Appendix 1: Estimation Model Specifications

Model dynamics were developed in collaboration for NPRB project 1307 between Dr. André E. Punt and Caitlin Allen Akselrud.

A.1.1 Basic dynamics

The dynamics of the modeled population account for mortality due to fishing and natural causes as well as growth, recruitment, and ageing at the end of the year. The model has an annual time-step that leads to the following equation for the population dynamics for an age-size cohort (Equation A.1.1a) and an age-cohort (Equation A.1.1b).[1]

$$N_{p,y,a,l} = \begin{cases} R_y\, \omega_{p,l} & \text{if } a = 0 \\ \sum_{l'} X_{p,l',l} N_{p,y-1,a-1,l'} e^{-Z_{p,y-1,a-1,l'}} & \text{if } 1 \le a < a_{\max} \\ \sum_{l'} \left(X_{p,l',l} N_{p,y-1,a_{\max}-1,l'} e^{-Z_{p,y-1,a_{\max}-1,l'}} + X_{p,l',l} N_{p,y-1,a_{\max},l'} e^{-Z_{p,y-1,a_{\max},l'}} \right) & \text{if } a = a_{\max} \end{cases} \quad \text{(A.1.1a)}$$

$$N_{p,y,a} = \begin{cases} R_y\, \omega_p & \text{if } a = 0 \\ N_{p,y-1,a-1} e^{-Z_{p,y-1,a-1}} & \text{if } 1 \le a < a_{\max} \\ N_{p,y-1,a_{\max}} e^{-Z_{p,y-1,a_{\max}}} + N_{p,y-1,a_{\max}-1} e^{-Z_{p,y-1,a_{\max}-1}} & \text{if } a = a_{\max} \end{cases} \quad \text{(A.1.1b)}$$

where $N_{p,y,a,l}$ is the number of animals of age *a* in platoon *p* at the start of year *y* that are in size-class *l*, $N_{p,y,a}$ is the number of animals of age *a* in platoon *p* at the start of year *y*, $a_{\max}$ is the maximum age-class, $X_{p,l',l}$ is the probability of an animal in platoon *p* growing from size-class *l*' to size-class *l* during a year, R_y is the total number of age-0 animals during year *y*, $\omega_{p,l}$ is the proportion of the total number of age-0 animals that settle to platoon *p* in size-class *l* (assumed to be time-variant), ω_p is the proportion of the total number of age-0 animals that settle to platoon *p*, $Z_{p,y,a,l}$ is the total mortality on animals of age *a* in platoon *p* that are in size-class *l* during year *y*, and $Z_{p,y,a}$ is the total mortality on animals of age *a* in platoon *p* during year *y*.

The total mortality during each year accounts for natural mortality and fishing mortality. The model allows for both age-specific and size-specific selectivity (or availability) patterns, with retention allowed to be age- as well as size-specific. Survival of discards is allowed through the parameter λ^f, which, if non-zero, allows for deaths of discarded individuals. Equations A.1.2a and A.1.2b respectively outline how total mortality is defined for the age-size- and age-structured population dynamics models. Equation A.1.2b allows for size-specific selectivity by integrating size-specific selectivity over the distribution for size-at-age (which is considered to be time-invariant) in the age-structured model.

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[1] The model allows for parameters and variables to depend on sex, but this is ignored in this presentation and in the example application.

$$Z_{p,y,s,a,l} = M + \sum_f \left\{ F_y^f (\Omega_{y,l}^f + [1-\Omega_{y,l}^f]\lambda^f) S_{y,l}^f (\tilde{\Omega}_{y,a}^f + [1-\tilde{\Omega}_{y,a}^f]\lambda^f) \tilde{S}_{y,a}^f \right\} \quad \text{(A.1.2a)}$$

$$Z_{p,y,a} = M + \sum_f \left\{ F_y^f (\tilde{\Omega}_{y,a}^f + [1-\tilde{\Omega}_{y,a}^f]\lambda^f) \tilde{S}_{y,a}^f \sum_l \phi_{p,a,l} (\Omega_{y,l}^f + [1-\Omega_{y,l}^f]\lambda^f) S_{y,l}^f \right\} \quad \text{(A.1.2b)}$$

where M is natural mortality (assumed to be independent of age, size and time), $S_{y,l}^f$ is the selectivity of fleet *f* on animals in size-class *l* during year *y*, $\tilde{S}_{y,a}^f$ is the selectivity of fleet *f* on animals of age *a* during year *y*, F_y^f is the fully-selected ($S_{y,l}^f / S_{y,a}^f \to 1$) fishing mortality due to fleet *f* during year *y*, $\Omega_{y,l}^f$ is the proportion of animals in size-class *l* that are retained by fleet *f* during year *y*, $\tilde{\Omega}_{y,a}^f$ is the proportion of animals of age *a* that are retained by fleet *f* during year *y*, $\phi_{p,a,l}$ is the proportion of fish of age *a* in platoon *p* that are in size-class *l*, and λ^f is the mortality rate for discarded animals by fleet *f*. Retention can only be a function of size or age, but not both. Note that for a size-structured model, total mortality is computed using Eqn A.1.2a, but with the terms related to age omitted.

The total number of age-0 animals during year y, R_y, is given by a Beverton-Holt stock-recruitment relationship, i.e.:

$$R_y = \frac{4hR_0 S_y / S_0}{(1-h) + (5h-1) S_y / S_0} e^{\varepsilon_y - \psi_y \sigma_R^2 / 2} \quad \text{(A.1.3)}$$

where *h* is the steepness of the stock-recruitment relationship, S_y is the reproductive output during year *y*, S_0 is the unfished reproductive output, R_0 is the number of age-0 animals at unfished equilibrium, ε_y is the recruitment deviation for year *y*, σ_R is the extent of variation in recruitment about the stock-recruitment relationship, ψ_y is the extent of bias-correction (a linear function that increases from 0 at a user-specified year to 1 between two user-specified years and then declines to 0 at a final user-specified year; Methot and Taylor, 2011). The bias-correction factor is needed to ensure that the expected recruitment is equal to the deterministic component of Equation A.1.3

The reproductive output during year *y* is given by Equation A.1.4a for the age-size-structured model and Equation A.1.4b for the age-structured model:

$$S_y = \sum_l w_l \sum_p \sum_a m_a N_{p,y,a,l} \qquad S_y = \sum_l m_l w_l \sum_p \sum_a N_{p,y,a,l} \quad \text{(A.1.4a)}$$

$$S_y = \sum_p \sum_a m_a \left(\sum_l \phi_{p,a,l} w_l \right) N_{p,y,a} \qquad S_y = \sum_p \sum_a \left(\sum_l \phi_{p,a,l} m_l w_l \right) N_{p,y,a} \quad \text{(A.1.4b)}$$

depending on whether maturity is a function of age (left equations) or size (right equations), where m_l is the proportion of animals in size-class *l* that are mature, m_a is the proportion of animals in age-class *a* that are mature, and w_l is the mean weight of an animal in size-class *l*.

The spawning biomass during the year is reported as it is a key output of stock assessments:

$$\tilde{S}_y=\sum_l w_l\sum_p\sum_a m_a N_{p,y,a,l}\, e^{-\tau Z_{p,y,a,l}} \qquad \tilde{S}_y=\sum_l m_l w_l\sum_p\sum_a N_{p,y,a,l}\, e^{-\tau Z_{p,y,a}} \tag{A.1.5a}$$

$$\tilde{S}_y=\sum_p\sum_a m_a\left(\sum_l \phi_{p,a,l} w_l\right)N_{p,y,a}\, e^{-\tau Z_{p,y,a,l}} \qquad \tilde{S}_y=\sum_p\sum_a\left(\sum_l \phi_{p,a,l} m_l w_l\right)N_{p,y,a}\, e^{-\tau Z_{p,y,a}} \tag{A.1.5b}$$

where τ is the time between the start of the year and when spawning takes place.

The proportion of the total number of age-0 animals that settle to platoon *p*, and size-class *l* is modeled using a gamma function, i.e.:

$$\omega_{p,l}=\frac{(\bar{L}_l)^{\alpha_R}\, e^{\bar{L}_{l'}/\beta_R}}{\sum_{l'}(\bar{L}_{l'})^{\alpha_R}\, e^{\bar{L}_{l'}/\beta_R}} \tag{A.1.6}$$

where α_R and β_R are the parameters that define the mean and variance of the gamma function, and $\bar{L}_l$ is the midpoint of size-class *l*. The range of size-classes to which animals can settle can be pre-specified.

The landed (retained) catches (superscript R) and discard (superscript D) mortality for animals of age *a* by fleet *f* during year *y* are given by the following equations (Equation A.1.7a for the age-size-structured population dynamics model and Equation A.1.7b for the age-structured population dynamics model):

$$C^{\mathrm{R},f}_{y,a,l}=\sum_p\frac{\tilde{S}^f_{y,a}\tilde{\Omega}^f_{y,a}S^f_{y,l}\Omega^f_{y,l}\,F^f_y}{Z_{p,y,a,l}}N_{p,y,a,l}(1-e^{-Z_{p,y,a,l}}) \tag{A.1.7a1}$$

$$C^{\mathrm{D},f}_{y,a,l}=\sum_p\frac{\lambda^f[1-\tilde{\Omega}^f_{y,a}\,\Omega^f_{y,l}]\tilde{S}^f_{y,a}S^f_{y,l}\,F^f_y}{Z_{p,y,a,l}}N_{p,y,a,l}(1-e^{-Z_{p,y,a,l}}) \tag{A.1.7a2}$$

$$C^{\mathrm{R},f}_{y,a,l}=\sum_p\frac{\tilde{S}^f_{y,a}\tilde{\Omega}^f_{y,l}\phi_{p,a,l}S^f_{y,l}\,\Omega^f_{y,l}F^f_y}{Z_{p,y,a}}N_{p,y,a}(1-e^{-Z_{p,y,a}}) \tag{A.1.7b1}$$

$$C^{\mathrm{D},f}_{y,a,l}=\sum_p\frac{\lambda^f\tilde{S}^f_{y,a}\phi_{p,a,l}[1-\tilde{\Omega}^f_{y,a}\,\Omega^f_{y,l}]S^f_{y,l}\,F^f_y}{Z_{p,y,a}}N_{p,y,a}(1-e^{-Z_{p,y,a}}) \tag{A.1.7b2}$$

The landed catch and discard mortality in weight are given by:

$$\tilde{C}^{\mathrm{R},f}_y=\sum_a\sum_l C^{\mathrm{R},f}_{y,a,l}\,w_l \qquad \tilde{C}^{\mathrm{D},f}_y=\sum_a\sum_l C^{\mathrm{D},f}_{y,a,l}\,w_l \tag{A.1.8}$$

A.1.2 Selectivity

The relationship between size/age and selectivity is governed by several functions[2].

[2] The dependence of selectivity on year and sex is dropped from this section for ease of presentation. The functions below are in terms of length but the age-specific functions are identical to the size-specific functions except that length is replaced by age.

A.1.2.1 Logistic function

Selectivity can be assumed to be a logistic function:

$$S_l^f = (1+\exp(-\ell n 19[\bar{L}_l - L_{50}^f]/[L_{95}^f - L_{50}^f]))^{-1} \tag{A.1.9}$$

where L_{50}^f is the size-at-50%-selectivity for fleet *f*, and L_{95}^f is the size-at-95%-selectivity for fleet *f*.

A.1.2.2 Double logistic

Selectivity can be a double logistic function (Methot and Wetzel, 2013). This function is composed of three segments: an ascending segment for small fish (asc), a flat top where selectivity equals 1.0 and a descending limb for large fish (dsc). The three sections are joined at two intersections using steep logistic functions j_1 and j_2:

$$\beta_{f,l} = asc_f\left(1 - j_{1,f,l}\right) + j_{1,f,l}\left(1 - j_{2,f,l} + dsc_f \cdot j_{2,f,l}\right) \tag{A.1.10a}$$

where:

$$asc_f = \psi_{5,f} - \left(1-\psi_{5,f}\right)\frac{\left(\exp[-(\bar{L}_l - \psi_{1,f})^2/\psi_{3,f}] - \exp[-(L_{\min} - \psi_{1,f})^2/\psi_{3,f}]\right)}{\left(1 - \exp[-(L_{\min} - \psi_{1,f})^2/\psi_{3,f}]\right)} \tag{A.1.10b}$$

$$dsc_f = 1 + (\psi_{6,f} - 1)\frac{\exp[-(\bar{L}_l - \psi_{2,f})^2/\psi_{4,f}] - 1}{\exp[-(\bar{L}_{\max} - \psi_{2,f})^2/\psi_{4,f}] - 1} \tag{A.1.10c}$$

$$j_{1,f,l} = \left(1 + \exp[-\frac{20(\bar{L}_l - \psi_{1,f})}{1 + \left|\bar{L}_l + \psi_{1,f}\right|}]\right)^{-1} \tag{A.1.10d}$$

$$j_{2,f,l} = \left(1 + \exp[-\frac{20(\bar{L}_l - \psi_{2,f})}{1 + \left|\bar{L}_l + \psi_{2,f}\right|}]\right)^{-1} \tag{A.1.10e}$$

where $L_{\min}$ is the mean size of fish in the smallest size bin, $L_{\max}$ is the mean size of fish in the largest size bin, $\psi_{1,f}$ is the size at which selectivity reaches 1.0 for fleet *f*, $\psi_{2,f}$ is the size where selectivity starts decreasing from 1.0 for fleet *f*, $\psi_{3,f}$ determines the slope of the ascending limb for fleet *f*, $\psi_{4,f}$ the slope of the descending limb for fleet *f*, and $\psi_{5,f}$ is the selectivity at $L_{\min}$ for fleet *f*, $\psi_{6,f}$ is the selectivity at $L_{\max}$ for fleet *f*.

A.1.3 Retention

The probability of being retained as a function of size is governed by a logistic curve where the asymptote, i.e.:

$$\Omega_{y,s,l}^f = \Omega_\infty^f(1+\exp(-\ell n 19[\bar{L}_l - L_{50}^{\Omega,f}]/[L_{95}^{\Omega,f} - L_{50}^{\Omega,f}]))^{-1} \tag{A.1.11}$$

where Ω_∞^f is the asymptotic probability of retention (the probability of selectivity for "infinitely long" animals, and $L_{50}^{\Omega,f}$ is the size-at-50%-retention for fleet *f*, and L_{95}^f is the size-at-95%-retention for fleet *f*.

A.1.4 Growth

A.1.4.1 Size-structured model

Growth in the size-structured and age-size-structured models is modeled using a size-transition matrix that depends on size and platoon, i.e.:

$$X_{p,l',l} = \int_{\bar{L}_l-\Delta L/2}^{\bar{L}_l+\Delta L/2} P_p(\ell\,|\bar{L}_{l'})d\ell \tag{A.1.12}$$

where the relationship between current size and growth increment is linear, i.e.:

$$\alpha_{\bar{L},p} = \alpha_p + \beta_p\bar{L} \tag{A.1.13}$$

where α_p and β_p determine the relationship between growth increment and size, and are computed from the asymptotic size and growth rate parameters. The distribution of growth increment about the mean growth increment can be normal, log-normal or gamma.

Growth in the age-structured model is based on the assumption that the size-at-age distribution is not impacted by size-specific selectivity, i.e.

$$\phi_{p,a,l} = \int_{\bar{L}_l-\Delta L}^{\bar{L}_l+\Delta L} \tilde{P}_p(\ell\,|\,a)d\ell \tag{A.1.14}$$

The relationship between age and size is based on the von Bertalanffy growth equation under the assumption that size-at-age is normally distributed, i.e.:

$$\tilde{P}_p(\ell\,|\,a) = \tfrac{1}{\sqrt{2\pi}\tilde{\sigma}} e^{-(\tilde{\alpha}_{p,a}-\ell)^2/(2\tilde{\sigma}^2)} \tag{A.1.15}$$

where $\tilde{\sigma}$ is the standard deviation of size-at-age, and $\tilde{\alpha}_{p,a}$ is the expected size for an animal in platoon *p* and of age *a*:

$$\tilde{\alpha}_{p,a} = L_0 + (\ell_{\infty,p} - L_0)(1-\mathrm{e}^{-\kappa_p a}) \tag{A.1.16}$$

where L_0 is the size of an animal of age 0, $\ell_{\infty,p}$ is asymptotic size for platoon *p*, and κ_p is the growth rate parameter for platoon *p*.

A.1.4.2 age-structured-only model

Growth in the age-structured-only model is based on the assumption that the size-at-age distribution is not impacted by size-specific selectivity, i.e.

$$\phi_{p,s,a,l} = \int_{\bar{L}_l-\Delta L}^{\bar{L}_l+\Delta L} \tilde{P}_{p,s}(\ell\,|\,a)d\ell \tag{A.1.17}$$

The relationship between age and size is based on the von Bertalanffy growth equation under the assumption that size-at-age is normally distributed, i.e.:

$$\tilde{P}_{p,s}(\ell\,|\,a) = \tfrac{1}{\sqrt{2\pi}\tilde{\sigma}_s} e^{-(\tilde{\alpha}_{p,s,a}-\ell)^2/(2\tilde{\sigma}_s^2)} \tag{A.1.18}$$

where $\tilde{\sigma}_s$ is the standard deviation of size-at-age for animals of sex *s*, and $\tilde{\alpha}_{p,s,a}$ is the expected size for an animal of sex *s* in platoon *p* and of age *a*:

$$\tilde{\alpha}_{p,s,a} = L_0 + (\ell_{\infty,p,s} - L_0)(1 - \mathrm{e}^{-\kappa_{p,s} a}) \tag{A.1.19}$$

where L_0 is the size of an animal of age 0.

A.1.5 Catches

The landed (retained) catches and discard mortality for animals of sex *s* and age *a* by fleet *f* during year *y* are given by the following Equations A1.20a for the age-size-structured model and A.1.20b for the age-structured-only model):

$$C^{\mathrm{R},f}_{y,s,a,l} = \sum_p \frac{\tilde{S}^f_{y,s,a}\tilde{\Omega}^f_{y,s,a} S^f_{y,s,l}\Omega^f_{y,s,l}\, F^f_y}{Z_{p,y,s,a,l}} N_{p,y,s,a,l}(1 - e^{-Z_{p,y,s,a,l}}) \tag{A.1.20a1}$$

$$C^{\mathrm{D},f}_{y,s,a,l} = \sum_p \frac{\lambda^f [1 - \tilde{\Omega}^f_{y,s,a}\ \Omega^f_{y,s,l}]\tilde{S}^f_{y,s,a} S^f_{y,s,l}\, F^f_y}{Z_{p,y,s,a,l}} N_{p,y,s,a,l}(1 - e^{-Z_{p,y,s,a,l}}) \tag{A.1.20a2}$$

$$C^{\mathrm{R},f}_{y,s,a,l} = \sum_p \frac{\tilde{S}^f_{y,s,a}\tilde{\Omega}^f_{y,s,l}\phi_{p,s,a,l} S^f_{y,s,l}\ \Omega^f_{y,s,l} F^f_y}{Z_{p,y,s,a}} N_{p,y,s,a}(1 - e^{-Z_{p,y,s,a}}) \tag{A.1.20b1}$$

$$C^{\mathrm{D},f}_{y,s,a,l} = \sum_p \frac{\lambda^f \tilde{S}^f_{y,s,a}\phi_{p,s,a,l}[1 - \tilde{\Omega}^f_{y,s,a}\ \Omega^f_{y,s,l}] S^f_{y,s,l}\ F^f_y}{Z_{p,y,s,a}} N_{p,y,s,a}(1 - e^{-Z_{p,y,s,a}}) \tag{A.1.20b2}$$

The landed catch and discard mortality in weight are given by:

$$\tilde{C}^{\mathrm{R},f}_{y,s} = \sum_a \sum_l C^{\mathrm{R},f}_{y,s,a,l}\, w_{s,l} \qquad \tilde{C}^{\mathrm{D},f}_{y,s} = \sum_a \sum_l C^{\mathrm{D},f}_{y,s,a,l}\, w_{s,l} \tag{A.1.21}$$

A.1.6 Initial conditions

The initial conditions correspond to unfished equilibrium in a pre-specified initial year. The population is then projected forward with stochastic recruitment (if recruitment deviations are estimated for the years before the first year with catches) and estimated fishing mortality rates by fleet so that the age- and size-structure at the start of the first year is not in equilibrium. This approach mimics the assessment of rock lobster, *Jasus edwardsii* off Tasmania (Punt and Kennedy, 1997) and which has also formed the basis for recent assessments of EBS Tanner crab (Stockhausen, 2015).

A.1.7 Objective function

The data potentially available for assessment purposes are catches (assumed known without error), indices of relative abundance, time-series of discards, time-series of effort values, size-frequency data, and conditional age-at-size data[3]. The following sections outline the contribution of each data source to the likelihood function. There is no contribution by the landed catch because this is assumed to be known exactly. The overall objective function is a weighted sum of each of the contributions to the likelihood function by each data source.

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[3] For ease of presentation the dependence on fleet is ignored here and the same symbols for residual standard errors and effective sample sizes are used across data sources.

A.1.7.1. Index data

The indices of abundance are assumed to be log-normally distributed about the model-predictions:

$$L_1 = \sum_y (\ell\mathrm{n}(\sigma_y) + \tfrac{1}{2\sigma_y^2}[\ell\mathrm{n}I_y - \ell\mathrm{n}\hat{I}_y]^2) \qquad \text{(A.1.22)}$$

where I_y is the index of abundance for *y*, $\hat{I}_y$ is the model-estimate corresponding to I_y (equation A.1.23a for the size- and age-size-structured model and equation A.1.23b for the age-structured model):

$$\hat{I}_y = \begin{cases} q_y \sum_{p,a,l} S^*_{y,a} S^*_{y,l} N_{p,y,a,l} e^{-\tilde{\tau} Z_{p,y,a,l}} & \text{for indexes in numbers} \\ q_y \sum_{p,s,a,l} w_l S^*_{y,a} S^*_{y,l} N_{p,y,a,l} e^{-\tilde{\tau} Z_{p,y,a,l}} & \text{for indexes in mass} \end{cases} \qquad \text{(A.1.23a)}$$

$$\hat{I}_y = \begin{cases} q_y \sum_{p,a} S^*_{y,a} N_{p,y,a} e^{-\tilde{\tau} Z_{p,y,a}} \sum_l \phi_{p,a,l} S^*_{y,l} & \text{for indexes in numbers} \\ q_y \sum_{p,a} S^*_{y,a} N_{p,y,a} e^{-\tilde{\tau} Z_{p,y,a}} \sum_l \phi_{p,a,l} S^*_{y,l} w_l & \text{for indexes in mass} \end{cases} \qquad \text{(A.1.23b)}$$

$S^*_{y,l}$ is the survey selectivity-at-size for animals in size-class *l* during year *y*, $S^*_{y,a}$ is the survey selectivity-at-age for animals of age *a* during year *y* (ignored for the size-structured model), q_y is the catchability coefficient for year *y*, $\tilde{\tau}$ is the time during the year corresponding to the index, and σ_y is the standard error of the logarithm of I_y. The catchability coefficient can either be estimated, given a prior, or set to a pre-specified value.

A.1.7.2 Discard

The data on discard are expressed as the mass or number of discards. These data are assumed log-normally distributed, i.e.:

$$L_2 = \sum_y (\ell\,\mathrm{n}\,\sigma_y + \tfrac{1}{2\sigma_y^2}[\ell\mathrm{n}J_y - \ell\mathrm{n}\hat{J}_y]^2) \qquad \text{(A.1.24)}$$

where J_y is the observed discard for year *y*, σ_y is the standard error of the logarithm of J_y, and $\hat{J}_y$ is the model-estimate of the discard for year *y*:

$$\hat{J}_y = \sum_s \tilde{C}^{\mathrm{D}}_{y,s} \qquad \text{(A.1.25)}$$

Equations A1.24 and A1.25 are modified if the data are recorded by sex rather than as sex-combined. Similarly, Equation A1.21 is modified to exclude the $w_{s,l}$ if the discard data are expressed in numbers rather than weight.

A.1.7.3 Effort data

Effort time-series are assumed to provide indices of fully-selected fishing mortality. Under the assumption that effort is log-normally distributed about fully-selected fishing mortality, the contribution of the effort data to the objective function is given by:

$$L_3=\sum_y(\ell\,\mathrm{n}\,\sigma_y+\tfrac{1}{2\sigma_y^2}[\ell\mathrm{n}E_y-\ell\mathrm{n}(\tilde{q}F_y)]^2) \tag{A.1.26}$$

where E_y is the effort for year *y*, F_y is the fully-selected fishing mortality during year *y*, and $\tilde{q}$ is the constant of proportionality between effort and fishing mortality.

A.1.7.4 *Size-composition data*

Size-composition data are assumed to be available for the retained component of the catch, the discarded component of the catch, or the total catch. The size-composition data are assumed to be multinomially distributed about the model predictions, i.e.:

$$L_2=\sum_y N_y^{\mathrm{Eff}}\sum_l P_{y,l}\ell\mathrm{n}(\hat{P}_{y,l}/P_{y,l}) \tag{A.1.27}$$

where $P_{y,l}$ is the observed proportion of the catch during year *y* that is in size-class *l*, $\hat{P}_{y,l}$ is the model-estimate corresponding to $P_{y,l}$:

$$\hat{P}_{y,l}=\begin{cases}\sum_a C^{\mathrm{R}}_{y,a,l}/\sum_{a'}\sum_{l'}C^{\mathrm{R}}_{y,a',l'} & \text{Retained catch}\\ \sum_a C^{\mathrm{D}}_{y,a,l}/\sum_{a'}\sum_{l'}C^{\mathrm{D}}_{y,a',l'} & \text{Discarded catch}\\ \sum_a (C^{\mathrm{D}}_{y,a,l}+C^{\mathrm{R}}_{y,a,l})/\sum_{a'}\sum_{l'}(C^{\mathrm{D}}_{y,a',l}+C^{\mathrm{R}}_{y,a',l'}) & \text{Total catch}\end{cases} \tag{A.1.28}$$

N_y^{Eff} is the effective sample size for year *y*. The symbols *a'* and *l'* are used to denote the indices over which summations occur. λ^f in Equations A.1.2a and A.1.2b is ignored when computing the model-predicted discard size-compositions because these size-compositions are sampled before animals are discarded.

A.1.7.5 Conditional age-at-size data

Conditional age-at-size data are assumed to be available for the retained component of the catch, the discarded component of the catch, or the total catch. These data are assumed to be multinomially distributed about the model predictions, i.e.:

$$L_3=\sum_y\sum_{l'}N_{y,l'}^{\mathrm{Eff}}\sum_a P_{y,a,l'}\,\ell\mathrm{n}(\hat{P}_{y,a,l'}/P_{y,a,l'}) \tag{A.1.29}$$

where $P_{y,a,l'}$ is the proportion of the catch in numbers during year *y* which is of age *a* given the catch is from a set of size classes, indicated in this section by l, $\hat{P}_{y,a,l'}$ is the model-estimate corresponding to $P_{y,a,l'}$:

$$
\hat{P}_{y,a,l'} = \begin{cases} \sum_{l \in l'} C^{\mathrm{R}}_{y,a,l} / \sum_{l'' \in l'} \sum_{a'} C^{\mathrm{R}}_{y,a',l''} & \text{Retained catch} \\ \sum_{l \in l'} C^{\mathrm{D}}_{y,a,l} / \sum_{l'' \in l'} \sum_{a'} C^{\mathrm{D}}_{y,a',l''} & \text{Discarded catch} \\ \sum_{l \in l'} (C^{\mathrm{R}}_{y,a,l} + C^{\mathrm{D}}_{y,a,l}) / \sum_{l'' \in l'} \sum_{a'} (C^{\mathrm{R}}_{y,a',l''} + C^{\mathrm{D}}_{y,a',l''}) & \text{Total catch} \end{cases} \tag{A.1.30}
$$

$N^{\mathrm{Eff}}_{y,l'}$ is the effective sample size for year y and size-grouping l. The symbols a' and l'' are used to denote the indices over which summations occur. For the purposes of this paper and following Thompson (2015), the aging data were pooled across size-classes.

A.1.8 Parameterization

The estimable parameters of the model determine natural mortality by sex and time, growth as a function of sex and platoon, maturity and the weight-size relationship as a function of sex, unfished average recruitment (R_0), the steepness of the stock-recruitment relationship (h), the annual recruitment deviations, survey and fishery selectivity by sex and time, and survey catchability.

The maturity function can be modelled using a logistic function or using a spline function while the weight-size relationship is given by:

$$
w_{s,l} = \tilde{\alpha}_s (\bar{L}_l)^{\tilde{\beta}_s} \tag{A.1.31}
$$

where $\tilde{\alpha}_s$ and $\tilde{\beta}_s$ are the parameters of the weight-size relationship for sex s.

A.1.9 Penalty terms

The objective function minimized to find the estimates of the model parameters includes a penalty on the recruitment deviations:

$$
P_1 = \frac{1}{2\sigma_R^s} \sum_{y=y_{R1}}^{y_{R2}} \varepsilon_y^2 \tag{A.1.32}
$$

where σ_R is the (assumed) extent of variation in log-recruitment, and y_{R1} and y_{R2} denote the years for which recruitment is estimated.

A penalty is placed on deviations in fishing mortality rate from the mean fishing mortality for those years for which fishing mortality is estimated because it is “missing.”