Essays on Index Investing

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Abstract

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This dissertation consists of two chapters about the economic consequences of index investing. The first chapter examines the impact of index investing on short selling. Short sellers convey negative information to securities lenders when borrowing shares. I model how this information generates novel interactions between institutional investors' equilibrium lending, trading, and governance decisions. Index funds lenders cannot trade on lending market information, allowing them to attract more shorting demand and thereby improve price efficiency—despite increasing lending fees. The second chapter explores how index investing impacts managerial compensation in a moral hazard setting. With index investing, effort is reflected not only in the manager's stock price but also in the stock prices of all other constituent firms through the synchronized asset demands of index investors. Thus, the prices of other constituent firms are positively related to and contain unique information about the manager's effort. The optimal contract puts a positive weight on the index's price to enhance the effort sensitivity of the manager's pay, not to reduce risk.

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Dedication

To my parents, George and Janet, and my fiancée, Molly.

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Chapter 1

Learning by Lending Securities

1.1 Introduction

Short sellers are informed traders who must borrow shares to execute their trading strategies. This observation implies that securities lenders can learn valuable information about a stock during the lending process. Specifically, a securities lender's valuation should drop the instant a short seller borrows shares, given the latter's reputation as a bearer of high-quality negative information.[1] How might securities lenders use this information? Recent empirical evidence is consistent with lenders using this information for both their trades (Honkanen (forthcoming) and Greppmair et al. (2024)) and governance decisions (Aggarwal, Saffi, and Sturgess (2015), Li and Zhu (2024)).[2] These uses of lending market information indicate that securities lending is no longer just an ancillary revenue stream for institutional investors. Rather, lending is tightly integrated with these investors' trading and governance strategies, highlighting the need for a formal analysis of how these activities interact in equilibrium.

In this paper, I model how an institutional investor's ability to obtain information by lending securities affects its lending, trading, and governance decisions. In doing so, this paper provides the first analysis of how these three core activities of an institutional investor interact in equilibrium. This framework helps reconcile recent empirical findings about the securities lending market and sheds new light on broader issues in financial markets and corporate governance. For example, I examine how the growth of index investing impacts price efficiency and corporate governance; whether the feedback effect between financial markets and real decisions discourages short selling; and whether there is an inherent conflict between securities lending and corporate governance.

I generate four main results. First, lower lending fees improve information quality but

[1] In a survey article on short selling, Reed (2013) refers to short sellers being informed traders as "one of the most robust findings of the literature."

[2] Honkanen (forthcoming) and Greppmair et al. (2024) show that active mutual funds reduce their holdings of lent-out stocks relative to similar funds that do not have those stocks on loan. Aggarwal, Saffi, and Sturgess (2015) and Li and Zhu (2024) demonstrate that high recall activity around a vote's record date is associated with subsequent votes against management; while these two governance studies do not directly test my hypothesis, their results are consistent with a learning channel.

also create a commitment problem. Lenders sell following a loan agreement, tempting them to recall shares to boost their selling price at the expense of short sellers. Higher fees may then be required to solve the commitment problem and thus enable lenders to attract short sellers. Second, passive lenders, such as index funds, cannot trade on lending market information. This restriction allows them to relax short-selling constraints enough to improve price efficiency—even though increased indexing also leads to higher lending fees. Third, securities lenders can engage in governance to increase firm value after learning from borrowing demand and the stock price. However, rather than reducing short-selling profits, this engagement can benefit short sellers because lenders compensate them with lower fees to maintain access to their information. Finally, because lenders lose the votes associated with lent-out shares, recalling them before important votes is generally considered good governance. Nevertheless, I demonstrate that recalling shares to vote can harm firm value, as this policy deters short sellers from borrowing, making lenders' votes less informed.

My model involves a securities lender ("he") and an informed short seller ("she") interacting in a lending market before submitting orders to a Kyle (1985) style market maker. Lending fees are endogenous and are chosen to maximize lender profits.[3] Higher lending fees increase revenue per loan but decrease lending volume; less obviously, they also decrease the information conveyed by the lending market. After learning whether the short seller borrowed one of his shares, the lender updates his valuation and decides whether to recall the share (if borrowed) before submitting his order to the market maker. If the lender recalls the share, the short seller cannot trade. The market maker observes the aggregate order flow and sets the price equal to the stock's expected value conditional on that order flow. I then allow the lender, as a large shareholder, to take a costly corrective action that affects firm value after he observes borrowing demand and the stock price.

I start by considering how lending and trading interact without governance. After learning that the short seller borrowed one of his shares, the lender always sells. This trade

[3] One can interpret this process as either the lender directly setting fees or an unmodeled lending agent setting fees on the lender's behalf.

lowers the short seller's profits by reducing her expected selling price. However, the lender's optimal trading direction conditional on no loan agreement depends on his private signal and the inference that he can make about the stock's payoff, given that there was no loan. Intuitively, since borrowing demand is a negative signal, the absence of borrowing demand must be a positive signal. The strength of this inference is decreasing in the fee because when fees are high, the lack of borrowing demand is more likely because of the short seller finding shorting too costly than the stock's payoff being high. Consequently, the lender's optimal fee is decreasing in his ex-ante (i.e., before observing borrowing demand) valuation of the stock. When the lender is optimistic about the stock, he plans on buying and does not expect much lending revenue. Therefore, the lender favors low fees to verify that his positive information is correct. A lender with a low valuation plans on selling regardless of the lending market outcome, so he prefers high fees to reduce trading competition from the short seller. This result predicts that high (low) fees and low borrowing demand precede sales (purchases) by the lender.

I then show that the lender's incentive to recall shares prevents him from having too low a fee. For example, mutual funds are required by the Investment Company Act of 1940 to be able to recall lent-out shares at will, which, if done, forces the short seller to purchase replacements.[4] As a result, after learning that the stock is overvalued through the lending market, the lender can camouflage his sale by recalling shares; Chuprinin and Ruf (2018) provide evidence for this practice. The prospect of a higher expected selling price tempts the lender to recall shares, but the short seller anticipates this action and will not borrow if she believes her lender will behave this way. As a result, the lender must commit to not recalling shares if he wishes to learn from the lending market; he does so by increasing fees. The lender relinquishes the fee when he recalls shares, so the foregone lending revenue is the cost of this action from his perspective. Thus, he loses the incentive to recall shares once the

[4]https://www.sec.gov/divisions/investment/noaction/1972/statestreet052272.pdf The implication is that lenders covered by the Investment Company Act of 1940 (e.g., mutual funds) must have this provision in their lending agreements.

fee is sufficiently high. This no-recall condition establishes a positive lower bound on the set of fees that a lender can charge while still attracting short sellers; the existence of this lower bound is, to my knowledge, new to the short-selling literature. My analysis shows that this constraint can considerably increase lending fees and discourage short selling. Surprisingly, this result predicts that lenders may have to increase fees to attract short sellers.

The previous two results show that the lender's ability to trade imposes substantial costs on short sellers, so I compare the short-selling constraints that emerge when the lender trades to a benchmark where he cannot. I refer to this benchmark lender as passive, which one can interpret as an index fund. Index funds do not have the discretion to trade on information and are prominent players in the securities lending market, making them an instructive benchmark.[5] Because they never trade, passive lenders aim to maximize expected lending revenue. I show that passive lenders' ability to commit to never trade on information learned during the lending process or strategically recall shares (without raising fees) can enable them to attract enough additional short sellers to improve price efficiency. The result that passive funds can enhance rather than hinder price efficiency, as is commonly conjectured, by relaxing short-sale constraints is novel in the theoretical literature on passive investing; von Beschwitz, Honkanen, and Schmidt (2022) and Palia and Sokolinski (2024) provide empirical support for this result.

This framework also helps reconcile the results from the growing empirical literature on how passive investing impacts short-sale constraints. First, Honkanen (forthcoming) and Greppmair et al. (2024) show that active securities lenders (i.e., those with the ability to trade) trade on information learned while lending. This result is, of course, part of the motivating evidence for my model. Second, Honkanen (forthcoming) and Palia and Sokolinski (2024) find that passive lending leads to higher fees. Third, von Beschwitz, Honkanen, and Schmidt (2022) and Palia and Sokolinski (2024) demonstrate that short-selling demand increases with passive ownership. My model's predictions are consistent with all three find-

[5]Blocher and Whaley (2016) document that lending revenue can exceed management fees for indexed ETFs.

ings. In effect, these papers say that the costs imposed by the active lenders' trades are more burdensome to short sellers than the higher fees of passive lenders. However, the economic forces that prevent active lenders from having fees sufficiently low to attract more short sellers remain unclear. My model provides an answer and says that an active lender's desire to recall shares or curb trading competition leads to equilibrium fees high enough to reduce short-selling demand relative to a passive lender.

I then address how lending market information impacts corporate governance.[6] Lenders can use this information, along with the stock price, to engage in better-informed governance and thereby improve firm value. Previous work, including an important contribution by Edmans, Goldstein, and Jiang (2015), finds that such improvements harm short sellers by forcing them to close out their positions at higher prices. However, I show that this value-improving governance can also benefit short sellers. Unlike other key stakeholders, such as management or the board, lenders interact with short sellers and can compensate them for the higher firm value by offering lower fees. A passive lender behaves this way when the short seller's information is sufficiently valuable to him, leading to lower fees and greater short-selling profits with this feedback effect between financial (and lending) markets and governance. The active lender would like to act similarly but cannot always do so because of the need to maintain high fees as a commitment device. This result suggests that the securities lending practices of index funds can help mitigate concerns about their relative reluctance to gather information for governance.

My final result applies this governance framework to an important decision that all lenders face: whether to recall shares to vote. When an institutional investor lends shares, it loses the votes associated with those shares for the duration of the loan. As a result, the SEC states that "if fund management has knowledge of a material vote with respect to the loaned securities, fund directors should recall the loan in time to vote the proxies."[7] However, it is

[6]Many mutual funds fulfill this dual role of blockholder and securities lender, particularly index funds. Evans, Ferreira, and Prado (2017) document that roughly two-thirds of passive funds and one-half of active funds lend securities.

[7]https://www.sec.gov/investment/divisionsinvestmentsecurities-lending-open-closed-end-investment-companieshtm

important to consider the informational consequences of such a decision. Like before, short sellers will not borrow from lenders who recall shares. Thus, recall leads to less information for the lender. Firm value can, therefore, decrease when the lender recalls shares to vote because this policy eliminates the lending market as a source of information; the comparison is between a smaller number of informed votes and a larger number of uninformed votes for the lender. The former scenario will improve firm value when the lender's augmented voting power from recall does not significantly increase the likelihood of the value-maximizing outcome and when the lender's optimal decision is unclear. Thus, it is not necessarily good governance for lenders to recall shares to vote.

Related Literature

I know only two other contemporaneous papers that model information spillovers during the securities lending process: Pankratov (2020) and Chen, Kaniel, and Opp (2024).[8] Pankratov (2020) solves a model where an uninformed lender sets fees to maximize profits. In doing so, the lender faces a trade-off between boosting lending revenue with higher fees and augmenting trading profits through lower fees and better information. My model nests this outcome as a special case and introduces three empirically motivated elements that (i) change our understanding of the relationships between the lender's various decisions and (ii) are required to reconcile empirical findings about the securities lending market. First and most importantly, I analyze how lenders can use lending market information for corporate governance; I demonstrate that this use of lending market information not only enhances our understanding of lender behavior but also reshapes our perspective on several key topics in the governance literature. Second, I generate the no-recall condition, which is required to reconcile the empirical findings discussed in the introduction and to produce the result that passive lenders can improve price efficiency by relaxing short-sale constraints. Third, I

[8] Baldauf and Mollner (2024) also model information leaks in a more general over-the-counter setting. See Duffie, Garleanu, and Pedersen (2002); Blocher, Reed, and van Wesep (2013); Atmaz, Basak, and Ruan (2023); and Sikorskaya (2024) for models of securities lending without information spillovers.

allow the lenders to condition their fees on their private information, which is necessary to produce the result that high lending fees forecast low subsequent returns (Jones and Lamont (2002) and Duong et al. (2017)).

Chen, Kaniel, and Opp (2024) study market power in securities lending and find evidence for market concentration and non-competitive fees. The authors then propose a model to explain why such market power exists. They compare two different lending market structures: an over-the-counter setting where lenders with market power set fees to maximize lending revenue (similar to my passive lender case) and a competitive, centralized market where borrowing demand is observed by all participants. In the former setting, we see elevated fees because of the lender's market power, but no information leakages because the lender does not trade. In the latter, competition drives fees to zero, but short sellers are worse off with this market structure because their borrowing demand is immediately revealed to all. A fundamental assumption in this model is that lenders in the over-the-counter setting do not trade on whatever information they learn from the lending market (or use the information for governance). My paper relaxes this assumption and endogenizes the quality of that information through the lender's fee choice. These changes are not just theoretical exercises: they reflect the observed behavior of securities lenders in recent empirical research. Honkanen (forthcoming) and Greppmair et al. (2024) demonstrate that active lenders rebalance their portfolios away from stocks on loan and find that the associated sales precede further price declines. Similarly, Chague, Giovannetti, and Herskovic (2023) show that Brazilian securities lenders leak lending market information to other clients.

This paper also contributes to the theoretical literature on how passive investing impacts price efficiency. There is no overwhelming consensus in this literature. For example, Bond and Garcia (2022) find that passive investors can improve price efficiency, while Coles, Heath, and Ringgenberg (2022) find that passive investors have no effect. On the other hand, Baruch and Zhang (2022) and Nurisso (2024) solve models where increased indexing harms price efficiency. However, none of the above papers explicitly analyze how the growth in passive investing impacts price efficiency through the securities lending market. I contribute to this literature by generating the novel theoretical result that passive funds can improve price efficiency by relaxing short-sale constraints. Von Beschwitz, Honkanen, and Schmidt (2022) and Palia and Sokolinski (2024) empirically show that passive lenders can improve the ability of prices to incorporate negative information through this channel. However, it is unclear whether passive lenders increase overall price efficiency if they facilitate the incorporation of negative information, but hinder the incorporation of positive information. My model provides conditions under which this channel increases overall price efficiency with passive lenders.

My corporate governance analysis contributes to the literature on financial market feedback. Edmans, Goldstein, and Jiang (2015) show that short-selling profits decline when a decision maker uses financial market prices to inform an intervention decision. I apply this framework to the case where the decision maker is a blockholder who lends securities, and I find conditions under which short-selling profits can increase with intervention because the lender can compensate short sellers with lower fees to acquire their information. Kunzmann and Meier (2018) provide evidence for this learning channel by showing that elevated short interest predicts CEO turnover (via shareholder activism). My analysis also contributes to the governance literature on recalling shares to vote (see Aggarwal, Saffi, and Sturgess (2015); Hu, Mitts, and Sylvester (2021); and Li and Zhu (2024)). I consider the informational consequences of share recall, and I show that firm value can increase when the lender commits to a policy of not recalling shares to vote: such a policy encourages more short selling, making the lender's smaller number of votes more informed.

Finally, this paper contributes to the literature on blockholder engagement (see Edmans and Holderness (2017) for a survey). In particular, my paper shows how securities lending can generate a voice vs. exit tradeoff for active securities lenders through the provision of new information (Bhide (1993) and Coffee (1991)); loan agreements incentivize both sales and intervention, but sales also give the lender less "skin in the game". By contrast, lending only boosts the voice of passive investors, which helps mitigate concerns about the willingness of index funds to acquire information for governance (Brav, Malenko, and Malenko (2023) and Kahan and Rock (2020)). My voting application interprets the cost of the lender's governance action as any cost associated with voting against management, perhaps because of business ties with the firm (Cvijanovic, Dasgupta, and Zachariadis (2016); Davis and Kim (2007); and Ashraf, Jayaraman, and Ryan (2012)) or public backlash (Bebchuk and Hirst (2019)). The empirical finding that passive funds are more prone to vote with management (Brav et al. (2022) and Heath et al. (2022)) suggests that these costs may be particularly salient for them. Lastly, the implication that short-selling activity induces shareholders to vote against management suggests an additional reason why management is typically so hostile to short sellers (Lamont (2012)).[9]

1.2 Model Setup

The model contains four agents: an informed speculator (H, "she"), a lender (L, "he"), noise traders, and a market maker. All agents are risk-neutral. There is a single firm whose stock is traded in a financial market; its payoff $v(\theta, a) \in \{0, 1\}$ is affected by an underlying state of the world θ and an action a taken by the lender.[10] This model has three main phases: lending, trading, and governing. In the lending phase, H can borrow shares from L for a fee f. Next, L, H, and the noise traders submit their asset demands to the market maker, who sets the stock's price $P(z)$ equal to its expected payoff conditional on the order flow z. Then, the lender, as a blockholder, can take a costly corrective action $a \in \{0, 1\}$ to improve the firm's value, where $a = 1$ indicates that L takes the action. After the governance phase, the firm's payoff $v(\theta, a)$ is revealed.

The state of the world θ equals zero or one with equal probability; I assume $v(1, a) \geq$

[9] Trading and voting is also examined in works by Levit, Malenko, and Maug (2023); Bar-Isaac and Shapiro (2020); Brav and Mathews (2011); and Meirowitz and Pi (2022).

[10] Since this payoff can be interpreted as a liquidating dividend for the firm, I will use the terms "payoff" and "firm value" interchangeably depending on the context.

$v(0, a)$ and strictly so when the lender does not take the corrective action (i.e., $a = 0$). I discuss how the lender's action affects this payoff more in Section 1.2.2.

H is perfectly informed about the realization of θ. L has access to a private signal $s \in \{0, 1\}$ of quality q about the realization of the state's value. Quality q means that

$$Pr[s = \theta|\theta] = q \in [0.5, 1)$$

I assume q is common knowledge and that $q < 1$ to focus on the case where the lender can learn from the lending market.[11] Notation-wise, I refer to the lender who sees signal s as L_s.

H faces a random opportunity cost of trading, c, that is uniformly distributed on $[0, \bar{C}]$ when $\theta = 0$ and on $[0, \hat{C}]$ when $\theta = 1$. In general, $\bar{C}$ may differ from $\hat{C}$. The realization of c is the speculator's private information. The speculator will only attempt a trade if her expected profits exceed c. One can interpret this situation as the speculator having a finite amount of capital that she can deploy in this period, with c representing the profits of her next best option.

I now describe the lending, trading, and governing phases in more detail.

1.2.1 Lending and Trading

I assume L owns enough shares to lend without constraining his subsequent ability to trade; this assumption implies that the lender owns $N \geq 2$ shares of the stock.[12] The lender's asset demand will be $D_L \in \{-1, 0, 1\}$ where 1 represents a purchase, 0 represents no trade, and -1 represents a sale (not a short sale). Passive lenders are a special case where D_L always equals zero. On the other hand, the informed speculator owns zero shares, so she must borrow shares from L to sell the stock short. Her demand is: $D_H \in \{-1, 0, 1\}$, but $D_H = -1$ signifies a short sale. H must therefore first borrow a share from L to have

[11] I discuss the perfect information case in Section 1.5.2.

[12] This assumption is motivated by existing regulations prohibiting mutual funds from lending more than 1/3 of their total asset value.

$D_H = -1$.

The lender chooses a fee $f \geq 0$ to maximize his expected profits conditional on his private signal s. Note that fees will always be non-negative; negative fees amount to the lender paying the speculator to borrow his shares, and this choice is never optimal for the lender.[13] This fee is a "take it or leave it" offer sent to the speculator H. The speculator borrows if and only if her expected selling profits minus f exceed c.[14] The lender's fee choice resembles a monopolist selecting the profit-maximizing point on a downward-sloping demand curve.[15] All else equal, a lower fee induces a broader set of opportunity costs to sell the stock short.

After observing whether H borrowed a share, the lender rationally updates his stock valuation. Denote the lending market signal as $b \in \{1, 0\}$ where $b = 1$ represents the outcome where H borrows the lender's share and $b = 0$ represents the outcome where H does not borrow the lender's share. Thus, L's trading information set contains the signals s and b. The lender can also recall the lent-out share between the lending and trading stages. If he recalls the share, L returns the fee to H, and H cannot trade. This outcome implies that H will make zero trading profits, so her total profits net of opportunity cost c will be negative. Thus, the informed speculator will not attempt to borrow if she expects L to recall the loan.

The financial market is a discrete version of Kyle (1985). Besides H and L, there is also noise trade. Aggregate noise demand takes values: $D_{NT} \in \{-2, -1, 0, 1, 2\}$ with each outcome having equal probability. I assume that the realization of noise demand is independent of all other random variables and that negative noise demand corresponds to *sales* and not

[13]With a negative fee, it is always optimal for H to borrow, regardless of the realization of θ. This outcome destroys the informativeness of the lending market signal and produces negative lending revenue. Thus, a negative fee can never be optimal.

[14]I rule out "manipulative" borrowing strategies for H. For example, H may want to borrow even if she knows that $\theta = 1$ to give L a false signal and trick him into selling, leading to a lower buying price for her. I make this assumption to focus on the fundamental case where H borrows to sell short. More generally, this strategy will be unprofitable for H when the decrease in her expected buying price following manipulative borrowing is less than f.

[15]There is strong evidence that securities lenders have market power. See Duffie, Garleanu, and Pedersen (2002), Kolasinski, Reed, and Ringgenberg (2013), and Chen, Kaniel, and Opp (2024).

short sales—only H shorts.[16] Noise traders also never lend.

All traders submit their demands to a risk-neutral, competitive market maker. The market maker observes the aggregate order flow

$$z \equiv D_H + D_L + D_{NT}$$

and sets the price equal to the expected value of $v(\theta, a)$ conditional on z. Thus, I define the price as:

$$P(z) \equiv E[v(\theta, a)|z] \tag{1.1}$$

Importantly, the market maker does not observe developments in the lending market (both fees and quantities) and, therefore, cannot distinguish regular sales from the more informative short sales. This inability to observe lending market outcomes allows the lender to trade profitably on the lending market signal b.[17] After the payoff $v(\theta, a)$ is revealed, all short sales are completed by H paying L the terminal payoff $v(\theta, a)$.

1.2.2 Governance

After $P(z)$ is generated, the lender can take corrective action a to improve firm value. Without loss of generality, I assume that the market maker announces the order flow z along with the price $P(z)$. This assumption allows the lender to condition his action directly on z, simplifying the exposition. As a result, L's governance information set contains the signals s, b, and z. The mapping between the lender's action and the stock's expected payoff is described in Table 1.1. When $\theta = 1$, firm value equals one regardless of the lender's action.

[16] Allowing noisy short sales would make $b = 1$ a less negative signal about firm value. However, this modification then raises the complication of what fees noise traders accept. If noise traders accept any fee, the lender optimally sets $f \to \infty$. My model, therefore, analyzes the extreme case where short-selling demand is perfectly informative about $\theta = 0$ rather than just highly informative. Note also that securities lenders seem able to identify informative short sales, which weakens the strength of this assumption (see Honkanen (forthcoming)).

[17] This assumption is justified as long as there is some gap in time between when the lender and borrower complete the loan and when this information becomes publicly available.

Table 1.1: Firm Value with Intervention

(θ, a)	$E[v(\theta, a)]$
(1, 1)	1
(1, 0)	1
(0,1)	ψ
(0,0)	0

When $\theta = 0$ and $a = 1$, firm value equals one with probability ψ and zero with probability $1 - \psi$. Firm value also equals zero when $\theta = 0$ and $a = 0$. In other words, $a = 1$ is a corrective action that improves firm value from zero to one with probability ψ when we are in the low state of the world ($\theta = 0$). The action has no effect in the high state of the world ($\theta = 1$). I assume choosing $a = 1$ costs the lender $k > 0$.

1.2.3 Timing

The timing of this game is as follows:

1. The state of the world θ is generated.

2. H learns the realizations of θ and c. L receives signal s.

3. **Lending Market**

   (a) L chooses a borrowing fee $f \geq 0$ and offers it to H.

   (b) H decides whether to borrow a share at fee f.

   (c) L learns whether his share was borrowed and updates his valuation of the asset.

   (d) L decides whether to recall the loan. If so, H must return the share and L must return the fee f. If recalled, H can no longer trade.

4. All traders submit their asset demands to the market maker. $P(z)$ is then generated through the pricing rule.

5. L chooses his governance action a and pays cost ak.

6. The realization of $v(\theta, a)$ is made public. All short sales are completed by the borrower paying the lender $v(\theta, a)$.[18]

1.2.4 Equilibrium Concept

My solution concept for this model is the Perfect Bayesian Equilibrium (PBE). An equilibrium requires (i) a trading strategy for H that maximizes expected profits given the price-setting rule, the fee, beliefs about L's signal realization, L's equilibrium trading, recall, and intervention strategies, and the realizations of θ and c; (ii) a fee-setting, trading, recall, and intervention strategy for L that maximizes expected profits given the price-setting rule, H's equilibrium trading strategy, and L's information set; and (iii) a price-setting rule for the market maker that follows Equation (1.1) given the strategies of H and L. H, L, and the market maker have rational expectations in that they update probabilities via Bayes' rule and have correct-in-equilibrium beliefs about the other players' strategies. Observe that L's information set is $\{s\}$ while lending, $\{s, b\}$ while trading, and $\{s, b, z\}$ when governing.

One can view the market maker in my model as the receiver in a multi-sender signaling game, so I will apply the notion of "unprejudiced beliefs" from Bagwell and Ramey's (1991) analysis of such games as my primary equilibrium refinement. This refinement requires the receiver to believe that off-equilibrium outcomes come from the fewest number of deviations from equilibrium play. In my model, a market maker with unprejudiced beliefs should view an off-equilibrium order flow as coming from a single trading deviation by H *or* L instead of by both simultaneously.[19]

[18] The fact that the lender is perfectly informed and returns the share at fundamental value implies zero credit risk. In practice, credit risk is usually minimal since the loans are over-collateralized and typically indemnified by the lender's lending agent.

[19] Given the structure of noise demand, off-equilibrium order flows can only be at the endpoints of the set of feasible order flows: $z = 4$ or $z = -4$. At these potential off-equilibrium order flows, H must either be buying or shorting, respectively, so unprejudiced beliefs are sufficient to pin down the price as $P(4) = 1$ and $P(-4) = 0$. In more general multi-sender signaling games, unprejudiced beliefs pin down which sender deviated, and further refinements, such as the intuitive criterion of Cho and Kreps (1987), determine which types of the deviating sender want to deviate. However, as I explained, further refinements are unnecessary in my setting. See Vida and Honryo (2021) for a discussion on the optimality of combining unprejudiced beliefs with the intuitive criterion in multi-sender signaling games.

1.3 Lending and Trading Equilibrium

I start by solving the model in a baseline case without governance; that is, the lender's action a is always set equal to zero in this section.[20] As a result, $v(\theta, 0) = \theta$, so I will refer to $v \equiv v(\theta, 0) = \theta$ as firm value.

I describe my solution procedure in Section 1.3.1. In Section 1.3.2, I solve for the equilibrium with a passive lender. In Section 1.3.3, I find all equilibria for the active lender. I then analyze the various equilibria and examine the implications for price efficiency in Section 1.3.4.

1.3.1 Solution Procedure

I solve the model under the following parametric assumptions.

Assumption 1.1. *$\hat{C}$ is low enough to ensure that the H always finds buying optimal when she learns that $v = 1$.*

Assumption 1.2. *I assume $\bar{C}$ is the maximum value of c that supports H always selling when $v = 0$ as an equilibrium trading strategy in the counterfactual setting where H already owns a share (and thus never needs to borrow) and L does not exist.*

Assumption 1.1 says that the exogenous cost of trade c is never high enough to deter H from buying when $v = 1$. This assumption allows me to focus on how the lending market impacts short selling and contrast those results with "frictionless" buying. As such, $\hat{C}$, the maximum opportunity cost when $v = 1$, does not play a role in the main analysis. Assumption 1.2 then facilitates a more exact understanding of how the lending market impacts outcomes by establishing a clear counterfactual benchmark: when H does not need to borrow shares to sell and the lender does not exist, the unique equilibrium has H always buy when $v = 1$ and always sell when $v = 0$.[21] The exogenous cost c alone does not deter

[20] This choice will be optimal whenever $k > (N + D_L)\psi$.

[21] See Proposition A1 in the appendix for more details.

$H's$ trades: it only does so when combined with lending market frictions and the lender's actions. My main results do not depend on these specific parameter restrictions, but they make the model much more tractable. I discuss robustness in Section 1.5.

I conjecture that H's trading strategy conditional on $v = 0$ follows a threshold rule. In a separating equilibrium (i.e., L_1 and L_0 trade differently or set different fees), H will short sell when $v = 0$ if and only if $c < c_s^*$ where $c_s^* \in [0, \bar{C}]$. The short-selling cutoff will generally depend on whether the lender is L_1 or L_0 in a separating equilibrium (hence the subscript s on the cutoff). In a pooling equilibrium (i.e., L_1 and L_0 have the same trading strategies and set the same fees), the cutoff does not depend on s and will be denoted as just c^*.

L's trading strategy will depend on whether H borrowed one of his shares (i.e., the realization of b) and the realization of his private signal s. Denote $D_L^s(b) \in \{-1, 0, -1\}$ as L's demand when he observes signals s and b.

Lemma 1.1. *L always sells when $b = 1$. That is, $D_L^s(b = 1) = -1$.*

The proof of Lemma 1.1 is straightforward: H only sells short when she knows $v = 0$, so L's valuation also becomes zero after observing $b = 1$. Thus, selling is always optimal for L since prices can never go below zero. However, L's optimal trading strategy following the lending market signal $b = 0$ is less obvious. I focus only on pure strategies, so $D_L^s(b = 0) \in \{-1, 0, 1\}$. I also assume the convention that the lender will not trade if indifferent about doing so for the rest of the paper.

I solve for all equilibria by guessing and verifying whether a pair of lender demands conditional on no loan agreement,$\{D_L^1(b = 0), D_L^0(b = 0)\}$, can be part of an equilibrium trading strategy for the lender. I then repeat this process for all possible pairs.

1.3.2 Passive Benchmark

I first solve the model in the benchmark case where the lender is passive. The passive lender's defining characteristic is that he does not trade, so D_L always equals zero (regardless of the realization of b). As a result, the passive lender also has no incentive to recall shares to improve his (nonexistent) trading profits. One can view the passive lender as an index fund that cannot deviate from its index.

I solve the model backward. First, given $H's$ conjectured trading strategy and the fact that the passive lender never trades, the market maker generates prices $P(z) = Pr[v = 1|z]$, where z is an integer between negative three and three. The prices are functions of the equilibrium short-selling thresholds c_0^* and c_1^*. Specifically,

$$P(z) = \begin{cases} 1 \text{ when } z = 3 \\ \frac{1}{2-(1-q)G(c_1^*)-qG(c_0^*)} \text{ when } z = 2 \\ \frac{1}{2} \text{ when } z \in \{1, 0, -1\} \\ 0 \text{ when } z \in \{-2, -3\} \end{cases} \tag{1.2}$$

where $G(x) \equiv \frac{x}{C}$ is the cumulative distribution function of the opportunity cost c when $v = 0$. Intuitively, $P(z) = 1$ means order flow z only occurs when $v = 1$ and $P(z) = 0$ similarly means that z only appears in equilibrium when $v = 0$. $P(z) = \frac{1}{2}$ implies that z is an order flow that can always appear, and $P(z) = \frac{1}{2-(1-q)G(c_1^*)-qG(c_0^*)}$ corresponds to a z that only appears when H does not short sell; that is, when $v = 1$, or $v = 0$ and $c > c_s^*$. Observe that since the prices are functions of conjectured equilibrium strategies, L and H take them as given when acting.

I must now derive the equilibrium values of c_1^* and c_0^*. Given L_s's choice of fee f_s, the *actual* short-selling thresholds c_1 and c_0 satisfy

$$c_1 = E[P(z)|D_H + D_L = -1] - f_1 \tag{1.3}$$

$$c_0 = E[P(z)|D_H + D_L = -1] - f_0 \tag{1.4}$$

Equations (1.3) and (1.4) will be identical in a pooling equilibrium. These equations say that H is indifferent between short selling at fee f_s and not trading when she knows that $v = 0$ and has opportunity cost $c = c_s$. All H with $c < c_s$ will find it optimal to short when $v = 0$, and all H with $c > c_s$ will find it optimal not to trade when $v = 0$. Observe also that H does not care about her lender's type (i.e., the realization of the lender's private signal s) per se because both L_1 and L_0 trade the same way after observing $b = 1$; consequently, only the observable fee matters for H's shorting decision.

Next, I solve for the passive lender's optimal fee. Since the passive lender never trades, he chooses the fee to maximize his expected lending revenue.

$$\max_{f_s} Pr[D_H = -1|s]f_s = Pr[v = 0|s] \left(\frac{E[P(z)|D_H + D_L = -1] - f_s}{\bar{C}} \right) f_s \tag{1.5}$$

where $Pr[v = 0|s]$ equals q for L_0 and $1 - q$ for L_1. Also observe that

$$E[P(z)|D_H + D_L = -1] = \frac{1}{5} \sum_{z=-3}^{1} P(z)$$

Solving (1.5) generates L_s's optimal fee, and I find that L_1 and L_0 optimally choose the same fee:

$$f^* \equiv f_1^* = f_0^* = \frac{E[P(z)|D_H + D_L = -1]}{2} \tag{1.6}$$

Since L_1 and L_0 set the same fee, trade the same way, and H's beliefs about the lender's type are irrelevant, the only equilibrium is a pooling equilibrium. Equation (1.6) also says the passive lender collects half the short seller's expected selling profits as a fee. I can then plug this optimal fee into Equations (1.3) and (1.4), set $c_s = c_s^*$, and solve for the equilibrium thresholds. I describe the passive equilibrium in the following proposition.

Proposition 1.1. *The unique equilibrium with a passive lender is a pooling equilibrium. The optimal fee maximizes expected lending revenue and takes half of the short seller's expected profits. Neither the equilibrium fee f^* nor the short-selling threshold c^* depend on the lender's private signal quality q.*

Proposition 1.1 describes the lending and financial market outcomes with a lender who sets fees to maximize expected lending revenue and never trades. The existing theoretical literature on securities lending typically assumes this sort of lender behavior. I now analyze what happens to these outcomes when the lender trades.

1.3.3 Active Lender

The solution procedure is quite similar when the lender trades. The only difference is that I now allow the lender to trade optimally instead of imposing $D_L = 0$. That is, the maker maker believes that L_s sells following a loan agreement (see Lemma 1.1) and demands $D_L^s(b = 0) \in \{-1, 0, 1\}$ following no loan agreement. The market maker also continues to believe that H always buys when $v = 1$ and will borrow from L_s and sell short when $v = 0$ if and only if $c < c_s^*$. The market maker then sets prices given these conjectures; again, the prices will be functions of the equilibrium values c_1^* and c_0^*.

It will be instructive to pause and examine how prices respond to changes in L's asset demand conditional on $b = 0$ to build intuition for future results. Figure 1.1 characterizes the mappings between prices and order flows in the three possible pooling equilibria: L buying (top row), not trading (middle row), and selling (bottom row) conditional on $b = 0$. Yellow dots correspond to a price of 1, red dots to a price of $\frac{1}{2-G(c^*)} \in [\frac{1}{2}, 1]$, purple dots to a price of $\frac{1}{2}$, and blue dots to a price of 0. Order flows that do not occur in equilibrium are unfilled.

As L's demand conditional on $b = 0$ declines, the set of order flows that can appear when $v = 1$ shifts downward. This outcome generates greater overlap between the sets of order flows that can occur when $v = 0$ and $v = 1$. As a result, order flows that previously *only* occurred when $v = 0$ can now appear when $v = 1$, which increases those order flows' prices. Consequently, among all prices that always occur in equilibrium ($z \in \{-4, ..., 2\}$), prices are weakly decreasing in $L's$ demand conditional on no loan agreement ($b = 0$). The intuition is broadly similar in a separating equilibrium.[22]

[22] The only exceptions for separating equilibria are $P(3)$ when we switch from $\{D_L^1(b = 0), D_L^0(b = 0)\} =$

Figure 1.1: Order Flows

$P(z)$ for different trading strategies conditional $b = 0$ for L. Yellow dots represent $P(z) = 1$, red dots represent $P(z) = \frac{1}{2-G(c^*)}$, purple dots represent $P(z) = \frac{1}{2}$, and blue dots represent $P(z) = 0$. Unfilled dots do not occur in equilibrium.

Next, I must solve for the equilibrium short-selling cutoffs given L's conjectured trading strategy. Given any fee choice f_s, the *actual* cutoffs are:

$$c_1 = E[P(z)|D_H + D_L = -2] - f_1 \tag{1.7}$$

$$c_0 = E[P(z)|D_H + D_L = -2] - f_0 \tag{1.8}$$

Equations (1.7) and (1.8) only differ from their passive counterparts (1.3) and (1.4) in that the expected selling price is now conditional on $D_H + D_L = -2$ rather than -1. This change follows from H's belief that the lender always sells after observing $b = 1$ (see Lemma 1.1). Also, the insight that H does not care about the lender's type, just the observable fee, continues to hold with an active lender because of this conjectured trading strategy for L.

$\{1, -1\}$ to $\{1, 0\}$ and $P(-2)$ when we switch from $\{D_L^1(b = 0), D_L^0(b = 0)\} = \{0, -1\}$ to $\{1, -1\}$ (and their mirror images); when L_1 and L_0 transition to following polar opposite trading strategies (e.g., $\{1, -1\}$), their "mistakes" have a large impact on prices. For example, $z = 3$ can only appear when $v = 1$ with probability q at $\{1, -1\}$, and can always appear when $v = 1$ at $\{1, 0\}$. In other words, L_0's mistake of selling when $v = 1$ reduces the association of $z = 3$ with $v = 1$, which lowers rather than raises the price.

I next examine L_s's stock valuation after the lending stage given an arbitrary fee f_s.

$$\begin{aligned} E[v|b=1,s] &= 0 \\ E[v|b=0,s=1] &= \frac{q}{q+(1-q)(1-G(c_1))} \\ E[v|b=0,s=0] &= \frac{(1-q)}{(1-q)+q(1-G(c_0))} \end{aligned} \tag{1.9}$$

Since the short seller is perfectly informed, outcome $b=1$ implies $v=0$; it is always optimal for the lender to sell in this scenario because prices are guaranteed to be no lower than zero. On the other hand, the outcome $b=0$ could come from a speculator who knows that $v=1$ or a speculator who knows that $v=0$ but has a high realization of c.

Lemma 1.2. *The lender's valuation of the asset conditional on $b=0$ is decreasing in f_s.*

With a high fee, fewer speculators short sell, making it more likely that the outcome $b=0$ comes from a speculator who knows $v=0$ but finds the fee too expensive.

Moving back to the fee-setting stage, L_s's objective is:

$$\begin{aligned} \max_{f_s} Pr[D_H=-1|s]\Big(E[P(z)|D_H+D_L=-2]+f_s\Big) \\ +\Big(1-Pr[D_H=-1|s]\Big)\Big(D_L^s(b=0)E[v-P(z)|b=0,s,D_L^s(b=0)]\Big) \end{aligned} \tag{1.10}$$

Equation (1.10) tells us that a higher fee has three effects on L's expected profits. First, there is the direct effect of a higher fee leading to greater lending revenue conditional on a short sale. Second, a higher fee reduces the likelihood of a short sale when $v=0$; that is, $Pr[D_H=-1|s]$ is decreasing in f_s. Third, as previously mentioned, higher fees decrease $E[v|b=0,s]$. This third effect lowers profits for lenders with $D_L^s(b=0)=1$, increases profits for lenders with $D_L^s(b=0)=-1$, and has no effect for lenders with $D_L^s(b=0)=0$. I characterize the optimal fees as a function of each possible trading strategy $D_L^s(b=0)\in\{-1,0,1\}$ below in Lemma 1.3.

Lemma 1.3. *Given some set of prices ($P(z)$ for $z \in \{-4, ..., 4\}$), each possible trading strategy $D_L^s(b=0) \in \{-1, 0, 1\}$ maps to a unique optimal fee $f_s^* \geq 0$. Furthermore, the fee associated with $D_L^s(b=0)=1$ is no higher than the fee associated with $D_L^s(b=0)=0$, and the fee associated with $D_L^s(b=0)=0$ is strictly lower than the fee associated with $D_L^s(b=0)=-1$.*

Specifically, these fees are:

$$f_s^* = \begin{cases} \max\left(\frac{-E[P(z)|D_H+D_L=1]}{2}, 0\right) = 0 \text{ } when \text{ } D_L^s(b=0)=1 \\ 0 \text{ } when \text{ } D_L^s(b=0)=0 \\ \max\left(\frac{E[P(z)|D_H+D_L=-1]}{2}, 0\right) = \frac{E[P(z)|D_H+D_L=-1]}{2} \text{ } when \text{ } D_L^s(b=0)=-1 \end{cases} \tag{1.11}$$

When $D_L^s(b=0)=1$, the lender's profits conditional on $b=0$ are strictly decreasing in f_s, so his optimal fee is pushed downward. Intuitively, by setting a lower fee, L's valuation conditional on $b=0$ increases because the outcome $b=0$ is more likely to come from a speculator who knows $v=1$. L, therefore, subsidizes short selling to improve the quality of his lending market signal. Conversely, his profits conditional on $b=0$ are strictly increasing in f_s when $D_L^s(b=0)=-1$, which pushes his optimal fee upwards. By setting a higher fee, L can kick short sellers out of the market and generate a higher expected selling price ($E[P(z)|D_H+D_L=-1]$ rather than $E[P(z)|D_H+D_L=-2]$) for himself.

I now incorporate strategic recall risk into my model as per the timeline given in Section 1.2.3. To summarize, the lender can recall the lent-out share after he and the borrower reach a loan agreement but before they submit their demands to the market maker. If the lender recalls the loan, he loses the fee, and the speculator cannot sell short. In actual lending markets, the lender would recall the loan after the speculator sold short, forcing her to buy. One can, therefore, equivalently interpret my timeline as the speculator submitting a sale order followed by a buy order when recalled and the market maker only observing the net impact of $H's$ trades on the order flow: zero.

Conditional on observing $b = 1$, an active lender will not want to recall the share if and only if the following no-recall condition on fees is satisfied:

$$\begin{aligned} E[P(z)|D_H + D_L = -2] + f &\geq E[P(z)|D_H + D_L = -1] \\ \implies f \geq \Delta E[P(z)] \equiv E[P(z)|D_H + D_L = -1] &- E[P(z)|D_H + D_L = -2] \end{aligned} \tag{1.12}$$

By not recalling, the lender expects to sell at $E[P(z)|D_H + D_L - 2]$ and collect fee f. By recalling, the lender loses the fee but expects to sell at the higher price $E[P(z)|D_H + D_L = -1]$. Note that

$$\Delta E[P(z)] = \frac{1}{5}\left(P(1) - P(-4)\right) > 0$$

As a result, $f = 0$ is inconsistent with the no-recall condition given by Equation (1.12), meaning that L_s cannot commit to not recalling shares at his optimal fee when he wants to buy or not trade conditional $b = 0$. H will anticipate being recalled and never attempt to short when L_s sets fees so low.[23] Thus, if the lender wants to attract short sales, he must raise his fee to satisfy Equation (1.12).

Lemma 1.4. *The no-recall constraint only binds for the active lender when he buys or does not trade conditional on $b = 0$. When the lender's optimal fee from (1.11) violates the no-recall condition, his constrained-optimal fee becomes $\Delta E[P(z)]$.*

Intuitively, L_s's expected profits are decreasing in $f \geq 0$ when he wants to buy or not trade conditional on no short sale, so the new optimal choice of fee for these types of lenders will be the minimum fee that satisfies (1.12): $f_s^* = \Delta E[P(z)]$. L_s will prefer this choice to a fee that violates (1.12) because the latter is equivalent to setting an infinitely high fee (in that they both lead to zero borrowing demand), which is suboptimal given that the lender's profits are decreasing in f. The constraint also does not bind when the lender intends to sell following $b = 0$ because he already sets a high fee.

[23] H earns zero trading profits when recalled, which is never above her opportunity cost $c \in [0, \bar{C}]$.

Now that we have solved for the optimal fees given L_s's planned trading strategy, we can plug the fee associated with the conjectured trading strategy into (1.7) and (1.8) and solve the resulting two-dimensional fixed-point problem for c_1^* and c_0^*. After solving this fixed-point problem, I verify whether the conjectured strategies are optimal. First, I check that L_s does not want to deviate to another trading strategy, holding the fee constant. Then, I check that L_s does not want to deviate to any other fee and trading strategy pair.

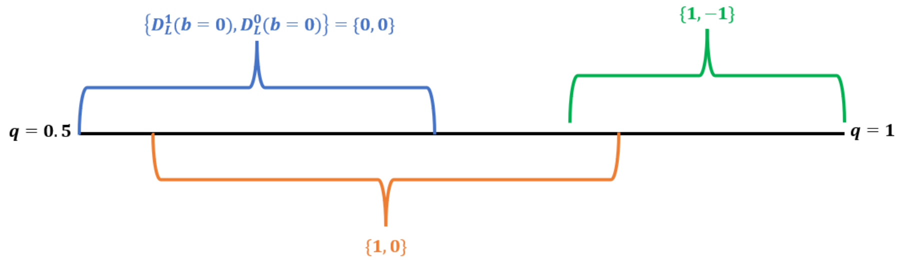
Proposition 1.2. *There are three possible active equilibrium:* $\{D_L^1(b=0), D_L^0(b=0)\} \in \Big\{\{0,0\},\{1,0\},\{1,-1\}\Big\}$.

Figure 1.2 displays the various equilibria as a function of q; the $\{0,0\}$ equilibrium exists for low values of q, the $\{1,0\}$ equilibrium exists for medium values of q, and the $\{1,-1\}$ equilibrium exists for high values of q. I give the exact intervals in the proof of this proposition in the appendix.

Starting with $\{0,0\}$, L's private signal is essentially just noise when q is near 0.5, so L finds it optimal to not trade conditional on $b=0$. However, as q increases, L_1's positive $s=1$ private signal more strongly complements the positive lending market signal $b=0$, making buying increasingly profitable. This development transitions us to the $\{1,0\}$ equilibrium. However, these two equilibria can also exist at the same value of q. When the market maker believes that L_1 is not trading, prices will be higher because of the increased overlap between the order flows that can occur after $v=0$ and those that can occur after $v=1$ (see the discussion surrounding Figure 1.1). As a result, L_1 may find it optimal not to trade conditional $b=0$ if the market maker believes that is the case (because of higher buying prices) while simultaneously being willing to buy if the market maker instead has that belief. Nevertheless, L_1's positive private signal eventually becomes strong enough to eliminate the $\{0,0\}$ equilibrium.

The story is similar when we transition to our final equilibrium: $\{1,-1\}$. As q increases, L_0's negative $s=0$ private signal dominates the positive lending market signal $b=0$, making it increasingly optimal for L_0 to sell. This outcome produces our final equilibrium:

Figure 1.2: Active Equilibrium



$\{D_L^1(b=0), D_L^0(b=0)\} = \{1,-1\}$. The $\{1,0\}$ and $\{1,-1\}$ equilibria can also overlap for reasons similar to the first case of overlap.

Corollary 1.1. *The expected equilibrium fee when $v = 0$ is always at least as high as the expected equilibrium fee when $v = 1$ and strictly higher when $\{D_L^1(b=0), D_L^0(b=0)\} = \{1,-1\}$ is the equilibrium.*

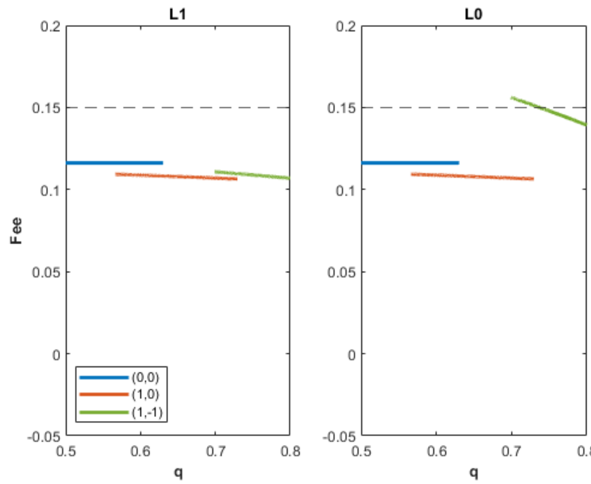
L_0 always chooses a fee no lower than L_1 and chooses one strictly higher when he sells conditional on $b = 0$. Thus, when $v = 0$, the lender is more likely to see signal $s = 0$ and set higher fees. Corollary 1.1 implies that higher short-selling fees predict lower subsequent returns, in line with the empirical literature.[24]

1.3.4 Analysis

I plot the equilibrium fees for L_1 and L_0 as a function of q in Figure 1.3; the dotted line represents the passive benchmark value.[25] Recall from (1.11) that the active lender (L_1 or L_0) wants to charge a fee of zero when he either buys or does not trade conditional on $b = 0$ and there is no recall risk. However, when recall risk is present, Figure 1.3 shows that these fees jump up and are instead at a level slightly below what the passive lender charges. Additionally, once L_0 sells conditional on $b = 0$, the fee increases further because the lender wants to deter trading competition.

[24] For example, see Duong et al. (2017) and Jones and Lamont (2002).
[25] In these graphs, $\bar{C} = 0.3$.

Figure 1.3: Equilibrium Fees



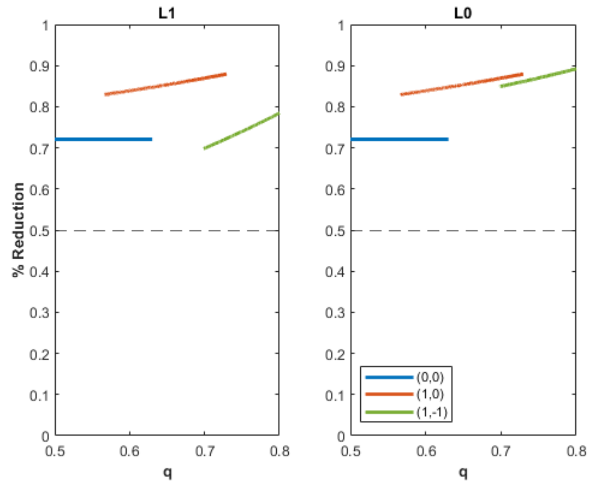
We see H's equilibrium reduction in selling because of the lending market in Figure 1.4. I define this measure as $1 - G(c^*)$ because H would always sell when $v = 0$ if endowed with a share (and the lender does not exist) given the definition of $\bar{C}$ (see Assumption 1.2). This expression is also a measure of equilibrium short-selling constraints: higher (lower) values indicate tighter (looser) constraints.

Corollary 1.2. *A passive lender always attracts more short selling than an active lender. That is, $G(c^*)$ is greater with a passive lender.*

The active lender wants to recall the loan when his fee is low, so he must raise it as a commitment device. As a result, the combined costs of increased fees and L copying H's trade exceed the cost of the passive lender's fee. This result explains the empirical finding that passive funds attract more short sellers despite charging higher fees (see Honkanen (forthcoming), Palia and Sokolinski (2024), and Beschwitz, Honkanen, and Schmidt (2022)). Active funds would ideally charge fees low enough to attract more short sellers than passive lenders, but they must charge higher fees to maintain their commitment to not recall shares. Finally, I look at price efficiency which I define as

$$Eff \equiv 1 - |v - E[P(z)|v]| \tag{1.13}$$

Figure 1.4: Equilibrium Percentage Reduction in Short Selling



The term $|v - E[P(z)|v]|$ measures the distance between the expected price conditional on the value and the value.[26] If prices are perfectly revealing, then this term equals zero. I subtract this absolute value from one so that my measure has the property that higher values indicate greater efficiency and lower values indicate lesser efficiency. As benchmarks, a completely uninformative price (i.e., unconditional expectation) has an efficiency measure of one-half, while a perfectly revealing price has an efficiency measure of one.

Proposition 1.3. *For all $q \in [0.5, 1)$, we can say the following about price efficiency:*

i Without the no-recall constraint, active efficiency is higher than passive efficiency for all $q \in [0.5, 1)$ where an active equilibrium exists.[27]

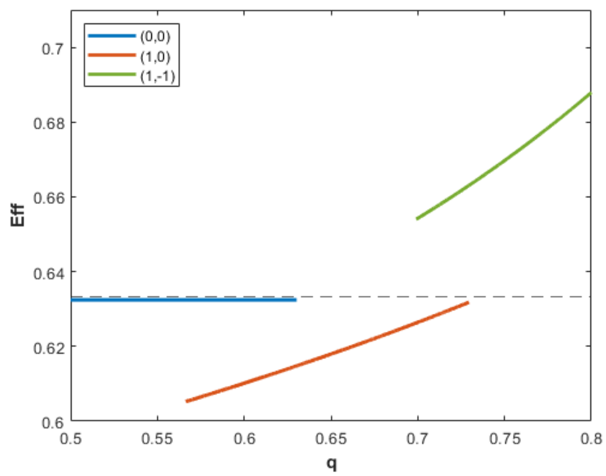
ii With the no-recall constraint, there exists some $q' \in (0.5, 1)$ such that for all $q < q'$, passive price efficiency is higher than active price efficiency in every equilibrium.

Proposition 1.3 says that price efficiency can be higher with a passive lender once I introduce recall risk. The desire to recall shares forces the active lender to raise fees as a commitment device; these higher fees, combined with the fact that the active lender front

[26]Note that the efficiency measure is the same regardless of whether $v = 1$ or $v = 0$. See the proof of Proposition 1.3 for details.

[27]The equilibria without the no-recall constraint are described by Proposition A2 in the appendix.

Figure 1.5: Equilibrium Price Efficiency



runs short sales, so exacerbate short-selling constraints that the passive lender who never trades can improve price efficiency. However, once the active lender becomes sufficiently well-informed, his trades improve price efficiency enough to counter the loss in information caused by the decline in short selling. In sum, passive lenders can improve price efficiency once they attract sufficiently more short sellers than active lenders, and this difference only becomes large enough when recall risk forces the active lender to increase his fees.

1.4 Corporate Governance and Securities Lending

The central theme of my paper is that borrowing demand is an informative signal about firm value for securities lenders. So far, I have looked at how lenders apply this information in their trades. However, information is valuable for corporate governance as well. The securities lenders in my model own several shares, so one can also interpret them as blockholders. In this section, I reintroduce governance as described in Section 1.2, so now firm value depends on both the state of the world θ and the lender's action a. The lender is a blockholder who can take a costly corrective action to improve firm value after learning about the realization of θ from the lending market and stock price. I interpret this action broadly to include any costly governance decision that can improve firm value: for example, voting against management, launching a proxy fight, or providing management with advice. I focus on the simplest case where the lender's private signal s is uninformative (i.e., $q = 0.5$). The main takeaways from this section do not depend on this assumption.

1.4.1 Intervention and Short-Selling Profits

In this section, I explore how lender intervention impacts equilibrium short selling. A 2015 paper by Edmans, Goldstein, and Jiang examines a similar question. They solve a feedback model where a manager learns from his stock price to make better decisions. The authors show that the manager's intervention endogenously increases short-sale constraints. To summarize their finding, short sellers learn that the state of the world is "bad," so they sell and drive down the stock price. The manager infers the state from the stock price and makes the efficient decision given that state, which increases firm value. This increase in firm value harms short sellers because they must close out their position at a higher price.

This result may change when the decision maker is a securities lender rather than a firm manager because the former can directly impact the short seller's profits through the lending fee. For example, by lowering the fee, the lender compensates the short seller for the higher close-out price post-intervention and improves the short seller's profits. However, without a formal analysis, it is difficult to predict whether the lender values the improvements in firm value enough to do so in equilibrium.

I start by analyzing the passive lender. Working backward, the lender decides whether to intervene based on the lending market signal b and the realized order flow z. He will intervene if and only if

$$N\Bigg(E[v(\theta, a = 1)|b, z] - E[v(\theta, a = 0)|b, z]\Bigg) > k \tag{1.14}$$

Equation (1.14) says that the lender will intervene when the expected increase in the payoff following intervention multiplied by the number of shares he owns N exceeds the cost of intervention k. The lender's valuation can be described by three regions of the variables b and z.

$$\begin{aligned} &\text{(i) } E[v(\theta, a)|b, z] = 1 \text{ when } b = 0 \text{ and } z = 3 \\ &\text{(ii) } E[v(\theta, a)|b, z] = \frac{1 + a(1 - G(c^*))\psi}{2 - G(c^*)} \text{ when } b = 0 \text{ and } z \in \{2, 1, 0, -1\} \\ &\text{(iii) } E[v(\theta, a)|b, z] = a\psi \text{ when } b = 1 \text{ or } (b = 0 \text{ and } z = -2) \end{aligned} \tag{1.15}$$

where c^* is H's equilibrium short-selling threshold, which in this case is $c^* = E[P(z)|D_H + D_L = -1] - f^* - a\psi$. Notice that, all else equal, intervention lowers the threshold (i.e., reduces short-selling profits) by increasing the short seller's expected close-out price from zero to $\psi > 0$. Region (i) corresponds to the case where L learns that $\theta = 1$, Region (ii) corresponds to the case where all the lender learns is that H did not borrow a share (i.e., $b = 0$), and Region (iii) corresponds to the case where L learns that $\theta = 0$.

Corollary 1.3. *The lender is more likely to intervene when* $b = 1$.

Corollary 1.3 follows from (1.15). Intervention is more likely when $E[v(\theta, a = 1)|b, z] - E[v(\theta, a = 0)|b, z]$ is large, and this difference will be greatest when the lender knows that $\theta = 0$ (i.e., he is in Region (iii)). The lending market outcome $b = 1$ guarantees that we are in Region (iii), while $b = 0$ only puts us in Region (iii) with only some probability.

The lender never intervenes in Region (i) because intervention does not impact firm value. I assume $N\psi > k$ so the lender intervenes in Region (iii) because if not, he never intervenes, and we return to the analysis from Section 1.3. There are two cases to analyze: first, the lender only intervenes in Region (iii), and second, the lender intervenes in both Regions (ii) and (iii). Denote $a_2 \in \{0, 1\}$ as the action the lender takes in Region (ii).

Taking the same steps as in Section 1.3, we work backward until we reach the lender's fee-setting problem conditional on his trading (always zero for the passive lender) and intervention strategies. The lender chooses fee f to maximize expected terminal wealth.[28]

$$\max_f Pr[b=1]\Bigg(N\psi + f - k\Bigg) + Pr[b=0, z=-2]\Bigg(N\psi - k\Bigg) + \tag{1.16}$$
$$Pr\left[b=0, z=\in\{2,1,0,-1\}\right]\Bigg(NE[v(\theta,a_2)|b=0, z\in\{2,1,0,-1\}] - a_2k\Bigg) +$$
$$Pr[b=0, z=3](N)$$

The first term corresponds to the event $b=1$ while the second term corresponds to the event where no short sale occurs, but the lender learns that $\theta=0$ from the price $P(-2)$. The third term represents the event where $b=0$ and the price provides no further information (i.e., Region (ii)), and the final term is the event where the price $P(3)$ reveals that $\theta=1$.

The solution to (1.16) is

$$f(\psi)^* = \begin{cases} \frac{E[P(z)|D_H+D_L=-1]}{2} - \frac{(4N+5)\psi-4k}{10} \text{ when } a_2=0 \\ \frac{E[P(z)|D_H+D_L=-1]-\psi}{2} \text{ when } a_2=1 \end{cases} \tag{1.17}$$

Equation (1.17) tells us that the lender reduces fees when he intervenes and that fees are lower when $a_2=0$. Intuitively, the passive lender charges lower fees when $a_2=0$ to improve the accuracy of the lending market signal because it allows him to learn when $\theta=0$ more frequently and, therefore, intervene to improve the value of his N shares. Note also that the passive lender lowers fees more than one-for-one with ψ in this case. Conversely, the lending market signal does not affect the lender's intervention decision when $a_2=1$; he always intervenes unless the price reveals that $\theta=1$. As a result, the passive lender's optimal fee when $a_2=1$ maximizes expected lending revenue, like in Section 1.3; he is unwilling to sacrifice lending revenue to improve the quality of a signal he does not use. To focus on how outcomes adjust to changes in ψ, I will assume the parameters N, k, and ψ are such that the fees given by (1.17) are positive. Otherwise, there is no room for the lender to compensate

[28]Equation (1.16) is the same objective function as (1.5). Terms involving the number of shares owned N were dropped from (1.5) because this parameter does not affect outcomes without governance.

short sellers, so their profits are guaranteed to decline in ψ.[29] Solving for the equilibrium given this fee choice generates the following result.

Proposition 1.4. *When the parameters N, k, and ψ induce the passive lender to select $a_2 = 0$, short-selling profits are increasing in ψ. When N, k, and ψ lead to $a_2 = 1$, short-selling profits are decreasing in ψ.*

The specific values of N, k, and ψ that support a_2 equaling zero or one as equilibrium intervention strategies are given in the appendix. In contrast to Edmans, Goldstein, and Jiang (2015), Proposition 1.4 says that short-selling profits increase when ex-post intervention is possible when $a_2 = 0$ is the lender's intervention strategy (i.e., he only intervenes in Region (iii)). Short-selling profits equal:

$$E[P(z)|D_H + D_L = -1] - f(\psi)^* - \psi$$

As ψ increases, H's expected *selling* profits (i.e., $E[P(z)|D_H + D_L = -1] - \psi$) decrease. Intuitively, her expected selling price does not increase as much as the actual value following intervention because the market maker does not know whether the intervention will happen; therefore, if fees do not adjust, short-selling profits decline when intervention occurs. However, short-sale constraints loosen when $a_2 = 0$ because the passive lender lowers the fee by more than the expected increase in firm value. Fees do not decline as much when $a_2 = 1$ because the lender's intervention decision no longer depends on the lending market signal: he always intervenes unless the stock price reveals that $\theta = 1$ for sure. As a result, fees do not drop by enough to increase short-selling profits. In sum, short-selling profits only increase with intervention when the lending market signal influences the lender's intervention decision.

In principle, the results are similar for the active lender. He, too, owns several shares of the firm and ideally wants to lower his fee to intervene more effectively when his intervention

[29] This assumption requires $-\frac{8\,k-6\,\psi-8\,N\,\psi-8\,k\,\psi+8\,N\,\psi^2+3}{10\,(\psi-2)} > 0$.

strategy is $a_2 = 0$. However, there is a caveat. Unlike the passive lender, the active lender may not be able to lower his fee to his new optimum—regardless of the specific values of N, k, and ψ—because of the no-recall constraint from the base model given by Equation (1.12). Proposition 1.5 then follows.

Proposition 1.5. *When the parameters N, k, and ψ induce the active lender to select $a_2 = 0$, the unique equilibrium trading strategy for L is to sell conditional on $b = 1$ and not trade conditional on $b = 0$. Furthermore, equilibrium short-selling profits are decreasing in ψ.*

I provide the values of N, k, and ψ that lead to $a_2 = 0$ in the proof of this proposition in the appendix. Like in Section 1.3.3, it is an equilibrium for the active lender with an uninformative private signal to sell when $b = 1$ and not trade otherwise. As a result, his optimal fee when $a_2 = 0$ can be shown to equal $f(\psi)^* = \frac{2N}{5}(k - N\psi) < 0$, which means in equilibrium, he must charge $f = \Delta E[P(z)] > 0$.[30] Consequently, the active lender cannot counter the decline in trading profits ($E[P(z)|D_H + D_L = -2] - \psi$) for the short-seller with lower fees, leading to lower equilibrium profits for the short seller.

The active lender also has a lower incentive to intervene because of his inclination to sell shares after observing $b = 1$. Selling after observing $b = 1$ is always an equilibrium trading strategy for the lender because, like H, he now knows that the state of the world is $\theta = 0$ but does not have to pay any fee to sell. Thus, if H finds it profitable to short the stock, L must also find it profitable to sell.[31] Because of this sale, the active lender owns $(N - 1)$ rather than N shares when deciding whether to intervene, making (1.14) less likely to be satisfied (after substituting $(N - 1)$ in for N). This result shows how the lending market can generate an exit vs. voice tradeoff for blockholders: information enhances voice (i.e., efficient intervention) yet also reduces "skin in the game" by encouraging exit.[32] This insight also suggests a new complementarity between liquidity and blockholder engagement: increased

[30] Or $f^* = 0$ if $\Delta E[P(z)] < 0$; see the proof for details.
[31] See the formal proof of this claim in Lemma A1 in the appendix.
[32] See Bhide (1993) and Coffee (1991) for an overview of this tradeoff. Conversely, Levit (2018) analyzes how the threat of exit can aid communication between shareholders and management.

liquidity induces more short selling, which makes the lender more informed.

Passive lenders, by contrast, experience the benefits of enhanced voice through better information without the downside (from a governance perspective) of greater incentives to exit. This insight suggests that the securities lending practices of index funds can ameliorate concerns about their disinclination to gather information for governance because of their diversified stakes in many firms (Brav, Malenko, and Malenko (2023), Kahan and Rock (2020)).

1.4.2 Application: Voting and Recall

Voting is one of the most important governance actions that a large shareholder can take. However, investors lose the votes associated with stocks on loan unless they are recalled before the vote's record date. In this sense, the foregone fee revenue is the opportunity cost of voting. The tradeoff between lending revenue and voting power is well known in both the academic literature (e.g., Aggarwal, Saffi, and Sturgess (2015) and Hu, Mitts, and Sylvester (2021)) and among policymakers.[33]

However, recalling shares to vote entails not only a loss of fee income, but also a loss of information because informed speculators will not borrow when they anticipate recall. While previous studies have convincingly argued that borrowing demand on the record date itself is likely because of so-called vote traders (i.e., those who borrow shares to increase their voting power) rather than informed short sellers, the decision to recall shares affects all borrowers, not just those who borrow on the record date.[34] Thus, informed speculators will not borrow shares before the record date if recall is expected, making the securities lender less informed about the firm's prospects and the importance of his vote. This loss of information ultimately leads to less informed decisions.[35]

[33]In 2019, the SEC provided guidance regarding the proxy voting responsibilities of investment advisors. This guidance explicitly mentions the opportunity cost of securities lending income as an acceptable justification for not voting. https://www.govinfo.gov/content/pkg/FR-2019-09-10/pdf/2019-18342.pdf

[34]See Christoffersen et al. (2007) and Aggarwal, Saffi, and Sturgess (2015) for empirical research on vote trading.

[35]This application focuses on the voting choice of the securities lender, not the borrower. I implicitly

I reinterpret my governance analysis to analyze this tradeoff between voting and lending. First, I interpret the action $a = 1$ as voting against management. The cost k then represents the lender's cost of antagonizing management following such a vote.[36] I further assume that the lender's corrective action now works with probability ψ_H when $\theta = 0$ following recall (or if there was no loan in the first place) and with probability $\psi_L < \psi_H$ if he has a share on loan. That is, having a share on loan makes the lender's intervention less likely to succeed. I interpret the difference $\psi_H - \psi_L > 0$ as the increase in the probability that the lender's preferred outcome occurs when he votes all of his shares.

This application generates a new no-recall constraint for the active and passive lender. The passive lender is not incentivized to recall shares for strategic trading purposes but may recall shares to increase his voting power. I assume $N\psi_H > k$ so the lender will always intervene in Region (iii) (i.e., when L knows $\theta = 0$) when the probability of success is ψ_h. If this condition fails, intervention never occurs, and we return to the baseline model of Section 1.3. Given this assumption, the no-recall constraints for the lenders are:

$$f > \begin{cases} N\psi_H - k - \max(N\psi_L - k, 0) \text{ when passive} \\ (N-1)\psi_H - k - \max((N-1)\psi_L - k, 0) + \Delta E[P(z)] \text{ when active} \end{cases} \tag{1.18}$$

Equation (1.18) says that the passive lender's fee must exceed the expected increase in terminal wealth following recall and intervention. The no-recall constraint for the active lender is similar, except that it incorporates his sale of the stock following a loan agreement; this trade lowers the active lender's ownership to $N - 1$ shares and reintroduces the usual $\Delta E[P(Z)]$ term, which represents his expected gain in trading profits following recall. Thus,

assume that the shorted share ends up in the hands of either the market maker or noise traders, who, for some reason, are not guaranteed to vote the same way as the lender (if they vote at all); hence, $\psi_L \leq \psi_H$. For research on the voting decisions of borrowers and the so-called empty voting phenomenon, see Hu and Black (2006) and Brav and Mathews (2011).

[36] Examples of these costs include harming other business ties with the company (such as management of the company's 401k plan) or making management less amenable to the lender's future suggestions. See Bebchuk and Hirst (2019) for a discussion on index funds being deferential to management and Cvijanovic, Dasgupta, and Zachariadis (2016) for evidence on how business ties increase pro-management voting.

the desire to recall shares to vote introduces a positive lower bound on fees for the passive lender and further raises the minimum fee for the active lender. If the lender's optimal fee violates Equation (1.18), then his constrained-optimum fee will be the fee that exactly satisfies this condition. The intuition for this result is the same as in the base model.

Even though not recalling shares is optimal for the lender (and thus also for his investors since there are no agency problems), it is an open question whether such a policy is good for firm value. Suppose the lender commits to a policy of not recalling shares. He must then decide whether to intervene in Region (iii) (i.e., when L knows $\theta = 0$) when $\psi = \psi_L$ and in Region (ii) (i.e., when all L knows is that $b = 0$). Denote a_3^{NR} as his decision in the former and a_2^{NR} as his decision in the latter. Now consider an otherwise identical lender whose policy is to recall his shares on the record date.[37] By assumption, this lender always intervenes in Region (iii) (since his probability of success is ψ_H), so his only decision is whether to intervene in Region (ii), which I will denote as a_2^R.[38] There are eight combinations of a_3^{NR}, a_2^{NR}, and a_2^R to check to see the impact of share recall on firm value. The firm's expected values with and without recall are:

$$E[v(\theta,a)|\text{no recall}] = \left(\frac{G(c^*)}{2}\right)\left(\psi_L a_3^{NR}\right) + \left(\frac{1-G(c^*)}{10}\right)(\psi_H) \tag{1.19}$$

$$+ \left(\frac{4(1-G(c^*))}{10} + \frac{4}{10}\right)\left(\frac{1 + a_2^{NR}(1-G(c^*))\psi_H}{2-G(c^*)}\right) + \left(\frac{1}{10}\right)(1)$$

$$E[v(\theta,a)|\text{recall}] = \left(\frac{1}{10}\right)\psi_H + \frac{8}{10}\left(\frac{1+a_2^R\psi_H}{2}\right) + \left(\frac{1}{10}\right)(1) \tag{1.20}$$

Equations (1.19) and (1.20) hold for both the active and passive lender (though with different values for c^*). The four terms in (1.19) correspond to those described in Equation (1.16): observing $b = 1$, observing $b = 0$ and learning that $\theta = 0$ from the stock price, observing $b = 0$ and not learning anything else from the stock price, and observing $b = 0$ and learning

[37] I assume the lender can commit to this suboptimal decision, perhaps because a regulatory change forces him to.

[38] Intervention still has no effect in Region (i) where L knows $\theta = 1$. Thus, it is never optimal for L to intervene here.

that $\theta = 1$ from the stock price. With recall (i.e., Equation (1.20)), there is no short selling, so there are only three terms that correspond to learning that $\theta = 0$ from the stock price, not learning anything from the stock price, and learning that $\theta = 1$ from the stock price.

Proposition 1.6. *Firm value can increase when the lender does not recall shares to vote. This result occurs when the parameters N, ψ_H, ψ_L, and k produce the following equilibrium governance choices: $a_3^{NR} = 1, a_2^{NR} = 0$, $a_2^R = 0$, and $5\psi_L > \psi_H$.*

See the proof in the appendix for conditions that trigger each intervention decision. Firm value improves with a policy of not recalling shares when $a_3^{NR} = 1, a_2^{NR} = 0$, $a_2^R = 0$, and $5\psi_L > \psi_H$; that is, when intervention occurs if and only if the lender knows $\theta = 0$. As a result, not recalling shares can improve firm value because the lender will learn when $\theta = 0$ more often through the lending market signal $b = 1$. Firm value will be higher in this case as long as ψ_L is not too much lower than ψ_H. This result suggests that a securities lender not recalling shares is good for firm value when voting those extra shares does not increase the probability of his desired voting outcome too much (i.e., $\psi_H - \psi_L$ is small) and when his optimal vote is not obvious (i.e., lending market information can influence his decision). By contrast, when $a_2^R = 1$, the recalling lender always intervenes unless the price reveals $\theta = 1$. As a result, intervention no longer occurs more frequently with a policy of not recalling shares, so reducing the probability of successful intervention for information that does not increase the frequency of intervention must harm firm value. However, it remains optimal for the lender to not recall shares because he is compensated for this loss through the fee and increased trading profits (if active).

Thus, there may be a conflict between maximizing fund value and maximizing the value of the firms in the fund for large shareholders who lend securities. To determine whether a regulation requiring securities lenders to recall shares before the record date is beneficial, one must trade off the lender's role as a fiduciary for his investors and the broader economic benefits from improved firm value for other stakeholders.

1.5 Discussion of Model Assumptions

This section discusses various tweaks to the model's assumptions. Since the changes focus on trading outcomes, I assume that no governance occurs in this section (i.e., $k > (N + D_L)\psi$ so $a = 0$, like in Section 1.3).

1.5.1 Maximum Cost of Trade: $\bar{C}$

Proposition 1.3 states that price efficiency can improve under the passive lender when he attracts enough additional short sellers. Mathematically, $G(c^*) = \frac{c^*}{\bar{C}}$ must be sufficiently higher under the passive lender to improve price efficiency. The parameter $\bar{C}$, as set in Assumption 1.2, ensures meaningful variation in $G(c^*)$ as lending market conditions (e.g., fees, lender trades, and lender information) change. Thus, when short-selling profits are greater under one type of lender, there will be a material increase in expected short-selling demand. I now wish to discuss what happens to short-selling demand and price efficiency when I generalize $\bar{C}$.

It will be instructive to consider the extreme cases first. Suppose $\bar{C} \to 0$, so there is no opportunity cost of trade. Then, the perfectly informed H will always be willing to trade in equilibrium regardless of lending market conditions, so $G(c^*) \approx 1$ for both the active and passive lender. As a result, there is no variation in expected short-selling demand between the active and passive lender, even if short-selling profits are relatively higher under the latter. We get an analogous result when $\bar{C} \to \infty$. Here, the expected opportunity cost of trade is prohibitive, so the probability of H trading is essentially zero; that is, $G(c^*) \approx 0$. Thus, even though short-selling is relatively more profitable under the passive lender, it generates no additional expected borrowing demand, since short-selling profits are always less than the opportunity cost. The intuition is similar when I relax Assumption 1.1 and allow changes in $\hat{C}$ to affect $H's$ purchases when $v = 1$.

Consequently, $\bar{C}$ must take an intermediate value to make the analysis non-trivial. In

Appendix B, I relax Assumptions 1 and 2, set $\hat{C} = \bar{C}$, and numerically solve for H's equilibrium buying and selling cutoffs; I then examine price efficiency as a function of $\bar{C}$. This analysis shows that price efficiency can be higher under the passive lender when $\bar{C}$ lies in an intermediate range (besides the other requirements outlined in Proposition 1.3). When $\bar{C}$ gets too big or too small, improved short-selling profits under the passive lender do not generate enough of an increase in $G(c^*)$ to allow him to improve price efficiency. Whether $\bar{C}$ (and for that matter, q) is indeed in this efficiency-improving range is ultimately an empirical question; however, there is evidence that supports the claim that short-selling demand is sensitive to costs, as an intermediate $\bar{C}$ predicts. For example, von Beschwitz, Honkanen, and Schmidt (2022) and Palia and Sokolinski (2024) show that short-selling demand for a stock increases with higher levels of passive ownership (similar to my Corollary 1.2), and Engleberg, Reed, and Ringgenberg (2018) show that short-selling demand decreases when short selling becomes riskier (i.e., more uncertainty about future fees and share availability). Finally, Mainardi (2023) constructs and estimates a structural model of short selling and finds that a 1% exogenous increase in lending fees leads to a decrease in shorting demand of between 0.98% and 4%.

1.5.2 Perfectly Informed Lender

The analysis assumes $q < 1$ to allow the active lender to learn something from the perfectly informed speculator. Let us now consider what happens when $q = 1$, so the lender is also perfectly informed. To be as general as possible, I will relax Assumptions 1 and 2 for this analysis. Denote y_s^* as H's equilibrium buying threshold and $\hat{G}(y) \equiv= \frac{y}{\hat{C}}$ as the associated CDF; that is, H buys when $v = 1$ if and only if $c < y_s^* \leq \hat{C}$. Naturally, L_1's optimal demand is now to buy and L_0's is to sell. In addition, L_1's optimal fee is undefined because H will never short sell when $v = 1$. As a result, short selling only occurs under L_0 (and H buying only occurs under L_1). L_0's fee-setting problem is unchanged from Section 1.3, so his optimal fee is still $f_0^* = \frac{E[P(z)|D_H+D_L=-1]}{2}$. I then solve the following two-dimensional fixed-point problem for y_1^* and c_0^*:

$$y_1^* = 1 - E[P(z)|D_H + D_L = 2] \tag{1.21}$$
$$c_0^* = E[P(z)|D_H + D_L = -2] - f_0^*$$

Despite already having perfect information, L_0's chosen fee will not always eliminate short-selling. The following corollary characterizes when short selling is eliminated in this setting.

Corollary 1.4. *Short selling will be eliminated (i.e., $G(c_0^*) = 0$) with a perfectly informed lender if and only if $\hat{G}(y_1^*) = 1$.*

The perfectly informed L_0 has the same fee-setting motivation as an imperfectly informed lender who sells conditional on $b = 0$, so he does not go out of his way to eliminate short selling: his optimal fee exhibits the same tradeoff between fee revenue and a better selling price. Short selling is only eliminated in equilibrium when the solution to the fixed-point problem characterized by (1.21) produces $\hat{G}(y_1^*) = 1$ (which happens when $\hat{C}$ is "small"). Intuitively, buying and selling are strategic substitutes for H: when $\hat{G}(y_1^*)$ increases, the set of order flows that can occur following $D_H = 0$ are less strongly associated with $v = 1$, so their prices decline. Since the set of order flows that can occur when $D_H \neq 0$ overlaps with this set, H's expected buying and selling prices fall, incentivizing further buying and less selling, and so on.

1.5.3 Lender Myopia

It is perhaps unsurprising that lenders behave so opportunistically in my static model. There is no future relationship or reputation to maintain, so active lenders will take advantage of the short seller to improve trading profits. As a result, fees must adjust, sometimes dramatically, to account for this behavior. It is then natural to ask whether active lenders would behave the same way in an infinitely repeated game, where the lender considers not only current period profits but also the present value of all future profits. Can a reduced continuation value for the lender upon the detection of front running or strategic recall deter such myopic behavior?

I argue that front running is unlikely to be deterred for two reasons. First, if the short seller's ex-post information set is only her trading price, perfect detection of front running is impossible in my model. For example, H expects an order flow between 1 and -3 when she shorts and L never trades. If L deviates by selling after observing $b = 1$, the resulting order flow will be between 0 and -4. The only way H can detect this deviation is if the price outcome is $P(-4)$ *and* $P(-4) \neq P(z)$ for $z \in \{1, 0, ..., -3\}$. However, with "unprejudiced" beliefs about off-equilibrium order flows (as discussed in Section 1.2.4), $P(-4)$ will always equal $P(-3)$, meaning that H cannot detect this deviation.[39]

It is also not clear that the short seller wants her lender to have such commitment power because fees may increase. If the lender commits to not trading, he becomes a passive lender and will set fees as described in Section 1.3.2. As shown in Figure 1.3, passive fees exceed active fees unless we are in the equilibrium where L_0 sells conditional on $b = 0$. It is, therefore, misleading to say that the lender is taking advantage of the short seller because the short seller's fees can decline with front running. A more accurate interpretation of this scenario is that the short seller sells her information to the lender for a reduced fee.

Unlike front running, the short seller can catch and will want to discourage strategic recall. The recall decision is a message sent directly from the lender to the borrower, so there is no ambiguity. Furthermore, recall always harms the short seller's profits. Applying standard results from game theory, the lender will not recall shares in an infinitely repeated game if and only if the following reduced-form condition is satisfied:

[39] Even if we expand $H's$ ex-post information set to include the specific realization of z, there is still only a $\frac{1}{5}$ probability of detection. Detection probability is almost certainly lower in actual financial markets due to the presence of many more traders who can alter the resulting order flow and the ability of the lender to shade his trades to avoid detection.

$$f + E[P(z)|D_H + D_L = -2] + \frac{\delta}{1-\delta}\pi_H > E[P(z)|D_H + D_L = -1] + \frac{\delta}{1-\delta}\pi_L$$
$$\implies f > \Delta E[P(z)] - \frac{\delta}{1-\delta}(\pi_H - \pi_L) \tag{1.22}$$

where δ is the lender's discount factor, π_H is the lender's expected per-period profit when he has never recalled shares, and π_L is the lender's expected per-period profit when he has recalled shares before. Loosely, H plays a "grim-trigger" strategy where she acts as if there is no recall risk so long as she has never been recalled in the past and only accepts fees that satisfy the static no-recall constraint (1.12) otherwise. Observe that $\pi_H \geq \pi_L$ because the lender can move his fee closer to the optimal level described in (1.11) if he never deviates (assuming the static no-recall constraint binds). If L cares sufficiently about the future (i.e., a large δ), the threat of a permanent switch to the "bad" recall-proof equilibrium from Proposition 1.2 is enough to deter recall, even when fees are low. I discuss which factors are likely to impact δ empirically in Section 1.6.

1.5.4 Cost of Lending

I have abstracted from lending costs in this model, though these costs can be quite substantial in practice. For example, Johnson and Weitzner (2024) show that, on average, 17.4% of gross lending revenue goes to costs, such as lending agent fees and cash collateral fees. These costs, therefore, put a wedge between the fee paid by the borrower and the lending profits of the lender. I want to examine how the existence of this wedge impacts the behavior of lenders in my model. To be concrete, suppose that the short seller continues to pay a fee f, but the lender only pockets ρf as profits, where $\rho \in (0, 1)$. Corollary 1.5 describes what happens to fees.

Corollary 1.5. *The passive lender's optimal fee does not change when $\rho < 1$. When the no-recall constraint does not bind, the active lender tilts his optimal fee in the direction that* *increases expected trading profits. When the no-recall constraint does bind, the active lender increases fees.*

The passive lender maximizes expected lending revenue (the probability of H selling short times the fee), so introducing a proportional cost does not change this optimum; it reduces the passive lender's profits without distorting economic activity. The active lender, on the other hand, sets fees by trading off lending and trading profits. Thus, the reduction in lending profits caused by $\rho < 1$ incentivizes him to put more weight on trading profits when choosing a fee, so fees decline when $D_L^s(b=0) \in \{1, 0\}$ and increase when $D_L^s(b=0) = -1$. However, when the active lender's no-recall constraint binds, $\rho < 1$ further increases that lower bound, leading to higher fees. Intuitively, the lender needs high lending profits to refrain from recalling shares, so when ρ declines, fees must rise commensurately to maintain that commitment.

1.6 Empirical Applications

1.6.1 Direct Predictions

My model makes predictions about how lenders' trades are affected by lending market information. First, active securities lenders sell stocks that they have on loan. This prediction aligns with the findings of Honkanen (forthcoming) and Greppmair et al. (2024). Second, my model makes new predictions about how fees relate to a lender's trades on stocks made available for loan but not taken up. Equation (1.11) states that high fees combined with low utilization predict lender sales, while low fees combined with low utilization predict lender purchases. The intuition is that lenders with low stock valuations expect high short-selling demand and raise fees to limit trading competition. Lenders with high valuations prefer to set low fees to learn from the lending market because they expect little lending revenue to begin with. Thus, high fees also predict low returns, which is consistent with much empirical evidence. However, in my model, high fees emerge before borrowing demand is realized rather than because of it (similar to the empirical findings of Duong et al. (2017)).

My model also predicts that strategic recall risk increases equilibrium fees. Active lenders are tempted to recall shares to improve their selling price and must raise fees as a sign of commitment. Passive lenders have no such incentive because they cannot trade. Even though strategic recall never happens in equilibrium in my model, this result relies heavily on the lender and borrower having rational expectations and, in particular, complete knowledge of the price-formation process. Suppose there is some uncertainty about what $\Delta E[P(z)]$ is; here, we should expect the borrower to make some "mistakes" and occasionally underestimate which fees satisfy the no-recall constraint: $f > \Delta E[P(z)]$. Thus, when mistakes are allowed, active funds should strategically recall shares more often than passive funds. Furthermore, by applying the infinitely repeated game logic of Section 1.5.3, those funds that have strategically recalled shares in the past should have to charge higher fees going forward to attract short-sellers.

My corporate governance analysis makes predictions about blockholder intervention. Corollary 1.3 states that the lender is more likely to intervene in a firm whose shares the he has on loan. Blockholder intervention takes various forms, such as advising management, launching a proxy fight, or making an informed vote. In particular, since short sales convey negative information, any intervention that follows should be against the status quo operations of the firm. There is some existing empirical evidence for this prediction. Aggarwal, Saffi, and Sturgess (2015) examine changes in stock-level lendable supply around record dates and show that recall activity (i.e., a decline in lendable supply) is associated with less support for management in the subsequent votes. Similarly, Li and Zhu (2024) show that fund-level recalls are associated with that same fund voting against management.

Proposition 1.4 implies that passive lenders lower fees when lending market information influences their intervention decisions. To test this prediction, one must identify instances when passive lenders seek more information for governance. For example, passive funds with a history of voting independently from proxy advisors or that do not have a one-size-fits-all governance strategy can be considered more engaged shareholders. As a result, these funds should be more willing to sacrifice lending revenue for information. Furthermore, Proposition 1.5 implies that similarly engaged active funds should be less likely to lower fees because the no-recall constraint often prevents them from doing so.

Proposition 1.6 states that a policy of not recalling shares to vote can improve firm value. As in the previous paragraph, this outcome arises when the lending market information can influence the lender's decision; my model predicts this situation occurs when the lender is an engaged voter whose optimal decision is unclear. This uncertainty could exist because the issue is complicated or because the lender lacks better sources of information. In addition, Proposition 1.6 implies that not recalling shares can improve firm value when the extra votes are unlikely to impact the outcome. For example, if the lender's remaining votes are already pivotal before recall, then $\psi_L = \psi_H$ and there is no downside to not recalling shares to vote. However, if the lender can substantially increase the likelihood of becoming pivotal following recall, then the cost of not recalling shares increases (since $\psi_H >> \psi_L$), making recalling shares the better policy for firm value.

1.6.2 Possible Empirical Extensions

I expect my results to be more salient for lenders with an in-house lending agent. I have abstracted from the lending agent in this paper, but in practice, shareholders deposit the shares they want to lend with a lending agent, who is then responsible for lending them out. This lending agent is often a third-party dealer bank. Funds may use a third party because establishing a lending program can be expensive. However, some large mutual fund families—such as Fidelity, BlackRock, and Vanguard—have their own internal lending operations. I would, therefore, expect in-house lenders to show greater care in tailoring fees to maximize fund (or fund family) profits as described in this paper compared to a third party. This conjecture also suggests that switching from a third-party to an in-house lending agent is about more than just costs: there are also revenue benefits from optimal fee setting. To what extent in-house vs. third-party lending agents differ in their fee setting is an interesting question for future research.

As mentioned in Section 1.5.3, the lender's time preference parameter δ from Equation (1.22) affects his willingness to recall shares in a repeated game. Specifically, he will not recall the share when: $f > \Delta E[P(z)] - \frac{\delta}{1-\delta}(\pi_H - \pi_L)$. The parameter δ has many empirical interpretations. First, one can view the discount factor δ as the probability of the game continuing for another period. Suppose there are two types of speculators: a high type who will continue playing next period with probability $\delta^h > 0$ and a low type who will continue with probability zero. One can view the high types as the lender's long-term customers, while the low types are one-off short sellers. Naturally, the lender will be less inclined to recall a high type's share because he expects future business from her. Thus, from Equation (1.22), we expect repeat customers to get lower fees. Suppose further that the speculator's type is only known by her current lender. High-type speculators would be hesitant to switch lenders because a new lender would not know her type and would therefore require a greater fee to abstain from recalling the loan. The implication that the incumbent is the only lender who can commit to not recalling loans at low fees is a novel source of market power in the short-selling literature. This prediction is analogous to those found in the banking literature on relationship lending.

Agency concerns may also influence the lender's discount factor from Section 1.5.3. Suppose the lender is a mutual fund. The fund manager cares about his fund's performance to the extent that it impacts his present and future compensation. As a result, the fund manager's optimal recall strategy (i.e., his δ) may differ from that of his investors. For example, two recent empirical papers show that fund manager pay is a concave function of fund revenue (Ibert et al. (2018) and Cen et al. (2024)). Such a relationship suggests that a fund manager will be more likely to act opportunistically and recall shares when current performance is weak. Similarly, Chevalier and Ellison (1998) show that inexperienced fund managers face concave incentives with respect to job security. These incentives should soften as the manager develops a longer history of strong performance. Thus, I expect inexperienced fund managers to have a lower δ and set higher fees to satisfy the no-recall constraint.

While an index fund manager cannot trade on information gained from the lending market, he should be able to pass it along to an active fund in the same family.[40] There is mixed empirical evidence for this conjecture: Honkanen (forthcoming) finds evidence for these within-family spillovers in the U.S., while Greppmair et al. (2024) find none in Germany. My model predicts that index funds that share lending market information with others will charge lower fees in equilibrium because short sellers will anticipate information leakage and require lower fees to borrow.

Finally, my result about share recall harming firm value relies on the lender losing information from the associated reduction in short selling. The impact of this information loss should be more severe when the time between the record and voting dates is shorter; with more time between these dates, the lender can better make up for the loss in information pre-record date with more aggressive information gathering (either through the lending market or otherwise) post-record date. Empirically, one can exploit cross-sectional differences in these timing gaps to understand how important this loss of information is for firm value. For example, the gap between the record and voting date is at most 48 hours in the United Kingdom and between 10 and 60 days in Delaware.[41]

1.7 Conclusion

In this paper, I model securities lending as an information acquisition problem, where lenders infer information about a stock from the level of borrowing demand. In my base model, lenders with high valuations set fees as low as possible to verify their information. In contrast, those with low valuations prefer to set high fees to eliminate trading competition.

[40] The index fund manager may be incentivized to do so because of pay linked to family performance. See Ibert et al. (2018) and Cen et al. (2024) for evidence of this compensation pattern. Furthermore, Chague, Giovannetti, and Herskovic (2023) find that Brazilian securities lenders leak lending market information to other clients.

[41] See Section 360B of the 2006 Companies Act for the UK and Del. Code Ann. tit. 8, § 213 for Delaware.

Lenders are also tempted to recall lent-out shares to disguise their trades when fees are low; as a result, they must raise fees as a sign of commitment to continue to attract short sellers. This higher fee reduces short selling and allows passive lenders, who never trade, to improve price efficiency.

I also analyze what happens when the securities lender acts as a blockholder who can intervene to improve firm value. In contrast to prior research, I show that short-selling profits can improve when a decision maker intervenes based on financial (and lending) market information when that decision maker is a securities lender. Unlike other interested agents (e.g., management, the board, and other non-lending blockholders), a securities lender can compensate the short seller's decline in trading profits with lower fees. I also show that firm value can increase when a lender does not recall shares to vote because short-selling demand increases under such a policy. This outcome makes the smaller number of votes that the lender does cast more informed.

Information leakage during the securities lending process is a relatively unexplored research area, perhaps because securities lending occurs in an opaque, over-the-counter market, making it difficult to find quality data. However, the SEC has recently pushed to increase transparency in securities lending, which may mitigate this problem: Rule 10c-1a (adopted in October 2023) requires lenders to report loan-level quantities and fees for the first time.[42] The availability of such granular data should spur further research in this area (assuming these rules survive legal challenges[43]). My model provides an organizing framework for motivating and interpreting future research on how securities lenders use lending market information and shows how a fund's lending, trading, and governing activities are all connected. Section 1.6 suggests some starting points for future empirical work.

[42] https://www.sec.gov/files/34-98737-fact-sheet.pdf

[43] Financial Times: Hedge fund groups sue SEC in effort to block short-selling rules

1.8 Appendix A: Proofs from Main Text

Supplementary Results

Result A1: *Price Calculations*

Since virtually every proof requires calculating prices, I will describe how to do so here in full detail for the baseline case where $a = 0$ (i.e., the prices from Section 1.3). I will not repeat these steps in the proofs to follow to avoid excessive repetition; one can calculate prices there by plugging the conjectured trading strategies into this generic procedure. Without loss of generality, I will calculate prices with an active lender; the passive lender is just a special case where D_L is always zero.

From the main text, the price that order flow z maps to is

$$P(z) = Pr[v = 1|z] = \frac{Pr[z|v = 1]Pr[v = 1]}{Pr[z|v = 1]Pr[v = 1] + Pr[z|v = 0]Pr[v = 0]} \tag{A1}$$

Observe that $Pr[v = 1] = Pr[v = 0] = 0.5$, so those terms can be eliminated from the pricing equation.

We can then apply the law of total probability a couple times to get

$$Pr[z|v = 1] = qPr[z|v = 1, s = 1] + (1 - q)Pr[z|v = 1, s = 0] \tag{A2}$$

$$\begin{aligned} Pr[z|v = 0] =& q\left[G(c_0^*)Pr[z|v = 0, s = 0, c < c_0^*] + (1 - G(c_0^*))\, Pr[z|v = 0, s = 0, c \geq c_0^*]\right] + \\ & (1 - q)\left[G(c_1^*)Pr[z|v = 0, s = 1, c < c_1^*] + (1 - G(c_1^*))\, Pr[z|v = 0, s = 1, c \geq c_1^*]\right] \end{aligned} \tag{A3}$$

where c_s^* is H's shorting cutoff. Recall that H always buys when $v = 1$ due to Assumption 1.1.

It is at this point that the conjectured trading strategies come into play. For the two $Pr[z|v = 0, s, c \leq c_s^*]$ probabilities, we know that $D_H = -1$ given her threshold trading strategy and that the lender's demand will equal $D_L^s(b=1)$. Denote $w_s^{-1} \equiv -1 + D_L^s(b=1)$. Then

$$Pr[z|v=0,s,c\leq c_s^*] = \begin{cases} \frac{1}{5} \text{ if } z \in \{w_s^{-1}, w_s^{-1}\pm 1, w_s^{-1}\pm 2\} \\ 0 \text{ otherwise} \end{cases} \tag{A4}$$

The $\frac{1}{5}$ probabilities come from the possible realizations of aggregate noise demand. Next we have the two cases where H does not trade (i.e., when $c \geq c_s^*$). Since there is no loan agreement, the lender's demand will equal $D_L^s(b=0)$. Denote $w_s^0 \equiv 0 + D_L^s(b=0)$ as the sum of the rational demands. Then

$$Pr[z|v=0,s,c\geq c_s^*] = \begin{cases} \frac{1}{5} \text{ if } z \in \{w_s^{0}, w_s^{0}\pm 1, w_s^{0}\pm 2\} \\ 0 \text{ otherwise} \end{cases} \tag{A5}$$

Lastly, we have the final two cases where H buys. Since there is no short sale, the lender's demand will still equal $D_L^s(b=0)$. Denote $w_s^1 \equiv 1 + D_L^s(b=0)$ as the sum of rational demands. We then have

$$Pr[z|v=1,s] = \begin{cases} \frac{1}{5} \text{ if } z \in \{w_s^{1}, w_s^{1}\pm 1, w_s^{1}\pm 2\} \\ 0 \text{ otherwise} \end{cases} \tag{A6}$$

We can now plug A4, A5, and A6 into A2 and A3, and A2 and A3 into A1 to get our price $P(z)$. We can repeat this procedure for all possible realizations of z to get our entire set of prices. Expected prices are then calculated by averaging over these $P(z)$'s. For example,

$$E[P(z)|D_H+D_L] = \frac{1}{5}\sum_{z=D_H+D_L-2}^{D_H+D_L+2} P(z) \tag{A7}$$

Proposition A1 *Suppose H is endowed with a share, so she never needs to borrow. Further, suppose L does not exist. Then H will always sell when $v=0$ if and only if $\bar{C}\leq 0.3$. If this* *condition is met, then the unique equilibrium in this setting is for H to always trade in the direction of her signal. Assumption 1.2 therefore implies $\bar{C} = 0.3$.*

Proof. There is no governance in this case, so I will refer to v as firm value in this proof (like in Section 1.3). I first calculate the maximum level of c that supports H always selling when $v = 0$ in this setting. With these trading strategies, I can calculate prices by following the procedure described in Result A1:

$$P(z) = \begin{cases} 1 \text{ when } z = 3 \\ \frac{1}{2-G(c^*)} \text{ when } z = 2 \\ \frac{1}{2} \text{ when } z \in \{1, 0, -1\} \\ 0 \text{ when } z \in \{-2, -3\} \end{cases}$$

$H's$ expected selling price is $E[P(z)|D_H + D_L = -1] = 0.3$. Thus, H will always sell when $v = 0$ if $\bar{C} \leq 0.3$. Note also that this result does not depend on the market maker's equilibrium conjecture c^*. Thus, this result combined with Assumption 1.1 implies that H always trading in the direction of her signal is the unique equilibrium in this counterfactual setting. □

Proofs of Section 1.3

Proof of Lemma 1.1

Proof. See text. □

Proof of Proposition 1.1

Proof. The fact that the equilibrium is pooling follows from the passive lender trading the same way (i.e., $D_L = 0$) and setting the same fee ($\frac{E[P(z)|D_H+D_L=-1]}{2}$). Deviations to any other fee are not optimal since H's beliefs about L's type are irrelevant. Given the optimal fee from (1.6) in the main text, we can plug into (1.3) (or (1.4)) and solve the resulting equation for c^*. Using the prices given in (1.2), $c^* = E[P(z)|D_H + D_L = -1] - \frac{E[P(z)|D_H+D_L=-1]}{2} = \frac{3}{20}$. This fraction is also the optimal fee. Since $\frac{3}{20}$ is a constant, it does not depend on s or q. □

Proof of Lemma 1.2

Proof. Follows immediately from (1.9) in the main text. The fee only enters the valuation through $G(c_s)$. Further, these valuations are increasing in $G(c_s)$ and $G(c_s) = \frac{E[P(z)|D_H+D_L=-2]-f_s}{\bar{C}}$ is decreasing in f_s. Thus, the valuations are decreasing in f_s. □

Proof of Lemma 1.3

Proof. Follows immediately from (1.10) and (1.11) in the main text. The only thing I need to show is that when the lender's optimal fee violates the non-negativity constraint, his constrained-optimum fee is zero; this outcome only happens when $D_L^s(b = 0) = 1$. The derivative of expected profits with respect to the fee is $Pr[v = 0|s]$ when $f < E[P(z)|D_H + D_L = -2] - \bar{C}$ (i.e., when H always shorts when $v = 0$),

$$\frac{Pr[v = 0|s]\left(\frac{-E[P(z)|D_H+D_L=1]}{2} - f\right)}{\bar{C}}$$

for all $E[P(z)|D_H + D_L = -2] - \bar{C} \leq f \leq E[P(z)|D_H + D_L = -2]$, and 0 for all $f > E[P(z)|D_H + D_L = -2]$ (i.e., when short selling is eliminated). Thus, expected profits are strictly decreasing in the range $f \in [0, E[P(z)|D_H + D_L = -2]]$ and constant thereafter when $E[P(z)|D_H + D_L = -2] - \bar{C} < 0$. Given how $\bar{C}$ is defined in Assumption 1.2, the inequality $E[P(z)|D_H + D_L = -2] - \bar{C} < 0$ will always hold. As a result, L will take the lowest possible fee that meets the non-negativity constraint, i.e., $f = 0$. □

There are nine possible pure-strategy equilibria without recall risk given Lemma 1.1. These potential equilibria represent the demands that L_1 and L_0 can have conditional on $b = 0$. In the supplemental proposition below, I solve for which of these pairs can be part of L's equilibrium trading strategy without recall risk.

Proposition A2 *Without recall risk, there is no pure-strategy equilibrium where, conditional on $b = 0$, L_1 either sells or has a lower asset demand than L_0. Additionally, $\{D_L^1(b = 0), D_L^0(b = 0)\} = \{0, -1\}$ is not an equilibrium. The remaining strategy pairs are equilibria without recall risk.*

Proof. First, I demonstrate that $\{D_L^1(b = 0), D_L^0(b = 0)\} = \{-1, -1\}$ is not an equilibrium by showing that selling conditional on $b = 0$ is unprofitable for L_1. Given these trading strategies, L_1 and L_0 set the same fee (see Lemma 1.3), so we will have $c_1^* = c_0^* \equiv c^*$ in this proposed pooling equilibrium. Result A1 then gives us prices as a function of c^*.

$$P(z) = \begin{cases} 1 \text{ when } z = 2 \\ \frac{1}{2-G(c^*)} \text{ when } z = 1 \\ \frac{1}{2} \text{ when } z \in \{0, -1, -2\} \\ 0 \text{ when } z \in \{-3, -4\} \end{cases}$$

We pin down the cutoff c^* by plugging the lender's optimal fee (see Equation (1.11)) into the indifference condition given by (1.7) (or (1.8)) from the main text and setting $c = c^*$.

$$E[P(z)|D_H + D_L = -2] - \frac{E[P(z)|D_H + D_L = -1]}{2} = c^*$$

where expected prices are calculated by averaging over the relevant $P(z)$'s as defined in (A7). Solving this equation for c^* gives us, $c^* = \frac{15-\sqrt{129}}{40}$. We can then plug this value into the prices. $L_1's$ expected selling profits conditional on $b = 0$ are:

$$\begin{aligned} &E[P(z)|b = 0, s = 1] - E[v|b = 0, s = 1] \\ = &\frac{qE[P(z)|D_H + D_L = 0] + (1-q)(1-G(c^*))E[P(z)|D_H + D_L = -1] - q}{q + (1-q)(1-G(c^*))} \\ &= \frac{3q + 15\sqrt{129}q - 5\sqrt{129} - 4\sqrt{129}q^2 - 132q^2 + 15}{(20(4q^2 + 7q - 5))} \end{aligned}$$

It is then straightforward to verify that this expresison is negative for all $q \geq 0.5$, so selling is not optimal for L_1 here.

$\{D_L^1(b = 0), D_L^0(b = 0)\} = \{\{-1, 0, \}, \{-1, 1\}, \{0, 1\}\}$ cannot be equilibrium trading strategies because L_1 cannot have a lower demand than L_0 in equilibrium. For example, consider trading strategy $\{D_L^1(b = 0), D_L^0(b = 0)\} = \{-1, 1\}$. This strategy says that L_0 wants to buy and L_1 wants to sell conditional on $b = 0$. If this pair was an equilibrium trading strategy, it would imply that L_0's expected profits from setting the optimal buying fee ($f = 0$, see Equation (1.11)) and then buying conditional on $b = 0$ are greater than those expected from deviating, setting the optimal selling fee ($\frac{E[P(z)|D_H+D_L=-1}{2}$), and then selling conditional on $b = 0$ (and, of course, always selling after $b = 1$). But if L_0 finds buying more profitable, so must L_1 because for a given trading strategy and associated optimal fee, his valuation of the asset conditional on $b = 0$ is always as least as great as L_0's. Thus, this strategy cannot occur in equilibrium. The reasoning is similar for the other two strategy pairs.

Next, I must rule out the equilibrium $\{D_L^1(b = 0), D_L^0(b = 0)\} = \{0, -1\}$. I do this by showing that L_1 will always want to deviate to buy. The prices are now:

$$P(z) = \begin{cases} 1 \text{ when } z = 3 \\ \frac{1}{1+(1-q)(1-G(c_1^*))} \text{ when } z = 2 \\ \frac{1}{1+(1-q)(1-G(c_1^*)+q*(1-G(c_0^*))} \text{ when } z = 1 \\ \frac{1}{2} \text{ when } z \in \{0, -1\} \\ \frac{1-q}{2-q} \text{ when } z = -2 \\ 0 \text{ when } z \in \{-3, -4\} \end{cases}$$

Given these prices and each lender's optimal fee (given their trading strategy), the two equilibrium indifference conditions are

$$E[P(z)|D_H + D_L = -2] = c_1^*$$
$$E[P(z)|D_H + D_L = -2] - \frac{E[P(z)|D_H + D_L = -1]}{2} = c_0^*$$

It is straightforward to see $c_1^* = \frac{2q-3}{5(q-2)}$. We can then plug this value into the prices and solve for c_0^* to get:

$$c_0^* = \frac{7\,q + q\,\sqrt{4\,q^2 + 24\,q + 9} - 2\,\sqrt{4\,q^2 + 24\,q + 9} - 6\,q^2 + 6}{20\,(2\,q - q^2)}$$

After taking into account that the off-equilibrium order flow $z = 4$ maps to a price of $P(4) = 1$ given our refinement of "unprejudiced beliefs"(see Section 1.2.4), we can plug these cutoffs into L_1's expected buying profit equation to get:

$$E[v|b = 0, s = 1] - E[P(z)|b = 0, s = 1] =$$
$$= \frac{q - qE[P(z)|D_H + D_L = 2] - (1-q)(1 - G(c^*))E[P(z)|D_H + D_L = 1]}{q + (1-q)(1 - G(c^*))}$$
$$- \frac{28\,q\,\sqrt{\kappa} - 516\,q - 72\,\sqrt{\kappa} + 152\,q^2 + 71\,q^3 - 22\,q^4 + 28\,q^2\,\sqrt{\kappa} - 11\,q^3\,\sqrt{\kappa} + 288}{10\,(2\,q + \sqrt{\kappa} + 3)\,(4\,q^3 + q^2 - 38\,q + 42)} \text{ with}$$
$$\kappa = 4\,q^2 + 24\,q + 9$$

This expression is positive for $q \in [.5, 1)$, so L_1 will always deviate.

I now examine conditions under which the remaining four possible trading strategies are equilibria given Assumptions 1.1 and 1.2 and Lemma 1.1. I will then verify that the conjecture is satisfied in each equilibrium. When I give the restrictions on q in decimal form, it is because I solve for them numerically.

First, I find prices for each proposed equilibrium by applying Result A1. I display them in Table 1.2; the four columns correspond to the conjectured equilibrium trading strategies. When an order flow is not reached in equilibrium, I apply the "unprejudiced beliefs"

Table 1.2: Prices Without Recall

	$\{0,0\}$	$\{1,1\}$	$\{1,0\}$	$\{1,-1\}$
P(4)	1	1	1	1
P(3)	1	$\frac{1}{2-G(c^*)}$	$\frac{1}{1+(1-q)(1-G(c_1^*))}$	$\frac{q}{q+(1-q)(1-G(c_1^*))}$
P(2)	$\frac{1}{2-G(c^*)}$	$\frac{1}{2-G(c^*)}$	$\frac{1}{1+q(1-G(c_0^*))+(1-q)(1-G(c_1^*))}$	$\frac{1}{1+(1-q)(1-G(c_1^*))}$
P(1)	$\frac{1}{2-G(c^*)}$	$\frac{1}{2-G(c^*)}$	$\frac{1}{1+q(1-G(c_0^*))+(1-q)(1-G(c_1^*))}$	$\frac{1}{1+q(1-G(c_0^*))+(1-q)(1-G(c_1^*))}$
P(0)	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
P(-1)	$\frac{1}{2}$	0	$\frac{1-q}{2-q}$	$\frac{1-q}{2-q}$
P(-2)	0	0	0	$\frac{1-q}{1+(1-q)G(c_0^*)}$
P(-3)	0	0	0	0
P(-4)	0	0	0	0

refinement to get the price (see Section 1.2.4).

I start with the $\{D_L^1(b=0), D_L^0(b=0)\} = \{0,0\}$ equilibrium. In this equilibrium, both L_1 and L_0 set fees equal to zero, which leads to the same short-selling cutoff: $c^* = E[P(z)|D_H + D_L = -2] = 0.2$. With these cutoffs, we can plug in to get equilibrium prices as a function of q. In this case, however, the prices do not depend on q because L_1 and L_0 behave identically. I now verify that the conjectured trading strategies are optimal. Both L_1 and L_0 value the asset at 0 after observing $b = 1$, and the expected selling price conditional on observing $b = 1$ is $E[P(z)|D_H + D_L = -2] = 0.2$. Thus, it is profitable for L to sell after observing $b = 1$. Holding fees constant, L_1's profits from deviating to buying conditional on $b = 0$ are:

$$E[v - P(z)|b = 0, s] =$$
$$\frac{Pr[v=1|s] - Pr[v=1|s]E[P(z)|D_H + D_L = 2] - Pr[v=0|s](1-G(c^*))E[P(z)|D_H + D_L = 1]}{Pr[v=1|s] + Pr[v=0|s](1-G(c^*))} \tag{A8}$$

$$= \frac{13\,q - 7}{10\,(2\,q+1)}$$

This expression is positive for all $q > \frac{7}{13}$. L_1's profits from deviating to selling are

$$E[P(z) - v|b = 0, s] =$$
$$\frac{Pr[v = 1|s]E[P(z)|D_H + D_L = 0] + Pr[v = 0|s](1 - G(c^*))E[P(z)|D_H + D_L = -1] - Pr[v = 1|s]}{Pr[v = 1|s] + Pr[v = 0|s](1 - G(c^*))} \tag{A9}$$

this term is negative for all $q \in [0.5, 1)$.

Next, I must verify that L_1 will not deviate to a different fee and trading strategy. Specifically, I must confirm that the expected profits of the conjectured equilibrium (at the fee-setting stage) exceed those expected from any other fee and trading strategy pair. This verification procedure is simplified by Lemma 1.3, which states that there is a unique optimal fee for every trading strategy. As a result, I only need to check that L_1 will not deviate to buying at the optimal buying fee or selling at the optimal selling fee (since he is already setting the optimal no-trade fee); any other fee and trading strategy pair is dominated by at least one of these three options (note also that H's beliefs about L's type are irrelevant, so there are no signaling benefits from deviating either). The optimal buying fee is the same as the optimal no-trade fee, so L_1 will not deviate as long as the buying profits defined by (A8) are negative. L_1 will deviate to selling at the optimal selling fee if

$$\begin{aligned} &Pr[v = 0|s]G(c^D)(E[P(z)|D_H + D_L = -2] + f_D)+ \\ &Pr[v = 1|s](E[P(z)|D_H + D_L = 0] - 1) + Pr[v = 0|s](1 - G(c^D))E[P(z)|D_H + D_L = -1] \end{aligned} \tag{A10}$$

is greater than his expected equilibrium profits as defined by plugging the optimal values into (1.10). Note that $c^D = E[P(z)|D_H + D_L = -2] - \frac{E[P(z)|D_H+D_L=-1]}{2}$ and $f_D = \frac{E[P(z)|D_H+D_L=-1]}{2}$. If this deviation is profitable, the subsequent act of selling must also be profitable because this higher deviant fee reduces profits conditional on $b = 1$ (and therefore must be made up when $b = 0$). In this case, the deviation profits equal $\frac{7}{32} - \frac{23q}{32}$, which is negative for all $q \geq 0.5$.

Following a similar procedure, $L_0's$ profits from deviating to buying and selling are $\frac{13q-6}{10(2q-3)}$ and $-\frac{37q-30}{20(2q-3)}$ respectively. $L_0's$ profits from deviating to the optimal selling fee and selling are $\frac{409q}{480} - \frac{1}{2}$. And again, since the optimal buying fee is the same as the optimal no-trade fee, L_0 will not deviate to buying with the optimal buying fee as long as the aformentioned $\frac{13q-6}{10(2q-3)} \leq 0$. There is no $q \in [.5, 1)$ that makes buying profitable, but selling will be profitable for all $q > \frac{30}{37}$ and selling at the selling optimal fee will be profitable for all $q > \frac{304}{473}$. Putting these results together,$\{D_L^1(b = 0), D_L^0(b = 0)\} = \{0, 0\}$ is an equilibrium for all $q \in [0.5, \frac{7}{13}]$.

Next, we move to the $\{D_L^1(b = 0), D_L^0(b = 0)\} = \{1, 1\}$ equilibrium. The procedure to verify this equilibrium is the same as above. We can calculate the short-selling cutoff $c^* = E[P(z)|D_H + D_L = -2] = 0.1$ using the fact that both L_1 and L_0 set $f^* = 0$ when they buy conditional on $b = 0$. We then plug this cutoff into the prices and verify that all conjectured trades are optimal. Once again, because $E[P(z)|D_H + D_L = -2] > 0$, L finds it optimal to sell conditional on $b = 1$. We can then plug the prices and equilibrium cutoff into the formula given by (A8); L_1's buying profits after observing $b = 0$ are $\frac{97q-46}{50(q+2)}$, which is positive for all $q \in [0.5, 1)$. Additionally, selling is always unprofitable for L_1 here, with profits of $-\frac{11(11q-2)}{50(q+2)}$. This expression can be calculated by plugging into (A9). Finally, his deviation profits from selling and setting the optimal fee for a seller are calculated by plugging into (A10). This expression equals $\frac{1481}{3000} - \frac{4481q}{3000}$, which is also negative for all q in the relevant range. L_0, on the other hand, has buying profits of $\frac{97q-51}{50(q-3)}$, which will be positive for $q \in [0.5, \frac{51}{97}]$. Within that range, selling yields negative profits: $-\frac{11(11q-9)}{50(q-3)}$. Lastly, L_0's deviation profits from selling with the optimal selling fee are $\frac{4481q}{3000} - 1$, which is negative for all $q \in [.5, \frac{3000}{4481}]$. Putting it all together, $\{D_L^1(b = 0), D_L^0(b = 0)\} = \{1, 1\}$ will be an equilibrium when $q \in [0.5, \frac{51}{97}]$.

Next, we address the $\{D_L^1(b = 0), D_L^0(b = 0)\} = \{1, 0\}$ equilibrium. Again, both L_1

and L_0 set $f^* = 0$, so they will generate the same equilibrium short-selling cutoff; this cutoff will therefore be the expected short-selling price: $c^* = E[P(z)|D_H + D_L = -2] = \frac{3\,q-4}{10\,(q-2)}$. Since this price is positive, L will always find selling optimal conditional on $b = 1$. With this cutoff, we can again plug into the expected prices and profit equations to express them as a function of q. After plugging into $L_1's$ profit equations, we can see that his buying profits conditional on $b = 0$ are $\frac{81\,q^5-1014\,q^4+4242\,q^3-7784\,q^2+6256\,q-1664}{10\,(45\,q^5-342\,q^4+922\,q^3-964\,q^2+128\,q+256)}$ while his selling profits are $\frac{27\,q^4-252\,q^3+726\,q^2-712\,q+112}{10\,(-9\,q^4+54\,q^3-98\,q^2+36\,q+32)}$. Selling is never profitable for him and buying will be profitable for all q greater than 0.511. Lastly, L_1's deviation profits from selling with the optimal selling fee are $-\frac{3717\,q^6-48717\,q^5+249936\,q^4-638880\,q^3+844288\,q^2-525632\,q+107008}{120\,(5\,q-8)\,(3\,q^2-14\,q+16)^2}$, which is also negative for all $q \in [0.5, 1)$. Similarly, $L_0's$ profits from deviating to buying and selling are $\frac{81\,q^5-1095\,q^4+4764\,q^3-8948\,q^2+7280\,q-1920}{10\,(45\,q^5-387\,q^4+1324\,q^3-2292\,q^2+2048\,q-768)}$ and $-\frac{27\,q^4-279\,q^3+918\,q^2-1168\,q+480}{10\,(9\,q^4-63\,q^3+164\,q^2-196\,q+96)}$ respectively. Buying is never profitable and selling is profitable once $q > 0.807$. Next, $L_0's$ deviation profits from selling and setting the optimal selling fee are $\frac{1197\,q^5-11520\,q^4+42792\,q^3-76032\,q^2+63680\,q-19456}{120\,(3\,q^2-14\,q+16)^2}$, which will be positive once $q > 0.703$. As a result, this strategy pair will be an equilibrium for all $q \in [0.511, .703]$.

We then move to the final equilibrium: $\{D_L^1(b = 0), D_L^0(b = 0)\} = \{1, -1\}$. Now L_1 and L_0 will set different fees. Following the same procedure as before, we get the following cutoff conditions:

$$E[P(z)|D_H + D_L = -2] = c_1^*$$
$$E[P(z)|D_H + D_L = -2] - \frac{E[P(z)|D_H + D_L = -1]}{2} = c_0^*$$

Solving this system yields two complicated expressions for c_1^* and c_0^*:

$$
\begin{aligned}
c_1^* =& -\frac{4\,q-\sqrt{33\,q^4-204\,q^3+472\,q^2-488\,q+196}-3\,q^2+2}{20\,(q^2-3\,q+2)}\\
c_0^* =& 2\,\sqrt{2}\,\sqrt{\tau(\kappa)}+188\,\sqrt{\kappa}\,q+\frac{6\,q^2-18\,q+12}{5\,\sqrt{\kappa}-50\,q+15\,q^2+50}+\frac{q-1}{5\,q-10}-112\,\sqrt{\kappa}\text{ with}\\
\tau(\kappa) =& \rho(\kappa)-5784\,q+7956\,q^2-6148\,q^3+2808\,q^4-720\,q^5+81\,q^6+166\,\sqrt{\kappa}\,q^2-78\,\sqrt{\kappa}\,q^3+\\
& 15\,\sqrt{\kappa}\,q^4+\frac{\sqrt{2}\,\sqrt{\rho(\kappa)+1888}}{40\,(q-2)\,(10\,\sqrt{\kappa}-10\,\sqrt{\kappa}\,q-344\,q+316\,q^2-132\,q^3+21\,q^4+3\,\sqrt{\kappa}\,q^2+148)}+\frac{1}{10}\\
\rho(\kappa) =& 52864\,\sqrt{\kappa}-19776\,\sqrt{\kappa}\,q-1290112\,q-2286960\,q^2+9452768\,q^3-13045136\,q^4+9703200\,q^5\\
& -4044672\,q^6+755856\,q^7+62532\,q^8-53676\,q^9+6993\,q^{10}-230904\,\sqrt{\kappa}\,q^2+430328\,\sqrt{\kappa}\,q^3\\
& -333540\,\sqrt{\kappa}\,q^4+119640\,\sqrt{\kappa}\,q^5-9702\,\sqrt{\kappa}\,q^6-5562\,\sqrt{\kappa}\,q^7+1215\,\sqrt{\kappa}\,q^8+752896\\
\kappa =& 33\,q^4-204\,q^3+472\,q^2-488\,q+196
\end{aligned}
$$

We can then check for possible trading deviations. The expressions for the lender's trading profits are extremely long and unwieldy but can be found by plugging c_1^* and c_0^* into the prices and then into Equations A8-A10. After plugging in, we see that buying is always profitable for L_1 while selling is always unprofitable. Deviation profits are also negative when L_1 sells with the optimal selling fee. Next, one can see that selling is profitable for L_0 for all $q > .6312$; over that same range, buying is unprofitable. Lastly, selling will be more profitable than deviating to not trading at the optimal no-trade fee once $q > .731$ and more profitable than buying at the optimal buying fee once $q > 0.6525$. More specifically, L_0's deviation profits from not trading at the optimal no trade fee are:

$$qG(c^D)(E[P(z)|D_H + D_L = -2] + f_D) \tag{A11}$$

and his deviation profits from buying at the optimal buying fee are:

$$qG(c^D)(E[P(z)|D_H + D_L = -2] + f_D)+$$
$$(1-q)(1 - E[P(z)|D_H + D_L = 2]) - q(1 - G(c^D))E[P(z)|D_H + D_L = 1] \quad \text{(A12)}$$

where $f_D = 0$ and $c^D = E[P(z)|D_H + D_L = -2]$. Thus, $\{D_L^1(b = 0), D_L^0(b = 0)\} = \{1, -1\}$ will be an equilibrium for all $q \in [.731, 1)$. It also remains profitable for L to sell after observing $b = 1$.

□

Proof of Lemma 1.4

Proof. To prove this claim, I first show that the lender's optimal fee violates (1.12) when he buys or does not trade conditional on $b = 0$. From (1.11), these lenders optimal fees are both $f^* = 0$. Then since $\Delta E[P(z)] = \frac{1}{5}(P(1) - P(-4)) > 0$, this optimal fee violates the no-recall constraint. Next, observe that the no-recall constraint never binds when the lender sets the optimal selling fee $\frac{E[P(z)|D_H+D_L=-1]}{2}$. From (12), the constraint will not bind if $\frac{E[P(z)|D_H+D_L=-1]}{2} > E[P(z)|D_H + D_L = -1] - E[P(z)|D_H + D_L = -2]$. By re-arranging, this amounts to $E[P(z)|D_H + D_L = -2] - \frac{E[P(z)|D_H+D_L=-1]}{2} > 0$. Observe that this condition equals the shorting threshold for H when $D_L^s(b = 0) = -1$. So, if $c_s^* > 0$, it must be satisfied. If $c_s^* \leq 0$, the lender was already attracting zero short sellers so increasing the fee to satisfy the condition has no impact on his profits. Thus, this lender will never need to deviate away from his optimal fee.

I must now show that the lender will choose $f^* = \Delta E[P(z)]$ when his optimal fee violates the no-recall constraints. I have already shown that the lender's expected profits are decreasing in f for all $f \in [0, E[P(z)|D_L + D_H = -2]]$ (and constant after that) when $D_L^s(b = 0) = 1$ in the proof of Lemma 1.3. As a result, this lender will choose the minimum fee that satisfies the no-recall constraint, $\Delta E[P(z)]$, as opposed to a fee that violates the constraint because such a fee is economically equivalent to one greater than $E[P(z)|D_L + D_H = -2]$ (in that all short selling is eliminated).

The intuition is similar for for lenders with $D_L^s(b=0)=0$ once I show that their profits are also strictly decreasing in the range $[0, E[P(z)|D_L + D_H = -2]$. First, note that $E[P(z)|D_L + D_H = -2] - \bar{C} < 0$ again given our definition of $\bar{C}$. Then, for $D_L^s(b=0)=0$, the first derivative of expected profits with respect to f is $\frac{-2fPr[v=0|s]}{\bar{C}}$ over the range $[0, E[P(z)|D_L + D_H = -2]$ and zero beyond that. Thus, L will choose the minimum fee satisfying (1.12),$\Delta E[P(z)]$, since any fee in violation of the no-recall condition is again equivalent to a fee greater than $E[P(z)|D_L + D_H = -2]$. □

Proof of Proposition 1.2

Proof. I start by ruling out the remaining six possible trading strategies. The process is fundamentally the same as Proposition A2; I find prices for each conjectured trading strategy and impose the optimal fee for a lender who has chosen that strategy. I then verify that L will optimally deviate from this proposed strategy. The only change is that now L sets $f^* = \Delta E[P(z)]$ instead of zero when buying or not trading conditional on $b = 0$.

The proof for the $\{D_L^1(b=0), D_L^0)\} = \{-1, -1\}$ trading strategy not being an equilibrium does not change from Proposition A2 because the no-recall condition does not bind (see Lemma 1.4). The trading strategies $\{D_L^1(b=0), D_L^0)\} \in \{\{-1,0\},\{-1,1\},\{0,1\}\}$ cannot be equilibria for the same reasons as given in the proof of Proposition A2. Then, we have to rule out $\{D_L^1(b=0), D_L^0)\} = \{1,1\}$ as an equilbirium. Under this proposed trading strategy, the lender's optimal fee will equal $\Delta E[P(z)]$. Thus, taking the prices from Table 2, the short-selling cutoff solves $c^* = \frac{c^*}{10c^*-6}$. There are two solutions to this equation mathematically, but the only solution within $[0, \bar{C}]$ is $c^* = 0$. Consequently, we have no equilibrium short selling under this conjecture, so L_0's expected buying profits become $\frac{2-4q}{5}$, which is less than or equal to zero for all $q \geq 0.5$.

Finally, we have the $\{0, -1\}$ trading strategy. L_1 sets the constrained fee $f_1^* = \Delta E[P(z)]$ while L_0 continues to set the optimal selling fee. Using the prices given in the proof of Proposition A2, L_1's short-selling cutoff will solve: $c_1^* = E[P(z)|D_H + D_L = -2] - \Delta E[P(z)]$.

The only solution within $[0, \bar{C}]$ is $c_1^* = \frac{2q + 10\sqrt{\frac{q^2}{25} + \frac{33}{100}} - 9}{10(q-2)}$. Next, I check to see when L_1 will deviate to buying. Using the usual formula from (A8), one can solve numerically to see that L_1 will deviate and buy once $q > 0.636$. Following the same process for L_0, we see that c_0^* solves $c_0^* = E[P(z)|D_H + D_L = -2] - \frac{E[P(z)|D_H + D_L = -1]}{2}$; the solution is $c_0^* = \frac{2q + 10\sqrt{\frac{q^2}{25} + \frac{33}{100}} - 9}{20(q-2)}$. I now check and see when L_0 prefers selling to deviating to not trading with the optimal no-trade fee ($\Delta E[P(z)]$). Plugging in to (A11), I see that this will only happen once $q > .679$. Thus, since L_1 will deviate to buying once $q > .636$ and selling is only optimal for L_0 once $q > .679$, the strategy pair $\{0, -1\}$ cannot be an equilibrium.

Next I demonstrate that the remaining strategies do exist in equilibrium. In the $\{0, 0\}$ equilibrium, the short-selling cutoff solves $c^* = E[P(z)|D_H + D_L = -2] - \Delta E[P(z)]$ since the no-recall constraint binds. Using the prices from Table 2, we can see that the solution is $c^* = \frac{4-\sqrt{10}}{10}$. Following the same procedure of verification from the proof of Proposition 1.2, L_1 will deviate to buying once $q > \frac{22\sqrt{10}}{523} + \frac{260}{523} \approx 0.6302$. Similarly, L_0 will deviate to selling once $q > \frac{206}{271} - \frac{4\sqrt{10}}{271} \approx .713$. L_1 never wants to sell and L_0 never wants to buy. This is verified by plugging into Equations (A8) and (A9). L_1's gain in profits from deviating to selling at the optimal fee are $\frac{3\sqrt{10}q}{10} - \frac{7q}{4} - \frac{\sqrt{10}}{10} + \frac{11}{20}$, which is negative for all $q \geq 0.5$. L_0's gains from deviating to selling at the optimal selling fee are $\frac{13q}{12} - \frac{\sqrt{10}q}{30} + \frac{\sqrt{10}}{15} - \frac{13}{15}$, which will be positive for all $q > \frac{4\sqrt{10}-52}{2\sqrt{10}-65} \approx 0.671$. These conditions are found by plugging into (A10). Thus, $\{0, 0\}$ will be an equilibrium for $q \in [.5, .6302]$.

The second equilibrium is $\{1, 0\}$. Following the same steps, the short-selling cutoff solves $c^* = E[P(z)|D_H + D_L = -2] - \Delta E[P(z)]$ since the no-recall constraint binds for L_1 and L_0. The only solution within $[0, \bar{C}]$ is $c^* = \frac{9q + \sqrt{33q^2 - 144q + 160} - 16}{20(q-2)}$. I then follow the usual procedure to verify that the lender will not deviate to another trading strategy. The exact closed-form expressions for buying and selling profits are quite long, so I omit them here. One can calculate them by plugging the above value for c^* into the prices found in Table 2 and then into Equations (A8-A10). L_1 will find buying profitable when $q > 0.5663$ and never finds selling profitable. L_0 will never find buying profitable and will find selling profitable once $q > 0.7479$. L_1 will also never deviate to selling at the optimal selling fee, and L_0 will deviate to selling at the optimal selling fee once $q > 0.7295$. So $\{1, 0\}$ is an equilibrium for $q \in [.5663, 0.7295]$.

Finally, there is the $\{1, -1\}$ equilibrium. Plugging into the prices listed in Table 2, we must solve $c_1^* = E[P(z)|D_H + D_L = -2] - \Delta E[P(z)]$ and $c_0^* = E[P(z)|D_H + D_L = -2] - \frac{E[P(z)|D_H+D_L=-1]}{2}$. The solutions cannot be expressed in closed form, so I will verify this equilibrium numerically. L_1 will find buying optimal once $q > 0.533$ and L_0 will find selling optimal once $q > .6530$. L_1 will never deviate to selling at the same fee or at the optimal selling fee. L_0 will never find buying optimal. L_0 also prefers selling at the optimal selling fee to buying at the optimal buying fee (this time $f_D = \Delta E[P(z)]$) once $q > .5758$ and to not trading at the optimal no-trade fee (again, $f_D = \Delta E[P(z)]$) once $q > 0.6991$ (plug into (A11) and (A12)). Thus, $\{1, -1\}$ is an equilibrium for $q \in [.6991, 1)$. In all these equilibrium, L will want to sell after observing $b = 1$ since $E[P(z)|D_H + D_L = -2] > 0$. □

Proof of Corollary 1.1

Proof. Given Proposition 1.2, when the equilibrium is $\{D_L^1(b = 0), D_L^0(b = 0)\} \in \{(0, 0), (1, 0)\}$, L_1 and L_0 set the same fee: $\Delta E[P(z)]$. Thus, the expected fee is always $\Delta E[P(z)]$. However, when the equilibrium is $\{1, -1\}$, the expected fee when $v = 1$ is $qf_1^* + (1 - q)f_0^*$, which is less than $(1 - q)f_1^* + qf_0^*$, the expected fee when $v = 0$ (since $f_1^* = \Delta E[P(z)] < f_0^* = \frac{E[P(z)|D_H+D_L=-1]}{2}$). We therefore expect higher fees when the value of the asset is 0. □

Proof of Corollary 1.2

Proof. This proof amounts to comparing the equilibrium values of $G(c^*)$ found with an active lender to that of the passive lender. Proposition 1.1 states that $G(c^*) = 0.5$ when the lender is passive. For the active lender, I take the equilibrium values calculated in the proof of Proposition 1.2 above. Observe that since L_1 and L_0 trade the same way conditional on $b = 1$, and since L_1 sets a fee no higher than L_0, I only need to compare the passive value to the value under L_1; if there is more short-selling under the passive lender than L_1, then there will also be more than under L_0.

First, the $\{D_L^1(b=0), D_L^0)\} = \{0,0\}$ equilibrium; in this case $c^* = \frac{4-\sqrt{10}}{10}$, so $G(c^*) = \frac{c^*}{\bar{C}} = \frac{\frac{4-\sqrt{10}}{10}}{.3} < 0.5$. Next, when $\{D_L^1(b=0), D_L^0)\} = \{1,0\}$, $c_1^* = \frac{9\,q+\sqrt{33\,q^2-144\,q+160}-16}{20\,(q-2)}$. In this case, $G(c_1^*) = \frac{c_1^*}{.3}$ is less than 0.5 for all $q \in [0.5, 1)$. Finally, we have the $\{1,-1\}$ equilibrium. There is not a closed-form expression for c_1^*, but one can numerically verify that the resulting $G(c_1^*)$ is always less than 0.5. □

Proof of Proposition 1.3

Proof. Price efficiency is defined as $1 - |v - E[P(z)|v]|$, and note that its value is the same whether $v = 1$ or $v = 0$. To see this, observe that the unconditional price $E[P(z)] = Pr(v = 1) = 0.5$. Then, by the law of total expectations

$$
\begin{aligned}
&E[P(z)] = E[P(z)|v=1]Pr(v=1) + E[P(z)|v=0]Pr(v=0) \\
\implies &0.5 = 0.5E[P(z)|v=1] + 0.5E[P(z)|v=0] \\
\implies &1 - |0 - E[P(z)|v=0] = 1 - |1 - E[P(z)|v=1]|
\end{aligned}
$$

So without loss of generality, I will focus on $1 - |1 - E[P(z)|v = 1]|$ as my efficiency measure because it is usually easier to calculate.

Applying Result A1 and taking expectations gives us the expected prices. We can then plug in the previously calculated equilibrium values of c_1^* and c_0^* to get our equilibrium expected prices. For the passive lender, our efficiency measure gives us $Eff_p = E[P(z)|D_H + D_L = 1] = \frac{19}{30}$.

I next move to the active lender without recall risk. Repeating the price calculations for the active equilibrium with trading strategy $\{D_L^1(b=0), D_L^0(b=0)\} = \{1,1\}$, we get $Eff_a = E[P(z)|D_H + D_L = 2] = \frac{33}{50} > \frac{19}{30}$. So this active equilibrium leads to greater efficiency. Similarly, the $\{D_L^1(b=0), D_L^0(b=0)\} = \{0,0\}$ equilibrium yields an efficiency value of $E[P(z)|D_H + D_L = 1] = 0.7$, which is also greater than the passive efficiency measure.

Next, we look at $\{D_L^1(b=0), D_L^0(b=0)\} = \{1, 0\}$; efficiency here equals $qE[P(z)|D_H + D_L = 2] + (1-q)E[P(z)|D_H + D_L = 1] = \frac{93\,q^3 - 580\,q^2 + 1208\,q - 832}{10\,(15\,q^3 - 94\,q^2 + 192\,q - 128)}$. The derivative of this efficiency expression with respect to q is $-\frac{21\,q^4 + 264\,q^3 - 1960\,q^2 + 3968\,q - 2560}{5\,(15\,q^3 - 94\,q^2 + 192\,q - 128)^2}$, which is positive for all $q \in [0.5, 1)$. Efficiency is therefore increasing in q in this equilibrium, so its minimum value occurs at $q = 0.5$. This minimum value equals $0.6738 > \frac{19}{30}$.

Lastly, efficiency equals $qE[P(z)|D_H + D_L = 2] + (1-q)E[P(z)|D_H + D_L = 0]$ in the $\{D_L^1(b=0), D_L^0(b=0)\} = \{1, -1\}$ equilibrium. The exact expression is too long to write out here, but it can be calculated by plugging the solutions to c_1^* and c_0^* into the expected prices as defined in the proof of Proposition A2. Then, by taking the derivative with respect to q, you can verify that this derivative is positive. Thus, efficiency is minimized at $q = 0.5$ where it equals 0.665, which is still greater than the passive efficiency measure.

Now I move onto the three active equilibria with recall. The relevant efficiency measures are now

$$\{D_L^1(b=0), D_L^0(b=0)\} = \{0, 0\} \implies Eff = E[P(z)|D_H + D_L = 1]$$
$$\{D_L^1(b=0), D_L^0(b=0)\} = \{1, 0\} \implies Eff = qE[P(z)|D_H + D_L = 2] + (1-q)E[P(z)|D_H + D_L = 1]$$
$$\{D_L^1(b=0), D_L^0(b=0)\} = \{1, -1\} \implies Eff = qE[P(z)|D_H + D_L = 2] + (1-q)E[P(z)|D_H + D_L = 0]$$

I then proceed by plugging in the equilibrium values of c_1^* and c_0^* into the prices listed in Table 2 and averaging as described in (A7) to obtain the equilibrium expected prices. Recall that passive efficiency is unchanged from Section 1.3 and it equals $\frac{19}{30} \approx 0.633$. Price efficiency under equilibrium $\{0, 0\}$ is $\frac{\sqrt{10}}{5} \approx 0.6325$, which is less than 0.633. Price efficiency under equilibrium $\{1,0\}$ is:

$$\frac{104\sqrt{\kappa} - 148\sqrt{\kappa}\, q - 1352\, q + 484\, q^2 + 15\, q^3 - 24\, q^4 + 71\sqrt{\kappa}\, q^2 - 12\sqrt{\kappa}\, q^3 + 1024}{(10\, q - 20)\,(160\, q + 5\sqrt{\kappa}\, q - 8\sqrt{\kappa} - 75\, q^2 + 12\, q^3 - 112)}$$

with $\kappa = 33q^2 - 144q + 160$

This expression is less than $\frac{19}{30}$ for all q that support this trading strategy as an equilibrium.

Efficiency will always be greater under the active lender than the passive lender when the active equilibrium is $\{1,-1\}$ (verified numerically). Therefore, whenever this equilibrium is not feasible– that is, when $q < .6530$–efficiency will be greater under the passive lender than an active lender, no matter what equilibrium trading strategy L has. □

Proofs of Section 1.4

Proof of Corollary 1.3

Proof. When $b = 1$, the lender knows that $\theta = 0$ for sure. As a result, the expected benefits from intervention are $N\psi$ (see (1.14) in the main text). When $b = 0$, the expected benefits of intervention are: $E[v(\theta,a)|b = 0, a = 1] - E[v(\theta,a)|b = 0, a = 0] = \frac{1}{10}N\psi + \frac{8}{10}N\left(\frac{(1-G(c))\psi}{2-G(c)}\right)$. Thus, expected benefits of intervention are higher when $b = 1$ since $\frac{(1-G(c))\psi}{2-G(c)} \leq \psi$. Since the cost of intervention is always k, intervention is expected to be more profitable when $b = 1$; thus the lender is more likely to intervene in this case. □

Proof of Proposition 1.4

Proof. Let's start with the case where $a_2 = 0$. Prices are as follows:

$$P(z) = \begin{cases} 1 \text{ when } z = 3 \\ \frac{1}{2-G(c^*)} \text{ when } z = 2 \\ \frac{1}{2}(1+\psi G(c^*)) \text{ when } z \in \{1, 0, -1\} \\ \psi \text{ when } z \in \{-2, -3\} \end{cases}$$

The short-selling cutoff is the solution to $c^* = E[P(Z)|D_H + D_L = -1] - f(\psi)^*$ where $f(\psi)^*$ equals $\frac{E[P(z)|D_H+D_L=-1]}{2} - \frac{(4N+5)\psi-4k}{10}$. The resulting short-selling cutoff is $c^* = \frac{8k+6\psi-8N\psi-3}{10\psi-20}$ when $\frac{8k+6\psi-8N\psi-3}{10\psi-20} \in (0, \bar{C})$. The derivative of this cutoff with respect to ψ is $\frac{dc^*}{d\psi} = \frac{16N-8k-9}{10(2-\psi)^2} > 0$. This expression is positive because we are assuming that intervention is desirable when θ is known to equal zero, which implies that $N\psi > k$. Since $\psi \in [0, 1]$, it must be true that $k \leq N$. We are also assuming that $N \geq 2$ to ensure the active lender can simultaneously lend and trade. These two restrictions are sufficient to show that this derivative is positive, which implies that short selling profits are increasing in ψ when $a_2 = 0$. If c^* is instead at a corner value of 0 or $\bar{C}$, then clearly the short-selling cutoff does not depend on ψ.

Furthermore, $a_2 = 0$ will be the optimal intervention choice here when $N\psi\left(\frac{1-G(c^*)}{2-G(c^*)}\right) < k$, where $G(c^*) = \frac{c^*}{\bar{C}}$ and $c^* = \frac{8k+6\psi-8N\psi-3}{10\psi-20}$.

Now suppose the parameters are such that $a_2 = 1$. Prices are now

$$P(z) = \begin{cases} 1 \text{ when } z = 3 \\ \frac{1+(1-G(c^*))\psi}{2-G(c^*)} \text{ when } z = 2 \\ \frac{1}{2}(1+\psi) \text{ when } z \in \{1, 0, -1\} \\ \psi \text{ when } z \in \{-2, -3\} \end{cases}$$

Following the same procedure as above, we can plug into $c^* = E[P(Z)|D_H+D_L = -1]-f(\psi)^*$

with the relevant prices and fees and solve to get c^*. The interior solution turns out to equal $\frac{3}{20}(1-\psi)$, which is clearly decreasing in ψ. As a result, short selling profits are decreasing in ψ when $a_2 = 1$. If c^* is instead at a corner value of 0 or $\bar{C}$, then the short-selling cutoff does not depend on ψ. Also, $a_2 = 1$ will be the optimal intervention choice here when $N\psi\left(\frac{1-G(c^*)}{2-G(c^*)}\right) - k = \frac{N\,\psi^2+(N-k)\,\psi-3\,k}{\psi+3} > 0$. □

Lemma A1: *When short-selling occurs in equilibrium, the unique equilibrium trading strategy for the lender conditional on $b = 1$ is to sell.*

Proof. First, I show that selling conditional on $b = 1$ is always an equilibrium trading strategy for L. Suppose such a trading strategy is conjectured for L. Then for H to short sell (and generate $b = 1$) it must be true that $E[P(z)|D_H + D_L = -2] - f - \psi - c \geq 0$ for some $f \geq 0$ and $c > 0$. L's subsequent selling profits then equal $E[P(z)|D_H + D_L = -2] - \psi$, which, as a result, must be strictly positive.

I now show that buying or not trading following $b = 1$ is never optimal. Let us start with $D_L(b = 1) = 1$. Suppose $\psi > \frac{k}{N} > \psi\left(\frac{1-G(c^*)}{2-G(c^*)}\right)$ for some arbitrary $G(c^*) \in [0,1]$, so $a_2 = 0$ is the lender's intervention strategy. As a result, there are four possible values for $P(z)$: $1, \frac{1}{2-G(c^*)}, 0.5(1 + G(c^*)\psi)$, and ψ. Observe that $\psi > 0.5(1 + G(c^*)\psi)$ if and only if $\psi > \frac{1}{2-G(c^*)}$. Suppose $\psi \leq \frac{1}{2-G(c^*)}$. Then, the lender's valuation following $b = 1$ equals the lowest possible price (ψ), so buying cannot be optimal. Now suppose $\psi > \frac{1}{2-G(c^*)}$. Then H cannot receive a selling price greater than her valuation (ψ) since $P(z)$ cannot equal one when H sells unless ψ itself equals 1. Consequently, H will never short sell, meaning that the signal $b = 1$ never occurs on the equilibrium path. Now suppose $\frac{k}{N} > \psi\left(\frac{1-G(c^*)}{2-G(c^*)}\right)$, so $a_2 = 1$ is the equilibrium intervention strategy. Then the four possible prices are $1, \frac{1+(1-G(c^*))\psi}{2-G(c^*)}, 0.5(1+\psi)$, and ψ. In this case, ψ is always the lowest possible price, so buying can never be optimal for a lender with a valuation equal to ψ.

Now, let us move to $D_L(b = 1) = 0$. When the parameters lead to a_2=0, both $E[P(z)|D_H + D_L = -1]$ and $E[P(z)|D_H + D_L = -2]$ will be weighted averages of ψ and $0.5(1 + G(c^*))\psi$, with each option getting strictly positive weights. $P(z) = \psi$ when $\theta = 0$ for sure, and $P(z) = 0.5(1 + G(c^*))\psi$ when the associated order flow z can always occur. The other two prices from the previous paragraph only occur when $b = 0$, so they are irrelevant. Furthermore, $E[P(z)|D_H + D_L = -1]$ (which is an equal-weighted average of $P(1)$ through $P(-3)$) will have a strictly greater weight on $0.5(1 + G(c^*))\psi$ than $E[P(z)|D_H + D_L = -2]$ (an equal-weighted average of $P(0)$ through $P(-4)$) because $P(1) = 0.5(1 + G(c^*))\psi$ and $P(-4) = \psi$. If $\psi < 0.5(1+G(c^*)\psi)$, then L will always deviate to selling because his expected selling price will be above his valuation ψ. On the other hand, if $\psi \geq 0.5(1 + G(c^*)\psi)$, then H will not short sell because her expected selling price will be below her valuation, so the signal $b = 1$ never appears on the equilibrium path. Using similar logic, when the parameters lead to $a_2 = 1$, the two expected selling prices will be weighted averages of $0.5(1+\psi)$ and ψ, with $0.5(1+\psi)$ taking the place of $0.5(1+G(c^*)\psi)$ from before. In this case, $\psi < 0.5(1+\psi)$ for sure, so L will always deviate and sell because his expected selling price will be greater than ψ. □

Proof of Proposition 1.5

Proof. Suppose that for any equilibrium with $G(c^*) \in [0, 1]$, the parameters ψ, k, and N are such that $\psi > \frac{k}{N} > \psi\left(\frac{1-G(c^*)}{2-G(c^*)}\right)$. As a result, the active lender optimally chooses intervention strategy $a_2 = 0$. I now prove that the only possible trading strategy for this lender conditional on $b = 0$ is $D_L(b = 0) = 0$.[44] The lender also sells following $b = 1$ due to Lemma A1.

I start by showing that $D_L(b = 0) = 1$ cannot be part of L's equilibrium trading strategy. With this conjectured trading strategy (and intervention strategy $a_2 = 0$) we get the following prices:

$$P(z) = \begin{cases} 1 \text{ when } z = 4 \\ \frac{1}{2-G(c^*)} \text{ when } z \in \{3, 2, 1\} \\ \frac{1}{2}(1 + \psi G(c^*)) \text{ when } z = 0 \\ \psi \text{ when } z \in \{-1, -2, -3, -4\} \end{cases}$$

[44] I suppress the private signal s subscript since this signal is uninformative

Moving back a step, an active lender with this trading and intervention strategy chooses his fee to maximize

$$
\begin{aligned}
\max_f &\frac{1}{2}G(c)\left((N-1)\psi + E[P(z)|D_H + D_L = -2] + f - k\right) + \\
&\frac{1}{10}(1-G(c))((N+1)\psi - E[P(z)|D_H + D_L = 1] - k) + \\
&\frac{4}{10}(2-G(c))\left((N+1)\left(\frac{1}{2-G(c)}\right) - \frac{E[P(z)|D_H + D_L = 2] + (1-G(c))E[P(z)|D_H + D_L = 1]}{2-G(c)}\right) + \\
&\frac{1}{10}((N+1)(1) - E[P(z)|D_H + D_L = 2])
\end{aligned}
$$

where $G(c) = E[P(z)|D_H + D_L = -2] - f - \psi$. The (unconstrained) solution is $-\frac{E[P(z)|D_H+D_L=-1]}{2} - \frac{2}{5}(N\psi - k) + \frac{\psi}{10}$. One can easily verify that this value is negative by noting that $N\psi > k$ (by assumption) and that $\frac{\psi}{10} < \frac{E[P(z)|D_H+D_L=-1]}{2}$ from the prices above (regardless of what $G(c^*)$ is). Thus, like in Section 1.3.3, the lender chooses $f^* = \Delta E[P(z)]$ when $\Delta E[P(z)] > 0$. I take this case first. By plugging in this fee choice, the only solution is $c^* = 0$. The lender's expected buying profits will be positive if and only if

$$1 + (1-G(c^*))\left(\frac{1}{5}\right)\psi - E[P(z)|D_H + D_L = 2] - (1-G(c^*))E[P(z)|D_H + D_L = 1] > 0 \quad \text{(A13)}$$

Plugging in $c^* = 0$ then shows that buying generates zero profits, so the lender will not choose this trading strategy. Suppose now that $P(-4) = \psi > P(1) = \frac{1}{2-G(c^*)}$ so that $\Delta E[P(z)] < 0$; the lender will then select $f^* = 0$. Unlike the baseline model from Section 1.3, this case is possible if intervention succeeds with a high enough probability, leading to a non-monotonic price function. A necessary condition for this to occur is $\psi > 0.5$. However, with this case, we instead get $c^* = \max\{-\frac{3(2\psi-1)}{10(3-\psi)}, 0\} = 0$ for all $\psi > 0.5$. As a result, $c^* = 0$ here too, leading once again to zero buying profits for the lender. Thus, $D_L(b = 0) = 1$ cannot be part of an equilibrium trading strategy.

The procedure is similar to rule out $D_L(b=0) = -1$. Prices are now:

$$P(z) = \begin{cases} 1 \text{ when } z = \in \{4,3\} \\ \frac{1}{2-G(c^*)} \text{ when } z \in \{2,1\} \\ \frac{1}{2}(1+\psi G(c^*)) \text{ when } z \in \{0,-1\} \\ \psi \text{ when } z \in \{-2,-3,-4\} \end{cases}$$

The lender will find selling conditional on $b = 0$ profitable in equilibrium when

$$E[P(z)|D_H+D_L = 0]+(1-G(c^*))E[P(z)|D_H+D_L = -1]-1-(1-G(c^*))\left(\frac{1}{5}\right)\psi > 0 \quad \text{(A14)}$$

which happens when $\frac{c^*(10c^*\psi-6\psi+3)}{3} < 0$. Thus, a necessary condition for selling to be optimal is $\psi > 0.5$ and $c^* > 0$. For any arbitrary choice of fee f, the equilibrium short-selling cutoff will solve $E[P(z)|D_H + D_L = -2] - f - \psi = c^* \implies c^* = \max\{-\left(\frac{10\,f+6\,\psi-3}{10\,(1-\psi)}\right), 0\}$. Observe that a necessary condition for $c^* > 0$ is $\psi < 0.5$, which means that it is impossible to have both $\psi > 0.5$ and $c^* > 0$, meaning that selling conditional on $b = 0$ can never be part of an equilibrium trading strategy for the lender.

Finally, we have the case where $D_L(b = 0) = 0$. With this trading strategy prices are:

$$P(z) = \begin{cases} 1 \text{ when } z = \in \{4,3,2\} \\ \frac{1}{2-G(c^*)} \text{ when } z = 1 \\ \frac{1}{2}(1+\psi G(c^*)) \text{ when } z \in \{0,-1,-2\} \\ \psi \text{ when } z \in \{-3,-4\} \end{cases}$$

The lender's (unconstrained) optimal fee is $-\frac{2}{5}(N\psi - k) < 0$, so the lender's optimal fee becomes $\max\{\Delta E[P(z)], 0\}$. Plugging this fee into H's short-selling cutoff and solving

$E[P(z)|D_H + D_L = -2] - f - \psi = c^*$ for c^* tells us that

$$c^* = \begin{cases} \frac{3\left(3\,\psi+\sqrt{\psi^2-8\,\psi+10}-4\right)}{10\,(2\,\psi-3)} \text{ when } \psi \leq 0.5 \\ 0 \text{ when } \psi > 0.5 \end{cases}$$

We can then plug this cutoff into the prices to get our equilibrium prices. I now verify that $D_L(b=0) = 0$ is optimal. First, suppose $\psi \leq 0.5$. Then, by plugging c^* into (A13), it is simple to verify that these buying profits are negative. Since the optimal buying fee is the same as the optimal no-trade fee, L will not deviate to buying holding fees constant or at the optimal buying fee. Then for selling, (A14) can be expressed as

$$-\frac{108\,\psi - 900\,c^2\,\psi + 1000\,c^3\,\psi + 300\,c^2 - 81}{90\,(5\,c-3)} \tag{A15}$$

for any $c \in [0, \bar{C}]$. It is straightforward to verify that this expression is negative for all $c \in [0, \bar{C}]$ (remembering from Proposition A1 that $\bar{C} = 0.3$) and $\psi \in [0, .5]$. Consequently, deviating and selling is also not optimal.

Now suppose $\psi > 0.5$ so $c^* = 0$ and $f^* = 0$. As a result, the deviant cutoff value of c^D will equal zero no matter what fee $f^D \geq 0$ the lender picks. Buying profits are then $\frac{2\psi-3}{10} < 0$, so deviating to buying is never optimal. Similarly, selling profits are $\frac{4\psi-3}{10}$, which will be positive for all $\psi > 0.75$.

To summarize, $D_L(b=0) = 0$ is an equilibrium trading strategy for all $\psi \leq 0.75$. When $\psi > 0.75$, there is no pure-strategy equilibrium. Intuitively, like in Figure 1.1 from the main text, higher values of $D_L(b=0)$ reveal the outcome $\theta = 0$ more often. Without intervention, this strategy leads to lower prices because $v(\theta = 0, a = 0) = 0$. But with intervention, $E[v(\theta = 0, a = 1)] = \psi$, so when ψ is sufficiently large, prices can increase rather than decrease when the lender's demand conditional on $b = 0$ increases.

Then for intervention, $a_2 = 0$ will be optimal when $\psi > \frac{k}{N} > \psi\left(\frac{1-G(c^*)}{2-G(c^*)}\right)$ for $G(c^*) = \frac{\frac{3\left(3\,\psi+\sqrt{\psi^2-8\,\psi+10}-4\right)}{10\,(2\,\psi-3)}}{\bar{C}}$ for $\psi \leq 0.5$ and when $\psi > \frac{k}{N} > \frac{\psi}{2}$ for $\psi > 0.5$.

Lastly, I show that c^* is decreasing in ψ. When $\psi \leq 0.5$, $\frac{dc^*}{d\psi} = -\frac{3\left(\sqrt{\psi^2-8\,\psi+10}-5\,\psi+8\right)}{10\,(2\,\psi-3)^2\,\sqrt{\psi^2-8\,\psi+10}} < 0$. Obviously, when $\psi > 0.5$, $c^* = 0$, so c^* does not depend on ψ. Thus, c^* is decreasing in ψ and strictly so when $\psi < 0.5$. □

Proof of Proposition 1.6

Proof. I can characterize the three intervention decisions for the passive lender as follows:

$$a_3^{NR} = 1 \iff N\psi_L > k \tag{A16}$$

$$a_2^{NR} = 1 \iff N\psi_H \left(\frac{1-G(c^*)}{2-G(c^*)}\right) > k \tag{A17}$$

$$a_2^{R} = 1 \iff N\psi_H \left(\frac{1}{2}\right) > k \tag{A18}$$

The conditions for the active lender are similar, but with N being replaced by $N-1$ in (A16) and N being replaced by $N + D_L^s(b=0)$ in (A17-A18). Proving this claim amounts to plugging in the eight possible combinations of $\vec{a} \equiv (a_3^{NR}, a_2^{NR}$, and $a_2^R)$ into Equations (1.19) and (1.20) from the main text and comparing the resulting expected values.

When $\vec{a} = (1,1,1)$, plugging in and subtracting the two expected values gives us $E[v|\text{no recall}] - E[v|\text{recall}] = -\frac{G(c^*)}{2}(\psi_H - \psi_L) < 0$. Now suppose $\vec{a} = (1,0,1)$. Then the difference in expected firm value is $\frac{5G(c^*)\psi_L - G(c^*)\psi_H - 4\psi_H}{10}$. I aim to show that this expression is negative. Re-arranging, the expression will be negative when $\psi_L < \psi_H\left(\frac{4+G(c^*)}{5G(c^*)}\right)$. Since $\psi_H > \psi_L$, I only need to show that $\left(\frac{4+G(c^*)}{5G(c^*)}\right) \geq 1$ to prove my claim, which it is for all $G(c^*) \in [0,1]$. Thus, firm value is lower with recall in this case too. Next, I consider $\vec{a} = (1,0,0)$, and the associated difference in expected firm value is $\frac{G(c^*)(5\psi_L - \psi_H)}{10}$. This expression will be positive if and only if $5\psi_L > \psi_H$. Next I consider $\vec{a} = (0,1,1)$, and the associated expected value difference is $-\frac{G(c^*)\psi_H}{2} < 0$; thus, recalling shares hurts value here too. When $\vec{a} = (0,0,1)$ the associated expected value difference is $-\frac{(G(c^*)+4)\psi_H}{2} < 0$. When $\vec{a} = (0,0,0)$, the associated expected value difference is $-\frac{G(c^*)\psi_H}{10} < 0$.

Observe also that these first six combinations of $(a_3^{NR}, a_2^{NR}$, and $a_2^R)$ can be generated with any equilibrium value of $G(c^*) \in (0,1]$ with the appropriate choices of the parameters N, k, and ψ. When $G(c^*) = 0$, the two cases are identical.

The last two cases are $(1,1,0)$ and $(0,1,0)$, but these cannot occur in equilibrium since $a_2^{NR} = 1$ implies $a_2^R = 1$ when we hold the number of shares owned by the lender constant across the recall and no recall policies. This follows from $0.5 \geq \left(\frac{1-G(c^*)}{2-G(c^*)}\right)$ for all $G(c^*) \in [0,1]$.

□

Proofs of Section 1.5

Proof of Corollary 1.4

Proof. Suppose the lender is perfectly informed (i.e., $q = 1$). In this case, the lender clearly wants to buy when $v = 1$ since he knows v and prices are always less than or equal to 1. The lender sells when $v = 0$ for similar reasons. Given this trading strategy, we can calculate prices in the usual fashion.

$$P(z) = \begin{cases} 1 \text{ when } z \in \{4,3,2 \\ \frac{1}{2-G(c_0^*)} \text{ when } z = 1 \\ \frac{1}{2} \text{ when } z = 0 \\ \frac{1-\hat{G}(y_1^*)}{2-\hat{G}(y_1^*)} \text{ when } z = -10 \text{ when } z \in \{-2,-3,-4\} \end{cases}$$

where y_1^* is H's buying threshold and $\hat{G}(y_1^*) = \frac{y_1^*}{\hat{C}}$. Short sellers only appear when $v = 0$, so L_0 is the only lender who sets a fee in equilibrium. L_0 chooses his fee f to maximize

$$G(c_0)(E[P(z)|D_H + D_L = -2] + f) + (1 - G(c_0))E[P(z)|D_H + D_L = -1]$$

where $G(c_0) = \frac{E[P(x)|D_H+D_L=-2]-f}{C}$. The solution to this problem is the same as for any other lender who sells conditional on $b = 0$, namely $f^* = \frac{E[P(x)|D_H+D_L=-1]}{2}$. Given this fee, the equilibrium short-selling cutoff can be rearranged as

$$\begin{aligned} G(c_0^*) &= \frac{1}{\bar{C}}\left(E[P(z)|D_H + D_L = -2] - \frac{E[P(x)|D_H + D_L = -1]}{2}\right) \\ &= \frac{1}{\bar{C}}\left(\frac{1}{20} + \frac{1-\hat{G}(y_1^*)}{20-10\hat{G}(y_1^*)} - \frac{1}{20-10G(c_0^*)}\right) \end{aligned} \tag{A19}$$

I now wish to demonstrate that $G(c_0^*) = 0$ is a solution to this equation if and only if $\hat{G}(y_1^*) = 1$.

First, suppose $G(c_0^*) = 0$. Then Equation (A19) simplifies to $0 = \frac{1-\hat{G}(y_1^*)}{\bar{C}(20-10\hat{G}(y_1^*))}$. This equation will only be satisfied when $\hat{G}(y_1^*) = 1$, proving the "if" portion. Now suppose that $\hat{G}(y_1^*) = 1$. Equation (A19) becomes $\bar{C}G(c_0^*) = \frac{1}{20} - \frac{1}{20-10G(c_0*)}$. Observe that $\frac{1}{20-10G(c_0^*)} > \frac{1}{20}$ for all $G(c_0^*) \in (0, 1]$, implying that those values cannot be an equilibrium since the left-hand side will be positive and the right hand side will be negative. When $G(c_0^*) = 0$ however, both the left and right hand sides equal zero, making that the unique solution. □

Proof of Corollary 1.5

Proof. First, start with the passive lender. Take Equation (1.5) from the main text, but now multiply the received fee f_s by ρ and solve the same problem. Since ρ is just a proportional constant, it does not affect the passive lender's optimal fee: it remains $\frac{E[P(z)|D_H+D_L=-1]}{2}$.

The idea is similar for the active lender: take Equation (1.10) from the main text but multiply the received fee by ρ. Solving that equation gives the following characterization of the lender's optimal fee as a function of his trading strategy.

$$f_s^* = \begin{cases} -\frac{E[P(z)|D_H+D_L=1]}{2\rho} + \frac{(\rho-1)E[P(z)|D_H+D_L=-2]}{2\rho} & when\ D_L^s(b=0) = 1 \\ \frac{(\rho-1)E[P(z)|D_H+D_L=-2]}{2\rho} & when\ D_L^s(b=0) = 0 \\ -\frac{E[P(z)|D_H+D_L=-1]}{2\rho} + \frac{(\rho-1)E[P(z)|D_H+D_L=-2]}{2\rho} & when\ D_L^s(b=0) = -1 \end{cases}$$

Then, by taking the derivative of these fees with respect to ρ, we can see how the optimal fee changes as costs change. The derivative of the first fee (when $D_L^s(b=0)=1$) is

$$\frac{E[P(z)|D_H+D_L=1]+E[P(z)|D_H+D_L=-2]}{2\rho^2}>0$$

the derivative of the second fee (when $D_L^s(b=0)=0$) is

$$\frac{E[P(z)|D_H+D_L=-2]}{2\rho^2}>0$$

, and the derivative of the third fee (when $D_L^s(b=0)=-1$) is

$$\frac{E[P(z)|D_H+D_L=-2]-E[P(z)|D_H+D_L=-1]}{2\rho^2}<0$$

Thus, when L buys or does not trade conditional on $b=0$, he tilts his optimal fee downward as ρ decreases (i.e, as he receives less fee revenue) while he tilts his optimal fee upward as ρ decreaes when $D_L^s(b=0)=-1$.

Now suppose the no-recall constraint binds. The constraint then requires the following fee:

$$f>\frac{\Delta E[P(z)]}{\rho}$$

so committing to not recall shares requires a greater fee as ρ declines. □

1.9 Appendix B: Numerical Analysis for Cost of Trading $\bar{C}$

In Section 1.5.1, I describe how the passive lender improving price efficiency depends on his ability to attract more short sellers (i.e., have a greater $G(c^*)$). I claimed that such a result occurs when $\bar{C}$ takes an intermediate value and q is low. I now numerically demonstrate this claim when the lender is uninformed (i.e., $q=0.5$). As a result, all equilibrium are pooling equilibrium. I also relax Assumption 1.1 and set $\hat{C} = \bar{C}$; thus, it is no longer guaranteed that H always buys when $v = 1$.

I start with the passive lender. Following the usual procedure, the three relevant expected prices are:

$$E[P(z)|D_H + D_L] = \begin{cases} \frac{12-5G(c^*)}{20-10G(c^*)} \text{when } D_H + D_L = 1 \\ \frac{3}{10} + \left(\frac{1}{2(2-G(c^*))}\right) + \left(\frac{1-\hat{G}(y^*)}{2(2-\hat{G}(y^*))}\right) \text{ when } D_H + D_L = 0 \\ \frac{8-5\hat{G}(y^*)}{20-10\hat{G}(y^*)} \text{when } D_H + D_L = -1 \end{cases}$$

where $G(c^*) = \frac{c^*}{\bar{C}}$ is the probability of H short-selling conditional on $v = 0$ and $\hat{G}(y^*) = \frac{y^*}{\bar{C}}$ is the probability of H buying conditional on $v = 1$. There will be three regions of outcomes in this setting: (1) $G(c^*) = \hat{G}(y^*) = 1$, (2) $\hat{G}(y^*) = 1$ and $G(c^*) < 1$, and (3) $\hat{G}(y^*) < 1$ and $G(c^*) < 1$. There is no case where $G(c^*) = 1$ and $\hat{G}(y^*) < 1$ because short selling requires paying fees while buying does not. The passive lender still sets fees equal to $f^* = \frac{E[P(z)|D_H+D_L=1]}{2}$, so the equilibrium buying and selling cutoffs are the lower of the solutions to the following fixed point problem and $\bar{C}$

$$1 - E[P(z)|D_H + D_L = 1] = y^* \quad \text{(A20)}$$

$$E[P(z)|D_H + D_L = -1] - f^* = c^* \quad \text{(A21)}$$

Solving, Region 1 prevails (i.e., $G(c^*) = \hat{G}(y^*) = 1$) for all $\bar{C} < \frac{3}{20}$, Region 2 for all $\frac{3}{20} \leq \bar{C} \leq \frac{3}{8}$, and Region 3 for all $\bar{C} > \frac{3}{8}$. In Region 2, $\bar{c^*} = \frac{3}{20}$ (i.e., the result from the base model), and in Region 3

$$c^* = -\frac{16\,\bar{C} - 160\,\bar{C}^2 + 20\,\sqrt{64\,\bar{C}^4 - \frac{192\,\bar{C}^3}{5} + \frac{236\,\bar{C}^2}{25} - \frac{6\,\bar{C}}{5} + \frac{1}{16}} + 5}{40\,\left(4\,\bar{C} - 1\right)}$$

and

$$y^* = \frac{8\,\bar{C} + \frac{16\,\bar{C}-160\,\bar{C}^2+20\,\sqrt{64\,\bar{C}^4-\frac{192\,\bar{C}^3}{5}+\frac{236\,\bar{C}^2}{25}-\frac{6\,\bar{C}}{5}+\frac{1}{16}}+5}{4\left(4\,\bar{C}-1\right)} - \frac{\bar{C}\left(16\,\bar{C}-160\,\bar{C}^2+20\,\sqrt{64\,\bar{C}^4-\frac{192\,\bar{C}^3}{5}+\frac{236\,\bar{C}^2}{25}-\frac{6\,\bar{C}}{5}+\frac{1}{16}}+5\right)}{4\,\bar{C}-1}}{40\,\bar{C}-5}$$

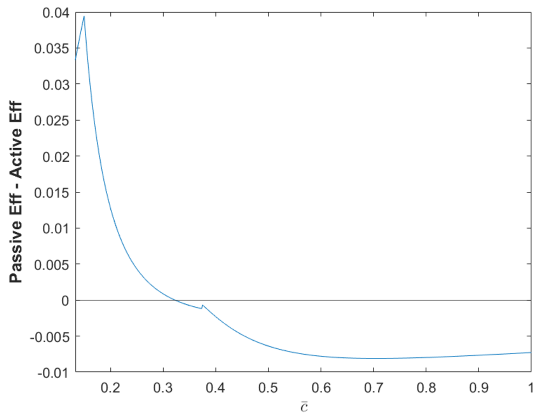
.

The analysis is similar for the active lender. Since I am continuing to impose the no-recall constraint, I will stick with the unique trading equilibrium from the base model where $\{D_L^1(b=0), D_L^0(b=0)\} = \{0,0\}$, which lends itself to a fee of $f^* = \Delta E[P(z)]$. Expected prices in this case are:

$$E[P(z)|D_H + D_L] = \begin{cases} \frac{14-5G(c^*)}{20-10G(c^*)} \text{when } D_H + D_L = 2 \\ \frac{6-2G(c^*)}{10-5G(c^*)} \text{when } D_H + D_L = 1 \\ \frac{2}{10} + \frac{2}{5(2-G(c^*))} + \frac{1-\hat{G}(y^*)}{5(2-\hat{G}(y^*))} \text{when } D_H + D_L = 0 \\ \frac{2}{10} + \frac{1}{5(2-G(c^*))} + \frac{1-\hat{G}(y^*)}{5(2-\hat{G}(y^*))} \text{when } D_H + D_L = -1 \\ \frac{3-2\hat{G}(y^*)}{10-5\hat{G}(y^*)} \text{when } D_H + D_L = -2 \end{cases}$$

Then the cutoffs are solved by a fixed point problem similar to (A20) and (A21) with the exception that the expected short-selling price is conditional on $D_H + D_L = -2$ rather than -1. In contrast to the passive lender, only Regions 2 and 3 exist in equilibrium with an active lender. Region 2 will exist for all $\bar{C} \in [\frac{2}{15}, \frac{\sqrt{13}+2}{15}]$, and Region 3 will exist for all $\bar{C} > \frac{\sqrt{13}+2}{15}$. There is no equilibrium when $\bar{C} < \frac{2}{15}$. Intuitively, when $\bar{C}$ gets small, both $\hat{G}(y^*)$ and $G(c^*)$ approach one, making the borrowing signal b extremely informative; thus, the active lender will want to deviate and buy when $b = 0$, destroying the $D_L(b=0) = 0$ equilibrium. However, $D_L(b=0) = 1$ is not an equilibrium either because when this is the conjectured trading strategy, prices become more revealing, so the only recall-proof fee completely eliminates all short selling, destroying the informativeness of signal b. As a result, there is no pure-strategy equilibrium when $\bar{C}$ is small. The selling cutoff in Region 2

Figure 1.6: Efficiency Difference



is $c^* = \bar{C} - \frac{\sqrt{100\,\bar{C}^2+1}}{10} + \frac{1}{10}$. The buying and selling cutoffs in Region 3 cannot be described in closed form, so I must solve them numerically.

Now that I have the active and passive equilibria as a function of $\bar{C}$, I will plot the difference between active and passive price efficiency. In both cases, my efficiency measure equals $E[P(z)|D_H + D_L = 1]$. [45]. Figure 1.6 shows that the efficiency difference between the passive and active lender reaches a peak around $\bar{C} = 0.19$ and then hits zero at around $\bar{C} = 0.35$. There is no active equilibrium for values $\bar{C} < \frac{2}{15}$, but we can also see the passive edge in efficiency decreasing to the left of this peak, so it would continue to decline towards negative values if we imposed the same (non-equilibrium) trading strategy on the active lender for these low values of $\bar{C}$. Thus, passive lending leads to greater efficiency when $\bar{C}$ is in a middle region where short-selling demand $(G(c^*))$ is sensitive to changes in lending profits, as described in Section 1.5.1. As $\bar{C}$ gets large, passive efficiency falls below active efficiency. Then, this difference approaches zero from below in the limit as the probability of $H's$ trades (buying and selling) heads to zero.

[45]Recall that efficiency is $1 - E[|v - E(P|v])| = \frac{1}{2}(1 + E[P(z)|v = 1] - E[P(z)|v = 0])$, and by the law of total expectations $1 - E[P(z)|v = 0] = E[P(z)|v = 1]$

Chapter 2

The Consequences of Index Investing on Managerial Incentives

2.1 Introduction

Properly incentivizing executives is one of the most important governance decisions faced by shareholders and their representatives on boards of directors. However, optimal compensation schemes derived from academic theory and those observed in practice often diverge. One of the most notable differences concerns relative performance evaluation (RPE): the classic analysis of Holmstrom (1982) states that optimal contracts should filter out common shocks to reduce the volatility of a manager's pay, enabling the implementation of stronger incentives. However, executive contracts implement RPE far less frequently than theory predicts—instead paying managers for good market or industry stock performance—giving rise to the "RPE puzzle" (e.g., Murphy (1999) and Jenter and Kanaan (2015)).[1] Many papers have tried reconciling such "payment for luck" with various extensions to the standard framework,[2] but it is not obvious whether such a compensation scheme is truly rewarding luck rather than performance once one considers one of the most prominent developments in financial markets over the past few decades: the rise of index investing.

Recent work by Chinco and Sammon (2024) estimates that around one-third of the U.S. stock market is held by index investors, underscoring the major shift away from active investing in recent years. Index investors purchase baskets of stocks in fixed proportions, so any information that induces the purchase of an index product should be transmitted to all constituent stock prices, regardless of which firm in the index generates the signal. Consider the recent experience of the "Magnificent Seven" stocks[3]; when discussing record inflows into S&P 500 ETF products in 2024, Vanguard's head of US ETF capital markets said, "People see a lot more in the news about the Magnificent Seven stocks. People want a piece of that, and all these stocks are in the S&P.[4]" Consequently, compensating the

[1] While the use of RPE appears to be growing (Bizjak et al. (2022)), it is still far from universal and many shocks are not filtered out. See the survey by Edmans, Gabaix, and Jenter (2017) for a full discussion on the state of RPE.

[2] See the literature review for a discussion of these papers.

[3] Alphabet, Amazon, Apple, Meta, Microsoft, Nvidia, and Tesla.

[4] https://www.ft.com/content/4a9b70d7-f956-4ac7-b730-7ee3cafbd9e1

Magnificent Seven CEOs for good index performance may not be rewarding luck, but rather, their good performance, as reflected in these other constituent stocks via the synchronized asset demands of index investors. However, whether such a channel is meaningful enough to dominate the traditional RPE contracting motivation in equilibrium is challenging to foresee without a formal model.

In this paper, I develop a joint model of contracting with moral hazard and financial markets with index investors to examine how optimal managerial compensation changes as the fraction of index investors increases. To my knowledge, this is the first paper to study the impact of index investing on managerial compensation. My baseline model contains two ex-ante identical firms (labeled 1 and 2) that make up a stock index. Each firm has a principal who must solve a standard managerial moral hazard contracting problem to maximize the expected payoff of their firm; in particular, the contracts can depend on both firms' stock prices. Stock prices are determined via a noisy rational expectations equilibrium model of financial markets (e.g., Grossman and Stiglitz (1980) or Hellwig (1980)), where some fraction of traders are active, and the rest are indexers. Index investors can only purchase the stocks in some fixed proportion, while active traders have complete portfolio discretion.

My main result is that the optimal contract should put a positive and increasing weight on the stock performance of other index constituents when indexing increases. The key mechanism is as follows. Manager 1's effort increases expectations about the payoffs of Firm 1 and the index (because Firm 1 is part of the index). Index investors then buy more of the index after receiving this good news, which pushes up both stock prices. As a result, *both* stock prices reflect the index investors' information about Manager 1's effort but also become more positively correlated. These two results have opposite implications for Manager 1's optimal contract: the increased sensitivity of Firm 2's stock price to Manager 1's effort suggests a positive weight on Firm 2's stock performance (effort sensitivity channel), while the increased correlation suggests a negative weight to hedge out common shocks (RPE or hedging channel). I find that the effort sensitivity channel dominates in equilibrium.

The intuition for this main result follows from how the financial market equilibrium adjusts as the fraction of index investors increases. Following an increase in index investing, fewer active traders remain to supply *relative* liquidity. For example, suppose liquidity demand for Stock 1 is negative, while liquidity demand for Stock 2 is positive. For these markets to clear, active traders must take the opposite side of these trades because indexers cannot trade against relative liquidity demand shocks; index investors can only supply *absolute*, not relative, liquidity. However, since traders are risk-averse, the remaining active traders will require greater compensation (i.e., more favorable prices) to supply this relative liquidity, meaning that liquidity demand has a greater price impact as indexing increases. The equilibrium prices' increased sensitivity to liquidity trades dampens the positive correlation induced by indexing (and thus also the benefit of RPE), which allows the effort sensitivity channel to dominate.

My model also predicts that under most parameters, the sensitivity of the manager's pay to his stock price decreases in the fraction of index investors. This result follows from the insight that as indexing increases, other index constituents' stocks will reflect relatively more of the market's information about the manager's effort—due to the synchronized demands of index investors. Moreover, my model predicts that managerial pay will become more highly correlated across firms as index investing increases. This finding arises naturally from my previous two results, which show that firms put less weight on their own firm's stock performance and more on other firms' as index investing increases. In fact, all executive pay will be identical in the limiting case with only indexers.

My baseline model assumes that the goal of each firm's principal is to maximize the expected payoff of its firm. However, the common ownership literature (e.g., Azar, Schmalz, and Tecu (2018)) suggests that the growth of indexing may change this objective. Index investors typically invest through an index fund or ETF managed by a large asset manager (e.g., Vanguard, BlackRock, or Fidelity), so stock ownership tends to become more concentrated as indexing increases. These diversified asset managers may then govern their firms to maximize portfolio value, which is not necessarily equivalent to maximizing the value of each firm in their portfolio. I include an extension where a common principal sets each manager's contract to maximize expected index payoffs. The optimal contract parameters with common ownership have the same sign as the separate owner case; all the common owner does is alter the magnitudes. As a result, any empirical evidence showing that the rise of index investing results in more highly correlated CEO pay is not necessarily evidence of anti-competitive common ownership activities; it can also be the optimal contract for separate value-maximizing principals.

Literature Review

My paper contributes to the executive compensation literature on relative performance evaluation as developed theoretically by Holmstrom (1979) and Holmstrom (1982). However, the lack of empirical support for this theory is a well-known puzzle, and several papers have proposed explanations. Jin (2002) and Garvey and Milbourn (2003) argue that RPE may be redundant if executives can hedge exposure to risk factors on their personal portfolios; Himmelberg and Hubbard (2000) argue that shocks to executive labor demand generate a positive correlation between aggregate stock returns and CEO pay; Oyer (2004) proposes a model where RPE may be sub-optimal when the CEO's outside option covaries positively with aggregate stock returns; Goplan, Milbourn, and Song (2010) argue that not filtering out industry shocks leads to executives choosing more appropriate firm exposure to industry conditions; and DeMarzo and Kaniel (2023) show that RPE may not be optimal when managers have "keeping up with the Joneses" preferences. Lastly, Edmans, Gosling, and Jenter (2023) survey UK directors and investors about executive compensation and find that fairness concerns are the most common explanation for the lack of RPE. I contribute to this literature by demonstrating that RPE can be suboptimal even in a textbook moral hazard framework once one considers the impact of index investing on equilibrium stock prices.

My paper also contributes to the literature on the real effects of financial markets—specifically, how financial market developments impact managerial incentives (see Bond, Goldstein, and Edmans (2012) for a survey). Holmstrom and Tirole (1993) and Diamond and Verrecchia (1982) are two classic papers on this subject. Holmstrom and Tirole (1993) examine whether a firm's stock price can be a useful contracting variable when profits are observable and contractible. They find that stock prices contain additional information, and the value of this information is increasing in market liquidity. Diamond and Verrecchia (1981) examine a moral hazard model where the stock price contains unique information for the principal above and beyond output. My main contribution to this literature is to analyze how well markets can monitor a manager's effort when index traders impact the informativeness of prices.

This paper also addresses the impact of index investing on equilibrium asset prices. Most theoretical papers in this literature rely on noisy rational expectations equilibrium (REE) models of financial markets as developed by Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), and Admati (1985). I use the Admati (1985) framework (a multi-asset version of Hellwig (1980)) for my financial market and add index investors and endogenous cash flows driven by managerial effort.

The three index investing papers most similar to mine are Bond and Garcia (2022), Buss and Sundaresan (2020), and Liu and Wang (2018). Bond and Garcia (2022) use a Diamond and Verrecchia (1981) style financial market to analyze the welfare consequences of a decline in the cost of index investing. They find that as the fraction of index investors increases, the price efficiency of the index declines and the price efficiency of individual stocks increases. The latter result occurs because the marginal trader who switches from active to index investing following a decrease in the cost of index investing is less informed than the remaining active traders. Thus, the average information quality for individual stocks increases when he leaves. Buss and Sundaresan (2020) develop a model with a feedback mechanism between price informativeness and corporate investment. They find that an increase in the fraction of passive investors lowers the firm's cost of capital because active investors hold fewer shares in equilibrium—and thus less risk. A lower cost of capital encourages investment, increasing the volatility of the firm's cash flows and incentivizing active investors to gather more information. Liu and Wang (2018) develop a model of index investors with endogenous information acquisition and find that the effect of index investors on price efficiency depends on how gathering information on one stock affects the costs of gathering information on additional stocks.

As the above papers suggest, there is no theoretical consensus on how an increase in indexing impacts price efficiency. Other papers on the subject include Lee (2020), who finds a positive relationship between indexing and price efficiency; Baruch and Zhang (2021), who find a negative relationship; and Coles, Heath, and Ringgenberg (2022), who find no relationship. Davies (2021) and Sammon (2022) also contribute to this literature. The latter uses theoretically motivated measures of price informativeness to find that price efficiency has declined over the last 30 years. This result suggests a negative relationship between index investing and price efficiency. My simple model of index investors also implies a negative relationship between index investing and price efficiency, but that is not the focus of my paper. One of my main contributions to this literature is endogenizing stock payoffs with a managerial moral hazard problem.

Finally, I contribute to the common ownership literature by examining an extension where a common owner sets contracts to maximize index value. Prior work (e.g., Gordon (1990), Aggarwal and Samwick (1999), Liang (2016), and Anton et al. (2022)) has shown that common owners may find RPE suboptimal. However, I take a step back and show that index investing—the primary driver of common ownership—endogenously alters asset prices, making RPE suboptimal even in the absence of common ownership. This finding suggests that common ownership may only be correlated with—as opposed to causing—the absence of RPE, with both phenomena driven by the rise of index investing.

2.2 Toy Model

I start by setting up and solving a simple contracting problem that demonstrates the intuition behind my optimal contract. There are two firms ($k \in \{1,2\}$) whose outputs (x_k) depend on their manager's effort (e_k) and noise (η_k). Formally, $x_k = e_k + \eta_k$ with $\eta_1, \eta_2 \sim N(0,1)$ and $corr(\eta_1, \eta_2) = 0$.

Firms cannot contract directly on output but must rely instead on a performance measure $P_k = e_k + \alpha e_j + \eta_k + \phi_k$ with $\alpha \in [0,1]$, $\phi_k \sim N(0,1)$, and $corr(\phi_1, \phi_2) = \rho$. Assume ϕ_k is independent of η_1 and η_2. Firm 1's performance measure equals its output plus a term that depends on the effort of Manager 2 (αe_j) and an additional noise term that is correlated across performance measures (ϕ). Thus, even though outputs are uncorrelated, the two firms' performance measures are linked by effort (through α) and a correlated noise term (with correlation ρ).

The principal is risk-neutral, the manager is risk-averse (specifically, CARA utility[5]), and contracts are linear in the two performance measures. For Firm 1, the contract is $w_1 = l_1 + m_1 P_1 + n_1 P_2$. The formal contracting problem for Firm 1 is:

$$\max_{l_1, m_1, n_1} E(x_1 - w_1)$$

$$\text{s.t. } E(w_1) - \frac{1}{2}e_1^2 - \frac{1}{2}Var(w_1) \geq 0 \tag{IR}$$

$$e_1 \in \underset{\tilde{e_1}}{\text{argmax}}\, E(w_1) - \frac{1}{2}\tilde{e_1}^2 - \frac{1}{2}Var(w_1) \tag{IC}$$

The principal maximizes the expectation of output minus the manager's wage. The manager requires non-negative expected utility to participate and chooses effort to maximize his expected utility given the contract. The manager's expected utility equals the expected wage minus a quadratic cost of effort term and a risk-aversion term. I will drop the firm subscripts in the following discussion because the solutions are the same for both firms.

[5] I set the manager's coefficient of absolute risk aversion to 1 for simplicity.

Lemma 2.1. *When $\alpha = 0$, the optimal contract sets $m^* > 0$ and sets n^* to have the opposite sign of $\rho = corr(\phi_1, \phi_2)$.*

We can interpret Lemma 2.1 as the classic RPE result. The positive sign on m^*—the contract weight on the firm's own performance measure—is straightforward, but the sign of n^* is the main RPE result; we set the sign of n^* to be opposite that of ρ to hedge common noise in the performance measures. By doing so, we reduce the volatility of the risk-averse manager's pay, which allows the principal to implement stronger incentives.

Lemma 2.2. *When $\alpha > \frac{\rho}{2}$, the optimal contracts sets $m^* > 0$ and $n^* > 0$.*

We still get $m^* > 0$ when $\alpha > 0$. However, we are now guaranteed to get $n^* > 0$ when $\alpha > \frac{\rho}{2}$. Note that α can be interpreted as the sensitivity of Firm 1's performance measure to Manager 2's effort divided by the sensitivity of Firm 1's performance measure to Manager 1's effort. We can also interpret $\frac{\rho}{2}$ as the covariance of the two performance measures divided by their common variance: $Var(P) \equiv Var(P_1) = Var(P_2) = 2$.

$$\alpha = \left(\frac{dP_k}{de_j}\right) / \left(\frac{dP_k}{de_k}\right) \text{ and } \frac{\rho}{2} = \frac{cov(P_1, P_2)}{Var(P)}$$

In the RPE model, we always set $n^* < 0$ when $\rho > 0$, but now we set $n^* > 0$ whenever $\alpha > \frac{\rho}{2} > 0$ even though doing so increases the manager's risk. The benefits of increasing the effort sensitivity of the manager's pay outweigh the costs of increased pay volatility.

In my full model, I endogenize the parameters α and ρ through a model of financial markets with index investors. We can then interpret my heretofore generic performance measures as stock prices, and as we will see, the synchronized asset demands of index investors create both the correlation between the two performance measures (ρ) and the presence of the other manager's effort (α) in each firm's performance measure. I then demonstrate conditions that result in the magnitudes of these parameters being such that $\alpha > \frac{\rho}{2}$ so $n^* > 0$.

2.3 Full Model

2.3.1 Firms

There are two firms: 1 and 2.[6] Each firm is controlled by a risk-neutral principal and managed by a risk-averse manager. The manager privately exerts costly effort to increase the mean cash flow of his firm. Denote firm $k's$ cash flow as x_k. The manager of firm k is paid a wage w_k as compensation for his effort. Each firm pays out the entirety of its cash flow net of managerial compensation (i.e., $x_k - w_k$) to its shareholders as a dividend. Financial market participants trade shares of each firm at the equilibrium price P_k. Assume each firm has one share outstanding. The details of the contracting problem and financial market will be explained in the following subsections.

2.3.2 Contracting Problem

Each firm's principal faces a moral hazard problem. The risk-neutral principal wants to maximize the expected dividends of the firm: $E(x_k - w_k)$. Each risk-averse manager chooses his firm's mean cash flow e_k at a personal cost of $\frac{1}{2}e_k^2$. The manager's choice of e_k is his private information. I assume the cash flows of the two firms are normally distributed, independent, and have identical variances. Specifically, let $x_k \sim N(e_k, 1)$.[7]

The principal induces effort with a contract. I focus on linear incentive schemes of the following form:

$$w_k = l_k + m_k P_k + n_k P_j$$

That is, the contract has a fixed component (l_k), a component based upon the firm's stock price (m_k), and a component based upon the other firm's stock price (n_k). I assume that the principal cannot contract on the actual cash flow x_k. This assumption can be rationalized

[6] All results generalize to an arbitrary number N firms. See Section 2.6.1 for details.

[7] When the cash flow's variance gets too small, market clearing restricts the principal to contracts that make the stock resemble a risk-free asset. This restriction generates all sorts of bizarre results such as putting a negative contract weight on the firm's own stock price. To avoid these unrealistic complications, I normalize the variance of the cash flow to one.

by supposing that the payoffs are for a long-term project and the manager must be paid immediately for current consumption.

The manager has CARA utility over $w_k - \frac{1}{2}e_k^2$ with risk aversion coefficient 1. He also requires an expected utility of at least 0 to participate. Therefore, the principal of firm k chooses (l_k, m_k, n_k) to maximize expected dividends subject to the manager participating and choosing the effort level that maximizes his expected utility (given the contract).

2.3.3 Financial Market

The financial market consists of three assets: a risk-free asset, the stock of Firm 1, and the stock of Firm 2. I assume the risk-free asset has a guaranteed price and payoff of \$1. There is an infinite supply of the risk-free asset and one share of each stock outstanding.

There exist a unit mass of agents $i \in [0, 1]$ with CARA utility over terminal wealth and a common coefficient of absolute risk aversion 1. All agents have the same starting wealth W_0. Agents choose portfolios to maximize their expected utility. I also assume there are noise traders whose aggregate demand for each asset (z_1, z_2 respectively) are i.i.d random variables with distribution $N(0, \tau_z^{-1})$. These random variables are independent of every other random variable in the model.

However, for mathematical convenience, I follow Bond and Garcia (2022) and analyze the financial market in terms of two synthetic assets rather than the original stock prices. Define the following synthetic asset prices:

$$\begin{bmatrix} P_{IN} \\ P_{LS} \end{bmatrix} = T \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, \text{with } T = \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{2.1}$$

I divide by $\sqrt{2}$ to keep the variances of the synthetic cash flows and noise demands equal to the originals. P_{IN} is the price of an index asset (proportional to $P_1 + P_2$) and P_{LS} is the price of a long-short asset (proportional to $P_1 - P_2$). We can similarly transform all other variables in the model: variables with subscript IN are for the index asset and variables with subscript LS are for the long-short asset. Also, observe that $Cov(x_{IN}, x_{LS}) = 0, Cov(z_{IN}, z_{LS}) = 0$, and $Cov(x_{IN}, z_{LS}) = Cov(x_{LS}, z_{IN}) = 0$. As a result, the two synthetic asset prices are uncorrelated.

The purpose of this transformation in Bond and Garcia (2022), and here, is to simplify the traders' asset demands. Index investors generate multiple sources of correlation in the original stock prices, which makes each trader's information set quite large. However, because the two synthetic assets have zero correlation, focusing on them simplifies the calculations considerably.

I assume an exogenous fraction of traders λ are active, and the remaining $1 - \lambda$ are indexed. I endogenize this fraction in Appendix D, but it does not impact my results. Active traders can purchase as much of the index and long-short assets as they want, while index investors are constrained to only trade the index asset. This restriction is equivalent to letting active traders freely trade the two original stocks while constraining index investors to purchase them in a fixed proportion.

Active traders receive independent noisy signals about both the index and long short assets. For example, let $s_{i1} = x_1 + \epsilon_{i1}$ be a signal for the Stock 1 and $s_{i2} = x_2 + \epsilon_{i2}$ be a signal for Stock 2 and $s_i = (s_{i1}, s_{i2})'$. Active investors see the two synthetic signals ($s_{i,IN}$ and $s_{i,LS}$) generated by Ts_i. Observe that these two synthetic signals are uncorrelated. Index investors only receive a signal about the index asset; that is, index trader i only observes the signal $s_{i,IN} = \frac{s_{i1}+s_{i2}}{\sqrt{2}}$. This setup is equivalent to giving active traders signals about both of the original assets while only giving index traders an aggregate signal. Let $\epsilon_{ik} \sim N(0, \tau_\epsilon^{-1})$ be the distribution of noise and assume that noise realizations are independent across assets k and agents i. [8] Further assume signal noise is independent of all other random variables in the model.

Equilibrium prices are reached through market clearing. Once the synthetic asset prices

[8]Note that the composite signals have the same variance as the signals for the two base assets.

are discovered, it is straightforward to invert Equation (2.1) to find the original stock prices
.

2.3.4 Timing

1. The principal and manager agree on a contract and publicly announce it. The manager exerts effort.
2. Investors make a rational inference about e_k given the contract. Denote this belief e_k^*.
3. Investors trade based upon their information sets, the managers' contracts, and their type (active or indexed). Equilibrium prices are generated.
4. The manager of each firm k receives wage w_k.
5. Gross payoffs x_k are realized.

Note that this timeline has the manager being paid before payoffs are realized. Thus, the manager's contract cannot depend on the actual realization of payoffs. Rather, it must depend on stock prices—a noisy signal for payoffs. One can interpret the manager being paid before x_k is realized as the firm borrowing amount w_k at the risk-free rate once equilibrium prices are realized.

2.4 Model Solution

I solve for the equilibrium using the usual "guess-and-verify" method for REE models of financial markets. Specifically, I first conjecture an equilibrium price function for each risky asset. Then, given these prices, the principal and agent find the optimal moral hazard contract. Third, given the conjectured prices, managerial contracts, their information, and their type, each trader selects their optimal portfolio. Finally, given the asset demands, prices are reached via market clearing. I will then have an equilibrium if the resulting prices match the form of the initial conjecture.

2.4.1 Financial Market Equilibrium

I make the following conjectures for the prices:

$$P_1 = A_1 + Bx_1 + Cx_2 + Dz_1 + Ez_2 \tag{2.2}$$
$$P_2 = A_2 + Bx_2 + Cx_1 + Dz_2 + Ez_1$$

Traders form correct-in-equilibrium beliefs about the efforts chosen by the managers given contracts w_1 and w_2. Denote these beliefs as e_1^* and e_2^*. These conjectures imply that all traders have identical prior beliefs about the payoffs of the assets. I allow the constant terms in the asset prices (A_1 and A_2) to be different because they depend on the equilibrium effort conjectures.[9]

We now must confirm that equilibrium prices will indeed be of the conjectured form. As discussed in Section 2.3, it will be convenient to transform the original assets into an index and a long-short asset. I will then use the law of one price to find the original asset prices. As a reminder, I define the synthetic prices as

$$\begin{bmatrix} P_{IN} \\ P_{LS} \end{bmatrix} = T \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, \text{with } T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

I define all other transformed variables similarly: variables with subscript IN are for the index asset, and variables with subscript LS are for the long-short asset. I conjecture that the prices of the synthetic assets will be linear in their own payoffs and noise demands. This conjecture implies that the original prices are of the form specified in Equation (2.2).

$$P_{IN} = A_{IN} + B_{IN}x_{IN} + C_{IN}z_{IN} \tag{2.3}$$
$$P_{LS} = A_{LS} + B_{LS}x_{LS} + C_{LS}z_{LS}$$

[9] Of course, the managers will choose the same effort in equilibrium given the symmetrical nature of the problem. As a result, we will see that the constant terms will also be the same in equilibrium.

Because the two synthetic assets have zero covariance, traders do not need to include information about the long-short asset in their demand for the index asset and vice-versa. Standard results about CARA utility imply that Trader i's demand for the index asset will be:

$$\begin{aligned} D(i, IN) =& \frac{E(x_{IN}|s_{i,IN}, P_{IN}) - w_{IN} - P_{IN}}{Var(x_{IN}|s_{i,IN}, P_{IN})} \\ &= e^*_{IN} + s_{i,IN}\tau_\epsilon + \rho^2_{IN}\tau_z(x_{IN} + \rho^{-1}_{IN}z_{IN}) - (1 + \tau_\epsilon + \rho^2_{IN}\tau_z)(w_{IN} + P_{IN}) \end{aligned}$$

where $\rho_{IN} = \frac{B_{IN}}{C_{IN}}$ and e^*_{IN} is the traders' correct-in-equilibrium belief about the mean payoff of the index. Notice that the wage is a constant to traders because they are conditioning on the price and know all the contract parameters. Demand for the long-short asset is defined similarly. Market clearing for each asset then requires:

$$\int_0^1 D(i, IN)di + z_{IN} = \frac{2}{\sqrt{2}}$$
$$\int_0^\lambda D(i, LS)di + z_{LS} = 0$$

Index asset demand from rational traders integrates from zero to one because both active and index investors trade it. Long-short asset demand integrates to λ because only active traders trade it. The supply of each synthetic asset comes from:

$$T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{2}} \\ 0 \end{bmatrix}$$

Plugging the asset demands into the market clearing conditions and re-arranging allows me to verify that the synthetic prices are indeed of the form conjectured in Equation (2.3). I then pre-multiply the synthetic price vector by T^{-1} to confirm my conjecture from Equation (2.2).

Contracts impact prices in two ways: directly through effort's effect on the mean payoff and indirectly through the price coefficients. It will be convenient to follow Holmstrom and Tirole (1993) and normalize the prices so that they only depend on the direct effect of contracting. Focusing on the direct effect highlights the key economic forces in my model and allows for sharper conclusions. I will discuss the results for the original prices later on, but as a preview, the signs on the optimal contract parameters are unchanged by this normalization.

From the prices (P_1, P_2) and contracting parameters $(l_1, l_2, m_1, m_2, n_1, n_2)$, we can create the following normalized prices:

$$\tilde{P}_1 = (1 + m_1)P_1 + n_1 P_2 + l_1$$
$$\tilde{P}_2 = (1 + m_2)P_2 + n_2 P_2 + l_2$$

These normalized prices can then be expressed as follows:

$$\tilde{P}_1 = \tilde{A}_1 + \tilde{B}x_1 + \tilde{C}x_2 + \tilde{D}z_1 + \tilde{E}z_2$$
$$\tilde{P}_2 = \tilde{A}_2 + \tilde{B}x_2 + \tilde{C}x_1 + \tilde{D}z_2 + \tilde{E}z_1 \text{ with}$$
$$\tilde{A}_1 = \frac{e_1^* - 2 + e_2^*}{(\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2} + \frac{e_1^* - e_2^*}{(\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2}$$
$$\tilde{A}_2 = \frac{e_1^* - 2 + e_2^*}{(\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2} - \frac{e_1^* - e_2^*}{(\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2}$$
$$\tilde{B} = \frac{\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon}{(\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2} + \frac{\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon}{(\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2}$$
$$\tilde{C} = \frac{\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon}{(\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2} - \frac{\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon}{(\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2}$$
$$\tilde{D} = \frac{\tau_\epsilon\,\tau_z + 1}{(\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2} + \frac{\tau_\epsilon\,\tau_z\,\lambda^2 + 1}{\lambda\,(\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2}$$
$$\tilde{E} = \frac{\tau_\epsilon\,\tau_z + 1}{(\tau_z\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2} - \frac{\tau_\epsilon\,\tau_z\,\lambda^2 + 1}{\lambda\,(\tau_z\,\lambda^2\,\tau_\epsilon{}^2 + \tau_\epsilon + 1)\,2}$$

It can be shown that $\tilde{B}, \tilde{C}$, and $\tilde{D} \geq 0$ and $\tilde{E} \leq 0$. The signs of $\tilde{B}$ and $\tilde{D}$ are straightforward: a firm's stock price increases following positive cash flow news or noise demand. The signs of $\tilde{C}$ and $\tilde{E}$ are consequences of indexing. Suppose Firm 1 has a large realization of x_1, so index investors receive a positive signal about the index asset. They know that $x_{IN} = \frac{x_1+x_2}{\sqrt{2}}$ is large, but do not know the relative contributions of each firm. As a result, they buy more of both firms through the index. Therefore, a large realization of x_1 leads to greater index demand, which increases Firm 2's stock price. Similarly, suppose Stock 1 has a lot of noise demand. Index investors see the index's price rise without any good cash flow news. Thus, they buy less of both stocks in the index, which drives down Firm 2's stock price.

Observe that the normalized prices are constructed from public information and equal the hypothetical asset prices from a model without the contracting problem (taking the mean payoff e_k^* as given). As a result, this normalization allows us to separate the financial market and contracting problem, so the principal can take the normalized prices as given when choosing a contract. Moreover, contracting on performance measures $\tilde{P}_1$ and $\tilde{P}_2$ is equivalent to contracting on P_1 and P_2 with the correct adjustments. Define the normalized contract for Firm 1 as $w_1 = \tilde{l_1} + \tilde{m_1}\tilde{P}_1 + \tilde{n_1}\tilde{P}_2$ with

$$\begin{aligned}\tilde{l_1} &= \frac{l_1 + l_1\, m_2 - l_2\, n_1}{m_1 + m_2 + m_1\, m_2 - n_1\, n_2 + 1} \\ \tilde{m_1} &= \frac{m_1 + m_1\, m_2 - n_1\, n_2}{m_1 + m_2 + m_1\, m_2 - n_1\, n_2 + 1} \\ \tilde{n_1} &= \frac{n_1}{m_1 + m_2 + m_1\, m_2 - n_1\, n_2 + 1}\end{aligned} \tag{2.4}$$

The compensation for Manager 2 is defined similarly (swap all the 1's and 2's).

2.4.2 Contracts

The principal's problem with the normalized performance measures is:

$$\max_{\tilde{l_k}, \tilde{m_k}, \tilde{n_k}} E(x_k - w_k) \tag{2.5}$$

$$\text{s.t. } E(w_k) - \frac{1}{2}e_k^2 - \frac{1}{2}Var(w_k) \geq 0 \tag{IR}$$

$$e_k \in \underset{\hat{e_k}}{\operatorname{argmax}}\, E(w_k) - \frac{1}{2}\hat{e_k}^2 - \frac{1}{2}Var(w_k) \tag{IC}$$

Taking the manager's first-order condition in constraint IC and solving gives us the following characterization of optimal effort:

$$e_k^* = \tilde{m_k}\tilde{B} + \tilde{n_k}\tilde{C}$$

Optimal effort equals the weight the contract puts on the manager's stock price ($\tilde{m_k}$) times the sensitivity of his stock price to his effort ($\tilde{B}$) plus the weight the contract puts on the other firm's stock price ($\tilde{n_k}$) times the sensitivity of the other stock price to his effort ($\tilde{C}$). Notice the positive incentive effect of putting a positive weight on the other firm's stock price. We know that $\tilde{C} > 0$, so greater effort by Manager 1 boosts Firm 2's stock price through $\tilde{C}$. Thus, setting $\tilde{n_k} > 0$ generates more effort, all else equal. With this choice of effort, we can plug back into the optimization problem. Specifically, we can take e_k^* and insert it into the objective function and IR constraint. We then bind the IR and substitute it into the objective function. The principal's new problem is

$$\max_{\tilde{m_k},\tilde{n_k}} e_k^* - \frac{1}{2}e_k^{*2} - \frac{1}{2}\left((\tilde{m_k}^2 + \tilde{n_k}^2)Var(\tilde{P}) + 2\tilde{m_k}\tilde{n_k}Cov(\tilde{P_1},\tilde{P_2})\right)$$

where $Var(\tilde{P}) = Var(\tilde{P_1}) = Var(\tilde{P_2})$ and does not depend on $\tilde{l_k}, \tilde{m_k}$, or $\tilde{n_k}$. Solving this maximization problem then gives us:

$$\tilde{m_k}^* = \Gamma\left(\tilde{B}Var(\tilde{P}) - \tilde{C}Cov(\tilde{P_1},\tilde{P_2})\right) \geq 0 \tag{2.6}$$

$$\tilde{n_k}^* = \Gamma\left(\tilde{C}Var(\tilde{P}) - \tilde{B}Cov(\tilde{P_1},\tilde{P_2})\right) \tag{2.7}$$

$$\text{with } \Gamma \equiv \frac{1}{Var(\tilde{B}\tilde{P_1} - \tilde{C}\tilde{P_2}) + Var(\tilde{P})^2 - Cov(\tilde{P_1},\tilde{P_2})^2} > 0$$

These contract parameters will be the same for both firms, which implies that $e_1^* = e_2^*$. Thus, the equilibrium asset prices will have identical constant terms (i.e., $\tilde{A}_1 = \tilde{A}_2 \equiv \tilde{A}$). Also, $\tilde{l_k}^*$ will be the value that binds the agent's participation constraint given $\tilde{m_k}^*$ and $\tilde{n_k}^*$.

Even before plugging in, we can tell that $\tilde{m_k}^*$ will be non-negative because $\tilde{B} \geq \tilde{C}$ and

$Var(\tilde{P}) \geq Cov(\tilde{P}_1, \tilde{P}_2)$. This result is intuitive: the manager's own stock price is increasing in his effort, so putting a positive weight on $\tilde{m}$ is good for incentives. The sign of $\tilde{n}$, however, is unclear at this stage, so we will need to plug in the price coefficients. However, we can say that $\tilde{n}$ will be positive if and only if the following condition is met:

Lemma 2.3. *The optimal choice of $\tilde{n}$ will be positive if and only if $\frac{\tilde{C}}{\tilde{B}} > \frac{Cov(\tilde{P}_1, \tilde{P}_2)}{Var(\tilde{P})}$*

Lemma 2.3 is a direct application of the condition $\alpha > \frac{\varrho}{2}$ (i.e., Lemma 2.2) from the toy model in Section 2.2; $\frac{\tilde{C}}{\tilde{B}}$ represents α and $\frac{Cov(\tilde{P}_1, \tilde{P}_2)}{Var(\tilde{P})}$ represents $\frac{\varrho}{2}$. There are two reasons to tie the manager's wage to the other stock price: doing so provides additional evidence of the manager's effort (effort sensitivity effect), and doing so can eliminate noise from the manager's wage (hedging effect). The effort sensitivity effect is captured by $\frac{\tilde{C}}{\tilde{B}}$, and the hedging effect is represented by $\frac{Cov(\tilde{P}_1, \tilde{P}_2)}{Var(\tilde{P})}$. The effort sensitivity effect measures how efficient (in terms of price responsiveness) it is to incentivize effort via the other firm's stock price relative to the manager's stock price. The hedging effect measures how providing these incentives affects the manager's risk. If $\tilde{C}$ is large relative to $\tilde{B}$, it will be efficient to incentivize effort by putting more weight on the other firm's stock price rather than just on the manager's stock price because it provides additional evidence of managerial effort. Additionally, suppose $Cov(\tilde{P}_1, \tilde{P}_2) > 0$. Then a larger covariance induces the principal to lower $\tilde{n}$ (and potentially make it negative) to hedge out common sources of noise in the manager's pay. The reverse will be true if the covariance is negative. If $\text{Var}(\tilde{P})$ is large, then the principal will rely less on the other stock for any level of covariance because tying the manager's compensation to another volatile stock will increase his risk. After plugging in all the relevant expressions, I get Proposition 2.1, which states that the optimal contract always puts a positive weight on the other firm's stock price when there are index investors.

Proposition 2.1. *When there are index investors (i.e., $\lambda \neq 1$), $\tilde{n}^* > 0$. When there are only active investors, $\tilde{n}^* = 0$.*

Proposition 2.1 claims that the effort sensitivity effect always dominates the hedging effect, so $n^* > 0$. The two effects are working in the same direction when $Cov(\tilde{P}_1, \tilde{P}_2) < 0$, so the claim is only in doubt when the covariance is positive.[10] In this case, the effort sensitivity effect dominates because noise traders dilute the benefits of hedging.

To see this result, note that the hedging effect depends on both the covariance and variance of the prices. I will address each of these items in turn. The covariance can be expressed as:

$$Cov(\tilde{P}_1, \tilde{P}_2) = 2\left(\tilde{B}\tilde{C} + \frac{\tilde{D}\tilde{E}}{\tau_z}\right) \tag{2.8}$$

Recall that $\tilde{B}, \tilde{C}, \tilde{D} \geq 0$ and $\tilde{E} \leq 0$. These signs us that the first term in the covariance $(2\tilde{B}\tilde{C})$ is positive and the second term $(2\frac{\tilde{D}\tilde{E}}{\tau_z})$ is negative. The first term is the cash flow component of the covariance, and it is increasing in the fraction of index investors. Intuitively, as index investing increases, Firm 1's payoff appears more prominently in Firm 2's stock price (and vice-versa), increasing the correlation between the stocks.

However, this positive correlation is mitigated by the second term: the noise trader component. This term is negative and decreasing in the fraction of index investors. Negativity follows immediately from $\tilde{D} \geq 0$ and $\tilde{E} \leq 0$, and it decreases in the fraction of index investors because of how market clearing interacts with trader risk aversion. To be more precise, one can interpret the rational traders submitting demand schedules (active or indexed) as suppliers of liquidity and the noise traders as demanders of liquidity. When rational traders are risk averse, they will require compensation (i.e., a more favorable price) to supply liquidity to noise traders—since doing so forces them to hold greater quantities of risky assets. It follows that keeping noise demand constant, a decrease in the number of liquidity suppliers should result in a higher price of liquidity (i.e., stock prices becoming more sensitive to noise trader demand), as each remaining supplier must bear relatively more risk. Consequently, as index investing increases, there will be fewer suppliers of *relative* liquidity; that is, fewer traders

[10]Whether the covariance is positive or negative depends on the parameters in my model—both can be achieved.

to accommodate the difference in noise demand between the two stocks. Index investors can supply absolute liquidity, but not relative liquidity. Thus, the increased compensation required by active traders to supply relative liquidity as indexing grows in popularity makes $\tilde{D}(\tilde{E})$ more positive (negative)—amplifying the negative noise trader component of the covariance.

Thus, the two terms in the covariance somewhat offset each other, preventing the covariance from ever getting too positive. This result can be seen in Figure 2.1 [11]. The covariance equals zero when there are only active investors and then slowly increases with the fraction of index investors. However, the noise trader component $(\frac{\tilde{D}\tilde{E}}{\tau_z})$ keeps the covariance from ever getting "too big" and eventually becomes so dominant that it drives the covariance negative.

We can see an analogous trend for the variance. The variance is defined as

$$Var(\tilde{P}) = \tilde{B}^2 + \tilde{C}^2 + \frac{\tilde{D}^2 + \tilde{E}^2}{\tau_z}$$

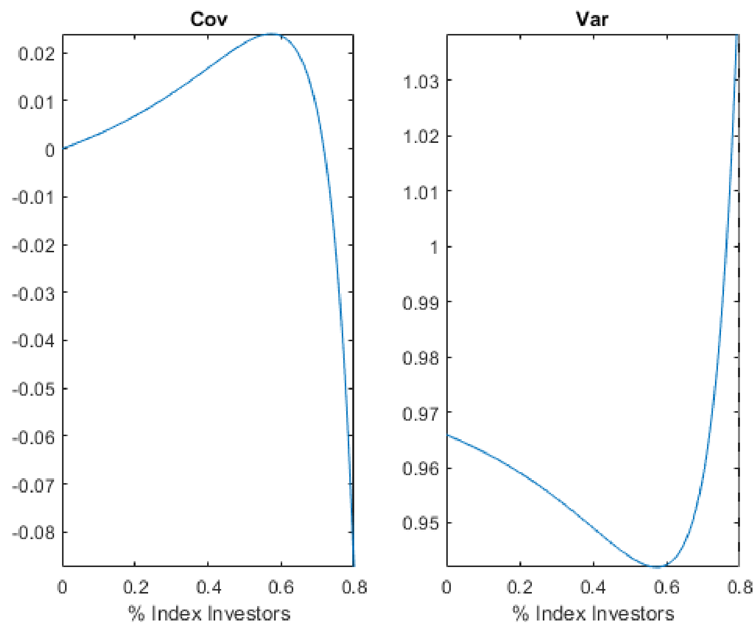
Initially, the most important component of the variance is $\tilde{B}^2$, the squared sensitivity of the stock price to its cash flow (see Figure 2.2 for an illustration of the magnitudes). As a result, the variance initially decreases as the fraction of index investors increases because $\tilde{B}$ is decreasing in the fraction of index investors. However, for the same reasons mentioned in the discussion of the covariance, the increased sensitivity of prices to noise demand prevents the variance from ever getting too small.

Finally, the effort sensitivity effect gets stronger as indexing increases. This can be seen by looking at a graph of the price coefficients in Figure 2.2. As the fraction of index investors increases, $\tilde{B}$ decreases while $\tilde{C}$ increases, so the ratio $\frac{\tilde{C}}{\tilde{B}}$ is increasing in the fraction of index investors. Intuitively, with more indexers, prices should reflect the firm's cash flow less and the cash flow of the other firm more.

Putting all these pieces together, we see that the increased sensitivity of prices to noise

[11] This example sets $\tau_\epsilon = 2$, and $\tau_z = 6$

Figure 2.1: Normalized Price Covariance and Variance

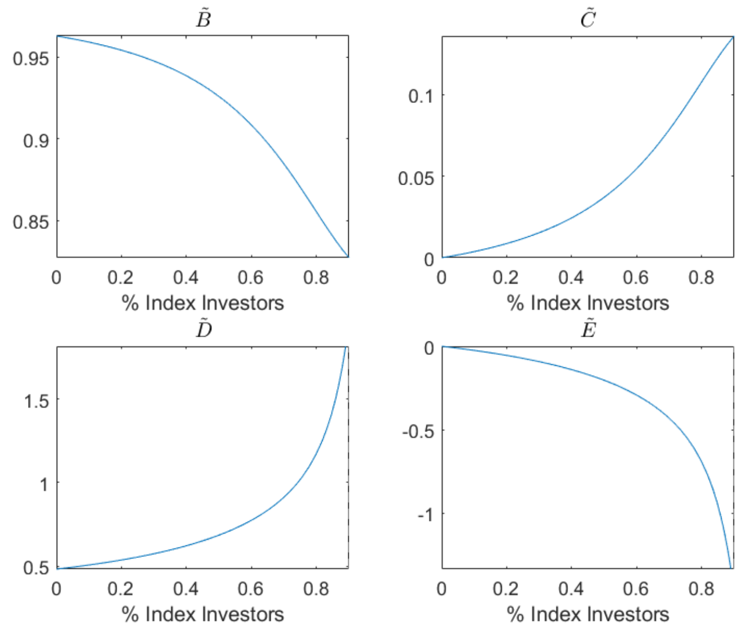


trader demands as indexing increases prevents the benefits of hedging from ever getting large enough to dominate the effort sensitivity effect (Figure 2.3), leading to $\tilde{n}^* > 0$ whenever there are index investors. We can see these optimal normalized contracting coefficients as a function of the fraction of index investors in Figure 2.4. The principals put equal weight on both firms' stock prices when there are only index investors because the market cannot distinguish between the two firms in any way. The principals then put greater (less) weight on their (the other firm's) stock price as more active traders appear because it more (less) strongly reflects their manager's effort.

The above discussion implies that trader risk aversion is an important property for Proposition. My baseline case where the traders' common coefficient of absolute risk aversion equals one is enough to ensure that $\tilde{n}^* > 0$. However, when this coefficient of absolute risk aversion approaches zero (i.e., approaches risk neutrality), the active traders will not demand enough compensation in equilibrium to sufficiently dampen the correlation, making $\tilde{n}^*$ negative. Corollary 2.1 formalizes this argument.

Corollary 2.1. *There is some level of trader risk aversion $\gamma^* > 0$ such that for all $\gamma > \gamma^*$*

Figure 2.2: Normalized Price Coefficients



we will have $\tilde{n^*} > 0$*.*

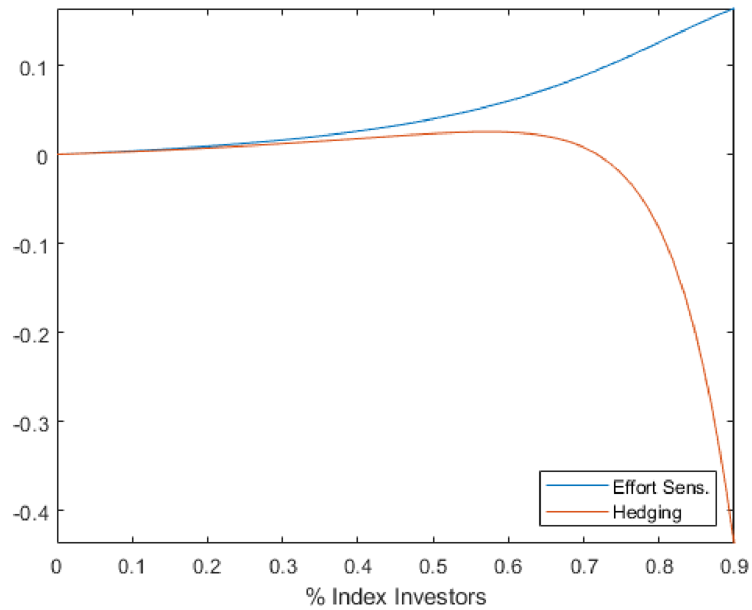
Interestingly, the manager's risk aversion has no impact on the signs of the contract parameters—it is only that of the traders that matters.

I can also derive the following results, whose empirical implications I will discuss in Section 2.7:

Proposition 2.2. *Comparitive statics for key outcomes:*

- $\tilde{n}^* \geq 0$ *and is increasing in the fraction of index investors.*
- $\tilde{m}^* \geq 0$*. Can be increasing or decreasing in the fraction of index investors.*
- *The wage covariance is positive and increasing in the fraction of index investors.*
- *Effort is positive and is decreasing in the fraction of index investors.*
- *The principal's expected profits are positive and decreasing in the fraction of index investors.*

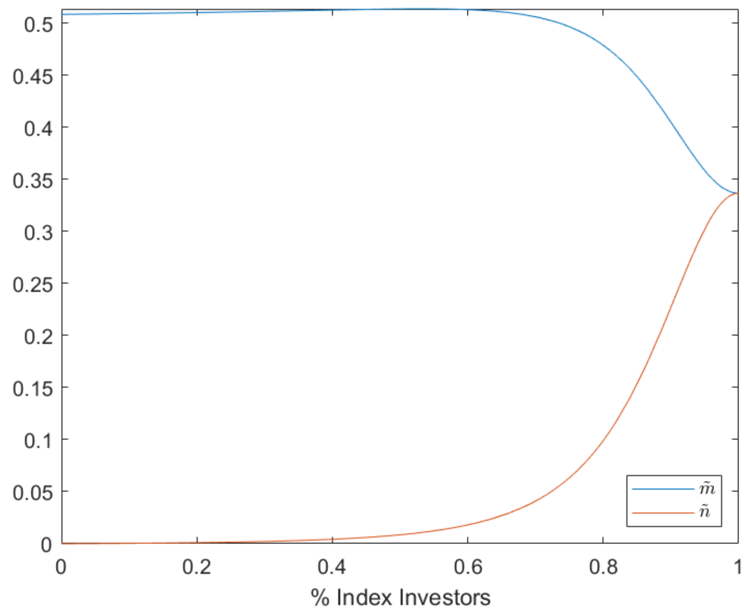
Figure 2.3: Effort Sensitivity and Hedging Effects



I have discussed the first point at length. The second point regarding the comparative statics of $\tilde{m}$ with respect to λ merits additional discussion. Under most parameters, $\tilde{m}$ decreases in the fraction of index investors because, with more index traders, the firm's stock price reflects managerial effort less intensely. However, price variance can also decrease with the fraction of index traders under certain parameters due to the magnitude of $\tilde{B}$—the most prominent coefficient in the pricing function (see Figures 2.1 and 2.2). Thus, when indexing reduces $\tilde{B}$, the variance declines, making stronger incentives—that is, a higher level of $\tilde{m}$—increasingly optimal as indexing grows. However, this effect only occurs when the traders' signal and noise demand is sufficiently precise, so under most parameters, $\tilde{m}$ decreases in the fraction of index investors.

Increased wage covariance is a direct consequence of firms more closely tying their manager's wage to the stock performance of the other firms. The final two points follow from price efficiency decreasing with the fraction of index investors in my model. With more index investors, the manager's effort is reflected less strongly in his stock price, more strongly in the other firm's stock price, and less overall. Thus, some information is "lost" with more

Figure 2.4: Normalized Contracting Parameters



index investors, which hurts incentives and profits.

2.4.3 Original Performance Measures

I now want to map $\tilde{m}$ and $\tilde{n}$ back into the original contract parameters m and n. Inverting the mappings given by Equation (2.4) gives us the following characterization of m and n in terms of $\tilde{m}, \tilde{n}$.

$$m_1^* = m_2^* \equiv m^* = \frac{\tilde{m}^2 - \tilde{m} - \tilde{n}^2}{2\tilde{m} - \tilde{m}^2 + \tilde{n}^2 - 1} \tag{2.9}$$

$$n_1^* = n_2^* \equiv n^* = \frac{-\tilde{n}}{2\tilde{m} - \tilde{m}^2 + \tilde{n}^2 - 1} \tag{2.10}$$

By undoing the normalization, we get the following proposition.

Proposition 2.3. *Optimal contract parameters based on the original prices have the same sign as those based on the normalized prices.*

The intuition for the original contract is largely the same as for the normalized contract.

We can also rewrite the original price coefficients as functions of the normalized price coefficients and the normalized contract parameters.

$$B = \tilde{B} - \tilde{m}\tilde{B} - \tilde{n}\tilde{C}$$

$$C = \tilde{C} - \tilde{m}\tilde{C} - \tilde{n}\tilde{B}$$

$$D = \tilde{D} - \tilde{m}\tilde{D} - \tilde{n}\tilde{E}$$

$$E = \tilde{E} - \tilde{m}\tilde{E} - \tilde{n}\tilde{D}$$

The price coefficients can be interpreted as the sum of the following three components. I'll use B as an example, but the rest of the coefficients follow the same pattern.

1. The normalized price coefficient ($\tilde{B}$)
2. The reduction in dividends caused by placing incentives on the firm's stock price ($-\tilde{m}\tilde{B}$)
3. The reduction in dividends caused by placing incentives on the other firm's stock price ($-\tilde{n}\tilde{C}$)

How does a marginal increase in Firm 1's cash flow affect P_1? First, there is the elementary effect of higher cash flows leading to higher dividends. $\tilde{B}$ represents this effect and measures how sensitive the price is to dividends in a world without the contracting problem. Second, because higher cash flows mean higher prices, dividends are reduced because the manager receives a higher wage. The manager's compensation is related to his firm's cash flow through $\tilde{B}\tilde{m} + \tilde{C}\tilde{n}$. This term negatively affects dividends. Therefore, higher cash flows have both positive and negative effects on the stock price, but in the end, $B > 0$, so the positives outweigh the negatives. This result does not necessarily follow for C, however, because the positive cash flow effect of $\tilde{C}$ is relatively small and the negative cash flow effect of $\tilde{m}\tilde{C} + \tilde{n}\tilde{B}$ is large. Thus, C can be positive or negative depending on the parameters. Nevertheless, this outcome does not overturn my result about the effort sensitivity effect dominating the hedging effect in the most empirically relevant case where $Cov(P_1, P_2) > 0$ (see Barberis et. al (2005) and Da and Shive (2017)). Recall that the covariance between the two original prices is

$$Cov(P_1, P_2) = 2\left(BC + \frac{DE}{\tau_z}\right)$$

We still have $D > 0$ and $E < 0$, so a necessary condition for a positive covariance given the other parameters is $C > 0$. Thus, if we focus on parameters that generate a positive covariance, the effort sensitivity still dominates the hedging effect because n^* is guaranteed to be positive by Proposition 2.3.

2.5 Common Ownership

I have so far ignored how the growth of index investing may change the principal's objective function. Index investors typically purchase an index through an index fund or an ETF managed by a large asset manager such as Vanguard or BlackRock. The common ownership literature examines how corporate governance changes as these asset managers build increasingly large stakes in many firms due to the growth of indexing. One view is that these asset managers try to maximize the value of their entire portfolio, which may be inconsistent with maximizing the value of each constituent firm. For example, individual firm value maximization may lead to inefficiently aggressive competition between firms from the asset manager's perspective. One hypothesized way to prevent such things from happening is to rely less on relative performance evaluation for managerial incentives or even to put a positive contract weight on a competing firm's stock price (Gordon (1990) and Aggarwal and Samwick (1999)).

My paper, therefore, offers a separate rationale for placing a positive contract weight on competitors' stock prices: it helps incentivize managerial effort. In my model, introducing a common principal that maximizes

$$E(x_1 + x_2 - w_1 - w_2)$$

would not alter the optimal contracts because the $x's$ only depend on their own manager's effort. I will now provide an extension where each manager's effort impacts the other firm's payoff. Define Firm k's new mean payoff as

$$\overline{e_k} = e_k + \beta e_j, \text{ for } k, j \in \{1, 2\}, k \neq j \text{ and } \beta \in (-1, 1) \tag{2.11}$$

So now the mean of x_1 is $\overline{e_1}$, which is impacted by both Manager 1's effort (through e_1) and Manager 2's effort (through e_2). The parameter β reflects how sensitive Firm 1's output is to Manager 2's effort (and vice-versa). My base model is, therefore, a specialized case with $\beta = 0$. A positive β describes a situation where the two managers' efforts complement each other. This situation could represent a manufacturer and a retailer; both firms benefit if the manufacturer makes higher quality products and the retailer becomes more effective at sales. We can interpret a negative β as a situation where the two firms are rivals competing for the same market share.

This modeling change only affects the equilibrium asset prices $(\tilde{P}_1, \tilde{P}_2)$ through changed mean payoffs. Importantly, price coefficients $\tilde{B}, \tilde{C}, \tilde{D}$, and $\tilde{E}$ are unchanged because they only depend on exogenous parameters (τ_ϵ, τ_z, and λ). Consequently, the variances and covariances of the prices are also unchanged.[12]

Given the unchanged asset prices, we can jump directly to the contracting problem.

[12] I focus on the normalized stock prices denoted with a tilde for tractability in this section.

2.5.1 Single Principals

A single principal maximizing his own firm's expected dividends solves:

$$\max_{\tilde{l_k},\tilde{m_k},\tilde{n_k}} E(x_k - w_k) \tag{2.12}$$

$$\text{s.t. } E(w_k) - \frac{1}{2}e_k^2 - \frac{1}{2}Var(w_k) \geq 0 \tag{IR}$$

$$e_k \in \underset{\hat{e_k}}{\text{argmax}}\, E(w_k) - \frac{1}{2}\hat{e_k}^2 - \frac{1}{2}Var(w_k) \tag{IC}$$

where now:

$$E(x_k) = e_k + \beta e_j$$

$$E(w_k) = E(\tilde{l_k} + \tilde{m_k}\tilde{P_k} + \tilde{n_k}\tilde{P_j})$$

$$= \tilde{l_k} + \tilde{m_k}(\tilde{A} + \tilde{B}(e_k + \beta e_j) + \tilde{C}(e_j + \beta e_k)) + \tilde{n_k}(b_{0j} + \tilde{B}(e_j + \beta e_k) + \tilde{C}(e_k + \beta e_j))$$

Without loss of generality, I will focus on the optimal contract for Manager 1. Manager 2's contract will take an analogous form. Manager 1's optimal choice of effort is then

$$e_1^* = \tilde{m_1}(\tilde{B} + \beta\tilde{C}) + \tilde{n_1}(\tilde{C} + \beta\tilde{B}) \tag{2.13}$$

The manager's effort appears in two additional places relative to the base model: through the x_2 terms in both his and Firm 2's stock prices. Holding the contract fixed with $\tilde{m}, \tilde{n} > 0$, a positive β induces more effort from the manager because his effort is more strongly reflected in the stock market. Conversely, a negative β generates less effort because effort lowers the payoff and expected stock price of Firm 2. The signs of the optimal contracting parameters are described by a condition analogous to Lemma 2.3:

Lemma 2.4. *The optimal choice of $\tilde{m}$ will be greater than or equal to zero if and only if $\frac{\tilde{B}+\beta\tilde{C}}{\tilde{C}+\beta\tilde{B}} \geq \frac{Cov(\tilde{P_1},\tilde{P_2})}{Var(\tilde{P})}$. The optimal choice of $\tilde{n}$ will be greater than or equal to zero if and only if $\frac{\tilde{C}+\beta\tilde{B}}{\tilde{B}+\beta\tilde{C}} \geq \frac{Cov(\tilde{P_1},\tilde{P_2})}{Var(\tilde{P})}$.*

The hedging effect is the same as before, but now the effort sensitivity effect considers how effort affects the other firm's cash flows. I assume throughout that the exogenous parameters are such that $Cov(\tilde{P}_1, \tilde{P}_2) > 0$ because that is the most empirically relevant case.

Suppose $\beta > 0$; then both $\tilde{m}$ and $\tilde{n}$ remain positive. In fact, the effort sensitivity effect is stronger than in the base case with $\beta = 0$. This result follows because a positive β increases Stock 2's sensitivity to Manager 1's effort more than Stock 1's; mathematically, $\frac{\tilde{C}+\beta\tilde{B}}{\tilde{B}+\beta\tilde{C}} > \frac{\tilde{C}}{\tilde{B}}$ because $\beta > 0$ and $\tilde{B} > \tilde{C} > 0$. Intuitively, because stock prices are more sensitive to their cash flow than the other firm's cash flow (i.e., $\tilde{B} \geq \tilde{C}$), the impact that effort has on Firm 2's cash flow through β is more strongly reflected in Firm 2's stock price than in Firm 1's.

The results are less clear cut when $\beta < 0$. I assume that $\beta \in (-\frac{\tilde{B}}{\tilde{C}}, 0)$ because if it were more negative, the manager's effort would be so destructive for the other firm that greater effort would actually reduce *his own* expected stock price, which seems unrealistic. If $\beta \in (-\frac{\tilde{B}}{\tilde{C}}, -\frac{\tilde{C}}{\tilde{B}})$, $\tilde{n}$ will trivially be negative because both the effort sensitivity and hedging effects call for a negative sign. The effort sensitivity effect wants a negative sign in this region because effort reduces the expected stock price of the other firm, and the hedging effect also wants a negative sign because the correlation between the two stock prices is positive. One can then show that if $\beta \in \left(-\frac{\tilde{C}}{\tilde{B}}, \frac{\tilde{C}var(\tilde{P})-\tilde{B}Cov(\tilde{P}_1,\tilde{P}_2)}{\tilde{C}Cov(\tilde{P}_1,\tilde{P}_2)-\tilde{B}var(\tilde{P})}\right)$, $\tilde{n}$ will be negative, and if $\beta \in \left(\frac{\tilde{C}var(\tilde{P})-\tilde{B}Cov(\tilde{P}_1,\tilde{P}_2)}{\tilde{C}Cov(\tilde{P}_1,\tilde{P}_2)-\tilde{B}var(\tilde{P})}, 0\right)$, $\tilde{n}$ will be positive. Basically, for a fixed hedging effect, a lower β makes a negative $\tilde{n}$ increasingly optimal because the manager's effort is reflected less positively in the other firm's stock price.

2.5.2 Common Owner

A common owner who maximizes joint dividends solves:

$$\max_{\hat{l},\tilde{m},\tilde{n}} E(x_1 + x_2 - w_1 - w_2) \tag{2.14}$$

$$\text{s.t. } E(w_k) - \frac{1}{2}e_k^2 - \frac{1}{2}Var(w_k) \geq 0 \text{ for } k \in \{1,2\} \tag{IR}$$

$$e_k \in \operatorname*{argmax}_{\hat{e_k}} E(w_k) - \frac{1}{2}\hat{e_k}^2 - \frac{1}{2}Var(w_k) \text{ for } k \in \{1,2\} \tag{IC}$$

Each manager's optimal effort choice is still given by Equation (2.13). The only substantive difference between the common owner and single principal problems is that the common owner considers how each manager's effort impacts the other firm's payoff in the $E(x_1 + x_2)$ portion of his objective function. Thus, when deciding on Manager 1's contract, the common owner considers $(1 + \beta)e_1^*$ instead of e_1^*. Let $\tilde{m_s}$ and $\tilde{n_s}$ be the optimal contracts for the single principal (as solved for in Section 2.5.1) and $\tilde{m_c}$ and $\tilde{n_c}$ be the optimal common owner contract. It turns out that there is a simple relation between the two:

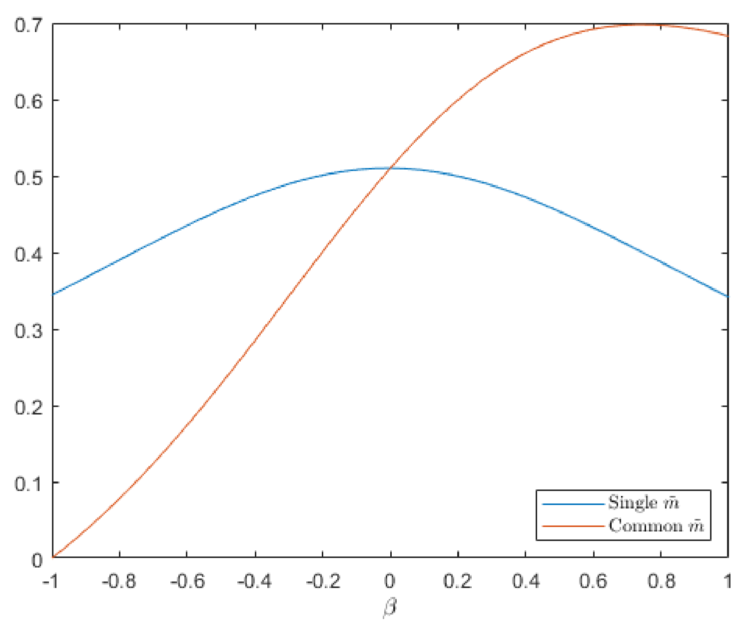
Proposition 2.4. *The optimal common owner contract parameters are the optimal single owner contract parameters multiplied by* $1+\beta$. *That is,* $\tilde{m_c} = (1+\beta)\tilde{m_s}$ *and* $\tilde{n_c} = (1+\beta)\tilde{n_s}$.

Thus, for $\beta > -1$, the common owner simply scales the single-owner contract up or down without changing the sign. A positive β means that each manager's effort generates a positive externality for the other firm that the single owner does not care about. The common owner wants to take advantage of this positive externality, which makes each contract more performance-sensitive in a positive direction. The opposite happens when $\beta \in (-1, 0)$. Each manager's effort generates a negative externality for the other firm, so the common owner makes the optimal contract less performance-sensitive, but with the same sign. In the extreme case where $\beta < -1$, the negative externality is so severe that it outweighs the positive effect the manager's effort has on his firm's performance, so the common owner flips the signs on the optimal contract parameters.

In sum, all common ownership does in the realistic case where $\beta \in (-1, 1)$ is change the magnitude of the single-owner contract without altering the sign. Common ownership is, therefore, not the driving force behind a positive sign on the other firm's stock price in the optimal contract; rather, the key is still the relative sizes of the effort sensitivity and the hedging effects. Thus, the results of my paper caution against interpreting the presence of a positive association between managerial compensation and the stock performance of peers as evidence of anti-competitive activities. A model in which each firm is a strict profit maximizer that wants to incentivize greater managerial effort can also generate such a result.

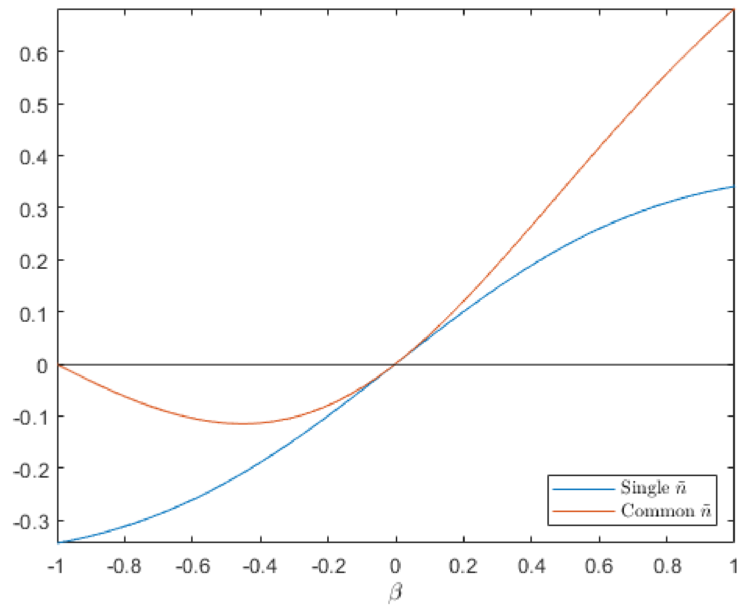
Plots of the optimal $\tilde{m}$ and $\tilde{n}$ for the single and common owner as a function of β are shown below. In these plots, the fraction of index investors is set at 33.5% to match the headline estimate from Chinco and Sammon (2024). In Figure 2.5, we see that the optimal $\tilde{m}$ is positive everywhere, but the common owner sets it higher when $\beta > 0$ and lower when $\beta < 0$. The optimal $\tilde{n}$ (see Figure 2.6) can be positive or negative depending on β, but the sign on the common owner's contract is always the same as the single owner's.

Figure 2.5: Single and Common Owner $\tilde{m}$



Finally, I address how common ownership addresses effort. Proposition 2.2 claims that

Figure 2.6: Single and Common Owner $\tilde{n}$



effort decreases in the fraction of index investors because traders' information about the manager's actions is reflected less strongly in financial market prices. However, this result may change if the growth in indexing also induces a common ownership motivation.

Corollary 2.2. *Equilibrium effort will be higher under common ownership whenever* $\beta > 0$.

The intuition for Corollary 2.2 is similar to Proposition 2.4; a common owner internalizes the manager's effort slipovers to a greater degree than separate owners and thus is more (less) willing to incentivize effort when β is positive (negative). Therefore, if $\beta > 0$, index investing can lead to greater equilibrium effort if it also induces common ownership.

2.6 Robustness

2.6.1 Number of Firms

It is tempting to think that the preeminence of the effort sensitivity effect in Proposition 2.1 is a consequence of there being two firms. With two firms, good aggregate cash flow signals can only come from two places, so indexed investors buy relatively large amounts of both firms. As a result, Firm $1's$ stock price will be quite sensitive to Firm $2's$ cash flows. It then stands to reason that index investors will further divide their asset demands as the number of firms increases, so Firm 1's stock price will be less sensitive to any other firm's cash flow. However, the effort sensitivity effect always dominates, even with an arbitrarily large number of firms.

Proposition 2.5. *Both $\tilde{m}^*$ and $\tilde{n}^*$ are greater than or equal to zero for all number of firms $N \geq 2$.*

The condition outlined in Lemma 2.3 is still decisive, even with N firms. Although the sensitivity of Firm 1's stock price to x_2 declines as the number of firms increases, so does the covariance-variance ratio and the sensitivity of Firm 1's stock price to its own cash flow. In the end, the effort sensitivity effect still dominates. However, even though $\tilde{n}^*$ is always positive, its magnitude is decreasing in N; the optimal contract puts less weight on each firm in the index, but that weight is always positive.

Even though the optimal contract puts less weight on the other firms' stock prices as N increases, it turns out that their combined weight is increasing in N. The principal will put the same weight, $\tilde{n}^*$, on all other index firms in the contract because they are ex-ante identical. Therefore, the principal puts a combined weight of $(N-1)\tilde{n^*}$ on all other firms. My claim is then formally expressed in the following lemma:

Lemma 2.5. *The combined weight that the optimal contract puts on all other index firms, $(N-1)\tilde{n^*}$, is increasing in N.*

The intuition for Lemma 2.5 comes from how index investors split their demands across index constituents. Suppose the manager of Firm 1 takes a good action and increases his firm's payoff. Index investors expect higher index payoffs and buy more of all index firms. When there are N=2 firms in the index, this demand is split between the two firms. One can crudely think of this as index investors assigning 50% of "the credit" for this signal to

Stock 1 and 50% to Stock 2. When there are N=100 firms in the index, the demand is split between one hundred firms, so index investors assign 1% of "the credit" to Stock 1 and 99% to the remaining stocks. As a result, they assign relatively more of "the credit" to other firms in the index as N increases. The optimal contract reflects this outcome by putting more combined weight on other firms' stock prices as N increases.

We can also see this result by looking at the stock prices when there are N firms.

$$\tilde{P}_k = \tilde{A} + \tilde{B}x_k + \tilde{C}(x_1 + ...x_{k-1} + x_{k+1} + ...x_N) + \tilde{D}z_k + \tilde{E}(z_1 + ...z_{k-1} + z_{k+1} + ...z_N)$$

Therefore, the sensitivity of firm $k's$ stock price to its cash flow is $\tilde{B}$, while the combined sensitivity of all other firms' stock prices to firm k's cash flow is $(N-1)\tilde{C}$.

These terms equal:

$$\begin{aligned} \tilde{B} &= \frac{\tau_z\tau_\epsilon^2 + \tau_\epsilon}{\tau_z\tau_\epsilon^2 + \tau_\epsilon + 1} + \frac{1}{N}\left(\frac{\tau_z\tau_\epsilon^2 + \tau_\epsilon}{\tau_z\tau_\epsilon^2 + \tau_\epsilon + 1} - \frac{\lambda^2\tau_z\tau_\epsilon^2 + \tau_\epsilon}{\lambda^2\tau_z\tau_\epsilon^2 + \tau_\epsilon + 1}\right) \\ (N-1)\tilde{C} &= \frac{N-1}{N}\left(\frac{\tau_z\tau_\epsilon^2 + \tau_\epsilon}{\tau_z\tau_\epsilon^2 + \tau_\epsilon + 1} - \frac{\lambda^2\tau_z\tau_\epsilon^2 + \tau_\epsilon}{\lambda^2\tau_z\tau_\epsilon^2 + \tau_\epsilon + 1}\right) \end{aligned} \tag{2.15}$$

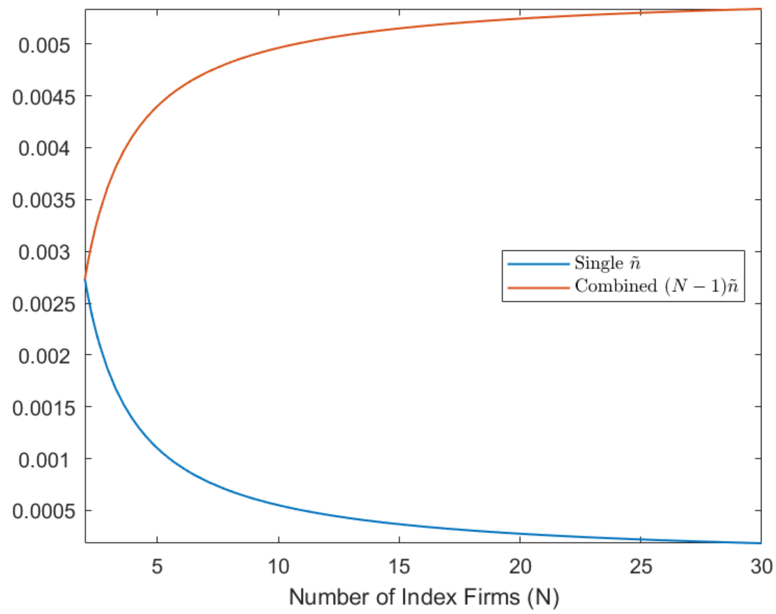
Since $\frac{\tau_z\tau_\epsilon^2+\tau_\epsilon}{\tau_z\tau_\epsilon^2+\tau_\epsilon+\tau_x} \geq \frac{\lambda^2\tau_z\tau_\epsilon^2+\tau_\epsilon}{\lambda^2\tau_z\tau_\epsilon^2+\tau_\epsilon+\tau_x}$, it is easy to see that $\tilde{B}$ is decreasing in N while $(N-1)\tilde{C}$ is increasing in N. That is, Firm 1's stock price becomes less sensitive to its cash flow, and the rest of the stock market becomes more sensitive to Firm 1's cash flow as N increases. These results can be seen graphically in Figure 2.7.[13]

2.6.2 Index Investor Information Set

The theoretical literature has not yet reached a consensus about the best way to model the information sets of index investors. Some papers (e.g., Bond and Garcia (2022)) let index investors condition their index demand on both private information and equilibrium

[13] The fraction of index investors is set to 33.5% to match the headline estimate from Chinco and Sammon (2024).

Figure 2.7: Single $\tilde{n}$ and Combined $(N-1)\tilde{n}$



prices, while on the other extreme, Coles et al. (2022) model index investors as only using prior information. I follow the former approach in this paper and allow index investors to have private information. Specifically, index investors in my model only receive private information about the cash flows of the index, while active investors receive private information about both index and relative cash flows. Gathering private information on index cash flows can be thought of as focusing on broad economic developments rather than on individual firms. One can, therefore, view an index investor in my model as someone who researches the state of the economy and arrives at some private conclusion. Or alternatively, one can view the index investors in my model as macro or industry specialists who gather information about a given sector of the economy rather than on firm-specific details. With this interpretation, index investors are not necessarily passive investors—even though the two terms are generally used interchangeably in the literature.

However, I will now examine what happens to my results if index investors have no private information. Suppose we model index investors as having no private information but rationally incorporating any public information embedded in the prices. Each index investor then demands:

$$D(i, IN) = \frac{E(x_{IN}|P_{IN}) - w_{IN} - P_{IN}}{Var(x_{IN}|P_{IN})}$$

Shocks to Firm 1 still impact Firm 2's stock price through the demands of index investors. Suppose x_1's realization is higher than expected. Index investors see a higher index price, but since they do not see a commensurate increase in cash flow expectations, they buy less of the index; this action lowers Firm 2's stock price. Therefore, Firm 2's stock price still reflects Manager 1's effort (albeit in the opposite direction), so the optimal contract will depend on it. However, the effort sensitivity effect now suggests placing a negative weight on the other firm's stock price. The hedging effect always implies a positive weight because the stock covariance is negative in this setting. In the end, the hedging effect dominates because both terms of the covariance (see Equation 2.8) are negative, making the covariance's magnitude more extreme and thus amplifying the benefits of hedging.

I can take this reasoning a step further. The minimum condition required for each firm's stock price to reflect the fundamentals[14] of the other is that index investors do not have perfectly inelastic demand for the index asset. As long as index investors have price-sensitive demand, each firm's stock price will somewhat reflect the fundamentals of the other through this demand. Private information, optimally incorporating all public information, or even downward sloping demand is not required—just that index demand is price sensitive.

The "no private information" conclusions also lead to some results that go against the empirical evidence. A model where index investors have no private information leads to a negative correlation between the two stock prices. In contrast, nearly all empirical work on index investing finds that indexing induces a positive correlation (e.g., Barberis et al. (2005) for S&P 500 index funds and Da and Shive (2017) for ETFs). A necessary condition for a positive correlation in my model is that index investors buy more of the index when its payoff is higher. This condition fails in the model with no private information because investors

[14]By fundamentals, I mean x and z.

cannot tell index price increases caused by cash flows versus noise demand apart, so they buy less of the index following a price increase. Providing index investors with a private signal about index cash flows can be enough to increase their demand for the index following a rise in one of the firm's cash flows. My results also do not depend on index investors having the same aggregate signal quality as active investors. I only require that the indexers' signals are not *too* much worse.

2.6.3 Endogenous Indexing

So far, the fraction of indexers $(1 - \lambda)$ has been an exogenous parameter. However, one can endogenize this parameter by having investors decide ex-ante whether to be active or indexed. To do so, I impose some cost $c > 0$ to being active; otherwise, all investors would become active. Such an assumption is typical in equilibrium models of index investing (e.g., Bond and Garcia (2022) and Coles, Heath, and Ringgenberg (2022)). I interpret this cost as an information acquisition cost since active investors receive both aggregate and relative payoff signals instead of just aggregate.

Investors must be indifferent ex-ante between being active (and thus having greater portfolio flexibility) and being indexed (to avoid paying the cost c) for there to be an interior fraction of indexers in equilibrium. I formally analyze the equilibrium fraction of indexers in Appendix D. I summarize the main conclusion of this analysis in the following proposition.

Proposition 2.6. *There are two types of equilibria: (1) with an interior fraction of index investors (i.e,* $\lambda \in (0, 1)$*) and (2) with no index investors (i.e,* $\lambda = 1$*). There does not exist an equilibrium with only index investors (i.e,* $\lambda = 0$*).*

In general, my proof follows closely that of Theorem 3 in the seminal work by Grossman and Stiglitz (1980), where they try to pin down the ratio of informed to uninformed traders. However, unlike in Grossman and Stiglitz (1980), there is no equilibrium where all the traders are of the less-informed type (i.e., index investors) in my model. This conclusion follows from the crucial fact that index investors are unable to supply relative liquidity to noise traders (see the discussion following Proposition 2.1). Because index investors cannot step in to supply liquidity when noise traders demand different quantities of Stocks 1 and 2, markets can only clear if there exists some positive mass of active traders who can. Thus, as the fraction of index investors increases, prices become more sensitive to noise trader demand to compensate active investors for taking on greater relative payoff risk. Thus, for any finite cost of being active $c > 0$, there will always be some trader who will deviate from indexed to active as the fraction of index investors approaches one—since the expected trading gains from doing so become arbitrarily large.

2.7 Empirical Predictions

My model's primary empirical prediction is that greater index investing should result in executive pay being more positively linked with index performance (for firms in an index). To my knowledge, there is no direct empirical test of this prediction, but there are related studies examining how the growth in common ownership—which is driven by the indexing—affects executive pay; Anton et al. (2022) and Liang (2016) both show that increases in common ownership lead to executive pay that loads more positively on peer performance.[15] One could test my specific hypothesis that indexing drives greater pay for peer (or index) performance by looking at the response of CEO pay in firms that are added or deleted from indexes; that is, using the standard "index reconstitution" identification strategy (see Appel, Gormley, and Keim (2024) for a discussion of this common identification strategy).

Second, my model predicts that within-index executive pay should become more highly correlated as indexing grows. While not a direct test, recent work by Jochem et al. (2021) and Cabezon (2021) demonstrates that CEO pay has become more uniform. In particular, Jochem et al. (2021) claim that increased reciprocal benchmarking is the primary cause

[15] Kwon (2016) finds the opposite result: common ownership is associated with more relative performance pay, so the empirical evidence is not unanimous.

of this trend and that the growth of index investing drives reciprocal benchmarking. This result is consistent with my model. However, their rationale for why index investing increases benchmarking is that index investors are more likely to rely on proxy advisors— who typically analyze compensation relative to peers—for voting recommendations. My paper suggests an additional channel for this phenomenon: index investing causes stocks in the same index to reflect the effort of all managers, so linking a manager's compensation to the index's performance improves incentives. In other words, there are two ways through which index investing can make contracts more similar: (1) the governance channel identified by Jochem et al. (2021) and (2) the asset pricing channel implied by my model. Which channel is more important is an interesting unanswered empirical question.

Additionally, my model predicts that the effect of indexing on pay-performance sensitivity should be ambiguous. Specifically, this sensitivity should decrease whenever the traders' signals and noise demand are noisy enough to ensure that price volatility increases in the fraction of index investors. There exists evidence consistent with this condition, as Coles, Heath, and Ringgenberg (2018) and Ben-David et al. (2018) show that stock volatility increases with indexing. Given that the real-world parameters appear to be such that stock volatility increases with index investing, my model predicts that pay-performance sensitivity should decline with greater index investing. Once again, the most direct evidence for my prediction comes from the common ownership literature: Anton et al. (2022) find that increases in common ownership are associated with a decline in the performance sensitivity of managerial pay.

As the previous few paragraphs show, most of the existing evidence for my model's predictions comes from the common ownership literature rather than directly through indexing. Future theoretical work can examine the relative importance of these channels on executive compensation (or governance more generally) by integrating more realistic financial markets into models of common ownership. Specifically, by having common ownership arise endogenously through the rise in index investing. Empirically, one can examine the differential impact of the asset pricing versus the common ownership effect of indexing by trying to isolate these two channels. One intriguing possibility for empirical work comes from the recent trend of asset managers instituting "pass-through voting," where end investors in a fund can independently cast votes instead of automatically delegating them to the fund. While few investors utilize these programs now,[16] if they become more popular going forward, we would see a reduction in effective common ownership while preserving the asset pricing effects of indexing. More specifically, my model predicts that common ownership has a second-order effect on executive compensation—the main driver is changing asset prices. Thus, holding indexing fixed, a decline in effective common ownership should not change the signs on the optimal contract parameters.

Conclusion

I develop a joint model of financial markets with index investors and contracting with moral hazard to examine how changes in the prevalence of indexing impact optimal managerial contracts. Index investors are constrained to purchase all risky assets in the same proportion, so information that affects their demand for the index is transmitted to all constituent stocks.

There are two (possibly conflicting) reasons to condition a manager's compensation on these other firms: (1) to increase the effort sensitivity of the manager's pay and (2) to hedge the manager's risks. The former reason is a consequence of the synchronized asset demands of index investors distributing information about the manager's effort to these stocks. The latter reason comes from the relative performance evaluation literature: the principal should utilize the non-zero correlation between the stocks to reduce the manager's exposure to common risks. I find that the effort sensitivity channel dominates the hedging channel because indexing also allows noise traders to play a more prominent role in determining

[16] For example, only around 10% of the dollars in BlackRock's index equity funds exercise voting choice. https://www.blackrock.com/corporate/about-us/investment-stewardship/blackrock-voting-choice

relative prices, which dampens the benefits of hedging.

Stock prices are a useful conditioning variable for managerial compensation because they are an easy-to-access and high-frequency performance measure. Thus, assuming prices are relatively efficient, they enable one to learn about actions privately taken by a manager. However, financial markets are constantly evolving, so firms must consider how these changes—such as the growth of index investing—impact the optimal incentive scheme for their manager.

2.8 Appendix C: Proofs from Main Text

Proof of Lemmas 2.1 and 2.2

Proof. Setting the first order condition for IC equal to 0 and solving for e_1 yields

$$e_1^* = m_1 + n_1\alpha$$

We can then bind the IR condition and plug it and e_1^* into the objective function to get (after some simplification):

$$\max_{m_1,n_1} = e_1^* - \frac{1}{2}e_1^{*2} - \frac{1}{2}Var(w_1)$$

where $Var(w_1) = m_1^2 Var(P_1) + n_1^2 Var(P_2) + 2m_1n_1Cov(P_1, P_2) = 2m_1^2 + 2n_1^2 + 2m_1n_1\rho$. We can then set the two first-order conditions with respect to m_1 and n_1 equal to zero to get a system of two equations for m_1 and n_1. Solving gives us:

$$m_1^* = \frac{2 - \alpha\rho}{\Gamma} > 0$$
$$n_1^* = \frac{2\alpha - \rho}{\Gamma}$$

where $\Gamma \equiv 2\alpha^2 - 2\alpha\rho - \rho^2 + 6 > 0$. Clearly, m_1^* is always positive given the restrictions on α and ρ. When $\alpha = 0$, we see that n_1^* will have the opposite sign of ρ, proving Lemma 2.1. Additionally, whenever $\alpha > \frac{\rho}{2}$, we are guaranteed to get $n_1^* > 0$, proving Lemma 2.2. □

Proof of Lemma 2.3

Proof. Follows immediately from Equation (2.7). □

Proof of Proposition 2.1

Proof. Plug the relevant price coefficients into Equation (2.7) and simplify. This gives us:

$$\tilde{n^*} = \frac{\tau_\epsilon \, \tau_z \, (1-\lambda^2) \, (\tau_z \, \lambda^2 \, {\tau_\epsilon}^2 + \tau_z \, {\tau_\epsilon}^2 + \tau_\epsilon + 1)}{(\tau_\epsilon \, \tau_z \, \lambda^2 + 1) \, (\tau_\epsilon \, \tau_z + 1) \, (4 \, \lambda^2 \, {\tau_\epsilon}^4 \, {\tau_z}^2 + 3 \, \lambda^2 \, {\tau_\epsilon}^2 \, \tau_z + 3 \, {\tau_\epsilon}^2 \, \tau_z + 2)} \tag{C1}$$

Since $\lambda \in [0, 1]$, it must be the case that $\tilde{n^*} > 0$ when $\lambda < 1$ and $\tilde{n^*} = 0$ when $\lambda = 1$.

Similarly, we can plug in and see that

$$\tilde{m}^* = \frac{K_1 \, \tau_\epsilon \, \tau_z}{K_2 \, (\tau_\epsilon \, \tau_z + 1)} \text{ where} \tag{C2}$$

$$\begin{aligned} K_1 =& 2 \, \lambda^4 \, {\tau_\epsilon}^5 \, {\tau_z}^3 + 2 \, \lambda^4 \, {\tau_\epsilon}^4 \, {\tau_z}^2 + 2 \, \lambda^4 \, {\tau_\epsilon}^3 \, {\tau_z}^2 + \lambda^4 \, {\tau_\epsilon}^2 \, \tau_z + 2 \, \lambda^2 \, {\tau_\epsilon}^4 \, {\tau_z}^2 \\ &+ 2 \, \lambda^2 \, {\tau_\epsilon}^3 \, {\tau_z}^2 + 2 \, \lambda^2 \, {\tau_\epsilon}^3 \, \tau_z + 4 \, \lambda^2 \, {\tau_\epsilon}^2 \, \tau_z + 2 \, \lambda^2 \, \tau_\epsilon \, \tau_z + \lambda^2 \, \tau_\epsilon + \lambda^2 + {\tau_\epsilon}^2 \, \tau_z + \tau_\epsilon + 1 > 0 \\ K_2 =& 4 \, \lambda^4 \, {\tau_\epsilon}^5 \, {\tau_z}^3 + 3 \, \lambda^4 \, {\tau_\epsilon}^3 \, {\tau_z}^2 + 4 \, \lambda^2 \, {\tau_\epsilon}^4 \, {\tau_z}^2 \\ &+ 3 \, \lambda^2 \, {\tau_\epsilon}^3 \, {\tau_z}^2 + 3 \, \lambda^2 \, {\tau_\epsilon}^2 \, \tau_z + 2 \, \lambda^2 \, \tau_\epsilon \, \tau_z + 3 \, {\tau_\epsilon}^2 \, \tau_z + 2 > 0 \end{aligned}$$

□

Proof of Corollary 2.1

Proof. Denote γ as each trader's coefficient of absolute risk aversion. Asset demands are adjusted by multiplying each trader's asset demand (D(i, IN) or D(i, LS)) by $1/\gamma$. Then, solving for prices in the usual manner, we can see that this change has no impact on $\tilde{B}$ and $\tilde{C}$ and that it alters $\tilde{D}$ and $\tilde{E}$ by replacing the 1 in the numerators with γ. That is, $\tilde{D} = \frac{\gamma + \tau_\epsilon \, \tau_z}{2 \, \tau_z \, {\tau_\epsilon}^2 + 2 \, \tau_\epsilon + 2} + \frac{\tau_\epsilon \, \tau_z \, \lambda^2 + \gamma}{2 \, \lambda \, (\tau_z \, \lambda^2 \, {\tau_\epsilon}^2 + \tau_\epsilon + 1)}$ and $\tilde{E} = \frac{\gamma + \tau_\epsilon \, \tau_z}{2 \, \tau_z \, {\tau_\epsilon}^2 + 2 \, \tau_\epsilon + 2} - \frac{\tau_\epsilon \, \tau_z \, \lambda^2 + \gamma}{2 \, \lambda \, (\tau_z \, \lambda^2 \, {\tau_\epsilon}^2 + \tau_\epsilon + 1)}$.

From there, the contracting problem remains unchanged, so the sign of $\tilde{n}^*$ still depends on the sign of $H(\gamma) \equiv \tilde{C} Var(\tilde{P}) - \tilde{B} Cov(\tilde{P}_1, \tilde{P}_2)$. Taking the derivative of this expression with respect to γ gives us

$$\frac{\tau_\epsilon \left(1-\lambda^2\right) \left(\lambda^4 \, {\tau_\epsilon}^4 \, {\tau_z}^3 + \lambda^4 \, {\tau_\epsilon}^3 \, {\tau_z}^2 + \lambda^2 \, {\tau_\epsilon}^4 \, {\tau_z}^3 + 3 \, \lambda^2 \, {\tau_\epsilon}^3 \, {\tau_z}^2 + \lambda^2 \, {\tau_\epsilon}^2 \, {\tau_z}^2 + 2 \, \lambda^2 \, {\tau_\epsilon}^2 \, \tau_z + \lambda^2 \, \tau_\epsilon \, \tau_z + {\tau_\epsilon}^3 \, {\tau_z}^2 + 2 \, {\tau_\epsilon}^2 \, \tau_z + \tau_\epsilon \, \tau_z + \tau_\epsilon + 1\right)}{\lambda^2 \, \tau_z \, (\tau_z \, \lambda^2 \, {\tau_\epsilon}^2 + \tau_\epsilon + 1)^2 \, (\tau_z \, {\tau_\epsilon}^2 + \tau_\epsilon + 1)^2} > 0$$

so $H(\gamma)$ is increasing in γ. Next, by plugging in $\gamma = 0$, we can see

$$H(0) = \frac{{\tau_\epsilon}^3\,\tau_z\,(1-\lambda^2)\,(\tau_z\,\lambda^2\,{\tau_\epsilon}^2 + \tau_z\,{\tau_\epsilon}^2 + 2\,\tau_\epsilon + 1)}{2\,(\tau_z\,\lambda^2\,{\tau_\epsilon}^2 + \tau_\epsilon + 1)^2\,(\tau_z\,{\tau_\epsilon}^2 + \tau_\epsilon + 1)^2} < 0$$

So $H(\gamma)$ is a continuous, increasing function with $H(0) < 0$ and $H(1) > 0$ (from the baseline model). Thus, by the intermediate value theorem, there must exist some $\gamma^* \in (0,1)$ where $H(\gamma^*) = 0$ and $H(\gamma) > 0$ for all $\gamma > \gamma^*$. Consequently, by Lemma 2.3, $\tilde{n}^* > 0$ for all $\gamma > \gamma^*$. □

Proof of Proposition 2.2

Proof. See the proof of Proposition 2.1 for the signs of $\tilde{m}^*$ and $\tilde{n}^*$. Then take the partial derivatives of (C1) and (C2) with respect to λ. The partial derivative of $\tilde{n^*}$ with respect to λ is immediately shown to be negative after simplifying, so $\tilde{n^*}$ is increasing in the fraction of index investors. The sign of the $\tilde{m}^*$ derivative is ambiguous. For example, the parameterization $(\tau_\epsilon, \tau_z, \lambda) = (5, 1, .9)$ yields a negative partial derivative while $(\tau_\epsilon, \tau_z, \lambda) = (1, 1, .9)$ yields a positive partial derivative.

The wage covariance can be expressed as:

$$Cov(w_1, w_2) = (\tilde{m}^{*2} + \tilde{n}^{*2})Var(\tilde{P}) + 2 * \tilde{m}\tilde{n}Cov(\tilde{P}_1, \tilde{P}_2) = \frac{2\,{\tau_\epsilon}^2\,\tau_z\,(\tau_z\,{\tau_\epsilon}^2 + 1)\,(1-\lambda^2)\,(\tau_z\,\lambda^2\,{\tau_\epsilon}^2 + 1)}{(4\,\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2 + 3\,\lambda^2\,{\tau_\epsilon}^2\,\tau_z + 3\,{\tau_\epsilon}^2\,\tau_z + 2)^2}$$

This expression is clearly positive and its partial derivative with respect to λ is

$$\frac{\partial Cov(w_1, w_2)}{\partial \lambda} = -\frac{4\,\lambda\,{\tau_\epsilon}^2\,\tau_z\,(\tau_z\,{\tau_\epsilon}^2 + 1)^2\,(4\,\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2 + \lambda^2\,{\tau_\epsilon}^2\,\tau_z + 5\,{\tau_\epsilon}^2\,\tau_z + 2)}{(4\,\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2 + 3\,\lambda^2\,{\tau_\epsilon}^2\,\tau_z + 3\,{\tau_\epsilon}^2\,\tau_z + 2)^3} < 0$$

Thus, the wage covariance is decreasing in the fraction of active investors, or equivalently, increasing in the fraction of index investors.

Equilibrium effort is defined in Section 2.4.2 as

$$e^* = \tilde{m}\tilde{B} + \tilde{n}\tilde{C} = \frac{{\tau_\epsilon}^2\,\tau_z\,(2\,\tau_z\,\lambda^2\,{\tau_\epsilon}^2 + \lambda^2 + 1)}{4\,\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2 + 3\,\lambda^2\,{\tau_\epsilon}^2\,\tau_z + 3\,{\tau_\epsilon}^2\,\tau_z + 2}$$

Taking the partial derivative of this expression with respect to λ and simplifying gives us:

$$\frac{\partial e^*}{\partial \lambda} = \frac{4\,\lambda\,{\tau_\epsilon}^2\,\tau_z\,\left(\tau_z\,{\tau_\epsilon}^2+1\right)^2}{\left(4\,\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2+3\,\lambda^2\,{\tau_\epsilon}^2\,\tau_z+3\,{\tau_\epsilon}^2\,\tau_z+2\right)^2} > 0$$

From Section 2.4.2, the principal objective function (i.e., expected profits) equal

$$\begin{aligned} E(x_k - w_k) &= e^* - \frac{1}{2}e^{*2} - \frac{1}{2}\Big((\tilde{m}^{*2} + \tilde{n}^{*2})Var(\tilde{P}) + 2\tilde{m}\tilde{n}Cov(\tilde{P}_1, \tilde{P}_2)\Big) \\ &= \frac{{\tau_\epsilon}^2\,\tau_z\,\left(2\,\tau_z\,\lambda^2\,{\tau_\epsilon}^2+\lambda^2+1\right)}{2\left(4\,\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2+3\,\lambda^2\,{\tau_\epsilon}^2\,\tau_z+3\,{\tau_\epsilon}^2\,\tau_z+2\right)} \end{aligned}$$

This expression is positive. We can once again take the partial derivative of this expression with respect to λ and see that it is positive. The partial dervative has the form:

$$\frac{\partial E(x_k - w_k)}{\partial \lambda} = \frac{2\,\lambda\,{\tau_\epsilon}^2\,\tau_z\,\left(\tau_z\,{\tau_\epsilon}^2+1\right)^2}{\left(4\,\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2+3\,\lambda^2\,{\tau_\epsilon}^2\,\tau_z+3\,{\tau_\epsilon}^2\,\tau_z+2\right)^2}$$

Thus, the principal's expected profits are decreasing in the fraction of index investors. □

Proof of Proposition 2.3

Proof. Insert the expressions for $\tilde{m}^*$ and $\tilde{n}^*$ given in (C1) and (C2) to the mappings given

Equations (2.9) and (2.10) and simplify. We get

$$m^* = \frac{\tau_\epsilon \tau_z K_3}{K_4} > 0$$

$$n^* = \frac{\tau_\epsilon\, \tau_z\, (1-\lambda^2)\, (4\,\lambda^2\, {\tau_\epsilon}^4\, {\tau_z}^2 + 3\,\lambda^2\, {\tau_\epsilon}^2\, \tau_z + 3\, {\tau_\epsilon}^2\, \tau_z + 2)\, (\tau_z\, \lambda^2\, {\tau_\epsilon}^2 + \tau_z\, {\tau_\epsilon}^2 + \tau_\epsilon + 1)}{K_4} > 0 \text{ where}$$

$$K_3 = 4\,\lambda^6\, {\tau_\epsilon}^9\, {\tau_z}^5 + 4\,\lambda^6\, {\tau_\epsilon}^8\, {\tau_z}^4 + 6\,\lambda^6\, {\tau_\epsilon}^7\, {\tau_z}^4$$

$$+ 6\,\lambda^6\, {\tau_\epsilon}^6\, {\tau_z}^3 + 2\,\lambda^6\, {\tau_\epsilon}^5\, {\tau_z}^3 + 3\,\lambda^6\, {\tau_\epsilon}^4\, {\tau_z}^2 + 4\,\lambda^4\, {\tau_\epsilon}^8\, {\tau_z}^4 + 6\,\lambda^4\, {\tau_\epsilon}^7\, {\tau_z}^4 + 4\,\lambda^4\, {\tau_\epsilon}^7\, {\tau_z}^3 + 12\,\lambda^4\, {\tau_\epsilon}^6\, {\tau_z}^3$$

$$+ 8\,\lambda^4\, {\tau_\epsilon}^5\, {\tau_z}^3 + 6\,\lambda^4\, {\tau_\epsilon}^5\, {\tau_z}^2 + 11\,\lambda^4\, {\tau_\epsilon}^4\, {\tau_z}^2 + 2\,\lambda^4\, {\tau_\epsilon}^3\, {\tau_z}^2 + 3\,\lambda^4\, {\tau_\epsilon}^3\, \tau_z + 5\,\lambda^4\, {\tau_\epsilon}^2\, \tau_z + 6\,\lambda^2\, {\tau_\epsilon}^6\, {\tau_z}^3$$

$$+ 2\,\lambda^2\, {\tau_\epsilon}^5\, {\tau_z}^3 + 6\,\lambda^2\, {\tau_\epsilon}^5\, {\tau_z}^2 + 11\,\lambda^2\, {\tau_\epsilon}^4\, {\tau_z}^2 + 2\,\lambda^2\, {\tau_\epsilon}^3\, {\tau_z}^2 + 6\,\lambda^2\, {\tau_\epsilon}^3\, \tau_z + 6\,\lambda^2\, {\tau_\epsilon}^2\, \tau_z + 2\,\lambda^2\, \tau_\epsilon + 2\,\lambda^2$$

$$+ 3\, {\tau_\epsilon}^4\, {\tau_z}^2 + 3\, {\tau_\epsilon}^3\, \tau_z + 5\, {\tau_\epsilon}^2\, \tau_z + 2\,\tau_\epsilon + 2$$

$$K_4 = 4\,\lambda^6\, {\tau_\epsilon}^{10}\, {\tau_z}^6 + 4\,\lambda^6\, {\tau_\epsilon}^9\, {\tau_z}^5 + 4\,\lambda^6\, {\tau_\epsilon}^8\, {\tau_z}^5 + 8\,\lambda^6\, {\tau_\epsilon}^7\, {\tau_z}^4 + \lambda^6\, {\tau_\epsilon}^6\, {\tau_z}^4 + 3\,\lambda^6\, {\tau_\epsilon}^5\, {\tau_z}^3 + 4\,\lambda^4\, {\tau_\epsilon}^9\, {\tau_z}^5$$

$$+ 4\,\lambda^4\, {\tau_\epsilon}^8\, {\tau_z}^5 + 4\,\lambda^4\, {\tau_\epsilon}^8\, {\tau_z}^4 + 8\,\lambda^4\, {\tau_\epsilon}^7\, {\tau_z}^4 + 2\,\lambda^4\, {\tau_\epsilon}^6\, {\tau_z}^4 + 8\,\lambda^4\, {\tau_\epsilon}^6\, {\tau_z}^3 + 9\,\lambda^4\, {\tau_\epsilon}^5\, {\tau_z}^3 + 3\,\lambda^4\, {\tau_\epsilon}^4\, {\tau_z}^2$$

$$+ 2\,\lambda^4\, {\tau_\epsilon}^3\, {\tau_z}^2 + 8\,\lambda^2\, {\tau_\epsilon}^7\, {\tau_z}^4 + \lambda^2\, {\tau_\epsilon}^6\, {\tau_z}^4 + 8\,\lambda^2\, {\tau_\epsilon}^6\, {\tau_z}^3 + 9\,\lambda^2\, {\tau_\epsilon}^5\, {\tau_z}^3 + 18\,\lambda^2\, {\tau_\epsilon}^4\, {\tau_z}^2 + 4\,\lambda^2\, {\tau_\epsilon}^3\, {\tau_z}^2$$

$$+ 8\,\lambda^2\, {\tau_\epsilon}^2\, \tau_z + 3\, {\tau_\epsilon}^5\, {\tau_z}^3 + 3\, {\tau_\epsilon}^4\, {\tau_z}^2 + 2\, {\tau_\epsilon}^3\, {\tau_z}^2 + 8\, {\tau_\epsilon}^2\, \tau_z + 4$$

□

Proof of Lemma 2.4:

Proof. The procedure to prove this is identical to that of Lemma 2.3. The only change is that the sensitivity of Firm 2's stock price to Manager 1's effort is now represented by $\tilde{C}+\beta\tilde{B}$ rather than $\tilde{C}$ and similarly for the sensitivity of Firm 1's stock price to Manager 1's effort (i.e., $\tilde{B}+\beta\tilde{C}$ rather than $\tilde{B}$).

□

Proof of Proposition 2.4

Proof. The manager's optimal effort is given by Equation (2.13). Binding the IR constraint and plugging into the common owner's objective function gives us:

$$\max_{\tilde{l},\tilde{m},\tilde{n}}(1+\beta)e_1^* + (1+\beta)e_2^* - \frac{1}{2}e_1^{*^2} - \frac{1}{2}e_2^{*^2} - \frac{1}{2}Var(w_1) - \frac{1}{2}Var(w_2)$$

with $e_k^* = \tilde{m}_k(\tilde{B}+\beta\tilde{C}) + \tilde{n}_k(\tilde{C}+\beta\tilde{B})$. Take the first-order conditions with respect to $\tilde{m}_1, \tilde{m}_2, \tilde{n}_1$, and $\tilde{n}_2$ to get a pair of symmetric systems of equations: one for $\tilde{m}_1$ and $\tilde{n}_1$, and another for $\tilde{m}_2$ and $\tilde{n}_2$.

$$\tilde{m}_k = \frac{\overline{B} + \overline{B}\,\beta - \text{cov}\,\tilde{n}_k - \overline{B}\,\overline{C}\,\tilde{n}_k}{\overline{B}^2 + \text{var}}$$
$$\tilde{n}_k = \frac{\overline{C} + \overline{C}\,\beta - \text{cov}\,\tilde{m}_k - \overline{B}\,\overline{C}\,\tilde{m}_k}{\overline{C}^2 + \text{var}}$$
$$\text{with } \overline{B} = \tilde{B} + \beta\tilde{C} \text{ and } \overline{C} = \tilde{C} + \beta\tilde{B}$$

Solving then gives us answers analogous to (2.6) and (2.7) with $\overline{B}$ in place of $\tilde{B}$ and $\overline{C}$ in place of $\tilde{C}$ and the entire thing multiplied by $(1+\beta)$. The only change for the single owner case is that the $\beta\overline{B}$ and $\beta\overline{C}$ terms in the expressions for $\tilde{m}_k$ and $\tilde{n}_k$ respectively are eliminated. Thus, the common owner contract is just the single owner solution multiplied by $(1+\beta)$. □

Proof of Corollary 2.2

Proof. Recall from (2.13) that equilibrium effort expressed as $e^* = \tilde{m}(\tilde{B}+\beta\tilde{C}) + \tilde{n}(\tilde{C}+\beta\tilde{B})$ and from Proposition 2.4 that $\tilde{m}_c = (1+\beta)\tilde{m}_s$ and $\tilde{n}_c = (1+\beta)\tilde{n}_s$. Thus, the ratio of effort under common and single ownership must therefore be $\frac{e_c}{e_s} = (1+\beta)$. As a result, equilibrium effort will be higher under common ownership whenever $\beta > 0$. □

Proof of Proposition 2.5

Proof. Following the same procedure as with $N = 2$ firms, we get the following price conjecture and resulting equilibrium coefficients:

$$\tilde{P}_k = b_0 + \tilde{B}x_k + \tilde{C}(x_1 + ...x_{k-1} + x_{k+1} + ...x_N) + \tilde{D}z_k + \tilde{E}(z_1 + ...z_{k-1} + z_{k+1} + ...z_N)$$

$$\tilde{B} = \frac{\tau_\epsilon\,(\tau_\epsilon\,\tau_z + 1)}{N\,(\tau_z\,{\tau_\epsilon}^2 + \tau_\epsilon + \tau_x)} + \frac{\tau_\epsilon\,(\tau_\epsilon\,\tau_z\,\lambda^2 + 1)\,(N-1)}{N\,(\tau_z\,\lambda^2\,{\tau_\epsilon}^2 + \tau_\epsilon + \tau_x)}$$

$$\tilde{C} = \frac{\tau_\epsilon\,(\tau_\epsilon\,\tau_z + 1)}{N\,(\tau_z\,{\tau_\epsilon}^2 + \tau_\epsilon + \tau_x)} - \frac{\tau_\epsilon\,(\tau_\epsilon\,\tau_z\,\lambda^2 + 1)}{N\,(\tau_z\,\lambda^2\,{\tau_\epsilon}^2 + \tau_\epsilon + \tau_x)}$$

$$\tilde{D} = \frac{\tau_\epsilon\,\tau_z + 1}{N\,(\tau_z\,{\tau_\epsilon}^2 + \tau_\epsilon + \tau_x)} + \frac{(\tau_\epsilon\,\tau_z\,\lambda^2 + 1)\,(N-1)}{N\,\lambda\,(\tau_z\,\lambda^2\,{\tau_\epsilon}^2 + \tau_\epsilon + \tau_x)}$$

$$\tilde{E} = \frac{\tau_\epsilon\,\tau_z + 1}{N\,(\tau_z\,{\tau_\epsilon}^2 + \tau_\epsilon + \tau_x)} - \frac{\tau_\epsilon\,\tau_z\,\lambda^2 + 1}{N\,\lambda\,(\tau_z\,\lambda^2\,{\tau_\epsilon}^2 + \tau_\epsilon + \tau_x)}$$

The manager's choice of effort will then be

$$e_k^* = \tilde{m}\tilde{B} + (N-1)\tilde{n}\tilde{C}$$

The above equation has the principal put the same weight on each of the other firms in the contract. Just like in the $N = 2$ case, this decision is optimal because all firms are ex-ante identical.

Then by binding the participation constraint and plugging into the objective, the principal's problem becomes

$$\max_{\tilde{m},\tilde{n}} e_k^* - \frac{1}{2}e_k^{*2} - \frac{1}{2}\left((\tilde{m}^2 + (N-1)\tilde{n}^2)var + 2((N-1)\tilde{m}\tilde{n} + \binom{N-1}{2}\tilde{n}^2)cov\right)$$

where

$$var = \tilde{B}^2 + (N-1)\tilde{C}^2 + \frac{\tilde{D}^2 + (N-1)\tilde{E}^2}{\tau_z}$$

$$cov = (N-2)\tilde{C}^2 + 2\tilde{B}\tilde{C} + \frac{(N-2)\tilde{E}^2 + 2\tilde{D}\tilde{E}}{\tau_z}$$

We can then take the first order conditions and solve the resulting system of equations for

$\tilde{n}$ and $\tilde{m}$. For any integer $N \geq 2$, we can see that the optimal contract weight will be

$$\tilde{n}^* = \frac{\tau_\epsilon\,\tau_z\,(1-\lambda^2)\,(\tau_z\,\lambda^2\,{\tau_\epsilon}^2+\tau_z\,{\tau_\epsilon}^2+\tau_\epsilon+1)}{(\tau_\epsilon\,\tau_z\,\lambda^2+1)\,(\tau_\epsilon\,\tau_z+1)\,(N+{\tau_\epsilon}^2\,\tau_z+N\,{\tau_\epsilon}^2\,\tau_z+2\,N\,\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2+(2\,N\,-1)\lambda^2\,{\tau_\epsilon}^2\,\tau_z)} \geq 0 \tag{C3}$$

$$\tilde{m}^* = \frac{\tau_\epsilon\,\tau_z\kappa}{(\tau_\epsilon\,\tau_z\,\lambda^2+1)\,(\tau_\epsilon\,\tau_z+1)\,(N+{\tau_\epsilon}^2\,\tau_z+N\,{\tau_\epsilon}^2\,\tau_z+2\,N\,\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2+(2\,N\,-1)\lambda^2\,{\tau_\epsilon}^2\,\tau_z)} \geq 0 \text{ with}$$

$$\begin{aligned}\kappa &= (1-\lambda^2)\tau_\epsilon+(N-1)\,\lambda^2+{\tau_\epsilon}^2\,\tau_z+N\,\lambda^2\,\tau_\epsilon+N\,\lambda^2\,{\tau_\epsilon}^3\,{\tau_z}^2+N\,\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2+N\,\lambda^4\,{\tau_\epsilon}^3\,{\tau_z}^2\\ &+N\,\lambda^4\,{\tau_\epsilon}^4\,{\tau_z}^2+N\,\lambda^4\,{\tau_\epsilon}^5\,{\tau_z}^3+N\,\lambda^2\,\tau_\epsilon\,\tau_z+2\,N\,\lambda^2\,{\tau_\epsilon}^2\,\tau_z+N\,\lambda^2\,{\tau_\epsilon}^3\,\tau_z+(N-1)\,\lambda^4\,{\tau_\epsilon}^2\,\tau_z+1\end{aligned}$$

□

Proof of Lemma 2.5

Proof. Multiply Equation (C3) by $N-1$ and take the partial derivative of the resulting expression with respect to N. This derivative can be expressed as:

$$\frac{\partial(N-1)\tilde{n}^*}{\partial N} = \frac{\tau_\epsilon\,\tau_z\,(2\,\tau_z\,{\tau_\epsilon}^2+1)\,(1-\lambda^2)\,(\lambda^4\,{\tau_\epsilon}^4\,{\tau_z}^2+\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2+\lambda^2\,{\tau_\epsilon}^3\,\tau_z+2\,\lambda^2\,{\tau_\epsilon}^2\,\tau_z+{\tau_\epsilon}^2\,\tau_z+\tau_\epsilon+1)}{(\tau_\epsilon\,\tau_z\,\lambda^2+1)\,(\tau_\epsilon\,\tau_z+1)\,(N+{\tau_\epsilon}^2\,\tau_z+N\,{\tau_\epsilon}^2\,\tau_z+2\,N\,\lambda^2\,{\tau_\epsilon}^4\,{\tau_z}^2+(2\,N\,-1)\lambda^2\,{\tau_\epsilon}^2\,\tau_z)^2}$$

This expression is non-negative. □

2.9 Appendix D: Endogenous Fraction of Index Investors

To endogenize the fraction of index investors, I will include a cost of being active $c > 0$ to my model. This cost can be thought of as the extra trading, information gathering, or information processing costs of dealing with multiple assets rather than just the index. Each investor must be indifferent between being an indexer and active ex-ante in order to have a positive fraction of both index and active traders ($\lambda \in (0,1)$). Define the ex-ante (T=2 in the timeline) expected utilities of being an active and index investor as $E(U_a)$ and $E(U_{in})$ respectively. To avoid complications with correlated prices, I will stick to using the synthetic assets in this analysis. My proof is similar to Theorem 3 in Grossman and Stiglitz (1980) and can be viewed as a special case of Proposition 3.1 in Admati and Pfleiderer (1987). To avoid clutter, I will use variables without subscripts to indicate the set containing that variable's value for the two synthetic assets. For example, $P \equiv \{P_{IN}, P_{LS}\}$.

Proposition D1 *The expected utilities of active (U_a) and index (U_{in}) traders can be expressed as:*

$$E(U_{in}) = -\sqrt{\frac{Var(x_{IN} - P_{IN}|s_i, P)}{Var(x_{IN} - P_{IN})}} \exp\left(-W_0 - \frac{E(x_{IN} - P_{IN})^2}{2Var(x_{IN} - P_{IN})}\right) \tag{D1}$$

$$E(U_a) = E(U_{in})\sqrt{\frac{Var(x_{LS} - P_{LS}|s_i, P)}{Var(x_{LS} - P_{LS})}} \exp\left(c - \frac{E(x_{LS} - P_{LS})^2}{2Var(x_{LS} - P_{LS})}\right) \tag{D2}$$

Proof. First, let's write down the expected utility of an active agent at the trading stage.

$$E(U_a|s_i, P) = -\exp\left(-w_0 + c - \frac{(E(x_{IN}|s_i, P) - P_{IN})^2}{2Var(x_{IN}|s_i, P)} - \frac{(E(x_{LS}|s_i, P) - P_{LS})^2}{2Var(x_{LS}|s_i, P)}\right)$$

We will now integrate over s_i and use the fact that (x, ϵ_i, z) are all independent to get:

$$E(U_a|P) = -\exp(-w_0+c)E\left(\exp\left(\frac{-(E(x_{IN}|s_i,P)-P_{IN})^2}{2Var(x_{IN}|s_i,P)}\right)|P\right)E\left(\exp\left(\frac{-(E(x_{LS}|s_i,P)-P_{LS})^2}{2Var(x_{LS}|s_i,P)}\right)|P\right) \tag{D3}$$

Now define

$$Y_k \equiv \frac{E(x_k - P_k|s_i, P)}{\sqrt{Var(E(x_k - P_k|s_i, P))}}$$

Note that Y_k is a normal random variable with variance 1. Also define:

$$\xi \equiv \frac{Var(E(x_k - P_k)|s_i, P))}{2E(Var(x_k - P_k|s_i, P))}$$

Then observe that

$$E\left(\exp\left(\frac{-(E(x_k|s_i,P)-P_k)^2}{2Var(x_k|s_i,P)}\right)|P\right) = E(\exp(-\xi Y_k^2)|P) \tag{D4}$$

Because Y_k is normal with variance 1, Y_k^2 is a non-central chi square random variable. Equation (D4) then defines the moment generating function, which equals (e.g., see Grossman and Stiglitz (1980) Equation (A21))

$$E(\exp(-\xi Y_k^2)|P) = \frac{1}{\sqrt{1+2\xi}}\exp\left(\frac{-E(Y_k|P)^2\xi}{1+2\xi}\right) \tag{D5}$$

Following this process for both $k = IN, LS$ and plugging into Equation (D3) gives us:

$$E(U_a|P) = -\sqrt{\frac{Var(x_{IN}-P_{IN}|s_i,P)}{Var(x_{IN}-P_{IN}|P)}}\sqrt{\frac{Var(x_{LS}-P_{LS}|s_i,P)}{Var(x_{LS}-P_{LS}|P)}}\exp(G) \text{ with}$$
$$G = -(W_0 - c) - \frac{E(x_{IN}-P_{IN}|P)^2}{2Var(x_{IN}-P_{IN}|P)} - \frac{E(x_{LS}-P_{LS}|P)^2}{2Var(x_{LS}-P_{LS}|P)} \tag{D6}$$

Now we want to integrate out prices, so repeat this process and the result is Equation (D2) for the ex-ante expected utility of the active agent. The derivation is identical for the indexed agent except that all terms involving the long-short synthetic asset disappear. The proposition then immediately follows. □

Corollary D1 *The ratio of expected utilities is*

$$\Lambda(\lambda) \equiv \frac{E(U_a)}{E(U_{in})} = \sqrt{\frac{Var(x_{LS} - P_{LS}|s, P)}{Var(x_{LS} - P_{LS})}} \exp\left(c - \frac{E(x_{LS} - P_{LS})^2}{2Var(x_{LS} - P_{LS})}\right) \tag{D7}$$

Proof. Follows immediately from dividing Equation (D2) by Equation (D1). □

It will now be helpful to note the following lemma.

Lemma D1 $\Lambda(\lambda)$ *is an increasing function of* λ

Proof. Note that $E(x_{LS} - P_{LS}) = 0$ because it is an equal-weighted long-short portfolio of two ex-ante identical assets. Thus, both its expected payoff and expected price are zero; it does not carry any risk-premium. So the only term in $\Lambda(\lambda)$ involving λ is

$$\sqrt{\frac{Var(x_{LS} - P_{LS}|s, P)}{Var(x_{LS} - P_{LS})}} = \left(\frac{\lambda^2\,\tau_z\,(\tau_z\,\lambda^2\,{\tau_\epsilon}^2 + \tau_\epsilon + 1)}{\lambda^4\,{\tau_\epsilon}^2\,{\tau_z}^2 + 2\,\lambda^2\,\tau_\epsilon\,\tau_z + \lambda^2\,\tau_z + 1}\right)^{\frac{1}{2}} \tag{D8}$$

It is straightforward to confirm that the RHS of Equation (D8) is increasing in λ by taking the partial derivative with respect to λ. □

Lemma D1 states that active traders become relatively worse off as the fraction of active traders increases. This result should be intuitive: relative prices become more efficient as the number of active traders increases, so gathering information becomes less profitable. Also, recall that utilities are negative in this model, so $\Lambda(\lambda) < 1 (> 1)$ means active traders are better (worse) off. Then, similar to Grossman and Stiglitz (1980), I can define different types of financial market equilibria as described in Proposition 2.6 of the main text.

Proof of Proposition 2.6

Proof. The first two parts of the proposition follow from Lemma D1 and the fact that higher levels of $\Lambda()$ indicate that active traders are worse off. More specifically, if $\lambda \in (0,1)$ and $\Lambda(\lambda) = 1$, then we have an interior equilibrium. If $\lambda = 1$ and $\Lambda(1) \leq 1$, then we have a no-indexing equilibrium. To achieve either, one simply has to appropriately vary the exogenous cost of actively trading c; higher values of c make the interior equilibrium more likely. To show that there is no equilibrium where all traders are indexers, observe that if $\lambda = 0$, then $Var(x_{LS} - P_{LS})^{-1} = 0$. As a result, $\Lambda(0) = 0$ so some indexers will find it profitable to deviate and collect information. Correcting relative prices becomes increasingly profitable as λ approaches zero, making it optimal for someone to always be active. □

Thus, the type of equilibrium is completely determined by $\Lambda(1)$. If $\Lambda(1) \leq 1$, we have an equilibrium with only active traders. If $\Lambda(1) > 1$, then we have an interior equilibrium with a positive fraction of both active and index investors. The equilibrium percentage of index investors can then be manipulated by changing the cost of active investing c. Therefore, we can essentially talk about λ as if it is exogenous because changing the exogenous c amounts to changing the equilibrium λ that results.

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