

Optimizing Renewable Energy Utilization Ratio with Model Predictive  
Control

Michael Hockman

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Committee:  
Thillainathan Logentitiran  
Jie Sheng

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Michael Hockman

University of Washington

**Abstract**

Optimizing Renewable Energy Utilization Ratio with Model Predictive Control

Michael Hockman

Chair of the Supervisory Committee:  
Thillainathan Logentirani  
Electrical Power Engineering

This work focuses on optimizing the performance of power networks by maximizing and optimizing the utilization of renewable energy sources (RESs). In order to accomplish this, a cooperative distributed model predictive control scheme is used in which each microgrid subsystem consists of a controllable load, an energy storage system (ESS), and a non-renewable controllable generator. This thesis will also be looking at methods of increasing the computational efficiency of previously established algorithms. The result is better utilization of available RESs while also keeping supply-demand balance satisfied all in a more computationally efficient manner than would be otherwise possible. Simulated results are promising, showing that the utilization of RESs in the network as a whole is increased while also preventing deep discharging of the ESSs. This demonstrates the feasibility of the project as a whole.

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# 1 Introduction

Renewable energy sources are quickly becoming a larger and larger part of our power grid. However, renewable distributed generators introduce new challenges that must be overcome while at the same time integrating this emerging technology with our existing grid while not effecting performance on the customer's end. This is a large and complicated task, and there are real and costly consequences if it isn't done correctly. The complication mainly comes from the variable nature of the renewable energy sources (RESs), and is best dealt with by maximizing the utilization of renewable powers as soon as they are generated. Therefore, computationally efficient algorithms for controlling the utilization of these renewable energy sources is of the utmost importance for transitioning smoothly into an era of renewable resources.

With this in mind, this paper will be contributing to the work started by Yigao Du et al.[2] in the creation of algorithms to optimize a network of cooperating distributed microgrids by using the concept of model predictive control and load control techniques.

Each MG operates independently but is able to use information from other MGs in the system to determine the optimal set of actions to take at any given time. This is important because renewable power has a variable output, and therefore must be used or stored immediately after it is generated.

The first goal of this paper will be developing an algorithm to determine the percentage of total power that needs to be shared among the microgrids. This term will be represented by the variable  $\alpha_i$ , where  $0 \leq \alpha_i \leq 1$ , which is the percentage of the total power that the  $i^{th}$  microgrid is taking in.

The second concept this paper will be introducing is the power provided to the  $i^{th}$  microgrid by the  $i^{th}$  controllable generator, represented by the variable  $P_i^C$ . Even though this paper focuses on maximizing the use of renewable energies, in the case that renewables aren't enough to satisfy the load, non-renewable generators need to be available to satisfy customer demand.

The third concept introduced in this paper is a more computationally efficient algorithm that takes advantage of matrix equations and the load

forecast. This will reduce the number of total steps needed for the algorithm to maximize the energy utilization ratio, defined in Equation (1), and allow for an optimization strategy to be calculated offline or in real time.

$$REUR = \frac{\sum_{i=1}^N P_i^{REU}(k)}{\sum_{i=1}^N P_i^L(k)} \quad (1)$$

Where  $P_i^{REU}(k)$  is the total amount of renewable energy utilized by the  $i^{th}$  microgrid at time  $k$ ,  $P_i^L(k)$  is the load power of in the  $i^{th}$  microgrid at time  $k$ , and  $N$  is the total number of microgrids in the system.

Assuming initial access to forecasted load and RES power data, the plan is to break the problem of maximizing renewable energy utilization up into 3 different problems, which are as follows.

1. Optimizing the power sharing coefficient  $\alpha_i \forall i$ , which represents the percentage of total excess power each microgrid gets. It is assumed that all excess power is split, and that microgrids that generate excess power borrow no power from other microgrids.
2. Computer power coordination scheduling, which determines the power going into each microgrids ESS at time  $k$ , which allows us to determine if the subsystem is charging or discharging and maximize use of RESs.
3. Compute the optimal load power,  $\mathbf{P}_{L,opt}$ , that is to be implemented using load control methods

Throughout all these steps, matrix equations will be created to simplify the algorithm and reduce computational complexity.

## 1.1 Microgrid Review

A microgrid is defined by United States Department of Energy Microgrid Exchange Group as a group of interconnected loads and distributed energy resources (DERs) within clearly defined electrical boundaries that acts as a single controllable entity with respect to the grid [19]. This means that it can be viewed as a local electric grid which is able to act either independently (islanded mode) or work together with other microgrids and the main grid (grid connected). An example of such a system is shown in the figure below.

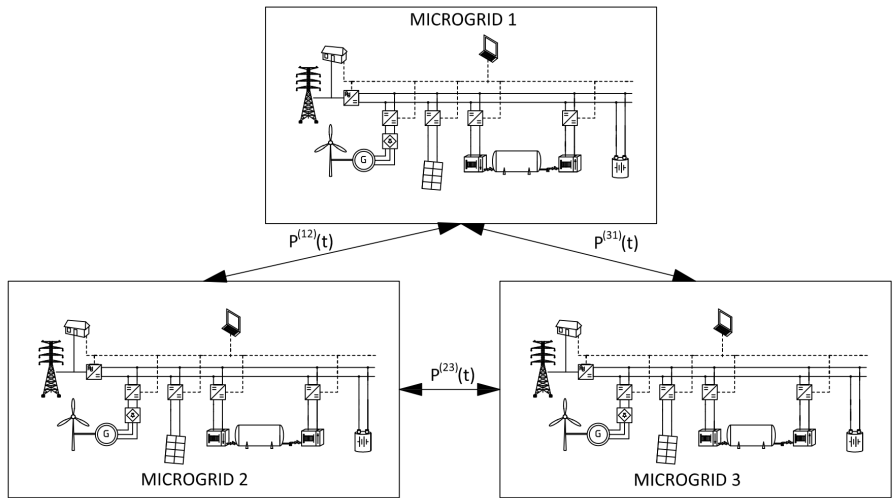


Figure 1: Typical Microgrid Scheme [3]

Each grid is typically made up of a load, energy storage system, and a controllable generator. For the purposes of this paper, each MG is defined as a system with an energy storage system (ESS) (such as batteries, fuel cells, and flywheels) a renewable energy source (RES) (such as solar, wind or hydro), and a load that must be satisfied. E

There are three main ways that the microgrid energy management system (MEMS) can control the network.

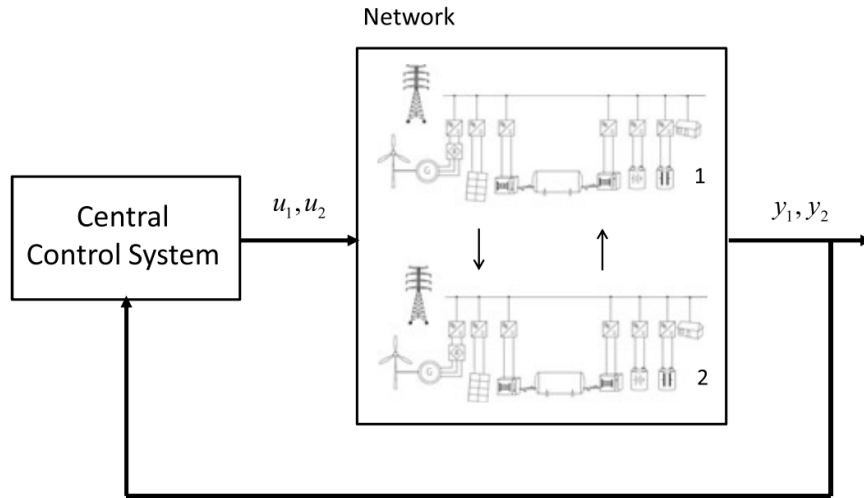


Figure 2: Centralized control architecture [3]

1. Using a method where one single aggregator controls all of the systems known as a centralized control scheme, as shown in Figure [2]. This method can become very computationally intensive as the size of the grid increases.
2. Using a method where the loads and distributed generators are split into subsystems known as microgrids, wherein each microgrid has a local aggregator that maximizes local constraints, known as decentralized control, shown in Figure [3]. This reduces the computational burden when dealing with large systems, but each microgrid is unable to access any information on neighboring microgrids, which can lead to the network as a whole behaving sub-optimally.
3. Using a distributed control scheme, shown in Figure [4], which is the same as the 2<sup>nd</sup> method except microgrids in the subsystem are able to access either partial or full information about each other. Using this, each microgrid subsystem is able to work cooperatively towards maximizing a global cost function, thereby improving system performance.

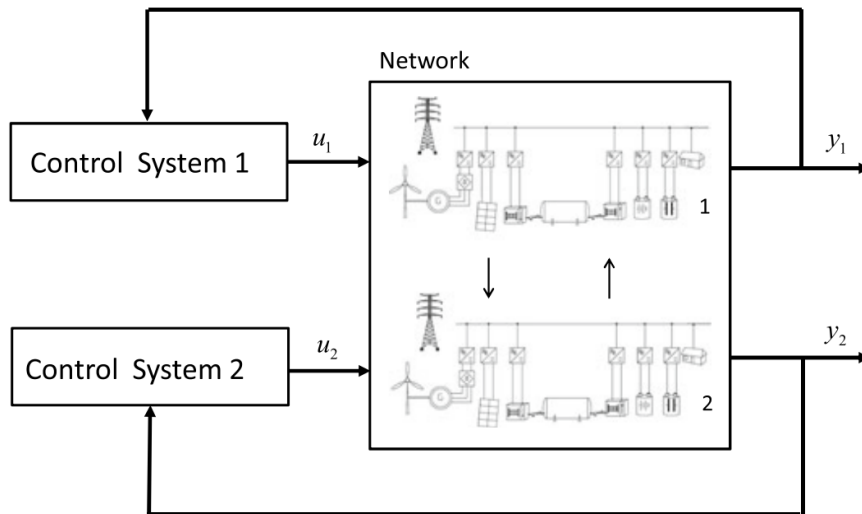


Figure 3: Decentralized control architecture [3]

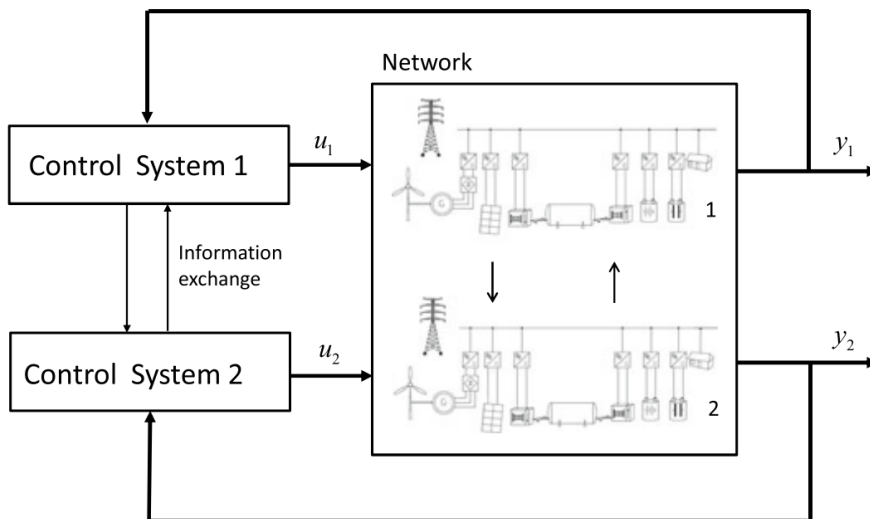


Figure 4: Distributed control architecture [3]

Overall, the trend of moving from a fully centralized electric grid to a more distributed electric grid as well as the difficulty in controlling renewable energy resources makes microgrids, and especially the control of renewable

power in microgrids, an important topic for this thesis.

Regardless of control architecture, a hierarchical control structure for all microgrids is typically used. In this system there is a primary, secondary, and tertiary level where different optimizations are done based on different time scales. Primary is the shortest time scale, while the tertiary level is the longest time scale. A summary of these levels is shown in Figure [5]

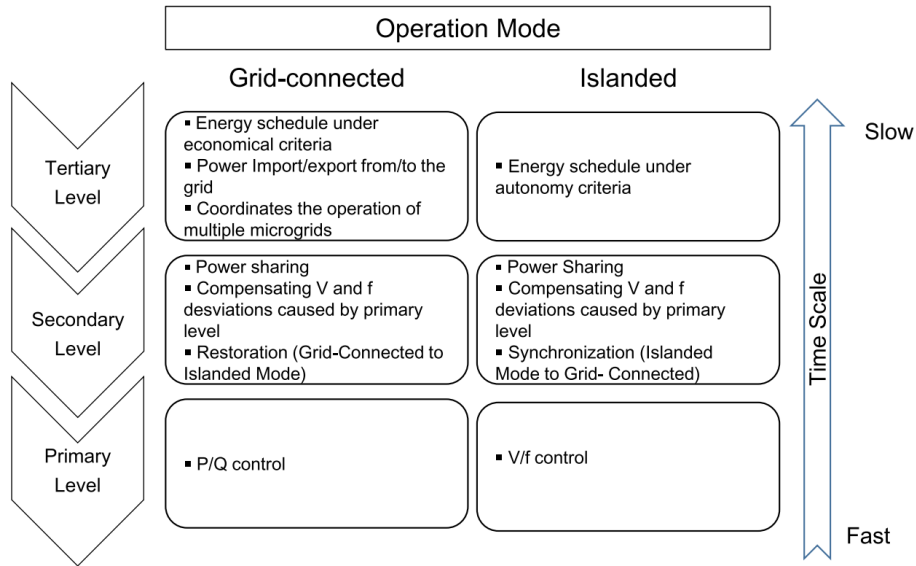


Figure 5: Hierarchical control levels[3]

The main point of interest for this thesis will be on development of an energy management system (EMS), which includes a mix of the secondary and tertiary levels.

## 1.2 MPC Review

Model predictive control (MPC) is an method of controlling and optimizing a process while staying within some predetermined constraints. A major advantage of this method is that it takes into account future states of the system it is controlling, which makes it a perfect fit for power systems where it is common to have access to forecast data which predicts values of load and renewable powers available at certain times.

To use MPC a set of equation in the follow form must be available

$$x(k + 1) = A(k)x(k) + B(k)u(k) \quad (2)$$

$$y(k) = C(k)x(k) + D(k)u(k) \quad (3)$$

Where  $x(k)$  is the state,  $y(k)$  is the output, and  $u(k)$  is the input control signal. For increased accuracy, it is usually written in incremental form with  $\Delta u(k)$  replacing  $u(k)$ . This allows the system of equations to be written as follows [3].

$$\begin{bmatrix} x(k + 1) \\ u(k) \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ u(k - 1) \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} \Delta u(k) \quad (4)$$

$$y(k + 1) = [C \quad 0] \begin{bmatrix} x(k) \\ u(k - 1) \end{bmatrix} \quad (5)$$

Which gives a complete dynamic model of the system. A cost function can then be defined as follows

$$J(N_p, N_c) = \sum_{j=1}^{N_p} \|y(k + j|k) - w(k + j)\|_R^2 + \sum_{j=1}^{N_c} \|\Delta u(t + j - 1)\|_P^2 \quad (6)$$

Where  $\|y(k + j|k) - w(k + j)\|_R^2 = [y(k + j|k) - w(k + j)]^T R [y(k + j|k) - w(k + j)]$ ,  $w(k)$  is the reference output at time  $k$ ,  $N_p$  is the prediction horizon, and  $N_c$  is the control horizon. Notice the second term puts a weighting to the change in the control variable, thus putting a limit on how quickly the input can be changed.

With the ability to calculate the dynamics of the system, it is now possible to predict the systems future states, and from there it is possible to find optimal values of the control variable to track a referenced output. A visual representation of this strategy is shown in Figure [6]

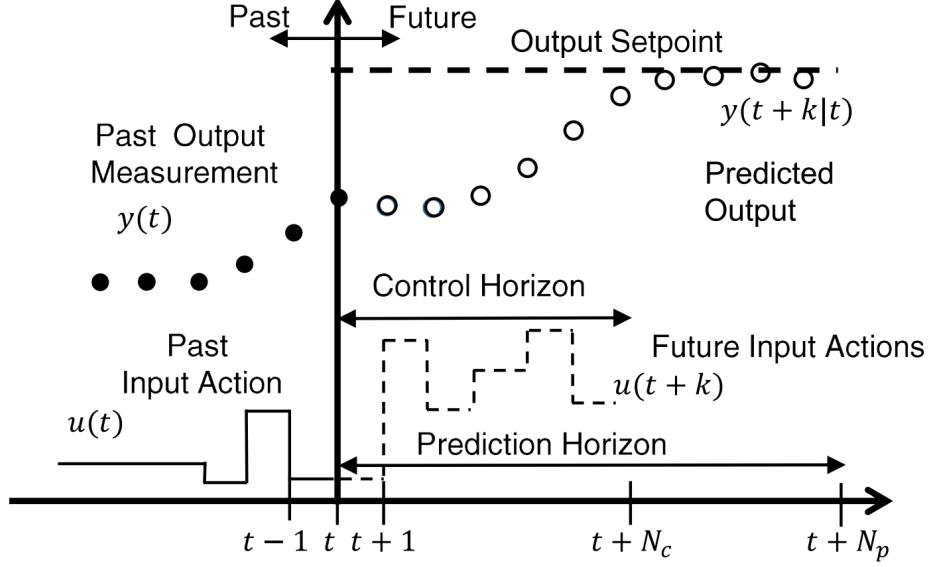


Figure 6: MPC Control Scheme [3]

The ability for MPC to control systems within certain bounds while taking future information into account makes it an ideal fit to work with microgrids. In particular, a method known as distributed model predicted control (DMPC) is an ideal technique to control a large number of microgrids with minimal complexity. A state-space representation for a DMPC network is shown in the formula below.

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{i \neq j} A_{ij}x_j(k) + \sum_{j=1}^N B_{ij}u_j(k) + \sum_{j=1}^N D_{ij}d_j(k) \quad (7)$$

This formula is similar to Equation (2) but state and input coupling from neighboring microgrids in the network are taken into consideration.

A key idea is each microgrid subsystem works together to maximize a global objective function, shown in the equation below. This is opposed to a decentralized approach where each subsystem focuses only on maximizing a local objective function.

$$J_{global} = \sum_{i=1}^N J_{local}^i(\mathbf{x}, \mathbf{u}) \quad (8)$$

Where  $\mathbf{x}$  and  $\mathbf{u}$  are the state and input variables respectively.

When using an iterative method to optimize the network, interconnection variables must be defined as shown in the equation below.

$$v_i(k) = [u_{1i}(k) \quad u_{2i}(k) \quad u_{3i} \quad \dots \quad u_{Ni}] \quad (9)$$

Where  $u_{ij}(k)$  is the power exchanged between microgrid subsystems  $i$  and  $j$ , calculated by microgrid  $i$ . In order for a solution to be reached,  $u_{ij}(k) = u_{ji}(k)$  must hold true.

### 1.3 Literature Review

Networks of distributed microgrids are optimized by controlling shiftable loads, energy storage systems, and distributed generators in [4]. They also take into account factors such as economic performance, minimization of harmful environmental effects, minimizing peak demand, and smoothing the curve.

The primary and secondary levels (see figure 5) are discussed and optimized in [5]. This allows for more precise control but at the cost of a more complex and computationally intensive algorithm. Similarly, these same topics, as well as tertiary level control methods, are discussed in [10]. Optimization of the secondary control level is focused on exclusively in [12].

Fuel cells and electrolyzer are used as an ESS in order to balance the renewable generators uncertain output [6]. It also takes factors such as machine degradation into account, which makes it stand out compared to other similar papers.

When it comes to the use of specific types of renewables, controlling microgrid networks that include photovoltaic (PV), wind, and hydro as RESs have been discussed in detail [7][8][11].

In recent years, many other control techniques using the distributed control architecture scheme have been proposed [13]-[16]. Similarly, comparisons between centralized and decentralized approaches have been made when simulating interactions between the microgrids and the electricity market [19]. Decentralized methods are still a relevant topic in some contexts as well

[17][18].

The idea of using a distributed control scheme and model predictive control to maximize the renewable energy utilization ratio of a network of microgrids was introduced in [1]. This is the topic that will be mainly discussed in this thesis, as the contents are extremely promising, and there is room for improvement with the addition of several other control techniques.

## 2 System Modeling and Problem Formulation

The power distribution system being described and modeled is a network of microgrid subsystems, each containing a controllable load, an ESS, a RES, and a controllable generator. The table below describes commonly used variable names when it comes to this topic.

Table 1: Variable Names and Descriptions

Variable	Description
$\mathbf{F}_P^L$	The load forecast power. Is a matrix of size N by $t_f$
$\mathbf{F}_P^{RES}$	Renewable energy source forecasted power. Is a matrix of size N by $t_f$
$\mathbf{P}_S(k)$	Forecasted mismatch power of the microgrids at time k. Is a column vector of size N.
$\mathbf{P}_P(k)$	All positive entries of $\mathbf{P}_S(k)$ . Is a column vector of size N.
$\mathbf{P}_N(k)$	All negative entries of $\mathbf{P}_S(k)$ . Is a column vector of size N.
$\mathbf{P}_{Shr}(k)$	The power being shared by the microgrids at time k. Is a column vector of size N. Is always the same as $\mathbf{P}_P(k)$ since all excess power is shared.
$\mathbf{P}_{Brw}(k)$	The power being borrowed by the microgrids at time k. Is a column vector of size N.
$P_i^L(k)$	Forecasted load power of microgrid i at time k.
$P_i^{RES}(k)$	Forecasted renewable energy source power of microgrid i at time k.
$P_i^S(k)$	Forecasted mismatch power of microgrid i at time k.
$\mathbf{F}_\alpha$	Forecasted need for power. Is a matrix of size N by $t_f$ . Entry (i, k) is the percentage of the systems excess power that is going to microgrid i at time k. In other words, the "need for power" of microgrid i at time k.
$\boldsymbol{\alpha}(k)$	Forecasted need for power at time k. Is a column vector of size N.
$\alpha_i(k)$	Forecasted need for power of microgrid i at time k.
N	Total number of microgrids in the system.
$t_f$	Length of time in hrs that the forecast goes on.

The system involves a network of N microgrids each capable of working in either islanded or grid connected mode. Each microgrid consists of a load, an energy storage system, and a renewable energy source, and is able to share the state of each of these components with other microgrids in the

subsystem, as is shown in figure 1.

## 2.1 Modeling of Load and RES

Throughout this thesis there will be an assumption that forecasted data  $\mathbf{F}_P^L(k)$  and  $\mathbf{F}_P^{RES}(k)$  is given ahead of time, and are defined as.

$$\mathbf{F}_P^L(k) = [\mathbf{P}_L(1) \quad \mathbf{P}_L(2) \quad \mathbf{P}_L(3) \quad \dots \quad \mathbf{P}_L(t_f)]$$

$$\mathbf{F}_P^{RES}(k) = [\mathbf{P}_{RES}(1) \quad \mathbf{P}_{RES}(2) \quad \mathbf{P}_{RES}(3) \quad \dots \quad \mathbf{P}_{RES}(t_f)]$$

Where each entry  $\mathbf{P}_L(k)$  and  $\mathbf{P}_{RES}(k)$  is defined as the load power and renewable energy source power respectively at time  $t = k$ , defined below.

$$\mathbf{P}_L(k) = [P_1^L(k) \quad P_2^L(k) \quad P_3^L(k) \quad \dots \quad P_N^L(k)]^T$$

$$\mathbf{P}_{RES}(k) = [P_1^{RES}(k) \quad P_2^{RES}(k) \quad P_3^{RES}(k) \quad \dots \quad P_N^{RES}(k)]^T$$

From this it can be seen that both matrices  $\mathbf{F}_P^L(k)$  and  $\mathbf{F}_P^{RES}(k)$  are of size  $N$  by  $t_f$ , which are the number of grids in the system and the total length of the forecast in hours respectively.

In this project it can be assumed that the forecast accurately predicts the RES and load power of the subsystems. However, in [2] real time measurements of the RES system are used to update the forecast and update the value to one more close to the actual value.

$$P_i^{RES}(k+1) = P_i^{RES}(k) + \alpha_r(P_i^r(k) - P_i^{RES}(k)) \quad (10)$$

Where  $P_i^{RES}(k+1)$  is the forecasted renewable power  $k$ ,  $P_i^r(k)$  is the actual renewable power at time  $k$ , and  $\alpha_r$  is a smoothing constant. However, based on previous stated assumptions, this equation will not be used in the state-space representation of the system dynamics.

Using the load and RES power, the mismatch power forecast is defined as

$$\mathbf{P}_S(k) = \mathbf{P}_{RES}(k) - \mathbf{P}_L(k) = \mathbf{P}_P(k) + \mathbf{P}_N(k) \quad (11)$$

Where the mismatch power,  $\mathbf{P}_S$ , has been split into positive and negative parts,  $\mathbf{P}_P$  and  $\mathbf{P}_N$ , respectively.

## 2.2 Energy Storage System

The energy storage system is the key to the microgrid being able to smoothly handle the intermittent nature of the renewable energy sources by storing excess energy at times of overproduction and using it during times of underproduction. The dynamics of this model is shown in the equation below.

$$E_i(k+1) = E_i(k) + \eta_i P_i^b(k) \quad (12)$$

$$E_{min} \leq E_i(k) \leq E_{max} \quad (13)$$

Where  $E_i(k)$  is the energy of the energy storage system,  $\eta_i$  is the charging efficiency, and  $P_i^b(k)$  is the power going into the energy storage system, all relevant to the  $i^{th}$  microgrid subsystem in the network.

## 2.3 Power Balance

Assume that each microgrid is working cooperatively in islanded mode, meaning they must each satisfy a local load, but are able to share excess power with neighboring microgrids. This means that load power can come from local ESS, RES, controllable generators, or from neighboring microgrids. Therefore, the following power balance equations can be defined.

$$P_i^b(k) = P_i^S(k) + P_{ij}(k) + P_i^C(k) \quad (14)$$

$$P_{ij}(k) = \alpha_i \sum_{j \neq i} P_j(k) \quad (15)$$

$$P_{ij}(k) \geq 0 \quad (16)$$

Where  $P_i^S(k)$  has been defined in Equation (11),  $P_i^C(k)$  is the power produced by the controllable generator,  $P_{ij}(k)$  is the power coming to microgrid i from microgrid j, and  $\alpha_i$  is the percentage of the total excess power being delivered to microgrid i.

## 2.4 System Modeling

A state-space model representing the dynamics of the system can be created using Equations (14) and (12) by defining the state variables  $x_i(k)$  as

$$x_i(k) = \begin{bmatrix} \frac{E_i(k+1)}{\eta_i(k)} \\ P_i^{RES}(k) \end{bmatrix} \quad (17)$$

Which allows for the following state-space representation to be defined

$$x_i(k+1) = Mx_i(k) + Qu_i(k) + A_i \sum_{j=1, j \neq i}^N x_j(k) + B_i \sum_{j=1, j \neq i}^N u_j(k) \quad (18)$$

Where

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad (19)$$

$$Q = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad (20)$$

$$A_i = \begin{bmatrix} 0 & \alpha_i & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (21)$$

$$B_i = \begin{bmatrix} -\alpha_i \\ 0 \end{bmatrix} \quad (22)$$

Subject to (16) and (13)

### 3 Proposed Distributed Model Predictive Control

A cooperative distributed control scheme was chosen in order to overcome the challenges that the uncertain nature of RESs by controlling the use of excess power in other microgrids in the subsystem. In order to accomplish this, each subsystem has to have information on other subsystems in the network, which is accomplished by implementing a local MA that has access to an augmented prediction model that takes all of the subsystems in the network into consideration. This augmented model can be written as

$$\bar{x}(k+1|k) = A\bar{x}(k|k) + B\bar{u}(k|k) \quad (23)$$

Where

$$\bar{x}(k) = [x_1(k|k) \quad x_2(k|k) \quad \dots \quad x_N(k|k)]^T$$

$$\bar{u}(k) = [x_1(k|k) \quad x_2(k|k) \quad \dots \quad x_N(k|k)]^T$$

Based off (18) the matrices A and B can be defined as follows.

$$A = \begin{bmatrix} M & A_1 & A_1 & \dots & A_1 \\ A_2 & M & A_2 & \dots & A_2 \\ A_3 & A_3 & M & \dots & A_3 \\ \dots & \dots & \dots & \dots & \dots \\ A_N & A_N & A_N & \dots & M \end{bmatrix} \quad (24)$$

$$B = \begin{bmatrix} Q & B_1 & B_1 & \dots & B_1 \\ B_2 & Q & B_2 & \dots & B_2 \\ B_3 & B_3 & Q & \dots & B_3 \\ \dots & \dots & \dots & \dots & \dots \\ B_N & B_N & B_N & \dots & Q \end{bmatrix} \quad (25)$$

Where M,  $A_i$ ,  $B_i$ , and Q are defined in equations (19), (21), (22), (20) respectively.

However, in order to use these formulas, methods of finding values for key variables must be determined, which will be explored in the remainder of this section.

### 3.1 Microgrid Power Sharing Coefficient

The power sharing coefficient for the  $i^{th}$  microgrid,  $\alpha_i(k)$ , is the total percentage of the excess power is being delivered to microgrid  $i$ . These values are stored in matrix  $\mathbf{F}_\alpha$ , where the  $(i, k)^{th}$  entry is the power sharing coefficient of MG  $i$  at time  $k$ , which represents the "Need for Power" of microgrid  $i$  at time  $k$ , and is represented with the following matrix

$$\mathbf{F}_\alpha = [\boldsymbol{\alpha}(1) \quad \boldsymbol{\alpha}(2) \quad \dots \quad \boldsymbol{\alpha}(t_f)] \quad (26)$$

Where

$$\boldsymbol{\alpha}(1) = \begin{bmatrix} \alpha_1(1) \\ \alpha_2(1) \\ \dots \\ \alpha_N(1) \end{bmatrix} \quad (27)$$

To find the power sharing coefficient  $\alpha_i$  the following cost function must be defined.

$$G(\boldsymbol{\alpha}(k)) = \sum_{i=1}^N [P_i^N(k) + \alpha_i(k) \sum_{n=1}^N P_n^P(k)]^2 \quad (28)$$

Subject to the following constraints

$$\sum_{i=1, P_i^S(k) < 0}^N \alpha_i = 1 \quad (29)$$

$$\alpha_{i, P_i^S(k) > 0} = 0 \quad (30)$$

Where  $\alpha_{i, P_i^S(k) < 0}$  are the values of  $\alpha$  in MGs where  $P_i^S(k) < 0$ ,  $P_i^N(k)$  is the deficit power, and  $P_i^P(k)$  is the excess of power with regards to the  $i^{th}$  microgrid.

### 3.2 Borrowed and Shared Power

Then matrix  $\mathbf{P}_{ij}(k)$  representing the power being sent from microgrid  $j$  coming to microgrid  $i$  at time  $k$  can then be created from the following equation

$$\mathbf{P}_{ij}(k) = \boldsymbol{\alpha}(k)\mathbf{P}_P(k)^T = \begin{bmatrix} \alpha_1(k) \\ \alpha_2(k) \\ \dots \\ \alpha_N(k) \end{bmatrix} [P_{P,1}(k) \quad P_{P,2}(k) \quad \dots \quad P_{P,N}(k)] \quad (31)$$

Where each column of  $\mathbf{P}_{ij}(k)$  represents the power being shared by microgrid  $j$  to all other microgrids  $i \neq j$ .

The borrowed and shared powers are variables unique to this thesis and are helpful in quickly calculating the battery power without resorting to chains of if/elseif statements.

Recall that each column of  $\mathbf{P}_{ij}(k)$  represents the power being shared by microgrid  $j$  to all other microgrids  $i \neq j$ . Therefore the sum of all these columns will result in a single column vector of size  $N$  where each entry represents the total power being borrowed by microgrid  $i$  at time  $k$ . This is exactly the desired variable,  $\mathbf{P}_{Borrow}(k)$  and can be written as follows.

$$\mathbf{P}_{Borrow}(k) = \mathbf{P}_{ij}(k)\mathbf{1}_{N,1} \quad (32)$$

Similarly, by summing up the rows of  $\mathbf{P}_{ij}(k)$  the shared power can be determined, which can be from the equation below

$$\mathbf{P}_{Shared}(k) = \mathbf{P}_{ij}(k)^T\mathbf{1}_{1,N} \quad (33)$$

Where  $\mathbf{1}_{N,1}$  and  $\mathbf{1}_{1,N}$  are column and row vectors, respectively, of size  $N$  filled with ones.

### 3.3 Energy in ESS

Using the results from the previous section, I am able to define the power going into the ESS as.

$$\mathbf{P}_b(k) = \mathbf{P}_S(k) + \mathbf{P}_{Borrow}(k) - \mathbf{P}_{Shared}(k) = [P_{b,1}(k) \quad P_{b,2}(k) \quad \dots \quad P_{b,N}(k)]^T \quad (34)$$

Equivalently

$$P_i^b(k) = P_i^S(k) + P_i^{Borrow}(k) - P_i^{Shared}(k) \quad (35)$$

From there the charging efficiency of microgrid  $i$  can be defined as

$$\eta_i(k) = \begin{cases} \eta_{ch} & P_{b,i}(k) > 0 \\ \frac{1}{\eta_{dch}} & P_{b,i}(k) < 0 \end{cases}$$

I can then define the following diagonal matrix

$$\boldsymbol{\eta}(k) = \begin{bmatrix} \eta_1(k) & 0 & \dots & 0 \\ 0 & \eta_2(k) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \eta_N(k) \end{bmatrix} \quad (36)$$

Next the energy level in the battery can be defined as

$$\mathbf{E}(k+1) = \mathbf{E}(k) + \boldsymbol{\eta} \mathbf{P}_b(k) \quad (37)$$

Any number of MPC solver tools could be used to control the system using the following state variables  $\mathbf{x}(k) = \mathbf{E}(k)$ ,  $\mathbf{u}(k) = \mathbf{P}_b(k)$ ,  $\mathbf{A} = \mathbf{I}$ , and  $\mathbf{B}(k) = \boldsymbol{\eta}(k)$ . However, in this form the control variable is the battery power instead of the load power.

The energy within each ESS is then determined by the following cost function, using (37), and is used to protect the ESS from damages that come with deep discharging.

$$J_{i,1}(E_i(k)) = (\min(E_i(k) - E_{min}, 0))^2 \quad (38)$$

Under the constraint

$$E_{min} \leq E_i(k) \leq E_{max} \quad (39)$$

### 3.4 Optimizing Load Power

Before optimal load power can be calculated, it is helpful to rewrite various formulas in terms of load power, so that a cost function can be written in terms of a single input variable, which is the load power in this case.

#### 3.4.1 Energy in ESS

The first in rewriting equation (37) is to expand it to the following form.

$$\mathbf{E}(k+1) = \mathbf{E}(k) + \boldsymbol{\eta}(k)[\mathbf{P}_{RES}(k) - \mathbf{P}_L(k) + \mathbf{P}_{Brw}(k) - \mathbf{P}_{Shr}(k)] \quad (40)$$

Alternatively, an explicit expression for the energy in the  $i^{th}$  microgrids ESS can be found as.

$$E_i(k+1) = E_i(k) + \eta_i(k)P_i^b(k) \quad (41)$$

One goal is to simplify the algorithm, which requires all equations to be written in terms of the input variables, which in this case is the load power  $P_i^L(k)$ . In order to do this, an expression for the battery power  $P_i^b(k)$  in terms of the load power must be established.

### 3.4.2 Borrowed and Shared Power

I can define a function  $\gamma_i(k)$  that is 1 when  $P_i^S(k) \geq 0$  and 0 when  $P_i^S(k) < 0$ , which can be written as

$$\gamma_i(k) = \begin{cases} 1 & P_i^{RES}(k) \geq P_i^L(k) \\ 0 & P_i^{RES}(k) < P_i^L(k) \end{cases} \quad (42)$$

I can then define the diagonal matrix  $\mathbf{\Gamma}(k)$  as

$$\mathbf{\Gamma}(k) = \begin{bmatrix} \gamma_1(k) & 0 & \dots & 0 \\ 0 & \gamma_2(k) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \gamma_N(k) \end{bmatrix} \quad (43)$$

I can then define  $\mathbf{P}_P(k)$  as

$$\mathbf{P}_P(k) = \mathbf{\Gamma}(k)\mathbf{P}_S(k) = \mathbf{\Gamma}(k)[\mathbf{P}_{RES}(k) - \mathbf{P}_L(k)] \quad (44)$$

$\mathbf{P}_N(k)$  can then be easily defined as

$$\mathbf{P}_N(k) = \mathbf{P}_S(k) - \mathbf{P}_P(k) = [\mathbf{I}_N - \mathbf{\Gamma}(k)][\mathbf{P}_{RES}(k) - \mathbf{P}_L(k)] \quad (45)$$

Therefore I can rewrite equation (31) by substituting in Equation (44) to get the following expression

$$\mathbf{P}_{ij}(k) = \boldsymbol{\alpha}(k)[\mathbf{P}_{RES}(k) - \mathbf{P}_L(k)]^T \mathbf{\Gamma}(k)^T \quad (46)$$

Since perfect knowledge of RES is assumed for the sake of optimization, and  $\boldsymbol{\alpha}(k)$  is assumed to be optimized in a previous step,  $\mathbf{P}_{ij}(k)$  can be written as purely a function of the control variable, which is the load power.

This means by combining equations (32), (33) and (46) the borrowed and shared power can be written as a function of the load power, as shown below.

$$\mathbf{P}_{Borrow}(\mathbf{P}_L(k)) = \boldsymbol{\alpha}(k)[\mathbf{P}_{RES}(k) - \mathbf{P}_L(k)]^T \boldsymbol{\Gamma}(k)^T \mathbf{1}_{N,1} \quad (47)$$

$$\mathbf{P}_{Share}(k) = \boldsymbol{\Gamma}(k)[\mathbf{P}_{RES}(k) - \mathbf{P}_L(k)] \boldsymbol{\alpha}(k)^T \mathbf{1}_{N,1} \quad (48)$$

### 3.4.3 Battery Power

By substituting equation (46) into equation (34) a new expression for  $\mathbf{P}_b(k)$  is able to be determined.

$$\mathbf{P}_b(k) = \mathbf{P}_{RES}(k) - \mathbf{P}_L(k) + \mathbf{P}_{Borrow}(\mathbf{P}_L(k)) - \mathbf{P}_{Share}(\mathbf{P}_L(k)) \quad (49)$$

Plugging in Equations (47) and (48) gives an expression for the power going into the microgrids ESS at time k in terms of only the input variable,  $\mathbf{P}_L(k)$ , and a known constant,  $\mathbf{P}_{RES}(k)$ .

$$\begin{aligned} \mathbf{P}_b(k) = & \mathbf{P}_{RES}(k) + \boldsymbol{\alpha}(k)\mathbf{P}_{RES}(k)^T \boldsymbol{\Gamma}(k)^T \mathbf{1}_{N,1} - \boldsymbol{\Gamma}(k)\mathbf{P}_{RES}(k)\boldsymbol{\alpha}^T \mathbf{1}_{N,1} \\ & + \boldsymbol{\Gamma}(k)\mathbf{P}_L(k)\boldsymbol{\alpha}^T \mathbf{1}_{N,1} - \mathbf{P}_L(k) - \boldsymbol{\alpha}(k)\mathbf{P}_L(k)^T \boldsymbol{\Gamma}(k)^T \mathbf{1}_{N,1} \end{aligned} \quad (50)$$

Where  $\mathbf{P}_b(k)$  is a column vector of size N where the  $i^{th}$  entry,  $P_i^b(k)$ , represents the power going into the  $i^{th}$  subsystems ESS.

To simplify the equation and separate the constants from the variables the following variable can be defined

$$\mathbf{H}(k) = \mathbf{P}_{RES}(k) + \boldsymbol{\alpha}(k)\mathbf{P}_{RES}(k)^T \boldsymbol{\Gamma}(k)^T \mathbf{1}_{N,1} - \boldsymbol{\Gamma}(k)\mathbf{P}_{RES}(k)\boldsymbol{\alpha}^T \mathbf{1}_{N,1} \quad (51)$$

### 3.4.4 Dynamic Model

By using equations (40) and (50) the equation for the energy in the  $i^{th}$  microgrids energy storage system can be written as.

$$\mathbf{E}(k+1) = \mathbf{E}(k) + \eta[\mathbf{H}(k) + \boldsymbol{\Gamma}(k)\mathbf{P}_L(k)\boldsymbol{\alpha}^T \mathbf{1}_{N,1} - \mathbf{P}_L(k) - \boldsymbol{\alpha}(k)\mathbf{P}_L(k)^T \boldsymbol{\Gamma}(k)^T \mathbf{1}_{N,1}] \quad (52)$$

Due to the complexity of this equations, a typical state-space representation cannot be formed from this equation. However, it serves the same purpose as it allows for  $\mathbf{E}(k+n|k)$  to be computed.

Two additional cost functions can be defined to aid in the control of the load power. The customer satisfactory level  $J_2(\mathbf{P}_L(k))$  and renewable energy utilization level  $J_3(\mathbf{P}_L(k))$  defined below.

$$J_2(\mathbf{P}_L(k)) = \text{sum}\{\min[\mathbf{P}_L(k) - \mathbf{P}_{L,fc}(k), 0]^2\} \quad (53)$$

$$J_3(\mathbf{P}_L(k)) = \text{sum}\{\min[\mathbf{P}_{RES}(k) - \mathbf{P}_L(k), 0]^2\} \quad (54)$$

Where  $\mathbf{P}_{L,fc}(k)$  is the forecasted load power at time k.

The first cost function can be rewritten in terms of the load power as follows by using Equation (52).

$$J_1(\mathbf{P}_L(k)) = \text{sum}\{\min[\mathbf{E}(\mathbf{P}_L(k)) - \mathbf{E}_{min}, 0]^2\} \quad (55)$$

The following constraints can then be added to ensure the optimized load power stays within operating bounds.

$$\mathbf{P}_{L,min} \leq \mathbf{P}_i^L(k) \leq \mathbf{P}_{L,max} \quad (56)$$

A total cost function can then be created by combining the cost functions in Equations (53), (54) and (55) and summing them up for each MG in the system.

$$J(\mathbf{P}_L) = \sum_{m=1}^{N_p} [\chi_1 J_1(\mathbf{P}_L(k+m|k)) + \chi_2 J_2(\mathbf{P}_L(k+m|k)) + \chi_3 J_3(\mathbf{P}_L(k+m|k))] \quad (57)$$

Where  $\chi_i$  are the weighting of  $i^{th}$  MGs cost function  $J_i$ .

### 3.4.5 Optimal Load Power

The previous sections have given the cost function and dynamics of the system, which are the two key components needed to optimize a system using model predictive control. Therefore, by implementing equation (52) to predict the energy, cost function (57) can be constructed which can then be minimized in order to find an optimal value for the load power at time k by using any number of optimization tools.

### 3.5 Controlled Generator Power

The values of  $P_i^C(k)$  could then be chosen in order to satisfy the equation

$$P_i^L(k) = P_i^{RES}(k) + P_i^C(k)$$

Which then allows for control and optimization of the flow of power in the MGs based off forecasted values.

### 3.6 Proposed Algorithm

With the results derived in the previous sections I am able to propose 3 algorithms that together form the complete energy management system. The first algorithm finds optimal values of the power sharing coefficient  $\alpha$  and is shown below.

#### Algorithm 1 - Power Sharing Coefficient Optimization

- 1: Initialize  $\mathbf{P}_L(k)$ ,  $\mathbf{P}_{RES}(k)$ ,  $\forall k 1, 2, 3, \dots, N$
- 2: Compute  $\mathbf{P}_S(k) = \mathbf{P}_{RES}(k) - \mathbf{P}_L(k)$
- 3: Compute  $\mathbf{\Gamma}(k)$  using Equations (43) and (44)
- 4: **for**  $i = 1$  **to**  $i = N$
- 5:     **if**  $P_i^S(k) < 0$
- 6:          $G_i(k) = [P_i^S(k) + \alpha_i(k) \sum_{n=1}^N P_n^P(k)]^2$
- 7:     **else**
- 8:         Set  $G_i(k) = 0$
- 9: **end for**
- 10: Compute cost function (28) using the formula  $G(\boldsymbol{\alpha}(k)) = \sum_{i=1}^N G_i(k)$
- 11: Compute value of  $\boldsymbol{\alpha}(k)$  by minimizing the cost function  $G(\boldsymbol{\alpha}(k))$  subject to constraints from equations (29) and (30).

The next proposed algorithm, shown below, is the power scheduling algorithm which calculates the power going into the ESSs and determines if each subsystem is charging/discharging.

#### Algorithm 2 - Power Coordination Scheduling

- 1: Initialize  $\mathbf{P}_L(k)$ ,  $\mathbf{P}_{RES}(k)$ ,  $\forall k 1, 2, 3, \dots, N$
- 2: Compute  $\mathbf{\Gamma}(k)$  using Equations (43) and (44)
- 3: Compute  $\mathbf{P}_b(k)$  using Equation (50)
- 4: If  $P_i^b(k) > 0$ , the  $i^{th}$  MGs ESS is charging, otherwise it is discharging, where  $P_i^b(k)$  is the  $i^{th}$  entry of  $\mathbf{P}_b(k)$

The final proposed algorithm is the distributed MPC, shown below, in which optimal values of the load power are calculated using iterative techniques.

### Algorithm 3 - Proposed Distributed MPC

- 1: Initialize  $\mathbf{P}_L(k)$ ,  $\mathbf{P}_{RES}(k)$ ,  $\forall k 1, 2, 3, \dots, N$
- 2: **for**  $k = 1$  to  $k = \text{tf}$  **do**
- 3:     **until** convergence
- 4:         Compute optimal  $\mathbf{P}_{L,opt}(k)$  by minimizing cost function (57)
- 5:         Send  $\mathbf{P}_{Shr}(k)$  to and receive  $\mathbf{P}_{Brw}(k)$  from neighboring MGs.
- 6:     Implement optimized load  $\mathbf{P}_L(k) = \mathbf{P}_{L,opt}(k)$
- 7:     Increment  $k = k+1$
- 8: **end for**

The algorithms are used sequentially in order to optimize the entire system. First the power sharing coefficient  $\alpha$  is found, which is then used to find the power going into the  $i^{th}$  MGs ESS  $\mathbf{P}_b(k)$ , which is then used to calculate the optimal load power  $\mathbf{P}_{L,opt}(k)$

## 4 Results

The proposed methodology has been applied to a simulated network of 3 microgrids, assuming the controllable power  $P_i^C(k) = 0 \text{ W } \forall k$ . Initial energy in the ESSs were defined to be  $E_i(0) = 40\text{pu}$ , the min and max energy was defined to be  $E_{min} = 5\text{pu}$  and  $E_{max} = 60\text{pu}$ , the charging efficiency was defined to be  $\eta_{ch} = 0.7$ ,  $\eta_{dch} = 0.65$ , and the control and prediction horizon used are  $N_c = 1$ ,  $N_p = 4$

From the following figure, it can be seen that load control methods were applied when power from RES weren't enough to cover the load.

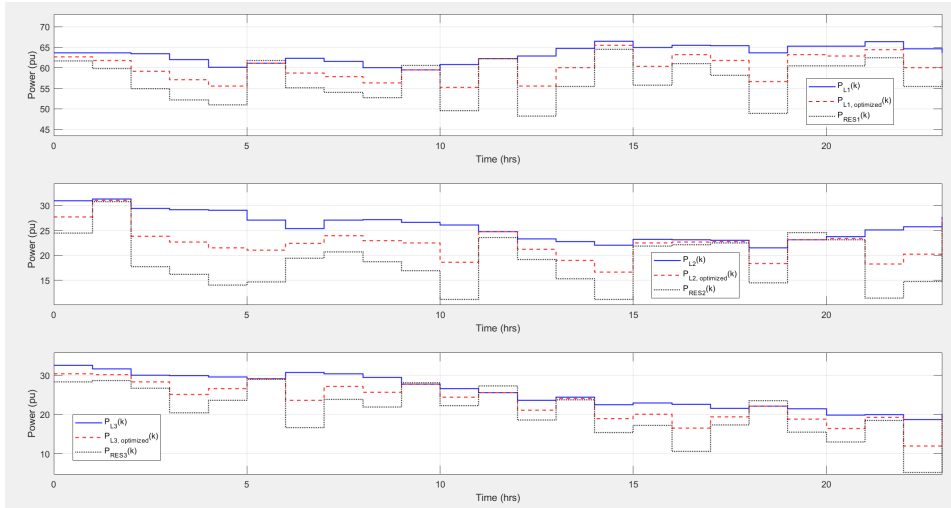


Figure 7: Optimized Load Power

As a result, a larger percentage of the total power drawn was supplied from RES, resulting in a higher value of the REUR, defined in Equation (1), as seen from figure 8.

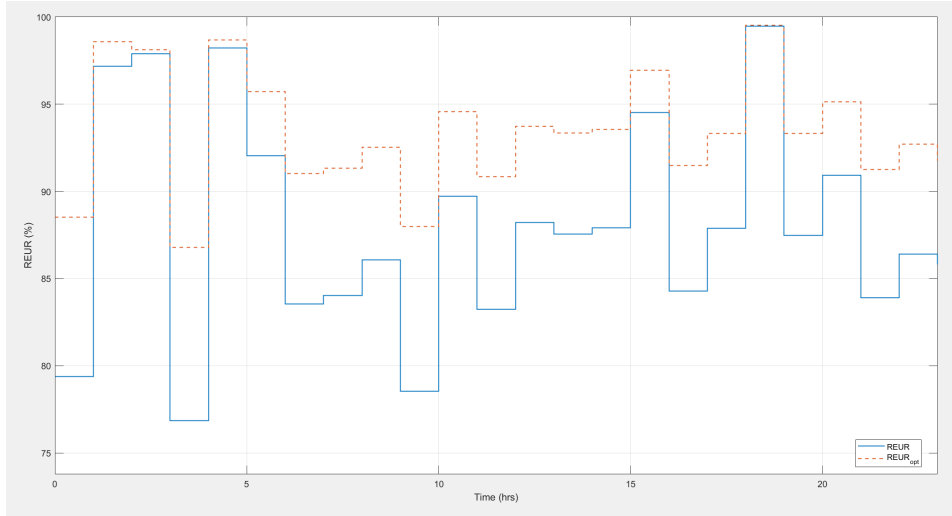


Figure 8: Optimized Renewable Energy Utilization Ratio

This increase in the utilization of available RESs is especially apparent from during the time  $5 \leq t \leq 17$ . This shows that the proposed methodology is successful in its goal of maximizing the utilization of RESs.

One thing to note is that if the load powers in all the microgrids are greater than the available renewable powers, the optimized REUR will be the same. Otherwise the optimized REUR will be greater than the originals, which guarantees the algorithms will produce equal or better utilization of RESs.

The constraints that model predictive control allows also ensures that the ESSs stay within acceptable bounds, shown in the figure below, which leads to an increased device lifetime and improved system performance.

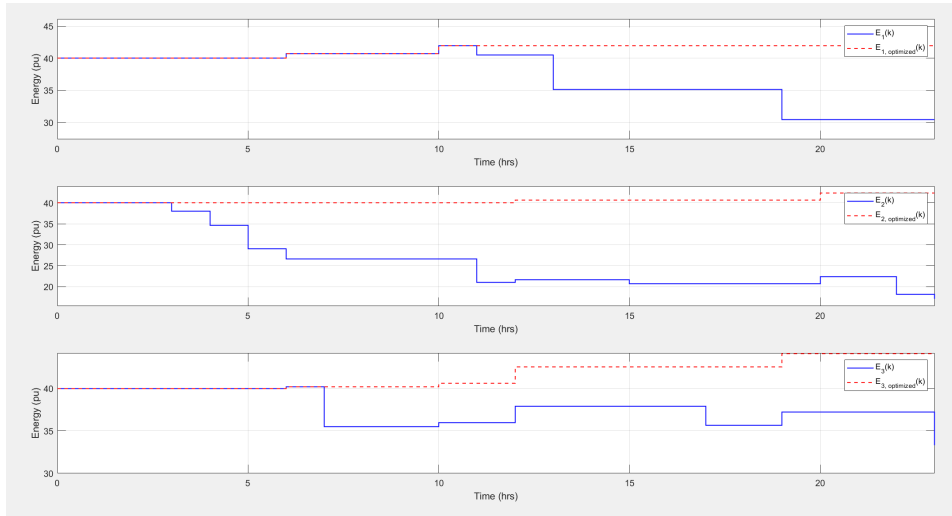


Figure 9: Optimized and Unoptimized Energy in ESS

Based on figures 8, 9, and 10 it can be determined that the proposed goal of maximizing the REUR while taking system constraints into consideration is a success.

## 5 Conclusions

Based off the results it can be seen that the performance of a network of distributed MGs working cooperatively using the proposed methodologies leads to an increased utilization of RESs. This increase in the REUR was able to occur while at the same time considering other factors such as the customer satisfaction level and deep over/undercharging of the energy storage system were also taken into account and optimized.

Further research to take into account the startup time of the DGs as well as use of optimization techniques to chose the values of  $P_i^C(k)$  could be employed to improve performance even further.

Results from [7], [8] and [11] could be incorporated in order to utilize different types of renewable energy sources, such as PV, wind, and hydropower. A specific ESS could be studied by implementing the fuel cell electrolyzer ESS formula shown in [6]. Results from [4] could be used to take other factors like economic performance and environmental impact. Overall, the research shows promising results while also showing lots of room for future improvements.

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