Measuring Stock Returns in the Presence of Transaction Uncertainty

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Abstract

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The asset pricing literature models uncertainty in assets' payoffs, but typically assumes that any quantity of an asset can be traded at observed prices. For an investor observing a price quote, there is uncertainty about the effective transaction price for an order of a given size. This *transaction uncertainty* encompasses uncertainty about depth at quoted prices and uncertainty about the slope of the price schedule for transactions executing at prices inferior to the quotes. For an order in excess of quoted depth, the difference between a transaction price and that quoted is referred to as *price impact*. In the presence of transaction uncertainty, realized investment return has a random component, due to price impact, that is distinct from the effects of uncertainty about firm performance. CRSP closing-price returns do not account for the transaction uncertainty component of returns. CRSP returns are shown to be an inadequate proxy for monthly returns with realistic transaction uncertainty. On a monthly horizon, transaction uncertainty *significantly* affects measurement of mean returns, the variance/covariance matrix of returns, and the covariance of returns with risk factors. A more appropriate way to measure returns is proposed, along with a practical means for correcting historical data sets. Evidence is presented that transaction uncertainty risk is systematic and examples of the impact on asset pricing tests are given. In particular, two studies of Amihud and Mendelson (AM: JFE 1986 and JF 1989) are revisited. AM model returns that correctly reflect expected liquidity costs. They predict cross-sectional differences in returns, induced by differences in liquidity cost and in investor holding period. Data limitations forced AM to use CRSP returns and average annual bid/ask spread in their tests. CRSP returns are shown to induce a spurious positive return/spread relation that mimics the investment horizon clientele effect predicted by AM. Better proxies for liquidity costs and for AM's returns can now be constructed using fitted values of spread and price impact. Tests using these alternate proxies obtain strikingly different results.

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Dedication

To Christine

Introduction

Stock returns calculated from CRSP closing prices ignore such aspects of realizable returns as the bid/ask spread and the buy/sell direction of transactions. A general belief, reflected in empirical practice, is that transaction details are insignificant at the monthly return horizon typical of asset pricing research. There is increasing interest in the application of microstructure to asset pricing,[1] but transaction-level data currently are available for less than 15 years. This paper shows that monthly CRSP returns are a poor proxy for realizable returns because of transaction price uncertainty. A more accurate and consistent way of measuring returns that accounts for bid/ask spreads and the price impact of trades is presented, along with evidence that using these corrected returns dramatically affects test results. A practical procedure is given for creating a long, historical data set of daily spread and price impact for any NYSE firm. Evidence also is presented that transaction uncertainty risk is systematic.

Investors must weigh the effects on realized returns of implementing an investment strategy. The finance literature has long-standing models in which investors react to uncertainty with respect to firm cash flows and discount rates. Reactions to uncertainties in the market mechanism have been considered less thoroughly.[2] For an investor observing a price quote, denote uncertainty about the effective transaction price for a trade of a given size as *transaction uncertainty*. Transaction uncertainty reflects uncertain depth at the quote and

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[1] Brennan and Subrahmanyam (1996); O'Hara (1999) speaks of the need to bring together microstructure and asset pricing.

[2] Alternative market mechanisms have been compared with respect to various measures, such as transparency (Pagano and Röell, 1996; Garfinkel and Nimalendran, 2000) or the ex post level of transaction costs (Huang and Stoll, 1996; Bessembinder and Kaufman, 1997).

uncertain price impact for orders in excess of the depth. A related uncertainty has been studied in the context of informed investors,[3] but any investor whose trades are large enough to impact prices faces transaction uncertainty.[4] This study focuses on large trades.[5]

A trader observes quoted prices and depths when placing an order, but learns the realized transaction price only after the order is filled. Price improvement, a trade executed within the quotes, primarily benefits small orders.[6] For an order in excess of quoted depth, the difference between a transaction price and that quoted will be referred to as *price impact*.[7] For large trades, realized investment return has a random component due to price impact that is distinct from the effects of uncertainty about firm performance. Transaction cost thus has a pre-determined component,[8] the quoted bid/ask spread, and an uncertain component, the price impact realized on a large trade. Traders would

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[3] Kyle (1985) characterizes equilibrium behavior in a batch trading market for an informed investor with noise traders and a market maker who can see total net order flow. Market price is linear in the net order flow - the informed trader's choice of trade size plus the realization of liquidity orders. The informed investor limits the size of successive orders in response to the expected effect of the order on market price.

[4] Hong and Rady (1999) extend Kyle's framework, making informed traders uncertain about the distribution of liquidity trades, and so of the expected price impact of their informed trades. As a result, trading volume and information efficiency depend upon the path of prices. Dennis and Weston (2001) find evidence that ownership by insiders and institutions is positively correlated with measures of informed trading. Their measures of informed trading include proportional bid/ask spread and price impact. These findings are consistent with the view that large traders drive the return generating process in stock markets.

[5] About 51% of the 1997 share volume on the NYSE involved trades greater than 10,000 shares (referred to as block trades). Trades in the upstairs market are negotiated, avoiding the transaction uncertainty of the downstairs market in exchange for a known price discount. The choice to transact in the upstairs market will reflect a belief by the investor that this certainty equivalent discount cost is lower than the cost in the downstairs market. By value, the downstairs market accounts for more than 80% of all trades, and for more than 70% of trades greater than 50,000 shares, soothe effects of limited liquidity is a major consideration for large traders. (Fact Book for the year 1997) (Cheng, M. & A. Madhavan, 1994).

[6] Price improvement occurs primarily for smaller order sizes (less than 10,000 shares) in actively traded securities. For block trades of NYSE stocks, only 7% of market orders receive a price improvement of about 1c per share. (Ross, K., J. Shapiro and K. Smith, 1996)

[7] Price impact is (transaction price – ask) for a buy and (bid – transaction price) for a sell.

[8] The term "pre-determined" will be used to describe information that is observable going into a trade.

not "work" large orders were they certain about realized transaction prices.

For large trades, the market maker's quoted price is merely an indicator of the price at which *part* of a desired order may transact. A large order is expected to complete execution neither in one trade, nor at the quoted price. Investors react to expected price impacts by spreading execution of large orders over time.[9] As an order is managed to execute over a longer period, however, there is greater risk that new cash flow information will arrive. Large traders face a trade-off between price impact risk due to their trade, when trading large amounts quickly, and asset cash flow risk due to information flowing into the market, when trading patiently.[10] In equilibrium, order execution is managed so that the expected increase in marginal utility from spreading trades over time, to reduce price impact, just balances the expected decrease in marginal utility from increased cash flow risk associated with information flow.

Most studies of market liquidity have looked at the behavior of those who *supply* liquidity, in the form of bid/ask quotes and limit orders. These papers have treated bid/ask spread as a cost known to traders when entering into a trade. In this sense, the literature has studied *pre-determined* transaction costs. This study will focus on

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[9] Chan and Lakonishok (1995) study the trades of 37 large investment management firms. In their sample, orders are split to execute over four or more days for greater than 50% of the dollar value of trades. Keim and Madhavan (1997) study the trades of 21 institutional investors. In their sample, buy orders execute on average in 1.80 days and sell orders execute on average in 1.65 days.

[10] In Foster and Viswanathan (1990), the informed trader's information advantage declines over time. The informed trader's trading aggressiveness varies from day to day, reflecting a trade-off between price impact and the information cost of delaying trade. Seppi (1990) shows that institutions may choose to place block orders, even when uninformed. He and Mamaysky (2001) find that an investor with an incentive to sell a large block of shares in a short time splits her trades in the face of price impact.

consumers of market liquidity and their behavior when facing *uncertainty* in the supply of liquidity.

The remainder of this paper is organized as follows. Section 1 describes the data. Section 2 presents the model of trading with transaction uncertainty. Section 3 shows how CRSP returns mismeasure realizable returns by not accounting correctly for bid/ask spread plus price impact. Test results establish the significance of return mismeasurement on a monthly horizon by showing that CRSP returns are a poor proxy for realizable buy-then-sell returns. Section 4 presents a practical procedure for correcting historical CRSP returns to account for transaction uncertainty. Test results show that corrected monthly returns are a good proxy for realizable buy-then-sell returns. The return correction factors use spread and price impact. In Section 5, models of spread and price impact are estimated using the TORQ data set. These models are then fitted to monthly data to create long, historical data sets of spread and price impact for correcting CRSP returns. Section 6 examines whether or not transaction uncertainty risk is systematic. The transaction uncertainty component of returns is found to covary with systematic risk factors. In Section 7, the tests of Amihud and Mendelson (AM: 1986, 1989) on the cross-section of expected returns are recreated. AM model returns that account for expected liquidity costs. They predict cross-sectional differences in returns, induced by differences in liquidity cost and in investor holding period. Data limitations forced AM to use CRSP returns and average annual bid/ask spread in their tests. Better proxies for liquidity costs and for AM's returns are constructed using fitted values of spread and price impact. AM's tests are revisited using the improved proxies. Results using the improved proxies differ markedly from results using CRSP returns. There is no clear support for the main prediction of the Amihud and Mendelson investment-horizon clientele model (1986). Section 8 lists other implications of transaction uncertainty. Section 9 concludes.

1. Data

The TORQ database (Trades, Orders, Reports and Quotes) provides transaction data for 144 NYSE securities from November 01, 1990 through to January 31, 1991. The securities in TORQ were chosen randomly so as to be distributed evenly among NYSE size quintiles. For this study only common stocks are retained. All REIT's, units, and closed-end funds are eliminated. As well, common stocks are eliminated that have many days with no market order transactions, including at least one stretch of more than five trading days with no transactions. For the remaining 98 stocks, all market orders after the open and their associated transactions are used. Bid-ask spread and mid-quote price are calculated from the ask and bid quote time series. Price impact is calculated using the prices of the transactions associated with a market order and the quote in effect at the time the order was placed.[11]

The Federal Reserve provides daily yield data on government and corporate bonds and on T-bills. Citibank provides monthly corporate bond yields. For daily data on NYSE common shares, CRSP provides volume, number of shares outstanding, closing price and AskHi and BidLo. CRSP also provides the daily level of the S&P500 index. The NYSE provides daily total NYSE share volume.

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[11] The TORQ data are screened for errors. A transaction is examined if the difference between its price impact and the mean price impact for that security exceeds five standard deviations of the price impact for that security. For the 98 firms, only 97 transactions are eliminated as erroneous.

2. Asset Pricing with Transaction Uncertainty

Consider a model with a perfectly liquid, riskless bond and a risky asset with finite liquidity. Traders submit market orders after observing quoted prices and depths.[12] For the risky asset, true depth at the quote is known only to the market maker. A random fraction of any market order executes at the quoted price. The remainder of the order executes at an average price that differs from the quote by a random amount. Traders, thus, face transaction uncertainty.

A trader seeks to maximize expected utility of lifetime consumption.

$$V(c) \equiv \max_{c_t} E_0\left[\sum_{t=0}^{\infty} \beta^t U(c_t)\right]$$

Because the risky asset has limited liquidity, orders are not guaranteed to execute at the quoted prices, $P_t + s_t$ for a buy or $P_t - s_t$ for a sell, where: $P_t \equiv$ Midpoint of bid and ask prices quoted before trade at t

$s_t \equiv$ half the bid/ask spread

A single order can execute in several transactions, and at different prices.

Assume:

1. Random fraction $(1-\tilde{x}_t)$ of an order to trade the risky asset executes at the quoted price. The remaining fraction of the order, $\tilde{x}_t$, executes

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[12] An in-depth study of the trading behavior of 21 institutions over two years found less than three percent of total trade value was placed as limit orders. (Keim, D. and A. Madhavan, 1997)

at the random weighted-average price $P_t + s_t + \tilde{\delta}_t$ if a buy and at $P_t - s_t - \tilde{\delta}_t$ if a sell,[13] where $\tilde{\delta}_t$ is the price impact of the trade.

2. The riskless bond can be bought or sold with no transaction uncertainty and no transaction cost.

Under the stated assumptions, a perturbation argument gives the Euler equations for consumption and return data.[14] The equations reflect the buy-then-sell strategy assumed to be followed by traders.[15]

$$0 = E_t\left[\beta \frac{U'(\tilde{c}_{t+1})}{U'(c_t)} \tilde{R}_{t+1} - 1\right] \tag{1}$$

$$0 = E_t\left[\beta \frac{U'(\tilde{c}_{t+1})}{U'(c_t)} \tilde{R}_{f,t+1} - 1\right] \tag{2}$$

where a realizable buy-then-sell return is:

$$\tilde{R}_{t+1} \equiv \frac{\tilde{P}_{t+1} - \left(\tilde{s}_{t+1} + \tilde{x}_{t+1}\tilde{\delta}_{t+1}\right)}{P_t + \left[s_t + E_t\left(\tilde{x}_t \tilde{\delta}_t\right)\right]} \tag{3}$$

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[13] Consider a 400 share buy order that executes as $q_{t1} = 100$ shares at quoted price $P_t + s_t = 10\,\frac{1}{8}$, $q_{t2} = 200$ shares at price $P_t + s_t + \tilde{\delta}_{t1} = 10\,\frac{1}{4}$, and $q_{t3} = 100$ shares at price $P_t + s_t + \tilde{\delta}_{t2} = 10\,\frac{3}{8}$. Price impact is:

$$\tilde{\delta}_t \equiv \sum_{k=2}^{3} \tilde{\delta}_{tk} * \tilde{q}_{tk} \Big/ \sum_{k=2}^{3} \tilde{q}_{tk} = \left(200 * \tfrac{1}{8} + 100 * \tfrac{1}{4}\right)/(200 + 100) = \tfrac{1}{6}$$

The realized price for the buy order is then:

$$\bar{P}_t \equiv (1 - \tilde{x}_t)(P_t + s_t) + \tilde{x}_t\left(P_t + s_t + \tilde{\delta}_t\right) = P_t + s_t + \tilde{x}_t \tilde{\delta}_t = 10\,\tfrac{1}{4} \quad \text{where} \quad \tilde{x}_t \equiv \sum_{k=2}^{3} \tilde{q}_{tk} \Big/ q_t = \tfrac{3}{4}$$

[14] See Appendix A for derivation.

[15] A buy-then-sell strategy is defined here as buying at t and selling at t+1. A different strategy changes details of Euler equation (3), but often will not change the finding that monthly CRSP returns are a poor

and: $U'(\tilde{c}_{t+1}) \equiv$ Marginal utility of consumption in period t+1

$\beta \equiv$ Time discount rate

$P_t \equiv$ Midpoint of bid and ask prices quoted before trade at t

$1 - \tilde{x}_t \equiv$ Fraction of order executed at quoted price during trade at t

$s_t \equiv$ Half the bid/ask spread

$\tilde{\delta}_t \equiv$ Price impact $($price for trade in excess of depth at quote $-$ quote$)$

$P_t \pm \left(s_t + \tilde{x}_t\, \tilde{\delta}_t\right) \equiv$ Realized price for order traded at t

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proxy for returns realizable by large traders following a consistent investment strategy. See Appendix F for a discussion of CRSP returns when used in alternative strategies.

3. Evaluating CRSP Returns as a Proxy for Realizable Returns

The transaction uncertainty component of stock prices makes CRSP closing-price returns differ from realizable returns. Here CRSP returns will be compared to realizable buy-then-sell returns. Monthly buy-then-sell returns measure the returns that would be realized with a one-month holding period. Other strategies may correspond to a longer holding period, in which the impact of transaction uncertainty may be reduced. None-the-less, results of a similar nature would be obtained when comparing CRSP returns to some other realizable return.

CRSP closing *prices* can be transaction prices inside, outside or at the quotes, or the midpoint of the closing quotes on days with no trades. A particular CRSP *return* can be a sell, followed by a buy, or a buy, followed by a quote midpoint, or some other combination. Hence, in a market with bid/ask spread and finite depth, CRSP returns do not measure the returns realized by an investor. Errors in measuring realized returns can be substantial.[16] Return measurement error is shown to be economically and statistically significant in sections 3.1 and 3.2.

3.1 CRSP Returns are Far From Realizable Returns: TORQ Evidence

In this section, we use TORQ data to compare realized returns with returns that correspond to CRSP measured returns. Over the 63 trading days in the TORQ data period, a closing buy price and a closing

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[16] Blume and Stambaugh (1983) consider the bid/ask bounce bias in computed daily returns introduced by using closing-price returns in place of unobservable "true" price returns. Their bias is exacerbated when portfolio returns are calculated as the arithmetic average of the individual daily returns in the portfolio. They propose the use of buy-and-hold returns in calculations. Conrad and Kaul (1993) use a similar analysis to show that cumulating biased monthly returns over several years cumulates the biases. Neither of these studies deals explicitly with price impact and the uncertainty it engenders. Nor do "true" price returns represent realizable returns.

sell price are constructed from the mid-quote price, half-spread and price impact associated with the last market order transaction of each day. The price of the last market order transaction is used as the CRSP closing price. Overlapping, monthly buy-then-sell and CRSP returns are constructed using these buy, sell and CRSP daily price series, where each month is taken to have 23 trading days. For example, return 1 uses days 1 and 24, whereas return 63 uses days 63 and 86. Buy-then-sell returns use a sell price divided by a 23-lag buy price.[17] CRSP returns use a closing price divided by a 23-lag closing price.

Table 1 shows summary statistics for three NYSE firms: Boeing (BA), Federal Express (FDX) and Texas Industries Inc. (TXI), and for the pooled data of 98 NYSE firms. Statistics for the individual firms are shown twice: for *all* market order transactions, and for *big* market order transactions, in which order size exceeds quoted depth or 9,999 shares. The term $(s_{t-1} + x_{t-1}\delta_{t-1})/[P_t + s_t + E_t(x_t\,\delta_t)]$ in Euler equation (3) measures the effect of transaction uncertainty on returns. BA and FDX are relatively large, liquid stocks, so the effect of transaction uncertainty tends not to be as big as it is for smaller firms, such as TXI.[18] Nonetheless, the mean differences between CRSP and big buy-then-sell returns for all three firms are economically large at 0.24%, 0.62% and 1.16% per month. For the 98 firms, the mean difference is even larger at 1.29%.

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[17] Over the TORQ period, measured buy-then-sell returns deviate from realizable, consistent buy-then-sell returns in one respect. No lagged buy price is available for the return denominator in the first 23 buy-then-sell returns. CRSP closing prices are used instead. Assuming a 50 percent chance that a CRSP closing price is a buy, 11 or 12 of these 23 prices tend to be buy prices, giving buy-then-sell returns, as desired. The remaining 11 or 12 returns will be sell-then-sell returns. Sell-then-sell returns are larger than realizable buy-then-sell returns. Associated return measurement errors make the TORQ buy-then-sell returns closer to CRSP returns, so subsequent tests of the difference in the two return series understate economic and statistical differences.

[18] The analysis by Blume and Stambaugh (1983) suggests that the return bias from using closing prices is proportional to the square of the proportional spread (Bid/ask spread divided by price). Focusing on all

Are CRSP returns statistically far from realizable returns? Parametric and non-parametric tests are used to evaluate CRSP returns as a proxy for realizable returns. In each test, realizable buy-then-sell returns are compared to proxy returns. The parametric test is a paired t-test of the difference in mean returns:[19]

$$\begin{aligned} H_{01} &: E\left[R_{\text{Buy-then-sell}} - R_{\text{Proxy}}\right] = 0 \\ H_{A1} &: E\left[R_{\text{Buy-then-sell}} - R_{\text{Proxy}}\right] \neq 0 \end{aligned} \tag{4}$$

The non-parametric test is the Wilcoxon signed rank test of the difference in returns:

$$\begin{aligned} H_{02} &: E\left[\text{signed rank}\left(R_{\text{Buy-then-sell}} - R_{\text{Proxy}}\right)\right] = 0 \\ H_{A2} &: E\left[\text{signed rank}\left(R_{\text{Buy-then-sell}} - R_{\text{Proxy}}\right)\right] \neq 0 \end{aligned} \tag{5}$$

Table 2 shows test results for 98 NYSE stocks. The difference between buy-then-sell and CRSP returns is transformed to remove autocorrelation and heteroskedasticity, then pooled.[20] The null hypothesis of no difference between buy-then-sell and CRSP returns is

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transactions, they assert that return bias is economically significant only for smaller firms, since the proportional spread tends to decline with firm size.

[19] The mean difference t-test is equivalent to a test of the mean difference in pricing errors for Euler equations (1) - (3) for risk-neutral traders. In this case, the ratio of marginal utilities is constant and the stochastic discount factor is the inverse of the riskless rate.

$$\alpha_{\text{Buy-then-sell}} - \alpha_{CRSP} = \left(\frac{R_{\text{Buy-then-sell}}}{R_F} - 1\right) - \left(\frac{R_{CRSP}}{R_F} - 1\right) = \frac{R_{\text{Buy-then-sell}} - R_{CRSP}}{R_F}$$

$$\Downarrow$$

$$\alpha_{\text{Buy-then-sell}} - \alpha_{CRSP} = 0 \quad \Leftrightarrow \quad R_{\text{Buy-then-sell}} - R_{CRSP} = 0$$

[20] Because of the overlapping observations, the individual returns have an MA(22) distribution. The differenced returns seem to be adequately modeled by an AR(1) process. The differenced returns are transformed to have a Gaussian distribution by subtracting the product of the estimated autocorrelation times the lagged difference. Heteroskedasticity is removed by dividing by the estimated standard deviation of each firm's transformed series.

rejected at the 0.1% level for both the parametric and non-parametric tests. CRSP returns are *far from* realizable returns. For these 98 NYSE firms, from November 1990 through January 1991, the mean difference between realized and CRSP monthly returns is more than 16% per annum.

CRSP monthly returns differ from realizable buy-then-sell returns. CRSP measures a buy-then-sell return only about one observation in four.[21] CRSP gross returns that do not measure a buy followed by a sell are larger than buy-then-sell returns, since the smallest ratio of closing prices is a sell divided by a buy. For the firms in Table 1, both mean and median return error is positive. Mismeasurement of realizable returns will affect estimates of return covariances, as well as mean returns. An application as basic as estimating mean-variance efficient portfolio weights is very sensitive to mismeasurement of return moments.[22]

3.2 CRSP Returns are Far From Realizable Returns: Simulation Evidence

Transaction prices can be simulated to investigate the affects of transaction uncertainty on the outcome of asset pricing tests that use returns measured at monthly or longer frequencies. The statistical significance of transaction uncertainty over different return horizons and for different levels of price impact, holding overall return constant, can be tested by varying the parameters of the simulation. Simulation is thus a powerful way of investigating whether or not CRSP returns are "far from" realizable returns.

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[21] A buy-then-sell return involves a buy price at the close of one period and a sell price at the next close. It seems reasonable to assume that period-to-period CRSP prices fall with equal probability on the bid or ask side of the spread midpoint. Roll (1984) used this assumption to estimate the effective spread.

[22] Best and Grauer (1991) show that finite sample estimates of portfolio weights are sensitive to mean returns. Britten-Jones (1999) develops a procedure for estimating sampling error in the measurement of mean-square efficient weights.

Appendix B describes the simulation. The effect of transaction uncertainty on returns is measured by $(s_{t+1}+x_{t+1}\delta_{t+1})/\left[P_t+s_t+E_t(x_t\delta_t)\right]$. The simulation is calibrated to set the mean and standard deviation of this transaction uncertainty measure and of CRSP returns close to those of Boeing in Table 1A. Table 3 shows summary statistics for the simulated firm.

Simulated monthly CRSP and buy-then-sell returns are compared using the parametric t-test and non-parametric Wilcoxon signed difference test. Table 4 presents test results for three levels of the transaction uncertainty measure $(s_{t+1}+x_{t+1}\delta_{t+1})/\left[P_t+s_t+E_t(x_t\delta_t)\right]$: 0.24% is the level calibrated to Boeing; 0.12% is a lower level of transaction uncertainty, and 0.06% is a near zero level. The mean and standard deviation of simulated CRSP monthly returns are held constant at 1.64% and 9.88% respectively. In all three cases, and for both tests, the null hypothesis of no difference between buy-then-sell and CRSP returns is rejected at the 1% level.

The TORQ and simulation results strongly suggest that CRSP returns indeed are statistically far from realizable buy-then-sell returns. Transaction uncertainty, therefore, *is* significant on a monthly basis.

4. Approximating Realizable Returns in the Presence of Transaction Uncertainty

Transaction uncertainty makes CRSP returns a poor proxy for realizable returns. A means of converting CRSP returns to reduce return measurement error will be presented next.

4.1 Categorizing Return Measurement Error

A CRSP closing price can represent a sell-initiated trade, a buy-initiated trade or a quote midpoint. CRSP returns, being random combinations of prices from these buy-trades, sell-trades and quotes, can be categorized into nine types for each permutation of a buy, sell or quote price divided by a buy, sell or quote price. A buy-then-sell return involves a sell-trade price divided by a lagged buy-trade price. The returns for other possible strategies are discussed later.

Assume the mid-quote price, P_t, half-spread, s_t and price impact, δ_t are the same for the last buy and sell transactions of period t. Represent a last buy, sell or mid-quote with superscripts. Thus buying at the end of period t, and selling at the end of period t+1 gives return $R^{S\ B}$. The buy-then-sell return is then:

$$R^{\text{Buy-then-sell}} \equiv R^{S\ B} \equiv R_{t+1} = \frac{P_{t+1} - s_{t+1} - \delta_{t+1}}{P_t + s_t + \delta_t}$$

Any CRSP return can be converted into a buy-then-sell return. For example, when the closing transactions for days t and t+1 are sells, the CRSP return is:

$$R^{S\,S} \equiv R^{CRSP}_{t+1} = \frac{P_{t+1} - s_{t+1} - \delta_{t+1}}{P_t - s_t - \delta_t}$$

Express the buy-then-sell return in terms of the CRSP return:

$$R^{\text{Buy-then-sell}} = R^{S\,S} * \left(\frac{P_t - s_t - \delta_t}{P_t + s_t + \delta_t} \right)$$

Conversion factors for the eight remaining return types are found by writing a buy-then-sell return in terms of the CRSP return, as in the example. Table 5 lists the categories of CRSP returns and exact conversion factors.

Notice that realizable buy-then-sell gross returns never exceed CRSP returns. The return measurement error from using a sell/sell CRSP return to represent a buy-then-sell return is:

$$R^{CRSP} - R^{\text{Buy-then-sell}} = R^{S\,S} * \left[\frac{2*(s_t + \delta_t)}{P_t + s_t + \delta_t} \right]$$

Since the correction factors in Table 5 are always between zero and one, and gross return is never negative, return measurement error is never negative. Assuming equal probabilities of buy or sell closing prices and ignoring non-trading, CRSP returns overstate buy-then-sell returns approximately 75% of the time.

Table 5 is problematic in that CRSP does not give enough information to distinguish all nine return types. For days with no transactions, CRSP reports the closing mid-quote price as negative, but for days with transactions, since buy/sell direction is not observable, the nine CRSP return types of Table 5 are reduced to four observable return types.[23] Since CRSP returns cannot be categorized exactly, only approximate correction of historical CRSP returns is possible.

4.2 Minimizing Mean Square Error

Correction factors that minimize the mean square difference between buy-then-sell and corrected returns are derived in Appendix C and are listed in Table 6. An approximation that minimizes mean square return error (MSE) seems consistent with the assumptions of classical regression models, used so widely in finance.

4.2.a MSE Corrected CRSP Returns are Close to Realizable Returns

Data limitations make our ability to correct for transaction uncertainty imperfect. Are corrected returns close to buy-then-sell returns? The TORQ and simulation data used to test CRSP returns are used with Table 6 to calculate corrected returns and to test them as a proxy for realizable returns. Table 7 shows test results for the TORQ data. The mean difference between corrected and realizable returns is 0.041%, compared to a mean difference of 1.29% between CRSP and realizable returns in Table 2. Neither the parametric nor the non-parametric test rejects the null hypothesis of no difference between corrected and realizable returns. Table 8 shows test results for the three simulated levels of transaction uncertainty. None of these tests show corrected returns to differ statistically from buy-then-sell returns.

The TORQ and simulation results of Section 4 suggest that corrected monthly returns are statistically close to realizable returns. The MSE correction factors, therefore, account well for transaction uncertainty.

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[23] Since quote data is unavailable over the 20+ year historical period of interest, buy/sell direction

CRSP returns are a poor proxy for buy-then-sell returns, and corrected returns are a good proxy, but the MSE correction factors use bid/ask spread and price impact, which are not readily available before 1984. An approach for using the correction factors developed in Table 6 is addressed in the next section.

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cannot even be inferred as in Lee and Ready (1991).

5. Estimating Models of Spread and Price Impact

A contribution of this paper is to present a practical procedure for producing long data sets of realizable returns. First, regression models of spread and price impact are used to construct fitted estimates of these transaction uncertainty variables. Then, the fitted data are used in the MSE minimizing correction factors to approximate realizable returns over the period covered by the CRSP daily stock files.

5.1 Explanatory Variables for Spread and Price Impact

Spread and price impact are modeled here using firm-specific and market-wide variables motivated by previous theoretical and empirical research.

5.1.a Determinants of the Bid/Ask Spread

The bid/ask spread has been modeled to reflect three components of a market maker's costs: order processing, inventory, and asymmetric information.[24] Using an inventory model, Stoll (1978) predicts that the spread will be proportional to the product of dollar volume with the variance of returns. Financing costs associated with holding inventory will be positively correlated with the riskless interest rate and with unexpected volume, which increases inventory positions. Easley and O'Hara (1987) predict that trade size will be positively correlated with information asymmetry. Engle and Lange (1997) suggest that high volume is associated with increased informed trading. Smith Bamber et al (1999) find more informed trading when volume is abnormally high. Using an option-based model for the spread, Copeland and Galai (1983)

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[24] Huang and Stoll (1997) provide a categorization of models and an approach for measuring the components of the spread.

predict that the spread increases with the level of stock price and return variance, and decreases with the level of trade volume.

5.1.b Determinants of Price Impact

Kraus and Stoll (1972) suggest that the price impact of large trades includes an inventory cost associated with distributing a trade among several counter-parties. This implies that price impact, like spread, reflects the costs of financing and the volume and size of trades. Kraus and Stoll (1972) also suggest that the price impact from large trades may be due to a downward-sloping demand for stocks. Kavajecz (1999) argues that market makers reduce quoted depth on the side of the market from which they fear adverse price movements. Together these suggestions imply that price impact, like spread, increases with measures of trade size and of asymmetric information.

5.1.b.i Further Implications of Spread and Price Impact

For a given transaction, the realized price includes spread and price impact. Both spread and price impact could be modeled with permanent and transient components. The inventory component of price impact is transient, but the information component has a permanent effect on price level.[25] Although both spread and price impact have components associated with information, the permanent information component in the quoted spread could be viewed as measuring the risk of informed trading. The permanent information component in the price impact could be viewed as measuring the flow of information about the security's value.

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[25] See for example Kraus and Stoll (1972).

An AR(1) model is one way to capture this permanent-plus-transient behavior. Interpreting the mid-quote price as the "intrinsic" price level, the permanent component of the price impact, by capturing the flow of information about value, is exactly the change in the intrinsic value of the stock. The permanent information component of the spread does not measure changes in the level of intrinsic value, but rather the probability and expected cost of informed trading.

A simultaneous equation relation can model the interrelation between price impact and mid-quote price. Here $a_1\ \delta_t$ is the permanent component of price impact, which is the change in intrinsic value.

$$\delta_{t+1} = a_0 + a_1\ \delta_t + a_2\ Inv_{t+1} + e_{t+1}$$

$$P_{t+1} - P_t = \alpha_0 + a_1\ \delta_t + \varepsilon_{t+1}$$

where: δ = price impact
P = mid-quote price
Inv = proxy for inventory effects

The error terms are assumed to be zero mean and uncorrelated with other variables.

The two equations can be combined to eliminate the δ_t term:

$$\delta_{t+1} = (a_0 - \alpha_0) + (P_{t+1} - P_t) + a_2\ Inv_{t+1} + (e_{t+1} - \varepsilon_{t+1})$$

The change in the "intrinsic value" of the stock reflects capitalization of information flow to the market, with the following empirical implications:

- The coefficient of a regression of price impact on the change in mid-quote price should have a magnitude of 1.
- In a regression of price impact on various variables, the change in mid-quote price should capture the impact of asymmetric

information, so coefficients for other proxies for information flow should not be significantly different from zero.

Alternatively, the transient component of price impact is captured by $\delta_{t+1} - (P_{t+1} - P_t)$, with the following empirical implication:

- Autocorrelation of the transient component of price impact, $\rho_{\delta_{t+1} - (P_{t+1} - P_t)}$, should not be significantly different from zero.

These implications will be investigated in a subsequent paper.

5.1.c Determinants of Half-Spread Plus Price Impact

The components of stock price are mid-quote price, spread and price impact. Price movements and returns must reflect changes in these components. An increase in the spread or price impact increases buy prices and decreases sell prices. Correlation between a market variable and measured price movement thus *may* be due to correlation between the market variable and the liquidity costs.

Karpoff (1987) presents clear evidence that trade volume is positively correlated with absolute price movements. Brennan and Subrahmanyam (1996) find that returns reflect pre-determined transaction costs, and that dollar trading volume explains a large portion of their transaction cost measure (1995). They find average trade size to be positively related to the standard deviation of firm returns (1998). Wang (1994) argues that informed traders offer liquidity traders a price concession as compensation for asymmetric information risk. This price concession makes volume positively correlated with absolute excess returns. Controlling for shares outstanding, Keim and Madhavan (1997) find that average transaction costs increase with trade size only for smaller orders. Lesmond et al (1999) find that higher liquidity cost is associated with days when there is little or no trading.

This theoretical and empirical work suggests that variables for modeling spread and price impact should include security-specific volume and trade size, dollar volume times return variance, measures of infrequent trading, price level and standard deviation of return, plus the riskless rate and measures of asymmetric information. The proxies for these variables are chosen to be consistent with this earlier work, while scaling to make the variables meaningful in a model meant to work across firms of very different size. For example, a given level of dollar volume surely will convey different information for a large, liquid firm than for a small, infrequently traded one. This concern is dealt with by using share turnover[26] (volume divided by shares outstanding) in the place of volume. Scaling has also been used to allow for the eventual fitting of these models over a long time period. Here the concern is with changes in scale of a variable across time. A firm that has a high average level of volume over the estimation period may have a much lower average level 20 years earlier. As in the cross-sectional case, the use of scaled predictor variables, such as turnover, means that a given scaled change in a predictor variable will have the same impact on the fitted value over the extended fitting period.

The proxy variables used are standardized volume, share turnover, excess turnover, dollar turnover, dollar turnover times return variance, price and the standard deviation of returns. AskHi is the highest sale price of the day. On no-trade days, AskHi is the closing ask quote. BidLo is the lowest sale price of the day. On no-trade days, BidLo

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[26] Lo and Wang (2000) suggest share turnover (volume/shares outstanding) as a measure of trading activity.

is the closing bid quote. Because AskHi-BidLo is zero when there is only one trade during the day, but non-zero otherwise, a regression intercept dummy is used that is one when AskHi-BidLo is zero. On days with no trade, price impact set to zero, and the spread is set to AskHi-BidLo. Slope dummies are formed using the product of the intercept dummy with the standard deviation of returns and with lagged AskHi-BidLo over price. Additional slope dummies were avoided to keep the number of predictor variables manageable. Because AskHi-BidLo increases with the amount of trade price variation during the day, it is used as a firm-specific proxy of asymmetric information and as a measure of information flow to the market.[27] AskHi-BidLo also increases with price impact. This variable is scaled, by price, by volume and by dollar volume, to provide three alternative information measures. It is assumed, for example, that the day's price range per unit trade volume increases as information asymmetry increases. Standardized volume on the NYSE is used as a proxy for the market-wide rate of information flow. The yield on a 6 month T-bill is used to measure inventory costs. The standard deviation of the S&P500 Index return and the corporate bond quality spread are used as proxies for market-wide risk. The one year to ten year government bond spreads are used to condition on the shape of the term structure.

TORQ transaction data provides the dependent variables. Long historical daily time series are available for the independent variables. Tables 9 and 10 provide descriptive statistics on the data.

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[27] AskHi-BidLo equals the spread on days with no trade. It will increase as share price moves throughout the day. Since AskHi-BidLo captures the range of price levels over the day, it will be large when there has been a large adjustment in share price.

5.2 Modeling Realized and Expected Spread and Price Impact

Euler equations (1) and (3) can be rearranged as follows:

$$1 = E_t\left\langle \beta \frac{U'(c_{t+1})}{U'(c_t)} \frac{P_{t+1}}{P_t + \left[s_t + E_t(x_t \delta_t)\right]} \right\rangle - E_t\left\langle \beta \frac{U'(c_{t+1})}{U'(c_t)} \frac{(s_{t+1} + x_{t+1}\delta_{t+1})}{P_t + \left[s_t + E_t(x_t \delta_t)\right]} \right\rangle \quad (6)$$

Equation (6) separates out the terms showing interaction between marginal utility and the mid-quote price and between marginal utility and the transaction uncertainty variables. The transaction uncertainty variables appear both as random variables and as conditional expected values. The MSE correction factors of Table 6 use both $s_{t+1} + x_{t+1}\delta_{t+1}$ and $E_t(s_t + x_t\delta_t)$. Different estimation techniques are used for $s_{t+1} + x_{t+1}\delta_{t+1}$ and $E_t(s_t + x_t\delta_t)$.

5.2.a Ordered Probit Estimation of $s + x\delta$

s_{t+1} and $x_{t+1}\delta_{t+1}$, are modeled so that corrected returns reflect any covariance between marginal utility and transaction uncertainty. An ordered probit model is used to capture the discrete nature of spread and price impact.[28] The ordered probit models explicitly account for both discreteness, such as tick size, and for the ordering of the discrete values of the dependent variable.

One consideration in implementing the model is the treatment of the dependent transaction data. Any firm on a given day could have no transactions, one transaction or multiple transactions. Days with no

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[28] Half-spread, s, varies by \$1/16, with the majority of observations in the sample falling between \$1/16 and \$3/16. Half-spread plus price impact, $s + x\delta$, mostly varies by \$1/16 because orders tend to transact completely at the quote or completely at prices inferior to the quote. The majority of observations in the sample fall between \$1/16 and \$5/16.

transaction are ignored, but an issue in the estimation is how to handle several observations per day of $s+x\delta$ when there is only one daily observation of the independent variables. A daily *dependent* time series is constructed by randomly choosing one transaction from those on any given day. Random selection is consistent with the Euler equation model, where we may consider the trader to be placing an order at a random time within a given day, assuming that a day is effectively an instant in time, relative to the trading period.

Table 11 shows regression results for panel data estimates of the models for s and δ. Results shown are the mean for 40 estimation runs. Each run used a different random selection of the dependent variables.

All but four of the 20 regressors are significant in the ordered probit regression for the spread. Only seven regressors are significant in the ordered probit regression for the price impact. The randomly drawn observations of the dependent variable reflect transaction uncertainty. Spread is much more certain than price impact. It is the proxies for information asymmetry that are insignificant for the spread regression, whereas they are significant for the price impact regression. The yield spreads and volume measures are insignificant for the price impact model. Lagged regressors have less power to explain price impact than spread.

5.2.b Weighted Least Squares Estimation of $E_t(s+x\delta)$

The conditional expectation, $E_t(s_t+x_t\delta_t)$, reflects information known to investors at the time orders are being placed for trade at time t. For example, the quoted spread is known and general conditions in the market may be used to condition expectations about the likely price impact for an order under consideration. Discreteness is less of an issue because the expectation can take on many values, even if $s+x\delta$ has only a few distinct values. The following model was estimated, using a weighted regression of panel data:

$$\bar{y}_t = X_t'\beta + e_t \tag{7}$$

For each firm, the dependent variable $\bar{y}_t$ is the mean of all day t observations of s or $x\delta$. Since there can be a different number of transactions from day to day, the number of observations of y_t in $\bar{y}_t$ varies, making $\bar{y}_t$ inherently heteroskedastic. The heteroskedasticity is corrected by weighting both $\bar{y}_t$ and X_t by the square root of the number of observations in day t. Days with no transactions are ignored. Table 12 shows regression results for panel data estimates of the models for $E_t(s)$ and for $E_t(\delta)$.

Ten of the 20 regressors are significant in the ordered weighted least squares regression for the spread. Ten also are significant in the weighted least squares regression for the price impact. The daily average observations of the dependent variable do not reflect transaction uncertainty as much as in the random draw case. Spread is not much more certain than price impact. Proxies for information asymmetry are insignificant for the spread regression here, as in the ordered probit case. Market-level regressors are also less significant here. As in the ordered probit case, the proxies for information asymmetry are significant for the price impact regression. The yield spreads and volume measures here are significant for the price impact model. Perhaps the average price impact is reflected better in the turnover measures for the day, whereas the price impact at a given time within the day can vary enough that average conditions explain little.

5.3 Returns in the Presence of Transaction Uncertainty: Fitted Data

These regression models make it possible to create data sets of daily spread and price impact for any NYSE common stock. The spread and price impact models are fitted to historical, end-of-month observations of CRSP daily time series. On days with no trade, spread and price impact can be observed directly in the CRSP data, rather than fitted from the models. AskHi-BidLo equals quoted closing bid/ask spread and price impact is zero on days with no trade. Given CRSP prices and these fitted spread and price impact variables, the correction factors of Table 6 are used to construct long, historical data sets of corrected returns. In rare instances, the fitted values of spread or price impact are large compared to the range found in the TORQ data. In these cases, the fitted values are capped below the TORQ maximum.[29]

Table 13 shows summary statistics for BA, FDX, TXI and the mean statistics for all 98 NYSE firms used from the TORQ database. Fitted data covers the period from July, 1962 to December, 2000. Mean monthly differences between CRSP and corrected returns are all economically significant: 0.64% for BA, 0.65% for FDX and 1.01% for TXI. These values are quite close to the mean values for the TORQ data shown in Table 1.

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[29] Less than 0.4% of fitted values of proportional spread and price impact exceed the maximum observed in TORQ. For proportional spread greater than 0.125 (the TORQ maximum), spread is set equal to P/10. For proportional price impact greater than 0.25 in magnitude (the TORQ maximum), the magnitude of price impact is set equal to P/6.

6 Is Transaction Uncertainty Risk Systematic?

Is transaction uncertainty systematic, in the sense of helping to explain cross-sectional differences in expected stock returns?[30] The literature on the bid/ask spread argues that spread includes components due to inventory risk and private information.[31] Similarly, the literature on price impact argues for components due to inventory and to information.[32] Since inventory and information seem like systematic factor risks, spread and price impact may be affected by factor risks. If so, then factor-induced cross-sectional differences in spread plus price impact may be a component of cross-sectional differences in expected stock returns.

Nonzero covariance between a transaction uncertainty variable and a systematic factor risk is a necessary condition for "priced" transaction uncertainty. Establishing this correlation is difficult because the time frame for the collection of most factor data (monthly or quarterly) is much longer than for stock transaction data, such as bid/ask spread (daily or intra-daily). Section 6.1 presents covariance estimates using monthly returns. The spread and price impact models of Section 5 are used to correct historical monthly CRSP returns for a large number of NYSE stocks. In effect, the time frame of the transaction uncertainty variables is adapted to that of the factors. In a complementary approach, Section 6.2 presents correlation estimates using daily returns. Risk factors that are observable on a daily basis,

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[30] Amihud and Mendelson (AM 1986) is a seminal contribution in this area. They show that clienteles form in which investors with longer expected holding periods are compensated for investing in assets with higher expected liquidity costs. Investors with shorter expected holding periods prefer assets with lower expected liquidity costs. AM do not look at the effects of uncertainty in liquidity costs.

[31] See for example Huang and Stoll (1996).

[32] See for example Kraus and Stoll (1972).

along with the observable transaction uncertainty variables of the TORQ data set are used. Finally, Section 6.3 outlines a possible approach for direct measurement of the covariance of transaction uncertainty variables with risk factors.

6.1 The Covariances of Risk Factors with Closing-Price and Corrected Returns on a Monthly Horizon

Given the data set of corrected monthly returns, three approaches are used to test for correlation with factors.

First, does the covariance of risk factors with CRSP returns differ from the covariance of factors with corrected returns?

$$\begin{aligned} H_{01}: &\quad Cov(risk\ factor, R_{CRSP} - Correction * R_{CRSP}) = 0 \\ H_{A1}: &\quad Cov(risk\ factor, R_{CRSP} - Correction * R_{CRSP}) \neq 0 \end{aligned}$$

The results of testing H_{01} are important because rejection would suggest that tests sensitive to the covariance of factors with returns will give different results when CRSP returns are used as a proxy, in place of realizable returns.

Second, rejection of H_{01} would not establish that transaction uncertainty variables are correlated with risk factors because a simple scale transformation of CRSP returns will change the covariance with a factor. Consider, for example, a correction factor equal to a constant, α :

$$\begin{aligned} Cov(\text{risk factor}, R_{CRSP} - \alpha * R_{CRSP}) &= Cov\left[\text{risk factor}, (1-\alpha) * R_{CRSP}\right] \\ &= (1-\alpha) * Cov(\text{risk factor}, R_{CRSP}) \\ &\neq 0 \quad \text{for} \quad \alpha \neq 1 \end{aligned}$$

If the correction factor indeed were simply a constant, then there could be significant *economic* implications for measurement of returns, but it is unlikely that statistical tests using corrected returns would differ *qualitatively* from tests using CRSP returns. To eliminate possible scale effects and to help focus on any impact of the return correction on correlation with factors, corrected returns are regressed on CRSP returns. The regression residuals should be purged of any information about the correlation of CRSP returns with factors. These residuals are tested for covariance with risk factors. Table 14 lists the average coefficients from firm-by-firm estimates of regression (8).

$$R_{Corrected} = a + b * R_{CRSP} + e \tag{8}$$

The average beta and R-squared both are close to one.

Rational investors weigh the covariance of stocks in their portfolio with systematic risk factors. A non-zero covariance of the regression residuals with risk factors would suggest that investors demand a return premium in the face of transaction uncertainty. Is the covariance of risk factors with these regression residuals zero?

$$H_{02}: Cov(\text{risk factor}, e) = 0 \qquad H_{A2}: Cov(\text{risk factor}, e) \neq 0$$

Third, another concern about the effects of the correction factors remains. The spread and price impact regression models of Section 5 include risk factors as explanatory variables. Suppose, however, that transaction uncertainty variables are uncorrelated with risk factors. Then there ought to be zero correlation between risk factors and any *linear* function of fitted values of the transaction uncertainty variables. Since the return correction factors are *non-linear* functions of the transaction uncertainty variables, the estimated covariance of risk factors with returns may change even when transaction uncertainty variables are uncorrelated with risk factors.[33] To address this concern, a form of the correction factors is constructed that is linear in proportional liquidity costs. Each of the four correction factors from Table 6 is expanded in powers of spread plus price impact. Keeping only the terms with the first power of liquidity cost, a common linearized correction factor is obtained:

$$C_{\text{Linear}} = 1 - \frac{(s+\delta)_t}{P_t^{\text{Close}}} - \frac{(s+\delta)_{t+1}}{P_{t+1}^{\text{Close}}}$$

The covariance tests are repeated using linearly-corrected returns. Table 14 lists the average coefficients from firm-by-firm regressions of the linearly-corrected returns on CRSP returns.

$$R_{Linear} = a_L + b_L * R_{CRSP} + e_L$$

The residuals from these regressions ought to be independent of scale effects and purged of any information about the correlation of CRSP returns with factors. Is the covariance of risk factors with these regression residuals zero?

$$H_{03} : Cov(\text{risk factor}, e_L) = 0 \qquad H_{A3} : Cov(\text{risk factor}, e_L) \neq 0$$

A number of different risk factors are used in these monthly tests. The S&P500 value-weighted return (constructed using closing-price returns) and an equal-weighted portfolio of 639 randomly chosen NYSE common stocks (constructed using corrected returns) are motivated by

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[33] Correlations of risk factors with corrected returns are unaffected if the transaction uncertainty variables are independent of risk factors.

single factor models, such as the CAPM.[34] The quality yield spread between BAA and AAA corporate bonds, the long term yield spread between 10 year and one year government bonds, and short term yield spread between one year and three month treasury bills are chosen to explain the magnitude of firms' expect cash flows and the rates used to discount them.[35] The quality spread factor conditions on the business cycle and proxies for bankruptcy risk. The two term spreads are meant to capture the shape of the term structure of interest rates. Because investor response to news makes price changes reflect *changes* in expectations, factor risk may also involve innovations to macroeconomic variables. The standardized innovations to the quality spread and term structure variables give three additional traded risk factors.

6.1.a Test Results

Table 15 gives results of testing for covariance of monthly returns with these five traded risk factors and three innovation risk factors. The first column of Panel A shows the covariance between CRSP returns and each of the factors. Only the covariance of CRSP returns with innovations in the long yield spread is zero.

H_{01} asserts that correcting returns to account for transaction uncertainty has no effect upon measured covariance with risk factors. For all eight factors, Panel A shows the difference in covariance when returns are corrected to be significant. Using paired t-tests, H_{01} is rejected at the 1% level for each factor. Further, the effect of the return correction does not seem to be a simple scale effect because the covariance estimate increases for some factors and decreases for others.

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[34] See for example Sharpe (1964).

[35] Chen, Roll and Ross (1986) propose both traded and non-traded factors. Only traded factors are tested here.

H_{02} asserts that projecting corrected returns onto CRSP returns leaves a residual component of corrected returns that is uncorrelated with factor risks. H_{03} asserts that projecting linearly-corrected returns onto CRSP returns leaves a residual component of linearly-corrected returns that is uncorrelated with factor risks. Panels A and B show the results of paired t-tests. H_{02} and H_{03} are rejected at the 1% level for all factors.

Conditional tests also reject all three hypotheses. The dividend yield on the S&P500 value-weighted index and the yield on the 6 month treasury bill are used as the public information set.

Because closing-price returns are used to calculate the S&P500 portfolio return, there are two concerns with using it as a risk factor in this context. Transaction uncertainty error in the returns of the index stocks may induce a spurious difference in the covariances measured with CRSP and with corrected returns. Further, if some of the stocks being tested for covariance with this factor happen to be in the index, then again perhaps a spurious difference in the covariances measured with CRSP and with corrected returns is induced The equal-weighted portfolio deals with both of these concerns. It is constructed using corrected returns and a stock is not included in the portfolio when that stock is being tested for covariance with the portfolio. Both hypotheses are rejected using either portfolio, so the concerns seem not to be an issue.

The test results in Table 15 are consistent with systematic transaction uncertainty risk and suggest that using monthly CRSP returns in place of corrected returns can have a strong effect on measured covariance with risk factors.

6.2 Correlations of Risk Factors with Closing-Price, Buy-then-Sell and Corrected Returns on a Daily Horizon

Consider CRSP, buy-then-sell and corrected returns. CRSP closing-price returns do not consistently reflect trading costs. Buy-then-sell returns are one example of returns that do. Corrected returns make CRSP returns more consistent with trading costs. Are covariances with risk factors affected if return calculations do not treat transaction uncertainty variables in a way that is consistent with a realizable investment strategy? If transaction uncertainty is a systematic risk, then covariances between risk factors and returns *will* depend upon the way that trading costs are treated in return calculations.

This section addresses a concern associated with measuring the covariance between risk factors and returns when corrected returns proxy for buy-then-sell returns. Suppose that transaction uncertainty variables are correlated with risk factors, but with a time variation that the constant coefficient models of Section 5 fail to capture. If so, the fitted values of spread and price impact will have time-varying errors that may affect the estimated correlation of risk factors with corrected returns.

Tests in this section cover the brief period of the TORQ data set, attempting to minimize the time variation of the model coefficients. Closing-price, buy-then-sell and corrected returns are calculated for 97 NYSE common stocks in TORQ. Mid-quote price, spread and price impact are observable in the TORQ data set, so buy-then-sell returns are not modeled. The closing-price and buy-then-sell returns use the mid-quote price, spread and price impact of the last transaction for each day. A buy price is calculated as mid-quote price plus spread plus price impact; a sell price as mid-quote price minus spread minus price impact. For each stock and for each day, the buy/sell order direction of the closing transaction is observable. Corrected returns use the closing-price return times a correction factor calculated using *fitted* values of spread and price impact. Daily observations are available for the same risk factors, except the equal weighted portfolio, as in the monthly-horizon tests of Section 6.1.

Factor loading estimates are made for closing-price, buy-then-sell and corrected returns using individual firm time series. If transaction uncertainty does *not* affect the covariances of returns with risk factors, then the cross-sectional correlation of the factor loadings for closing-price and buy-then-sell returns should be close to 1. If the correction process does *not* induce spurious changes in the factor loadings, then the cross-sectional correlation of the factor loadings for closing-price and corrected returns also should be close to 1. If both sets of correlations are significantly less than 1, then there is further evidence consistent with transaction uncertainty as a systematic risk. If only the correlation of closing-price and corrected loadings is significantly less than 1, then there is evidence consistent with the idea that the correction process creates a spurious correlation with factors.

$$H_{04} : Correlation\left(\text{factor loading}_{\text{return}}^{\text{Closing-price}}, \text{factor loading}_{\text{return}}^{\text{Transaction cost-consistent}}\right) = 1$$

$$H_{A4} : Correlation\left(\text{factor loading}_{\text{return}}^{\text{Closing-price}}, \text{factor loading}_{\text{return}}^{\text{Transaction cost-consistent}}\right) < 1$$

Table 16 presents estimates of the cross-sectional correlation between factor loadings for closing-price and for buy-then-sell daily returns, for closing-price and for corrected daily returns, and for corrected and buy-then-sell returns. Using a bootstrapped distribution of the estimates, 95% confidence intervals for the correlations are also presented.

H_{04} asserts that, whether or not return calculations account for liquidity costs, cross-sectional differences in estimated factor loadings are maintained. This appears not to be the case when either closing-price or corrected returns proxy for transaction cost-consistent returns. For all seven daily factors, the correlation between factor loadings is less than one at the 5% level: for closing-price return and buy-then-sell return factor loadings, for closing-price return and corrected return factor loadings, and for buy-then-sell return and corrected return factor loadings.

The test results in Table 16 are consistent with systematic transaction uncertainty risk. Results suggest that using closing-price returns in place of realizable returns can affect tests that are sensitive to the relative magnitude of factor loadings. It appears that correcting returns to minimize mean square measurement error may not create a better proxy for this application. An alternative correction, tailored to this situation, may perform better.

6.3 Measuring Covariance of Intrinsic Returns with Risk Factors

One can interpret the mid-quote price as the "intrinsic" underlying value of the stock and P_{t+1}/P_t as the intrinsic value return. Appendix D, Estimation of Transaction Uncertainty Risk, shows that the covariance of CRSP closing-price returns with risk factors equals the covariance of intrinsic value returns with risk factors.

$$Cov\left(risk\ factor_{t+1}, R_{t+1}^{CRSP}\right) = Cov\left(risk\ factor_{t+1}, R_{t+1}^{intrinsic}\right) \qquad (9)$$

A CRSP price equals the mid-quote price plus or minus a spread plus price impact:

$$P^{CRSP}_{Buy-initiated} = P + (s + \delta) \quad \text{or} \quad P^{CRSP}_{Sell-initiated} = P - (s + \delta)$$

Because covariance is linear in its arguments, the random buy/sell pattern in CRSP prices washes out any covariance with risk factors of the spread plus price impact component of CRSP prices, while leaving the covariance with the intrinsic value. This result is interesting in that CRSP returns may be useful for addressing certain research questions that focus on intrinsic value.[36]

CRSP returns can be used to measure the covariance with risk factors of the mid-quote component of returns. Is there other price data that would be useful to measure the covariance with risk factors of the transaction uncertainty component of returns? Appendix D shows that the covariance of risk factors with returns calculated from the AskHi and BidLo time series will depend upon the covariance of the risk factors with the transaction uncertainty. Use of the AskHi and BidLo time series to directly estimate the systematic component of transaction uncertainty is left to a future paper.

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[36] However, the mid-quote price series is affected by transaction uncertainty because investors adjust the size and timing of orders to trade-off between transaction uncertainty risk and information flow risk. An "intrinsic" value cannot really be separated from the effects of trading frictions.

7. The Effects of Systematic Risk, Residual Risk, Size, Bid/Ask Spread and Price Impact on Asset Returns

Asset types can differ by the liquidity cost component of the return expected when that asset type is used in a given trading strategy. Does expected liquidity affect the investment strategy used with a particular asset? Does liquidity cost help explain cross-sectional differences in the expected returns that are observed in markets?

One might argue that investors are interested in their returns net of transaction costs. Somebody wanting to invest funds over night is unlikely to purchase real estate because of the high fees and high expected liquidity cost. But how does the expected return on real estate compare with that on an asset with low liquidity cost? If these expected returns differ, is the difference related to costs?

One might argue further that the price of an illiquid asset will fall until some investors find the asset's return to be attractive relative to the return on a comparable liquid asset. How might one formalize these arguments and do the intuitions behind them reflect equilibrium market behavior? Amihud and Mendelson (hereafter AM) have made major contributions to these questions through their studies of the premium offered by assets of different liquidity and the relation between liquidity costs and investors' holding periods.

In their 1986 paper, AM model asset payoffs as a fixed cash flow per unit of time, $\$d_t$. Assets differ in the size of this cash flow and in the proportional liquidity cost, s_t/P_t, where s_t is the half-spread and P_t is the mid-quote price.[37] Since the model abstracts from differences in risk, all investors choose a portfolio of assets that maximizes the present value of expected cash flows over their expected holding period. Investors differ by expected holding period, $1/\mu_j$. Differences in holding period mean that the per-period liquidity cost of the assets differs across investor types. Marginal liquidity costs are highest for investors with the shortest holding period; they end up being the marginal investors for the most liquid assets. AM's Proposition 1 predicts that investment-horizon clienteles result. Investors differing in expected holding period end up differing in the weights of their optimal portfolios. The most illiquid assets are held preferentially by investors with the longest expected holding period.

AM's model ignores price impact and transaction uncertainty, using spread as the total liquidity cost. Because they abstract from uncertainty in the cash flows or spreads, they do not use a time subscript. AM's one-period return realized on asset i by investor j is: [38]

$$r_i = \frac{\$d_i - 2\mu_j s_i}{P_i + s_i}$$

Since AM are considering the transaction price from buying the asset, our treatment includes price impact in realized returns:

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[37] Where possible, the symbols from this paper are used in place of those of Amihud and Mendelson.

[38] In AM's notation, the realized return is:

$$r_i = \left(d_i - \mu_j S_i\right)/V_i \qquad \text{where } S_i \text{ is the relative spread and } V_i \text{ is the ask price}$$

This is not quite a "realized" return because the numerator uses the expected holding period, rather than the realized holding period.

$$r_{i,t+1} = \frac{\$d_t - \mu_i\left[(s+\delta)_{i,t+1} + (s+\delta)_{i,t}\right]}{P_t + (s+\delta)_{i,t}}$$

Because low liquidity assets are most valued by all investors, in equilibrium expected realizable returns increase with liquidity cost. In this equilibrium, investors with longer (shorter) horizons expect to obtain higher realized returns by holding securities with higher (lower) proportional bid/ask spreads.

Since holding periods are not observable, AM look at the one-period return gross of the sell-time liquidity cost:[39]

$$r_{i,t+1}^{\text{Gross}} = \frac{\$d_t}{P_t + (s+\delta)_{i,t}}$$

AM's Proposition 2 predicts that expected gross returns, over any holding period, increase with percentage liquidity costs, but that the increase gets smaller as costs get larger.

In their 1986 and 1989 papers, AM tested this increasing, concave return/spread relation. At that time, AM lacked data that would allow them to account consistently for transaction costs in returns. Using mean annual spreads and monthly CRSP closing-price returns as a proxy for gross returns, they obtained support for their predictions about returns observed in the market. Fitted values of spread and price impact now make it possible to revisit AM's tests using better proxies for gross returns. In this section, I will examine the

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[39] AM's Gross return is their realized return, less the liquidity cost term in the numerator:

$$r_i^{\text{Gross}} = d_i / V_i$$

sensitivity of AM's test results to their use of closing-price returns. I will focus primarily on the tests in AM's 1989 paper.

In the 1989 paper, AM test Merton's 1987 extension of the CAPM. Merton predicts that the expected return on an asset is an increasing function of systematic risk, residual risk and market capitalization, and is a decreasing function of the availability of public information about the asset. AM argue that correlations among the explanatory variables make it important to use all these variables when estimating the expected return model.

AM's return has the form of a buy-then-sell return in which the liquidity cost of the sell is prorated over the expected holding period. AM's gross return contains only the liquidity cost of entering a strategy. In this paper, we consider several alternative proxies. One, which we label as Gross return, is a one period buy-then-sell with no cost for selling. This is one measure of the one-period return to an infinite buy-and-hold in that buying costs are recognized, but selling costs are not. Corrected buy-then-sell returns are a one-period return in which both buying and selling are recognized. As a proxy that lies between these two extremes, a one-period measure of a six-period holding period is formed from a linear combination of five parts Gross return and one part corrected return.

AM use the returns for 49 portfolios of NYSE stocks. Stocks are sorted by lagged proportional bid/ask spread, then evenly allocated into seven portfolios. Each of the seven is split into seven sub-portfolios based on market beta. Table 18 shows the correlations among the factors in the Merton model and the returns on the 49 portfolios.

Correlations are shown for each return type over the entire data period, 1972/09 – 2000/12, and for the two half-periods.

The correlations of returns with the other variables are graphed in Figure 1. The correlations of returns with residual risk and with firm capitalization change from negative to positive as return type changes from CRSP to Gross to corrected. The correlations of returns with systematic risk, spread, and with the squares of spread and price impact all change from positive to negative as return type changes from CRSP to Gross to corrected. The correlation of returns with price impact becomes increasingly negative. It seems that the greater the weight of liquidity cost in the return measure, the more correlations with the other variables are shifted. That the sign of the correlations can change, depending upon the return proxy, leads us to expect differences in the sign of coefficients in the regression tests that follow.

The correlation of systematic risk with the other variables is small, except for the correlation with residual risk (positive) and with firm capitalization (negative). This suggests that firms with high systematic risk tend to have high total risk, and is consistent with small firms being risky. The correlation of residual risk with firm capitalization is about -13%, with spread is about 12% and with price impact is –10%. This suggests that non-market risks affect the components of liquidity cost, but that total liquidity cost is insensitive to non-market risks. Firm capitalization has strong, negative correlations with the linear and squared measures of spread and price impact. This suggests that regressions of returns on capitalization will get biased results if there is no conditioning on liquidity costs. Spread has a strong, positive correlation with price impact, suggesting that the two components of liquidity cost have common determinants.

7.1 Tests Using Alternative Measures of Return

AM's 1989 paper uses their 1986 model as a basis to test Merton's predictions. As a proxy for the availability of public information, AM use asset liquidity, measured as the mean annual bid/ask spread, scaled by monthly closing price. CRSP monthly returns proxy for gross returns. To measure systematic and residual risk, they use an equally-weighted portfolio of CRSP returns as their market portfolio.

AM find that expected returns depend positively on systematic risk and spread, but not significantly upon residual risk or size. The positive dependence of expected return on liquidity cost is support for the prediction of their Proposition 2. But are AM's results sensitive to the limitations of the data that was available to them?

7.1.a Analytical Behavior of Return Measures with Respect to Liquidity

Since AM predict a positive, concave relation between expected Gross returns, r^{Gross} and proportional liquidity costs, θ, we have the following hypotheses:

$$\text{H7.1: } \frac{dE_t\left(r^{\text{Gross}}\right)}{d\theta} \leq 0 \qquad \text{H7.1A: } \frac{dE_t\left(r^{\text{Gross}}\right)}{d\theta} > 0$$

$$\text{H7.2: } \frac{d^2E_t\left(r^{\text{Gross}}\right)}{d\theta^2} \geq 0 \qquad \text{H7.2A: } \frac{d^2E_t\left(r^{\text{Gross}}\right)}{d\theta^2} < 0$$

Appendix E looks at derivatives of expected returns with respect to proportional liquidity cost. For AM's Gross returns, the first derivative is positive only if the first derivative of *mid-quote* returns is positive and greater than Gross returns. The second derivative of Gross returns is negative only if the second derivative of *mid-quote* returns is less than twice the first derivative of Gross returns. That is, AM's liquidity-clientele effect holds for Gross returns only if mid-quote returns also increase with liquidity costs. In this sense, AM are predicting that mid-quote returns depend upon liquidity costs. Viewing mid-quote returns as intrinsic returns, this implies that intrinsic returns do not exist independent of market frictions.

On the other hand, if mid-quote returns are invariant with respect to liquidity costs, then expected Gross returns and expected buy-then-sell returns are shown to decrease with liquidity cost, the decrease getting smaller as costs get larger. This is the mirror image of the behavior predicted by AM and reflects the decrease in return that one would expect were trading costs to increase, all else equal. Thus, for there to be any evidence of a liquidity-clientele effect, expected mid-quote returns must increase with liquidity cost by enough to outweigh the mechanical decrease in returns due to the added liquidity cost.

Suppose CRSP returns are used as a proxy for Gross returns. Expected CRSP returns are shown to increase with proportional liquidity cost, *even if Gross returns do not.* Hence, tests using CRSP returns cannot distinguish between a possible liquidity-clientele effect and a mechanical increase in closing-price returns. With our data, this distinction can be made.

7.1.b Empirical Behavior of Return Measures with Respect to Liquidity

AM's 1989 tests are recreated, first using CRSP returns, and then using Gross and corrected buy-then-sell returns. Gross returns measure a buy-and-hold return with an infinite holding period. Monthly buy-then-sell returns measure a one-month buy-and-hold return.

Somewhere along this continuum lie realistic realizable returns for holding periods greater than one month. Both extremes and a linear combination with a six-month holding period (referred to hereafter as R6) are tested against the above hypotheses. As well, quarterly CRSP and buy-then-sell returns are tested to investigate whether or not the liquidity cost component of returns affects test results beyond a monthly holding period. The tests use three measures of liquidity: spread, price impact, and the squares of spread and price impact. Table 17 provides statistics on the data used. Table 19 gives test results - for monthly OLS regressions (panel A), for monthly GLS regressions (panel B), and for quarterly OLS and GLS regressions (panel C).

7.1.b.i Data Preparation

Following AM 1986, betas for individual firms are estimated for each return type.

$$R_i - R_{t\text{-}bill} = \alpha_i + \beta_i \left(R_M^{EW} - R_{t\text{-}bill} \right) + \varepsilon_i$$

The market model regressions use 60 months of excess returns and an equally-weighted market portfolio formed from the excess returns of all stocks in the sample,[40] except the particular stock being estimated. All stocks are divided equally among seven portfolios based upon proportional bid/ask spread. Each spread portfolio is divided equally among seven equally-weighted portfolios based upon beta. These portfolios use return observations 61 – T_j for each stock. Thus, a stock must have at least 61 months worth of observations to be included in the sample.

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[40] Because stocks differ in the period for which observations are available, the market portfolio is comprised of different numbers of stocks from month to month.

Portfolio market model regressions are then estimated using 60 months of excess returns and an equally-weighted market portfolio formed from the excess returns of all stocks in the sample.

$$R_p - R_{t\text{-}bill} = \alpha_p + \beta_p \left(R_M^{EW} - R_{t\text{-}bill} \right) + \varepsilon_p$$

The portfolio regressions determine the systematic and residual risks for each of the 49 portfolios. The average spread, price impact and market capitalization of each portfolio is then calculated. Overall, this two-stage estimation process uses the first 60 months of observations for each individual stock, then the next 60 months of observations for each portfolio. The subsequent tests use 340 months of observations (1972/09 - 2000/12) for each portfolio.

7.1.b.ii Econometric Considerations

For each return type - CRSP, Gross and corrected buy-then-sell - three regressions are run. Portfolio excess returns are regressed on systematic risk, β, residual risk, σ, log of firm capitalization, $Size$, and year dummies, DY. The three regressions differ in the regressor used as a measure of liquidity: spread, s/P^{CRSP}, both spread and price impact, δ/P^{CRSP}, or spread, price impact and their squares. Each regression uses pooled data and has year dummy regressors.

$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 \text{Size}_p + \gamma_4 s/P_p + \varepsilon_p$$
$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 \text{Size}_p + \gamma_4 s/P_p + \gamma_5 \delta/P_p + \varepsilon_p$$
$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 \text{Size}_p + \gamma_4 s/P_p + \gamma_5 \delta/P_p + \gamma_6 \left(s/P_p \right)^2 + \gamma_7 \left(\delta/P_p \right)^2 + \varepsilon_p$$

The fitted values of half-spread and price impact likely are measured with error. Since half-spread and price impact appear as regressors, measurement error induces an errors in variables problem; the regressors and the regression error term will be contemporaneously correlated. Further, Gross and buy-then-sell returns are calculated using fitted values of half-spread and price impact. When these return types appear as the dependent variable in the regressions, the correlation of the regressors and the error term becomes complicated. The OLS estimation process is meant to separate variation in the dependent variable that is due to the regressors from variation that is due to the error term. In an errors in variables situation, OLS estimation cannot clearly separate the two sources of variation in the dependent variable. As a result, OLS coefficient estimates are biased, even asymptotically, and the variance of the error term is underestimated. AM's tests using average annual spread also suffer from measurement error.

Let $X = X^* + \Delta$ be the measured regressors, such that $Q^* = \text{plim}\frac{1}{T}X^{*\prime}X^*$ and $\Sigma_{\Delta\Delta}$ is the variance/covariance matrix of the measurement errors. Then:

$$\text{plim}\, b = \left[Q^* + \Sigma_{\Delta\Delta}\right]^{-1} Q^* \beta$$

One way to reduce the estimation bias is to reduce the variance of the error terms. Assuming measurement errors to be uncorrelated in the cross-section, the spread-beta portfolios used in the tests diversify away much of the error term variance. This diversification benefit occurs both for the liquidity cost regressors and when estimating the systematic and residual risk regressors.

7.1.b.iii Empirical Results

Repeating the treatment used by AM, the GLS regressions account for possible heteroskedasticity and correlations across portfolios. Return autocorrelation is assumed to be insignificant. This means that the variance/ covariance matrix of the true residuals, V, is assumed to be block diagonal. The common diagonal block, U, is the 49 x 49 positive definite matrix formed using the 340 month-by-month (113 quarter-by-quarter) OLS residuals. The data is then pre-multiplied by D, where $D' * D = V^{-1}$ is the Cholesky decomposition of V^{-1}. The GLS estimates use the transformed data.

Table 19 presents the regression results for the tests that follow.

Pooled Regression 1

Regression 1 recreates the test performed by AM in their 1989 paper. Using monthly CRSP returns, OLS and GLS coefficients have the same signs and similar magnitudes to those obtained by AM. The OLS and GLS coefficients for proportional spread both are positive and significant. Looking at alternative measures of return, the OLS spread coefficients for Gross, R6 and buy-then-sell returns are all negative and significant. For all regressions and all return types, the coefficients for measures of liquidity cost get progressively more negative going from CRSP returns to Gross returns to R6 returns to buy-then-sell returns. The GLS spread coefficients for Gross, R6 and buy-then-sell returns are insignificant. Neither the OLS nor the GLS results can reject the null hypothesis that Gross returns decrease with increasing spread. The progressively more negative spread coefficient obtained as one switches from Gross returns to R6 returns to corrected buy-then-sell returns is consistent with Appendix E. A negative spread coefficient is also consistent with Brennan, Chordia and Subrahmanyam (BCS 1996).[41] The positive coefficient obtained using CRSP closing-price returns is consistent with both AM's liquidity clientele model and with return mismeasurement due to liquidity costs. The results using quarterly returns are consistent with those for monthly returns, except that significance levels generally are lower.

Pooled Regression 2

The sensitivity of AM's results to the choice of liquidity cost measure is tested in regression 2. AM use mean *annual* spread in their tests. It is likely that the correlation is low between this lagged mean annual spread and the actual end of month bid/ask spread. The fitted models for spread and price impact provide *daily* observations for use here, which should lead to more powerful tests. Price impact provides a second measure of liquidity cost. Moreover, price impact reflects the transaction uncertainty component of returns much more than bid/ask spread, which is known at the time of placing an order. Regression 2 uses end-of-holding period spread and price impact.

For each type of return, the signs of the spread coefficients match those of regression 1. The price impact coefficients are all negative and highly significant. The magnitude of the price impact coefficients increases moving from CRSP to Gross to R6 to buy-then-sell returns. The spread coefficient for CRSP returns is positive and for buy-then-sell returns is negative. For Gross and R6 returns, the spread coefficients lie between the positive CRSP value and the negative buy-then-sell value. One possibility is that the different return types have different weights

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[41] BCS use returns risk-adjusted by Connor and Korajczyk (CK in JFE 1988) principal component factors. Although preliminary tests find very different CK factors when buy-then-sell returns replace CRSP returns, the complex interaction of return mismeasurement on expected returns and factor estimates is difficult to analyze. Further study is required.

of the opposing effects of liquidity. In Gross returns, there is little liquidity cost in the return, so the liquidity-clientele effect dominates. In buy-then-sell returns, there is a large liquidity cost in the return, so the liquidity cost effect dominates. In any case, this regression gives no definitive evidence for or against AM's Proposition 2. This regression does give support for price impact as an important component of liquidity costs. That price impact coefficients are significantly negative for all return types, including CRSP returns, suggests that the transaction uncertainty inherent in price impact has a fundamentally different effect on expected returns than does the relatively certain bid/ask spread. This is particularly interesting given the high, positive correlation between spread and price impact.

Pooled Regression 3

AM's 1986 prediction of a *non-linear* relation between expected return and liquidity cost is tested in regression 3. In addition to spread and price impact, the squares of spread and price impact are added as regressors to capture any non-linear behavior.

As in regression 2, for each type of return, the price impact coefficients are all negative and highly significant. For the spread, only the CRSP return coefficient remains positive. The coefficients for Gross, R6 and buy-then-sell returns are all negative. For all three regressions, the t-statistics of the spread coefficients show a u-shaped pattern. The t-statistic is relatively high for CRSP and buy-then-sell returns and lower for Gross and R6 returns. This suggests a situation in which two opposing effects have different weights for the different return types.

The coefficient for squared spread is negative for CRSP returns and positive for the other return types. None of the GLS coefficients are significant. The squared price impact coefficients are all positive.

Regression 3 provides support for a non-linear relation between returns and the price impact component of liquidity cost. The evidence for a non-linear spread effect is weak, however. Using CRSP returns, the squared spread coefficient is indistinguishable from zero. Gross, R6 and buy-then-sell returns all have a convex relation with liquidity cost for both spread and price impact. Once one controls for non-linearity in the return-liquidity relation, and uses a return measure that treats liquidity cost consistently, as in Gross or buy-then-sell returns, there is little to no support for the hypotheses derived from AM's liquidity-clientele theory.

Systematic Risk, Residual Risk and Firm Capitalization

Regressions 1 through 3 give consistent results for systematic risk, residual risk and firm capitalization. Expected returns increase as systematic risk increases and decrease as residual risk or size increases.

7.2 Tests Using Fama-MacBeth Regressions

Chen and Kan (CK 1995) point out that AM's test procedure constrains the market risk premium to be constant over the entire 20 year test period. CK argue that the bid-ask spread may be correlated with time variation in the risk premium. Further, they find evidence that time variation in beta coefficients is correlated with the spread. CK recommend Fama-MacBeth (F-M 1973) regressions so that the risk premium is free to vary each month and the significance of the spread coefficient is not affected by any correlation between spread and time-series errors in beta estimation.

Eleswarapu and Reinganum (ER 1993) extend AM's data period by ten years and use a portfolio formation procedure that eliminates fewer small firms than with AM's procedure. Using Fama-MacBeth regressions, they are able to test for seasonal patterns in the return/liquidity cost relationship.

Using the portfolios formed following AM's procedures, the following Fama-MacBeth regressions are run:

$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 \text{Size}_p + \gamma_4 s/P_p + \varepsilon_p$$
$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 \text{Size}_p + \gamma_4 s/P_p + \gamma_5 \delta/P_p + \varepsilon_p$$
$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 \text{Size}_p + \gamma_4 s/P_p + \gamma_5 \delta/P_p + \gamma_6 (s/P_p)^2 + \gamma_7 (\delta/P_p)^2 + \varepsilon_p$$

Following ER, the regressions are run for all observations together and for the observations from each month. Table 20 lists the regression results for all observations (Panel A) and for January only (Panel B).

In F-M regression 1, the liquidity cost proxy is spread. For all observations, the spread coefficient is significant – positive for CRSP returns and negative for Gross, R6 and corrected returns. On a monthly basis, however, spread is indistinguishable from zero in each month for CRSP returns. For Gross, R6 and corrected returns, the spread coefficient is negative in each month and significant in most cases.

In F-M regression 2, the liquidity cost proxies are spread and price impact. For all observations with CRSP returns, the spread coefficient is positive, but insignificant and the price impact coefficient is negative, but insignificant. For Gross, R6 and corrected returns, both coefficients are negative and significant. On a monthly basis for CRSP returns, spread is indistinguishable from zero each month, and price impact is significant and negative only in December. For Gross, R6 and corrected returns, the spread coefficient is negative each month and significant in many cases. In contrast to the pooled regressions of Section 7.1, the price impact coefficient mostly is indistinguishable from zero.

In F-M regression 3, the squares of spread and price impact are added as regressors to test for a nonlinear return/liquidity cost relation. For all observations, the spread and price impact both are negative. For CRSP returns, only price impact is significant. For Gross, R6 and corrected returns, both are significant near the 5% level. Neither squared term is significant. On a monthly basis for CRSP returns, spread is indistinguishable from zero and price impact is negative and significant only in January. For Gross, R6 and corrected returns, the spread coefficient is negative and significant about half the time. The price impact coefficient is negative and significant only in January. The squared terms are mostly insignificant.

Summary and Comparison to Other Research on Returns and Liquidity Costs

Other research has obtained results inconsistent with AM's model. Chen and Kan (1995) use Fama-MacBeth (1973) regressions of monthly closing-price returns on beta and spread. They find a coefficient for spread that is positive, but indistinguishable from zero. Their results are consistent with liquidity cost having, at best, a second order effect on expected returns.[42] In Table 20, the spread coefficient is found to be significant using Fama-MacBeth regressions.

Eleswarapu and Reinganum (ER 1993) find a positive and significant return/spread relationship only in January. They suggest a not-yet-understood, pervasive January seasonal puzzle as the source of these results. In Table 20, the January spread coefficient is found to be insignificant for CRSP returns and significant, but negative, for corrected returns. No evidence is found to support monthly seasonality in the return/liquidity cost relation.

Brennan, Chordia and Subrahmanyam (BCS 1997) obtain a negative coefficient for proportional bid-ask spread in regressions of monthly, risk-adjusted returns on non-risk factors. They suggest that a combination of their regressors may proxy for lagged return. They then explain the negative coefficient as consistent with negative autocorrelation of returns induced by infrequent trading.

Suppose, however, that the negative return-liquidity cost relationship found here and in BCS is not illusory. It is not clear why investors would purchase high liquidity cost assets if they do not receive an offsetting compensation. Developing a model to replace AM's 1986 model would be a significant contribution to the Finance literature.

In summary, evidence is presented here that using some return alternatives that account consistently for liquidity costs gives markedly different results than those obtained using CRSP closing-price returns.

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[42] Vayanos (1998) constructs an equilibrium asset pricing model with proportional transaction costs. He finds that transaction costs have a small effect on asset prices because investors adjust their holding

Price impact is found to have a different effect upon expected returns than does spread. The return-liquidity relation seems to be non-linear.

AM's 1986 model is very important as an early work relating returns to an asset characteristic unrelated to firm payoffs. Empirical support in the literature is mixed. The empirical results presented here are preliminary. A more comprehensive investigation of the return/liquidity cost relationship is left to future research.

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period to offset the impact of costs. Vayanos also predicts the return/liquidity cost relation to be nonlinear and suggests that regressions of returns on liquidity cost include an interaction term for cost and risk.

8. Implications

The ability to use corrected monthly returns to account for transaction uncertainty has implications for practitioners and academics. Research findings in many areas of asset pricing could be re-evaluated using returns that are corrected to be a good proxy for the returns appropriate to each study. Asset pricing models can be re-interpreted from the perspective of priced transaction uncertainty risk. Some specific implications are listed below.

Return Variance Bounds

CRSP returns overstate the magnitude of realized returns. He and Modest (1995) and Luttmer (1996) estimate the impact of *pre-determined* transaction costs on variance bounds. The procedure in this study should allow the testing of variance bounds, accounting for possible correlations with market factors implicit in transaction uncertainty.

Time-varying Expected Returns

Since investors trade-off between payoff risk and transaction uncertainty risk, estimates of time-varying expected returns that do not account for this equilibrium behavior by investors may be inaccurate. Previous studies of the components of security returns may not account adequately for transaction uncertainty. For example, Conrad, Kaul and Nimalendron (CKN 1991) assume that returns have a component, due to bid/ask error, that is uncorrelated in the cross-section. Table 15 gives evidence that the return mismeasurement due to spread plus price impact covaries with systematic risk factors. This implies that CKN's assumption does not hold. They further assume that the bid/ask error term is contemporaneously uncorrelated with the expected return term. With corrected returns, both terms are functions of the current price, implying that this assumption does not hold.

Cross-autocorrelation in the Moments of Returns

Investors trade-off between payoff risk and transaction uncertainty risk. In response to systematic news, investors will trade less aggressively in large transaction uncertainty firms than in small because the marginal gains from faster trading will be offset by larger expected price impact. If firm size is negatively correlated with transaction uncertainty, as suggested by the correlations in Table 18, then the prices of large firms may well incorporate news more quickly. Thus, cross-autocorrelation in stock returns might result as investors exhibit cross-sectional differences in trading aggressiveness to match cross-sectional differences in transactional uncertainty in stocks.

Using Ask/Ask (or Bid/Bid) Returns

This study shows that using CRSP returns introduces an error in measuring returns that can affect test outcomes. Quote return series are often used to eliminate the effects of bid/ask bounce on time series tests. But Bid-bid or ask-ask returns do not reduce the magnitude of the return measurement error relative to using CRSP returns. For example, if one is using an ask-ask return series to proxy for buy-then-sell returns, then the return measurement error is:

$$
\begin{aligned}
R_{t+1}^{\text{Sell Buy}} - R_{t+1}^{\text{Ask Ask}} &\equiv \frac{P_{t+1}-\left(s_{t+1}+\bar{\delta}_{t+1}\right)}{P_t+\left(s_t+\bar{\delta}_t\right)} - \frac{P_{t+1}-s_{t+1}}{P_t-s_t} \\
&= \frac{-2\left[P_{t+1}-\left(s_{t+1}+\bar{\delta}_{t+1}\right)\right]s_t - P_{t+1}\left(\bar{\delta}_{t+1}+\bar{\delta}_t\right)}{\left[P_t+\left(s_t+\bar{\delta}_t\right)\right]\left(P_t-s_t\right)}
\end{aligned}
$$

It may be worthwhile to investigate in different test situations whether or not quote return series actually improve the conclusions drawn.

Day-of-the-Week Effects and Other Seasonalities

Seasonalities may reflect patterns in buying or selling near the close, rather than any real change in returns. It may be that apparent excess returns are artifacts of inconsistent return measurement. Chordia, Roll and Subrahmanyam (1999) find evidence of day-of-the-week seasonalities in various measures of market liquidity, including proportional spread.

Mutual Fund Performance Benchmarking

The performance of funds and their managers is of interest, both to academics and to investors. Transaction uncertainty seems especially relevant in this area, given that the focus of this paper is on the returns realizable by large investors. Not just individual returns, but indices might be corrected for benchmarking purposes.

9. Conclusions

CRSP monthly returns are economically and statistically far from buy-then-sell returns. CRSP gross returns are larger than buy-then-sell returns approximately 75% of the time. Using alternative data sets and both parametric and non-parametric tests, the hypothesis that CRSP returns and realizable buy-then-sell returns have the same distribution is strongly rejected.

A problem with correcting for deficiencies in CRSP returns has been the lack of transaction data over a long enough time period. This paper presents a means of modeling the time series of a firm's spread and price impact using CRSP volume, closing price, AskHi and BidLo. For any NYSE firm, fitted spread and price impact now can be constructed to cover the entire period over which daily CRSP data are available.

CRSP returns can be corrected over a long period (30 or more years) to provide a good proxy for realizable buy-then-sell returns. Corrected monthly returns are economically and statistically close to buy-then-sell returns. Using alternative data sets and both parametric and non-parametric tests, the hypothesis that corrected returns and realizable buy-then-sell returns have the same distribution cannot be rejected.

Amihud and Mendelson's tests of their investment horizon clientele model (1989) are found to be inconclusive in two ways. First, a spurious positive relation between expected CRSP returns and transaction costs means that AM's test results may be unrelated to a clientele effect. Second, a recreation of AM's tests finds that the results are sensitive to the choice of return proxy. The clientele model is not supported in the tests using three alternatives to closing-price returns.

Evidence suggests that transaction uncertainty risk is systematic. Separate calculations of the covariance of risk factors with CRSP returns and with corrected returns are done using TORQ data, simulated data and fitted data. In each case, parametric and non-parametric tests find CRSP covariances to differ from corrected covariances. Transaction uncertainty variables are found to covary with risk factors, when tested using either TORQ data or fitted data.

A theoretical result suggests that the covariance of CRSP returns with risk factors measures the covariance of midpoint "intrinsic" returns with risk factors.

In summary, transaction costs, as measured by spread and price impact, should be accounted for in tests that use a monthly return horizon.

List of References

Amihud, Y. and H. Mendelson, 1986, Asset pricing and the bid-ask spread, *Journal of Financial Economics* 17, 223-249.

Amihud, Y. and H. Mendelson, 1987, Trading mechanisms and stock returns: an empirical investigation, *Journal of Finance* 42, 533-553.

Amihud, Y. and H. Mendelson, 1989, The Effects of beta, bid-ask spread, residual risk, and size on stock returns, *Journal of Finance* 44, 479-486.

Andrews, D., 1991, Heteroskedasticity and autocorrelation consistent covariance matrix estimation, *Econometrica* 59, 817-858.

Bessembinder, H. and H. Kaufman, 1997, A comparison of trade execution costs for NYSE and NASDAQ-listed stocks, *Journal of Financial and Quantitative Analysis* 32, 287-310.

Best, M. and R. Grauer, 1991, On the sensitivity of mean-variance efficient portfolios to changes in asset means: some analytical and computational results, *Review of Financial Studies* 4, 315-342.

Blume, M. and R. Stambaugh, 1983, Biases in computed returns: an application to the size effect, *Journal of Financial Economics* 12, 387-404.

Brennan, M., T. Chordia and A. Subrahmanyam, 1997, A re-examination of some popular security return anomalies, Working Paper.

Brennan, M. and A. Subrahmanyam, 1995, Investment analysis and price formation in securities markets, *Journal of Financial Economics* 38, 361-381.

Brennan, M. and A. Subrahmanyam, 1996, Market microstructure and asset pricing: on the compensation for illiquidity in stock returns, *Journal of Financial Economics* 41, 441-464.

Britten-Jones, M., 1999, The sampling error in estimates of mean-variance efficient portfolio weights, *Journal of Finance* 54, 655-671.

Campbell, J., A. Lo and C. MacKinlay, *The Econometrics of Financial Markets*, Princeton: Princeton University Press, 1997.

Chan, L. and J. Lakonishok, 1995, The behavior of stock prices around institutional trades, *Journal of Finance* 50, 1147-1174.

Chen, N. and R. Kan, 1995 ,Expected return and the bid-ask spread, Working Paper.

Cheng, M. and A. Madhavan, 1994, In search of liquidity: block trades in the upstairs and downstairs markets, NYSE Working Paper 94-02.

Chordia, T., R. Roll and A. Subrahmanyam, 1999, Market liquidity and trading activity, Working Paper.

Cohen, K., S. Maier, R. Schwartz and D. Whitcomb, 1978, The returns generation process, returns variance, and the effect of thinness in securities markets, *Journal of Finance* 33, 149-167.

Connor, G. and R. Korajczyk, 1988, Risk and return in an equilibrium APT application of a new test methodology, *Journal of Financial Economics* 21, 255-289.

Conrad, J. and G. Kaul, 1993, Time-variation in expected returns, *Journal of Business* 61, 409-425.

Conrad, J., G. Kaul and M. Nimalendron, 1991, Components of short-horizon individual security returns, *Journal of Financial Economics* 29, 365-384.

Conrad, J. and G. Kaul, 1993, Long-term market overreaction or biases in computed returns, *Journal of Finance* 48, 39-63.

Constantinides, G., 1982, Intertemporal asset pricing with heterogeneous consumers and without demand aggregation, *Journal of Business* 55, 253-268.

Copeland, T. and D. Galai, 1983, Information effects on the bid-ask spread, *Journal of Finance* 38, 1457-1469.

Dennis, P. and J. Weston, 2001, Who's informed? An analysis of stock ownership and informed trading, Working Paper.

Easley, D. and M. O'Hara, 1987, Price, trade size, and information in securities markets, *Journal of Financial Economics* 19, 69-90.

Eleswarapu, V. and M. Reinganum, 1993, The seasonal behavior of the liquidity premium in asset pricing, *Journal of Financial Economics* 34, 373-386.

Engle, R. and J. Lange, 1997, Measuring, forecasting and explaining time varying liquidity in the stock market, NBER Working Paper No. W6129.

Fama, E. and J. MacBeth, 1973, Risk, return, and equilibrium: empirical tests, *Journal of Political Economy* 71, 607-636.

Fact Book for the year 1997, NYSE, New York, 1997.

Foster, F. and S. Viswanathan, 1990, A theory of the intraday variations in volume, variance and trading costs in securities markets, *Review of Financial Studies* 3, 593-624.

Garfinkel, J. and M. Nimalendran, 2000, Market structure and trader anonymity: an analysis of insider trading, Working Paper.

Greene, W., *Econometric Analysis*, 3rd ed. New Jersey: Prentice Hall, 1997.

Hansen, L., 1982, Large sample properties of the generalized method of moments estimators, *Econometrica* 50, 1029-1054.

Hasbrouck, J., G. Sofianos and D. Sosebee, 1993, New York Stock Exchange systems and trading procedures, NYSE Working Paper 93-01.

Hasbrouck and D. Sosebee, 1992, Orders, trades, reports and quotes at the New York Stock Exchange, NYSE Working Paper 92-01.

Hausman, J., A. Lo and C. MacKinlay, 1992, An ordered probit analysis of transaction stock prices, *Journal of Financial Economics* 31, 319-379.

He, H. and H. Mamaysky, 2001, Dynamic trading policies with price impact, Yale ICF Working Paper 00-64.

He, H. and D. Modest, 1995, Market frictions and consumption-based asset pricing, *Journal of Political Economy* 103, 94-117.

Hong, H. and S. Rady, 1999, Strategic trading and learning about liquidity, Working Paper.

Huang, R. and H. Stoll, 1996, Dealer versus auction markets: a paired comparison of execution costs on NASDAQ and NYSE, *Journal of Financial Economics* 41, 313-357.

Huang, R. and H. Stoll, 1997, The components of the bid-ask spread: a general approach, *Review of Financial Studies* 10, 995-1034.

Jegadeesh, N., 1990, Evidence of predictable behavior of security returns, *Journal of Finance* 45, 881-898.

Jegadeesh, N. and S. Titman, 1993, Returns to buying winners and selling losers: implications for stock market efficiency, *Journal of Finance* 48, 65-91.

Karpoff, J., 1987, The relation between price change and trading volume: a survey, *Journal of Financial and Quantitative Analysis* 22, 109-126.

Kavajecz , K., 1999, A specialist's quoted depth and the limit order book, *Journal of Finance* 54, 747-771.

Keim, D. and A. Madhavan, 1997, Transaction costs and investment style: an inter-exchange analysis of institutional equity trades, *Journal of Financial Economics* 46, 265-292.

Kraus, A. and H. Stoll, 1972, Price impacts of block trading on the New York Stock Exchange, *Journal of Finance* 27, 569-588.

Kyle, A., 1995, Continuous auctions and insider trading, *Econometrica* 53, 1315-1336.

Lee, C. and M. Ready, 1991, Inferring trade direction from intraday data, *Journal of Finance* 46, 733-747.

Lesmond, D., C. Trzcinka and J. Ogden, 1999, A new estimate of transaction costs, *Review of Financial Studies* 12, 1113-1141.

Lo, A. and A. MacKinlay, 1990, When are contrarian profits due to stock market overreaction?, *Review of Financial Studies* 3, 175-208.

Lo, A. and J. Wang, 2000, Trading volume: definitions, data analysis, and implications of portfolio theory, *Review of Financial Studies* 13, 257-300.

Luttmer, E., 1996, Asset pricing in economies with frictions, *Econometrica* 64, 1439-1467.

Madhavan, A. and M. Cheng, 1996, The upstairs market for large block transactions: analysis and measurement of price effects, *Review of Financial Studies* 9, 1-36.

Madhavan, A. and M. Cheng, 1997, In search of liquidity: block trades in the upstairs and downstairs markets, *Review of Financial Studies* 10, 175-203.

Merton, R., 1987, A simple model of capital market equilibrium with incomplete information, *Journal of Finance* 42, 483-510.

Newey, W. and K. West, 1987, A simple, positive definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703-708.

O'Hara, M., 1999, Making market microstructure matter, *Financial Management* 28, 83-90.

Pagano, M. and A. Röell, 1996, Transparency and liquidity: a comparison of auction and dealer markets with informed trading, *Journal of Finance* 51, 579-611.

Roll, R., 1984, A simple implicit measure of the effective bid-ask spread in an efficient market, *Journal of Finance* 39, 1127-1139.

Ross, K., J. Shapiro and K. Smith, 1996, Price improvement of SuperDot market orders on the NYSE, NYSE Working Paper 96-02.

Scholes, M. and J. Williams, 1977, Estimating betas from nonsynchronous data, *Journal of Financial Economics* 5, 309-328.

Seppi, D., 1990, Equilibrium block trading and asymmetric information, *Journal of Finance* 45, 73-94.

Sharpe, W., 1990, Capital asset prices: a theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-442.

Siegel, A. and C. Morgan, *Statistics and Data Analysis An Introduction 2'nd ed.*, NY: Wiley, 1996.

Smith Bamber, L., O. Barron and T. Strober, 1999, Differential interpretations and trading volume, *Journal of Financial and Quantitative Analysis* 34, 369-386.

Stoll, H., 1978, The supply of dealer services in securities markets, *Journal of Finance* 33, 1133-1151.

Vayanos, D., 1998, Transaction costs and asset prices: a dynamic equilibrium model, *Review of Financial Studies* 11, 1-58.

Wang, J., 1994, A model of competitive stock trading volume, *Journal of Political Economy* 102, 127-168.

Appendix A: Derivation of Euler Equations

Consider a model with a risky asset and a perfectly liquid riskless bond. An agent seeks to maximize expected utility of lifetime consumption.

$$V(c) \equiv \max_{c_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t \, U(c_t) \right]$$

Assume:

- An order to trade the risky asset at time t may execute in several transactions, and at different prices. Only random fraction $(1-\tilde{x}_t)$ executes at the quoted price, $P_t \pm s_t$ (where s_t is the half-spread, and P_t is the quote midpoint). The remaining fraction of2q an order, $\tilde{x}_t$, executes at random price $P_t + s_t + \tilde{\delta}_t$ for buys and at $P_t - s_t - \tilde{\delta}_t$ for sells, where $\tilde{\delta}_t$ is the price impact. This random price equals the weighted-average of transactions at non-quote prices.
- The bond can be bought or sold without transaction uncertainty.

Perturb off the optimal consumption path, $\{c_t\}$. For the optimal path, a small shift of consumption between periods will not change expected utility. The perturbation studied here involves the following composite action:

- Purchase ξ extra shares of the risky asset at time t. Consumption at time t is *reduced* below optimal by the cost of the extra shares.

- Sell ξ extra shares of the risky asset at time t+1 to return to the optimal portfolio. Consumption at time t+1 is *increased* above optimal by the amount received for the extra shares.

Consider the utility terms affected by the perturbation:

$$\begin{aligned} J(\xi) \equiv E_t \Big\langle \beta^t U\Big\{c_t - \xi\Big[(1-\tilde{x}_t)(P_t + s_t) + \tilde{x}_t\big(P_t + s_t + \tilde{\delta}_t\big)\Big]\Big\} + \\ \beta^{t+1} U\Big\{\tilde{c}_{t+1} + \xi\Big[(1-\tilde{x}_{t+1})\big(\tilde{P}_{t+1} - \tilde{s}_{t+1}\big) + \tilde{x}_{t+1}\big(\tilde{P}_{t+1} - \tilde{s}_{t+1} - \tilde{\delta}_{t+1}\big)\Big]\Big\}\Big\rangle \\ = E_t \Big\langle \beta^t U\Big[c_t - \xi\big(P_t + s_t + \tilde{x}_t\tilde{\delta}_t\big)\Big] + \beta^{t+1} U\Big[\tilde{c}_{t+1} + \xi\big(\tilde{P}_{t+1} - \tilde{s}_{t+1} - \tilde{x}_{t+1}\tilde{\delta}_{t+1}\big)\Big]\Big\rangle \end{aligned}$$

Along the optimal path:

$$0 = \left.\frac{\partial J(\xi)}{\partial \xi}\right|_{\xi=0} = E_t\Big[-\big(P_t + s_t + \tilde{x}_t\tilde{\delta}_t\big)\,\beta^t U'(c_t) + \big(\tilde{P}_{t+1} - \tilde{s}_{t+1} - \tilde{x}_{t+1}\tilde{\delta}_{t+1}\big)\,\beta^{t+1} U'(\tilde{c}_{t+1})\Big] \quad \text{(A1)}$$

where:

- c_t (Optimal consumption) and ...
- P_t+s_t (Quoted price) are known at time t.
- $(1-\tilde{x}_t)$ (Fraction of order executed at the quote) and ...
- $\big(P_t + s_t + \tilde{x}_t\tilde{\delta}_t\big)$ (Average realized price) is known at $t+\varepsilon$ for $0<\varepsilon<1$.

Since P_t and s_t are known when the order at t is placed, we can rearrange (A1) as:

$$0 = E_t\left\langle \beta\,\frac{U'(\tilde{c}_{t+1})}{U'(c_t)}\,\frac{\tilde{P}_{t+1} - \big(\tilde{s}_{t+1} + \tilde{x}_{t+1}\tilde{\delta}_{t+1}\big)}{P_t + \big[s_t + E_t\big(\tilde{x}_t\tilde{\delta}_t\big)\big]} - 1 \right\rangle \qquad \text{(A2)}$$

Define the realizable stock return over the period as:

$$\tilde{R}_{t+1} \equiv \frac{\tilde{P}_{t+1} - \left(\tilde{s}_{t+1} + \tilde{x}_{t+1}\tilde{\delta}_{t+1}\right)}{P_t + \left[s_t + E_t\left(\tilde{x}_t \tilde{\delta}_t\right)\right]} \tag{A3}$$

When investors face transaction uncertainty, as they do in stock markets, (A3) gives the stock return that should be used in tests of asset pricing because the realizable return consistently reflects the interaction between uncertain cash flows and uncertain liquidity costs.

Substitute (A3) into (A2):

$$0 = E_t\left\langle \beta \frac{U'(\tilde{c}_{t+1})}{U'(c_t)} \tilde{R}_{t+1} - 1 \right\rangle \tag{A4}$$

There is no transaction uncertainty for the perfectly liquid riskless bond:

$$0 = E_t\left[\beta \frac{U'(\tilde{c}_{t+1})}{U'(c_t)} \tilde{R}_{f,t+1} - 1 \right] \tag{A5}$$

Equations (A3), (A4) and (A5) are the Euler equations.

The discounted ratio of marginal utilities, $\beta U'(\tilde{c}_{t+1})/U'(c_t)$, is often termed the stochastic discount factor.

Appendix B: Simulating Returns that Reflect Transaction Uncertainty

The simulated return on midpoint price, R, and half-spread plus price impact, $s+x\delta$, both are distributed log normally, with approximate means $\mu_{s+x\delta}$ and μ_R. Changes in $s+x\delta$ and R have idiosyncratic components and a systematic component. The idiosyncratic variations have standard deviations $\sigma_{s+x\delta}$ and σ_R. Systematic variation is driven by one factor with a normally distributed instantaneous growth, ε_f. Systematic changes in the half-spread plus price impact, $s+x\delta$, and in midpoint return, R, come from covariances, $\sigma_{f,s+x\delta}$ and $\sigma_{f,R}$, with the factor. The midpoint price, P, is generated from the gross midpoint return using an initial price of \$40. The above conditions are summarized below:

$$\varepsilon_f \overset{iid}{\sim} N(0,1) \qquad \text{systematic variation}$$

$$\varepsilon_R \overset{iid}{\sim} N(0,1) \text{ and } \varepsilon_{s+x\delta} \overset{iid}{\sim} N(0,1) \qquad \text{idiosyncratic variations}$$

$$R \equiv \exp\left(\mu_R + \sigma_R * \varepsilon_R + \sigma_{f,R} * \varepsilon_f\right) \qquad \text{gross midpoint return}$$

$$s + x\delta \equiv \exp\left(\mu_{s+x\delta} + \sigma_{s+x\delta} * \varepsilon_{s+x\delta} + \sigma_{f,s+x\delta} * \varepsilon_f\right) \qquad \text{half-spread + price impact}$$

$$P_{t+1} \equiv P_t * R_{t+1} \quad \text{for } P_0 = 40 \qquad \text{midpoint price}$$

The half-spread plus price impact, $s+x\delta$ and mid-quote price, P, are simulated for 5300 “trading days.” These daily values are sampled every 22 trading days to give 240 “end-of-month” observations. Thus half-spread plus price impact is scaled to daily transactions, while return is monthly. Prices were calculated as $P^B = P + (s + x\delta)$ for buy-initiated trades and as $P^S = P - (s + x\delta)$ for sell-initiated trades. Closing prices,

P^{CRSP}, use a ±1 buy/sell indicator, I, to determine trade direction. I is distributed Bernoulli with probability 1/2. Transaction prices are truncated at increments of \$1/8; $s+x\delta$ is truncated at increments of \$1/16.[43] Simulated CRSP returns use closing prices divided by lagged closing prices. Simulated realizable buy-then-sell returns use sell prices divided by lagged buy prices.

Parameters of the simulation are calibrated to Boeing (ticker BA). Mean and standard deviation of transaction uncertainty measure $(s_{t+1}+x_{t+1}\delta_{t+1})/(P_t+s_t+x_t\delta_t)$ are matched to the TORQ data period, 901101-910131, by varying the means, μ_R and $\mu_{s+x\delta}$, the standard deviations, σ_R and $\sigma_{s+x\delta}$, and the covariances, $\sigma_{I,R}$ and $\sigma_{I,s+x\delta}$. Mean and standard deviation of CRSP monthly returns are matched for the extended data period, 6207-9712. The transaction uncertainty measure $(s_{t+1}+x_{t+1}\delta_{t+1})/(P_t+s_t+x_t\delta_t)$ tends to be smaller for a large firm, like Boeing, than for smaller firms. By calibrating the spread plus price impact data to Boeing, tests using the simulated data to gauge the effects of transaction uncertainty will be conservative in their assumption about the magnitude of the transaction uncertainty component of returns.

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[43] For trades with price impact, δ, the fraction of the trade executing off the quote, x, is always taken to be 1. In the TORQ sample, the distribution of the fraction of the trade executing off the quote was found to closely approximate a unit step function: x was generally either very close to 0 (the order executed completely at the quote) or very close to 1.

Appendix C: Corrected Returns that Minimize Mean Square Measurement Error

Because buy/sell direction is not observed in CRSP data, only approximate corrections can be made to CRSP returns. Assume that buy-initiated trades are as likely as sell-initiated trades. Assume further that a day with no trades (-1 x mid-quote price is reported) followed by a day with a closing sell-initiated trade is as likely as a day with no trades followed by a day with a closing buy-initiated trade. Recall that S/B represents buying at the end of period t, then selling at the end of t+1. Table 6 lists the categories of CRSP returns and the operational correction factor that minimizes mean square error relative to a buy-then-sell return.

Derivation of the correction factor, C, is given for a CRSP return comprising a mid-quote price followed by a trade. The remaining correction factors are derived in a similar manner.

Let: $\Pr\left[\text{quote followed by sell}\right] \equiv \Pr\left[{}^{S}\!/\!{}_{Q}\right] = \Pr\left[\text{quote followed by buy}\right] \equiv \pi$

Then mean square error for the CRSP return relative to a buy-then-sell return is:

$$
\begin{aligned}
MSE &= E\left[\left(R^{S/B} - R^{S/Q} * C\right)^2 + \left(R^{S/B} - R^{B/Q} * C\right)^2\right] \\
&= \pi * \left(R^{S/B} - R^{S/Q} * C\right)^2 + \pi * \left(R^{S/B} - R^{B/Q} * C\right)^2 \\
&= \pi * \left\{\left[\left(R^{S/Q}\right)^2 + \left(R^{B/Q}\right)^2\right] * C^2 - 2 * R^{S/B} * \left(R^{S/Q} + R^{B/Q}\right) * C + 2 * \left(R^{S/B}\right)^2\right\}
\end{aligned}
$$

$$\frac{dMSE}{dC} = 0 \quad \Rightarrow \quad \text{Correction factor.} \quad C = \frac{R^{S/B} * \left(R^{S/Q} + R^{B/Q} \right)}{\left[\left(R^{S/Q} \right)^2 + \left(R^{B/Q} \right)^2 \right]}$$

Verifying a minimum: $\frac{d^2 MSE}{dC^2} = 2 * \pi * \left[\left(R^{S/Q} \right)^2 + \left(R^{B/Q} \right)^2 \right] > 0$

From Table 5:

$$R^{S/B} = R^{S/Q} * \left(\frac{P_t}{P_t + s_t} \right) = R^{B/Q} * \left[\left(\frac{P_t}{P_t + s_t} \right) * \left(\frac{P_{t+1} - s_{t+1} - \delta_{t+1}}{P_{t+1} + s_{t+1} + \delta_{t+1}} \right) \right]$$

Substituting for $R^{S/Q}$ and $R^{B/Q}$ and rearranging:

$$C = \frac{P_t * P_{t+1} * \left(P_{t+1} - s_{t+1} - \delta_{t+1} \right)}{\left(P_t + s_t \right) * \left[P_{t+1}^2 + \left(s_{t+1} + \delta_{t+1} \right)^2 \right]}$$

The operational form of this correction factor uses CRSP prices. Using the CRSP closing price in place of the (unobservable) quote midpoint does not affect the value of the correction factor appreciably, as long as the half-spread plus price impact is less than five percent of the price:

$$C \approx \frac{P_t^{Close} * P_{t+1}^{Close} * \left(P_{t+1}^{Close} - s_{t+1} - \delta_{t+1} \right)}{\left(P_t^{close} + s_t \right) * \left[\left(P_{t+1}^{Close} \right)^2 + \left(s_{t+1} + \delta_{t+1} \right)^2 \right]}$$

The rest of Table 6 is derived in a similar manner.

Appendix D: Estimation of Transaction Uncertainty Risk

CRSP daily data: closing price, AskHi and BidLo, include components for spread plus price impact. Can these time series be used directly to estimate the covariance with risk factors of the transaction uncertainty variables?

Model a CRSP closing-price return as: $R_{t+1}^{CRSP} \equiv \dfrac{P_{t+1} + I_{t+1}\left(s + x\delta\right)_{t+1}}{P_t + I_t\left(s + x\delta\right)_t}$

where:

$P_t^{CRSP} \equiv$ CRSP closing price for trade at t

$P_t \equiv$ Midpoint of bid and ask prices quoted before trade at t

$I_t \equiv$ Buy/Sell indicator variable $\overset{IID}{\sim}\left[+1 \text{ for buys}, -1 \text{ for sells}, \Pr(\text{buy}) = \tfrac{1}{2}\right]$

$s_t \equiv$ Half the bid/ask spread

$\delta_t \equiv$ Price impact (price for trade in excess of depth at quote − quote price)

$1 - x \equiv$ Fraction of order going through at quote

The Transaction Uncertainty Components of CRSP Returns Do Not Covary with Risk Factors

Define:

$R_{t+1}^{Intrinsic} \equiv P_{t+1}/P_t$ to be the intrinsic value return

$\theta_t \equiv \left(s + x\delta\right)_t / P_t$ to be the proportional transaction uncertainty

Then divide numerator and denominator by P_t and rearrange:

$$
\begin{aligned}
R_{t+1}^{CRSP} &\equiv \frac{R_{t+1}^{Intrinsic}\left(1 + I_{t+1}\theta_{t+1}\right)}{\left(1 + I_t\theta_t\right)} \\
&= R_{t+1}^{Intrinsic}\left(1 + I_{t+1}\theta_{t+1}\right)\left(1 - I_t\theta_t + \theta_t^2 - I_t\theta_t^3 + \cdots\right) \\
&= R_{t+1}^{Intrinsic}\left(1 + \frac{I_{t+1}\theta_{t+1}}{1 + I_t\theta_t} - \frac{I_t\theta_t}{1 + I_t\theta_t}\right)
\end{aligned}
$$

and for risk factor, f:

$$Cov\left(f_{t+1}, R_{t+1}^{CRSP}\right) = Cov\left(f_{t+1}, R_{t+1}^{Intrinsic}\right) + Cov\left(f_{t+1}, R_{t+1}^{Intrinsic} \frac{I_{t+1}\theta_{t+1} - I_t\theta_t}{1 + I_t\theta_t}\right)$$

Now consider Cov(X,I Y) for random variables (X,Y) and recall that I is independent of (X,Y) and has zero mean. Then:

$$\begin{aligned} Cov(X, IY) &= E(X\,IY) - E(X)E(IY) \\ &= \left[E(I)E(XY) + Cov(I, XY)\right] - E(X)\left[E(I)E(Y) + Cov(I, Y)\right] \\ &= 0 \end{aligned}$$

Hence: $Cov\left(f_{t+1}, R_{t+1}^{CRSP}\right) = Cov\left(f_{t+1}, R_{t+1}^{Intrinsic}\right)$

Thus, covariance of the intrinsic return with risk factors is measured using CRSP returns. Note that this result depends critically upon the assumed zero mean for the buy/sell indicator, I.

The Transaction Uncertainty Components of AskHi and BidLo Returns Do Covary with Risk Factors

AskHi and BidLo are the order statistics of a firm's price time series. Use superscripts, $\_^{aH}$ to indicate a time series associated with AskHi and $\_^{bL}$ to indicate a time series associated with BidLo. For example, I_t^{aH} is the indicator variable for buys versus sells for the AskHi time series and P_t^{aH} is the mid-quote price at the time of the AskHi trade for period t.

The unconditional probability of any closing price being a buy is assumed to be 1/2. Given that a buy-initiated price is $P+(s+\delta)$, whereas a sell-initiated price is $P-(s+\delta)$, it seems reasonable to assume that

AskHi$_t$, the highest trade price on day t, has a greater than 1/2 probability of being a buy and that BidLo$_t$ has a less than 1/2 probability of being a buy. Assuming symmetry in the distributions of AskHi and BidLo, define:

$$\pi \equiv \Pr\left[I_t^{aH}=1\right]=1-\Pr\left[I_t^{bL}=1\right]>1/2$$

This means that $E\left(I_t^{aH}\right)=-E\left(I_t^{bL}\right)=2\pi-1>0$.

Now consider covariance of AskHi and BidLo returns with a risk factor.

$$\begin{aligned}
Cov\left(\mathrm{f}_{t+1},\frac{AskHi_{t+1}}{AskHi_t}\right) &\equiv Cov\left[\mathrm{f}_{t+1},\frac{P_{t+1}^{aH}+I_{t+1}^{aH}\left(s+\delta\right)_{t+1}^{aH}}{P_t^{aH}+I_t^{aH}\left(s+\delta\right)_t^{aH}}\right] \\
&= Cov\left[\mathrm{f}_{t+1},\frac{R_{t+1}^{aH}\left(1+I_{t+1}^{aH}\theta_{t+1}^{aH}\right)}{\left(1+I_t^{aH}\theta_t^{aH}\right)}\right] \\
&= Cov\left(\mathrm{f}_{t+1},R_{t+1}^{aH}\right)+Cov\left[\mathrm{f}_{t+1},\frac{R_{t+1}^{aH}\left(I_{t+1}^{aH}\theta_{t+1}^{aH}-I_t^{aH}\theta_t^{aH}\right)}{\left(1+I_t^{aH}\theta_t^{aH}\right)}\right]
\end{aligned}$$

Since $E\left(I_t^{aH}\right)$ is non-zero, the second covariance term is non-zero. A similar analysis can be performed for the BidLo return series.

Because of the non-zero mean of their buy/sell indicators, AskHi and BidLo returns *do* reflect the covariance of transaction uncertainty with risk factors, even though CRSP returns do *not* reflect the covariance of transaction uncertainty with risk factors. This means it should be possible to estimate the transaction component of returns using the AskHi and BidLo time series. This idea will be pursued in a future paper.

Appendix E: Expected Returns and Liquidity Costs

Amihud & Mendelson "Market-Observed" Gross Returns Decrease with Increasing Liquidity Costs

In their 1986 paper, Amihud and Mendelson model expected returns as a function of liquidity costs and investors' holding periods. AM's Proposition 2 states, "In equilibrium, the observed (Gross) return is an increasing and concave piecewise-linear function of the (relative) spread." That is, given two otherwise identical securities, investors will demand a return premium to hold the less liquid security. AM define Gross return as:

$$r_{t+1}^{\text{Gross}} \equiv \frac{\text{period}_{t+1} \text{ frictionless cash flow}}{\text{P}_t + (s+\delta)_t}$$

where P_t is mid-quote price and $(s+\delta)_t$ is quoted half-spread plus price impact. Their model does not explicitly consider price impact, but they are modeling the *purchase* of a security for some particular holding period. AM's Gross return is not realizable in that it ignores the liquidity costs of exiting the holding period.

Treating the holding period cash flow as the difference in mid-quote price over the holding period gives:

$$r_{t+1}^{\text{Gross}} \equiv \frac{\text{P}_{t+1} - \text{P}_t}{\text{P}_t + (s+\delta)_t} = \frac{r_{t+1}^{Mid}}{1+\theta_t}$$

where: $r_{t+1}^{Mid} \equiv P_{t+1}/P_t - 1$ is intrinsic value or mid-quote return

$\theta_t \equiv (s+\delta)_t / P_t$ is proportional liquidity cost

Consider the effect of liquidity cost on expected Gross returns. Dropping subscripts and taking derivatives with respect to θ gives:

$$\frac{dE\left(r^{\text{Gross}}\right)}{d\theta} = E\left[\frac{dr^{\text{Mid}}}{d\theta}\frac{1}{(1+\theta)} - \frac{r^{\text{Mid}}}{(1+\theta)^2}\right] = E\left[\frac{1}{(1+\theta)}\left(\frac{dr^{\text{Mid}}}{d\theta} - r^{\text{Gross}}\right)\right] \qquad \text{(A6)}$$

$$\begin{aligned}\frac{d^2E\left(r^{\text{Gross}}\right)}{d\theta^2} &= E\left[\frac{d^2r^{\text{Mid}}}{d\theta^2}\frac{1}{(1+\theta)} - 2\frac{dr^{\text{Mid}}}{d\theta}\frac{1}{(1+\theta)^2} + 2\frac{r^{\text{Mid}}}{(1+\theta)^3}\right] \\ &= E\left\{\frac{1}{(1+\theta)}\left[\frac{d^2r^{\text{Mid}}}{d\theta^2} - 2\frac{dr^{\text{Mid}}}{d\theta}\frac{1}{(1+\theta)} + 2\frac{r^{\text{Mid}}}{(1+\theta)^2}\right]\right\}\end{aligned}$$

Substituting from (A6):

$$\frac{d^2E\left(r^{\text{Gross}}\right)}{d\theta^2} = E\left\{\frac{1}{(1+\theta)}\left[\frac{d^2r^{\text{Mid}}}{d\theta^2} - 2\frac{dr^{\text{Gross}}}{d\theta}\right]\right\} \qquad \text{(A7)}$$

AM assume that $E\left(r^{\text{Gross}}\right) > 0$ and *predict* that the first derivative is positive and the second derivative is negative. For the *first* derivative of Gross returns to be positive, the first derivative of *mid-quote* returns must be positive (on the order of r_{t+1}^{Gross}).[44] Otherwise, Gross returns decrease as liquidity costs increase.

In AM's model, the increase in expected Gross returns gets smaller as liquidity costs get bigger. For the *second* derivative of Gross returns to be negative, the second derivative of mid-quote returns must be less than twice the *first* derivative of Gross returns. If the first derivative of Gross returns is positive, then the second derivative of mid-quote returns can be positive. If the first derivative of Gross returns is negative, then the second derivative of mid-quote returns must be negative.

Hence, AM's prediction that Gross returns are an increasing, concave function of proportional liquidity cost implies that a similar relation holds between mid-quote returns and liquidity cost.

$$\frac{dE\left(r^{\text{Gross}}\right)}{d\theta}>0 \;\Rightarrow\; \frac{dE\left(r^{\text{Mid}}\right)}{d\theta}>E\left(r^{\text{Gross}}\right)$$

$$\frac{d^2E\left(r^{\text{Gross}}\right)}{d\theta^2}<0 \Rightarrow \frac{d^2E\left(r^{\text{Mid}}\right)}{d\theta^2}<2\frac{dE\left(r^{\text{Gross}}\right)}{d\theta}$$

On the other hand, if mid-quote returns are fixed with respect to liquidity costs, then Gross returns decrease as liquidity costs increase and the decease gets smaller as liquidity costs increase.

CRSP Returns Increase Mechanically with Increasing Liquidity Costs

Individual CRSP Returns

Ignoring non-trading, CRSP returns can be buy-then-buy, buy-then-sell, sell-then-buy or sell-then-sell. Assume that a closing price can be buy-initiated or sell-initiated with equal probability. Then the expected CRSP return is:

$$E_t\left(r_{t+1}^{CRSP}\right)=\frac{E_t}{4}\left[\frac{P_{t+1}+(s+\delta)_{t+1}}{P_t+(s+\delta)_t}+\frac{P_{t+1}-(s+\delta)_{t+1}}{P_t+(s+\delta)_t}+\frac{P_{t+1}-(s_{t+1}+\delta)_{t+1}}{P_t-(s+\delta)_t}+\frac{P_{t+1}+(s+\delta)_{t+1}}{P_t-(s+\delta)_t}\right]-1$$

$$=E_t\left\{\frac{P_{t+1}\,P_t}{\left[P_t+(s+\delta)_t\right]\left[P_t-(s+\delta)_t\right]}\right\}-1$$

$$=E_t\left[\frac{P_{t+1}\,P_t-P_t^2+(s+\delta)_t^2}{P_t^2-(s+\delta)_t^2}\right]$$

Express CRSP returns in terms of mid-quote returns and liquidity cost:

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[14] Assume that covariance terms are small.

$$E_t\left(r_{t+1}^{CRSP}\right) = E_t\left(\frac{r_{t+1}^{Mid} + \theta_t^2}{1-\theta_t^2}\right)$$

Consider the effect of liquidity cost on expected CRSP returns. Dropping subscripts and taking derivatives with respect to θ gives:

$$\frac{dE\left(r^{CRSP}\right)}{d\theta} = E\left\{\frac{1}{\left(1-\theta^2\right)}\left[\frac{dr^{Mid}}{d\theta} + 2\theta + \frac{2\theta\left(r^{Mid}+\theta^2\right)}{1-\theta^2}\right]\right\}$$

Simplifying:

$$\frac{dE\left(r^{CRSP}\right)}{d\theta} = E\left[\frac{1}{\left(1-\theta^2\right)}\left(\frac{dr^{mid}}{d\theta} + \frac{2\theta + 2\theta r^{mid}}{1-\theta^2}\right)\right]$$

$$\begin{aligned}\frac{d^2E\left(r^{CRSP}\right)}{d\theta^2} &= E\left[\frac{1}{\left(1-\theta^2\right)}\frac{d^2r^{Mid}}{d\theta^2} + \frac{4\theta}{\left(1-\theta^2\right)^2}\frac{dr^{Mid}}{d\theta} + \frac{2\left(1+r^{Mid}\right)}{\left(1-\theta^2\right)^2} + \frac{8\theta^2\left(1+r^{Mid}\right)}{\left(1-\theta^2\right)^2}\right] \\ &= E\left\{\frac{1}{\left(1-\theta^2\right)}\left[\frac{d^2r^{Mid}}{d\theta^2} + \frac{4\theta}{1-\theta^2}\frac{dr^{Mid}}{d\theta} + \frac{\left(2+8\theta^2\right)\left(1+r^{Mid}\right)}{1-\theta^2}\right]\right\}\end{aligned}$$

Since $0 < E(\theta) < 1$ and $\theta^2 << 1$:

$$\frac{dE\left(r^{CRSP}\right)}{d\theta} \approx E\left[\frac{dr^{mid}}{d\theta} + 2\theta\left(1+r^{Gross}\right)\right] \tag{A8}$$

$$\frac{d^2E\left(r^{CRSP}\right)}{d\theta^2} \approx E\left\langle\frac{d^2r^{Mid}}{d\theta^2} + 4\theta\frac{dr^{Mid}}{d\theta} + 2\right\rangle \tag{A9}$$

AM assert that expected Gross returns increase with proportional liquidity costs. Since $0 < E(\theta) < 1$ and $E\left(r^{\mathrm{Gross}}\right) > 0$, the first derivative of CRSP returns is positive, unless the first derivative of mid-quote returns is sufficiently negative. Thus, expected Gross returns *may* increase with liquidity costs, but (A6) indicates that CRSP returns increase with liquidity cost, even if AM's investment-horizon clientele effect does *not* hold.

In AM's model, the increase in expected Gross returns gets smaller as liquidity costs get bigger. With CRSP returns, the second derivative of mid-quote return must be *strongly* negative (on the order of -2) for any such effect to show up.

$$\frac{dE\left(r^{\text{CRSP}}\right)}{d\theta} > 0 \Rightarrow \frac{dE\left(r^{\text{Mid}}\right)}{d\theta} > -2E(\theta)$$

$$\frac{d^2E\left(r^{\text{CRSP}}\right)}{d\theta^2} < 0 \Rightarrow \frac{d^2E\left(r^{\text{Mid}}\right)}{d\theta^2} < -2$$

On the other hand, if mid-quote returns are fixed or decrease with respect to liquidity costs, the spurious increase in expected CRSP returns gets bigger as liquidity costs get bigger.

Portfolios of CRSP Returns

AM use equally-weighted portfolios of CRSP returns in their tests. Because the portfolios are linear in the returns, the above analysis for individual CRSP returns holds for the portfolios.

$$\frac{\partial E\left(r^{\text{CRSP Portfolio}}\right)}{\partial\theta_n} = \frac{1}{N}E\left\{\sum_{n=1}^{N}\frac{1}{\left(1-\theta_n^2\right)}\left[\frac{dr_n^{\text{Mid}}}{d\theta_n} + \frac{2\theta_n + 2\theta_n r_n^{\text{Mid}}}{\left(1-\theta_n^2\right)}\right]\right\} > 0 \quad \text{for} \quad 1 \leq n \leq N$$

CRSP portfolio returns increase with increasing proportional liquidity costs, even when there is no investment-horizon clientele effect.

Buy-then-sell Returns Decrease with Increasing Liquidity Costs

Consider expected buy-then-sell returns:

$$E_t\left(R_{t+1}^{Buy-then-sell}\right) = E_t\left[\frac{P_{t+1}-\left(s_{t+1}+\delta_{t+1}\right)}{P_t+\left(s_t+\delta_t\right)}\right]$$
$$= E_t\left[\frac{R_{t+1}^{Mid}\left(1-\theta_{t+1}\right)}{1+\theta_t}\right]$$

Assume $\theta_{t+1} = \phi\theta_t + e_{t+1}$ where $\phi\theta_t$ is the permanent component of θ_{t+1} and the transient component, e_{t+1}, is zero-mean and independent of r^{Mid} and θ_t. Then

$$E_t\left(R_{t+1}^{Buy-then-sell}\right) = E_t\left[\frac{R_{t+1}^{Mid}\left(1-\phi\theta_t\right)}{1+\theta_t}\right]$$

Dropping subscripts and taking derivatives with respect to θ gives:

$$\frac{dE\left(R^{Buy-then-sell}\right)}{d\theta} = E\left\{\frac{1}{(1+\theta)}\left[\frac{dR^{Mid}}{d\theta}\left(1-\phi\theta\right)-\phi R^{Mid} - R^{Buy-then-sell}\right]\right\} \qquad \text{(A10)}$$

Taking the second derivative with respect to θ gives:

$$\frac{d^2E\left(R^{Buy-then-sell}\right)}{d\theta^2} = E\left\{\frac{1}{(1+\theta)}\left[\frac{d^2R^{Mid}}{d\theta^2}\left(1-\phi\theta\right)-2\phi\frac{dR^{Mid}}{d\theta} - \frac{dR^{Buy-then-sell}}{d\theta}\right]\right\}$$
$$- E\left\{\frac{1}{(1+\theta)^2}\left[\frac{dR^{Mid}}{d\theta}\left(1-\phi\theta\right)-\phi R^{Mid} - R^{Buy-then-sell}\right]\right\}$$

(A11)

Assume $0 < E(\theta) << 1$ and simplify (A10) and (A11):

$$\frac{dE\left(R^{Buy-then-sell}\right)}{d\theta} \approx E\left[\frac{dr^{Mid}}{d\theta} - R^{Mid}\left(1+\phi\right)\right] \qquad \text{(A12)}$$

$$\frac{d^2E\left(R^{Buy-then-sell}\right)}{d\theta^2} \approx E\left[\frac{d^2r^{Mid}}{d\theta^2}\left(1+\phi\right) - \frac{dr^{Mid}}{d\theta}\left(2+3\phi\right) + \phi\left(1+\phi\right)R^{Mid}\right] \qquad \text{(A13)}$$

Since $0<\phi<1$, for the first derivative of buy-then-sell returns to be positive, the first derivative of *mid-quote* returns must be *strongly* positive (on the order of $1+\phi$). Otherwise, buy-then-sell returns decrease as liquidity costs increase.

For the *second* derivative of buy-then-sell returns to be negative, the *first* derivative of *mid-quote* returns must be positive and/or the second derivative of mid-quote return must be *strongly* negative (on the order of $-\phi$).

$$\frac{dE\left(r^{\text{Buy-then-sell}}\right)}{d\theta} > 0 \;\Rightarrow\; \frac{dE\left(r^{\text{Mid}}\right)}{d\theta} > 1+\phi$$

$$\frac{d^2E\left(r^{\text{Buy-then-sell}}\right)}{d\theta^2} < 0 \Rightarrow \; \frac{dE\left(r^{\text{Mid}}\right)}{d\theta} > 0 \;\text{ and/or }\; \frac{d^2E\left(r^{\text{Mid}}\right)}{d\theta^2} < -\phi$$

Summary of Expected Returns and Liquidity Costs

The Amihud and Mendelson model predicts that expected Gross returns increase with liquidity costs, the increase getting smaller as costs get bigger. We find in (A6) and (A7) that AM's model for Gross returns holds only if it is strongly mirrored in mid-quote returns. We find in (A8) and (A9) that using CRSP returns as a proxy for Gross returns gives inconclusive test results because mismeasurement of returns can make expected returns *appear* to increase with liquidity costs. We find in (A12) and (A13) that using buy-then-sell returns as a proxy for Gross returns should give more accurate test results than does CRSP returns. The results of testing these alternative predictions are shown in Tables 19 and 20.

Because of the linearity of the portfolio formation process, portfolios of CRSP or buy-then-sell returns behave the same way as individual returns when liquidity costs increase.

Appendix F: Transaction Uncertainty and Different Kinds of Traders

Return Horizon Affects the Significance of Market Frictions

The magnitude of the effects of any market friction depends upon the details of the study being considered. For example, in a study using a monthly return horizon, one clearly should be more concerned with the effects of market frictions than in a study using an annual return horizon. I argue that transaction uncertainty has a major effect on measured returns and on estimates of the moments of these returns when one is doing a study with a monthly horizon.

CRSP Returns versus Buy-Then-Sell Returns

CRSP returns (in their daily file, monthly file and for use in index calculations) are calculated as:

$$r_{t+1}^{CRSP} = \frac{P_{t+1}^{\text{Closing}} + Dividend_{t+1}}{P_t^{\text{Closing}}} - 1 \tag{A14}$$

This is the form of a buy-then-sell return (with measurement error due to transaction uncertainty costs). As shown earlier, historical CRSP returns can be corrected to perform much better as a measure of monthly buy-then-sell returns.

CRSP Returns versus Short Sale Returns

A short seller wants a sell-then-buy return. A short sale return is:

$$r_{t+1}^{Short} = 1 - \frac{P_{t+1} + (s+\delta)_{t+1} + Dividend_{t+1}}{P_t - (s+\delta)_t}$$

The equivalent closing-price return has the form:

$$\begin{aligned} r_{t+1}^{Proxy} &= 1 - \frac{P_{t+1}^{Closing} + Dividend_{t+1}}{P_t^{Closing}} \\ &= -r_{t+1}^{CRSP} \end{aligned}$$

As in the buy-then-sell case, a study using closing prices can only expect to observe a sell-then-buy return about one time in four. The other 75% of the time, the return measured by the closing price proxy will overstate the returns realizable by the short sale strategy.

Using (negative) CRSP returns to proxy for realizable short sale returns will lead to the same mismeasurement problems as in the buy-then-sell case. As in the buy-then-sell case, returns can be corrected to improve the measurement of short sale returns.

CRSP Returns versus Buy-and-Hold Returns

A buy-and-hold strategy with periodic rebalancing incurs liquidity costs less than those from liquidating at the rebalancing times. The rebalancing period "return", like an intrinsic return, is not realizable. Thinking of a return as the wealth at time t+1 divided by the wealth at time t, it perhaps makes most sense to measure rebalancing-time wealth as mid-quote prices, less liquidity costs associated with rebalancing. Mid-quote price is not provided in the CRSP data set,[45] but in a large portfolio, the sum of CRSP prices will approximate the sum of the mid-quote prices. The random buy/sell direction in CRSP closing prices will tend to cancel out the liquidity costs:[46]

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[45] The closing mid-quote price is provided on non-trading days.

[46] This is the argument made by Blume and Stambaugh (1983).

$$\text{CRSP wealth, } W^{CRSP} \equiv \sum_n P_n^{CRSP} = \sum_n \left[P_n + I_n \left(s + \delta \right)_n \right]$$
$$\approx \sum_n P_n \equiv W, \quad \text{mid} - \text{quote wealth}$$

Using wealth calculated this way doesn't completely solve the problem of calculating rebalancing returns because the liquidity costs of rebalancing are ignored. Define Δ_{tn} as portfolio rebalancing cost.[47] The rebalancing return is then:

$$\frac{W_{t+1} - \Delta_{t+1}}{W_t + \Delta_t} = \frac{W_{t+1} \;\; W_t - \Delta_{t+1} \;\; W_t}{1 + \Delta_t \;\; W_t}$$
$$\approx \left(\frac{W_{t+1}}{W_t} - \frac{\Delta_{t+1}}{W_t} \right) \left(1 - \frac{\Delta_t}{W_t} \right)$$
$$\approx \frac{W_{t+1}}{W_t} + \left(\frac{W_{t+1}}{W_t} + 1 \right) \frac{\Delta_t}{W_t} \approx \frac{W_{t+1}^{CRSP}}{W_t^{CRSP}} - \left(\frac{W_{t+1}^{CRSP}}{W_t^{CRSP}} + 1 \right) \frac{\Delta_t}{W_t^{CRSP}}$$

Rebalancing costs should be significantly less than the liquidation cost, so the magnitude of the error introduced by ignoring rebalancing costs is probably economically small. However, the rebalancing costs, and so the error will vary across portfolios based upon the volatility of prices in each portfolio. Studies that ignore rebalancing costs and use portfolios sorted on criteria that are correlated with liquidity may find spurious statistical patterns. For example, Blume and Stambaugh (1983) find that only part of the size effect is explained by their return calculation procedure when using closing-price returns Using the models of spread and price impact from this paper, rebalancing costs can be accounted for and spurious predictability should be purged.

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[47] Less than liquidation costs.

CRSP Returns versus Intrinsic Returns

Interpreting a stock's mid-quote price as the "intrinsic value," the return of interest is:

$$r_{t+1}^{Intrinsic} = \frac{P_{t+1}^{Mid-quote}}{P_t^{Mid-quote}} - 1$$

A closing-price proxy will be too large 25% of the time, too small 25% of the time, but close to the desired return 50% of the time. The measured first moment is:

$$E\left[r_{t+1}^{Proxy}\right] = E\left[\frac{P_{t+1}^{Mid-quote} + I_{t+1}\left(s + x\delta\right)_{t+1}}{P_t^{Mid-quote} + I_t\left(s + x\delta\right)_t} - 1\right]$$

where $I_t \equiv +1\left(-1\right)$ for a buy $\left(\text{sell}\right)$. Defining $\theta_t \equiv \left(s + x\delta\right)_t / P_t^{Mid-quote}$ gives:

$$E\left[r_{t+1}^{Proxy}\right] = E\left[\frac{\left(r_{t+1}^{Intrinsic} + 1\right)\left(1 + I_{t+1}\theta_{t+1}\right)}{1 + I_t\theta_t} - 1\right]$$

Assume an equal probability of a buy or a sell in any period, and no correlation of buy/sell direction with other variables. This means that the buy/sell indicator has zero mean, as does the product of the indicator with any other variables. Expanding the denominator and taking expectations of all terms involving the indicator variable gives:

$$\begin{aligned}
E\left[r_{t+1}^{Proxy}\right] &= E\left[\left(r_{t+1}^{Intrinsic} + 1\right)\left(1 + I_{t+1}\theta_{t+1}\right)\left(1 - I_t\theta_t + \theta_t^2 - I_t\theta_t^3 + \cdots\right) - 1\right] \\
&= E\left[\left(r_{t+1}^{Intrinsic} + 1\right)\left(1 + \theta_t^2 + \theta_t^4 + \cdots\right) - 1\right] \\
&= E\left[r_{t+1}^{Intrinsic}\left(1 + \theta_t^2 + \theta_t^4 + \cdots\right) + \left(\theta_t^2 + \theta_t^4 + \cdots\right)\right] \\
&= E\left(\frac{r_{t+1}^{Intrinsic}}{1 - \theta_t^2}\right) + E\left(\frac{\theta_t^2}{1 - \theta_t^2}\right)
\end{aligned}$$

Thus, the expected return of the proxy over-states the expected intrinsic return. As with rebalanced returns, the return error varies cross-sectionally with firm-specific transaction uncertainty. Any variable correlated with transaction uncertainty will seem to explain cross-sectional variation, suggesting that researchers using CRSP returns should be careful in this respect.

The monthly return error seems to be economically small, however. Using the average value of $\theta_t \equiv (s + x\delta)_t / P_t^{Mid-quote}$ from Table 17, mean CRSP monthly returns are 0.4400%, compared to mean mid-quote returns of 0.4354%. See Appendix D for further discussion of CRSP returns in studies of intrinsic value.

In an analysis that considers bid/ask spread, but ignores price impact and transaction uncertainty, Blume and Stambaugh (1983) derive an expression similar to equation (A12) for the bias in closing-price returns relative to mid-quote returns:

$$E\left[r_{t+1}^{Proxy}\right] = E\left(r_{t+1}^{Intrinsic}\right) + \sigma^2 \qquad \text{where} \qquad \sigma^2 \equiv Var\left(\frac{S_t}{P_t^{Mid-quote}}\right)$$

Using NYSE and AMEX stocks, they estimate that closing price daily returns overstate mid-quote returns by approximately 50% for small stocks.[48]

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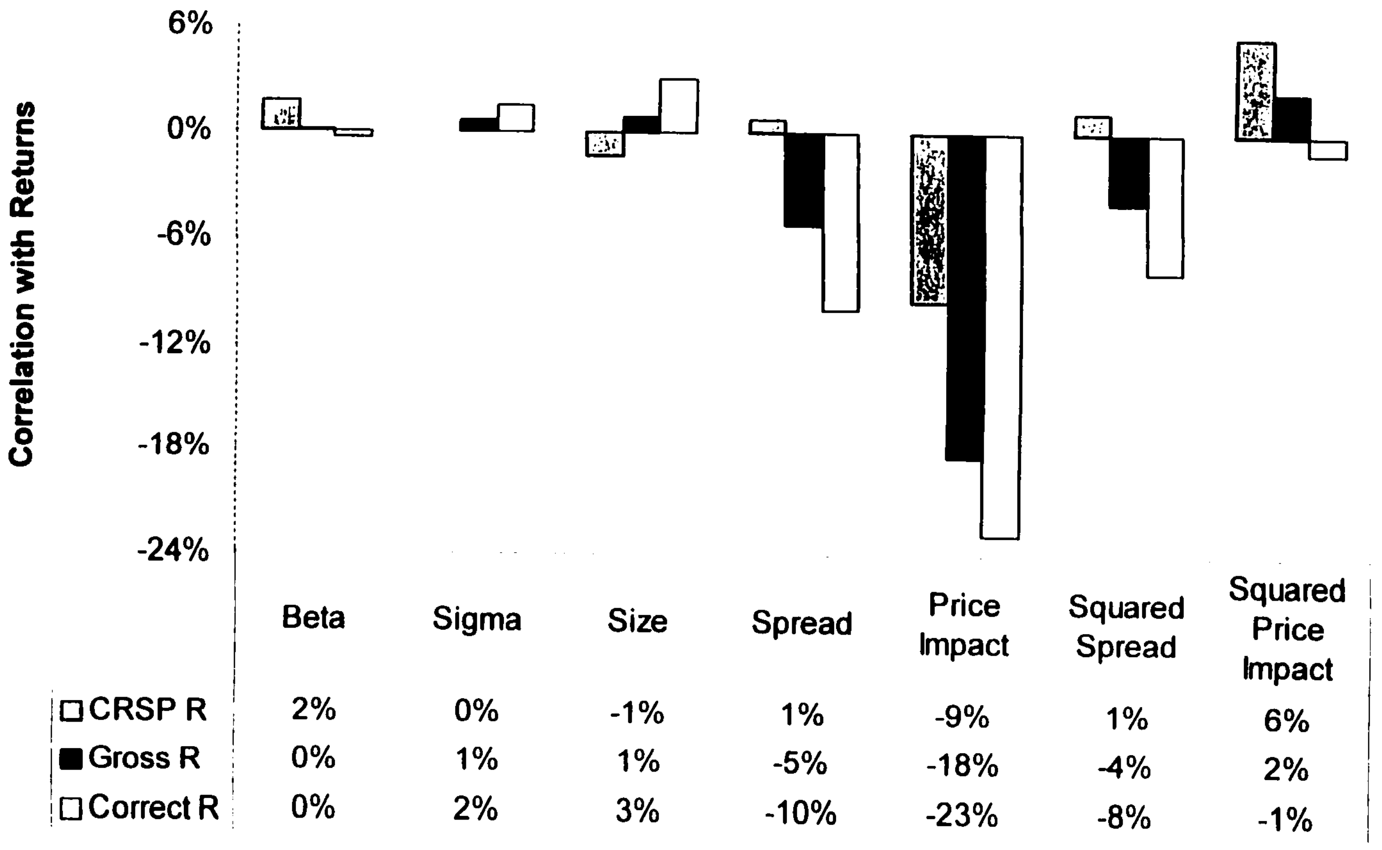
[48] Blume and Stambaugh (1983 page 391) find an average daily closing-price return of 0.141% and a bias term of 0.051%.

Appendix G: Figures



	Beta	Sigma	Size	Spread	Price Impact	Squared Spread	Squared Price Impact
☐ CRSP R	2%	0%	-1%	1%	-9%	1%	6%
■ Gross R	0%	1%	1%	-5%	-18%	-4%	2%
☐ Correct R	0%	2%	3%	-10%	-23%	-8%	-1%

Figure 1: Correlation with Returns

Appendix H: Tables

Table 1: Summary Statistics During the TORQ Period

TORQ market order prices, and their associated quotes are used to calculate the statistics of Table 1. The last transaction price of the day becomes the CRSP price. If from a buy order, this price also serves as the closing buy price. Otherwise, a closing buy price is constructed, using the closing bid price plus the price impact of the last transaction. Similarly, if the closing transaction is from a sell order, its price also serves as the closing sell price. Otherwise, a closing sell price is constructed, using the closing ask price minus the price impact of the last transaction. For big transactions, a big buy (sell) price is constructed using the closing bid (ask) price plus (minus) the price impact of the last big transaction.

$$\text{Big transactions} \equiv \text{order size} > \left(\text{quoted depth} \quad \text{OR} \quad 9{,}999 \text{ shares}\right)$$

These daily buy, sell and closing prices are used to calculate overlapping monthly returns. Each "month" has 23 trading days. For example, the first closing-price return uses CRSP closing prices for days 1 and 24; the second closing-price return uses CRSP closing prices for days 2 and 25.

On days with no big transactions, the associated buy and sell prices from all market order transactions are used. On these days with no big transactions, the substitute transaction will have either no price impact, or price improvement. Returns calculated with these prices understate the difference, $R_{\text{CRSP}} - R^{\text{All trans'}}_{\text{buy\&hold}}$. When no buy and sell prices are available, the prices from the previous trading day are used.

$$\begin{aligned} R_{CRSP} &\equiv \frac{\left[P_{t+1} + I_{t+1}\left(s_{t+1} + \delta_{t+1}\right)\right]}{\left[P_t + I_t\left(s_t + \delta_t\right)\right]} - 1 && \text{is the CRSP closing} - \text{price return} \\ &= \frac{\text{Closing Price}(\text{day t+23})}{\text{Closing Price}(\text{day t})} - 1 \\ R_{\text{Buy-then-sell}} &\equiv \frac{\left[P_{t+1} - \left(s_{t+1} + \delta_{t+1}\right)\right]}{\left[P_t + \left(s_t + \delta_t\right)\right]} - 1 && \text{is the realizable buy} - \text{then} - \text{sell return} \\ &= \frac{\text{Sell Price}(\text{day t+23})}{\text{Buy Price}(\text{day t})} - 1 \end{aligned}$$

$P_t \equiv$ Midpoint of bid and ask prices quoted before trade at t

$\delta_t \equiv$ Price impact (transaction price − quote price)

$s_t \equiv$ Half the bid/ask spread

$I_t \equiv$ Buy/sell indicator = 1 if a trade is buy-initiated, -1 if sell-initiated

Table 1 (continued)

Panel A Boeing During TORQ Three Month Period

BA		Mean	Std	Skew	Min	Median	Max
P^{Daily}_{CRSP}	($)	45.65	1.53	0.54	42.62	45.38	49.38
$R^{\text{Monthly}}_{CRSP}$	(%)	0.99	5.01	-0.49	-12.18	1.40	10.58
$R^{\text{Monthly}}_{\text{CRSP}} - R^{\text{All trans'ns}}_{\text{Buy-then-sell}}$	(%)	0.10	0.28	1.04	-0.54	0.00	1.12
$R^{\text{Monthly}}_{\text{CRSP}} - R^{\text{Big trans'ns}}_{\text{Buy-then-sell}}$	(%)	0.24	0.95	-1.08	-4.00	0.27	2.58
$Volume^{Daily}_{CRSP}$	$(\text{sh}/10^5)$	10.95	7.18	0.00	2.60	9.21	51.18
$\$Vol^{Daily}_{CRSP}$	$(\$\text{sh}/10^6)$	499.7	320.0	0.00	120.3	413.8	2181.
All Transactions: Transactions associated with all market orders							
$(s+x\delta)/P$	(%)	-0.00	0.19	11.33	-4.10	0.00	6.16
x	(%)	43.87	49.23	0.25	0.00	0.00	100.0
Price Impact, δ	($)	-0.05	0.10	5.60	-2.00	0.00	2.75
Half-spread, s	($)	0.09	0.03	1.09	0.00	0.06	0.50
$Volume^{Daily}_{TORQ}$	$(\text{sh}/10^5)$	2.13	1.06	0.00	0.56	1.88	6.17
Big Transactions: Order Size > (quoted depth or 9,999 shares)							
$(s+x\delta)/P$	(%)	0.24	0.77	2.79	-4.10	0.00	6.16
x	(%)	50.58	48.44	-0.02	0.00	53.33	100.0
Price Impact, δ	($)	0.05	0.34	2.58	-2.00	0.00	2.75
Half-spread, s	($)	0.09	0.06	4.94	0.00	0.06	0.50
$Volume^{Daily}_{TORQ}$	$(\text{sh}/10^5)$	0.21	0.15	0.00	0.01	0.18	0.64

Table 1 (continued)

Panel B Federal Express During TORQ Three Month Period

FDX		Mean	Std	Skew	Min	Median	Max
P^{Daily}_{CRSP}	$(\$)$	34.09	2.55	1.46	29.88	33.50	42.75
$R^{Monthly}_{CRSP}$	$(\%)$	-0.57	10.49	0.39	-22.98	-2.27	26.67
$R^{Monthly}_{CRSP} - R^{All\ trans'ns}_{Buy\text{-}then\text{-}sell}$	$(\%)$	0.24	0.53	0.33	-1.84	0.00	2.09
$R^{Monthly}_{CRSP} - R^{Big\ trans'ns}_{Buy\text{-}then\text{-}sell}$	$(\%)$	0.62	1.41	1.14	-3.11	0.36	5.75
$Volume^{Daily}_{CRSP}$	$(sh/10^5)$	2.26	1.16	0.00	0.65	1.88	5.99
$\$Vol^{Daily}_{CRSP}$	$(\$sh/10^6)$	77.39	41.34	0.00	22.07	66.22	209.7
All Transactions: Transactions associated with all market orders							
$(s+x\delta)/P$	$(\%)$	0.02	0.38	5.84	-1.22	0.00	5.74
x	$(\%)$	59.07	49.04	-0.37	0.00	100.0	100.0
Price Impact, δ	$(\$)$	-0.07	0.15	3.43	-0.75	-0.12	1.88
Half-spread, s	$(\$)$	0.12	0.04	0.51	0.00	0.12	0.50
$Volume^{Daily}_{TORQ}$	$(sh/10^5)$	0.33	0.15	0.00	0.08	0.31	0.70
Big Transactions: Order Size > (quoted depth or 9,999 shares)							
$(s+x\delta)/P$	$(\%)$	0.36	0.81	1.92	-0.77	0.00	4.15
x	$(\%)$	54.74	48.21	-0.20	0.00	91.38	100.0
Price Impact, δ	$(\$)$	0.05	0.28	1.38	-0.75	0.00	1.25
Half-spread, s	$(\$)$	0.12	0.07	2.04	0.06	0.12	0.50
$Volume^{Daily}_{TORQ}$	$(sh/10^5)$	0.09	0.11	0.00	0.01	0.05	0.56

Table 1 (continued)

Panel C Texas Industries Inc During TORQ Three Month Period

TXI		Mean	Std	Skew	Min	Median	Max
P^{Daily}_{CRSP}	$(\$)$	13.10	3.82	-5.87	-14.38	14.12	15.62
$R^{\text{Monthly}}_{CRSP}$	$(\%)$	-0.24	22.04	-0.03	-34.56	-2.54	33.33
$R^{\text{Monthly}}_{\text{CRSP}} - R^{\text{All trans'ns}}_{\text{Buy-then-sell}}$	$(\%)$	1.07	1.67	1.74	-2.24	0.82	7.47
$R^{\text{Monthly}}_{\text{CRSP}} - R^{\text{Big trans'ns}}_{\text{Buy-then-sell}}$	$(\%)$	1.16	1.91	1.15	-2.42	0.82	7.47
$Volume^{Daily}_{CRSP}$	$(\text{sh}/10^5)$	0.16	0.19	2.55	0.00	0.11	0.99
$\$Vol^{Daily}_{CRSP}$	$(\$\text{sh}/10^6)$	2.14	2.50	2.63	-0.00	1.70	12.59
All Transactions: Transactions associated with all market orders							
$(s+x\delta)/P$	$(\%)$	0.13	0.57	2.72	-1.11	0.00	3.33
x	$(\%)$	37.16	47.68	0.52	0.00	0.00	100.0
Price Impact, δ	$(\$)$	-0.04	0.10	0.68	-0.33	0.00	0.50
Half-spread, s	$(\$)$	0.12	0.06	1.33	0.00	0.12	0.50
$Volume^{Daily}_{TORQ}$	$(\text{sh}/10^5)$	0.03	0.03	1.49	0.00	0.03	0.14
Big Transactions: Order Size > (quoted depth or 9,999 shares)							
$(s+x\delta)/P$	$(\%)$	0.55	0.91	0.81	-1.09	0.00	2.85
x	$(\%)$	42.20	46.63	0.25	0.00	0.00	100.0
Price Impact, δ	$(\$)$	0.02	0.13	-0.15	-0.33	0.00	0.25
Half-spread, s	$(\$)$	0.12	0.07	3.06	0.06	0.12	0.50
$Volume^{Daily}_{TORQ}$	$(\text{sh}/10^5)$	0.03	0.03	1.98	0.00	0.02	0.14

Table 1 (continued)

Panel D 98 NYSE Firms During TORQ Three Month Period

(Mean of Individual Firm Statistics)

98 NYSE Firms	Mean	Std	Skew	Min	Median	Max
$R_{\text{CRSP}}^{\text{Monthly}}$ (%)	2.69	9.79	0.11	-15.37	2.06	24.05
$R_{\text{Buy-then-sell}}^{\text{Big trans'ns}}$ (%)	1.39	9.54	0.11	-16.22	0.62	22.09
$R_{\text{CRSP}}^{\text{Monthly}} - R_{\text{Buy-then-sell}}^{\text{Big trans'ns}}$ (%)	1.29	2.25	0.64	-3.92	1.04	8.26
s/P (%)	0.35	0.44	1.01	0.00	0.19	1.61
δ/P (%)	-0.04	1.02	0.45	-2.65	-0.04	3.02
$(s+x\delta)/P$ (%)	0.28	0.99	1.41	-1.75	-0.01	3.71
Price Impact, δ ($)	0.00	0.17	0.42	-0.48	-0.00	0.63
Half-spread, s ($)	0.12	0.04	1.05	0.06	0.11	0.26

Table 2: TORQ Data - Do CRSP and Realizable Returns Differ?

Tests of Data for Firms in TORQ

TORQ data covers only a three month period, so 63 overlapping monthly returns are calculated using 23 transaction days per "month". Overlapping the monthly returns induces high autocorrelation in the return series. To test the mean and median of the return differences, the realizable returns are first subtracted from the corrected returns. Autocorrelation is reduced by differencing, but mean first-order autocorrelation of the differenced series is still 8.55%, with a standard error of 1.76%. This autocorrelation in the differenced return series is removed by transforming the series (See Kennedy 1998).

Let $\Delta R_t \equiv R_t^{CRSP} - R_t^{\text{Realizable}} = a + e_t$ where $e_t = \rho e_{t-1} + u_t$ and $u_t \sim N(\mu, \sigma^2)$

Then $\Delta R_t^{\text{Transformed}} \equiv \Delta R_t - \rho \Delta R_{t-1} = a(1-\rho) + u_t$

The transformed return series has an autocorrelation of 0.31%, with a standard error of 0.24%. Cross-sectional heteroskedasticity is removed by dividing the transformed return differences for each firm by the estimated standard deviation. Finally, these homoskedastic, non-autocorrelated return differences are scaled to have the same overall mean as the raw differenced returns.

Table 2 shows the mean of the transformed difference between CRSP returns and buy-then-sell returns, estimated for pooled firm data using the generalized method of moments; Newey-West standard errors, with no moving average terms, are used to calculate the t-statistic (shown in parentheses). The null hypothesis of no difference between CRSP and realizable buy-then-sell returns is rejected at better than the 0.1% level.

Under the null for the non-parametric Wilcoxon signed rank test, the return differences are symmetrically distributed about zero. Table 2 shows the signed rank z-statistic for the difference between CRSP returns and realizable buy-then-sell returns. The two-sided p-value for the z-statistic is vanishingly small, indicating rejection of the null hypothesis at better than the 0.1% level – there is a difference between CRSP and buy-then-sell returns.

Results for 98 NYSE Firms

6174 pooled observations	$R_{\text{CRSP}}^{\text{Monthly}} - R_{\text{Buy-then-sell}}^{\text{Monthly}}$ (%)
Mean Difference in Firm Returns (t-statistic)	1.291 (39.56)
Signed Rank z-statistic for $\left(R_{\text{CRSP}}^{\text{Monthly}} - R_{\text{Buy-then-sell}}^{\text{Monthly}}\right)$	36.84

Table 3: Summary Statistics from Simulating Prices and Returns that Reflect Bid/Ask Spread and Price Impact

Table 3 shows mean statistics for 500 runs of 240 months of simulated transactions data. Parameters of the simulation are adjusted, as described in Appendix C, to match, on average, the mean and standard deviation of Boeing's CRSP returns, $R^{\text{Monthly}}_{\text{CRSP}}$, over the period 6207 - 9912, and the level and standard deviation of Boeing's transaction uncertainty, $(s+\delta)_{t+1}/(P+s+\delta)_t$, over the period 901101 - 910131.

Calibrated to BA		Mean	Std	Skew	Min	Max	Autocorr1
$P^{\text{Monthly}}_{\text{CRSP}}$	($)	471.00	541.30	1.18	30.10	2317.0	0.99
$R^{\text{Monthly}}_{\text{CRSP}}$	(%)	1.64	9.92	0.37	-23.67	36.23	-0.02
$R^{\text{Monthly}}_{\text{Buy-then-sell}}$	(%)	1.14	9.76	0.23	-24.80	32.10	0.01
$R^{\text{Monthly}}_{\text{Corrected}}$	(%)	1.12	9.90	0.14	-28.17	32.11	0.00
$s+\delta$	($)	0.19	0.54	6.69	0.00	6.07	-0.00
$\frac{(s+\delta)_{t+1}}{(P+s+\delta)_t}$	(%)	0.25	0.90	7.40	0.00	10.78	0.06

Table 4: Simulation Data - Do CRSP Returns Differ from Realizable Returns?

Tests of Data for a Simulated Firm

Because the asset pricing literature typically has 20 or more years of data for tests that use a monthly horizon, 20 years of "monthly" returns are simulated for these tests. The generalized method of moments is used to estimate the mean of the difference between CRSP returns and realizable buy-then-sell returns. Newey-West standard errors, with no moving average terms, are used to calculate t-statistics (shown in parentheses). Table 4 shows that the mean difference between CRSP returns and realizable buy-then-sell returns is significantly different from zero at better than the 1% level.

Under the null for the non-parametric Wilcoxon signed rank test, the return differences are symmetrically distributed about zero. Table 4 shows the mean signed rank z-statistics (standard errors in parentheses) for the difference between CRSP returns and realizable buy-then-sell returns. The two-sided p-values for the z-statistics are very small, indicating rejection of the null hypothesis at better than the 1% level – there is a difference between CRSP returns and realizable buy-then-sell returns.

Table 4 shows test results for a range of levels of the transaction uncertainty measure, $(s_{t+1} + x_{t+1}\delta_{t+1}) / \left[P_t + s_t + E_t(x_t\delta_t)\right]$. In each case, the mean and standard deviation of returns are maintained at 1.64% and 9.88% respectively. Results shown are the average for 500 simulation runs of 240 months (20 years) of data. Rejection of the null hypotheses of no difference in mean returns is consistent over a wide range of transaction uncertainty in returns.

(Cross-sectional Mean of Time-series Statistics)

Measure of Transaction Uncertainty	$(s_{t+1} + x_{t+1}\delta_{t+1}) / \left[P_t + s_t + E_t(x_t\delta_t)\right]$		
	0.24%*	0.12%	0.06%
Mean$\left(R_{\text{CRSP}}^{\text{Monthly}} - R_{\text{Buy-then-sell}}^{\text{Monthly}}\right)$(%) (t-statistic)	0.4887 (4.28)	0.2586 (4.30)	0.1390 (3.98)
Signed Rank z-statistic for $\left(R_{\text{CRSP}}^{\text{Monthly}} - R_{\text{Buy-then-sell}}^{\text{Monthly}}\right)$ (standard error)	10.0423 (0.5500)	8.4141 (0.6768)	6.3511 (0.7159)

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[*] Mean level of transaction uncertainty for Boeing (See Table 1A)

Tables 5: Theoretical Conversion of CRSP Returns into Realizable Buy-then-Sell Returns

On days with no trades, a trade with no price impact is assumed to have occurred. For example, $P_{t+1}-s_{t+1}$ replaces P_{t+1} in the numerator of a CRSP return when there was no trade on day $t+1$. The assumption of no price impacts for non-trade days tends to overstate the gross returns that would have been realized had a large trade actually occurred on the days in question.

Categories of CRSP Returns	**Converting CRSP to Buy-then-Sell**
$R^{S/B} \equiv \frac{P_{t+1}-s_{t+1}-\delta_{t+1}}{P_t+s_t+\delta_t}$	$R^{buy\&hold} \equiv R^{S/B} \equiv P_{t+1}^{Sell} / P_t^{Buy}$
$R^{S/S} \equiv \frac{P_{t+1}-s_{t+1}-\delta_{t+1}}{P_t-s_t-\delta_t}$	$R^{S/S}\left(\frac{P_t-s_t-\delta_t}{P_t+s_t+\delta_t}\right)$
$R^{B/B} \equiv \frac{P_{t+1}+s_{t+1}+\delta_{t+1}}{P_t+s_t+\delta_t}$	$R^{B/B}\left(\frac{P_{t+1}-s_{t+1}-\delta_{t+1}}{P_{t+1}+s_{t+1}+\delta_{t+1}}\right)$
$R^{B/S} \equiv \frac{P_{t+1}+s_{t+1}+\delta_{t+1}}{P_t-s_t-\delta_t}$	$R^{B/S}\left(\frac{P_t-s_t-\delta_t}{P_t+s_t+\delta_t}\right)\left(\frac{P_{t+1}-s_{t+1}-\delta_{t+1}}{P_{t+1}+s_{t+1}+\delta_{t+1}}\right)$
$R^{Q/B} \equiv \frac{P_{t+1}}{P_t+s_t+\delta_t}$	$R^{Q/B}\left(\frac{P_{t+1}-s_{t+1}}{P_{t+1}}\right)$
$R^{Q/S} \equiv \frac{P_{t+1}}{P_t-s_t-\delta_t}$	$R^{Q/S}\left(\frac{P_t-s_t-\delta_t}{P_t+s_t+\delta_t}\right)\left(\frac{P_{t+1}-s_{t+1}}{P_{t+1}}\right)$
$R^{S/Q} \equiv \frac{P_{t+1}-s_{t+1}-\delta_{t+1}}{P_t}$	$R^{S/Q}\left(\frac{P_t}{P_t+s_t}\right)$
$R^{B/Q} \equiv \frac{P_{t+1}+s_{t+1}+\delta_{t+1}}{P_t}$	$R^{B/Q}\left(\frac{P_t}{P_t+s_t}\right)\left(\frac{P_{t+1}-s_{t+1}-\delta_{t+1}}{P_{t+1}+s_{t+1}+\delta_{t+1}}\right)$
$R^{Q/Q} \equiv \frac{P_{t+1}}{P_t}$	$R^{Q/Q}\left(\frac{P_t}{P_t+s_t}\right)\left(\frac{P_{t+1}-s_{t+1}}{P_{t+1}}\right)$

S Trade occurred, initiated by a sell order.
B Trade occurred, initiated by a buy order.
Q No trade that day; – mid-quote price reported.

Tables 6: Operational Conversion of CRSP Returns into Realizable Buy-then-sell Returns

When converting CRSP returns, for days with no trades, a trade with no price impact is assumed to have occurred. For example, $P_{t+1} - s_{t+1}$ replaces P_{t+1} in the numerator of a CRSP return when there was no trade on day $t+1$. The assumption of no price impacts for non-trade days tends to overstate the gross returns that would have been realized had a large trade actually occurred on the days in question. In this sense, tests of the difference between CRSP and corrected returns understate the difference between CRSP and realizable buy-then-sell returns.

Categories of CRSP Returns that are Observable	Converting CRSP Returns to Buy-then-Sell Returns
$R^{T\,T} \equiv \frac{P_{t+1} \pm (s_{t+1} + \delta_{t+1})}{P_t \pm (s_t + \delta)_t}$	$R^{T\,T} \left\langle \frac{P_t^{Close}\ P_{t+1}^{Close}\ \left(P_t^{Close} - s_t - \delta_t\right)\left(P_{t+1}^{Close} - s_{t+1} - \delta_{t+1}\right)}{\left[\left(P_t^{Close}\right)^2 + (s_t + \delta_t)^2\right]\left[\left(P_{t+1}^{Close}\right)^2 + (s_{t+1} + \delta_{t+1})^2\right]} \right\rangle$
$R^{Q\,T} \equiv \frac{P_{t+1}}{P_t \pm (s_t + \delta_t)}$	$R^{Q\,T} \left\langle \frac{P_t^{Close}\ \left(P_t^{Close} - s_t - \delta_t\right)\left(P_{t+1}^{Close} - s_{t+1}\right)}{P_{t+1}^{Close}\left[\left(P_t^{Close}\right)^2 + (s_t + \delta_t)^2\right]} \right\rangle$
$R^{T\,Q} \equiv \frac{P_{t+1} \pm (s_{t+1} + \delta_{t+1})}{P_t}$	$R^{T\,Q} \left\langle \frac{P_t^{Close}\ P_{t+1}^{Close}\ \left(P_{t+1}^{Close} - s_{t+1} - \delta_{t+1}\right)}{\left(P_t^{close} + s_t\right)\left[\left(P_{t+1}^{Close}\right)^2 + (s_{t+1} + \delta_{t+1})^2\right]} \right\rangle$
$R^{Q\,Q} \equiv \frac{P_{t+1}}{P_t}$	$R^{Q\,Q} \left\langle \frac{P_t^{Close}\ \left(P_{t+1}^{Close} - s_{t+1}\right)}{\left(P_t^{Close} + s_t\right)\ P_{t+1}^{Close}} \right\rangle$

T Trade occurred, but trade direction is uncertain.

Q No trade that day; – mid-quote price reported.

$R^{T\,Q} \equiv P_{t+1}^{Trade} / P_t^{Quote}$

Table 7: TORQ Data – Do Corrected and Realizable Returns Differ?

Tests of TORQ Data

The TORQ data cover only a three month period, so 63 overlapping monthly returns are calculated using 23 transaction days per "month". Overlapping the monthly returns induces high autocorrelation in the return series. To test the mean and median of the return differences, the realizable returns are first subtracted from the corrected returns. Autocorrelation is reduced by differencing, but mean first-order autocorrelation of the differenced series is still 9.77%, with a standard error of 1.95%. This autocorrelation in the differenced return series is removed by transforming the series (See Kennedy 1998).

Let $\Delta R_t \equiv R_t^{Corrected} - R_t^{\text{Realizable}} = a + e_t$ where $e_t = \rho e_{t-1} + u_t$ and $u_t \sim N(\mu.\sigma^2)$

Then $\Delta R_t^{Transformed} \equiv \Delta R_t - \rho \Delta R_{t-1} = a(1-\rho) + u_t$

Autocorrelation of the transformed return series is 0.14%, with a standard error of 0.25%. Cross-sectional heteroskedasticity is removed by dividing the transformed return differences for each firm by the estimated standard deviation. Then, these homoskedastic, non-autocorrelated return differences are scaled to have the same overall mean as the raw differenced returns.

Table 7 shows the mean of the transformed difference between CRSP returns and realizable buy-then-sell returns, estimated for pooled firm data using the generalized method of moments; Newey-West standard errors, with no moving average terms, are used to calculate the t-statistic (shown in parentheses). With a zero t-statistic, the null hypothesis of no difference between CRSP and buy-then-sell returns cannot be rejected.

Under the null for the non-parametric Wilcoxon signed rank test, the return differences are symmetrically distributed about zero. Table 7 shows the signed rank z-statistic for the difference between CRSP returns and realizable buy-then-sell returns. The two-sided p-value for the z-statistic is 9% – there is weak evidence supporting a difference between CRSP and realizable buy-then-sell returns.

Results for 98 NYSE Firms

6174 pooled observations	$R_{\text{Corrected}}^{\text{Monthly}} - R_{\text{Buy-then-sell}}^{\text{Monthly}}$ (%)
Mean Difference in Firm Returns (t-statistic)	-0.0410 (1.40)
Signed Rank z-statistic for $\left(R_{\text{Corrected}}^{\text{Monthly}} - R_{\text{Buy-then-sell}}^{\text{Monthly}}\right)$	0.471

Table 8: Simulation Data - Do Corrected Returns Differ from Realizable Returns?

Because the asset pricing literature typically has 20 or more years of data for tests that use a monthly horizon, 20 years of "monthly" returns are simulated for these tests. The generalized method of moments is used to estimate the mean of the difference between corrected returns and buy-then-sell returns. First order autocorrelation of the return differences is -45%, so Newey-West standard errors, with one moving average term, are used to calculate t-statistics (shown in parentheses). Table 8 shows that the mean difference between CRSP returns and realizable buy-then-sell returns is not significantly different from zero.

Under the null for the non-parametric Wilcoxon signed rank test, the return differences are symmetrically distributed about zero. Table 8 shows the mean signed rank z-statistic for the difference between corrected returns and realizable buy-then-sell returns (standard errors in parentheses). The null hypothesis of no difference between corrected and realizable buy-then-sell returns cannot be rejected.

Table 8 shows test results for a range of levels of the transaction uncertainty measure. $\left(s_{t+1}+x_{t+1}\delta_{t+1}\right)/\left[P_t+s_t+E_t\left(x_t\,\delta_t\right)\right]$ In each case, the mean and standard deviation of returns are maintained at 1.64% and 9.88% respectively. Results are the average means for 500 simulation runs of 240 months of data. Failure to reject the null hypotheses of no difference in mean returns is consistent over this wide range of transaction uncertainty in returns.

(Cross-sectional Mean of Time-series Statistics)

Measure of Transaction Uncertainty	$\left(s_{t+1}+x_{t+1}\delta_{t+1}\right)/\left[P_t+s_t+E_t\left(x_t\,\delta_t\right)\right]$		
	0.24%[*]	0.12%	0.06%
Mean$\left(R^{\text{Monthly}}_{\text{Corrected}}-R^{\text{Monthly}}_{\text{Buy-then-sell}}\right)$(%) (t-statistic)	-0.0187 (0.35)	-0.0062 (0.21)	-0.0027 (0.16)
Signed Rank z-statistic for $\left(R^{\text{Monthly}}_{\text{Corrected}}-R^{\text{Monthly}}_{\text{Buy-then-sell}}\right)$ (standard error)	-0.0217 (0.3834)	-0.0181 (0.3582)	0.0076 (0.3560)

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[*] Mean level of transaction uncertainty for Boeing (See Table 1A)

Table 9: Summary Statistics for Data Used to Estimate Regression Models: Left Hand Side Variables

Statistics cover 98 NYSE firms over the period 10/01/1990 - 01/31/1991. There are up to 63 observation days per firm. Days with no transactions are excluded.

Daily mean half-spread, s, and price impact, δ, are obtained for each firm from TORQ quote and transaction data. Quotes and half-spread are those quoted at the time of an order. Price impact is the difference between transaction execution price and quoted price. Only "big" transactions, as defined in Table 1, are considered.

For each firm on a given trading day, one transaction is drawn randomly from all the day's transactions. The ordered probit model is estimated using the spread and price impact of these randomly drawn transactions. Using different random draws, the probit model was estimated 40 times and the coefficient estimates averaged. Table 9 shows statistics for the data used in one of the estimates.

Each observation for the weighted least squares (WLS) models is the mean of all the transactions a firm has on a given day. Observations are multiplied (weighted) by the square root of the number of transactions in the day to correct for heteroskedasticity.

The regressions are:[49]

$$\log(\text{half spread} - \$1/64) = X\beta + \varepsilon$$

$$\text{price impact} = X\gamma + \mu$$

Probit: Each Observation is a Random Draw from a Firm's Daily Transactions

3699 pooled observations	**Mean**	**Std**	**Skew**	**Min**	**Median**	**Max**
Half-Spread ($)	0.115	0.063	3.036	0.016	0.125	1.000
Price Impact ($)	0.004	0.268	1.382	-2.625	0.000	3.000

Weighted LS: Each Observation is the Mean of a Firm's Daily Transactions

3699 pooled observations	**Mean**	**Std**	**Skew**	**Min**	**Median**	**Max**
Half-Spread ($)	0.182	0.122	3.149	0.016	0.153	1.908
Price Impact ($)	0.014	0.329	0.972	-4.619	0.000	5.438

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[49] For the WLS spread regressions, the dependent variable is log(half-spread – 1/64). When the model is estimated using log(half-spread), the distribution of fitted values includes spreads of zero. Using log(half-spread – 1/64) produces a distribution of fitted values that is strictly positive and otherwise similar to the TORQ sample.

Table 10: Summary Statistics for Data Used to Estimate Regression Models: Right Hand Side Variables

Statistics cover 10/01/1990 - 01/31/1991 using daily CRSP data and NYSE files. There are up to 63 observation days for each of 98 NYSE firms. Days with no transactions are excluded. AskHi is the highest sale price of the day. On no-trade days, AskHi is the closing ask quote. BidLo is the lowest sale price of the day. On no-trade days, BidLo is the closing bid quote. An excess value is a daily value, less the mean value for days -22 to -1. Market statistics use the NYSE daily share volume, S&P500 daily index return, the six month T-bill secondary market, annualized yield, the one year and 10 year government bond annualized yields, and Moody's BAA and AAA corporate bond annualized yields.

Share turnover (t/o) is volume divided by shares outstanding. aH-bL is askHi-bidLo. Vol is share volume. L1(•) is lag one of its argument. D(•) is a dummy variable, set to one when the argument is true; set to zero otherwise. Long yield spread is the yield difference between 10 year and one year government bonds. Quality yield spread is the yield difference between BAA and AAA corporate bonds. P is closing price. P*t/o is dollar turnover.

Results for 98 NYSE Firms

3699 pooled observations		**Mean**	**Std**	**Skew**	**Min**	**Median**	**Max**
Normalized NYSE volume		0.11	1.43	-1.06	-7.67	-0.06	4.74
Std(market return)	(%)	1.01	0.23	0.09	0.68	1.02	1.44
T-bill 6 month yield	(%)	6.99	0.35	-0.15	6.45	7.04	7.43
Long yield spread	(%)	1.19	0.23	0.36	0.87	1.17	1.63
Quality yield spread	(%)	1.37	0.06	-0.05	1.24	1.37	1.49
Share turnover (t/o)	(%)	0.31	0.57	1.05	+0.00	0.17	15.1
Excess share turnover	(%)	0.03	0.44	1.07	-2.74	-0.02	12.6
Std(share turnover)	(%)	0.21	0.30	0.56	+0.00	0.13	3.18
Std(firm return)	(%)	2.38	1.84	4.58	0.60	1.98	22.04
D(vol>0)*(aH-bL)/P		2.98	3.11	6.15	0.00	2.19	56.82
D(vol>0)*(P*t/o)/(aH-bL)	(10^{-3})	113.2	147.3	5.74	0.83	73.64	2511
D(vol>0)*t/o/(aH-bL)	$(10^{-3}\ \$^{-1})$	6.14	12.19	8.93	0.06	3.05	233.6
D(aH-bL=0)		0.01	0.10	9.58	0.00	0.00	1.00
D(aH-bL=0)*Lag(aH-bL)/P		0.02	0.28	18.01	0.00	0.00	7.27
D(aH-bL=0)*std(firm return)	(%)	0.03	0.30	18.19	0.00	0.00	8.92
Log L1(P)	($)	3.09	0.76	-1.10	-0.90	3.18	4.84
Var(firm return) * Lag(P) * t/o	$(\%^2\ \$)$	0.05	0.14	12.66	0.00	0.02	3.14
L1(P) * t/o	$(10^{-3}\ \$)$	71.72	104.7	5.72	0.10	37.19	2169
L1[D(vol>0)*(P*t/o)/(aH-bL)]	(10^{-3})	115.6	157.3	7.13	0.00	75.24	3368
L1[D(aH-bL=0)]		0.02	0.14	7.06	0.00	0.00	1.00

Table 11: Ordered Probit Regression Models of Spread & Price Impact

The probit model is an unobservable equation: $y^* = X\beta + \varepsilon^*$ for $\varepsilon^* \sim N(0, \sigma_\varepsilon^2)$
and an observable series for the dependent variable:

$$\begin{aligned} y &= y_1 && \text{for } y^* < 0 \\ &= y_n && \text{for } 0 \le y^* < \alpha_{n-1} \qquad n = 2, \ldots, N \\ &= y_{N+1} && \text{for } \alpha_N \le y^* \end{aligned}$$

where: y^* is an unobservable, continuous dependent variable for a particular firm
y_n is the observable, discrete dependent variable
β is assumed to be constant across firms and across time

Some y's are placed in a coarser "bin" to ensure enough observations for estimation of each discrete y value modeled. For example, observations of half-spread of $3/16 and larger are combined in one bin, centered at the weighted average value of the bins. Coefficient estimates are shown, with heteroskedasticity-consistent t-statistics in parentheses. Coefficients are the average of 40 estimates of the ordered probit model, using independent random draws to construct the dependent variables.

Center of y bins ($)		0.0622 0.1248 0.2081		-0.2090 -0.0001 0.2502	
98 NYSE Firms (Pooled)		Half-Spread		Price Impact	
Normalized NYSE volume		-0.0664	(0.06)	-1.2231	(1.17)
Std(market return)	(%)	-0.4828	(3.21)	-0.0201	(1.48)
T-bill 6 month yield	(%)	-48.2781	(3.21)	-40.7060	(2.74)
Long yield spread	(%)	49.4608	(4.10)	30.1643	(2.52)
Quality yield spread	(%)	89.8840	(4.71)	16.8550	(0.88)
Share turnover (t/o)	(%)	-200.4910	(3.48)	46.7430	(0.81)
Excess share turnover	(%)	-0.0742	(4.78)	0.0125	(0.71)
Std(share turnover)	(%)	0.1246	(9.98)	-0.0112	(0.69)
Std(firm return)	(%)	0.1093	(7.35)	0.0088	(0.54)
D(vol>0)*(aH-bL)/P		-3.5904	(1.70)	-0.3991	(0.20)
D(vol>0)*(P*t/o)/(aH-bL)	(10^{-3})	0.2450	(0.25)	3.1657	(3.92)
D(vol>0)*t/o/(aH-bL)	(10^{-3} $\$^{-1}$)	0.2676	(0.84)	-0.2569	(1.07)
D(aH-bL=0)		-34.4040	(5.94)	-0.6018	(0.18)
D(aH-bL=0)*Lag(aH-bL)/P		2.9128	(4.81)	-0.9110	(2.27)
D(aH-bL=0)*std(firm return)	(%)	10.7754	(0.67)	23.1628	(2.12)
Log L1(P)	($)	-63.0309	(5.39)	5.3316	(0.30)
Var(firm return) * Lag(P) * t/o	($\%^2$ $)	0.2462	(5.01)	0.1149	(2.42)
L1(P) * t/o	(10^{-3} $)	1.2939	(5.12)	-0.4187	(1.68)
L1[D(vol>0)*(P*t/o)/(aH-bL)]	(10^{-3})	-2.2027	(4.99)	0.6308	(1.77)
L1[D(aH-bL=0)]		-0.3905	(3.03)	-0.1354	(0.97)
Normalized NYSE volume		0.9816	(7.06)	0.5044	(4.62)

Table 12: Weighted Regression Models of Mean Spread and Price Impact

The model estimated is: $\bar{y} = X\,\beta + \bar{\varepsilon}$ pooling data for all firms

Table 10 describes the independent variables, X. Table 9 describes the dependent variables, $\bar{y}$. Heteroskedasticity-consistent t-statistics are given in parentheses.

98 NYSE Firms (Pooled)		**Log(Half-Spread)**		**Price Impact**	
Intercept		-2.3782	(4.33)	-0.2936	(1.12)
Normalized NYSE volume		13.0927	(1.60)	0.0164	(0.00)
Std(market return)	(%)	-12.9494	(1.18)	-13.4632	(3.63)
T-bill 6 month yield	(%)	9.6004	(1.11)	7.0638	(2.24)
Long yield spread	(%)	18.2559	(1.53)	10.2238	(2.31)
Quality yield spread	(%)	-29.3013	(0.87)	-23.9512	(1.74)
Share turnover (t/o)	(%)	-40.2353	(2.45)	9.1761	(3.13)
Excess share turnover	(%)	70.2840	(5.80)	-6.2808	(2.73)
Std(share turnover)	(%)	68.0335	(5.76)	-1.6770	(0.80)
Std(firm return)	(%)	-1.7688	(1.26)	0.6503	(2.17)
D(vol>0)*(aH-bL)/P		-1.2494	(1.02)	0.4963	(2.54)
D(vol>0)*(P*t/o)/(aH-bL)	(10^{-3})	1.0354	(2.85)	-0.0438	(0.77)
D(vol>0)*t/o/(aH-bL)	(10^{-3} \$$^{-1}$)	-45.3758	(-4.22)	-0.5740	(0.99)
D(aH-bL=0)		0.5988	(2.25)	-0.1890	(0.92)
D(aH-bL=0)*Lag(aH-bL)/P		0.1705	(0.04)	21.6655	(2.55)
D(aH-bL=0)*std(firm return)	(%)	7.2898	(0.59)	-16.3370	(1.43)
Log L1(P)	(\$)	-0.1050	(2.64)	0.0354	(3.23)
Var(firm return) * Lag(P) * t/o ($\%^2$ \$)		0.9875	(3.31)	-0.1517	(3.64)
L1(P) * t/o	(10^{-3} \$)	-0.7892	(3.39)	0.0584	(1.03)
L1[D(vol>0)*(P*t/o)/(aH-bL)](10^{-3})		0.0213	(0.26)	-0.0656	(1.78)
L1[D(aH-bL=0)]		0.5712	(7.58)	0.2197	(1.74)
	Adjusted R^2	84.39		8.19	

Table 13: Summary Statistics for Corrected CRSP Returns

Using historical values of the regressors, daily half-spread, s, and half-spread plus price impact, $s+\delta$, are fitted to the models in Tables 11 and 12. Using historical CRSP prices and the fitted values of half-spread and price impact in the correction factors of Table 6, monthly corrected returns are calculated for the 98 NYSE firms from the TORQ data.

BA: 62/07 - 00/12		**Mean**	**Std**	**Skew**	**Min**	**Median**	**Max**
$R^{\text{Monthly}}_{CRSP}$	(%)	0.81	11.32	-0.59	-56.07	1.11	48.44
$R^{\text{Monthly}}_{Corrected}$	(%)	0.16	11.38	-0.61	-56.07	0.45	41.57
$(s+x\delta)/P$	(%)	0.39	0.68	1.90	-1.56	0.23	4.39
Price Impact, δ	($)	0.05	0.17	0.65	-0.50	0.04	0.76
Half - spread, s	($)	0.10	0.03	2.63	0.06	0.09	0.30

FDX: 78/04 - 00/12		**Mean**	**Std**	**Skew**	**Min**	**Median**	**Max**
$R^{\text{Monthly}}_{CRSP}$	(%)	0.92	11.78	-0.51	-51.39	0.33	37.48
$R^{\text{Monthly}}_{Corrected}$	(%)	0.28	11.71	-0.53	-51.38	-0.08	37.83
$(s+x\delta)/P$	(%)	0.40	0.51	1.23	-0.79	0.24	2.15
Price Impact, δ	($)	0.08	0.19	0.75	-0.49	0.05	0.78
Half - spread, s	($)	0.10	0.04	5.80	0.06	0.09	0.53

TXI: 64/06 - 00/12		**Mean**	**Std**	**Skew**	**Min**	**Median**	**Max**
$R^{\text{Monthly}}_{CRSP}$	(%)	0.64	10.57	-0.06	-52.79	0.00	38.24
$R^{\text{Monthly}}_{Corrected}$	(%)	-0.37	10.75	-0.04	-52.89	-0.57	40.17
$(s+x\delta)/P$	(%)	0.64	1.05	-0.02	-5.64	0.50	4.84
Price Impact, δ	($)	0.05	0.19	0.56	-0.55	0.03	0.76
Half - spread, s	($)	0.13	0.09	8.27	0.05	0.11	1.22

98 NYSE Firms		**Mean**	**Std**	**Skew**	**Min**	**Median**	**Max**
$R^{\text{Monthly}}_{CRSP}$	(%)	0.74	11.02	-0.34	-49.96	0.40	47.32
$R^{\text{Monthly}}_{Corrected}$	(%)	-0.28	11.32	-0.31	-50.81	-0.50	47.22
$(s+x\delta)/P$	(%)	0.60	1.47	0.97	-4.69	0.39	7.78
Price Impact, δ	($)	0.03	0.18	0.14	-0.68	0.02	0.68
Half - spread, s	($)	0.12	0.13	6.01	0.05	0.10	1.50

Table 14: Regressions to Remove Possible Scaling Effects of Return Correction

For 639 NYSE firms, fitted spread and price impact are used with CRSP returns to calculate corrected returns and linearly corrected returns. The corrected returns are then regressed, firm-by-firm, on the CRSP returns. Any sensitivity of CRSP returns to risk factors should be purged from the regression residuals. The net impact on factor sensitivity resulting from the conversion from CRSP to corrected returns should be captured in the residuals.

Table 14 shows the mean coefficients and statistics from the 639 regressions.

639 Firms	Mean Coefficient			
	Corrected Returns		Linearly-Corrected Returns	
Intercept	-0.008	(7.06)	-0.008	(6.95)
R_{CRSP}	0.999	(155.04)	1.000	(155.11)
R^2	0.959		0.959	

Table 15: Tests of Covariance with Risk Factors

The covariance of CRSP monthly returns and of corrected returns with a risk factor is calculated for five traded risk factors. The S&P500 return is the value-weighted return for the S&P500 index. An equal-weighted portfolio of the corrected returns for the 639 stocks (EW Return) is intended to deal with concerns about using the S&P500 return, which is mismeasured because it is calculated using CRSP returns. The EW Return differs slightly for each of the 639 firms. The EW Return used as a risk factor for firm k contains the returns of the 638 stocks, excluding the return of firm k. The short term spread (Short Spread) is the difference between the mean monthly yields for 1 year and three month treasury bills. The long term spread (Long Spread) is the difference between the mean monthly yields for 10 year and one year government bonds. The quality spread is the difference in monthly yields between BAA and AAA corporate bonds.

Because the factor risk may include innovations to the risk factors, as well as their levels, the covariance is also tested using the standardized values of the spread risk factors as three additional risk factors. These spread factors are standardized by using the twelve lagging observations to calculate a mean value and standard deviation.

Both conditional and unconditional tests of covariance are performed using pooled data for 639 firms. Newey-West standard errors, with no moving average terms, are used to calculate t-statistics (shown in parentheses). The conditioning variables are the dividend yield on the S&P500 value-weighted index and the yield on 6 month t-bills. Both conditioning variables are detrended by subtraction the 12 month lagged mean.

Panel A shows GMM estimates of the covariances of the factors with CRSP monthly returns and the results of two-sample t-tests for whether or not correcting CRSP monthly returns makes a non-zero change in the covariance of the returns with risk factors.

Panel B shows results of two-sample t-tests for whether or not corrected returns, less returns fitted from a regression on CRSP returns, have a non-zero covariance with risk factors. Regression residuals from Table 14 are used. The fitted returns should capture any scale effects and correlation of CRSP returns with the factors, so the correlation of the residuals is zero under the null hypothesis.

Panel C shows results of two-sample t-tests for whether or not linearly-corrected returns, less returns fitted from a regression on CRSP returns, have a non-zero covariance with risk factors. If the transaction uncertainty variables are uncorrelated with the factors, then linear correction of returns should not induce spurious correlation with the factors. Regression residuals from Table 14 are used. The fitted returns should capture any scale effects and correlation of CRSP returns with the factors, so the correlation of the residuals is zero under the null hypothesis.

Table 15 (continued)

Panel A – Covariance with Risk Factors: CRSP versus Corrected Returns

131,488 Observations	$Cov[factor, R_{CRSP}]$ (Covariance x 10^6)	$Cov[factor, R_{Corrected} - R_{CRSP}]$ (Covariance x 10^6)	
Factor Levels	**Unconditional**	**Unconditional**	**Conditional**
S&P500 Return	1,941	49.18	65.99
	(104.0)	(12.8)	(17.1)
EW Return	2,748	269.5	266.2
	(106.8)	(55.0)	(54.3)
Short Spread	-1.068	-1.531	-1.659
	(10.0)	(45.1)	(48.8)
Long Spread	2.821	4.359	3.930
	(10.4)	(60.2)	(54.3)
Quality Spread	2.936	-1.649	-1.570
	(24.6)	(48.1)	(45.8)
Factor Innovations	**Unconditional**	**Unconditional**	**Conditional**
Short Spread	-11,180	-405.3	-1,897
	(23.1)	(3.3)	(15.3)
Long Spread	229.5	-3,656	-2,828
	(0.4)	(26.6)	(20.5)
Quality Spread	11,160	-2,837	-1,504
	(19.7)	(18.8)	(10.0)

Table 15 (continued)

Panel B – Covariance with Risk Factors: CRSP versus Residuals of Corrected Returns Regressed on CRSP Returns

131.488 Observations	$Cov\left[factor, R_{Corrected} - \left(a + b * R_{CRSP}\right)\right]$ (Covariance x 10^6)	
Factor Levels	**Unconditional**	**Conditional**
S&P500 Return	47.25	64.60
	(13.1)	(17.9)
EW Return	255.9	256.7
	(55.3)	(55.4)
Short Spread	-1.295	-1.410
	(42.5)	(46.3)
Long Spread	4.292	4.020
	(65.0)	(60.9)
Quality Spread	-1.184	-1.131
	(37.8)	(36.1)
Factor Innovations	**Unconditional**	**Conditional**
Short Spread	-578.1	-1,937
	(5.0)	(16.8)
Long Spread	-3,219	-2,537
	(25.4)	(20.0)
Quality Spread	-2,819	-1.661
	(20.2)	(11.9)

Table 15 (continued)

Panel C – Covariance with Risk Factors: CRSP versus Residuals of Linearly-Corrected Returns Regressed on CRSP Returns

131,488 Observations	$Cov\left[factor, R_{\substack{\text{Linearly-}\\\text{Corrected}}} - \left(a + b * R_{\text{CRSP}}\right)\right]$	
	(Covariance x 10^6)	
Factor Levels	**Unconditional**	**Conditional**
S&P500 Return	47.25	64.60
	(13.2)	(18.0)
EW Return	257.8	258.6
	(55.9)	(56.0)
Short Spread	-1.291	-1.404
	(42.9)	(46.7)
Long Spread	4.316	4.054
	(66.1)	(62.1)
Quality Spread	-1.164	-1.122
	(37.4)	(36.0)
Factor Innovations	**Unconditional**	**Conditional**
Short Spread	-552.7	-1,890
	(4.8)	(16.6)
Long Spread	-3,189	-2,504
	(25.4)	(19.9)
Quality Spread	-2,755	-1,626
	(19.9)	(11.7)

Table 16: Correlation of Factor Loadings for Closing-Price Returns with Factor Loadings for Buy-then-Sell and Corrected Returns

Closing-price, buy-then-sell and corrected returns calculated from daily TORQ data (01/11/1990 – 31/01/1991) are used to estimate factor loadings for 97 NYSE firms from the following time-series equation:

$$R - E(R) = \mu + \gamma\left[\text{Factor} - E(\text{Factor})\right] + \varepsilon$$

The corrected returns use mean half-spread, s, and price impact, δ, fitted to coefficients from the probit and weighted least squares regressions of Tables 11 and 12.

Table 16 shows the sample estimate of the cross-sectional correlation of the factor loadings for different return types. A 95% confidence interval (shown in parentheses) is derived from a bootstrapped distribution, using 5,000 iterations.

Correlation	$\rho\left(\gamma_{\text{Closing-price}}, \gamma_{\text{Buy-then-sell}}\right)$	$\rho\left(\gamma_{\text{Closing-price}}, \gamma_{\text{Corrected}}\right)$	$\rho\left(\gamma_{\text{Corrected}}, \gamma_{\text{Buy-then-sell}}\right)$
S&P500 Value Wtd	0.91 (0.81, 0.96)	0.83 (0.67, 0.94)	0.87 (0.78, 0.94)
Short Yield Spread	0.68 (0.44, 0.88)	0.56 (0.19, 0.86)	0.69 (0.43, 0.86)
Long Yield Spread	0.51 (0.21, 0.89)	0.42 (0.11, 0.83)	0.83 (0.52, 0.96)
Quality Yield Spread	0.49 (0.29, 0.87)	0.32 (0.19, 0.83)	0.87 (0.45, 0.96)
Short Spread Innovation	0.78 (0.66, 0.86)	0.70 (0.45, 0.86)	0.63 (0.32, 0.88)
Long Spread Innovation	0.81 (0.60, 0.93)	0.79 (0.57, 0.92)	0.59 (0.30, 0.84)
Quality Spread Innovation	0.33 (0.22, 0.78)	0.21 (0.17, 0.77)	0.94 (0.58, 0.98)

Table 17: Summary Statistics for Data to Test Effects of Beta, Residual Risk, Size, Bid/Ask Spread and Price Impact on Stock Returns

Daily CRSP data (1972/09 - 2000/12) is used to form 49 portfolios from 656 NYSE firms. Mean half-spread, s, and price impact, δ, are fitted to coefficients from the probit and weighted least squares regressions (Tables 11 and 12). Gross and corrected returns use factors that minimize the mean square error between the desired return type and the converted CRSP return. Size is the log of CRSP monthly closing price times number of shares outstanding. Statistics are calculated for each portfolio, then averaged across portfolios.

	Mean	**Std**	**Skew**	**Min**	**Q1**	**Q2**	**Q3**	**Max**
$R^{\text{Monthly}}_{\text{CRSP}}$ (%)	0.44	6.93	0.12	-28.75	-3.68	0.33	4.42	30.22
$R^{\text{Monthly}}_{\text{Gross}}$ (%)	-0.19	7.12	0.16	-29.16	-4.38	-0.23	3.90	31.89
$R^{\text{Monthly}}_{\text{Corrected}}$ (%)	-0.62	7.31	0.10	-29.42	-4.94	-0.66	3.63	31.47
β_{CRSP}	0.95	0.14	-0.07	0.65	0.85	0.96	1.06	1.25
β_{Gross}	0.95	0.14	-0.00	0.65	0.84	0.95	1.06	1.25
$\beta_{\text{Corrected}}$	0.95	0.14	-0.06	0.65	0.85	0.95	1.05	1.24
σ_{CRSP} (%)	4.33	0.62	0.28	3.17	3.90	4.27	4.76	5.77
σ_{Gross} (%)	4.37	0.63	0.22	3.15	3.97	4.32	4.83	5.78
$\sigma_{\text{Corrected}}$ (%)	4.45	0.64	0.22	3.21	3.85	4.41	4.87	5.88
s/P (%)	0.54	0.30	1.31	0.16	0.31	0.46	0.71	1.82
δ/P (%)	0.68	0.97	0.49	-2.58	0.06	0.51	1.20	4.42
Size (log \$)	14.81	1.44	0.25	11.59	13.74	14.79	15.79	18.72

Table 18: Correlations Among Variables in Amihud & Mendelson Study

Panel A - CRSP Returns

Correlations are for 72/09 – 00/12 and for sub-periods 72/09 – 86/10 and 86/11 – 00/12. Variables are measured monthly and 49 portfolios are formed using CRSP returns.

The variables are portfolio excess return, R, (average monthly return on the stocks in the portfolio in excess of the six month T-bill rate), systematic risk, β, non-systematic risk, σ, average log capitalization, *Size*, average proportional bid/ask spread, s/P, and price impact, δ/P, and their squares. β and σ are estimated by regressing portfolio excess monthly returns on excess returns of the equally-weighted market portfolio for the 60 months preceding the current month.

Correlation	(x100)	β	σ	*Size*	s/P	δ /P	$(s/P)^2$	$(\delta/P)^2$
1972 – 2000	R	1.8	-0.1	-1.4	0.7	-9.4	1.2	5.6
1987 – 2000		1.8	-1.1	-2.6	4.6	-10.1	4.2	7.4
1972 – 1986		1.3	1.1	-2.7	2.1	-10.5	3.0	7.8
1972 – 2000	β		3.0	-4.7	-0.4	-0.1	-1.8	0.0
1987 – 2000			10.9	-5.8	8.9	0.9	7.8	2.4
1972 – 1986			9.6	-2.4	-3.6	-0.2	-3.6	1.9
1972 – 2000	σ			-12.9	12.4	-10.5	8.6	0.9
1987 – 2000				-6.2	22.7	0.8	21.5	7.2
1972 – 1986				-5.8	1.7	-25.6	1.0	-7.0
1972 – 2000	*Size*				-51.6	-12.3	-43.1	-25.6
1987 – 2000					-31.2	-3.6	-28.5	-13.0
1972 – 1986					-35.5	6.0	-29.9	-14.1
1972 – 2000	(s/P)					48.7	95.4	56.7
1987 – 2000						40.1	98.4	25.4
1972 – 1986						41.5	96.4	54.9
1972 – 2000	δ /P						48.5	28.7
1987 – 2000							41.4	-23.3
1972 – 1986							41.3	30.3
1972 – 2000	$(s/P)^2$							58.9
1987 – 2000								26.8
1972 – 1986								55.3

Table 18 (continued)

Panel B - Gross Returns

Correlations are for 72/09 – 00/12 and for sub-periods 72/09 – 86/10 and 86/11 – 00/12. Variables are measured monthly and 49 portfolios are formed using CRSP returns.

The variables are portfolio excess return, R, (average monthly return on the stocks in the portfolio in excess of the six month T-bill rate), systematic risk, β, non-systematic risk, σ, average log capitalization, *Size*, average proportional bid/ask spread, s/P, and price impact, δ/P, and their squares. β and σ are estimated by regressing portfolio excess monthly returns on excess returns of the equally-weighted market portfolio for the 60 months preceding the current month.

Correlation	(x100)	β	σ	*Size*	s/P	δ/P	(s/P)^2	(δ/P)^2
1972–2000	R	0.1	0.6	0.8	-5.2	-18.4	-4.0	2.4
1987–2000		1.0	-1.5	-1.7	1.1	-15.0	1.1	8.1
1972–1986		0.2	2.5	-2.8	-2.7	-20.1	-1.2	4.3
1972–2000	β		3.5	-2.5	-0.5	0.7	-1.9	-0.1
1987–2000			12.5	-3.2	10.1	0.6	9.4	1.5
1972–1986			8.3	-1.4	-3.7	0.7	-3.7	1.4
1972–2000	σ			-13.3	14.0	-10.6	9.8	1.6
1987–2000				-2.5	22.1	2.4	20.9	6.5
1972–1986				-7.7	5.1	-25.2	3.6	-5.8
1972–2000	*Size*				-51.6	-12.2	-43.2	-25.4
1987–2000					-32.1	-4.6	-29.7	-12.8
1972–1986					-35.0	6.2	-29.7	-14.1
1972–2000	(s/P)					48.8	95.4	56.5
1987–2000						40.3	98.5	25.1
1972–1986						41.5	96.3	54.8
1972–2000	δ/P						48.4	29.2
1987–2000							41.5	-23.1
1972–1986							41.2	30.7
1972–2000	(s/P)^2							58.6
1987–2000								26.3
1972–1986								55.2

Table 18 (continued)

Panel C - Corrected Returns

Correlations are for 72/09 – 00/12 and for sub-periods 72/09 – 86/10 and 86/11 – 00/12. Variables are measured monthly and 49 portfolios are formed using CRSP returns.

The variables are portfolio excess return, R, (average monthly return on the stocks in the portfolio in excess of the six month T-bill rate), systematic risk, β, non-systematic risk, σ, average log capitalization, *Size*, average proportional bid/ask spread, s/P, and price impact, δ/P, and their squares. β and σ are estimated by regressing portfolio excess monthly returns on excess returns of the equally-weighted market portfolio for the 60 months preceding the current month.

Correlation	(x100)	β	σ	*Size*	s/P	δ/P	(s/P)^2	(δ/P)^2
1972 – 2000	R	-0.4	1.5	3.1	-9.9	-22.9	-7.8	-1.1
1987 – 2000		1.9	-1.6	-1.8	-2.2	-17.6	-2.1	7.1
1972 – 1986		0.3	4.4	-2.6	-5.0	-23.6	-3.0	1.3
1972 – 2000	β		9.8	-2.8	0.3	1.3	-1.1	0.3
1987 – 2000			21.4	-3.8	10.9	0.9	10.3	2.2
1972 – 1986			13.2	-2.4	-2.3	0.9	-2.8	2.0
1972 – 2000	σ			-13.0	11.9	-10.1	7.9	1.9
1987 – 2000				-4.1	22.0	0.4	20.5	6.8
1972 – 1986				-8.6	2.9	-23.6	1.8	-4.8
1972 – 2000	*Size*				-51.9	-12.4	-43.5	-25.3
1987 – 2000					-32.9	-4.2	-30.4	-13.0
1972 – 1986					-35.4	6.2	-30.1	-14.2
1972 – 2000	(s/P)					48.8	95.4	56.5
1987 – 2000						40.4	98.5	24.7
1972 – 1986						41.5	96.3	54.9
1972 – 2000	δ/P						48.4	29.7
1987 – 2000							41.5	-22.7
1972 – 1986							41.2	31.7
1972 – 2000	(s/P)^2							58.7
1987 – 2000								26.0
1972 – 1986								55.2

Table 19: The Effects of Beta, Residual Risk, Size, Spread and Price Impact on Stock Returns – Pooled Regressions

To test Merton (1987), returns in excess of the T-bill rate are regressed on beta, residual risk, size and liquidity cost. Merton predicts positive coefficients. Amihud and Mendelson's Proposition 2 (AM 1986) predicts that expected *Gross* returns (holding period cash flows divided by buying transaction price) increase with liquidity costs. AM use proportional bid/ask spread to proxy liquidity cost. Price impact is added here as an additional measure of illiquidity. The squares of half-spread and of price impact are included as regressors to test Amihud and Mendelson's prediction that expected returns increase less for less liquid assets.

AM use monthly CRSP returns to proxy for Gross returns. Here, Gross returns and corrected returns also are constructed using fitted values of spread and price impact. A fourth return, R6, is constructed as a linear combination of five parts Gross return and one part corrected return. This return proxies the unrealized monthly return for an investor with a 6 month holding period.

For each return type (CRSP, Gross, corrected), three regressions are performed, using the particular return type as the dependent regression variable, and to form an equally-weighted market portfolio for calculating independent regression variables beta and sigma. This equally-weighted portfolio of the stock returns is used in a 60 month moving window to estimate beta for each stock. All regressions use pooled time series, cross-sectional data with yearly intercept dummies, DY. The three regressions have the form:

$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 Ln(\text{size})_p + \gamma_4 s/P_p + \varepsilon_p$$

$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 Ln(\text{size})_p + \gamma_4 s/P_p + \gamma_5 \delta/P_p + \varepsilon_p$$

$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 Ln(\text{size})_p + \gamma_4 s/P_p + \gamma_5 \delta/P_p + \gamma_6 (s/P_p)^2 + \gamma_7 (\delta/P_p)^2 + \varepsilon_p$$

Regression results are shown in Table 19. Newey-West standard errors are used in calculating t-statistics (shown in parentheses). For monthly returns, OLS estimates appear in Panel A; GLS estimates appear in Panel B. Panel C lists OLS and GLS estimates for quarterly CRSP and corrected returns.

Table 19 (continued)

Panel A: OLS Regressions – Monthly Returns

Returns	β	σ	*Size*	s/P	$(s/P)^2$	δ/P	$(\delta/P)^2$	Adj.R^2
CRSP	0.007	-0.190	-0.001	0.376				4.64
	(3.22)	(2.31)	(3.13)	(3.27)				
AM Gross	0.006	-0.211	-0.001	-0.453				6.57
	(2.55)	(2.35)	(2.81)	(3.31)				
R6	0.005	-0.187	-0.001	-0.564				7.08
	(2.24)	(2.05)	(2.92)	(4.05)				
Corrected	0.002	-0.087	-0.001	-1.030				9.50
	(0.67)	(0.96)	(3.26)	(7.23)				
CRSP	0.007	-0.211	-0.001	0.439		-0.277		4.79
	(3.46)	(2.56)	(2.77)	(3.73)		(3.94)		
AM Gross	0.008	-0.322	-0.001	-0.219		-0.903		8.02
	(3.43)	(3.61)	(1.81)	(1.57)		(11.91)		
R6	0.007	-0.312	-0.001	-0.300		-0.983		8.76
	(3.23)	(3.45)	(1.79)	(2.11)		(12.73)		
Corrected	0.005	-0.227	-0.001	-0.705		-1.269		12.15
	(1.90)	(2.59)	(1.69)	(4.86)		(15.54)		
CRSP	0.007	-0.242	-0.001	0.532	-4.138	-0.290	3.982	4.87
	(3.42)	(2.92)	(2.11)	(1.90)	(0.65)	(4.12)	(2.50)	
AM Gross	0.007	-0.279	-0.001	-0.748	9.077	-0.911	3.659	8.10
	(3.01)	(3.25)	(2.47)	(2.54)	(1.28)	(11.95)	(2.26)	
R6	0.006	-0.261	-0.001	-0.887	10.489	-0.990	3.352	8.83
	(2.80)	(3.00)	(2.57)	(2.88)	(1.40)	(12.72)	(1.98)	
Corrected	0.004	-0.136	-0.001	-1.559	17.711	-1.257	0.200	12.26
	(1.62)	(1.59)	(2.94)	(4.13)	(1.91)	(15.19)	(0.10)	

Table 19 (continued)

Panel B: GLS Regressions – Monthly Returns

Returns	β	σ	*Size*	s/P	$(s/P)^2$	δ/P	$(\delta/P)^2$	Adj.R^2
CRSP	0.006	-0.584	-0.002	0.739				4.16
	(0.78)	(2.48)	(3.36)	(3.73)				
AM Gross	0.006	-0.537	-0.002	0.239				5.57
	(0.75)	(2.30)	(3.32)	(1.11)				
R6	0.005	-0.523	-0.002	0.192				5.67
	(0.56)	(2.19)	(3.30)	(0.86)				
Corrected	0.006	-0.295	-0.002	-0.244				6.16
	(0.71)	(1.17)	(3.74)	(1.13)				
CRSP	0.008	-0.456	-0.001	0.590		-0.221		4.18
	(2.10)	(3.57)	(2.44)	(3.69)		(2.75)		
AM Gross	0.010	-0.587	-0.001	0.171		-0.749		6.18
	(2.46)	(4.57)	(1.74)	(1.02)		(8.62)		
R6	0.010	-0.581	-0.001	0.101		-0.784		6.43
	(2.29)	(4.46)	(1.55)	(0.59)		(8.80)		
Corrected	0.005	-0.455	-0.001	-0.335		-1.031		8.34
	(1.12)	(3.65)	(1.45)	(1.88)		(10.91)		
CRSP	0.010	-0.523	-0.001	0.838	-6.680	-0.235	3.803	4.45
	(2.52)	(4.05)	(2.08)	(2.33)	(0.94)	(2.90)	(2.21)	
AM Gross	0.008	-0.586	-0.001	-0.227	5.289	-0. 733	4.136	6.26
	(1.98)	(4.40)	(2.11)	(0.60)	(0.70)	(8.28)	(2.32)	
R6	0.008	-0.580	-0.001	-0.316	6.104	-0.778	3.773	6.51
	(1.83)	(4.28)	(2.12)	(0.80)	(0.77)	(8.52)	(2.00)	
Corrected	0.004	-0.461	-0.001	-0.839	8.947	-1.021	2.025	8.62
	(0.96)	(3.57)	(2.47)	(1.79)	(0.92)	(10.70)	(0.97)	

Table 19 (continued)

Panel C: OLS and GLS Regressions – Quarterly Returns

Returns	β	σ	*Size*	s/P	$(s/P)^2$	δ/P	$(\delta/P)^2$	Adj.R^2
CRSP	0.003	-0.041	-0.001	0.138				11.23
OLS	(1.04)	(0.52)	(2.05)	(0.88)				
CRSP	0.000	-0.263	-0.002	-0.009				11.18
GLS	(0.00)	(1.26)	(1.74)	(0.03)				
Corrected	-0.003	0.055	-0.000	-1.020				18.97
OLS	(0.77)	(0.55)	(0.65)	(5.96)				
Corrected	-0.009	-0.208	-0.001	-0.429				12.75
GLS	(1.29)	(1.01)	(0.66)	(1.30)				
CRSP	0.003	-0.045	-0.001	0.166		-0.068		11.25
OLS	(1.08)	(0.57)	(1.97)	(1.04)		(0.65)		
CRSP	0.005	-0.111	-0.001	0.114		0.108		11.67
GLS	(1.56)	(1.26)	(1.54)	(0.57)		(0.93)		
Corrected	-0.001	-0.038	0.000	-0.607		-0.957		20.72
OLS	(0.20)	(0.38)	(0.24)	(3.62)		(7.36)		
Corrected	-0.012	0.181	0.001	-0.379		-0.813		19.04
GLS	(2.42)	(1.37)	(1.82)	(1.59)		(6.07)		
CRSP	0.003	-0.065	-0.001	0.455	-9.057	-0.121	2.810	11.30
OLS	(1.12)	(0.80)	(1.31)	(1.22)	(1.27)	(1.06)	(1.11)	
CRSP	-0.002	-0.178	-0.000	0.675	-14.864	0.080	3.540	12.74
GLS	(0.62)	(1.83)	(0.25)	(1.61)	(1.85)	(0.66)	(1.51)	
Corrected	-0.001	-0.002	0.000	-1.284	12.415	-1.019	4.632	20.87
OLS	(0.39)	(0.02)	(0.43)	(3.27)	(1.46)	(7.95)	(1.56)	
Corrected	-0.010	0.110	0.001	-0.961	11.254	-1.003	3.407	21.26
GLS	(2.33)	(0.91)	(1.08)	(1.88)	(1.13)	(8.63)	(1.19)	

Table 20: The Effects of Beta, Residual Risk, Size, Bid/Ask Spread and Price Impact on Stock Returns – Fama / MacBeth Regressions

To test Merton (1987), returns in excess of the T-bill rate are regressed on beta, residual risk, size and liquidity cost. Merton predicts positive coefficients. Amihud and Mendelson's Proposition 2 (AM 1986) predicts that expected *Gross* returns (holding period cash flows divided by buying transaction price) increase with liquidity costs. AM use proportional bid/ask spread to proxy liquidity cost. Price impact is added here as an additional measure of illiquidity. The squares of half-spread and of price impact are included as regressors to test Amihud and Mendelson's prediction that expected returns increase less for less liquid assets.

AM use monthly CRSP returns to proxy for Gross returns. Here, Gross returns and corrected returns also are constructed using fitted values of spread and price impact. A fourth return is constructed as a linear combination of five parts Gross return and one part corrected return. This return proxies the unrealized monthly return for an investor with a 6 month holding period.

For each return type (CRSP, Gross, corrected), three regressions are performed, using the particular return type as the dependent regression variable, and to form an equally-weighted market portfolio for calculating independent regression variables beta and sigma. This equally-weighted portfolio of the stock returns is used in a 60 month moving window to estimate beta for each stock. Monthly cross-sectional data is used to perform Fama-MacBeth regressions. The three regressions have the form:

$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 Ln(\text{size})_p + \gamma_4 s/P_p + \varepsilon_p$$

$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 Ln(\text{size})_p + \gamma_4 s/P_p + \gamma_5 \delta/P_p + \varepsilon_p$$

$$R_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 \sigma_p + \gamma_3 Ln(\text{size})_p + \gamma_4 s/P_p + \gamma_5 \delta/P_p + \gamma_6 (s/P_p)^2 + \gamma_7 (\delta/P_p)^2 + \varepsilon_p$$

Regression results are shown in Table 20. Fama-MacBeth standard errors are used in calculating t-statistics (shown in parentheses).

Table 20 (continued)

Panel A: Fama-MacBeth Regressions – All Observations

Returns	β	σ	*Size*	s/P	$(s/P)^2$	δ/P	$(\delta/P)^2$
CRSP	0.007	0.009	-0.001	0.299			
	(1.99)	(0.14)	(1.91)	(2.50)			
AM Gross	0.007	-0.040	-0.000	-0.558			
	(2.17)	(0.67)	(0.67)	(4.15)			
R6	0.007	-0.023	-0.000	-0.418			
	(2.09)	(0.35)	(0.86)	(3.17)			
Corrected	0.003	0.037	-0.000	-0.974			
	(0.80)	(0.57)	(0.05)	(6.15)			
CRSP	0.006	-0.004	-0.001	0.227		-0.096	
	(1.89)	(0.07)	(1.71)	(1.16)		(0.67)	
AM Gross	0.007	-0.055	-0.000	-0.795		-0.439	
	(2.13)	(0.85)	(0.79)	(3.80)		(2.35)	
R6	0.007	-0.036	-0.000	-0.668		-0.374	
	(2.02)	(0.56)	(0.96)	(3.16)		(2.05)	
Corrected	0.005	-0.005	-0.000	-0.835		-0.502	
	(1.53)	(0.08)	(0.71)	(3.93)		(3.03)	
CRSP	0.006	-0.024	-0.001	-0.367	48.467	-0.933	-17.896
	(1.76)	(0.37)	(1.56)	(0.48)	(1.37)	(2.23)	(0.36)
AM Gross	0.008	-0.054	-0.000	-1.782	56.555	-1.142	23.416
	(2.17)	(0.81)	(1.32)	(2.19)	(1.35)	(2.12)	(0.43)
R6	0.007	-0.041	-0.000	-1.382	44.630	-0.916	-29.764
	(2.01)	(0.61)	(1.37)	(1.73)	(1.06)	(1.81)	(0.51)
Corrected	0.005	-0.009	-0.001	-1.811	54.110	-0.841	11.431
	(1.43)	(0.14)	(1.94)	(2.20)	(1.37)	(1.69)	(0.25)

Table 20 (continued)

Panel B: Fama-MacBeth Regressions – January Observations

Returns	β	σ	*Size*	s/P	$(s/P)^2$	δ/P	$(\delta/P)^2$
CRSP	-0.012	-0.042	-0.000	-0.214			
	(1.12)	(0.20)	(0.29)	(0.59)			
AM Gross	-0.004	-0.923	0.000	-1.026			
	(0.40)	(3.82)	(0.28)	(1.77)			
R6	-0.005	-0.800	0.000	-0.930			
	(0.46)	(3.56)	(0.15)	(1.69)			
Corrected	0.003	0.037	-0.000	-0.974			
	(0.80)	(0.57)	(0.05)	(6.15)			
CRSP	-0.013	-0.077	-0.000	-0.418		-0.508	
	(1.18)	(0.34)	(0.26)	(1.12)		(1.61)	
AM Gross	-0.004	-0.869	0.000	-1.631		-0.563	
	(0.44)	(3.69)	(0.29)	(1.99)		(1.59)	
R6	-0.005	-0.769	0.000	-1.573		-0.695	
	(0.54)	(3.42)	(0.18)	(2.04)		(1.87)	
Corrected	0.005	-0.005	-0.000	-0.835		-0.502	
	(1.53)	(0.08)	(0.71)	(3.93)		(3.03)	
CRSP	-0.014	-0.029	0.000	-2.025	81.253	-2.087	-92.348
	(1.25)	(0.12)	(0.23)	(0.96)	(1.15)	(2.10)	(2.09)
AM Gross	0.008	-0.674	0.000	-7.120	217.99	-3.132	-114.84
	(0.75)	(2.71)	(0.03)	(1.98)	(1.58)	(1.89)	(2.12)
R6	-0.009	-0.563	0.000	-6.979	221.20	-3.362	-115.40
	(0.79)	(2.40)	(0.08)	(1.92)	(1.57)	(1.94)	(2.07)
Corrected	0.005	-0.009	-0.001	-1.811	54.110	-0.841	11.431
	(1.43)	(0.14)	(1.94)	(2.20)	(1.37)	(1.69)	(0.25)

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PUBLICATIONS

Farnsworth. H. et al. 2002. Performance evaluation with stochastic discount factors. *Journal of Business* 75. 473-504.