-1} - (1-\theta) \Pi_{HF,t}^{*\gamma} \right] \\
&+ \lambda_{3t} \left[\Pi_{HF,t}^* - \left(\frac{\gamma}{\gamma-1} \right) \frac{F_{1t}}{F_{2t}} \right] \\
&+ \lambda_{4t} \left[F_{1t} - \frac{\Delta_{H,t}^\psi Y_{H,t}^{\psi+2}}{A_{F,t}^{\psi+1} G_t^{\chi(\psi+1)}} - \theta \beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^\gamma F_{1t+1} \right] \\
&+ \lambda_{5t} \left[F_{2t} - Y_{H,t} + G_t - \theta \beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma-1} F_{2t+1} \right] \\
&+ \lambda_{6t} \left[(1-\theta) \Pi_{HF,t}^{*(1-\gamma)} + \theta \Pi_{HF,t}^{\gamma-1} - 1 \right] \\
&+ \lambda_{7t} \left[\Pi_t - \Pi_{HF,t}^{1-a} (\Pi_{FF,t}^{*a} (dE_t)^a) \right] \\
&+ \lambda_{8t} \left[dE_t - \frac{\Pi_{HF,t}}{\Pi_{FF,t}} \left(\frac{C_{t-1}}{C_t} \right)^{\frac{1}{a}} \left(\frac{Y_{H,t}}{Y_{H,t-1}} \right)^{\frac{1}{a}} \right]
\end{aligned}$$

$$\begin{aligned}
C_t : \frac{1}{C_t} + \lambda_{1t} \frac{1}{1-a} C_t^{\frac{a}{1-a}} - \lambda_{4t} \theta \beta \left(\frac{1}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^\gamma F_{1t+1} + \lambda_{4t-1} \theta \beta \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-1} \Pi_{HF,t}^\gamma F_{1t} \\
- \lambda_{5t} \theta \beta \left(\frac{1}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma-1} F_{2t+1} + \lambda_{5,t-1} \theta \beta \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-1} \Pi_{HF,t}^{\gamma-1} F_{2t} \\
+ \lambda_{8t} \frac{1}{a} \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^*} \right) \frac{C_{t-1}^{\frac{1}{a}}}{C_t^{\frac{1+a}{a}}} \left(\frac{Y_{H,t}}{Y_{H,t-1}} \right)^{\frac{1}{a}} - \lambda_{8t+1} \left(\frac{\Pi_{HF,t+1}}{\Pi_{FF,t+1}^*} \right) \left(\frac{Y_{H,t+1}}{Y_{H,t}} \right)^{\frac{1}{a}} \frac{C_t^{\frac{1-a}{a}}}{C_{t+1}^{\frac{1}{a}}} = 0
\end{aligned}$$

$$\Delta_{H,t} : - \left(\frac{Y_{H,t}}{A_{F,t} G_t^\chi} \right)^{1+\psi} \Delta_{H,t}^\psi + \lambda_{2t} - \lambda_{2t+1} \theta \Pi_{HF,t+1}^\gamma - \lambda_{4t} \psi \frac{\Delta_{H,t}^{\psi-1} Y_{H,t}^{\psi+2}}{A_{F,t}^{\psi+1} G_t^{\chi(\psi+1)}} = 0$$

$$\begin{aligned}
Y_{H,t} : - \left(\frac{\Delta_{H,t}}{A_{F,t} G_t^{\chi a}} \right)^{1+\psi} Y_{H,t}^\psi - \lambda_{1t} (C_t^*)^{\frac{1-a}{a}} - \lambda_{4t} \frac{(\psi+2) \Delta_{H,t}^\psi Y_{H,t}^{\psi+1}}{A_{F,t}^{\psi+1} G_t^{\chi(\psi+1)}} - \lambda_{5t} - \lambda_{8t} \frac{1}{a} \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^*} \right) \left(\frac{C_{t-1}}{C_t} \right)^{\frac{1}{a}} \frac{Y_{H,t}^{\frac{1-a}{a}}}{Y_{H,t-1}^{\frac{1}{a}}} \\
+ \lambda_{8t+1} \left(\frac{1}{a} \right) \left(\frac{\Pi_{HF,t+1}}{\Pi_{FF,t+1}^*} \right) \left(\frac{C_t}{C_{t+1}} \right)^{\frac{1}{a}} \frac{Y_{H,t+1}^{\frac{1}{a}}}{Y_{H,t}^{\frac{1+a}{a}}} = 0
\end{aligned}$$

$$\Pi_t : + \lambda_{4t-1} \theta \beta \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-2} \Pi_{HF,t-1}^\gamma F_{1t} + \lambda_{5t-1} \theta \beta \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-2} \Pi_{HF,t-1}^{\gamma-1} F_{2t} + \lambda_{7t} = 0$$

$$\begin{aligned}
\Pi_{HF,t} : - \lambda_{2t} \theta \beta \Pi_{HF,t}^{\gamma-1} \Delta_{t-1} - \lambda_{4t-1} \theta \beta \gamma \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-1} \Pi_{HF,t}^{\gamma-1} F_{1t} - \lambda_{5t-1} \theta \beta (\gamma-1) \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-1} \Pi_{HF,t}^{\gamma-2} F_{2t} \\
+ \lambda_{6t} \theta (\gamma-1) \Pi_{HF,t}^{\gamma-2} - \lambda_{7t} (1-a) \Pi_{HF,t}^{-a} \Pi_{FF,t}^{*a} \left(\frac{E_t}{E_{t-1}} \right)^a - \lambda_{8t} \frac{1}{\Pi_{FF,t}^*} \left(\frac{C_{t-1}}{C_t} \right)^{\frac{1}{a}} \left(\frac{Y_{H,t}}{Y_{H,t-1}} \right)^{\frac{1}{a}} = 0
\end{aligned}$$

$$dE_t - \lambda_{7t} a \Pi_{HF,t}^{1-a} \Pi_{FF,t}^{*a} (dE_t)^{a-1} + \lambda_{8t} = 0$$

$$G_t : \chi \left(\frac{\Delta_{H,t} Y_{H,t}}{A_{F,t}} \right)^{1+\psi} \frac{1}{G_t^{\chi(1+\psi)+1}} + \lambda_{4t} \chi(\psi+1) \frac{\Delta_{H,t}^\psi Y_{H,t}^{\psi+2}}{A_{F,t}^{\psi+1} G_t^{\chi(1+\psi)+1}} + \lambda_{5t} = 0$$

$$\Pi_{HF,t}^* : +\lambda_{2t}(1-\theta)\gamma\Pi_{HF,t}^{*(-\gamma-1)} + \lambda_{3t} + \lambda_{6t}(1-\theta)(1-\gamma)\Pi_{HF,t}^{*-\gamma} = 0$$

$$F_{1t} : -\lambda_{3t} \left(\frac{\gamma}{\gamma-1} \right) \left(\frac{1}{F_{2t}} \right) + \lambda_{4t} - \lambda_{4t-1} \theta \beta \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-1} \Pi_{HF,t}^\gamma = 0$$

$$F_{2t} : -\lambda_{3t} \frac{\gamma}{\gamma-1} \left(\frac{F_{1t}}{F_{2t}^2} \right) + \lambda_{5t} - \lambda_{5t-1} \theta \beta \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-1} \Pi_{HF,t}^{\gamma-1} = 0$$

$$\lambda_{1t} : C_t = Y_{H,t}^{1-a} (C_t^*)^a$$

$$\lambda_{3t} : \Pi_{HF,t}^* = \left(\frac{\gamma}{\gamma-1} \right) \frac{F_{1t}}{F_{2t}}$$

$$\lambda_{4t} : F_{1t} = \frac{\Delta_{H,t}^\psi Y_{H,t}^{\psi+2}}{A_{F,t}^{\psi+1} G_t^{\chi(\psi+1)}} + \theta \beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^\gamma F_{1t+1}$$

$$\lambda_{5t} : F_{2t} = Y_{H,t} - G_t + \theta \beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma-1} F_{2t+1}$$

$$\lambda_{6t} : (1-\theta)\Pi_{HF,t}^{*(1-\gamma)} + \theta\Pi_{HF,t}^{\gamma-1} = 1$$

$$\lambda_{7t} : \Pi_t - \Pi_{HF,t}^{1-a} (\Pi_{FF,t}^{*a} (dE_t)^a)$$

$$\lambda_{8t} : dE_t - \frac{\Pi_{HF,t}}{\Pi_{FF,t}} \left(\frac{C_{t-1}}{C_t} \right)^{\frac{1}{a}} \left(\frac{Y_{H,t}}{Y_{H,t-1}} \right)^{\frac{1}{a}}$$

Steady State:

- $\lambda_1 = (a-1)C^{\frac{1}{a-1}}$
- $-\frac{Y_H^{1+\psi}}{G^\chi} + (1-\theta)\lambda_2 - \lambda_4 \frac{\psi Y_H^{\psi+2}}{G^{\chi(\psi+1)}} = 0$
- $\frac{Y_H^\psi}{G^{\chi(\psi+1)}} + \lambda_1 (C^*)^{\frac{a}{1-a}} + \lambda_4 \frac{(\psi+2)Y_H^{\psi+1}}{G^{\chi(\psi+1)}} + \lambda_5 = 0$
- $\lambda_4 \theta \beta F_1 + \lambda_5 \theta \beta F_2 + \lambda_7 = 0$
- $-\lambda_2 \theta \beta \Pi_{HF,t}^{\gamma-1} \Delta - \lambda_4 \theta \beta \gamma \Pi_{HF,t}^{\gamma-2} F_1 - \lambda_5 \theta \beta (\gamma-1) \Pi_{HF,t}^{\gamma-3} F_2 + \lambda_6 \theta \beta (\gamma-1) \Pi_{HF,t}^{\gamma-2} - \lambda_7 (1-a) + \lambda_8 = 0$
- $-\lambda_7 a \Pi_{HF,t}^{1-a} (dE)^{a-1} + \lambda_8 = 0$
- $-\lambda_7 a + \lambda_8 = 0$

- $\Pi_{HF} = dE \Rightarrow \Pi_{HF}^{1-a}(dE)^{a-1} = \Pi_{HF}^{1-a+a-1} = \Pi_{HF}^0 = 1$
- $\frac{1}{G^{\chi(1+\psi)+1}} \left[\chi Y_H^{\psi+2} + \lambda_4 \chi (\psi + 1) Y_H^{\psi+2} \right] + \lambda_5 = 0$
- $F_1 = \frac{Y_H^{\psi+2}}{(1-\theta\beta)G^{\chi(1+\psi)}}$
- $F_2 = \frac{Y_H - G}{1-\theta\beta} \frac{F_1}{F_2} = \frac{Y_H^{\psi+2}}{(Y_H - G)G^{\chi(\psi+1)}}$
- $\frac{\gamma-1}{\gamma} = \frac{Y_H^{\psi+2}}{(Y_H - G)G^{\chi(\psi+1)}}$
- $Y_H^{\psi+2} = m(Y_H - G)G^{\chi(\psi+1)}$
- $\lambda_2(1 - \theta)\gamma + \lambda_3 + \lambda_6(1 - \theta)(1 - \gamma) = 0$
- $-\lambda_3 \frac{\gamma}{\gamma-1} \frac{1}{F_2} + \lambda_4(1 - \theta\beta) = 0$
- $-\lambda_3 \frac{\gamma}{\gamma-1} \frac{F_1}{F_2^2} + \lambda_4(1 - \theta\beta) = 0$
- $C = Y_H^{1-a}(C^*)^a$

D- Sticky Price Equilibrium with Informality

With Lump Sum Tax:

When the public good is financed through a lump sum tax on the formal sector, the equilibrium of the economy are as follow:

$$\begin{aligned}
 C_t^{\frac{1-2\alpha}{(1-\alpha)(1-a)}} &= \left(\frac{Y_{H,t}}{1-\alpha} \right) \left(\frac{Y_{I,t}}{\alpha} \right)^{\frac{-\alpha}{(1-\alpha)(1-a)}} (C_t^* \Theta_t^{*\alpha})^{\frac{a}{1-a}} \\
 Y_{I,t} &= A_{I,t} (\delta G_t)^\chi \alpha^{\frac{1}{1+\mu}} \\
 \Delta_{p_h,t} &= \theta \Pi_{HF,t}^\gamma \Delta_{p_h,t-1} + (1-\theta) \Pi_{HF,t}^{*(-\gamma)}; \\
 \Pi_{HF,t}^* &= \left(\frac{\gamma}{\gamma-1} \right) \frac{F_{1t}}{F_{2t}} \\
 F_{1t} &= \frac{\Delta_{p_h,t}^\psi Y_{H,t}^{\psi+2}}{(1-\alpha)v^\psi A_t^{F(1+\psi)} G_t^{\chi(1+\psi)}} + \theta\beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^\gamma F_{1,t+1} \\
 F_{2t} &= Y_{H,t} + \theta\beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t}^{\gamma-1} F_{2,t+1} \\
 1 &= (1-\theta) \Pi_{HF,t}^{*(1-\gamma)} + \theta \Pi_{HF,t}^{\gamma-1} \\
 \Pi_t &= \Pi_{I,t}^\alpha \Pi_{F,t}^{1-\alpha} \\
 \Pi_{I,t}^{1-\alpha} &= \left(\frac{G_{t-1}}{G_t} \right)^\chi \left(\frac{C_t}{C_{t-1}} \right) \left(\frac{A_{i,t-1}}{A_{i,t}} \right) \Pi_{F,t}^{1-\alpha} \\
 \Pi_{F,t} &= (\Pi_{HF,t})^{(1-a)} (\Pi_{FF,t}^*)^a (\Delta E_t)^a \\
 \Delta E_t &= \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^*} \right) \left(\frac{C_t}{C_{t-1}} \right)^{\frac{2\alpha-1}{a(1-\alpha)}} \left(\frac{Y_{H,t}}{Y_{H,t-1}} \right)^{\frac{1}{a}} \left(\frac{Y_{I,t}}{Y_{I,t-1}} \right)^{\frac{\alpha}{a(1-\alpha)}}
 \end{aligned}$$

With Distortionary Tax:

The equilibrium of the economy when the public good is financed through a unit tax on formal output:

$$\begin{aligned}
C_t^{\frac{1-2\alpha}{(1-\alpha)(1-a)}} &= \left(\frac{Y_{H,t}}{1-\alpha} \right) \left(\frac{Y_{I,t}}{\alpha} \right)^{\frac{-\alpha}{(1-\alpha)(1-a)}} (C_t^* \Theta_t^{*\alpha})^{\frac{a}{1-a}} \\
Y_{I,t} &= A_{I,t} (\delta G_t)^\chi \alpha^{\frac{1}{1+\mu}} \\
\Delta_{ph,t} &= \theta \Pi_{HF,t}^\gamma \Delta_{ph,t-1} + (1-\theta) \Pi_{HF,t}^{*(-\gamma)}; \\
\Pi_{HF,t}^* &= \left(\frac{\gamma}{\gamma-1} \right) \frac{F_{1t}}{F_{2t}} \\
F_{1t} &= \frac{\Delta_{ph,t}^\psi Y_{H,t}^{\psi+2}}{(1-\alpha) \nu^\psi A_t^{F(1+\psi)} G_t^{\chi(1+\psi)}} + \theta \beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^\gamma F_{1,t+1} \\
F_{2t} &= Y_{H,t} - G_t + \theta \beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t}^{\gamma-1} F_{2,t+1} \\
1 &= (1-\theta) \Pi_{HF,t}^{*(1-\gamma)} + \theta \Pi_{HF,t}^{\gamma-1} \\
\Pi_t &= \Pi_{I,t}^\alpha \Pi_{F,t}^{1-\alpha} \\
\Pi_{I,t}^{1-\alpha} &= \left(\frac{G_{t-1}}{G_t} \right)^\chi \left(\frac{C_t}{C_{t-1}} \right) \left(\frac{A_{i,t-1}}{A_{i,t}} \right) \Pi_{F,t}^{1-\alpha} \\
\Pi_{F,t} &= (\Pi_{HF,t})^{(1-a)} (\Pi_{FF,t}^*)^a (\Delta E_t)^a \\
\Delta E_t &= \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^*} \right) \left(\frac{C_t}{C_{t-1}} \right)^{\frac{2\alpha-1}{a(1-\alpha)}} \left(\frac{Y_{H,t}}{Y_{H,t-1}} \right)^{\frac{1}{a}} \left(\frac{Y_{I,t}}{Y_{I,t-1}} \right)^{\frac{\alpha}{a(1-\alpha)}}
\end{aligned}$$

The Ramsey Planner chooses the optimal tax and inflation by solving the following problem:

$$\begin{aligned}
Max \sum_0^{\infty} \log(C_t) &- \left(\frac{1+x^{\frac{1}{\psi}}}{1+\psi} \right) \left(\frac{\Delta_{H,t} Y_{H,t}}{v A_{f,t} G_t^{\chi}} \right)^{1+\psi} - \frac{\alpha}{1+\mu} \\
&+ \lambda_{1t} \left[(1-\alpha) C_t^{\frac{1-2\alpha}{\alpha}} - Y_{H,t} Y_{I,t}^{-\frac{\alpha}{1-\alpha}} (C_t^*)^{\frac{\alpha}{1-\alpha}} (\Gamma_t^*)^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} \right] \\
&+ \lambda_{2t} \left[Y_{I,t} - A_{I,t} (\delta G_t)^{\chi} \alpha^{\frac{1}{1+\mu}} \right] \\
&+ \lambda_{3t} \left[\Delta_{H,t} - \theta \Pi_{HF,t}^{\gamma} \Delta_{H,t-1} - (1-\theta) \Pi_{HF,t}^{*(-\gamma)} \right] \\
&+ \lambda_{4t} \left[\Pi_{HF,t}^* - \left(\frac{\gamma}{\gamma-1} \right) \frac{F_{1t}}{F_{2t}} \right] \\
&+ \lambda_{5t} \left[F_{1t} - \frac{\Delta_{H,t}^{\psi} Y_{H,t}^{\psi+2}}{(1-\alpha) v^{\psi} A_{F,t}^{\psi+1} G_t^{\chi(\psi+1)}} - \theta \beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma} F_{1t+1} \right] \\
&+ \lambda_{6t} \left[F_{2t} - Y_{H,t} + G_t - \theta \beta \left(\frac{C_t}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma-1} F_{2t+1} \right] \\
&+ \lambda_{7t} \left[(1-\theta) \Pi_{HF,t}^{*(1-\gamma)} + \theta \Pi_{HF,t}^{\gamma-1} - 1 \right] \\
&+ \lambda_{8t} \left[\Pi_t - \Pi_{I,t}^{\alpha} \Pi_{F,t}^{1-\alpha} \right] \\
&+ \lambda_{9t} \left[\Pi_{I,t}^{1-\alpha} - \left(\frac{G_t-1}{G_t} \right)^{\chi} \left(\frac{C_t}{C_{t-1}} \right) \left(\frac{A_{I,t-1}}{A_{I,t}} \right) \Pi_{F,t}^{1-\alpha} \right] \\
&+ \lambda_{10t} \left[\Pi_{F,t} - \Pi_{HF,t}^{1-a} (\Pi_{FF,t}^*)^a (dE_t)^a \right] \\
&+ \lambda_{11t} \left[dE_t - \frac{\Pi_{HF,t}}{\Pi_{FF,t}^*} \left(\frac{C_t}{C_{t-1}} \right)^{\left(\frac{2\alpha-1}{a(1-\alpha)} \right)} \left(\frac{Y_{H,t}}{Y_{H,t-1}} \right)^{\frac{1}{a}} \left(\frac{Y_{I,t}}{Y_{I,t-1}} \right)^{\left(\frac{\alpha}{a(1-\alpha)} \right)} \right]
\end{aligned}$$

$$\begin{aligned}
C_t : & \frac{1}{C_t} + \lambda_{1t} \left(\frac{1-2\alpha}{1-\alpha} \right) C_t^{\left(\frac{1-2\alpha}{\alpha} - 1 \right)} - \lambda_{5t} \theta \beta \left(\frac{1}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma} F_{1t+1} + \lambda_{5t-1} \theta \beta \left(\frac{C_{t-1}}{C_t^2} \right) \Pi_t^{-1} \Pi_{HF,t}^{\gamma} F_{1t} \\
&- \lambda_{6t} \theta \beta \left(\frac{1}{C_{t+1}} \right) \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma-1} F_{2t+1} + \lambda_{6t-1} \theta \beta \left(\frac{C_{t-1}}{C_t^2} \right) \Pi_t^{-1} \Pi_{HF,t}^{\gamma-1} F_{2t} \\
&+ \lambda_{9t} \left(\frac{1}{C_{t-1}} \right) \left(\frac{G_{t-1}}{G_t} \right)^{\chi} \left(\frac{A_{I,t-1}}{A_{I,t}} \right) \Pi_{F,t}^{1-\alpha} \\
&- \lambda_{9t+1} \left(\frac{C_{t+1}}{C_t^2} \right) \left(\frac{G_t}{G_{t+1}} \right)^{\chi} \left(\frac{A_{I,t}}{A_{I,t+1}} \right) \Pi_{F,t+1}^{1-\alpha} \\
&- \lambda_{11t} \left(\frac{2\alpha-1}{a(1-\alpha)} \right) \frac{C_t^{\left(\frac{2\alpha-1}{a(1-\alpha)} - 1 \right)}}{C_{t-1}^{\left(\frac{2\alpha-1}{a(1-\alpha)} \right)}} \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^*} \right) \left(\frac{Y_{H,t}}{Y_{H,t-1}} \right)^{\frac{1}{a}} \left(\frac{Y_{I,t}}{Y_{I,t-1}} \right)^{\left(\frac{\alpha}{a(1-\alpha)} \right)} \\
&+ \lambda_{11t+1} \left(\frac{2\alpha-1}{a(1-\alpha)} \right) \frac{C_{t+1}^{\left(\frac{2\alpha-1}{a(1-\alpha)} \right)}}{C_t^{\left(\frac{2\alpha-1}{a(1-\alpha)} + 1 \right)}} \left(\frac{\Pi_{HF,t+1}}{\Pi_{FF,t+1}^*} \right) \left(\frac{Y_{H,t+1}}{Y_{H,t}} \right)^{\frac{1}{a}} \left(\frac{Y_{I,t+1}}{Y_{I,t}} \right)^{\left(\frac{\alpha}{a(1-\alpha)} \right)} = 0
\end{aligned}$$

$$\Delta H, t : -(1 + x^{\frac{1}{\psi}}) \left(\frac{Y_{H,t}}{vA_{F,t}G_t^{chi}} \right)^{1+\psi} \Delta_{H,t}^{\psi} + \lambda_{3t} - \lambda_{3t+1} \theta \Pi_{HF,t+1}^{\gamma} - \lambda_{5t} \psi \frac{\Delta_{H,t}^{\psi-1} Y_{H,t}^{\psi+2}}{(1-\alpha)v^{\psi} A_{F,t}^{\psi+1} G_t^{\chi(\psi+1)}} = 0$$

$$\begin{aligned} Y_{H,t} : & -(1 + x^{\frac{1}{\psi}}) \left(\frac{\Delta_{H,t}}{vA_{F,t}G_t^{\chi}} \right)^{1+\psi} Y_{H,t}^{\psi} - \lambda_{1t} (Y_{I,t})^{\left(\frac{-\alpha}{\alpha}\right)} (C_t^*)^{\left(\frac{a}{1-a}\right)} (\Gamma_t^*)^{\left(\frac{\alpha-a}{1-a}\right)} \alpha^{\frac{\alpha}{\alpha}} \\ & - \lambda_{5t} \frac{(\psi+2)\Delta_{H,t}^{\psi} Y_{H,t}^{\psi+1}}{(1-\alpha)v^{\psi} A_{F,t}^{\psi+1} G_t^{\chi(\psi+1)}} \\ & - \lambda_{6t} \\ & - \lambda_{11t} \frac{1}{a} \frac{Y_{H,t}^{\frac{1-a}{a}}}{Y_{H,t-1}^{\frac{1}{a}}} \frac{\Pi_{HF,t}}{\Pi_{FF,t}^*} \frac{C_t}{C_{t-1}} \frac{2\alpha-1}{a(1-\alpha)} \frac{Y_{I,t}}{Y_{I,t-1}} \frac{\alpha}{a(1-\alpha)} \\ & + \lambda_{11t+1} \frac{1}{a} \frac{Y_{H,t+1}^{\frac{1}{a}}}{Y_{H,t}^{\frac{1+a}{a}}} \frac{\Pi_{HF,t+1}}{\Pi_{FF,t+1}^*} \frac{C_{t+1}}{C_t} \frac{2\alpha-1}{a(1-\alpha)} \frac{Y_{I,t+1}}{Y_{I,t}} \frac{\alpha}{a(1-\alpha)} = 0 \end{aligned}$$

$$\begin{aligned} Y_{I,t} : & \lambda_{1t} \frac{\alpha}{(1-\alpha)(1-a)} Y_{H,t} Y_{I,t}^{\frac{-\alpha}{(1-\alpha)(1-a)}-1} (C_t^*)^{\frac{a}{1-a}} (\Gamma_t^*)^{\frac{\alpha a}{1-a}} \alpha^{\frac{\alpha}{(1-\alpha)(1-a)}} + \lambda_{2t} \\ & + \lambda_{11t} \frac{\alpha}{a(1-\alpha)} \frac{Y_{I,t}^{\frac{\alpha}{a(1-\alpha)}-1}}{Y_{I,t-1}^{\frac{\alpha}{a(1-\alpha)}}} \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^*} \right) \left(\frac{C_t}{C_{t-1}} \right)^{\frac{2\alpha-1}{a(1-\alpha)}} \left(\frac{Y_{H,t}}{Y_{H,t-1}} \right)^{\frac{1}{a}} \\ & + \lambda_{11t+1} \frac{\alpha}{a(1-\alpha)} \frac{Y_{I,t+1}^{\frac{\alpha}{a(1-\alpha)}}}{Y_{I,t}^{\frac{\alpha}{a(1-\alpha)}+1}} \left(\frac{\Pi_{HF,t+1}}{\Pi_{FF,t+1}^*} \right) \left(\frac{C_{t+1}}{C_t} \right)^{\frac{2\alpha-1}{a(1-\alpha)}} \left(\frac{Y_{H,t+1}}{Y_{H,t}} \right)^{\frac{1}{a}} = 0 \end{aligned}$$

$$\Pi_t : \lambda_{5t-1} \theta \beta \frac{C_{t-1}}{C_t} \Pi_t^{-2} \Pi_{HF,t}^{\gamma} F_{1t} + \lambda_{6t-1} \theta \beta \frac{C_{t-1}}{C_t} \Pi_t^{-2} \Pi_{HF,t}^{\gamma-1} F_{2t} + \lambda_{8t} = 0$$

$$\Pi_{F,t} : -\lambda_{8t} (1-\alpha) \Pi_{I,t}^{\alpha} \Pi_{F,t}^{-\alpha} - \lambda_{9t} (1-\alpha) \left(\frac{G_{t-1}}{G_t} \right)^{\chi} \left(\frac{C_t}{C_{t-1}} \right) \frac{A_{I,t-1}}{A_{I,t}} \Pi_{F,t}^{-\alpha} + \lambda_{10t} = 0$$

$$\Pi_{I,t} : -\lambda_{8t} \alpha \Pi_{I,t}^{\alpha-1} \Pi_{F,t}^{1-\alpha} + \lambda_{9t} (1-\alpha) \Pi_{I,t}^{-\alpha} = 0$$

$$\begin{aligned} \Pi_{HF,t} : & -\lambda_{3t} \theta \gamma \Pi_{HF,t}^{\gamma-1} \Delta_{H,t-1} - \lambda_{5t-1} \theta \beta \gamma \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-1} \Pi_{HF,t}^{\gamma-1} F_{1t} - \lambda_{6t-1} \theta \beta (\gamma-1) \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-1} \Pi_{HF,t}^{\gamma-2} F_{2t} \\ & + \lambda_{7t} \theta (\gamma-1) \Pi_{HF,t}^{\gamma-2} - \lambda_{10t} (1-a) \Pi_{HF,t}^{-a} (\Pi_{FF,t}^*)^a (dE_t)^a \\ & - \lambda_{11t} \frac{1}{\Pi_{FF,t}^*} \left(\frac{C_t}{C_{t-1}} \right)^{\frac{2\alpha-1}{a(1-\alpha)}} \frac{Y_{H,t}}{Y_{H,t-1}} \frac{1}{a} \left(\frac{Y_{I,t}}{Y_{I,t-1}} \right)^{\frac{\alpha}{a(1-\alpha)}} = 0 \end{aligned}$$

$$\Pi_{HF,t}^* : \lambda_{3t} \gamma (1-\theta) (\Pi_{HF,t}^*)^{-\gamma-1} + \lambda_{4t} + \lambda_{7t} (1-\theta) (1-\gamma) (\Pi_{HF,t}^*)^{-\gamma} = 0$$

$$F_{1t} : -\lambda_{4t} \left(\frac{\gamma}{\gamma-1} \right) \frac{1}{F_{2t}} + \lambda_{5t} - \lambda_{5t-1} \theta \beta \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-1} \Pi_{HF,t}^{\gamma} = 0$$

$$F_{2t} : \lambda_{4t} \left(\frac{\gamma}{\gamma-1} \right) \left(\frac{F_{1t}}{F_{2t}^2} \right) + \lambda_{6t} - \lambda_{6t-1} \theta \beta \left(\frac{C_{t-1}}{C_t} \right) \Pi_t^{-1} \Pi_{HF,t-1}^{\gamma-1} = 0$$

$$\begin{aligned} G_t : & \chi (1 + x^{\frac{1}{\psi}}) \frac{\Delta_{H,t} Y_{H,t}^{1+\psi}}{vA_{F,t}} \frac{1}{G_t^{\chi}} - \lambda_{2t} \chi A_{I,t} \delta^{\chi} G_t^{\chi-1} \alpha^{\frac{1}{1+\mu}} + \lambda_{5t} \chi (\psi+1) \frac{\Delta_{H,t}^{\psi} Y_{H,t}^{\psi+2}}{(1-\alpha)v^{\psi} A_{F,t}^{\psi+1} G_t^{\chi(\psi+1)+1}} \\ & + \lambda_{9t} \chi \frac{G_{t-1}^{\chi}}{G_t^{\chi+1}} \left(\frac{C_t}{C_{t-1}} \right) \left(\frac{A_{I,t-1}}{A_{I,t}} \right) \Pi_{F,t}^{1-\alpha} \\ & + \lambda_{9t+1} \chi \frac{G_t^{\chi-1}}{G_{t+1}^{\chi}} \left(\frac{C_{t+1}}{C_t} \right) \left(\frac{A_{I,t}}{A_{I,t+1}} \right) \Pi_{F,t+1}^{1-\alpha} + \lambda_{6t} = 0 \end{aligned}$$

$$dE_t : -\lambda_{10t} a \Pi_{HF,t}^{1-a} (\Pi_{FF,t}^{*a} (dE_t)^{a-1} + \lambda_{11t} = 0$$

Steady State:

- $C = \left(\frac{Y_H}{1-\alpha}\right)^{\frac{(1-\alpha)(1-a)}{1-2\alpha}} \left(\frac{Y_I}{\alpha}\right)^{\frac{\alpha}{2\alpha-1}} (C^* \Gamma^* \alpha)^{\frac{a(1-\alpha)}{1-2\alpha}}$
- $Y_I = (\delta G)^\chi \alpha^{\frac{1}{1+\mu}}$
- $\Delta_H = \frac{(1-\theta)(\Pi_{HF}^*)^{-\gamma}}{1-\theta \Pi_{HF}^\gamma}$
- $\Pi_{HF}^* = \left(\frac{\gamma}{\gamma-1}\right) \left(\frac{F_1}{F_2}\right)$
- $F_1(1 - \theta\beta \Pi_{HF}^{\gamma-1}) = \frac{\Delta_H^\psi Y_H^{\psi+2}}{(1-\alpha)v^\psi G^{\chi(\psi+1)}}$
- $F_2(1 - \theta\beta \Pi_{HF}^{\gamma-2}) = Y_H - G \Rightarrow F_2 = \frac{Y_H - G}{1 - \theta\beta \Pi_{HF}^{\gamma-2}}$
- $\Pi = \Pi_I^\alpha \Pi_F^{1-\alpha}$
- $\Pi_I^{1-\alpha} = \Pi_F^{1-\alpha} \Rightarrow \Pi_I = \Pi_F$
- $\Pi_F = \Pi_{HF}^{1-a} (dE)^a \Pi_F^a$
- $dE = \Pi_{HF}$

Chapter 2

Unemployment and Migration in a Currency Union

2.1 Introduction

Labor mobility has been discussed as a precondition for the optimality of a currency union (Mundell 1961). The idea is very straightforward: if one region in the union is in a recession and another region is enjoying better economic conditions, then unemployed workers from the region in recession can relocate to the booming region, improving welfare for the entire union. High labor mobility within the US, a currency union, is cited as the reason for improvement of economic conditions during some past periods of regional recessions. For instance Table 2.1 below reports some labor statistics (unemployment rates and share of employment) for the state of Massachusetts during and after a period of regional economic slowdown in the late eighties. While the rest of the US enjoyed relatively higher economic activity, Massachusetts was going through difficult economic times (with the unemployment rate peaking at around 8.8%). By 1996 the region had recovered and achieved full employment, but the number of employed, as measured by the state's share in US total employment, never reached the pre-bust levels. Even though unemployment was down to the same level as 1986, the employment

share in 1996 was lower than the 1986 figure. This indicates that the unemployed agents simply relocated to different parts of the country for better job prospects. Low levels of labor mobility within the euro area (Bonin et al. [2008]) are seen by many scholars as the reason for the recent episodes of extended stagnation experienced by some countries in the union.

Table 2.1: Labor Mobility and Unemployment Krugman [2012]

Year	MA Share in US Employment	MA Unemployment	US Unemployment
1986	2.70	4.0	7.0
1991	2.48	8.8	6.8
1996	2.43	4.6	5.4

The question that this paper intends to answer is whether or not labor mobility within a monetary union can improve economic conditions across the union. The theoretical literature about labor mobility within a currency union is very sparse. Farhi and Werning [2014] and Dmitriev and Hoddenbagh [2015] were arguably the first to formally study this topic in a theoretical model. Farhi and Werning [2014] show that labor mobility does not necessarily improve the welfare of the union after agents relocate to different parts. In their model, agents who relocate achieve higher economic welfare, but their relocation does not necessarily improve welfare for those who stayed behind. In their view, labor mobility does not necessarily improve economic welfare for the union. Dmitriev and Hoddenbagh [2015] allow for labor mobility in a dynamic model and conclude that it eliminates some distortions in the economy, namely real wage rigidity and the lack of risk sharing (if financial autarky). Farhi and Werning [2014] develop a very stylized static model of a currency union. Dmitriev and Hoddenbagh [2015] develop a dynamic model but do not explicitly analyze the welfare impact of labor mobility.

I set up a New Keynesian (NK), dynamic stochastic general equilibrium (DSGE) that

extends the benchmark model developed by Abbritti and Mueller [2013]. The model consists of a simple currency union with country members Home and Foreign. Both country members' economies are characterized by labor market frictions as developed by Mortensen and Pissarides [1994] and introduced in a NK model by Blanchard and Galí [2010]. The extension I introduce is a constant migration rate from Foreign (the country member with higher long run unemployment rate and fewer job prospects) to Home (the country with lower long run unemployment rate and more job prospects for unemployed agents). One can think of Greece or Spain being the Foreign country and Germany or the United Kingdom being the Home country. As a matter of fact, I calibrate the model to match labor market moments of Germany and Greece. I only consider one way migration (from Foreign to Home) for simplicity and tractability. Furthermore, I take monetary policy as given and study the impact of migration on short run dynamics and the transmission of aggregate shocks across country members of the monetary union.

I find that labor migration increases the job finding rate in the country of origin (Foreign) and the decreases job finding rate in the host country (Home). This causes Home unemployment to rise and Foreign unemployment to fall. The relocating agents increase the size of the pool of job seekers in Home and decrease the size of the pool of job seekers in Foreign. With a constant numbers of job openings, the rise of the number of job seekers increases unemployment in Home and the fall in the number of job seekers lowers unemployment in Foreign. I also find that migration of unemployed agents increases welfare for the host country and decreases welfare for the country of origin. On average, the union's welfare improves because the welfare gains in Home outweigh the losses in Foreign. In Home, higher unemployment means lower welfare, but a lower job finding rate means lower hiring costs. Lower hiring costs mean that firms increase employed labor and thus output expands and consumption also expands. The positive impact of the increase in consumption is larger than the negative impact of the increase in labor effort (employment and unemployment). The opposite is true in Foreign. It is important to note that neither nominal wages nor real

wages are equalized across country members. This is due to the fact that there is no perfect labor mobility, but rather an exogenous constant rate of migration, hence the features and distortions creating diverging wages are still present.

This paper contributes to the literature on labor mobility within currency unions. The main contribution to that literature is the inclusion of unemployment as a motive for migration. Past studies of labor mobility have ignored unemployment (Dmitriev and Hoddenbagh [2015], Baglioni et al. [2015] and Farhi and Werning [2014]). And the few studies that have considered unemployment have ignored migration of unemployed agents between country members of the union. This paper attempts to close this gap by studying unemployment in a currency union with an exogenous migration rate. A significant amount of the debate regarding the current euro crisis is centered around the level of unemployment in different parts of the union (i.e: high unemployment in Spain and low unemployment in Germany), and no model of currency union formally incorporates unemployment.

Section 2 briefly reviews the literature on labor market friction and labor mobility in a currency union. Section 3 lays out the model. Section 4 presents the steady state of the model and the long run effect of the exogenous migration rate on union wide unemployment. Section 5 compares the short dynamics of the union with labor migration and the economy with no labor migration. Section 6 concludes.

2.2 In the Literature

The optimal currency area (OCA) literature pioneered by Mundell [1961] extensively documents factor (i.e capital and labor) mobility as a precondition for an optimal currency area. Though the idea has been around for a long time, relatively few formal models have been developed to explain how labor mobility helps with macroeconomic adjustment.

Farhi and Werning [2014] develop a very stylized static model of a currency union and conclude that labor mobility does not necessarily improve welfare cross the union. On

one hand, if the region experiencing an economic downturn has specialized production and consumption (non-traded), then relocating agents achieve higher welfare, but those who stay do not. On the other hand, if the consumption is not specialized in the region experiencing a downturn, then relocating agents improve the welfare of those who stay. Contrary to Farhi and Werning [2014], Dmitriev and Hoddenbagh [2015] develop a dynamic model, but both of these studies do not include unemployment. Dmitriev and Hoddenbagh [2015] conclude that labor mobility can act as a substitute to fiscal integration within the currency union. Baglioni et al. [2015] study labor mobility in a currency union within a stylized model and conclude that labor mobility is not a precondition for the union to be optimal. Particularly, they show that costless labor mobility does not necessarily improve welfare. Mandelman and Zlate [2012] study migration and unskilled labor from Mexico to the US and the patterns of remittances sent back to Mexico by those migrants. The model documents and explains short run dynamics and aggregate shock transmission in the presence of migration of unskilled labor. The closest model to the one I develop in this paper is in Abbritti and Mueller [2013]. The authors develop a simple model of a currency union with labor market friction, which gives rise to an involuntary unemployment. The model in this paper can be thought of as a merger between Mandelman and Zlate [2012] and Abbritti and Mueller [2013].

Although I do not study optimal policy in this paper, it is worth mentioning the vast literature on optimal fiscal and monetary policy in a currency union, which has extensively been studied in recent years. Benigno [2004] characterizes the optimal monetary policy in a currency union in which country members are characterized by different degrees of nominal rigidity. Ferrero [2009] and Galí and Monacelli [2008] lay out the optimal conduct of both fiscal and monetary policy in a currency union. All concluded that price stability at the union level is optimal and fiscal policy should be used for country specific counter-cyclical stabilization. The literature in the area has not incorporated unemployment or labor mobility in their studies. Blanchard and Galí [2010] explain the importance of unemployment in discussing monetary policy, which has been the main criticism and shortcoming of traditional

New-Keynesian models. Along the lines of Blanchard and Galí [2010], Faia [2009] studies the optimal Ramsey monetary policy with labor market frictions. There is a departure from price stability in the presence of labor market frictions. In what follows I lay out the elements of the model I develop in order to study the implications of labor migration.

2.3 The Model

2.3.1 Households

The household in country j maximizes its lifetime utility:

$$E_0 \sum \beta^t \left(\log C_t^j - \frac{\chi}{1+\phi} (L_t^j)^{1+\phi} \right) \quad (2.1)$$

where the consumption baskets in each country is defined as $C_t = \left(\frac{C_{H,t}}{1-\alpha} \right)^{1-\alpha} \left(\frac{C_{F,t}}{\alpha} \right)^\alpha$ and $C_t^* = \left(\frac{C_{H,t}^*}{1-\alpha} \right)^{1-\alpha} \left(\frac{C_{F,t}^*}{\alpha} \right)^\alpha$. $L_t = N_t + \kappa U_t$ is the total labor effort by members of the representative household. This captures the fact that the household derives a dis-utility from from employed and unemployed agents. N_t is employment and U_t is unemployment. A star (*) denotes variables for the Foreign country and H, F denotes the country of origin (i.e $C_{H,t}$ is the consumption bundle produced in Home and consumed in Home; $C_{H,t}^*$ is the consumption bundle produced in Home and consumed in Foreign).

The budget constraint for the Foreign country is the following equation:

$$P_t^* C_t^* + E_t (Q_{t,t+1}^* D_{t+1}^*) + \delta^*(1-v)N_t^* f_{e,t} = W_t^* N_t^* + D_t^* \quad (2.2)$$

where W_t^* is the Nash equilibrium bargained wage, which I'll return to later in the paper. $\delta^*(1-v)$ represents the fraction of agents who lost their jobs and decided to relocate to Home. $\delta^*(1-v)N_t^* f_{e,t}$ is the cost associated with relocating unemployed household members, which is expressed in terms of bundles of Foreign goods. And $f_{e,t} = \varepsilon_t f_e$ where ε_t is the shock to

the fixed migration cost (i.e random subsidies that lower the cost of moving or restrictions that increase the cost of migration). This is modeled after Mandelman and Zlate [2012]. The total cost of relocation (paid by the Foreign household) is $F_t^* = \delta^*(1 - v)N_t^*f_{e,t}^*$ and is expressed in terms of units of Foreign (produced) goods. Finally for simplicity, I assume that financial markets are complete. The (Foreign) households' optimality conditions are:

$$\chi C_t^* (L_t^*)^\phi = \frac{W_t^* - \delta^*(1 - v)f_{e,t}^*}{P_t^*} \quad (2.3)$$

$$\beta \left(\frac{C_t^*}{C_{t+1}^*} \right) \left(\frac{P_t^*}{P_{t+1}^*} \right) = \frac{1}{R_t} = Q_{t,t+1} \quad (2.4)$$

The optimality conditions for the representative household in Home is standard and exactly the same as the above condition when $v = 1$, meaning no migration.

2.3.2 Relative Prices and International Risk Sharing

The terms of trade between the Home and Foreign countries are given by:

$$S_t = \frac{P_{F,t}}{P_{H,t}} \quad (2.5)$$

The Law of One Price holds across the union, so $P_{F,t} = P_{F,t}^*$ and $P_{H,t} = P_{H,t}^*$. The aggregate prices in the Home and Foreign countries are respectively $P_t = P_{H,t}^{1-\alpha} P_{F,t}^\alpha$ and $P_t^* = (P_{H,t}^*)^\alpha (P_{F,t}^*)^{1-\alpha}$. These can be rewritten as a function of the terms of trade as $P_t = P_{H,t} S_t^\alpha$ and $P_t^* = P_{F,t} S_t^{-\alpha}$. The real exchange rate is then given by $Q_t = P_t^*/P_t = S_t^{1-2\alpha}$. In complete asset markets, each household has access to a complete set of contingent claims, traded internationally. It follows that Home and Foreign consumption co-move with respect to the real exchange rate:

$$C_t = \Upsilon Q_t C_t^* \quad (2.6)$$

2.3.3 Firms and the Labor Market

The Labor Market

At the beginning of every period, a fraction N_t of any given household members are employed and the remaining members $1 - N_t$ are unemployed. Of those who are employed, a fraction δ lose their job by the end of the period. In the Foreign country a fraction $(1 - \nu)$ of those that lose their jobs relocate to the Home country to find employment. In Home, the pool of jobless members consists then of those unemployed plus those who arrive from Foreign.

Timing and Flow:

The pool of employed in country i consists of newly hired agents and those who remained employed last period. This pool evolves as:

$$N_{i,t} = (1 - \delta_i) N_{i,t-1} + H_{i,t} \quad (2.7)$$

The size of the pool of newly hired agents (the difference between employment at time t and employment at the end of time $t-1$) is also given by the above equation: $H_{i,t} = N_{i,t} - (1 - \delta_i) N_{i,t-1}$. The pool of jobless agents looking for employment at the end of each period is given by:

$$U_{i,t} = (Lf_{i,t} - N_{i,t-1}) + \delta_i N_{i,t-1} = Lf_{i,t} - (1 - \delta_i) N_{i,t-1} \quad (2.8)$$

However, the aggregate pool of jobless looking for work in each country is different than the above equation due to migration. A fraction of this pool from Foreign relocates to Home and the pool of unemployed agents looking for employment in Home is given by:

$$U_t^0 = [Lf_{ss} - (1 - \delta) N_{t-1}] + (1 - \nu) [Lf_{ss}^* - (1 - \delta^*) N_{t-1}^*] \quad (2.9)$$

Home labor markets also depends on Foreign labor market conditions. The size of the total pool of jobless looking for work in Foreign (from Home and Foreign) is:

$$U_t^{*0} = v [Lf_{ss}^* - (1 - \delta^*) N_{t-1}^*] \quad (2.10)$$

With migration, the labor force in both countries changes. As the labor increases in Home with migration, it falls in Foreign.

$$Lf_t = Lf_{ss} + (1 - v) [Lf_t^* - (1 - \delta^*) N_{t-1}^*] \quad (2.11)$$

$$Lf_t^* = \frac{Lf_{ss}^* + (1 - v) (1 - \delta^*) N_{t-1}^*}{(2 - v)} \quad (2.12)$$

The labor force in foreign decreases by the number of unemployed who relocate.

Hiring Costs:

Every firm in both countries must pay to hire new workers; the cost of hiring one worker is:

$$G_{i,t} = \Gamma_i A_{i,t} x_{i,t}^\gamma \quad (2.13)$$

Total hiring costs for firm m in country i is $G_{i,t} H_{i,t}(m)$. There is no uncertainty in hiring costs and the costs are expressed in terms of domestic goods. The (per hire) cost can be expressed as $G_{i,t} = \Gamma_i A_{i,t} \left(\frac{H_{i,t}}{E_{i,t}}\right)^\gamma$. Furthermore, $x_{i,t} = \left(\frac{H_{i,t}}{U_{i,t}^0}\right)$ is a measure of labor market tightness in country i , also referred to as the job finding rate. For each country the job finding rate can expressed as:

$$x_t = \frac{N_t - (1 - \delta) N_{t-1}}{[Lf_t - (1 - \delta) N_{t-1}] + (1 - v) [Lf_t^* - (1 - \delta^*) N_{t-1}^*]} \quad (2.14)$$

$$x_t^* = \frac{N_t^* - (1 - \delta) N_{t-1}^*}{v [Lf_t - (1 - \delta^*) N_{t-1}^*]} \quad (2.15)$$

For Home, the hiring cost is now a function of both Home and Foreign labor market characteristics.

2.3.4 Market Clearing Conditions

Focusing on goods market clearing for Foreign, output must equal the additions of Foreign demand for Foreign goods, Home demand for Foreign goods, the total hiring cost, the cost of relocation to Home and the cost of price adjustment (all expressed in terms of bundles of Foreign goods). Firm m in Foreign faces the following aggregate demand for its good:

$$Y_t(m)^* = \left(\frac{P_{F,t}(m)}{P_{F,t}} \right)^{-1} (C_{F,t} + C_{F,t}^* + G_t^* H_t^* + F_t^* + \Psi_t^*) \quad (2.16)$$

Using the definition of aggregate output, $Y_t = \left(\int_0^1 (Y_t(m))^{\frac{\epsilon-1}{\epsilon}} dm \right)^{\frac{\epsilon}{\epsilon-1}}$, the market clearing condition becomes:

$$Y_t^* = C_t^* S_t^{-\alpha} + G_t^* H_t^* + F_t^* + \Psi_t^* \quad (2.17)$$

Expressed in terms of Home consumption $Y_t^* = C_t S_t^{\alpha-1} + G_t^* H_t^* + F_t^* + \Psi_t^*$. A similar market clearing condition for the Home country can be derived:

$$Y_t = C_t S_t^\alpha + G_t H_t + \Psi_t$$

2.4 The Constrained Efficient Allocations

I first look at the equilibrium in the absence of all nominal rigidity and monopolistic power. The social planner takes the labor market frictions as given and allocates resources accordingly, by maximizing the union-wide welfare with respect to the resource constraints in both Home and Foreign. The union-wide welfare is simply the sum of Home and Foreign

welfare functions:

$$U_t + U_t^* = \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{\chi}{1+\phi} (L_t)^{1+\phi} + \log C_t^* - \frac{\chi^*}{1+\phi} (L_t^*)^{1+\phi} \right) \quad (2.18)$$

The resource constraints are as defined earlier. The constrained efficient allocation of consumption is to equalized consumption across Home and Foreign: $C_t = C_t^*$. Given a job finding rate x , the constrained efficient level of employment is:

$$N_e = \frac{x(1 + (1 - \nu)[1 - (1 - \delta^*)N_e^*])}{\delta + (1 - \delta)x} \quad (2.19)$$

$$N_e^* = \frac{\nu x^*}{\delta + (1 - \delta^*)\nu x^*} \quad (2.20)$$

The constrained optimal job finding rate is given by the solution of the following system of equations:

$$\begin{aligned} \chi C_t N_t^\phi &= [1 - \Gamma(\gamma + 1)x_t^\gamma] + \dots \\ \Gamma\left(\frac{A_{t+1}}{A_t}\right) \left(\frac{C_t}{C_{t+1}}\right) &[-(\gamma + 1)(1 - \delta)x_{t+1}^\gamma + \gamma\nu(1 - \delta)x_{t+1}^{\gamma+1}] + \dots \\ \Gamma^*\left(\frac{A_{t+1}^*}{A_t^*}\right) \left(\frac{C_t}{C_{t+1}^*}\right) &\gamma(1 - \nu)(1 - \delta)x_{t+1}^{*(\gamma+1)} \end{aligned}$$

$$\begin{aligned} \chi^* C_t^* N_t^{*\phi} &= [1 - \Gamma^*(\gamma + 1)x_t^{*\gamma}] + \dots \\ \Gamma^*\left(\frac{A_{t+1}^*}{A_t^*}\right) \left(\frac{C_t^*}{C_{t+1}^*}\right) &[-(\gamma + 1)(1 - \delta^*)x_{t+1}^{*\gamma} + \gamma\nu^*(1 - \delta^*)x_{t+1}^{*\gamma+1}] \end{aligned}$$

where both N_t and N_t^* are functions of job finding rates x_t and x_t^* . The constrained

optimal rate of migration v is then the following equation:

$$v = \frac{\left[1 + \left(\frac{(1-(1-\delta)N_{t-1})}{(1-(1-\delta^*)N_{t-1}^*)} \right) \right]}{\left[1 + \left(\frac{C_t^*}{C_t} \right) \left(\frac{\Gamma^*}{\Gamma} \right) \left(\frac{A_t}{A_t^*} \right) \left(\frac{\gamma}{\gamma^*} \right) \left(\frac{x_t^\gamma}{(x_t^*)^{\gamma^*}} \right) \right]} \quad (2.21)$$

2.5 Wage Bargaining

Here I focus on wage bargaining in Foreign and relegate wage bargaining in Home to the appendix at the end of the paper. The value of an employed household member in the Foreign country V_t^{*E} is:

$$V_t^{*E} = \widetilde{W}_t^* - \chi^* C_t^* L_t^{*\phi} + \beta E_t \left\{ \frac{C_t^*}{C_{t+1}^*} [(1 - \delta^* v(1 - x_{t+1}^*)) V_{t+1}^{*E} + \delta^* v(1 - x_{t+1}^*) V_{t+1}^{*U}] \right\} \quad (2.22)$$

The value of the employed is now a function of v , and when there is no migration ($v = 1$) the equation reduces to the ones found in the literature on labor market frictions (Blanchard and Galí [2010]). The value of an unemployed household member in Foreign V_t^{*U} is also a function of migration in addition to the cost of migration to Home:

$$V_t^{*U} = -\delta^*(1 - v)f_{e,t}^* + \beta E_t \left\{ \frac{C_t^*}{C_{t+1}^*} [x_{t+1}^* V_{t+1}^{*E} + (1 - x_{t+1}^*) V_{t+1}^{*U}] \right\} \quad (2.23)$$

Combining the employed and unemployed values, the households' surplus can be expressed as:

$$V_t^{*E} - V_t^{*U} = \widetilde{W}_t^* - \chi^* C_t^* L_t^{*\phi} + \delta^*(1 - v)f_{e,t}^* + \dots \\ (1 - \delta^* v) \beta E_t \left\{ \frac{C_t^*}{C_{t+1}^*} [(1 - x_{t+1}^*) (V_{t+1}^{*E} - V_{t+1}^{*U})] \right\} \quad (2.24)$$

The value of an employee to a firm (continued match) V_t^{*F} is just the hiring cost in terms of bundles of the Foreign good:

$$V_t^{*F} = \left(\frac{P_{F,t}^*}{P_t^*} \right) G_t^* = A_t^* \Gamma^* S_t^\alpha x_t^{*\gamma} \quad (2.25)$$

This surplus will be split between the firm and the workers. through bargaining Furthermore, if μ is the relative bargaining power of the workers then:

$$V_t^{*E} - V_t^{*U} = \mu V_t^{*F} = \mu^* A_t^* \Gamma^* S_t^\alpha x_t^{*\gamma} \quad (2.26)$$

The Nash wage schedule in the Foreign country is given by the following three conditions:

$$\begin{aligned} \widetilde{W}_t^* &= \mu^* A_t^* \Gamma^* S_t^\alpha x_t^{*\gamma} + \chi^* C_t^* L_t^{*\phi} - \delta^* (1 - v) f_{e,t}^* - \dots \\ & (1 - \delta^* v) \beta E_t \left\{ \frac{C_t^*}{C_{t+1}^*} [(1 - x_{t+1}^*) (\mu^* A_{t+1}^* \Gamma^* S_{t+1}^\alpha x_{t+1}^{*\gamma})] \right\} \end{aligned} \quad (2.27)$$

2.6 Price Setting

2.6.1 Flexible Prices

Under flexible prices, the firms set their prices as a mark-up ($\varrho = \frac{\epsilon}{\epsilon-1}$) over their marginal cost.

$$P_{H,t}(i) = \varrho MC_t P_{H,t} \quad (2.28)$$

where MC_t is the real marginal cost in terms of bundles of the domestic goods. The (supply-side) flexible price equilibrium condition in Home is given by:

$$MC_t^r = \frac{\widetilde{W}_t}{A_t} S_t^\alpha + \Gamma x_t^\gamma - \beta (1 - \delta) E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{S_t}{S_{t+1}} \right)^\alpha \left(\frac{A_{t+1}}{A_t} \right) \Gamma x_{t+1}^\gamma \right\} \quad (2.29)$$

Note that in equilibrium, assuming symmetry, $P_{H,t}(j) = P_{H,t}$ and $MC_t = \frac{1}{\varrho}$. Replacing marginal cost in equation 2.29 with the previous expression, the supply-side equilibrium is given by the following condition:

$$\frac{\widetilde{W}_t}{A_t} S_t^\alpha = \frac{1}{\varrho} - \Gamma x_t^\gamma + \beta (1 - \delta) E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{S_t}{S_{t+1}} \right)^\alpha \left(\frac{A_{t+1}}{A_t} \right) \Gamma x_{t+1}^\gamma \right\} \quad (2.30)$$

Under flexible prices, monetary policy has is neutral, hence I introduce nominal rigidity (sticky prices) so that monetary policy plays a stabilizing role.

2.6.2 Sticky Prices

I introduce price stickiness à la Rottemberg. Each period, each firm faces a quadratic cost of price adjustment. Each firm m in country j chooses $P_{j,t}(m)$ to maximize its lifetime profits. Firm m in the Home chooses the optimal price by solving the following maximization problem:

$$\begin{aligned} \underset{P_{H,t}(m)}{\text{Max}} E_t \sum_0^{\infty} Q_{t,t+k} \{ & P_{H,t+k}(m) Y_{H,t+k}(m) - MC_{H,t+k}^r Y_{H,t+k}(m) P_{H,t+k} - \dots \} \\ & \left\{ \dots \frac{\psi}{2} \left(\frac{P_{H,t+k}(m)}{P_{H,t+k-1}(m)} - 1 \right)^2 Y_{H,t+k} P_{H,t+k} \right\} \end{aligned}$$

using the fact that all firms get to reset their prices and thus choose the same optimal price, which must satisfy the following equation:

$$\begin{aligned} (1 - \epsilon) + \epsilon MC_{H,t}^r - \psi \left(\frac{P_{H,t}}{P_{H,t-1}} - 1 \right) \left(\frac{P_{H,t}}{P_{H,t-1}} \right) + \dots \\ \dots \psi Q_{t,t+1} \left(\frac{P_{H,t+1}}{P_{H,t}} - 1 \right) \left(\frac{Y_{H,t+1}}{Y_{H,t}} \right) \left(\frac{P_{H,t+1}}{P_{H,t}} \right)^2 = 0 \end{aligned}$$

The previous equation can then be expressed in terms of inflation to obtain the (supply-side) sticky price equilibrium, and is represented by the following equation (the Philips curve):

$$(1 - \epsilon) + \epsilon MC_{H,t}^r - \psi (\Pi_{H,t} - 1) \Pi_{H,t} + \psi Q_{t,t+1} (\Pi_{H,t+1} - 1) \left(\frac{Y_{H,t+1}}{Y_{H,t}} \right) \Pi_{H,t+1}^2 = 0 \quad (2.31)$$

These conditions are similar for both countries. The following equations describe the situation in the Foreign country:

$$MC_t^{*r} = \frac{\widetilde{W}_t^*}{A_t^*} S_t^{-\alpha} + \Gamma^* x_t^{*\gamma} - \beta (1 - \delta^*) E_t \left\{ \left(\frac{C_t^*}{C_{t+1}^*} \right) \left(\frac{S_t}{S_{t+1}} \right)^{-\alpha} \left(\frac{A_{t+1}^*}{A_t^*} \right) \Gamma^* x_{t+1}^{*\gamma} \right\} \quad (2.32)$$

\widetilde{W}_t^* is the Nash bargained wage given by equation 2.27.

2.7 Real Wage Rigidity

Without real wage rigidity, unemployment remains constant. To induce inefficient fluctuations in the unemployment, I follow the literature and introduce real wage rigidity. Different approaches have been used to model real wage rigidity and all of them have their drawbacks. Here I follow Hall [2005] and Blanchard and Galí [2010] and assume that the real wage follows a wage norm:

$$\begin{aligned} W_t^r &= \left(\widetilde{W}_t\right)^{1-\nu} (W_{ss})^\nu \\ W_t^{*r} &= \left(\widetilde{W}_t^*\right)^{1-\nu^*} (W_{ss}^*)^{\nu^*} \end{aligned}$$

The real wages are a Cobb-Douglas aggregation of the Nash bargained wage and the wage norm. The wage norm in each country is simply the steady state level of the Nash bargained wages, hence in the steady state, the real wage is the same as the Nash bargained wage.

2.8 Monetary Policy

The equilibrium conditions of this simple currency union model are described in the Appendix but, to close the model a monetary policy rule must be specified. I do not characterize the optimal monetary policy, but instead I follow Faia [2009] and Baglioni et al. [2015]³ and set the monetary policy rule as:

$$R_t = \left(\frac{1}{\beta}\right) \left(\frac{Y_t}{Y_{ss}}\right)^{\phi_y} \left(\frac{\Pi_t}{\Pi_{ss}}\right)^{\phi_\pi} \quad (2.33)$$

All of the variables are averages of those of the country members, and the central bank conducts monetary policy for the whole union. Y_{ss} is the union-wide average steady level

of output and Π_{ss} is the union-wide average steady level of inflation, which I assume to be equal to one.

2.9 Calibration

Each time period corresponds to a quarter. I set the parameters that characterize the preferences equal to the common values used in the literature.

$$\beta = 0.96 \quad \phi = 5$$

This corresponds to a long run interest rate of about 4% and a Frisch elasticity of 0.2. I assume these parameters to be uniform across both countries in the union.

Many other estimates have been reported in the literature, but alternative estimates have no impact on the results of this paper since the focus is not monetary policy but instead labor mobility.

Below I describe the calibration strategy used to pin down the parameters describing the labor markets in each country. Table 2.3 in the Appendix summarizes all of the parameters used in the simulation exercise.

2.9.1 Euro Area: Germany and Greece

I calibrate the model so that the steady matches the labor characteristics of two countries in the euro area. One country has a lower steady state unemployment rate and a higher job finding rate. I calibrate the Home country to match Germany and Foreign to match Greece. Germany has the lowest unemployment rate in the euro area and Greece, the highest unemployment rate. The calibration is such that when there is no labor mobility ($v = 1$), the long unemployment rates match those of Germany (Home) and Greece (Foreign). The characteristics of the labor market in both countries are described below.

Germany: the unemployment rate in Germany is $U = 5\%$ and the employment rate stands at 73.91% ¹. This implies a labor force of 78.91% . I take the job finding rate from Hobijn and Sahin [2007] and set $x_{ss} = 20\%$ (a monthly job finding rate of 7%). Given the unemployment and the job finding rates, the separation rate is $\delta = 0.013$. I follow Blanchard and Galí [2010] and assume that the total hiring cost is about 1% of GDP, and the value of Γ is reported in Table 2.3.

Greece: the unemployment rate in Greece is about $U^* = 26.05\%$ and the employment rate is 49.75% , which implies a labor force participation rate of 75.80% . Hobijn and Sahin [2007] reports a job finding rate of $x_{ss}^* = 0.151$ for Greece. This is consistent with a monthly job finding rate of 5.3% . The separation rate imputed from these is $\delta^* = 0.056$. I also assume that the total hiring cost is about 1% of GDP.

The monthly job finding rates and separation rates taken from Hobijn and Sahin [2007] are then translated to cumulative quarterly job-finding rates as:

$$x_q = x_m + (1 - x_m)x_m + (1 - x_m)^2x_m$$

where x_m is the monthly job finding rate. And given the job finding rate and the employment rate, the separation rate for both countries is calculated as:

$$\delta = \frac{x [(Lf - N) + (1 - v)(Lf^* - (1 - \delta^*)N^*)]}{(1 - x)N} \quad (2.34)$$

$$\delta^* = \frac{x^*v(Lf^* - N^*)}{(1 - x^*v)N^*} \quad (2.35)$$

The migration rate parameter is free to vary, and I chose two different values in the simulations. The first is such that the job finding rates are equalized across both country and unemployed from Greece have no longer an incentive to move to in order to increase their job prospects. Under this scenario $v = 0.97$, so only 3% of the unemployed members relocate every period. The second calibration is such that there is convergence in unemployment rates

¹From FRED (St Louis FED)

across the countries, hence long run unemployment rates are equalized. In this case $v = 0.79$, which means that every period about 30% of the unemployed agents in Greece relocate to Germany. A higher rate of migration is required to achieve convergence in unemployment compared to convergence in job-finding rate.

ECB's Monetary Policy: As for the parameters in the monetary policy rule, I simply use the specification in Fourcans and Vranceanu [2002] and set $\phi_\pi = 1.16$ and $\phi_y = 0.18$.

2.10 Short Term Dynamics

In this section, I compare the short run dynamics adjustments of the economies when Foreign experiences a negative productivity shock under the two different rates of migration mentioned above. Figure 2.12 displays the impulse responses of some select variables to the negative shock in Foreign (Greece). With a negative productivity shock, output shrinks and unemployment increases in Foreign. Home unemployment also rises due to a portion of the unemployed migrating from Home. The union average unemployment also rises. Except for Home, unemployment rises less with migration, especially when the migration rate is such that long run unemployment is equalized across the two countries.

The negative shock causes a decrease in the job-finding rate in both countries. Again the fall is lessened by migration. Inflation behave in the same manner regardless of the presence of migration. This result is surprising since the Nash bargained wage is a function of the cost of relocation, so the marginal cost faced by the firm would also be affected by this cost. The migration rate and migration cost both affect the marginal cost, so if their effect cancel each other then inflation would react the same regardless of migration.

Given the dampening effect that migration has on these variables at the union level, one should expect an improvement in the union's average welfare due to migration.

2.11 Migration and Welfare

In this section I explore the impact of migration on welfare in both countries. I compare welfare by calculating the compensation required in terms of consumption such that the representative household is indifferent between living in an environment with labor mobility and one with no labor mobility. The consumption equivalence percentage Δ solves the following equation:

$$W_{migration} = \frac{U \left[\left(1 + \frac{\Delta}{100}\right) C_{nomigration}, L_{nomigration} \right]}{(1 - \beta)} \quad (2.36)$$

where W is the welfare with labor mobility and the function U on the right hand side is the period utility when there is no labor mobility. I calculate Δ for both country members. When the migration rate is calibrated so that the job-finding rates are equalized in the long run, the welfare loss in Home (Greece) due to labor migration is $\Delta = -0.11\%$. This loss decreases to $\Delta = -0.06\%$ when the migration is such that long run unemployment rates are equalized. There are welfare gains for Foreign (Germany), the host country. The welfare gains are $\Delta = 3.18\%$ when v is such that $x = x^*$ and $\Delta = 0.67\%$ when the migration rate is such that long run unemployment rate are equalized. The welfare gains and losses associated with migration are summarized in Table 2.2

Table 2.2: Welfare Gain/Loss

	Home (Germany)	Foreign (Greece)
$v = 0.793$ (s.t $Un = Un^*$)	3.18%	-0.11%
$v = 0.97$ (s.t $x = x^*$)	0.67%	-0.06%

On average, the union's welfare increases in both calibrations.

2.12 Conclusion

This paper revisited the topic of labor mobility in a currency union and in particular attempted to answer the question of whether labor mobility helps the union's economy adjust faster to aggregate shocks in the short run, and thus improving welfare across the union. This question has been studied extensively in recent years. While some studies have concluded that labor mobility improves welfare, others have concluded the opposite. These past studies have all ignored unemployment and the role of labor market characteristics in determining labor migration and its impact on the economy. In this paper, I set up a simple model of a currency union with two country members characterized by different labor markets frictions. I calibrate the model to match the labor market moments of the economies of Germany and Greece. Every period, a constant fraction of unemployed agents in Greece relocate to Germany. I first calibrate the rate of migration such that the job finding rate is equalized across country members in the long run. In an alternative calibration, I pick the rate of labor migration so that the job-finding rates are equalized in the long run. Under both calibrations I find that labor mobility decreases unemployment in the country of origin but increases the job finding rate, and welfare falls. The exact opposite happens in the host country: unemployment rises the job finding rate falls, and welfare improves. Assuming that households are identical across country members, then on balance, labor migration is welfare improving for the union as a whole since the welfare gains in the host country outweigh the losses in the country of origin.

This paper provide opportunities for several extensions. First of all, the natural continuation of this work would be the study of the optimal fiscal and monetary policy, which would provide insight into how limited migration, as it is this paper, influences the conduct of the optimal policy. A second extension, which I plan to tackle in future research, is the possibility of different consumption baskets for migrants in the host country. This extension could prove to be welfare improving for both the country of origin of the migrants and also for the host country. Finally, one could also estimate the parameters of the model developed

here instead of calibrating it to the moments. I also intend to pursue this in the future.

Tables

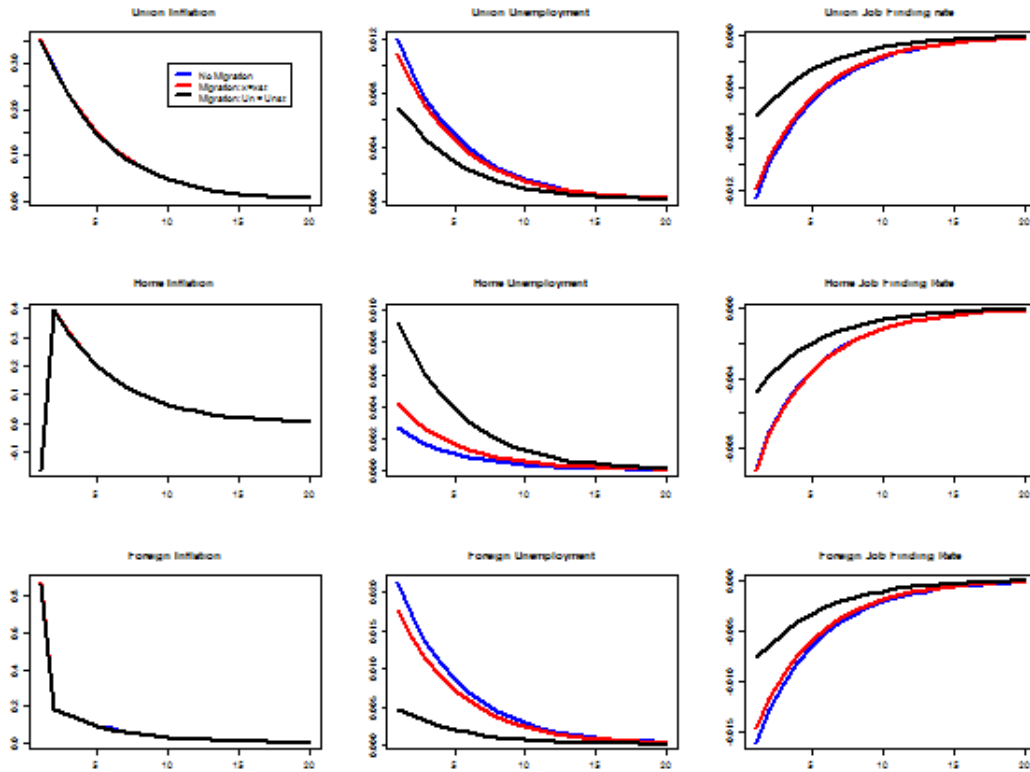
Table 2.3: Calibration

	Parameter	Value
Openness	α	0.2
Discount factor	β	0.96
Cost of Price adjustment (Home)	ψ	0.025
Cost of Price adjustment (Foreign)	ψ^*	0.025
Bargaining power (Home)	μ	1
Bargaining power (Foreign)	μ^*	1
Separation rate (Home)	δ^*	0.013
Separation rate (Foreign)	δ^*	0.056
Coef. of autocor. of output (Home)	ρ_F	0.9
Coef. of autocor. output (Foreign)	ρ_I	0.9
Goods elasticity (Home)	ϵ	6
Goods elasticity (Foreign)	ϵ^*	6
Labor Cost (Home)	Γ	7.93 (3.70)
Labor Cost (Foreign)	Γ^*	0.46 (0.67)
Labor cost elasticity wrt to job-finding rate (Home)	γ	1
Labor cost elasticity wrt to job-finding rate (Foreign)	γ^*	1
Labor effort dis-utility (Home)	χ	2.16 (3.11)
Labor effort dis-utility (Foreign)	χ^*	18.03 (10.15)
Elasticity of labor effort wrt unemployment (Home)	κ	0.8
Elasticity of labor effort wrt unemployment (Foreign)	κ^*	0.8

When two numbers are reported, the first one is for the calibration such that unemployment is equalized in the long run.

Figures

Figure 2.1: Negative Productivity Shock in the Foreign Country



Appendix

Equilibrium Conditions of the Model Economy

The economy of the union is characterized by the following equations.

- Variables: $Y_{H,t}; Y_{F,t}; C_t; C_t^*; N_t; N_t^*; x_t; x_t^*; \widetilde{W}_t, \widetilde{W}_t^*; W_t^r; W_t^{*r}; U_t; U_t^*; G_t; G_t^*; MC_t^r; MC_t^{*r}$

- Goods Market Clearing:

$$\begin{aligned}
 Y_t &= C_t S_t^\alpha + G_t H_t + \frac{\psi}{2} (\Pi_{H,t} - 1)^2 Y_t \\
 Y_t^* &= C_t^* S_t^{-\alpha} + G_t^* H_t^* + F_t^* + \frac{\psi^*}{2} (\Pi_{F,t} - 1)^2 Y_t^* \\
 &= C_t S_t^{\alpha-1} + G_t^* H_t^* + F_t^* + \frac{\psi^*}{2} (\Pi_{F,t} - 1)^2 Y_t^* \\
 S_t &= \frac{Y_t - G_t H_t - \frac{\psi}{2} (\Pi_{H,t} - 1)^2 Y_t}{Y_t^* - G_t^* H_t^* - F_t^* - \frac{\psi^*}{2} (\Pi_{F,t} - 1)^2 Y_t^*}
 \end{aligned}$$

- Production Technology:

$$Y_{H,t} = A_t N_t$$

$$Y_{F,t} = A_t^* N_t^*$$

- Aggregate hiring and employment:

$$H_t = N_t - (1 - \delta) N_{t-1}$$

$$H_t^* = N_t^* - (1 - \delta^*) N_{t-1}^*$$

- Hiring cost:

$$G_t = \Gamma A_t x_t^\gamma$$

$$G_t^* = \Gamma^* A_t^* x_t^{*\gamma}$$

- Jobless and looking for employment (in Home and Foreign):

$$U_t = (1 - (1 - \delta)N_{t-1}) + (1 - \nu) (1 - (1 - \delta^*)N_{t-1}^*)$$

$$U_t^* = \nu (1 - (1 - \delta^*)N_{t-1}^*)$$

- Job finding rate:

$$x_t = \frac{H_t}{U_t} = \frac{N_t - (1 - \delta) N_{t-1}}{(1 - (1 - \delta)N_{t-1}) + (1 - \nu) (1 - (1 - \delta^*)N_{t-1}^*)}$$

$$x_t^* = \frac{H_t^*}{U_t^*} = \frac{N_t^* - (1 - \delta^*) N_{t-1}^*}{\nu (1 - (1 - \delta^*)N_{t-1}^*)}$$

- Wage Setting (Nash bargained wage):

$$\widetilde{W}_t = \mu A_t \Gamma S_t^{-\alpha} x_t^\gamma + \chi C_t L_t^\phi - \beta (1 - \delta) E_t \left\{ \frac{C_t}{C_{t+1}} [(1 - x_{t+1}) (\mu A_{t+1} \Gamma S_{t+1}^{-\alpha} x_{t+1}^\gamma)] \right\}$$

$$\widetilde{W}_t^* = \mu^* A_t^* \Gamma^* S_t^{\alpha} x_t^{*\gamma} + \chi^* C_t^* L_t^{*\phi} - \delta^* (1 - \nu) f_{e,t}^* - (1 - \delta^* \nu) \beta E_t \left\{ \frac{C_t^*}{C_{t+1}^*} [(1 - x_{t+1}^*) (\mu^* A_{t+1}^* \Gamma^* S_{t+1}^{\alpha} x_{t+1}^{*\gamma})] \right\}$$

- Real Wage Rigidity:

$$W_t^r = (\widetilde{W}_t)^{1-\nu} (W_{ss})^\nu$$

$$W_t^{*r} = (\widetilde{W}_t^*)^{1-\nu^*} (W_{ss}^*)^{\nu^*}$$

- Price Setting:

$$(1 - \epsilon) + \epsilon MC_{H,t}^r - \psi (\Pi_{H,t} - 1) \Pi_{H,t} + \psi Q_{t,t+1} (\Pi_{H,t+1} - 1) \left(\frac{Y_{H,t+1}}{Y_{H,t}} \right) \Pi_{H,t+1}^2 = 0$$

$$(1 - \epsilon^*) + \epsilon^* MC_{F,t}^r - \psi^* (\Pi_{F,t} - 1) \Pi_{F,t} + \psi^* Q_{t,t+1} (\Pi_{F,t+1} - 1) \left(\frac{Y_{F,t+1}}{Y_{F,t}} \right) \Pi_{F,t+1}^2 = 0$$

- Marginal Costs:

$$MC_t^r = \frac{W_t^r}{A_t} S_t^\alpha + \Gamma x_t^\gamma - \beta (1 - \delta) E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{S_t}{S_{t+1}} \right)^\alpha \left(\frac{A_{t+1}}{A_t} \right) \Gamma x_{t+1}^\gamma \right\}$$

$$MC_t^{*r} = \frac{W_t^{*r}}{A_t^*} S_t^{-\alpha} + \Gamma^* x_t^{*\gamma} - \beta (1 - \delta^*) E_t \left\{ \left(\frac{C_t^*}{C_{t+1}^*} \right) \left(\frac{S_t}{S_{t+1}} \right)^{-\alpha} \left(\frac{A_{t+1}^*}{A_t^*} \right) \Gamma^* x_{t+1}^{*\gamma} \right\}$$

The Model Steady State

The economy of the union is characterized by the following equations.

- Zero inflation steady state: $\Pi_{ss} = \Pi_{H,ss} = \Pi_{F,ss} = 1$
- Variables: $Y_{H,ss}; Y_{F,ss}; C_{ss}; C_{ss}^*; N_{ss}; N_{ss}^*; x_{ss}; x_{ss}^*; W_{ss}; W_{ss}^*; W_{ss}^r; W_{ss}^{*r}; U_{ss}; U_{ss}^*; G_{ss}$
 $G_{ss}^*, MC_{ss}^r; MC_{ss}^{*r}$
- Households' conditions:

$$\chi C_{ss} (L_{ss})^\phi = W_{ss}^r \quad (2.37)$$

$$\chi^* C_{ss}^* (L_{ss}^*)^\phi = W_{ss}^{*r} - \delta^*(1 - v) \quad (2.38)$$

$$\beta = \frac{1}{R_{ss}} = Q_{ss} \quad (2.39)$$

- Goods Market Clearing:

$$Y_{ss} = C_{ss} S_{ss}^\alpha + G_{ss} H_{ss}$$

$$\begin{aligned} Y_{ss}^* &= C_{ss}^* S_{ss}^{-\alpha} + G_{ss}^* H_{ss}^* + F_{ss}^* \\ &= C_{ss} S_{ss}^{\alpha-1} + G_{ss}^* H_{ss}^* + F_{ss}^* \end{aligned}$$

$$S_{ss} = \frac{Y_{ss} - G_{ss} H_{ss}}{Y_{ss}^* - G_{ss}^* H_{ss}^* - F_{ss}^*}$$

- Production Technology:

$$Y_{H,ss} = N_{ss} = C_{ss} S_{ss}^\alpha + G_{ss} H_{ss}$$

$$Y_{F,ss} = N_{ss}^* = C_{ss}^* S_{ss}^{-\alpha} + G_{ss}^* H_{ss}^* + F_{ss}^*$$

- Aggregate hiring and employment:

$$H_{ss} = N_{ss} - (1 - \delta)N_{ss} = \delta N_{ss}$$

$$H_{ss}^* = N_{ss}^* - (1 - \delta^*)N_{ss}^* = \delta^* N_{ss}^*$$

- Hiring cost:

$$G_{ss} = \Gamma x_{ss}^\gamma$$

$$G_t^* = \Gamma^* x_{ss}^{*\gamma}$$

- Job finding rate:

$$x_{ss} = \frac{\delta N_{ss}}{(1 - (1 - \delta)N_{ss}) + (1 - v)(1 - (1 - \delta^*)N_{ss}^*)}$$

$$x_{ss}^* = \frac{\delta^* N_{ss}^*}{v(1 - (1 - \delta^*)N_{ss}^*)}$$

- Jobless and looking for employment (in Home and Foreign):

$$U_{ss} = (1 - (1 - \delta)N_{ss}) + (1 - v)(1 - (1 - \delta^*)N_{ss}^*)$$

$$U_{ss}^* = v(1 - (1 - \delta^*)N_{ss}^*)$$

- Wage Setting (Nash bargained wage):

$$W_{ss} = \mu \Gamma S_{ss}^{-\alpha} x_{ss}^\gamma + \chi C_{ss} N_{ss}^\phi - \beta(1 - \delta)(1 - x_{ss})(\mu \Gamma S_{ss}^{-\alpha} x_{ss}^\gamma)$$

$$W_{ss}^* = \mu^* \Gamma^* S_{ss}^{\alpha} x_{ss}^{*\gamma} + \chi^* C_{ss}^* N_{ss}^{*\phi} - \delta^* v F_{ss}^* - (1 - \delta^*(1 - v))\beta(1 - x_{ss}^*)\mu^* \Gamma^* S_{ss}^{\alpha} x_{ss}^{*\gamma}$$

- Real Wage Rigidity:

$$W_{ss}^r = W_{ss}$$

$$W_{ss}^r S_{ss}^\alpha = \mu \Gamma x_{ss}^\gamma + \chi C_{ss} N_{ss}^\phi S_{ss}^\alpha - \beta(1 - \delta)(1 - x_{ss})(\mu \Gamma x_{ss}^\gamma)$$

$$W_{ss}^{*r} = W_{ss}^*$$

$$W_{ss}^{*r} S_{ss}^{-\alpha} = \mu^* \Gamma^* x_{ss}^{*\gamma} + \chi^* C_{ss}^* N_{ss}^{*\phi} S_{ss}^{-\alpha} - \delta^*(1 - v)F_{ss}^* - (1 - \delta^*v)\beta(1 - x_{ss}^*)\mu^* \Gamma^* x_{ss}^{*\gamma}$$

- Price Setting:

$$MC_{ss}^r = \frac{\epsilon - 1}{\epsilon} = \varrho$$

$$MC_{F,ss}^r = \frac{\epsilon^* - 1}{\epsilon^*} = \varrho^*$$

- Marginal Costs:

$$\begin{aligned} MC_{ss}^r &= W_{ss}^r S_{ss}^\alpha + \Gamma x_{ss}^\gamma - \beta(1 - \delta)\Gamma x_{ss}^\gamma \\ &= \mu \Gamma x_{ss}^\gamma + \chi C_{ss} N_{ss}^\phi S_{ss}^\alpha - \beta(1 - \delta)(1 - x_{ss})(\mu \Gamma x_{ss}^\gamma) + \Gamma x_{ss}^\gamma - \beta(1 - \delta)\Gamma x_{ss}^\gamma \\ \frac{1}{\rho} &= (1 + \mu)\Gamma x_{ss}^\gamma + \chi C_{ss} N_{ss}^\phi S_{ss}^\alpha - \beta(1 - \delta)\Gamma x_{ss}^\gamma [1 + \mu(1 - x_{ss})] \end{aligned}$$

$$W_{ss}^{*r} = W_{ss}^*$$

$$\begin{aligned} MC_{ss}^{*r} &= W_{ss}^r S_{ss}^{-\alpha} + \Gamma^* x_{ss}^{*\gamma} - \beta(1 - \delta^*)\Gamma^* x_{ss}^{*\gamma} \\ &= \mu^* \Gamma^* x_{ss}^{*\gamma} + \chi^* C_{ss}^* N_{ss}^{*\phi} S_{ss}^{-\alpha} - \delta^*(1 - v)F_{ss}^* - (1 - \delta^*v)\beta(1 - x_{ss}^*)\mu^* \Gamma^* x_{ss}^{*\gamma} + \dots \\ &\quad \dots \Gamma^* x_{ss}^{*\gamma} - \beta(1 - \delta^*)\Gamma^* x_{ss}^{*\gamma} \\ \frac{1}{\rho^*} &= (1 + \mu^*)\Gamma^* x_{ss}^{*\gamma} + \chi^* C_{ss}^* N_{ss}^{*\phi} S_{ss}^{-\alpha} - \delta^*(1 - v)F_{ss}^* \dots \\ &\quad \dots - (1 - \delta^*v)\beta(1 - x_{ss}^*)\mu^* \Gamma^* x_{ss}^{*\gamma} - \beta(1 - \delta^*)\Gamma^* x_{ss}^{*\gamma} \end{aligned}$$

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