Application of model predictive control on wind farm incorporated with dual battery energy storage system

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Abstract

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The research work in this thesis focuses on the application of Model Predictive Control (MPC) to Dual Battery Energy Storage System (dual BESS) so that the reference power from the actual wind farm power can be tracked with satisfactory performance. The control strategy considers certain practical constraints including the power delivered/extracted from each battery, as well as the state of charge on each battery. The operation of the two batteries is categorized into two modes. In Mode 1, the first battery charges and the second battery discharges; and Mode 2 deals with the opposite, i.e., the second battery charges and the first battery discharges. The power from the battery is treated as the control signal which is an optimized computation results given real-time by MPC.

The significance of this work contains mainly two aspects. First, the two-mode operation of batteries removes the problem of overloading battery. Second, the actual wind power data is used to verify the dual battery integrated wind farm performance using MPC, thereby tracking the reference power by reducing the battery switching times and extending the life of the battery.

TABLE OF CONTENTS

LIST OF FIGURES

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DEDICATION

I dedicate this work to my parents.

Chapter 1. INTRODUCTION

Nowadays the need for renewable energy sources is increasing, as the natural resources are depleting. The main sources for renewable energy include photovoltaic or solar systems, wind farms or turbines, geothermal energy systems, and hydroelectric systems, just to name a few. It is evident that for expansion of renewable energy sources across the world, increasing the penetration of the corresponding renewable energy sources has significant importance. In this work one such renewable energy source penetration, i.e, wind farms, is considered. Wind farms are also called wind power plants, which is a collection of a group of wind turbines placed in an area to generate wind energy in a collective effort. As a matter of fact, there is a lot of uncertainty in the wind speed in any part of the world [17][18]. Therefore, predicting or forecasting of wind power, if done, will lead to some margin of error and a mismatch between the supply and demand [16]. Not only that, but this may also lead to poor power quality because of the fluctuations in power. If the penetration of wind farms increases with this kind of power fluctuations, then there is a high chance for the grid to collapse and there may be damage to the electrical equipment used by the household. To prevent such irregularities, we need to track the supply to meet the demand and reduce the fluctuations of the power generated. To mitigate these irregularities, a control strategy must be applied on the system with appropriate methodology.

Model predictive control is an efficient strategy control strategy which considers practical constraints and computes in real-time an optimal solution in a receding horizon fashion. This control strategy can be used on distributed energy storage systems and can meet the desired objectives as shown in [7][8]. Distributed energy storage systems are the collection of energy storage systems including electric battery energy storage systems, and hydrogen battery energy storage systems, just to name a few. These are a part of one microgrid or a standalone system distributed to various sources.

In this paper the focus is on two battery energy storage systems on one wind farm where some practical constraints are imposed. The constraints include the state of charge of the battery, and the power delivered/extracted from the battery as mentioned in [16]. This makes the control strategy an optimization problem with multiple inequality constraints. To solve such optimization problems, MPC is going to be designed and implemented. Although there exist a variety of solutions to constrained MPC problems, Lagrange's multipliers, Karush Kuhn Tucker conditions and quadratic programming methods are used in this research.

This thesis proposed a constrained MPC solution to dual BESS so that the reference power from the actual wind farm power can be tracked with satisfactory performance. The operation of the two batteries is categorized into two modes. In Mode 1, the first battery charges and the second battery discharges; and Mode 2 deals with the opposite, i.e., the second battery charges and the first battery discharges. The power from the battery is treated as the control signal which is an optimized computation results given real-time by MPC. The significance of this work contains mainly two aspects. First, the two-mode operation of batteries removes the problem of overloading battery. Second, the actual wind power data is used to verify the dual battery integrated wind farm performance using MPC, thereby tracking the reference power by reducing the battery switching times and extending the life of the battery.

The contribution of this research is to formulate the given problem by clearly modeling the system mathematically and solving the problem with appropriate methods. MATLAB simulation results are provided to verify the effectiveness of our method.

In the remaining of this thesis, related work will be briefly discussed in Chapter 2, followed by Chapter 3, where some background on MPC and mathematical methods to constrained optimization problems are introduced. Chapter 4 will propose the constrained MPC solution to dual BESS systems and provide supporting results via MATLAB simulation to verify the effectiveness of the approach. Finally, in Chapter 5, conclusions and future work are presented.

Chapter 2. RELATED WORK

In this chapter, the work related to MPC on distributed energy storage system is first presented, then the work related to applications of battery energy storage systems, and finally on the wind farm integrated with battery energy storage systems using model predictive control.

2.1 MODEL PREDICTIVE CONTROL (MPC) ON DISTRIBUTED ENERGY STORAGE SYSTEM

Distributed energy storage systems (DESS) have gained popularity as the renewable energy sources came into picture. As these systems have several applications to benefit the supply from renewable energy resources and sometimes, they are the backup options. After embedding DESS into the renewable energy architecture some control strategy is required to coordinate the entire microgrid. Some of these strategies are discussed in this section. Among them [4] uses distributed model predictive control (DMPC). The system is treated as two layers, the upper one corresponding to the optimal scheduling of power with distribution network operator. The lower layer is responsible for tracking supply power to the forecasted load. For this a method is proposed based on the conditional probability and the control strategy explained above. In another example [5]the information exchange mechanism is designed with the help of DMPC. All the components in a microgrid exchange the information with each other and they try to meet the demand by adjusting their corresponding supply. The author claims that the cost of maintenance comes down if this method is used and the closed loop stability of the system is improved. To investigate an example with variety in DESS like [6]. In this paper applies model predictive control on a hybrid energy storage system with diesel generators, electric energy storage system and hydrogen energy storage system. The model predictive algorithm’s main objective is to achieve track the demand by maintaining state of charge and the level of hydrogen within their limits. The additional constraint present is the power supply since this is a standalone system and there is no abundant power supply. Another example of this kind is [7]. This paper develops some control strategies to supervise the operation and maintain the system, other control strategies supervise iterative distributed actions taken by sub systems. Here all the control strategies used are model predictive control. The system is a wind turbine, a photovoltaic generator which is combined with the DC power grid. To each renewable energy source, a battery energy storage system is attached. Some examples try to use more than one control strategies like [8]. In this paper a combination of the control techniques is used, namely the model predictive control and adoptive droop control. The objective set in this paper is to manage the system frequency irregularities with the help of energy storage systems and electric vehicles. The mode predictive control regulates the system frequency using energy storage systems in large scale integration. Adoptive droop control takes the electric vehicles help using vehicle to grid technologies in contributing to system frequency regulation. Another such work in which modes of operation are defined is [9]. In this paper a voltage control scheme for distributed generation is proposed using model predictive control. This proposed strategy has two modes of operation, one is the preventive mode which is in a normal condition and the other is the corrective mode when there is a serious deviation in the voltage. In this work a small portion of this is taken into consideration, for example one storage system among these DESS is selected and the performance of control strategy is investigated.

2.2 APPLICATIONS OF BATTERY ENERGY STORAGE SYSTEM (BESS)

Battery energy storage systems had come into limelight because of their mobility and ease of configuration. In this section the applications of BESS are discussed briefly. From the literature [10], details of the energy management and control methods on grid using battery energy storage system (BESS) can be inferred. Initially the paper elaborates on the materials used in large scale battery energy storage systems. The configuration of the BESS with the respective material along with the rated capacity and the main functions performed by these batteries are explained briefly. Then a comparison of the batteries with the different materials is made and the usage of these batteries in appropriate applications is made. Later the usage of battery energy storage systems is shown across the globe. Then comes the important aspect of the paper that is the applications of the BESS in different application scenarios. The main three application scenarios are on the power supply side, at the power grid side and at the power distribution side. The major applications at the power supply side are smoothing output fluctuations using some excess power from the BESS. Frequency modulation of the ancillary service using some power from BESS and standby power supply where BESS can be a reliable source. The second part discusses the power grid side applications those are peak regulation and frequency modulation by providing some power to grid, power flow distribution optimization, power quality improvement, when there is a delay in transmission equipment upgrade BESS can be used as a backup option to meet the demand. Relieve congestion in transmission equipment using some power from BESS to meet the demand, load tracking or renewable energy ramping control, sometimes used as a system backup and can also be considered as a virtual power plant. The third part discusses at power distribution side, some of them are peak load shifting using power from BESS, vehicle to grid applications where battery can be charged using electrical vehicles, community energy storage using BESS, power supply reliability can be maintained using BESS, Demand electricity charge management using BESS. This work focuses on tracking forecasted power from the supply side and smoothing application.

2.3 WIND FARM INTEGRATED WITH BESS USING MPC

Model predictive control has gained popularity because of its efficient performance in real world applications. In this section the applications of model predictive control on wind farm integrated with battery energy storage system are discussed. The work [11], uses open loop control optimal control scheme to smooth the output power as much as possible and track the forecasted power on an hourly basis. The operational constraints are placed on the control strategy such as the state of charge limits, current charge discharge limits, and lifetime. To give a unique opinion to relate control strategy to selling cost of energy. The paper [12] presents a control strategy to manage the amount of net energy generation sold to the market from a wind farm integrated with a battery energy storage system. Here the model predictive control strategy is used with practical system constraints. The method used here is to store the energy in off-peak conditions and sell the energy in peak conditions in compliance with the Australian National Electricity Market. The paper presents a comparison of the performance of the given control strategy under different scenarios in terms of key performance index and earnings. To share a glimpse on protection of BESS. The work [13], develops a methodology for high performance, risk-averse battery storage control design. The paper addresses the problem of control performance and optimistic shortfall. This method is based on two aspects. First one is for the battery system model is enabled by calculating the upper and lower bound on the global optimal solution. The second aspect is based on underestimating the capacity and giving control decisions for normal variations. Another similar idea is presented in [14]. This work develops a control strategy to minimize the operational cost by optimally using the battery capacity and smooth the transient power through short term dispatching. For this the lifetime cost function of a battery is defined along with the dispatching cost function with the constraints on the state of charge. Coming to tracking the forecasted power, the paper [15] presents a scenario where a wind farm is working together with a dual BESS to track the supply power to the forecasted power. To achieve this a method is proposed where the first battery is discharged when the actual wind power is less than the forecasted power. The second battery is charged when the actual wind power is greater than the forecasted power. These batteries switch their roles when they exceed their state of charge limits. This is because the battery, when operated within the limits of its state of charge, the lifetime of the battery increases. This reduces the cost of replacing the battery energy storage system with the new one. Now integrating model predictive control into it. The paper [16] works on a wind farm integrated with a battery energy storage system. Initially the wind power is forecasted using a probabilistic method with the history of the wind farm data. Then a control strategy using model predictive control is proposed to track the actual wind power to the forecasted wind power. Some constraints are added to the model predictive control relating to the battery energy storage system like state of charge on the battery and power extracted/delivered to the battery. This paper discusses a similar idea with two BESSs. Two modes of operations are assumed to propose a methodology, focusing only on tracking the forecasted power and not on the forecasting part of it. In the next chapter some background related to the methodology used in this paper is explained.

Chapter 3. BACKGROUND

This chapter will start with a brief introduction to Model Predictive control, followed by the types of constraints on it, then Lagrange's Multipliers are introduced, later KKT conditions and finally quadratic programming is explained.

3.1 Model predictive control (MPC)

Model predictive control is a control strategy which follows the method of computing a trajectory in which the decision variable is the manipulated variable u, which is the control signal. Through this it tries to optimize the future behavior of the plant output y. To describe it in a simpler way this control strategy chooses two windows, which have fixed sizes. The first window is the prediction horizon with a size of Np, and the second window is the control horizon with a size of Nc. At some instant k, let the control signal in control horizon be $u(k), u(k+1), u(k+2), \ldots, u(k+Nc-1)$ and let the outputs corresponding to prediction horizon be $y(k+1), y(k+2), y(k+3), \ldots., y(k+Np)$. Now the model predictive control tries to optimize the future behavior of output y in prediction horizon (which are the outputs defined above in prediction horizon) using the control signal in control horizon (which are the control signals defined above in control horizon). These control signals are estimated by minimizing a cost function which has two terms. First term relevant to error and the second term relevant to the control signal. At every instant this control strategy, the control strategy runs this optimization and takes the first value of the control signal into consideration for the plant. This is the process in which the model predictive control works. Reference of this material is taken from [2]. In the next section constraints on model predictive control are elaborated.

3.2 MODEL PREDICTIVE CONTROL WITH CONSTRAINTS

What makes MPC a powerful tool is its capability in dealing with constraints. As mentioned in [2], it is very important to check the constraints on the practical systems before implementing the actual control method. In the following example, we consider a system described by continuous-time state space model as below:

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \quad (1)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (2)$$

In Equations (1) and (2), $x_1(t)$ *and* $x_2(t)$ are two states, $y(t)$ is the output of the system. Since MPC by nature is a discrete-time control strategy, the continuous-time model needs to be converted into discrete-time. Assuming the sampling period Δt=0.1sec, a discrete-time state space model can be derived:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.9801 & 0.0993 \\ -0.3973 & 0.9801 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.0993 \\ -0.0199 \end{bmatrix} u(k) \quad (3)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \quad (4)$$

In Equations (3) and (4), $x_1(k)$ *and* $x_2(k)$ are the two states, and $y(k)$ the system output, all in discrete domain at the k^{th} sampling instant $t = k \cdot \Delta t$,. Now applying MPC and doing simulation through MATLAB, the closed-loop system will perform as Figure 1.1 shows. It indicates that the control signal amplitude is almost reaching 40, which might be too high for practical systems, and causes damage to actuators.

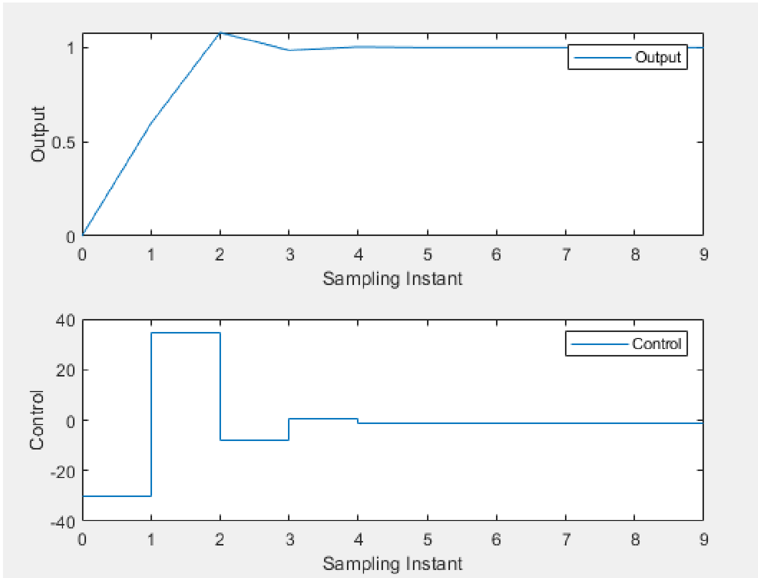

Figure 1.1. Output and control signal for example without saturation.

Now suppose a practical system has restrictions on the operation range of its control signal, say, between -25 and 25. If we are ignoring the constraints during the design, instead, we let the control signal saturate beyond the limits, i.e., the control signal becomes 25 if its greater than 25 and -25 if it less than -25, then the output becomes as shown in Figure 1.2.

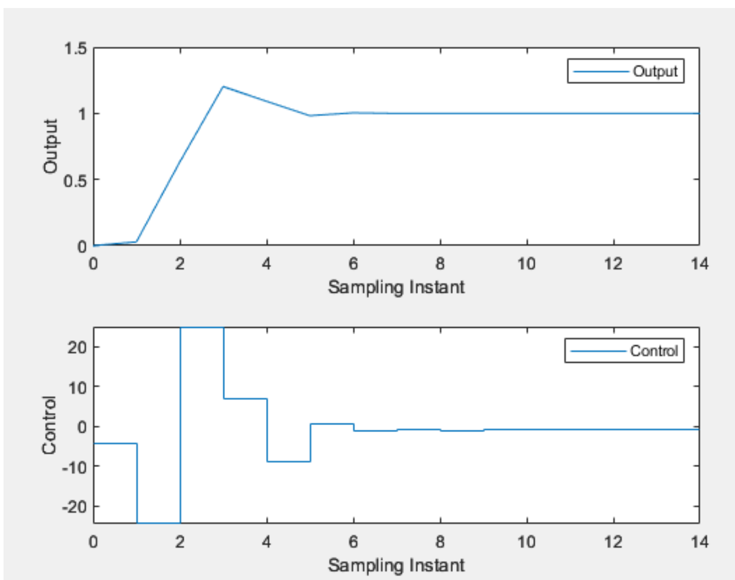

Figure 1.2. Output and control signal for example with saturation.

When the control signal reaches saturation, the output becomes oscillatory, and the performance of the closed loop deteriorates as it can be observed in Figure 1.2. As the performance parameters like the peak overshoot has increased by a good amount (1.0785 to 1.2021), the settling time has increased from 4 instances to 9 instances, other parameters like rise time are increased. Hence, it is evident that if the control signal constraints are not considered during the design, the closed loop performance deteriorates. More details on the design with or without constraints can be found in [2].

Regarding the constraints, there are mainly three types of constraints which can be formulated into MPC optimization.

3.2.1 *Constraints incremental variation of control signal*

For a single input and single output system, let $\Delta u(k)$ be the incremental control signal, Δu_{min} *and* Δu_{max} the minimum and maximum limits on the incremental control signal, respectively, the constraints can be represented using the following inequality equation (5):

$$\Delta u_{min} \leq \Delta u(k) \leq \Delta u_{max} \quad (5)$$

For a multi-input and multi output system, the upper limits on the incremental control signal can be grouped into a vector as shown in (6):

$$[\Delta u_{1max} \quad \Delta u_{2max} \quad \Delta u_{3max} \quad \dots \quad \Delta u_{nmax}], \quad (6)$$

and the same for the lower limits on the incremental control signal, as shown in (7):

$$[\Delta u_{1min} \quad \Delta u_{2min} \quad \Delta u_{3min} \quad \dots \quad \Delta u_{nmin}]. \quad (7)$$

The constraints defined on the incremental control signals $[\Delta u_1 \quad \Delta u_2 \quad \Delta u_3 \quad \dots \quad \Delta u_n]$ can be written separately by the following inequality Equations (8)-(11):

$$\Delta u_{1min} \leq \Delta u_1(k) \leq \Delta u_{1max} \quad (8)$$

$$\Delta u_{2min} \leq \Delta u_2(k) \leq \Delta u_{2max} \tag{9}$$

$$\Delta u_{3min} \leq \Delta u_3(k) \leq \Delta u_{3max} \tag{10}$$

$$\vdots$$

$$\Delta u_{nmin} \leq \Delta u_n(k) \leq \Delta u_{nmax} \tag{11}$$

We note that inequality Equations (8)-(11) will be grouped later into a compact form using vector representations. More details on the design with or without constraints can be found in [2]. We have adopted most of the same notation as that in [2].

3.2.2 *Constraints amplitude of control signal*

For a single input and single output system, let $u(k)$ represent the control signal, u_{min} *and* u_{max} the minimum and maximum limits on the control signal, respectively, the constraints can then be written as an inequality equation (12):

$$u_{min} \leq u(k) \leq u_{max} \tag{12}$$

For a multi-input and multi output system, the upper limits on the control signal can be grouped into a vector as shown in (13):

$$[u_{1max} \quad u_{2max} \quad u_{3max} \quad \cdots \quad u_{nmax}], \tag{13}$$

and the same for the lower limits on the control signal, as shown in (14):

$$[u_{1min} \quad u_{2min} \quad u_{3min} \quad \cdots \quad u_{nmin}]. \tag{14}$$

The constraints defined on the control signal $[u_1 \quad u_2 \quad u_3 \quad \cdots \quad u_n]$ can be written separately by the following inequality Equations (15)-(18):

$$u_{1min} \leq u_1(k) \leq u_{1max} \tag{15}$$

$$u_{2min} \leq u_2(k) \leq u_{2max} \tag{16}$$

$$u_{3min} \leq u_3(k) \leq u_{3min} \tag{17}$$

$$\vdots$$

$$u_{nmin} \leq u_n(k) \leq u_{nmin} \tag{18}$$

Again, Equations (15)-(18) can be grouped into a compact form using vector representations. More details on the design with or without constraints can be found in [2].

3.2.3 *Constraints on output or state variables*

Constraints can also happen on output or state variable. For a discrete-time multi-input and multi output system, at the k^{th} sampling time, the i^{th} output $y_i(k)$, and the i^{th} state variable $x_i(k)$, might be limited as shown in the following inequality Equation (19) and (20):

$$y_{imin} \leq y_i(k) \leq y_{imax} \tag{19}$$

$$x_{imin} \leq x_i(k) \leq x_{imax} \tag{20}$$

Here $y_{imin}\ and\ y_{imax}$ are the minimum and maximum limits on the output, and $x_{imin}\ and\ x_{imax}$ are the minimum and maximum limits on the state variables. Next when we discuss Lagrange's multipliers, more details relevant to the constraints on output and state variables will be presented and explained.

3.3 LAGRANGE MULTIPLIERS

Lagrange multipliers are a technique to find the maximum or minimum of a multivariable function. Let us consider a two-variable one-constraint optimization problem with variables (x_1, x_2), equality constraints $\emptyset(x_1, x_2)$, and cost function $Y(x_1, x_2)$.

$$Max/Min\ Y(x_1, x_2) \tag{21}$$

Subject to,

$$\emptyset(x_1, x_2) = 0 \tag{22}$$

To find the optimal, differential of $\emptyset(x_1, x_2)$ can be derived as below:

$$d\emptyset = \left(\frac{\partial\emptyset}{\partial x_1}\right) dx_1 + \left(\frac{\partial\emptyset}{\partial x_2}\right) dx_2 \tag{23}$$

Then the optimal solution can be found through the following conditions:

$$\emptyset = 0 \tag{24}$$

$$d\emptyset = 0 \tag{25}$$

$$\left(\frac{\partial\emptyset}{\partial x_1}\right) dx_1 + \left(\frac{\partial\emptyset}{\partial x_2}\right) dx_2 = 0 \tag{26}$$

$$dx_1 = \frac{-\left(\frac{\partial\emptyset}{\partial x_2}\right) dx_2}{\left(\frac{\partial\emptyset}{\partial x_1}\right)} \tag{27}$$

$$dY = \left(\frac{\partial Y}{\partial x_1}\right) dx_1 + \left(\frac{\partial Y}{\partial x_2}\right) dx_2 \tag{28}$$

$$dY = -\left(\frac{\partial Y}{\partial x_1}\right) \frac{\left(\frac{\partial\emptyset}{\partial x_2}\right) dx_2}{\left(\frac{\partial\emptyset}{\partial x_1}\right)} + \left(\frac{\partial Y}{\partial x_2}\right) dx_2 \tag{29}$$

Define λ in (30), then Equation (29) can be re-written as (31):

$$\lambda = \frac{\left(\frac{\partial Y}{\partial x_1}\right)}{\left(\frac{\partial\emptyset}{\partial x_1}\right)} \tag{30}$$

$$dY = (\frac{\partial Y}{\partial x_2} - \lambda \frac{\partial\emptyset}{\partial x_2}) dx_2 \tag{31}$$

So now the optimal solution can be obtained by solving the following two Equations (32) and (33):

$$dY = 0 \tag{32}$$

$$\frac{\partial Y}{\partial x_2} = \lambda \frac{\partial\emptyset}{\partial x_2} \tag{33}$$

Note that the definition of the Lagrange multiplier λ is given by:

$$\nabla Y - \lambda \nabla\emptyset = 0, \tag{34}$$

which can also be re-written as,

$$\lambda = \frac{\nabla Y}{\nabla \emptyset}. \tag{35}$$

Comparing and (35), λ in (30) is the Lagrange multiplier. If λ got solved, consequently the cost function with constraints can be solved.

Lagrange's multiplier is also called sensitivity coefficient or shadow price; it gives a measure of the change of output according to the change of constraints present. In the next section Karush Kuhn Tucker conditions are briefed.

3.4 KARUSH KUHN TUCKER CONDITIONS

Continue with the two-variable one-constraint optimization problem in section 3.3, with cost function Y and constraints $\emptyset$ and λ be Lagrangian multiplier. The Lagrangian function, L, is defined below:

$$L = \nabla Y - \lambda \nabla \emptyset \tag{36}$$

$$\nabla L = 0 \tag{37}$$

Here if we can solve the Lagrangian function, we can then also solve the optimization problem. Also including inequality constraints, the optimization problem can be formulated as follows. Let $Y(x_1, x_2, \dots, x_n)$ the cost function, $x_1, x_2, \dots, x_n$ being state variables, $\emptyset_i(x_1, x_2, \dots, x_n)$ being equality constraints, and $\psi_i(x_1, x_2, \dots, x_n)$ being inequality constraints. The optimization is then:

$$Min\, Y(x_1, x_2, \dots, x_n) \tag{38}$$

Subject to,

$$\emptyset_1(x_1, x_2, \dots, x_n) = 0 \tag{39}$$

$$\emptyset_2(x_1, x_2, \dots, x_n) = 0 \tag{40}$$

$$\vdots \tag{41}$$

$$\emptyset_m(x_1, x_2, \ldots, x_n) = 0 \quad (42)$$

and
$$\psi_1(x_1, x_2, \ldots, x_n) \geq 0 \quad (43)$$

$$\psi_2(x_1, x_2, \ldots, x_n) \geq 0 \quad (44)$$

$$\vdots \quad (45)$$

$$\psi_k(x_1, x_2, \ldots, x_n) \geq 0 \quad (46)$$

Assume the Lagrange's multipliers for equality constraints are $\lambda_1, \lambda_2, \ldots, \lambda_m$ and Lagrange's multipliers for equality constraints are $u_1, u_2, \ldots, u_k$. Now the Lagrangian Equation (36) becomes,

$$L = \nabla Y - \sum_{i=1}^{m} \lambda_i \nabla \emptyset_i - \sum_{r=1}^{k} u_r \nabla \psi_r \quad (47)$$

$$\emptyset_i = 0, \forall\ i \in 1,2, \ldots, m \quad (48)$$

$$\psi_r = 0, \forall\ r \in 1,2, \ldots, k \quad (49)$$

There are some new conditions that can be added, those are Karush Kuhn Tucker conditions:

$$u_r \psi_r = 0, \forall\ r \in 1,2, \ldots, m \quad (50)$$

$$u_r \geq 0, \forall\ r \in 1,2, \ldots, k \quad (51)$$

In condition (51), either $u_r = 0$ or $\psi_r = 0$. If $u_r = 0$ then there is no involvement of the inequality ψ_r in the optimization. Since the inequality constraint becomes non-binding. If $\psi_r = 0$ it is an equality constraint, which cannot happen. There is a complementarity involved in this, that's the reason to call the above equations complimentary slackness conditions. Condition (51) indicates that u_r cannot be less than zero. If there are any u_r less than zero, then the corresponding ψ_r need not be considered for optimization. If u_r is greater than zero, then the corresponding $\psi_r$is binding. Next, Hildreth quadratic programming is explained.

3.5 HILDRETH QUADRATIC PROGRAMMING

Hildreth quadratic programming is a sequential approach to solve the dual problem in quadratic programming. Let us consider the optimization problem as follows.

$$\text{Min } J = \frac{1}{2}x'Ex + x'F \tag{52}$$

$$Mx \leq \gamma \tag{53}$$

Here in (53), λ is the Lagrangian multiplier. Now the dual problem is obtained from (52) and (53) as:

$$max_{\lambda\geq 0} min_x [\frac{1}{2}x'Ex + x'F + \lambda'(Mx - \gamma)] \tag{54}$$

If this is an unconstrained problem, then x can be solved:

$$x = -E^{-1}(F + M'\lambda) \tag{55}$$

Introduce two terms H and K defined in (57) and (58), respectively, dual problem (54) can be re-written as,

$$max_{\lambda\geq 0}(-\frac{1}{2}\lambda'H\lambda - \lambda'K - \frac{1}{2}\gamma'E\gamma) \tag{56}$$

$$H = ME^{-1}M' \tag{57}$$

$$K = \gamma + ME^{-1}F \tag{58}$$

Remove the negative sign in (56), the dual problem is equivalent to the following,

$$min_{\lambda\geq 0}(\frac{1}{2}\lambda'H\lambda + \lambda'K + \frac{1}{2}\gamma'E\gamma) \tag{59}$$

To solve this minimization of dual problem (59), Hildreth’s quadratic programing algorithm is used. In this algorithm the direction vectors are assumed to be the same as basis vectors. If $\lambda \geq 0$ then the objective function is treated as a quadratic function to minimize with single component

λ_i. If $\lambda < 0$, then λ_i is set to zero. Then this is repeated for λ_{i+1}, till all the Lagrange's multipliers are done to complete one cycle. These cycles are repeated, and the Lagrange's multipliers gets updated from λ^m to λ^{m+1}, till the convergence is reached. The convergence criteria can be set on the difference between the current cycle Lagrange's multipliers λ_i^{m+1} to the previous cycle Lagrange's multipliers λ_i^m. The results are obtained with good accuracy if the convergence criteria are slow, that is the difference is set to a low number like 10^{-8}. Below the updating rule is shown as follows.

$$\lambda_i^{m+1} = \max\,(0, w_i^{m+1}) \tag{60}$$

$$w_i^{m+1} = -\frac{1}{h_{ii}}\left(k_i + \sum_{j=1}^{i-1} h_{ij}\lambda_j^{m+1} + \sum_{j=i+1}^{n} h_{ij}\lambda_j^{m}\right) \tag{61}$$

Now the decision variable at optimal point is given as shown below as follows.

$$x = -E^{-1}(F + M'\lambda^*) \tag{62}$$

Here, λ^* is the Lagrangian multiplier obtained from the Hildreth quadratic programming. More details explaining Hildreth quadratic programming algorithm can be found in [2].

Chapter 4. APPLICATION USING MODEL PREDICTIVE CONTROL

In this chapter, the MPC strategy is going to be applied to a wind farm incorporated with battery energy storage system. An overview of the system is first given, followed by the derivation of the state space model for the system. Then the modes of operations are defined, and cost function along with constraints are developed. The implementation of MPC will be shown in cases both without and with constraints. MATLAB simulation will be discussed and the effectiveness of proposed MPC strategy will be verified via simulation results.

4.1 OVERVIEW

In this section an overview of the system is defined. The configuration of the wind farm integrated with the dual BESS is shown in Figure 4.1.

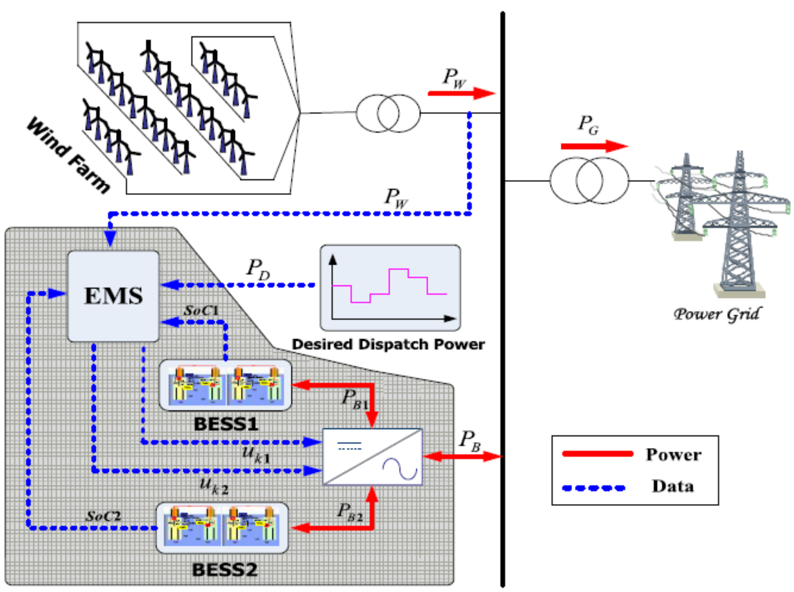

Figure 4.1. Configuration of wind farm with dual BESS [1].

Here the output from the wind farm is denoted as P_w, the forecasted power or dispatch order power is denoted as P_D. P_{B1} and P_{B2} are power delivered/extracted from the battery and P_G is the total power from the integrated wind farm with dual BESS configuration.

Initially the forecasted power is predicted from the history of the wind farm data, here this power is assumed to be known. The actual power from the wind farm and this forecasted power is sent into the Energy Management System which determines our control strategy. This control strategy decides the power delivered/extracted from the batteries to generate total output power which smooths and tracks the forecasted power.

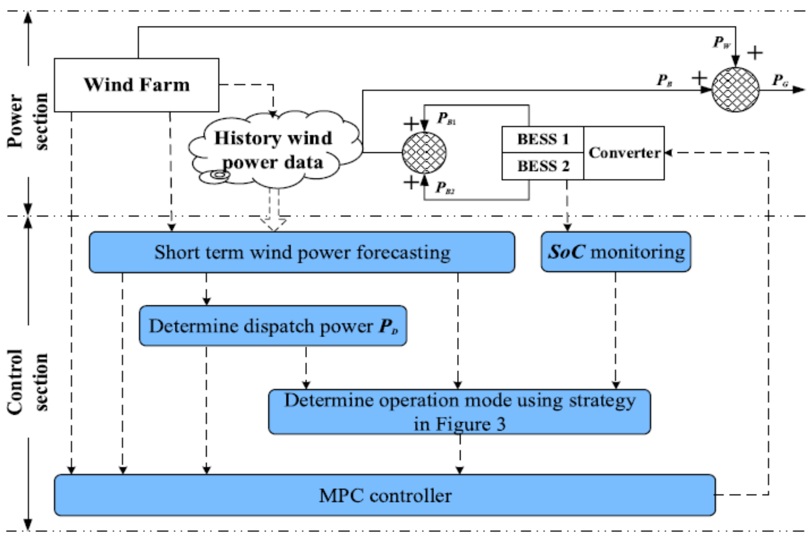

Figure 4.2. Dispatch scheme of wind farm with dual BESS [1].

The dispatch scheme for this wind farm with dual BESS is shown in Figure 4.2; it is divided into two sections: one is the power section, and the other the control section. The control section is taken care of by the Energy Management System containing the short-term wind power forecasting which gives forecasted power and is assumed to be known in our work. Using this power and the state of charge (SoC) from the SoC monitoring at that moment, the mode of operation is determined. After mode of operation is known, the appropriate model predictive control strategy is applied with specific constraints.

Note that we adopted the same structure of system and most of the notation as that in [1]. We are not formulating the problem and solving the MPC optimization in the same way as that in [1] though. Differences showing our idea and research efforts will be presented later in relevant subchapters.

4.2 STATE SPACE MODEL

The state space model is a mathematical description of a physical system. We formulate the given problem with the known equations first, then come to the state space model for MPC design.

From the power balance, total output power P_G can be represented in terms of actual wind farm power P_w and the battery power P_B in discrete time format as:

$$P_G(k+1) = P_B(k) + P_w(k) \tag{63}$$

From the energy balance, each battery energy can be represented in terms of the battery power in discrete time format as:

$$E_{B1}(k+1) = E_{B1}(k) - \Delta T_B P_{B1}(k) \tag{64}$$

$$E_{B2}(k+1) = E_{B2}(k) - \Delta T_B P_{B2}(k) \tag{65}$$

Here P_G is the total output power, E_{B1} and E_{B2} are the Energy from each battery, and ΔT_B is sampling time. Note that P_{B1} and P_{B2} are the power delivered/extracted from the BESS1 and BESS2, which are used as control signals. The data is sampled for every 5 min, i.e., ΔT_B=5 min=1/12 Hrs.

The state space model has the following form:

$$X(k+1) = AX(k) + B_1U(k) + B_2d(k) \tag{66}$$

$$Y(k) = CX(k) \tag{67}$$

$$\begin{bmatrix} x1(k) \\ x2(k) \\ x3(k) \end{bmatrix} = \begin{bmatrix} P_G(k) \\ E_{B1}(k) \\ E_{B2}(k) \end{bmatrix} \tag{68}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{69}$$

$$B_1 = \begin{bmatrix} 1 & 1 \\ -\Delta T_B & 0 \\ 0 & -\Delta T_B \end{bmatrix} \tag{70}$$

$$B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{71}$$

$$C = [1 \quad 0 \quad 0] \tag{72}$$

$$Q = [B_1 \quad AB_1] \tag{73}$$

The controllability matrix Q is full rank; hence the system is controllable. More details on the system modelling can refer to [1].

4.3 MODES OF OPERATION

As shown in Equations (64) and (65), the two batteries output power is used as control signal in the control strategy. The batteries are operated in two modes of operation, to reduce the switching times. In Mode 1, the first battery charges and second battery discharges; in Mode 2, the first battery discharges and the second battery charges. To decide the mode of operation, the state of charge (SoC) is considered, and SoC for any battery can be calculated as:

$$SoC(k+1) = SoC(k) + \left(\frac{1}{E_{rat}}\right) \int_k^{k+1} P_b \, dx \tag{74}$$

Here in Equation (74), $SoC(k+1)$ is the state of charge at the $(k+1)^{\text{th}}$ sampling instant, $SoC(k)$ is the state of charge at the k^{th} sampling instant, E_{rat} is the rated capacity of the battery and P_b is the power delivered/extracted from the battery. Specifically, the algorithm can be explained using the flowchart shown in Figure 4.3.

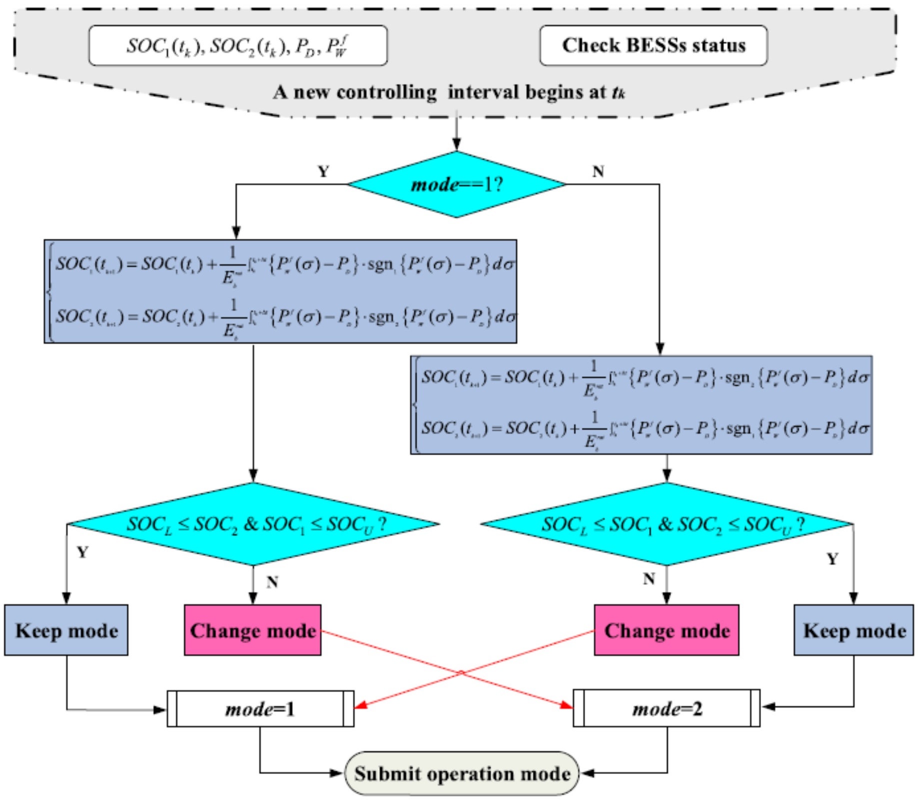

Figure 4.3. Flowchart for determining mode of operation [1].

In Figure 4.3, the state of charge of both batteries will be first estimated, then for the new time instant $t_{k+1} = (k+1)\Delta T_B$ mode is verified, if it is Mode1. Assume that state of charge for BESS1 and BESS2 as SoC_1 and SoC_2, respectively, P_w actual wind power, and P_d the forecasted power. Then the state of charge on two batteries can be computed as:

$$SoC_1(t_{k+1}) = SoC_1(t_k) + \left(\frac{1}{E_{rat}}\right) \int_k^{k+1} (P_w(x) - P_d).sgn_1(P_w(x) - P_d)\, dx \qquad (75)$$

$$SoC_2(t_{k+1}) = SoC_2(t_k) + \left(\frac{1}{E_{rat}}\right) \int_k^{k+1} (P_w(x) - P_d).sgn_2(P_w(x) - P_d)\, dx \qquad (76)$$

Similarly, if it is Mode 2, then the state of charge on two batteries are computed as follows:

$$SoC_1(t_{k+1}) = SoC_1(t_k) + (\frac{1}{E_{rat}}) \int_k^{k+1} (P_w(x) - P_d).\, sgn_2(P_w(x) - P_d)\, dx \qquad (77)$$

$$SoC_2(t_{k+1}) = SoC_2(t_k) + (\frac{1}{E_{rat}}) \int_k^{k+1} (P_w(x) - P_d).\, sgn_1(P_w(x) - P_d)\, dx \qquad (78)$$

Here,

$$sgn_1(x) = \begin{cases} 1, x > 0 \\ 0, x \leq 0 \end{cases} \qquad (79)$$

$$sgn_2(x) = \begin{cases} 0, x > 0 \\ -1, x \leq 0 \end{cases} \qquad (80)$$

Then constraints on the SoC are verified, and the decision is made to keep or change the mode of operation, then this cycle is repeated throughout the process. SoC equations obtained here are logical, for example the power flowing out of the system in Mode 1 is from BESS2, which is consistent with the definition of Mode 1 where BESS2 discharges. So, the energy is deducted from the BESS2. In Mode 1, the power will flow into BESS1 of the system, since, in Mode 1, it is BESS1 that charges. The opposite is true in Mode 2. We note that this part of operation adopts the same strategy as that in [1].

4.4 COST FUNCTION WITH CONSTRAINTS

The model predictive control algorithm is used to match the plant output to the reference output. To achieve this an optimization is performed on the cost function in a prediction window Np and the control signal is estimated in the control window Nc. This process is repeated from the start of the process to the end of the process, at every sampling instant $t_k = (k)\Delta T_B$. The cost function contains the quadratic terms of the error between the actual output $yact$ and the predicted output

$yref$ and the control signal u or the rate of control signal Δu. Here we use a cost function as defined in Equation (81):

$$J = \sum(yref - yact)^2 + \sum u^2 \quad (81)$$

Note that the output of the plant is the summation of the actual wind power and the power output from the two batteries, as shown in Equation (63). The control signal is the power output from the two batteries, thus Equation (81) can be re-written as:

$$J = \sum_{i=1}^{Np}(P_G(k+i|k) - P_D(k+i|k)) + \sum_{i=1}^{Nc} u(k+i|k)^2 \quad (82)$$

Here Np and Nc are the prediction and control horizons, respectively. Notation $(k+i|k)$ refers to the prediction at $k+i$ given time k.

4.4.1 *SoC constraints*

There are two types of constraints defined on the system, as we will explain below.

First, to increase the lifetime of a battery, the battery must be operated within proposed state of charge limits at any given instant k. This helps in reducing battery costs.

$$S_{min} \leq \frac{E(k)}{CB} \leq S_{max} \quad (83)$$

In Equation (83), S_{min} and S_{max} are the minimum and maximum limits on the state of charge, $E(k)$ is the remaining charge at the instant k and CB is the rated capacity of the battery.

4.4.2 *Charging and discharging constraints on Power*

Second, for each battery charging and discharging constraints on power are imposed according to the mode of operation. Let P_{B1} and P_{B2} are power delivered/extracted from battery. The maximum power limit on the battery is P_{max}.

In first mode of operation,

$$0 \leq P_{B1} \leq P_{max} \tag{84}$$

$$-P_{max} \leq P_{B2} \leq 0 \tag{85}$$

In second mode of operation,

$$-P_{max} \leq P_{B1} \leq 0 \tag{86}$$

$$0 \leq P_{B2} \leq P_{max} \tag{87}$$

Reference of this material is taken from [1]. In the next section implementation of MPC without constraints is explained.

4.5 IMPLEMENTATION OF MPC WITHOUT CONSTRAINTS

Let $x(k+1|k) \quad x(k+2|k) \quad x(k+3|k) \quad \ldots \quad x(k+Np|k)$ be states at instant k over the entire prediction horizon, $u(k|k) \quad u(k+1|k) \quad u(k+2|k) \quad \ldots \quad u(k+Nc-1|k)$ be the control signal over the entire control horizon at an instant k.

Also let $y(k+1|k) \quad y(k+2|k) \quad y(k+3|k) \quad \ldots \quad y(k+Np|k)$ be outputs at instant k over the entire prediction horizon, $d(k|k) \quad d(k+1|k) \quad d(k+2|k) \quad \ldots \quad d(k+Nc-1|k)$ be the measured disturbance signals which are the forecasted power over the entire control horizon at an instant k.

Our state space model is:

$$x(k+1) = Ax(k) + B_1u(k) + B_2d(k) \tag{88}$$

$$y(k) = Cx(k) \tag{89}$$

Then at instant k, the output values over the prediction horizon can be found as:

$$y(k+1|k) = Cx(k+1|k)$$

$$= CAx(k) + CB_1u(k) + CB_2d(k)$$

$$y(k+2|k) = Cx(k+2|k)$$

$$= C*(Ax(k+1|k) + B_1u(k+1|k) + B_2d(k+1|k))$$

$$= CAx(k+1|k) + CB_1u(k+1|k) + CB_2d(k+1))$$

$$= CA^2x(k) + CAB_1u(k) + CB_1u(k+1|k) + CAB_2d(k) + CB_2d(k+1))$$

$$y(k+3|k) = Cx(k+3|k)$$

$$= C*(Ax(k+2|k) + B_1u(k+2|k) + B_2d(k+2|k))$$

$$= CA^3x(k) + CA^2B_1u(k) + CAB_1u(k+1|k) + CB_1u(k+2|k) + CA^2B_2d(k) + CAB_2d(k+1) + CB_2d(k+2)$$

$$\vdots$$

$$y(k+Np|k) = CX(k+Np|k)$$

$$= C*(AX(k+Np|k) + B_1U(k+Np|k) + B_2d(k+Np|k))$$

$$= CA^{Np}X(k) + CA^{Np-1}B_1U(k) + CA^{Np-2}B_1U(k+1|k) + \cdots + CA^{Np-Nc}B_1U(k+Nc-1|k) + CA^{Np-1}B_2d(k) + CA^{Np-2}B_2d(k+1) + \cdots + CA^{Np-Nc}B_2d(k+Nc-1) \quad (90)$$

Let us define,

$$Y(k) = Gx(k) + HU(k) + LD(k) \quad (91)$$

$$Y(k) = \begin{bmatrix} y(k+1|k) \\ y(k+2|k) \\ y(k+3|k) \\ \vdots \\ y(k+Np-1|k) \end{bmatrix} \quad (92)$$

$$U(k) = \begin{bmatrix} u(k|k) \\ u(k+1|k) \\ u(k+2|k) \\ \vdots \\ u(k+Nc-1|k) \end{bmatrix} \quad (93)$$

$$D(k) = \begin{bmatrix} d(k) \\ d(k+1) \\ d(k+2) \\ \vdots \\ d(k+Nc-1) \end{bmatrix} \tag{94}$$

From the above derivation,

$$G = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{Np} \end{bmatrix} \tag{95}$$

$$H = \begin{bmatrix} CB_1 & 0 & \dots & 0 \\ CAB_1 & CB_1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{Np-1}B_1 & CA^{Np-2}B_1 & \dots & CA^{Np-Nc}B_1 \end{bmatrix} \tag{96}$$

$$L = \begin{bmatrix} CB_2 & 0 & \dots & 0 \\ CAB_2 & CB_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{Np-1}B_2 & CA^{Np-2}B_2 & \dots & CA^{Np-Nc}B_2 \end{bmatrix} \tag{97}$$

Let J be the cost function, Y_r and Y be the reference and actual outputs, U be the control signal. Let us see the cost function to minimize for every instance to get the appropriate control signal as follows,

$$J = (Y_r - Y)'Q(Y_r - Y) + U'RU \tag{98}$$

The cost function is comprised of two terms. The first one is relevant to the error between the actual output and the reference output, and the second term is relevant to the control signal. Here the matrices Q and R are the penalties for the error and the control signal.

$$J = (Y_r - Y)'Q(Y_r - Y) + U'RU$$

$$= (Y_r - Gx(k) - HU(k) - LD(k))'Q(Y_r - Gx(k) - HU(k) - LD(k)) + U'RU$$

$$= (Y_r - Gx(k) - LD(k))'Q(Y_r - Gx(k) - LD(k)) - 2U'H'Q\left(Y_r - Gx(k) - LD(k)\right)$$
$$+ U'H'QHU + U'RU$$

$$= (Y_r - Gx(k) - LD(k))'Q(Y_r - Gx(k) - LD(k)) - 2U'H'Q\left(Y_r - Gx(k) - LD(k)\right) + U'(H'QH + R)U \tag{99}$$

To find the minimum of the cost function, taking the derivative with respect to U and equating it to zero,

$$\frac{\partial J}{\partial U} = 0 \tag{100}$$

$$-2H'Q(Y_r - Gx(k) - LD(k)) + 2(H'QH + R)U = 0$$

$$U = inv(H'QH + R)H'Q(Y_r - Gx(k) - LD(k)) \tag{101}$$

$$Q = \begin{bmatrix} q_w & 0 & \dots & 0 \\ 0 & q_w & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & q_w \end{bmatrix} \tag{102}$$

$$R = \begin{bmatrix} r_u & 0 & \dots & 0 \\ 0 & r_u & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & r_u \end{bmatrix} \tag{103}$$

The first two elements of this represent the control signals from BESS one and BESS two. So, the first two elements are considered and plugged into the state space equation to find the state variables and is then followed by the output at a particular instant. Finally, this procedure is repeated for all the instances to get the output. Reference of this material is taken from [1]. In the next section the implementation of MPC with constraints is discussed.

4.6 IMPLEMENTATION OF MPC WITH CONSTRAINTS

In this section implementation of MPC with constraints is discussed. After finding G, H, L from section 4.5 , we try to impose the following constraints on the cost function so that our system is efficient and economically feasible. Now the problem becomes as follows.

$$J = (Y_r - Y)'Q(Y_r - Y) + U'RU \tag{104}$$

Subject to,

In mode 1,

$$SoC_{1min} \leq \frac{x_2(k+i|k)}{CB_1} \leq SoC_{1max} \tag{105}$$

$$SoC_{2min} \leq \frac{x_3(k+i|k)}{CB_2} \leq SoC_{2max} \tag{106}$$

$$0 \leq u_1(k+i|k) \leq P_{max} \qquad 107)$$

$$-P_{max} \leq u_2(k+i|k) \leq 0 \tag{108}$$

In mode 2,

$$SoC_{1min} \leq \frac{x_2(k+i|k)}{CB_1} \leq SoC_{1max} \tag{109}$$

$$SoC_{2min} \leq \frac{x_3(k+i|k)}{CB_2} \leq SoC_{2max} \tag{110}$$

$$-P_{max} \leq u_1(k+i|k) \leq 0 \tag{111}$$

$$0 \leq u_2(k+i|k) \leq P_{max} \tag{112}$$

$$Q = \begin{bmatrix} q_w & 0 & \cdots & 0 \\ 0 & q_w & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & q_w \end{bmatrix} \tag{113}$$

$$R = \begin{bmatrix} r_u & 0 & \dots & 0 \\ 0 & r_u & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \cdots & r_u \end{bmatrix} \quad (114)$$

Here, the rated capacity of both the batteries CB_1 and CB_2 are same as CB. The minimum limits on state of charge for both the batteries SoC_{1min} and SoC_{2min} are same as SoC_{min} and the maximum limits on the state of charge for both the batteries SoC_{1max} and SoC_{1max} are same as SoC_{max}. For illustration purpose, here $N_p = N_c$. Reference of this material is taken from [1].

4.6.1 *Constraints for amplitude of control signal*

In mode 1,

When constraints on the amplitude of first control signal u_1 are applied,

$$0 \leq u_1(k) \leq P_{max}$$

$$u_1(k) \leq P_{max} \quad (115)$$

$$-u_1(k) \leq 0 \quad (116)$$

$$0 \leq u_1(k+1|k) \leq P_{max}$$

$$u_1(k+1|k) \leq P_{max} \quad (117)$$

$$-u_1(k+1|k) \leq 0 \quad (118)$$

$$\vdots$$

$$0 \leq u_1(k+Nc-1|k) \leq P_{max}$$

$$u_1(k+Nc-1|k) \leq P_{max} \quad (119)$$

$$-u_1(k+Nc-1|k) \leq 0 \quad (120)$$

A compact form can be derived as below:

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_1(k+1) \\ u_1(k+2) \\ \vdots \\ u_1(k+Nc-1) \end{bmatrix} \leq \begin{bmatrix} P_{max} \\ P_{max} \\ P_{max} \\ \vdots \\ P_{max} \end{bmatrix} \tag{121}$$

And

$$\begin{bmatrix} -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_1(k+1) \\ u_1(k+2) \\ \vdots \\ u_1(k+Nc-1) \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{122}$$

When we combine both the equations (121) and (122), a larger compact form can be formed:

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_1(k+1) \\ u_1(k+2) \\ \vdots \\ u_1(k+Nc-1) \end{bmatrix} \leq \begin{bmatrix} P_{max} \\ P_{max} \\ P_{max} \\ \vdots \\ P_{max} \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{123}$$

Similarly, when constraints on the amplitude of second control signal u_2 are applied, say,

$$-P_{max} \leq u_2(k) \leq 0$$

$$u_2(k) \leq 0 \tag{124}$$

$$-u_2(k) \leq P_{max} \tag{125}$$

$$-P_{max} \leq u_2(k+1|k) \leq 0$$

$$u_2(k+1|k) \leq 0 \tag{126}$$

$$-u_2(k+1|k) \leq P_{max} \tag{127}$$

$$\vdots$$

$$-P_{max} \le u_2(k+Nc-1|k) \le 0$$

$$u_2(k+Nc-1|k) \le 0 \tag{128}$$

$$-u_2(k+Nc-1|k) \le P_{max} \tag{129}$$

Compact forms again can be derived as below:

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} u_2(k) \\ u_2(k+1) \\ u_2(k+2) \\ \vdots \\ u_2(k+Nc-1) \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{130}$$

And

$$\begin{bmatrix} -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_2(k) \\ u_2(k+1) \\ u_2(k+2) \\ \vdots \\ u_2(k+Nc-1) \end{bmatrix} \le \begin{bmatrix} P_{max} \\ P_{max} \\ P_{max} \\ \vdots \\ P_{max} \end{bmatrix} \tag{131}$$

And a larger compact form combining both (130) and (131):

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_2(k) \\ u_2(k+1) \\ u_2(k+2) \\ \vdots \\ u_2(k+Nc-1) \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ P_{max} \\ P_{max} \\ P_{max} \\ \vdots \\ P_{max} \end{bmatrix} \tag{132}$$

Combining (123) and (132) form Mode 1, we obtained:

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_1(k+1) \\ u_2(k+1) \\ u_1(k+2) \\ u_2(k+2) \\ \vdots \\ u_1(k+Nc-1) \\ u_2(k+Nc-1) \end{bmatrix} \leq \begin{bmatrix} P_{max} \\ 0 \\ P_{max} \\ \vdots \\ P_{max} \\ 0 \\ 0 \\ P_{max} \\ \vdots \\ 0 \\ P_{max} \end{bmatrix} \tag{133}$$

In mode 2,

When constraints on the amplitude of first control signal u_1 are applied,

$$-P_{max} \leq u_1(k) \leq 0$$

$$u_1(k) \leq 0 \tag{134}$$

$$-u_1(k) \leq P_{max} \tag{135}$$

$$-P_{max} \leq u_1(k+1|k) \leq 0$$

$$u_1(k+1|k) \leq 0 \tag{136}$$

$$-u_1(k+1|k) \leq P_{max} \tag{137}$$

$$\vdots$$

$$-P_{max} \leq u_1(k+Nc-1|k) \leq 0$$

$$u_1(k+Nc-1|k) \leq 0 \tag{138}$$

$$-u_1(k+Nc-1|k) \leq P_{max} \tag{139}$$

Compact forms can be derived, similar to that for Mode 1:

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_1(k+1) \\ u_1(k+2) \\ \vdots \\ u_1(k+Nc-1) \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{140}$$

And

$$\begin{bmatrix} -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_1(k+1) \\ u_1(k+2) \\ \vdots \\ u_1(k+Nc-1) \end{bmatrix} \leq \begin{bmatrix} P_{max} \\ P_{max} \\ P_{max} \\ \vdots \\ P_{max} \end{bmatrix} \tag{141}$$

And a combined version for (140) and (141):

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_1(k+1) \\ u_1(k+2) \\ \vdots \\ u_1(k+Nc-1) \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ P_{max} \\ P_{max} \\ P_{max} \\ \vdots \\ P_{max} \end{bmatrix} \tag{142}$$

Still for Mode 2, when constraints on the amplitude of second control signal u_2 are applied,

$$0 \leq u_2(k) \leq P_{max}$$

$$u_2(k) \leq P_{max} \tag{143}$$

$$-u_2(k) \leq 0 \tag{144}$$

$$0 \leq u_2(k+1|k) \leq P_{max}$$

$$u_2(k+1|k) \leq P_{max} \tag{145}$$

$$-u_2(k+1|k) \leq 0 \tag{146}$$

$$\vdots$$

$$0 \leq u_2(k+Nc-1|k) \leq P_{max}$$

$$u_2(k+Nc-1|k) \leq P_{max} \tag{147}$$

$$-u_2(k+Nc-1|k) \leq 0 \tag{148}$$

We can derive the following compact form,

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} u_2(k) \\ u_2(k+1) \\ u_2(k+2) \\ \vdots \\ u_2(k+Nc-1) \end{bmatrix} \le \begin{bmatrix} P_{max} \\ P_{max} \\ P_{max} \\ \vdots \\ P_{max} \end{bmatrix} \tag{149}$$

And

$$\begin{bmatrix} -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_2(k) \\ u_2(k+1) \\ u_2(k+2) \\ \vdots \\ u_2(k+Nc-1) \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{150}$$

And a combined version for equations (149) and (150):

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_2(k) \\ u_2(k+1) \\ u_2(k+2) \\ \vdots \\ u_2(k+Nc-1) \end{bmatrix} \le \begin{bmatrix} P_{max} \\ P_{max} \\ P_{max} \\ \vdots \\ P_{max} \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{151}$$

So, all constraints in Mode 2 can now be re-written in one inequality expression:

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_1(k+1) \\ u_2(k+1) \\ u_1(k+2) \\ u_2(k+2) \\ \vdots \\ u_1(k+Nc-1) \\ u_2(k+Nc-1) \end{bmatrix} \le \begin{bmatrix} 0 \\ P_{max} \\ 0 \\ \vdots \\ 0 \\ P_{max} \\ P_{max} \\ 0 \\ \vdots \\ P_{max} \\ 0 \end{bmatrix} \tag{152}$$

4.6.2 *Constraints on state variable*

When constraints on the second state variable are applied,

$$SoC_{min} \leq \frac{x_2(k+1|k)}{CB} \leq SoC_{max}$$

$$CB.SoC_{min} \leq x_2(k+1|k) \leq CB.SoC_{max}$$

$$x_2(k+1|k) \leq CB.SoC_{max}$$

$$-x_2(k+1|k) \leq -CB.SoC_{min}$$

$$x_2(k) + \Delta T_B.u_1(k) \leq CB.SoC_{max}$$

$$-x_2(k) - \Delta T_B.u_1(k) \leq -CB.SoC_{min}$$

$$u_1(k) \leq \left(\frac{1}{\Delta T_B}\right).(CB.SoC_{max} - x_2(k)) = x_{2ul} \quad (153)$$

$$-u_1(k) \leq \left(\frac{1}{\Delta T_B}\right).(x_2(k) - CB.SoC_{min}) \ = x_{2ll} \quad (154)$$

$$SoC_{min} \leq \frac{x_2(k+2|k)}{CB} \leq SoC_{max}$$

$$CB.SoC_{min} \leq x_2(k+2|k) \leq CB.SoC_{max}$$

$$x_2(k+2|k) \leq CB.SoC_{max}$$

$$-x_2(k+2|k) \leq -CB.SoC_{min}$$

$$x_2(k+1) + \Delta T_B.u_1(k+1) \leq CB.SoC_{max}$$

$$-x_2(k+1) - \Delta T_B.u_1(k+1) \leq -CB.SoC_{min}$$

$$x_2(k) + \Delta TB.u_1(k) + \Delta T_B.u_1(k+1) \leq CB.SoC_{max}$$

$$-x_2(k) - \Delta TB.u_1(k) - \Delta T_B.u_1(k+1) \leq -CB.SoC_{min}$$

$$u_1(k) + u_1(k+1) \leq \left(\frac{1}{\Delta T_B}\right).(CB.SoC_{max} - x_2(k)) = x_{2ul} \quad (155)$$

$$-u_1(k) - u_1(k+1) \leq \left(\frac{1}{\Delta T_B}\right).(x_2(k) - CB.SoC_{min}) \ = x_{2ll} \quad (156)$$

$$\vdots$$

$$SoC_{min} \leq \frac{x_2(k+Nc|k)}{CB} \leq SoC_{max}$$

$$u_1(k) + u_1(k+1) + \cdots + u_1(k+Nc-1) \leq \left(\frac{1}{\Delta T_B}\right).(CB.SoC_{max} - x_2(k)) \ = x_{2ul} \qquad (157)$$

$$-u_1(k) - u_1(k+1) - \cdots - u_1(k+Nc-1) \leq \left(\frac{1}{\Delta T_B}\right).(x_2(k) - CB.SoC_{min}) \ = x_{2ll} \qquad (158)$$

So,

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_1(k+1) \\ u_1(k+2) \\ \vdots \\ u_1(k+Nc-1) \end{bmatrix} \leq \begin{bmatrix} x_{2ul} \\ x_{2ul} \\ x_{2ul} \\ \vdots \\ x_{2ul} \end{bmatrix} \qquad (159)$$

And

$$\begin{bmatrix} -1 & 0 & \dots & 0 \\ -1 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ -1 & -1 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_1(k+1) \\ u_1(k+2) \\ \vdots \\ u_1(k+Nc-1) \end{bmatrix} \leq \begin{bmatrix} x_{2ll} \\ x_{2ll} \\ x_{2ll} \\ \vdots \\ x_{2ll} \end{bmatrix} \qquad (160)$$

When constraints on the third state variable are applied,

$$SoC_{min} \leq \frac{x_3(k+1|k)}{CB} \leq SoC_{max}$$

$$CB.SoC_{min} \leq x_3(k+1|k) \leq CB.SoC_{max}$$

$$x_3(k+1|k) \leq CB.SoC_{max}$$

$$-x_3(k+1|k) \leq -CB.SoC_{min}$$

$$x_3(k) + \Delta T_B.u_2(k) \leq CB.SoC_{max}$$

$$-x_3(k) - \Delta T_B.u_2(k) \leq -CB.SoC_{min}$$

$$u_2(k) \leq \left(\frac{1}{\Delta T_B}\right).(CB.SoC_{max} - x_3(k)) = x_{3ul} \qquad (161)$$

$$-u_2(k) \leq \left(\frac{1}{\Delta T_B}\right).(x_3(k) - CB.SoC_{min}) \ = x_{3ll} \tag{162}$$

$$SoC_{min} \leq \frac{x_3(k+2|k)}{CB} \leq SoC_{max}$$

$$CB.SoC_{min} \leq x_3(k+2|k) \leq CB.SoC_{max}$$

$$x_3(k+2|k) \leq CB.SoC_{max}$$

$$-x_3(k+2|k) \leq -CB.SoC_{min}$$

$$x_3(k+1) + \Delta T_B.u_2(k+1) \leq CB.SoC_{max}$$

$$-x_3(k+1) - \Delta T_B.u_2(k+1) \leq -CB.SoC_{min}$$

$$x_3(k) + \Delta T_B.u_2(k) + \Delta T_B.u_2(k+1) \leq CB.SoC_{max}$$

$$-x_3(k) - \Delta T_B.u_2(k) - \Delta T_B.u_2(k+1) \leq -CB.SoC_{min}$$

$$u_2(k) + u_2(k+1) \leq \left(\frac{1}{\Delta T_B}\right).(CB.SoC_{max} - x_3(k)) = x_{3ul} \tag{163}$$

$$-u_2(k) - u_2(k+1) \leq \left(\frac{1}{\Delta T_B}\right).(x_3(k) - CB.SoC_{min}) \ = x_{3ll} \tag{164}$$

$$\vdots$$

$$SoC_{min} \leq \frac{x_3(k+Nc|k)}{CB} \leq SoC_{max}$$

$$u_2(k) + u_2(k+1) + \cdots + u_2(k+Nc-1) \leq \left(\frac{1}{\Delta T_B}\right).(CB.SoC_{max} - x_3(k)) = x_{3ul} \tag{165}$$

$$-u_2(k) - u_2(k+1) - \cdots - u_2(k+Nc-1) \leq \left(\frac{1}{\Delta T_B}\right).(x_3(k) - CB.SoC_{min}) \ = x_{3ll} \tag{166}$$

So,

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} u_2(k) \\ u_2(k+1) \\ u_2(k+2) \\ \vdots \\ u_2(k+Nc-1) \end{bmatrix} \leq \begin{bmatrix} x_{3ul} \\ x_{3ul} \\ x_{3ul} \\ \vdots \\ x_{3ul} \end{bmatrix} \tag{167}$$

And

$$\begin{bmatrix} -1 & 0 & \dots & 0 \\ -1 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ -1 & -1 & \dots & -1 \end{bmatrix} \begin{bmatrix} u_2(k) \\ u_2(k+1) \\ u_2(k+2) \\ \vdots \\ u_2(k+Nc-1) \end{bmatrix} \le \begin{bmatrix} x_{3ll} \\ x_{3ll} \\ x_{3ll} \\ \vdots \\ x_{3ll} \end{bmatrix} \tag{168}$$

When constraints on all state variables, say, (159), (160), (167) and (168), are combined,

$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 0 & \dots & 1 \\ -1 & 0 & & \dots & 0 \\ 0 & -1 & & \dots & 0 \\ \vdots & \vdots & & \dots & \vdots \\ 0 & 0 & & \dots & -1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_1(k+1) \\ u_2(k+1) \\ u_1(k+2) \\ u_2(k+2) \\ \vdots \\ u_1(k+Nc-1) \\ u_2(k+Nc-1) \end{bmatrix} \le \begin{bmatrix} x_{2ul} \\ x_{3ul} \\ x_{2ul} \\ \vdots \\ x_{2ul} \\ x_{3ul} \\ x_{2ll} \\ x_{3ll} \\ \vdots \\ x_{2ll} \\ x_{3ll} \end{bmatrix} \tag{169}$$

4.6.3 *Hildreth's quadratic programming*

When constraints on all amplitude of control signal (152) and state variables (169) are combined.

Reference of this material is taken from [2] .

$$\begin{bmatrix} 1 & 0 & \dots & 0 & \\ 0 & 1 & \dots & 0 & \\ \vdots & \vdots & \dots & \vdots & \\ 0 & 0 & \dots & 1 & \\ -1 & 0 & \dots & 0 & \\ 0 & -1 & \dots & 0 & \\ \vdots & \vdots & \dots & \vdots & \\ 0 & 0 & \dots & -1 & \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 0 & \dots & 1 \\ -1 & 0 & \dots & 0 & \\ 0 & -1 & \dots & 0 & \\ \vdots & \vdots & \dots & \vdots & \\ 0 & 0 & \dots & -1 & \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_1(k+1) \\ u_2(k+1) \\ u_1(k+2) \\ u_2(k+2) \\ \vdots \\ u_1(k+Nc-1) \\ u_2(k+Nc-1) \end{bmatrix} \leq \begin{bmatrix} 0 \\ P_{max} \\ 0 \\ \vdots \\ 0 \\ P_{max} \\ P_{max} \\ 0 \\ \vdots \\ P_{max} \\ 0 \\ x_{2ul} \\ x_{3ul} \\ x_{2ul} \\ \vdots \\ x_{2ul} \\ x_{3ul} \\ x_{2ll} \\ x_{3ll} \\ \vdots \\ x_{2ll} \\ x_{3ll} \end{bmatrix} \tag{170}$$

Suppose the MPC design is to optimize a cost function defined as,

$$J = \frac{1}{2} U'.Hhil.U + U'.f \tag{171}$$

Subject to,

$$Acons.U \leq b \tag{172}$$

And the optimization will apply Hildreth's quadratic programming algorithm as shown below, where the cost function (173) is equivalent to (171).

$$J = -2U'H'Q\big(Y_r - Gx(k) - LD(k)\big) + U'(H'QH + R)U \tag{173}$$

$$Acons = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 0 & \dots & 1 \\ -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \tag{174}$$

$$b = \begin{bmatrix} 0 \\ P_{max} \\ 0 \\ \vdots \\ 0 \\ P_{max} \\ P_{max} \\ 0 \\ \vdots \\ P_{max} \\ 0 \\ x2ul \\ x3ul \\ x2ul \\ \vdots \\ x2ul \\ x3ul \\ x2ll \\ x3ll \\ \vdots \\ x2ll \\ x3ll \end{bmatrix} \tag{175}$$

$$Hhil = 2(H'QH + R) \tag{176}$$

$$f = -2H'Q\left(Y_r - Gx(k) - LD(k)\right) \tag{177}$$

In the next section the simulation studies are described.

4.7 SIMULATION RESULTS AND DISCUSSION

In this section, the simulation studies are described. Here a wind farm with dual battery energy storage systems is simulated in MATLAB 2023 environment run on a i5 8GB personal computer using a MATLAB script. The data is collected from Kaggle database [1], the data is sampled in 5-minute intervals. The first two days of data were considered for actual wind farm power in simulation studies. The reference power is assumed to be known, as it varies randomly between -20MW and 20MW from the actual wind farm data. The dataset is scaled to match wind farm data with rated capacity of 140 MW. The two batteries used have a rated capacity of 30 MWH each. The battery energy storage system is assumed to be ideal; it has no losses. The constraints on the state of charge of each battery are assumed to be 0.1 and 0.9 for minimum and maximum limits. Initially, the state of charge on the first battery is assumed to be 0.15 and state of charge on the second battery is assumed to be 0.85. The maximum limit on the power delivered/extracted from any battery is 20 MW. For MPC, the prediction horizon and the control horizon are equal (Nc=Np=6), qw and ru are taken as 0.8 and 0.2. The data is taken from reference paper [1].

The actual power from wind farm and the reference power are shown in one Figure 4.4. The actual power from wind farm is between 0 and 140 MW, whereas the reference power is between 0 and 160 MW.

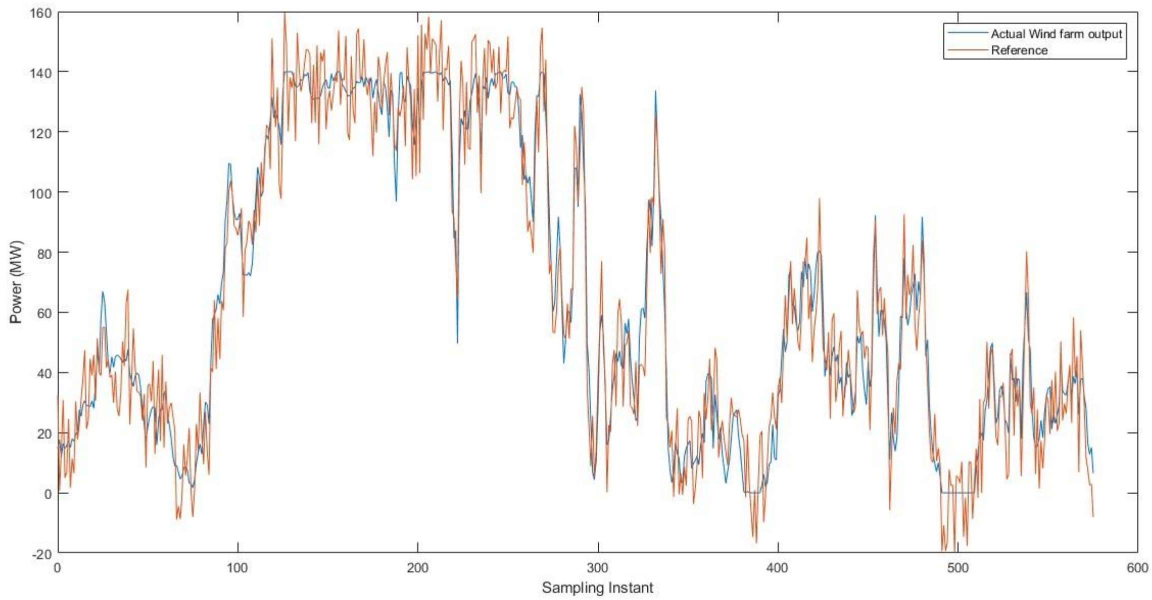

Figure 4.4. Reference power and actual wind farm power.

4.7.1 *Implementation of MPC without constraints*

The model described in 4.5 is simulated in MATLAB. State of charge on both the batteries is shown in Figure 4.5.

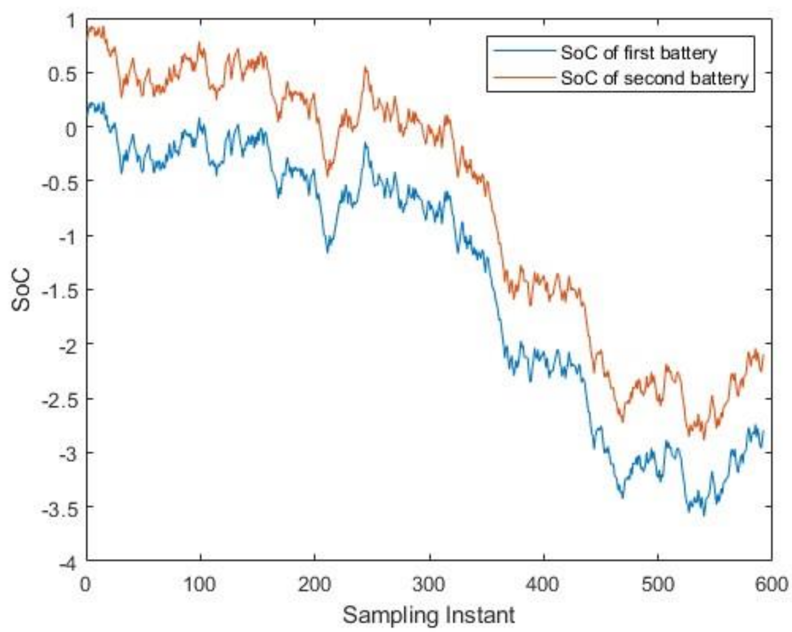

Figure 4.5. State of charge of both batteries without constraints.

It can be observed that the state of charge of the batteries is between -4 and 1. This is out of the limits of state of charge of the batteries and can cause damage to both the battery energy storage systems. Control signals from battery energy storage systems are shown below. It can be inferred from both Figure 4.6 and Figure 4.7, that the control signals are similar.

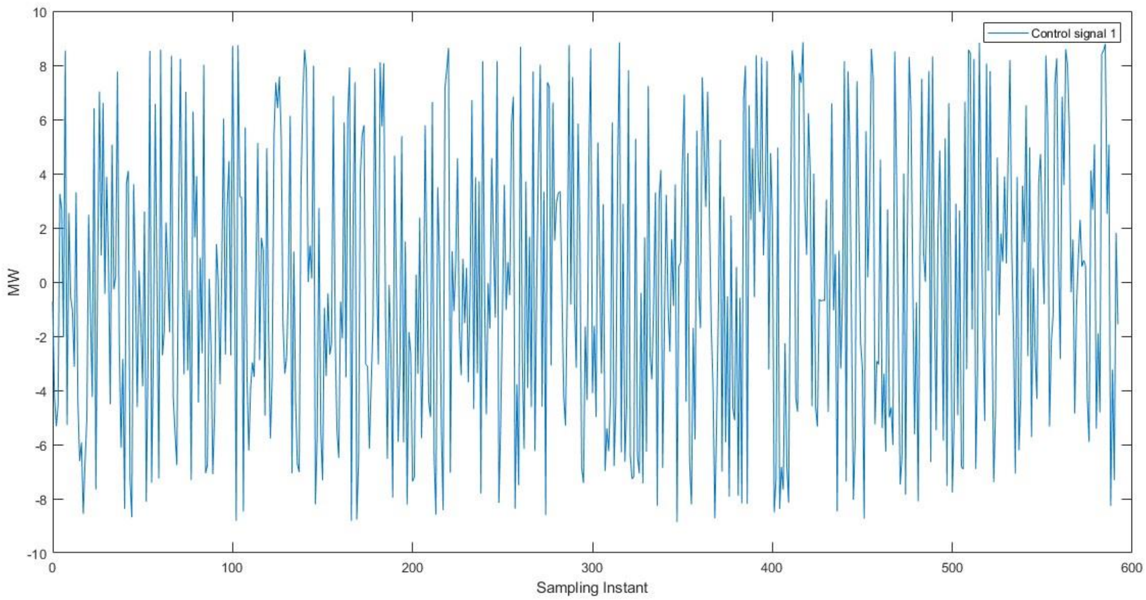

Figure 4.6. Control signal from BESS1 without constraints.

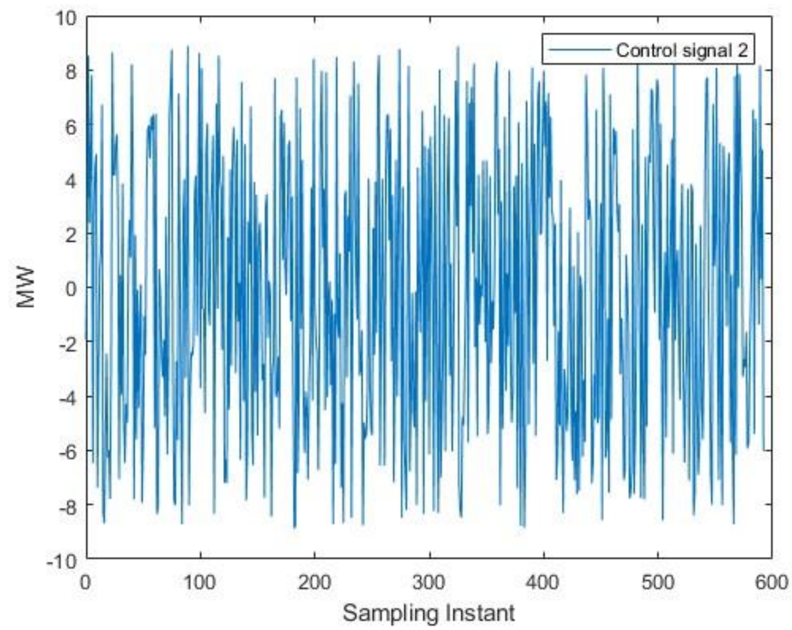

Figure 4.7. Control signal from BESS2 without constraints.

The control signals are between -10 and 10. Also, the power delivered/drawn from the batteries, that is the control signals are also within their limits. Output power after applying control strategy, reference power to track and actual wind farm output power is shown in Figure 4.8.

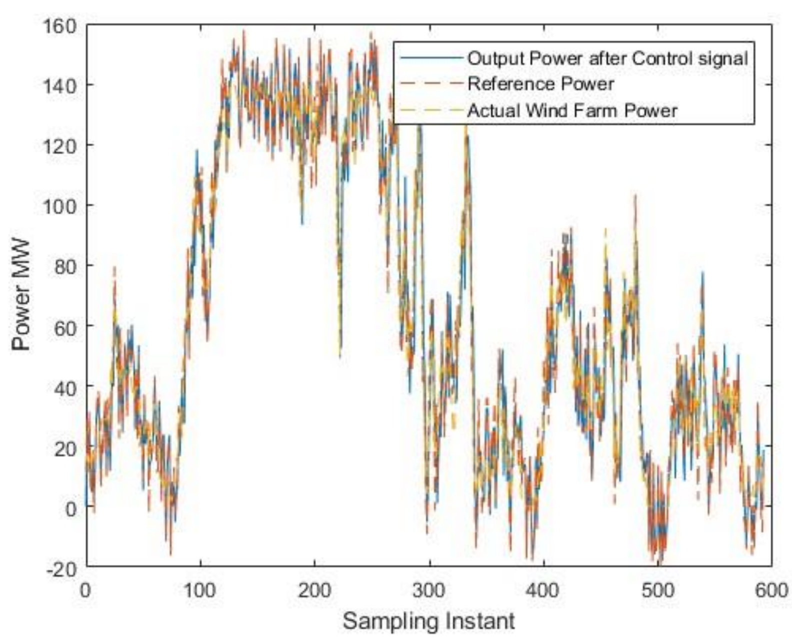

Figure 4.8. Output power, reference power and actual wind farm power without constraints.

It can be inferred here that the output power after control signal is tracking the reference power without much deviation. Even though the output is tracking the reference signals appropriately in Figure 4.8. When constraints on the state of charge of the batteries are not imposed as discussed in section 4.5, it can be observed in Figure 4.5 that the state of charge of the batteries is exceeding their limits. This can cause damage to the batteries and increase the cost of the system by replacing them with new ones.

4.7.2 *Implementation of MPC with constraints*

The model described in section 4.6 is simulated in MATLAB. State of charge on both the batteries is shown below. On contrary to the result obtained in the Figure 4.5, when the state of charge on both the batteries is well within the limits, when constraints are imposed on them as discussed in section 4.6.

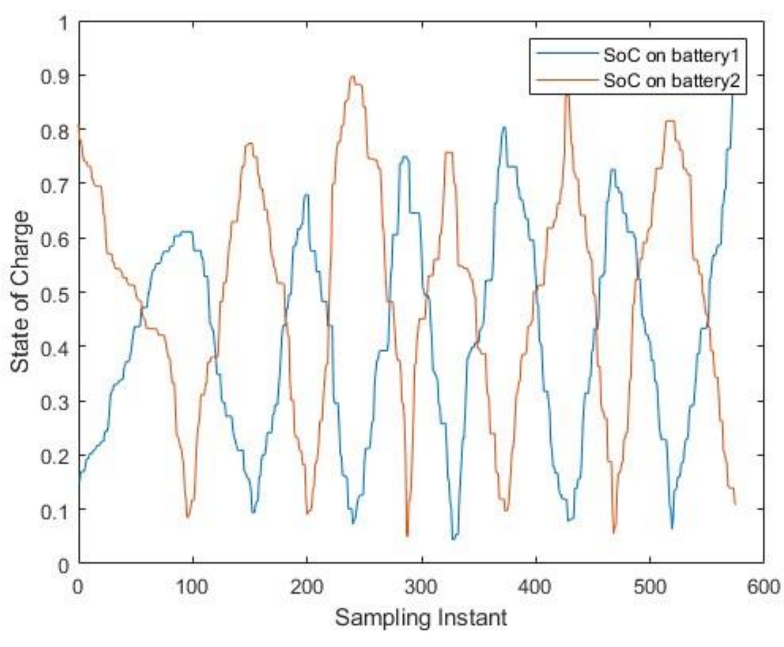

Figure 4.9. State of charge of both batteries with constraints.

Control signals from battery energy storage systems in mode1 and mode2 are shown below. Also, the power delivered/drawn from the batteries, that is the control signals are also within their limits.

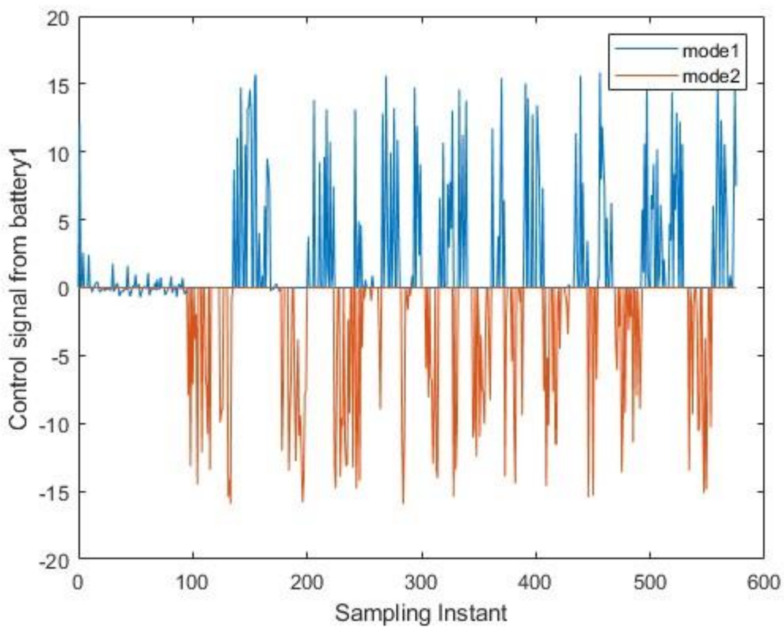

Figure 4.10. Control signal from BESS1 with constraints.

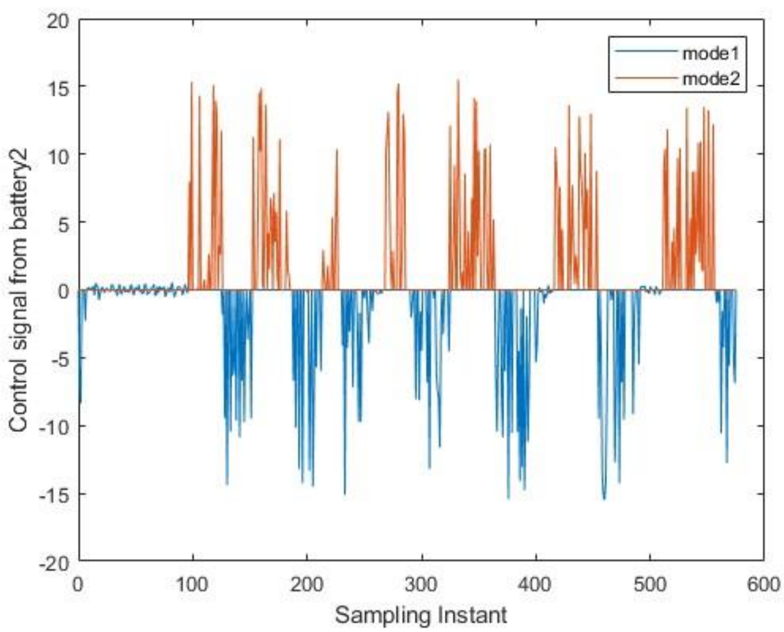

Figure 4.11. Control signal from BESS2 with constraints.

From the control signals with the constraints, it can be observed that in mode1 the first battery charges with control signal greater than zero and second battery discharges with control signal less than zero as discussed in 4.6.1. In mode2 the second battery charges with control signal greater than zero and the first battery discharges with control signal less than zero. This reduces the switching times of the batteries which increases the lifetime of the batteries. Output power after applying control strategy, reference power to track and actual wind farm output power is shown in Figure 4.12.

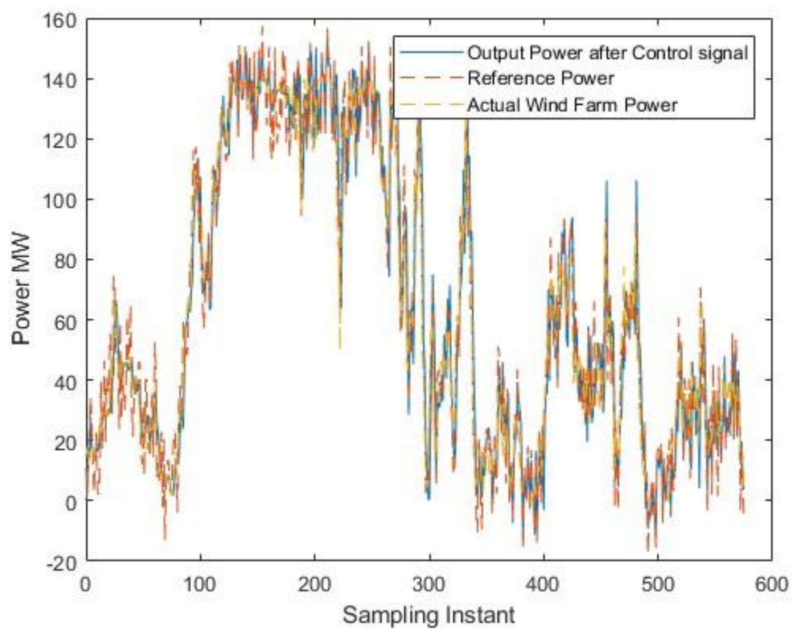

Figure 4.12. Output power, reference power and actual wind farm power with constraints.

It can be inferred here that the output power after control signal is tracking the reference power without much deviation. In the next chapter, a conclusion is given.

Chapter 5. CONCLUSION

In this thesis, constrained MPC has been designed and applied to the wind farm incorporated with dual battery energy storage systems. Simulation results showed that the design had met the objectives of maintaining the state of charge of two batteries within the limits and the power delivered/extracted from the batteries within the limit. In addition, the system tracks the reference power without much deviation, as expected. As a comparison, both unconstrained MPC and constrained MPC have been investigated and simulated. It verifies that with constraints, the control strategy can secure both the state of charge of batteries and power delivered/extracted behave within their limits. This has greatly reduced the switching times between operation modes and thus increases the lifetime of batteries.

Regarding the future application, the presented strategy can be tried on other hybrid systems such as battery energy storage systems with photovoltaic. Future studies also include developing a strategy to forecast the power and use more efficient optimization strategies.

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