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Tyler Mundt

# Geometric Scaling of Cylindrical Rotating Detonation Rocket Engine Combustors

Tyler Mundt

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Reading Committee:

Carl Knowlen, Chair

Mitsuru Kurosaka

Robert Breidenthal

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Aeronautics and Astronautics

University of Washington

**Abstract**

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Tyler Mundt

Chair of the Supervisory Committee:  
Carl Knowlen, Research Associate Professor  
Aeronautics and Astronautics

A Rotating Detonation Rocket Engine (RDRE) is a propulsion device employing detonation waves moving circumferentially around an annular channel to consume axially-fed propellants. Theoretically, this device provides benefits with respect to combustion pressure loss and thermodynamic efficiency when compared to deflagration-based combustors, although dominant design characteristics are not well identified. The relationships between geometric parameters, performance, and wave dynamics in particular are not thoroughly explored. This dissertation aims to provide empirical relations between combustor geometry, feed conditions, and engine operation, as well as correlation to thermodynamic parameters calculated with chemical equilibrium codes, including detonation cell size.

The experimental effort included over 400 tests across six engine configurations utilizing gaseous methane-oxygen propellant and flat-faced impinging injectors, and focused on two studies with

separate variations in combustor geometry. The first concentrated on radius of curvature effects through changes in the cylindrical combustor's outer diameter, where larger engine scales were linked to higher wave counts during operation as mass flux and equivalence ratio were varied. The second study encompassed the consequences of altering the annular gap width at the smallest diameter scale of the original study and included a coreless configuration as the upper limit. Gap width primarily affected wave stability, denoted by the presence or absence of a counter wave, and detonative operating range. Combustor axial pressures in both studies were found to be dominated by feed conditions rather than geometry.

For further comparisons across both studies, values for detonation cell size and energy flux based on combustion heat release were calculated with chemical equilibrium codes. Numerous relations between these calculated variables, combustor geometry, and experimental inputs and outputs were then explored across all six engine configurations. Mass flux combined with the ratio between the inner and outer radius, the latter of which was related to other geometric ratios, provided boundary estimates for operating modes, and the ratio between the inner radius and detonation cell size presented insight for wave stability. Combustor pressure showed a close relation to energy flux, although some geometric variance was still present. Finally, lessons from both the listed and additional comparisons were applied to experiments outside the original mass flux and equivalence ratio sweeps in an attempt to predict RDRE operation based solely on combustor geometry and fill conditions.



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# 1. INTRODUCTION AND FUNDAMENTALS

## 1.1. ROTATING DETONATION ENGINE

A Rotating Detonation Engine (RDE) utilizes continuously-spinning detonation waves within an annular channel to combust reactants fed into one end of the channel. An example of a standard design and the annular flow field can be seen in Figure 1.1 [1]. A RDE is considered a Pressure Gain Combustor (PGC), which, unlike traditional deflagrative combustor designs, theoretically exhibits a total pressure increase over the combustor. If realized, a PGC would lead to a reduction in the demands, and therefore size and weight, of supporting hardware in propulsion systems, including turbopumps and compressors, and holds potential efficiency advantages [2].

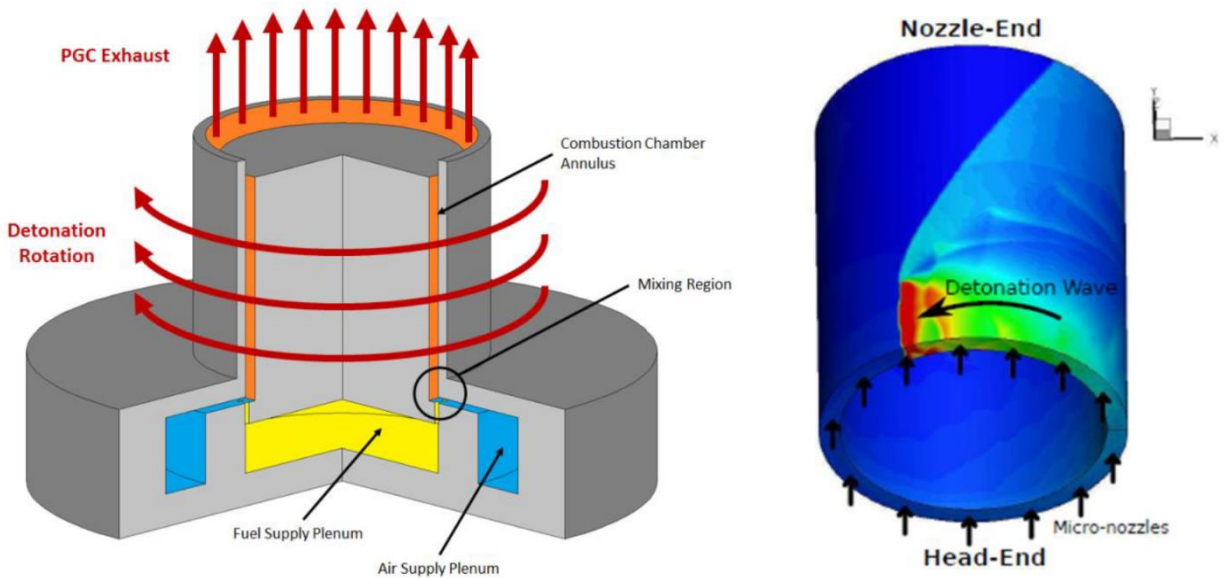


Figure 1.1: Characteristic RDE Design, reproduced from [1].

RDEs have a myriad of potential applications for in-duct burning, including stationary power generation, gas turbine propulsion, and rocket engines. The third of which, commonly referred to as RDREs (Rotating Detonation Rocket Engines), will be the focus application for the included work. Recent RDE advances include stable operation with non-cylindrical geometries

[3], the first successful in-flight demonstration of an RDRE within the environment of space [4], and testing of a 7000 lbf thrust class engine at NASA Marshall Space Flight Center [5]. Even so, the benefits of RDREs expressed by theory have not been realized, and the fundamentals of design and performance are not well identified [6].

In an effort to advance understanding of governing principles of RDRE design, a pair of experimental studies utilizing six RDRE configurations over three diameter scales have been conducted to explore scaling laws associated with cylindrical annulus sizing. Additionally, both studies would provide insight into operational limits and dynamics at small diameter scales, with potential application to satellite propulsion systems. The first study focused on radius of curvature effects by varying the combustor's outer diameter while holding annular gap width and injection characteristics constant. The second study was conducted at the smallest diameter scale of the first study, and concentrated on annular gap width changes over four configurations, including one without a core. As the same injector was used across this study, configuration changes were accomplished by replacing the inner core. Over 400 data points of steady RDRE operation were collected and the resulting performance and wave dynamics will be compared against the variation in geometric parameters and changes in thermodynamic properties, including detonation cell size, along the selected test matrix.

Derivation of one-dimensional, constant-area heating and one-dimensional steady detonations will be presented in the remainder of this section as a theoretical basis, in addition to a literature review of RDE dynamics, existing scaling studies, and related phenomena. Then, Section 2 will discuss the design of the facility and engines used to generate experimental data, as well as a description of the experimental procedure and data processing. Experimental results will be presented in Section 3, including example temporal data, the results from each engine

configuration, and initial data comparisons within each scaling study. Finally, Section 4 will cover additional thermodynamic and geometric analysis, including calculation of pertinent variables, such as detonation cell size.

## 1.2. ONE-DIMENSIONAL THEORY

This subsection details a brief derivation of heat addition and detonation model for the one-dimensional case. For this section, books by Liepmann and Roshko [7] and Lee [8] were used as reference material.

### 1.2.1. HEAT ADDITION IN A CONSTANT-AREA DUCT

Consider a one-dimensional constant-area duct without body or viscous forces as shown in Figure 1.2. Then, impose a control volume within which heat addition occurs between upstream station 1 and downstream station 2.

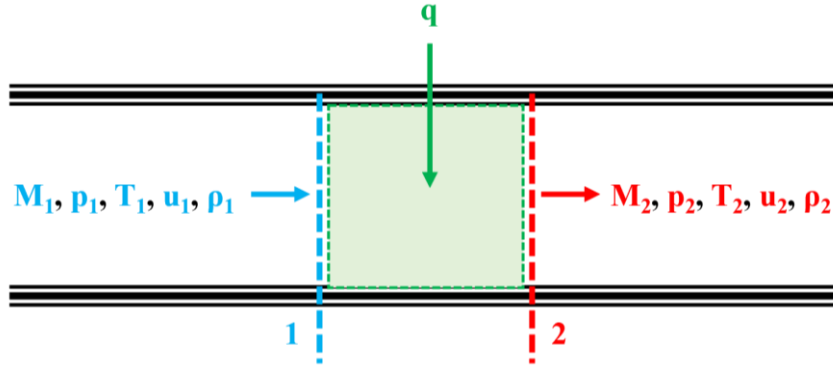


Figure 1.2: Control volume for one-dimensional constant-area heat addition.

For simplicity, the assumption that the fluid is perfect, and thereby has constant specific heat capacities, will be applied. Furthermore, it will be assumed the composition of the fluid does not change across the heat addition. The continuity, momentum, and energy equations as applied to this control volume can be written as:

$$\rho_1 u_1 = \rho_2 u_2 \tag{1.1}$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \quad (1.2)$$

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2} \quad (1.3)$$

Where  $\rho$  is density,  $u$  is velocity,  $p$  is pressure, and  $h$ , enthalpy, is equivalent to  $c_p T$  where  $c_p$  is the specific heat capacity at constant pressure and  $T$  is temperature. Next, the equation of state for a perfect gas and the definition of Mach number are included:

$$p = \rho R_{gas} T \quad (1.4)$$

$$M = \frac{u}{\sqrt{\gamma R_{gas} T}} \quad (1.5)$$

Where  $R_{gas}$  is the specific gas constant,  $M$  is the Mach number, and  $\gamma$  is the heat capacity ratio. After a series of substitutions and rearrangements amongst these five equations, a relation between heat release ( $q$ ), gas properties, and the upstream and downstream Mach numbers is reached:

$$\frac{q}{c_p T_1} = \left(\frac{M_2}{M_1}\right)^2 \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right)^2 \left(1 + \frac{\gamma - 1}{2} M_2^2\right) - \left(1 + \frac{\gamma - 1}{2} M_1^2\right) \quad (1.6)$$

Applying known upstream conditions and heat release, the resultant  $M_2$  can be determined. A plot of  $M_2$  based on the lefthand side of Equation 1.6, which is also known as the normalized heat release (represented as  $Q$ ), and a  $M_1$  at  $\gamma = 1.4$  can be seen in Figure 1.3.



c

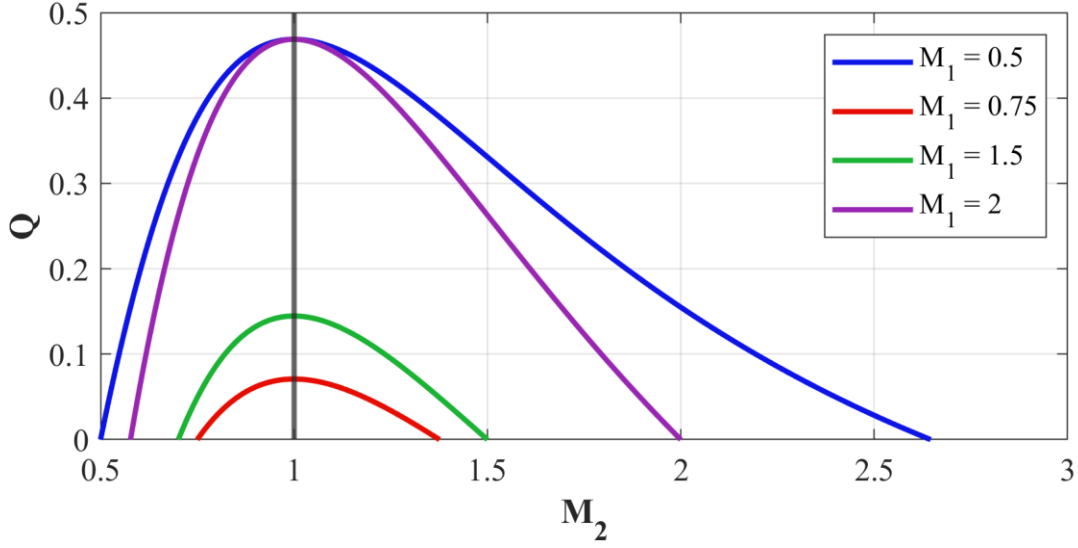


Figure 1.3: Downstream Mach number trend based on upstream Mach number and heat release in one-dimensional, constant-area duct heating.

In the case of zero heat release, a discontinuous solution appears that represents an adiabatic shock. In the contours shown, the maximum heat release for a given  $M_1$  corresponds to a downstream state where  $M_2$  is sonic, also known as thermal choking. In the case of either subsonic or supersonic upstream flow, heat addition moves the downstream Mach closer to this condition. According to a modified steady state analysis by Foa and Rudinger [9], if additional heat is added after reaching a thermally choked state downstream, the upstream conditions will be affected and adjust the incoming Mach number farther away from unity and onto a contour with a higher capacity for heat addition.

The incoming Mach number  $M_1$  associated with thermal choking is called the Chapman-Jouguet Mach number, or  $M_{CJ}$ . A solution can be found for  $M_{CJ}$  by applying the sonic  $M_2$  condition to Equation 1.6, which produces:

$$Q \equiv \frac{q}{c_p T_1} = \frac{(M_{CJ}^2 - 1)^2}{2(\gamma + 1)M_{CJ}^2} \quad (1.7)$$

This result has two solutions for a given heat addition. The first,  $M_{CJ}^-$ , represents the subsonic solution, while  $M_{CJ}^+$  represents the supersonic solution. A plot of these solutions with respect to normalized heat addition is shown in Figure 1.4.

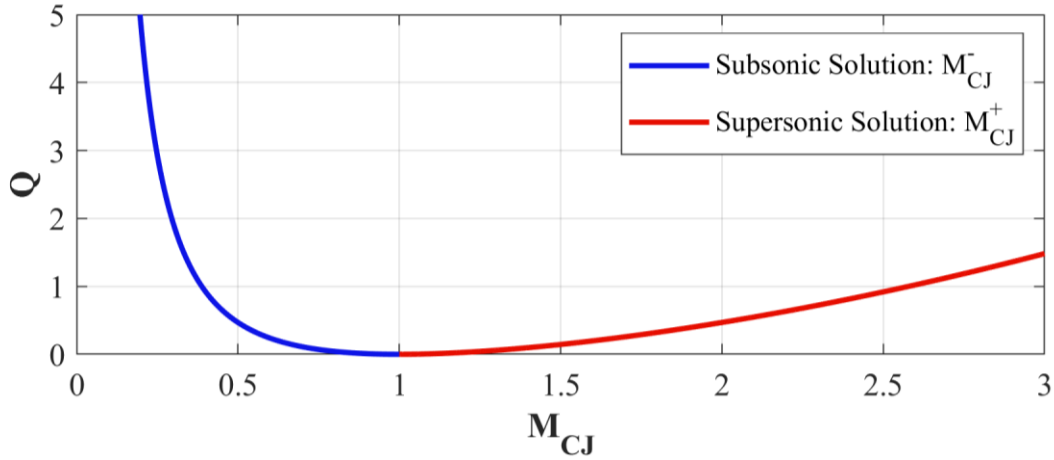


Figure 1.4: Subsonic and supersonic solutions for  $M_{CJ}$  with variation in heat release.

A standard form of entropy change for a perfect gas is given by:

$$\frac{s_2 - s_1}{R_{gas}} = \frac{\gamma}{\gamma - 1} \ln \left( \frac{T_2}{T_1} \right) - \ln \left( \frac{p_2}{p_1} \right) \quad (1.8)$$

Where  $s$  is entropy. The temperature and pressure ratios within this equation were found intermediately in the derivation of Equation 1.6 in terms of Mach number, and can be written as:

$$\frac{T_2}{T_1} = \left( \frac{M_2}{M_1} \right)^2 \left( \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \right)^2 \quad (1.9)$$

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \quad (1.10)$$

Combining Equations 1.8 through 1.10 allows entropy generation to be described by the heat capacity ratio, upstream Mach number, and downstream Mach number. Though application of Equation 1.6 itself, one of these Mach numbers can be replaced by normalized heat addition, as was done for Figure 1.5. Here, inputs of  $Q$  and supersonic  $M_1$  were used to calculate the entropy rise for a  $\gamma$  of 1.4. The contours represent collections of solutions calculated with the same value of normalized heat addition, and do not represent the paths of the heat addition process. For each

value of  $Q$ , the solution representing thermal choking (where the intermediately calculated  $M_2$  from Equation 1.6 was unity, and thereby  $M_1 = M_{CJ}^+$ ) is indicated by a circle and always had the lowest entropy generation for the specified normalized heat addition. It is important to note that while the thermally choked solutions had the lowest entropy generation when compared to other supersonic initial Mach numbers, within the heat addition process the thermal choke point, where  $M_2 = 1$ , represents a maximum in entropy generation for the process itself.

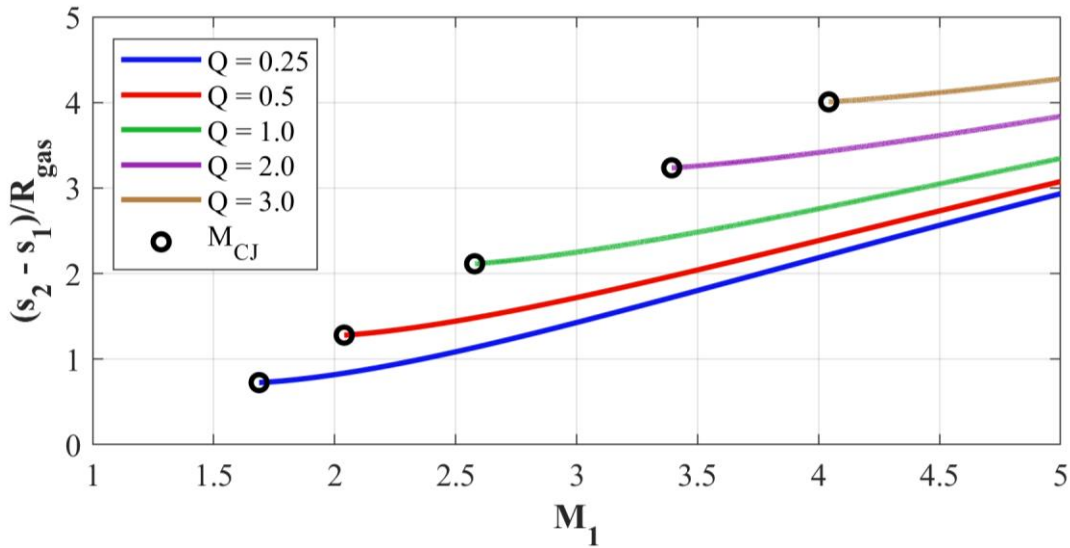


Figure 1.5: Solutions for entropy generation from heating an initially supersonic flow in a constant-area duct.

### 1.2.2. ZND DETONATION MODEL

The analysis of the previous subsection does not consider the structure between station 1 and 2. Continuing with a focus on a supersonic initial condition, a shock traveling at  $M_{CJ}$  is placed at station 1 and the thermally choked state is imposed on station 2. The thickness of a shock is on the order of the mean free path, which is the average distance between collisions of molecules. As such, the shock would be insignificant to the chemical reaction, which occurs over significantly longer length scales, and thus could be treated as adiabatic. Therefore, the temperature and pressure rise generated by the shock would initiate the chemical reaction at some distance downstream of

the shock. This chemical reaction would occur under subsonic post-shock conditions, then accelerate to the thermally choked state, which represents the end of the chemical reaction. The pressure difference between the products and reactants would then drive the structure forward into fresh reactants. This is called the ZND detonation model, and is named after Zel'dovich, von Neumann, and Döring who each proposed the model independently within a few years of each other. A sketch of this model in the shock reference frame can be seen in Figure 1.6. The von Neumann (VN) state is that immediately after the adiabatic shock, and the thermal choke point is also commonly reference as the Chapman-Jouguet (CJ) state.

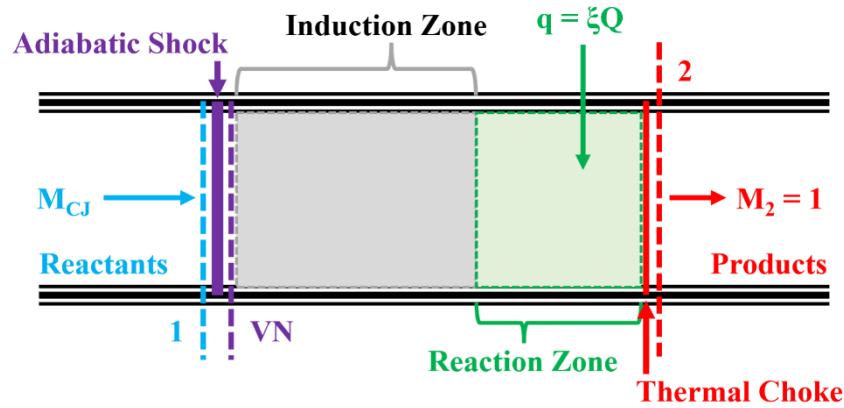


Figure 1.6: Basic structure of a ZND detonation model from shock reference frame.

Derivation of the conditions within the structure closely follows that for heating already presented. One key replacement is made to the energy equation by  $q = \xi Q$ , where  $\xi$  is between 0 and 1 and represents the progress of the chemical reaction. Computation is then performed piecewise, starting with calculation of the von Neumann state by the adiabatic shock relations. This result is then fed into the derived heating relations. Example trends in Mach number, pressure, and temperature with respect to the reaction progress variable  $\xi$  for a selection of detonation wave Mach numbers are shown in Figure 1.7. Here, initial conditions of 1 atm and 300 K were used.

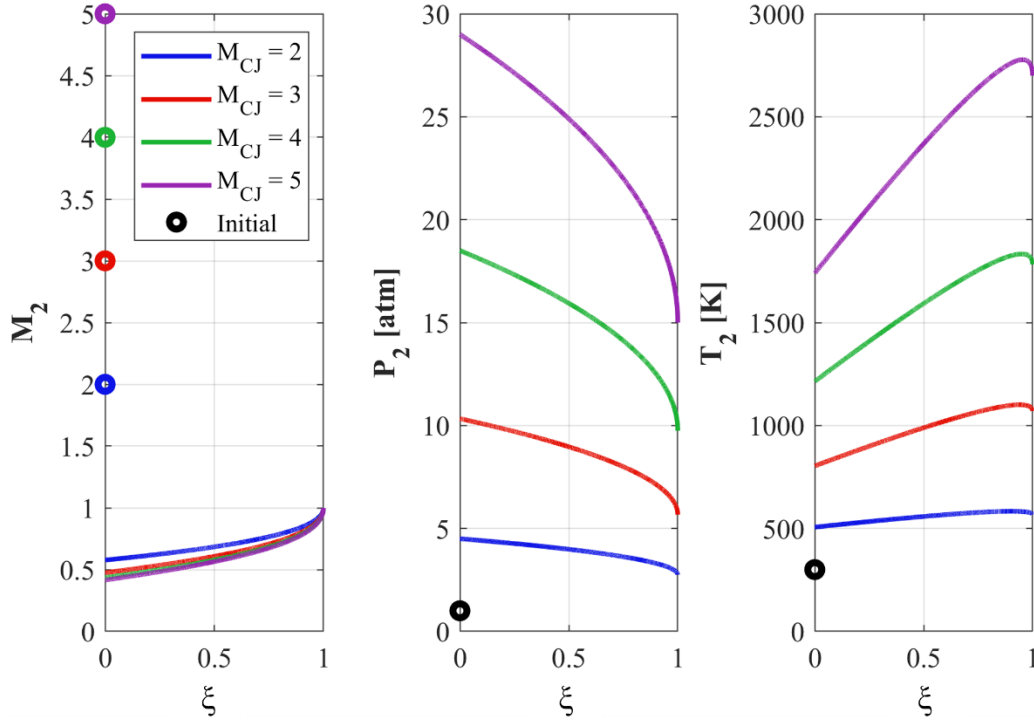


Figure 1.7: Trends in Mach number (left), pressure (middle), and temperature (right) with advancement of combustion progress variable for a ZND detonation model.

When time and distance scales are of interest, derivation can instead be accomplished using the differential forms of the conservation equations. In this case, a reaction rate law is required to describe the chemical progress variable, with the simplest form represented by a single-step Arrhenius equation:

$$\frac{d\xi}{dt} = k(1 - \xi)e^{-\frac{E_a}{R_{gas}T}} \quad (1.11)$$

The pre-exponential factor,  $k$ , and activation energy,  $E_a$ , are constants. With this reaction rate law, activation energy has the largest effect on the ZND structure itself.

Entropy generation for a ZND detonation can be computed from application of Equations 1.8 through 1.10 to the post-shock von Neumann state (where  $M_1$  is replaced by  $M_{VN}$  and  $M_2 = 1$ ) since the shock is treated as adiabatic. The ZND model utilizes a thermal choke as the end state of the post-shock heat addition, and its resulting entropy generation is lower than overdriven one-dimensional detonations (where  $M_1 > M_{CJ}$  and the end state does not result in a thermal choke)

with the same value of nondimensional heat addition, which echoes the results of Figure 1.5 for supersonic heat addition. As RDREs are often compared to traditional rocket combustors, which utilize subsonic heat addition, evaluation of the entropy generation of a ZND detonation versus thermally choked deflagration is also a pertinent comparison. This result is displayed in Figure 1.8 and shows the theoretical entropy generation of a ZND detonation is lower than thermally choked deflagration for every value of nondimensional heat release.

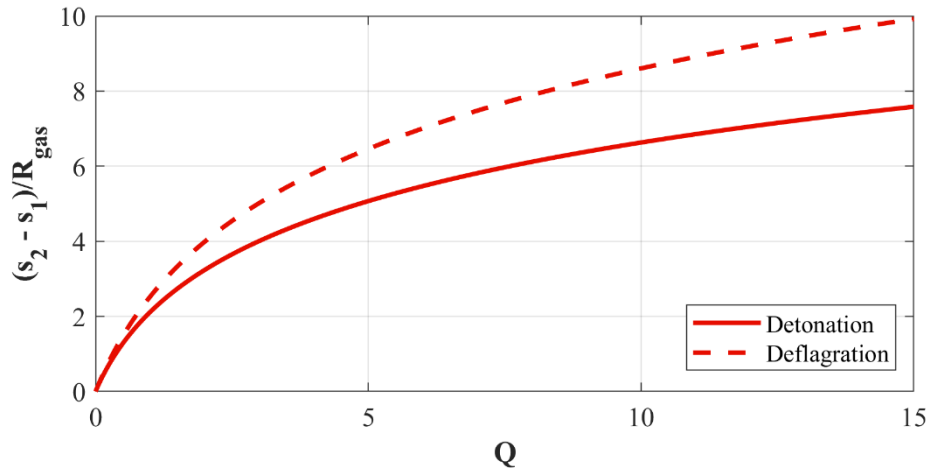


Figure 1.8: Entropy rise comparison between one-dimensional thermally choked deflagration and detonation.

### 1.3. PHYSICAL DETONATIONS

While a ZND detonation is a useful model, detonation waves are neither one-dimensional nor planar. The wave front constantly changes with time, leaving a cellular structure behind in soot foil testing, as can be seen in Figure 1.9 [10]. These cells were produced by the interaction of transverse waves with the shock front. Even so, Chapman-Jouguet detonation wave speeds ( $D_{CJ}$ , related to  $M_{CJ}$  by Equation 1.5) from the ZND model agree reasonably well with the mean velocity seen in experiments, and this value is still used in initial calculations. Empirical data on cell size exists for various mixtures, with the results of Schumaker et al. [11] using methane and oxygen at elevated pressures relevant to the engines discussed in the experimental effort of this paper.

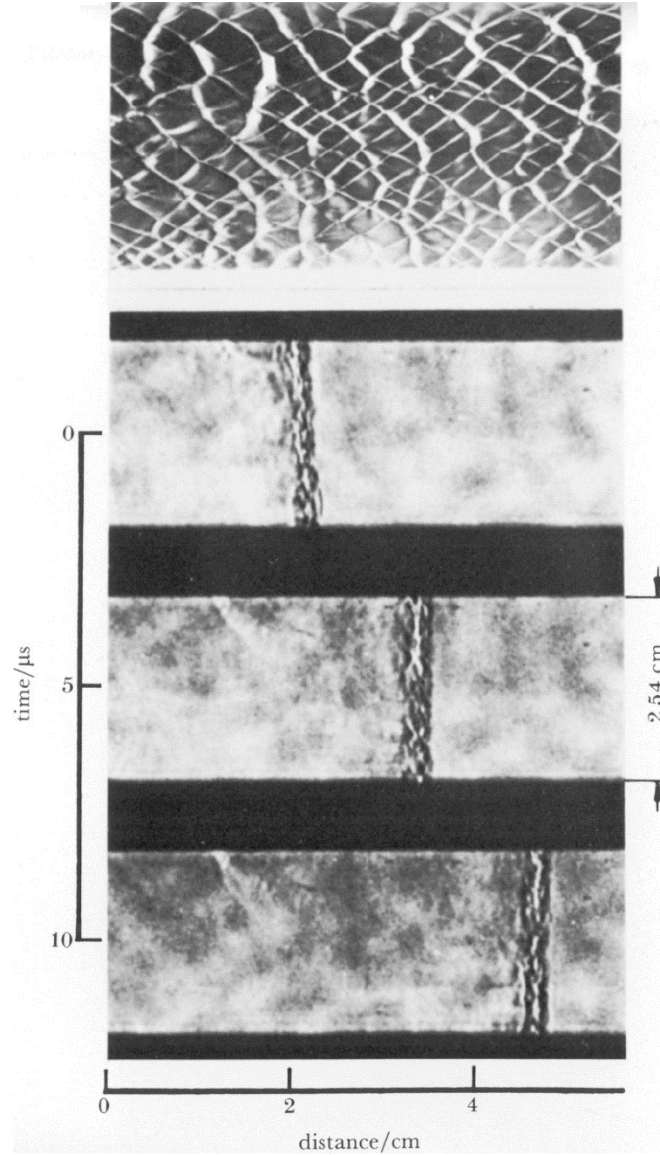


Figure 1.9: Schlieren record of self-sustained detonation wave in rectangular channel with resultant soot print above. Reproduced from [10].

When detonations propagate in a curved duct, the wave speed is seen to decrease, such as in the U-bend tube tests with propane-air of Frolov et al. [12]. A temporary 15 – 20% reduction in wave speed was seen through the curved sections, with complete decay of the detonation wave seen in some runs at the smallest radius of curvature tested.

Within curved channel sections, a cellular structure is still witnessed, except with a radial cell size variation [13, 14]. Pan et al. [13] show this distribution experimentally and numerically for rectangular curved channels, with the decreased cell size seen along the outer wall and larger cell size seen along the inner wall attributed to the effects of geometric compression and expansion, respectively. Similar variation is seen by Nakayama et al. [14], with the ratio between the inner radius and the normal detonation cell width highlighted as an indicator for stable detonation propagation through the curved channel. In these two-dimensional experiments, raising either fill pressure or inner radius while holding the other constant led to better stability of the detonation wave.

Detonation cell size has often been linked to minimum dimensions and operating range limitations in RDEs, including Bykovskii et al. [15]. Like much of the previously discussed work with curved channels, the experimental work of Hencel et al. [16] with detonations in planar curved channels was motivated by its application to RDEs. Tests with gaseous hydrogen-air mixtures across changes in radius of curvature, channel width, and detonation cell size were conducted, with detonation cell size manipulated through equivalence ratio. For stable detonations, a higher ratio between inner and outer radii was preferred and the ratio of channel width to detonation cell width was highlighted. Experimental work in an ethylene-oxygen RDE by Kawasaki et al. [17] showed differences in wave attachment as the inner radius was varied, with the wave eventually detaching from the inner core as the inner radius was reduced. Comparison of the ratio between the calculated unburned mixture layer height to detonation cell size with the ratio of measured to ideal specific impulse led to determination of a critical inner radius of their test series for efficient thrust generation.



#### 1.4. PRESSURE GAIN COMBUSTION

An example of the detonation wave structure in an “unrolled” state is shown in Figure 1.10 with injection occurring along the bottom surface. Following the labeling convention of Kailasanath [18], from which this figure is reproduced, the detonation wave (A) travels to the right consuming the fresh reactants (G) provided by choked injectors. The pressure behind the detonation wave is sufficient to block the injectors for a region following the detonation wave (F). An oblique shock wave (B) and a slip line (C) that separates new and old products follow the detonation wave. A secondary shock (D) also trails the primary structure, but varies with injection parameters. Burning not associated with detonation can occur where products and fresh reactants meet (E), leading to a performance loss.

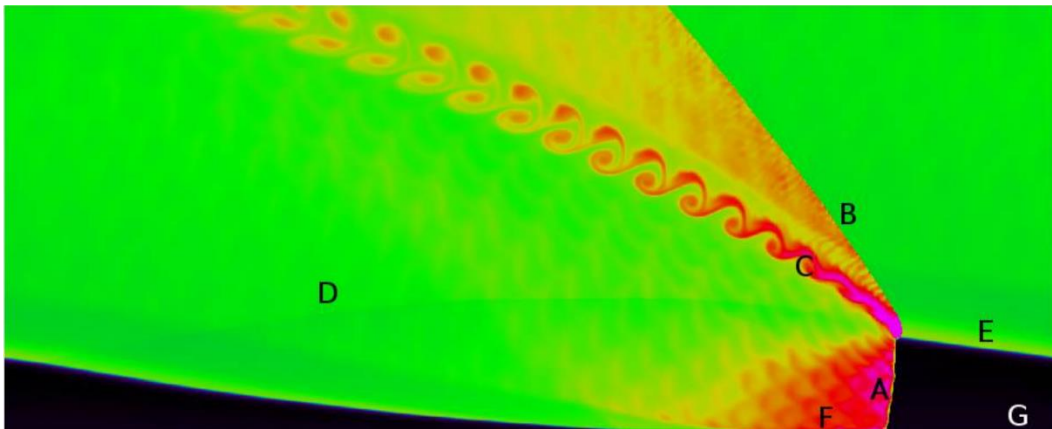


Figure 1.10: Example of an "unrolled" RDE wave structure. The labels represent the (A) detonation wave, (B) oblique shock, (C) slip line, (D) secondary shock, (E) mixing region, (F) region of blocked injectors, and (G) fresh reactants. Reproduced from [18].

Thermodynamic advantages are regularly mentioned as a motivation for pressure gain combustion, with a cycle analysis by Heiser and Pratt [19] on Pulse Detonation Engines (PDEs) regularly referenced. Like RDEs, pulse detonation engines use detonation waves to combust reactants, but are operated intermittently as the detonation travels axially and requires the combustion chamber to be purged and refilled in a synchronized fashion. As detonation waves

travel quickly, heat addition from combustion is close to that of a constant-volume process and is therefore commonly approximated by the Humphrey cycle. Heiser and Pratt instead propose a modification that replaces the heat addition process with a ZND detonation wave. A comparison of the ideal Humphrey, PDE, and Brayton cycles in the  $T$ - $s$  plane is shown in Figure 1.11. For the PDE cycle, the process from 3 to 3a was the adiabatic shock, and 3a to 4 was the heat addition and subsequent expansion described by the ZND model. The PDE cycle showed the most promise, especially against the Brayton cycle. When irreversibilities were added to model real thermodynamic cycles, the difference rapidly closed as the cycle static temperature ratio ( $T_3/T_0$ ) rose and the PDE cycle was eventually overtaken. Furthermore, it was observed that the real Humphrey cycle tended to overpredict thermal efficiency when used to estimate the real PDE cycle.

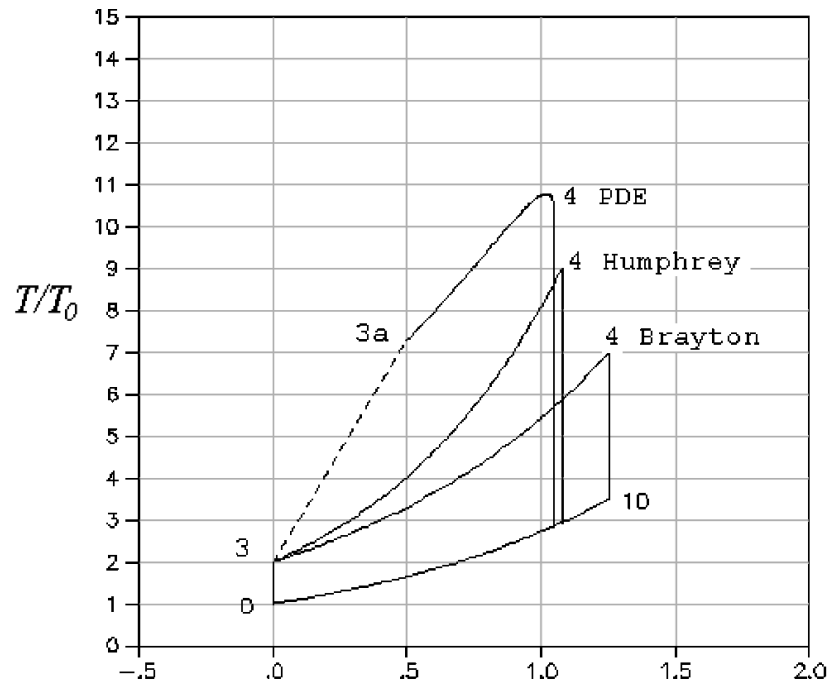


Figure 1.11:  $T$ - $s$  diagram comparing ideal thermodynamic cycles. Reproduced from [19].

For estimating total pressure gain teased by PGCs, a method referred to as Equivalent Available Pressure (EAP) was proposed by Kaemming and Paxson [20]. It used averaged

properties of the unsteady phenomena common in PGCs to imitate a steady flow, and was intended to represent the ability of the combustor's exit flow to provide work. From this, the result would be comparable to that of a standard combustor at matching mass flow and heat release. For usage with experimental data, derivation was done from the thrust equation and utilized static pressure and thrust measurements with a sonic exit flow assumption. Feleo et al. [21] discussed potential issues with the choked exit assumption, especially at low mass flux, based on uncertainty analysis of the measured values. Application of EAP and an alternative calculation called the Channel CTAP method was conducted by Fievisohn et al. [22] and comes to the same conclusion. A correction for low flow rates was then proposed that brings the two methods into agreement.

## 1.5. RDE SCALING AND DESIGN IN LITERATURE

The following subsections highlight the results of both experimental and numerical efforts to determine the influence of design characteristics on RDE operation, including injection design, combustor geometry, and exit conditions.

### 1.5.1. INJECTOR AND PLENUM CONSIDERATIONS

Propellant injection plays a strong role in RDE performance, as illustrated by Wyatt et al. [23] with experiments in an ethylene-nitrous oxide engine. Over different injection designs on the same combustor parameters, differences were seen in both wave speed and detonable mass flow rate range. The results were attributed to mixing quality, and a similar conclusion is drawn in numerical results from Yoshidomi et al. [24] where higher injector counts increased wave speed and pressure gain. The effects of mixing on performance were directly investigated by Bigler et al. [25] by comparing impinging and misaligned injectors in the same combustor. The misaligned injector with inferior mixing had lower performance and produced fewer waves that traveled at a lower speed, but exhibited no changes in detonative operating limits with respect to equivalence ratio.

Interaction of a detonation wave over a single RDE injector at different levels of mixing has been explored in detonation channel tests by Redhal et al. [26]. Mixing was found to affect the structure of the wave as it passed, and connections were discussed to counter-propagating waves and the detonation wave standoff distance seen during nominal RDE operation.

As discussed for region (F) in Figure 1.10, the pressure behind the detonation wave is sufficient to block injectors for a time after the wave passes. An investigation of characteristic timescales in RDREs by Bennewitz et al. [27] demonstrated a coupling between detonation performance and the recovery time of the injectors. Interactions between the plenums and detonation wave have also been seen experimentally, such as by Feleo et al. [28], even with nominally choked injectors. Paxson and Miki [29] explored the influence of backflow through the injectors numerically, and found injectors with low diodicity, which allowed more backflow, exhibited lower performance. A further exploration of this result has been performed numerically by Keller et al. [30] on an additively manufactured, high-diodicity impinging injector design, with production of a cold flow test article underway.

A numerical investigation by Schwer and Kailasanath [31] showed increased performance as the ratio between total injection area and annulus area was increased. This simulation did not include the effects of backflow through the injectors. The effect of the injector area ratio on RDE operation was later studied experimentally by the University of Washington using radial injectors in a 76 mm diameter methane-oxygen engine [32]. Reported performance metrics included specific impulse based on thrusting pressure, taken from the upstream wall of the combustor, and the pressure ratio across the injectors. As the injector area ratio was increased, the pressure ratio across the injectors also rose, leading to an increase in injector-plenum coupling and reduction in specific

impulse. In terms of wave dynamics, variations were influenced by the injector area ratio and equivalence ratio, but were relatively insensitive to mass flux changes.

### 1.5.2. COMBUSTOR GEOMETRY

As mentioned previously, stable detonative operation of an RDE has been achieved with non-cylindrical annuli, including the tri-arc channel demonstrated by Lee et al. [3]. Another design is that of a disk, or radial, RDE. In this configuration, the detonation still moves circumferentially, but reactants enter radially inwards from the outer radius and products exit the engine axially through the center. Thus, the detonation wave extends downstream radially, rather than axially as described for a cylindrical RDE. Successful experiments ranging from ethylene-air with an attached turbocharger turbine to assess power production [33] to methane-oxygen with an aerospike for a rocket application have been conducted [34]. Several numerical studies also exist, such as that by Kumazawa et al. [35] exploring effects of combustor dimensions on wave propagation.

Even amongst cylindrical RDE designs, alterations from the standard annulus are common. One configuration of particular interest is that of a coreless, also known as hollow, RDE where detonative operation occurs without the enforcement of a narrow annulus. Experiments in ethylene-oxygen engines by Yokoo et al. [36] have shown comparable specific impulse performance to RDEs with an annulus at a 20 mm diameter scale. Stable detonative operation has also been explored in hydrogen-air engines by Anand et al. [37], where improvements to wave speed were seen compared to annular RDEs. They also theorized operational comparisons to tangential instabilities that occur in rocket combustors. Effects of diameter changes in coreless methane-air engines have been explored by Liu et al. [38]. Here, wave speeds very close to the CJ

speed were seen and the reduction in diameter was related to the deterioration of both the wave structure and operating range.

The effect of radius of curvature on the operation of an RDE with an annulus has also been investigated. A numerical study in two-dimensional channels by Kim and Choi [39] showed higher wave stability at increased radius of curvature. Experiments by Ishii et al. [40] demonstrated a methane-oxygen engine with a larger outer diameter, but the same channel width and mass flux, operated at a higher wave number. Furthermore, the mass flux at which wave number transitions occurred was different across the two outer diameter scales. The smaller diameter combustor also exhibited a larger increase in wave speed with mass flux across the same wave number than the larger engine.

Zhdan et al. [41] discuss the effect of combustor length on an RDE and find it has little effect on operation, as long as the combustor is greater than a minimum required for the transverse detonation wave to be self-sustaining. Testing with a methane-oxygen RDRE with convergent nozzles by Bennewitz et al. [42] at two different lengths showed no change in operability, but small increases in thrust and wave speeds were seen in the shorter configuration. In addition, the shorter configuration saw a reduction in counter-propagating activity. These results were confirmed in a consequent numerical study by Ross et al. [43], where wave dynamics varied with chamber length in constricted engines.

The impacts of alterations to the annular gap width have been explored experimentally by Bilger et al. [44] with a 76.2 mm diameter methane-oxygen RDRE. At constant mass flow rate, the smallest gap width tested showed the highest performance, but had the lowest operating range with respect to equivalence ratio. When instead scaled by mass flux, the largest gap width emerged with the highest chamber pressure and comparable thrust to the baseline configuration, while the

smallest gap width showed universally lower performance. Loss mechanisms for the smallest width, including heat loss to the increased combustor surface area and higher plenum pressures demonstrating an increase in injector-plenum coupling, were discussed. Experiments by Brophy et al. [45] corroborate these results. Here, variation of annular gap width in a hydrogen-air RDE at constant mass flux revealed the largest width to have the highest pressure gain. Heat transfer losses were again discussed, with the combustor volume-to-surface area ratio per unit length highlighted as an important value.

### 1.5.3. BACK PRESSURE EFFECTS

In an RDE tested by Shepard et al. [46], the combustor experienced pressurization during startup due to the facility's setup. The resulting ignition transients demonstrated wave dynamics and modes of operation that differed from the steady-state operation. At high mass flow rates, detonation was seen immediately, but some tests at low flow rates showed a pulsing mode before establishing the expected transverse waves. Overall, the ignition transient did not affect the results of steady-state operation. In a pair of tests presented by Yokoo et al. [47] with a 20 mm diameter ethylene-air RDE, the run with higher back pressure exhibited lower wave speed for an identical combustion chamber pressure. These observations coincide with previous numerical work by Paxson and Schwer [48] on RDEs with an exit throat, where reducing the exit throat area (and thus increasing back pressure to the combustor) led to increased performance within a specified range. Further reduction led to detonation wave instability and eventual failure. Later simulation work by Nordeen and Schwer [49] connected this to resonance effects that reduced the allowed detonation height.

Direct testing of back pressure effects on an RDE with a constant area annulus was performed at the University of Washington by Koch et al. [50] on a 154 mm diameter methane-

oxygen engine. At the same mass flux and equivalence ratio, variation of the back pressure led to two distinct modes of operation. In the first, the flow was choked near the exit of the annulus and both wave frequencies and combustor pressures were insensitive to changes in back pressure. As back pressure was raised, the annulus eventually unchoked and operation was coupled with back pressure. Within this sensitive regime of operation, there was a deterioration of wave frequencies and a pressure rise from mass accumulation in the downstream dump tank. This pressure rise was reflected in combustor pressure time history, and is shown in Figure 1.12. As the engine used radial injectors, pressure from the thrusting plane was collected through the front endwall and used to normalize combustor pressures. A critical pressure ratio between the back pressure and this front endwall pressure was established to separate the two modes of back pressure sensitivity.

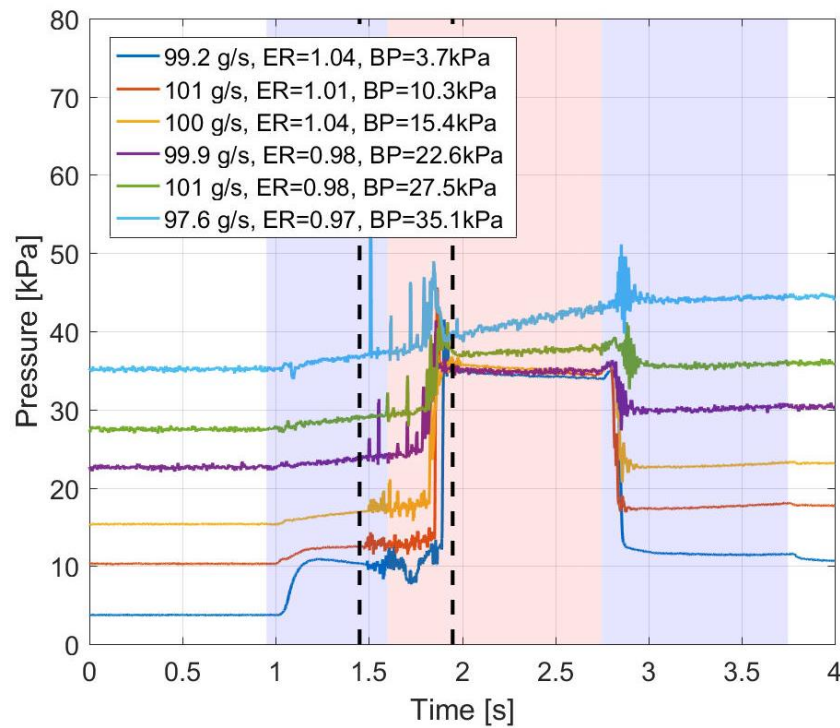


Figure 1.12: Pressure history from a single combustor axial station. Back pressure sensitivity is demonstrated by the 28 and 35 kPa tests. Reproduced from [50].



#### 1.5.4. ADDITIONAL TOPICS

In some of the work already discussed, the presence of counter-propagating waves was reported during steady-state operation. In a study by Batista et al. [51], the effects of wave collisions on a methane-oxygen RDRE were explored. During collisions, a pressure and temperature spike were seen that boosted engine performance, but also increased injector recovery time. During wave recovery, engine performance was reduced. As wave recovery occurred over significantly longer time scales than collisions, engine performance was dominated by non-collision sections and suffered an overall performance loss compared to a strictly co-rotating wave mode. An overall performance increase was theorized should the sum of collision times be greater than that of their corresponding recovery due to a large enough wave count, but would be closer to a standing wave mode for this to occur in a stable fashion.

The 76 mm diameter engine configuration discussed in the rest of this paper closely follows an RDRE tested by the Air Force Research Laboratory (AFRL). Both engines have been tested as part of the experimental effort for a Computational Fluid Dynamics (CFD) validation workshop across several universities. Example CFD results compared to the generated data can be seen in the work by Strakey and Ferguson [52], as well as Pal et al. [53], with results of the experimental effort to be published in the near future. The test matrix used in this paper for the 76 mm engine was driven by this venture, and then scaled down for use in the other engine configurations to be discussed. In addition, a subset of the presented data for the smaller engines scales was published in Shock Waves [54].

## 2. EXPERIMENT DESIGN

Two geometric scaling studies were planned to determine scaling laws for RDREs using empirical data and explore the operability and characteristics of engines at diameter scales of 76.2 mm and below. Design of engine configurations focused on variations in a single combustor dimension for each study, and work to maintain injector mixing characteristics. Following a description of the supporting facility and test stand, this section discusses the resultant engine designs in detail. Afterwards, the operating procedure and data processing methods used to generate experimental data will be reviewed.

### 2.1. FACILITY DESIGN

The University of Washington Rotating Detonation Engine Lab has successfully conducted testing with cylindrical RDEs ranging in scale from 10 mm to 154 mm outer diameter using both gaseous methane-oxygen and hydrogen-oxygen mixtures. The indoor facility captures combustion effluent during experiments and can set sub-atmospheric starting conditions within the combustor. While the test stand does not provide thrust measurements, the rear of the mounted RDE is optically accessible by a high-speed camera. The facility is also equipped with both low- and high-speed data acquisition systems and its gas delivery system can reliably provide flow rates between 8 and 525 g/s of stoichiometric methane-oxygen to the mounted engine. A labeled image of the test stand with the 154 mm engine mounted can be seen in Figure 2.1.

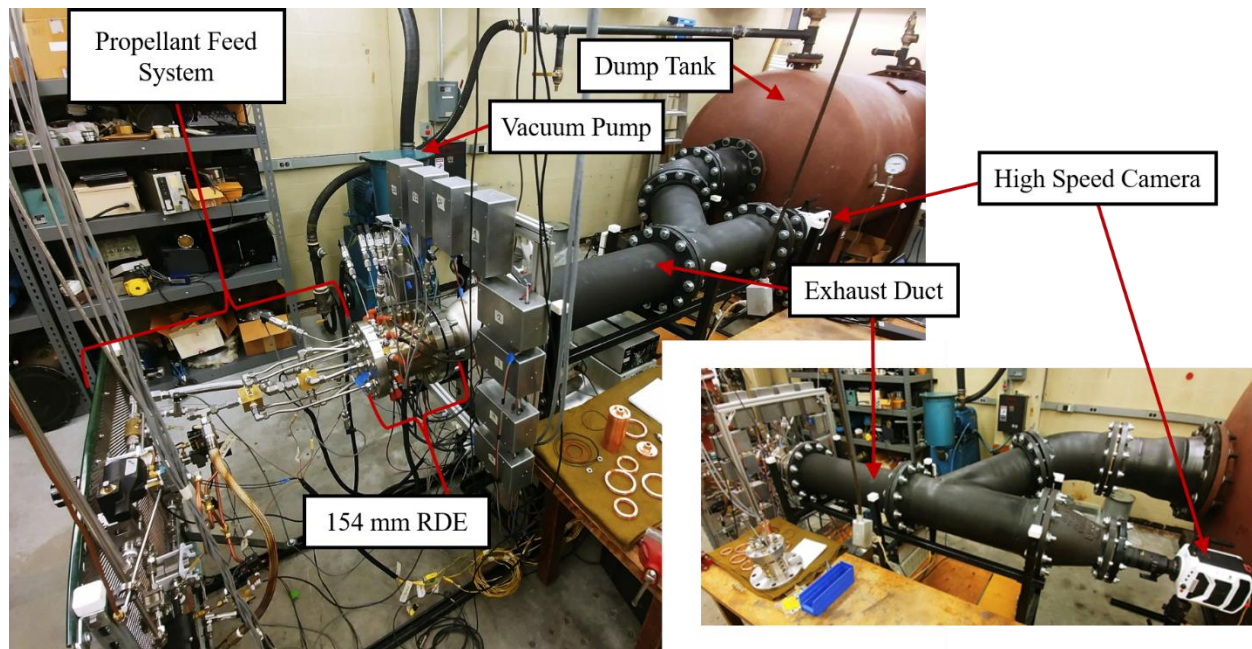


Figure 2.1: University of Washington Rotating Detonation Engine Lab test stand with 154 mm engine mounted.

For all experimental data presented in this paper, a mixture of gaseous methane and gaseous oxygen was used. The methane was Chemically Pure (CP) grade with 99.0% purity. Gaseous nitrogen was used to purge supply lines of fuel and oxidizer after each run, as well as provide a pre-purge in larger engine configurations. All gases were delivered to the system by a suite of between two to six T-style cylinders purchased from Linde Gas & Equipment (formerly Praxair, Inc.). Control of the regulators and ball valves was accomplished with 760 kPa air supplied by the building, which was dual-purposed to convectively cool the combustor annulus in-between runs.

RDEs were mounted to a 0.25 m inner diameter exhaust duct where combustion products were allowed to freely expand. During operation, the exhaust was captured by a 4 m<sup>3</sup> dump tank equipped with 500 kg of aluminum plates to condense combustion products. After each experiment, the dump tank contents were purged by a Kinney KT-300 vacuum pump, which was also used to set sub-atmospheric back pressure as low as 2 kPa prior to engine operation. The exhaust duct also featured a wye section near the dump tank with a straight branch aligned with

the RDE's axis to allow the capture of combustor wave dynamics through the exit of the annulus by a high-speed camera. A Phantom v12.11 camera recorded these dynamics at a frame rate of 240,000 frames per second through a 12.7 mm thick polycarbonate window. The window was placed 2.5 m from the mounting plane of the RDEs, and the camera was outfitted with a Nikon 70 to 200 mm zoom lens to maximize the resolution of captured video.

Static pressure and temperature data from both the engines and gas delivery system were sampled at 2 kHz by a National Instruments 64-channel data acquisition system (DAQ), with thermocouple signals amplified prior to the DAQ. Piezoelectric dynamic pressure sensors, when used, were sampled by a secondary National Instruments DAQ at a rate of 1.25 MHz. As this high-speed DAQ was shared amongst research groups, not all runs contained dynamic pressure data.

A piping and instrumentation diagram for the gas delivery system can be found in Figure 2.2. Delivery of the fuel and oxidizer were handled independently, with reactants first meeting within the combustor. Each reactant supply line is complemented by independent nitrogen supplies for purging. To accommodate the intended flow rate and equivalence ratio ranges of methane-oxygen reactants over various engine scales, the fuel delivery system is primarily composed of 12.7 mm outer diameter stainless steel tubing, and the oxidizer delivery system is dominated by 19.1 mm stainless steel tubing with Monel plumbing used near actuated components, including the pneumatically-actuated ball valves and regulator.

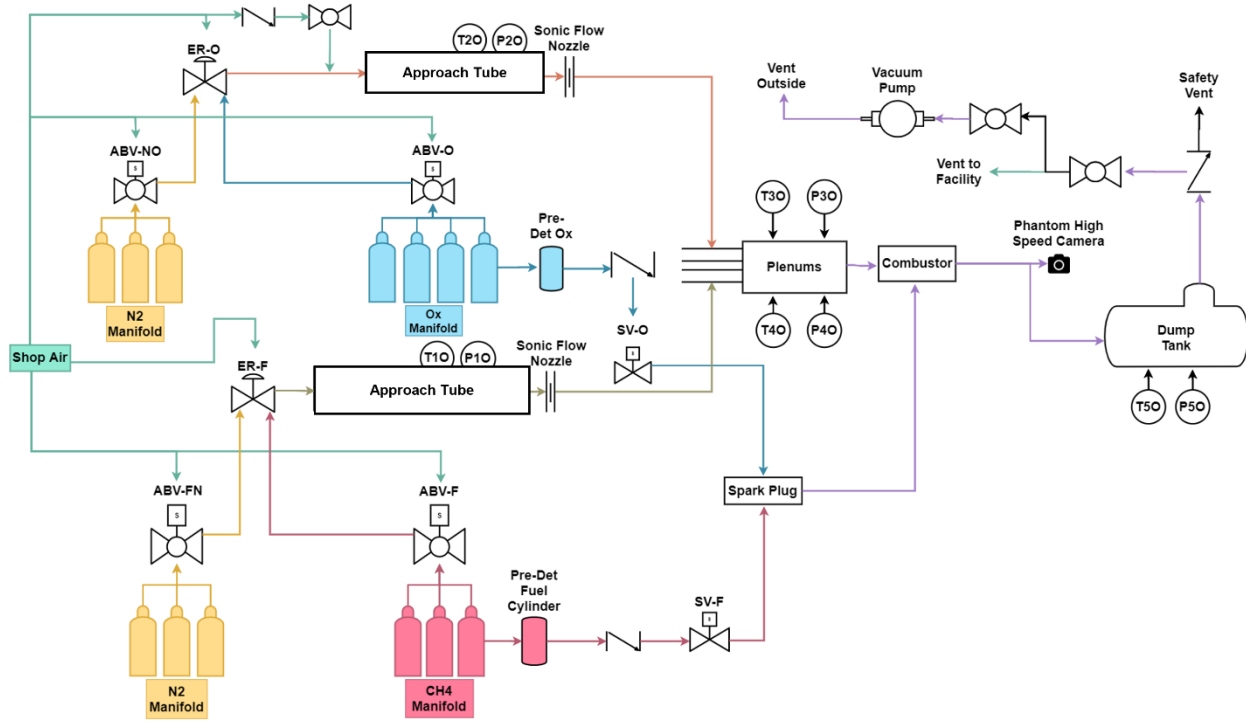


Figure 2.2: Piping and Instrumentation Diagram of the University of Washington Rotating Detonation Engine Lab gas delivery system.

Control of mass flow rate was governed by Flow-Dyne critical flow nozzles of varying minimum areas. These nozzles act as choked orifices, with flow rate linearly related to the pressure measured upstream. The nozzles were also insensitive to downstream pressure changes to a pressure ratio across the nozzle as low as 1.2 [55]. For the oxidizer supply, nozzles with throat diameters of 4.20 and 2.18 mm were used, while the fuel nozzles used throat diameters of 2.79 and 1.27 mm. Pressure and temperature taps, monitored by NoShok 615 Series pressure transducers and OMEGA Type-K exposed bead thermocouples respectively, were located upstream of the nozzles on approach tubes sized for the larger critical nozzle set. Control of the upstream pressures, and thus the flow rates, were governed by TESCO 44-4000 Series air load regulators controlled by ER5000 electropneumatic actuator units. The ER5000s utilized the building air supply and a hand-tuned PID loop to maintain predetermined setpoints for the pressures upstream of the critical nozzles during engine operation and minimize their settling time during engine startup.

Ignition for the engines was accomplished by a spark plug ignited pre-detonator system. Auxiliary 1000 cc supply tanks were filled manually from the propellant delivery system, then isolated during engine operation. Nominal fill levels were 1380 kPa for methane and 4140 kPa for oxygen. During engine startup, the reactants were injected by matching Parker Series 9 miniature calibrant valves into opposing ends of a stainless steel cross. Check valves were placed upstream of the calibrant valves and were sized such that the calibrant valves acted as choked orifices, and thus the ignition system nominally operated at an equivalence ratio of 0.95. The cross was connected to the engine by a stainless steel tube with an inner length-to-diameter ratio of 50 through one of the radial instrument ports in the combustor. To ignite the engine, the calibrant valves were left open for 200 msec to completely fill the cross and attachment tube. Then, 5 msec after closing, an NGK BUHW-2 surface gap spark plug was fired at a rate of at least 500 Hz for 100 msec. Ignition of the engine as reliably witnessed by the high-speed camera within 10 to 20 msec from the command signal, and an example image series of ignition for a 76 mm outer diameter engine can be seen in Figure 2.3.

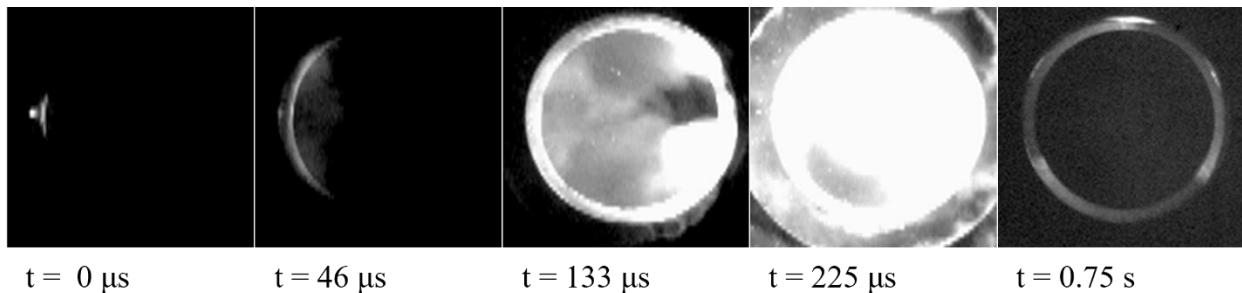


Figure 2.3: Example ignition from 76 mm outer diameter engine.

## 2.2. ENGINE DESIGN

The first study scaled the engines by the outer diameter of the combustor annulus, thereby focusing on influence from radius of curvature. Three size scales of 76.2, 50.8, and 25.4 mm were designed, with combustor length equaling the outer diameter for each. Across all configurations, annulus

width was kept constant at 5 mm with the intent to provide a constant ratio between annular gap size and detonation cell width. Injectors were a flat-faced impinging design, and would maintain inclination angles, impingement height, and impingement diameter with respect to the combustor outer diameter. For this study, the area ratio of total injection area to annulus area would also be kept constant to generate similar coupling between the plenums and combustor. Tests would also be performed at identical annulus mass flux conditions to keep plenum pressures constant across the configurations.

The second scaling study, focusing on annular gap width variation, utilized the 25 mm scale. Four gap widths were designed, including a coreless configuration and the 25 mm engine used in the previous study. A new injector would not be designed for this study, and thus the injection-to-annulus area ratio varied with each configuration. As in the first study, combustor length would be kept the same as the combustor's outer diameter. Furthermore, test points would be scaled according to annulus exit area to maintain mass flux.

For the purposes of this paper, a naming convention has been created to clearly reference the six different configurations, especially within data plots. This convention follows the format XX-FI-Y, with "XX" denoting the outer diameter of the annulus rounded to the nearest millimeter, "FI" indicating a flat-faced impinging injector was used, and "Y" denoting the width of the annular gap in millimeters. To demonstrate this, a summary of combustor and injector dimensions is provided in Table 2-1. A cutaway computer-aided design (CAD) image of the 76-FI-5 configuration is shown in Figure 2.4 with labels for common engine features and dimensions. All CAD images contain 25.4 mm reference lines to provide scale. Engineering drawings for the manufactured components of all engine configurations are included in Appendix A.

Table 2-1: Primary design dimensions and features for each configuration.

|                                        | <b>76-FI-5</b> | <b>51-FI-5</b> | <b>25-FI-5</b> | <b>25-FI-3</b> | <b>25-FI-7</b> | <b>25-FI-12.7</b> |
|----------------------------------------|----------------|----------------|----------------|----------------|----------------|-------------------|
| <b>Annulus Outer Diameter</b>          | 76.2 mm        | 50.8 mm        | 25.4 mm        | 25.4 mm        | 25.4 mm        | 25.4 mm           |
| <b>Annular Gap</b>                     | 5.0 mm         | 5.0 mm         | 5.0 mm         | 3.0 mm         | 7.0 mm         | 12.7 mm           |
| <b>Annulus Length</b>                  | 76.2 mm        | 50.8 mm        | 25.4 mm        | 25.4 mm        | 25.4 mm        | 25.4 mm           |
| <b>Injector Pairs</b>                  | 72             | 72             | 48             | 48             | 48             | 48                |
| <b>Fuel Injector Diameter</b>          | 0.79 mm        | 0.64 mm        | 0.51 mm        | 0.51 mm        | 0.51 mm        | 0.51 mm           |
| <b>Oxidizer Injector Diameter</b>      | 1.24 mm        | 0.99 mm        | 0.81 mm        | 0.81 mm        | 0.81 mm        | 0.81 mm           |
| <b>Injection-to-Annulus Area Ratio</b> | 0.110          | 0.110          | 0.110          | 0.164          | 0.0856         | 0.0684            |
| <b>Fuel Injector Feed</b>              | Radial         | Radial         | Axial          | Axial          | Axial          | Axial             |
| <b>Oxidizer Injector Feed</b>          | Radial         | Radial         | Radial         | Radial         | Radial         | Radial            |



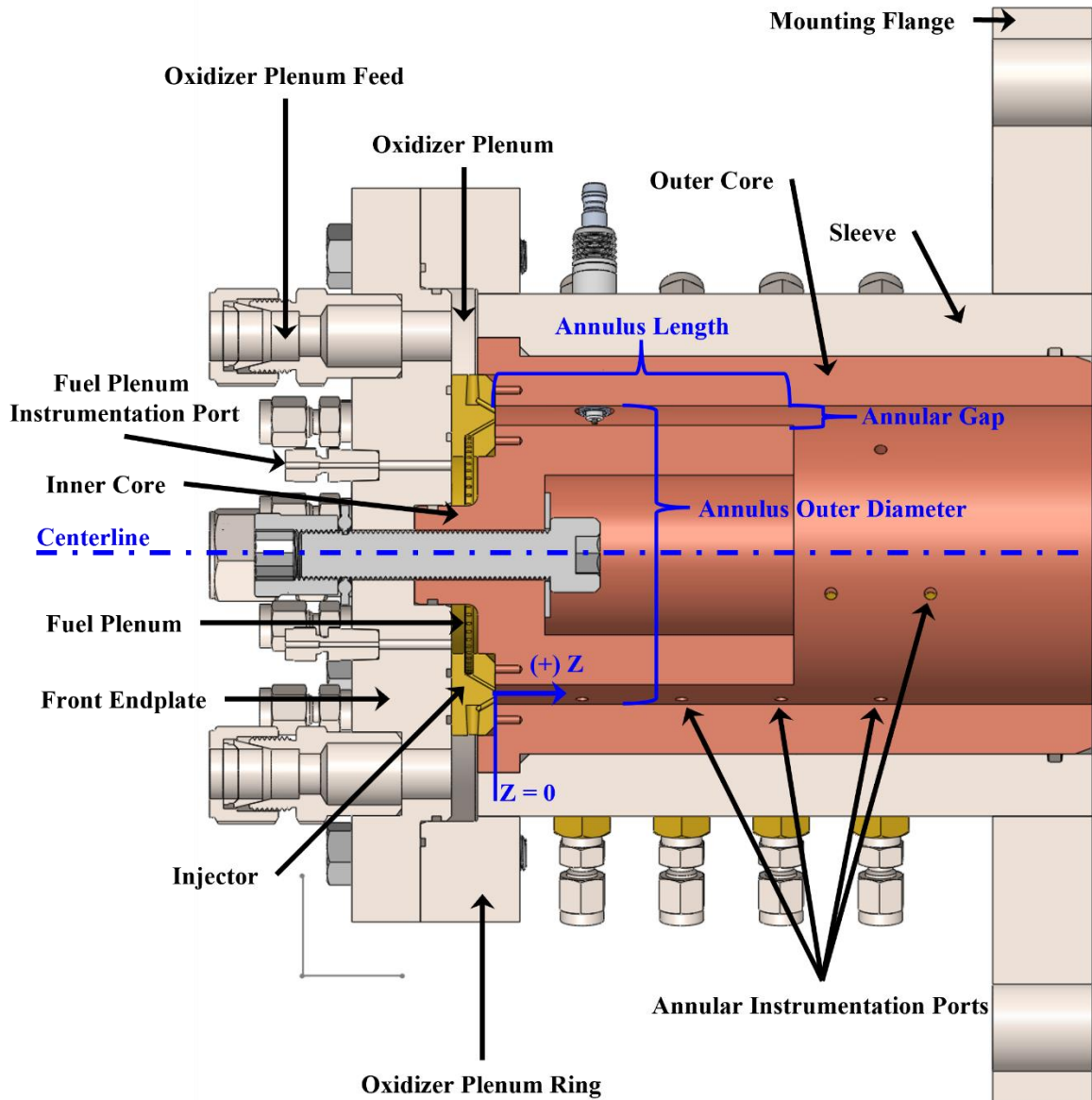


Figure 2.4: Cutaway of 76-FI-5 configuration with labels for part names and dimensions.

Material selection was identical across all configurations. 304 stainless steel was used for the front endplate, oxidizer ring, sleeve, mounting flange, and tube fittings. Instrument adapters and injection components were made from 360 brass, and fasteners were made from alloy steel. An exploded view of the 25-FI-5 with material callouts can be seen in Figure 2.5.

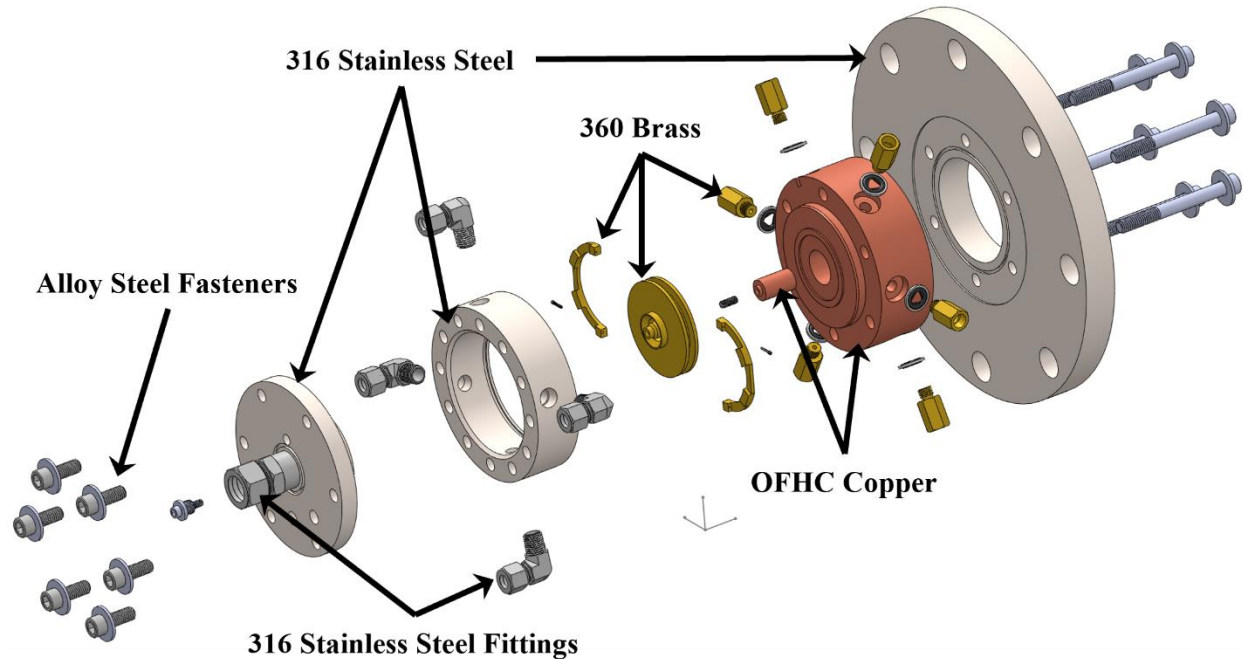


Figure 2.5: Exploded view of 25-FI-5 configuration with material callouts.

### 2.2.1. 76-FI-5 DESIGN

The exterior components of the 76-FI-5 configuration were originally designed for, and operated with, radial injectors and a six spark plug wave generator ignition system similar to that used in a 154 mm scale engine [56]. The 76 mm engine mounted in its original configuration can be seen in Figure 2.6. The spark plug ports were placed near the face of the injector, and had effects on high-speed video processing, which will be discussed in a later section. Due to its modularity, installation of a replacement injector and inner core for the 76-FI-5 configuration was possible without modification of existing parts, and can be seen in Figure 2.7. This configuration contained 24 instrument ports along a 152 mm long outer core. Although not all ports were used, this necessitated the use of a stainless steel shell to provide additional strength to the outer wall to support the 30 access holes in the annulus. This engine configuration was used as the basis for test conditions and design of the remaining engine scales.

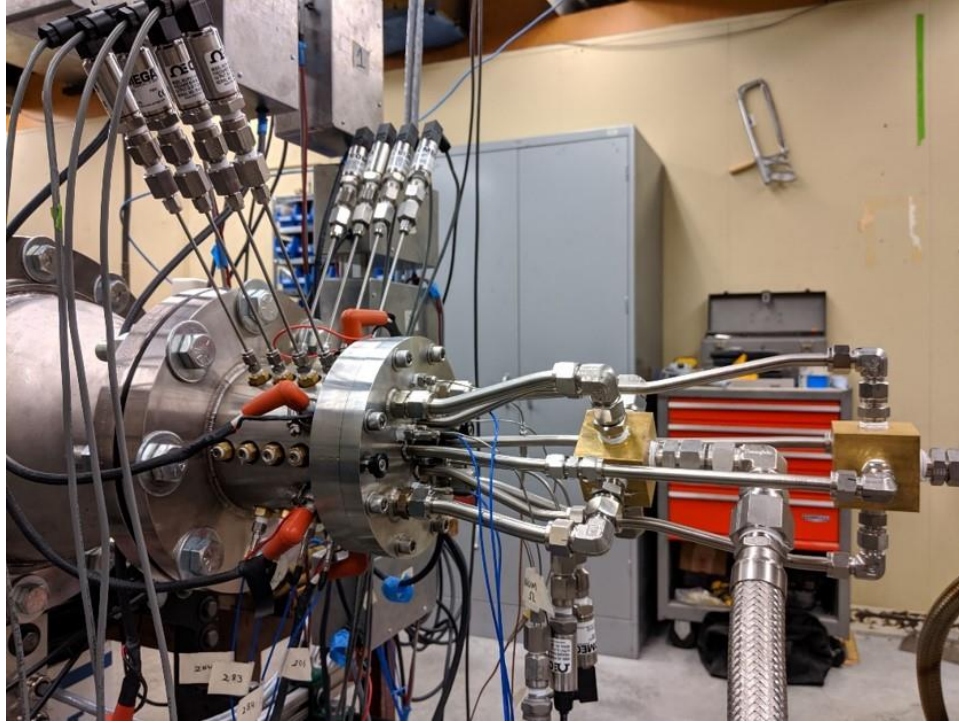


Figure 2.6: 76 mm engine mounted for radially-injected, wave-generator ignited testing.

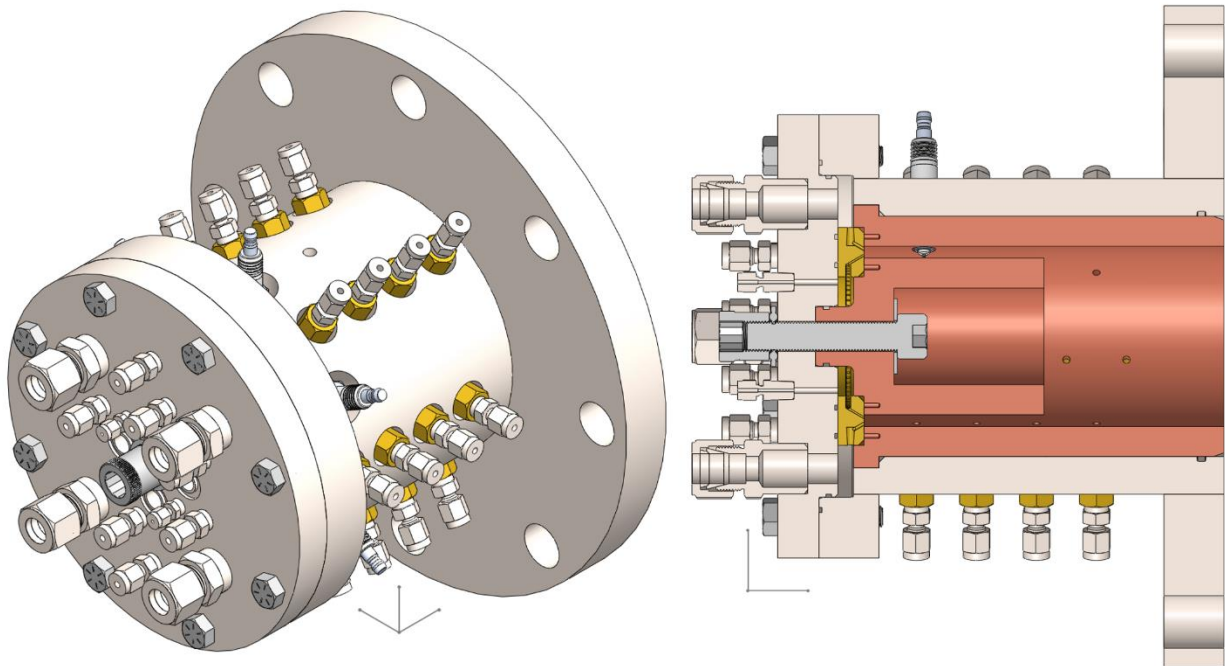


Figure 2.7: Isometric (left) and cutaway side (right) views of 76-FI-5 configuration.

### 2.2.2. 51-FI-5 DESIGN

Major alterations when designing the 51 mm engine included a radially-fed oxidizer plenum and collection of fuel plenum data from a 25.4 mm tee immediately upstream of the centralized feed. The 51 mm design also minimized annular pressure stations to reduce the influence of ports on wave dynamics, and thus did not require a stainless steel sleeve on the outer core. The final design for the 51-FI-5 configuration can be found in Figure 2.8.

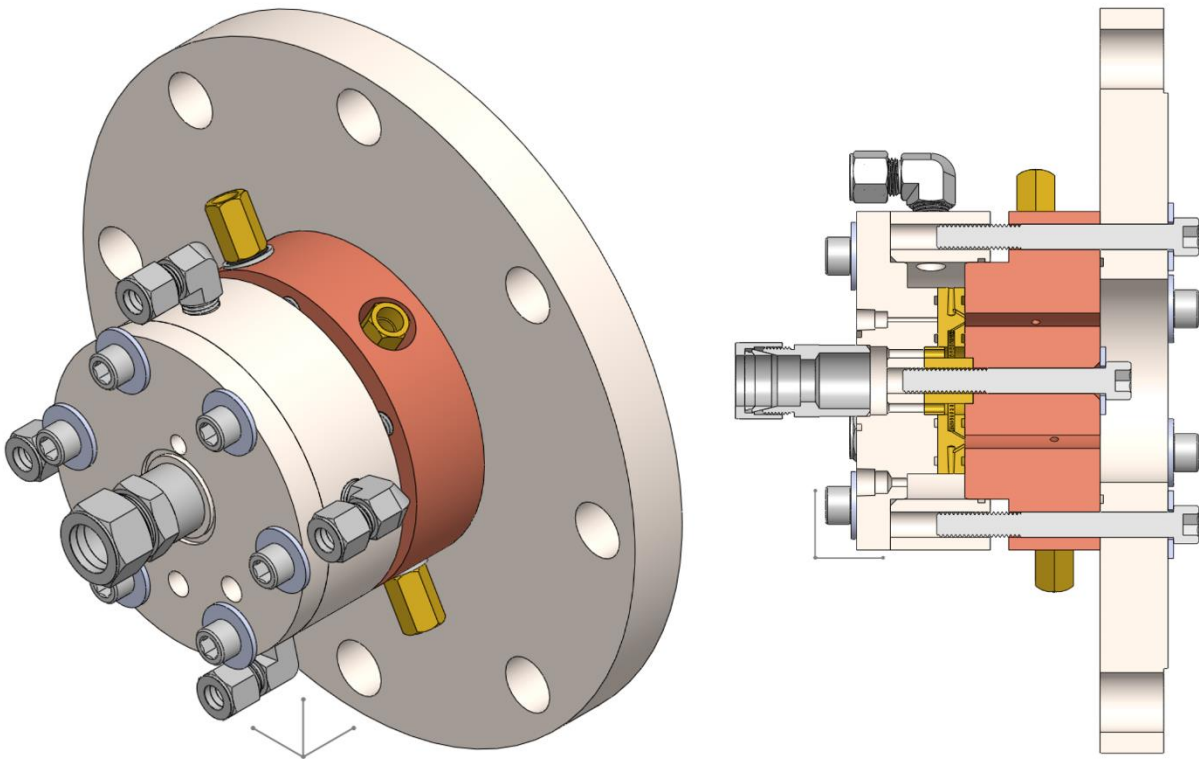


Figure 2.8: Isometric (left) and cutaway side (right) views of 51-FI-5 configuration.

### 2.2.3. 25 MM DESIGNS

The design for the 25 mm configurations utilized the same front endplate, oxidizer ring, and mounting flange as the 51-FI-5 configuration. The 25 mm engine was also designed with the same exterior diameter, and the thicker outer core allowed the engine to regain annular pressure stations compared to the 51 mm design. Reusing the front endplate led to modifications of the injector

design, prompting the need for an intermediate support ring. Another departure from previous engines is the removal of the inner core as a sealing surface, which permitted core changes without the need for leak testing. The resulting design, specifically the 25-FI-5 configuration, is shown in Figure 2.9. The same injector was used during all 25 mm testing, which required the 25-FI-3 inner core to have an initial chamfer to prevent fuel injector blockage. In order to minimize effects on mixing, this chamfer started at the same diameter as the 25-FI-5 inner core and terminated axially at the injector impingement height. This, as well as the design details of the other 25 mm engine cores, can be found in Appendix A. For the 25-FI-12.7 configuration, a small brass plug was installed to provide a flat injector surface for coreless testing.

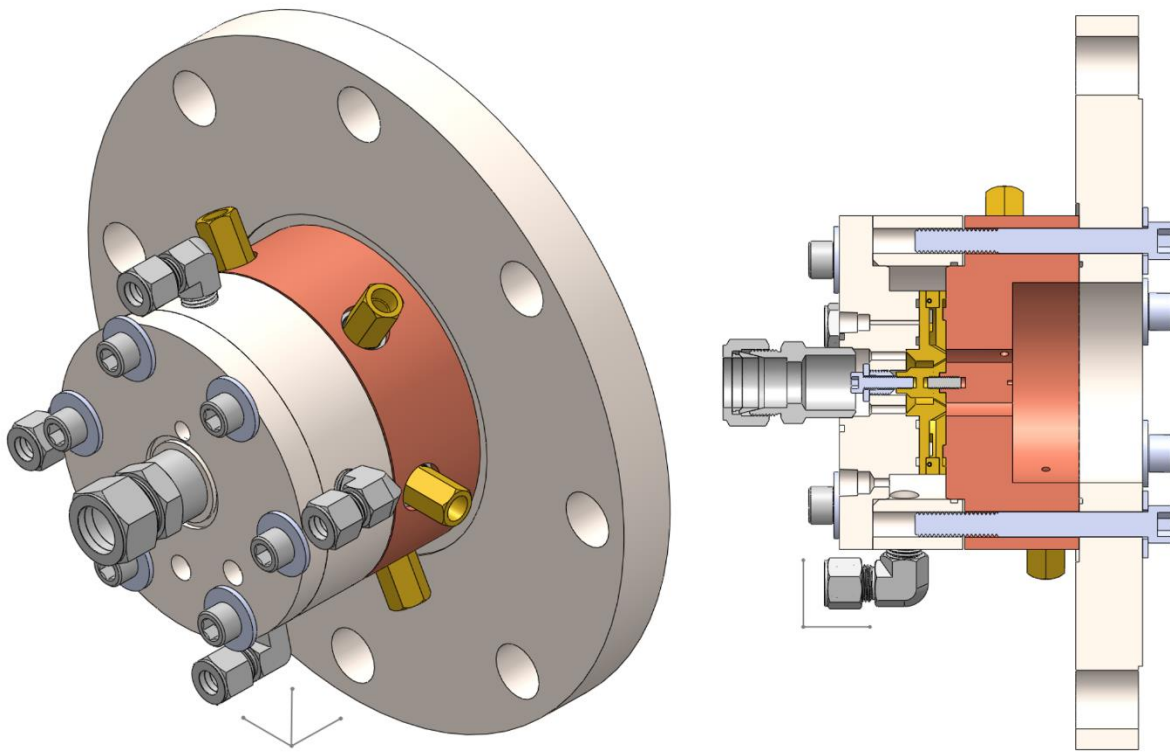


Figure 2.9: Isometric (left) and cutaway side (right) views of 25-FI-5 configuration.

#### 2.2.4. INJECTOR DESIGN

As mentioned previously, all injectors used a flat-faced impinging design. The number and diameter of injection holes for each configuration is summarized in Table 2-1, located at the beginning of this subsection. In each injector, an oxidizer-to-fuel area ratio of 2.50 was chosen to produce nominally identical plenum pressures under stoichiometric conditions for the planned propellants of methane and oxygen. The total injection area was scaled to maintain a ratio of 0.110 between the total injection area and annular combustor exit area for configurations with a 5 mm gap. As the same injector was used in all 25 mm configurations, this area ratio varied with the variations in gap width.

Across all configurations, both oxidizer and fuel injection holes were inclined 30 degrees from the combustor's axis. Furthermore, the impingement height was kept constant at 2.16 mm from the injector face. Each injector used an impingement diameter 5 mm smaller than the outer diameter of the configuration's annulus. A CAD image comparing the three injectors used is shown in Figure 2.10.



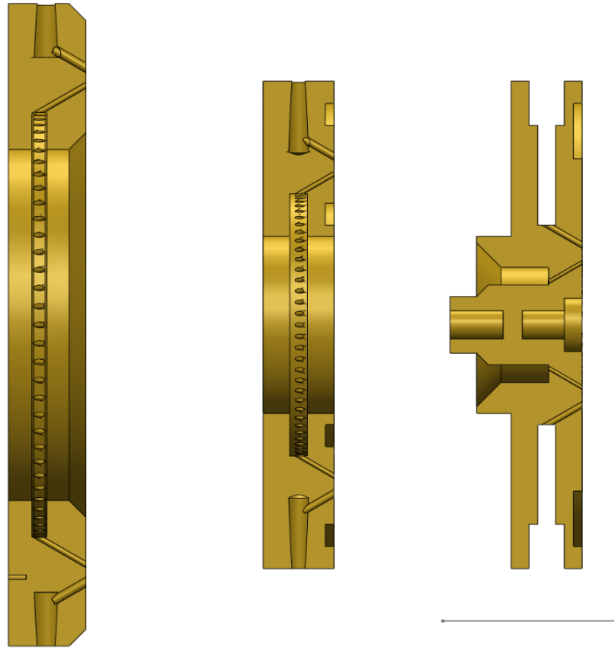


Figure 2.10: Comparison of injectors for 76 mm (left), 51 mm (middle), and 25 mm (right) configurations.

Scaling the injection design from the 51 mm to the 25 mm required a reduction in the number of injectors, but radial spacing between injector pairs along the impingement diameter was kept constant at 2.34 mm. In order to adapt to the front endplate design for the 51 mm engine, the 25 mm engine's fuel injectors were fed axially rather than radially. Furthermore, the 25 mm oxidizer injectors were fed directly by the plenum, whereas the larger diameter configurations utilized a design where a single radial feed hole fed two oxidizer injectors.

## 2.2.5. ENGINE INSTRUMENTATION

The quantity and placement of instrument ports along the outer core varied from engine to engine. A summary of the locations, including axial distance from injector face, clocking with reference to the top of the engine, and port diameter is included in Table 2-2. All standoff tubes used for annular pressure stations had an internal length-to-diameter ratio of 145.

Annular static pressures, including the front wall pressure in the 76 mm engine, were collected by OMEGA PX319-500A5V pressure transducers. Plenum pressures across all

configurations were collected with OMEGA PX319-1KA5V transducers. Plenum temperatures were collected by OMEGA Type-K exposed bead thermocouples, and a surface-mount Type-K was used for exterior engine temperature. The pair of high-speed piezoelectric dynamic pressure sensors were PCB 111A24 sensors and will often be referenced by their manufacturer name (PCB) in this paper. When used, one PCB was always located in the oxidizer plenum. The second PCB was used in the fuel plenum for the 76-FI-5 configuration, but was either unused or installed in an annular instrument port in other configurations due to fuel plenum access restrictions.



Table 2-2: Annular instrumentation locations by configuration.

| <b>76-FI-5</b>                  |                       |                 |                 |                        |
|---------------------------------|-----------------------|-----------------|-----------------|------------------------|
| <b>Location</b>                 | <b>Axial Distance</b> | <b>Clocking</b> | <b>Diameter</b> | <b>Notes</b>           |
|                                 | <b>mm</b>             | <b>deg.</b>     | <b>mm</b>       |                        |
| Front Wall                      | 0.00                  | 357.5           | 0.8             | Pressure               |
| Annulus                         | 22.2                  | 330             | 3.2             | Pressure with standoff |
| Annulus                         | 34.9                  | 30              | 3.2             | Pressure with standoff |
| Annulus                         | 47.6                  | 330             | 3.2             | Pressure with standoff |
| Annulus                         | 60.3                  | 30              | 3.2             | Pressure with standoff |
| Annulus                         | 73.0                  | 330             | 3.2             | Pressure with standoff |
| Annulus                         | 73.0                  | 210             | 3.2             | Pre-detonator port     |
| 76 mm Duct                      | 85.7                  | 30              | 3.2             | Pressure with standoff |
| 76 mm Duct                      | 98.4                  | 330             | 3.2             | Pressure with standoff |
| <b>51-FI-5</b>                  |                       |                 |                 |                        |
| <b>Location</b>                 | <b>Axial Distance</b> | <b>Clocking</b> | <b>Diameter</b> | <b>Notes</b>           |
|                                 | <b>mm</b>             | <b>deg.</b>     | <b>mm</b>       |                        |
| Annulus                         | 26.7                  | 150             | 3.2             | Pressure with standoff |
| Annulus                         | 33.7                  | 330             | 3.2             | Pressure with standoff |
| Annulus                         | 33.7                  | 210             | 3.2             | Pre-detonator port     |
| Annulus                         | 40.6                  | 30              | 3.2             | Pressure with standoff |
| <b>All 25-FI Configurations</b> |                       |                 |                 |                        |
| <b>Location</b>                 | <b>Axial Distance</b> | <b>Clocking</b> | <b>Diameter</b> | <b>Notes</b>           |
|                                 | <b>mm</b>             | <b>deg.</b>     | <b>mm</b>       |                        |
| Annulus                         | 16.5                  | 270             | 2.4             | Pressure with standoff |
| Annulus                         | 18.5                  | 330             | 2.4             | Pressure with standoff |
| Annulus                         | 20.3                  | 150             | 2.4             | Pressure with standoff |
| Annulus                         | 22.4                  | 30              | 2.4             | Pressure with standoff |
| Annulus                         | 22.4                  | 210             | 3.2             | Pre-detonator port     |
| 76 mm Duct                      | 38.1                  | 90              | 3.2             | Pressure with standoff |

### 2.3. OPERATING PROCEDURE

At the start of a test series, all equipment was powered, and sensor operation was verified with a dry run. Propellant lines were then pressurized, and the supply tanks for the ignition system were topped off, if necessary. Prior to each run, settings for valve timing, ER5000 pressure setpoints, and regulator air pre-load were established, and the dump tank was evacuated to the desired starting back pressure. After arming the system, the run was then executed according to the imported timing matrix. While data was saved after the run, the contents of the dump tank were evacuated, and the engine was convectively cooled by air passing through the oxidizer supply lines. The next run would not be conducted until the exterior of the combustor fell below a pre-determined temperature threshold near ambient.

During a run, control of the primary 2 kHz DAQ's input and output was accomplished with MATLAB. The Phantom high-speed camera and ER5000 control were governed by software provided by their respective manufacturers that ran in parallel to the primary DAQ. The secondary 1.25 MHz DAQ operated from a simple LabVIEW script. Timing of all components was synchronized with trigger signals from the primary DAQ according to a timing matrix imported before the run. This timing matrix also regulated the on and off timing of all valves and solenoids, including the ignition system. An example timing can be seen in Figure 2.11 for the 76-FI-5 configuration. A custom power distribution box used the low-voltage trigger signals to provide 12 VDC, 24 VDC, and 115 VAC to each valve and solenoid as required.

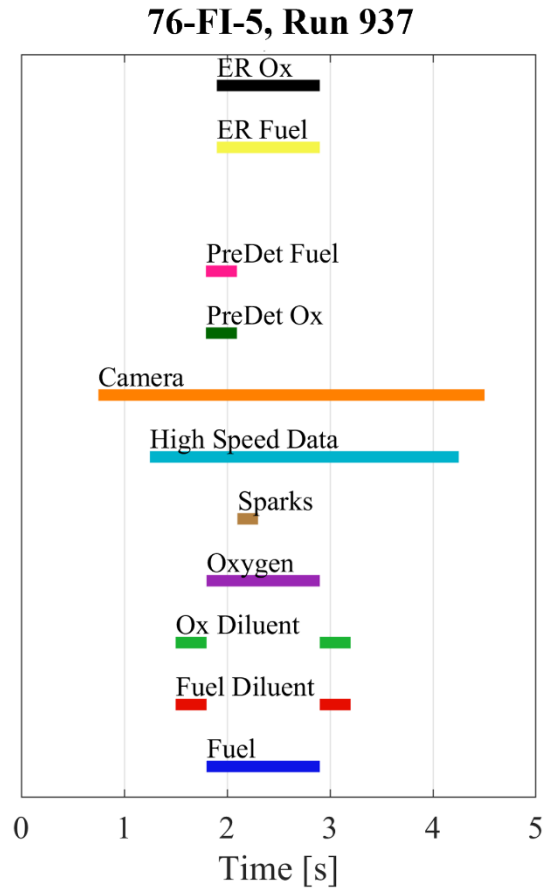


Figure 2.11: Example run timing.

Pressure setpoints for the ER5000s were calculated based on cold flow data generated at the beginning of each configuration's test series. This cold flow data was produced with the working gases of methane and oxygen, and was also used to calibrate injector discharge coefficients. A separate series of cold flows conducted on the propellant delivery system at installation generated the original PID gain settings for each ER5000. After pressure setpoints were input during setup, the regulator air domes were manually pre-loaded to match the setpoints in order to minimize settling time during startup.

Over time, adjustments were made to maximize ignition reliability and duration of steady operation. Primarily, these alterations were composed of a balance between ignition and valve timing with the proportional gain of the ER5000s. A demonstration of these improvements across

a single configuration can be seen in the plenum pressure cold flow data shown in Figure 2.12. Burn duration for hot fires was always longer than 700 msec, with steady plenum pressure traces required for at least the last 100 msec of the burn. Thus, these improvements notably reduced the time required to generate the reported data. Another major alteration was removal of the nitrogen pre-purge from both propellant lines in 25 mm configurations to combat unreliable ignition seen during initial 25-FI-5 testing.

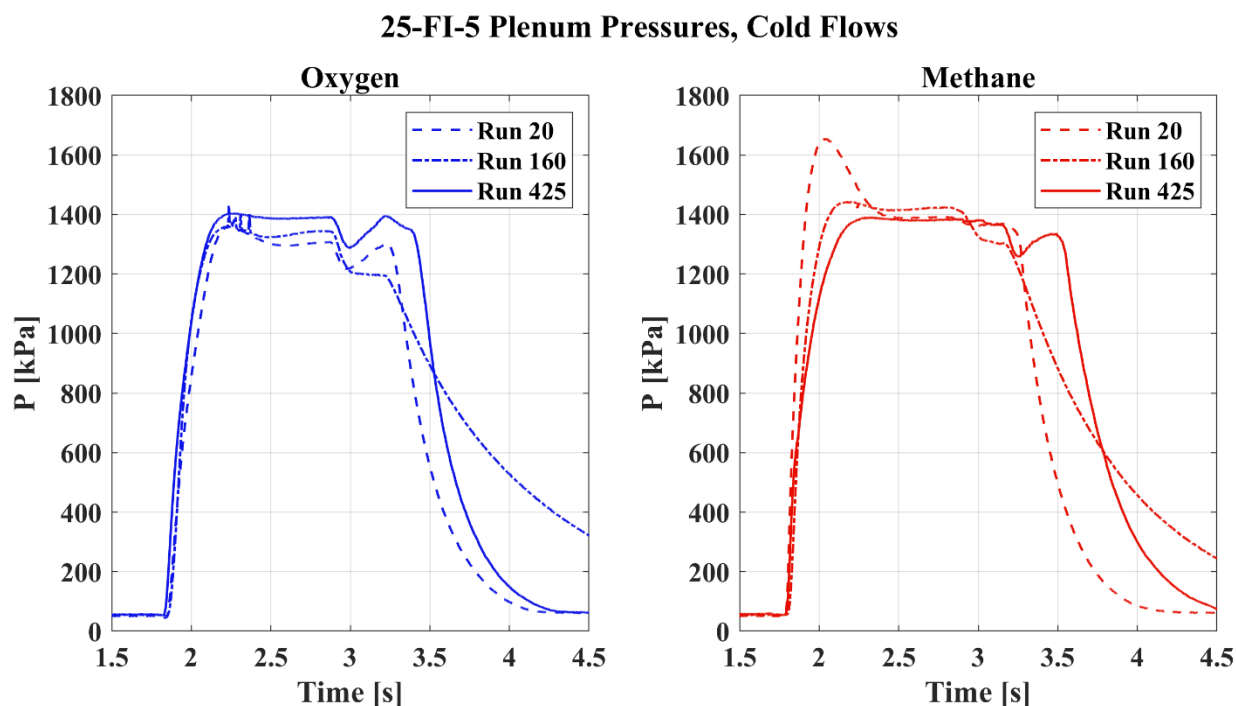


Figure 2.12: Evolution of plenum pressure traces in 25-FI-5 configuration due to timing and PID changes.

After the run, the raw voltage signals of all pressure and temperature traces were saved automatically, while high-speed video sections were saved manually along with their associated beginning and ending frames. For a run with detonation, video sections included ignition, steady-state operation at the end of the run, and the full duration of combustion from ignition until the propellant supply valves are closed. Additional sections would also be saved if wave count, wave stability, or operating mode (e.g., detonation to deflagration) changes were witnessed.

## 2.4. DATA PROCESSING

Calibrations of pressure transducer signals were completed according to data provided by their respective manufacturers. Afterwards, all pressure traces were normalized to each other based upon average data taken before any digital triggers or valve actuation in each individual run. To accomplish this, every channel was averaged over the first 500 msec of data collection. Each average was then compared to that measured from the dump tank, and a constant offset was applied to the full time history. In this way, every pressure signal had the same average over the first 500 msec of a single run. The dump tank pressure was measured with an OMEGA PX319-050A5V transducer, which reported absolute pressure over a 50 psi (345 kPa) range. This transducer was of the same series, and thus percent accuracy, as most transducers mounted to the engines, but had the smallest range and therefore highest pressure accuracy in the entire system. As the dump tank pressure transducer was an absolute pressure sensor, all reported pressures are also absolute.

As mentioned previously, thermocouples passed through an amplifier before being read by the DAQ. This was accomplished by a TCA-MS-K-8-A4 amplifier from The Sensor Connection. This unit converted all thermocouple input signals to linearized, independent analog outputs. Normalization, as described for pressure signals, was not done on thermocouple data.

Mass flow rate for each reactant was calculated using the choked mass flow equation [57]:

$$\dot{m} = C_d \frac{A_{CN} P}{\sqrt{T R_{gas}}} \sqrt{\gamma} \left( \frac{\gamma + 1}{2} \right)^{-\frac{\gamma+1}{2(\gamma-1)}} = C_d \frac{A_{CN} P}{\sqrt{T R_{gas}}} C_{FF} \quad (2.1)$$

Here, pressure,  $P$ , and temperature,  $T$ , were measured in the approach tube to the critical nozzle, and  $R_{gas}$  represents the specific gas constant. The critical nozzle throat area,  $A_{CN}$ , varied from 1.82 to 55.2 mm<sup>2</sup> depending on the critical nozzle installed. The discharge coefficient,  $C_d$ , was provided by the manufacturer for the two largest critical nozzles. This set was then used to calibrate the smaller set using nitrogen cold flows. The variable  $C_{FF}$  represents the critical flow

factor and was calculated using real gas effects based on the measured  $P$  and  $T$  with a lookup table generated by REFPROP [58].

No filtering was applied during processing, and therefore significant noise was seen in thermocouple traces, and thus mass flow rate data, during startup corresponding to noise generated by the ignition spark plug. When reporting the results from a single run, average values from the last 100 msec of the run were used, with the command to close propellant valves acting as the ending timestamp. For derived values, such as mass flow rate, this averaging was done after calculation and not on the intermediate values.

Extraction of wave dynamics from high-speed video data was conducted in a similar fashion to that described in Ref. [58]. For most configurations, the inner and outer diameters of the annulus were automatically identified during image processing, but the 25-FI-12.7 configuration did not have an inner core and consequently had a virtual annulus imposed. This was done by allowing the outer diameter to be discovered normally, then applying a 3 mm gap before binning. When reporting wave speed, the circumference of the outer diameter of the annulus was used as the length scale.

When counter waves were seen in these combustor configurations during steady operation, the counter wave number was always equal to that of the primary wave number. However, even during steady operation the counter waves, especially those with relatively low luminosity compared to the primary waves, would often fade away and restart. In order to apply a quantitative threshold to this phenomenon, the peaks of the power spectrum generated from the video  $x$ - $t$  data from the last 100 msec of the burn were recorded and normalized to the power associated with the primary co-rotating wavefronts. Comparing these relative powers with visual inspection across all configurations, a threshold of 58% was established, counter wave systems below this threshold

classified as “weak,” indicating the inconsistent fading in and out behavior previously described. Counter wave systems above this threshold were designated as “strong,” indicating the counter wave(s) were always present during steady operation, and often had a pixel luminosity on the order of the primary waves. Examples of high-speed camera images and frequency analysis for each operating mode, including wave modes with and without counter-rotation, are included in Appendix B.

The high-speed piezoelectric dynamic pressure sensor data were split into sections based upon the timing matrix for the run. On the burn section between the cessation of ignitor excitation and closing command of reactant valves, a fast Fourier transform (FFT) with a Hanning window was used to convert time data to the frequency domain. A spectrogram was used to verify stability, then a peak-finding routine, similar to the one used in counter wave strength computation, was applied to the power spectrum from the last 100 msec of the burn to generate reported values.

### 3. EXPERIMENTAL RESULTS

For each configuration, tests were conducted along sweeps of constant mass flux (243 kg/s/m<sup>2</sup>) and equivalence ratio (1.15) with gaseous methane and oxygen. Scaling test points in this manner led to similar plenum pressures across the radius of curvature study, and scaled plenum pressures with the injector-to-annulus area ratio for the gap width study. Additional test points were conducted as time and resources allowed to explore transitions in modes of operation and generate data along additional sweeps of equivalence ratio or mass flux. This section will detail the results obtained from experiments, including a sample run, the results from each configuration, and initial scaling results based on the geometric alterations of each study. Finally, the effects of variations in back pressure seen in the 25-FI-5 configuration will be discussed.

In reported hot fires on the cross of constant equivalence ratio and mass flux, at least two runs exceeding a burn time of 700 msec and exhibiting steady plenum pressures during the last 100 msec were completed per condition. While the operation and data processing closely followed that described in the previous section, some variations existed across the configurations and are summarized in Table 3-1. Starting back pressures were selected in order to produce back pressure insensitive results for the majority of data on the cross.

Table 3-1: Experimental summary and differences across configurations tested.

| <b>Configuration</b> | <b>Reported Hot Fires</b> | <b>Video Gap Calculation</b> | <b>Core Losses</b> | <b>Nitrogen Pre-Purge Duration</b> | <b>Nominal Starting Back Pressure</b> |
|----------------------|---------------------------|------------------------------|--------------------|------------------------------------|---------------------------------------|
| <b>76-FI-5</b>       | 63                        | Auto                         | 0                  | 0.3 s                              | 62 to 80 kPa                          |
| <b>51-FI-5</b>       | 108                       | Auto                         | 0                  | 0.3 s                              | 44 or 83 kPa                          |
| <b>25-FI-5</b>       | 115                       | Auto                         | 3                  | None                               | 50 kPa                                |
| <b>25-FI-3</b>       | 30                        | Auto                         | 1                  | None                               | 50 kPa                                |
| <b>25-FI-7</b>       | 9                         | Auto                         | 1                  | None                               | 50 kPa                                |
| <b>25-FI-12.7</b>    | 82                        | 3 mm                         | N/A                | None                               | 50 kPa                                |



### 3.1. SINGLE RUN EXAMPLE

In order to demonstrate the data processing methods described in the previous section, as well as the conversion of temporal data to reported values, the results of a single run on a single engine will be presented in its entirety. The selected run is from the 76-FI-5 configuration and utilized the timing matrix already shown in Figure 2.11. This run, referenced as 937, was chosen since it coincides with a test point at the intersection of the two cross legs tested. It also includes dynamic pressure sensor data and is representative of data from all configurations. To improve visual quality, digital low-pass filters were applied to the temporal data shown in this subsection. Pressure plots utilized a 100 Hz cutoff frequency, and temperature and mass flow rate plots used 50 and 20 Hz cutoff frequencies, respectively. These filters were not used when averaging or in data reported outside of this subsection.

The pressures upstream of the critical nozzles and resultant mass flow rates are shown in Figure 3.1, and demonstrate steady operation, especially during the 100 msec averaging region, which was between 2.8 and 2.9 seconds for this run. Figure 3.2 shows pressures within the engine itself, including those in the plenums and combustor. The combustor pressures have a stair-step shape with descending pressure levels associated with increasing axial position that is indicative of detonation. The last two axial stations at 85.7 and 98.4 mm were outside of the combustor in a 76 mm cylindrical expanded section after the inner core terminates. Temperature traces are shown in Figure 3.3. Orifice temperatures exhibited an initial spike due to adiabatic compression at the orifice, then were only quasi-steady within the averaging region. Even so, the variation in the averaging region ultimately had little effect on flow rate. As discussed previously, noise from the spark plug firing can be seen in the region between 2.1 and 2.3 seconds in accordance with the timing matrix.

76-FI-5, Run 937

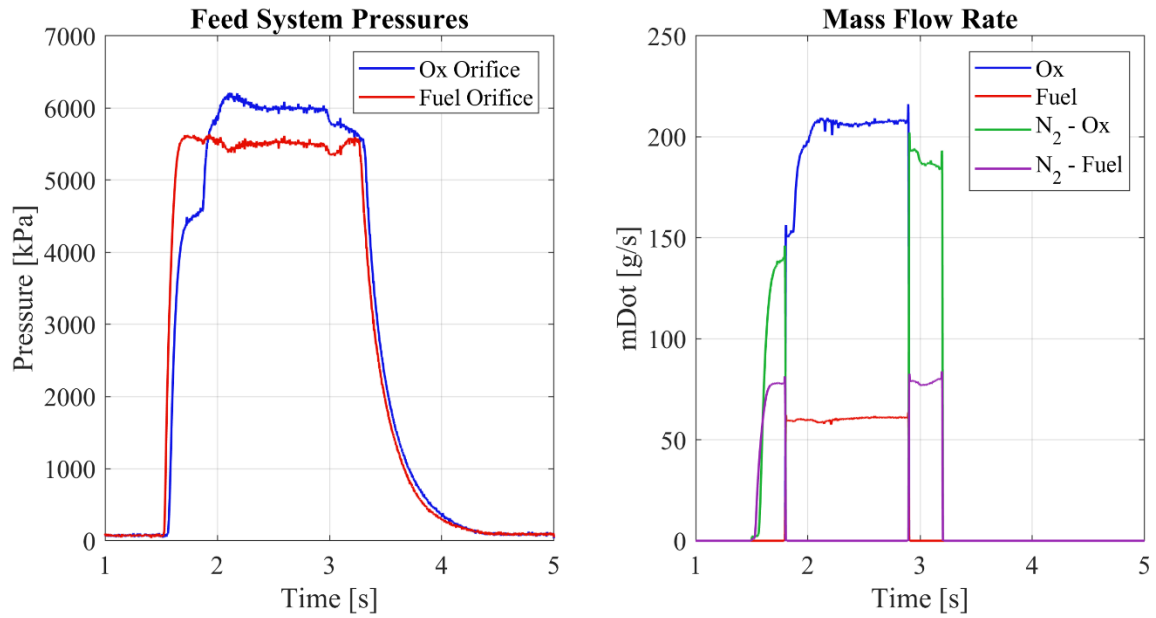


Figure 3.1: Feed system pressure (left) and mass flow rate (right) data for example run.

76-FI-5, Run 937

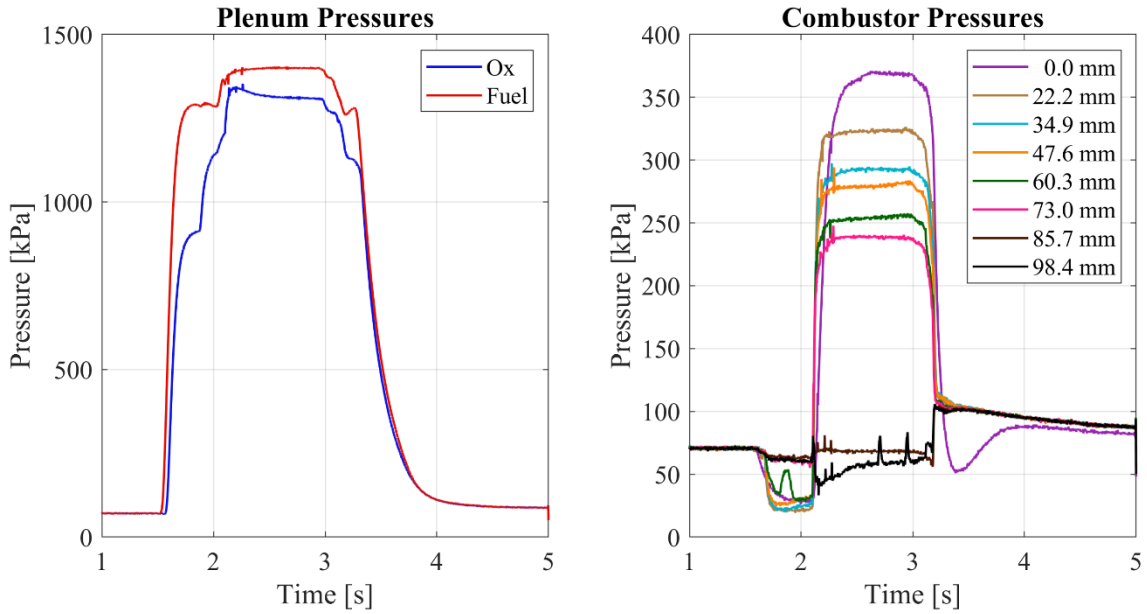


Figure 3.2: Plenum pressure (left) and combustor axial pressure (right) data for example run.

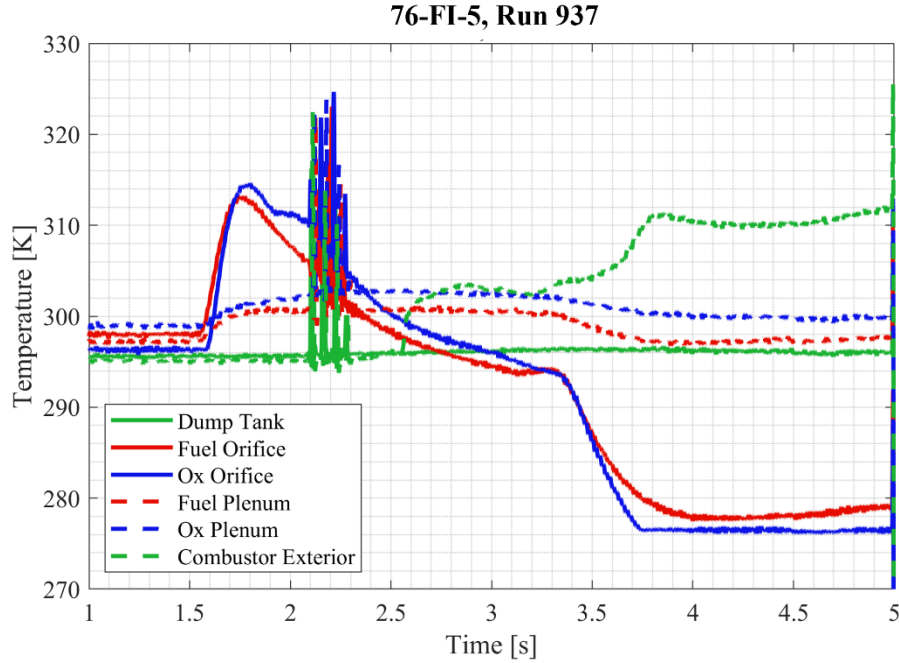


Figure 3.3: Thermocouple data for example run.

A single frame from the high-speed camera at the end of the run is shown in Figure 3.4. This run had two co-rotating waves for the entire duration and are located at 3 and 9 o'clock in the frame. The six evenly-spaced bright spots were associated with the spark plug ports near the front wall of this engine. Figure 3.5 demonstrates the x-t data taken from the last 2 msec of the burn and showcases the removal of stationary bright spots from wave interaction with spark plug ports through background subtraction. The two-dimensional FFT of Figure 3.6 reported a fundamental of two waves with a passing frequency of 15.84 kHz, in addition to the first harmonic. Finally, Figure 3.7 demonstrates the stability of the dominant wave frequency over the full duration of the burn.

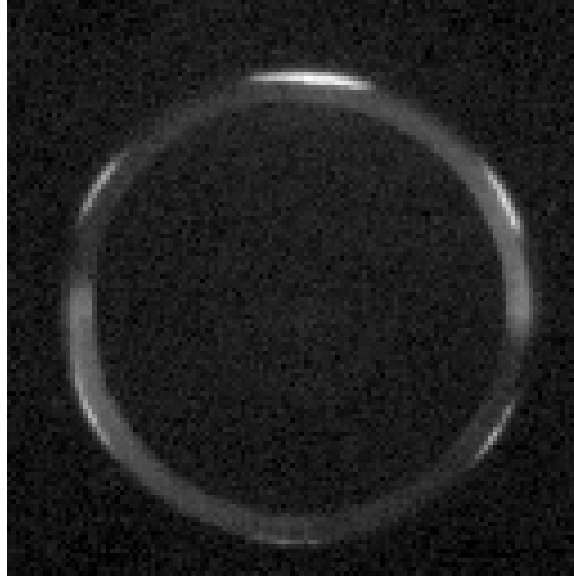


Figure 3.4: Camera frame from example run with two co-rotating waves at 3 and 9 o'clock.

**76-FI-5, Run 937**

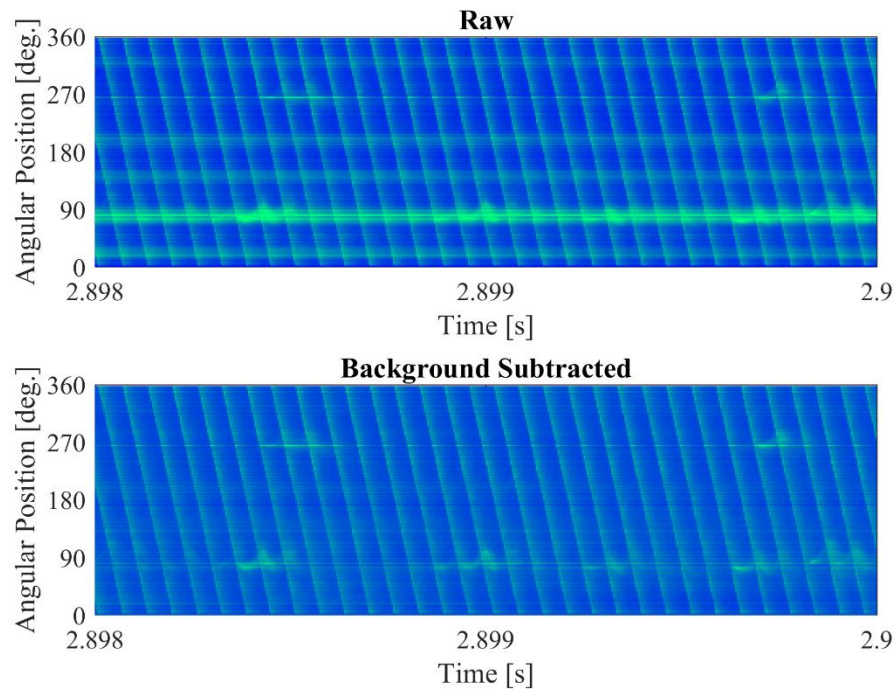


Figure 3.5: Camera x-t data from the last 2 msec of example run.

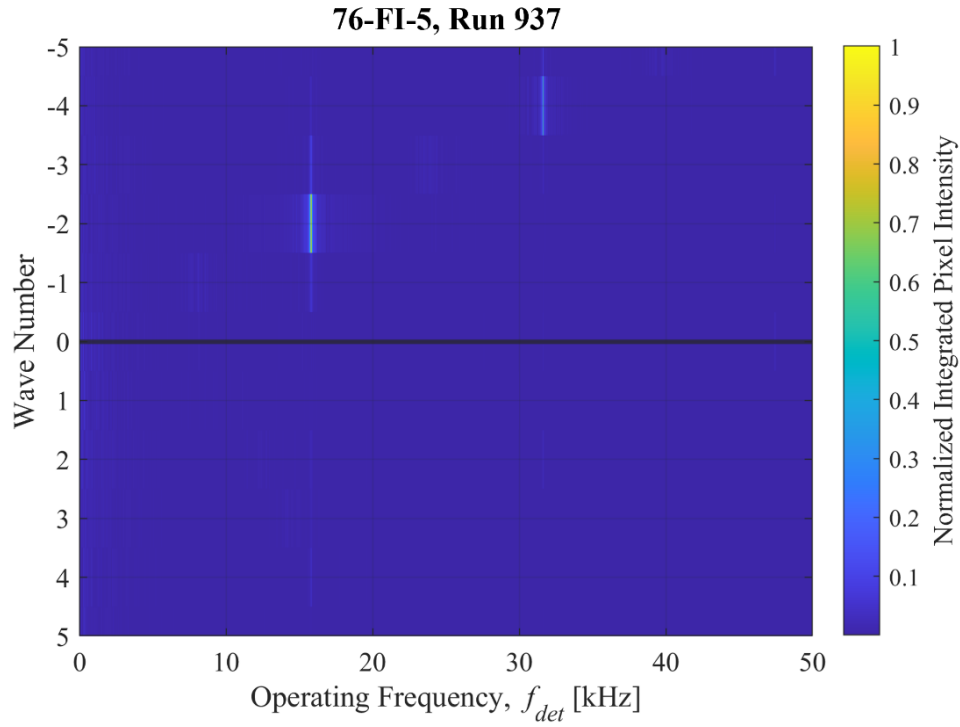


Figure 3.6: Two-dimensional FFT of high-speed video data from example run.

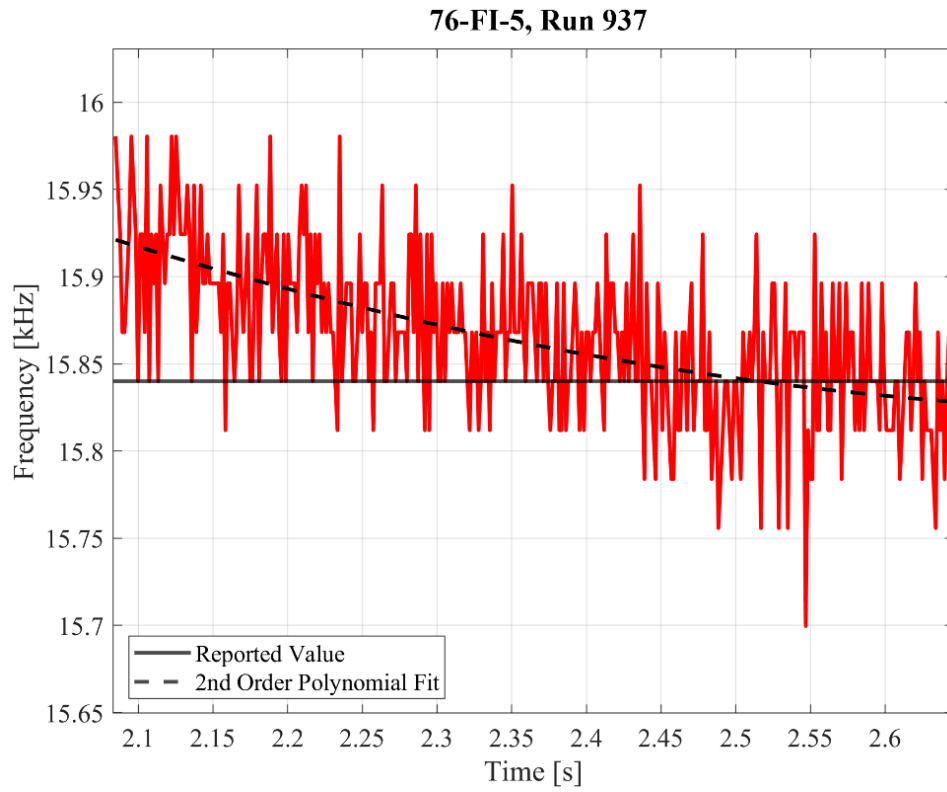


Figure 3.7: Evolution of dominant video frequency over full burn in example run.

An example of the raw high-speed piezoelectric dynamic pressure sensor (PCB) signals can be seen in Figure 3.8. A spectrogram taken from the designated burn section is shown in Figure 3.9, and shows excellent stability of the frequency response in both plenums. The power spectrum associated with the data collected during the last 100 msec of the burn is shown in Figure 3.10, and circles denote the solutions found by the peak finder routine. As seen in the peaks for the oxidizer plenum, structural frequencies sometimes dominated those of the wave system, and a lower frequency threshold was often used to ensure comparison with the reported video frequency. Overall, the dominant frequency from the PCBs closely followed that of the wave system in the combustor as seen by the high-speed camera.

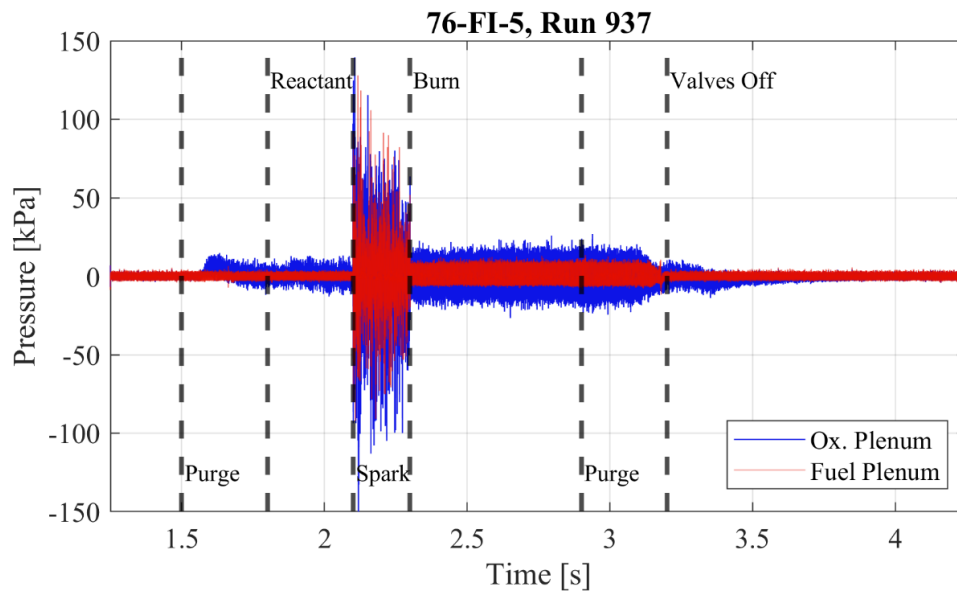


Figure 3.8: Example raw data from PCB sensors.

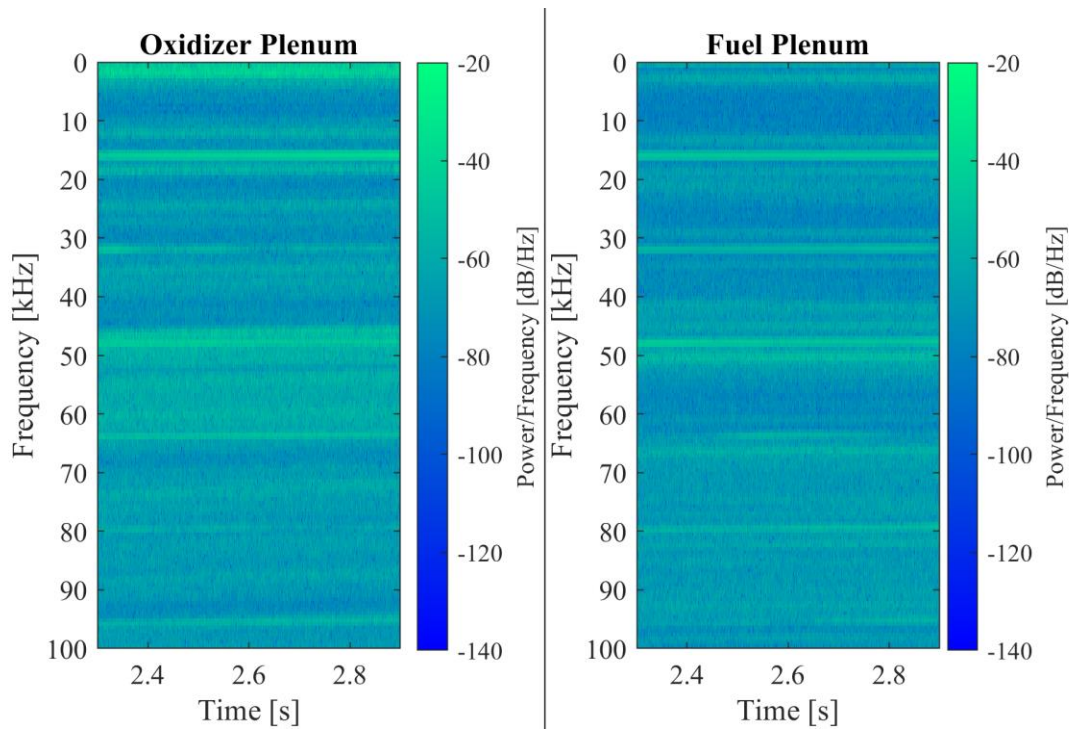


Figure 3.9: Spectrograms of PCB data during burn for example run.

### 76-FI-5, Run 937

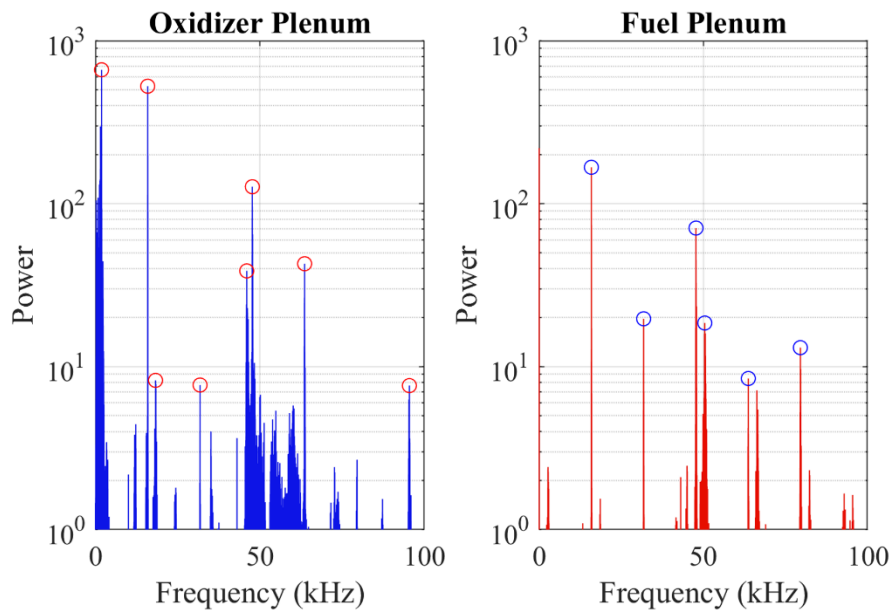


Figure 3.10: Power spectrum of PCB data from example run with peak finder routine results highlighted.

The reported results for the example run are given below in Table 3-2 based on averaging over the last 100 msec of the burn for the data presented in Figure 3.1 through Figure 3.10.

Table 3-2: Reported results for Run 937 of 76-FI-5 configuration.

|                                      |                            |                                       |            |
|--------------------------------------|----------------------------|---------------------------------------|------------|
| <b>Mass Flux</b>                     | 240.21 kg/s/m <sup>2</sup> | <b>Equivalence Ratio</b>              | 1.176      |
| <b>Mass Flow Rate</b>                | 268.65 g/s                 | <b>Burn Duration</b>                  | 0.696 sec  |
| <b>Back Pressure</b>                 | 100.2 kPa                  | <b>Initial Engine Temperature</b>     | 293 K      |
| <b>Wave Number</b>                   | 2                          | <b>Counter Rotation</b>               | None       |
| <b>Video Frequency</b>               | 15.84 kHz                  | <b>Periphery Speed</b>                | 1893 m/s   |
| <b>Chapman-Jouguet Speed</b>         | 2471 m/s                   | <b>Fraction of CJ Speed</b>           | 0.766      |
| <b>Oxygen Plenum Frequency (PCB)</b> | 15.89 kHz                  | <b>Methane Plenum Frequency (PCB)</b> | 15.89 kHz  |
| <b>Oxygen Orifice Pressure</b>       | 5991.5 kPa                 | <b>Methane Orifice Pressure</b>       | 5500.0 kPa |
| <b>Oxygen Orifice Temperature</b>    | 297 K                      | <b>Methane Orifice Temperature</b>    | 295 K      |
| <b>Oxygen Plenum Pressure</b>        | 1309.5 kPa                 | <b>Methane Plenum Pressure</b>        | 1399.4 kPa |
| <b>Oxygen Plenum Temperature</b>     | 303 K                      | <b>Methane Plenum Temperature</b>     | 301 K      |
| <b>Pressure at z = 0.0 mm</b>        | 368.5 kPa                  | <b>Pressure at z = 22.2 mm</b>        | 323.3 kPa  |
| <b>Pressure at z = 34.9 mm</b>       | 292.3 kPa                  | <b>Pressure at z = 47.6 mm</b>        | 281.1 kPa  |
| <b>Pressure at z = 60.3 mm</b>       | 255.4 kPa                  | <b>Pressure at z = 73.0 mm</b>        | 238.4 kPa  |
| <b>Pressure at z = 85.7 mm</b>       | 68.4 kPa                   | <b>Pressure at z = 98.4 mm</b>        | 59.5 kPa   |

### 3.2. FULL DATA SET BY CONFIGURATION

After the processing for each individual run was completed, repeated runs were averaged together. The constituents of each combined data point had individual mass flow rates and equivalence ratios within 2.5% of the group's average. Furthermore, each exhibited identical wave dynamics in terms of wave number, counter wave presence, and strength designation. An example of this reduction in data is shown across Figure 3.11 and Figure 3.12, where data from 63 runs was reduced to 21



averaged points for the 76-FI-5 configuration. In the data presented, whenever counter waves were present, the counter wave count always matched that of the primary system.

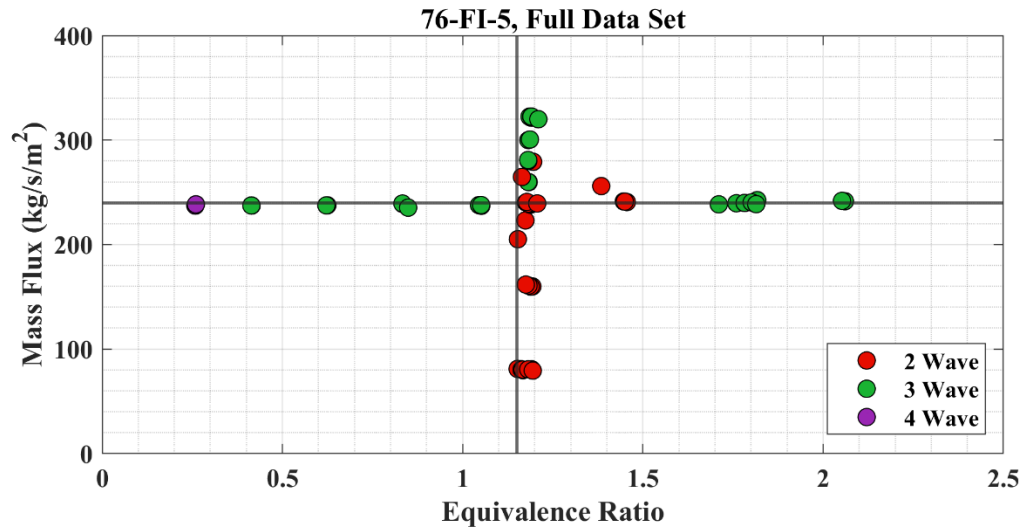


Figure 3.11: Full data set for 76-FI-5 configuration.

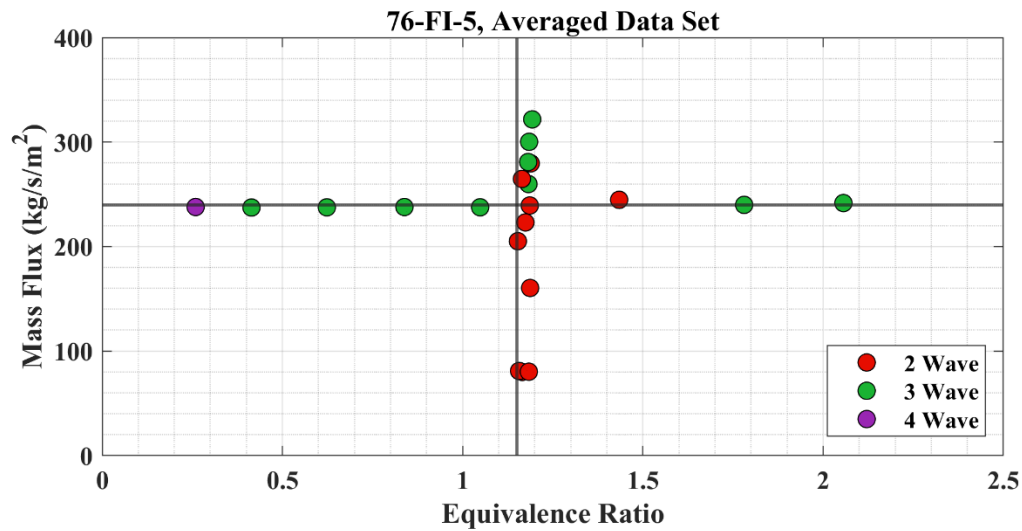


Figure 3.12: Reduced data set from averaging repeat runs for 76-FI-5 configuration.

### 3.2.1. 76-FI-5 RESULTS

Wave number results based on mass flux and equivalence ratio for the 76-FI-5 configuration is displayed in the previous subsection in Figure 3.12. This configuration did not display counter wave activity in any of the selected test conditions. The maximum wave number of this configuration, and of all data presented, was four waves, recorded near an equivalence ratio of

0.25. Wave number increased with mass flux, transitioning from two to three waves between 250 and 300 kg/s/m<sup>2</sup>. Within this transition region, both two- and three-wave modes were seen with repeated test conditions. With equivalence ratio, wave number was maximum at low and high fill conditions, with a minimum of two waves seen between equivalence ratios of 1.15 and 1.4.

### 3.2.2. 51-FI-5 RESULTS

The data set obtained for the 51-FI-5 configuration encompasses the largest collection of non-repeated test conditions, as well as the most variety in wave dynamics. Like with the 76-FI-5 configuration, the highest wave counts were seen in high flux, low equivalence ratio fill conditions. The transition to deflagrative operation was seen earliest in high equivalence ratio tests, with the transition occurring at lower equivalence ratio values as mass flux was reduced. Counter waves, regardless of strong or weak designation, appeared near transitions to deflagration and changes in wave number. A transition in wave number from one to two waves was seen in the equivalence ratio sweep of 1.15 near the mass flux of 243 kg/s/m<sup>2</sup> like in the 76-FI-5 configuration (where the transition was between two and three waves), but at a slightly lower flux.

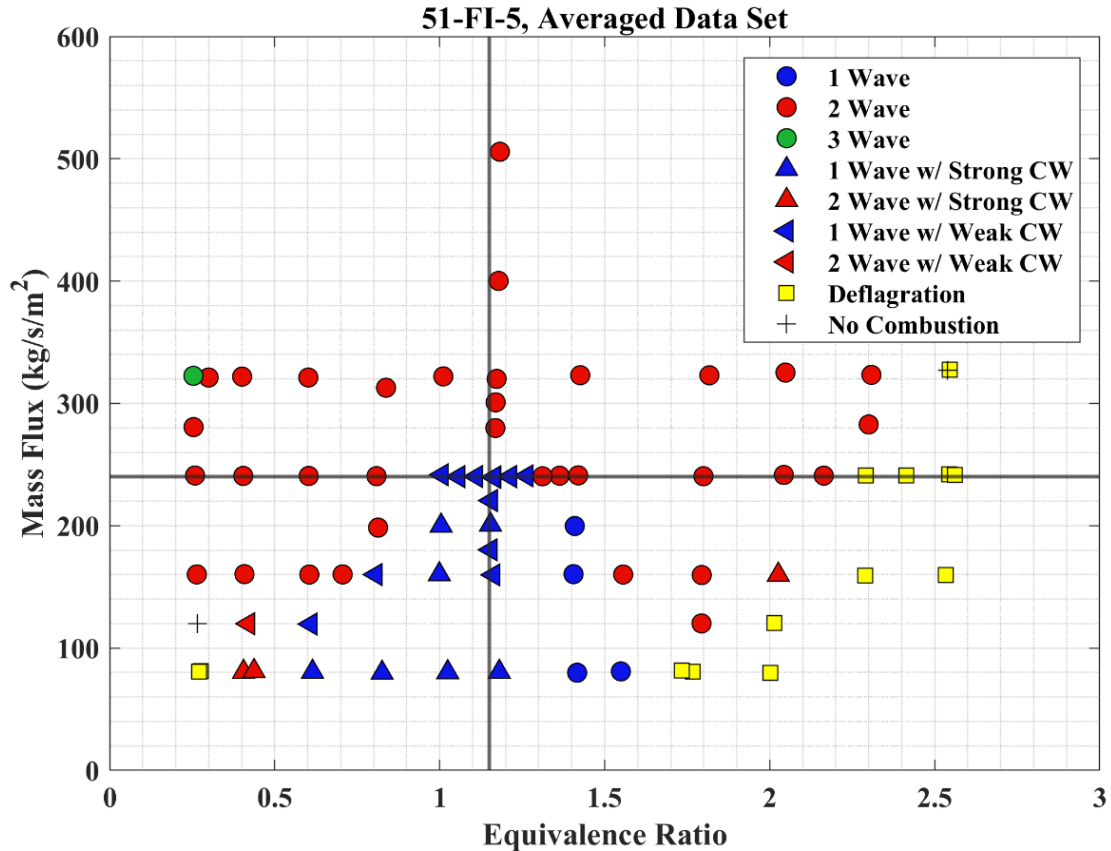


Figure 3.13: Reduced data set from averaging repeat runs for 51-FI-5 configuration.

### 3.2.3. 25 MM RESULTS

Inner core failures were seen in 25 mm configurations from inadequate cooling between runs. These failures led to an incomplete data set with the 25-FI-7 configuration, as replacement cores for the 25-FI-5 configuration were prioritized since results would be used in both scaling studies. The 25-FI-7 results are shown in Figure 3.14, which only showed one-wave operation along a partial equivalence ratio sweep. In order to combat core failure, different core mounting methods were explored to maximize thermal mass. Repeated runs with the different inner cores of the same exterior dimensions showed no discernable changes in results. A comparison of core post-fire conditions, including intermediate failures, is shown in Figure 3.15. Scorch marks from repeated igniter firing can be seen on the larger cores.

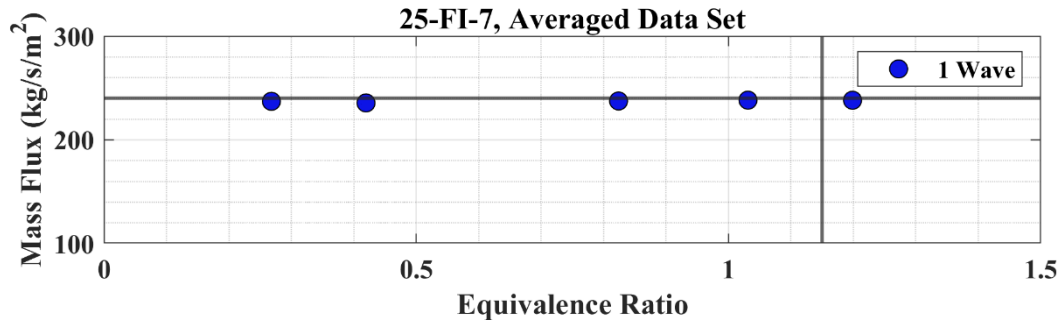


Figure 3.14: Reduced data set from averaging repeat runs for 25-FI-7 configuration.



Figure 3.15: Post-fire condition of inner cores, including core failures.

Where the 51 mm and 76 mm configurations with the same annular gap width had the highest wave counts at low equivalence ratio, high flux fill conditions, the 25-FI-5 configuration demonstrated steady one-wave operation within the matching quadrant, as seen in Figure 3.16. Reducing flux led to either the appearance of a weak counter wave, or cessation of detonative operation. Along the constant mass flux sweep of  $243 \text{ kg/s/m}^2$ , detonative, one-wave operation was maintained up to an equivalence ratio near 1.8. At sufficiently low mass flux on the constant equivalence ratio sweep of 1.15, a repetitive pulsing mode appeared. This pulsing mode was characterized with the high-speed camera by a regular, bright luminosity within the annulus, followed by a severe darkening during a period of filling, akin to the longitudinal pulsing operation

of a pulse detonation engine. The average frequency of the pulsing was near 16.6 kHz for the included runs.

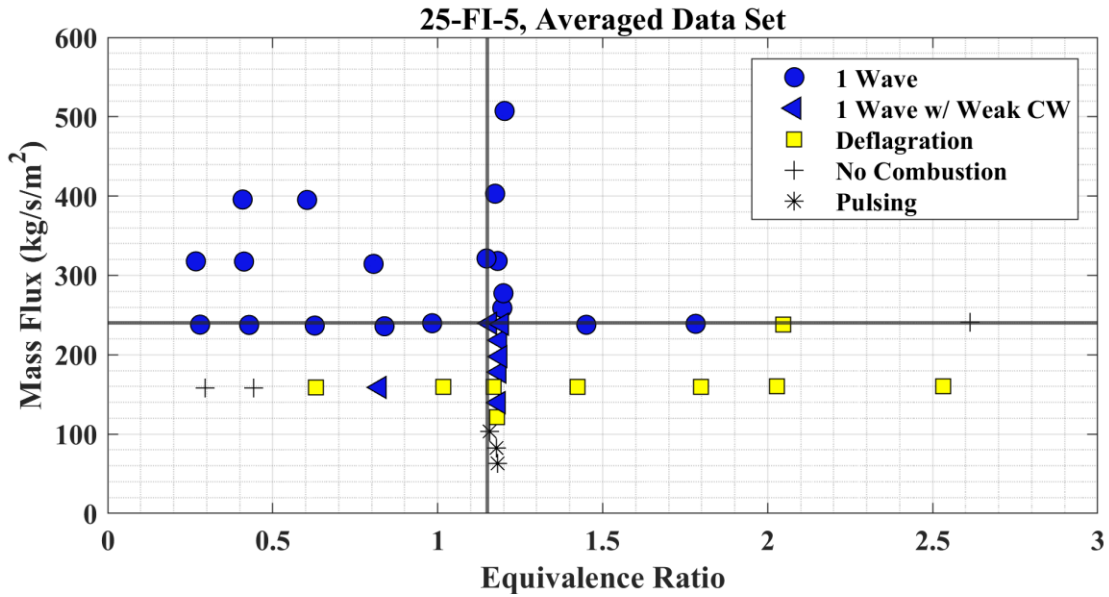


Figure 3.16: Reduced data set from averaging repeat runs for 25-FI-5 configuration.

The mass flux and equivalence ratio sweeps for the 25-FI-3 configuration are shown in Figure 3.17. Results near the intersection of the sweeps, and at high flux, demonstrated steady one-wave operation. Near the highest equivalence ratio tested of 2.5, deflagrative operation was seen, and a weak counter wave appeared at the lowest flux condition tested. The lowest equivalence ratios along the constant mass flux of 243 kg/s/m<sup>2</sup> demonstrated chaotic and unsteady wave behavior, with multiple co- and counter-rotating waves present, and has been classified here as unstable. A stable one-wave system with a constant, strong counter wave was seen at the boundary between the unstable and one-wave operating modes.

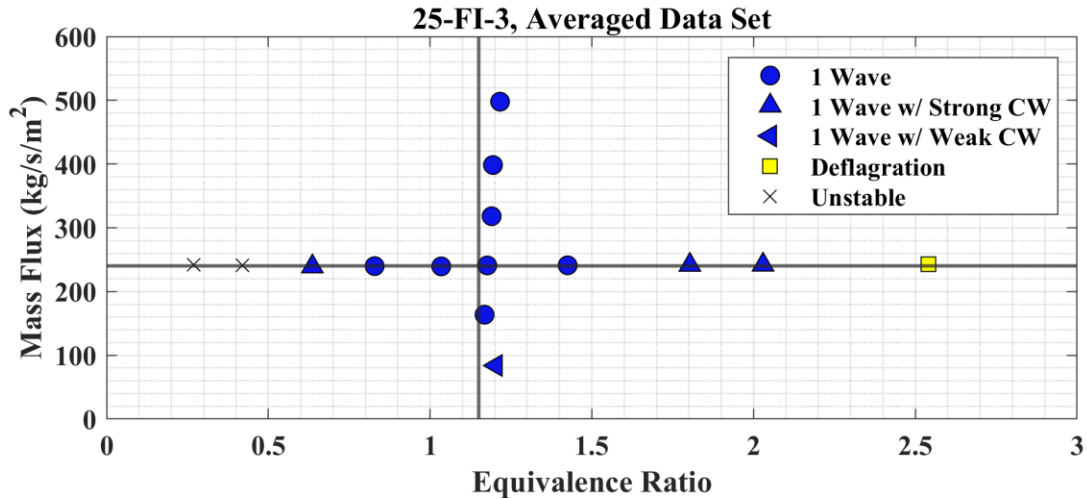


Figure 3.17: Reduced data set from averaging repeat runs for 25-FI-3 configuration.

Experiments with the coreless 25 mm configuration, shown in Figure 3.18, did not exhibit counter wave activity, and primarily demonstrated steady one-wave or deflagrative operating modes. A longitudinal pulsing mode, similar to that seen in the 25-FI-5 engine at low mass flux, was seen in a low flux, high equivalence ratio test condition. The coreless configuration operated with one wave above 300 kg/s/m<sup>2</sup>, and first lost detonative operation at low equivalence ratio when mass flux was reduced to the 243 kg/s/m<sup>2</sup> equivalence ratio sweep. Further reduction of mass flux reported only deflagrative operation, except near the equivalence ratio values of 0.4 and 0.6.

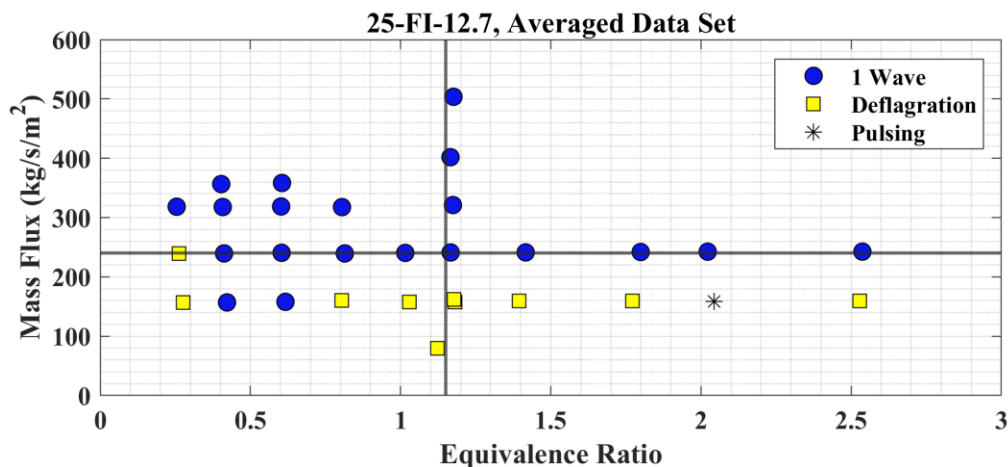


Figure 3.18: Reduced data set from averaging repeat runs for coreless 25-FI-12.7 configuration.

### 3.3. EXPERIMENTAL SCALING COMPARISONS

For this subsection, results grouped by the geometric changes in each scaling study are presented. Compared data only includes test conditions along the constant mass flux sweep of  $243 \text{ kg/s/m}^2$  or constant equivalence ratio (ER) sweep of 1.15, since they were shared by all configurations. Trends in mass flow rate, combustor pressure, wave frequency, wave speed, and finally wave speed compared to the Chapman-Jouguet (CJ) detonation speed, with respect to wave dynamics, are included. All configurations have a pressure station near 65% of the combustor's length, and thus this station was used for combustor pressure comparisons. Wave speed used the outer circumference of the respective combustor for the length scale. Finally, the CJ wave speed was estimated based on post-injection pressures, which will be discussed in detail in Section 4.

#### 3.3.1. RADIUS OF CURVATURE, CONSTANT EQUIVALENCE RATIO

Data pertaining to radius of curvature scaling along the cross leg of constant equivalence ratio is compiled in Figure 3.19 through Figure 3.22. Like in the results of individual configurations, wave number increased with mass flux and counter waves were common near transitions between one and two waves. Wave number also increased with radius of curvature (Figure 3.19). In the three engine scales of 76, 51, and 25 mm a pressure station was located at 62.4%, 66.3%, and 65.0% of their respective combustor lengths. At these stations, recorded pressure values were insensitive to both the wave mode and radius of curvature, provided the operating mode was detonative (Figure 3.20). These pressure results also rose linearly with mass flux and were similar to each other at each mass flux condition, with their variations attributable to the difference in exact percent axial location. The dominant wave frequency also increased with mass flux, and discontinuously increased when additional waves were added (Figure 3.21). Conversely, the wave speed,  $D_{spin}$ , which applies the appropriate length scale and wave number, is shown to decrease as additional

waves were added, although reported values still climb with increasing mass flux (Figure 3.22). As CJ speed has a strong dependence on equivalence ratio, especially compared to the influence of pre-detonation pressure, a comparison of the wave speed to the CJ speed provides no new trends compared to the wave speed itself since equivalence ratio is held constant in these plots. For this subset of data, the ratio between the wave speed and CJ speed fell within the range of 0.50 to 0.80.

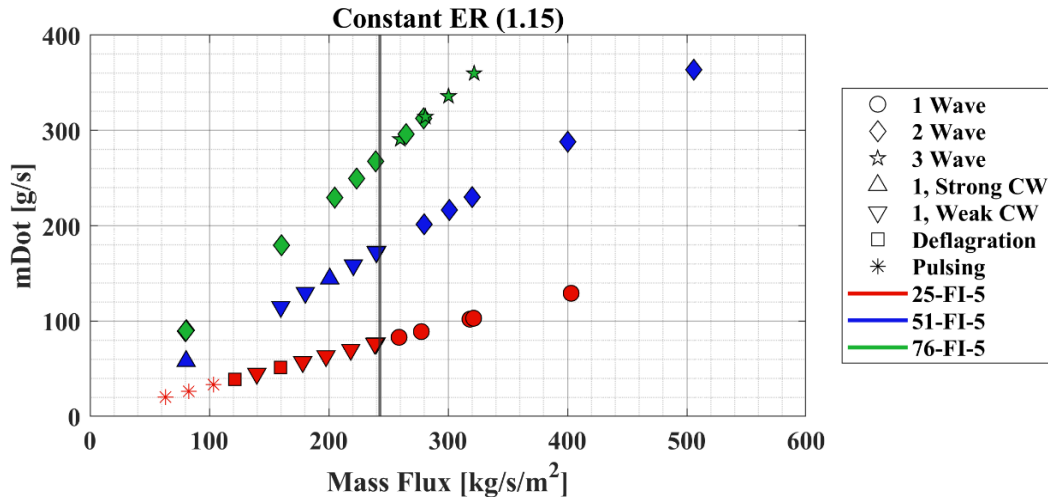


Figure 3.19: Radius of curvature data, constant equivalence ratio, mass flow rate variation.

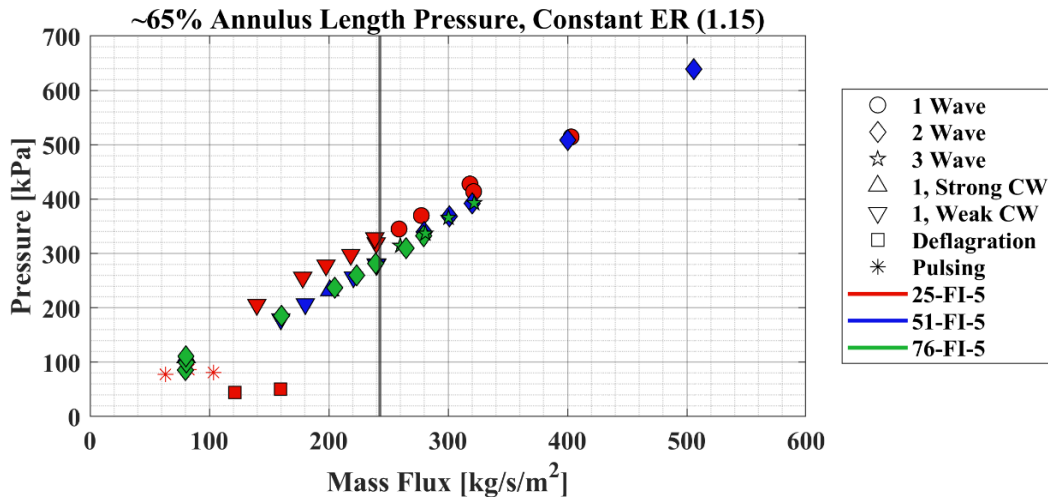


Figure 3.20: Radius of curvature data, constant equivalence ratio, combustor pressure variation.



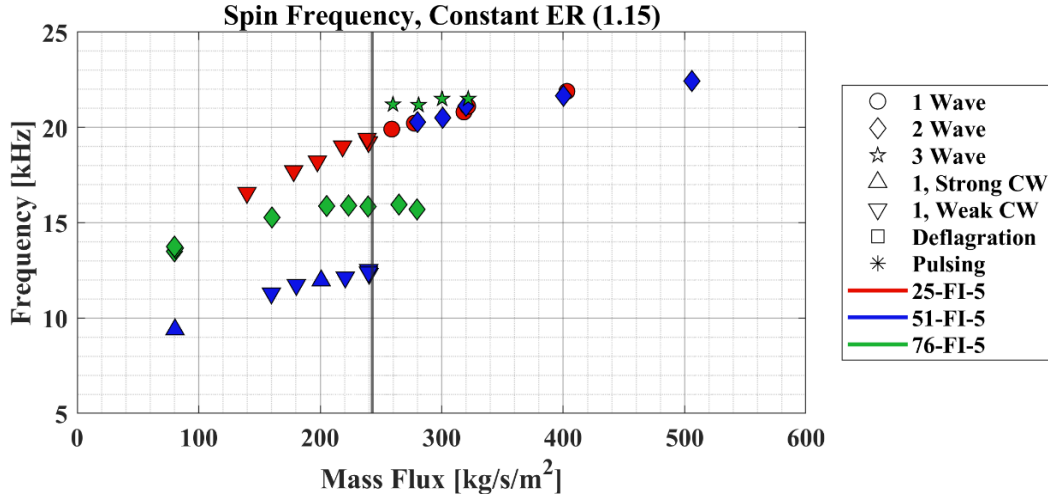


Figure 3.21: Radius of curvature data, constant equivalence ratio, video frequency variation.

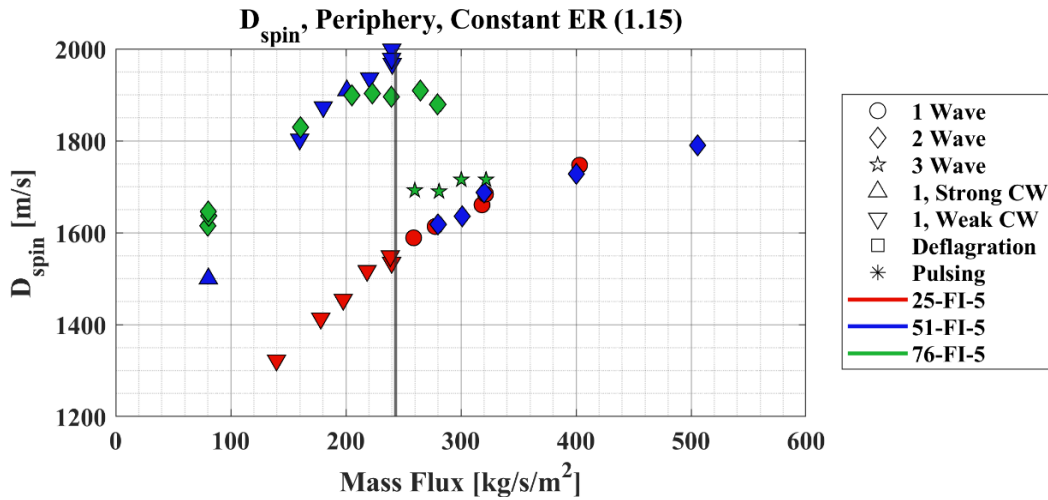


Figure 3.22: Radius of curvature data, constant equivalence ratio, wave speed variation.

### 3.3.2. RADIUS OF CURVATURE, CONSTANT MASS FLUX

Radius of curvature data within the constant mass flux sweep is shown in Figure 3.23 through Figure 3.27. Between equivalence ratios of 1 and 1.4, the wave count was either at a minimum or exhibited a counter wave when only one wave was present in the neighboring test conditions (Figure 3.23). In the same region, combustor pressures produced a maximum for each configuration (Figure 3.24). Non-detonative operation was seen at high equivalence ratios, with the boundary moving closer to stoichiometric as radius of curvature was reduced. Strictly co-rotating wave systems were seen at equivalence ratios lower than stoichiometric. Wave frequency

followed the same discontinuity trends discussed in the previous subsection across wave number discontinuities, but the data was comparatively constant with equivalence ratio (Figure 3.25). A local maximum in wave frequency for each configuration occurred near an equivalence ratio of 0.6. The location of these maxima also translated to wave speed, but discontinuities from wave number changes placed global wave speed maxima near an equivalence ratio of 1.15 in the 51 and 76 mm engines (Figure 3.26). When the wave speed was compared to the CJ speed, results from equivalence ratios lower than stoichiometric were elevated and the local maxima in this ratio were found to be closer to an equivalence ratio of 0.4 (Figure 3.27). As equivalence ratio was increased further, the ratio of wave speed to CJ speed monotonically decreased (neglecting the discontinuous jumps from wave number changes).

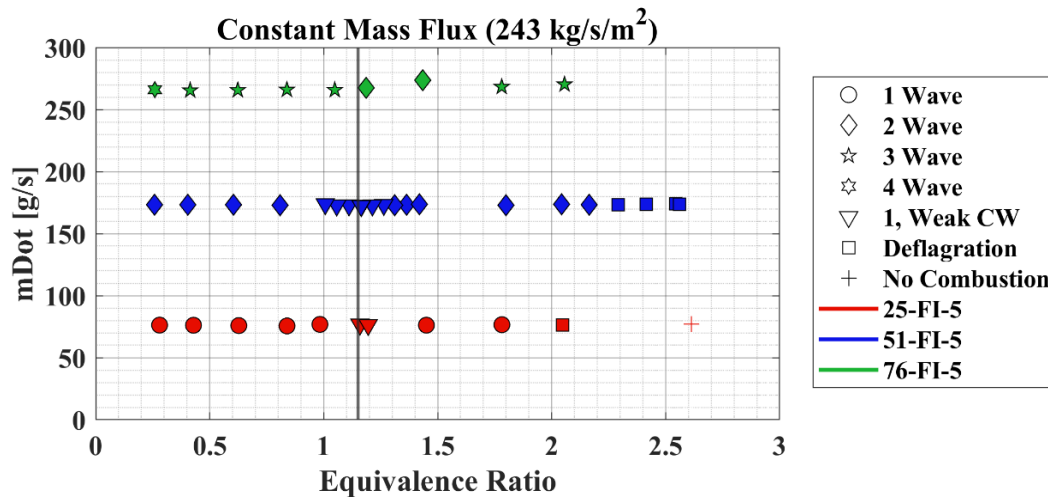


Figure 3.23: Radius of curvature data, constant mass flux, mass flow rate variation.

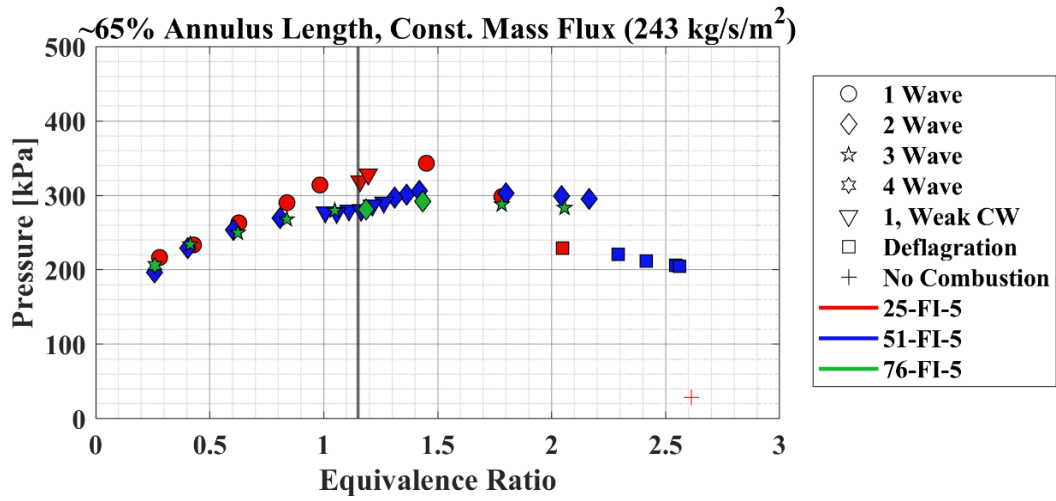


Figure 3.24: Radius of curvature data, constant mass flux, combustor pressure variation.

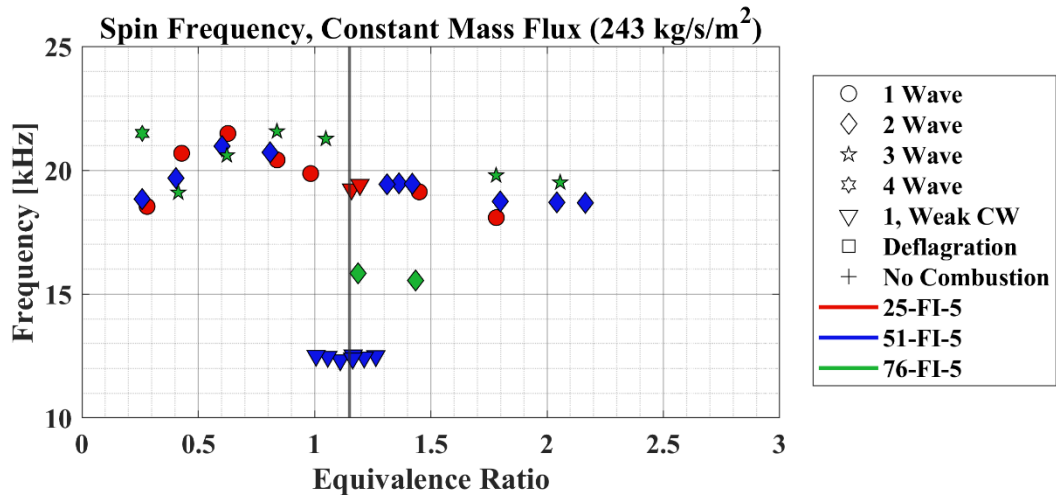


Figure 3.25: Radius of curvature, constant mass flux, video frequency variation.

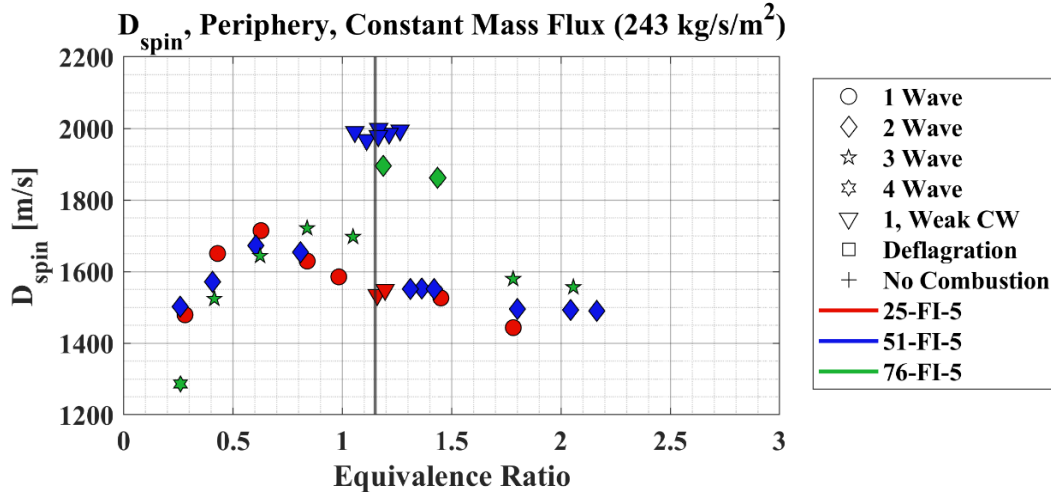


Figure 3.26: Radius of curvature data, constant mass flux, wave speed variation.

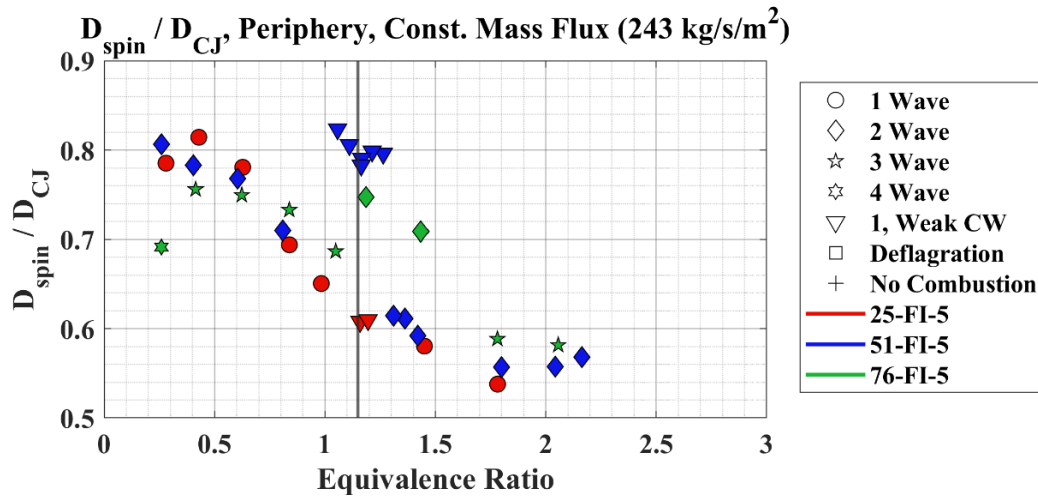


Figure 3.27: Radius of curvature data, constant mass flux, wave speed versus CJ speed variation.

### 3.3.3. ANNULAR GAP WIDTH, CONSTANT EQUIVALENCE RATIO

Annular gap width scaling data along the constant equivalence ratio sweep of 1.15 is displayed in Figure 3.28 through Figure 3.30. As this data set only includes data from the 25 mm scale, the outer circumference is the same for each configuration. Furthermore, all detonative results had one wave for the primary system. Therefore, a wave frequency comparison is omitted as the result is simply a scaling of the results of wave speed to the range of 16 to 24 kHz. As in Subsection 3.3.1, comparison of wave speed to CJ speed provides no new information and is also excluded, and varied between 0.50 and 0.80.

In this data subset, better wave stability, denoted by the absence of a counter wave, is seen with higher mass flux (Figure 3.28). Combustor pressure, which is now reported at a matching axial location, is again independent of the wave dynamics and increased with mass flux (Figure 3.29). For configurations with cores, wave speed increased with mass flux, but was comparatively constant for the coreless configuration (Figure 3.30).

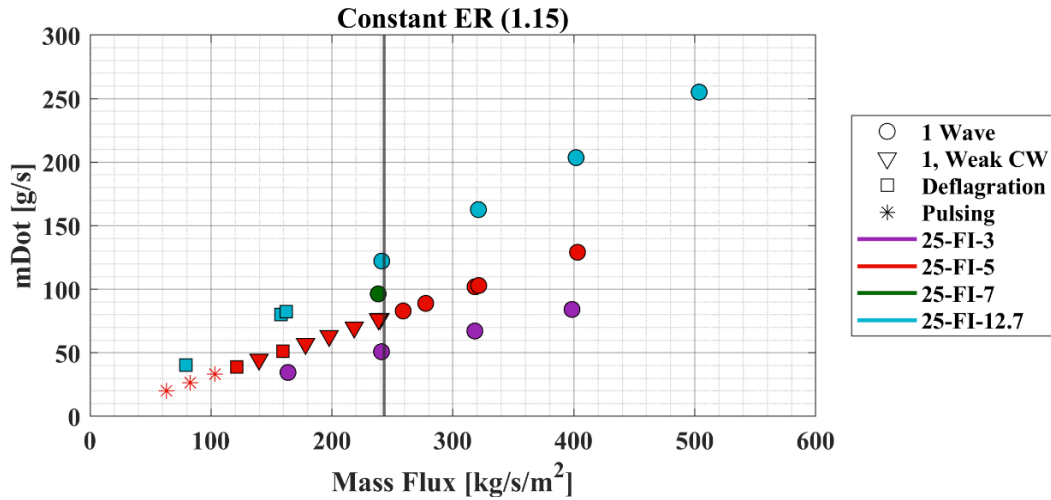


Figure 3.28: Annular gap width data, constant equivalence ratio, mass flow rate variation.

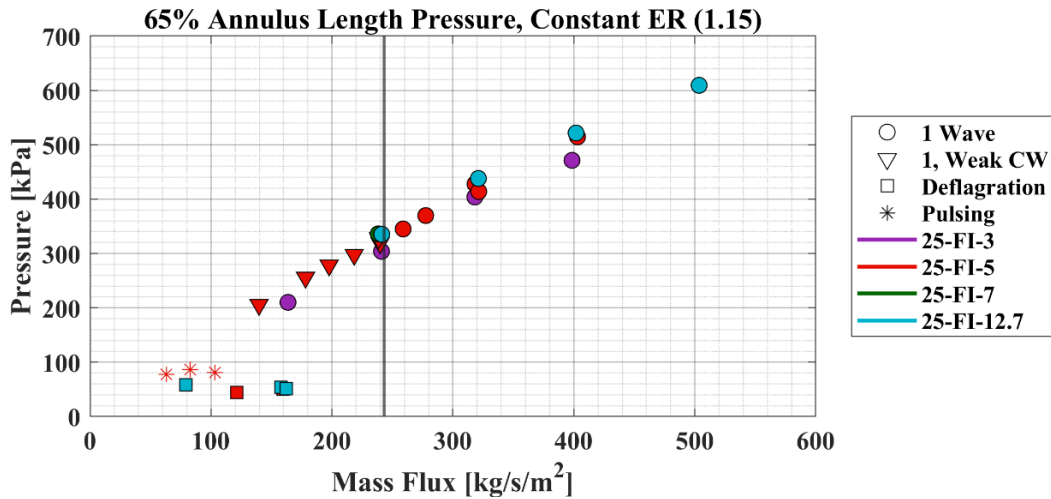


Figure 3.29: Annular gap width data, constant equivalence ratio, combustor pressure variation.

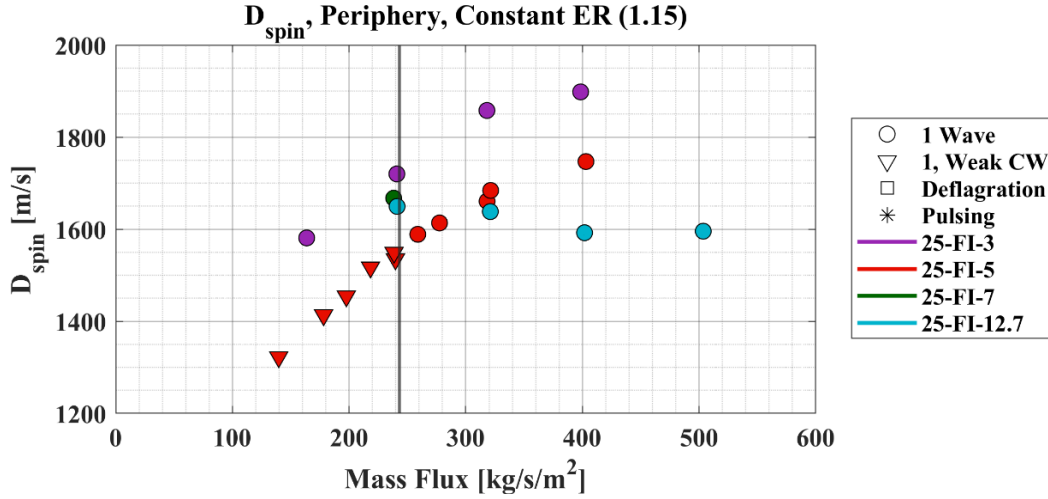


Figure 3.30: Annular gap width data, constant equivalence ratio, wave speed variation.

### 3.3.4. ANNULAR GAP WIDTH, CONSTANT MASS FLUX

Comparison of various parameters along the constant mass flux sweep of  $243 \text{ kg/s/m}^2$  within the annular gap width study are shown in Figure 3.31 through Figure 3.34. As in the previous subsection, comparison of wave frequency is omitted as it merely scales the wave speed results to the range of 17 to 29 kHz. Overall, wave stability was insensitive to changes in equivalence ratio unless nearing a detonative limit, with the exception of weak counter wave present in the 25-FI-5 configuration at an equivalence ratio near 1.15 (Figure 3.31). The coreless configuration, 25-FI-12.7, was unique in exhibiting no counter wave phenomena across the full equivalence ratio sweep. Combustor pressure data overlaid across configurations, with local maxima again seen in the data set at slightly fuel-rich equivalence ratios (Figure 3.32). The coreless configuration also exhibited a pressure spike at an equivalence ratio of 0.6.

In terms of wave speed, all configurations reported progressively slower speeds as the equivalence ratio increased from 0.8 (Figure 3.33). When equivalence ratio was lowered from this mark in the coreless configuration, reported wave speeds elevated and separated from the rest of the data. In fact, the result at an equivalence ratio near 0.4 was associated with the highest wave

frequency recorded amongst all test conditions and configurations. 25-FI-7 also demonstrated an increase in wave speed as equivalence ratio was reduced beyond 0.8, but not as drastically. The 25-FI-5 configuration reached its wave speed maximum near an equivalence ratio of 0.6, then slowed for lower equivalence ratios. The smallest annular gap width, 25-FI-3, produced a weak counter wave near the equivalence ratio of 0.6, then ceased detonative operation. Comparing wave speed to the CJ speed propagated most of these trends (Figure 3.34). Maxima for all configurations occurred under fuel-lean conditions, and monotonically decreased from the configuration's maximum with increasing equivalence ratio. Large annular gap width, low injector-to-annulus area ratio configurations approached the CJ speed with their maxima. In fact, the frequency maximum seen in coreless operation at an equivalence ratio near 0.4 exceeds the computed CJ speed.

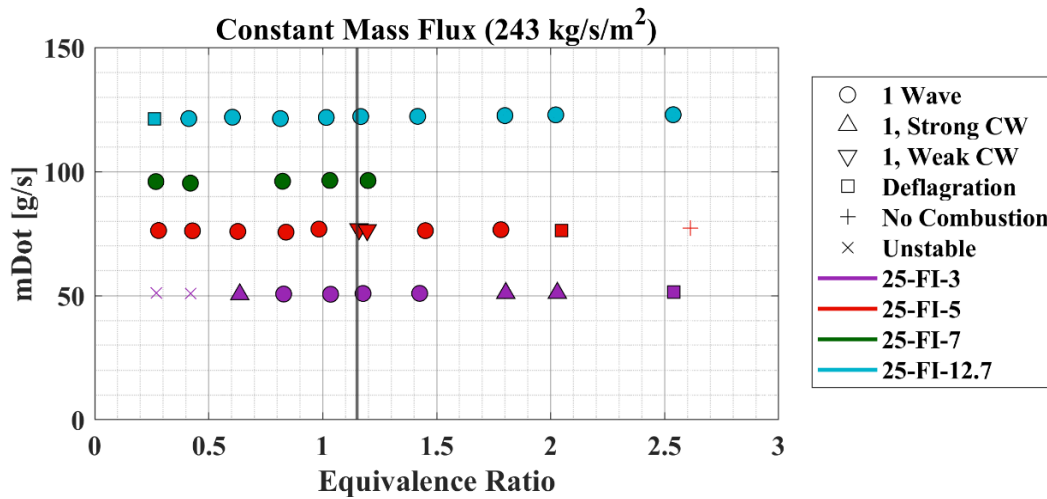


Figure 3.31: Annular gap width data, constant mass flux, mass flow rate variation.

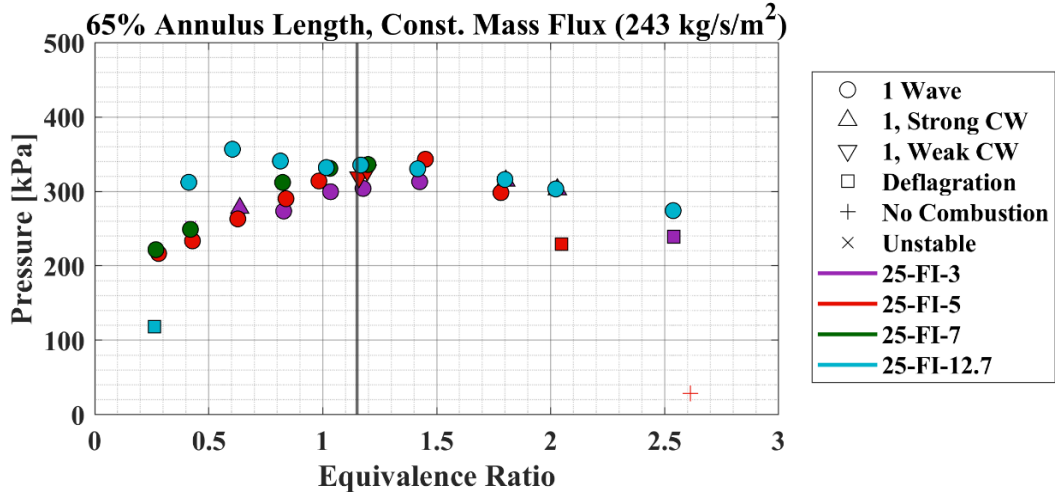


Figure 3.32: Annular gap width data, constant mass flux, combustor pressure variation.

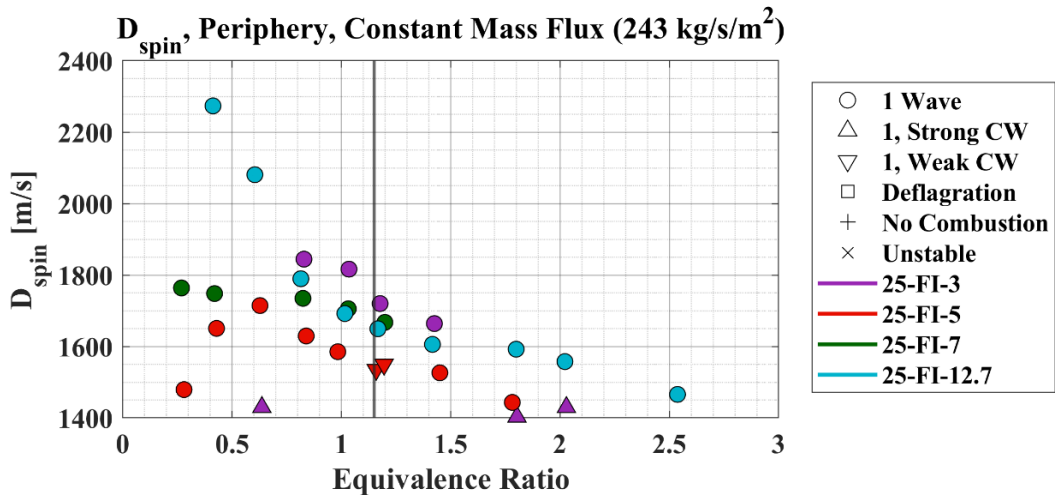


Figure 3.33: Annular gap width data, constant mass flux, wave speed variation.

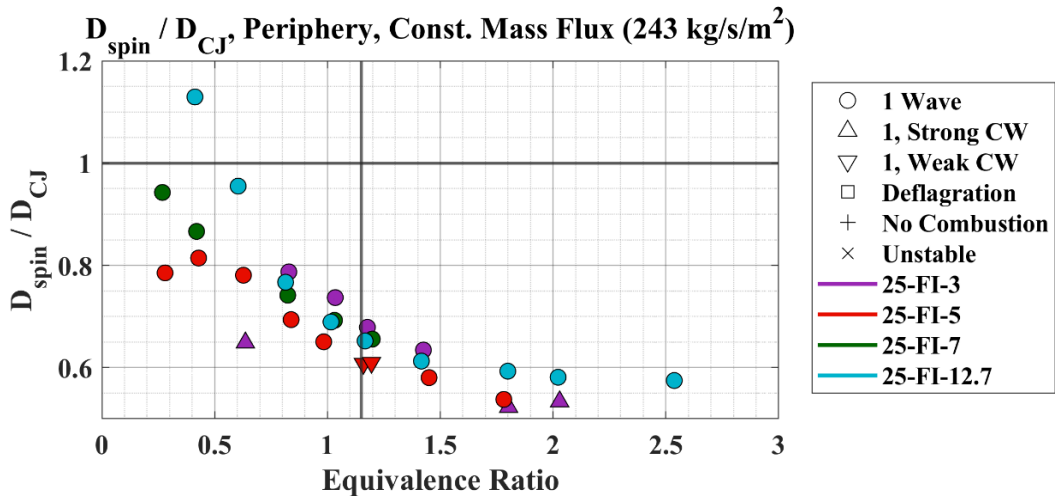


Figure 3.34: Annular gap width data, constant mass flux, wave speed versus CJ speed variation.



### 3.4. BACK PRESSURE EFFECTS

Initial testing with 25 mm configurations at low mass flux exhibited operation influenced by back pressure. To further explore this influence, back pressure sweeps at progressively lower mass flux were conducted in the 25-FI-5 configuration. For this test series, three runs were conducted at a single mass flux under atmospheric, half atmosphere, and near-vacuum back pressure conditions. Equivalence ratio was kept constant throughout the series. The results of this test series are displayed in Table 3-3.

Table 3-3: 25-FI-5 back pressure sweep results.

| 3 Run Average         |                                  |      | Wave Result at Back Pressure of... |              |                        |
|-----------------------|----------------------------------|------|------------------------------------|--------------|------------------------|
| Mass Flow Rate<br>g/s | Mass Flux<br>kg/s/m <sup>2</sup> | Phi  | 3.5 kPa                            | 52 kPa       | 97 kPa                 |
| 102.29                | 319.20                           | 1.19 | 1 wave                             | 1 wave       | 1 wave                 |
| 89.53                 | 279.40                           | 1.19 | 1 wave                             | 1 wave       | 1 wave                 |
| 83.36                 | 260.10                           | 1.18 | 1 wave                             | 1 wave       | 1, weak CW             |
| 76.57                 | 239.00                           | 1.17 | 1 wave                             | 1, weak CW   | 1, weak CW             |
| 70.51                 | 220.10                           | 1.17 | 1, weak CW                         | 1, weak CW   | 1, weak CW             |
| 63.76                 | 199.00                           | 1.18 | 1, weak CW                         | 1, weak CW   | 1, weak CW             |
| 57.36                 | 179.00                           | 1.17 | 1, weak CW                         | 1, weak CW   | 1, weak CW             |
| 51.38                 | 160.30                           | 1.16 | Deflagration                       | 1, weak CW   | 1, weak CW             |
| 44.96                 | 140.30                           | 1.17 | Deflagration                       | 1, weak CW   | 1, weak CW             |
| 38.97                 | 121.60                           | 1.16 | Deflagration                       | Deflagration | 1, weak CW             |
| 33.06                 | 103.20                           | 1.16 | Deflagration                       | Pulsing      | Unsteady<br>Detonation |
| 26.48                 | 82.60                            | 1.17 | Deflagration                       | Pulsing      | Unsteady<br>Detonation |
| 20.23                 | 63.10                            | 1.18 | Deflagration                       | Pulsing      | 1 wave                 |

Sensitivity to ambient back pressure for this configuration began between a mass flux of 260 and 280 kg/s/m<sup>2</sup>, with stable one-wave operation persistent at higher mass flux conditions. As mass flux was lowered, continued degradation of wave stability, denoted by the appearance of a weak counter wave, was witnessed, eventually leading to non-detonative operation. The pulsing

mode exhibited regular radial surges across the face of the inner core, and the unsteady detonation label represents irregular cessation and regeneration of weak multi-wave systems.

Outside of wave dynamics, back pressure sensitivity is most readily determined from the combustor axial pressure profiles. Furthermore, this metric may be preferred as back pressure sweeps at low mass flux in the 76-FI-5 configuration demonstrated minimal changes in wave dynamics, including no change in wave count, while having marked changes in axial pressure profiles as the combustor became back pressure sensitive [60]. The axial pressure profiles of a collection of low mass flux tests from a subset of the 76-FI-5 results, previously published in [60], are shown in Figure 3.35a. At the  $80 \text{ kg/s/m}^2$  test condition, increasing back pressure generated a shallower axial pressure profile and lower front endwall pressure. When normalizing these results to the front endwall pressure, shown in Figure 3.35b, the back pressure insensitive profiles of  $180 \text{ kg/s/m}^2$ , 98.1 kPa back pressure and  $80 \text{ kg/s/m}^2$ , 42.6 kPa back pressure, closely overlaid each other. This phenomenon also applied to other feed conditions, as long as back pressure insensitivity was maintained.

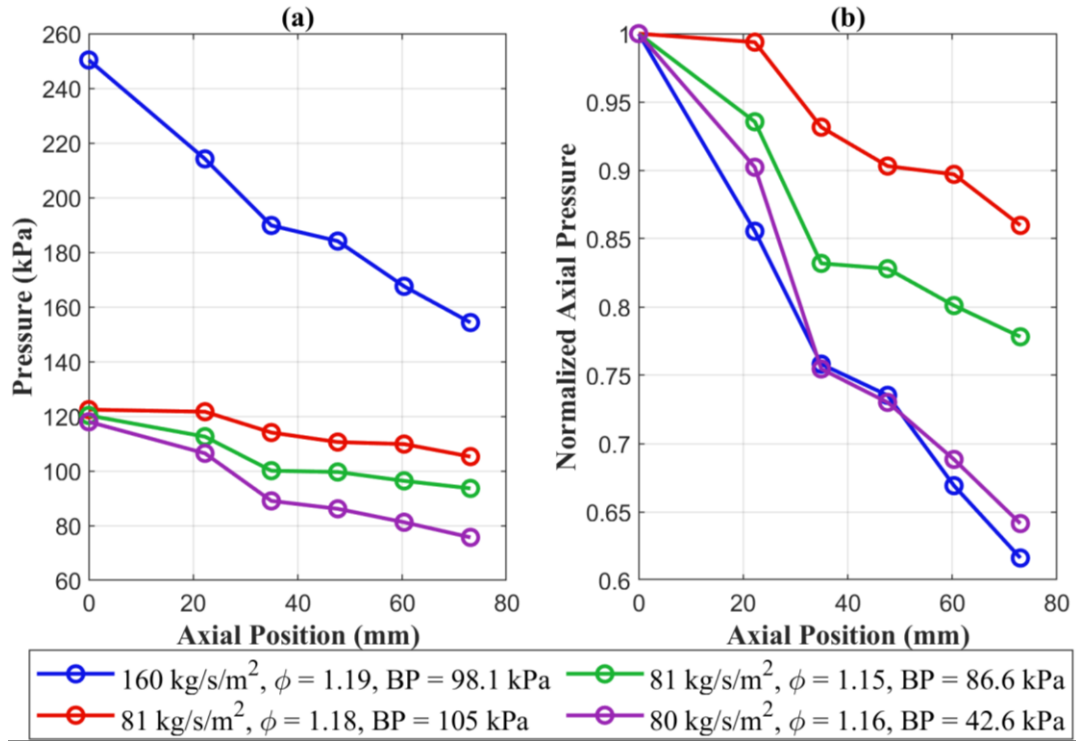


Figure 3.35: Low flux 76-FI-5 (a) axial pressure profiles and (b) axial pressure profiles normalized to front endwall pressure. Reproduced from [60].

## 4. SCALING ANALYSIS

Further analysis and comparisons of the experimental results are presented in this section, including the processes for computing thermodynamic properties. Focus will be placed on the data from the equivalence ratio and mass flux sweeps discussed in Sections 3.3.1 through 3.3.4 unless otherwise noted.

### 4.1. THERMODYNAMIC CALCULATIONS

#### 4.1.1. PRE-DETONATION PRESSURE DETERMINATION

In order to calculate detonation cell size and other thermodynamic properties, an input pressure is required in addition to the gas mixture. As only the 76-FI-5 configuration possessed a pressure tap prior to the detonation (through the front endwall), a basis for computing the pre-detonation pressure for the other configurations was needed. In pressure data already presented, such as in Figure 3.20 and Figure 3.29, pressures recorded at a station at or near 65% of the combustor length were predominantly influenced by the fill conditions, rather than either the wave mode or geometry. Mass flux demonstrated the strongest influence, with combustor pressures rising linearly as it was increased. Figure 4.1 shows the values recorded at the 65% pressure stations for detonative operating modes within the equivalence ratio sweeps for all configurations, where the largest differences were seen across configurations. Each individual configuration reported maxima near an equivalence ratio of 1.4, with a shallow decrease in pressure as equivalence ratio is increased with a relatively sharper decline in pressure as equivalence ratio is decreased. The coreless configuration acts as the outlier with a pressure rise at low equivalence ratio and maxima near 0.6, while also reporting the highest pressures at most test conditions. Variation in pressure at each test condition ranged from 6 to 16% when neglecting the coreless configuration, which had a maximum deviation from the 76 mm configuration of 42% at the coreless maxima at  $\phi = 0.6$ .

Some of the deviations between configurations can be attributed to small feed condition differences, in addition to not all configurations having a pressure station at exactly 65% of the combustor length. Further similarity in combustor pressures was discussed for Figure 3.35, where axial pressure profiles normalized to their respective front endwall pressures in the 76-FI-5 configuration coalesced a 41% variation in the 65% length pressure station across the equivalence ratio sweep (based on the lowest pressure of the configuration) to only a 7% variation in normalized pressure. As a final note, scaling test conditions by mass flux and maintaining injector characteristics led to similar pressure levels in the feed plenums when compared across configurations.

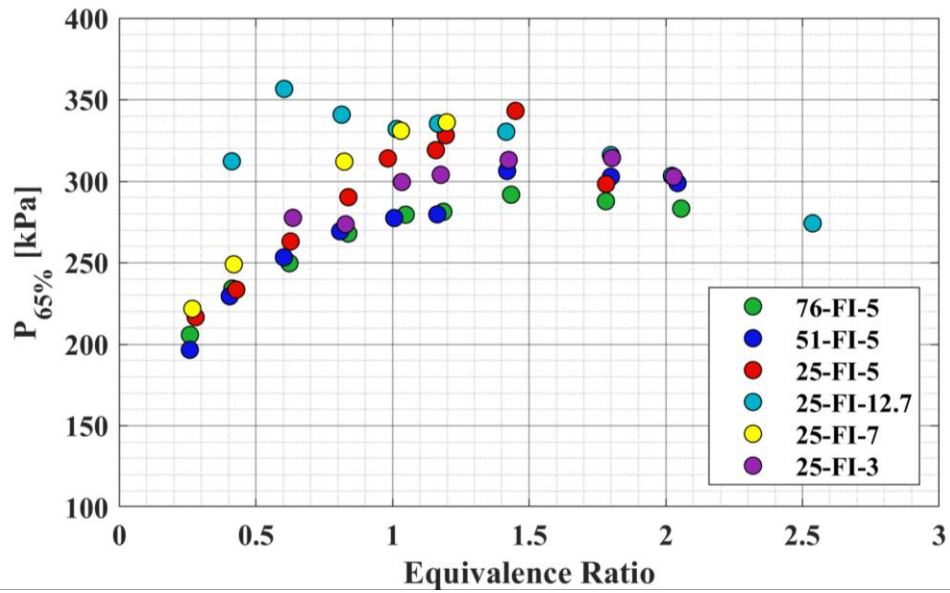


Figure 4.1: Combustor pressures at the 65% combustor length station across all configurations for detonative operating modes in the equivalence ratio sweep.

Motivated by this series of statements, a pressure map from the 65% length station to the front endwall pressure ( $p_0$ ) based on the 76-FI-5 configuration was created based on inputs of mass flux and equivalence ratio, then applied to the other configurations. Utilizing a second-order polynomial fit, an  $R^2$  value of 0.998 was achieved, and the fit itself is described by:

$$p_0 = -531.2 + 2.965 m_{flux} + 526.6 \phi + .0009336 m_{flux}^2 - 1.418 m_{flux} \phi - 54.73 \phi^2 \quad (4.1)$$

Where  $\phi$  represents equivalence ratio.

#### 4.1.2. DETONATION CELL SIZE CALCULATION

In order to calculate detonation cell size, the chemical kinetics software Cantera [61] was utilized, along with the Shock and Detonation Toolbox from Caltech's Explosion Dynamics Laboratory [62]. The required inputs for calculating detonation cell size were the pre-detonation pressure, temperature, and gas mixture, as well as the reaction mechanism. A range of pertinent pre-detonation pressures were calculated as described in the previous subsection, which thereby represented the mass flux influence on pre-detonation conditions. As both fuel and oxidizer plenum temperatures averaged near 300 K across the full data set and this temperature was regularly used as an initial condition for calculations in the references found in Section 1, the initial temperature was held constant at 300 K. The mixture of methane-oxygen was altered over the range of equivalence ratio covered in experiments and was assumed premixed. The GRI-Mech 3.0 combustion mechanism was selected due to its established accuracy and capabilities with mixtures pertinent to gaseous RDE operation, including methane-oxygen [63].

Calculation of the detonation cell size utilized the ZND model, and related induction time to detonation cell size through the methods described by Westbrook [64]. In doing so, induction time was defined as the time where the rate of temperature increase was at a maximum, which typically translated to the time it took the temperature to complete half of its total increase. After converting this to an induction length based on the post-shock gas velocity, the result was multiplied by a factor of 29, which represents the average relation between detonation cell size and induction length. To reach this factor, the average for unconfined hemispherical detonations over various fuel-air and fuel-oxygen mixtures based on experiments relating critical tube diameter

and induction length was computed, then combined with the number of transverse waves needed to initiate such a detonation at the critical tube diameter.

The resulting detonation cell sizes based on equivalence ratio sweeps at four different initial pressures can be seen in Figure 4.2. Not all intended cases reached convergence, and were either omitted or repeated with a slightly altered equivalence ratio. The expected trends were witnessed, with minimum cell width at slightly fuel rich conditions and cell size decreasing with the increase in initial pressure. The results also agree with those seen experimentally, such as the experimental results of Schumaker et al. of methane-oxygen at elevated pressures [11], although the calculated results tend to underestimate cell size, which could be attributed to the selected combustion mechanism. During intermediate calculations, the CJ detonation velocity based on equivalence ratio and the initial pressure was also calculated, and is shown in Figure 4.3. These values were used in comparisons of the previous section, such as in Figure 3.34.

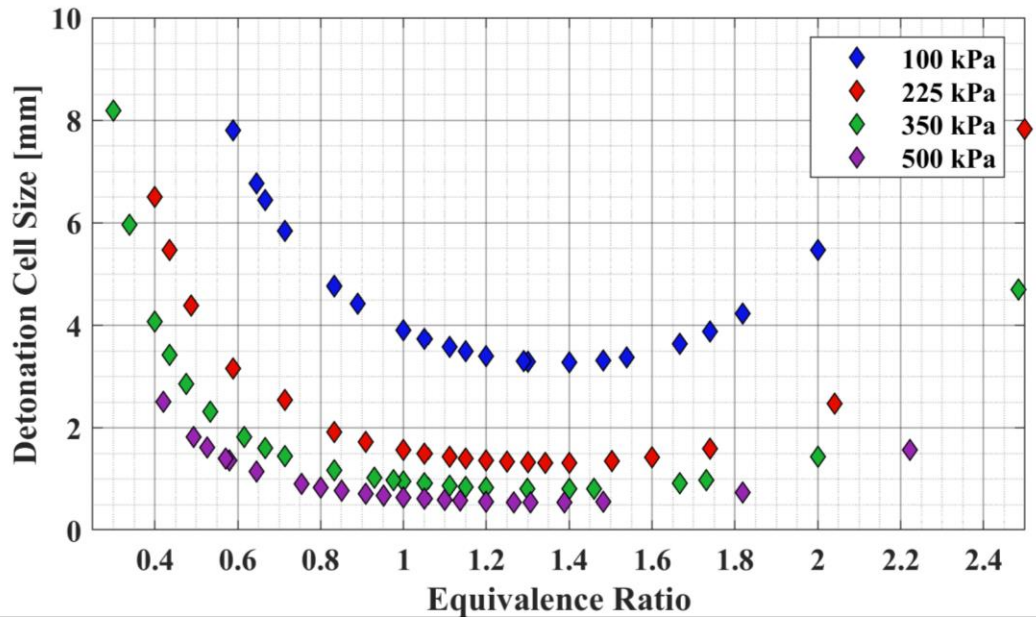


Figure 4.2: Calculated detonation cell size based on equivalence ratio and initial pressure for a methane-oxygen mixture.

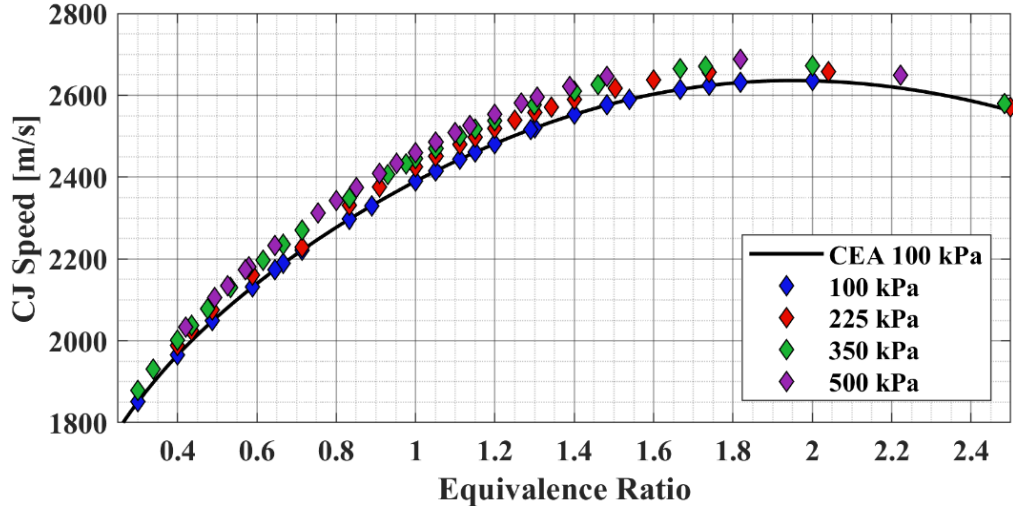


Figure 4.3: Calculated CJ detonation velocity based on equivalence ratio and initial pressure for a methane-oxygen mixture.

#### 4.1.3. HEAT RELEASE CALCULATION

For calculation of heat release, NASA's Chemical Equilibrium Applications (CEA) program was employed to generate the mass fractions at equilibrium [65]. Results were generated in the Chapman-Jouguet Detonation mode, and, as with detonation cell size, sweeps of pressure and equivalence ratio were performed with a constant initial temperature of 300 K. Query points, particularly in equivalence ratio, were selected near the test conditions used in experiments. To calculate heat release, the following equation was applied across the species in the reactants and products:

$$q = \sum_{\text{Reactants}} \frac{X_i \Delta h_{f,i}}{MW_i} - \sum_{\text{Products}} \frac{X_i \Delta h_{f,i}}{MW_i} \quad (4.2)$$

Where the heat release  $q$  is in the units of kJ per kilogram propellant mixture,  $X_i$  is the species mass fraction,  $\Delta h_{f,i}$  is the standard enthalpy of formation, and  $MW_i$  is the molecular weight. As oxygen,  $O_2$ , is an element in its standard state,  $\Delta h_{f,O_2}$  is equal to zero and the reactant summation only includes methane, with  $X_{CH_4}$  determined by the equivalence ratio. The standard enthalpy of formation and the molecular weight for each species of the products were retrieved from NIST



Chemistry Webbook [66]. The calculated heat addition based on equivalence ratio and pressure can be seen in Figure 4.4, alongside spline curves used to handle interpolation for experimental deviations in equivalence ratio. After computing a heat release value for each run, results were then multiplied by mass flux to generate the energy flux used in analysis.

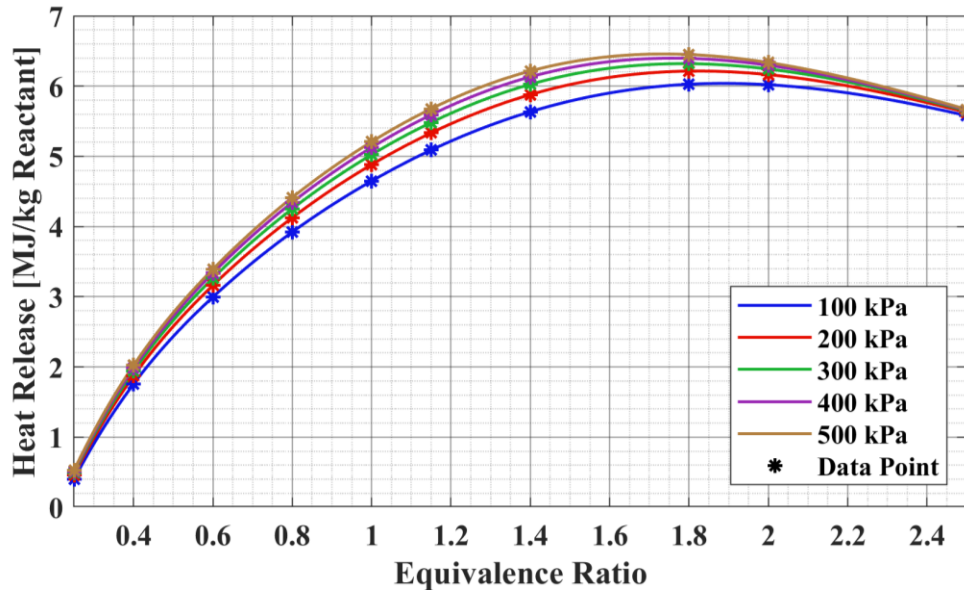


Figure 4.4: Calculated heat release based on CEA results with varied fill conditions.

#### 4.1.4. FILL HEIGHT DERIVATION

After a detonation wave has passed a circumferential position and consumed the propellant, fresh reactants begin to refill the combustor (section G of Figure 1.10). The maximum fill height achievable by the reactants is based on the axial velocity of the injected propellants and the time until the detonation wave returns. The velocity of the injected propellants can be found through rearrangement of the conservation of mass with the ideal gas law (Equation 1.4) applied to replace density, resulting in:

$$u = \frac{\dot{m} R_{gas} T_0}{p_0 A_e} \quad (4.3)$$

Where  $\dot{m}$  represents mass flow rate and  $A_e$  represents the annulus exit area, and whose ratio is defined as mass flux ( $m_{flux}$ ). The time,  $t$ , until the next detonation wave is governed by:

$$t = \frac{2\pi r_0}{N D_{spin}} \equiv \frac{1}{f} \quad (4.4)$$

Where  $r$  represents outer radius and  $N$  is the wave count of the primary system. Due to the fact  $D_{spin}$  of the included data utilized outer diameter in its calculation, the outer radius would be appropriate here. In doing so, it should be noted  $t$  becomes equivalent to the inverse of operating frequency,  $f$ . Combining Equations 4.3 and 4.4 nets the fill height,  $h$ , which is written as:

$$h = u * t = \frac{m_{flux} R_{gas} T_0}{p_0} \frac{1}{f} \quad (4.5)$$

This fill height equation closely follows that in other publications, such as Ikema et al. [67]. As  $p_0$  was calculated in a previous section based on fill conditions, fill height is a function of mass flux, equivalence ratio, and operating frequency of the primary system. A plot of the calculated fill height compared to equivalence ratio for all runs in the mass flux and equivalence ratio sweeps is shown in Figure 4.5. Fill height ranged from 2 to 7 mm over the included runs, with majority of the data between 2 and 4 mm (which corresponds primarily to the equivalence ratio sweeps). When comparing one wave operation to its counterparts with a counter wave, counter wave operation always showed a larger fill height. This indicates one of the purposes of a counter wave is to consume reactants not processed by the primary wave, which is in line with the counter wave presence near transition points. In such cases, there was not enough propellant to support the presence of an additional wave, leading to a weaker counter wave. Mixing effects may contribute to the inconsistent appearance of the counter wave as well as its relative strength to the primary system, but was not explored within the two experimental studies.

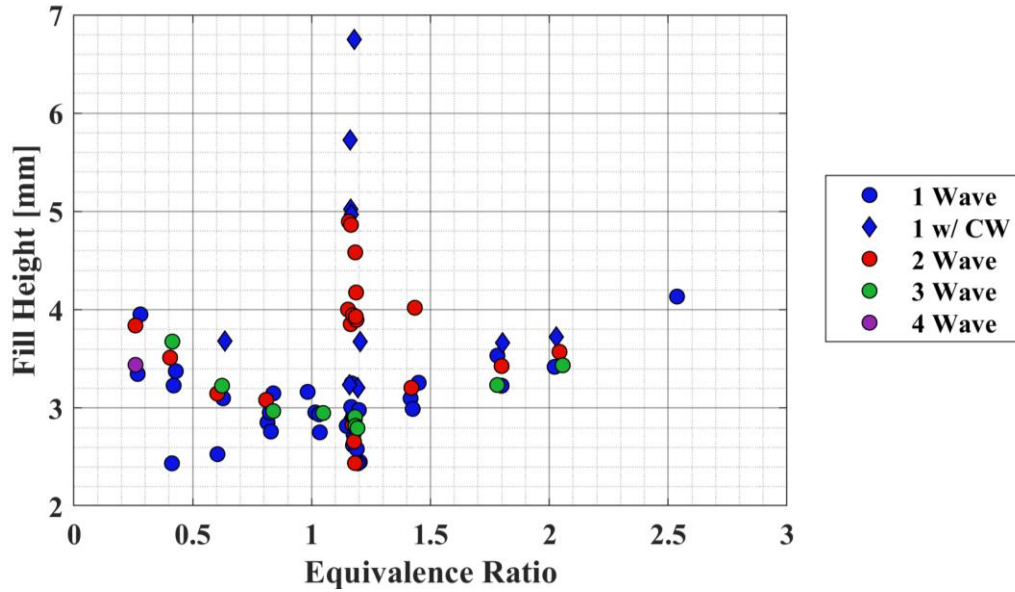


Figure 4.5: Calculated fill height of detonative runs

#### 4.2. THERMODYNAMIC AND GEOMETRIC COMPARISONS

Utilizing the results already described in this section, values for initial pressure, detonation cell size, energy flux, and fill height were calculated for each run. The following subsections will compare these across all engines to various geometric values and ratios, including some discussed in the literature review of Section 1. Pertinent geometry for the engine configurations is presented in Table 4-1, and can be used to cross-reference when desired. It should be noted that some of these ratios are linearly related to each other due to the design of the engines. In particular, both the ratio of gap width to the outer radius ( $r_o$ ) and the ratio of annulus volume ( $V$ ) to the surface area ( $S_A$ ) multiplied by the length ( $L$ ) decreased linearly as the ratio between inner and outer radius ( $r_i / r_o$ ) increased.

Injector-to-annulus-exit-area ratio had been previously cited as having influence on engine operation, but was not a focus of the experimental effort. This ratio was intentionally held constant for the radius of curvature study, but was varied during the gap width study as a consequence of reusing the same injector across the four configurations tested. As such, influence of injector-to-

annulus-exit-area ratio is best seen in Sections 3.3.3 and 3.3.4, with the concession that center-of-gap impingement was not maintained within the gap study.

Table 4-1: Pertinent geometric ratios of engine configurations.

| Configuration | $r_i$<br>(mm) | $r_i / r_o$ | Gap<br>Width / $r_o$ | $r_i /$<br>Gap Width | $r_o /$<br>Gap Width | $V /$<br>( $S_A * L$ ) | $A_{inj} /$<br>$A_e$ |
|---------------|---------------|-------------|----------------------|----------------------|----------------------|------------------------|----------------------|
| 76-FI-5       | 33.1          | 0.87        | 0.13                 | 6.62                 | 7.62                 | 0.033                  | 0.110                |
| 51-FI-5       | 20.4          | 0.80        | 0.20                 | 4.08                 | 5.08                 | 0.049                  | 0.110                |
| 25-FI-3       | 9.7           | 0.76        | 0.24                 | 3.23                 | 4.23                 | 0.059                  | 0.164                |
| 25-FI-5       | 7.7           | 0.61        | 0.39                 | 1.54                 | 2.54                 | 0.098                  | 0.110                |
| 25-FI-7       | 5.7           | 0.45        | 0.55                 | 0.81                 | 1.81                 | 0.138                  | 0.086                |
| 25-FI-12.7    | 0.0           | 0.00        | 1.00                 | 0.00                 | 1.00                 | 0.250                  | 0.068                |

#### 4.2.1. RADIUS TO GAP WIDTH

The inner radius was the only combustor dimension altered across all six engine configurations tested. As such, normalizing this dimension to other length scales, including gap width and detonation cell size, would provide insight that spans all presented data. A comparison of the former example ( $r_i /$  gap width) to mass flux, seen in Figure 4.6, provided demarcations in operating mode, with the region corresponding to low mass flux ( $< 200 \text{ kg/s/m}^2$ ) and low  $r_i /$  gap width ( $< 2.5$ ) containing nearly all deflagrative results seen in the mass flux and equivalence ratio sweeps. In a similar fashion, test conditions containing a counter wave occurred within a bordering region with slightly larger thresholds of mass flux ( $< 280 \text{ kg/s/m}^2$ ) and the ratio of inner radius to gap width ( $< 4.8$ ). Increasing along either axis led to higher wave stability and higher wave counts. Both the deflagration and counter wave regions are indicated in Figure 4.6, but the previously listed boundaries act as rough estimates since the results were too separated in both geometry and mass flux to estimate definitive limits. It should be noted the ratio of inner radius to gap width was proportional to the ratio between inner and outer radius, which was cited repeatedly in Section 1.3 as an indicator for wave stability.

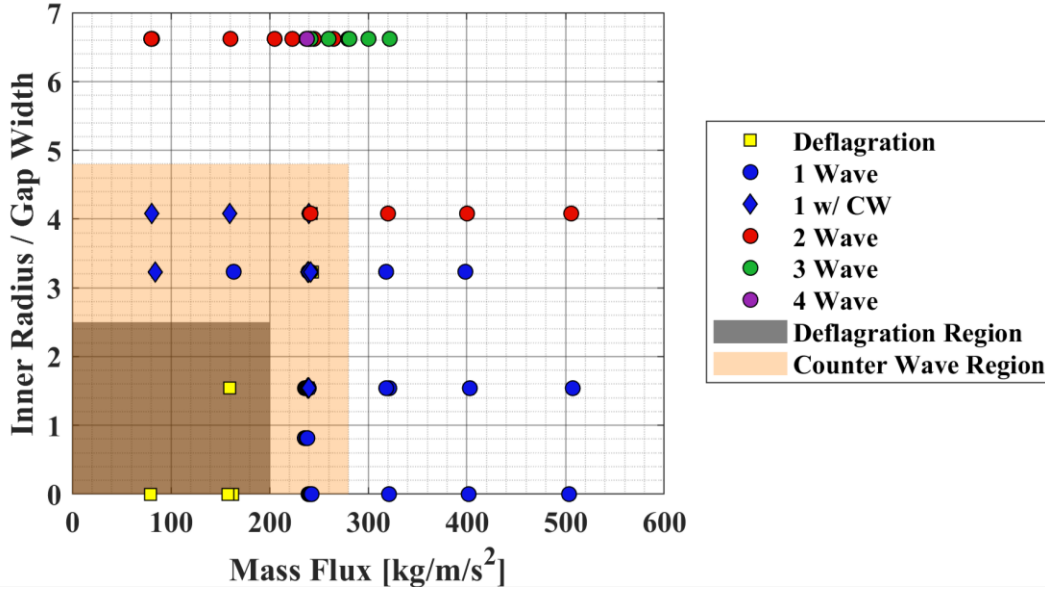


Figure 4.6: Comparison of the ratio of inner radius to gap width with mass flux provides demarcations in operating mode.

Each engine had a unique value for the ratio between inner radius and gap width, and thus wave speed ranges can be shown across all configurations simultaneously. As the calculation of wave speed utilized the circumference of the outer radius though, the proper comparison would be with the ratio of outer radius to gap width, which is shown in Figure 4.7. If the ratio of inner radius to gap width was instead used, the results would simply be scaled on the y-axis from 0 to 7.62 to the range of 0 to 6.62, per Table 4-1. As previously discussed, the 25-FI-7 configuration (with a  $r_o$  / gap width of 1.81) has limited data due to core failure, and thus has an abnormally small wave speed range. Each engine had a near-identical range of equivalence ratios tested, but the 76-FI-5 (with a  $r_o$  / gap width of 7.62) had a significantly smaller maximum for mass flux due to feed system limitations. All engines operated within similar ranges of wave speed, with the coreless engine exhibiting the highest wave speeds and largest spread. This plot also echoes the results from Section 3.3, where wave count had a geometric requirement and counter waves primarily appeared near wave count transitions. Figure 4.8 compares the ratio of outer radius to gap width to the ratio of wave speed to CJ speed, and maintains the properties discussed for wave speed. The

bulk of the data falls within 1400 to 1900 m/s wave speed, which correlated to 50% to 80% of the CJ speed.

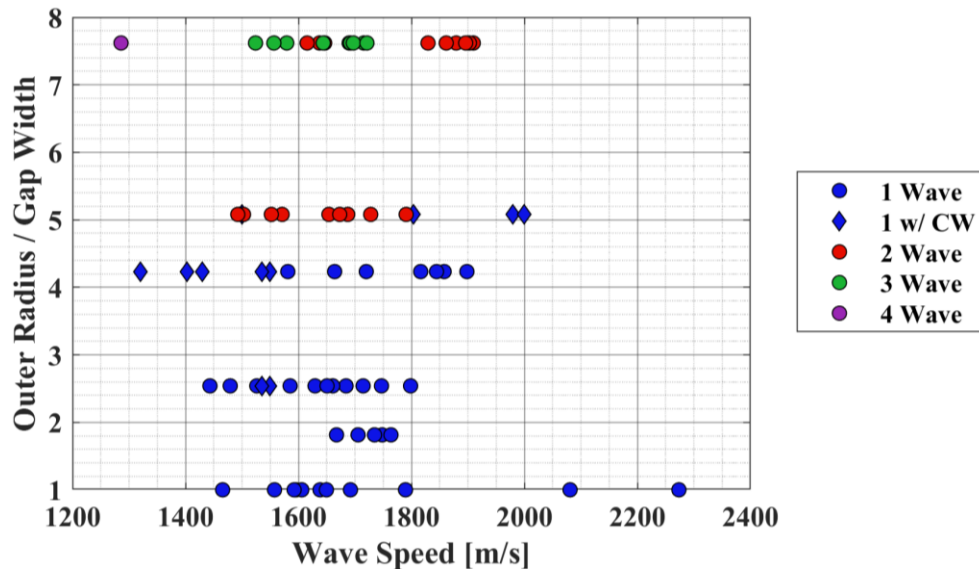


Figure 4.7: Comparison of the ratio of outer radius to gap width with wave speed.

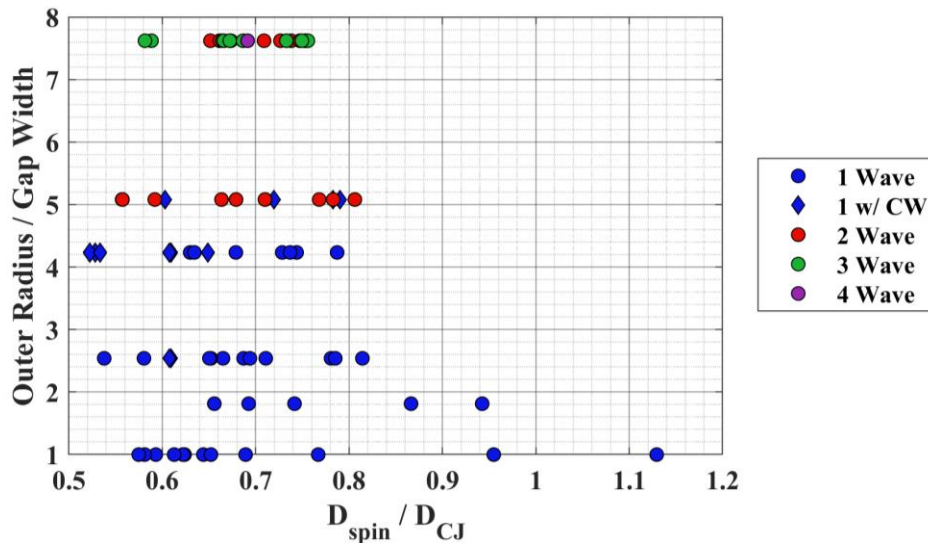


Figure 4.8: Comparison of the ratio of outer radius to gap width with ratio between wave speed and CJ speed.

Repeating the six-engine comparison for combustor pressure, seen in Figure 4.9, provided no new information as each configuration had a similar pressure spread directly related to the range of mass flux tested. As discussed earlier, the coreless configuration stood as an outlier when compared to other configurations at individual test conditions, such as transitioning to deflagrative

operation at a higher mass flux threshold, but the recorded range of pressures remained comparable over the combined mass flux and equivalence ratio sweeps.

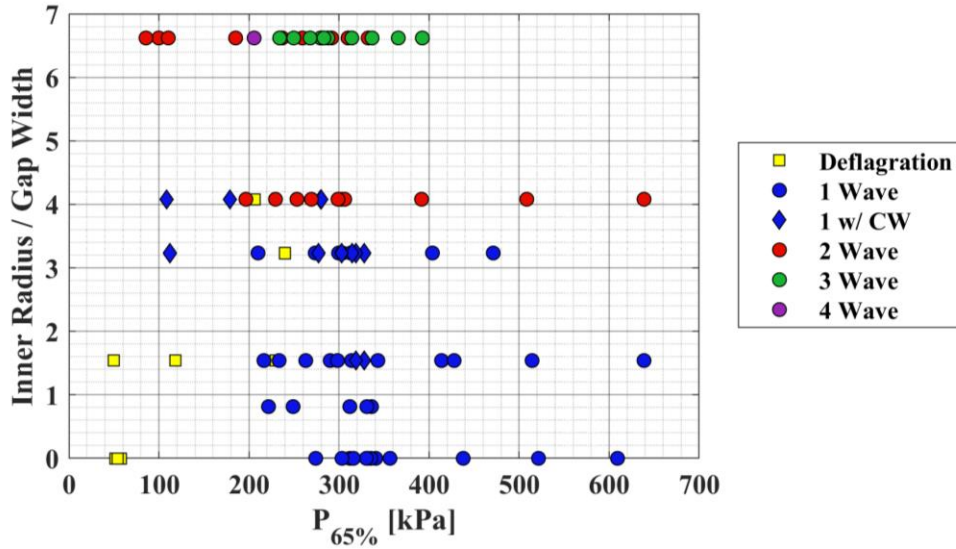


Figure 4.9: Comparison of ratio between inner radius and gap width to pressure recorded at the 65% combustor length station.

After reviewing the relation of experimental outputs such as pressure and wave dynamics to the ratio of inner radius to gap width, it is pertinent to continue the multi-configuration comparisons with the variables calculated in Section 4.1. The spread of fill height in each engine was similar, and comparison across engine configurations provides no new information from what was already discussed for the comparison of fill height to equivalence ratio in Figure 4.5. For detonation cell size, shown in Figure 4.10, the majority of test conditions, regardless of configuration, were less than 2 mm. From this plot alone, it is unclear if detonation cell size influenced wave count as results were interspersed amongst each other, especially compared with changes associated with the combustor geometry changes. Deflagrative operating modes tended to have larger cell sizes, implying detonation cell size had some operating range influence.

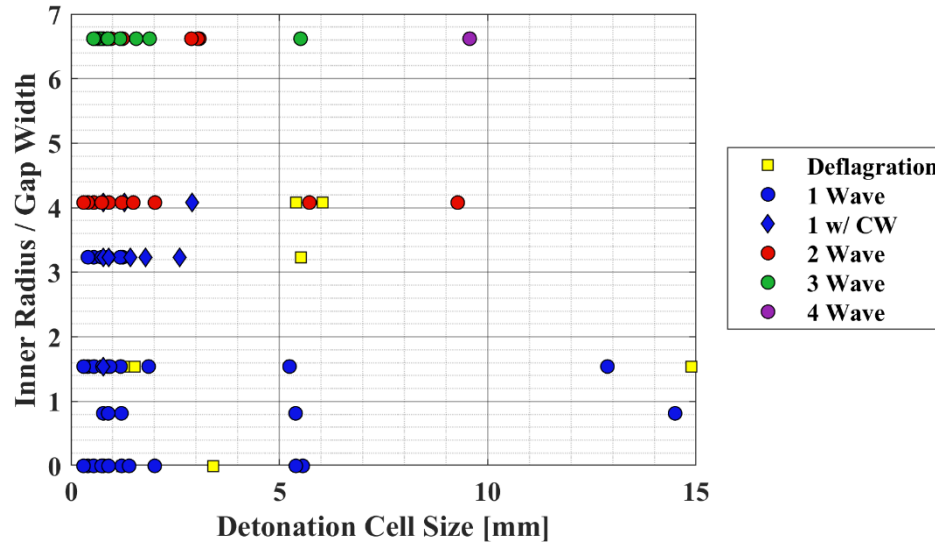


Figure 4.10: Comparison of the ratio of inner radius to gap width with calculated detonation cell size.

The final calculated variable to compare across configurations is energy flux, which is displayed in Figure 4.11. The trends in spread were similar to that seen for combustor pressure in Figure 4.9, which was driven by mass flux. Due to the mass flux influence on both combustor pressure and energy flux, comparison of these two variables is of interest and will be explored in Section 4.2.4. Deflagration cases tended to be on the lower end of the energy flux conditions tested, but did not apply as strictly as the results of mass flux seen in Figure 4.6.



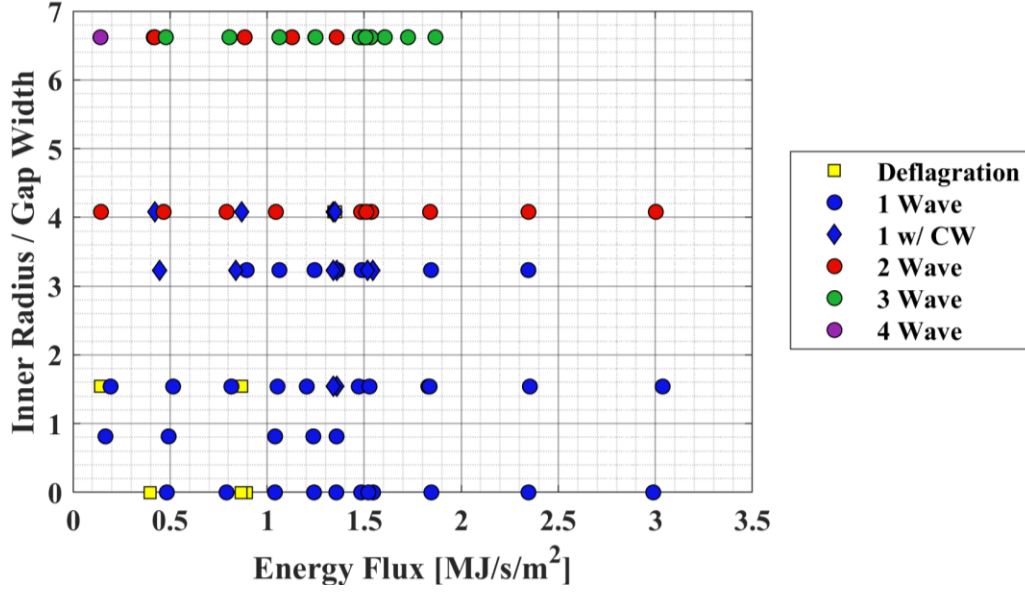


Figure 4.11: Comparison of the ratio of inner radius to gap width with calculated energy flux.

#### 4.2.2. INNER RADIUS TO DETONATION CELL SIZE

When comparing inner radius to detonation cell size,  $\lambda$ , boundaries for regions pertaining to wave stability appeared and are shown in Figure 4.12. Straight, diagonal lines of constant  $r_i / \lambda$  can be drawn such that three regions appear: one that contains all non-detonative runs ( $r_i / \lambda < 6$ ), another that includes all counter-wave activity ( $6 \leq r_i / \lambda \leq 26$ ), and only stable wave operation ( $r_i / \lambda > 26$ ). Increasing inner radius or reducing cell size through fill conditions, by raising mass flux and thereby fill pressure for instance, provided higher wave stability. While these boundaries appear in different locations, the stability trends and boundary presence follow that previously discussed by Nakayama et al. [14] for detonation propagation in curved channels. One significant deviation from their results was the presence of stable wave modes within the counter-wave and non-detonative regions, and as such the ratio  $r_i / \lambda$  only acted as guidance for, and not a rule of, wave stability in these RDREs. Comparing  $r_i / \lambda$  to equivalence ratio, displayed in Figure 4.13, reiterates the previous points, while also showing the influence of one of the fill conditions. This

plot also demonstrates that operating mode is not solely controlled by inner radius and fill conditions, but is still strongly influenced by them.

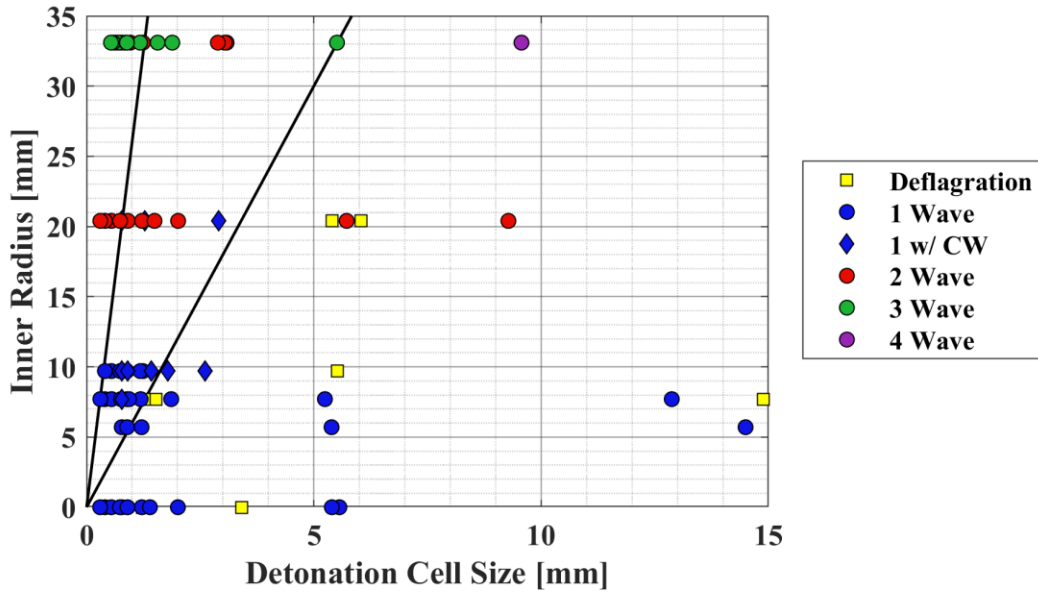


Figure 4.12: Comparison of inner radius to detonation cell size, with region boundaries corresponding to wave stability drawn in black.

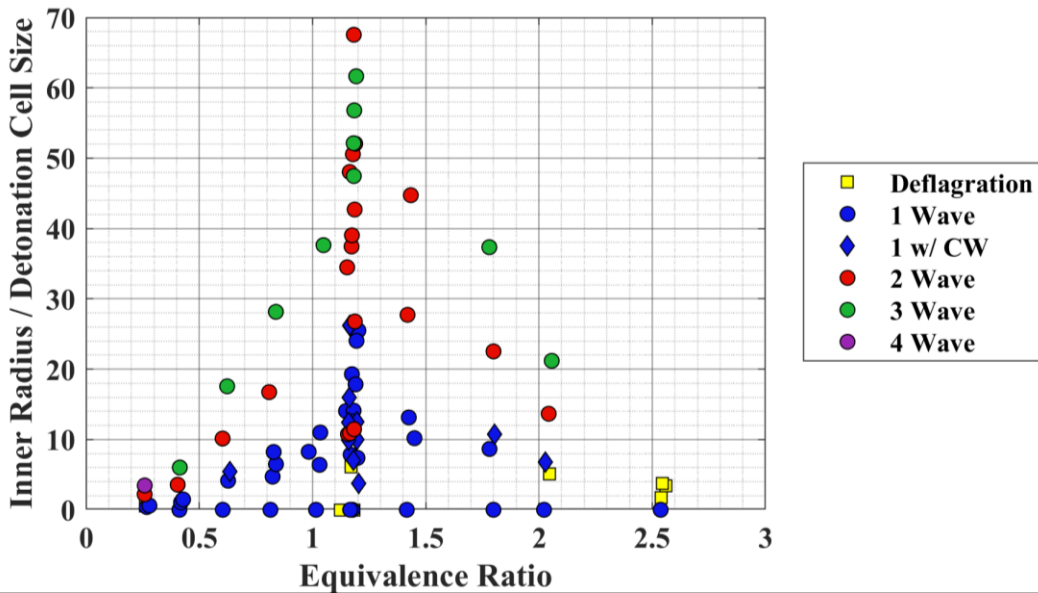


Figure 4.13: Ratio of inner radius to detonation cell size compared to equivalence ratio.

Comparison of the ratio of inner radius to detonation cell size to wave dynamics, specifically the ratio of wave speed to CJ speed so that both axes are dimensionless quantities, is displayed in Figure 4.14. Overall, results from all six configurations were interspersed and tests

with differing operating modes overlapped. Larger wave counts tended to have larger  $r_i / \lambda$ , although this was primarily driven by the influence of inner radius on wave count as the detonation cell sizes for most experiments were comparable.

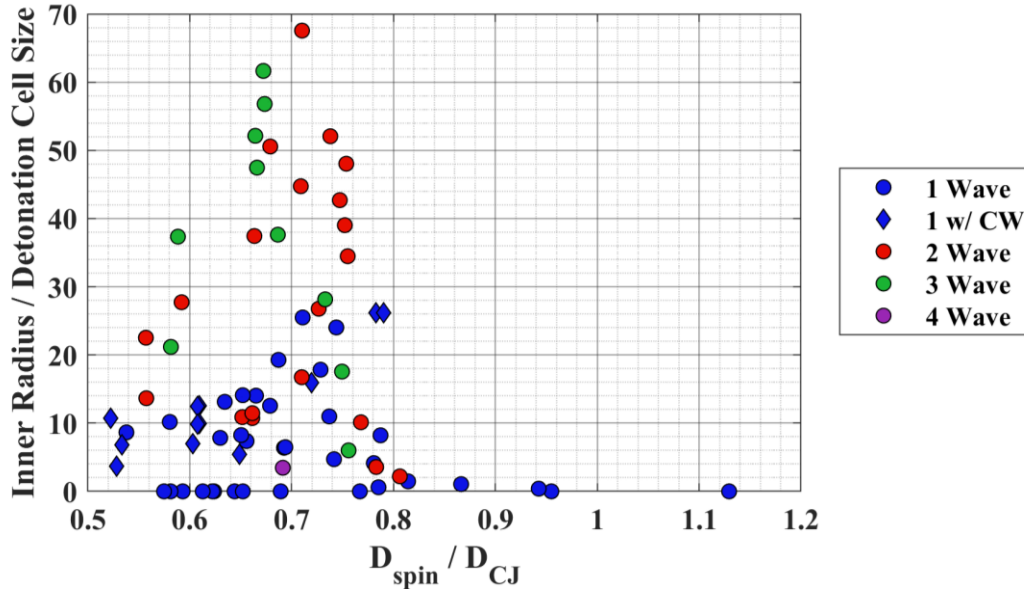


Figure 4.14: Comparison of the ratio of inner radius to detonation cell size with the ratio of wave speed to CJ speed.

As already seen in this subsection, as well as the previous one, inner radius has an influence on combustor dynamics, and is the only combustor dimension varied amongst all configurations. Gap width was only varied across four configurations, being held constant within the radius of curvature study. This statement is of particular importance as the ratio between gap width and detonation cell size was expected to influence operation. By scaling test conditions by mass flux in the radius of curvature study, the intent had been to maintain the ratio of gap width to detonation cell size across configurations. Computing this ratio for only the radius of curvature data, shown in Figure 4.15, demonstrates this goal was achieved as the results of each test condition closely match. Due to this overlap, comparison of detonation cell size to gap width over the full data set, rather than inner radius as seen in Figure 4.13, becomes more difficult to achieve, while also providing no new insights or trends.

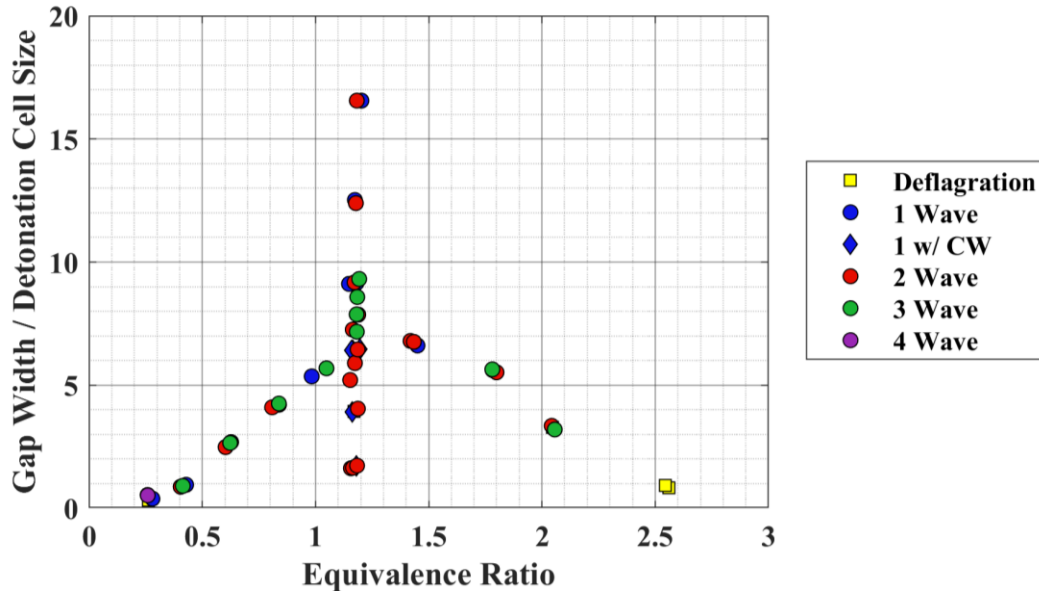


Figure 4.15: Comparison of the ratio of gap width to detonation cell size with equivalence ratio for radius of curvature study shows a constant gap width to detonation cell size ratio was achieved by scaling test conditions by mass flux.

#### 4.2.3. FILL HEIGHT TO DETONATION CELL SIZE

The fill heights calculated for the primary equivalence ratio and mass flux sweeps has already been presented in Figure 4.5. As both wave speed and pressure are inputs for computing fill height, comparisons to these experimental outputs simply demonstrate their expected influence as outlined by Equations 4.4 and 4.5. Relating fill height to combustor geometry, such as the ratio of the inner radius and gap width used previously, only shows that each configuration operated with a similar spread of fill heights, and reiterates that larger fill heights were seen with operating modes possessing a counter wave.

A comparison of fill height to detonation cell size, another calculated length scale, is shown in Figure 4.16. Overall, little relation is seen between the two values, as the computed fill heights cover a large range of detonation cell sizes. As in Figure 4.5, runs with a counter wave exhibited a larger fill height. In addition, counter waves only appeared at smaller detonation cell sizes.

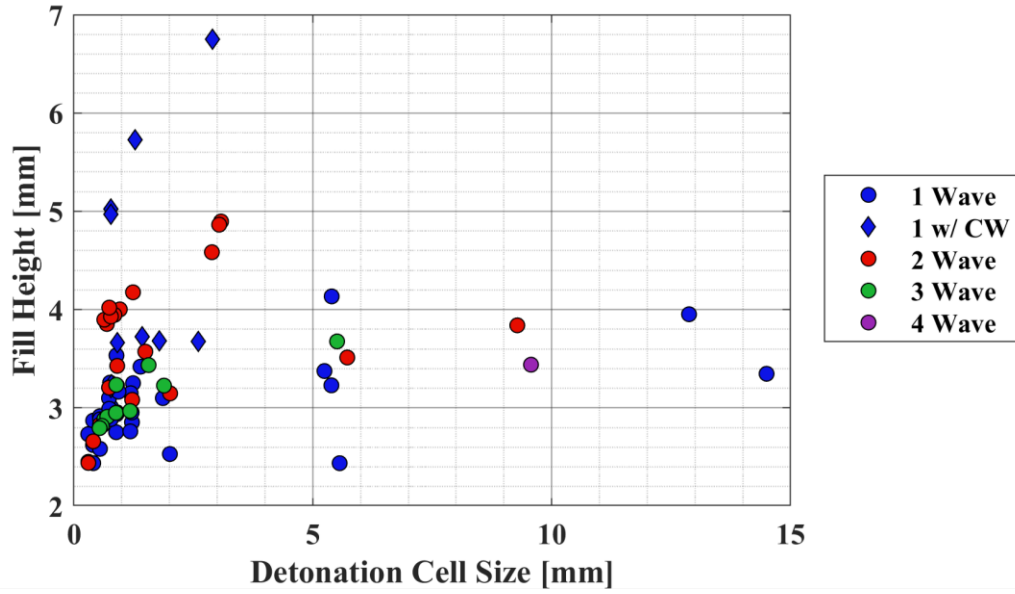


Figure 4.16: Comparison of calculated fill height to detonation cell size.

Since both detonation cell size and fill height were computed from fill conditions, a relation between the two may appear if the influence of the experimental outputs on fill height were removed. Multiplying fill height by operating frequency provides such a result, which acts as a fill velocity based on mass conservation and was originally shown in Equation 4.3. Comparison of this fill velocity to detonation cell size is shown in Figure 4.17 and shows the fill velocity increases with detonation cell size, with a shallower rise after a detonation cell size of 5 mm. Although the scatter is greatly reduced from Figure 4.16, the comparison of fill velocity to detonation cell size does not provide any clearer guidance on wave stability or operating mode than the previous comparison with fill height.

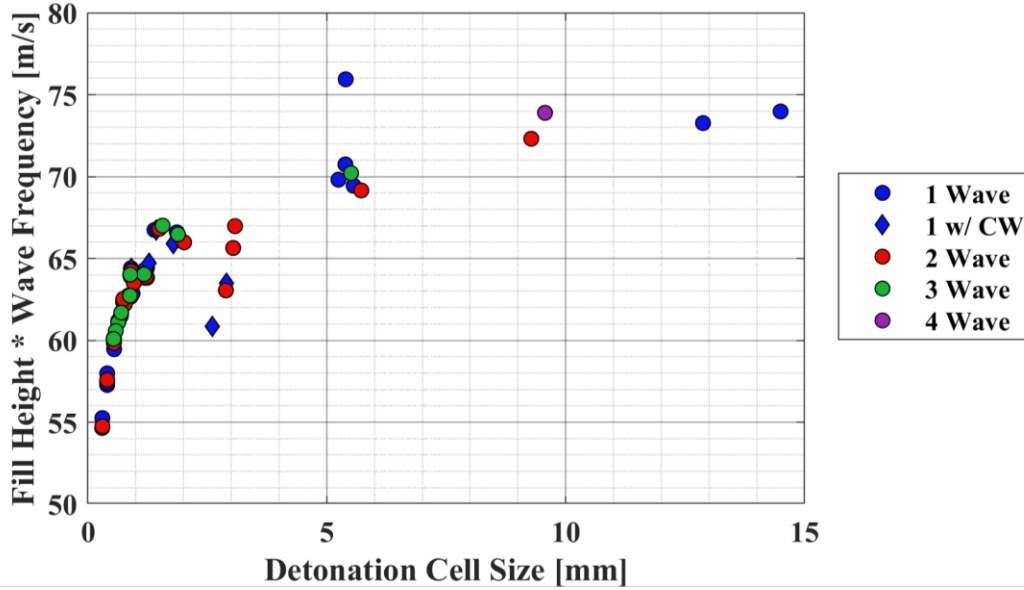


Figure 4.17: Fill velocity versus detonation cell size.

The ratio of fill height to detonation cell size had been cited to have implications on efficient thrust generation, through a comparison of recorded specific impulse to that calculated from CEA [17]. Although these engines did not have thrust data and therefore such analysis could not be directly applied, the results could be applied to the theorized boundary of  $h / \lambda \geq 4.5$ . Figure 4.18 displays the ratio of fill height to detonation cell size on a per-engine basis utilizing the ratio of inner radius to gap width used previously. The spread of  $h / \lambda$  was similar for each configuration, and every engine had results above the theorized efficient thrust limit of  $h / \lambda \geq 4.5$ . Results above this limit did not represent the majority of data and tended to have lower wave counts and no counter waves, but as with much of the data presented this acted as a guideline and not a rule.

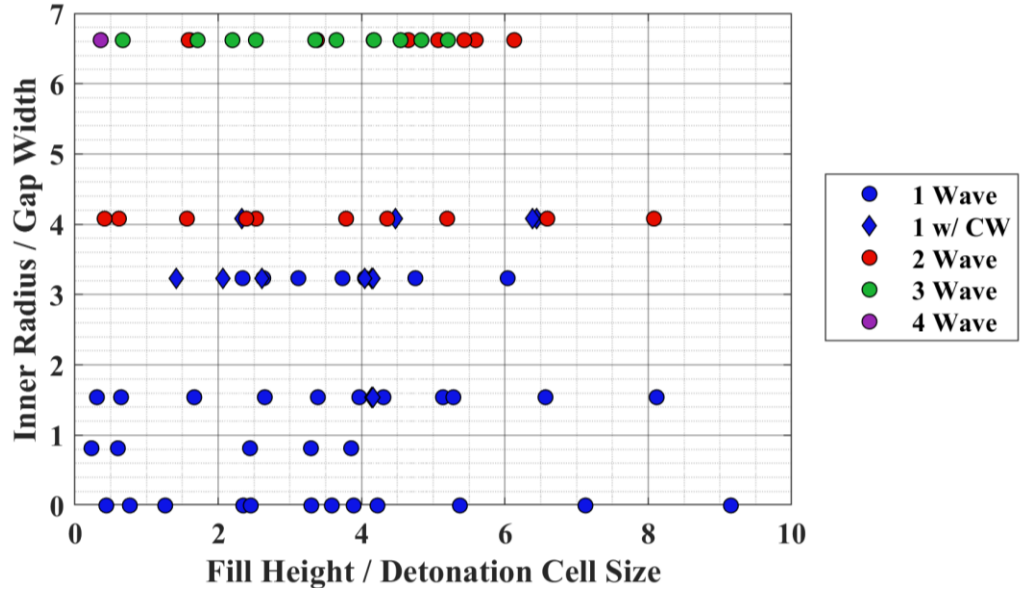


Figure 4.18: Comparison of the ratio of inner radius to gap width with ratio of fill height to detonation cell size.

#### 4.2.4. ENERGY FLUX

The energy flux calculated for experiments across all six engine configurations are shown in Figure 4.19 based on equivalence ratio and demonstrates the influence of the fill conditions on the computation of energy flux, with the vertical spread at an equivalence ratio of 1.15 due to mass flux. As the fill conditions were ultimately the only constituents for computing energy flux (since pre-detonation pressure was calculated from fill conditions), there was no separation of results due to geometry or wave dynamics. It is important to note that the energy flux calculated for deflagrative results in this section represents the energy flux that would have been expected had the operating mode been detonative instead, and not the actual energy flux based on a deflagrative heat release process. Energy flux results, when focusing on the equivalence ratio trend, had a peak in the equivalence ratio range of 1.4 to 1.8, which coincides with the peak in combustor pressure with equivalence ratio seen previously, such as in Figure 4.1, demonstrating a close relation between the two. If energy flux is displayed on a per-wave basis, as in Figure 4.20, results become separated based on operating mode, with higher wave counts possessing less energy flux per wave.

Furthermore, the energy flux of each wave was less sensitive to changes in equivalence ratio at larger wave counts.

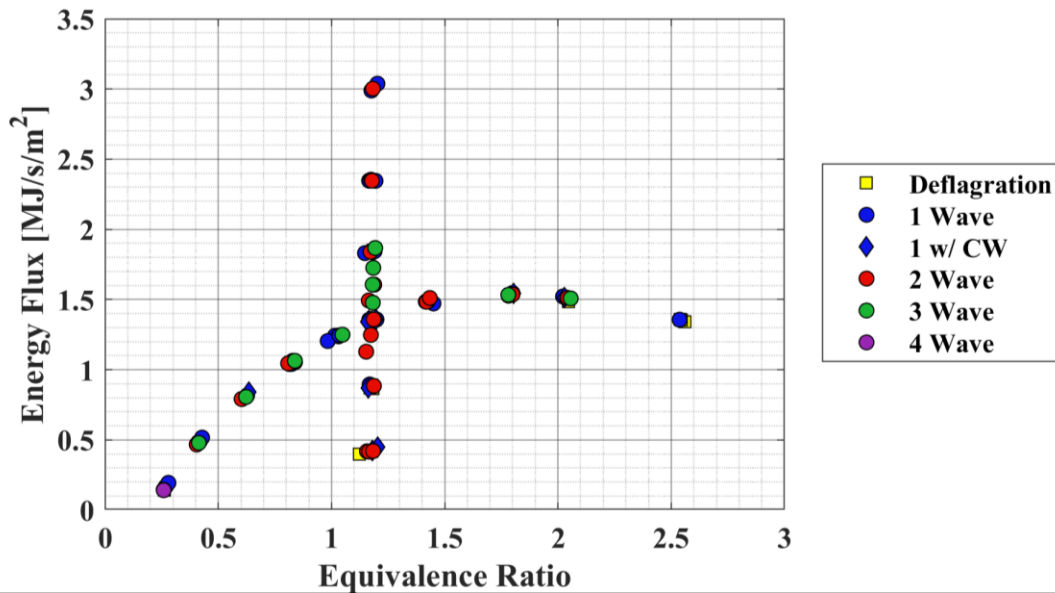


Figure 4.19: Calculated energy flux versus equivalence ratio.

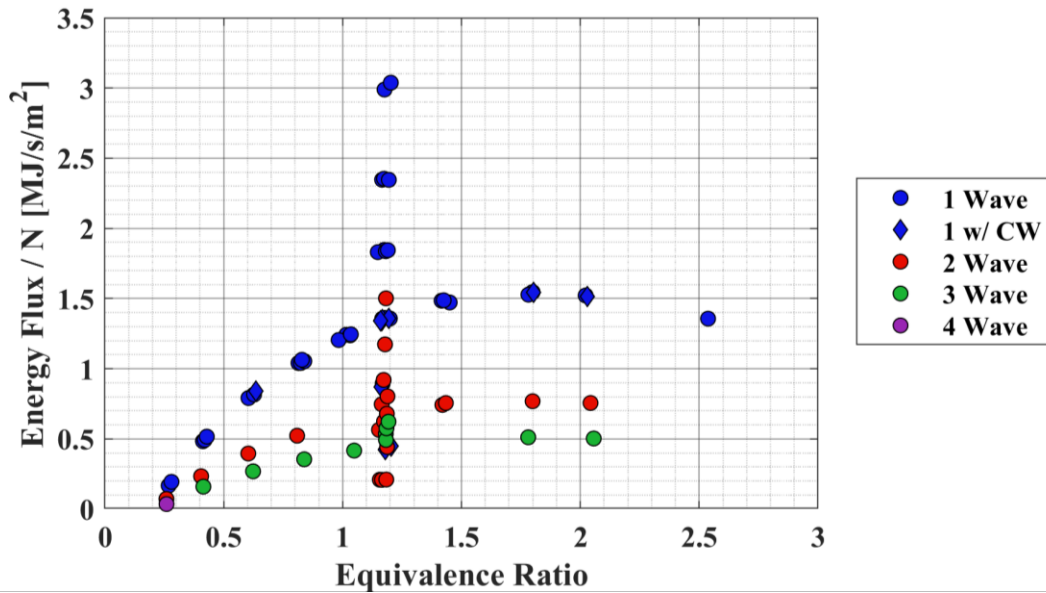


Figure 4.20: Per-wave energy flux versus equivalence ratio.

As mentioned for the previous plots, energy flux had a similar fuel-rich peak with equivalence ratio as combustor pressure, prompting a comparison between energy flux and combustor pressure that is shown in Figure 4.21. Combustor pressure monotonically increased



with energy flux regardless of operating mode, and deflagration results reported lower pressures than expected for detonative results at the same energy flux. Some variation in pressure for the same energy flux was seen due to geometry variations, which would be expected due to the variance in combustor pressure with geometry shown in Figure 4.1. As these geometric variations also coincided with operating mode changes, lower wave counts tended to exhibit a higher pressure for a constant energy flux. Even with these relatively small differences seen from different geometric configurations, energy flux and thereby combustor pressure were dominated by the fill conditions. Figure 4.22 demonstrates the relation of combustor pressure with per-wave energy flux, and maintained the monotonic increase of pressure seen above. Displaying results in such a fashion tightened the scatter within each operating mode, while also increasing linearity and separating each mode into almost distinct regions.

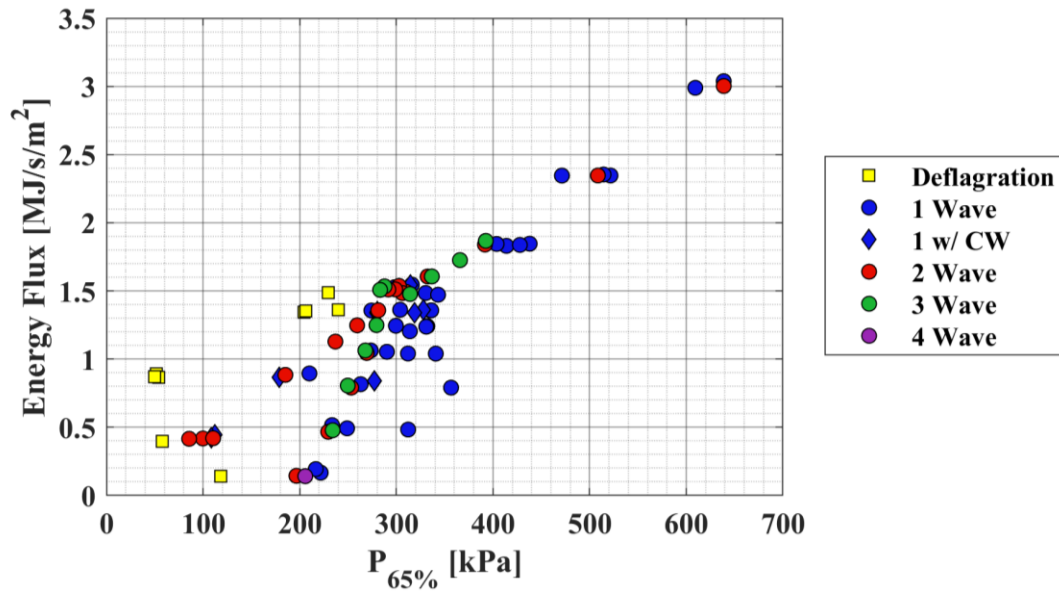


Figure 4.21: Comparison of energy flux with combustor pressure recorded at the 65% combustor length station.

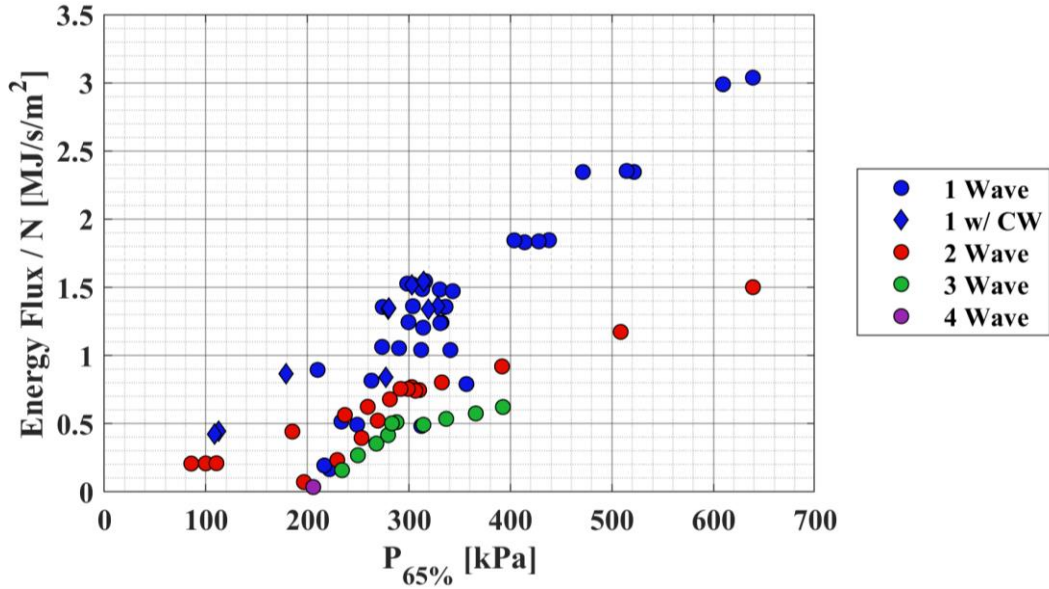


Figure 4.22: Per-wave energy flux versus combustor pressure recorded at the 65% combustor length station.

Comparison of energy flux to the wave speed is provided in Figure 4.23, where a global correlation between the two variables is not present within the scatter of data. Focusing on each operating mode independently though, a positive trend between wave speed and energy flux is witnessed. Operating modes with more than one wave were only present in the 51 and 76 mm engines, the latter of which had the lowest mass flux ceiling due to feed system limitations. As such, the full energy flux and wave speed ranges possible for the three- and four-wave operating modes is not covered with presented data. Even so, each operating mode falls within a similar range of energy flux and wave speed, reinforcing a geometric element drives the wave count due to interdependency of  $N$  and  $r_o$  in the computation of wave speed. When wave speed is normalized by CJ speed, a negative trend within each operating mode is observed with energy flux, as seen in Figure 4.24. Furthermore, the maximum fraction of CJ speed reported within each operating mode (as heat flux approaches zero) seems to decrease with increasing wave count. As there was a significantly larger pool of one-wave data within the results, it is difficult to make this statement

with certainty. Repeating these comparisons with the per-wave energy flux maintains the discussed trends, but separates the data in a fashion similar to Figure 4.22.

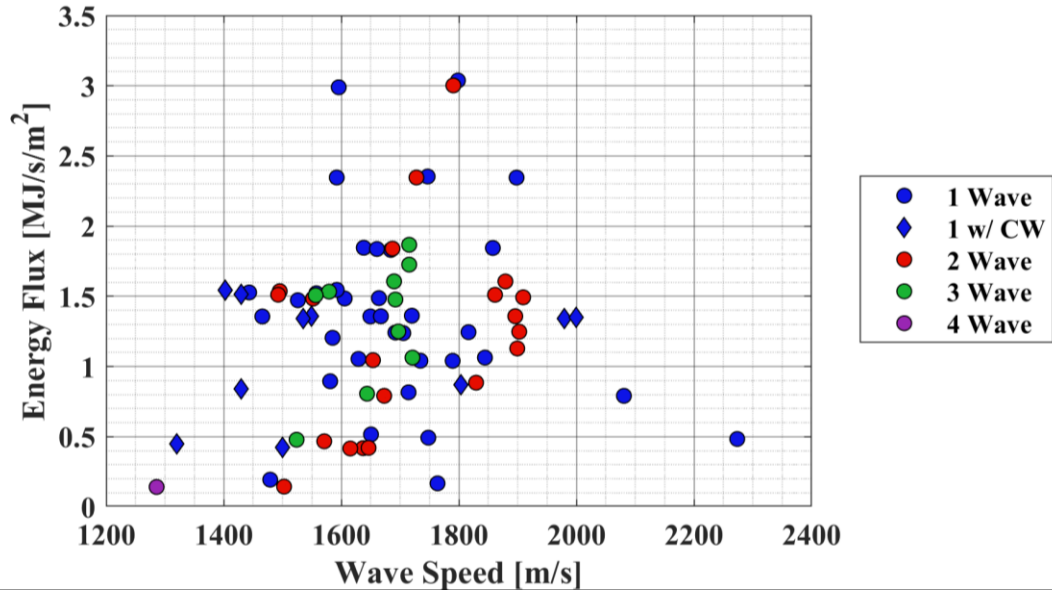


Figure 4.23: Comparison of energy flux with wave speed.

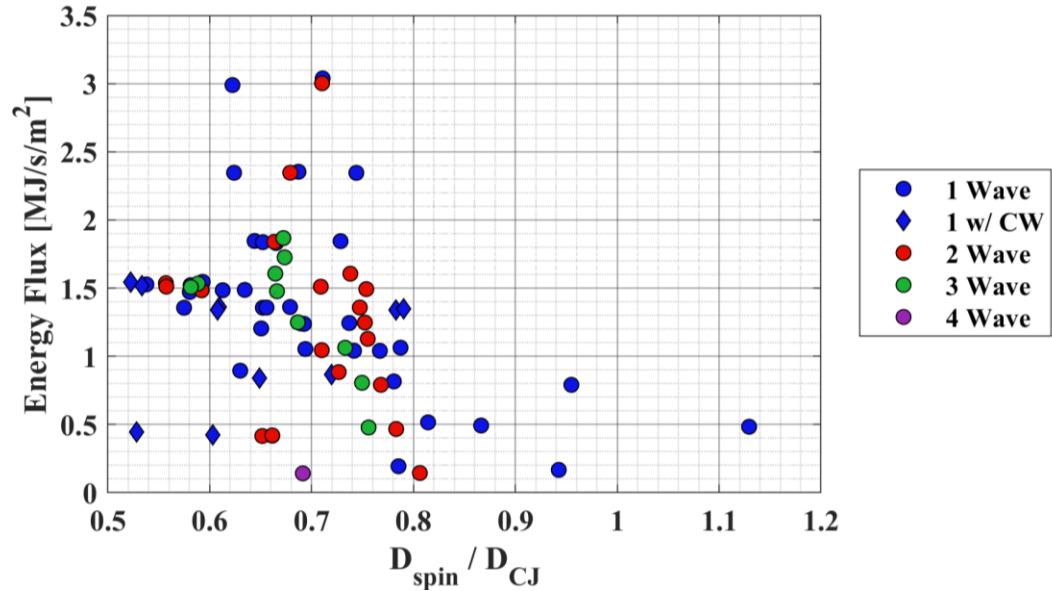


Figure 4.24: Comparison of energy flux with ratio of wave speed to CJ speed.

The previous four plots compared energy flux to pressure and wave dynamics, and since the latter two variables are contributors to the computation fill height, a relation may also exist between energy flux and fill height. Due to the fact fill height incorporates both wave speed and

wave number, a comparison between the per-wave energy flux, rather than energy flux itself, is presented in Figure 4.25. As a whole, the fill height and per-wave energy flux were inversely proportional, and the mass flux ceiling for the 76-FI-5 configuration again acts a limitation for presenting the full range of results for higher wave counts. Increasing either fill height or per-wave energy flux while holding the other constant tended to lower the wave count.

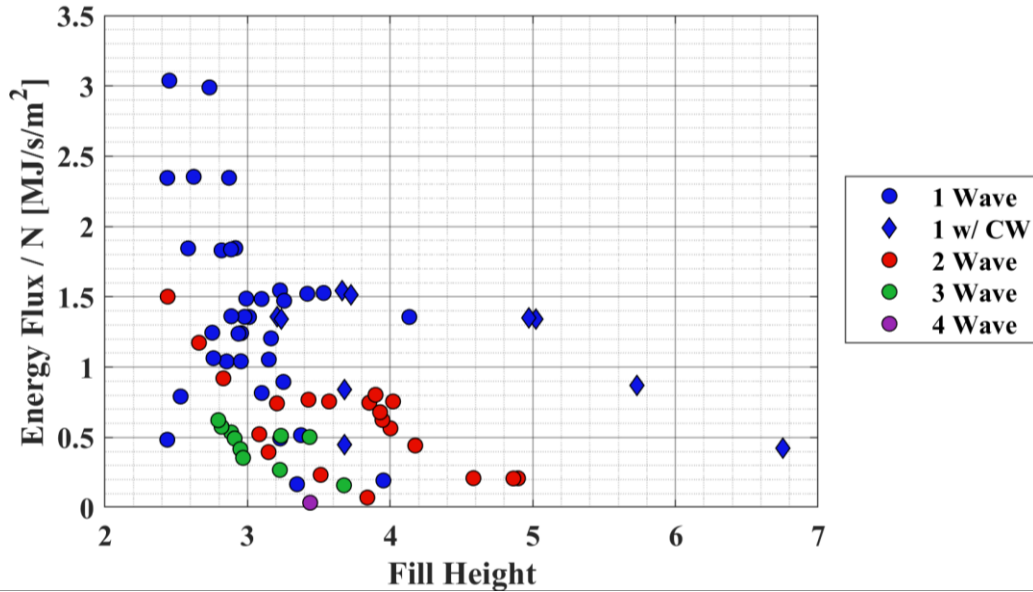


Figure 4.25: Comparison of per-wave energy flux with fill height.

As both energy flux and detonation cell size were computed from fill conditions, each test condition would have a unique value for each that was independent of both geometry and operating mode. This is demonstrated in Figure 4.26, where energy flux and detonation cell size were inversely proportional and test conditions repeated across configurations overlapped with little scatter. The decrease in detonation cell size with increasing energy flux correlates with the increase of wave speed seen in Figure 4.23. Although not a direct relation, CJ speed rises as detonation cell size shrinks (comparing Figure 4.2 and Figure 4.3). Altering the y-axis to per-wave energy flux, shown in Figure 4.27, separates data based on operating mode. An increase in per-wave energy flux still led to a decrease in detonation cell size, but if cell size was instead held constant the wave

count decreases. Similarly, lower per-wave energy flux was associated with larger detonation cell sizes and higher wave counts. When combined with the detonation cell size comparisons of previous sections, this indicates detonation cell size has influence over not only detonative limits, but operating mode as well.

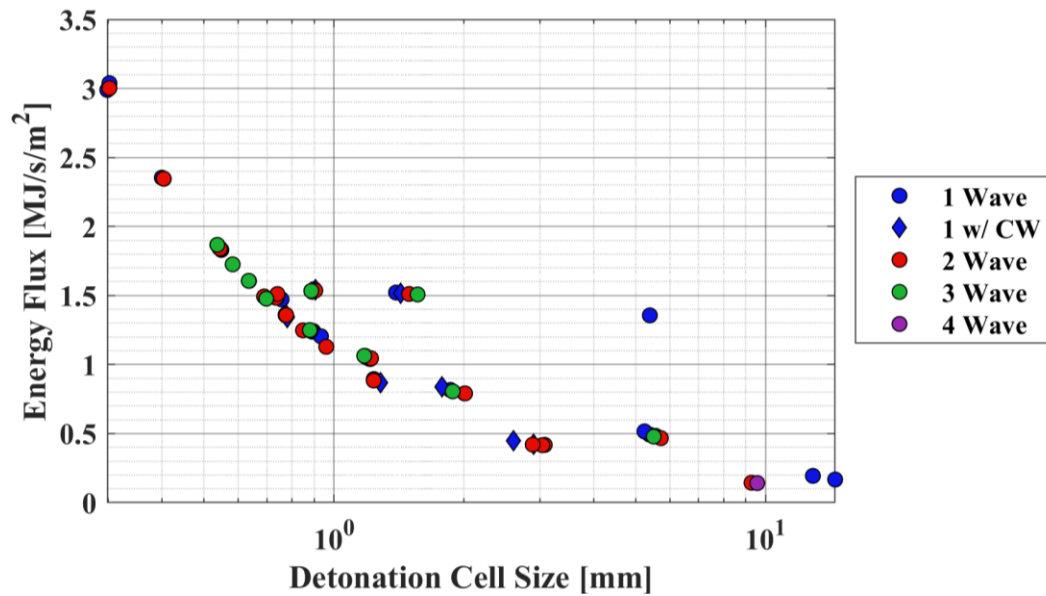


Figure 4.26: Relationship between calculated detonation cell size and energy flux.

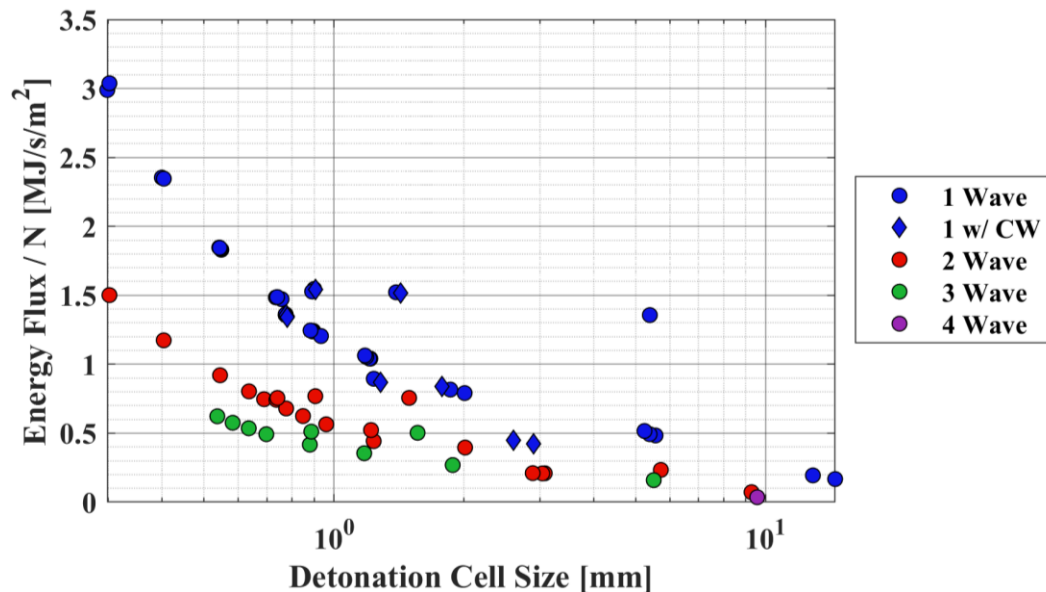


Figure 4.27: Comparison of energy flux per wave with detonation cell size.

With flat-faced impinging injectors, the flow enters the combustor with axial and, in the case of unbalanced injectors, radial components of velocity. As the detonation waves travel around the circumference of the combustor, the associated heat addition process would also impart a tangential velocity component, much like a bladeless compressor. For traditional compressors, the relation between work addition and angular momentum change of a blade row is described by the Euler turbine equation, which, for a constant radius, can be written as:

$$P_{comp} = \dot{m} \Omega r \Delta u_{\theta} \quad (4.6)$$

Where  $P_{comp}$  is the work input by the compressor,  $\Omega$  is the angular velocity of the blades, and  $\Delta u_{\theta}$  is the change in circumferential velocity. For an RDRE, work is provided by the heat addition process, represented by the heat release,  $q$ , multiplied by mass flow rate to be in the units of power (energy per second). In addition,  $\Omega r_0$  describes the circumferential velocity of the blade tips, which would be represented by wave speed. Finally, the initial circumferential velocity is zero and the final velocity in the lab frame could be described by the pressure at the von Neumann state immediately behind the adiabatic shock. Combining all of these modifications creates:

$$\dot{m} q \propto \dot{m} D_{spin} u_{VN} \quad (4.7)$$

The velocity at the von Neumann state was an intermediate output during the Cantera modeling for detonation cell size, and was near 270 m/s for most test conditions.

Evaluation of the left and right hand sides of Equation 4.7 for experiments within the radius of curvature study is shown in Figure 4.28. The data within the mass flux sweep is dispersed along the positive, diagonal trend, while equivalence ratio results for each engine branch away from this trend with fuel-rich conditions at higher heat release values. Based on the range of the axes, around 9% of the power from heat release is directed to angular momentum change, and this momentum change was primarily driven by mass flow rate, rather than equivalence ratio. Furthermore, as  $u_{VN}$  was nearly constant for the presented results, this statement also applies to wave speed. As mass

flux had a significantly larger influence on wave speed than equivalence ratio in Section 3.3, this result is unsurprising. Increasing mass flow rate also led to higher wave counts, echoing a similar discussion for fill height.

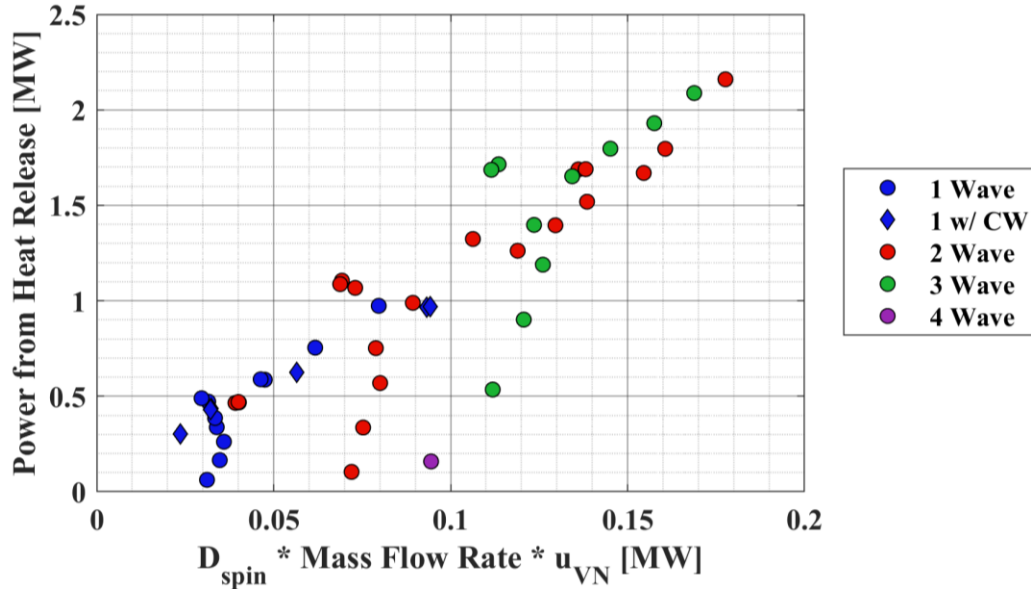


Figure 4.28: Power generated by the heat release process compared to angular momentum change induced by detonation wave propagation for radius of curvature data.

For the above comparison, data from the gap width study had been removed as those configurations had variations in both the impingement location relative to the center of the annular gap and the injector-to-annulus exit area ratio. Inclusion of this data, as done in Figure 4.29, does not disturb the discussed trends, and is primarily distributed within the lower left-hand quadrant due to the influence of mass flow rate.

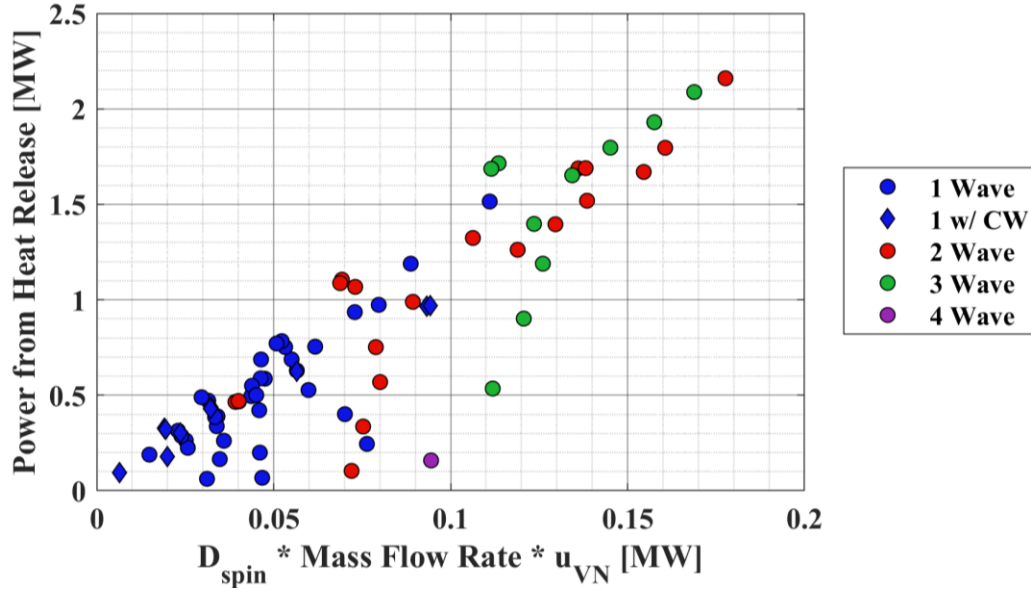


Figure 4.29: Power generated by the heat release process compared to angular momentum change induced by detonation wave propagation for all six configurations.

#### 4.3. APPLICATION OF RESULTS

Over the previous sections, the influence of several variables and ratios on engine operation have been discussed, including driving variables for operating mode, wave speed, and combustor pressure. Utilizing these empirical lessons to predict experimental outputs from only geometry and fill conditions is desirable to guide and optimize future RDRE designs. In order to test the applicability of the previously discussed lessons, they will be applied to the data not utilized thus far in Section 4, such as the deflagration limits and counter-wave activity in the 25 and 51 mm testing (e.g., Figure 3.13) and compared to their experimental results.

The new data set includes the results not part of the original mass flux and equivalence ratio sweeps. This “off-sweep” data set includes 138 test conditions across three engine configurations, of which 43 were deflagrative and 28 exhibited counter wave activity. Pre-detonation pressure, detonation cell size, and energy flux were calculated for each experiment as described by their respective portions of Section 4.1.



Operating mode was driven by a combination of geometry and fill conditions, notably mass flux, as was discussed for Figure 4.6. Replication of this plot for the off-cross data is presented in Figure 4.30, and the increase in wave count and wave stability with both mass flux and the ratio of inner radius to gap width are maintained. Every run with a counter wave was within the proposed counter wave region, but the same could not be said for deflagrative results. Approximately 12% of deflagrative results were outside of the proposed mass flux and inner radius to gap width ratio limits and were associated with operative limits with equivalence ratio in the 51-FI-5 configuration. In addition, 15% of the detonative results, primarily with a counter wave, were present within the deflagrative region. Overall, the ratio of inner radius to gap width and mass flux were still strong indicators of primary wave count, but inclusion of equivalence ratio effects would be needed to improve prediction of operating mode.

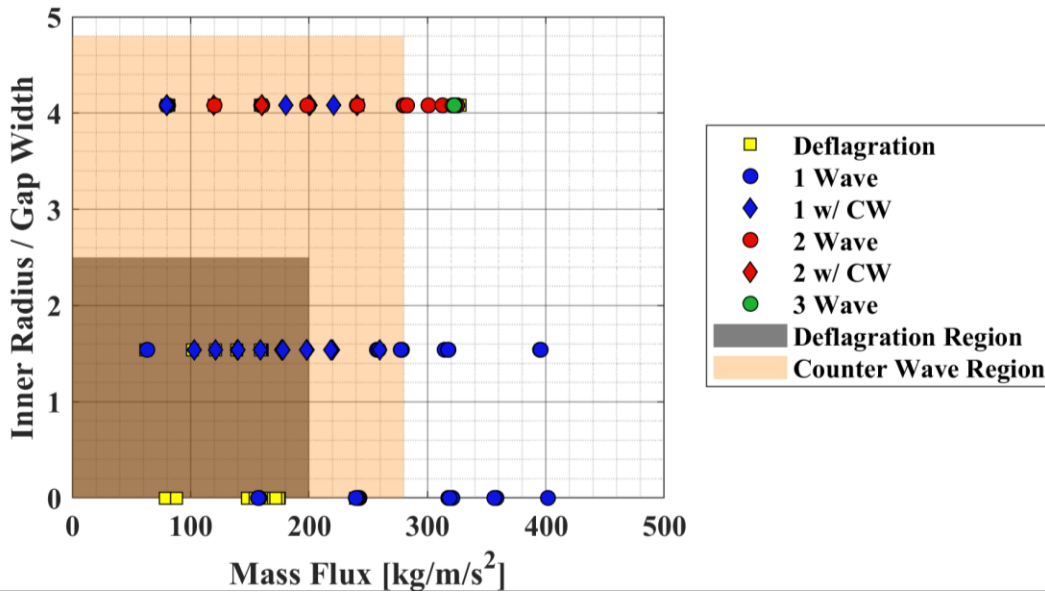


Figure 4.30: Distribution of off-sweep data based on the ratio of inner radius to gap width and mass flux.

For wave stability, which was defined by the presence of a counter wave, the ratio of inner radius to detonation cell size provided rough boundaries for counter-wave presence and deflagration limits in Figure 4.12. Application of the same comparison to off-cross data is shown

in Figure 4.31, and shows similar scatter to the sweep data across regions. For the suggested boundaries, 16% of deflagrative results were above  $r_i / \lambda = 6$ , while only 7% of counter wave results exceeded  $r_i / \lambda = 26$ . The ratio of inner radius to detonation cell size included both geometric and fill condition influence, and predicts the presence of deflagrative and counter wave results reasonably well. The largest drawback though, as discussed for Figure 4.12, is the scatter of solely co-rotating wave systems into both regions, meaning this ratio only implies a result could be deflagrative, and not that it will be. For instance, utilizing this ratio as the only indicator of operating mode would render the coreless configuration as a strictly deflagrative combustor, which is demonstrably incorrect.

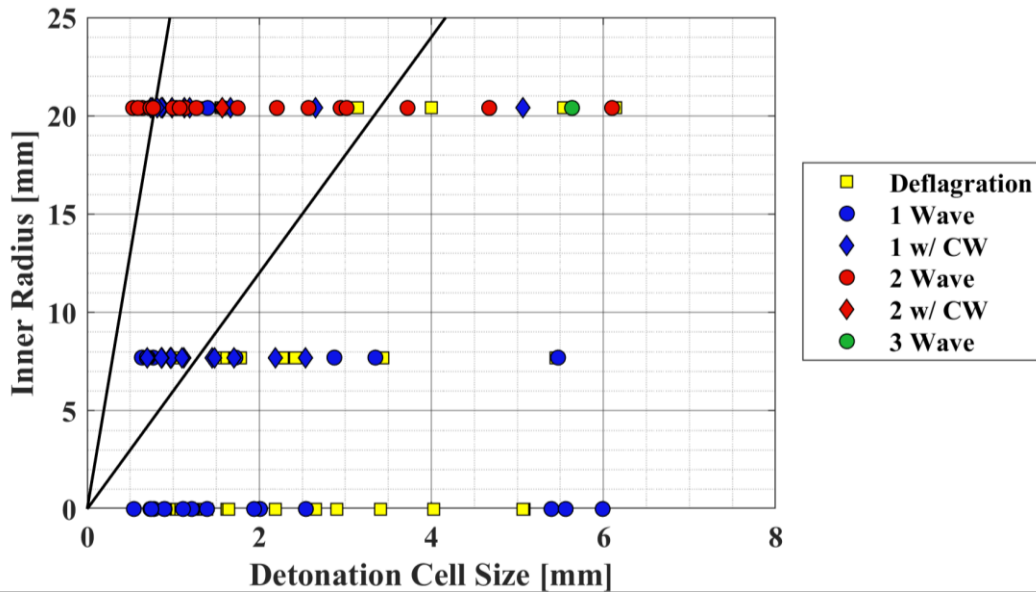


Figure 4.31: Relation of off-sweep data with inner radius and detonation cell size.

For predicting combustor pressure, a method for determining pre-detonation pressure was presented at the beginning of the section based on fill conditions. It is important to note the pressure calculation did not account for geometric variations in combustor pressure, as discussed for Figure 4.1, which was within 16% of the 76-FI-5 results for most pressure data. When discussing back pressure sensitivity in Section 3.4, combustor pressure distributions normalized by front endwall

pressure in the 76-FI-5 configuration overlaid with minimal variation. The pressure at the 65% combustor length station in this configuration was approximately 73% of the front endwall pressure (Figure 3.35). Applying this reduction to the calculations pre-detonation pressure for detonative results in the off-sweep data showed combustor pressures at the 65% station were underpredicted by an average of 8%, which was smaller than the variance due to geometric differences. Including a third input variable in the computation of  $p_0$  to represent geometry, preferably a ratio in order to be dimensionless, may serve to reduce the prediction error.

The computation of wave speed requires the experimental output of wave number as an input, and therefore wave frequency presents itself as a better option for prediction of wave dynamics. Fill height provides a starting point, as it is computed from properties based on the initial reactant mixture, and operating frequency (see Equation 4.5). Based on Figure 4.5, fill height had minimal variation over large changes in geometry, operating mode, and fill conditions, especially when the elevated results associated with counter wave modes are removed. Within the mass flux and equivalence ratio sweeps fill height had an average value of 3.25 mm. Holding fill height constant at this value and solving Equation 4.5 for wave frequency provided results remarkably close to recorded values in the off-sweep data. On average, wave frequency was overpredicted by 5%, with the largest deviations seen at extreme equivalence ratios. As the CJ speed was calculated from fill conditions using chemical equilibrium codes and the majority of wave speeds were within the range of 50 to 80% of this value, combining CJ speed, outer radius, and wave frequency would provide a path for predicting the wave number. Exploration of the influence of injection and mixing characteristics on fill height was outside the scope of included experiments, and would be necessary before applying to other configurations.

## 5. CONCLUSION

A sizeable collection of experiments were conducted across six RDREs with flat-faced impinging injectors using gaseous-methane oxygen propellant. The culmination of comparisons between feed conditions, geometry variations, experimental outputs, and results from chemical equilibrium modeling then provided empirically-driven insight into the operation and scaling of RDRE combustors.

With respect to geometry, variations in radius of curvature were associated with changes in wave number and frequency, and thereby wave speed. Alterations to annular gap width also affected wave speed, as well as wave stability (denoted by counter wave presence) and the range of detonative operation. Combustor pressure was slightly influenced by both geometric variations, but was dominated by adjustments to fill conditions, especially mass flux. When the combustor was back pressure insensitive, normalizing combustor axial pressure distributions to pre-detonation pressure and pressure station placement to combustor length presented similar profiles independent of both geometry and fill conditions.

Chemical equilibrium results from Cantera and CEA based on experiment fill conditions provided values for detonation cell size and energy flux, respectively. Detonation cell size was associated with detonative limits and operating mode, especially when compared to other variables (such as inner radius of the combustor). Energy flux had a strong relationship with both detonation cell size and combustor pressure, while also exhibiting influence on wave speed when the operating mode was held constant.

Calculation of fill height of fresh propellant showed little variation with either geometry or fill conditions. As such, fill height was useful in predicting operating frequency, which could be extended to wave count through the application of a Chapman-Jouguet detonation velocity derived

from chemical equilibrium codes due to the fact majority of reported wave speeds were within 50 to 80% of this value.

Although fill conditions dominated the operational trends, a balance with combustor geometry was required to describe the minutia of performance, wave stability, and operational limits. Regardless of geometry, wave stability and performance were both most readily improved through an increase in mass flux. Across the 400+ experiments, the coreless RDRE configuration reported some of the highest combustor pressures and wave speeds at comparable test conditions, while also avoiding complications associated with inner core loss due to overheating seen at the small engine scale.

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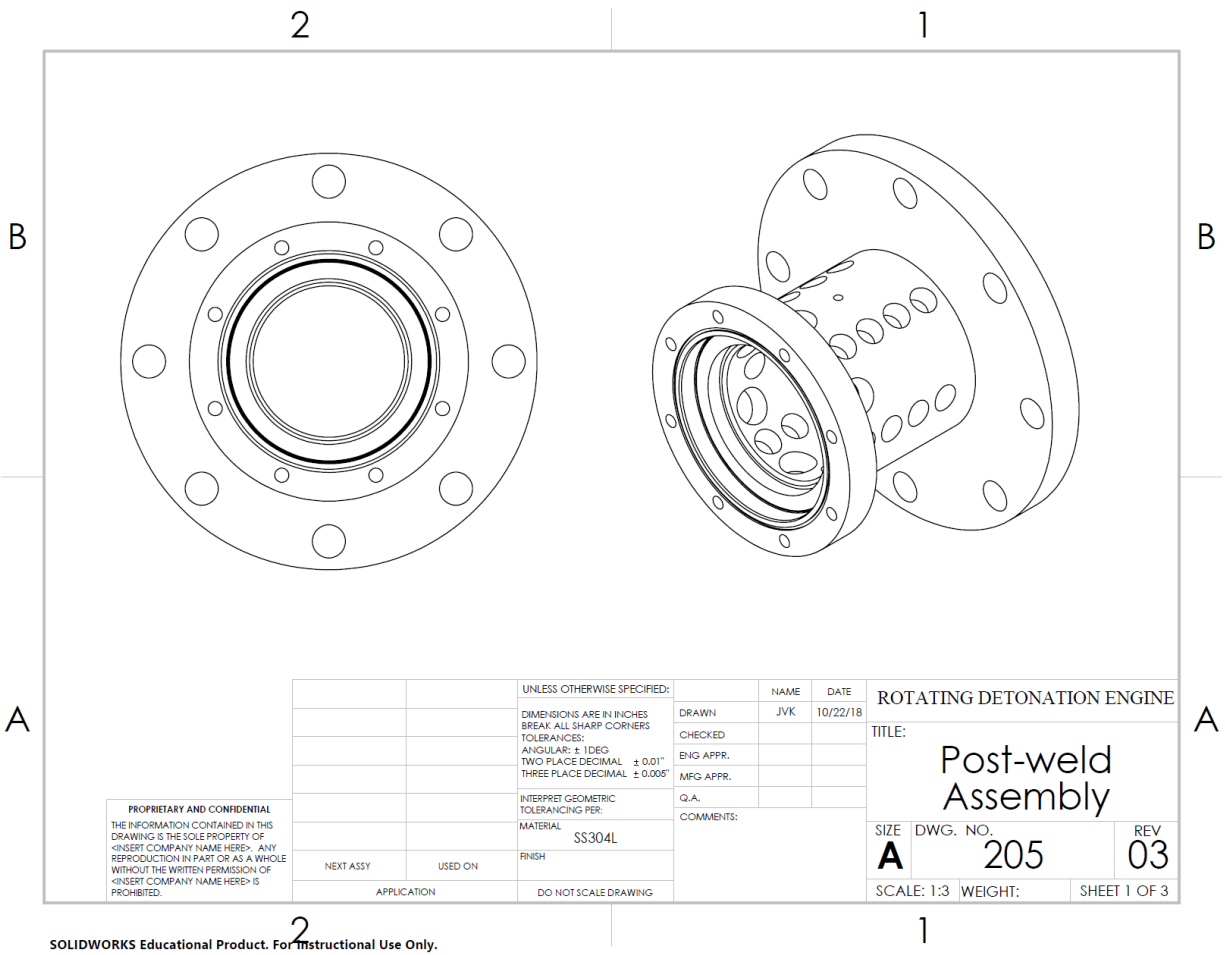
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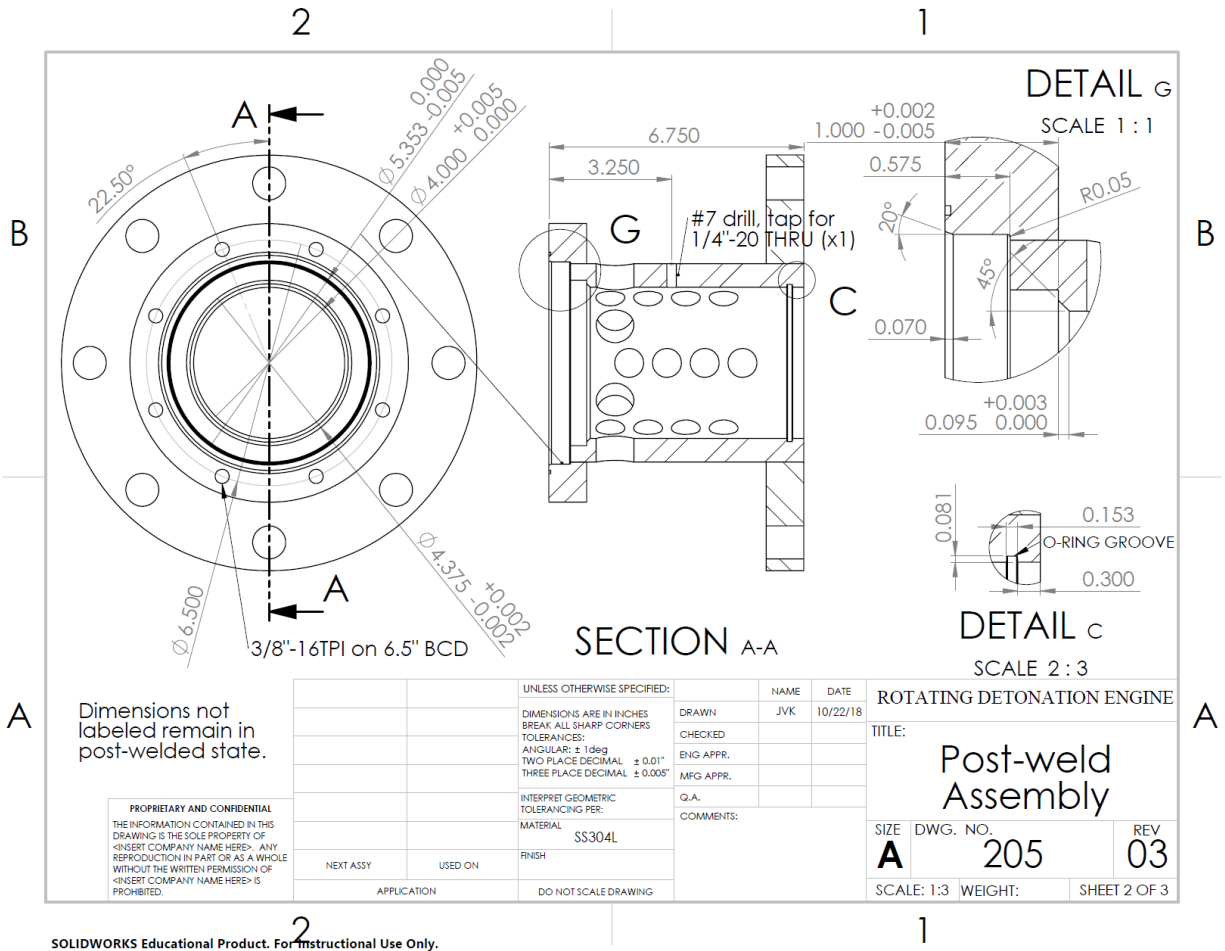
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- [65] B. McBride and S. Gordon, Computer Program for Calculation of Complex Chemical Equilibrium Compositions and Applications II. Users Manual and Program Description, <https://ntrs.nasa.gov/citations/19960044559>.
- [66] P.J. Linstrom and W.G. Mallard, Eds., **NIST Chemistry WebBook, NIST Standard Reference Database Number 69**, National Institute of Standards and Technology, Gaithersburg MD, 20899, <https://doi.org/10.18434/T4D303>.
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- [68] Lightfoot, M., Danczyk, S., Watts, J., and Schumaker, S. (2011). Accuracy and Best Practices for Small-Scale Rocket Engine Testing.

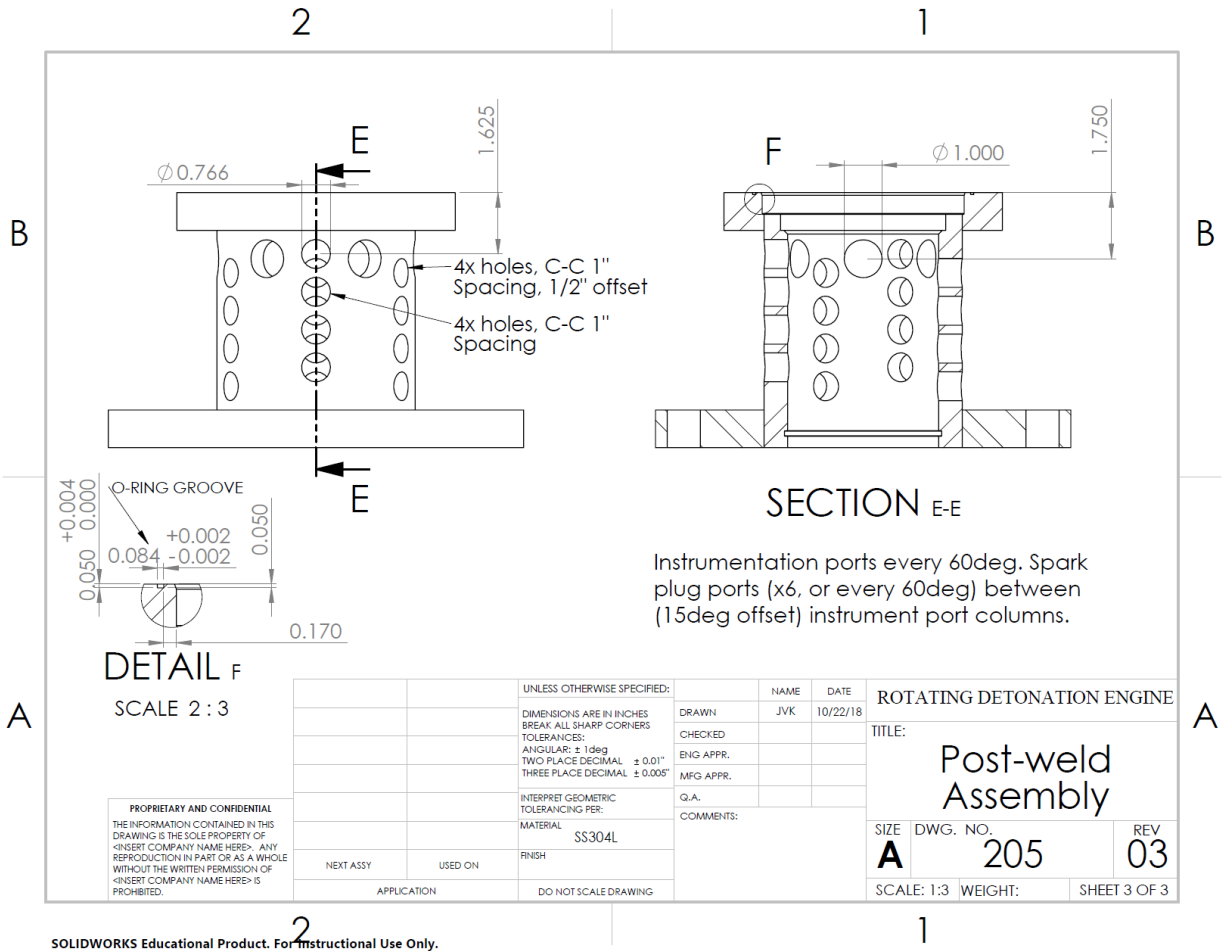
APPENDIX A: ENGINE DRAWINGS

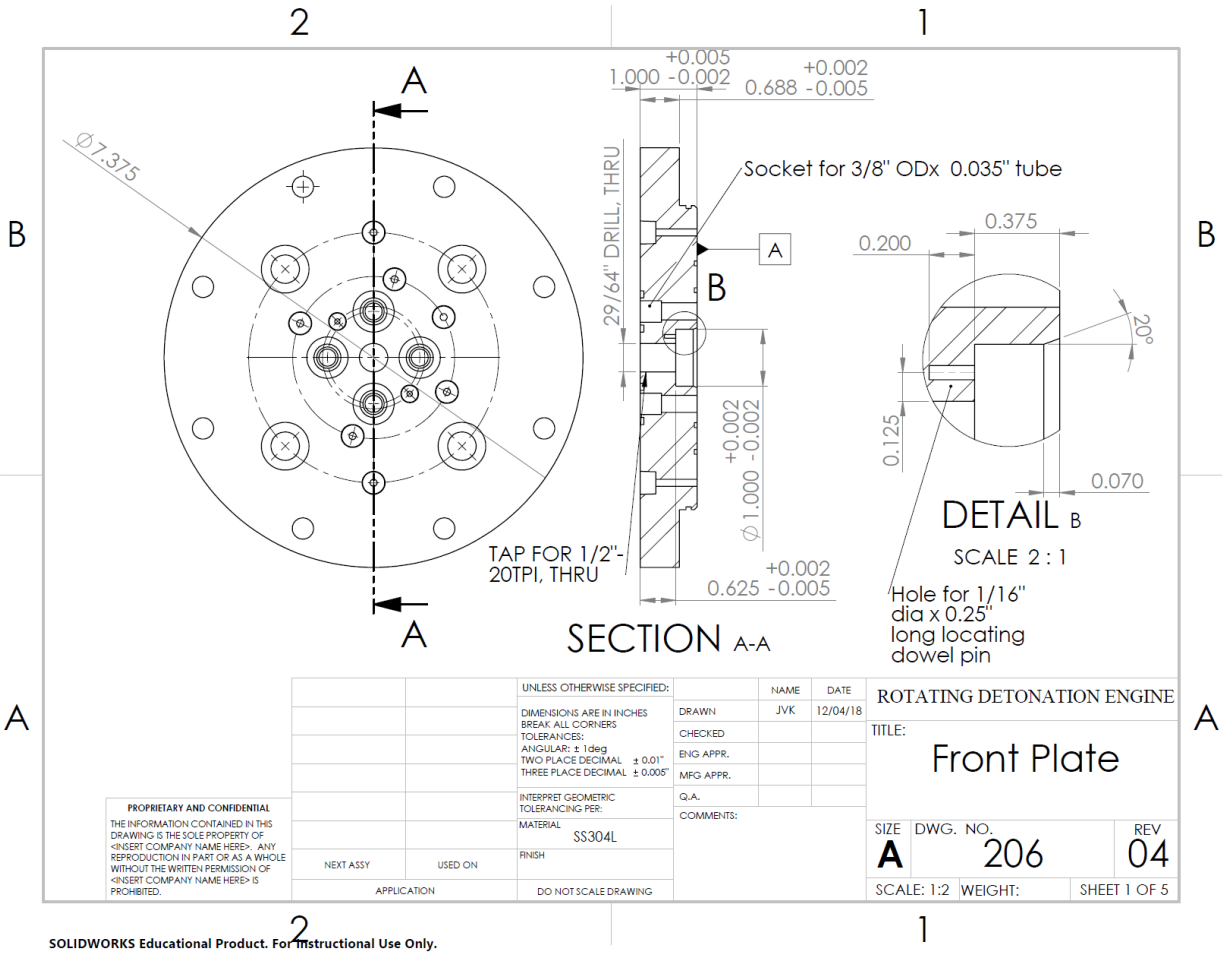
A.1: 76-FI-5

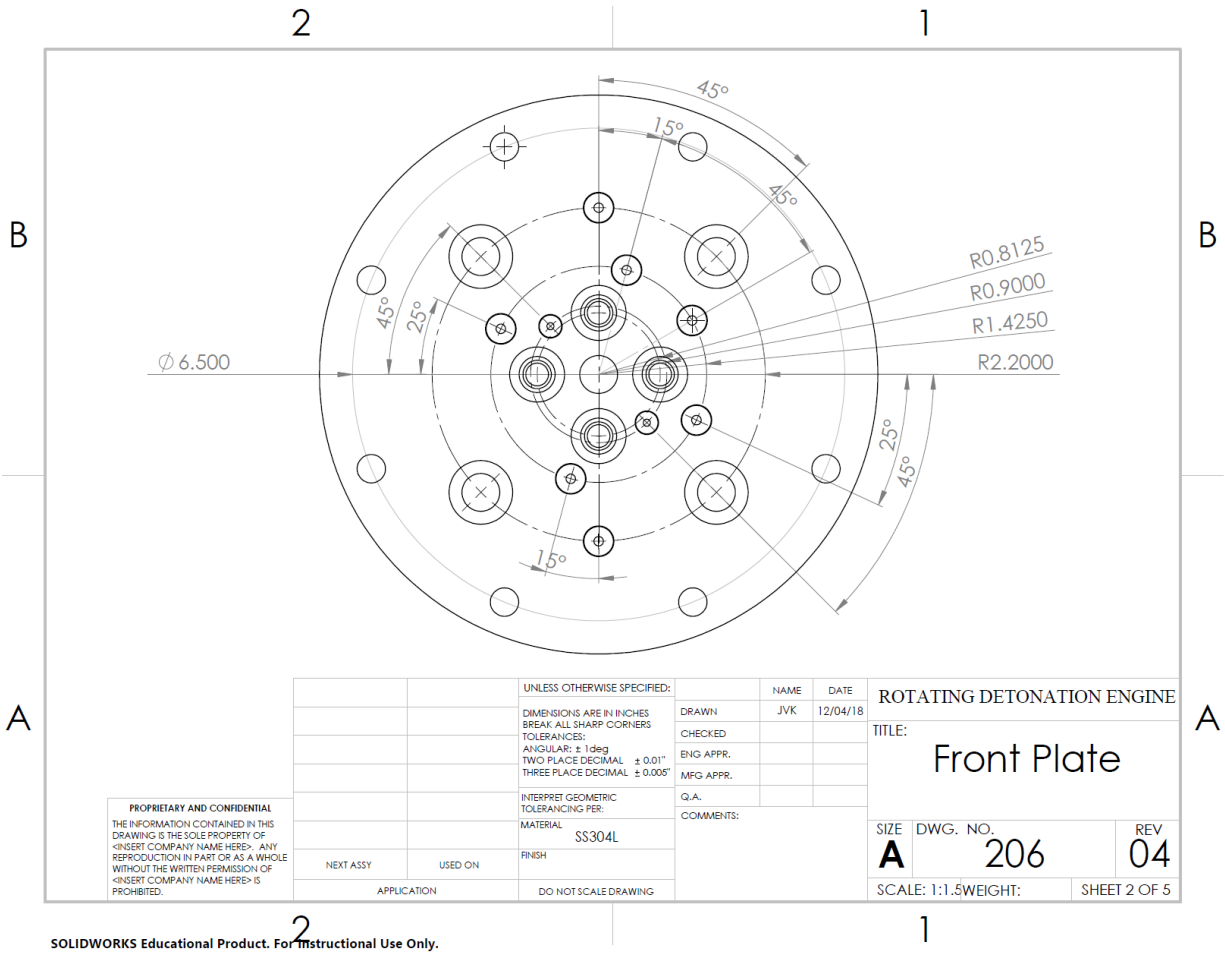
Design for the 76-FI-5 configuration utilized components from an existing 76 mm engine with radial injection.

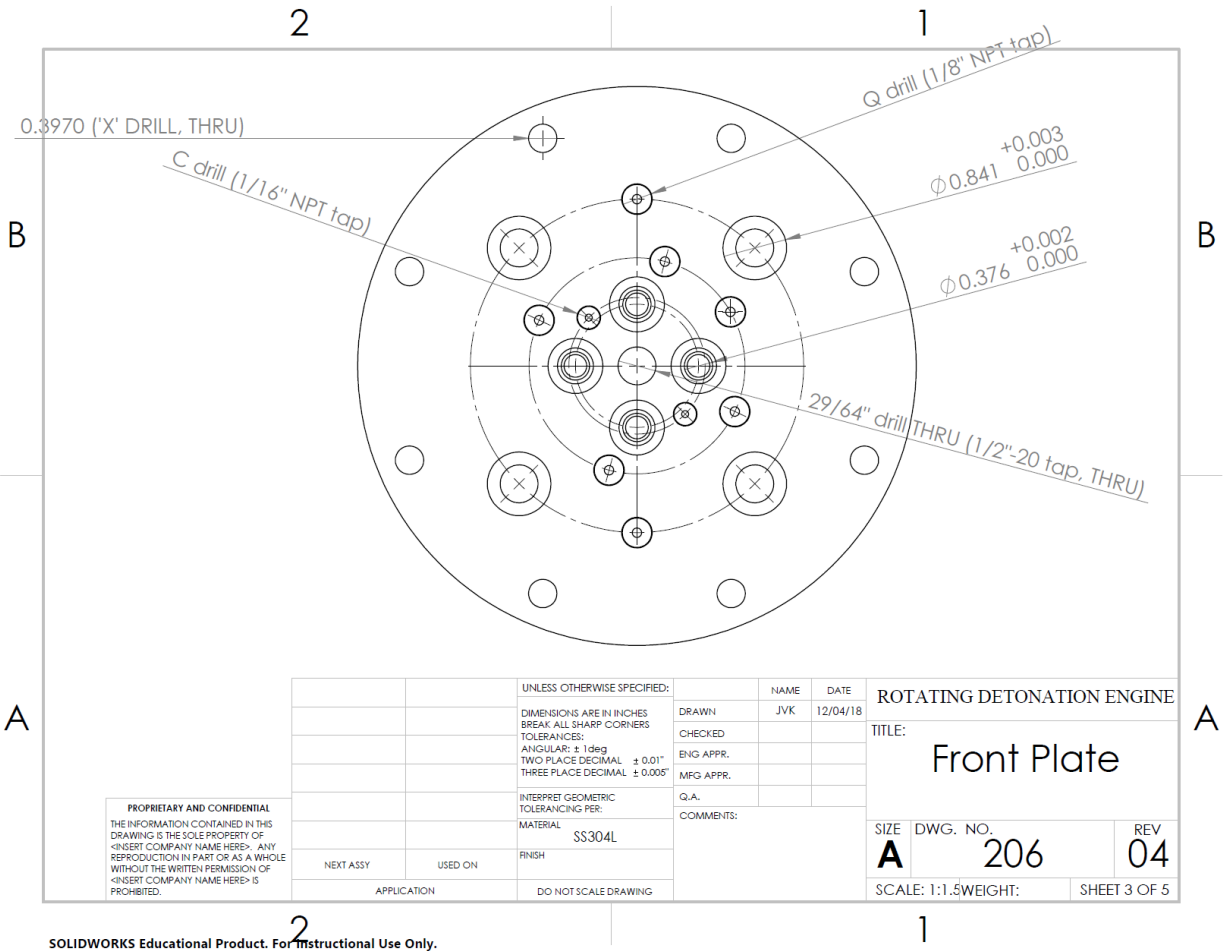




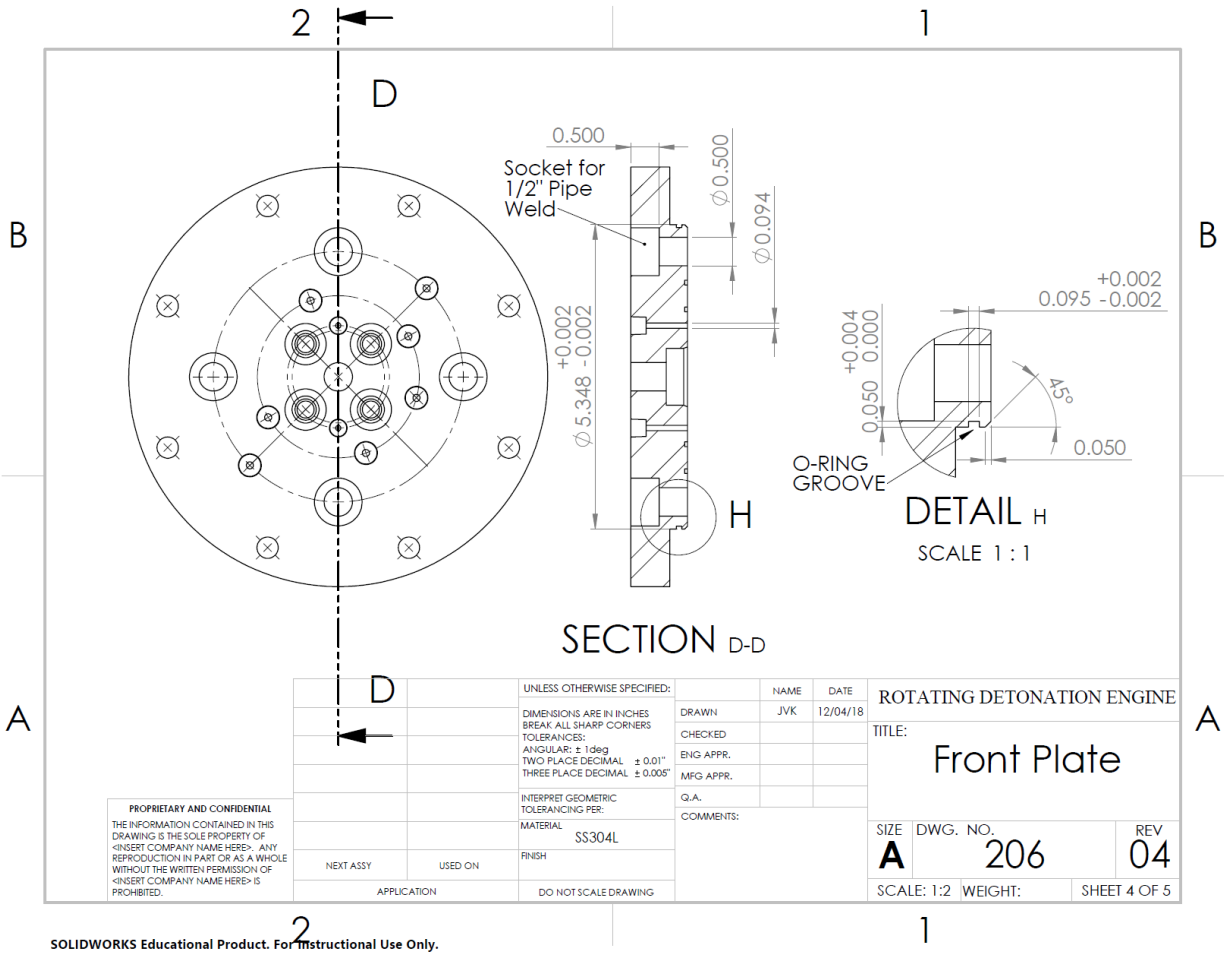


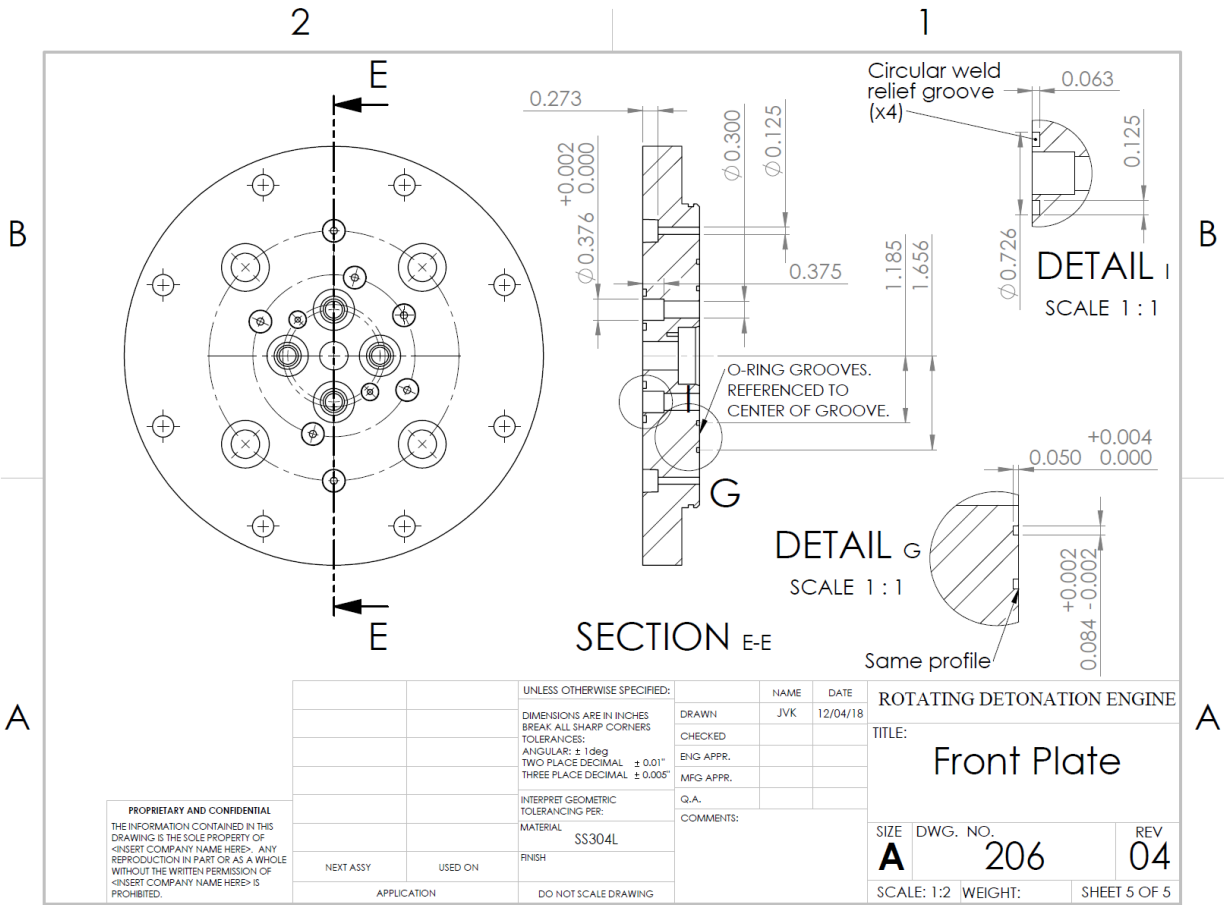


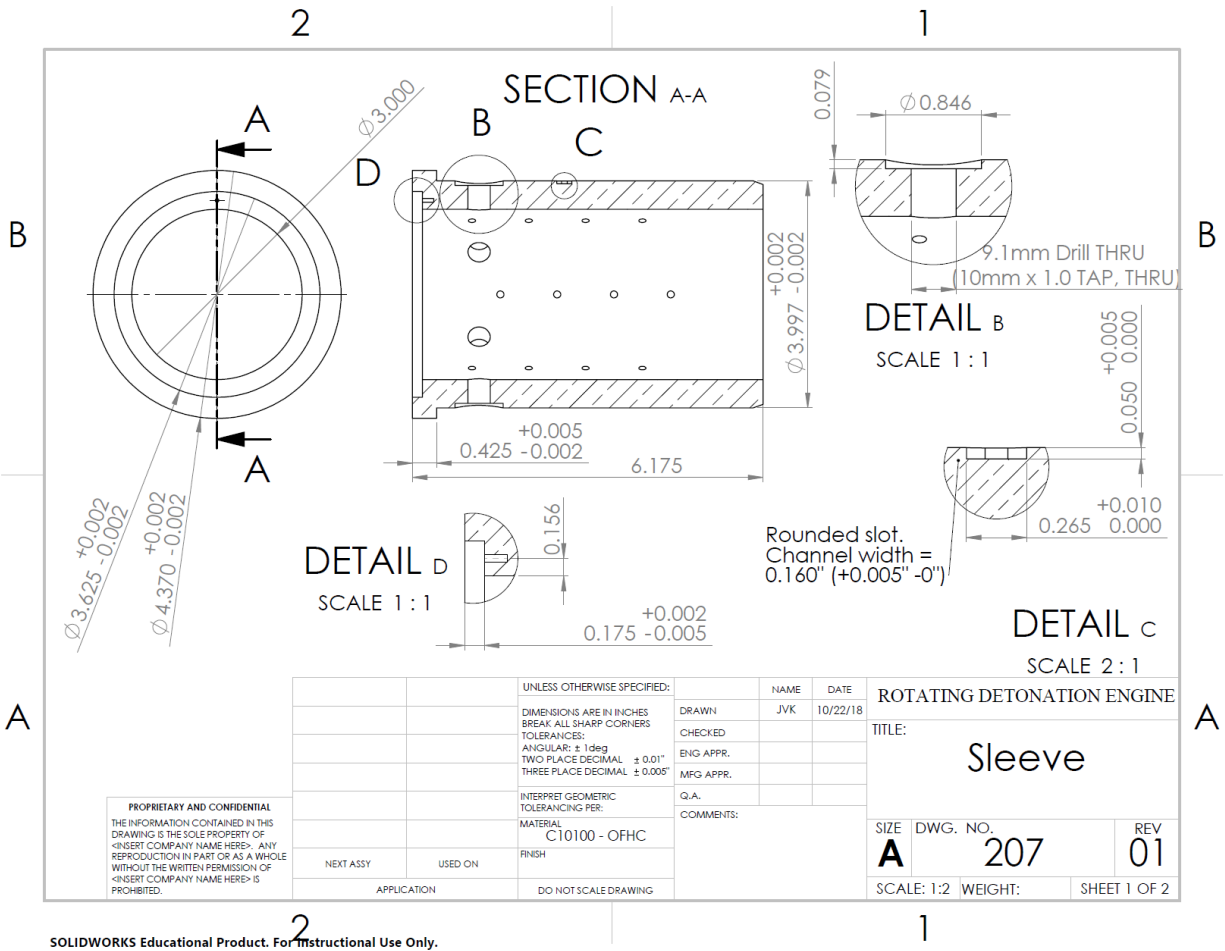


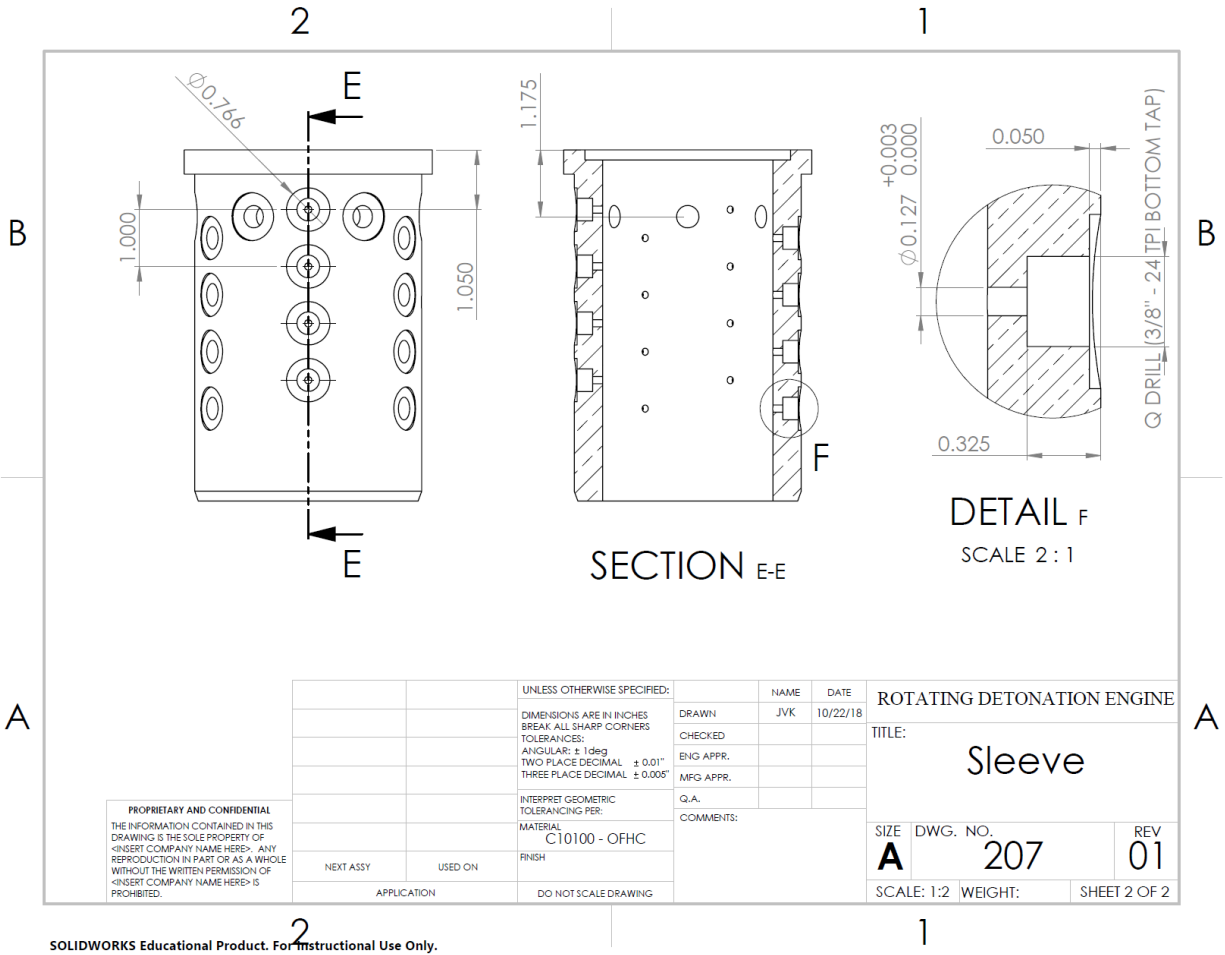


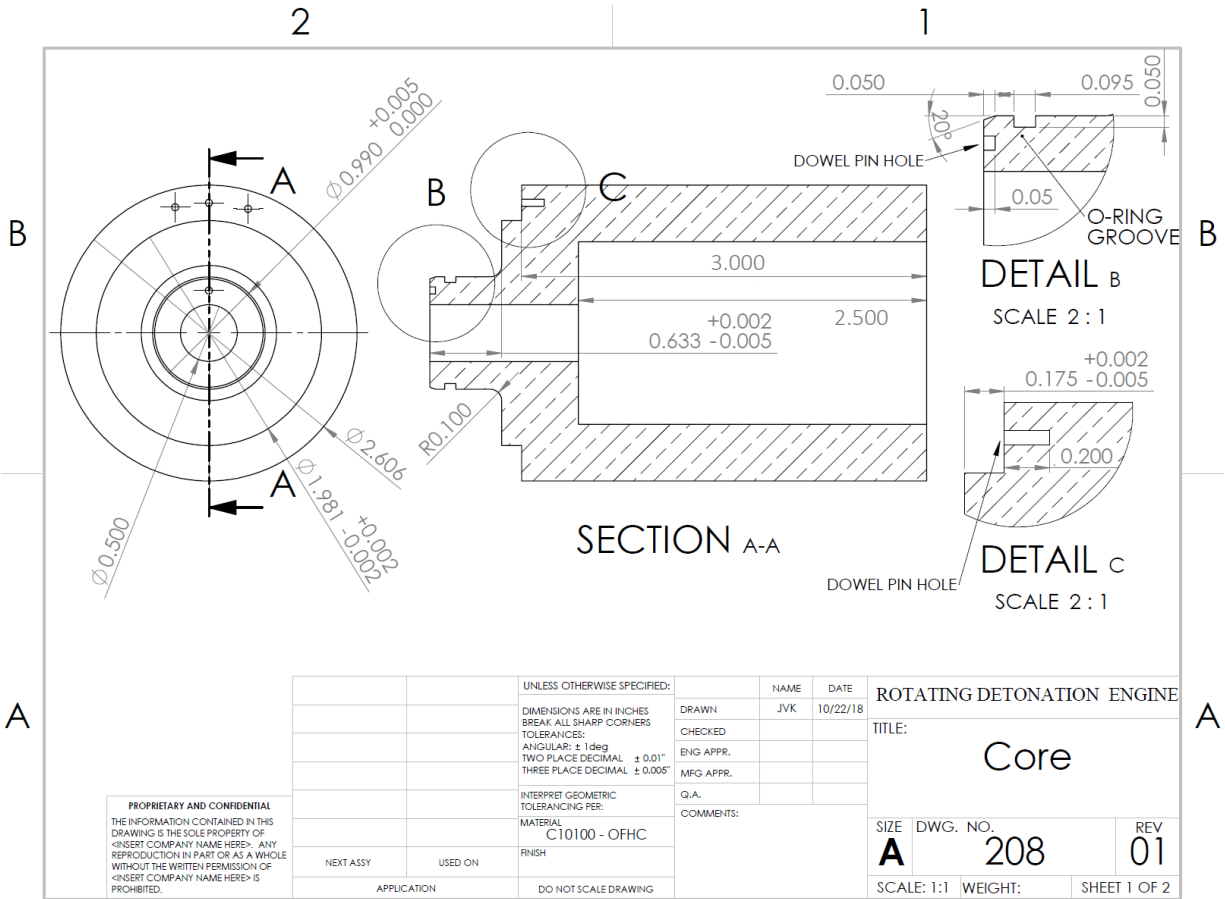


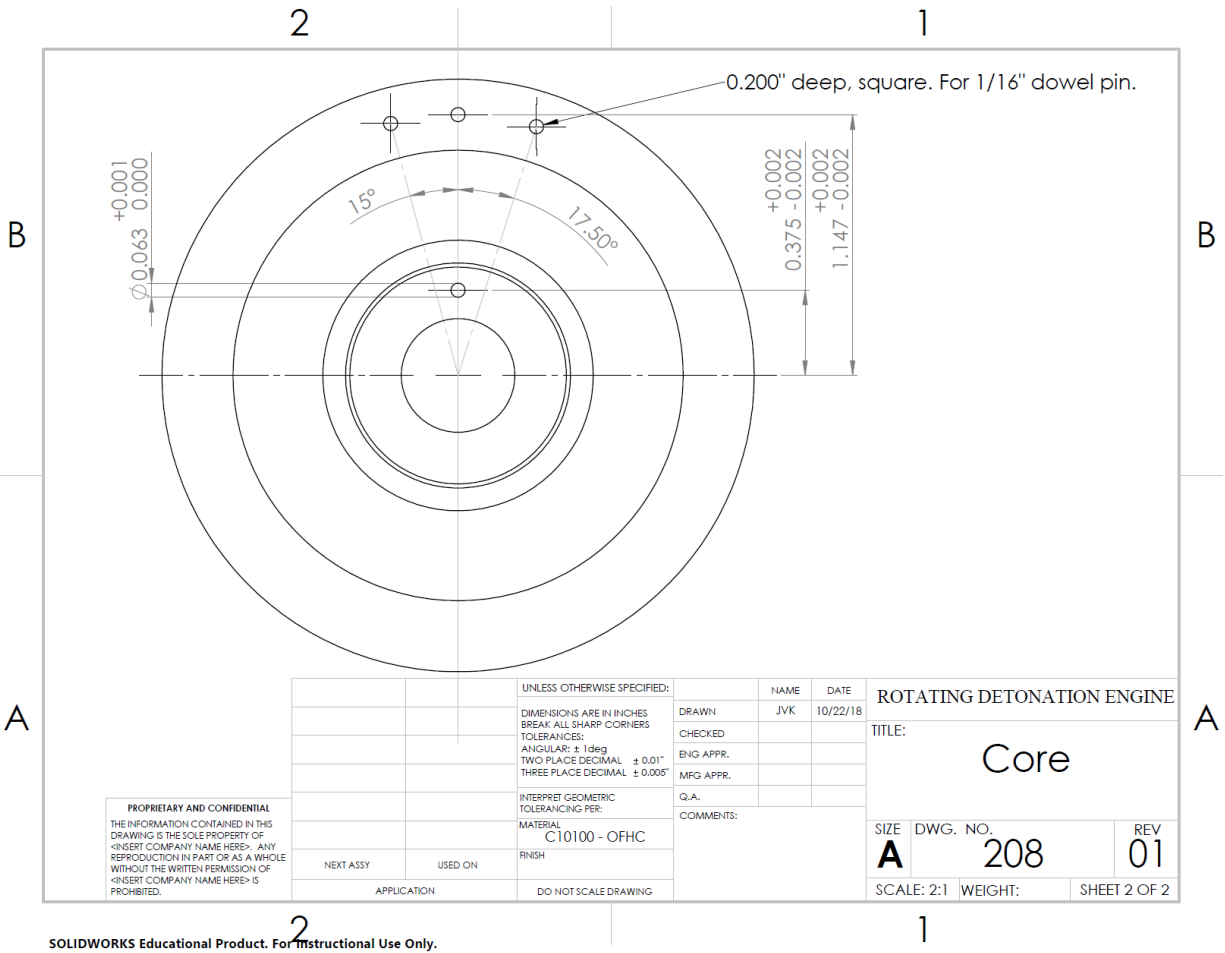




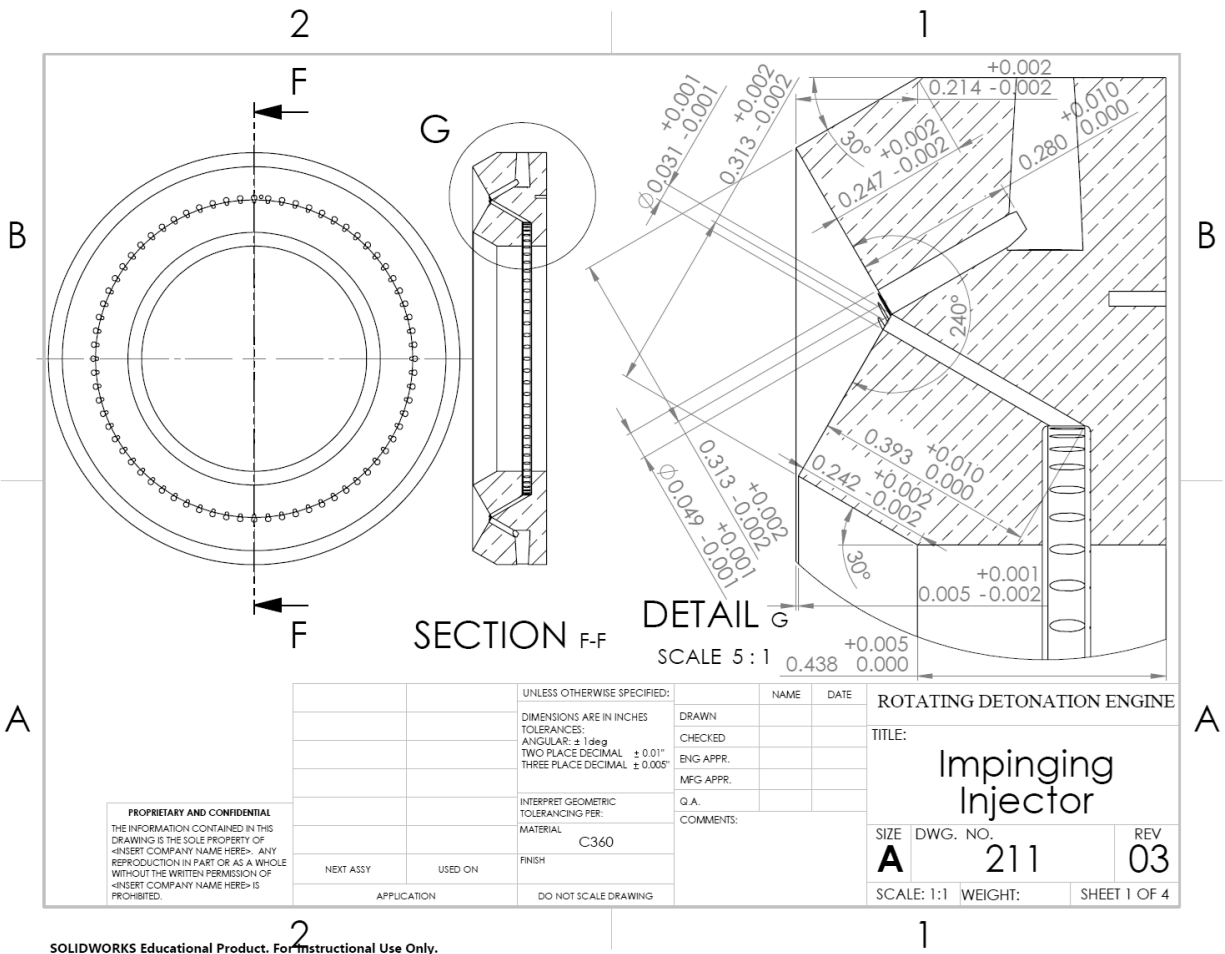


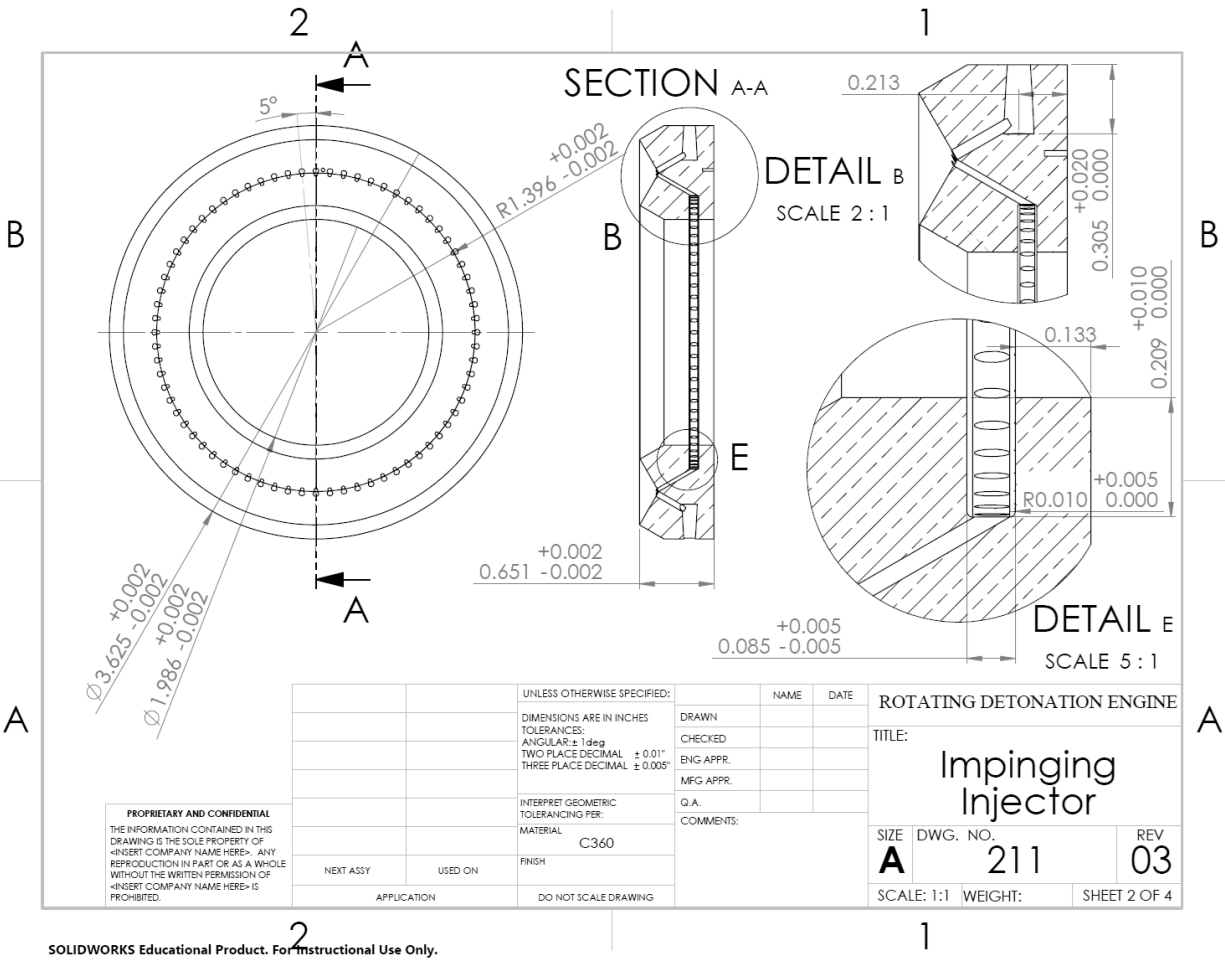






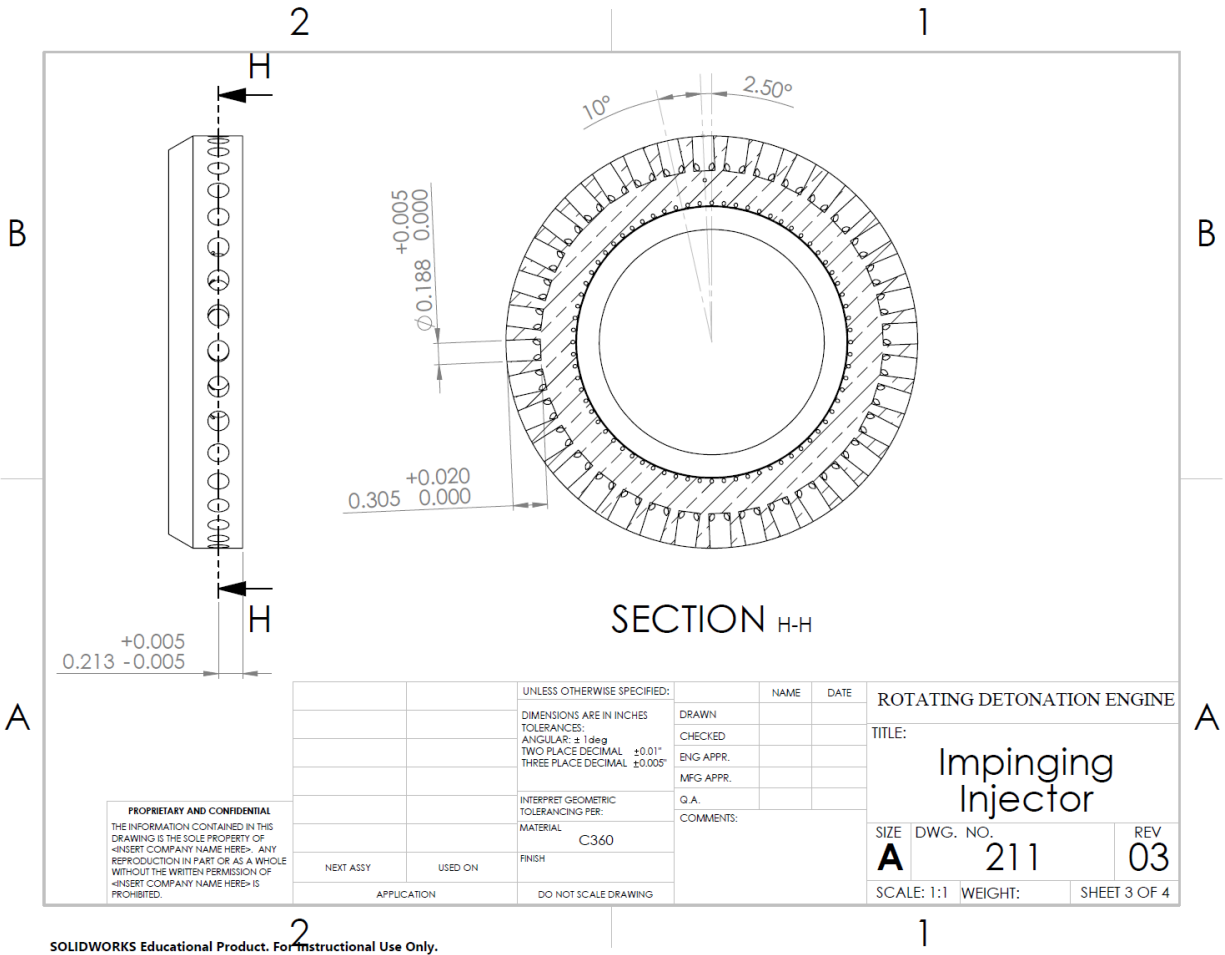
V-groove on injector face acts allows drilling of orifices without specialized tooling and was removed in the final injector drawing on Page 128.

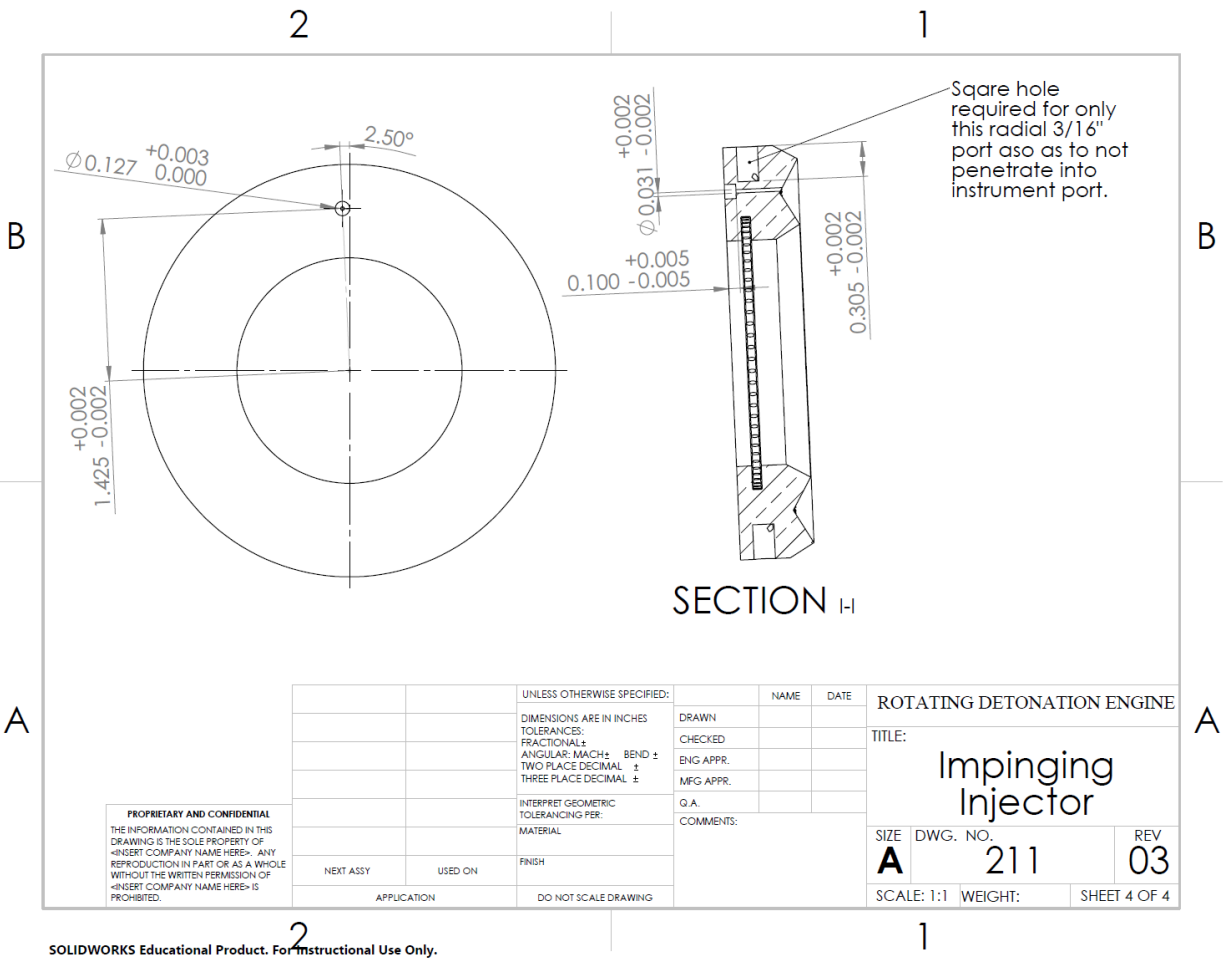


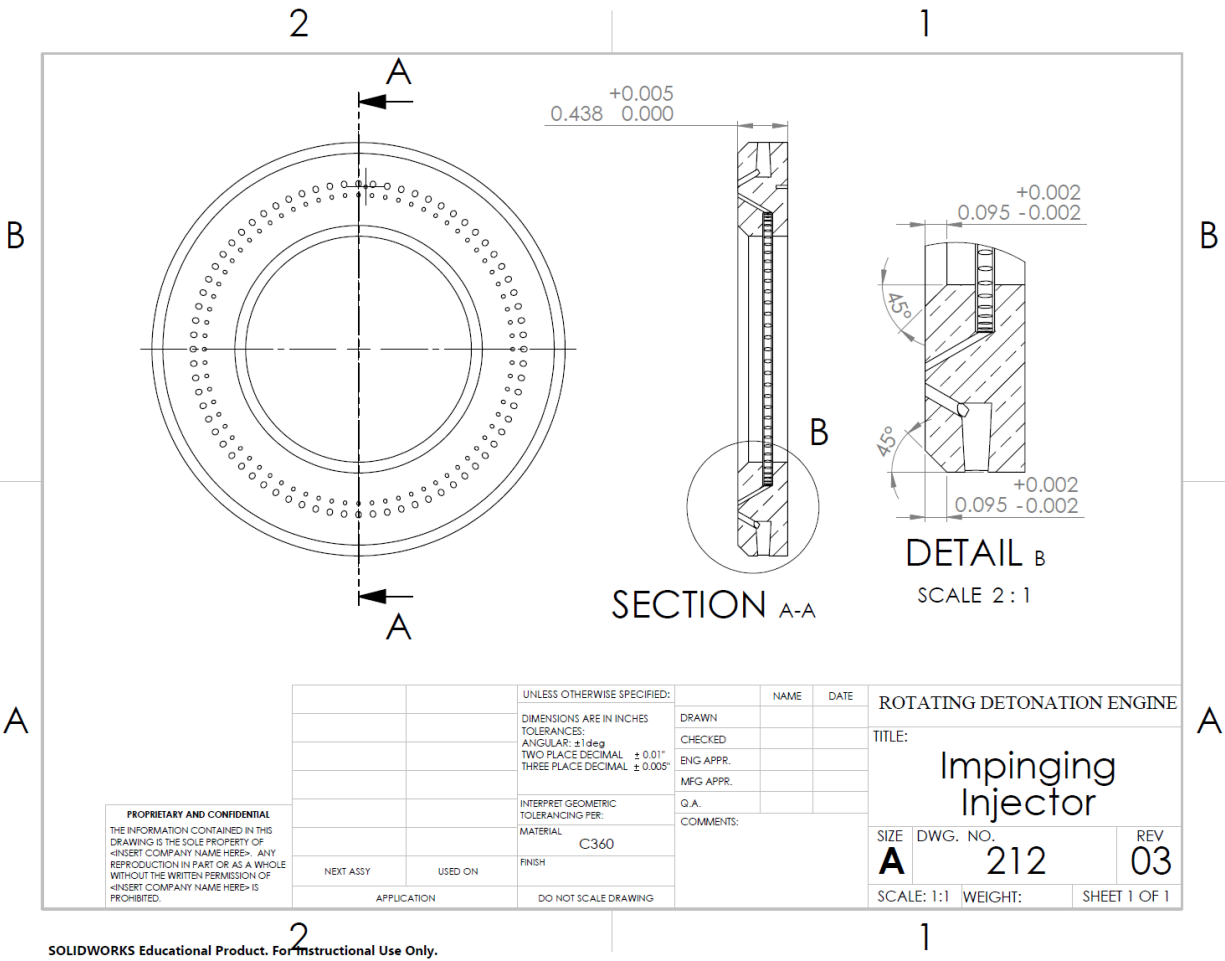


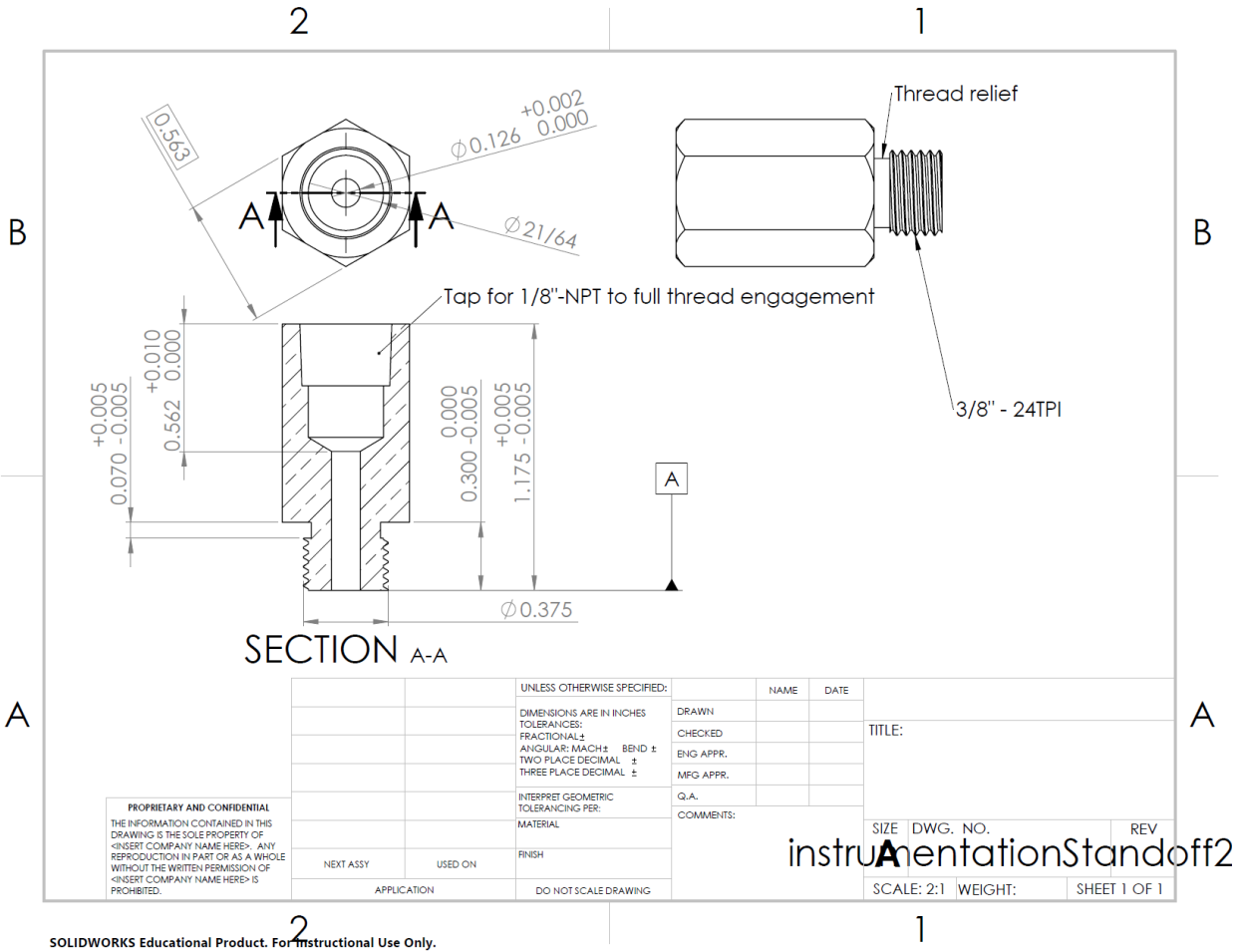
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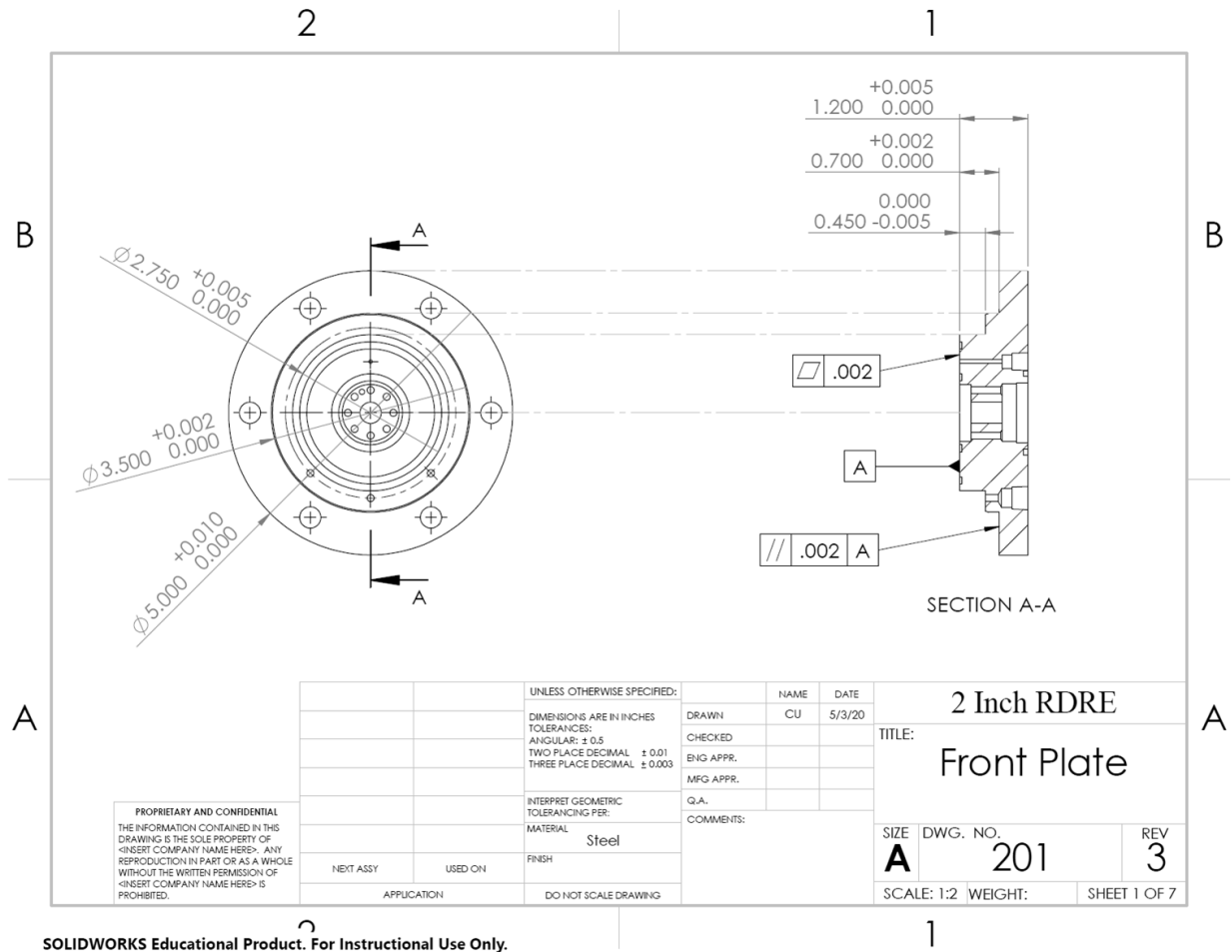


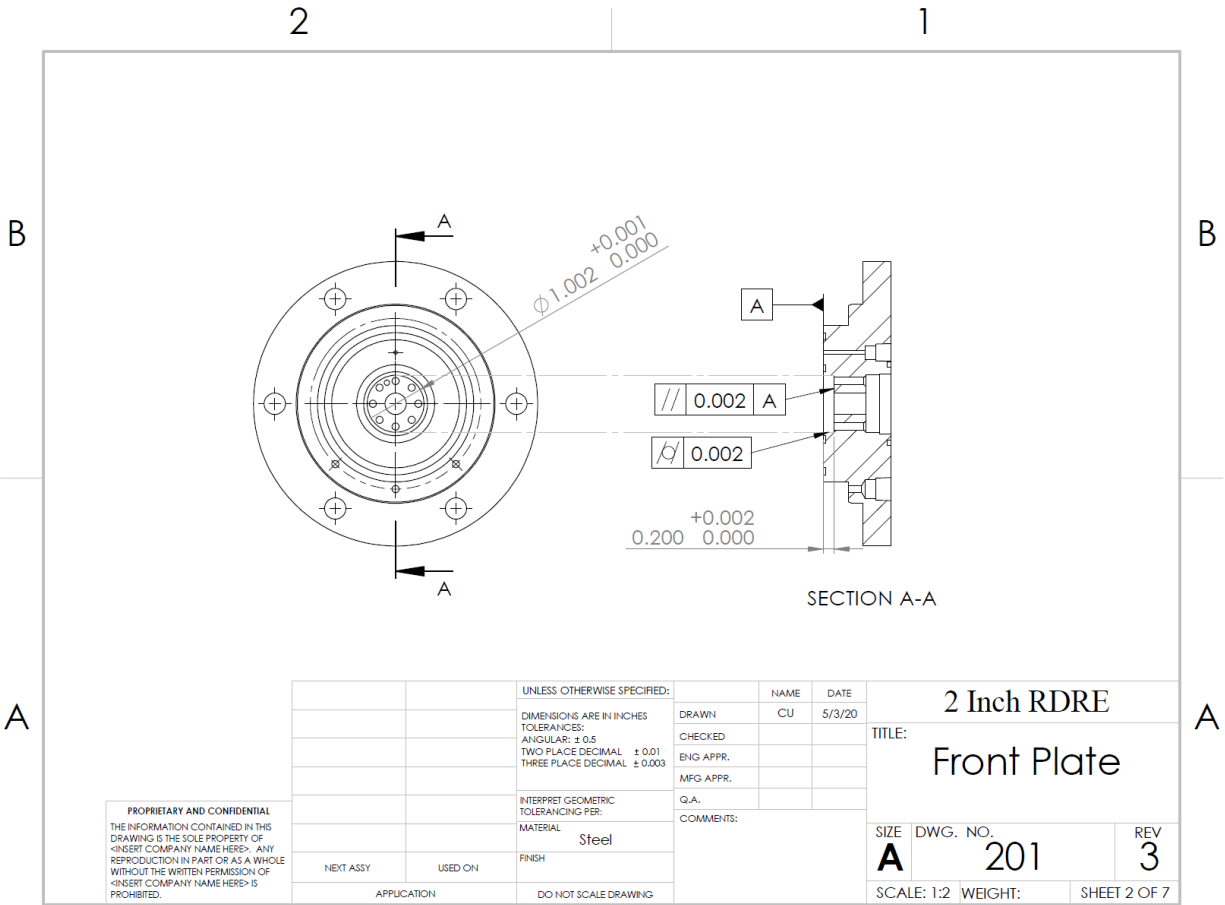


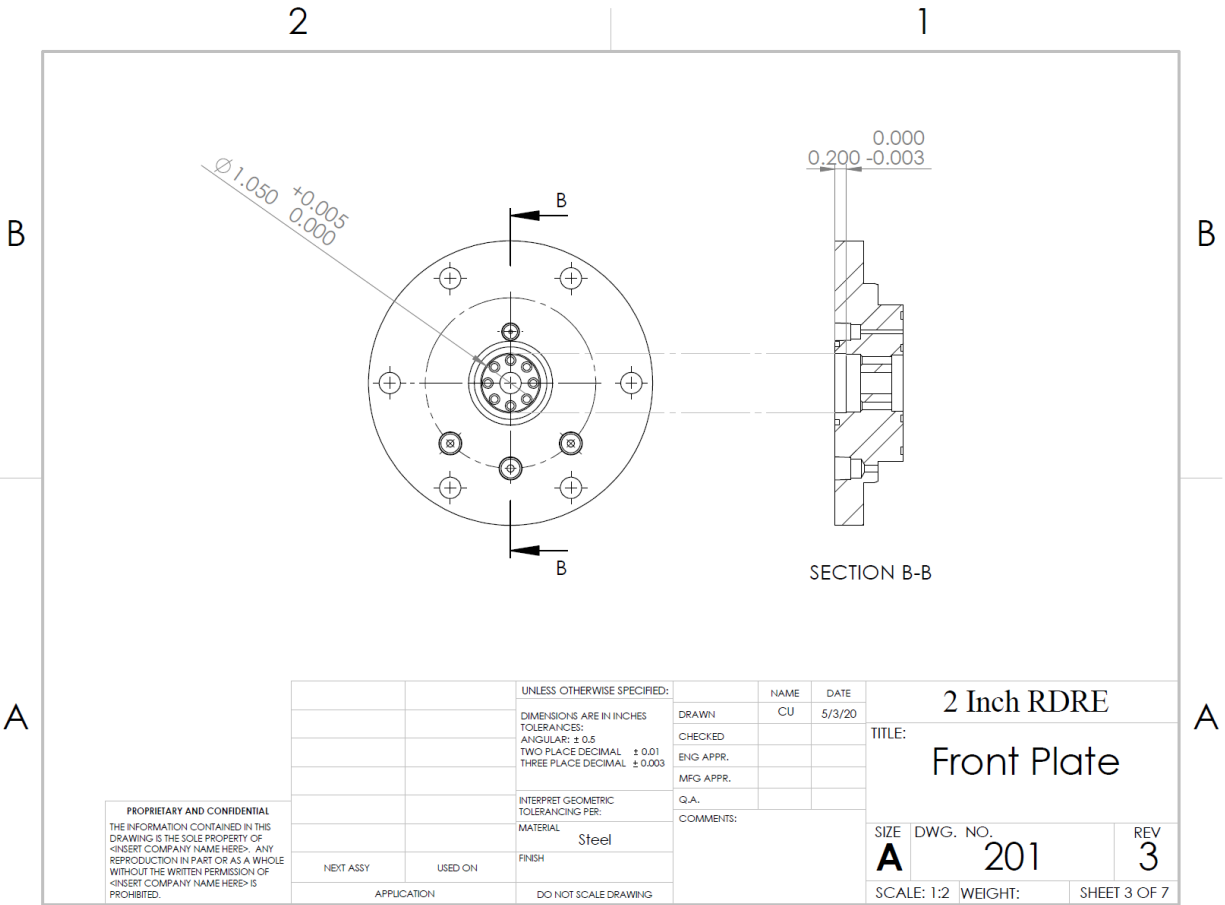


## A.2: 51-FI-5

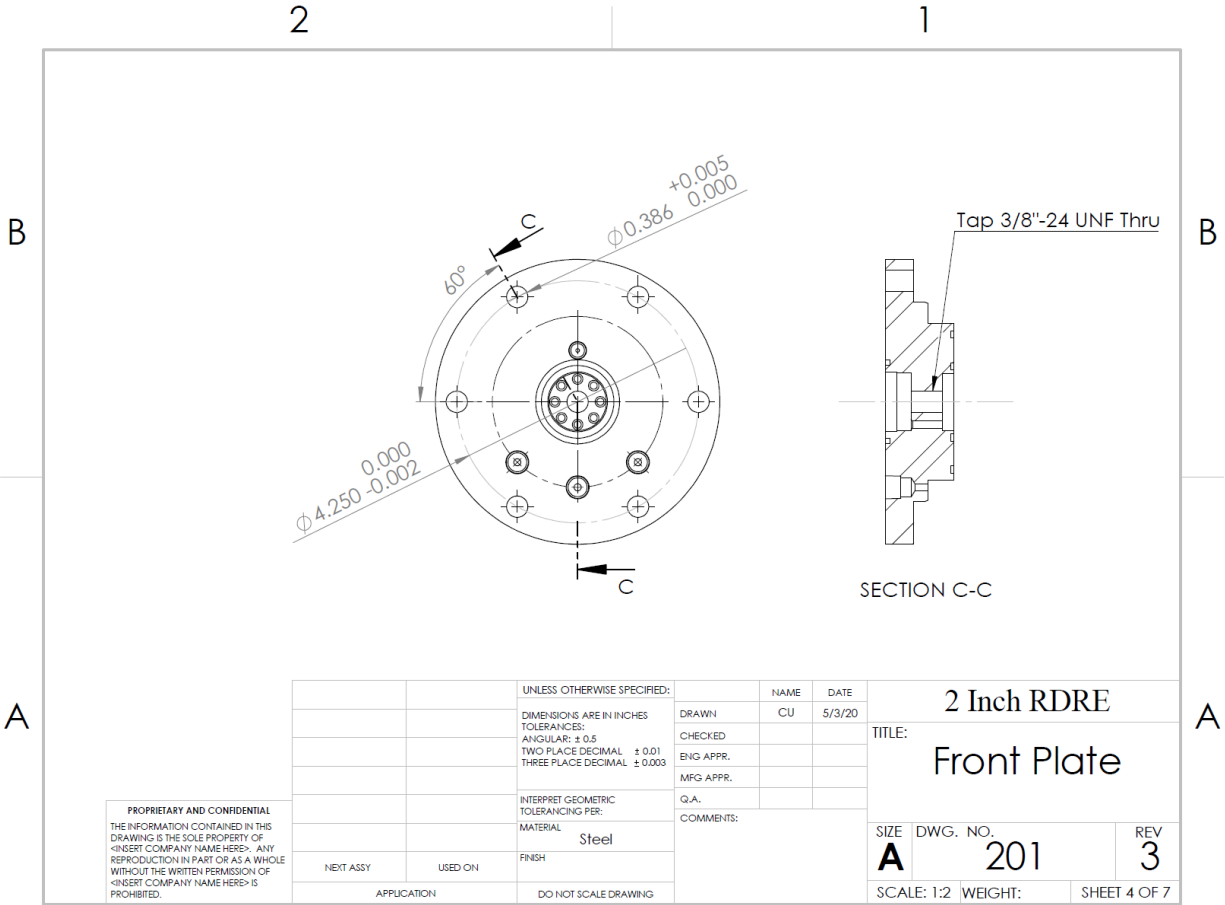
Instrumentation standoffs were the only machined components re-used from the 76 mm configuration.





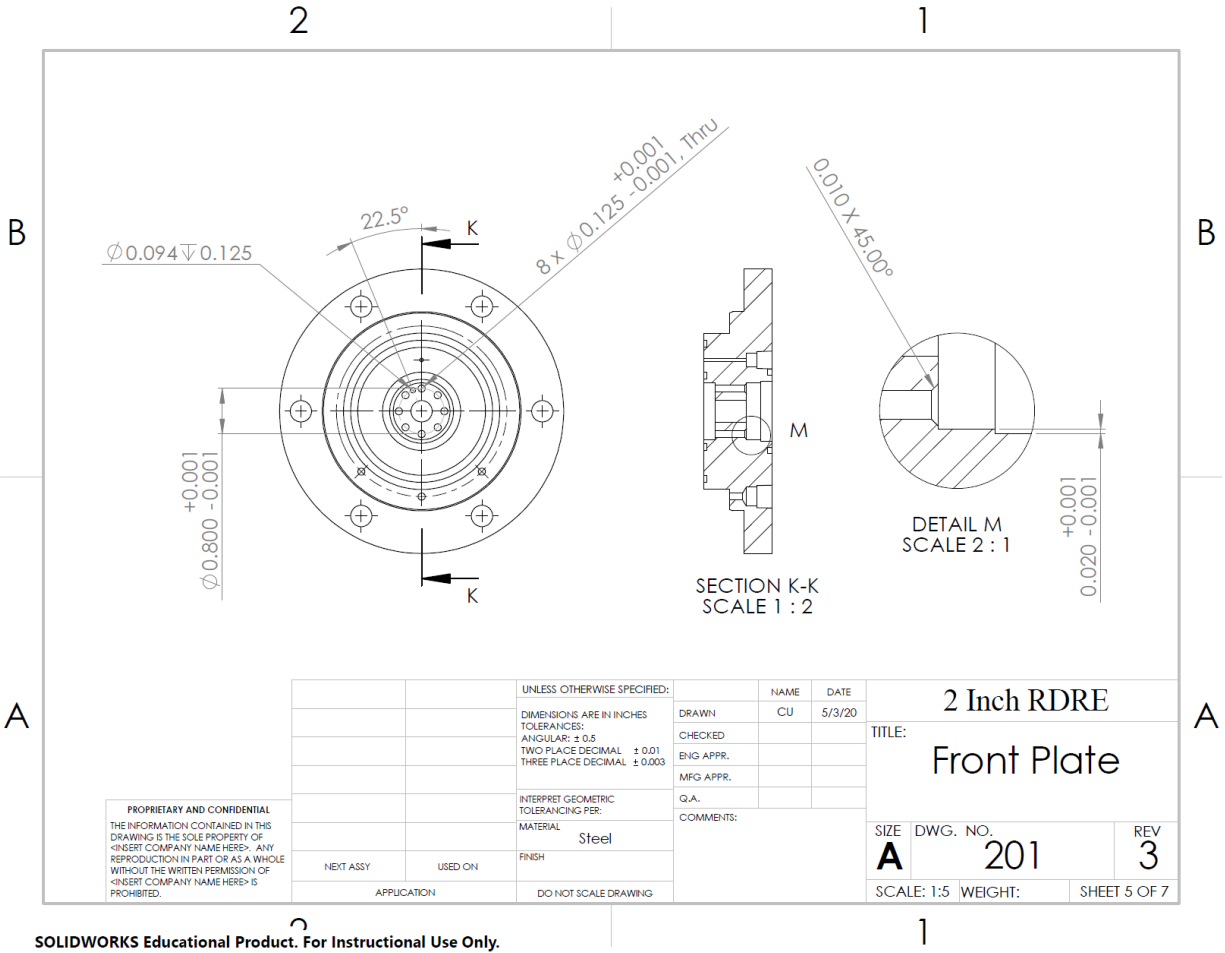


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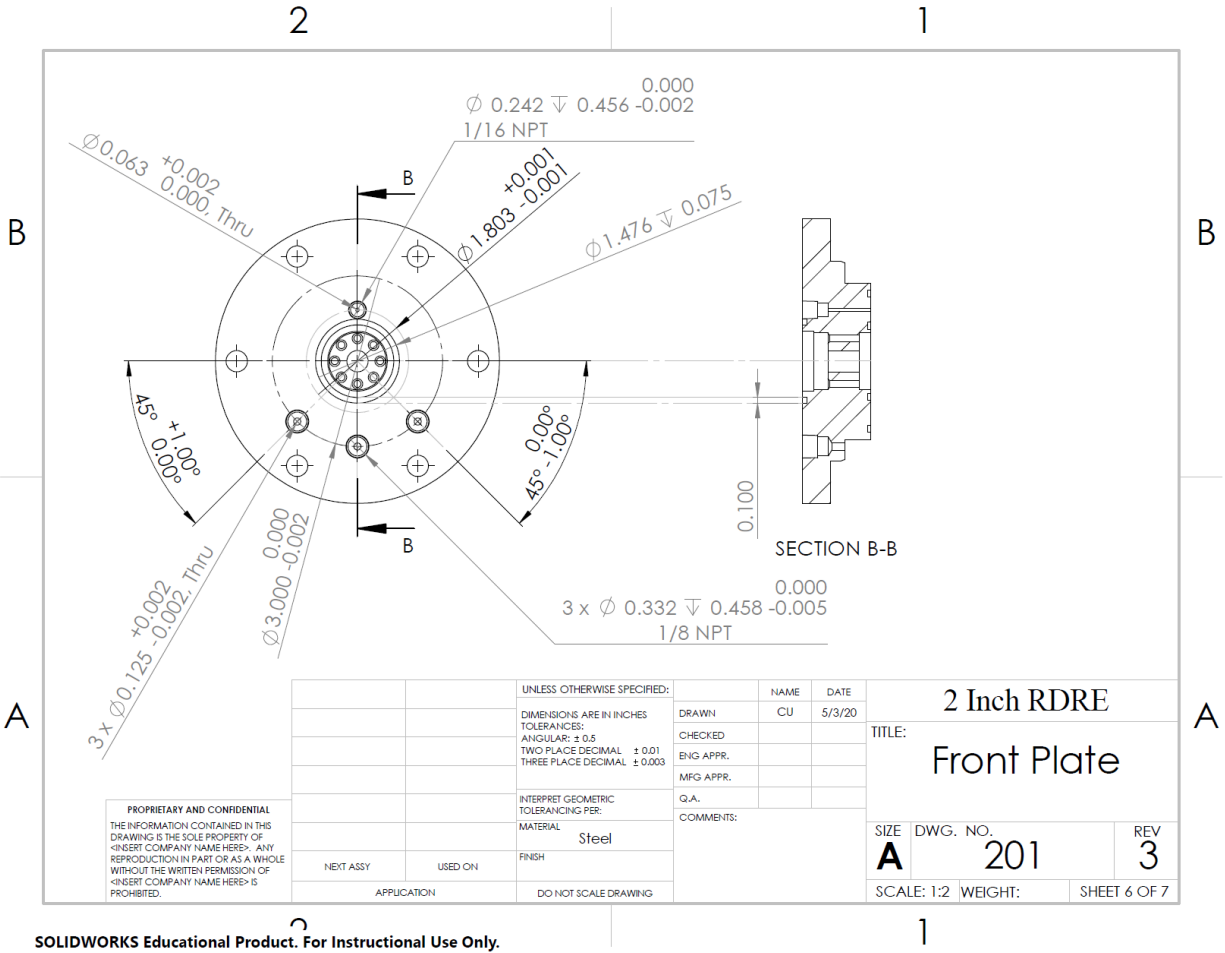


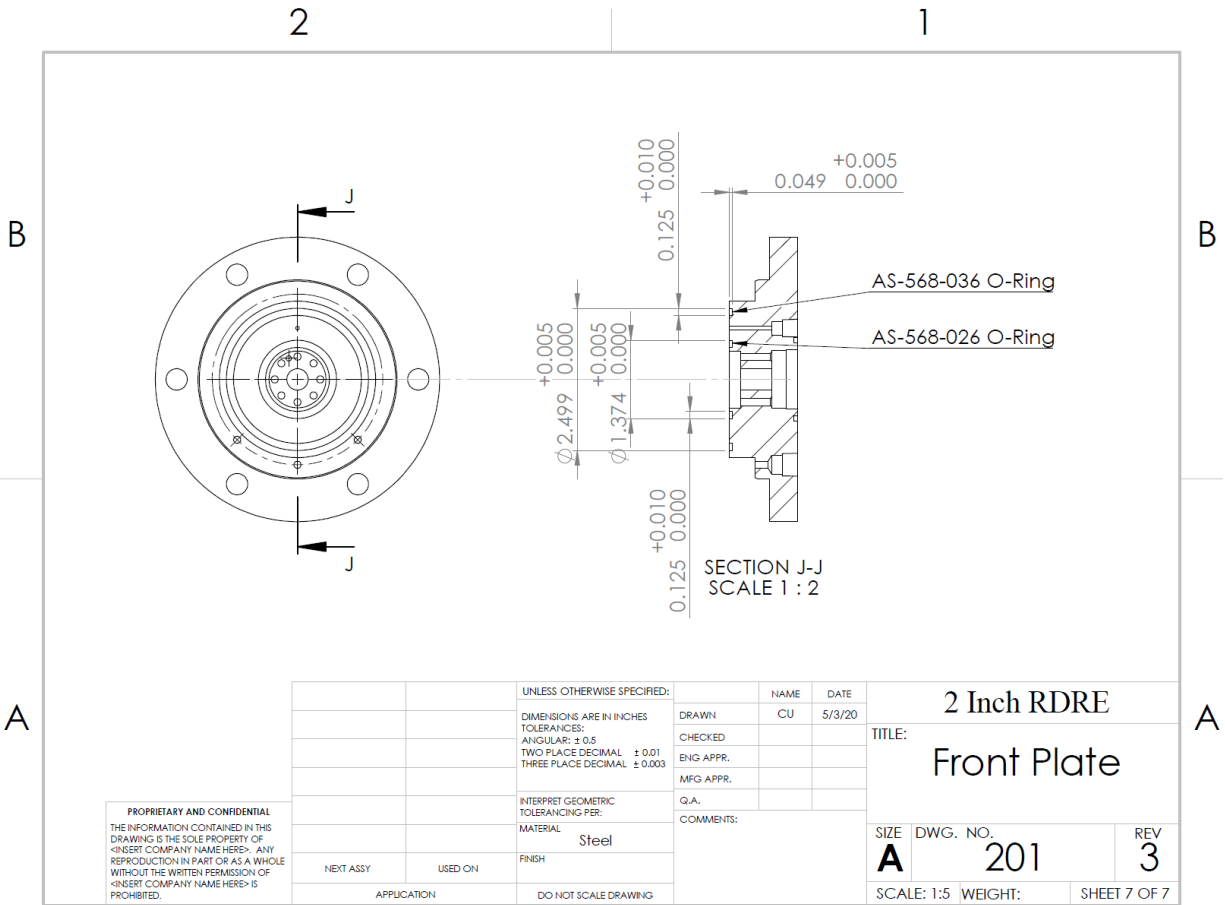
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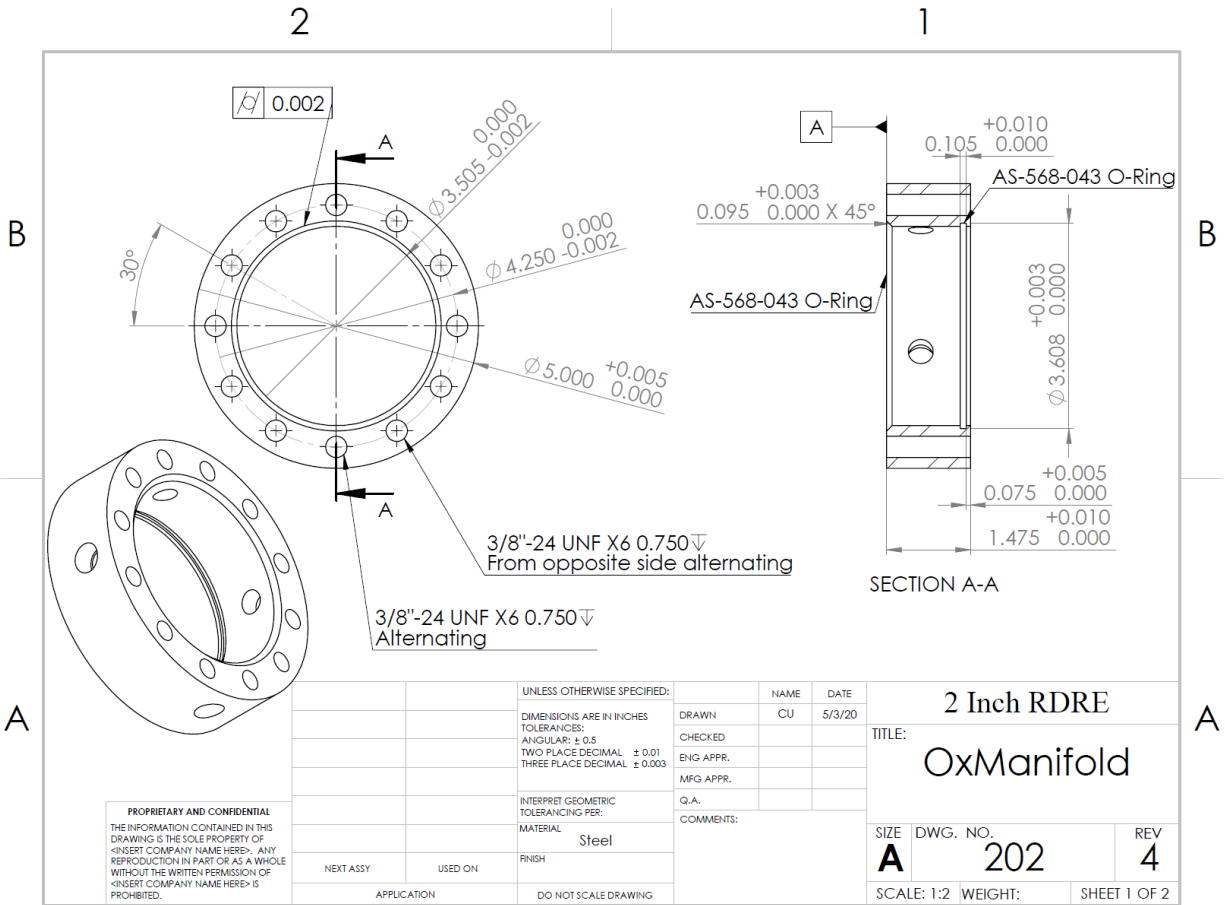
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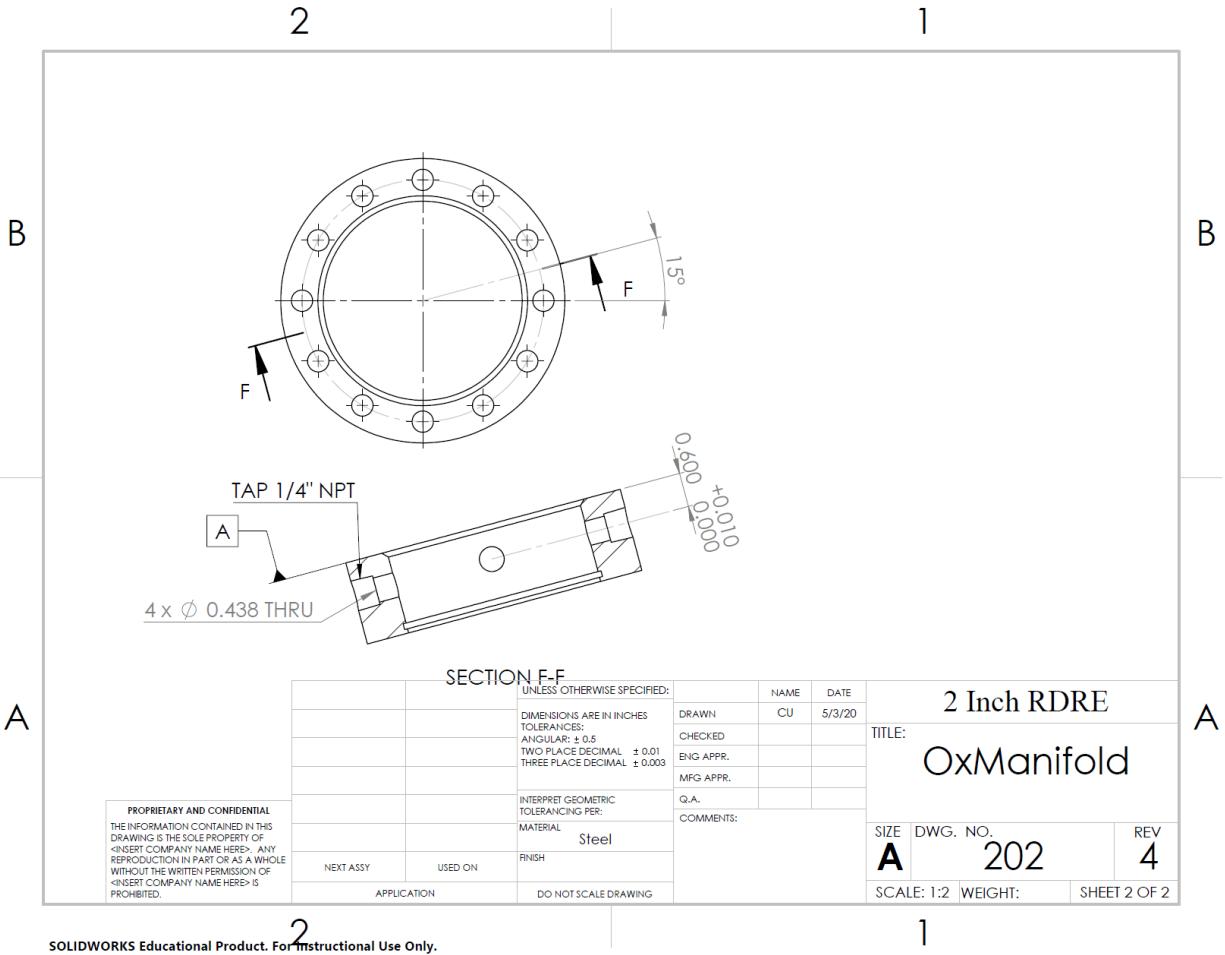


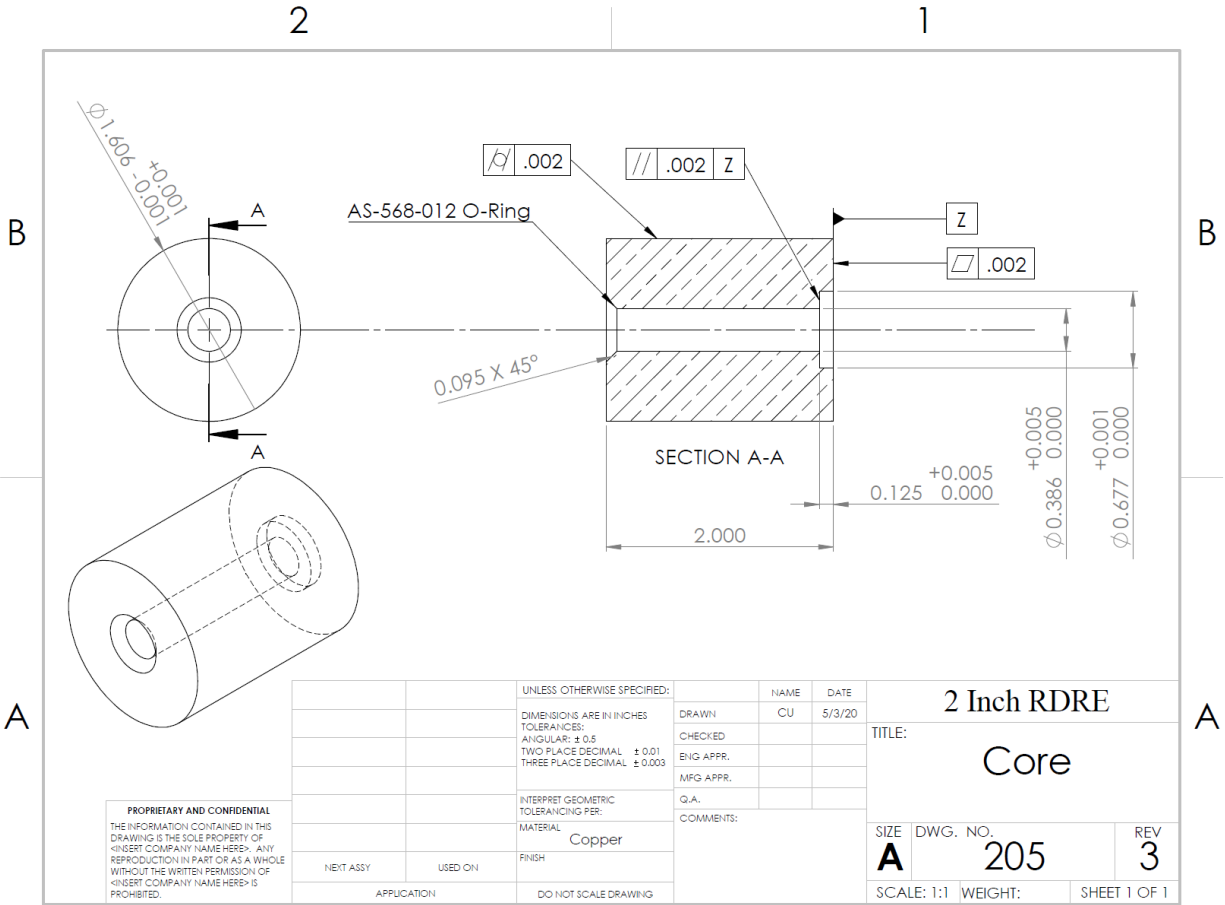


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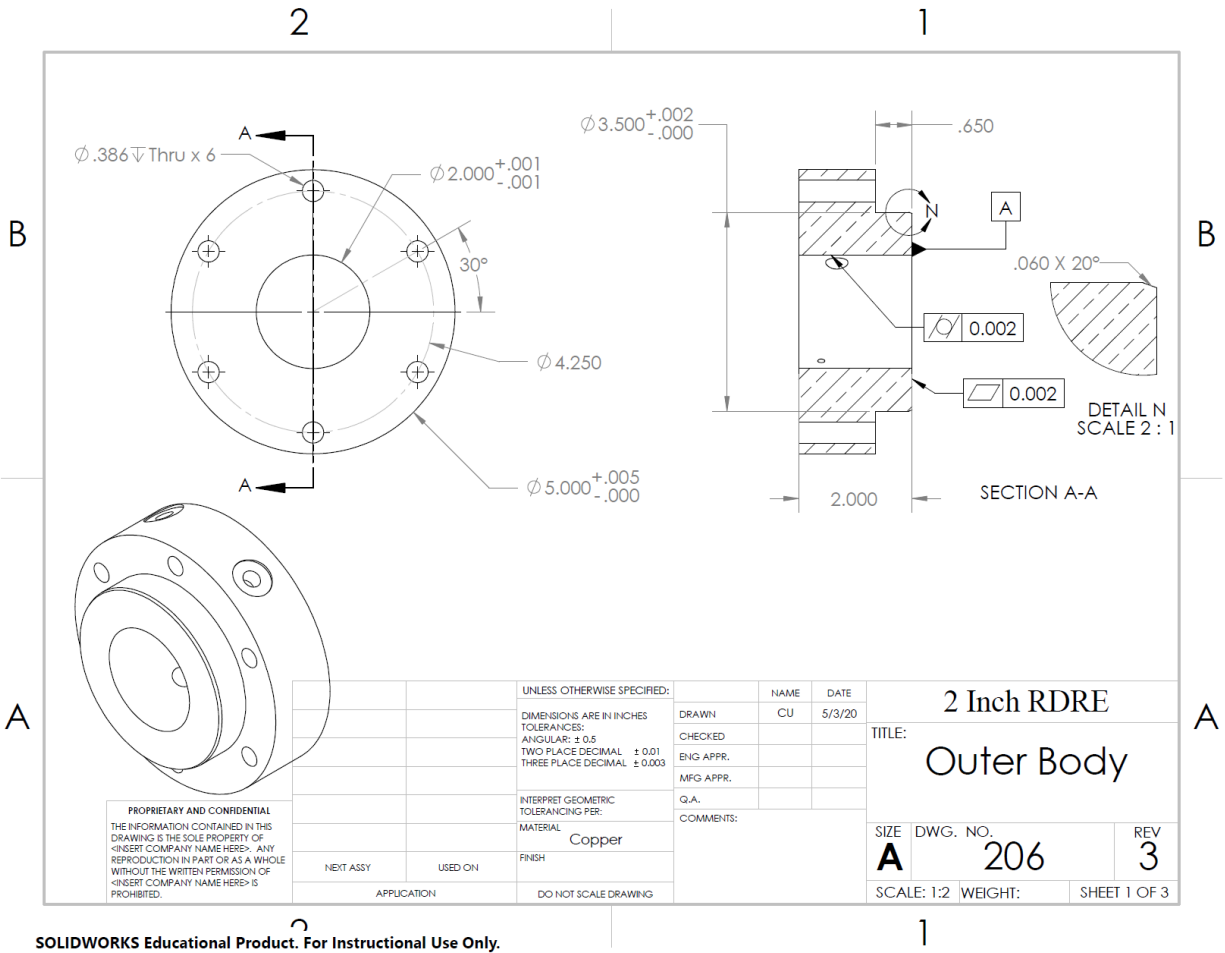
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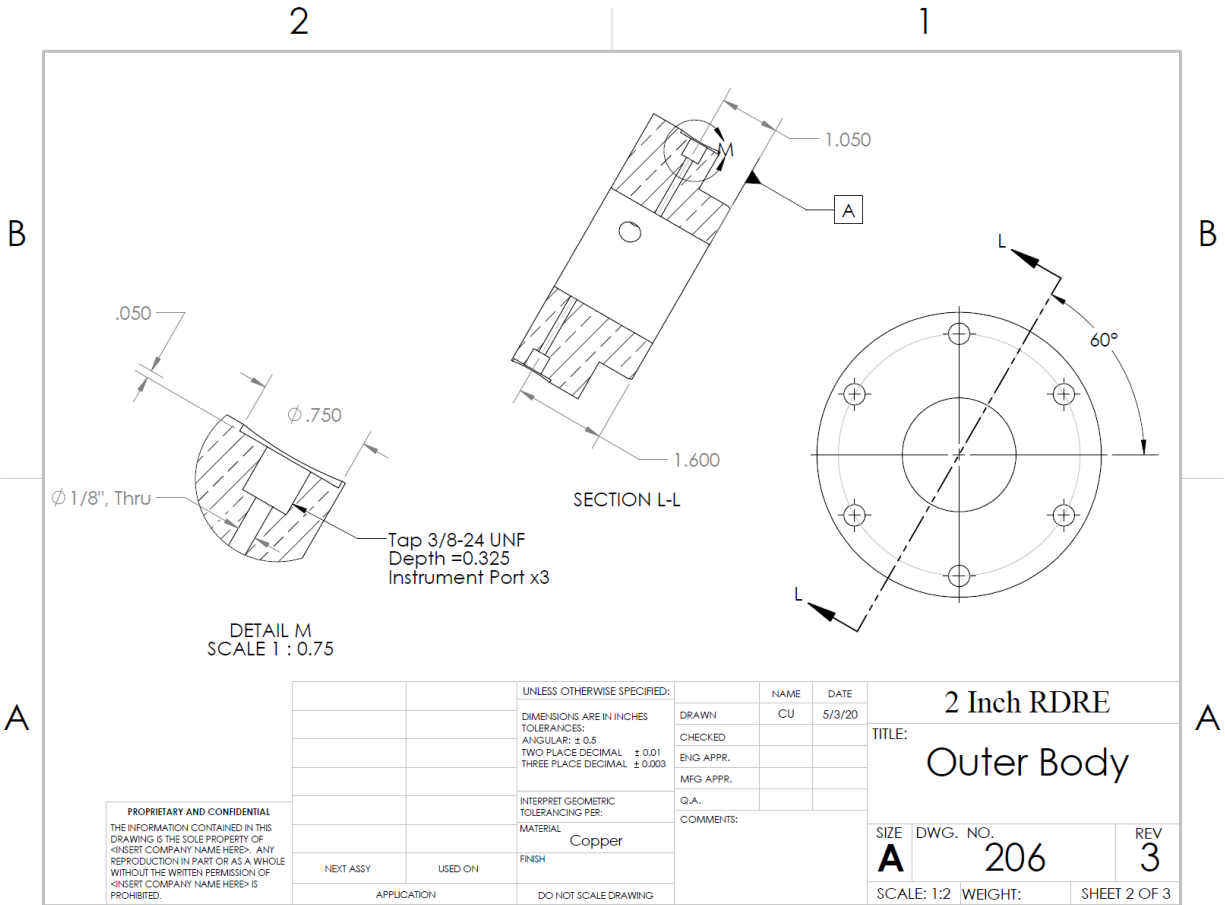


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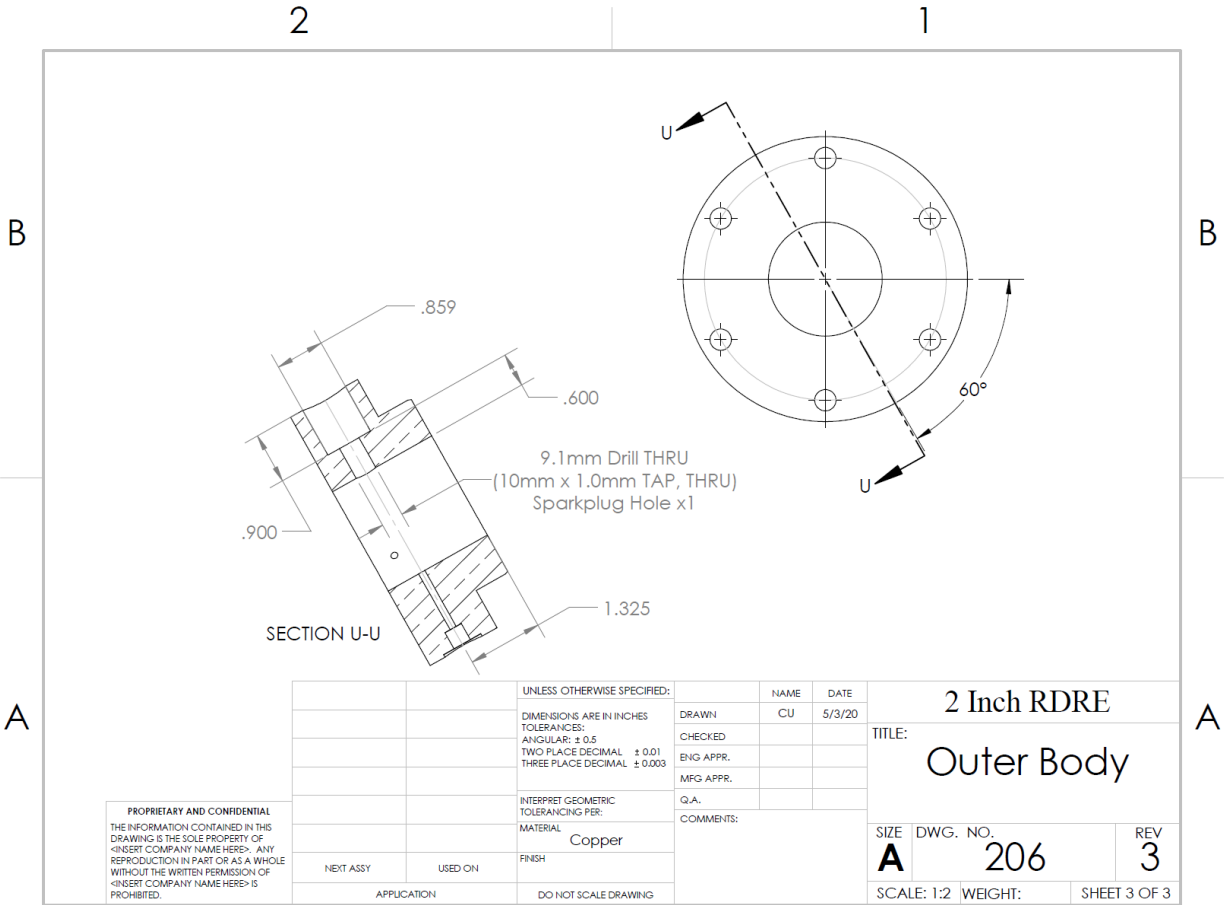
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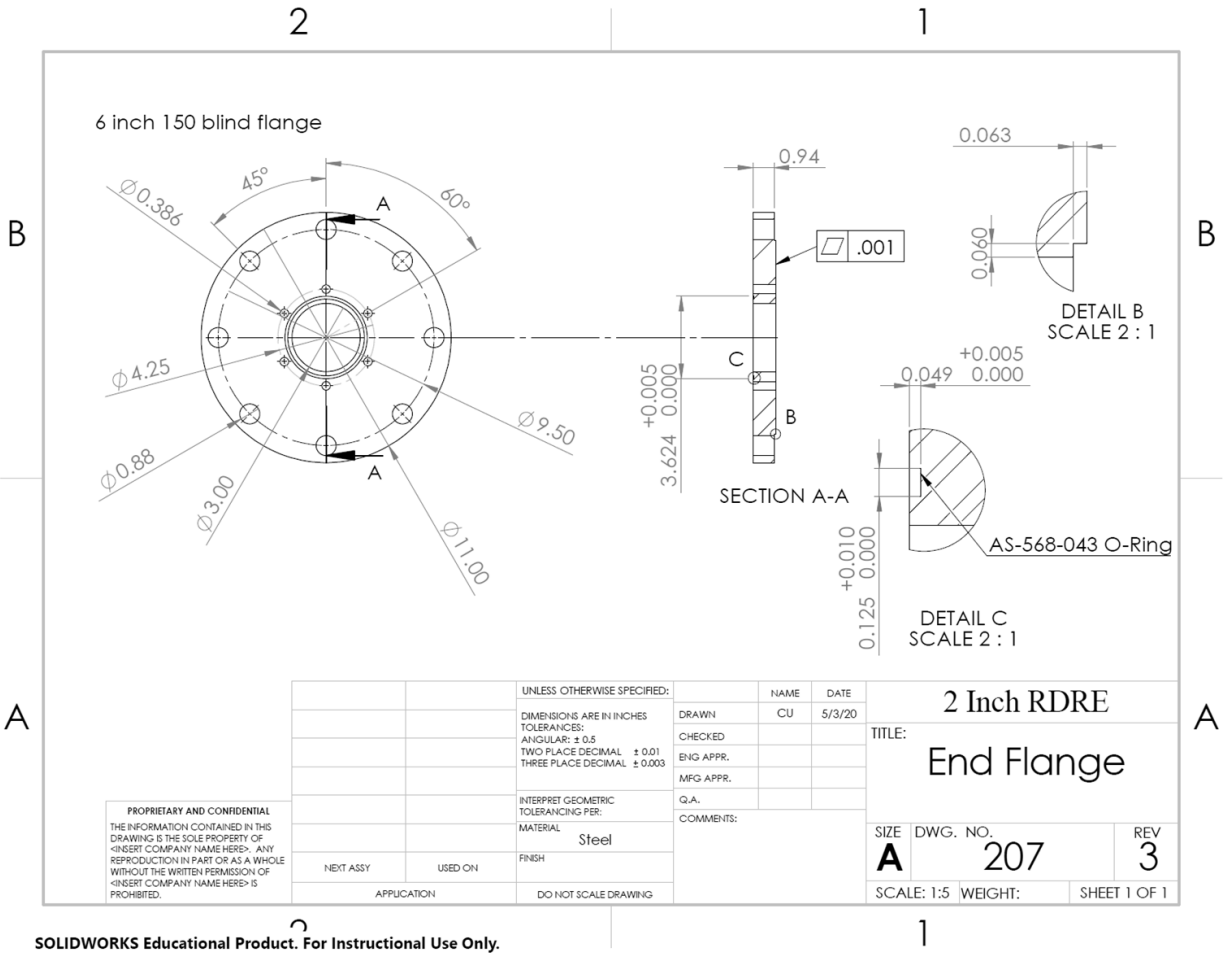


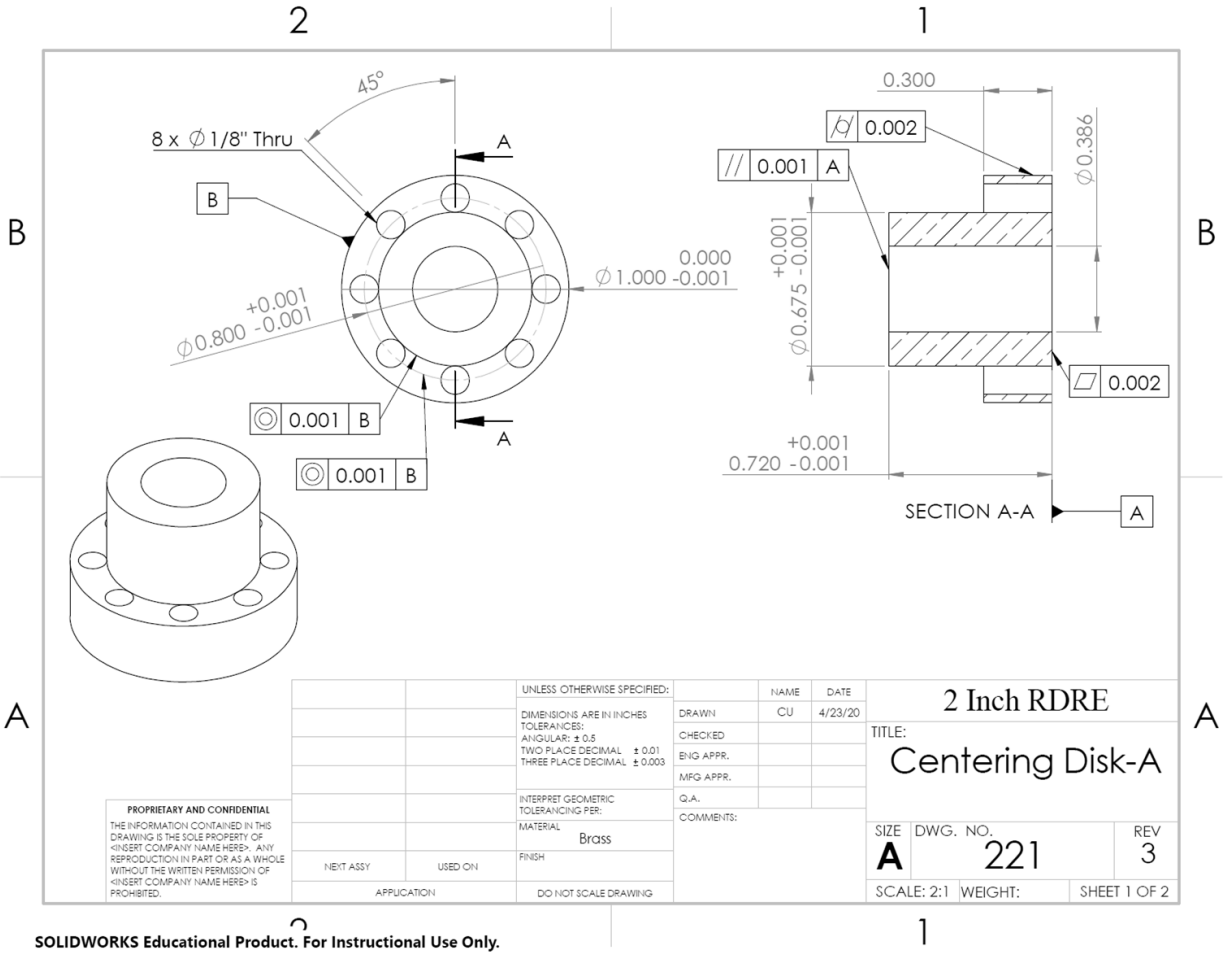
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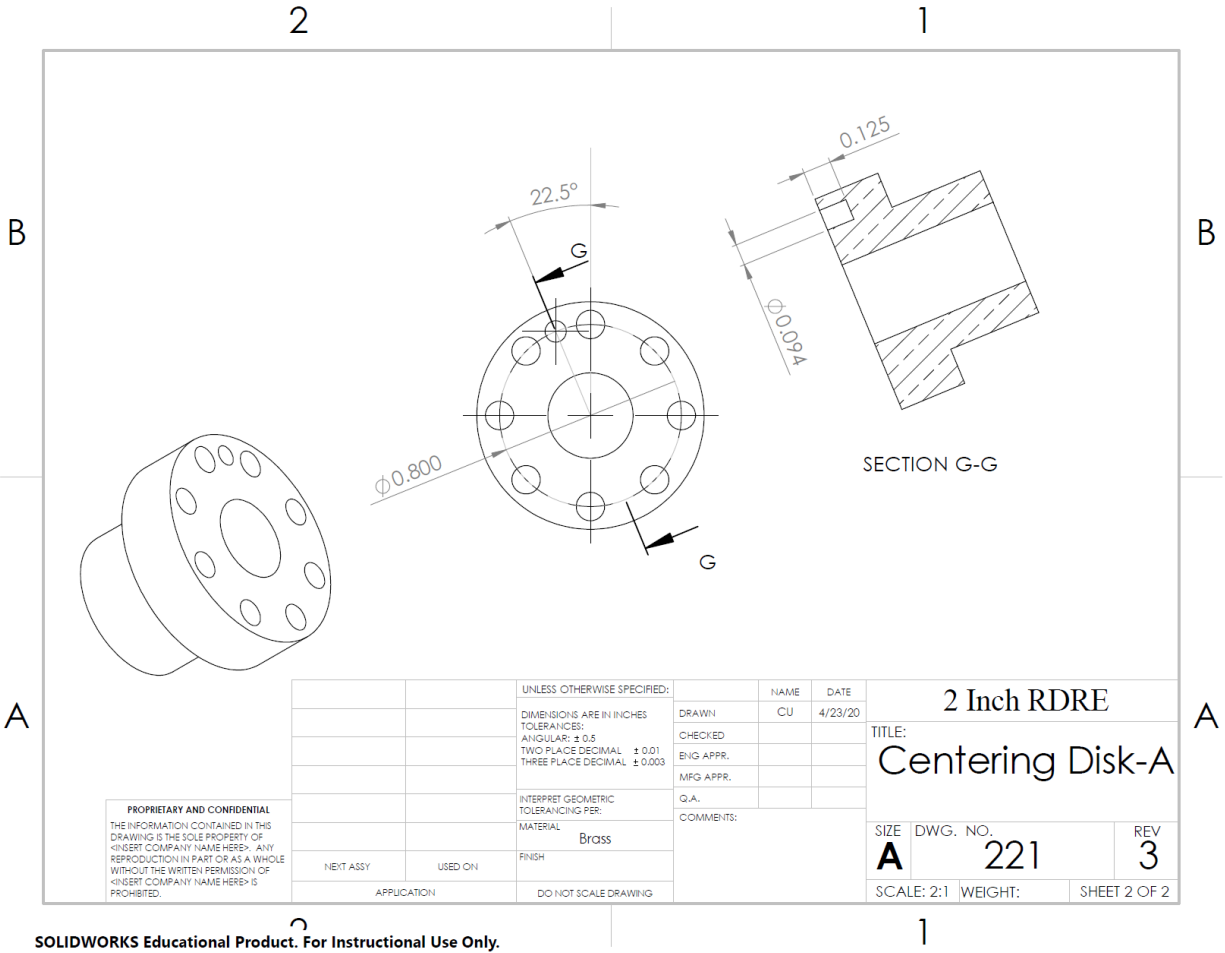




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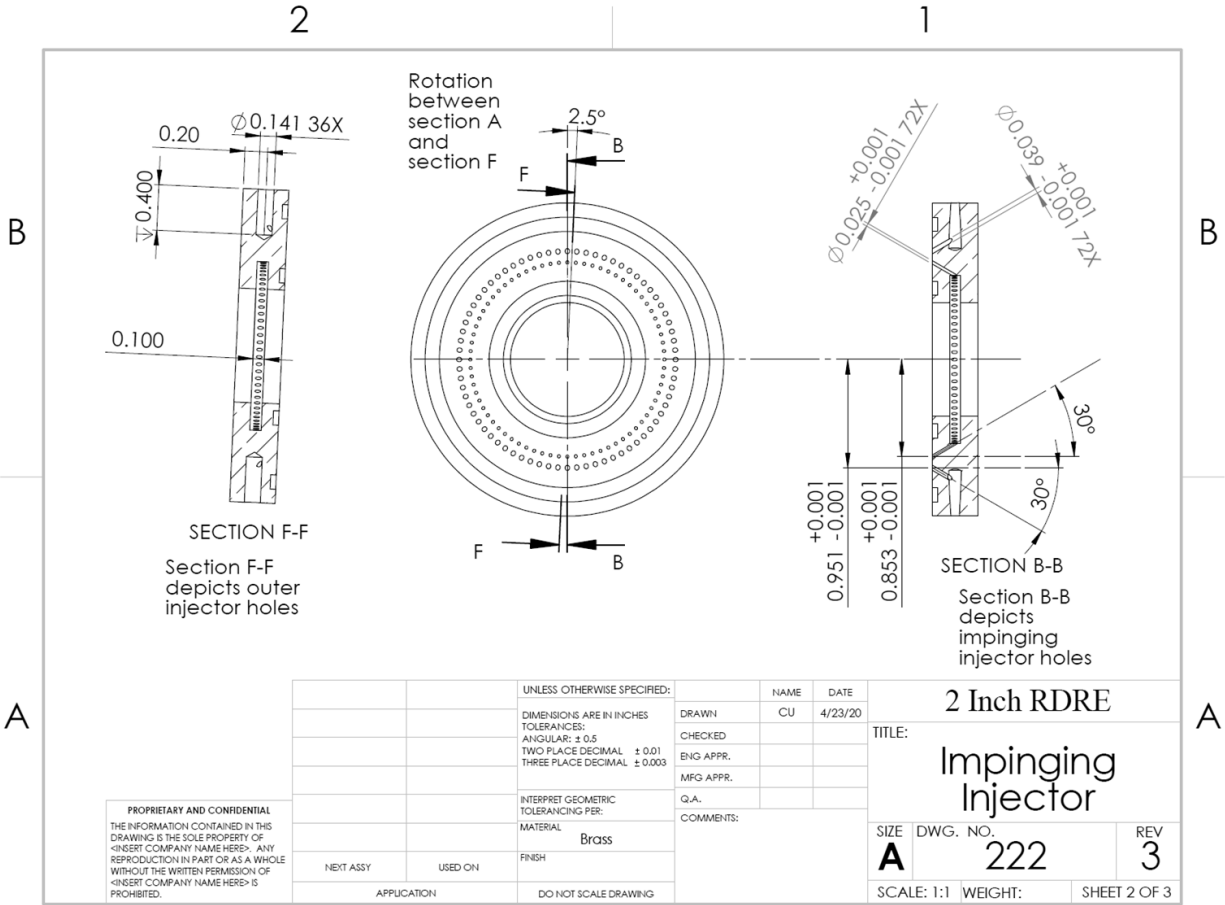


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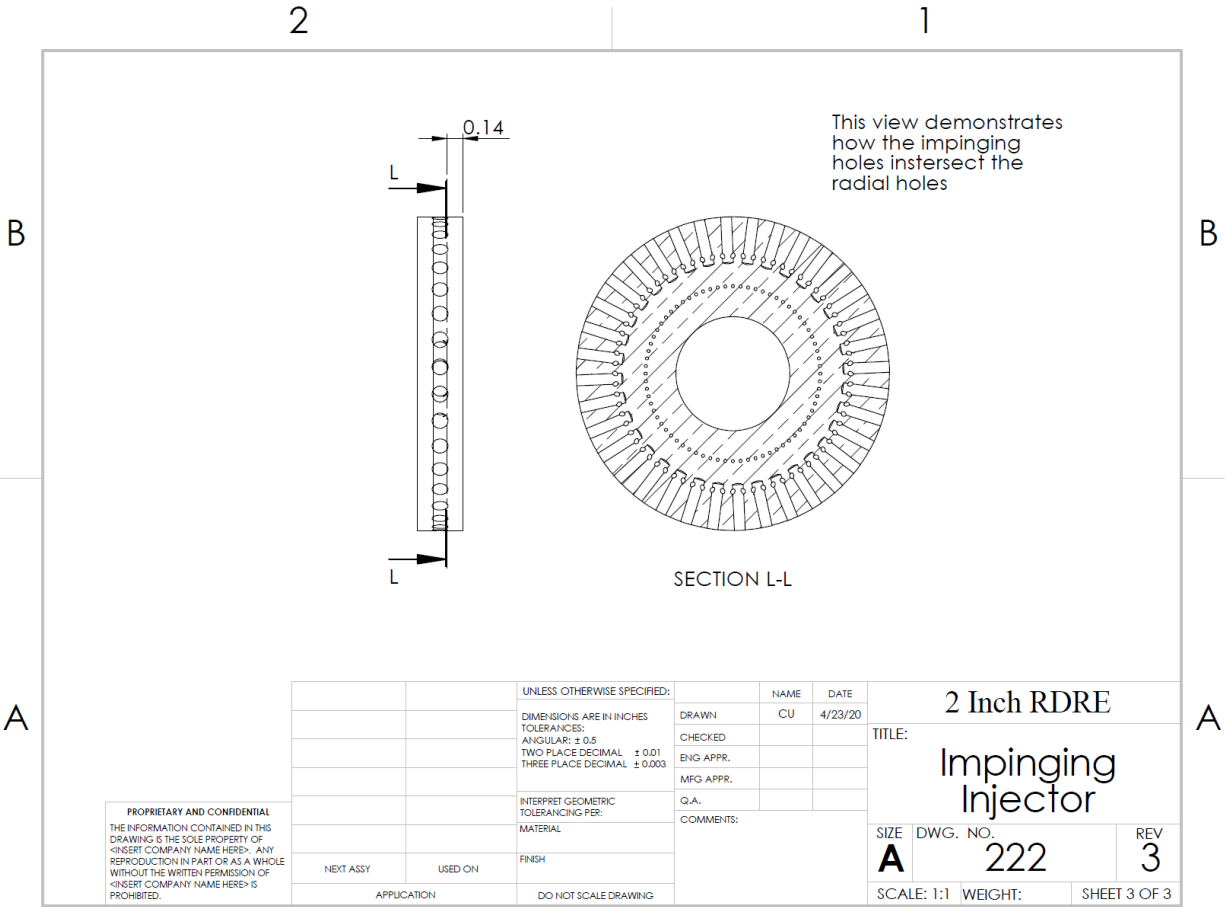


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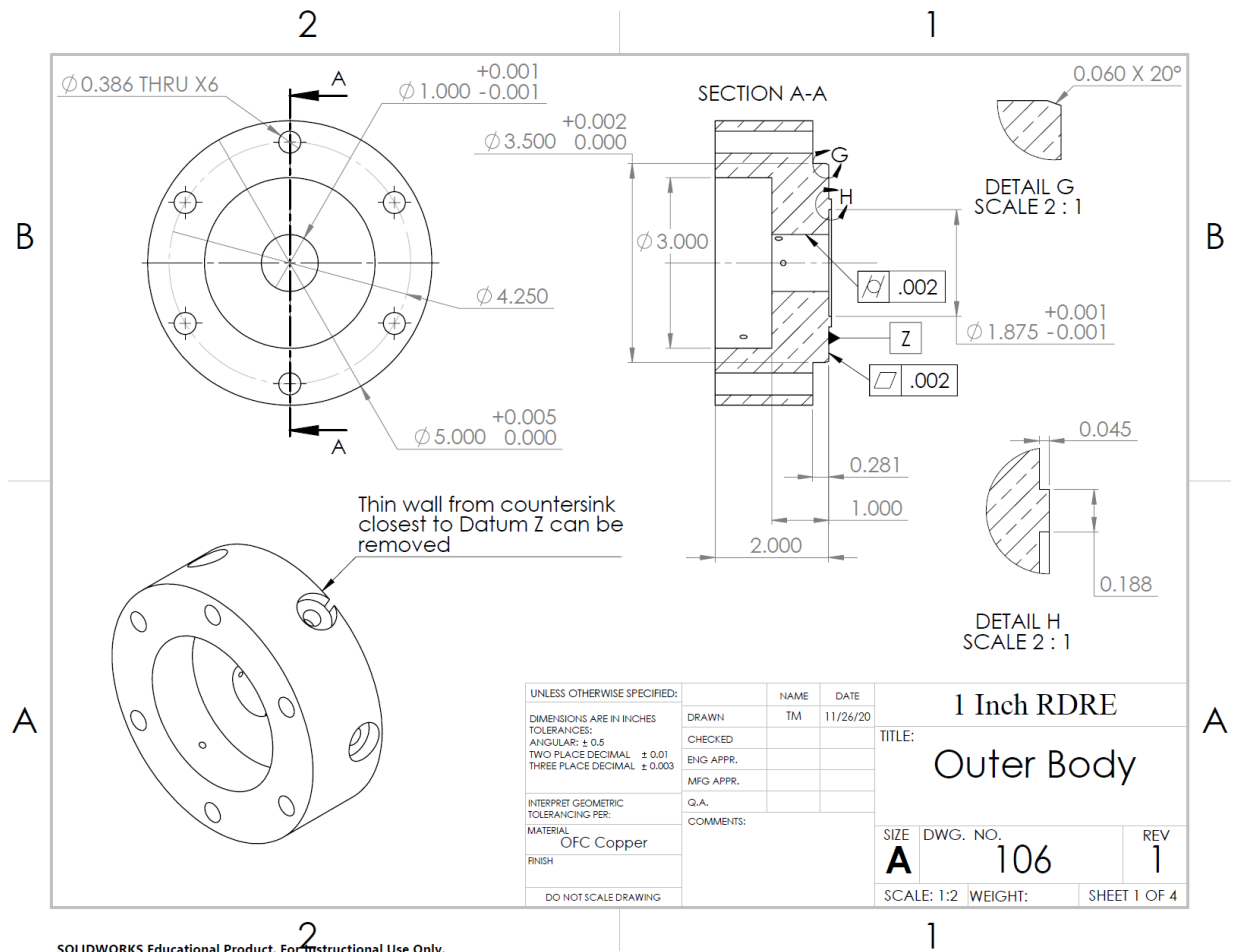


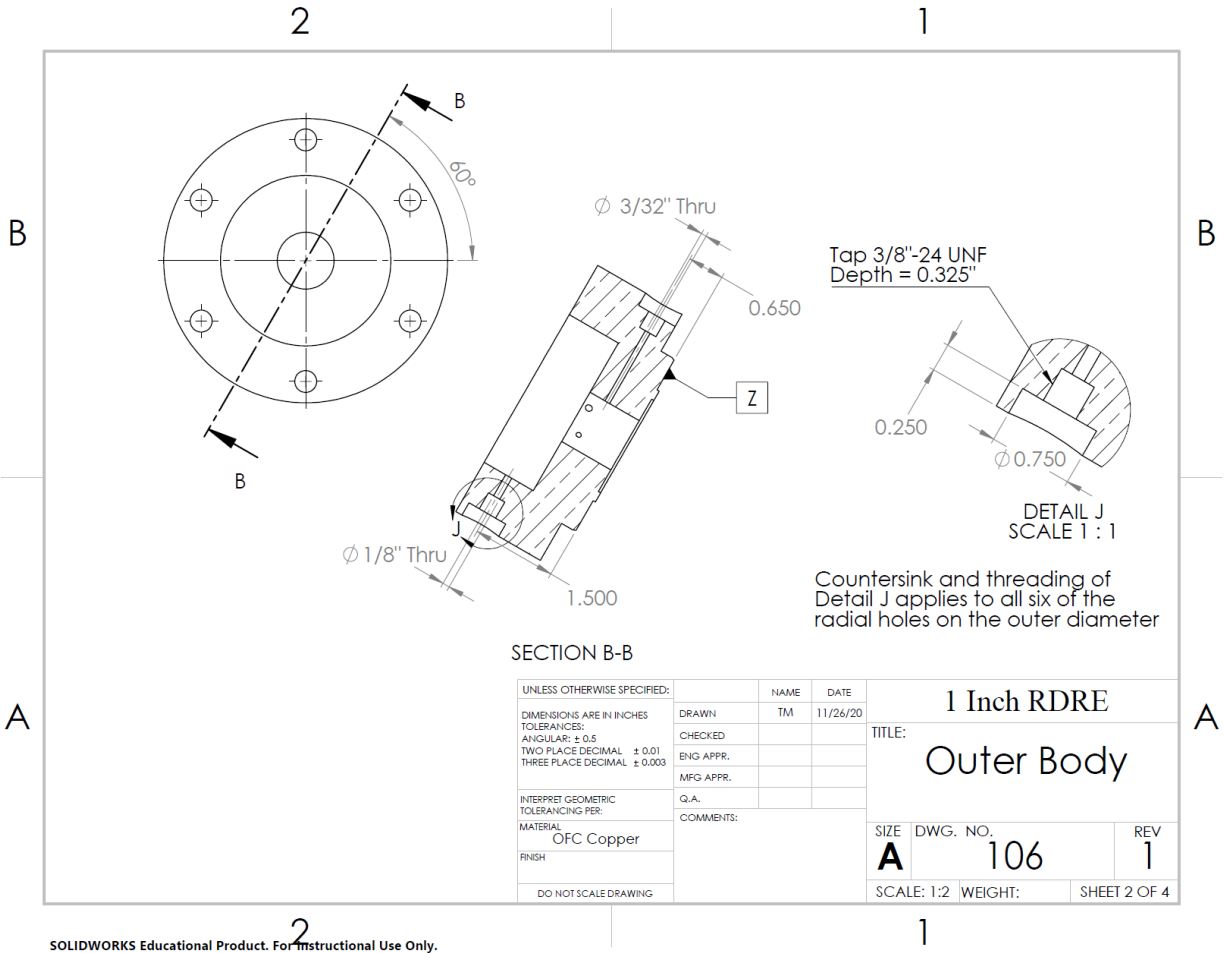
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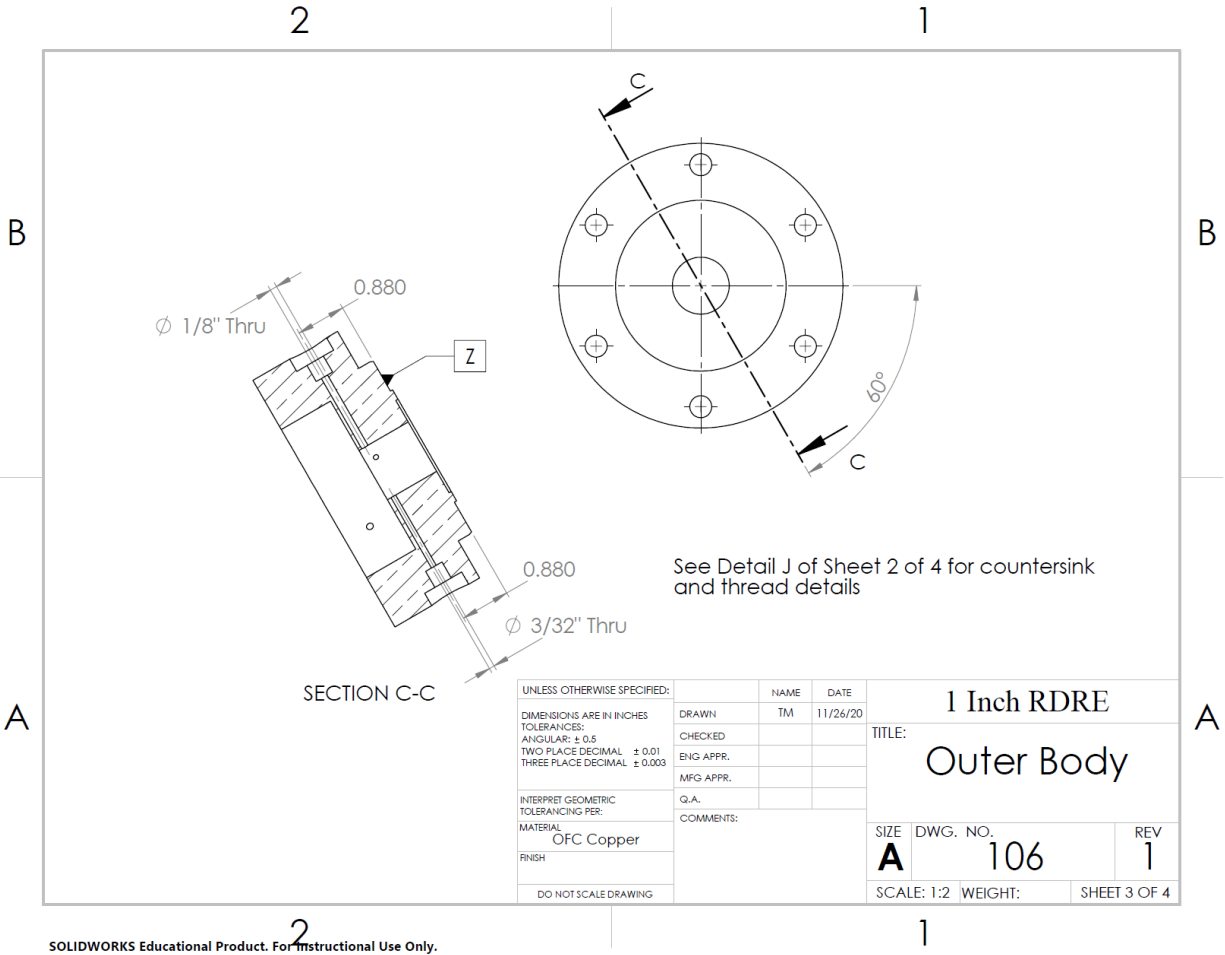
### A.3: 25-FI-5

The 25-FI-5 configuration utilized the stainless steel components of the 51-FI-5 configuration, which includes the front plate, oxidizer manifold ring, and mounting flange. All new components for this configuration are shown below.

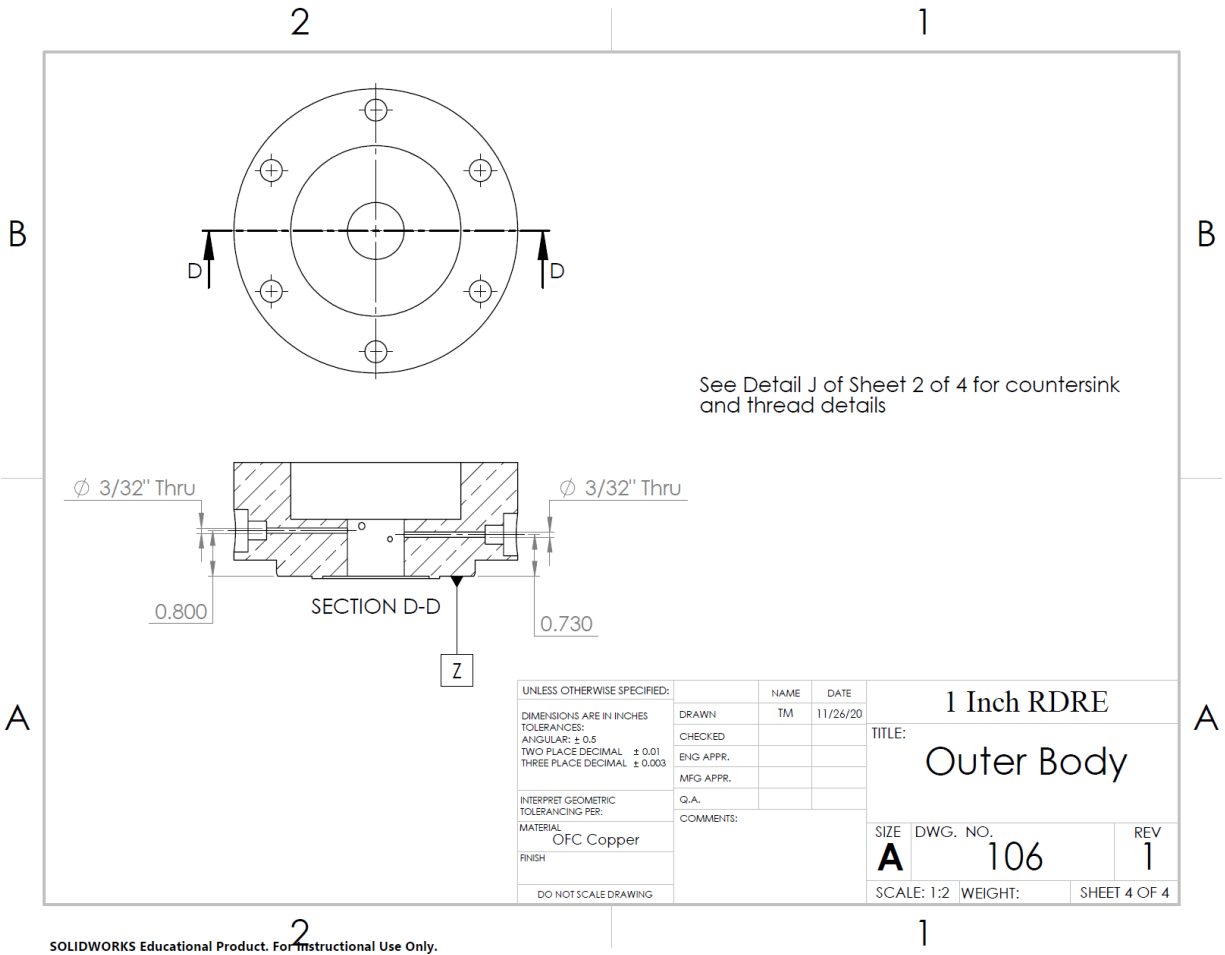


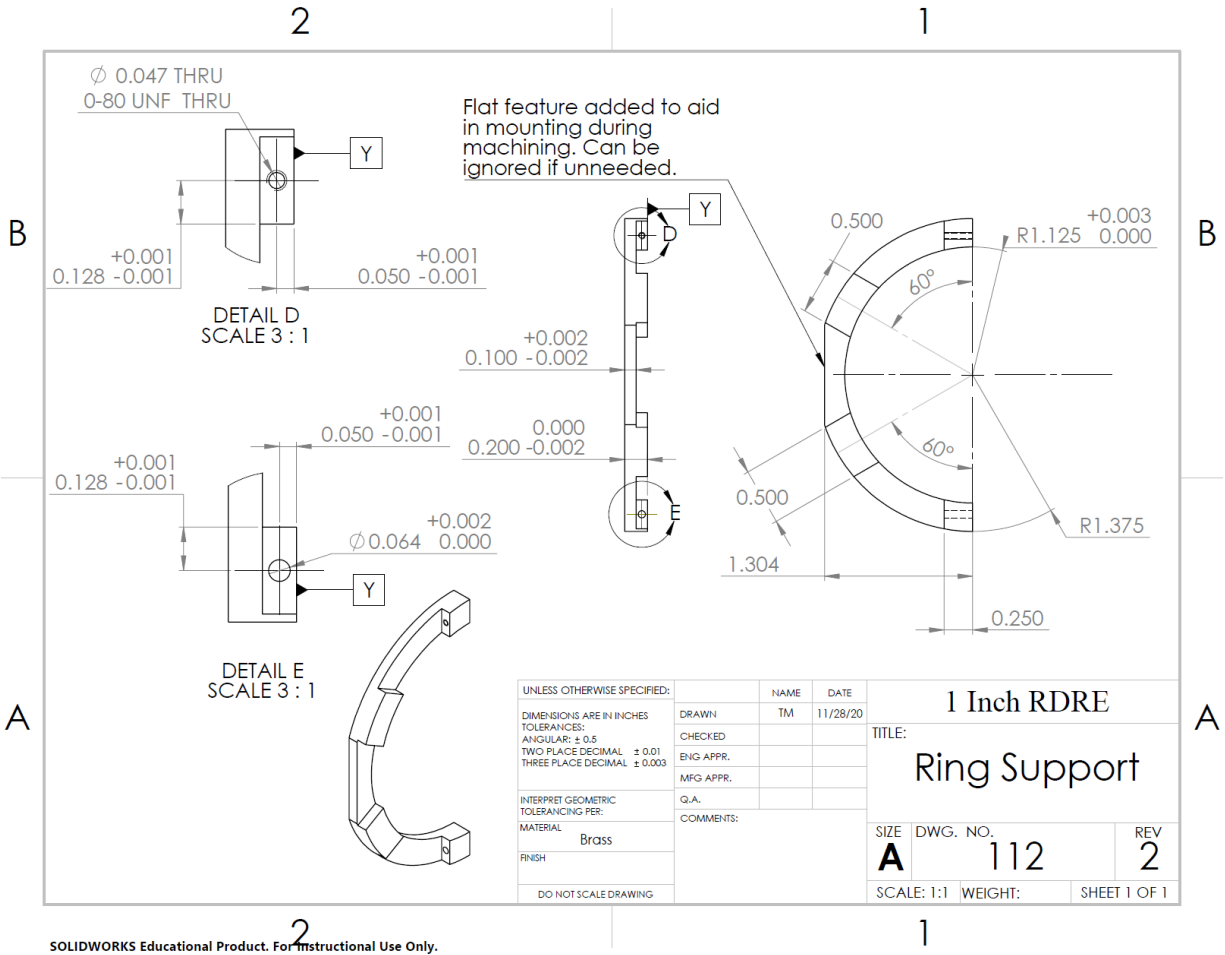


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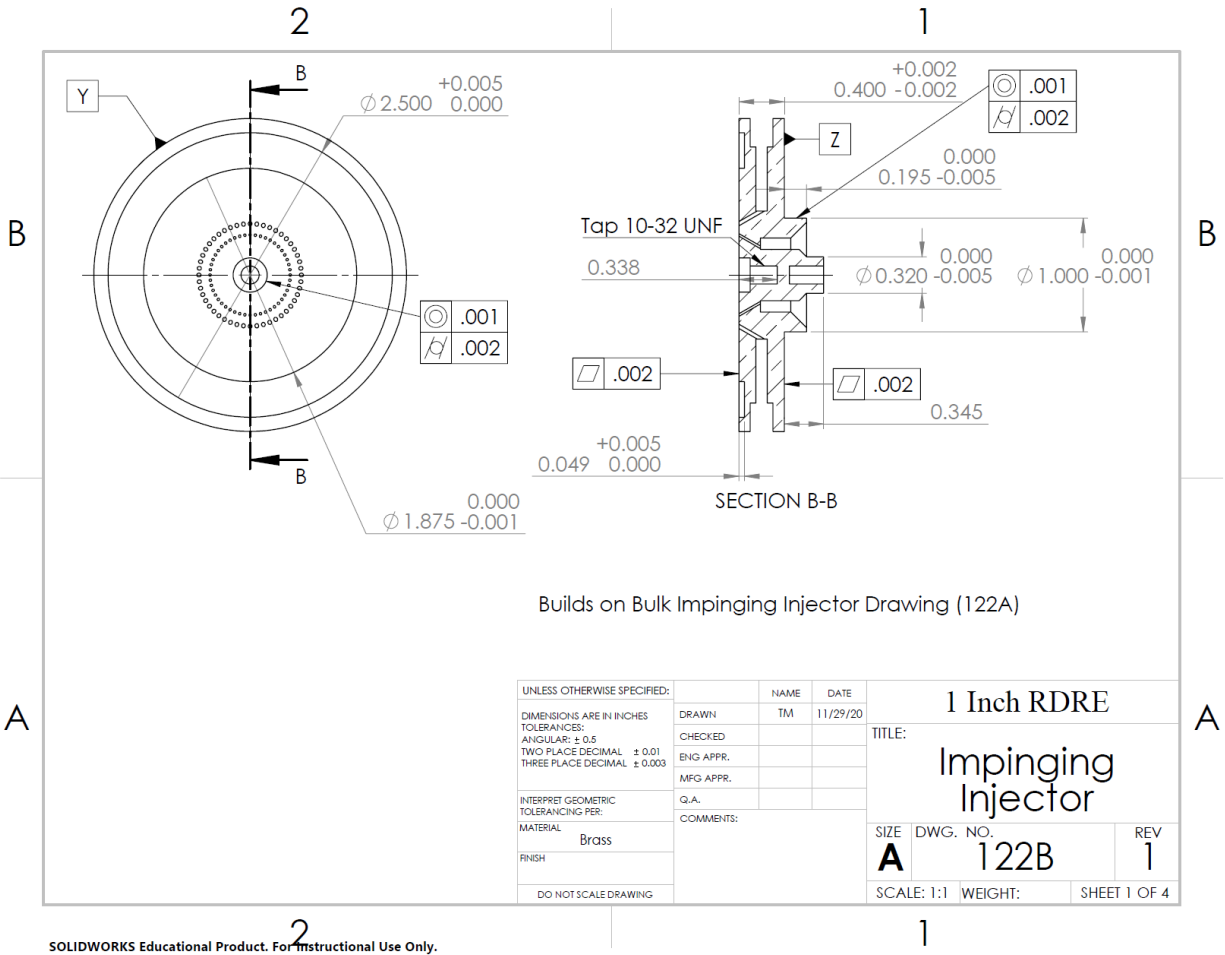
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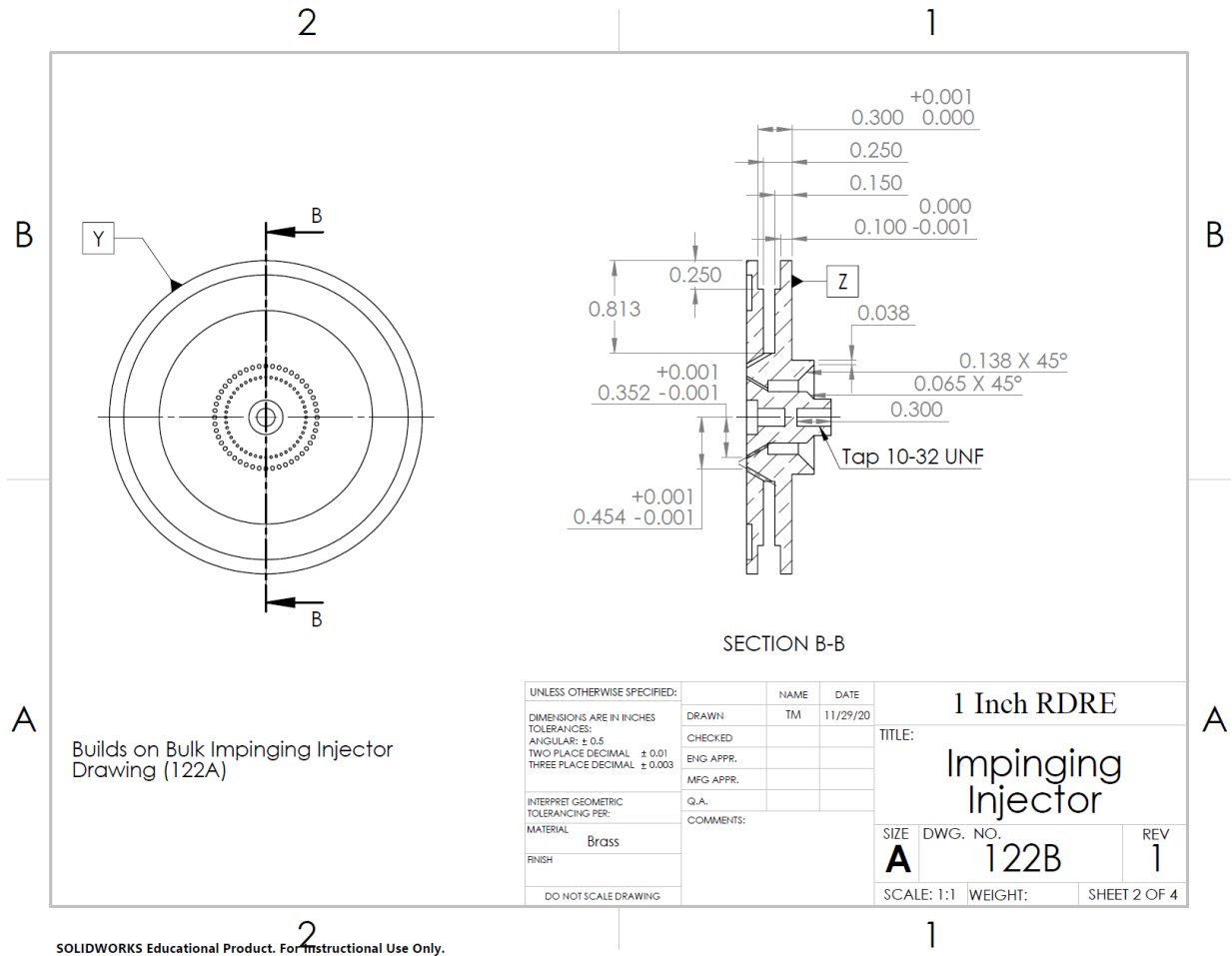
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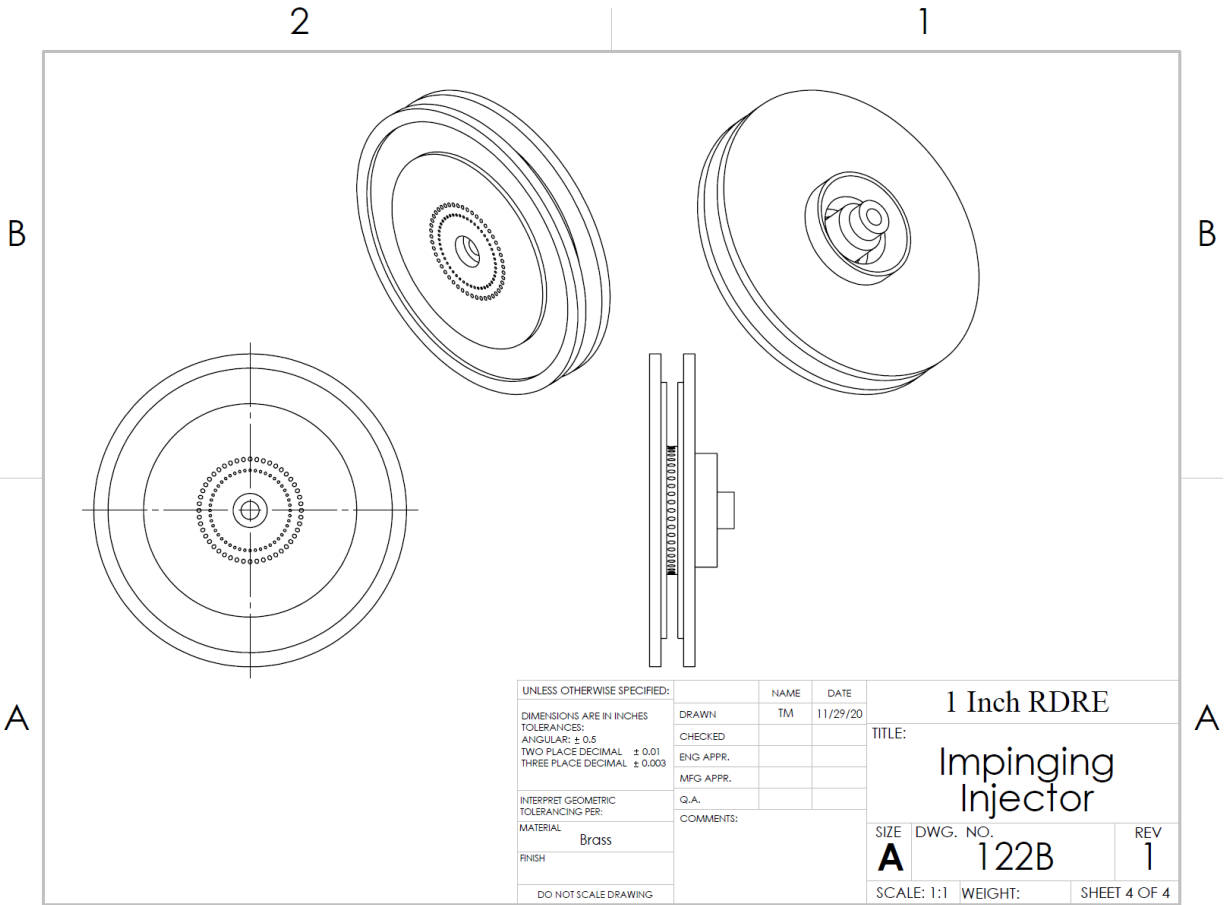
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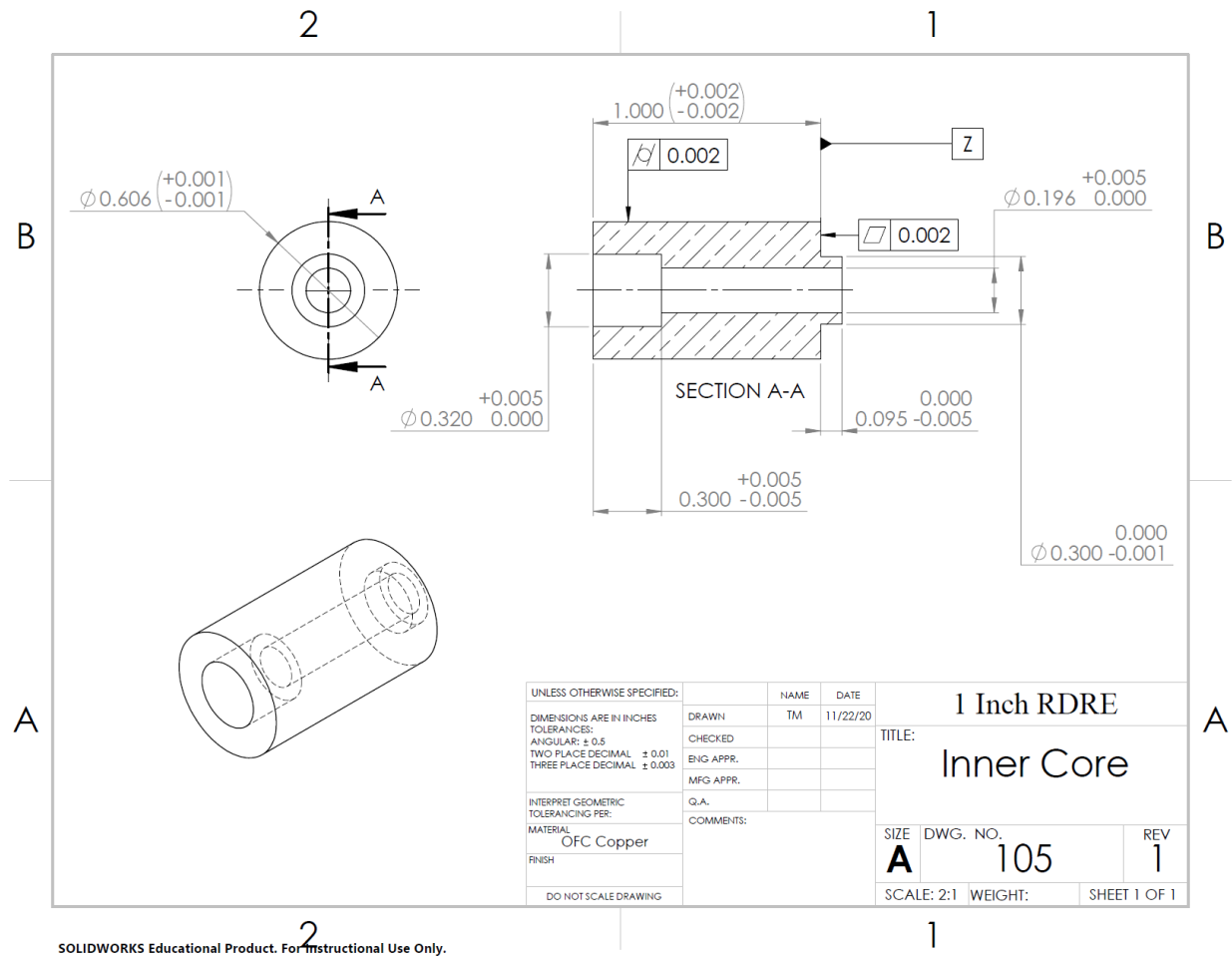




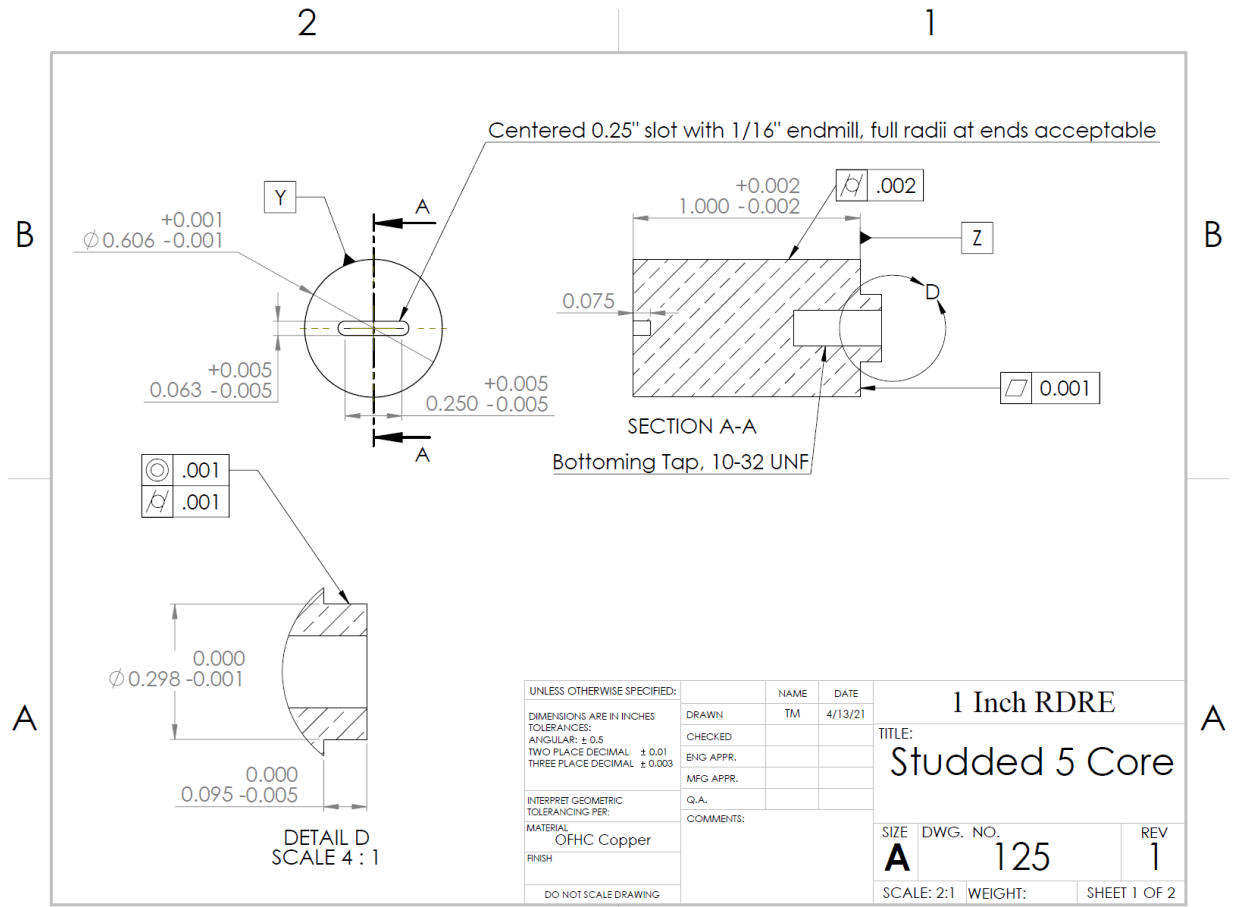
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Three different core designs with the same outer dimensions, but different mounting styles, were used during the course of 25-FI-5 testing due to failures from overheating. Core drawings are presented in order from Revision (Rev.) 1 through 3, as shown in Figure 3.15.

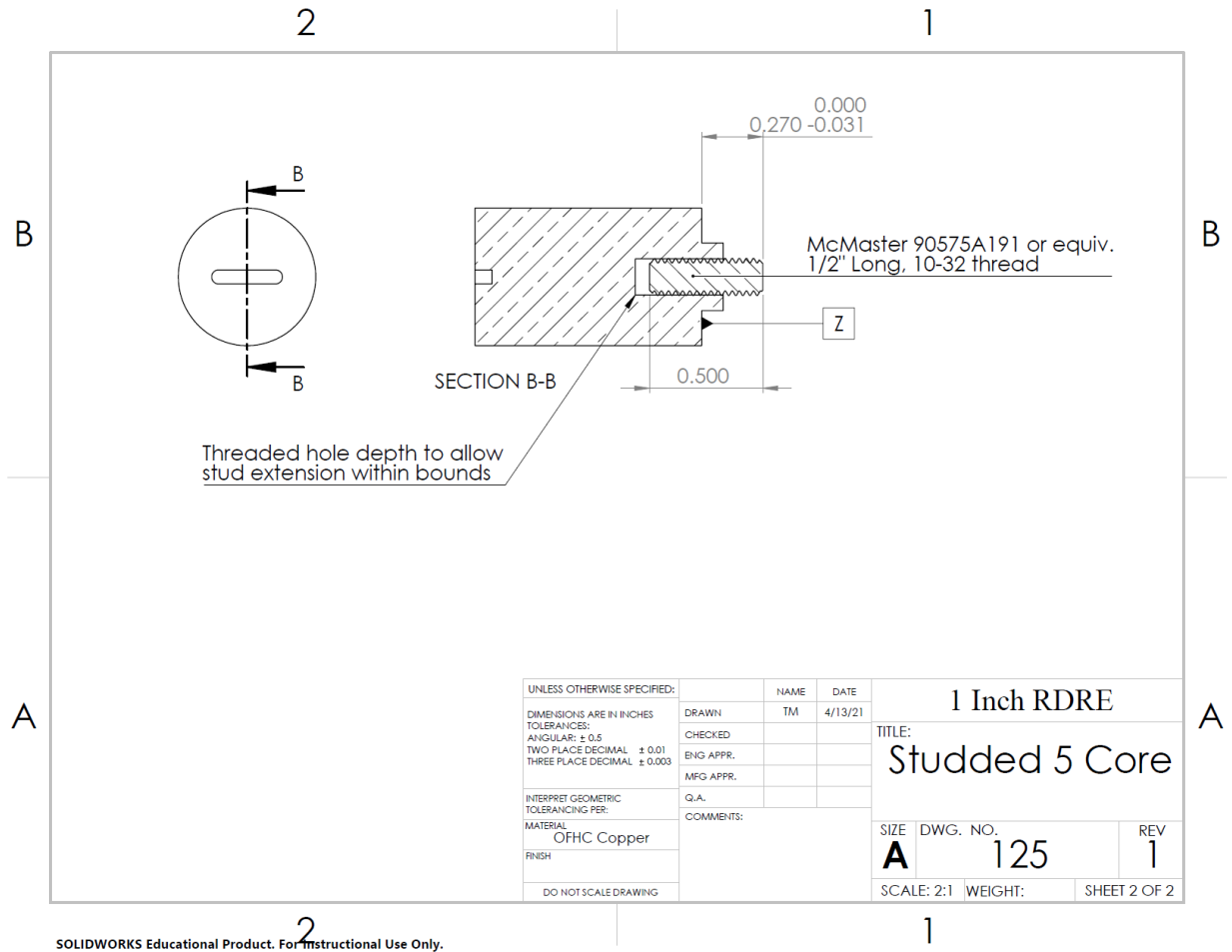
### **Revision 1: Through-bolt mounting**



## Revision 2: Stud mounting

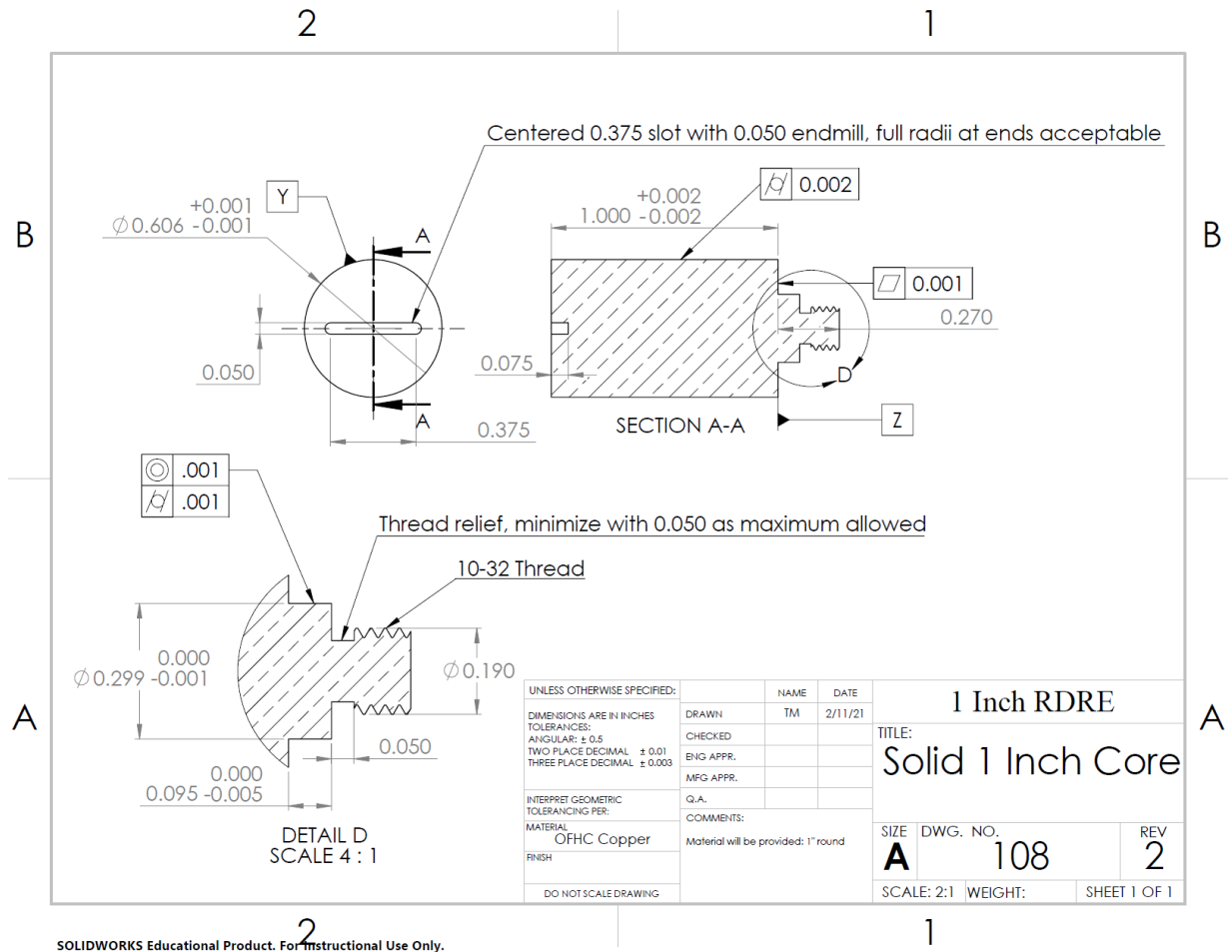


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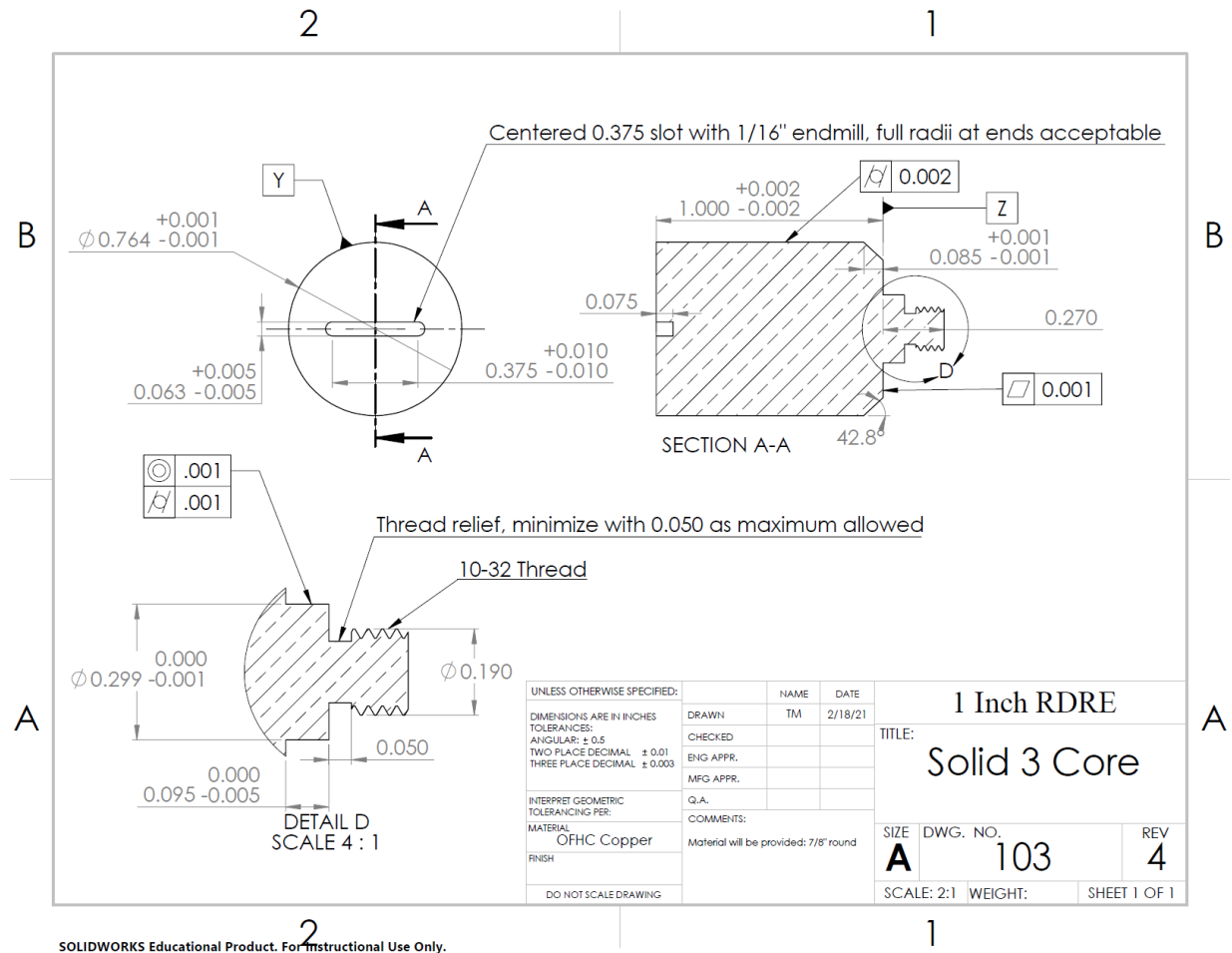
|                                      |  |           |      |             |                |
|--------------------------------------|--|-----------|------|-------------|----------------|
| UNLESS OTHERWISE SPECIFIED:          |  | NAME      | DATE | 1 Inch RDRE |                |
| DIMENSIONS ARE IN INCHES             |  | DRAWN     | TM   | 4/13/21     | TITLE:         |
| TOLERANCES:                          |  | CHECKED   |      |             | Studded 5 Core |
| ANGULAR: ± 0.5                       |  | ENG APPR. |      |             |                |
| TWO PLACE DECIMAL ± 0.01             |  | MFG APPR. |      |             |                |
| THREE PLACE DECIMAL ± 0.003          |  | Q.A.      |      |             |                |
| INTERPRET GEOMETRIC TOLERANCING PER: |  | COMMENTS: |      | SIZE        | DWG. NO.       |
| MATERIAL                             |  |           |      | A           | 125            |
| FINISH                               |  |           |      |             | REV            |
| DO NOT SCALE DRAWING                 |  |           |      | SCALE: 2:1  | 1              |
|                                      |  |           |      | WEIGHT:     | SHEET 2 OF 2   |

### Revision 3: Machined threads, solid core



## A.4: 25-FI-3

The 25-FI-3 configuration reused all components from the 25-FI-5 configuration except for the inner core, which was replaced with the design shown below.

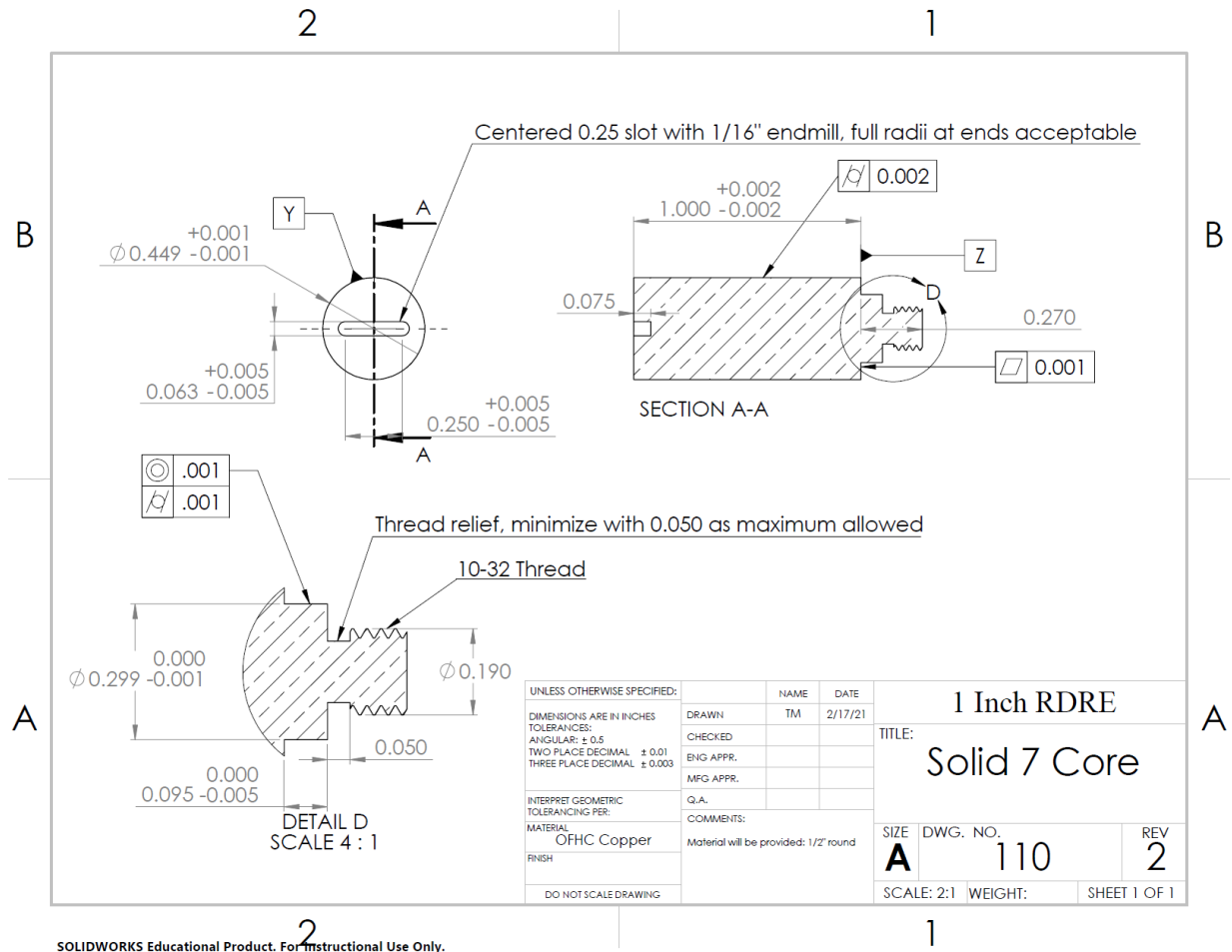


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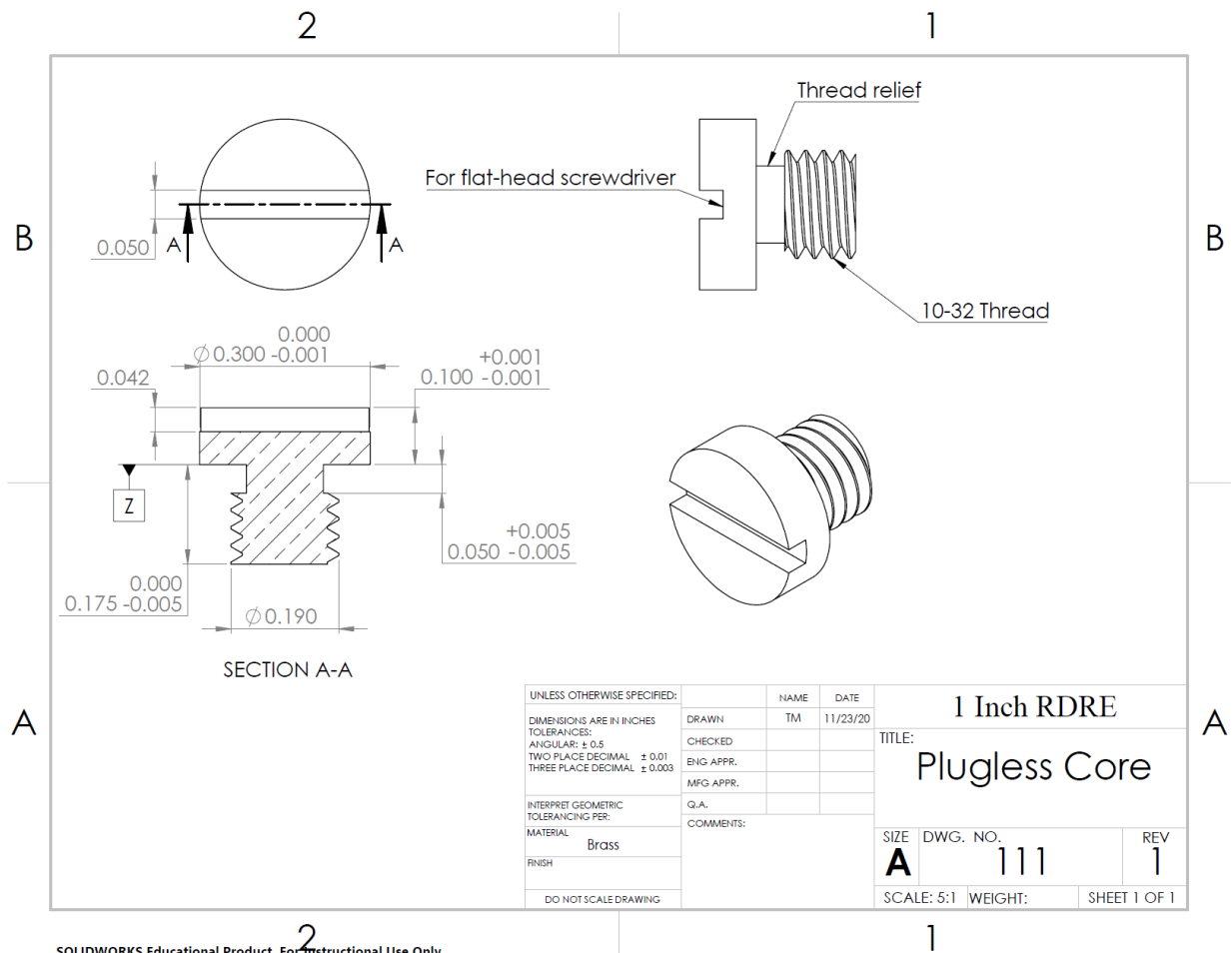
## A.5: 25-FI-7

The 25-FI-7 configuration reused all components from the 25-FI-5 configuration, except for the inner core, which was replaced with the design shown below.



## A.6: 25-FI-12.7

As with the other 25 mm configurations, the 25-FI-12.7 configuration reused all components from the 25-FI-5 configuration except for the inner core. Instead, a small plug, shown below, was used to fill in the recess used to attach and center the cores of the other 25 mm configurations.

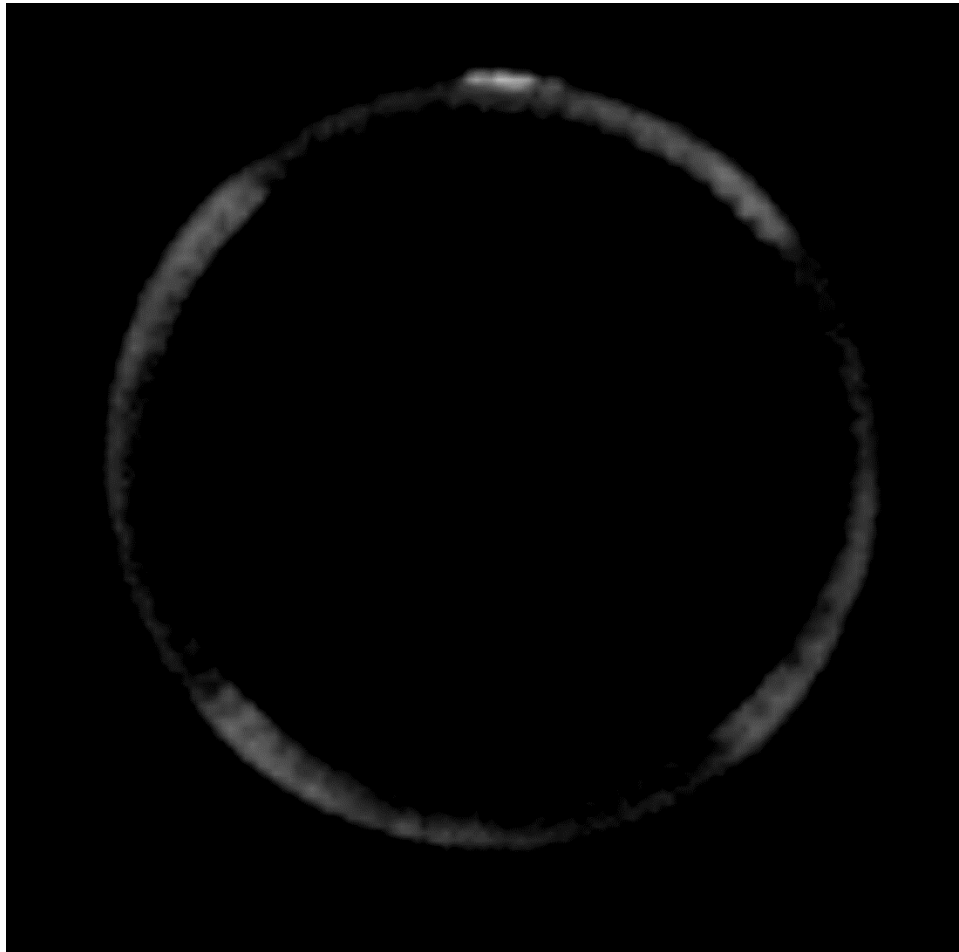


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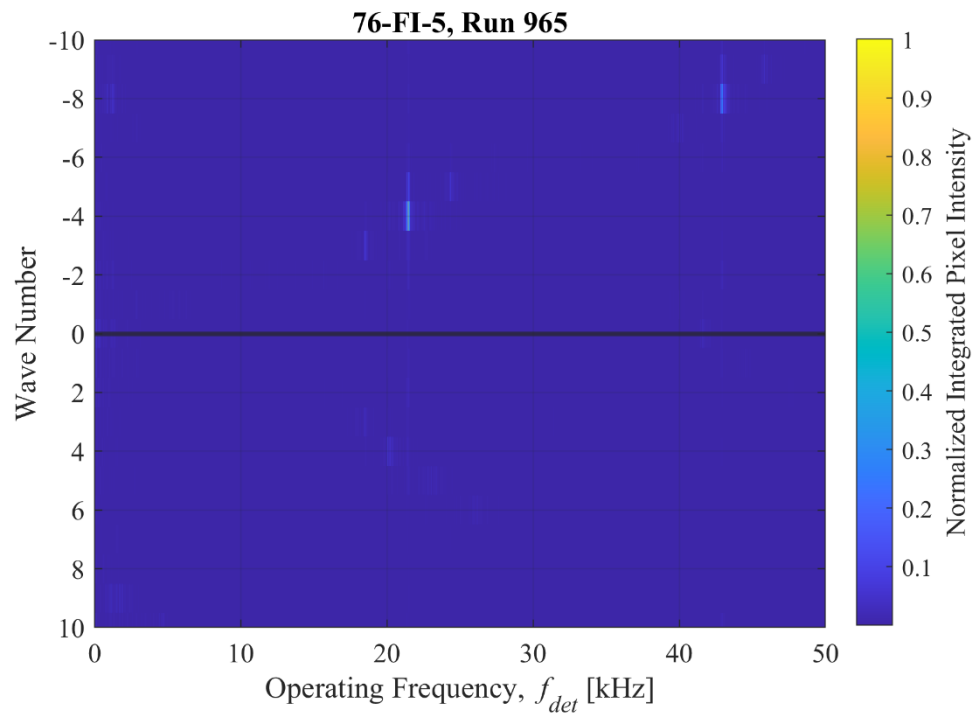
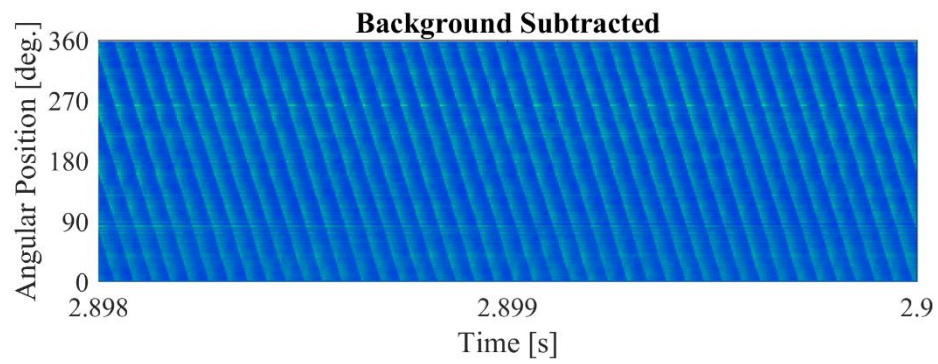
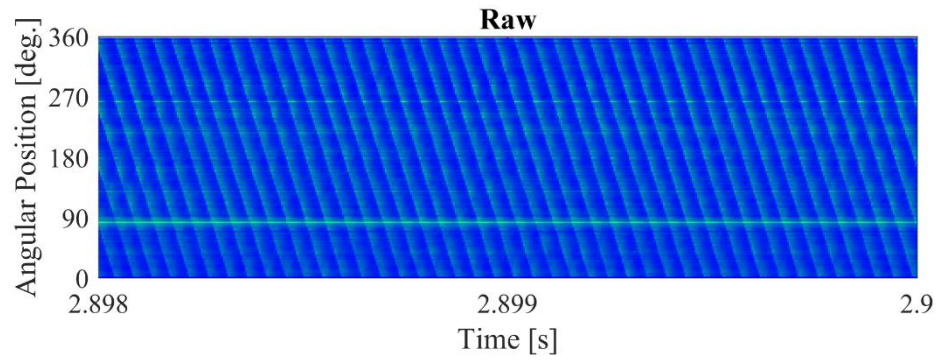
## APPENDIX B: WAVE MODE EXAMPLES

For each operating mode, an example camera frame, x-t subsection from the end of the run, and two-dimensional FFT will be presented, as was done in Section 3.1.

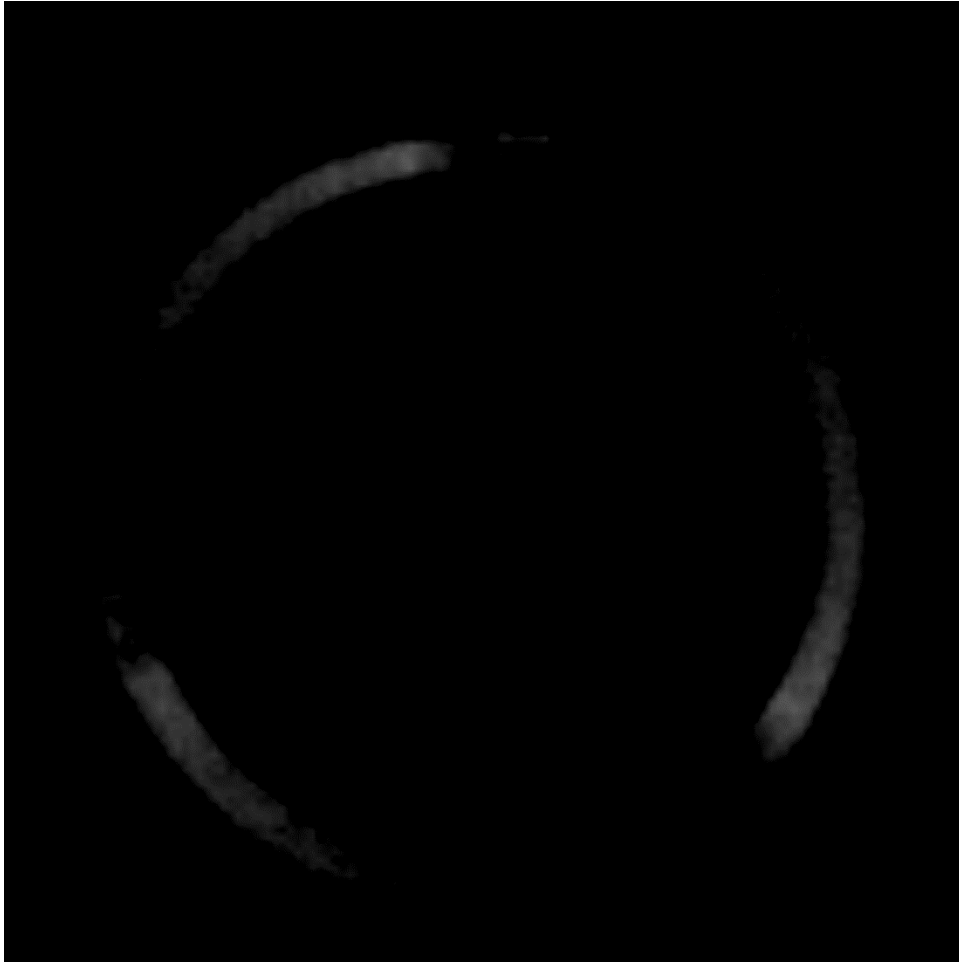
### B.1: 4-WAVE (76-FI-5)



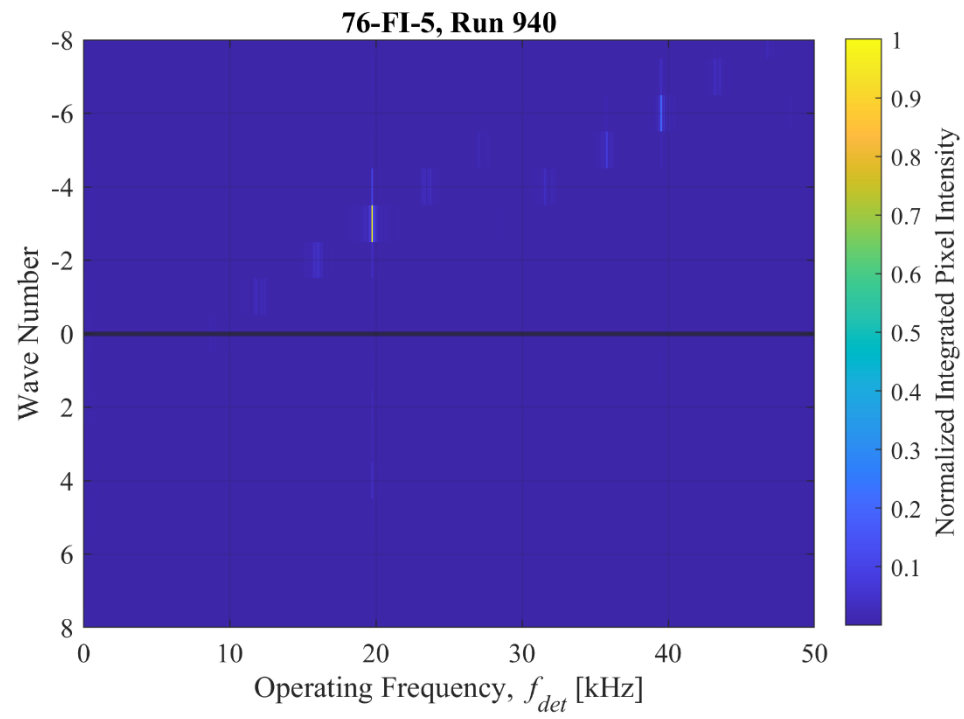
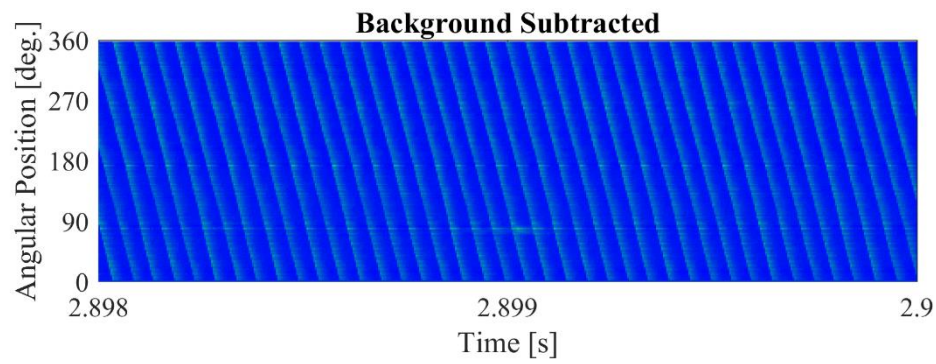
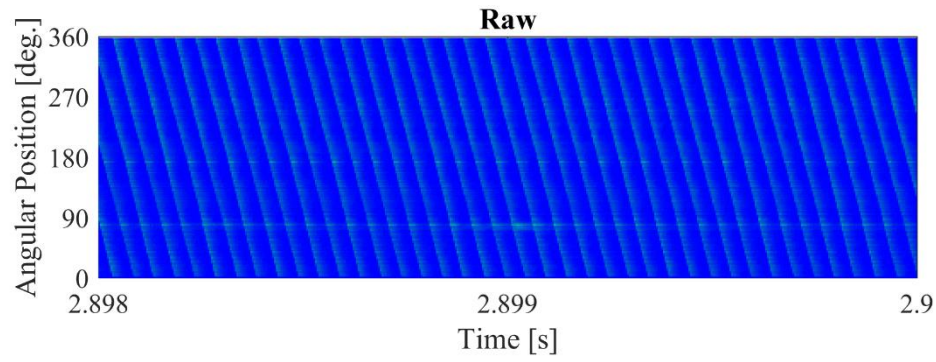
### 76-FI-5, Run 965



B.2: 3-WAVE (76-FI-5)



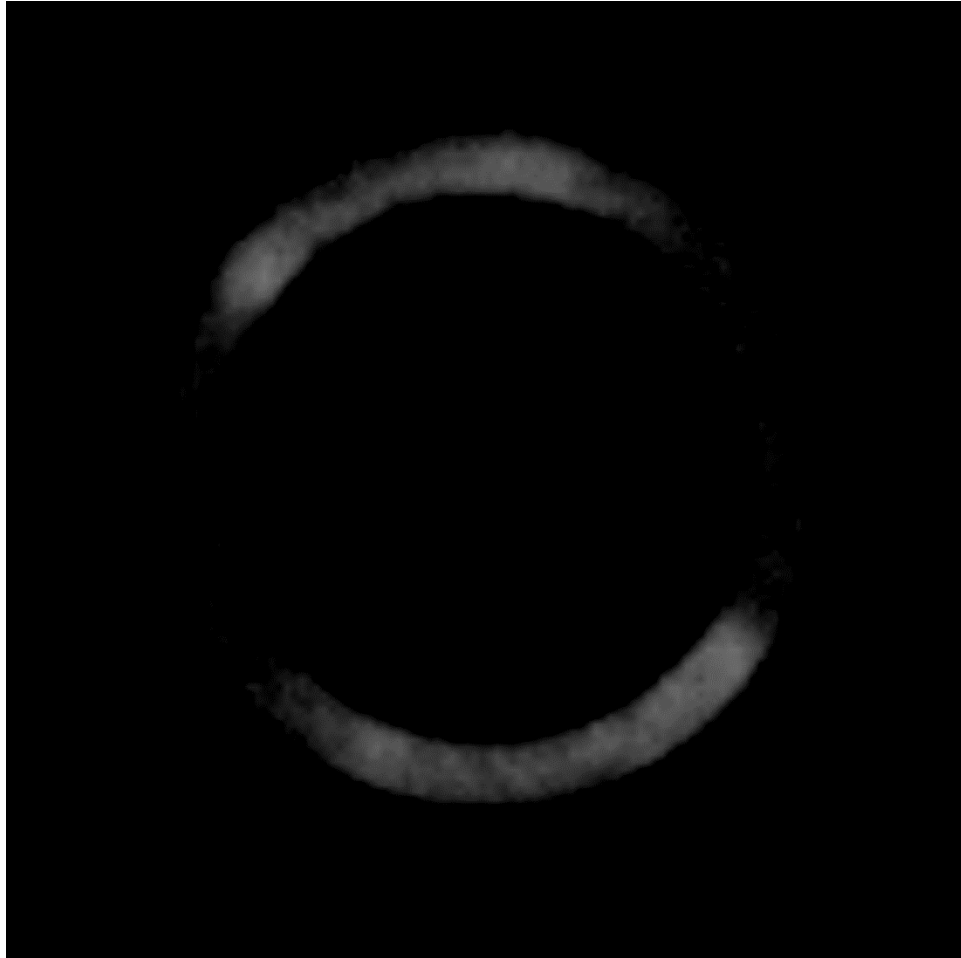
### 76-FI-5, Run 940



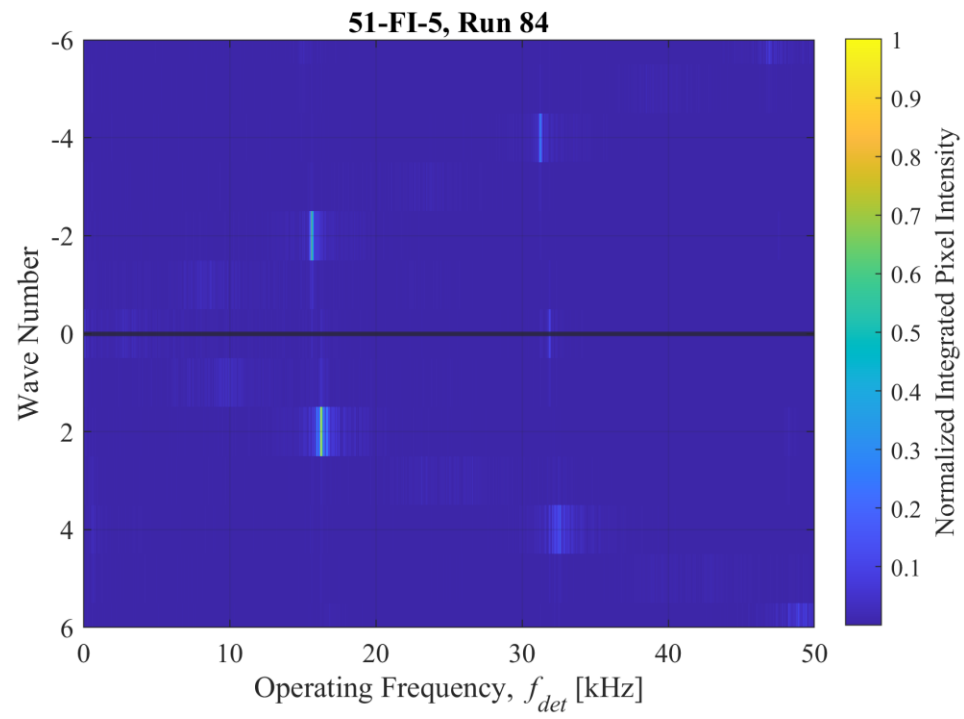
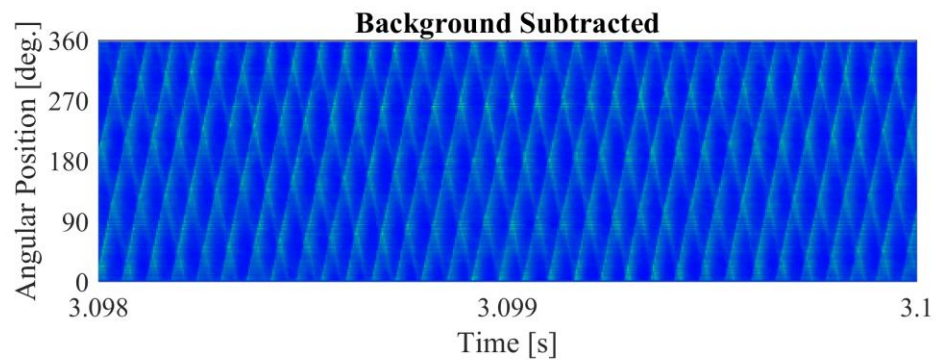
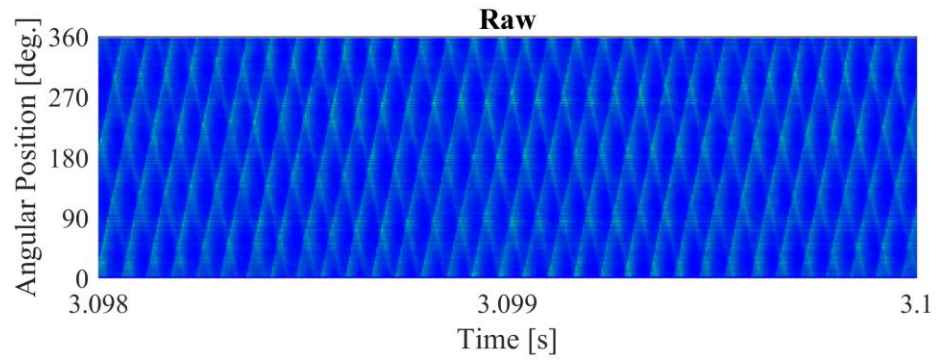
### B.3: 2-WAVE (76-FI-5)

Already presented in Figure 3.4 through Figure 3.6 of Section 3.1.

### B.4: 2-WAVE WITH STRONG COUNTER WAVES (51-FI-5)

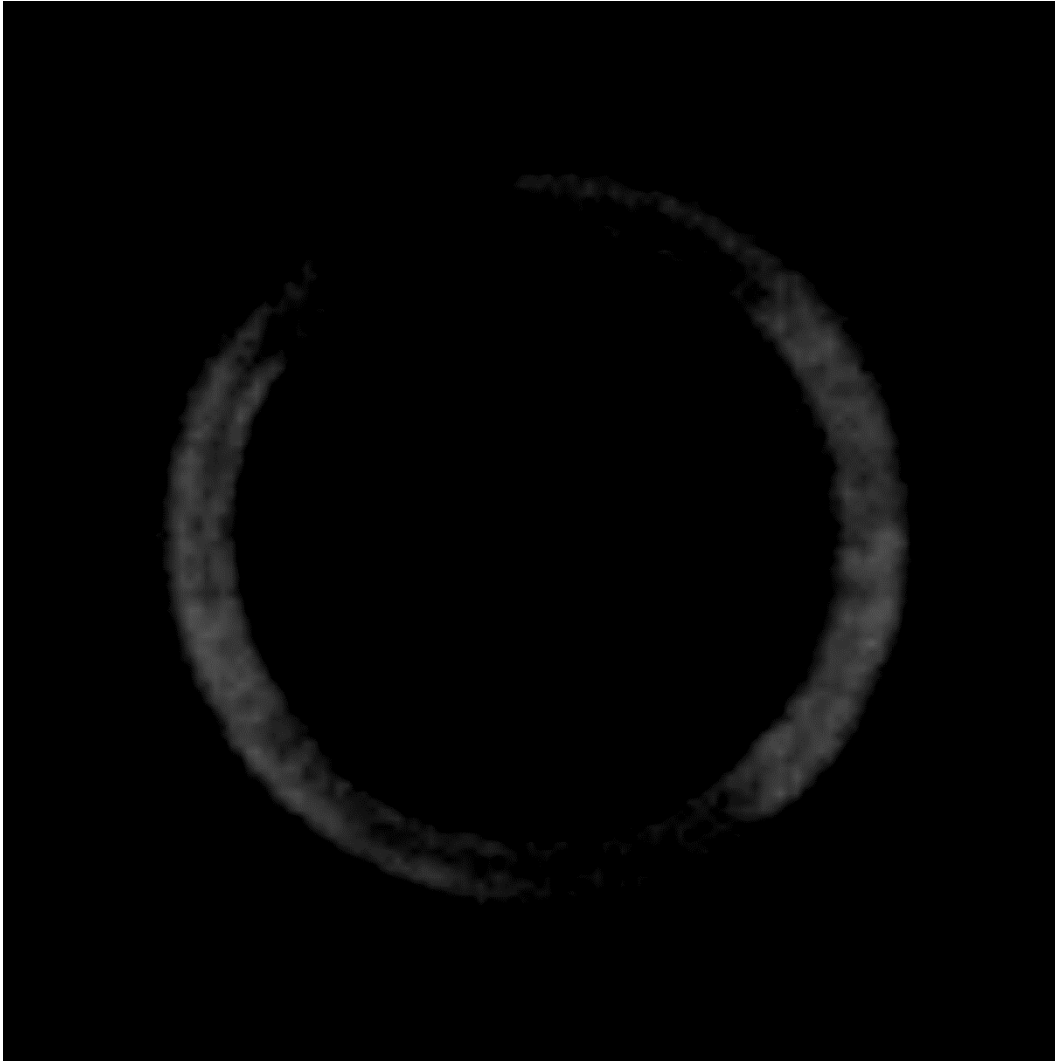


### 51-FI-5, Run 84

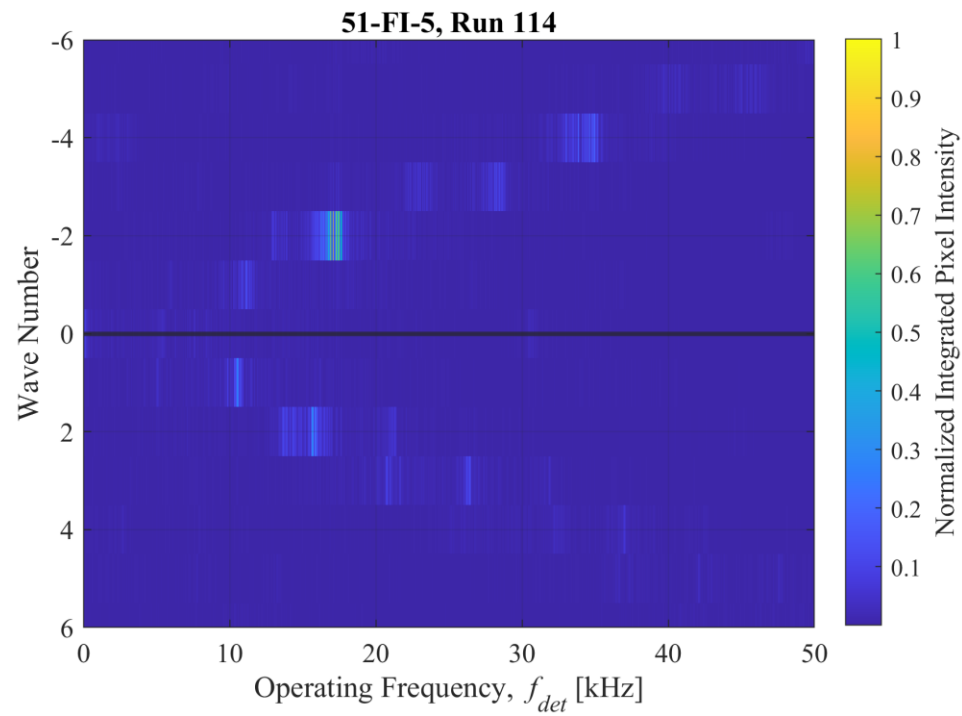
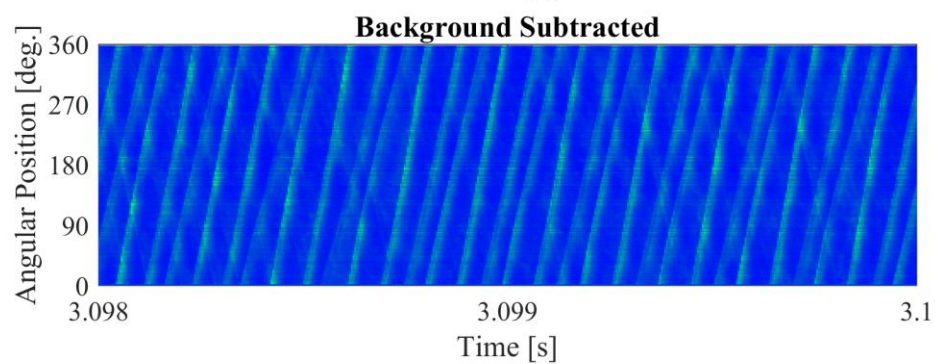
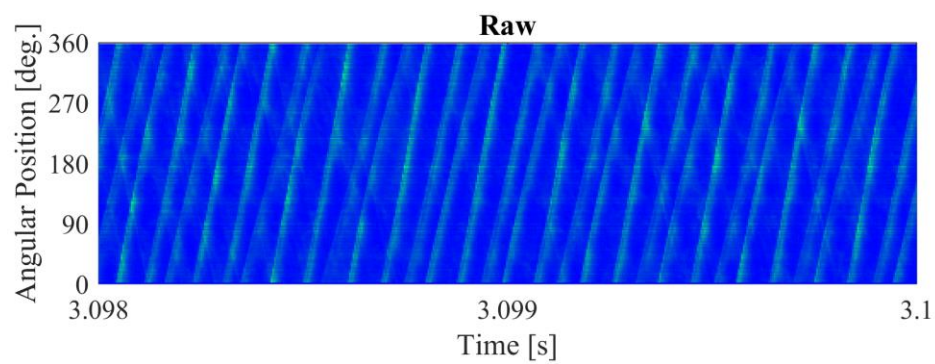




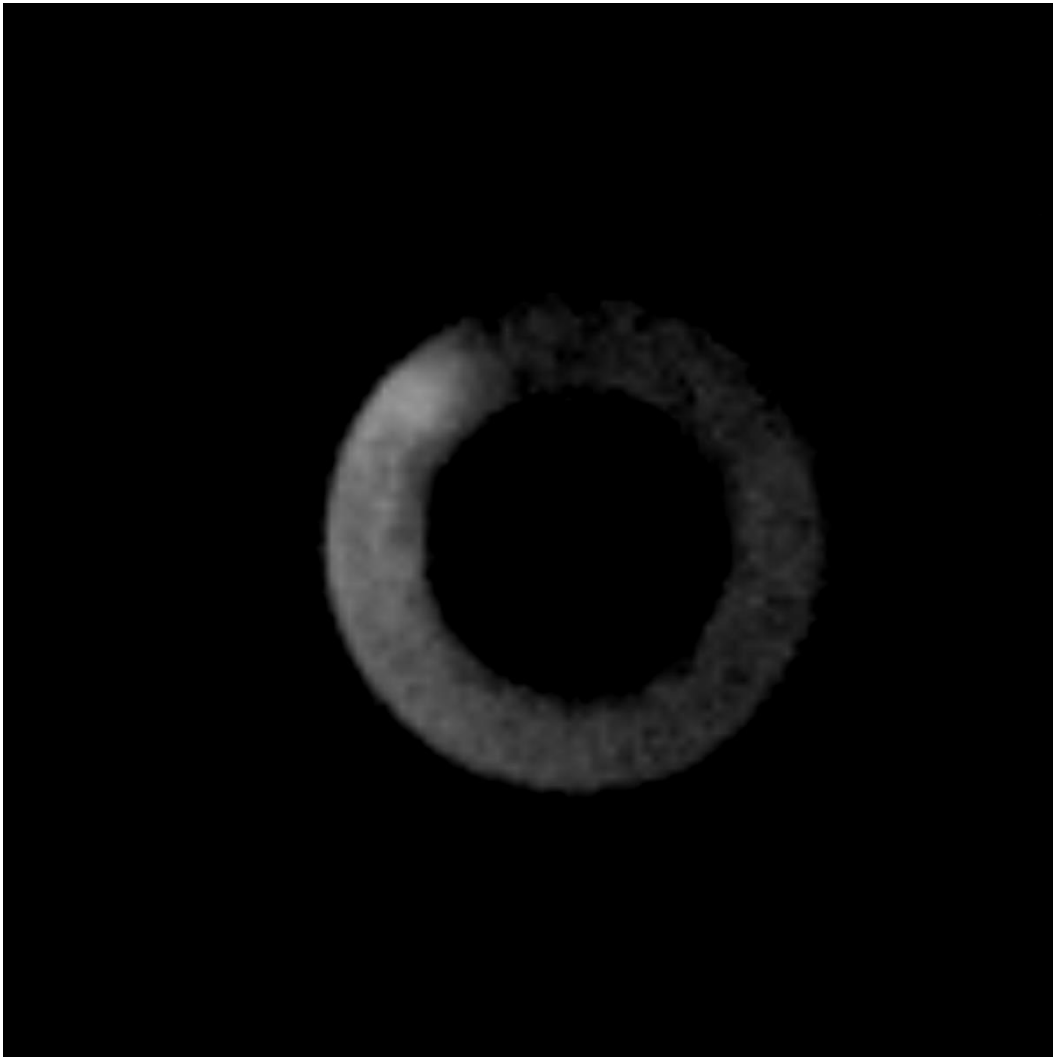
### B.5: 2-WAVE WITH WEAK COUNTER WAVES (51-FI-5)



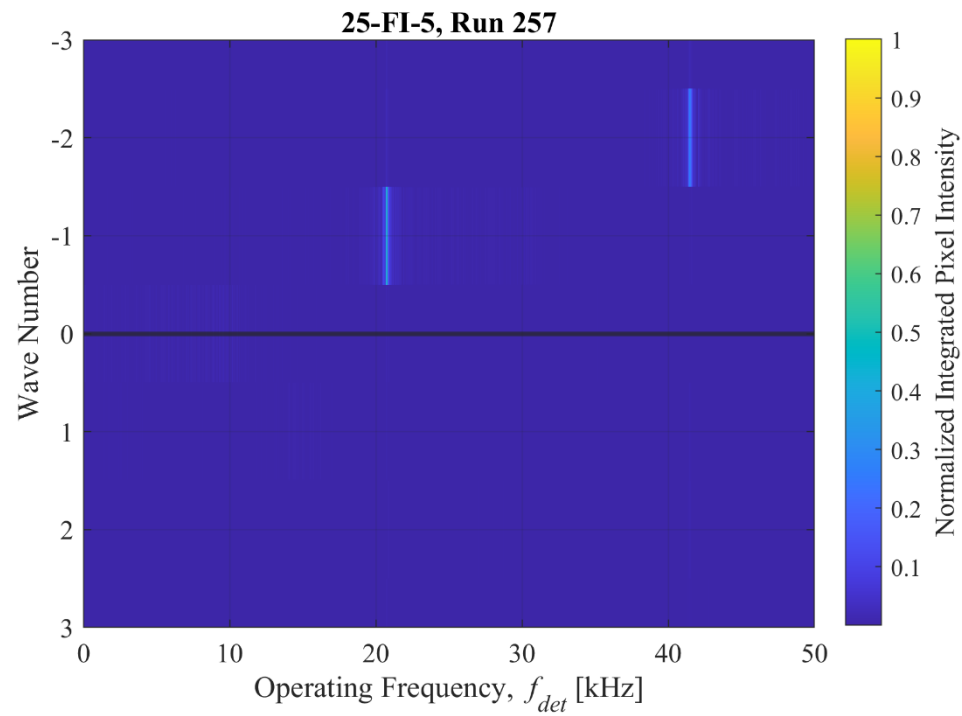
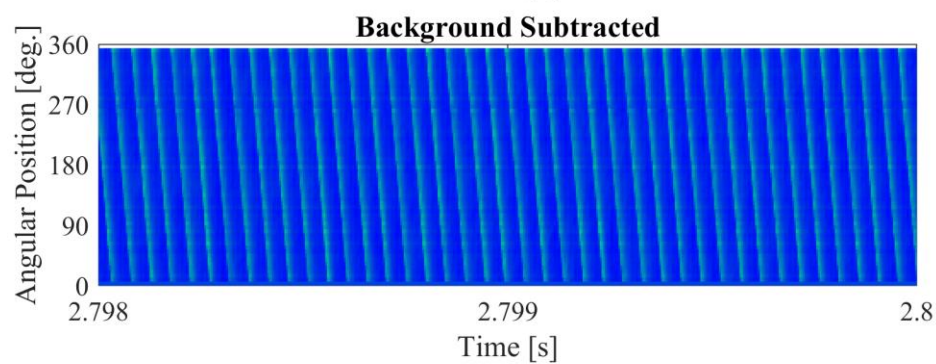
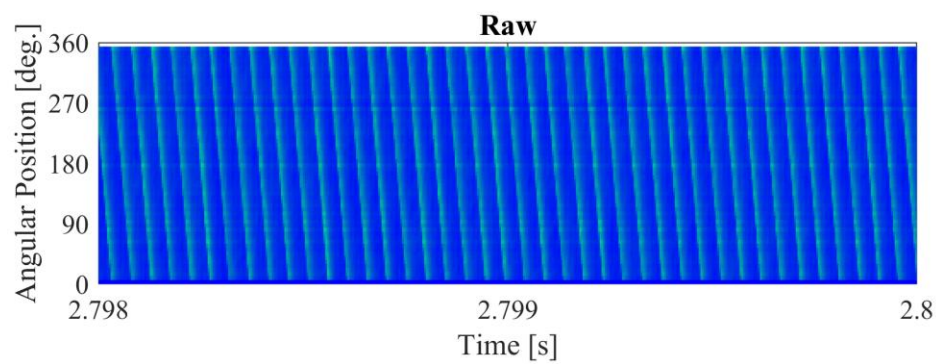
### 51-FI-5, Run 114



B.6: 1-wave (25-FI-5)



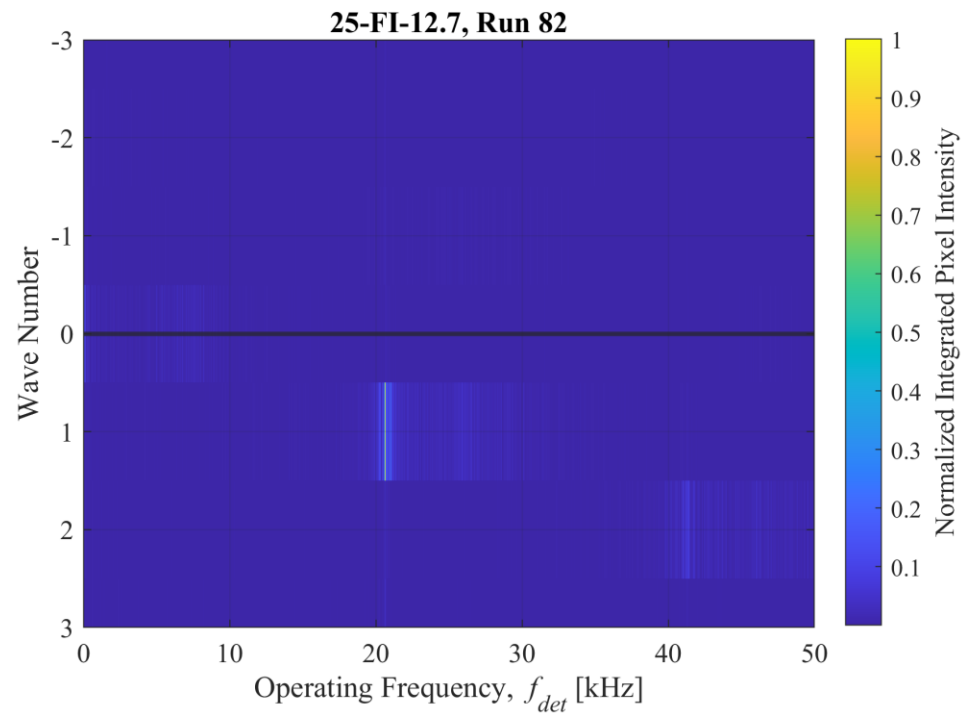
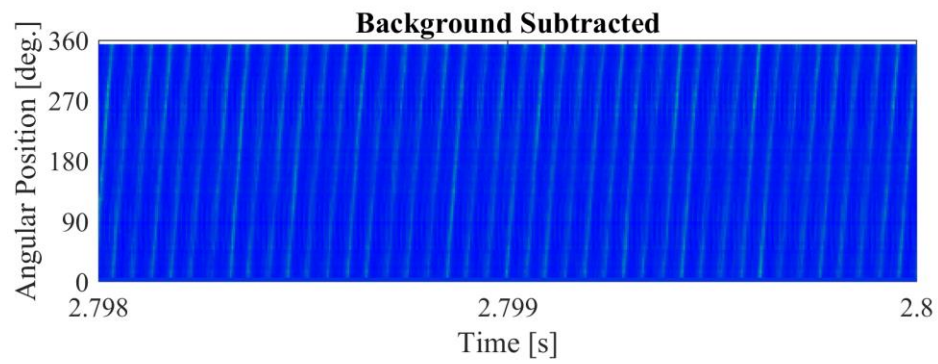
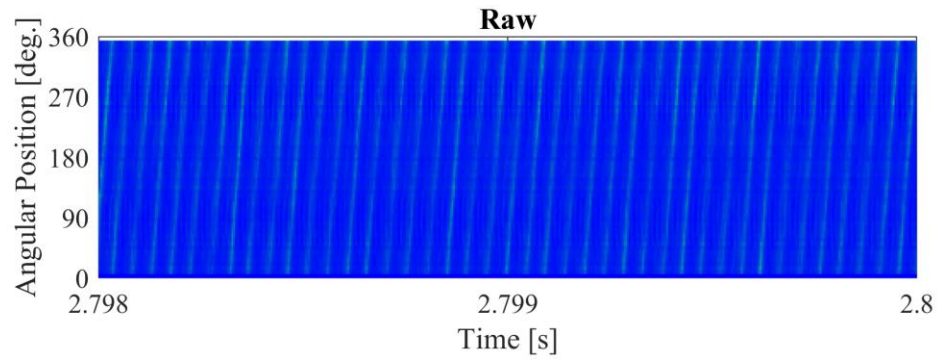
### 25-FI-5, Run 257



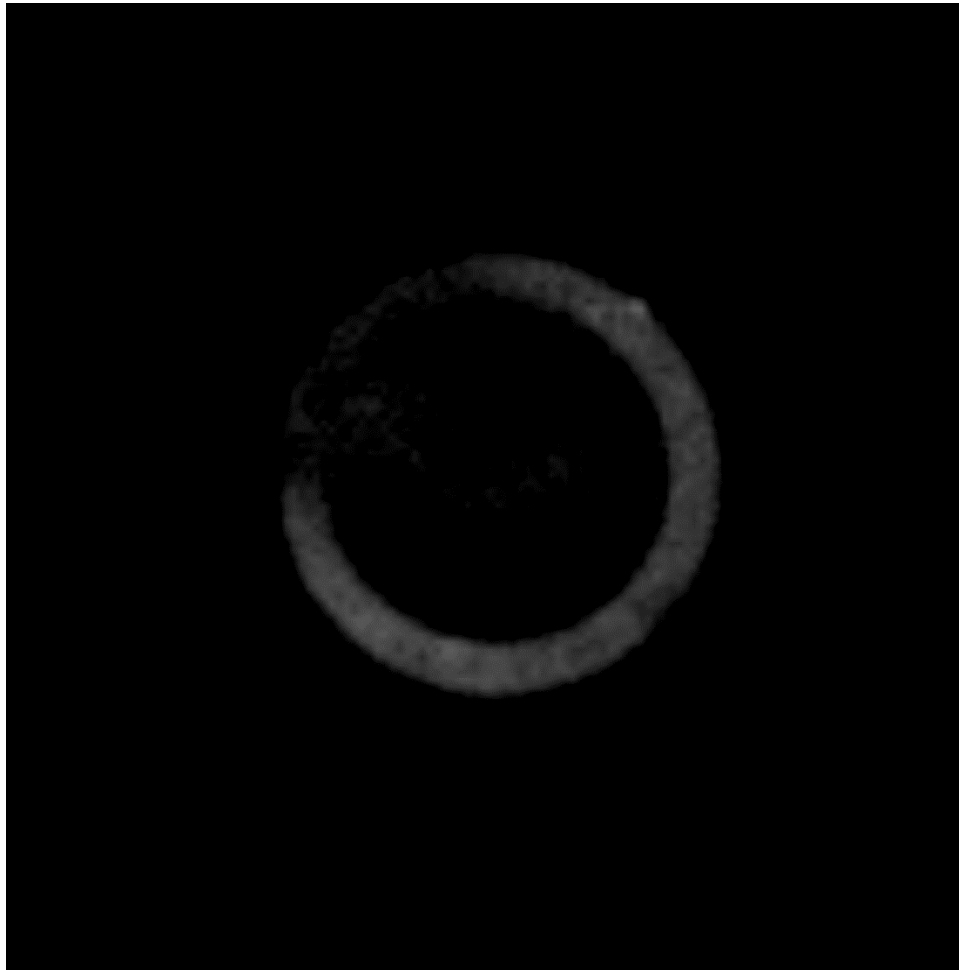
B.7: 1-WAVE (25-FI-12.7)



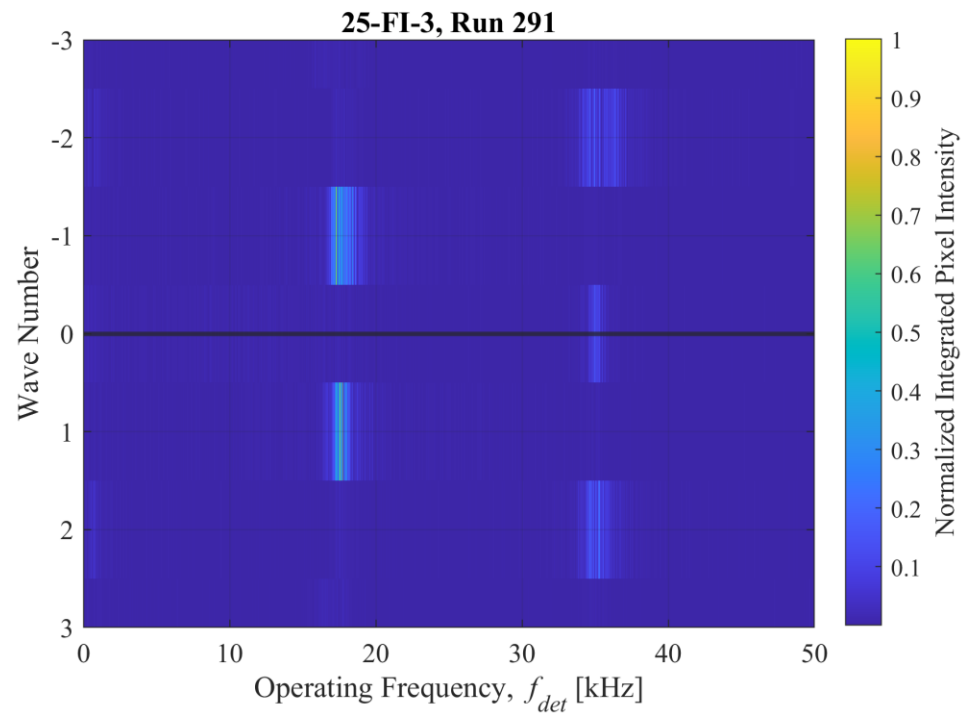
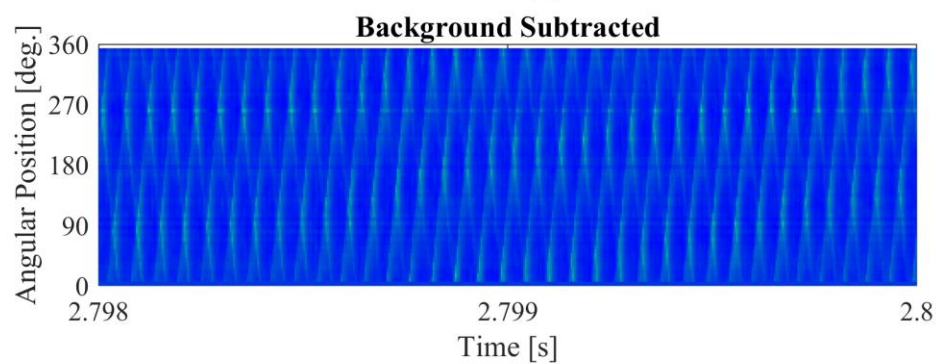
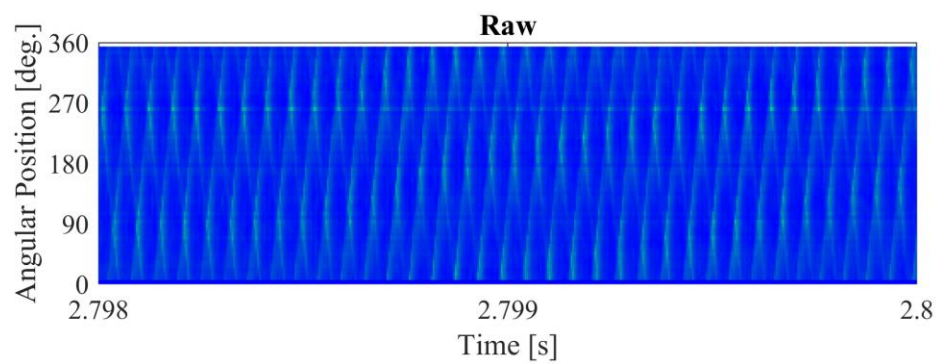
### 25-FI-12.7, Run 82



B.8: 1-WAVE WITH STRONG COUNTER WAVE (25-FI-3)



### 25-FI-3, Run 291

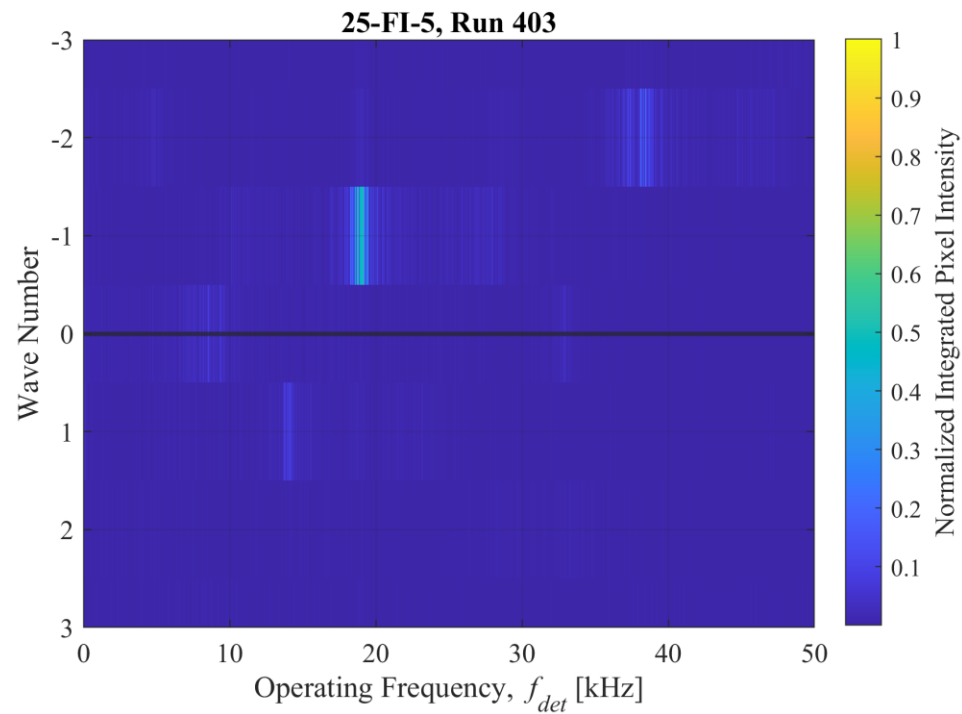
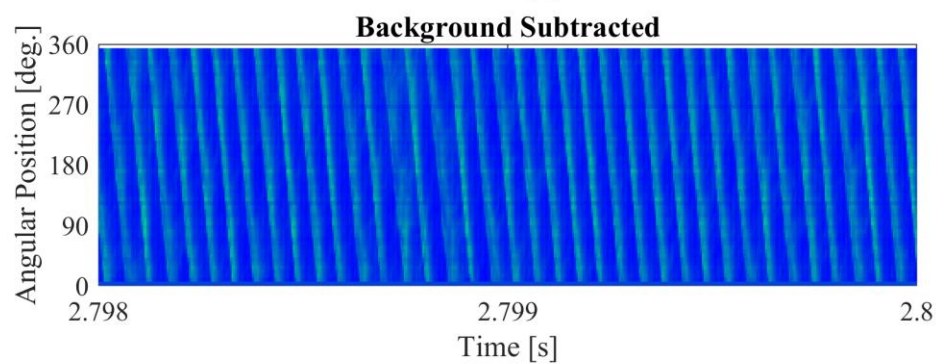
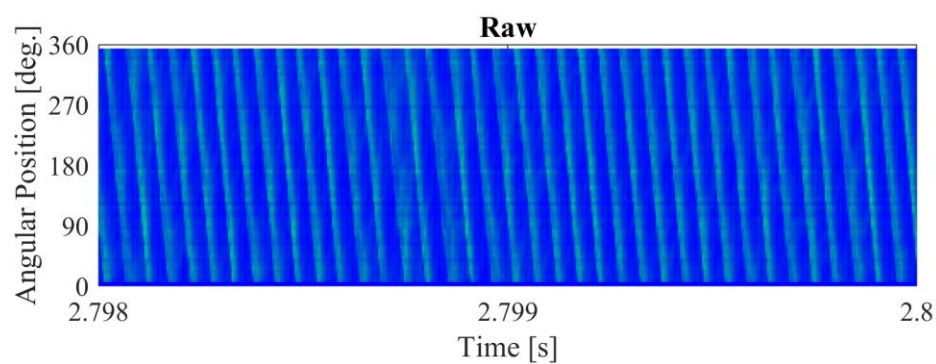




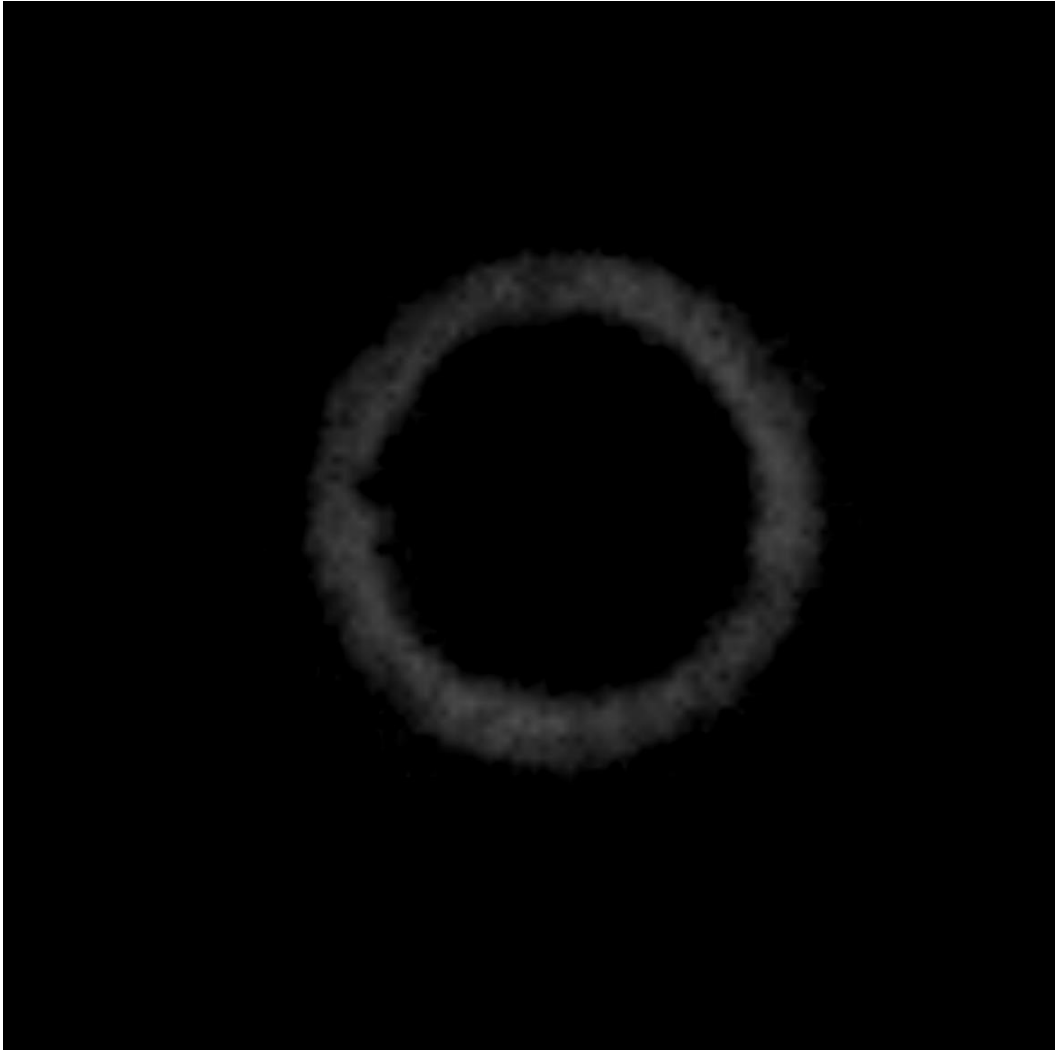
B.9: 1-WAVE WITH WEAK COUNTER WAVE (25-FI-5)



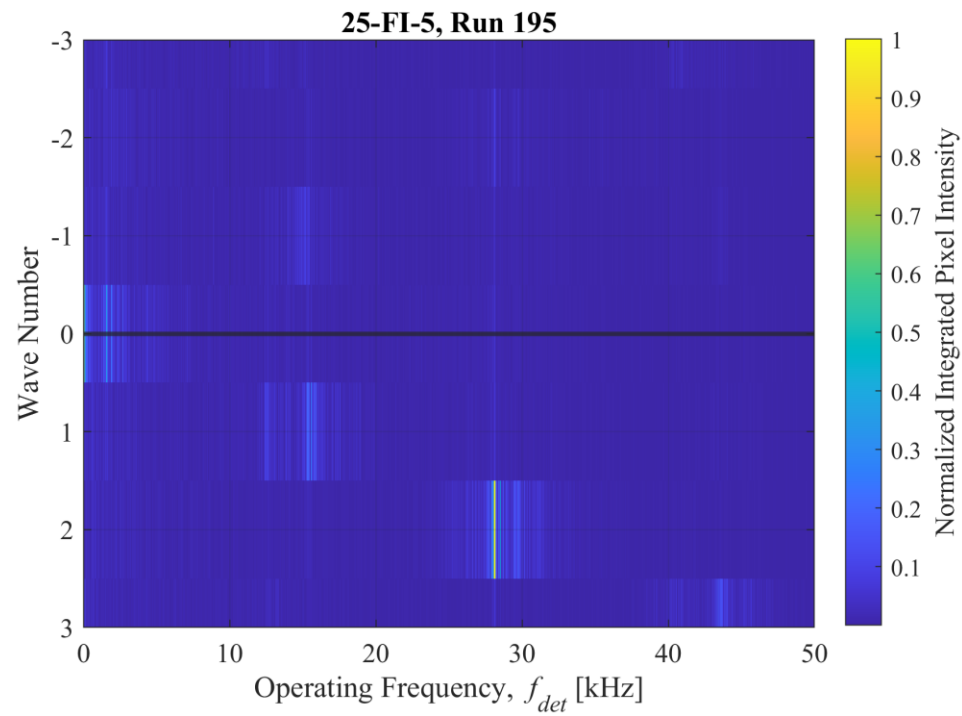
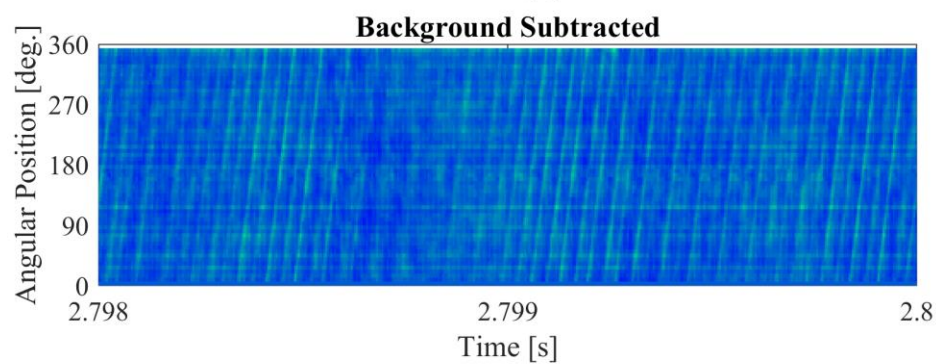
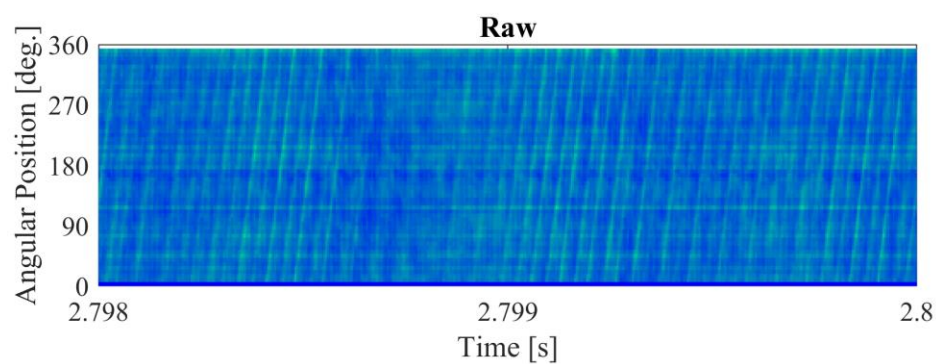
# 25-FI-5, Run 403



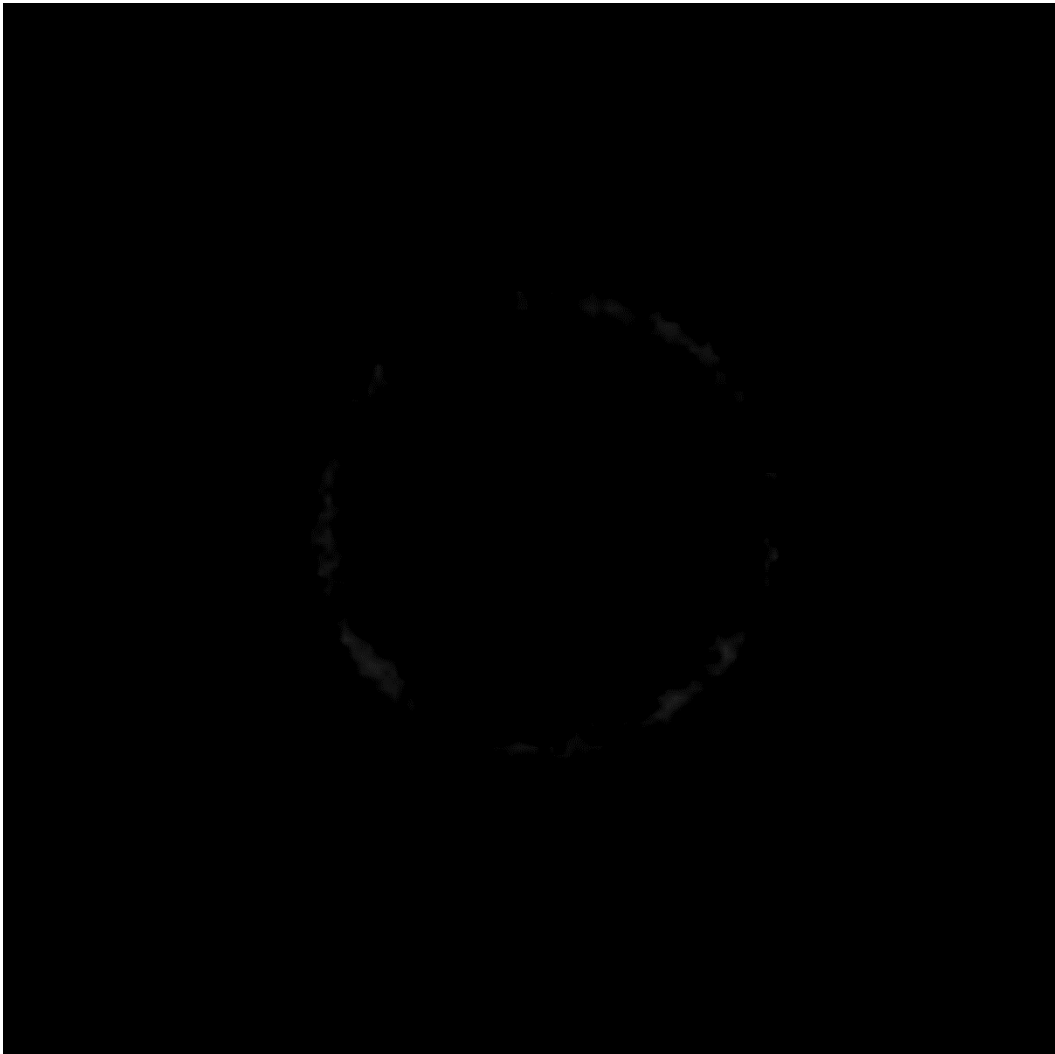
B.10: DEFLAGRATION (25-FI-5)



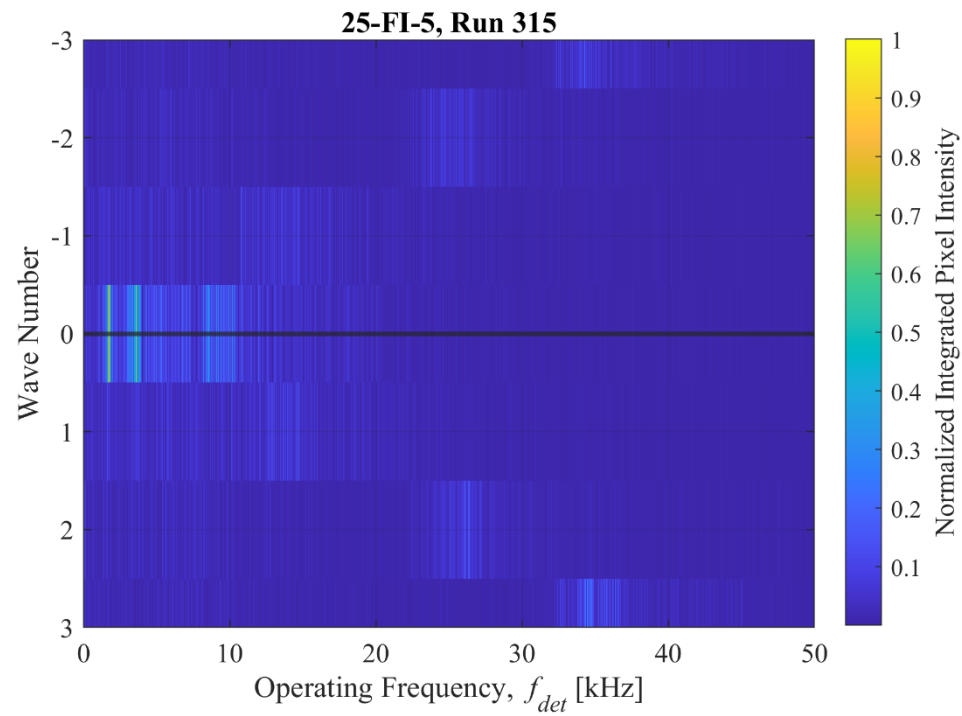
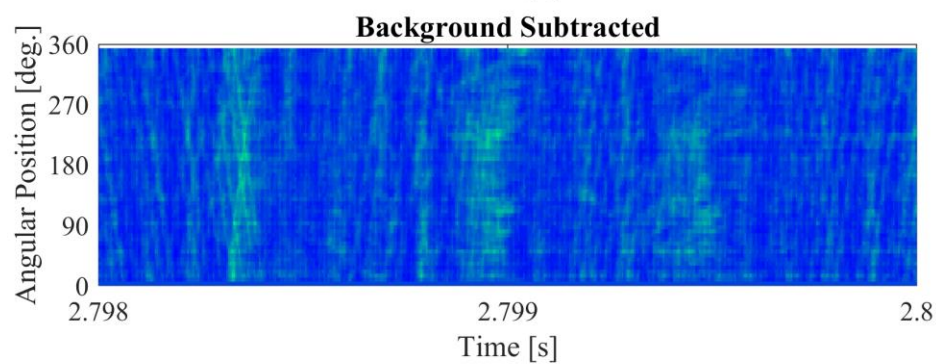
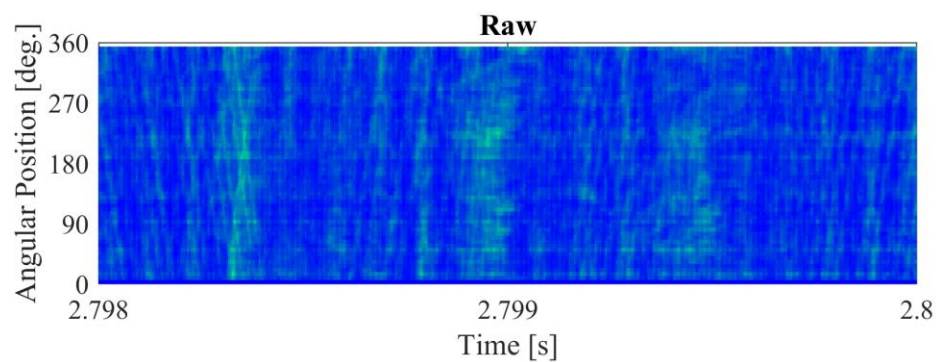
### 25-FI-5, Run 195



B.11: UNSTABLE (25-FI-3)

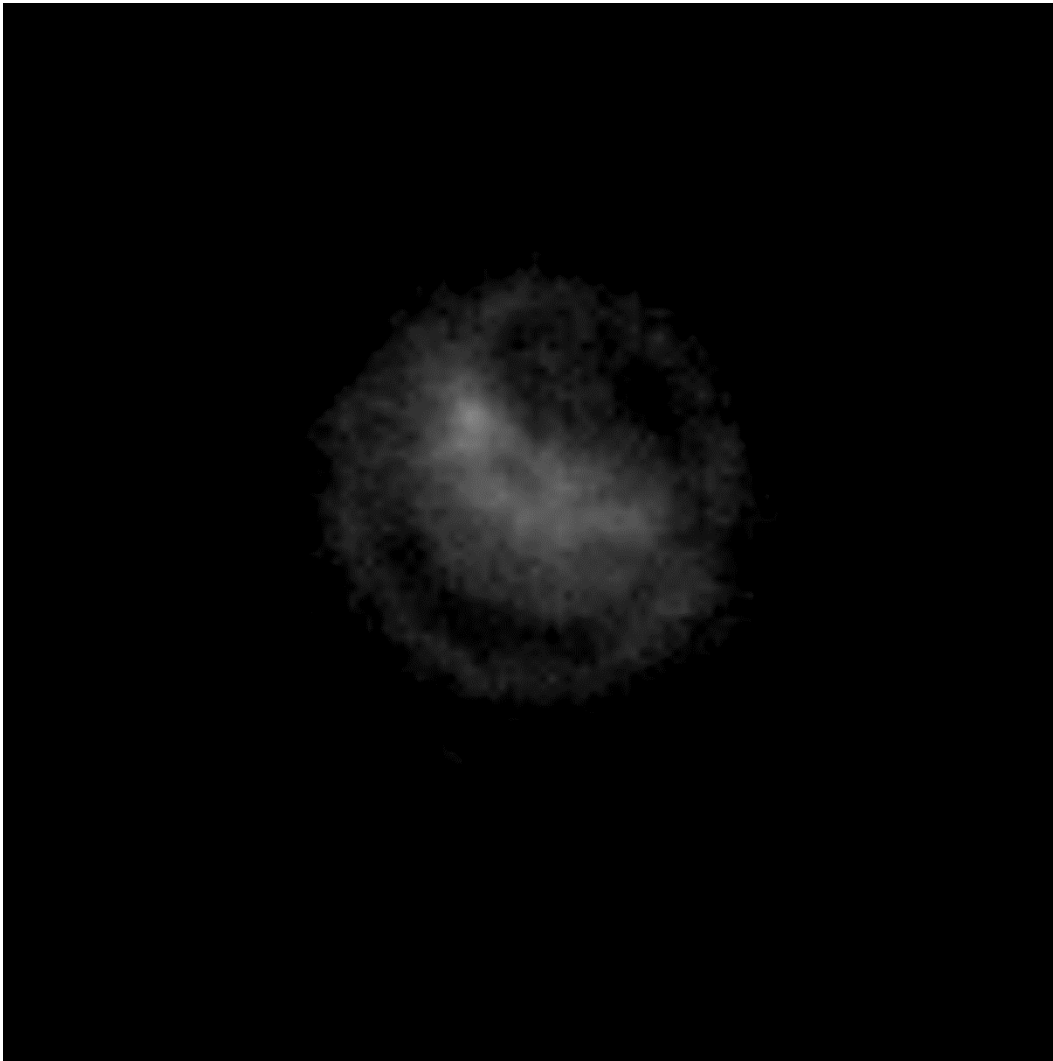


### 25-FI-5, Run 315



### B.12: PULSING (25-FI-5)

Due to the periodic changing diameter of luminosity, described video processing methods cannot be applied to this mode directly without significant modification, and thus x-t and FFT images are not yet available for this operating mode.



## APPENDIX C: UNCERTAINTY ANALYSIS

Calculation of measurement uncertainties for the 76-FI-5 configuration follows that described by Lightfoot et al. [68] and are presented below in Table 5-1. As every configuration utilized the same feed system, pressure and temperature probes, high speed camera, and data acquisition systems, these results apply to all experimental data presented. The large uncertainty reported for the mass flow rates and equivalence ratio was due to high frequency noise present in the thermocouple channels. If this uncertainty source was neglected and the average temperature value instead used, which is in line with the data processing described by Section 2.4, the individual oxidizer and fuel mass flow rates were each  $\pm 1.0\%$ . Consequently, the total mass flow rate and equivalence ratio uncertainties were then both below  $\pm 1.5\%$ .

Table 5-1: Measurement uncertainties for 76-FI-5 configuration

| Ox. Mass Flow Rate  | Fuel Mass Flow Rate  | Total Mass Flow Rate | Equivalence Ratio   |             |
|---------------------|----------------------|----------------------|---------------------|-------------|
| $\pm 2.4\%$         | $\pm 2.8\%$          | $\pm 2.4\%$          | $\pm 6.1\%$         |             |
| Ox. Plenum Pressure | Fuel Plenum Pressure | Axial Pressures      | Operating Frequency | Wave Speed  |
| $\pm 0.4\%$         | $\pm 0.5\%$          | $\pm 1.6\%$          | $\pm 0.7\%$         | $\pm 0.9\%$ |