# Multiple Regression Inference of Yield for Douglas-fir Plantations in the Pacific Northwest

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# Acknowledgment

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# Introduction

## Yield Models

Growth and yield models provide information for forest management decisions. They are the production functions that provide support for all timber management. In the United States, early estimates of forest growth and yield were based upon the concept of “normal” yield tables. In this study, yield is used as a general term implying the accumulation of the total amount of wood capable of being harvested at a certain time. Normal is defined as a stand where trees were visually asses as fully occupying the growing space. The concept of normality was essential to represent growth and yield information in graphical and tabular forms. These graphs and tables represented yield over time classified by site from plots established over a range of age and site classes for pure, even-aged stands. In light of the limitations imposed by the concept of normality, its ambiguous definition became a topic of criticism (Nelson and Bennett, 1965), and empirical yield tables were constructed as alternatives. The use of empirical linear equations in growth and yield studies is fairly common and has several advantages. The solution to the equations is unique, easily obtained, and rather robust, even when assumptions implicit in the method are violated (Vanclay 1994). MacKinney and Chaiken (1939) published one of the first yield equations fitted by linear regression. They assumed that yield is a function of site index (SI) and stand density. Also Reineke (1927) provided a model which expresses the relationships between age, dominant height, average diameter at breast height, basal area, stocking, and total volume under bark. Many of these whole stand models are sufficient for even-aged plantations.

Building on the empirical yield concept in even-age stands, a yield function required variables include stand age, site index, and stocking. Once the predictor variables have been defined, the technique of multiple regression may be used to obtain a relationship between yield and the predictor variable in the form of y=f(age, SI, stocking). This study builds on this function to include geographic variables of latitude, longitude, and elevation.

## Predictor Variables

Douglas-fir (*Psuedotsuga mensizii* [Mirb.] Franco) has one of the broadest ranges, from northern British Columbia and Alberta to northern Mexico, of any North American conifer. Much of its distribution is over extremely dissected terrain, and the species exhibits a plethora of genetic differentiation. Adaptive patterns of genetic variation occur among Douglas-fir even within the local region's populations (Hermann and Lavender; n.d.). Local differentiation extends from longitudinal differences as well. Moreover, genetic differences as a result of altitude exist and have influential effects on local climate; generally, temperature decreases and precipitation increases with elevation. Extending predictor variables to include latitude, longitude, and elevation may account for growth differences resulting from geographic variations.

Site index has been commonly used to represent the potential capability of a site to produce wood products; therefore reflective of site conditions.  In other words, the height of a tree is dependent on the quality of the site. However, site index is only an indirect and a rather imperfect measure of site quality.  Site index can only be directly observed at the time the dominant and codominant trees in the stand are at the index age (McQuilkin and Rogers, 1978). At any other age, the height that the site trees will reach, or did reach, at the index age must be estimated (Monserud, 1984; Berguson et al., 1994). Because of the inherent inaccuracy of site index as a measure of site quality, it is common to find that the measured site index of an area changes from one time to another.  This does not mean that site quality has changed, only that the measure of site quality has changed. Also sources for site index can vary resulting in a range of values (Curtis et al., 1982). More importantly, site index models have not been known to show accurate characterization of early stand development when tree growth can be strongly influenced by short term climatic variations (Lauer and Kush, 2010). Latitude, longitude, and elevation are variables that can further enhance the use of site indices as a measure of site quality.

Slope and aspect are additional variables to consider in evaluating site quality based on topographic features (Blanco et. al., 2006; Jason et al., 2011).  For this study, plots are located on flat surfaces; for this reason, slope and aspect were not considered for predictor variables.

**Density Assessment**

When the Stand Management Cooperative (SMC) was established one of the goals was to address concerns about the lack of long-term data representing relatively wide spacings of intensively managed plantations. Stands that have received early stocking control are important due to the regional scarcity of data from low-density stands (Maguire, et al., 1991). Yield is regulated by planting density. The typical planting spacings for coastal Douglas-firs are based on a variety of criteria, including forecasts of tree survival, growth, and stability (Talbert and Marshall, 2005). These criteria often are viewed as components of stand vigor, and one goal of selecting the proper spacing is to enable crop trees to maintain their vigor until a subsequent thinning or other treatment. Also spacing can determine the timing and intensity of resource competition among individual trees. There are limited nutritional and spatial resources available to support tree growth on a forest site with increasing competition among individual trees as they grow in size; as trees get larger, their resource requirements are compounded. Spacing allows for control over the timing, and therefore tree size, at which these competitive interactions occur (Long et al, 2004). Future thinning also regulates yield as a method of spacing control. Thinning regulates production by opening growing space and shifting growth to standing trees.

**Objectives**

The objectives of this study are to: (1) build yield models that can be applicable to western Oregon, western Washington, and southwestern British Columbia, (2) use the models to assess yield due to early density control, and (3) to examine the effects of subsequent thinning on stands that have had early density control.

# Methods

## Study Location

The SMC has three study projects within the Pacific Northwest that differ based on silvicultural goals. The data for this study is representative of the 23 Douglas-fir Type I installations. The Type I installations are located (Figure 1) in western Oregon, western Washington, and Vancouver Island in British Columbia (Maguire et al. 1991). The geographic range of Oregon and Washington is within the western hemlock, *Tsuga heteropylla*, zone (Franklin and Dryness, 1973) and the coastal western hemlock and coastal Douglas-fir biogeoclimatic zones in British Columbia (Green et al., 1988).


Figure 1. Location of SMC Type I Installations. Adapted from Maguire, et al. (1991).

**Experimental Design**

There are 23 pure (> 90% by stem count) Douglas-fir Type I installations available for analysis. Within each installation there are seven plots for a total of 167 plots. Each plot is 1.1 acres (Figure 2) and consists of a 0.5 acre square measurement plot surrounded on all sides by a 30.5 foot buffer strip. An additional buffer was added to one side for destructive wood testing (Maguire et al. 1991). The objective of the SMC installations was to provide data for regional responses rather than site-specific assessments. Therefore, no replication of treatment regimes exists within an installation; treatment regimes were instead replicated across installations in the region.

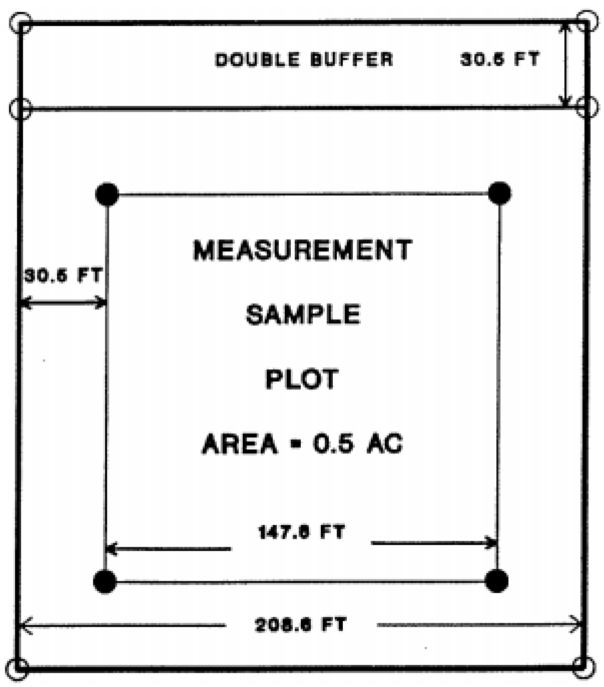


Figure 2. Installation Type I plot design. Adapted from Maguire, et al. (1991).

Density treatment regimes were composed of early spacing in stands that had not experienced substantial inter-tree competition by the year when plots were established. Seven plots were created on each installation and randomly assigned one of the treatment regimes shown in Table 1. Early spacing was systematically performed in the establishment year to yield a range of density levels. Initial density when a plot was established was defined as initial stems per acre (ISPA), one-half original density as ISPA/2, and one-fourth original density as ISPA/4. This produced a wide range of initial Trees Per Acre (TPA) across the installations in the region.

Thinning regimes were based on Curtis' (1982) relative density as indicated in Table 1. Subsequent thinning was conducted in a subset of the ISPA plots and minimal thinning was conducted in ISPA/2 plots. There are 26 plots that received a thinning in the data.

After establishment of plots, TPA varied from 66-844, so plots were placed into 100, 300, 500, and 700 initial TPA classes to study the effects of early density control on yield. A key to abbreviated attributes and their units are in Table 2, and the average and range of stand attributes for initial TPA classes are summarized in Table 3. Measurements were taken every four years. Data currently span the range of ages from five to 37 years.

Table 1. Nomenclature of the seven Type I Installation density management plots, Curtis' (1982) relative density (RD) thinning regime triggers, and mean and range of plot TPA after establishment.

| Treatment | Thinning Triggers | Initial TPA average (range) |
|---|---|---|
| 1. ISPA/4; no thin | | 125 (66-178) |
| 2. ISPA/2; light thin | RD55 -> RD35 | 226 (130-332) |
| 3. ISPA/2; no thin | | 234 (132-332) |
| 4. ISPA; repeated thin | RD55 -> RD35; RD55 -> RD40; RD60 -> RD40 | 490 (266-844) |
| 5. ISPA; minimal thin | RD55 -> RD35 | 497 (224-704) |
| 6. ISPA; no thin | | 490 (260-742) |
| 7. ISPA; heavy thin | RD45 -> RD30; RD50 -> RD35, RD55 -> RD40 | 471 (260-720) |

Overall initial TPA average: 330 (66-844)

**Response Variables**

There are seven multiple regression models for volume yield that have been designed for this study. These are regional averages of basal area per acre (BA), cubic-foot volume including top (CVT/ac), cubic-foot volume to four and six inch diameter inside bark per acre (CV4 and CV6), Scribner board-foot volume to four and six inch diameter inside bark per acre (BF4 and BF6), and quadratic mean diameter (QMD). Cubic-foot volumes are calculated by dividing the tree into 16 feet log segments above a 12 inch stump, using a taper equation (Hann et al., 2011) to estimate inside-bark diameters to the segment ends and using Smalian's (2011) formula. Board-foot volumes are based on scaling factors from 1982 (Anon) that uses a taper function for small end diameter and log length (Ceder, 2012). The calculated tree volumes on each plot were totaled and converted to per acre volumes which are summarized in Table 3. It should be noted that many trees failed to have any logs meeting the four or six inch minimum top diameter limits and therefore had zero volume.

Table 2. Key to attributes used in analyses.

| **Variables** | |
|---|---|
| AGE | Total age (year) |
| RD | Relative density (Curtis, 1982) |
| TPA | Initial trees per acre |
| SI | Site index; feet at 50 year (Bruce, 1981) |
| LAT | Latitude (decimal degrees) |
| LONG | Longitude (decimal degrees) |
| ELEV | Elevation (meters above sea level) |
| QMD | Quadratic mean diameter (inch) |
| BA | Basal area (feet$^2$ per acre) |
| THIN | value is 1 for measurements recorded sfter first thinning, otherwise 0 |
| CVT | Cubic foot volume including top (per acre) |
| CV4 | Cubic foot volume to 4 inch top (per acre) |
| CV6 | Cubic foot volume to 6 inch top (per acre) |
| BF4 | Scribner board foot volume to 4 inch top (per acre) |
| BF6 | Scribner board foot volume to 6 inch top (per acre) |

Table 3. Mean and range of plot attributes by initial TPA class (ranges are in parentheses).

| | Initial Trees Per Acre | | | |
|---|---|---|---|---|
| **Stand Variables** | 100 | 300 | 500 | 700 |
| Total age (year) | 20.5 (5-37) | 20 (5-37) | 20.26 (8-33) | 22(10-35) |
| RD | 19.94 (1.05 -0.75) | 31.23 (1.82-67.43) | 37.94 (3.16-74) | 40.98 (4-71) |
| SI | 132 (86.55-163) | 136.44 (91.39-164.76) | 144.6 (115.2-164.6) | 129.95(86-147) |
| Latitude (decimal degrees) | 45.98 (43.73-.10) | 46.3 (43.73-50.1) | 46.44 (42.92-49.06) | 46.41 (44.24-9.06) |
| Longitude (decimal degrees) | 123.3 (121.6-6.4) | 123.1 (121.6-126.4) | 122.8 (121.6-124.3) | 122.5 (121.6-23.7) |
| Elevation (meter) | 1147 (300-3800) | 1157 (300-3800) | 1012 (300-2200) | 1572 (300-3800) |
| QMD (inch) | 8.67 (1.3-18.6) | 7.45 (1-14.12) | 6.95 (1.21-14) | 6.06(1.1-11.9) |
| BA ($ft^2$/acre) | 65.26 (1.26-2.70) | 92.3 (1.85-222.71) | 105 (3.5-227) | 105.12 (4.3-216.2) |
| CVT ($ft^3$/acre) | 1023 (36-4717) | 832 (51-4151.82) | 762 (87-2646) | 638 (55.81-2325) |
| CV4 ($ft^3$/acre) | 941 (0-4664) | 708.86 (0-3874) | 603 (0-2578) | 463.203 (0-2241) |
| CV6 ($ft^3$/acre) | 801 (0-4557) | 552 (0-3119) | 438 (0-2421) | 318 (0-1994) |
| BF4 (Scribner bd ft/acre) | 3013 (0-17864) | 2592 (0-14960) | 2337 (0-10232) | 1906 (0-9488) |
| BF6 (Scribner bd ft/acre) | 2784 (0-18598) | 2112 (0-13238) | 1740 (0-9758) | 1316 (0-8574) |

## Predictor Variables

Predictor variables for variable density yield models traditionally are some function of age, site index, and a density metric. In this study latitude, longitude, and elevation, which would enable predictions of the response variables at locations throughout the Figure 1 region, were also included. These covariates are hypothesized to account for differences in site quality that site index may not capture such as soil, climate, and genetic variation. Stocking variables are initial TPA, RD, and whether or not thinning had occurred. Initial TPA is required due to variability in stocking at plot establishment. RD is included for two reasons: (a) stands were thinned to a certain RD to represent the intended treatment regime at establishment, and (b) RD was used to determine the timing of subsequent thinning. More importantly, these variables were chosen because they are stand level variables commonly available through forest inventory data collection.

## Model Formulations

Response variables were analyzed using multiple linear regressions in R (R, 2012). First, the response variables were predicted with the following additive model:

[Model 1]

$$\text{Response} = b_0 + b_{1*}\,\text{AGE} + b_{2*}\,\text{TPA} + b_{3*}\,\text{RD} + b_{4*}\,\text{THIN} + b_{5*}\,\text{SI} + b_{6*}\,\text{LAT} + b_{7*}\,\text{LONG} + b_{8*}\,\text{ELEV} + e$$

Where:

Response = QMD, BA, CVT, CV4, CV6, BF6, or BF4,

$b_0$ - $b_8$ are constant parameters to be estimated,

*e* is the error term,

and other variables are as previously listed in Table 2.

However, there are underlying assumptions of linearity, independence, homoscedasticity, and normality that must be met (Zar, 1999) for multiple regressions to be considered best. When one or more of the assumptions are violated, the errors can be forced to be normal through transformations of the predictor and/or the response variables. To normalize the error distribution of Model 1, additive main effects were raised to the second power and interactions were added. Raising the power and adding interactions allowed for the model to be curvilinear and formed a second-order model, which is not shown. The second-order model is the same as Model 1, but includes the square of each variable except for the THIN binary variable. Further, the second-order model is more flexible because it can take a variety of functional forms and is a good estimation of the true response surface (Myers, 1971). A Box-cox power transformation in the MASS package of R (Fox and Weisberg, 2011; Box and Cox 1964) was applied to the response variable to also normalize the error term.

The predictors of the transformed multiple regression models were then removed using backwards elimination. The least significant variable was dropped, one at a time, until only significant variables were left; significance was based on a t-test at $\alpha = 0.05$. Backwards elimination was chosen as the preferable method because stepwise regression available in R combines a forward and backward process that was considered unfavorable when main predictors have been chosen beforehand. Also, backwards elimination has been shown to perform as well as the method of comparing all subsets of regressions (Berk, 1978). If a main effect became insignificant during the backwards elimination process, the variable was not removed unless all of its interactions have been removed.

**Model Fitting and Assessment**

Goodness-of-fit was assessed as follows. First, the coefficient of determination, $R^2$, gives the proportion of variability of the response variable that accounts for the predictor variables (Steel and Torrie, 1960). R-squared values equaling one indicate that the model explains all variability in the response variable. However, $R^2$ can be misleading when used to compare the fit of regression involving different numbers of predictors. An increase in the number of predictor variables will usually increase $R^2$, even when the true values of the new regression coefficients are zero (Draper, 1984). Therefore, the assumption of normality and homoscedasticity were graphically screened by plotting the predicted values and their residuals. Residual review was used at every step of model formulation. This plot is more capable of revealing trends than a plot of observed versus fitted values (Vanclay, 1994). A good fit by a model should not express any pattern or shape. A pattern of well scattered, homogenous, residuals indicates that random errors are independent, normally distributed, and with constant variance. Third, quantile-quantile (q-q) plots were examined to determine if the residuals fit within a 95% confidence envelope for the theoretical error distribution. The q-q plot (Figure 3) provides a visual comparison of the studentized residuals to a theoretical quantile (Fox and Weisberg, 2011), in this study the t-distribution. A perfect one-to-one fit would show a straight 45 degree line (Scott, n.d.). A q-q plot also works well for graphically exposing outliers as shown in Figure 3.

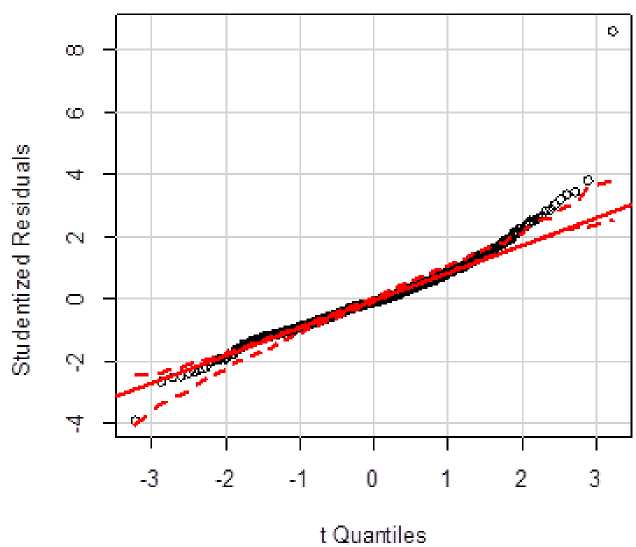


Figure 3. Example q-q plot of residuals from a fitted model.

Backwards elimination produced reduced models for the response variables which were analyzed for normal residuals and outliers; they were refitted again if necessary. The removal of outliers can improve accuracy of estimates (Osborne et al., 2001), but care was taken to keep removal of outliers to a minimum. A point was considered an outlier if it appeared to have an overly high influence on the predicted fit of the model. A Bonferroni p-value, which tests the significance of the studentized residuals of the suspected outlier, was calculated. If the Bonferroni p-value was significant, then an influence plot in the “car” package of R was examined for its Cook’s distance (Fox, 1997; Pedhazur, 1997). Cook’s distance considers an observation as having an influence when $D > 4/n$, where $D$ is based on an F-statistic for the extreme sample and *n* is the number of observations in the model (Bollen and Jackman, 1990). The larger the Cook's distance is, the larger the influence of an observation.

**Assessing Early Spacing and Thinning Effects**

Thinning effects from the first thin were assessed only for heavy thinnings, which is RD 45 down to RD 30. The 300 TPA class had six total measurements after the first thinning. Two of the six measurements were minimal thinning measurements taken at age 33, so there were not enough measurements for assessing minimal thinning. Limiting thinning observations to only heavy thinning resulted in four observations for the 300 TPA class, 14 observations for the 500 TPA class, and 11 observations for the 700 TPA class. These observations were used in predicting RD by age. There were no plots in the 100 TPA class that were thinned, so only the 300, 500, and 700 TPA classes provided material for analysis on the effects of a first thin at each respective density level. The 100 TPA class was used for examining the performance of stands planted at wide spacing without thinning.

To assess effects of early density control and first thinning effects, the fitted regressions were used to predict average yield for varying initial TPA. In estimating yield, predictor variables were set to average conditions and a changing relative density. The RD was predicted by age for each TPA class. Predicting the change in RD at each age was necessary because thinning was based on RD triggers as shown in Table 1. Table 4 shows the change in RD, and Table 5 shows the age at the first thinning and the time elapsed since thinning.

The estimated volumes are presented graphically and as yield tables on a per-acre basis. The yield tables are divided into three parts: (1) per acre estimation based on no thinning, (2) estimation after the first thinning, and (3) percent gained from thinning; percent gained was calculated as:

$$((\text{Yield}_{\text{thinned}} - \text{Yield}_{\text{unthinned}}) / \text{Yield}_{\text{unthinned}})*100.$$

Though some treatment regimes call for repeated thinnings in the Type Is, only two plots had received a second thinning; therefore, this study only examined the effects after the first thinning.

Table 4. Relative density used in estimating mean volume production. Empty spaces in the unthinned (top) mean trees were older when the first measurement was taken. Also in the thinned table, empty spaces mean no thinning for the 100 TPA level and measurements are not available for the other levels.

Unthinned RD

| Age | 100 | 300 | 500 | 700 |
|---|---|---|---|---|
| 5 | 1.05 | 1.05 | | |
| 9 | 3.77 | 8.56 | 9.75 | |
| 13 | 8.79 | 17.56 | 24.76 | 22.27 |
| 17 | 15.00 | 27.16 | 37.00 | 36.82 |
| 21 | 22.09 | 35.91 | 44.70 | 46.48 |
| 25 | 28.17 | 42.57 | 49.00 | 52.57 |
| 29 | 31.85 | 45.98 | 52.27 | 57.72 |
| 33 | 31.89 | 45.10 | 53.93 | 61.69 |

Thinned RD

| Age | 100 | 300 | 500 | 700 |
|---|---|---|---|---|
| 5 | | | | |
| 9 | | | | |
| 13 | | | | |
| 17 | | | | |
| 21 | | 34.00 | 36.37 | |
| 25 | | 43.00 | 40.75 | 39.45 |
| 29 | | 50.00 | 43.40 | 42.42 |
| 33 | | | 44.46 | 44.22 |

Table 5. Minimum age at which first thinning occurred for each TPA class and years since thinning.

| | TPA | | | |
|---|---|---|---|---|
| | 100 | 300 | 500 | 700 |
| Age at thinning | NA | 16 | 17 | 19 |
| Years since thinning | NA | 17 | 16 | 14 |

100 TPA was not thinned.

# Results and Discussion

## Model Formulations

Homoscedasticity is commonly violated when productivity rate varies over time (Fortin, et al., 2007; Yang, et al., 1992); therefore, Model 1 is an incorrect model as shown in Figure 4. In reducing the lack of fit and trying to find the correct model, the inclusion of interaction and quadratic terms resulted in a better fit. However, analyses of residuals for each response variable fitted to the second-order model resulted in an upward tilting of the variance for small volume values and a widening variance with increasing volume. Figure 5 is an example of the upward tilt shown in a QMD fitted model. A cubic term was considered and tested for significance, but including the third power produced a reduction in fit when compared to the quadratic model. This was observed for all seven response models. Therefore, a Box-cox power transformation was applied to normalize the response variables and to normalize the error terms. Passing each response variable as a function of the predictors through the Box-cox function showed a maximum log-likelihood of lambda ($\lambda$) at approximately 0.4 power (Figure 6) for all seven variables. To avoid introducing bias in back transformations (Newman, 1993; Rothery, 1988), a 0.5 power, or square root transformation, was used. A power of 0.4 is not substantially less skewed than a power of 0.5. All other response variables suggested a square root transformation as well. After the Box-cox transformation, the full model took the form of Model 2. Figure 7 summarizes the steps this study took in reaching the final fitted models.

[Model 2]

$$\text{Response}^{1/2} = b_0 + b_{1*}\,\text{AGE} + b_{2*}\,\text{TPA} + b_{3*}\,\text{RD} + b_{4*}\,\text{THIN} + b_{5*}\,\text{SI} + b_{6*}\,\text{LAT}$$
$$+ b_{7*}\,\text{LONG} + b_{8*}\,\text{ELEV} + b_{9*}\text{AGE}^2 + b_{10*}\text{TPA}^2 + b_{11*}\,\text{RD}^2 + b_{12*}\,\text{SI}^2 + b_{13*}\,\text{LAT}^2$$
$$+ b_{14*}\,\text{LONG}^2 + b_{15*}\,\text{ELEV}^2 + b_{16*}\,\text{AGE*TPA} + b_{17*}\,\text{AGE*RD} + b_{18*}\,\text{AGE*SI}$$
$$+ b_{19*}\,\text{AGE*THIN} + b_{20*}\,\text{AGE*LAT} + b_{21*}\,\text{AGE*LONG} + b_{22*}\,\text{AGE*ELEVATION}$$
$$+ b_{23*}\,\text{TPA*RD} + b_{24*}\,\text{TPA*SI} + b_{25*}\text{TPA*THIN} + b_{26*}\text{TPA*LAT} + b_{27*}\text{TPA* LONG}$$
$$+ b_{28*}\text{TPA*ELEVATION} + b_{29*}\text{RD*SI} + b_{30*}\text{RD*THIN} + b_{31*}\text{RD*LAT}$$
$$+b_{32*}\text{RD*LONG} + b_{33*}\text{RD*ELEVATION} + b_{34*}\text{SI*THIN} + b_{35*}\text{SI*LAT}$$
$$+ b_{36*}\text{SI*LONG} + b_{37*}\text{SI*ELEV} + b_{38*}\text{THIN*LAT} + b_{39*}\text{THIN*LONG}$$
$$+ b_{40*}\text{THIN*ELEV} + b_{41*}\text{LAT*LONG} + b_{42*}\text{LAT*ELEV} + b_{43*}\text{LONG*ELEV} + e$$

Where :

Response = QMD, BA, CVT, CV4, CV6, BF6, or BF4,

$b_0 – b_{43}$ are constant parameters to be estimated,

*e* is the error term, and

other variables are as previously listed in the key of Table 2 in the Methods section.

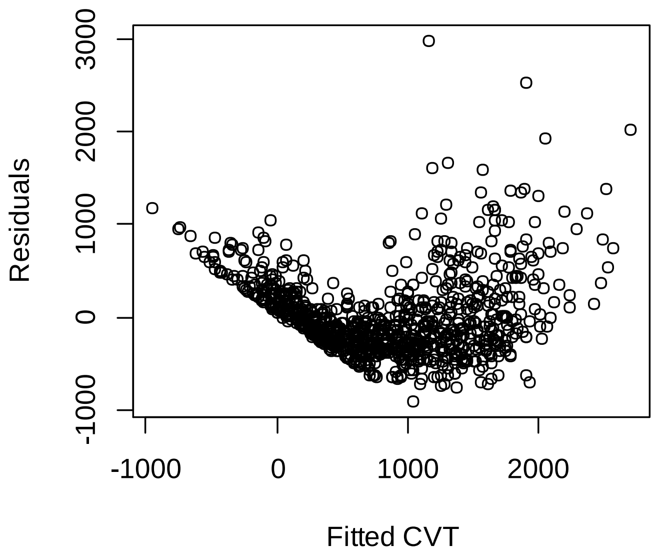


Figure 4. Residuals of Model 1 for CVT.

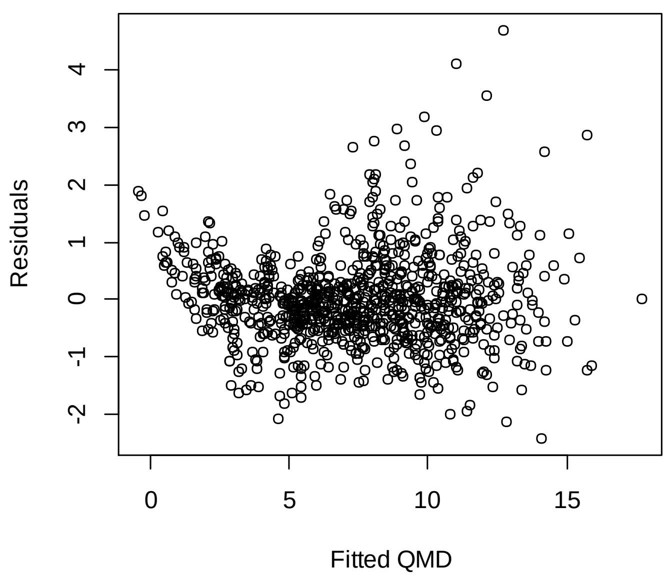


Figure 5. QMD residuals of Model 1 plus quadratic terms and interactions before a square root transformation of the dependent variable.

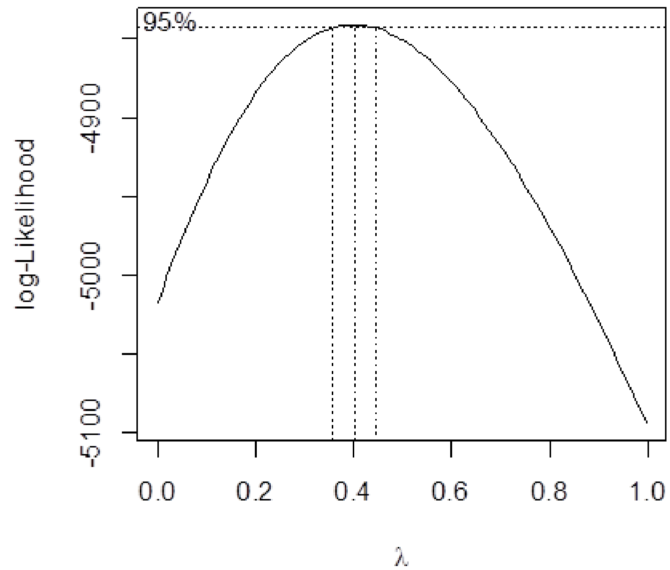


Figure 6. A graph of the Box-cox transformation of CV6.

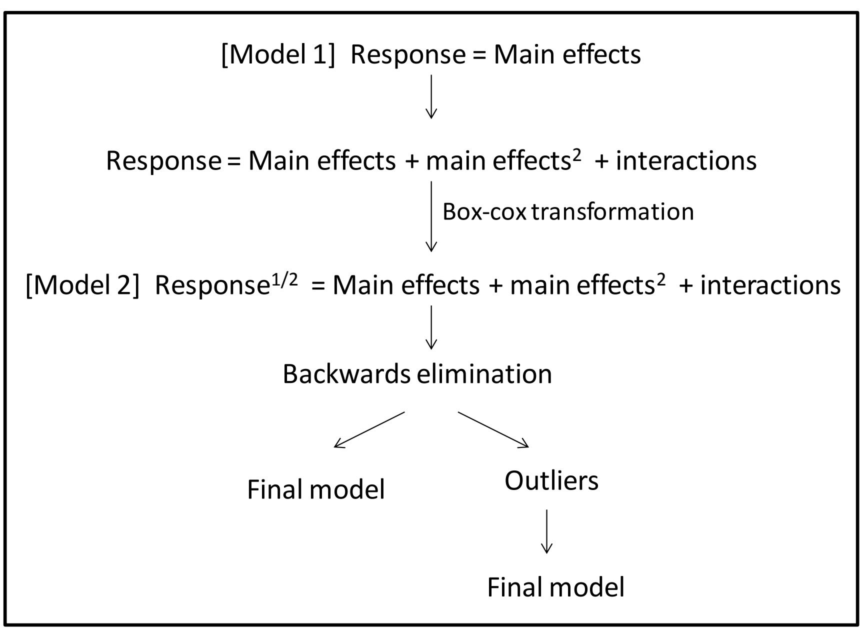


Figure 7. Steps in model fitting.

**Model Fitting and Assessment**

All regression models explained at least 88% of the yield's variance. All main effects were kept in all models. The number of terms maintained in the models varied from 29 in the BA model to 37 in the CV4 model out of 43 possible combinations of variables, including intercept, per regression. BA and CV4 had 99% and 93% of its variance explained, respectively. The p-values for the F-statistics in all seven models are significant at $p < 0.05$, which indicated that the models are suitable in order to explain the response variable's variance. Estimated coefficients for CV4 and fit statistics of the final model are given in Table 6 and the fit statistics for other models are given in Appendix A.

Table 6. Estimated CV4 model coefficients and associated fit statistics.

CV4

| Parameters | Estimate | Std. Error | t value | Pr(>\|t\|) | Sig. |
|---|---|---|---|---|---|
| (Intercept) | 5.37E+03 | 3.60E+03 | 1.493 | 0.135965 | |
| AGE | -3.39E+01 | 6.20E+00 | -5.47 | 6.38E-08 | *** |
| TPA | -1.97E+00 | 2.56E-01 | -7.694 | 5.13E-14 | *** |
| RD | 2.24E+01 | 3.25E+00 | 6.889 | 1.30E-11 | *** |
| SI | -1.52E+01 | 3.38E+00 | -4.487 | 8.50E-06 | *** |
| THIN | -8.47E+01 | 1.97E+01 | -4.308 | 1.89E-05 | *** |
| LAT | 1.39E+02 | 3.12E+01 | 4.465 | 9.40E-06 | *** |
| LONG | -1.25E+02 | 5.84E+01 | -2.143 | 0.03244 | * |
| ELEVATION | 4.39E-01 | 1.79E-01 | 2.457 | 0.014273 | * |
| AGE^2 | 2.03E-02 | 6.23E-03 | 3.255 | 0.001191 | ** |
| TPA^2 | 1.11E-04 | 7.69E-06 | 14.445 | < 2e-16 | *** |
| SI^2 | 4.10E-03 | 8.51E-04 | 4.82 | 1.78E-06 | *** |
| LAT^2 | -4.18E-01 | 9.48E-02 | -4.411 | 1.20E-05 | *** |
| LONG^2 | 6.05E-01 | 2.44E-01 | 2.477 | 0.013484 | * |
| ELEVATION^2 | -4.79E-06 | 9.71E-07 | -4.933 | 1.02E-06 | *** |
| AGE:TPA | 2.57E-03 | 2.94E-04 | 8.735 | < 2e-16 | *** |
| AGE:RD | -3.93E-02 | 3.53E-03 | -11.144 | < 2e-16 | *** |
| AGE:SI | 3.22E-02 | 3.21E-03 | 10.035 | < 2e-16 | *** |
| AGE:THIN | -4.54E-01 | 1.47E-01 | -3.086 | 0.002109 | ** |
| AGE:LONG | 2.48E-01 | 4.89E-02 | 5.075 | 5.04E-07 | *** |
| AGE:ELEVATION | 2.05E-04 | 5.96E-05 | 3.431 | 0.000638 | *** |
| TPA:RD | -1.38E-03 | 1.16E-04 | -11.921 | < 2e-16 | *** |
| TPA:SI | 1.05E-03 | 1.27E-04 | 8.22 | 1.06E-15 | *** |
| TPA:THIN | -2.91E-02 | 4.79E-03 | -6.084 | 1.97E-09 | *** |
| TPA:LAT | 3.63E-03 | 6.05E-04 | 5.994 | 3.34E-09 | *** |
| TPA:LONG | 1.23E-02 | 1.89E-03 | 6.513 | 1.44E-10 | *** |
| TPA:ELEVATION | 1.31E-05 | 2.37E-06 | 5.552 | 4.08E-08 | *** |
| RD:SI | -1.14E-02 | 1.59E-03 | -7.166 | 2.05E-12 | *** |
| RD:THIN | 1.73E-01 | 8.17E-02 | 2.122 | 0.03422 | * |
| RD:LONG | -1.54E-01 | 2.58E-02 | -5.989 | 3.45E-09 | *** |
| RD:ELEVATION | -1.28E-04 | 3.21E-05 | -3.995 | 7.20E-05 | *** |
| SI:LONG | 1.12E-01 | 2.73E-02 | 4.089 | 4.86E-05 | *** |
| SI:ELEVATION | -7.00E-05 | 2.25E-05 | -3.112 | 0.001938 | ** |
| THIN:LAT | 2.29E+00 | 4.20E-01 | 5.454 | 6.95E-08 | *** |
| LAT:LONG | -8.05E-01 | 2.10E-01 | -3.828 | 0.000141 | *** |
| LAT:ELEVATION | -1.90E-03 | 3.72E-04 | -5.097 | 4.50E-07 | *** |
| LONG:ELEVATION | -2.70E-03 | 1.30E-03 | -2.077 | 0.038179 | * |

Residual standard error: 3.621 on 670 degrees of freedom

Multiple R-squared: 0.9342, Adjusted R-squared: 0.9307

F-statistic: 264.3 on 36 and 670 DF, p-value: < 2.2e-16

Significance levels are: *** p< 0.001, ** p< 0.01, * p<0.05.

Note: 210 zero $ft^3$ values removed form model fitting.

$$\mathbf{CV4^{1/2}} = 5.37E{+}03 + -3.39E{+}01*AGE + -1.97E{+}00*TPA + 2.24E{+}01*RD + -1.52E{+}01*SI + -8.47E{+}01*THIN + 1.39E{+}02*LAT + -1.25E{+}02*LONG + 4.39E{-}01*ELEV + 2.03E{-}02*AGE^2 + 1.11E{-}04*TPA^2 + 4.10E{-}03*SI^2 + -4.18E{-}01*LAT^2 + 6.05E{-}01*LONG^2 + -4.79E{-}06*ELEV^2 + 2.57E{-}03*AGE{:}TPA + -3.93E{-}02*AGE{:}RD + 3.22E{-}02*AGE{:}SI + -4.54E{-}01*AGE{:}THIN + 2.48E{-}01*AGE{:}LONG + 2.05E{-}04*AGE{:}ELEV + -1.38E{-}03*TPA{:}RD + 1.05E{-}03*TPA{:}SI + -2.91E{-}02*TPA{:}THIN + 3.63E{-}03*TPA{:}LAT + 1.23E{-}02*TPA{:}LONG + 1.31E{-}05*TPA{:}ELEV + -1.14E{-}02*RD{:}SI + 1.73E{-}01*RD{:}THIN + -1.54E{-}01*RD{:}LONG + -1.28E{-}04*RD{:}ELEV + 1.12E{-}01*SI{:}LONG + -7.00E{-}05*SI{:}ELEV + 2.29E{+}00*THIN{:}LAT + -8.05E{-}01*LAT{:}LONG + -1.90E{-}03*LAT{:}ELEV + -2.70E{-}03*LONG{:}ELEV$$

The residuals of all models did not express any obvious irregular patterns. The q-q plots indicated normal distributions within the 95% confidence envelope fairly well. Figure 8 shows the q-q and residuals plots of the CV4 model. The plots for other response variables are in Appendix B. The q-q plots indicate that all extreme observations are dealt with effectively. Of all volume models, the BF4 model performed the best at an adjusted $R^2 = 94\%$, a homogenous residual plot, and straightest normal q-q plot. BA has a surprising adjusted $R^2 = 99\%$, but appears to be poorest in residuals distribution with a small, but noticeable curve in the tail of the q-q plot.

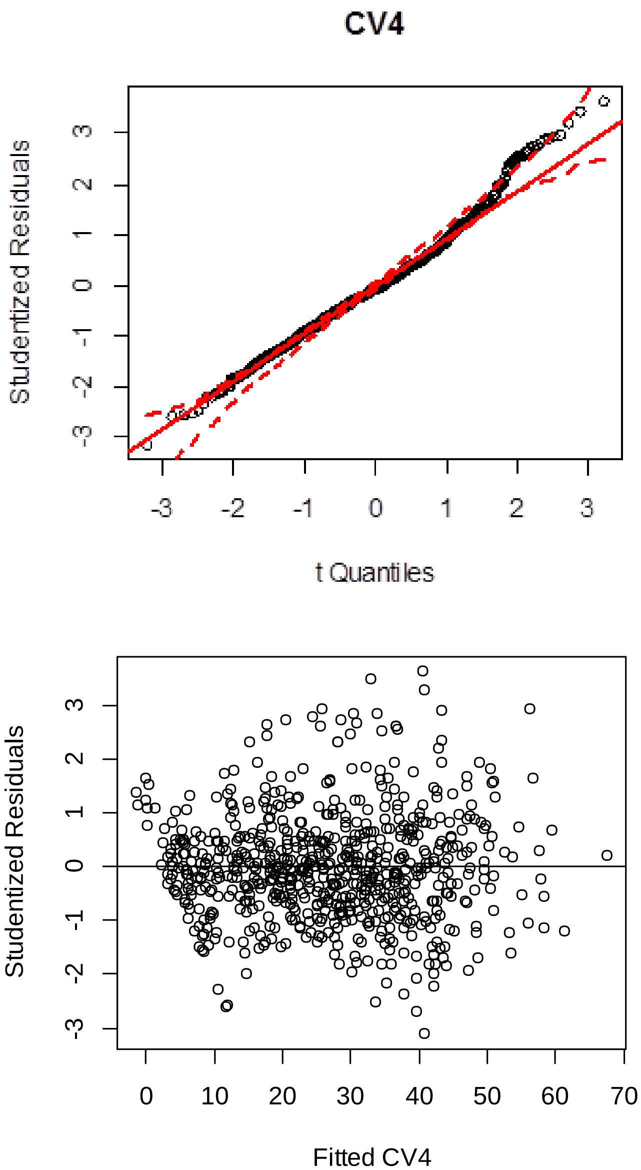


Figure 8. Q-Q plot and the residuals of the final model of CV4.

Figure 9 shows an outlier diagnostic plot of the BF4 model. Point 147 appears as an extraneous observation in both a Bonferroni p-value and the Cook's distance plot. This point has a significant Bonferroni value ($p<0.05$). Also, its Cook's distance is about 0.04, which is larger than the cutoff at D= 0.005. Point 147 is installation 706, plot 7, and measurement number 5. Point 147 is a plot growing at 300 meters above sea level (MASL), whereas the average elevation is 1175 MASL. When compared to the average for other stands at age 29 and elevation of 300 MASL, point 147 has 6954 BF4, height of 77 feet (ft), SI 138, whereas the average at the same age and elevation is 3908 BF4, height of 61 ft, and a SI of 120. Observation 147 consistently appeared as an outlier for other models as well, so it was removed from all model fitting. Other points in the diagnostic plots appeared as possible outliers, but they did not produce a significant Bonferroni p-value.

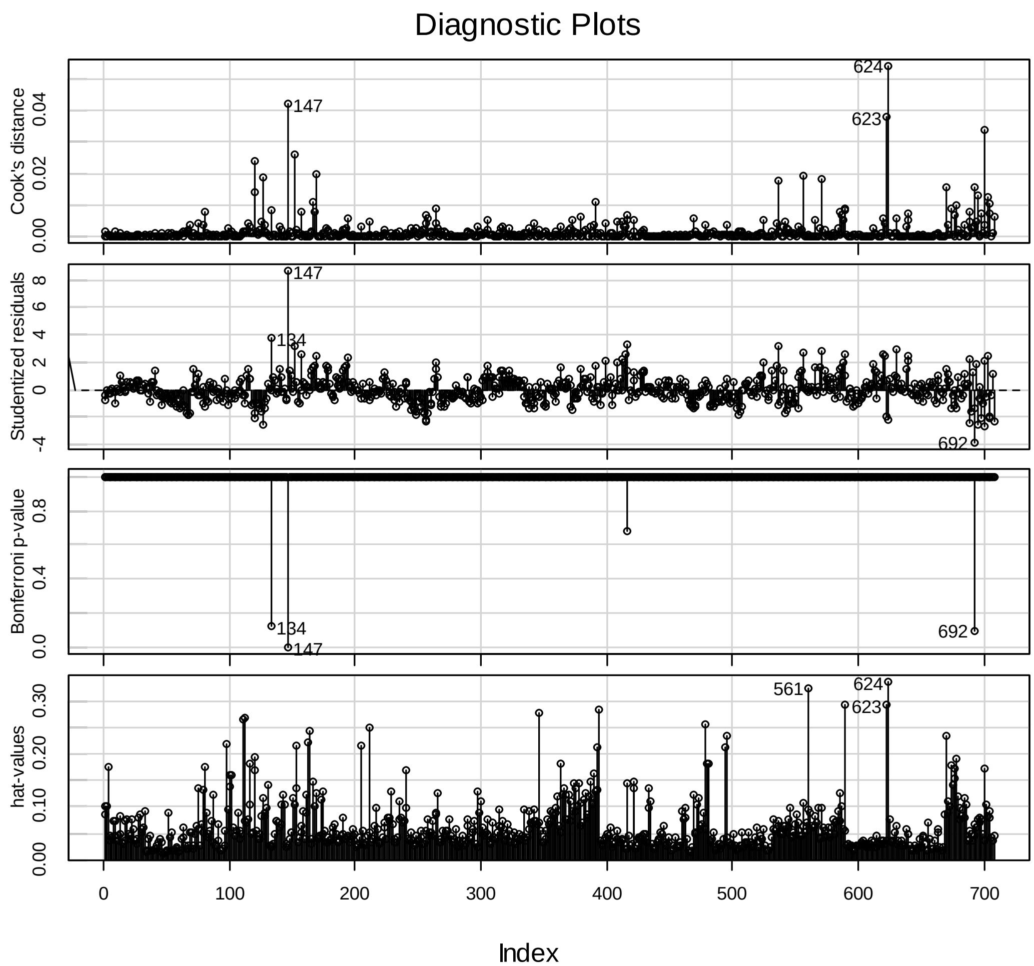


Figure 9. Outlier diagnostic for BF4. Point 147 shows influence appearing in both a Bonferroni p-value plot and a relatively large Cook's distance.

## Model Fitting and Assessment Discussion

Installation 733 was removed from the final fitting of the models. Most data from this installation was isolated from all other observations, and its volumes were consistently much lower. The SI at installation 733 is 74 ft., an average height of 20 ft. and elevation of 60 MASL. The average height of other stands is 46 ft and the lowest SI, not including installation 733, is 86.5 ft at an elevation of 300 MASL. Additionally, Installation 733 is at 50 degrees north, where it is growing near the Douglas-fir limit of 55 degrees north (Hermann and Lavender; n.d.). Installation 733 is likely in a location of unfavorable growth conditions.

Trees that failed to have any 16 ft log segments meeting the minimum diameter requirements were assigned a zero volume. In CV4 and BF4 about 23% of observations were removed and about 37% of observations were removed in CV6 and BF6. See Table 7 for minimum values after zeros were removed. Zero volume trees were excluded from the regression; therefore, the degrees of freedom varied considerably between CV4, CV6, BF4, and BF6 models. Degrees of freedom for other models, QMD, BA, and CVT varied as a result of the removal of insignificant terms.

Table 7. Minimum values after zeros were removed and percent of observations removed.

| Attributes | CV4 | CV6 | BF4 | BF6 |
|---|---|---|---|---|
| AGE | 11 | 15 | 11 | 15 |
| RD | 5.60 | 8.59 | 5.60 | 8.59 |
| QMD | 3.70 | 3.70 | 3.70 | 3.70 |
| BA | 11.60 | 24.99 | 11.60 | 24.99 |
| Tree volume | 4.79 | 10.75 | 22.00 | 42.00 |
| % of observations removed | 22.88 | 37.04 | 22.88 | 37.04 |

Minimum for all other attributes are not affected by removing tree volumes below zero.

In the BF4 model, latitude is not significant at α = 0.05, but is retained because its interactions are significant. In the BA model, site index is highly insignificant, but retained due to its significant interactions as well. Removal of these insignificant main effects would be in violation of the backwards hierarchical modeling process. Due to the large proportion of zeros, fitting was extremely skewed if these observations were not removed (see Figure 10).

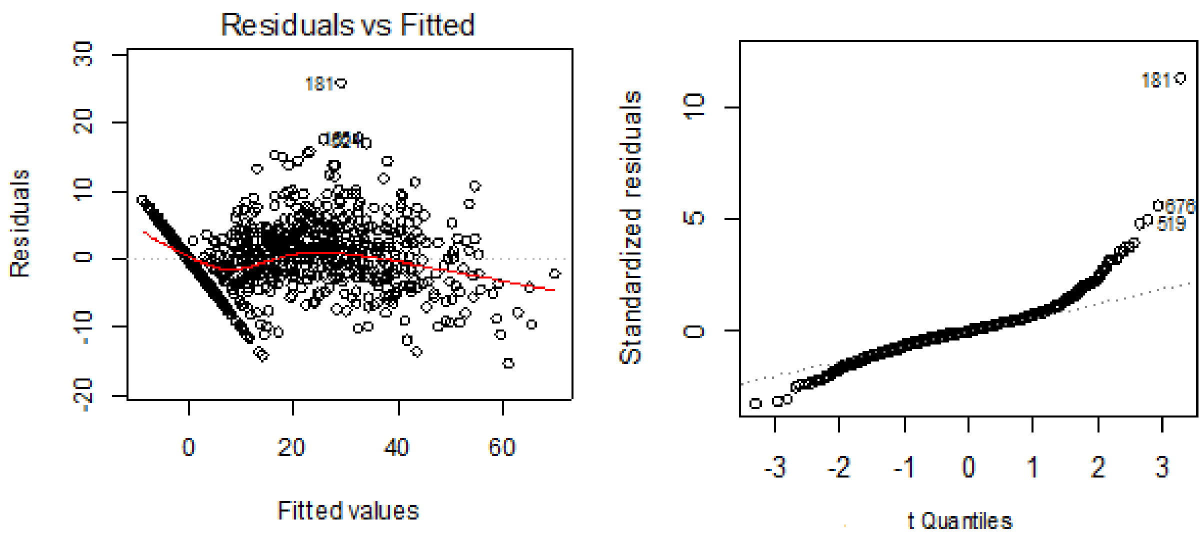


Figure 10. Impact of zero volume trees on CV6 values. Including them reduced the accuracy of the fit. The red line is a loess fit of the residuals. The diagonal line of residual points in the left plot is comprised entirely of zeros.

A possible exception to normality may be in the BA model in which residuals have a slight curvature driven by younger stands (Figure 11). The q-q plot shows that the majority of the fit is fairly normal and a plot of fitted against observed values shows a good fit. However, there does exist a bit of a curvature at the top tail of the q-q plot. The curvature may result in an overestimate of BA for younger stands and caution should be taken. A cubic term was not able to fix this problem and would run into possibilities of overfitting as well as a model with an excessively large number of parameters.

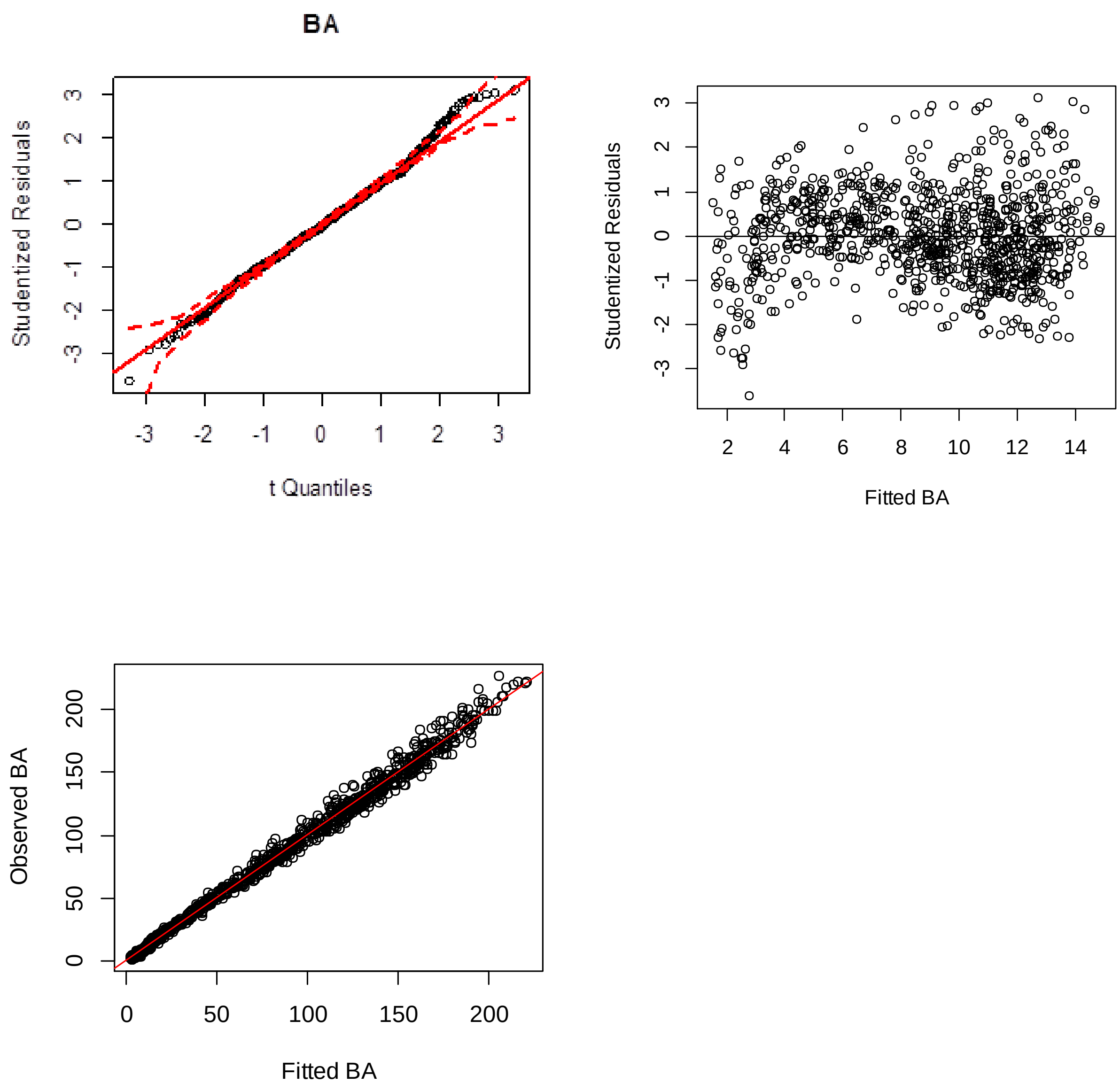


Figure 11. Q-Q plot, residuals, and an observed vs. fitted of BA.

**Assessing Early Spacing Effects and Discussion**

All density levels performed as expected by age 33. In the BA model, the 100 TPA stands had the lowest basal area per acre while 500 and 700 TPA stands had the largest basal area per acre. The inverse is shown in QMD, where the smallest basal area translates into the largest QMD (Figure 12). Basal area is a commonly used indicator of stocking; thus, as stocking increases there is a decrease in diameter. Graphs of all volume measures, CVT, CV4, CV6, BF6, and BF4 follow the same trend as QMD in which the largest volumes per acre are in the 100 TPA stands and the least volume per acre is in the 700 TPA stands. For example, in Table 8, CVT at 100 TPA can yield over 3000 $ft^3$/acre, while 700 TPA has about 48% less CVT at about 1500 $ft^3$/acre. See Appendix C for graphs and tables of all other models.

At 100 TPA stands were able to accumulate more volume and continued to be the spacing level with the most production of all other levels. At this level of stocking there is very little competition for resources and the trees are able to continue growing freely. In the 500 and 700 TPA stands, trees are in more competition for space and nutrients resulting in slower growth, and therefore, less volume. Li (2005), in a previous study of the Type Is found density effects on volume growth and yield to be statistically significant. Curtis (2006) in the study of Douglas-fir volume growth trends and Hoyer et al. (1996) from a study in the Levels-of-Growing-Stock found similar results in volume trend due to spacing controls.

However, Figure 12 reveals that below age 15, the 500 level had a better girth performance. Scott et al. (1992) have reported that in seven to nine year-old plantations of coastal Douglas-fir there is an increase in both height and diameter with increasing density. With time, as trees become large enough to begin competition, size and spacing effect is observed to revert back to expected relationship of increased spacing will result in increased diameter.

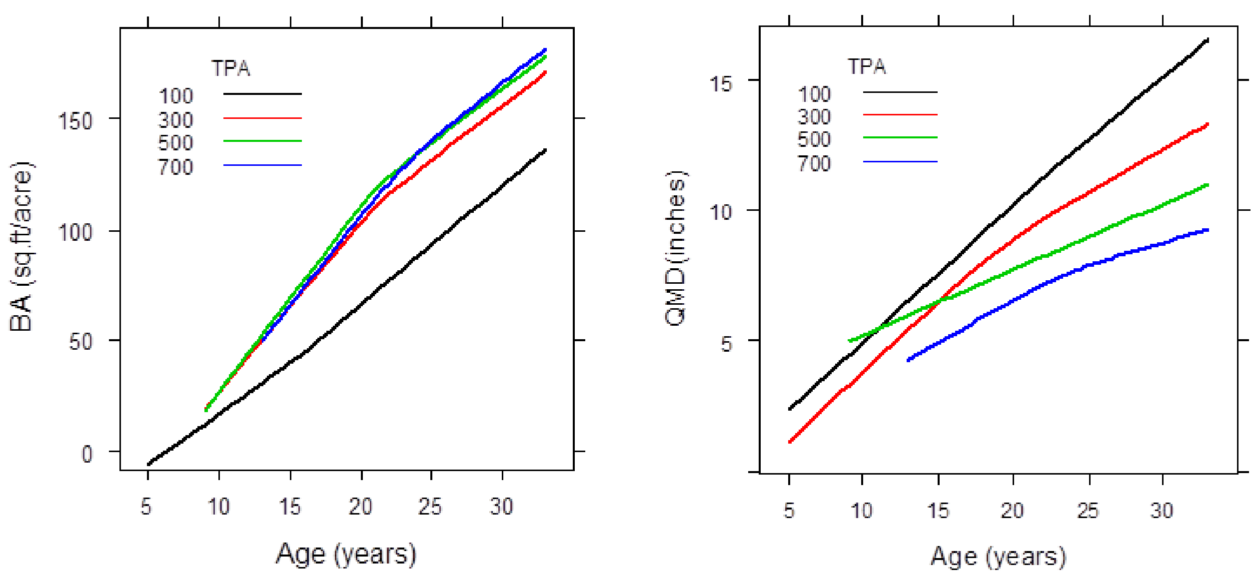


Figure 12: Plots of unthinned BA and QMD.

Table 8. Yield estimates for CVT. Top: yield estimates without thinning, middle: thinned estimates, bottom: percent gain if thinning occurred.

CVT ($ft^3$/acre)

Average conditions

| SI | THIN | LAT | LONG | ELEVATION |
|---|---|---|---|---|
| 137 | 0 | 46 | 123 | 1180 |

Unthinned

| | Density at establishment | | | |
|---|---|---|---|---|
| Age | 100 | 300 | 500 | 700 |
| 5 | 103.45 | 92.55 | | |
| 9 | 185.74 | 166.61 | 287.83 | |
| 13 | 357.03 | 304.12 | 331.91 | |
| 17 | 654.51 | 537.02 | 449.64 | 579.39 |
| 21 | 1130.46 | 889.15 | 654.52 | 651.89 |
| 25 | 1773.09 | 1365.81 | 958.49 | 848.58 |
| 29 | 2523.39 | 1945.43 | 1381.96 | 1149.09 |
| 33 | 3277.61 | 2586.41 | 1952.26 | 1587.51 |

Thinned

| | Density at establishment | | | |
|---|---|---|---|---|
| Age | 100 | 300 | 500 | 700 |
| 5 | | | | |
| 9 | | | | |
| 13 | | | | |
| 17 | | | | |
| 21 | | 1290.88 | 796.51 | |
| 25 | | 1904.48 | 1119.81 | 982.66 |
| 29 | | 2686.52 | 1567.81 | 1337.56 |
| 33 | | | 2168.20 | 1847.62 |

% Gain

| | Density at establishment | | | |
|---|---|---|---|---|
| Age | 100 | 300 | 500 | 700 |
| 5 | | | | |
| 9 | | | | |
| 13 | | | | |
| 17 | | | | |
| 21 | | 31 | 18 | |
| 25 | | 28 | 14 | 14 |
| 29 | | 28 | 12 | 14 |
| 33 | | | 10 | 14 |

**Assessing Thinning Effects and Discussion**

The first thinning had the largest effect on QMD in the 300 TPA class (see Figure 13; graphs and tables for other models are given in Appendix C). Thinning the 300 TPA class resulted in more space and resources for CVT, CV4, BF4 and BF6 almost immediately after the first thin and production rose to meet the unthinned 100 TPA class. The increase in volume seems to have slowed before age 30 (17 years after thinning), and production may be falling below that of 100 TPA class. Tassissa and Burkhart (1997) and Peltola et al. ( 2002) found that the diameter growth response at the stem base is immediate. This effect is greater with heavier thinning (Tassissa and Burkhart, 1997), which may partially explain the large volume change in the 300 TPA level. In Figure 14, the first thin in the 300 TPA of CV6 did not allow for production to catch up to the 100 TPA class. Production may start to follow the same trend as the unthinned 300 TPA stands in the future because CVT is showing a continuous increase in total production.

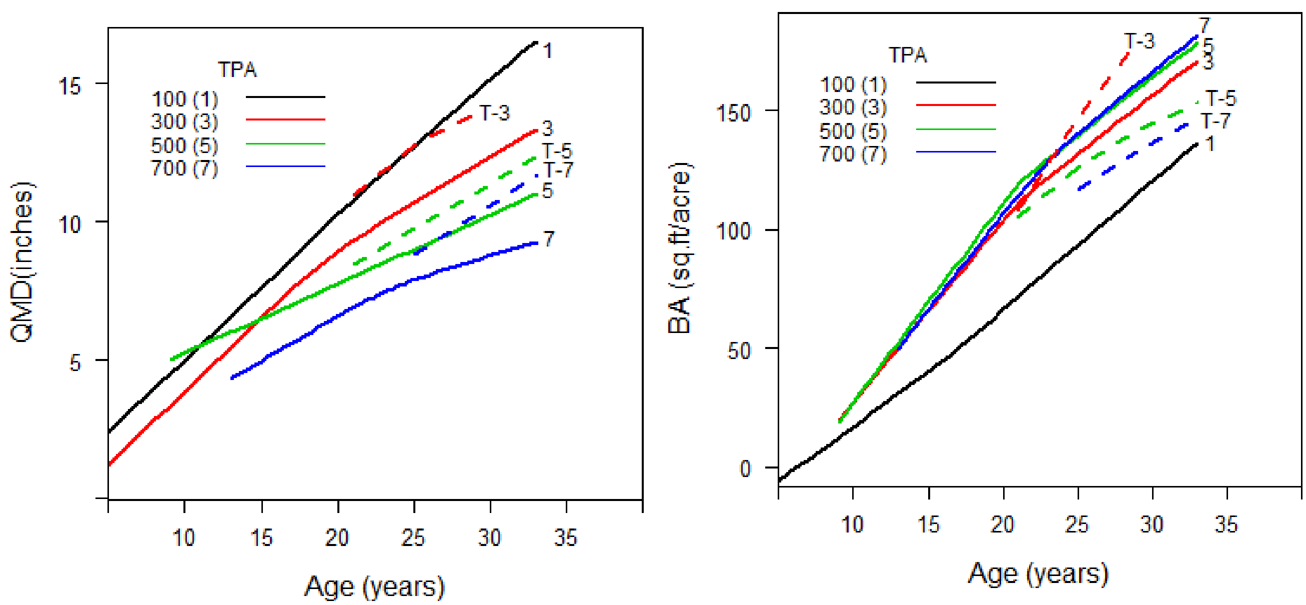


Figure 13. BA and QMD by age for each unthinned (100 (1) – 700 (7)) and thinned (T-3, T-5, T-7) TPA class.

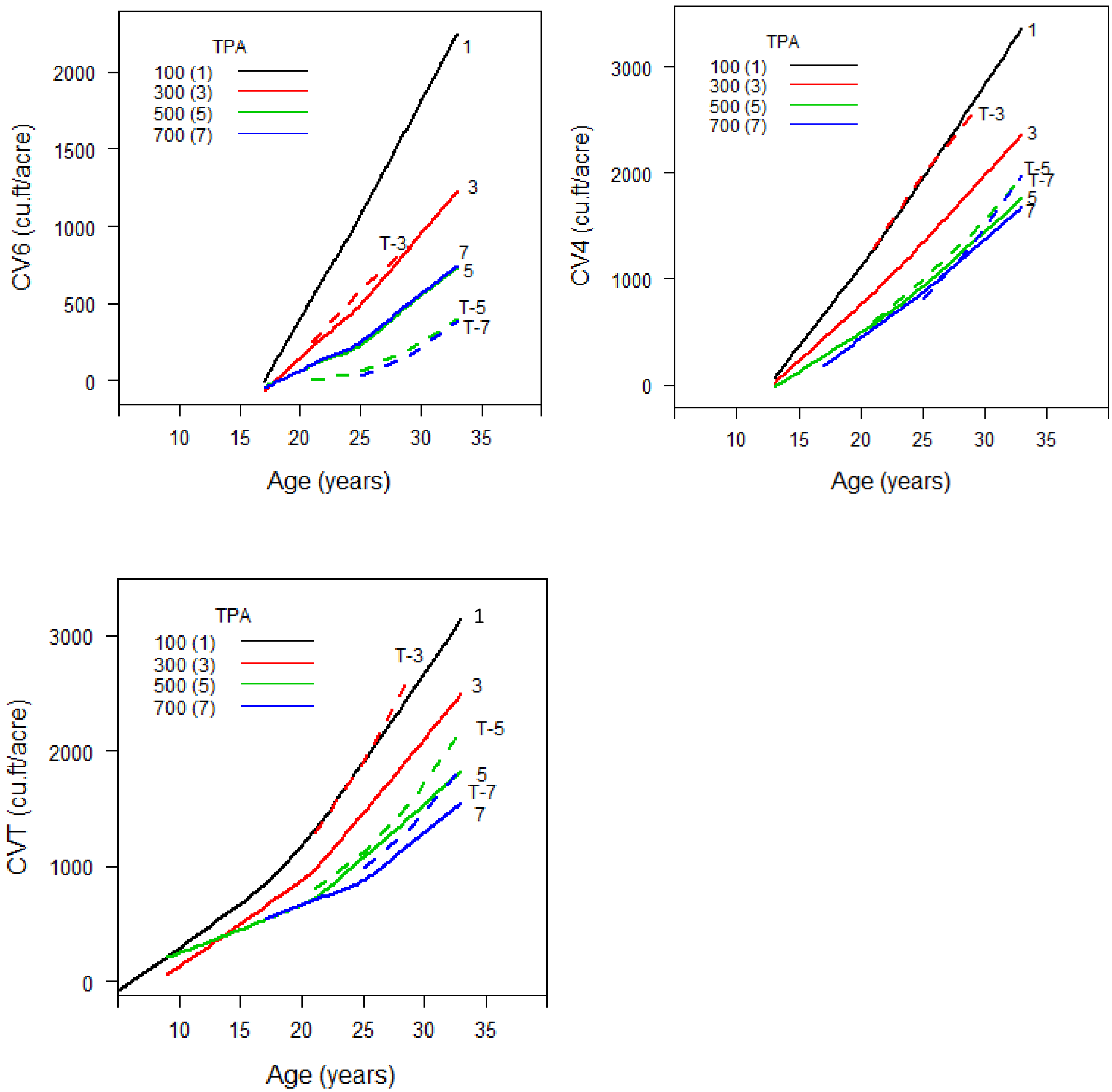


Figure 14. CVT, CV4, and CV6 by age for each unthinned (100 (1) – 700 (7)) and thinned (T-3, T-5, T-7) TPA class.

Figure 15 shows the responses in BF4 and BF6 to the first thin, exhibiting components directly linked to changes in growth allocation and indirectly linked to board foot scaling rules. At the 300 level, BF6 starts to slow and BF4 increased at about 5 years after the first thin. The shift in production from the top at six inches in diameter to four inches in diameter shows a change in growth allocation. Kantavichai (2011) found that although mass gains from thinning were still evident 12 years after a thin, response was lower and there was a shift in mass distribution up the stem. Generally, thinning increases stem growth by reducing competition for light, nutrients, and water. Trees respond to thinning by building a larger crown which produces more photosynthate and is exported down the tree permitting increased diameter growth. The increased diameter growth is usually observed first in the lower bole and then proceeds up the stem (Tasissa and Burkhart 1997). The maximum lower bole responses were at seven to nine years after thinning, but towards the top of the stem, the maximum response was delayed to 10 to 12 years after thinning (Peltola, 2002). Also, certain attributes of the Scriber Log Rule may be acting in conjunction with these growth responses. For example, the Scribner log rule volume for any diameter assumes a cylindrical log so a smaller diameter log, even though it might be longer than a larger diameter log, could produce smaller volume.

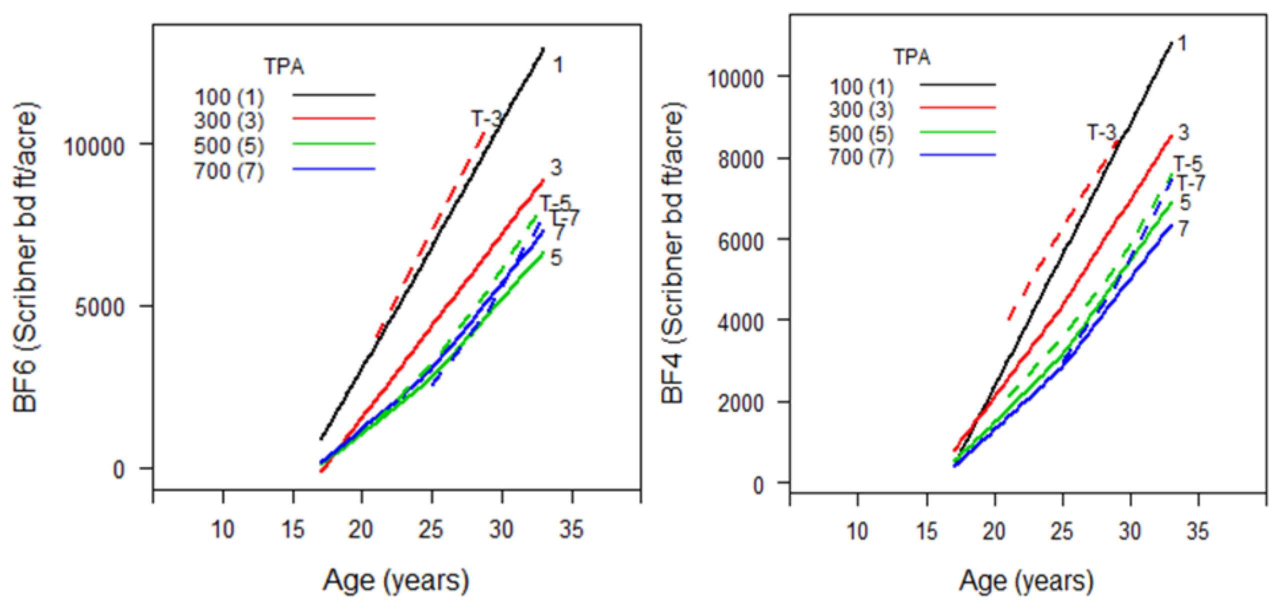


Figure 15. BF4 and BF6 by age for each unthinned (100 (1) – 700 (7)) and thinned (T-3, T-5, T-7) TPA class.

Thinning effects from the first thin for the 500 and 700 TPA classes appeared to be minimal. The yield trends in CV4, BF4, and BF6 are all very similar; there is not much deviation in total volume between the different volume measures. For example, in Table 9 the first thinning in the 500 TPA and 700 TPA classes produced only a 7% and 25% increase, respectively, in BF4 over the unthinned counterparts. Whereas, at the 300 level there was a 64% increase in board foot from first thinning. Furthermore, in total volume (CVT), thinning at the 500 and 700 levels allowed for more volume to accumulate but not much more when compared to the 300 level.

Table 9. Yield estimates for BF4. Top: yield estimates without thinning, middle: thinned estimates, bottom: percent gain if thinning occurred.

BF4 (Scribner bd ft/acre)

Average conditions

| SI | THIN | LAT | LONG | ELEVATION |
|---|---|---|---|---|
| 137 | 0 | 46 | 123 | 1187 |

Unthinned

| | Density at establishment | | | |
|---|---|---|---|---|
| Age | 100 | 300 | 500 | 700 |
| 5 | | | | |
| 9 | | | | |
| 13 | | | | |
| 17 | 1174 | 847 | 594 | 480 |
| 21 | 3136 | 2440 | 1700 | 1469 |
| 25 | 5620 | 4352 | 3089 | 2797 |
| 29 | 8223 | 6364 | 4826 | 4444 |
| 33 | 10821 | 8621 | 7031 | 6474 |

Thinned

| | Density at establishment | | | |
|---|---|---|---|---|
| Age | 100 | 300 | 500 | 700 |
| 5 | | | | |
| 9 | | | | |
| 13 | | | | |
| 17 | | | | |
| 21 | | 4014 | 2126 | |
| 25 | | 6286 | 3578 | 2999 |
| 29 | | 8442 | 5349 | 4914 |
| 33 | | | 7603 | 7470 |

% Gain

| | Density at establishment | | | |
|---|---|---|---|---|
| Age | 100 | 300 | 500 | 700 |
| 5 | | | | |
| 9 | | | | |
| 13 | | | | |
| 17 | | | | |
| 21 | | 64 | 25 | |
| 25 | | 44 | 16 | 7 |
| 29 | | 33 | 11 | 11 |
| 33 | | | 8 | 15 |

The first thinning in the 500 and 700 TPA actually resulted in a CV6 production loss of at least 50%. Thinning reduced the production when growing space is left vacant; therefore, the total volume is reduced at first and will eventually increase above that of the unthinned stand as spatial conditions are now more favorable for increased wood production. Here, the 500 and 700 levels may still be recovering from the initial loss due to thinning. Though, in QMD plots, diameter continues to grow with an increasing trend; in which, the 300 level appears to be forming a concaved upwards behavior. The response to thinning in the two denser stands may be showing a delayed response and may require more than the elapsed 14 years after thinning to show more noticeable changes. At the more dense stands trees are spending more time building a larger crown to produce more photosynthate which can then be expended on stem girth. Some studies even reveal that thinning may not increase total volume and, for short rotations (20 years or less), may decrease total cubic foot volume (Mann, 1952; Smith, 1967; Wakeley, 1969).

It should be noted that the time elapsed since the first thinning is still too soon to show definitive responses. Not enough samples from minimal thinnings were available for analysis across all TPA class even for heavy thinnings the numbers of observations are small. So care should be taken with interpretations. Especially for the 300 level when the change in RD is predicted through measurements taken from 2 plots (measured twice after first thinning). The large differences observed between the 300 TPA to the 500 and 700 TPA may not be typical. Observations for the 300 level may be unique to these two plots. The results can be used as a general idea of what to expect from heavy thinning studies after wide early spacings.

# Conclusion

Multiple regressions were formulated with predictor variables of latitude, longitude, and elevation, stand stocking, age, and site index. The additive model of these predictors resulted in incorrect models. Through the use of quadratic terms and interactions, model errors were shown to be more normal and more homoscedastic. However, a Box-cox transformation of the response variables showed that a square-root transformation would be a better fit. The results of which are seven response models whose error terms are normally distributed.

Models were assessed for normality using plots of studentized residuals by fitted values, q-q plots, and fit was assessed by $R^2$ value. The normal error distribution permitted the use of backwards step regression with a t-test at $\alpha=0.05$ in deciding which predictor variables to keep in the models. Models with all significant predictor variables were then assessed for normal distribution again, which revealed an outlier point. Outliers were examined based upon Bonferroni p-values and Cook's distance. The developed seven multiple regression models were for response variables of CVT, CV4, CV6, BF4, BF6, QMD, and BA.

The seven models were plotted by age and relative density for four TPA classes in order to examine the effects of early density controls and the effects of subsequent heavy first thinnings on volume yield. The assessment concluded that early density controls resulted in highest to lowest productivity in the order of 100 TPA, 300 TPA, 500 TPA, and 700 TPA. However, 500 and 700 TPA exhibited very minimal differentiation. In assessing effects from the first thin, the 300 TPA class was shown to have the most increased volume. The first thinning may increase size quickly, but in some situations resulted in a much faster reduction in production. The first thinning in the 500 and 700 levels showed an overall minimal increase in volume and diameter size due to a longer delay in allocating growth from the crown to the stem.

The models produced in this study rely only on stand level information and are intended to augment models such as Forest Vegetation Simulator (FVS). FVS is an individual-tree, distance-independent forest stand projection program capable of predicting growth and yield for defined geographic regions. Furthermore, not many models exist for stands of young (< 40 years) Douglas-firs. The models produced in this study can be of use in assessing and understanding predictions made by other regional growth models because these new empirical yield models are devised from recently collected data by the SMC for the Washington, Oregon, and southwest British Columbia region.

## Future Study Recommendations

- Refitting is necessary once more data are collected for thinned stands to make thinning assessment more definite.
- Though the current models seem to perform well for the observed data, a model cross-validation through another dataset should be performed to examine the models performance when applied to other stands. This can also be done with a subset of the data if data from other stands are not available.
- Care should be taken about the extrapolation of results past the available data range. Since a quadratic term was used in the predictor variables, timing of when production will reach an asymptotic maximum representing the time when stands start to decline due to competition induced mortality should be looked into. Even so, this should not be of much concern in managed stands.
- The fitting of the response variable BA may be improved through nonlinear regression.
- The site index (SI) values calculated from Bruce's 1981 equation range from 86-165 ft. The average observed SI of 137 ft, seems high. However, dominant trees are shown to be fairly tall. This can also be seen in Hanus et al. (1999), where Bruce's SI values from 1309 plots of 187 Douglas-fir installations were calculated. Hanus et al. calculated SI to be in the range of 52-162 with an average SI of 105.7 ft. However, other SI calculations should be considered and compared to examine which method of SI is most representative of the trees in this study.

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# APPENDIX A: Model Fit and Statistics

Table A.1. Estimated coefficients for BA and associated statistics.

BA

| Parameters | Estimate | Std. Error | t value | Pr(>\|t\|) | Sig. |
|---|---|---|---|---|---|
| (Intercept) | -3.73E+00 | 1.18E+01 | -0.317 | 0.75094 | |
| AGE | -1.25E-01 | 4.48E-02 | -2.785 | 0.005461 | ** |
| TPA | -4.42E-02 | 1.09E-02 | -4.044 | 5.72E-05 | *** |
| RD | 3.39E-01 | 1.14E-02 | 29.88 | < 2e-16 | *** |
| SI | 4.91E-04 | 3.25E-03 | 0.151 | 0.879909 | |
| THIN | -3.05E+01 | 8.61E+00 | -3.546 | 0.000412 | *** |
| LAT | 1.52E+00 | 4.45E-01 | 3.407 | 0.000685 | *** |
| LONG | -2.20E-01 | 3.41E-02 | -6.452 | 1.81E-10 | *** |
| ELEVATION | -3.76E-02 | 3.99E-03 | -9.412 | < 2e-16 | *** |
| TPA^2 | 3.71E-06 | 4.10E-07 | 9.033 | < 2e-16 | *** |
| RD^2 | -1.14E-03 | 8.34E-05 | -13.703 | < 2e-16 | *** |
| LAT^2 | -1.78E-02 | 4.80E-03 | -3.7 | 0.000228 | *** |
| AGE:TPA | 7.11E-05 | 1.54E-05 | 4.622 | 4.37E-06 | *** |
| AGE:RD | -9.60E-04 | 2.03E-04 | -4.736 | 2.53E-06 | *** |
| AGE:SI | 7.38E-04 | 1.42E-04 | 5.193 | 2.57E-07 | *** |
| AGE:LAT | 2.07E-03 | 8.05E-04 | 2.573 | 1.03E-02 | * |
| AGE:ELEVATION | 1.25E-05 | 3.24E-06 | 3.865 | 0.000119 | *** |
| TPA:RD | -7.97E-05 | 8.37E-06 | -9.512 | < 2e-16 | *** |
| TPA:SI | 3.29E-05 | 6.34E-06 | 5.192 | 2.57E-07 | *** |
| TPA:LAT | 1.99E-04 | 3.51E-05 | 5.669 | 1.94E-08 | *** |
| TPA:LONG | 2.07E-04 | 7.87E-05 | 2.623 | 8.85E-03 | ** |
| TPA:ELEVATION | 6.73E-07 | 1.25E-07 | 5.373 | 9.91E-08 | *** |
| RD:SI | -2.45E-04 | 7.70E-05 | -3.183 | 1.51E-03 | ** |
| RD:ELEVATION | -7.43E-06 | 1.63E-06 | -4.566 | 5.68E-06 | *** |
| SI:ELEVATION | -4.73E-06 | 8.30E-07 | -5.702 | 1.61E-08 | *** |
| THIN:LAT | 1.38E-01 | 3.28E-02 | 4.208 | 2.84E-05 | *** |
| THIN:LONG | 1.99E-01 | 6.29E-02 | 3.161 | 1.63E-03 | ** |
| LAT:ELEVATION | 5.60E-05 | 1.09E-05 | 5.13E+00 | 3.62E-07 | *** |
| LONG:ELEVATION | 2.90E-04 | 3.07E-05 | 9.43E+00 | < 2e-16 | *** |

Residual standard error: 0.2634 on 889 degrees of freedom
Multiple R-squared: 0.9943, Adjusted R-squared: 0.9941
F-statistic: 5533 on 28 and 889 DF, p-value: < 2.2e-16
Significance levels are: *** p< 0.001, ** p< 0.01, * p<0.05.

**BA** $^{1/2}$ = -3.73E+00 + -1.25E-01*AGE + -4.42E-02*TPA + 3.39E-01*RD + 4.91E-04*SI + -3.05E+01*THIN + 1.52E+00*LAT + -2.20E-01*LONG + -3.76E-02*ELEV + 3.71E-06*TPA$^2$ + -1.14E-03*RD$^2$ + -1.78E-02*LAT$^2$ + 7.11E-05*AGE:TPA + -9.60E-04*AGE:RD + 7.38E-04*AGE:SI + 2.07E-03*AGE:LAT + 1.25E-05*AGE:ELEV + -7.97E-05*TPA:RD + 3.29E-05*TPA:SI + 1.99E-04*TPA:LAT + 2.07E-04*TPA:LONG + 6.73E-07*TPA:ELEV + -2.45E-04*RD:SI + -7.43E-06*RD:ELEV + -4.73E-06*SI:ELEV + 1.38E-01*THIN:LAT + 1.99E-01*THIN:LONG + 5.60E-05*LAT:ELEV + 2.90E-04*LONG:ELEV

Table A.2. Estimated coefficients for QMD and associated statistics.

QMD

| Parameters | Estimate | Std. Error | t value | Pr(>\|t\|) | Sig. |
|---|---|---|---|---|---|
| (Intercept) | 2.89E+02 | 1.22E+02 | 2.362 | 0.018402 | * |
| AGE | -7.20E-01 | 2.16E-01 | -3.339 | 0.000877 | *** |
| TPA | -4.45E-02 | 7.76E-03 | -5.728 | 1.39E-08 | *** |
| RD | 3.63E-01 | 1.05E-01 | 3.444 | 0.0006 | *** |
| SI | -2.47E-01 | 1.06E-01 | -2.336 | 0.019727 | * |
| THIN | -1.99E+01 | 4.91E+00 | -4.05 | 5.56E-05 | *** |
| LAT | 9.25E-01 | 2.65E-01 | 3.488 | 0.000511 | *** |
| LONG | -4.48E+00 | 1.92E+00 | -2.328 | 0.020125 | * |
| ELEVATION | -2.41E-02 | 2.32E-03 | -10.369 | < 2e-16 | *** |
| AGE^2 | 7.26E-04 | 2.20E-04 | 3.294 | 0.001025 | ** |
| TPA^2 | 3.24E-06 | 2.57E-07 | 12.619 | < 2e-16 | *** |
| RD^2 | -1.57E-04 | 5.70E-05 | -2.755 | 0.005999 | ** |
| SI^2 | 6.77E-05 | 2.60E-05 | 2.609 | 0.009236 | ** |
| LAT^2 | -1.09E-02 | 2.88E-03 | -3.767 | 0.000176 | *** |
| LONG^2 | 1.63E-02 | 7.43E-03 | 2.187 | 0.029031 | * |
| AGE:TPA | 5.83E-05 | 1.15E-05 | 5.077 | 4.69E-07 | *** |
| AGE:RD | -1.38E-03 | 2.02E-04 | -6.797 | 1.97E-11 | *** |
| AGE:SI | 7.90E-04 | 1.21E-04 | 6.554 | 9.51E-11 | *** |
| AGE:LAT | 1.06E-03 | 4.46E-04 | 2.37 | 0.018018 | * |
| AGE:LONG | 4.78E-03 | 1.71E-03 | 2.803 | 0.00517 | ** |
| AGE:ELEVATION | 1.10E-05 | 2.11E-06 | 5.21 | 2.36E-07 | *** |
| TPA:RD | -3.42E-05 | 5.65E-06 | -6.054 | 2.09E-09 | *** |
| TPA:SI | 2.81E-05 | 4.02E-06 | 6.992 | 5.36E-12 | *** |
| TPA:THIN | -5.18E-04 | 1.79E-04 | -2.89 | 0.003949 | ** |
| TPA:LAT | 1.31E-04 | 1.98E-05 | 6.583 | 7.90E-11 | *** |
| TPA:LONG | 2.42E-04 | 5.71E-05 | 4.234 | 2.54E-05 | *** |
| TPA:ELEVATION | 5.21E-07 | 7.52E-08 | 6.923 | 8.51E-12 | *** |
| RD:SI | -3.27E-04 | 6.32E-05 | -5.179 | 2.77E-07 | *** |
| RD:LONG | -1.98E-03 | 8.33E-04 | -2.382 | 0.017411 | * |
| RD:ELEVATION | -5.91E-06 | 1.09E-06 | -5.442 | 6.84E-08 | *** |
| SI:LONG | 1.81E-03 | 8.53E-04 | 2.122 | 0.034136 | * |
| SI:ELEVATION | -1.77E-06 | 5.99E-07 | -2.96 | 0.003163 | ** |
| THIN:LAT | 1.07E-01 | 1.88E-02 | 5.671 | 1.92E-08 | *** |
| THIN:LONG | 1.25E-01 | 3.60E-02 | 3.471 | 0.000545 | *** |
| LAT:ELEVATION | 2.95E-05 | 6.38E-06 | 4.62 | 4.41E-06 | *** |
| LONG:ELEVATION | 1.86E-04 | 1.79E-05 | 10.408 | < 2e-16 | *** |

Residual standard error: 0.1424 on 882 degrees of freedom

Multiple R-squared: 0.9544, Adjusted R-squared: 0.9525

F-statistic: 526.9 on 35 and 882 DF, $p < 2.2e-16$.

Significance levels are: *** $p< 0.001$, ** $p< 0.01$, * $p<0.05$.

**$QMD^{1/2}$** = 2.89E+02 + -7.20E-01*AGE + - 4.45E-02*TPA + 3.63E-01*RD + -2.47E-01*SI + -1.99E+01*THIN + 9.25E-01*LAT + -4.48E+00*LONG + -2.41E-02*ELEV + 7.26E-04*$AGE^2$ + 3.24E-06*$TPA^2$ + -1.57E-04*$RD^2$ + 6.77E-05*$SI^2$ + -1.09E-02*$LAT^2$ + 1.63E-02*$LONG^2$ + 5.83E-05*AGE:TPA+ -1.38E-03*AGE:RD + 7.90E-04*AGE:SI + 1.06E-03*AGE:LAT + 4.78E-03*AGE:LONG + 1.10E-05*AGE:ELEV + -3.42E-05*TPA:RD + 2.81E-05*TPA:SI + -5.18E-04*TPA:THIN + 1.31E-04*TPA:LAT + 2.42E-04*TPA:LONG + 5.21E-07*TPA:ELEV + -3.27E-04*RD:SI + -1.98E-03*RD:LONG + -5.91E-06*RD:ELEV + 1.81E-03*SI:LONG + -1.77E-06*SI:ELEV + 1.07E-01*THIN:LAT + 1.25E-01*THIN:LONG + 2.95E-05*LAT:ELEV + 1.86E-04*LONG:ELEV

Table A.3. Estimated coefficients for CVT and associated statistics.

CVT

| Parameters | Estimate | Std. Error | t value | Pr(>\|t\|) | Sig. |
|---|---|---|---|---|---|
| (Intercept) | 8.13E+02 | 3.73E+02 | 2.182 | 0.029343 | * |
| AGE | -2.21E+01 | 5.84E+00 | -3.778 | 0.000168 | *** |
| TPA | -1.11E+00 | 2.13E-01 | -5.23 | 2.12E-07 | *** |
| RD | 9.62E+00 | 2.83E+00 | 3.4 | 0.000703 | *** |
| SI | -1.12E+01 | 2.15E+00 | -5.189 | 2.62E-07 | *** |
| THIN | -5.62E+01 | 2.10E+01 | -2.671 | 0.007697 | ** |
| LAT | 3.30E+01 | 7.19E+00 | 4.594 | 4.97E-06 | *** |
| LONG | -1.20E+01 | 2.47E+00 | -4.848 | 1.47E-06 | *** |
| ELEVATION | 2.71E-02 | 7.47E-03 | 3.629 | 0.000301 | *** |
| AGE^2 | 2.44E-02 | 3.55E-03 | 6.866 | 1.24E-11 | *** |
| TPA^2 | 8.78E-05 | 5.71E-06 | 15.372 | < 2e-16 | *** |
| RD^2 | 1.84E-03 | 7.54E-04 | 2.442 | 0.014808 | * |
| SI^2 | 2.98E-03 | 7.30E-04 | 4.082 | 4.87E-05 | *** |
| LAT^2 | -3.53E-01 | 7.73E-02 | -4.564 | 5.72E-06 | *** |
| ELEVATION^2 | -1.98E-06 | 3.60E-07 | -5.501 | 4.94E-08 | *** |
| AGE:SI | 2.80E-02 | 2.56E-03 | 10.953 | < 2e-16 | *** |
| AGE:LONG | 1.48E-01 | 4.69E-02 | 3.161 | 0.001623 | ** |
| AGE:ELEVATION | 2.27E-04 | 5.54E-05 | 4.099 | 4.52E-05 | *** |
| TPA:RD | -1.49E-03 | 7.25E-05 | -20.513 | < 2e-16 | *** |
| TPA:SI | 7.55E-04 | 1.06E-04 | 7.133 | 2.04E-12 | *** |
| TPA:THIN | -2.12E-02 | 4.66E-03 | -4.552 | 6.05E-06 | *** |
| TPA:LAT | 3.51E-03 | 5.60E-04 | 6.266 | 5.79E-10 | *** |
| TPA:LONG | 6.54E-03 | 1.58E-03 | 4.15 | 3.64E-05 | *** |
| TPA:ELEVATION | 7.76E-06 | 2.12E-06 | 3.665 | 0.000262 | *** |
| RD:SI | -9.63E-03 | 1.40E-03 | -6.897 | 1.01E-11 | *** |
| RD:LONG | -6.19E-02 | 2.27E-02 | -2.727 | 0.006518 | ** |
| RD:ELEVATION | -1.13E-04 | 2.76E-05 | -4.097 | 4.57E-05 | *** |
| SI:LONG | 8.13E-02 | 1.73E-02 | 4.708 | 2.90E-06 | *** |
| THIN:LAT | 1.50E+00 | 4.54E-01 | 3.31 | 0.000969 | *** |
| LAT:ELEVATION | -5.22E-04 | 1.63E-04 | -3.201 | 0.001416 | ** |

Residual standard error: 4.188 on 887 degrees of freedom
Multiple R-squared: 0.8932, Adjusted R-squared: 0.8897
F-statistic: 255.9 on 29 and 887 DF, p-value: < 2.2e-16
Significance levels are: *** p< 0.001, ** p< 0.01, * p<0.05.

**$CVT^{1/2}$** = 8.13E+02 + -2.21E+01*AGE + -1.11E+00*TPA + 9.62E+00*RD + -1.12E+01*SI + -5.62E+01*THIN +3.30E+01*LAT + -1.20E+01*LONG + 2.71E-02*ELEV + 2.44E-02*$AGE^2$ + 8.78E-05*$TPA^2$ + 1.84E-03*$RD^2$ + 2.98E-03*$SI^2$ + -3.53E-01*$LAT^2$ + -1.98E-06*$ELEV^2$ + 2.80E-02*AGE:SI + 1.48E-01*AGE:LONG + 2.27E-04*AGE:ELEV + -1.49E-03*TPA:RD + 7.55E-04*TPA:SI + -2.12E-02*TPA:THIN + 3.51E-03*TPA:LAT + 6.54E-03*TPA:LONG + 7.76E-06*TPA:ELEV + -9.63E-03*RD:SI + -6.19E-02*RD:LONG + -1.13E-04*RD:ELEV + 8.13E-02*SI:LONG + 1.50E+00*THIN:LAT + -5.22E-04*LAT:ELEV

Table A.4. Estimated coefficients for CV4 and associated statistics.

CV4

| Parameters | Estimate | Std. Error | t value | Pr(>\|t\|) | Sig. |
|---|---|---|---|---|---|
| (Intercept) | 5.37E+03 | 3.60E+03 | 1.493 | 0.135965 | |
| AGE | -3.39E+01 | 6.20E+00 | -5.47 | 6.38E-08 | *** |
| TPA | -1.97E+00 | 2.56E-01 | -7.694 | 5.13E-14 | *** |
| RD | 2.24E+01 | 3.25E+00 | 6.889 | 1.30E-11 | *** |
| SI | -1.52E+01 | 3.38E+00 | -4.487 | 8.50E-06 | *** |
| THIN | -8.47E+01 | 1.97E+01 | -4.308 | 1.89E-05 | *** |
| LAT | 1.39E+02 | 3.12E+01 | 4.465 | 9.40E-06 | *** |
| LONG | -1.25E+02 | 5.84E+01 | -2.143 | 0.03244 | * |
| ELEVATION | 4.39E-01 | 1.79E-01 | 2.457 | 0.014273 | * |
| AGE^2 | 2.03E-02 | 6.23E-03 | 3.255 | 0.001191 | ** |
| TPA^2 | 1.11E-04 | 7.69E-06 | 14.445 | < 2e-16 | *** |
| SI^2 | 4.10E-03 | 8.51E-04 | 4.82 | 1.78E-06 | *** |
| LAT^2 | -4.18E-01 | 9.48E-02 | -4.411 | 1.20E-05 | *** |
| LONG^2 | 6.05E-01 | 2.44E-01 | 2.477 | 0.013484 | * |
| ELEVATION^2 | -4.79E-06 | 9.71E-07 | -4.933 | 1.02E-06 | *** |
| AGE:TPA | 2.57E-03 | 2.94E-04 | 8.735 | < 2e-16 | *** |
| AGE:RD | -3.93E-02 | 3.53E-03 | -11.144 | < 2e-16 | *** |
| AGE:SI | 3.22E-02 | 3.21E-03 | 10.035 | < 2e-16 | *** |
| AGE:THIN | -4.54E-01 | 1.47E-01 | -3.086 | 0.002109 | ** |
| AGE:LONG | 2.48E-01 | 4.89E-02 | 5.075 | 5.04E-07 | *** |
| AGE:ELEVATION | 2.05E-04 | 5.96E-05 | 3.431 | 0.000638 | *** |
| TPA:RD | -1.38E-03 | 1.16E-04 | -11.921 | < 2e-16 | *** |
| TPA:SI | 1.05E-03 | 1.27E-04 | 8.22 | 1.06E-15 | *** |
| TPA:THIN | -2.91E-02 | 4.79E-03 | -6.084 | 1.97E-09 | *** |
| TPA:LAT | 3.63E-03 | 6.05E-04 | 5.994 | 3.34E-09 | *** |
| TPA:LONG | 1.23E-02 | 1.89E-03 | 6.513 | 1.44E-10 | *** |
| TPA:ELEVATION | 1.31E-05 | 2.37E-06 | 5.552 | 4.08E-08 | *** |
| RD:SI | -1.14E-02 | 1.59E-03 | -7.166 | 2.05E-12 | *** |
| RD:THIN | 1.73E-01 | 8.17E-02 | 2.122 | 0.03422 | * |
| RD:LONG | -1.54E-01 | 2.58E-02 | -5.989 | 3.45E-09 | *** |
| RD:ELEVATION | -1.28E-04 | 3.21E-05 | -3.995 | 7.20E-05 | *** |
| SI:LONG | 1.12E-01 | 2.73E-02 | 4.089 | 4.86E-05 | *** |
| SI:ELEVATION | -7.00E-05 | 2.25E-05 | -3.112 | 0.001938 | ** |
| THIN:LAT | 2.29E+00 | 4.20E-01 | 5.454 | 6.95E-08 | *** |
| LAT:LONG | -8.05E-01 | 2.10E-01 | -3.828 | 0.000141 | *** |
| LAT:ELEVATION | -1.90E-03 | 3.72E-04 | -5.097 | 4.50E-07 | *** |
| LONG:ELEVATION | -2.70E-03 | 1.30E-03 | -2.077 | 0.038179 | * |

Residual standard error: 3.621 on 670 degrees of freedom

Multiple R-squared: 0.9342, Adjusted R-squared: 0.9307

F-statistic: 264.3 on 36 and 670 DF, p-value: < 2.2e-16

Significance levels are: *** p< 0.001, ** p< 0.01, * p<0.05.

Note: 210 zero $ft^3$ values removed form model fitting.

$$\mathbf{CV4^{1/2}} = 5.37E{+}03 + -3.39E{+}01*AGE + -1.97E{+}00*TPA + 2.24E{+}01*RD + -1.52E{+}01*SI + -8.47E{+}01*THIN + 1.39E{+}02*LAT + -1.25E{+}02*LONG + 4.39E{-}01*ELEV + 2.03E{-}02*AGE^2 + 1.11E{-}04*TPA^2 + 4.10E{-}03*SI^2 + -4.18E{-}01*LAT^2 + 6.05E{-}01*LONG^2 + -4.79E{-}06*ELEV^2 + 2.57E{-}03*AGE{:}TPA + -3.93E{-}02*AGE{:}RD + 3.22E{-}02*AGE{:}SI + -4.54E{-}01*AGE{:}THIN + 2.48E{-}01*AGE{:}LONG + 2.05E{-}04*AGE{:}ELEV + -1.38E{-}03*TPA{:}RD + 1.05E{-}03*TPA{:}SI + -2.91E{-}02*TPA{:}THIN + 3.63E{-}03*TPA{:}LAT + 1.23E{-}02*TPA{:}LONG + 1.31E{-}05*TPA{:}ELEV + -1.14E{-}02*RD{:}SI + 1.73E{-}01*RD{:}THIN + -1.54E{-}01*RD{:}LONG + -1.28E{-}04*RD{:}ELEV + 1.12E{-}01*SI{:}LONG + -7.00E{-}05*SI{:}ELEV + 2.29E{+}00*THIN{:}LAT + -8.05E{-}01*LAT{:}LONG + -1.90E{-}03*LAT{:}ELEV + -2.70E{-}03*LONG{:}ELEV$$

Table A.5. Estimated coefficients for CV6 and associated statistics.

CV6

| Parameters | Estimate | Std. Error | t value | Pr(>\|t\|) | Sig. |
|---|---|---|---|---|---|
| (Intercept) | 1.20E+04 | 4.63E+03 | 2.594 | 0.009729 | ** |
| AGE | -5.67E+01 | 9.05E+00 | -6.267 | 7.53E-10 | *** |
| TPA | -2.48E+00 | 3.32E-01 | -7.474 | 3.15E-13 | *** |
| RD | 2.69E+01 | 4.05E+00 | 6.637 | 7.76E-11 | *** |
| SI | -1.63E+01 | 4.18E+00 | -3.893 | 0.000112 | *** |
| THIN | -8.78E+01 | 2.35E+01 | -3.736 | 0.000207 | *** |
| LAT | 1.90E+02 | 3.27E+01 | 5.812 | 1.05E-08 | *** |
| LONG | -2.44E+02 | 7.55E+01 | -3.235 | 0.001289 | ** |
| ELEVATION | 7.79E-02 | 1.19E-02 | 6.518 | 1.64E-10 | *** |
| TPA^2 | 1.51E-04 | 1.03E-05 | 14.604 | < 2e-16 | *** |
| SI^2 | 4.69E-03 | 1.09E-03 | 4.306 | 1.97E-05 | *** |
| LAT^2 | -6.28E-01 | 1.20E-01 | -5.252 | 2.16E-07 | *** |
| LONG^2 | 1.11E+00 | 3.11E-01 | 3.582 | 0.000371 | *** |
| ELEVATION^2 | -3.35E-06 | 5.64E-07 | -5.931 | 5.38E-09 | *** |
| AGE:TPA | 2.60E-03 | 3.59E-04 | 7.231 | 1.64E-12 | *** |
| AGE:RD | -3.69E-02 | 4.31E-03 | -8.564 | < 2e-16 | *** |
| AGE:SI | 3.17E-02 | 3.94E-03 | 8.051 | 5.24E-15 | *** |
| AGE:THIN | -3.35E-01 | 1.52E-01 | -2.205 | 0.027863 | * |
| AGE:LAT | 8.08E-02 | 2.84E-02 | 2.848 | 0.004567 | ** |
| AGE:LONG | 4.13E-01 | 6.79E-02 | 6.073 | 2.36E-09 | *** |
| AGE:ELEVATION | 3.12E-04 | 7.99E-05 | 3.906 | 0.000106 | *** |
| TPA:RD | -1.58E-03 | 1.46E-04 | -10.768 | < 2e-16 | *** |
| TPA:SI | 1.39E-03 | 1.67E-04 | 8.325 | 6.88E-16 | *** |
| TPA:THIN | -3.86E-02 | 5.78E-03 | -6.684 | 5.80E-11 | *** |
| TPA:LAT | 4.39E-03 | 7.90E-04 | 5.562 | 4.19E-08 | *** |
| TPA:LONG | 1.55E-02 | 2.43E-03 | 6.38 | 3.81E-10 | *** |
| TPA:ELEVATION | 1.49E-05 | 3.08E-06 | 4.835 | 1.73E-06 | *** |
| RD:SI | -1.37E-02 | 1.96E-03 | -6.978 | 8.80E-12 | *** |
| RD:LONG | -1.88E-01 | 3.22E-02 | -5.849 | 8.54E-09 | *** |
| RD:ELEVATION | -1.31E-04 | 3.99E-05 | -3.282 | 0.001095 | ** |
| SI:LONG | 1.20E-01 | 3.36E-02 | 3.562 | 0.0004 | *** |
| SI:ELEVATION | -6.77E-05 | 2.52E-05 | -2.683 | 0.007524 | ** |
| THIN:LAT | 2.57E+00 | 5.07E-01 | 5.082 | 5.15E-07 | *** |
| LAT:LONG | -1.08E+00 | 2.13E-01 | -5.073 | 5.38E-07 | *** |
| LAT:ELEVATION | -1.40E-03 | 2.42E-04 | -5.787 | 1.21E-08 | *** |

Residual standard error: 4.237 on 542 degrees of freedom

Multiple R-squared: 0.9081, Adjusted R-squared: 0.9024

F-statistic: 157.6 on 34 and 542 DF, p-value: < 2.2e-16

Significance levels are: *** p< 0.001, ** p< 0.01, * p<0.05.

Note: 340 zero $ft^3$ values removed form model fitting.

**$CV6^{1/2}$** = 1.20E+04+ -5.67E+01*AGE + -2.48E+00*TPA + 2.69E+01*RD + -1.63E+01*SI + -8.78E+01*THIN + 1.90E+02*LAT + -2.44E+02*LONG + 7.79E-02*ELEV + 1.51E-04*$TPA^2$ + 4.69E-03*$SI^2$ + -6.28E-01*$LAT^2$ + 1.11E+00*$LONG^2$ + -3.35E-06*$ELEV^2$ + 2.60E-03*AGE:TPA + -3.69E-02*AGE:RD + 3.17E-02*AGE:SI + -3.35E-01*AGE:THIN + 8.08E-02*AGE:LAT + 4.13E-01*AGE:LONG + 3.12E-04*AGE:ELEV + -1.58E-03*TPA:RD + 1.39E-03*TPA:SI + -3.86E-02*TPA:THIN + 4.39E-03*TPA:LAT + 1.55E-02*TPA:LONG + 1.49E-05*TPA:ELEV + -1.37E-02*RD:SI + -1.88E-01*RD:LONG + -1.31E-04*RD:ELEV + 1.20E-01*SI:LONG + -6.77E-05*SI:ELEV + 2.57E+00*THIN:LAT + -1.08E+00*LAT:LONG + -1.40E-03*LAT:ELEV

Table A.6. Estimated coefficients for BF4 and associated statistics.

BF4

| Parameters | Estimate | Std. Error | t value | Pr(>\|t\|) | Sig. |
|---|---|---|---|---|---|
| (Intercept) | 4.65E+03 | 7.25E+02 | 6.413 | 2.68E-10 | *** |
| AGE | -6.60E+01 | 1.28E+01 | -5.165 | 3.18E-07 | *** |
| TPA | -3.64E+00 | 5.20E-01 | -7.002 | 6.14E-12 | *** |
| RD | 4.28E+01 | 7.10E+00 | 6.024 | 2.80E-09 | *** |
| SI | -3.86E+01 | 4.22E+00 | -9.142 | < 2e-16 | *** |
| THIN | -1.03E+02 | 3.82E+01 | -2.695 | 0.007205 | ** |
| LAT | 2.49E+01 | 1.38E+01 | 1.801 | 0.072155 | . |
| LONG | -3.90E+01 | 5.05E+00 | -7.718 | 4.28E-14 | *** |
| ELEVATION | 6.63E-02 | 1.60E-02 | 4.152 | 3.72E-05 | *** |
| AGE^2 | 4.61E-02 | 1.17E-02 | 3.94 | 9.02E-05 | *** |
| TPA^2 | 1.59E-04 | 1.45E-05 | 10.956 | < 2e-16 | *** |
| SI^2 | 9.89E-03 | 1.70E-03 | 5.803 | 1.00E-08 | *** |
| LAT^2 | -3.37E-01 | 1.45E-01 | -2.327 | 0.020246 | * |
| ELEVATION^2 | -5.22E-06 | 7.25E-07 | -7.199 | 1.63E-12 | *** |
| AGE:TPA | 3.83E-03 | 5.53E-04 | 6.929 | 9.94E-12 | *** |
| AGE:RD | -5.61E-02 | 6.57E-03 | -8.549 | < 2e-16 | *** |
| AGE:SI | 7.18E-02 | 6.24E-03 | 11.497 | < 2e-16 | *** |
| AGE:THIN | -6.23E-01 | 2.57E-01 | -2.421 | 0.015728 | * |
| AGE:LAT | 1.18E-01 | 5.11E-02 | 2.309 | 0.021242 | * |
| AGE:LONG | 4.23E-01 | 9.28E-02 | 4.555 | 6.21E-06 | *** |
| AGE:ELEVATION | 5.34E-04 | 1.09E-04 | 4.905 | 1.17E-06 | *** |
| TPA:RD | -2.23E-03 | 2.15E-04 | -10.347 | < 2e-16 | *** |
| TPA:SI | 1.85E-03 | 2.46E-04 | 7.491 | 2.15E-13 | *** |
| TPA:THIN | -3.72E-02 | 8.97E-03 | -4.15 | 3.75E-05 | *** |
| TPA:LAT | 9.33E-03 | 1.71E-03 | 5.468 | 6.41E-08 | *** |
| TPA:LONG | 2.23E-02 | 3.67E-03 | 6.072 | 2.11E-09 | *** |
| TPA:ELEVATION | 2.67E-05 | 4.58E-06 | 5.834 | 8.43E-09 | *** |
| RD:SI | -2.31E-02 | 3.26E-03 | -7.087 | 3.46E-12 | *** |
| RD:LAT | -6.47E-02 | 2.67E-02 | -2.42 | 0.01577 | * |
| RD:LONG | -2.73E-01 | 5.10E-02 | -5.352 | 1.20E-07 | *** |
| RD:ELEVATION | -3.38E-04 | 5.96E-05 | -5.662 | 2.21E-08 | *** |
| SI:LAT | 4.40E-02 | 1.91E-02 | 2.304 | 0.021529 | * |
| SI:LONG | 2.68E-01 | 3.41E-02 | 7.856 | 1.57E-14 | *** |
| THIN:LAT | 3.10E+00 | 8.32E-01 | 3.732 | 0.000206 | *** |
| LAT:ELEVATION | -1.26E-03 | 3.50E-04 | -3.596 | 0.000347 | *** |

Residual standard error: 6.473 on 671 degrees of freedom
Multiple R-squared: 0.9452, Adjusted R-squared: 0.9424
F-statistic: 330.8 on 35 and 671 DF, p-value: < 2.2e-16
Significance levels are: *** p< 0.001, ** p< 0.01, * p<0.05.
Note: 210 zero bd.ft. values removed form model fitting.

$$BF4^{1/2} = 4.65E{+}03 + -6.60E{+}01*AGE + -3.64E{+}00*TPA + 4.28E{+}01*RD + -3.86E{+}01*SI + -1.03E{+}02*THIN + 2.49E{+}01*LAT + -3.90E{+}01*LONG + 6.63E{-}02*ELEV + 4.61E{-}02*AGE^2 + 1.59E{-}04*TPA^2 + 9.89E{-}03*SI^2 + -3.37E{-}01*LAT^2 + -5.22E{-}06*ELEV^2 + 3.83E{-}03*AGE{:}TPA + -5.61E{-}02*AGE{:}RD + 7.18E{-}02*AGE{:}SI + -6.23E{-}01*AGE{:}THIN + 1.18E{-}01*AGE{:}LAT + 4.23E{-}01*AGE{:}LONG + 5.34E{-}04*AGE{:}ELEV + -2.23E{-}03*TPA{:}RD + 1.85E{-}03*TPA{:}SI + -3.72E{-}02*TPA{:}THIN + 9.33E{-}03*TPA{:}LAT + 2.23E{-}02*TPA{:}LONG + 2.67E{-}05*TPA{:}ELEV + -2.31E{-}02*RD{:}SI + -6.47E{-}02*RD{:}LAT + -2.73E{-}01*RD{:}LONG + -3.38E{-}04*RD{:}ELEV + 4.40E{-}02*SI{:}LAT + 2.68E{-}01*SI{:}LONG + 3.10E{+}00*THIN{:}LAT + -1.26E{-}03*LAT{:}ELEV$$

Table A.7. Estimated coefficients for BF6 and associated statistics.

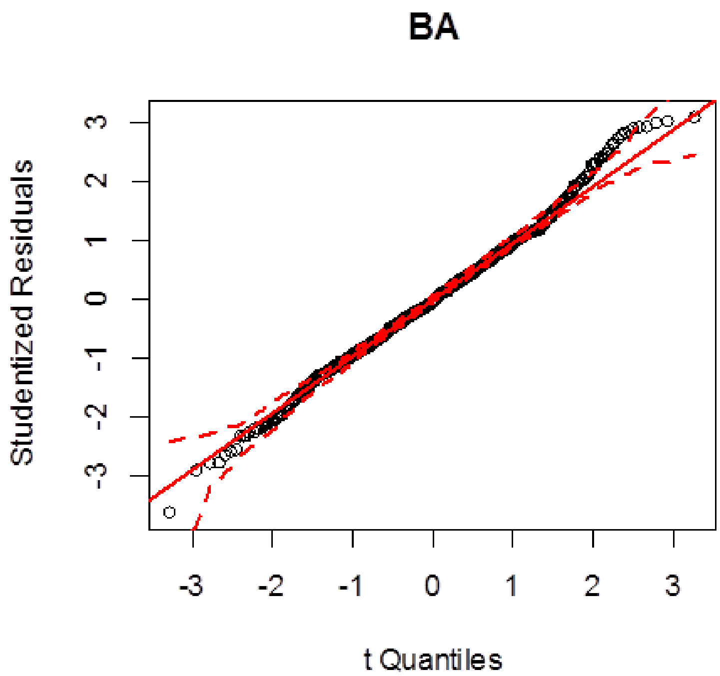
BF6

| Parameters | Estimate | Std. Error | t value | Pr(>\|t\|) | Sig. |
|---|---|---|---|---|---|
| (Intercept) | 2.73E+04 | 8.04E+03 | 3.402 | 0.000719 | *** |
| AGE | -1.08E+02 | 1.65E+01 | -6.567 | 1.20E-10 | *** |
| TPA | -4.74E+00 | 6.06E-01 | -7.827 | 2.61E-14 | *** |
| RD | 4.92E+01 | 7.18E+00 | 6.86 | 1.88E-11 | *** |
| SI | -3.98E+01 | 6.99E+00 | -5.698 | 1.99E-08 | *** |
| THIN | -1.77E+02 | 4.33E+01 | -4.082 | 5.14E-05 | *** |
| LAT | 2.07E+02 | 5.81E+01 | 3.572 | 0.000386 | *** |
| LONG | -4.59E+02 | 1.29E+02 | -3.566 | 0.000395 | *** |
| ELEVATION | -1.11E+00 | 1.45E-01 | -7.617 | 1.16E-13 | *** |
| TPA^2 | 2.56E-04 | 1.92E-05 | 13.356 | < 2e-16 | *** |
| SI^2 | 8.74E-03 | 1.85E-03 | 4.718 | 3.03E-06 | *** |
| LAT^2 | -9.91E-01 | 2.19E-01 | -4.527 | 7.35E-06 | *** |
| LONG^2 | 1.82E+00 | 5.23E-01 | 3.47 | 0.000561 | *** |
| AGE:TPA | 4.04E-03 | 6.48E-04 | 6.229 | 9.38E-10 | *** |
| AGE:RD | -5.62E-02 | 8.08E-03 | -6.951 | 1.04E-11 | *** |
| AGE:SI | 6.79E-02 | 6.92E-03 | 9.806 | < 2e-16 | *** |
| AGE:LAT | 1.79E-01 | 5.37E-02 | 3.325 | 0.000945 | *** |
| AGE:LONG | 7.69E-01 | 1.24E-01 | 6.222 | 9.82E-10 | *** |
| AGE:ELEVATION | 6.77E-04 | 1.45E-04 | 4.678 | 3.66E-06 | *** |
| TPA:RD | -2.94E-03 | 2.71E-04 | -10.841 | < 2e-16 | *** |
| TPA:SI | 2.47E-03 | 2.99E-04 | 8.264 | 1.08E-15 | *** |
| TPA:THIN | -6.91E-02 | 1.09E-02 | -6.36 | 4.27E-10 | *** |
| TPA:LAT | 8.78E-03 | 1.50E-03 | 5.834 | 9.27E-09 | *** |
| TPA:LONG | 3.02E-02 | 4.45E-03 | 6.777 | 3.19E-11 | *** |
| TPA:ELEVATION | 2.83E-05 | 5.43E-06 | 5.213 | 2.64E-07 | *** |
| RD:SI | -2.49E-02 | 3.55E-03 | -7.016 | 6.82E-12 | *** |
| RD:LONG | -3.45E-01 | 5.73E-02 | -6.026 | 3.10E-09 | *** |
| RD:ELEVATION | -2.82E-04 | 6.55E-05 | -4.304 | 1.99E-05 | *** |
| SI:LONG | 2.98E-01 | 5.75E-02 | 5.183 | 3.08E-07 | *** |
| THIN:LAT | 4.71E+00 | 9.39E-01 | 5.018 | 7.08E-07 | *** |
| LAT:LONG | -9.83E-01 | 3.76E-01 | -2.613 | 0.009213 | ** |
| LONG:ELEVATION | 8.94E-03 | 1.18E-03 | 7.575 | 1.55E-13 | *** |

Residual standard error: 8.045 on 545 degrees of freedom
Multiple R-squared: 0.9152, Adjusted R-squared: 0.9104
F-statistic: 189.7 on 31 and 545 DF, p-value: < 2.2e-16
Significance levels are: *** p< 0.001, ** p< 0.01, * p<0.05.
Note: 340 zero bd.ft. values removed form model fitting.

**BF6**$^{1/2}$ = 2.73E+04 + -1.08E+02*AGE + -4.74E+00*TPA + 4.92E+01*RD + -3.98E+01*SI + -1.77E+02*THIN + 2.07E+02*LAT + -4.59E+02*LONG + -1.11E+00*ELEV + 2.56E-04*TPA$^2$ + 8.74E-03*SI$^2$ + -9.91E-01*LAT$^2$ + 1.82E+00*LONG$^2$ + 4.04E-03*AGE:TPA + -5.62E-02*AGE:RD + 6.79E-02*AGE:SI + 1.79E-01*AGE:LAT + 7.69E-01*AGE:LONG + 6.77E-04*AGE:ELEV + -2.94E-03*TPA:RD + 2.47E-03*TPA:SI + -6.91E-02*TPA:THIN + 8.78E-03*TPA:LAT + 3.02E-02*TPA:LONG + 2.83E-05*TPA:ELEV + -2.49E-02*RD:SI + -3.45E-01*RD:LONG + -2.82E-04*RD:ELEV + 2.98E-01*SI:LONG + 4.71E+00*THIN:LAT + -9.83E-01*LAT:LONG + 8.94E-03*LONG:ELEV

# Appendix B: Q-Q and Residuals for the Models in Appendix A



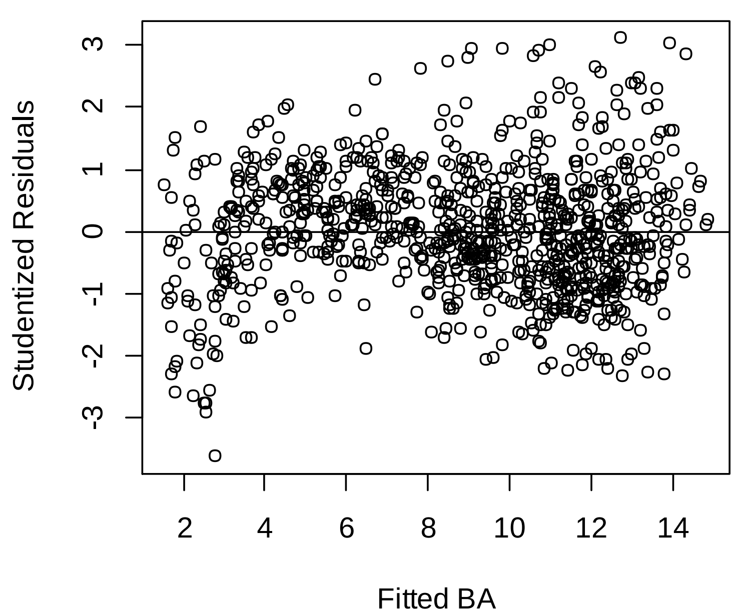




Figure B.1. Q-Q plot and studentized residuals of fitted BA.

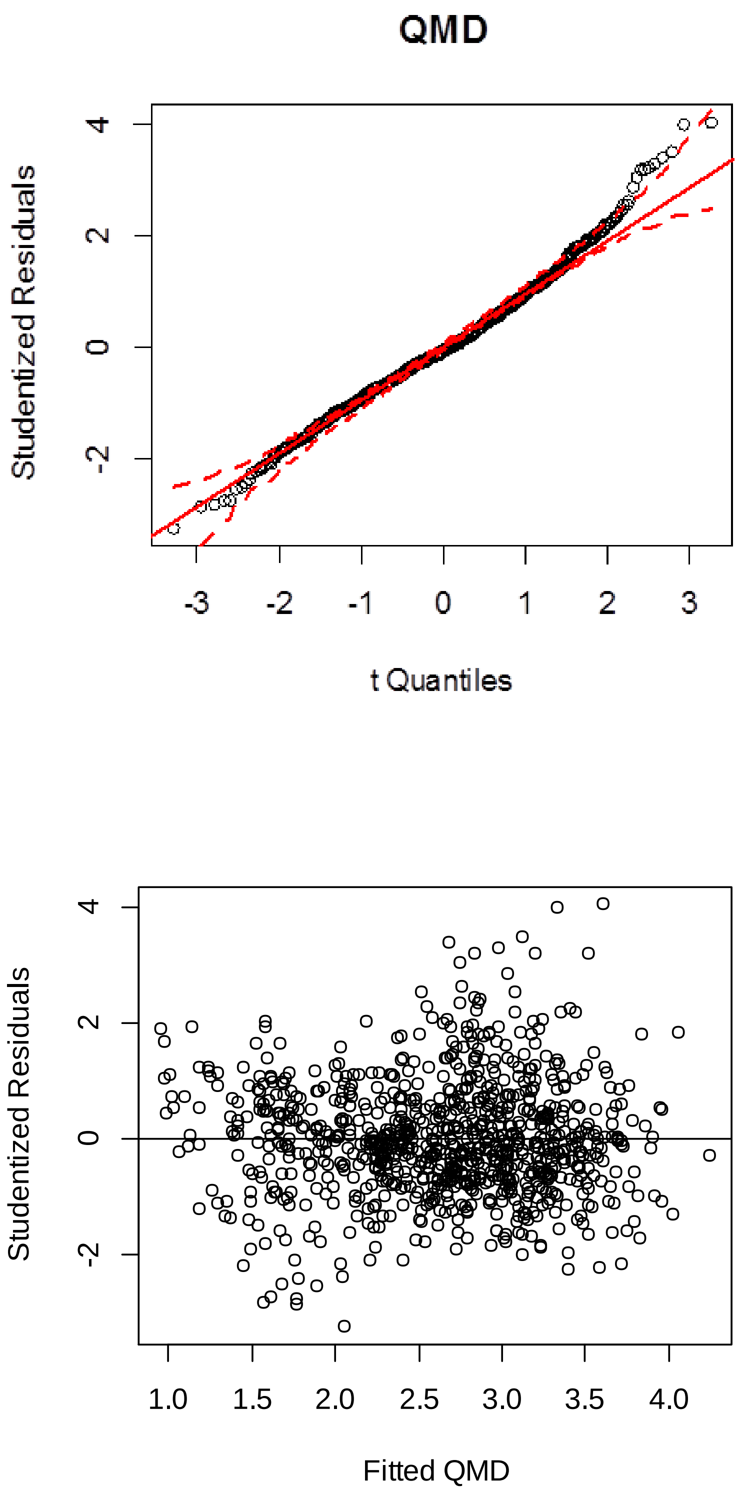



Figure B.2. Q-Q plot and studentized residuals of fitted QMD.

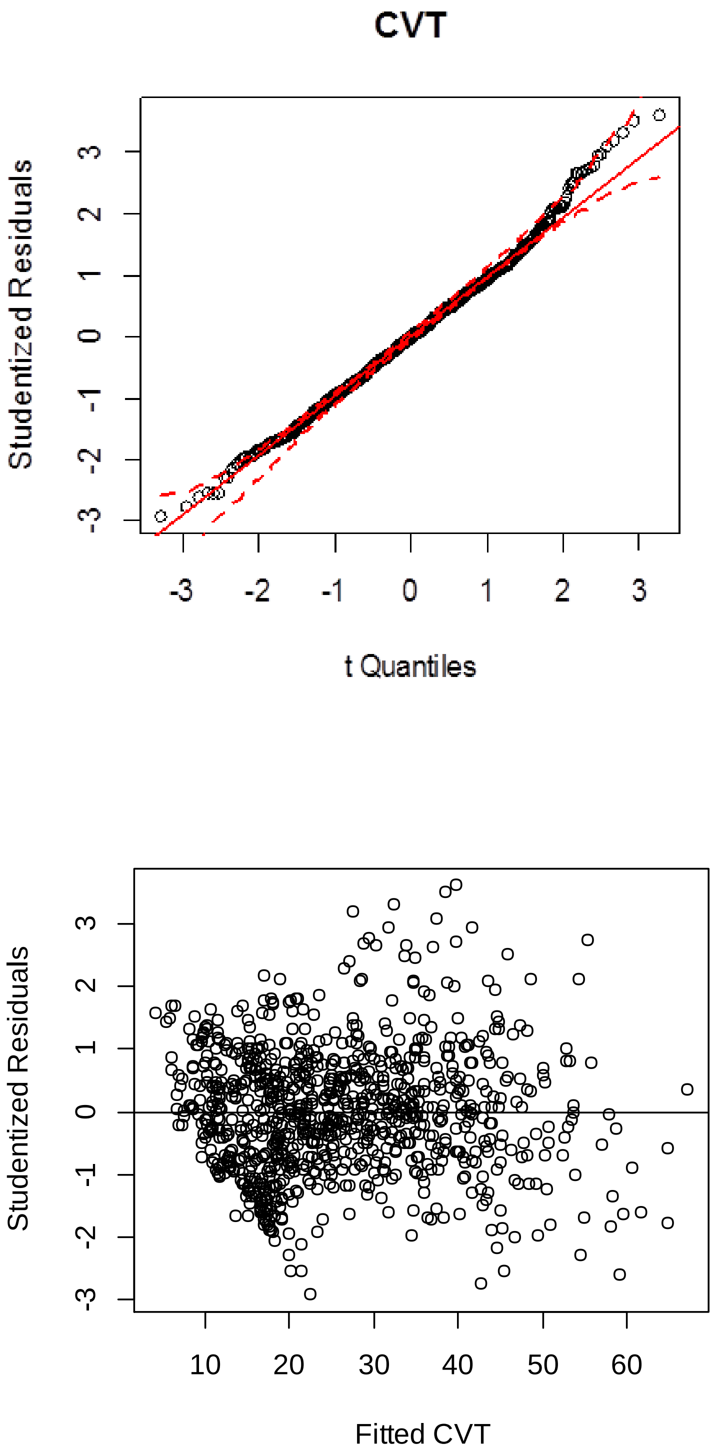



Figure B.3. Q-Q plot and studentized residuals of fitted CVT.

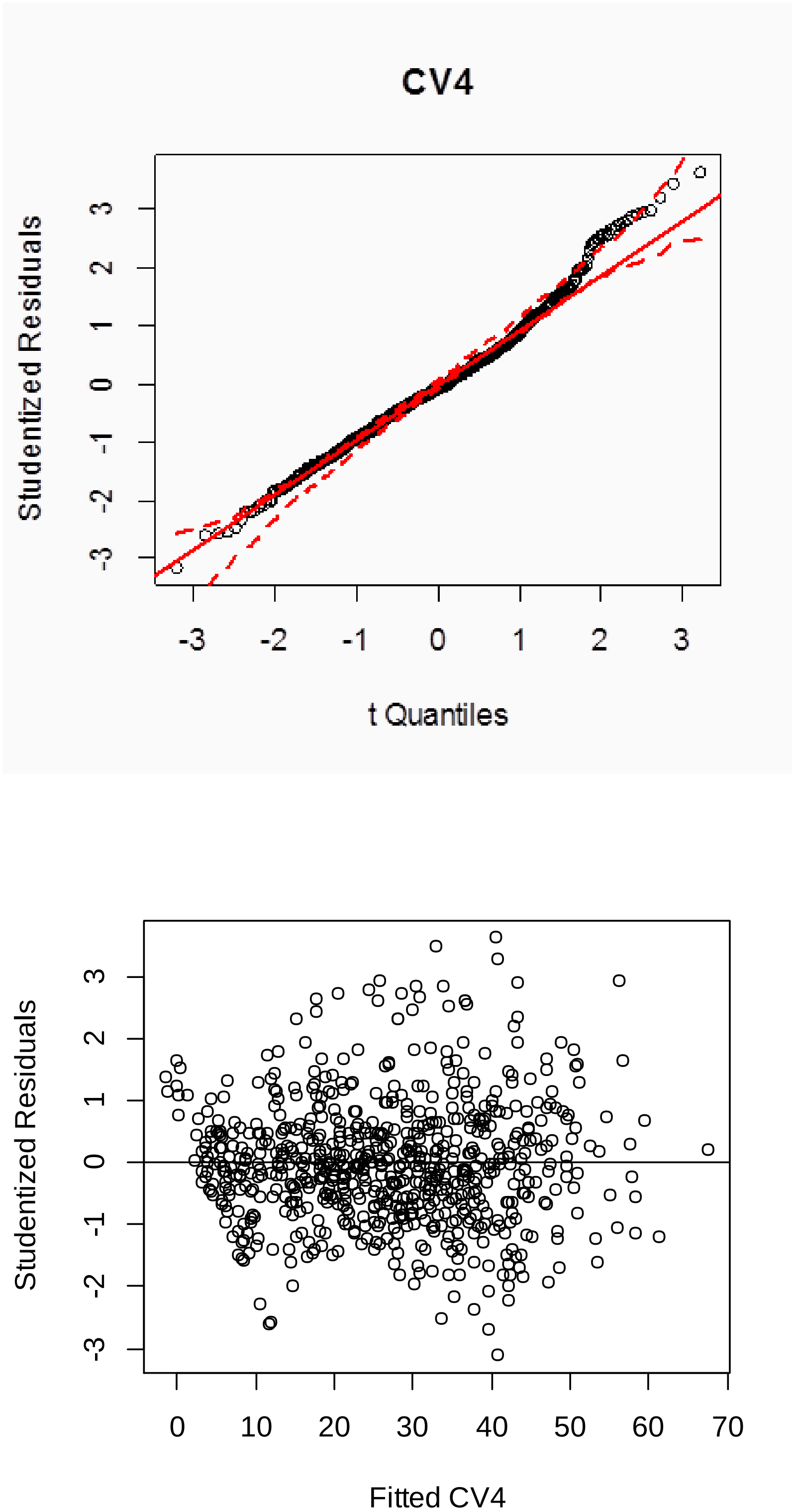
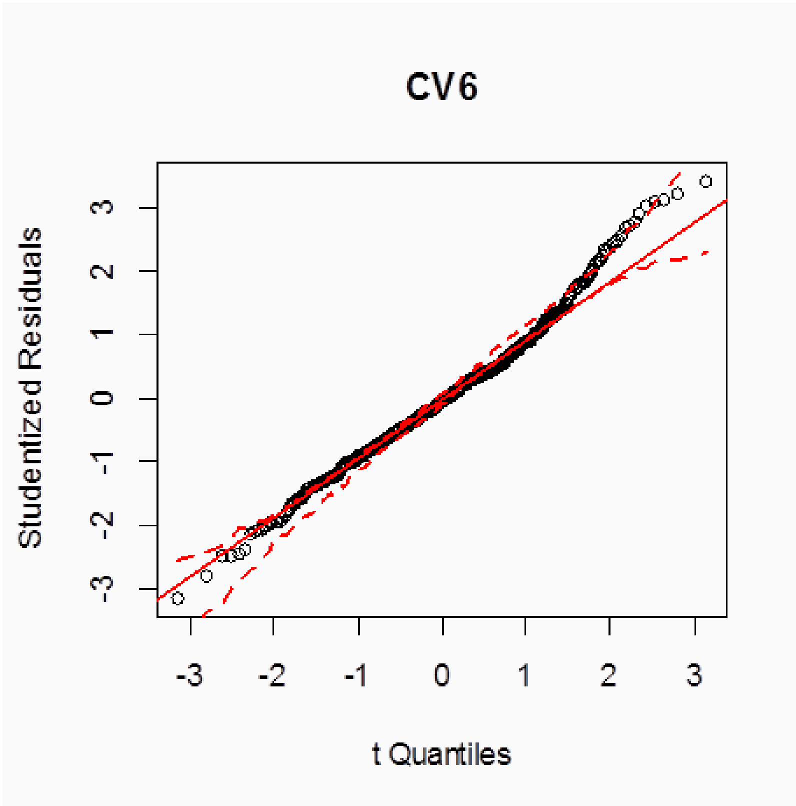



Figure B.4. Q-Q plot and studentized residuals of fitted CV4.



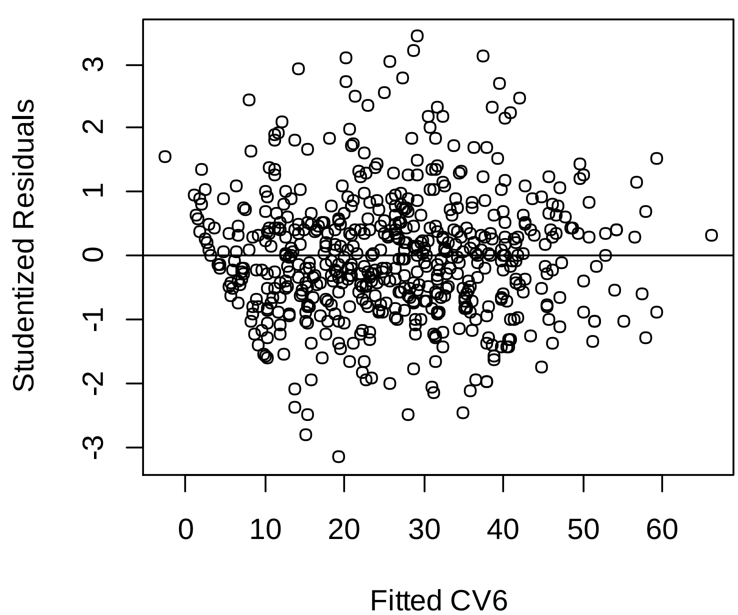
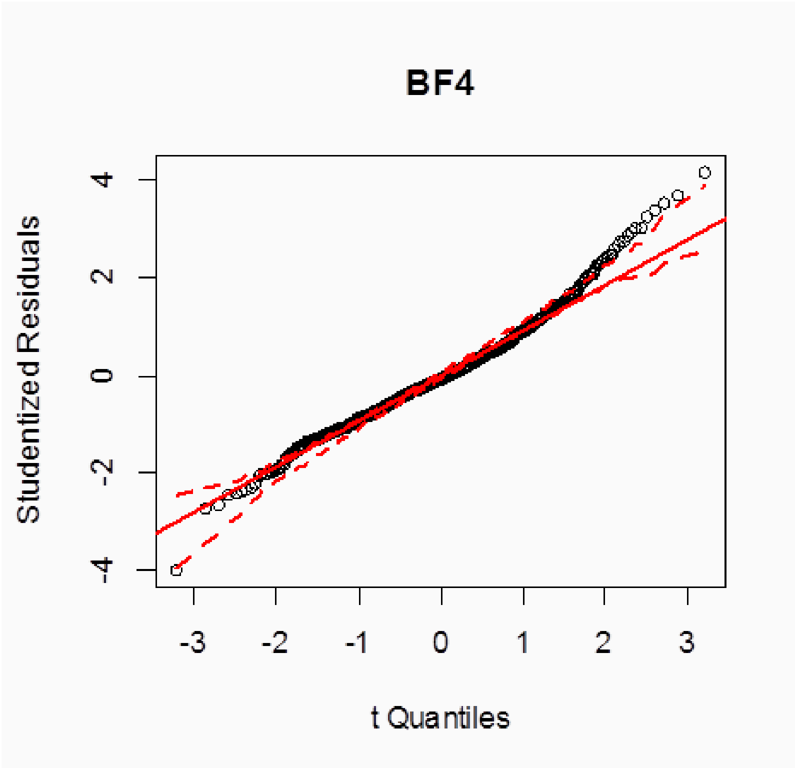




Figure B.5. Q-Q plot and studentized residuals of fitted CV6.



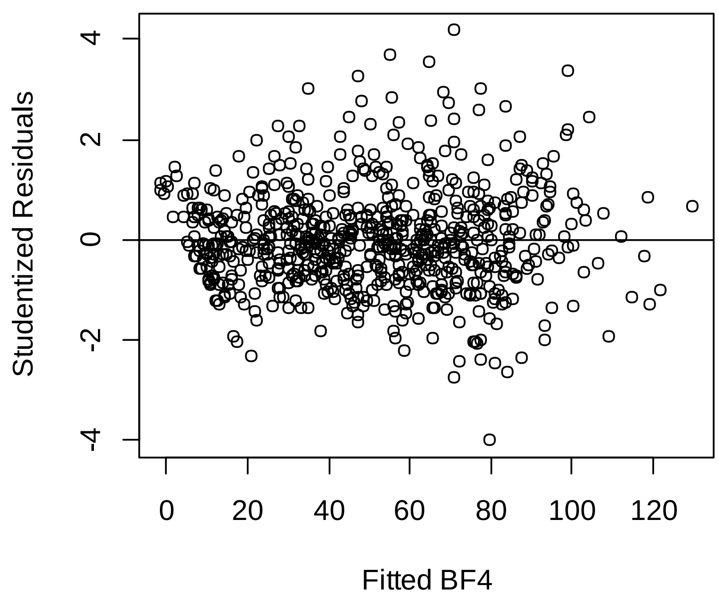
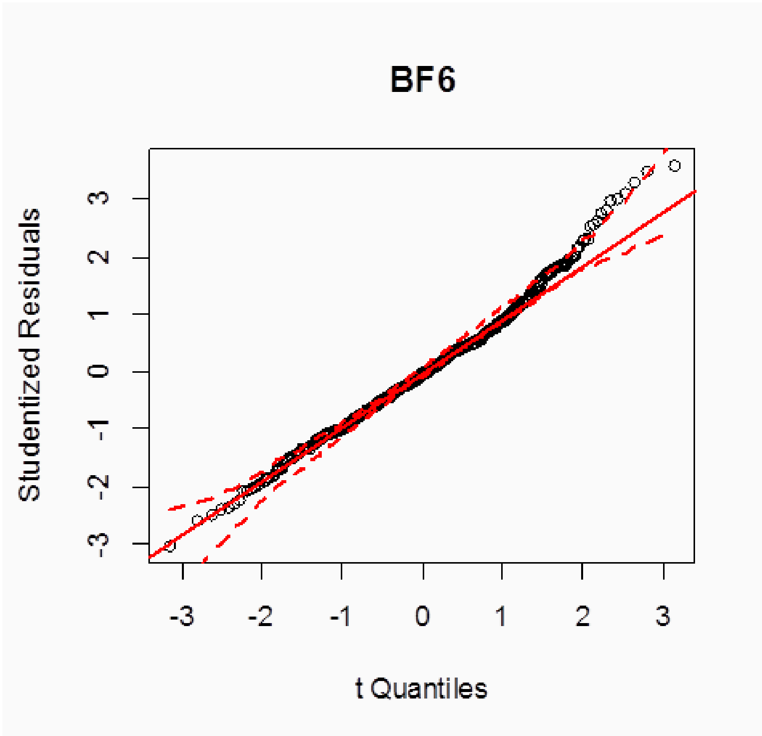




Figure B.6. Q-Q plot and studentized residuals of fitted BF4.



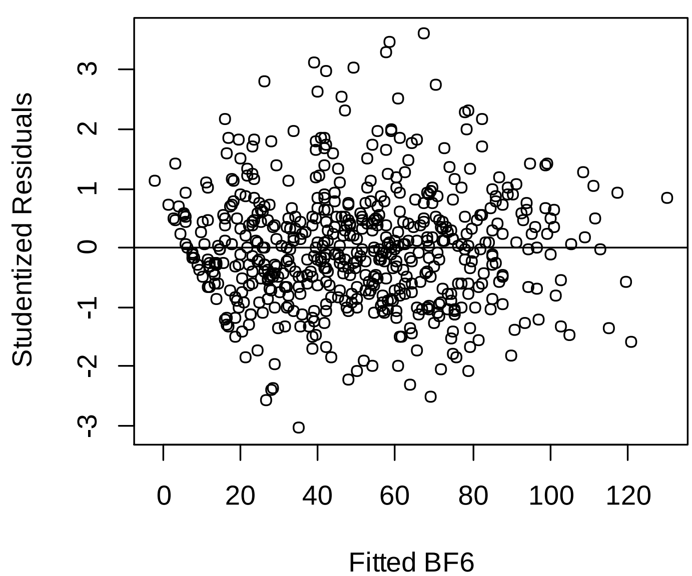




Figure B.7. Q-Q plot and studentized residuals of fitted BF6.

# APPENDIX C: Yield Tables and Trends

Note: Solid lines in plots are unthinned yield and dotted are thinned.

Table C.1. BA yield by age and TPA.

BA ($ft^2$ /acre)

| Average conditions | | | | |
|---|---|---|---|---|
| SI | THIN | LAT | LONG | ELEVATION |
| 137 | 0 | 46 | 123 | 1180 |

| | Unthinned | | | | Thinned | | | % Gain | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Density at establishment | | | | Density at establishment | | | Density at establishment | | |
| Age | 100 | 300 | 500 | 700 | 300 | 500 | 700 | 300 | 500 | 700 |
| 5 | 3.63 | 1.91 | | | | | | | | |
| 9 | 9.14 | 13.81 | 13.68 | | | | | | | |
| 13 | 22.09 | 38.33 | 52.28 | 43.59 | | | | | | |
| 17 | 43.11 | 72.76 | 94.26 | 87.47 | | | | | | |
| 21 | 72.07 | 109.00 | 124.58 | 120.41 | 107.81 | 104.69 | | -1 | -16 | |
| 25 | 100.85 | 139.15 | 145.08 | 143.38 | 147.25 | 125.82 | 116.14 | 6 | -13 | -19 |
| 29 | 121.38 | 157.64 | 161.11 | 163.02 | 178.87 | 141.63 | 132.86 | 13 | -12 | -18 |
| 33 | 127.18 | 161.03 | 173.02 | 179.08 | | 152.68 | 146.60 | | -12 | -18 |

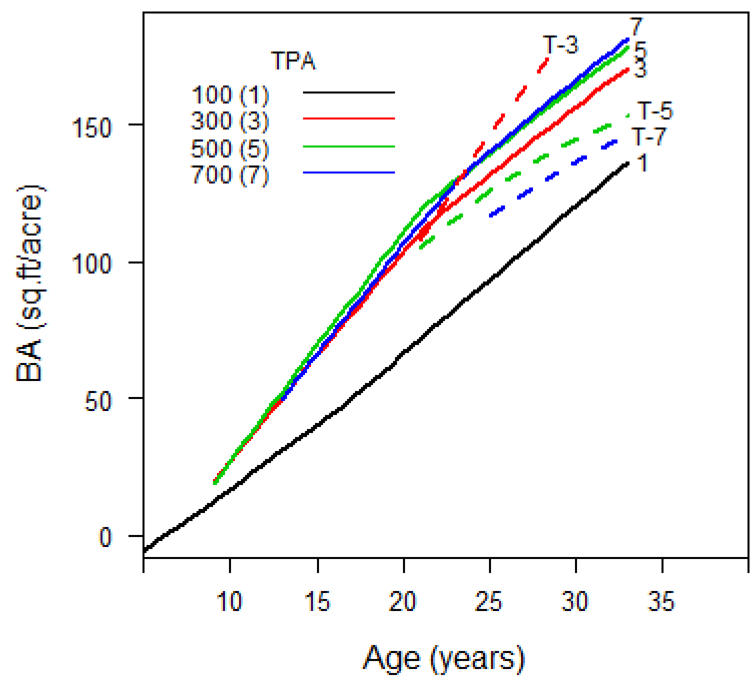



Figure C.1. BA by age for each unthinned (100 (1) – 700 (7)) and thinned (T-3, T-5, T-7) TPA class.

Table C.2. QMD yield by age and TPA.

QMD (inch)

| Average conditions | | | | |
|---|---|---|---|---|
| SI | THIN | LAT | LONG | ELEVATION |
| 137 | 0 | 46 | 123 | 1180 |

| Unthinned | | | | | Thinned | | | % Gain | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Density at establishment | | | | Density at establishment | | | Density at establishment | | |
| Age | 100 | 300 | 500 | 700 | 300 | 500 | 700 | 300 | 500 | 700 |
| 5 | 2.90 | 1.35 | | | | | | | | |
| 9 | 4.20 | 3.02 | 2.23 | | | | | | | |
| 13 | 6.19 | 5.24 | 4.71 | 4.14 | | | | | | |
| 17 | 8.57 | 7.59 | 6.73 | 6.07 | | | | | | |
| 21 | 11.12 | 9.57 | 8.02 | 7.27 | 10.96 | 8.46 | | 14 | 6 | |
| 25 | 13.33 | 11.01 | 8.95 | 8.11 | 12.74 | 9.77 | 8.80 | 16 | 9 | 9 |
| 29 | 15.06 | 12.05 | 9.78 | 8.74 | 13.89 | 11.01 | 10.19 | 15 | 13 | 17 |
| 33 | 16.50 | 13.13 | 10.68 | 9.31 | | 12.38 | 11.74 | | 16 | 26 |

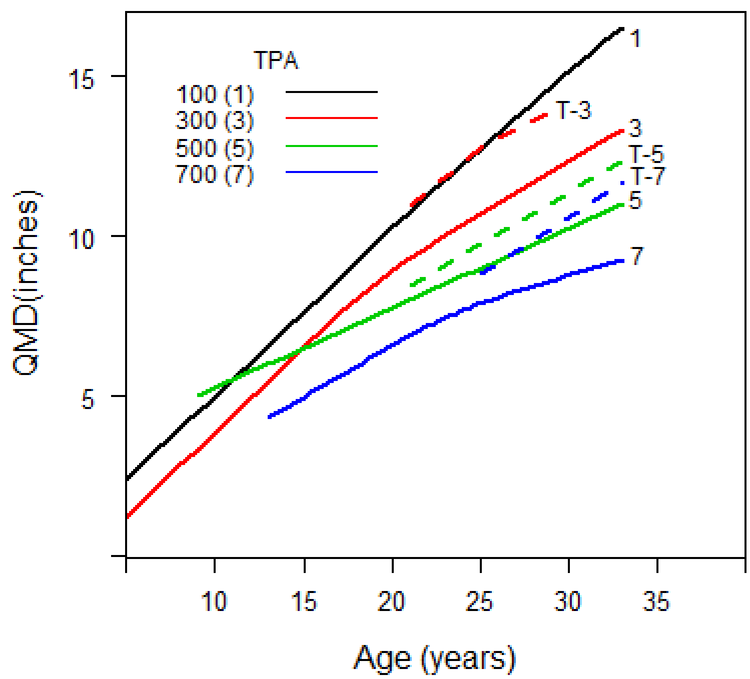



Figure C.2. QMD by age for each unthinned (100 (1) – 700 (7)) and thinned (T-3, T-5, T-7) TPA class.

Table C.3. CVT yield by age and TPA.

CVT (ft$^3$/acre)

| Average conditions | | | | |
|---|---|---|---|---|
| SI | THIN | LAT | LONG | ELEVATION |
| 137 | 0 | 46 | 123 | 1180 |

| | Unthinned | | | | Thinned | | | % Gain | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Density at establishment | | | | Density at establishment | | | Density at establishment | | |
| Age | 100 | 300 | 500 | 700 | 300 | 500 | 700 | 300 | 500 | 700 |
| 5 | 103.45 | 92.55 | | | | | | | | |
| 9 | 185.74 | 166.61 | 287.83 | | | | | | | |
| 13 | 357.03 | 304.12 | 331.91 | | | | | | | |
| 17 | 654.51 | 537.02 | 449.64 | 579.39 | | | | | | |
| 21 | 1130.46 | 889.15 | 654.52 | 651.89 | 1290.88 | 796.51 | | 31 | 18 | |
| 25 | 1773.09 | 1365.81 | 958.49 | 848.58 | 1904.48 | 1119.81 | 982.66 | 28 | 14 | 14 |
| 29 | 2523.39 | 1945.43 | 1381.96 | 1149.09 | 2686.52 | 1567.81 | 1337.56 | 28 | 12 | 14 |
| 33 | 3277.61 | 2586.41 | 1952.26 | 1587.51 | | 2168.20 | 1847.62 | | 10 | 14 |

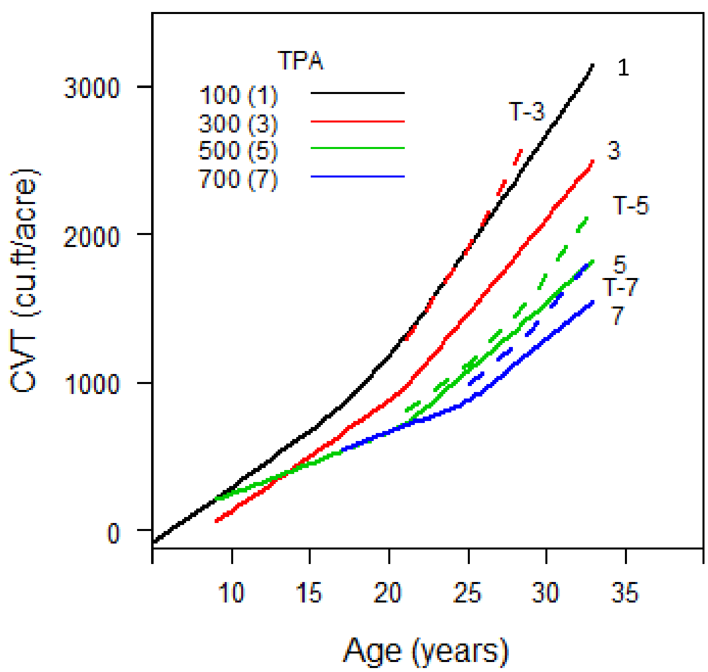



Figure C.3. CVT by age for each unthinned (100 (1) – 700 (7)) and thinned (T-3, T-5, T-7) TPA class.

Table C.4. CV4 yield by age and TPA.

CV4 (ft$^3$ /acre)

| Average conditions | | | | |
|---|---|---|---|---|
| SI | THIN | LAT | LONG | ELEVATION |
| 137 | 0 | 46 | 123 | 1186 |

Unthinned

| Age | Density at establishment 100 | 300 | 500 | 700 |
|---|---|---|---|---|
| 5 | | | | |
| 9 | | | | |
| 13 | 151.94 | 71.76 | 22.21 | |
| 17 | 586.19 | 379.08 | 230.47 | 194.93 |
| 21 | 1187.02 | 827.55 | 529.61 | 502.29 |
| 25 | 1949.99 | 1335.58 | 890.18 | 858.93 |
| 29 | 2685.99 | 1832.16 | 1315.44 | 1265.23 |
| 33 | 3342.93 | 2380.87 | 1800.43 | 1687.71 |

Thinned

| Age | Density at establishment 300 | 500 | 700 |
|---|---|---|---|
| 5 | | | |
| 9 | | | |
| 13 | | | |
| 17 | | | |
| 21 | 1279.78 | 592.43 | |
| 25 | 1984.88 | 992.95 | 812.41 |
| 29 | 2556.72 | 1438.23 | 1321.81 |
| 33 | | 1969.33 | 1972.70 |

% Gain

| Age | Density at establishment 300 | 500 | 700 |
|---|---|---|---|
| 5 | | | |
| 9 | | | |
| 13 | | | |
| 17 | | | |
| 21 | 55 | 12 | |
| 25 | 49 | 12 | -5 |
| 29 | 40 | 9 | 4 |
| 33 | | 9 | 17 |

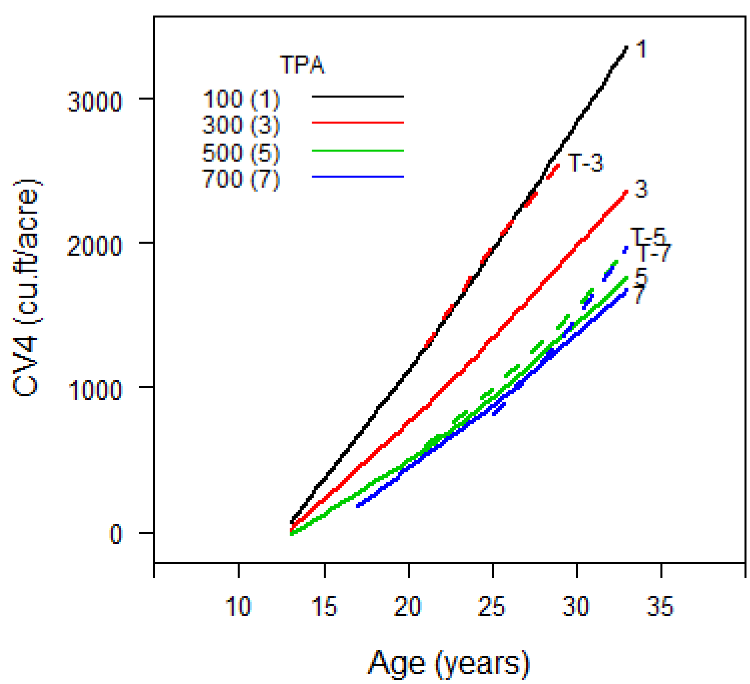



Figure C.4. CV4 by age for each unthinned (100 (1) – 700 (7)) and thinned (T-3, T-5, T-7) TPA class.

Table C.5. CV6 yield by age and TPA.

CV6 (ft$^3$ /acre)

| Average conditions | | | | |
|---|---|---|---|---|
| SI | THIN | LAT | LONG | ELEVATION |
| 137 | 0 | 46 | 123 | 1180 |

Unthinned

| | Density at establishment | | | |
|---|---|---|---|---|
| Age | 100 | 300 | 500 | 700 |
| 5 | | | | |
| 9 | | | | |
| 13 | | | | |
| 17 | 42.74 | 0.27 | 32.33 | 17.46 |
| 21 | 437.44 | 148.50 | 27.40 | 37.69 |
| 25 | 1058.83 | 474.32 | 190.42 | 214.32 |
| 29 | 1692.90 | 848.26 | 449.86 | 473.20 |
| 33 | 2242.21 | 1243.84 | 789.30 | 786.02 |

Thinned

| Density at establishment | | |
|---|---|---|
| 300 | 500 | 700 |
| | | |
| | | |
| | | |
| | | |
| 250.59 | 0.97 | |
| 584.31 | 65.63 | 27.33 |
| 868.56 | 199.74 | 157.57 |
| | 392.40 | 387.53 |

% Gain

| Density at establishment | | |
|---|---|---|
| 300 | 500 | 700 |
| | | |
| | | |
| | | |
| | | |
| 69 | -96 | |
| 23 | -66 | -87 |
| 2 | -56 | -67 |
| | -50 | -51 |

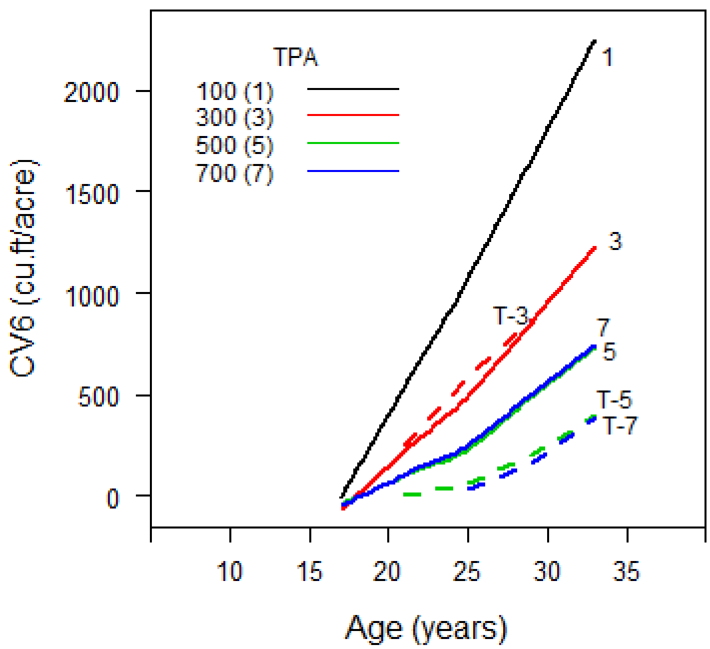



Figure C.5. CV6 by age for each unthinned (100 (1) – 700 (7)) and thinned (T-3, T-5, T-7) TPA class.

Table C.6. BF4 yield by age and TPA.

BF4 (Scribner bd ft/acre)

| Average conditions | | | | |
|---|---|---|---|---|
| SI | THIN | LAT | LONG | ELEVATION |
| 137 | 0 | 46 | 123 | 1187 |

| | Unthinned | | | | Thinned | | | % Gain | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Density at establishment | | | | Density at establishment | | | Density at establishment | | |
| Age | 100 | 300 | 500 | 700 | 300 | 500 | 700 | 300 | 500 | 700 |
| 5 | | | | | | | | | | |
| 9 | | | | | | | | | | |
| 13 | | | | | | | | | | |
| 17 | 1174 | 847 | 594 | 480 | | | | | | |
| 21 | 3136 | 2440 | 1700 | 1469 | 4014 | 2126 | | 64 | 25 | |
| 25 | 5620 | 4352 | 3089 | 2797 | 6286 | 3578 | 2999 | 44 | 16 | 7 |
| 29 | 8223 | 6364 | 4826 | 4444 | 8442 | 5349 | 4914 | 33 | 11 | 11 |
| 33 | 10821 | 8621 | 7031 | 6474 | | 7603 | 7470 | | 8 | 15 |

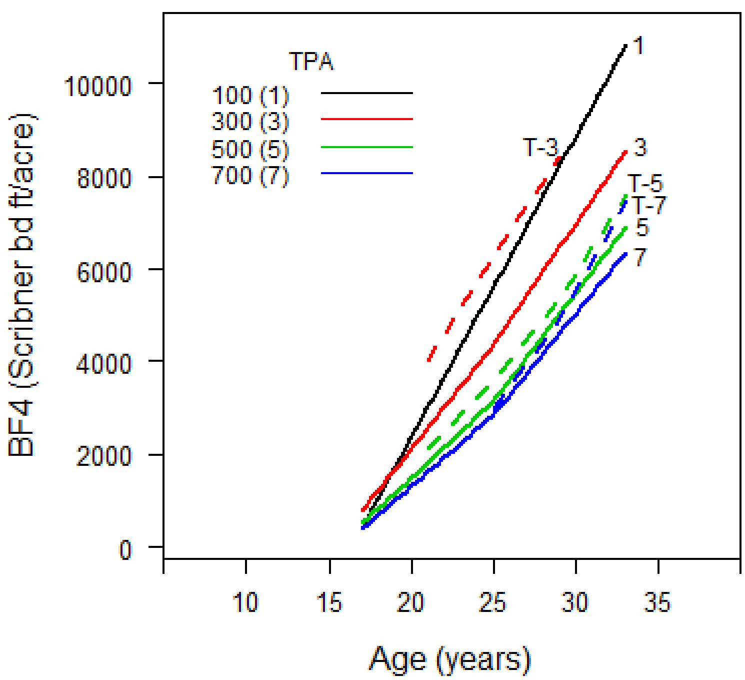



Figure C.6. BF4 by age for each unthinned (100 (1) – 700 (7)) and thinned (T-3, T-5, T-7) TPA class.

Table C.7. BF6 yield by age and TPA.

BF6 (Scribner bd ft/acre)

| Average conditions | | | | |
|---|---|---|---|---|
| SI | THIN | LAT | LONG | ELEVATION |
| 137 | 0 | 46 | 123 | 1180 |

Unthinned

| | Density at establishment | | | |
|---|---|---|---|---|
| Age | 100 | 300 | 500 | 700 |
| 5 | | | | |
| 9 | | | | |
| 13 | | | | |
| 17 | 1018 | 428 | 209 | 290 |
| 21 | 3533 | 2124 | 1224 | 1328 |
| 25 | 6795 | 4367 | 2709 | 2942 |
| 29 | 10117 | 6647 | 4590 | 5009 |
| 33 | 12812 | 8901 | 6767 | 7422 |

Thinned

| Density at establishment | | |
|---|---|---|
| 300 | 500 | 700 |
| | | |
| | | |
| | | |
| | | |
| 4038 | 1428 | |
| 7331 | 3218 | 2550 |
| 10546 | 5459 | 4816 |
| | 8169 | 7750 |

% Gain

| Density at establishment | | |
|---|---|---|
| 300 | 500 | 700 |
| | | |
| | | |
| | | |
| | | |
| 90 | 17 | |
| 68 | 19 | -13 |
| 59 | 19 | -4 |
| | 21 | 4 |

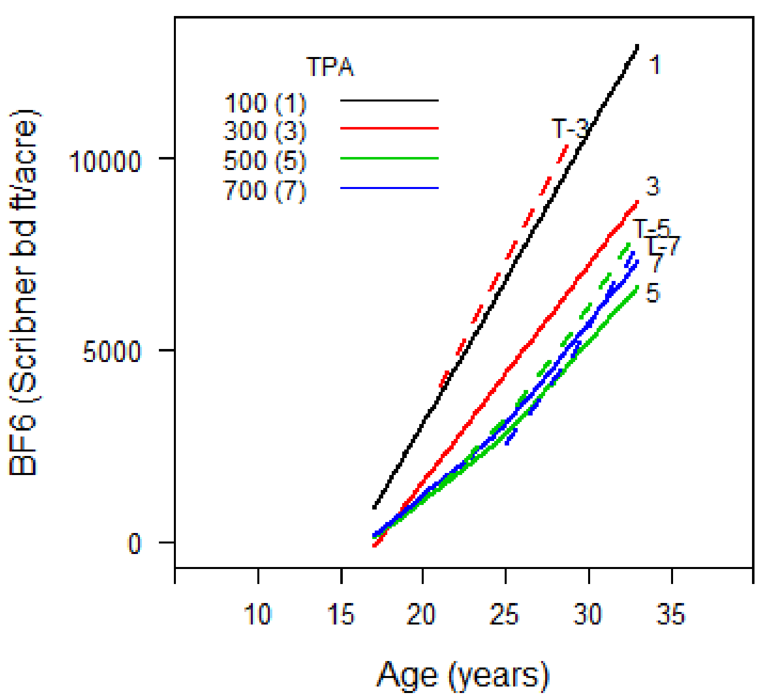



Figure C.7. BF6 by age for each unthinned (100 (1) – 700 (7)) and thinned (T-3, T-5, T-7) TPA class.