

Effects of near wall vortex perturbation in an array on the induced velocity and pressure

Aditya Sridhar

A thesis submitted in partial fulfillment
of the requirements for the degree of

Master of Science

in

Aeronautics and Astronautics

University of Washington

2016

Committee members:

James C. Hermanson

Carl Knowlen

Program Authorized

To offer Degree

Aeronautics and Astronautics

©Copyright [2016]

Aditya Sridhar

University of Washington

Abstract

Effects of near wall vortex perturbation in an array on the induced velocity and pressure

Aditya Sridhar

Chair of the Supervisory Committee:

Prof. James C. Hermanson

William E. Boeing Department of Aeronautics and Astronautics

When stationary streamwise vortices are introduced in a turbulent boundary layer, previous flow visualization studies revealed that the flow can be partially relaminarized. To study this phenomenon, and the wall pressure signature of the vortices for detecting unstable departures from equilibrium in active flow control systems, it is important to understand the behavior of vortices near the wall, as well as their locally-induced velocity and pressure gradients. A relatively simple potential flow field with multiple point vortices is calculated, using the method of images, where for each vortex, a virtual vortex of equal strength and distance from the wall, but with opposite rotation, was introduced in the domain.

The induced velocity and coefficient of pressure on the wall are sinusoidal and dependent on the vortex position and the array configuration; i.e., the span of the vortices (i.e., the distance between the vortex cores) and the height of the vortices from the wall. The effects of center-pair vortex perturbation in different directions on the induced velocity and pressure coefficient on the wall was also investigated.

Table of Contents

		Page no.
List of Figures		6
Chapter 1:	Introduction	
1.1	Background	9
1.2	Theory	9
Chapter 2:	Problem solving approach	
2.1	Algorithm and flowchart of the code composed in MATLAB	11
2.2	Perturbation distance effects	18
Chapter 3:	Results	21
Chapter 4:	Analysis and Discussions	44
Chapter 5:	Conclusions, future study and applications	
5.1	Conclusions	48
5.2	Potential applications	48
5.3	Future study	49
References		50
Appendix	MATLAB codes	51

List of tables

Table no.	Label	Page no.
Table 2.1	List of symbols used in the code	11
Table 3.1	Listing of all cases considered	21

List of figures

Figure number		Label	Page no.
Figure	1	Vortex array and its image	11
Figure	2.1	A representation of nine-pair vortex array and its image in a stationary arrangement	14
Figure	2.2	Induced velocity normalized with respect to the height	15
Figure	2.3	Induced velocity normalized with respect to the span	15
Figure	2.4	Coefficient of pressure normalized with respect to height	17
Figure	2.5	Coefficient of pressure normalized with respect to span	17
Figure	2.6	Induced velocities on the wall with different perturbation values	19
Figure	2.7	Coefficient of pressure on the wall with different perturbation values	20
Figure	2.8	Induced horizontal velocity values v/s the perturbation distance	21
Figure	2.9	Coefficient of pressure values v/s the perturbation distance	21
Figure	3.1	No perturbation case (case 1)	24
Figure	3.2	Induced velocity normalized with <i>height</i> (no perturbation)	25
Figure	3.3	Induced velocity normalized with <i>span</i> (no perturbation)	25
Figure	3.4	Coefficient of pressure normalized with <i>height</i> (no perturbation)	26
Figure	3.5	Coefficient of pressure normalized with <i>span</i> (no perturbation)	26
Figure	3.6	Perturbation towards east case (case 2)	27
Figure	3.7	Induced velocity normalized with <i>height</i> (perturbed east)	28
Figure	3.8	Induced velocity normalized with <i>span</i> (perturbed east)	28
Figure	3.9	Coefficient of pressure normalized with <i>height</i> (perturbed east)	29
Figure	3.1	Coefficient of pressure normalized with <i>span</i> (perturbed east)	29
Figure	3.11	Perturbation towards west case (case 3)	30
Figure	3.12	Induced velocity normalized with <i>height</i> (perturbed west)	31
Figure	3.13	Induced velocity normalized with <i>span</i> (perturbed west)	31
Figure	3.14	Coefficient of pressure normalized with <i>height</i> (perturbed west)	32
Figure	3.15	Coefficient of pressure normalized with <i>span</i> (perturbed west)	32
Figure	3.16	Perturbation towards north case (case 4)	33
Figure	3.17	Induced velocity normalized with <i>height</i> (perturbed north)	34
Figure	3.18	Induced velocity normalized with <i>span</i> (perturbed north)	34
Figure	3.19	Coefficient of pressure normalized with <i>height</i> (perturbed north)	35
Figure	3.20	Coefficient of pressure normalized with <i>span</i> (perturbed north)	35
Figure	3.21	Perturbation towards north-east case (case 5)	36

Figure	3.22	Induced velocity normalized with <i>height</i> (perturbed north-east)	37
Figure	3.23	Induced velocity normalized with <i>span</i> (perturbed north-east)	37
Figure	3.24	Coefficient of pressure normalized with <i>height</i> (perturbed north-east)	38
Figure	3.25	Coefficient of pressure normalized with <i>span</i> (perturbed north-east)	38
Figure	3.26	Perturbation towards south case (case 6)	39
Figure	3.27	Induced velocity normalized with <i>height</i> (perturbed south)	40
Figure	3.28	Induced velocity normalized with <i>span</i> (perturbed south)	40
Figure	3.29	Coefficient of pressure normalized with <i>height</i> (perturbed south)	41
Figure	3.3	Coefficient of pressure normalized with <i>span</i> (perturbed south)	41
Figure	3.31	Perturbation towards south-east case (case 7)	42
Figure	3.32	Induced velocity normalized with <i>height</i> (perturbed south-east)	43
Figure	3.33	Induced velocity normalized with <i>span</i> (perturbed south-east)	43
Figure	3.34	Coefficient of pressure normalized with <i>height</i> (perturbed south)	44
Figure	3.35	Coefficient of pressure normalized with <i>span</i> (perturbed south)	44
Figure	4.1	Induced velocity plot (normalized with <i>height</i>) for all cases with $h/s = 1/2$	45
Figure	4.2	Coefficient of pressure (normalized with <i>height</i>) for all cases with $h/s = 1/2$	46
Figure	4.3	Induced velocity plot (normalized with <i>height</i>) for all cases with $h/s = 1/8$	47
Figure	4.4	Coefficient of pressure (normalized with <i>height</i>) for all cases with $h/s = 1/8$	47
Figure	4.5	Close up of Induced velocity plot for all cases with $h/s = 1/8$	48
Figure	4.6	Close up of coefficient of pressure plot for all cases with $h/s = 1/8$	48
Figure	5.1	A schematic section of a Dielectric-barrier-discharge (DBD) plasma actuator	49

ACKNOWLEDGEMENTS

I wish to thank my advisor, Professor Robert E. Breidenthal for his foresight, availability, and patience while I was working on this thesis. I am also very grateful for his assistance, guidance and encouragement throughout the two years of my M.S studies. The success of the thesis was largely due to the expertise on this flow control and stationary vortices passed on to me by Dr. Giovanni Nino. I greatly appreciate his optimistic outlook. The frequent meetings and discussions helped me to develop a deeper understanding of the research topic.

I would also like to thank Prof. James C. Hermanson and Prof. Carl Knowlen who are members of my thesis supervisory committee. Their inputs on the research and this paper have been valuable.

I owe special thanks to several of my fellow graduate students who assisted me and helped me through my research. I am grateful to Kaifu Hinkleman, my fellow colleague in our research group, for sharing his knowledge, time and work on this topic with me.

I would also like to thank the faculty, students and staff of the Department of Aeronautics and Astronautics, University of Washington for providing a stimulating environment and, invaluable knowledge and support.

Chapter 1

Introduction

1.1 Background

Experiments and research performed by Balle and Cotel [1,2,3,4,5] have shown that the entrainment rate of large scale vortices on a body depends on if they are stationary. They define the stationarity of the vortex as the ratio of the rotational and translational velocities of the vortex. Dawson, Bauer and Breidenthal [4,5] further conducted experiments to show the impact of stationary large-scale vortices on the relaminarization of turbulent boundary layer using flow visualization. The experiments consisted of studying stationary vortices in a flow over a wavy wall, where the vortices were held stationary in the troughs due to the wavy nature of the wall. The experiments revealed that the flow resembled Kelvin's cat's eyes flow, which also proved the relaminarization capability of the stationary vortices.

To implement this relaminarization technique over a flat plate instead of a wavy wall and to maintain the stationary vortices stationarity, the introduction of a uniform flow in opposing direction of the vortex's transverse motion would be required. The purpose of this research is to investigate the induced velocity due to the vortices on a flat wall and the corresponding pressure signatures on the wall. This research also aims at investigating the effects of one pair of vortex being perturbed in different directions and the perturbation effects on the induced velocity and pressure coefficient.

1.2 Theory

To solve the problem of vortex near a wall this research used the method of images. This method is used to find solutions to solve Laplace's equation, which satisfy the boundary condition at the wall or plane of symmetry.

The flow field created by singularities in the presence of solid boundaries or walls was simulated using the method of images. An image vortex with the identical strength but opposite sign as the original vortex was created and was positioned on the opposite side of the wall. The distance of both the vortices from the wall was the same.

Fig.1 is a sketch showing a vortex array and its image vortex configuration.

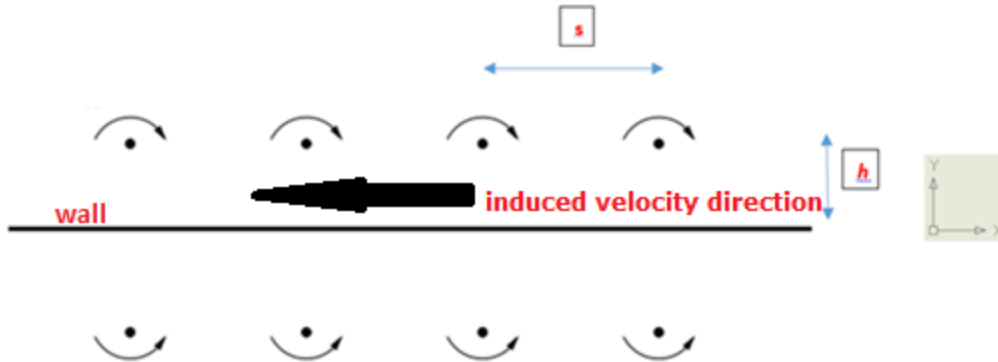


Figure 1: A sketch of a vortex array and its image placed on both sides of the wall

For an isolated pair of vortices, the pair moves parallel to the wall with velocity given by

$$\Gamma/4\pi h,$$

Where Γ the vortex circulation and h is the height above the wall.

The image system is a doubly infinite row and complex variables as below are introduced as

$$w(z) = \Phi + i\Psi \quad .. (1)$$

$$z = x + iy$$

where, $w = w(z)$ is the complex potential.

For an array of vortices the complex potential equation [6]:

$$w = -\frac{i\Gamma}{2\pi} \log \frac{z-i(s-h)}{z-ih} + W(z) \quad ..(2)$$

Where, s is the span or horizontal distance between the vortices in the array.

Differentiating Eq. (2) with respect to dz gives:

$$\frac{dw}{dz} = -\frac{\Gamma}{2\pi} \left[\frac{s-2h}{(z-is+h)(z-ih)} \right] \quad ..(3)$$

Where, $u - iv = dw/dz$, u is the horizontal velocity i.e. velocity parallel to the x -axis as indicated in Fig. 1, and v is the vertical velocity, i.e., the velocity parallel to the y -axis as indicated in Fig. 1.

Chapter 2

Problem solving approach

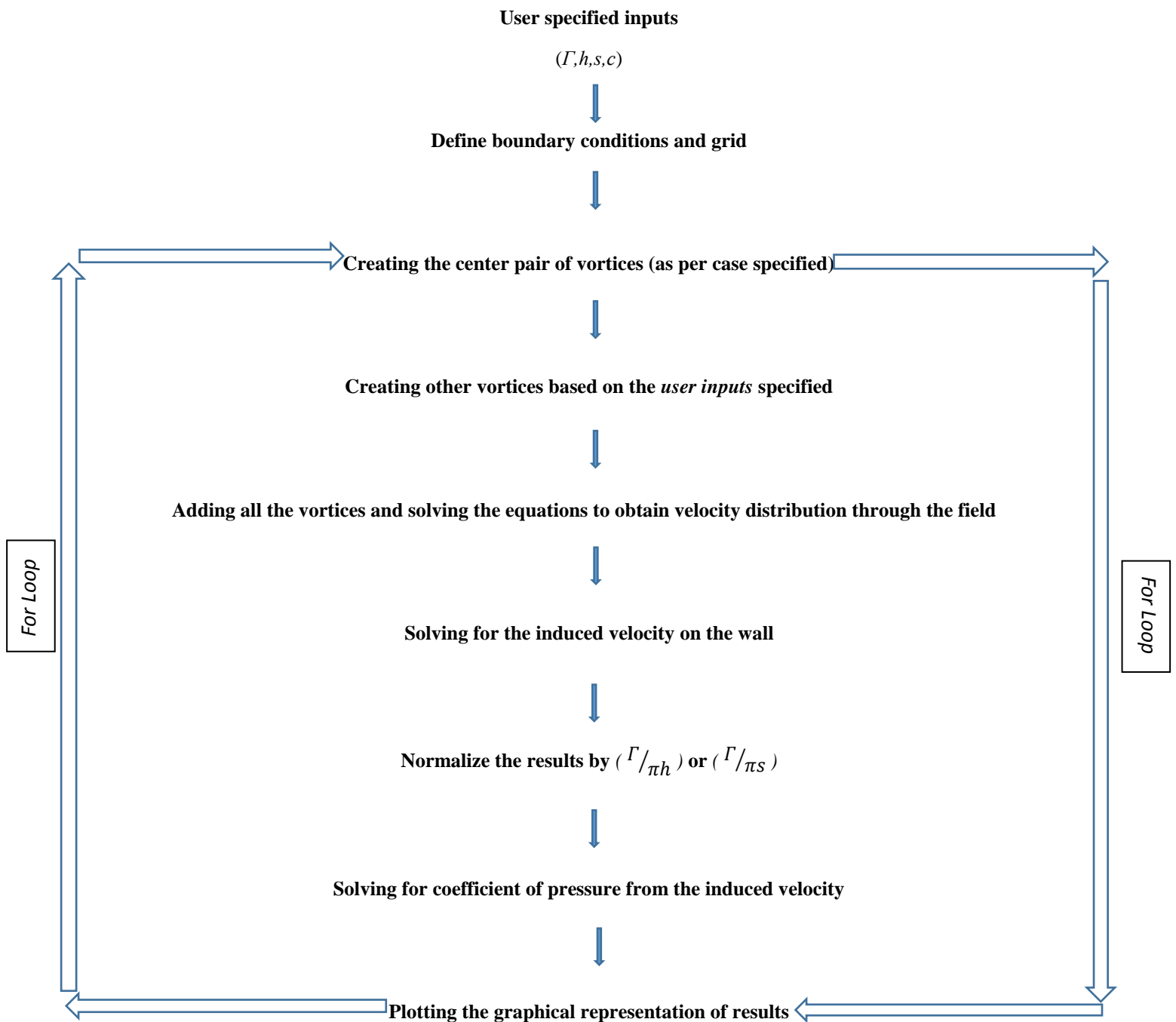
The code is modified for generating pairs of counter-rotating vortices. The code contains user-input data comprising of parameters including the strength of vortex circulation (Γ), the horizontal separation between vortex arrays (s), the number of vortex pairs (c), the vertical distance between vortex pairs (h).

2.1 Algorithm and flowchart of the code composed in MATLAB

Table 2.1 List of symbols used in the code

Variable	Parameter
Γ	Strength of vortex
H	Height of the vortices from wall
S	Span or distance between the adjacent vortices
C	Number of adjacent vortex pairs

Code Algorithm flowchart:



To understand the behavior of the vortices and the effects of induced velocity on the wall due to the array configuration, the problem is set to be solved in several steps. As shown in Fig. 2.1, all the vortices are in an unperturbed configuration, where the span s and height h between all vortices is held constant. This is an idealized configuration where the vortex motions and instabilities due to the variations in induced velocities when any pair is displaced is neglected.

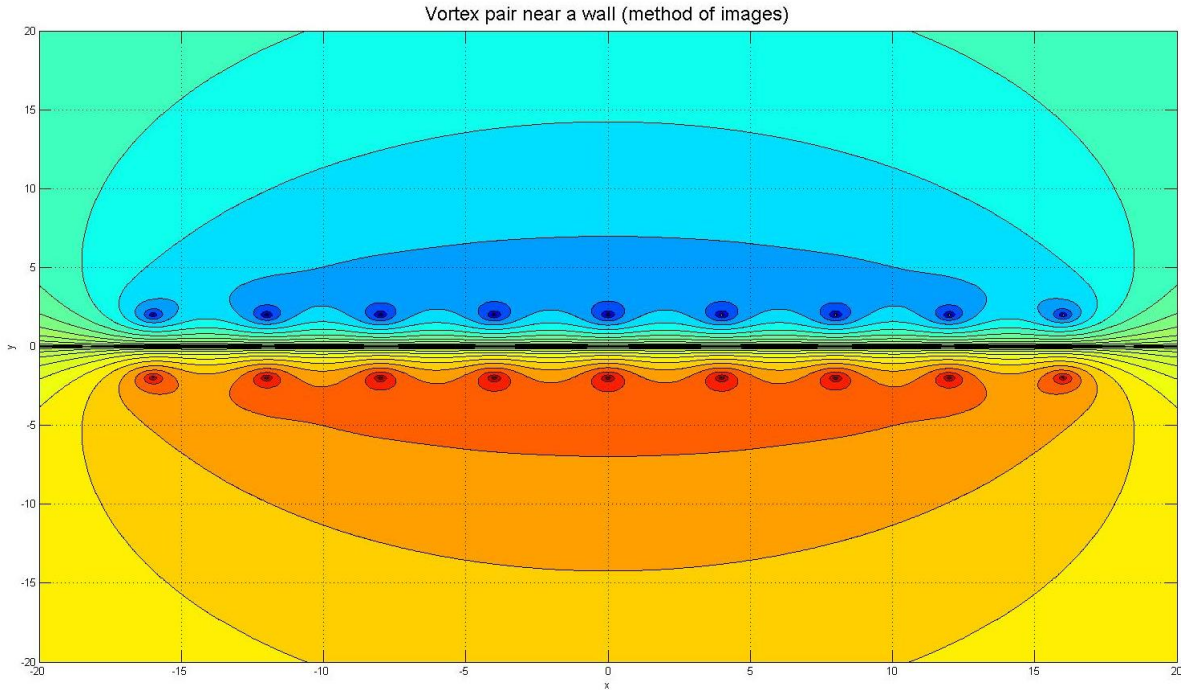


Figure 2.1 A representation of nine-pair vortex array and its image in a stationary arrangement. All the vortices are at their fixed positions. All vortices above the wall (blue region) counter-clockwise rotation while the image array (red) have clockwise rotation sense.

Once the induced velocity is obtained in this configuration, the height between the vortex arrays is changed to obtain different values of the aspect ratio h/s to compare the effects of vortex position on the induced velocity. The induced velocity is normalized by either the circulation combined with either the height between the wall and vortex array ($\Gamma/\pi h$) or with the span of the vortices ($\Gamma/\pi s$).

In the limit when the height is low and the span is high, each vortex in the array only interacts with its counter-rotating image. In the other limit when the height is much greater than the span, the vortex interacts with its neighbor and sees minimal interaction with its image. Thus when the height is low, the normalization with respect to the height (h) is more appropriate. When h is of higher value, normalization with respect to the span (s) would be more appropriate. While calculating the induced velocity, viscous effects are neglected in all cases.

Shown below in Fig. 2.2 and Fig. 2.3 are the induced velocity plots for the vortex configuration shown in Fig. 2.1. The blue line indicates the induced velocity on the wall for an array in a configuration where the h/s ratio is equal to 1.2. The red line indicates the induced velocity for an array with h/s equal to 1/3. When the array configuration parameter h/s is 1/3, the height is small as compared to the span. In other words, the vortices are closer to the wall than their neighboring vortices, hence normalizing with respect to the height gives us an accurate indication of the induced velocity. When the array configuration is $h/s = 1.2$, the height is greater than the span between the vortices, thus the induced velocity is primarily due to the interaction of the adjacent vortices.

The velocity u on the wall is normalized as:

$$U = \Gamma / \pi X \quad ..(4)$$

where, X is the height (h) or the span (s)

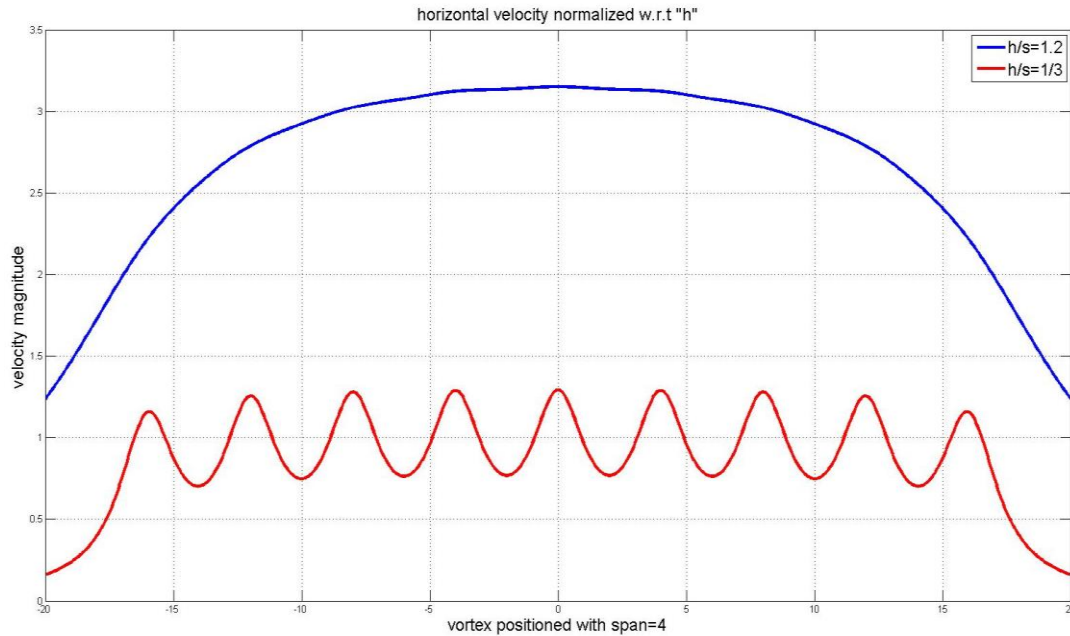


Figure 2.2: Induced velocity profile on the wall normalized with $\Gamma/\pi h$

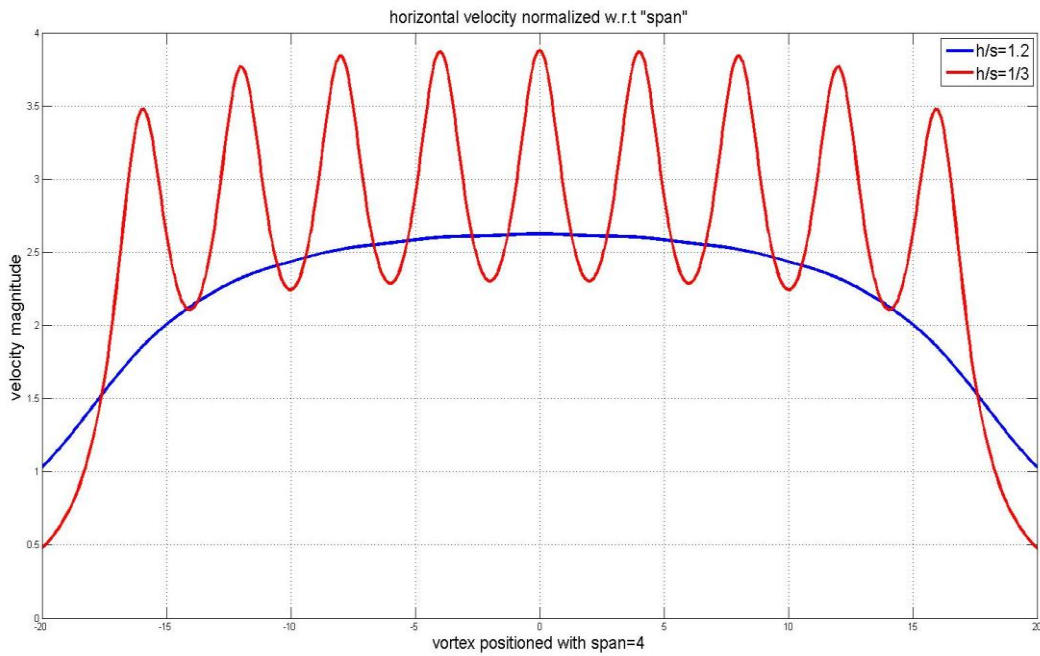


Figure 2.3: Induced velocity on the wall normalized with $\Gamma/\pi s$.

When the array configuration has the value $h/s = 1.2$, the height is greater than the span between the vortices, thus the induced velocity is primarily due to the interaction of the adjacent vortices. In other words, the vortices are far too away from the wall than their neighboring vortices, hence normalizing with respect to the span gives us an accurate indication of the induced velocity.

As can be seen in Figs. 2.2 and 2.3, increasing h/s affects the velocity distribution. At small h/s , i.e. when the vortex arrays are close to wall, the induced horizontal velocity on the wall fluctuates greatly, with the velocity just between the vortices being high and nearing zero between adjacent pairs. When h/s is large, the effect of the vortex on its image tends to decrease. The induced velocity on the wall is now mainly due to the effects of vortices adjacent to each other.

The coefficient of pressure, C_p , is defined as:

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho\left(\frac{\Gamma}{\pi X}\right)^2} \quad \dots (5)$$

Where X is the *height* or the *span* depending on the normalization method used, Γ is the vortex circulation, and P is the static pressure

The total pressure P_t is given by

$$P_t = P + q \quad \dots (6)$$

Where q is the dynamic pressure and P is static pressure.

Substituting Eq. (6) into Eq. (5), we have

$$C_p = \frac{P_t - q - P_\infty}{\frac{1}{2}\rho\left(\frac{\Gamma}{\pi X}\right)^2} \quad \dots (7)$$

$$C_p = \frac{q_\infty - q}{\frac{1}{2}\rho\left(\frac{\Gamma}{\pi X}\right)^2} \quad \dots (8)$$

where, $q_\infty = 0$

and $q = \frac{1}{2} \rho u^2$

Thus, the coefficient of pressure can be obtained by using the normalized and non-dimensional velocity U as stated in Eq. 4

$$C_p = -\left(\frac{u}{\Gamma/\pi X}\right)^2 \quad \dots (9)$$

Thus as shown in Figs. 2.4 and 2.5, the nature of the C_p curve is same as that of velocity as seen in Figs. 2.2 and 2.3, only inverted. At the places where velocities are high, the C_p will be low and vice versa.

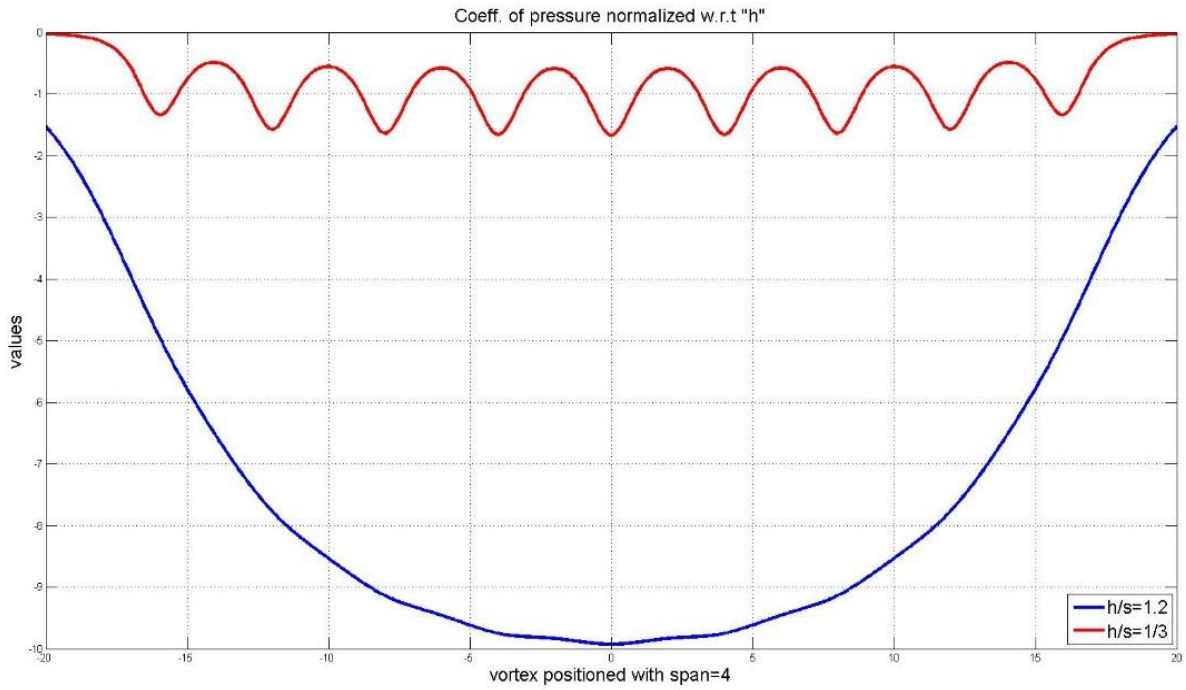


Figure 2.4: Coefficient of Pressure normalized with $\Gamma/\pi h$

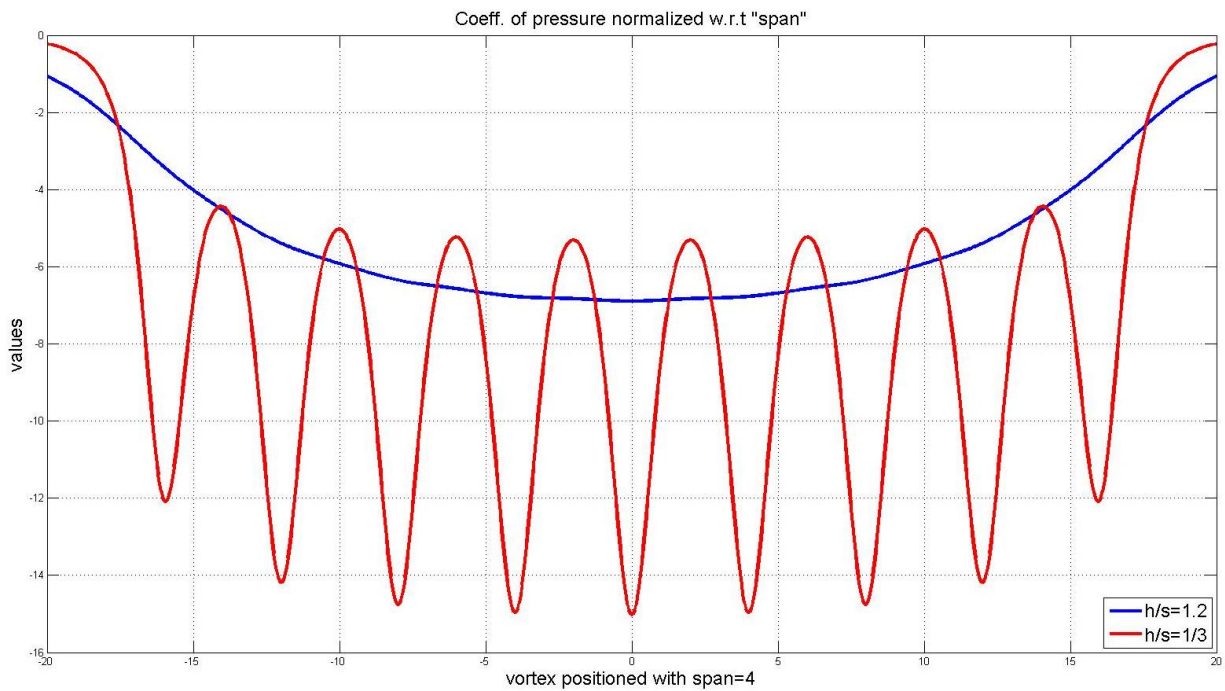


Figure 2.5: Coefficient of Pressure normalized with the $\Gamma/\pi s$

With the comparison cases now available for the steady case, the center pair of vortices is then displaced to new locations (a total of 6 cases) while keeping all the other vortices stationary. The process of varying the aspect ratio h/s is then repeated for the given case. This study is focused on the instantaneous velocity field associated with vortex displacement and does not consider any subsequent motion of the vortex cores.

2.3 Perturbation distance effects

The distance or amount by which the vortex is perturbed has significant effects on the induced velocity on the wall and the corresponding pressure coefficient. Consider the case where the center vortex in the array and its image are being moved closer to the wall, while the other vortices in the array remain fixed at their original position. The closer the vortex moves towards the wall, the higher will be the local induced velocity on the wall, and lower corresponding pressure coefficient. Fig. 2.6 and Fig. 2.7 show the effects of perturbing the vortex by perturbing the vortex location by various values.

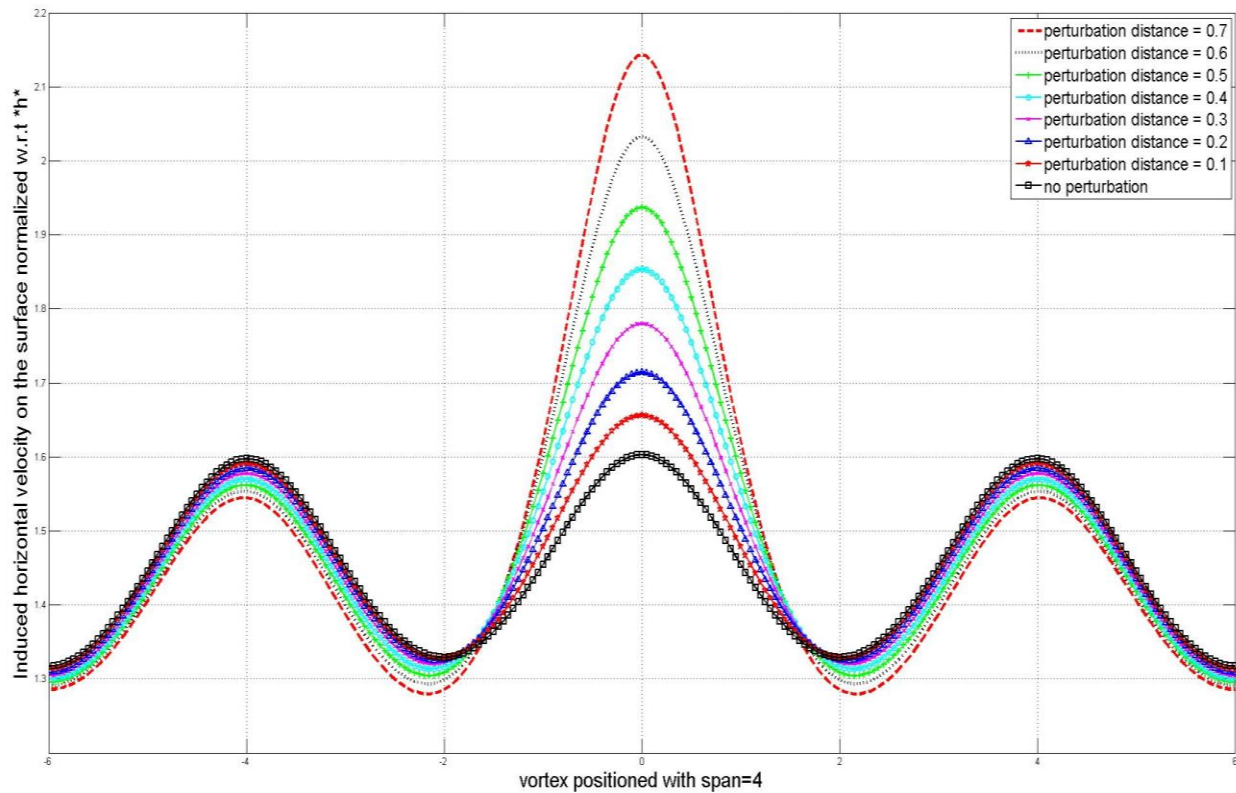


Figure 2.6: Induced velocities on the wall with different perturbation cases. The figure indicates that for the case where the vortices are moved closer to the wall, there is a higher induced velocity at a point on the wall directly between the original vortex and its image.

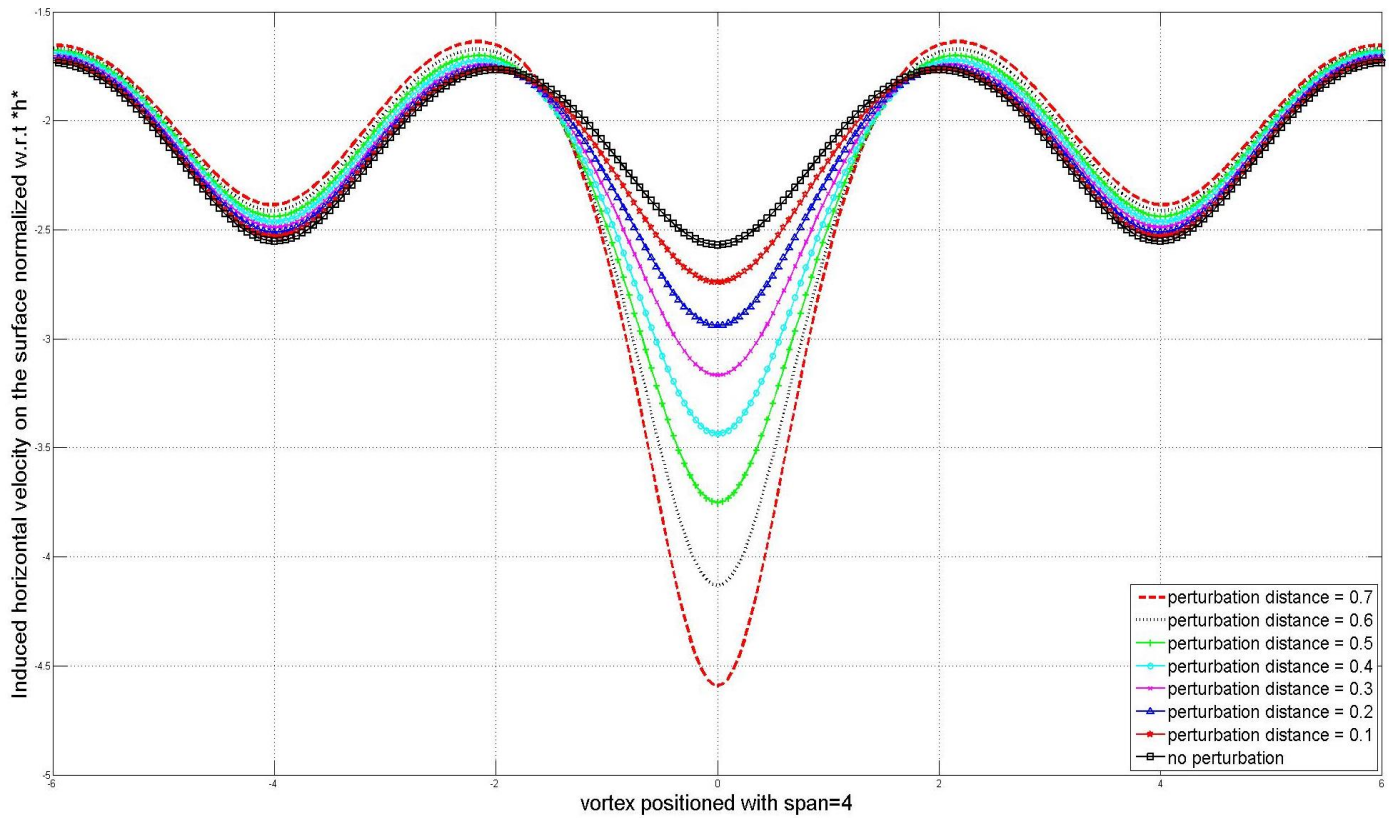


Figure 2.7: The corresponding coefficient of pressure on the wall for the perturbation cases shown in Fig. 2.6.

From the above figures, it is clear that the greater the perturbation the higher the change in induced velocity and pressure coefficient there will be. To illustrate this, consider the same case of moving both the vortex and its image closer to the wall, while keeping the rest of the vortices in a fixed position. Figures. 2.8 and 2.9 below indicate the change in the induced velocity and pressure coefficient with respect to the perturbation distance.

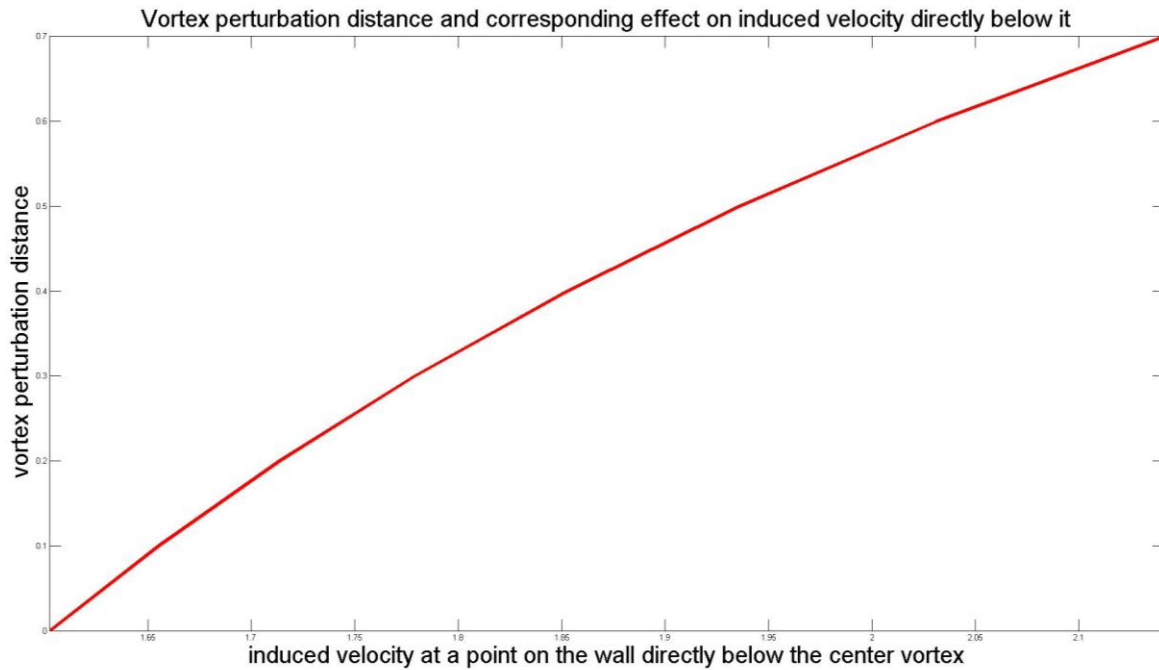


Figure 2.8: Induced velocity (normalized by $\Gamma/\pi h$) values versus the perturbation distance

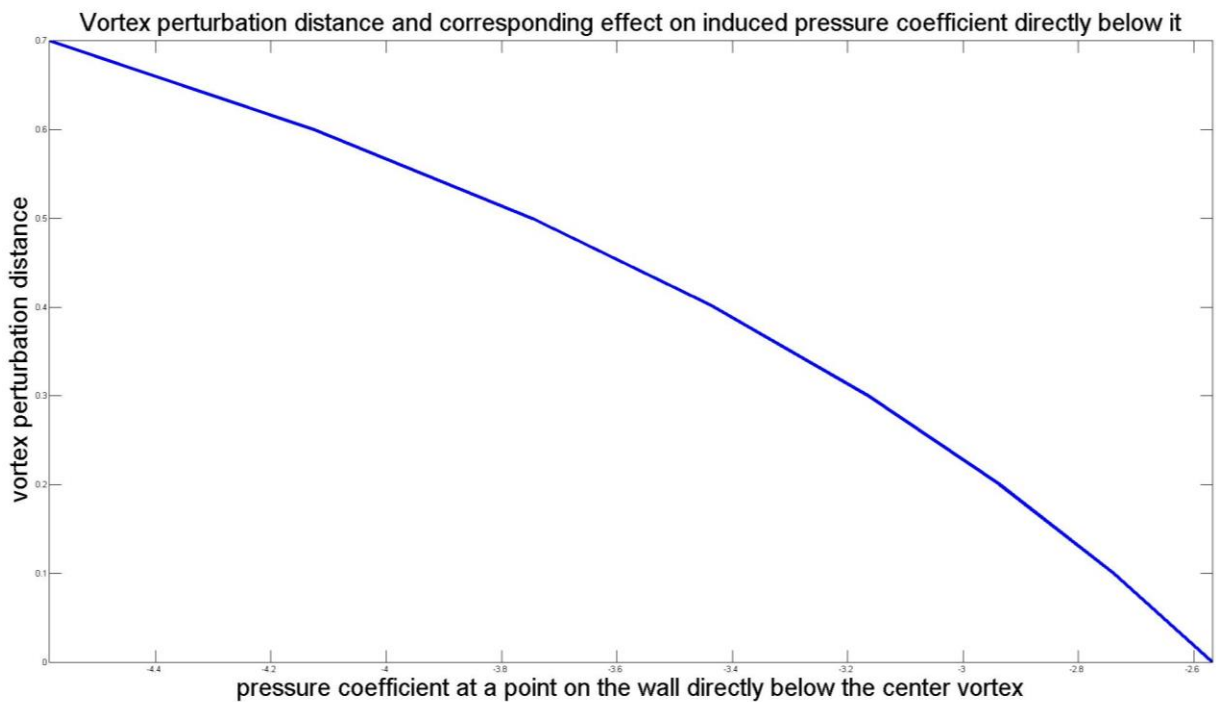


Figure 2.9: Coefficient of pressure values (normalized by $\Gamma/\pi h$) versus the perturbation distance

Chapter 3

Results

A more detailed study of perturbing the vortices with values beyond those shown in the previous chapter can be done to get a better understanding of the perturbation effects. By perturbation, it means that the vortices are displaced to a different position and direction in different cases. While designing a control system, as a part of the future studies, it may be useful to study different cases of perturbation values to determine the frequency response of the system and corresponding pressure signatures.

To understand the perturbation effects on the induced velocity and pressure on the wall when the vortex is perturbed in different directions, a total of six cases have been studied in this paper. The perturbation distance chosen arbitrarily for the center pair of vortices in all the cases has been held constant at 0.5. The strength (circulation) Γ of all the vortices is assumed to be constant. The array consists of a total of nine pairs of vortices where the fifth pair is considered as the center pair.

A detailed list of all cases and the vortex perturbation directions considered, and the corresponding effect on the induced velocity magnitude and pressure coefficient is summarized in Table 3.1 below.

Table 3.1: Listing of all cases considered

Case no.	Designation	Vortex position	Induced velocity magnitude on the wall surface	Pressure coefficient (C_p) on the wall surface
1	Center	No perturbation from regular vortex array	Symmetric baseline	Symmetric baseline
2	East	Center vortex pair moved east	Induced velocity magnitude in the region between the center vortex pair shifts east	Pressure coefficient in the region between the center vortex pair shifts east
3	West	Center vortex pair moved west	Induced velocity magnitude in the region between the center vortex pair shifts west	Pressure coefficient in the region between the center vortex pair shifts west
4	North	Center vortex pair moved away from wall (original vortex moved north, image vortex moved south)	Induced velocity magnitude in the region between the center vortex pair decreases	Pressure coefficient in the region between the center vortex pair increases

Case no.	Designation	Vortex position	Induced velocity on the plate surface	Pressure coefficient (C_p)
5	North-east	Center vortex pair moved away and east from wall (original vortex moved north-east, image vortex moved south-east)	Induced velocity magnitude in the region between the center vortex pair decreases and shifts east	Pressure coefficient in the region between the center vortex pair increases and shifts east
6	South	Center vortex pair moved closer to wall (original vortex moved south, image vortex moved north)	Induced velocity magnitude in the region between the center vortex pair increases	Pressure coefficient in the region between the center vortex pair decreases
7	South-east	Center vortex pair towards wall and east from wall (original vortex moved south-east, image vortex moved north-east)	Induced velocity magnitude in the region between the center vortex pair increases and shifts east	Pressure coefficient in the region between the center vortex pair decreases and shifts east

To summarize, the assumptions made in this analysis are as follows:

1. The entire vortex array and its image array positions are taken to be fixed, and any subsequent vortex motion is not considered;
2. The strength Γ of all the vortices is identical and constant;
3. Any viscous effects are neglected.

Case 1:

No perturbation

All the vortices in this case are regularly spaced, as shown Fig. 3.1 with the span between adjacent vortices and the height between all the vortices with respect to its image is identical.

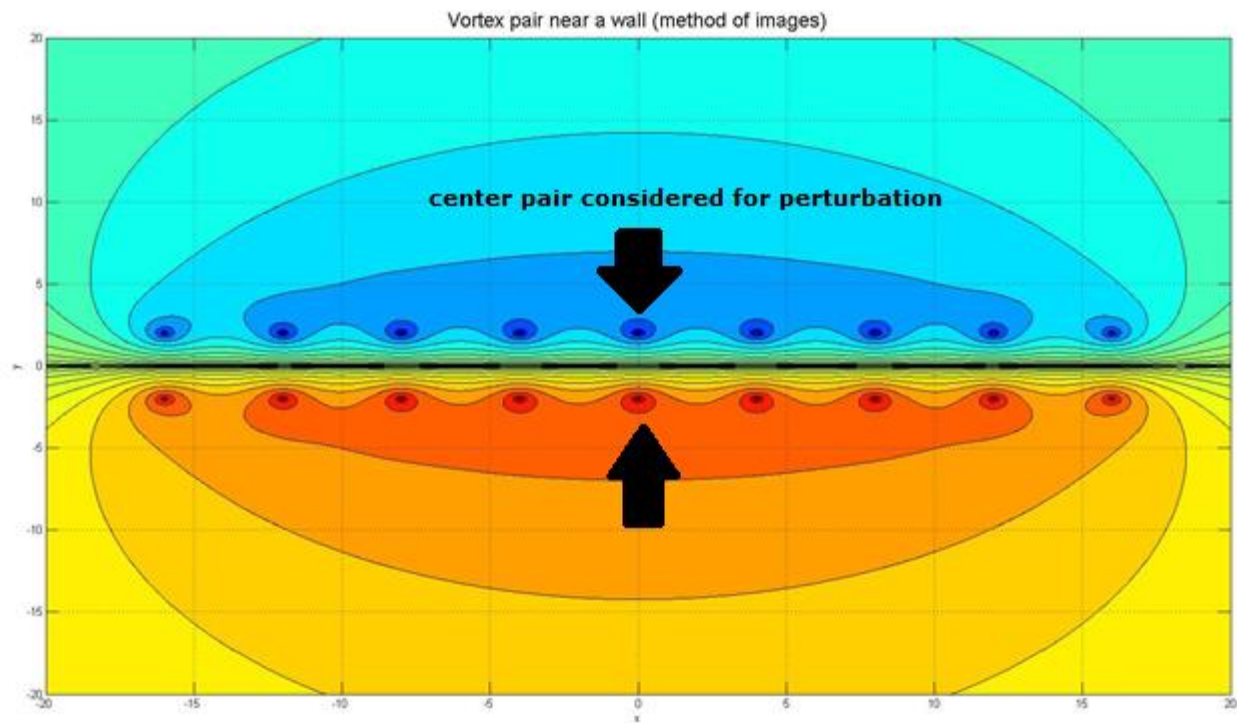


Figure 3.1: No perturbation case (case 1). A representation of vortex array and its image in a stationary arrangement. All the vortices are at their fixed positions. All vortices above the wall (blue region) are rotating counter-clockwise while the image array (red) are rotating clockwise.

The nature of the induced velocities on the wall, as shown below in Figs. 3.2 and 3.3, would be expected from a regular vortex array. The closer the vortices are to the wall the stronger the induced velocity in the region right between the vortices on the wall. The different normalizations ($\Gamma/\pi h$ or $\Gamma/\pi s$) show similar trends. When normalized by $\Gamma/\pi h$, the array configurations with $h/s < 1$ are in a reasonable velocity magnitude range. When the normalized by $\Gamma/\pi s$, array configurations with $h/s > 1$ are in a reasonable velocity magnitude range.

The effects of perturbation effects on induced velocity and pressure coefficient on the wall is discussed in greater detail in Chapter 4 below.

Velocity profiles:

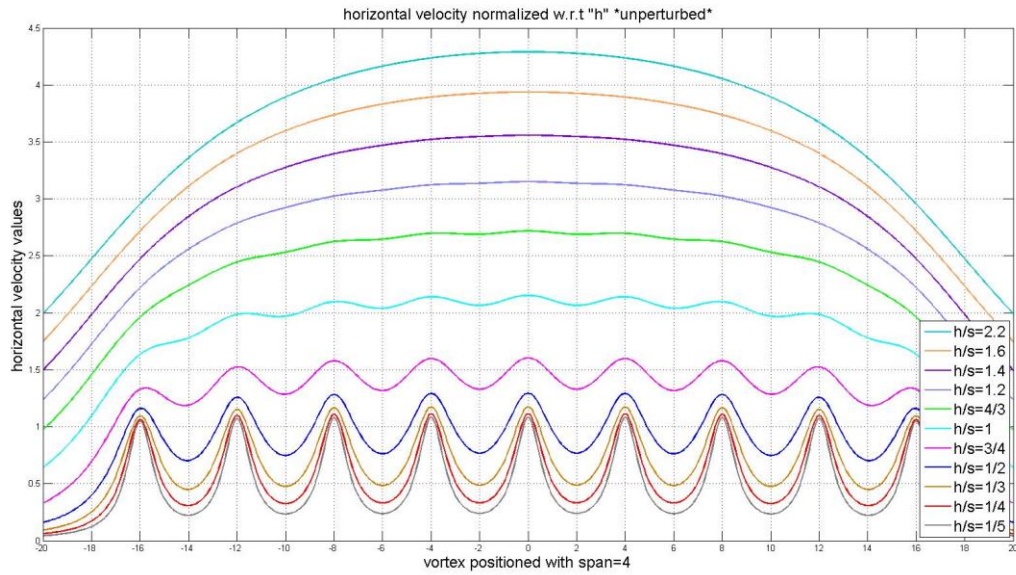


Figure 3.2: No perturbation case (case 1). The induced velocity is normalized with $\Gamma/\pi h$

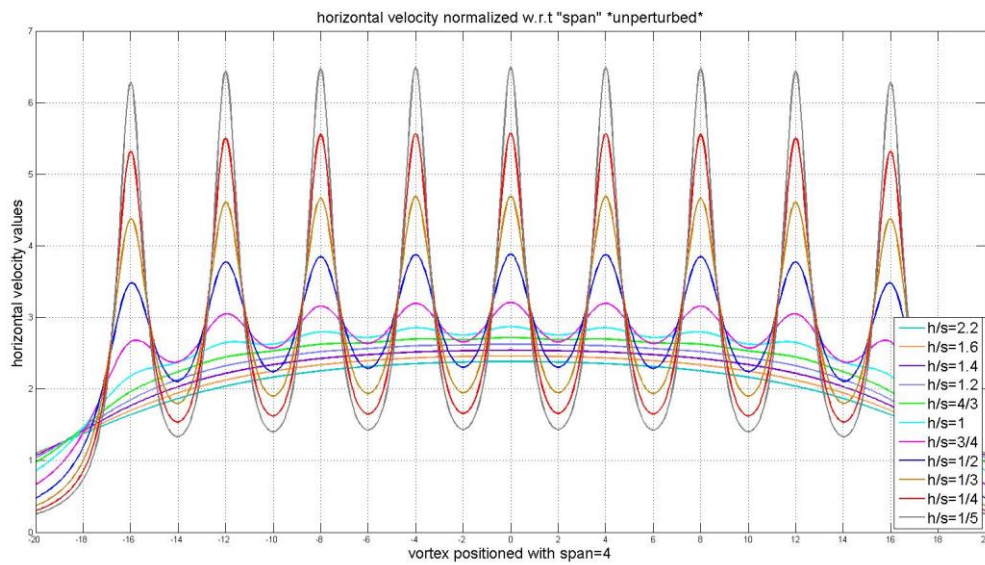


Figure 3.3: No perturbation case (case 1). The induced velocity normalized with $\Gamma/\pi s$

Coefficient of Pressure profiles:

We see similar trends as the velocity, but reversed, which is an artifact of the definition of the pressure coefficient, as stated in Eq. 9.

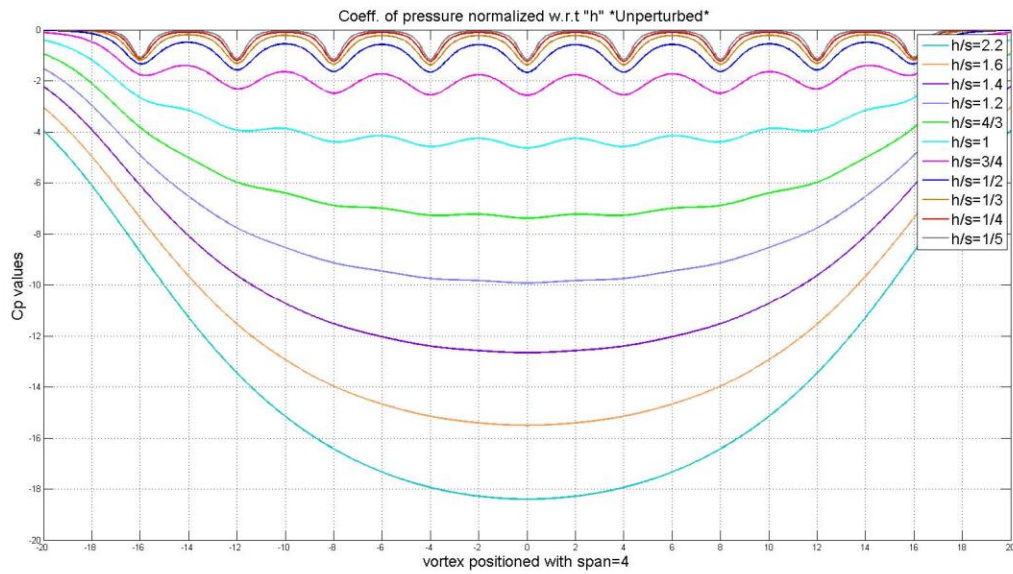


Figure 3.4: No perturbation case (case 1). Coefficient of Pressure normalized with $\Gamma/\pi h$

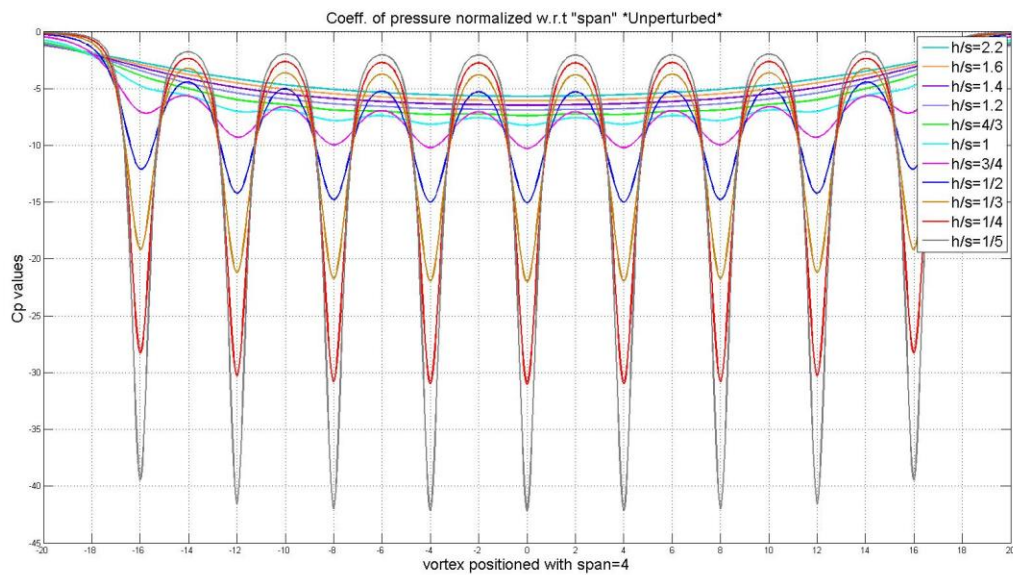


Figure 3.5: No perturbation case (case 1). Coefficient of Pressure normalized with $\Gamma/\pi s$

Case 2: Center vortex pair in the top array vortex perturbed east

The center vortex pair is perturbed from the initial position to the right (east) of the domain as shown Fig. 3.6.

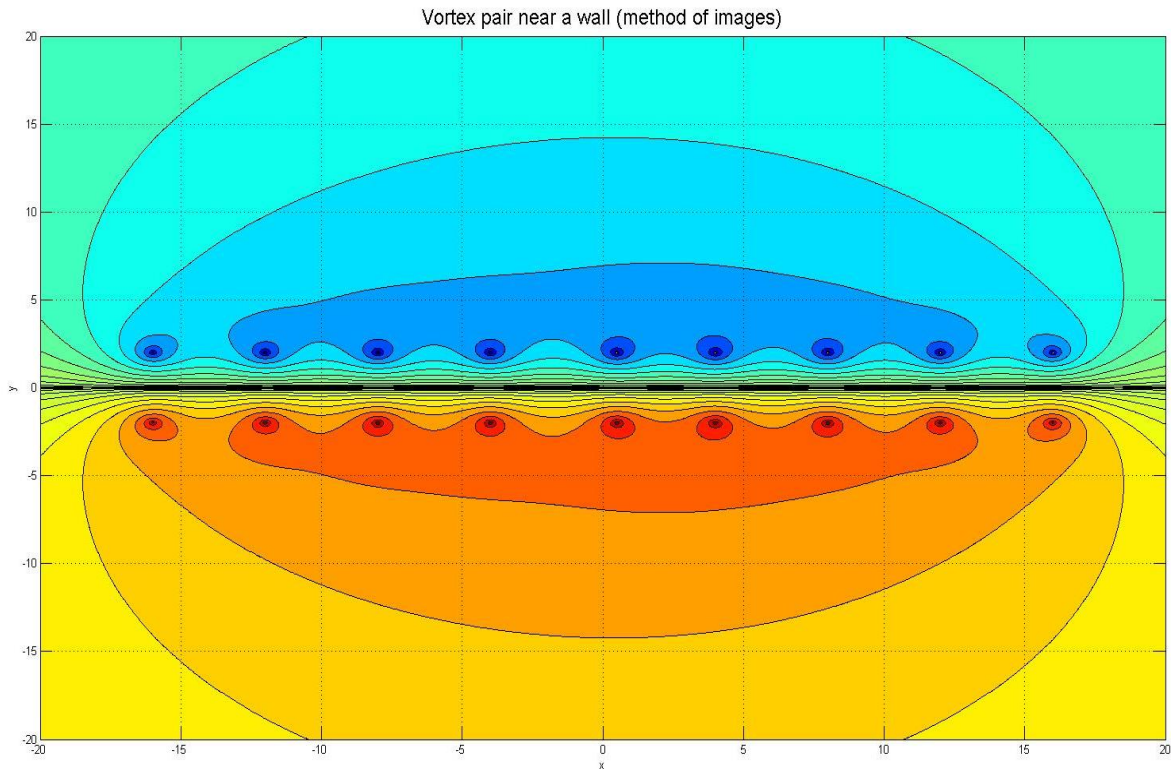


Figure 3.6: Perturbation towards east case (case 2). A representation of vortex array and its image in case where the center pair is perturbed east. All vortices above the wall (blue region) are rotating counter-clockwise while the image array (red) are rotating clockwise.

The nature of the induced velocities on the wall are shown below in Figs. 3.7 and 3.8.

The velocity peak or the high velocity region on the wall right between the center pair of vortices moves east as the vortices are perturbed east, thus shifting the high velocity region and introducing a low velocity region between the center pair and the pair on the left. This is observed since in this idealized case, the vortex pairs surrounding the center pair are assumed to be fixed at their positions.

Velocity profiles:

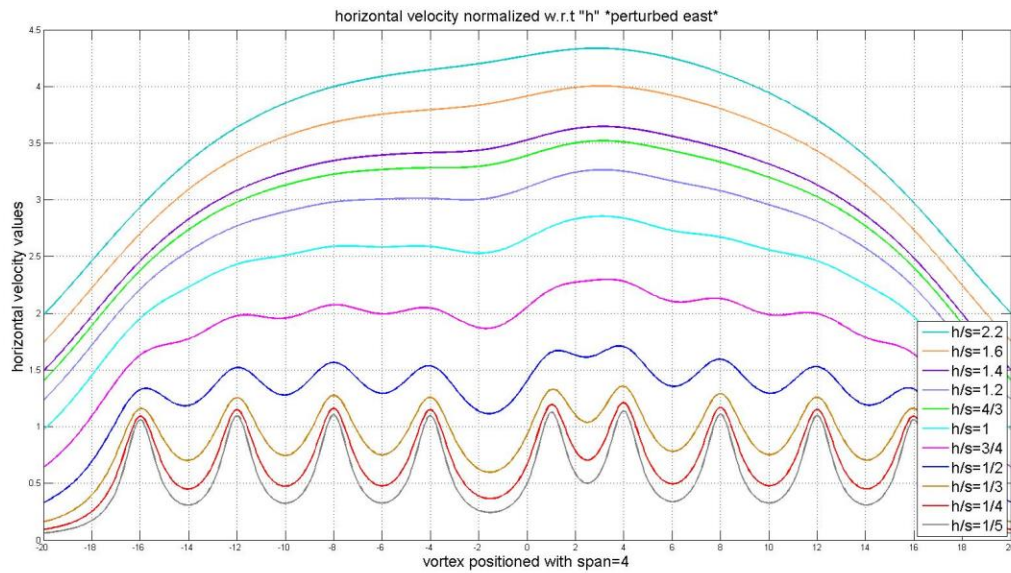


Figure 3.7: Perturbation towards east case (case 2). Induced velocity normalized with $\Gamma/\pi h$

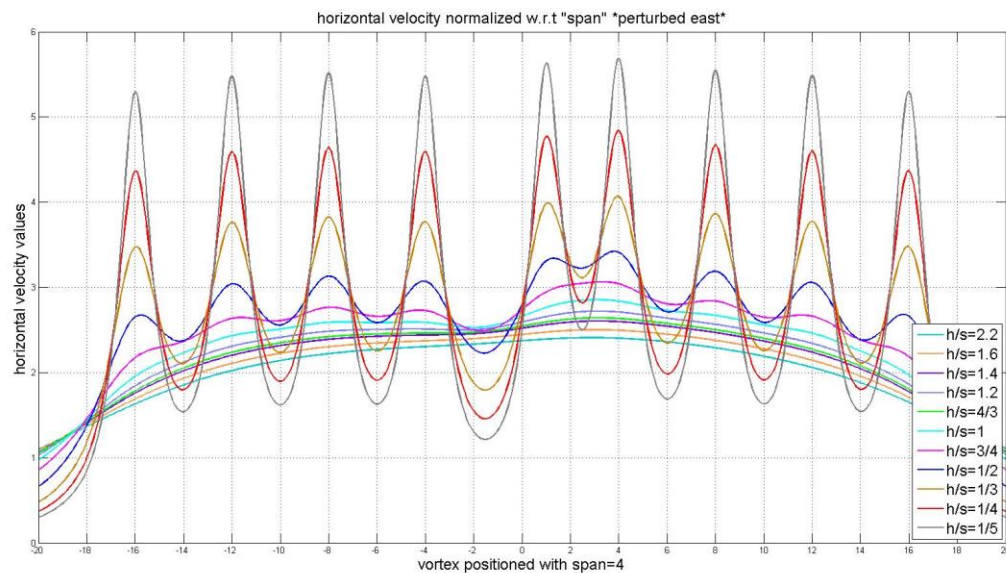


Figure 3.8: Perturbation towards east case (case 2). Induced velocity normalized with $\Gamma/\pi s$

Coefficient of Pressure profiles:

The coefficient of pressure signature on the wall for the east-perturbed case is shown in Figs. 3.9 and 3.10. As discussed earlier there similar, but reversed trends as the velocity, which is an artifact of the definition of the pressure coefficient. The nearby region around vortices of center pair are impacted by the perturbation. The distant vortices remain unaware of the perturbation and hence there is no effect on pressure signature on the wall between the vortices towards the end-sides of the wall. The difference in pressure signature fades as we move away from the center pair.

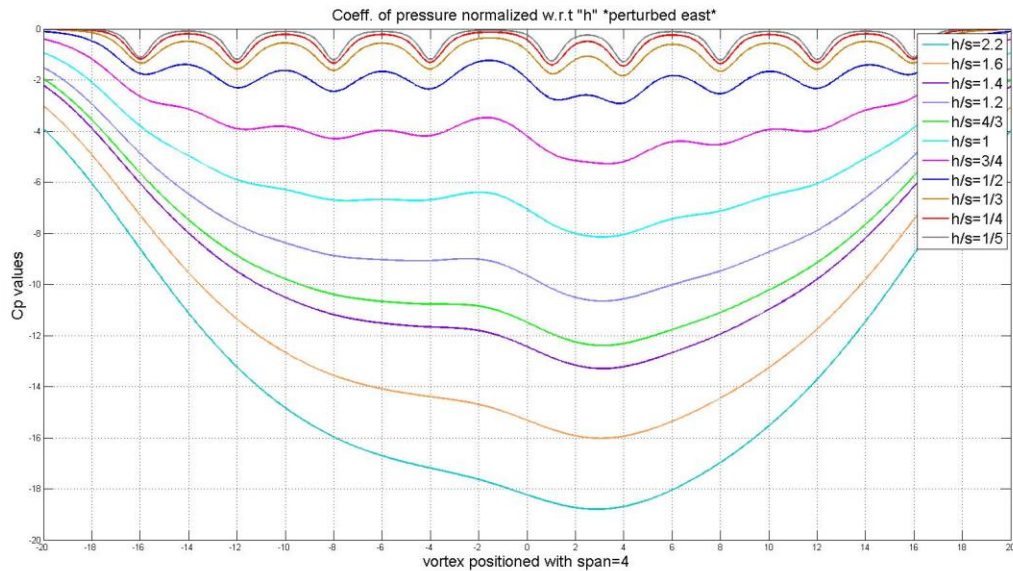


Figure 3.9: Perturbation towards east case (case 2). Coefficient of Pressure normalized with $\Gamma/\pi h$

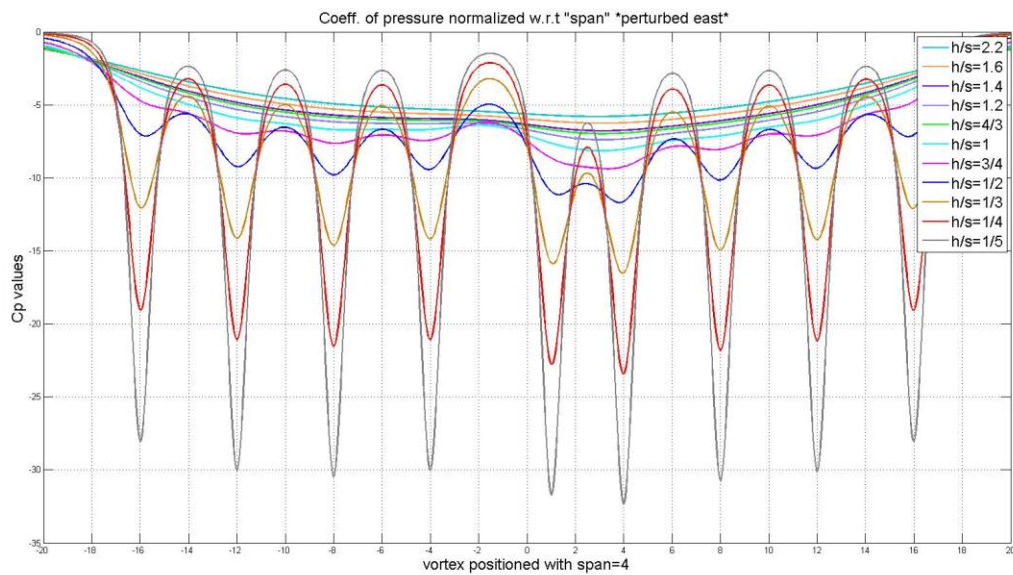


Figure 3.10: Perturbation towards east (case 2) case. Coefficient of Pressure normalized with $\Gamma/\pi s$

Case 3: Center vortex pair in the top array vortex perturbed west

The vortex and its image is moved to the left (west) as shown in Fig. 3.11.

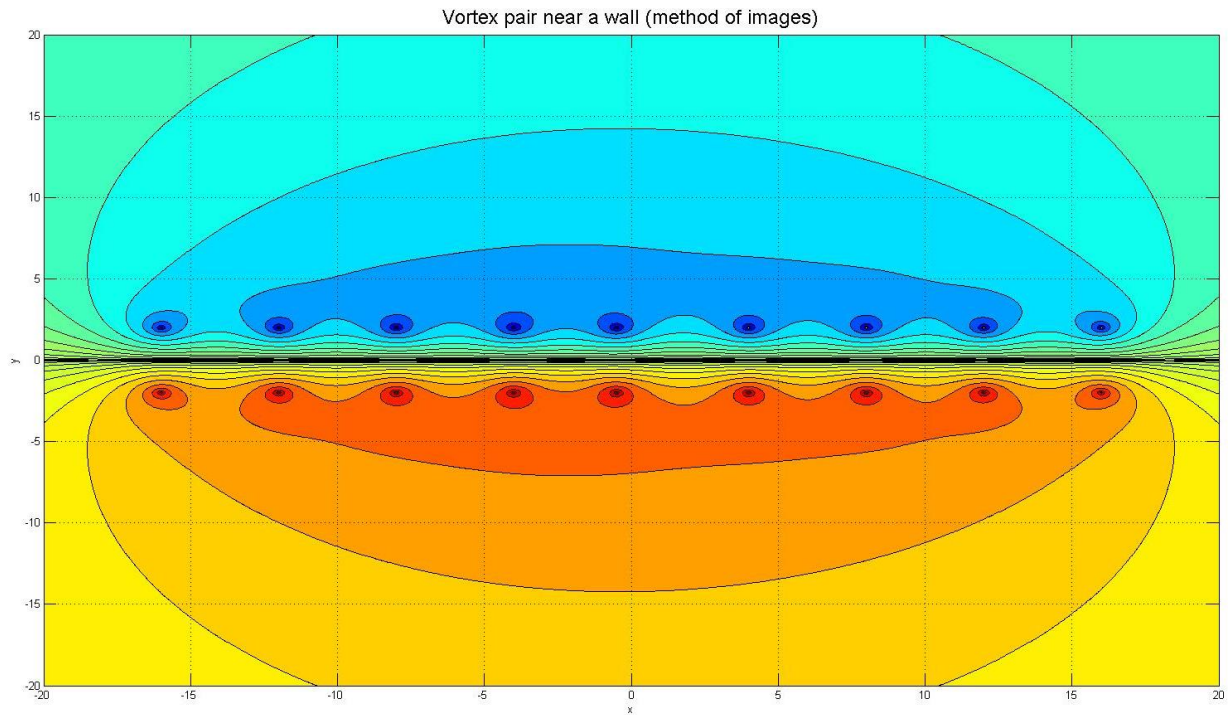


Figure 3.11: Perturbation towards west case. A representation of vortex array and its image in case where the center pair is perturbed west. All vortices above the wall (blue region) are rotating counter-clockwise while the image array (red) are rotating clockwise.

The induced velocity profiles on the wall in this case is similar to the previously discussed west-perturbation case shown below in Figs. 3.12 and 3.13. The high velocity region on the wall right between the center pair of vortices moves towards the west introducing a low velocity region between the center pair and the pair on the right.

Velocity profiles:

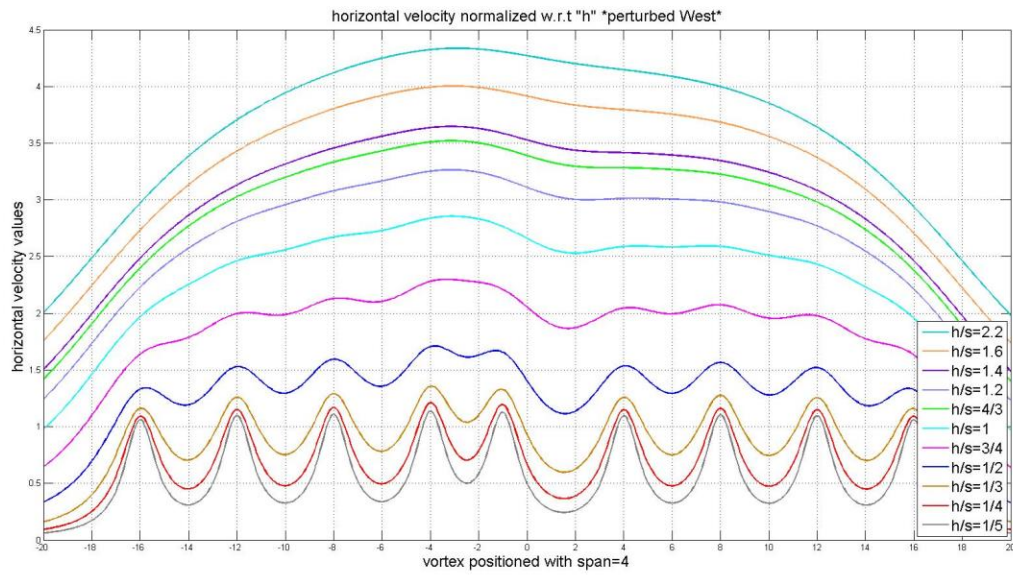


Figure 3.12: Perturbation towards west case (case 3). The induced velocity normalized with $\Gamma/\pi h$

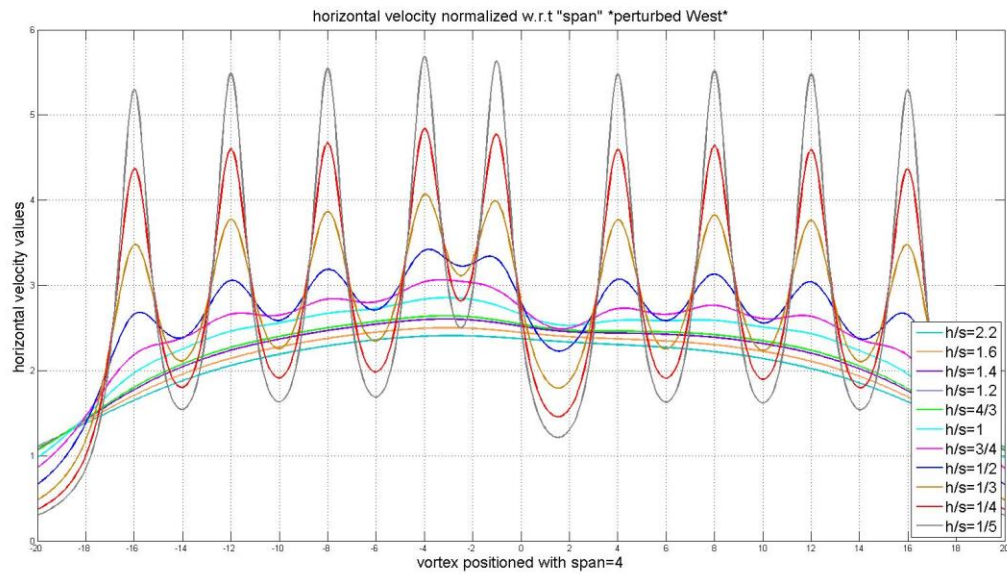


Figure 3.13: Perturbation towards west case (case 3). The induced velocity normalized with $\Gamma/\pi s$

Coefficient of Pressure profiles:

As discussed in the previous case (case 2), a similar trend is observed in pressure coefficient signatures.

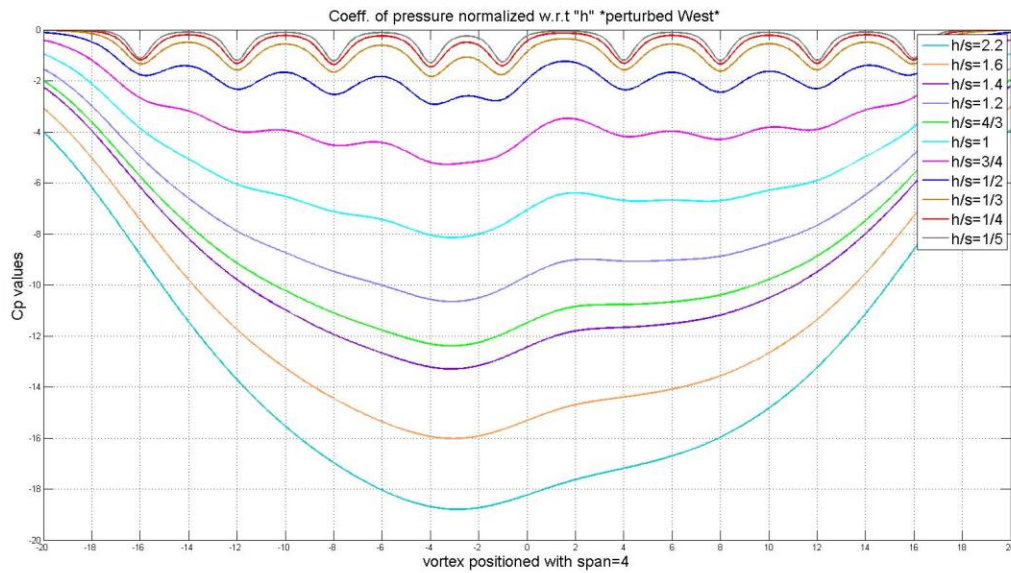


Figure 3.14: Perturbation towards west case (case 3). Coefficient of Pressure normalized with $\Gamma/\pi h$

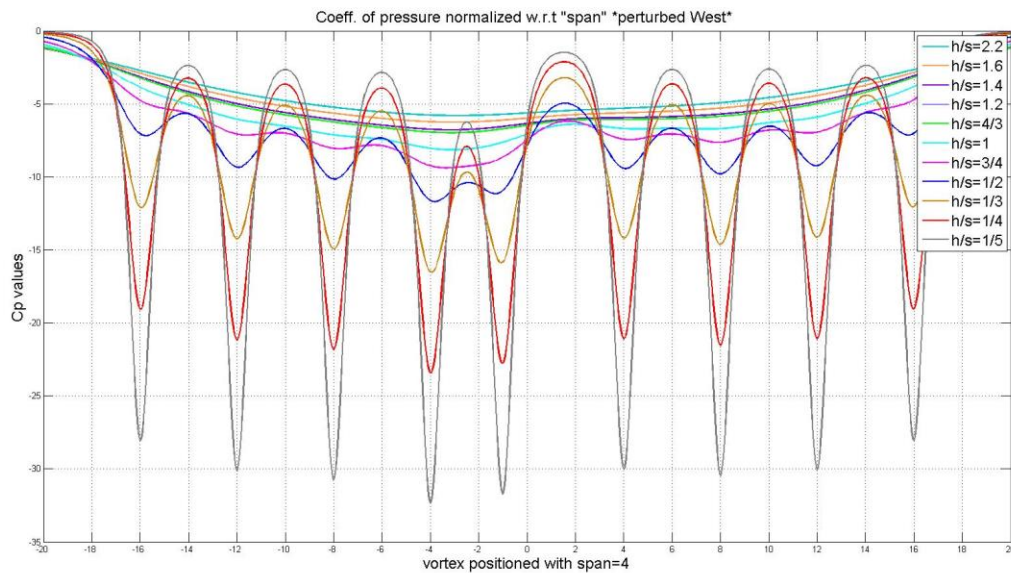


Figure 3.15: Perturbation towards west case (case 3). Coefficient of Pressure normalized with $\Gamma/\pi s$

Case 4: North

The vortex is moved away from the wall (north), thus the image vortex moves away from the wall too (south) as shown Fig. 3.16.

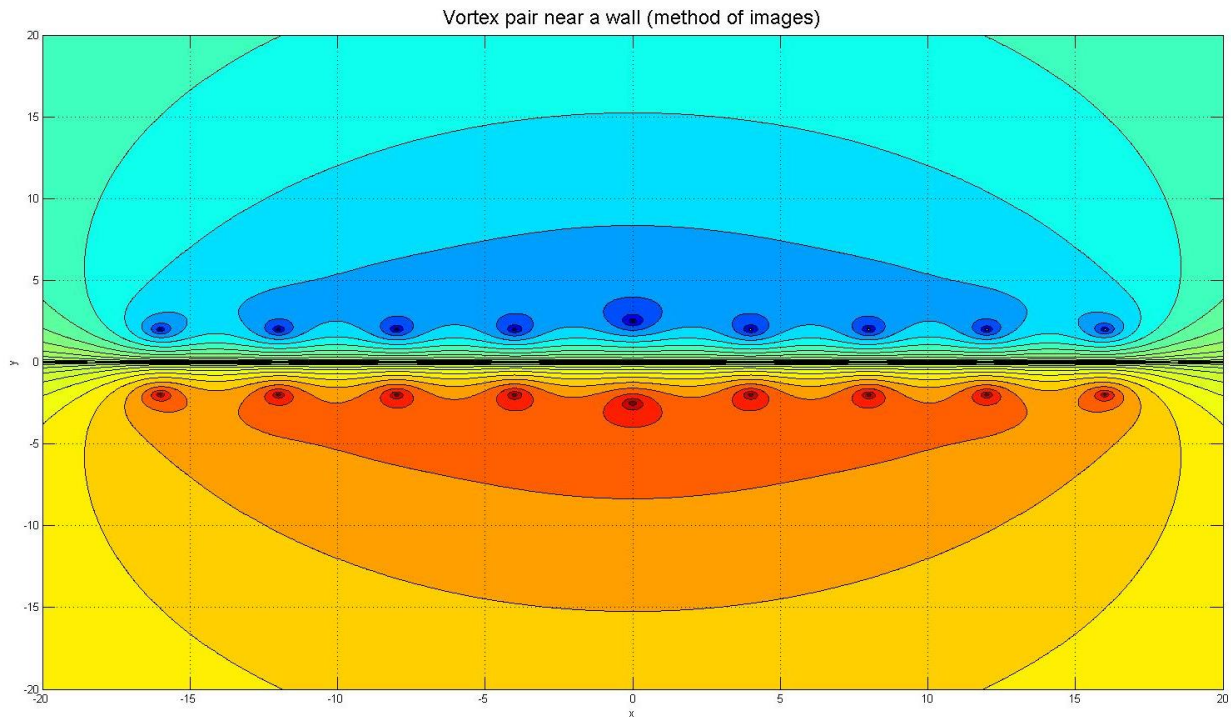


Figure 3.16: Perturbation towards north case. A representation of vortex array and its image in case where the vortex in center pair is perturbed north, while its image is perturbed south. All vortices above the wall (blue region) are rotating counter-clockwise while the image array (red) are rotating clockwise.

The nature of the induced velocities on the wall are shown below in Figs. 3.17 and 3.18.

The induced velocity in the region between the perturbed center vortex pair on the wall is reduced as the vortices are moved away from the wall. As stated in the previous chapters, since the induced velocity is a function of the height of the vortices from the wall, this behavior is expected. The further the vortex is moved away from the wall the lower the induced velocity will be on the wall.

Velocity profiles:

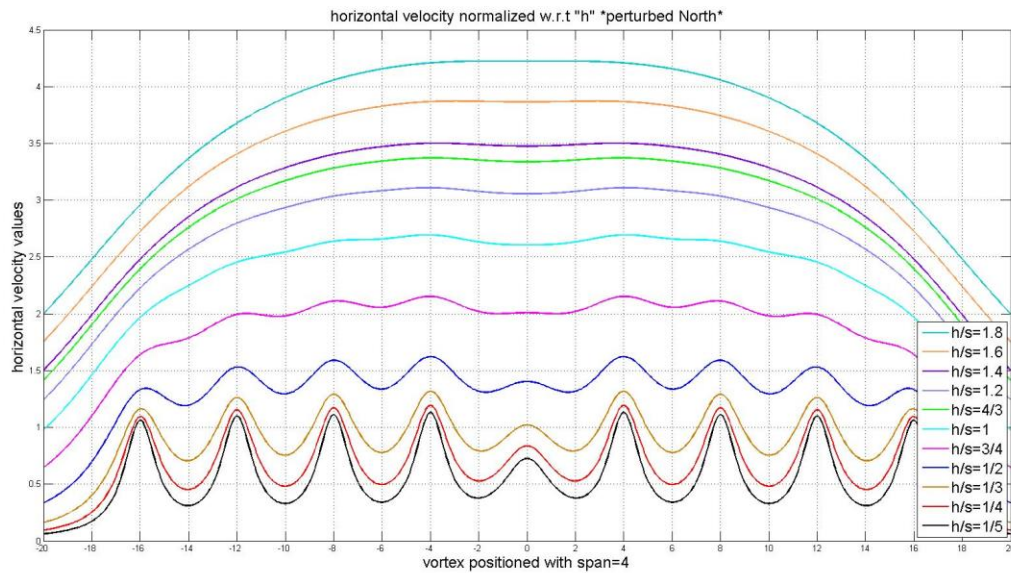


Figure 3.17: Perturbation towards north case (case 4). The induced velocity normalized with $\Gamma/\pi h$

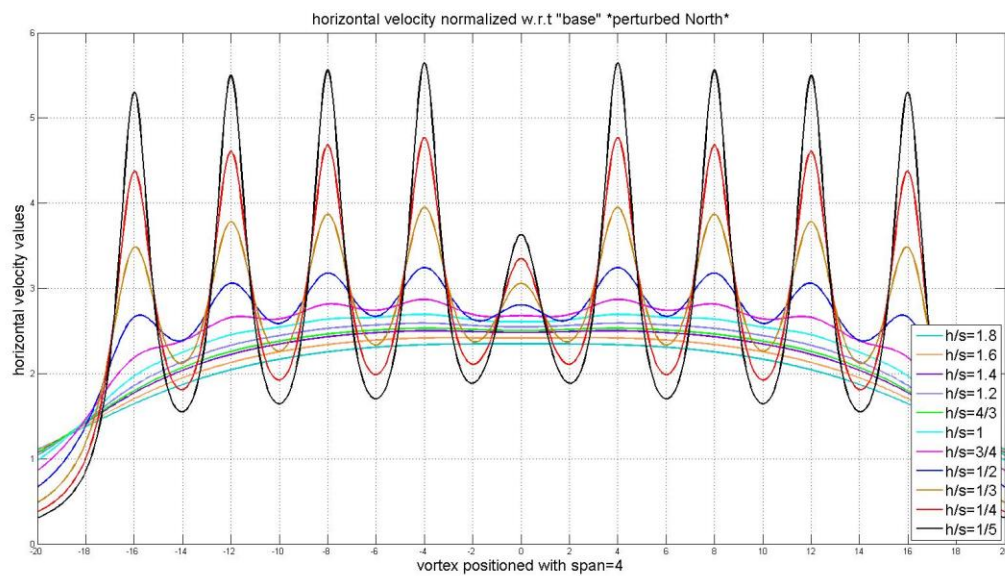


Figure 3.18: Perturbation towards north case (case 4). The induced velocity normalized with $\Gamma/\pi s$

Coefficient of Pressure profiles:

The coefficient of pressure signature on the wall for the north-perturbed case is shown in Figs. 3.19 and 3.20. The induced pressure coefficient in the region between the perturbed center vortex pair on the wall is reduced as the vortices are moved away from the wall.

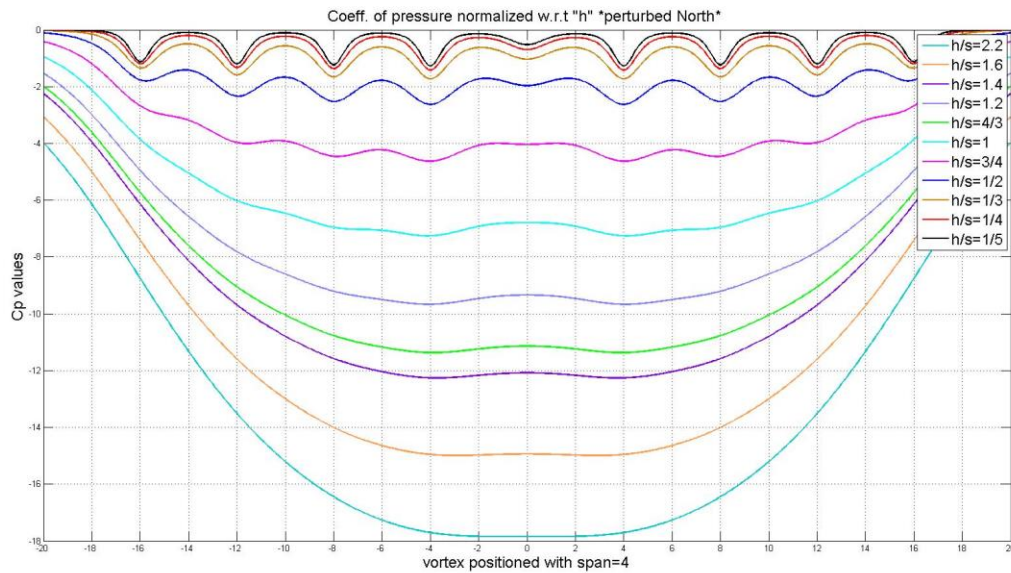


Figure 3.19: Perturbation towards north case (case 4). Coefficient of Pressure normalized with $\Gamma/\pi h$

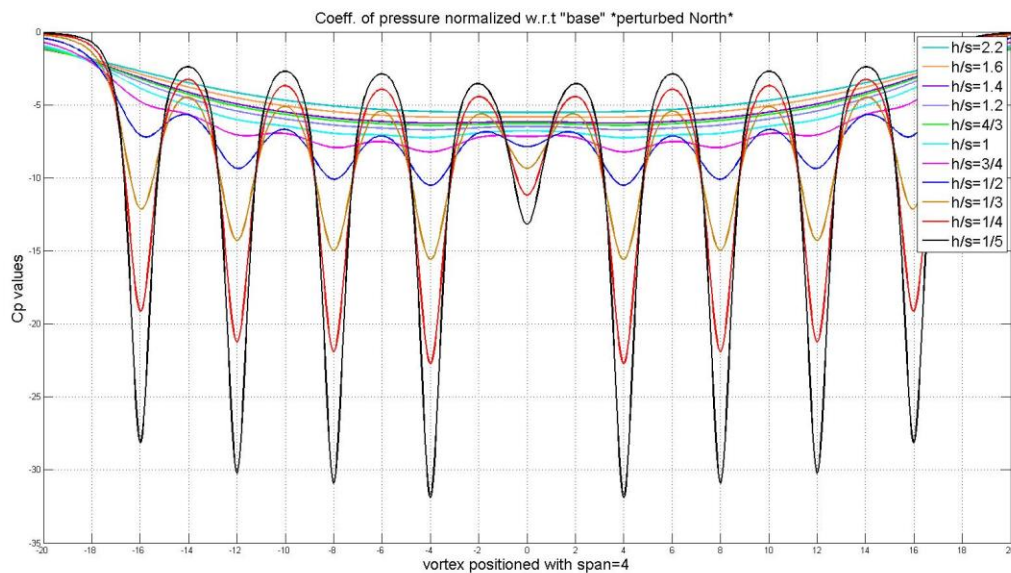


Figure 3.20: Perturbation towards north case (case 4). Coefficient of Pressure normalized with $\Gamma/\pi s$

Case 5: NorthEast:

The vortex is moved north from the wall and towards the east, thus the image vortex moves south from the wall towards the east as shown Fig. 3.21.

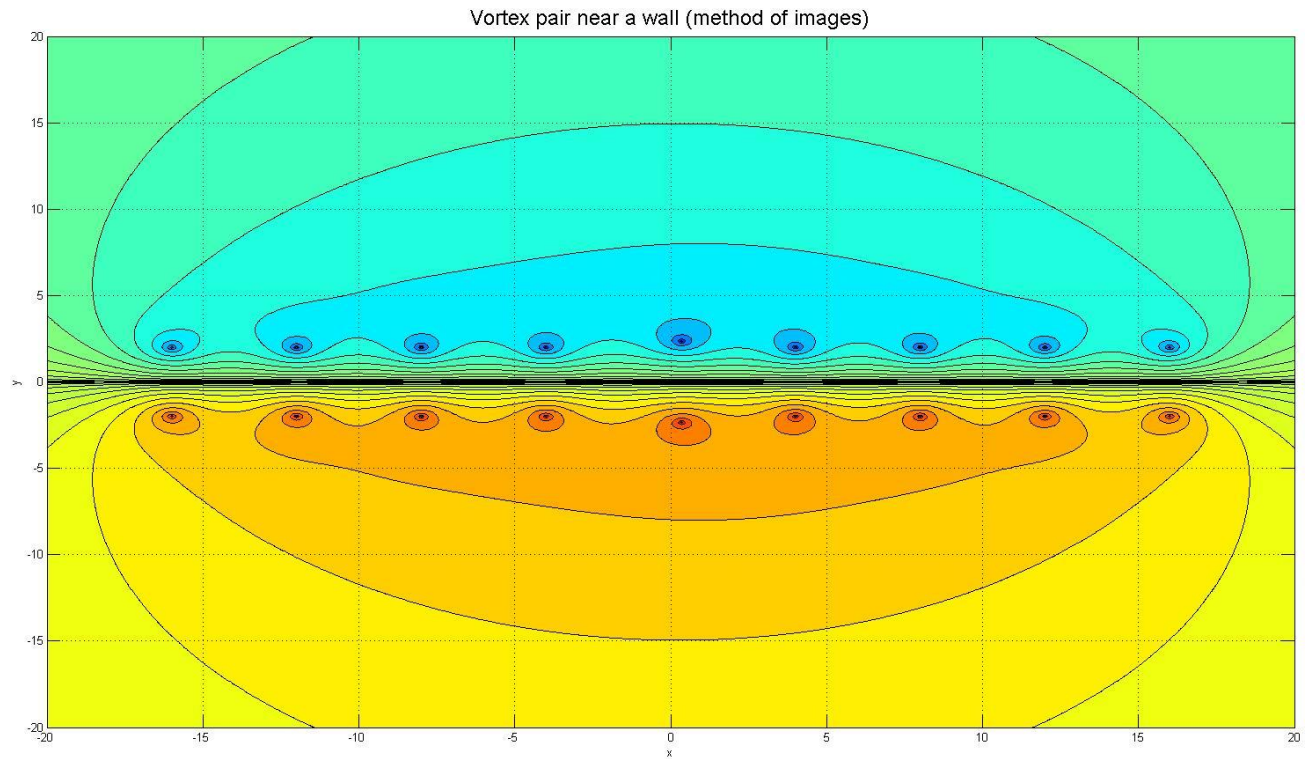


Figure 3.21: Perturbation towards north-east case. A representation of vortex array and its image in case where the vortex in center pair is perturbed north-east, while its image is perturbed south-east. All vortices above the wall (blue region) are rotating counter-clockwise while the image array (red) are rotating clockwise.

The nature of the induced velocities on the wall are shown below in Figs. 3.22 and 3.23.

The induced velocity in the region between the perturbed center vortex pair on the wall is reduced and also moved right as the vortices are moved. The behavior of induced velocity is a combined effect of case 4 (north-perturbation) and case 2 (east perturbation), since the other vortices are fixed in this idealized setup.

Velocity profiles:

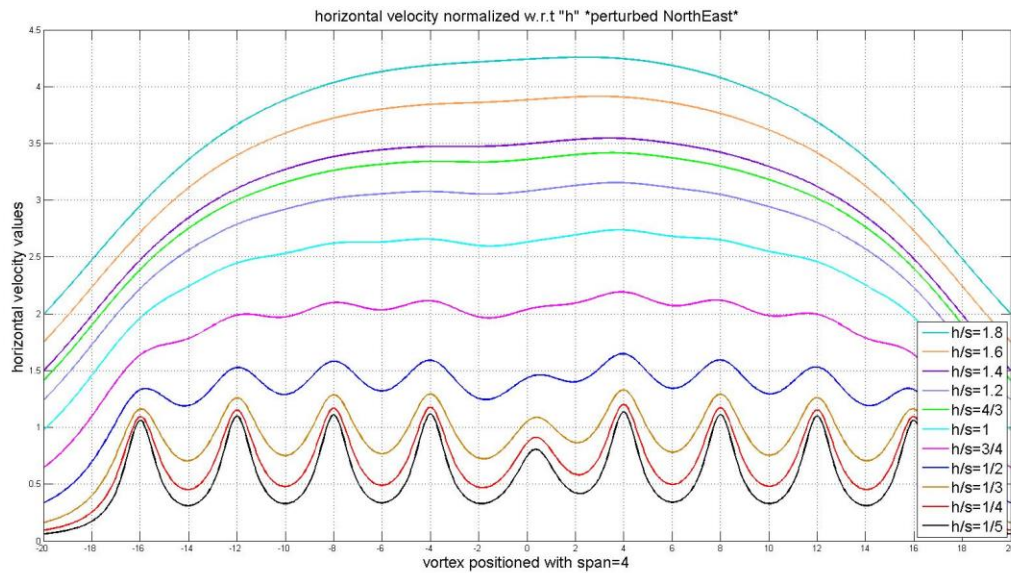


Figure 3.22: Perturbation towards north-east case (case 5). The induced velocity normalized with $\Gamma/\pi h$

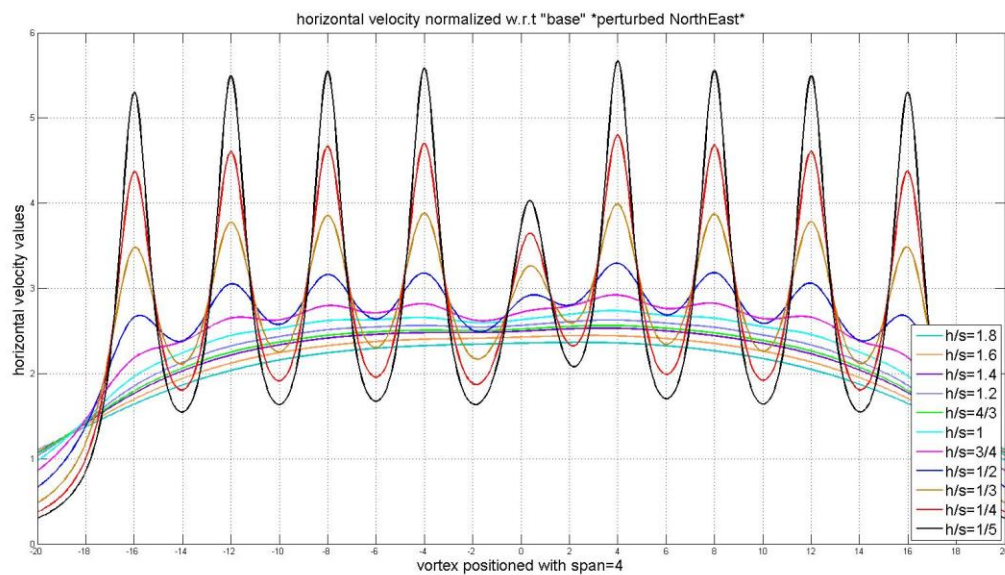


Figure 3.23: Perturbation towards north-east case (case 5). Induced velocity normalized with $\Gamma/\pi s$

Coefficient of Pressure profiles:

The coefficient of pressure signature on the wall for the north-east-perturbed case is shown in Figs. 3.24 and 3.25. Similar to the induced velocities behavior, the induced pressure coefficient is a combined effect of case 4 (north-perturbation) and case 2 (east perturbation), since the other vortices are fixed and this is an idealized setup.

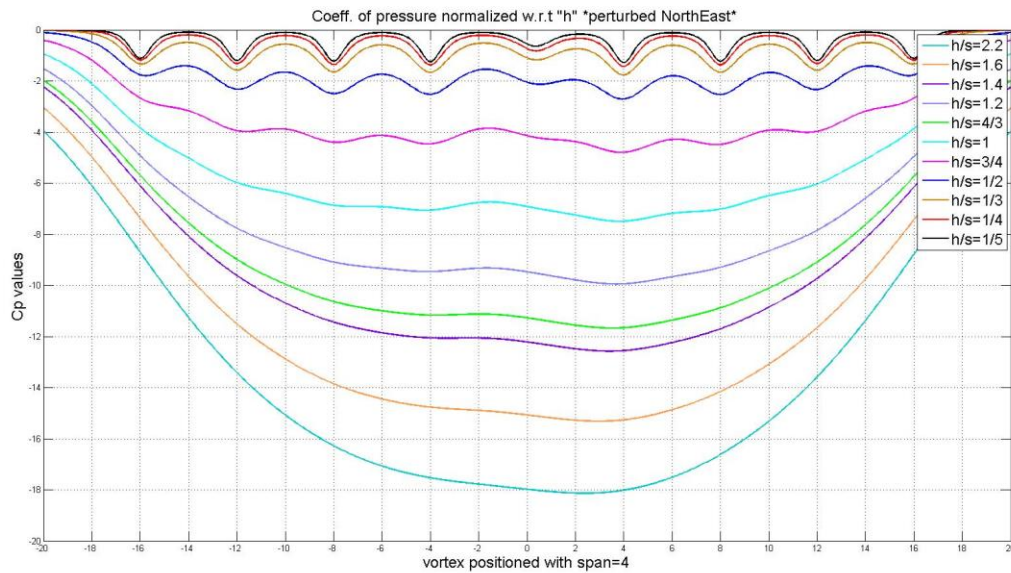


Figure 3.24: Perturbation towards north-east case (case 5). Coefficient of pressure normalized with $\Gamma/\pi h$

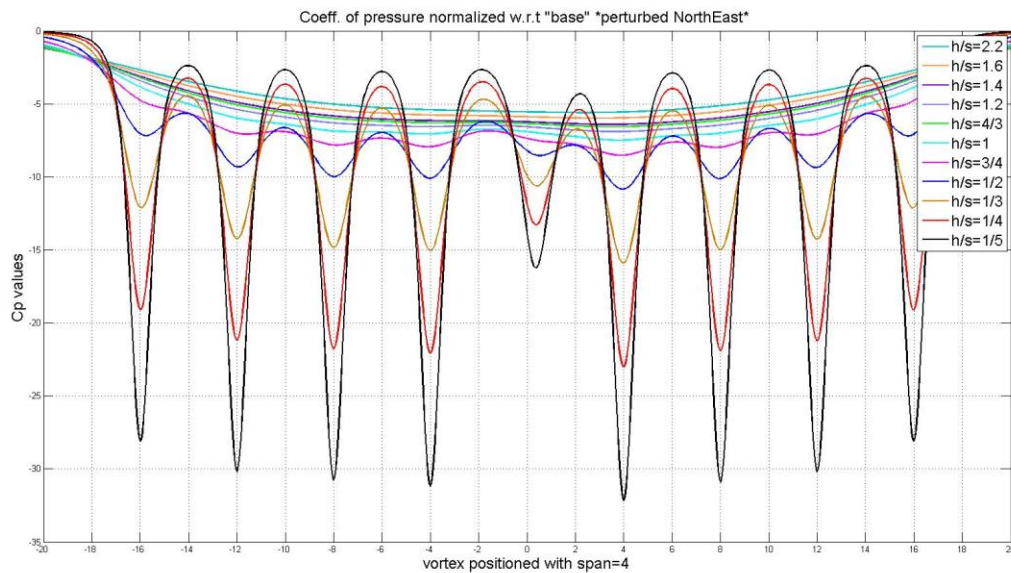


Figure 3.25: Perturbation towards north-east case (case 5). Coefficient of Pressure normalized with $\Gamma/\pi s$

CASE 6: South

The vortex is moved closer to the wall (south) and thus its image vortex moves closer to the wall (north) as shown Fig. 3.26.

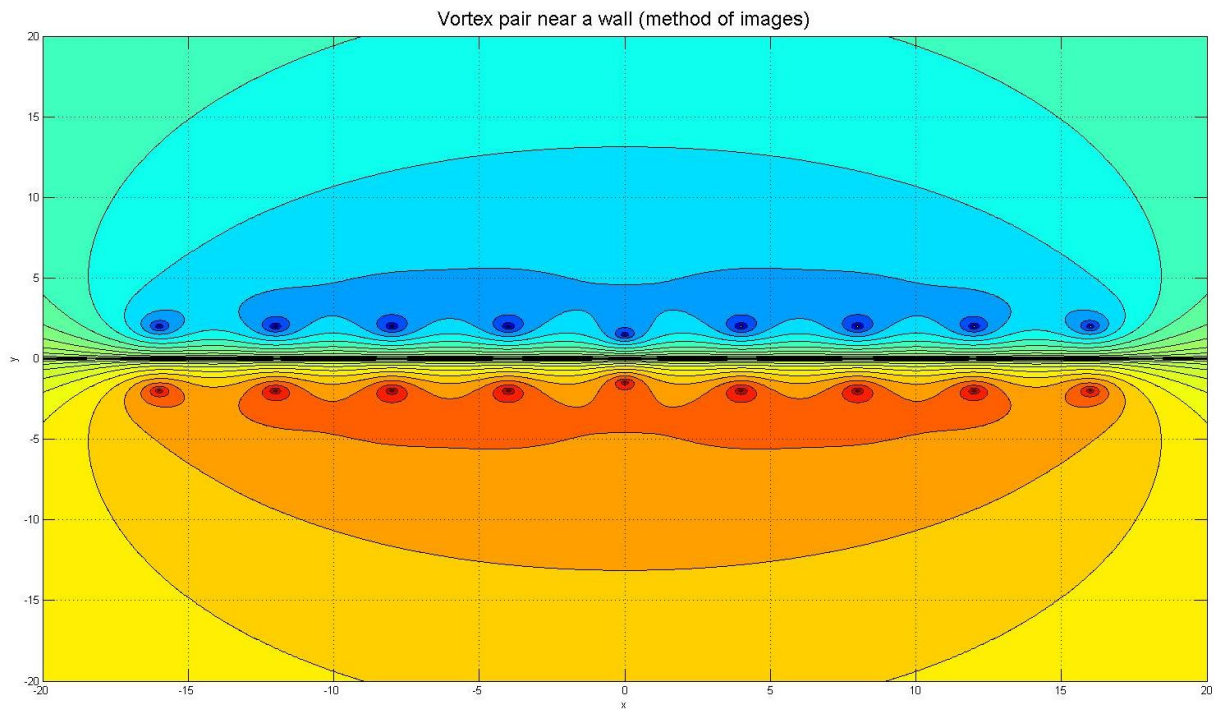


Figure 3.26: Perturbation towards south case. A representation of vortex array and its image in case where the vortex in center pair is perturbed south, while its image is perturbed north. All vortices above the wall (blue region) are rotating counter-clockwise while the image array (red) are rotating clockwise.

The nature of the induced velocities on the wall are shown below in Figs. 3.27 and 3.28.

The induced velocity in the region between the perturbed center vortex pair on the wall is increased as the vortices are moved towards the wall. The more the vortex moved towards from the wall the higher the induced velocity will be on the wall.

Velocity profiles:

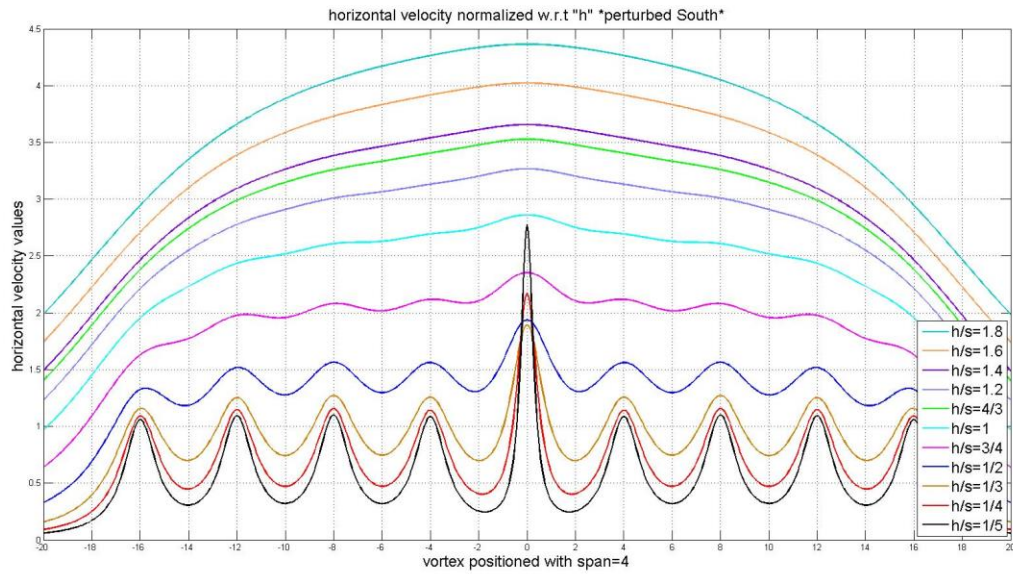


Figure 3.27: Perturbation towards south case (case 6). The induced velocity normalized with $\Gamma/\pi h$

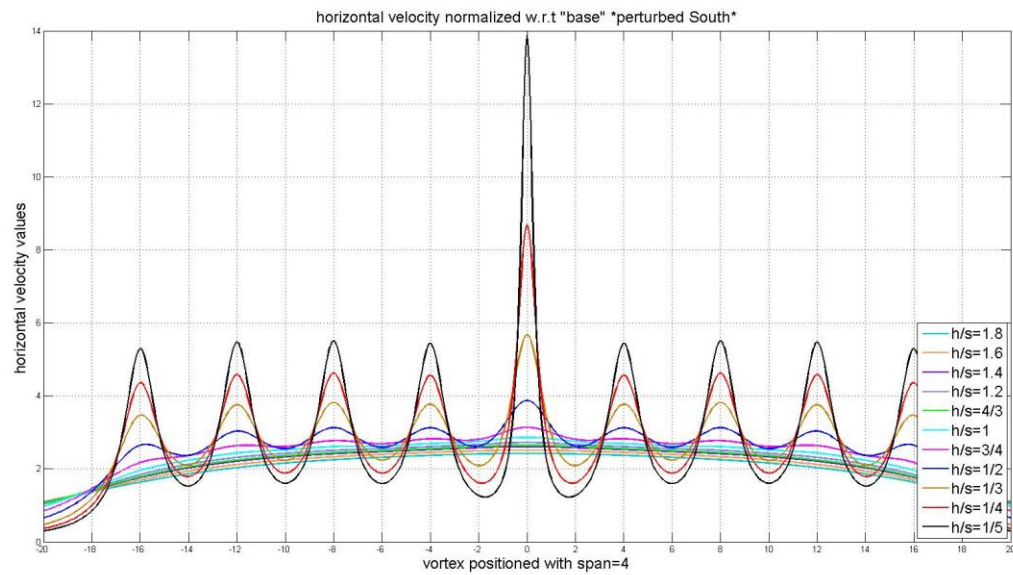


Figure 3.28: Perturbation towards south case (case 6). The induced velocity normalized with $\Gamma/\pi s$

Coefficient of pressure profiles:

The coefficient of pressure signature on the wall for the south-perturbed case is shown in Figs. 3.29 and 3.30. The induced pressure coefficient in the region between the perturbed center vortex pair on the wall is increased as the vortices are moved towards the wall. This case shows similarities to case 4 (north case), but only reversed.

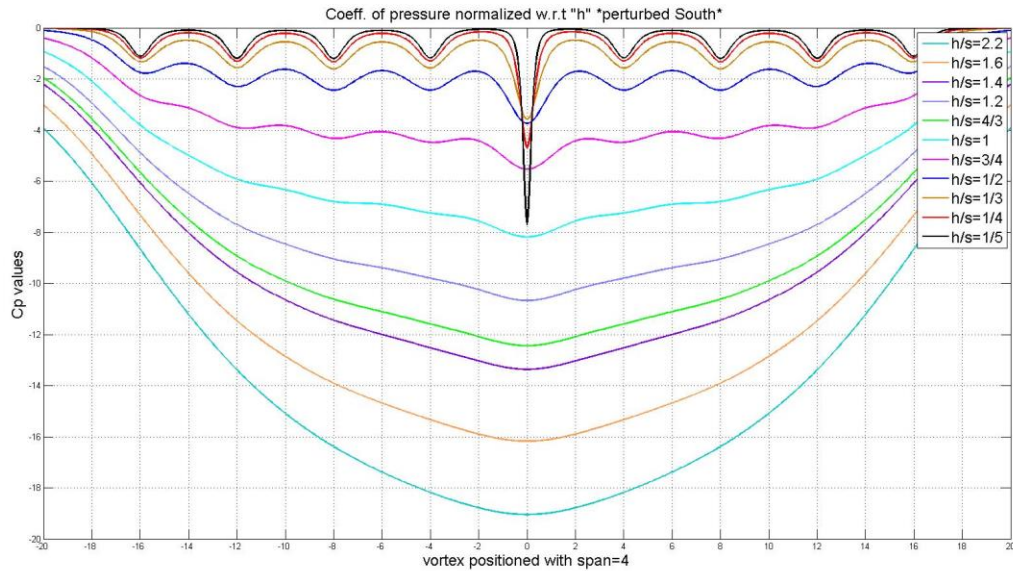


Figure 3.29: Perturbation towards south case (case 6). Coefficient of Pressure normalized with $\Gamma/\pi h$

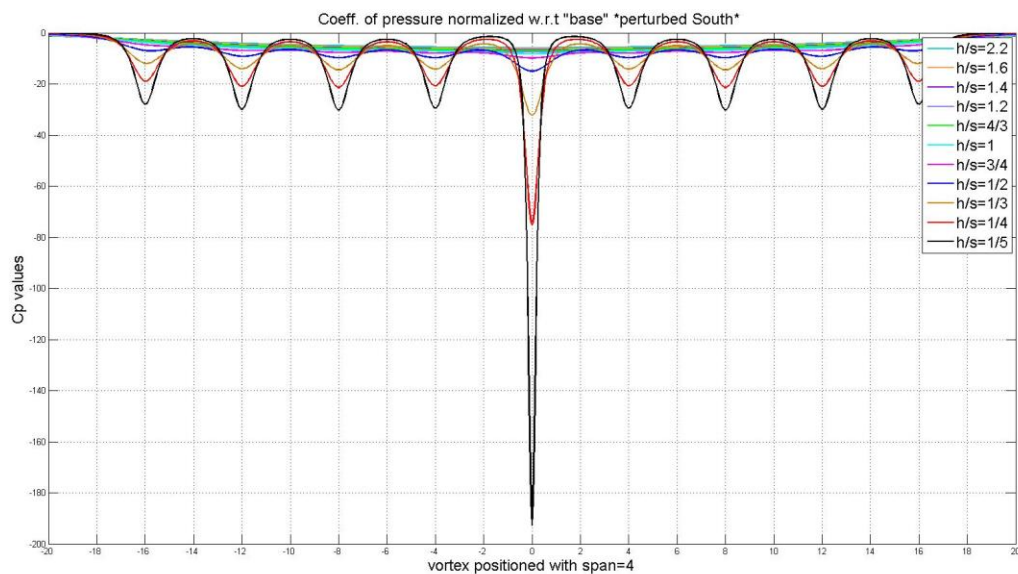


Figure 3.30: Perturbation towards south case (case 6). Coefficient of Pressure normalized with $\Gamma/\pi s$

Case 7: South East

The vortex is moved closer to the wall (south) towards the east, thus the image vortex moves closer to the wall (north) towards the east as shown Fig. 3.31.

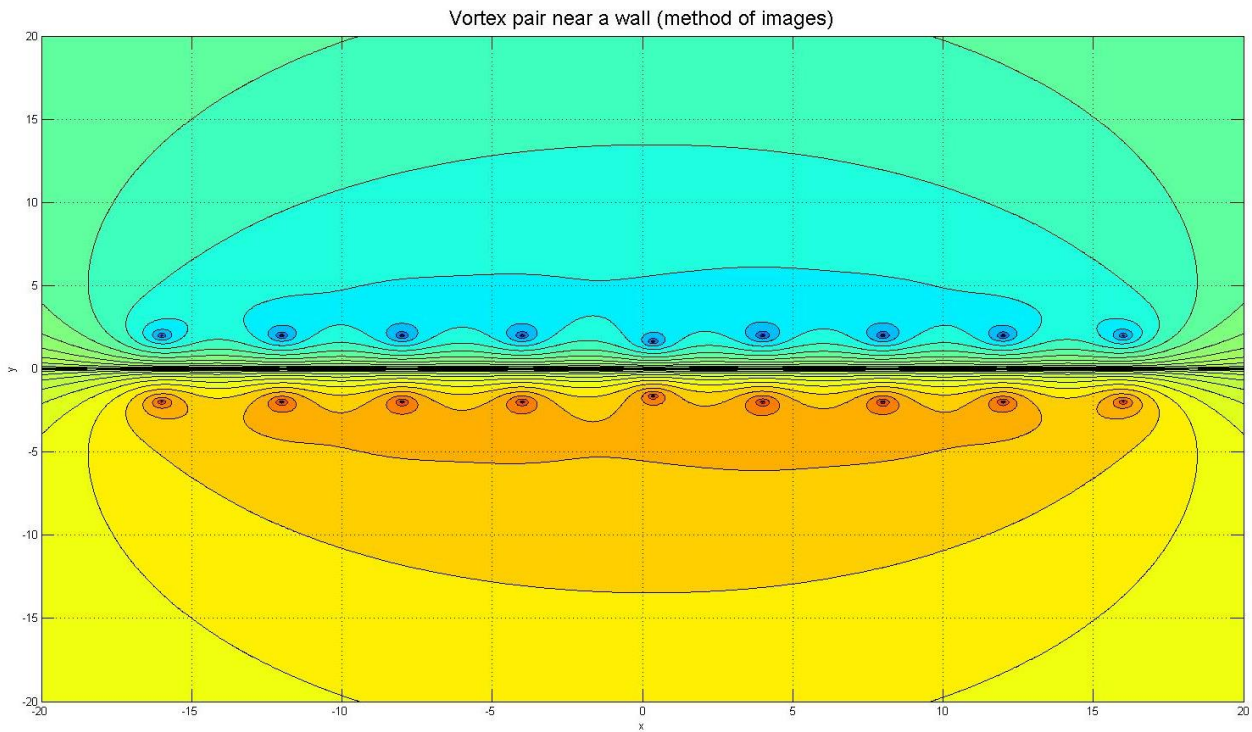


Figure 3.31: Perturbation towards south-east case. A representation of vortex array and its image in case where the vortex in center pair is perturbed south-east, while its image is perturbed north-east. All vortices above the wall (blue region) are rotating counter-clockwise while the image array (red) are rotating clockwise.

The nature of the induced velocities on the wall are shown below in Figs. 3.32 and 3.33.

The induced velocity in the region between the perturbed center vortex pair on the wall is increased and moved east as the vortices are moved. The more the vortex moved towards from the wall the higher the induced velocity will be on the wall and more will be the high velocity region shift.

Velocity profiles:

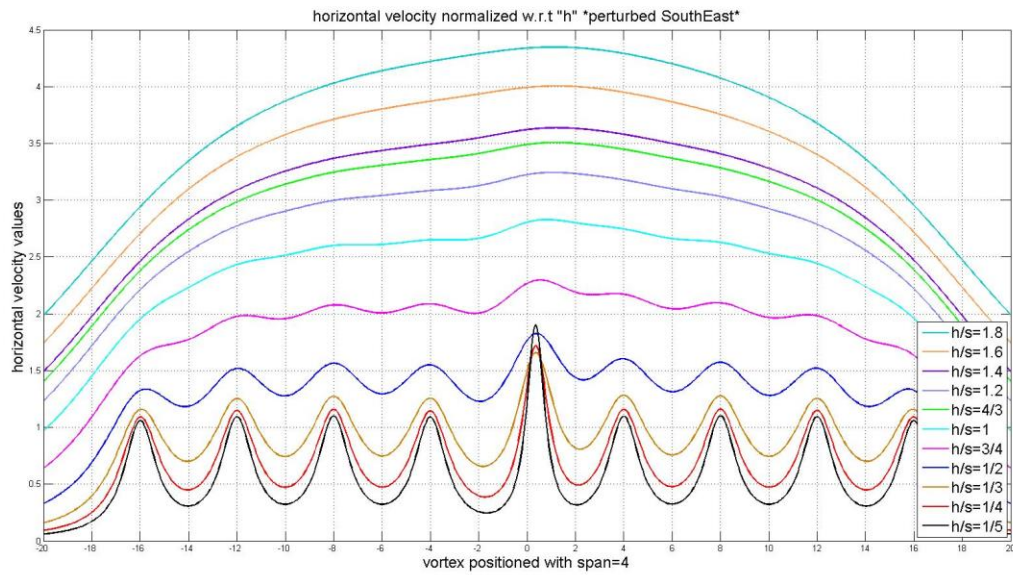


Figure 3.32: Perturbation towards south-east case (case 7). The induced velocity normalized with $\Gamma/\pi h$

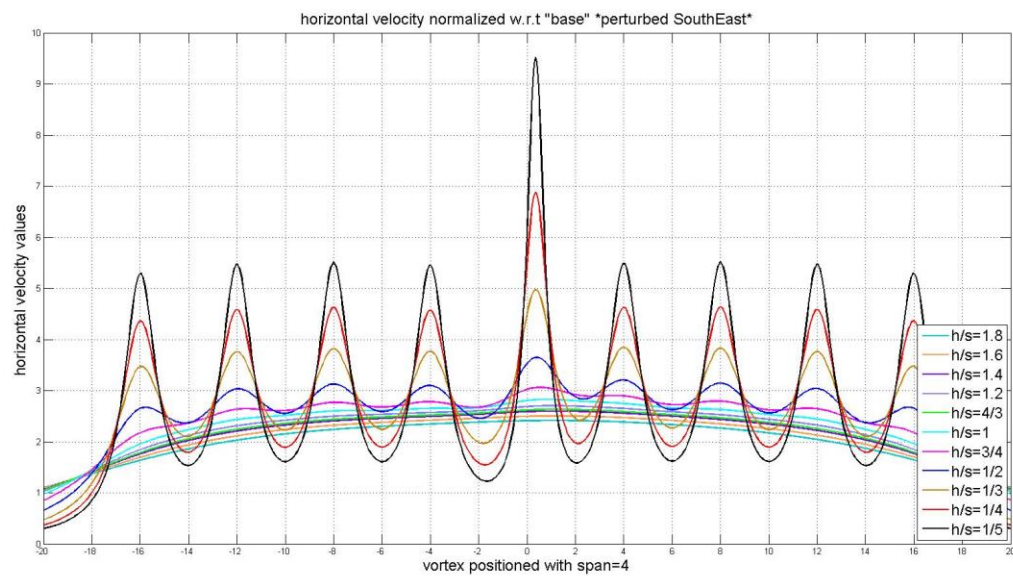


Figure 3.33: Perturbation towards south-east case (case 7). The induced velocity normalized with $\Gamma/\pi s$

Coefficient of Pressure profiles:

The coefficient of pressure signature on the wall for the south-east-perturbed case is shown in Figs. 3.34 and 3.35. Similar to the induced velocities behavior, the induced pressure coefficient is a combined effect of case 6 (south-perturbation) and case 2 (east perturbation), since the other vortices are fixed and this is an idealized setup.

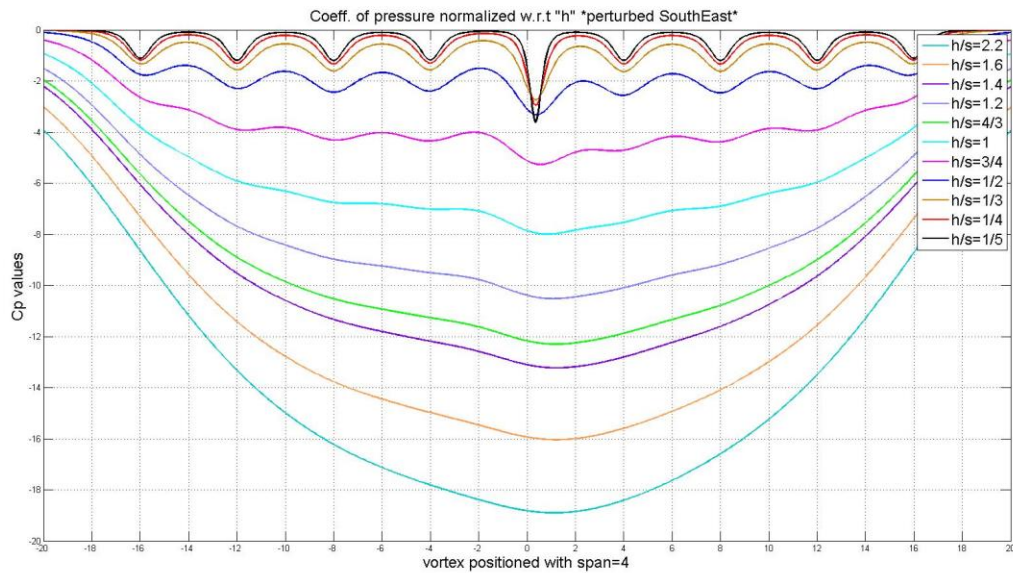


Figure 3.34: Perturbation towards south-east case (case 7). Coefficient of pressure normalized with $\Gamma/\pi h$

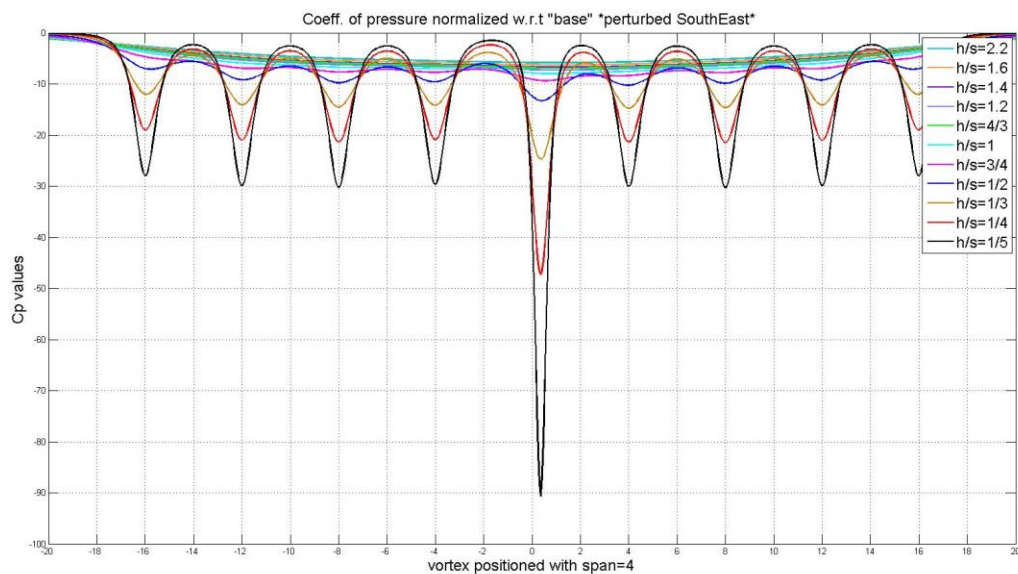


Figure 3.35: Perturbation towards south-east case (case 7). Coefficient of Pressure normalized with $\Gamma/\pi s$

To study this in an experimental setup, the vortex position and strength may be determined by using high sensitivity and high frequency response pressure sensors. Simulating the effects on vortex perturbation on the coefficient of pressure, the expected results would be to see an increase, decrease or shift of high pressure regions, similar to velocity. The corresponding coefficient of pressure for constant h/s ratio is shown in Fig. 4.2:

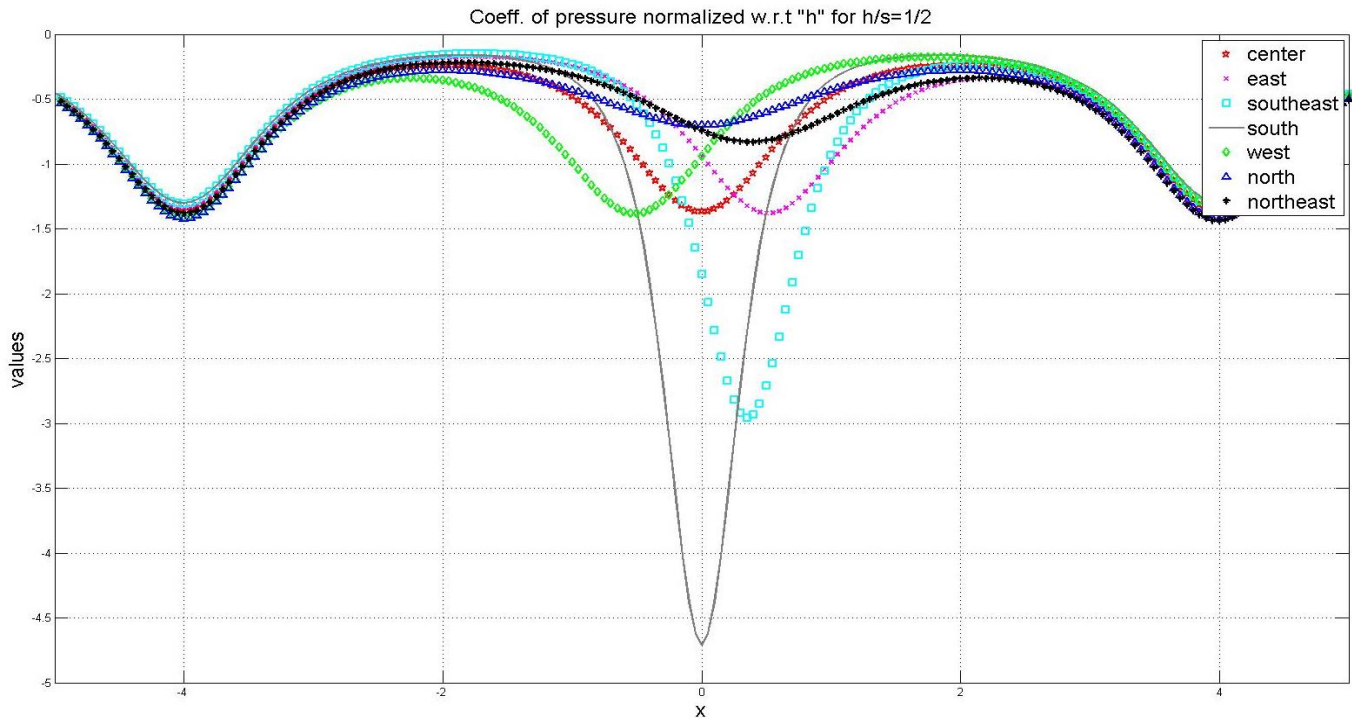


Figure 4.2: The corresponding pressure coefficient on the wall, normalized w.r.t the height of between the vortices and the wall for all seven cases of vortex perturbation

For a case where the parameter h/s is very small, meaning the span of the vortices is large compared to the height, the induced velocity would be high right in between the counter rotating vortex pair on the wall, and go to almost zero or a negligible value in between two neighboring or adjacent co-rotating pairs. This is observed since when the vortex pairs are placed far enough away the induced velocity on the wall is only due to the interaction between the counter rotating pair of vortices. The adjacent pairs are then too far apart to have an effect on the induced velocity at the wall for the values of separation and vortex height considered here. This is observed in a case where the array configuration parameter $h/s = 1/8$. As shown below in Fig. 4.3 and Fig. 4.4, the induced velocity is high in the wall region lying directly between the counter rotating vortex pair. The contribution to the induced velocity by the adjacent vortex pairs is negligible since they are too far away for the interaction to occur.

When observed closely, as indicated in Fig. 4.5 and Fig. 4.6, the effects of center pair vortex perturbation on the induced velocity on the wall is confined to a small region on the wall directly between the center pair. Again, this is observed since the induced velocity in the considered array parameter h/s is mainly due to the interaction of the counter rotating pair.

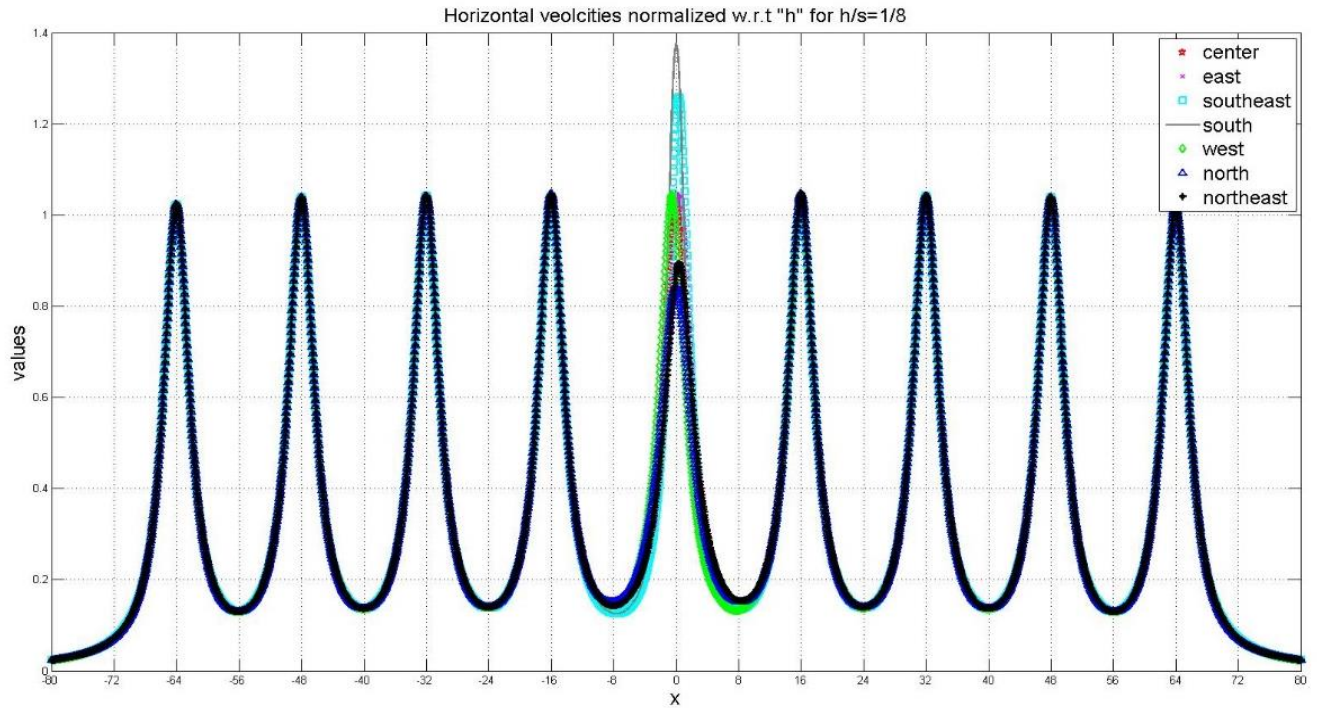


Figure 4.3: The induced horizontal velocity on the wall for $h/s = 1/8$

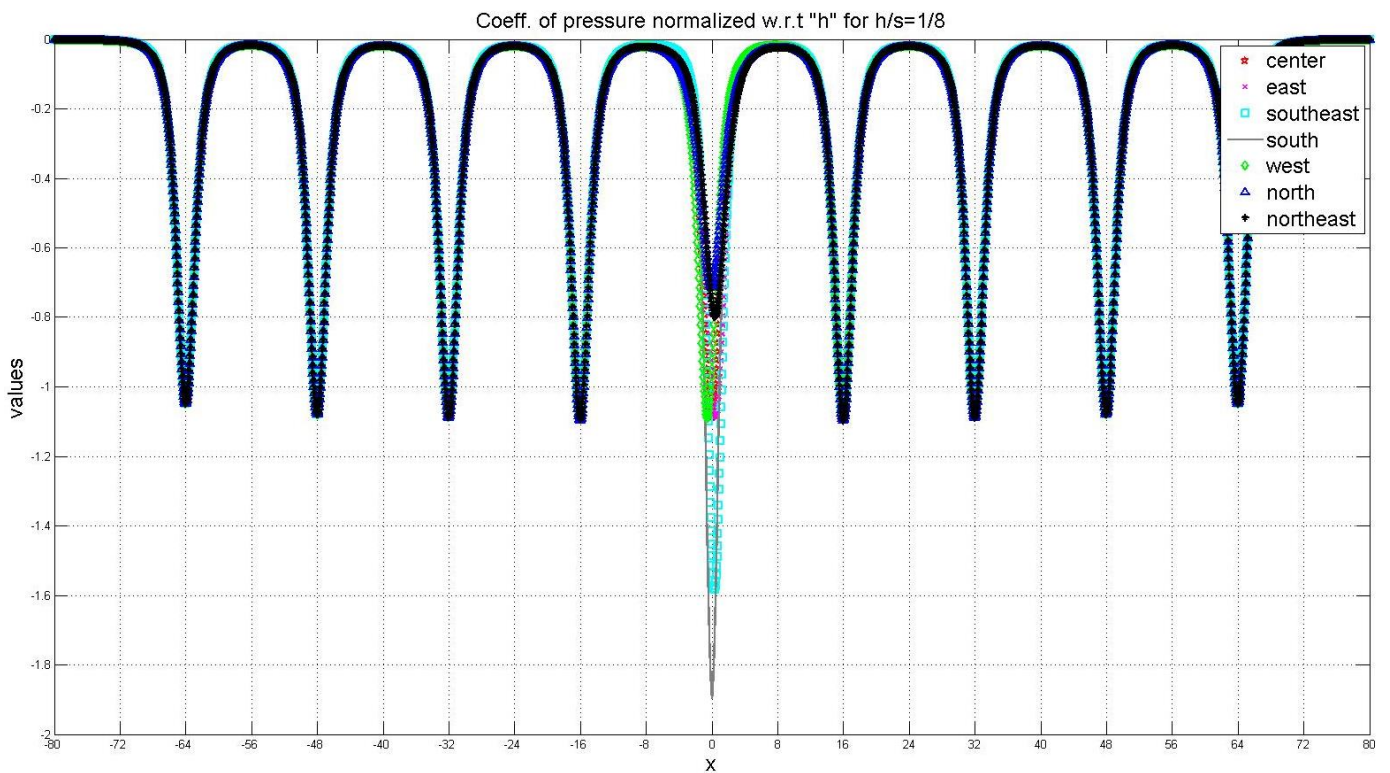


Figure 4.4: The corresponding coefficient of pressure on the wall for $h/s = 1/8$

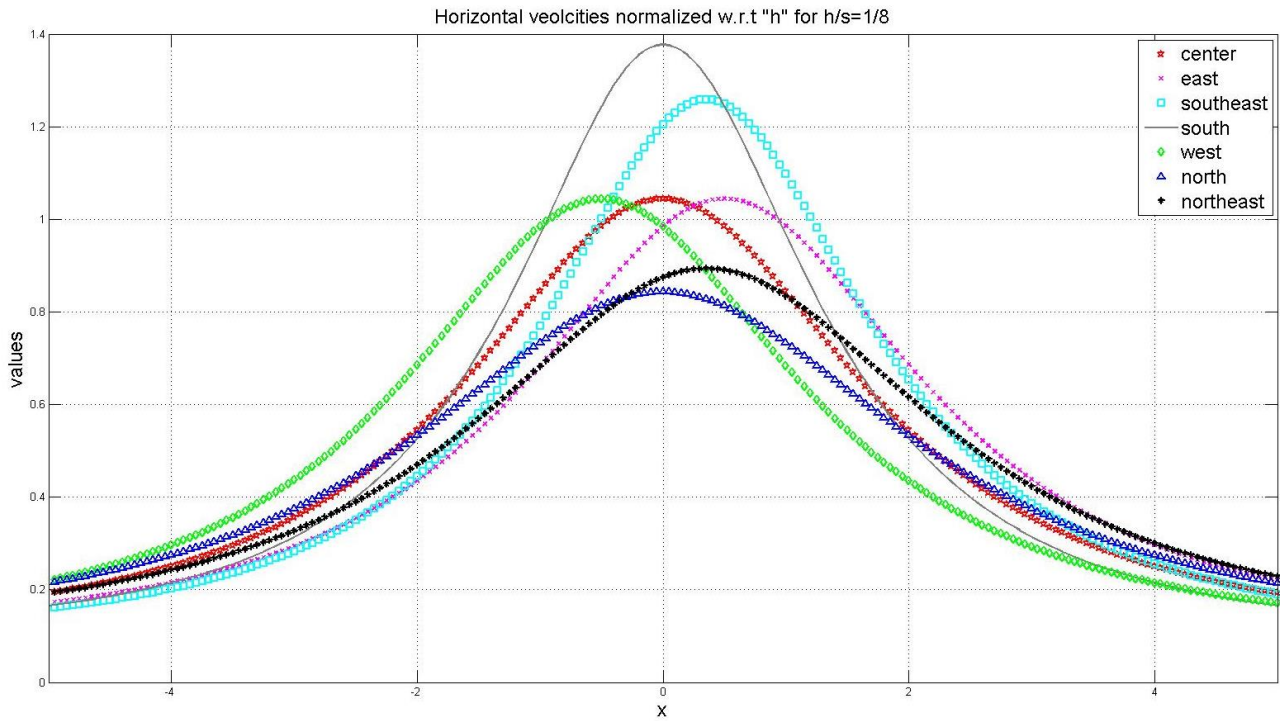


Figure 4.5: Close up of the induced horizontal velocity on the wall for $h/s = 1/8$

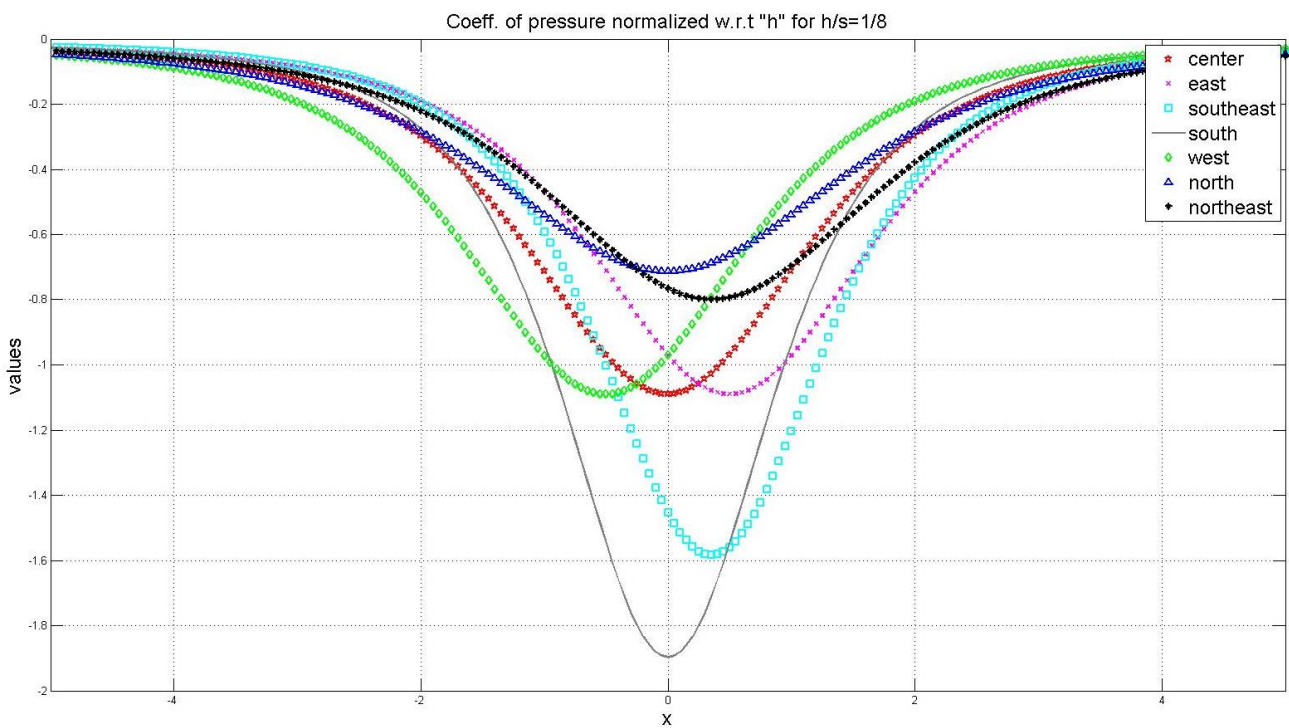


Figure 4.6: Close up of the corresponding coefficient of pressure on the wall for

$$h/s = 1/8$$

Chapter 5

Conclusions, future study and applications

5.1 Conclusions

The results show that the induced velocity is largely dependent on the amount of perturbation of the vortex and its proximity to the wall. The closer the vortex gets to the wall the higher the induced velocity and lower the pressure coefficient. By understanding the pressure signature of the vortices for detecting unstable departures from equilibrium in active control systems, it is important to understand the vortices near the wall, as well as their locally-induced velocity and pressure gradients.

5.2 Potential applications:

The results of this research forms a basis for development of a control system consisting of pressure sensors, microprocessor, and actuators.

Plasma jets or ionic wind generators are potential candidates for the actuators. They have no moving parts and a relatively high frequency response. Arrays of such actuators could be mounted flush to a flat plate, on which a nominally zero pressure gradient turbulent boundary layer develops upstream of the array. However, such actuators can only generate relatively weak forces on the fluid, placing a premium on a high bandwidth for the active control system. A typical plasma actuator is shown in Fig. 5.1.

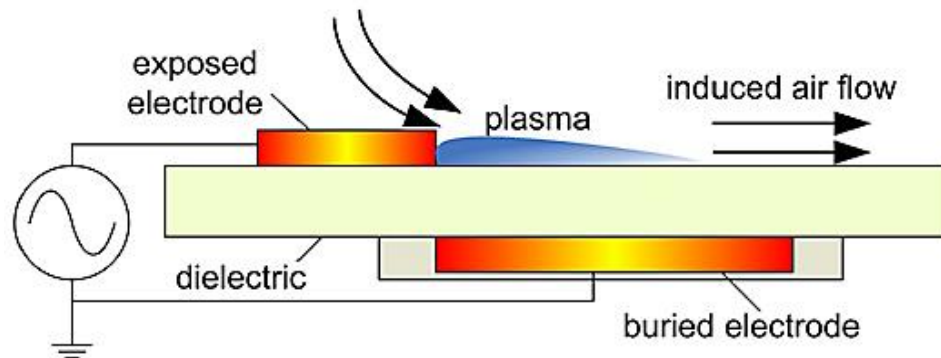


Figure 5.1: A schematic section of a Dielectric-barrier-discharge (DBD) plasma actuator [7]

A DBD plasma is used to induce a flow close to a surface of the plate. The DBD plasma actuator consists of two electrodes separated by a dielectric barrier, Fig.5.1. When a high voltage alternating current is applied, the air close to the exposed electrode is ionized. The ions collide with the surrounding neutral air particles so as to transfer their momentum to the air. Therefore, the plasma actuator can be thought of as imposing a localized body force to the surrounding air. This localized induced velocity would be such that its magnitude and direction is just enough to push the vortex back to its initial position and hold it stationary in that position, thus maintaining the stationarity of the entire array.

To achieve this feat, a control system consisting of the pressure sensors, microprocessors, and actuators could be designed. The pressure sensors in the control system would read the pressure signatures on the wall and detect vortex departures from initial position. Once the pressure changes on the wall or plate surface due to the vortex displacement is read by the pressure sensor, the signal can be used as an input for the control system to activate the plasma actuators. Based on the displacement magnitude of the vortex and the intensity of the pressure on the wall, the control system can determine the amount magnitude and direction of the velocity that would be added in a direction opposite to that of the vortex's motion. This would help the vortex push back to its position and make it stationary.

The control system needs to have a high frequency response pressure sensors to read the pressure signatures on the wall surface. The higher the frequency response of the sensor, the better the efficiency of the system. It is necessary for the pressure sensors to detect to departure of the vortex from its equilibrium position as quickly as possible, in order for the control system to act promptly. If there is an excessive time delay in detecting the departure, the vortices would have moved too far away from their equilibrium positions, and it would require more power to nudge the vortices back into their nominal, equilibrium positions.

5.3 Future study:

The skin friction drag and induced drag due to the stationary vortices array have yet to be determined. When the flow is relaminarized, the local drag will be reduced considerably as compared to turbulent flow.

To make the vortex array stationary, the time it takes for array to become unstable when one pair of vortex in the array is perturbed needs be determined. The instability time can be used as a requirement to determine the frequency response of the pressure sensor and the overall response time of the control system.

References

- [1]. A.J. Cotel and R.E. Breidenthal. Persistence theory of stratified entrainment. In Fourth International Symposium on Stratified Flows, Grenoble, 1994.
- [2]. G.J. Balle. Stationary vortices and persistent turbulence in karman grooves. *Journal of Turbulence*, 3(033), 2002.
- [3]. A.J. Cotel, J.A. Gjestvang, N.N. Ramkhelwan, and R.E. Breidenthal. Laboratory experiments of a jet impinging on a stratified interface. *Experiments in Fluids*, 23:155-160, 1997.
- [4]. A.J. Cotel and R.E. Breidenthal. A model of stratified entrainment using vortex persistence. *Applied Scientific Research*, 57:349-366, 1997.
- [5]. R.E. Breidenthal. Turbulent stratified entrainment and a new parameter for surface fluxes. *Recent Res. Devel. Geophysics*, 2:61-65, 1999
- [6]. P.G. Saffman. *Vortex Dynamics* (1992). Chapter 7, 7.2 Vortices near walls.
- [7]. http://www.temasek-labs.nus.edu.sg/program/program_aeroexperimental_highlight5.php

Appendix

The Code:

%Code to study perturbation effects of vortices counter rotating vortices

% Notations and symbols

%%% w -> $w(z)$ complex potential

%%% k -> circulation strength

%%% h -> vertical distance

%%% b -> horizontal distance

%%% $z = x + iy$

%%% $u - iv = dw/dz$

%%% $u = \text{real}(dw)$

%%% $v = -\text{imag}(dw)$

k = 1; %strength of circulation

base = 2; %horizontal separation between vortices

count = 4; %number of vortices around center pair

h = .5; %vertical distances between vortex array

wdisp = 0.2; %dx - amount to move the vortex

%% Define the grid here

ref = max(base,h);

if count == 0

 xlist=(-2*ref):(0.05):(2*ref);

else

 xlist=(-(count+1)*ref):(0.05):((count+1)*ref);

end

```

[x,y]=meshgrid(xlist,xlist);
z=x+1i*y;
b = 0;
%% Creating the center pair of vortex

%Case 1
w = -1i*k/(2*pi)*log10((z-1i*(b-h))./(z-1i*h)); %case 1 center
dw = (-k/(2*pi)*(b -2*h))./((z-1i*(b-h)).*(z-1i*h)); %case 1

%Case 2
% w = -1i*k/(2*pi)*log10((z-1i*(b-h)-wdisp)./(z-1i*h-wdisp)); %case 2 east
% dw = -k/(2*pi)*(b -2*h)./((z-1i*(b-h)-wdisp).*(z-1i*h-wdisp)); %case 2 east

%Case 3
% w = -1i*k/(2*pi)*log10((z-1i*(b-h)+wdisp)./(z-1i*h+wdisp)); %case 2 west
% dw = -k/(2*pi)*(b -2*h)./((z-1i*(b-h)+wdisp).*(z-1i*h+wdisp)); %case 2 west

%case 4
% w = -1i*k/(2*pi)*log10((z-1i*(b-h-wdisp))./(z-1i*(h+wdisp)));
% dw = -k/(2*pi)*(b -2*h)./((z-1i*(b-h-wdisp).*(z-1i*(h+wdisp)))); %North

w_final = 0;
dw_final = 0;

```

```

%% creating the other vortices around the center pair
for ic = 1:count
    wtemp1 = li*k/(2*pi)*log10((z-li*(b-h)+base*ic)/(z-li*h+base*ic));
    wtemp2 = li*k/(2*pi)*log10((z-li*(b-h)-base*ic)/(z-li*h-base*ic));
    dw1 = -k/(2*pi)*(b-2*h)/((z-li*(b-h) + base*ic).*(z-li*h+base*ic));
    dw2 = -k/(2*pi)*(b-2*h)/((z-li*(b-h) - base*ic).*(z-li*h-base*ic));
    w_final = w_final - wtemp2 - wtemp1; % Sure this will be subtracted?
    dw_final = dw_final - dw1 - dw2;
    clear [wtemp1,wtemp2]
end
w_final = w_final + w;
dw_final = dw_final + dw;

% % h1 = figure;
u = real(dw_final); %horizontal velocity throughout the domain
v = -imag(dw_final); %vertical velocity throughout the domain

U1= (u(440,:)); %horizontal velocity on the wall
C1= -(U1).^2; %Pressure coefficient on the wall

contourf(x,y,v,(-1:0.05:1).^3);
title( 'VERTICAL VELOCITY')

```

```

figure(2)
contourf(x,y,u,(-1:0.05:1).^3);
xlabel('x');ylabel('y');
title('horizontal velocity')
shading interp
colormap hot
colorbar
axis square;
grid on;

figure(3)
plot(xlist,u(200,:),r');xlabel(x);ylabel('horizontal velocity variation along x')
figure(4)
plot(u(:,200),xlist,r');xlabel(y);ylabel('horizontal velocity variation along y')

%% %plot for horizontal velocity at y=0
figure
plot(xlist,U1,r','LineWidth',1.5);
xlabel(x);ylabel('horizontal velocity variation along x');
title('horizontal velocity vortex unperturbed base=10 constant h=1');
axis tight;
ax.Xcolor='k';ax.Ycolor='k';grid on; grid minor;

% plot for Coeff of Pressure at y=0
whitebg(figure,'w');
plot(xlist,C1,r','LineWidth',1.5);
xlabel(x);ylabel('values');
axis tight;
title ('Coeff. of pressure vortex unperturbed count=4 base=10'); grid on

```