Prospective Teachers' Affective Experiences of an Inquiry-Oriented Mathematics for Elementary School Teachers Course

Noelle Conforti Preszler

A dissertation
submitted in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy

University of Washington

2014

Reading Committee:

Elham Kazemi, Chair

Susan Bobbitt Nolen

Deborah Kerdeman

Program Authorized to Offer Degree:
College of Education

University of Washington

Abstract

Prospective Teachers' Affective Experiences of an Inquiry-Oriented Mathematics for Elementary School Teachers Course

Noelle Conforti Preszler

Chair of the Supervisory Committee:
Professor Elham Kazemi
College of Education

As mathematics education literature emphasizes the importance for students to be actively engaged in the learning of mathematics, research has articulated how teachers can implement such courses and teacher education has encouraged prospective teachers to take on such teaching methods. However, little is known about how students, particularly undergraduates, psychologically experience the inquiry-oriented math classroom. This study contributes to such literature by exploring students', specifically prospective teachers', affective experiences of an inquiry-oriented Mathematics for Elementary School Teachers content course, including influences of their existing affective experiences with mathematics and their anticipated future mathematics teaching. The self-determination theory constructs of competence, relatedness, and autonomy are leveraged to carefully attend to the affective experiences of students, making direct connections between the participatory and cognitive demands of the inquiry-oriented classroom and students' affective experiences. Characterization of the classroom in this study revealed that

students were expected to discuss mathematics with others, construct their own solution strategies, be intellectually courageous, and to let their thinking develop over time. Three major findings around prospective teachers' affective experiences in mathematics were revealed: (1) students expanded the ways in which they felt competent in mathematics while participating in an inquiry-oriented course; (2) students' new sense of relatedness in math class stemmed from the community fostered within an inquiry-oriented course; and (3) students valued having the autonomy to pursue their own mathematical reasoning. In addition, there were two ways that students' affective experiences within this course appeared to promote their developing teaching practices: (1) students felt more positive about teaching mathematics after participating in an inquiry-oriented math course; and (2) experiences within this inquiry-oriented classroom fostered prospective teachers' desires to support student inquiry in their future mathematics teaching practices. These results suggest that prospective teachers had an influential experience within this inquiry-oriented math course, one that might positively affect their relationships with mathematics as a discipline and their developing mathematics teaching practices.

Table of Contents

List of Figures

List of Tables

ACKNOWLEDGEMENTS

This dissertation would not have been possible without the support of many. First, my deepest gratitude goes to my advisor, Elham Kazemi. Even having had years to find the words, nothing comes close to describing the support, both professional and personal, that you have given me. You are supremely dedicated, encouraging, and confidence-inspiring. From the first time we talked I wanted to learn from and work with you. I am grateful for the opportunity.

I also owe a great debt of thanks to my "math" advisor, Fred Adler, who supported me both when I decided to leave the math world for the math education world and when I decided to return to the math world for employment. You taught me many things, but two stand out: never apologize for your work and acronyms and sarcasm have a (great) place in academia.

I also thank my committee members, Sue Nolen, Debby Kerdeman, and Judith Arms. Sue, thank you for helping me attend to precision in the final drafts. Debby, I will forever be grateful for the early stages meeting in which you asked me to "talk about my ideas." I still have the pages of notes you took while you listened, with my research question boxed and starred at the end. Thank you for the opportunity to talk without expectation.

I have had the great pleasure of working alongside two of the best post-doctoral scholars I am sure I will ever know. Rachel Phillips, you are so much more than a mentor to me, you are a true friend. You supported and encouraged my work and development as a teacher and scholar in ways no one else could. I can only hope to someday emulate the way you leveraged my strengths, allowing me to contribute meaningfully to our research work together while realizing my own potential. Lynsey Gibbons, your constant check-ins via email and phone kept me going during the most difficult times.

To the UW Mathematics Education Research Group, all those MERGers out there who came before me and softened the ground, leaving signposts along the way, and all those MERGers who came after me, looking to me as a leader and mentor, I thank you. Words do not express the gratitude I have for the support you have provided me over the years. Thank you: Allison, Maria, Teresa, Becca, Helen, Keri, Biz (not a MERGer, but still), Kate, Leslie, and Liz.

My sincerest thanks goes out to the Math for Elementary School Teachers instructor and students who not only allowed me into their daily classroom lives as if it was not at all the slightest inconvenience, but went above and beyond to encourage me with their enthusiasm for this work.

Thank you to my family, immediate and extended, for their constant support. If you ever let me talk about my work, or ramble on about math education, you played a role in shaping this dissertation. To my brother, Byron, for constantly asking for updates and genuinely caring about my progress, without asking when I would finish the degree. And to my dad, for making me feel like I can do anything, and telling me that I am good at everything (even making pottery...).

Jason. Without your endless support and constant encouragement, this work might never have come to completion. Thank you for letting me talk, even when you had no idea what I was talking about, and for your genuine interest in my scholarly work. You are my best friend, an inspirational math teacher, and my most treasured colleague. I love you.

I also have to thank Duchess, who spent more time with me than anyone else during the final months of this process. Thank you for sleeping next to me on the couch during long writing sessions, and for learning how to encourage breaks without jumping on my laptop (I still miss that “m” key sometimes...). Let's go play—finally!

DEDICATION

For Dad. We are always together.

To Jason. Two and two thirds PhDs is enough.

CHAPTER 1: INTRODUCTION

Statement of the Problem

The mathematics education literature overwhelmingly emphasizes that students should be actively involved in the construction of mathematical knowledge, rather than simply passive receivers of such knowledge (e.g., Kazemi & Stipek, 2001; Lampert, 1990; Martino & Maher, 1999; National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; Sherin, 2002; Yackel, 2002). Research on inquiry-oriented classrooms that aim to provide an environment conducive to this has focused a great deal on the role of the teacher in creating and managing such a classroom (Franke, Kazemi, & Battey, 2007). However, a growing body of literature has explored students' experiences in mathematics classrooms (e.g., Hintz, 2011; Zavala, 2012), including a few studies focused on students' perceptions of mathematics in various types of classrooms (e.g., Boaler & Greeno, 2000), but little research has been conducted on how students make sense of their *affective* experiences in inquiry-oriented classrooms aiming for high quality mathematics instruction. Students' affective experiences might be particularly relevant here as inquiry-oriented classrooms set new, demanding expectations on student participation in the mathematics classroom.

Although there does appear to be a link between inquiry-oriented classrooms and higher quality student engagement with mathematics documented in the literature, few, if any, connections are made to motivation frameworks that could help articulate the affective aspects of students' engagement in these mathematics classrooms. One goal of this study is to begin to fill in that gap, to use motivation constructs from self-determination theory (Ryan & Deci, 2000) to document how students are affectively experiencing the inquiry-oriented mathematics classroom,

particularly as those experiences relate to the cognitive and participatory demands of such a classroom. This study focuses specifically on articulating prospective elementary school teachers'[1] affective experiences in the inquiry-oriented undergraduate mathematics classroom, and the potential effects of those experiences on the prospective teachers' future teaching practices. Prospective teachers are the particular focus here because their experiences in such a course, along with their other experiences as a student, have been shown to help shape their future teaching practices (e.g., Ball, 1988).

Research Questions

As such, this study addresses the following research questions:

1. How do prospective elementary school teachers understand their affective experiences of competence, relatedness, and autonomy (self-direction) over the course of participating in an inquiry-oriented mathematics content course for teachers?
   a) How do students describe their existing affective experiences of mathematics?
   b) How do students make sense of their affective experiences in relation to the cognitive and participatory expectations and demands of the current inquiry-oriented classroom?
2. What influences do prospective teachers anticipate these affective experiences will have on their future mathematics teaching practice?

Conceptual Framing

I conceptually framed my study of students' affective experiences in an inquiry-oriented classroom within a sociocultural perspective. I worked to understand the classroom microculture

1 I use "prospective teachers" to emphasize my focus on adult learners planning to become teachers. I use the word "students" to discuss previous research on K-16 students who are not prospective teachers (but in some cases are adults) and to refer to the prospective teachers within this study as students in the class being studied.

within which students were experiencing mathematics in order to get a full picture of students' affective experiences. I argue that contextualizing students' experiences is essential to understanding how they feel about their experiences of mathematics and to connecting those feelings to particular student demands that are present in an inquiry-oriented mathematics classroom.

In order to carefully attend to students' affective experiences, I utilize constructs from the motivation framework of self-determination theory to articulate the types of affective experiences, both positive and negative, that students are having in the inquiry-oriented mathematics classroom. Specifically, I utilize the psychological needs of competence, relatedness, and autonomy, to both fully characterize students' feelings using meaningful terms within the motivation literature, and to make connections between students' experiences and the cognitive and participatory demands of an inquiry-oriented mathematics classroom.

Major Results

The inquiry-oriented classroom in which this study took place fostered student engagement with mathematics in a variety of ways. Specifically, characterization of the classroom revealed that students were expected to discuss mathematics with others, construct their own solution strategies, be intellectually courageous, and to let their thinking develop over time. This engagement with mathematics was typically supported by having students consistently work in small groups. Analysis of the prospective teachers' descriptions of their experiences with mathematics before and during this course suggest that they had more positive affective experiences within this inquiry-oriented course than in their past courses. Three major findings around prospective teachers' psychological needs satisfaction in mathematics were revealed in this study: (1) students expanded the ways in which they felt competent in

mathematics while participating in an inquiry-oriented course; (2) students' new sense of relatedness in math class stemmed from the community fostered within an inquiry-oriented course; and (3) students valued having the autonomy to pursue their own mathematical reasoning. In addition, it was clear that prospective teachers were developing their mathematical teaching practices while engaged in this course. Specifically, there were two ways that students' affective experiences within this course appeared to promote their developing teaching practices: (1) students felt more positive about teaching mathematics after participating in an inquiry-oriented math course; and (2) experiences within this inquiry-oriented classroom fostered prospective teachers' desires to support student inquiry in their future mathematics teaching practices. These results suggest that prospective teachers had an influential experience within this inquiry-oriented math course, one that had the potential to transform both the ways they considered themselves in relation to mathematics as a subject and how they considered mathematics teaching and learning.

Organization of the Dissertation

This dissertation is organized in the following way. In Chapter 2, I further describe the rationale for this study, articulating the need for research focusing on affective experiences of students in inquiry-oriented mathematics classrooms, reviewing the literature that situates this study. Chapter 3 articulates the conceptual framing for the study, highlighting the sociocultural nature of the study and, in particular, how I utilized the motivation framework of self-determination theory to inform my study of students' affective experiences. In Chapter 4, I describe the study methodology, including details of data collection and analysis, and give an introduction to the focal students and the classroom at the heart of this study. Chapter 5 characterizes the inquiry-oriented nature of the classroom, including the instructor's design of the

course and the observed implementation of the course, providing the details necessary to situate the focal student experiences, which are described in Chapter 6. The case studies described in Chapter 6 articulate how the focal students in this study affectively experienced this inquiry-oriented Mathematics for Elementary School Teachers course, specifically addressing the change over time indicated by the research questions, and the developing teaching practices of the prospective teachers within this study. Finally, in Chapter 7, I discuss study results, including cross-case analysis of the focal student case studies, and highlight implications relevant for teaching and learning of mathematics and future research, particularly around the implementation and affordances of inquiry-oriented mathematics.

CHAPTER 2: RATIONALE AND SITUATING LITERATURE

This study of students' affective experiences of an inquiry-oriented undergraduate mathematics classroom is situated within literature that characterizes inquiry-oriented classrooms and inquiry-oriented learning in general, literature around affect in mathematics education, and literature around elucidating student experiences, particularly at the undergraduate level and experiences of future teachers. This chapter focuses on synthesizing these bodies of literature, articulating how they contribute to and demonstrate the need for research into how students make sense of the affective experiences they have while participating in an inquiry-oriented math content course for prospective teachers.

Inquiry-Oriented Mathematics

There is broad consensus in the mathematics education literature that students should rigorously construct mathematical knowledge in the classroom, rather than passively receive their mathematical knowledge from teachers (e.g., Kazemi & Stipek, 2001; Lampert, 1990; Martino & Maher, 1999; NCTM, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; Sherin, 2002; Yackel, 2002). In this study, I use the phrase *inquiry-oriented* to characterize mathematics classrooms in which teaching and learning is designed to support students as they engage in constructing mathematical knowledge. Inquiry-oriented classrooms are places where students are actively involved in classroom learning by discussing reasoning, justifying strategies, suggesting approaches, asking questions, evaluating the ideas of others, and much more.

The notion that students should be seen as knowledgeable resources in the classroom, charged with engaging in meaningful ways with the content they are learning, is not new. Indeed, since the turn of the 20th century, John Dewey (e.g., 1897, 1997) has promoted this type

of teaching and learning. Dewey (1997) suggests that students are more than mindless information receivers. "In schools, those under instruction are too customarily looked upon as acquiring knowledge as theoretical spectators, minds which appropriate knowledge by direct energy of intellect" (Dewey, 1997, p. 140). According to Dewey, students are to play an active role in thinking about material, using material, developing it for themselves, and taking ownership over the material in a way that allows them to do much more than accumulate knowledge for display. He argues that students must go beyond following rote procedures without personal understanding of content or purpose to develop genuine understanding of material and aims. When students are no longer following rote procedures simply to satisfy teacher demands, they are engaging in authentic understanding of the utilities of subject matter.

Fundamentally, for Dewey (1997), the goal of education "is to develop their [students'] ability to think" (p. 152). That is, accumulation of skills and knowledge considered only from a list of school subjects to be learned is not, in the end, successful education. Dewey says, "...ultimately the value of knowledge is subordinate to its use in thinking. For we live not in a settle and finished world, but in one which is going on, and where our main task is prospective..." (p. 151). According to Dewey, students should be capable of functioning in a changing world, one which cannot be anticipated beforehand or met with a stale list of acquired facts. Rather, knowledge and skills must be used actively in thought, to meet the needs of any current situation.

Dewey provides a philosophical foundation for what it means for learning to be genuinely educative. Ultimately, students should be capable of active, reflective thinking, and should be able to use the knowledge and skills from their education in attaining meaningful goals and life purposes. Mathematics classrooms that can be characterized as inquiry-oriented position

students in just the ways that Dewey calls for. In mathematics classrooms that support inquiry-oriented learning, teachers must support students as they are expected go beyond simply answering questions correctly, helping them to make and justify conjectures (Lampert, 1990), explain their reasoning (Kazemi & Stipek, 2001), and generalize beyond individual problem solutions (Martino & Maher, 1999). Similarly, students and teachers must work together to negotiate the sociomathematical norms that guide what "counts" as acceptable mathematical argument, reasoning, and justification in their particular classroom (Yackel & Cobb, 1996). In inquiry-oriented classrooms, students are seen as valuable resources of mathematical knowledge and ideas, and as a result, are given more autonomy over their participation, rather than being asked to simply follow rote procedures (Boaler & Greeno, 2000). Students are asked to contribute more to the learning process and teachers must be willing to learn from their students (Sherin, 2002) and should have both deep understanding of mathematical concepts and their students' conceptual understanding (Yackel, 2002) in order to facilitate inquiry-oriented classrooms. In short, the inquiry-oriented math classroom is a place where students: (1) approach mathematics conceptually (rather than rotely), which redefines competence in the classroom to go beyond displaying correct answers, (2) play an active role as members of a mathematical community, and (3) are given the autonomy in the classroom necessary to do so.

Inquiry-oriented classrooms place high demands on students, to go beyond passively receiving knowledge to become active, engaged mathematical thinkers. I argue that these demands necessitate that attention be paid to students' affective experiences in the classroom, not only in addition to, but in relation to, their cognitive and participatory experiences.

Affect in Mathematics Education

Indeed, important goals relating to students' affective experiences in mathematics are

interwoven throughout the Principles and Standards articulated by the NCTM and in the Common Core Standards for Mathematical Practice. While learning mathematics content, students are expected to also reflect on their abilities and understanding, become confident in those abilities and persevere when required, developing the independence to take control of their own learning (NCTM, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Although affect[2] itself, or the experience of feelings and emotions, is generally a well researched topic, research on affect in mathematics education has had little influence on curriculum or instruction, as affect is frequently considered separate from mathematical thinking (McLeod, 1992; Zan, Brown, Evans, & Hannula, 2006). As a result, calls for affective research in mathematics education suggest that connections be made between the cognitive and affective aspects of the mathematics classroom (McLeod, 1992; Zan et al., 2006). Like others (e.g., Gresalfi, 2009; Turner, Warzon, & Christensen, 2011), I take up this call, and consider students' affective experiences with mathematics to be an integral part of their learning of and engagement with mathematics. In addition, McLeod (1992) suggests that as mathematics classrooms are reformed to be more inquiry-oriented, students and teachers will have "intense affective reactions" (p. 591) to these new ways of engaging with mathematics that will need to be documented. Documenting *the relationship between* students' affective and cognitive experiences is important for capturing a full picture of students' experiences in the mathematics classroom, and is a major goal of this study.

Inquiry-Oriented Mathematics at the Undergraduate Level

There is limited empirical research on undergraduate mathematics teaching practices

2 I use the term affect to refer to students' psychological needs as defined in self-determination theory. Other researchers use the term in different ways. For example, Phillip (2007) includes emotions, attitudes, and beliefs.

(Speer, Smith, & Horvath, 2010), particularly related to inquiry-oriented teaching, in part because learner-centered math and science teaching is much less prevalent at the undergraduate level (Walczyk & Ramsey, 2003). The limited research in this area has shown that college faculty rarely teach mathematics classes in student-centered ways that foster engagement and encourage students to develop their own mathematical knowledge (Rasmussen & Kwon, 2007; Walczyk & Ramsey, 2003; Walczyk, Ramsey, & Zha, 2007). Rather, "lecture – recitation – evaluation is alive and well in college classrooms" (Walczyk & Ramsey, 2003, p. 579), even when the institution emphasizes teaching over research.

However, in recent years federal funding has been dedicated to projects intended to promote learner-centered teaching at all levels, including for undergraduates and prospective math and science teachers (Walczyk & Ramsey, 2003). A large mixed-methods study recently undertaken by the Assessment and Evaluation Center for Inquiry-Based Learning in Mathematics (2011) to explore the outcomes of inquiry-oriented learning at the college level provided a great deal of information including, among other things: characteristics of such classes, a plethora of information about learning gains (both self-assessed and test-based), and reports of affective gains like "confidence, enjoyment, and interest" (p. 122). A subset of students were briefly interviewed, but the focus was on how students described the practices of the inquiry-oriented classroom, and no attempt was made to link affective comments to the cognitive experiences students also described in these interviews. As a result, the affective comments articulated in the report are shallow, indicating general enjoyment or confidence gains.

The study also provides a thorough review of the small body of existing research on inquiry-oriented teaching at the college level, for both "math track" and prospective K-12 teachers. Generally speaking, more attention has been paid to student achievement and

understanding (both of content knowledge and the nature of mathematics) as it relates to inquiry-oriented classroom experiences (Assessment & Evaluation Center for Inquiry-Based Learning in Mathematics, 2011). A small subset of these studies of inquiry-oriented mathematics have focused on student attitudes and beliefs, but that focus has been limited to gains in confidence and ideas about the teaching and learning of mathematics, and has not focused on how students affectively experience the classrooms or the effects of those experiences (Assessment & Evaluation Center for Inquiry-Based Learning in Mathematics, 2011). In short, there is much more to learn about students' experiences of undergraduate mathematics courses.

The Importance of Prospective Teachers' Experiences as Students

This study focused specifically on an undergraduate mathematics content course for future elementary school teachers. Since inquiry-oriented classrooms have been shown to benefit student engagement and learning, there is interest in training future teachers to be capable of fostering such environments (Walczyk & Ramsey, 2003). Indeed, K-12 teachers are expected to foster inquiry-oriented teaching and learning in their classroom, encouraging students to be active participants in the classroom (NCTM, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). For example, the Common Core Standards for Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) articulate types of expertise that teachers should be fostering in their students. Among other things, students should be able to (a) “make sense of problems,” analyzing problem situations to find entry points, plan solution strategies, revising their thinking as necessary; and (b) “construct viable arguments and critique the reasoning of others,” which involves communicating a justifiable solution strategy to others and considering other strategies that are presented. Just from these two brief examples, we can see the kinds of

rigorous mathematical thinking and learning that teachers are being asked to develop with their students. The expectations go far beyond making sure students get correct answers or check off lists of procedural skills, though computational fluency is also an important goal.

Of additional importance to consider here is the knowledge that prospective teachers' beliefs and ideas about mathematics and teaching are shaped by their own personal experiences as students (Ball, 1988; Calderhead, 1991; Pajares, 1992; Zeichner, Tabachnick, & Densmore, 1987). Ball (1988) articulates clearly the notion that prospective teachers have had countless hours in the classroom as students before they become teachers, and during this time they have had experiences that shaped how they think *and feel* about school, mathematics, and the roles teachers and students play in the classroom. Additionally, Ball (1988) makes an important note about how prospective teachers' past experiences might influence their teaching practices:

> If they were successful in mathematics, prospective teachers are likely to approve of the patterns they saw and thus be uninterested in alternative ways of teaching. If they struggled, they may aspire to teach differently. Even if they are critical of their own past teachers for teaching badly and for making them feel stupid, they may lack alternative models. (p. 46)

I have found these distinctions to be true in both my personal teaching experience with prospective teachers and within this study. Documenting the classroom experiences of future teachers matters significantly because those experiences will influence how the prospective teachers will eventually teach. If teachers shape their practice based on their own experiences as students, and it is expected that they teach in inquiry-oriented ways, it makes sense to consider how prospective teachers think about their experiences as math students, particularly in relation to their developing teaching practice.

Prospective teachers' experiences within a Math for Elementary School Teachers course are not only important because their experiences as students shape their future teaching, but also because their experiences as students in this particular course are intended to provide them with specialized content knowledge required for teaching (Ball, Thames, & Phelps, 2008). Some examples this type of knowledge include: "giving or evaluating mathematical explanations," "asking productive mathematical questions," and "selecting representations for particular purposes" (Ball et al., 2008, p. 400). This means prospective teachers need to learn successfully in their math content courses for teachers. Thus, prospective teachers' experiences of this course are important for both their own mathematical learning and their future teaching practice. Rather than research prospective teachers experiences from an outside perspective, I will focus on understanding how the prospective teachers themselves make sense of their affective experiences in the inquiry-oriented classroom.

Considering Students' Perspectives on Their Own Experiences

In one of the limited number of examples researching students' perceptions of the classroom environment, Boaler and Greeno (2000) compared discussion-oriented AP calculus classrooms with didactic AP calculus classrooms. This study informs my own because it explores the differences between traditional and inquiry-oriented classrooms, does so from the perspective of the learner, and goes beyond documenting students' cognitive experiences.

They found that students in traditional classrooms, which they called didactic classrooms, did not equate mathematical success to ability, but instead, a willingness to accept the procedural nature of mathematics that required obedience, compliance, perseverance, and a willingness to be frustrated (Boaler & Greeno, 2000). In contrast, students interviewed from classrooms centered around discussion spoke about being active learners who think about why they solve

problems the way they do, emphasizing how this helps them understand mathematical concepts (Boaler & Greeno, 2000). Some students in the didactic mathematics classrooms lacked a sense of belonging, either because they were unwilling to give up their autonomy to follow procedures or because they wanted a more meaningful connection with mathematics content (Boaler & Greeno, 2000). These students described how mathematics did not fit with their perceived identity of themselves. Boaler and Greeno make the case that presenting mathematics as procedural knowledge positions students to simply memorize and recite someone else's ideas, rather than view themselves as individuals who can discover, shape, and construct mathematical knowledge.

Although Boaler and Greeno focus largely on high school students' perceptions of mathematics, their comparison of these two types of classrooms also reveals details about students' personal experiences, which are worthy of further investigation. Mainly, that inquiry-oriented classrooms can both redefine what being competent in mathematics means to students and have the potential to alter students' sense of whether they belong in mathematics. Knowing more about how students make sense of their own affective experiences in the inquiry-oriented mathematics classroom will further elucidate our understanding of inquiry-oriented teaching and learning.

The Research Questions

Little research has unpacked *how students understand* their affective experiences in inquiry-oriented mathematics classrooms, particularly as these experiences link to cognitive experiences in such classrooms or how students believe those experiences might influence their future participation in the current course, other math work, and future teaching. Additionally, exploration of students' perceptions of their affective experiences in an inquiry-oriented course

can reveal connections between their current and past affective experiences in mathematics, which may or may not have been in inquiry-oriented mathematics courses. Figure 1 illustrates the possible connections that are explored in this study. In particular, this study addresses the following research questions:

1. How do prospective elementary school teachers understand their affective experiences of competence, relatedness, and autonomy (self-direction) over the course of participating in an inquiry-oriented mathematics content course for teachers?
   a) How do students describe their existing affective experiences of mathematics?
   b) How do students make sense of their affective experiences in relation to the cognitive and participatory expectations and demands of the current inquiry-oriented classroom?
2. What influences do prospective teachers anticipate these affective experiences will have on their future mathematics teaching practice?

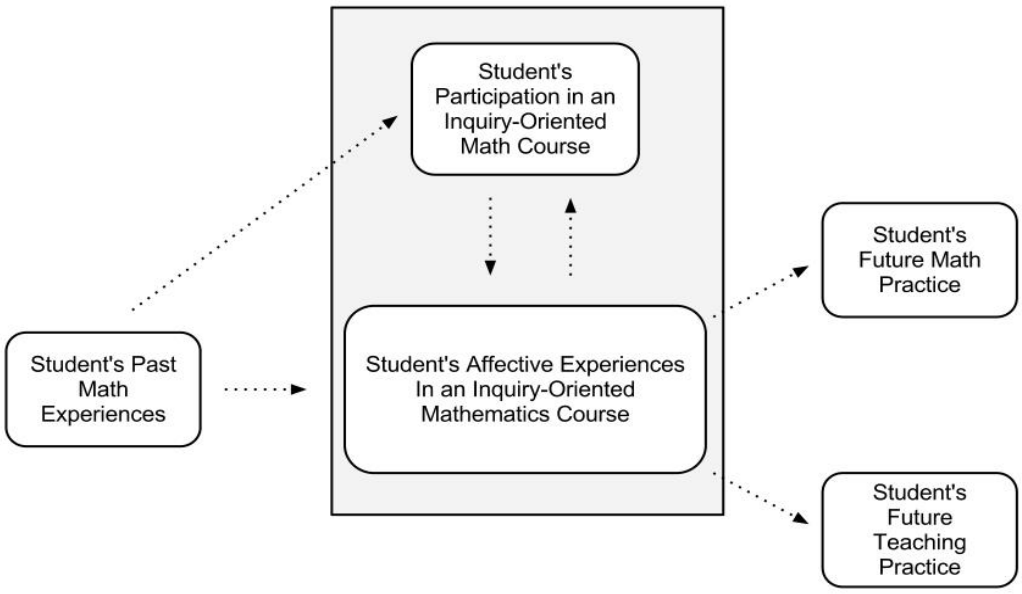


Figure 1. Potential connections to be explored in this study.

Conclusion

This chapter reviewed the literature that helps rationalize the need for study of prospective teachers' affective experiences in inquiry-oriented mathematics courses. In particular, I defined the term inquiry-oriented, discussing generally what inquiry-oriented classrooms look like and existing research of this type of environment. I also emphasized the importance of students' affective experiences, particularly for future teachers, and how important it is to consider the effects of future teachers' experiences as students. In the following chapter, Chapter 3, I will articulate the conceptual framing for this study, explaining how I used the constructs of competence, relatedness, and autonomy from self-determination theory to carefully attend to the students' affective experiences, making links between those affective experiences and the participatory and cognitive demands placed upon students in the inquiry-oriented math classroom.

Chapter 3: Conceptualizing Students' Affective Experiences in Inquiry-Oriented Mathematics Classrooms

This chapter articulates how I conceptually framed my study of students' affective experiences in an inquiry-oriented classroom. I begin by discussing the importance of a sociocultural perspective to my study, highlighting the role the classroom microculture plays in students' experiences and the necessity of understanding that role in order to more completely understand students' experiences within that influential context. I then explain how I utilize the motivation framework of self-determination theory (SDT) to carefully characterize the types of affective experiences students might be having in the inquiry-oriented mathematics classroom. In particular, I articulate how I link constructs from self-determination theory, feeling competent, related, and autonomous, to the cognitive and participatory demands of such a classroom.

Taking a Sociocultural Perspective

Sociocultural approaches focus on the relationship between the context and the individual, suggesting that the individual cannot be understood outside of their context (Luria, 1979; Nolen & Ward, 2008; Rogoff, 2003; Wertsch, 1988). That is, individual development can only be fully understood through understanding the larger cultural or community practices (Luria, 1979; Rogoff, 2001; Wertsch, 1988). In contrast, cognitive and motivational psychologists, such as the researchers using self-determination theory, to be utilized here and described below, frequently consider motivational constructs as characteristics of individuals, which can be manipulated by changing the social context (Nolen, Ward, & Horn, 2011). This perspective acknowledges the influence of the environment on the individual, but stops short of considering how the individual fits into and potentially influences their larger community or context.

In this study, a sociocultural approach will be taken, with the inquiry-oriented classroom considered to be a microculture in which students participate and develop. As a result, understanding that culture, putting students and their experiences in their appropriate context, is essential to understanding how students make sense of their affective experiences while participating within that culture. That is, we need to know how the classroom is organized, what is expected of students, how students participate and engage with mathematics and other students in the course, how their past experiences shape their current participation, and much more, in order to fully understand the relationship between the inquiry-oriented mathematics classroom and the student experience. Knowing this type of information will allow for consideration of how and to what extent the classroom environment influences students' experiences and feelings, and allow for consideration of how students' experiences and feelings in turn influence the classroom environment, and potentially the experiences of other students in the classroom.

In Chapter 5, the Mathematics for Elementary School Teachers classroom will be characterized, particularly with respect to the ways the classroom was intended and implemented to be inquiry-oriented. That chapter will therefore provide rich descriptions of the instructor's views and intentions, student participation and interaction, and the classroom in general, so that prospective teachers' experiences, which will be described in Chapter 6, can be situated within the classroom microculture in which they occurred.

Self-Determination Theory as a Lens to Study of Students' Affective Experiences

Self-determination theory, a widely used approach to studying human motivation and personality, provides a framework for characterizing students' affective classroom experiences. I will suggest that although SDT is a theory that speaks to human development in a general, universal sense, constructs from the theory will allow me to attend carefully to the ways in which

students affectively experience the inquiry-oriented mathematics classroom.

A basic assumption of self-determination theory is that human beings are innately intrinsically motivated, meaning they have a propensity to pursue something for its own sake (Ryan & Deci, 2000). As a result, self-determination research has focused on the factors that hinder or prevent individuals' natural tendencies toward intrinsic motivation (Ryan & Deci, 2000). Results from such research suggest that the satisfaction of three basic psychological needs---autonomy, competence, and relatedness---fosters motivation, improves performance, and enhances mental health. "Simply stated, motivation, performance, and development will be maximized within social contexts that provide people the opportunity to satisfy their basic psychological needs for competence, relatedness, and autonomy" (Deci, Vallerand, Pelletier, & Ryan, 1991, p. 327-8). Autonomy is defined as "...self-initiating and self-regulating one's own actions" (Turner et al., 2011, p. 721), competence as "understanding how to achieve certain outcomes and feeling efficacious in performing the required actions" (Turner et al., 2011, p. 720), and relatedness is defined as "the need to feel belongingness and connectedness with others" (Ryan & Deci, 2000, p. 73). It is the fulfillment of these three psychological needs that forms the backbone of self-determination theory.

Aspects of social context that have been shown to facilitate intrinsic motivation include choice (under some circumstances, see Katz & Assor, 2007), acknowledgment of feelings, and chances for self-direction, while imposed goals, threats, deadlines, and, in some cases, tangible rewards based on performance have been shown to undermine intrinsic motivation, as they imply an external locus of control (Ryan & Deci, 2000). Expression of intrinsic motivation is also fostered by feelings of relatedness or security, a sense of social belonging (Ryan & Deci, 2000). Regarding competence, Ryan and Deci (2000) specify "optimal challenges, effectance-promoting

feedback, and freedom from demeaning evaluations" (p. 70) as events that incite feelings of competence. SDT research also suggests that intrinsic motivation will not be promoted by feelings of competence unless a sense of autonomy, or freedom, is also felt (Ryan & Deci, 2000). Autonomy-supportive classrooms also result in higher learning and well-being than controlling classrooms, particularly when learning is conceptual or creative (Ryan & Deci, 2009).

Because self-determination research has shown that student learning and well-being is improved by fostering feelings of competence, relatedness, and autonomy in the classroom, SDT has been used to structure changes to the classroom environment and changes to teachers' behavior. For example, in their study on teachers' beliefs about motivation in mathematics, Turner et al. (2011) introduced a series of classroom strategies intended to help teachers foster competence, relatedness, and autonomy in their middle school mathematics classrooms. The SDT constructs were introduced to teachers as tangible strategies for improving their teaching, such as giving students meaningful feedback (competence), grouping students to facilitate belongingness (relatedness), and encouraging students to be active classroom contributors (autonomy). Similarly, a second example from Reeve and Halusic (2009) illustrates how teacher questions about carrying out autonomy-supportive teaching can be transformed into autonomy-supportive teaching techniques. These examples illustrate how SDT research results on environments that satisfy students' psychological needs have been appropriated for use in the classroom, emphasizing what teachers might do to create environments where students' competence, relatedness, and autonomy needs are more likely to be met.

Traditionally, self-determination research on determining classroom characteristics that facilitate intrinsic motivation has been carried out using quantitative research methods. Experimental procedures have allowed researchers to directly manipulate and measure how

social context variables affect individuals' self-motivation and integration of external motivations (Ryan & Deci, 2000). For example, students might participate in one of several experimentally designed class sessions and take a survey intended to assess their feelings of autonomy, competence, and/or relatedness alongside their perception of how deeply they engaged with the classroom material (for an example, see Vansteenkiste, Simone, Lens, Sheldon, & Deci, 2004).

While research utilizing self-determination theory has been used to suggest classroom characteristics that promote learning and fulfillment, SDT also provides a lens through which students' affective experiences in the classroom can be explicated. In addition to asking students *whether* they experience feelings of autonomy, relatedness, and competence in certain environments, SDT allows for investigation of *how* students understand their affective experiences and what sense they make of experiencing such feelings in the classroom. More specifically, the use of SDT constructs will allow me to investigate what it means to students to experience feelings of autonomy, competence, and relatedness in the mathematics classroom, why they believe they experienced those feelings, and how those feelings affected them in the moment and might affect them in their future classroom and teaching experiences.

Using Self-Determination Theory to Study Students' Affective Experiences in the Inquiry-Oriented Mathematics Classroom

Self-determination theory provides a valuable characterization of the psychological needs and feelings of learners. Generally speaking, review of the mathematics education literature shows that the psychological needs of competence, relatedness, and autonomy as defined in SDT are closely related to how inquiry-oriented mathematics classrooms are designed. That is, inquiry-oriented classrooms, which redefine competence, foster a mathematical community, and provide students with more autonomy over their participation in the classroom to increase

student engagement, are ideal places to study whether and how students' psychological needs are being met. SDT provides a framework which puts the focus on students' feelings and affective experiences in these classrooms. Utilizing the self-determination constructs to study the ways in which students affectively experience the inquiry-oriented mathematics classroom will complement existing research that documents students' cognitive and participatory experiences in such classrooms. Figure 2 shows how this study conceptualizes this idea, with cognitive and participatory demands placed on students shown in solid ovals, with associated affective experiences in dotted lines around the outside.

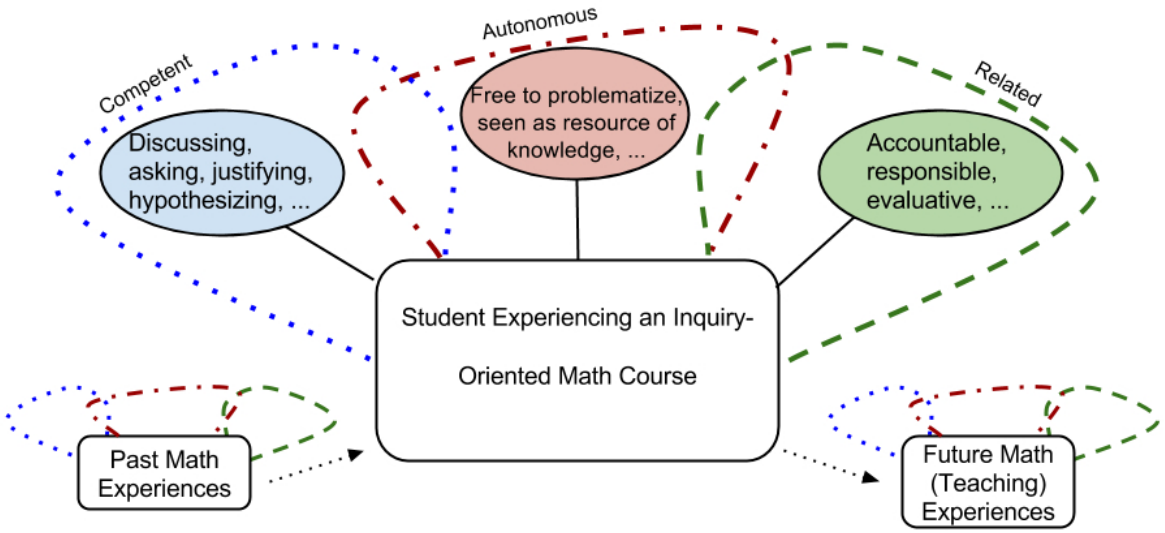


Figure 2. Conceptual framing of students' participatory and affective experiences in an inquiry-oriented mathematics classroom.

Notice that there may be overlap in students' feelings (ie: if a student feels a sense autonomy related to being seen a source of knowledge, they may also feel competent, or able to express their knowledge) and that in this study, students' past affective experiences may influence their current experiences and their current experiences may influence their future experiences of mathematics The details of these connections will now be discussed in more detail.

Experiencing Competence

In inquiry-oriented mathematics classrooms, competence is redefined to be more than giving correct answers or following rote procedures, students are expected to be "making conjectures, abstracting mathematical properties, explaining their reasoning, validating their assertions" (Lampert, 1990, p. 32-33). For example, when students are expected to explain their thinking by justifying their reasoning or encouraged to generalize their arguments, they are more engaged conceptually with the mathematics (Kazemi & Stipek, 2001; Martino & Maher, 1999; Webb, Franke, Chan, De, Freund, & Battey, 2008). In addition to cognitively having more meaningful engagement with mathematics, in these types of situations students have the potential to *feel* competent even if they cannot fully follow a procedure or provide a correct final answer. Students might feel competent if they can make suggestions, justify particular steps in the process, or try to explain their reasoning. Additionally, students might feel less competent as they are asked to participate in unfamiliar, cognitively demanding ways, particularly if they have past success in traditional mathematics courses. By asking students how they felt during various situations in which they had the opportunity to show competence, we can understand what it means to students to be able to complete required tasks for participation (Ryan & Deci, 2000). That is, when students are given more opportunities to demonstrate competence, how do they see, interpret, and take advantage of those opportunities? How do students describe feeling competent and how do they believe feeling (or not feeling) competent affects them?

Experiencing Relatedness

Students are an important part of the classroom community while participating in inquiry-oriented mathematics classes. While developing the ability to use their own mathematical knowledge, students also aid in the determination of sociomathematical norms

(Yackel & Cobb, 1996), helping to hold others accountable to both classroom and mathematical norms, evaluating others' ideas and reasoning. While this type of participation helps students become "...aware of, and draw on, their own intellectual capabilities when making mathematical decisions and judgements" (Yackel & Cobb, 1996, p. 473), it also provides students a chance to feel like they belong in the mathematics classroom. That is, students might feel a sense of relatedness if they are asked to play a role in establishing and maintaining the structure or quality of the classroom community. By asking students how they felt when they participated in situations intended to foster classroom norms or community, we can learn about what it means to students to feel (or not feel) a sense of social belonging (Ryan & Deci, 2000). When students are given opportunities to participate in the mathematical classroom community, how do they describe feeling as if they belong and how do they believe feeling (or not feeling) connected to the classroom, their classmates, or mathematics affects them?

Experiencing Autonomy

Although the preceding sections describe students' participation in a mathematical community and ways that competence is redefined in the inquiry-oriented classroom, they also suggest that students in inquiry-oriented classrooms are given more autonomy, or self-direction, over their classroom participation. For instance, when competence is more than answering questions correctly or following prescribed procedures, students are able to engage in their own problem solving methods, seeking solutions, determining justifications required. That is, they are given the freedom to conceptually engage in the mathematics. Generally speaking, students are given the space to show their mathematical competence in many ways, which may also increase their sense of belonging in the mathematics classroom, as it opens the classroom to various acceptable forms of participation. Additionally, students are given the chance to play

meaningful community roles, evaluating acceptable mathematical arguments and negotiating classroom norms, rather than simply reporting answers for teacher verification.

The above describes situations during which students might be feeling autonomous, or self-directed, but students might not experience increased autonomy or freedom to choose in a positive way (Katz & Assor, 2007). For example, increased autonomy might be intimidating or uncomfortable for students who believe that only experts, like teachers, can generate strategies for solving problems. By asking students how they felt during situations in which they were given the chance to act in self-directed ways, we can understand what it means to students to feel a sense of autonomy (Ryan & Deci, 2000). That is, how do students interpret opportunities for self-direction and take advantage of those opportunities? How do students describe being given autonomy and how do they believe feeling (or not feeling) autonomous affects them? Table 1 provides a summary of the hypothesized connections between students' cognitive and participatory experiences and their affective experiences, showing students' potential cognitive

Table 1

Hypothesized Connections Between Participatory and Affective Experiences

Inquiry-oriented mathematics classroom	
Participatory experiences Students might...	Affective experiences Students might *feel*...
Discuss, ask questions, make proposals, justify arguments, etc.	Competent, able to complete required tasks for participation. Lacking competence, being asked to participate in unfamiliar ways.
Hold themselves and others accountable to norms, evaluate others' strategies, etc.	Related, as if they belong in the mathematics classroom. Isolated, as if they do not fit in.
Be seen as mathematical resources, be given freedom to problematize and help establish and uphold norms, etc.	Autonomous, freedom to be self-directed in their actions (can be positively or negatively experienced).

and participatory experiences in the left hand column, and potential corresponding affective experiences on the right.

Conclusion

This chapter explained the importance of understanding the classroom microculture in which students experienced mathematics to the framing of this study on their affective experiences of inquiry-oriented mathematics. It also articulated how constructs from self-determination theory, feelings of competence, relatedness, and autonomy, provide a deliberate way of exploring the types of affective experiences students might be having. Careful articulation of students' affective experiences is particularly important to providing in-depth explanations of how and why students experience certain feelings, creating a link between the cognitive classroom expectations and students' affective experiences. The next chapter will describe how the study was conducted, the methodology behind the study design and the collection and analysis of data.

CHAPTER 4: METHODOLOGY

A qualitative approach is ideal for this study because the central aim is to understand how undergraduate students in an inquiry-oriented mathematics content class for teachers "make sense of their world and the experiences they have in the world" (Merriam, 2009, p. 13). That is, the study explores students' affective experiences in the natural classroom environment and attempts to understand the meanings that students attribute to those experiences. More specifically, the study takes the form of a qualitative case study because it provides an "in-depth description and analysis of a bounded system" (Merriam, 2009, p. 40), students' experiences in a single undergraduate-level inquiry-oriented mathematics course for prospective teachers. According to Merriam (2009), one special feature of qualitative case study is its heuristic nature, its ability to "illuminate the reader's understanding of the phenomenon under study" (p. 44). This feature is particularly relevant here as the purpose is to elucidate students' affective experiences in inquiry-oriented mathematics classrooms, experiences that typically tend to remain unexplored in the background of such classrooms. This chapter will describe the methodology behind the study design and the collection of data, and will describe how the data was analyzed to characterize how the Math for Elementary School Teachers course in the study was inquiry-oriented and how the students in the study affectively experienced the course.

Setting

The study took place in a Mathematics for Elementary School Teachers content course for prospective elementary school teachers at Great Valley Community College[3] (GVCC) in the Pacific Northwest. This particular course, which focuses on problem-solving techniques, number theory, set theory, and numeration systems, is the first of three such content courses for

3 Names of all schools, instructors, and students are pseudonyms.

prospective elementary teachers offered at GVCC. This particular Math for Elementary School Teachers course was chosen for this study because it is taught in an inquiry-oriented manner, and the nature of the course was confirmed through discussions with the course instructor prior to the study. There were 21 students enrolled in the course, which met Monday through Friday for 50 minutes each day. Students in the course also worked together with frequency outside of class. In fact, it was not unusual for most of the class and the instructor to be gathered casually, talking about both math and life in general, near the classroom leading up to class on any given day.

The instructor of the course, Scott, has been a mathematics instructor at Great Valley Community College for over 15 years, was a middle school teacher for 24 years before that, and focuses specifically on the mathematical content preparation of future teachers, getting involved with programs to recruit teachers at the community college level and helping to design statewide transfer degrees for students planning to become elementary teachers. The course and classroom environment itself will be characterized fully in Chapter 5 in order to place students' experiences within the appropriate context.

Participants

Purposeful sampling (Merriam, 2009; Patton, 2003) was used to select students who were enrolled in the course with the intent of becoming an elementary school teacher and willing to reflect upon and discuss their affective experiences in mathematics. Additionally, the focal students represent, to the extent possible, a maximum variation sampling (Merriam, 2009), in order to document the variety of student backgrounds and experiences that might be taking place during students' time in the course. Maximum variation sampling is also useful in identifying shared or central experiences of students in the course across different backgrounds (Merriam, 2009). As a result, the study attempts to show both typical and unique student experiences.

Eight students were selected as focal students, and the six students described in this dissertation completed the entire study. Table 2 gives a brief description of the mathematical background of each focal student, illustrating the variety of background experiences of these six focal students. The race and gender distribution of the focal students represented the diversity of the class as a whole, which consisted of mostly white students and only two males. All focal students were female, one was African-American, and the rest were white. The following section gives a more detailed description of each student's mathematics background, utilizing information obtained during initial meetings with students and their math autobiographies, written for class. This is intended to be only a brief introduction to the types of students in the study.

Table 2

Summary of Focal Students' Mathematical Backgrounds

Focal student	Mathematical experience background
Jessica	A student with traditional[4], mostly negative experiences of math.
Morgan	A home-schooled student with mostly negative experiences of math.
Bahira	An "average" student, with traditional, neutral experiences of math.
Katie	A student returning to school to change careers, with traditional, mixed experiences of math.
Cheryl	A student with traditional, mostly positive experiences of math.
Samantha	A student with inquiry-oriented, mostly positive experiences of math.

Introductory Descriptions of the Focal Students

Jessica, a student in her mid-twenties, who began her schooling in England as a small child, described her past math experiences as a "difficult battle." The transition to American schools was difficult for Jessica due to concepts being taught in different grades. After her transition, Jessica's descriptions of her math classroom experiences are very traditional,

4 By traditional, I mean lecture-based math experiences, during which students listen to a teacher and take notes.

involving teachers lecturing at the chalk board and flash card memorization work. For Jessica, a pivotal negative experience during middle school, which resulted in her placement in a remedial course, colors the lack of effort she put in to her mathematics learning throughout high school. Having had a good experience the previous quarter at Great Valley Community College, where students worked on problems together with the teacher before class, Jessica enters this class with the hope that inquiry-oriented mathematics will "increase her comfort level" with mathematics.

Morgan, a home-schooled eighteen year old, is a Running Start student in her second quarter at Great Valley Community College. Despite having a positive math experience last quarter at GVCC, Morgan describes "loathing" mathematics, as she spent her childhood trying to learn math on her own using various textbooks, curricula, and computer programs. This experience led Morgan to "thrive" in a course with any kind of teacher instruction, but she still describes math as "tricky," and comes into this class looking for ways to improve her knowledge for teaching mathematics, which she describes as her "weakest link" as a future teacher.

Bahira, a self-described "average" math student who is 19 years old, entered this course sure she wants to serve students in some way, promoting education, but unsure of whether she really wants to teach. Bahira's past math experiences are traditional, lecture-based experiences, but during elementary school she remembers feeling average and being "pleased" with her abilities. During middle school, Bahira has strong memories of being tracked, and while she was placed on the track appropriate for her grade level, she distinctly remembers feeling disappointed that she was not placed with her friends in the higher track. She told me she has worked to convince herself that she is a very good student, but that math is "just not [her] subject," and is generally neutral in her feelings around math. She is taking this Math for Elementary School Teachers course to determine whether she can see herself as a K-12 teacher.

Katie, a post-baccalaureate student in her mid-forties and parent of two, returned to school to take this course as a prerequisite for a Master's in Teaching certification program. Having grown up the child of two university professors, Katie has a strong value of education, and sees herself as a smart, capable person. Despite this, she described failing calculus her freshman year in college, where she originally planned to major in math. This experience shook her confidence, but she later earned a 4.0 in the same course with a different instructor. Her classroom experiences with mathematics were all accelerated, traditional, lecture-based, but her calculus experiences led her to believe that how material is presented matters significantly. Her passion for teaching children, especially those who might not "get it" right away from a teacher's explanation, lead her to believe in the potential of inquiry-oriented teaching methods.

Cheryl, a student in her mid-twenties in the process of transferring to a local university, described her math experiences as positive, and has both Montessori school experience and traditional math experience. She has traditional ideas about success in mathematics, describing her enjoyment of timed tests, getting high scores on exams, and valuing her correct answers over showing or explaining her work. Cheryl already feels confident in her ability to teach elementary school mathematics because she has successfully tutored an 11 year old boy, and is in this course largely to find ways to make math "fun and easy to learn."

Samantha, a student in her early twenties, describes math as her "favorite subject," but is quick to emphasize she does not consider herself a "math genius." Having grown up in elementary classrooms that largely utilized inquiry-oriented methods, Samantha recalls thinking at the time that her teachers were "mean" for teaching the students multiple ways to consider addition, subtraction, multiplication, and division. Now, she looks back and values how much she learned in classrooms that valued inquiry on the part of students. Samantha spent a lot of

time “playing school” as a child, is passionate about teaching, and believes she will be an especially good math teacher. She entered this class already believing inquiry-oriented approaches are “one of the best ways to have children learn.”

Data Collection

Data collection occurred in six phases over the course of the class, tracking students' experiences over time: (1) selecting students/introduction to the inquiry-oriented environment, (2) initial interviews, (3) classroom observations, (4) follow-up interviews, (5) a final interview, and (6) a focus group discussion. Figure 3 shows the data collection timeline. Collecting data from several different sources, including surveys, student writing, interviews, and observations, is one way I attempted to ensure internal validity, as it provides opportunities for triangulation of data (Merriam, 2009).

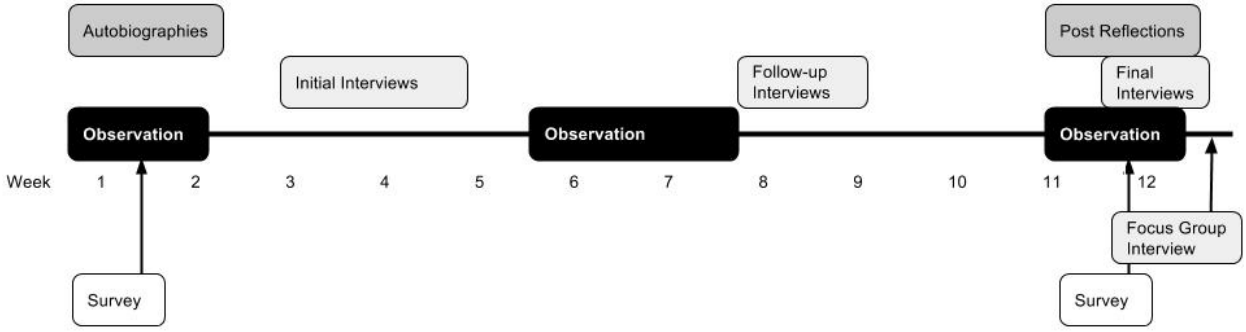


Figure 3. Timeline of data collection during a one quarter Math for Elementary Teachers course.

Introduction to the Inquiry-Oriented Environment

In order to select focal students and to observe how the course instructor set up the classroom environment to be inquiry-oriented, I observed the first week of class sessions. I also observed two weeks in the middle of the course, and a few classes at the end of the course to see the course wrap up. These observations were used to characterize the inquiry-oriented nature of

the course, to collect fieldnotes which informed a rich description of the classroom environment.

During the first week of the course, all class members wrote a mathematical autobiography. Potential focal students also submitted their mathematical autobiographies to me at the beginning of the course, which provided detailed information about students' previous mathematics experiences and their feelings about those experiences, so that a maximum variation sampling could be created as described above. Autobiographies, which informed my initial interviews with the focal students, also included students' reflections about their understanding and expectations of the current inquiry-oriented course, which were based on initial descriptions of the course given by the syllabus, the first week of class, or other sources, such as descriptions of such a classroom from their instructor[5].

Interviewing the course instructor. To aid in characterizing the inquiry-oriented environment, I interviewed the instructor of the course twice. An initial interview near the beginning of the course helped me understand how Scott generally structures the course to be inquiry-oriented, what that means to him, and why he thinks it is important to structure this particular course around inquiry. A follow-up interview near the end of course allowed for more detailed questions about particular classroom practices revealed during observations, and for the two of us to reflect on how the course was implemented. These interviews lasted around 45 minutes each, and were audio recorded and transcribed for analysis. Instructor interview protocols are given in Appendices A and B. The instructor's perspectives on the course are integrated into the classroom characterization in Chapter 5.

Basic Psychological Needs Scale

All 21 students in the course completed two short surveys that captured their feelings of

5 Math autobiographies are frequently assigned in Math for Elementary School Teachers courses, and it is not unusual for them to include a reflective component about the upcoming course in relation to past experiences.

competence, relatedness, and autonomy in math courses in general (at the beginning of course) and in this math course in particular (at the end of the course). The survey was adapted from the Basic Need Satisfaction at Work Scale[6], with the word "work" essentially being replaced with "math class" or "class." Each survey consisted of 21 questions on three subscales: competence (6 questions), relatedness (8 questions), and autonomy (7 questions). Sample items include: for competence, "When I was in class I often did not feel very capable"; for relatedness, "I got along with people in class"; and for autonomy, "I was free to express my ideas and opinions in class." In order to understand the degree to which students' needs were being satisfied, students were asked to rate their feelings about math by deciding how true each statement was for them on a 7-point scale (1 being not true at all, 7 being very true). The full prompt and list of survey questions, including which questions belong to each subscale, are given in Appendices C and D.

Initial Individual Interviews

The second phase of the study consisted of individual semi-structured interviews with the focal students, which took place during the third and fourth weeks in the course and focused on articulating students' expectations of the current course through the lens of their prior mathematical experiences. Students were asked to discuss their expectations and conceptions of the current, inquiry-oriented course. For example, students were asked to talk about how they expect to participate in the course, what they expected engagement in mathematics to look like, and how those expectations made them feel. Students were also asked to describe their past mathematical experiences, building on their mathematical autobiography given. Specifically, students were asked to reflect upon their affective experiences during their past mathematics experiences, to describe when and how they felt (or did not feel) competent, as if they belonged,

6 This survey and other questionnaires used by Ryan and Deci and others doing SDT work are publicly available from selfdeterminationtheory.org.

and self-directed, and to express how those experiences have come to currently affect their feelings or experiences with mathematics. All interviews in this study were semi-structured in order to attempt to gain similar data from all interviewees, so questions were essentially predetermined, but there was flexibility in the use or order of questions in each interview as necessary (Merriam, 2009), particularly to discuss students' individual experiences as relevant. These initial interviews were typically about 40 minutes long, and were audio recorded and transcribed for analysis. See Appendix E for the initial interview protocol.

Classroom Observations

The third phase of data collection consisted of classroom observations of the focal students engaging with mathematics in their content course for teachers (see Appendix F). During the fifth, sixth, and seventh weeks of the course, I observed full coverage of two topics, numerations systems and addition and subtraction, in order to gather data on focal student participation. Sessions were chosen to intentionally fall after students had had significant time in the course, and in consultation with the course instructor in order to observe typical classroom engagement. As such, over the two weeks that I observed, I was able to see a wide variety of classroom activities that typically take place, but also get a sense for the general and typical routine. My role in these observations is best described as "observer as participant" (Merriam, 2009), my presence was known in the classroom, but my role peripheral. The students were aware that I had taught an equivalent course at a local university. In interviews it was clear they considered me like an instructor, but also like a student, and sometimes referred to me as a "math person." They knew my deep interest in their classroom experiences stemmed from my own work in preparing teachers. Classroom observations were video and audio recorded and fieldnotes were taken, both for the purposes of creating a rich description of the inquiry-oriented

classroom culture, adding to what was observed at the beginning of the course, and to provide footage for use in stimulated recall portions of the follow-up interviews. Specifically, the main focus of observations was to identify potential participatory and cognitive events during which students might experience feeling competent, related, or autonomous, in order to facilitate stimulated recall during follow-up interviews. Afterwards, I selected a range of experiences for each focal student to watch and discuss with me. The video served to both support student reflection upon their affective experiences and to make direct connections between those experiences and the participatory demands of the inquiry-oriented mathematics classroom (see the next section for more on the stimulated recall).

Follow-up Individual Interviews

The fourth phase of the study consisted of individual semi-structured follow-up interviews with the focal students, focusing on students' affective experiences in the current, inquiry-oriented course. These interviews took place as quickly as possible after classroom observations, during the end of week seven and into week eight of the course. Students were asked to generally describe how they have (or have not) felt competent, as if they belonged, and self-directed during the course, and to tell me about any experiences that have stood out for them during the course[7]. In order to aid in these reflections, beyond only answering interview questions that allowed students to highlight situations that had stood out for them during the course, each student watched four or five video clips (around 12 minutes total) of class sessions with me and thought aloud (Patton, 2003) about what they were thinking and how they were feeling during those classroom experiences. This served to connect students' affective

7 Students kept required journals at the beginning of the course, which I asked them to review before this interview, but around this time the required nature of the journals was dropped after mutual agreement between the students and the instructor.

experiences to their cognitive and participatory experiences observed in the classroom that might have been facilitating (or failing to facilitate) feelings of competence, autonomy, and relatedness.

For example, during my observations students organized and presented group projects on different numeration systems, so students were asked to watch clips of their group planning time and clips of their group presentation and to reflect on how they felt at the time. In this particular situation, it was important for me to note whether the students felt competent and related, as they carried out this group project. It was also important to attend to all aspects of the classroom microculture that might have influenced the students' affective experiences in the chosen clips. Interactions with content, specific tasks, peers, the instructor, and classroom resources all had the potential to influence how the student experienced the inquiry-oriented course, and I emphasized to the students that we could talk about any and all of these influences during our discussion of the video clips. My goal was to understand the affective experience of students within the classroom environment as fully as possible, so while I did ask specific questions, I allowed them to direct the focus of the conversation so that I could understand their perspective on the clips.

Students were also asked to consider their affective experiences in the current inquiry-oriented course as they relate to their experiences in previous mathematics courses, particularly if those courses were taught in more traditional ways. Questions of this nature built upon individual responses in the initial interview. Finally, students were asked to briefly talk about how the inquiry-oriented nature of the course might influence their future teaching, which was the primary focus of our final interview. Follow-up interviews were roughly an hour long, and were audio recorded and transcribed for analysis. See Appendix G for the protocol.

Final Individual Interviews

The fifth phase of the study consisted of final individual semi-structured interviews with

the focal students near the end of the course, during the eleventh, and final, week of the course and into finals week, week 12 of the quarter. These interviews focused on bringing together the story of students' affective experiences throughout the course and articulating how students expect their affective experiences in the current course might affect them in the future. Students were asked to reflect upon their overall experiences or transformations throughout the current, inquiry-oriented course, to describe any pivotal affective moments they had. In addition, students were asked to think about which aspects of the course will affect their own teaching practice, and how those aspects of the course made them feel as a student, particularly with regard to feelings of competence, relatedness, and autonomy. Students were also asked whether and how this course has changed their view of mathematics, their view of themselves in relation to mathematics, and how they will move forward with mathematics. Final interviews were informed by a written post-reflection that students were asked to complete in the course, which they also sent to me. In the short reflection, students were asked to read and comment on their autobiography from the first week of the course. These final interviews were also connected to previous interviews, wrapping up any individual loose ends or questions. Final interviews were about 40 minutes long, and were audio recorded and transcribed for analysis. The final interview protocol is given in Appendix H.

Focus Group Interview

The final phase of data collection was a single semi-structured focus group interview with all of the focal students after final individual interviews, during finals week (see Appendix I). This interview was a chance for the students to discuss their affective experiences in the inquiry-oriented mathematics course together, something they often requested I organize during the course of the study. Hearing others' reflect on their experiences in the mathematics

classroom gave students a chance to expand upon previous thoughts on their experiences from prior interviews, and the focal students were eager for this conversation. It was clear during the discussion that students typically did not discuss the differences between their experiences in traditional and inquiry-oriented mathematics classes or how their current experiences in an inquiry-oriented course might affect them in the future on their own, but students might have gained new understanding of their experiences in discussing their course experiences together (Merriam, 2009). The focus group interview was 90 minutes long and was video and audio recorded to keep track of specific participants' comments. Parts of the interview were transcribed for analysis, serving as a source for triangulation with other interview data.

Data Analysis

My aim was to articulate prospective elementary teachers' understandings of their affective experiences in inquiry-oriented mathematics courses, particularly relating to their feelings of competence, relatedness, and autonomy and how those feelings relate to the cognitive and participatory demands of such a course. As part of elucidating students' affective experiences of the inquiry-oriented classroom, this study also documents how students understand their existing affective experiences of mathematics. In addition, the study focuses on how prospective teachers expect their current affective experiences to influence their future practice of and teaching of mathematics. This section describes how the data was analyzed in order to characterize the inquiry-oriented nature of this Math for Elementary School Teachers course, describe how individual focal students understand their affective experiences within this course, and to look across focal student experiences to consider similarities and differences between student experiences. Preliminary analysis of the data was ongoing during the study, as data collected affected where attention needed to be directed and generated follow-up questions

that were asked of students or the course instructor in order to pursue initial hypotheses. Throughout the study, analytic notes were put into memos[8] focused on hunches related to various categories of future findings (Merriam, 2009), such as classroom characteristics, individual student narratives, or cross-case findings. Member checks in the form of questions or discussion were used during follow-up interviews to ensure I was accurately capturing students' experiences during the course (Miles & Huberman, 1994).

Classroom Characterization

The analysis undertaken to characterize the classroom as inquiry-oriented focused primarily on understanding how the course instructor designed the course to be inquiry-oriented, in what ways that design was implemented in the classroom, and how the entire classroom population rated their math experiences of competence, relatedness, and autonomy. Specifically, to understand the instructor's course design, instructor interview transcripts were coded for this instructor's particular description of inquiry-oriented teaching and learning, which included understanding the major characteristics a course needs to have to be inquiry-oriented, the benefits for students, and the role of the student and teacher in inquiry-oriented courses. The instructor's descriptions in interviews were triangulated with the syllabus, the autobiography and post-reflection assignment sheets, and observations of how the instructor introduced the nature of the course to students during the first few introductory class sessions.

In order to document how the instructor's inquiry-oriented course design was implemented, course activities, fieldnotes, and videos of class sessions were analyzed for common themes around the typical class structure, including expectations placed on students in each class session and the role the instructor played. I created a table of all course activities that

8 Memos of this nature were kept in document files. Formal coding of data (transcripts, fieldnotes, course documents) was undertaken using Dedoose, a web-based analysis software program.

occurred during my mid-course observations, describing what students and the instructor did during that activity. For example, I noted whether students were working individually or in groups, what their participation looked like, and the role the instructor played. I then looked across this table to identify the common ways the students and instructor participated in this course. Finally, a rich description of a typical classroom activity was written, in order to situate the reader within the classroom context for a discussion of how the important inquiry-oriented characteristics played out within the classroom.

The basic need satisfaction survey, which was given to all students at the beginning of the course asking about prior math experiences, and at the end of the course asking about experiences in this course, helps situate the focal students within the classroom population. Recall that these surveys consisted of 21 questions on three subscales: competence, relatedness, and autonomy. Scores were calculated by averaging the responses, given on a 7-point scale, on each subscale. If students did not answer all questions, the subscale averages were calculated using only the questions answered. Additionally, three matched-pairs *t*-tests were performed to determine whether the students reported significantly higher need satisfaction in the current inquiry-oriented course. The characterization of the classroom is undertaken in Chapter 5.

Focal Student Experiences

Coding of data. The first major task in describing individual focal students' affective experiences was to carry out an initial reading of the interview transcripts, student writing collected, and observation fieldnotes. During this reading I asked questions of the data in order to identify places where students described feeling competent, related, and autonomous (such as, "When and how do students describe feeling competent?") (Emerson, 1995; Merriam 2009). To identify responses to the first research question (Merriam, 2009), particularly subparts (a) How

do students describe their existing affective experiences of mathematics? and (b) How do students make sense of their affective experiences in relation to the cognitive and participatory expectations and demands of the current inquiry-oriented classroom?, initial coding of the data focused on students' past and current experiences, with subcodes of competence, relatedness, and autonomy at each time point coming from the psychological needs articulated in the conceptual framing. As a reminder, data on past experiences largely came from mathematical autobiographies and initial student interviews, and the course observations and follow-up interviews focused on how the focal students were affectively experiencing the current course. However, I did not restrict time point codes to information only from those sources, I coded based on the time point to which the student was referring. To answer the second research question, What influences do prospective teachers anticipate these affective experiences will have on their future mathematics teaching practice?, a future teaching code was used to characterize any comments around students' own future teaching practice. Data used to answer this question also came from across all sources, but primarily came from the final interviews and post-reflections.

During the initial coding of data, additional codes helped identify the aspects of the classroom microculture (peers, content, teacher) that were described by students as playing a role in their affective experiences. Codes also helped to distinguish between positive and negative affective experiences, though neither was favored or excluded, as the goal was to document students' understandings of their own experiences. I was also open to tracking general emotions, to help me understand how students described their general feelings about mathematics. After initial coding of the entire data set, an analytic code list was finalized, and a second, focused coding of the data was undertaken (Emerson, 1995). At this point, data analysis shifted to

become more deductive (Merriam, 2009), in that the categories for coding were more firm, and data was being categorized rather than used to create categories.

In coding for student experiences around the psychological needs of competence, relatedness, and autonomy, I coded to reflect the definitions of these terms as used in self-determination theory. That is, when coding for student experiences (or lack) of competence, I looked for places where students talked about feeling capable of and knowing what it would take to be successful in class. When this came up, students and I often talked about what they thought it meant to be successful in this and their prior math experiences, which helped put context around why they felt or did not feel competent. When I coded for relatedness, I tagged places where students described experiencing, or not experiencing, a sense of social belonging or a feeling of fitting in or not fitting in. In the process of coding for relatedness, I noticed that students also discussed a sense of identification with mathematics as a discipline. This does not fit with the definition of relatedness as in SDT, but was an important experience that some students mentioned during our discussions of belonging in math class. As a result, I created an identification with mathematics code. Finally, in coding for autonomy, I made note of instances when students talked about having a sense of self-direction, specifically around their learning of mathematics. These were times when students felt, or did not feel, a freedom to be self-directed in their mathematical thinking. Table 3 gives definitions and examples of these codes.

Creating the individual case studies. To create the individual case studies, a narrative was written of each student's affective experiences (Merriam, 2009). All data relevant to the individual student at hand was gathered, sorted chronologically in order to tell students' stories over the course of the quarter, and then analyzed to tell a meaningful story around that student's affective experiences. For example, all coded segments for a student's past experience were

Table 3

Main Codes Used to Characterize Student Experiences

Code	Definition	Examples
Competence	Mention of feeling or not feeling capable of doing what is expected or required for success in mathematics.	"I was able to show it, demonstrate it, and in a way that they could understand better than the way Scott was showing it up on the board... ." "Half the time though I wasn't sure how to explain how I got my answer from my head to the sheet of paper."
Relatedness	Times when students experience or do not experience a sense of social belonging in class.	"I enjoyed the class atmosphere because it was comfortable and we had many opportunities to work with one another and get to know each other's names." "Because I didn't get it and the students around me might have got it or didn't, it just kind of felt like an alone in the crowd situation."
Identification with Math	Instances when students identify or do not identify with mathematics as a discipline, like being or not being a "math person."	"I now have an input on the math world. It is not just me and then everybody else in the math world. I am a part of it." "I began to tell myself that I am a really good student, but math is just not my subject."
Autonomy	Places where students experience feeling or not feeling self-directed in their mathematical thinking/learning.	"This class really gives you the opportunity to really figure it out whichever way is best for you." "There was always, step 1, do this, step 2, do this. I would put the example on one side and put my paper on the other with the question..."

collected, and then sorted by the subcodes of competence, relatedness, and autonomy. I then looked through the excerpts to see, for example, how a student described feeling competent in their past experiences with mathematics. This process involved more than "concrete description" (Merriam, 2009, p. 188), as the constructs of competence, relatedness, and autonomy were used to organize how each student understands their affective experiences in their past mathematical

experiences, makes sense of their affective experiences in relation to the current class, and anticipates their current experiences influencing them in the future. To summarize, narratives were focused on the time points highlighted in the research questions, the three basic psychological needs of competence, relatedness, and autonomy, and making direct connections between the cognitive and participatory demands of the inquiry-oriented environment and students' affective experiences. As I made claims about how or when a student described feeling competent, related, and autonomous, I frequently checked the data on that student for confirming and disconfirming evidence of my characterizations of their experiences. I did not attempt to simplify student experiences, and where appropriate, I discuss the nuances of those experiences.

Cross-Case Analysis

After the development of the individual focal student case studies, cross-case analysis was undertaken to more generally answer the two main research questions: (1) How do prospective elementary teachers understand their affective experiences of competence, relatedness, and autonomy (self-direction) over the course of participating in an inquiry-oriented mathematics content course for teachers?; (2) What influences do prospective teachers anticipate these affective experiences will have on their future mathematics teaching practice? To do so, the following analysis questions were addressed across the individual case studies:

- How and when do students feel competent, related, autonomous?
- How are these feelings linked to the cognitive and participatory demands of the inquiry-oriented mathematics classroom?
- How to students' describe linking their affective experiences to their future teaching?
- Do answers to the above questions depend on the students' background or past experiences in mathematics?

This phase of the analysis became even more theoretical, considering a "more conceptual overview" (Merriam, 2009; Miles & Huberman, 1994) of the individual case studies. Considering the case study narratives, all transcripts and student writing, claims were generated to answer these questions and to put together an overall discussion of students' affective experiences over the course of this class.

Making these overall claims involved considering patterns and themes that emerged within and between the student cases, to understand both common and unique student experiences. Data was searched for both confirming and disconfirming evidence of the claims to ensure that claims reflected what was actually demonstrated by the data (Miles & Huberman, 1994). For example, as I noticed that students were describing feeling competent in new ways during this course than their previous math experiences, I examined individual student cases for evidence this was demonstrated or not demonstrated. The end result is higher level claims about how prospective elementary teachers make sense of their affective experiences in an inquiry-oriented mathematics classroom, with attention paid to variations in experiences.

Conclusion

This chapter described the study design and research methodology used, including an introduction to the course in which the study was situated and to the focal students at the heart of the study, a description of the data collection process, and how that data was analyzed to understand how students made sense of their affective experiences in this Math for Elementary School Teachers course. Multiple sources of data were utilized in this study, including student writing for course assignments, classroom observations, and interviews, in order to understand how students' affective experiences are connected to their cognitive and participatory experiences in this inquiry-oriented course and how they evolved over time in the course. The

next chapter characterizes the particular Math for Elementary School Teachers course in this study, articulating the ways in which the course was designed and implemented to be inquiry-oriented, with the aim of understanding the environment in which students' experiences were situated.

Chapter 5: Characterization of the Inquiry-Oriented Mathematics for Elementary School Teachers Course

The goal of this study is to document how prospective teachers make sense of their affective experiences in an inquiry-oriented math content course. In taking a sociocultural perspective to frame this study, I argue that it is essential to understand the classroom culture in which students are affectively experiencing mathematics. In order to situate the individual case studies that will follow in the next chapter, this chapter is dedicated to elucidating the inquiry-oriented nature of the Math for Elementary School Teachers course in this study. This involves analysis of how the instructor of the course intended for the course to be inquiry-oriented, and what that means to him, and an analysis of how the course was implemented to be inquiry-oriented, which includes a rich description of a course activity followed by an articulation of the typical expectations placed on student participation and the instructor role. The chapter concludes with a discussion of the psychological needs survey, which helps characterize the affective experiences of the entire class population.

The Instructor's Inquiry-Oriented Course Design

The instructor of the course, Scott, and I discussed his course design twice during this study, and I had access to all course documents, such as the syllabus, assignments, handouts, and the online course management system which he used to organize the course material for students. In this section, I present analysis of this data in order to understand how Scott designed this Math for Elementary School Teachers course to be inquiry-oriented, which is part of the larger characterization of the classroom in which this study was situated.

When Scott discusses inquiry-oriented teaching and learning of mathematics, he contextualizes this type of teaching on a spectrum, and emphasizes the increased responsibility

of learners in inquiry-oriented courses. He notes the latter by saying that some students "resist" taking a more active role in the classroom, complaining, "You are not teaching me!" Scott places direct instruction, where teachers "pretty much lecture to students," on one end of the teaching spectrum, and inquiry-oriented teaching and problem-based learning on the other end. He gave an example of direct teaching around area and perimeter, where an instructor would start the lesson by giving students the formal definitions of these terms, perhaps from a textbook. This is in contrast to the first activity in his Math for Elementary School Teachers course, which will be discussed in detail later in this chapter, in which Scott gave the students an activity intended to foster discussion around ideas related to area and perimeter, before a discussion focused on formalization of such terms and when and how they might be used. Scott explains that fostering inquiry involves "giving students some activity where they are not sure of the answer," so that it is necessary for students to "interact with each other, or with you [the instructor], and with things like manipulatives" to come up with a solution strategy.

It is important to note that Scott contrasts inquiry-oriented learning with problem-based learning, another commonly used form of learning that gets students actively involved in doing mathematics within the classroom. According to Scott, problem-based learning is more directed than inquiry-oriented learning, giving students a list of questions or directions that is essentially "leading students down a track" to the answer[9]. Scott suggests that if students "pretty much know where they are headed ahead of time," the activity could still be of high quality, but is perhaps not being implemented in a way that fosters students' own sense of inquiry. He emphasizes the importance of not "wrapping it up all of the time," but leaving things "hanging" for students to think about and come back to in future classes.

9 This is Scott's interpretation of problem-based learning. In either approach it is possible to lead students through steps as Scott discusses, which would decrease the inquiry nature of the task.

To understand how Scott makes sense of an inquiry-oriented classroom, it is helpful to think about how he describes what students should be doing in the classroom, what their engagement with mathematics looks like, and the role he takes on as the instructor as well. When talking about his expectations for students, Scott emphasizes that he expects them to "dive in," spending "most of the time" working in groups on activities. Occasionally, Scott says he gives students time to think about problems alone before engaging in group work or the entire class discusses something together, but most often, students are "engaged in an activity," which means they are "in and out" of groups, "discussing and talking with each other." An important aspect of this engagement is the preparation that Scott expects the students to take on. He expects that they will "read the textbook, gather other sources, realizing that they are their own learner." Thus, students are expected to take control of their own learning, but Scott emphasizes that through the group work he and the other students in the class "are there to help."

Interestingly, Scott describes this classroom relationship as "a family," saying "This class is going to become a family. We are going to get together to compare solutions." In Scott's description of his expectations for students, there is a large emphasis on communication. He want students to be talking a lot with each other and with him, both inside and outside of class. He emphasizes this by having discussion forums online where students can talk with each other about classwork, homework, or really anything at all, outside of class time. Included in that talking, Scott heavily emphasizes that students should be asking questions. Getting the students in this course to ask a lot of questions would be Scott's "ideal" sense of engagement. In his words, "putting yourself out there" is one of the hardest parts of an inquiry-oriented course.

The syllabus for Scott's Math for Elementary School Teachers course is in many ways a typical course syllabus, with textbook details, course components and grading, and policies, but

it also includes a full page of "student attributes" where much of what he described in our interviews was communicated to students in the course (see Appendix J). A table outlines "evidence of achievement" for each of four attributes: (a) demonstrates intellectual engagement; (b) takes responsibility of own learning; (c) perseveres when faced with time consuming or complex tasks; and (d) pays attention to detail. When Scott talks about his expectations of students, he is very clear that he expects them to be doing a large amount of work, characterizing this from the student perspective as students appearing to do more work than he does in the classroom. That is not actually the case, as Scott spends a great deal of time on his classes, but it might appear this way in class because students are doing most of the knowledge construction.

The instructor role that Scott describes centers around being a "guide" who is "trying to put all of the pieces together" for students. His first major responsibility is to come up with activities that will engage students in inquiry, which he notes is "the hardest part." While students are working activities, Scott is "floating" around the room, asking questions, fostering thinking, and providing them with materials they need to carry out their solution approach. He supports students' approaches, by "rounding up stuff that they want to use to solve whatever problem" is at hand. Another part of putting all of the pieces together is highlighting important information that comes up during group work, making certain pieces of knowledge or misconceptions public knowledge. Scott notes that he often "asks students to come up and talk about what they were talking about [with their group], whether they are on the right track or the wrong track." He talks about fostering classroom discourse around discussing ideas, even incorrect ideas, so that all students can benefit when misconceptions come up during small group work on activities.

Indeed, another important aspect of Scott's role as the instructor is "creating a classroom

atmosphere where kids will put themselves out there," asking questions and discussing ideas even if they are not sure they are correct. One characteristic of his teaching that helps him do this is highlighted several times in interviews. He goes out of his way to show students his own thinking process, how he revises his thinking, acknowledging that he does not instantly know everything, and is engaging in mathematics in the same ways he expects of his students, saying, "I've got to be willing to do it myself as well." For example, he mentions willingness to show students how he would "approach a problem," "thinking out loud," even when he is "flustered," noting a significance in "modeling that sort of thing for students."

Scott designs this Math for Elementary School Teachers course to be inquiry-oriented in very intentional ways, and his motives for doing so are centered around his belief that this type of learning is best for students. Specifically, Scott believes that when students genuinely engage with mathematics, which involves "muddling through," "taking stuff you know from the past," "taking stuff other people are telling you," and "making sense of it for yourself," they "appreciate" math more, "remember" more, and find mathematics to be more "meaningful." In particular, Scott focuses on helping his students, who are prospective teachers, understand that how they teach mathematics will affect their future students. He believes that engaging prospective teachers in their own inquiry-oriented learning experiences will help the teachers understand this idea because most prospective teachers have "learned through our public school system" that math is "something to be memorized," "not creative," and "discovered ahead of time." Without experiencing this type learning first hand as students, Scott believes it is difficult for prospective teachers to "buy in" to fostering this type of learning for their students. He says, flat out, "I want these future elementary teachers to change the way the kids think about math."

In addition, for Scott, because his students are future teachers, there is an added pressure

to help them learn to be “comfortable” with mathematics. He wants his students to, at the very least, not be afraid of mathematics, so that they have the confidence to take on teaching mathematics in inquiry-oriented ways, noting that it is “safer” and “easier” to not use inquiry-oriented learning because students have a much larger contribution to the class sessions in inquiry-oriented classrooms, which can lead to unpredictability. As a result, students' affective experiences are an important part of this course for Scott. He notes that ”feelings are a big part” of prospective teachers' mathematical learning because he does not want elementary teachers in schools who are “fearful of mathematics.” Scott emphasizes that teachers may certainly question whether they “have everything they need to know” to teach mathematics, because this type of question leads to inquiry on the part of the teacher, to continued, active development of one's teaching practice, but that fear of mathematics would prohibit that development. His way of articulating this notion to his students is to emphasize that they will always be “becoming a teacher,” they will never be “there.”

During our second interview, Scott and I were able to discuss the course in retrospect, considering how things went, with particular regard to the inquiry-oriented design of the course. He noted that he was particularly pleased that most students expressed feeling “more comfortable” with mathematics and “more competent with themselves and their ability to do [math].” For Scott, this competence is closely linked to the communication within the classroom. One way he measures student competence is by their comfort level with discussing their ideas with others, even while knowing their thinking might be incorrect, that they are “comfortable telling someone else their idea.” In this reflection on the course he is pleased to realize that every student in this course got up and discussed something in front of the entire class at some point during the quarter. Additionally, Scott reiterated that students expect

instructors to “lecture over a section” in math classes, but that he hopes this type of course “convinces all students” that students “learn best” in inquiry-oriented type courses where they are asked to engage meaningfully in mathematics, rather than “just rotely.” While he acknowledges that this convincing is a difficult undertaking, particularly in only one quarter, at the end of this one course Scott is enthusiastic about the benefits inquiry can afford students.

To summarize, Scott talks about expecting students to (a) discuss mathematics with others; (b) construct their own solution strategies; (c) be intellectually courageous[10]; and (d) let their thinking develop over time. His role as the instructor involves (a) providing tasks and resources; (b) acting as a guide; (c) encouraging perseverance and student generation of knowledge; and (d) facilitating share-outs of misconceptions and reasoning. In the next section of this chapter, the implementation of his inquiry-oriented course design is analyzed in detail, to provide a rich description of the classroom environment in which prospective teachers were engaging with mathematics.

Implementation of the Inquiry-Oriented Course

Now that the instructor's intended inquiry-oriented course design has been articulated, it is essential to discuss how this design was implemented during the actual teaching of the course. In this section, I will focus on providing an analysis which characterizes the ways in which the Math for Elementary School Teachers course in this study was carried out. In particular, I will discuss how the inquiry-oriented nature of the course was introduced to students, the first classroom activity that students engaged with, the typical class routine and culture that developed by the middle of the course, and the opportunity students were given to reflect upon their experience at the end of the course.

10 Note that I use this phrase similarly to Lampert (1990), who attributes the phrase to Polya (1954).

Introducing Inquiry and Managing Expectations: Math Autobiographies

The first assignment students were given in this course, which was emailed to them before the first day of class, involved writing their mathematical autobiography and considering the inquiry-oriented nature of this course. On the assignment sheet, students were told the course would emphasize inquiry-oriented learning, and they were asked to watch at least one of three videos[11] about this type of learning and think about how it relates to their past experiences in math and their views about how they think math should be taught or how they plan to teach. The videos, which show teachers, researchers, and other leaders talking about the benefits they were afforded by learning within a classroom environment that focused on inquiry, were chosen by Scott to given students another way to understand the type of classroom environment he tries to foster, which he described to me during his interviews as was articulated in the preceding section. Autobiographies were between two and three pages long and were to address the following questions:

- What are your earliest memories of doing mathematics? Were they good, bad or indifferent?
- What incidents, family members, and/or teachers may have had a strong impact (positive or negative) on your feelings toward math?
- How do you feel about your abilities in mathematics?
- How do you feel about your abilities to teach mathematics to K-8 students?
- Think about the ways that you learned mathematics in your own K-12 career. Would you say that inquiry-oriented approaches were used?
- What are your current feelings about this class? Do you have any concerns? What do

11 The videos students could watch are available here: http://www.inquirybasedlearning.org/page=Video_about_IBL

you hope to gain?

The autobiography assignment introduced and emphasized Scott's dedication to the inquiry-oriented nature of his Math for Elementary School Teachers course, while also allowing students to consider both their own mathematical learning experiences and the feelings associated with those experiences. In writing this assignment, students were able to express their feelings about this course, getting their concerns and hopes out on paper and therefore more at the forefront of their thoughts, a typical use for autobiographies in teacher education courses (Alvine, 2001). In addition, the autobiography assignment also began to allow students to reflect upon the impacts of certain kinds of teaching and learning, since they are asked to try to link their feelings to memories and incidents from the past. Thus, this assignment not only set the stage for the inquiry-oriented course work to come, but also set the stage for the prospective teachers to consider their own developing teaching practices, which is consistent with newer ways that autobiographies are being used in teacher education (Alvine, 2001).

Diving Right In: The First Days of the Course

In the spirit of Scott's claim that he expects students to "dive in" to the inquiry work in this course, the very first thing students did in the course was engage in a group work activity, the Lake Activity, that embodied Scott's ideas around inquiry-oriented learning. By 20 minutes into the first class, after a very brief introduction to the importance of the series of courses for teachers, the online course management system, and other details like the textbook, students were working on the Lake Activity. This goal of this activity was to get students immediately into the mathematics, and do so in a way that quickly allowed them to see the participatory and cognitive expectations that would be placed on them and the role that Scott would play as the instructor during this course, while also introducing the course content around problem solving.

Students were given a picture of two lakes drawn to the same scale, shown in Figure 4, and asked to individually consider "How might you determine which lake is the larger of the two?" Students were encouraged on the handout: "Be creative—your ideas can be practical, theoretical, and even a bit far-fetched." After a minute of silent working, Scott enthusiastically said, "Maybe you have an idea and you don't know whether it's right or wrong. Put it down!"

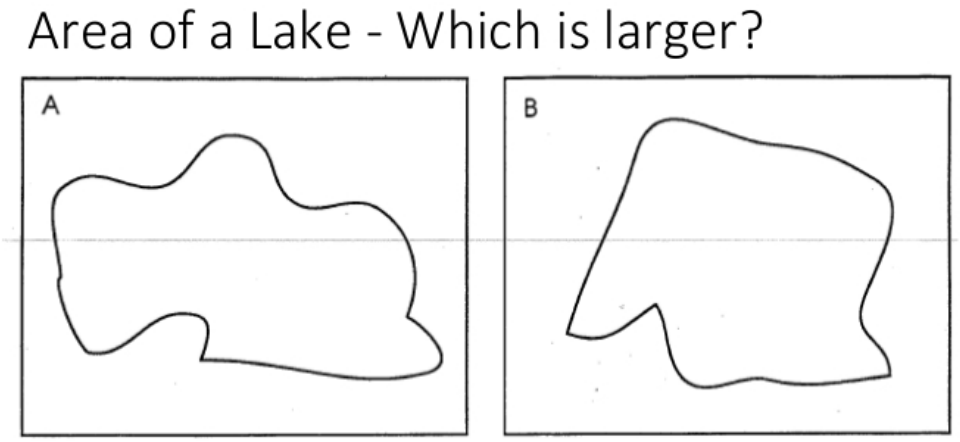


Figure 4. The image given on the Lake Activity handout.

In class, students were given around five minutes to consider this task on their own. At that time, they were directed to come together in groups of two or three students to "compare ideas" and add additional ideas that the group could come up with together. During this time, Scott "floated" around the classroom, and could be heard saying things like, "Can you say how you know that?," encouraging students to justify their thinking to one another. After working in small groups for around two minutes, Scott interrupted the class to point out that a "great question," whether they need to know the depth of the lakes, had come up. He clarified that they could certainly think about volume, but that he is intending this task to be about surface area, and students resume their work. Five minutes later, groups were asked to share the "most creative"

idea that they came up with in a whole class share-out. During this time a wide range of ideas were eagerly shared, such as making a clay mold of the lakes and forming a familiar shape like a square to calculate area, paddling around the lake in a boat, laying a string around the drawing of the lake, and putting the lake on a grid and cutting pieces off and moving them around to form a shape they could more easily use to calculate the area. Even after Scott signaled the end of class, students continued to share their ideas. This ended the first class session, and students were encouraged to discuss this activity with other people before the next day's class.

The next day, groups got back together to reflect on the share-out and other ideas that came up at home to consider whether they might have even more approaches, and to define the terms area and perimeter. Students could use Geoboards to consider differences and relationships between area and perimeter. Scott wandered around the room, engaging with various groups. The students were enthusiastic to continue discussing these ideas, and periodically there were audible outbursts of joy as students continued their engagement on this task. After about 30 minutes of working on this activity, Scott brought the class back together, and displayed a list he had been keeping of the various ideas that had been suggested. He used these ideas to transition to a wrap-up about the ways in which this problem was an introduction to the type of engagement that will be undertaken through the course, which ends the second day of class. This is the list of key ideas that Scott emphasized around this activity:

- There isn't always a formula for solving problems. Math often involves creativity.
- Many of us have built in misconceptions about mathematics that we might realize need further exploration in this course.
- Many heads can be better than one!
- It helps to attach meaning to math terms. It's hard to remember things if they don't

have meaning attached, if you haven't played with things.

- Often times there are many solutions to a problem.
- Some solutions are theoretical, some are practical.
- Problems don't always have quick solutions—perseverance is important.
- Solutions to problems can grow over time.
- Inquiry-oriented activities are often more meaningful and more memorable than typical exercises or scripted problems.

Students spent almost two full class days thinking about The Lake Activity, as a way to understand the nature of this course. The above list reiterates many of the ways Scott verbalized expecting his students to engage with mathematics during this class, and the description of the Lake Activity illustrates how he introduced these ideas to the students in the course. The timing of this activity is important here, as the first days of a class sets the tone for the classroom environment, instructor expectations, and levels of student engagement (Patrick, Turner, Meyer, & Midgley, 2003). Patrick et al. (2003) showed that teachers' patterns of "motivational and organizational discourse" (p. 1521) held steady from the beginning of a course to the end, and that students in supportive classrooms, where enjoyment of math was communicated and expectations for and belief in all students were high, reported less avoidance of mathematics. During this activity students in Scott's course had the opportunity to engage meaningfully with mathematics, discussing ideas with others, revising their thinking, justifying their ideas, and presenting their ideas to the class. In addition, Scott floated around the room, engaging students in conversation, encouraging creativity, highlighting important questions, concerns, or misconceptions, and he facilitated a share-out of the major take aways from this activity.

On the third day of the course, Scott introduced the syllabus. After outlining some of the

basic details of the course, such as homework assignments, details of the book, and calculators, he took another opportunity to explain his inquiry philosophy to the students.

Scott: We learn things mostly by doing. You can learn by listening if it's interactive, but most of what we learn is by doing. Let's think of something other than math that you have taught somebody else.

Student: I teach skiing and snowboarding.

Scott: I'm going to take a lesson from you and guess what you do. You probably teach it in a room that looks like this one, but maybe up on a mountain. We're in a room like this and I need paper, pencil, and you stand up in front. I won't need my snowboard, so I won't bring it. I mean, I have to learn to snowboard before I can get up on it, right? Maybe we can watch some videos, you can show me on your snowboard, we probably take a quiz, and then there's a final. If you pass with a 2.5 or better you can finally go snowboard.

Class: ((Laughter.))

Scott: No? So, tell me what you do.

Student: You're out on the snow.

Scott: On the first day?!

Student: Yeah.

Scott: That's scary!

Student: I would tell you to get on your snowboard, but you aren't on the hill yet. It's flat and you learn what the board is.

Scott: Can you describe what else you are doing?

Student: I'd show students something using my snowboard and make sure they understand how to do it. I'd let them try it or I would come over and help them.

The students clearly connected to the point Scott is trying to make, as they laugh when he describes learning to snowboard in the ways typical of math instruction. After this discussion, they return to the syllabus, discussing exams, projects, and other aspects of the course. It is clear as he took time for this example that Scott considers the inquiry-oriented nature of the course to be important, but also clear from how he situates the example within the syllabus overview that the inquiry-oriented nature of the course is a regular part of the course characteristics.

The Routine: Typical Engagement in Course Activities

After Scott and the class had had several weeks together to establish a routine, I returned and observed seven 50 minute class sessions during weeks six, seven, and eight of the 12 week course. I observed and video recorded the class sessions so that I could watch and discuss focal students' participation with them during interviews, and I wanted to document and analyze the course culture to help situate focal students' individual case studies within the classroom. The seven class sessions that I observed spanned two course topics, numeration systems and addition and subtraction, and allowed me to understand the various and typical ways that students and Scott engaged with mathematics together during this course.

Throughout the course, students were constantly put into groups of three or four, both randomly (e.g., by drawing playing cards) and as assigned by Scott. In order to facilitate video recording during my observations, Scott assigned my six focal students into two groups of four, though students were unaware they were placed with other focal students in my study at the

time[12]. Between the two lessons class groups were shuffled, but the focal students were still within two groups. In addition, it was standard practice for the students to receive notes packets for each chapter of the textbook that they were covering within the course (for an example, see Appendix K). These packets were partially filled in guides Scott created to help students take notes on the key aspects of the assigned textbook readings. This was something Scott had not used before, but his intention was that students would read relevant material before class and that this packet might help them take good notes since he did not lecture on key terms and ideas in class. Students were not required to take these notes; some students used them and others did not. In class, students were given corresponding activity packets, which had the details of the tasks and activities that they would undertake together in class for a given topic (for an example, see Appendix L). These packets were heavily used by students, and sometimes group copies were turned in to Scott for informal evaluation and comment.

During the seven 50 minute class sessions that I observed during the middle of the course, I saw students participate in a wide variety of activities, such as planning and giving presentations, problem solving, playing games, and working arithmetic problems. Table 4 lists the activities and gives a general description of student and teacher engagement during those activities chronologically. Their study of numeration systems focused on the group presentations, and the work in bases other than 10 facilitated understanding of place value, which continued to be an emphasis as the class moved into their study of the operations of addition and subtraction. While the classroom activities took a wide range of forms, the expectations placed on student participation were quite consistent throughout the activities. The instructor role that

12 Recall the study started with eight focal students, but two did not complete the study. This group assignment was not noticeable to the students since Scott frequently assigned groups in random ways. Students later found out, during the focus group interview, that they were placed with other focal students on those days. Also note that all focal students had given me permission to tell the instructor that they were participating in my study.

Table 4

Activities and Engagement That Took Place During Classroom Observations

Activity	Student engagement	Instructor engagement
Plan and present group presentations with posters on different numeration systems	Worked together to create material to present, decided who will present what, created posters, each student presented a portion of each presentation	Answered and asked questions, gathered resources/materials for students, highlighted key ideas during presentations
Use base block manipulatives to learn how to represent numbers and count in various bases other than 10	Worked together, used manipulatives, shared out counting in different bases in a whole class discussion	Encouraged discussion and use of the base block manipulatives, highlighted key ideas during share-out
Find the smallest whole number difference between two five digit numbers made by using the digits zero through nine once	Worked in groups to make and test conjectures, reasoned about and justified solutions, revised thinking, persisted	Encouraged conjectures and persistence, told groups whether they had found the smallest number
Understand models for subtraction (take-away, missing-addend, comparison)	Individually wrote word problems, shared during class discussion	Facilitated a discussion around the types of subtractions, highlighted key differences
Play a base five addition and subtraction dice game with manipulatives	Worked together to keep track of manipulatives for each student's total points in base five	Encouraged students to work together to determine how to use base five blocks, asked questions to highlight key ideas for students to notice
Work together on base five addition and subtraction problems outside of the game using a “standard algorithm”	Worked primarily individually, came together in groups with questions or struggles, those that understood worked with those who did not to make sure they understood	Encouraged discussion between students, answered questions only if necessary, monitored student “help” to ensure they were facilitating learning, not giving answers
Categorize mental strategies for addition and subtraction	Worked primarily individually, discussed inconsistent answers with the group to come to consensus	Encouraged discussion, prompted students to use the textbook to put formal labels on strategies
Take a clicker quiz	Put in individual quiz answer using a clicker, talked with others to justify their answer, revised thinking if desired	Encouraged discussion, showed responses, facilitated share-out of answer justifications

Scott took on was also consistent. In addition to engagement described in Table 4, the instructor always "floated" around the room during activities when he was not facilitating a whole class discussion or share-out. He was also always keeping an eye out for correctness, and following through with questions to encourage further thinking if needed.

The following section gives a rich description of the Smallest Difference subtraction activity in which the class engaged on day four of this set of my observations. In the class, this activity served as the first activity in the unit on addition and subtraction, and bridged with the previous section on numeration systems because the activity required students to think about place value. This activity was specifically chosen for inclusion here because it serves as a typical course example, encompassing most of the ways in which students were regularly expected to participate within the course and the usual ways in which Scott interacted with the students. I will use the description of this activity as a reference point from which I will articulate the expectations placed on students, recount their participation, discuss the instructor's role, and generally describe the inquiry-oriented nature of this course.

Subtraction activity: Smallest Difference. As the class wraps up their study of different numeration systems, Scott is at the front of the room on his tablet computer, which is connected to the projection screen as usual. Students are seated in six groups of three or four students each around the room. Bahira, Morgan, Samantha, and Laura[13] are seated in Group 1 on the front right hand side of the classroom. Katie, Jessica, Cheryl, and Annie are in Group 3, seated in the back right hand side of the classroom. Students are putting materials and papers away as Scott transitions to their lesson on addition and subtraction. He puts the Smallest Difference activity, shown in Figure 5, up on the screen and reads the question out loud to the students.

13 Laura and Annie started out as focal students, but did not complete the study.

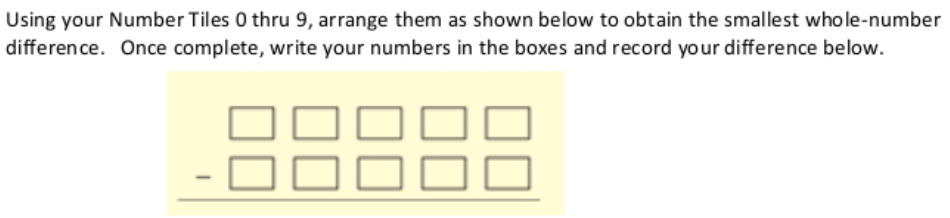


Figure 5. The Smallest Difference activity

The class listens carefully and focuses on Scott as he explains they do not need to use number tiles as indicated in the original handout, but can instead use their mini white boards. After Scott reads the problem, deliberately and slowly, he says, "Go!," which prompts the students to spring into action. In Group 1, the four students quickly huddle around a single white board in the middle of their group space with Bahira writing, as shown in Figure 6. Samantha and Bahira repeat, quietly to themselves, "the smallest whole number difference..." while the group starts thinking.

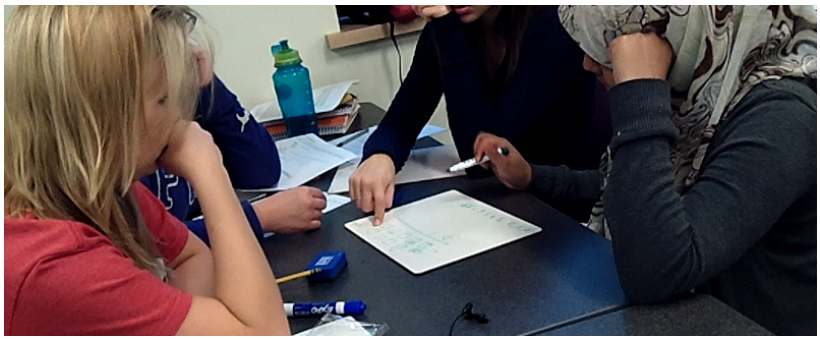


Figure 6. Group 1 working on the Smallest Difference task.

Over in Group 3, Scott's, "Go!', has caused Katie to move quickly. She says to herself, "I did this already...," while flipping through her textbook. Then, she directs Cheryl on what to put down on their white board.

Katie: Nine zero, one eight, seven six, five four three two.

Jessica: Cheater McCheaterson!

Katie: Try that. Hold it up. That's going to equal one, one, one, one, one. That might be it, but I don't know.

Cheryl: ((Writes it down and holds up the white board for Scott.))

Jessica: But, can't--

Katie: That's the smallest whole number difference.

Jessica: We can't make a smaller number?

Scott: Did your group agree on that?

Katie: No, I totally bullied them into it.

Jessica: She did!

Scott: That's the right idea, but it's not the smallest. ((Walks away.))

Jessica: Okay, so this is what I was thinking...

Group 3 regroups and starts talking about the problem. Jessica's suggestion gets them back to having ones in the difference, so she keeps brainstorming ideas out loud. Katie says, "Try it," giving each group member their own white board and markers, and they work to write down ideas as shown in Figure 7.

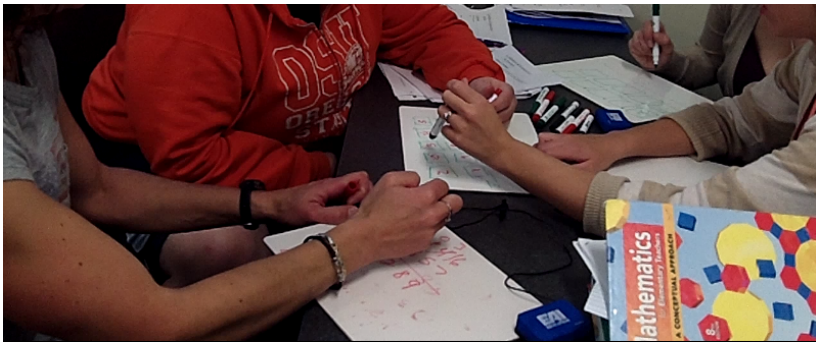


Figure 7. Group 3 working on the Smallest Difference task.

Cheryl shouts out to check with Scott to see if they can use negative numbers, and he responds, "whole numbers," and she tells the group, "Okay, so no negative numbers." Katie is still struggling, saying to herself, "It's smaller than one, one, one, one, one? How can that be?" Then, she engages the group in her thinking.

Katie: There's not going to be any zeros, you guys.

Cheryl: Because you can't use numbers over again.

Katie: But one, one, one, one, one is not *correct*. What's smaller than that?

Katie then rereads the problem out loud to the group, and they think for a few minutes without writing very much down. Then, Jessica gets an idea.

Jessica: But, what if all of...((Make crossing out motions across her white board.))

Katie: I see what you're saying!

Jessica: You're gonna borrow! We want this—it's just a matter of getting there.

This spurs Cheryl and Katie to start writing on their own white boards. Annie, who has not said anything up to this point, leans in, and says, "The BBB Rule, or whatever it was called in third grade? I never got that. I suck at the whole borrowing thing." This gets Jessica's attention and she and Annie discuss borrowing together while Katie and Cheryl move ahead on their own white boards. After a few minutes, Cheryl holds up an answer to Scott, and he replies, "It's smaller than your last one. Talk it over with your group." Katie then holds up her white board for the group, and tries to explain her reasoning to them, describing how she tried to put smaller numbers on the top and larger numbers on the bottom. She also notes that Scott just said they can "go smaller," perhaps in response to his comment to Cheryl, and the group keeps working. Even though they work individually on their white boards, they monitor what others are writing and talk to one another, highlighting important realizations, "This is a good idea, to put the

smaller numbers above the larger."

About five minutes into the activity, Scott gets the class' attention, so that Group 4 can share a solution. Jessica says, "I don't want the answer!," covering her eyes, but the class turns their attention to the front of the room. A member of Group 4 holds up their white board, showing a difference of 469. Scott says, "I would announce that that is small, but it's not the smallest." Katie writes down Group 4's work, saying, "That's a good idea, let's keep messing with that!" The group then huddles more around Cheryl's white board, discussing how they might be able to use ideas from that solution as an example to model, particularly with regard to keeping the zeros in the left most place values. A few minutes later, the group hovers around Katie's white board and working together they come up with the difference of 289. They announce this to Scott by shouting to him across the room, and he asks them to come to the front to share the numbers leading to the difference of 289. Katie goes to the front of the room, and Cheryl shouts their numbers to her as she fills in the boxes on the tablet projection for the class to see, as shown in Figure 8.

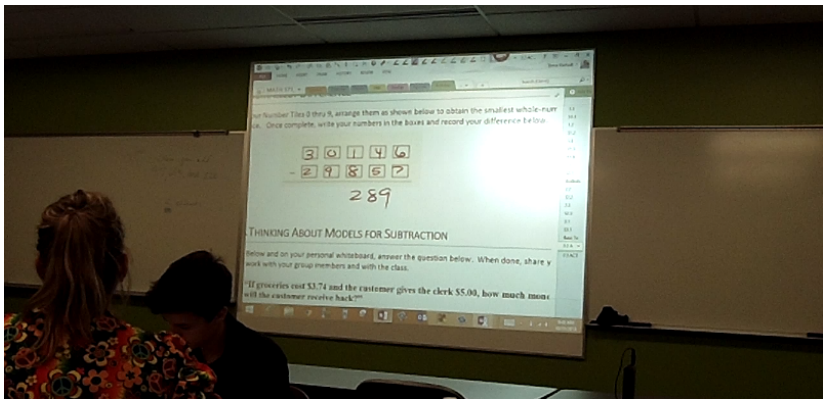


Figure 8. Group 3 shares an answer to the Smallest Difference problem.

Scott: I would like to announce that, 289, is...not even in the top six. *However*, it is in the right hundreds range. The smallest number is in the

two-hundreds.

Before Katie even returns to her group, they are revitalized and determined to keep trying.

Annie: So, let's look at what we did.

Katie: I tried to make this difference the biggest. ((Pointing to the pair of numbers in each place value.)) With the biggest number on the bottom and the smallest number on the top. We want these differences to be *huge*, where we have to borrow a lot.

The group talks about whether they should leave certain parts of their solution and move other numbers, continuing to build off their current work. They quickly get down to 278, and keep working, undeterred when they try something that gets them a larger difference.

Meanwhile, in Group 1, they start by trying out some numbers, thinking that larger numbers should be to the left, but quickly move to wondering whether the difference in individual place values can be zero.

Samantha: I was thinking the nine here and the eight here. ((Point to the top left and bottom left squares.))

Morgan: Is there any way we can get zero here? ((Pointing to the difference below those squares.))

Samantha: No, we can't use the same numbers.

Morgan: Oh.

Bahira: ((Writing the numbers Samantha had suggested, looks at Morgan.)) Wait, what were you saying?

Morgan: ((Thinking.))

Samantha: She wants to get this to be zero. ((Pointing to the difference.))

At this point, the students sit in silence for a few minutes and then start suggesting pairs of numbers that produce differences of one. Laura suggests, "We could try two and one," and Bahira adds, "Or six and seven." Samantha then quickly says, "We have one, one, one, one, one. We have that option," but the group then goes quiet into thought again. Scott comes near the group, standing behind Samantha, but does not say anything. Another minute passes, and Laura suggests putting a zero in any place on the top row with a non-zero number below it.

Samantha: Oh! You're right!

Bahira: ((Quiet, but still writing the group's ideas.))

Morgan: I hate having tricks like that.

Scott: Ooh... ((Tone indicating they're on to something, he walks away.))

Samantha: Oh! It's all small numbers up here. ((Pointing to the top row.))

Laura: ((To Bahira, pointing at white board.)) So, cross this out, so we can borrow.

At this point, Laura and Morgan start making a large number of suggestions to Bahira very quickly. She tries to write what they are saying, but does not talk. Samantha is quiet, thinking to herself, and Morgan starts writing on her own white board. The group comes around to discussing that the smaller numbers should be to the left, and how that might work if they borrow across in each place value, "borrowing all the way through." Their thoughts are brief and quiet. About five minutes into their work, Scott has Group 4 share their answer of 469, as described above. Group 1 looks up briefly, and then Laura says, "I think we're good." and they go back to their own work. Another minute later, Laura says, "I need to write it down," and the members of Group 1 move to their individual white boards, each of them deep in thought and writing down different number combinations. They occasionally say things out loud, but are not discussing the

problem together at this point, and they barely look up when Group 3 shares their solution of 289, though the work on Group 1's white boards shows they are in the range of the smallest differences, with answers like 311.

About twelve minutes after the start of the Smallest Difference activity, the class period is coming to an end. Scott says, "So far, one group has gotten it, and they are *not* going to tell." He mentions that sometimes it's hard to think while everyone is hovering around, so he encourages them to take the problem home, and to come in tomorrow with the next activity on the sheet ready to share. Katie runs up to the front of the room and asks Scott about 249, a difference she has just gotten. She returns to the group and says, "This is only two away. We are close!" She and Jessica clean up and talk about how they want to find the answer, not have someone tell them. As the students circle the problems Scott has asked them to do at home and start to pack up their materials, Samantha is still hovering closely over her white board with her back to Scott. The other students start to leave around her, but she appears on the verge of a breakthrough, saying, "Hang on, hang on, oh!, that's zero, and then, no … ."

Characterization of the classroom culture. The above description of students and instructor engaging together in a classroom activity helps facilitate a discussion of how Scott implemented his preceding description of an inquiry-oriented classroom. In coordination with Table 4, which showed all of the course activities observed during two topics over seven days of the course, this description of the Smallest Difference task helps elucidate how students were expected to (a) discuss mathematics with others; (b) construct solution strategies; (c) be intellectually courageous; and (d) let their thinking develop over time.

Discussing mathematics with others. In this activity, we see students engaged in group work, a routine aspect of this course. They are expected to communicate their ideas to other

students in their group, making suggestions for how to proceed. For example, Group 1 works together on this activity to get their ideas started, suggesting numbers to try, whether they can make the difference zero in each place value, and eventually coming to idea of regrouping. In addition to simply sharing or suggesting ideas, students are expected to justify their reasoning to other students, explaining how they came up with their idea or why their idea makes sense or is reasonable. We see this as Katie holds up her white board for her group, describing what she was thinking about when she arrived at a difference of 289. In the Smallest Difference activity, we see this justification more clearly in small groups, but do not see this in the larger class setting. It is important to note that students often justified their thinking to or shared their ideas with the entire class, through large group discussions or brief moments of sharing at the board or tablet projection, whereas in this activity that share-out focused more on answers.

Discussion expectations also involved encouraging other students in the course, seeking their ideas or addressing questions or confusions. At one point early in Group 1's discussion, Bahira specifically stops the conversation to ask Morgan to repeat her idea about making the difference zero, trying to understand the suggestion. In addition, Jessica and Annie have a side conversation about how regrouping works when Annie says she "never understood borrowing" in school. During other activities observed in the course, it was not unusual to see students talk in pairs within groups about questions that one student had or even to move off from the group and work some problems together on a white board on the wall. Communicating mathematical ideas through justification, reasoning, and questioning was one of the major participatory expectations for students in this course.

Constructing solution strategies. Intricately linked to the communication demands of the course is the notion that students were also expected to generate solution strategies and

approaches individually and within groups, seeing themselves and other group members as mathematical authorities with valuable knowledge. That is, students were expected to generate ideas for how to solve or reason through problems based on information given, their prior knowledge and experience, and the course reading assignments, rather than follow prescribed procedures or set algorithms invented by others. Though students were sometimes given introductions to content or terminology, especially through textbook reading assignments, during these observations they were never given predetermined approaches and asked to replicate them to gain understanding or for practice. The Smallest Difference activity in itself is designed to foster problem solving and strategy generated by students. There is no associated procedure or process give with the original problem that students are given. Though they were asked to read the book chapter about addition and subtraction before this class, it is clear from the way they jump in and suggest ideas and eventually come around to using the idea of regrouping that this approach was not prescribed to them.

Although it was a clear expectation in the course that students learn to see themselves and others as having mathematical authority and freedom to try their own approaches, students did occasionally look to the instructor or textbook for confirmation of ideas or suggestions that might show they were on the right track. In this activity, we see Katie make use of both of these sources of knowledge. When released to work on the Smallest Difference activity, she immediately starts flipping through her textbook to an example she remembered reading. We see Jessica disapprove of Katie's use of the book to find the answer, as she jokingly calls Katie a "Cheater McCheaterson." Then, Katie urgently calls Scott over for confirmation that her difference of 11111 is the smallest difference, without consulting with her group at all before doing so, but readily admitting she did not do so. Scott turns the question back to the group, a

typical instructor strategy that will be discussed below. Later, members of Group 3 continue to hold white boards up for Scott to check their answer, and he does give them a yes or no response. During the course observations, students rarely looked to Scott as the only math authority in the room, but it did happen on a few other occasions. Scott sometimes addressed questions from individual students, if they were clarifying in nature or simple, but if the students were supposed to be working as groups, and the question was about whether an approach or argument was valid, he usually deflected the question back to the group for consideration, as he did several times during the Smallest Difference activity.

Being intellectually courageous. The Smallest Difference activity is an excellent example of the ways in which Scott expected the students in the course to try out ideas and test hypotheses, being willing to make mistakes and revise their thinking based on attempts or new ideas. We see students immediately going to work trying out certain number combinations during this activity, making progress in getting smaller and smaller differences by first trying numbers, realizing that regrouping can lead to zeros in certain places, and then working through their understanding of how regrouping can help them to make the difference between the two numbers smallest. The students work on mini white boards during this activity, which are used in virtually every class session in the course. The white boards facilitate the revision of thinking by making it easy for students to erase their ideas and try out new approaches. This activity shows a particularly ideal use of white boards for this purpose, though students used them consistently in class sessions and focal students even talked, during interviews, about how they enjoyed the freedom the white boards allowed for revision of work.

Another aspect of being intellectually courageous that Scott emphasizes in the course is being willing to ask questions. One place this is clear in the Smallest Difference activity is when

Morgan says, “Is there any way we can get zero here?” It is clear she has not yet figured out how to do that, but is willing to make a suggestion by asking this question. Similarly, Annie is willing to openly admit to her group that she does not understand how borrowing works. It was a very regular occurrence for students to ask other group members for help during these course observations. As noted in the discussing mathematics section, where I emphasized that students were expected to address others' questions, students were expected to be willing to admit their confusions and work through them in discussions with their group members. The risk of making mistakes and having to reconsider their approach is something that became routine for students in this Math for Elementary School Teachers course, as they repeatedly show their willingness to work through solution approaches as they did during the Smallest Difference activity.

Letting thinking develop over time. The Smallest Difference activity is a perfect example of what Scott meant in interviews when he said that in inquiry-oriented learning things are not always wrapped up neatly in the end. This activity is left open for students to consider, there is no whole class share-out or wrap up of the main mathematical points of the activity, which is primarily to facilitate understanding of place value and regrouping in the standard subtraction algorithm. Indeed, when I interviewed students in the week following this activity, almost all of them mentioned it, talking about how they left class still thinking about how to find the smallest whole number difference and what the answer might be. Even as students are cleaning up and leaving the room, Samantha is clinging to her white board saying, “hang on, hang on,” not wanting the class to end. In interviews, students described talking to each other at later dates, bringing up the problem with family and friends (both to challenge them and to see if they could come up with a smaller answer than the students had in class), and trying to investigate the problem using other resources such as books and websites. The activity clearly

had an impact on students, and engaged them in deep, long term mathematical thinking around place value in subtraction, appreciating that not every problem has a quick, neat solution process.

The instructor role. The description of the Smallest Difference activity also helps illuminate the ways in which the instructor, Scott, engaged with students during this course. We can see how he enacted the instructor role described earlier in this chapter, which included: (a) providing tasks and resources; (b) acting as a "guide" that "floats around the room"; (c) encouraging perseverance and student generation of knowledge, pushing questions back to small groups; and (d) facilitating share-outs of key misconceptions or important ideas and student thinking. During the Smallest Difference activity we particularly see the second and third aspects of the role Scott described for himself.

During this activity Scott encourages engagement with his enthusiasm for the task, challenges groups to consider their own ideas carefully, and deflects questions back to the group for consideration. At the beginning of the activity group work, we see an example of how Scott deflects a question back to the group for consideration. Katie does not consult her group, but rather asks Scott to consider her answer of 11111, and he responds by asking whether everyone agreed that that was even a viable option to propose for consideration. In this way, he not only suggests that the group has valuable knowledge to be utilized in considering their response, but also encourages them to think carefully about the solutions, not just proposing ideas without careful consideration. In deflecting this question, he simultaneously encourages deeper reasoning and communication within the group. During the course of this activity, Scott keeps the students engaged and excited about the task. He introduces the task in an almost race-like competition (which may partly contribute to Katie's sense of urgency), and stimulates engagement by keeping the whole class appraised of other groups' progress, noting when groups

have found differences that are close to the smallest. The task is designed so that the students do not need a lecture on material beforehand in order to engage meaningfully. As a result, during this activity, Scott motivates both enthusiasm for mathematics and serious problem solving, checking in with the groups without directing their thought process.

One thing that Scott routinely did during class activities that is not as present in this particular activity is facilitate share-out of key ideas. In this activity, there were two times when students were asked to come in front of the class to present ideas on Scott's tablet projection, which was typical. However, in this activity those share-outs focused only on answers. Quite often, in the class sessions observed, individual students were asked to share *and justify* their own thinking or their group's thinking in brief moments when key ideas or misconceptions were addressed in whole class discussions. Scott's actions during the course activities align with the role of "guide" that he described for himself during his interviews with me. Although we do not see him explicitly facilitate knowledge sharing in this activity, around place value or the subtraction algorithm for example, it certainly shows an example of an activity that fostered his desired engagement on the part of students, encouraging inquiry, courageousness, and communication.

This section has characterized the classroom culture that developed as the students and instructor worked together to engage deeply with mathematics in this inquiry-oriented course for prospective teachers. During my follow-up interviews, which took place immediately following these seven class days of observation, all of the focal students confirmed that the lessons and activities that I observed during that time were typical of the course engagement they had experienced up to that point. In doing so they highlighted the emphasis on engaging in mathematics through group work, coming up with their own solution strategies, and talking with

and presenting ideas to others and the whole class during discussions. At the time, some focal students noted it was unusual that they made more "formal" presentations with posters during the numerations systems lesson than the usual class share-out or discussion, but I did see students give a couple of similar formal presentations using posters or computers later in the course.

Self-Evaluating: Post-Reflections

At the end of the course, students were assigned to write a two page post-reflection, during which they reread their math autobiography and discussed the following questions:

- After re-reading your autobiography, have any of your thoughts changed?
- Have your views about your own abilities in doing mathematics changed in any way?
- Have your views about how you will teach mathematics changed in any way?
- Have your views about teaching changed in any way? Do you still want to teach? Have you thought more about what setting you want to teach in? What grade level?

This reflection allowed the students the opportunity to evaluate their experiences over the course of their enrollment in this class, thinking about both their past experiences, from their autobiography, and their recent experiences within the course in relation to their future teaching of mathematics. This assignment is one way that Scott implemented his desire for his prospective teachers to think carefully about the benefits of inquiry-oriented teaching and learning for students and how their experiences as students in Math for Elementary School Teachers might influence their teaching. Along with the preceding characterization of this course as inquiry-oriented, the entire class was surveyed as part of this study on students' affective experiences in mathematics. This survey, discussed in the next section, helps to situate the affective experiences of the focal students by placing those students within the larger classroom population.

Psychological Needs Satisfaction of the Whole Class

All 21 students in the course completed a 21 question survey about the satisfaction of their basic psychological needs of competence, relatedness, and autonomy, twice during the quarter. The first time students took the survey, they were focused on reporting their feelings during their past math classes. The second time, students were asked to focus on only their experiences in the current math course. Table 5 shows the class mean on each of three subscales, and the mean scores for each focal student on each subscale. Higher numbers indicate more satisfaction of the need in question.

Table 5

Mean Scores for Subscales of the Self-Determination Theory Basic Psychological Needs Scales

	Competence		Relatedness		Autonomy	
	Past math courses	Current math course	Past math courses	Current math course	Past math courses	Current math course
Whole class	4.79	6.32	4.35	5.77	3.95	5.83
Jessica	5.17	7.00	5.88	6.75	5.00	6.29
Morgan	4.67	6.83	6.25	6.50	4.29	6.86
Bahira	4.33	4.33	4.14	5.14	3.50	5.00
Katie	3.17	6.83	4.50	6.13	3.43	6.00
Cheryl	5.50	6.50	3.50	5.88	5.29	6.14
Samantha	4.80	6.33	3.50	6.25	4.71	6.00

Note. Original scores were given on a 7-point scale (1 “not at all true” to 7 “very true”).

The Cronbach's alpha for the total need satisfaction scale on the initial survey was .79 and .86 on the second survey. The alphas for the competence, relatedness, and autonomy subscales on the initial survey were .51, .86, and .59, and on the second survey, .62, .84, and .55, respectively.

The results show that students reported feeling more competent, related, and autonomous during their participation in this inquiry-oriented course than they reported for their past

mathematics courses. Indeed, three matched-pairs *t*-tests indicate that students reported significantly higher feelings of competence ($M = 6.32$, $SD = .62$) in the current, inquiry-oriented course than they reported feelings competence in their previous mathematics courses ($M = 4.79$, $SD = .77$), $t(20) = 7.28$, $p < .001$, significantly higher satisfaction of their need to feel related in the current course ($M = 5.77$, $SD = .96$) than in the their previous courses ($M = 4.35$, $SD = 1.08$), $t(20) = 6.04$, $p < .001$, and reported feeling significantly more autonomous in the inquiry-oriented course ($M = 5.83$, $SD = .69$) than in their past math courses ($M = 3.95$, $SD = .91$), $t(20) = 7.96$, $p < .001$. Table 6 summarizes the results from these *t*-tests.

Table 6

Matched-Pairs t-Tests Comparing Current and Past Need Satisfaction of the Whole Class

	Subscale means		*t*	*df*
	Past math courses	Current math course		
Competence	4.79 (.77)	6.32 (.62)	7.28***	20
Relatedness	4.35 (1.08)	5.77 (.96)	6.04***	20
Autonomy	3.95 (.91)	5.83 (.69)	7.96***	20

Note. Original scores, given on a 7-point scale (1 "not at all true" to 7 "very true"), represent the degree to which a student experienced feelings of competence, relatedness, and autonomy on questions from three subscales. Shown here, are the mean scores on each subscale. Standard deviations are shown in parentheses below these mean scores. ***p < .001.

The survey also results help illustrate how the focal students in this study were representative of the course population as a whole, in general, individually following the trends just discussed[14]. They typically reported lower degrees of need satisfaction in past math courses, which were described in student writing and interviews as largely traditional, lecture-based courses, and higher fulfillment of each need in the current, inquiry-oriented course. Focal

14 Focal students volunteered to discuss these feelings with me over the course of their enrollment in this class, which may contribute to their scores for this course being slightly higher than the class average.

students' survey responses will be discussed further in each of the individual case studies articulated in the following chapter.

The preceding characterization of the classroom microculture associated with this course gives some possible explanations for why students might report feeling competent, related, and autonomous. For example, students were regularly given a variety of opportunities to demonstrate competence, such as discussing problems with their groupmates, presenting or reporting strategies or final products, and persevering while tackling complex problems. Additionally, the instructor of the course heavily promoted the idea of the classroom community as a family, learning together, which was taken up by students both inside and outside of the classroom. Although the course activities were communicated by a set handout for each unit, which could be interpreted as a more directed management of student work, the students were routinely encouraged to genuinely engage in mathematical problem solving, taking ownership over devising their solution approaches and revising their thinking as necessary.

The individual focal student case studies, which follow in Chapter 6, allowed me to expand upon and carefully attend to what students mean when they report and talk about experiencing these feelings, when students report feeling these needs satisfied, and how they describe their affective experiences being linked to the participatory and cognitive demands of the inquiry-oriented classroom. Additionally, the individual case studies provide insight into prospective teachers' developing mathematics teaching practices, as they also discussed with me how these affective experiences might shape their future teaching of mathematics and what they hoped their students would experience in the their classrooms. In short, the case studies allow me to document the details behind how and why students experience their feelings, and to understand how students themselves make connections between the course microculture and

their affective experiences.

Conclusion

In this chapter I have described how the Math for Elementary School Teachers course in which this study was designed and implemented as inquiry-oriented. I have focused on the conscious decisions the instructor, Scott, made to promote the active involvement of students in mathematical knowledge construction and to foster a strong sense of community in the classroom, the observations of the course that allowed for interpretation of how those design efforts were implemented within the actual classroom environment, and the overall feelings reported by the entire classroom population from which the focal students were selected. In the next chapter, Chapter 6, I will put forth the individual focal student case studies, which articulate how the prospective teachers in this inquiry-oriented course make sense of their past mathematical experiences, their current affective experiences as influenced by the cognitive and participatory demands of an inquiry-oriented math course, and how they anticipate these affective experiences influencing their future teaching of mathematics.

CHAPTER 6: FOCAL STUDENT CASE STUDY FINDINGS

The focal student case studies that make up this chapter articulate the ways in which the six focal students make sense of their affective experiences in mathematics. Specifically, the case studies describe how the focal students understand their past affective experiences in mathematics, how they related their current affective experiences to the demands of an inquiry-oriented math class, and how they believe their affective experiences will shape their future teaching of mathematics. As such, the case studies in this chapter are arranged considering three primary time focuses: students' past affective experiences of mathematics, their current experiences within this course, and their anticipations of how their affective experiences will influence them in the future[15]. Each case study also includes a brief discussion of the student's psychological needs survey responses and a summary discussion of the student case.

Each case begins with a general introduction to the focal student, expanding upon the brief introduction given in Chapter 4. This description is intended to characterize the focal student as a person, inside and outside the classroom to the extent possible, to give a sense for who they are and their overall story during this quarter. Then, the narrative centers on the focal student's past experiences of mathematics, focusing on their experiences, or their lack of experiences, of feeling competent, related, and autonomous. As a reminder, data on past experiences largely came from mathematical autobiographies and initial student interviews.

The next section of the case studies examines the ways students made sense of their participation within the current course and the aspects of the course that they described as playing an important role in their affective experiences of the course. This included students'

15 It is important to emphasize the uniqueness of each student's experience and their conversations with me. While I attempted to gather data around competence, relatedness, and autonomy for each student with regards to each time point, my goal was to tell students' stories from their perspective. As such, I did not push them to focus on certain feelings or instances. Information attained about each psychological need is presented here if it was present in the data from the student, but the relative focus on certain feelings varies from student to student.

comments on the content, the course structure, and interactions with the course instructor and other students and the class. Although all interview and writing data contributed to this section, course observations and follow-up interviews were the primary sources of this information.

Next, the case study will shift to discuss how the focal student anticipates their experiences within this inquiry-oriented course to shape their future teaching of mathematics, which was primarily discussed in final interviews. Connections are made to the affective experiences the student had during the course, and how they envision their future mathematics classroom, from what they want their students to be doing and feeling to why they articulate their teaching practice in the ways they do.

Each case study will conclude with a brief discussion of the focal student's survey responses and a summary of the case, which will help situate each student as part of the maximum variation sample I tried to attain to tell the stories of a wide range students. The focal students represent a diverse set of mathematical backgrounds typically present within a Math for Elementary School Teachers course. As such, their stories help articulate both the common and disparate experiences that might occur during the course, based on how those prior experiences prepared students for their participation within this course.

Jessica: A Student with Traditional, Mostly Negative Experiences of Math

Jessica is a student in her mid-twenties, in her last year at Great Valley Community College, and is looking forward to transferring to a local state university to major in education. She works at Starbucks and describes herself as “bubbly and loud,” also emphasizing that she is an “overachiever” who “really enjoys school” now that she has a goal. She told me about how she had changed her mind several times about what she wanted to do, gone to several colleges, and then took about six years off before coming to GVCC. During that time she worked at two

different KinderCare child care centers. In the first center, the focus was on high quality teaching of children, but Jessica left the second because she did not like how the curriculum was being implemented. After she left the second job, not working with children helped her realize how much she wanted to teach, so she returned to school at GVCC.

Although Jessica is passionate about teaching, she has had strong, negative experiences with mathematics. Her past math classrooms were entirely traditional, and her lack of success in those classrooms is what makes Jessica hopeful about inquiry-oriented teaching and learning. Jessica's experience in one previous math course at GVCC helped her see that she could be successful in mathematics, if she worked hard. In this course, Jessica takes on a leadership role, is very active in volunteering answers, and values her high exam scores. She entered the course knowing she could be a good teacher, and also knowing she did not want to lecture like her previous teachers. As such, she was very open to the transformative experience that she went through in just a few short months, leaving this course wanting to be a leader in education, teaching in inquiry-oriented ways, and looking forward to taking more mathematics courses during her teacher training preparation.

Jessica's Past Mathematics Experiences

Jessica transitioned to education in the United States after starting her education in England as a small child, which was not easy for her because the content was taught in a different order in each country. She remembered feeling like math was a "weak subject" for her and that this time was "the start of a long and problematic struggle in mathematics." Jessica described herself as an "outsider" in mathematics because she "struggled so much." She used questions to try to belong, but talked about a noticeable lack of community in her prior math classes, noting that it felt like it was "everyone all for themselves." Those past math experiences

were “always lecture-based,” where “the teacher would just stand up there and solve problems very quickly on the chalk board and expect the class to know how to do it all right away.” In addition to not fostering a sense of community, the courses did not provide Jessica opportunities to develop her own strategies in mathematics. She remembered only being told “do it this way” in most of her classes.

As mentioned, when she struggled in mathematics, Jessica attempted to ask questions, but her most pivotal negative memory comes from an instance when she tried to ask her ninth grade teacher a question. The teacher told Jessica “to use that thing between [her] ears and stop asking stupid questions,” and transferred her to “the slower math class.” Jessica described this event as “degrading and embarrassing.” After that, Jessica described her coursework as “slow moving and not challenging,” telling me she “checked out” and had no desire to put in effort to learn, so she “took shortcuts” like using calculators or looking answers up online whenever possible.

Preceding this course, Jessica took another math class at GVCC that improved her outlook on mathematics because the teacher fostered a sense of community and strategy sharing outside of class, meeting with students before class to work problems on the board. Jessica experienced newfound confidence in her abilities, working hard, solving problems on the board in front of other students, meeting with the teacher before every class session. This meeting time also allowed Jessica to experience a sense of community and belonging in a math class, and experience having the autonomy to show her own reasoning “in a mathematical way,” not a particular way specified by the instructor. This class was one of the only times Jessica talked about feeling competent in a past math course because she was able explain ideas in front of other students and work problems on the white board.

It is important to note that whenever Jessica talked about her past experiences in

mathematics, she mentioned her grades. While telling me about her previous college math experiences, she only described how she did not get grades "good enough to move on," or "barely passing," and even while sharing this one positive, recent experience, she told me the 3.0 she earned could have be higher if she had worked harder. Still, that class did show Jessica that if she "puts in enough effort," she can succeed in mathematics, fostering a sense of perseverance, and opening her mind to the possibility that mathematics can be taught without lecturing. As a result, she is willing to "use resources" or "walk away and come back" to problems she is trying to solve, "rather than just give up." Entering this course, she wrote in her autobiography that she is "no longer intimidated by math."

Jessica's Experiences During Math for Elementary School Teachers

This class built on Jessica's positive experience in her last math class at GVCC. That course taught her that if she puts in effort, she can succeed in math, and this course took that further, showing her new ways to teach math. In this course, Jessica describes herself as "definitely one to raise my hand and make comments or ask lots of questions" and "always put in [her] two cents." She went on to tell me that she takes on leadership roles in groups as well: "If we have a paper to turn in, I'll be the one that is writing on the actual one that we are turning in. I am always talking and helping people understand it or keeping track of stuff." I saw the class role she described during my observations, noticing that she consistently talked or explained, and rarely listened to or encouraged other students to share their ideas. Jessica only talked about feeling competent when she was showing what she knew, suggesting that competence is about individual demonstration of knowledge for Jessica. She also talked frequently about getting "full credit on all assignments" and how she had earned perfect scores on every test in the class until the last one, expressing disappointment about getting an 86 percent. In addition, Jessica disliked

that during clicker quizzes students were expected to discuss their answers together because she thought there should be more individual accountability for understanding, suggesting they be "silent, with no talking, here is the question, put in your answer."

At the beginning of the course, Jessica told me that group work helps her "not feel alone," knowing other people are struggling, and she liked that people from class worked together in "study groups" after class. However, when I asked Jessica whether she felt like she belonged in this class, she told me she worries that students in the class see her as "harsh" or "mean." When we watched a clip of her group working on their numeration systems presentation and preparation, she expressed a great deal of anger that her groupmates were not prepared. During the observation, I saw Jessica manage the entire planning and presentation, redoing the posters that the group created in class at home, and then doing most of the presentation because her groupmates did not know what was on the new posters she had created. When I interviewed the other students in this group, they were upset that Jessica thought they were unprepared and "took over," telling me that they had done the assigned reading for that day and were ready to contribute to the presentation planning. Jessica complained about having to "teacher" her group during this particular activity, and at other times complained about having to "spend ten minutes explaining something to another classmate when they should already know it from the reading."

During our conversations, Jessica told me that the "goal" was to come to class "prepared," "understanding what we are going to go over," and then in class to "do activities to make it more concrete." This may explain why Jessica was often disappointed that other students in her groups did not know enough. She expected them to fully know the material in advance, and they expected to learn the material in class. Indeed, Jessica told me, "I don't like how in some classes it is like you come to class to learn." Jessica did mention that she is "not

always as confident as people think," and that sometimes she "thinks aloud" to get "feedback." However, my observations and interviews with other students suggest that other students in the course do not interpret her thinking out loud in that way, instead interpreting it, as two groupmates put it, as "taking over." In retrospect, Jessica also "felt bad about interjecting" during other students' part of their group presentation, but felt it was necessary to get correct information presented to the whole class. It seemed that Jessica's strong focus on feeling and showing competence as giving correct answers, in discussions, in groups, and on assessments, somewhat hindered her ability to learn from and work with others in this course.

Having the autonomy to pursue her own solution strategies and be seen as having valuable mathematical contributions also strongly affected Jessica's feelings about math. She told me, "this class really gives you the opportunity to really figure it out whichever way is best for you," emphasizing that "as long as you can explain how you got there" you are demonstrating understanding. Being given this type of freedom was closely linked to Jessica's realization that "math class doesn't have to be a boring lecture." At the end of this course, Jessica described "enjoying" math, *wanting* to go "above and beyond," to "dig deeper" to "engage more" with mathematics. She was "fascinated" by her turn around about mathematics, noticing that even though she "strongly disliked math" just few short months ago, at the end of the course she has "a new passion for it" and sees "how math can be fun."

Jessica's Future as a Mathematics Teacher

Jessica entered the course knowing she didn't want to be "the typical lecturer," and looked forward to this class giving her "another perspective" on how to teach that might be "more effective" for students. Interestingly, at the beginning of this course, Jessica wrote in her autobiography that she hoped that inquiry-oriented teaching and learning would help her know

how to teach students “so that they will not rely on a teacher, but instead depend on themselves.” There is a noticeable emphasis on student independence. Over the course of the quarter, Jessica frequently talked about how letting students come up with their own solution strategies and try their own ideas is a “great experience” that will “give them deeper understanding.”

To be more specific, Jessica frequently talked about solving problems in “whichever way is best” for the individual student, noting that “failing and failing and getting back up and trying again is more of a valuable learning experience than someone telling you how to do it.” She closely linked her experiences in this course, being given “tools” like manipulatives, pictures, or algebra, to come up with her own solution strategies to how she hopes to teach kids, giving them “tools for their toolboxes.” Jessica also frequently talked about her practicum site, a third grade classroom, giving specific details of a variety of ways she was applying what she had learned in this class to help students learn math. She had the chance to observe a teacher first hand, who primarily lectured to students, and this what she told me about that:

> She will sit there with her document camera in the front of the room and be like, “This is how you do it.” Then she will show them again, “This is how you do it.” Then she will have them do a couple, and then she will say, “Okay, this is how you are supposed to do it, guys.” I am, thinking in my head, I'm sitting in the back of the room observing and I'm thinking, I learned so much stuff in my math class, I just want to tell her, “You are trying to teach your kids multiplication. Let's play with some blocks, let's build some arrays.”

Although Jessica can describe in detail how she wants to teach elementary students, and felt confident in this course, she worries about her future in “harder” mathematics like calculus. She is considering getting a math endorsement, but her fear stems from her thinking that more advanced mathematics cannot be taught in inquiry-oriented ways.

Jessica's feelings about mathematics and about teaching mathematics shifted over a short period of time, and she acknowledged this in our final interview, “Sometimes I think it only takes one teacher to really change your mind on how you perceive math.” She discussed this in relation to both her own experience and how she hopes to positively influence her future students. Jessica is aware that just a short time ago she thought she “wasn't a math person,” and that she needs to be sensitive to future students who might be thinking that about themselves, but strongly feels that success in math is just about effort and those students “just need to try harder.” By the end of this course, Jessica told me she has a “burning desire to pursue more math,”and wrote in her autobiography about wanting to be a “leader in [her] profession.”

Jessica's Survey Responses

Jessica's responses to the Basic Psychological Needs Scales, shown in Table 7, align with the above story of her experiences based on her interview and writing data, showing her past experiences in mathematics not fostering as much satisfaction of her needs to feel competent, related, and autonomous as this current class.

Table 7

Jessica's Mean Scores for Subscales of the Basic Psychological Needs Survey

	Competence		Relatedness		Autonomy	
	Past math courses	Current math course	Past math courses	Current math course	Past math courses	Current math course
Whole class	4.79	6.32	4.35	5.77	3.95	5.83
Jessica	5.17	7.00	5.88	6.75	5.00	6.29

Note. Original scores, given on a 7-point scale (1 “not at all true” to 7 “very true”), represent the degree to which a student experienced feelings of competence, relatedness, and autonomy on questions from three subscales. Shown here, are the mean scores on each subscale.

Two observations are interesting to note with regard to her survey responses. Initially,

Jessica rated her past courses as fostering a feeling of relatedness that is much higher (5.88 out of 7) than the class average (4.35 out of 7). This could be in part because of the experiences she described in her last class at Great Valley Community College. Although that course was not as inquiry-oriented as this Math for Elementary School Teachers course, the instructor fostered a sense of community, encouraging students to work together before class. As mentioned above, this did give Jessica hope for the sense of community that can be created during a mathematics course. Additionally, it is of note that Jessica's competence subscale mean for this inquiry-oriented course is a 7, which means that she rated every question relating to how competent she felt is this course as highly as possible. Given that Jessica came into the course as one of the focal students with the most negative past experiences with mathematics, describing her math journey as a "difficult battle," this is quite astounding. The high rating of her feelings of competence supports the above described positive experiences within this course, which prompted her to come to see herself as not only capable in mathematics, but as a future teacher leader in mathematics.

Jessica Summary

Jessica's experiences of this course dramatically increased her confidence in her abilities to do mathematics and her desire to learn and teach mathematics. Table 8 summarizes the key ideas to remember about her case. Over the course of the quarter, Jessica established strong positive feelings about her own competence, and frequently discussed the importance of hard work and effort in math class, something she had not been willing to put into her past coursework. She linked her feelings of competence to grades during each of our interviews, referencing specific numerical grades in past courses, and on individual tests throughout this quarter, calling herself a "4.0 student in this course" many times. The emphasis Jessica placed

on being able to explain and talk in class, indicates that she also considers this an important part of showing her competence in the classroom. Additionally, that focus on her independent abilities appeared to contribute to feelings of hesitance when we discussed whether she felt related in this class, and course observations suggested that she sometimes did not work effectively within groups to hear other students' ideas. Interestingly, she still rated her sense of relatedness high on the psychological needs survey, which might be because she enjoyed working in study groups after class, doing homework with other students, something she had only experienced once before in math class.

Table 8

Notable Takeaways About Jessica's Affective Experiences Over Time

Characteristics of student experiences	Past math experiences	Current math experiences
Traditional/non-traditional	Traditional	Inquiry-oriented
Positive/negative	Mostly negative	Mostly positive
Feelings of competence	Low, linked to grades, being able to work problems in front of others	High, linked to grades, about individual demonstration of understanding
Feelings of relatedness	Rarely felt until recent class experience working problems with other students	Enjoyed working outside of class with others, hesitant about fit during group work due to her strong leadership
Feelings of autonomy	Rarely felt until recent class experience working problems on the board before class	Valued autonomy and chances to develop perseverance
Feelings about teaching mathematics	Only knew she did not want to be a lecturer	Passionate about teaching, sees herself as future leader
Described characteristics of future classroom	Students not dependent on teacher	Wants to support productive struggle, student generation of solution strategies
Other student specific notes	Significant negative experience of being transferred to a lower math class for asking question	Felt bad about not giving other students in the class support and always taking over.

Jessica left this course with confidence that math class need not be comprised of "boring lectures," articulating clearly how she wants students to be able to generate their own solution strategies and struggle productively to find understanding, rather than be given procedures directly from teachers and be limited to following strict predetermined approaches. I do wonder about how Jessica will implement inquiry-oriented teaching in the future, and how her understanding of it might change over the course of her teacher training. She is focused and passionate about such teaching, but appeared to miss opportunities to give other students in this course that freedom by taking over group discussions, always recording the group work herself, and underestimating their preparedness. It would be interesting to follow up with Jessica to see whether she allows her future students to have autonomy over their mathematical reasoning, the very aspect of this course that appeared to boost her sense of her own competence.

Morgan: A Home-Schooled Student with Mostly Negative Experiences of Math

Morgan is an 18 year old, home-schooled, Running Start student, taking courses at GVCC while she finishes her senior year of high school and applies to colleges. For Morgan, Running Start is a chance to get used to working with a teacher in a classroom, a "stepping stone to a university." It was important to her that I know she is a "hard worker" who "likes to get things done quickly and the first time," though she noted she will persevere if necessary. Her experiences with mathematics are largely negative, isolated experiences related to her home-schooling background. Morgan describes herself as "a loving and encouraging person," and always had a smile on her face when we talked. I found out during our interviews that Morgan is a gymnast and gymnastics coach.

During this course, Morgan was one of the quieter students, though she was always willing to share her ideas, work at the board to demonstrate when necessary, and quick to help

other students when they had questions during group work. My observations of her suggest that she was consistently on top of the material, and she told me she filled in the notes pages Scott handed out, but she was more of a quiet, confident student who sometimes let other more vocal students take leadership roles. Morgan's dislike of mathematics comes from her classification of math as "tricky," which she defines as "when there is not a solution readily apparent." Before entering this course, Morgan took one previous math course at GVCC, her first with an actual teacher. That course helped foster a "willingness" to help other people learn math, "so they won't have to struggle like I did, hating it." Although Morgan had success and enjoyed this course, she leaves, intending to major in elementary education in college, confident she can "be good at math" and "appreciates" it, but clear that she still does not enjoy math.

Morgan's Past Mathematics Experiences

Morgan described feeling "very discouraged" and "negative" about having to teach herself mathematics during her childhood home-schooling years. She goes so far as to write in her mathematical autobiography that before she came to GVCC she was "loathing" mathematics. In her early childhood, Morgan studied mathematics using "visual aids" like blocks and Legos with her mother, but as soon as she could read well, by fourth grade, she was tasked to learn mathematics on her own, either in the kitchen or her bedroom, while her mom taught her younger brother. During her childhood, Morgan "dreaded" math, telling me, "No matter what I did or how hard I worked, I would fail."

Morgan also described her home-schooling experience as "boring," in that it was the same process year after year of reading textbooks, or watching videos of a teacher read textbooks, and trying to solve problems on her own. She described the process as one in which she had little autonomy over solution strategies:

> There was always, step 1, do this, step 2, do this. I would put the example on one side and put my paper on the other with the question and then go, step 1, what do I need to do?, that, okay. I think it was very set in stone or do this, do this, do this.

After describing this process in our first interview, Morgan told me that she is a gymnast, and that "school was just an extra thing that I had to do" in order to get to what she wanted to do, which was gymnastics. She never felt like she belonged in school, but felt most strongly that math and science were not for her, telling me, "I would always leave math for the very end."

When I asked Morgan what made her feel competent in mathematics, she talked about wishing she had had other people to study with, that maybe if she understood a concept that someone else did not, she would know she was competent. However, not having a class, she judged her competence only based on correct or incorrect answers, which were indicated by "the red marks on the side of the page." Taking courses at the community college caused Morgan to worry because it takes her "longer to process things than most people," indicating that she also sometimes measures her competence by speed.

Morgan was surprised by her positive experience in an algebra course she had taken the previous quarter at GVCC. She had been scared the course would "murder" her GPA, but in the end, she "thrived": "Having a teacher both explaining the process by word of mouth and drawing examples on the white board stuck in my mind so much easier than just reading a textbook." For Morgan, having any teacher, even one who focused primarily on "showing [the students] how to do all the steps," was an improvement from the days when she was expected to learn by reading textbooks. After the instructor showed students how to solve problems, she would do examples while asking students to fill in the necessary steps. When we talked about whether Morgan had autonomy to generate her own solution strategies in this class, she told me that she did not, but

that if students did not solve things as shown by the instructor "It probably wouldn't work. It is simply not going to be the correct answer," suggesting that the instructor showed the only viable solution path. Although Morgan described this course as mostly traditional lecture, she noted that the instructor of this course was "very okay with questions," which allowed students to be comfortable asking questions. This experience set Morgan on a new track, improving her confidence in math, and opening her eyes to the fact that she could "actually do it."

Morgan's Experiences During Math for Elementary School Teachers

This Math for Elementary School Teachers class increased Morgan's confidence in her math abilities. During the class she felt competent when she could work productively in a group, which included both asking questions and helping others when she understood something and they did not. During our interviews, Morgan constantly referred to math as "tricky," by which she means some work is required to find the solution. This reference is less about whether she can do it, and more about whether she wants to: "I think [this class] has shed a new light on how I can dig deep on problems, but that doesn't mean I enjoy it."

Morgan's confidence in her abilities was closely linked to being able to explain her reasoning to others, but she was sometimes hesitant to take on a leadership role. During my classroom observations, I saw her work in a group where Jessica "took control," constantly directing what others were doing. Morgan and I watched video clips and talked about how she felt during those class sessions. Morgan emphasized how prepared she was for class, and how confident she was in her ability to complete her part of a group presentation and to help others with their questions. However, she deferred to Jessica in the group work, noting that she let Jessica remake a poster, thinking, "If you want to put forth that work, you can." Morgan explained this in part by using her age, mentioning that she is "only a senior in high school," but

it was clear that she was confident in her abilities and this is perhaps more of a reflection of her general math participation. When asked to describe her participation generally speaking, Morgan said: "I don't really say much unless somebody asks or I have a question. I love, at times, I love being the person in charge, but when it comes to math class, I'll let somebody take over that job." Knowing this, it is not surprising that Morgan let Jessica lead the group.

However, Morgan told me she "likes" when other students come to her for help, and I did get to observe her "love" of being in front of the class and talking in front of people. During my class observations, I saw Morgan work problems on the white board during a demonstration with Scott. When we watched the clip of this together, Morgan lit up, remembering how much she enjoyed "working through" her "own thoughts" "in front of everybody." This experience speaks to both Morgan's competence and her sense of belonging, as she also emphasized that the classroom environment was "really comfortable." Having a teacher and a classroom full of students to work with is new for Morgan, and this course certainly affected how she felt about belonging in the math classroom. She liked working in groups "to hear their ideas," and was "most frustrated" when working alone on homework. In our final interview, she told me, "I now have an input on the math world. It is not just me and then everybody else in the math world. I am a part of it." She noted that she is "not the best," but this sense of identifying with mathematics at all is a big shift for Morgan.

When Morgan talked about relatedness, she also talked about how having classmates helped her see she is "not the only one who needs to work out a problem different from others." She emphasized that in this class she had the "freedom" to work problems in ways that made sense to her and to "not be discouraged when [she] did not pick up ideas as fast as others." Although she talked about the fact that it was "easier" to follow the old rules she learned from

her childhood than come up with her own ideas, the autonomy Morgan was given to find her own solution strategies and in her own time greatly affected both her confidence in her math abilities and her sense of belonging in math class.

Morgan's Future as a Mathematics Teacher

At the beginning of this course, Morgan's motivation to learn math was not for herself, but to help kids, to ensure they do not dislike mathematics like she did. She repeatedly told me that she does not have a "passion" or "heart" for mathematics, but very much wants her students to be able to enjoy learning mathematics. Morgan has always had a passion for teaching kids, as a gymnastics coach, but this desire to help students learn math is new, a result of her recent success at GVCC. Before, she would not have wanted to "force" math on someone else. For Morgan, this class was about learning to "work through" the "frustrations" she had as a kid learning mathematics, so that she can help her future students.

During the middle of this course, Morgan realized that she teaches gymnastics much like Scott was teaching this course, asking students for input, to think through approaches to decide what might be ideal. This connection was a "cool" breakthrough in Morgan's thinking about her teaching of mathematics. At this midway point, Morgan told me her latest test score suggested that she could "explain everything, and that is what you have to do as a teacher." By our final interview, there was a notable shift in the way she talked about mathematics. The inquiry-oriented nature of the course benefited Morgan, as she emphasized what it meant for her to be "guided," not following rules: "For me, I don't remember facts for history or dates. But if I experience it and I come to a conclusion, then I am going to be more willing to remember it and remember the circumstances that were placed around it." As a result, she wants kids to "come to their own conclusions" in the future math classes she teaches.

Importantly, Morgan also discussed how challenging inquiry-oriented teaching is for teachers. Despite her difficulties with mathematics and the fact that "it is not [her] favorite thing," she is willing to "take the initiative" to teach this way in the future. She wants her students "to feel comfortable to ask questions and to be involved in the class," and talked about how she will use manipulatives because they made her feel "comfortable" in this class. Morgan also told me at the end of last interview that she realized this quarter that "you are worth more than your grades," explaining that kids "just care if you know your stuff and if you know how to teach it," not whether you are "top of the line." Although she told me she still only "appreciates" math, and does not "like it," Morgan captured a significant shift in her feelings about mathematics in her post-reflection, writing:

> Before, I never thought I knew enough or was good enough at mathematics to teach. I was also scared that my distaste for the subject would be evident to the kids and then they too would dislike the subject. But now, I am excited to teach and share math with others.

This excitement will transfer to Morgan's annual summer job as a nanny, as she is now "excited to see what they have to do" and "what we can figure out together."

Morgan's Survey Responses

Examination of Morgan's psychological needs survey responses, shown in Table 9, illustrate her general feelings of increased competence, relatedness, and autonomy in this course as opposed to her past math experiences, which was typical of the class averages. However, Morgan has interesting high ratings of relatedness for both her past math classes and this course, and her rating of her sense of autonomy in this course is quite high. Morgan's high relatedness ratings could be linked to her experiences as a home-schooled student. Her most recent math experience before this course, a course taken at GVCC, was her first course with a classroom of

students and an instructor. This was a pivotal experience for her, as she emphasized in her autobiography and interviews how she "thrived" with an actual teacher, in contrast to the isolated self-teaching she described undertaking while being home-schooled. Additionally, this course in particular helped her value her own thinking and ways of learning. She describes the "freedom" she felt to find her own approach to problems, much in contrast to learning directly from a textbook with limited solution approaches shown.

Table 9

Morgan's Mean Scores for Subscales of the Basic Psychological Needs Survey

	Competence		Relatedness		Autonomy	
	Past math courses	Current math course	Past math courses	Current math course	Past math courses	Current math course
Whole class	4.79	6.32	4.35	5.77	3.95	5.83
Morgan	4.67	6.83	6.25	6.50	4.29	6.86

Note. Original scores, given on a 7-point scale (1 "not at all true" to 7 "very true"), represent the degree to which a student experienced feelings of competence, relatedness, and autonomy on questions from three subscales. Shown here, are the mean scores on each subscale.

Morgan Summary

Morgan had a positive experience in this inquiry-oriented course, which strongly affected her feelings about mathematics. Key ideas to remember about Morgan are shown in Table 10. Much of Morgan's positive experience of this course is centered around having a classroom community, something she lacked during her home-schooling years. Specifically, her feelings of competence increased in this classroom, and were intimately linked to learning with other students in groups, explaining ideas to other students, and seeing herself in relation to those other students, knowing she was not the only one with questions or struggles. Morgan was able to actively participate in the course community and increase her confidence in her own

mathematical abilities and her excitement around teaching mathematics. Additionally, she mentioned benefiting from having the autonomy to find her own solution strategies in this classroom, a marked changed from the procedural step following she described from her home-schooling work. Although she still struggled to take on leadership roles occasionally in this math class, despite describing her confidence in her knowledge and mentioning she likes to be a leader, Morgan's feelings about mathematics have shifted greatly during this course.

Table 10

Notable Takeaways About Morgan's Affective Experiences Over Time

Characteristics of student experiences	Past math experiences	Current math experiences
Traditional/non-traditional	Home-schooled	Inquiry-oriented
Positive/negative	Mostly negative	Mostly positive
Feelings of competence	Low, only judged by checking answers, often wrong	Higher, linked test scores and to sense of community
Feelings of relatedness	Low until recent experience of having teacher and classmates	High, valued having students to work and struggle with
Feelings of autonomy	Felt little autonomy, described following textbook steps	Valued autonomy, gave her freedom to try own ideas
Feelings about teaching mathematics	Worried she will transfer his dislike of math to students	Excited to teach math, despite the fact that she still does not love it herself
Described characteristics of future classroom	--	Wants students to come to their conclusions
Other student specific notes	--	Acknowledged that inquiry-oriented teaching is challenging, but worthwhile

In addition to valuing the chance to generate her own ideas, and work with other students in groups to do so, Morgan has experienced enough in this course to know that she wants her future students in mathematics to have the chance to figure things out for themselves in a guided

situation. Morgan leaves the course still not liking or loving mathematics, but with an appreciation of it and an enthusiasm for teaching it that is new to her. Entering the course, she was motivated to learn more math so that she could avoid passing her dislike of the subject on to her students. At the end of the course, she has a different motivation, a small passion for helping students engage in mathematics in meaningful ways that will foster their enjoyment of math. Exposure to a new way of teaching and learning mathematics has helped Morgan move beyond tolerating mathematics to developing confidence in and excitement around the subject.

Bahira: An "Average" Student, with Traditional, Neutral Experiences of Math

Bahira is a 19 year old student who entered this Math for Elementary School Teachers course not completely sure she wanted to teach. As such, enrolling in this course was one way she was considering whether that was the right career path for her. When I asked Bahira to describe herself, she focused on her role as the eldest child in her family and that she is "a reader" who "loves foreign languages and cultures." Bahira's parents are from Africa, so she grew up speaking two languages, though she and her three younger siblings were born in the United States. She is the first member of her family to graduate high school and go to college.

Bahira consistently described herself as "average" or someone who "doesn't go the extra mile" in mathematics, and generally speaking she was okay accepting that label in her past math courses and again in this course. Specifically, she often noted that she did not seem as excited about this class as the other students, and this contributed to her decision that teaching is likely not the profession for her, despite feeling comfortable as a member of the class community. By the time of our final interview, she had changed her major to psychology, and was talking about other jobs in which she could serve students in student development or counseling.

It is important to note that, in the beginning of the course, Bahira labeled herself as "not a

math person," a label that she no longer supported at the end of the course. Indeed, by the end of the course, she no longer believed that some people were good at math and others were not, but that she could be successful if she was willing to put in effort. It was the latter part, the willingness to put in effort, that she said she simply did not possess.

Bahira's Past Mathematics Experiences

Bahira is "indifferent towards math," not liking it or disliking it, describing herself as an "average" student, and generally feeling "pleased" with her abilities. She has memories of being tracked for the first time in middle school, which confirmed her "average" label of herself. Being placed on the "at grade level track" was somewhat troubling for Bahira, but only because most of her friends were placed in the "higher track." She saw those students as "smart in general," not specifically in math, and wanted to be placed with them because it was "cool" to be smart, but "was not too upset" about being "average." When I asked her to tell me about a time she felt competent in math class, she could not recall any specific memories of feeling that way. She did remember not feeling competent when other students knew "how to solve a problem" and she was "still struggling with it."

Bahira's past math experiences were mostly traditional, lecture-based experiences. When describing most of her classes, she told me that the teacher would give a lecture and then most of the class was students individually working problems. Interestingly, Bahira always felt self-directed in the methods she could use to solve problems in her math classes, telling me: "My teachers just wanted to know that I could solve a problem and that was the grade I would get on my test, just an answer that reflected that I knew the concept."

Bahira often described loud and disruptive students in her prior math classes, and that behavior was incompatible with the ways that Bahira viewed herself. She did not fit in in these

types of classrooms because the “goofing off” was “distracting” and she “wasn't learning” in those kinds of classes where the “teacher wasn't in control.” The only times Bahira described feeling like she belonged in math class was if she could “talk to the teacher.” Being comfortable asking the teacher for help was a key aspect of relatedness for Bahira. One course, ninth grade algebra, was enjoyable for Bahira, which she attributed to a teacher who was “funny” and “passionate about math” and to having a “best friend” in the class who “worked hard,” which in turned “motivated” Bahira to also work hard. This was one time when a sense of social belonging in the course helped Bahira work hard to achieve success in and enjoy math.

While struggling through most of her high school math classes, “barely passing,” Bahira started to think she was simply “weak” in math: “I began to tell myself that I am a really good student, but math is just not my subject.” Her dad, who always “motivates” her to “overachieve” started “reaffirming” this, accepting grades less than As in mathematics, while still expecting only As in every other subject. At the start of this class, Bahira confirmed that while “math is a really important subject,” she “doesn't put a lot of effort into it,” telling me: “I just kind of learn what I need to, to do my homework and to pass the class.”

Bahira's Experiences During Math for Elementary School Teachers

Bahira did not feel a strong sense of autonomy in this Math for Elementary School Teachers course, though she did tell me: “Rarely is there a time when [Scott] says there is only one way to solve this problem, and I want you to show me only that one way. You should experiment yourselves and work on how you got to the solution-- that is accepted.” Interestingly, however, she described feeling restricted by being “expected to use manipulatives” in class during several of our conversations. Bahira's feelings around autonomy may be influenced by the fact that she experienced autonomy in her mathematical thinking during her

past math courses, so it was something she had come to expect.

In this course, Bahira's sense of relatedness was linked to her lack of passion for teaching. Despite deciding during this course that she did not want to become a teacher, Bahira "enjoyed the class atmosphere because it was comfortable and we had many opportunities to work with one another and get to know each other's names." She enjoyed a sense of community in getting to know other students, but told me, "I feel like I belong only in the sense that I know the classmates and I can talk to them." This was one way Bahira made the distinction between herself and the other students in the class who were, in her mind, more passionate about teaching than she was. She continued, "I do feel a sense of not belonging, just because I don't think I am passionate about the subject or the class compared to everyone else. I am not motivated to go the extra mile." Bahira repeatedly told me that she did not "go the extra mile" in this class, which to her included doing outside reading and research. She told me in our final interview, "I only engaged in math when it was necessary for the grade." My classroom observations supported this idea. Bahira was always engaged in group work, sometimes took the initiative to record her group's thinking, and shared her ideas in small groups and presented with her group when asked. She was, by most accounts, just doing what was necessary and nothing more.

In addition, Bahira's experience of the classroom community was also linked to her feelings about her own competence in mathematics. She did think that working in groups with other students was "interesting" and "helpful," but she also told me she liked working on her own and "didn't really enjoy the group work." She said, "I don't usually have a problem with group work, but there are times when I like to just think calmly about a problem and just work on my own and have my own space and my own thoughts." Despite telling me that she probably would not take another math class focused on group work, Bahira did see benefits of group work

for future teachers, to "have a strong community." Notably, Bahira wrote in her post-reflection that she was "inspired" by her classmates because they "kept going and trying" even though sometimes "they did not find the math to be easy." She wrote:

> That [perseverance] reinforced my belief that just because math might be difficult for me, that does not mean I will not be able to learn it. I believe it all depends on how much effort you put not only inside the class but outside of the class as well.

Ultimately, the classroom community, and sense of belonging that Bahira felt in that community, had a strong affect on both her views about mathematics and her own potential math abilities, and also contributed to her decision not to become a teacher.

In summary, Bahira had another neutral experience with mathematics in taking this course. However, the way she judges who can be competent in mathematics did change significantly over the quarter. Initially, Bahira told me she was "not a math person," and that competence "means getting a good score on a test or homework," which she "sometimes did." However, in her post-reflection she wrote:

> I believe math is frustrating because you can only move up once you have grasped the understanding of the basics and the foundations of the lessons before the new one. For that reason I think I was quick to think that I could not be good at math even though I think the problem stemmed from me not mastering the previous lessons. I feel that I could have worked harder in this class but overall I believe I was able to learn a lot not just about math and teaching math, but just about me in general.

At the end of the course, Bahira is able to "disagree" with her initial statement that she is "not a math person," and attribute competence in mathematics more to effort than natural ability, even as she is not interested in putting forth effort in mathematics.

Bahira's Future as a Mathematics Teacher

Bahira entered this course unsure of whether she wanted to become a teacher. During our first interview, she was open to considering teaching as a career, and looked forward to learning about how kids "approach math at a young age." She was considering becoming an elementary school teacher or a middle or high school English teacher. By the time we met for our middle interview, Bahira was "97 to 98 percent sure" she would not become an elementary school teacher. Despite that, she "enjoyed being able to relate the material [she] was learning in class directly to the second grade class [she] was volunteering in." Through this experience she realized that teaching elementary school mathematics is more complex that she initially thought: "I realized that it is not as easy as I thought. Actually, it can be really challenging. I learned that just because we might know how to solve an equation does not necessarily mean you will be able to effectively teach it to others."

During the week of our middle interview, Bahira had been reflecting about whether she wanted to teach in order to register for the following quarter. It was at this time she started noticing that Scott, the other students in this course, and students in another education course she was taking, were more passionate than she was about math and teaching: "I just didn't want to admit to myself, just looking around at my classmates in this class and in my education class, and just seeing how they are passionate about it, and they are involved in their engagement." She told me that if she was really meant to teach, she would "enjoy class" and "go above and beyond the work required to just get a good grade." She concluded: "The class is just not for me." By the time our final interview came around, she had confirmed these ideas, "not just in math," but in teaching in general, and had changed her major to psychology.

Despite deciding not to pursue teaching, Bahira was changed by taking this course. She

was responsive to the "different approaches to perceiving and teaching math," seeing how they were applied in a second grade classroom, and describing how she would use skills from this course to "better relate" to her "younger relatives that are in elementary school," knowing how to "effectively" help them with their homework. Perhaps her most significant realization was that she no longer felt right in labeling herself as "not a math person," because no such label is appropriate. She wrote in her post-reflection: "Even though I feel a bit intimidated about my future math classes, this class really gave me the confidence to believe that despite one's struggles in math, the learning of math is not inclusive to only a select group of people." Bahira knows that she can be successful in math, if she puts in the effort, but also knows she does not like math enough to put forth that kind of effort.

Bahira's Survey Responses

Bahira's scores on the Basic Psychological Needs Scales, shown in Table 11, are similar to the class averages in many ways, but do show some interesting responses that should be commented upon. Notably, her degree of experiencing competence remains unchanged from her past math courses to this inquiry-oriented course, and her reports of feeling competent, related, and autonomous in this current course are lower than both the class average and the other focal students in the study. That Bahira's mean feelings of competence remain unchanged, at 4.33 on a 7-point scale, is actually not surprising. Self-described as "average" in mathematics over the course of her past experiences, this feeling is one that stuck with Bahira throughout this Math for Elementary School Teachers course. Although she reports some increased feelings of relatedness and autonomy, this sense of averageness in math appears to have influenced Bahira strongly in this course. The scores reflect her interview comments that she enjoyed the students in the course, and felt comfortable, but only somewhat enjoyed the group work and inquiry-oriented

style of the course.

Table 11

Bahira's Mean Scores for Subscales of the Basic Psychological Needs Survey

	Competence		Relatedness		Autonomy	
	Past math courses	Current math course	Past math courses	Current math course	Past math courses	Current math course
Whole class	4.79	6.32	4.35	5.77	3.95	5.83
Bahira	4.33	4.33	4.14	5.14	3.50	5.00

Note. Original scores, given on a 7-point scale (1 “not at all true” to 7 “very true”), represent the degree to which a student experienced feelings of competence, relatedness, and autonomy on questions from three subscales. Shown here, are the mean scores on each subscale.

Bahira Summary

Bahira's experience in this course, summarized in Table 12, ended up continuing the rather neutral experiences of mathematics that have characterized her math education to this point. In the end, Barhira enjoyed the classroom atmosphere, and saw connections between the course work and her volunteer work in a second grade classroom, but felt that she lacked the passion necessary to become an educator. Notably, Bahira felt a sense of community in that she knew students in the course and enjoyed working with them to an extent, but also saw that they went “above and beyond” to notice applications of material outside of class, do extra reading, or other things that generally demonstrated how the course was strongly influencing their drive to teach. Since she was not feeling this sense of “passion,” Bahira felt like an outsider in the class, and decided that this missing passion was a sign that she should not become a teacher. The course did, however, influence Bahira's views on how someone can become competent in mathematics, and who can be competent. At the beginning of the course she was comfortable saying that math was just “not for” her, suggesting that some people are simply naturally not

good at math and that is okay. By the end of the course, Bahira herself acknowledged a shift in that thinking, throwing off that label, and instead noting that she could be good at math, if she was willing to put forth effort. Ultimately, she decided she was not willing to do that, but seemed to feel better about making a choice not to pursue math because she does not enjoy it, rather than be forced out of mathematics because she was not capable.

Table 12

Notable Takeaways About Bahira's Affective Experiences Over Time

Characteristics of student experiences	Past math experiences	Current math experiences
Traditional/non-traditional	Traditional	Inquiry-Oriented
Positive/negative	Neutral, leaning negative	Neutral, leaning positive
Feelings of competence	Lacking, described self as not a math person	Improved, linked to effort, not innate ability
Feelings of relatedness	Lacking, unless she was comfortable asking the teacher for help	Felt belonging in the community, but not as passionate about teaching
Feelings of autonomy	Felt freedom in reasoning and solution approaches	Felt restricted by required use of manipulatives
Feelings about teaching mathematics	Unsure she wants to teach	Sure she does not want to teach mathematics. Acknowledges that teaching is a complex task.
Described characteristics of future classroom	--	--
Other student specific notes	Eager to support students in her work in some way, but unsure how	Excited to use what she learned to help younger family members in mathematics

Katie: A Student Returning to School to Change Careers, with Traditional, Mixed Experiences of Math

Katie is a 46 year old mother of two, taking this Math for Elementary School Teachers course as a prerequisite for a Master's in Teaching program, which she was accepted to just after

this study concluded. As such, Katie already has an undergraduate degree in psychology, and is unique from the other focal students in this respect. When asked to describe herself, Katie focuses on her long past with education. Raised by two university professors, Katie has always been told she was "smart" and "destined for higher education." She refers to this "expectation," set by her parents, as a "godsend," something that kept her on track and persevering through difficult times. Katie has had mixed affective experiences with mathematics, feeling both successful and experiencing failure, but those experiences all took place in traditional classrooms, where her teachers would lecture on material and then expect students to work problems on their own. Her past experiences of failure in mathematics have made Katie open to the inquiry-oriented nature of this class.

In this course, Katie worked hard, putting in a significant number of hours reading the textbook to prepare for class, and very often took on a leadership role within her groups. Other students noticed that she seemed well prepared and often relied on her for consideration of their questions or to confirm their thinking. Despite this, Katie left the course feeling both positive and apprehensive. She throughly enjoyed the course and felt successful, and is admitted to a teacher education program, but links her successful and positive experience in this course to the instructor and his role in structuring the classroom environment. As a result, despite being able to articulate specific characteristics she'd like her future math classroom to have, Katie is concerned that at any point in time she might encounter mathematics presented in a way that she will not be able to understand.

Katie's Past Mathematics Experiences

When discussing her past experiences of mathematics, Katie's descriptions of her feelings swing from "love" and "beaming with pride" to "horrible experience." In her descriptions of her

early feelings around mathematics, Katie focuses on her feelings of competence, describing how she was first or almost first "every time" her class took timed tests. In fact, Katie characterizes feeling competent when she either gets things quickly or "on her own," meaning she can visually see, such as during timed tests, that she is finishing before most of the other students or she "sees" a problem and thinks "Oh, I got this!" Katie feels "excited" when she knows how to solve problems and can do so quickly.

When asked in more detail about how her past courses were structured, Katie talked about how inquiry was not supported or encouraged, and for her, this is closely related to a lack of autonomy and community belonging. She remembers not being given the autonomy to pursue her own solution approaches, but being forced to follow certain procedures and being told she "didn't include all of the right steps" even though "all the work is there." She notes there was never discussion of "what works for you" or support of multiple approaches generated by student thought in her past courses. In fact, group work was essentially non-existent in her past math courses, classes consisted of "just board work and homework." While reflecting on geometry, a class in which Katie struggled to write good proofs, she noted that because of the lack of discussion, she was "feeling very, very isolated." She said, "Working together? Math was always like, I got my answer, I get it, don't look, because if you don't know it, you shouldn't be looking at mine." Interestingly, she never noticed the lack of community until she was the struggling student in the classroom. In her geometry course, Katie remembers thinking back to her elementary school timed tests and noting, "It never occurred to me, that person who must be struggling and that must feel awful, because there was no group. It was all individual timed." Katie describes this geometry experience as "just a tumble" in her math journey, as she went on to success in trigonometry and pre-calculus in high school, bolstered by the knowledge of her

general “intelligence” that was fostered by her parents.

Until she reached college, aside from her “tumble” in geometry, Katie described “loving” math, getting answers quickly and easily, and always being in the accelerated courses. In her freshman calculus course, her “horrible” math experience, Katie felt that her desire to “understand what in the real world corresponds to” the calculus they were learning was not welcome in the class. She remembered a close friend telling her she didn't need to understand “*why* things were” just “how to use them,” suggesting little freedom to make sense of the mathematics in individually meaningful ways. Despite seeking outside help from her friend, getting tutoring, and a lot of extra study time, Katie failed the course. Having entered college intending to be a math major, she then switched her major. When she retook calculus, still required for her new major and taught from the same textbook, her new instructor “questioned his English skills” so he presented multiple approaches to each problem. Katie realized she only needed to understand one of the approaches, and earned a 4.0 in this course. Her experiences in calculus, her most recent math experiences to this point, significantly color the way Katie thinks about herself in relation to mathematics. Katie described a positive, sustaining confidence in her math abilities before her calculus experience. However, at the beginning of this course Katie wrote in her autobiography:

> I left math feeling I was truly good at math, but frightened to death still of things I cannot seem to understand. I know I can fail or ace a given math topic at the drop of a hat, and I don’t know which result I’ll get: failure or achievement.

Although Katie was feeling positive about this current course at the time of her initial interview, she entered this course knowing she had her “own fears to address” and with some heavy concerns and negative feelings about mathematics on her mind.

Katie's Experiences During Math for Elementary School Teachers

Katie described her experience in this course as "wonderful," repeatedly telling me how "confidence building" it had been. Her experiences of competence have changed from her early childhood memories, which focused largely on speed. While Katie mentioned contributing answers as one thing that makes her feel competent, and that she could easily take on a leadership role, she emphasized backing off to let other students contribute, considering their ideas. Interestingly, Katie attributed this not to classroom culture, but to her age and maturity:

> The more mature me, I've learned to keep my mouth closed because there is a lot to learn. I want to raise my hand so bad and sometimes I just can't help it, but I'm just excited. But if I just zip my lip for a minute and listen, other people have the same answer and 80 percent of the time, I would say, have an even more full understanding of what I was going to say, a little different perspective on it that never occurred to me.

In my course observations, I saw how Katie allowed space for other students to share their thinking, and as a result, other students in the course often turned to her for consideration of their ideas during small group work. Katie appeared to enjoy that leadership role.

However, that speed related competence still lingered for Katie. A pivotal moment she brought up in both our middle and final interview involved her not being able to quickly understand a cereal box puzzle that Scott showed the class one day. Scott intended this as a quick "look what I saw" moment, but Katie deeply struggled with not being able to understand it because Scott did not explicitly say that in class: "If he had said, this isn't a measurement of your ability to do math, this is for fun, please don't spend a bunch of time trying to figure it out and I only did because I like the cereal or whatever." She needed to have permission not to get this task right away, and without that, she lingered on the task for several days afterwards. This is the

only time Katie mentioned not feeling competent in this course.

Katie also mentioned feeling competent when other students asked for her opinion, which they did more because she "backed off" and did not always offer it right away, "I think when people ask me, if I am able to explain a way that they get, that makes me feel really good." My interviews revealed that Katie spent significant time reading and working through the text and homework, and sometimes other students mocked her for this, though she noted it was "not mean spirited." Katie also revealed to me that her drive comes from fear:

> I can honestly say I could not have worked any harder on the material. I think I mentioned before that is all fear based. It is all fear based as are so many things, I think, in my life -- fear of failure and how much hinges on this being a little microcosm of whether I am going to be able to teach math.

Katie' achievement is closely linked to her fear of failure, despite high scores and being respected by other students in the course, and her strong desire to become a mathematics teacher.

Katie also links this aspect of her competence, helping others, to her relationship within the classroom community. Because she is a mother, she works ahead of much of the class, getting things done very early. So, while Katie is "honored" when people ask for her opinion, she justifies this by saying they know she "already has stuff done." Despite continually tempering her achievement as just getting things done early, Katie did value the community in this classroom, and felt that her views of her own abilities were "bolstered by the trust" her peers placed in her during the course, noting that she formed "lasting connections" and "relationships," something that she never had in her previous math courses. My observations of Katie's interactions with her classmates, in small groups and before and after class, suggest that she was one of the students most respected for their understanding of the course content. Other students

constantly turned to her for help, and the other focal students frequently referenced enjoying working with her or getting help from her in their interviews with me.

In this class, Katie felt supported to try her own solution approaches. She talked about how Scott supported the students by saying things like, "That's interesting, let's think about that," noting that Scott's "message" is "keep thinking about it, keep thinking about it." Katie also very positively reflected on the fact that "different strategies were all okay" and that "seeing things in your own way" was supported. However, she made it clear that she needed instructor "permission to not have to immediately grasp" things right away, as evidenced by the cereal box incident mentioned above. She considered autonomy in student thinking one of Scott's "messages" in this course, but clearly needed him to specify when that was the case, not generalizing it to all engagement with math.

Katie's Future as a Mathematics Teacher

Despite having a "wonderful" experience in this course, as described above, and deciding while taking this course that she will purse a math endorsement in addition to her elementary certification, Katie left this course with lingering fear and hesitance with regard to her own math abilities. She was quite attached to Scott, attributing much of her success in the class to him. She referenced Scott's role repeatedly in interviews and writing assignments, saying things like: "This experience was that I worked hard in a great setting, with a great instructor" and "This class was like the perfect storm of good things. You have Scott, which you don't even need to go into. Then you have, for me, a textbook that walks you through it." More than once Katie goes so far as to mention wanting to carry Scott in her pocket, so he can teach any future math class she has to take. So, although she does recognize the classroom structure as influential in her positive experience, she continues to feel, as she did at the beginning of the course, that much of

that structure is out of her hands and in the hands of the instructor. She is still nervous, and is not clear that her confidence gained in this course will transfer to future math courses she needs to take to complete her teaching certification: "I don't know what I'm going to do without that confidence building crutch of [Scott] and this class and this setting. I want to take that with me through the next hoops, through the next couple of years. I can't, but I want to."

Upon entering this course, Katie's views of teaching mathematics centered around how professors present material to students, a result of her calculus experiences. In her math autobiography, she talked about how she would present problems in many ways to students, saying, "People are wired differently, particularly for math." Her comments, though emphasizing multiple approaches, also emphasize that material is presented by a teacher *to* students. At the end of the course, however, there are noticeable changes in the way she talks about what her future teaching will look like, and she can clearly connect these changes to her experiences as a student within this course. She said,

> I would like to think that I can get to a point where my students will feel in my class the way I felt in Scott's class. That the material was presented at about the right pace, in a way that they could understand, that they participated in that learning process, that I was available to them through different avenues, that they are encouraged to contribute and that their contribution was valued. That's ideal, I think for math. The setting was ideal in my mind for math, versus sitting there and getting the information presented to you and going home and doing the homework and coming back and testing on it.

Katie is able to specifically describe what she hopes her students will do *and feel* in the classes she teaches in the future, highlighting student contribution, and specifically referencing her own feelings and experiences in this particular class as the "ideal" for her future students.

Katie's Survey Responses

The responses that Katie gave on the surveys of the satisfaction of her psychological needs in her past and current math classes, shown in Table 13, characterize her experiences as quite typical, when compared to the average ratings of all students enrolled in this course.

Table 13

Katie's Mean Scores for Subscales of the Basic Psychological Needs Survey

	Competence		Relatedness		Autonomy	
	Past math courses	Current math course	Past math courses	Current math course	Past math courses	Current math course
Whole class	4.79	6.32	4.35	5.77	3.95	5.83
Katie	3.17	6.83	4.50	6.13	3.43	6.00

Note. Original scores, given on a 7-point scale (1 "not at all true" to 7 "very true"), represent the degree to which a student experienced feelings of competence, relatedness, and autonomy on questions from three subscales. Shown here, are the mean scores on each subscale.

Like most of the focal students in this study, Katie described past negative mathematics experiences relating to each of these psychological needs. In addition, her very influential negative experience, failing calculus her freshman year in college, had connections to each of these needs. She describes mainly her lack of competence, as indicated by the F, or 0.0, she received in the course, which may contribute to her very low competence subscore mean in her past math courses. However, her discussion of this experience involved her feeling a lack of autonomy, as she felt she was not allowed to learn the way she was wanted to, understanding how and why to do things. This affected her to the extent that she even changed her major. Fortunately, Katie's descriptions of her experiences in this course, reflected in her much higher survey scores, suggest that she had a significantly positive experience in this inquiry-oriented course for prospective teachers.

Katie Summary

Katie is a case of a student returning to school to become a teacher and of a student who has strong, mixed feelings about mathematics. Some key points to remember about her are shown in Table 14. Her story is complicated much more than the other focal students' by extreme positive and negative experiences of mathematics, but she also opened up to me more than most of the other focal students. She saw me as a mathematical authority, sometimes asking me for advice about teaching or her future program after we had finished our interviews.

Table 14

Notable Takeaways About Katie's Affective Experiences Over Time

Characteristics of student experiences	Past math experiences	Current math experiences
Traditional/non-traditional	Traditional	Inquiry-oriented
Positive/negative	Positive, until college	Positive
Feelings of competence	Related to speed, high until college calculus	High, includes giving space for sharing, answering questions
Feelings of relatedness	Fit in until she failed, no prior experience with community in the classroom	Fit in, took on leadership role, valued community
Feelings of autonomy	Little autonomy to pursue own solution strategies	Values autonomy to try own solution approaches
Feelings about teaching mathematics	Open to potential of inquiry-oriented methods	Positive, but worried about own ability to pass future math classes
Described characteristics of future classroom	Presenting multiple methods to students	Wants students involved in the learning process
Other student specific notes	Influential failing and passing of calculus, attributed to presentation of material	Not confident can succeed in math without good teacher and specific classroom structure

Katie's initial descriptions of feeling competent, in her previous mathematics courses, centered around speed or understanding something quickly and before others. After this course,

Katie spoke about demonstrating competence mostly by allowing other students to share their ideas and by listening to learn something new. This is a noticeable shift that Katie attributed largely to her age and maturity, suggesting often that if she were younger she would likely just be focused on sharing her ideas and showing what she knows. Regardless of whether this shift came from maturity or from the structure of the class, Katie felt good about being able to help other students and about learning from the other students' ideas. These feelings of competence would not be possible without the community structure within this class. Katie often took on leadership roles, but the nature of the group work structure allowed her to focus on ensuring every group member's understanding.

Initially, in her autobiography, Katie talked about teaching by presenting multiple approaches to students, so that they can connect with at least one strategy, emphasizing the role of the teacher in presenting mathematical material. Over her time in this course, her perspective on this changed. At the end of the course, she emphasized how she wants students to "participate in the learning process," describing how she plans to value student contributions. It was clear from our interviews that Katie felt she gained mathematical confidence in this course, but her focus on the future is still tainted with lingering fear, so it appears that her confidence is not necessarily generalizable to all mathematical engagement. Her frequent comments about wanting to carry Scott around in her pocket to teach all of her future math courses, indicate how heavily she relied on his leadership and how much of her success she attributed to him.

Katie's story brings up many questions, particularly related to what it takes for students, especially prospective teachers, to overcome a negative experience with mathematics. Katie only experienced one negative course, but it was significant enough to affect her confidence for decades, lingering even after this strongly positive experience. Since Katie is headed quickly

into her teacher education program after this study, I am convinced that a longitudinal study of prospective teachers' affective experiences is needed, and would contribute even more to our understanding of the role of affect in teachers' developing practices.

Cheryl: A Student with Traditional, Mostly Positive Experiences of Math

Cheryl is in her mid-twenties and was in the process of transferring to a nearby university at the end of this study, where she will complete her degree and a Master's in Teaching program. When asked to describe herself, Cheryl focused only on her desire to teach, regardless of what is being taught, telling me how she taught her dad to ice skate and snowboard, was a ski and snowboard instructor, and has tutored a child in various subjects. She concluded: "This is my calling. This is what I want to do, is teach. I just love helping people figure out how to do something. That is a big part of me." That strong desire to teach played an important role in Cheryl's experience in Math for Elementary School Teachers, as most of her comments about the course and her feelings were tightly linked to her future as a teacher.

Even at the beginning of this course, Cheryl considered herself good at mathematics, and particularly noted that she already felt confident in her ability to teach elementary school math. She told me several times that she thought this class would be "easy-peasy" since it would just be "relearning" content she already knew. At that time, Cheryl held traditional views about what it means to be successful in math, referencing grades frequently and talking about her success on timed tests. Additionally, Cheryl admitted to struggling to see the point in showing her work in her past math classes, highlighting the importance of simply being able to get correct answers. Over the course of this class, Cheryl was surprised by initial low scores on exams, and came to appreciate the need to explain her thinking or justify her reasoning, particularly when she thought about her future as a teacher. Cheryl sees herself as a teacher, and despite entering this

course feeling ready to go teach, she described learning a great deal from her experiences that will influence her future teaching of mathematics.

Cheryl's Past Mathematics Experiences

Cheryl attended a Montessori school for her elementary school years, in which she remembered taking a lot of timed tests and feeling "self-directed" in the pace at which she covered material. She "loved and hated" the structure of using timed tests to advance because she liked to work through the material at her own, quick pace, but also felt the pressure of needing to show what you know quickly on the test. After she left Montessori school and moved to middle school, Cheryl emphasized having little autonomy to use her own strategies to solve problems in math class, calling it "one way teaching." If you didn't understand that "one way," the teacher would just keep explaining it again in the same way. Cheryl's dad would work with her at home to show her multiple ways to solve problems, something that was only supported inside one of Cheryl's math classes throughout middle and high school, and this encouraged her to continue to enjoy mathematics. This lack of autonomy influenced Cheryl's sense of competence, as she described that in order to "get things right," she had to have "the correct answer and the way [the teacher] wanted it, with the same set up. If it wasn't the same set up, you were wrong." As a result, even if she understood the material, Cheryl felt restricted to needing to understand the teacher's exact approach in order to feel competent.

In both her Montessori school experiences and her later traditional school experiences, Cheryl linked her feelings of competence in mathematics to the grades she was earning and the speed at which she could understand things. In her memories, she referenced specific grades from tests, often 100 or 102 percent, "low" scores like 87, or being "in the top of the class," and she was particularly proud that she " was able to get to division at the end of first grade," despite

the fact that she “totally forgot and had to start all over” the next year. Interestingly, however, when Cheryl talked about the importance of being fast, the examples she used were grounded in number sense, demonstrating that while she did value speed, she also had flexibility to work with numbers mentally that allowed her to be efficient.

In addition to linking her feelings of competence to grades, Cheryl also discussed struggles she had had with teachers wanting her to show her work in math class. She talked about wanting to simply solve the problems, and thought giving a correct answer was enough, often showing “a little 'tude” to the teacher in response. She said:

> My teachers use to hate it when I would solve math problems in my head and write down the answer without solving the problem on paper. Half the time though I wasn’t sure how to explain how I got my answer from my head to the sheet of paper.

Interestingly, Cheryl felt motivated to show her work in classes where she thought her solution strategies and knowledge were valued. In one class in particular, she felt this strongly:

> I always felt competent in what I was doing and that my work was quality work, because I understood it. The way I set it up was the way I understood it and he was able to understand it, in the sense that, okay, this is one way to do it.

That is, solution approaches were valued as long as they were valid, and were not required to meet a set lay out prescribed by the teacher.

Cheryl talked very little about whether she felt a sense of belonging in her math classes. For her, this was simple. She felt related, when she “could kick back while still learning math,” emphasizing her desire to be comfortable, make jokes, and learn the material at the same time. One middle school class stood out in particular when Cheryl discussed whether she belonged, as she described how it had the above characteristics and that the teacher supported multiple

approaches to solving problems. The class seemed to support all three of Cheryl's psychological needs very well, as she described belonging, getting high scores, and feeling supported to solve problems using approaches that worked best for her. There was only one standout class, Geometry, where Cheryl remembered not feeling related, also the one class she did not feel competent in. She said, "Because I didn't get it and the students around me might have got it or didn't, it just kind of felt like an alone in the crowd situation." While her sense of belonging required a comfortable atmosphere where she could be herself, making jokes and having fun, it notably included learning the content successfully as well. Cheryl admitted in her autobiography that none of her past experiences in math supported inquiry, emphasizing that she felt a strong focus on learning for standardized tests, but she was open to inquiry-oriented teaching "because it allows students to learn problem solving, critical thinking skills, and how to organize their thoughts in appropriate ways."

Cheryl's Experiences During Math for Elementary School Teachers

Cheryl felt related in this course, telling me, "I feel accepted. I feel valued as either a group member or as a member of the whole class." In past courses, this sense of belonging was related to being herself, which included the ability to joke around and be comfortable. In my course observations, Cheryl's personality was very evident. She often made "sarcastic jokes or comments," and felt comfortable shouting questions to Scott from across the room. Cheryl also felt that her ideas and solution approaches were valued in this class, mentioning how she enjoyed having conversations with group members and classmates where she could express her ideas but also get ideas from other students, and perhaps revising her ideas based on new information. Having her ideas valued also meant that multiple solution approaches were accepted and that it was "less frustrating" because "one way of doing it is not the absolute," referring to her past

math experiences with little autonomy over reasoning.

Cheryl continued to feel competent in this course, and while she did continue to link her feelings of competence to homework and test scores, which she told me started out low in this course, she also frequently related her competence to her future teaching. For example, she often mentioned being able share her ideas with others or being able to teach something as measures of when she felt competent with material. She described a moment she felt particularly competent: "I was able to show it, demonstrate it, and in a way that they could understand better than the way Scott was showing it up on the board... ." In my observations, I saw Cheryl working with another student, separately from their group, to understand arithmetic in base five. She worked hard to guide the student, not giving them the answer, but helping them come to their own understanding. Cheryl and I watched a clip of this and she remembered feeling "good" and having "fun" while helping this student. That said, there were also times when she worked hard to learn things for herself, and times when she was "just too tired" to participate thoroughly, and described letting other students "figure things out" for her.

Cheryl repeatedly mentioned the progress she was making in explaining her reasoning. In our second interview, she was proud that her homework scores had improved, telling me: "I've gotten more comfortable with explaining and showing how to do my work rather than just doing the work and expecting him to know that I know what I'm doing." In our final interview, she actually said, "Well, I won't just be doing math in my head. It gets it done fast, but I can't necessarily expect others to do that, so I am going to have to continue learning how to show it." Indeed, this change extended beyond just needing to justify her reasoning to earn points on assignments to her future teaching. She articulated this change clearly in her post-reflection:

Elementary math is supposed to be easy-peasy for an adult like me. At least that is what I

> thought at the beginning of the course. Although adding, subtraction, division, and multiplication are just drilled into my head and I just know them, I never realized just how hard it would be to try and teach the how and why of them. I just knew how to do them and never really thought about it. The class really made me think about just how and why math comes together like it does especially for those that are just learning about math. I definitely don't want to turn students away from math because I refuse or did not know how to show them many ways of doing a certain thing.

This deep reflection on her progress shows a significant shift in the value Cheryl placed on the importance of explaining and justifying her mathematical reasoning.

Cheryl's Future as a Mathematics Teacher

Cheryl entered the course confident in her ability to teach K-8 students, having successfully tutored an 11 year old boy in various subjects. Her focus on writing down or explaining her mathematical reasoning was grounded more in her future as a teacher than increasing her own math abilities. She said, "I have a hard time explaining in words what I understand," but admitted she was motivated in this course because "I know a lot of [teaching] is you have to be able to tell that student how you did it, or show your steps in words." She repeatedly referred to this course as a "refresher course on everything you have learned in math from the beginning." As such, since Cheryl was quite confident in her own math abilities and her abilities to teach, in this class she focused a great deal on learning to explain.

During our second interview, Cheryl had made some progress in achieving this goal, but admitting wishing that she did not need this skill quite so much to teach. She told me, "I wish I could tell a kid, "This is how it is and that's how it is done. You just do it," but it doesn't work out that way." When asked to expand on that idea, she talked about how this "would mean that

all kids are the same" and would "make things go a lot faster." Despite that, Cheryl felt that group work was "helpful" for going from "not explaining to explaining." By the end of the course, Cheryl wrote in her post-reflection:

> I have learned how to explain how I come about an answer and how I can better explain it to a peer or a child. The importance of learning just that will take me far on how capable I will be in teaching children, since many of them will not learn the same way.

She is able to articulate clearly the benefits this course will have on her future teaching, noting particularly how being able to explain will make her a better teacher, and also noting how "boring" and ineffective the class would have been if it was "just lecture."

In our final interview, Cheryl mentioned that she does not "put too much emotion into math," but did want to "make it fun" for her students, and could articulate the specific ways in which she hoped her future students engaged with mathematics. She described that she would be "more patient" with them, encourage "multiple ways," and would like for them to have discussions in groups to share ideas. Interestingly, Cheryl did mention that her classes would not be as "unstructured" as this Math for Elementary School Teachers course because students would have to "follow procedures from the book." This notion came from observations Cheryl had made in some of her practicum classrooms where she got the sense that teachers had little freedom in structuring their lessons. However, Cheryl did mention that she would be willing to "rebel" against this kind of rigid structure in order to show how much students were learning because "results speak louder than words."

Cheryl's Survey Responses

Cheryl's survey responses, shown in Table 15, indicate an overall increase in her feelings of competence, relatedness, and autonomy in this inquiry-oriented Math for Elementary School

Teachers course compared to her previous math courses, which were largely traditional. While her responses generally follow the trend shown by the whole class means, Cheryl does have noticeably higher ratings of feeling competent and autonomous in her previous math courses.

Table 15

Cheryl's Mean Scores for Subscales of the Basic Psychological Needs Survey

	Competence		Relatedness		Autonomy	
	Past math courses	Current math course	Past math courses	Current math course	Past math courses	Current math course
Whole class	4.79	6.32	4.35	5.77	3.95	5.83
Cheryl	5.50	6.50	3.50	5.88	5.29	6.14

Note. Original scores, given on a 7-point scale (1 “not at all true” to 7 “very true”), represent the degree to which a student experienced feelings of competence, relatedness, and autonomy on questions from three subscales. Shown here, are the mean scores on each subscale.

Indeed, Cheryl entered this course feeling quite confident in her abilities, and she was the only focal student to express her readiness to head straight into elementary math teaching. Her measures of competence entering the course were traditional, however, stemming from her success in the form of grades and speed in her previous math courses. Cheryl does report a higher sense of autonomy in her past classes than the class average, which is not surprising since she does describe a positive memory of students' multiple approaches being encouraged and supported in one of her most memorable math classes in middle school. Despite already experiencing high degrees of satisfaction related to competence and autonomy in mathematics, Cheryl's interviews, writings, and survey results suggest she had important positive affective experiences within this inquiry-based course for prospective teachers.

Cheryl Summary

Cheryl is an example of a student who felt ready to teach elementary school before taking

this math content course for teachers. She has had primarily positive past experiences with mathematics, that continued within this course, and were often linked to her math abilities and her sense of belonging in class. Her experiences are summarized in Table 16.

Table 16

Notable Takeaways About Cheryl's Affective Experiences Over Time

Characteristics of student experiences	Past math experiences	Current math experiences
Traditional/non-traditional	Montessori and traditional	Inquiry-oriented
Positive/negative	Mostly positive	Mostly positive
Feelings of competence	High, linked to grades, speed, and getting correct answers	High, now also linked to ability to explain/justify reasoning
Feelings of relatedness	Rare, only when comfortable showing personality in class	Felt comfortable and valued as a group member and classmate
Feelings of autonomy	In Montessori, related to pace. In traditional, related to multiple accepted strategies	Valued autonomy as related to supporting learning to explain reasoning
Feelings about teaching mathematics	Confident before taking this course	More confident in ability to explain
Described characteristics of future classroom	Wants learning to be fun	Wants to support student inquiry, but expresses need to also follow book procedures
Other student specific notes	Identifies strongly as a teacher, not just of mathematics	Realizes difficulty of inquiry-oriented teaching, willing to push for it in schools

Despite linking her competence frequently to test scores, both now and in the past, she focused much of her energy in this course on learning how to articulate her mathematical reasoning, an idea that she had resisted improving until she saw direct benefits for her teaching. Seeing the value in explaining her reasoning was the major shift in thinking around mathematics that occurred for Cheryl in this course.

In her past math experiences, Cheryl felt a sense autonomy. First, in Montessori school,

she felt autonomy over the pace in which she could learn mathematics, and later, she experienced a sense of autonomy over her solution approaches in middle school. That latter experience linked to the benefits she saw in group work for her mathematics students. Cheryl valued having the autonomy to use her own methods in middle school, and after experiencing group work in this course, she valued being able to learn to articulate her thinking by talking with other students. At the end of this study she was also able to explain that she wanted her students to work in groups to discuss ideas and see and generate multiple ways to solve problems. Cheryl leaves this course with continued confidence in her abilities, improved methods for explaining her thinking, and a willingness to foster inquiry in her future mathematics courses.

Samantha: A Student with Inquiry-Oriented, Mostly Positive Experiences of Math

Samantha is in her early twenties, but describes herself as "really a kid at heart," frequently mentioning her "love of anything Disney." Working in a daycare, she also knows "about all the latest little kids shows." Samantha acknowledges that she is typically "bubbly and outgoing," which is reflected in our conversations, during which she talks very quickly, but wanted to make sure I know that she is quite shy and needs to "warm up to people at first." Math is Samantha's "favorite subject," and she is generally confident in her abilities, but does not consider herself a "math genius," meaning she does not necessarily consider herself to be the very best math student. However, she was consistently placed in accelerated math courses, reaching AP Calculus her senior year. Samantha grew up "playing school" with her younger cousins. She was always the teacher, giving them math problems to solve, and this experience is what makes her believe she will be a good teacher, "especially in math."

Samantha wrote in her autobiography that her past math classes did utilize inquiry-oriented methods, as she remembers learning multiple approaches to problems, working

problems on the board and discussing them, and other specific hands on activities. The one negative math experience she mentioned was in AP Calculus, where the focus was "on the test, not the math." She entered this Math for Elementary School Teachers course excited to learn about how to teach math, and had to rethink her notion of the course when she realized it was not a teaching methods course. This disappointed Samantha because she is eager to learn how to teach, and confident that she will be a good mathematics teacher, but worries that she does not know enough about how to teach mathematics to children well.

Samantha's Past Mathematics Experiences

Although math is Samantha's favorite subject, she told me that considers herself just "pretty good" at math, not "a genius when it comes to math." She was consistently placed in accelerated courses through high school, and enjoys doing math problems when she is "bored out of her mind." Samantha did consider herself more competent than many of her peers during middle and high school, noting, she "was really good and would get things the first time," and repeatedly mentioned that she struggled to be patient when her teachers would "go over things 5,000 times for the one kid who just did not understand." She also felt competent when she could work problems on the board, explaining her reasoning "in front of the entire class."

One time that Samantha remembered not feeling competent in math was when she took AP Calculus her senior year. When we talked about this experience, she said:

> To be good at math, you need to have critical thinking. Alright, what are my options, how do I figure it out? You have to lay it out. You can't just know the answer. You know the steps to get to the answer, and I would just pretend I know the answer. I'm more of a guesser.

In this discussion during our first interview, Samantha started to characterize herself as someone

who does not work through mathematics problems like someone who is "good at math." She clarified this by adding, "Good math students are willing to take the time to figure out the answer and they know the equations to get the answer." So, despite the fact that she enjoys working math problems simply for enjoyment and in her free time, Samantha considered herself not to be one of these good math students.

Samantha felt a sense of belonging in her past math courses because she was "really good at math," letting me know she "skipped algebra and went straight to geometry" because "[she] worked hard." The AP Calculus course where she did not feel competent was also the one time she described not feeling like she belonged in her math class. She attributed this to the fact that the course was "fast-paced" and "more about the test than about the math," remembering how the teacher moved quickly to cover material for the AP exam rather than answer questions for understanding.

It is of interest that in some of her math courses, Samantha remembers her classes utilizing inquiry-oriented methods and related this to the variety of ways she learned to solve problems. When she reflected on her elementary school math classes, Samantha remembered learning multiple ways to think about arithmetic operations and was excited to tell me how she still remembered those various methods. At the time, she felt little autonomy over her learning, because her "mean" teacher "made" the students learn multiple approaches. In retrospect, she values the flexibility she has in solving problems as a result of learning many approaches. She also talked about how much she remembered from her childhood math courses as a result of approaching problems in inquiry-oriented ways that allowed exploration rather than following formulas, like creating shapes out of graph paper or using string to explore circumference. In middle and high school though Samantha also felt limited in the ways she was allowed to

approach problems: “It was more here is a problem, here is the equation to do it, solve it.” Despite having memories of doing hands on activities, and occasional times when students would share problems on the board and justify their reasoning, Samantha described having little autonomy in her solution approaches during past math courses. In our first interview, Samantha was already excited that this class “is more thinking rather than teaching.”

Samantha's Experiences During Math for Elementary School Teachers

In the beginning of this course, Samantha thought this Math for Elementary School Teachers content course was actually a teaching methods course. As such, she described feeling “excited” to learn about how to teach math to kids, and focused her attention on contributing ideas around how kids would understand the mathematics. After she realized the course was about learning content knowledge for teaching, she described herself as “awesome possum” at “everything” that had been covered in class because it was “like middle school all over again.” Specifically, Samantha frequently talked about how important it was in this class to be willing to “do a lot of thinking” and “fight about math.”

To Samantha, the “thinking” aspect was about digging deep into problems, being willing to find a solution, and was something she “loved” about the course. Indeed, when the groups worked the Smallest Difference activity described in Chapter 5, Samantha was still working as students left the class for the day, and described in detail how she text messaged this problem to friends and family members to continue the thinking around it after class. This “thinking” was in contrast to having a teacher tell students how to solve problems. It was an aspect of this course that strongly influenced Samantha. She very often relayed excitement around having autonomy while saying things like “It is me thinking all for myself,” “We were allowed to use whatever method we wanted,” and “It sounds weird, but I love how it is new concepts and thinking about

why does this work and how does it work." Samantha's enthusiasm for feeling this sense of autonomy was not just related to her individual work. In our final interview, she said that it was "awesome" that "everyone was learning together instead of a teacher telling you."

The "fighting" Samantha frequently mentioned was about feeling competent in sharing her ideas with her classmates. She was willing to share ideas in small groups, though sometimes she liked to work on her own before doing so, and she was also willing to share her reasoning in front of the whole class. In fact, I observed Samantha's first sharing experience in this course, during the fourth day of class. Even at that early stage, she was willing to present her group's idea around a problem the class had worked. It is also important to note that during this share-out Samantha was presenting reasoning that was not completely correct, and had no hesitation to "argue" her ideas fervently and then revise her thinking in front of the entire class.

Importantly, Samantha also linked her own feelings of competence in this course to her ability to help others. Since this course was for future teachers, Samantha felt like she needed to "be a teacher, to help people." At times, she struggled to help other students understand course concepts, and those moments stood out to her as times when she was "not confident enough to try to help others." She also felt "mean" because she struggled to "stay calm" when she was trying to explain material to other students in the course that was "not that hard" for her. Despite those moments, Samantha felt related in this course, particularly because she tried "to contribute a lot," which meant both helping other people and having other people help her. She described the relationships with her classmates by saying, "Our whole class is like a big team. We try and help each other. We are really close and I love it."

It is interesting to think about Samantha's participation with respect to the numeration systems group project that she undertook with Jessica and Morgan, which we discussed in our

second interview. Samantha was "mad" that Jessica "took over" because she thought the other group members were unprepared. Samantha described how she prepared for that class session, watching YouTube videos and doing research for a few hours. Interestingly, despite feeling confident in her knowledge that day and her general math abilities, Samantha deferred to Jessica, remaining quiet for most of the planning session after making a small attempt to be heard at the beginning of the class. When I asked her about why she stayed quiet, she said: "I was done. Whatever, she can do it." There were clearly complicated group dynamics at play here[16], but it is interesting that Samantha backed off, considering how eager and confident she usually was at other times. In our final interview, Samantha described her typical participation: "I try to help a lot, but if someone knows more, I'll let them talk. Especially with Jessica. I love her, but she read the book and did more, so she knew it better." Notably, she referenced working with Jessica when I asked her to describe her typical participation, a question that was not linked to the above group activity in any way, in which Samantha was very clear that she had done significant outside preparation and was confident she knew the material.

Samantha's Future as a Mathematics Teacher

Samantha wrote in her autobiography that she thought that "inquiry methods work best for learning," emphasizing how much more she retained when these types of methods were used in her elementary school math classes. She also entered this course eager to teach mathematics. Her strong disappointment in realizing this was a content for teaching course, rather than a methods course, was an indicator of how much she wanted to get into teaching. However, in her autobiography, Samantha worried that "the whole teaching process might be too much" for her. She clarified that she had felt "frustrated" and questioned whether she could be a good teacher

16 The reader might be interested to know the group was presenting the Mayan system, which sources explain in conflicting ways. Scott expected this complex system to require significant group discussion to understand.

when students in this class had trouble understanding material that she had tried to explained to them. Her strong feelings of competence make her worried that she will be "over-frustrated" working with students. This was a fear she retained throughout the course, as a result of continued difficulties explaining what she knew to students who were struggling.

Samantha left the course confirming her original assertion that "the inquiry approach works the best to help children understand and have fun learning what they are being taught." She emphasized her enthusiasm for learning multiple approaches to solving problems, which were generated through group work, suggesting that students may learn better if allowed to explore multiple approaches: "You are not getting it this way? Here, let's try it this way and maybe this way will help you better." Samantha's experiences of this course, particularly around her feelings of autonomy, helped her articulate why she thinks inquiry-oriented methods are best and the particular ways she wants her future students to engage with mathematics: "There is going to be a lot of students working with students. I want them to be excited to learn the math. I want them to not feel obligated to do it just one way. I want them to think of their own ways to learn it, too." Samantha wants her future students to be so "excited" that they "teach everyone else," like their family members and friends, what they are learning in math class.

Samantha's Survey Responses

Considering Samantha's position as one of the only focal students to describe having strong positive feelings related to mathematics at the beginning of this course, her psychological needs survey scores, shown in Table 17, are somewhat surprising. Like the other focal students, her responses follow the general trend of showing increased experiences of competence, relatedness, and autonomy in this inquiry-oriented Math for Elementary School Teachers course over her prior mathematics experiences.

Table 17

Samantha's Mean Scores for Subscales of the Basic Psychological Needs Survey

	Competence		Relatedness		Autonomy	
	Past math courses	Current math course	Past math courses	Current math course	Past math courses	Current math course
Whole class	4.79	6.32	4.35	5.77	3.95	5.83
Samantha	4.80	6.33	3.50	6.25	4.71	6.00

Note. Original scores, given on a 7-point scale (1 "not at all true" to 7 "very true"), represent the degree to which a student experienced feelings of competence, relatedness, and autonomy on questions from three subscales. Shown here, are the mean scores on each subscale.

Interestingly, Samantha is the only one of the focal students to describe her past experiences with mathematics as having characteristics of inquiry-oriented classrooms. Yet, her initial survey responses suggest that her psychological needs were only sometimes filled. Her survey scores do indicate increased fulfillment of all three of her needs in this course, and her interviews and writing submissions suggest that her confidence did improve even more during her enrollment in this course, she had a strong sense and value of autonomy, and felt a strong sense of belonging.

Samantha Summary

Samantha entered and exited this course claiming that math was her "favorite subject," and having a passion for helping others achieve their potential in mathematics and see the "fun" that she sees in the subject. Table 18 gives a summary of her experiences. In her autobiography, Samantha declared that she was "not a genius" when it comes to math, and this declaration was about the fact that she typically did not know and readily use formulas to solve mathematical problems. She distinguished herself from students who appeared to know how to formally solve problems following steps or using procedures by saying she often "just guessed." In this course though, Samantha "loved thinking" for herself, which meant spending significant time working

Table 18

Notable Takeaways About Samantha's Affective Experiences Over Time

Characteristics of student experiences	Past math experiences	Current math experiences
Traditional/non-traditional	Some traditional, some inquiry-oriented	Inquiry-oriented
Positive/negative	Mostly positive	Mostly positive
Feelings of competence	High, but felt not as good as people who use procedures	High, valued thinking more in judging competence, likes sharing and justifying ideas
Feelings of relatedness	High, linked to competence	Fit in, valued community
Feelings of autonomy	After elementary school, felt limited, only one approach was supported	Strong, highly valued both individual thinking and learning together in groups
Feelings about teaching mathematics	Excited to teach, feels especially confident about teaching math	Excited, even more confident, worries about having patience with struggling students
Described characteristics of future classroom	Believed inquiry-oriented methods to be best for learning	Value of inquiry methods confirmed, wants students to think for themselves, and work together

out viable solution approaches. It may be that she initially considered this as less mathematical. Her past experiences suggest she always enjoyed thinking about math problems, as she mentioned she would work problems in her spare time for fun. She was frequently very confident in her abilities, saying she was "awesome possum," or "very, very good" at tasks from class. In addition, her post-reflection at the end of the course had no such hesitation about her lack of "math genius." Even when reflecting on challenges she faced in the course, Samantha reported that once she came to her own understanding of material, she was eager to help others learn.

Samantha had long thought that inquiry-oriented methods were best for student learning,

particularly around student retention of knowledge, and this class confirmed for her that her suspicions were true. In particular, at the end of this course, she can talk articulately about the fact that she wants students to be generating their own knowledge, engaging in the "thinking" that she considers most enjoyable, rather than receiving knowledge directly from an instructor. For Samantha, this notably includes group work, which is an integral part of developing a sense of community in the math classroom. Samantha's positive experiences with inquiry-oriented methods, both in her elementary school past and in this course, "boosted" her already high confidence in her mathematical abilities and her ability to teach mathematics.

Conclusion

The individual student case studies presented in this chapter are at the heart of understanding how prospective teachers make sense of their affective experiences of mathematics over enrollment in an inquiry-oriented math course. Particular attention was paid to how students described their past experiences, and the connections that could be made between affective experiences of students and the cognitive and participatory demands placed on students in inquiry-oriented mathematics courses. As the participants in this study were prospective teachers, each case also focused on how focal students expect their experiences to influence their teaching. In adhering to that structure, I attempted to present each student's journey throughout this Math for Elementary School Teachers course, working to understand how each student was making sense of their affective experiences over the course of the quarter. In this way, I began to articulate the common and disparate experiences of the various students. In the final chapter of this dissertation, I will discuss a cross-case analysis of these individual focal student case studies, exploring the similarities and differences among and within the prospective teachers' affective experiences of mathematics, and the larger implications and significance of this study.

CHAPTER 7: CROSS-CASE DISCUSSION AND IMPLICATIONS

This concluding chapter will focus on synthesizing the results of this study, highlighting implications for both teaching and future research related to inquiry-oriented mathematics. I begin this chapter with a brief reminder of the rationale for this study, calling attention to the need to link students' affective experiences in mathematics with the cognitive and participatory expectations that are placed on students in inquiry-oriented classrooms. I also reiterate the affordances of the self-determination theory based conceptual framework that I have chosen to use to make such connections. The majority of this chapter is dedicated to a discussion of the cross-case analysis of the individual case studies that were presented in the preceding chapter. The dissertation concludes with a discussion of study limitations, future directions, and significance.

The goal of this study was to understand how students make sense of their affective experiences of an inquiry-oriented math content course for teachers. Specifically, I wanted to be able to expand on limited results that suggest that students enjoy these types of courses more or that these types of classes can increase student confidence, to understand how the nature of the inquiry-oriented math classroom might foster such positive affective experiences and what those feelings mean to students, particularly if those students are future teachers. In short, I wanted to understand in detail what students feel they gain affectively from inquiry-oriented classrooms, to add to what research suggests students gain cognitively from such classrooms.

There are notably higher expectations placed on students in inquiry-oriented classrooms. Students are constructing arguments, justifying their mathematical reasoning, persevering to solve complex problems, and generally playing an active role in their mathematical learning. Indeed, Chapter 5 articulated the ways in which this particular inquiry-oriented class embodied

these participatory and cognitive expectations for student engagement that go far beyond absorbing material presented by a teacher. Students were expected to actively participate in constructing mathematical knowledge during this inquiry-oriented Math for Elementary School Teachers course. Specifically, students were expected to *discuss mathematics with others*, which involved communicating and justifying their ideas and reasoning to other students and answering other students' questions, both in small group and whole class discussions. This discussion was closely linked to the expectation that students *construct solution strategies* using resources and their own knowledge, seeing themselves as valuable members of the community, with mathematical knowledge to contribute. This is in contrast to being expected to simply assimilate existing procedures or rules. Students were also asked to *be intellectually courageous*, which meant being willing to suggest ideas, ask questions, and risk making mistakes, revising thinking as necessary. Finally, students were expected to *let their thinking develop over time*, realizing that it was acceptable to spend significant time working problems and to leave problems without a neatly presented solution and revisit them after having time to think. The cross-case analysis in this chapter will help show how this inquiry-oriented environment, which placed high demands on students to actively engage with mathematics within the classroom, contributed to the satisfaction of students' psychological need to feel competent, related, and autonomous.

In my conceptual framing of this study, I argue that as students engage in these new, demanding ways, their affective experiences matter significantly. I am not the first to suggest that students' affective experiences should be considered as integral to their mathematical learning (e.g., Gresalfi, 2009; Turner et al., 2011). Math experiences in general are often psychologically loaded, and these students' experiences were no exception, as was demonstrated by the detailed recounting of both positive and negative experiences in their mathematical

autobiographies. In addition, new, inquiry-oriented ways of engaging with mathematics also resulted in affective responses, as some students felt strongly a renewed sense of confidence in mathematics or struggled with the importance of being of able to explain their reasoning. I used self-determination theory to frame my study of student affect because focusing on the constructs of competence, relatedness, and autonomy allowed me to carefully attend to certain aspects of students' affective experiences which have been shown to foster intrinsic motivation and general well-being (Ryan & Deci, 2000). I wanted to go beyond shallow responses given by students in other similar research to understand what is behind students' reported feelings, to see how specific characteristics of the inquiry-oriented math classroom influence student affect. In the following cross-case analysis, I discuss what I discovered about the links between students' affective experiences and the participatory demands of the inquiry-oriented math classroom. In addition, I discuss the ways this study suggests those feelings within this course are linked to the prospective teachers' developing mathematics teaching practices.

Cross-Case Analysis of Students' Affective Experiences of Mathematics

Cross-case analysis of the individual case studies allows for consideration of the common and disparate themes that appear within and across the experiences of the focal students in this study. The quantitative psychological needs survey results, articulated in Table 5 in Chapter 5, showed that students did feel more competent, related, and autonomous in this course than in their previous mathematics courses[17]. The following discussion of the cross-case analysis, which draws on the individual case studies from the previous chapter, expands on those results to discuss how students made sense of their experiences of competence, relatedness, and autonomy in this course, making links to both what students understood about their past experiences of

17 While I don't know the backgrounds of all the students in the course, the vast majority of the focal students had traditional, lecture-based math experiences before taking this course.

mathematics and the participatory demands of this particular math course. I also articulate findings around the developing mathematical teaching practices of the students, which are closely linked to the prospective teachers' experiences of this Math for Elementary School Teachers course. I will begin by discussing three major findings around prospective teachers' psychological needs satisfaction in mathematics: (1) students expanded the ways in which they felt competent in mathematics while participating in an inquiry-oriented course; (2) students' new sense of relatedness in math class stemmed from the community fostered within an inquiry-oriented course; and (3) students valued having the autonomy to pursue their own mathematical reasoning.

Students Expanded the Ways in Which They Felt Competent in Mathematics While Participating in an Inquiry-Oriented Course

Over the course of participating in this inquiry-oriented math class, students increased both the degree to which and the ways in which they described feeling competent in mathematics. The quantitative survey results show that students generally felt more competent in this course than their previous math courses. However, the survey only allows students to indicate whether they felt competent (for example, students rated how true this statement was for them: When I am in class I often do not feel very capable), not when or how those feelings were arising in relation to their mathematical work. The individual narratives allow for additional consideration of when and how students experienced competence in this inquiry-oriented math course. When students discussed their feelings of competence in their past math classes, it was not unusual for those feelings to be linked with traditional measures of competence like speed, grades, or correct answers, which were sometimes put on the board for display. By the end of the course, however, students were describing their experiences of competence more broadly,

relating to their ability to explain, to help other students, or to allow space for other students to share their ideas. The participatory and cognitive demands of this inquiry-oriented classroom provided these new ways of feeling competent, and results suggest that students took up those opportunities to engage more broadly with mathematics. In this section I will focus on two new ways that students described feeling competent: (1) in relation to working mathematics along side other students, and (2) in being able to explain their reasoning to others.

One way students described experiencing feelings of competence in this course was in relation to learning mathematics with other students, linking their competence to the group work nature of the course. Morgan, Bahira, and Samantha are examples of students who felt competence in this way. In their previous experiences, these students lacked opportunities to think deeply about mathematics with other learners. Due to her home-schooling experiences, Morgan could only judge her competence by the grading of her answers, which led her to remembering "red marks" all down the side of her paper. Even at the beginning of this class, she hypothesized that having other people to work with would help her feel competent, if she could show she knew something that other students did not. Bahira entered the course describing herself repeatedly as "average" in mathematics, which she judged by test scores and her placement in middle and high school. In past courses and in this course, she described doing only what was required to get "the grade," and felt that she could justify her averageness in mathematics by convincing herself that she is "not a math person." Interestingly, Samantha entered this course feeling quite competent in mathematics, talking about how she was always placed in advanced classes, but described herself as "not a genius." Importantly, she talked about how she often "guessed" when solving math problems, and that people who were truly good at mathematics used procedures or had a more organized strategy for solving problems. She said,

"Good math students are willing to take the time to figure out the answer and they know the equations to get the answer." To Samantha, students who followed procedures or knew equations to use were more competent in mathematics than she was.

In this course, students were consistently given opportunities to do mathematics with other students during class time. Rather than sit quietly and follow procedures, students were expected to engage deeply with mathematical content, consider ideas with others, and leverage their own knowledge. As a result, students had opportunities to feel competent, or capable of doing what is required, in math by doing any of those things. For example, Morgan talked about how much she enjoyed "working through" her "own thoughts" "in front of everybody" when Scott asked her to demonstrate some ideas on the white board. Near the end of the course, Morgan still talked about test scores, but indicated that her high scores on them indicated she "could explain everything," suggesting a link to understanding, not just a final score. For Morgan, having other students to consider mathematics with was important to increasing her confidence in her mathematics capabilities. Bahira thought group work was "helpful" in this class and found it to be "inspiring" because the other class members persevered even when math was "not easy" for them. This contributed to Bahira's shift in attributing her competence in mathematics not to innate ability, but rather to effort. At the end of the course, she wrote in her post-reflection: "Just because math might be difficult for me, that does not mean I will not be able to learn it. I believe it all depends on how much effort you put not only inside the class but outside of the class as well." During her participation in this course, Samantha was also able to shift her views of her competence to include when she was "thinking" about mathematics, something she described frequently and "loved" because it was about "everyone learning together." In this class, Samantha also felt competence when she could justify her thinking to

others, which she referred to as “fighting” about mathematics. Though she never explicitly stated it, it appears that over time in this course, Samantha valued her thinking process, rather than considering it less valuable than using equations or following procedures, saying frequently in our later interviews, “I love thinking for myself...instead of a teacher telling you.” Morgan, Bahira, and Samantha's experiences of competence show that they attributed their improved feelings of competence to learning mathematics within the classroom community, something that was not typically supported in their prior, traditional experiences of mathematics.

Katie, Cheryl, Samantha, and Jessica talked about feeling competent when they could explain their ideas to others. For Jessica, this was focused around her own mathematical knowledge and finding ways to show what she knew. In a different way, Katie, Cheryl, and Samantha linked their feelings of competence in this course to being able to explain their reasoning to other students, but emphasized how their feelings of competence were not only about their own understanding, but also about how that understanding was linked to their future teaching. Both Katie and Cheryl described feeling strongly competent in most of their prior mathematics courses, though both have a single stand-out negative experience described within their individual cases. Past feelings of competence in math were linked to speed and grades for both students. Katie talked about feeling competent when she “got” things “on her own” or when she finished timed tests before other students. Like Katie, Cheryl described feeling competent when she finished timed tests quickly, during her Montessori school years, and she also felt competent in her ability to get answers to math problems, which often resulted in high scores on tests. Cheryl did struggle to explain her reasoning, noting, “I have a hard time explaining in words what I understand.”

In this course, Katie, Cheryl, and Samantha took up opportunities to explain their

reasoning to others, and it was in doing so that they expressed feeling particularly competent in this course. Katie specifically described how she worked hard to allow space for other students, noting that they would often ask for her opinion, which was something I observed during my course observations. She reflected on this by saying, "I think when people ask me, if I am able to explain a way that they get, that makes me feel really good." She felt competent when other students asked for her assistance or opinion. In addition, when we talked about this, Katie told me that this class is a "little microcosm" of whether she is going to be able to teach math. This suggests that her competence around being able to help others is not only linked to her own capabilities to learn the class material, but also to her capabilities as a math teacher. Importantly, Katie left this course feeling quite unsure about her abilities for success in future mathematics courses, which I discussed in detail in her individual case. Nevertheless, Katie left the course having experienced feeling competence in mathematics in new ways.

In this class, Cheryl worked hard to learn to explain her thinking. She reflected on her progress, which she measured by her ability to explain concepts to other students: "I was able to show it, demonstrate it, and in a way that they could understand better than the way Scott was showing it up on the board... ." Cheryl was primarily interested in improving her explaining because it was important for teaching, saying, "I know a lot of [teaching] is you have to be able to tell that student how you did it, or show your steps in words." Although Cheryl did tell me that she was getting better at explaining her reasoning to Scott for credit on homework and other classwork, she always emphasized this underlying connection to her future teaching as a motivation. Samantha also linked her own feelings of competence in this course to her ability to help others, which she directly connected to teaching, saying she felt the need to "be a teacher, to help people" in this class as long as she understood the material first herself. This led to times

when she felt less competent in class because occasionally she was "not confident enough to try to help others."

Like Cheryl, Katie, and Samantha, Jessica felt competent when she could explain her mathematical thinking or ideas. However, she did not link this to her future teaching; her feelings of competence were only about individually showing what she knew. In her very recent past, she was one of the students that would have labeled herself "not a math person," lacking competence in math. She described her past experiences, having been placed in lower level courses where teachers had low standards, by telling me she "checked out" and "took short cuts." However, Jessica entered this course with a prior experience at GVCC that suggested to her that effort mattered significantly in mathematics. As a result, in this course, Jessica put in a great deal of effort to learn mathematics. She consistently took on leadership roles, being the "one who is writing," "putting in [her] two cents," or taking over group presentations. Jessica only described feeling competent when she was actively showing what she knew, which included explaining her thinking in various ways and also earning full credit on almost every exam and homework[18]. This class gave Jessica many opportunities to show what she knew, allowing her to personally feel very competent and as if she could demonstrate that competence, but in the next section I will discuss how this strong need to show her competence appeared to influence her lack of relatedness in this course.

For some students, feelings of competence were closely linked with being able to justify their thinking or answer questions from other students, and for some, this was also strongly linked to their future as a teacher of mathematics. The nature of this inquiry-oriented class

18 It is possible that Jessica emphasized this demonstration of knowledge because I asked her whether she felt she had *demonstrated* competence in this class, rather than just felt competent. However, after hearing her response, I did follow up with asking her whether she felt competent at any other times.

provided many opportunities for students to explain or justify their thinking to others, in small group work, during whole class share-outs and presentations, and even outside of class as students frequently chose to continue working together.

The above discussion articulates how the inquiry-oriented nature of this course fostered new ways that students could experience feelings of competence in mathematics, in contrast to the limited experiences of competence most had in their past math courses. Table 19 gives examples of how students felt competent, or not competent, in their prior mathematics experiences.

Table 19

Examples of Students' Prior Experiences of Competence in Mathematics

	Prior experiences of competence	Interpretation
Morgan:	"It was mostly just the red marks on the side of the page that you have to do this over again..."	Felt less competent knowing she answered questions incorrectly.
Bahira:	"I just thought I was an average student. I was in the average math class."	Judged competence using tracking placement.
Samantha:	"Good math students are willing to take the time to figure out the answer and they know the equations to get the answer."	Felt less competent than people who used procedures to solve problems.
Katie:	"I get it and I get it really quickly, or, oh, I got this -- visually, when I immediately recognize it as something that I understand how to do."	Felt competent because she was quick in various ways.
Cheryl:	"I always felt competent in what I was doing and that my work was quality work, because I understood it."	Felt competent in getting answers that she understood, but less competent explaining.
	"I have a hard time explaining in words what I understand."	
Jessica:	Did not get grades "good enough to move on" or "barely passing."	Felt less competent when getting unsatisfactory grades.

It is also worth mentioning here that some of the focal students, notably Bahira, Samantha, and Jessica mentioned that being asked to put a correct solution on the board was an occasional, though not frequent, way they felt competent in their past math classes. Samantha's quote here describes the idea they were each getting at: "Just showing that I knew what I was doing, like in front of the entire class, here is a problem and I'm going to go up and show you I know how to do it." This solution display was always about showing a correct answer, not discussing mathematics or sharing strategies. Since students seemed to rarely experience this in their prior courses, which were largely traditional, it did not come up often in this study. However, it might be worth investigating in future work, particularly exploring how students experience this presentation of their answers on the board differently from explaining mathematics to others in small groups, which they described as promoting feelings of competence in this course.

This class afforded all students more ways to feel competent in mathematics, shown in Table 20, influencing students with both negative and positive experiences in past courses. Students with primarily traditional past experiences that were used to only measuring success by checking answers, like Morgan, or who formerly focused solely on speed, like Katie, benefited from experiencing increased opportunities to feel mathematical competence. However, even students like Cheryl and Samantha, who were confident in their mathematical abilities prior to taking this course, took up new opportunities to feel mathematically capable, and talked about increasing their confidence even further. Regardless, each of the focal students expressed feeling competence in new ways during their time in this inquiry-oriented class compared to their past classes, and most of the students expressed feeling more competent in this course than in their prior mathematics courses.

Table 20

Examples of Students' Experiences of Competence in This Inquiry-Oriented Math Class

	Current experiences of competence	Interpretation
Morgan:	"I enjoyed working through my own thoughts in front of everybody."	Feels competent when she can think about what she knows with other students.
Bahira:	"That [perseverance] reinforced my belief that just because math might be difficult for me, that does not mean I will not be able to learn it."	Feels capable when she sees other students working hard.
Samantha:	"We will sit there and it is not an argument, but it is more of a whose is right, why are we right, and okay, I believe you. Yeah, show that you understand it."	Feels competent when she can discuss and justify with others.
Katie:	"I think when people ask me, if I am able to explain a way that they get, that makes me feel really good."	Feels competent when answering questions from others, linked to future teaching.
Cheryl:	"I was able to show it, demonstrate it, and in a way that they could understand better than the way Scott was showing it up on the board... ."	Feels competent when showing ideas to others, linked to future teaching.
Jessica:	"I'm definitely one to raise my hand and make comments or ask lots of questions and always put in my two cents."	Feels competent when showing what she knows.

Students' New Sense of Relatedness in Math Class Stemmed from the Community Fostered Within an Inquiry-Oriented Course

The inquiry-oriented nature of this Math for Elementary School Teachers course, which emphasized actively learning mathematics together with fellow students, allowed students to feel a sense of social belonging in math class that was new for most. Quantitative survey results revealed that students felt more related in this course than their previous math classes, and in this section I will articulate how and why students' felt that sense of belonging. The majority of the

focal students in this study described past mathematics experiences that did not involve a sense of community in the classroom. As a result, most students in this study did not feel a strong sense of relatedness during their previous math classes. Regardless of their prior experiences with belonging or community within math class, each focal student in this study described feeling a sense of social belonging in this course, and those feelings arose from the community that was fostered within this inquiry-oriented course.

I start this section by articulating how students described not feeling a sense of relatedness in their previous mathematics classes. Three of the focal students talked about the lack of community within their prior mathematics classes, which contributed to their lack of feeling a sense of social belonging[19]. Morgan's home-schooling experiences readily lent themselves to this lack of community, as she described doing math alone in the kitchen or her bedroom. Katie said she had never worked with other students in mathematics: "Working together? Math was always like, I got my answer, I get it, don't look, because if you don't know it, you shouldn't be looking at mine." At times when she struggled in mathematics, Katie remembered this lack of community as more apparent to her, suggesting that if there had been a social aspect to learning mathematics, she might have been more successful during times of struggle. When Jessica described trying to ask questions as a way to belong in her prior mathematics courses, she said that it felt like it was "everyone all for themselves." These students highlight a lack of community that at times memorably influenced their lack of relatedness in mathematics. In particular, when they struggled with mathematics, students felt a noticeable lack of places to turn for learning support.

19 The other students only discussed belonging or not belonging in math class in a way that I classify as identifying or not identifying with math as a discipline, discussed in a later section, not as a presence or lack of relatedness as defined in self-determination theory.

In contrast to these past experiences, all of the focal students told me they felt a sense of social belonging in this inquiry-oriented course, and each one discussed how the community that was fostered as part of the mathematical learning within this course promoted those feelings. Students valued the sense of community in both a general sense, in that they got along with others in the class, and because it fostered their understanding of the mathematical content.

Many of the students expressed the latter, valuing the classroom community because it supported their mathematical learning. Cheryl felt "accepted" and "valued as a group member and classmate" and Samantha felt like she was part of a "team" that she participated with by "contributing a lot" in terms of both asking and answering questions. She described the relationships with her classmates by saying, "Our whole class is like a big team. We try and help each other. We are really close and I love it." Similarly, Morgan said she appreciated realizing she was "not the only one who needs to work out a problem different from others," emphasizing how knowing that other students also needed time to consider problems helped her have a sense of belonging. Katie also expressed her sense of belonging as related to her competence, noting that her "confidence" in her abilities was "bolstered by the trust" her peers placed in her during the course. In these comments, students illustrate how the course design fostered a community that in turn helped them successfully learn mathematics. Specifically, Katie talks about how her confidence improved when students trusted her to evaluate their ideas or consider their thinking. Both being willing to ask questions, or trust others, and answer questions from other students were key aspects to participating in this inquiry-oriented course. In addition, being valued as a contributing member of the "team" or group is a common feeling shown here, which is linked to the participatory demand that students discuss mathematics together and generate their own mathematical ideas.

Some students expressed their sense of relatedness as more generally linked to the sense of community, but less specifically to their mathematical abilities. For example, Jessica hesitated to say she fit in during group work because she felt that other students might think she was "mean" for always taking on a leadership role, something she did because she was confident in her math knowledge. However, she did note that she valued working with other students outside of class on homework. Additionally, Bahira primarily "enjoyed the class atmosphere because it was comfortable and we had many opportunities to work with one another and get to know each other's names" telling me "I feel like I belong only in the sense that I know the classmates and I can talk to them." This was in part because she felt less "passionate" about teaching than she perceived the other students in this course to be. These students still felt related in the class, but on a more personal level, rather than connected to their mathematical learning. Note, however, that those feelings are still linked to mathematical work, such as doing homework together or working together in class.

Most of the focal students experienced a strong shift in their feelings of relatedness in math class while enrolled in this inquiry-oriented class, and each of the students experienced some sense of belonging as part of the classroom community. Table 21 summarizes how students felt a sense of relatedness in this class, highlighting that all students linked their sense of belonging in this class to learning mathematics together, even if the degree to which that was true varied. That is, this kind of participation in mathematics helped them have a place to contribute ideas or to answer questions from other students as well as simply connect with other students. The group work nature of the course made it necessary for students to work together to learn mathematics, and students in the course strongly felt that sense of "family" that Scott described wanting to foster within this classroom.

Table 21

How Students Experienced Relatedness in This Inquiry-Oriented Math Class

	Experiences of relatedness	Interpretation
Cheryl:	"I feel accepted. I feel valued as either a group member or as a member of the whole class."	Felt valued in a mathematical sense, as a groupmate or class member.
Samantha:	"Our whole class is like a big team. We try and help each other. We are really close and I love it."	Emphasizes closeness around working together to learn math.
Morgan:	"I found out I am not the only one who needs to work out a problem different from others."	Social learning of math allowed her to be comfortable taking time needed to learn.
Katie:	"I felt good about working with my peers... . My views about my abilities in math have certainly been bolstered by the trust my peers placed in me."	Social relationships and learning of math led to increased feelings of competence.
Bahira:	"I primarily enjoyed the class atmosphere because it was comfortable and we had many opportunities to work with one another and get to know each other's names"	Appreciated knowing students in class, which was a result of working together.
Jessica:	"It is really nice to know that people in class know my name, and people will sit down together and work on problems [after class]." "Sometimes I wonder if I come across too harsh with some of my classmates."	Appreciated students knowing her, and working homework together outside of class, but was hesitant to say she felt related during in class group work.

Identification with mathematics as a discipline. While exploring students' feelings of relatedness, I noticed that each student thought not only about belonging in the social sense, as relatedness is defined in self-determination theory, but also about whether they identified with mathematics as a discipline. For all focal students, whether they described feeling a sense of

belonging or not, ideas about "belonging," in the general sense of the word[20], were also linked to their feelings of mathematical competence. I will first discuss how this idea of identifying or not identifying with mathematics characterized students' past experiences, and then consider how their identification with the subject changed while participating in this course.

Bahira, Jessica, and Morgan each described feeling like they were "not a math person" in their past experiences, noting that they did not fit in. In particular, Bahira felt that she did not fit in in math class unless she could ask her teachers questions, and she used the "not a math person" label to justify her outsider status in mathematics, which was linked to test scores that were lower than she would have normally wanted. Jessica similarly described herself as a person who "asks a lot of questions," and her memorable experience of being transferred to a lower math course for asking a question influenced her sense of identification with mathematics. In that "slow moving" class, she "checked out," not feeling like mathematics was a subject in which she should put forth effort. Morgan did not feel like any school subject was for her when she was growing up, preferring to be at the gym, but emphasized her particular "dread" of mathematics. She told me, "No matter what I did or how hard I worked, I would fail." These students describe feeling like an outsider when it came to mathematics, particularly linking this identity to their lack of competence. In some ways, the "not a math person" label became a way for students to rationalize their inability to succeed in mathematics. That is, their lack of competence mattered less if they did not identify as someone who should be, or could be, successful in mathematics anyway.

In contrast, Katie, Cheryl, and Samantha described feeling like they fit in in most of their mathematics classes, and that sense of identity was fostered by their feelings of competence in

20 I use belong and fit in in this section to describe how students talked about whether it was appropriate for them to be in math class. That is, when they felt they identified as someone who was supposed to be doing math.

mathematics. Specifically, Katie described the only time she felt a lack of belonging in mathematics was when she failed calculus her freshman year. This was particularly influential for her because she put in a great deal of effort, and was planning to major in mathematics. Her failure in that course influenced her identification with math so strongly that she changed her major. Cheryl's sense of identity in math class was related to feelings of comfort, that she could be her funny and sarcastic self, while also succeeding in mathematics. She told me she fit in when she "could kick back while still learning math." Samantha too felt a sense of identification with mathematics in her prior classes, telling me she knew she belonged in math class because she was "really good at math." The one time she described not fitting in was in AP calculus, which she failed, noting that the focus was only on the test, not learning the mathematics. These three students talked about their sense of identifying with mathematics in conjunction with their positive feelings of competence. These experiences, and the experiences of Bahira, Jessica, and Morgan, who typically described feeling less competent in mathematics, suggest that in their past classes, students partly attributed their sense of fitting in or not fitting in to their math abilities.

Two of the three students who did not identify with mathematics at the beginning of this course, shifted the way they discussed their identification with the subject at the end of the course. Morgan valued the community in this classroom because it helped her feel not as separated from mathematics as she had previously. At the end of the course, Morgan said: "I now have an input on the math world. It is not just me and then everybody else in the math world. I am a part of it." The collaborative work she did in this course helped her change the way she labeled herself. In a similar way, Bahira came to disregard these kinds of labels at all. In her post-reflection, she wrote: "I wrote in my math autobiography that I felt that my difficulties in math were not because I was a bad student but because math was not the subject

for me. However, now I disagree with that statement." She continued, "This class really gave me the confidence to believe that despite one's struggles in math, the learning of math is not inclusive to only a select group of people." Bahira does not make a strong statement about her own math abilities, but she does now believe that anyone is capable of learning mathematics. Although Jessica did not return to discussing labels, she did write this in her post-reflection: "I find it fascinating that even though for so many years I strongly disliked math, I now have a new born passion for it." This and other discussions with her show clearly that she has positive feelings about her abilities to learn math at the end of this course.

These findings around identification with mathematics as a discipline, that came out of discussing relatedness and belonging with students, suggest that students' views of their own math abilities and their identification as "not a math person" are linked to a lack of general belonging in the math classroom. These findings also suggest that inquiry-oriented mathematics classrooms have the potential to change who students identify as capable in mathematics.

Students Valued Having the Autonomy to Pursue Their Own Mathematical Reasoning

The expectation that students pursue their own mathematical thinking, rather than follow prescribed mathematical procedures is one that is not always positively experienced by students in math class, particularly if they prefer to have teachers tell them what to do. It is possible that students could say, for example, on the psychological needs survey that they are "free to express ideas and opinions in class," but may not actually enjoy or value doing so. Prospective teachers in this inquiry-oriented Math for Elementary School Teachers course not only felt more autonomous, as indicated by the psychological needs survey, but those feelings of autonomy were positively valued by most of the focal students. Many students described lacking feelings of autonomy in their prior mathematics courses, noting feelings of frustration that there was little

freedom for or support of student generated ideas. Some students had isolated past experiences of autonomy, which encouraged them to understand the importance of being able to pursue their own mathematical thoughts and to be supported in showing their mathematical reasoning in ways that made sense for them. This course did allow all of the prospective teachers to feel self-directed in their mathematical reasoning, which gave them experiences as students of mathematics that changed the way they discussed mathematics teaching and learning in the later stages of this study.

Students' past experiences of autonomy were mixed, and I begin this section by looking into the different ways that students describe lacking autonomy in math class. Morgan, Katie, and Samantha described never feeling autonomous in their prior mathematics coursework. Specifically, Morgan described in detail how she followed the book steps on her homework when she was teaching herself mathematics at home. She lined up her paper with the book, carefully following the step by step methods prescribed in her textbook. Even in her prior course at GVCC, she talked about how she enjoyed the fact that the teacher "showed students" how to do each mathematical step, still focusing on procedures. Notably, at the beginning of this course she suggested that it "probably wouldn't work" to solve problems in ways different from what the teacher demonstrated. Katie specifically remembered lacking a feeling of autonomy as she told me she remembered teachers telling her she "didn't include all of the right steps" even though she thought "all the work is there." Similarly, even Samantha, who enjoyed her mathematics experiences, said she remembered being "forced" to learn multiple approaches to problems, not appreciating the affordances of that learning until later in life. Additionally, she described her middle and high school experiences by saying, "It was more here is a problem, here is the equation to do it, solve it." These students describe how their math courses did not provide them

opportunities to develop their own mathematical ideas, but instead, forced them to follow prescribed methods for approaching problems. For these three students, the freedom to pursue mathematical reasoning in their own ways, strongly encouraged in this inquiry-oriented class, was a brand new experience, one they took up in positive ways.

Like the above students, Jessica and Cheryl sometimes lacked feelings of self-direction in their mathematical learning. However, they did experience autonomy in some ways. Jessica described being told "do it this way" in most of her classes, but did experience some autonomy in her prior math class at GVCC. Specifically, Jessica told me that in that class she only need to show her reasoning "in a mathematical way," which contributed to her feelings of competence within that course. Cheryl talked about having autonomy over her pace of mathematics learning in her Montessori school days, but when she talked about middle school and beyond, she described most classes as consisting of "one way teaching." She expanded upon this idea, telling me she had to have "the correct answer and the way [the teacher] wanted it, with the same set up. If it wasn't the same set up, you were wrong." Her dad showed her multiple approaches at home, and her favorite teacher also supported the students to justify their own reasoning, not requiring them to follow a certain approach. This was the only time Cheryl described being willing to explain her reasoning, something that she acknowledged she always needed to work on in mathematics. All of the prospective teachers in this course had, often frustrating, past experiences of not feeling self-directed in their mathematical learning. There is a pattern evident in their descriptions of classrooms where they lacked autonomy over their mathematical thinking, which centers around not only seeing just one mathematical approach, but also being forced to follow and show exact steps predetermined by the textbook or their teacher.

In this course, there was strong consensus among the prospective teachers around the

value of feeling autonomous in their mathematical reasoning. For most students, having autonomy over their mathematical thinking helped foster their feelings of competence and belonging in mathematics. In addition, for Jessica, her own autonomy was consistently linked to her future teaching of mathematics. Notably, however, Katie did not experience autonomy as positively as the other students, expressing a desire for Scott to tell her when it was appropriate to take longer to understand something. The following paragraphs discuss the positive ways that students in this course experienced autonomy over their mathematical thinking.

Among the students that connected their feelings of autonomy to their feelings of competence and belonging, Morgan emphasized that "freedom" to work problems in ways that made sense to her in this class, encouraged her to "not be discouraged when [she] did not pick up ideas as fast as others." Here she emphasized not worrying about needing time to understand mathematics, something that could make her feel less competent or isolated in math class. In a similar sense, Cheryl valued her feelings of autonomy because in this class "one way of doing it is not the absolute," which made math class "less frustrating" for her. In this class, Cheryl focused on learning to justify her thinking, something she was unlikely to do in previous courses where only "one way" was accepted as correct. Samantha "loved" how class was "me thinking all for myself," mentioning frequently that the "thinking" was what she enjoyed most about this course. There was excitement in her voice when she mentioned that in class students "were allowed to used whatever method" they wanted, and she also acknowledged how unusual it might seem to "enjoy not having a teacher telling you." For Samantha and others, autonomy was a significant part of their enjoyment of this class, particularly as it related to their ability to demonstrate competence.

For Jessica, autonomy was also closely related to how she envisioned her own future

mathematics classroom. She told me that "this class really gives you the opportunity to really figure it out whichever way is best for you," emphasizing how this both helped her be able to explain her understanding and to realize that "math class doesn't have to be a boring lecture." As a result, she wanted to learn more math, indicating new willingness to put in significant effort to learn mathematics in order to be the teacher she wanted to be. Other students also talked about the influence their feelings of autonomy had on their forming teaching practices, but they did so only when we talked about their teaching, while Jessica made this connection more readily, not distinguishing her own experience from her ideas about teaching. Other students' ideas about how autonomy relates to their teaching will be explored further in a later section.

Interestingly, although Katie also valued how "different strategies were all okay" and that "seeing things in your own way" was supported, she was also somewhat hesitant about this idea. Even at the end of the course, she talked about needing "permission to not have to immediately grasp" the ideas that were being covered in class. This new freedom to work at her own pace to generate her own math ideas is still somewhat uncomfortable to Katie at the end of this course. Each of the prospective teachers in this course valued the freedom they were expected to take up in order to develop and use solution strategies and mathematical reasoning that made sense to them. However, despite the fact that this was one of the most consistently upheld norms of this inquiry-oriented class, not all of the students left this course feeling completely comfortable with the productive struggle that this type of student engagement fostered. Notably, for Katie, the one student with a negative experience of autonomy in this study, this may relate to feeling a lack of competence unless it is explicit that a problem is challenging and may not be immediately grasped. Though, I would argue that in this class, Scott always intended for students to engage deeply with mathematics and that time limits for understanding were rarely, if ever, imposed.

Bahira's experiences of autonomy in this course stood out differently from the other focal students who planned to become teachers, and deserves some consideration here. She did acknowledge that Scott promoted freedom for students to pursue problem solving in their own ways, telling me, "Rarely is there a time when [Scott] says there is only one way to solve this problem, and I want you to show me only that one way." However, she did not feel a strong sense of autonomy in her work in this class. In particular, she felt "expected to use manipulatives," which in her mind directed how she could solve problems. In describing her past experiences, Bahira told me she always felt self-directed in her learning, so this sense of restriction could come from the fact that she was used to feeling autonomous in her math classes. In addition, her decision not to teach could be influencing her desire not to use manipulatives.

Most of the prospective teachers in this Math for Elementary School Teachers course were positively influenced by their feelings of autonomy, which were fostered by the inquiry-oriented nature of the classroom. The students describe valuing the freedom they were allowed to pursue their own ideas about mathematics, working in groups to consider student generated solution strategies, and co-constructing the mathematical knowledge they were learning within the course. In addition, this sense of autonomy played a large role in the prospective teachers' discussions of their ideas about what mathematics instruction can and should look like, notably influencing the ways in which they discussed their developing teaching practices. This influence will be discussed further in a following section.

The above cross-case analysis findings build on the quantitative survey findings, which revealed that students' psychological needs were more satisfied in this inquiry-oriented course than in their previous mathematics courses. The case studies allow for detailed articulation of the ways in which students took up those affective experiences, and how those experiences were

situated within the inquiry-oriented nature of the course. Experiences of competence, relatedness, and autonomy influenced the students in generally positive ways, improving their confidence in mathematics, fostering a new sense of belonging, and encouraging their mathematical thinking and development. Students' affective experiences of this course also influenced their developing teaching practices in two important ways: (1) students felt more positive about teaching mathematics after participating in an inquiry-oriented math course; and (2) experiences within this inquiry-oriented classroom fostered prospective teachers' desires to support student inquiry in their future mathematics teaching practices. In the next section I will elaborate these findings, which are inextricably linked to the participatory and affective experiences of the prospective teachers in this course.

Students Felt More Positive About Teaching Mathematics After Participating in an Inquiry-Oriented Math Course

The prospective teachers[21] in this study entered this mathematics course with varying degrees of confidence in their ability to teach mathematics and varying enthusiasm for teaching the subject. After participating in this inquiry-oriented math course, each focal student talked about having either more confidence in their ability to teach math or more enthusiasm for teaching math. The students that entered the course with confidence in their ability to teach math left the course talking about their higher confidence in their math teaching abilities, while those students who entered feeling unsure about teaching math expressed enthusiasm or excitement for teaching math at the end of the course.

The two students who entered the course with confidence in their teaching abilities,

21 In the following discussions I generally do not include Bahira when I say "the prospective teachers" since she had decided not to teach by the time we really discussed future teaching, and therefore did not discuss future teaching with me in interviews. However, I do still discuss her feelings and experiences when appropriate.

Cheryl and Samantha, are the two students who described having the most positive experiences and feelings of competence in their past mathematics courses. Cheryl identified more as a teacher generally speaking, while Samantha particularly emphasized that she thought she would be a good math teacher. Despite their already high confidence, both Cheryl and Samantha expressed increased confidence in their teaching abilities at the end of this course. For Cheryl, this increased confidence focused on her improved ability to explain her mathematical reasoning. During the quarter, she worked particularly hard to use the group work in the class to develop this skill, as she knew it was important for teaching. She said, "I have learned how to explain how I come about an answer and how I can better explain it to a peer or a child." Samantha said that her confidence was "boosted" in this course and she was particularly enthusiastic about opportunities she had had to "be a teacher" to classmates. Samantha's positive personal experience within this class confirmed her preexisting notions that inquiry-oriented learning was "best for students," so it is no surprise that at the end of the course she was even more enthusiastic about teaching, if possible. Both Cheryl and Samantha talked specifically about how inquiry-oriented aspects of the course, like explaining to others, led them to feel even more confident in their math teaching abilities at the end of this course.

The other four students in this study entered the course with various reasons they were hesitant about teaching mathematics, ranging from concern about their dislike of math to not knowing how to teach math. At the end of the course, each of these students talked more positively about teaching mathematics. Their comments are linked to both increased confidence in their own general math abilities, and also focus on a new passion or excitement for teaching mathematics that was previously not present.

Morgan entered this course "loathing" mathematics, worried she was not "good enough at

mathematics to teach it," and deeply concerned that her future students would dislike mathematics because of her "distaste for the subject." In the beginning she was motivated in this course by that concern. Over time, however, Morgan gained confidence in her math abilities. She talked about being able to work through her "frustrations," noting that the community within the classroom helped her feel "comfortable" and that the "freedom" she felt to explore her own ideas increased her confidence in her own math abilities. At the end of the class, she wrote in her post-reflection: "But now, I am excited to teach and share math with others." This shows a significant shift in her enthusiasm for teaching mathematics.

Like Morgan, Jessica entered the course with a negative history with mathematics, but had slightly more confidence in her potential as a teacher, writing in her autobiography, " I feel as though I will be a great educator." Specifically, Jessica was hoping for "another perspective" on math teaching different from "just lecturing." Like her confidence in her own math abilities, Jessica's confidence in her teaching grew quickly within this course. By our second interview, Jessica was already reflecting on suggestions she had made to a practicing teacher. Although she was just volunteering, and it was still early in this course, Jessica had the confidence to use ideas from class, particularly around manipulatives, in another teacher's classroom. At the end of the study, Jessica had undergone a transformation, emphasizing wanting to be a teacher "leader" and a "burning desire" to learn more mathematics.

While Bahira decided not to become a teacher during this course, she has an interesting perspective to consider here. While she is now sure she does not want to become a teacher, of any subject, she is clear in our interviews and her post-reflection writing that that decision was not based on lack of confidence. Rather, her confidence in her math potential did increase in this course, as discussed earlier in this chapter, but she made the decision not to teach because she

lacked the "passion" for it that she saw in her classmates from Math for Elementary School Teachers and another education course she was taking. Despite deciding that she not did want to pursue teaching, Bahira spoke passionately about using the ideas she had learned in this course to help her younger family members enjoy and experience success in mathematics. She told me that this class had exposed her to "different approaches" to teaching mathematics than the traditional experiences she had as a child, and had taught her "effective" strategies for working with kids learning mathematics. For these three students who entered this course with less confidence or enthusiasm for teaching mathematics, this course served to provide opportunities to increase their own general confidence in their mathematical abilities, which in turn helped foster excitement and confidence for teaching math to children.

Katie stands out a bit differently in this discussion. While she had a positive experience in this course, describing the course often as "wonderful," and decided to pursue a math endorsement with her teaching credential, her confidence is heavily loaded with worry. I found this surprising given the success and respect she earned within this class. When we talked about her future teaching, Katie focused only on her concerns around finishing the math courses she needs to get credentialed. She referred to Scott as a "confidence building crutch," which suggests that she is not confident in her math abilities without Scott, even wishing he could teach the rest of the math classes she needs. While I cannot know for sure why Katie links her confidence in her future math classes to the course instructor, given the data I have from her it does seem likely that her past calculus experience is playing a role. She attributed both her failure and her success in calculus to the way the material was presented, which was controlled by the instructor. At the end of this course she does not express confidence or a lack of confidence in her ability to teach mathematics, she only focuses on her ability to learn

mathematics to complete her teaching certification.

Other students did express concerns about teaching at the end of the study, but these concerns were actually a result of feeling more confident in teaching mathematics. Notably, Samantha worried that she will get "frustrated" with students who do not understand, and struggle to show them the "patience" they need from her to learn. Cheryl expressed concern around being allowed by administrators to carry out inquiry-oriented methods in her classroom. Although Jessica does not directly link this to her teaching, she expressed concern around her strong leadership during group work, regretting "interjecting" when other members of her group were sharing. These are some examples of how that confidence in mathematics and mathematics teaching ability started to present the prospective teachers with important new challenges.

Looking across the student experiences within this inquiry-oriented math content course for teachers, it is clear that each student had changes in their confidence or enthusiasm related to mathematics teaching (see Table 22). Two of the students, Cheryl and Samantha, were already confident in their abilities to teach, and this course served to increase that confidence even more as they were given opportunities to practice explaining ideas to other students. Importantly, even students like Morgan and Jessica, who entered the class with very strong negative experiences in mathematics, left this course with excitement and confidence, respectively, around teaching mathematics to children. For initially less confident students, subsequent increases in confidence were linked to increased feelings of their own competence in math. While there are still concerns in the prospective teachers' minds, like getting frustrated with students, I argue that the inquiry-oriented classroom fostered generally positive feelings around teaching math and that the fact that students are bringing these concerns to light shows that they are already actively reflecting on their teaching abilities and practices, something that will hopefully serve them

positively as they move forward in their teaching development.

Table 22

Changes in Students' Feelings about Teaching Mathematics

	Previous math experiences	Feelings about teaching math	
		Beginning of course	End of course
Jessica	Negative	Only knew did not want to lecture.	Passionate about teaching, sees herself as future leader. Confident using ideas from class at classroom observation sites.
Morgan	Negative	Worried she will transfer her dislike of math to students.	Excited to teach math, despite the fact that she still does not love it.
Bahira	Neutral	Unsure whether she wants to teach.	Sure she does not want to teach. Excited to use what she learned to help younger family members.
Katie	Mixed	Open to the potential benefits of inquiry-oriented teaching methods.	Positive, but worried about her own ability to pass future math classes.
Cheryl	Positive	Confident before taking this course.	More confident in her ability to explain her reasoning.
Samantha	Positive	Excited to teach, feels especially confident about teaching math.	Excited, even more confident, worries about having patience with struggling students.

Experiences Within This Inquiry-Oriented Classroom Fostered Prospective Teachers' Desires to Support Student Inquiry in Their Future Mathematics Teaching Practices

By the end of their time in this inquiry-oriented Math for Elementary School Teachers course, each focal student articulated some key characteristics that they wanted to have in their future mathematics classroom. In particular, every prospective teacher mentioned wanting their students to be involved in the knowledge construction that would occur during class, having space to generate their own ideas and pursue their own solution strategies, a key characteristic of

an inquiry-oriented classroom. More specifically, the autonomy that these prospective teachers experienced within this inquiry-oriented course strongly affected how they thought about the mathematics classroom environment, and what they expressed wanting for their future students' mathematics experiences.

Although the words the prospective teachers used to talk about this idea of fostering autonomy differed, the notion of allowing students freedom to think in their own ways and play an active role in the construction of mathematical knowledge, as opposed to passively receiving knowledge from a teacher, was at the heart of each prospective teacher's discussion about their future teaching. For example, Morgan said students should "come to their own conclusions," while Katie referred to students being "encouraged to contribute." Their comments focused on giving students the space to generate their own ideas and contribute those ideas to the class. The other prospective teachers spoke similarly, but put slightly more emphasis on encouraging students to use multiple approaches, emphasizing allowing freedom for exploration of a variety of ideas. Samantha said students should "not feel obligated to do it just one way," Jessica mentioned letting students use "whichever way is best," and Cheryl talked about how she would "encourage multiple ways" to solve problems.

The prospective teachers' emphasis on fostering autonomy for student engagement with mathematics was linked to their experiences of both competence and relatedness within this inquiry-oriented course. In particular, several of the students made connections between their sense of autonomy and their feelings of competence. For example, Morgan talked about how being "guided" rather than "following rules" was something that reinforced her retention of material. Jessica expressed this connection in a different way, emphasizing that having the freedom to pursue her own mathematical ideas fostered her sense of perseverance, and this was

key for her own competence because she realized that she could be successful in math if she put in effort. Katie also linked autonomy for her students to their competence, telling me that allowing students to contribute would help them feel like they could "understand," which was how she felt in this course.

In addition, some of the students made connections between their sense of autonomy and their feelings of belonging. Samantha spoke passionately about how the class was "a team," emphasizing how working together with other students was something that helped foster the "thinking" she consistently "loved" about this course. In a similar way, Cheryl emphasized how group work helped her learn to explain her own thinking. She had already been used to having some autonomy to pursue her own solution strategies, but the community in this class combined with that autonomy is what helped her learn to justify her reasoning in writing or words.

At the beginning of the study, not all students could articulate specifics about what they wanted their mathematics classroom to be like. However, some of the prospective teachers did express ideas they had about their future teaching practice. These initial thoughts on their future teaching varied widely, and appeared to largely be shaped by particular students' personal experiences with mathematics. The shifts in the ways the prospective teachers talked about their teaching practices over the course of the study are worth further exploration, and are summarized in Table 23.

At the beginning of this course, Katie discussed wanting to "present" multiple solution strategies to her future students. This desire related to her personal experiences of failing, and later passing, calculus, which she attributed to the way the material was presented by the instructor. In our final interview, she said she wanted her classroom to be a place where students "are encouraged to contribute and that their contribution was valued" going on to say that this is

"ideal" "versus sitting there and getting the information presented to you." By the end of the course, her focus had shifted significantly, suggesting she now valued student contributions and no longer talked just about presenting material *to* students.

Table 23

Prospective Teachers' Characterizations of Their Future Mathematics Classrooms

	Characterization of their future math classroom	
	Beginning of course	End of course
Jessica	Wants students not to be dependent on the teacher for success.	Wants to support productive struggle, student generation of solution strategies.
Morgan	--	Wants students to come to their own conclusions.
Katie	Plans to present multiple solution methods *to* students.	Wants students involved in the learning process.
Cheryl	Wants learning math to be fun for students.	Wants to support student inquiry, but expresses need to also follow textbook procedures.
Samantha	Believes inquiry-oriented methods to be best for student learning.	Value of inquiry methods confirmed, wants students to think for themselves, and work together.

All of the students were at least open to idea of inquiry-oriented learning, after watching the videos and writing about the idea in their autobiography, and none expressed concern about the nature of the inquiry-oriented course itself. Although I cannot say for sure, it seems that this openness might have been fostered by students' past negative experiences with other teaching approaches. It is also notable that a few of the students already knew they did not want to "just lecture," as Jessica put it, before they entered this course. Specifically, Jessica talked about wanting to encourage her students to "be independent from the teacher," indicating that she

wanted them to have a strong sense of "perseverance" so they could succeed in mathematics even faced with a teacher that was not very good. This emphasis came directly from her own experiences within classrooms where teachers had low expectations and she put in little effort as a result, and it was my sense that she wanted students to be somewhat protected from similar situations. However, at the end of the course, Jessica had more concrete ways to talk about how she would support her students, emphasizing the "deeper understanding" that comes from productive struggle and wanting students to use "whichever way is best for them."

Going even further, Samantha came in to this course already believing that "inquiry methods work best for learning," which was in direct relation to her experiences with inquiry-oriented learning as a child. She had vivid memories of being actively engaged with mathematics, and knew she remembered material more from such experiences. Samantha's experiences within this course confirmed those ideas, as she had very positive experiences "thinking" for herself during the course. Samantha is also one of the focal students that emphasized the role group work played in developing that thinking, telling me her classroom would have "a lot of students working with students." In Samantha's case, it seems that inquiry-oriented experiences as a student significantly solidified her desire to use inquiry-oriented methods in her own teaching.

Cheryl was only able to tell me that she wanted to make learning math "fun" for students at the beginning of this course. Interestingly, she was the only student that mentioned that her future classroom would not be as "unstructured" as this class, talking about how she would expect her students to generate their own ideas and work together, but also to learn to follow procedures from the textbook. For Cheryl, this was linked to requirements she had seen placed on teachers at her practicum site, as she told me about how they were required to have their

lesson plans and timing written out and approved. Despite this, she wrapped that conversation up by saying that she was "kind of a rebel" and would be willing to do what was best for students and use students' positive learning results to help convince administrators that inquiry-oriented methods were worth using.

Despite the backgrounds that students entered the course with, or the way they viewed teaching at the beginning of the term, each of the focal students articulated an emphasis on actively involving their future students in their own learning of mathematics at the end of this study. Some shifted away from considering lecture-based teaching and some confirmed their strong beliefs in inquiry-oriented mathematics. Regardless, leaving this course, the prospective teachers are focused on creating inquiry-oriented classroom spaces where students are not receiving knowledge directly from a teacher, but are engaged in generating their own ideas about mathematics, justifying their thinking, and pursuing their own solution strategies.

Discussion and Implications

The above cross-case analysis helps make sense of the affective experiences of competence, relatedness, and autonomy that prospective teachers were feeling during an inquiry-oriented Math for Elementary School Teachers course. Additionally, the analysis helps make sense of how those affective experiences influenced the prospective teachers' developing mathematical teaching practices. Three major findings around prospective teachers' psychological needs satisfaction in mathematics were discussed: (1) students expanded the ways in which they felt competent in mathematics while participating in an inquiry-oriented course; (2) students' new sense of relatedness in math class stemmed from the community fostered within an inquiry-oriented course; and (3) students valued having the autonomy to pursue their own mathematical reasoning. Students in this study often had traditional experiences of

competence, like test scores, speed, or getting correct answers. During this course they showed and felt competence in a variety of ways, such as by discussing ideas with others, asking questions, and revising their thinking, to name just a few. Relatedly, the sense of community that grew within this inquiry-oriented classroom was particularly influential in fostering students' sense of belonging. For many, it was the first time they had worked with others in a mathematics course, and for some it provided a sense of belonging that they had always lacked in math class. Finally, and arguably most importantly to the students, there was a sense of autonomy, or freedom, in this classroom, that not only allowed, but encouraged students to see themselves as mathematical authorities, worthy of contributing ideas, pursuing solution strategies, and generating mathematical knowledge.

Self-determination theory asserts that if students' psychological needs to feel competent, related, and autonomous are met, the students should be more intrinsically motivated (Ryan & Deci, 2000). As discussed above, both the quantitative and qualitative findings in this study do suggest that students' needs were satisfied in this course. Additionally, there is evidence that students were intrinsically motivated to engage with mathematics in this course. For example, students often gathered to work on mathematics with other students from class outside of class time. On all days when I was at GVCC for observations or interviews, close to half the class was usually gathered around tables near the classroom, typically talking about topics or assignments from class. This appeared as more of a social time to engage casually with the course content, talking about how they approached certain problems from the homework or a lingering question from class. In addition, I usually saw students gathered after class more formally working on homework together. They appeared to intentionally chose with frequency to remain immediately after the already daily class sessions to work together on mathematics. There were also times in

class that students showed intrinsic motivation to engage with course content. For example, during the Smallest Difference activity, Samantha was reluctant to leave when class was over, continuing to work the problem. She also told me during interviews that she text messaged that problem to her friends and family to work on with her, and other students mentioned engaging with that task outside of class as well. Overall, students did appear to be engaging with mathematics on a level that seemed to go beyond what was needed for basic success in the course, suggesting they were intrinsically motivated to learn mathematics in this class.[22]

This analysis also explored two ways that students' affective experiences within this course appeared to promote their developing teaching practices: (1) students felt more positive about teaching mathematics after participating in an inquiry-oriented math course; and (2) experiences within this inquiry-oriented classroom fostered prospective teachers' desires to support student inquiry in their future mathematics teaching practices. In particular, the classroom environment was a place where students came to feel confident as learners *and teachers* of mathematics, practicing and developing their ability to explain their reasoning and answer questions of fellow students. At the end of the study, the prospective teachers were able to articulate what they hoped their teaching practices and future mathematics classrooms would look like, particularly emphasizing their desire to support student autonomy as it had been supported for them in this course.

Students' affective experiences were closely linked to the participatory and cognitive demands of the course. Students were expected to engage with mathematics in demanding ways, that were new to most members of the classroom, and although those expectations were high, it

22 Because these students are future teachers, they may already be motivated to engage in this class. As a result, I try to limit my examples to instances when students appeared motivated to engage with the course content, rather than motivated by their excitement around teaching math.

appears that the course culture promoted satisfaction of students' need to feel competent, related, and autonomous. In telling the transformative experiences that students had during this course in their own words, I hoped to particularly highlight the often missing link between students' cognitive and participatory experiences and their affective experiences. Understanding students' affective experiences has given a more complete picture of the affordances of inquiry-oriented learning, going beyond understanding the cognitive benefits of student engagement with math in such classrooms to understanding how an inquiry environment can affect prospective teachers' psychological relationships with mathematics and their developing teaching practices.

Limitations

A few limitations of this study are important to acknowledge. One limitation of this study is that it focused primarily on students' experiences in the inquiry-oriented mathematics classroom. Interviews with Scott revealed that he thought deeply about both the design and benefits of inquiry-oriented instruction for teachers. He also thought consciously about students' affective experiences within his course. It could be fruitful to consider teachers' perspectives on the affective experiences that students have in inquiry-oriented courses and how those experiences might or might not influence their teaching of the course. In addition, this study is limited to undergraduate students in mathematics content courses designed for future teachers. It is possible that students in such courses have different experiences from students in mathematics content courses not specialized for future teachers as students planning to become teachers may have more invested in learning mathematical content directly related to their future careers compared to students taking introductory mathematics courses to satisfy college requirements, or students taking mathematics courses for other career paths, such as pre-medicine students or mathematics majors. Finally, this study was not designed to be longitudinal. It is possible that

students were unable to foresee or articulate how their affective experiences in this inquiry-oriented mathematics course will affect them in the future. It is also likely that their notions of high quality teaching and learning of mathematics will continue to develop throughout their teacher training. As these focal students continue on through their training to become elementary school teachers and eventually take on their own classrooms, it would be of interest to continue exploring how their experiences as a students in inquiry-oriented mathematics classrooms influences their teaching practices.

Future Directions

As noted in the previous section, limitations of this study suggest future directions for this research. In addition, some of the findings articulated within the study may warrant further investigation as well. First, focusing more specifically on the experience and role of the instructor in this type of inquiry-oriented class may reveal insight into the value the instructor places on prospective teachers' affective experiences in learning mathematics. Here, Scott told me how important it was to him that future teachers had opportunities to get past their fears around mathematics. For Scott, this is linked to the effectiveness of teachers in the elementary school classroom. One reason he told me he uses inquiry-oriented teaching in classes for future teachers is to help with his concern that teachers fear mathematics. He wants them to gain comfort in asking questions and engaging with mathematics. Future study of the instructor's perspective on students' affective experiences could articulate whether students' experiences of the course align with how prepared the instructor sees the future teacher from an affective perspective. In addition, conversations with the instructor about specific students could further our understanding of how, or whether, teachers attempt to directly influence individual students' affective experiences of their courses.

Longitudinal study of the focal students in this research could provide opportunities to follow up on interesting affective experiences that surfaced during the original study, particularly around how students move forward in their learning of mathematics and inquiry-oriented teaching and learning. For example, in this study, Katie left the course with a positive attitude about teaching elementary mathematics, but had lingering concerns around her own mathematical abilities. As she moves ahead in her teacher training, she already knows she has more math classes she needs to take. As she engages in these classes, will she experience the courses positively, as she did here, or have another negative experience like her college calculus course? In addition, the larger issue that arises from Katie's story is that the success she experienced in Math for Elementary School Teachers was, from her perspective, a result of the circumstances of this course, circumstances that may or may not be present in her future mathematics coursework. Although other students in the course and I saw Katie as successful because of the effort she put in to the course, she appeared to not attribute her success to her own effort, suggesting frequently that she needed a certain environment or instructor to replicate her success here. As such, there are questions to be investigated. Under what circumstances do students perceive inquiry-oriented learning as a general way of engaging with mathematics that they can utilize in any situation? What can an instructor do to ensure that students see inquiry-oriented learning as a general way to engage with mathematics, rather than only as an instructor imposed structure?

A second interesting case that implores follow up is Jessica. Jessica left the course feeling very positive about her own mathematical abilities, but during this course, she failed to work successfully in groups to encourage other students' autonomy. This is concerning since Jessica plans to become a teacher, and will be encouraged to teach in inquiry-oriented ways. At

this point, it is not clear whether Jessica will be able to give her own students autonomy over their mathematical thinking, even though she valued it as a student herself. How can instructors of future teachers encourage their students to utilize inquiry both as students of mathematics and as teachers of mathematics?

The findings around competence in this study are very rich, with many places for further exploration of students' experiences of competence, particularly in an inquiry-oriented math course. For example, both Morgan and Bahira told me they felt more capable in mathematics as a result of working with other students in class. In addition, they both told me that just because they feel more capable in mathematics, does not mean they enjoy mathematics more. However, Morgan still wants to teach elementary school, and is actually excited to teach mathematics, while Bahira has decided not to teach. What factors went in to each students' decision? What makes Morgan want to teach mathematics even though she does not enjoy it? What types of relationships exist between feeling competent in mathematics and enjoying mathematics?

Additionally, this study revealed that some students felt competent when they could explain mathematics to others. While I make some effort to parse when this is related to feeling capable in mathematics as opposed to feeling capable as a teacher of mathematics, there is additional work that could be done here. The students in this study plan to become elementary school teachers, and this course is focused on that goal. It would be interesting to consider this intricacy in more detail in an inquiry-oriented math course not for teachers. That is, do students in inquiry-oriented math courses generally feel competent when they can explain or justify their reasoning, a major component of an inquiry-oriented course, or is this finding a product of my subjects being future teachers of mathematics?

In this study it was also difficult to separate feeling competent, defined in

self-determination theory as feeling capable of doing what is required for certain outcomes, from demonstrating competence, which may be interpreted as showing what you know (in part because of how interview questions were sometimes asked). That is, it is possible that students shared times when they felt they could outwardly demonstrate what they knew, rather than when they felt capable of doing what was required (even if they could not or did not necessarily demonstrate that capability). It is possible that more detailed analysis of competence within this data and study of literature around mathematical competence can further elucidate this distinction. The students and I also sometimes talked about what it meant to be competent in math class or how they knew what it meant to be capable. Further data analysis could explore in more detail how students knew what was needed to be capable and who decides what counts as capable in math class.

A fourth line of future investigation involves considering more closely how and when students identify with mathematics as a discipline, and the differences between social belonging, or relatedness, and this identification. While it was not surprising that some students told me they were "not a math person," I was surprised that these labels came up during our discussions of belonging in math class, as I was originally focused on the social sense of belonging as used in the definition of relatedness from self-determination theory. As a result, further analysis or study might help elucidate whether they felt a sense of relatedness while not identifying with mathematics as a discipline, or whether not identifying with mathematics precluded opportunities to feel socially related in math class. Through my research on relatedness, I found that inquiry-oriented experiences changed how some students, such as Morgan, labeled themselves as no longer "not a math person." In addition, Bahira changed her notion that people could even be labeled in that way at all, suggesting that those categories are no longer relevant. Knowing now

that inquiry-oriented student experiences can influence identification with the discipline, future research in inquiry-oriented math classrooms could focus more specifically on when and how students in inquiry-oriented classrooms identify with mathematics, how they determine whether someone is or is not a math person, and whether they believe such labels should even exist[23]. This study of students' experiences of the inquiry-oriented math class has revealed many further researchable questions.

Significance

Inquiry-oriented mathematics classrooms appear to promote fulfillment of students' need to feel competent, related, and autonomous, though little qualitative research has investigated how students affectively experience inquiry-oriented classrooms. The conceptual framework outlined here suggests one way to explore students' experiences of inquiry-oriented mathematics classrooms, while also making connections between the cognitive and participatory experiences of students in these classrooms and their affective experiences. Understanding more about how students affectively experience inquiry-oriented math classrooms helps articulate what students might gain psychologically from participating in such classrooms, which will contribute to the literature motivating the use of inquiry-oriented teaching and learning approaches.

This study also extends traditional self-determination results, which use quantitative research methods to identify correlations between classroom environments and an individual's perceived competence, relatedness, and autonomy. That is, in addition to asking questions about *whether* certain classroom environments fulfill students' psychological needs, I asked *how* and *why* questions that focused on the ways in which students experienced competence, relatedness, and autonomy within a classroom intended to foster the satisfaction of those needs. This helps

23 These labels and questions are closely linked to fixed and growth mindset work done by Dweck (2006).

articulate how students themselves characterized their affective experiences in the classroom and what those experiences meant to students.

As noted previously, inquiry-based teaching methods are still emerging at the undergraduate level (Walczyk & Ramsey, 2003). I found this to be true based on my focal students' experiences, and I was able to capture students' experiences in one of their first exposures to college-level inquiry-oriented teaching, and for many students, their first exposure to inquiry-oriented teaching altogether. As a result, this study provides insight into how students understand the differences between traditional and inquiry-oriented mathematics experiences, particularly if they have taken traditionally taught mathematics courses prior to, or during, their college careers.

Carefully elucidating students' affective experiences, connecting those experiences to students' participatory and cognitive experiences, and doing so from students' own perspectives, has two clear benefits. First, inquiry-oriented classrooms have the potential to foster renewed, positive dispositions towards mathematics. Students in this study, even those with deep seeded negative experiences of mathematics, experienced the inquiry-oriented classroom in an influentially positive way, not only enjoying the course, but articulating clearly the numerous positive experiences they had and how those feelings influenced their participation in and experience of this Math for Elementary School Teachers course. Second, these affective experiences contribute to the shaping of prospective teachers' mathematics teaching practices. Experiencing an inquiry-oriented classroom as a student exposes prospective teachers to a teaching approach that may be new to them, which may help them articulate and justify how they specifically hope their own students will experience mathematical learning in their future math classroom.

Inquiry-oriented mathematics classrooms dramatically change the role of the student in the mathematics classroom, and therefore have the potential to change students' perceptions of both mathematics and themselves, facilitating transformative student experiences, such as those described here. Documenting the transformative experiences that students, particularly those identifying as prospective teachers, undergo during enrollment in an inquiry-oriented mathematics course and how students make sense of that transformative process was the major goal of this study. Characterizing these experiences in students' own words, from their own perspectives, allows for a deep exploration of prospective teachers' changing relationships with mathematics and insight into the development of their mathematics teaching practices.

References

Alvine, L. B. (2001). Shaping the teaching self through autobiographical narrative. *The High School Journal*, *84*(3), 5-12.

Assessment & Evaluation Center for Inquiry-Based Learning in Mathematics (2011). *Evaluation of the IBL Mathematics Project: Student and Instructor Outcomes of Inquiry-Based Learning in College Mathematics*. (Report to the Educational Advancement Foundation and the IBL Mathematics Centers) Boulder, CO: University of Colorado, Ethnography & Evaluation Research.

Ball, D. L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics*, *8*(1), 40-48.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching what makes it special?. *Journal of Teacher Education*, *59*(5), 389-407.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematical worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 45-82). Stamford, CT: Ablex.

Calderhead, J. (1991). The nature and growth of knowledge in student teaching. *Teaching and Teacher Education*, *7*(5), 531-535.

Deci E.L, Vallerand, R.J., Pelletier, L.G., & Ryan, R.M. (1991). Motivation and education: The self-determination perspective. *Educational Psychologist, 26*, 325-346.

Dewey, J. (1897). My pedagogic creed. *School Journal, 54*, 77-80**.** Retrieved from http://dewey.pragmatism.org/creed.htm

Dewey, J. (1916/1994/1997). *Democracy and education: An introduction to the philosophy of education.* New York, NY: Free Press.

Dweck, C. (2006). *Mindset: The new psychology of success*. New York, NY: Random House.

Emerson, R.M., Fretz, R.I., & Shaw, L.L. (1995). Processing fieldnotes: Coding and memoing. In R.M. Emerson, R.I. Fretz, & L.L. Shaw, *Writing ethnographic fieldnotes* (pp. 142-168). Chicago, IL: University of Chicago Press.

Franke, M.L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F.K. Lester, Jr., (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). Charlotte, NC: Information Age Publishing.

Gresalfi, M. S. (2009). Taking up opportunities to learn: Constructing dispositions in mathematics classrooms. *The Journal of the Learning Sciences*, *18*(3), 327-369.

Hintz, A. (2011). Understanding students' experiences as listeners during mathematical discussion. *Canadian Journal of Science, Mathematics and Technology Education, 11*(3), 261-272.

Katz, I., & Assor, A. (2007). When choice motivates and when it does not. *Educational Psychology Review, 19(4)*, 429-442.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary School Journal, 102*, 59-80.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29-63.

Luria, A. R. (1979). Cultural differences in thinking. In M. Cole & S. Cole (Eds.), *The making of mind* (58-80). Cambridge, MA: Harvard University Press.

Martino, A.M., & Maher, C.A. (1999). Teacher questioning to promote justification and generalization in mathematics: What research practice has taught us. *Journal of*

Mathematical Behavior, 18(1), 53-78.

McLeod, D.M. (1992). Research on affect in mathematics education: A reconceptualization. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 575-596). New York, NY: Macmillan.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco, CA: Jossey-Bass.

Miles, M., & Huberman, M.A. (1994). *Qualitative data analysis: A sourcebook of new methods* (2nd ed.). Beverly Hills, CA: SAGE.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards*. Washington, DC: Authors.

Nolen, S. B., & Ward, C. J. (2008). Sociocultural and situative approaches to studying motivation. In M. L. Maehr, S. Karabenick & T. Urdan (Eds.), *Social psychological perspective on motivation and achievement* (Vol. 15, pp. 425-460). London: Emerald Press.

Nolen, S.B., Ward, C.J., & Horn, I.S. (2011). Motivation, engagement, and identity: Opening a conversation. In D.M. McInerney, R.A. Walker, & G.A.D. Liem (Eds.), *Sociocultural theories of learning and motivation: Looking back, looking forward* (pp. 109-135). Charlotte, NC: Information Age Publishing.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307-332.

Patrick, H., Turner, J., Meyer, D., & Midgley, C. (2003). How teachers establish psychological environments during the first days of school: Associations with avoidance in mathematics. *The Teachers College Record*, *105*(8), 1521-1558.

Patton, M. Q. (2003). *Qualitative research and evaluation methods* (3rd ed.). Thousand Oaks, CA: SAGE.

Philipp, R.A. (2007). Mathematics teachers' beliefs and affect. In F.K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Charlotte, NC: Information Age Publishing.

Polya, G. (1954). *Induction and analogy in mathematics*. Princeton, NJ: Princeton University Press.

Reeve, J.M., & Halusic, M. (2009). How K-12 teachers can put self-determination theory principles into practice. *Theory and Research in Education, 7*(2), 145-154.

Rogoff, B. (2003). Chapter 1: Orienting concepts and the ways of understanding the cultural nature of human development. In *The cultural nature of human development* (3-24). New York, NY: Oxford University Press.

Ryan, R.M. & Deci, E.L. (2000). Self-determination theory and the facilitation of intrinsic motivation, social development, and well-being. *American Psychologist, 55*(1), 68-78.

Ryan, R.M. & Deci, E.L. (2009). Promoting self-determined school engagement: Motivation, learning, and well-being. In K.R. Wentzel & A. Wigfield (Eds.), *Handbook of motivation at school* (pp. 171-196). New York, NY: Routledge.

Sherin, M.G. (2002). A balancing act: Developing a discourse community in a mathematics classroom. *Journal of Mathematics Teacher Education, 5*, 205-233.

Speer, N.M., Smith, J.P., & Horvath, A. (2010). Collegiate mathematics teaching: An

unexamined practice. *Journal of Mathematical Behavior, 29*, 99–114.

Turner, J.C., Warzon, K.B., & Christensen, A. (2011). Motivating mathematics learning: Changes in teachers' practices and beliefs during a nine-month collaboration. *American Educational Research Journal, 48*(3), 718-762.

Vansteenkiste, M., Simone, J., Lens, W., Sheldon, K.M., &, Deci, E.L. (2004). Motivating learning, performance, and persistence: The synergistic effects of intrinsic goal contents and autonomy-supportive contexts. *Journal of Personality and Social Psychology, 87(2), 246-260*.

Walczyk, J.J., & Ramsey, L.L. (2003). Use of learner-centered instruction in college science and mathematics classrooms. *Journal of Research in Science Teaching, 40*, 566-584.

Walczyk, J.J., Ramsey, L.L., & Zha, P. (2007). Obstacles to instructional innovation according to college science and mathematics faculty. *Journal of Research in Science Teaching, 44*, 85-106.

Webb, N. M., Franke, M. L., Chan, A., De, T., Freund, D., & Battey, D. (2008). The role of teacher instructional practices in student collaboration. *Contemporary Educational Psychology*, *33*(3), 360-381.

Wertsch, J. V. (1988). L. S. Vygotsky's "new" theory of mind. *American Scholar, 57,* 81-90.

Yackel, E. (2002). What we can learn from analyzing the teacher's role in collective argumentation. *Journal of Mathematical Behavior, 21*, 423-440.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Zan, R., Brown, L., Evans, J., & Hannula, M.S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63*(2), 113-121.

Zavala, M.R. (2012). *Race, language, and opportunities to learn : the mathematics identity negotiation of Latino/a youth.* (Doctoral dissertation). University of Washington, Seattle, WA.

Zeichner, K. M., Tabachnick, B. R., & Densmore, K. (1987). Individual, institutional, and cultural influences on the development of teachers' craft knowledge. In J. Calderhead (Ed.). *Exploring teachers' thinking* (pp. 21-60). London: Cassell.

Appendix A: Initial Instructor Interview Protocol

1. Can you describe what inquiry-oriented teaching and/or learning means to you?
   - What makes a course inquiry-oriented?

2. How is your current course inquiry-oriented?
   - What is your role as the teacher in an inquiry-oriented course?
   - What do you expect of students in an inquiry-oriented course?

3. Describe a typical day in your Math for Elementary Teachers course.
   - What is your role in the classroom?
   - What does student participation look like?
   - How is the class structured in terms of group work, discussion?

4. Why do you think it's important for math classes to be inquiry-oriented?
   - What benefits does it provide to students?
   - Are there reasons why it's important for this particular class to be inquiry-oriented?

5. Is there anything else you think I should know about the nature of this course?

APPENDIX B: FOLLOW-UP INSTRUCTOR INTERVIEW PROTOCOL

1. Tell me a little bit about how you think this course went, particularly relating to the inquiry-based nature of how you try to teach it.
   - What has gone particularly well this quarter?
   - Has anything not gone as well as you hoped?

2. What are that major things you hope students take away from being in this course?
   - Do you think these students are indeed getting that?
   - How do you know?

3. Do you think you give your students opportunities to feel a sense of autonomy related to how they learn and approach math in this class?
   - In what ways?
   - Do you think they take up those opportunities? Do you think they feel that autonomy?

4. Do you think you give your students chances to feel competent in this class?
   - In what ways?
   - Do you think they take up those opportunities? Do you think they feel competent?

5. Do you think you give your students opportunities to feel a sense of belonging in this class?
   - In what ways?
   - Do you think they feel as if they belong? Why or why not?

6. I heard from my interviews with students that you dropped the journal requirement. Can you tell me about that?
   - What did you hope students would get out of the journals?
   - Why did you make the decision to stop collecting them?

7. During my observations of your course I noticed that you used those little white boards a lot. I was hoping you could tell me why you use them. What you think they afford you (or the students).

8. Is there anything else you think we should discuss about the course or about students' experiences in this course?

APPENDIX C: PRE-SURVEY

BASIC NEED SATISFACTION IN *PREVIOUS* MATH CLASSES[24]

Name: _____
(Note: Names will be blacked out and participants will be assigned codes immediately after surveys are collected).
Interested in participating further as a focal student? YES NO
If YES, list best way to contact:_____

The following questions concern your feelings about your ***previous*** math classes. Please indicate how true each of the following statement is for you given your experiences in your ***past math classes***. Remember that your teacher will never know how you responded to the questions. Please use the following scale in responding to the items. **Circle your choice for each question**.

When I was in my Previous Math Classes...	**Not at all true**			**Somewhat true**			**Very true**
I felt like I could make a lot of input into deciding how my work got done.	1	2	3	4	5	6	7
I really liked the people I was in class with.	1	2	3	4	5	6	7
I did not feel very competent when I was in math class.	1	2	3	4	5	6	7
People in class told me I am good at what I do.	1	2	3	4	5	6	7
I felt pressured in class.	1	2	3	4	5	6	7
I got along with people in class.	1	2	3	4	5	6	7
I pretty much kept to myself when I was in class.	1	2	3	4	5	6	7
I was free to express my ideas and opinions in class.	1	2	3	4	5	6	7
I considered the people I was in class with to be my friends.	1	2	3	4	5	6	7
I was able to learn interesting new skills in class.	1	2	3	4	5	6	7
When I was in class, I had to do what I was told.	1	2	3	4	5	6	7

24 Survey adapted from the Basic Psychological Needs Scales, available at http://selfdeterminationtheory.org/

When I was in my Previous Math Classes...	Not at all true			Somewhat true			Very true
Most days I felt a sense of accomplishment from participating in class.	1	2	3	4	5	6	7
My feelings were taken into consideration in class.	1	2	3	4	5	6	7
In class I did not get much of a chance to show how capable I was.	1	2	3	4	5	6	7
People in class cared about me.	1	2	3	4	5	6	7
There were not many people in class that I was close to.	1	2	3	4	5	6	7
I felt like I could pretty much be myself in class.	1	2	3	4	5	6	7
The people I was in class with did not seem to like me much.	1	2	3	4	5	6	7
When I was in class I often did not feel very capable.	1	2	3	4	5	6	7
There was not much opportunity for me to decide for myself how to go about participating in class.	1	2	3	4	5	6	7
People in class were pretty friendly towards me.	1	2	3	4	5	6	7

Scoring Information. Form three subscale scores by averaging item responses for each subscale after reverse scoring the items that were worded in the negative direction. Specifically, any item that has (R) after it in the code below should be reverse scored by subtracting the person's response from 8. The subscales are:

Autonomy: 1, 5(R), 8, 11(R), 13, 17, 20(R)
Competence: 3(R), 4, 10, 12, 14(R), 19(R)
Relatedness: 2, 6, 7(R), 9, 15, 16(R), 18(R), 21

APPENDIX D: POST-SURVEY

BASIC NEED SATISFACTION IN *CURRENT* MATH CLASS (MATH FOR ELEMENTARY SCHOOL TEACHERS)[25]

Name: _____
(Note: Names will be blacked out and assigned codes immediately after surveys are collected).

The following questions concern your feelings about your ***current*** math class, Math for Elementary School Teachers. Please indicate how true each of the following statement is for you given your experiences in ***this math class***. Remember that your teacher will never know how you responded to the questions. Please use the following scale in responding to the items. **Circle your choice for each question.**

When I am in my Current Math Class (Math for Elementary School Teachers)...	**Not at all true**			**Somewhat true**			**Very true**
I feel like I can make a lot of input into deciding how my work gets done.	1	2	3	4	5	6	7
I really like the people I am in class with.	1	2	3	4	5	6	7
I do not feel very competent when I am in math class.	1	2	3	4	5	6	7
People in class tell me I am good at what I do.	1	2	3	4	5	6	7
I feel pressured in class.	1	2	3	4	5	6	7
I get along with people in class.	1	2	3	4	5	6	7
I pretty much keep to myself when I am in class.	1	2	3	4	5	6	7
I am free to express my ideas and opinions in class.	1	2	3	4	5	6	7
I consider the people I am in class with to be my friends.	1	2	3	4	5	6	7
I have been able to learn interesting new skills in class.	1	2	3	4	5	6	7
When I am in class, I have to do what I am told.	1	2	3	4	5	6	7
Most days I feel a sense of accomplishment from participating in class.	1	2	3	4	5	6	7

25 Survey adapted from the Basic Psychological Needs Scales, available at http://selfdeterminationtheory.org/

When I am in my Current Math Class (Math for Elementary School Teachers)...	**Not at all true**			**Somewhat true**			**Very true**
My feelings are taken into consideration in class.	1	2	3	4	5	6	7
In class I do not get much of a chance to show how capable I am.	1	2	3	4	5	6	7
People in class care about me.	1	2	3	4	5	6	7
There are not many people in class that I am close to.	1	2	3	4	5	6	7
I feel like I can pretty much be myself in class.	1	2	3	4	5	6	7
The people I am in class with do not seem to like me much.	1	2	3	4	5	6	7
When I am in class I often do not feel very capable.	1	2	3	4	5	6	7
There is not much opportunity for me to decide for myself how to go about participating in class.	1	2	3	4	5	6	7
People in class are pretty friendly towards me.	1	2	3	4	5	6	7

Scoring Information. Form three subscale scores by averaging item responses for each subscale after reverse scoring the items that were worded in the negative direction. Specifically, any item that has (R) after it in the code below should be reverse scored by subtracting the person's response from 8. The subscales are:

Autonomy: 1, 5(R), 8, 11(R), 13, 17, 20(R)
Competence: 3(R), 4, 10, 12, 14(R), 19(R)
Relatedness: 2, 6, 7(R), 9, 15, 16(R), 18(R), 21

APPENDIX E: INITIAL FOCAL STUDENT INTERVIEW PROTOCOL

1. You've told me a bit about yourself in relation to mathematics in your math autobiography, I'd like to start this interview by having you tell me about yourself in general, not necessarily related to mathematics.
   - Probe: How would you describe yourself?
   - Probe: What are your goals? Why are you in this course?

2. What were your past experiences in math classes like?
   - Probe (as needed): In your autobiography, you said […], can you [clarify and/or expand upon …]?

3. In your past math classes, did you feel like you were able to demonstrate competence?
   - Probe (as needed)
     - Can you give an example of a time you did (or did not) feel that way?
     - What do you think "counted" as being competent in your past classes?

4. In your past math classes, did you feel like you were self-directed in your learning?
   - Probe (as needed):
     - Do you feel like you had the freedom to approach math in your own ways?
     - Can you give an example of a time you did (or did not) feel that way?

5. In your past math classes, did you feel as if you belonged?
   - Probe (as needed)
     - Can you give an example of a time you did (or did not) feel that way?
     - What do you think would have made you feel like you belonged?

6. What do you expect this current math class to be like?
   - How do you think it will it be similar or different from other math classes you have taken?

7. How do you expect to participate? What will engagement with mathematics look like?
   - How do these expectations make you feel?
   - Probe (as needed):
     - In your autobiography, you said […], can you [clarify and/or expand upon …]?

8. Is there anything else you'd like me to know about your prior mathematics experiences or your expectations for the current course?

APPENDIX F: CLASSROOM OBSERVATION PROTOCOL

DATA SUMMARY
Data Collection Event: *(e.g.: mid-quarter classroom observation)*
Primary Purpose: *(e.g.; characterize the classroom, gather video for interviews)*
Topic: *(e.g.: place value)*
Date/Time:
Documents/Artifacts: *Keep track of all consent forms, surveys, handouts, etc.*
Cameras and Audio Recorders: *Make note of where cameras and audio recorders are located, and what/who they are focused on.*
Camera 1: [filename (ex: Observation_2013_xx_xx_C1)], description
Audio Recorder 1: [filename (ex: Observation_2013_xx_xx_A1)], description

Notes on Video/Sound: *Record any important information about video or audio (problems, etc).*
Classroom Map: **C***reate a map of the classroom, include locations of teacher and focal students, and camera locations.*

SESSION OVERVIEW
This section is created after the session, to summarize the key events. This is meant to be a quick reference to help me find my way through the fieldnotes and videos.

FIELDNOTES
This section is created in real-time, and tries to capture as much of what happened as possible, and then is later filled in using audio and video. This should include: Participants verbal contributions, including the teacher, Notes on expression, body language, movement, Notes on feelings/emotion/affect, if obvious.

QUESTIONS TO GUIDE OBSERVATIONS:
For the purpose of documenting and characterizing the inquiry-based nature of the course:

- How is the course structured to foster high quality math learning? To be inquiry-based?
- How do the teacher and students participate?
- In what ways might the structure of the course or the teacher's actions support students' feelings of competence, relatedness, and autonomy?

For observations of focal student participation to be use for stimulated recall:

- What does the focal students' participation in the course look like?
  1. How do they participate (do they talk, listen, ask questions, etc)?
  2. How do they feel about the course/math/participating (if obvious)?
- In what ways might students be feeling (or not feeling) autonomous, competence, and/or a sense of belonging?
- What does students' participation look like when such feelings might arise?
- Identify and record such instances for use in follow-up interviews.

APPENDIX G: FOLLOW-UP FOCAL STUDENT INTERVIEW PROTOCOL

1. Generally speaking, what has your experience of Math for Elementary Teachers been like?
   - (*If needed*) How have you been *feeling* about your experiences in class?

2. Can you describe any pivotal experiences, positive or negative, from class that have stood out for you?
   - Probe: Perhaps you wrote about these times in your class journal?
   - Probe: How did you feel in that moment?

3. What do you think about having to keep a journal in this class? Has it helped you reflect on your experiences? What do you typically write about?

---

4. Have you felt like you have been able to demonstrate competence in this class?
   - Probe: Can you give an example of a time you did (or did not) feel that way?
   - Probe: What are some accepted ways to demonstrate competence in this class?

5. Have you felt a sense of autonomy related to your mathematical thinking and learning in this class? Note: By autonomy I mean the freedom to be self-directed
   - Probe: Can you give an example of a time you did (or did not) feel that way?
   - Probe: What motivates you to participate in this class?

6. Do you feel as if you belong in this class?
   - Probe: Can you give an example of a time you did (or did not) feel that way?

Stimulated Recall:

7. *I have some clips of you participating in class. I'd like to watch them together, so that you can tell me what you were thinking and feeling during these moments. Your feelings may be influenced by the content, the task, your groupmates, the teacher. Those are all things I'd like to hear about if applicable.*
   - Probes (as needed, depending on the focus of the clip):
     - What was it about that moment that made you feel that way?
     - How were your interactions with the teacher or other students contributing to your feelings?
     - How were your interactions with the course content contributing to your feelings?
     - How were your interactions with the specific task contributing to your feelings?
     - What does feeling [competent or related or self-directed] mean to you?

8. Were these class sessions I observed typical of the course? Why or why not?

9. Was your participation that we just watched typical of your participation in this course? Why or why not?

---

10. What does it mean to be engaged with mathematics in this class?
    - What kinds of participation are expected of you?
    - Probe: How is that different from your previous math experiences?

11. How do you think these participatory experiences will influence your future teaching practice?

12. What is the role of the teacher in this type of math class?

---

13. How do your affective experiences (feelings of competence, relatedness, and autonomy) in this class compare to those in your previous math classes?
    - Probe: Do you think your past experience influence your participation or feelings in this class?

14. How do you think your affective experiences in this class will influence your future teaching practice?

Optional follow-ups or lead-ins from first interview:

15. In your first interview, you said […], can you [clarify and/or expand upon …]?

16. What do you think about the clicker quizzes?

17. What do you think about the little white boards?

18. Is there anything else you'd like me to know about how you are feeling about this math class?

APPENDIX H: FINAL FOCAL STUDENT INTERVIEW PROTOCOL

1. How have you been experiencing Math for Elementary School Teachers? Has anything changed since the last time we talked?

2. Have there been any pivotal experiences in class that particularly stood out for you?
   - Probe (as needed): Why does this moment stand out? How were you feeling? Why?

3. What did your participation end up being like in this course? How did you engage with mathematics?

4. How do you think your past participatory experiences in math influenced your participation in this course?

5. What is the role of the teacher in an inquiry based class like this?

6. How will your participatory experiences in this course influence your future teaching practice?

7. How will your participatory experiences in this course influence your future engagement with mathematics?

---

8. What were your affective experiences like in this course?

9. How do your affective experiences in this course compare to those in your previous math courses?

10. How did your past affective experiences with math influence your affective experiences in this course?

11. How will your affective experiences in this course influence your future teaching practice?

12. Which aspects of this course will particularly affect your teaching? Why?
    - Probe: How did this (these) aspect(s) of the course make you feel as a student?
    - Probe: What do you want your future math students' participation to look like?

13. How will your affective experiences in this course influence your future engagement with mathematics?

14. Did this course change how you think about yourself in relation to mathematics? How?

15. What did you write about in your Post-Reflection? Is there anything else you'd like me to know about your experiences of Mathematics for Elementary School Teachers?

APPENDIX I: FOCUS GROUP INTERVIEW PROTOCOL

1. Let's talk about what you were expecting this course to be like back at the beginning of the quarter. What did you expect your participation and experiences in this class to be like?
   - Probes (as needed):
     - Does anyone have anything to add?
     - Did anyone have different ideas?

2. How did your past experiences influence your expectations of this course?
   - What were your past math experiences like, in general?
   - Probes (as needed):
     - Does anyone have anything to add?
     - Did anyone have different ideas?

3. What did your participatory experiences end up being like in this class?
   - Probes (as needed):
     - Does anyone have anything to add?
     - Did anyone have different ideas?

4. How would you describe your affective experiences in this course?
   - Probes (as needed):
     - Does anyone have anything to add?
     - Did anyone have different ideas?

5. Did any particular moments or experiences stand out for you?
   - Probes (as needed):
     - Does anyone have anything to add?
     - Did anyone have different ideas?

6. How do you expect your affective experiences in this course to affect you in the future?
   - How will these experiences influence your **teaching**?
   - How will these experiences influence your **relationship with or study of math**?
   - Probes (as needed):
     - Does anyone have anything to add?
     - Did anyone have different ideas?

7. Is there anything else anyone would like to add about their affective experiences in this Math for Elementary Teachers course?

APPENDIX J: STUDENT ATTRIBUTES FROM THE COURSE SYLLABUS

Student Attributes

Success in college depends on a student's ability to respond to the challenges presented by new problems and new ideas. In addition to the process and content standards that follow, the attributes described below are crucial to success in college-level courses, both in mathematics and in other disciplines.

Attributes	Evidence of Achievement
Demonstrates intellectual engagement	Perceives mathematics as a way of understanding — a view that mathematics must make sense, and is not a sequence of algorithms to be memorized. Actively explores new ideas, posing questions about their meaning, significance, and implications. Recognizes patterns–as well as deviations–from previously learned patterns in data, diagrams, symbols, and words. Appreciates that abstraction and generalization are important sources of the power of mathematics. Is willing to take risks and be challenged as part of the learning process. Contributes to and benefits from group problem-solving activities.
Takes responsibility for own learning	Attends nearly every class session and when absent, seeks ways to learn the material covered in class. Conscientiously prepares work assigned for class. Examines and learns from his or her errors and seeks help when needed. Takes advantage of available resources — class time, notes, textbook, assignments, tutoring services, supplemental materials. Sets aside the time necessary to be successful.
Perseveres when faced with time-consuming or complex tasks	Is willing to work on problems that require time and thought, particularly problems that cannot be solved by mimicking a previously seen example. Successfully completes tasks that require organizing and implementing multiple steps, concepts, or techniques. Recognizes when an approach is unproductive and makes logical modifications to that approach or switches to another approach. Is convinced that effort is an important component of success in mathematics.
Pays attention to detail	Correctly follows all parts of oral and written directions without needing additional reminders. Makes few notational errors, e.g., accidentally changing digits, dropping or altering algebra symbols, incorrectly positioning points on a grid, etc.

APPENDIX K: SAMPLE NOTES PAGE

3.2 Addition & Subtraction NOTES

1 DEFINITIONS

Define each of the following:

sum

addend

difference

algorithm

2 MODELS FOR ADDITION ALGORITHMS

Using 345 + 278, show each of these methods:

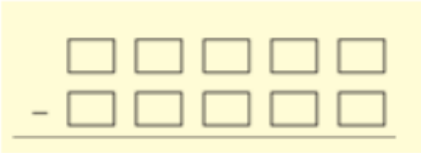
A. **Partial Sums**

B. **Left–to–Right** or **Scratch Method**

3.2 Addition & Subtraction ACTIVITIES

1 SMALLEST DIFFERENCE

Using your Number Tiles 0 thru 9, arrange them as shown below to obtain the smallest whole-number difference. Once complete, write your numbers in the boxes and record your difference below.



2 THINKING ABOUT MODELS FOR SUBTRACTION

1. Below and on your personal whiteboard, answer the question below. When done, share your work with your group members and with the class.

   "If groceries cost $3.74 and the customer gives the clerk $5.00, how much money will the customer receive back?"

2. Now, write your own word problem that involves subtraction. Write it below and on your whiteboard. When done, compare your problem with your group members and with the class.

3. Finally, compare your answer to #1 & #2 above with others in the class and classify your solution as one of the three models for subtraction: Take-Away, Missing Addend, or Comparison