ESSAYS ON BUSINESS CYCLES
AND ENDOGENOUS GROWTH

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Abstract

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This dissertation explores the nexus between asset and credit market cycles, short-run fluctuations, and growth. What factors contribute to slow and incomplete recoveries from major crises? Why are some economies more prone to such dynamics than others and what lessons does it offer for policymakers? These are among the questions that I explore in my research.

In the first chapter, I document that persistent fluctuations in trend growth – medium frequency cycles – tend to be more volatile and negatively skewed in emerging as opposed to developed small open economies. I argue that this evidence can be understood as stemming from the non-linear interaction between credit cycles, occasionally binding collateral constraints, and innovation-driven endogenous growth. Negative shocks are highly detrimental to productivity growth in vulnerable economies that are prone to sudden stops, but this is not the case in economies with deep financial markets where agents are more often able to optimally borrow to offset temporary negative income shocks.
The second chapter studies the long-term effects of housing market boom-and-bust cycles. I first examine the relationship between the dynamics of the housing market, household debt, and economic activity in a historical panel of 50 countries. I show that housing market crashes robustly predict slower future output growth, most of which is explained by slower total factor productivity growth. Notably, the magnitude of this relation is increasing in the measure of preexisting household debt. To interpret these stylized facts, I construct a two-agent (borrower-saver) dynamic general equilibrium model with an occasionally binding collateral constraint tied to housing equity. Productivity grows endogenously in the model through forward-looking innovation investment. When the preexisting level of debt is sufficiently high, negative housing demand shocks cause the collateral constraint to bind and trigger deleveraging. The endogenous slowdown in TFP growth emerges as one of the adjustment margins during this process, prolonging the real effects of a crisis. The initial shock is amplified by a negative feedback loop between deleveraging, borrowers’ housing wealth, and growth. I use the calibrated model to identify implications for the policy response during episodes of household deleveraging. Measures that reduce the debt burden of borrowers are effective in alleviating the short-run and persistent effects of deleveraging. In terms of monetary policy, the endogenous response of productivity growth warrants a greater focus on short-run output stabilization as opposed to inflation stabilization.

Finally, in the third chapter (joint with Fabio Ghironi) we study the macroeconomic consequences of trade policy uncertainty emphasizing its negative effects on productivity growth. To that end, we build a small open economy model with nominal rigidity, innovation-driven endogenous growth, and time-varying volatility of domestic import tariffs. Several conclusions emerge: import tariff uncertainty shocks act as aggregate supply shocks; they cause a temporary improvement of the current account along with the real exchange rate
appreciation in the medium run. In addition, an increase in import tariff uncertainty causes a sharp decline in the introduction of new intermediate products, which is detrimental to productivity growth and prolongs the effect of the shock. The size of these persistent effects — relative to short-term effects — is much larger for tariff uncertainty shock than for tariff level shocks. We show that endogenous risk premia in equity and bond markets is the key channel transmitting the shock to the broader economy and study role monetary policy in shaping it.
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Chapter 1

MEDIUM-TERM CYCLES: THE ROLE OF OCCASIONALLY BINDING CONSTRAINTS

1.1 Introduction

In addition to the regular short-run business cycle fluctuations, economies tend to oscillate between periods of robust growth and relative stagnation over longer horizons. These medium-frequency cycles are deemed to be the result of the endogenous response of productivity growth to business-cycle shocks.\(^1\) Characteristics of medium-frequency cycles vary across countries and over time. In particular, they tend to be larger in amplitude and more negatively skewed in emerging rather than developed small open economies (figure 1.1 and table 1.1).\(^2\)

What factors can account for the greater amplitude and negative skewness of medium-term fluctuations of output in emerging, as compared to developed, small open economies? In this paper, I argue that this evidence can be understood through the non-linear interaction between credit cycles, occasionally binding collateral constraints, and endogenous growth. Historically, emerging economies are more prone to volatile credit cycles that end with sudden stops and deleveraging. At the peak of the credit boom, when the economy is close to its leverage constraint, negative shocks cause the constraint to bind. The resulting decline in credit leads to a sharp drop in R&D spending, business and product creation: activities that drive productivity growth over the long run but require paying significant upfront sunk costs. As such, deleveraging not only exacerbates the construction in the short-run but also

\(^{1}\) See Comin and Gertler (2006), among others.

\(^{2}\) I decompose the data using the optimal one-sided bandpass filter of Fitzgerald and Christiano (2003). As in Comin and Gertler (2006), I define the medium-term cycle as fluctuations with frequencies from 0 to 50 years, which consists of business cycles (from 0 to 8 years) and the medium-term component (from 8 to 50 years).
weights down the growth dynamics in the medium run through the endogenous productivity mechanism. Since collateral constraints bind only occasionally following adverse shocks, the economy exhibits negative skewness. This dynamics contrasts developed open economies with resilient financial markets that more often remain unconstrained to borrow from the rest of the world to smooth temporary negative income shocks.

I begin my analysis providing new evidence on the dynamics of recessions and recoveries that occur at high and low points of credit cycles based on a panel of 43 small open economies. I show that recessions that are associated with credit boom and bust episodes tend to be not only deeper in the short-run but also more protracted with output and consumption persistently lagging relative to their initial trajectories. Almost half of this persistent decrease in economic activity can be directly attributed to the dynamic of utilization-adjusted total factor productivity (TFP). In contrast, recoveries from recessions that occur in “tranquil” times, when credit is not elevated relative to the long-run trend, tend to follow a more conventional mean-reverting path with some debt accumulation along the way.

In the second part of the paper I present a dynamics general equilibrium model that I use to interpret these empirical observations. The model builds on the standard real business cycle small open economy framework extending it in two dimensions. First, productivity in the economy grows endogenously through forward-looking investment in the creation of new intermediate products, akin to the innovation-driven growth model of Romer (1990). Returns on productivity-improving activities vary over the business cycle, leading to oscillations in productivity growth.

Second, borrowing from the rest of the world is subject to an occasionally binding collateral constraint tied to the value of the capital shock, as in the literature on emerging market crises pioneered by Mendoza (2010). This feature allows explaining financial crises – periods when the constraint binds – as rare events nested within the regular business cycle. When the economy is close to its debt limit, negative shocks cause the collateral constraint to bind triggering deleveraging that amplifies the downturn. As such, occasionally binding collateral constraints introduce asymmetry in the link between credit and economic activity. Endogenous productivity growth inherits these amplification and asymmetry properties. The response of productivity growth to negative shocks is more pronounced during economic downturns exacerbated by deleveraging as compared to “regular” recessions or expansions.

This paper contributes to several strands of literature. The empirical exercise that I
conduct adds to the literature on the macroeconomic effects of credit cycles and financial distress. In this dimension the closest paper is Jordà et al. (2013), which also studies how past credit accumulation affects the dynamics of recessions. Unlike their work, I study a broader variety of macroeconomic indicators, their dynamics over a longer horizon, and factors that can account for it. I also contribute to the theoretical literature on sudden stops. Aguiar and Gopinath (2007) famously noted that shocks to trend growth — rather than transitory fluctuations around a stable trend — are an important feature of business cycles in emerging markets. My framework offers a view that these drastic swings in trend growth in emerging countries can be seen as an endogenous outcome of sudden stops, rather than an exogenous shock.

The paper relates to the literature on the interconnectedness between business cycles and endogenous growth. The closet reference is Queraltó (2013) who study the transmission of world interest rate shocks in a small open economy with endogenous growth and frictional financial intermediaries. This paper differs in its treatment of financial frictions but more substantively, it focuses on the role of non-linearities that arise due to occasionally binding collateral constraints and how they interact with endogenous growth. Finally, the present work relates to the literature on non-linear effects of financial frictions.

The rest of the paper is organized as follows. Section 2 discusses motivating empirical evidence. Section 3 describes the model. Section 4 summarizes the calibration, the simulation technique, and the main results. Section 5 concludes.

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3 Cerra and Saxena (2008), Romer and Romer (2017)

4 The non-exhaustive list includes, Benigno et al. (2013), Benigno et al. (2016), Bianchi and Mendoza (2018a), Devereux et al. (2017), Devereux and Yu (2017), Korinek (2008), Mendoza (2010).


6 See Maffezzoli et al. (2015), Guerrieri and Iacoviello (2017), Jensen et al. (2016) on the implications of households and entrepreneurs being occasionally financially constrained; as well as contributions by Aknc and Queralto (2017), Holden et al. (2019) focusing on the frictional banking sector.
1.2 Empirical motivation

1.2.1 Recessions and debt overhang: cross-country evidence

In this section, I study the association between post-recession recovery dynamics and the level of pre-existing private debt. In particular, I document that recessions that happen in periods when private debt is above its long-run trend tend to be not only deeper in the short run, but also have a very persistent negative effect on the level of economic activity, almost a half of which can be attributed to a slowdown in the TFP growth.

To that end, I study a panel of 43 small open economies. The unbalanced panel of macroeconomic indicators consists of annual observations from 1950 to 2017, and includes the following variables: (1) real per capita GDP; (2) real per capita consumption; (3) real per capita capital; (4) real per capita investment; (5) utilization-adjusted TFP; (6) current account to GDP; and (7) real exchange rate index. Appendix A summarizes the data and its sources.

In addition, to assess the prevailing credit market conditions I employ the data from the novel IMF Global Debt Database, Mbaye et al. (2018). I use this data to calculate private credit-to-GDP gaps. Similarly to output gaps, this variable is defined as a deviation of the credit-to-GDP ratio from its long-run trend. The long-run trend is commonly extracted using the one-sided HP filter with the smoothing parameter of $4 \cdot 10^5$ for quarterly observations. As common in the literature, I use credit-to-GDP gaps as a proxy for credit-market imbalances in my analysis.

In the absence of a comprehensive database on the timing of recessions, I proceed as follows. First, I calculate annual growth rates of real per-capita GDP and identify years when the growth rate was negative as recessionary periods. Whenever growth is negative for several consecutive years, the first year in the sequence is labeled as a beginning of a recession. I use the database of Laeven and Valencia (2013) to exclude recessions associated with wars. The result is an indicator variable $\{1_{i,t}^{\text{recession}}\}$ of a recession onset in county $i$ in year $t$. Finally, I separate the resulting sample of events into two groups: the ones that occurred when the credit-to-GDP gap was below 0% and above 10% (low-debt and high-debt.

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7 See, for example, Basel Committee on Banking Supervision (2010). The smoothing parameter of $4 \cdot 10^5$ for quarterly observations corresponds to $4 \cdot 10^5 / 4^4$ for annual observations, according to Martone (2002).
recessions henceforth). This approach identifies 51 low-debt and 36 high-debt recessions.

I proceed estimating average recovery paths of the above-listed variables using Jordà (2005) local projections. Instead of extrapolating impulse responses from the estimated multivariate system, this model-free approach involves estimating following dynamic cross-country panel:

$$\Delta_h y_{i,t+h} = \alpha_i^h + \alpha_t^h + \beta_1^h D_{i,t} + \gamma_1^h \mathbb{1}_{i,t}^\text{banking} + \varepsilon_{i,t}^h$$

The dependent variable is the country $i$’s $h$-period log difference of the response variable: $\Delta_h y_{i,t+h} = \log(Y_{i,t+h}) - \log(Y_{i,t})$; and the perturbation variable is recession indicator $D_{i,t}$ discussed above. Estimating this relation at different horizons $h$ produces a set of coefficients $\{\beta_h^h\}_{h=1:H}$ that can be interpreted as an average recession path across the considered sample of events, controlling for country fixed effects ($\alpha_i^h$) and year fixed effects ($\alpha_t^h$), and the fact that some recessions in the sample coincide with systemic banking crises, captured by the variable $\mathbb{1}_{i,t}^\text{banking}$ based on the dates from Laeven and Valencia (2013).

Figure 1.2 shows estimated responses of GDP, consumption, investment, and capital, all in real per-capita terms, for high-debt and low-debt recessions. Recoveries after low-debt recessions, on average, exhibit mean reversion with the short-run consumption response being smoother than the output response. On the other hand, the effect of high-debt recessions is very persistent and remains significant beyond 5 years after the recession onset. Also, the responses of consumption and output are of comparable magnitudes.

What factors can account for this persistence? Figure 1.3 shows more responses of relevant variables. Two important observations emerge. First, high-debt recessions, as opposed to low-debt recessions, are associated with the gradual decline in utilization-adjusted TFP. The magnitude of this effect is such that I can directly explain up to a half of the persistent decline in output. Second, the dynamics of private debt, the current account, and the real exchange rate differ considerably across the two groups of recessions. Recessions that occur after credit-intensive booms are associated with gradual deleveraging with the debt-to-GDP ratio falling for up to 4%. There is also evidence of external adjustment: the sharp on-impact improvement in the current account of almost 4% and the real exchange rate depreciates in the medium run. On the other hand, low-debt recessions are associated with an increase in the debt-to-GDP ratio in the short run, a muted response of the current account, and no statistically significant movement of the real exchange rate.
1.2.2 Persistent effects of financial distress

What makes low-debt and high-debt recessions so different and why the lion’s share of the persistent decline in the output level is due to the low productivity growth? My argument is as follows. Contractionary shocks in a high-debt environment make agents liquidity-constrained by reducing the value of their collateral. The resulting deleveraging not only exacerbates the short-run effect of the contractionary shock but also has generates a persistent effect by reducing productivity-enhancing investment and growth in the medium-run. Crucially, this effect is state-dependent and materializes to the full extent only when economy is in high-debt financially fragile state.

To support the proposed mechanism, in this section I present an evidence of comovement between financial conditions, private credit, measures of real activity, and TFP. For this purpose, fist employ the narrative quantitative measure of financial distress from Romer and Romer (2017). I use the variable as indicative of periods when economies become credit-constrained, as well as the severity of these episodes. The measure, which is an unbalanced panel that covers 30 countries from 1967 to 2017, is added to the database from the previous section. Responses to the financial distress shock are estimated by local projections as follows:

$$\Delta h y_{i,t+h} = \alpha^h_i + \alpha^h_t + \beta^h D_{i,t} + X_{i,t} \Gamma + \varepsilon^h_{it},$$

where \{D_{i,t}\} in the index of financial distress in country \(i\) in year \(t\), \(\alpha^h_i\) and \(\alpha^h_t\) are country and year fixed effects, respectively. Vector \(X_{i,t}\) is a set of macroeconomic controls, which includes three lags of (1) financial distress index, (2) log of the response variable, (3) log private debt to GDP, and (4) current account to GDP. In addition, to account for the possible effect of an exchange-rate regime, I include fixed exchange rate indicators from Ilzetzki et al. (2019). The setting implies that the financial distress shock is identified with short-run restrictions: it is assumed not to react to the above macroeconomic variables contemporaneously.

Figure 1.4 shows responses to a one point increase in the financial distress shock. To put this shock in context of the scale use by Romer and Romer (2017), the value of the financial distress index in the US in 2008 was 11.5. The shock is associated with deleveraging with the debt-to-GDP gap falling by about 2% over the course of 7 years, as well a an improvement in the current account together with a sharp depreciation of the real exchange rate. Consistently with the previous results, the decline in output is very persistent and up to a half of it can
be directly attributed to the dynamics of utilization-adjusted TFP.

The second exercise is a VAR model that provides an evidence supporting the conjecture that credit-market disruptions lead to persistent output losses by negatively affecting the pace of productivity-improving activities and hence the level of measured TFP. This exercise comes with a caveat: the cross-country data that could be used as a proxy for the scope of innovation activities is limited, especially at high frequencies. For this reason, I resort to the US data that offer the longest quarterly series of R&D spending. Although in general the transmission of shocks in the US is likely to somewhat different than in other, smaller, countries, I believe that this exercise highlights relations between variables that are not unique to the US economy.

The VAR model includes variables ordered as follows: (1) financial conditions index, (2) total employment, (3) real GDP, (4) utilization-adjusted TFP, (5) real R&D spending, (6) private credit to GDP. The data is quarterly spanning from Q1 1971 to Q3 2018. The TFP measure is the utilization-adjusted TFP of Fernald (2012) that takes out the effect of short-run fluctuations in factor utilization. The measure of financial conditions is the Chicago Feds National Financial Conditions Index (NFCI), which is a weighted average of a 105 measures of financial activity that intends to capture changes in financial-market risk, credit availability, and leverage. More specifically, I employ the adjusted version of this index (ANFCI) that isolates a component of financial conditions orthogonal to economic conditions, which motivates ordering this variable first. The of other variables implies that TFP reacts to R&D expenditure and real GDP reacts to TFP with a one-period lag. Credit is ordered last as financial markets tend to quickly react to changes in the real activity. Total employment is ordered before real GDP since the labor-market dynamics lags with respect to the business cycle. Variables except for the ANFCI enter the model in log-levels due to the potential cointegration between them. As noted by Kilian and Lütkepohl (2017) estimating a level VAR is an alternative to estimating a VECM model, which may be more robust to possible (near) cointegration of unknown form among the variables.

Figure 1.5 presents IRFs to a worsening of financial conditions (a one standard deviation increase in ANFCI). Short-term tightening of financial conditions is contractionary, it’s associated with a drop in R&D spending and a decrease in the private-sector debt-to-GDP ratio. Notably, not all of the initial contraction is fully recovered in the medium-run. Even 15 years after the shock 0.01% decrease in the real GDP, 0.004% decrease in TFP, and 0.006%
decrease in employment remain. In addition, Appendix A presents responses to an identified R&D spending shock. An increase in R&D spending is expansionary and has a very persistent effect in the real activity. Over the course of 60 quarters, an R&D shock of 0.6% leads to a 0.2% increase in the level of real GDP, 0.05% increase in the utilization-adjusted TFP, and a 0.1% increase in the total employment.

1.3 The model

This section lays out a real business cycle small open economy with capital accumulation extended in two dimensions. First, the model features financial frictions. Borrowing from the rest of the world is subject to an occasionally binding collateral constraint tied to the value of capital stock, as in Mendoza (2010), among others. Second, productivity in the economy growth endogenously through introduction of new intermediate products as in Romer (1990). Figure 1.6 summarizes the structure of the model.

1.3.1 Production

Final good sector

The final good is produced by perfectly competitive firms using homogeneous labor, $L_t$, capital, $K_t$, and a CES basket of intermediate products $X_t = \left[ \int_0^{N_t} x_t(\omega) \frac{1}{\nu} d\omega \right]^\nu$, where $\nu$ is the elasticity of substitution between varieties. The aggregate production function takes the following Cobb-Douglas form:

$$Y_t = Z_t \left( (\psi_t) L_t^{1-\alpha} \right)^{1-\xi} X_t^{\xi} = Z_t \left( (\psi_t K_t)^{\alpha} L_t^{1-\alpha} \right)^{1-\xi} \left( \int_0^{N_t} x_t(\omega) \frac{1}{\nu} d\omega \right)^{\nu \xi}$$

where $\psi_t$ is a lognormally distributed capital quality shock, which at the beginning of period $t$ determines the effective amount of physical capital in possession of the firm, $\psi_t K_t$. Following the finance literature, this shock has become a common way to introduce an exogenous shock.

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For instance Merton (1973)
variation to the total capital stock value.\textsuperscript{9}

Firms are subject to a working capital requirement: a fraction $\zeta$ of the current-period wage bill needs to be financed in advance via an intra-period borrowing from households at a rate $R^F_t$. This is a reduced-form way to cause household financial frictions to also have supply side implications through labor demand. Other than that, the problem is standard. Given input prices a representative firm maximizes its discounted profit stream:

$$\max \left\{ x_{t+j}(\omega), L_{t+j}, K_{t+j} \right\} \sum_{j=0}^{\infty} \beta_{t,t+j} \left[ Y_{t+j} - R^K_{t+j} K_{t+j} - (1 + \zeta(R^F_t - 1))W_{t+j}L_{t+j} - \int_0^{N_t} p^F_{t+j}(\omega)x_{t+j}(\omega)d\omega \right],$$

which implies the following input demands:

$$W_t \left[ 1 + \zeta(R^F_t - 1) \right] = (1 - \alpha)(1 - \xi)\frac{Y_t}{L_t} \quad (1.1)$$

$$R^K_t = \alpha(1 - \xi)\frac{Y_t}{\psi_tK_t} \quad (1.2)$$

$$p^F_t(\omega) = \xi\frac{Y_t}{X_t}x_t(\omega)^{1-\nu}$$

Firms are implicitly owned by households, so the future stream of profits is discounted by the households stochastic discount factor $\beta_{t,t+j} = \beta^j \frac{U_{t+j}}{U_t}$.

\textit{Intermediate good sector}

The intermediate sector is populated by a mass $[0, N_t]$ of monopolistically competitive firms, each operating a roundabout technology that requires $A^{-1}$ units of the domestic good to produce a unit of the intermediate good. One should not take this setup literally. The correct interpretation of this formal description is that the forgone final good is never manufactured. The resources that would have been used to produce the forgone output are used instead to manufacture intermediate goods.

\textsuperscript{9}See, for instance, Gertler and Karadi (2011). Gertler et al. (2012) provide an explicit microfoundation for this shock.
Each firm maximizes its real profit subject to the production sector demand:

$$\max_{\{p_{t+j}(\omega)\}_{j=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta_{t,t+1} (p_{t+j}(\omega)x_{t+j} - A^{-1}x_{t+j}(\omega)) \text{ s.t. } p_{t}(\omega) = \xi \frac{Y_t}{X_t} x_t(\omega)^{1-\nu}$$

In a symmetric equilibrium the optimal quantity of the intermediate good ($x_t$), the firm’s profit ($d_t$), and its relative price ($p_x^t$) are the following:

$$x_t = \left( \frac{A \xi}{\nu} \right)^{\frac{1}{1-\xi}} Z_t^{1-\xi} N_t^{\frac{\xi-1}{1-\xi}} K_t^\alpha L_t^{1-\alpha} \quad (1.3)$$

$$d_t = \frac{\nu - 1}{\nu} p_x^t x_t = \frac{\nu - 1}{A} x_t \quad (1.4)$$

$$p_x^t = \nu A^{-1}$$

Positive profit in this sector motivates entry. To open a firm, an entrepreneur needs to pay an sunk entry cost that consists of the cost of buying a blueprint of a new product from innovators at a price $p_b^t$. New firms finance entry by selling shares of their equity to entrepreneurs. Free entry pins down the equilibrium value of an intermediate firm, which should be equal to the entry cost: $v_t = p_b^t$.

Innovators

The sector of innovators involves inventing blueprints for new types of intermediate goods. The sector is populated with the unbounded mass of potential innovators. Let $S_t$ be the total innovation spending and $\phi_t^i$ be the innovators individual productivity parameter, which each innovator takes as given. The individual production function blueprints of intermediate goods is then $N_t^{\epsilon_i} = \phi_t^i S_t^i$. However, the aggregate innovators productivity $\phi_t$ depends on the existing stock of knowledge, measured by the number of existing intermediate goods, $N_t$. As in Romer (1990), this knowledge spillover externality is responsible for the existence of the balanced growth path in the model. In line with Comin and Gertler (2006), I include a congestion externally $N_t^\rho S_t^{1-\rho}$ that allows to control for the aggregate elasticity of blueprints output with respect to innovation spending. The resulting aggregate innovators productivity is:

$$\phi_t = \phi_t = \frac{N_t}{N_t^\rho S_t^{1-\rho}}$$
where $S_t = \int S^i di$. The aggregate production function of innovators is then $N_t^e = \phi N_t \left( \frac{S_t}{N_t} \right)^\rho$. Perfectly competitive innovators set the price of blueprints $p^b_t$ to maximize their expected discounted stream of profit:

$$\max_{\{S^i_{t+j}\}_{j=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta_{t,t+j} \left( p^b_{t+j} \phi_{t+j} S^i_{t+j} - S^i_{t+j} \right)$$

The optimal price then is $p^b_t = \phi_t^{-1}$. Together with the intermediate sector free-entry condition this leads to the following expression that pins down intermediate firm value:

$$v_t = \phi_t^{-1} \left( \frac{S_t}{N_t} \right)^{\rho-1}$$

(1.5)

The model is calibrated at annual frequency, which motivates the assumption of no time-to-build lag: newly invented blueprints can be used for production within the same period. In each period existing varieties face a constant probability of becoming obsolete, $\delta_N$. All of the above imply the following law of motion for the total number of varieties $N_t = (1 - \delta_N)N_{t-1} + N^e_t$ so that the productivity growth rate is:

$$g_t = \frac{N_t}{N_{t-1}} = \phi \left( \frac{S_t}{N_t} \right)^\rho + (1 - \delta_N)$$

(1.6)

R&D expenditure $S_t$ vary over the business cycle that causes fluctuations in the growth rate. The sensitivity of growth to R&D expenditure is controlled by the parameter $\rho$.

### 1.3.2 Households

The economy is populated by a measure 1 of households. The representative household consumes; invests in capital that is rented out to firms at the rate $R^K_{t+1}$; supplies labor $L_t$; borrows from the rest of the world in domestic consumption units at the given rate $R_t$ for its own use ($B_t$) and to provide inter-period working-capital loans to firms ($B^F_t$) at the rate $R^F_t$. Finally, households hold shares in a mutual fund of intermediate firms. In each period $t$ the household buys $e_{t+1}$ shares in a mutual fund of $N_t$ currently active firms at the price of $v_t$ per share; receives the dividend income from currently owned firms $d_t$; and receives the return on the shares purchased in the previous period, adjusted for the probability of a firm
going out of business, \( \delta_N \).

The utility-maximization problem of a representative household is as follows:

\[
\begin{aligned}
\max & \{ C_t, e_t, L_t, K_t+1, B_t, B_{t+1} \} \\
\text{s.t.} & \quad C_t + e_t + L_t + R_t B_t = W_t L_t + R_t^K \psi_t K_t + e_t N_t d_t + \\
& \quad e_t \{ N_{t-1} (1 - \delta_N) \} + B_{t+1} + (R_t^F - 1) B_t^F \\
& \quad K_{t+1} = (1 - \delta_K) \psi_t K_t + (1 - AC_{t,t}) I_t \\
& \quad B_{t+1} + B_t^F \leq \rho q_t K_{t+1}
\end{aligned}
\]

I resort to GHH preference of Greenwood et al. (1988), which is widely used in the small open economy analysis. This preference does away from the wealth effect in labor supply and isolates the labor supply decisions from the dynamics of consumption, saving, and portfolio choices and improves the performance of open-economy real business cycle models.\(^\text{10}\) As in Queralto (2019), the disutility of labor is governed by the following process:\(^\text{11}\)

\[
\Upsilon_t = \Upsilon_{t-1}^{\rho_{\Upsilon}} N_t^{1-\rho_{\Upsilon}}
\]

The parameter \( \rho_{\Upsilon} \) determines the responsiveness of the disutility of labor to changes in productivity growth. This formulation ensures that the BGP with constant hours exists, but the medium-run swings in growth do not excessively affect labor supply.\(^\text{12}\)

External borrowing is limited to a fraction \( \rho \) of the capital stock value. Effectively, this parameter sets the maximum loan-to-value ratio and can be linked to the liquidity of the collateral. It can also be interpreted as the degree of domestic financial market development. As Kiyotaki and Moore (1997), I do no explicitly model the origins of this constraint. A nat-

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\(^\text{10}\) See contributions of Mendoza (1991) and Raffo (2008), among others.

\(^\text{11}\) Jaimovich and Rebelo (2009) suggest a similar preference that allows parameterizing the short-run wealth effect on the labor supply

\(^\text{12}\) One way to interpret this feature of the preference is as follows. As noted by Benhabib et al. (1991), the GHH preference can be interpreted as a reduced form of an economy with home production. The disutility of work then consists of the forgone output in home production. This disutility increases as productivity improvements in the formal sector spill over to the home production. However, to the extent this process takes time, the disutility of labor exhibits inertia.
ural interpretation, however, should be that due to the imperfect enforceability of contracts, the ability of households to borrow is bounded by a fraction of their collateral value that can be seized by creditors in a case of default. However, the model abstracts from endogenous default and debt is always repaid. Capital accumulation is subject to a convex adjustment cost, which implies an upward-sloping supply curve of capital and gives rise to endogenous fluctuations in the price of capital. Changes in the capital stock valuation cause endogenous fluctuations of the borrowing limit.\footnote{An alternative way that brings about endogenous fluctuations in the collateral price is the assumption that there are physical constraints to the supply of the collateralizable asset. This approach is common in the literature on the macro effects of the housing market. See, for instance, Iacoviello (2005), Liu et al. (2013), Ferrero (2015), Liu et al. (2016).}

Denote Lagrange multipliers with respect to the budget constraint, the law of motion for capital, and the borrowing constraint as $\lambda_t$, $q_t$ and $\mu_t$, respectively. The consumption Euler equation and the complementary slackness condition for the borrowing constraint are then the following:

\begin{align}
E_t [\beta_{t,t+1} R_{t+1}] &= 1 - \frac{\mu_t}{\lambda_t} \tag{1.10} \\
\left[\rho q_t K_{t+1} - B_{t+1} - B_F^t \right] \mu_t &= 0, \quad \mu_t \geq 0 \tag{1.11}
\end{align}

Binding collateral constraint ($\mu_t > 0$) introduces an external financing premium in consumption and labor demand. Effectively, households become more impatient when the constraint binds, as it distorts optimal borrowing and the consumption level is suboptimally low. Moreover, binding collateral constraint increases the effective cost of labor for firms through the working capital requirement: $R_F^t = \mu_t + 1$. As Bianchi and Mendoza (2018a) I assume that working capital incurs no explicit interest rate payments, so at the unconstrained steady state the factor shares are not distorted by this feature of the model.

The next two first order conditions describe capital accumulation:

\begin{align}
q_t &= 1 + q_t (AC_{t,t} + AC_{I,t}^t I_t) - E_t \left( \beta_{t,t+1} q_{t+1} AC_{I,t+1}^t I_{t+1} \right) \tag{1.12} \\
q_t &= E_t \left( [\beta_{t,t+1} (1 - \delta_K) q_{t+1} + R_{t+1}^K] \psi_{t+1} + \frac{\mu_t}{\lambda_t} q_{t+1} \right) \tag{1.13}
\end{align}

The first is the standard condition describing fluctuations of Tobin’s $q$ due to the presence of investment adjustment costs. The second condition is the capital supply that takes into
account the role of capital as collateral. Binding collateral constraint affects the current capital supply through two channels. The direct effect of $\mu_t > 0$ is positive. When credit-constrained, households have an incentive to accumulate more capital to relax the borrowing constraint. The second, indirect, effect works through the households stochastic discount factor. As noted above, $\mu_t > 0$ effectively makes households more impatient which, other factors fixed, reduces their willingness to accumulate capital. As noted by Mendoza (2010), for plausible values of $\varrho$ the latter effect dominates. Binding borrowing constraint causes the collateral demand to fall, capital price to decrease, which further tightens the constraint in a classic Fisherian debt deflation vicious circle.

From the first order condition with respect to shares of intermediate firms equity it follows that:

$$v_t = d_t + \mathbb{E}_t [\beta_{t,t+1}(1 - \delta_N)v_{t+1}]$$  \hspace{1cm} (1.14)

When iterated forward, it implies that in equilibrium the current firm value, $v_t$, equals to the future discounted profit stream. Factoring in the growth equation (3.20) along with the free-entry condition $v_t = p_t^b = \phi_t^{-1}$ results in:

$$g_t = \phi^{2\mu - 1} \sum_{j=0}^{\infty} (1 - \delta_N)^j \mathbb{E}_t [\beta_{t,t+j} d_{t+j}] + (1 - \delta_N)$$  \hspace{1cm} (1.15)

The productivity growth rate is determined by the future expected discounted stream of profits from investing in business creation. Finally, the labor supply condition is standard:

$$\chi_t L_t^\ell = W_t$$  \hspace{1cm} (1.16)

### 1.3.3 Symmetric equilibrium and market clearing

A representative household owns all intermediate firms, so $e_{t+1} = e_t = 1$. Under symmetric equilibrium all intermediate good producers are alike, so $x_t(\omega) = x_t$, $p_t(\omega) = p_t$, and $d_t(\omega) = d_t, \forall \omega$. Since some of the final good sector output is used by the intermediate sector, the correct measure of real GDP in the model is output of the final sector net intermediate consumption. Final output equals to consumption, capital and R&D investment, net
investment income and the net change in the foreign asset position:

\[ Y_t^{GDP} = Y_t - N_t \frac{X_t}{A} = C_t + I_t + S_t - (B_{t+1} - (1 + R_t)B_t) \]

Equilibrium choices of intermediate producers along with other optimality conditions allow to express the production-sector output as follows:

\[ Y_t = N_t \frac{\xi^{(\nu-1)}}{1-\xi} \left( \frac{A\xi}{\nu} \right)^{\frac{\xi}{\nu}} Z_t^{\frac{1}{1-\xi}} \mu_t^{\frac{\xi}{\nu-1}} K_t^{\alpha} L_t^{1-\alpha} \]  \hspace{1cm} (1.17)

This expression makes it clear that the following condition on structural parameters needs to be satisfied for growth to take a labor-augmenting form: \( \frac{\xi^{(\nu-1)}}{1-\xi} = 1 - \alpha \). To close the model and to avoid the non-stationarity of the net foreign asset position, I assume that the world interest rate is debt-elastic:

\[ R_t = R + \psi_B \left[ e^{\left( \frac{B_t}{N_t^t} - b \right)} - 1 \right] \]

This assumption pins down the level of borrowing in the non-stochastic steady state. The model features endogenous growth with all variables growing at the same rate \( g_t = \frac{N_t}{N_{t-1}} \) on the balanced growth path. For solution purposes, I stationarize the model defining productivity-adjusted lower-case counterparts for the variables that exhibit growth, e.g. \( y_t = \frac{Y_t}{N_t} \), \( c_t = \frac{C_t}{N_t} \), \( k_t = \frac{K_t}{N_t} \), etc.

*Equilibrium definition:* equations (1-14, 16, 17) determine 16 endogenous variables \( (y_t, c_t, b_{t+1}, R_t, \chi_t, L_t, w_t, R_t^K, k_{t+1}, i_t, q_t, v_t, s_t, d_t, x_t, g_{t+1}) \) as a function of endogenous states \( (k_t, i_{t-1}, \chi_{t-1}, b_t) \) and exogenous states. Table 1.2 summarizes all equilibrium conditions.

### 1.4 Simulations

#### 1.4.1 Calibration

The model is calibrated at annual frequency. I set the relative risk aversion to \( \gamma = 2 \), a common in the real business cycle literature. I calibrate innovators productivity \( \phi \) to match the 1% annual TFP growth rate on the BGP. The steady-state real interest rate is then

\( 14 \) Schmitt-Grohé and Uribe (2003) presents a summary of other ways to stationarize the NFA position for models solved with perturbation methods. An alternative model-based approach to obtain a stationary distribution of assets as in Mendoza (2010) is more accurate for studying nonlinear effects and the effects of precautionary savings, but requires a global solution method.
pinned down by the savers discount rate as follows: 
\[ R = g^\sigma / \beta. \]
I set the Frisch elasticity of labor supply to \( \frac{1}{\epsilon} = 4 \), consistent with the King and Rebelo (1999) calibration. The capital share is set to the conventional value of \( \alpha = 0.33 \). The intermediate goods share is set to \( \xi = 0.5 \), as in Comin and Gertler (2006). The elasticity of substitution between varieties of intermediate goods is then set to satisfy the condition necessary for growth to be of labor-augmenting form \( \xi(\nu - 1)/(1 - \xi) = 1 - \alpha. \)

Following Comin and Gertler (2006), I set the annual rate of intermediate product obsolescence to 3% (\( \delta_N = 0.03 \)), and the elasticity of innovators output with respect to R&D expenditure to \( \rho = 0.8 \). Annual capital depreciation is set to the conventional value of 10% (\( \delta_K = 0.1 \)). The following four parameters are borrowed from Mendoza (2010): capital adjustment cost \( \phi_K = 2.5 \); maximum loan-to-value ratio \( \varrho \); working capital requirement \( \zeta = 0.25 \); and the steady-state debt-to-GDP ratio \( b = 0.85 \). Finally, the debt-elastic interest rate parameter is set to be very low so that this feature does not compromise the dynamics of endogenous variables in the short-run. I choose \( \psi_B = 0.0007 \), as suggested by Schmitt-Grohé and Uribe (2003). Table 1.3 summarizes the choice of parameters.

### 1.4.2 Solution method

The model features an occasionally binding collateral constraint that generates non-linearities and poses a computational difficulty since the model cannot be solved using standard perturbation methods. One way to tackle this issue is to resort to value function iteration or other global solution methods that allow to fully account for the non-linearities and precautionary behavior linked to the possibility that the constraint may become binding in the future. However, these methods are computationally demanding and are not easily scalable due to the curse of dimensionality. The simplest alternative solution was introduced by Guerrieri and Iacoviello (2015) and involves using a piecewise-linear solution. This method builds on the insight that occasionally binding constraints can be handled as different regimes of the same model. Under one regime, the occasionally binding constraint is slack. Under the other regime, the same constraint is binding. The piecewise linear solution method involves linking the first-order approximation of the model around the same point under each regime. However, just like any linear solution, this method does not allow capturing the effects of uncertainty and so to account for precautionary behavior. I use a similar approach developed by Holden (2019) and implemented as an extension to Dynare: Dynare-
OBC. Unlike the Guerrieri and Iacoviello (2015) method, its compatible with higher-order approximations and by integrating over future uncertainty allows to capture some of the precautionary behavior.

In my solution technique, I deviate from the existing literature that mostly relies on global methods. I do it for simplicity, as I focus on illustrating the key qualitative properties of the model, for which the solution method is likely to be less crucial. First is the state dependence induced by occasional financial frictions: the dynamic response to shocks may be very different depending on the pre-existing level of debt. Second, the model produces occasional financial crises nested within normal business cycle fluctuations and need not to rely on the exceptionally large shock to explain them. These crises are not only deeper than regular contractions but also are characterized by abnormally low productivity growth that leaves a permanent footprint on the level of output.

1.4.3 Simulation results

*High-debt and low-debt economy*

The first simulation illustrates the state dependence that exists in the model due to the presence of the occasionally binding constraint. Specifically, I show that the effect of temporary negative income shocks on endogenous productivity depends significantly on the preexisting level of debt in the economy relative to the upper bound for the loan-to-value-ratio. This effect is much larger in situations when the level of debt is high and the shock causes the borrowing constraint to bind triggering deleveraging. For illustrative purposes I consider a one-time unanticipated productivity shock $\varepsilon^Z_t$. Exogenous productivity follows the standard autoregressive process: $\log(Z_t) = (1 - \rho_Z) \log(Z) + \rho_Z \log(Z_{t-1}) + \varepsilon^Z_t$.

Figure 1.7 shows responses to a 1% negative productivity shock with the persistence parameter $\rho_Z = 0.15$, which means that the shock dies out after about two years. I consider two scenarios: when the preexisting level of debt is 50% and 99% of the upper bound for borrowing (low-debt and high-debt cases henceforth). In the low-debt scenario, the collateral constraint is slack and the multiplier with respect to the collateral constraint remains zero. In this case, the economy responds like a standard small open economy RBC model. The

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15 Available at https://github.com/tholden/dynareOBC
shock causes a temporary contraction in economic activity. Driven by the consumption-smoothing motive, the economy runs a current account deficit and accumulates net foreign debt to offset a temporary fall in income. As a result, the fall in consumption is almost twice lower than the fall in output.

The response of the economy is significantly different in the high-debt regime when the negative productivity shock causes the collateral constraint to bind by reducing the price of capital. As per equation (1.13), when the constraint binds, the price of capital falls further. This effect can be interpreted as the outcome of the fire sale of capital as debtors strive to satisfy the tightened borrowing constraint. This endogenous fall in Tobin’s q further tightens the collateral constraint exacerbating deleveraging setting in motion a classic Fisherian debt-deflation amplification mechanism. As the result of being temporary credit constrained, the economy runs a temporary current account surplus and accumulates net foreign assets in stark contrast to the low-debt scenario. Moreover, binding constraint affects the supply side of the economy by increasing the effective cost of working capital for firms and hence reducing their labor demand, as implied by the equation (2.20). This working-capital effect amplifies the fall in output.

The endogenous growth mechanism of the model makes measured productivity to respond to business-cycles shocks. Indeed, in both scenarios a negative shock causes a temporary slowdown in productivity growth adding persistence to responses of other endogenous variables. However, this effect is much more pronounced in the high-debt regime when the collateral constraint binds and the economy undergoes a phase of deleveraging. The reason is straightforward: households deleverage through a combination of a fall in consumption, capital investment, and innovation. The consumption-smoothing motive causes deleveraging to disproportionately affect capital investment and innovation prolonging the negative effect of the shock on output. To be more specific, the binding collateral constraint reduces innovation spending through two channels, as per equation (1.15). First, it amplifies the cyclical downturn and hence reduces incentives to invest in innovation due to lower returns: the market-size effect. Second, according to the equation (1.10), it temporarily reduces the stochastic discount factor of households. Other factors equal, lower discount factor decreases entrepreneurs incentives to innovate because of the higher discounting of future profits: the discounting effect. Moreover, atomistic agents do not internalize positive externalities of private R&D spending on aggregate growth. The endogenous decline in TFP in the high-debt
case is more than five times greater than in the low-debt case. For comparison, I also plot the response of a counterfactual economy without endogenous in the low-debt regime. As one can see, the presence of endogenous growth does not significantly affect the dynamics of the economy in the low-debt regime keeping it close to the standard RBC benchmark.

An average financial crisis

How does an economy end up in the high-debt state where contractionary shocks have especially disruptive effects? To illustrate this, as standard in the literature, I run a 100k periods stochastic simulation, identify periods when the borrowing constraint binds, and then consider the median dynamics of variables within a ten-period window centered around the first period when the constraint binds. The economy is subject to persistent log-normally distributed productivity, world interest rate, and capital quality shocks:

\[
\log(Z_t) = (1 - \rho_Z) \log(Z) + \rho_Z \log(Z_{t-1}) + \varepsilon_t^Z
\]
\[
\log(R_t) = (1 - \rho_R) \log(R) + \rho_R \log(R_{t-1}) + \varepsilon_t^R
\]
\[
\log(\psi_t) = \rho_\psi \log(\psi_{t-1}) + \varepsilon_t^\psi
\]

I borrow the parameters of productivity and interest rate processes from Mendoza (2010) and set \( \rho_Z = 0.537, \rho_R = 0.572, \sigma_Z = 0.0134, \sigma_R = 0.0196. \) The capital-quality shock parameters are the following: \( \rho_\psi = 0.5 \) and \( \varepsilon_\psi = 0.01. \)

As figure 1.8 suggests, periods leading to a crisis are – on average – characterized by low world interest rates and low exogenous productivity, both of which cause the economy to run current account deficits and gradually accumulate foreign debt. Things go south when the world interest rate sharply increases and the economy is hit by a negative capital quality shock, which decreases the effective value of the county’s capital stock that determines its borrowing capacity. The rest of the dynamics is consistent with the description in the previous section. The financial crisis is characterized by a sharp decrease in the price of capital, current account reversal, and an overall deep decline in economic activity.
1.5 Conclusion

This paper contributes to the emerging literature on the interconnectedness between business cycle fluctuations and long-run growth. I argue that rare and deep contractions exacerbated by financial frictions result in a sizable persistent output loss. In my analysis, I point to the state-dependence of this effect: the degree of endogenous productivity losses crucially depends on the pre-existing level of debt. What are the main lessons from this simple exercise? First and foremost, the real costs of financial crises may be significantly larger than commonly perceived if at least a fraction of the observed post-crisis productivity dynamics is endogenous to the crisis event. The potential endogeneity of these productivity losses rises stakes for policy, which can not only smooth out transitory shocks but also indirectly affect the supply side of the economy in the long run.
Figure 1.1: Medium-term cycles of real per-capita GDP, developed and emerging countries

[Note: the data is decomposed using the optimal one-sided bandpass filter of Fitzgerald and Christiano (2003). As in Comin and Gertler (2006), I define the medium-term cycle as fluctuations with frequencies from 0 to 50 years, which consists of business cycles (from 0 to 8 years) and the medium-term component (from 8 to 50 years).]
Table 1.1: Standard deviation and skewness of the medium-term component of fluctuations

[cited on page and 1]

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<th>Advanced economies</th>
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<tbody>
<tr>
<td>$\sigma_Y$</td>
<td>$\sigma_{TFP}$</td>
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<tr>
<td>AUS</td>
<td>3.29</td>
</tr>
<tr>
<td>AUT</td>
<td>2.56</td>
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<td>BEL</td>
<td>3.63</td>
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<td>CAN</td>
<td>3.60</td>
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<td>CHE</td>
<td>2.92</td>
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<td>DNK</td>
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<td>ESP</td>
<td>6.81</td>
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<td>FIN</td>
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<td>HKG</td>
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<td>IRL</td>
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<td>ISR</td>
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<td>NLD</td>
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<td>NOR</td>
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<td>NZL</td>
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<td>PRT</td>
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<tr>
<td>SWE</td>
<td>4.06</td>
</tr>
<tr>
<td>mean</td>
<td>4.58</td>
</tr>
<tr>
<td>median</td>
<td>4.05</td>
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Note: the medium-term component is defined as fluctuations with frequencies from 8 to 50 years, using the optimal one-sided bandpass filter of Fitzgerald and Christiano (2003). Country classification is according to MSCI, G7 economies are excluded. $Y$ is real per-capita GDP, $TFP$ is utilization-adjusted TFP from Brizhatyuk (2020a).
Figure 1.2: Recession paths conditional on the preexisting debt-to-GDP gap

[cited on page 5]

Note: Cross-country panel local projections. Responses of level variables contingent on the debt-to-GDP value at the onset of a recession. Shaded areas correspond to 90% confidence bounds. Standard errors are clustered at the country level.
Figure 1.3: Recession paths conditional on the preexisting debt-to-GDP gap, cont’d

Note: Cross-country panel local projections. Responses of level variables contingent on the debt-to-GDP value at the onset of a recession. Shaded areas correspond to 90% confidence bounds. Standard errors are clustered at the country level.
**Figure 1.4:** Responses to the financial distress shock

[cited on page 6]

*Note:* Cross-country panel local projections. Responses of level variables to the financial distress shock of Romer and Romer (2017). Shaded areas correspond to 90% confidence bounds. Standard errors are clustered at the country level.
Figure 1.5: US VAR, responses to the financial conditions shock

VAR(2), IRFs of (1) Adjusted NFCI; (2) Total employment; (3) Real GDP; (4) Utilization-adjusted TFP; (5) Real R&D expenditure; (6) Total credit to private non-financial sector, % of GDP. Shaded areas correspond to 90% confidence bound. Responses to variables other than NFCI are expressed in log deviations.
Figure 1.6: Baseline model flow chart
[cited on page 8]
**Table 1.2: Model summary**

[cited on page 15]

| 1. Resource constraint | \( y_t = c_t + s_t + i_t + \frac{x_t}{A_t} \) |
| 2. Capital supply | \( q_t = \mathbb{E}_t \left[ \beta_{t+1} (1 - \delta_K) q_{t+1} + R^K_{t+1} \psi_{t+1} + g_{t+1} q_t \right] \) |
| 3. Capital demand | \( R^K_t = \alpha (1 - \xi) \frac{w}{\psi k_t} \) |
| 4. Capital accumulation | \( k_{t+1} g_{t+1} = (1 - \delta_K) \psi_t k_t + (1 - AC_{I,t}) i_t \) |
| 5. Tobin’s q | \( q_t = 1 + g_t (AC_{I,t} + AC'_{I,t} i_t) - \mathbb{E}_t \left[ \beta_{t+1} g_{t+1} AC'_{I,t+1} i_{t+1} \right] \) |
| 6. Labor supply | \( w_t = \nu_t L_t \) |
| 7. Disutility of labor | \( \nu_t = (\nu_{t-1}/g_t)^{\rho T} \) |
| 8. Labor demand | \( w_t [1 + \zeta \mu_t] = (1 - \alpha)(1 - \xi) \frac{w}{E_t} \) |
| 9. Consumption Euler | \( \mathbb{E}_t [\beta_{t+1} R_{t+1}] = 1 - \mu_t \) |
| 10. Borrowing constraint | \( [\rho q_t k_{t+1} - b_{t+1} - b_{F,t+1}] \mu_t = 0, \ \mu_t \geq 0 \) |
| 11. Equity demand | \( v_t = d_t + \mathbb{E}_t [\beta_{t+1} (1 - \delta_N) v_{t+1}] \) |
| 12. Equity supply (free entry) | \( v_t = \phi^{-1} s_t^{1-\rho} \) |
| 13. Growth rate | \( g_t = \phi s_t^\rho + (1 - \delta_N) \) |
| 14. Final output | \( y_t = \left( \frac{A_t}{\nu} \right)^{1-\xi} Z_t^{\frac{1}{1-\xi}} (\psi_t k_t)^\alpha (L_t)^{1-\alpha} \) |
| 15. Intermediate output | \( x_t = \left( \frac{A_t}{\nu} \right)^{1-\xi} Z_t^{\frac{1}{1-\xi}} (\psi_t k_t)^\alpha L_t^{1-\alpha} \) |
| 16. Intermediate firm profit | \( d_t = (\nu - 1) \frac{s_t^\rho}{A} \)

*Note:* where appropriate, lower-case letters denote stationary counterparts of original variables, i.e. \( c_t = \frac{C_t}{N_t} \).
Table 1.3: Structural parameters
[cited on page 16]

<table>
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<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<td>Inverse elasticity of intertemporal substitution</td>
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<td>Conventional</td>
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<td>$R$</td>
<td>Steady-state world interest rate</td>
<td>1.04</td>
<td>Conventional</td>
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<td>$\beta$</td>
<td>Discount factor</td>
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<td>$g^\gamma / R$</td>
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<td>$\epsilon$</td>
<td>Inverse elasticity of labor supply</td>
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<td>King and Rebelo (1999)</td>
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<tr>
<td>$\rho_\chi$</td>
<td>Disutility of labor inertia</td>
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<td></td>
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<td>$\alpha$</td>
<td>Capital share</td>
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<tr>
<td>$\xi$</td>
<td>Intermediate good share</td>
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<td>Comin and Gertler (2006)</td>
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<td>$\nu$</td>
<td>Intermediate good elasticity of substitution</td>
<td>1.6</td>
<td>BGP condition: $\frac{\xi(\nu-1)}{1-\xi} = 1 - \alpha$</td>
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<td>$\delta_N$</td>
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<td>$\psi_B$</td>
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<td>Schmitt-Grohé and Uribe (2003)</td>
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<td>Steady-state world debt-to-gdp ratio</td>
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<td>$\rho$</td>
<td>Maximum loan-to-value ratio</td>
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<td>Mendoza (2010)</td>
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<td>Working capital requirement</td>
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<td>Mendoza (2010)</td>
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<td>$\rho$</td>
<td>R&amp;D output elasticity</td>
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<td>Comin and Gertler (2006)</td>
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<td>R&amp;D productivity</td>
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<td>Annual growth rate = 1%</td>
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<td>$Z$</td>
<td>Steady-state final sector productivity</td>
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<td>Steady state GDP = 1</td>
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<td>$A$</td>
<td>Intermediate sector productivity</td>
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<td>Normalization</td>
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Figure 1.7: 1% negative productivity shock

[cited on page 17]
**Figure 1.8:** An average financial crisis

[Note: ten-period windows centered around the periods when the collateral constraint binds (“financial crisis”). Median paths over 100k period simulation. Shaded areas correspond to 64% confidence intervals. Bottom right panel presents a median path of exogenous variables driving the dynamics.]
Chapter 2

HOUSING MARKET CYCLES, PRODUCTIVITY GROWTH, AND HOUSEHOLD DEBT

2.1 Introduction

Recoveries from financial crises tend to be slow and incomplete (e.g. Cerra and Saxena 2008, Reinhart and Rogoff 2009). This became especially evident in the years after the global financial crisis of 2007-2008 as growth in many affected economies remained slow considerably longer than forecasters anticipated (figure 2.1). Popular explanations of the lackluster rebound from the crisis involve shortfalls in aggregate demand and prolonged private deleveraging (e.g. Blanchard et al. 2015, Lo and Rogoff 2015, Anzoategui et al. 2019b). In particular, the Global Financial Crisis marked the end of the global household debt cycle that accompanied a rapid increase in housing prices worldwide (figure 2.2). The collapse of housing markets during the crisis had significant negative effects on the economy, including large drops in consumption, employment, and young-firm activity (Mian et al. 2013, Mian and Sufi 2014, Davis and Haltiwanger 2019).

This paper contributes to the debate on the causes of persistent effects of financial crises by focusing on housing market boom-and-bust cycles. The sheer size of the housing and mortgage markets makes their interplay with the macroeconomy important. The majority of citizens in advanced countries own housing, it accounts for a lion’s share of household wealth, and mortgage debt is the largest part of household debt.\footnote{Countries with the largest increases in household debt and housing prices in the years leading to the crisis tended to experience the biggest declines in consumption and growth once the cycle reversed (IMF 2012, Glick and Lansing 2010).}

\footnote{For example, the home ownership rate in the US in Q3 2019 was 64.7%. In 2011 the median share of mortgages in household credit was about 70 percent across mostly developed countries, according to Cerutti et al. (2015). Among the G7 economies in 2010, the share of housing wealth in the total national wealth was in the range of 20-50%, according to Piketty and Zucman (2014).}
I first provide new cross-country evidence on the dynamics of recessions and subsequent recoveries associated with housing market crashes. Using the local projections method developed by Jordà (2005), I estimate responses of key aggregate variables to turning points in housing market cycles conditioning on the size of the house price decline, a broad set of controls, country and year fixed effects, as well as country-specific trends. Three conclusions emerge. First, an average housing market crash is associated with household deleveraging. Second, the associated decline in economic activity is sizable and very persistent. For example, 10 years after the onset of a crash, the elasticity of output and consumption levels to the house price decline remains at 0.07 and 0.16, respectively. To put these estimates in perspective, the average house price decline during the 3-year period after the peak across studied events is 21%. The above elasticities then imply a decline in the level output and consumption of 1.47% and 3.36%, respectively, a decade after the beginning of the crash. A growth accounting decomposition suggests that this persistent negative effect on the level of economic activity is largely driven by a medium-run decline in capital stock and utilization-adjusted total factor productivity (TFP). For an average housing crash in the panel, my results suggest the associated decline in the level of labor productivity of 1.26% a decade after the beginning of a crash. Finally, rapid accumulation of household debt during the boom phase of a housing market cycle predicts a deeper recession and a shallower recovery after the crash. A variety of checks confirms that the conclusions are robust across estimation strategies and samples. The results are not driven by the global financial crisis or other systemic financial crises that sometimes coincide with housing market crashes. Similar responses of household debt and TFP growth to house price boom-and-bust cycles also emerge in a panel VAR identified with short-run restrictions.

The cross-country results are reinforced by the evidence of the persistent effect of the 2007 – 2010 housing market crash in the US on productivity growth. The rich US data allows to leverage the regional variation in the exposure to the housing market boom-and-bust cycle to estimate its effect more accurately. Consistently with the cross-country results, I document a significant positive correlation between the house price decline and the productivity growth since the pre-crisis peak across US metropolitan statistical areas. To alleviate endogeneity concerns, I adopt two instrumental variables proposed in the literature: the topology-based housing supply elasticity index of Saiz (2010) and the sensitivity instrument of Guren et al. (2018) that captures systematic differences in city-level exposure to regional house price cycles. Depending on the specification, the elasticity of cumulative 2007 – 2017
labor productivity growth to the decline in house prices during 3 years from the peak in Q1 2007 falls in the range of 0.12 to 0.32. Given these elasticities, a back-of-the-envelope calculation using the aggregate US data suggests that the 2007 – 2010 housing market crash lowered the cumulative labor productivity growth in the decade after 2007 by as much as 4.6%. As such, the detrimental effect of the housing market crash on the post-crisis productivity growth can account for more than 40% of the gap between the actual level of real per-capita GDP in 2017 and its pre-crisis trend.

In the second part of the paper, I interpret the empirical observations through the lens of a quantitative dynamic general equilibrium model, which I use to explore the channels through which the crisis propagated and to perform counterfactuals. The model combines elements from the literature on deleveraging with borrower-saver heterogeneity (Eggertsson and Krugman 2012), the literature on the role of collateral constraints tied to housing wealth (Iacoviello 2005, Guerrieri and Iacoviello 2017), and the literature on endogenous growth (Romer 1990, Comin and Gertler 2006). In particular, the model features representative borrower and saver households, occasionally binding collateral constraints tied to housing equity, endogenous growth driven by the introduction of new products, and nominal rigidity. Monetary policy conducted through interest rate setting subject to a zero-lower-bound constraint.

My modeling strategy is guided by the empirical evidence. First, the persistent decline in utilization-adjusted TFP in the aftermath of housing market crashes motivates the inclusion of the endogenous growth mechanism. Second, the evidence of household deleveraging and a significant interaction between preexisting household debt and the house price decline motivates my focus on household mortgage debt. As in standard quantitative macro models, inclusion of physical capital as a factor of production, subject to endogenous utilization, improves the ability of the framework to replicate the data. Finally, I abstract from the role of housing in production and the construction sector of the economy, assuming a fixed housing supply. As documented by Davis and Heathcote (2007), most of the fluctuations in house prices are driven by fluctuations in prices of residential land, of which there is ultimately a limited supply, and not by the price of structures.

Within this framework, I explore the aggregate effects of negative housing wealth shocks. Motivated by evidence from the existing literature, I resort to a housing preference (demand) shock as a source of exogenous variation in the price of housing. Liu et al. (2013) identified
this shock as the one that drives most of the observed fluctuations in land prices and as important for generating empirically relevant comovement between land prices and investment. Furthermore, the estimates of Guerrieri and Iacoviello (2017) suggest that about 70% of the US consumption decline during the Great Recession can be attributed to housing demand shocks. More recently, Kaplan et al. (2019) argue that demand-side factors, but not changes in credit conditions, were the dominant force behind the U.S. housing market boom-and-bust cycle around the Great Recession.

When the preexisting level of debt is sufficiently high, negative demand shocks trigger the collateral constraint and cause deleveraging: credit-constrained households must reduce their spending to satisfy a lower borrowing limit. Under nominal rigidity in the short run, deleveraging leads to a sharp demand-driven contraction. However, over time, deleveraging affects the economy's potential output as it slows the pace of capital and firm-creation investment. The shock thus acts like an aggregate demand shock in the short run, and like an aggregate supply shock in the medium run.

The calibrated model is successful in accounting for the empirical comovement between aggregate variables associated with housing market crashes. I calibrate parameters of the mortgage market and the balanced growth path (BGP) productivity growth to an average across economies in the cross-country panel under study. In addition, I directly use the empirical impulse responses to discipline several quantitative parameters of the model through impulse-response matching. The negative feedback loop between deleveraging, borrowers net worth, and growth appears to be strong enough to explain the entirety of the estimated aggregate dynamics associated with housing market crashes. The endogenous growth mechanism embedded in the model is key to its success by generating the empirically-relevant persistence in the responses of capital and TFP.

Specifically, I identify and illustrate four key channels of shock propagation that shape the general equilibrium response to housing demand shocks. Under nominal rigidity, borrowers deleveraging results in a demand-driven recession in the short run as the real interest rate does not adjust enough to cause savers to pick up the slack. I refer to this as the aggregate demand channel. This is especially pronounced when monetary policy is constrained by the zero lower bound, which significantly amplifies the effect of the shock. This short-run effect has the potential to leave deep scars on the level of economic activity through the productivity growth channel. The basic insight is that producer entry and product introduction is a form
of investment, which responds to current and expected market conditions just like investment in physical capital does. Hence, changes in aggregate demand and credit availability affect entry and productivity growth. This is especially true for large recessions that occur in a high-leverage environment. Finally, two additional channels amplify the above effects. The first is a negative feedback loop between deleveraging and borrowers housing wealth: *Fisherian debt deflation channel*. The initial negative shock causes the collateral constraint to bind and trigger deleveraging. The resulting weak demand then exacerbates the damage to borrowers balance sheets and causes further deleveraging. The second is a negative feedback loop between future expected growth and current consumption: the *expected income growth channel*. Downward revisions in growth expectations weigh down current demand, which in turn further suppresses growth.

I conclude my study by employing the calibrated model to perform several policy counterfactuals. First, I ask how sensitive is the aggregate welfare cost of the shock to the stance of monetary policy. Both in the baseline model and in the counterfactual where without endogenous variations in growth, the welfare cost of the shock is most responsive to the strength of policy reaction to cyclical changes in output. Strong inflation targeting, in contrast, does little or nothing to improve welfare. In fact, for some combination of parameter values stronger reaction to inflation is welfare-reducing. The comparison to the counterfactual where endogenous variations in growth are shut down, shows that the endogenous productivity growth mechanism warrants stronger focus of monetary policy on the short-run output stabilization.

Turning to fiscal policy, I explore the effects of a policy that redistributes from savers to borrowers. In a narrow view, this policy can be considered as a debt forgiveness program. More broadly, it can be interpreted as a budget-neutral policy that shifts the tax burden from savers to borrowers. I demonstrate that this policy is effective in alleviating the negative feedback loop between deleveraging, asset prices, and productivity growth.
2.1.1 Contribution to the existing literature

My cross-country analysis adds to the existing literature on the real effects of financial and asset market cycles. The empirical strategy I follow is closest to Jordà et al. (2015), who studied the short-run output dynamics following housing and equity-market crashes in a panel of developed countries. However, my analysis importantly differs in its scope and focus. I significantly expand the number of events in the study and explore the dynamics of a broader set of macroeconomic variables. More substantively, I focus on the persistent dynamics of aggregate variables and identify their potential drivers. Using the harmonized cross-country data on household and corporate debt, I emphasize the role of household indebtedness.

Furthermore, my empirical analysis is also related to the existing literature that demonstrated that the deterioration of household balance sheets during the 2006-09 housing market collapse in the US played a significant role in the sharp decline in employment and consumption (Mian et al. 2013, Mian and Sufi 2014). The evidence from US metropolitan statistical areas that I present suggests that the implications of the housing market’s crash for the US economy extend far beyond the contemporaneous effect; rather, the legacy of the crash lingers to this day.

Finally, this paper also relates to the broader literature that explores the nexus between the housing market dynamics and business dynamism. Recent contributions provide evidence of a causal relationship between homeowner housing wealth and young-firm activity (Davis and Haltiwanger 2019), as well as on the probability of homeowners becoming entrepreneurs (Corradin and Popov 2015, Schmalz et al. 2017). According to such work, a significant fall in housing prices causes a slowdown in startup activity. Periods when entry is especially weak consequently result in a missing generation of firms, which may have a very persistent effect on output and measured productivity (Gourio et al. 2016).

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4 Andersen et al. (2014) and Bunn et al. (2015) identify similar patterns in the data from Denmark and U.K. respectively. Overall, changes in household debt correlate stronger with growth than changes in corporate debt, both in developed and developing countries, as documented by Bahadir and Gumus 2016 and Mian et al. 2017.

5 To put this statement in context, the U.S. establishment entry rate plummeted by 26% between 2006 and 2009, according to the CENSUS Business Dynamics Statistics.
This paper theoretically relates to several bodies of existing literature. Similar to the literature on deleveraging crises pioneered by Eggertsson and Krugman (2012), recession in my model is a result of a reduction in the borrowing capacity of debtor households. Differently from this literature, however, the borrowing limit is not exogenous, but it is tied to the borrowers’ housing wealth determined in general equilibrium. This feature connects the paper to the body of literature on the macroeconomic effects of home equity-based borrowing started by Iacoviello (2005). The implications of treating the borrowing limit as endogenously determined are far from trivial. This approach allows the model to account for the amplification effect through the two-way interactions between deleveraging, borrower housing wealth, and economic activity. This effect is state-dependent, shaped by the policy response, and very important quantitatively.

Empirically, episodes of household deleveraging during housing market crashes are associated with very persistent declines in economic activity. I interpret this observation building on insights from the literature on the interconnectedness between business cycles growth. Much of the recent theoretical literature on this topic builds on the seminal contribution of Comin and Gertler (2006). In particular, particularly large contractions and slow recoveries in many countries after the Global Financial Crisis have motivated research on the role endogenous growth and financial shocks in generating such persistence. So far, the existing

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6 See also Benigno and Romei (2014), Benigno et al. (2020), Guerrieri and Lorenzoni (2017), and Korinek and Simsek (2016).

7 Other important contributions include Ferrero (2015), Jensen et al. (2019), Justiniano et al. (2015), Liu et al. (2013), Liu et al. (2016), Midrigan and Philippon (2018), Iacoviello and Minetti (2006), and Iacoviello and Neri (2010).


9 See later contributions by Bianchi et al. (2019b), Comin et al. (2014a), Correa-López and de Blas (2018), Croce et al. (2012), Garga and Singh (2020), Gornemann (2015), Holden (2016), Moran and Queralto (2018), Queralto (2019). Recently, endogenous growth mechanisms have also been used in the finance literature. By generating a small but persistent endogenous productivity component, this feature has been shown to improve the asset-pricing implications of dynamic general equilibrium models, see Bocola and Gornemann 2013, Gavazzone and Santacreu 2020, Guerrón-Quintana et al. 2019, and Kung 2015 among others.

10 Fernald et al. (2017) and Gordon (2015) point out that the productivity growth slowdown in the U.S. economy that followed the crisis has started before the Great Recession and argue that the dynamics in the recent years is the continuation of a secular trend. Similar observations have been made about European countries. In this context, the question is not why there seems to be a secular decline in output growth, but whether the global financial crisis has accelerated an existing trend.
literature to date has either focused on financial frictions that directly affect financing of innovations (Queralto 2019, Guerron-Quintana and Jinnai 2019, Ikeda and Kurozumi 2018) or remained agnostic about the source of the financial shock all together, treating it as exogenous (Anzoategui et al. 2019b). I contribute to this debate by investigating the persistent effects of household deleveraging generated by negative housing demand shocks.

I approach the issue of persistent effects of business-cycle fluctuations with a particular focus on the house equity-based borrowing. This is complementary to the alternative mechanisms proposed in the literature. Persistent effects of temporary shocks may also stem from the labor market dynamics (Blanchard and Summers 1987, Acharya et al. 2018); purely from self-fulfilling expectations of low growth (Benigno and Fornaro 2018). A number of papers emphasize the role of firm dynamics. Ates and Saffie (2020) document that firms born during the credit shortage are fewer, but more productive. Schmitz (2017) focuses on how tight financial conditions cause small and young innovating firms reduce their R&D resulting in R&D misallocation.\(^{11}\)

Finally, this work also contributes to the literature on the non-linear effects of occasionally binding constraints (OBCs). The closest reference is Guerrieri and Iacoviello (2017), who show that a model with an OBCs tied to housing wealth makes it possible to account for the asymmetry in the link between housing prices and consumption growth during the latest housing market cycle in the US\(^ {12}\)

**Outline.** The rest of the paper is organized as follows. Section 2.2 discusses the empirical evidence on short-run and persistent effects of housing crashes. Section 2.3 presents the model. Section 2.4 discusses calibration and the ability of the model to account for the empirical evidence. 2.5 explores the key mechanisms driving the dynamics. Section 2.6 draws implications for monetary and fiscal policy. Section 2.7 concludes.

\(^{11}\) See also Garcia-Macia (2015), Knowles (2018), and Kozlowski et al. (2019).

\(^{12}\) OBCs are also a central ingredient of small open economy models designed to study sudden stops and macroprudential policy, see Akinci and Chahrour (2018), Benigno et al. (2016), Bianchi and Mendoza (2018b), Korinek (2018), and Mendoza (2010). The models in these papers account for financial crises — periods when credit constraints of borrowers bind — as rare events nested within the regular business cycle.
2.2 Evidence on the effects of housing market crashes

In this section, I first provide a comprehensive cross-country account of dynamics of recessions and subsequent associated with housing market crashes. I then show that the conclusions of the cross-country analysis are consistent with the evidence from US Metropolitan Statistical Areas in the aftermath of the Great Recession.

2.2.1 Housing market cycles worldwide

On average, what is the dynamics of recessions and subsequent recoveries associated with a housing market crash? In this section, I explore a cross-country panel of real house price indexes, identify turning points of housing market cycles and estimate the associated dynamics of a number of macroeconomic indicators, controlling for the magnitude of the price decline and a rich set of macroeconomic indicators.

My empirical approach is similar to the event analysis of Jordà et al. (2015). I find studying large and sudden house price declines that were preceded by explosive growth appealing for several reasons. First, as emphasized by Guerrieri and Iacoviello (2017), the relation between house prices and economic activity can be highly asymmetrical and state-dependent due to the presence of occasionally binding collateral constraints tied to housing wealth. This evidence motivates my focus on periods when household financial frictions, which are at the center of my interest, matter.\textsuperscript{13} In addition, during the studied periods the house price dynamics is more likely to be an important independent factor affecting the macroeconomy and is less likely to be subject to reverse causality from macroeconomic fundamentals to housing prices. Nevertheless, one should be cautious to place a strong causal interpretation on the results of this section. Rather, they illustrate the equilibrium comovement between variables of interest during periods of house price decline. In the second part of the paper, I develop a dynamics general equilibrium model that allows to sheds some light on the mechanisms and gauge how much of the empirical comovements between variables can be accounted for by the house price decline alone.

\textsuperscript{13} On average, the studied events are preceded by accelerated credit growth, both household and firm. See summary statistics in the appendix for details.
Defining housing market boom-and-bust cycles

I start with constructing a database of turning points of housing market cycles, which I then use to study the associated dynamics of macroeconomic variables. For that purpose, I collect a comprehensive cross-country data set of aggregate real housing price indexes. It combines series from BIS, the Dallas FED, OECD, and Jorda et al. (2016) Macrohistory databases. The resulting unbalanced panel consists of annual observations covering 50 countries from 1950 to 2017.

For each country, I determine periods of sudden and large declines in house prices. My strategy consists of two steps. First, the aggregate housing price index needs to be sufficiently elevated relative to the long-run trend defined using a one-sided HP filter. Second, the price index needs to sharply fall from the peak. As a rule of thumb, I use the threshold of at least 10% decline in the first three years after the peak, although the results are robust to choosing a different threshold. To put this threshold in perspective, the US aggregate real housing price index fell by around 14% during the first three years from the peak in 2006. This second step intends to filter out “soft landing” situations when the rapid house price growth was not followed by an equally rapid correction. Figure B.1 presents an example of this definition applied to the UK, the US, Singapore, and South Africa. The UK, for instance, has experienced 3 major housing market cycles in postwar history with turning points in 1973, 1989 and 2007.

The above procedure identifies 63 events in total, 39 of which happened before the Global Financial Crisis; 43 events have taken place in Europe, 6 in Americas, 12 in Asia and Oceania, and 2 in Africa. On average, the price decline continues for 5 years and reaches the magnitude of -31% with the two-thirds of the price decline occurring in the first three years. Table 2.1 lists all identified events along with some descriptive statistics. To estimate the responses of macro variables to a housing market crash, I construct a variable \( \Delta_3 P_{crash}^i \) that marks the three-year price decline from the peak of the identified housing market boom-and-bust cycles in country \( i \). I use this variable as an exogenous perturbation for the impulse response estimations in the next section.

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14 I use the smoothing parameter of 400000/4 for annual observations for this procedure to be symmetric to the definition of credit cycles by the BIS.
Impulse responses to a housing market crash by local projections

I ask: (1) how long-lasting is the effect of recessions associated with housing market crashes on the level of output, consumption, and investment? (2) What factors drive the observed dynamics of output? (3) Does household and/or corporate pre-existing indebtedness amplify this dynamics? To address these questions, I complement the cross-country housing price data with information on the dynamics of key macro-economic variables, such as output, consumption, investment, household/corporate debt, and measures of productivity. Table B.2 includes the full list of variables with summary statistics and data sources.

The dynamic responses of variables are estimated by local projections (Jordà 2005). This method involves running a series of regressions of a variable of interest on the lagged shock, which contrasts approaches that rely on estimating an autoregressive model. This model-free approach is attractive because of its flexibility. However, as noted by Ramey (2012), this flexibility comes at a price of efficiency. Responses at each horizon are estimated independently from the rest, which may result in wider confidence bounds and erratic oscillations of impulse responses.\footnote{Barnichon and Brownlees (2019) suggest improving the precision of local-projection impulse responses using a method based on penalized B-splines. The use of this method would not change the main conclusions of the analysis and is beyond the scope of the paper.} This feature should be kept in mind when interpreting the results.

The baseline specification is the following dynamic cross-country panel:

$$\Delta_h y_{i,t+h} = \alpha_i^h + \alpha_t^h + \beta^h \Delta_3 p_{i,t}^{crash} + X_{i,t} \Gamma + \varepsilon_{i,t}^h$$ \hspace{1cm} (2.1)

The dependent variable is the country $i$’s $h$-period log difference of the response variable: $\Delta_h y_{i,t+h} = \log(Y_{i,t+h}) - \log(Y_{i,t})$; and the perturbation variable is the three-year house price decline $\Delta_3 p_{i,t}^{crash}$ during the identified events of interest. Estimating this relation at different horizons $h$ produces a set of coefficients $\{\beta^h\}_{h=1:H}$ that can be interpreted as an $H$-period dynamics of variable $Y$ associated with a 1% house price decline, conditional on the set of controls described below.

The baseline specification includes a rich set of controls and fixed effects. In particular, I include country fixed effects ($\alpha_i^h$) to control for time-invariant country-level characteristics; year fixed effects ($\alpha_t^h$) to control for common shocks such as the Global Financial Crisis. A variety of other country-specific factors is accounted for with a set of macroeconomic controls.
$X_{i,t}$. It includes values at the peak of a housing market cycle and one lag of (1) the response-variable growth rate; (2) real per-capita investment growth, (3) GDP-deflator inflation rate, (4) net exports to GDP ratio, (5) real house price growth rate. In addition, to control for the possible effect of an exchange-rate regime, I include fixed exchange rate indicators from Ilzetzki et al. (2019). Many but not all housing market crashes coincide with broad financial crises. I include Laeven and Valencia (2013) indicators of systemic banking and currency crisis to account for that. Finally, I use investment to GDP ratio to control for the overall investment intensity of an economy.

On average, how different the path of the economy is, conditional on the set of controls described above, if there is a housing marker crash? Figure 2.3 presents the resulting impulse responses. Several key observations emerge. First, the studied events are associated with a rapid house price decline lasts for the first 3 year and is associated with a broad decline in the economic activity. At the though in year 3 the 1.3% decline in house price is associated with a decline in consumption larger than output (-0.2% and -0.28% respectively) and a -0.6% decline in investment. Second, the decline in the economic activity is associated with households deleveraging: the household debt-to-GDP gap falls by about 0.2 per 1.3% decline in house prices. The non-financial corporate debt burden falls as well but the change is less significant and the magnitude is twice as low.

The negative effect on the levels of output and consumption is very persistent and lasts longer than conventional business cycles. My results suggest that the sluggish dynamics of capital accumulation and productivity growth is likely to be responsible. Figure 3 also shows impulse responses of capital, labor, and measures of productivity. Responses of the capital stock, total factor productivity, and labor productivity are very persistent. Employment-to-population ratio, on the other hand, exhibits a stronger mean reversion dynamics.

Note that I use utilization-adjusted total factor productivity. The persistent response of productivity then is unlikely to be driven by time-varying factor utilization. Appendix B.3.1 provides details on the utilization adjustment procedure, which follows Imbs (1999). This method is an alternative to the approach of Basu et al. (2006) that utilizes industry-level data to also control for nonconstant returns to scale and aggregation effects. Unfortunately, cross-country data limitations do not allow to account for these factors.

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16 Comin and Gertler (2006) refer to the frequencies from 2 to 32 quarters as the business cycle component.

17 See other applications of this method in Taylor et al. (2020) and Levchenko and Pandalai-Nayar (2018).
The role of credit imbalances

Rapid growth of household – but not corporate – debt during the boom is associated with a deeper recession during the crash. This result is complimentary to the one of Jordà et al. (2015) who show that recession associated with housing market crashes tend to be deeper when preceded by below-average bank lending growth. I revisit this question using the novel IMF Global Debt database that contains separate data on household and non-financial corporate debt.\textsuperscript{18} I use this data to calculate household and non-financial corporate debt-to-GDP gaps, denoted $\hat{B}_{HH}$ and $\hat{B}_{F}$ respectively. Similarly to output gaps, this variable is defined as a deviation of the debt-to-GDP ratio from its long-run trend, defined using the one-sided HP filter.\textsuperscript{19} In my analysis, I use debt-to-GDP gaps as a measure credit intensity of expansions preceding housing market crashes relative to the secular trend of financialization. Figure B.2 shows an example of U.S. household and non-financial corporate debt-to-GDP gaps along with the cyclical dynamics of the aggregate house price. As this visual suggests, the cyclical dynamics of the US housing market correlates more closely with the dynamics of household, rather than corporate, indebtedness.

I extend the baseline specification by adding interactions between the price decline and the debt-to-GDP gaps to formally test whether preexisting credit imbalances affect the transmission of house price declines:\textsuperscript{20}

\begin{equation}
\Delta_{h}y_{i,t+h} = \alpha_{i}^{h} + \alpha_{t}^{h} + (\beta_{HH}\hat{B}_{i,t}^{HH} + \beta_{F}\hat{B}_{i,t}^{F})\Delta_{3p_{crash}} + \hat{B}_{i,t}^{HH} + \hat{B}_{i,t}^{F} + X_{i,t}\Gamma + \varepsilon_{it}^{h} \tag{2.2}
\end{equation}

To assess the relative role of household and corporate debt, I then calculate marginal effects of the house price decline at different values of household and corporate debt-to-GDP gaps. For illustrative purposes I consider the two scenarios when household credit gap is 5% and corporate credit gap is 0% and the other way around. Figure 2.4 shows the corresponding conditional impulse responses for output, consumption, and employment. The main message

\textsuperscript{18} See Mbaye et al. 2018 for the database description. Compared to the existing counterparts, the database is more than double in the cross-sectional dimension and employs a broader and more consistent definition of debt across countries and time.

\textsuperscript{19} Following the BIS definition the smoothing parameter is set to $4 \cdot 10^{5}/4^{4}$. This variable has recently became popular among policymakers as a reduced-form way to capture credit imbalances. For instance, it is used in the Basel III regulatory framework as a guide for setting countercyclical capital buffers.

\textsuperscript{20} Debt data is available for only 50 events in the sample.
from this result is the following. The same house price decline is associated with a deeper recession and a slower recovery when household — but not corporate — debt-to-GDP gap is high. In other words, a rapid preceding expansion of household debt tends to exacerbate the effect of housing market crashes.\footnote{This evidence is consistent with the results of Mian et al. (2017) who uncover the (unconditional) detrimental effect of increases in the household debt on growth and employment in the medium run.}

Robustness and additional results

The results of the cross-country analysis are robust to alternative empirical specifications. \textbf{Figure B.5} presents results of several checks. I estimate the following alternatives in addition to the baseline specification: (1) excluding post-2007 observations; (2) including 4 lags of controls instead of 2; (3) excluding macro controls; (4) excluding macro controls and year fixed effects. I specifically want to point out that the baseline results are not simply driven by the events associated with the Global Financial Crisis: the estimated dynamics of variables is very similar for the sample that excludes years from 2007 onward. These checks also make clear that the large set of macroeconomic controls and year fixed effects mop up a sizable amount of variables response. The indicator of systemic banking and currency crises plays a particularly prominent role in this respect. The results are then not likely to be driven by financial crises that sometimes coincide with housing market crashes.

Appendix B also gathers some additional impulse responses to a housing market crash. \textbf{Figure B.2} illustrates that housing market crashes are associated with a short-run improvement in the trade balance, driven primarily by a decline in imports; and a mild inflationary effect. The effect on the real exchange rate and the GDP-deflator inflation rate is ambiguous.

Finally, \textbf{Figure B.4} provides a complementary panel VAR evidence of comovement between house prices, TFP growth, and household debt. House price shocks are identified with short-run restrictions with variables ordered as follows: utilization-adjusted TFP, household debt-to-GDP gap, house price index. Although I find the local projection approach more appealing for my purposes, results from the panel VAR based on the same data are broadly with the discussed earlier in the section. A temporary acceleration of house price growth leads to household debt accumulation. Consequently, the reversion of the housing market cycle is associated with household deleveraging and a persistent decline in the TFP level.
2.2.2 Evidence from US states and Metropolitan Statistical Areas

The cross-country evidence of the decline in productivity growth associated with housing market crashes is consistent with the labor productivity dynamics across U.S. states and Metropolitan Statistical Areas (MSAs). Figure 2.5 shows a negative correlation between the magnitude of the housing market crash (peak to trough) and the cumulative growth of real GDP per worker from 2007 to 2017 across states and MSAs. Naturally, one should be cautious to interpret this result causally. Unobserved factors may have been driving both housing prices and productivity growth.

I adopt two instrumental variables proposed in the literature. The first is the Saiz (2010) housing supply elasticity index, which captures geographical and regulatory constraints to construction. This variable has been widely used in the literature and is argued to generate variation in house price growth uncorrelated with factors that might otherwise be driving the housing market dynamics. As an additional check I use the regional sensitivity instrument developed by Guren et al. (2018). This instrument exploits the fact that house prices in some cities are systematically more sensitive to regional housing cycles than in others. By explicitly controlling for local macroeconomic factors, the authors construct an instrument stronger than the more commonly used housing supply elasticity index. Another advantage of this instrument is its availability for a larger number of MSAs. Figure B.6 B plots first-stage relationships.

The baseline specification for the cross-section of MSAs is the following:

\[
\Delta_{2007}^{2017} \log \left( \frac{Y}{L} \right)_i = \alpha + \eta \Delta_{2007}^{2010} \log \hat{P}^H_i + X'_i \Gamma + \varepsilon_i
\]

(2.3)

where \( \Delta \log \left( \frac{Y}{L} \right) \) is the log difference of labor productivity measured as real GDP per worker; \( \Delta \log \hat{P}^H_i \) is the instrumented log difference of the house price index; and \( X'_i \) is the vector of controls that includes 2006 GDP shares of (a) mining, quarrying, and oil and gas

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22 See Mian et al. 2013, Mian and Sufi 2014, Giroud and Mueller 2017, Davis and Haltiwanger 2019, among others. The use of housing supply elasticity instrument, although common in the literature, has been a subject of controversy lately, see, for instance, a discussion in Davidoff (2013).

23 Note the non-linear relationship between the house price decline measures and the Saiz (2010) housing supply elasticity: the relationship is stronger for low-elasticity areas. Following Kaplan et al. (2016), in addition to the linear first-stage specification I consider a quadratic polynomial in the housing supply elasticity index as an instrument.
extraction; (b) construction; (c) manufacturing; (d) retail trade; as well as (e) 2006 per capital personal income.

The parameter of interest is $\eta$: the elasticity of labor productivity growth to the decline in house price during the housing market crash. Figure 2.6 reports $\eta$ across a number of specifications. Depending on the specification, elasticity of the cumulative labor productivity growth (2007 – 2017) to the decline in house price (Q1 2007 – Q1 2010) ranges from 0.12 to 0.32. For illustrative purposes, let me pick elasticity of 0.2 within this range. One way to put this estimate in perspective is to consider aggregate data. In 2017, US real per capita GDP was about 10% below the pre-crisis trend (figure 2.1). How much of this gap can be explained by the negative effect of the housing market crash?

The US aggregate house price fell by 14.4% from Q1 2007 to Q1 2010, according to the all-transaction index from FHFA. The elasticity of 0.2 then implies that the loss of labor productivity in the decade since 2007 driven by the fall in house prices during the 2007-2010 crash is equal to 2.88%. In other words, almost 30% of the gap between the actual US real GDP and the pre-crisis trend can be accounted for by the negative effect of the housing market crash on labor productivity growth. This result is not driven by the location-specific industry structure or the four states most exposed to the boom-and-bust cycle (California, Florida, Nevada, Arizona).

Appendix B gathers some additional results. In addition to the house price decline I also employ the housing net worth shock constructed by Mian and Sufi (2011). This variable measures the percentage decline in household net worth due to the house price decline from 2006 to 2009. Figure B.8 reports the results. A similar conclusion emerges. Elasticity of the cumulative labor productivity growth (2007 – 2017) to the housing net worth shock (2006 – 2009) ranges from 0.04 to 1.01, let me pick a somewhat conservative value of 0.3. A drop of housing value between 2006 and 2009 equals to 8.7% of total household net worth, according to the estimate of Mian et al. (2013). Given the elasticity of 0.3, this housing net worth shock then implies a 2.61% labor productivity loss from 2007 to 2017. Finally, the relationship between the house price dynamics and the labor productivity growth is asymmetric: the boom phase of the latest US housing market cycle was not associated with an increase in productivity growth, as Table B.9 shows.
2.3 The model

In this section, I develop the dynamic general equilibrium model that I will use for the exercises in the remainder of the paper. The core of the model is a Woodford (2003) cashless economy with capital accumulation and monetary policy conducted through interest rate setting. This framework is extended along two dimensions. First, instead of a representative household the model features two agents: borrowers and savers that differ in their discount factors. The accumulation of household debt in the US and other developed countries in the recent decades increasingly has been financed by savings of rich households, as Mian et al. (2020) suggest. Both saver and borrower households supply labor, trade risk-free bonds, and hold housing, which generates a utility flow. Borrowers are subject to an occasionally binding collateral constraint tied to their housing wealth (Kiyotaki and Moore 1997, Iacoviello 2005). Borrowers also have an access to investment opportunities: they accumulate capital and finance product creation. As suggested by Eggertsson and Krugman (2012), borrowers should not necessarily be interpreted as “liquidity-constrained poor”, as is common in the literature. Instead, they can be broadly interpreted as those who have an access to investment opportunities and are in need of external funding, which is constrained by the debt limit. In my model, this assumption makes holders of capital and equity levered. As a result, investment and product creation become additional margins of adjustment to deleveraging along with consumption spending. Second, the model features productivity growth through expanding variety of intermediate products, broadly interpretable as horizontally differentiated innovations (Romer 1990, Comin and Gertler 2006). Figure 2.7 presents a flow chart of the model that summarizes its key participants and their interactions.

2.3.1 Households

There are two types of households: savers and borrowers denoted by a superscript \( H \in \{S, B\} \). A common way in the literature to motivate borrowing and lending is to assume

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24 This variety-based approach differs from the quality ladder growth models of Grossman and Helpman (1991) and Aghion and Howitt (1992), where endogenous growth takes a form of repeated quality improvements over the pre-fixed number of varieties. However, as Grossman and Helpman (1991) note, these two frameworks result in very similar reduced forms.
that savers are more patient than borrowers $\beta_S > \beta_B$.\footnote{An alternative but conceptually similar way to ensure that borrowers do not self-finance in the long run is to assume that they are finitely-lived. See, for example, the formulation of bankers problem in Gertler and Karadi (2011).}

Each household gains utility from consumption and the stock of housing in its possession, and disutility from labor: $U(C_t^H, L_t^H) + G(h_t^H)$. I assume Greenwood et al. (1988) period utility function in consumption and labor: $U(C_t^H, L_t^H) = \left(\left(C_t^H - \frac{\gamma_t(L_t^H)^{1+\epsilon_L}}{1+\epsilon_L}\right)^{1-\sigma} - 1\right)/(1 - \sigma)$, where the aggregate consumption is a CES basket of differentiate retail goods $C_t^H = \left(\int_0^1 C_t^H(j)^{\frac{\sigma-1}{\sigma}} dj\right)$, and the two parameters, $\sigma$ and $\epsilon_L$, are the inverse elasticity of intertemporal substitution and the inverse Frisch elasticity of labor supply respectively. The GHH preference abstracts from the wealth effect in labor supply, this assumption allows to avoid a counterfactual dynamics of borrowers labor supply during periods when they are credit-constrained. As in Queralto (2019), the disutility of labor is governed by the following process:\footnote{Jaimovich and Rebelo (2009) suggest a similar preference that allows to parameterize the short-run wealth effect on the labor supply.}

$$\gamma_t = \gamma_{t-1} N_t^{1-\rho_T}$$

The parameter $\rho_T$ determines the responsiveness of disutility of labor to changes in productivity growth. This formulation insures that the BGP with constant hours exists, but the medium-run swings in growth do not excessively affect labor supply.\footnote{One way to interpret this feature of the preference is as follows. As noted by Benhabib et al. (1991), the GHH preference can be interpreted as a reduced form of an economy with home production. The disutility of work then consists of the forgone output in home production. This disutility increases as productivity improvements in the formal sector spill over to the home production. However, to the extent this process takes time, disutility of labor exhibits inertia.}

As common in the literature, utility from housing is assumed to be separable from consumption and labor. I assume that it takes the standard CRRA form $G(h_t^H) = \kappa_t \varphi_t (h_t^H)^{1-\epsilon_h-1}$, where $\epsilon_h$ is the inverse elasticity of housing demand; $\kappa_t$ is the weight of housing in the total period utility, which is allowed to exhibit trend to ensure that the marginal rate of substitution between consumption and housing is constant on the balanced growth path; and $\varphi_t$ is a housing preference shock.
Saver households

Saver households supply labor $L_t^S$; earn wage $W_t$; consume the aggregate basket of goods $C_t^S$; trade nominal risk-free bonds $B_{t+1}^S$; and adjust their housing stock $h_t^S$ at a price $P_t^h$ per unit. The representative household maximizes its expected discounted lifetime utility subject to the budget constraint:

$$\max_{\{C_{t+j}^S, L_{t+j}^S, h_{t+j}^S, B_{t+1}^S\}_{j=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta_t^j \left( U(C_{t+j}^S, L_{t+j}^S) + \kappa_t \vartheta_t G(h_{t+j}^S) \right) \quad \text{s.t.}$$

$$C_t^S + P_t^h (h_t^S - h_{t-1}^S) + (1 + r_{t-1}) \frac{B_t^S}{P_t} = W_t L_t^S + \frac{B_{t+1}^S}{P_t} \quad (2.5)$$

From now on, let $\lambda_t^H$ denote the household $H$’s Lagrange multiplier with respect to the budget constraint, and $\Lambda_{t,t+1}^H = \beta_h \frac{\lambda_t^H}{\lambda_t^S}$ denote the households stochastic discount factor. The intertemporal optimality condition, labor supply, and housing demand implied by the saver household’s optimization problem are as follows:

$$\mathbb{E}_t \left( \Lambda_{t,t+1}^S \frac{1 + r_t}{\Pi_{t+1}} \right) = 1 \quad (2.6)$$

$$W_t = \Upsilon_t(L_t^S)^{\epsilon_L} \quad (2.7)$$

$$P_t^h = \mathbb{E}_t \left( \Lambda_{t,t+1}^S P_{t+1}^h \right) + \kappa_t \vartheta_t \frac{(h_t^S)^{-\epsilon_h}}{\Lambda_t^S} \quad (2.8)$$

The first two conditions are standard. Equation (2.8) is the housing demand condition, which implies that the current real hosing price ($P_t^h$) equals to the expected discounted lifetime stream of utility from housing expressed in the units of the consumption good.

Borrower households

Similarly to savers, borrower households supply labor, consume, trade nominal risk-free bonds, and demand housing. Moreover, they have an access to investment opportunities. They accumulate capital and, as in Bilbiie et al. (2012), hold shares in a mutual fund of intermediate firms. In each period $t$ the household buys $\iota_{t+1}$ shares in a mutual fund of $N_t + N_{e,t}$ firms (already operating and new entrants) at the price $v_t$ per share; receive the dividend income from currently owned firms $d_t$, as well as the return on the shares purchased
in the previous period. Finally, as Greenwood et al. (1988) I endogenize capital utilization by assuming that the depreciation rate of capital is an increasing function of utilization. This quantitative feature is important for capturing the short-run dynamics of measured TFP.

The amount these households can borrow from savers ($B_t^B$) is subject to a Kiyotaki and Moore (1997) type occasionally binding collateral constraint: the household cannot borrow more than a fraction $m$ of the current value of its housing stock $P_t^h h_t^B$. Note that I do not impose that the household is constrained per se. Whether the collateral constraint binds or not would be determined in equilibrium depending on the state of the economy. As in Guerrieri and Iacoviello (2017), the borrowing limit does not reset within one period, parameter $\rho_B$ governs the degree of its persistence. This quantitative feature allows to capture the empirically-relevant gradual adjustment of the borrowing capacity in response to changes in borrowers housing wealth.29

The full program of borrowers takes the following form:

\[
\max \left\{ C_t^B, L_t^B, h_t^B, B_{t+1}^B \right\}
\]  

\[
E_t \sum_{j=0}^{\infty} \beta^j (U(C_{t+j}^B, L_{t+j}^B) + \kappa_t \delta_t G(h_{t+j}^B))
\]  

s.t.

\[
C_t^B + I_t + P_t^h (h_t^B - h_{t-1}^B) + (1 + r_t) \frac{B_t^B}{P_t} + \ell_{t+1} v_t (N_t + N_{e,t}) =
\]

\[
\ell_t (v_t + d_t) N_t + W_t L_t^B + P_t K_t + \frac{B_{t+1}^B}{P_t} + \text{div}_t
\]

\[
\frac{B_{t+1}^B}{P_t} \leq \rho_B \frac{B_t^B}{P_t-1} + (1 - \rho_B) m P_t^h h_t^B
\]

\[
K_{t+1} = (1 - \delta_K(u_t)) K_t + (1 - AC_{t,t}) I_t
\]

\[
\delta_K(u_t) = \delta_K + c_1(u_t - 1) + \frac{c_2}{2} (u_t - 1)^2
\]

28 Following the existing literature, I do no explicitly model the origins of this constraint. A natural interpretation, however, should be that due to the imperfect enforceability of contracts, the ability of households to borrow is bounded by a fraction of the value of their collateral assets that can be seized by creditors in a case of default. The parameter $m$ can be narrowly interpreted as the maximum loan-to-value (LTV) ratio. It can also be linked to the degree of country’s financial markets development, as suggested by some of the existing literature.

29 A natural interpretation of this feature is the implicit existence of multi-period credit contracts, as in Kydland et al. (2016).
Let $\chi_t$ be the Lagrange multiplier with respect to the borrowing constraint (2.10). The intertemporal optimality condition and the complimentary slackness condition are then:

$$
\mathbb{E}_t \left( \Lambda_{t,t+1}^B \frac{1 + r_t - \rho_B \chi_{t+1}}{\Pi_{t+1}} \right) = 1 - \chi_t \tag{2.13}
$$

$$
\left( \frac{B_{t+1}^B}{P_t} - \rho_B \frac{B_t^B}{P_{t-1}} - (1 - \rho_B) m P_t^h B_t^B \right) \chi_t = 0, \quad \chi_t \geq 0
$$

When the collateral constraint binds ($\chi_t > 0$) it creates an endogenous wedge between the real interest rate and the borrowers intertemporal marginal rate of substitution in consumption. In other words, the consumption path of the credit-constrained borrowers deviates from the one predicted by the real interest rate dynamics.

Next, borrowers housing demand is:

$$
P_t^h = \mathbb{E}_t \left( \Lambda_{t,t+1}^B P_{t+1}^h \right) + \kappa_t \partial_t \left( \frac{h_t^B}{\lambda_t^B} \right)^{\epsilon_h} \chi_t (1 - \rho_B) m P_t^h \tag{2.14}
$$

The expression is symmetric to savers housing demand, equation (2.8), except for the last term. Its interpretation is the following: the direct effect of borrowers being credit-constrained ($\chi_t > 0$) is that they value a marginal unit of housing more since it has an additional benefit of relaxing their borrowing limit. However, the general equilibrium effect of the binding borrowing constraint on their housing demand is the opposite. I postpone the detailed discussion of this effect until section (2.5.3).

The first order condition with respect to share holdings $\iota_{t+1}$ implies the following expression:

$$
v_t = (1 - \delta_N) \mathbb{E}_t \left( \Lambda_{t,t+1}^B (d_{t+1} + \nu_{t+1}) \right), \tag{2.15}
$$

When iterated forward, it produces the standard firm value equation: $v_t = \mathbb{E}_t \sum_{j=0}^\infty \Lambda_{t,t+j}^B (1 - \delta_N)^j d_{t+j}$. The present firm value equals to its expected discounted profit stream, accounting for the fact that each period a firm faces an exogenous probability of exiting the market $\delta_N$, as discussed further.
The optimality conditions for accumulation and utilization of capital are standard:

\[ q_t = E_t \left( \Lambda_{t,t+1}^B ((1 - \delta_{K,t}) q_{t+1} + R_{t+1}^K) \right) \]  
(2.16)

\[ q_t = 1 + q_t (AC_{I,t} + AC'_{I,t} I_t) - E_t \left( \Lambda_{t,t+1}^B q_{t+1} AC'_{I,t+1} I_{t+1} \right) \]  
(2.17)

\[ R_t^K = c_1 + c_2 (u_t - 1) \]  
(2.18)

where \( q_t \) is Tobin’s q: the Lagrange multiplier with respect to the capital law of motion (2.11). Capital investment is assumed to be subject to a standard quadratic adjustment cost \( AC_{I,t} = \frac{\psi_K}{2} \left( \frac{\tilde{I}_t}{I_{t-1}} - 1 \right)^2 \), where \( \psi_K \) governs the size of the adjustment cost and \( g \) in the BGP growth rate.

Finally, the labor supply condition is symmetric to the one of savers:

\[ W_t = \Upsilon_t (L_t^B)^{\epsilon_L} \]  
(2.19)

### 2.3.2 Production

The production structure of the economy consists of two upstream and two downstream sectors. The upstream sectors are the production sector that employs labor, capital and a basket of intermediate good, and a sector of intermediate-good suppliers. The downstream sectors are the wholesale and retail sectors that differentiate the production-sector good and distribute it to final consumers. Moreover, a sector of innovators invents blueprints of intermediate goods. Nonrivalry of ideas generated by innovators is the source of endogenous growth in the economy. Given the focus of the paper, I abstract from the role of housing in production.

**Production sector**

The production sector is populated by perfectly competitive firms that employ homogeneous labor supplied by both households, \( L_t = L_t^S + L_t^B \), effective capital, \( \tilde{K}_t = u_t K_t \), and a CES basket of intermediate products with elasticity of substitution \( \nu \). \( X_t = \left[ \int_0^{N_t} x_t(\omega) \frac{1}{\nu} d\omega \right]^{\nu} \). Positive externalities in the innovation sector, as discussed further, cause the mass of intermediate products, \( N_t \), to expand over time. This brings about efficiency gains to diversity.
implied by the CES aggregator and increases the measured TFP. As in Comin and Gertler (2006), the aggregate production function takes the following form:

\[ F_t = Z_t \left( \tilde{K}_t^\alpha L_t^{1-\alpha} \right)^{1-\xi} X_t^\xi = Z_t \left( \tilde{K}^\alpha L^{1-\alpha} \right)^{1-\xi} \left( \int_0^{N_t} x_t(\omega) \frac{1}{\nu} d\omega \right)^{\nu \xi} \]

Given input prices, the representative firm maximizes its expected profit stream (expressed in units of the consumption good):

\[
\max_{\{x_{t+j}(\omega), L_{t+j}, K_{t+j}\}_{j=0}^\infty} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[ p_t^F F_{t+j} - R_t^K K_{t+j} - W_{t+j} L_{t+j} - \int_0^{N_t} p_t^\tau(\omega) x_{t+j}(\omega) d\omega \right]
\]

The problem implies the following input demands:

\[
W_t = p_t^F (1 - \alpha)(1 - \xi) F_t L_t
\]

\[
R_t^K = p_t^F \alpha(1 - \xi) \frac{F_t}{K_t}
\]

\[
p_t^\tau(\omega) = p_t^F \xi F_t \frac{1}{X_t} x_t(\omega)^{1-\nu}
\]

where \( p_t^F = \frac{P_t^F}{P_t} \) and \( p_t^\tau(\omega) = \frac{P_t^\tau(\omega)}{P_t} \) are relative prices of the production-sector and the intermediate-sector goods respectively.

**Intermediate-good sector**

The intermediate sector is populated by a mass \([0, N_t]\) of monopolistically competitive firms, each operating a roundabout technology that requires \( A^{-1} \) units of the domestic good to produce a unit of the intermediate good. One should not take this setup literally. The correct interpretation of this formal description is that the forgone final good is never manufactured. The resources that would have been used to produce the forgone output are used instead to manufacture intermediate goods.

Each intermediate-sector firm maximizes its real profit subject to the production sector
demand:

\[
\max_{p_t^x(\omega)} \left[ (p_t^x(\omega) - A^{-1})x_t(\omega) \right] \text{ s.t. } p_t^x(\omega) = p_t^F \frac{F_t}{X_t} x_t(\omega)^{\frac{1-\nu}{\nu}}
\]

In a symmetric equilibrium the optimal quantity of the intermediate good \((x_t)\), the firm’s profit \((d_t)\), and its relative price \((p_t^x)\) are the following:

\[
x_t = \left( \frac{A\xi}{\nu} \right)^{\frac{1-\xi}{1-\xi}} (p_t^F Z_t)^{\frac{1}{1-\xi}} N_t^{\frac{\nu-1}{1-\xi}} K_t^\alpha L_t^{1-\alpha}
\]

\[
d_t = \frac{\nu - 1}{\nu} p_t^x x_t = \frac{\nu - 1}{A} x_t
\]

\[
p_t^x = \nu A^{-1}
\]

Positive profit in this sector motivates entry. To open a firm, an entrepreneur needs to pay an sunk entry cost that consists of the cost of buying a blueprint of a new product from innovators at a price \(p_t^b\). New firms finance entry by selling shares of their equity to entrepreneurs. Free entry pins down the equilibrium value of an intermediate firm, which should be equal to the entry cost: \(v_t = p_t^b\).

**Wholesalers**

Each monopolistically competitive wholesale firm \(j \in (0,1)\) purchases homogeneous production-sector good \(F_t\) at the price \(P_t^F\) and produces a differentiated variety sold to retailers. Following Rotemberg (1982), wholesale-good prices are sticky due to the presence of a quadratic price adjustment cost \(AC_{p,t} = \psi_p \left( \frac{P_t(j)}{\Pi_{t-1}(j)} - 1 \right)^2 Y_t\), where parameter \(\psi_p \geq 0\) governs the strength of nominal rigidity and \(\Pi\) is the steady-state inflation rate.

Each wholesaler sets the price \(P_t(j)\) to maximize the future expected profit stream \(d_t^w(j)\),
subject to the production function, the retailers demand, and the price adjustment cost:

$$\max_{\{P(j)_{t+k}\}_{k=0}^{\infty}} \mathbb{E}_{t} \sum_{k=0}^{\infty} \Lambda_{t,t+k}^{B} d^{u}_{t}(j) = \mathbb{E}_{t} \sum_{k=0}^{\infty} \Lambda^{B}_{t,t+k} \left[ \frac{P_{t+k}(j)}{P_{t}} Y^{r}_{t+k}(j) - \frac{P^{F}_{t+k}}{P_{t}} F_{t+k}(j) - AC_{p,k}(j) - \Gamma \right], \quad \text{s.t}$$

$$Y_{t}(j) = F_{t}(j)$$

$$Y_{t}(j) = \left( \frac{P_{t}(j)}{P_{t}} \right)^{-\eta} Y_{t}$$

$$AC_{p,t}(j) = \frac{\psi_{p}}{2} \left( \frac{P_{t}(j)}{P_{t-1}(j)^{\Pi}} - 1 \right)^{2} Y_{t}$$

The problem implies that the optimal price is a time-varying markup $\mu_{t}$ over the price of the production-sector good $P^{F}_{t}$:

$$P_{t}(j) = \mu_{t} P^{F}_{t}$$

$$\mu_{t} = \frac{\eta}{(\eta - 1) + \psi_{p} \frac{\Pi}{\Pi - 1} - \psi_{p} \mathbb{E}_{t} \Lambda_{t,t+1}^{B} \left( \frac{\Pi^{t+1}}{\Pi} - 1 \right) \frac{\Pi^{t+1}}{\Pi} Y^{t+1}_{t}} \quad (2.24)$$

When prices are flexible, $\psi_{p} = 0$, the markup is constant over the business cycle $\mu = \frac{\eta}{\eta - 1}$. Given the optimal price choice, the real period profit of a wholesaler $j$ is $d^{w}_{t}(j) = \left( 1 - \frac{1}{\mu} \right) Y_{t}(j) - AC_{p,t}(j) - \Gamma$. To ensure zero steady state profit in this sector and rule out entry, I assume that production involves a fixed cost $\Gamma = \frac{1}{\eta} Y$.

**Retailers**

Firms in the retail sector are perfectly competitive and demand varieties of the wholesale good $Y_{t}(j)$ to produce the final consumption-investment good. The final good is a CES aggregate of wholesale varieties $Y_{t} = \left( \int_{0}^{1} Y_{t}(j) \frac{\eta}{\eta - 1} dj \right)^{\frac{1}{\eta - 1}}$. The corresponding aggregate price index and demands are standard: $P_{t} = \left( \int_{0}^{1} P_{t}(j) \frac{1}{1-\eta} dj \right)^{1-\eta}$ and $Y^{d}_{t}(j) = \left( \frac{P_{t}(j)}{P_{t}} \right)^{-\eta} Y_{t}$.

**Innovators**

The sector of innovators involves inventing blueprints for new types of intermediate goods. The sector is populated with the unbounded mass of potential innovators. Let $S_{t}$ be the
total innovation spending and \( \phi_t^i \) be the innovators’ individual productivity parameter. The individual production function blueprints of intermediate goods is then \( N_{et}^i = \phi_t^i S_t^i \).

In aggregate, however, the technology coefficient \( \phi_t \) depends on the existing stock of knowledge, measured by the number of existing intermediate goods, \( N_t \). As in Romer (1990), this knowledge spillover externality is responsible for the existence of the balanced growth path in the model. Moreover, in line with Comin and Gertler (2006), I include congestion externally \( N_t^\rho S_t^{1-\rho} \) that allows to control for the aggregate elasticity of blueprints output with respect to innovation spending. The resulting aggregate innovators productivity is:

\[
\phi_t = \phi \frac{N_t}{N_t^\rho S_t^{1-\rho}},
\]

where \( S_t = \int \mathcal{S}_t^i di \). The aggregate production function of innovators is then \( N_{et} = \phi N_t \left( \frac{S_t}{N_t} \right)^\rho \).

Investors maximize their expected profit stream subject to a quadratic adjustment costs in innovation spending:

\[
\max_{\{S_{t+j}^i\}} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}^B \left( p_{t+j}^b S_{t+j}^i - (1 + AC_{S,t+j}) S_{t+j}^i \right)
\]

The first-order condition is:

\[
\phi_t p_t^b = 1 + AC_{S,t} + AC_{S,t}^d S_t^i - \mathbb{E}_t \left( A_{t,t+1}^B AC_{S,t+1}^d S_{t+1}^i \right)
\]

I assume the standard quadratic adjustment cost \( AC_{S,t} = \frac{\psi s}{2} \left( \frac{S_t^i}{S_{t-1}^i g} - 1 \right)^2 \), where \( g \) is the growth rate of the economy on the balanced growth path.

As in Bilbiie et al. (2012), I there is a time-to-build lag: newly invented blueprints are adopted with a one-period lag. At each period existing varieties of the intermediate good face a constant probability of becoming obsolete \( \delta_N \). The resulting law of motion for the total number of intermediate-good varieties is \( N_{t+1} = (1 - \delta_N)(N_t + N_{et}) \), where \( N_{et} = \int \mathcal{N}_{et}^i di \).

The positive knowledge spillover externality in the innovation sector gives rise to variety-

\[30\text{ I abstract from endogenous adoption of technologies as in Comin and Gertler (2006), Anzoategui et al. (2019b), and Correa-López and de Blas (2018), among others. The inclusion of this feature would allow to capture the cyclicality of adoption, but would not alter the main conclusions.}\]
driven endogenous growth, which rate equals to:

\[ g_{t+1} = \frac{N_{t+1}}{N_t} = (1 - \delta_N) \left( 1 + \phi \left( \frac{S_t}{N_t} \right) \right) \]

(2.27)

The endogenous growth rate \( g_t \) varies over the business cycle depending on the level of innovation spending \( S_t \), which is determined in general equilibrium.

### 2.3.3 Monetary policy

Monetary policy is conducted through interest rate setting. The policy rate \( r_t \) is governed by a Taylor rule that allows for an occasionally binding zero lower bound constraint:

\[
1 + r_t = \max \left[ 0; (1 + r_{t-1})^{\rho_r} \left( 1 + \left( \frac{GDP_t}{GDP_{BGP}} \right)^{\phi_Y} \left( \frac{\Pi_t}{\Pi} \right)^{\phi_{\Pi}} \right)^{1-\rho_r} \right] \quad (2.28)
\]

Parameters \( \phi_Y \geq 0 \) and \( \phi_{\Pi} > 0 \) govern the policy response to changes in GDP and inflation; \( \rho_r \in [0, 1) \) determines the degree of interest rate smoothing; \( \tilde{r}_t \) an exogenous disturbance to the policy rate. Note that the above rule responds to changes in the domestic GDP relative to its balanced growth path. In other words, the monetary authority does not respond to endogenous changes in the productivity growth, which is consistent with how central banks tend to conduct monetary policy.

### 2.3.4 Symmetric equilibrium

In a symmetric equilibrium all retailers are alike, so \( P_t(j) = P_t \) and \( Y_t(j) = Y_t = F_t, \forall j \), and the relative price of the production-sector is the inverse of the wholesale markup \( p_t^F = \frac{1}{\mu_t} \). Similarly, a representative borrower owns all intermediate firms \( \iota_{t+1} = \iota_t = 1 \), each of which is alike \( x_t(\omega) = x_t, p_t^F(\omega) = p_t^F, d_t(\omega) = d_t, \forall \omega \).

Since some output is used by the intermediate sector, the correct measure of real GDP in the model economy is the retail-sector output net intermediate consumption. Real GDP then equals to the sum of the total consumption, capital investment, firm creation spending
(sunk entry cost), and the total adjustment cost spending:

\[ GDP_t = Y_t - N_t \frac{x_t}{A} = C_t^S + C_t^B + I_t + c_t N_{e,t} + AC_{p,t} + AC_{S,t} + AC_{I,t} \]

Equilibrium choices of intermediate producers along with other optimality conditions allow to express the production-sector output as follows:

\[ Y_t = N_t \left( \frac{\xi}{\nu} \right)^{\frac{1}{1-\xi}} \left( \frac{A}{\nu} \right)^{\frac{1}{1-\xi}} Z_t^{\frac{1}{1-\xi}} K_t^\alpha L_t^{1-\alpha} \]  

This expression makes it clear that the following condition on structural parameters needs to be satisfied for growth to take a labor-augmenting form: \( \frac{\xi(\nu-1)}{1-\xi} = 1-\alpha \). In this case, real GDP in the model economy simplifies to \( Y_{t}^{GDP} = \Omega_t Z_t^{\frac{1}{1-\xi}} K_t^\alpha (N_t L_t)^{1-\alpha} \), where \( \Omega_t = \left( \frac{A}{\nu \mu_t} \right)^{\frac{1}{1-\xi}} - \left( \frac{A}{\nu \mu_t} \right)^{\frac{A}{\nu \mu_t}} \). Now, define the Solow residual as \( TPF_t = \frac{Y_t^{GDP}}{(K_t^\alpha L_t^{1-\alpha})} \). The following model-consistent output decomposition then holds true (expressed in log differences):

\[ \Delta Y_t^{GDP} = \Delta TPF_t + \alpha \Delta K_t + (1-\alpha) \Delta L_t \]

\[ \Delta TPF_t = \Delta \Omega_t + \alpha \Delta u_t + (1-\xi)^{-1} \Delta Z_t + (1-\alpha) \Delta N_t \]

The measured TFP in the model is driven by four components. The first one relates to the fact that the wholesale-sector markup distorts the quantity of the intermediate good produced; the second component is the short-run variations in the measured TFP due to time-varying capital utilization; the third is the exogenous TFP component subject to stationary shocks. These three terms drive stationary fluctuations in the measured TFP. The last term is the innovation effect stemming from accumulation of the stock of intermediate goods, which drives fluctuations in the trend growth.

Finally, the credit, housing, and labor markets clear:

\[ B_t^B + B_t^S = 0 \]  
\[ h_t^B + h_t^S = 1 \]  
\[ L_t^B + L_t^S = L_t \]
Equilibrium definition: equations (4-32) determine 29 endogenous variables \((y_t, c_t^B, c_t^S, b_{t+1}^B, b_{t+1}^S, \chi_t, x_t, L_t^S, L_t^B, L_t, v_t, w_t, R_t^K, k_{t+1}, i_t, q_t, \Pi_t, \mu_t, r_t, h_t^B, h_t^S, p_t^h, v_t, s_t, \phi_t, d_t, g_{t+1}, u_t, \delta_{K,t})\) as a function of endogenous states \((b_t^B, b_t^S, v_{t-1}, k_{t-1}, r_{t-1}, h_{t-1}^B, h_{t-1}^S, s_{t-1}, g_t)\) and exogenous states \((\vartheta_t, Z_t, \tilde{r}_t)\). Table 2.3 lists all equilibrium conditions of the model expressed in terms of stationary lower-case variables that remain constant on the balanced growth path, e.g. \(y_t = \frac{Y}{N_t}\), \(i_t = \frac{I}{N_t}\), and \(c^B = \frac{C^B}{N_t}\).

2.4 Calibration and simulation

2.4.1 Calibrated parameters

The model is calibrated at quarterly frequency. To the extent possible, the parameter choice is informed either directly by the data or by the existing estimates in the literature.

I first describe the structural parameters of the household sector. I set the relative risk aversion to \(\sigma = 2\), as common in the literature. I calibrate innovators productivity \(\phi\) to match the 0.8% annual TFP growth rate on the BGP, an average across countries in the empirical exercise, i.e. \(g = 1.008^{1/4}\). The steady-state real interest rate is then pinned down by the savers discount rate as follows: \(R = \frac{1+r}{\Pi} = \frac{g}{\beta}\). I chose \(\beta^S\) to replicate the steady-state annual real interest rate of 4%, which implies \(\beta^S = 0.9968\). The borrowers discount factor should be lower than the savers discount factor. Under this assumption, the collateral constraint binds in the deterministic steady state as impatient households choose to borrow as much as possible. The difference between the two discount factors determines the steady-state shadow value of the borrowing constraint as follows: \(\chi = \frac{\beta^S-\beta^B}{\beta^S} \left(1 - \frac{\rho_B\beta^B}{\Pi g^\sigma}\right)\); it also determines how often the collateral constraint binds over the business cycle. I choose \(\beta^B = 0.9963\) so that the borrowers are only slightly more impatient than savers and hence become credit-constrained only occasionally.

I assume the Frisch elasticity of labor supply of \(\frac{1}{\epsilon_L} = 4\), consistent with the King and Rebelo (1999) calibration. The price elasticity of housing demand choice is based on the results from Hanushek and Quigley (1980) who provide estimates of this elasticity in the range of -0.2 to -0.9 in the long-run and around -0.1 in the short-run (within a year). Given my focus on short- and medium-run fluctuations I assume relatively inelastic demand and
set $\epsilon_h = 5$. Calibration of the following two parameters relies on the estimates from Warnock and Warnock (2008). I set the loan-to-value ratio (LTV henceforth) to $m = 0.75$, which is close to an average LTV ratio across European countries. The steady-state weight of housing in the utility function $\kappa$ is calibrated to set the steady-state mortgage debt-to-GDP ratio to 55%, an average for 23 developed countries over 2001-2005.

I now turn to the structural parameters of the production side of the economy. The capital share is set to the average value across the sample of studied events $\alpha = 0.4$ (Penn World Table v. 9.1 estimates). The share of intermediate goods is set to $\xi = 0.5$, consistent with the existing literature. I set the elasticity of substitution between intermediate inputs then is pinned down by the BGP requirement $\frac{\xi(\nu-1)}{1-\xi} = 1 - \alpha$, which implies $\nu = 1.6$, as in Comin and Gertler (2006) who motivate the low value of this parameter by the specialized nature of intermediate products. Next, I set the elasticity of substitution between varieties of the final investment-consumption good to $\eta = 11$ implying a steady-state markup of 10%, a conventional choice in the literature. Based on equivalence results of Born and Pfeifer (2016), the quadratic price adjustment cost parameter is set to $\psi_p = 120$ to replicate, in a linearized setting, the slope of the Phillips curve derived using Calvo stickiness with an average price duration of about a year, which is close to direct estimates of Gál and Gertler (1999).

To normalize the steady-state capital utilization to 1, I set $c_1 = \frac{\delta_K B R}{\beta S} - 1 - \delta_K$. I set the value of quarterly capital depreciation standard in the literature $\delta_K = 0.025$. Following Bilbiie et al. (2012), I set the intermediate firm exit rate $\delta_N = 0.025$. This value is based on the Bernard et al. (2010) estimate of the minimum production destruction rate, measured as a market share. It is also consistent with the Caballero and Jaffe (1993) estimate of technological obsolescence rate. As in Comin and Gertler (2006), the elasticity of innovators output to expenditure is set to $\rho = 0.8$. The steady-state aggregate productivity level $Z$ is chosen to normalize the steady-state GDP to unity.

Finally, I assume 2% steady-state annual inflation, and choose the conventional parameters of the Taylor rule: $\phi_Y = 0.25$, $\phi_\pi = 1.5$, $\rho_r = 0.7$. Table 2.4 summarizes parameters of the baseline model.

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2.4.2 Structural shock and solution method

I use the structural model described in the previous section to explore the mechanism and draw policy implications of a negative shock to housing prices. This scenario is simulated as a result of a negative housing preference (demand) shock \( \varepsilon_t^\vartheta \) that affects the housing preference parameter governed by a standard AR(1) process \( \ln(\vartheta_t) = (1 - \rho_\vartheta) \ln(\vartheta) + \rho_\vartheta \ln(\vartheta_{t-1}) + \varepsilon_t^\vartheta \). I resort to this shock as an exogenous disturbance to the house price based on the evidence from the existing literature. In particular, Liu et al. (2013) identify this shock as the one that drives most of the fluctuations in the U.S. land prices and is crucial for generating the empirical comovement between land prices and investment. Later estimates of Guerrieri and Iacoviello (2017) suggest that about 70% of the consumption decline during the Great Recession in the U.S. can be traced back to housing demand shocks.32 This shock should be treated as reflecting not only pure changes in the taste for housing but also other unmodeled factors that shift housing demand. In the context of housing market crashes the former can be interpreted as a sudden realization that the explosive housing market dynamics is unsustainable and the asset is overvalued, akin to the work of Minsky (1986) that got back in vogue since the Global Financial Crisis. Other factors central for housing demand and about which the model is silent include mortgage credit availability.

The model features two sources of non-linearities: the zero lower bound constraint on the policy rate and the collateral constraint. The presence of these two occasionally binding constraint (OBC henceforth) poses a computational difficulty since the model cannot be solved using standard perturbation methods. One way to tackle this issue is to resort to policy/value function iteration or other global solution methods that allow to fully account for the non-linearities and precautionary behavior linked to the possibility that the constraint may become binding in the future. However, these methods are computationally demanding and are not easily scalable due to the curse of dimensionality. The simplest alternative solution was introduced by Guerrieri and Iacoviello (2015) and involves using a piecewise-linear solution. This method builds on the insight that occasionally binding constraints can be handled as different regimes of the same model. Under one regime, the occasionally binding constraint is slack. Under the other regime, the same constraint is binding. The piecewise linear solution method involves linking the first-order approximation of the model

\[ \text{See also the study of sources and consequences of US housing market fluctuations in Iacoviello and Neri (2010).} \]
around the same point under each regime. However, just like any linear solution, this method does not allow to capture the effects of uncertainty and so to account for precautionary behavior. I use a similar approach developed by Holden (2019) and implemented as an extension to Dynare: DynareOBC. Unlike the Guerrieri and Iacoviello (2015) method, its compatible with higher-order approximations and by integrating over future uncertainty allows to capture some of the precautionary behavior.

2.4.3 Accounting for the empirical evidence

**IRF matching estimation**

I first investigate the ability the model to account for the empirical evidence from section 2.2. For that purpose, I conduct a “crisis” experiment by hitting the model economy with a series of negative housing preference shocks $\varepsilon^\vartheta_t$ calibrated to match the empirical dynamics of the aggregate housing price index. Autocorrelation of the shock process is set to 0.978, close to the estimate of Iacoviello and Neri (2010) who find housing preference shocks to be very persistent.

Given the sequence of negative housing demand shocks, I estimate the remaining five parameters of the model $P = \{\rho_b, \psi_N, \psi_K, \rho_\Upsilon, c_2\}$ to minimize the distance between empirical (local-projection) and theoretical (model-based) impulse responses, as in Christiano et al. (2005). These are parameters that govern the borrowing limit inertia ($\rho_b$), innovation spending adjustment cost ($\psi_N$), investment adjustment cost ($\psi_K$), the disutility of labor inertia ($\rho_\Upsilon$), and the responsiveness of capital depreciation to utilization ($c_2$). Before proceeding I want to emphasize that the overall quantitative predictions of the model are robust to the choice of the above parameters.

Formally, the problem is to minimize the weighted distance between the two IRFs:

$$\min_P \left( \Sigma^{DSGE}(P) - \Sigma^{LP} \right) \Omega^{-1} \left( \Sigma^{DSGE}(P) - \Sigma^{LP} \right)'$$

33 Available at https://github.com/tholden/dynareOBC

34 For the purpose of estimation I consider the sequence of shocks that hits the economy at the deterministic steady state at which the collateral constraint binds. In the later sections I explicitly explore the role of non-linearities induced by the occasionally binding collateral constant.
where $\Sigma^{DSGE}(P)$ denotes the mapping between the estimated parameters of the model and the theoretical impulse responses; $\Sigma^{LP}$ is a vector of empirical impulse responses; and $\Omega$ is the weighing matrix. As common in the literature, the weighting matrix includes standard deviations of the empirical impulse responses on the main diagonal, thus putting a larger weight on matching empirical impulse responses that are estimated with more precision. For estimation, I use empirical impulse responses of output, consumption, investment, capital, labor, TFP, and labor productivity. I exclude the IRF of the household debt-to-GDP ratio because of the problematic mapping between this variable in the model and the data. Firstly, this ratio in the data includes all household debt, not only mortgage debt. Moreover, it combines debt held by all households, not only those who do not hold enough liquid assets and are likely to become credit-constrained, the group I focus on in my analysis. Since the model is calibrated at the quarterly frequency, I average theoretical impulse responses to make them comparable with empirical responses of annual frequency.

The estimation results in the following parameter values. The borrowing limit inertia $\rho_b = 0.67$, which is close to the estimate of Guerrieri and Iacoviello (2017); adjustment costs parameters for capital and innovation spending are $\psi_N = 0$ and $\psi_K = 2$ respectively. The low value of the capital utilization parameter $c_2 = 0.075$ suggests that this component of the model is important for capturing the short-run dynamics of the Solow residual. Finally, disutility of labor exhibits a significant degree of inertia, $\rho_T = 0.98$, implying that in the short run the wealth effect in labor supply is weak.

**Impulse responses**

Overall, the model accounts for the empirical evidence well. Immediate results of IRF matching are presented on Figure 2.8. Two things are worth noting. First, the model predicts a somewhat weaker response of capital investment than in the data. The reason is straightforward: the theoretical framework abstracts from any amplification mechanisms that pertain capital accumulation directly. Second, the short-run response in labor is stronger than in the data. One possible reason being that the empirical response of labor is likely to be underestimated, since it does not account for changes in working hours.

Figure 2.9 shows a broader set of impulse responses associated with the IRF-matching experiment (baseline simulation henceforth). Given the perfectly inelastic housing supply, a
sequence of negative demand shocks $\vartheta_t$ leads to a sharp decrease in the equilibrium house price. As a result, borrowers housing wealth falls reducing their borrowing capacity. Note that borrowers housing wealth falls by more than the house price. Although the housing preference shock is aggregate, in general equilibrium the housing stock is reallocated towards savers. This effect additionally contributes to the worsening of borrowers balance sheets, see section 2.5.3 for a detailed discussion.

When borrowers housing wealth falls and the collateral constraint binds, they are forced to reduce spending to meet the lower debt limit. Under nominal rigidity, the reduction in spending leads to a demand-driven recession in the short run. Judging by the dynamics of debt and the borrowing constraint multiplier, the active phase of deleveraging lasts for about 20 quarters. However, it leaves a long-lasting scarring effect on the level of consumption and output due to its detrimental effect on the pace of innovation and capital accumulation. Figure 2.10 presents the decomposition of output and TFP dynamics. Consistent with the empirical evidence, the decline in the economic activity at medium-run horizons is driven largely by the negative effect on the level of capital stock and TFP. The model also allows to shed some light on the relative contributions of factors driving measured TFP. A significant part of a short-run response of the measured TFP, more than a half during the first year, is driven by a decrease in capital utilization and the markup distortion.

The shock is inflationary in the medium run. Although inflation is not significantly affected on impact, in about 8 quarters, as the acute phase of deleveraging is over, it persistently overshoots the steady-state level. Intuitively, this medium-run inflationary effect is driven by a persistent decrease in the capital stock and TFP, both of which push marginal costs up. The effect disappears if one shuts down the endogenous response of TFP and capital by setting the respective adjustment costs to arbitrary high values.

The role of household indebtedness. The model is successful in accounting for the empirical interaction between household indebtedness and house price declines outlined in section 2.2.1. Figure 2.11 compares the baseline simulation to counterfactual scenarios under various values of pre-existing household debt-to-GDP and loan-to-value (LTV) ratios. Consistent with the evidence, higher initial household debt-to-GDP ratio magnifies the response of an economy to a negative housing price shock, at both short- and medium-run horizons.

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35 In general, borrowers also deleverage by supplying more labor, but the assumption of GHH utility eliminates this effect.
Perhaps more surprisingly, increasing the LTV ratio — holding the debt-to-GDP ratio the same — mostly plays the short-run amplification role. Figure B.10 presents a detailed view of the welfare cost of the studied scenario under different calibrations of the credit market in both the baseline model and in the model where the endogenous growth mechanism is shut down. By generating the persistent effect of the shock — as well as amplifying the on-impact effect as discussed further — the endogenous growth mechanism magnifies the welfare cost of the shock approximately by a factor of 4 to 6. The welfare loss seems to be slightly more sensitive to increasing debt levels, rather than LTV ratios. Overall, these results suggest that countries with deeper credit markets are more susceptible deleveraging shocks.

The role of occasionally binding collateral constraint. The occasionally binding collateral constraint is one of the two sources of non-linearities in the model. It causes the effects of housing demand shocks to be state- and sign-dependent. To illustrate this property, Figure 2.12 compares the responses to a large negative and positive housing demand shock. The stark difference between the two scenarios is driven by the fact that the collateral constraint amplifies the effects of negative shocks. Positive shocks, on the other hand cause the constraint to become slack and irrelevant. The occasionally binding collateral constraint generates asymmetry in the link between house prices, economic activity and growth. This state- and sign-dependent dynamics was the reason why in the cross-country empirical exercise of section 2.2 I focused on periods of large and rapid declines in house prices. Collateral constraints tied to housing wealth are more likely to play a prominent role for macroeconomic dynamics during these periods.

In what follows, I focus on the case of negative shocks that cause the collateral constraint to bind. The complementary paper Brizhatyuk (2020b) focuses on the role of the occasionally binding collateral constraints in generating asymmetric and state-dependent fluctuations in growth. The second source of non-linearities, the zero lower bound on nominal interest rate, is discussed in section 2.5.1.

36 Please refer to appendix B.3.5 for the discussion of welfare calculation.
2.5 Exploring the mechanism

I identify and illustrate four main channels that shape the general equilibrium response to negative housing demand shocks.

2.5.1 Aggregate demand channel

Under nominal rigidity borrowers deleveraging results in a demand-driven recession in the short run: the *aggregate demand channel*. When a negative housing preference shock causes the collateral constraint (2.10) to bind forcing borrowers to reduce spending to meet the lower credit limit. It might be tempting to think that the aggregate implications of this reduction in borrowing are of a second-order importance. After all, in a closed-economy context debt is money we owe to ourselves, so the implications of deleveraging may be mostly redistributional. However, the downward revision in the borrowing limit causes a temporary decrease in the natural rate, determined by the dynamics of the savers stochastic discount factor in the flexible price economy. Under nominal rigidity, the real interest rate may fail to adjust accordingly allow savers pick up most of the slack.

Doing away with nominal rigidity significantly reduces the real effect of the shock. Panel (a) of figure 2.13 compares the baseline simulation to the flexible-price counterfactual by setting the price adjustment cost to zero ($\psi_p = 0$). When prices are flexible, the real interest rate fully adjusts. As a result, consumption of savers increases on impact fully offsetting the fall in consumption of credit-constrained borrowers. The mild negative effect on the economy that remains is driven by supply-side factors. Even in a flexible-price economy, credit-constrained borrowers still decrease investment and innovation spending. The associated decrease in the marginal product of labor then lowers households labor supply.

Binding zero lower bound on the policy rate (ZLB henceforth) amplifies the effect of this channel by orders of magnitude. To illustrate it, I proceed as follows. I first append a risk-premium shock $a_t$ to the savers intertemporal optimality condition (2.6): $\mathbb{E}_t \left( \Lambda_{t,t+1}^{S} \frac{1+r_t}{1+r_{t+1}} \right) = 1 + a_t$. As common in the literature, I use this shock to simulate a situation when the ZBL
A temporary increase in $a_t$ reduces the natural rate and causes the policy rate to fall accordingly. I then calculate the effect of a baseline sequence of negative housing preference shocks contingent on the ZLB binding for the first 8 quarters. Panel (b) of figure 2.13 presents the result. Binding ZLB significantly amplifies the effect of the shock. The amplification is driven by a larger decrease in the aggregate demand when the policy rate in constrained by the ZLB, which is reminiscent of the Eggertsson and Krugman (2012) discussion of the implications of household deleveraging under nominal rigidity and monetary policy constraints.

### 2.5.2 Productivity growth channel

The endogenous slowdown in productivity growth, the *productivity growth channel*, emerges as an additional margin of adjustment to the shock due to a combination of two forces. Forward-looking innovation spending falls as returns on this investment are temporary lower. Moreover, the consumption-smoothing motive of credit-constrained borrowers make them reduce investment by more than consumption when deleveraging. To facilitate illustration, throughout this section I abstract from adjustment costs in innovation spending by setting $\psi_N = 0$.

I first illustrate the contribution of this channel by shutting down endogenous fluctuations in TFP growth. Panel (a) of figure figure 2.14 compares the baseline simulation to the counterfactual where the endogenous growth mechanism is shut down by setting the innovation spending adjustment cost to an arbitrary high value ($\psi_S = 10^5$). As a result, consumption and output recover quickly and fully. Moreover, shutting down the endogenous growth mechanism lowers the medium-run inflationary effect of the shock. Finally, the on-impact effect of the shock is muted. This effect has to do with the fact that changes in expected income growth affect present consumption. I discuss this channel further in section

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37 See Eggertsson et al. (2003), Eggertsson (2008), and Christiano et al. (2016), among others. This shock can be interpreted as a reduced-form way to capture the temporary increase in the agents desire to save. As Fisher (2015) has shown, it can also be interpreted as a structural shock to the demand for safe and liquid assets. To a first-order approximation, this shock is isomorphic to a savers discount factor shock.

38 Formally, the effect of a shock conditional on binding ZLB is calculated as a difference between responses of variables in the following two simulations. (1) Baseline: a savers discount factor shock at $t = 0$ that causes the ZLB bind for a chosen number of periods. (2) Counterfactual: a savers discount factor shock at $t = 0$ that causes the ZLB bind for a chosen number of periods and a shock of interest at $t = 1$. 

2.5.4.

What is the relative size of factors that cause a fall in productivity growth after a negative housing preference shock? Recall that entry of intermediate firms — and ultimately innovation spending — is financed by selling equity to households. To illustrate the equity market dynamics, I linearise the relevant equilibrium conditions to get the model-consistent linear equity supply and demand curves. Please refer to appendix B.3.3 for derivations. The result is as follows:

\begin{align}
\text{Equity supply: } & \nu_t = (1 - \rho)s_t \\
\text{Equity demand: } & \nu_t = \mathbb{E}_t \left( A_{v1} d_{t+1} + A_{v2} \nu_{t+1} - R_{t+1} \right) - A^B_{A1} \chi_t + A^B_{A2} \mathbb{E}_t (\chi_{t+1} - R_{t+1} - \Pi_{t+1}) \\
& \text{Discounted next-period return} \quad \text{Collateral constraint wedge}
\end{align}

Given general equilibrium outcomes, these curves determine the current-period equity price $\nu_t$ and the amount of innovation spending financed $s_t$, in percentage deviations from the deterministic steady state. In the absence of innovation spending adjustment costs the supply curve does not shift. The equity demand shifts due to a combination of two factors. changes in returns from firm ownership, which consist of future expected stream of profit. Moreover, since these are borrowers who have an access to the equity market, their investment decisions are affected when they are credit-constrained ($\chi_t > 0$).

Panel (b) of figure figure 2.14 shows the equity market dynamics consistent with the simulation on the panel (a). As a result of a negative housing preference shock, the equity market equilibrium moves from the steady state at point A to an equilibrium with lower innovation spending as equity demand falls at $t = 5$ (point B). How much of this demand shift is directly due to the effect of binding collateral constraint vs the? The dashed line plots the equity demand curve at $t = 5$ excluding, in a partial equilibrium sense, the effect of the collateral constraint wedge.

At this point of time, about two-thirds of the shift in the equity demand curve is directly driven by the collateral effect, the rest of it is driven by demand factors.
2.5.3 Fisherian debt deflation channel

Two more channels amplify the initial effect of the shock. First is the negative feedback loop between deleveraging and borrowers housing wealth: *Fisherian debt deflation.* At the core of this channel is the pecuniary externality: credit-constrained borrowers do not internalize the effect of their individual spending reduction on aggregate housing wealth that ultimately affects their borrowing capacity. The initial deleveraging then exacerbates the damage to borrowers balance sheets and causes further deleveraging. I provide a glimpse into the strength of this channel with two experiments.

Although the housing preference shock is aggregate, binding collateral constraint drives a sizable asymmetry in the dynamics of borrowers and savers housing demand. Similarly to the previous section, I use linearized equilibrium conditions to illustrate this point. Please refer to appendix B.3.2 for details. Let the san-serif font denote percentage deviations of variables from the deterministic steady state. The resulting linear housing demand curves are as follows:

Savers demand: \[ p^h_t = A_{h1}^S h^B_t + A_{h2}^S \vartheta_t - A_{h2}^S \lambda^S_{t+1} + A_{h3}^S E_t (p^h_{t+1} + g_{t+1} - R_{t+1}) \]  

(2.35)

Borrowers demand: \[ p^h_t = -A_{h1}^B h^B_t + A_{h2}^B \vartheta_t - A_{h2}^B \lambda^B_{t+1} + A_{h3}^B E_t (p^h_{t+1} + g_{t+1} - R_{t+1}) \]  

(2.36)

\[ -A_{h4}^B \chi_t + A_{h5}^B E_t (\chi_{t+1} - R_{t+1} - \Pi_{t+1}) \]

Collateral constraint wedge

These model-consistent linear demand curves determine the aggregate real house price \( p^h_t \) and the allocation of housing towards borrowers \( h^B_t \) given the aggregate housing preference shock \( \vartheta_t \) and general equilibrium outcomes (expressed in percentage deviations from the deterministic steady state). What causes these demand curves to shift? Common factors are the aggregate preference shock and changes in the expected next-period return. Demands are

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39 Lately, this channel has been most widely discussed in the literature on emerging market crises, see for instance Mendoza (2010). However, it is standard for any model where the borrowing capacity is linked to the relative price of collateral, see the original discussion in Fisher (1933).
also affected by household-specific wealth effects: when the shadow value of consumption is high, the valuation of housing in consumption units is low and vice versa. Under the baseline particularization the wealth effect on the housing demand is small. Finally, the asymmetry between borrowers and savers housing demands is mainly driven by the effect of binding collateral constraint: the collateral constraint wedge. Simulations shows that for plausible parameter values the effect of this wedge on borrowers housing demand is negative: other things equal, binding collateral constraint causes borrowers to decrease their housing demand more than savers.

The negative equilibrium effect of binding collateral constraint on borrowers housing wealth is large and it significantly exacerbates deleveraging. Panel (b) of figure 2.15 plots the model-consistent demand curves implied by equations (2.35) and (2.36). The left panel shows the initial steady-state housing market equilibrium at \( t = 0 \) (point A). The middle panel presents a snapshot of the housing market equilibrium in the baseline simulation at \( t = 5 \). The shock pushes both demand curves down and causes the equilibrium price to sharply fall (point B). Importantly, the borrowers housing demand falls by more. As a result, borrowers housing wealth suffers not only because the house price falls, but also because the housing stock is being redistributed towards savers. The lion’s share of this outcome is driven the collateral constraint wedge: the dashed line shows, in a partial equilibrium sense, the shift of borrowers demand curve excluding this effect (point C). The right panel plots the dynamics of borrowers and savers housing wealth in the baseline simulation.

Panel (a) of figure 2.15 shows the contrafactual dynamics of the economy where the effect of housing reallocation towards savers on the borrowing limit is shut down. In other words, equation (2.10) is modified as follows:  
\[
\frac{B_t}{P_t} \leq \rho_B m P^h h^B,
\]
where \( h^B \) is steady-state housing of borrowers. This exercise allows to mute some — but not all — of the effect of the Fisherian debt deflation channel. As the resulting impulse responses show, the reallocation of housing that arises endogenously as a result of borrowers deleveraging plays a quantitatively significant role in shaping the overall response of the economy.

### 2.5.4 Expected income growth channel

The second amplification force is the expected income growth channel: lower expected growth decreases current spending through intertemporal substitution. As such, the endoge-
nous productivity growth mechanism is not only generates the persistent effect of deleveraging, but also amplifies the short-run response.\textsuperscript{40}

To illustrate the driving forces of borrowers and savers consumption growth, I linearise their first-order conditions with respect to bond holdings, please refer to B.3.4 for details. Growth of marginal utility of consumption ($E_t \lambda_{t+1}$) is driven by the following factors:

Savers: $E_t \tilde{\lambda}^S_{t+1} - \tilde{\lambda}^S_t = -E_t R_{t+1} + \sigma g_{t+1}$

Borrowers: $E_t \tilde{\lambda}^B_{t+1} - \tilde{\lambda}^B_t = -E_t R_{t+1} + \sigma g_{t+1} - A_{A_1} X_t + A_{A_2} E_t (X_{t+1} - R_{t+1} - \Pi_{t+1})$

Note that productivity growth has a positive effect on the growth of marginal utility of consumption and the magnitude of this effect governed by the elasticity of intertemporal substitution. When growth is expected to be slower, consumption today decreases. As a result, the expected growth of the marginal utility of consumption increases. The above equations allow me to assess, in a partial equilibrium sense, the role productivity growth relative to other factors. For the interpretation of the results below, note that the model features GHH preference so the marginal utility of consumption also depends on labor.

Figure 2.16 plots the decomposition. In general equilibrium, the real interest rate falls, which, other else equal, has a positive impact on the growth of marginal utility of consumption. However, for borrowers, the effect of binding collateral constraint, which works in the opposite direction, outweighs the real interest rate effect causing the marginal utility of consumption to fall. The decline in the marginal utility of consumption is then exacerbated by the expectation of slower next-period growth by more than twofold.

### 2.6 Policy scenarios

I conclude highlighting important implications for monetary and fiscal policy.

\textsuperscript{40} See also Benigno and Fornaro (2018) for the discussion of how the self-fulfilling negative feedback loop between the aggregate demand and expected growth can cause the economy to be stuck in a low-growth equilibrium.
Monetary policy. Housing demand shocks — and more broadly shocks that manifest in deleveraging — warrant stronger focus of monetary policy on output stabilization. As section 2.5.1 illustrates, in the short run housing demand shocks propagate similarly to aggregate demand shocks. Naturally, their effects (at both short- and long-run horizons) are shaped by monetary policy. My question is of positive nature: I do not attempt to access optimal monetary policy. Rather, taking that the policy rate follows the Taylor rule as given, I want to access the sensitivity of the welfare cost of the baseline simulation to parameters of the rule. Please refer to appendix B.3.5 for the discussion of welfare calculations.

I simulate responses to a series of negative housing preference shocks as discussed in section 2.4.3 under various values of the policy rate response to inflation (\(\phi_\pi\)) and cyclical variations of output (\(\phi_Y\)). Moreover, I compare the results of the baseline model to the ones of the alternative where the endogenous growth mechanism is shut down by setting the R&D adjustment cost parameter to an arbitrary high value \(\psi_N = 10^5\). Figure 2.17 illustrates the sensitivity of the welfare cost of the crisis to parameters of the Taylor rule. Three things stand out. First, in both cases the welfare cost of the shock is strictly decreasing in the strength of policy response to output. Stronger response to inflation, on the other hand, is not welfare-improving. Under certain combination of parameters it may even exacerbate the welfare loss. Second, the welfare cost of the crisis is often larger for savers than for borrowers, especially for the cases when the policy rate response to output is relatively weak. Finally, the endogenous growth mechanism magnifies the effect of the shock by as much as 5 times and further emphasizes the importance of output stabilization relative to inflation stabilization in the studied scenario.\(^{41}\)

Fiscal policy. How effective fiscal policy can be in mitigating the damage to borrowers balance sheets and resulting deleveraging due to a decline in house prices? I offer a glimpse into this question by studying the following policy. Suppose the government finances its expenditure \(G_t\) by levying lump-sum taxes of savers and borrowers, \(T_t^S\) and \(T_t^B\) respectively. The policy then involves shifting the tax burden from borrowers to savers by \(\Delta_t\). This can be interpreted as a debt relief program or, more generally, as a revenue-neutral tax reform that benefits borrowers. The government budget constraint and the modified budget constraints

\(^{41}\) This is broadly consistent with results of Garga and Singh (2020) and Ikeda and Kurozumi (2018) who study optimal monetary policy in business cycles models with endogenous growth.
of savers and borrowers take the following form:

\[
G_t = T_t^B + T_t^S
\]  

(2.39)

\[
C_t^S + P_t^B (h_t^S - h_{t-1}^S) + (1 + r_{t-1}) \frac{B_t^S}{P_t} = W_t \ell^S_t + \frac{B_t^{S+1}}{P_t} - (T_t^S + \Delta_t)
\]  

(2.40)

\[
C_t^B + I_t + P_t^K (h_t^K - h_{t-1}^B) + (1 + r_{t-1}) \frac{B_t^B}{P_t} + u_{t+1} v_t (N_t + N_{e,t}) =
\]  

(2.41)

\[
t_t (v_t + d_t) N_t + W_t L_t^B + P_t^K u_t K_t + \frac{B_t^{B+1}}{P_t} + div_t - (T_t^S - \Delta_t)
\]

Debt relief measures implemented amid the crisis are very effective in alleviating both immediate and persistent effects of deleveraging.\(^{42}\) Figure 2.18 presents baseline IRF-matching simulation along with the policy intervention counterfactual. The policy is a transfer \(\Delta_t\) equivalent to a 0.25% of borrowers’ steady-state debt burden. As in Guerrieri and Iacoviello (2017) the transfer is governed by an AR(1) process with a persistence coefficient equal to 0.5. The success of the policy is based on two factors. First, in the economy where borrowers are credit-constrained Ricardian equivalence no longer holds. As a result, a transfer towards borrowers is expansionary. Moreover, this policy strikes at the heart of amplification mechanisms that operate in the economy and are responsible for the lion’s share of the resulting negative effect. The policy effectively alleviates the negative feedback loop between borrowers deleveraging, their housing wealth, and growth expectations (the debt-deflation channel and the expected income channel). Panel (b) shows the net effect on the policy and clearly indicates that the debt write-down relaxes the borrowers’ credit constraint. As such, it offsets the immediate decrease in consumption and improves the medium-run trajectory of the economy by decreasing the decline in innovation and capital investment: the two components of the aggregate demand that are affected by deleveraging the most.

\(^{42}\) The result is consistent with empirical estimates of Auclert et al. (2019) who document the large and highly persistent effect of U.S. debt forgiveness measures on non-tradable employment. However, it is important to keep in mind that the ultimate effect of such measures depends on their pass-through to households. See Piskorski and Seru (2018) for the discussion of the role of housing market and housing finance rigidities.
2.7 Conclusion

Why recoveries from some recessions are particularly slow and incomplete? I contribute to this debate by studying the effects of housing market boom-and-bust cycles. First, using cross-country data, I explore the dynamics of recessions and recoveries associated with housing markets crashes. Such events are robustly associated a decline in consumption, output, and utilization-adjusted TFP that lasts longer than regular business-cycle fluctuations. Next, I built a dynamics general equilibrium model with borrower and saver households, occasionally binding collateral constraints tied to housing wealth, endogenous growth through forward-looking investment, and nominal rigidity to study the channels through which declines in house prices affect the macroeconomy. The model successfully accounts for the empirical comovements between variables; highlights the importance of endogenous innovation in generating persistent effect of the shock; and illustrates the key amplification mechanisms. I then use the model to study several policy scenarios. House price shocks that trigger delivering warrant stronger focus of monetary policy on output stabilization, and even more so in the presence of endogenous response in the productivity growth. Fiscal intervention that alleviates the debt burden of borrowers is effective in offsetting a large fraction of the shock when implemented during the crisis.

This paper opens a number of promising avenues for future research. The presented U.S. state-level and MSA-level evidence have illustrated the persistent regional divergence that can occur when member states are subject to asset market boom-and-bust cycles of different intensity. Thinking beyond the U.S. economy, a similar pattern has been even more vivid in the Eurozone in recent years. An interesting question then is how regional and supraregional policies can be designed and coordinated to alleviate this problem. A two-country open-economy extension of the theoretical framework of this paper would be suitable to explore this question. In terms of policy, the present work focused on a simple scenario of a transfer from savers to borrowers. A study of a broader set of more realistic fiscal policy measures, such as tax- and/or borrowing-financed increase in government spending, is warranted. I am working on the extensions to address these issues.
Figure 2.1: US real per capita GDP
[cited on page 32 and 47]
Figure 2.2: Housing market boom and bust cycles across countries

[cited on page 32]

Note: real housing prices are from various sources, see description in the text; household debt to GDP ratios are from IMF Global Debt Database; real per-capita GDP growth is from World Bank.
Table 2.1: Sample of housing market crashes
[cited on page 41]

<table>
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<th>Country code</th>
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<th>First 3 years</th>
<th>Peak to trough</th>
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<td>GBR</td>
<td>1973</td>
<td>1977</td>
<td>-23.5%</td>
<td>-28.9%</td>
<td>SGP</td>
<td>1996</td>
<td>1998</td>
<td>-32.2%</td>
<td>-33.9%</td>
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<tr>
<td>GBR</td>
<td>1989</td>
<td>1996</td>
<td>-21.9%</td>
<td>-30.0%</td>
<td>SRB</td>
<td>2010</td>
<td>2013</td>
<td>-29.4%</td>
<td>-29.4%</td>
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<tr>
<td>GBR</td>
<td>2007</td>
<td>2012</td>
<td>-16.0%</td>
<td>-23.0%</td>
<td>SVK</td>
<td>2008</td>
<td>2012</td>
<td>-21.4%</td>
<td>-26.1%</td>
</tr>
<tr>
<td>GRC</td>
<td>2007</td>
<td>2017</td>
<td>-15.1%</td>
<td>-44.8%</td>
<td>SVN</td>
<td>2011</td>
<td>2014</td>
<td>-20.6%</td>
<td>-20.6%</td>
</tr>
<tr>
<td>HKG</td>
<td>1981</td>
<td>1984</td>
<td>-46.8%</td>
<td>-46.8%</td>
<td>SWE</td>
<td>1979</td>
<td>1985</td>
<td>-26.3%</td>
<td>-34.8%</td>
</tr>
<tr>
<td>HKG</td>
<td>1997</td>
<td>2003</td>
<td>-42.2%</td>
<td>-57.3%</td>
<td>SWE</td>
<td>1990</td>
<td>1993</td>
<td>-30.4%</td>
<td>-30.4%</td>
</tr>
<tr>
<td>HRV</td>
<td>1999</td>
<td>2002</td>
<td>-14.3%</td>
<td>-14.3%</td>
<td>THA</td>
<td>2006</td>
<td>2009</td>
<td>-30.1%</td>
<td>-30.1%</td>
</tr>
<tr>
<td>HRV</td>
<td>2009</td>
<td>2015</td>
<td>-18.6%</td>
<td>-24.0%</td>
<td>USA</td>
<td>2006</td>
<td>2012</td>
<td>-14.2%</td>
<td>-25.8%</td>
</tr>
<tr>
<td>HUN</td>
<td>2006</td>
<td>2013</td>
<td>-16.7%</td>
<td>-36.7%</td>
<td>ZAF</td>
<td>1984</td>
<td>1987</td>
<td>-39.4%</td>
<td>-39.4%</td>
</tr>
<tr>
<td>IRL</td>
<td>2006</td>
<td>2012</td>
<td>-30.4%</td>
<td>-46.1%</td>
<td>ZAF</td>
<td>2007</td>
<td>2012</td>
<td>-16.1%</td>
<td>-19.1%</td>
</tr>
<tr>
<td>ISL</td>
<td>2007</td>
<td>2010</td>
<td>-32.3%</td>
<td>-32.3%</td>
<td>Median</td>
<td>5 years</td>
<td></td>
<td>-21.1%</td>
<td>-30.6%</td>
</tr>
</tbody>
</table>

Note: unbalanced panel of 50 countries, 1950-2017. Housing market boom-and-bust cycles are identified in 43 countries. The sample consists 63 events: 39 before 2006 and 24 during/after the GFC.

Housing market bubbles are defined as periods when the aggregate housing price index (1) deviates from the long-run trend by more than one standard deviation and (2) declines of at least 10% within the first three years from the peak.
Figure 2.3: An average housing market crash

[cited on page 43]

Note: cross-country panel estimation, controlling for country and year fixed effects, country-specific trends, and preexisting macroeconomic conditions. Responses of variables per 1% on-impact decrease in the aggregate house price.

Variables: (1) Real per-capita GDP; (2) Real per-capita consumption; (3) Real per-capita investment; (4) Real house price index; (5) Household debt-to-GDP gap; (6) Corporate non-financial debt-to-GDP gap; (7) Employment-to-population ratio; (8) Real per-capita capital stock; (9) Total factor productivity, utilization-adjusted.

Responses are estimated by local projections and are expressed in log deviations times 100. Shaded areas correspond to 90% confidence intervals. Standard errors are clustered at the country level.

See details in section 2.2.1
Figure 2.4: An average housing market crash, the role private debt

[Figure showing charts of variables over time]

Note: cross-country panel estimation, controlling for country and year fixed effects, country-specific trends, and preexisting macroeconomic conditions, as well as corporate and household credit-to-GDP gaps at the onset of the crash. Responses of variables per 1% on-impact decrease in the aggregate house price conditional on credit-to-GDP gaps.

Variables: (1) Real per-capita GDP; (2) Real per-capita consumption; (3) Real per-capita capital stock; (4) Total factor productivity, utilization-adjusted; (5) Employment-to-population ratio.

Responses are estimated by local projections and are expressed in log deviations times 100. Shaded areas correspond to 90% confidence intervals. Standard errors are clustered at the country level.

See details in section 2.2.1
Figure 2.5: US housing market crash and labor productivity growth

![Graph showing the US housing market crash and labor productivity growth.](image)

Figure 2.6: Elasticity of labor productivity growth to house price decline

![Graph showing the elasticity of labor productivity growth to house price decline.](image)

Note: Baseline specification is in red, see details in table 2.
Table 2.2: Elasticity of labor productivity growth to house price decline

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆ house price 2007-2010</td>
<td>0.18***</td>
<td>0.16**</td>
<td>0.17**</td>
<td>0.27***</td>
<td>0.18***</td>
<td>0.19</td>
<td>0.24</td>
<td>0.28***</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.03)</td>
<td>(0.07)</td>
<td>(0.16)</td>
<td>(0.14)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.05***</td>
<td>0.05***</td>
<td>0.05***</td>
<td>0.06***</td>
<td>0.05***</td>
<td>0.05***</td>
<td>0.05***</td>
<td>0.06***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Observations</td>
<td>383</td>
<td>250</td>
<td>250</td>
<td>380</td>
<td>325</td>
<td>209</td>
<td>209</td>
<td>322</td>
</tr>
<tr>
<td>Specification</td>
<td>OLS</td>
<td>IV1</td>
<td>IV2</td>
<td>IV3</td>
<td>OLS</td>
<td>IV1</td>
<td>IV2</td>
<td>IV3</td>
</tr>
<tr>
<td>Controls</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Excluding CA, FL, NV, AZ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|                                | (9)       | (10)      | (11)      | (12)      | (13)      | (14)      | (15)      | (16)      |
| ∆ house price 2007-2010        | 0.12***   | 0.13      | 0.13      | 0.28***   | 0.14**    | 0.29      | 0.32*     | 0.32***   |
|                                | (0.04)    | (0.10)    | (0.09)    | (0.05)    | (0.06)    | (0.22)    | (0.18)    | (0.09)    |
| GDP construction share, 2006   | -0.77**   | -0.93*    | -0.91*    | -0.23     | -0.03     | 0.56      | 0.60      | 0.42      |
|                                | (0.34)    | (0.56)    | (0.52)    | (0.38)    | (0.43)    | (0.71)    | (0.68)    | (0.46)    |
| Constant                       | 0.06      | -0.02     | -0.02     | 0.05      | 0.08      | -0.09     | -0.10     | 0.05      |
|                                | (0.06)    | (0.10)    | (0.10)    | (0.06)    | (0.07)    | (0.12)    | (0.13)    | (0.07)    |
| Observations                   | 312       | 200       | 200       | 310       | 257       | 160       | 160       | 255       |
| Specification                  | OLS       | IV1       | IV2       | IV3       | OLS       | IV1       | IV2       | IV3       |
| Controls                       | +         | +         | +         | +         | +         | +         | +         | +         |
| Excluding CA, FL, NV, AZ       |           |           |           |           |           |           |           |           |

Robust standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: labor productivity is defined as real GDP per worker. State and MSA output and employment data is from BEA. House price growth is based on FHFA all-transactions house price indexes. Controls: 2006 shares in GDP of (a) mining, quarrying, and oil and gas extraction; (b) construction; (c) manufacturing; (d) retail trade; as well as (e) 2006 per capital personal income. Reduced sample explains four states that were affected by the housing market crash the most.

Figure 2.7: Baseline model flow chart

[ cited on page 48 ]
<table>
<thead>
<tr>
<th>Table 2.3: Model summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>[cited on page 60]</td>
</tr>
<tr>
<td>1. Final good market clearing</td>
</tr>
<tr>
<td>2. Savers budget constraint</td>
</tr>
<tr>
<td>3. Intermediate good output</td>
</tr>
<tr>
<td>4-5. Savers/borrowers bond demand</td>
</tr>
<tr>
<td>6. Collateral constraint</td>
</tr>
<tr>
<td>7. Credit market clearing</td>
</tr>
<tr>
<td>8-9. Savers/borrowers labor supply</td>
</tr>
<tr>
<td>10. Disutility of labor</td>
</tr>
<tr>
<td>11. Labor demand</td>
</tr>
<tr>
<td>12. Labor market clearing</td>
</tr>
<tr>
<td>13. Capital supply</td>
</tr>
<tr>
<td>14. Tobin’s q</td>
</tr>
<tr>
<td>15. Capital law of motion</td>
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<tr>
<td>16-17. Capital utilization</td>
</tr>
<tr>
<td>18. Capital demand</td>
</tr>
<tr>
<td>19. Savers housing demand</td>
</tr>
<tr>
<td>20. Borrowers housing demand</td>
</tr>
<tr>
<td>21. Housing market clearing</td>
</tr>
<tr>
<td>22. Equity demand</td>
</tr>
<tr>
<td>23. Equity supply (free entry)</td>
</tr>
<tr>
<td>24. Innovators productivity</td>
</tr>
<tr>
<td>25. Intermediate firms profit</td>
</tr>
<tr>
<td>26. Growth rate</td>
</tr>
<tr>
<td>27. Markup</td>
</tr>
<tr>
<td>28. Taylor rule</td>
</tr>
<tr>
<td>29. Final output</td>
</tr>
</tbody>
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Table 2.4: Structural parameters
[cited on page 61]

<table>
<thead>
<tr>
<th>Calibrated parameters:</th>
<th>Source / target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_S$ Savers discount factor</td>
<td>0.9968  4% annual real interest rate</td>
</tr>
<tr>
<td>$\beta_B$ Borrowers discount factor</td>
<td>0.9963 $\beta_B = \beta_S - \varepsilon$</td>
</tr>
<tr>
<td>$\sigma$ Relative risk aversion</td>
<td>2 Conventional</td>
</tr>
<tr>
<td>$1/\epsilon_L$ Frisch elasticity of labor supply</td>
<td>4 King and Rebelo (1999)</td>
</tr>
<tr>
<td>$1/\epsilon_h$ Price elasticity of housing demand</td>
<td>0.2 Hanushek and Quigley (1980)</td>
</tr>
<tr>
<td>$m$ Max leverage</td>
<td>0.75 Warnock and Warnock (2008)</td>
</tr>
<tr>
<td>$\alpha$ Capital share</td>
<td>0.4 Data median, PWT 9.1</td>
</tr>
<tr>
<td>$\frac{\nu}{\nu-1}$ Intermediate-good elasticity of subst.</td>
<td>1.6 BGP requirement $\frac{\xi(\nu-1)}{\nu-\xi} = 1 - \alpha$</td>
</tr>
<tr>
<td>$\eta$ Retail-good elasticity of subst.</td>
<td>11 10% steady-state markup</td>
</tr>
<tr>
<td>$\xi$ Intermediate good share</td>
<td>0.5 Comin and Gertler (2006)</td>
</tr>
<tr>
<td>$1/A$ Intermediate sector marginal cost</td>
<td>1 Normalization</td>
</tr>
<tr>
<td>$\rho$ Innovation output elasticity</td>
<td>0.8 Comin and Gertler (2006)</td>
</tr>
<tr>
<td>$\delta_K$ Steady state capital depreciation</td>
<td>0.025 Conventional</td>
</tr>
<tr>
<td>$\delta_N$ Intermediate sector exit rate</td>
<td>0.025 Bilbiie et al. (2012)</td>
</tr>
<tr>
<td>$\phi_y; \phi_{\pi}; \rho_r$ Taylor rule parameters</td>
<td>0.25; 1.5; 0.7 Carare and Tchaidze (2005)</td>
</tr>
<tr>
<td>$\phi$ Innovators productivity</td>
<td>0.11 Annual TFP growth = 0.8% (data median, PWT 9.1)</td>
</tr>
<tr>
<td>$\kappa$ Share of housing in utility</td>
<td>0.03 Mortgage debt to GDP = 0.55, Warnock and Warnock (2008)</td>
</tr>
<tr>
<td>$\bar{Z}$ Final sector productivity</td>
<td>1.74 Normalization, $Y_{GDP} = 1$</td>
</tr>
<tr>
<td>$\psi_p$ Price adjustment cost</td>
<td>120 4-quarter average Calvo price ridigity equivalent</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimated parameters:</th>
<th>IRF matching</th>
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<tbody>
<tr>
<td>$\rho_b$ Borrowing limit inertia</td>
<td>0.67</td>
</tr>
<tr>
<td>$\rho_T$ Disutility of labor inertia</td>
<td>0.98</td>
</tr>
<tr>
<td>$\psi_K$ Investment adjustment cost</td>
<td>2</td>
</tr>
<tr>
<td>$\psi_N$ Innovation adjustment cost cost</td>
<td>0</td>
</tr>
<tr>
<td>$c_2$ Capital utilization responsiveness</td>
<td>0.075</td>
</tr>
</tbody>
</table>
Figure 2.8: Baseline simulation: model vs evidence
[cited on page 64]

Note: (1) Data: baseline local projection responses to a housing market crash; (2) Model: responses to a series of negative housing preference shocks calibrated to match the empirical real house price decline.

See details in section 2.4.3
Figure 2.9: Baseline simulation, extended set of impulse responses

[cited on page 64]

Note: impulse responses to a series of negative housing preference shocks calibrated to match the empirical real house price decline, as in figure 8. Inflation rate and the policy rate are annualized.

See details in section 2.4.3.
Figure 2.10: Baseline simulation, model-based decompositions

Note: model-based decompositions of the output and TFP dynamics in figure 8, see details in section 2.3.4.

Left panel, growth accounting: $\Delta Y_t = \Delta TFP_t + \alpha \Delta K_t + (1 - \alpha) \Delta L_t$

Right panel, Solow residual: $\Delta TFP_t = \underbrace{\Delta \Omega_t}_{\text{Markup effect}} + \underbrace{\alpha \Delta \nu_t}_{\text{Utilization effect}} + \underbrace{(1 - \alpha) \Delta N_t}_{\text{Innovation effect}}$
Figure 2.11: Baseline simulation, the role household debt

(a) Sensitivity to the initial debt to GDP ratio

(b) Sensitivity to the initial loan-to-value ratio

Note: counterfactual responses to the baseline in figure 9. Inflation rate and the policy rate are annualized.
Figure 2.12: Sign-dependent effects of housing preference shocks
[cited on page 66]

Note: baseline simulation and responses to a symmetric sequence of positive shocks. Inflation rate and the policy rate are annualized.
Figure 2.13: Baseline simulation, aggregate demand channel

(a) The effect of nominal rigidity  

(b) The effect of a binding zero lower bound

Note: counterfactual responses to the baseline in figure 9. Panel (a): flexible price counterfactual by setting $\psi_p = 0$. Panel (b): net effect of the housing preference shock contingent on a savers discount rate shock that makes the zero lower bound bind for the first 8 quarters. See details in section 2.5.1.
**Figure 2.14:** Baseline simulation, endogenous growth channel

(a) No endogenous response of productivity growth [cited on page 69]

(b) Model-consistent equity market equilibrium

*Note:* counterfactual responses to the baseline in figure 9. Panel (a): counterfactual without growth response by setting $\psi_N = 10^5$. Panel (b): intermediate-firm equity market dynamics consistent with the baseline simulation. See details in appendix B.3.3 and section 2.5.2.
Figure 2.15: Baseline simulation, Fisherian debt deflation channel

(a) No housing reallocation effect in the borrowing constraint [cited on page 71]

(b) Model-consistent housing market equilibrium

Note: counterfactual responses to the baseline in figure 9. Panel (a): counterfactual by fixing the housing quantity in the borrowing limit $B^B_{t+1}/P_t \leq \rho_B m P^h_t h^B$. Panel (b): housing market dynamics consistent with the baseline simulation. See details in appendix B.3.2 and section 2.5.3.
Figure 2.16: Baseline simulation, expected income growth channel

Expected growth of marginal utility of consumption [cited on page 72]

Note: Decomposition of the marginal utility of consumption growth, baseline simulation in figure 9.

Savers: $E_t g^{S}_{X_{t+1}} = E_t \hat{\lambda}^S_{t+1} - \lambda^S_t = -E_t R_{t+1} + \sigma g_{t+1}$
Borrowers: $E_t g^{B}_{X_{t+1}} = E_t \hat{\lambda}^B_{t+1} - \lambda^B_t = -E_t R_{t+1} + \sigma g_{t+1} - A^{B}_{11}X_t + A^{B}_{12}E_t(X_{t+1} - R_{t+1} - \Pi_{t+1})$

See details in appendix B.3.4 and section 2.5.4.
Figure 2.17: Baseline simulation, welfare cost and monetary policy

(a) Model with endogenous growth (% of steady-state cons.)  [cited on page 73]

(b) Model without endogenous growth (% of steady-state consumption)

Note: counterfactual scenarios to the baseline in figure 9 under different parameters of the Taylor rule: $1 + r_t = (1 + r_{t-1})^{\rho_r} \left( (1 + r) \left( \frac{y^GDP}{y^GDP} \right)^{\phi_Y} \left( \frac{\Pi_t}{\Pi} \right)^{\phi_\Pi} \right)^{1-\rho_r}$. The welfare cost is expressed in steady-state consumption equivalent under different values of parameters that govern the policy rate reaction to output ($\phi_Y$) and inflation ($\phi_\Pi$). In all cases the interest rate inertia is set to the baseline value of $\rho_r = 0.7$. Dashed lines mark baseline values of parameters. See details in section 2.6.
Figure 2.18: Baseline simulation, debt relief policy

(a) Baseline simulation vs policy  [cited on page 74]

(b) Net effect of the policy

<table>
<thead>
<tr>
<th>Y</th>
<th>C</th>
<th>L</th>
<th>Debt</th>
<th>OBC multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Baseline" /></td>
<td><img src="image2" alt="Debt relief policy" /></td>
<td><img src="image3" alt="Baseline" /></td>
<td><img src="image4" alt="Debt relief policy" /></td>
<td><img src="image5" alt="Baseline" /></td>
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</table>

<table>
<thead>
<tr>
<th>Util.-adjusted TFP</th>
<th>K</th>
<th>Inflation</th>
<th>Policy rate</th>
<th>Housing pref. shock</th>
</tr>
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<tbody>
<tr>
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<td><img src="image8" alt="Debt relief policy" /></td>
<td><img src="image9" alt="Baseline" /></td>
<td><img src="image10" alt="Debt relief policy" /></td>
<td><img src="image11" alt="Baseline" /></td>
</tr>
</tbody>
</table>

Note: counterfactual scenario to the baseline in figure 9. Inflation rate and the policy rate are annualized. Debt relief policy consists of a temporary transfer from savers to borrowers equivalent to a 0.25% of borrowers steady-state debt burden implemented at the time of the shock. See details in section 2.6.
Chapter 3

SCARRING EFFECTS OF TRADE POLICY UNCERTAINTY

with Fabio Ghironi

3.1 Introduction

The recent surge in the global trade policy uncertainty driven by the US protectionism is unprecedented (figure 3.1). The average level of the trade policy uncertainty index of Baker et al. (2016) since 2016 has increased by around 500% from its average level in 2000-2015. During the same period, the average contribution of trade policy uncertainty to the US stock market volatility has increased more than five-fold from 2% to 11%, according to Baker et al. (2019). Policymakers, investors, and executives routinely cited the trade policy uncertainty as one of the biggest threats to the economy at the time.\textsuperscript{1} The research on aggregate effects of economic uncertainty that followed the seminal contribution of Bloom (2009) supports this point of view. Empirically, economic uncertainty, various measures of which are procyclical and fluctuate considerably over time, appears to be a quantitatively important driver of business cycles (figure 3.2).\textsuperscript{2}

This paper contributes to this literature by studying the effects of import tariff uncertainty on macroeconomic fluctuations. To that end, we develop a small New Keynesian open-economy model deviating from the standard setting in two ways. First, import tariff is subject not only to standard level shocks but also to second-moment shocks (uncertainty

\begin{footnotesize}
\textsuperscript{1} For instance, see the World Economic Outlook update by IMF (2019): “[G]lobal trade, investment, and output remain under threat from policy uncertainty, as well as from other ongoing trade tensions. Failure to resolve differences and a resulting increase in tariff barriers would lead to higher costs of imported intermediate and capital goods and higher final goods prices for consumers. Beyond these direct impacts, higher trade policy uncertainty and concerns over escalation and retaliation would lower business investment, disrupt supply chains, and slow productivity growth.”

\textsuperscript{2} See also Bloom et al. (2018), Gilchrist et al. (2014), and Jurado et al. (2015), among others.
\end{footnotesize}
shocks). To assess the effect of uncertainty shocks, we solve the model by perturbation methods based on a third-order approximation of its equilibrium conditions. Second, total factor productivity (TFP) in the model economy grows endogenously through forward-looking investment in the introduction of new products financed by equity. This feature provides a channel through which temporary uncertainty shocks can generate persistent effects on economic activity. The empirical literature indeed supports this relation. Specifically, the seminal contribution by Ramey and Ramey (1995) points out the robust negative relation between macroeconomic volatility and growth. To further motivate this modeling choice, we estimate a quarterly US VAR that includes the following seven variables: a measure of economic uncertainty, real GDP, TFP, R&D spending, trade balance to GDP, real exchange rate, and the S&P 500 stock market index. As common in the literature, we use the CBOE S&P 100 volatility index (VXO) as a proxy to economic uncertainty and identify the uncertainty shock by short-run restrictions ordering this variable first. Figure 3.3 shows the responses to a stock market volatility shock. Although the increase in volatility is very short-lived and subsides after less than 10 quarters the corresponding decline in the GDP level is very persistent and about half of it can be attributed to the decline in the total factor productivity. Moreover, the shock is associated with the improvement of the trade balance and appreciation of the real exchange rate.

We use the model as a laboratory for an in-depth study of the effects and propagation channels of domestic import tariff uncertainty shocks. Simulations suggest that import tariff uncertainty shocks work as aggregate supply shocks: they are contractionary and inflationary. In the short run, they cause the current account to improve, whereas the real exchange rate depreciates on impact and then appreciates in the medium-run. Despite the uncertainty shock being transitory, it leaves a persistent scarring effect on the level of output and consumption by endogenously slowing down productivity growth. The results are robust to the type of nominal rigidity, or lack of thereof.

Our exercise highlights the crucial role of endogenous risk premia in shaping the dynamics. An increase in import tariff uncertainty causes foreign-currency bonds to become riskier relative to domestic bonds. As a result, the demand for foreign-currency bonds decreases, and the current account temporarily improves. A similar logic applies product creation

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3 See also Aghion et al. (2010), Badinger (2010), Martin and Ann Rogers (2000), and Mobarak (2005), among others.
financed by selling equity to households. The uncertainty shock increases the equity risk premium leading to a fall in the stock market value and a decrease in productivity-enhancing investment.

We conclude exploring the role of monetary policy in shaping the effects of the shock. The conduct of monetary policy affects the dynamics mainly through influencing the stochastic properties of firm equity returns. The Taylor rule with more focus on inflation stabilization, as opposed to output stabilization, introduces more equity risk and amplifies the productivity loss associated with the shock. Doing away with independent monetary policy and pegging the nominal exchange rate to the foreign currency eliminates this additional risk and dampens the long-run effect of the shock.

This paper relates to several strands of literature. First, we contribute to the literature on the effects of protectionism and trade policy uncertainty. Second, we contribute to the literature that studies the effects of uncertainty shocks on economic activity. Finally, we contribute to the literature on the relation between business cycles and endogenous growth. Two papers are the closest to ours. Bonciani and Oh (2019) also consider the relation between uncertainty shocks and endogenous growth. Unlike the present work, their study focuses on the role of endogenous growth in amplifying the effects of demand uncertainty shocks in a closed-economy model with Epstein-Zin preference. Caldara et al. (2020) also study imports tariff uncertainty shocks using a dynamic general equilibrium model. This paper compliment theirs by focusing on the risk-premium channel of uncertainty shocks, its effect on product creation, and their interactions with monetary policy.

The rest of the paper is organized as follows. Section 2 lays down the model. Section 3 discusses calibration, simulation technique, and baseline results. Section 4 explores the main channels of the transmission mechanism of import tariff uncertainty shocks. Section 5 concludes.

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3.2 The model

In this section we develop a small open economy model that we use to study macroeconomic effects of import tariff uncertainty shocks. The model features nominal price and wage rigidity with exports prices set in the domestic currency. International financial markets are incomplete and the home economy borrows from the rest of the world in foreign currency. Finally, the economy features endogenous growth through introduction of new varieties of intermediate products as in Romer (1990) and Comin and Gertler (2006). Figure 3.4 summarizes the structure of the model. Below we describe in detail the problems facing agents in the model economy.

3.2.1 Households

The home economy is populated with households \( h \in [0, 1] \), each of which consumes a basket of Home and Foreign goods, and supplies labor. International asset markets are incomplete, non-contingent nominal bonds denominated in Foreign currency \( B_t^*(h) \) are the only internationally traded asset. Households also trade non-contingent nominal bonds denominated in Home currency \( B_t(h) \), which are only traded domestically. To pin down the steady-state net foreign asset position and stationary responses of the economy to temporary shocks, we follow Turnovsky (1985) and assume a quadratic cost of adjusting Foreign bond holding \( AC_t^{B^*}(h) = \psi_B \left( \frac{B^*(h)_{t+1}}{P_t N_t} \right)^2 N_t rer_t \) denominated in foreign consumption units.\(^7\)

Households are implicit owners of all firms in the economy. In particular, at time \( t \) each household buys \( \iota_{t+1}(h) \) shares in a mutual fund of \( N_t + N_{et} \) existing intermediate firms at the price of \( v_t \) per share and receives dividend income, \( d_t(h) \) on shares from the previous period. Entrants at time \( t \) start producing and paying dividends only in time \( t+1 \). In each period existing firms and entrants are subject to an exit shock with probability \( \delta_N \).

As in Erceg et al. (2000), each household exert monopolistic power over the imperfectly substitutable labor variety it supplies. Households set the nominal wage \( W_t(h) \) subject to firms demand \( L_t(h) = (W_t(h)/W_t)^{-\theta_L} L_t \) and the quadratic adjustment cost of adjusting the

\(^7\) Since the economy exhibits growth, the bond adjustment cost is scaled by the stock of intermediate products \( N_t \), which determines the BGP growth rate.
nominal wage $AC^W_t(h) = \frac{\psi^\Pi}{2} \left( \frac{W_t(h)}{W_{t-1}(h)\Pi_t} - 1 \right)^2 \frac{W_t(h)}{P_t} L_t(h)$, where $\psi^\Pi \geq 0$ governs the degree of nominal wage rigidity and $\Pi^W_t$ is the steady-state gross inflation rate.

A generic household $h$ maximizes its intertemporal utility

$$\max_{\{C_{t+k}(h), L_{t+k}(h), W_{t+k}(h)\}} \sum_{k=0}^{\infty} \beta^k U(C_{t+k}(h), L_{t+k}(h))$$

subject to the following period budget constraint and firms labor demand:

$$C_t(h) + \iota_{t+1}(h) v_t(N_t + N_{et}) + \frac{B_{t-1}(h)}{P_t} (1 + i_t) + \frac{\varepsilon_t B^*_t(h)}{P_t} (1 + i_t^*) + AC^B_t(h) + AC^W_t(h) =$$

$$= \frac{W_t(h)}{P_t} L_t(h) + \iota_t(h) (v_t + d_t) N_t + \frac{B_t(h)}{P_t} + \frac{\varepsilon_t B^*_t(h)}{P_t} + d_t^w(h) + T(h)$$

$$L_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\theta_L} L_t,$$

where $i_t$ and $i_t^*$ are Home and Foreign nominal interest rates, $d_t^w(h)$ are profits from owing domestic wholesale firms, and $T$ is the lump-sum rebate of the collected import tariffs and bond holding adjustment costs, which the households takes as given.

In order to simplify the notation, we anticipate symmetry of the equilibrium across households and omit the index $h$ below. First order conditions with respect to Home and Foreign bonds imply:

$$\mathbb{E}_t \left( \beta_{t, t+1} \frac{1 + r_t}{\Pi_{t+1}} \right) = 1$$

(3.1)

$$\mathbb{E}_t \left( \beta_{t, t+1} \frac{rer_t + 1 + r_t^*}{rer_t + \Pi_{t+1}^*} \right) = 1 - \psi_B \frac{B^*_t}{P^*_t N_t}$$

(3.2)

where $\beta_{t, t+1} = \beta^{U^*_t C_{t+1}}_{C_t}$ is the households stochastic discount factor and $rer_t = \frac{r_t}{P_t}$ is the real exchange rate. Equity demand following from the first order condition with respect to share holdings $\iota_{t+1}$ is:

$$v_t = (1 - \delta) \mathbb{E}_t [\beta_{t, t+1} (d_{t+1} + v_{t+1})]$$

(3.3)

When iterated forward, this expression implies that the present firm value equals to its expected discounted profit stream, adjusted for the fact that in each period a firm faces an exogenous probability of exiting the market $\delta$: $v_t = \mathbb{E}_t \sum_{k=1}^{\infty} \beta_{t, t+k} (1 - \delta)^k d_{t+k}$. 
Finally, households’ monopolistic power in labor supply and the nominal wage adjustment cost imply a time-varying markup \( \mu^W_t \) that introduces a wedge between the real wage and the marginal rate of substitution between consumption and labor:

\[
\frac{W_t}{P_t} = \mu^W_t \frac{U'_t}{U'_C_t} \\
\mu^W_t = \frac{\theta_L}{(\theta_L - 1) \left(1 - \psi_W \left(\frac{\Pi^W_t}{\Pi^W - 1}\right)^2\right) + \psi_W \left(\frac{\Pi^W_t}{\Pi^W - 1}\right) \Pi^W_t - \psi W E_t \left[\frac{\beta_{t+1}}{\Pi_{t+1}} \left(\frac{\Pi^W_{t+1}}{\Pi^W_{t+1} - 1}\right) \Pi^W_{t+1} \frac{2 L_{t+1}}{L_t}\right]}
\]

(3.4)

(3.5)

### 3.2.2 Production

**Final good basket**

The final consumption-investment basket is a composite of Home and Foreign goods:

\[
Y_t = \left[a^\theta (Y^H_t)^{\theta_H-1} + (1-a)^\theta (Y^F_t)^{\theta_F-1}\right]^{\frac{1}{\theta_H-1}},
\]

where \( Y^H_t = \left(\int_0^1 Y^H_t(j)^{\theta_H-1}d_j\right)^{\frac{1}{\theta_H-1}} \) is the aggregate Home good sold domestically, \( Y^F_t = \left(\int_0^1 Y^F_t(i)^{\theta_F-1}d_i\right)^{\frac{1}{\theta_F-1}} \) is the aggregate Foreign good, \( a > 0.5 \) is the home-bias parameter, \( \theta \) governs the elasticity of substitution between Home and Foreign goods, and \( \theta_H \) governs the elasticity of substitution between varieties of these goods. Let \( P^H_t \) denote the domestic price of the aggregate Home good, \( P^F_t \) denote the dock price of the aggregate Foreign good denominated in the domestic currency, and \( \tau_t \) denote the domestic import tariff. The intratemporal optimization problem associated with the above basket then implies the aggregate price index \( P_t = \left[a^\theta (P^H_t)^{1-\theta} + (1-a)(\tau_t P^F_t)^{1-\theta}\right]^{\frac{1}{1-\theta}} \), and demands for Home and Foreign goods are \( Y^H_t = a \left(\frac{P^H_t}{P_t}\right)^{-\theta} Y_t \), and \( Y^F_t = (1-a) \left((1+\tau_t)\frac{P^F_t}{P_t}\right)^{-\theta} Y_t \) respectively.

From now on, we denote foreign variables with a star and use lower-case price indices to denote relative prices, i.e. \( \pi^H_t = \frac{P^H_t}{P_t} \). Let \( \epsilon_t \) be the nominal exchange rate, then \( \text{rer}_t = \frac{\epsilon^* P^*}{\epsilon P} \) is the real exchange rate. Also assume that the law of one price (LOP henceforth) holds, i.e.
\( P_t^F = \epsilon_t P_t^{F*} \). Demand functions and the aggregate price index can be simplified as follows:

\[
Y_t^H = a(p_t^H)^{-\theta} Y_t \\
Y_t^F = (1 - a)(\tau_t p_t^{F*} rer_t)^{-\theta} Y_t \\
a(p_t^H)^{1-\theta} + (1 - a)(\tau_t p_t^{F*} rer_t)^{1-\theta} = 1
\]

(3.6)  
(3.7)  
(3.8)

Wholesalers

Each monopolistically competitive wholesale firm \( j \in (0, 1) \) purchases homogeneous production sector good \( F_t \) at the price \( P_t^G \) and produces a differentiated variety sold to both home and foreign markets. Prices are sticky in the currency of the producer. That is, wholesalers set prices in domestic currency for the home of foreign markets, \( P_t^H(j) \) and \( P_t^{H*}(j) \) respectively, subject to nominal frictions. Following Rotemberg (1982), prices are sticky due to the presence of a quadratic price adjustment cost \( AC_t^P(j) = \frac{\psi_P}{2} \left( \frac{P_t^H(j)}{P_{t-1}(j)H^*} - 1 \right)^2 \frac{P_t^H(j)Y_t^H(j)}{p_t} \) for the domestic market and \( AC_t^{P*}(j) = \frac{\psi_{P*}}{2} \left( \frac{P_t^{H*}(j)}{P_{t-1}(j)H^{**}} - 1 \right)^2 \frac{P_t^{H*}(j)Y_t^{H*}(j)}{p_t} \) for the foreign market. Adjustment costs are paid in units of the aggregate basket, parameters \( \psi_P \geq 0 \) and \( \psi_{P*} \geq 0 \) govern the degree of nominal rigidity, \( \Pi^H \) and \( \Pi^{H*} \) are steady-state home-good inflation rates for the home and foreign markets.

Each wholesaler sets prices for the domestic and foreign markets to maximize the future expected stream of its period profits \( d_t^P(j) \):

\[
\max_{\left\{ P_t^H(j)_{t+k}, P_t^{H*}(j)_{t+k} \right\}} \mathbb{E}_t \sum_{k=0}^{\infty} \beta_{t,t+k} \left[ \frac{P_t^H(j)_{t+k}}{P_t} Y_{t+k}(j) + \frac{P_t^{H*}(j)_{t+k}}{P_t} Y_{t+k}^{H*}(j) - \frac{P_t^G}{P_t} F_{t+k}(j) - AC_{t+k}(j) - \Gamma \right],
\]

subject to demands \( Y_t^H(j) = (P_t^H(j)/P_t^H)^{-\theta} Y_t^H \) and \( Y_t^{H*}(j) = (P_t^{H*}(j)/P_t^{H*})^{-\theta^{H*}} Y_t^{H*} \), where the price for the foreign market is \( P_t^{H*} = P_t^{H*}(j)/\epsilon_t \); the resource constraint \( Y_t^H(j) + Y_t^{H*}(j) = F_t(j) \); and price adjustment costs \( AC_t(j) = AC_t^P(j) + AC_t^{P*}(j) \). To ensure zero steady state profit in this sector and rule out entry, I assume that production involves a fixed cost \( \Gamma = \frac{1}{\theta^H} (p^H Y^H + p^{H*} Y^{H*}) \).

Conjecturing a symmetric equilibrium where all wholesale firms are alike, we drop the index \( j \) below. The optimal relative price on the domestic and the foreign markets is a
time-varying markup over the real marginal cost $p^G_t$:

$$ p^H_t = \mu^H_t p^G_t $$

$$ p^{H^*}_t = \mu^{H^*}_t \frac{p^G_t}{rer_t} $$

where markups on the home and foreign markets are

$$ \mu^H_t = \frac{\theta_H}{(\theta_H - 1) \left(1 - \frac{\psi_p}{2} \left(\frac{\Pi^H_H}{\Pi^{H^*}_{H^*}} - 1\right)^2\right) + \psi_P \frac{\Pi^H_H}{\Pi^{H^*}_{H^*}} \left(\frac{\Pi^H_H}{\Pi^{H^*}_{H^*}} - 1\right) - \psi_P E_t \left[\frac{\beta_t}{\Pi^{H^*}_{t+1} - 1}\right]} $$

$$ \mu^{H^*}_t = \frac{\theta_{H^*}}{(\theta_{H^*} - 1) \left(1 - \frac{\psi_{P^*}}{2} \left(\frac{\Pi^{H^*}_{H^*}}{\Pi^{H^*}_{H^*}} - 1\right)^2\right) + \psi_{P^*} \frac{\Pi^{H^*}_{H^*}}{\Pi^{H^*}_{H^*}} \left(\frac{\Pi^{H^*}_{H^*}}{\Pi^{H^*}_{H^*}} - 1\right) - \psi_{P^*} E_t \left[\frac{\beta_t}{\Pi^{H^*}_{t+1} - 1}\right]} $$

When $\psi_p = 0$ wholesale prices are flexible and the markup is constant over the business cycle $\mu^H = \frac{\theta_H}{\theta_H - 1}$, and $\mu^{H^*} = \frac{\theta_{H^*}}{\theta_{H^*} - 1}$. Home good inflation for both home and foreign markets can be expressed as follows:

$$ \Pi^H_t = \frac{p^H_t}{p^H_{t-1}} = \frac{p^H_t}{\Pi^H_t} \Pi_t $$

$$ \Pi^{H^*}_t = \frac{p^{H^*}_t}{p^{H^*}_{t-1}} = \frac{p^{H^*}_t}{rer_t} \Pi_t $$

**Domestic producers**

A representative firm in this perfectly competitive sector operates a production function that combines a unit mass of differentiated labor, $L_t = \left(\int_0^1 L_t(h)^{(\theta_L - 1)/\theta_L} dh\right)^{\theta_L/(\theta_L - 1)}$ and bundle of differentiated intermediate product $x_t(\omega) \in [0, N_t]$, $X_t = \left(\int_0^{N_t} x_t(\omega)^{(\theta_x - 1)/\theta_x} d\omega\right)^{\theta_x/(\theta_x - 1)}$.

$$ F_t(L_t, X_t) = Z_t L_t^G X_t^{1-\alpha} = Z_t L_t^G \int_0^{N_t} x_t(\omega)^{1-\alpha} d\omega $$

For growth to be of labor-augmenting form, the elasticity of substitution between varieties of the differentiated intermediate input should equal the inverse of labor share $\theta_x = \frac{1}{\alpha}$. 

As described further, positive externalities in developing blueprints of intermediate goods generate balanced growth. Following Romer (1990) endogenous growth model, the expanding mass of intermediate products, \( N_t \), brings about efficiency gains to diversity implied by the CES aggregator and increases the measured TFP.

Given input prices the representative firm maximizes its profit:

\[
\max_{x_t(\omega), L_t} \left[ \frac{P_t^G}{P_t} Z_t L_t^\alpha \int_0^{N_t} x_t(\omega)^{1-\alpha} d\omega - \int_0^{N_t} \frac{P_t^x(\omega)}{P_t} x_t(\omega) d\omega - \frac{W_t}{P_t} L_t \right]
\]

Resulting input demands are the following:

\[
\frac{W_t}{P_t} = p_t^G \alpha Z_t L_t^{\alpha-1} \int_0^{N_t} x_t(\omega)^{1-\alpha} d\omega \tag{3.16}
\]

\[
p_t^x(\omega) = p_t^G (1 - \alpha) Z_t L_t^\alpha x_t(\omega)^{-\alpha}
\]

In addition, the CES labor aggregator \( L_t \) implies demand for an individual labor variety of a standard form: \( L_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\theta_L} L_t \).

**Intermediate good sector**

The intermediate sector is populated with monopolistically competitive firms \( \omega \in [0, N_t] \), each of which requires \( A^{-1} \) units of the aggregate final good to produce a unit of intermediate good. One should not take this setup literally. The correct interpretation of this formal description is that the forgone final good is never manufactured. The resources that would have been used to produce the forgone output are used instead to manufacture intermediate goods.

In order to simplify the notation, we anticipate symmetry of the equilibrium across intermediate firms and omit the index \( \omega \) from now on. Each intermediate sector firm maximizes profit subject to production sector’s demand:

\[
\max_{\{P_t^x\}} \sum_{j=0}^\infty \beta_{t+1+j} \left[ \frac{P_t^{x,j}}{P_t^{x,j}} x_{t+j} - \frac{P_t^{H,j}}{P_t^{x,j}} x_{t+j} - A \right] \quad \text{s.t.} \quad p_t^x = p_t^G (1 - \alpha) Z_t L_t^\alpha x_t^{-\alpha}
\]

From this problem it follows that the optimal quantity of a generic intermediate good, \( x_t \),
its price, $p_t^x$, and the firm’s real profit, $d_t$, are the following:

$$x_t = \left( A(1 - \alpha)^2 Z_t / \mu_t^H \right)^{\frac{1}{\alpha}} L_t \tag{3.17}$$

$$d_t = \alpha p_t^x x_t = \frac{\alpha}{(1 - \alpha)A} p_t^H x_t \tag{3.18}$$

$$p_t^x = \frac{1}{1 - \alpha} \frac{p_t^H}{A}$$

Positive profit in this sector motivates entry. To open a firm an entrepreneur needs to buy a blueprint for a new variety at the price of $p_t^b$ from innovators, which is financed by selling equity shares to households. Free entry to this sector implies that in equilibrium firms value equals the blueprint price $v_t = p_t^b$

Innovators

The sector of innovators involves inventing blueprints for new types of intermediate goods. The sector is populated with the unbounded mass of potential innovators. Let $S_t$ be the total innovation spending and $\phi_t^i$ be the innovators individual productivity parameter, which each innovator take as given. The individual production function blueprints of intermediate goods is then $N_t^i = \phi_t^i S_t^i$. However, the aggregate innovators productivity $\phi_t$ depends on the existing stock of knowledge, measured by the number of existing intermediate goods, $N_t$. As in Romer (1990), this knowledge spillover externality is responsible for the existence of the balanced growth path in the model. In line with Comin and Gertler (2006), I include a congestion externally $N_t^\rho S_t^{1-\rho}$ that allows to control for the aggregate elasticity of blueprints output with respect to innovation spending. The resulting aggregate innovators productivity is:

$$\phi_t = \phi \frac{N_t}{N_t^\rho S_t^{1-\rho}},$$

where $S_t = \int S_t^i di$. The aggregate production function of innovators is then $N_{et} = \phi N_t \left( \frac{S_t}{N_t} \right)^\rho$. Perfectly competitive innovators set the price of blueprints $p_t^b$ to maximize their expected discounted stream of profit:

$$\max_{\{s_{t+j}^i\}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{t+j} \left( p_{t+j}^b \phi_{t+j} S_{t+j}^i - S_{t+j}^i \right)$$
The optimal price then is \( p^b_t = \phi_t^{-1} \). Together with the intermediate sector free-entry condition this leads to the following expression that pins down intermediate firm value:

\[
v_t = \phi_t^{-1} \left( \frac{S_t}{N_t} \right)^{\rho-1}
\]

(3.19)

As in Bilbiie et al. (2012), we assume a time-to-build lag: newly invented blueprints are adopted with a one-period lag. At each period existing varieties of the intermediate good face a constant probability of becoming obsolete \( \delta_N \). The resulting law of motion for the stock of intermediate goods is \( N_t = (1 - \delta)(N_{t-1} + N_{et}) \), where \( N_{et} = \int N_{et}^i di \). The positive knowledge spillover externality in the innovation sector gives rise to variety-driven endogenous growth, which rate equals to:

\[
g_t = \frac{N_t}{N_{t-1}} = (1 - \delta) \left( 1 + \phi \left( \frac{S_t}{N_t} \right)^\rho \right)
\]

(3.20)

The endogenous growth rate \( g_t \) varies over the business cycle depending on the level of innovation spending \( S_t \), which is determined in general equilibrium.

3.2.3 Monetary policy

The economy operates under the flexible exchange rate with monetary policy conducted through interest rate setting. The domestic policy rate is governed by the following Taylor rule:

\[
1 + i_t = \left( \frac{1 + i_{t-1}}{1 + \bar{i}} \right)^{\rho_i} \left( \frac{\Pi_t}{\Pi} \right)^{\phi_{\Pi}} \left( \frac{GDP_t}{GDP_t^{BGP}} \right)^{\phi_Y} \left( \frac{\phi_Y}{1 - \rho_i} \right),
\]

(3.21)

Parameters \( \phi_Y \geq 0 \) and \( \phi_{\Pi} > 0 \) govern the policy response to changes in GDP and inflation; \( \rho_i \in [0, 1] \) determines the degree of interest rate smoothing. Note that the above rule responds to changes in the domestic GDP relative to its balanced growth path. In other words, the monetary authority does not respond to endogenous changes in the productivity growth, which is consistent with how central banks tend to conduct monetary policy.
3.2.4 Rest of the world

The rest of the world is modeled in a reduced form, with foreign variables taken by the small open economy as given. In particular, the foreign real interest rate is $\frac{1 + \pi^*_t}{\Pi_t}$; the foreign-good price in foreign currency is $p^{F*}$; and the foreign economy is assumed to supply as much traded good as demanded by the home economy.

The aggregate consumption-investment basket in Foreign is assumed to have a symmetric form $Y^*_t = \left[a^{\frac{1}{\theta}}(Y^F_t)^{\frac{\theta-1}{\theta}} + (1 - a)^{\frac{1}{\theta}}(Y^H_t)^{\frac{\theta-1}{\theta}}\right]^\theta$, which implies that the exports demand for Home is $Y^H_t = (1 - a) \left(p^{H*}_t\right)^{-\theta} Y^*_t$. Given the real exchange rate definition and the LOP assumption it can be rearranged as:

$$Y^H_t = (1 - a) \left(p^{H*}_t\right)^{-\theta} Y^*_t \quad (3.22)$$

Finally, the foreign economy is assumed to be growing at a constant rate equal to the steady-state growth rate of the home economy $g^* = g$. For simplicity, we assume no technological spillovers between Home and Foreign. As such, any fluctuations in the growth rate in Home lead to permanent differences in levels. Define the ratio of Home to Foreign intermediate good varieties as $n_t = \frac{N^*_t}{N_t}$, then:

$$\frac{n_t}{n_{t-1}} = \frac{N^*_t}{N^*_{t-1}} \frac{N_{t-1}}{N_t} = g^* \ \ g_{t-1} \quad (3.23)$$

3.2.5 Closing the model

The description of equilibrium conditions of the model is concluded with the following accounting identities. The aggregate basket of domestic and imported goods is used for consumption, investment in product creation, and paying adjustment costs:

$$Y_t = C_t + S_t + AC^W_t + AC^P_t + AC^{P*}_t \quad (3.24)$$

The domestic good is demanded both from home and foreign economies:

$$GDP_t = F_t - N_t \frac{x_t}{A} = Y^H_t + Y^{H*}_t \quad (3.25)$$
As described in appendix C.2.1, equilibrium conditions imply that the dynamics of net foreign assets is governed by the following accounting identity:

\[
p_t^F Y_t^F - p_t^H Y_t^H = \frac{B_t^*}{P_t^*} - \frac{1 + i_{t-1}^*}{\Pi_{t-1}^*} \tag{3.26}
\]

Finally, a representative household owns all intermediate firms \( \iota_{t+1} = \iota_t = 1 \), and is being rebated collected foreign bond adjustment costs along with domestic import tariff \( T = \tau_t p_t^r Y_t^F + AC_t^B \). Domestic bonds are in zero net supply \( B_t = 0 \) \( \forall t \).

Equilibrium conditions allow to simplify the production function as \( F_t = \Omega N_t Z_t^{\frac{1}{\alpha}} \left( \mu_t^G \right)^{\frac{\alpha - 1}{\alpha}} L_t \), where \( \Omega = A^{\frac{1-\alpha}{\alpha}} \left( 1 - \alpha \right) \frac{2(1-\alpha)}{\alpha} \). Changes in domestic output are ultimately driven by the following factors:

\[
\Delta F_t = \Delta TFP_t + \Delta L_t
\]
\[
\Delta TFP_t = \underbrace{\frac{1}{Q} \Delta Z_t}_{\text{TFP shock}} + \underbrace{\frac{\Delta N_t}{\alpha}}_{\text{Innovation}} - \underbrace{\frac{1 - \alpha}{\alpha} \Delta \mu_t^G}_{\text{Markup}}
\]

The model features endogenous growth at the rate of \( g_{t+1} = \frac{N_{t+1}}{N_t} \). To solve the model using perturbation techniques, I define stationary lower-case counterparts to the variables that exhibit growth as \( y_t = \frac{Y_t}{N_t}, c_t = \frac{C_t}{N_t} \). Note that real wage \( \frac{W_t}{P_t} \) exhibits growth due to endogenous productivity improvements. As such, I define detrended real wage as \( w_t = \frac{W_t}{P_t N_t} \), from which it follows that nominal wage inflation can be expressed as

\[
\Pi_t^W = \frac{w_t}{w_{t-1} g_{t-1} \Pi_t}. \tag{3.27}
\]

**Equilibrium definition**: equations (1-27) determine 27 endogenous variables \( (y_t, x_t, y_t^H, y_t^H^*, y_t^F, f_t, c_t, \Pi_t, \Pi_t^H, \Pi_t^H^*, \Pi_t^W, p_t^H, p_t^H^*, p_t^F, p_t^H, p_t^H^*, s_t, v_t, d_t, g_t, n_t, L_t, w_t, \mu_t^W, b_t^*, \tau_t, \iota_t) \) as a function of endogenous states and exogenous disturbances. Table 3.1 lists all equilibrium conditions of the economy.
3.3 Model calibration and simulation

3.3.1 Calibration

We assume a standard period utility separable in consumption and labor \( U(C_t, L_t) = \ln(C_t) - \lambda \frac{L_t^{1+\epsilon_L}}{1+\epsilon_L} \) with Frisch elasticity of labor supply \( \epsilon_L = 1 \). Steady-state real interest rate equals to \( R = \beta/g^\gamma \). We assume the steady-state real interest rate of 4%, BGP growth rate of 1%, and set the households discount factor to \( \beta = 0.9927 \) accordingly. Elasticity of substitution between home and foreign goods \( \theta = 1.5 \), a common choice in the RBC literature, and home bias parameter is set to \( a = 0.65 \). Following Schmitt-Grohé and Uribe (2003), the foreign bond adjustment cost parameter is set to a very low value of \( \psi_B = 0.00074 \), which ensures the stationarity of the net foreign asset position but does not significantly interfere with the broader model dynamics.

Moving to the production side of the economy, we set labor share to the conventional value of \( \alpha = 0.7 \). Elasticity of substitution between intermediate goods is then set to \( \theta_x = \alpha^{-1} \) so that growth takes the labor-augmenting form. The elasticity of substitution between varieties of labor \( \theta_L = 11 \) to set the steady-state nominal wage markup at 10%, one other conventional choice in the literature. Following the same consideration, the elasticity of substitution between home good varieties for home and foreign markets is \( \theta_H = \theta_H^* = 11 \). Based on the results of Born and Pfeifer (2016), we set the nominal wage and price adjustment costs to \( \psi_W = \psi_P = \psi_{P^*} = 120 \) to replicate, in a linearized setting, the slope of the Phillips curve derived using Calvo stickiness with an average price duration of about a year. As in Ghironi and Melitz (2005), product exit rate is set to \( \delta_N = 0.025 \). The concavity of the innovation production function is \( \rho = 0.9 \), as in Kung (2015). Productivity of innovators is calibrated to match the annual TFP growth of 1%, which implies \( \phi = 0.144 \).

Finally, we assume the steady-state inflation rate of 2% and set the parameters of the Taylor rule to \( \phi_{\Pi} = 1.5, \phi_Y = 0.1, \) and \( \rho_i = 0.7 \). Table 3.2 summarizes the choice of parameters.
3.3.2 Shocks and the solution method

Home and foreign import tariffs are governed by the following process that allows for both level and volatility shocks:

\[
\tau_t = (1 - \rho_t) + \rho_t \tau_{t-1} + \sigma^\tau_t \varepsilon^\tau_t \\
\sigma^\tau_t = (1 - \rho_{\sigma^\tau})\sigma^\tau + \rho_{\sigma^\tau}\sigma^\tau_{t-1} + \sigma^\sigma \varepsilon^\sigma_t
\]

We borrow parameters of the import tariff process from Barattieri et al. (2018) and set \(\sigma^\tau = 0.0176, \rho = 0.56\). The import tariff shock \(\varepsilon^\tau_t\) then involves a 1.75% on-impact increase in tariff that returns to its initial level in about 8 quarters. Import tariff uncertainty shocks are innovations to the variance of the import tariff shock \(\varepsilon^\sigma_t\). As such, in what follows we study effects a temporary increase in variance of the tariff shock, holding the level of import tariff constant. For illustrative purposes, we consider a large but short-lived uncertainty shock setting \(\sigma^\sigma = 5\sigma^\tau\) and \(\rho_{\sigma^\tau} = 0.75\).

We solve the model by perturbation methods based on a third-order approximation of its equilibrium conditions. As discussed in Fernández-Villaverde et al. (2011), studying independent effects of uncertainty shocks requires at least a third-order approximation of policy functions, as the first-order approximation features certainty equivalence and the second-order approximation captures the effect of uncertainty only through its interaction with level shocks. Furthermore, higher-order approximation terms shift the ergodic distribution of endogenous variables of the model away from the deterministic steady state. As such, we follow previous literature and calculate impulse responses in deviations from the stochastic steady state, defined as a point in the state-space where agents choose to remain in the absence of shocks but incorporating the risk of shocks in the future.

\[\text{Both processes are defined in levels, as opposed to logs, to avoid the problem of non-existing moments, as discussed in Andreasen (2010), and to prevent volatility shock from affecting the average import tariff level via Jensen's inequality.}\]

\[\text{Higher order approximations often generate explosive dynamics even when the corresponding linearized system is stable. To alleviate this problem we employ the pruning method that involves eliminating terms of order higher than the approximation order when the system is iterated forward. Pruning was first introduced by Kim et al. (2008) for second-order approximations and then generalized by Andreasen et al. (2017).}\]
3.3.3 Baseline simulation

Import tariff level shock

We first study an unexpected increase in the domestic import tariff in order to compare predictions of the model with the existing results in the literature. Figure 3.5 shows the effect of a temporary increase in home import tariff. The import tariff shock acts as a negative supply shock: output falls and CPI inflation increases. Moreover, there is a short-lived improvement in the current account. To understand the inflationary effect of this shock recall the CPI index:

\[
P_t = [a(P_t^H)^{1-\theta} + (1-a)(\tau_t\varepsilon_t\tau_t^*)^{1-\theta} \frac{1}{1-\theta}]
\]

Higher tariff \(\tau_t\) increases import prices faced by domestic consumers. This effect is only partially mitigated by appreciation of the nominal exchange rate. As a result, domestic CPI inflation increases.

The effect of the import tariff shock on output is shaped by three factors: (i) reduction in real income, which is contractionary; (ii) expenditure-switching towards domestic goods as imported goods become relatively more expensive, which is expansionary; and (iii) the response of monetary policy that faces a trade-off between stabilizing inflation and output. In general equilibrium, the first effect dominates the second and output falls. The reduction in real income stems primarily from a fall in investment in product creation. Since it requires both domestically-produced and imported components, an increase in import tariff makes investment in product creation more costly. Appendix C shows the effect of a tariff increase in an economy without product creation investment, in which case the contractionary effect of the shock disappears. The above results largely follow the conclusions of Barattieri et al. (2018).

Import tariff uncertainty shock

Figure 3.6 shows responses of the baseline economy to an import tariff uncertainty shock. Similarly to the tariff level shock, it is contractionary and inflationary raising the trade-off between stabilizing inflation and output for monetary policy. The sharp initial increase in
CPI inflation is primarily driven by on-impact depreciation of the real exchange rate, while the continuing inflationary effect is due to the increase in the domestic-good price level.

The contractionary effect is shaped by several factors. First, an increase in uncertainty renders foreign-exchange borrowing riskier and generates inefficient capital outflow. However, the associated depreciation of the real exchange rate induces expenditure-switching towards domestic goods and partially offsets the contractionary effect of capital outflow. Second, there is an on-impact fall in innovation spending, which is very significant relative to the decline in consumption. As we discuss further, the response of innovation spending is primarily driven by the endogenous increase in equity risk. Under nominal rigidity, the equilibrium is demand-determined in the short-run so that the on-impact fall in innovation spending exacerbates the immediate contraction in the economic activity. Finally, over time the initial drop in innovation spending translates into a persistent decline in the TFP level, which prolongs the negative effect of the shock on consumption and output as well as causes the persistent appreciation of the real exchange rate in the medium-run. We explore these channels in more detail in the following sections.

Finally, appendix C illustrates that both short-run and persistent negative effects of the shock are very sensitive to its persistence. For instance, increasing the persistence parameter to $\rho_{\sigma_r} = 0.85$ from the baseline value of 0.75 amplifies responses to the shock by more than twofold.

3.4 Exploring the mechanism

3.4.1 The role of endogenous growth

Figure 3.7 compares the baseline simulation to a counterfactual with no endogenous growth. The first immediately obvious difference is the lack of persistent negative effect on the level of output and consumption as well as the medium-run appreciation of the real exchange rate, all of which are driven by the endogenous decline in productivity growth in the baseline simulation. The difference in the short-run responses is driven by a combination of demand and supply forces. Recall that the aggregate demand for home-produced goods comes from domestic consumers, domestic innovators, and the rest of the world. Shutting
down endogenous growth excludes the aggregate demand component – innovation spending – that contracts the most in the baseline economy. Although domestic consumption still decreases due to precautionary saving, it is compensated by an increase in the export demand increases due to the real exchange rate depreciation, causing the aggregate demand to increase. Moreover, an increase in uncertainty induces precautionary labor supply: other factors equal, households increase their labor supply when the marginal utility of consumption goes up.

The role of this precautionary labor supply becomes clear when you do away with the wealth effect by assuming Greenwood et al. (1988) preferences with period utility function $U_t(C_t, L_t) = \left(\left(C_t - \chi \Upsilon_t \frac{L_t^{1+\epsilon_L}}{1+\epsilon_L}\right)^{1-\gamma} - 1\right)/(1-\gamma)$, where $1/\gamma$ is the elasticity of intertemporal substitution and $1/\epsilon_L$ is the Frisch elasticity of labor supply. As in Queralto (2019), the disutility of labor is governed by the following process: $\Upsilon_t = \Upsilon_t^{\rho_T N_t^{-\rho_T}}$ with $\rho_T$ governs its responsiveness to changes in productivity growth. This setting ensures that the BGP with constant hours exists but limits the wealth effect in labor supply due to medium-run swings in growth.\(^{10}\) We set $\rho_T = 0.95$ so that the short-run wealth effect in labor supply is minimal. Appendix C presents responses to an import tariff uncertainty shock under the GHH preference. Doing away with the wealth effect in labor supply not only amplifies responses to the shock in the baseline model but also preserves the contractionary effect when the endogenous growth mechanism is shut down.

### 3.4.2 The role of convexity of import demand

An import tariff uncertainty shock is contractionary. However, it generates an unrealized expectation of expansion. Figure 3.8 illustrates this by comparing the actual dynamics of variables with the dynamics of their expected next-period values. To understand this effect recall import demand implied by the CES aggregator, equation (3.7): $Y_t^F = (1-a)(\tau_t \rho_t^F r_{er_t})^{-\delta} Y_t$. Since the elasticity of substitution between home and foreign goods is

---

\(^{10}\) As noted by Benhabib et al. (1991), the GHH preference can be interpreted as a reduced form of an economy with home production. The disutility of work then consists of the forgone output in-home production, which increases in the long-run as productivity improvements in the formal sector spill over to home production. However, to the extent this process takes time, the disutility of labor exhibits inertia. Jaimovich and Rebelo (2009) suggest a similar preference specification that allows us to parameterize the short-run wealth effect on the labor supply.
\( \theta > 0 \), the imports demand is convex in import tariff. Therefore, other factors fixed, higher import tariff uncertainty increases expected imports demand by Jensen’s inequality similarly to what would happen due to a decrease in an expected tariff level. In general equilibrium, this effect dominates and causes an expected — but unrealized — expansionary effect. This feature would be important for understanding the dynamics of forward-looking variables discussed below.

3.4.3 The role of time-varying risk premia

Uncertainty shocks introduce time-varying risk premia in asset returns. The three relevant assets in the present model are domestic bonds, foreign-currency bonds, and intermediate firm equity. An increase in import tariff uncertainty causes foreign-currency bonds and equity to become riskier relative to domestic bonds, which contributes to an improvement in the country’s net current account and to a decrease in product creation investment.

To illustrate the current account effects this channel, we assume log-normality and express demand for foreign-exchange bonds, equation (3.2), as follows:

\[
\begin{align*}
(\text{Var}_t \Delta c_{t+1} - \mathbb{E}_t \Delta c_{t+1} - g_t) + (R^*_t + \mathbb{E}_t \Delta \text{rer}_{t+1}) - \text{Cov}_t(\Delta c_{t+1}, \Delta \text{rer}_{t+1}) & \approx 0, \\
\text{Discounting} & \text{Foreign exchange risk premium}
\end{align*}
\]

where we took into account that the foreign interest rate \( R^*_t \) follows a deterministic process and used sans-serif font to denote log-deviations of variables from the steady state, see appendix C.2.2 for details. The demand for foreign-exchange bonds is affected by the foreign exchange risk premium: conditional covariance between consumption growth and the change in the real exchange rate. A positive covariance means that the asset tends to yield high returns in states of nature when consumption is high thus not providing a good hedge against consumption fluctuations. For this reason, the demand for foreign-currency bonds is decreasing in the foreign exchange risk premium.

How does the foreign-exchange risk premium respond to an import tariff uncertainty shock? Panel (a) of figure 3.9 reports relevant impulse responses. The shock increases the foreign exchange risk premium and makes foreign bonds riskier. To understand this recall that import tariff level shocks cause the real exchange rate to appreciate so that foreign bonds are a bad hedge against this shock. An increase in import tariff uncertainty
amplifies this property and decreases the demand for foreign bonds. As a result, the current account temporarily improves as the economy decreases holdings of foreign debt. In general equilibrium, this reduction in foreign debt causes a combination of a fall in consumption and a fall product creation investment.

The dynamics real exchange rate and current account can be further interpreted through the lens of the following equality:

$$(R^*_t + \mathbb{E}_t \Delta \text{rer}_t) - \mathbb{E}_t R_{t+1} \approx \text{Cov}_t (\Delta c_{t+1}, \Delta \text{rer}_{t+1}) - \text{Cov}_t (\Delta c_{t+1}, R_{t+1}).$$

which is implied by equations (3.1) and (3.2) under the assumption of log-normality, see appendix C.2.2. The left hand side of the expression is the difference in ex ante real returns between foreign and home bonds, which equals to zero when the uncovered interest rate parity (UIP) holds. The right hand side represents the hedging property of foreign-currency bonds relative to domestic bonds, measured by conditional covariances of returns with consumption growth. Deviations from the UIP arise when one asset is a better hedge against the consumption risk than the other.

Panel (b) of figure 3.9 visualizes deviation from the UIP in response to an import tariff uncertainty shock based on equation (3.29). The shock causes foreign bonds to become more risky than domestics bonds. As a result, the real exchange rate is expected to depreciate by more than predicted by the UIP so that foreign bonds deliver a higher expected return to compensate for risk.

A similar logic applies to product creation financed by selling equity to households. Using equation (3.3) and the definition of the risk-free rate $R^f_t$ excess return on equity can be expressed as follows:

$$\mathbb{E}_t R^\text{equity}_{t+1} - R^f_{t+1} = \text{Cov}_t (\Delta c_{t+1}, R^\text{equity}_{t+1}),$$

where $R^\text{equity}_{t+1} = (v_{t+1} + d_{t+1})/v_t$ denotes equity return, see appendix A.3 for details. Panel (a) of figure 3.10 illustrates that the shock increases equity risk premium. Higher risk requires equity to exhibit excess return to compensate for it. Current firm price then $v_t$ falls to deliver higher expected return. The shock also causes an increase in the implied equity return volatility calculated in annualized percentage term as follows $\text{VXO}_t = 100\sqrt{4\text{Var}_t (R^\text{equity}_{t+1})}$.
by about 3%.

How large is the contribution of risk to the firm value dynamics? Assuming log-normality, current firm value can be expressed as follows:

\[
v_t = \left( \text{Var}_t \Delta c_{t+1} + \mathbb{E}_t \Delta c_{t+1} - g_t \right) + \mathbb{E}_t P_{t+1}^e - \text{Cov}_t \left( \Delta c_{t+1}, P_{t+1}^e \right),
\]

(3.31)

where \( P_{t+1}^e = v_{t+1} + d_{t+1} \) denotes the next-period payoff. Panel (b) of Figure 3.10 plots the dynamics of firm value components in response to an import tariff volatility shock. The expected next-period payoff increases due to the anticipated expansionary effect, as discussed in section 3.4.2. However, this effect is roughly canceled out by a decrease in the stochastic discount factor of households that experience a temporary fall in consumption. As a result, the majority of the realized decrease in firm value is due to the increased equity risk.

Finally, risk premium wedges affect not only the responses of the economy to shocks but also the stochastic steady state. Our higher-order solution of the model allows us to capture how risk affects the ergodic distribution of endogenous variables. Appendix C includes the comparison between deterministic and stochastic steady states. At the stochastic steady state, compared to the deterministic steady state, the economy improves its net foreign asset position as holding foreign debt is risky. As a result, consumption is higher and the labor supply is lower due to the wealth effect. Even a moderate long-run import tariff risk that we consider in the baseline simulation has a non-negligible effect on endogenous variables.

Note that the mechanisms described in this section do not rely on nominal rigidity. As pointed out by Basu and Bundick (2017) uncertainty shocks are often counterfactually expansionary in closed-economy models without nominal rigidity since precautionary saving leads to an increase in investment. Nominal rigidity makes equilibrium demand-determined in the short run and causes precautionary savings to be contractionary. Price and/or nominal wage rigidity hence is deemed to be a necessary ingredient for uncertainty shocks to have an empirically-relevant negative effect on output. Our exercise shows that this logic does not necessarily hold in the open-economy context. Figure 3.11 compares the responses of the economy to an import tariff uncertainty shock under various assumptions about nominal rigidity. Although rigid prices and/or nominal waged do amplify the effects of the shock by rendering the equilibrium demand-determined in the short run, the results remain qualitatively the same even in the flexible economy.
3.4.4 The risk channel of monetary policy

One of the channels through which policy can influence real allocations is by systematically affecting stochastic properties of asset returns and hence relevant risk premia.\(^\text{11}\) In this section we focus on the role of monetary policy and exchange-rate regimes.

What is the role of exchange-rate arrangements in shaping the effects of the import tariff uncertainty shock? We conclude that the fixed exchange rate regime exacerbates the short-run contractionary effects of the shock but alleviates the persistent endogenous decline in the TFP level. From a technical standpoint, the fixed exchange rate regime is implemented as follows. When pegging the exchange rate, the home country forgoes its monetary policy autonomy and the Taylor rule (3.21) no longer applies. Instead, as pointed out by Benigno et al. (2007), the fixed exchange rate regime is implemented by the home central bank credibly committing to the following reactive rule:

\[
1 + i_t = (1 + i^*_t) f \left( \frac{\varepsilon_t}{\bar{\varepsilon}} \right),
\]

where the function \(f(\cdot)\) is continuous, monotone non-decreasing, differentiable, strictly increasing in a neighborhood of \(\varepsilon_t = \bar{\varepsilon}\), and \(f(1) = 1\). In equilibrium, the nominal exchange rate is fixed, \(\varepsilon_t = \varepsilon_{t-1} = \bar{\varepsilon}\), and the dynamics of the real exchange rate is solely driven by the cross-country inflation differential and the home interest rate is pegged to the foreign interest rate:

\[
\frac{\text{rer}_t}{\text{rer}_{t-1}} = \frac{\Pi^*_t}{\Pi_t} \frac{\varepsilon_t}{\varepsilon_{t-1}} = \frac{\Pi^*_t}{\Pi_t}.
\]

**Figure 3.12** compares responses of the baseline flexible exchange rate economy to the fixed exchange rate counterfactual. The immediate contractionary effect of the shock is exacerbated relative to the baseline. The reason lies in the dynamics of the real exchange rate. In the baseline flexible exchange rate economy the real exchange rate depreciates in the short run. Under producer-currency pricing the depreciation of the real exchange induces expenditure-switching towards domestic goods, which, other factors equal, boosts domestic demand and output. The fixed exchange rate regime diminished this expansionary channel and increases the on-impact decline in output. Similarly, the muted short-run response of the real exchange rate under the fixed exchange rate regime causes the response of

\(^{11}\) See for example discussions by Benigno et al. 2011, Hassan et al. 2016, and de Groot (2014)
CPI inflation to be much flatter than in the baseline economy. Finally, the fixed exchange rate regime eliminates the risk differential between home and foreign bonds by rendering
\[ \text{Cov}_t(\Delta c_{t+1}, \Delta \text{rer}_{t+1}) = \text{Cov}_t(\Delta c_{t+1}, R_{t+1}) = \text{Cov}_t(\Delta c_{t+1}, \Pi_{t+1}) \]. As a result, the UIP violation described in equation (3.29) disappears.

Although amplified in the short-run, the effects of the import tariff uncertainty shock is smaller in the medium-run under the fixed exchange rate regime. Recall that the persistent effect of the shock is driven by the endogenous decrease in TFP, the lion’s share of which is due to the equity risk channel described by equations (3.30) and (3.31). Under the fixed exchange rate regime the equity risk increases by less than in the baseline economy hence the persistent endogenous productivity loss in smaller.

We conclude by assessing the role of parameters of the monetary policy rule, equation (3.21), when the exchange rate is flexible. Our results suggest that a greater focus on inflation stabilization, as opposed to output stabilization, exacerbates the endogenous decrease in productivity. Figure 3.13 compares responses under different values of the interest rate response to CPI inflation \( \phi_{\Pi} \) and deviations of output from its balanced growth path \( \phi_Y \). Greater values of \( \phi_{\Pi} \) magnify the equity risk and drive the result.

### 3.5 Conclusion

Trade tensions, even when not resulting in tariff increases, are detrimental to the economy by generating policy uncertainty. We study trade policy uncertainty in a dynamic general equilibrium framework and conclude that it is inflationary, contractionary, and causes a temporary improvement in the current account. Although short-lived, economic uncertainty shocks can generate long-lasting effects by impeding productivity growth. We emphasize the key role of endogenous risk premia in transmitting uncertainty shock to the real economy.

The present setting can be extended along several dimensions. First, a two-country small open economy model would allow us to study the effects of foreign imports tariff uncertainty, a scenario relevant for small open economies that are subject to the global trade policy uncertainty. Second, one other margin of adjustment to trade policy uncertainty that is likely to be important is the endogenous selection of heterogeneous firms in and out of exporting, which can be modeled along with Ghironi and Melitz (2005).
Figure 3.1: US trade policy uncertainty

[Note: the trade policy uncertainty index is from Baker et al. (2016); the trade policy equity market volatility is from Baker et al. (2019)]
Figure 3.2: US economic uncertainty

[cited on page 97]
Figure 3.3: US VAR, responses to stock market volatility shock

[Note: 7-variable VAR, (1) CBOE S&P 100 volatility index; (2) real GDP; (3) total factor productivity; (4) real R&D spending; (5) trade balance to GDP; (6) real exchange rate index; (7) S&P 500 index. Volatility shock identified with short-run restriction with VXO index ordered first. Impulse responses are in log deviations times 100, except for trade balance to GDP, which is in levels. Shaded areas correspond to 68% and 90% confidence intervals.]
FIGURE 3.4: Baseline model flow chart

[cited on page 100]
### Table 3.1: Model summary

<table>
<thead>
<tr>
<th>1-2. Market clearing</th>
<th>$y_t = c_t + s_t + AC_t^w + AC_t^p + AC_t^{p*}$, $f_t - \frac{\sigma}{\rho} = y_t^H + y_t^{H*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Net foreign assets</td>
<td>$p^<em>_t y^</em><em>t - p^<em>_t y^</em><em>t = b_t^* - \frac{1 + \phi}{\psi</em>{t-1}} b</em>{t-1}^*/g_t$</td>
</tr>
<tr>
<td>4. Domestic bond Euler</td>
<td>$E_t \left( \beta_{t,t+1}^* \frac{1 + \phi}{\psi_{t+1}} \right) = 1$</td>
</tr>
<tr>
<td>5. Foreign bond Euler</td>
<td>$E_t \left( \beta_{t,t+1} \frac{r_{t+1}}{r_{t+1}} \frac{1 + \phi}{\psi_{t+1}} \right) = 1 - \psi B b_t^*$</td>
</tr>
<tr>
<td>6. Labor demand</td>
<td>$w_t = p^G_t \alpha Z_t L_t^{\alpha - 1} x_t^{1-\alpha}$</td>
</tr>
<tr>
<td>7. Labor supply</td>
<td>$w_t = \mu^W_t U^L_t / U^L_t$</td>
</tr>
<tr>
<td>8. Wage markup</td>
<td>$\mu^W_t = \frac{1}{(\theta_L - 1) (1 - \phi/\psi) (\frac{\phi}{\psi} - 1)} + \psi G^W_t (\frac{\phi}{\psi} - 1) - \psi E_t \left[ \frac{\phi_{t-1} + \psi}{\psi_{t-1}^W} \left( \frac{n_{t-1}^W}{n_{t-1}^W - 1} \right) \right]$</td>
</tr>
<tr>
<td>9. Wage inflation</td>
<td>$\Pi^W_t = \frac{w_t}{w_{t-1}} g_t \Pi_t$</td>
</tr>
<tr>
<td>10-11. Optimal price</td>
<td>Home market: $p^H_t = \mu^H_t p^G_t$, Foreign market: $p_t^{H*} = \mu^{H*}<em>t \frac{p^G_t}{r</em>{t-1}}$</td>
</tr>
<tr>
<td>12-13. Inflation</td>
<td>Home market: $\Pi^H_t = \frac{p^H_t}{r_{t-1}} \Pi_t$, Foreign market: $\Pi^{H*}_t = \frac{\Pi^{H*}<em>t}{r</em>{t-1}}$</td>
</tr>
<tr>
<td>14-15. Price markups</td>
<td>$\mu_t^H = \frac{1}{(\theta_H - 1) \left( 1 - \phi P_r^W (\frac{\phi}{\psi} - 1) \right) + \psi P_r^W (\frac{\phi}{\psi} - 1) - \psi E_t \left[ \frac{\phi_{t-1} + \psi}{\psi_{t-1}} \left( \frac{n_{t-1}^W}{n_{t-1}^W - 1} \right) \right]}$</td>
</tr>
<tr>
<td></td>
<td>$\mu_t^{H*} = \frac{1}{(\theta_{H*} - 1) \left( 1 - \phi P_r^W (\frac{\phi}{\psi} - 1) \right) + \psi P_r^W (\frac{\phi}{\psi} - 1) - \psi E_t \left[ \frac{\phi_{t-1} + \psi}{\psi_{t-1}} \left( \frac{n_{t-1}^W}{n_{t-1}^W - 1} \right) \right]}$</td>
</tr>
<tr>
<td>16. Equity demand</td>
<td>$v_t = (1 - \delta) E_t [\beta_{t,t+1}(d_{t+1} + v_{t+1})]$</td>
</tr>
<tr>
<td>17. Equity supply</td>
<td>$\phi s_t^{\rho-1} v_t = 1$</td>
</tr>
<tr>
<td>18. Profit</td>
<td>$d_t = \frac{1}{(1-\alpha)p^H_t x_t}$</td>
</tr>
<tr>
<td>19. TFP growth</td>
<td>$g_t = (1 - \delta) (1 + (\phi(s_t)^\rho)$</td>
</tr>
<tr>
<td>20. Intermediate output</td>
<td>$x_t = (A(1-\alpha)^2 Z_t / \mu^H_t)^{\frac{1}{\alpha}} L_t$</td>
</tr>
<tr>
<td>21. Home good output</td>
<td>$f_t = Z_t L_t^{\alpha} x_t^{1-\alpha}$</td>
</tr>
<tr>
<td>22. Relative prices</td>
<td>$a(p_H^t)^{1-\theta} + (1-a)(\tau_0 p^*<em>t r</em>{t-1})^{1-\theta} = 1$</td>
</tr>
<tr>
<td>23. Exports demand</td>
<td>$y_t^{H*} = (1 - a) (p_t^{H*})^{-\theta} y_t^*$</td>
</tr>
<tr>
<td>24. Imports demand</td>
<td>$y_t^p = (1 - a) (\tau_0 p_t^{H*} r_{t-1})^{-\theta} y_t$</td>
</tr>
<tr>
<td>25. Home-good demand</td>
<td>$y_t^H = a (p_t^H)^{-\theta} y_t$</td>
</tr>
<tr>
<td>26. Taylor rule</td>
<td>$\frac{1 + \phi}{\psi} = \left( \frac{1 + \phi}{\psi} \right)^{\rho H} \left( \Pi_t / \Pi_t^{\phi H} (gdp_t / gdp_t^{\phi H})^{(1-\rho H)} \right)$</td>
</tr>
<tr>
<td>27. Productivity gap</td>
<td>$\frac{\phi_t}{\phi_{t-1}} = \frac{1}{\phi_{t-1}}$</td>
</tr>
</tbody>
</table>
## Table 3.2: Structural parameters

[cited on page 110]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source / target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9927</td>
<td>4% annual real interest rate</td>
</tr>
<tr>
<td>$\epsilon_L$</td>
<td>Inverse of Frisch elasticity of labor supply</td>
<td>1</td>
<td>Conventional</td>
</tr>
<tr>
<td>$a$</td>
<td>Home bias</td>
<td>0.65</td>
<td>Conventional</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labor share</td>
<td>0.7</td>
<td>Conventional</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Innovation output elasticity</td>
<td>0.9</td>
<td>Kung (2015)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Intermediate sector exit rate</td>
<td>0.025</td>
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<td>1.5</td>
<td>Backusus et al. (1994)</td>
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<td>$\theta_x$</td>
<td>Elasticity of substitution, intermediate goods</td>
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<td>Elasticity of substitution, labor varieties</td>
<td>11</td>
<td>10% steady-state wage markup</td>
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<tr>
<td>$\theta_H$</td>
<td>Elasticity of substitution, retail goods</td>
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<td>10% steady-state price markup</td>
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<td>$\psi_P$</td>
<td>Domestic price adjustment cost</td>
<td>120</td>
<td>4-quarter average Calvo price ridigity equivalent, home</td>
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<tr>
<td>$\psi_P^*$</td>
<td>Exports price adjustment cost</td>
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<td>4-quarter average Calvo price ridigity equivalent, foreign</td>
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<td>$\psi_W$</td>
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<td>120</td>
<td>4-quarter average Calvo nominal wage ridigity equivalent</td>
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<td>Schmitt-Grohé and Uribe (2003)</td>
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<td>$\phi_y; \phi_x; \rho_r$</td>
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<td>0.1; 1.5; 0.7</td>
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<td>$\phi$</td>
<td>Innovators productivity</td>
<td>0.144</td>
<td>Annual TFP growth = 1%</td>
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<td>$1/A$</td>
<td>Intermediate sector marginal cost</td>
<td>1</td>
<td>Normalization</td>
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<td>$\chi$</td>
<td>Disutility of labor scale</td>
<td>0.754</td>
<td>Normalization, $L = 1$</td>
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<tr>
<td>$\bar{Z}$</td>
<td>Final sector productivity</td>
<td>2.25</td>
<td>Normalization, $GDP = 1$</td>
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</tbody>
</table>
Figure 3.5: Responses to a home import tariff shock

[cited on page 112]

Note: impulse responses in log deviations from the stochastic steady state, unless otherwise noted. Inflation rate and the real interest rate are annualized.
Figure 3.6: Responses to a home import tariff uncertainty shock

Note: impulse responses in log deviations from the stochastic steady state, unless otherwise noted. Inflation rate and the real interest rate are annualized.
Figure 3.7: Responses to a home import tariff uncertainty shock, the role of endogenous growth

[cited on page 113]

Note: impulse responses in log deviations from the stochastic steady state, unless otherwise noted. Inflation rate and the real interest rate are annualized.
Figure 3.8: Responses to a home import tariff uncertainty shock, the role of convexity of imports demand

[cited on page 114]

Note: the figure compares actual responses of variables to responses of their next-period expected values. The difference is driven primarily by the convexity if the import demand in tariff: $Y_t^F = (1-a)(\tau_t p_t^F \text{ rer}_t)^{-\theta} Y_t$. An increase in the tariff uncertainty increases the expected import demand and generated an expected — but unrealized — expansion.
Figure 3.9: Responses to a home import tariff uncertainty shock, the role of foreign exchange risk

(a) Net foreign asset dynamics [cited on page 116]

(b) Violation of the uncovered interest rate parity

Note: (a) plots the current account effect of FX risk
\[
\text{CA to GDP} = \left( \text{Var}_t \Delta c_{t+1} - \mathbb{E}_t \Delta c_{t+1} - g_t \right) + \left( R^*_{t+1} + \mathbb{E}_t \Delta \text{rer}_{t+1} \right) - \text{Cov}_t \left( \Delta c_{t+1}, \Delta \text{rer}_{t+1} \right) \approx 0
\]

(b) plots UIP violation
\[
\text{Im tariff } \sigma = \left( R^*_{t+1} + \mathbb{E}_t \Delta \text{rer}_{t+1} \right) - \mathbb{E}_t R_{t+1} \approx \text{Cov}_t \left( \Delta c_{t+1}, \Delta \text{rer}_{t+1} \right) - \text{Cov}_t \left( \Delta c_{t+1}, R_{t+1} \right)
\]
Figure 3.10: Responses to a home import tariff uncertainty shock, the role of equity risk

(a) Equity risk premium

(b) Firm value decomposition

Note: (a) excess equity return over the risk-free real rate $E_t R_{equity}^{t+1} - R_{t+1}^t = \text{Cov}_t \left( \Delta c_{t+1}, R_{equity}^{t+1} \right)$, where $R_{equity}^{t+1} = \frac{v_{t+1} + d_{t+1}}{v_t}$,
Implied equity return volatility is in annualized percentage terms $VXO_t = 100 \sqrt{4 \text{Var}_t (R_{equity}^{t+1})}$

(b) plots the firm value decomposition $v_t = \left( \text{Var}_t \Delta c_{t+1} - E_t \Delta c_{t+1} - g_t \right) + \frac{E_t P_{equity}^e_{t+1}}{P_{equity}^e_{t+1}} - \text{Cov}_t \left( \Delta c_{t+1}, P_{equity}^e_{t+1} \right)$, where $P_{equity}^e_{t+1} = v_{t+1} + d_{t+1}$
Figure 3.11: Responses to a home import tariff uncertainty shock, the role of nominal rigidity

Note: impulse responses in log deviations from the stochastic steady state, unless otherwise noted. Inflation rate and the real interest rate are annualized.
Figure 3.12: Responses to a home import tariff uncertainty shock, fixed exchange rate regime
[cited on page 118]

Note: impulse responses in log deviations from the stochastic steady state, unless otherwise noted. Inflation rate and the real interest rate are annualized.
Figure 3.13: Responses to a home import tariff uncertainty shock, the role of Taylor rule parameters

(a) Productivity growth response under different Taylor rule parameters [cited on page 119]

(b) Equity risk response under different Taylor rule parameters

Note: monetary policy is conducted through interest rate setting

The risk-free nominal interest rate is governed by the following rule

\[
\frac{1 + i_t}{1 + i_t} = \left(1 + i_{t-1} \frac{1 + i_t}{1 + i_t}\right)^{\rho_i} \left(\frac{\Pi_t}{\Pi_t} \phi_{\Pi} \left(\frac{GDP_t}{GDP_{BGP}}\right)^{\phi_Y}\right)^{1 - \rho_i}
\]

The interest rate smoothing parameter is held at \(\rho_i = 0.7\) in each scenario.
BIBLIOGRAPHY


10. Annicchiarico, B. (2016). Innovation, Growth, and Optimal Monetary Policy. [Cited on page 3.]


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A.1 Appendix

Table A.1: Country sample
[cited on page 4]

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Note: ISO 3166-1 alpha-3 codes. 42 small open economies.
Table A.2: Summary statistics

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<td>0.023</td>
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<td>∆I</td>
<td>1967</td>
<td>0.038</td>
<td>0.042</td>
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<tr>
<td>5.</td>
<td>∆TFP_{adj}</td>
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<td>0.006</td>
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<td>∆B</td>
<td>1838</td>
<td>0.026</td>
<td>0.023</td>
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<td>7.</td>
<td>Credit gap</td>
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<td>1.614</td>
<td>1.055</td>
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<td>8.</td>
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<td>log(RER)</td>
<td>2289</td>
<td>4.556</td>
<td>4.562</td>
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Private credit-to-GDP gap is defined as a deviation of the private debt-to-GDP ratio from the long-run HP trend. According to the BIS definition, the smoothing parameter of the HP trend is 400000 for quarterly data (corresponds to 400000/4^4 for annual data).
Figure A.1: US VAR, responses to an R&D shock

VAR(2), IRFs of (1) Adjusted NFCI; (2) Total employment; (3) Real GDP; (4) Utilization-adjusted TFP; (5) Real R&D expenditure; (6) Total credit to private non-financial sector, % of GDP. Shaded areas correspond to 68% and 95% confidence bounds and are obtained by bootstrap with 50000 replications. Responses to variables other than NFCI are expressed in log deviations.
Appendix B

CHAPTER 2

B.1 Data appendix

World Bank, 1960-2018  data.worldbank.org/indicator
GDP per capita, constant LCU  NY.GDP.PCAP.KN
Households and NPISHs final consumption expenditure, constant LCU,  NE.CON.PRVT.KN
Gross fixed capital formation, constant LCU  NE.GDI.FTOT.KN
GDP deflator  NY.GDP.DEFL.ZS

Penn World Table version 9.1, 1950-2017  rug.nl/ggdc/productivity/pwt
Output-side real GDP at chained PPPs (in mil. 2011 USD)  rgdpo
Population (in millions)  pop
Number of persons engaged (in millions)  emp
TFP at constant national prices (2011=1)  rtfpna
Capital stock at constant 2011 national prices (in mil. 2011 USD)  rnna
Share of merchandise exports at current PPPs  csh_x
Share of merchandise imports at current PPPs  csh_m
Share of labour compensation in GDP at current national prices  labsh

Household debt, loans and debt securities, percent of GDP
Nonfinancial corporate debt, loans and debt securities, percent of GDP

Jordà-Schularick-Taylor Macrohistory Database  http://www.macrohistory.net/data
House prices (nominal index, 1990=100)  hpnom
Total loans to households (nominal, local currency) \( thh \)
Total loans to business (nominal, local currency) \( tbus \)
Consumer prices (index, 1990=100) \( cpi \)

**Aggregate real housing price indexes, other sources**

- BIS real residential property indices [bis.org/statistics/pp_selected.htm]
- Dallas FED International House Price Database [dallasfed.org/institute/houseprice]
- OECD real house price indices [stats.oecd.org/Index.aspx?DataSetCode=HOUSE_PRICES]

**Laeven and Valencia (2013) [sites.google.com/site/laevenl/codes]**

Systemic Banking Crises Database

**Ilzetzki et. al. (2019) [carmenreinhart.com/data/browse-by-topic/topics/11]**

Exchange rate regime classification

**Bruegel [bruegel.org/publications/datasets]**

Real effective exchange rates

**Table B.1: Country sample**

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<td>POL</td>
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<td>ZAF</td>
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*Note:* List of 50 countries with available aggregate housing price indices, ISO 3166-1 alpha-3 codes. Countries for which no housing bubbles have been identified are highlighted.
<table>
<thead>
<tr>
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<td>0.002</td>
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<td>π_{GDP} deflator</td>
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<td>10.</td>
<td>N/X/Y</td>
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<td>-0.021</td>
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<td>12.</td>
<td>∆B_{F}</td>
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<td>0.017</td>
<td>0.016</td>
<td>0.094</td>
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</table>

Credit-to-GDP gaps:

<p>| | | | |</p>
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<td>\hat{B}_{HH}</td>
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<tr>
<td>14.</td>
<td>\hat{B}_{F}</td>
<td>1590</td>
<td>0.020</td>
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Conditional on the housing market crash at \( t \), \( 1_{i,t}^{crash} = 1 \):

<p>| | | | |</p>
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<td>15.</td>
<td>\Delta_{t} P_{housing,t+3}</td>
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<tr>
<td>16.</td>
<td>\hat{B}_{t,HH}</td>
<td>50</td>
<td>3.600</td>
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<tr>
<td>17.</td>
<td>\hat{B}_{t,F}</td>
<td>50</td>
<td>5.821</td>
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</table>

Note: unbalanced panel of 50 countries, 1950-2018. (1) Real GDP per capita; (2) Real consumption expenditure per capita; (3) Real investment per capita; (4) Employment-to-population ratio; (5) Real capital stock per-capita; (6) TFP index at constant national prices; (7) Real output per worker; (8) National real housing price index; (9) GDP-deflator inflation rate; (10) Net exports to GDP ratio; (11) Household debt, loans and debt securities, % of GDP; (12) Nonfinancial corporate debt, loans and debt securities, % of GDP. Where applicable, the IMF Global Debt Database data is extended for the earlier years by the banking lending data from the Jordà-Schularick-Taylor Macrohistory Database. (13) Household debt-to-GDP gap; (14) Firm debt-to-GDP gap;

Conditional on the housing market crash at \( t \): (15) Real housing price index decline in the first 3 years from the peak; (16) Household debt-to-GDP gap at the peak of the housing market cycle; (17) Firm debt-to-GDP gap at the peak of the housing market cycle.
Figure B.1: Housing market boom-and-bust cycles definition

[cited on page 41]

Note: definition similar to Jordà et al. (2015). Blue shaded areas correspond to a 1 st. dev. bound around the one-sided HP trend (smoothing parameter = $\frac{400000}{4^4}$).

Housing market crashes are defined as periods when (1) the aggregate housing price index deviates from the long-run trend by more than one standard deviation (marked by gray shaded areas) and (2) exhibit the price decline of at least 10% within the first three years from the peak.
**Figure B.2:** US household and non-financial corporate debt to GDP ratios

[Note: A debt-to-GDP gap is defined as a deviation of the debt-to-GDP ratio from the long-run HP trend (smoothing parameter of $10^5/4^3$ for annual observations). The data on household and non-financial corporate debt comes from the IMF Global Debt Database and includes both loans and debt securities.]
### B.2 Additional results

**Figure B.3:** An average housing market crash, additional results

[cited on page 45]

*Note:* cross-country panel estimation, controlling for country and year fixed effects, country-specific trends, and preexisting macroeconomic conditions.

Variables: (1) Trade balance to GDP; (2) Exports to GDP; (3) Imports to GDP; (4) Real exchange rate index; (5) GDP deflator index; (6) Real GDP per worker; (7) Solow residual.

Responses are estimated by local projections and are expressed in log deviations times 100 (see details in the text). Shaded areas correspond to 90% confidence intervals. Standard errors are clustered at the country level.
Note: cross-country panel VAR controlling for country fixed effects. House price shocks are identified with short-run restrictions with house price index ordered last. The system includes 4 lags of variables based on the Akaike information criterion.

Variables: (1) log utilization-adjusted TFP; (2) household debt-to-GDP gap; (3) percentage deviation of the aggregate house price index from the long-run trend. Both variables (2) and (3) are defined relative to the one-sided HP trend with a smoothing parameter $10^5/4^3$.

Shaded areas correspond to 90% confidence intervals obtained with bootstrap. Standard errors are clustered at the country level.
Figure B.5: Impulse responses to a housing market crash, robustness

Note: cross-country panel estimation, the baseline specification (1) controls for country/time fixed effects, country-level trends, and macroeconomic conditions. Alternatives: (2) Baseline, excluding observations from 2007; (3) 4 lags of macro controls instead of 2; (4) No macro controls; (5) No macro controls and year fixed effects.

Responses are estimated by local projections and are expressed in log deviations times 100 (see details in the text). Shaded areas correspond to 95% confidence intervals of the baseline IRFs. Standard errors are clustered at the country level.
Figure B.6: US MSA-level evidence, first stage regressions

(a) Saiz (2010) housing supply elasticity instrument [cited on page 46]
(linear and quadratic first-stage regressions)

(b) Guren et al. (2018) regional sensitivity instrument

Note: labor productivity is defined as real GDP per worker. State and MSA output and employment data is from BEA. Housing price growth is based on the Federal Housing Finance Agency all-transactions house price indexes. Housing net worth shock is from Mian and Sufi (2014).
**Figure B.7:** Housing net worth shock and labor productivity growth

**Figure B.8:** Elasticity of labor productivity growth to housing net worth shock

*Note:* Baseline specification is in red, see details in table B.3.
Table B.3: Elasticity of labor productivity growth to housing net worth shock

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<td>∆ housing net worth 2006-2010</td>
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<td>0.38**</td>
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Robust standard errors in parentheses,  *p < 0.10, **p < 0.05, ***p < 0.01

Note: labor productivity is defined as real GDP per worker. State and MSA output and employment data is from BEA. Housing net worth shock is defined as a percent change in household net worth between 2006 and 2009 that comes from the collapse in house prices, source: Mian and Sufi (2014). Controls: 2006 shares in GDP of (a) mining, quarrying, and oil and gas extraction; (b) construction; (c) manufacturing; (d) retail trade; as well as (e) 2006 per capital personal income. Reduced sample explides four states that were affected by the housing market crash the most.

Figure B.9: House price boom and labor productivity growth

Table B.4: Elasticity of labor productivity growth to house price growth

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| Robust standard errors in parentheses, * p < 0.10, ** p < 0.05, *** p < 0.01

Note: labor productivity is defined as real GDP per worker. State and MSA output and employment data is from BEA. House price growth is based on FHFA all-transactions house price indexes. Controls: 2006 shares in GDP of (a) mining, quarrying, and oil and gas extraction; (b) construction; (c) manufacturing; (d) retail trade; as well as (e) 2006 per capital personal income. Reduced sample explides four states that were affected by the housing market crash the most. IV 1: Saiz (2010) housing supply elasticity instrument, linear first stage; IV 2: Saiz (2010) housing supply elasticity instrument, quadratic first stage; IV 3: Guren et al. (2018) regional sensitivity instrument.
Figure B.10: Baseline simulation, welfare cost and debt burden

(a) Model with endogenous growth response

(b) Model without endogenous growth response

Note: counterfactual scenarios to the baseline in figure 9 under different levels of pre-existing household indebtedness. Dashed lines mark baseline values of debt-to-GDP and LTV ratios.
B.3 Derivations

B.3.1 Utilization-adjusted TFP

I follow the approach of Imbs (1999) who employs a partial-equilibrium version of a model from Burnside and Eichenbaum (1996). The aggregate production function is assumed to be constant returns to scale in effective capital and labor services:

\[ Y_t = Z_t(u_t K_t)\alpha(e_t L_t)^{1-\alpha} = Z_t u_t^\alpha e_t^{1-\alpha} K_t^\alpha L_t^{1-\alpha} \]

where \( u_t \) is capital utilization rate and \( e_t \) is labor effort. Capital utilization is endogenized by assuming that it affects capital depreciation: \( \delta_t = \delta u_t^\phi \). Firms labor is assumed to be predetermined within one period, while the labor effort \( e_t \) can be adjusted instantaneously against wage changes. The firm’s period optimization problem then can be written as follows:

\[
\max_{K_t,u_t,e_t} \left[ Z_t(u_t K_t)^\alpha(e_t L_t)^{1-\alpha} - w(e_t)L_t - (r_t + \delta u_t^\phi)K_t \right]
\]

which yields the following first-order conditions:

\[
\frac{\alpha Y_t}{K_t} = r_t + \delta u_t^\phi \\
\frac{\alpha Y_t}{u_t} = \delta \phi u_t^{\phi - 1} K_t \\
(1 - \alpha) \frac{Y_t}{e_t} = w'(e_t)L_t
\]

Combining equations (B.1) and (B.2), and that at the steady state \( u = 1 \) yields:

\[
 u_t = \left( \frac{Y_t}{K_t} \right)^{\frac{\delta}{1+\delta}}
\]
where \( Y/K \) is the steady state capital-to-output ratio. Turning to households, they solve the following optimization problem:

\[
\max \left\{ C_t + \beta \left( \frac{C_t - L_t^{1+\epsilon} - e^{1+\psi}}{1+\epsilon - 1} \right)^{1-\sigma} \right\}_{j=0}^{\infty} \quad \text{s.t.} \quad C_t \leq w(e_t) L_t
\]

The two margins of labor supply enter the utility function separately. The form of the utility function implies no wealth effect in labor supply. This assumption matches the utility function choice in the general equilibrium model. Moreover, the wealth effect on labor effort is likely to be muted at annual frequency of the data. The first-order condition with respect to labor effort is then the following:

\[
w'(e_t) L_t = \frac{u'_e}{u'_C} = e^\psi
\]

Now, to proceed let me make two assumptions regarding the utility function. First, the labor effort enters the utility function. Combining equations (B.5) and (B.3), and assuming the steady state effort \( e = 1 \) yields:

\[
e_t = \left( \frac{Y_t}{Y} \right)^{\frac{1}{1+\psi}}
\]

Equations (B.4) and (B.6) are used to construct measures of capital and labor utilization. The steady-state values of output, consumption, and capital are determined using a one-sided HP filter with a smoothing parameter of 100. I set the parameter that governs elasticity of effort with respect to wage to the average value across OECD countries according to Imbs (1999): \( \psi = 0.1 \), although the results are robust to different values of this parameter. I set the two remaining parameters to \( r = 0.04 \) and \( \delta = 0.1 \), standard values in the RBC literature (annual calibration). The total utilization component of the Solow residual then equals to \( u_t^{\alpha} e_t^{1-\alpha} \), where I use the time-varying labor share from Feenstra et al. (2015).

As a validation exercise, I compare the resulting changes in factor utilization for the U.S. with the widely-used measure based on Basu et al. (2006) methodology. As figure 29 demonstrates, the two series exhibit very strong correlation, so do its components. For the sake of simplicity, the structural model in the second part of the paper specifies only
endogenous utilization of capital. However, the model parameters are estimated to match the dynamics of TFP adjusted for both labor effort and capital utilization. As such, the model setup should be interpreted as a proxy for both margins of utilization. Finally, figure 30 demonstrates that the utilization-adjusted TFP implied by the model is consistent with the data.

**Figure B.11: US factor utilization**

*Note:* annual changes in U.S. factor utilization according to (1) Imbs (1999) methodology (author’s calculations) and (2) Basu et al. (2006) methodology (annual data from [https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tpf](https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tpf)). Correlation between the two measures = 0.87
Note: baseline IRF-matching simulation in figure 8, additional results. Labor productivity is defined as GDP per worker.

B.3.2 Housing market equilibrium

For illustrative purposes, I linearize equilibrium conditions of the housing market around the deterministic steady state at which the collateral constraint binds. The housing market equilibrium is determined by the following households demands (written in terms of stationary variables) and the market clearing condition:

\[
\begin{align*}
    p_t^h &= \mathbb{E}_t \left( \Lambda_{t,t+1}^S p_{t+1}^h g_{t+1} \right) + \kappa \vartheta_t \left( \frac{h_t^S}{\lambda_t^S} \right)^{-\epsilon_h} \\
    p_t^B &= \mathbb{E}_t \left( \Lambda_{t,t+1}^B p_{t+1}^h g_{t+1} \right) + \kappa \vartheta_t \left( \frac{h_t^B}{\lambda_t^B} \right)^{-\epsilon_h} + \chi_t (1 - \rho_B) mp_t^h \\
    1 &= h_t^S + h_t^B
\end{align*}
\]

where \( p_t^h = \frac{p_t^h}{N_t^h} \) and \( \tilde{\lambda}_t^h = \lambda_t^h N_t^\sigma \). The above equations are linearized around the deterministic steady state using sans-serif font to denote percentage deviations from the steady state (i.e.
\[ x_t = \frac{z_t - x}{x} \)

\[ p^h_t = \Lambda^S g \mathbb{E}_t (\Lambda^S_{t,t+1} + p^h_{t+1} + g_{t+1}) + \frac{\kappa}{\lambda^S p^h(h^S)^{\epsilon_h}} \left( \vartheta_t - \tilde{\lambda}^S_t + \epsilon_h h^B_t \right) \]

\[ p^B_t = \Lambda^B g \mathbb{E}_t (\Lambda^B_{t,t+1} + p^h_{t+1} + g_{t+1}) + \frac{\kappa}{\lambda^B p^h(h^B)^{\epsilon_h}} \left( \vartheta_t - \tilde{\lambda}^B_t - \epsilon_h h^B_t \right) + \chi(1 - \rho_B)m(\chi + p^B_t) \]

\[ h^S_t = -\frac{h^B_t}{h^S} h^B_t \]

Given other general equilibrium outcomes, the two demand curves along with the market clearing condition determine the current housing price \( p^h_t \) and its quantity held by borrowers and savers, \( h^B_t \) and \( h^S_t \) respectively, in terms of percentage deviations from the deterministic steady state.

Housing demands can be further simplified by substituting linearized first order conditions with respect to bonds for borrowers and savers, equations 2.13 and 2.6 in the main text. The original FOCs are as follows:

\[ \mathbb{E}_t \left( \Lambda^S_{t,t+1} \frac{1 + r_t}{\Pi_{t+1}} \right) = 1 \]

\[ \mathbb{E}_t \left( \Lambda^B_{t,t+1} \frac{1 + r_t - \rho_B\chi_{t+1}}{\Pi_{t+1}} \right) = 1 - \chi_t \]

Denote the gross real interest rate as \( R_t = \frac{1 + r_t}{\Pi_t} \). Linearizing around the deterministic steady state where the collateral constraint binds and simplifying taking into account that \( \chi = \frac{\beta^S - \beta^B}{\beta^S - \beta^B\rho_B} \Pi \) and \( R = g^\sigma / \beta^S \) one can get:

\[ \mathbb{E}_t \Lambda^S_{t,t+1} = - \mathbb{E}_t R_{t+1} \]

\[ \mathbb{E}_t \Lambda^B_{t,t+1} = - \mathbb{E}_t R_{t+1} - A^B_{\Lambda_1} \chi_t + A^B_{\Lambda_2} \mathbb{E}_t (\chi_{t+1} - R_{t+1} - \Pi_{t+1}) \]

Collateral constraint wedge

where \( A^B_{\Lambda_1} = \frac{\chi}{1 - \chi} \) and \( A^B_{\Lambda_2} = \left( \frac{R\Pi}{\rho_B \chi} - 1 \right)^{-1} \) are positive constants. Finally, substituting this
result into the linearized housing demand delivers the final expressions:

\[
\begin{align*}
\text{Eq. (B.7)} & \quad p_t^h = A_{h1}^S h_t^B + A_{h2}^S \delta_t - A_{h2}^S \lambda_{t+1}^S + A_{h3}^S E_t \left( p_{t+1}^h + g_{t+1} - R_{t+1} \right) \\
\text{Eq. (B.8)} & \quad p_t^h = -A_{h1}^B h_t^B + A_{h2}^B \delta_t - A_{h2}^B \lambda_{t+1}^B + A_{h3}^B E_t \left( p_{t+1}^h + g_{t+1} - R_{t+1} \right) - A_{h4}^B \tilde{\chi}_t + A_{h5}^B E_t (\tilde{\chi}_{t+1} - R_{t+1} - \Pi_{t+1})
\end{align*}
\]

where \( A_{h1}^S = \epsilon_h^B A_{h2}^S \), \( A_{h2}^S = \kappa \), \( A_{h3}^S = g^{1-\sigma} \beta \); and \( A_{h1}^B = \epsilon_h A_{h2}^B \), \( A_{h2}^B = \frac{\kappa \rho}{\rho (1-\rho_B) m_{\chi}} \), \( A_{h3}^B = \frac{\rho - \sigma \beta}{1-(1-\rho_B) m_{\chi}} \), \( A_{h4}^B = A_{t,t+1} \tilde{\chi}_t + A_{h5}^B E_t (\tilde{\chi}_{t+1} - R_{t+1} - \Pi_{t+1}) \) are positive constant.

\[\text{B.3.3 Equity market equilibrium}\]

For illustrative purposes, I linearise equilibrium conditions of the equity market around the deterministic steady state at which the collateral constraint binds. The dynamics of the equity market that finances innovation in determined by households demand, equation (2.15) and the blueprint price \( p_t^b \), determined by equation (2.26), along with the free-entry condition that equalizes the firm values and the blueprint price (entry cost):

\[
\begin{align*}
v_t & = (1 - \delta_N) E_t \left( A_{t,t+1} B_t^B (dt_{t+1} + v_{t+1}) \right) \\
\phi_t p_t^b & = 1 + AC_{S,t}^i + AC_{S,t+1}^i - E_t \left( A_{t,t+1} AC_{S,t+1}^i \right) \\
v_t & = p_t^b
\end{align*}
\]

To simplify derivations, abstract from adjustment costs in innovation spending \( (\psi_N = 0) \). Using the definition of the time-varying innovators productivity \( \phi_t \), equation (3.2.2), the system can be simplified as follows:

\[
\begin{align*}
v_t & = (1 - \delta_N) E_t \left( A_{t,t+1} B_t^B (dt_{t+1} + v_{t+1}) \right) \\
v_t & = \phi_t^{-1} \tilde{s}^{1-\rho}_t
\end{align*}
\]
Taking into account that \( E_t \Lambda^B_{t+1} = -E_t R_{t+1} - A^B_{\Lambda_1} \chi_t + A^B_{\Lambda_2} E_t (\chi_{t+1} - R_{t+1} - \Pi_{t+1}) \), the linearized counterparts of the above conditions are:

\[
\nu_t = E_t (A_{v1} d_{t+1} + A_{v2} v_{t+1} - R_{t+1}) - A^B_{\Lambda_1} \chi_t + A^B_{\Lambda_2} E_t (\chi_{t+1} - R_{t+1} - \Pi_{t+1}) \tag{B.9}
\]

\[
\nu_t = (1 - \rho) s_t \tag{B.10}
\]

where \( A_{v1} = \frac{d}{v+d}, \ A_{v2} = \frac{v}{v+d}, \ A^B_{\Lambda_1} = \frac{\chi}{1-\chi}, \) and \( A^B_{\Lambda_2} = \left( \frac{RIH}{\rhoBH} - 1 \right)^{-1} \) are positive constants.

### B.3.4 Marginal utility of consumption dynamics

Start with linearized first order conditions with respect to bonds for borrowers and savers, described in the previous subsections and note that households stochastic discount factor can be expressed as \( E_t \Lambda^H_{t+1} = E_t \tilde{\lambda}^H_{t+1} - \sigma g_{t+1}, \ H \in \{S, B\} \)

\[
E_t \tilde{\lambda}^S_{t+1} = E_t \tilde{\lambda}^S_t - \tilde{\lambda}^S_t = -E_t R_{t+1} + \sigma g_{t+1} \tag{B.11}
\]

\[
E_t \tilde{\lambda}^B_{t+1} = E_t \tilde{\lambda}^B_t - \tilde{\lambda}^B_t = -E_t R_{t+1} + \sigma g_{t+1} - A^B_{\Lambda_1} \chi_t + A^B_{\Lambda_2} E_t (\chi_{t+1} - R_{t+1} - \Pi_{t+1}) \tag{B.12}
\]

where \( A^B_{\Lambda_1} = \frac{\chi}{1-\chi} \) and \( A^B_{\Lambda_2} = \left( \frac{RIH}{\rhoBH} - 1 \right)^{-1} \) are positive constants.

In frictionless economy that exhibits growth the dynamics of marginal utility of consumption depends not only on the expected real interest rate but also on growth expectations.
B.3.5 Aggregate welfare cost of the crisis

Recall the lifetime utility of a household $H \in \{S, B\}$:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta_j^t \left( U(C^H_{t+j}, L^H_{t+j}) + \kappa_t \vartheta_t G(h^H_{t+j}) \right),$$  
where

$$U(C^H_t, L^H_t) = \left( \left( c^H_t - \nu_t \frac{(L^H_t)^{1+\epsilon_L}}{1 + \epsilon_L} \right)^{1-\sigma} - 1 \right) / (1 - \sigma)$$

$$G(h^H_t) = \kappa_t \vartheta_t \frac{(h^H_t)^{1-\epsilon_h} - 1}{1 - \epsilon_h}$$

Period utilities can be rewritten in terms of stationary variables and the stock of knowledge $N_t$ as follows: $U(c^H_t, L^H_t) = N_t^{1-\sigma} \left( \left( c^H_t - \nu_t \frac{(L^H_t)^{1+\epsilon_L}}{1 + \epsilon_L} \right)^{1-\sigma} - 1 \right) / (1 - \sigma)$ and $G(h^H_t) = N_t^{1-\sigma} \kappa_t \vartheta_t \frac{(h^H_t)^{1-\epsilon_h} - 1}{1 - \epsilon_h}$. Lifetime utility of each household then can be expressed recursively in terms of stationary variables and productivity growth rates:

$$W^H_H = \left( \left( c^H_t - \nu_t \frac{(L^H_t)^{1+\epsilon_L}}{1 + \epsilon_L} \right)^{1-\sigma} - 1 \right) / (1 - \sigma) + \kappa_t \vartheta_t \frac{(h^H_t)^{1-\epsilon_h} - 1}{1 - \epsilon_h} + \beta^{H} W^H_{t+1}$$  \quad (B.13)

Finally, the aggregate welfare then is the weighted sum of welfare of savers and borrowers $W_t = \gamma_B W^B_t + \gamma_S W^S_t$. The baseline case assumes that each of the types of households is of the same mass: $\gamma_B = \gamma_S = 0.5$.

The aggregate lifetime utility of households across different scenarios is calculated numerically using the second-order approximation of the model. As common in the literature, I compare the welfare loss/gain across different scenarios by calculating the equivalent variation in steady-state consumption: the percentage by which the steady-state consumption of households would have to be changed in order to achieve the same welfare as in the scenario of interest. Formally, for household $H$ the welfare loss/gain in steady-state consumption units $\Delta_{Wh}$ is calculated as $W^H_t = W^H((1 - \Delta_{Wh})c^H_t, L^H_t, h^H_t)$, where $W^H_t$ is the welfare of the household under the scenario of interest and $W^H$ is its steady-state welfare.
Appendix C

CHAPTER 3

C.1 Additional results

Table C.1: Deterministic vs stochastic steady state
[cited on page and 117]

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Figure C.1: Responses to a home import tariff shock, the role of innovation spending

[cited on page 112]

Note: impulse responses in log deviations from the stochastic steady state, unless otherwise noted. Inflation rate and the real interest rate are annualized.
Figure C.2: Responses to a home import tariff uncertainty shock, the role of shock’s persistence

[cited on page 113]

Note: impulse responses in log deviations from the stochastic steady state, unless otherwise noted. Inflation rate and the real interest rate are annualized. $\sigma_t = (1 - \rho_{\sigma}) \sigma_t + \rho_{\sigma} \sigma_{t-1} + \sigma^e \epsilon_t$
Figure C.3: Responses to a home import tariff uncertainty shock, GHH preference

[Note: impulse responses in log deviations from the stochastic steady state, unless otherwise noted. Inflation rate and the real interest rate are annualized.]
C.2 Derivations

C.2.1 Net foreign assets law of motion

Start with the households budget constraint:

\[ C_t + v_t N_t + AC_t^{B*} + AC_t^{W} = \frac{W_t}{P_t} L_t + d_t N_t + \varepsilon_t \left( \frac{B_t^*}{P_t} - (1 + i_{t-1}^*) \frac{B_{t-1}^*}{P_t} \right) + d^*_t + T \]

Taking into account equilibrium the lump-sum transfer \( T = \tau p_t^F Y_t^F + AC_t^{B*} \), innovation spending \( S_t = v_t N_t \), and wholesale firms profit \( d^*_t = (p_t^H - p_t^G)(Y_t^H + N_t x_t/A) + (p_t^{H+} - p_t^G)Y_t^{H*} - AC_t^P - AC_t^{P*} - \Gamma \) one can simplify it as follows:

\[ C_t + S_t + AC_t^{W} = \frac{W_t}{P_t} L_t + d_t N_t + \varepsilon_t \left( \frac{B_t^*}{P_t} - (1 + i_{t-1}^*) \frac{B_{t-1}^*}{P_t} \right) + p_t^H(Y_t^H + N_t x_t/A) + p_t^{H+}Y_t^{H*} - p_t^G(Y_t^H + N_t x_t/A + Y_t^{H*}) - (AC_t^P + AC_t^{P*} + \Gamma) + \tau p_t^F Y_t^F \]

Then, use the final good market clearing \( Y_t = C_t + S_t + AC_t^{W} + AC_t^{P} + AC_t^{P*} + \Gamma \) to get:

\[ Y_t = \frac{W_t}{P_t} L_t + d_t N_t + \varepsilon_t \left( \frac{B_t^*}{P_t} - (1 + i_{t-1}^*) \frac{B_{t-1}^*}{P_t} \right) + p_t^H(Y_t^H + N_t x_t/A) + p_t^{H+}Y_t^{H*} - p_t^G(Y_t^H + N_t x_t/A + Y_t^{H*}) + \tau p_t^F Y_t^F \]

Total spending on the aggregate consumption good inclusive of the import tariff is \( P_t Y_t = P_t^H Y_t^H + (1 + \tau_t) P_t^F Y_t^F \), which allows to simplify the expression further:

\[ p_t^F Y_t^F - p_t^{H+}Y_t^{H*} = \varepsilon_t \left( \frac{B_t^*}{P_t} - (1 + i_{t-1}^*) \frac{B_{t-1}^*}{P_t} \right) + \frac{W_t}{P_t} L_t + d_t N_t + p_t^H N_t x_t/A - p_t^G(Y_t^H + N_t x_t/A + Y_t^{H*}) \]

Substitute in the domestic good market clearing condition \( F_t = Y_t^H + Y_t^{H*} + N_t x_t/A \)

\[ p_t^F Y_t^F - p_t^{H+}Y_t^{H*} = \varepsilon_t \left( \frac{B_t^*}{P_t} - (1 + i_{t-1}^*) \frac{B_{t-1}^*}{P_t} \right) + \frac{W_t}{P_t} L_t + d_t N_t + p_t^H N_t x_t/A - p_t^G F_t \]

Production sector firms operate on a perfectly competitive market with zero profits, which implies \( p_t^G F_t = N_t p^* x_t + \frac{W_t}{P_t} L_t = N_t (d_t + p_t^H x_t/A) + \frac{W_t}{P_t} L_t \). Use this identity to cancel out
terms:

\[ p_t^F Y_t^F - p_t^{H_h} Y_t^{H^*} = \varepsilon_t \left( \frac{B_t^*}{P_t^*} - (1 + i_{t-1}^*) \frac{B_{t-1}^*}{P_{t-1}^*} \right) \]

Finally, use the definition of the real exchange rate \( rer_t = \frac{\varepsilon_t P_t^*}{P_t^r} \) and the fact that \( p_t^{H_h} = rer_t p_t^{H^*} \) and \( p_t^F = rer_t p_t^{F^*} \) to get:

\[ p_t^{F^*} Y_t^F - p_t^{H^*} Y_t^{H^*} = \frac{B_t^*}{P_t^*} - \frac{1 + i_{t-1}^* B_{t-1}^*}{\Pi_t^* P_{t-1}^*} \]

(C.1)

### C.2.2 Foreign exchange risk

In what follows we assume log-normality and use the fact that for an arbitrary log-normal variable \( x_t \) the following holds true: \( \log(\mathbb{E}_t x_t) = \mathbb{E}_t(\log x_t) + \frac{1}{2} \text{Var}_t(\log x_t) \).

Start with the Euler equation for foreign currency bonds (3.2) and the assumption of log utility in consumption. Take logs:

\[ \log \beta - \log g_t + \log \mathbb{E}_t \left( \frac{c_t}{c_{t+1}} \frac{rer_{t+1}}{rer_t} R_{t+1}^* \right) = \log(1 - \psi b_{t+1}^*) \approx 0, \]

which under the assumption of log-normality becomes:

\[ \log \beta - \log g_t + \mathbb{E}_t \log \left( \frac{c_t}{c_{t+1}} \frac{rer_{t+1}}{rer_t} R_{t+1}^* \right) + \frac{1}{2} \text{Var}_t \log \left( \frac{c_t}{c_{t+1}} \frac{rer_{t+1}}{rer_t} R_{t+1}^* \right) \approx 0 \]

\[ \log \beta - \log g_t - \mathbb{E}_t \log \left( \frac{c_{t+1}}{c_t} \right) + \mathbb{E}_t \log \left( \frac{rer_{t+1}}{rer_t} \right) + \mathbb{E}_t \log R_{t+1}^* + \frac{1}{2} \left( \text{Var}_t \log \left( \frac{c_{t+1}}{c_t} \right) + \text{Var}_t \log \left( \frac{rer_{t+1}}{rer_t} \right) + \text{Var}_t \log R_{t+1}^* \right) + \text{Cov}_t \left( \log R_{t+1}^*, \log \left( \frac{rer_{t+1}}{rer_t} \right) \right) - \text{Cov}_t \left( \log \left( \frac{c_{t+1}}{c_t} \right), \log \left( \frac{rer_{t+1}}{rer_t} R_{t+1}^* \right) \right) \approx 0 \]
The fact that $E_t(\log x_t) = \log(E_t x_t) - \frac{1}{2} \text{Var}_t(\log x_t)$ allows to simplify the expression further:

$$
\log \beta - \log g_t - \log E_t \left( \frac{c_{t+1}}{c_t} \right) + \log E_t \log \frac{\text{rer}_{t+1}}{\text{rer}_t} + \log E_t \log R^*_{t+1} + \text{Var}_t \log \frac{c_{t+1}}{c_t} +
\text{Cov}_t \left( \log R^*_{t+1}, \log \left( \frac{\text{rer}_{t+1}}{\text{rer}_t} \right) \right) - \text{Cov}_t \left( \log \frac{c_{t+1}}{c_t}, \log \left( \frac{\text{rer}_{t+1}}{\text{rer}_t} R^*_{t+1} \right) \right) \approx 0
$$

Using sans serif font to denote log deviations from the steady state (e.g. $E_t x_{t+1} = \log E_t x_{t+1} - \log x$), we arrive at the following expression:

$$
(\text{Var}_t \Delta c_{t+1} - E_t \Delta c_{t+1} - g_t) + (E_t R^*_{t+1} + E_t \Delta \text{rer}_{t+1}) +
\text{Cov}_t (R^*_{t+1}, \Delta \text{rer}_{t+1}) - \text{Cov}_t (\Delta c_{t+1}, \Delta \text{rer}_{t+1} + R^*_{t+1}) \approx 0
$$

Finally, if the foreign real interest rate follows a deterministic process we get:

$$
(\text{Var}_t \Delta c_{t+1} - E_t \Delta c_{t+1} - g_t) + (R^*_{t+1} + E_t \Delta \text{rer}_{t+1}) - \text{Cov}_t (\Delta c_{t+1}, \Delta \text{rer}_{t+1}) \approx 0 \quad (C.2)
$$

Similarly, the assumption of log-normality allows to transform the Euler equation for domestic bonds (3.1) as follows:

$$
\log \beta - \log g_t - E_t \log \left( \frac{c_{t+1}}{c_t} \right) + E_t \log R_{t+1} +
\frac{1}{2} \left( \text{Var}_t \log \left( \frac{c_{t+1}}{c_t} \right) + \text{Var}_t \log R_{t+1} \right) - \text{Cov}_t \left( \log \left( \frac{c_{t+1}}{c_t} \right), \log R_{t+1} \right) = 0
$$

$$
\log \beta - \log g_t - E_t \log \left( \frac{c_{t+1}}{c_t} \right) + \log E_t R_{t+1} - \text{Cov}_t \left( \log \left( \frac{c_{t+1}}{c_t} \right), \log R_{t+1} \right) = 0
$$

$$
(\text{Var}_t \Delta c_{t+1} - E_t \Delta c_{t+1} - g_t) + E_t R_{t+1} - \text{Cov}_t (\Delta c_{t+1}, R_{t+1}) = 0 \quad (C.3)
$$

Take the difference between (C.2) and (C.3) to show that UIP violation is linked to the
relative risk premia:

\[
(\mathbb{E}_t R^*_t + \mathbb{E}_t \Delta \text{rer}_t) - \mathbb{E}_t R_{t+1} \approx \text{Cov}_t (\Delta c_{t+1}, \Delta \text{rer}_t) - \text{Cov}_t (\Delta c_{t+1}, R_{t+1}) \tag{C.4}
\]

### C.2.3 Equity risk

Start with a first-order condition for equity holdings (3.3) and take logs:

\[
\log(\beta (1 - \delta)) - \log g_t + \log \mathbb{E}_t \left( \frac{c_t}{c_{t+1}} R^\text{equity}_{t+1} \right) = 0,
\]

where \(R^\text{equity}_{t+1} = \frac{d_{t+1} + r_{t+1}}{v_t} \). Assuming log-normality and using sans serif font to denote log deviations from the steady state we get:

\[
\log(\beta (1 - \delta)) - \log g_t - \mathbb{E}_t \log \left( \frac{c_{t+1}}{c_t} \right) + \mathbb{E}_t \log R^\text{equity}_{t+1} + \frac{1}{2} \left( \text{Var}_t \log \left( \frac{c_{t+1}}{c_t} \right) + \text{Var}_t \log R^\text{equity}_{t+1} \right) - \text{Cov}_t \left( \log \left( \frac{c_{t+1}}{c_t} \right), \log R^\text{equity}_{t+1} \right) = 0
\]

\[
(\text{Var}_t \Delta c_{t+1} - \mathbb{E}_t \Delta c_{t+1} - g_t) + \mathbb{E}_t R^\text{equity}_{t+1} - \text{Cov}_t \left( \Delta c_{t+1}, R^\text{equity}_{t+1} \right) = 0
\]

The risk-free rate \(R^f_{t+1}\) is such that \(\text{Var}_t \Delta c_{t+1} - \mathbb{E}_t \Delta c_{t+1} - g_t + R^f_{t+1} = 0\) and so

\[
\mathbb{E}_t R_{t+1} - R^f_{t+1} = \text{Cov}_t (\Delta c_{t+1}, R_{t+1}), \tag{C.5}
\]

from which it follows that the excess return on equity is

\[
\mathbb{E}_t R^\text{equity}_{t+1} - R^f_{t+1} = \text{Cov}_t (\Delta c_{t+1}, R^\text{equity}_{t+1}) \tag{C.6}
\]

Similarly, equation (3.3) can be used to decompose the current firm value \(v_t\) as follows:

\[
v_t = (\text{Var}_t \Delta c_{t+1} - \mathbb{E}_t \Delta c_{t+1} - g_t) + \mathbb{E}_t P^e_{t+1} - \text{Cov}_t (\Delta c_{t+1}, P^e_{t+1}) \tag{C.7}
\]

where \(P^e_{t+1} = v_{t+1} + d_{t+1}\) denotes the \(t + 1\) payoff from holding equity, which includes the equity price and the dividend.