Essays on Risk-Return Relation and Asset Pricing

Changhun Choi

A dissertation
submitted in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy

University of Washington

2025

Reading Committee:

Chang-Jin Kim, Chair

Eric Zivot

Jing Tao

Program Authorized to Offer Degree:
Economics

University of Washington

Abstract

Essays on Risk-Return Relation and Asset Pricing

Changhun Choi

Chair of the Supervisory Committee:
Chang-Jin Kim
Department of Economics

In this dissertation, I empirically examine stock market valuations, with a particular focus on the risk-return relation and the respective roles of cash flow and discount rate news.

In Chapter 1, I empirically investigate the risk-return relation under the investors' subjective volatility expectations, which deviate from the rational expectations. I first derive the objective risk premium under the slow-moving subjective volatility expectations based on the theoretical model of Lochstoer and Muir (2022) and show that the slow-moving volatility expectation generates a lead-lag specification in the objective risk-return relation. Then, I develop and estimate an empirical model by employing the log-linear present value framework. The empirical results using U.S. monthly excess stock returns suggest that the slow-moving feature of volatility expectations and the lead-lag structure in the objective risk-return relation are both significantly identified from the data. The parameter estimates suggest that while the objective risk-return relation can be negative, the subjective risk-return relation remains strongly positive, aligning with the key prediction and assumption in Lochstoer and Muir (2022). Moreover, I find that incorporating subjective expectations that deviate from rational expectations helps explain the variation in the Sharpe ratio.

In Chapter 2, I examine the subjective risk-return relation from the observed stock return data in the presence of information rigidity in investors' volatility expectations. Based on the present-value approach of Campbell and Shiller (1988), I develop an empirical model for excess stock returns by introducing the sticky information model of Mankiw and

Reis (2002) into aggregate subjective volatility expectations, while using realized volatility to capture time-varying risk. The estimation results based on U.S. monthly excess stock returns and realized volatility suggest that a significant information rigidity component and a positive and statistically significant subjective risk-return relation are identified from the observed stock return data. Meanwhile, the restriction among parameters implied by the present-value approach is rejected, indicating that other factors may influence stock return variations. I suggest that investors' overextrapolative belief may help explain the rejection of the restriction. I also find a state-dependent information rigidity: it increases during a period with lower macroeconomic volatility. Consistent with the findings of Coibion and Gorodnichenko (2015), the estimation results indicate that the degree of information rigidity increased during the Great Moderation period.

Chapter 3 explores the regime dependency and time variation of the relative importance of cash flow news and discount rate news in explaining excess stock return variance. To this end, I apply the variance decomposition method of Campbell and Ammer (1993) to the threshold VAR (TVAR) and the time-varying parameter VAR with stochastic volatilities (TVP-VAR-SV). To identify the regimes in the stock market, I use the Chicago Fed's financial condition index and investor sentiment index constructed by Baker and Wurgler (2006). The variance decomposition results using TVAR suggest that the contribution of discount rate news increases during tight financial conditions or high investor sentiment regimes. The result of TVP-SV-VAR indicates that cash flow news has become more important than discount rate news after the 1990s. I propose possible explanations for the results. First, the regime-dependent relative importance may be associated with the change in attention allocations of investors and the asymmetric stock return predictability across the regimes. Second, the reversal of the relative importance after the 1990s may be attributed to the less volatile discount rate news caused by increased information rigidity and changes in the return-earnings relationship after the onset of the Great Moderation.

TABLE OF CONTENTS

Page

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGMENTS

First and foremost, I would like to express my deepest gratitude to my PhD supervisor, Professor Chang-Jin Kim, for his unwavering guidance, intellectual stimulation, and constant support throughout my graduate studies. His invaluable advice helped shape my initial ideas into well-defined research topics for my dissertation. Without his patience and mentorship, I truly believe I would not have achieved my goals.

I am also deeply indebted to my committee members. The thoughtful feedback and encouragement from Professor Eric Zivot and Professor Jing Tao broadened my perspective and deepened my understanding of the subject. I am especially grateful to my GSR member, Professor Thomas Gilbert, for his insightful comments and generosity.

My appreciation extends to many individuals who have directly or indirectly contributed to this study. I would like to thank my colleagues for their valuable feedback and encouragement. I am also grateful to the departmental staff, including Michelle, Kim, and others, for their kindness and support.

Lastly, I wish to express my heartfelt gratitude to my family. Without their unwavering love and encouragement, I could not have overcome the many challenges along this journey. Their support has been my greatest source of strength, and for that, I am forever grateful.

DEDICATION

To my dear wife, So Young, and my wonderful son, Yonjoon,

whose love and support are always by my side

Chapter 1

EMPIRICAL INVESTIGATION OF RISK-RETURN RELATION: WITH THE SLOW-MOVING SUBJECTIVE BELIEF ON MARKOV-SWITCHING VOLATILITY

1.1 Introduction

The relationship between the expected return and ex-ante volatility (the risk-return relation) has been a fundamental topic in asset valuation research. Because asset prices reflect investors' expectations of future market outcomes, it is crucial to understand how investors form their subjective expectations. The conventional approach assumes the rational expectations of investors, and asset pricing models based on the rational expectations predict a positive risk-return relation[1]. However, the existence of a positive risk-return relation has not been invariably found in existing empirical literature. On the contrary, many empirical studies have reported weak or even negative risk-return relations.

One stylized fact about stock return is that the empirical results for risk-return relation are sensitive to model specifications. Because the expected return and the volatility are latent variables that should be estimated, different approaches and specifications lead to different results of estimated risk-return relation. For example, some studies find a positive risk-return relation using a symmetric GARCH model (e.g., French et al. (1987), and Chou (1988)), while the relation turns to negative when asymmetric volatility is taken into account (e.g., Nelson (1991) and Glosten et al. (1993)). Guo and Whitelaw (2006) emphasize the possibility of omitted variable bias in estimating the risk-return relation and argue that omitting the hedge component in expected return can be attributable to the empirically weak or negative risk-return relation. Additionally, the choice of conditioning variable used to estimate expected return and volatility can impact the empirical result; Harvey (2001) shows that the estimated risk-return relation is influenced by the choice of conditioning

---

[1] These models include ICAPM of Merton (1980), the habit model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), and the time-varying disasters model of Wachter (2013).

variables, and Ludvigson and Ng (2007) and Lettau and Ludvigson (2010) argue that the relatively small amount of conditioning variables to model expected return and volatility is a primary reason for the conflicting empirical findings.

Another stylized fact about stock return is that the Sharpe ratio estimated from data exhibits large countercyclical variations. Since the Sharpe ratio reflects the price of risk, asset pricing models with a constant price of risk (e.g., Sharpe (1964) and Lintner (1965)) cannot explain the considerable variation of the Sharpe ratio. Leading rational expectations asset pricing models explain this with countercyclical risk aversion (e.g., Campbell and Cochrane (1999)) or the time-varying consumption volatility (e.g., Bansal and Yaron (2004))[2]. However, Lettau and Ludvigson (2010) document that the leading rational expectations asset pricing models that allow variation in the price of the risk do not appropriately account for the dynamics of the Sharpe ratio estimated from the data. Recently, Nagel and Xu (2022) demonstrate that subjective belief dynamics deviating from the rational expectations can generate a volatile Sharpe ratio by weakening the strong positive relationship between the expected return and volatility.

A fundamental feature of the rational expectations is the alignment of objective expectations, which can be estimated by econometricians through ex-post data and a true data-generating process, with the subjective expectations of economic agents. However, there has been growing empirical evidence that casts doubt on this assumption. Starting from Manski (2004), researchers have paid attention to survey data that are assumed to represent the economic agent's subjective belief. Several empirical studies using survey data have reported rejections of the rational expectations hypothesis in forecasting macroeconomic variables (e.g., Coibion and Gorodnichenko (2015), Bordalo et al. (2020), and Landier et al. (2020)) as well as financial variables (e.g., Bacchetta et al. (2009), Greenwood and Shleifer (2014), and Adam et al. (2017)). In light of the empirical findings from survey data, researchers have attempted to develop models that can account for asset

[2]In the of Campbell and Cochrane (1999), the effective risk-aversion increases as consumption approaches the reference level, which generally happens in recessions, and this can generate the countercyclical variation of the Sharpe ratio. The model of Bansal and Yaron (2004) generates time-variations of the Sharpe ratio because the risk premium varies as the consumption volatility fluctuates, and the risk premium rises in periods of high economic uncertainty.

pricing phenomena by incorporating agents with subjective beliefs that deviate from the rational expectations (e.g., Barbris et al. (2015), Nagel and Xu (2022), and Lochstoer and Muir (2022)).

In particular, Lochstoer and Muir (2022) emphasize the role of the market participants' sticky and biased beliefs about volatility in explaining several stylized facts in the stock market, including a weak or negative risk-return relation and a negative contemporaneous correlation between realized returns and the shock to volatility (the discount rate effect of volatility). They present an asset pricing model in which investors slowly update the volatility expectations due to the information rigidity in expectations and their overestimation of the persistence of volatility. According to their model, an increase in volatility leads not only to a decrease in the current stock price but also to a further decline in stock prices in the short term (i.e., a hump-shaped stock price response), and the recovery process for the stock price takes longer than what is predicted under the rational expectations. Due to these price movements, the relationship between the objective risk premium and expected volatility (the objective risk-return relation) can be weak or negative even when investors subjectively require a higher return for a higher volatility expectation (the positive subjective risk-return relation). The price movement in their model has an important implication: the objective risk premium remains high even after the volatility shock subsides, indicating a lead-lag relationship between the objective risk premium and expected volatility. Lochstoer and Muir's (2022) findings suggest that empirical models that do not incorporate investors' subjective beliefs that are inconsistent with the rational expectations may be misspecified. Therefore, it is crucial to explore whether the slow-moving volatility expectations can be detected from stock return data and to assess how these expectations influence the estimation of the objective risk-return relation and the time variation of the Sharpe ratio.

Meanwhile, the findings of Lochstoer and Muir (2022) rely on parameter calibration that supports the assumption of a strongly positive subjective risk-return relation. However, empirical investigations using survey data have produced mixed results on this relationship. For instance, Nagel and Xu (2023) report a positive correlation between subjective return variance expectations and subjective excess return expectations, while Giglio et al. (2021)

find a negative relationship between subjective crash risks and subjective return expectations. Moreover, Jo et al. (2022) observe that the subjective risk-return relation varies depending on the assets being compared. Given these mixed results, it is essential to examine whether there is empirical evidence supporting the critical assumptions made by Lochstoer and Muir (2022).

This paper contributes to the existing literature by proposing an empirical model of the objective risk-return relation that accounts for agents' subjective volatility expectations deviating from the rational expectations. Through the analysis of U.S. monthly excess stock return data, I aim to examine whether the empirical results support the predictions and assumptions made by Lochstoer and Muir (2022). Additionally, I explore how incorporating the slow-moving volatility expectations influences the response of stock prices to volatility innovation and the variation of the Sharpe ratio over time.

Building on Lochstoer and Muir's (2022) theoretical model, I derive a specification for the objective risk premium, which captures the relationship between the objective risk premium and the objective expected volatility under investors' slow-moving volatility expectation. The model specifies that the objective risk premium is a function of its own lag as well as the current and lagged objective expected volatility, aligning with previous studies that emphasize the critical lead-lag interaction between the conditional mean and volatility of stock returns (e.g., Whitelaw (1994), Brandt and Kang (2004), Ludvigson and Ng (2007), and Lettau and Ludvigson (2010)). By incorporating the specification for the objective risk premium into the log-linear present value framework of Campbell and Shiller (1988), I derive an empirical model of the objective risk-return relation that also accounts for the discount rate of volatility, as suggested by Campbell and Hentschel (1992) and Kim, Morley, and Nelson (2004). To model the market volatility, I adopt a Markov-switching specification following previous studies, including Schaller and van Norden (1997), Kim, Nelson, and Startz (1998), Mayfield (2004), and Kim, Morley, and Nelson (2004), that successfully employed a Markov-switching specification to model monthly stock return volatility.[3]

The main empirical results obtained from U.S. monthly excess stock returns data can

[3]Moreover, Hamilton and Susmel (1994) document that most of ARCH dynamics vanish at the monthly return horizon and only the Markov-switching regime changes persist over a more prolonged period.

be summarized as follows. First, I identify a significant slow-moving volatility expectation component from the excess return data. Second, I find a significant lead-lag relationship between the objective risk premium and expected volatility. I observe a negative contemporaneous effect of the objective expected volatility on the objective risk premium, supporting the prediction of Lochstoer and Muir (2022). At the same time, I find that the lagged objective expected volatility has a significant positive impact on future objective risk premium, indicating a positive subjective risk-return relation. This highlights the potential misspecification issue in empirical models under the rational expectations assumption. Third, the model's parameter estimates support the main assumptions and predictions in Lochstoer and Muir (2022). Specifically, an impulse-response analysis based on the parameter estimates exhibits a hump-shaped price response to volatility innovation. Finally, incorporating the slow-moving subjective volatility expectations in estimating the risk-return relation improves the model's explanatory power and helps explain the dynamic behavior of the U.S. aggregate stock market Sharpe ratio. Compared to the model with the rational expectations, the log-likelihood value of the model with slow-moving volatility expectation increases significantly, and the Sharpe ratio exhibits clear countercyclical patterns with sufficient time variations.

The rest of this paper is organized as follows. In Section 1.2, I derive the objective risk-return relation under the slow-moving subjective volatility expectations and present the formal derivation of the empirical model used in this chapter. Section 1.3 reports the empirical findings, and I conclude in Section 1.4.

1.2 An Empirical Model of risk-return Relation in the Presence of Slow-moving Subjective Volatility Expectations

1.2.1 The Objective Risk-return Relation under Investors' Slow-moving Volatility Expectations

Lochstoer and Muir (2022) construct a theoretical model that cam simultaneously explain the weak or negative objective risk-return relation and the negative contemporaneous correlation between realized excess returns and volatility shocks. A key ingredient in their model is that investors have slow-moving subjective beliefs regarding the dynamics of stock return volatility. The realized volatility and the objective and subjective expectations of the volatility in their model are given as follows:

$$\sigma_{t+1}^2 = \bar{v} + \rho(\sigma_t^2 - \bar{v}) + v_{t+1}, \tag{1.1}$$

$$E_t[\sigma_{t+1}^2] = \bar{v} + \rho(\sigma_t^2 - \bar{v}), \tag{1.2}$$

$$E_t^S[\sigma_{t+1}^2] = \bar{v} + \lambda(1-\phi)\sum_{j=0}^{\infty}\phi^j(\sigma_{t-j}^2 - \bar{v}), \tag{1.3}$$

where σ_{t+1}^2 is a realized variance of dividend growth innovation, ρ is the persistence parameter of the variance, and superscript S on the expectations operator denotes the subjective expectations. A vital component of the subjective volatility expectations is that it depends not only on the most recent realization of variance but also farther lags of realized variance when ϕ is greater than zero. This feature reflects the slow-moving nature of investors' volatility expectations, in contrast to the objective expectations expressed in equation (1.2). Here, $\phi(\in(0,1))$ measures the degree of the slow-moving expectations, and λ is a parameter related to the agent's subjective belief about the persistence of the realized variance. The slow-moving expectation of their model comes from two sources: information rigidity and the investors' overestimation of the persistence of volatility process.[4] ϕ is zero when there is no information rigidity, and it increases as the degree of information rigidity and the investors' belief about the persistence of volatility increase. The subjective and objective

[4] Information rigidity represents the situation that only a fraction of the agents update expectation every period, and the investors' overestimation of the persistence of volatility means that the investors believe that a persistence parameter of variance is larger relative to the true persistence parameter.

volatility expectations become equivalent if $\phi = 0$ and $\lambda = \rho$.[5]

The specifications for subjective and objective risk premiums in their model are:

$$E_t^S[r_{t+1}] = a_0 + a_1 E_t^S[\sigma_{t+1}^2], \tag{1.4}$$

$$E_t[r_{t+1}] = E^S[r_{t+1}] + b(E_t^S[\sigma_{t+1}^2] - E_t[\sigma_{t+1}^2]), \tag{1.5}$$

where r_{t+1} is excess stock returns, a_1 is an increasing function of the risk aversion coefficient, and b is a function of the risk aversion coefficient, the elasticity of intertemporal substitution, ϕ, and λ.

There is a positive subjective risk-return relation if a_1 is positive, which means investors require a higher return for a higher volatility expectation. In equation (1.5), there are two channels through which volatility increase affects the objective risk premium. One channel operates through the subjective risk premium, while the other stems from the difference between the subjective and objective volatility expectations (the expectation error). If $a_1 > 0$ and $b > 0$ as the calibration in Lochstoer and Muir (2022), the two channels affect the objective risk premium in opposite directions.[6] The increase in volatility positively affects the objective risk premium as it raises the subjective volatility expectations ($E_t^S[\sigma_{t+1}^2]$) and the subjective risk premium ($E_t^S(r_{t+1})$). Meanwhile, due to the slow-moving nature of subjective volatility expectations ($\phi > 0$), investors adjust their future volatility expectations less in response to new information compared to the objective expectation ($E_t^S[\sigma_{t+1}^2] < E_t[\sigma_{t+1}^2]$). This results in a negative term in parentheses in equation (1.5), which, in turn, negatively impacts the objective risk premium. These opposing effects of volatility increases on the objective risk premium contribute to the weak or negative objective risk-return relation in their model.

To develop an empirical model based on the objective risk-return relation, I derive a relation between the objective risk premium and the objective volatility expectation in the presence of the investors' slow-moving volatility expectation. Equations (1.2) and (1.3) imply the following relationship between the subjective volatility expectations and objective

[5]Lochstoer and Muir (2022) set $\rho = 0.71$, $\phi = 0.5$, and $\lambda = 0.8$ in the benchmark calibration

[6]The benchmark calibration for a_1 and b in Lochstoer and Muir (2022) are 0.48 and 1.06 respectively.

volatility expectations:

$$E_t^S[\sigma_{t+1}^2] = \left(1 - \frac{\lambda}{\rho}\right)\bar{v} + (1-\phi)\frac{\lambda}{\rho}\sum_{j=0}^{\infty}\phi^j E_{t-j}[\sigma_{t-j+1}^2]. \tag{1.6}$$

Then, by substituting equation (1.4) into equation (1.5) and replacing $E_t^S[\sigma_{t+1}^2]$ with equation (1.6), equation (1.5) can be rewritten to express the relationship between the objective risk premium and the objective volatility expectation:[7]

$$\begin{aligned} E_t[r_{t+1}] =& c_0 + c_1 E_t[\sigma_{t+1}^2] + c_2 \sum_{j=1}^{\infty}\phi^j E_{t-j}[\sigma_{t-j+1}^2] \\ =& c_0 + c_1 E_t[\sigma_{t+1}^2] + c_2 \frac{\phi}{(1-\phi L)} E_{t-1}[\sigma_t^2], \end{aligned} \tag{1.7}$$

where $c_0 = a_0 + (a_1 + b)(1 - \frac{\lambda}{\rho})\bar{v}$, $c_1 = (a_1 + b)(1-\phi)\frac{\lambda}{\rho} - b$, $c_2 = (a_1 + b)(1-\phi)\frac{\lambda}{\rho}$, and L refers to the lag operator.

By multiplying both sides of equation (1.7) by $(1-\phi L)$, the objective risk-return relation in the presence of the slow-moving volatility expectation of investors can be expressed as follows:

$$E_t[r_{t+1}] = \beta_0 + \phi E_{t-1}[r_t] + \beta_1 E_t[\sigma_{t+1}^2] + \beta_2 E_{t-1}[\sigma_t^2], \tag{1.8}$$

where $\beta_1 = c_1$, and $\beta_2 = \phi b$. This specification is similar to that of literature emphasizing a lead-lag interaction between conditional mean and volatility of stock returns (e.g., Whitelaw (1994), Brandt and Kang (2004), Ludvigson and Ng (2007), and Lettau and Ludvigson (2010)). Equation (1.8) shows that the slow-moving subjective volatility expectations gives rise to the lead-lag specification of the objective risk premium, while the motivation for using the lead-lag specification in previous literature is primarily empirical.

The coefficient β_1 seizes a contemporaneous effect of the objective volatility expectation on the objective risk premium, and its sign depicts the objective risk-return relation. It can be either positive or negative depending on the relative magnitude of the aforementioned two effects of the volatility increase on the objective risk premium. When the volatility at period t (σ_t^2) increases, assuming the positive subjective risk-return relation, investors update the expectation on the future volatility and require a higher subjective risk premium.

---

[7] Refer to Appendix A.1 for a detailed derivation of equations (1.6) and (1.7).

This leads to a drop in the current stock price (the discount rate effect of volatility) and increases the objective risk premium at the period t. Meanwhile, there is a predictable decline in the stock price in the next period due to the slow-moving volatility expectation. Because the increase in volatility is partially reflected in investors' expectation of future volatility, they will perceive a positive discount rate shock in the next period (at period t+1) as they realize that the volatility is higher than their expectation. The stock price is predicted to drop further in the next period, which decreases the objective risk premium at period t.

The coefficient β_2 captures an effect of lagged objective volatility expectation on the future objective risk premium, which can be interpreted as an effect of the delayed adjustment of the subjective volatility expectations on the future objective risk premium. The sign of β_2 is related to the subjective risk-return relation. β_2 is positive when there is a positive subjective risk-return relation. When volatility at the period t-1 (σ^2_{t-1}) increases the objective volatility expectation ($E_{t-1}[\sigma^2_t]$) rises. In the presence of the slow-moving subjective volatility expectations, there will be a delayed increase of the subjective volatility expectations in the next period ($E^S_t[\sigma^2_{t+1}]$). This delayed adjustment of volatility expectation leads to an increase in the subjective risk premium ($E^S_t[r_{t+1}]$) and the objective risk premium ($E_t[r_{t+1}]$) in the next period under the positive subjective risk-return relation. Thus, $E_{t-1}[\sigma^2_t]$ is positively related to $E_t[r_{t+1}]$, and β_2 is expected to be positive.

Under the rational expectations assumption, where ϕ and β_2 are zero, the objective risk-return in equation (1.8) is reduced to:

$$E_t[r_{t+1}] = \beta_0 + \beta_1 E_t[\sigma^2_{t+1}], \tag{1.9}$$

which is the specification that has been broadly used in the empirical literature testing Merton's ICAPM.

1.2.2 Derivation of an Empirical Model of the risk-return relation under the Slow-moving Subjective Volatility Expectations

Following Campbell and Hentschel (1992) and Kim, Morley, and Nelson (2004) that account for the discount rate effect of volatility (the volatility feedback effect) in estimating the risk-return relation, I incorporate the objective risk premium specification in equation (1.8) into the log-linear present value framework.

I assume that the news about dividends is subject to two-state Markov-switching volatility reflecting previous studies (e.g., Schaller and van Norden (1997), Kim, Nelson, and Startz (1998), Mayfield (2004), and Kim, Morley, and Nelson (2004)) that successfully employed a Markov-switching specification to model monthly stock return volatility.

$$\begin{aligned} &\varepsilon_{t+1} \sim N(0, \sigma^2_{S_{t+1}}), \\ &\sigma^2_{S_{t+1}} = \sigma^2_0 + (\sigma^2_1 - \sigma^2_0)S_{t+1}, \quad \sigma^2_0 < \sigma^2_1, \\ &Pr[S_{t+1} = 0 | S_t = 0] = p_{00}, \quad Pr[S_{t+1} = 1 | S_t = 1] = p_{11}, \end{aligned} \tag{1.10}$$

where S_{t+1} is a first-order Markov-switching state variable that has discrete values of 0 or 1 according to the predominant volatility regime, p_{00} and p_{11} are the transition probabilities describing the evolution of S_{t+1}. S_{t+1} is 0 for the low-volatility regime and 1 for the high-volatility regime. The transition probabilities imply the following dynamics of S_{t+1} and $\sigma^2_{S_{t+1}}$:

$$\begin{aligned} S_{t+1} &= \bar{S} + \rho S_t + \eta_{t+1}, \\ \sigma^2_{S_{t+1}} &= \bar{\sigma}^2 + \rho \sigma^2_{S_t} + v_{t+1}, \end{aligned} \tag{1.11}$$

where $\bar{S} = 1 - p_{00}$, $\bar{\sigma}^2 = (1 - p_{11})\sigma^2_0 + (1 - p_{00})\sigma^2_1$, $\rho = p_{00} + p_{11} - 1$ represents the persistence of S_{t+1}, η_{t+1} denotes the innovation to S_{t+1}, and $v_{t+1} = (\sigma^2_1 - \sigma^2_0)\eta_{t+1}$ is the innovation to volatility.

With the Markov-switching volatility, the objective risk premium in equation (1.8) can be expressed as

$$\mu_t = \beta_0 + \phi \mu_{t-1} + \beta_1 E_t[\sigma^2_{S_{t+1}}] + \beta_2 E_{t-1}[\sigma^2_{S_t}], \tag{1.12}$$

where μ_t=$E_t[r_{t+1}]$, and $E_t[\sigma^2_{S_{t+1}}] = \sigma^2_0 + (\sigma^2_1 - \sigma^2_0)Pr[S_{t+1} = 1 | I_t]$.

As discussed in Kim, Morley, and Nelson (2004), the log-linear present value framework of Campbell and Shiller (1988) allows us to decompose the realized excess stock return into the objective risk premium, the discount rate effect of volatility, and the news about dividends:

$$r_{t+1} = \mu_t + f_{t+1} + \varepsilon_{t+1}, \tag{1.13}$$

where f_{t+1} is the term for discount rate effect of volatility:

$$f_{t+1} = -\sum_{j=1}^{\infty} \kappa^j (E_{t+1} - E_t)[r_{t+1+j}], \tag{1.14}$$

and κ is a constant slightly less than 1 (0.997) that comes from the log-linearization.

The discount rate effect of volatility explains the contemporaneous correlation between the realized excess return and the volatility in the data through the effect of unexpected changes in volatility on the investors' revisions on the future expected returns. Thus, it can be expressed as

$$f_{t+1} = \delta v_{t+1}, \tag{1.15}$$

where δ captures a contemporaneous effect of the volatility innovation on the realized excess return. The sign of δ provides indirect evidence for the subjective risk-return relation. For example, in the presence of a positive subjective risk-return relation, a positive volatility innovation leads to an increase in the investors' expectations of future volatility and discount rates. This increase in discount rates, in turn, drives down the current stock price and affects the realized excess return negatively, which implies a negative sign of δ.

After multiplying both sides of equation (1.13) by $(1 - \phi L)$ and substituting related terms using equations (1.12) and (1.15), I obtain an equation for the excess stock return that can be used to estimate objective risk-return relation under the slow-moving subjective volatility expectations:

$$\begin{aligned} r_{t+1} = &\beta_0 + \phi r_t + \beta_1 E_t[\sigma^2_{S_{t+1}}] + \beta_2 E_{t-1}[\sigma^2_{S_t}] \\ &+ \delta v_{t+1} - \phi \delta v_t + \varepsilon_{t+1} - \phi \varepsilon_t, \quad \varepsilon_{t+1} \sim N(0, \sigma^2_{S_{t+1}}). \end{aligned} \tag{1.16}$$

In estimating the empirical model, I employ a restriction on the parameters of the model that is implied by the log-linear present value model (equation (1.14)). Because $\mu_{t+j} = E_{t+j}[r_{t+1+j}]$, the law of iterated expectation implies $E_{t+1}[r_{t+1+j}] = E_{t+1}[\mu_{t+j}]$.

Then using equations (1.11) and (1.12), I obtain the term $(E_{t+1} - E_t)[r_{t+1+j}]$ in equation (1.14) as

$$\begin{aligned}(E_{t+1} - E_t)[r_{t+1+j}] =& \beta_1 \rho v_{t+1} \quad for \quad j = 1, \\ =& \phi(E_{t+1} - E_t)[r_{t+j}] + \beta_1 \rho^j v_{t+1} + \beta_2 \rho^{j-1} v_{t+1} \quad for \quad j \geq 2,\end{aligned}$$

Then, substituting the above equation into equation (1.14) results in the following constraint among the parameters of the model:[8]

$$\delta = -\frac{\kappa\rho}{(1-\kappa\rho)(1-\kappa\phi)}(\beta_1 + \kappa\beta_2). \tag{1.17}$$

I utilize the Kalman filter and the maximum likelihood method in estimation. However, v_t, which depends on the Markov-switching state variables, prevents us from using the standard Kalman filter to evaluate the likelihood function. Thus, I employ the approximation method proposed by Kim (1994) to evaluate the likelihood value by casting the model in equation (1.16) to the following state-space model:

Measurement Equation

$$r_{t+1} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} r_{t+1} \\ \varepsilon_{t+1} \end{bmatrix}. \tag{1.18}$$

$$(r_{t+1} = H\xi_{t+1})$$

Transition Equation

$$\begin{bmatrix} r_{t+1} \\ \varepsilon_{t+1} \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 E_t[\sigma^2_{S_{t+1}}] + \beta_2 E_{t-1}[\sigma^2_{S_t}] + \delta v_{t+1} - \phi\delta v_t \\ 0 \end{bmatrix} + \begin{bmatrix} \phi & -\phi \\ 0 & 0 \end{bmatrix} \begin{bmatrix} r_t \\ \varepsilon_t \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \varepsilon_{t+1}. \tag{1.19}$$

$$(\xi_{t+1} = A_{t+1} + F\xi_t + C\varepsilon_{t+1})$$

When investors have the rational expectations on future volatility, the objective risk premium follows the specification in equation (1.9), where ϕ and β_2 are imposed to zero

[8] Refer to Appendix A.2 for a detailed derivation.

in equation (1.8). Thus an empirical model for the risk-return relation under the rational expectations can be derived by imposing $\phi = \beta_2 = 0$ in equation (1.16):

$$r_{t+1} = \beta_0 + \beta_1 E_t[\sigma^2_{S_{t+1}}] + \delta v_{t+1} + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma^2_{S_{t+1}}), \tag{1.20}$$

and the constraint among the parameters can be also obtained by setting $\phi = \beta_2 = 0$ in equation (1.17):

$$\delta = -\frac{\kappa\rho}{(1-\kappa\rho)}\beta_1. \tag{1.21}$$

It is worth noting that equations (1.20) and (1.21) align with the model and the constraint of Kim, Morley, and Nelson (2004). In their research, they assume the rational expectations of investors and examine the risk-return relation of the U.S. aggregate stock market by employing the log-linear present value model.

1.3 Empirical Findings

1.3.1 Data Description

I use monthly excess stock returns on the market portfolio. The excess stock return (r_{t+1}) is constructed using the log returns (including capital gains as well as dividend yields) for a value-weighted portfolio of all NYSE listed stocks in excess of the log one-month U.S. Treasury bill yields. The data are collected from the Center for Research in Securities Prices (CRSP) and the Board of Governors of the Federal Reserve System. I choose the start of the sample period as January 1956 in consideration of the extraordinary impact of events such as World War II (1945), the Korean War (1953), and the pegging of Treasury-bill interest rates preceding the Treasury-Fed Accord (1951), and set the end of the sample period as December 2019 to ensure the estimation result free from the effect of the recent pandemic.

1.3.2 Estimation Results

To evaluate how the slow-moving volatility expectations of investors affect the estimated objective risk-return relation, I estimate the model with the rational expectations as well as the model with the slow-moving subjective volatility expectations. Moreover, I also estimate two models without imposing the restrictions in equations (1.17) and (1.21) to test the validity of the parameter restriction.

The model with Rational Expectations on Volatility

Table 1.1 summarizes the estimation results for the model when investors have the rational expectations on future volatility. In general, the result with imposing the restriction and the result without the restriction are qualitatively the same.

β_1 is estimated to be positive for both estimations (with and without imposing the restriction). A noteworthy aspect is that when the restriction $\delta = -\frac{\kappa\rho}{(1-\kappa\rho)}\beta_1$ is imposed, the estimate becomes larger in magnitude and statistically significant. The estimate of β_1 is 0.069 and is statistically significant at a 1% level when the restriction is imposed. In contrast, it is 0.031 and is not statistically significant when the restriction is not imposed and δ is treated as an additional parameter to be estimated. This result is consistent with the finding of Kim, Morley, and Nelson (2004) that consideration of the discount rate effect of volatility can provide a more effective way to capture the true sign of the risk-return relation. Similar to the result in Table 1.1, they find that the parameter for risk-return relation is estimated to be significantly positive when the restriction is imposed, while they obtain a negative or a positive but insignificant estimate of the risk-return relation when the restriction is not imposed.

The estimates of δ are negative for both estimations and statistically significant at 1% level. These results confirm the stylized fact that excess returns and volatility innovations have contemporaneously negative correlations and are consistent with the discount rate effect of volatility, which can be interpreted as indirect evidence of a positive risk-return relation.

The restriction implied by the log-linear present value model is not rejected as indicated by the likelihood ratio test results reported in the last row of Table 1.1. The likelihood ratio statistic is 2.364 with a p-value of 0.124. Thus, the result of the restricted model supports a positive relationship between the objective risk premium and the objective volatility expectation.

Table 1.1: Estimation Results under the Rational Expectations

	Restricted Estimation		**Unrestricted Estimation**	
β_0	-0.368	(0.320)	0.125	(0.442)
β_1	0.069	(0.023)	0.031	(0.033)
δ	-0.297	(0.071)	-0.333	(0.077)
σ_0^2	8.710	(1.082)	8.449	(0.960)
σ_1^2	32.143	(5.083)	30.873	(4.544)
p_{00}	0.956	(0.015)	0.950	(0.016)
p_{11}	0.857	(0.048)	0.826	(0.053)
lnL	-2127.187		-2126.005	
LR statistic	2.364	(0.124)		

i) The restriction of restricted estimation result is $\delta = -\frac{\kappa\rho}{(1-\kappa\rho)}\beta_1$.

ii) In the parentheses are the standard errors

iii) The standard error of δ in the restricted estimation is calculated using the delta method.

iv) LR statistic refers to the likelihood ratio test statistic for the restriction for δ. In the parentheses is the p-value.

The model with the Slow-moving Subjective Volatility Expectations

Table 1.2 summarizes the estimation results for the model when investors have slow-moving volatility expectations. The estimates of newly introduced parameters due to the slow-moving volatility expectation, ϕ and β_2, are statistically significant, suggesting that it is crucial to account for the subjective volatility expectations inconsistent with the rational expectations in estimating risk-return relation. Similar to the results of the model with the rational expectations, the results with the restriction and those without the restriction are qualitatively the same. All parameter estimates have the same signs and similar statistical significance in both estimations. Also, as the large p-value for the likelihood ratio tests indicates, the restriction $\delta = -\frac{\kappa\rho}{(1-\kappa\rho)(1-\kappa\phi)}(\beta_1 + \kappa\beta_2)$ is not rejected. Thus, I provide implications of the parameter estimates based on the restricted estimation result.

The estimate of ϕ, which measures the degree of slow-moving expectation, is 0.793 and is statistically significant at all conventional levels.[9] This result suggests that the slow-moving subjective volatility expectations component is significantly identified from the market return data. Because ϕ should be zero when there is no information rigidity, the estimate of ϕ provides evidence of information rigidity in the subjective volatility expectations.

The estimate of β_1, whose sign denotes the objective risk-return relation, is -0.129 and is statistically significant at 5% level. This result is consistent with the prediction of the model in Lochostoer and Muir (2022) that the objective risk-return relation can be weak or even negative in the presence of slow-moving volatility expectations. The significantly negative estimate implies that the effect of an increase in volatility on the objective risk premium through the expectation error outweighs that through the subjective risk premium.

The estimate of β_2, which captures the effect of lagged volatility expectation on the objective risk premium through investors' delayed adjustment of the subjective volatility expectations, is 0.171 and is statistically significant at 1% level. This result indicates that there is a significant lead-lag relation between the objective risk premium and expected volatility. Moreover, the positive estimate implies that the objective risk premium remains high after the volatility shock subsides, which is not predicted under the rational expectations assumption. The statistically significant estimates of ϕ and β_2 imply a possibility of misspecification problem in the empirical model with the rational expectations. This is evident in the notable improvement in the log-likelihood value of the model that incorporates slow-moving volatility expectations.

The estimate of δ, which measures the contemporaneous effect of the volatility innovation on the realized excess return, is -0.388 and statistically significant at 1% level, suggesting that a positive volatility innovation leads to an immediate decline in the current stock price. Combined with the positive estimate of β_2, the negative estimate δ provides evidence

[9]The estimate of ϕ is greater than the benchmark calibration, 0.5, of Lochstoer and Muir (2022). It is close to their estimate of ϕ using the survey data, 0.85.

in support of the positive subjective risk-return relation. I provide more discussion about the subjective risk-return relation in Section 1.3.3.

Table 1.2: Estimation Results under the Slow-moving Expectation

	Restricted Estimation		**Unrestricted Estimation**	
ϕ	0.793	(0.058)	0.808	(0.054)
β_0	-0.399	(0.201)	-0.486	(0.239)
β_1	-0.129	(0.064)	-0.106	(0.061)
β_2	0.171	(0.075)	0.155	(0.069)
δ	-0.388	(0.098)	-0.370	(0.094)
σ_0^2	8.576	(0.743)	8.671	(0.761)
σ_1^2	31.206	(4.895)	31.706	(4.952)
p_{00}	0.951	(0.012)	0.951	(0.012)
p_{11}	0.713	(0.062)	0.705	(0.065)
lnL	-2115.334		-2114.986	
LR statistic	0.696	(0.404)		

i) The restriction of restricted estimation result is $\delta = -\frac{\kappa\rho}{(1-\kappa\rho)(1-\kappa\phi)}(\beta_1 + \kappa\beta_2)$.

ii) In the parentheses are the standard errors

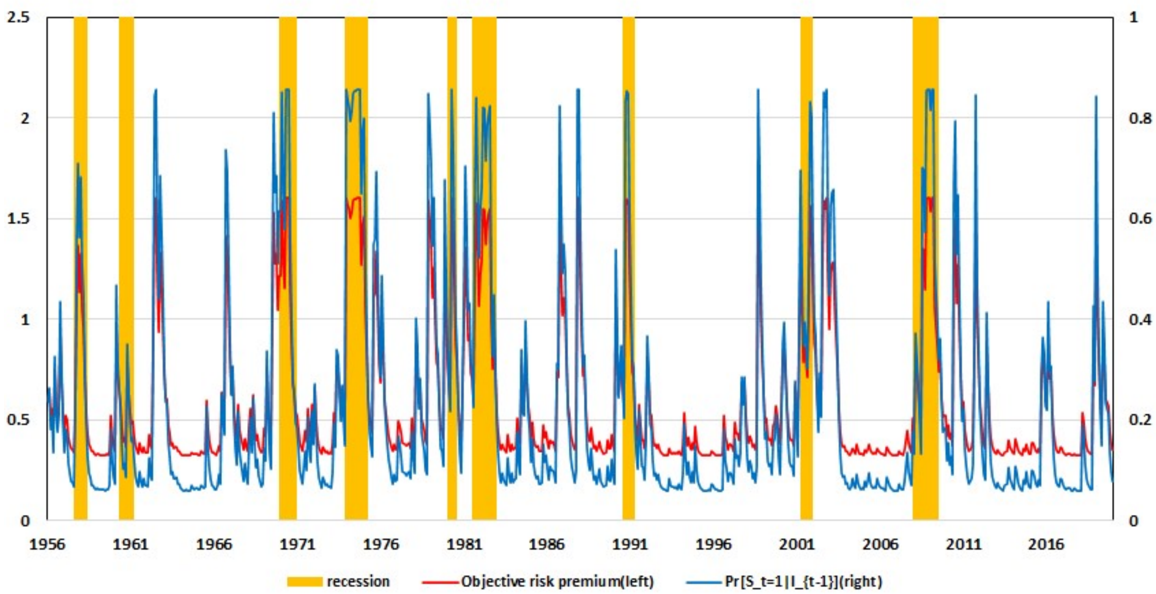
iii) The standard error of δ in the restricted estimation is calculated using the delta method.

iv) LR statistic refers to the likelihood ratio test statistic for the restriction for δ. In the parentheses is the p-value.

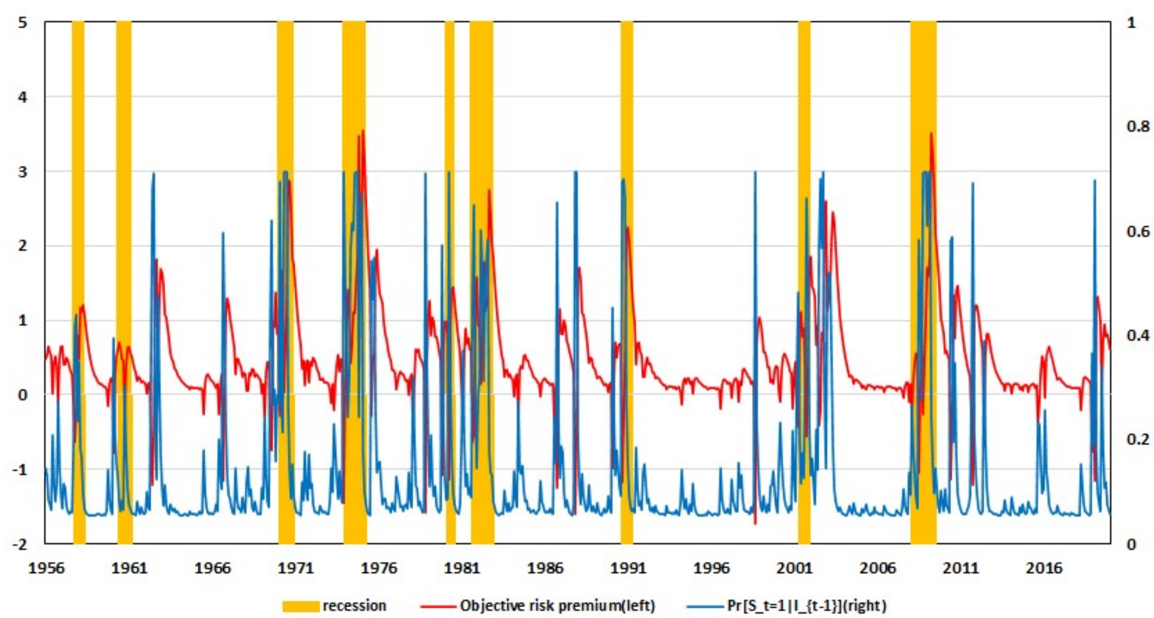
Combining the estimates of ϕ, β_1, and β_2, I examine the unconditional relationship between the objective risk premium and the objective expected volatility (the unconditional objective risk-return relation). Equation (1.12) implies that an expression for the unconditional objective risk-return relationship is $\frac{\beta_1+\beta_2}{(1-\phi)}$. The unconditional risk-return relation implied by the estimates of ϕ, β_1, and β_2 is 0.203 with a standard error of 0.09, indicating an unconditionally positive objective risk-return relationship. This result is consistent with the positive risk-return relation estimated from the model with the rational expectations.

Figure 1.1 presents the estimated objective risk premium ($E_t[r_{t+1}]$) and the conditional probability of being in a high-volatility regime ($Pr[S_{t+1} = 1|I_t]$) in which the NBER recession periods are also overlaid as shaded areas. First, the conditional probability of being in a high-volatility regime seems to capture not only historical financial crises but also economic contraction periods effectively.[10] Generally, the conditional probability of being in a high-volatility regime is high during recessions and low during economic expansions. This countercyclical movement is consistent with the finding that the aggregate stock market volatility is related to macroeconomic factors (e.g., Schwert(1989), Hamilton and Lin (1996), Campbell and Diebold (2009), and Kim and Nelson (2014)). Second, the estimated objective risk premium exhibits countercyclical movement, which has been noted by previous literature (e.g., Fama and French (1989) and Ferson and Harvey (1991)). Generally, it rises during economic contractions, falls during economic expansions, and reaches the highest point near business cycle troughs and its lowest point near peaks. Lastly, I confirm that the slow-moving volatility expectation generates the lead-lag interaction in the objective risk-return relation. Compared to the rational expectations case, it is evident that the volatility leads the objective risk premium. The conditional probability of being in a high-volatility regime rises first, whereas the objective risk premium increases gradually and reaches its peak later than the conditional probability.

[10] For instance, the 1987 stock market crash, the 1998 LTCM crisis and Russian default, and the European debt crisis during the early 2010s.



(a) Model under Rational Expectations



(b) Model under Slow-moving Expectation

Figure 1.1: Estimated Objective Risk Premium and the Conditional Probability of being in a High-Volatility Regime

1.3.3 Estimation Results and Key Assumptions and Predictions in Lochstoer and Muir (2022)

The Positive Subjective Risk-return Relation

Lochstoer and Muir (2022) assume a strong positive subjective risk-return relation. However, studies examining the subjective risk-return relation using survey data report different results. For example, Giglio et al. (2021) examine the relationship between the subjective probabilities of a stock market crash and the subjective expected return and find a significant negative subjective risk-return relation. Jo et al. (2022) investigate the relationship between respondents' overall perception of relative risk and expected return across assets, and find mixed evidence on the subjective risk-return relation; the subjective risk-return relations are significantly negative for all pairs among risky assets, but the relationship becomes positive for a pair of a risky asset and a risk-free asset.[11] On the other hand, Nagel and Xu (2023) document that subjective perceptions of risk measured by subjective return variance or crash risk are positively correlated with subjective excess return expectations.

The signs of β_2 and δ are related to the subjective risk-return relation in the model of this chapter. When there is a positive subjective risk-return relation, δ is expected to be negative, and β_2 is expected to be positive; an increase in volatility raises the subjective required return, which causes a drop in the current stock price ($\delta < 0$), and the delayed adjustment of volatility expectation due to the slow-moving expectation raises the subjective and the objective risk premium in the next period ($\beta_2 > 0$). In contrast, under a negative subjective risk-return relation, β_2 and δ are expected to be negative and positive, respectively. This is because investors react to an increase in volatility by requiring a lower return which leads to a rise in the current stock price ($\delta > 0$), and the delayed adjustment of volatility expectation decreases the subjective required return and objective risk premium in the next period ($\beta_2 < 0$). Thus, the significantly positive estimate of β_2 and the significantly negative estimate of δ (for both restricted and unrestricted estimations) provide empirical support

[11] They consider savings and government bonds as risk-free assets, and real estate, gold, stocks, and cryptocurrency as risky assets.

the positive subjective risk-return relation assumption in Lochostoer and Muir (2022).

Impulse Response of Stock Price to Volatility Innovation

The primary mechanism in the model of Lochstoer and Muir (2022) that generates a weak or negative risk-return relation is that an initial increase in volatility not only causes an immediate decline in the current stock price but also leads to further declines in stock prices in subsequent periods in the short term; the stock price has a hump-shaped response to the innovation to volatility. I show that the model with the slow-moving volatility expectation can generate an impulse response consistent with the main mechanism in their model.

Figure 1.2 illustrates the impulse responses of stock prices to the volatility innovation (v_{t+1}) from the restricted estimation results of the model with slow-moving volatility expectation and the model with the rational expectations.[12]. The impulse responses of the models with the rational expectations (red line) and the model with the slow-moving expectations (blue line) exhibit a clear difference. Under the rational expectations, when there is a positive volatility innovation, the stock price declines at impulse due to the discount rate effect, and it starts to rise from the subsequent periods. On the other hand, under the slow-moving volatility expectations, the impulse response displays a hump-shaped pattern. Similar to the model with the rational expectations, the stock price immediately declines in response to a positive volatility innovation. However, the stock price continues to decline in the short term and starts to rise after three periods of the impulse. It can also be observed that, in the model with slow-moving expectations, it takes a longer period for the effect of the initial volatility increase on the stock price to fully subside.

1.3.4 The Variation of Sharpe Ratio and the Slow-moving Volatility Expectation

The Sharpe ratio is the ratio of the conditional expected excess returns to the conditional standard deviation of excess returns. Because the Sharpe ratio measures the expected excess return per unit of risk, it can be interpreted as a price of the risk for an asset. Generally, the

[12]By assuming that the volatility innovation does not affect the risk-free rate and the dividend growth, I consider the cumulative sum of the impulse response of excess return as the impulse response of the stock price to volatility innovation.

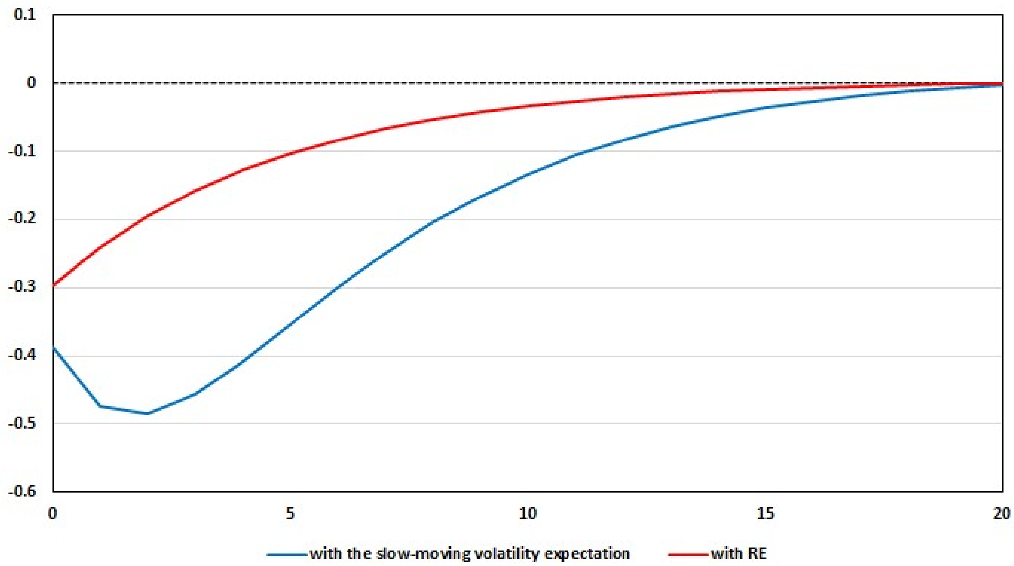


Figure 1.2: Responses of the Stock Price to a Positive Volatility Innovation

estimated Sharpe ratio of the U.S. aggregate stock market exhibits countercyclical movement and substantial time variations (e.g., Ludvigson and Ng (2007), Lettau and Ludvigson (2010), Tang and Whitelaw (2011)).

Because classical asset pricing models with a constant price of risk (e.g., Sharpe (1964) and Lintner (1965)) cannot explain the sizeable countercyclical variation of the Sharpe ratio, literature explains the time-variation of estimated Sharpe ratio with countercyclical risk aversion (e.g., Campbell and Cochrane (1999)) or time-varying consumption volatility (e.g., Bansal and Yaron (2004)). However, Lettau and Ludvigson (2010) show that leading rational expectations asset pricing models in which the price of risk varies over time do not appropriately account for the dynamics of empirically estimated Shape ratio. For example, the Sharpe ratio implied by the model of Campbell and Cochrane (1999) is countercyclical but shows an extremely moderate time-variation compared to the empirically estimated Sharpe ratio, and the model of Bansal and Yaron (2004) produces a Sharpe ratio that is volatile but negatively correlated with the empirically estimated one.[13]

[13]For a quarterly basis, the Sharpe ratio implied by Campbell and Cochrane (1999) model only varies from

Recently, Nagel and Xu (2022) attribute the failure of leading rational expectations asset pricing models to generate a volatile Sharpe ratio to the assumption of a strongly positive risk-return relation. The strong co-movement of the conditional return and volatility dampens the variability of the Sharpe ratio, while uncorrelated or negatively correlated conditional mean and volatility of stock return in data give rise to substantial time variation of the Sharpe ratio. They demonstrate that subjective belief dynamics deviating from rational expectations can generate a volatile Sharpe ratio by weakening the strong positive relationship between the objective expected return and volatility.

Figure 1.3 illustrates the estimated conditional Sharpe ratio from the model with slow-moving volatility expectations (blue line) and that of the model with the rational expectations (red line) in which NBER recession periods are highlighted as shaded areas. The conditional Sharp ratio is higher during economic expansions and lower during recessions in both models. However, the countercyclical pattern is more evident in the model with the slow-moving expectations. In the model with slow-moving expectations, the Sharpe ratio increases during recessions, whereas in the model with the rational expectations, it begins to decrease before the troughs of the business cycle.

The two estimated Sharpe ratios are starkly different in terms of variability. The Sharpe ratio of the model with the slow-moving expectations shows considerable countercyclical variation compared to the model with the rational expectations. Consistent with the finding of Lettau and Ludvigson (2010), the Sharpe ratio of the model with the rational expectations only varies from 0.1 to 0.3. On the other hand, the model with the slow-moving expectation generates the Sharpe ratio ranges from -0.3 to 0.8[14][15]. Additionally, the standard deviation

0 to 0.3. In contrast, the empirically estimated Sharpe ratio varies from -0.45 to 1.76 and its standard deviation is approximately five times larger.

[14]The range of Sharpe ratio is similar to those of previous literature, which estimates Sharpe ratio by conditioning returns and realized variances onto predetermined variables. For example, the monthly Sharpe ratio of Tang and Whitelaw (2011) ranges from -0.2 to 0.9. Additionally, when I convert the monthly Sharpe ratio to a quarterly basis through time aggregation, the range is -0.5 to 1.4, which is comparable to the quarterly Sharpe ratio estimated in Lettau and Ludvigson (2010).

[15]The Sharpe ratio is estimated to be negative sometimes, and this can be attributable to the negative estimates of risk premium. The negative risk premium is also reported in previous literature using the predetermined variables to estimate the risk premium (e.g., Lettau and Ludvigson (2010) and Tang and Whitelaw (2011)). Also, it is not necessarily inconsistent with the asset pricing models in which the covariance of consumption growth and the stochastic discount factor changes over time (e.g., Whitelaw

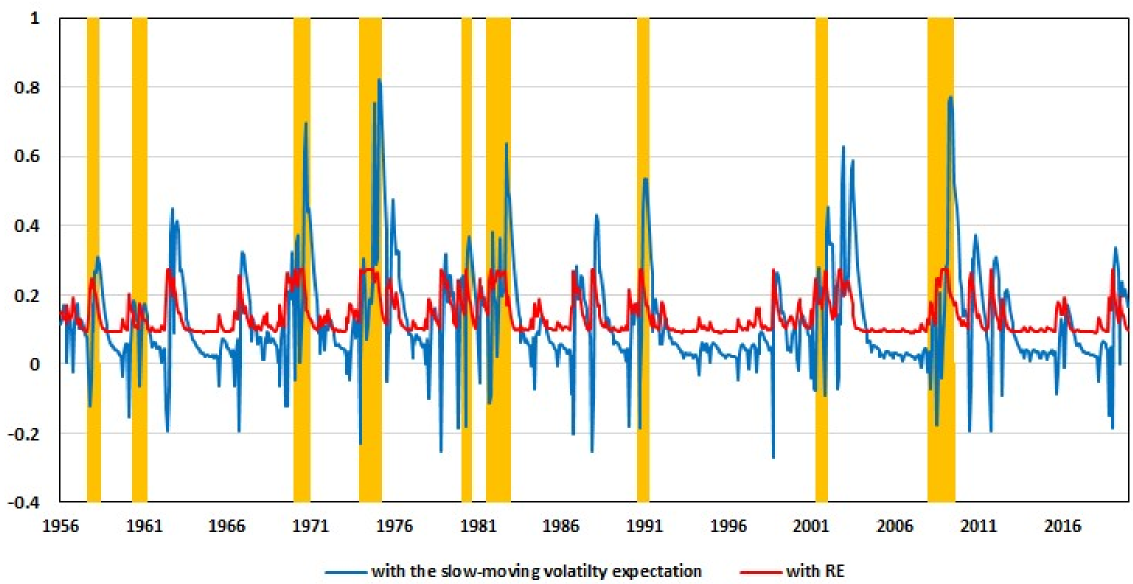


Figure 1.3: Time Variation of Estimated Sharpe Ratio

of the Sharpe ratio in the model with slow-moving expectations (0.15) is approximately three times larger than that in the model with rational expectations (0.05). Therefore, the slow-moving volatility expectations can generate the Sharpe ratio with large countercyclical variations by weakening the positive objective risk-return relation.

1.4 Conclusion

In this chapter, I revisit the risk-return relation by proposing an empirical model incorporating the slow-moving subjective volatility expectations in Lochostoer and Muir (2022). First, I identify the significant slow-moving volatility expectations component from the market return data. Second, I find a significant lead-lag relationship between the objective risk premium and expected volatility, and demonstrate that the objective risk-return relation can be negative, even when the subjective risk-return relation is positive. Third, the estimation results are consistent with the critical assumptions and predictions in the Lochstoer and Muir (2022) model, including the positive subjective risk-return relation and

(2000)).

the hump-shaped response of stock prices to volatility innovation. Finally, by comparing the estimation results of the model with the slow-moving volatility expectation to the model with the rational expectations, I find that incorporating the subjective volatility expectations departing from the rational expectations not only enhances the model's explanatory power but also helps explain the dynamic behavior of the Sharpe ratio.

Chapter 2

AN EMPIRICAL MODEL OF STOCK RETURNS UNDER INFORMATION RIGIDITY: DOSE MARKET RETURN DATA CONTAIN INFORMATION ABOUT INFORMATION RIGIDITY AND SUBJECTIVE RISK-RETURN RELATION?

2.1 Introduction

Full-information rational expectations, which align objective expectations with investors' subjective expectations, have been the bedrock of asset valuation research for the past few decades. However, it has been documented that theoretical asset pricing models based on the full-information rational expectations have limitations in explaining several asset pricing phenomena in the data. One such phenomenon is the empirical risk-return relation (the relationship between the risk premium and the investor's risk perception). Earlier asset pricing models based on the full-information rational expectations predict a positive risk-return relation.[1] However, empirical investigations of the risk-return relation have not yielded consistent results.[2]

Because both expected return and volatility are latent variables for econometricians, the empirical risk-return relation is sensitive to model specifications. One potential source of misspecifications regarding the risk-return relation is the deviation of investors' subjective expectations from the full-information rational expectations, which has been actively discussed in the literature. In contrast to the prediction of the full information rational expectation, a growing body of research using survey data has reported systematic pre-

[1]For example, ICAPM of Merton (1980), the habit model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), and the time-varying disasters model of Wachter (2013).

[2]Some empirical studies find a positive risk-return relation (e.g., French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), Guo and Whitelaw (2006), Pastor et al. (2008), and Nyberg (2012)), whereas many other studies find a weak or negative risk-return relation (e.g., Campbell (1987), Nelson(1991), Glosten, Jagannathan and Runkle (1993), and Brandt and Kang (2004))

dictability of forecast errors in both macroeconomic and financial variables.[3] One finding in the literature on the predictability of forecast errors is that consensus-level forecasts tend to underreact to new information, while individual-level forecasts tend to exhibit overreaction to new information (e.g., Coibion and Gorodnichenko (2015), Bouchaud et al. (2019), Bordalo et al. (2020), and Kohlhas and Walther (2021)).

Alternative models for the formation of economic agents' subjective expectations have been proposed to explain the characteristics observed in survey data on subjective expectations. The underreaction of average forecasts to new information has been explained by models incorporating information rigidity, which deviate from the full-information assumption (e.g., Mankiw and Reis (2002) and Woodford (2003)). For example, in the sticky information model proposed by Mankiw and Reis (2002), agents update their information sets infrequently due to information processing costs. The noisy information model of Woodford (2003) assumes that agents continuously update their information sets but adjust their expectations gradually due to the noisy signals. On the other hand, to account for the overreaction of individual-level forecasts to new information, researchers have focused on deviations from rationality stemming from behavioral biases. One such bias is agents' overextrapolative belief. Under this bias, agents perceive the persistence of a stochastic process to be higher than its true value, leading them to place excessive weight on recent changes when forming future expectations (e.g., Fuster et al. (2010), Angeletos et al. (2021), and Lochstoer and Muir (2022)).[4]

When investors' expectations deviate from full-information rational expectations, a discrepancy arises between the objective and subjective risk-return relation. This discrepancy leads to a misspecification problem in earlier empirical models of the risk-return relation, which were derived under the assumption of full-information rational expectations. Thus,

[3]For macroeconomic variables such as inflation and the unemployment rate, Coibion and Gorodnichenko (2015), Bordalo et al. (2020), and Angeletos et al. (2021) report predictability of forecast errors. Bacchetta et al. (2009), Greenwood and Shleifer (2014), Adam et al. (2017), Bouchaud et al. (2019), and Bordalo et al. (2018) find the predictability of forecast errors in financial variables such as aggregate stock market returns, credit spreads, and corporate earnings.

[4]Literature documents that overconfidence of economic agents also explains the overreaction of individual forecasts. Under overconfidence, economic agents overestimate the accuracy of the information they acquire (e.g., Daniel et al. (1998) and Hirshleifer et al. (2011)).

recent theoretical asset pricing models explicitly account for deviations from full-information rational expectations to explain several asset pricing phenomena, including the empirically weak or negative objective risk-return relation. For example, Lochstoer and Muir (2022) demonstrate that the objective risk-return relation can be weak or negative even under a positive subjective risk-return relation. In their model, investors have slow-moving subjective volatility expectations due to information rigidity and a misperception of the persistence of volatility, leading to a partial adjustment of aggregate subjective volatility expectations and stock prices to changes in volatility. As investors, on average, realize that the volatility is higher (lower) than they had expected, there is an additional decrease (increase) in stock prices in the short term, which induces a weak or negative objective risk-return relation.[5] By distinguishing the subjective and objective risk-return relation, the theoretical models successfully explain the empirically controversial risk-return relation, based on the positive subjective risk-return relation assumption.

However, empirical studies using survey data have yielded inconclusive results on the relationship between subjective return expectations and risk perceptions (the subjective risk-return relation). For the intertemporal subjective risk-return relation, Nagel and Xu (2023) use subjective stock return variance as a proxy for the subjective risk perception and find a significantly positive subjective risk-return relation. In contrast, Giglio et al. (2021) find that the subjective probability of market crash (the survey respondent's subjective probability that the one-year stock return is below -30 percent) is a better measure of the subjective risk perception, and show that it is negatively related to the subjective return expectation. For the cross-sectional subjective risk-return relation, Jensen (2023) finds that investors' subjective expected return is higher for stocks that they subjectively perceive as more risky. Jo et al. (2022) compare subjective expected return and risk perception across several asset classes and find evidence for a negative subjective risk-return relation.[6]

---

[5] Using a model in which a representative agent learns about the payout growth with gradually fading memory, Nagel and Xu (2022) show that the decoupling of objective and subjective expectations leads to a negative correlation between objective risk premium and conditional return variance, which helps explain a large countercyclical variation of the Sharpe ratio estimated from data.

[6] Jo et al. (2022) consider savings and government bonds as risk-free assets, and real estate, gold, stocks, and cryptocurrency as risky assets. They find that the subjective risk-return relation is significantly negative for all pairs among risky assets, but the relationship becomes positive for a pair of a risky asset

Meanwhile, this inconsistent evidence regarding the subjective risk-return relation may be attributable to potential limitations of survey data. Because survey data are usually collected from small samples and specific groups, such as CFOs and professional forecasters, they may not represent the beliefs of all market participants. Moreover, due to survey respondents' confusion, carelessness, or dishonesty, survey data may contain measurement error: the reported belief in surveys is a noisy measure of the survey respondent's true belief, which may cause attenuation bias in the coefficient estimate of empirical studies.[7] In contrast, stock return data observed in the market not only reflects all market participants' expectations but is also free from measurement errors. This motivates me to examine the subjective risk-return relation and the predictions of theoretical models that incorporate subjective expectations deviating from full-information rational expectations using the observed return data.

In this chapter, I develop an empirical model for excess stock returns that can directly estimate the subjective risk-return relation and examine features of investors' subjective expectations from observed stock return data, particularly in cases where subjective expectations deviate from full-information rational expectations. Our empirical model is inspired by the volatility feedback models of Campbell and Hentschel (1992) and Kim, Morley, and Nelson (2004), as it is derived from the present-value approach of Campbell and Shiller (1988), with the risk premium modeled as a function of the volatility of cash flow news. However, the approach in this chapter departs from the rational expectations assumption by incorporating factors that capture key features of subjective expectations observed in survey data. Specifically, to address the underreaction of average forecasts to new information, I introduce information rigidity in the formation of investors' subjective volatility expectations. This is done by adopting the sticky information model of Mankiw and Reis (2002), where only a fraction of investors update their information sets and expectations about future volatility based on new information in each period.

---

and a risk-free asset.

[7]For example, Giglio et al. (2021) find that investors' asset allocation is strongly related to subjective expected return in survey data. However, the sensitivity of portfolio allocation to the subjective expected return is significantly lower than the prediction of frictionless asset pricing models. They show that measurement error in subjective return expectation can partly explain the low sensitivity.

Another deviation I make from the previous literature is in the modeling of time-varying risks. In contrast to the previous literature that extracts information about the time-varying nature of risks from a single observation of squared returns, I utilize the realized volatility measure to capture the time-varying risks. For example, Campbell and Hentschel (1992) and Kim, Morley, and Nelson (2004) assume GARCH and Markov-switching volatility specifications, respectively. However, voluminous literature suggests that the realized volatility measure, constructed from higher frequency data, provides more accurate information about the current level of volatility than a single observation of squared returns. Moreover, incorporating this realized measure into volatility modeling significantly improves forecasting performance. (e.g., Anderson et al. (2003), Dobrev and Szerszen (2010), Hansen et al. (2012), and Christoffersen et al. (2014)). Given these findings, I use the monthly realized volatility, constructed by summing the daily squared stock returns, to model time-varying risk

The main empirical findings, based on U.S. monthly excess stock returns and realized volatility, can be summarized as follows. First, I identify a significant information rigidity component from the excess stock return data, which leads to a substantial enhancement in the model's explanatory power compared to the model under the full-information rational expectations. The estimation result suggests that approximately 40% of investors do not adjust their expectations on future volatility using the new information each month. Second, accounting for the information rigidity in subjective volatility expectations, I find a positive and statistically significant subjective risk-return relation. In contrast to the significantly positive subjective risk-return relation, the parameter estimates indicate a weak objective risk-return relation that is positive but statistically insignificant. The delayed adjustments in volatility expectations and stock prices, driven by information rigidity, play a crucial role in explaining this weak objective risk-return relation. These results align with the theoretical predictions of Lochoster and Muir (2022), which suggest that the objective risk-return relation can be weak or even negative, despite investors subjectively demanding higher returns for higher risks. Third, I find that the initial response of excess stock return to volatility innovation implied by data is larger than that implied by the present-value approach, indicating the potential existence of other factors influencing excess returns that are not considered in the model. I propose that the overextrapolative belief, a behavioral bias used to explain the overreaction of individual-level forecasts in survey data, may account for the larger response observed in the data. Lastly, I find evidence for state-dependent information rigidity. Consistent with the findings of Coibion and Gorodnichenko (2015), the information rigidity in subjective volatility expectation becomes greater during the Great Moderation, a period specified by a substantial decline in macroeconomic volatility since the early or mid-1980s.

The rest of this paper is organized as follows. In Section 2.2, I present assumptions and the formal derivation of an empirical model for excess return under information rigidity. Section 2.3 reports the empirical findings, and I conclude in Section 2.4.

2.2 An Empirical Model of Stock Returns under Information Rigidity in Subjective Expectation on Realized Volatility

To estimate the subjective risk-return relation directly from the observed stock return, it is necessary to derive an analytical expression for the subjective risk premiums and volatility expectations. To do this, I assume that investors use Campbell-Shiller's (1988) present-value framework with their subjective expectations to value stocks, following the approaches of Katz et al. (2017) and Gomez-Cram (2022).

When the subjective expectation is applied to Campbell and Shiller's (1988) present-value decomposition, it can be shown that excess stock returns are decomposed into the subjective risk premium, the subjective volatility feedback effect, and news about cash flow:

$$r_{t+1} = E_t^*(r_{t+1}) + f_{t+1}^* + \varepsilon_{t+1}, \tag{2.1}$$

where r_{t+1} is excess stock returns, $E_t^*(r_{t+1})$ is the subjective risk premium, and f_{t+1}^* is the subjective volatility feedback term that reflects revisions in future expected returns:

$$f_{t+1}^* = -(E_{t+1}^* - E_t^*)\sum_{t=1}^{\infty} \kappa^j r_{t+1+j}, \tag{2.2}$$

where κ is a constant slightly less than 1 that comes from the log-linearization. ε_{t+1} reflects the news about cash flow.[8]

[8] As Campbell (1991) demonstrates, when Campbell and Shiller's (1988) present-value decomposition is

2.2.1 Assumptions of the Model

#1: Volatility of cash flow news and its relation with the realized volatility

Assuming that the aggregate risk in the stock market is captured by the conditional volatility of the return on a market portfolio, I first model the time-varying volatility of the stock return. Specifically, I abstract from the subjectiveness in the investors' expectation of future cash flow and treat news about cash flow ε_{t+1} as an exogenous shock which has time-varying volatility as in Campbell and Hentschel (1992) and Kim, Morley, and Nelson (2004).[9] In modeling the time-varying volatility of the cash flow news, I employ the realized measure of volatility, which is known to be far more informative about the current level of volatility than a single observation of squared return.[10] I assume that the volatility of news about cash flow is a linear function of the realized volatility of excess stock return:

$$\varepsilon_{t+1} \sim N(0, h_{t+1}), \tag{2.3}$$

$$h_{t+1} = \alpha_0 + \alpha_1 x_{t+1}, \quad \alpha_0 > 0,\ \alpha_1 > 0, \tag{2.4}$$

where h_{t+1} is the volatility of cash flow news and x_{t+1} realized volatility.

I assume that the log of realized volatility evolves according to an AR(1) process.

$$lnx_{t+j} = \phi_0 + \phi_1 lnx_{t+j-1} + v_{t+j}, \quad v_{t+j} \sim i.i.d.N(0, \sigma_v^2), \tag{2.5}$$

where ϕ_1 lies between 0 and 1, and the innovation to the log of realized volatility v_{t+j} is assumed to be uncorrelated with ε_{t+j}, i.e., $cov(\varepsilon_{t+j}, v_{t+j}) = 0$. To calculate the forecast of

---

applied to excess returns, ε_{t+1} can be interpreted as news about dividends in addition to news about the risk-free rate. Therefore, news about cash flow in this chapter refers to collective news about dividends and risk-free rates.

[9] In other words, investors use subjective expectations in calculating discount rate news and use full-information rational expectations in calculating cash flow news. I believe this assumption is plausible because volatility is essentially latent, whereas cash flow itself is observable. Therefore, compared to the cash flow, it is more challenging for investors to acquire and incorporate information about the volatility of cash flow into their forecasts. In addition, Katz et al. (2017) document that it is reasonable to assume that stickiness in discount rate expectations may be larger than that in cash flow expectations because forecasting the discount rate involves macro-inflation forecasting, a factor not typically considered by investors when predicting firm-level cash flow.

[10] Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002) show that, under suitable conditions, realized volatility is an unbiased and efficient estimator of return volatility.

realized volatility, I linearize $x_{t+j} = exp(lnx_{t+j})$ around the unconditional mean of lnx_{t+j}, $\frac{\phi_0}{1-\phi_1}$:

$$x_{t+j} = \bar{x} + \phi_1(x_{t+j-1} - \bar{x}) + \bar{x}v_{t+j}, \tag{2.6}$$

where $\bar{x} = exp(\frac{\phi_0}{1-\phi_1})$.

#2: Law of motion for the aggregate subjective volatility expectation

To derive a closed-form expression for the subjective risk premium, it is necessary to specify the form of the subjective expectations. I depart from the assumption of full-information rational expectations by introducing information rigidity into the subjective expectations. I incorporate information rigidity using the sticky information model proposed by Mankiw and Reis (2002), where only a fraction of economic agents update their expectations about the future in each period: a fraction λ of investors do not update their beliefs about future volatility each period, while a fraction $1 - \lambda$ of investors update their beliefs about future volatility using the rational expectations..[11]

In the presence of information rigidity, the law of motion for realized volatility in equation (2.6) implies the following aggregate subjective expectation on the realized volatility:

$$E_t^*(x_{t+j}) = \bar{x} + (1 - \lambda)\phi_1^j \frac{1}{1 - \lambda\phi_1 L}(x_t - \bar{x}), \tag{2.7}$$

where λ, which lies between 0 and 1, represents the degree of information rigidity and L is the lag operator. Then, using the relationship between the volatility of cash flow news and the realized volatility in equation (2.4), the aggregate subjective expectation on the volatility of cash flow news is obtained as follows:[12]

$$E_t^*(h_{t+j}) = \alpha_0 + \alpha_1\bar{x} + (1 - \lambda)\alpha_1\phi_1^j \frac{1}{1 - \lambda\phi_1 L}(x_t - \bar{x}). \tag{2.8}$$

When all investors update their beliefs to the new information each period ($\lambda = 0$), the aggregate subjective expectation coincides with the full-information rational expectations.

[11]The sticky information model of Mankiw and Reis (2002) has been widely used in asset pricing literature on the effect of subjective expectations deviating from the full-information rational expectations, including Katz et al. (2017), Lochstoer and Muir (2022), and Gomez-Cram (2023).

[12]Refer to Appendix B.1 for a detailed derivation.

#3: Subjective risk-return relation

Finally, I assume that the subjective risk premium for a given period $t+j$ is a linear function of the aggregate subjective expectation on the volatility of cash flow news:

$$E_t^*(r_{t+j}) = \gamma_0 + \gamma_1 E_t^*(h_{t+j}), \quad for \quad j = 1, 2, \cdots, \tag{2.9}$$

where the sign of γ_1 indicates the subjective risk-return relation. I adopt the subjective risk premium specification in equation (2.9) following previous literature on the risk-return relation. For example, Campbell and Hentschel (1992) and Kim, Morley, and Nelson (2004) also model the risk premium as a function of the expectation of volatility of cash flow news to estimate the risk-return relation under the full-information rational expectations framework.[13] In addition, Lochstoer and Muir (2022) derive the subjective risk premium as a linear function of the subjective expectation of the volatility of cash flow news in their theoretical model, which incorporates Epstein-Zin preferences and sticky and biased subjective expectations.

2.2.2 Deriving an Empirical Model

Given the assumptions presented in the preceding subsection, I develop an empirical model for excess stock return with the volatility feedback effect by deriving analytical expressions for the components in equation (2.1).

The Subjective Risk Premium as a function of Realized Volatilities

The linear relationship between the subjective risk premium and the aggregate subjective volatility expectation in equation (2.9), combined with the law of motion for the aggregate subjective volatility expectation in equation (2.8), yields the following expression for the

[13] Since there is no difference between the objective and the subjective expectation under the rational expectations assumption, the specification in their models can be also considered as a subjective risk-return relation.

subjective risk premium:

$$E_t^*(r_{t+j}) = \gamma_0 + \gamma_1(\alpha_0 + \alpha_1\bar{x}) + \gamma_1(1-\lambda)\alpha_1\phi_1^j \frac{1}{1-\lambda\phi_1 L}(x_t - \bar{x}), \quad for \quad j = 1, 2, \cdots . \tag{2.10}$$

In equation (2.10), the subjective risk premium depends not only on the most recent realized volatility but also on all past realized volatilities. This implies that the aggregate subjective risk premium adjusts gradually to a change in realized volatility in the presence of information rigidity ($\lambda > 0$). $E_t^*(r_{t+1})$, the first component of equation (2.1), can be obtained by setting $j = 1$ in equation (2.10).

The Subjective Volatility Feedback Effect as a function of Realized Volatilities

The subjective volatility feedback term f_{t+1}^*, the second term in the right-hand side of equation (2.1), can be derived by combining equations (2.2) and (2.10):[14]

$$f_{t+1}^* = \delta\bar{x}v_{t+1} + \delta\frac{\lambda\phi_1}{1-\lambda\phi_1 L}\bar{x}v_t, \tag{2.11}$$

where

$$\delta = -\gamma_1(1-\lambda)\alpha_1\phi_1\frac{\kappa}{1-\kappa\phi_1}. \tag{2.12}$$

Two important points should be noted regarding equation (2.11). First, given that κ is slightly less than 1 and both λ and ϕ_1 are positive but less than 1, the sign of δ (the volatility feedback effect) depends on the sign of γ_1 (the subjective risk-return relation). Thus, a positive (negative) subjective risk-return relation implies a negative (positive) coefficient δ. Second, in contrast to the volatility feedback term in the previous literature assuming the full-information rational expectations, equation (2.11) includes an additional term, $\delta\frac{\lambda\phi_1}{1-\lambda\phi_1 L}\bar{x}v_t$. This additional term represents a delayed volatility feedback effect due to information rigidity, indicating that the current excess return is influenced not only by the unexpected changes in the current period's volatility but also by past unexpected changes in volatility.[15]

[14] Refer to Appendix B.2 for the detailed derivation of equations (2.11) and (2.12)

[15] Note that under the full-information rational expectations where $\lambda = 0$, the second term in the right-hand side of equation (2.11) collapses to zero.

Given a positive subjective risk-return relation ($\gamma_1 > 0$), a positive volatility shock at period t ($v_t > 0$) results not only in a contemporaneous drop in the stock price but also in further declines in the stock price in the near future. To elaborate further, an unexpected increase in volatility during a given period drops the current stock price immediately as the discount rate increases by the upward revision of the aggregate subjective expectation of future volatility. When only a fraction of investors update their beliefs about future volatility ($\lambda > 0$), however, the initial adjustment of aggregate subjective volatility expectation and the stock price is partial. Then, there will be a positive update in the aggregate volatility expectation in the next period even without additional volatility shocks, as investors, on average, realize that the volatility is higher than they had expected. This delayed adjustment of aggregate volatility expectation acts as a positive discount rate shock in the next period, leading to further drops in the stock price. The subsequent decline in stock prices suggests that the initial increase in volatility is associated with lower excess returns in the near future. This delayed price adjustment, caused by the delayed volatility feedback effect, is a key factor contributing to the weak or negative empirical objective risk-return relation.

Implied Objective Risk Premium

To examine how the deviation from the full-information rational expectations affects the objective risk-return relation, I derive an expression for the objective risk premium implied by the model. After substituting equations (2.10) and (2.11) into equation (2.1) and rearranging terms, equation (2.1) can be rewritten as

$$r_{t+1} = \mu_t + \delta \bar{x} v_{t+1} + \varepsilon_{t+1}, \tag{2.13}$$

where $\mu_t \equiv E_t(r_{t+1})$ is the objective risk premium which has the following dynamics:

$$\mu_t = \beta_0 + \lambda \phi_1 \mu_{t-1} + \beta_1 (x_t - \bar{x}) + \lambda \phi_1 \delta \bar{x} v_t, \tag{2.14}$$

where $\beta_0 = (1 - \lambda\phi_1)[\gamma_0 + \gamma_1(\alpha_0 + \alpha_1 \bar{x})]$ and $\beta_1 = \gamma_1(1 - \lambda)\alpha_1\phi_1$.[16]

[16] Refer to Appendix B.3 for a derivation of equations (2.13) and (2.14).

The objective risk-return relation of the model is $(\beta_1 + \lambda\phi_1\delta)\frac{1}{\alpha_1\phi_1}$.[17] Given that $\alpha_1 > 0$ and $0 < \phi_1 < 1$, the objective risk-return relation has the same sign with $\beta_1 + \lambda\phi_1\delta$. The direction of objective risk-return relation is determined by two parts: the subjective risk-return relation and the delayed subjective volatility feedback effect. If there is a positive subjective risk-return relation ($\gamma_1 > 0$), an increase in realized volatility at time t has a positive effect on the objective risk premium because it decreases the stock price at period t through the discount rate effect, and $\beta_1 > 0$ reflects this effect. Meanwhile, the delayed volatility feedback due to information rigidity works in the opposite direction ($\lambda\phi_1\delta < 0$), as it causes a further decline in the stock price at period $t+1$, which has a negative effect on the objective risk premium. Therefore, the delayed subjective volatility feedback effect may mask the effect of positive subjective risk-return relation ($\beta_1 > 0$), and the sign of objective risk-return relation, $\beta_1 + \lambda\phi_1\delta$, can be either positive or negative.[18]

The Volatility Feedback Model of Stock Returns in the presence of Information Rigidity

Finally, by combining equations for excess return and the objective risk premium in equations (2.13) and (2.14) with equations for volatility outlined in equations from (2.3) to (2.5), I obtain

$$
\begin{aligned}
r_{t+1} &= \mu_t + \delta\bar{x}v_{t+1} + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, h_{t+1}) \\
\mu_t &= \beta_0 + \lambda\phi_1\mu_{t-1} + \beta_1(x_t - \bar{x}) + \lambda\phi_1\delta\bar{x}v_t \\
h_{t+1} &= \alpha_0 + \alpha_1 x_{t+1} \\
lnx_{t+1} &= \phi_0 + \phi_1 lnx_t + v_{t+1}, \quad v_{t+1} \sim i.i.d.N(0, \sigma_v^2), \qquad (2.15)
\end{aligned}
$$

where $\beta_0 = (1-\lambda\phi_1)[\gamma_0 + \gamma_1(\alpha_0 + \alpha_1\bar{x})]$, $\beta_1 = \gamma_1(1-\lambda)\alpha_1\phi_1$, and $\delta = -\gamma_1(1-\lambda)\alpha_1\phi_1\frac{\kappa}{1-\kappa\phi_1}$. Equation (2.15) represents an empirical model of stock return and realized volatility accounting for volatility feedback effect and information rigidity. Notably, the model in equation

[17] From equations (2.4), (2.6) and (2.14), we have $\frac{\partial E_t(r_{t+1})}{\partial x_t} = \beta_1 + \lambda\phi_1\delta$ and $\frac{\partial E_t(h_{t+1})}{\partial x_t} = \alpha_1\phi_1$. Then, the objective risk-return relation $\frac{\partial E_t(r_{t+1})}{\partial E_t(h_{t+1})} = \frac{\partial E_t(r_{t+1})}{\partial x_t}\frac{\partial x_t}{\partial E_t(h_{t+1})} = (\beta_1 + \lambda\phi_1\delta)\frac{1}{\alpha_1\phi_1}$.

[18] The mechanism generating the weak or negative objective risk-return relation in the model of this chapter is consistent with that of Lochstoer and Muir (2022). In their theoretical model, investors' sticky and biased volatility expectation leads to delayed adjustments of stock price which generates the weak or negative objective risk-return relation.

(2.15) reduces to the volatility feedback model under the full-information rational expectations model when λ is set to zero.

2.3 Empirical Findings

2.3.1 Data Description

I utilize monthly excess stock market returns (r_t) and realized volatility (x_t) data spanning from January 1952 to December 2019. These monthly data are constructed from daily excess stock market returns. The daily stock market return represents the value-weighted portfolio return of all NYSE, AMEX, and NASDAQ listed stocks, including both capital gains and dividend yields. The risk-free rate is a one-month U.S. Treasury bill rate. Because the daily risk-free rates are not available, following Guo and Whitelaw (2006) and others, I assume that the risk-free rate is constant within each month and compute the daily risk-free rate by dividing monthly observations by the number of trading days in the respective month. The daily excess stock market return is the daily stock market return in excess of the daily risk-free rate.

The monthly excess stock market return is calculated as the sum of daily excess stock returns, and the monthly realized volatility is defined as the sum of squared daily excess stock market returns within a month:

$$x_t = \sum_{k=1}^{\tau_t} r_{t,k}^2, \tag{2.16}$$

where $r_{t,k}$ is the excess stock return of day k in month t and τ_t is the number of trading days in month t.[19] Depending on the researcher, the realized volatility is often defined as the sum of squared deviations from the within-month mean of daily excess returns. However, I do not subtract the mean from each daily return because adjusting the realized volatility for the within-month mean excess return does not significantly affect the results.

The sample period begins in January 1952, considering the extraordinary impact of events such as World War II (which ended in 1945) and the pegging of Treasury bill interest

[19]Following Campbell et al. (2001) and Guo (2002), I replace the realized volatility of October 1987 with the largest realized volatility observed in the preceding sample period to mitigate the potential distortion caused by the one-time event of the market crash in October 1987.

rates prior to the Treasury-Fed Accord in 1951. I conclude the sample period in December 2019 to avoid any potential influence from the recent pandemic on estimation results. The data are collected from the Center for Research in Securities Prices (CRSP) and the Board of Governors of the Federal Reserve System.

2.3.2 Estimation Results and Implications

For model estimation, I employ the Kalman filter and the maximum likelihood estimation method by casting the model in equation (2.15) into the following state-space model:

Measurement Equation

$$\begin{bmatrix} r_{t+1} \\ lnx_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \delta\bar{x} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mu_t \\ \varepsilon_{t+1} \\ lnx_{t+1} \\ v_{t+1} \end{bmatrix}. \tag{2.17}$$

$$(\tilde{y}_{t+1} = H\eta_{t+1})$$

Transition Equation

$$\begin{bmatrix} \mu_t \\ \varepsilon_{t+1} \\ lnx_{t+1} \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1(x_t - \bar{x}) \\ 0 \\ \phi_0 \\ 0 \end{bmatrix} + \begin{bmatrix} \lambda\phi_1 & 0 & 0 & \lambda\phi_1\delta\bar{x} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \phi & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \varepsilon_t \\ lnx_t \\ v_t \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1} \\ v_{t+1} \end{bmatrix}, \tag{2.18}$$

$$(\eta_{t+1} = A_t + F\eta_t + Ce_{t+1}, \quad e_{t+1} \sim i.i.d.N(0, \Omega_{t+1}))$$

where $\Omega_{t+1} = \begin{bmatrix} \alpha_0 + \alpha_1 x_{t+1} & 0 \\ 0 & \sigma_v^2 \end{bmatrix}$.

Table 2.1 summarizes the estimation results for the model with information rigidity and the model with full-information rational expectations, where λ is set to zero.

First, I identify a statistically significant information rigidity component from excess market return data. The estimate of λ, the parameter representing the degree of information rigidity, is 0.41 and is statistically significant at the 1% level. This implies that approximately 40% of investors do not update their expectations of future volatility each month. Consistent with this result, accounting for information rigidity in subjective volatility expectations helps explain the variation in stock returns. The likelihood ratio statistic for the null hypothesis of $\lambda = 0$ is 7.10 with a p-value less than 1%, suggesting that ignoring information rigidity may induce a misspecification problem.

Table 2.1: Estimation Results of with and without Information Rigidity

	Information Rigidity ($\lambda > 0$)	**Rational Expectations** ($\lambda = 0$)
λ	0.41 (0.14)	
γ_0	0.00 (0.00)	0.00 (0.00)
γ_1	9.04 (2.61)	4.82 (0.94)
α_0	0.00 (0.00)	0.00 (0.00)
α_1	1.00 (0.09)	1.00 (0.09)
ϕ_0	-2.12 (0.18)	-2.12 (0.17)
ϕ_1	0.69 (0.03)	0.69 (0.03)
σ_v	0.67 (0.02)	0.67 (0.02)
δ	-11.96 (1.55)	-10.77 (1.49)
$\beta_1 + \lambda\phi_1\delta$	0.35 (1.30)	
lnL	762.06	758.51
LR statistic	7.10 (<0.01)	

i) In the parentheses are the standard errors

ii) $\delta = -\gamma_1(1-\lambda)\alpha_1\phi_1\frac{\kappa}{1-\kappa\phi_1}$ and $\beta_1 = \gamma_1(1-\lambda)\alpha_1\phi_1$. The standard errors of δ and $\beta_1 + \lambda\phi_1\delta$ are calculated using the delta method.

iii) LR stat refers to the likelihood ratio test statistic for the null hypothesis $\lambda = 0$ with the p-value in the parentheses.

Second, in the model with information rigidity, the estimate of γ_1 is 9.04 and is statistically significant at the 1% level. This significantly positive estimate of γ_1 indicates a positive subjective risk-return relation. In the model with full-information rational expectations ($\lambda = 0$), the estimate is 4.82 and is also statistically significant at all conventional levels, which is consistent with the findings of Kim, Morley, and Nelson (2004).[20] The lower magnitude of γ_1 estimate under the full-information rational expectations may be attributed to the omitted variable bias. The objective risk premium in equation (2.14) reveals that the terms $\lambda\phi_1\mu_{t-1}$ and $\lambda\phi_1\delta\bar{x}v_t$ are omitted in the risk-return relation under full-information rational expectations ($\lambda = 0$). The omitted variable bias depends on the correlation between x_t and the omitted variables, μ_{t-1} and v_t, as well as the coefficients of those variables. The positive λ and negative δ estimates suggest a downward bias in the γ_1 estimate in the model under the full-information rational expectations.

Third, in both models, δ $(= -\gamma_1(1-\lambda)\alpha_1\phi_1\frac{\kappa}{1-\kappa\phi_1})$, which represents the volatility feedback effect, are estimated to be significantly negative. It is -11.96 in the model with information rigidity and -10.77 in the model with the full-information rational expectations. These results confirm a stylized fact about the aggregate stock return: excess returns and volatility innovations are negatively correlated.

Lastly, consistent with the theoretical prediction of Lochstoer and Muir (2022), the parameter estimates indicate that the objective risk-return relation can be weak or negative due to information rigidity, even when the subjective risk-return relation is significantly positive. While positive γ_1 implies the positive subjective risk-return relation, the objective risk-return relation is weak in the models with information rigidity. The sign of objective risk-return relation under information rigidity is consistent with $\beta_1 + \lambda\phi_1\delta$: β_1 $(= \gamma_1(1-\lambda)\alpha_1\phi_1)$ reflects the effect of positive subjective risk-return relation, and $\lambda\phi_1\delta$ reflects the delayed volatility feedback effect (delayed adjustment of stock price) on the objective risk-return relation. The estimate of $\beta_1 + \lambda\phi_1\delta$ implied by the parameter estimates is 0.35 with a standard error of 1.30, indicating a positive but statistically insignificant objective risk-return relation.

[20] Kim, Morley, and Nelson (2004) show that considering the volatility feedback effect in estimating the risk-return relation can provide a more effective way to capture the true sign of the risk-return relation.

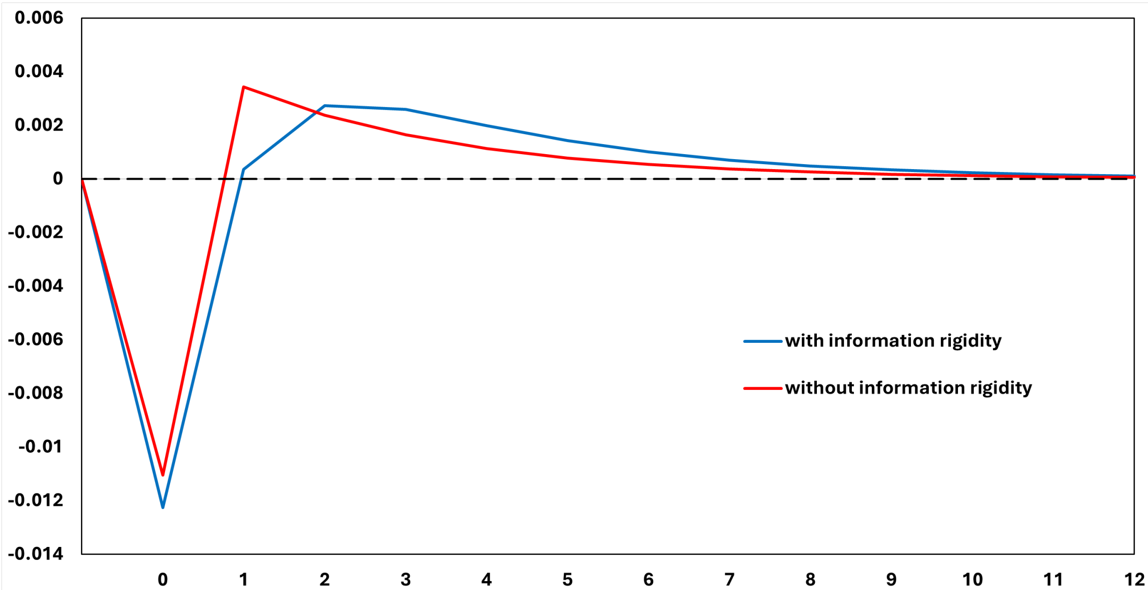


Figure 2.1: Impulse response of the excess return to a Positive innovation to Realized Volatility

The influence of the delayed volatility feedback effect on stock prices can also be confirmed through an impulse response analysis. The impulse responses of excess return to innovations in log realized volatility ($\frac{\partial r_{r+1+j}}{\partial v_{t+1}}$) for both models are illustrated in Figure 2.1. Reflecting the positive subjective risk-return relation, the immediate effect of an increase in the realized volatility on the excess returns (when $j = 0$) is negative for both models, indicating that an increase in volatility during a given period decreases the current stock price immediately. However, the impulse responses for $j \geq 1$ reveal a distinct difference between the two models. Under the full information rational expectations (illustrated by the red line), the impulse response of excess returns turns significantly positive, suggesting that stock price starts to rise significantly in the following period because all investors have already updated their expectations. In contrast, under information rigidity (illustrated by the blue line), the impulse response is nearly zero at $j = 1$, but becomes highly positive in later periods, suggesting that stock prices do not increase much in the next period. In the presence of information rigidity, investors who did not update their volatility expectations perceive a positive discount rate shock in the next period, which partly masks the increase in stock price at $j = 1$.

The estimation results indicate the significant information rigidity in investors' volatility expectations and demonstrate that accounting for information rigidity helps identify the positive subjective risk-return relation. I can also identify the wedge between the positive subjective risk-return relation and the weak objective risk-return relation. However, because these results are derived under the restriction on the feedback parameter δ, I cannot be certain that the model fully captures the information embedded in market return data regarding factors that influence stock price determination. Thus, to examine whether market return data provide additional information about other factors influencing stock valuation, I estimate the model without imposing the restriction (unrestricted estimation) in equation (2.12). The unrestricted estimation can be done by adding an additional parameter to equation (2.12):

$$\delta = -\gamma_1(1-\lambda)\alpha_1\phi_1\frac{\kappa}{1-\kappa\phi_1} + \delta_2, \tag{2.19}$$

where δ_2 is the additional parameter to be estimated and indicates that stock return data provide additional information about stock price determination when it is significantly different from zero.

Table 2.2 presents the unrestricted estimation results of both models. I generally find qualitatively similar results to the restricted estimation. Information rigidity in volatility expectation (λ) is significantly identified from the data. I find a significantly positive subjective risk-return relation (γ_1) and a negative but statistically insignificant objective risk-return relation ($\beta_1 + \lambda\phi_1\delta$).

However, the estimate of δ_2 is -9.89 and is statistically significant at 1% level, and the likelihood ratio statistic for the null hypothesis of $\delta_2 = 0$ is 13.45 with a P-value value less than 1%. This result indicates that the restriction in our model is not fully supported by data and thus suggests that market return data contain other information about stock valuation. The estimate of δ ($= -\gamma_1(1-\lambda)\alpha_1\phi_1\frac{\kappa}{1-\kappa\phi_1} + \delta_2$) is -15.99 and is larger in magnitude than that of the restricted estimation -11.96. In other words, there would exist other factors that could magnify the contemporaneous reaction of excess return (or stock price) to the volatility innovation (the volatility feedback effect).

One such factor not considered in the model is the investors' overextrapolative belief about the persistence of volatility: the perceived persistence of a stochastic process by investors is higher than the actual persistence (e.g., Lochostoer and Muir (2022) and Afrouzi et al. (2023)).[21] Intuitively, when investors perceive the persistence of realized volatility to be greater than the actual persistence (ϕ_1), any unexpected change in volatility will have a prolonged impact on their expectations. Consequently, investors anticipate that future volatility will remain elevated for an extended period following a positive innovation in volatility, leading to a greater increase in discount rates and a sharper decline in current stock prices.

2.3.3 Sub-sample Analysis: Variation of the Degree of Information Rigidity

I investigate whether the degree of information rigidity in subjective volatility expectations varies with prevailing economic conditions. By subsample analyses, I provide evidence of state-dependent information rigidity, consistent with previous studies using survey data.

The literature on information rigidity points out that the degree of information rigidity (λ) may vary in response to changes in economic conditions, as incentives for agents to collect and incorporate additional information into their expectation change. Intuitively, agents may be less attentive to new information during relatively tranquil times, when macroeconomic volatility declines. Consequently, there should be a higher degree of information rigidity in such periods. One such change in economic conditions documented in the literature is the onset of the Great Moderation, a period characterized by a substantial decline in macroeconomic volatility since the early or mid-1980s. Consistent with this anticipation, Coibion and Gorodnichenko (2015) find that the degree of information rigidity in inflation expectations, extracted from survey data, increased from the mid-1980s to the

---

[21] Empirical studies also find evidence for the overextrapolative belief in the economic agents' perceived persistence. For example, Lochostoer and Muir (2022) find that the persistence of stock return volatility implied by CFO survey data (0.96) is much higher than that of realized variance (0.71). In an experimental setting, Afrouzi et al. (2023) find that the bias becomes severe when the underlying process is less persistent.

Table 2.2: Estimation Results of the Model without imposing the Restriction on δ

	Information Rigidity ($\lambda > 0$)		**Rational Expectations** ($\lambda = 0$)	
λ	0.41	(0.14)		
γ_0	0.00	(0.00)	0.01	(0.00)
γ_1	5.13	(2.85)	0.78	(1.55)
α_0	0.00	(0.00)	0.00	(0.00)
α_1	0.91	(0.09)	0.92	(0.09)
ϕ_0	-2.12	(0.17)	-2.12	(0.17)
ϕ_1	0.69	(0.03)	0.69	(0.03)
σ_v	0.67	(0.02)	0.67	(0.02)
δ_2	-9.89	(3.14)	-13.04	(3.89)
δ	-15.99	(2.25)	-14.63	(2.11)
$\beta_1 + \lambda\phi_1\delta$	-2.65	(1.60)		
lnL	768.78		765.54	
LR statistic	13.45	(<0.01)	14.18	(<0.01)

i) In the parentheses are the standard errors

ii) $\delta = -\gamma_1(1-\lambda)\alpha_1\phi_1\frac{\kappa}{1-\kappa\phi_1} + \delta_2$.

iii) The standard errors of δ and $\beta_1 + \lambda\phi_1\delta$ are calculated using the delta method.

iv) LR stat refers to the likelihood ratio test statistic for the null hypothesis $\delta_2 = 0$ with the p-value in the parentheses.

mid-2000s, but declined after 2008.[22]

I examine whether the decline in macroeconomic volatility during the Great Moderation affects the degree of information rigidity in subjective volatility expectations in the stock market. First, I use a dummy variable, D_t, which is assigned as 1 from January 1984 to December 2007, and 0 for all other periods. Then the state-dependent degree of information

[22] Hur (2018) and Mertens and Nason (2020) also find similar time-varying patterns of information rigidity in inflation expectation, corroborating findings in Coibion and Gorodnichenko (2015).

rigidity λ_{D_t} is defined as

$$\lambda_{D_t} = \lambda_0 + \theta_\lambda D_t,$$

where λ_0 represents the degree of information rigidity outside the Great Moderation period, and θ_λ captures the change in the degree of information rigidity during the Great Moderation period.

Table 2.3: Estimation Results of the Model Employing a the Great Moderation Dummy

	Restricted Estimation		**Unrestricted Estimation**	
λ_0	0.34	(0.15)	0.33	(0.15)
θ_λ	0.19	(0.14)	0.24	(0.19)
γ_0	0.00	(0.00)	0.01	(0.00)
γ_1	8.77	(2.42)	4.90	(2.62)
α_0	0.00	(0.00)	0.00	(0.00)
α_1	1.00	(0.09)	0.91	(0.09)
ϕ_0	-2.12	(0.18)	-2.12	(0.17)
ϕ_1	0.69	(0.03)	0.69	(0.03)
σ_v	0.67	(0.02)	0.67	(0.02)
δ_2			-9.97	(3.05)

i) In the parentheses are the standard errors

Table 2.3 reports the restricted and unrestricted estimation results of the model with the Great Moderation dummy variable D_t. Consistent with the findings of previous literature about the macroeconomic variable forecast, I find that the degree of information rigidity in subjective volatility expectations rises during the Great Moderation period. The degree of information rigidity outside the Great Moderation period, λ_0 is estimated to be $0.33 \sim 0.34$, and it increases to $0.54 \sim 0.57$ during the Great Moderation period. The increase in degree of information rigidity is marginally significant. Under $H_0 : \theta_\lambda = 0$ and $H_1 : \theta_\lambda > 0$,

θ_λ estimates are statistically significant at 10% levels in both restricted and unrestricted estimation.

I further explore the variation of λ through a rolling analysis and also find evidence supporting the increase in information rigidity during the Great Moderation.[23] Figure 2.2 displays the estimated λ over a fixed rolling window of 25 years with the horizontal axis representing the end of the estimation sample. I obtain similar patterns of the degree of information rigidity in both restricted and unrestricted estimations. The λ estimate is low during the 1970s and begins to rise as the sample approaches the 1980s. It exhibits a higher level from the mid-1980s through the 1990s, and starts to decline during the 2000s. The decline in information rigidity in the early 2000s coincides with the burst of the dot-com bubble, and the decline around the mid-2000s coincides with the onset of the Great Recession, both of which are periods with increased macroeconomic volatility.

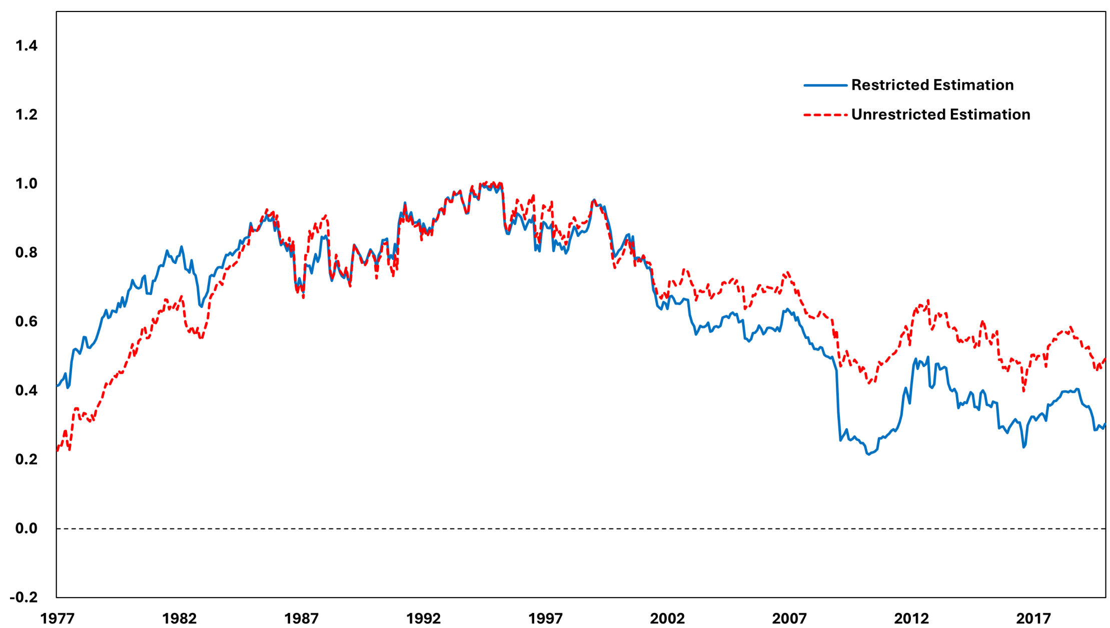


Figure 2.2: Time-variation of the Degree of Information Rigidity

[23] To focus on how λ estimate varies for different sample periods, I fix ϕ_0, ϕ_1, as σ_v as the full sample estimates and estimate other parameters.

2.4 Conclusion

This chapter proposes an empirical model that can directly estimate the subjective risk-return relation from observed excess return data, accounting for deviations in investors' subjective volatility expectations from full-information rational expectations. The model incorporates the sticky information framework proposed by Mankiw and Reis (2002) with the volatility feedback models developed by Campbell and Hentschel (1992), and Kim, Morley, and Nelson (2004), and utilizes realized volatility to capture the time-varying nature of volatility. I empirically identify a positive subjective risk-return relation as well as information rigidity from monthly excess stock return data. The empirical results suggest that objective risk-return relation can be estimated to be weak or negative, even when subjective risk-return relation is significantly positive. I also observe the potential presence of overextrapolative beliefs regarding the persistence of volatility and find evidence that the degree of information rigidity increases during the Great Moderation.

Chapter 3

STATE-DEPENDENT RELATIVE IMPORTANCE OF THE DISCOUNT RATE NEWS AND THE CASH FLOW NEWS IN EXPLAINING STOCK RETURN VARIATIONS

3.1 Introduction

Because asset price is equivalent to the discounted sum of cash flows, discount rates and cash flows are the two fundamental components of asset valuation. Consequently, investigating which of the two components is more influential in explaining asset price variation has become crucial for understanding how the financial market operates and for theoretical asset pricing modeling.[1] Campbell and Shiller's (1988) log-linearize approximation framework sheds light on investigating the relative importance of cash flows and discount rates by showing that unexpected stock returns can be decomposed into news about cash flows (NCF) and news about discount rates (NDR).

Researchers have employed the framework of Campbell and Shiller (1988) to study the influence of NDR and NCF on the stock return variation. One popular approach employed in previous literature is the VAR-based decomposition method introduced by Campbell (1991) and Campbell and Ammer (1993). In their approach, the expected return and NDR are directly modeled via VAR, and NCF is extracted as the residual. Then, stock return variance can be decomposed into the variances of each news and the relevant covariances. Binsbergen and Koijen (2010), and Choi et al. (2017) propose a latent variable approach in which expected returns and expected dividend growth are modeled as latent, based on Campbell and Shiller's (1988) framework.

In general, for the postwar periods, the literature emphasizes that the news about future discount rates plays a more important role than the news about future cash flows in explain-

[1] For instance, Campbell and Cochrane (1999) put more attention on the discount rate side by modeling the time-varying risk aversion, while Bansal and Yaron (2004) focus more on the cash flow side by introducing the long-run risk.

ing U.S. stock return variation. Meanwhile, the literature on stock return predictability has provided evidence for parameter instability, suggesting that the parameters (and therefore the stock return predictability) may vary over time. For example, Ang and Bakaert (2007) show that coefficients of predictive regression for stock return change according to the choice of the sample period. Pastor and Stambaugh (2001) and Kim et al. (2005) find evidence for structural breaks in coefficients of predictive regression, and Dangl and Halling (2012) show that models with time-varying coefficients dominate models with constant coefficients. In addition, the literature highlights that the VAR-based variance decomposition approach has a limitation: its results are sensitive to the choice of sample period and state variables (e.g., Chen and Zhao (2009)). Because the variance decomposition result depends on the parameter estimates of model, such as coefficients and variances, the previous empirical evidence for the parameter instability implies that the relative importance of NDR and NCF can vary within and across the sample period. Therefore, previous empirical results that do not account for potential parameter variations may provide misleading conclusions about the relative importance of NDR and NCF in explaining stock return variation.

In this chapter, I examine the variations in the relative importance of NCF and NDR that may arise from changes in the economic and financial market environment. For this purpose, I consider two VAR models in which coefficients and variances are allowed to vary. The first model is the threshold VAR (TVAR) model proposed by Alessandri and Mumtaz (2017), in which the coefficients and variances shift across stock market regimes identified by a threshold variable representing market environments. I consider two threshold variables: the Chicago Fed's Financial Conditions Index to capture financial market tightness, and the Investor Sentiment Index constructed by Baker and Wurgler (2006) to gauge overall sentiment in the financial market. The second model is the time-varying parameter VAR with stochastic volatilities (TVP-VAR-SV) proposed by Primiceri (2005), in which the coefficients and variances evolve according to random walk processes. Bayesian approach is employed to estimate the posterior distribution of parameters of both models. Then, using the posterior mean of parameters, the variance of unexpected stock returns in each regime or in each period is decomposed into the variances of NCF and NDR, along with their relevant covariances, following Campbell (1991) and Campbell and Ammer (1993). The relative importance of NCF and NDR is assessed by their variances relative to the variance of unexpected return.

Following Campbell and Ammer (1993), the excess stock return, the real interest rate, the dividend-price ratio, the relative bill rate, the change in bill rate, and the term spread are used as state variables. The threshold variables are used as additional predictors for the stock return in TVAR. The estimation results of both VAR models for the postwar period indicate that the relative importance of NCF and NDR exhibits apparent regime dependency and time variation. First, when I use the financial condition index as a threshold variable of TVAR, the contribution of NDR in explaining the excess stock return variance is greater than NCF in the tight financial condition regime, while NCF contributes similarly or more to the variance of excess stock returns than NDR in the loose financial condition regime. Second, when the investor sentiment index is employed as a threshold variable, the contribution of NDR is higher than that of NCF in the high sentiment regime, while this pattern reverses in the low sentiment regime. Lastly, the variance decomposition result using TVP-VAR-SV indicates that NCF has become relatively more important than NDR since the early 1990s. Before the 1990s, NDR generally explains most of the variations of unexpected excess return. The share of the variance of NDR begins to decline, while that of NCF starts to increase from the 1980s, leading to the reversal of the relative importance of NCF and NDR in the early 1990s. Interestingly, this reversal occurred during the period known as the Great Moderation, characterized by declining macroeconomic volatility since the mid-1980s.

I propose potential explanations for the variance decomposition results. First, the change in attention allocation of investors can explain the increase in the importance of NDR during the tight financial condition regime. Investors are more likely to pay attention to information about aggregate shocks rather than information about idiosyncratic shocks during bad times such as recessions and financial crises. Because information about aggregate shocks is more likely to affect the discount rates, NDR becomes relatively more important in the tight financial condition regime. Second, the asymmetric stock return predictability across the low and high sentiment regimes may explain the greater contribution of NDR in the high investor sentiment regime. In the presence of unsophisticated investors, stock prices may deviate from those implied by fundamentals. Due to the increased participation and activeness of unsophisticated investors during high sentiment periods, mispricing is more prevalent during high sentiment periods. This more prevalent and substantial mispricing during the high sentiment regime provides additional stock return predictability and contributes to the increased relative importance of NDR. Lastly, the reversal in the relative importance of NCF and NDR in the early 1990s may be related to increased information rigidity in the expectations of economic agents following the onset of Great Moderation (e.g., Coibion and Gorodnichenko (2015)). With the decreased macroeconomic volatility, investors tend to be less attentive to new information and revise their expectations less frequently. This leads to a less volatile NDR, which results in a relatively lower contribution of NDR to the variance of unexpected excess stock returns.

The rest of the chapter is organized as follows. Section 3.2 presents how the unexpected stock return is decomposed by using TVAR and TVP-VAR-SV models. Section 3.3 presents the variance decomposition results and offers possible explanations for the findings. Section 3.4 concludes.

3.2 Stock Return Variance Decomposition using TVAR and TVP-VAR-SV

3.2.1 VAR Based Variance Decomposition in Campbell and Ammer (1993)

According to the present value model of Campbell and Shiller (1988), the unexpected excess stock return can be expressed in terms of revision of the expected future dividend growth, real interest rate, and future excess stock return.

$$e_{t+1} - E_t(e_{t+1}) = (E_{t+1} - E_t)\left[\sum_{j=0}^{\infty}\rho^j \Delta d_{t+1+j} - \sum_{j=0}^{\infty}\rho^j r_{t+1+j} - \sum_{j=1}^{\infty}\rho^j e_{t+1+j}\right], \tag{3.1}$$

where, e_{t+1} is the excess stock return, Δd_{t+1} is the dividend growth rate, and r_{t+1} is the real interest rate. E_t denotes the rational expectations conditional on all the information up to period t. The decomposition in equation (3.1) can be rewritten as

$$\eta_{t+1} = \eta_{d,t+1} - \eta_{r,t+1} - \eta_{e,t+1}, \tag{3.2}$$

where, $\eta_{t+1} \equiv e_{,t+1} - E_t(e_{t+1})$ is the unexpected excess return, $\eta_{d,t+1} \equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j}$ depicts news about future dividend, $\eta_{r,t+1} \equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r_{t+1+j}$ depicts news about future interest rate, and $\eta_{e,t+1} \equiv (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j e_{t+1+j}$ represents news about discount rate.

Equation (3.2) indicates that the variance of the unexpected excess stock return can be decomposed into the sum of variances of three news and the relevant covariances.

$$\begin{aligned} Var(\eta_{t+1}) =& Var(\eta_{d,t+1}) + Var(\eta_{r,t+1}) + Var(\eta_{e,t+1}) \\ & - 2Cov(\eta_{d,t+1}, \eta_{r,t+1}) - 2Cov(\eta_{d,t+1}, \eta_{e,t+1}) + 2Cov(\eta_{r,t+1}, \eta_{e,t+1}). \end{aligned} \tag{3.3}$$

To obtain each element of the right-hand side of equation (3.2), it is necessary to calculate the discounted sum of the revisions in expectations. Campbell and Ammer (1993) propose a method for deriving the revisions in expectations based on a reduced-form VAR model. In particular, they assume a first-order VAR specification for a vector of state variables, suppressing the constant term, as follows:

$$Z_{t+1} = \Gamma Z_t + u_{t+1}, \quad u_{t+1} \sim N(0, \Omega) \tag{3.4}$$

where Z_{t+1} is an $n \times 1$ vector of state variables with the excess stock return and interest rate as its first and second elements, respectively. Then, j-step ahead prediction of the excess stock return and the interest rate at period t are

$$E_t e_{t+1+j} = s_1' \Gamma^{j+1} Z_t, \tag{3.5}$$

$$E_t r_{t+1+j} = s_2' \Gamma^{j+1} Z_t, \tag{3.6}$$

where s_1 is a selection vector whose first element is equal to one and zero otherwise, and s_2 is a vector with one at the second element and zero otherwise. Equations (3.5) and (3.6) imply that the news about the discount rate and the news about the interest rate are calculated as follows:

$$\begin{aligned} \eta_{e,t+1} &= s_1' \sum_{j=1}^{\infty} \rho^j \Gamma^j u_{t+1} \\ &= s_1' \rho \Gamma (I_n - \rho \Gamma)^{-1} u_{t+1}, \end{aligned} \tag{3.7}$$

$$\begin{aligned}\eta_{r,t+1} &= s_2' \sum_{j=0}^{\infty} \rho^j \Gamma^j u_{t+1} \\ &= s_2'(I_n - \rho\Gamma)^{-1} u_{t+1}.\end{aligned} \tag{3.8}$$

Since the excess stock return is the first element of Z_{t+1}, the unexpected excess stock return η_{t+1} is

$$\eta_{t+1} = s_1' u_{t+1}, \tag{3.9}$$

and the news on the future dividend can be extracted as residual using equations (3.7), (3.8), and (3.9).

$$\begin{aligned}\eta_{d,t+1} &= \eta_{t+1} + \eta_{e,t+1} + \eta_{r,t+1} \\ &= [s_1' + s_1'\rho\Gamma(I_n - \rho\Gamma)^{-1} + s_2'(I_n - \rho\Gamma)^{-1}]u_{t+1}.\end{aligned} \tag{3.10}$$

3.2.2 Models for investigating the Change in Relative Importance of NCF and NDR

To investigate changes in the relative importance of NCF and NDR, I employ models that allow the parameters of the VAR model to vary across different market environments or over time: the threshold VAR (TVAR) and the time-varying parameter VAR with stochastic volatilities (TVP-VAR-SV).

Variance Decomposition based on TVAR

To analyze the regime-dependent relative importance of NCF and NDR, I employ the TVAR of Alessandri and Mumtaz (2017). Then, equation (3.4) can be expressed within the TVAR framework as follows:

$$\begin{aligned}Z_{t+1} =& \left(c_0 + \Gamma_0 Z_t + \Omega_0^{1/2}\varepsilon_{t+1}\right)(1 - S_{t+1}) \\ &+ \left(c_1 + \Gamma_1 Z_t + \Omega_1^{1/2}\varepsilon_{t+1}\right)S_{t+1}, \quad \varepsilon_{t+1} \sim i.i.d.\ N(0,1),\end{aligned} \tag{3.11}$$

where S_{t+1} is a variable representing the market regime that is determined by the level of threshold variable relative to an unobserved threshold z^*. In particular, S_{t+1} is 0 when the threshold variable is less than the threshold, and is 1 otherwise:

$$\begin{aligned} S_{t+1} &= 0 \quad if \quad z_{i,t+1-d} \leq z^* \\ S_{t+1} &= 1 \quad if \quad z_{i,t+1-d} > z^*, \end{aligned} \tag{3.12}$$

where $z_{i,t+1-d}$ is the threshold variable which is a lag of one of the state variables in Z_{t+1}, and d is the delay parameter which is set to 1.[2] Note that, given z^* and d, TVAR is simplified to two VAR models defined over different samples depending on $S_{t+1} = 0$ or $S_{t+1} = 1$.

Following Alessandri and Mumtaz (2017), I employ the Bayesian method, the Gibbs sampling with the Metropolis-Hastings algorithm, to estimate the parameter of TVAR. The coefficients and covariance matrices (c_0, c_1, Γ_0, Γ_1, Ω_0, Ω_1) can be drawn from their posterior distribution via the Gibbs sampling, and threshold (z^*) can be drawn from its posterior distribution via a random-walk Metropolis-Hastings algorithm. I employ a natural conjugate prior (a Normal prior for the VAR coefficients and an inverse Wishart prior for the covariance matrix) for the VAR parameters implemented via dummy observations. I assume a Normal prior for the threshold.[3]

Let X_i and Y_i be the matrices of the right-hand side and left-hand side variables of VAR for each regime $i = 0$ or 1, respectively. Additionally, X_D and Y_D denote the dummy observations used to implement the natural conjugate prior. Then, the conditional posterior distributions of the VAR coefficients $B_i = [c_i, \Gamma_i]$ and the covariance matrix Ω_i for each regime are defined as

$$\begin{aligned} &vec(B_i)|\Omega_i, Y_i \sim N(vec(\tilde{B}_i), \Omega_i \otimes (\tilde{X}_i'\tilde{X}_i)^{-1}) \\ &\Omega_i|B_i, Y_i \sim IW(\tilde{S}_i, \tilde{T}_i), \end{aligned} \tag{3.13}$$

where $\tilde{X} = [X_i; X_D]$, $\tilde{Y} = [Y_i; Y_D]$, $\tilde{B} = (\tilde{X}_i'\tilde{X}_i)^{-1}\tilde{X}_i'\tilde{Y}_i$, $\tilde{S}_i = (\tilde{Y}_i - \tilde{X}_i\tilde{B}_i)'(\tilde{Y}_i - \tilde{X}_i\tilde{B}_i)$, and $\tilde{T}_i$ is the number of rows of matrix $\tilde{Y}_i$.

After we draw the parameters of TVAR from their posterior distributions, we can examine whether the relative importance of NCF and NDR depends on regimes by using the

[2]Chen and Lee (1995) show that the delay parameter d has a Multinomial posterior distribution and can be drawn from Gibbs samplar. However, I set $d = 1$ to align with the VAR(1) specification used in this chapter and to simplify the estimation process.

[3]See Appendix C.1 for the details of priors employed for TVAR.

posterior mean of parameters. The elements of unexpected excess stock return in equation (3.2) for each regime can be derived as follows:

$$\begin{aligned} \eta_{e,t+1} &= s_1'\rho\Gamma_i(I_n - \rho\Gamma_i)^{-1}u_{i,t+1}, \\ \eta_{r,t+1} &= s_2'(I_n - \rho\Gamma_i)^{-1}u_{i,t+1}, \\ \eta_{d,t+1} &= [s_1' + s_1'\rho\Gamma_i(I_n - \rho\Gamma_i)^{-1} + s_2'(I_n - \rho\Gamma_i)^{-1}]u_{i,t+1}, \end{aligned} \tag{3.14}$$

where $u_{i,t+1} = \Omega_i^{1/2}\varepsilon_{t+1}$.

Variance Decomposition based on TVP-VAR-SV

The time-variation of the relative importance of NCF and NCR is examined by the TVP-VAR-SV of Primiceri (2005):

$$Z_{t+1} = c_{t+1} + \Gamma_{t+1}Z_t + u_{t+1}, \quad u_{t+1} \sim N(0, \Omega_{t+1}). \tag{3.15}$$

The reduced-form covariance matrix Ω_{t+1} has the following triangular reduction:

$$A_{t+1}\Omega_{t+1}A_{t+1}' = \Sigma_{t+1}\Sigma_{t+1}', \tag{3.16}$$

where A_{t+1} is the lower triangular matrix

$$A_{t+1} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \alpha_{21,t+1} & 1 & \ddots & \\ \vdots & \ddots & \ddots & 0 \\ \alpha_{n1,t+1} & \cdots & \alpha_{nn-1,t+1} & 1 \end{bmatrix}, \tag{3.17}$$

and Σ_{t+1} is the diagonal matrix

$$\Sigma_{t+1} = \begin{bmatrix} \sigma_{1,t+1} & 0 & \cdots & 0 \\ 0 & \sigma_{2,t+1} & \ddots & \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{n,t+1} \end{bmatrix}. \tag{3.18}$$

The triangular reduction of Ω_{t+1} implies that the heteroscedastic shocks u_{t+1} can be rewritten as

$$u_{t+1} = A_{t+1}^{-1}\Sigma_{t+1}\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim i.i.d.N(0, I_n), \tag{3.19}$$

which allows us to write equation (3.15) as follows:

$$Z_{t+1} = X'_{t+1}B_{t+1} + A^{-1}_{t+1}\Sigma_{t+1}\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim i.i.d.N(0, I_n), \tag{3.20}$$

where, $X'_{t+1} = I_n \otimes [1,\ Z'_t]$ and $B_{t+1} = vec([c_{t+1},\ \Gamma_{t+1}]')$. Coefficients and volatilities of the model are assumed to evolve as a random walk process as follows:

$$B_{t+1} = B_t + \nu_{t+1}, \tag{3.21}$$

$$\alpha_{t+1} = \alpha_t + \xi_{t+1}, \tag{3.22}$$

$$ln\sigma_{t+1} = ln\sigma_t + \omega_{t+1}, \tag{3.23}$$

where α_{t+1} is vector obtained by stacking the non-zero off-diagonal elements of A_{t+1} by rows and σ_{t+1} is the vector of the diagonal elements of Σ_{t+1}. In addition, all the innovations of the model are jointly normally distributed as follows:

$$\begin{bmatrix} \epsilon_{t+1} \\ \nu_{t+1} \\ \xi_{t+1} \\ \omega_{t+1} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{bmatrix} \right), \tag{3.24}$$

where Q, S, and W are positive definite matrices. Additionally, S is assumed to be block diagonal as in Primiceri (2005).

I choose Normal priors for time-varying coefficient (B_{t+1}), simultaneous relations (A_{t+1}), and volatilities ($ln\sigma_{t+1}$). Inverse-Wishart priors are used for the covariance matrices (Q, S, W). The means, variances, scales, and degrees of freedom for the Normal and inverse-Wishart priors are set by using a training sample (60 pre-sample observations).[4] Given the priors, we can draw parameters of TVP-VAR-SV from their posterior distribution via Gibbs sampling. Let B_T, A_T, and Σ_T be the entire history of B_{t+1}, A_{t+1}, and Σ_{t+1}, respectively. Then, due to the normality assumption of the innovations, we can draw B_T (given A_T and Σ_T) and A_T (given B_T and Σ_T) by using the algorithm proposed by Carter and Kohn (1994). Because of the non-linear and non-normal state-space form, I employ the method

[4]See Appendix C.2 for the details of priors employed for estimation of TVP-VAR-SV.

of Kim et al. (1998) to draw Σ_T. Lastly, the covariance matrices in equation (3.24) can be drawn from the inverse-Wishart distribution.[5].

We can examine the time-variation of the relative importance of NCF and NDR by using the posterior mean of B_{t+1}, A_{t+1}, and Σ_{t+1}. The elements of unexpected excess stock return in equation (3.2) at each period can be derived as follows:

$$
\begin{aligned}
\eta_{e,t+1} &= s_1' \rho \Gamma_{t+1} (I_n - \rho \Gamma_{t+1})^{-1} A_{t+1}^{-1} \Sigma_{t+1} \varepsilon_{t+1}, \\
\eta_{r,t+1} &= s_2' (I_n - \rho \Gamma_{t+1})^{-1} A_{t+1}^{-1} \Sigma_{t+1} \varepsilon_{t+1}, \\
\eta_{d,t+1} &= [s_1' + s_1' \rho \Gamma_{t+1} (I_n - \rho \Gamma_{t+1})^{-1} + s_2' (I_n - \rho \Gamma_{t+1})^{-1}] A_{t+1}^{-1} \Sigma_{t+1} \varepsilon_{t+1}.
\end{aligned}
\tag{3.25}
$$

3.3 Empirical Results on the Variance Decomposition of U.S. Excess Stock Return

3.3.1 Data Description

I decompose the variance of U.S. monthly excess stock return. For both models (the TVAR and the TVP-VAR-SV), I use the six variables employed by Campbell and Ammer (1993): the excess stock return, the real interest rate, the dividend-price ratio, the relative bill rate, the change in bill rate, and the term spreads.

The excess stock return is the CRSP value-weighted portfolio (all listed stocks in S&P 500) stock return in excess of the risk-free rate (the 1-month Treasury bill rate). The real interest rate is calculated as the 1-month Treasury bill rate minus the inflation rate. To circumvent a misleading result due to the strong seasonality of dividends, the dividend-price ratio is smoothed as the ratio of the 12-month moving sum of dividends to the current stock price. The relative bill rate is defined as the difference between the 3-month Treasury bill rate and its 1-year lagged moving average, and the change in bill rate is the change in three-month bill rate. Finally, the term spread is calculated as the difference between 10-year and 3-month Treasury bill rates.

For the estimation of TVAR, I also use two threshold variables that may identify changes in the stock market conditions: the Chicago Fed's National Financial Condition Index (NFCI) and investor sentiment index (S_{BW}) constructed by Baker and Wurgler (2006).

[5] See Primiceri (2005) for the details of the estimation procedure.

NFCI is chosen to measure the degree of financial distress and S_{BW} is employed to represent the market-wide level of investor sentiment.[6][7] Considering that NFCI and S_{BW} may contain the business cycle information, I also estimate TVAR with the adjusted National Condition Index (ANFCI) and the orthogonalized investor sentiment index (S_{BW}^{or}) that are uncorrelated with real economic conditions.

The sample period for TVAR is dependent on the availability of threshold variable data. It spans from January 1971 to December 2019 when the financial condition index is used, and from July 1965 to December 2019 when the investor sentiment index is used as a threshold variable. The sample period for TVP-VAR-SV runs from January 1953 to December 2019. The data are collected from Amit Goyal's website, Jeffery Wurgler's website, and FRED.

3.3.2 Variance Decomposition Results using TVAR

Variance Decomposition using Financial Condition Index as a Threshold Variable

Table 3.1 summarizes the variance decomposition results using the first-order TVAR when the financial condition index (NFCI or ANFCI) is used for a threshold variable. The state variables are excess stock returns, real interest rates, log dividend price ratio, relative bill rates, term spreads, change in bill rates, and the financial condition index. It reports the variance share of each component in equation (3.3) relative to the unexpected excess stock return within each regime, based on the posterior mean of the estimated parameters. For comparison, I also present the variance decomposition using the VAR with constant parameters (the constant parameter VAR).

Consistent with the previous literature on decomposing the postwar U.S. stock return, the variance of NDR relative to the unexpected excess stock return is larger than that of

---

[6] Chicago Fed's National Financial Condition Index (NFCI) is a weekly indicator of comprehensive financial conditions in money markets, debt and equity markets, and the traditional and shadow banking systems, and is constructed using dynamic factor analysis. The adjusted National Financial Condition Index (ANFCI) isolates a component of financial conditions uncorrelated with economic conditions.

[7] Baker and Wurgler (2006) construct a composite index for investor sentiment by extracting the first principal component of six individual sentiment proxies: the closed-end fund discount, the share turnover, the number of IPOs, the first-day return of IPOs, the equity share in new issues, and the dividend premium. The orthogonalized index removes business cycle information by regressing the six individual proxies on macroeconomic variables.

Table 3.1: Variance Decomposition of Excess Stock Return: TVAR, Financial Condition

	Threshold VAR		Constant VAR
	NFCI $\leq z^*$	NFCI $> z^*$	
$Var(NCF)$	0.53	0.10	0.33
$Var(NDR)$	0.12	0.78	0.52
$Var(real\ rates)$	0.03	0.03	0.03
$-2cov(NCF, NDR)$	0.31	0.03	0.02
$-2cov(NCF, real\ rates)$	-0.03	-0.03	0.05
$2cov(NDR, real\ rates)$	0.04	0.09	0.05
	Threshold VAR		Constant VAR
	ANFCI $\leq z^*$	ANFCI $> z^*$	
$Var(NCF)$	0.26	0.10	0.31
$Var(NDR)$	0.27	0.78	0.39
$Var(real\ rates)$	0.03	0.03	0.03
$-2cov(NCF, NDR)$	0.32	-0.02	0.16
$-2cov(NCF, real\ rates)$	0.02	-0.03	0.06
$2cov(NDR, real\ rates)$	0.10	0.15	0.04

i) The ratios of the variances of cash flow news, discount rate news, real interest rate news, and their covariances relative to the variance of unexpected excess returns are reported.

ii) The sample runs from 1971M1 to 2019M12.

NCF when the variance of unexpected excess stock return is decomposed using the constant parameter VAR. When NFCI is included as one of the state variables in the VAR model, the variance of NCF accounts for 33% of the variance of excess stock return, while the variance of NDR contributes 52%. When NFCI is replaced by ANFCI, the variance shares of NCF and NDR are 31% and 39%, respectively. In both cases, the contribution of the real interest rate is negligible.

Meanwhile, the variance decomposition results using TVAR indicate that the relative importance of NCF and NDR in explaining stock return variations changes based on the prevailing financial conditions. With NFCI as the threshold variable, NCF appears more important than NDR when the financial condition is relatively loose (NFCI is below the threshold value): the variance share of NCF is 53%, while that of NDR is 12%. Conversely, the relative importance reverses when the financial condition becomes tighter (NFCI is above the threshold value): the variance share of NCF decreases to 10%, while that of NDR increases to 78%. When ANFCI is used as the threshold variable, the contributions of NCF and NDR are similar during periods of loose financial conditions. During periods of tight financial conditions, I observe a similar pattern in the relative importance of NCF and NDR as when NFCI is used as the threshold variable. In summary, the variance decomposition of excess stock return using TVAR suggests that the importance of NDR increases as financial conditions tighten.

Variance Decomposition using Investor Sentiment Index as a Threshold Variable

Table 3.2 summarizes the variance decomposition results using the TVAR with the investor sentiment index (S_{BW} or S_{BW}^{or}) as the threshold variable. Similar to the results using financial condition indices, the decomposition result of the constant parameter VAR indicates a greater importance of NDR. Depending on which sentiment index is included in the VAR, the variance share of NDR ranges from 38% to 41%, while that of NCF ranges from 33% to 34%.

I find a specific pattern in the relative importance of NCF and NDR from the variance decomposition results using TVAR. Employing S_{BW} as the threshold variable, NCF appear more important than NDR during the period of low sentiments (S_{BW} is below the threshold value): the variance share of NCF is 65% whereas that of NDR 19%. The contributions of NCF and NDR reverse during the periods of high sentiment regimes (S_{BW} is above the threshold value): the variance share of NCF decreases to 12%, while that of NDR increases to 89%. The decomposition using S_{BW}^{or} as the threshold variable yields similar results, showing relatively higher contributions of NCF during low sentiment regimes and relatively higher contributions of NDR during high sentiment regimes.

Table 3.2: Variance Decomposition of Excess Stock Return: TVAR, Investor Sentiment

	Threshold VAR		Constant VAR
	$S_{BW} \leq z^*$	$S_{BW} > z^*$	
$Var(NCF)$	0.65	0.12	0.34
$Var(NDR)$	0.19	0.89	0.41
$Var(real\ rates)$	0.05	0.03	0.06
$-2cov(NCF, NDR)$	0.17	-0.21	0.16
$-2cov(NCF, real\ rates)$	-0.07	0.01	0.11
$2cov(NDR, real\ rates)$	0.02	0.15	-0.09
	Threshold VAR		Constant VAR
	$S^{or}_{BW} \leq z^*$	$S^{or}_{BW} > z^*$	
$Var(NCF)$	0.73	0.34	0.33
$Var(NDR)$	0.23	0.93	0.38
$Var(real\ rates)$	0.03	0.03	0.06
$-2cov(NCF, NDR)$	-0.03	-0.43	0.18
$-2cov(NCF, real\ rates)$	0.00	0.09	0.10
$2cov(NDR, real\ rates)$	0.04	0.04	-0.05

i) The ratios of the variances of cash flow news, discount rate news, real interest rate news, and their covariances relative to the variance of unexpected excess returns are reported.

ii) The sample runs from 1965M7 to 2019M12.

One limitation of the VAR-based variance decomposition method is its sensitivity to the choice of predictor variables. For example, Chen and Zhao (2009) find that, for the postwar periods, the variance of NCF becomes larger than the variance of NDR when the book-to-market ratio is used as a substitute for the dividend-price ratio or the equity-price ratio.

Table 3.3: Variance Decomposition of Excess Stock Return using the Book-to-Market Ratios

	Financial Condition Index			
	NFCI $\leq z^*$	NFCI $> z^*$	ANFCI $\leq z^*$	ANFCI $> z^*$
$Var(NCF)$	0.63	0.42	0.58	0.43
$Var(NDR)$	0.10	0.69	0.29	0.74
	Investor Sentiment Index			
	$S_{BW} \leq z^*$	$S_{BW} > z^*$	$S_{BW}^{or} \leq z^*$	$S_{BW}^{or} > z^*$
$Var(NCF)$	1.02	1.06	1.11	0.81
$Var(NDR)$	0.10	0.90	0.16	0.70

i) The ratios of the variances of cash flow news and discount rate news relative to the variance of unexpected excess returns are reported.
ii) State variables of TVAR are the excess stock returns, the real interest rates, the book-to-market ratios, the relative bill rates, the term spreads, the change in bill rates, and the threshold variable.

Table 3.3 presents the shares of NCF and NDR variance in the unexpected excess return variance when replacing the dividend-price ratio with the book-to-market ratio. When the financial condition index is used as the threshold variable, replacing the dividend-price ratio with the book-to-market ratio does not significantly affect the variance decomposition result. For both financial condition indices (NFCI and ANFCI), the variance of NCF contributes more than that of NDR in explaining stock return variation during loose financial regimes, while this relationship is reversed during tight financial regimes. In contrast, when I use the investor sentiment index as the threshold variable, I obtain a result aligning with the findings of Chen and Zhao (2009). The variance share of NCF is greater than that of NDR in both sentiment regimes. However, it is worth noting that in the high sentiment regime, the variance share of NDR increases significantly, and the gap between the variance shares of NDR and NCF narrows compared to the low sentiment regime. Thus, the importance of NDR rises comparatively in the high sentiment regime, which aligns with the results obtained using the dividend-price ratio in some respects.[8]

3.3.3 Variance Decomposition Results using TVP-VAR-SV

I investigate the time-variation of the relative importance of NCF and NDR by employing a first-order TVP-VAR-SV. One concern regarding the estimation of TVP-VAR-SV is that computational challenges increase as the number of state variables increases. Considering this, I exclude the threshold variables from the state variables in TVP-VAR-SV.

Panel (a) of Figure 3.1 presents the time-variation of the variance of NCF and NDR relative to the unexpected stock return variance from 1953 to 2019. The relative variances of NCF and NDR do not display stable patterns, which is consistent with the findings of studies that address the dependency of the relative importance of NCF and NDR (or the stock return and dividend growth predictability) on the selection of the sample period. For example, Bernanke and Kuttner (2005) find that the relative importance of NCF increases when the sample period begins after the 1990s. For the full sample (from 1973 to 2002), NDR and NCF contribute 76% and 24.5% of the variance of unexpected excess returns, respectively. However, when restricting the sample to the period from 1989 to 2002, the contributions of NDR and NCF change to 38% and 31.9%, respectively, indicating a greater role for NCF in explaining return variations in the later period. They argue that the increase in the relative importance of NCF for the sample from 1989 to 2002 is attributable to the decrease in the predictability of stock returns.[9]

One notable feature of the time variation in the relative importance of NCF and NDR, as presented in panel (a) of Figure 3.1, is that NCF becomes more important than NDR after the 1990s. From the 1950s to the 1980s, the variance share of NDR is generally larger than

---

[8] To examine whether including the threshold variable as one of the state variables of VAR affects the relative importance of NCF and NDR, I also estimate TVAR where the threshold variable is not an element of state variables but is only used for identifying financial market regimes and obtain qualitatively similar results. See Appendix C.3 for the detailed variance decomposition results.

[9] Some studies document changes in stock return and dividend predictability as well as the relative importance of NCF and NDR during the postwar period. Chen (2009) finds that the unpredictable stock returns during the prewar period become predictable in the postwar period, whereas the significant predictability of dividend growth during the prewar period disappears in the postwar period. Chen and Zhao (2009) find that the relative variance of NCF and NDR changes depending on whether prewar samples (from 1929 to 1951) are included.

that of NCF, with more than half of the variance of unexpected excess returns assigned to the variance of NDR during the 1970s. Then, the variance share of NDR begins to decline, while that of NCF starts to increase from the early 1980s, leading to the reversal of the relative importance of NCF and NDR in the early 1990s. The timing of the reversal coincides with the period known as the Great Moderation, characterized by declining macroeconomic volatility from the 1980s to the 2000s.

To examine the sensitivity of the results to the choice of predictor variables, I also estimate the TVP-VAR-SV by replacing the dividend-price ratio with the book-to-market ratio. Panel (b) of Figure 3.1 presents the variance decomposition result using the book-to-market ratio as a predictor variable. Except for the first two decades, replacing the dividend-price ratio with the book-to-market ratio does not significantly affect the pattern of the relative importance of NCF and NDR.[10] NDR is the main determinant of the variance of excess stock return during the 1970s, and the reversal of the relative importance is observed during the mid-1980s.

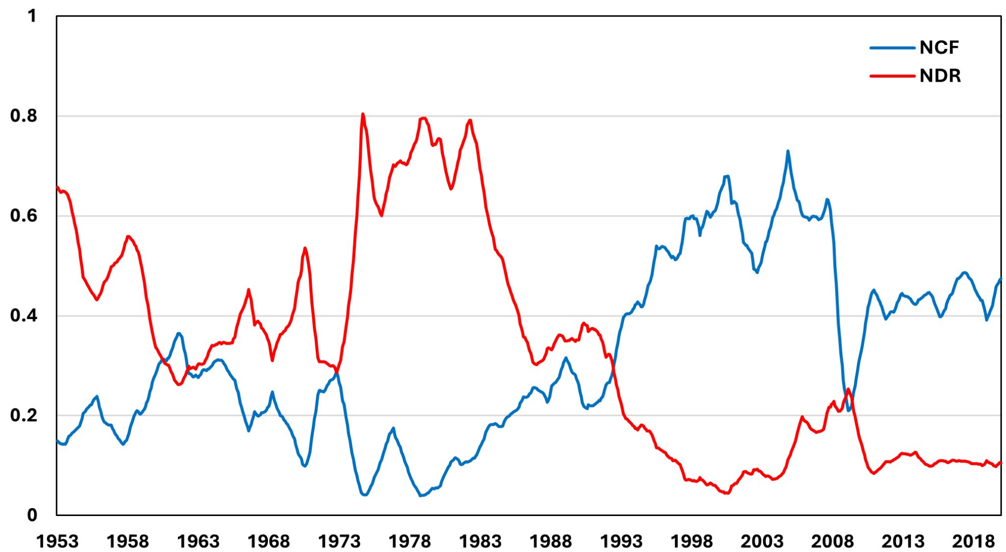
3.3.4 Possible Explanations for the Change in the Relative Importance of NCF and NDR

The results using TVAR indicate that NDR becomes relatively more important in explaining the variance of unexpected excess stock return during the tight financial condition regimes and the high sentiment regimes. From the result using TVP-VAR-SV, I find that NCF becomes a main determinant of the variance of unexpected excess stock return after the Great Moderation. In this section, I present possible explanations for these results.

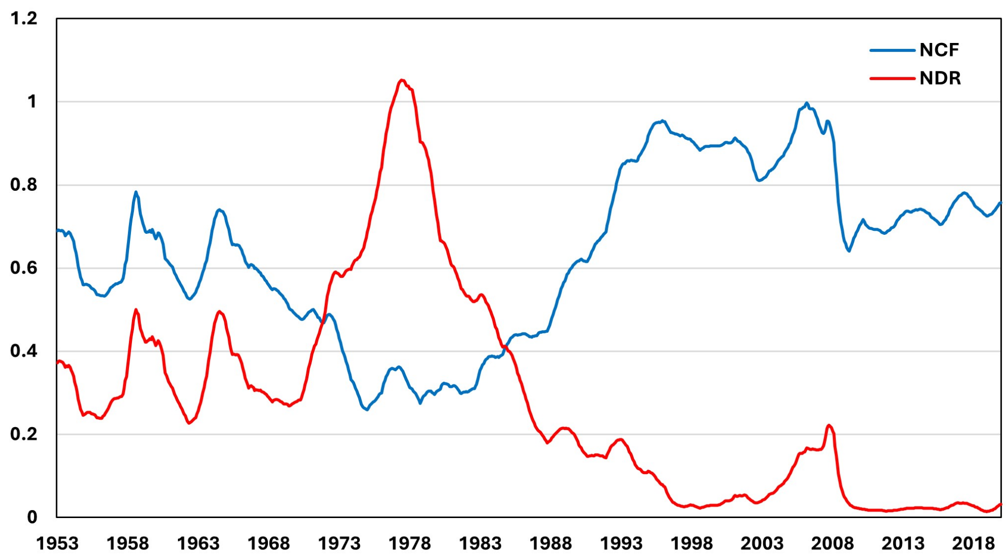
Increasing Relative Importance of NDR in Tight Financial Condition Regime

What kind of information investors pay more attention to in different economic states may help explain the relatively higher contribution of NDR in tight financial condition regimes. In the presence of information processing constraints, economic agents need to prioritize which information they pay more attention to. (e.g., Sims (2003) and Mackowiak and Wiederholt (2009)).

[10] During the 1950s and 1960s, NCF contributes relatively more than NDR in explaining excess stock return variation, which contrasts with the result using the dividend-price ratio.



(a) Dividend-Price ratio as a predictor



(b) Book-to-Market ratio as a predictor

Figure 3.1: Time Variation of the Relative Variance of NCF and NDR

Recently, Kacperczyk et al. (2016) show that how investors allocate their attention to different types of information depends on the business cycle. One of the main predictions from their work is that investors' optimal attention allocation for a certain shock increases in its volatility. Because aggregate shocks become more volatile during recessions compared to expansions, this prediction implies that investors pay more attention to aggregate shocks than to idiosyncratic shocks during recessions.[11] Tsiakas et al. (2020) provide empirical evidence that is consistent with the prediction of Kacpercyk et al. (2016). They show that variables containing stock-specific information predict stock returns well in expansions and financial recovery periods, while variables containing aggregate information have better predictability in recessions and financial contraction periods.

Meanwhile, NDR is more closely related to information about aggregate shocks compared to NCF, as investors need to predict macroeconomic variables, such as the inflation rate, when setting discount rates (e.g., Katz et al. (2017)). Therefore, considering that information about aggregate shocks may draw more attention from investors during tight financial condition regimes, the larger contribution of NDR in such periods may be associated with the investors' increased attention to aggregate shocks.

Increasing Relative Importance of NDR in High Inversor Sentiment Regime

The asymmetric predictability of stock returns due to the mispricing in the stock market may explain the larger contribution of NDR during high sentiment regimes. In the presence of unsophisticated investors who are relatively naive and tend to be over-optimistic or over-pessimistic about the market's prospects than rational investors, the stock prices can deviate from their intrinsic values (e.g., De Long et al. (1990), Daniel et al. (1998), and Brown and Cliff (2005)).

Several studies document that unsophisticated investors are more likely to participate in the stock market and trade more actively during high sentiment periods, which implies that the influence of unsophisticated investors on stock price increases during those periods (e.g., Lemmon and Portniaguina (2006), and Yu and Yuan (2011)). Moreover, due to the short sale impediments, the mispricing can be more substantial during high sentiment periods, and overvaluation is expected to be more prevalent than undervaluation (e.g., Stambaugh

[11] To buttress this prediction, Kacpercyk et al. (2016) also provide empirical evidence for the significant increase in the aggregate risk during recessions.

et al. (2012)).

The mispricing is corrected as the economic fundamentals are revealed. Given that stock prices tend to deviate more from the intrinsic value during high sentiment periods compared to low sentiment periods, the correction of mispricing is more significant and concentrated during high sentiment periods. This implies a more pronounced negative relationship between investor sentiment and future stock return, leading to a stronger stock return predictability in high sentiment regimes. Thus, these asymmetric mispricing and stock return predictability lead to differences in the relative importance of NDR and NCF across different sentiment regimes.

The stronger stock return predictability during high sentiment periods is also supported by a theoretical asset pricing model. Barberis et al. (2015) explain several features of prices and returns with a theoretical asset pricing model in which some investors irrationally form beliefs about future stock prices by extrapolating past price changes. They show that the stock return predictability of the dividend-price ratio stems from the overvaluation of stock prices in the presence of irrational investors, and the predictive power becomes stronger when there are more irrational investors in the market.

The Reversal of the Relative Importance after the 1990s

Firstly, as Bernanke and Kuttner (2005) documented, the decline in stock return predictability may explain the increase in the relative importance of NCF after the 1990s. Hsu et al. (2023) also find that asset return predictability of macroeconomic volatility declines during the Great Moderation.[12] Because NCF is extracted as residuals in the VAR-based decomposition approach, the decline in stock return predictability results in an increase in the relative importance of NCF.

The increase in information rigidity during the Great Moderation may also explain the reversal of the relative importance of NCF and NDR. Economic agents form their expectations by collecting and processing new information. The degree of information rigidity

[12] Using calibration exercises of an equilibrium model, Hsu et al. (2023) find that an improved monetary policy (more responsive to inflation and output), a structural change in the relation between output and inflation (a larger slope in the Phillips Curve), and changes in dynamics of shocks account for the decline in the predictability during the Great Moderation.

varies as the incentives for investors to collect and process additional information change. Intuitively, investors tend to pay less attention to new information during stable times when macroeconomic volatility declines, leading to greater information rigidity in expectations. Consistent with this intuition, Coibion and Gorodnichenko (2015) find empirical evidence that the degree of information rigidity regarding inflation expectations during the Great Moderation is higher than in other periods. Considering that the discount rate is closely related to macroeconomic expectations, the increase in information rigidity results in smaller revisions in discount rates. This, in turn, leads to less volatile NDR and reduces the relative importance of NDR during periods of low macroeconomic volatility.

Lastly, the shift in the return-earning relationship (i.e., the relationship between aggregate stock returns and aggregate earnings) may be a reason for the increased contribution of NCF after the 1990s. Recently, the literature provides evidence that the return-earning relationship turns from negative to positive during the early 1990s (e.g., Kim et al. (2020) and Curtis et al. (2024)).[13] Curtis et al. (2024) show that the relative importance of NCF in explaining stock return variation increases after the return-earnings relationship turns positive. They argue that the primary driver of this shift is improved inventory management, facilitated by advancements in information technology, which is considered one of the causes of the Great Moderation.

3.4 Conclusion

In this chapter, I examine the state dependency and time variation in the relative importance of cash flow news and discount rate news in explaining excess stock return variance. To do this, I employ a threshold VAR (TVAR) and a time-varying parameter VAR with stochastic volatilities (TVP-VAR-SV), both of which allow for variations in coefficients and variances.

I find evidence supporting the state-dependent and time-varying relative importance of the cash flow news and the discount rate news. First, the stock return variance decomposition results using TVAR suggest that the contribution of discount rate news increases during periods of tight financial conditions or high investor sentiment regimes. I propose

[13]Kim et al. (2020) report the timing of shift as 1994, and Curtis et al. (2024) identify the timing of shift as the fourth quarter of 1991.

changes in investors' attention allocations and asymmetric stock return predictability as explanations for these results. Second, the results using the TVP-SV-VAR model indicate that cash flow news has become more important than discount rate news in explaining stock return variation since the 1990s. This reversal may be explained by the less volatile discount rate news caused by increased information rigidity and changes in the return-earnings relationship after the onset of the Great Moderation.

BIBLIOGRAPHY

[1] Klaus Adam, Albert Marcet, and Johannes Beutel. Stock price booms and expected capital gains. *American Economic Review*, 107(8):2352–2408, 2017.

[2] Hassan Afrouzi, Spencer Y Kwon, Augustin Landier, Yueran Ma, and David Thesmar. Overreaction in expectations: Evidence and theory. *The Quarterly Journal of Economics*, 138(3):1713–1764, 2023.

[3] Piergiorgio Alessandri and Haroon Mumtaz. Financial conditions and density forecasts for us output and inflation. *Review of Economic Dynamics*, 24:66–78, 2017.

[4] Torben G Andersen, Tim Bollerslev, Francis X Diebold, and Heiko Ebens. The distribution of realized stock return volatility. *Journal of financial economics*, 61(1):43–76, 2001.

[5] Torben G Andersen, Tim Bollerslev, Francis X Diebold, and Paul Labys. Modeling and forecasting realized volatility. *Econometrica*, 71(2):579–625, 2003.

[6] Andrew Ang and Geert Bekaert. Stock return predictability: Is it there? *The Review of Financial Studies*, 20(3):651–707, 2007.

[7] George-Marios Angeletos, Zhen Huo, and Karthik A Sastry. Imperfect macroeconomic expectations: Evidence and theory. *NBER Macroeconomics Annual*, 35(1):1–86, 2021.

[8] Philippe Bacchetta, Elmar Mertens, and Eric Van Wincoop. Predictability in financial markets: What do survey expectations tell us? *Journal of International Money and Finance*, 28(3):406–426, 2009.

[9] Malcolm Baker and Jeffrey Wurgler. Investor sentiment and the cross-section of stock returns. *The journal of Finance*, 61(4):1645–1680, 2006.

[10] Marta Bańbura, Domenico Giannone, and Lucrezia Reichlin. Large bayesian vector auto regressions. *Journal of applied Econometrics*, 25(1):71–92, 2010.

[11] Ravi Bansal and Amir Yaron. Risks for the long run: A potential resolution of asset pricing puzzles. *The journal of Finance*, 59(4):1481–1509, 2004.

[12] Nicholas Barberis, Robin Greenwood, Lawrence Jin, and Andrei Shleifer. X-capm: An extrapolative capital asset pricing model. *Journal of financial economics*, 115(1):1–24, 2015.

[13] Ole E Barndorff-Nielsen and Neil Shephard. Econometric analysis of realized volatility and its use in estimating stochastic volatility models. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 64(2):253–280, 2002.

[14] Ben S Bernanke and Kenneth N Kuttner. What explains the stock market's reaction to federal reserve policy? *The Journal of finance*, 60(3):1221–1257, 2005.

[15] Tim Bollerslev, Robert F Engle, and Jeffrey M Wooldridge. A capital asset pricing model with time-varying covariances. *Journal of political Economy*, 96(1):116–131, 1988.

[16] Pedro Bordalo, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer. Overreaction in macroeconomic expectations. *American Economic Review*, 110(9):2748–2782, 2020.

[17] Pedro Bordalo, Nicola Gennaioli, Rafael La Porta, and Andrei Shleifer. Diagnostic expectations and stock returns. *The Journal of Finance*, 74(6):2839–2874, 2019.

[18] Jean-Philippe Bouchaud, Philipp Krueger, Augustin Landier, and David Thesmar. Sticky expectations and the profitability anomaly. *The Journal of Finance*, 74(2):639–674, 2019.

[19] Michael W Brandt and Qiang Kang. On the relationship between the conditional mean and volatility of stock returns: A latent var approach. *Journal of Financial Economics*, 72(2):217–257, 2004.

[20] Gregory W Brown and Michael T Cliff. Investor sentiment and asset valuation. *The Journal of Business*, 78(2):405–440, 2005.

[21] John Y Campbell. Stock returns and the term structure. *Journal of financial economics*, 18(2):373–399, 1987.

[22] John Y Campbell. A variance decomposition for stock returns. *The economic journal*, 101(405):157–179, 1991.

[23] John Y Campbell and John H Cochrane. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of political Economy*, 107(2):205–251, 1999.

[24] John Y Campbell and Ludger Hentschel. No news is good news: An asymmetric model of changing volatility in stock returns. *Journal of financial Economics*, 31(3):281–318, 1992.

[25] John Y Campbell, Martin Lettau, Burton G Malkiel, and Yexiao Xu. Have individual stocks become more volatile? an empirical exploration of idiosyncratic risk. *The journal of finance*, 56(1):1–43, 2001.

[26] John Y Campbell and Robert J Shiller. The dividend-price ratio and expectations of future dividends and discount factors. *The review of financial studies*, 1(3):195–228, 1988.

[27] Sean D Campbell and Francis X Diebold. Stock returns and expected business conditions: Half a century of direct evidence. *Journal of Business & Economic Statistics*, 27(2):266–278, 2009.

[28] Chris K Carter and Robert Kohn. On gibbs sampling for state space models. *Biometrika*, 81(3):541–553, 1994.

[29] Stefano Cassella and Huseyin Gulen. Extrapolation bias and the predictability of stock returns by price-scaled variables. *The Review of Financial Studies*, 31(11):4345–4397, 2018.

[30] Long Chen. On the reversal of return and dividend growth predictability: A tale of two periods. *Journal of Financial Economics*, 92(1):128–151, 2009.

[31] Long Chen and Xinlei Zhao. Return decomposition. *The Review of Financial Studies*, 22(12):5213–5249, 2009.

[32] Kwang Hun Choi, Chang-jin Kim, and Cheolbeom Park. Regime shifts in price-dividend ratios and expected stock returns: A present-value approach. *Journal of Money, Credit and Banking*, 49(2-3):417–441, 2017.

[33] Ray Yeutien Chou. Volatility persistence and stock valuations: Some empirical evidence using garch. *Journal of applied econometrics*, pages 279–294, 1988.

[34] Peter Christoffersen, Bruno Feunou, Kris Jacobs, and Nour Meddahi. The economic value of realized volatility: Using high-frequency returns for option valuation. *Journal of Financial and Quantitative Analysis*, 49(3):663–697, 2014.

[35] Olivier Coibion and Yuriy Gorodnichenko. Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review*, 105(8):2644–2678, 2015.

[36] Asher Curtis, Chang-Jin Kim, and Hyung Il Oh. A structural break in the aggregate earnings–returns relation. *Journal of Financial Econometrics*, page nbae015, 2024.

[37] Thomas Dangl and Michael Halling. Predictive regressions with time-varying coefficients. *Journal of Financial Economics*, 106(1):157–181, 2012.

[38] Kent Daniel, David Hirshleifer, and Avanidhar Subrahmanyam. Investor psychology and security market under-and overreactions. *the Journal of Finance*, 53(6):1839–1885, 1998.

[39] J Bradford De Long, Andrei Shleifer, Lawrence H Summers, and Robert J Waldmann. Noise trader risk in financial markets. *Journal of political Economy*, 98(4):703–738, 1990.

[40] Dobrislav Dobrev and Pawel Szerszen. The information content of high-frequency data for estimating equity return models and forecasting risk. 2010.

[41] Eugene F Fama and Kenneth R French. Business conditions and expected returns on stocks and bonds. *Journal of financial economics*, 25(1):23–49, 1989.

[42] Wayne E Ferson and Campbell R Harvey. The variation of economic risk premiums. *Journal of political economy*, 99(2):385–415, 1991.

[43] Kenneth R French, G William Schwert, and Robert F Stambaugh. Expected stock returns and volatility. *Journal of financial Economics*, 19(1):3–29, 1987.

[44] Jeffrey C Fuhrer. Intrinsic expectations persistence: Evidence from professional and household survey expectations. In *Intrinsic Expectations Persistence: Evidence from Professional and Household Survey Expectations: Fuhrer, Jeffrey C.* [Sl]: SSRN, 2019.

[45] Eric Ghysels, Pedro Santa-Clara, and Rossen Valkanov. There is a risk-return trade-off after all. *Journal of financial economics*, 76(3):509–548, 2005.

[46] Stefano Giglio, Matteo Maggiori, Johannes Stroebel, and Stephen Utkus. Five facts about beliefs and portfolios. *American Economic Review*, 111(5):1481–1522, 2021.

[47] Lawrence R Glosten, Ravi Jagannathan, and David E Runkle. On the relation between the expected value and the volatility of the nominal excess return on stocks. *The journal of finance*, 48(5):1779–1801, 1993.

[48] Roberto Gómez-Cram. Late to recessions: Stocks and the business cycle. *The Journal of Finance*, 77(2):923–966, 2022.

[49] Robin Greenwood and Andrei Shleifer. Expectations of returns and expected returns. *The Review of Financial Studies*, 27(3):714–746, 2014.

[50] Mark Grinblatt and Matti Keloharju. Sensation seeking, overconfidence, and trading activity. *The Journal of finance*, 64(2):549–578, 2009.

[51] Hui Guo. Stock market returns, volatility, and future output. *Review-Federal Reserve Bank of Saint Louis*, 84(5):75–84, 2002.

[52] Hui Guo and Robert F Whitelaw. Uncovering the risk–return relation in the stock market. *The Journal of Finance*, 61(3):1433–1463, 2006.

[53] James D Hamilton and Gang Lin. Stock market volatility and the business cycle. *Journal of applied econometrics*, 11(5):573–593, 1996.

[54] James D Hamilton and Raul Susmel. Autoregressive conditional heteroskedasticity and changes in regime. *Journal of econometrics*, 64(1-2):307–333, 1994.

[55] Peter Reinhard Hansen, Zhuo Huang, and Howard Howan Shek. Realized garch: a joint model for returns and realized measures of volatility. *Journal of Applied Econometrics*, 27(6):877–906, 2012.

[56] Campbell R Harvey. The specification of conditional expectations. *Journal of Empirical Finance*, 8(5):573–637, 2001.

[57] Alex Hsu, Francisco Palomino, and Liang Qian. Gone with the vol: A decline in asset return predictability during the great moderation. *Management Science*, 69(5):3025–3047, 2023.

[58] Joonyoung Hur. Time-varying information rigidities and fluctuations in professional forecasters' disagreement. *Economic Modelling*, 75:117–131, 2018.

[59] Theis Ingerslev Jensen. Subjective risk and return. *Available at SSRN 4276760*, 2023.

[60] Chanik Jo, Chen Lin, and Yang You. Subjective risk-return trade-off. *Available at SSRN 4096443*, 2022.

[61] Marcin Kacperczyk, Stijn Van Nieuwerburgh, and Laura Veldkamp. A rational theory of mutual funds' attention allocation. *Econometrica*, 84(2):571–626, 2016.

[62] Michael Katz, Hanno Lustig, and Lars Nielsen. Are stocks real assets? sticky discount rates in stock markets. *The Review of Financial Studies*, 30(2):539–587, 2017.

[63] Chang-Jin Kim. Dynamic linear models with markov-switching. *Journal of econometrics*, 60(1-2):1–22, 1994.

[64] Chang-Jin Kim, James C Morley, and Charles R Nelson. Is there a positive relationship between stock market volatility and the equity premium? *Journal of Money, Credit and banking*, pages 339–360, 2004.

[65] Chang-Jin Kim, James C Morley, and Charles R Nelson. The structural break in the equity premium. *Journal of Business & Economic Statistics*, 23(2):181–191, 2005.

[66] Chang-Jin Kim and Charles R Nelson. Has the us economy become more stable? a bayesian approach based on a markov-switching model of the business cycle. *Review of Economics and Statistics*, 81(4):608–616, 1999.

[67] Chang-Jin Kim, Charles R Nelson, and Richard Startz. Testing for mean reversion in heteroskedastic data based on gibbs-sampling-augmented randomization. *Journal of Empirical finance*, 5(2):131–154, 1998.

[68] Jaewoo Kim, Bryce Schonberger, Charles Wasley, and Hunter Land. Intertemporal variation in the information content of aggregate earnings and its effect on the aggregate earnings-return relation. *Review of Accounting Studies*, 25:1410–1443, 2020.

[69] Sangjoon Kim, Neil Shephard, and Siddhartha Chib. Stochastic volatility: likelihood inference and comparison with arch models. *The review of economic studies*, 65(3):361–393, 1998.

[70] Yunmi Kim and Charles R Nelson. Pricing stock market volatility: does it matter whether the volatility is related to the business cycle? *Journal of Financial Econometrics*, 12(2):307–328, 2013.

[71] Alexandre N Kohlhas and Ansgar Walther. Asymmetric attention. *American Economic Review*, 111(9):2879–2925, 2021.

[72] Rafael La Porta. Expectations and the cross-section of stock returns. *The Journal of Finance*, 51(5):1715–1742, 1996.

[73] Michael Lemmon and Evgenia Portniaguina. Consumer confidence and asset prices: Some empirical evidence. *The Review of Financial Studies*, 19(4):1499–1529, 2006.

[74] Martin Lettau and Sydney C Ludvigson. Measuring and modeling variation in the risk-return trade-off. *Handbook of financial econometrics: Tools and techniques*, pages 617–690, 2010.

[75] John Lintner. Security prices, risk, and maximal gains from diversification. *The journal of finance*, 20(4):587–615, 1965.

[76] Lars A Lochstoer and Tyler Muir. Volatility expectations and returns. *The Journal of Finance*, 77(2):1055–1096, 2022.

[77] Sydney C Ludvigson and Serena Ng. The empirical risk–return relation: A factor analysis approach. *Journal of financial economics*, 83(1):171–222, 2007.

[78] Bartosz Maćkowiak and Mirko Wiederholt. Optimal sticky prices under rational inattention. *American Economic Review*, 99(3):769–803, 2009.

[79] N Gregory Mankiw and Ricardo Reis. Sticky information versus sticky prices: a proposal to replace the new keynesian phillips curve. *The Quarterly Journal of Economics*, 117(4):1295–1328, 2002.

[80] Charles F Manski. Measuring expectations. *Econometrica*, 72(5):1329–1376, 2004.

[81] E Scott Mayfield. Estimating the market risk premium. *Journal of Financial Economics*, 73(3):465–496, 2004.

[82] Margaret M McConnell and Gabriel Perez-Quiros. Output fluctuations in the united states: What has changed since the early 1980's? *American Economic Review*, 90(5):1464–1476, 2000.

[83] Elmar Mertens and James M Nason. Inflation and professional forecast dynamics: An evaluation of stickiness, persistence, and volatility. *Quantitative Economics*, 11(4):1485–1520, 2020.

[84] Robert C Merton. On estimating the expected return on the market: An exploratory investigation. *Journal of financial economics*, 8(4):323–361, 1980.

[85] Stefan Nagel and Zhengyang Xu. Asset pricing with fading memory. *The Review of Financial Studies*, 35(5):2190–2245, 2022.

[86] Stefan Nagel and Zhengyang Xu. Dynamics of subjective risk premia. *Journal of Financial Economics*, 150(2):103713, 2023.

[87] Daniel B Nelson. Conditional heteroskedasticity in asset returns: A new approach. *Econometrica: Journal of the econometric society*, pages 347–370, 1991.

[88] Henri Nyberg. Risk-return tradeoff in us stock returns over the business cycle. *Journal of Financial and Quantitative Analysis*, 47(1):137–158, 2012.

[89] L'uboš Pástor and Robert F Stambaugh. The equity premium and structural breaks. *The Journal of Finance*, 56(4):1207–1239, 2001.

[90] Giorgio E Primiceri. Time varying structural vector autoregressions and monetary policy. *The Review of Economic Studies*, 72(3):821–852, 2005.

[91] Huntley Schaller and Simon Van Norden. Regime switching in stock market returns. *Applied Financial Economics*, 7(2):177–191, 1997.

[92] G William Schwert. Why does stock market volatility change over time? *The journal of finance*, 44(5):1115–1153, 1989.

[93] William F Sharpe. Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, 19(3):425–442, 1964.

[94] Christopher A Sims. Implications of rational inattention. *Journal of monetary Economics*, 50(3):665–690, 2003.

[95] Robert F Stambaugh, Jianfeng Yu, and Yu Yuan. The short of it: Investor sentiment and anomalies. *Journal of financial economics*, 104(2):288–302, 2012.

[96] James H Stock and Mark W Watson. Has the business cycle changed and why? *NBER macroeconomics annual*, 17:159–218, 2002.

[97] Yi Tang and Robert F Whitelaw. Time-varying sharpe ratios and market timing. *The Quarterly Journal of Finance*, 1(03):465–493, 2011.

[98] Ilias Tsiakas, Jiahan Li, and Haibin Zhang. Equity premium prediction and the state of the economy. *Journal of Empirical Finance*, 58:75–95, 2020.

[99] Christopher M Turner, Richard Startz, and Charles R Nelson. A markov model of heteroskedasticity, risk, and learning in the stock market. *Journal of Financial Economics*, 25(1):3–22, 1989.

[100] Jules H Van Binsbergen and Ralph SJ Koijen. Predictive regressions: A present-value approach. *The Journal of Finance*, 65(4):1439–1471, 2010.

[101] Jessica A Wachter. Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance*, 68(3):987–1035, 2013.

[102] Robert F Whitelaw. Time variations and covariations in the expectation and volatility of stock market returns. *The Journal of Finance*, 49(2):515–541, 1994.

[103] Robert F Whitelaw. Stock market risk and return: An equilibrium approach. *The Review of Financial Studies*, 13(3):521–547, 2000.

[104] Michael Woodford. Imperfect common knowledge and the effects of monetary policy. In *Knowledge, Information, and Expectations in Modern Macro-economics: In Honor of Edmund S. Phelps*, ed. Philippe Aghion, Roman Frydman, Joseph Stiglitz, and Michael Woodford. Princeton University Press, 2003.

[105] Jianfeng Yu and Yu Yuan. Investor sentiment and the mean–variance relation. *Journal of Financial Economics*, 100(2):367–381, 2011.

Appendix A

EMPIRICAL INVESTIGATION OF RISK-RETURN RELATION: WITH THE SLOW-MOVING SUBJECTIVE BELIEF ON MARKOV-SWITCHING VOLATILITY

A.1 Derivation of Equations (1.6) and (1.7)

This section shows a detailed derivation of equations (1.6) and (1.7). Equation (1.2) implies that

$$\sigma^2_{t-j} - \bar{v} = \frac{1}{\rho}(E_{t-j}[\sigma^2_{t-j+1}] - \bar{v}]). \tag{A.1}$$

By substituting $\sigma^2_{t-j} - \bar{v}$ in equation (1.3) using equation (A.1), we obtain

$$\begin{aligned}
E^S_t[\sigma^2_{t+1}] =& \bar{v} + \lambda(1-\phi)\sum_{j=0}^{\infty}\phi^j(\sigma^2_{t-j} - \bar{v}) \\
=& \bar{v} + \lambda(1-\phi)\sum_{j=0}^{\infty}\phi^j\frac{1}{\rho}(E_{t-j}[\sigma^2_{t-j+1}] - \bar{v}]) \\
=& \bar{v} - \frac{\lambda}{\rho}(1-\phi)\sum_{j=0}^{\infty}\phi^j\bar{v} + \frac{\lambda}{\rho}(1-\phi)\sum_{j=0}^{\infty}\phi^j E_{t-j}[\sigma^2_{t-j+1}] \\
=& \left(1 - \frac{\lambda}{\rho}\right)\bar{v} + \frac{\lambda}{\rho}(1-\phi)\sum_{j=0}^{\infty}\phi^j E_{t-j}[\sigma^2_{t-j+1}],
\end{aligned} \tag{A.2}$$

where $E^S_t[\sigma^2_{t+1}] = E_t[\sigma^2_{t+1}]$ when $\phi = 0$ and $\lambda = \rho$.

Then, by substituting equations (1.4) and (A.2) into equation (1.5), we can derive equation (1.7) as follows:

$$
\begin{aligned}
E_t[r_{t+1}] =& E^S[r_{t+1}] + b(E_t^S[\sigma_{t+1}^2] - E_t[\sigma_{t+1}^2]) \\
=& a_0 + a_1 E_t^S[\sigma_{t+1}^2] + b(E_t^S[\sigma_{t+1}^2] - E_t[\sigma_{t+1}^2]) \\
=& a_0 + (a_1 + b)E_t^S[\sigma_{t+1}^2] - bE_t[\sigma_{t+1}^2] \\
=& a_0 + (a_1 + b)\left(1 - \frac{\lambda}{\rho}\right)\bar{v} + (a_1 + b)(1 - \phi)\frac{\lambda}{\rho}\sum_{j=0}^{\infty}\phi^j E_{t-j}[\sigma_{t-j+1}^2] - bE_t[\sigma_{t+1}^2] \\
=& a_0 + (a_1 + b)\left(1 - \frac{\lambda}{\rho}\right)\bar{v} + \left[(a_1 + b)(1 - \phi)\frac{\lambda}{\rho} - b\right]E_t[\sigma_{t+1}^2] \\
& + (a_1 + b)(1 - \phi)\frac{\lambda}{\rho}\sum_{j=1}^{\infty}\phi^j E_{t-j}[\sigma_{t-j+1}^2] \\
=& c_0 + c_1 E_t[\sigma_{t+1}^2] + c_2\sum_{j=1}^{\infty}\phi^j E_{t-j}[\sigma_{t-j+1}^2], \qquad \text{(A.3)}
\end{aligned}
$$

where $c_0 = a_0 + (a_1 + b)(1 - \frac{\lambda}{\rho})\bar{v}$, $c_1 = (a_1 + b)(1 - \phi)\frac{\lambda}{\rho} - b$, and $c_2 = (a_1 + b)(1 - \phi)\frac{\lambda}{\rho}$.

A.2 Derivation of Equation (1.17)

To obtain a restriction on the parameters of the model implied by the log-linear present value model, we first need to compute the term $(E_{t+1} - E_t)[r_{t+1+j}]$ in equation (1.14). Because $\mu_{t+j} = E_{t+j}[r_{t+1+j}]$, we can compute the this terms as $(E_{t+1} - E_t)[\mu_{t+j}]$ by the law of iterated expectation(LIE).

Firstly, from equation (1.12), it can be shown that

$$
\begin{aligned}
E_{t+1}[\mu_{t+j}] =& \mu_{t+1} \quad for \quad j = 1, \\
=& \beta_0 + \phi\mu_{t+j-1} + \beta_1 E_{t+1}[\sigma_{S_{t+j+1}}^2] + \beta_2 E_{t+1}[\sigma_{S_{t+j}}^2] \quad for \quad j \geq 2, \\
E_t[\mu_{t+j}] =& E_t[\mu_{t+1}] \quad for \quad j = 1, \\
=& \beta_0 + \phi E_t[\mu_{t+j-1}] + \beta_1 E_t[\sigma_{S_{t+j+1}}^2] + \beta_2 E_t[\sigma_{S_{t+j}}^2] \quad for \quad j \geq 2. \qquad \text{(A.4)}
\end{aligned}
$$

Then, $(E_{t+1} - E_t)[\mu_{t+j}]$ is

$$
\begin{aligned}
(E_{t+1} - E_t)[\mu_{t+j}] =& \beta_1 (E_{t+1} - E_t)[\sigma^2_{S_{t+2}}] \quad for \quad j = 1, \\
=& \phi(E_{t+1} - E_t)[\mu_{t+j-1}] \\
&+ \beta_1 (E_{t+1} - E_t)[\sigma^2_{S_{t+j+1}}] + \beta_2 (E_{t+1} - E_t)[\sigma^2_{S_{t+j}}] \quad for \quad j \geq 2, \quad \text{(A.5)}
\end{aligned}
$$

which requires us to compute $(E_{t+1} - E_t)[\sigma^2_{S_{t+j+1}}]$.

In calculating $(E_{t+1} - E_t)[\sigma^2_{S_{t+1+j}}]$, I assume that $E_{t+s}[\sigma^2_{t+s}]$ is approximately the same with $\sigma^2_{S_{t+s}}$ for all s≥0. With this assumption and equation (1.10), we have

$$
\begin{aligned}
(E_{t+1} - E_t)[\sigma^2_{S_{t+j+1}}] =& \rho^j (E_{t+1} - E_t)[\sigma^2_{S_{t+1}}] \\
=& \rho^j (\sigma^2_{S_{t+1}} - \bar{\sigma}^2 - \rho \sigma^2_{S_t}) \\
=& \rho^j v_{t+1}. \qquad \text{(A.6)}
\end{aligned}
$$

By combining equations (A.5) and (A.6), we can derive the term $\kappa^j (E_{t+1} - E_t)[r_{t+1+j}]$ in equation (1.14) as

$$
\begin{aligned}
\kappa(E_{t+1} - E_t)[r_{t+2}] &= \kappa \rho \beta_1 v_{t+1}, \\
\kappa^2 (E_{t+1} - E_t)[r_{t+3}] &= \kappa^2 \phi (E_{t+1} - E_t)[r_{t+2}] + \kappa^2 \rho^2 \beta_1 v_{t+1} + \kappa^2 \rho \beta_2 v_{t+1}, \\
\kappa^3 (E_{t+1} - E_t)[r_{t+4}] &= \kappa^3 \phi (E_{t+1} - E_t)[r_{t+3}] + \kappa^3 \rho^3 \beta_1 v_{t+1} + \kappa^3 \rho^2 \beta_2 v_{t+1}, \\
&\vdots \\
\kappa^s (E_{t+1} - E_t)[r_{t+s+1}] &= \kappa^s \phi (E_{t+1} - E_t)[r_{t+s}] + \kappa^s \rho^s \beta_1 v_{t+1} + \kappa^s \rho^{s-1} \beta_2 v_{t+1}, \\
&\vdots \qquad \text{(A.7)}
\end{aligned}
$$

Then, summing up all the equations for $\kappa^j (E_{t+1} - E_t)[r_{t+1+j}]$ yields

$$
\sum_{j=1}^{\infty} \kappa^j (E_{t+1} - E_t)[r_{t+1+j}]) = \kappa \phi \sum_{j=1}^{\infty} \kappa^j (E_{t+1} - E_t)[r_{t+1+j}]) + \frac{\kappa \rho}{1 - \kappa \rho} \beta_1 v_{t+1} + \frac{\kappa^2 \rho}{1 - \kappa \rho} \beta_2 v_{t+1}, \qquad \text{(A.8)}
$$

$$
\begin{aligned}
&\rightarrow \sum_{j=1}^{\infty} \kappa^j (E_{t+1} - E_t)[r_{t+1+j}]) = \frac{\kappa \rho}{(1 - \kappa \rho)(1 - \kappa \phi)} (\beta_1 + \kappa \beta_2) v_{t+1}, \\
&\rightarrow f_{t+1} = -\frac{\kappa \rho}{(1 - \kappa \rho)(1 - \kappa \phi)} (\beta_1 + \kappa \beta_2) v_{t+1}, \qquad \text{(A.9)}
\end{aligned}
$$

from which I obtain the constraint among the parameters the model:

$$\delta = -\frac{\kappa\rho}{(1-\kappa\rho)(1-\kappa\phi)}(\beta_1 + \kappa\beta_2). \tag{A.10}$$

Note that when $\phi = 0$ and $\beta_2 = 0$ (the rational expectations case), the constraint in equation (A.10) becomes

$$\delta = -\frac{\kappa\rho}{(1-\kappa\rho)}\beta_1. \tag{A.11}$$

Appendix B

AN EMPIRICAL MODEL OF STOCK RETURNS UNDER INFORMATION RIGIDITY: IS THE SUBJECTIVE RISK-RETURN RELATIONSHIP POSITIVE IN THE STOCK MARKET?

B.1 Derivation of the Aggregate Subjective Volatility Expectation

Under the assumption that a fraction of $1-\lambda$ investors update their beliefs each period, the aggregate subjective expectation for the volatility of cash flow news is

$$E_t^*(h_{t+j}) = (1-\lambda)F_t(h_{t+j}) + \lambda E_{t-1}^*(h_{t+j}), \tag{B.1}$$

where $E_t^*(h_{t+j})$ is the aggregate subjective expectation about h_{t+j} at the end of period t, $F_t(h_{t+j})$ is the expectation about h_{t+j} by investors who update their expectation at the end of period t.

When we iterate the equation (B.1) backwardly, $E_t^*(h_{t+j})$ can be represented as a function of all the past subjective expectations of investors:

$$E_t^*(h_{t+j}) = (1-\lambda)\sum_{k=0}^{\infty}\lambda^k F_{t-k}(h_{t+j}), \quad for \quad k = 0, 1, 2, \cdots, \tag{B.2}$$

which requires us to calculate the forecast of realized volatility x_{t+j} by the investors who update their beliefs at the end of period $t-k$.

We use equations (2.4) and (2.6) to calculate $F_{t-k}(h_{t+j})$. Since equation (2.4) indicates $F_{t-k}(h_{t+j}) = \alpha_0 + \alpha_1 F_{t-k}(x_{t+j})$, and equation (2.6), under the assumption that investors who update their belief follow the rational expectations, implies $F_{t-k}(x_{t+j}) = \bar{x} + \phi_1^{k+j}(x_{t-k} - \bar{x})$, we have the following expression for $F_{t-k}(h_{t+j})$:

$$\begin{aligned} F_{t-k}(h_{t+j}) &= \alpha_0 + \alpha_1 F_{t-k}(x_{t+j}) \\ &= \alpha_0 + \alpha_1\bar{x} + \alpha_1\phi_1^{k+j}(x_{t-k} - \bar{x}). \end{aligned} \tag{B.3}$$

Then, by substituting equation (B.3) into equation (B.2), we obtain the aggregate subjective volatility expectation as a function of all the previous realized volatilities as in equation (2.7):

$$
\begin{aligned}
E_t^*(h_{t+j}) &= (1-\lambda)\sum_{k=0}^{\infty}\lambda^k F_{t-k}(h_{t+j}) \\
&= \alpha_0 + \alpha_1\bar{x} + (1-\lambda)\alpha_1\phi_1^j \frac{1}{1-\lambda\phi_1 L}(x_t - \bar{x}),
\end{aligned} \tag{B.4}
$$

where L is the lag operator.

B.2 Derivation of the Subjective Volatility Feedback Effect

To derive an expression for the subjective volatility feedback effect and a restriction among parameters implied by the present value model, we first need to compute the term $(E_{t+1}^* - E_t)(r_{t+1+j})$ in equation (2.2).

From equation (2.10), we obtain $E_{t+1}^*(r_{t+1+j})$ and $E_t^*(r_{t+1+j})$ as

$$
\begin{aligned}
E_{t+1}^*[r_{t+1+j}] =&\gamma_0^* + \gamma_1(1-\lambda)\alpha_1\phi_1^j \frac{1}{1-\lambda\phi_1 L}(x_{t+1} - \bar{x}), \\
E_t^*[r_{t+1+j}] =&\gamma_0^* + \gamma_1(1-\lambda)\alpha_1\phi_1^{j+1} \frac{1}{1-\lambda\phi_1 L}(x_t - \bar{x}),
\end{aligned} \tag{B.5}
$$

from which we get the expression for $(E_{t+1}^* - E_t)(r_{t+1+j})$ as

$$
\begin{aligned}
E_{t+1}^*[r_{t+1+j}] - E_t^*[r_{t+1+j}] =&\gamma_1(1-\lambda)\alpha_1\phi_1^j \frac{1}{1-\lambda\phi_1 L}[x_{t+1} - \bar{x} - \phi_1(x_t - \bar{x})] \\
=&\gamma_1(1-\lambda)\alpha_1\phi_1^j \frac{1}{1-\lambda\phi_1 L}\bar{x}v_{t+1},
\end{aligned} \tag{B.6}
$$

where we use equation (2.6) to achieve the second equality.

By substituting equation (B.6) into equation (2.2), we have

$$\begin{aligned}
f^*_{t+1} =& -\left(E^*_{t+1} - E^*_t\right)\sum_{j=1}^{\infty}\kappa^j r_{t+1+j} \\
=& -\sum_{j=1}^{\infty}\kappa^j\gamma_1(1-\lambda)\alpha_1\phi_1^j\frac{1}{1-\lambda\phi_1 L}\bar{x}v_{t+1} \\
=& -\gamma_1(1-\lambda)\alpha_1\left[\sum_{j=1}^{\infty}(\kappa\phi_1)^j\right]\frac{1}{1-\lambda\phi_1 L}\bar{x}v_{t+1} \\
=& -\gamma_1(1-\lambda)\alpha_1\frac{\kappa\phi_1}{1-\kappa\phi_1}\frac{1}{1-\lambda\phi_1 L}\bar{x}v_{t+1}. \qquad \text{(B.7)}
\end{aligned}$$

Then, by defining $\delta = -\gamma_1(1-\lambda)\alpha_1\frac{\kappa\phi_1}{1-\kappa\phi_1}$, we obtain the expression for the subjective volatility feedback effect as

$$\begin{aligned}
f^*_{t+1} =& \delta\frac{1}{1-\lambda\phi_1 L}\bar{x}v_{t+1} \\
=& \delta\bar{x}v_{t+1} + \delta\frac{\lambda\phi_1}{1-\lambda\phi_1 L}\bar{x}v_t. \qquad \text{(B.8)}
\end{aligned}$$

B.3 Derivation of the Implied Objective Risk premium

After substituting the subjective risk premium in equation (2.10) and the subjective volatility feedback effect in equation (2.11) into equation (2.1), we obtain

$$r_{t+1} = \gamma^*_0 + \gamma_1(1-\lambda)\alpha_1\phi_1\frac{1}{\lambda\phi_1 L}(x_t - \bar{x}) + \frac{\lambda\phi_1}{1-\lambda\phi_1 L}\delta\bar{x}v_t + \delta\bar{x}v_{t+1} + \varepsilon_{t+1}. \qquad \text{(B.9)}$$

Because the objective risk premium $E_t(r_{t+1})$ can be defined as a part of the excess stock return that is known to econometricians at the end of period t, by defining $\mu_t \equiv E_t(r_{t+1})$, we can rewrite equation (B.9) as

$$r_{t+1} = \mu_t + \delta\bar{x}v_{t+1} + \varepsilon_{t+1}, \qquad \text{(B.10)}$$

where,

$$\mu_t = \gamma^*_0 + \gamma_1(1-\lambda)\alpha_1\phi_1\frac{1}{\lambda\phi_1 L}(x_t - \bar{x}) + \frac{\lambda\phi_1}{1-\lambda\phi_1 L}\delta\bar{x}v_t. \qquad \text{(B.11)}$$

Then, by multiplying $(1-\lambda\phi_1 L)$ on both sides of equation (B.11), we obtain the following dynamics for the implied objective risk premium:

$$\mu_t = \beta_0 + \lambda\phi_1\mu_{t-1} + \beta_1(x_t - \bar{x}) + \lambda\phi_1\delta\bar{x}v_t, \tag{B.12}$$

where $\beta_0 = (1-\lambda\phi_1)[\gamma_0^* + \gamma_1(\alpha_0 + \alpha_1\bar{x})]$, and $\beta_1 = \gamma_1(1-\lambda)\alpha_1\phi_1$.

Appendix C

DOES THE RELATIVE IMPORTANCE OF THE DISCOUNT RATE NEWS AND THE CASH FLOW NEWS CHANGE WITH MARKET STATE AND TIME?

C.1 Priors for the TVAR

I employ a natural conjugate prior for the parameters of threshold VAR using artificial dummy observations. Given the dummy observations, Y_D and X_D, the priors for coefficient and variance-covariance matrix are

$$\begin{aligned} p(B_0|\Omega_0) &\sim N(vec(b_0), \Omega \otimes (X_D' X_D)^{-1}) \\ p(\Omega_0) &\sim IW(S_0, T_D - K), \end{aligned} \tag{C.1}$$

where T_D is the length of the dummy observations, K is the number of coefficients in each equation of VAR, and b_0 and S_0 are OLS estimates of the coefficients and the sum of squared residuals such that

$$\begin{aligned} b_0 &= (X_D' X_D)^{-1}(X_D' Y_D) \\ S_0 &= (Y_D - X_D B_0)'(Y_D - X_D B_0). \end{aligned} \tag{C.2}$$

Following Banbura et al. (2007), for a VAR(P) with N variables, Y_D and X_D can be generated as follows:

$$Y_D = \begin{bmatrix} \frac{diag(\gamma_1\sigma_1 \cdots \gamma_N\sigma_N)}{\lambda} \\ 0_{N(P-1)\times N} \\ diag(\sigma_1 \cdots \sigma_N) \\ 0_{1\times N} \end{bmatrix}, \; X_D = \begin{bmatrix} \frac{diag(1\cdots P)\otimes diag(\sigma_1\cdots\sigma_N)}{\lambda} & 0_{NP\times 1} \\ 0_{N\times NP} & 0_{N\times 1} \\ 0_{1\times NP} & \delta \end{bmatrix}, \tag{C.3}$$

where γ_i and σ_i for $i = 1, \cdots, N$ are the OLS estimates for the coefficient of first lag and standard error obtained by individual AR(1) regressions, λ is a hyperparameter controls the overall tightness of the prior, and δ controls the tightness of the prior on constants (intercept terms) of VAR. Following Alessandri and Mumtaz (2017), I set $\lambda = 0.1$ and $\delta = 0.0001$.

I assume a Normal prior for the threshold value z^*, $z^* \sim N(\bar{z}, \bar{v})$. The prior mean $\bar{z}$ is set as the sample mean of the threshold variable, $\frac{1}{T}\sum_{t=1}^{T} Z_t$. The prior variance $\bar{v}$ is set as 10, following Alessandri and Mumtaz (2017).

C.2 Priors for the TVP-VAR-SV

Following Primiceri (2005), I assume Normal priors are employed for B, A, and $log\sigma$. To calibrate the prior distributions, I used the first 60 observations (from January 1948 to December 1952) as a training sample.

$$\begin{aligned} B_0 &\sim N(\hat{B}_{OLS}, 4V(\hat{B}_{OLS})), \\ A_0 &\sim N(\hat{A}_{OLS}, 4V(\hat{A}_{OLS})), \\ log\sigma_0 &\sim N(log\hat{\sigma}_{OLS}, I_n), \end{aligned} \tag{C.4}$$

where $\hat{B}_{OLS}$ and $V(\hat{B}_{OLS})$ are the OLS estimates for coefficients and their variance, $\hat{A}_{OLS}$ and $V(\hat{A}_{OLS})$ are obtained by using OLS residuals, and $log\sigma$ is the logarithm of the OLS estimates of the standard errors in a constant parameter VAR using the training sample.

I employ inverse-Wishart distributions for the variance of innovations, Q, S, and W.

$$\begin{aligned} Q &\sim IW(k_Q^2 \cdot 60 \cdot V(\hat{B}_{OLS}), 60), \\ W &\sim IW(k_W^2 \cdot I_n, 4), \\ S_1 &\sim IW(k_S^2 \cdot 2 \cdot V(\hat{A}_{1,OLS}), 2), \\ S_2 &\sim IW(k_S^2 \cdot 3 \cdot V(\hat{A}_{2,OLS}), 3), \\ S_3 &\sim IW(k_S^2 \cdot 4 \cdot V(\hat{A}_{3,OLS}), 4), \\ S_3 &\sim IW(k_S^2 \cdot 5 \cdot V(\hat{A}_{4,OLS}), 5), \\ S_3 &\sim IW(k_S^2 \cdot 6 \cdot V(\hat{A}_{5,OLS}), 6), \end{aligned} \tag{C.5}$$

where S_1 to S_5 denote the five blocks of S, and A_1 to A_5 are five corresponding blocks of $\hat{A}_{OLS}$. Because 6 variables are included in the state vector of the TVP-VAR-SV, there are 5 blocks in S and A. For the hyperparameters controlling the scale matrix of inverse-Wishart distribution, I use values used in Primiceri (2005): $k_Q = 0.01$, $k_W = 0.01$, and $k_S = 0.1$.

C.3 TVAR Variance Decomposition Result Excluding the Threshold variables from the State Variables

Table C.1: Variance Decomposition Result using the Dividend-Price ratio

	Financial Condition Regimes			
	NFCI $\leq z^*$	NFCI $> z^*$	ANFCI $\leq z^*$	ANFCI $> z^*$
$Var(NCF)$	0.31	0.09	0.30	0.10
$Var(NDR)$	0.21	0.78	0.24	0.65
$Var(real\ rates)$	0.03	0.04	0.03	0.03
$-2cov(NCF, NDR)$	0.35	-0.05	0.37	0.09
$-2cov(NCF, real\ rates)$	0.02	-0.04	0.00	-0.02
$2cov(NDR, real\ rates)$	0.08	0.18	0.06	0.14
	Investor Sentiment Regimes			
	$S_{BW} \leq z^*$	$S_{BW} > z^*$	$S_{BW}^{or} \leq z^*$	$S_{BW}^{or} > z^*$
$Var(NCF)$	0.41	0.09	0.54	0.09
$Var(NDR)$	0.19	0.58	0.10	0.51
$Var(real\ rates)$	0.02	0.02	0.02	0.02
$-2cov(NCF, NDR)$	0.43	0.13	0.35	0.17
$-2cov(NCF, real\ rates)$	-0.04	0.00	-0.02	0.01
$2cov(NDR, real\ rates)$	-0.01	0.18	0.01	0.19

i)The ratios of the variances of cash flow news, discount rate news, real interest rate news, and their covariances relative to the variance of unexpected excess returns are reported.
ii) The state variables are the excess stock return, the real interest rate, the dividend-price ratio, the relative bill rate, the term spread, and the change in bill rate.

Table C.2: Variance Decomposition Result using the Book-to-Market ratio

	Financial Condition Regimes			
	NFCI $\leq z^*$	NFCI $> z^*$	ANFCI $\leq z^*$	ANFCI $> z^*$
$Var(NCF)$	0.87	0.37	0.87	0.38
$Var(NDR)$	0.02	0.80	0.02	0.91
$Var(real\ rates)$	0.02	0.04	0.02	0.03
$-2cov(NCF, NDR)$	0.08	-0.37	0.10	-0.46
$-2cov(NCF, real\ rates)$	-0.04	-0.03	-0.04	-0.03
$2cov(NDR, real\ rates)$	0.04	0.19	0.03	0.17
	Investor Sentiment Regimes			
	$S_{BW} \leq z^*$	$S_{BW} > z^*$	$S_{BW}^{or} \leq z^*$	$S_{BW}^{or} > z^*$
$Var(NCF)$	1.01	0.51	1.06	0.49
$Var(NDR)$	0.03	0.35	0.04	0.35
$Var(real\ rates)$	0.02	0.03	0.03	0.04
$-2cov(NCF, NDR)$	0.05	-0.04	-0.03	-0.04
$-2cov(NCF, real\ rates)$	-0.10	-0.02	-0.09	-0.01
$2cov(NDR, real\ rates)$	-0.01	0.17	-0.02	0.17

i)The ratios of the variances of cash flow news, discount rate news, real interest rate news, and their covariances relative to the variance of unexpected excess returns are reported.
ii) The state variables are the excess stock return, the real interest rate, the book-to-market ratio, the relative bill rate, the term spread, and the change in bill rate.