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**Pre-Service Teachers' Mathematics Language and Reflection in the Context
of an Early Childhood Mathematics Methods Course**

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Abstract

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Preschool teachers are expected to engage young children in challenging and supportive mathematics learning. Rich and responsive language experiences in mathematics support children's language acquisition and engagement related to mathematics, however, such engaging experiences may be minimally available to many young children. Professional development programs designed for in-service teachers have developed approaches which help teachers in their classrooms develop better mathematics teaching skills. Another important group to prepare for effective early mathematics teaching is pre-service teachers. The current study examined a pre-service early mathematics course at a university and the nature of pre-service teachers' mathematics language, responsiveness, and reflection, and whether and how these key elements of practice changed. A multiple-case qualitative design was used to follow the practices and reflections of four pre-service teachers. Findings indicate that during the course pre-service teachers' frequency of mathematics language increased and the responsiveness of their language to children increased. They also engaged in "productive reflection" (Bayat, 2010) that was generative of improved practice and demonstrated improved beliefs related to children's abilities and their own practices teaching mathematics. Recommendations for the design of professional development which supports pre-service teachers' mathematics practices are provided. A range of materials may support pre-service teachers' confidence and the use of a broader range of mathematics language, and teacher educators should select materials that support pre-service teachers' implementation of new practices. Reflection should be carefully scaffolded to achieve specific aims.

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DEDICATION

I dedicate this dissertation to my grandmother, Joyce Ista Boyd. She (and her twin sister Janet) attend the University of Washington from 1939 – 1942, and earned her B.S. in Nutrition. She always taught me the importance of education and showed me how to be a life-long learner. She also taught me how to believe in myself, no matter what challenges arose. I will always love and remember you Grandma Joyce, this one's for you!

Chapter 1. INTRODUCTION

When does math start? Mathematics learning begins early in life and much earlier than many might be aware. Infants as young as three months old are sensitive to spatial categories of above/below and right/left, and orient their gaze to objects when they are moved into these spatial categories (Quinn, 1994). Later, toddlers 16-17 months old are able to locate objects using enclosures and landmark cues (Hermer & Spelke, 1996), meaning that they can locate a much loved toy knowing that it is always *behind* the couch at grandma's house. This foundation for geometric understanding is clearly related to space and the location of objects in space. By the preschool years, children are able to follow directions related to navigation and space, identify common figures (shapes) that they have experience with in their environments, and arrange shapes to make new two dimensional figures and three dimensional structures (NCTM, 2000).

Early mathematics skills are strong predictors of learning outcomes across domains and for years to come (Duncan, Dowsett, Claessens, Magnusson, Huston, Klebanov, Pagani, Feinstein, Engel, Brooks-Gunn, Sexton, & Duckworth, 2007), and mathematics skills, which begin to develop early in life, are critical for success in school and adult life. In particular, mathematics skills are part of the STEM fields and support participation in science, technology, engineering learning in school and participation in these fields in the workforce. For these reasons and more, National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM) have advised the early learning field to increase and improve young children's mathematics learning opportunities for young children (2003, 2010).

However, many young children enter kindergarten without the mathematics skills expected for their age (WA Kids Report, 2014), and once behind in mathematics, may struggle to catch up (Clements & Sarama, 2008). This may be related to a lack of mathematical learning opportunities before kindergarten (Early, Iruka, Ritschie, Barbarin, Winn, Crawford, Frome, Clifford, Burchinal, Howes, Bryant, Pianta, 2010). For example, Early et al. (2010) found that only 6% of the day on average is spent in any form of math learning, compared to literacy at 14% of the day. They also found that children from low-income families, Latino/a children, and African American children were more likely to spend more time in “no coded learning activity” or teacher directed activity, and less time in scaffolded learning opportunities. These findings suggest that fewer “scaffolded teaching interactions”, and lack of opportunities to practice higher order thinking skills may contribute to ongoing differences in the mathematics skills of low-SES and mid- and high-SES children. Furthermore, when teachers do include mathematics, they embed math learning in other activities, for example, singing a song with mathematical element, yet not clearly talking with children about the mathematical concepts. Young children are also less likely to engage in interactions focused on mathematical problem solving (Clements, 2007). This is problematic because children may benefit most from focused and direct math interactions facilitated by teachers. Therefore, indirect and embedded mathematics may miss the mark when it comes to effectively building early math skills.

It is critical that preschool environments provide more supportive and challenging mathematics learning opportunities for children, and it is critical that preschool teachers be able to provide such opportunities. Unfortunately, preschool teachers enter the field with little mathematics preparation. Many early childhood educators have limited coursework, if any, in mathematics in pre-service teacher preparation courses (Horn, Hyson, & Winton, 2013), further

limited field-based experiences teaching mathematics, and few opportunities to attend in-service trainings on early mathematics. Consequently, teachers may not be aware of the range of mathematics content appropriate for children (Copley, 2004), nor how to teach it. Without preparation and ongoing professional development in mathematics, teachers may maintain practices and beliefs that limit the quality of mathematical learning opportunities they provide (Ginsburg & Ertle, 2008).

Scholarship, research, and experience indicate that certain teacher *practices* and *beliefs* support early mathematics. *Practices* include the frequency and responsiveness of teacher's mathematics language used with young children (Cohrssen, Church, & Tayler, 2014; Rudd, Lambert, Satterwhite, & Smith, 2009). Teachers' mathematical language is positively related to, if not predictive of, young children's math outcomes (Klibanoff et al., 2006; Pruden et al., 2011), and early language mediates children's math knowledge (Ginsburg, Lee, & Boyd, 2008). In early childhood, children engage in similar types of math related play naturally, regardless other background factors such as income and race, yet early on differences emerge related to mathematics language, and children with stronger mathematics language skills are more likely to perform well on early mathematics assessments than children with limited mathematics language skill (Ginsburg, Lee, & Boyd, 2008). Hearing and using mathematically rich language helps children acquire skills necessary for communicating their mathematical thinking, and building on this thinking with others (Mathematics Learning in Early Childhood, 2009). Responsive language from adults also reinforces children's sustained engagement and provides contingent language (McWilliams & Aguilar, 2013). It offers timely mathematics language that children can map onto events and build into their own language repertoires (Pianta et al, 2008).

Responsive mathematics language can support extended time children spend engaged with materials or concepts, and therefore provide more time for learning math related concepts.

Teachers' *beliefs* about children and themselves also inform the types and qualities of mathematics interactions they provide. Specifically, expectations of children's abilities factor in how they provide mathematics learning opportunities. Children can engage in more complex math thinking and problem solving than often assumed by adults (D. Clements & J. Sarama, 2000). When teachers are trained in children's mathematical development, they have more accurate expectations of children, and can more appropriately respond to their efforts and guide them to more complex mathematics thinking (Jacobson & Lehrer, 2000). Teachers' accurate expectations and responsive interactions with young children are essential to providing challenging yet supportive mathematics learning. Such practices should be the focus of professional development designed for early childhood educators. Teachers' confidence in their ability to teach mathematics is another important belief. Preschool teachers report less confidence in their mathematics skills compared to other content areas (Copley, 1999; Chen & McCray, 2012). Confidence can also affect the likelihood of teachers being receptive to new practices (Bandura, 1993). Teachers are more apt to implement new mathematics practices in early learning classrooms when steps are taken to boost their comfort and confidence (Chen & McCray, 2013).

Clearly, early childhood educators need support in developing the skills and beliefs that will enhance their mathematics teaching. A number of professional development efforts have successfully provided such support. Teachers' mathematical language can be improved with effective professional development (Rudd et al., 2009). A number of projects have found that with effective professional development experiences, teachers improve their skills in leading

effective mathematics lessons, discussions, and attitudes towards mathematics (Chen & McCray, 2012; D. H. Clements & Sarama, 2008; Rudd et al., 2009). However, much of this work has examined support for practicing teachers (Chen & McCray, 2012; D. H. Clements & Sarama, 2008; Rudd et al., 2009). Less research exists on the math language, and potential changes in language, of pre-service teachers. Ginsburg and colleagues (2008) recommend professional development at the pre-service level as a part of an overall plan to improve early mathematics education. Course work and field experiences in mathematics for pre-service teachers need to prepare them to enter the field better equipped to provide young children with important opportunities to build mathematical skills. Additionally, frameworks for professional development specific to early learning, but not mathematics, may inform the design of early mathematics courses. The Head Start National Center for Quality Teaching and Learning has adopted an intentional teacher framework (Hamre, Downer, Jamil, & Pianta, 2012) in the design of courses (Joseph & Brennan, 2013). The framework addresses teachers' content knowledge, beliefs, ability to see target practices, reflect on practice, and improve over time, in addition to beliefs and practices.

In light of the relevance of early mathematics learning and the challenges facing early childhood educators, it is imperative that institutions of higher education provide courses that prepare teachers to engage young children in mathematics. Furthermore, specific elements of professional development effective for in-service teachers may usefully inform the design of such early mathematics methods courses. The application of such elements to early mathematics courses have not yet been, but should be, assessed to determine their effectiveness in developing key teaching practices and beliefs for pre-service teachers.

The current study aimed to contribute to the literature on early childhood educators' mathematics professional development by assessing pre-service teachers' practices, reflections, and beliefs in the context of an early mathematics methods course. The course included elements of effective mathematics professional development as well as elements of the intentional teaching framework. The study used qualitative multiple-case methods to describe whether and how pre-service teachers' mathematics language practices, reflections, and beliefs changed as they engaged in learning and implementing key practices, followed by video feedback and analysis. Results suggest that the course, to varying degrees, was effective at increasing the frequency of pre-service teachers' mathematical language use, the responsiveness of this language, engagement in reflection generative of improved practice, demonstrated improved expectations of children, and improved confidence in their math teaching practices.

The following chapters will more fully describe the research context and the study's methodology, findings, and implications. Chapter 2 provides a literature review of research related to the core concepts in the study, including tools for early mathematics learning, the role of early educators in this process, the elements of professional development that effectively support early educators' mathematics teaching. Chapter 3 describes the research questions and the methodology used to answer them, specifically a mixed-method multiple-case study which pairs quantitative and qualitative analysis of data to address change and relationships between reflection and practice. Results from the study are then presented in Chapter 4, with a description of the extent to which practices (language and responsiveness) changed, followed by description of reflection, changes in reflection, and the relationship between reflection and practice vis-à-vis goals. Finally, Chapter 5 discusses the results' contribution to the early mathematics research and professional development field, and the studies limitations.

Chapter 2. LITERATURE REVIEW AND CONCEPTUAL FRAMING

The following review will focus on elements of young children's mathematical learning, followed by trends in preschool teachers' mathematics teaching practices, effective approaches to mathematics teaching professional development, and finally a conclusion and theory of change that guided the current study. I begin with tools for children's learning in early childhood mathematics. This section focuses on three tools, which are mathematical language, engagement in mathematics play and learning, and materials supportive of mathematics learning. I then turn to teachers and professional development to reach children's learning goals. Lastly, I present a theory of change describing the process by which teachers develop these skills and that guided the current study.

2.1 EARLY CHILDHOOD MATHEMATICAL TOOLS FOR LEARNING

2.1.1 *Language*

Children's ability to access and use mathematical language is a fundamental resource for engagement in mathematical problem solving and communication (Ginsburg, Lee, & Boyd, 2008). In general, the variability and complexity of parent language with young children is predictive of children's early language production (Hoff & Naigles, 2002; Huttenlocher, Vasilyeva, Cymerman, & Levine, 2002). This general finding applies to domain specific language as well. For instance, children build a mathematics vocabulary through their language experiences with others, and the extent to which adults (teachers and parents) use math language with young children is positively related to their math learning outcomes (Ginsburg et al., 2008; Mathematics, 2009; Szechter & Liben, 2004).

Few studies address how young children's mathematics language develops. More focus on parental inputs than on teacher inputs. This body of research has demonstrated links between adult mathematics language, children's mathematics language, and children's later mathematics knowledge (Pruden, Levine, & Huttenlocher, 2012; Szechter & Liben, 2004). For example, children's use of **spatial language** is positively related to parents' use of spatial language with them at earlier ages (Pruden, Levine, & Huttenlocher, 2012). Spatial language is a domain specific type of math language which includes terms like "I'll sit *next* to you", "let's push the ball *through* the tube", and "hide *under* the table". Similarly, parent language during book reading that focuses on spatial-graphic representational understanding is significantly correlated with children's performance on spatial measures and scene ordering tasks (Szechter & Liben, 2004). Teacher use of verbal scaffolding during block play, an activity in which children engage in **geometric and spatial thinking**, also leads to children's more complex block building, compared to children who did not receive verbal scaffolding during play (Gregory, Kim, & Whiren, 2003).

In preschool and child care settings, **number related language** spoken by teachers over the school year correlates positively with children's gains in assessments of number knowledge (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). Number related language includes phrases like "Sure, all three of you can help me" and "Nine, what comes after nine". Children in classrooms where teachers used more number related language made greater gains across mathematics domains (including number, shape, and part/whole relationships) than children in lower math language classrooms. This trend is consistent across low- and middle-SES children.

2.1.2 *Engagement*

Engagement has been defined in several ways in the early learning literature. McWilliam and Aguilar (2013) define engagement as “The amount of time children spend interacting with the environment, in a developmentally and contextually appropriate manner, at different levels of competence” (p.102). Engagement also has been defined as transactional, meaning that the quality of child participation is related to the quality of adult interaction and vice versa (Mahoney & Neville-Smith, 1996). Engagement can include “mastery behavior”, in which child behavior is categorized by the amount of manipulation of objects and involvement in specific tasks (Messer, Rachford, McCarthy, & Yarrow, 1987); and as “behavior engagement”, including child interest, social initiative, and object initiative (Marfo, 1991). In other studies, engagement has been left to be defined by teachers participating in research and identifying desired behaviors of children in teacher planned activities (Wilczenski, Sulzer-Azeroff, Feldman, & Fajardo, 1987). Additionally, different levels of engagement also have been identified. These range from low-engagement (i.e. passive or inappropriate interaction with others or the environment) to high level engagement (i.e. persistent work on task, pretend play) (McWilliam, Scarborough, & Kim, 2003). Engagement, then, has been operationalized as an observable behavior, or set of behaviors, demonstrated by children, which can include interactions with the environment, adults, and peers in desired and appropriate ways, and also can range from low to high levels.

Young children engage in mathematical thinking and learning in a variety of ways. How they engage, for how long they engage, and what factors are associated with their engagement are important research questions addressed to some extent in prior scholarship. Young children’s engagement in mathematics has been as associated with their involvement in active play with given materials in the classroom accompanied by their speech and/or actions that have a

mathematical focus (Chien et al., 2010; Early et al., 2010; Edens & Potter, 2013; Ginsburg, Lin, Ness, & Seo, 2003). From this perspective, children engage in a broad array of mathematical activities naturally during their play. Specifically, studies interested in identifying how children meaningfully engage in mathematics have found that children demonstrate engagement often as they participate in many classroom activities, including book reading (Ginsburg & Seo, 1999), mapping the classroom (Ginsburg & Amit, 2008), block play (Ginsburg, 2008), and shapes (D. Clements & J. Sarama, 2000). For example, during block play children can explore ideas related to geometry, including the composition of shape, symmetry, as they construct buildings. Or, while reading, children may engage in making mathematical connections between ideas in the book, such as preparing a feast for ten people, and their experiences with number and measurement when preparing for meals with their families. Additionally, children may represent mathematical ideas in books through their own drawings – drawing a visual representation of a story (like three lines for the three little bears), a process that can then support children’s reasoning about objects (Hintz & Smith, 2013; Mathematics, 2009).

Children engage in mathematics domains to varying degrees and lengths of time. Ginsburg et al. (2003) reported that Chinese and American children are both likely to engage in enumeration, magnitude, and pattern and shape in free play at child care, and less likely to engage in classification and dynamics. However, there are differences regarding amount of time spent in math activity, with Chinese children averaging 69% of observation time engaged in math activity and American children averaging 44% of their time in math activity (Ginsburg et al., 2003). In a study in Sweden, Edins and Potter (2013) found that children chose mathematically related activities approximately 29% of the time, with activities ranging from block play to art. In observation of individual children, the researchers found that those children

who engage in private speech (talking to themselves as they engaged in math activity) also performed better on assessments of math skills. The link identified in this study between language, engagement, and mathematics outcomes resonates with Ginsburg et al.'s (2003) assertion that language is a critical part of early mathematics as it is required to “express and justify mathematical thinking” (p. 5).

2.1.3 *Materials*

Research results indicate that manipulatives support mathematically related play and thinking of young children (Ginsburg et al., 2003; Gregory et al., 2003; Zambrzycka, 2014). Manipulatives help children make representations of their math learning. Opportunities to explore various representations with manipulatives can lead to more flexible thinking and connections across ideas and content (Kazemi, 2003). Blocks may be an especially important material for early mathematics play and learning. Children across diverse cultures and socio-economic backgrounds engage in more mathematics activity when playing with Legos or blocks verses other materials (such a books or dress up) (Ginsburg, Lin, Ness, Seo, 2003). Chen et al. (2012) asserted that children are able to develop greater understanding of various shape attributes as they play and work with real figures, such as blocks, and are asked to describe shapes and attributes in their own words. Teachers' use of blocks has also been found amenable to professional development programs. According to Siegler (2009) and Siegler and Ramani (2008), play with games, especially linear board games, has been found to promote early number skills.

Young children with access to blocks and Legos engage in more geometry related play and language than those with less access (Ginsburg et al., 2003). Other math manipulatives as well, with good teacher support, give children opportunities to represent their mathematical

thinking and support development of abstract thinking (Clements & Sarama, 2000; Kazemi, 2003). Considering the relationship between materials and children's mathematical learning, it stands to reason that different materials might support teachers' mathematical teaching in different ways and to different extents. Pre-service teachers may try a variety of math materials in their process of learning about benefits of each. Research has addressed how in-service teachers take up mathematical practices with the support of specific materials such as books (Chen et al., 2012) or blocks (Gregory et al., 2003). However, similar research addressing how pre-service teachers may use such materials in mathematics interaction with young children is rare. This study aims to extend the knowledge base to include pre-service teachers.

2.2 EARLY CHILDHOOD EDUCATORS AND MATHEMATICS PROFESSIONAL DEVELOPMENT

Above, I described language, engagement, and materials as tools which support mathematics learning for young children, and reviewed research which identified relationships between these three tools and children's mathematical development. Now, I turn to the specific role early childhood educators play in providing language rich interactions, as well as two critical teacher beliefs. The following will describe the nature of preschool teachers' mathematical language.

2.2.1 *Teachers' Mathematical Language*

Teachers provide mathematical language to different extents across content domains (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Lee & Ginsburg, 2007; Rudd, Lambert, Satterwhite, & Smith, 2009; Sarama & Diabase, 2004). In observational and self-

report studies of preschool teachers, language related to number is most frequently used by teachers, followed by spatial sense, and measurement (Klibanoff et al., 2006; Rudd et al., 2009). Less frequently do teachers use, or report using, language related to geometry, pattern, and operations (Rudd et al., 2009; Sarama & Diabase, 2004). Literature on early educators' tendency to emphasize number skills over other types of mathematics (Sarama and Diabase, 2004; Lee and Ginsburg, 2007) found that when teachers were asked about which math topics they taught children, the topics least popular were "making shapes" and "measuring", with only 14-16% of teachers reporting doing either of these.

Even within domains, such as number, teacher math talk can vary (Klibanoff et al., 2006). Observations of teachers' number language with children in preschool classrooms, broken into nine coding categories, showed a wide range in relative frequencies across categories. Categories included counting, cardinality, equivalence, nonequivalence, number symbols, conventional nominative, ordering, calculation, and place holding. The results showed that cardinality constituted the most language at 48% of number utterances, followed by number symbols in second place at 17%, down to place holding, only making up 1% of number related utterances.

The lack of math language provided in specific domains (such as geometry and measurement) is an important factor to consider regarding children's early math knowledge and learning. Early educators' tendency to emphasize number skills over other types of mathematics may have an impact on the mathematics learning of young children (Sarama & Diabase, 2004; Lee & Ginsburg, 2007). Clements and Sarama (2011) reported that young children's competence in geometry skills is largely related to their experiences with geometry concepts and language. With potentially few opportunities to engage in domain specific language around geometry,

measurement and other domains, children may be missing out on important and foundational mathematics learning opportunities. Additionally, these behaviors may reflect the early educators' beliefs about what math is - early educators may often think of mathematics in terms of arithmetic, focusing on counting, cardinality and one-to-one correspondence, and give less attention to other areas of mathematics like geometry, measurement and algebra (Lee and Ginsburg, 2007). Professional development opportunities which emphasize talk in specific domains that include but go beyond number are needed, as well as studies to explore the potential differences and changes in teacher talk across mathematics domains.

Professional development supports geared towards early childhood educators have also provided a specific term to capture the special type of language they are to use. *Mathematize*, or *mathematizing*, has been defined in several ways, including as “a process of inquiring about, organizing, and constructing meaning with a mathematical lens” (Hintz & Smith, 2013, p. 104) and “teaching math concepts through children's play and everyday experiences by bringing out the math in what they are doing” (High Five Mathematize, 2010, p. 5). The current study has used the latter definition in the course materials which pre-service teachers encountered, however, the former definition was also introduced to pre-service teachers.

2.2.2 *Teachers' Responsive Language*

Language adults use that is responsive to children's early mathematics engagement increases young children's sustained engagement. In early learning scholarship, several types of responsive language identified include individualized “elaboratives”, that is, adding language to what children say or do, and “information”, or, providing timely information related to the activities in which children are engaged (McWilliam, Scarborough, & Kim, 2003).

Mathematics discussion prompts, such as asking children to share their thinking, providing “wait

time” thoughts (Kazemi, 2003), and verbal scaffolding during play (Gregory et al., 2003), support engagement in mathematical thinking and discourse. In observations of an expert teacher of kindergarten children, a number of effective elements emerged which further supported child engagement, including provision of clear instruction relating to mathematics content, emphasis on correct use of math concept related language, extensive content knowledge, and opportunities for students to plan (Ginsburg & Amit, 2008). Such strategies led to children articulating connections between mathematical ideas and visually representing their thinking. In another example, early educators trained to pause before responding to young children during mathematically based play interactions were found to have more attuned responses, thus supporting children’s engagement and also improving the emotional support in interactions (Corhssen, Church, & Tayler, 2014)

At the elementary level, elements of responsive mathematics discussions may also look like those recommended for younger children. For example, in a study of teacher’s implementation of Cognitively Guided Instruction in a geometry unit with fourth graders, teachers who more often used the strategy of “revoicing” (defined as expanding, refining, or highlighting important ideas in students comments) were described as better at facilitating students thinking (Jacobson & Lehrer, 2000). This is a strategy not unlike that of providing “elaboratives” identified in the early learning literature by McWilliams et al. (2003). Students in these classrooms scored better on assessments of their transformational knowledge than students of teachers who employed the revoicing technique less frequently.

Responsive mathematical language usage is attuned to children’s interest and adds relevant and more expansive mathematics language. It is also emotionally supportive, and can improve the quality of relationship between teacher and child. The literature has identified

strategies teachers can use to engage in responsive interactions in classrooms, yet observed these in the context of in-service teaching and professional development. For pre-service teachers, less is known about the extent to which they may engage in responsive mathematics interactions, a line of inquiry which the current study addressed.

2.2.3 *Teachers' Expectations*

The NAEYC and NCTM (2010) joint position paper provided six principles of early childhood mathematics education, the first of which is equity. The paper defined equity as high expectations and strong supports to all children required to achieve excellence in early mathematics education. High expectations are a part of what other mathematics education organizations list as a core component of quality teaching (Ginsburg et al., 2008; National Research Council, 2009). However, literature on the expectations of in-service and pre-service teachers of young children related to mathematics reveals generally low expectations (Clements & Sarama, 2011; Lee & Ginsburg, 2007). Generally, teachers may be resistant to the idea that children can and should engage in challenging mathematical thinking in preschool (Lee & Ginsburg, 2007), and may tend to address mathematics learning indirectly and with limited depth in the classroom (Chien, Howes, Burchinal, Pianta, Ritchie, Bryant, & Barbarin, 2010; Early, Iruka, Ritchie, Barbarin, Winn, Crawford, Frome, Clifford, Burchinal, Howes, Bryant, & Pianta, 2010). Specifically, many teachers have low expectations of young children's geometry knowledge, and may repeatedly ask children to identify shapes (after children have done this successfully), without then inviting them to demonstrate more complex skills such as transforming or composing shapes (Clements & Sarama, 2011). Pre-service teachers' expectations and beliefs about children's capability may also be especially low for children in poverty (Lee & Herner-Patnode, 2010).

The NAEYC and NCTM position paper pairs high expectations with strong support, echoing the work of researchers in culturally responsive teaching such as Gay (2000), who described teachers who care as those who “honor their [students] humanity, hold them in high esteem, expect high performance from them, and use strategies to fulfill their expectations” (p. 46). Gay (2000) provided examples of teachers who are “warm demanders” as being especially effective in educating ethnic minority students (Gay, 2000). “Warm demanders” develop meaningful relationships with students, make clear and consistent demands for high performance, and establish classrooms characterized by warmth and support. These practices lead to higher academic and social success of students (Klienfeld, 1975). Delpit (2012) also asserted that successful teachers of African American children believe in their ability to meet high expectations, that poverty is not an excuse, and that they will do whatever it takes to help students succeed.

Young children benefit from engaging, challenging, and supportive interactions, and gain on measures of academic and cognitive ability when they experience instruction that challenges their higher-order thinking skills, uses performance feedback, and uses rich vocabulary (Pianta, La Paro, & Hamre 2008). Performance expectations are communicated in the depth and complexity of questions teachers ask children, time and support provided to help children articulate their ideas, and learning opportunities that deeply explore topics relevant and interesting to children. Teacher relationships with and expectations of children have been well-documented contributors to child outcomes, both positive and negative (Gay, 2000; Saft & Pianta, 2001). Professional development on equity in early mathematics education should promote teachers’ high expectations and supportive approaches. Given evidence that pre-service teachers may be at risk for holding low expectations, it is important to address this group

specifically in the research to identify approaches that effectively support their high expectations of young children's mathematical abilities.

2.2.4 *Teachers' Confidence*

Teachers confidence is powerful driver of their behavior and ability to develop new skills (Bandura, 1993; Tomkins, 1962, 1963; Skatterbol, 2010). Gibson and Dembo (1984) have found relationship between teachers' high self-efficacy and spending more classroom time on learning, more support to students having difficulty, and providing more positive feedback for student accomplishments. Likewise, there is an association between low teacher self-efficacy with less focus on learning and more focus on managing classrooms, giving up on students who are struggling, and providing negative feedback when students fail (Bandura, 1993). Given that teacher interactions which are rich in positive feedback and high press academic instruction are related to improved student outcomes (Delpit, 2012; Gay, 2000; Kazemi, 2003; Pianta et al., 2008), it is not surprising that teachers' beliefs of self-efficacy predict student outcomes in mathematics and language achievement over the year (Ashton & Webb, 1986; Bandura, 1993).

Teachers' goal setting is also informed by self-efficacy: the stronger an individual believes in their capability, the more challenging are their goals and the greater is their commitment to completing them. Not only this, but performance is achieved as individuals carry out goals with a sense of confidence in their capabilities. Ability to implement certain practices is not just based on knowledge and skill in the practice itself; ability also involves skill in managing emotions in real situations which can impede effectiveness (for example, when a novice teacher panics when asked a difficult question by a student). While an individual may have knowledge and skill related to practices, self-efficacy is what enables them to bounce back and use their knowledge and skills well in challenging moments (Bandura, 1993).

Clear and challenging goals can promote motivation (Locke & Latham, 1990; Bandura, 1993). By comparing performance to goals, one can determine the level to which they are satisfied with accomplishment of goals, and continue by setting yet more challenging goals. Self-efficacy can be threatened, however, as teachers attempt challenging goals and then are not satisfied with their performance. This can lead to concerns about their abilities to meet children's needs, and, if others are aware of these failures, teachers may feel shame. Social support which views these events as opportunities to learn, however, can mediate these emotional outcomes, reducing fear of failure and shame and increasing self-efficacy as teachers realize they are capable of learning from their experiences (Skatterbol, 2010). Therefore, while clear and challenging goals can be motivating, it is important that teachers discuss their progress on goals in a socially supportive environment.

Unfortunately, early childhood teachers often report low levels of confidence in their ability to teach young children mathematics (Copley, 1999; Sarama & Diabase, 2003). Considering the above described relationships between self-efficacy and teacher behaviors, as well as goal setting, low confidence or self-efficacy among early childhood educators may pose a threat to their ability to implement new mathematics practices, and to their ability to learn from ongoing experience. Approaches that might promote self-efficacy, however, are also provided, and these include learning from new experiences and social support. Additionally, familiar materials (such as books or blocks) have been found to support early educators' confidence in their ability to implement mathematics lessons (Chen & McCray, 2012).

2.3 ELEMENTS OF PROFESSIONAL DEVELOPMENT FOR MATHEMATICS TEACHING

Given growing understanding of how mathematical thinking develops in young children and the tools which support it (Clements & Sarama, 2000, 2000; Froebel, 1895; Pusey, 2003; van

Hiel 1999), as well as evidence that early mathematics understanding supports a range of other math and literacy outcomes (Duncan et al., 2007), it is imperative that early childhood educators engage children in powerful mathematics learning. Recent national focus on preparing students for success in the STEM fields (Clements & Sarama, 2000) lends urgency to this work, highlighting a need for professional development to promote effectiveness in mathematics teaching in early childhood, specifically in the area of geometric knowledge and skills.

Several professional development programs focused on mathematics in early learning shed light on effective teaching practice. Additionally, research with elementary teachers presents professional development programs that lead to teacher and child outcomes in mathematics. A review of research on professional development in each of these arenas follows.

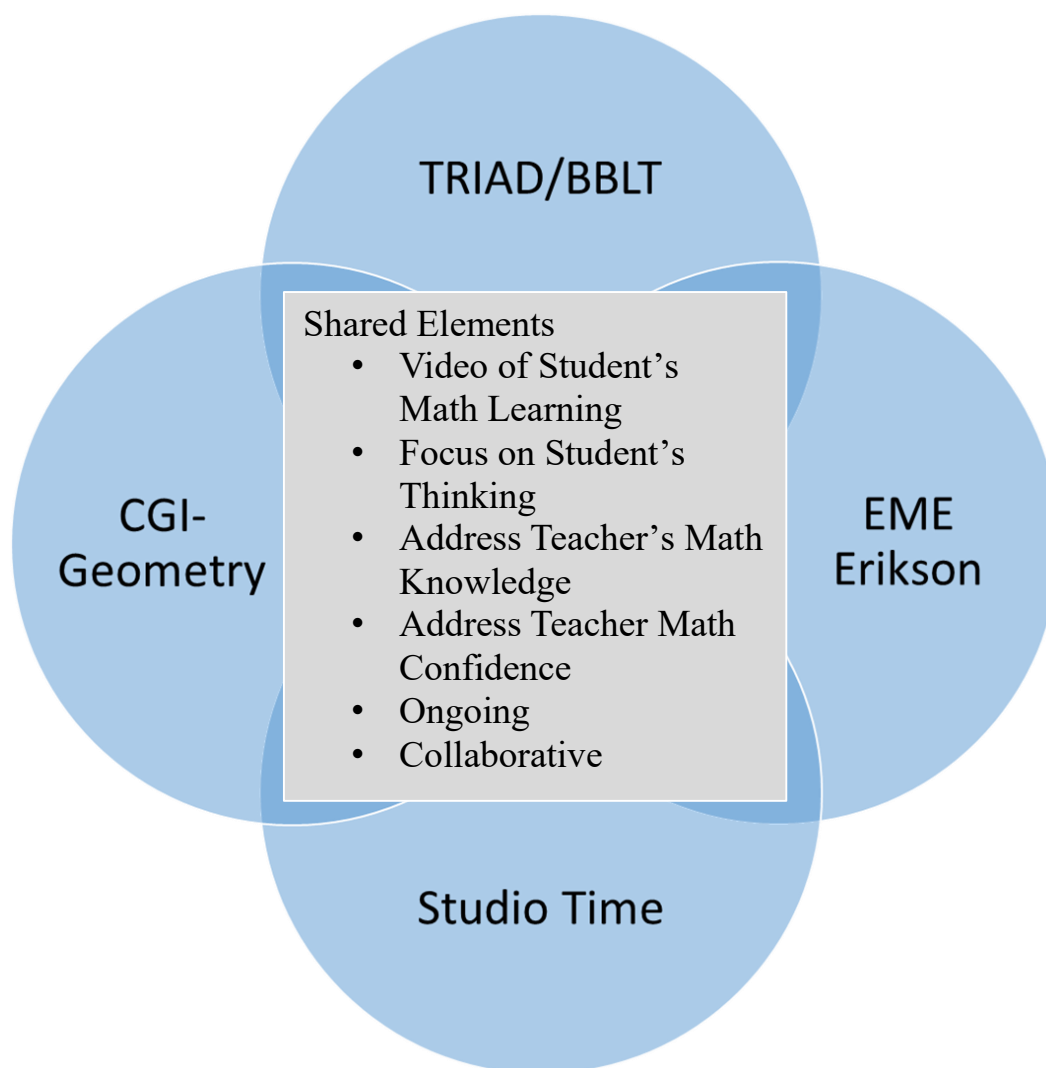


Figure 1 Shared Elements of In-Service Early Mathematics Professional Development Programs

2.3.1 Technology-enhanced, Research-based, Instruction, Assessment, and professional Development

Technology-enhanced, Research-based, Instruction, Assessment, and professional Development (TRIAD) is a professional development program designed for early educators that has demonstrated gains in the geometry performance of children whose teachers participated in it (Clements & Sarama, 2011). This is perhaps the only research at this time that provides links between a professional development program and preschool children's gains in geometry

learning. TRIAD consists of several components, including between five and eight full day trainings per year for teachers (the program lasts two years), with monthly coaching in between. These day long trainings have a theoretical base in the learning trajectories developed by Clements & Sarama (2009), and are guided by the specific goal to create multifaceted, practice related, and peer driven interactions and learning for teachers. In these trainings, teachers participate in discussions and activities which introduce and clarify how children learn mathematics, make links to theory, and connect back to practice. The second component is use of the Building Blocks Learning Trajectories web application (BBLT), which teachers are introduced to in training, and then can use ongoing throughout the year. Through BBLT, teachers gain access to descriptions, videos, and commentaries that illustrate the learning trajectories. Clements & Sarama (2011) suggest that video clips are a powerful resource for because teachers can view video first to see exemplary curriculum activity, and then later to study children's thinking and development. Finally, mentors, trained in the TRIAD model, work with teachers in their classrooms each month, and use the learning trajectories to guide discussion with teachers on teaching and children's learning.

The TRIAD/BBLT model has been evaluated, teacher curriculum implementation fidelity and child outcomes were compared in 36 preschool classrooms between children whose teachers used TRIAD/BBLT, those using an alternate preschool mathematics curriculum, and those using no specific mathematics curriculum. Teachers were found to implement the BBLT curriculum with fidelity. In classrooms with the curriculum, children outperformed those in the other two conditions on nearly every measure of mathematics knowledge, including scoring significantly higher on completing correct constructions of shapes.

2.3.2 *Early Mathematics Education Project at Erikson Institute*

The Early Mathematics Education Project at Erikson Institute (EME) (Chen & McCray, 2012) is designed to help preschool and kindergarten teachers improve their attitudes, knowledge, and practices in teaching young children mathematics. Evaluation of the program found gains of an additional three to five months of math learning for every year children spent in the classrooms of teachers who participated, and greatest gains occurred for children initially most behind in mathematics. EME evolved from survey results of preschool and kindergarten teachers in Chicago Public schools indicating that teachers were unsure of their own math knowledge, did not feel confident teaching young children math, and desired professional development to improve their math teaching with young children. In response to these needs, EME was developed and became a voluntary in-service professional development program for early educators in Chicago Public Schools.

Chen and McCray (2012, 2012) provide a conceptual framework for the program, the *Whole Teacher Approach*, that they see as critical to understanding EME's success. The *Whole Teacher Approach* is distinct from other professional development approaches which typically only addresses teacher's knowledge, in that it also focuses on the development of teachers positive attitudes and in class practices related to mathematics. The simultaneous focus on knowledge, attitude, and practice is supported through the above described components of EME, and leads to improvements for teachers and children.

EME has three components: learning labs, on-site coaching, and classroom implementation. Learning labs occur every five to eight weeks over the course of the school year and consist of interactive instructional sessions focused on developing teachers content knowledge related to the "big ideas" of mathematics. In learning labs, teachers participate in

activities that are centered on hands-on experiences with math learning, as well as discussion and problem solving with peers. Learning labs and activities serve to help teachers develop in depth content knowledge, while addressing their attitudes about math. Through them “teachers see themselves as learners”, and “teachers who are anxious about mathematics become more comfortable and confident” (p. 116).

The second component of EME, on-site coaching, consists of 3 on-site visits from a coach over the course of the school year, between learning lab sessions. Coaches engage with teachers in a cycle of planning, implementation, and analysis of teacher’s and children’s math activity, which makes teacher practice also a source of teacher learning. Sometimes, however, as teachers take responsibility in planning their own math interactions, they may not accurately present the big ideas of math to children. In response to this problem, EME coordinators created “research lessons”, developmentally appropriate lesson plans that addressed Big Ideas which teachers could implement, and then analyze with their coach. These “research lessons” were typically based on books that would be well known to teachers, and coaches first tried the lessons with children, and then taught teachers how to do the “research lesson” before the teacher implemented it. The authors believe this addition better supported teachers’ development of math knowledge.

The third component of EME is implementation. By using familiar books as the foundation of Research Lessons, an “in” was created with teachers that drew on their positive attitudes and comfort with reading books. This “in” improved teachers’ attitudes towards implementing math lessons in their practice and their beliefs that they could do so, supporting the ongoing implementation of these research lessons.

2.3.3 “Studio” Time

“Studio” time is an in-service professional development approach by which teachers meet together to review video tape of themselves, or colleagues, in classroom episodes in order learn about students’ development and learning, and make plans for increasing learning. Higgins (2013) describes the use of “studio” time to support learning about the developmental levels of geometry among students, as well as plan for ways to increase “students use of mathematical language and vocabulary” (p. 490) for elementary teachers from multiple schools in a district (teachers from kindergarten through grade five participated). “Studio” sessions occurred in three session cycles, and there were a total of 5 whole “studio” session cycles over the school year, for a total of 15 “studio” session meetings over the school year. Each whole “studio” session cycle focused on a different age group and topic related to mathematics, and the second cycle focused on a unit in geometry for second grade students. The process started with an instructional coach gathering a video recording of a teacher’s facilitation of students discussing what shapes are, and debating ideas about shapes and things that are not shapes (some of the students, for instance, did not believe circles were shapes because they did not have points). This video was brought the “studio” session and shared with other teachers, leading to a discussion about the levels of geometric understanding these students were demonstrating.

Through their discussion of the students understanding in the video, teacher’s own understanding of geometry increased as they clarified their knowledge of van Hiele’s levels of geometric thinking, and defined the difference between polygons and other types of shapes. Through their assessment of students geometrical understanding using the van Hiele levels, teachers also came to see differences in students thinking not as “right” or “wrong”, but as indicating different developmental levels, related to their experience with geometry. Some

teacher's also came to realize that they had been using mathematically incorrect language with children, and this might be impacting children's understanding of shape (for instance, some teachers had been telling students that squares and rectangles were two different types of shapes, rather than explaining to students that squares are a special type of rectangle, and that both are quadrilaterals).

While observations of teachers' interactions with children following "studio" sessions were not provided in this study, teachers reported that as a result of "studio" sessions they believed that the use of vocabulary and definitions was very important in geometry teaching so that children can develop shared and accurate understandings of geometry terms and concepts. Higgins concludes by encouraging the use of "studio" sessions as a means of supporting professional growth which leads to deeper understanding of mathematics and new perspectives on practices with students that will effectively engage them in mathematics.

2.3.4 *Cognitively Guided Instruction - Geometry*

Cognitively Guided Instruction (CGI) is a program in which teachers use their knowledge of individual children's thinking to guide instructional decisions. Jacobson & Lehrer (2000) describe research in which teachers participated in CGI focused on increasing teachers understanding students thinking about geometry through workshops. Workshops occurred over the course of three years, and added up to approximately 60 hours of professional development. Half of the workshop time involved teachers solving hands-on measurement and geometry problems themselves. The remaining time focused on helping teachers understand students thinking about solving the same or similar problems through discussion and viewing video of children solving problems. In workshops, teachers were encouraged to explore and identify ways to facilitate discussions of measurement and space with children.

Instructional practices from two teachers participating in this program were compared to two teachers who participated in a similar program, however, the focus was on arithmetic, not geometry. Student's test scores on understanding of transformational geometry concepts were also measured and compared. All four teachers implemented the same geometry unit, called "Geometry in Design" (Watt & Shanahan, 1994) in which students learn about transformational geometry by constructing a quilt using transformations of quilt tiles made of square and rectangle shapes.

Results, from analysis of classroom discussion and scores from student assessments, showed that teachers participating in the geometry focused professional development workshops had more in-depth conversations with students about their transformational thinking, and more often used the strategy of "revoicing", in which teachers expand, refine, or highlight important ideas in students' comments (p.86). Jacobson and Lehrer (2000) describe these teachers as better at facilitating students thinking, and attribute this to their experiences with the geometry focused professional development. Students in these classroom scored better on assessments of their transformational knowledge, than students of teachers who received professional development in arithmetic only. These student gains were sustained: in follow up assessments one month later, scores remained stable. This finding indicates that domain specific professional development, in this case the domain of geometry, makes a difference in teaching and student outcomes. Teachers specific professional development related to geometry prepared them for more effective and cognitively guided geometry interactions with students compared to teachers training in another mathematics domain.

Across these four professional development programs aimed at mathematics for young children either in preschool or early elementary grades there are several common aims and

design elements. These include the aims of addressing young children's mathematical thinking and development, addressing teachers' content knowledge in specific mathematics domains (geometry in these cases), and addressing teachers' confidence. These aims were accomplished with the design elements of video observation to target child understanding as well as teacher knowledge, and the use of ongoing and collaborative settings to support teacher confidence. Such aims and elements proved effective in improving teachers' knowledge, confidence, and practices, and in most of the cases children's mathematics achievement. These are therefore instructive examples for the design of professional development geared towards early mathematics.

2.4 INTENTIONAL TEACHING FRAMEWORK: KNOW, SEE, DO, REFLECT, IMPROVE

In combination with the above aims and elements of effective in-service early mathematics professional development is a framework specific to early childhood teacher professional development which further guided the development of a one quarter early mathematics methods course. The Intentional Teaching framework was used by the Head Start National Center for Quality Teaching and Learning to guide adult learning (Joseph & Brennan, 2013). This framework is simplified as: Know, See, Do, Reflect and Improve (KSDRI). KSDRI outlines a process for adult learning and development by which teachers move from acquiring knowledge about early childhood development and evidence based teaching practices (know) through lecture, readings, video exemplars, and discussion. Observation skills are honed and assessed as teachers engage in observing and identifying key child developmental progressions and teaching practice in videos, case studies and other observations, this is the "See". These first two components of the intentional teaching cycle provide a foundation for the adult learner to enact the teaching practices in real world settings, that is, practice, or "Doing", is a necessary

part of development. Reflection follows practice and allows teachers to look back on the actual situation of teaching young children, and compare their experience with the previously learned theory about child development and their own ability to employ teaching strategies. This sort of reflection is analogous to reflection-on-practice, which Schon (1991) describes as a more basic type of reflection after practice, in contrast to reflection-in-practice, a more advanced reflection which is concurrent with professional practice. Reflection can also be scaffolded to encourage demonstration of productive reflection characteristics (Bayat, 2010), and support pre-service teacher confidence as they attempt and reflect on new practices (Calderhead, 1989). The overall process is expected to lead to Improvement, that is, increased knowledge, greater ability to see aspects of child development and teaching technique, more frequent implementation of effective teaching practices, and more keen reflection.

The Know, See, Do, Reflect and Improve framework draws in some ways from Shulman's (1987) often cited conception of teacher knowledge. Shulman (1987) emphasizes pedagogical content knowledge and describes it thus: "Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue" (p. 8). Shulman (1987) frames the goal of teacher education as "not to indoctrinate or train teachers to behave in prescribed ways, but to educate teachers to reason soundly about their teaching as well as perform skillfully" (p. 13). Teachers develop such pedagogical reasoning and action through a process of connecting their pedagogical content *knowledge* to ongoing reasoning and decision making to inform *improved practice*. This process, or cycle, begins with comprehension (of both ideas and educational purposes); transformation of comprehended ideas and/or purposes into teaching interactions (this includes sub-processes of preparation, representation, selecting instructional methods, adaptation of ideas to characteristics

of learners as a group, and individualizing adaptations to specific learners); instruction, the “observable performance of a variety of teaching acts” (p. 17); evaluation, including interactive checking for students understanding as well as formative assessment; reflection, the setting in which teachers look back on practice to learn from experience by comparing teaching to the intended outcomes; and, finally, new comprehension, in which, by engaging in the above phases of the process, teachers arrive at refined comprehension which leads into yet more transformation, instruction, etc. Teachers should be able to engage in each aspect of the process, and teacher education, then, should afford opportunities with each aspect of the process. The Know, See, Do, Reflect, Improve framework applies many aspects of the process of developing pedagogical content knowledge earlier described by Shulman, yet frames them for use in the early learning field.

2.4.1 *Know*

Teachers’ knowledge of content and child development is a necessary component of effective teaching (Hamre et al., 2015; Shulman, 1987). Teacher’s knowledge of mathematics content in specific domains, such as geometry, as well as how children might think within a given domain is positively associated with their implementation of effective mathematics discussions with children (Jacobson & Lehrer, 2000). Professional development programs for teaching mathematics take knowledge as a starting point with teachers. What teachers should know and how to develop knowledge are both important aspects. Chen & McCray (2012) propose that teachers need three sorts of knowledge: “knowledge about what to teach (content knowledge)”, “knowledge about how to teach (instructional methods)”, and “knowledge about who is taught” (understanding of child development) (p. 10). Other mathematics professional development programs also focus on content knowledge (Clements & Sarama, 2011; Jacobsen & Lahrer, 2000),

and knowledge of children's developmental stages in mathematical thinking (Clements & Sarama, 2011; Higgins, 2013). Characteristics of professional development which effectively develop teacher knowledge include interactive and sustained training (repeated sessions over several months or years) which may use case studies (Jacobson & Lehrer, 2000), presenting and discussing mathematical definitions of "big ideas" (Clements & Sarama, 2011; Chen & McCray, 2012), and learning and identifying children's learning sequences within given domains using video (Higgins, 2013).

2.4.2 *See*

Teachers also need opportunities to observe and identify effective teaching when they see it. Teachers can also be trained to identify children's abilities along a trajectory of learning from watching video of children at various stages (Clements & Sarama, 2011). Social learning theory (Bandura, 1986) suggests people learn how to behave, for example, during classroom interactions, through watching others. Skill in correctly assessing effective interactions of others via observation has also been related to demonstration of those same effective interactions (Moreno & Valdez, Hamre et al., 2015). The Building Blocks TRIAD professional development approach includes video observation of teachers and children as a part of ongoing professional development in which teachers can return to demonstration videos to recalibrate their understanding of teaching approaches and child development. Video was also used to guide group discussion of children's geometry thinking in Studio Time (Higgins, 2013), and Cognitively Guided Instruction (Jacobson & Lehrer, 2000) professional development, as described above.

2.4.3 *Do*

Learning happens not only through acquiring knowledge of concepts, or observations of others practices, but also through one's own experiences (Kolb et al., 2000). This may be especially important to consider in developing pre-service teachers understanding and ability to teach effectively. Teachers' field experiences during their preparation programs and other opportunities to practice skills can contribute to teachers' development of knowledge and skill (Darling-Hammond, 2010), and students in teacher preparation courses value these experiences and believe they will help their later in-service practices (Boyd et al., 2009). Teachers develop the ability to demonstrate skills over time (Hamre et al., 2015). Early on, they may practice skills individually, perhaps with a peer or small group of children. Later, they may be able to embed these skills seamlessly in classroom teaching. They are helped in doing this by have clear examples and specific skills to enact.

2.4.4 *Reflect*

Atkins & Murphy (1993) summarize reflection in three steps: awareness of a problem in practice (usually aroused by uncomfortable feelings or thoughts), critical analysis of the situation, and development of new perspective on the situation. In addition to these steps, there are both cognitive and affective skills needed to engage in the reflection process, including self-awareness, and the ability to describe, critically analyze, synthesize, and evaluate a given situation (Schon, 1991). Schon (1987) describes reflection as something that happens in two ways. Reflection-on-action refers to a professionals thinking about their thinking and actions after the fact in a way that leads to new understanding of practice and one's responsibility for new thinking and acting (Atkins, 1993). Second, Reflection-in-action refers to the often unconscious decisions made while actually engaged in a particular practice (for example, reading

a story book to a group of preschool children). Because these decisions are made in the moment and the practitioner may not be totally aware of each decision, this reflection-in-action can be difficult to articulate. Schon (1991) also uses the term “reflective practice” to highlight how reflection is connected to action, and more specifically, professional practice, thus leading to the term “reflective practice” to emphasize the focus of reflection on professional practice.

Reflection facilitates professionals learning from and through their own practice (Schon, 1987, 1991). Within each profession, practitioners “face unique and complex situations which are insolvable by technical rational approaches alone” (Schon, 1991). In the work of early childhood educators, there is an ongoing need to address the many complex problems that arise in practice, as well as implement best practices as defined by a professional, professional associations, governing bodies, and research. Reflective practice, as a process by which teachers engage in self-awareness, analysis, and new understanding about any number of challenges in their work, allows the teacher to propel their own ongoing professional development, typically in a group with other reflective practitioners. Research with professionals working with young children has found that they benefit from this process by having improved relationships with children and families, increased empathy and understanding of others, and improved practices with children (Gilkerson, 2004; Skatterbol, 2010; Virmani & Ontai, 2010).

Video feedback may be especially helpful in promoting reflection, and several studies have demonstrated how video feedback in the professional development of teachers of young children (Corcadden, Bell, & McCullagh, 2010; Bayat, 2010; Fukkink & Tavecchio, 2010; Rosaen, Lundeberg, Cooper, Fritzen, & Terpstra, 2008). Video feedback has been used as tool to improve pre-service teacher’s reflective capacities, as well as deepen knowledge of teaching in specific content areas (Bayat, 2010; Corcadden, Bell, & McCullagh, 2010; Rosaen et al., 2008).

In Bayat's study (2010), early education pre-service teachers experienced a course in which video recording and dialogue journals were utilized to improve pre-service teacher's self-analysis of their interactions with children using the Pre-K CLASS (Pianta et al. 2008) to focus on particular interactions. Similarly, pre-service elementary teacher students in the United Kingdom participated in video feedback by form of recording and reviewing video of themselves teaching a lesson, and then using multimedia technology programs to editing and annotating the video to highlight connections between practice and theory (Corscadden et al., 2010). Another pre-service teacher education program compared student's reflections on the same teaching event from memory and then with the aid of a video recording to explore added benefits of video review in the reflection process (Rosaen et al., 2008). Across these studies, video feedback led to increased specificity in students' observations (Corscadden, 2010; Rosaen et al., 2008), and to what Bayat (2010) describes as "productive reflection", in which students integrate "ideas about multiple aspects of teaching, such as learner and learning, subject matter, instruction, and assessment" (p. 161). Pre-service teachers across these studies also reported the benefits of using the various forms of video feedback in helping them recognize the positive practices they were doing (Corscadden et al., 2010; Rosaen et al. 2008) , as well as specific practices they wanted to improve (Bayat, 2008). Additionally, when comparing teachers written reflections from memory verses those written after watching video of the same lesson, Rosaen and colleagues (2008) found that students' reflections on teaching interactions became more focused on instructional aspects of the interaction (compared to behavior management aspects), and that pre-service teachers became more focused on the children's contributions in the lesson.

2.4.5 *Improve*

Changes in teachers' knowledge, ability to see, practices, and reflection are expected to occur as they engage in these very activities in coherent professional development. The Head Start National Center for Quality Teaching and Learning has identified two types of improvement expected to occur in professional development built on the above components. "I"mprovement refers to "students' demonstration of meeting overall course objectives", and "i"mprovement relates to smaller changes along the way during courses. Change or improvement can be measured in various ways, but should be related to the essential elements of the intentional teacher. Improvement, especially the smaller changes along the way, coupled with feedback can also be an important part of building students' belief in their ability to develop new and sometimes challenging practices (Bandura, 1993). Attention to how teachers develop skills and knowledge also is an important part of understanding the effectiveness of course elements and courses overall.

2.5 CONCLUSION AND THEORY OF CHANGE

Against the above backdrop, the following theory of change guided the design of professional development evaluated in the current study:

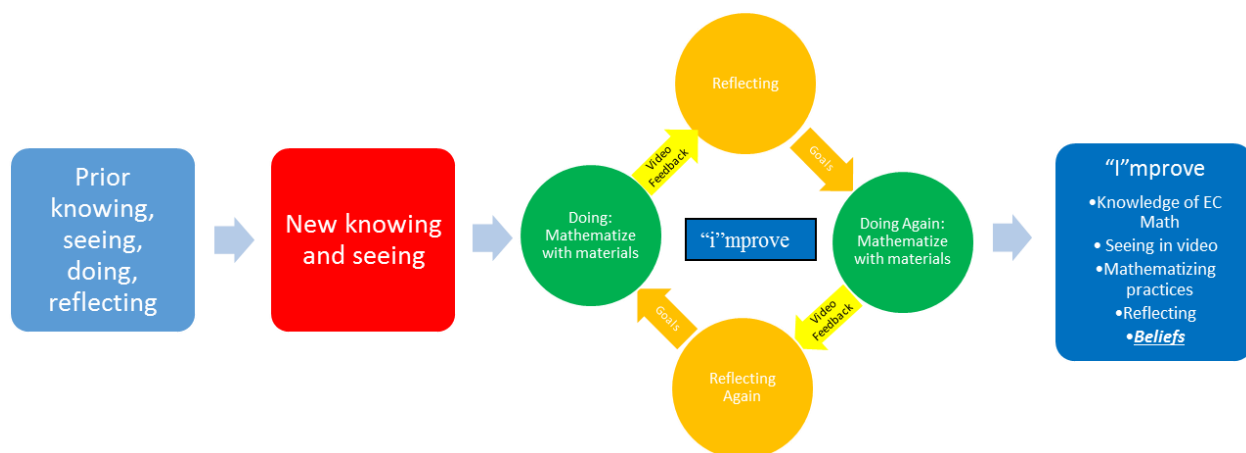


Figure 2 Theory of Change

Previous research provides evidence for the types of professional development that improve mathematics teaching and learning in early education and elementary settings (Clements & Sarama, 2012; Higgins, 2013; Jacobson & Lehrer, 2000). Key elements that each of these programs share are that teachers engage in reflection with others, professional development occurs over time (between three months to one year), video is used (of their own classrooms, or of other classrooms) to understand children's mathematical thinking, and teachers learning is linked to theory of children's mathematical learning. For the studies focused on professional development with preschool teachers specifically (Clements & Sarama, 2012; Chen & McCray, 2012) authors also cited the importance of attending to teachers' attitudes towards mathematics, and employed strategies to build teachers comfort and confidence with teaching mathematics. Additionally, the Know, See, Do, Reflect, and Improve framework provides a guide for the broad range of skills possessed by effective and intentional teachers and the process by which they can develop these skills.

The above findings and frameworks offer an important guide for new programs desiring to support early educators in their mathematics teaching and learning. The current study will use a holistic view of teacher development (Chen & McCray, 2012), which offers opportunities to for teachers to develop their knowledge as well as their ability to identify key features in mathematical interactions, practice these features, and reflect upon their experiences to integrate knowledge and prepare for change. Additionally, peer and instructor learning communities will support confidence, while video observation of children is expected to support accurate expectations of children's abilities. While the professional development itself will address all

of these elements, the current study will focus on the specific changes in practice, reflection, expectations, and confidence that might occur.

Given that much of the research on professional development of early educators in mathematics is focused on in-service teachers, this study aims to validate their strength with pre-service teachers. While studies demonstrating how pre-service teachers reflect provide insight on this cognitive aspect of their development (Bayat, 2010; Corscadden et al., 2010; Rosaen et al, 2008) there is less literature on their practices, especially related to math. This study will address how pre-service teachers take up opportunities to apply concepts learned in class, their reflections on experience, how they further respond in new experiences and reflection in multiple cycles, and how important beliefs like expectations and confidence change in the process. In doing so, the study will bring together multiple strands of inquiry on practice, reflection, and pre-service teachers' beliefs about children and themselves.

Chapter 3. METHODS

3.1 RESEARCH PARADIGM

The purpose of this study was to assess the intentional teaching framework as applied in a pre-service math course. Pre-service teachers' practice of mathematizing interactions with young children were observed and analyzed in terms of the frequency and responsiveness of mathematizing language. Pre-service teacher reflections after practice, in response to viewing video of their interactions with children, were observed for indicators of productive and unproductive reflection. Pre-service teachers' demonstrations of practice and reflection over time points were compared to determine whether, in fact, change, and specifically, Improvement, could be detected. Did their practice change in regard to frequency and responsiveness of

mathematizing? Did their reflections become more productive? Findings provided insight as to how pre-service teachers engaged in aspects of the Do and Reflect in the Know, See, Do, Reflect framework in this specific ECE Math course, and whether change occurred in terms of Improvement.

3.2 RESEARCH QUESTIONS

1. Did pre-service teachers engage in mathematics talk in their interactions with young children? If so, at what frequency?
 - a. Were there differences in frequencies of math language across classification, magnitude, enumeration, spatial relations, pattern, part/whole, and geometry? If so, what differences exist?
 - b. Did the frequency of mathematics language change during professional development? Did the frequency of specific mathematics domains change?
2. Did pre-service teachers provide responsive mathematics language to children? If so, at what rate?
 - a. Did the rate of responsiveness change during professional development?
 - b. Were there differences in pre-service teachers' rate of responsiveness when using mathematically focused materials (Magna-Tiles and math games) verses "free play" materials (open-ended toys and blocks)?
3. How did PSTs reflect on mathematics practice?
 - a. Did pre-service teachers engage in productive reflection? If so, how?

- b. Did reflections demonstrate change in pre-service teacher expectations and confidence teaching over time? If so, how?
- c. What relationship, if any, was there between PST's goals set in reflection and subsequent mathematics practices in video observations?

3.3 RESEARCH DESIGN

A multiple-case study design was used to examine four cases of pre-service teacher practice (mathematizing) and reflection in the context of the ECE Math course. This approach allowed for examination of multiple cases through qualitative data analysis over an extended time period (Lee, 2014), and is appropriate to describe the experiences and practices of pre-service teachers over an extended period of time and various stages of professional development (Rudd et al., 2009). Multiple data sources for each case presented multidimensional perspectives on practice and reflection, the phenomena of interest, and included video observations, participant reflections, and pre- and post-surveys of confidence in teaching early math skills. These data allowed for exploration of pre-service teachers' practice and reflection in the context of their interactions with children and the real world context of their ECE Math course. Within case comparisons explored whether and how frequency and responsiveness of mathematizing and productivity of reflection changed between time points in the course. Cross case comparisons examined whether and how individual patterns and themes in each case were similar or different from others.

3.3.1 *Design Selection and Alternatives Considered*

Several research designs were considered for the current study. Because this study focused on the individual experiences of a small set of pre-service teachers regarding a specific

set of behaviors and intervention, elements from case study, Applied Behavior Analysis, and design based research were considered. A final determination to follow the multiple-case study design was made because case study elements include the examination of multiple cases through qualitative data collected over an extended time period (Lee, 2014). Multiple data sources for each participant present multidimensional perspectives on the phenomena of interest (math language and reflection), and include multiple video observations and multiple participant reflections. These data allow for exploration of pre-service teachers' math language and reflection in the context of their interactions with children and the real world context of their ECE Math course.

Applied Behavior Analysis design elements were also relevant, particularly related to the studies focus on math speech, conceived of as a specific behavior, whether and how it changes over time, and whether and how it changes in relationship with intervening events (experience with the mathematics course elements). The study follows an ABBA design in that the first video observation presents a pre-intervention observation of math speech, followed by the introduction of intervention activities intended to increase mathematizing (including lecture on mathematizing, provision of math materials to use with children, and guided video feedback and reflection) for the two middle observations, followed finally by the last video observation in which materials and reflection are not involved. This study focused on the relationship between mathematizing speech and the intervention activities, particularly whether and how materials and reflection relate with changes in mathematizing speech. However, the data available does not allow for a full applied behavior analysis with multiple subject design because there is only one pre-intervention observation and one post-intervention observation, which is not enough to create

a stable baseline or make claims about the sustained effect of the intervention over multiple observations following the intervention.

Design based research elements were also considered. This study focused not only in whether or not intervening features (materials, reflection) relate to change in math speech, but also the additional limitations/affordances these designed activities bring to the endeavor of pre-service preparation for mathematics. However, data in the current study only account for one iteration of course implementation, whereas design based studies typically include multiple iterations of the design and outcomes. Therefore, this approach was not used in the current study.

3.4 SETTING

The study took place in the context of an undergraduate Early Childhood Mathematics Course. The course was an open elective and twelve students from an Early Childhood Family Studies major and other majors enrolled. Content addressed core mathematics domains learning and development (number, geometry, and measurement), interaction strategies like mathematizing, and curriculum and assessment. The course followed the Know, See, Do, Improve framework for pre-service teacher learning, and content areas were divided into modules that addressed each component of the framework. For example, the module on geometry and spatial sense included readings and lecture which outlined the development of early geometry and spatial sense (Know), viewing video and reading case examples of geometry and spatial sense learning and teaching (See), an assignment in which students engaged young children in learning geometry and spatial sense skills, which was video recorded (Do), and a reflection assignment, in which students viewed their video, identified strengths and

challenges, and planned for improved practice (Improve). During the course students also used the Coaching Companion Campus Edition to upload and share video observations of their teaching with peers and the instructor. The Coaching Companion Campus Edition is a web-based application developed at the University of Washington in which students and teachers can upload and annotate video, providing an opportunity for video feedback and reflection as a part of course assignments.

The course focused specifically on mathematizing, a topic addressed in several sessions, and through homework assignments. Assignments included video observation. During the course, students submitted four video observations of their interactions with one or two children ages three to five. The first and fourth videos were of free play and did not have a specific math focus. In the second and third videos students were instructed to use math materials (magnatiles) and focus on mathematizing the play interaction with the children. After the second and third videos students participated in video feedback, reflection, and discussion in communities of practice via the Coaching Companion. A more complete explanation of course experiences will be given in the Procedures section.

3.5 PARTICIPANTS

Twelve undergraduate students in the Early Childhood Mathematics course were invited to participate in the study. Of the 12 students, eight chose to participate in the study. Seven of the eight students were in their fourth year, one was in her first year at the university. Six of the eight were in the Early Childhood Family Studies undergraduate major, while the other two were majoring in other areas. Participation in the study was voluntary. Of the eight students who consented to participate in the study, four also obtained parental consent for the children they

worked with in videos to also be included in the study, and they also completed all of the course requirements (video submissions, reflection and feedback submissions) that comprised the data for the current study. These four student cases, therefore, are the final set that will be considered in the present study. Each of these four participants were students in an early childhood and family studies major, and each participant was between 18 – 25 years of age at the time of the study. Henceforth, students will be referred to as pre-service teachers (or PSTs).

Ji hye (pseudonym) was a fourth year pre-service teacher. Ji hye is Korean. Although Ji hye was placed in a service learning site, she was not allowed to video record there. She therefore recruited her cousin, Yu jin (pseudonym), to participate in the video observations. Yu jin is also Korean, and was three years old during the study. All four video observations were completed in Yu jin's home, either in the living room or dining room.

Melissa (pseudonym) was the only first year pre-service teacher, and she is European American. Melissa reported having no prior teaching experience, and was not in a service learning placement at the time. The instructor arranged for her to complete video observation assignments with a child whose parents had given permission for participation. This child, Odessa (pseudonym), is a European American girl who was three years old at the time of the study. All four video observations were conducted at Odessa's home.

Eun ji (pseudonym) was another fourth year pre-service teacher. Eun ji is Korean. Eun ji participated in a service learning placement in a local preschool, and recruited two children to participate in her video observations over the quarter. These were two boys, Derek and Mark (pseudonyms). Each boy was four years old, and each European American. All videos were recorded at the preschool, two of the videos were in the classroom during free play while other children could be heard, but not pictured in the video. Two of the videos were recorded either

when the classroom was empty or in the hallway outside the classroom. Only Eun ji, Derek, and Mark could be heard and seen in these videos.

Tara (pseudonym) was a fourth year pre-service teacher at the time of the study. She is European American. Tara requested help in finding children to interact with to complete course video assignments. The instructor matched her with two children whose parent had agreed to participate in the course assignments and study. The children were two girls, Faith and Jasmine (pseudonyms), who were five and three years old, respectively, at the time of the study. Both girls are African American. Tara, Faith, and Jasmine met four times over the quarter, two times at Tara's house, and two times at Faith and Jasmine's home. Faith's mother reported she attended preschool, and Jasmine attended child care.

Table 1 Participants

Name	Gender	Ethnicity	Age
Group 1, Setting: Home			
Ji hye	Female	Korean	18-25
Yu jin	Female	Korean	3
Group 2, Setting: Home			
Melissa	Female	European American	18-25
Odessa	Female	European American	3
Group 3, Setting: Preschool			
Eun ji	Female	Korean	18 - 25
Derek	Male	European American	4
Mark	Male	European American	4
Group 4, Setting: Home			
Tara	Female	European American	18 -25
Faith	Female	African American	5
Jasmine	Female	African American	3

3.6 TIMEFRAME

Data were collected over the 10 weeks of the ECE Mathematics Course. In week one, pre-service teachers were introduced to the course and study and given the option to participate in the study. Those who wanted to participate returned signed consent forms as they left the class. Week one also consisted of completion of a pre-survey of their confidence and beliefs related to teaching mathematics. All, regardless of study participation, completed the same in class and homework assignments over the following weeks. Over the following week, pre-service teachers recruited and obtained parental consent for a child with whom they could practice mathematics interactions over the quarter. The researcher/instructor helped students who could not arrange this by arranging pairings with families with young children who were willing to participate in the course and the research. Small tri-pods and video recording devices were provided to each pre-service teacher by the college for use in the course, which they used to capture video data. In three out of the four cases, the student teachers simply set up the tripod nearby their planned location of interaction, and the camera remained stationary throughout the observation. In one case, the student teacher had the children's parent hold the video recording device during several of the observations.

In week two, pre-service teachers engaged in their first video observation, video recording themselves in a free play activity with their selected children for five to ten minutes, and then uploading this video to the Coaching Companion. In their first video observation, pre-service teachers were informed that free play could consist of any child chosen activity, with no particular curricular focus. This video served as a baseline observation of mathematizing speech in a general setting.

In week three pre-service teachers were introduced to content related to mathematizing and geometry, provided with Magna-tiles™, and assigned to complete another five to ten minute video observation, this time practicing mathematizing children's play with Magna-tiles™. Pre-service teachers completed this assignment over the following week, and submitted their video to the Coaching Companion.

In week four, pre-service teachers viewed their second video observation with the Magna-tiles™. They completed an individual reflection assignment in response to their video with instructor provided prompts designed to promote productive reflection (such as “What do you notice about the child's math learning?”, “What do you notice about your math teaching?”) (Appendix D). These prompts were based on the work of van Es (2002) with the goal of scaffolding PSTs noticing aspects of children's mathematics learning and their teaching. The desired outcome of these prompts for reflection was that PSTs would write what Bayat (2010) refers to as “productive reflections”, that is, reflections which focus on children as individual learners, on children mathematics content knowledge, on children's thinking and learning (including mathematics learning), and the quality of PST's mathematics instruction. “Productive reflection” is in contrast to “unproductive reflection”, which focuses on general descriptions of the lesson and listing PST's opinions and preferences. PSTs were also prompted to describe changes they plan to make in their future interactions with children which promote mathematics learning and teaching. Students also participated in peer learning communities, in which a group of three to four peers provided feedback on each other's videos. Feedback was scaffolded with prompts provided by the instructor.

In week five, pre-service teachers again gathered five to ten-minute video observations of their work with children, focused on their Mathematizing practice with the use of Magna-tiles™

or other math materials they had been introduced to in the course. These videos were again uploaded to the Coaching Companion, and in week six, pre-service teachers participated in a second round of reflection on their video, responding to the same instructor provided prompts, and peer learning communities.

In week eight, pre-service teachers collected and submitted a final five to ten-minute video observation, this time returning to the free play format. No reflection on the video on peer learning community was conducted following this final video observation. In week ten, a post-survey of pre-service teacher confidence teaching mathematics was administered during the last session of the course.

3.7 DATA COLLECTION

3.7.1 *Video Observations*

Sixteen total video observations were collected for data analysis (four for each pre-service teacher case). Each video was approximately five to ten minutes long. All videos were collected in the Coaching Companion, then transferred for analysis to InqScribe software. InqScribe is a video and transcript analysis software application which supports development of transcripts, as well as analysis of video, audio, and transcript data. The application is ideal for use by qualitative researchers to “Analyze and code video with time stamped tags”, according to the InqScribe website (www.inqscribe.com).

Table 2 Video Log

Video Name	Length	Context	Participants	Materials
<i>Eun Ji Video 1</i>	9:58	Playing with interlocking blocks in classroom	Eun Ji Derek Mark	Blocks
<i>Eun Ji Video 2</i>	10:00	Constructing with Magna-tiles in classroom	Eun Ji Derek Mark	Magna-Tiles
<i>Eun Ji Video 3</i>	9:02	Comparing length with cars in school hallway	Eun Ji Derek Mark	Open-Ended
<i>Eun Ji Video 4</i>	9:34	Constructing with unit blocks in classroom	Eun Ji Derek Mark	Blocks
<i>Melissa Video 1</i>	10:00	Playing with toys and games in child's room	Melissa Odessa	Open-Ended
<i>Melissa Video 2</i>	9:34	Arranging shapes with Magna-tiles in living room	Melissa Odessa	Magna-Tiles
<i>Melissa Video 3</i>	10:00	Playing math games at kitchen table	Melissa Odessa	Math Games
<i>Melissa Video 4</i>	10:00	Creating with pipe cleaners at kitchen table	Melissa Odessa	Open-Ended
<i>Ji Hye Video 1</i>	10:00	Arranging parquetry blocks in living room	Ji Hye Eun Jin	Blocks
<i>Ji Hye Video 2</i>	10:00	Arranging and constructing with Magna-tiles in living room	Ji Hye Eun Jin	Magna-Tiles
<i>Ji Hye Video 3</i>	9:59	Reading pattern strips, playing math games in living room	Ji Hye Eun Jin	Math Games
<i>Ji Hye Video 4</i>	9:53	Reading pattern strips, playing in living room	Ji Hye Eun Jin	Math Games
<i>Tara Video 1</i>	10:00	Playing with dress up in child's room	Tara Faith Jasmine	Open-Ended
<i>Tara Video 2</i>	10:00	Arranging and constructing with Magna-tiles in living room	Tara Faith Jasmine	Magna-Tiles
<i>Tara Video 3</i>	10:00	Playing math games in living room	Tara Faith Jasmine	Math Games
<i>Tara Video 4</i>	9:04	Playing with dress up in child's room	Tara Faith Jasmine	Open-Ended

3.7.2 *Transcripts*

Sixteen total video observation transcripts were created for data analysis (one for each video in each pre-service teacher case). Transcripts of video dialogue were written verbatim for each video. Rev.com, an online transcription service, wrote transcripts of spoken language. Rev.com is used by a number of researchers in the same college as the current study's researcher, and is an efficient and inexpensive way to obtain video transcripts. Rev.com's website assures 99% accuracy of transcripts (www.rev.com). Rev.com also reports that its services are appropriate for a broad range of clients, including academic researchers.

3.7.3 *Reflective Entries*

Eight total reflective entries were collected for data analysis (two for each pre-service teacher case) via the Coaching Companion. Entries were written and contained responses to reflection prompts (see Appendix D).

Table 3 Reflection Log

Participant	Reflection	Video Reviewed	Number of Words	Materials
<i>Eun Ji Video 1</i>	9:58	Playing with interlocking blocks in classroom	Eun Ji Derek Mark	Blocks
<i>Eun Ji Video 2</i>	10:00	Constructing with Magna-tiles in classroom	Eun Ji Derek Mark	Magna-Tiles
<i>Melissa Video 1</i>	10:00	Playing with toys and games in child's room	Melissa Odessa	Open-Ended
<i>Melissa Video 2</i>	9:34	Arranging shapes with Magna-tiles in living room	Melissa Odessa	Magna-Tiles
<i>Ji Hye Video 1</i>	10:00	Arranging parquetry blocks in living room	Ji Hye Eun Jin	Blocks
<i>Ji Hye Video 2</i>	10:00	Arranging and constructing with Magna-tiles in living room	Ji Hye Eun Jin	Magna-Tiles

<i>Tara Video 1</i>	10:00	Playing with dress up in child's room	Tara Faith Jasmine	Open-Ended
<i>Tara Video 2</i>	10:00	Arranging and constructing with Magna-tiles in living room	Tara Faith Jasmine	Magna-Tiles

3.8 INSTRUMENTS AND ANALYSIS

3.8.1 *Early Mathematics Language Coding System*

In order to answer first research question (Did pre-service teachers engage in mathematics talk in their interactions with young children? If so, at what frequency?) the Early Mathematics Language Coding System (EMLCS) (see Appendix A) was developed. The EMLCS is a coding scheme designed to measure frequency of language across mathematics domains. The EMLCS was developed based on a similar coding scheme originally created by Ginsburg, Lin, Ness, & Seo (2003) and later refined by Austin, Blevins-Knabe, & Lokteff (2013). The original coding schemes were developed to document the presence and frequency of math engagement of young children in international settings (Ginsburg et al., 2003), and in home care settings (Austin et al., 2013). Codes in original versions spanned six mathematics domains (classification, magnitude, enumeration, pattern, spatial relationships, part/whole relationships). The coding scheme in the present study created an additional domain, geometry, which in previous schemes was combined with spatial relationships. Geometry was added to better align the instrument with expected language used by pre-service teachers in the ECE Mathematics course, which focused specifically on geometry and the use of Magna-Tiles™, a material chosen by the researcher specifically to support geometry language.

The study used the codes to track presence and frequency of mathematics language across domains. Utterance level coding occurred, and the presence of math language in a domain was counted when it occurred in an utterance. Utterances often contained instances of math language from more than one domain, each of which was counted (Klibanoff et al, 2006). For example, when a pre-service teacher said, “Now it's a square. It looks like a square, look it's just like these. See if you put this on top you won't be able to see it because it matches up,” this would be considered an utterance containing shape (“Now it’s a square”, “It looks like a square”, “it matches up”), and spatial relations (“if you put this on top”), for a total of two domain instances of math language in one utterance.

A 20% sample of videos and transcripts were double coded by the researcher and a graduate student trained on the coding schemes. Codes were compared to estimate interrater reliability. The agreement between raters was checked to determine the interrater reliability, using a Kappa statistic, (κ), analogous to Rudd et al. (2009). The Kappa statistic is useful to establish agreement while accounting for random agreement in qualitative research in which two coders independently assign categories to items (Cohen, 1960). Interrater reliability ranged from (κ)= 0.75 to (κ)= 0.89 for overall mathematics codes. Individual kappa statistics for each category are reported in Appendix B. Transcripts were coded for presence and frequency of mathematics related language using the EMLCS. Frequencies were tabulated and graphed across mathematics domains, time points, and material categories. Visual analysis was conducted to compare domain specific and overall math language frequencies differences by time point and material within each case. This led to determinations as to whether and to what extent time and material conditions related to the frequency and domains of mathematics language.

3.8.2 *Mathematizing Opportunity Ratio Tool*

To answer the second research question (Did pre-service teachers provide responsive mathematics language to children? If so, at what rate?) the Mathematizing Opportunity Ratio Tool (MORT) (see Appendix C) was developed. The MORT is designed to capture the ratio of teachers' mathematized speech to children's math engagement behaviors. Child math engagement behaviors are defined as looking, touching/gesturing, and speech opportunities which a teacher could related to mathematical ideas or objects. Mathematized responses are defined as adult response with mathematical language related to the mathematics engagement behaviors presented by the child (known as opportunities). When children demonstrate a mathematics engagement behavior, this is an opportunity for the pre-service teacher to respond with a mathematizing comment. The number of opportunities children present and the mathematizing responses create a ratio of opportunity to response, called the "Mathematizing Opportunity Ratio". The number of pre-service teacher responses can be divided by the number of opportunities to further create an "opportunity to mathematize take up" percentage, indicating the proportion of opportunities pre-service teachers took up in response to children's behaviors which present an opportunity for mathematizing. The MORT is based on a similar tool called the *Examination of the Implementation of Embedded Intervention, through Observation* (EIEIO) (McWilliam & Scott, 2001), designed to capture the number of opportunities children have to learn skills in embedded instruction and how frequently teachers appropriately respond to children engagement behaviors.

Videos were analyzed, and ten second intervals were coded. This time was determined based on the need to have enough time to observe an opportunity and a potential opportunity take up. In the 10 seconds a code was assigned to children for presence of the opportunity definition

(look, touch, talk), and the pre-service teacher for presence of response (mathematizing) or a teacher initiated mathematizing comment. Ten second segments were used because even with opportunity definitions, opportunities were nearly continuous (such as “look”), and it was necessary to brake time segments apart in which codes and opportunity/response could be assigned.

A 20% sample of videos and transcripts were double coded by the researcher and a graduate student trained on the MORT. Codes were compared to estimate interrater reliability. The agreement between raters was checked to determine the interrater reliability, using a Kappa statistic, (κ), analogues to Rudd et al. (2009). Interrater of (κ)= 1.0 was achieved for all categories except children’s look and children’s touch, which were so infrequent that a kappa statistic could not be determined. Individual Kappa statistics for each category are reported in Appendix D.

Table 4 Engagement and Response Definitions

Child Math Engagement Behavior	Definition
Looking	Child looks at a math related object with interest
Touching	Child touches or manipulates objects with mathematical focus, or using physical movements (gestures) with or without an object to explore or explain a mathematical concept
Talking	Child talks about math idea/uses math words (see definitions in language coding tool)
Adult Mathematized Response	Definition
Mathematizing	Talking about mathematical concepts in relation to what children are doing/saying

Each video was analyzed for instances of child mathematics engagement and pre-service teachers mathematizing responses, resulting in a ratio of child engagement to pre-service teacher mathematizing response. Ratios across each video per participant will be analyzed and compared for patterns in change across time points, and differences in ratios between interactions with math materials compared to free play videos.

3.8.3 *Reflection Coding and Analysis*

Reflective journal entries were qualitatively analyzed using codes based on characteristics of productive and unproductive reflection, in line with previous research on characteristics of pre-service teacher reflection (Bayat, 2010) (Appendix E). Two reflective entries in each case were examined to identify presence of characteristics of productive and unproductive reflection. Within case comparison of productive/unproductive codes will identify whether codes in either category change between the first and second reflection. A consistency check of 25% of entries was conducted by the author and a graduate student which established agreement in coding journals along characteristics of productive reflection.

A second round of analysis on reflections was conducted with a focus on content related to plans for future practice. Written plans for future practice will be identified and compared to demonstration of practice in subsequent video to identify whether and how plans identified in reflection journals were later enacted in practice.

Chapter 4. RESULTS

PSTs took up mathematics language and made changes in terms of frequency and responsiveness with young children. They also engaged in reflection that was productive and, in most cases, generated goals which informed subsequent practice. These results will be explained in more detail to follow. The first section addresses PSTs mathematical language in terms of frequency and responsiveness. For this section, the primary data sources are PSTs videos and transcripts of video. First, I describe the frequency of PSTs mathematical language overall and change across time point. I then describe the rate of responsiveness of this language and change across time point. Finally, I present results of analysis of difference between materials for frequency and responsiveness of language.

The second section of the results address PSTs reflection. Here, the primary data source is their written reflection journals. In this section I first describe whether and how PSTs journals included characteristics of “productive reflection” (Bayat, 2010). I then turn to whether and how PST’s reflections changed. Here I focus on two beliefs: their expectations of children and confidence in their own practice. Finally, at the end of this section, I address PST’s goals set in reflection. Here, I triangulate between reflection journal data, video data, and transcript data to determine whether and how PST’s goals related to subsequent practice. The chapter then concludes with a summary of results.

4.1 PRE-SERVICE TEACHERS’ MATHEMATICAL LANGUAGE

4.1.1 *Mathematical Language Domains and Frequency*

Transcripts were coded for presence of mathematics speech in PST and child utterances across seven mathematics domains. Coding resulted in frequency counts for mathematics language for each transcript of video, and provided total mathematics language frequencies for PSTs and children, as well as the frequency of mathematics and in each domain. PSTs engaged in mathematics talk with young children with varying frequency across mathematics domains and time points. The amount of math language provided ranged from 24 to 115 instances per observation, with an average of 62.75 ($SD = 31.13$) math language instances per observation. Language by mathematics domain varied, with PSTs providing math language in as few as four and as many as six (out of seven) domains in a given video. Examples of language in each domain are given in Table 5. Pre-service teachers most frequently provided enumeration language, constituting 33% of total mathematical language, followed by shape at 20%, spatial relations at 19%, and magnitude at 18% of total instances. The remaining three domain categories (pattern, 7%; part/whole, 2%; classification, 1%) combined accounted for 10% of mathematics language instances. Table 6 presents the frequency of mathematics language across domains.

Table 5 Mathematical Language Type Examples

Mathematical Language Type	Example
Classification	“Let’s put all the squares in one pile and all the triangles in another pile.”
Magnitude	“There’s a lot here”
Enumeration	“How many green beans are there?”
Patterning	“What comes next in the pattern?”
Spatial Relations	“Let’s put the small blocks in the bucket.”
Part/whole	“Of the group of beans, about half are green.”
Shape	“If we put these two triangles together, we can make a square!”

Table 6 Frequency of Mathematics Speech Instances Across Domains

Mathematics Domain	Raw number across observations	Proportion of total instances	Number of observations with domain type
<i>Enumeration</i>	329	0.33	16
<i>Shape</i>	200	0.20	14
<i>Spatial Relations</i>	186	0.19	16
<i>Magnitude</i>	184	0.18	16
<i>Pattern</i>	74	0.07	3
<i>Part/Whole</i>	16	0.02	6
<i>Classification</i>	15	0.01	7

Cross case review of PSTs provided another view of the variation in frequency and type of mathematical language. Table 7 presents frequencies from each case at each video observation time point. Across time points, frequency of mathematics language changed, and in the majority of cases this change was extreme, especially between the first and second video observations. To varying extents, this was then followed by a tapering in mathematics language frequency in videos three and four. Three of the pre-service teacher cases show a greater range of mathematics language frequencies across videos. Eun Ji ranged from a low of 36 instances of mathematics language in her initial observation to a high of 115 instances in her second observation, a difference of 79 instances. Ji Hye ranged from a low of 31 instances of mathematics language in video 1 to a high of 115 instances in video 2, a difference of 84 instances. Tara had a low of 28 mathematics language instances in video 1 and a high of 102 instances in video 2, a difference of 74 instances. In contrast, while there was change in the frequency of mathematics language in Melissa's video observations, this change was less extreme. Melissa's first video contained 32 mathematics language instances, followed by 51 instances in her second video observation, a difference of 19 instances, a much narrower range of

change compared to the average range of change of 79 instances for Eun Ji, Ji Hye, and Tara. Based on the spike of mathematics language frequency for three of the four PSTs, it appears that in the second video observation, these PSTs may have hyper implemented the recently learned strategy of mathematizing, which they had been introduced to in the course the week prior to the video observation. On the other hand, Melissa's increase in mathematics language was more measured.

In three of the four cases there was an increase in mathematics language instances between videos one and four, the first and last video observations. The average range of difference between videos one and four was 29.5 instances of mathematics language across all four PSTs. That is, on average, there was an increase of 29.5 mathematics language instances between PST's first and final video observations. The range of difference for Eun Ji between videos one and four was an increase of 34 mathematics language instances. The range of difference for Melissa between videos one and four was an increase of 20 mathematics language instances. The range of difference for Sang Eun was an increase of 68 mathematics language instances between videos one and four. In contrast to these three cases, Tara's is one case in which there was very minimal change, with a decrease of four mathematics language instances between video one and four.

Table 7 Mathematical Language Across Video Observations

Pre-Service Teacher	Video 1	Video 2	Video 3	Video 4
<i>Eun Ji</i>	36	115	59	70
<i>Melissa</i>	32	51	45	52
<i>Ji Hye</i>	31	115	76	99
<i>Tara</i>	28	102	69	24

4.1.2 *Mathematical Language Responsiveness*

Video data were analyzed and coded every ten seconds for the presence of child mathematizing opportunity behaviors (looking, touching, and/or gesturing towards objects, math related speech, and general speech) and whether PSTs responded using mathematical speech. Pre-service teachers mathematized response rate ranged from 23% to 84%, with an average response rate of 57% ($SD = 19.14\%$). Of the child mathematics engagement behaviors, PSTs responded to children's mathematics speech at the highest rate, with an average response rate of 50% ($SD = 19.21\%$), compared to an average response rate of 31% ($SD = 17.18\%$) to children's non-speech math engagement behaviors, and 15% ($SD = 7.64\%$) to children's general speech.

Table 8 Rate of Mathematizing Across Videos

Pre-service Teacher	Video 1	Video 2	Video 3	Video 4
<i>Eun Ji</i>	0.43	0.73	0.65	0.54
<i>Tara</i>	0.23	0.84	0.63	0.43
<i>Ji Hye</i>	0.26	0.67	0.76	0.84
<i>Melissa</i>	0.35	0.53	0.71	0.60
Rate	0.32	0.69	0.69	0.6

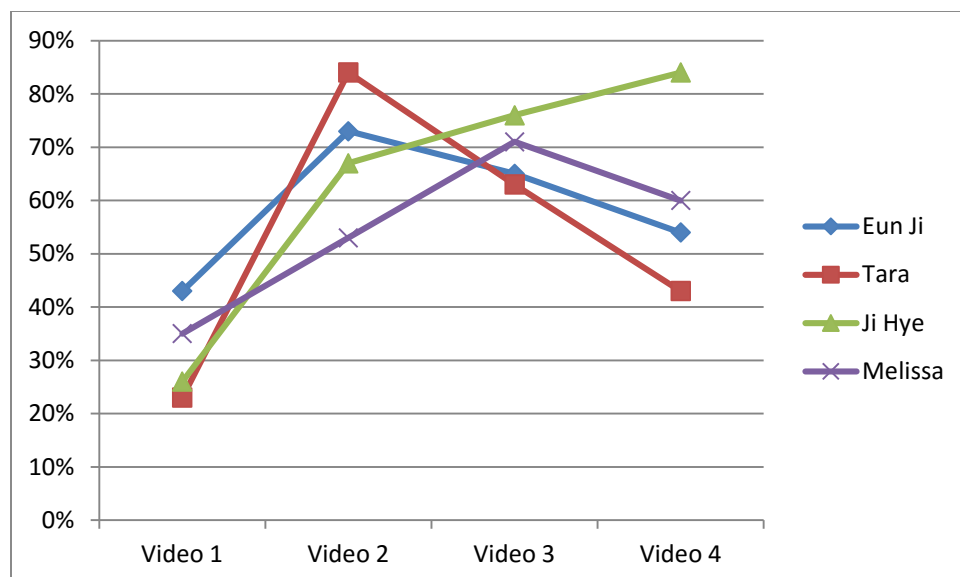


Figure 3 Graph of Pre-servie Teacher Mathematizing Rates Across Videos

PST's response rates changed across time points, and in all four cases their response rates increased between videos one and four, the first and last video observation. This is presented in Table 8. The average range of difference between videos one and four was 29% change in response rates. That is, on average, PSTs provided mathematical language responses to child engagement 29% more in video four than in video one. The range of difference for each PST is as follows: Eun Ji increased by 11%, Melissa's by 25%, Sang Eun by 58%, and Tara by 20%.

These findings are interesting to compare with findings on mathematics language frequency because the trends do not always follow the same path. For example, in Tara's case, mathematical language frequency was lower in video four verses video one, indicating a decrease in mathematics language, while her response rate in video four was higher than in video one, an increase. It appears that two changes in language interaction can be observed: while there may be fewer mathematical words provided, the interaction is more responsive in terms of children's mathematical engagement. Tara picked up on the children's cues and followed them

with mathematical language more frequently than in her first video, even though she used fewer math terms as she did so. This indicates an interaction more aligned with child interests, an aspect of teacher-child interaction associated with sustained child engagement and learning (Gregory et al., 2003; McWilliam & Scott, 2001).

In other cases, such as Ji hye's, both response rates and math language frequency increased over time. Ji hye's case demonstrates the greatest change in frequency of math language and rate of responsiveness relative to her peers. Sixty-eight more mathematics language instances are observed in her final video than her first, and her response rate increases 58%, from 26% in video one to 84% in video four. Ji hye manages to more frequently respond to her student's mathematics engagement, and also use more mathematical language as she does so.

Eun Ji and Melissa follow a similar pattern as Ji Hye, providing mathematical language more frequently over time and responding more frequently to children's mathematical engagement. However, the range of difference for Eun Ji and Melissa is narrower than Ji Hye's. Thirty-four more mathematical language instances were observed in Eun Ji's fourth video than her first, and her response rate increased by 11%, from 43% to 54%. Melissa had 20 more mathematical language instances in video four compared to video one, and an increased response rate of 25%, from 35% to 60%.

4.1.3 *Mathematical Language Frequency, Domains, and Responsiveness Across Materials*

Next, I considered the relationship between materials and math language frequency and responsiveness. Specific materials, such as blocks and board games, are associated with improved teacher math talk (Greggory et al., 2003) and gains in children's math skills (Siegler & Ramani, 2009). I was therefore interested in whether and how materials in the study would

affect pre-service teacher's mathematics language. PSTs and children played with a variety of materials across video observations. These materials fit into four categories: Magna-Tiles, mathematics games, blocks, and open-ended. As Figure 8 illustrates, overall, the frequency of mathematics language was highest with Magna-Tiles, followed by mathematics games, blocks, and open-ended materials. The frequency of mathematics words in by domain varied between materials. For example, while Magna-Tiles prompted more enumeration language followed by shape language on average, on the other hand, blocks prompted more shape language, followed by spatial relations on average (see Table 9). The response rate of PSTs was different between materials, with the highest response rate, on average, associated with mathematics games, followed by Magna-Tiles, open-ended, and blocks (see Figure 4.3). The specifics of mathematics language frequency and response rates for each material will follow.

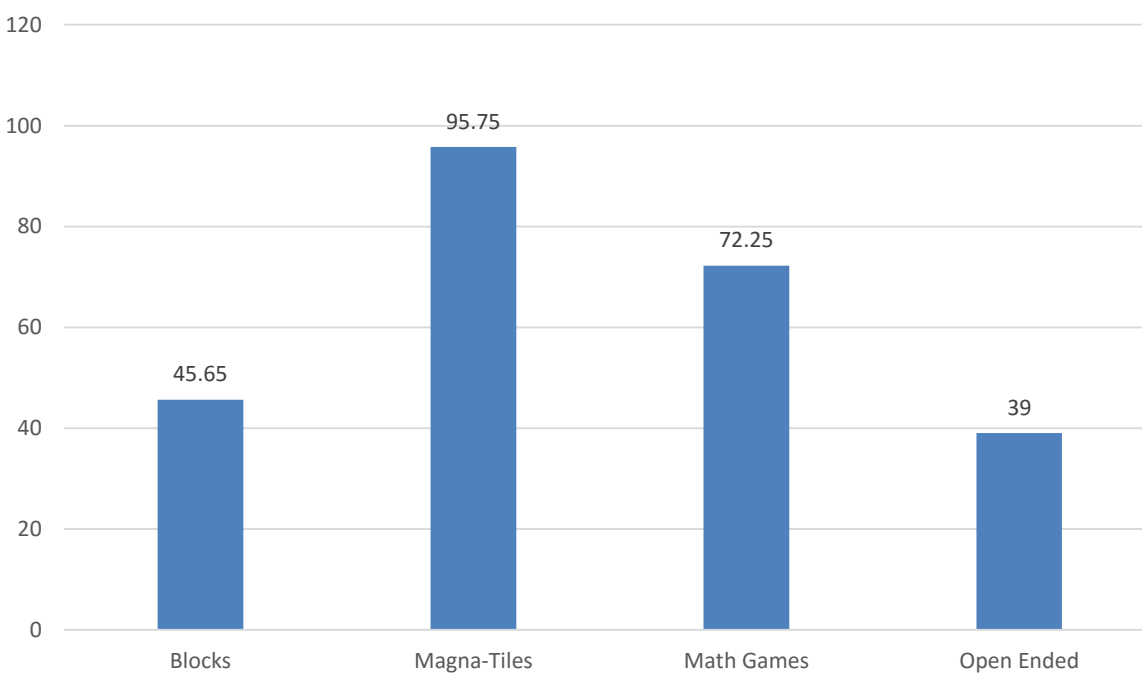


Figure 4 Mathematical Language Frequency by Material

Magna-Tiles

Magna-Tiles were used in video two for each PST, as they were prompted to use this material in their interaction with children, and as such were in four of the sixteen total videos. Magna-Tiles were selected because they are a type of block, and can be used to build a variety of 2-dimensional designs and 3-dimensional structures. They are light weight and easy for PSTs to transport to and from visits with children, and their translucent multi-color design makes them visually appealing, and therefore were expected to pique young children's interest. Overall, the frequency of mathematics language was the highest with Magna-Tiles compared to other materials, with an average of 95.75 mathematics language instances per video. Of the mathematics language instances, enumeration and shape language were most common, each constituting 31% of the total mathematics language instances, followed by magnitude at 21%, spatial relations at 13%, classification at 3% and part/whole at 1%. Pattern was not observed.

Mathematics Games

Mathematics Games were observed in four of the sixteen videos. Mathematics Games included "The Green Bean Game" and "Animal Dots", both of which target counting and cardinality skills, as well as "The Pattern Strip", an exercise in which children predict the next unit in a pattern, and then make their own pattern. Mathematics games were introduced to the PSTs in the course in week five by the instructor. Overall, the frequency of mathematics language was 72.25 instances per video, less than Magna-Tiles, and more than blocks and open ended materials. Enumeration was the most common mathematics domain during games, constituting 40% of the mathematics language, followed by pattern at 26%, spatial relations at 14%, magnitude at 10%, shape at 7%, part/whole at 2%, and classification at 1%. Mathematics games were the only category of materials in which language from each of the seven mathematics domains was observed.

Blocks

Of the sixteen total video observations across all PSTs, three videos had blocks as the primary materials. Blocks included interlocking flat blocks and wooden unit blocks. PSTs and children typically played with these materials in unstructured and child directed ways, and in two of the three videos with blocks, the children had selected this material themselves and the PST joined in their play. In the other video, it is not clear whether the child selected blocks or the PST provided them, yet, play in this video is primarily child directed. There was an average of 45.66 mathematics language instances per video, and PSTs used language from five of the seven mathematics domains. In order of frequency these are shape at 30%, spatial relations at 24%, enumeration at 22%, magnitude at 21%, and part/whole relationships at 3%. Classification and pattern were not observed.

Open-Ended Materials

Open-ended materials were also primarily used during free play videos. Of the sixteen total video observations across all PSTs, five had open ended materials as the primary materials. Open-ended materials included dress up clothes and props such as kitchen items, as well as art materials including pipe cleaners, beads, and markers. PSTs and children typically played with these materials in unstructured and child directed ways. There was an average of 39 mathematics language instances per video with open ended materials, and PSTs used language from six of the seven domains. In order of frequency, these include enumeration at 33%, spatial relations at 31%, magnitude at 24%, shape at 11%, part/whole relationships at 1%. One instance of classification was observed (0.05%). Pattern was not observed.

Table 9 Average Mathematical Language Frequency Across Materials

Material	Mean Words Per Video	Mathematics Domain	Avg. Frequency
Math Games	72.25	Enumeration	29.00
		Pattern	18.50
		Spatial Relations	10.50
		Magnitude	7.50
		Shape	5.00
		Part/Whole	1.25
		Classification	0.50
Magna-Tiles	95.75	Enumeration	29.75
		Pattern	0.00
		Spatial Relations	12.75
		Magnitude	19.75
		Shape	29.50
		Part/Whole	1.00
		Classification	3.00
Open-Ended	39.00	Enumeration	12.80
		Pattern	0.00
		Spatial Relations	12.00
		Magnitude	9.20
		Shape	4.20
		Part/Whole	0.60
		Classification	0.20
Blocks	45.66	Enumeration	10.00
		Pattern	0.00
		Spatial Relations	11.00
		Magnitude	9.66
		Shape	13.66
		Part/Whole	1.33
		Classification	0.00

Rate of Responsiveness Across Materials

Mathematics Games

Mathematics games were observed in four of the sixteen videos, and as described above included “The Green Bean Game”, “Animal Dots”, and “The Pattern Strip”. Overall, at 74%, math games had the highest average response rate compared to other materials (see Figure 9). PSTs responded with mathematics language to children’s engagement behaviors most frequently when playing math games compared to other materials. As presented in Table 10, the rate at which PSTs responded to different engagement behaviors also varies. PSTs were most responsive to children’s mathematical speech with an average response rate of 57%, then to children’s non-verbal engagement with a response rate of 48%, followed by general speech with a 19% response rate when playing mathematics games.

Magna-Tiles

Magna-Tiles were used in video two for each PST, as they were prompted to use this material in their interaction with children, and as such were in four of the sixteen total videos. Overall, there was an average response rate of 69% in Magna-Tiles video observations. Similar to math games, PSTs were most responsive to children’s mathematical speech, the average response rate of 63%, then to children’s non-verbal engagement with a response rate of 37%, followed by general speech with a 11% response rate.

Open-Ended Materials

Open-ended materials were primarily used during free play videos. Of the sixteen total video observations across all PSTs, five had open ended materials as the primary materials. There was an average response rate of 45% when PSTs and children used open ended materials. There was a higher rate of response to children’s mathematics speech at 36%, followed by non-verbal engagement at 24%, and then children’s general speech at 14%.

Blocks

Of the sixteen total video observations across all PSTs, three videos had blocks as the primary materials. There was an average response rate of 41% when PSTs and children played with blocks. There was a higher rate of response to children's mathematics speech at 36%, but unlike the other three materials types, PSTs had a higher rate of response to general speech, at 19%, than to non-verbal engagement at 13%.

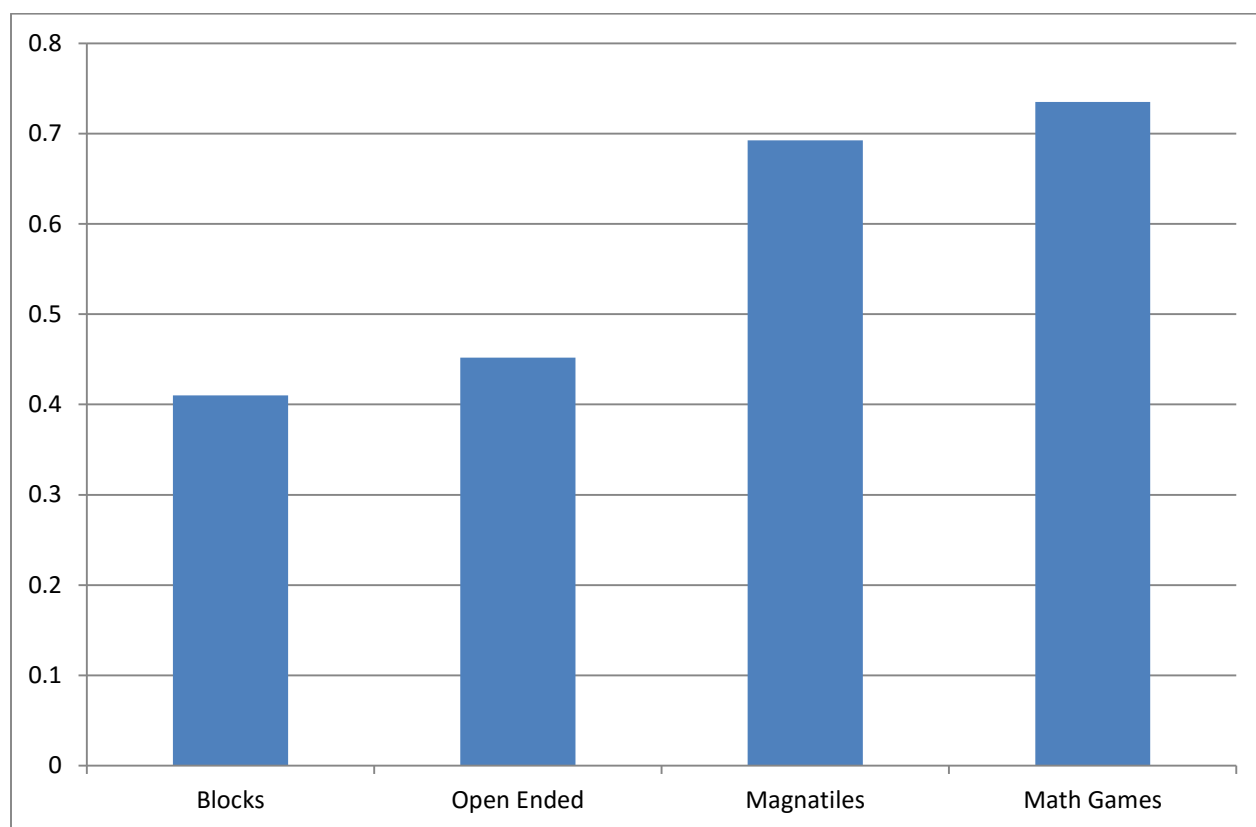


Figure 5 Average Response Rate Across Materials

Table 10 Average Resonse Rate by Material and Child Engagement

Material	Overall Average Response Rate	Child Engagement Behaviors	<i>n</i>	Avg. Response Rate to Behavior
Math Games	74%	Math Speech	136	57%
		Gesture/Look	238	48%
		General Speech	68	19%
Magna-Tiles	69%	Math Speech	144	63%
		Gesture/Look	234	37%
		General Speech	62	11%
Open Ended	45%	Math Speech	133	36%
		Gesture/Look	280	24%
		General Speech	127	14%
Blocks	41%	Math Speech	79	51%
		Gesture/Look	174	13%
		General Speech	69	19%

4.2 PRE-SERVICE TEACHERS' REFLECTION

Focus now turns to analysis of reflection journals at two time points in the course.

Overall, results indicate that scaffolded video reflection led to three important outcomes for pre-service teachers: productive reflection, changes in reflection, and goals generative of improved practice. An initial question was whether PSTs would engage in “productive reflection”. Content analysis of PSTs journals resulted in findings which indicate that, overall, PSTs engaged in productive reflection to varying extents. Each PST did respond to prompts and in fact did write about specific learners, and as they did so, highlighted young children’s mathematics knowledge as well as the process of mathematics learning. PSTs also frequently commented on the quality

of their teaching, focus on the quality of their mathematics language, as well as the use of materials, and the relationship between specific strategies and child learning.

While PSTs generally engaged in productive reflection across their two reflection journal entries, notable changes were observed between their first and second reflection journal entries. Over time, PSTs reflections included greater focus on children as individual and multi-dimensional learners. As PSTs focus shifted to emphasize children's capabilities, they also included more specific examples of children's abilities. PSTs practice also changed between reflection one and reflection two, and they commented on changes in their own practice, as well as increased confidence in reflection two.

PSTs also made plans for change to varying degrees in reflection, and many plans for change were realized in subsequent observations of practice. Plans for change, which can also be thought of as goals, fell into three dimensions: improved mathematizing, addressing specific mathematics content domains, and managing interactions. There were also shifts in goals between reflection one and two, with more goals set related to mathematizing in reflection one, and more goals related to mathematics domains in reflection two. Notably, the majority of goals set by PSTs were realized and observed in subsequent practice, and at times commented on by PSTs themselves in following reflections. PST's reflection was not only productive in the sense that it met criteria of "productive reflection", but it in fact *produced* changes in practice aligned with course learning goals and best practice for early mathematics teaching.

The following will explore these three major findings. First, whether and how PSTs demonstrate "productive reflection" will be described. Next, change in reflections related to PSTs expectations and confidence is addressed. Finally, findings on goals, their dimensions, and whether and how they led to change in practice are presented.

4.2.1 *Demonstrations of Productive Reflection*

Focus on Children's Capabilities and Knowledge

PSTs consistently observed and commented on young children's mathematics knowledge and capabilities, with a total of 16 occurrences across the eight journals. PSTs typically focused on children's capabilities and knowledge in two areas: mathematics language and specific mathematics domains. Two of the four PSTs called out children's mathematics language and with specific quotes as evidence of children's mathematics language. These quotes point to children's ability to provide mathematics language for their own play, and PSTs also commented on the value and importance of this language. For example, in her first reflection Eun ji comments that "The children constantly counts and use some math terms such as 'bigger', 'taller', 'too big vs too small' and 'too heavy' while building." She later adds that "I will try to talk less and encourage the children to talk more", indicating her value of child talk and desire to support children's mathematical language. In her first reflection, Melissa also points out how Odessa uses mathematics language in her play, "Odessa says, 'Let's make a huge triangle and a huge square.' She proceeds to put several shapes together to make one big triangle and one big square," and briefly comments later, "She was also very good at naming the different shapes." In these cases, mathematics language stands out to PSTs as a valuable capability children already possess.

Children's knowledge related to specific mathematics contents was also a consistent focus of PST comments. Mathematics domains PSTs commented on included geometry, enumeration, and pattern. Observations across domains at times highlighted capabilities and at other times children's difficulty or misunderstanding of certain content. Observations were typically objective and descriptive accounts of children's behaviors demonstrating mathematics knowledge or skills not yet present.

Of the mathematics domains, PSTs called out children's geometry skills the most, with each PST making observations in this domain. Comments addressed children's ability with identifying, sorting, transforming and composing geometric shapes. For example, Eun ji notes that "The child was able to tell me that when the big square is tilted, it can be a diamond" and "When I asked the child that I need a rectangle piece, he put two squares together and gave it to me." Melissa also calls out how Odessa "...proceeds to put several shapes together to make one big triangle and one big square." Sometimes PSTs commented on children's difficulty with geometry. For example, Ji hye notices Yu jin's understanding of geometric shape, calling out the child's seeming trouble with triangles "She was confused with triangles. She understood squares every time I asked her, but was not fully understanding the triangular shapes." Tara also points out that the child she was working with was challenged to find shape alternatives to build a structure, writing that "Faith showed that she did not easily find substitutions when certain shapes did not work."

Other domains called out in reflection were enumeration and pattern, with statements like Eun ji's: "Another thing that stood out to me was that the child was able to count over 30 without any problem." Tara briefly addressed magnitude and enumeration in her first reflection when she comments that "Faith had a difficult time with the concept of "how many more" even though her counting and simple operations are fairly perfect." Tara later writes about Faith's skills in enumeration and part/whole relationships in her second reflection, observing that Faith "... put the numbers in order and then made columns of the same fractions all on her own." Ji hye and Melissa both comment on the limited understanding of pattern children demonstrate. Melissa, for example, writes that "At first, Odessa is confused as says things like "let me show you what a pattern is" and then says things such as 'my pattern is a giraffe!'"

PSTs attended to children's skills and knowledge across domains, with observations related to geometry cited by each PST, while other domains were discussed, though not as consistently across PSTs. This may be due to the materials PSTs used in observations: all of the video two observation had Magna-Tiles, creating a setting which readily drew out children's geometric knowledge and capabilities, whereas as in video three PSTs used a variety of materials, allowing for other domains to be observed.

Focus on Child Interests and Child Lead in Interactions

Another theme in PST noticing was attention to children's interests and concern with following children's lead. Statements related to this theme occurred 11 times and were found in at least one journal entry of each PST. Children's interests seem to surprise PSTs at times, and PSTs appeared to be reminded, by observing the children in their videos, to make children's interests a more central in future interactions. PSTs noticed what children enjoyed, children's goals in play (which at times were not the same as PSTs), that children had important ideas to add to the mathematics learning, and that following children's lead could improve the quality of interactions and the mathematics learning.

Children's excitement and engagement was commented on across reflections from each PST, and they noticed both when children were engaged and excited about mathematics activities, and when they appeared bored or disengaged. Sometimes, PSTs saw that children eagerly engaged in activities, especially the more structured and game like activities in video three. For example, in video three, Eun ji invites Derek to count and compare lines of trains, a material he enjoys, and in the following reflection notes that "[Derek] was very excited when I asked him, 'Can you make the trains all line up in a long, straight line and see how long it will be?' He lined most of the trains in a straight line and counted the number of trains." Melissa also noticed Odessa's interest in the materials she has brought and her engagement in the activity

when she wrote, "Odessa really enjoyed rolling out the beans and counting how many pink colored beans there were in order to figure out how many spot she should put on her cheetah." There were other times when activities did not seem to spark as much interest, and, at least upon video review, PSTs were aware of this. Ji hye, reflecting on her efforts to do a patterning activity late in the lesson with Yu jin, states that, "She quickly became disengaged in the activity. (I think it happened because I did an activity for 15 minutes prior to this moment; she could have been tired from it)". She reasons that the timing of the activity may have detracted from Yu jin's ability to engage. Such observations and reflection are good and indicate that Ji hye is seeing the child's engagement in relation to her teaching decisions. With this awareness, she can make different decisions to support engagement. Tara also notices that, even though she had a clear learning goal related to composing shapes, this was not enough to "capture" Faith's interest. She later describes a different approach that promoted engagement, which was to scaffold children's exploration of shapes instead of directing it, and writes, "Instead, it was better to ask them questions about the shapes they used while building whatever they wanted." These examples demonstrate how PSTs noticed what contributed to and detracted from engagement and, especially when realizing children weren't engaged, considered better approaches to support engagement.

Another important aspect of PSTs attending to children's interests was that it prompted them to both notice how they were following children's leads, and also to prioritize this for future practice. PSTs were sometimes not aware of children's interests and lead during interactions, but upon video review, became attuned to them. For example, Eun ji notices, after viewing her video, that Yu jin was sharing ideas about rectangles that Eun ji did not pick up on during the interaction. She writes "I noticed I missed a teaching moment when she mentioned

Rectangle. We didn't talk about the rectangle but she knew it. I would have asked her where she heard it and explained her what that is." Melissa was aware of Odessa's ideas and leadership both during the interaction and when reviewing the video, and, while she celebrated Odessa's lead, she also felt it caused them to jump from one topic to another in a way that prevented focused exploration. She writes "While Odessa was very good at suggesting things such as sorting all the shapes by color, she would usually get side-tracked and begin something else."

Focus on Challenges with Mathematizing

Challenges with mathematizing were a consistent comment made across PSTs. PSTs commented on the ease and effectiveness of their mathematizing as well as their comfort with mathematizing. The main challenges PSTs mentioned were talking too much, having trouble asking effective questions, not knowing how to mathematize certain situations, and providing explanations to children that were confusing rather than helpful.

Talking too much was a challenge addressed by three of the four PSTs. PSTs felt , especially in their first reflection after watching video two, that they overran children with incessant questioning and a lack of focus on children's ideas. For example, Ji hye writes that "I thought I was definitely aware of how she interacted with me while filming, but now I think I was leading most of the time rather than following the child's lead," and concludes that "I will definitely make her lead the play and will use more descriptive language instead of having myself talking too much for the next video." Eun ji also becomes aware of how much she talks and commit to reduce this in future practice:

From this experience, I learned that I talked too much throughout the video. I constantly asked questions about shapes and other math ideas. During my next math interaction with these children, I will try to talk less and encourage the children to talk more. Instead

of constantly asking questions, I think it is also important for me to sit and just watch these children explore the magna tiles. What they build, how they build, how they interact with each other is also important rather than just talking about math terms.

Ji hye and Eun ji, while somewhat concerned about their performance, identify key practices that they can implement in future practice, including describing children's actions, quietly observing children's play, and encouraging children's talk.

PSTs also wrote about their challenge to readily provide language around certain mathematics content or activities. Children at time presented questions and misunderstandings that PSTs did not know how to address, such as when Derek and Mark incorrectly think that only equilateral triangles count as triangles, rejecting other triangle types. In this instance, Eun ji freezes, unsure of how to explain and describes her thinking, "I never had this interaction with children around me. Therefore, I somewhat panicked, thinking, 'How do I explain that all these are triangles?' For now, I told them we will talk about it later." At other times, PSTs were not sure how to mathematize in various and flexible ways as children engaged in different activities. Melissa describes how, during open ended play with the Magna-Tiles, Odessa's play often went beyond Melissa's comfort zone with mathematizing: "I was very good at asking Odessa to count how many we had stacked, or laid out next to each other, but I did not ask very many other sorts of questions. Odessa wanted to build things such as castles, and rocket-ships, and I found these types of play a little difficult to introduce math into."

A third mathematizing challenge was difficulty providing effective explanations of mathematical concepts. This was true of several domains, but especially geometry. Tara, Eun ji, and Ji hye each describe how they attempted to explain geometric concepts, but had difficulty providing clear explanations, at time resulting in confusion for children. For example, Tara

writes in reflection one, "I notice my teaching about the sides of the square may have been too abstract until I specifically said there were 4 equal sides. It seems like she was not quite ready for the concept." Similarly, Eun ji wrote that, "I don't think I made it clear enough when explaining how triangle and square are different. I could have restated the numbers of sides after counting the sides together. She was still confused with the different shapes because of that."

While noticing how explanations may not have been effective, PSTs also identified new ways to improve practice. In Tara's case, she realized this during the interaction and adjusted instruction, while Eun ji noticed after instruction and made plans for change in the future.

Noticing Effective Mathematizing

PSTs also noticed mathematizing moves that were effective, and often comments included the mathematizing move and its outcome with children. Mathematizing moves included effective prompts, question asking, explanations, descriptions of phenomena, and wait time. Prompting that was effective usually included wait time and lead to children's enthusiastic engagement, responses that were on topic with the prompt. Tara comments on this sort of prompt, writing that "Prompting Jasmine to count and allowing wait time helped me to understand how far she is able to count in order." She also notes that open ended prompting is more effective, in contrast to specific directions: "Asking the girls to build something specific 'i.e. only use triangles' did not work well. Instead, it was better to ask them questions about the shapes they used while building whatever they wanted."

PSTs also used questions to challenge children to demonstrate skills. PSTs viewed these questions as effective when children responded appropriately and enthusiastically. For example, Eun ji wrote "The child in the orange was very excited when I asked him, 'Can you make the

trains all line up in a long, straight line and see how long it will be?'. He lined most of the trains in a straight line and counted the number of trains."

PSTs noticed when they effectively explained mathematical concepts. For example, Melissa successfully explains patterns to Odessa, evidenced in Odessa's creating her own patterns. Melissa writes, "I finally explain to Odessa that a pattern is something that repeats such as 'blue yellow, blue yellow.' Odessa finally begins to understand what a pattern is and is excited to show her mom the pattern that she made using round color stickers." Another strategy that seemed to work when helping children understand pattern was stating the pattern more than once to help children remember the sequence. Ji hye saw this as helpful for Yu jin when she commented, "I restated the pattern to refresh it because she was disengaged for a moment. It allowed her to guess what'll be the next pattern."

Another type of mathematizing that one PST called out was describing the child's play as it happened. Eun ji comments that this description in fact prompted the child's mathematical reasoning and language, something Eun ji had not planned. "When I said, 'It fell down, I don't know' when what the girl was building collapsed, she said 'may be it's too big.' The question I unintentionally asked provoked her thoughts." Descriptive statements elicited children's reasoning, which also led problem solving, such as when Eun ji notices that "When the structure kept collapsing, I said "may be it's too heavy" and she put the tiles somewhere else."

4.2.2 *Changes in Reflection*

Important shifts occurred between reflections one and two. Changes between reflection one and two included more multi-dimensional observations of children's skills and abilities, more attention to children's interests and lead, more comments on effective practices, and greater confidence in abilities to teach mathematics. Some change appeared related to change in

practice itself. As PSTs made new efforts in their third video observation, they noticed changed practices and commented in reflection in a new way. The quality of reflection also changed, with PSTs providing more evidence for their statements in reflection two, as well as more detailed descriptions of interactions and interpretation of events.

Shift in Focus on Children's Capabilities and Knowledge

PSTs produced more multi-dimensional and detailed reflections of children's capabilities and knowledge in reflection two. In short, they became better noticers of children. Reflections shifted from evaluating children's performance in either positive or negative terms in reflection one, to multi-dimensional observations of children's skills and challenges related to specific mathematics domains in reflection two.

For example, Eun ji's comments in reflection one are primarily brief and predominately positive evaluations of children's skills, such as "The child was able to tell me that when the big square is tilted, it can be a diamond". In contrast, her second reflection included observations of children's skills as well as challenges. Eun ji also provides some interpretation as to why a specific mistake occurred. This is exemplified in her observation regarding Derek's trouble with counting accurately to 30: "Another thing that stood out to me was that the child was able to count over 30 without any problem. Though he had some moments when he counted the number of objects wrong, it wasn't because of his ability to count up, but because he skipped a train while pointing at each one to count." Eun ji first focuses on the child's ability to count, then follows with a description of an error in counting, which she interprets by attributing it to inaccuracy in one-to-one correspondence when counting to high numbers.

Tara makes a similar shift from comments, which focus on Faith's inabilities in reflection one to attention to her capabilities as well as challenges in reflection two. In her second

reflection journal, Tara highlights Faith and Jasmine's capabilities more than in reflection one, with statements such as, "Our game went very well when Faith was in charge of writing the fractions down. She put the numbers in order and then made columns of the same fractions all on her own," and "Faith surprised me with her knowledge of fractions." While Tara's comments in reflection one were primarily deficit oriented (such as "Faith showed that she did not easily find substitutions when certain shapes did not work"), they shifted to focus on Faith's abilities (in writing and general knowledge of fractions) in reflection two. An important factor that may help explain this shift is the change in Tara's teaching approach between videos two and three, in which she moved from teacher directed to allowing more child direction. Her shift in practice led to more opportunities for Faith to demonstrate her knowledge, creating more opportunities, in turn, for Tara to notice and comment on Faith's capabilities.

Shift in Focus on Children's Interests and Child Lead

In each PST's first reflection they commented to some extent on changes they'd like to make to promote children leading play in some way. As they attempted to do this in the subsequent interactions, children responded with new engagement behaviors. PSTs then noticed this in reflection two, leading to the basic finding that new experiences led to changes in reflection. More specifically, PSTs noticed and commented on children's interest and following children's leads more in their second reflection compared to their first. This shift may also be related to change in PSTs ability to notice children's interests and role in interactions. They may have become more attuned to notice children's interests as they had repeated experiences doing so in reflection.

Melissa considers how she could engage Odessa to maintain focus on an activity for more time in her first reflection. She plans to do this by "asking more follow up questions" and helping Odessa complete tasks. After video three, in which she implemented new strategies,

Melissa reflects that Odessa continued to lead the play, yet sustained focus on activities because the materials were interesting and activities had more structure. Melissa noticed Odessa's interests in video two and in her next visit brought materials she thought would draw Odessa to the activities. Melissa's change in materials maintained Odessa's role in leading the play, yet provided choices between structured activities (math games) with interesting materials so that Odessa was interested to complete games. Odessa's role as play leader was maintained, but focused to productive mathematics engagement by the activity and materials arrangement provided by Melissa.

This change in experience and practice likely contributed to shifts in Melissa's reflection. When asked what she notices about Odessa in her second reflection, Melissa writes about the interest Odessa had in various activities, such as the green bean game or pattern strip. This is contrast to reflection one, when Melissa's comments focused on Odessa's various skills, especially with mathematizing, and inability to focus. This shift indicates a new awareness of how mathematics activities interest and motivate Odessa to engage in mathematical thinking.

Comments in Eun ji's second reflection also point to changes in her practice, the main change being to prioritize children's interest in activities and then work to mathematize as children actively engaged. She articulates that this was in fact accomplished in her second reflection when she writes, "Both children seemed to be interested in the toys (Magna-Tiles, turtle figures, and trains) which I was able to talk 'math' with." In this video observation and second round of reflection, Eun ji calls out how the main child, Derek, responds to her prompts enthusiastically, "The child in the orange was very excited when I asked him, 'Can you make the trains all line up in a long, straight line and see how long it will be?' He lined most of the trains in a straight line and counted the number of trains." The provision of materials and Eun ji's

focused prompts appear to contribute to Derek's sustained engagement, in this instance demonstrated by his focus on counting all the trains up to the number 33. While Eun ji prompts much of the mathematics activity, she deems Derek's interest and excitement as a marker of the success of this interaction. Whereas in her first reflection she was somewhat puzzled as the children had different goals in the play than she had (they wanted to "just play" with the Magna-Tiles while she wanted to "do math") in this reflection she notices Derek's interest in the materials, and how her intentional prompting of mathematics talk and use of interesting materials supported this interest.

Ji hye goes from limited awareness of Yu jin's interests or contributions in her first reflection to including several comments on Yu jin's interests in reflection two. In this first reflection Ji hye shows beginning awareness of Yu jin having a role in offering ideas to the interaction when she points out that Yu jin in fact accurately named rectangles when she correctly identified the rectangle, something Ji hye had not noticed until after the interaction. She comments that "I thought I was interacting with the child but now that I watched the video, I noticed I missed a teaching moment when she mentioned Rectangle. We didn't talk about the rectangle but she knew it. I would have asked her where she heard it and explained her what that is." Upon reviewing the video, Ji hye appears to become aware of the idea's Yu jin might have brought to the interaction, and then makes plans for the next observation, stating "Moreover, I will definitely make her lead the play and will use more descriptive language instead of having myself talking too much for the next video."

In Ji hye's next reflection entry, she includes several comments about Yun ji's interest and engagement related to completing activities, noticing, for example, that, "She quickly became disengaged in the activity. (I think it happened because I did an activity for 15 minutes

prior to this moment; she could have been tired from it).” At several points Ji hye calls out how Yu jin is or is not attending to the activities provided her by Ji hye, usually in relationship to Ji hye’s teaching moves, “I did not put the beans aside, and allowed her to be distracted by them,” and “I restated the pattern to refresh it because she was disengaged for a moment.” In this reflection, unlike the first, Ji hye reflects on Yun ji’s level of engagement, interest, and the possible role of Ji hye’s structuring of the activity on interest and engagement. This may indicate a shift in Ji hyes’ attention during reflection, now more attuned to observing what Yu jin is interested in and how this relates to Yu jin’s engagement in mathematics learning. Another important consideration here is that Yu jin is at times disinterested in activities as Ji hye present them. So, while Ji hye was more aware of this child’s level of interest, it may have been due to her increased efforts to maintain Yu jin’s attention during the interaction.

When it comes to noticing and describing children’s interests and role in leading interactions, Tara moves from shorter and general statements in her first reflection to descriptions that are more specific and highlight child competency with mathematical skills. Comments related to child interest in Tara’s first reflection include statements like, “Making different shapes out of the sides of the shapes went well for learning purposes, but did not capture her interest well” and the observation that “... it was better to ask them questions about the shapes they used while building whatever they wanted.” While Tara is aware that the interaction is improved when children have some say in what they are doing, she does not appear to have prioritized this in the interaction, and only acknowledges it in the reflection. Nonetheless, Tara makes sense of her experience, concluding that following children’s lead is a more effective way of engaging them in mathematics than directing them in how to play.

On the other hand, in her second reflection Tara describes with more detail (compared to reflection one) an activity in which Faith, the older of the two girls, is “in charge” and how this is a benefit to the game play: "Our game went very well when Faith was in charge of writing the fractions down. She put the numbers in order and then made columns of the same fractions all on her own." In the context of noticing Faith’s lead, Tara points out specific mathematics skills Faith had mastered, and also in the reflection acknowledges “Faith surprised me with her knowledge of fractions.” Tara notices and describes Faith’s mathematical capabilities in a more specific and positive way than in her description of child skills in the first reflection. This may indicate a shift in Tara’s orientation to the children; she now sees Faith’s contributions, and sees them as valuable.

Shift in Expectations

Between reflection one and two, PSTs made more comments on children’s skills which surprised them or exceeded their expectations. In reflection one, only one of the four PSTs made comments like this, however, by reflection two, all four PSTs noticed capabilities that surpassed what they thought children capable of. Children disrupted PSTs expectations along several lines, most consistently regarding the level of mathematics knowledge children already possessed. PSTs also commented on children’s ability to learn new concepts as surprising.

No PST commented on children’s mathematical ability exceeding their expectations in reflection one, however, there was a clear shift in reflection two, in which three PSTs pointed out children’s abilities to count with skill and ease, compose shapes, and knowledge of fractions. Example comments include "One thing that struck out and was surprised by was that [Mark] remembered what I had taught him in the previous video. When I asked both the children, ‘Do you remember what we can do if we don’t have enough squares?’ [Mark] thought about it for a little and had his a-ha moment. He picked up the two right triangles in front of him and put them

together” (Eun ji), “Faith surprised me with her knowledge of fractions” (Tara), and “A lot of the play that we completed was basic knowledge such as counting, which I believe Odessa already has a really good grasp on” (Melissa). By reflection two, PSTs seemed more attuned to children’s capabilities, and more interested in setting up learning opportunities in which children could demonstrate knowledge. Their willingness to let children do more led to pleasant surprises.

Two PSTs also commented on how children’s interest and persistence in learning new skills exceeded their expectations. Ji hye notices that while she is prompting Yu jin to predict which shape will come next in the pattern, the child “Another thing that stood out was that she could self-correct her answer. I am not so certain if I gave her a clue, since I tried to be plain when restating her answer...”. Melissa also discusses Odessa’s eagerness and ability to learn about patterns, a concept Melissa was not sure Odessa would grasp. She writes, “At first, I thought that maybe patterns was a little bit of a challenging concept for a three-year-old, but I decided to try and explain to her a couple more times what a pattern was, and I am sure glad I did! Odessa finally was able to construct more and more challenging patterns, and identify them as well.” While PSTs had some preconceived notions of children’s abilities to learn new skills, they tested these boundaries with children and found that, in these two cases, children were capable of learning new mathematics concepts in ways that surprised PSTs.

Shift in Confidence

Another change between reflections one and two was increased confidence in some cases, or at least, fewer statements related to low confidence in other cases. PST’s outlook on their own performance seemed to improve by their second reflection, and they reported better feelings about their ability to mathematize, pleasure with carrying out teaching plans, and more attention to successful rather than unsuccessful interactions. One PST, Melissa, wrote

specifically about her sense of confidence, and how it increased between reflection one and two. One of several comments in reflection one that conveyed her concerns with mathematizing was “Odessa wanted to build things such as castles, and rocket-ships, and I found these types of play a little difficult to introduce math into.” In contrast, Melissa reports feeling of accomplishment and confidence regarding mathematizing when she writes “This session, I feel a lot more confident about my mathematizing” and “I felt really accomplished at the end of the session, because I taught Odessa what a pattern was.”

Other PSTs wrote less specifically, yet there are indicators in their reflections that denote changes in confidence, in the direction of increased confidence. For example, Tara shifts from a predominance of statements focused on interactions that did not go “well” in her first reflection, to a predominance of statements on successful interactions in her second reflection. She is seeing herself in terms of abilities and successes as she teaching children more over time. Ji hye also makes a subtle shift between reflections, reducing the number of negative statements about her own performance from two such comments in her first to only one in her second reflection. Eun ji also does not call out her feelings related to practice, per se, but a shift can be noticed in her sense of preparation to guide learning. Whereas in her first reflection she discusses feelings of panic at not being prepared to guide children in a clarifying conversation about types of triangles, by reflection two she does not voice such concerns. Instead she points out how she planned and successfully carried out instruction related to measurement and comparison, writing that “I was able to focus on comparison in this video as I planned. We compared the size of the turtles then we compared the length of the trains put together by each child.” Whereas reflection one was characterized with PST’s concerns about their skills in mathematizing and management,

reflection two was characterized by focus on mathematics content, comments on effective teaching moves, and attention to children's contributions.

4.2.3 *Pre-service Teachers' Goals and Improved Practice*

In addition to noticing a variety of characteristics of children, themselves, and the interactions in reflection, the majority of PSTs also identified a variety of goals for future practice. Goals typically were responses to PST's observations of their implementation of mathematics teaching practices, especially mathematizing, as well as other targets, such as introducing children to specific mathematics topics or activities. However, in one case, very few if any goals were identified in reflection. Most PSTs set multiple goals in each of their reflection journals, and goals generally fell into three dimensions: Mathematizing, Mathematics Domains, and Managing Interactions. Examples of the goals set in each dimension will be discussed, as well as whether and how goals were realized. The intention here is to extend the idea of "productive reflection" proposed in previous literature by Bayat (2010) and others (Corscadden et al., 2010; Rosaen et al. 2008). While productive reflection has been previously defined in terms of its content and focus, this study is also interested in whether and how reflection produces change in practice, and proposes changed practice be another marker of productive reflection.

Table 11 Dimensions of Goals, Totals, and % Completed

Dimension	Total # of Goals	Total # of Goals Completed or Partially Completed	% of Goals Completed or Partially Completed
Mathematizing	8	7	86%
Mathematics Domains	7	4	57%
Managing Interactions	6	4	67%

Mathematizing

Goals related to mathematizing were most common among PSTs. This is especially true in their first reflection, in which three of the four PSTs set a total of six goals or plans for change on mathematizing. Overall in this dimension eight total goals were set (between the two reflections), and PSTs followed through on their plans in seven of these eight goals (86%). These included goals such as “During my next math interaction with these children, I will try to talk less and encourage the children to talk more” (Eun ji), “Learn to speak in mathematical language, mathematizing everyday situations will come more naturally to me” (Melissa), and “I will ... use more descriptive language instead of having myself talking too much for the next video” (Ji hye). There is evidence to indicate that PSTs met goals. Using an example from above, Eun ji did in fact talk less and the children did indeed talk more in her next interaction, demonstrated by the reduction in her speech from 869 total words in video two to 486 words in video three, and the increase in children’s speech from words 570 in video two to 632 words in video three. Eun ji also notices and comments on this change in her reflection after video three when she writes “I also noticed that I was talking less than my previous video. In my previous video, I felt like I was talking all the time. But in this video, I did a little less talking and let children explore the toys.” Both of the other above listed goals were also completed, with similar evidence from subsequent observation and reflection. There was one plan for change relating to mathematizing set by Tara that was vague, “I will try to be more clear and have thought-out answers when they ask unrelated questions.” It was therefore difficult to observe this as having been completed in future interactions. Other than this, all mathematizing goals were completed or at least partially completed.

Mathematics Domains

There were a total of seven goals set by PSTs in the Mathematics Domains dimension, and there is evidence that four of the seven (57%) were completed by PSTs. There was an increase in these goals from reflection one, in which only two Mathematics Domains goals were set, to reflection two, in which five were set. Goals usually included PST's intention to provide more challenging mathematics content, or to provide content from a broader range of domains. Some examples of goals in this dimension include "I will remember this and work on counting up to higher number as well as work on another domain element" (Eun ji), "For next session, I hope to challenge Odessa a little more in her thinking. A lot of the play that we completed was basic knowledge such as counting, which I believe Odessa already has a really good grasp on. I would like to teach her more about geometry and use the magna tiles a bit more" (Melissa), and "I will use more complex pattern for the next video. It was a test drive for her to get to know what pattern is like and I will use more complicated pattern strips as I scaffold her" (Ji hye).

Above half (57%) of the goals in this dimension were completed or partially completed. An example of a completed goal, drawn from the previous paragraph, is that Ji hye did in fact introduce Yu jin to a more complex pattern in their following interaction. There is evidence of this change in the video that followed this reflection, in which Ji hye starts with an ABAB pattern strip and then introduces a AABBAABB pattern strip, a pattern with a longer unit. This appears to be a pattern she thinks of as more "complex", and which does appear to be more challenging for this child, who relies on scaffolding and prompts from Ji hye to complete the pattern. An example of a goal that was not met in the Mathematics Domain dimension is Eun ji's goal to engage Derek in counting to a "higher number" than 33, which he successfully counted to in the previous interaction. Upon review of subsequent video, no counting was observed, and numbers were only mentioned several times. A possible reason this goal was not

carried out was lack of focus on enumeration in the following video. In this video, Derek spent the majority of the time building with unit blocks, and Eun ji spent much of the time mathematizing with a focus on geometry words. It did not appear that Eun ji set out to engage Derek in more counting opportunities, and did not take block building as an opportunity for this either. It seems that setting the goal may not be enough, PSTs need to intentionally plan how they will address specific domains either with materials, activities, or mathematizing strategies that work across a range of materials and activities.

Managing Interactions

There were six goals related to Managing Interactions, and four of the six goals (67%) set were completed or partially completed. There was a noticeable decline in the number of goals from reflection one, in which five related goals were set across PSTs, to reflection two, in which only one goal was set in this dimension. Managing Interactions refers to goals related to teaching arrangements that are not necessarily related to mathematics, but are focused on the management of interactions in general, such as increasing child lead, engaging multiple children, and sustaining engagement in activities. Goals in this dimension covered a range of topics, and each PST seemed to set goals around management in response to very specific observations or concerns. PSTs made plans in this dimension such as “Next time I am in a math interaction with these two children, I will definitely work with both children” (Eun ji), “Next time I teach math to Odessa, I really want to encourage her to complete the activities she begins” (Melissa), and “Moreover, I will definitely make her lead the play ... for the next video” (Ji hye).

PSTs followed up on these goals in various ways. Melissa did indeed shift her teaching approach to support Odessa’s sustained engagement in activities by bringing more interesting materials, and providing activities that have a mathematical aim while allowing Odessa to “tweak” the games, thus maintaining child lead. Observation of her following video shows

Odessa enthusiastically choosing and playing mathematics games provided by Melissa, and the subsequent reflection also points to the carrying out of this plan:

I was happy because I feel as though I still let Odessa guide our play for the most part. She was choosing activities or revising the activities I suggested in order to suite her desires. I didn't feel as though I was forcing Odessa to sit down and 'do math' but I felt as though she learned, or at least practiced, some math skills.

As in other dimensions, some goals related to Managing Interactions were not accomplished. In both reflections one and two, Eun ji determines to engage both children more equally in interactions. Observations of practice in video three (following reflection one) and video four (following reflection 2) do not present evidence of this goals completion or partial completion. In video three, the proportion of questions directed to Mark (the child who received relatively little attention in video two) vs. Derek decreased from 47% of the questions to 10% in video three, and then 6% in video four, indicating that the goal of more evenly asking questions to both boys was not realized. Additionally, Eun ji acknowledges that her attention is not evenly shared in her second reflection: "One thing that did not go very well is that I seem to be focusing my interaction with only one child, when I had two children with me. [Mark] seems to be playing by himself (but he did play alone very well)." It is surprising that despite having set this goal twice, and having twice observed its absence, Eun ji continued in this practice. One possible explanation is that of the two children, Derek is observed to be more talkative. Eun ji is often responding to his comments and questions, while Mark plays somewhat more quietly.

Responsiveness of Goals and Differences between PSTs

Goals were an indicator of PST's responsiveness to the learning needs of children and identifying ways to improve their interactions with children. They arose out of PST's reflection on their interactions and were plans for future change, what Schon (1991) might call reflection

for action. This type of reflective and responsive practice was carried out in different ways by each PST because their responses and actions were born out of specific experiences with individual children. While goals could be fit into three dimensions, why PSTs arrived at these goals and how they carried them out was individualized. Therefore, why and how each of the three PSTs set goals will be considered on a case by case basis, to examine their individual characteristics and differences.

Melissa and Odessa present a unique pair among the PSTs and children they worked with. Odessa, relative to the other children, is very exuberant in her play, and Melissa comments several times on Odessa's great enthusiasm in play, yet also sees Odessa's tendency to go from one activities to another as a barrier to in depth mathematics learning. It is in response to her observation, and determination that more sustained play would be optimal, that she sets a goal to provide Odessa with more questions and activities that will sustain her engagement. While Melissa values Odessa's skills and lead in the interaction, she also wants Odessa to focus on one mathematics activity at a time, and for a longer period of time. Melissa therefore responds to Odessa's need for choice and direction in play, as well as her own need for more focused and sustained attention, by introducing a selection of math games and materials. These materials are selected based on Melissa's knowledge of Odessa's interests (for example, Melissa found out that Odessa enjoyed coloring, and brought markers), as well as games Melissa had learned how to play and thought would be interesting and provide a mathematical focus for Odessa.

Eun ji demonstrates responsiveness to Mark and Derek in a slightly different way. This grouping is unique because there are two children for Eun ji to interact with, both of whom readily participate, but need varying amounts of time to respond to her questions. In her first reflection, Eun ji notices that when she asked Derek a question she only pauses briefly before

redirecting her question to Mark, who quickly supplies an answer. Eun ji responds to this observation and her thinking that Derek may need more time to respond to questions by planning to allow for more wait time in her next interaction. Eun ji values the children's contributions (as she writes it is important to observe and them and what they bring to the interaction instead of constantly mathematizing), and seeing that her interactions do not always leave time for their contributions, she determines to change this practice. Another instance of responsive goal setting is in her second reflection, when Eun ji notices the higher than expected skill level with numeracy demonstrated by Derek, and responds to this by planning to challenge him to count higher, as well as work on other domains where his skills might not be as strong. She identifies ways to respond to Derek's skill, first by building on it (with more numeracy opportunities), and by pivoting from it to another domain with which he may have less experience. Eun ji plans to change instruction in response to what she knows Derek's knowledge and capabilities. In both cases, planning to provide more wait time and adjusting instruction, Eun ji carries out her responsive goals. In the subsequent videos, she is seen providing ample wait time when she asks Derek to count all the trains in video three, and she pivots from a focus on counting to focus on geometry and spatial sense in video four.

There is an interesting contrast between what Ji hye and Melissa respond to with children. Whereas Melissa wanted to find ways to bring a focus and direction to the play with Odessa, Ji hye seeks to increase child direction with Yu jin. This is in response to her observation that she in fact missed a learning opportunity, when Yu jin correctly identifies a rectangle, and Eun ji does not respond but continues talking about squares. She responds to this observation in two ways, by planning to allow Yu jin to lead the play in the next interaction, and by shifting from "talking too much" to using "descriptive language". It seems Ji hye is intending

to be less directive or instructional, and more responsive in her language and interaction style going forward. Ji hye also responds to her noticing of Yu jin's mathematical capabilities, observing that while the little girl easily identified squares, identifying triangles presented a challenge. Ji hye responds by planning for additional opportunities to explore triangles in two ways: by looking at only triangles (not with other shapes), and by comparing the lengths of the sides of various triangles. Ji hye also implements these responsive goals, by talking less and asking more questions that elicit Yu jin's thinking as well as providing more opportunities to play with triangle Magna-Tiles in video three.

Goals and plans for change were founded in PST's experiences with children, and combined their observations of children's capabilities and learning needs, as well as PST's ideas of how they should engage children in mathematics. Their goals were individualized based on the children, and the type of follow up goals and interactions, materials, and activities varied based on what each PST thought would be most supportive for the individual children they worked with, while also identifying practices they deemed important and then improving these practices.

As indicated by Table 12, there were also notable shifts in goals between reflections. Goals set in the first reflection more often fell in the Mathematizing or Managing Interactions dimensions. There is a noticeable shift in reflection two, though, with the majority of goals falling in the Mathematics Content dimension. This shift in goal setting follows PSTs shift in focus. Whereas in their initial reflections they more frequently noticed missed mathematizing opportunities and unsuccessful interactions with children, in their second reflection, focus turned to success in mathematizing, and more nuanced noticing of children's mathematical skills and thinking. Based on more attuned awareness of children's mathematical thinking, teachers

became interested in introducing new and more challenging math concepts to children, demonstrated in their goals in this second reflection.

Table 12 Dimensions of Goals and Shifts Between Reflections

Dimension	Reflection 1	Reflection 2
Mathematizing	6	2
Mathematics Content	2	5
Managing Interactions	5	1

Chapter 5. DISCUSSION

I now turn to discuss the study's major results. This study demonstrates how PSTs can develop new skills with professional development designed to enhance their mathematical language practices, engagement in productive reflection, and beliefs about children and themselves. Overall, PSTs increased the rate of responsiveness and diversity of mathematics language in mathematics interactions with young children, and they made movement towards practices related to improving children's mathematics learning. Pre-service teachers also engaged in productive reflection, with shifts towards more multi-dimensional and detailed commenting on children's mathematics thinking and learning, as well as improved expectations and confidence. Most PSTs also set goals which were responsive to children's learning needs and their assessment of the effectiveness of their own mathematizing practices. Importantly, goals set in reflection were carried out more than half the time in subsequent interactions. Preservice teachers engaged in new practices related to goals, including more responsive and child led interactions, activities PSTs believed would present more challenge for children, and addressing a broader range of mathematical domains in activities.

This study extends the current research on a) the types of mathematics language PSTs can provide during interactions with young children, b) how mathematics language, in terms of domains and responsiveness, can change during the course of professional development, c) PST's capacity to engage in productive reflection, and d) the important relationship between reflection, goal setting, and improved practice.

5.1 MATHEMATICS LANGUAGE

The types of mathematical language PSTs provided during interactions with young children identified in this study were consistent in some ways and departed in other ways from previous research. There is consistency with Rudd et al.'s (2008) finding that teachers engage in number related speech at higher rates relative to other domains such as measurement (referred to as magnitude in the current study), with enumeration being the most frequent type of mathematics language in my study. However, shape was the second most frequent language type in this study, at 20% of the mathematics language instances, whereas it was only 1.2% of instances in the Rudd et al. (2008) findings. This difference may be due to the materials PSTs used in the current study, which included Magna-Tiles, selected to support PST shape language, and points to how features of professional development can make a difference in supporting specific outcomes.

Considering previous findings that early educators tend to emphasize numeracy over other mathematics domains (Sarama & Diabase, 2004; Lee & Ginsburg, 2007), the current findings highlight that this is not a fixed aspect of teacher practice. PST's language shifted to include shape language at nearly the same rates as numeracy language when using Magna-Tiles, and greater parity across domains when either Magna-Tiles or mathematical games were used. PSTs also identified in reflection their desire to address domains in addition to numeracy. With opportunity to reflect on video recordings of practice, it appears that PSTs became aware of imbalanced attention across domains and were capable of intentionally incorporating domains such as shape and pattern.

5.2 RESPONSIVENESS AND EXPECTATIONS: PROMISING PRACTICES FOR CULTURALLY RESPONSIVE TEACHING

Pre-service teachers also became more responsive in the provision of mathematics language during the course of professional development. Change in responsiveness may have been especially related to video feedback and PST's observation, especially in their first reflection journals, that they often talked over children or were not attuned to children's interests. Increased awareness and subsequent sensitivity to children's interests is manifested in later practice, with response rates increasing an average of 28% across PSTs between their first and final video observations. The findings suggest that video feedback and scaffolded reflection were effective means of increasing PST awareness of children's interests and thinking, planning for interactions that are responsive to children, and actual implementation of these changes. This is aligned with previous research on video feedback as a means of improving attunement between adults and children during interactions (Fukkink & Tavecchio, 2010), yet extends the literature by demonstrating this concept in the context of a mathematics course for early childhood PSTs. Video feedback is an effective means of increasing in-service caregivers' sensitive interactions with infants and toddlers (Fukkink & Tavecchio, 2010), but the relationship between video feedback, reflection, and change in mathematics teaching practices for early childhood PSTs had not been previously addressed in the research. Studies addressing video feedback and PSTs learning and development have focused on the quality of reflection of ability to analyze actions as the target outcome of video feedback. The current study and finding provide a view of how video feedback can support reflection, raising PSTs awareness of children's thinking and interests, and then translating their awareness into new actions with children.

This change indicates that PSTs can develop responsive skills related to child engagement in mathematics, such as providing verbal scaffolding while children play (Gregory et al., 2002), “revoicing” children’s mathematical utterances (Jacobson & Lehrer, 2000), and providing “wait time” (Cohrssen, Church, & Taylor, 2013; Kazemi, 2003). Responsiveness requires better attunement between PSTs actions and comments and children’s interests and thinking. In order to be responsive, PSTs must become more aware of children’s individual strengths, strategies, interests, and needs for support. Other early learning scholars have framed responsiveness as a dimension of teacher behavior called “regard for student perspectives” (Pianta et al., 2008) which is associated with increased student “motivation and desire to learn” (Pre-K CLASS Dimensions Guide, 2011).

Changes in responsiveness are important because research elsewhere (Moses & Cobb, 2001; Varelas, Martin, & Kane, 2013) indicates how responsiveness may promote positive child identities and dispositions towards learning mathematics as it builds on children’s interests. As PSTs attuned their responses to children, they communicated that children had valuable knowledge and approaches to learning mathematics. This is an important development for the early learning educator, as many teachers in fact may undermine children’s approaches and dispositions to mathematics learning with narrow conceptions of what is “appropriate” mathematics (Anderson & Gold, 2006). Young children benefit from responsive interactions not only because they sustain engagement and learning in the moment, but because they also reinforce a positive identity towards mathematics, communicating to children that they are active and important participants in mathematics learning and interactions (Varelas, Martin, & Kane, 2013). When teachers build upon children’s ideas and interests, they not only respond to the child as an individual, but also as a member of a cultural community within and outside of the

classroom. Within the classroom, teachers are signaling that children have a vital role in mathematics discussion. Children are also members of cultural communities outside of the classroom, and therefore much of the ways they engage in mathematics learning are informed by these other communities (Anderson & Gold, 2006). Teachers' ability to notice and supportively respond to these culturally informed modes of engagement has several important potential affects: it strengthens the relationship between the teacher and the child, strengthens the child's identity as a math doer, and strengthens their cultural identity as it relates to mathematics (Varelas, Martin, & Kane, 2013).

While the current study did not address which child engagement behaviors were specifically culturally informed and how they might have been so, it assumed that these were behaviors children had acquired in various cultural contexts (home, preschool, play groups). Therefore, it stands to reason that the more teachers responded to engagement behaviors, the more they were supporting children's cultural approaches to mathematics, therefore their cultural and mathematical identities. This line of reasoning may have important implications for the professional development of teachers to support not only their mathematics teaching, but their culturally responsive mathematics teaching. The current study did not compare how PSTs engaged in responsive mathematics interactions with children across cultural groups to identify whether and how there may be biases in responsiveness that promote cultural mathematical identities for some children and undermine them for others. This would certainly be an important next phase of research, and would likely well inform future designs of professional development which seek to address culturally responsive early mathematics teaching.

5.3 EXPECTATIONS

PSTs also became more aware of children's capabilities, leading to higher expectations, and provision of more challenging mathematics learning. This was demonstrated as PSTs revised their mathematics activities to include more "advanced" concepts in order to challenge children, for example, when Eun ji determined to incorporate more complex patterns in her subsequent lesson, having realized Yun ji had mastered the simple ABAB pattern. Increased responsiveness and expectations demonstrate PST development of characteristics aligned with culturally responsive teaching (Gay, 2000; Delpit, 1995, 2002). These characteristics are *caring* and *high expectations*, and are interrelated: teachers who truly care about their students "honor their [students] humanity, hold them in high esteem, expect high performance from them, and use strategies to fulfill their expectations" (Gay, 2000, p. 46). Culturally responsive teaching characteristics provide a framework of teacher behaviors, approaches, and dispositions that enable them to engage diverse students in learning. While the current study did not explicitly explore aspects of student or teacher culture, it did provide a window into how PSTs can develop two requisite skills of culturally responsive teaching. This suggests that the professional development approach in this study, which engaged PSTs in carefully noticing children through video feedback and reflection, may be effective in enhancing skills relevant to both better mathematical instruction as well as cultural responsiveness.

5.4 CONFIDENCE AND SELF-EFFICACY

Overall, PST confidence improved between their first and second reflections. This shift seemed related to their successful implementation of new practices as well as materials which they felt supported more successful mathematics teaching. This extends the research on

teachers' confidence by demonstrating how it can change, and elements that may contribute to increased confidence. Mathematics PD interventions have prioritized the support of teacher confidence as an important outcome (Clements & Sarama, 2009; Chen & McCray, 2012). Chen & McCray (2009) identify teacher attitudes as one of three components of the Whole Teacher Approach, and use strategies to support confidence, including mentoring, peer learning groups, and math lessons utilizing familiar materials like books. The findings from the current study indicate that additional elements – materials as well as noticing successes in video observation, can also be utilized in professional development to promote confidence.

This adds to the extensive research and theorizing about self-efficacy, defined as “people’s beliefs about their capabilities to exercise control over their own level of functioning and over events that affect their lives” (Bandura, 1993, p. 118). Seeing oneself increase ongoing effectiveness supports self-efficacy, in contrast to seeing oneself fail or seeing others surpass one’s on level of attainments, which are associated with lower-self efficacy (Bandura & Jourden, 1991). PSTs in the current study had two opportunities to observe their own practice, and in each one responded to prompts to identify three successes as well as one missed opportunity. While in the first reflection, PSTs seemed more focused on the challenges in the interaction, by their second observation they had the opportunity to set goals and then see themselves successfully implement these goals in most cases. The presence of goals helped shape not only subsequent practice, but also influenced how they focused their observation – by attending specifically to noticing the completion of goals. This noticing was typically of the successful completion of goals, and therefore was likely supportive of increased self-efficacy. PSTs change in confidence is a noteworthy finding with important implications. Whereas previous studies described early childhood educator’s confidence with mathematics in static

terms (it's either low or high) (Copley, 1999; Cross et al., 2009; Lee & Ginsburg 2007), analysis of PSTs comments indicate that confidence can change with new experiences. Important factors in PSTs change in confidence were materials and successful implementation of teaching plans. These factors point to the idea that confidence is not fixed, and conditions can be set by teacher educators which scaffold mathematics interactions and increasing confidence. Additionally, PSTs themselves support their own confidence by reflecting with video on how their plans for practice are effective. This is likely related to their sense of efficacy, that is, their belief that, even if they did not succeed in a previous interaction, they are capable of making the appropriate changes going forward, and then seeing the effectiveness of these changes. Both practice and reflection set the stage for changes in confidence, with repeated opportunities for practice, PSTs know they have more opportunities to improve, and with reflection, they have the venue to thoughtfully respond to previous experience. Confidence, then, is dynamically generated through planning, experience, and reflecting on successful implementation.

5.5 REFLECTION AND PRACTICE

This study also adds information on the nature of goal setting in the context of reflection. Though reflection is a core professional practice in education (Bayat, 2008) and in professional fields generally (Schon, 1991), research (outside of the current study) which links early childhood pre-service teachers' reflection to subsequent change in practice is yet to be identified. This study has made a contribution to the research on reflection by exploring plans for improvement (goals) and completion specifically, analyzing PST's subsequent practices through the lens of the goals set in previous reflection. Having done so, it was found that more than half of the time, PSTs carried out the goals they set, particularly related to mathematics language. This finding affirms and specifically demonstrates how reflection can be a productive space for

the setting of goals that are likely to be carried out by PSTs. Reflection can have multiple purposes, primary purposes identified in the literature include integration of ideas and learning from experience in and of itself (Bayat, 2008; Schon, 1991), developing collaborative relationships with peers in peer reflection settings (van Es, 2002), and improved practice (Schon, 1991). However, these purposes, or outcomes, are not equally attended to in the research on pre-service teacher development. While studies demonstrate change in teacher knowledge integration (Bayat, 2008; Corscadden & Bell, 2010) and the development of peer reflection (van Es, 2002), there is less focus on goals and changes in practices that may have occurred following reflection. PST's plans for future action can provide valuable windows into how they are prioritizing various aspects of mathematics teaching. Their attempts at these plans can demonstrate for themselves the possibility of change and growth, as well as new activity upon which to reflect and plan yet again for change. It is important to reinforce for PSTs this important process in professional development by making planning for change, and implementation of plans, as an explicit purpose of reflection. This purpose does not replace knowledge integration or ability to engage in peer reflection, rather it reinforces these two elements by linking them with goals and observable change in practice which can be reflected upon again, supporting yet another round of knowledge integration, peer support, and goal setting. Additionally, goal setting and observation of goal accomplishment can support self-efficacy. Clear and challenging goals promote motivation (Locke & Latham, 1990; Bandura, 1993), and by comparing observed performance to identified goals, one focuses on accomplishments and the mastery process. This sets the stage for positive self-beliefs. As PSTs notice their successes, even if they are not total but prompt further refinement of goals and practice, they visualize themselves as capable of change.

5.6 RECOMMENDATIONS FOR PROFESSIONAL DEVELOPMENT

The current study demonstrates student learning in the context of a course built on the Know See Do Reflect Improve framework. Results indicate that students did improve their mathematics language with young children, as well as in their ability to notice young children's mathematical capabilities and needs. As PSTs became more aware of children's mathematical thinking, they responded with plans for practice responsive to children. While results are promising and indicate how the KSDRI framework has been effective, recommendations for future professional development using this framework to promote high quality mathematics interactions are also relevant. These include using materials strategically to promote confidence and culturally responsive teaching, prompting goal setting in reflection more clearly, and developing tools to readily assess mathematics interactions which can be used by PSTs, instructors, and researchers.

5.6.1 *Prompts for Goal Setting within Reflection*

If planning for change is to be an explicit part of reflection, prompts for effective planning are needed. While the current study prompted PSTs to identify changes they would like to make in their next interaction with children, they did not always do this, or at other times, provided unclear or unrealistic plans difficult to carry out. Plans for action based on PST's own experiences and conclusions, while perhaps incomplete, unclear, or unrealistic, are based on "problems of practice" which relate to their knowledge of best practice in early mathematics education, and their desire to achieve these best practices. Goal setting theory (Bandura, 1993) asserts that individuals are most likely to follow through on goals which are aligned with their values and immediate concerns. Reflection is productive when it provides the venue for PSTs to notice (that is, surface what they believe to be important in mathematics interactions), consider

whether any “problems of practice” exist in their noticing’s, and then, when prompted, *and* identify plans for alternative action in the future. A limitation of the current study in this regard is that scaffolds did not always elicit goals clear enough to be observed (by the researcher or the PST) in later practice. Therefore, it is recommended, especially for PSTs and/or others unfamiliar with setting clear and measureable goals, to have specific prompts. In pre-service course on positive behavior guidance, prompts like “What are some personal goals you have, to guidance skills, that you hope to accomplish by the end of your lab experience (what are some areas/skills you would like to see improvement)?”, and “What steps will you take to accomplish these goals?” elicited goal setting. At a later point in the course, prompts like “List your mid-semester goals and describe your progress in reaching each goal” (e.g., Have you accomplished the goal or do you feel like you could use more work in that area?), “What were your greatest challenges in reaching your guidance goals?”, and “What area/skill do you feel like you learned the most or made the most improvement over the semester?” prompted reflection on implementation of goals (McFarland & Saunders, 2009). Similar prompts might be adopted and altered in the context of an early mathematics course.

5.6.2 *Promoting self-efficacy in PST Training*

Characteristics of learning environments that promote self-efficacy include viewing “ability as an acquirable skill” (Bandura, 1993, p. 125), moving away for competitive comparison, and focusing on self-progress and accomplishment. While PSTs engaged in noticing self-progress and accomplishments via video feedback, the course did not specifically address the idea that skill in mathematics language is acquirable with practice and effort. While the researcher hopes this was apparent to the PSTs, and it might have been implied in the course activities which were designed to promote acquisition of new skills through information sharing,

practice, and reflection, it might be important to foreground their experiences with children by clearly explaining that PSTs own ability can change. Such foregrounding should be written into the course syllabus and also be incorporated in the lectures in the early part of the course, as well as reminders during reflection assignments.

5.6.3 *Materials*

Materials specifically can be capitalized on in professional development to scaffold teachers' language in specific domains. Materials also support confidence: by using familiar and fun materials with clear mathematical objectives, teachers are cued as to what sorts of mathematical language to use. In the current study, PSTs and children played with a variety of materials, which were categorized as open ended materials, blocks, mathematics games, and Magna-Tiles. Magna-Tiles were provided to PSTs with directions to use in mathematics plan, and mathematical games were also supplied in the course, which some PSTs then used play with children. While the Magna-Tiles were associated with increased frequency of shape language, PSTs reported greater confidence and self-perceived success when using mathematical games. Other professional development programs have also relied on familiar materials, such as books, as a means of building teachers confidence in leading mathematics lessons (Chen & McCray, 2012). Books and games are both materials most early childhood educators are familiar and comfortable with and make a good foundation upon which to build new skills, especially in the area of mathematics where educators may otherwise lack confidence. Mathematical games may provide extra support as they usually have rules and clear learning goals, so teachers playing them know what steps to follow and where to focus children's attention. It may be, though, that while it is beneficial to start with materials which highly scaffold interactions, as comfort develops, educators can shift to materials and activities in which they identify more of the

learning goals and plan for mathematical discussions. For early childhood pre-service teacher educators, it might be helpful to think of a range of mathematical materials to intentionally choose from. These materials should support teachers at varying levels of confidence and fluency in mathematical discussions, and address diverse mathematical domains.

Figure 10 presents a hierarchy of materials and their relationship to teacher and child interactions. At the bottom level are mathematics games and learning activities. This is at the foundational level because it provides the most support to teachers who may or may not be comfortable with mathematics and guiding mathematics discussions. Games and specific learning activities, such as Chutes and Ladders, provide the purpose of the activity (i.e. to reach the end of the game board), and the mathematical focus (i.e. counting). Teachers then can build on these scaffolds to engage children in game play and math learning.

The next level is books with a mathematical focus. Books with math focus are considered to provide some scaffolding for teacher and child math discussions, however, not as much as games. Teachers have to read books and plan to highlight mathematical components when reading with children. Such books may explicitly call out math elements, for example *Chicka Chicka 1,2,3* (Bill Martin et al., 2014), which engages the readers in counting to 100, or may have mathematical ideas embedded for example *The Very Hungry Caterpillar* (Eric Carl, 1994) which follows a pattern which may be highlighted by the readers but is not explicitly named in the book. Books provide adults and children many opportunities to mathematize, yet may require some intentional planning on teachers part, putting them at the second level of the hierarchy.

At the third level is materials with a math focus. These are materials teachers may frequently provide in classrooms for math “enrichment”, such as blocks, magna-tiles, and small

counters. Such materials are often associated with mathematics engagement, yet teachers may be challenged to fluently engage in mathematizing with such materials. Such materials may require additional language planning on teachers' part, and close observation of children's use of materials to provide responsive math language. The benefit of such materials is that children can use them in unstructured and child directed play, and the expectation is that teachers then provide responsive math language.

At the highest level are open-ended materials with no specific math focus. This top level represents the goal of mathematizing: teachers should ideally have the competence and fluency to engage in mathematically rich discussions with children in nearly any environment, with or without specific materials. This is what I like to call "math in the wild", and may include toys and materials in the environment not usually associated with math, such as dress up or art, or outside the classroom, such as plants at the park, observations of vehicles or transit, and beyond. Teachers need to have fluent and flexible use of mathematics language to mathematize at this level. They may need to prepare for conversations by identifying mathematical concepts inherent in every day play ahead of time, or in the moment (for example, discussing size when trying on clothes in dress up or pointing out patterns during art activities). Teachers fluency at this level means they are able to mathematize in almost any situation, and are consistently engaging young children in mathematics discussion throughout the day, not limited to lessons on math or specific materials. Teachers at this level may also draw connections from math specific games, books, and materials to children's everyday play and experiences, increasing the relevance of these math specific materials.

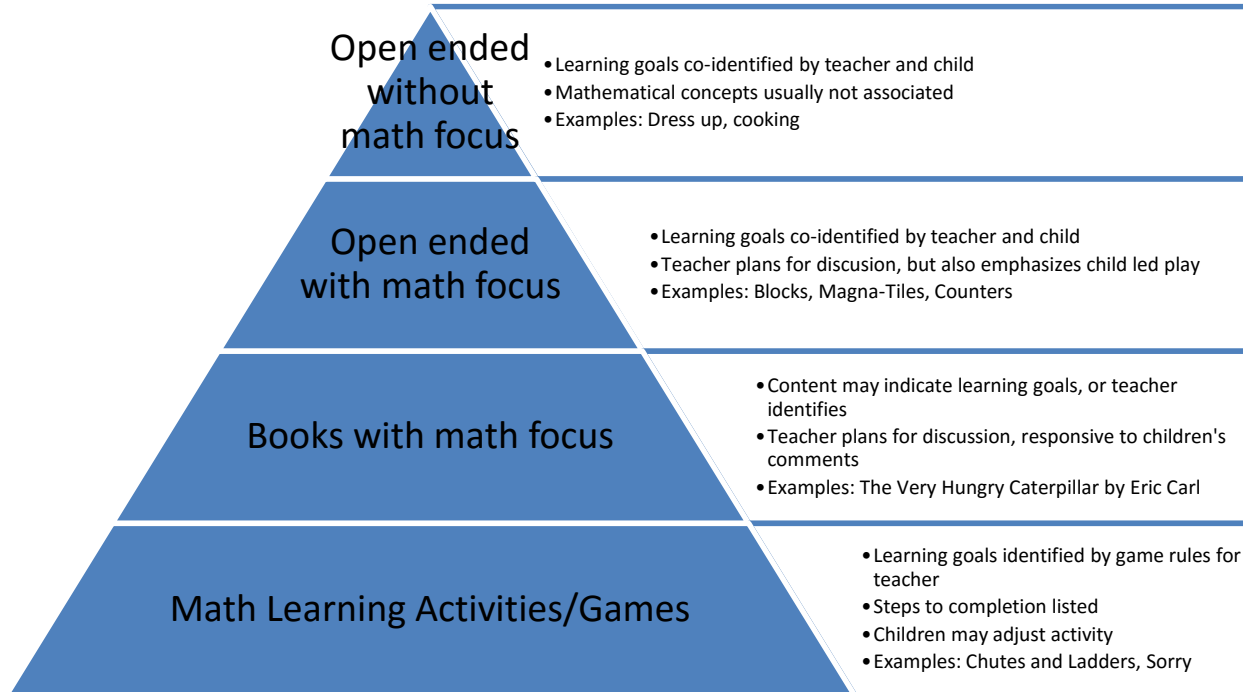


Figure 6 Materials to Support PST Confidence and Practice

Furthermore, consideration of materials at any level may help teachers plan for culturally relevant and inclusive learning opportunities for young children (Anderson & Gold, 2006; Moses & Cobb, 2001). On one hand, some children may have a variety of materials at home with which they are familiar and make a strong starting point for mathematics learning at school as well. When teachers provide mathematics language related to materials already present in children's homes and which they enjoy, they can join children's culturally based play and draw out the mathematical ideas inherent in it. From the current study, Tara's interactions with Jasmine and Faith present a nice example. During their first and fourth video observations, Tara and the two young girls engaged in dramatic play, using dress up clothing and props to pretend to cook a meal (in video one) and later, to play as princesses (video four). While these materials may be at the top tier in terms of challenge for mathematizing, Tara became more responsive in

her provision of mathematics language, demonstrated by the 20% increase in her rate of responsiveness in video one (23%) to video four (43%). This example shows how a PST can be increasingly responsive while children engage in culturally based play, reinforcing both the value of this play as well as providing language around its mathematical components. With this view of materials in mind, future iterations of this course and similar professional development should consider the role of materials as both a support for teacher confidence, as well as a support to culturally responsive mathematics teaching. Materials may or may not serve both purposes simultaneously. Materials that are associated with increased teacher confidence (games and books) typically also keep the teacher as the primary actor in the mathematics interaction. With these sort of materials, teachers provide the goals and the structure for mathematics talk, unless or until children learn how to use such materials independently. On the other hand, materials which may be more culturally based (such as open ended materials in the home such as dress up or blocks) leave the door open for children to drive the focus of play and discussion, a situation which may be uncomfortable for teachers who may not have flexible skills in mathematizing. Therefore, it might be helpful to clearly articulate to teachers the various roles materials can play in engaging young children in mathematics, and how they themselves can draw on certain materials (which they may or may not feel comfortable with) to move towards more culturally responsive practices as they gain fluency in mathematizing with a variety of materials.

There may also be important implications for providing early mathematics learning opportunities to low-income children. Such children may have limited access to many of the materials listed in this study prior to attending preschool or child care. These children may not be familiar books, blocks, games, or even open-ended materials in the home, or be used to playing with these in the ways many teachers expect. Here, we have an opportunity and a duty

to prepare teachers to mathematize with these children. When teachers are skilled mathematizers, they are able to transcend the need for specific materials, and help children notice what is mathematical around them no matter where they are. Teachers also can become aware of the various types of engagements these children might demonstrate. Whereas a child familiar with a specific material might ask questions and act on materials in expected ways, children who are using classroom materials (such as blocks or games) for the first time might engage with them in ways that teachers do not identify as mathematical (or desirable). It is important for teachers to notice young children's first attempts with materials and provide responsive mathematizing.

5.6.4 *Assessing Mathematical Interactions and Reflection*

Tools are needed to measure the quality of mathematics interactions in early learning settings. In the present study measures were developed to assess the change in language and responsiveness of PSTs over time. While these measures provided a valuable lens, they may not be feasible outside of research settings. For the college instructor tasked with assessing many students in short periods of time, more streamlined measures are needed. To this end, it is important to consider if appropriate measures exist, and if not, how available measures might be modified in order for teacher educators working gauge the mathematics language quality of PSTs. A possible recommendation is the Mathematics Scan (M-Scan) (Berry, Rimm-Kaufman, Ottmay, Walkowiak, & Merrit, 2010) developed to assess teachers' implementation of mathematics teaching and learning standards set forth by NCTM. Whereas the current study focused on two discrete dimensions of interaction – language provision and responsiveness – the M-Scan provides a more holistic view of Mathematics teaching. However, the M-Scan is targeted at elementary age instruction, and important developmental differences between

preschool age children and older children should be considered in how to assess teacher/child engagement in mathematics. Another tool, the Classroom Observation of Early Mathematics (Clements & Sarama, 2010) is designed to measure the environment and mathematics activities which teachers provide. This tool is designed for use in preschool classrooms and requires three or more hours of observation of teachers and children in the classroom. A measure similar to either the M-Scan or COEMET which is designed to observe preschool teachers and children in somewhat shorter time segments would be a valuable tool for teacher educators who desire to provide timely assessment and feedback of teacher's skills. It would also provide a specific framework for teachers to observe their own practice.

5.7 DIRECTIONS FOR FUTURE RESEARCH

This study raises new questions. First, the study did not focus analysis on children's mathematics language, and whether and how did this change along with PSTs. This line of inquiry is the logical compliment to current results, and could describe the effectiveness of the course in terms of change in children's language. The math language of adults is so vitally important because it is a strong predictor of children's own math language, and subsequent performance on math assessments. Whether and to what extent children's math language changed over time and with the use of materials would clarify findings on PSTs, potentially confirming and possibly complicating current findings. For example, while not the focus of this study, it was found that one child's math language diminished noticeable between video 1 and video 4. While this was not the case for all children, it would be important to unpack this potentially troubling shift in language for the child, and what this might mean in relation to the changes observed for the teacher working with him.

Another question raised, but not answered, by the study was what is the right amount of teacher language in general? What is the right amount of math language? PSTs themselves wondered about this in reflection, and worked to strike a balance between providing mathematics language and facilitating young children's math talk and play. Previous research by Klibanoff et al. (2006) found that higher rates of teacher math talk correlated with children's math gains over the school year. However, it might be possible that too much teacher provision of math talk inhibits children as active participants in math learning, and could potentially work against gains for children. I would like to study whether there are thresholds of optimal math language frequency from teachers. Additionally, this study did not address the quality of math language outside of responsiveness. Math educators and scholars identify specific "talk moves" that teachers can use to engage children in meaningful mathematics conversations (Kazemi, 2003; Hintz & Smith, 2013). This study did not analyze math language in terms of quality, and I would like to analyze this or new data again with a quality lens. Another direction for research is the refinement and validation study of Mathematizing Opportunity Ratio Tool (MORT). Here, the question is *do response rates relate to child math gains?* While the tool afforded analysis of responsiveness and measured change across time points, it is not known if the rate of responsiveness relates with children's math learning, and moreover, if there is an optimal rate.

Finally, I would like to apply several of the coding tools for use with college instructors and students. I would like to adapt both the Early Mathematics Language Coding System (EMLCS) and MORT for instructor and student use in the Coaching Companion. To do so will require some simplification of coding schemes and development and addition of coding schemes in the Coaching Companion. Instructors and students could then code their own video and track

data regarding mathematics domains in math lessons and responsiveness. Such access to quick coding would put analysis in the hands of the college students and instructors outside the research context. Instructors and college student could then review video and produce data about practice. For the instructor data would ideally guide lectures and coursework. For the college student, data would support noticing the content and quality of interactions, and ideally guide goal setting.

5.8 LIMITATIONS

There are several limitation of the current study which may pose threats to internal and external validity. First, the selection of participants was based on their completion of consents and course assignments used in data analysis. This reduced the potential participant pool by half (originally, eight students had agreed to participate), and these students who worked so hard to complete assignments could represent a sub-group of highly motivated pre-service teachers. Additionally, each of the PSTs in the study indicated they were between 20 and 25 years of age, whereas the age of teachers varies widely in the field. These limitations to the participant group make it difficult to make claims about how pre- and in-service teachers more generally would respond to the professional development. Future studies should the address of professional development on mathematics language using larger samples of pre- and in-service teachers from a wider age range. Additionally, it is advised that research measures to not rely wholly on PSTs submission of course work.

Another limitation is in the use of a multiple-case design in which all participants experienced professional development simultaneously and there was no control group. It was not possible, therefor, to differentiate between change that might have occurred due to maturation verses due to the professional development, and difficult to make causal links elements of

professional development to changes in PST behavior. Yet again, continued research on this topic should include quasi-experimental treatment and control designs with two sections of the same course with design differences, or a multiple baseline design.

5.9 CONCLUSION

While future research will be needed which addresses recommended changes in the design of professional development, as well as the incorporation of new approaches not envisaged by the current study, the results indicate that professional development designed to address pre-service teachers' knowledge, noticing, practice, and reflection can set the stage for change in the frequency and responsiveness of pre-service teachers' mathematics language with young children. Results further indicate shifts in pre-service teachers' reflection and beliefs can also occur, bringing together multiple elements of pre-service teacher change together in one study. This study contributes a multi-dimensional description of pre-service teacher change, and adds to the existing literature by describing outcomes of a professional development program designed for pre-service early childhood educators.

APPENDIX

Appendix A. Early Mathematics Language Coding System

Code/Sub code	Definition	Example
Classification	Systematic arrangement of groups according to clear criteria	“Let’s put all the squares in one pile and all the triangles in another pile.”
Magnitude	Description of a magnitude or comparison of two or more items to evaluate relative magnitude	"There's a lot here"
Saying quantity or magnitude words	This involves describing the global quantity or magnitude of objects	“This is a huge pile of blocks”
Empirical matching	Here the adult or child makes a direct comparison/match of concrete objects	Teacher, “I have four cars, can you make yours like mine?” Child lines up four more cars next to the set the teacher has presented.
Comparison without quantification (the er words)	The adult or child engages in magnitude in an approximate way, without exact quantification	“You are way bigger than your baby sister”
Comparison with quantification	The adult or child compares dimensions using quantitative words. The child or adult may estimate the quantity or measure it exactly.	As two children are building a structure, one says "we need one more," indicating that the line of blocks was too short by one.
Qualitative comparison	The child or adult makes a comparison with an attempt at quantification, but one that is inexact.	“It looks like there are about five or six fewer cheerios in your bowl than mine.”
Enumeration	Numerical judgement or quantification	
Saying number words	the child or adult simply says a number word	“I’m four years old”
Counting	The child or adult overtly counts objects or says the number words without counting objects	“One, two, three”

Subitizing /estimation	Without having counted, the child uses a number word to designate the cardinal value of a set. The child or adult could have subitized the value - that is, perceived the number without counting - or the child could have estimated the cardinal values, there is no way for us to tell. In either case, the context makes it clear that the child is not simply producing a wild guess or randomly producing a number word.	Teacher: "How many green beans are there?" Child without counting correctly answers "Three!"
Reading/writing numbers	The child or adult read numbers and/or writes numbers	A child writing their age on a picture of themselves
Dynamics	Exploration of the process of change or transformation	Teacher, "We had six cars, but now we have two, how did it get to be two?" Child, "Benny took four away!"
Extra codes for Enumeration		
Number total - all	Number total - when number used to refer to how many or all	Teacher "How many children are here today?" Child counts children, "Twelve!"
Number one - identifier	Number one identifier - often the word one is used to distinguish which object. It is not clear that this is a number meaning	Teacher, "Which outfit do you want to try in dress up?" Child, "This one."
Number zero	Number zero - when number words are used to mean there's is nothing	Teacher, "We're all at the table, how many chairs are left empty?" Child, "Zero".
Number position	When adult or child refers a number position in order relative to another number/other numbers	Teacher, "Can you point out what number comes after 12 on the chart?" Child points, saying "Thirteen."
Number time	When adult or child refer to a time of day using number words	"My mommy said she's picking my up at 3."
Number question	When adult or child ask about how many of a given object	"How many cookies are there?"
Number measure	When adult or child count standard (inches, feet) or non-standard (i.e. hand width, steps) units of measurement.	"It's about 10 steps from the door to the play ground."

Ordering with numbers (Boyd, 2014)	When adult or child use ordinal counting to determine/assign who/what is first, second, third, etc.	“I’m first in line, then your second, then Matt is third.”
Patterning	Objects are arranged in a regular, rule governed manner	Child arranges blocks in a repeated ABAB pattern of square, rectangle, square, rectangle
Spatial Relations	Exploration of positions, directions, and distances in space	
Around	So as to surround or envelop	“I’m going to put walls around the cars.”
By	Close to, next to.	“Come sit by me.”
In	From the outside to a point within; into.	“Let’s put the small blocks in the bucket.”
On	In a position above, but in contact with and supported by, upon	“I put the girl (figure) on top of the tower.”
Out	In a direction away from the inside.	“Let’s take the small blocks out of the bucket.”
Over	In, at, or to a position up from; higher than, above	“We’ll have to step over the puddle so our feet don’t get wet.”
Under	In, at, or to a position down from; lower than; below	“I’m standing under the umbrella.”
Up	From a lower to a higher place' away from or out of the ground	“We’re climbing up the stairs.”
Part/whole	Says part of whole (Blevins-Knabe et al, 2000)	“Of the group of beans, about half are green.”
7 - Shape	Exploration of spatial forms	
Symmetry	Exploration of symmetrical relationship, involving a correspondence in size, shape, and relative position of parts on opposite sides of a dividing line, median plane, or axis.	“Hey, when I cut this square in half, it’s the same on both sides,
Figure identification	The child or adult's behavior indicate recognition of particular shapes	“That’s a triangle”
Shape matching	The child or adult uses geometric properties of shape to complete a task or solve a problem.	“Let’s put all the triangles together”
Shape composition	The child or adult uses geometric forms to compose a different shape (whether a new shape or simply a larger version of the parts of the shape). (New code added by Boyd,	“If we put these two triangles together, we can make a square!”

	2014, based on Clements and Sarama, 2003.)	
Shape attribute description	The child or adult describes the shape according to its parts (i.e. how many angles, sides, etc.) (New code added by Boyd, 2014, based on Clements and Sarama, 2003.)	“See the pointy angles? There are three, that’s what makes it a triangle.”
Shape comparative description	The child or adult describes the shape by using comparison. (New code added by Boyd, 2014, based on Clements and Sarama, 2003.)	“Yeah, that’s the door block” (the rectangular block)

Appendix B. Kappa Statistic for Mathematics Language Codes

Kappa Statistic (2-raters)			
Domain	Transcript 1	Transcript 2	Transcript 3
Math Words	0.75	0.90	0.89
Classification	0.00	0.00	0.00
Magnitude	0.62	0.53	0.74
Enumeration	0.90	0.92	0.85
Patterns	0.00	0.00	0.00
Spatial Relations	0.75	0.81	0.71
Part/Whole	0.00	0.00	0.80
Shape	0.82	0.41	0.95

Appendix C. Mathematizing Opportunity Ratio Tool Coding Guidelines

Purpose:

The purpose of the Mathematizing Opportunity Ratio Tool (MORT) is to observe the rate at which teachers or other adults mathematize (provide mathematical language) in response to opportunities presented by children.

Procedure:

1. Code ten second segments. Start each segment at the zero and go to the nine second mark (e.g. 00:00 - 00:09, 00:10 - 00:19, 00:20-00:29).
2. In each segment code for each Opportunity that occurs (math engagement - look, touch, gesture, speech) presented by children by marking 1 in the column under the appropriate Opportunity category and in the appropriate time segment row. Mark each opportunity behavior in a 10-second segment only one time.
3. In each segment code for Mathematizing provided by teacher, either in response to a specific opportunity behavior, or that is initiated by the teacher. Code by marking 1 in the appropriate column next to the related opportunity behavior, or if teacher initiated, in the Mathematizing T.I. column in the appropriate time segment row.

Opportunity Code Definitions: Child Math Engagement Behaviors

Look – the child looks at object with interest (May or may not yet have clear mathematical focus).

Touch/gesture - The child actively touches (not passively holding an object that they forgot about) or manipulates objects. Objects may have clear mathematical focus, or child may identify mathematical focus (i.e. tapping each item as they count). May also include gesture, when the child uses physical movements with or without an object to communicate about a mathematical idea.

Speech - Math Specific – The child talks about mathematical concepts and/or uses math related language.

Speech - General - The child talks using general/not math specific language.

Mathematizing Code Definitions: Responsive and Teacher Initiated

Responsive Mathematizing Speech - Teacher utterance about mathematical concepts in response to child mathematics engagement behavior(s). There may be more than one instance of math language in a single response (e.g. *I like the big tower you are making* (Instance of magnitude), *it has all the squares stacked on top of each other* (Instance of geometry and spatial awareness)).

Teacher Initiated Mathematizing Speech - Teacher utterance about mathematical concepts which is initiated by teacher and not in response to observable child engagement. Similar to above definition, there may be more than one instance of math language in a single teacher initiation.

Coding Rules

- A. Each 10 second segment may have more than one engagement behavior, code 1 for each engagement behavior observed.
- B. Each 10 second segment may have more than one response, code 1 for the response observed in the column next to the highest engagement behavior to which it was a response. For example, in a 10 second interval the child may look at and touch an object, to which the

teacher responds, "You have the big cup"; code Look - 1, Touch - 1, Response to Touch - 1 (coding the response with the highest level of engagement). Later in the 10 second segment the child may say "Yeah, Big cup", and the adult responds, "Where is the small cup?", here you would code Math Talk - 1, Response to Math Talk - 1. Therefore there is a maximum of three child engagement per child per interval and 3 response codes per adult per interval (even if the adult responds multiple times to one type of child engagement in a given segment, you would only code this one per child and once per adult).

- C. If children are engaged in more than one engagement behavior and the teacher responds, code the response with the highest level engagement behavior. For example, if a child is looking at and touching math objects and the teacher responds, the response would be coded with touching (Look - 1, Touch - 1, Response to Touch - 1).
- D. If a communication act in response to the child began within one second of the previous window, count this in the previous window.
- E. If a teacher provides a mathematizing statement that is not in response to a child engagement behavior, code this as a 1 in the bright green "Mathematizing - T.I." (T.I. = Teach Initiated) column. For example, if the child is playing with a toy, and the teacher introduces a new game (which the child was not previously looking at, touching, or talking about) and says, "I'd like to play this counting game with you."

Appendix D. Kappa Statistic for Mathematical Opportunity Ratio Tool Codes

Kappa Statistic (2-raters)			
Behavior	Video 1	Video 2	Video 3
LookOp	1	1	1
LookRes	NA	1	NA
TouchGesOp	1	1	1
TouchGesRes	NA	1	1
MathSpOp	1	1	1
MathSpRes	1	1	1
GenSpOp	1	1	1
GenSpRes	1	1	1

Appendix D. Reflection Prompts

Students responded to reflective prompts during and after viewing their video observations.

1. What do you notice about the children in this lesson?
2. What do you notice about yourself in this lesson?
3. What worked well in this lesson?
4. What did not work?
5. How will you use what you have learned to inform your next mathematics interaction with children?

Appendix E. Productive Reflection Codes (adapted from Bayat, 2008)

Characteristics of Productive Reflection:

1. Comments on a specific learner or several learners
2. Comments on child's mathematics thinking
3. Comments on mathematics content knowledge
4. Comments on quality of mathematics instruction

Characteristics of Unproductive Reflection:

1. Comments on basic description of what occurred in teaching (could be math related or not)
2. Comments on listing preferences or opinions (could be math related or not)

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