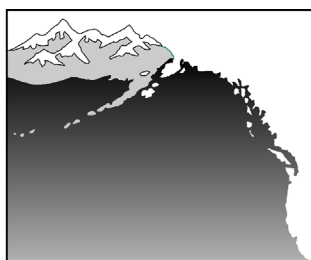


A Bayesian Version of the NIWA Two-Stock Hoki Model

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Key Words

age-structured model, Bayesian analysis, estimation methods, model complexity, stock assessment

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Abstract

A Bayesian implementation of the National Institute for Water and Atmospheric Research (NIWA) hoki (*Macruronus novaezelandiae*) model (Cordue 1999) is described. This implementation, termed the UW/Seafic implementation, is based on the documentation and data provided in Cordue (1999) with minor differences. Overall model fit for the UW/Seafic model implementation is similar to that reported by Cordue. Differences in model fit occur primarily in data series that use age composition data from juvenile hoki.

The model estimator was changed from a least-squares formulation to a maximum likelihood formulation to implement Bayesian methods to describe the posterior distributions of key parameters. Posterior distributions of the biomass trajectories and their associated confidence bounds show little effect of the inclusion of data in the model; that is, there is little attenuation in the width of the confidence bounds over the historical trajectory. This is interpreted as evidence that the model structure and the assumed bounds impose constraints on model output or that the data are not providing much information to the model.

The high penalty weights imposed by the assumption that the survey proportionality constants should be similar among areas for the same survey type have a large effect on model biomass estimates. Biomass estimates for the western region hoki are nearly doubled when these penalty weights are relaxed while the biomass estimates for the eastern region hoki are about 20% smaller. The sensitivity of important model estimates on an untestable assumption is a poor attribute for this model.

The authors conclude that the NIWA two-stock multi-area hoki model is unnecessarily complex and over-parameterized. Insufficient data exist to estimate the nearly 200 model parameters, and the estimates for most of these parameters are not well determined. The elaborate model structure allows inclusion of previously omitted data in a fashion consistent with current hypotheses of hoki population dynamics and stock structure. However, whether these additional data and the increased complexity of the model have improved the quality and precision of the stock abundance estimates is unclear.

Introduction

For the 1998 hoki (*Macruronus novaezelandiae*) stock assessment, Cordue (1999) from the National Institute for Water and Atmospheric Research (NIWA) introduced a two-stock, multi-area model specifically designed for assessing New Zealand hoki with the intention of including a large amount of auxiliary data that had previously been unutilized. These modeled dynamics adhere closely to current hypotheses about the life history, stock structure,

and movement patterns of the hoki stocks. The model structure comprises six regions and nine annual time periods. Age- and sex-specific migration between regions is modelled explicitly. This leads to a highly complex model, requiring nearly 200 parameters to describe the migration rates and the selectivities to the fisheries and to the trawl surveys.

There are many useful aspects of the NIWA model: in particular, the two-stock structure allows hoki from both the eastern and western regions to explicitly occupy the

Chatham Rise. Catches from this area can be appropriately allocated between the stocks, and age-specific trawl survey abundance data can be used in a consistent fashion. However, the model structure likely is overly complex and, consequently, the model is over-parameterized. To conduct analyses using this model, bounds are placed on all model parameters and many of the parameter values are at one of their bounds after fitting. Whether enough information exists in the data to estimate all the model parameters with sufficient precision for the purpose of the model is unclear.

To investigate properties of the NIWA two-stock model, we independently coded the model structure using AD Model Builder software (Otter Research Ltd. 1994) and following the description provided in Cordue (1999). Stock reconstructions were conducted with this model, which we term the University of Washington/New Zealand Seafood Industry Council (UW/Seafic) implementation, using the data presented in Cordue (1999). The parameter estimates were compared with those reported by Cordue to ascertain that our implementation of the model was consistent with the NIWA implementation. The estimation method was then reformulated from a least-squares estimation to likelihood-based estimation in order to use Markov Chain Monte Carlo methods (MCMC) to estimate the posterior distributions of the model parameters. The primary purpose of this work was to determine to the extent to which the parameter estimates were a result of information in the data rather than a result of the priors (i.e., the bounds).

Model Implementation

Model Structure

A detailed description of the NIWA two-stock hoki model is presented in Cordue (1999). Our version of the model is implemented as described in that paper with a few minor changes described in this report. One source of differences is the bounds placed on certain parameters. In our model, stock-specific bounds on the B_0 (virgin biomass) parameter were not specified; rather a bound of 20,000–5,000,000 metric tons (mt) for each stock was used. Bounds on the trawl survey proportionality constants (q) were not used, whereas the NIWA implementation had region-specific bounds for these constants. Also, a minimum level for the maximum exploitation rate in the pre-spawning fisheries was not specified. Maximum exploitation rates of 0.8 and 0.6 for the pre-spawning season and spawning season fisheries, respectively, were specified as in the NIWA analysis.

Perhaps the most significant difference between the UW/Seafic implementation of the two-stock model and

that used by NIWA is that we did not specify ageing error in the fit to fishery age-composition data because this aspect of the model was not documented in Cordue (1999). The NIWA analysis included ageing error assumptions for the otolith based age-composition data, but not for the MIX-based age-composition data (P.L. Cordue, NIWA, Wellington, New Zealand, pers. comm. 1999),

To compare the UW/Seafic and NIWA implementations of the hoki model, we fit our model to the hoki data using the least-squares estimator described by Cordue (1999). As in the Cordue analysis, the model was fit separately to the western region CPUE and acoustics data series because these data sets show contradictory population biomass trends.

Likelihood Formulation and MCMC Analysis

The values of all independent parameters in the NIWA two-stock model are constrained within bounds specified for each parameter. From a Bayesian perspective, these bounds imply uniform prior distributions for the model parameters. That is, all values within the bounds are equally probable and values outside the bounds have a zero probability. These prior distributions, in conjunction with the likelihood described below, were used to estimate the posterior distributions of model parameters.¹

The posterior distributions of the model parameters were estimated using Markov Chain Monte Carlo (MCMC) simulation (Gelman et al. 1995). The MCMC is a method for approximating the posterior distribution for parameters of interest in a Bayesian framework. Markov Chain simulation simulates a random walk in the parameter space that converges with a stationary distribution that is the joint posterior distribution. The AD Model Builder software implements MCMC using a version of the Metropolis-Hastings algorithm (Gelman et al. 1995). The software algorithm begins the Markov chain at the maximum of the joint posterior distribution and uses the inverse Hessian at the maximum to set an appropriate movement scale for the Markov process.

Estimation of the joint posterior distribution requires an estimate of the likelihood of the model parameters given the data observations, whereas Cordue (1999) used least-squares estimation. Therefore, we reformulate the weighted sums of squares function to use likelihood estimation for the MCMC analysis. An implicit assumption in our approach is that the weightings used by Cordue for each data source in the least squares function define the appropriate

¹A list of the priors and the estimated posterior distributions is presented in Tables 5 to 8 for most model parameters.

relative weightings (i.e., are proportional to the variances) for each data type.

The weighted sums of squares function for the data observations is (Cordue 1999, Appendix 4) as follows:

$$SSQ_{obs} = \sum_{k \in K} w_k [\ln(X_k) - \ln(P_k)]^2$$

where k = index of all observed values (individual biomass indices or individual proportions or numbers-at-age and sex),

X_k = k th observation

P_k = k th predicted value, and

w_k = k th relative weight, respectively.

The process for calculating weights for each observation is described in Cordue (1999, page 62). Two “penalty” terms are added to the SSQ term so that the model fit conforms to prior belief about the behaviour of the system. The first is that the average year-class strength for each stock Y_s is equal to one (1). The penalty term is

$$SSQ_y = m_y \sum_{s \in stocks} (\ln(Y_s))^2$$

where $stock = \{eastern, western\}$ and a weighting, $m_y = 5$, was used in the 1998 assessment. The second penalty function results from the assumption that the proportionality constants for a given survey method should be similar for the two regions (eastern and western). The penalty term is

$$SSQ_q = m_q \sum_{i \in methods} [\ln(q_{i,e}) - \ln(q_{i,w})]^2$$

where $methods = \{acoustics, Tangaroa, Shinkai Maru, Amatal Explorer\}$, w denotes the western region and e denotes the eastern region. A penalty weight, m_q , of one (1) was used in the 1998 assessment. The objective function (f) minimized in the least-squares estimation is then

$$f = SSQ_{obs} + SSQ_y + SSQ_q.$$

If we assume that the w_k are proportional to the variance of observation k , and that m_y and m_q are proportional to the variance in the average year-class strength and the difference in the log of the proportionality constants, the negative log-likelihood ($-\ln L$) of the observations, up to an additive constant, is

$$-\ln L = 0.5 (n_{obs} + 6) \ln (SSQ_{obs} + SSQ_y + SSQ_q)$$

where n_{obs} is the total number of data observations and the six additional observations result from the two stocks in

the penalty term associated with average year-class strength and from the four survey types in the penalty term associated with the proportionality constants. Note that this formulation corresponds with the concentrated likelihood where the residual variance for all the weighted residuals is estimated as

$$\hat{\sigma}^2 = \frac{SSQ_{obs} + SSQ_y + SSQ_q}{n_{obs} + 6}.$$

This negative log-likelihood formulation was used in the MCMC analysis. One million MCMC simulations were conducted from which 5,000 point estimates were sampled to approximate the posterior distributions of the dependent and independent model parameters.

Model Results

Least-Squares Estimates

The total sums of squares and the sums of squares from each data source are shown in Table 1 for both the NIWA and the UW/SeaFIC implementations of the two-stock model. Results are from the model fits to the western region acoustics data. The total sums of squares value for the UW/SeaFIC implementation of the model is higher than that of the NIWA implementation (37.573 versus 35.273). The largest difference in the sums of squares components is in the fit to the Chatham Rise (E_HM) R/V *Tangaroa* survey data based on MIX ages (6.091 versus 3.856). The Chatham Rise R/V *Tangaroa* survey data are fit twice in the model: to the MIX-based ages (age classes 1-6+) and to the otolith-based ages (age classes 6-10+ for males and 6-11+ for females). The numbers of fish aged 6 and older are higher for the otolith-based ages than for the MIX-based ages. The ageing error assumption used in the NIWA model implementation seems to allow a better fit to the contradictory data from this survey.

The differences in the constraints on virgin biomass, trawl survey proportionality constants, and the maximum spawning season exploitation rates between the two implementations of the two-stock model do not appear to cause significant differences in the least-squares fits. Our estimates for these parameters were all within the constraints used in the NIWA analysis. In general, model parameter estimates that we obtained are similar to those reported by Cordue (1999). The estimated values for a subset of the model parameters are listed in Table 2 for the NIWA and UW/SeaFIC model implementations.

There is a large difference between the NIWA and UW/

TABLE 1. Values of the component sums of squares for all data sources (see Cordue 1999 for a description of the data sources) and the total sums of squares for the two-stock model fit to the acoustics data series. Results are shown for the NIWA implementation as presented in the 1998 stock assessment (P.L. Cordue, NIWA, Wellington, New Zealand, pers. comm. 1999) and for the UW/Seafic sums of squares implementation described in this document. Where the age classes fit for a data source differ between males and females, the last age class for females is shown in brackets.

| Data source | Age classes | Sums of squares | |
|-----------------------------------|-------------|-----------------|-----------|
| | | NIWA | UW/Seafic |
| Western Acoustics | | 0.9051 | 0.6562 |
| Eastern Acoustics | | 1.2213 | 1.4324 |
| Southland Trawl | 1-2 | 1.2925 | 1.3701 |
| Western spawn season catch | 3-11 (12) | 2.6560 | 2.0395 |
| Eastern spawn season catch | 3-11 (12) | 1.1965 | 1.8023 |
| Chatham Rise catch | 2-6+ | 4.3141 | 3.3301 |
| Sub-Antarctic catch | 2-6+ | 2.1059 | 2.1303 |
| W_HM Tangaroa Dec | 1 | 0.3813 | 0.2667 |
| W_HM Tangaroa Dec | 2-11+ (12+) | 1.6994 | 1.6158 |
| W_HM Tangaroa Sep | 1 | 1.0382 | 1.0661 |
| W_HM Tangaroa Sep | 2-13+ (14+) | 2.0728 | 1.9408 |
| W_HM Tangaroa Apr/May | 1 | 0.3253 | 0.4987 |
| W_HM Tangaroa Apr/May | 2-11+ (12+) | 0.8518 | 1.1151 |
| W_HM Shinkai Mar/Apr | 3-6+ | 0.7791 | 1.1598 |
| W_HM Shinkai Oct/Nov | 3-6+ | 0.3643 | 0.7118 |
| W_HM A maltal Oct/Nov | 2-6+ | 1.0810 | 1.1367 |
| W_HM A maltal Jul/Aug | 2-6+ | 0.5597 | 0.4869 |
| E_HM Tangaroa Jan | 1-6+ | 3.8564 | 6.0907 |
| E_HM Tangaroa Jan | 6-10+ (11+) | 1.6063 | 1.4314 |
| E_HM Shinkai Mar | 1-6+ | 0.7427 | 1.1012 |
| E_HM Shinkai July | 2-6+ | 5.7889 | 5.4741 |
| E_HM A maltal Nov/Dec | 2-6+ | 0.2575 | 0.2805 |
| Total for all data sources | | 35.096 | 37.137 |
| Total including penalty functions | | 35.273 | 37.573 |

TABLE 2. Estimates of some model parameters from the NIWA and UW/Seafic two-stock model fits to the acoustics and CPUE time series. Parameter estimates from the UW/Seafic model implementation are presented for both the sums of squares and log-likelihood estimation methods. Virgin biomass is in thousands of metric tons (mt).

| Model parameter | NIWA | | UW/Seafic | | | |
|------------------------------|-----------------|------|-----------------|-------|----------------|-------|
| | Sums of squares | | Sums of squares | | Log-likelihood | |
| | Acoustic | CPUE | Acoustic | CPUE | Acoustic | CPUE |
| Virgin biomass – eastern | 400 | 280 | 396 | 277 | 397 | 276 |
| Virgin biomass – western | 1,430 | 905 | 1,360 | 1,062 | 1,360 | 1,058 |
| Prop. to south corridor | 0.11 | 0.11 | 0.70 | 0.70 | 0.70 | 0.70 |
| Prop. in first wave home | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 |
| Prop. in first spawning wave | 0.22 | 0.10 | 0.16 | 0.10 | 0.16 | 0.10 |
| Prop. spawning – eastern | 0.70 | 0.70 | 0.84 | 0.70 | 0.84 | 0.70 |
| Prop. spawning – western | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 |

Seafic estimate for the parameter “proportion to south corridor,” a parameter for the proportion of western juvenile hoki that migrate around the south end of the South Island to the Chatham Rise rearing grounds. The NIWA estimate for this parameter is 0.11 compared with the UW/Seafic estimate of 0.70 (Table 2). Although the difference between these estimates is large, the degree of difference is not reflected in the estimates for other key parameters such as B_0 . This implies that differences in this parameter are not crucial to the behaviour of the overall model and are likely not crucial to the estimates of the derived parameters of management importance for these stocks. This difference has been highlighted as an example of the high level of unnecessary complexity in the NIWA model. If the route that juvenile hoki use to migrate to the Chatham Rise does not affect model fit and parameter estimates, it may be better to remove this part of the model structure.

Likelihood-Based Estimates and MCMC Analysis

The estimated residual variance for the weighted residuals ($\hat{\sigma}^2$) is 0.0413 for the likelihood estimation fit to

the acoustics data and 0.0403 for the fit to the CPUE data. The standard deviations (SDs) for the various data series can be estimated conditionally on the assumption that the relative weights for the data observations (w_k) and the penalty weights (m_q, m_y) are proportional to the true variance of their respective quantities. The ratio of the estimated residual variance to the relative weighting is then an estimate of the variance for each quantity. The SDs (square root of the variances), based on the average relative weights for each data series, are shown in Table 3. The values for the data observations range from 0.20 for the acoustics data to 1.62 for the eastern pre-spawning season fishery (Chatham Rise) age-composition data. For the penalty functions, the implied SD for the average year-class strength equalling one is 0.091, and the SD is 0.203 for the equivalence of the survey proportionality constants.

The squared residuals, averaged by age class and survey, are presented in Table 4. In general, the magnitudes of the mean-squared residuals are lower for older age classes. Mean-squared residuals are particularly high for ages 1 and 2. The largest individual residuals are generally from very

TABLE 3. The average weighting for data observations in each survey series (w_k averaged over all observations in the series) and the implied average SD for the data series.

| Data source | Average weight per data observation | Estimated SD |
|-------------------------------|-------------------------------------|--------------|
| Western Acoustics | 1.0 | 0.2033 |
| Eastern Acoustics | 1.0 | 0.2033 |
| Southland Trawl | 0.075 | 0.7423 |
| West. spawn season catch | 0.0263 | 1.2531 |
| East spawn season catch | 0.0158 | 1.6178 |
| Chatham Rise catch | 0.04 | 1.0164 |
| Sub-Antarctic catch | 0.04 | 1.0164 |
| W_HM Tangaroa Dec (age 1) | 0.0435 | 0.9749 |
| W_HM Tangaroa Dec | 0.0435 | 0.9749 |
| W_HM Tangaroa Sep (age 1) | 0.037 | 1.0563 |
| W_HM Tangaroa Sep | 0.037 | 1.0563 |
| W_HM Tangaroa Apr/May (age 1) | 0.0435 | 0.9749 |
| W_HM Tangaroa Apr/May | 0.0435 | 0.9749 |
| W_HM Shinkai Mar/Apr | 0.125 | 0.5749 |
| W_HM Shinkai Oct/Nov | 0.125 | 0.5749 |
| W_HM A maltal Oct/Nov | 0.1 | 0.6428 |
| W_HM A maltal Jul/Aug | 0.05 | 0.9091 |
| E_HM Tangaroa Jan (MIX) | 0.0461 | 0.9467 |
| E_HM Tangaroa Jan (otolith) | 0.0474 | 0.9340 |
| E_HM Shinkai Mar | 0.0833 | 0.7042 |
| E_HM Shinkai July | 0.05 | 0.9091 |
| E_HM A maltal Nov/Dec | 0.1 | 0.6428 |

TABLE 4. Estimates of the mean squared residuals— $(\ln(X_k) - \ln(P_k))^2$ —averaged within age class and survey, from the two-stock model analysis fit to the western acoustics data series. Legend: W – western home ground; E – eastern home ground; T – R/V *Tangaroa*; SM – M/V *Shinkai Maru*; AE – M/V *Amaltal Explorer*.

| Survey | Mean-squared residual by age class | | | | | | | | | | | | | Survey |
|------------------------|------------------------------------|-------|------|------|------|------|------|------|------|------|------|------|------|--------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | mean |
| E T Jan (MIX) | 1.68 | 2.01 | 0.76 | 1.65 | 1.73 | 1.14 | | | | | | | | 1.50 |
| E T Jan (otolith) | | | | | | 0.40 | 0.56 | 0.31 | 0.31 | 0.89 | | | | 0.49 |
| E SM | 1.29 | 3.61 | 0.82 | 0.15 | 0.10 | 0.64 | | | | | | | | 1.10 |
| Southland Trawl | 1.70 | 0.56 | | | | | | | | | | | | 1.13 |
| W SM Mar/Apr | | | 2.41 | 1.47 | 0.20 | 0.56 | | | | | | | | 1.16 |
| W T Apr/May | | 0.80 | 0.25 | 0.09 | 2.51 | 0.22 | 0.42 | 0.32 | 0.53 | 0.05 | 0.76 | | | 0.60 |
| E SM July | | 51.59 | 1.02 | 1.58 | 0.05 | 0.50 | | | | | | | | 10.95 |
| W AE Jul/Aug | | 2.01 | 0.63 | 2.11 | 0.09 | 0.02 | | | | | | | | 0.97 |
| W T Sep | | 11.27 | 3.10 | 1.90 | 0.92 | 0.63 | 0.32 | 0.41 | 0.37 | 0.23 | 0.29 | 0.49 | 0.64 | 2.10 |
| E AE Nov/Dec | | 0.40 | 0.14 | 0.05 | 0.01 | 0.81 | | | | | | | | 0.28 |
| W SM Oct/Nov | | | 1.33 | 0.02 | 0.01 | 1.49 | | | | | | | | 0.71 |
| W AE Oct/Nov | | 0.67 | 0.07 | 0.61 | 0.92 | 0.62 | | | | | | | | 0.58 |
| W T Dec | | 0.95 | 1.46 | 0.55 | 0.26 | 0.22 | 0.55 | 0.41 | 0.65 | 0.37 | 0.36 | | | 0.60 |
| Chatham Rise catch | | 0.19 | 4.11 | 1.60 | 0.93 | 0.36 | | | | | | | | 1.44 |
| Sub-Antarctic catch | | 0.42 | 2.15 | 0.13 | 1.17 | 0.29 | | | | | | | | 0.83 |
| West spawn catch | | | 1.03 | 0.51 | 0.30 | 0.22 | 0.17 | 0.36 | 0.22 | 0.31 | 0.19 | | | 0.40 |
| East spawn catch | | | 0.83 | 1.12 | 0.67 | 0.41 | 0.61 | 0.64 | 0.15 | 0.21 | 0.21 | | | 0.55 |
| W T Dec | 1.01 | | | | | | | | | | | | | 1.01 |
| W T Apr/May | 2.88 | | | | | | | | | | | | | 2.88 |
| W T Sep | 14.39 | | | | | | | | | | | | | 14.39 |
| Mean – all data series | 2.39 | 2.70 | 1.35 | 0.92 | 0.82 | 0.46 | 0.44 | 0.44 | 0.28 | 0.37 | 0.27 | 0.49 | 0.64 | 0.90 |
| Western Acoustics | | | | | | | | | | | | | | 0.08 |
| Eastern Acoustics | | | | | | | | | | | | | | 0.19 |

small observations. For example, the observed and predicted values for the number of age 2 males in the 1992 R/V *Tangaroa* survey in the western home grounds are 0.0016 and 0.2189, resulting in a squared residual of 23.97.

Posterior distributions for the time-trajectories of spawning stock biomass (SSB) and relative year-class strength (YCS) are summarized by medians and 80% probability intervals (Figs. 1–3). The wide uniform distribution of the posterior for the spawning stock biomass, which does not attenuate through the trajectory period, suggests that little information is gained from all the acquired data as it is incorporated in the late 1980s and early 1990s. However, the range in the spawning stock biomass for the early years of the analysis is likely restricted because of the model assumption that the stocks are in deterministic equilibrium in the initial year and recruitment continues to be deterministic up to the 1975 year-class.

Correlations between the eastern and western proportionality constants for each survey method (Fig. 4) are high,

as would be expected given the penalty function on these model parameters. This penalty function has a large effect on constraining the model fits, so an MCMC analysis with a lower penalty weight on this function was conducted. A penalty weight of 0.1 was used, which corresponds with an implied SD of 0.643 for this sensitivity analysis.

With a high penalty weight on the proportionality constants, the paired eastern and western values tend to be quite similar. However, with a lower penalty weight, this pattern changes markedly (Fig. 4). The q values from the three survey vessels and the acoustics survey are generally higher for the eastern region than for the western region. This results in significantly higher biomass estimates (SSB) for the western region and slightly lower biomass estimates for the eastern region (Fig. 1). The uncertainty in current SSB increases for the western region and, somewhat counter-intuitively, decreases for the eastern region.

The posterior distributions of model parameter values, as estimated from the 5,000 MCMC samples, are shown

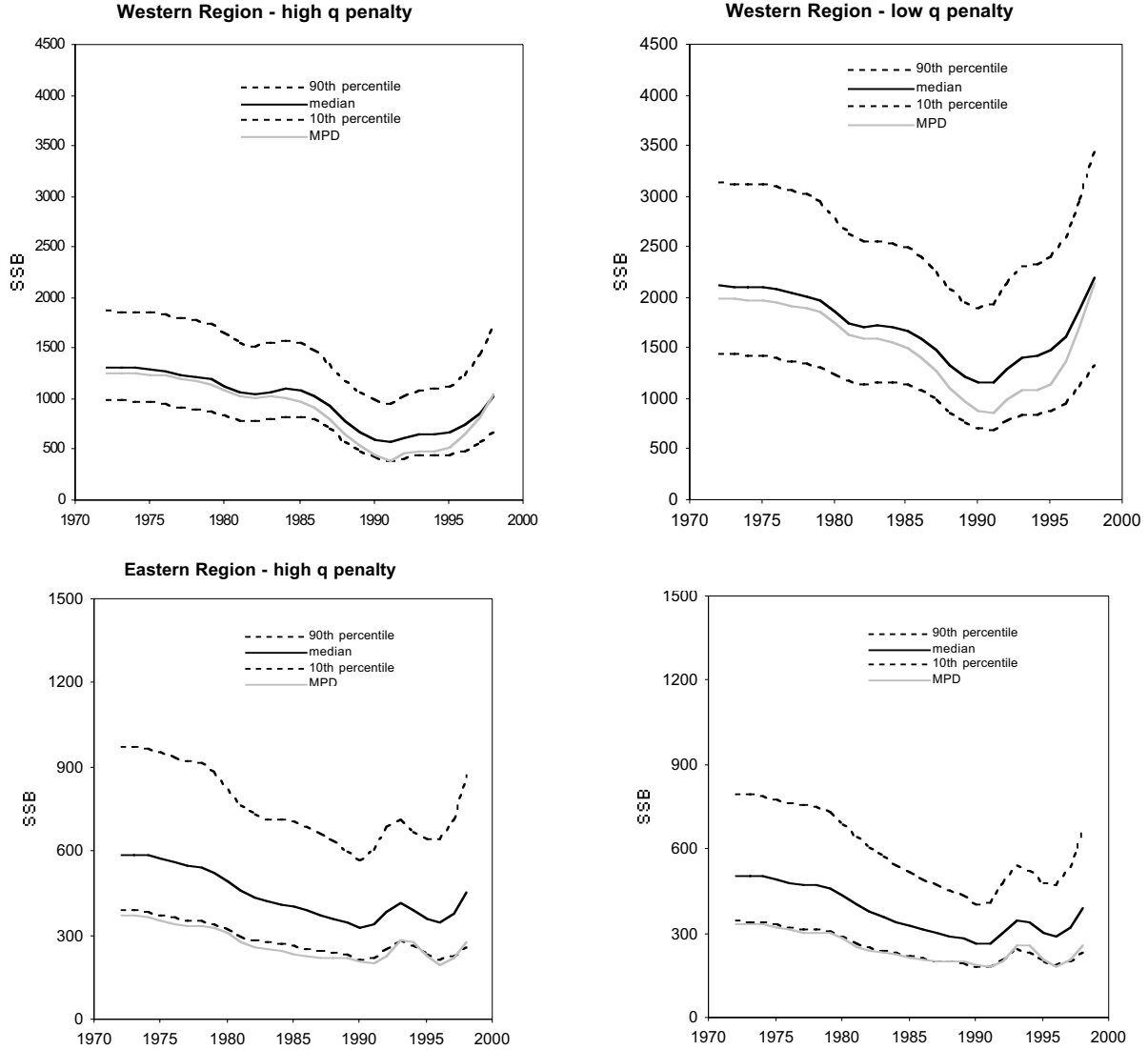


FIGURE 1. Median, 10th percentile, and 90th percentile of the MCMC distribution and the mode of the posterior (MPD) of spawning stock biomass for the eastern and western region fits to the acoustics data for high and low penalty weights on the survey proportionality constants (q).

in tabular form in Tables 5 to 8. The results presented are from the fit to the western acoustics data with the high penalty weight on the proportionality constant penalty function. The equivalent parameter estimates were usually similar for the MCMC simulations, with the low penalty weight on the proportionality constants for the reasons described in the following paragraph.

In general, the posterior distributions of model parameters either tend to be similar to their priors (i.e., uniform within the bounded range) or they tend to contain most of their density at one of their bounds. For example, the posterior distributions of the pre-spawning season fishery selectivity parameters for age classes 6 and older are similar to their priors. This is not surprising, given that the age-

composition data from these fisheries has a plus group that aggregates fish aged 6 and older. Hence, there is probably little information in the data to estimate age-specific selectivity for the older age classes.

The posterior distributions of selectivity and maturity parameters for age 2 and age 3 hoki in the eastern region all contain most of their density at one of their bounds (Fig. 5). This would suggest that either the priors (i.e., bounds) are inappropriate or that the model is misspecified in some way. For the western region, posterior distributions of these parameters are more similar to their priors (Fig. 6). The difference in the form of the posterior distributions between the two regions may result from the higher complexity in the model structure for the western stock. With the current model

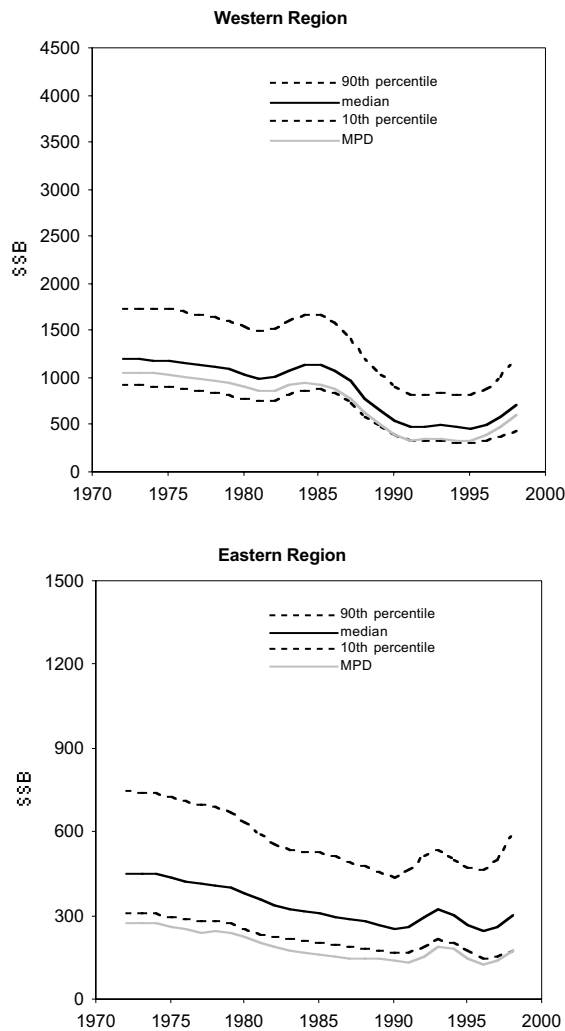


FIGURE 2. Median, 10th percentile, and 90th percentile of the MCMC distribution and the mode of the posterior (MPD) of spawning stock biomass (SSB) for the eastern and western region fits to the CPUE data (high penalty weights on the survey proportionality constants [q]).

structure, eastern stock larvae recruit to the northern corridor, and migrate from there to the eastern home ground. Mature eastern stock hoki migrate annually from the eastern home ground to their spawning ground. Larvae from the western stock recruit to both the northern and the southern corridors and migrate from these two areas to the eastern home ground. From the eastern home ground they migrate to the western home ground and mature fish in this area annually migrate to the spawning ground. Thus, there are substantially more parameters estimated that describe the age-specific migration rates of western stock hoki between these regions. This is likely to cause greater uncertainty in some of the parameter estimates for the western

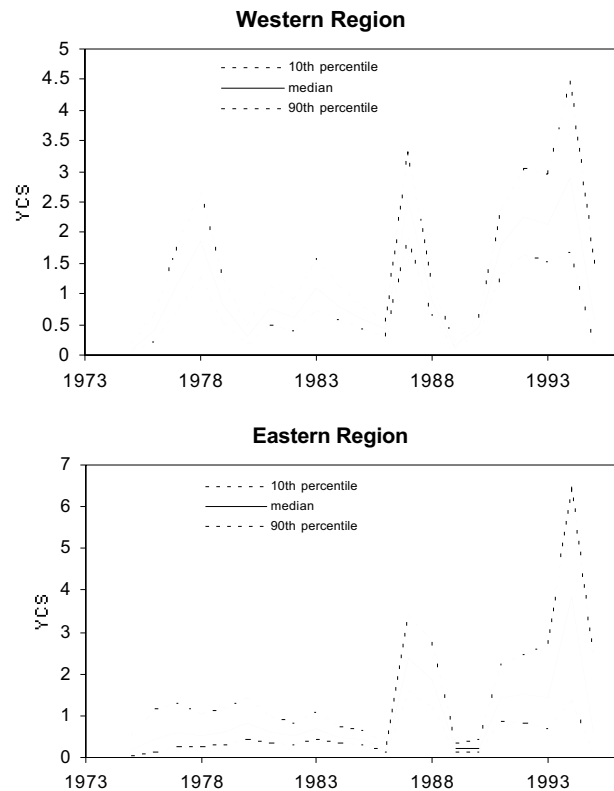


FIGURE 3. Distribution of relative year-class strengths from MCMC samples for the western and eastern hoki regions. Results are from the fits to the west coast acoustics data using high penalty weights for the survey proportionality constants (q).

stock because the number of fish in a specific region that are vulnerable to the fishery or to the trawl survey gear, or which can potentially mature, is dependent on the total migration parameters that determine the distribution of a year-class at any time.

Discussion and Conclusions

The UW/Seafic implementation of the two-stock multi-area hoki model differs from the NIWA implementation in some respects, most notably in that the ageing error parameterization was not included. The documentation of the NIWA model is clear and comprehensive (Cordue 1999); however, there may be additional minor differences between the two implementations. The impact of the model differences appears to be negligible as parameter estimates from the two implementations are very similar. The authors believe that the aspects of model behaviour investigated and described in this manuscript are not affected by these small differences in model implementation.

We believe that the NIWA two-stock multi-area hoki

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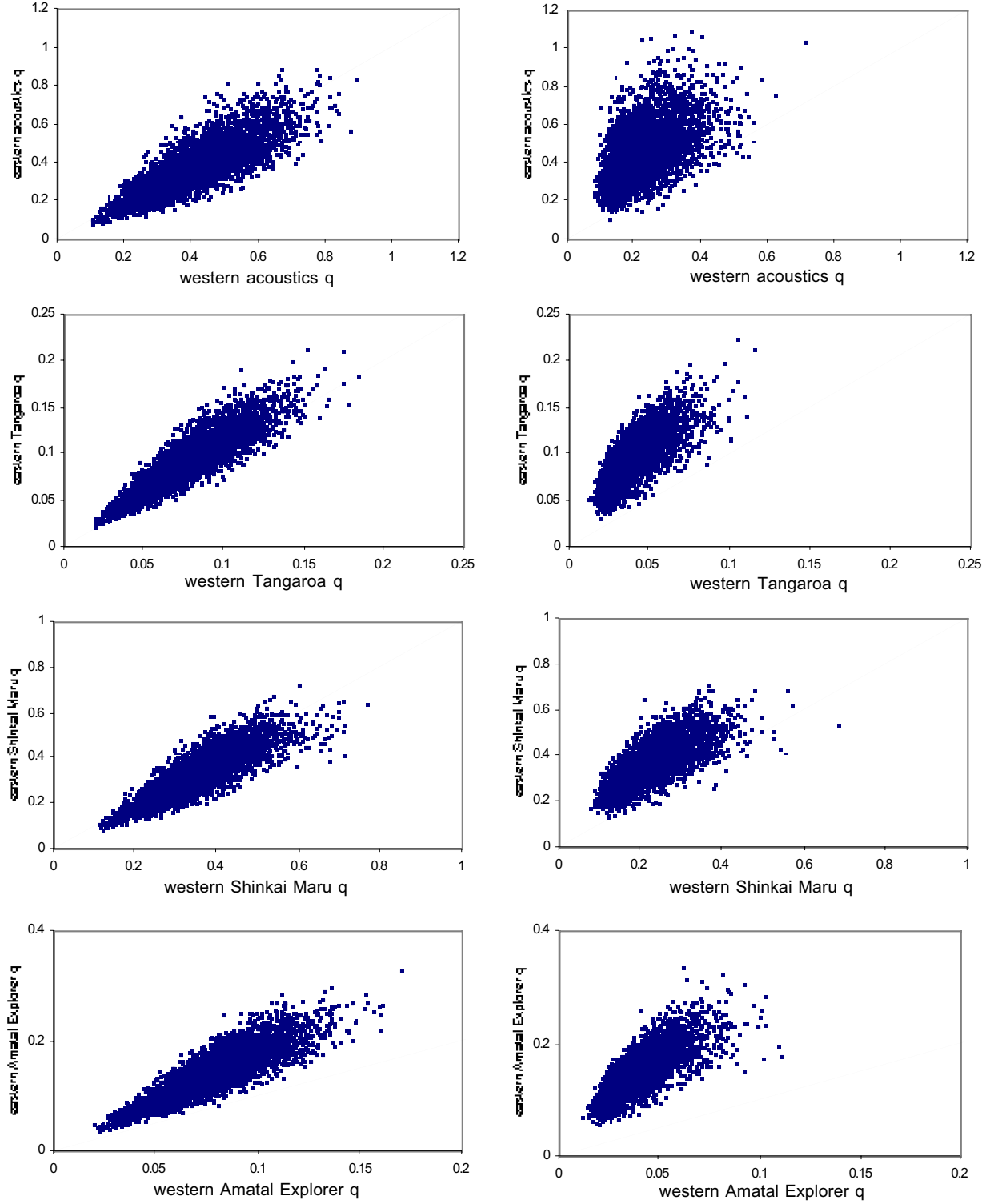


FIGURE 4. Estimated values of the q parameters for the survey pairs that have penalty weights on their differences for the 5,000 MCMC samples. Panels on the left are from the “high” penalty weight trials (\bullet), and on the right from the “low” penalty weight trials (\circ).

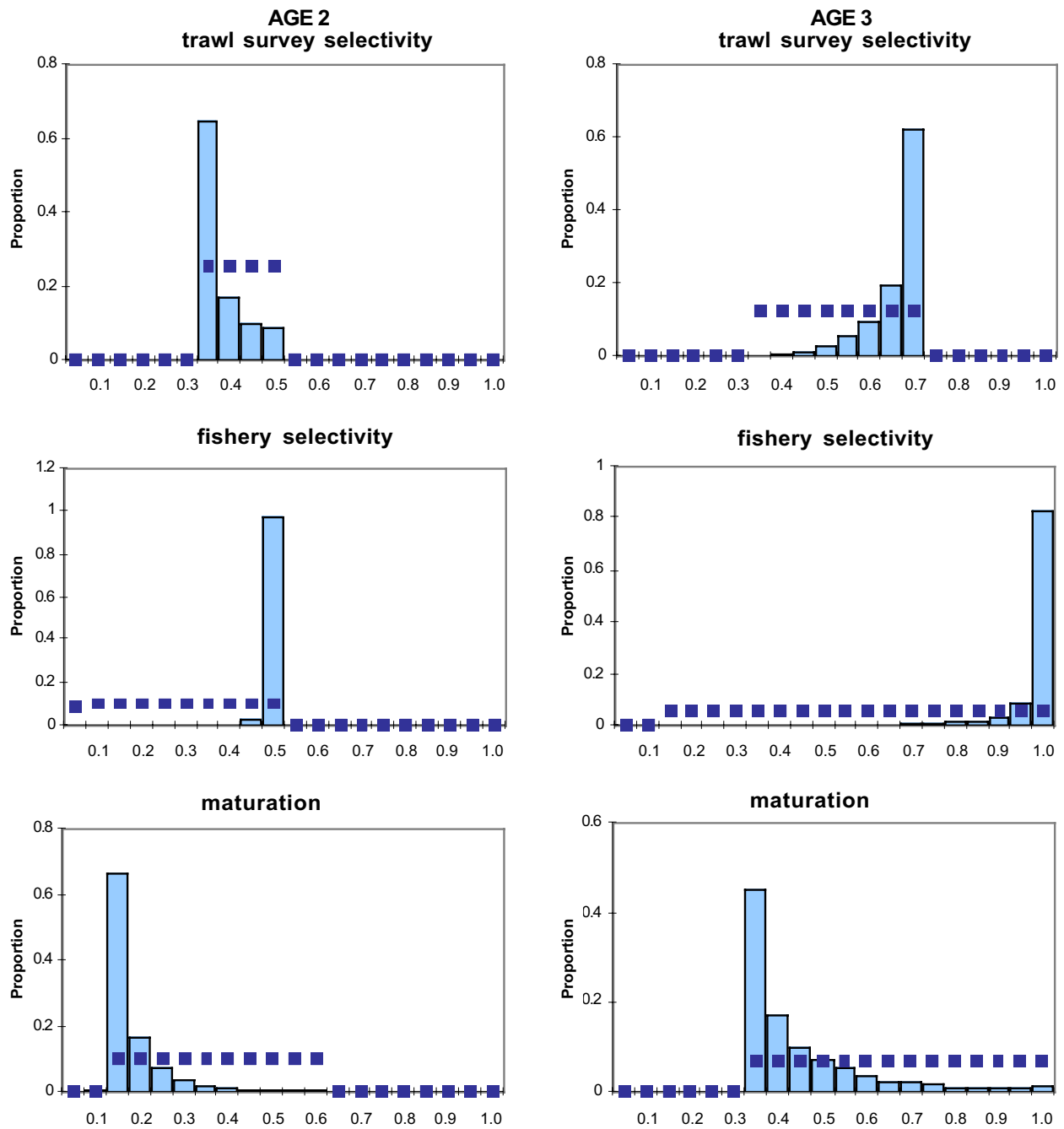


FIGURE 5. Prior (points) and posterior (bars) probability distributions of selected age 2 and age 3 model parameters for the eastern hoki region/stock.

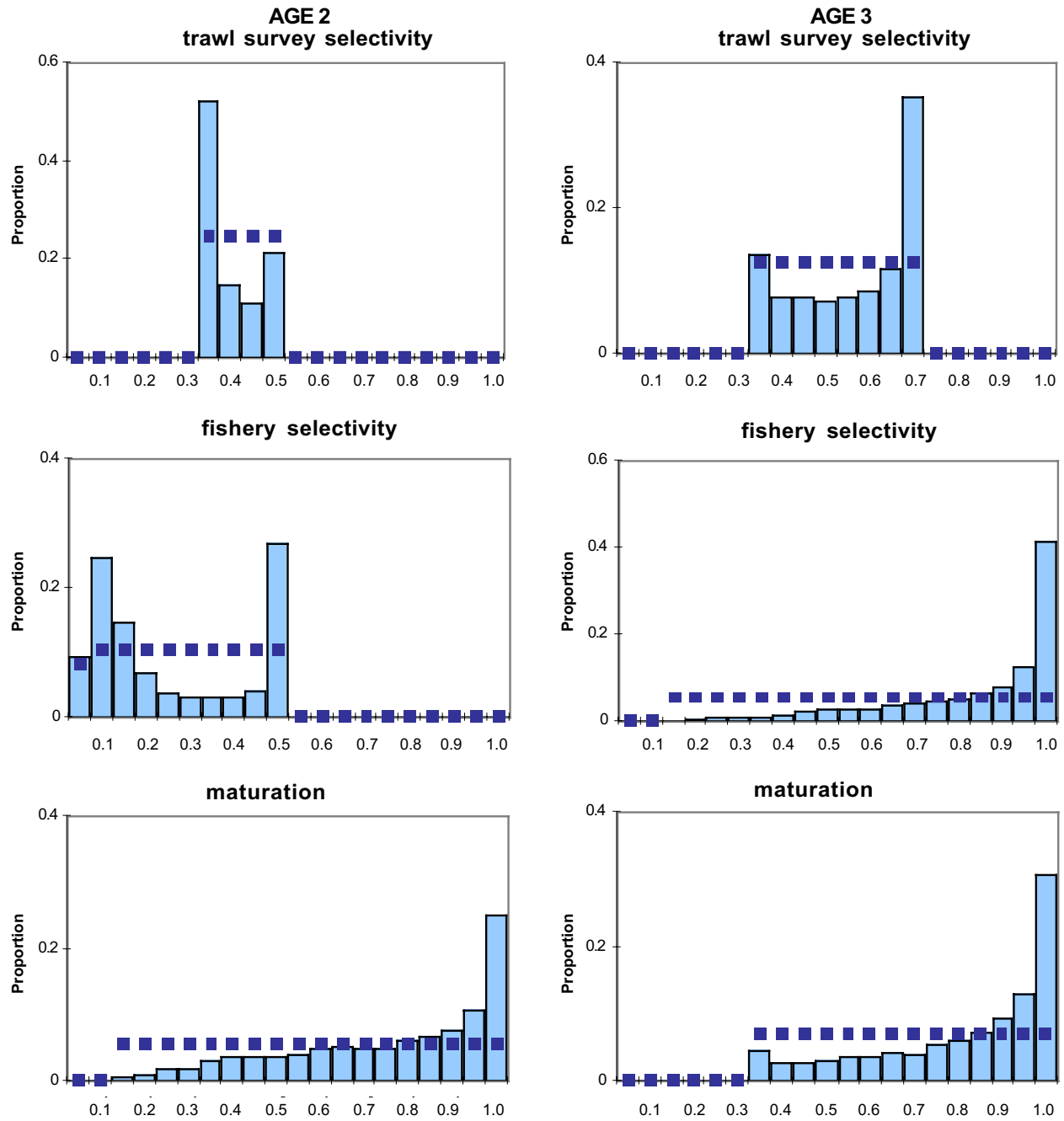


FIGURE 6. Prior (points) and posterior (bars) probability distributions of selected age 2 and age 3 model parameters for the western hoki region/stock.

TABLE 5. The uniform priors (i.e., bounds) and posterior distributions of model parameter values from MCMC samples of the NIWA two-stock model fit to the acoustics data series (see Cordue 1999 for a description of model parameters).

| Parameter | Stock | Bounds | Proportion of MCMC simulations in range | | | | | | | | | |
|-----------|-------|----------|---|------|------|------|------|------|------|------|------|------|
| | | | >0.1 | >0.2 | >0.3 | >0.4 | >0.5 | >0.6 | >0.7 | >0.8 | >0.9 | |
| | -age | | ≤0.1 | ≤0.2 | ≤0.3 | ≤0.4 | ≤0.5 | ≤0.6 | ≤0.7 | ≤0.8 | ≤0.9 | ≤1.0 |
| n_nurs | 1 | 0.01-1.0 | | 0.01 | 0.03 | 0.13 | 0.20 | 0.16 | 0.09 | 0.06 | 0.05 | 0.27 |
| | 2 | 0.3-1.0 | | | | 0.54 | 0.15 | 0.09 | 0.05 | 0.05 | 0.04 | 0.09 |
| s_nurs | 1 | 0.1-1.0 | | 0.46 | 0.23 | 0.15 | 0.10 | 0.04 | 0.02 | | | |
| | 2 | 0.3-1.0 | | | | 0.27 | 0.18 | 0.18 | 0.15 | 0.13 | 0.07 | 0.02 |
| p_south | | 0.1-0.7 | | 0.52 | 0.22 | 0.10 | 0.05 | 0.04 | 0.08 | | | |
| wave_hm | | 0.05-0.4 | | 0.01 | 0.06 | 0.93 | | | | | | |
| hm_og | 1 | 0.05-0.3 | 0.31 | 0.12 | 0.57 | | | | | | | |
| - male | 2 | 0.1-0.4 | | 0.75 | 0.14 | 0.11 | | | | | | |
| | 3 | 0.2-0.5 | | | 0.13 | 0.21 | 0.65 | | | | | |
| | 4 | 0.3-0.6 | | | | 0.56 | 0.21 | 0.23 | | | | |
| | 5 | 0.4-0.7 | | | | | 0.52 | 0.21 | 0.27 | | | |
| | 6 | 0.5-0.8 | | | | | | 0.41 | 0.22 | 0.37 | | |
| | 7 | 0.6-0.9 | | | | | | | 0.39 | 0.21 | 0.40 | |
| | 8 | 1.0 | | | | | | | | | | 1 |
| | | | | | | | | | | | | |
| hm_og | 1 | 0.05-0.3 | 0.79 | 0.20 | 0.01 | | | | | | | |
| - female | 2 | 0.1-0.4 | 0.01 | 0.98 | 0.01 | | | | | | | |
| | 3 | 0.2-0.5 | | | 0.76 | 0.13 | 0.11 | | | | | |
| | 4 | 0.3-0.6 | | | | 0.80 | 0.13 | 0.07 | | | | |
| | 5 | 0.4-0.7 | | | | | 0.69 | 0.17 | 0.14 | | | |
| | 6 | 0.5-0.8 | | | | | | 0.55 | 0.21 | 0.24 | | |
| | 7 | 0.6-0.9 | | | | | | | 0.45 | 0.22 | 0.33 | |
| | 8 | 1.0 | | | | | | | | | | 1 |
| | | | | | | | | | | | | |
| wave_sp | | 0.1-0.7 | | 0.70 | 0.18 | 0.08 | 0.03 | 0.01 | 0.01 | | | |
| spawn_p | E | 0.7-1.0 | | | | | | | | 0.67 | 0.20 | 0.12 |
| | W | 0.7-1.0 | | | | | | | | 0.96 | 0.03 | |

TABLE 6. The uniform priors (i.e., bounds) and posterior frequency distributions of model parameter values from MCMC samples of the NIWA two-stock model fit to the acoustics data series (see Cordue 1999 for a description of model parameters).

[illegible]

TABLE 7. The uniform priors (i.e., bounds) and posterior frequency distributions of trawl survey selectivity parameters from MCMC samples from the NIWA multi-stock model fit to the acoustics data series (see Cordue 1999 for a description of model parameters).

| Region | Age | Bounds | Proportion of MCMC simulations in range | | | | | | | | | | | |
|-------------------|-----|----------|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | | | | | | | | | | | | | | |
| | | | <=0.1 | <=0.2 | <=0.3 | <=0.4 | <=0.5 | <=0.6 | <=0.7 | <=0.8 | <=0.9 | <=1.0 | <=1.1 | <=1.2 |
| E-m | 1 | 0.01-0.2 | | 1.00 | | | | | | | | | | |
| | 2 | 0.3-0.5 | | | | 0.82 | 0.18 | | | | | | | |
| | 3 | 0.3-0.7 | | | | 0.01 | 0.04 | 0.15 | 0.81 | | | | | |
| | 4 | 0.4-1.2 | | | | | 0.13 | 0.11 | 0.12 | 0.12 | 0.11 | 0.11 | 0.11 | 0.20 |
| | 5 | 0.5-1.2 | | | | | | 0.44 | 0.16 | 0.11 | 0.08 | 0.06 | 0.05 | 0.09 |
| | 6 | 0.6-1.2 | | | | | | | 0.44 | 0.17 | 0.11 | 0.09 | 0.07 | 0.11 |
| | 7 | 0.7-1.2 | | | | | | | | 0.24 | 0.14 | 0.14 | 0.16 | 0.31 |
| | 8 | 0.7-1.2 | | | | | | | | 0.33 | 0.16 | 0.13 | 0.14 | 0.24 |
| | 9 | 0.8-1.2 | | | | | | | | | 0.60 | 0.17 | 0.11 | 0.12 |
| E-f | 10 | 1.0 | | | | | | | | | | 0.53 | 0.47 | |
| | 1 | 0.01-0.2 | | 1.00 | | | | | | | | | | |
| | 2 | 0.3-0.5 | | | | 0.87 | 0.13 | | | | | | | |
| | 3 | 0.3-0.7 | | | | | 0.02 | 0.11 | 0.86 | | | | | |
| | 4 | 0.4-1.2 | | | | | 0.36 | 0.18 | 0.13 | 0.10 | 0.06 | 0.05 | 0.04 | 0.07 |
| | 5 | 0.5-1.2 | | | | | | 0.29 | 0.16 | 0.13 | 0.11 | 0.09 | 0.09 | 0.13 |
| | 6 | 0.6-1.2 | | | | | | | 0.32 | 0.15 | 0.12 | 0.12 | 0.11 | 0.18 |
| | 7 | 0.7-1.2 | | | | | | | | 0.15 | 0.11 | 0.13 | 0.18 | 0.44 |
| | 8 | 0.7-1.2 | | | | | | | | 0.14 | 0.10 | 0.13 | 0.17 | 0.45 |
| W-m | 9 | 0.8-1.2 | | | | | | | | | 0.21 | 0.14 | 0.20 | 0.45 |
| | 10 | 0.8-1.2 | | | | | | | | | 0.26 | 0.16 | 0.19 | 0.39 |
| | 1 | 0.01-0.2 | 0.15 | 0.85 | | | | | | | | | | |
| | 2 | 0.3-0.5 | | | | 0.67 | 0.33 | | | | | | | |
| | 3 | 0.3-0.7 | | | | 0.21 | 0.15 | 0.17 | 0.47 | | | | | |
| | 4 | 0.4-1.2 | | | | | 0.17 | 0.15 | 0.13 | 0.13 | 0.12 | 0.09 | 0.08 | 0.13 |
| | 5 | 0.5-1.2 | | | | | | 0.07 | 0.05 | 0.06 | 0.08 | 0.09 | 0.12 | 0.53 |
| | 6 | 0.6-1.2 | | | | | | | 0.62 | 0.17 | 0.09 | 0.05 | 0.04 | 0.03 |
| | 7 | 0.7-1.2 | | | | | | | | 0.38 | 0.18 | 0.13 | 0.11 | 0.20 |
| W-f | 8 | 0.7-1.2 | | | | | | | | 0.33 | 0.16 | 0.14 | 0.13 | 0.24 |
| | 9 | 0.8-1.2 | | | | | | | | | 0.39 | 0.18 | 0.16 | 0.26 |
| | 10 | 1.0 | | | | | | | | | | 0.52 | 0.48 | |
| | 1 | 0.01-0.2 | 0.18 | 0.82 | | | | | | | | | | |
| | 2 | 0.3-0.5 | | | | 0.36 | 0.64 | | | | | | | |
| | 3 | 0.3-0.7 | | | | 0.09 | 0.15 | 0.21 | 0.56 | | | | | |
| | 4 | 0.4-1.2 | | | | | 0.04 | 0.04 | 0.05 | 0.06 | 0.08 | 0.10 | 0.14 | 0.49 |
| | 5 | 0.5-1.2 | | | | | | 0.04 | 0.05 | 0.07 | 0.09 | 0.11 | 0.16 | 0.49 |
| | 6 | 0.6-1.2 | | | | | | | 0.36 | 0.16 | 0.13 | 0.11 | 0.09 | 0.15 |
| southern corridor | 7 | 0.7-1.2 | | | | | | | | 0.26 | 0.15 | 0.14 | 0.15 | 0.31 |
| | 8 | 0.7-1.2 | | | | | | | | 0.15 | 0.11 | 0.14 | 0.18 | 0.43 |
| | 9 | 0.8-1.2 | | | | | | | | | 0.28 | 0.17 | 0.18 | 0.36 |
| | 10 | 0.8-1.2 | | | | | | | | | 0.21 | 0.16 | 0.20 | 0.42 |
| m | 1 | 0.1-1.2 | | 0.08 | 0.10 | 0.11 | 0.12 | 0.12 | 0.11 | 0.09 | 0.08 | 0.06 | 0.06 | 0.07 |
| m | 2 | 0.1-1.2 | | 0.01 | 0.03 | 0.06 | 0.09 | 0.11 | 0.12 | 0.12 | 0.12 | 0.12 | 0.11 | 0.11 |
| f | 1 | 0.1-1.2 | | 0.08 | 0.15 | 0.20 | 0.13 | 0.10 | 0.11 | 0.07 | 0.11 | 0.05 | | |
| f | 2 | 0.1-1.2 | | 0.02 | 0.04 | 0.08 | 0.09 | 0.11 | 0.13 | 0.12 | 0.11 | 0.11 | 0.10 | 0.09 |

TABLE 8. The uniform priors (i.e., bounds) and posterior frequency distributions of pre-spawning season fishery selectivity parameters from MCMC samples from the NIWA multi-stock model fit to the acoustics data series (see Cordue 1999 for a description of model parameters).

| Region | Age | Bound | Proportion of MCMC simulations in range | | | | | | | | | | | | | |
|--------|-----|-----------|---|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | | | >0.1 | >0.2 | >0.3 | >0.4 | >0.5 | >0.6 | >0.7 | >0.8 | >0.9 | >1.0 | >1.1 | >1.2 | >1.3 | |
| | | | ≤0.1 | ≤0.2 | ≤0.3 | ≤0.4 | ≤0.5 | ≤0.6 | ≤0.7 | ≤0.8 | ≤0.9 | ≤1.0 | ≤1.1 | ≤1.2 | ≤1.3 | ≤1.5 |
| E-m | 1 | 0.01-0.05 | 1.00 | | | | | | | | | | | | | |
| | 2 | 0.01-0.5 | | | | | 0.99 | | | | | | | | | |
| | 3 | 0.1-1.0 | | | | | | | 0.01 | 0.02 | 0.05 | 0.91 | | | | |
| | 4 | 0.4-1.5 | | | | | | | 0.01 | 0.01 | 0.02 | 0.03 | 0.05 | 0.07 | 0.10 | 0.71 |
| | 5 | 0.6-1.5 | | | | | | | 0.01 | 0.01 | 0.01 | 0.02 | 0.04 | 0.07 | 0.11 | 0.73 |
| | 6 | 0.7-1.5 | | | | | | | | 0.09 | 0.07 | 0.06 | 0.07 | 0.08 | 0.10 | 0.53 |
| | 7 | 0.7-1.5 | | | | | | | | 0.22 | 0.11 | 0.09 | 0.08 | 0.08 | 0.09 | 0.33 |
| | 8 | 0.8-1.4 | | | | | | | | | 0.25 | 0.12 | 0.11 | 0.11 | 0.12 | 0.28 |
| | 9 | 0.9-1.2 | | | | | | | | | | 0.40 | 0.22 | 0.38 | | |
| E-f | 1 | 0.01-0.05 | 1.00 | | | | | | | | | | | | | |
| | 2 | 0.01-0.5 | | | | 0.01 | 0.99 | | | | | | | | | |
| | 3 | 0.1-1.0 | | | | | | | 0.01 | 0.02 | 0.05 | 0.90 | | | | |
| | 4 | 0.4-1.5 | | | | | | | 0.01 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.08 | 0.75 |
| | 5 | 0.6-1.5 | | | | | | | | 0.01 | 0.02 | 0.02 | 0.04 | 0.06 | 0.10 | 0.76 |
| | 6 | 0.7-1.5 | | | | | | | | 0.09 | 0.06 | 0.06 | 0.07 | 0.08 | 0.10 | 0.54 |
| | 7 | 0.7-1.5 | | | | | | | | 0.25 | 0.11 | 0.09 | 0.08 | 0.08 | 0.08 | 0.32 |
| | 8 | 0.8-1.4 | | | | | | | | | 0.25 | 0.11 | 0.10 | 0.09 | 0.10 | 0.35 |
| | 9 | 0.8-1.4 | | | | | | | | | 0.28 | 0.13 | 0.10 | 0.11 | 0.11 | 0.26 |
| | 10 | 0.8-1.2 | | | | | | | | | 0.29 | 0.16 | 0.17 | 0.37 | | |
| W-m | 1 | 0.01-0.05 | 1.00 | | | | | | | | | | | | | |
| | 2 | 0.01-0.5 | 0.34 | 0.22 | 0.07 | 0.06 | 0.31 | | | | | | | | | |
| | 3 | 0.1-1.0 | | | 0.02 | 0.03 | 0.05 | 0.05 | 0.07 | 0.09 | 0.15 | 0.54 | | | | |
| | 4 | 0.4-1.5 | | | | | | | 0.01 | 0.01 | 0.02 | 0.03 | 0.06 | 0.09 | 0.11 | 0.65 |
| | 5 | 0.6-1.5 | | | | | | | 0.01 | 0.01 | 0.03 | 0.03 | 0.05 | 0.09 | 0.11 | 0.66 |
| | 6 | 0.7-1.5 | | | | | | | | 0.26 | 0.12 | 0.09 | 0.08 | 0.09 | 0.08 | 0.30 |
| | 7 | 0.7-1.5 | | | | | | | | 0.20 | 0.10 | 0.08 | 0.08 | 0.08 | 0.09 | 0.37 |
| | 8 | 0.8-1.4 | | | | | | | | | 0.27 | 0.13 | 0.12 | 0.11 | 0.12 | 0.26 |
| | 9 | 0.9-1.2 | | | | | | | | | | 0.40 | 0.22 | 0.38 | | |
| W-f | 1 | 0.01-0.05 | 1.00 | | | | | | | | | | | | | |
| | 2 | 0.01-0.5 | | 0.04 | 0.11 | 0.19 | 0.66 | | | | | | | | | |
| | 3 | 0.1-1.0 | | | 0.01 | 0.02 | 0.02 | 0.03 | 0.05 | 0.07 | 0.13 | 0.66 | | | | |
| | 4 | 0.4-1.5 | | | | | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.03 | 0.04 | 0.07 | 0.09 | 0.69 |
| | 5 | 0.6-1.5 | | | | | | | 0.03 | 0.02 | 0.04 | 0.04 | 0.05 | 0.07 | 0.10 | 0.65 |
| | 6 | 0.7-1.5 | | | | | | | | 0.26 | 0.11 | 0.09 | 0.08 | 0.08 | 0.09 | 0.30 |
| | 7 | 0.7-1.5 | | | | | | | | 0.24 | 0.10 | 0.09 | 0.08 | 0.08 | 0.09 | 0.33 |
| | 8 | 0.8-1.4 | | | | | | | | | 0.28 | 0.12 | 0.10 | 0.08 | 0.09 | 0.33 |
| | 9 | 0.8-1.4 | | | | | | | | | 0.28 | 0.12 | 0.11 | 0.11 | 0.12 | 0.26 |
| | 10 | 0.8-1.2 | | | | | | | | | 0.33 | 0.17 | 0.16 | 0.34 | | |

model is unnecessarily complex and is over-parameterized. Insufficient data exist to estimate all model parameters and the estimates of most of the parameters are not well determined. This elaborate model structure allows the inclusion of data that had previously not been fit in hoki stock assessment models. However, whether these additional data and the increased complexity of the model have improved the quality and precision of the stock abundance estimates is unclear.

High residual variances for the model fits to age class 1 and age class 2 data (mean-squared residuals of 2.39 and 2.70, respectively) suggest that little information is gained by their inclusion in the model. These data may contribute more noise than signal with respect to the relative year-class strengths. If these data were excluded from the model fit, two of the regions (the northern and southern corridors) and their associated migration parameters would be eliminated from the model structure. A more parsimonious model may lead to greater precision in parameter estimates.

Posterior distributions of model parameter values tend to be either similar to their priors (i.e., uniform within the bounded range) or aggregated at one of their bounds. Neither of these conditions is satisfactory. A posterior distribution that is similar to its prior distribution suggests the data provide little information to alter the value of the parameter. Posterior distributions that are dense at one extreme of the prior suggest that the bounds have excluded plausible values of the parameter or that the model is misspecified in some way.

The penalty function for the equality of the proportionality constants between the eastern and western regions has a significant impact on the estimates of stock abundance. When the penalty weight on this function was decreased from the value used in the 1998 stock assessment (1.0) to a value of 0.1, the biomass estimates for the western stock increased by approximately 100% and the estimates for the eastern region decreased by approximately 20%. Although the posterior distributions of spawning stock biomass were wider for the western region with the lower penalty weight, these distributions were narrower for the eastern region, suggesting a more consistent fit to the data for that stock. This penalty function clearly has a significant impact on the assessment and should therefore be examined in greater detail in future assessments. For instance, it may not be realistic for the proportionality constants to be similar in the different areas: the size and type of bottom may be very different between areas or the behaviour of acoustic methods may differ considerably between Cook Strait and the west coast South Island. The extreme sensitivity of important model estimates on an untestable assumption is a poor attribute for such a model.

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