INVESTIGATION OF STUDENT UNDERSTANDING OF HYDROSTATICS AND THERMAL PHYSICS AND OF THE UNDERLYING CONCEPTS FROM MECHANICS

by

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A dissertation submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

University of Washington

1999

Program Authorized to Offer Degree: Physics

UMI Number: 9937617

Doctoral Dissertation

University of Washington
Graduate School

This is to certify that I have examined this copy of a doctoral dissertation by

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Abstract

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Chairperson of the Supervisory Committee: Professor Lillian C. McDermott
Department of Physics

This dissertation describes an investigation of student understanding of hydrostatics and thermal physics in first- and second-year university physics courses. We found that after standard instruction many students have serious conceptual and reasoning difficulties with this material. Some of these difficulties seemed to be connected to underlying difficulties with concepts from introductory mechanics. We have examined the extent to which difficulties with mechanics affect student performance on qualitative questions involving hydrostatic pressure, buoyancy, and the first law of thermodynamics. Evidence is presented that many of the conceptual difficulties identified are due in part to incorrect ideas that students have about introductory mechanics and in part to many students' inability to apply correct ideas from mechanics in new contexts. The results of this research have been used to develop supplementary curriculum. The instructional materials have been assessed and proven to be effective.

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ACKNOWLEDGMENTS

I wish to acknowledge all the current and past members of the Physics Education Group for their support, their camaraderie, and their uncanny ability to ask the right questions. In particular I wish to recognize my advisor, Lillian McDermott, for her guidance and mentorship and her unflagging commitment to the field of physics education research. Paula Heron has been a creative collaborator, a skilled editor, and most of all a good friend. Peter Shaffer is as thorough a human as I know, and his questioning of assumptions and attention to detail have shaped my outlook as well as my work. Stamatis Vokos has taught me how to be in many places at one time and to bring good cheer and deep thought to all of them. My collaboration with Chris Kautz throughout this project has been tremendously rewarding. The other members of the group have been good friends and colleagues, and have given me a newfound appreciation for the subtleties of hyphens and italics.

I wish to thank the faculty who allowed us access to students in their courses for the research and curriculum development in this dissertation. At the University of Washington, thanks are due to Professors Bardeen, Thouless, den Nijs, Boynton, Rehr, Vilches, Heckel, Van Dyck, Seidler, Rothberg, Puff, and Adelberger. Much of the curriculum development was carried out in courses taught by Mark McDermott, Chuck Robertson, Rick Muirhead, and Aurel Bulgac. Also helpful have been Joe Redish, Mel Sabella, and Michael Wittman at the University of Maryland, Eric Mazur and Catherine Crouch at Harvard University, David Elmore at Purdue University, Gary Gladding at the University of Illinois, and Bob Endorf at the University of Cincinnati.

My deepest thanks go to my family and friends. My wife, Kirsten Lorentzen, has been extremely patient and has provided love and support that a lifetime of words and deeds could not recognize. Fortunately, I will have a lifetime with her in which to try. My parents, Janet and Leslie Loverude, and my sister, Jennifer Loverude, have been continuing sources of inspiration, support, and love.

DEDICATION

To Kirsten:

Camerado, I give you my hand! I give you my love more precious than money,
I give you myself before preaching or law;
Will you give me yourself? Will you come travel with me?
Shall we stick by each other as long as we live?

– Walt Whitman

INTRODUCTION TO THE DISSERTATION AND CONTEXT FOR RESEARCH AND CURRICULUM DEVELOPMENT

INTRODUCTION

This dissertation describes an investigation of student understanding of hydrostatics and thermal physics. This work is part of an ongoing study of the teaching and learning of physics by the Physics Education Group at the University of Washington. One of the major goals of this project is to contribute to the development of a research base on the conceptual and reasoning difficulties that students have with topics in introductory physics. The work described here and the strategies for research and curriculum development have built upon the efforts of other members of the group.

In the course of this investigation, what began as an effort to identify student difficulties with topics in hydrostatics and thermal physics has evolved to include a study of the effects of student understanding of mechanics in these new contexts. This evolution was the result of the realization that many of the difficulties that students have in understanding hydrostatics and thermal physics are due in part to an underlying failure to apply correctly concepts and principles from introductory mechanics. The structure of this dissertation reflects this evolution. In each context, we first describe the specific difficulties that students have with the topic and then examine student understanding of the mechanics concepts that underlie these difficulties. In some cases, this work in the contexts of liquids and gases led to the documentation of difficulties with concepts from introductory mechanics that had not been previously described in the research literature.

THE PHYSICS EDUCATION GROUP AT THE UNIVERSITY OF WASHINGTON

The work described in this dissertation was carried out within the Physics Education Group, a research group within the Department of Physics at the University of Washington. The Physics Education Group's activities include research, curriculum development, and instruction. Because the group's activities provide an important part of the context in which this work was done, we briefly describe each of these endeavors and the way in which they relate to this dissertation.

Research

The bulk of this dissertation describes research on student understanding of physics. Broadly speaking, the research questions that we seek to answer include: (1) is the standard treatment of a topic in the textbook or in lecture sufficient for students to gain a *functional understanding* of the topic (that is, the ability to apply the topic in contexts other than the one in which its was originally learned), and (2) if not, what modifications to the course can help students to develop a functional understanding? To these questions we add, for the purpose of this dissertation, a third question: to what extent is student understanding of ideas from the introductory mechanics course necessary and sufficient for students to develop a functional understanding of the concepts in the new contexts of hydrostatics and thermal physics? The specific methods of research employed in this study are described in some detail later in this chapter.

Development and Assessment of Curriculum

Work by the Physics Education Group and other researchers in identifying student conceptual difficulties has led to a handful of inescapable conclusions. First of all, instruction in which students are passive learners often fails to address the conceptual difficulties with which many students enter the course and often contributes to the development of new misconceptions.[1] Results have shown that, for many qualitative problems, student performance is essentially the same before and after traditional lecture instruction.[2] Second, instruction in which students are actively engaged in the construction of ideas can lead to significant improvements in students' conceptual understanding.[3] Therefore, an important part of this project has been the development of curriculum that can intellectually engage students and address common student difficulties.

As least as important as the development of curriculum is the systematic assessment of its effectiveness with the population of students for whom it is intended. In this study, as in others carried out by our group, this assessment relies upon the analysis of the results of student responses to written problems on course examinations and/or ungraded post-tests. When possible, we compare the performance on these problems of students who have completed standard instruction in a topic and of students who have completed instruction using the curriculum that we have developed. In some cases, we also compare the performance of students before instruction to that after instruction. By analyzing student performance after use of the curriculum, we can determine to what extent the curriculum has helped students to develop their ability to reason and to answer conceptual problems. This analysis helps us to decide whether the curriculum has successfully addressed the difficulties that we have identified or whether it needs modification. We go through an iterative cycle of research, development of curriculum, testing of curriculum, further research, and further development of curriculum. Using this iterative process, the Physics Education Group has developed curricular materials that are designed to develop conceptual understanding, address student difficulties, and improve reasoning skills. Two primary curriculum development projects have emerged from this work, *Physics by Inquiry*[4] and *Tutorials in Introductory Physics.*[5]

Physics by Inquiry

Physics by Inquiry is a laboratory-based curriculum that was originally developed to prepare precollege teachers to teach physics and physical science as a process of inquiry.[6] The curriculum has also been used with students who are seeking to major in science or engineering but whose previous training has left them underprepared for the introductory physics sequence, and with liberal arts students as part of an elective science course.[7] The curriculum includes several *modules*, each of which is a self-contained instructional sequence that begins with exploratory observations and guides students through the steps necessary to build a complete conceptual model for a topic. Work in this dissertation has included some examination of students after completion of parts of *Physics by Inquiry*.

Tutorials in Introductory Physics

Tutorials in Introductory Physics is a curriculum that is designed to supplement a traditional textbook in a large lecture-based introductory course.[8] The curriculum is composed of a series of *tutorial sequences*. Each tutorial sequence includes a *pretest*, a *tutorial worksheet*, and *tutorial homework*. The pretest is a short qualitative written problem that tests student understanding on the topic of the tutorial (see also research methods, below) and serves to elicit student difficulties by having students commit to a written response.[9] The tutorial worksheets are a series of carefully structured written and experimental tasks that guide students through the reasoning necessary to develop a conceptual understanding of a topic. In tutorials, students typically confront common conceptual difficulties and are then asked to resolve contradictions between their own understanding and observations. Tutorial instructors do not lecture but rather ask students questions in semi-Socratic dialogues intended to guide the students to answer their own questions. Tutorial homework helps students to extend, reflect upon, and generalize the concepts learned in the tutorial worksheets. In all cases, questions based on tutorial materials are covered on course examinations.

Instruction

A crucial part of the research and curriculum development activities of the Physics Education Group is the use of instructional materials with the students for whom the materials are designed. As a part of the Physics Department, the Physics Education Group participates extensively in the department's program of instruction, including introductory physics courses, special courses for teachers, and a teaching seminar for graduate teaching assistants. This participation allows for the testing of curriculum with the students for whom it is intended.

Courses for teachers

An important part of the instructional program of the Physics Education Group is the teaching of special courses for precollege teachers.[10] These courses use *Physics by Inquiry*, and are taught during the summer (primarily for inservice teachers) as well as during the academic year (for both inservice and preservice teachers). Although this study has concentrated on students in introductory courses, we have tested some of the curriculum in courses for preservice teachers. These courses are useful for curriculum development because there is typically a relatively small number of students and the instructional staff can work closely with students to determine the effectiveness of particular exercises and strategies within the instructional materials.

Introductory physics courses

The tutorial curriculum described above is used in the calculus-based introductory physics course at the University of Washington. Typically tutorials are used in small-group sections of twenty to twenty-five students that meet once weekly. Within the tutorial sections, students work in groups of four through carefully structured worksheets. Instruction based on small-group tutorials is discussed further in several other works by the Physics Education Group.[11]

Much of the development and assessment of the curriculum described in this dissertation has taken place in the context of courses that are not part of the tutorial project and do not include small-group sections of any kind. In such courses we have found that it is possible to adapt the instructional strategies used in the small-group tutorials for use in the lecture hall. In these cases, we call the instructional approach *Interactive Tutorial Lectures* (ITLs). Our intent was to develop materials that could be used in either the large-group setting or in courses that used tutorials in the small-group format described above.

In ITLs, students work together collaboratively in the lecture hall in groups of 2 or 3. The structure is provided by worksheets that consist of a series of questions and exercises that are expressly intended to help students confront and resolve common conceptual and reasoning difficulties. Often students are asked to make predictions about the results of a demonstration or calculation. The lecture instructor and one or more teaching assistants circulate and engage students in informal semi-Socratic dialogues similar to the ones in the small-group tutorial setting. At several key points, the instructor calls the class to order. At these checkpoints, the instructor may show a demonstration or outline some of the expected answers for the previous part of the ITL. The students are instructed to proceed from there until the next checkpoint. The checkpoints are more important in the ITLs than in small-group tutorials because in the large lecture class it is not possible for the instructors to reach all the students. Another difference between the tutorials and the ITLs is that, in the latter, it is not possible to have students perform some of the simple experiments that can play an important role in helping them to confront and resolve incorrect ideas. Therefore, many of these experiments are replaced by demonstrations.

RESEARCH METHODS

We use a variety of research methods in our investigations of conceptual understanding. Typically we strive to use each of these methods whenever possible in order to obtain a more complete picture of student difficulties. A more general summary of the methods used by the Physics Education Group in particular and by physics education researchers in general can be found in other work by our group.[12]

In all cases, our research has been performed with university students who are currently enrolled in physics courses or have recently completed such courses. We have typically presented students with a physical situation and asked them either to compare the values of physical quantities or to predict what would happen if the physical situation were changed in some way. In this way, we hope to avoid testing for simple declarative knowledge.

Informal observations

Much of the initial direction for our research into student understanding has come as a result of informal interactions with students during the course of instruction or office hours. In many cases, the questions posed by students or the responses given by students to questions on homework or examination problems suggest the existence of conceptual difficulties. Although these informal observations are difficult to document or reproduce, they nonetheless serve an important purpose in the initial stages of formulating specific research questions.

Individual student interviews

Individual student interviews are one of the most important research tools used by our group.[13] These interviews are similar in nature to those used by Piaget in his investigations of the cognitive development of children.[14] In these interviews, student volunteers are asked a series of questions and asked to respond to the questions while 'thinking aloud.' The interviews often involve simple physical demonstrations, about which students are asked to make predictions; in these cases the interviews are described as *individual demonstration interviews.* In other cases, the interviews have involved computer simulations or paper-and-pencil activities. The interviews typically have a predetermined protocol, with a series of specific questions that are asked of all volunteers. However, the interviews are open-ended, such that the interviewer is free to ask additional questions to probe certain ideas more deeply. In some cases, the interviews have included questions that we envision using in instructional materials, and student responses are analyzed to determine the effectiveness of the questions.

Typically, the students who participate in interviews are performing at or above the mean in their respective courses. The fact that the students are typically above average means that interviews are often not a good gauge of the prevalence of difficulties in the class as a whole. However, these students' responses are useful in that the better students are typically able to articulate their beliefs in detail and are often more able to reflect upon what they do and do not know. In addition, if the better students in a class have a certain conceptual difficulty, it is likely that the students who are less able will have similar difficulties.

Written problems

In addition to interviews, our group commonly uses written problems to investigate student understanding. Typically the written problems that we use are short-answer conceptual problems. Often the questions involve ranking the values of several quantities, predicting whether a quantity will increase, decrease, or remain the same when a situation is changed, or making a prediction of the result of a physical experiment. Only rarely do these problems require students to perform a calculation or determine a numerical answer. In virtually all of these problems, students are asked to explain their reasoning; in other words, they are asked to explain what physical principles or logical arguments they are using to answer the question. Students work independently on the problems. In some cases, due to the constraints of courses, we have posed multiple-choice versions of problems. Typically these are problems for which we also have data on student responses to free-response versions.

The written problems are always asked as part of a physics course that covers the topic that is covered by the problem. These written problems can take several forms:

Course examinations. In some cases, the problems are posed on course examinations. In these cases, we work with the course instructor to ensure that the problems are representative of the material covered in the course. These problems can therefore be considered to be testing the knowledge of students after instruction. Such problems are asked after standard instruction as well as after students have completed curriculum that we have developed.

Ungraded quizzes after instruction. Often we use written problems as ungraded quizzes after students have completed instruction in a topic. Often these are used by instructors as practice examination questions. In these cases, the instructor typically discusses the results of the questions immediately after the students have handed in the ungraded quizzes. Again, in these cases, the problems are given after instruction on the material covered by the questions.

Pretests–ungraded quizzes before research-based instruction. Ungraded quizzes that are used before research-based instruction are typically referred to as *pretests.* Typically we begin a tutorial sequence (see Section B) with a short (~10 minute or less) pretest, which serves to elicit student reasoning and document student understanding before research-based instruction. Depending on the course and the preference and pace of the instructor, pretests may take place before, during, or after lecture instruction on a topic.

Diagnostic quizzes before instruction. Before instruction in some topics, we have administered diagnostic quizzes that are designed to probe whether students have mastered material that is considered to be prerequisite to the course in which the quiz is administered. For example, in Chapters 4 and 6, we describe a series of problems that test student understanding of Newton's second law that we have administered in courses in hydrostatics and thermal physics, and in Chapter 9 we describe problems designed to test whether students understand the definition of mechanical work. We have used tests of this sort as a means of documenting the understanding that students have before instruction in an attempt to determine what level of knowledge is necessary for understanding of topics in both hydrostatics and thermal physics.

CONTEXT FOR RESEARCH

All of the research described in this dissertation has been carried out in the context of physics courses taught at the college level. These courses include several introductory-level courses at the University of Washington, introductory courses at other universities, upper-level undergraduate physics courses at the University of Washington, and laboratory-based physical science courses for precollege teachers.

Introductory courses at the University of Washington

The bulk of the work in this dissertation has been done in the context of first- and second-year physics courses at the University of Washington. Most of the students are studying the course material for the first time at the university level. In most cases, these introductory physics courses are service courses that students take to fulfill the requirements for majors in the natural sciences, mathematics, or engineering. Only a small percentage of the students are physics majors.

Algebra-based introductory physics course at the University of Washington

The algebra-based introductory course at the University of Washington is a fairly standard non-calculus physics course, based on a commonly-used textbook.[15] The course catalog describes the course as follows: "Basic principles of physics presented without use of college-level mathematics. Suitable for students majoring in technically oriented fields other than engineering or the physical sciences." The course includes four 50-minute lectures each week and no small-group recitation or tutorial sections. The course is accompanied by an optional three-hour weekly laboratory that, at the time of the study, included no experiments on hydrostatics or the thermodynamics of gases.

Calculus-based introductory physics course at the University of Washington

The calculus-based introductory course at the University of Washington is based on a commonly-used textbook.[16] The course catalog describes the first-year course as follows: "Basic topics in mechanics, electricity and magnetism, and electromagnetic waves, optics, and waves in matter." Typically, the course is taken by students majoring in engineering, the physical sciences, and mathematics. The course includes three 50-minute lectures each week, a required three-hour laboratory, and a weekly 50-minute tutorial (see above).

Second-year thermal physics

The course in thermal physics is taught in the second year and is primarily intended for physics and engineering majors. It uses the same textbook as the first-year introductory calculus-based physics sequence. The course catalog describes the course as follows: "Introduction to heat, thermodynamics, elementary kinetic theory, and the physics of continuous media." Traditionally, the first two weeks of this course are devoted to the study of fluid statics, with some instructors also briefly covering Bernoulli's equation and fluid dynamics. The course includes three 50-minute lectures each week and no small-group recitation or tutorial sections. There is no accompanying laboratory.

Introductory courses at other universities

Through collaborations with colleagues at other universities, we have been able to pose written questions in introductory courses at those universities. As in the introductory courses at the University of Washington, most of the students in these courses are in their first or second year of college and are studying the course material for the first time at the university level. Few are physics majors. In this study, we describe results from the standard algebra-based and calculus-based introductory physics courses at the University of Maryland.

In addition, as part of the development of *Tutorials in Introductory Physics,* several colleges and universities have agreed to serve as *pilot sites.* Faculty at the pilot sites are typically not researchers in physics education but actively collaborate with the Physics Education Group to implement the tutorials and share the results of pretest and post-test questions. In the case of the curriculum development that is described in this dissertation, several pilot sites have served as test sites for tutorials in hydrostatics and thermal physics that are under development. In this study, we describe results from calculus-based introductory courses at several pilot sites: Harvard University, the University of Illinois, and Purdue University.

OVERVIEW OF DISSERTATION

The main body of this dissertation describes the research, curriculum development, and assessment of curriculum that form the bulk of this study. It includes three parts: Part IA, Part IB, and Part II. The first two parts examine two aspects of student understanding of hydrostatics. Part IA focuses on student understanding of pressure in a static liquid. Part IB includes student understanding of buoyancy and Archimedes' principle. In Part II we examine student understanding of work in the context of the first law of thermodynamics.

In the introductory section of each of these three parts there is an examination of the relevant research literature. We also describe what is typically taught in the courses that we have examined in order to show what is usually expected of students what level of understanding is considered reasonable by most instructors. Each part contains a description of our initial research to identify student difficulties in the context that is . This description is followed by a discussion of the relationship of the difficulties identified in the new context to student understanding of ideas from introductory mechanics. Each part concludes with a description of the development of the curriculum that we designed to address specific difficulties and our assessment of its effectiveness. The dissertation also includes several appendices. Some of these include details of student responses to problems that have been omitted in the chapters for the sake of clarity. Also included are several versions of the curriculum that we have developed.

Part IA: Pressure in a static liquid

In Part IA, we examine student understanding of the idea of pressure in a static liquid. Chapter 1 includes a description of the major categories of written problems that we have used to probe student understanding. In Chapter 2 we describe our research to identify common student difficulties with the idea that the pressures at points at the same level in a static liquid are equal. Chapter 3 examines student difficulties in reasoning with the idea of the gradient of pressure. Chapter 4 involves an investigation of the relationship of difficulties with introductory mechanics to the difficulties described in Chapters 2 and 3. In Chapter 5, we describe a series of tutorials designed to address student difficulties with pressure in a liquid and present data from post-tests in order to assess the effectiveness of the tutorials. Appendices relevant to Part IA include A and B, which present details of student responses to pressure problems, and C, D and E, which present versions of the tutorial sequence that we have developed to address student difficulties with pressure.

Part IB: Buoyancy and Archimedes' principle

In Part IB, we examine student understanding of buoyancy and Archimedes' principle. Chapter 6 contains the text of a paper that has been prepared for publication. This paper describes our investigation of student understanding of Archimedes' principle and the buoyant force and our assessment of the mechanics knowledge that is necessary for understanding of the behavior of objects in a liquid. Chapter 7 includes a description of the development of a tutorial sequence on buoyancy and the assessment of the effectiveness of the instructional strategies. There are four appendices relevant to Part IB. These include Appendix F, in which we describe the analysis of a set of individual student interviews on the Cartesian diver, and Appendix G, in which we present details of student responses to written problems on buoyancy and the buoyant force. In Appendices H and I, we present initial and current versions of the tutorial sequence that we have developed to address student difficulties with buoyancy and Archimedes' principle.

Part II: Work and the first law of thermodynamics

Part II of the dissertation concerns student understanding the idea of work and the first law of thermodynamics. As in the second part, some of the research in this chapter has been described in a paper being prepared for publication. Chapter 8 includes the text of the paper, in which we examine the ability of students to apply the concept of work to processes in thermal physics and the relationship of this ability to student understanding of work in the context of mechanics. Chapter 9 includes a description of the curriculum that we have developed to address student difficulties with work in thermal physics and reports on the results of our assessment of this curriculum. There are three appendices relevant to Part II. In Appendix J, we present details of student responses to written problems requiring that students spontaneously apply the first law of thermodynamics. In Appendix K we present details of student responses to problems that ask specifically about the quantities in the first law of thermodynamics (*e.g.*, the work done on a gas sample). Appendix L includes the current version of the tutorial sequence that we have developed to address difficulties with work and the first law of thermodynamics.

PART IA: IDENTIFYING AND ADDRESSING STUDENT DIFFICULTIES WITH HYDROSTATIC PRESSURE

INTRODUCTION

The first part of this dissertation involves our work in identifying and addressing student difficulties with hydrostatic pressure. In this preface to Part IA, we discuss the level of understanding that is typically expected of students in the courses in the study. Following this discussion we present a brief summary of previous research on student understanding of hydrostatic pressure.

WHAT STUDENTS ARE TYPICALLY TAUGHT

Our investigation of student understanding has been driven in part by the standard presentation of hydrostatic pressure in introductory physics courses. In most cases, this presentation is connected quite closely to the equation for pressure in an incompressible liquid, with the expectation that students will be able to use this equation to analyze simple physical situations, like a mercury barometer or open-end manometer. In this dissertation, we focus on student ability to reason with the equation and to apply it to simple situations. As a result, we do not focus on several other issues pertaining to pressure, such as the 'isotropy' of pressure forces, the fact that liquids can in general push but not pull, or the inability of liquid to support shear forces. In some cases these ideas will arise in the discussion of pressure tasks.

One of the first concepts that students are taught in hydrostatics is that the pressure at points at the same level in a static liquid are equal. This idea is often demonstrated in lecture by consideration of two points at the same level with no intervening container walls. A carefully chosen free-body diagram drawn for a small amount of water can help to show that the pressure at the points must be equal. The pressures at points X and Y in Figure IA-

1, for example, can be shown to be equal by application of Newton's second law to the water within the rectangular region bounded by the dotted line. The force on the left surface of the water in this region must be equal to the force on the right surface, since the net force on the water is zero (the water is at rest). These forces can be related to the pressures at points *X* and *Y*, respectively, by dividing by the appropriate areas.[17] Since the areas of the two side surfaces are equal, the pressures at points *X* and *Y* must be equal.

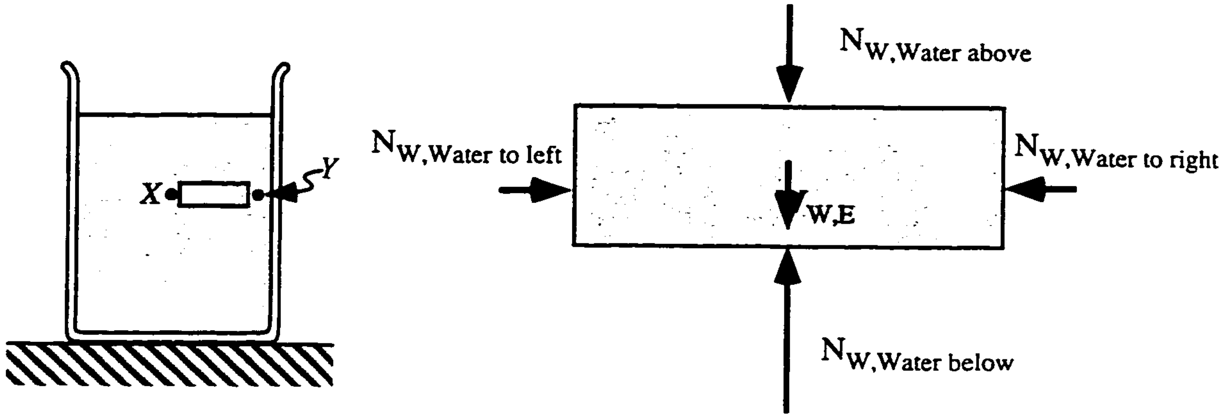


Figure IA-1: A free-body diagram that can be used to illustrate that pressures are equal at points at the same height in a static liquid. The pressures at points *X* and *Y* can be shown to be equal by applying Newton's second law to the water in the region outlined by the dotted line.

A similar argument can be applied to find that the pressures at points *R* and *S* in Figure IA-2 are equal. However, the fact that these pressures are equal may seem paradoxical, because there are different amounts of liquid above the two points. Free-body diagrams for small amounts of water near *R* and *S* show that all the forces on the two elements of water are identical, except the forces on the top surfaces of the elements (In the diagrams shown in Figure IA-2, only vertical forces are shown). For the element of water near *R*, the downward force is exerted by the water above the element. For the element near *S*, on the other hand, the downward force is exerted by the container wall. Because all the other forces on the two free-body diagrams are identical, and the elements are at rest, the magnitude of the force on the element near *R* by the water above must be equal to the magnitude of the force on the element near *S* by the container wall.[18] This comparison leads to an important observation: container walls exert forces in a static situation.

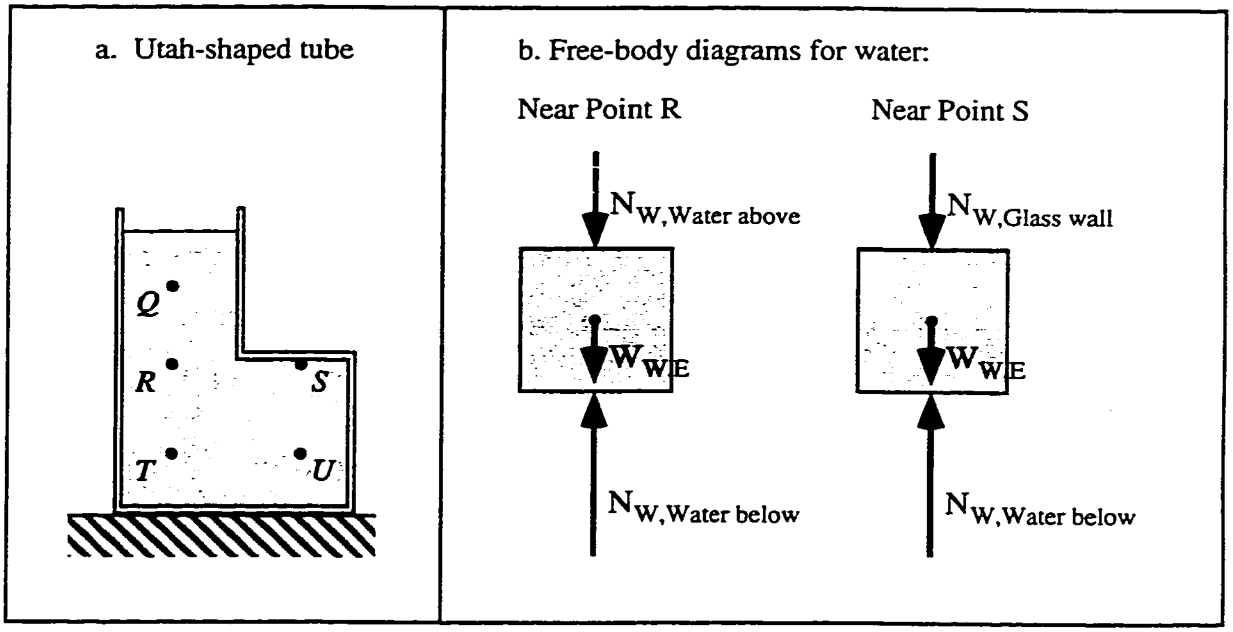


Figure IA-2: a. A tube that presents a difficult pressure comparison.
b. Free-body diagrams for small elements of water near points *R* and *S* in the Utah-shaped tube at left.

This approach of applying Newton's second law to an element of a static liquid is often used in lecture or textbooks to derive a functional relationship of pressure to depth. In most cases, students are asked to consider a free-body diagram for a small element of liquid (see Figure IA-3). There are vertical forces exerted on this element of liquid by the liquid above, the liquid below, and the earth (horizontal forces have been neglected).

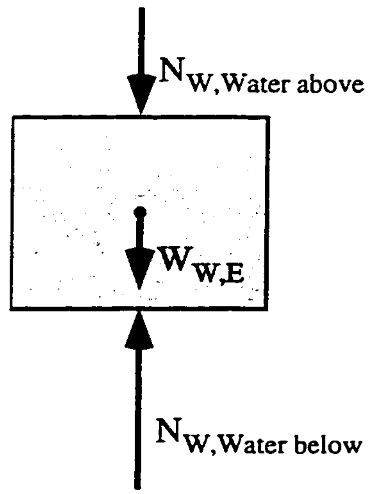


Figure IA-3: A free-body diagram showing the vertical forces acting on a small piece of a liquid at rest. The sum of the forces on the liquid must be zero. The magnitudes of the forces by the water above and below can be related to the pressures at the top and bottom of the piece of water.

The sum of the forces on this piece of liquid must be zero because the liquid is at rest. The magnitude of the contact forces can be related to the products of the pressures at the top and bottom surfaces and the cross-sectional area A,

$$P_{top}A + Mg = P_{bottom}A.$$

Qualitative examination of this equation leads to the observation that there is a pressure gradient. In other words, the pressures are not equal at the top and bottom of the element of liquid; rather, the pressure at the bottom is greater. In particular, for an incompressible fluid, where the density of the fluid can be assumed to be independent of depth:

$$P_{top} + \rho g h = P_{bottom}.$$

If the top surface of the element is taken to be at the *free surface* of the liquid (*i.e.*, the surface of the liquid that is in contact with the atmosphere), one can derive the well-known equation relating pressure to depth:

$$P = P_0 + \rho g h, \qquad (1)$$

where P_0 is equal to atmospheric pressure and h is the vertical distance *below* the free surface. A similar free-body diagram can be drawn for any volume element in a liquid at rest, so the result that the pressure is greater at lower points in the liquid must be true in general. A more general reading of equation *(1)* allows the pressure at any point to be related to a reference pressure, where the vertical distance h is measured from the level at which the pressure is equal to the reference pressure P_0. For students to reason with equation *(1)*, they must recognize that the height h in the equation is measured *downward* from the free surface of the liquid. In other words, for two points in a liquid, the lower point will have a greater value of h, and a greater pressure.

Equation *(1)* can be used to analyze a mercury barometer (see Figure IA-4). In a standard barometer, a column of mercury is at rest in a glass tube that is open to a reservoir of mercury that is in turn open to the atmosphere. The pressure at the open surface of the mercury reservoir is equal to atmospheric pressure. Consider a point X within the tube that is at the same height as the open surface. The pressure at point X is also equal to atmospheric pressure. Applying equation *(1)*, the pressure at any point above X is less than atmospheric pressure by an amount $\rho g \Delta h$, where Δh is the height difference between the point and point X. If a point at the top of a mercury column of height is chosen, the pressure at that point is equal to $P_0 - \rho g H$, where H is the height of the column.

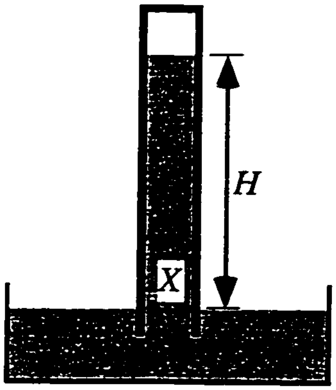


Figure IA-4: A mercury barometer.

If the pressure at the top of the column is known, this analysis can be used to determine a numerical value for atmospheric pressure. Standard treatments of a barometer assume that there is vacuum above the mercury column, so that the pressure at the top of the column is zero.[19] The difference in pressure between the top of the column and point X is $\rho g H$, which must be equal to the difference between zero and atmospheric pressure. In practice, the height of a mercury column is often used as a measure of atmospheric pressure. The SI unit 1 torr is defined as the pressure that would give $H = 1\text{mm}$ for a mercury column.

PREVIOUS RESEARCH ON STUDENT UNDERSTANDING OF HYDROSTATIC PRESSURE

The published literature holds only a few examples of systematic investigations of student understanding of pressure in a static liquid. Much of this research has been done with precollege students.

Driver and Engel Clough posed a series of pressure and force problems to precollege students.[20] Many of these tasks involved comparing forces or pressures in simple physical situations. For example, students were asked to predict whether the pressure on the back of a fish in an aquarium was greater than, less than, or equal to that on the nose of the fish. Most students gave responses consistent with an intuitive understanding of the idea that pressure increases with depth, with the fraction answering in this way increasing with age. Driver and Engel Clough also reported that many students associated pressure with downward forces, but often not with horizontal or upward forces. Giese performed a study using very similar tasks that corroborated these results.[21]

Kariotoglou and Psillos interviewed a number of 13-14 year old students in Greece.[22] They posed problems similar to those used by Driver and Engel Clough, but asked additional questions designed to determine whether students associated the pressure with a point in space, with the liquid at that point, or with an object at that point. For example, students were asked whether they preferred the statement, "point A has pressure" or the statement, "there is pressure at point A." In a related question, students were asked about the pressure at a point on the back of a diver, and then asked if the pressure at that location would remain the same if the diver left the water. Kariotoglou and Psillos identified three 'models' that students have for pressure: the "packed crowd" model, in which pressure is related to density and level of confinement of a liquid; the "pressing force" model, in which pressure has many characteristics of forces (for example, students associated a direction with pressure); and the accepted model, which they labeled as the "liquidness" model, in which pressure is associated with a point in a liquid and depends on the depth rather than the volume of liquid in the container.

Camacho and Cazares posed a series of questions in written problems and interviews centering on the contexts of U-tubes and a Cartesian diver.[23] For example, students were shown a U-tube capped on one side, with different water levels in the two tubes. Students were asked to explain why the water levels were different in the two tubes. Several correct and incorrect student conceptions were identified as being commonly held, including the ideas that pressure at a point is due to the weight of liquid above that point, and that "pressure is [only] exerted at the bottom of a liquid."

OUTLINE OF PART IA

Part IA includes five chapters. Chapter 1 includes descriptions of the most important categories of written problems that we have used to probe student understanding. Chapters 2 and 3 describe our work to identify student difficulties with hydrostatic pressure. For the sake of brevity, this work is divided into one chapter describing student responses for pressures at points at the same depth in a liquid and one chapter describing corresponding responses for problems requiring use of the pressure gradient. Chapter 4 includes a summary of our efforts to investigate the importance of concepts from mechanics in student understanding of pressure. Chapter 5 contains a description of the development and assessment of curriculum intended to address the conceptual difficulties described in the preceding chapters. Appendices pertaining to Part IA include Appendix A, which includes a detailed description of student responses to the problems used to probe student understanding of points at the same level, Appendix B, which includes a similar description for students responses to problems involving use of the pressure gradient, and Appendices C, D and E, which include the complete text of the curriculum described in Chapter 5.

1. PROBLEMS USED TO INVESTIGATE STUDENT DIFFICULTIES WITH HYDROSTATIC PRESSURE

1.1 INTRODUCTION

The first part of this dissertation involves our work in identifying and addressing student difficulties with hydrostatic pressure. A crucial part of this study has been the development of written problems designed to probe student conceptual understanding of pressure. In this chapter we describe the major categories of problems that we have used.

During informal interactions with students we found that after standard instruction many seemed to have difficulty in correctly applying the equation $P = P_0 + \rho g h$. Many students gave responses that suggested an interpretation of hydrostatic pressure in terms of the weight of liquid above a point. This interpretation can be correct in many situations; for example, in a container with vertical walls and uniform cross-section A, the gauge pressure at a depth h is numerically equal to the weight of the liquid above the depth h divided by A. However, there are situations in which such an interpretation of pressure can lead to incorrect answers; the hydrostatic paradox is one example of such a situation that is well-known in the instructional literature.[24] Arons identifies another such example in which a liquid mixture separates.[25] We sought in part to probe whether this interpretation of pressure is indeed used by students in cases in which it leads to incorrect answers. We also sought to identify whether the shape and orientation of containers and the presence of multiple liquids affected student responses.

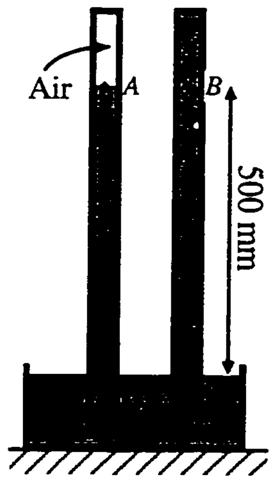
In order to probe these issues and to assess the effects of instruction, we have developed several written problems. In this section we describe the general features of several important types of problem that we have used. These problems have been posed in many different physics courses in the form of free-response ungraded diagnostic quizzes or course examination problems, typically after the completion of standard instruction in hydrostatics. In nearly all cases, the students have been asked both to give an answer and to explain their reasoning. Below we describe three main types of problems, below, along with two variations upon these problems that elicited significantly different reasoning.

1.2 MULTIPLE-BAROMETER PROBLEMS

We have used several written problems in which students are asked to consider barometer-like situations, in which several inverted glass tubes, completely or partly filled with one or more liquids, rest in an open reservoir of liquid. We have named these problems *Multiple-Barometer* problems.

1.2.1 Problem description

In each Multiple-Barometer problem, students answer questions about the pressures at various points in a system of glass tubes inverted in a fluid reservoir. Many of the tubes are different from standard barometers in that they contain a quantity of gas atop the fluid column or multiple immiscible liquids in the tube.[26] For example, in the problem shown in Figure 1-1, two tubes are filled with mercury to the same height. Above the left mercury column is a small amount of air. The right mercury column is capped at a height above the point of interest with no air between the mercury and the top of the tube. Students are typically asked to rank the pressures at several points and to compare the pressure at a point to atmospheric pressure. We have also posed problems in which students are shown a single inverted tube like the left tube in Figure 1-1 and given the height of the mercury column. In these problems, students are asked to compare the pressure at the top of the column or in the space above the column to atmospheric pressure. Other examples of Multiple-Barometer problems can be found in Chapters 2, 3, and 4 and Appendices A and B.



Consider the following arrangement of glass tubes. The top of each tube is closed; the bottom end is open to a mercury reservoir that is open at the top. In the left hand tube the space above the mercury column contains air at an unknown pressure. The atmospheric pressure is at its normal value of 1 atm = 760 torr.

Is the pressure of the gas in the left tube greater than, less than, or equal to atmospheric pressure? Explain your reasoning.

Is the pressure at point B greater than, less than, or equal to the pressure at point A? Explain your reasoning.

Figure 1-1: One version of the Multiple-Barometer problem. In this version, there is air above point *A* and mercury above point *B*.

1.2.2 Correct answers: pressure ranking

In any Multiple-Barometer problem, a correct pressure ranking can be found by recognizing that the labeled points are at the same vertical distance from the free surface and are located in the same liquid. Thus, the pressures at points *A* and *B* in Figure 1-1 are equal.

1.2.3 Correct answers: pressure compared to atmospheric

In any of the versions of the Multiple-Barometer problem, the pressure at a point can be compared to atmospheric pressure in two ways. First, one can draw a free-body diagram for the mercury column. Since the net force on each column is zero, the magnitude of the downward contact force on the column by the substance above the mercury must be less than the magnitude of the upward contact force on the column by the substance below (for a column of nonzero weight). Since the cross-sectional area of the column is fixed, the pressure at the top of the column must be less than the pressure at the bottom of the column. As the pressure at the bottom of the mercury column is equal to atmospheric pressure, the pressure at the top must be less than atmospheric.

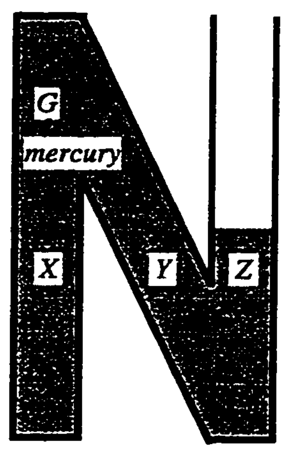
A simpler method for making these pressure comparisons involves using the equation $P = P_0 + \rho gh$ to find that the pressure at the point in question is ρgh less than atmospheric pressure. In the case of the problem shown in Figure 1-1, students must also recognize that the pressure at point A is the same as the pressure of the gas above the mercury column. In this case, the pressure of the gas must be less than atmospheric pressure by 500 mm Hg. Students using this method must recognize that the h in $P = P_0 + \rho gh$ is measured downward from a reference point. Thus, the pressure will be less than atmospheric at any points in the mercury that are above the free surface.

1.3 "LETTER TUBE" PROBLEM

We have used a series of problems in which students are asked to compare the pressures at various points within a single tube containing one or more liquids. As the initial version of this problem involved a tube in the shape of a letter of the alphabet, we chose the name *Letter Tube* problem to describe this family of problems. These problems have in some cases included straight segments of tubes and in other cases included curved segments (see Section 1.6 of this chapter).

1.3.1 Problem description

In the various versions of the Letter Tube problem, students are shown a static situation in which a tube is filled with one or more liquids. Students are asked either to rank the pressures at small numbers of points or to compare the pressures at pairs of points. In particular, the points that are chosen include some that at the same level in the same liquid but in different parts of the container. In some cases, students have been asked to compare the pressures at various points to atmospheric pressure. One example of a Letter Tube problem is the N-tube (see Figure 1-2).



An N-shaped glass tube with its right vertical leg open at the top is partly filled with mercury as shown at right. Points *X*, *Y* and *Z* are at the same height.

Rank the pressures at points G, X, Y, and Z. Explain how you determined this ranking.

Figure 1-2: The N-tube is representative of the Letter Tube problems.

1.3.2 Correct answers

In all cases, correct comparison of pressures in a Letter Tube problem involves use of the relationship of pressure to depth within a uniform fluid. The pressure increases with greater vertical distance from the free surface of the liquid and decreases with distance above the free surface. For example, the correct ranking of pressures in the N-tube is $P_X = P_Y = P_Z > P_G$. Points *X, Y* and *Z* are at the same vertical distance below the free surface of the liquid and point *G* is above them all. Other versions of the problem can be answered in similar fashion.

In cases in which students are asked to compare pressures to atmospheric, students can use reasoning similar to that described in the context of the Multiple-Barometer problem shown in Part 1.2 of this section. Students can either consider free-body diagrams for imaginary pieces of water, or they can apply the formula $P = P_0 + \rho gh$. For example, since point *G* in Figure 1-2 is above the free surface, the pressure at point *G* is less than atmospheric pressure.

1.4 DIFFERENT-DIAMETER U-TUBE PROBLEM

Unlike the Multiple Barometer problems and the Letter Tube problems, some of the problems that we have posed ask students to make an inference about the water level in a container based on their knowledge of pressure. The problem of this type that we have used most frequently is the *Different-Diameter U-tube* problem.

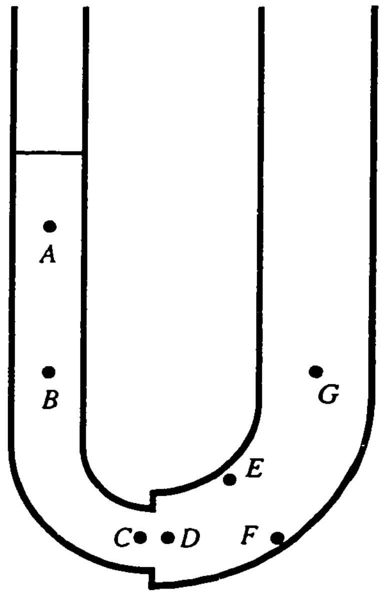
1.4.1 Problem description

In the Different-Diameter U-tube problem (see Figure 1-3), students are given a diagram showing a U-tube in which the two legs of the tube have different diameters. The water level in the left leg is shown, and students are asked to sketch the location of the water surface in the right leg. Students are then asked to rank the pressures at various points in the tube. In some versions of the problem, we have included the *Partition* task. In this task, students are asked to state whether the pressures at various points will change if a partition is added between the left and right legs of the tube (see Figure 2-5 in Chapter 2).

1.4.2 Correct answers

Correct analysis of the water level question requires use of the fact that the pressures at points at the same level are equal. In this case, the pressure at point C is equal to that at point D. The pressure at any free surface of the water must be equal to atmospheric pressure. The pressure at point C is greater than the pressure at the water surface in the left leg by an amount $\rho g h_C$, where h_C is the vertical distance from the water surface to point C. The variation of pressure with depth does not depend on the width of the column, so the pressure at point D must be greater than the pressure at the water surface in the right leg by the same amount $\rho g h_C$, and the level of the water surface in the right leg must be the same as that in the left leg. The observation characterized by the familiar phrase "water seeks its own level" is consistent with this analysis.[27]

A correct ranking of the pressures uses the idea that pressure at various points within a fluid is related to the vertical height of those points; thus the pressures at points C, D, and F are the greatest, the pressure at point E is next greatest, the pressures at points B and G are next, and the pressure at point A is least.



Consider a U-tube that is open at the top on both sides. The diameter of the right tube is one and a half times that of the left tube. The U-tube is filled with water such that the water level on the left is as shown in the diagram.

Sketch the water level on the right and explain how you determined your answer.

Rank the pressures at the points indicated in the diagram. If at any point there is no water, state that explicitly. Explain your reasoning.

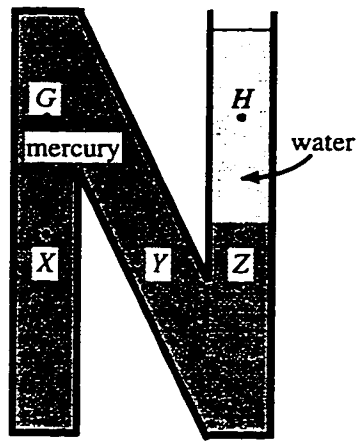
Figure 1-3: The Different-Diameter U-tube problem. In some versions of the problem, the Partition Task, shown in Figure 2-8 in Chapter 2, was added to the problem. In other versions, students were asked only to compare the pressure at point *C* to that at *D* and the pressure at point *E* to that at *F*. (In that version, the points have different labels. See Figure 2-7 in Chapter 2.)

1.5 PROBLEMS WITH TWO LIQUIDS

In several versions of the Letter Tube and Barometer problems, we posed questions in which students were asked to compare the pressures at two points that were at the same height but located within two liquids with different densities.

1.5.1 Problem statement

An example of a problem in which students compare the pressures of two points at the same level in liquids of different densities is the N-tube problem, shown in Figure 1-4. This problem has been used to supplement the single-liquid N-tube problem on course examinations. Similar questions have been asked in the context of Multiple-Barometer problems and other Letter Tube problems, including the Capped-U tube.



An N-shaped glass tube with its right vertical leg open at the top is partly filled with mercury as shown at right. Points *X*, *Y* and *Z* are at the same height. Water is poured into the opening on the right. The density of water is less than the density of mercury. Point *H* is at the same height as point *G*.

Determine whether the pressure at point H is greater than, less than, or equal to the pressure at point G. Explain your reasoning.

Figure 1-4: The N-tube problem with a second liquid.

1.5.2 Correct answer

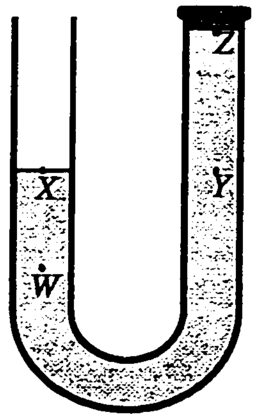
The reasoning used to compare the pressures at points *G* and *H* in the N-tube shown in Figure 1-4, or any two points at the same height in liquids of different densities, is quite difficult and requires considering the magnitude of the pressure gradient. The pressures at points X and Z are equal, and the pressures at points G and H are less. However, since the magnitude of the pressure gradient is greater in mercury than in water, the pressure at G is much less than that at X, and the pressure at H is only slightly less than that at Z. Algebraically, the pressure at point G is $P_Z - \rho_{Hg} g h_{GZ}$. The pressure at point H is $P_Z - \rho_{Water} g h_{GZ}$. Since $\rho_{Water} < \rho_{Hg}$, a smaller amount is subtracted from P_Z to find the pressure at H than is subtracted to find the pressure at G. The pressure at point G is therefore less than the pressure at point H.

1.6 CAPPED U-TUBE PROBLEM

Some problems that we originally considered to be similar to Letter Tube problems described above elicited sufficiently different reasoning that we have classified them separately. The different responses elicited by these problems are described in more detail later in Chapters 2 and 3.

1.6.1 Problem description

The Capped U-tube and similar problems involve curved tubes in the shape of roman letters with one end of the tube capped. See Figure 1-5.



A U-shaped tube (height ~ 0.5 meters) is partly filled with water as shown at right. The right end of the tube is closed at the top, but the left end is open to the atmosphere. There is no air between the rubber stopper and the water surface on the right-hand side.

Rank the pressures at points W, X, Y, and Z. Explain your reasoning.

Is the pressure at point Z greater than, less than, or equal to atmospheric pressure? Explain your reasoning.

Figure 1-5: The Capped U-tube problem.

1.6.2 Correct answers

As with the problems described above, students can compare the pressures at the points in the Capped U-tube problem using the equation $P = P_0 + \rho g h$. For the tube shown in Figure 1-5, a correct pressure ranking is $P_W > P_X = P_Y > P_Z$. Since the pressure at point X is equal to atmospheric pressure, the pressure at point Z must be less than atmospheric pressure.

2. IDENTIFYING STUDENT DIFFICULTIES IN COMPARING PRESSURES OF POINTS LOCATED AT THE SAME LEVEL IN A STATIC LIQUID

2.1 INTRODUCTION

Analysis of a static liquid reveals that the pressures are equal at points at the same level in the liquid. This idea is implicit in the equation for pressure $P = P_0 + \rho gh$, where P_0 is the atmospheric pressure, ρ is the density of the liquid, and h is the vertical distance measured downward from the free surface. That the pressures are equal at points at the same level is assumed to be self-evident by many instructors and textbook authors. One author states, for example, "$P = P_0 + \rho gh$. This shows clearly that the pressure in a liquid increases with depth but is the same at all points at the same depth."[28] In this chapter we examine two questions. First, to what extent do students recognize that two or more points that are at the same level in a static liquid have the same pressure? Second, what ideas lead students to make incorrect comparisons of the pressures at points at the same level in a static liquid? The fraction of students that do compare pressures correctly seems to vary depending on the specific situation, but we have observed some patterns, which we discuss in detail.

2.2 STUDENT SUCCESS WITH PRESSURE COMPARISONS FOR POINTS AT THE SAME LEVEL IN THE SAME LIQUID

In this section, we examine student responses on Multiple Barometer, Letter Tube, and Different-Diameter U-tube problems for those points that are located at the same level. (Analysis of student responses for points at different levels will be described in Chapter 3.) For a description of these problems, see Chapter 1.

2.2.1 Summary of student responses

We have examined student responses to pressure comparison questions in a variety of first- and second-year university physics courses after standard instruction. These include Physics 115, the algebra-based introductory course at the University of Washington, Physics 11, the calculus-based introductory physics course at Harvard University, Physics 152, the calculus-based introductory course at Purdue University, Physics 113, the second-year course in hydrostatics and thermal physics at the University of Illinois, and Physics 224, the second-year course in hydrostatics and thermal physics at the University of Washington. A summary of responses given by students in these courses to several problem types is shown in Table 2-1. The results from section to section of the same course have been very similar and are thus grouped in the table. Although a handful of the students who answered correctly either gave no reasoning or gave incorrect reasoning, nearly all correct answers have included correct reasoning, so we have shown only the percentage of students answering such pressure comparisons correctly. A more detailed analysis of student responses, showing the specific problems and the responses in individual sections of the same course, can be found in Appendix A.

Table 2-1: Student responses to pressure comparisons for points at the same level in a liquid after standard instruction. In each case, the number shown in the percentage of students answering correctly. See also Appendix A.

	Percentage of students answering correctly:				
Category of problem:	Phys. 115[AB] Washington	Phys. 11[CB] Harvard	Phys. 152[CB] Purdue	Phys. 113[2Y] Illinois	Phys. 224[2Y] Washington
Letter Tubes, one liquid	65% ($N = 310$)	-	-	-	75% ($N = 73$)
Barometers, one liquid	60% ($N = 64$)	-	-	-	50% ($N = 56$)
Different-diameter tubes	30% ($N = 110$)	55% ($N = 138$)	60% ($N > 1000$)	-	80% ($N = 98$)
Barometers or Letter Tubes w/ two liquids	45% ($N = 120$)	-	-	65% ($N = 182$)	45% ($N = 56$)
Capped U-tube (or similar)	20% ($N = 109$)	25% ($N = 138$)	40% ($N > 1000$)	-	70% ($N = 74$)

All percentages rounded to the nearest 5%. [AB]Algebra-based. [CB]Calculus-based. [2Y]Second-year. [MC]Multiple-choice problem.

The table shows that student responses varied somewhat depending on the specifics of the physical situation, especially in the algebra-based courses.[29] In response to Letter Tube problems and Barometer problems involving a single liquid, about two-thirds of the students in the algebra-based course correctly stated that the pressures were equal at the points at the same level. This fraction was slightly better for the more advanced second-year course. Among both groups, student performance on Letter Tube problems was slightly better than that on Barometer problems. In those courses in which we have posed problems in which students compare pressures at points at the same height with different liquids above, students have had somewhat more difficulty with these problems than with the single-liquid problems.

As the last row of the table shows, the Capped U-tube problem and some other problems that included curved tubes with one closed end seemed to elicit quite different responses than other tubes. Students in the first-year courses were considerably less successful in correctly responding to the pressure comparisons on these problems than on other Letter Tube problems. This difference in success rates was smaller among the students in the second-year course. The difference in response patterns suggests that the shape of these tubes elicit conceptual difficulties that straight tubes do not. The different types of responses elicited by these situations will be discussed in detail in Section 2.4 of this chapter. In addition, a comparison of student responses on these problems to those on other pressure ranking tasks can be found in Appendix A.

2.2.2 Commentary on results

Although the idea that the pressures at points at the same level in a static liquid are equal is a simple and fundamental consequence of Newton's second law, it seems that many students have difficulty in correctly applying this idea. The particular physical situation that students are asked to consider seems to affect student responses. In the following sections we examine the reasoning given by students in support of incorrect answers.

2.3 INCORRECT BELIEF THAT PRESSURE IS DUE SOLELY TO THE WEIGHT OF THE MATERIAL ABOVE A POINT

One of the most common incorrect beliefs that we identified is the idea that the pressure at a point is due solely to the weight of the substance above the point, which is incorrect in cases in which parts of the container exert downward forces on the liquid.[30] This reasoning, which we have named *'weight-above'* reasoning, seems to be very prevalent in responses given by students in certain types of physical situations. We show three examples of physical situations in which this type of reasoning is given by a significant fraction of students. In the first two situations, students are asked to rank pressures. In the third situation, students are asked to predict the water level in two connected tubes. See also Chapter 4 for a related problem.

2.3.1 Pressure comparison on Multiple-Barometer problem with multiple liquids

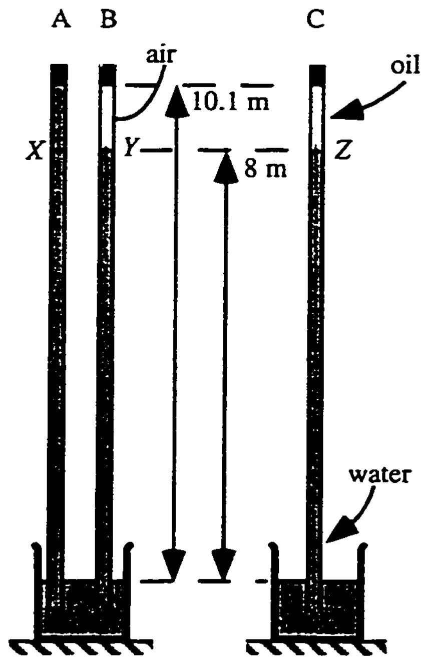
The incorrect belief that the pressure is due solely to the weight of the substance above a point is particularly evident in student responses to problems with multiple liquids.

2.3.1.1 Population tested

We posed Multiple-Barometer problems with multiple liquids on course examinations after standard instruction in two sections of Physics 224 at the University of Washington.

2.3.1.2 Student responses

Slightly different versions of the Multiple-Barometer problem were used with the two sections. Student responses in the two sections were quite similar. Slightly less than half of the students answered correctly, with just over one third giving correct reasoning. The criterion for correct reasoning was that an answer include either a reference to the height of the points or the equation $P = P_0 + \rho gh$. Some of the students who answered correctly also noted that the tops of the tubes exert different forces on the liquid in the three tubes.



Three students have constructed water barometers using long plastic tubes which are sealed at the top with rubber stoppers. **All heights in the problem are given with respect to the surface of the water in the buckets. The bottom of each stopper is at a height of 10.1 m.**

In A's tube, the water is in contact with the rubber stopper at the top of the tube. In B's tube, there is air trapped at the top of the tube. The bottom of the air bubble is at a height of 8.0 m. In C's tube, there is oil on top of the water. The oil-water interface is at a height of 8.0 m and the top surface of the oil is in contact with the stopper at the top of the tube. The density of the oil is 900 kg/m^3.

Rank the pressures at the three points X, Y and Z. If any pressures are equal, state that explicitly. Explain your reasoning

Figure 2-1: An example of a Multiple-Barometer problem with multiple liquids that elicits 'weight-above' reasoning.

In the version of the problem shown in Figure 2-1, for example, one-third of the class gave the ranking $P_X > P_Z > P_Y$, stating that the pressure is greatest at the point with water above and the least at the point with air above. All of the students who gave this answer referred either to the density or to the weight of the material above the three points. One such student wrote, "since they are all at the same depth relative to the cork, the ranking depends only on the stuff above–...water is the heaviest, ...oil is next, and the air is the lightest." A handful of students used similar reasoning to support other incorrect answers, often making additional errors that led to a different ranking. As is shown in Table 2-2, incorrect answers based on 'weight-above' reasoning were given by approximately the same number of students as the correct response.

Table 2-2: Responses given by students in the second-year thermal physics course to the pressure comparison task on two versions of the Multiple-Barometer problem (see Figure 2-1).

	Physics 224 Washington Su'95 ($N = 17$)	Physics 224 Washington Au'95 ($N = 39$)
Correct (all pressures equal)	45%	45%
with correct reasoning	30%	40%
Incorrect rankings based on 'weight-above' reasoning	30%	40%
Other / blank	25%	20%

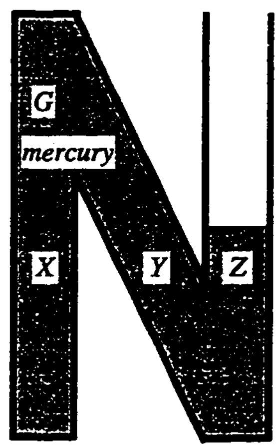
All percentages rounded to the nearest 5%.

2.3.1.3 Discussion of student responses

The failure of students to recognize that the pressures at the points shown are equal is based on the incorrect belief that pressure is due solely to the weight of the material above a point. Answers of this sort suggest that many students do not recognize that container walls can exert forces that will influence the pressure at a point.

2.3.2 Pressure comparison in the N-tube problem with a single liquid

In responses to some problems, students based their pressure ranking on the weight of the liquid above the points in question, but gave responses that suggested a very literal interpretation of the word 'above.' Some of these students drew vertical lines to the container wall and used the length of these lines as a measure of the amount of liquid above the points; their pressure ranking was then based on this amount of liquid. As an example, we show student responses to the N-tube problem (see Figure 2-2), but we have seen similar responses in other Letter Tube problems.



An N-shaped glass tube with its right vertical leg open at the top is partly filled with mercury as shown at right. Points *X*, *Y* and *Z* are at the same height.

Rank the pressures at points *G*, *X*, *Y*, and *Z*. Explain how you determined this ranking.

Figure 2-2: The N-tube problem.

2.3.2.1 Populations tested

The N-tube problem was used as a midterm examination problem in one section of Physics 224 and as an ungraded quiz in one section of Physics 115 at the University of Washington. In both cases, the problem was administered after the completion of standard lecture instruction on hydrostatics. For the purposes of this section, we focus on the rankings of pressures for the points at the same level, i.e., points *X*, *Y*, and *Z*.

2.3.2.2 Student responses

Student responses to the N-tube problem after standard instruction in the two courses are shown in Table 2-3. Although a majority of students stated that the pressures would be equal at the points at the same level, a significant fraction ranked the pressures incorrectly.

Table 2-3: Student ranking of points at the same level in the N-tube problem after standard instruction.

	Physics 115 Washington Wi'97 ($N = 201$)	Physics 224 Washington Wi'96 ($N = 23$)
Correct ranking ($P_X = P_Y = P_Z$)	65%	60%
Incorrect (P_X, P_Y, and P_Z not all equal)	30%	40%
based on 'weight-above' reasoning	15%	30%
Other / Blank	<5%	0%

All percentages rounded to the nearest 5%.

In both courses, a particular wrong answer was given by many students: $P_X \geq P_Y > P_Z$.[31] Approximately 25% of the students in the section of Physics 115 and seven of the nine students in Physics 224 who answered incorrectly gave this ranking. Nearly all of these students gave the same type of explanation, based on the amount of liquid directly above the three points. In many cases the students also referred to the equation $P = P_0 + \rho gh$ to support their ranking. For example, one student drew the lines shown in Figure 2-3, labeling the distance h_Y as the vertical distance from point *Y* to the wall directly above it. This student wrote, "*Z* has least pressure because it is closest to the surface of the liquid (*h* is minimum) it has less fluid above it than any other point. Because of the angled leg in the middle, *Y* has less fluid above it than *G* or *X* . . . all [pressures] vary with *h* . . . ρgh varies with *h*." Many students were less explicit but stated that their pressure ranking was based on the amount of liquid directly above the points: "Pressure depends on depth and gravity, where there is more mercury straight above the point there is more pressure."

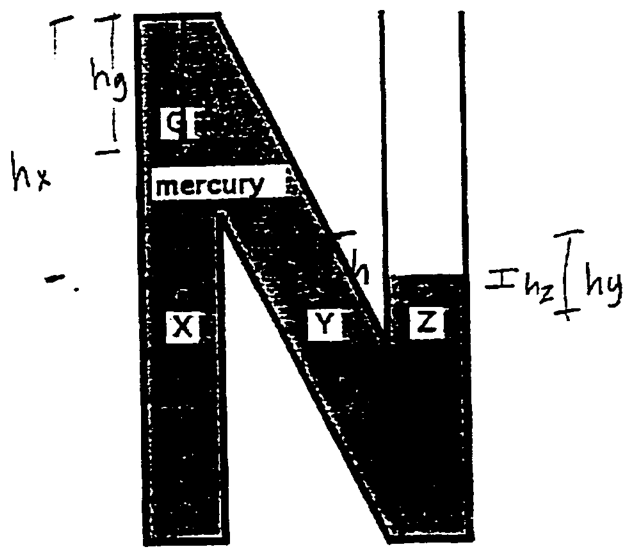


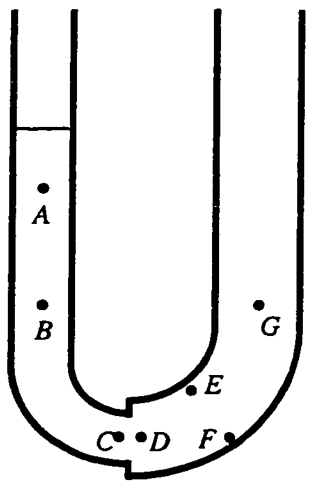
Figure 2-3: A student response to the N-tube problem.

2.3.2.3 Discussion of student responses

As with student responses to the Multiple Barometers problems, these incorrect responses suggest that many students do not understand the forces exerted by container walls in the presence of a static liquid. In particular, many students fail to recognize that the forces exerted by the liquid above a point may be greater than the weight of the liquid and that container walls can exert forces of varying size depending on the situation.

2.3.3 Water level comparison in Different-Diameter U-tube problem

In the previous two problems, students ranked pressures based on the incorrect idea that the pressure at a point is due solely to the weight of the material above the point. In order to probe this idea, we developed a problem in which students who used this idea would make an incorrect prediction about the behavior of the liquid. In this section we describe student responses to the water level task in the Different-Diameter U-tube problem (see Figure 2-4). A correct response to this problem is based on the fact that the pressure at the surface of the water in each leg of the tube must be atmospheric, and that the surfaces must therefore be at the same level.



Consider a U-tube that is open at the top on both sides. The diameter of the right tube is one and a half times that of the left tube. The U-tube is filled with water such that the water level on the left is as shown in the diagram.

Sketch the water level on the right and explain how you determined your answer.

Figure 2-4: The Different-Diameter U-tube problem.

2.3.3.1 Populations tested

The Different-Diameter U-tube problem has been administered as an ungraded written quiz to students in the following courses: Physics 115, the algebra-based introductory course at the University of Washington; Physics 224, Physics 11, the introductory calculus-based physics course at Harvard University; and Physics 152, the introductory calculus-based physics course at Purdue University. In all sections of Physics 115 and Physics 224 at the University of Washington, as well as in the section of Physics 152 at Purdue, students had completed standard lecture instruction on pressure. The students in Physics 11 at Harvard had not yet had lectures on pressure, but had been assigned reading on the topic.

2.3.3.2 Student responses

In each of these classes, there were two common responses to the question in which students were asked to draw the water level in the wider side of the U-tube. Many students correctly drew the water level at the same level, but a large fraction drew the water level lower in the wider tube. Responses for the different classes are summarized in Table 2-4. The idea that the water level should be lower in the right tube was sufficiently persistent that 10% of the students in a section of Physics 224 gave this answer even after having just seen a lecture demonstration showing connected tubes with liquid levels at the same height.[32]

Table 2-4: Answers to the water level question in the Different-Diameter U-tube problem in four different courses. Several sections are combined in the cases of the courses at the University of Washington.

	Physics 115 Washington 3 sections ($N = 236$)	Physics 11 Harvard Au'96 ($N = 138$)	Physics 152 Purdue Sp'98 ($N > 1000$)	Physics 224 Washington 3 sections ($N = 98$)
Correct (water levels at same height)	45%	30%	45%	70%
Incorrect (water level lower in wider tube)	50%	70%	55%	20%
Other / Blank	10%	<5%	~0%	10%

All percentages rounded to the nearest 5%.

Reasoning in support of correct answers. Students who answered correctly gave a variety of reasons to support their answer. Some students gave very common-sense responses: "intuition." One student argued that, "in an open system, water will always 'equilibrate' and find its own level." Other students thought about the pressure at the water surface, arguing, for example, "the U-tube is exposed to air, even [though] the tube is different size, I think that the pressure on both opening is the same 1 atm."

'Weight-above' reasoning. A large number of students who drew the water level lower in the right leg of the tube supported their incorrect answer by referring to pressure. Many of these students made an explicit connection between pressure and the weight or mass of the water in the two sides. For example, one student wrote, "pressure ~ mass of two sides' water must equal." The distinction between pressure and force was difficult for some students. One wrote "pressure on left side of tube has to equal to pressure on right side…air pressure the same, so volume of H_2O has to equal. So since diameter is greater, level is lower." This student drew the sketch shown in Figure 2-5 and showed the water level lower in the wider side of the tube.

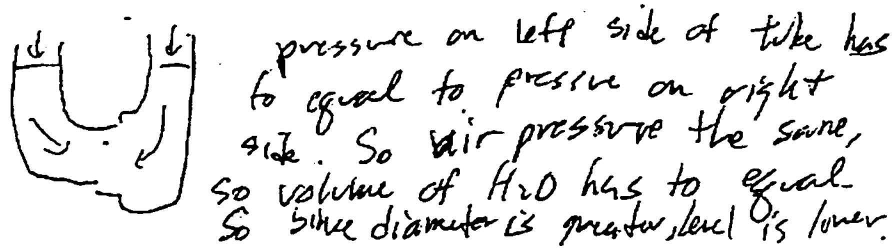


Figure 2-5: A student response to the Different-diameter U-tube problem. This student sketched the water level lower in the wider side of the tube.

Another group of students who drew the water level lower in the wider leg of the tube justified their responses based on the mass or volume of water in the two sides of the tube, but without an explicit connection between pressure and weight. For example, one student in this category wrote, "the weight of the water on both sides of the tube will be equal and so the level on the right will be lower." Although few of the students in this category used the term 'pressure,' the reasoning given by these students otherwise seems to be quite similar to the responses described above. Some students in this category referred explicitly to the gravitational forces on the water in the two tubes: "force on one side = force on the other; F = mg (pressure from atmosphere as well on greater surface area), equal m of water on each side thus lower water level on the right."

Reasoning based on incorrect force arguments. Not all incorrect responses were based on the confusion between pressure and weight. Some students instead used incorrect reasoning based on the forces acting on the top surfaces of the two water columns. These students typically noted that the pressure at the top of the two columns was equal to atmospheric, but that the column with the greater area would lead to a larger force on the water by the atmosphere. From this correct analysis, these students drew the incorrect conclusion that the larger force on the right surface would lead to the liquid being displaced downward on this side: "Since the surface area of the right side is greater than the left, more force will be exerted on it and it will be lower."

This idea seems to be based on a direct comparison of the force on the two open surfaces, without any consideration of the forces exerted by the earth or by container walls. In some cases, students seem to be incorrectly relating the "net" force by the atmosphere (*i.e.*, the difference between the forces exerted on the two top surfaces) to the final positions of the water surfaces. This difficulty may be related to the incorrect association of net force with displacement or position that we describe in Chapter 6. Further discussion of this type of reasoning in other contexts can be found in Section 2.4.3 of this chapter and in Section 4.4 of Chapter 4, in which we relate student difficulties with pressure to difficulties in identifying forces and applying Newton's second law.

2.3.3.3 Discussion of student responses

The student responses to the water level comparison on the Different-Diameter U-tubes problem showed that many students fail to differentiate the concepts of pressure, force, and weight. In particular, a large number of students drew the water level lower in the right leg of the U-tube based on the assumption that the weight of the water in the two legs of the U-tube should be equal.

2.3.4 Summary of 'weight-above' reasoning

In all three of the examples that we have shown, students made incorrect predictions consistent with an incorrect belief that the pressure at a point is due solely to the weight of the material above the point, sometimes with the additional consideration of cross-sectional area. Although this reasoning can lead to correct answer in some situations, student responses suggest a tendency to generalize this result to situations in which the reasoning is not sufficient. These incorrect responses suggest in addition that students are not considering the existence or variable magnitude of the forces exerted on the liquid by container walls.

2.4 INCORRECT BELIEF THAT PRESSURE VARIES WITH DISTANCE 'ALONG THE TUBE'

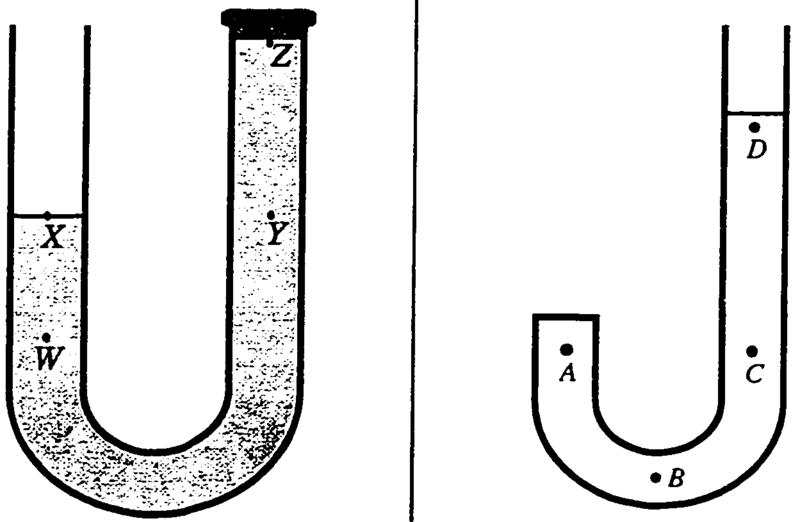
In Section 2.2 of this chapter, we showed that students performed at a lower level on problems involving curved tubes with a closed end than on problems with straight segments of tubes. The problems with curved tubes seemed to elicit different reasoning. In many cases, students referred either to the formula $P = P_0 + \rho g h$ or to the liquid pressing on a point, but determined the relevant 'h' or amount of liquid by measuring along the wall of the tube containing the liquid. Examples of this reasoning, which we have named *'along the tube'* reasoning, will be shown below.

2.4.1 Pressure comparisons in the Capped U-tube and J-tube

Both the Capped U-tube problem, shown in Figure 2-6a, and the J-tube problem, shown in Figure 2-6b, were developed to complement the N-tube and other Letter Tube problems. We believed that all of these problems were equivalent and that student responses for points in these tubes could be matched to those for corresponding points in the N-tube. In particular, both problems include points that are located at the same level that have different amounts of liquid direct above them. As demonstrated in the last section, that feature seemed to elicit 'weight-above' reasoning.

2.4.1.1 Populations tested

The J-tube has been administered to students as an ungraded quiz in one section of Physics 11 at Harvard University. At the time they answered the J-Tube problem, the students had been assigned reading on hydrostatics, but the lecturer had not covered the subject. The Capped-U Tube has been administered to students as an ungraded quiz in several courses, including one section of Physics 115 at the University of Washington and one section of Physics 152 at Purdue University. In Physics 115, the variation of pressure with depth had been introduced but instruction had not been completed at the time of the pretest. The students in Physics 152 had completed lecture instruction on the topic. Both the J-tube and the Capped U-tube were used as pretests for research-based instruction.



a. The Capped U-tube

b. The J-tube

A tube (height ~ 0.5 meters) is partly filled with water as shown. One end of the tube is closed at the top, but the other is open to the atmosphere. There is no air between the cap and the water surface on the closed side.

Rank the pressures at the marked points. Explain your reasoning.

Figure 2-6: (a) The Capped U-tube and (b) the J-tube. These problems elicited significantly different responses in many cases than did other Letter Tubes. Students often ranked the pressures based on their distance 'along the tube,' giving the ranking $P_Z > P_Y > P_W > P_X$ for the Capped U-tube.

2.4.1.2 Student responses

A summary of student responses to problems of this type is shown in Table 2-5. As was suggested by the results shown in Table 2-1, the success rate of students in several courses was considerably less on the curved and capped tubes than on other Letter Tube or Barometer problems with a single liquid. Fewer than half of the students correctly ranked the pressures at the two points at the same level, as compared to over 60% on other problems. In addition to the difference in correct answers, the nature of incorrect responses was quite different. Less than 10% of the students responded in a manner consistent with the pressure being related solely to the water directly above a point (*e.g.*, $P_Y \geq P_W > P_X > P_Z$ in the Capped U-tube).[33]

Instead, up to 50% of the students gave rankings consistent with the position 'along the tube' (*e.g.*, $P_Z > P_Y > P_W > P_X$ in the Capped U-tube). Students who gave this type of ranking often referred to measuring along the walls of the tube. One such student wrote, "points farthest away in the distance of the tube from water level feel greatest pressure." We interpret the phrase 'in the distance of the tube' as meaning the distance as measured 'along the tube.' One of the students at Purdue who gave this ranking drew the arrows shown in Figure 2-7. He wrote, "$Z > Y > W > X$. This is because there is pressure exerted on X which exerts a larger pressure on W and that builds and builds."

A related form of reasoning given by students involved the amount of water pushing on a point, similar to the 'weight-above' reasoning described in Section 2.3. However, rather than the amount of water directly above a point, this type of reasoning was based on the amount of water between that point and the open end of the tube: "There is more volume of water putting pressure on the points as they go into the J-tube." Several students suggested that the system behaved like a straight tube: "closed one end tube acts like a straight system with [point] Z therefore being deepest and pressure increases with depth." One student using this argument wrote, "since left end [of the J-tube] is closed, it counts as the bottom depth of water."

In addition to those students who gave the most common incorrect answer based on measuring along the tube, about five percent of the class gave the reverse of this ranking (*e.g.*, $P_X > P_W > P_Y > P_Z$ in the Capped U-tube). Few of these students explained their answers, but those that did typically mentioned the atmosphere as the source of the pressure and argued that the pressure decreased as distance from the free surface increased. One student, in response to the J-tube, argued, "the force is applied by atmospheric pressure, $D > C > B > A$." Another wrote, "$D > C > B > A$ because air is pushing directly on D." Although these students seem to view the atmosphere as the source of the pressure and neglect the weight of the liquid, their response also seems to be based on the idea that the pressure varies along the curvature of the tube.

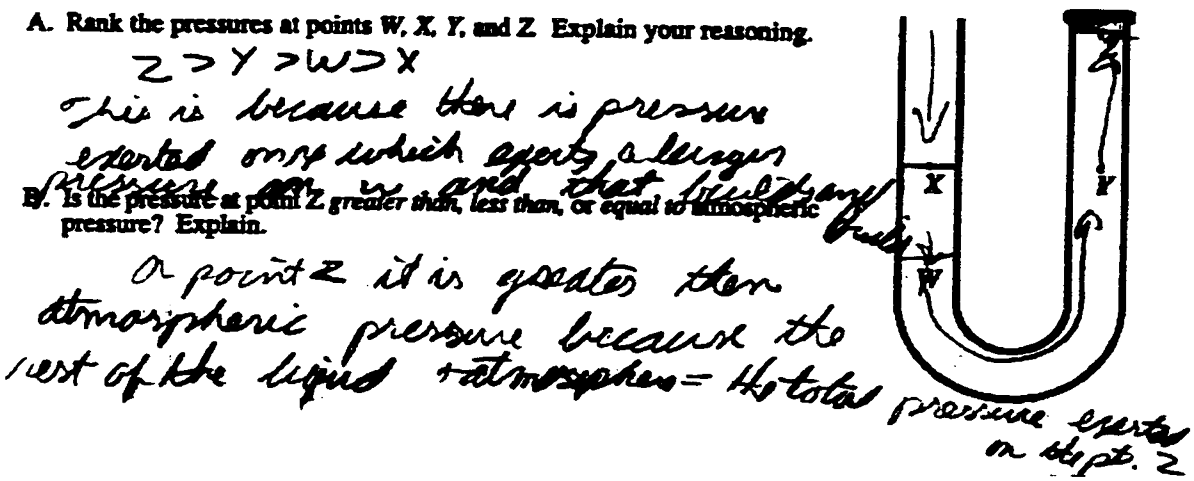


Figure 2-7:A student drew this diagram in support of a pressure ranking consistent with 'along the tube' reasoning. The written response reads: "$Z > Y > W > X$. This is because there is pressure exerted on X which exerts a larger pressure on W and that builds and builds."

Table 2-5: Student responses to capped and curved Letter Tube problems showing the prevalence of the 'along the tube' reasoning.

	Physics 115 Washington Sp'98 ($N = 109$)	Physics 11 Harvard Au'96 ($N = 138$)	Physics 152 Purdue Sp'98 ($N > 1000$)
Correct	20%	25%	40%
'Along the tube' or reverse	40%	45%	20%
Other / blank	40%	30%	40%

All percentages rounded to the nearest 5%.

2.4.2 Pressure comparisons in the N-tube problem reexamined

Student responses to the J-tube problem were very different from what we expected based on responses to previous versions of the Letter Tube problems. Therefore we decided to re-examine student responses to the N-tube problem to search for similar reasoning. We discovered that the most common incorrect ranking on the N-tube problem is also consistent with 'along the tube' reasoning: point Z is nearest the surface, and points Y, G, and X are successively further along the tube (see Figure 2-8). We decided to examine student reasoning for the N-tube to determine how many students might have been using 'along the tube' reasoning.

We found that all of the students in Physics 224 who gave the ranking $P_X > P_G > P_Y > P_Z$ used reasoning in which they explicitly referred to the water above a point. Many drew sketches in which they marked the distance from points to the container wall directly above. In the Physics 115 section, the reasoning was more varied. However, only a very small number of students made explicit statements suggesting that they were using the 'along the tube' reasoning. In fact, even if one assumes that all students who gave the ranking $P_X > P_G > P_Y > P_Z$ but did not give reasoning were using 'along the tube' reasoning, this group still includes less than 5% of the class.

It seems that the curved tubes in the Capped U-tube and J-Tube problems suggest the 'along the tube' reasoning to students in a way that the N-tube does not. We believe that the critical difference for students lies in the smooth curves of the Capped U-tube and J-tube. In student responses to interview problems with curved tubes, we have seen a tendency for students to think of the entire body of liquid in a curved tube as a single object, free to move along the curved 'length' of the tube. It is possible that the students do not perceive the liquid in the N-tube as free to move in the same way. We discuss these responses further in Chapter 4. As the Capped U-tube and similar problems seem to elicit both 'along the tube' reasoning and 'weight-above' reasoning and to allow us to distinguish student reasoning based on the pressure ranking, we have chosen to use these problems as pretests in the instructional materials that we have developed. Further discussion of these materials can be found in Chapter 5. In addition, the locations of points on future editions of the N-tube problem will be modified to allow us to distinguish the different forms of student reasoning.

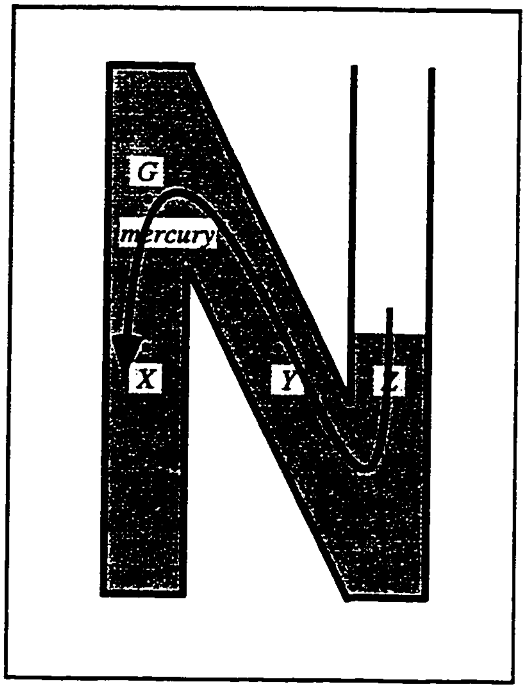


Figure 2-8: A possible interpretation of the pressure ranking $P_X > P_G > P_Y > P_Z$ in the N-tube problem, in terms of 'along the tube' reasoning.

2.4.3 'Along the tube' reasoning in the different-diameter U-tube problem

We modified the pressure ranking in the Different-Diameter U-tube problem so that we could probe further for the idea that the pressure varies with distance along the container wall. In the pressure ranking, students are asked to compare pressures at two points that are located at different heights in the right-hand leg of the tube, but are located along a line perpendicular to the curvature of the tube (see Figure 2-9). Thus both points can be thought of, in some sense, as being the same distance from the water surface as measured along the length of the tube. This form of the Different-Diameter U-tube problem was given at Harvard in Physics 1 and Physics 11 and at Purdue in Physics 152.

Approximately 25% of the students in the Physics 11 at Harvard, 30% of those in Physics 1 at Harvard, and 35% of those in Physics 152 at Purdue said that the pressures at these two points are equal. Several of the students who gave this answer drew lines like the one shown in Figure 2-9. Responses of this sort provide additional evidence that some students believe that the pressure in a curved tube varies along the curvature of the tube.[34]

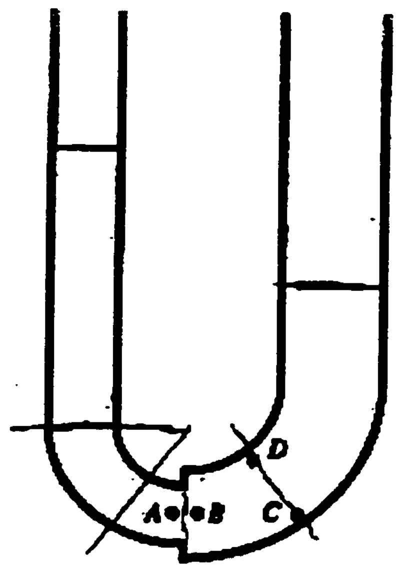


Figure 2-9: Lines drawn by some students in responses to the Different-Diameter U-tube problem. This student stated that the pressures at points *C* and *D* are equal.

2.4.4 Summary of 'along the tube' reasoning

We have shown several different situations in which students made incorrect pressure comparisons based on a tendency to associate pressure with the distance from the surface as measured along the container walls. Some of these students seem to interpret the h in $P = P_0 + \rho gh$ as the distance between a point and the water surface as measured along the tube, parallel to the container walls. For example, many students found that points *C* and *D* in Figure 2-9 had the same pressure, suggesting that these students believed that these points were at the same 'height' for the purpose of calculating pressure. In the J-tube and

Capped U-tube problems, students applied this type of reasoning to predict that the pressures at two points at the same level were different, reasoning that one point was farther 'along the tube' and that therefore the pressure there was larger. A handful of students turned this reasoning around and argued that the point that was farther from the surface of the liquid would have the smaller pressure.

Although the pressures of points in the N-tube could be analyzed in the same way, very few students did so. Similarly, we saw few students reasoning in this way on Multiple-Barometer problems, in which the liquid is contained in straight tubes. These observations suggest that the curved shape of the tubes is important in eliciting this reasoning. As we discuss in Chapter 4, we believe that this reasoning is connected to the tendency of students to think about the forces acting on large, curved pieces of water. For example, students in the Different-Diameter U-tube problem often referred to a U-shaped piece of water, and compared the downward forces on the two top surfaces to determine the final position of this piece of water, neglecting the forces on the water by the container as well as the gravitational force on the water.

2.5 INCORRECT BELIEF THAT PROXIMITY TO CONTAINER WALLS AFFECTS PRESSURE

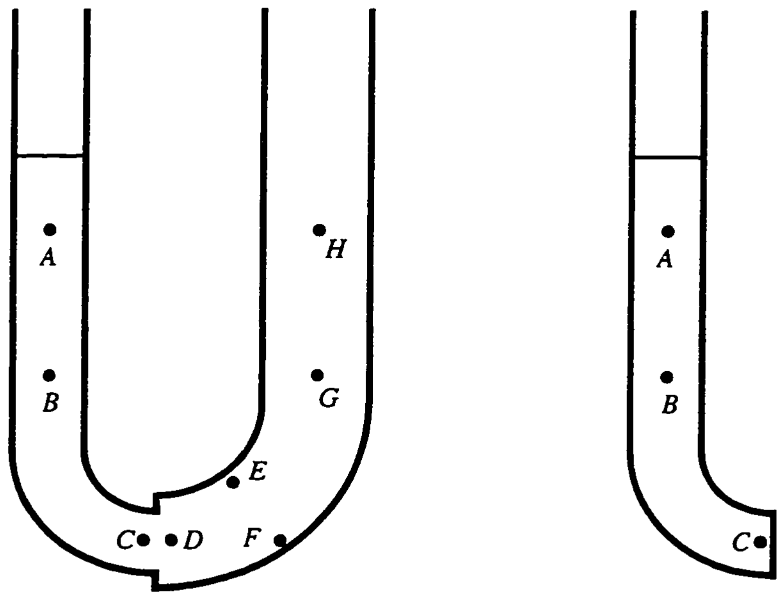
We have identified a second incorrect form of reasoning with pressure that involves the shape of the container. For some students, if a point in a liquid is near a container wall, the pressure at that point is different than that at points at the same level that are farther from the walls. This type of reasoning seems to be similar to that of the 'packed crowd' model the Kariotoglou and Psillos have identified.[35] This 'packed crowd' model is somewhat anthropomorphic, in that students compare liquid in confined spaces to people in a crowded stadium, who 'feel more pressurised.' Similarly, students reasoning on the basis on proximity to container walls often state that a liquid desires freedom or seeks to escape from a container. Although Kariotoglou and Psillos associate the 'packed crowd' model with variable liquid density, we have not probed this aspect of student reasoning.

2.5.1 Pressure comparisons in the J-tube problem

In addition to the 'along the tube' reasoning described above, some students' responses to the J-tube problem shown in Figure 2-6b are consistent with incorrect reasoning based on the proximity of points to the container walls. On this problem, nearly 10% of the class gave reasoning of this sort to support the response that the pressure at point *A*, which has a wall above it, is greater than the pressure at point *C*, which does not. For example, some students said that the pressure at point *A* would be greater because the wall was preventing the water in that end from reaching the same level as the water in the right end. One such student wrote, "[the water at] *A* wants to be same level as *D* but can't, force exerted on it from walls of tube." Other students simply referred to the proximity of the container wall: "water at same level [has] equal pressure, but since *A* is a closed end, more pressure."

2.5.2 Pressure predictions in the partition task

In the Different-Diameter U-tube problem, we have included in some versions of the question a task in which students consider the effect of inserting a partition between the two sections of the U-tube (see Figure 2-10). Students are asked whether the pressures at points *A*, *B*, and *C* will increase, decrease, or remain the same when the partition is inserted. This question can be answered using the equation $P = P_0 + \rho g h$ and noting that the vertical distances from points to the free surface of the liquid do not change. Therefore, the pressures at the points remain the same when the partition is added. The fact that the pressure has not changed means that the partition must exert a force on the water to its left of the same magnitude and direction as the force that was previously exerted by the water to the right of the partition. Many of the students who answered correctly referred to this force in their answer.



Suppose a wall is now inserted between the left and the right tubes, and the tube on the right is removed. The water level remains the same in the left tube.

After the wall has been inserted, would the pressure at each of the following points *increase, decrease* or *stay the same*? Explain your reasoning in each case.

- Point *A*
- Point *B*
- Point *C*

Figure 2-10: The Partition Task in the Different-Diameter U-tube problem.

The partition task has been included as part of the Different-Diameter U-tube problem on an ungraded quiz in a section of Physics 115. These students had completed standard instruction on pressure. Only one-third of the students answered the partition task correctly. Most of the students who answered incorrectly stated that the pressure at point *C* would change but that the pressure at point *A* would remain the same. The students who argued that the pressure at point *C* would change were divided nearly equally into those who said that the pressure would increase and those who said that it would decrease. A summary of student responses is shown in Table 2-6.[36] Students who answered that the pressure would decrease are discussed in Section 2.6 of this chapter.

Almost all of the students who answered that the pressure at point *C* would increase gave reasoning based on the proximity of point *C* to the container wall. One such student wrote that the pressure at point *C* would "increase...trying to escape." Another wrote, "the wall would be a barrier...so all that pressure would accumulate at point *C*."

Table 2-6: Student responses to the partition task (see Figure 2-10) in a section of Physics 115 after standard instruction.

	Physics 115 Washington Au'95 ($N = 69$)
Correct (P_A, P_C stay the same)	35%
Incorrect, P_C increases	30%
Incorrect, P_C decreases	25%
Other incorrect	15%

All percentages rounded to the nearest 5%.

2.5.3 Summary of reasoning based on proximity to container walls

As with many of the other incorrect beliefs that we have described in this chapter, reasoning based on proximity to container walls reflects in part a failure to understand the forces exerted on and by container walls. For example, students in the J-tube problem argued that the wall above point *C* led to a greater pressure at this point, failing to recognize that the wall would exert a downward force of the same magnitude and direction as the water at the corresponding level on the other side of the tube.

2.6 INCORRECT BELIEF THAT CONTAINER WALLS CAN 'ISOLATE' POINTS

We identified another difficulty that students have in reasoning about the effect of container walls on hydrostatic pressure. Many students seem to believe that points beneath a container wall have pressure related to the liquid above but are not affected by atmospheric pressure. In some cases, students also argue that a container wall can isolate points from the pressure due to some amount of liquid. We have named this type of reasoning *'isolation'* reasoning. This reasoning is similar in some ways to the proximity reasoning described in Section 2.5, in that students make errors based on the presence of container walls. One difference is that whereas students using proximity reasoning tend to overestimate the pressure at a point that is near a container wall, students using 'isolation' reasoning tend to underestimate the pressure at such a point. This idea was elicited by two of the problems that we have administered: the Capped U-tube problem and the Partition task in the Different-Diameter U-tube problem.

2.6.1 Pressure comparisons in the Capped U-tube problem

The Capped U-tube, in addition to eliciting the 'along the tube' reasoning described previously, seemed to elicit isolation reasoning from some students. In particular, several students gave a pressure ranking in which two points at the same level have different pressures and supported their ranking by stating that the cap on the right end of the U-tube would isolate points in that leg of the U-tube from the effects of the atmosphere (see Figure 2-6a). One such student wrote that $P_X > P_Y$ and stated that at point *X*, "air pressure [is] pushing down" but that at point *Y* there is "no air pressure."

Some students, in using this type of reasoning, explicitly referred to the formula for hydrostatic pressure. These students looked at the equation $P = P_0 + \rho gh$ and divided it into two terms, one (P_0) due to the atmosphere and one (ρgh) due to the liquid above a point. Some of these students gave the incorrect pressure ranking $P_W > P_X > P_Y > P_Z$, arguing, for example, that the points *Y* and *Z* in the right leg are beneath the stopper and therefore isolated from the effect of the atmosphere. These points, they argued, should only include the term ρgh, whereas points X and W in the left leg had the additional pressure term P_0 due to the effects of atmospheric pressure. One student who gave this ranking stated,"[Points] Z and Y do not have atmospheric pressure on them. Y is same height as X without atmospheric pressure."

2.6.2 The different-diameter U-tube problem: the partition task

The partition question in the Different-Diameter U-tube problem (see Figure 2-10) generated some incorrect responses that seemed to be based on 'isolation' reasoning. Typically these students argued that the pressure at point C would decrease when the partition was added.

Most of these students seemed to be arguing based on the idea that the water would no longer be pushing from the right leg of the tube. One student wrote, "[the pressure at point C will] decrease → the pressure from the water on the right is gone." Another, who answered that the pressure would decrease at both points B and C, wrote, "before there was pressure from below because of the weight of the water on the right, but now there is just pressure from above." These students seemed not to recognize that the partition could exert a force as well, and that in fact the force exerted by the partition would be the same as the force exerted by the water before the partition was inserted.

A very small fraction of the students who answered that the pressure at point C would increase when the partition was added used 'isolation'-like reasoning, but had additional difficulties. One such student seemed to be thinking that the forces exerted by the water to the left and right of the junction would somehow cancel each other without the partition in place; he wrote that the pressure at both points would increase when the partition was added, arguing that there would now be "no counteracting forces" by the water on the right.

2.6.3 Summary of 'isolation' reasoning

The reasoning described above is another example of a failure of students to understand the forces exerted by container walls on a static liquid. Many students gave answers suggesting that the presence of a wall would isolate the liquid from the effects of the atmosphere or of some body of liquid, and seemingly neglected the fact that the wall itself could and would exert a force. These responses are in some ways reminiscent of student responses on tasks involving tension such as those discussed by McDermott, Shaffer, and Somers.[37] In their study, they found that some students argued that the tension in a massless string attached to weights on both ends is different from the tension in a massless string that is attached to a similar weight at one end but to a wall or other 'passive' object on the other end. In both cases, a lack of understanding of the passive forces exerted by objects leads many students to make incorrect predictions.

2.7 DISCUSSION

As has been demonstrated in this chapter, many students fail to develop a functional understanding of a basic idea of hydrostatics, that the pressure is the same at all points located at the same level in a static liquid. A variety of incorrect beliefs lead students to make incorrect pressure comparisons. These difficulties seem to be sufficiently strongly held that they either result in students reasoning incorrectly with the equation $P = P_0 + \rho g h$ or failing altogether to recognize its applicability.

Many of the specific difficulties that we identified seem to be related to incorrect ideas about the effects of container walls. For example, many students believe that the pressure at a point is determined solely by the weight of the liquid above the point. These students fail to recognize that forces by the container can contribute to the pressure. In other problems, students state that the pressure at a given level is greater for a point near a container wall.

Student responses to the pressure ranking tasks seemed to vary considerably depending on the physical shape of the container presented in a problem. Containers that appear to be similar to a physicist do not always lead students to reason in a similar manner. For example, the N-tube problem seemed to be particularly effective at eliciting the student response that the pressure is a function of the water directly above a point, but the J-tube and Capped U-tube elicited different types of responses, in particular the idea that the distance from the free surface should be measured 'along the tube.' A summary of this finding is presented in Table 2-7.

Table 2-7: A summary of the forms of incorrect reasoning elicited by various pressure problems.

	'Weight-above'	'Along the tube'	'Proximity'	'Isolation'
N-tube	yes	very little	no	no
Capped U-tube and J-tube	yes	yes	yes	yes
Diff.-Diameter U-tube	yes	yes	partition	partition
Multiple-Barometer	yes	no	no	no

'Partition' refers to the partition task (see Figure 2-10).

These difficulties suggest that many students do not understand the interaction between a static liquid and the walls of its container. In particular, it seems that many students either do not understand that container walls are capable of exerting forces or fail to recognize that the magnitude of those forces can and will vary depending on the pressure. Many of the difficulties that students have with pressure seem to be related to similar difficulties with topics in introductory mechanics that have been reported by our group and other researchers. For example, the failure to recognize forces exerted by the container walls is similar in many ways to the failure of students to understand 'passive' forces in mechanical situations, like those of a string attached to a wall or a book at rest on a table.

We explore the connections between student difficulties with mechanics and student performance on hydrostatics tasks in Chapter 4.

The fact that students do not understand that the pressures at points at the same level are equal casts serious doubt on students' ability to understand other ideas in hydrostatics, to say nothing of situations in which a fluid is moving. In the following chapter, we examine student performance on tasks that involve the gradient of pressure.

3. IDENTIFYING STUDENT DIFFICULTIES WITH THE GRADIENT OF PRESSURE IN A STATIC FLUID

3.1 INTRODUCTION

In Chapter 2, we showed that many students fail to understand that points at the same level in a liquid have the same pressure. In this chapter, we describe several difficulties that students have in applying the idea of the gradient of pressure in a static fluid. In none of these cases have we explicitly used the term 'gradient.' Rather, students have been asked to rank the pressures at several points or to compare the value of the pressure at a given point to atmospheric pressure. Even some of those students who correctly rank the pressures at points at the same level have difficulty in comparing the pressures at points at different levels, in comparing the pressure at a point to atmospheric pressure, and in comparing the pressures at points at the same level in liquids of different densities.

In examining students' responses to a variety of problems, we have been able to make some generalizations about students' reasoning, which will be described with examples of student responses. As the responses given by students seemed to depend on the nature of the problem that was posed, we have organized this chapter based on the three main categories of questions that we have asked. In each category, we describe several specific conceptual difficulties and incorrect beliefs, beginning with the most basic errors and continuing with increasingly difficult concepts. Some of the student difficulties described in Chapter 2 suggest a misunderstanding of the pressure gradient as well. These difficulties are also discussed in this chapter.

3.2 DIFFICULTIES IN PRESSURE RANKINGS FOR POINTS AT DIFFERENT HEIGHTS

In Chapter 2, we showed the responses given by students on tasks requiring pressure rankings for points located at the same height in a static liquid. Many of the problems that we described also included points located at different heights in the liquid. Some students gave responses to these pressure rankings that suggested a failure to understand the gradient of pressure.

3.2.1 Populations tested

We have examined student responses to pressure comparison questions in a variety of first- and second-year university physics courses after standard instruction. These include Physics 115, the algebra-based introductory course at the University of Washington, Physics 1, the algebra-based introductory physics course at Harvard University, Physics 11, the calculus-based introductory physics course at Harvard University, Physics 152, the calculus-based introductory course at Purdue University, Physics 113, the second-year course in hydrostatics and thermal physics at the University of Illinois, and Physics 224, the second-year course in hydrostatics and thermal physics at the University of Washington. In the following sections, we will briefly summarize student responses and combine responses from students in several sections of the same course. A more detailed analysis of student responses, showing the specific problems and the responses in individual sections of the same course, can be found in Appendix B.

3.2.2 Incorrect belief that pressure is equal at all points in a static liquid

In a typical course, students are taught that there is a pressure gradient in a static liquid (*i.e.*, that the pressure is not the same at all points in the liquid). We have tested student understanding of this idea using Letter Tube and Multiple-Barometer problems (see Chapter 1 for more complete descriptions of these families of problems). In these problems, we have asked students to rank the pressures at several points in a liquid, including points at different heights (see, for example, the problems in Figure 3-2 and

Figure 3-3). We have found that there are usually a few students who state that the pressures at all points in the same liquid are equal in response to written problems. The percentage of students giving responses of this type on written problems are shown in Table 3-1. In most cases, fewer than 5% of the students made this error. The reasoning given by students in support of this incorrect answer is described below.

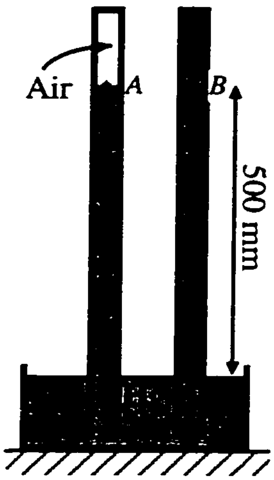
Table 3-1: Student responses suggesting that the pressures at all points in a liquid are equal.

	Physics 115[AB] Washington three sections (*N* = 449)	Physics 1[AB] Harvard Sp'98 (*N* = 120)	Physics 11[CB] Harvard Au'96 (*N* = 138)	Physics 224[2Y] Washington four sections (*N* = 117)
Instruction	Standard	Standard	Reading only	Standard
All pressures equal (incorrect)	~5%	10%	~5%	~0%

All percentages rounded to the nearest 5%. [AB]algebra-based. [CB]calculus-based. [2Y]second-year.

Students answering that the pressures at all points are equal have given reasoning that suggests several incorrect beliefs about pressure. One student wrote that the pressures at several points were all equal because "water is incompressible," possibly confusing pressure with density.[38] Other students base their answer on the perceived source of the pressure, arguing that all the pressures are equal, because "the tube is open to the atmosphere..." In problems like the Capped U-tube (see Figure 3-3), some students refer to the fact that the system is closed: "sealed system, all [pressures are] atmospheric."

The most common form of reasoning given by students who say that all points have the same pressure is based on an incorrect interpretation of Pascal's principle. Pascal's principle states that if there is a change in pressure at one point in a static liquid, the same change will be experienced at other points in the liquid.[39] Many students give responses suggesting that they interpret this principle as describing the pressures at various points rather than the changes in pressure. For example, one student in response to the Multiple-Barometer problem shown in Figure 3-1 wrote, "The pressure of the gas in the left tube is equal to the atmospheric pressure because in the static fluid, pressure is uniform throughout according to Pascal's principle." Although they were only asked to compare points at the same height, approximately 10% of the students in a section of Physics 115 spontaneously made similar statements in comparing the pressures at points *A* and *B* in this problem: "since both points *A* & *B* are within the fluid, [their] pressures are equal...the pressure of the Hg is the same no matter where you test it in the container."



Consider the following arrangement of glass tubes. The top of each tube is closed; the bottom end is open to a mercury reservoir that is open at the top. In the left hand tube the space above the mercury column contains air at an unknown pressure. The atmospheric pressure is at its normal value of 1 atm = 760 torr.

Is the pressure of the gas in the left tube greater than, less than, or equal to atmospheric pressure? Explain your reasoning.

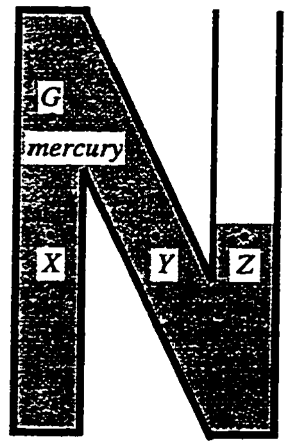
Is the pressure at point B greater than, less than, or equal to the pressure at point A? Explain your reasoning.

Figure 3-1: A Multiple-Barometer problem.

3.2.3 Incorrect belief that pressure increases with height in a static liquid

In addition to recognizing the existence of a pressure gradient in a static fluid, students are expected to know the direction of the gradient. In other words, they are expected to recognize that the pressure increases with depth. In responses to a variety of written questions, we have found that the majority of students recognize that pressure increases with depth. This result is consistent with those of Driver and Engel Clough, who reported that a large fraction of precollege students recognized that pressure increases with depth, with the fraction increasing up to 87% in the case of 16-year olds.[40] However, we have found that a handful of students give answers consistent with the belief that the pressure is less at points that are deeper in a liquid. In some cases, this incorrect idea seems to be based on incorrect application of the equation $P = P_0 + \rho g h$.

The responses of students to Letter Tube problems on which they are asked to rank pressures are shown in Table 3-2. The results from students in several sections were similar and have been combined. A detailed summary of responses and the problems posed can be found in Appendix B. For each problem, 5-10% of the students who correctly ranked the pressures for points at the same level have given rankings in which points at a greater height in the liquid have a greater pressure. For example, in the N-tube problem shown in Figure 3-2, 10% of the students in one section of Physics 115 stated that the pressure at point *G* was greater than the pressures at points *X*, *Y*, and *Z*.



An N-shaped glass tube with its right vertical leg open at the top is partly filled with mercury as shown at right. Points *X*, *Y* and *Z* are at the same height.

Rank the pressures at points G, X, Y, and Z. Explain how you determined this ranking.

Figure 3-2: The N-tube problem.

The reasoning given by these students suggests that some believe that positive h in the formula $P = P_0 + \rho g h$ is measured upward.[41] One student, in response to the N-tube problem, gave the ranking $P_G > P_X = P_Y = P_Z$, writing, "$P = P_0 + \rho g h$; if h is greater, pressure is greater." On the same problem, another student wrote, "because [point] G has $h_2 > h_1$, thus $P_G > P_X, P_Y, P_Z$," also writing the equation $P = P_0 + \rho g h$. Other students did not refer to the equation but made similar statements: "*X, Y, Z* are all at the same level, same pressure. *G*, the pressure is a lot higher than *X, Y, Z*, the height of *G* is the reason why."

Table 3-2: The fraction of students stating incorrectly that points at a greater height have greater pressure.

	Physics 115[AB] Washington 2 sections ($N = 311$)	Physics 1[AB] Harvard Sp'98 ($N = 120$)	Physics 11[CB] Harvard Au'96 ($N = 138$)	Physics 224[2Y] Washington two sections ($N = 70$)
Instruction	Standard	Standard	Reading only	Standard
Pressure *increases* with height (incorrect)	10%	10%	10%	~5%

All percentages rounded to the nearest 5%. [AB]algebra-based. [CB]calculus-based. [2Y]second-year.

This type of incorrect response seems to be due to simple misuse of an equation. However, students who have even a superficial conceptual understanding of hydrostatic pressure should recognize that the answer that pressure increases with height is incorrect.

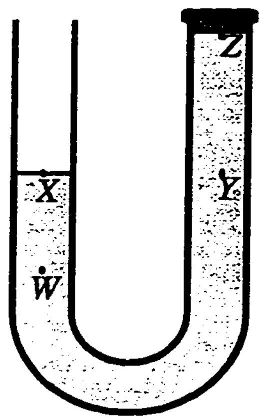
3.3 DIFFICULTIES IN COMPARING PRESSURE TO ATMOSPHERIC

We have posed problems in which students are asked to compare the pressures at various points to atmospheric pressure. These problems include the Letter Tube and Multiple-Barometer problems described in previous chapters. We have found that many students fail to apply the idea of a pressure gradient in making these comparisons. These difficulties are particularly evident in cases in which students are asked to compare the pressure at a point above the free surface of the liquid to atmospheric pressure. As we show below, student responses for this type of pressure comparison were often very different from those in pressure ranking tasks described in the previous section.

3.3.1 Populations tested

problems in which students were asked to compare the pressure of a point above the free surface to the value of atmospheric pressure have been asked of students in two sections of

Physics 115, five sections of Physics 224, one section of Physics 113 at the University of Illinois, and one section of Physics 152 at Purdue University. These problems were administered either on course examinations or on ungraded quizzes after standard instruction on hydrostatic pressure. We have posed problems in which there is a gas above a column of liquid (see the Multiple-Barometer problem in Figure 3-1), and students are asked to compare the pressure of the gas to atmospheric pressure. In other problems, students are asked to compare the pressure at a point in a liquid that is above the free surface to atmospheric pressure.[42] For example, students are asked to compare the pressure at point *Z* in the Capped U-tube problem in Figure 3-3 to atmospheric pressure.



A U-shaped tube (height ~ 0.5 meters) is partly filled with water as shown at right. The right end of the tube is closed at the top, but the left end is open to the atmosphere. There is no air between the rubber stopper and the water surface on the right-hand side.

Rank the pressures at points W, X, Y, and Z. Explain your reasoning.

Is the pressure at point Z greater than, less than, or equal to atmospheric pressure? Explain your reasoning.

Figure 3-3: The Capped U-tube problem. Students are asked to rank pressures and compare the pressure at a point that is located above the free surface to atmospheric pressure.

3.3.2 Summary of student responses

A summary of student responses to such problems is shown in Table 3-3. Details of the problems and of responses in various sections of the same course can be found in Appendix B. Student reasoning is described below.

Table 3-3: Student responses from two sections of Physics 115 for the comparison of the pressure at points above a free surface to atmospheric pressure P_0.

	Physics 115[AB] Washington 2 sections (N = 170)	Physics 152[CB] Purdue Sp'98 (N = 903)[MC]	Physics 113[SY] Illinois Sp'98 (N = 182)[MC]	Physics 224[2Y] Washington 5 sections (N = 155)
Instruction	Standard	Standard	Standard	Standard
Less than P_0 (correct)	40%	65%	45%	65%
with correct reasoning	15%	n/a	n/a	45%
Equal to P_0	20%	20%	35%	10%
Greater than P_0	35%	15%	20%	20%

[MC]multiple-choice problem. [AB]algebra-based. [CB]calculus-based. [2Y]second-year.
All percentages rounded to the nearest 5%.

3.3.3 Incorrect reasoning given by students

We have analyzed the reasoning given by students who answered incorrectly. There were several categories of incorrect reasoning. Some students reasoning based on surface features of the barometer, most notable the height of the mercury column. Other incorrect forms of reasoning included incorrect application of the equation for hydrostatic pressure, incorrect application of Newton's Second Law, incorrect arguments based on the properties of gases, and 'isolation' reasoning similar to that described in Chapter 2. Each is described in detail below.

3.3.3.1 Incorrect reasoning based on surface features of barometer

In several courses, the most common incorrect answer to these problems has been that the pressure at a point at the top of a liquid column is greater than atmospheric pressure. Again, many of the students who gave this answer seemed to misapply the condition for equilibrium. For example, in response to a Multiple-Barometer problem similar to the one shown in Figure 3-1, one student wrote, "the pressure in the closed end is greater than 1 atm since 1 atm = 760 mm and the Hg has only risen 700 mm, the pressure inside the closed end must be greater than 1 atm if the Hg cannot reach a height = 1 atm." Another student supported this answer by writing, "if the liquid level was at its standard height of 760 mm then this would imply atmospheric pressure." Even very good students made this error; one of the top students in a section of Physics 224 wrote, "the column is shorter than 760 mm, indicating it has been compressed by air at the top of the tube."

Some of these students may be attempting to apply Newton's Second Law to the liquid column. If so, their answers are consistent with a belief that the downward force on the mercury must be greater than the upward force because the liquid is displaced downward from its standard, 'equilibrium' height of 760 mm. Although few of these students explicitly state that there is a net force on the column of liquid, these responses are consistent with the incorrect reasoning that we describe in Chapter 6, in which students argue that the displacement or position of an object is related to the net force on the object.

A handful of students also compared the height of the mercury column to its 'standard' height of 760 mm but used this incorrect reasoning to support a correct answer. Most of these students gave reasoning based on an incorrect understanding of the working of a barometer and the contents of the space above the mercury column. One such student wrote simply, "the pressure in the left tube is less than atmospheric pressure because the column of mercury inside the tube is less than 760 mm Hg." This answer suggests an incorrect belief that if the height of the column were equal to 760 mm Hg, the pressure on top of the column would be equal to atmospheric pressure, rather than zero.

3.3.3.2 Incorrect application of Newton's second law

In some of the courses, the most common incorrect answer has been to say that the pressure (at point A in Figure 3-1, for example) is equal to atmospheric pressure. Most of these answers seemed to be based on an incorrect application of Newton's second law to the liquid. Many of these students attempted to answer based on the forces acting on a column of liquid, or, in some cases, based on the pressures at various locations. For example, one student stated, "the pressure of [point] A has to equal atmospheric pressure or the tubes would not be in equilibrium (*i.e.*, the level would be changing.)" Another wrote, "If the mercury is at equilibrium, then both of the quantities (the air and the Hg) are at the same pressure."

The reasoning given by these students is consistent with an incorrect application of Newton's second law to the liquid column that neglects the weight of the liquid. For a massless liquid, the magnitudes of the upward and downward contact forces exerted on a column of liquid at rest would be equal. However, if the liquid has nonzero mass and the column is at rest, the magnitude of the upward contact force on the bottom of the liquid must be greater than that of the downward force on top. In many cases the students argued that the pressures at the top and bottom had to be equal without referring to the weight of the liquid, suggesting that they had not considered all of the forces acting on the liquid.

3.3.3.3 Incorrect application of the equation for hydrostatic pressure

A small fraction of students answered using reasoning similar to that described in Section 3.2.3 of this chapter, misinterpreting the 'h' in the equation $P = P_0 + \rho gh$. Most of these students concluded that the pressure at points above the free surface of the liquid would be greater than atmospheric pressure. For example, in response to the Multiple-Barometer problem shown in Figure 3-1, one student stated that the pressure at point A would be greater than atmospheric pressure, "because atm pressure would be along the surface of the mercury at the bottom. Point A is up much higher; $P = \rho gh$." In response to the Capped U-tube (see Figure 3-3), a student wrote, "[the pressure at point Z is] greater than [atmospheric], because $P_Z = P_0 + \rho gh$ where $h > 0$."

In some cases, students' incorrect application of the equation for hydrostatic pressure was complicated by 'along the tube' reasoning of the type described in Section 2.4 of the previous chapter. For example, on the Capped U-tube problem (see Figure 3-3), one student argued that the pressure at point Z would be greater than atmospheric, "since the right end tube is closed, pressure increases with h." This student had also previously given the pressure ranking $P_Z > P_Y > P_W > P_X$, which indicates 'along the tube' reasoning. Another student wrote the equation $P = P_0 + \rho gh$ to support her answer that the pressure at point Z is greater than atmospheric, having previously argued, "the pressure increases as [you?] go down the tube" and given the common incorrect pressure ranking $P_Z > P_Y > P_W > P_X$. (See Chapter 2 for details on this form of incorrect reasoning.)

3.3.3.4 Incorrect arguments based on gas properties

Some students gave incorrect answers to these problems and supported their answers with incorrect arguments based on the properties of the gas in the tubes rather than on the basis of the properties of the liquid. In most of the courses in this study, the topic of gases is not covered before fluid statics. It should be noted, however, that most of the students had previously studied the ideal gas law, either in other physics courses, or, more commonly, in introductory chemistry courses.

Incorrect arguments based on confinement or compression of gas

Some students argued that the gas above the liquid column was confined and so must be at a pressure greater than atmospheric. For example, in response to the barometer problem shown in Figure 3-1, one student wrote, "[the pressure at] $A >$ atm, because the air is trapped and the pressure of the trapped air is greater than that of atmospheric pressure, which then puts a greater pressure at point A." Arguments of this sort are reminiscent of the incorrect models for hydrostatic pressure described in Chapter 2.

Incorrect belief that all gas is at atmospheric pressure

In addition to the students who decided that the pressure of the gas above the liquid column was greater than atmospheric, a handful of students argued that any point where a liquid is in contact with a gas must be at atmospheric pressure. One student wrote, "The pressure at point A is equal to atmospheric pressure or 1 atm. Atmospheric pressure is when the surface of a liquid comes in contact with air...."

3.3.3.5 Incorrect belief that container walls can 'isolate' points from effect of atmosphere

In the Capped U-tube problem (see Figure 3-3), some students gave the correct answer that the pressure at point *Z* is less than atmospheric pressure, but gave incorrect reasoning that seems similar to the 'isolation' reasoning described in Chapter 2. For example, one student answered that the pressure at point *Z* is less than atmospheric pressure, writing, "because of the rubber stopper, [there is] no atmospheric pressure acting on water in right side of tube." Some of these students may have been focusing on the part of the problem statement that reads, 'there is no air between the stopper and the water surface on the right side.' Another wrote, "[pressure at] $Z < P_{atm}$ because no air is involved."

3.3.4 Summary

Many students gave answers that suggested that they did not have a complete understanding of the dependence of pressure on depth. Few students recognized the need to subtract ρgh from the value of atmospheric pressure to find the pressure at a point above a free surface. Interestingly, the problems requiring the comparison of pressure to atmospheric seem to elicit different student responses than those requiring pressure rankings. For example, a comparison of the results in Table 3-2 to those in Table 3-3 reveals that whereas only about 10-15% of students give answers consistent with no gradient or with a reversed gradient in pressure rankings, the fraction doing so in problems requiring comparisons to atmospheric pressure is 30-55%.

Many of the errors made by students also reflect a failure to apply Newton's second law to the liquid. For example, many students said that the pressures at the top and bottom of a column of liquid are equal, because the liquid is at rest. Responses of this sort suggest that students attempted to apply Newton's second law, but neglected some of the relevant forces (*e.g.*, the weight of the liquid). Other students gave answers suggesting that they did not even consider the consistency of their responses with Newton's second law. These responses suggest that some student difficulties with the gradient of pressure may be connected to underlying difficulties with mechanics.

Many of the students' responses seemed to be specific to a particular physical situation. In responses to the Multiple-Barometer problem, students often compared the height of a mercury column to 760 mm and answered based on this comparison. For example, students stated that the pressure of the gas in the space above a mercury column must be at a pressure greater than atmospheric because the liquid column is displaced from its standard height of 760 mm. The Capped U-tube did not seem to elicit this type of reasoning, perhaps because students could not compare the height of the water column in the U-tube to any perceived 'natural' height. The Capped U-tube, on the other hand, elicited from many students the 'along the tube' reasoning described in Chapter 2. Despite the differences in the reasoning given by students answering incorrectly, the fraction of students answering correctly was quite similar on the two problem types, suggesting that it is the students who answer incorrectly who are distracted by surface features of the problem.

3.4 DIFFICULTIES IN COMPARING THE PRESSURES AT POINTS AT THE SAME LEVEL IN LIQUIDS OF DIFFERENT DENSITIES

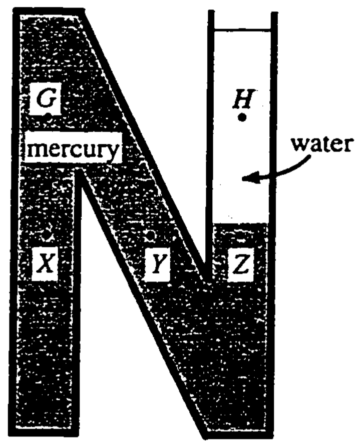
In order to further probe student understanding of the gradient of pressure, we posed problems in which students are asked to compare pressures at points at the same level in liquids of different densities (see Figure 3-4). This type of task requires students to reason using differences in pressure, therefore requiring knowledge of the quantities that influence the magnitude of the pressure gradient.

3.4.1 Populations tested

We have posed several different problems in which students have been asked to compare the pressures of two points at the same height in liquids of different densities. Problems asking students to make this comparison have been given to one section of Physics 115, six sections of Physics 224, and one section of Physics 113, the second-year course in hydrostatics and thermal physics at the University of Illinois. In all of the problems the points were located in tubes of the same diameter, except in the problem posed in Physics 115, in which the tube containing the more dense liquid had twice the diameter (see Figure 3-5). In some of the sections of Physics 224, the course instructor had emphasized this type of reasoning in lecture, often explicitly covering a similar situation.

3.4.2 Summary of student responses

The pressure comparison between pairs of points at the same height but located in different liquids proved to be extremely difficult for students. For example, when the N-tube problem shown in Figure 3-4 was given to a section of Physics 224 on a midterm examination, only one student of the 23 who took the exam answered correctly with correct reasoning. Even in three sections of Physics 224 in which the lecture instructor emphasized this type of reasoning, fewer than a third of the students were able to answer correctly with correct reasoning.[43] By far the most common incorrect answer has been to say that the pressure in the more dense liquid is greater (see Section 1.4 of Chapter 1 for an example of the correct reasoning for this task).



An N-shaped glass tube with its right vertical leg open at the top is partly filled with mercury as shown at right. Points *X*, *Y* and *Z* are at the same height. Water is poured into the opening on the right. The density of water is less than the density of mercury. Point *H* is at the same height as point *G*.

Determine whether the pressure at point H is greater than, less than, or equal to the pressure at point G. Explain your reasoning.

Figure 3-4: The N-tube problem with liquids of different densities.

Table 3-4: Student responses to problems requiring comparison of pressures at points at the same height in liquids of different densities. In the problem given in Physics 115, the two points were in tubes of different cross-sectional area (see Figure 3-5).

	Phys 115^{AB} Washington Au'95 (*N* = 120)	Phys 224^{2Y} Washington 3 sections (*N* = 95)	Phys 224^{2Y} Washington 3 sections (*N* = 94)	Phys 113^{2Y} Illinois Au'98 (*N* = 182)^{MC}
Instruction	Standard	Standard	Standard w/ emphasis	Standard
Pressure less in denser liquid (correct)	35%	15%	40%	15%
with correct reasoning	~0%	5%	30%	N/A
Pressure greater in denser liquid	20%	60%	40%	60%
Pressures equal	30%	15%	15%	30%
Other / blank	15%	15%	10%	none

All percentages rounded to nearest 5%. ^{MC}multiple-choice problem ^{AB}algebra-based ^{2Y}second-year

3.4.3 Incorrect reasoning given by students

We examined student reasoning for this problem. We found several incorrect forms of reasoning, one of which led to the correct answer.

3.4.3.1 Incorrect reasoning based on the cross-sectional area

Although more than one third of the students in Physics 115 correctly stated that the pressure was greater at the point with greater density, only two of these students gave correct reasoning. Most of those who answered correctly with incorrect reasoning referred to the areas of the two tubes, stating that the point in the less-dense fluid has a greater pressure because is located in the tube with smaller diameter (see Figure 3-5). Most of these students had also answered incorrectly for points at the same level, like points 3 and 6 in the diagram, suggesting a rote use of the formula $P = F / A$.

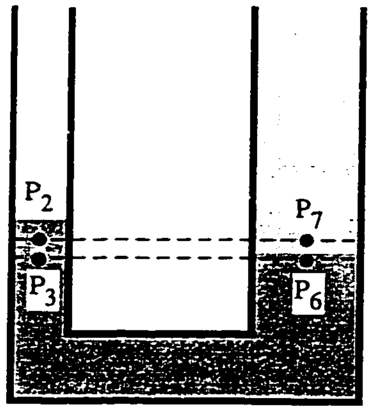


Figure 3-5: A two-liquid U-tube problem given in one section of Physics 115. In this problem, the tube containing the less-dense liquid (oil, in this case) also had twice the diameter of the tube containing the denser fluid (mercury).

In other cases, the height of liquid above the points was the determining factor: "The points located in mercury in the left tube have less pressure b/c there is less liquid on top of them. In the right tube the depths are greater, therefore these points will have greater pressure as they get deeper." These students typically answered that the pressure at point 3 in the narrow tube would be greater than the pressure at point 6 in the wider tube, also due to the height of the liquid column above the points.

3.4.3.2 'Weight-above' reasoning

The most common incorrect answer on this pressure comparison has been that the pressure is greater in the denser liquid. Students who give this answer typically focus on the weight or density of the liquid above the point. We therefore categorize these students as using the 'weight-above' reasoning described in Chapter 2, in which the pressure at a point is due solely to the weight of the material above a point. In response to the problem in Figure 3-4, one such student wrote, "The pressure at point G is greater than at point H...since the weight of the water above point H is less than the weight of mercury above

G, point *H* has less pressure." As in the examples shown in Chapter 2, students who give this reasoning seemingly failed to recognize the force exerted by the container wall above point *G*.

Some of the students who gave this answer gave very brief explanations, writing, for example, "the pressure at point *C* is less than the pressure at point *B* because the density of oil is less than the density of water." Many also referred to the equation $P = P_0 + \rho gh$ in their response, "*H* is less than *G*, while their depth may be the same, the density of each medium is different, ρgh determines the pressure, and ρ being less with point *G* and h being constant, *P* must be less."

3.4.3.3 Pressure related to height alone

Some the students answered that the pressures at points *G* and *H* in Figure 3-4 are equal. Most of the students who gave this answer did not give much explanation. Many simply stated that the points were at the same height and would therefore have the same pressure. One such student wrote, "my rankings are based solely on height. Start at the top of the page and work down, from lowest to highest pressure." Another student explicitly rejected the importance of density: "Pressure is dependent on height, density, and gravity. Oil is less dense but has greater heights applied to counter. Any point at the same height should have the same pressure." These students seem to have overgeneralized the result that the pressure depends only on depth for a uniform liquid.

Those students who did give more detailed explanations to support this answer tended to use the term 'equilibrium.' For example, one student wrote, "both levels are in equilibrium with the weights of oil and water and P_{atm}. Same height, same pressure." Another student wrote, "Equal. If not, net force $\neq 0$ and [there will be] motion." One of these students had previously responded that the pressures at all points in the N-tube were equal, suggesting that he did not understand the existence of a pressure gradient in the first place.

3.5 SUMMARY

As in the previous chapter, we have seen that many students fail to apply correctly the equation $P = P_0 + \rho gh$ to qualitative questions asking for simple pressure comparisons. Students hold a variety of incorrect beliefs about the gradient of pressure in a liquid. Some students state that all pressures are equal in a static liquid, incorrectly citing Pascal's principle to support their answer. Others believe that pressure increases with height rather than depth, seemingly misinterpreting the quantity h in the equation $P = P_0 + \rho gh$. As shown in Chapter 2, some students believe that the direction of the gradient is dependent on the shape of the container in which a liquid is located. All of these difficulties seem to be related to the mathematical formalism of hydrostatics.

We posed a variety of questions that do not ask explicitly about the pressure gradient, but that are most easily answered by using this idea. For example, a student could compare the pressure at a point in a liquid to atmospheric pressure based on application of Newton's second law to the free-body diagram for a carefully chosen portion of the liquid, but a faster solution can be obtained by comparing the vertical level of the point to that of the free surface of the liquid. Similarly, the pressures at two points at the same level in liquids of different densities could be compared using free-body diagrams, but a simpler solution uses the fact that the pressure gradient is proportional to the density of the liquid. In these problems, we have found that few students reason successfully with the pressure gradient.

Many of the incorrect responses that we have seen are based on incorrect application of Newton's second law to portions of a liquid at rest. For example, many students say that the pressure at a point at the top of a column of mercury in a barometer must be equal to atmospheric pressure. In many cases, this incorrect answer is based on the incorrect belief that the upward and downward contact forces on the column must be equal, an analysis that neglects the weight of the mercury. These responses suggest that whereas some of the difficulties that students have in reasoning with the pressure gradient are particular to the context of liquids, some may be related to difficulties with Newtonian mechanics. In the following chapter, we will examine these connections in more detail.

4. RELATING STUDENT UNDERSTANDING OF FORCES TO STUDENT UNDERSTANDING OF PRESSURE IN A STATIC LIQUID

4.1 INTRODUCTION

In Chapters 2 and 3 we described student difficulties with pressure and the gradient of pressure, respectively. Many of the difficulties with these concepts seemed to be due to fundamental difficulties with forces; for example, some difficulties with pressure seemed to stem from a failure to recognize that container walls could exert forces. In this chapter we describe our investigation of student understanding of forces exerted on and by static liquids. We also examine the extent to which student understanding of forces is related to student ability to solve pressure problems.

We discuss the results from problems that deal explicitly with forces in hydrostatic situations, as well as problems that deal with forces in purely mechanics contexts. We also examine the relationship of student responses on both these types of problems to student performance on hydrostatics problems involving pressures. In several courses, we have done student-by-student comparisons, matching responses given by students on mechanics problems to those given by the same students on pressure problems, in an effort to determine which ideas from mechanics are necessary and which are sufficient for student success on tasks involving pressure.

4.2 WRITTEN PROBLEMS USED TO IDENTIFY STUDENT DIFFICULTIES

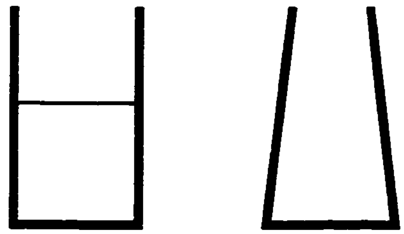
In this section we describe several additional written problems that we have used to identify student difficulties. In all cases, the problems were free-response, and students were asked to give an answer and to explain their reasoning. The problems appeared either on ungraded diagnostic quizzes or on course examination problems. The discussion includes the problems described below as well as others that were described in Chapter 1.

4.2.1 The Hydrostatic Paradox problem

The hydrostatic paradox is well-known to experienced instructors and is often addressed in textbooks.[44] We have posed problems based on the hydrostatic paradox in order to probe the 'weight-above' reasoning described in Chapter 2.

4.2.1.1 Problem Statement

In the *Hydrostatic Paradox* problem, students are presented with two containers of different shape. We have used several versions of this problem, which have in common containers with the same base area, but one has a varying vertical cross-section. For example, in the problem shown in Figure 4-1, the cross-section of the right container decreases with height. Students are asked to compare the pressures at the bottoms of the two containers and then to compare the force exerted on the water by the bottom of the container to the weight of the water in the container.



Two differently shaped containers with bases of equal area are shown at right. They contain the same volume of water.

Sketch the height of the water surface in container 2.

Determine whether the gauge pressure at the bottom of container 2 ($P_{bot,2}$) is greater than, less than or equal to the gauge pressure at the bottom of container 1 ($P_{bot,1}$). Explain.

Draw a free-body diagram for the water in container 2. Label each force, and identify the object on which it is exerted and the object exerting the force.

Is the force that the bottom of container 2 exerts on the water in container 2 greater than, less than or equal to the weight of the water in container 2? Explain.

Figure 4-1: One version of the Hydrostatic Paradox problem.

4.2.1.2 Correct answers

In the example, the vessels contain the same volume of water, so the depth of water will be greater in container 2. Therefore, by the equation $P_{gauge} = \rho gh$, the gauge pressure will be greater at the bottom of container 2. The 'paradox' lies in the apparently contradictory observations that the weight of the water in container 1 is equal to that in container 2, but the force exerted on the bottom of container 1 by the water is not equal to the corresponding force in container 2.

A free-body diagram for the water in container 2 is shown in Figure 4-2. There are forces on the water by the earth, by the bottom of the container, and by the slanted sides of the container (although the pressure varies along these sides, only two vectors are shown for the forces by the sides).[45] The contact forces exerted by the various container walls are each perpendicular to the corresponding wall.

The magnitude of the force exerted on the water by the bottom of the second container can be compared to the weight of the water in two ways. One method is based on the free-body diagram for the water (see Figure 4-2). Application of Newton's second law to the water shows that the force on the bottom of container 2 by the water is greater than the weight of the water in container 2.

The second way to answer the Hydrostatic Paradox problem involves using the pressure comparison. In the first container, the gauge pressure times the area of the bottom surface is equal in magnitude to the weight of the water. The gauge pressure at the bottom of the second container is greater than that at the bottom of the first, and the area of the bottom is the same, so the force on the bottom of the second container by the water is greater than the force on the bottom of the first container by the water. As the magnitude of the force on the bottom of the first container is equal to the weight of the water, and there is the same amount of water in each container, the magnitude of the force on the bottom of the second container (and, by Newton's third law, that of the force on the water by the bottom) must be greater than the weight of the water in that container.[46]

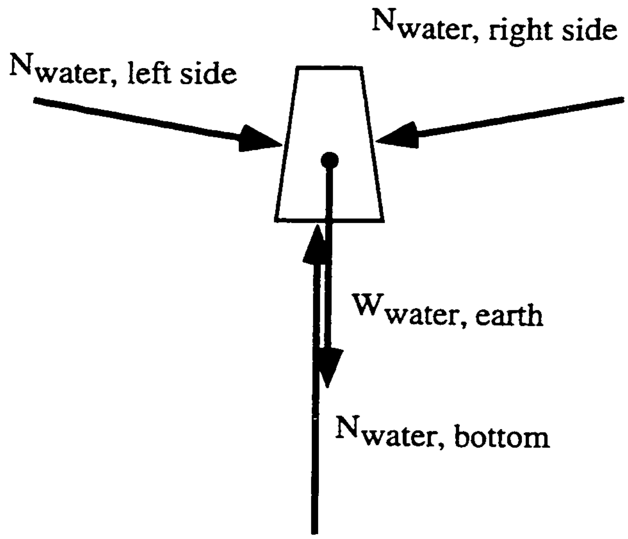


Figure 4-2: A free-body diagram for the water in container 2 in the Hydrostatic Paradox problem shown in Figure 4-1. Application of Newton's second law requires that the force on the water by the bottom of the container must be greater than the weight of the water.

4.2.2 The Two Books problem

Many of the difficulties that students had in answering problems involving hydrostatic pressure seemed to stem from lingering confusion about concepts that are widely assumed to be understood by students before the study of hydrostatics begins. For example, in Chapter 3 we have shown results that suggest that many students incorrectly apply Newton's second law to the liquid column in a barometer. In order to investigate these difficulties further and attempt to determine whether the difficulties were specific to the context of hydrostatics or were lingering and unresolved difficulties from mechanics instruction, we posed several mechanics problems before instruction in hydrostatics. We developed several problems designed to test the specific concepts that seemed to give students problems in hydrostatics, in particular the difference between the weight force and contact forces and the application of Newton's Second Law in static situations. These problems were designed to probe student understanding of concepts from mechanics in contexts that did not involve hydrostatic forces.

4.2.2.1 Problem statement

The Two Books problem is shown in Figure 4-3. In the problem, two books are stacked on a table. Students are asked to draw free-body diagrams for the two books and label the forces acting on the books. In some versions of this problem, the masses of the two books are equal; in others, the top book is more massive than the bottom book.

Two books are stacked on top of each other on a level table as shown in the diagram at right. The mass of the upper book is greater than the mass of the lower book.

In the spaces provided below, draw a separate free-body diagram for each book (i.e., draw vectors to represent the forces acting on the book).

Figure 4-3: The Two Books problem.

4.2.2.2 Correct answer

Correct free-body diagrams for the books in the Two Books problem are shown in Figure 4-4. Each diagram shows a weight force exerted by the Earth. The upper book includes a normal force exerted by the lower book. On the free-body diagram for the lower book, there is an upward normal force exerted by the table and a downward normal force exerted by the upper book. This latter force is correctly described as a normal force rather than a weight force for several reasons. First, this force is part of a Newton's third law force pair with the normal force on the upper book by the lower book; therefore, the two forces are the same type of interaction and should be labeled accordingly. Second, the force by the upper book on the lower book is a contact force whose magnitude may vary, whereas the weight force is a non-contact force exerted by the Earth whose magnitude is always equal to *mg,* where *m* is the mass of the object in question. Student diagrams were examined for the correct number and direction of forces. We also examined the labeling of forces, checking whether students described the force on the lower book by the upper book as the weight of the upper book, and whether they described the weight force of the upper book as being applied on the upper book or on the lower book.

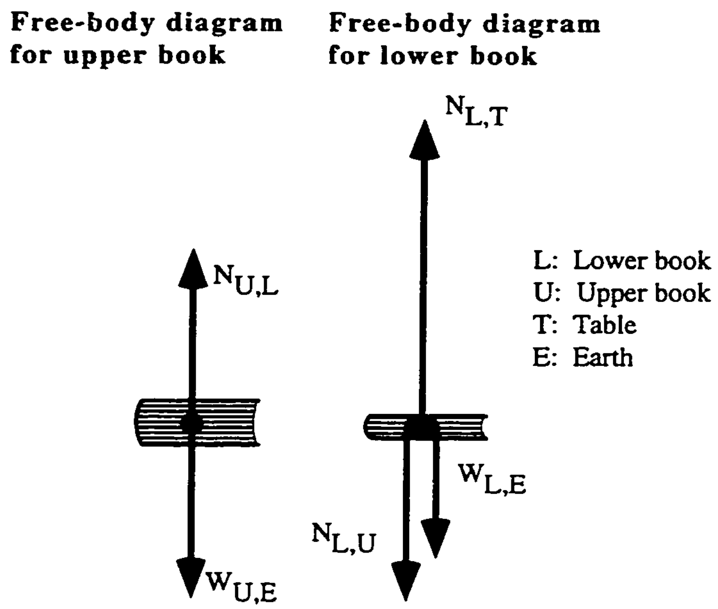


Figure 4-4: Correct free-body diagrams for the version of the Two Books problem shown in Figure 4-3.

4.3 STUDENT DIFFICULTIES IN IDENTIFYING FORCES ACTING ON OR BY A LIQUID

An important prerequisite to the analysis of static and dynamic systems is the ability to identify forces. In a static liquid, students must identify forces acting on a body of liquid, whether by the earth, the atmosphere, other bodies of liquid, or container walls. We have found that some students have difficulty in identifying one or more of these categories of forces. In several cases, we have evidence that suggests that failure to identify forces is related to the difficulties with pressure that we have described.

4.3.1 Failure to consider forces exerted by container walls

We have posed problems in which students are asked to draw free-body diagrams for a portion of the liquid in a container. In many cases, we have found that students failed to identify forces exerted by container walls. Our analysis suggested that some of the difficulties encountered by students in correctly ranking pressures may be due to failure to consider forces exerted by container walls. Therefore we have attempted to measure the prevalence of this difficulty by supplementing existing problems with questions that explicitly ask students about forces exerted on the liquid.

4.3.1.1 Example from Hydrostatic Paradox problem

The version of the Hydrostatic Paradox problem shown in Figure 4-1 was posed on a course examination in a section of Physics 224, the second-year thermal physics course at the University of Washington, after standard instruction in hydrostatics. Students were asked to compare the gauge pressures at the bottom of the two containers and to draw a free-body diagram for the water in the container with varying cross-section. Students were then asked explicitly to compare the magnitude of the force exerted on the water by the bottom of the container to the weight of the water.

Nearly all the students correctly recognized that the pressure at the bottom of the second container would be greater than that at the bottom of the first container. Only two students answered incorrectly, both saying that the pressures would be equal based on the weight of the water in the containers. However, only slightly more than half of the students correctly stated that the force on the water by the container would be greater than the weight of the container. The remainder of the students seem not to have recognized the 'paradox' as such; these students seemed to answer the two questions independently without recognition that their answers were contradictory.

About a third of the students answered incorrectly that the force on the water by the container would be equal to the weight of the water. All of these students had also made an error in drawing the free-body diagram for the water, and in particular all had made errors involving the forces exerted on the water by the container walls. The most common error was to neglect forces exerted by the walls entirely. Other students drew these forces but drew them in the wrong direction, either drawing horizontal arrows or showing forces acting upward, parallel to the surface of the container wall rather than perpendicular to the walls. A handful of students drew the forces exerted on the walls by the water in addition to those acting on the water. Whereas all of the students who had drawn correct free-body diagrams correctly compared the force on the bottom to the weight, only about one-third of those who had made errors with the forces by the container walls made this comparison correctly.[47] These results are summarized in Table 4-1.

Table 4-1: A comparison of student free-body diagrams for the water in the right container in Figure 4-1 to responses for the comparison between the forces on the container bottom to the weight of the water. Each row shows the responses to the force comparison given by those students who gave a particular answer on the free-body diagram for the water. Only the two most common force comparisons are shown, so that the numbers in each row do not add up to 100%.

	Physics 224 Washington Au'95 ($N = 35$)	
	Correct comparison (Force on bottom > weight of water)	Incorrect response: Force on bottom = weight of water
Drew correct forces on water ($N = 10$)	100%	0%
Omitted forces by side walls ($N = 11$)	35%*	45%
Drew forces by side walls in wrong direction ($N = 7$)	45%*	30%
Included forces *on* side walls ($N = 7$)	45%	45%

All percentages rounded to the nearest 5%. *See footnote 3.

4.3.1.2 Example from Multiple-Barometer problem

In one section of Physics 224, we asked the Multiple-Barometer problem shown in Figure 4-5 on a course examination. In this problem, students rank the pressures at points *X*, *Y*, and *Z* and then draw a free-body diagram for the oil at the top of the rightmost water column. The pressures at points *X*, *Y*, and *Z* are all equal because the points are located in the water at the same level relative to the free surface of the water. A correct free-body diagram for the oil is shown in Figure 4-5. The forces acting on the oil include the weight of the oil, exerted by the earth, the upward contact force exerted on the oil by the water below, and the downward contact force exerted on the oil by the stopper.[48]

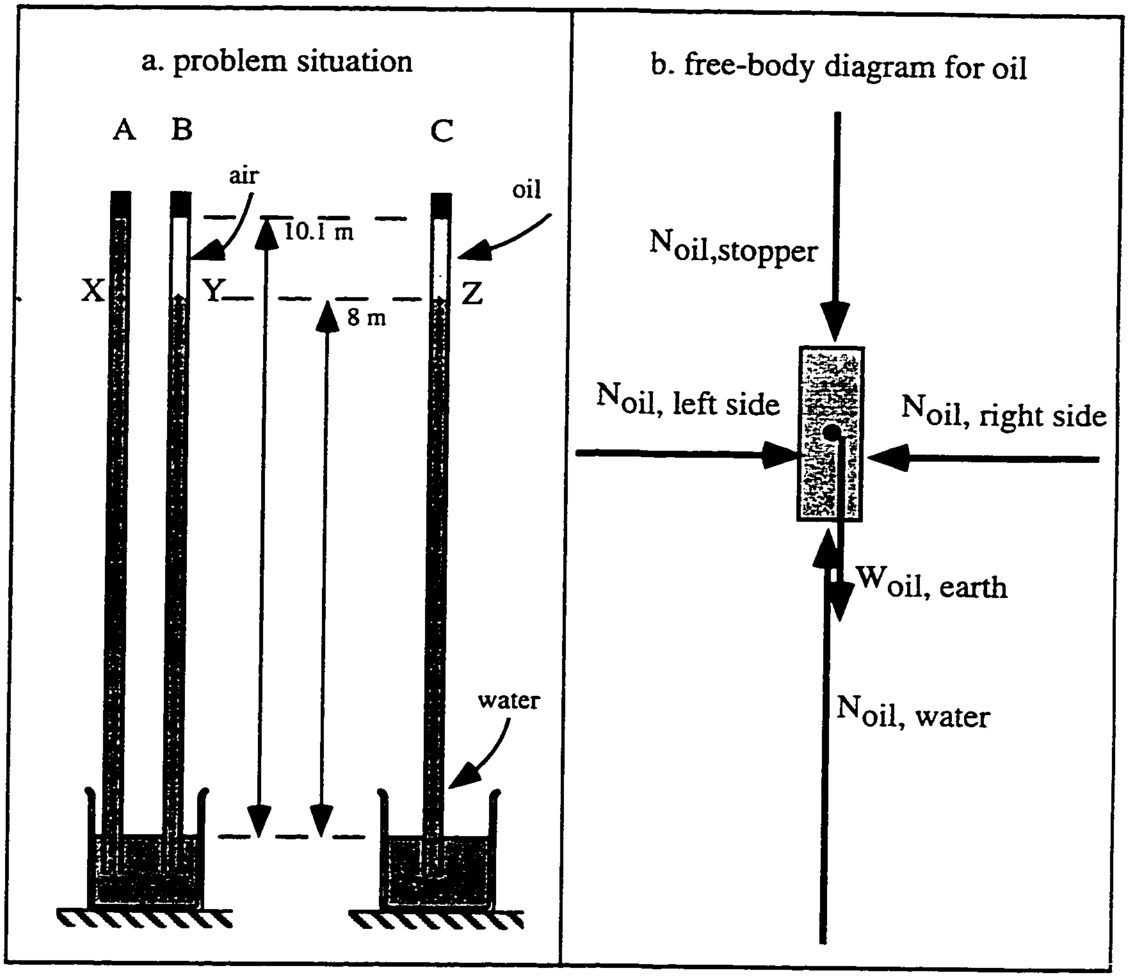


Figure 4-5: a. A Multiple-Barometer problem that was asked on a course examination in Physics 224. b. A correct free-body diagram for the oil above point *Z*.

As was described in Section 2.3 of Chapter 2, approximately 40% of the students correctly ranked the pressures at points *X*, *Y*, and *Z*. Another 40% gave responses based on the weight of the material above these points. When we examined students' free-body diagrams for the oil above the column, we found that students' responses for the pressure ranking were closely related to their responses to the free-body diagram question. Among those students who drew correct free-body diagrams, or free-body diagrams that were correct except for the labels of forces, nearly two-thirds answered the pressure ranking question correctly. Among the students who failed to draw the force on the oil by the stopper, fewer than one third answered correctly. In this group of students, nearly half gave an answer based solely on the weight of the material above the three points. These responses are summarized in Table 4-2.

Table 4-2: A comparison of student responses to the free-body diagram (FBD) for the oil above the right column in Figure 4-5 to student responses to the pressure ranking for points *X*, *Y*, and *Z*. Each row shows the responses to the pressure ranking of those students who gave a particular answer on the FBD for the oil. Only the two most common pressure rankings are shown, so that the numbers in each row do not add up to 100%.

	Physics 224 Washington Au'95 ($N = 39$)	
	Ranking of pressures at points *X*, *Y*, and *Z*	
	Correct ranking	Incorrect ranking: 'weight-above'
FBD is correct or correct except for labels ($N = 15$)	60%	25%
FBD shows no force exerted by stopper ($N = 11$)	25%	45%
FBD includes forces exerted *by* oil ($N = 6$)	35%	35%
Other errors / blank ($N = 7$)	45%	0%

All percentages rounded to the nearest 5%.

As the table shows, the students who drew correct or mostly correct free-body diagrams were more successful than the other students in ranking the pressures at the three points. Students who neglected the force exerted by the stopper on the oil were more likely to give the most common incorrect pressure ranking (that based on the weight of the material above the respective points). In addition, we have evidence that the students who drew correct free-body diagrams were not typically the best students in the class, at least as measured by course grade. In fact, the students who neglected to include the force by the stopper in their free-body diagrams had an average course grade that was above the mean for the course, suggesting that their incorrect responses cannot be accounted for simply by poor overall academic performance.[49]

4.3.1.3 Commentary

In both the Hydrostatic Paradox problem and the Multiple-Barometer problem, many of the students drew incorrect free-body diagrams for a piece of liquid. In both problems, the most common errors involved the forces exerted on the liquid by the container walls. Many students omitted these forces and some students drew them acting in the wrong direction. In both problems, a significant number of students drew forces exerted *by* the liquid on the container walls on the free-body diagram for the liquid. Some of these students drew the forces by the liquid instead of those on the liquid by the walls, and others drew both forces, possibly due to difficulties with Newton's third law.

On both of the problems, students who made errors in drawing the free-body diagrams for the liquid were less successful on pressure problems and more likely to give common incorrect answers. In particular, the students who made errors with the forces exerted by the container were more likely to give answers consistent with the idea that pressure is due solely to the weight of liquid above a point.

4.3.2 Difficulties with weight force in mechanical contexts compared to difficulties in hydrostatics

We have seen that many students make errors in solving pressure problems that are connected to difficulties with the weight force. Students emphasize the role of the weight force acting on a portion of liquid without considering the forces exerted by gases or by container walls, and conclude that the pressure at a point is solely due to the weight of the material above the point. In addition, we and other researchers have found that many students, when considering two objects in a stack (see, for example, Figure 4-3), draw free-body diagrams showing the weight of the upper object as a force that acts on the lower object.[50] We have attempted to determine whether there is a connection between these difficulties. We will present results in which we have compared students responses on the Two Books problem shown in Figure 4-3 to responses by the same students on problems in which they were asked to rank pressures at several points.

In two sections of Physics 115, we posed the Two Books problem as part of an ungraded diagnostic quiz on the first day of the course, after students had completed the mechanics portion of the introductory algebra-based course. Several days later, we posed a problem in which students were asked to rank pressures at points with different amounts of liquid above. One class was given a Barometer problem; the other, a Letter Tube problem. The students in the two courses performed similarly on the Two Books problem as well as on the pressure problems, so we have combined results from the two courses.[51]

We analyzed the free-body diagrams drawn by students for the Two Books problem. For each force that they depicted, students were asked to describe the type of force, the object exerting the force, and the object on which the force is exerted. Many students drew diagrams in which they labeled one or more forces in a manner that suggested a belief that the weight of an object is a contact force acting on whatever is below the object. For example, one student drew a downward vector on the free-body diagram for the upper book and described this vector as the "gravitational force on lower book by upper book." Students also often drew a vector on the free-body diagram for the lower book that they described as, for example, "force of gravity from lower book exerted on table."

In each of the two sections, approximately 30% of the students described the weight force of either the upper or lower book as a force acting on the object below. In each case, the students giving this answer were less successful on pressure ranking tasks and more likely to give pressure rankings consistent with the 'weight-above' reasoning described in Chapter 2 (see Table 4-3). Although this result is by no means conclusive, the results suggest that the incorrect force labels given by many students on the Two Books problem are connected to the incorrect reasoning based on the belief that the pressure is due solely to the weight of the material above a point.

Table 4-3: A summary of student responses on pressure ranking problems compared to student responses on the Two Books problem. Each row shows the responses to the pressure ranking given by those students who gave a particular answer on the Two Books problem. Only the two most common pressure rankings are shown, so that the numbers in each row do not add up to 100%.

	Physics 115 Washington two sections ($N = 169$)	
	Ranking of pressures	
	Correct ranking	Incorrect ranking consistent with 'weight-above'
Correct or mostly correct free-body diagrams ($N = 107$)	70%	15%
Incorrect: weight acting on object below ($N = 49$)	45%	30%

All percentages rounded to the nearest 5%.

We wondered whether the students who answered the mechanics problem correctly were simply better students. In order to answer this question, we obtained records of the course grade in Physics 115 received by each of these students and compared the mean grade received by the students who answered the mechanics problem correctly to that of those who did not. We have found that the students in the two categories had final course grades that were very similar.[52] These results suggest that the students that make the error with the weight force on the mechanics problem are not simply the weakest students academically.

4.4 STUDENT DIFFICULTIES IN APPLYING NEWTON'S SECOND LAW TO THE LIQUID IN A CURVED TUBE

We found in several different pressure problems that students gave responses suggesting that they were misapplying the equilibrium condition for a static liquid. These responses were particularly dramatic in the case of curved tubes. For example, in Chapter 2 we showed that many students who answered the Different-Diameter U-tube problem incorrectly gave reasoning that suggested that they were using an argument based on a comparison of the forces acting on the surfaces of the liquid in the two legs of the tube. (See Section 2.4.3 for more details.) In order to probe student understanding of situations of this sort, we developed a protocol for a series of individual demonstration interviews in a similar context. In this protocol, which we have named the *U-tube Interview Protocol,* we asked students to predict the final levels of water in a U-tube in various configurations. (For more on individual demonstration interviews, see the Introduction.)

4.4.1 Interview protocol

The U-tube Interview Protocol involves a series of predictions concerning a U-tube that is partially filled with liquid. We ask students to consider several different states of the U-tube. In each state, students are asked to predict the liquid levels in the two sides of the U-tube and to rank the pressure at several points in the liquid. For example, students are asked to consider a situation in which water is poured into a U-tube that is open at both ends. They are asked to predict the relative heights of the water surfaces in the two legs of the tube and rank the pressures at several points in the tube. The interviewer then places a cap in one end of the U-tube and asks students to predict the final water levels if more liquid were poured in the uncapped end (see Figure 4-6 for the water levels before additional liquid is poured). Students also consider a situation with two liquids, in which oil is poured into the uncapped end, and a situation in which liquid is removed from the uncapped end with a syringe. Depending on the amount of time that students spend on the initial tasks, the interviewer may or may not pose the later questions.

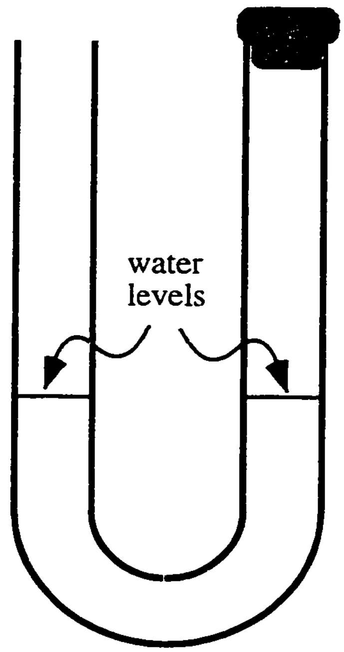


Figure 4-6: A task in the U-tube Interviews. Students were shown this tube and asked what would result if water were poured in the left end of the tube.

4.4.2 Population

We interviewed * students from Physics 224. All of the students were interviewed in the last two weeks of the quarter, after they had completed instruction on hydrostatics.

4.4.3 Tendency of students to consider forces acting on a curved piece of liquid

In responding to the interview tasks, nearly all of the students gave responses in which they seemed to consider forces acting on a system composed of a U-shaped piece of liquid that is free to move along the tube. These students, in trying to explain their predictions, related the final position of the water to the forces acting on the water, but typically considered only the forces applied at the ends of the column. None correctly described the forces acting on this U-shaped piece of water by the container, and several failed to consider the weight of the water.

For example, when asked to consider the situation in which water was poured into the open U-tube, one student correctly predicted that the liquid level would be the same at both ends. To justify her prediction, she said:

> S: I can sort of think of [the water] as a whole thing. If you push here [indicates top of right tube], it'll come up...the more you push here, the more pressure ... [indicates top of water in left tube] it'll move up at the same time due to the pressure.

This student did not mention either the gravitational force on the liquid or any force by the container wall. In a later situation, in which water was added to the U-tube after the other end U-tube was sealed, this student predicted that the water surface in the capped end would be lower:

> S: The force is greater here, so, ... it'll push it down further here [indicates right surface] than it's getting pushed down here [indicates left surface].

Again, it seems that this student is considering only the forces on the top surfaces of the water in the U-tube. Her response suggests in addition that she is relating the relative magnitudes of these two forces to the displacement of the liquid 'along the tube.' Interestingly, in both cases this reasoning led the student to make correct predictions. Thus, although the student is using incorrect reasoning, there is no conflict between predictions and observations that would cause her to modify her beliefs.

Another student made a similar statement in the case of the open U-tube but, instead of comparing the forces on the top surfaces of the liquid, seemed to compare the two downward gravitational forces on the water in the two legs of the tube:

> S_2: There's a force on all the liquid that's in here, pulling it down. If there was more height on this side [indicates right side of tube], there'd be more liquid, so there would be more force pushing this [right] side down than this [left] side, and so it would have to move until there's an equal amount of force on each side, and that would occur when they're at the same level.

This student later was asked about the U-tube with one end sealed. After the student stated that the added liquid would exert a force, the interviewer asked for clarification:

I: When you say this liquid element will supply a force, on what object will that force act?

S_2: On the liquid below it, and, I guess, all the way around.

We interpret the phrase 'all the way around' to reflect the belief that the liquid in the capped end of the U-tube can be considered to be 'beneath' the liquid in the uncapped end, similar to some of the student responses that we classified as being part of the 'along the tube' reasoning described in Chapter 2.

Several of the students gave responses similar to those given by these two students, suggesting that they were considering the whole body of liquid as a single system. An argument based on forces can account for the relative positions of the water surfaces in the U-tube, but such an argument must treat only parts of the water in the tube rather than the whole body of water. A complete analysis must also include consideration of forces exerted by the container walls and gravitational forces. The downward forces that these students have described do not allow a complete description of either the motion or the position of the water.

4.4.4 Interpretation of responses

The responses given by these students to the interview problems suggest an interpretation for the 'along the tube' reasoning described in Chapter 2. It seems that students are considering a system that includes a curved 'column' of water and imagining that this system is free to move along the length of the tube. They treat only forces on the ends of this system as being important. Then, students incorrectly apply either the equilibrium condition or the equation for pressure vs. depth, choosing a coordinate system that extends along the curvature of the container.[53]

Reasoning that is similar to this 'along the tube' reasoning can be found in other contexts. We will give two examples that are presented in many classes and that may contribute to student confusion. In both cases, students are encouraged to neglect some of the forces acting on a system that is topologically similar to the liquid in a Capped U-tube.

Hydraulic lift. Most textbooks and instructors use the hydraulic lift as an example of the applicability of hydrostatics to real-world situations. In a typical discussion, students consider a lift, with a large mass (*e.g.*, an automobile) on the end of the lift with the greater area. Students are often asked to find the force that must be applied to the small surface of the lift in order to lift the automobile (see Figure 4-7). Rarely, if ever, do textbooks or instructors consider forces by container walls in the hydraulic lift, or the different heights of the small and large surfaces. Rather, the hydraulic fluid in the lift is seemingly treated as a system that moves away from the greater force (or pressure). In some cases, the force applied to the small surface is referred to as the 'input' force and the force on the large surface the 'output' force, which is not incorrect, but perhaps further suggests the idea that the force is 'transmitted.'

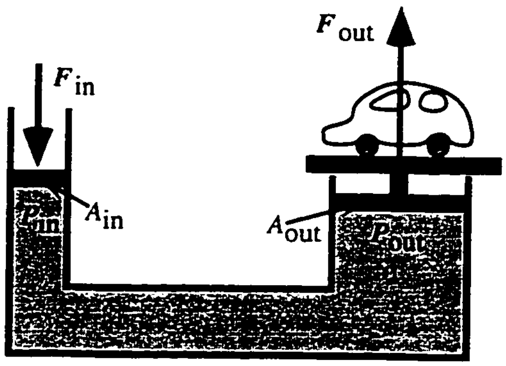


Figure 4-7: Sketch of diagram from textbook showing the hydraulic lift.[54]

Atwood's Machine. The Atwood's machine is an example of multiple-particle mechanics that is commonly used in the introductory physics course. In the typical treatment, students are given the masses of two objects connected to a massless string that is across a massless and frictionless pulley, and are asked to find the acceleration of the objects. In the Atwood's machine, students draw free-body diagrams for the two blocks, and relate the motion of the two blocks by considering the kinematical constraint of the string. However, students often do not consider the free-body diagram for the string. In addition to the forces on the string by the objects at its two ends, there is an upward force by the pulley. Research has shown that many students argue that the force by one of the objects (typically the one with greater mass) is 'transmitted' by the string to the other object.[55] Such an argument fails to consider the forces on the string, rather treating it as an object that moves in the direction of the greater force.

In both of these examples, reasoning that is similar to the 'along the tube' reasoning is often explicitly modeled by textbooks and instructors. Students may not recognize the approximations that are made (*e.g.*, that the mass of the hydraulic fluid or string is small compared to other relevant quantities and thus may be neglected), and often fail to understand the role of the constraining forces. Although it is unlikely that these examples explicitly cause students to make the errors that we see in the Capped U-tube and Different-Diameter U-tube problems, they may unwittingly reinforce students' tendency to use this type of reasoning. As we have seen, students often neglect passive forces, particularly those exerted by container walls.

4.5 IMPLICATIONS FOR INSTRUCTION

This chapter provides evidence for connections between the difficulties that students have with forces in relatively simple contexts and the difficulties that they have with concepts from hydrostatics. Most of the students who have difficulty with forces in purely mechanical contexts perform very poorly on hydrostatics problems that require them to apply their understanding. However, there is also evidence that a good knowledge of mechanics is not sufficient for success in hydrostatics. Many students who seem to have mastered the relevant concepts from mechanics often have difficulty in applying these concepts to the new contexts of hydrostatics. Specific difficulties with the ideas of hydrostatics complicate students' ability to apply their conceptual understanding to these new contexts. For example, most of the students in both the Hydrostatic Paradox problem and the Multiple-Barometer problem drew free-body diagrams that

These results suggest two important generalizations with implications for instruction in hydrostatics. First, students need to have a good background in the relevant ideas from mechanics (*e.g.*, identifying and labeling forces). Second, students need help in applying their mechanics skills to the new context of liquids. They need to resolve difficulties with the new forces that they encounter. They need assistance in thinking about passive forces exerted on liquids by container walls and practice in relating the magnitudes of forces to the pressures at various points.

These generalizations have informed the development of the curriculum that we have developed to address student difficulties with pressure. As we will describe in the following two chapters, we have developed a series of tutorials in hydrostatics. The first of the tutorials reviews ideas from mechanics, helping students to practice identifying forces and applying Newton's laws in a context that is designed to make connections between mechanics and hydrostatics. The second tutorial applies these ideas to the context of pressure, attempting to address the conceptual difficulties described in Chapters 2 and 3. The development, assessment, and continuing modification of these tutorials will be described in detail in Chapter 5. A third hydrostatics tutorial, which covers buoyancy, is described in Chapter 7.

5. ADDRESSING STUDENT DIFFICULTIES WITH PRESSURE AND FORCES IN FLUIDS

5.1 INTRODUCTION

Based on the research described in Chapters 2, 3, and 4, we have developed a set of instructional materials designed to address student difficulties with pressure and forces in liquids. These materials have been developed in an iterative cycle of research, curriculum development, and instruction that includes testing of student performance before and after instruction. In this chapter we describe a tutorial sequence designed to address student difficulties with pressure and forces in a static, incompressible fluid. Assessment of the curriculum based on the performance of students on post-tests after completion of the materials is presented along with comparison to results after standard instruction. A revision of the instructional materials based on the results of this assessment is along with assessment of the revised version.

5.2 CRITERIA USED TO ASSESS STUDENT UNDERSTANDING

In order to assess the effectiveness of our instructional materials, we have posed various written problems after instruction. These post-tests are typically the same as those described in the relevant chapters (2 and 3) and appendices (A and B). We will briefly summarize the criteria that we have used for correct understanding and describe common student difficulties that the curriculum is designed to address.

5.2.1 Student ability to rank pressures at points in a liquid of uniform density

Within this category we include both the ability to rank the pressures at points at the same level in a liquid, as described in Chapter 2, and the ability to compare the pressures at points at different levels, as described in Chapter 3. The problems we have used to test this ability span a range of physical situations, including the Letter Tube and Multiple-Barometer problems described in Chapter 1.

5.2.1.1 Criteria for correct answers and reasoning

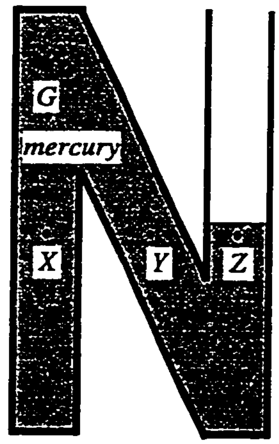
In addition to giving correct pressure rankings, we expect students to explain their reasoning using either the equation $P = P_0 + \rho gh$ or a verbal description of the fact that pressure depends on vertical distances only. Correct answers with clearly incorrect reasoning were not considered correct.

5.2.1.2 Incorrect reasoning used by students after standard instruction

After standard instruction, students used several forms of incorrect reasoning. The instructional materials that we have developed are designed to address many of these incorrect ideas.

'Weight-above' reasoning. After standard instruction, we found that many students gave incorrect answers consistent with the idea that the pressure at a point is due solely to the weight of the material above the point. For example, on Multiple-Barometer problems in which points at the same level are located below substances of different densities, students ranked the pressures based on the density of the substances above the points. As we have shown in Chapter 4, this incorrect reasoning is connected to the failure to recognize the forces applied by container walls.

Height measured to container walls. A variation of the preceding idea that the pressure is due to the weight of the material above was found in problems involving tubes of unusual shapes, like the N-tube problem shown in Figure 5-1. Some students measured directly upward from points to the nearest container wall, attributing the pressure at these points solely to the liquid directly above those points. This reasoning is also connected to a failure to recognize that the container walls exert forces on the liquid.



An N-shaped glass tube with its right vertical leg open at the top is partly filled with mercury as shown at right. Points *X, Y* and *Z* are at the same height.

Rank the pressures at points G, X, Y, and Z. Explain how you determined this ranking.

Figure 5-1: The N-tube problem.

Pressure varies along the tube. In curved tubes, some students measured along the length of the tube, answering as though the tube had been straightened. Most of these students answered that the pressure increases as distance from the free surface increases, although a few came to the reverse conclusion, that pressure decreases as distance from the free surface increases.

Proximity and isolation. Two incorrect approaches that suggest failure to recognize the forces exerted by container walls include 'proximity' and 'isolation.' Some students argued that the pressure at a point near the closed end of a tube will be greater than at a point at the same level that is not. Other students reasoned incorrectly that a point beneath a stopper will have a smaller pressure than a point at the same level that is beneath a free surface, because the stopper will 'isolate' the first point from the pressure contribution from the atmosphere.

Neglect or reversal of gradient. Some students failed to recognize the existence of the pressure gradient in a liquid, stating instead that the pressures at all points in a liquid are equal. In other cases, students reversed the gradient, arguing that pressure increased with height, often based on an incorrect interpretation of the equation $P = P_0 + \rho gh$.

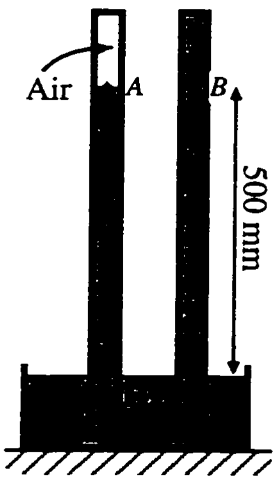
5.2.2 Student ability to compare pressures to atmospheric

In this category we describe problems that probe student ability to recognize that the pressure at a point in a liquid that is above the free surface of a liquid is less than atmospheric pressure.

5.2.2.1 Criteria for correct answers and reasoning

In problems including Multiple-Barometer problems (see Figure 3-1) or the Capped U-tube problem (see Figure 5-3), we ask students to determine whether the pressure at a point is greater than, less than, or equal to atmospheric pressure. Student reasoning was counted as correct if it included a reference either to the level of a point relative to the free surface (*e.g.*, "less than because [the pressure at point] *X* is equal to atmospheric and [the pressure at] *Z* < [that at] *X*.") or to a change in the system that would allow a comparison to atmospheric pressure (*e.g.*, "less than because if the stopper was removed the water would go to equal levels and *Z* would equal atmospheric pressure").

Some students gave correct answers but used clearly incorrect reasoning. For example, some students argued that the pressure at a point above the free surface of a liquid is less than atmospheric using reasoning similar to the 'isolation' reasoning that we have described in Chapter 2. These students said that a stopper or container wall isolates the point in question from the effects of atmospheric pressure, so that the pressure at the point is due solely to the liquid above the point. Such arguments were applied to point *Z* in the Capped U-tube in Figure 5-3, which is beneath the stopper. Some of these students in their explanation explicitly referred to dividing the two terms in the equation $P = P_{atm} + \rho gh$, arguing that the pressure is less at points beneath a stopper than at points beneath a free surface due to the absence of the P_{atm} term.

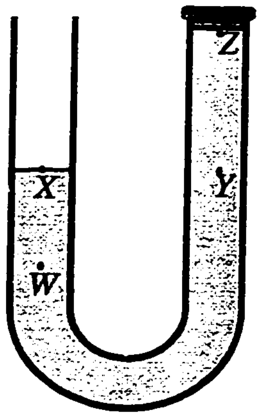


Consider the following arrangement of glass tubes. The top of each tube is closed; the bottom end is open to a mercury reservoir that is open at the top. In the left hand tube the space above the mercury column contains air at an unknown pressure. The atmospheric pressure is at its normal value of 1 atm = 760 torr.

Is the pressure of the gas in the left tube greater than, less than, or equal to atmospheric pressure? Explain your reasoning.

Is the pressure at point B greater than, less than, or equal to the pressure at point A? Explain your reasoning.

Figure 5-2: A Multiple-Barometer problem.



A U-shaped tube (height ~ 0.5 meters) is partly filled with water as shown at right. The right end of the tube is closed at the top, but the left end is open to the atmosphere. There is no air between the rubber stopper and the water surface on the right-hand side.

Rank the pressures at points W, X, Y, and Z. Explain your reasoning.

Is the pressure at point Z greater than, less than, or equal to atmospheric pressure? Explain your reasoning.

Figure 5-3: The Capped U-tube problem.

5.2.2.2 Incorrect reasoning used by students

We found several common incorrect forms of reasoning, which we describe here briefly. These ideas are described in more detail in Chapter 3.

Pressure is equal throughout a liquid of uniform density. Some students' incorrect answers were based on the idea that the pressure is the same throughout a liquid.

Pressure is equal to atmospheric because the liquid is at rest. In some cases, students seemed to be reasoning based on the application of an incorrect equilibrium condition for the liquid. For, example, in the Multiple Barometers problem shown in Figure 5-2, students said that the pressure at the top of the liquid column in the left tube must be equal to atmospheric because the liquid is at rest. This reasoning suggests an incorrect application of Newton's second law for the liquid, as it neglects the weight of the liquid and, in some cases, the forces exerted by the container.

Pressure comparisons based on other incorrect equilibrium ideas. Several students' answers were based on other incorrect applications of Newton's second law. For example, some students, in responding to problems in which the height of a mercury column is less than the height corresponding to atmospheric pressure, said that the pressure at a point at the top of the column must be greater than atmospheric because the liquid has been pushed down below its expected height.

Confinement arguments. Some students based answers on the idea that the liquid in a closed end of a tube is confined and therefore has a pressure greater than atmospheric, even if the points in question are above the free surface of the liquid.

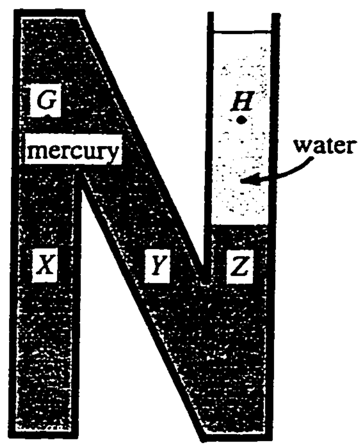
Along the tube reasoning. Many students used the idea that the pressure increases or decreases along the length of a curved tube that is capped at one end.

5.2.3 Student ability to rank pressures at points at the same level in liquids of different densities

In addition to the two categories above, we believe that students with a thorough knowledge of the relationship of pressure to depth should be able to rank the pressures at two points at the same level but in liquids of different densities.

5.2.3.1 Criteria for correct answers and reasoning

This type of problem proved to be the most difficult for students. Our criterion for correct reasoning required that students refer to the difference in pressure between two points in each of the two liquids. To answer correctly, students should choose a point or points where the pressures are the same in the two liquids (usually at the bottom of the two columns in connected tubes) and reason that the amount that the pressure decreases in the less dense liquid is less than the amount that the pressure decreases in the more dense liquid. Hence, the pressure in the less dense liquid will be greater than the pressure at a point at the same level in the more dense liquid. For example, in the N-tube problem shown in Figure 5-4, the pressure difference from point Z to point H is less than the pressure difference from the point X to point G. Since the pressures at points X and Z are equal, the pressure at point G is less than the pressure at point H.



An N-shaped glass tube with its right vertical leg open at the top is partly filled with mercury as shown at right. Points X, Y and Z are at the same height. Water is poured into the opening on the right. The density of water is less than the density of mercury. Point H is at the same height as point G.

Determine whether the pressure at point H is greater than, less than, or equal to the pressure at point G. Explain your reasoning.

Figure 5-4: The N-tube problem with two liquids.

5.2.3.2 Incorrect reasoning used by students

There are several incorrect forms of reasoning used by students in making incorrect comparisons of the pressures at two points at the same height in different liquids. We briefly describe the most common incorrect ideas.

Pressure based on density and/or weight of material above. The most common form of incorrect reasoning has been based on the idea that the pressure is related to density. Most students in this category offered little reasoning beyond the statement that the pressure would be greater in the liquid of greater density and a reference to the equation $P = P_{atm} + \rho gh$. Those students who did offer more reasoning typically referred to the weight of the material above.

Pressure based on height alone. Some students answered incorrectly that the pressures at the two points would be equal based on the fact that the points were at the same height, without referring to the density at all. A handful of students supplemented this idea with statements about the fact that the liquid was at rest or at equilibrium.

Other surface features. Other students based their incorrect answers on any number of surface features, including the cross-sectional area of the tubes including the points, the shape of the tubes, or the presence or absence of stoppers.

5.3 TUTORIAL SEQUENCE ON *REVIEW OF NEWTON'S' LAWS* AND *PRESSURE IN A LIQUID*

We have developed a series of tutorial materials designed to address the difficulties describe in Chapters 2 and 3 and reviewed above. The materials are designed to help students to apply the ideas of Newtonian mechanics in the context of liquids and to address the conceptual difficulties that many students seemed to have. The curriculum has been developed over the course of several years and has been tested extensively with several populations of students. We will describe the initial versions of two tutorial sequences that we have used on the topic of pressure in a liquid. A third tutorial sequence on the topic of Archimedes' principle will be described in Chapter 7.

5.3.1 *Tutorial Sequence:* Review of Newton's Laws

The first of these two tutorial sequences is actually shorter than most of the those developed by the Physics Education Group. It includes a pretest and a two-page tutorial worksheet that serve to review the basics of free-body diagrams and Newton's second and third laws. The text of the tutorial sequence is reproduced in Appendix C.

5.3.1.1 Pretest

The pretest for *Review of Newton's Laws* is the Two Books problem described in Chapter 4.[56] Two books are stacked on a table. Students are asked to draw a free-body diagram for each book and label each force to indicate the type of force, the object exerting the force, and the object on which it is exerted. In some cases, students have been asked to rank the magnitudes of the forces on the free-body diagrams.

5.3.1.2 Instructional sequence

The instructional sequence in this tutorial draws in part upon our experience with the tutorial *Forces* from *Tutorials in Introductory Physics.*[57] Because the emphasis in hydrostatics is on vertical forces, we chose to bypass the initial situation in *Forces,* in which several horizontal forces are exerted on a block, and move directly to a situation that more closely parallels the reasoning that students must use in analyzing a liquid at rest. We decided to ask students to consider stacks of paper at rest on a table, a situation that is a discrete analog of the continuous mass distribution in a liquid.

Free-body diagrams for stacks of paper. Students are asked to consider a yellow stack of paper placed on top of a blue stack. Students draw a free-body diagram for the upper stack and one for the lower stack. For each force, students are asked to describe the type of force, the object exerting the force, and the object on which the force is exerted. Students are then asked to rank the magnitudes of the forces that they have drawn, explicitly referring to Newton's second and third laws when appropriate.

Weight of the object above. As we described in Chapter 4, many students give answers consistent with the belief that the weight of an object is a contact force exerted on the object below, rather than a non-contact force exerted on the object by the Earth. Therefore, students are asked to consider a dialog between two students, in which one student expresses the correct idea that there is a normal force on the lower stack by the upper, and the second student responds that the force in question is the weight of the upper stack, adding that a normal force must always be opposite to gravity. Students are asked to respond to this dialog and explain which student is correct.

Free-body diagrams for individual sheets of paper. After considering the stacks, students are asked to draw a free-body diagram for a single sheet of yellow paper near the middle of the upper stack and one for a single sheet of blue paper near the middle of the lower stack. For each free-body diagram, students are asked to rank the forces shown on the diagram.

Bridge to pressure. In the final questions of the tutorial, students are asked to compare the upward normal force on the lower (yellow) sheet of paper to the upward normal force on the upper (blue) sheet of paper, and to generalize their response. Students are asked again about the net force on the paper and then asked about the sum of the forces on the sheet of paper by the surrounding sheets of paper. These questions are analogous to questions that students will encounter in the context of a static liquid.

5.3.1.3 Homework

The tutorial sequence *Review of Newton's Laws* does not typically have associated homework. As the tutorial is typically offered as a brief review of mechanics for students who are about to study hydrostatics, the homework focuses on the context of liquids.

5.3.2 *Initial Version of Tutorial Sequence:* Pressure in a Liquid

The second of the tutorials in the hydrostatics sequence is *Pressure in a Liquid.* In this tutorial, students consider free-body diagrams for layers of liquid and use these diagrams to develop the relationship between pressure and depth. They are asked to confront some common difficulties and reason with the equation $P = P_0 + \rho gh$. The text of the tutorial, along with the pretest, is reproduced in Appendix D.

5.3.2.1 Pretest

The pretest for the initial version of *Pressure in a Liquid* has included two types of problems that we have described previously. The first page has typically included a Letter Tube problem, in which students are asked to rank the pressures at several points in a fluid of uniform density. The second page has posed a Different-Diameter U-tube problem, in which students are asked to predict the water level in the wider leg of the tube given the water level in the narrow leg. In some versions of this pretest, we have asked students to rank the pressures at various points in the U-tube as well.

5.3.2.2 Instructional sequence

In several different sections of Physics 224, the second-year course in hydrostatics and thermal physics at the University of Washington, we tested early versions of *Pressure in a Liquid*. The versions that were used varied slightly, depending on the constraints of the class and the preference of the lecture instructor. Here we describe the final form of this initial version, which was used in two consecutive quarters. This instructional sequence is representative of the instructional strategies used in earlier iterations of the tutorial, though in some cases the questions posed were slightly different.

Blocks of liquid. In the initial version of *Pressure in a Liquid,* students were asked to consider a U-shaped tube of rectangular cross section. Students imagine that the liquid in the tube is broken into five cubical blocks of liquid (see Figure 5-5) and draw free-body diagrams for the blocks.

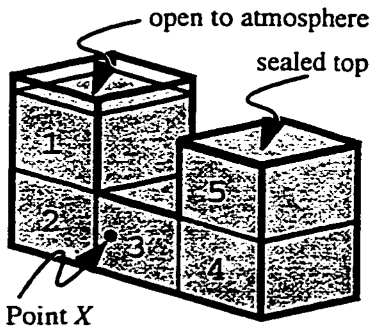


Figure 5-5: A diagram from the initial version of *Pressure in a Liquid*, showing water in a U-shaped tube of rectangular cross section.

Existence of forces. As we described in Chapters 2 and 4, many students had difficulties recognizing the existence of forces exerted by container walls. We sought to confront these difficulties. In the worksheet, students discuss the existence of horizontal forces, imagining that a small hole were drilled in the side of the container (for example, at point *X* in Figure 5-5) and predicting the motion of the water next to the hole. To help confront the incorrect idea that container walls exert no downward forces, we asked students to compare the free-body diagrams for two cubes of water at the same height, one of which has water above and one of which has a container wall (*e.g.*, blocks 2 and 3 in Figure 5-5).

Pressure and pressure differences. In the second section of *Pressure in a Liquid,* students are asked to reason with pressures based on their free-body diagrams for the cubes of water. They compare the pressures at points at the same level, including one directly under a container wall and one under liquid. Finally, students are asked to generalize their answers by articulating a rule for comparing pressures.

Application of pressure vs. depth to U-tubes. In this section of the tutorial, students are shown a U-tube that is nearly filled with water. Students are asked to rank the pressures at various points in the tube, including two points that are at the same distance from the water surface as measured 'along the container,' namely points *B* and *C* in Figure 5-6. The worksheet then states that the right tube is sealed such that there is no air between the water surface and the cap. Students are asked whether the pressures will change at selected points in each leg of the tube. This exercise is designed to elicit the 'proximity' and 'isolation' reasoning described in Chapter 2. Students who believe, for example, that the stopper blocks the atmospheric pressure may say that the pressure at points *E* and *F* decrease when the stopper is added. They can then discuss this idea with their partners or with tutorial instructors. Once students have recognized that the pressure does not change, they are asked how the force exerted by the cap on the water in the right leg compares to that exerted by the atmosphere on the water in the left leg.

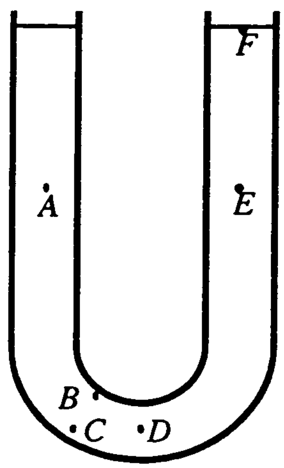


Figure 5-6: In the tutorial *Pressure in a Liquid,* students are asked to rank pressures in this U-tube.

Liquid removed from U-tube. In the last section of the tutorial *Pressure in a Liquid,* students are told that water is removed from the left leg of the tube (while the right leg is still capped) and asked whether the pressure at selected points will change. They are asked to identify points at which the pressure is equal to atmospheric pressure and then asked about the pressures at points above these points. At this point students are asked where the pressure would be equal to zero, if anywhere, and to use this answer to predict whether the water level would drop in the right leg. After students have made their prediction, the demonstration is shown and students are asked to resolve any inconsistencies between their prediction and observation. Often, after this demonstration, a second demonstration is shown in which a barometer is constructed using water rather than mercury.

5.3.2.3 Homework

Due to the constraints of the courses in which we were working, no tutorial homework was assigned with the initial version of the tutorial *Pressure in a Liquid.* In some cases, students were given homework assignments based on the tutorial *Buoyancy* that is described in Chapter 7.

5.4 ASSESSING STUDENT UNDERSTANDING AFTER *PRESSURE IN A LIQUID*

The initial versions of *Review of Newton's Laws* and *Pressure in a Liquid* were used with several sections Physics 224, the second-year course in hydrostatics and thermal physics at the University of Washington. In most of these cases, the materials were used in the lecture hall in the form of interactive tutorial lectures (ITLs). A more complete description of ITLs can be found in the Introduction to this dissertation. After the ITLs, we included written post-test problems on course examinations in order to assess the effectiveness of the instructional materials. As these post-tests have concentrated on pressure, we will describe them as primarily testing the effectiveness of *Pressure in a Liquid* and testing to a lesser extent *Review of Newton's Laws.* The post-tests included problems similar to those described in section 5.2 of this chapter for identifying student difficulties with pressure and the pressure gradient.

5.4.1 Post-test results

In this section we compare student responses on a post-test problem to responses given after standard instruction. Several specific difficulties are examined to determine the effectiveness of the curriculum.

5.4.1.1 Student ability to rank pressures at points at the same level

Student responses to problems in which they were asked to rank the pressures at points at the same level in a liquid are shown in Table 5-1. The responses given by students after standard instruction are shown for the purpose of comparison.[58] The results after standard instruction include different sections of the course from those after the tutorial, and none of the students saw these problems twice. On both Letter Tube and Multiple-Barometer problems, students were more successful after the initial version of *Pressure in a Liquid* than after standard instruction.[59] On both types of problem, the incidence of the common incorrect answer that the pressure is due to the weight of the material above has decreased. As in Chapter 2, the reasoning given by students is not shown in the table, but nearly all of the students who answered correctly used correct reasoning based on the height of the points or the equation $P = P_0 + \rho gh$. Only 5-10% of the students answered correctly without correct reasoning, and most of these students included no reasoning whatsoever. Almost none of the students who answered correctly gave obviously incorrect reasoning.

Table 5-1: Student pressure rankings for points at the same level in the same liquid. Results are shown in sections after standard instruction only and after the initial version of the tutorial *Pressure in a Liquid.*

	Physics 224 Washington 2 sections ($N = 73$)	Physics 224 Washington 2 sections ($N = 35$)	Physics 224 Washington 2 sections ($N = 56$)	Physics 224 Washington Wi'97 ($N = 33$)
Instruction	Standard	*Pressure in a Liquid*	Standard	*Pressure in a Liquid*
Type of problem	Letter tube[58]	Letter tube[59]	Multiple-Barometer	Multiple-Barometer
Correct	75%	95%	50%	80%
Not all equal	25%	5%	45%	15%
Other/Blank	~0%	0%	~5%	~5%

All percentages rounded to the nearest 5%.

5.4.1.2 Student ability to compare pressures to atmospheric

We have also examined student responses to problems in which students are asked to compare to atmospheric the pressures at points in a fluid above the free surface of the liquid. A summary of these responses are shown in Table 5-2. Corresponding results for students who had completed standard instruction are shown in the first column of the table. As the table shows, both groups of students were equally likely to answer correctly, with nearly identical fractions giving correct reasoning. The only difference between the results seems to be that the pattern of incorrect answers was different. Whereas in each of the standard sections, more students argued that the pressure was greater than atmospheric, more of the students who had completed the initial version of *Pressure in a Liquid* responded that the pressure was equal to atmospheric. This difference may be due to the fact that most of the problems given after standard instruction asked about the pressure in the space above the column and the problem given after *Pressure in a Liquid* asked about the pressure at a point at the top of the column. In section 5.4.2 of this chapter, we discuss how this result and other research suggested modification of the tutorial.

Table 5-2: Student responses to problems requiring the comparison of the pressure at a point above a free surface to atmospheric pressure (P_0).

	Physics 224 Washington 5 sections ($N = 116$)	Physics 224 Washington Wi'97 ($N = 33$)
Instruction	Standard	*Pressure in a Liquid* ITL
Problem type	Barometer	Barometer
Correct (Less than P_0)	65%	65%
with correct reasoning	50%	50%
Equal to P_0	10%	20%
Greater than P_0	20%	5%

All percentages rounded to the nearest 5%.

5.4.1.3 Student ability to rank pressures at points at the same level in liquids of different densities

As we have noted previously, this type of problem proved to be extremely difficult for students. Whereas fewer than 5% of students in three sections of Physics 224 had answered this type of problem correctly with correct reasoning after standard instruction, approximately 10% of the students in a section of Physics 224 did so after the initial version of *Pressure in a Liquid.* Based on this evidence, the instructional sequence seems to have done little to address student ability to answer this problem.

5.4.2 Summary of assessment of initial version of *Pressure in a Liquid*

Our impression was that the initial version of *Pressure in a Liquid* was too long. Many students spent a great deal of time completing the analysis of the liquid in the rectangular U-tube, which require drawing several free-body diagrams. Therefore, relatively few students were able to complete the section of the tutorial dealing with the U-tubes and the comparison of pressures to atmospheric pressure. These observations are consistent with the data shown above, in that although students were somewhat more successful on tasks involving pressure comparisons, they performed at the same level as students who had had only standard instruction on problems in which they were asked to compare pressures to atmospheric. These results led to the revision of the tutorial.

5.5 REVISED TUTORIAL SEQUENCE *PRESSURE IN A LIQUID*

We revised the tutorial *Pressure in a Liquid* in an effort to address student difficulties with pressure, weight, and the forces exerted by container walls in a shorter sequence of activities. Our goal was that students would have more time to work through the reasoning needed to compare the pressure at a point to atmospheric pressure. In addition, we developed homework exercises designed to address these ideas further, and added a question to the pretest in which students were asked to compare to atmospheric the pressure at a point above the free surface of the liquid.

5.5.1 Pretest

The pretest for the modified version of *Pressure in a Liquid* is similar to the pretest (described above) used with the initial version. Again, the first page includes a Letter Tube problem, in particular a Capped U-tube, in which students are asked to rank the pressures at several points in a fluid of uniform density. In addition, students are asked to compare the pressure at one or more points to atmospheric pressure. The latter question was added to the pretest in order to help students to recognize the importance of this comparison and to begin their thinking process. We chose to used a Capped U-tube problem on the first page in order to document the prevalence of 'along the tube' reasoning before the tutorial. As in the original pretest, the second page poses a Different-Diameter U-tube problem, in which students are asked to predict the water level in one leg of the tube given the water level in the other leg.

5.5.2 Instructional sequence

The revised instructional sequence for *Pressure in a Liquid* includes many tasks that are similar to those in the initial version of the tutorial. We will summarize the differences. First, to save time, we chose a simpler geometry for the initial free-body diagrams that students are asked to draw. In the new version, students consider layers of liquid enclosed by the container rather than the cubes of liquid with water to either side that they examine in the initial version. Secondly, the questions in this section make very explicit connections between force and pressure. For example, students were asked which force and which area they should use to find the pressure at a given point. Finally, the exercises that address the idea that the pressure is due solely to the weight of the liquid above have been condensed into a single page, involving a context that we refer to as the 'Utah tube,' since the tube is shaped like the state of Utah. A detailed description of the instructional sequence follows. (The complete text of the pretest, tutorial, and homework is shown in Appendix E.)

Free-body diagrams for layers of water. Students are first asked to consider the water in a beaker. The water is divided into three imaginary layers (see Figure 5-7). Students are asked to draw a free-body diagram for each layer of water, neglecting forces exerted by the atmosphere. For each force that they identify, students are asked to describe the type of force, the object exerting the force, and the object on which the force is exerted. Because we have found that many students have difficulty in deciding which force to use in determining the pressure at the bottom of a layer, students are asked to identify explicitly the surface on which each contact force is applied. Finally, students rank the magnitudes of these forces.

Horizontal forces. In Chapter 4, we described situations in which students failed to consider the horizontal forces exerted on a static liquid by container walls. In order to confront this incorrect idea, the tutorial asks students to predict what would happen if holes were opened near the center of each of the three layers of liquid. Students are then shown a demonstration in which such holes are opened and liquid pours out of the holes. Although we do not emphasize the relative magnitudes of these horizontal forces at this time, students often attempt to connect the magnitudes of horizontal forces to those of the vertical forces they ranked in the previous exercise.

Connection of force to pressure. In the earlier version of the tutorial and in the responses to the force comparison tasks in Chapter 3, we found that many students have difficulty in relating force, pressure, and area. To help students to make these connections, the tutorial asks students which force and which area they could use to determine the pressure at the bottom of the middle layer. A hint refers the students to their free-body diagrams for the layers, in which they identify the surface on which contact forces are applied, and asks explicitly which force or forces are exerted at the bottom of the middle layer. Students are then asked to describe how they could use a free-body diagram to determine the pressure at a point at the center of the middle layer.

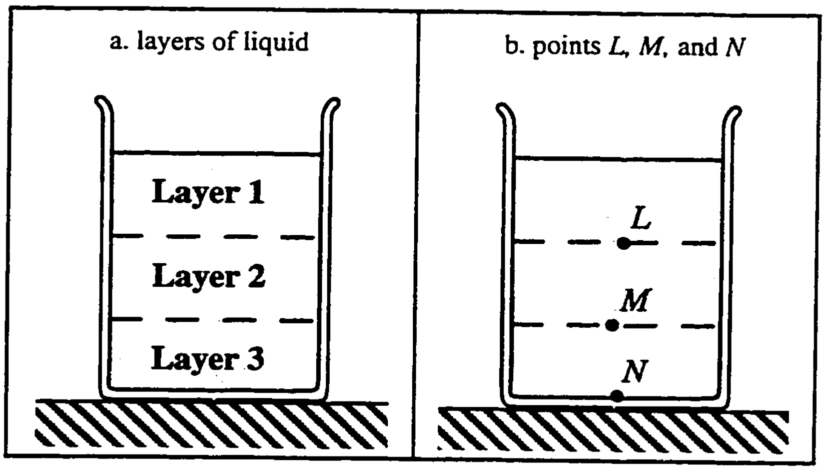


Figure 5-7: a. The layers of liquid in the revised version of *Pressure in a Liquid*.
b. The points L, M, and N that students consider in relating their free-body diagrams to the equation $P = P_0 + \rho gh$.

Forces by the atmosphere. Although there is not time to dwell on the difference between gauge pressure and absolute pressure, students are asked how their free-body diagrams would differ if they were to consider forces exerted by the atmosphere. They recognize that there would be a downward force on the top layer of the liquid by the atmosphere and that the magnitudes of the other vertical forces would increase.

Pressure ranking and application of the equation $P = P_0 + \rho gh$. In Chapter 3, we showed that several students reversed the pressure gradient, often based on incorrect application of the equation $P = P_0 + \rho gh$. Therefore, students are asked at this point to rank the pressures at three points, *L, M,* and *N,* at the bottom of the three layers (see Figure 5-7). We then ask students to connect their pressure ranking to the free-body diagrams that they drew for the three layers of water. Finally, students are asked whether their ranking is consistent with the equation $P = P_0 + \rho gh$. Students are given a hint that asks them to interpret $h = 0$ in this equation. Our interactions with students in the tutorial suggest that for many students, this question provokes a discussion of the equation and forces students to relate their qualitative understanding to the mathematical formalism.

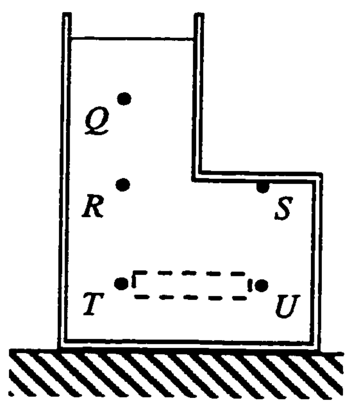


Figure 5-8: The revised version of *Pressure in a Liquid* includes an analysis of the liquid in the 'Utah tube,' so named because of the shape of the tube.

The 'Utah tube.' As we described in Chapter 2, a very common incorrect belief is that the pressure is due solely to the weight of the liquid above a point. Therefore, we developed an exercise to helps students confront this difficulty. In these questions, students consider the 'Utah tube' shown in Figure 5-8. Students draw a free-body diagram for the volume of water outlined by the dotted line; this volume of water has one end below the 'shelf' of the Utah tube and one end below the free surface. The students are asked to compare the magnitudes of the horizontal forces on this volume of water, and to use their answer to compare the pressures at the two ends of this volume of water.

Confronting 'weight-above' reasoning. Students are then asked to rank the pressures at several points and to discuss a student dialogue. One of the students in the dialogue articulates the common incorrect answer that the pressure is due solely to the weight of the water above a point and concludes that the pressures at the two points at the same height are different because point *S*, which is beneath the shelf, has less liquid above it. The second student agrees that the two pressures are different, citing the equation $P = P_0 + \rho gh$. We have found that, in this context, many students readily recognize that the idea that the pressure is due solely to the weight of the material above contradicts their analysis of the horizontal forces on the small volume of water. Thus, this exercise seems to provoke many students to discuss the relationship of pressure to depth in a static liquid. Many students recognize at this point, with the help of a tutorial instructor, that their dilemma can be resolved by thinking of forces exerted by the container.

Application of Pressure in a Liquid to U-tubes. In the final page of the tutorial, students are shown a series of U-tubes. This sequence of questions is nearly identical to that in the initial version; students rank the pressures at points in the tube, consider the addition of a stopper to seal the right end, and then predict what will happen to the water level in the capped end when water is removed from the left end of the tube. Unlike previous versions, students are not asked questions that seek to resolve this last issue in the tutorial; rather, these questions are moved to the homework.

5.5.3 Homework

Three problems are posed in the tutorial homework *Pressure in a Liquid.*

Liquid removed from U-tube. Students work through the reasoning needed to explain why the water level in the right end of the capped U-tube remains at the same location when water is removed from the left end (see Figure 5-9a). A brief statement reminds the students of the results of the demonstration shown in class, in which water is removed from the left end of the tube but the water level on the right end does not change. Students answer several qualitative questions and then determine the value of the absolute pressure at point *F* and the height of water that would result in zero pressure at the top of the right tube.

Pressure compared to atmospheric. Students are shown a W-shaped tube in which four points are marked, including points at the same level as the free surface, below the free surface, and above the level of the free surface (see Figure 5-9b). Students are asked to compare the pressure at each point to atmospheric pressure.

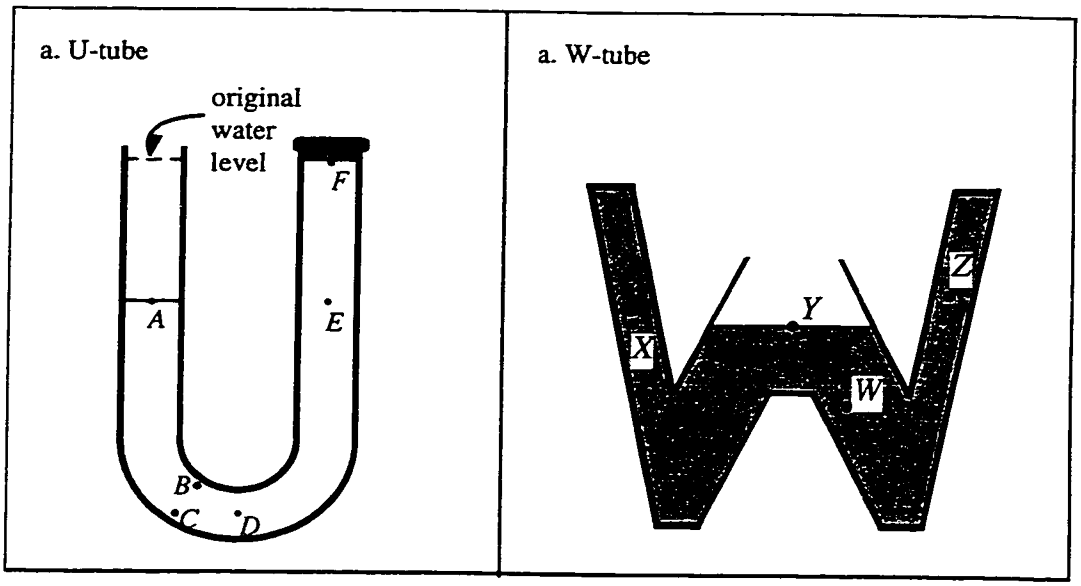


Figure 5-9: The diagrams for two of the problems in the tutorial homework accompanying *Pressure in a Liquid.*

Two-liquid problem. Students are shown a U-tube that contains two liquids, water and oil. The top surfaces of the water in both legs of the tube are shown, but the top surface of the oil in the left leg is not (see Figure 5-10). Students are asked to compare the pressure at the water surface in the left leg to that at a point at the same height in the right leg. Then they are asked to compare the difference in pressure between pairs of points in the oil and in the water that are separated by the same vertical distance and use their result to decide if the top surface of the oil is above, at the same height as, or below the top surface of the water.

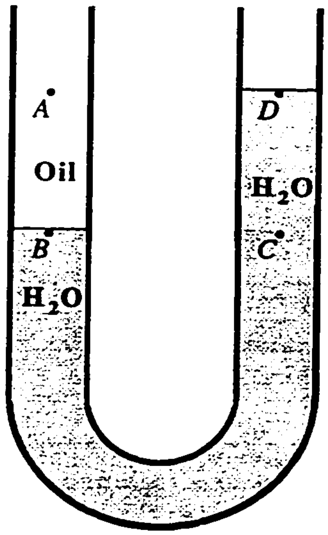


Figure 5-10: The diagram for the third problem in the tutorial homework accompanying *Pressure in a Liquid.*

5.6 ASSESSING STUDENT UNDERSTANDING AFTER THE REVISED VERSION OF *PRESSURE IN A LIQUID*

Due in part to the preferences of the instructors, the revised version of these materials were not used in Physics 224, and thus comparison of the revised version of the tutorial sequence to the initial version is not possible. In this section we present results from sections of Physics 115 that had completed standard instruction and sections that had completed the revised version of the tutorial sequence. We used the *Review of Newton's Laws* and the revised version of *Pressure in a Liquid* as part of a series of ITLs in several sections of Physics 115. The students spent one class period completing *Review of Newton's Laws* and approximately two class periods completing *Pressure in a Liquid.* After the ITLs, we again included several post-tests on course examination problems in order to assess the effectiveness of the instructional strategies in the tutorial. We found that students performed considerably better than after standard instruction.

5.6.1 Student ability to rank pressures at points in the same liquid

We have given as post-tests several versions of problems, both Letter Tube and Barometer problems, in which students are asked to compare the pressures at points at the same height in a liquid. In many cases, we have also asked students to rank the pressures at points at different heights in a liquid of uniform density.

5.6.1.1 Pressures at points at the same level

As shown in Chapter 1, students after standard instruction often have difficulty in determining that points at the same level in a liquid have the same pressure. On several problems (see Appendix A for details), we have found that approximately 60% of students in the algebra-based course were able to rank such points correctly, with approximately 40% giving correct reasoning. (As described above, correct reasoning was considered to include either use of the equation $P = P_0 + \rho g h$ or simply a reference to the relative height, depth, or level of the points.) After the modified version of *Pressure in a Liquid*, we posed a Barometer problem on a midterm examination and an N-tube problem on the final examination in one section of the algebra-based course. The responses given by these students and those of the students who had completed standard instruction are shown in Table 5-3. The students who had completed the modified tutorial were much more successful, with more than 80% giving correct answers with correct reasoning. The fraction of students stating that points have different pressures based on the material above or other incorrect forms of reasoning is also significantly lower after completion of the modified version of *Pressure in a Liquid.*

Table 5-3: Student responses to problems in which they are asked to compare the pressures at points at the same level in a liquid after standard instruction and after the revised version of *Pressure in a Liquid.*

	Physics 115 Washington 3 sections (*N* = 374)	Physics 115 Washington Sp'97 (*N* = 200)	Physics 115 Washington Sp'97 (*N* = 201)
Instruction	Standard	*Pressure in a Liquid* w/Utah tube	*Pressure in a Liquid* w/Utah tube
Type of problem	Letter or Barometer	Barometer	N-tube
Correct	65%	90%	90%
w/correct reasoning	40%	85%	80%
Not all equal	35%	10%	5%
Other/Blank	0	<5%	~0%

All percentages rounded to the nearest 5%.

5.6.1.2 Difficulties with the gradient of pressure

In Chapter 2, we have shown that students seem to have a variety of difficulties with the concept of pressure gradient. We have examined the incidence of these difficulties on a particular Letter Tube problem (the N-tube, shown in Figure 5-1), after standard instruction and after *Pressure in a Liquid.* In Table 5-4, we have shown the responses of only the subset of *N'* students who had correctly ranked the pressures at points at the same level. The performance of the two groups of students is very similar, but those who had completed the tutorial were slightly more likely to answer correctly. Although the differences in results between the two groups are very small, the fraction of students having these difficulties is also small. In addition, the fact that we are comparing only those students who had correctly ranked the pressures at points at the same level suggests that we may be comparing only the better students in the standard course to a group that includes nearly all of the students in the course that had completed *Pressure in a Liquid.*

Table 5-4: Pressure rankings for points at different levels given by those N' students who correctly ranked the pressures at points at the same level in two sections of the algebra-based introductory course.

	Physics 115 Washington Wi '97 ($N = 201$)	Physics 115 Washington Sp '97 ($N = 201$)
Instruction	Standard instruction	rev. *Pressure in a Liquid*
Students answering correctly for points at same level	60% ($N' = 133$)	90% ($N' = 185$)
Responses for points at different levels:		
Correct: pressure increases with depth	80%	90%
Incorrect: pressure the same at all levels	~0%	~0%
Incorrect: pressure increases with *height*	15%	10%

All percentages rounded to the nearest 5%.

5.6.1.3 Pressure in curved tubes and 'along the tube' reasoning

As shown in Section 2.4 of Chapter 2, curved tubes that are capped on one end seemed to elicit somewhat different reasoning from students than did other Letter Tube problems. In particular, students tended to give reasoning that suggested that the pressure increases with distance from the open end of the tube rather than depending on vertical distance. This issue was not explicitly addressed in the tutorial, but students were led through a series of steps to develop a correct model.

In Table 5-5, we show results from a section of Physics 115 after some instruction on pressure and in the same section after the revised version of *Pressure in a Liquid* including the Utah tube exercise. The problems used were a Capped U-tube (see Figure 5-3) in the first case and a topologically similar J-tube in the second. Students were much more successful in correctly ranking the pressures after completing the tutorial. In addition, the fraction of students giving pressure rankings based on the 'along the tube' reasoning (including those who reverse the most common 'along the tube' ranking) has dropped from over 40% to less than 5%.

For comparison purposes, we have included results from the graduate and undergraduate Teaching Assistants at Purdue University who took the pretest as part of their teaching seminar. The students in the algebra-based course who had completed *Pressure in a Liquid* performed at essentially the same level as the Teaching Assistants. We regard the performance of Teaching Assistants as a good goal for the performance that can be expected from students in the introductory course.[60]

In addition to noting the responses given by students before and after the modified instruction, we have found it instructive to compare student responses on curved tube problems to those on other pressure problems. Before modified instruction, students in Physics 115 seem to think of these problems in very different ways, as suggested by the very different success rates on these problems (25% correct on curved tube problems compared to 60% on other pressure problems). Even on the same ungraded quiz, we have seen students give reasoning based on height in one problem and based on 'along the tube' reasoning in another problem (see Appendix A for a detailed analysis). After completing the revised version of *Pressure in a Liquid,* students in the Physics 115 course gave very similar responses on the curved tube problem and on other pressure problems. This result suggests that the instructional strategies used in *Pressure in a Liquid* may successfully discourage students from giving responses based on surface features of the problem.

Table 5-5: Student responses to the problems that elicit 'along the tube' reasoning in two sections of Physics 115 before and after an ITL using *Pressure in a Liquid.* Responses of TAs at Purdue University before the tutorial are shown for comparison.

	Physics 115 Washington Sp'98 (N = 109)	Physics 115 Washington Sp'98 (N = 127)	TA Seminar Purdue Sp'98 (N = 18)
Instruction	Some pressure vs. depth	rev. *Pressure in a Liquid*	*N/A*
Correct w/correct	20%	90%	85%
'Along the tube' or reverse	40%	~0%	~5%
Other/blank	40%	10%	10%

All percentages rounded to the nearest 5%.

5.6.2 Student ability to compare pressures to atmospheric

In Chapter 3, we described the failure of many students to successfully compare the pressure at a point in a fluid above the free surface of a liquid to atmospheric pressure. In Table 5-6, we show results from several sections of Physics 115 after instruction with the modified version of *Pressure in a Liquid.* (Student responses after standard instruction are shown in the first column, which is reproduced from Chapter 2.) As is shown in the table, students were significantly more successful in making these pressure comparisons after the modified version of *Pressure in a Liquid.*

The Spring '98 section of Physics 115 performed much better than the Spring '97 section. In part this difference may be due to the difference in the problems. The Spring '97 class was given a Barometer problem; the Spring '98 class, a Letter Tube. As we described in Chapter 2 and Appendix B, the patterns of student responses to the pressure comparison problems after standard instruction were very similar, but the two types of problem seemed to elicit somewhat different patterns of reasoning. In particular, many students responding to the Barometer problem compared the height of the mercury column in the barometer to 760 mm Hg in order to compare the pressure to atmospheric, whereas such a comparison was not possible in the Letter Tube problem. The fact that the difference is observed after the tutorial and not after standard instruction is difficult to interpret, and may suggest that the problems are less similar than our results had led us to believe.

These results suggest that the revised version of *Pressure in a Liquid* is at least somewhat more successful than the initial version. For the initial version there was not improvement in student performance on tasks requiring a comparison of pressure to atmospheric. After the modified version there was some improvement in student performance on these tasks. However, the improvement is less dramatic than that on the problems shown above in which students rank pressures at points in a liquid, and the results on the Multiple-Barometer problem are not much better than those after standard instruction. This result, along with feedback from instructors at pilot sites, suggests that there is still room for improvement in the instructional strategies used in *Pressure in a Liquid,* particularly those in which students learn to apply the pressure gradient.[61]

Table 5-6: Responses given by students on problems requiring a comparison of the pressure at a point in a liquid above the free surface of the liquid to atmospheric pressure (P_0) after standard instruction and after the modified version of *Pressure in a Liquid.*

	Physics 115 Washington two sections (N = 170)	Physics 115 Washington Sp'97 (N = 200)	Physics 115 Washington Sp'98 (N = 126)	TA prep Purdue Sp'98 (N = 18)
Instruction	Standard	rev. *Pressure in a Liquid*	rev. *Pressure in a Liquid*	N/A
Problem type	Barometer / Letter Tube	Barometer	Letter Tube	Letter Tube
Less than P_0 (correct)	40%	45%	80%	90%
with correct reasoning	15%	35%	65%	55%
Equal to P_0	20%	50%	5%	5%
Greater than P_0	35%	5%	15%	5%
Other/blank	<5%	<5%	~0%	none

All percentages rounded to the nearest 5%.

5.6.3 Student ability to rank pressures at points at the same level in liquids of different densities

As we described previously, very few students are able to rank correctly the pressures at points at the same level in liquids of different densities. In Table 5-7 we show results from two sections of Physics 115, one after standard instruction, and one after the revised version of *Pressure in a Liquid.* Neither group was very successful in comparing the pressures at points at the same height in liquids of different densities. Although the students who had completed standard instruction were more likely to give the correct answer, nearly all of these students gave incorrect reasoning based on the cross-sectional area of the tube containing the denser liquid (see discussion in Section 3.4 of Chapter 3).

About 10% of the students after the modified tutorial gave correct reasoning. Therefore, we believe that the students in the class that had completed *Pressure in a Liquid* did better than those in the class after standard instruction, despite the difference in percentage of correct answers. However, the students after *Pressure in a Liquid* were still largely unsuccessful on this problem. Although we do not have comparable data on the responses of Teaching Assistants to this problem, anecdotal evidence suggests that many graduate students and faculty find this problem to be quite difficult.

Table 5-7: Responses given by students in two sections of Physics 115 in comparing the pressures at two points at the same level in liquids of different densities. Correct reasoning has increased but only slightly.

	Physics 115 Washington Au'95 (N = 120)	Physics 115 Washington Sp'97 (N = 200)
Instruction	Standard	*Pressure in a Liquid*
Pressure less in denser liquid (correct)	35%	15%
with correct reasoning	~0%	10%
Pressure greater in denser liquid	20%	75%
Pressures equal	30%	10%
Other / blank	15%	<5%

All percentages rounded to the nearest 5%.

5.7 DISCUSSION

This chapter illustrates the iterative cycle of curriculum development that typifies the curriculum development model of the Physics Education Group. We developed curriculum based on research into student difficulties with pressure. In this case, our findings included research in which we investigated the relationship of student understanding of forces in mechanics contexts to student performance on pressure problems (see Chapter 4).

Based on our findings that many student difficulties seemed to be connected to difficulties in (1) identifying the forces acting on bodies of liquid and (2) applying Newton's second law to these bodies, we developed instructional sequences in which students practiced these skills. In the tutorials *Review of Newton's Laws* and *Pressure in a Liquid,* students draw free-body diagrams for several different systems that are at rest. The tutorials include purely mechanical systems, like stacks of paper, and hydrostatics systems, like layers of water. Students apply Newton's second law to these systems and use their results to make qualitative comparisons of the magnitudes of the upward and downward contact forces on the system. Based on our finding that many students give answers consistent with the incorrect belief that the pressure is due solely to the weight of the material above a point, we have included exercises to help students distinguish between the weight force on an object and the contact forces on the object.

A crucial part of the development of curriculum is the testing of the materials with the students for whom they are intended. We have tested these tutorial sequences with students in the introductory algebra-based course and with the more sophisticated students in the second-year course. In order to determine whether the materials have effectively addressed the difficulties that we have found, we have posed post-test problems on course examinations. Based on the results of these post-tests after early versions of the materials, we have modified the curriculum and tested the effects of the modifications. The results after these modifications suggest that the revised version of the tutorial can help students to resolve many of their difficulties with pressure and the gradient of pressure. In some cases, after completing the materials, students at the introductory level perform at a level that is similar to that of teaching assistants who have had no special instruction. However, on some of the more difficult problems, there is still room for improvement.

PART IB: IDENTIFYING AND ADDRESSING STUDENT DIFFICULTIES WITH BUOYANCY

INTRODUCTION

The second part of this dissertation describes our work in identifying and addressing student difficulties with buoyancy and Archimedes' principle. We begin by examining what students are taught as a guide to the level of understanding typically expected by instructors in the courses that we have examined. We then summarize previously published research that pertains to student understanding of sinking and floating and the related concepts of mass, volume and density.

WHAT STUDENTS ARE TYPICALLY TAUGHT

When an object is immersed in a liquid, there is a force on each surface of the object by the liquid. For surfaces that are small enough that the surface is approximately flat and the pressure is nearly uniform, the force exerted by the liquid is normal to the surface and has magnitude equal to the product of the area of the surface and the pressure at the center of the surface. If the liquid is in a gravitational field, the pressure in the liquid increases with depth, such that the pressure at the bottom surface of an object is greater than the pressure at the top surface. Thus, the sum of the forces on the object by the surrounding liquid is directed upward. This sum is defined as the *buoyant force.*

For example, consider a cubical block that is completely submerged in a liquid. There are contact forces on the six faces of the cube by the liquid. The four horizontal forces are of equal magnitude. The magnitude of the contact force on the bottom surface of the cube is greater than that on the top surface. The magnitude of the sum of the forces can be related to the pressure at the top and bottom surfaces as well as the area of these surfaces:

$$\Sigma F_{\text{liquid}} = P_{\text{bottom}} A - P_{\text{top}} A.$$

$$= \rho g h_{\text{bottom}} A - \rho g h_{\text{top}} A,$$

where ρ is the density of the liquid and h is measured downward from the top surface of the liquid (so that $h_{\text{bottom}} > h_{\text{top}}$). Noting that the quantity ($h_{\text{bottom}} A$ - $h_{\text{top}} A$) is equal to the volume of the cube, we write,

$$B = \rho g V_{\text{cube}},$$

where B is the magnitude of the buoyant force on the object.

Archimedes' principle states that the magnitude of the buoyant force on an object is equal to the weight of the liquid displaced by the object.[62] For the completely submerged cube above, the volume of liquid displaced is equal to the volume of the cube, and the weight of that liquid is $\rho g V_{\text{cube}}$. In an incompressible liquid, the pressure increases linearly with depth, so the different in pressure between the top and bottom of an object does not change as the object moves to a greater depth; thus, the buoyant force depends only on the volume of the object and the density of the surrounding liquid.

PREVIOUS RESEARCH ON STUDENT UNDERSTANDING OF BUOYANCY

The bulk of the previous research on student understanding of buoyancy has been conducted with pre-college students. We will discuss some work on student understanding of physical quantities such as volume and density that are important prerequisites for understanding buoyancy, studies conducted with students at the elementary level and at the college level. We will also describe studies of precollege students' understanding of floating and sinking.

Student understanding of density and volume

Some of the initial work on understanding of the behavior of objects placed in liquids was performed by Piaget and his followers.[63] Among the tasks used for assessment of the cognitive development of children was one titled *Conservation of Volume.* Students in the study were shown two identical test tubes filled with equal amounts of water and two metal cylinders of equal volume but different mass. The subjects were then told that each cylinder would be placed in one of the test tubes and asked whether the increase in water level caused by the heavier cylinder would greater than, less than, or equal to the increase in the water level caused by the lighter cylinder. Subjects were asked to explain. Responses were classified based on whether subjects correctly predicted the outcome of the experiment and whether subjects who made incorrect predictions could modify their explanations after seeing the results of the experiment. The Conservation of Volume task was used by Renner *et. al.* in a study of over 1000 secondary students in Oklahoma, of whom nearly 50% made incorrect predictions.[64,65] In a related study with 185 first-year college students and 22 second- and third-year law students, Renner and Lawson reported that approximately 30% of the freshmen and 3 of the 22 law students made incorrect predictions on the Conservation of Volume task.[66]

A similar task was used by McKinnon in a study of 143 college students.[67] Two clay balls of equal volume were presented to a student in an interview setting. One ball was deformed into a cylinder. The student was asked to compare the weight of the two pieces of clay after this change and to predict the relative increase in water level when both pieces of clay were added to a graduated cylinder. Twenty-four of the 143 students said that the weight of the clay had changed when it was deformed, and fifteen predicted that the two objects would displace different amounts of water. In addition, some students who said that the volume of two clay balls were different predicted that they would displace the same amount of water, arguing that the weights were still the same. In a study of college students in a course designed for students whose preparation in math and science was inadequate for mainstream university science courses, Rosenquist reported that as many as 50% had difficulty in distinguishing between mass and volume, as measured by tasks including a water displacement task similar to the Conservation of Volume task.[68]

Student understanding of sinking and floating

Sinking and floating is commonly taught in precollege science classes. There has been some research on young students' understanding of the topic. Driver and her colleagues have studied understanding of sinking and floating behavior among precollege students. This work and that of others is summarized in two reviews.[69][70] Students at this level often state that whether an object sinks or floats depends on the weight of the object, without considering the volume. In addition, many students focus on specific features of objects, like air trapped inside or holes in the object, and make predictions based on these features. Because of the age level of these students, most of these studies have concentrated on the use of density as a predictor of sinking and floating and very few have probed student understanding of sinking and floating in terms of forces.

At the undergraduate level, McKinnon studied student understanding of density as it related to buoyancy in a group of first-year college geology students. These students were asked to explain why a large, heavy cube floated whereas a small, light one sank. Only one third of the students recognized that the ratio of mass and volume is characteristic of a given material, and fewer than one sixth were able to successfully relate the density of the cubes to their sinking and floating behavior.[71] Duckworth reported on a course for inservice teachers in which a sinking and floating was a primary topic. The students in this course had a great deal of difficulty developing a conceptual understanding of sinking and floating, with the idea of enclosed air being a particularly persistent incorrect belief for some.[72]

OVERVIEW OF PART IB

Part IB of the dissertation includes two chapters. The first, Chapter 6, is a draft of a paper that is in preparation for publication. This chapter describes the process of identifying student difficulties with buoyancy and Archimedes' principle as well as our efforts to identify some of the underlying student difficulties with principles from mechanics. Chapter 7 describes the development and assessment of curriculum to address student difficulties with buoyancy. Several appendices include information relevant to Part IB, including Appendix F, which includes a description of an initial set of interviews in hydrostatics, Appendix G, which includes a detailed summary of student responses to problems that we have used to identify difficulties with buoyancy, and Appendices H and I, which include initial and modified versions of the tutorial sequence that we developed to address student difficulties with buoyancy.

6. IDENTIFYING STUDENT DIFFICULTIES WITH ARCHIMEDES' PRINCIPLE

This chapter has been prepared for submission to the *Physics Education Research Supplement* to the *American Journal of Physics.* This work has built upon a set of interviews on buoyancy that are described in Appendix F. Additional details on student responses to several of the written problems described in this chapter can be found in Appendix G.

IDENTIFYING STUDENT DIFFICULTIES WITH ARCHIMEDES' PRINCIPLE

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6.1 INTRODUCTION

The Physics Education Group at the University of Washington is engaged in an ongoing investigation of student understanding of hydrostatics. Our object has been to determine to what extent students can apply the concepts they have studied to real-world situations and to establish a research base for the development of curriculum, both by our group and by others.[73]

In this study, we have investigated student understanding of hydrostatics at the level of introductory university physics courses. In a typical introductory course, the variation of hydrostatic pressure with depth is first derived. The buoyant force is then introduced as the sum of the hydrostatic forces on an object and is related to the weight of fluid displaced. For the purpose of this paper, we examine the ability of students to apply Archimedes' principle and Newton's laws to problems in which objects are partially or completely submerged in a liquid.[74]

Early in the investigation we interviewed seven students from a second-year course in hydrostatics and thermal physics at the University of Washington.[75] The students were volunteers and had final course grades at or above the mean for the class. The interviews took place after all instruction in hydrostatics was completed and after a course examination covering the topic. In the interviews, we asked students to predict and explain the behavior of a Cartesian diver. The diver, an object of variable density that can be made to sink or float in response to pressure changes in a liquid, is a popular demonstration used to illustrate Archimedes' and Pascal's principles.[76] It had been used in the course in which the interview volunteers were enrolled, although the apparatus used in the interviews was significantly different in appearance than that shown in lecture.

Although they had seen a similar demonstration in class, only two of the seven students we interviewed correctly predicted the behavior of the diver. None of the students was able to give a correct explanation after seeing the outcome of the demonstration. A few referred to having seen the demonstration. The responses given by the students suggested several incorrect beliefs about buoyancy and pressure. However, their level of confusion was such that we were unable to interpret many of the comments made. In order to probe student understanding further, we developed a series of written questions and interview tasks that involved simpler situations. In this article, we describe some specific student difficulties that we have identified.

At the University of Washington, the investigation has involved students in the introductory algebra-based course and in the second-year course described above. We will refer to these courses as "introductory" and "second-year," respectively. The introductory course is a standard non-calculus physics course for life science majors. Approximately half the students in this course enroll in an associated laboratory course, which until very recently had no experiments involving hydrostatics. Students enrolled in the second-year course have completed introductory calculus-based physics, which, at the University of Washington, does not cover hydrostatics. About 30% of these students are physics majors, and the course is typically taught with more mathematical sophistication than either of the introductory courses.[77] The second-year course has no laboratory component. The investigation has also involved introductory physics courses at other universities.

This work has built upon work that was done in the context of special science courses for precollege teachers, in which a laboratory-based curriculum on sinking and floating was developed on the basis of research findings.[78] In that curriculum, Archimedes' principle is developed through experimental observation without introducing the concepts of pressure and force. Students predict and explain sinking and floating on the basis of mass, density, and volume.

6.2 IDENTIFICATION OF STUDENT DIFFICULTIES WITH BUOYANCY

Our group's research methods have been described in several papers. Typically we seek to identify student difficulties based on analysis of responses to written problems as well as student responses in individual demonstration interviews.[79] The problems that we have used involve simple physical situations. Students are asked to compare the values of quantities (*e.g.*, the magnitude of the buoyant force) in various situations, or to predict the results of an experiment or of a change in the physical system.

6.2.1 Five Blocks problem

During the course of the Cartesian diver interviews, several students predicted (either spontaneously or in response to follow-up questions by the interviewer), that objects of different densities would come to rest completely submerged at different depths in a liquid. We decided to try to develop a simple question that would elicit this belief. The question we developed is referred to as the *Five Blocks* problem.

6.2.1.1 Description

In the problem, students are shown a sketch of five blocks. The problem states that the blocks are of the same size and shape but that the masses of the blocks increase from M_1 to M_5. (See Figure 6-1) The students are told that each block is held approximately halfway down in an aquarium full of water and released. The final positions of blocks 2 and 5 are shown: block 2 barely floats and block 5 rests on the bottom of the tank. Students are asked to sketch the positions of the other three blocks and to explain their answer.

Five blocks of the same size and shape but different masses are shown at right. The blocks are numbered in order of increasing mass (i.e. $m_1 < m_2 < m_3 < m_4 < m_5$).

All the blocks are held approximately halfway down in an aquarium filled with water and then released. The final positions of blocks 2 and 5 are shown.

On the diagram, sketch the final positions of blocks 1, 3, and 4. Explain your reasoning.

(Assume that the water is incompressible.)

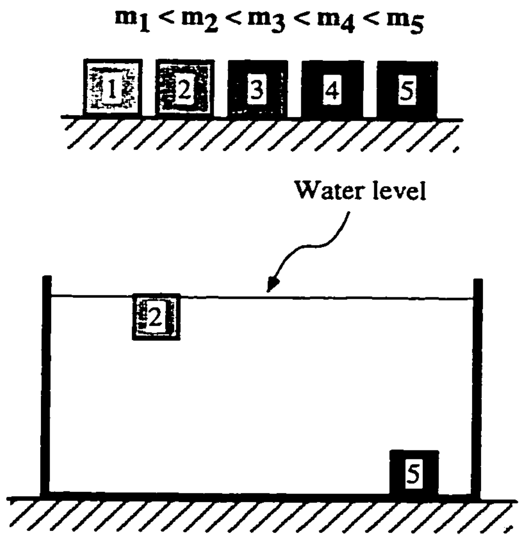


Figure 6-1: The written version of the Five Blocks problem.

A qualitatively correct answer to the Five Blocks problem can be determined either by considering buoyant forces or by simply comparing the density of each block to that of water. The approach using buoyant forces was typically emphasized in the classes in which the problem was given, and we will describe only this method in detail. However, in our analysis of student responses, either approach was considered to be correct.

Students can use the given information (including the final positions of blocks 2 and 5) to infer how the weight of each block compares to the buoyant force on the block when released underwater. When released, the blocks are completely submerged. Each block therefore displaces the same amount of water and the buoyant forces are all equal. Block 2 barely floats and, therefore, when fully submerged has a buoyant force that is very slightly greater than its weight. Therefore, blocks with smaller weight will have a net force upward and those with a greater weight will likely have a net force downward. Thus block 1 will float (with a smaller portion of its volume submerged). For blocks 3 and 4, the most likely result is that each will sink. It is possible that block 3 may have weight exactly equal to the buoyant force and remain at rest in the middle of the tank when released. In our analysis of student responses, any potentially correct answer that was supported by correct reasoning was considered to be correct, including those with block 3 remaining in the middle of the tank.[80,81] (See Fig. 2.)

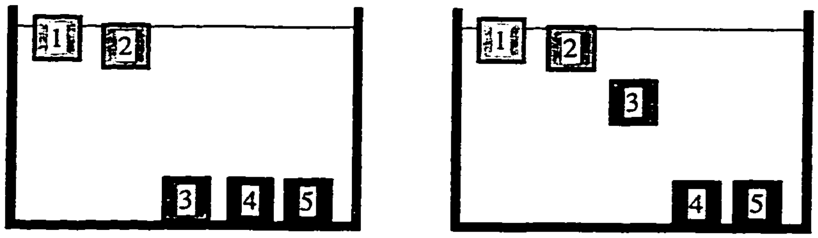
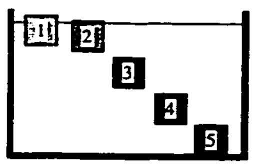


Figure 6-2: Possible correct responses to the Five Blocks problem.

6.2.1.2 Results

The Five Blocks problem has been given to students enrolled in the introductory course and in the second-year course. Typically it has been an ungraded problem administered in lecture. In some classes the problem has been administered before any instruction; in others, it has been given after the completion of traditional lecture instruction on hydrostatics. In both the introductory and the second-year course, the percentage of correct responses and of common incorrect responses were virtually identical before and after standard instruction. The amount of instruction also did not seem to affect the distribution of types of reasoning used by students.[82] Therefore the data have been combined. A summary of these results can be found in Table 6-1.

Table 6-1: Summary of student responses to the Five Blocks problem.

	Introductory Washington 3 sections (*N* = 371)	Second-year Washington 2 sections (*N* = 55)
Correct	20%	55%
Incorrect: 	80%	45%

All percentages rounded to the nearest 5%.

Relatively few students have given a qualitatively correct answer to the Five Blocks problem, ranging from less than 20% in the introductory course to approximately 50% of in the second-year course. The most common incorrect response in all cases has been to show blocks 3 and 4 at intermediate positions in the water, as shown in Figure 6-3. This answer has been given by essentially all of the students who answered incorrectly. We refer to it as the 'descending line' answer.

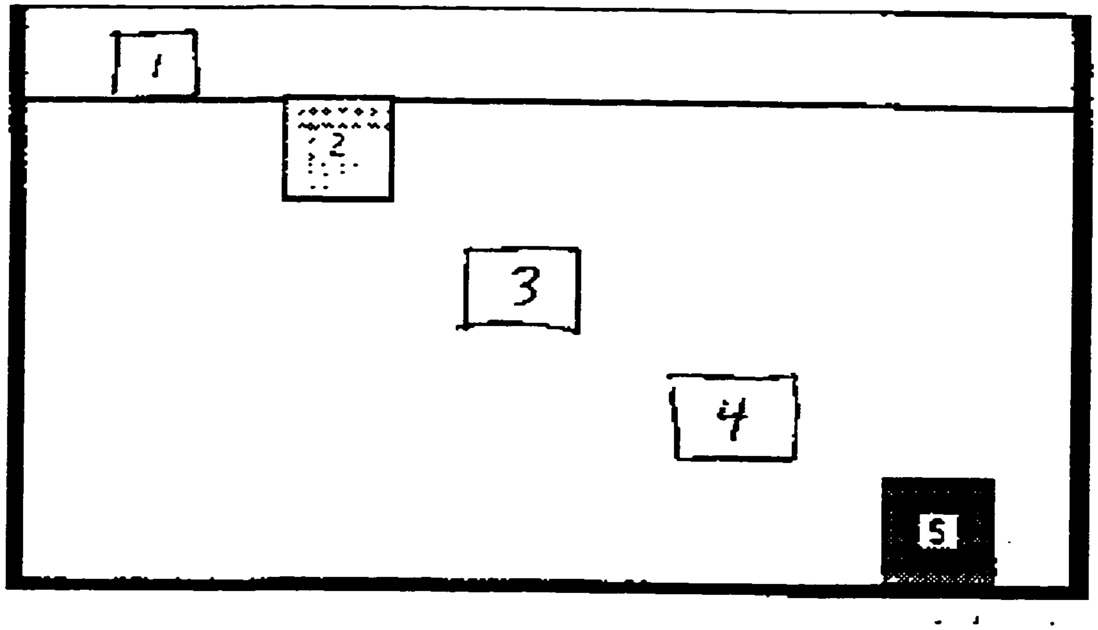


Figure 6-3: The incorrect 'descending line' response to the Five Blocks problem.

In addition to the written version, we have used the Five Blocks problem as an interview task. Five small blocks of equal volume but different materials were selected such that two of the blocks would float and three would sink. The students were shown that block 2 barely floats and that block 5 sinks and then asked to predict the final positions of the remaining blocks. Half of the twelve students that we interviewed gave the incorrect 'descending line' response. The responses to the interview questions suggest that the difficulties elicited by the problem are persistent; even after further probing by the interviewers, none of these students changed their answer, instead insisting on the correctness of the 'descending line' response. One student (who had not seen the outcome) said, "I assume that you set it up to do that ... to show that you can have a block that has a certain mass and certain density that will [rest] at different points in the water. To show that it doesn't just float or just sink." When students were shown the outcome of experiment after the interview, most were quite surprised.

As we have noted, students could approach the Five Blocks problem by considering either forces or density. In the introductory course, slightly fewer than one-third of the students approached the problem using forces. Another third of the class referred to the densities of the blocks. The remainder of the students gave very brief reasoning that simply referred to the mass or weight of the blocks. Interestingly, the success rates of all these groups of students were essentially the same.[83] The amount of prior instruction on buoyancy did not seem to affect the prevalence of either approach. In the following section, we will describe specific conceptual difficulties that became apparent from the analysis of the reasoning given by students.

6.2.1.3 Specific difficulties identified

Most of the students who answered the Five Blocks problem incorrectly can be grouped into two categories according to the reasoning they used: students who may have recognized that the buoyant force on each block is the same when released under water but who failed to apply correctly Newton's second law to predict the subsequent motion of the blocks, and students who failed to relate the buoyant force on a submerged block to its volume and therefore failed to recognize that the buoyant force on each block is the same. Each of these categories is discussed below.

Failure to apply Newton's second law correctly

Many students referred to forces in their explanations, but seemed to believe that a block could remain at rest with a non-zero net force acting on it. For example, a student who drew block 3 in a final position in the middle of the tank wrote, "Its weight (mg) is greater than #2 so it will sink a bit farther down - it is not quite as much as the buoyant force." Because of the prevalence of this type of response, we decided to modify the question to ask the students to draw free-body diagrams as part of their explanation.

On one version we asked students to draw a free-body diagram for block 3 at its final position. (As in the case of the interviews, additional questions of this nature did not seem to lead many students to change their initial answer.) This version was given to students in one section of the introductory course after they had completed instruction on pressure but had not yet begun instruction on Archimedes' principle. About 80% of the students gave the incorrect descending line answer. Many drew free-body diagrams that suggested a non-zero net force on the block. Although we did not ask students to specify the net force on the block or to compare the magnitudes of the forces acting on the block, about 30% were quite explicit in indicating a non-zero net force, as in the case of the student who drew the diagram in Fig. 4.[84] This diagram suggests that the comparison of the weight and buoyant force is being used to determine the final *position* of the block, not to predict whether it accelerates to the surface or to the bottom of the tank.

Previous research in the context of mechanics has found that many students associate the net force on an object with its velocity, rather than its acceleration.[85] The prevalence of free-body diagrams like the one in Fig. 4 suggests that many students relate the net force on an object to its *displacement* from the point where it is released or, equivalently, to its final *position*. For example, one student wrote, "Block 4 has greater force down than 3 so it sinks further." Several students gave very detailed and non-Newtonian responses: "...the buoyancy force produced by the displaced water is equal on all of the boxes. Therefore due to the boxes' various masses their ability to counteract the buoyancy force will increase $1 \rightarrow 5$. Therefore each will be farther submerged in H_2O."

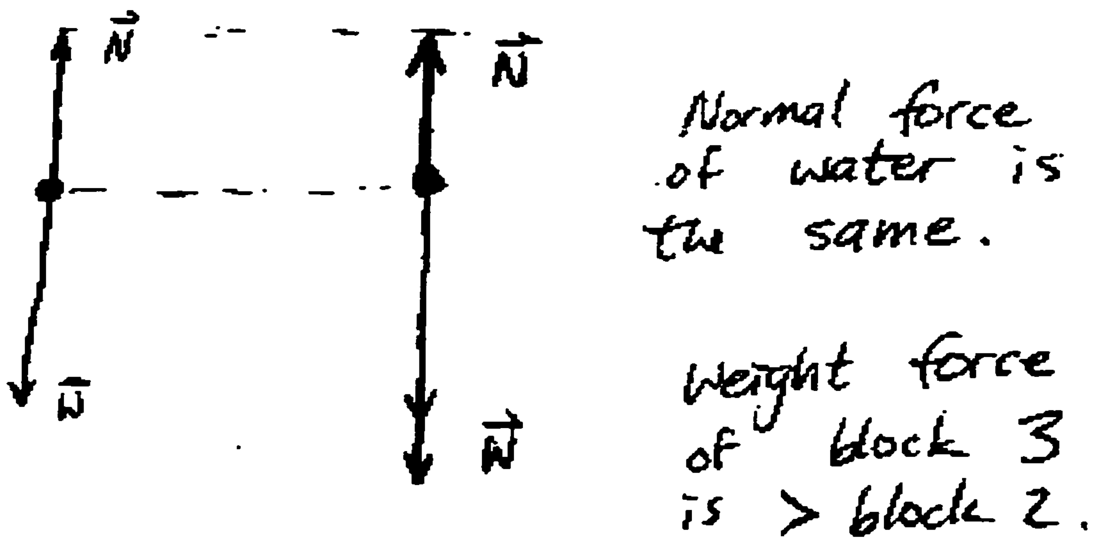


Figure 6-4: Student free-body diagrams for blocks 2 and 3 on the Five Blocks problem, Physics 115. This student drew the sketch in Figure 3. Note that the free-body diagram for block 3 is inconsistent with the position of block 3 in the sketch.

In addition to those students who explicitly applied an incorrect equilibrium condition, many gave very superficial reasoning in support of the descending line answer, often referring simply to the weights of the blocks. For example, one student wrote, "[block 3] had to be deeper than 2 since the block is heavier." Another wrote, "Block 4 is heavier than 3 but lighter than 5 so it descends somewhere halfway between 3 and 5." These responses are consistent with a belief that a block of a given mass or density has a unique equilibrium level below the water surface and that the block will seek that level. We and other researchers have seen similar associations in other contexts.[86,87] It seems that for some students this belief is so strong that it prevents correct application of Newton's second law. All of the students in our study had completed a previous course in mechanics, but many failed to apply the condition that the net force is zero on an object at rest in this context. Later in this paper we discuss our investigation into student understanding of equilibrium in purely mechanical contexts.

Failure to relate the buoyant force to the volume of a submerged object

Many students who referred to forces in their responses failed to recognize that the buoyant force on each of the blocks is the same at the moment they are released, when each block is completely submerged in the aquarium. Instead, they seemed to relate the buoyant force to the mass or density of a block, rather than the volume of water displaced.

Some students were very explicit in relating the buoyant force to the mass. For example, one student wrote, "assuming that the masses increase in equal intervals, it seems likely that the buoyant forces would also increase linearly." Other students reached the opposite conclusion: "[block 3] is heavier than block 2, so the buoyant force acting on it is less – so [it] should be farther down in the water than block 2." Surprisingly, students in both of these categories gave the descending line answer to the Five Blocks problem.

As mentioned earlier, we have given versions of the Five Blocks problem that ask for free-body diagrams. On one version, students were asked (1) to draw free-body diagrams for blocks 2 and 5 at the instant of their release and (2) to rank the magnitudes of all the vertical forces on their two diagrams. Free-body diagrams were considered to be correct if they included two forces: the gravitational force and the buoyant force. (It is also possible to draw forces by the water on the individual faces of the block, but few students did so.) A correct ranking has the weight of block 5 greater than the two buoyant forces, which are equal, and which are in turn greater than the weight of block 2.

This version has been given in two sections of the introductory course. Instruction on Archimedes' principle had been completed in one class, but not yet started in the other. The results were strikingly similar, as shown in Table 6-2. In each class, only about 25% of the students indicated that the two buoyant forces would be equal. About 50% made explicit statements indicating that the buoyant forces on the two blocks would not be equal, either by stating that one was larger or by stating that there was no buoyant force on one of the blocks. Most of these students suggested that the buoyant force on the lighter block would be greater.

Table 6-2: Student free-body diagrams for the Five Blocks problem; a comparison of the buoyant forces acting on blocks 2 and 5 at the instant they are released. One class had completed traditional instruction and the other had not yet heard lectures on Archimedes' principle, though they had completed research-based instruction on the topic of pressure in liquids.

	Introductory Washington Au '95 (N = 136)	Introductory Washington Sp '98 (N = 109)
Instruction	Standard	No instruction
Drew B on both	80%	90%
$B_2 = B_5$ (correct)	25%	25%
$B_2 > B_5$	30%	40%
$B_2 < B_5$	5%	5%
Other / no ranking	20%	20%
Drew B on 2 only	10%	<5%
Drew no buoyant forces	<5%	0%
Other/Blank	10%	~5%

All percentages rounded to the nearest 5%.

In addition to the widespread idea that the buoyant force on an object depends on the mass, many students gave answers that seemed to suggest that the buoyant force increases with depth. For example, one student wrote, "If the blocks have the same volume & different masses their densities will be different and they will float in different depths --> until the normal force = mass of block." Another student drew the sketch shown in Figure 6-5 to support the descending line answer. This sketch, in which the arrows that represent the buoyant force have increasing length with increasing depth, also strongly suggests a belief that the buoyant force depends on depth. Although the prevalence of the descending line answer suggests that many students believe the buoyant force on a completely immersed object increases with depth, most students did not articulate this belief clearly in their responses. Therefore, we developed another written problem, called the *Buoyant Force* problem, to test for this idea explicitly.

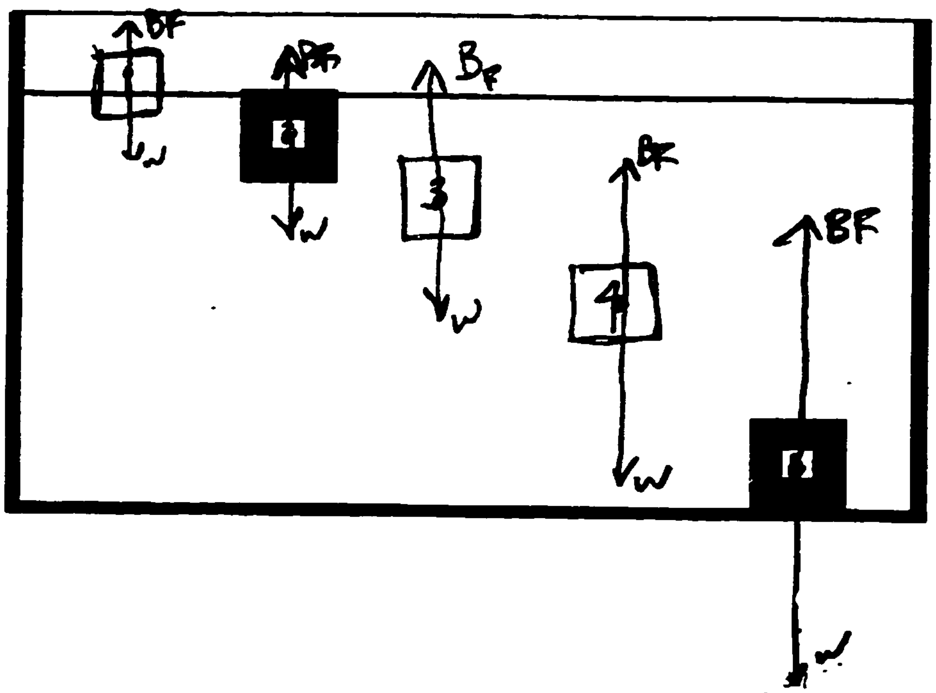


Figure 6-5: A response to the Five Blocks problem suggesting difficulties with the buoyant force.

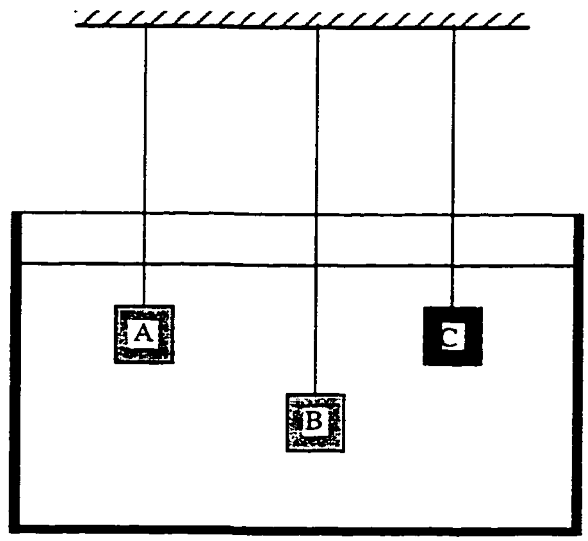
6.2.2 Buoyant Force problem

The standard definition of the buoyant force is the sum of all forces on an object by the surrounding fluid. In several contexts we have found that student beliefs are not consistent with this definition, often reflecting a lack of understanding of the relationship between the buoyant force and pressure. The belief that the buoyant force increases with depth seems to reflect this confusion. We saw evidence of another aspect of this confusion during the

Cartesian diver interviews. Some students, observing that the Cartesian diver sank after the container was squeezed, drew free-body diagrams that showed a buoyant force and an additional 'pressure force.' This 'pressure force' invariably pointed downward, and was described by some students as the weight of the water above the diver.[88] The Buoyant Force problem is also intended to gauge the prevalence of this belief as well as the tendency to relate the buoyant force to mass and/or depth.

6.2.2.1 Description

Several versions of the Buoyant Force problem have been given. The most common one is shown in Fig. 6. In this version, three blocks of equal volume are shown suspended from strings and completely submerged in a tank of water. The students are told that all the blocks have been observed to sink. There are two blocks of the same mass at different depths and two blocks of different mass at the same depth. In other versions, the blocks are less dense than water and thus are attached to strings connected to the bottom of the tank. In all cases, there is a pair of blocks of the same mass at different depth and a pair of blocks of different mass at the same depth. In Part 1 of the problem, students are asked to rank the magnitudes of the buoyant forces acting on all the blocks. The correct response is that magnitudes of the buoyant forces on the three submerged blocks are the same because they all displace the same volume of water.



Three cubical blocks of equal volume are suspended from strings. Blocks A and B have the same mass and block C has less mass. Each block is lowered into a fish tank to the depth shown in the figure below.

Rank the buoyant force acting on each block from largest to smallest. If any buoyant forces are equal indicate that explicitly. Explain.

Rank the tension in each string from largest to smallest. Explain.

Figure 6-6: The Buoyant Force problem. Students are asked to compare the buoyant forces on two blocks with the same volume and mass that are located at different depths and to compare the buoyant forces on two blocks with equal volume at the same depth that have different masses.

It is possible to rank the buoyant forces correctly in spite of a belief that an additional pressure force acts on the blocks due to the water above. Therefore, as Part 2 of the problem, students are asked to rank the tensions in the strings. Because the buoyant forces are equal, the ranking of the tensions is the same as the ranking of the weights.

6.2.2.2 Results

We have administered variations of the Buoyant Force problem in several sections of the introductory course and several sections of the second-year course. In most cases, the problem was given either as an examination question or as an ungraded quiz.[89]

In the sections of the introductory course in which the Buoyant Force problem was given after standard instruction, between 10% and 30% of the students gave a correct ranking for all three buoyant forces. A slightly higher fraction, between 15% and 35%, gave a correct ranking for the three tensions. The students in the second-year course were generally more successful in responding to these questions, with approximately half ranking the buoyant forces correctly and a third ranking the tensions correctly, but the students in this course made errors similar to those given by students in the introductory course. Below we discuss separately student responses for the ranking of the buoyant forces on the two blocks of the same mass at different depths and for the ranking of the buoyant forces on the two blocks of different mass at the same depth.

6.2.2.3 Specific difficulties identified

The specific difficulties elicited by the Buoyant Force problem are: (1) the tendency to relate the buoyant force on a submerged object to its mass, (2) the tendency to relate the buoyant force on a submerged object to its depth, (3) the incorrect belief that a force by the liquid acts in addition to the buoyant force and (4) failure to relate the buoyant force on a floating block to the volume of displaced fluid. We also found that the task of ranking the tensions elicited some responses that suggested difficulties with string tension and other mechanical concepts that seemed to be unrelated to the context of buoyancy.[90] These difficulties are not discussed here.

Tendency to relate the buoyant force on a submerged object to its mass

In their rankings, many students indicated that the buoyant forces on the blocks of different mass submerged to the same depth (blocks A and C in Fig. 6) are different. These results are summarized in Table 6-3.

Table 6-3: Student comparisons after standard instruction for the buoyant forces acting on blocks of equal volume but different mass that rest at the same depth. As shown in Figure 6-6, block A has a greater mass than block C.

	Introductory Washington Wi' 97 ($N = 201$)	Second-year Washington two sections ($N = 37$)
Correct ($B_A = B_C$)	25%	55%
$B_A > B_C$	15%	25%
$B_A > B_C$	45%	15%
Other/Blank/ Unclear	20%	<5%

All percentages rounded to the nearest 5%.

About 15% of the students in the introductory course and 20% of the students in the second-year course answered that the *less massive* block would have a greater buoyant force. Many responses suggested a failure to distinguish the buoyant force, a well-defined physical concept, from the more general term 'buoyancy.' For example, one student wrote, "All occupy the same volume, all displace same H_2O. A & B however, are more dense than C: weight less, less buoyant." Another wrote, "Buoyancy (or force of pull upward) can be looked at as the tendency for an object to be able to float. Since C has the lowest density it will float the easiest."

About 40% of the students in the introductory course and 15% of the students in the second-year course answered that the *more massive* block would have a greater buoyant force. Students giving this answer used a variety of incorrect forms of reasoning. In some cases these students cited the water displaced by the blocks. One student wrote, "the buoyant force on C is less than that of A & B - it displaces less water due to its smaller mass." Later in this paper we discuss the failure of some students to recognize that submerged objects of the same volume displace the same amount of water. Other students did not refer to displaced liquid but may have incorrectly generalized the result from the case of floating objects that the buoyant force is always equal to the weight of the block. For example, one wrote, "F_B=mg thus A & B are equal and since C has less mass, smaller F_B." Finally, some students seem to use Newton's third law incorrectly to answer this question. One such student wrote, "the buoyant force is equal and opposite to the force that the block exerts on the water. Since block C exerts a smaller force on the water ... the buoyant force also exerts less force."

Tendency to relate the buoyant force on a submerged object to its depth

In their rankings, many students indicated that the buoyant forces on the blocks depended on the depth to which they were submerged. These results are summarized in Table 6-4. Many of these students made explicit references to pressure in their reasoning. For example, a student wrote, "F_B of B is greatest because the pressure on B is the greatest." Many students simply asserted that the buoyant force on the lower block would be greater because the block was deeper in the water.

Table 6-4: Student responses to the buoyant force comparison between blocks of equal mass at different depths after traditional instruction. As shown in Figure 6-6, blocks A and B have equal mass, but block B is submerged to a greater depth.

	Introductory Washington Wi '97 ($N = 201$)	Second-year Washington two sections ($N = 37$)
Correct ($B_A = B_B$)	40%	50%
$B_B > B_A$	25%	25%
$B_A > B_B$	15%	10%
Other/Blank/ Unclear	20%	05%

All percentages rounded to the nearest 5%.

Incorrect belief that a force by the liquid acts in addition to the buoyant force

Student responses to the tension comparison question are shown in Table 6-5. In all classes the most common incorrect response, given by about 30% to 50% of the students, was to state that the tension is greater in the string supporting the lower block. Most of the students who responded in this way supported their answer with statements about the amount of water above the blocks. For example, one student wrote, "Since all buoyant forces are equal, I will ignore them. Since [block] B is the lowest, it has the most downward force on it from the water. $T_B > T_A$." These students seemingly failed to identify the buoyant force as the sum of *all* forces on an object by the surrounding fluid.

Table 6-5: Student responses to the tension comparison question, shown in Figure 6-6, after standard instruction. In each class, the most common incorrect response was that the tension in the string supporting block B was greater than that in the string supporting block A, typically due to the extra water above block B.

	Introductory Washington Wi '97 ($N = 201$)	Second-year Washington two sections ($N = 37$)
Correct ($T_A = T_B > T_C$)	25%	20%
All rankings that include $T_B > T_A$	35%	45%
Other	15%	20%
Blank	25%	15%

All percentages rounded to the nearest 5%.

Failure to relate the buoyant force on a floating object to displaced volume

Initially, the Buoyant Force problem was intended to probe student understanding of the buoyant forces on submerged objects. We limited the focus in part because it appeared that some students were generalizing the well-known result for floating objects to submerged objects, stating that the buoyant force is equal to the weight in all cases. However, informal discussions with students convinced us that some also have difficulty in determining the buoyant force on a floating object. In particular, the tendency to relate equilibrium position to the net force led some students to conclude that the buoyant force on a floating block is *greater* than that on a submerged blocks of the same size, because the floating block has come to rest at a *higher level* than the submerged blocks.[91] We decided to ask a variation of the Buoyant Force problem in which a fourth block of the same volume as the other three is shown floating. The floating block displaces a smaller volume of water and thus experiences a smaller buoyant force.

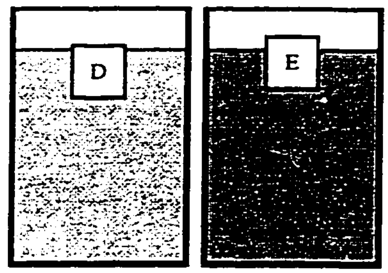
We found that many students (more than 30%) claimed that the buoyant force on the floating block is equal to that on a submerged block of the same volume. Most of these students based their answers on the volumes of the blocks and the equation for the buoyant force $F_B = \rho g V$, apparently failing to recognize that the V in this equation is the volume of liquid displaced and not the volume of the block itself. One student wrote, "the buoyant forces are equal because they have the same size, shape, mass and volume. They both will displace the same amount of water. $F_B = \rho_F g V$ => same volume same buoyant force." Other difficulties with displaced volume will be described further in Section III.

6.2.3 Floating Cubes problem

In order to probe student difficulties with the buoyant force on floating objects, we developed an additional problem. We refer to it as the *Floating Cubes* problem.

6.2.3.1 Description

In the problem, shown in Figure 6-7, two identical cubes are floating in liquids of different densities. Cube D is shown barely floating in its liquid, while cube E rests at a greater height in its liquid. Students are asked to compare the buoyant forces in the two blocks. An answer to this problem requires use of Newton's second law. Since the blocks are at rest, the net force on each must be zero. Therefore the buoyant force on each must be equal to its weight. Since the two cubes have the same weight, the buoyant forces acting on the two cubes have equal magnitude. Although students were not asked to do so, they could deduce that since the floating cubes displace the same weight of liquid (Archimedes' principle), then a smaller volume displaced must correspond to a liquid of greater density.



Two identical cubes (D and E) are placed in different liquids with different densities.

Is the buoyant force on cube D greater than, less than, or equal to the buoyant force on cube E?

Figure 6-7: The Floating Cubes problem. Students must use Newton's second law to compare the buoyant forces acting on cubes D and E.

6.2.3.2 Results

We have administered this problem on a course examination after traditional instruction in the introductory algebra-based course at the University of Maryland. In this course, hydrostatics is covered in the last two weeks of the first semester, after instruction on mechanics. The instructor for the course reported covering Archimedes' principle, albeit briefly, but felt that the question was fair. About 20% of the students answered correctly that the two buoyant forces are equal. Specific difficulties are described below.

6.2.3.3 Specific difficulties identified

The specific difficulties elicited by the Floating Cubes problem are the tendency to relate the net force on an object to its position and the failure to consider the density of the displaced liquid. Both are inconsistent with Newton's second law. In the first case, it seems that students apply an incorrect equilibrium condition. In the second case, the rote use of formulas leads to an answer inconsistent with Archimedes' principle and with Newton's second law, an inconsistency not noted by students.

Tendency to relate the net force on an object to its position

The most common incorrect answer, given by about 55% of the students, was that the buoyant force on cube E is greater than that on cube D, which floats at a lower level. In many cases, students gave reasoning based on the position of the objects, stating that the buoyant force on cube E is greater because it has been pushed farther upward than cube D. For example, one student wrote, "the weight of the cubes are the same, which means it has to be the buoyant force which is causing the difference in depth of the block. Since E is higher, it is being pushed up more which means its buoyant force is greater."

Failure to consider density of displaced liquid

About 25% of the students stated that the buoyant force on cube D is greater than that on cube E. They typically referred to the greater volume of liquid displaced by cube D, without considering the relative densities of the liquids. For example, one wrote, "we know the buoyant force on D is greater because more amount of liquid is displaced." On the Buoyant Force problem, many students claimed that the buoyant force on the floating block is the same as that on a submerged block of the same size. It seems that on the Floating Cubes problem, many students were able to identify the correct displaced volume, but failed to consider the different densities of the liquids. The rote use of formulas without an understanding of all the variables has been documented in many other contexts.[92]

6.3 IDENTIFICATION OF STUDENT DIFFICULTIES WITH CONCEPTS REQUIRED FOR STUDY OF BUOYANCY

Many of the difficulties that students had in answering the problems described above seemed to stem from lingering confusion about concepts that are widely assumed to be understood by students before the study of hydrostatics begins. As described earlier, many students failed to apply Newton's second law correctly to an equilibrium situation. Others seemed unable to relate the volume of water displaced by either fully or partially submerged objects to the objects' own volume.

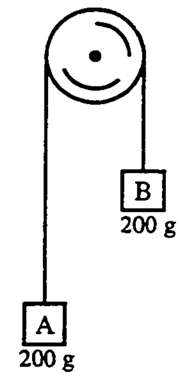
6.3.1 Difficulties with forces

In the questions on buoyancy, many students apparently failed to recognize that there is no net force acting on an object at rest. Many of these students instead used an intuitive notion of equilibrium, relating the forces on an object to its position. It was not clear whether this difficulty arose in applying force concepts in a different context from that in which they had been introduced (*i.e.*, introductory mechanics), or whether the concepts had not been mastered even in the original context. In order to investigate student understanding of the equilibrium condition, we decided to pose questions in contexts that did not involve hydrostatic forces. The questions that are referred to as the *Atwood's Machine* problem and the *Springs* problem.

The questions were given in a section of the introductory course at the University of Washington, before any instruction in hydrostatics. The students had completed the first quarter of the course, in which they studied mechanics in a traditional lecture setting. (This course has no small sections and, in particular, it does not involve the research-based curriculum *Tutorials in Introductory Physics,* which is described elsewhere.[93,94])

6.3.1.1 Atwood's Machine problem

In this problem, students are shown two identical blocks that are connected by a light string that runs over a frictionless pulley. The blocks are held in place, with one lower than the other (see Figure 6-8). The students are asked to predict what will happen to the blocks when released. To answer, they need to recognize both that the weights of the blocks are the same and that the (essentially massless) string exerts a force of the same magnitude on each. Therefore, the net force on each block is the same; given the constraint that the length of the string is fixed, the only possible result is that the blocks do not move.[95]



Two identical blocks are connected by a massless string that runs over an ideal pulley (*i.e.*, ignore friction and the mass of the pulley). Block B is initially held so that it is above block A and so that neither block can move.

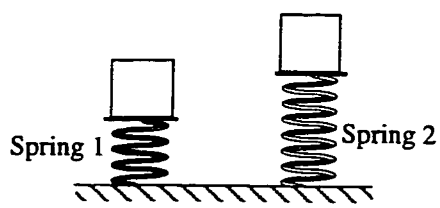
Predict the subsequent motions of blocks A and B after they are released. Explain.

Figure 6-8: The Atwood's Machine problem. This problem was used along with the Springs problem (see Figure 6-9) to assess student understanding of Newton's second law.

About 65% of the students answered correctly that the blocks would remain at rest. Most of the remaining students predicted that the blocks would move until they reach the same level. Students who made this prediction typically argued that the lower block would seek to be at the same height as the higher block: "Block A will move up (and B will move down) until they reach the same level. Having the same mass, the blocks will reach an equilibrium." Other students included references to forces but came to the same conclusion: "Block B will fall down as Block A rises until they rest at an equal height. The same forces are acting on equal objects therefore they will rest at equal positions." The reasoning given by these students, in which they referred to the tendency of blocks of the same weight to seek to be at the same height, was reminiscent of some responses to the Five Blocks problem, in which a number of students who gave the incorrect descending line answer stated that a heavier block would sink farther.[96]

6.3.1.2 Springs problem

In this problem, students are shown two identical blocks that are supported on two non-identical vertical springs (See Figure 6-9). Both blocks are at rest, with one at a greater height than the other. Students are asked to compare the forces exerted by the springs. To answer correctly, they need to recognize that since the net force on each block is zero and the weights are the same, then, by Newton's second law, the forces exerted by the springs must also be the same.



Two blocks are at rest on springs as shown. The *blocks* are identical but the *springs* are different.

Is the force on the left block by spring 1 greater than, less than, or equal to the force on the right block by spring 2? Explain.

Figure 6-9: The Springs problem. This problem was used along with the Atwood's Machine problem (see Figure 6-8) to assess student understanding of Newton's second law.

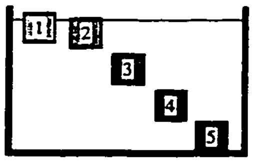
About 50% of the students answered correctly. About 25% argued that the force exerted by the spring on the higher block is greater than that on the lower block. The reasoning used by these students suggested the non-Newtonian association of net force with the displacement or position of a object. For example, one student wrote, "if blocks have the same mass and one is elevated higher into the air then it must have more upward force applied to it than the other one." Another wrote, "the masses of the blocks are the same but block 2 is higher, meaning that it is being pushed harder by the spring." Some students have been very explicit. For example, one wrote, "I would suspect that since the springs are not yielding to the ground, then the spring forces are greater than the gravitational force, and that the spring force on the right spring is greater than the one to the left." These responses are again reminiscent of some we had seen in the case of the Five Blocks problem. In that case, some students claimed that although the buoyant forces on the blocks are the same, the ones with greater weight would rest lower in the water.

About 15% of the students answered that the force exerted by the spring was greater on the lower block. Some seem to have used Hooke's law to relate the forces exerted by the two springs, despite the statement in the problem that the springs are different.

6.3.1.3 Correlation between mechanics difficulties and buoyancy difficulties

In the introductory classes in which the mechanics problems were given as ungraded quizzes, we also administered some of the buoyancy questions described earlier, including the Five Blocks problem. A summary of the responses given by students on the mechanics problems and their responses to the Five Blocks problem is shown in Figure 6-6. Perhaps unsurprisingly, students who answer both the Atwood's Machine and Springs problems incorrectly are very unlikely to answer the Five Blocks problem correctly.

Table 6-6: Responses to the mechanics problems shown in Figure 6-8 and Figure 6-9 compared to responses to Five Blocks problem.

	Responses on Atwood's and Springs problems		
	Both correct	One correct	Neither correct
	($N = 39$)	($N = 40$)	($N = 20$)
Five blocks correct	25%	25%	05%
'Descending line' 	70%	75%	95%

All percentages rounded to the nearest 5%.

To be able to interpret such results, we examined the performance of students on three course examinations. The first two exams included problems that required knowledge of mechanics, and, on average, the students who answered both mechanics problems incorrectly performed below the mean on the first two exams. The third exam covered topics from electricity and magnetism and was the least dependent on mechanics of the three exams. On that exam, the students who answered both mechanics problems incorrectly had a mean score that was less than two points below the class mean, with a standard error of the mean of approximately four points. We conclude that, on tasks that are not mechanics-dependent, the students who answered the mechanics problems incorrectly perform at a level similar to the students in the course as a whole. These students are not simply the weakest students academically.

It is not surprising that students who do not understand Newton's second law in purely mechanical contexts are not in a position to apply this law in the context of liquids. However, the prevalence of this difficulty my be surprising to some instructors. It is also significant that some of the students who can apply Newton's laws correctly in purely mechanical contexts do not seem able to apply them in situations involving hydrostatic forces. The responses to the Buoyant Force problem in particular suggest that the forces encountered in the study of hydrostatics seem to give students additional difficulty. These results have implications for instruction in introductory mechanics as well as hydrostatics.

6.3.2 Difficulties with volume

In addition to the difficulties with mechanics described above, some students seemed to have difficulties with volume, a concept that is generally regarded as a prerequisite for college physics.

6.3.2.1 Responses from interviews

The students who incorrectly answered the Five Blocks interview task all discussed displaced liquid in their responses. In each case, relating the amount of water displaced by a submerged object to the volume of the object was not a trivial step. One student's difficulty is illustrated by the following exchange:

I How did you decide for each of the blocks the amount of water displaced?

S By their mass. If one has a greater mass than another, then it's going to have greater density... greater volume? No, just greater density; these all have the same volume.

...

I So which property of any of the blocks are you looking at when you are deciding about the displaced water?

S Density.

Not surprisingly, this student concluded that the buoyant force on block 4 would be greater than the buoyant force on block 3, which has an equal volume but smaller mass. Another student said, "I get confused when I'm thinking about the volume and the mass. So I can't think of the displacement as a volume, I have to think of it as a mass."

The results of the interviews suggested that the idea of displaced liquid is more difficult for college students than many instructors assume.[97] We decided to ask a written question, called the *Displaced Water* problem, to try to determine the prevalence of this difficulty.

6.3.2.2 Displaced Water problem

The Displaced Water problem is shown in Fig. 9. Students are told that two blocks of the same size and shape are lowered into graduated cylinders with identical initial water levels. One block is aluminum; the other, brass. The aluminum block is lowered farther than the brass block. The final water level in the cylinder containing the aluminum block is shown. Students are asked to state how the final water levels in the two cylinders will compare and to sketch the water level in the cylinder containing the brass block. A second question asks students to compare the volume of water displaced by the aluminum block to the volume of water displaced by the brass block. The correct answers are that the final water levels are the same and that both blocks displace the same volume of water. We asked these two seemingly identical questions in part because some students in interviews had failed to relate the volume of water displaced and the change in water level.

An aluminum block, block A, and a brass block, block B, are attached to strings and lowered into identical graduated cylinders. The blocks are the same size and shape, but block B has a greater mass. After block A is lowered into the cylinder, the water level is as shown. Block B is lowered to the level shown at right.

The initial water levels in the cylinders are the same.

Is the water level in the cylinder containing block B higher than, lower than, or the same height as the water level in the cylinder containing block A? Explain.

Is the volume of water displaced by block A greater than, less than, or equal to the volume of water displaced by block B? Explain.

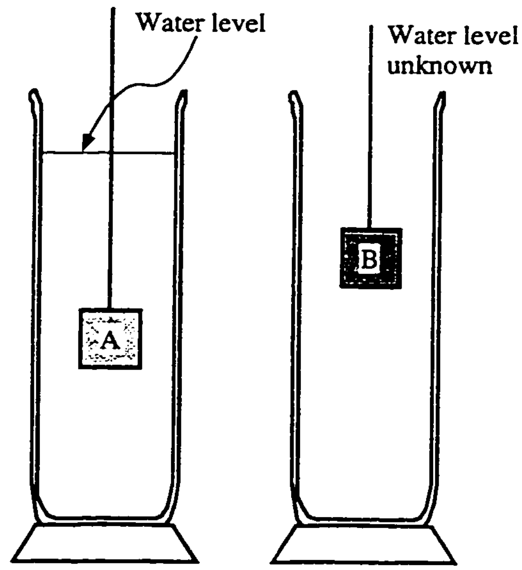


Figure 6-10:The Displaced Water problem.

The problem was administered in a section of the introductory course at the University of Washington. Instruction on the relationship between pressure and depth had been completed but buoyancy had not yet been introduced.

Just over two thirds of the students correctly stated that the water levels would rise the same amount in the two cylinders. About 20% stated that the water level would rise more in the cylinder containing the heavier block. A small number of students (less than 5% of the class) answered that the water level would rise more in the cylinder containing the lighter block, typically referring to the greater depth to which this block was lowered. In addition, five percent of the class either responded that they could not tell which water level would rise more or left this problem blank while moving on to others on the same quiz.

Students who answered incorrectly revealed their confusion among the concepts of mass, volume, and density. Some argued on the basis of mass: "since B has a higher mass, the volume of water displaced will be more, so the water level will be higher."

Others focused on density to come to the same conclusion: "volume displaced by A [aluminum] is less than volume displaced by B [brass] because water displacement is proportional to the object's density." Some students used the term 'amount' without specifying whether it referred to mass or volume: "[the brass will] be higher because [it] is heavier and the amount of water displaced is equal to B's weight."[98]

The second part of the Displaced Water problem asks explicitly about the volume of liquid displaced. Between 5% and 10% of the class gave contradictory answers to these two questions. For example, one student predicted a higher water level in the cylinder containing the heavier block "because block B displaces more water because it is heavier" but answered correctly for the displaced volume question, arguing "it is equal to, because both have the same volume."

6.4 CONCLUSION

The results reported in this paper suggest that many students do not develop a functional understanding of buoyancy after standard instruction. Serious conceptual difficulties with Archimedes' principle are both prevalent and persistent. For example, many students believe that objects of different densities can remain at rest at different depths beneath the surface of a liquid. Our results suggest that standard instruction does not affect the prevalence of this belief. Students also have difficulties with the buoyant force. The majority of students are unable to relate the magnitude of the buoyant force on a completely submerged object to the volume of the object. Instead, many students believe that the buoyant force depends on the depth to which an object is submerged or that it depends on the mass of the object. Many students also fail to understand that the buoyant force is the sum of the forces on an object by the surrounding liquid.

Our results suggest that many of these difficulties can be traced to difficulties with mechanics and with other concepts required for an understanding of buoyancy. Students have specific difficulties with Newton's second law that are particularly evident in the context of buoyancy, including the incorrect belief that the force on an object is related to the position or displacement of the object. Other prerequisite ideas, including the operational definition of volume and the ability to distinguish mass from volume, are difficult for a significant number of students. Not surprisingly, students who have such difficulties perform very poorly on the buoyancy problems that we have posed. However, the even among students who demonstrate an understanding of mechanics in contexts that do not involve fluids, many have difficulty applying these concepts to problems involving hydrostatics.

We have begun to develop instructional materials based on the results of this research. Preliminary results suggest that the materials are able to address many of these conceptual difficulties. The ongoing development and assessment of these materials will be reported in future articles by our group.

7. ADDRESSING STUDENT DIFFICULTIES WITH BUOYANCY

7.1 INTRODUCTION

Based on the research described in Chapter 6, we have developed a tutorial sequence designed to address student difficulties with Archimedes' principle, the buoyant force, and the application of Newton's Second Law to an object in a liquid. We have tested this curriculum with students in several different courses and assessed student performance on post-tests. Based on the post-test results, we have revised the curriculum. This cycle of iterative development reflects the Physics Education Group's model for research-based curriculum development. In this chapter, we describe the criteria for student understanding, the instructional strategies used in the curriculum, and our efforts to assess the effectiveness of the instructional materials.

7.2 CRITERIA FOR STUDENT UNDERSTANDING

In order to assess the effectiveness of the tutorial sequence, we have posed written post-tests. Most of these post-tests have previously been described in Chapter 6 and Appendix G. Here we briefly summarize the criteria that we have used for correct answers and reasoning and describe student difficulties that the curriculum is designed to address.

7.2.1 Student ability to rank the buoyant forces acting on completely submerged blocks

Much of the research that we have done has focused on the ability of students to rank the buoyant forces on completely submerged blocks. For example, in the Buoyant Force problem shown in Figure 7-1, students are shown three completely submerged blocks in a tank of water and asked either to rank the buoyant forces acting on the blocks or to compare the buoyant forces on pairs of blocks. As shown in the figure, the blocks that students are asked to consider have the same volume but different masses and positions.

The blocks are typically attached to strings that either extend out of the water tank or connect to the bottom of the tank.

7.2.1.1 *Criteria for correct answers and reasoning*

Students should be able to recognize that, by Archimedes' principle, the buoyant force on a block depends on the weight of liquid displaced by the block. Since each of the blocks in Figure 7-1 is completely submerged, each displaces a volume of water equal to its own volume. Thus the buoyant forces are equal. Our criterion for correct reasoning requires a reference to the volume of liquid displaced by the blocks or to the volumes of the blocks.

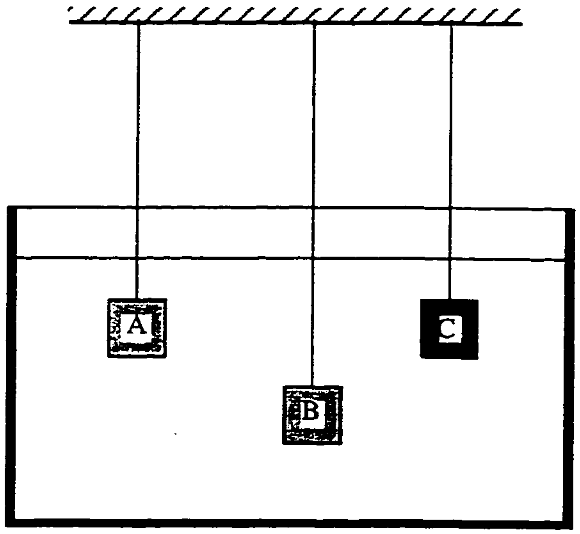
7.2.1.2 *Incorrect reasoning used by students*

We found several forms of incorrect reasoning that were commonly used by students. We will give brief descriptions of each; more complete descriptions and examples of incorrect responses are given in Chapter 6. Very few students gave completely correct answers with incorrect reasoning. Some students gave partly correct answers, revealing incorrect reasoning. For example, some students correctly stated that the buoyant forces on blocks A and B in Figure 7-1 are equal, but referred to the mass of the blocks rather than the volume in their reasoning. In nearly all cases, these students incorrectly compared the buoyant force on block C to those on A and B. Some students answered similarly for depth, correctly comparing the buoyant forces on A and C but then stating that B has a greater buoyant force.

Buoyant force increases with depth. A common response given by students was that the buoyant force on a submerged object will increase with increasing depth. Many of these students reasoned based on the fact that pressure increases with depth, suggesting a failure to recognize that the buoyant force is the sum of the forces on an object by the surrounding liquid. A handful of students reasoned that the lower block would displace a greater mass of water, based either on the belief that the density of the liquid increases with depth or the belief that the volume displaced liquid is related to the distance beneath the surface.

Buoyant force increases with mass. Many students, in comparing submerged blocks of the same volume, stated that the buoyant force on the more massive block is greater. In some cases this response was based on the incorrect idea that the more massive block displaces a greater volume of liquid. In other cases, students incorrectly applied the formula $B = \rho g V$ for the magnitude of the buoyant force, using the density of the block rather than the liquid.

Less dense objects are 'more buoyant.' Some students answered based on an intuitive notion of the buoyant force. In comparing two submerged blocks of the same volume, they argued that the less dense block is 'more buoyant' and thus is acted on by a buoyant force of greater magnitude.



Three cubical blocks of equal volume are suspended from strings. Blocks A and B have the same mass and block C has less mass. Each block is lowered into a fish tank to the depth shown in the figure below.

Rank the buoyant force acting on each block from largest to smallest. If any buoyant forces are equal indicate that explicitly. Explain.

Rank the tension in each string from largest to smallest. Explain.

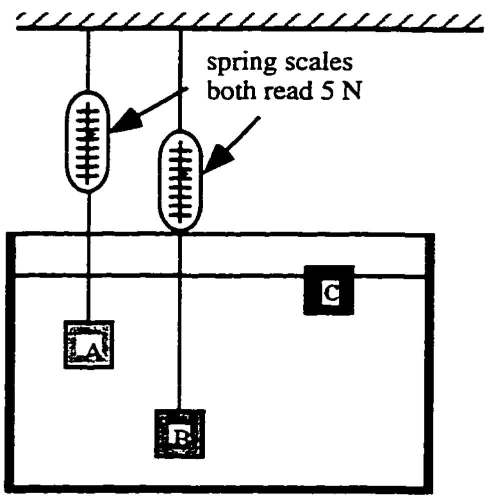
Figure 7-1: One version of the Buoyant Force problem. Students are asked to compare the buoyant forces on blocks with the same volume and mass that are located at different depths and to compare the buoyant forces on blocks with equal volume at the same depth that have different masses.

7.2.2 Student ability to relate the buoyant force on a floating block to displaced volume

We have asked students to compare the buoyant force on a floating block to that on a submerged block of the same volume. See, for example, the Spring Scales problem shown in Figure 7-2. The floating block has the same volume as others in the problem, but is only partly submerged.

7.2.2.1 Criteria for correct answers and reasoning

Students are expected to use Archimedes' principle to relate the magnitude of the buoyant force to the weight of the liquid displaced. For example, in the Spring Scales problem shown in Figure 7-2, the buoyant force on block C is less than that on block A, because block A displaces a greater volume of water. Our criteria for correct reasoning is that a response include either a reference to the volume of liquid displaced by the object or a statement that the object is only partly submerged.



Three blocks of the same size and shape are placed in a tank of water as shown below. The masses of the blocks are unknown. Blocks A and B are suspended from springs attached to spring scales. Both spring scales read 5 N. Block C is floating as shown.

Is the magnitude of the buoyant force on block B greater than, less than, or equal to the magnitude of the buoyant force on block A? Explain.

Is the magnitude of the buoyant force on block C greater than, less than, or equal to the magnitude of the buoyant force on block B? Explain.

Is the mass of block A greater than, less than, or equal to the mass of block B? Explain.

Figure 7-2:The Spring Scales problem with a floating block.

7.2.2.2 Incorrect reasoning used by students

Most of the students who answered this problem correctly used correct reasoning. The reasoning given by students in support of incorrect responses, as well as incorrect reasoning given by some students in support of the correct response, is described briefly below. For a more detailed description of these errors, see Chapter 6.

Buoyant force depends on object mass rather than displaced volume. Some students made statements about the mass of the objects to justify their answers for the buoyant force comparison. For example, some students argued that the buoyant force on the floating block (block C) in Figure 7-2 is greater than those on the submerged blocks because the floating block has a smaller mass. Other students gave the correct answer, but used incorrect reasoning based on the mass. These students inferred correctly that the mass of the floating block in Figure 7-2 is less than the masses of the other blocks, but then stated that the buoyant force on the floating block is smaller due to its smaller mass. These students are not considered to have answered correctly with correct reasoning.

Buoyant force depends on object volume rather than displaced volume. Many students stated that the buoyant force on the floating block in Figure 7-2 is equal to the buoyant forces on the other blocks because the blocks all have the same volume. In some cases these students referred to the equation for the magnitude of the buoyant force, $B = \rho g V$, but seemingly failed to recognize that the relevant volume in this equation is the volume of liquid displaced rather than the object's volume.

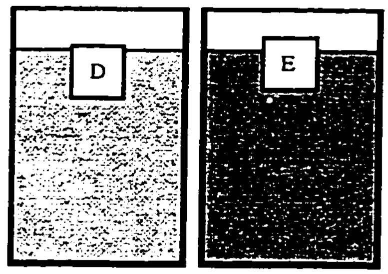
Buoyant force related to object's position. Many students stated that the buoyant force on the floating block (block C) in Figure 7-2 is greater than those on the submerged blocks. In many cases these students gave reasoning based on the position of the objects, stating that the buoyant force on the floating block is greater because the floating block has been pushed upward farther than the submerged block.

7.2.3 Student ability to apply the equilibrium condition to a block in a liquid

The final category of tasks involve problems in which a correct solution requires identifying the forces acting on an object and applying Newton's second law. For example, in the Floating Cubes problem shown in Figure 7-3, students are asked to compare the buoyant force acting on cube D to that acting on cube E. The tension task in the Buoyant Force problem (Figure 7-1) and the mass question in the Spring Scales problem (Figure 7-2) also test this understanding.

7.2.3.1 Criteria for correct answers and reasoning

An answer to the Floating Cubes problem requires use of Newton's second law. Since the two cubes are at rest, the net force on each must be zero, so the buoyant force must be equal to the weight force. The cubes are identical, so their weight forces are equal. Therefore the buoyant forces acting on the two cubes have equal magnitude. By Archimedes' principle, the cubes displace the same weight of water. (Since a smaller volume of liquid is displaced by cube E, the density of the liquid in the right container must be greater than that in the left container.)



Two identical cubes (D and E) are placed in different liquids with different densities.

Is the buoyant force on cube D greater than, less than, or equal to the buoyant force on cube E?

Figure 7-3: The Floating Cubes problem. Students must use Newton's second law to compare the buoyant forces acting on cubes D and E.

7.2.3.2 Incorrect reasoning used by students

Nearly all of the students who answered these problems correctly gave correct reasoning. The reasoning given by students in support of incorrect responses, as well as the reasoning given in support of the correct response that we classified as incorrect, is described briefly below. For a more detailed description of these errors, see Chapter 6.

Buoyant force related to object's position. Many students stated that the buoyant force on cube E in Figure 7-3 is greater than that on cube D. In many cases these students gave reasoning that seemed to be based on the position of the objects, stating that the buoyant force on cube E is greater because it has been pushed farther upward than cube D.

Buoyant force related to volume rather than mass of displaced liquid. Students who stated that the buoyant force on cube D in Figure 7-3 is greater than that on cube E typically referred to the greater volume of liquid displaced by cube D, without considering the relative densities of the liquids.

7.3 DEVELOPMENT AND ASSESSMENT OF *BUOYANCY*

We developed a tutorial sequence titled *Buoyancy* that is designed to address the difficulties that we have described. In this section we describe the development and assessment of one version of this tutorial sequence, which we will call *the initial version of Buoyancy.*[99]

7.3.1 Overview of instructional approach

In analyzing student responses to the Cartesian diver interviews described in Appendix F and the problems described in Chapter 6, we found that many students had difficulty in distinguishing the buoyant force from the related idea of hydrostatic pressure. We also found that many students predicted that objects of the same volume but different mass would come to rest at different equilibrium positions beneath the surface of an incompressible liquid, as shown in the responses to the Five Blocks problem. These responses suggested a need for instruction designed to help students connect the buoyant force to pressure as well as to the motion of an object in a liquid. We therefore developed a tutorial sequence intended to help students to apply Newtonian mechanics to predict the behavior of an object in a liquid. In addition, our experience in working with students was that many did not understand the connection between the forces by the water on the six surfaces of the cube and the single buoyant force described in Archimedes' principle. Therefore, we included exercises designed to make this connection.

7.3.2 Description of tutorial sequence: initial version of *Buoyancy*

The sequence of the initial version of *Buoyancy* reflects the research described in Chapter 6. The activities and exercises are intended to address the common student difficulties described there and help students to connect buoyant force and pressure.

7.3.2.1 Pretest

The pretest for the initial version of *Buoyancy* includes a written version of the Five Blocks problem (see Chapter 6). In addition to drawing the final positions of the blocks, students are asked to draw free-body diagrams for a floating block and a sinking block at the instant they are released underwater.

7.3.2.2 Instructional sequence

The instructional sequence in the initial version of *Buoyancy* is based on the observation that many students have difficulty in relating the buoyant force acting on an object to the pressure. Therefore, we asked students to determine the magnitude and direction of the buoyant force in a two-step process. First, students identify contact forces on the block by the surrounding water and relate the magnitude of each force to the pressure at a point on the appropriate surface of the object. Second, students construct the vector sum of the forces on the block by the surrounding water.

Floating block released underwater. The initial situation that students are asked to consider concerns a cubical block. The tutorial states that the block has been observed to float in water. Students imagine that the block is held in the center of a beaker and released. They are asked to predict the subsequent motion of the block and then draw a free-body diagram for the block. In this version of *Buoyancy,* students were given oral directions to draw the contact forces acting on each of the six faces of the cube rather than a single buoyant force acting on the cube. After drawing the free-body diagram, students are asked to compare the magnitudes of the forces they have drawn based on the relationship between pressure and depth in the liquid. They then check whether these magnitudes are consistent with the motion of the block.

Floating block in equilibrium position. In the next section of the worksheet, students consider the floating block in its equilibrium position. They draw a free-body diagram for the block and check that it is consistent with the motion of the block. After doing so, students are asked to compare the free-body diagrams for the block in its initial and final positions, and to compare the upward force on the block in the two cases. Finally, students imagine that a small weight is added to the block (the text directs student to consider the weight and block as a single system). Students compare the level at which the block floats to the level in the previous situation.

Sinking block. Students are next asked to consider a block that is of greater mass than the floating block but has the same size and shape. Students draw a free-body diagram for this block at the instant it is released in the center of the beaker. Students are asked explicitly to compare the forces diagram to the one drawn for the less massive block. We expected that this step would help students to recognize that the buoyant forces on the two blocks are the same, but that the different motions of the blocks result from the different relationships of the weight and buoyant force.

Buoyant force. At this point, the term buoyant force is introduced and defined as the sum of the forces on the block by the water. Students are asked whether the forces on the top and bottom surfaces would change and whether the buoyant force would change if the block were released from a greater depth.

7.3.2.3 Homework

The constraints of the course were such that no homework exercises were included in this version of the *Buoyancy* tutorial sequence.

7.3.3 Assessing student understanding after initial version of *Buoyancy*

In this section, we examine student responses on post-tests. After using the interactive tutorial lecture *Buoyancy*, a version of the Buoyant Force problem was posed on a course examination in order to assess the success of the instructional strategies. We have compared student responses on this problem to those on similar problems in other courses after standard instruction.

7.3.3.1 Populations in the study

We have posed problems in several different sections of Physics 115, the algebra-based introductory physics course at the University of Washington, and one section of Physics

121, the corresponding course at the University of Maryland. We have evidence to suggest that these large lecture courses do not vary significantly in student ability from quarter to quarter. In addition, we have seen that the students in the course at the University of Maryland have performed at a very similar level to the students in the course at the University of Washington.[100] Therefore, we believe that the performance of these students is a good measure for assessing the effectiveness of the curriculum.

One section of Physics 115 at the University of Washington completed a series of interactive tutorial lectures (ITLs) that included the initial version of *Buoyancy*. (For a description of instructional strategies used in ITLs, see the Introduction to the dissertation.) In this section, we asked a version of the Buoyant Force problem on the final examination for the course. The student responses on this problem will be compared to those previously shown from other sections of Physics 115 and the section of Physics 121 at Maryland after standard instruction.

7.3.3.2 Student responses to buoyant force comparison for submerged blocks

Student responses to the buoyant force comparisons after the initial version of *Buoyancy* are shown in Table 7-1. A majority of the students correctly ranked all the buoyant forces after the ITL, as compared to fewer than a third after standard instruction. In addition, the fraction of students giving answers consistent with the belief that the buoyant force on a completely submerged object depends on depth is significantly less in the class that had completed *Buoyancy*. However, nearly half of the class still gave answers consistent with the incorrect belief that the buoyant force on a completely submerged object depends on the mass of the object. The emphasis on the buoyant force as the sum of the forces on the block by the surrounding water seems to have made a difference. However, these results suggest a need for revision of the instructional strategies in order to address persistent student difficulties with the buoyant forces on blocks with different masses.

Table 7-1: Summary of buoyant force comparisons for several algebra-based introductory courses. Since some students gave answers in which the buoyant force depends both on mass and on depth, the sum of the rows is greater than 100% for each class

	Physics 115 Washington Wi'97 ($N = 201$)	Phys 121 Maryland Sp'98 ($N = 126$)	Physics 115 Washington Sp'97 ($N = 201$)
Instruction:	Standard	Standard	*Buoyancy*
Correct (all equal)	10%	30%	55%
Incorrect for blocks of different mass	60%	60%	40%
Incorrect for blocks at different depth	40%	35%	15%

All percentages rounded to the nearest 5%.

7.4 DEVELOPMENT AND ASSESSMENT OF REVISED VERSION OF *BUOYANCY*

In responses to the Buoyant Force problem after the initial version of *Buoyancy*, many students still incorrectly predicted that the buoyant forces on the two submerged blocks with equal volume but different masses would be different. Some students in their answers made incorrect statements about the mass or volume of water displaced, suggesting that this concept was still a source of difficulty. In particular, some students gave answers consistent with the belief that the heavier block displaced a greater mass or volume of water (see also Section X of Chapter 6). Several students had made similar mistakes in the Five Blocks interviews described in Chapter 6. Therefore, we added exercises to the tutorial designed to elicit and confront this difficulty.

7.4.1 Description of modifications in revised version of *Buoyancy*

The structure of the tutorial sequence in the revised version of Buoyancy is quite similar to that in the initial version. We have made several modifications, which are detailed below.

7.4.1.1 Modifications to pretest

Since some students seemed to have difficulty with the idea of displaced water, finding that the water displaced by a completely submerged object depends on the mass rather than volume of the object, we developed a written problem to elicit this difficulty on the pretest for the revised version of *Buoyancy.* In this problem, the Displaced Water problem, two blocks of equal volumes but different masses are lowered to different depths in separate graduated cylinders of liquid. Students are asked to compare the amount by which the liquid levels increase in the two containers and the volume of liquid displaced by the two blocks. Students' responses to this problem are described in Chapter 6.

7.4.1.2 Modifications to instructional sequence

We made several additional changes in preparing the revised version of *Buoyancy.* These changes were based on the results from the post-test problem and our experiences in working with students in the earlier versions of *Buoyancy.*

Modifications to analysis of initial free-body diagram. The improvement in student ability to rank the buoyant forces on the blocks lowered to different depths suggested that the approach in the initial version was successful in addressing the distinction between pressure and buoyant force. Therefore, we formalized this approach. In the written instructions accompanying the initial free-body diagrams, students are asked to draw the contact forces acting on the six surfaces of the cube rather than a single buoyant force. After students rank the magnitudes of these contact forces, using the relationship between pressure and depth, they construct the vector sum of the forces on the block by the surrounding water. Students compare this vector to the net force on the block and to the weight of the block. Our experience in earlier versions had been that many students confused the buoyant force with the net force, and that others incorrectly generalized from floating blocks that the magnitude of the buoyant force on an object is always equal to the weight of the object.

Modification to sequence of activities. Since students still seemed to have difficulty in comparing the buoyant forces acting on two completely submerged blocks of equal volume but different mass, we deleted entirely the section of the tutorial in which students consider the floating block in its equilibrium position. Now, immediately after considering the floating block, students apply a similar analysis to a block of the same volume that sinks. Students draw a free-body diagram for this block at the instant of its release and again draw a vector representing the sum of the forces on the block by the surrounding water. In an effort to confront the incorrect belief that the buoyant force depends on the mass of a completely submerged object, we added questions that ask students to compare the sum of the forces by the surrounding water on the sinking block to the corresponding sum for the floating block.

Definition of buoyant force. As in the previous version, students imagine that the sinking block is released from a greater depth. In the revised version, students construct the sum of the forces on the block by the water. A short statement then defines the buoyant force as the sum of the forces on an object by the surrounding fluid(s). Students are asked to generalize their answers and state whether the buoyant force on a completely submerged object depends on the mass of the object or the depth at which the object is located.

Displaced liquid and Archimedes' principle. At this point the tutorial attempts to elicit student difficulties in determining the volume of displaced water. The tutorial states that a block of aluminum causes an increase in volume reading of 3 cm^3 when dropped into a graduated cylinder. Students are then asked to predict the increase in volume reading when a block of brass with the same volume but greater mass is dropped into a second graduated cylinder. In the ITL version of *Buoyancy,* a demonstration is then performed. In tutorials, one or two sets of equipment for testing this prediction are placed in the tutorial room, so that students can test their prediction and confront and resolve any incorrect ideas. This task is followed by a statement of Archimedes' principle. Finally, students respond to a student statement articulating the incorrect idea that the buoyant force on an object is always equal to the weight of the object.

7.4.1.3 Homework

Unlike the previous versions of *Buoyancy,* the revised version does include associated homework exercises. The two exercises that we have used are described briefly.

Density problem. In this exercise, students compare three blocks of equal volume: one that floats, one that sinks, and one that is at rest underwater. Students apply Archimedes' principle and Newton's second law to compare the mass, volume, and density of the three objects to the mass, volume, and density of the water displaced by the objects.

Tension problem. In this exercise, students are shown two objects of irregular cross-section. The objects have the same volume but different shapes (see Figure 7-4). Students are asked to respond to three student statements, only one of which is correct. The incorrect statements articulate incorrect responses that are commonly given by students. One states that the buoyant forces on the two objects are the same but that an additional force due to the weight of the liquid above is greater on one object due to its larger cross-sectional area on top. The other incorrect statement refers to the fact that the second object extends farther down into the liquid, stating that the buoyant force is greater on this object due to the greater pressure at the bottom of the object.

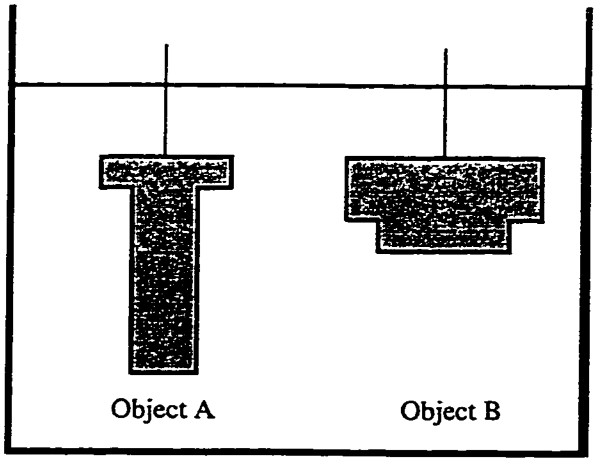


Figure 7-4: The tension problem from the homework accompanying the revised version of *Buoyancy*.

7.4.2 Assessing student understanding after the revised version of *Buoyancy*

As with the previous versions of *Buoyancy*, the revised version has been tested as part of a series of ITLs in the Physics 115 course at the University of Washington. We have assessed student performance on post-test questions after the ITLs in an effort to determine whether the instructional approach successfully addresses student difficulties. For this version of *Buoyancy*, we have used several problems to probe student understanding.

7.4.2.1 Populations in the study

Two sections of Physics 115 at the University of Washington have completed the revised version of *Buoyancy*. For one section, we used a version of the Buoyant Force problem on a course midterm examination. The student responses on this problem will be compared to those previously shown from other sections of Physics 115 and the section of Physics 121 at Maryland.

7.4.2.2 Student responses to buoyant force rankings for submerged blocks

Student responses to the Buoyant Force problem after the revised version of *Buoyancy* are shown in Table 7-2, along with student responses after standard instruction, and after the first version of *Buoyancy.* More than two-thirds of the students answered correctly. In addition, fewer than 20% of the students give answers consistent with the common incorrect beliefs that the buoyant force depends on the mass or depth. The success of students is dramatically improved after the revised version of Buoyancy. In particular, the exercises designed to address the common incorrect idea that the buoyant force depends on the mass of a completely submerged object seem to have been quite successful.

Table 7-2: Summary of responses on the Buoyant Force problem (see Figure 7-1) after standard instruction and after initial and revised versions of *Buoyancy.* Since some students gave answers in which the buoyant force depends both on mass and on depth, the sum of the rows is greater than 100% for each class.

	Physics 115 Washington Wi'97 ($N = 201$)	Phys 121 Maryland Sp'98 ($N = 126$)	Physics 115 Washington Sp'97 ($N = 201$)	Physics 115 Washington Sp'98 ($N = 130$)
	Standard	Standard	*Buoyancy* init.	*Buoyancy* rev.
Correct (all equal)	10%	30%	55%	70%
Incorrect for blocks of different mass	60%	60%	40%	20%
Incorrect for blocks at different depth	40%	35%	15%	15%

All percentages rounded to the nearest 5%.

7.4.2.3 Student responses to buoyant force rankings including floating blocks

A second test of student understanding of the buoyant force involves buoyant force comparisons between submerged and partially submerged blocks of the same volume. We have asked students to make these comparisons on problems including the Spring Scales problem and the Buoyant Force problem (see Figure 7-2 and Figure 7-1, respectively). Responses given by students after standard instruction at the University of

Maryland and after the revised version of the *Buoyancy* ITL at the University of Washington are shown in Table 7-3.

These results show a small improvement in the frequency of correct answers, and a significant decrease in the frequency of the most common incorrect answer given by students who had completed standard instruction. The most common incorrect answer given by students after the revised version of Buoyancy is that the buoyant forces on the two blocks are the same because the blocks have the same volume. This difficulty was less common after standard instruction. It is possible that the revised version of *Buoyancy,* with its increased emphasis on submerged objects, has increased the likelihood of students concluding that the buoyant force on an object depends only on the volume of the object rather than on the volume of liquid displaced.

Table 7-3: Student responses to buoyant force comparisons for submerged (block A) and partially submerged (block C) blocks of the same volume (see Figure 7-2). Results are shown after standard instruction and after the revised version of *Buoyancy.*

	Phys 121 Maryland Au'98 (N = 161)	Physics 115 Washington Sp'98 (N = 130)
Instruction:	Standard	*Buoyancy* rev.
Correct ($B_C < B_A$)	50%	65%
Incorrect ($B_C > B_A$)	30%	5%
Other	15%	30%

All percentages rounded to the nearest 5%.

7.5 DISCUSSION

The results of research have been used to develop a tutorial sequence on buoyancy and Archimedes' principle. This sequence is designed to address the most common student difficulties described in Chapter 6. As with all curriculum produced by the Physics Education Group, *Buoyancy* has undergone a series of revisions based on analysis of student responses to post-test problems after the tutorial. In particular, the initial version of *Buoyancy* was largely unsuccessful in addressing student difficulties with the dependence of the buoyant force on the mass of a completely submerged object. This failure led us to include a revised instructional sequence designed to confront the common difficulty that the amount of liquid displaced by a completely submerged object depends on the mass of the object rather than the volume.

We have found that student performance after completing the revised version of *Buoyancy* is significantly better than that after standard instruction on a variety of qualitative problems. In particular, students seem to be much more successful in applying Archimedes' principle to completely submerged objects, and somewhat more successful in applying it to partially submerged objects. This success illustrates the improvements that are possible when the development of curriculum is guided by the results of research. However, there is some reason to believe that the revised emphasis on submerged blocks may have led a few students to overgeneralize the result that the buoyant force on an object depends on the volume of the object. We are continuing to assess student understanding and will continue to develop this tutorial sequence based on our results. In addition, we are examining the possibility of modifying instruction in the laboratory associated with Physics 115 in order to address this persistent difficulty.

PART II: IDENTIFYING AND ADDRESSING STUDENT DIFFICULTIES WITH WORK AND THE FIRST LAW OF THERMODYNAMICS

INTRODUCTION

In Part II of this dissertation, we describe an investigation of student understanding of the first law of thermodynamics as applied to ideal gas systems.

WHAT STUDENTS ARE TYPICALLY TAUGHT

In thermal physics, the first law of thermodynamics is perhaps the most important and fundamental physical law. For a system whose particle number does not vary, it is commonly written

$$\Delta E_{int} = Q_{to} + W_{on}$$

i.e., the change in internal energy of a system is equal to the heat transfer to the system plus the work done on the system.[101] Both the work done on the system and the heat transfer to the system are scalar quantities; a positive sign corresponds to energy transfer to the system in question. The sign of the work done on one object by a second can be determined by applying the definition of work

$W =$ **Error!**

where $\vec{F}$ is the force applied and $d(s,\vec{\ })$is a differential displacement of the point of application of the force.

In the courses discussed in this study, students are expected to apply the first law to ideal gas processes. For an ideal gas, the internal energy of the gas is given by

$$E_{int} = N \frac{n_{dof}}{2} k_B T$$

where N is the number of gas particles, n_{dof} is the number of degrees of freedom, k_B is the Boltzmann constant, and T is the temperature of the gas. Thus, students are able to make predictions about the temperature of an ideal gas based on the first law; if the internal energy of the gas increases, the temperature does as well.

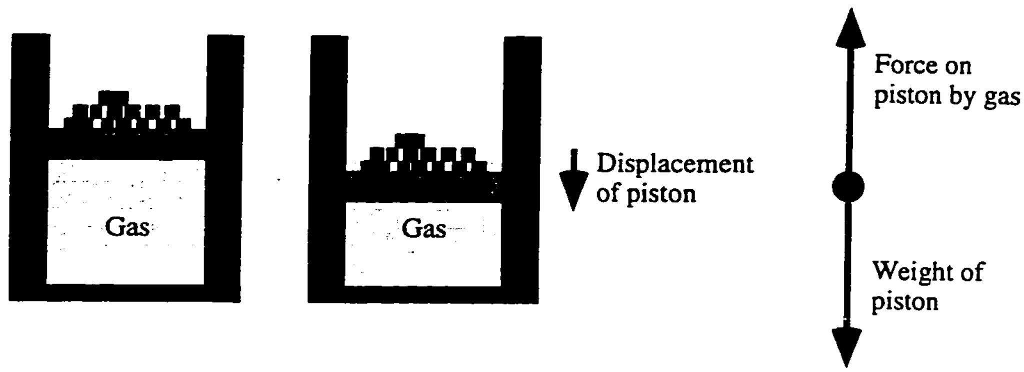


Figure II-5: The work done in an ideal gas process can be related to the force on the piston by the gas and the displacement of the piston.

In the first law of thermodynamics, the change in internal energy is equal to the sum of the work done on the gas and the heat transfer to the gas. The internal energy of the gas is a state function, and that the change in internal energy depends only on the initial and final states of the gas rather than the specific details of the process in question. Both the work done and the heat transfer do, however, depend on the process.

Students are expected to be able to determine the sign and absolute value of the work done on a gas in a process. In particular, for an ideal gas process, the sign of the work can be found by applying the definition of work. Consider, for example, a gas contained in a vertical cylinder closed by a massive piston . (See Figure II-5.) A free-body diagram for the piston shows that there must be a force on the piston by the gas. By Newton's third law, there is also a downward force on the gas by the piston. In a process in which the piston moves, the sign of the work can be determined by comparing the direction of the displacement vector for the point of application of the force to the direction of the force and applying the definition of work:

$$W = \int \vec{F} \bullet \vec{ds}$$

In the compression, the force on the gas points downward and the displacement of the point of application of the force (i.e., the piston surface) also points downward. These vectors are parallel, so the dot product is positive and the work on the gas is positive. [102]

To determine the absolute value of the work, the force on the gas can be related to the pressure and the displacement to the change in volume of the gas. Assume that the piston closing the cylinder in Figure II-5 has area *A*. Relating the force on the piston by the gas to the pressure of the gas, and the displacement of the point of application to the differential change in the height of the piston *dx* (where positive *x* is assumed to be upward) yields the following expression:

$$W = -\int PA\, dx$$

Noting that the product of the area and the differential change in the height of the piston *dx* is equal to the differential change in volume *dV*, the work can be rewritten as

$$W = -\int_{V_i}^{V_f} P\, dV$$

If the process is represented on a *PV* diagram (*i.e,* a graph of pressure vs. volume for the process), the last equation shows that the absolute value of the work done is equal to the area under the curve representing the process.

PREVIOUS RESEARCH ON STUDENT UNDERSTANDING OF THE FIRST LAW OF THERMODYNAMICS

Much of the previous work on student understanding of thermal physics has been conducted with pre-college students and has focused on student understanding of heat and temperature, typically in the context of calorimetry. Some researchers have examined student ability to apply the concept of work in purely mechanical situations. Several studies have examined

Student understanding of heat and temperature

Some of the initial work on understanding of concepts in thermal physics has been performed with precollege students. In most cases, this work has concentrated on calorimetry with liquids and solids, in which heat transfer is the only means of energy transfer. For example, Erickson performed some initial studies in which children aged 11-16 were asked to explain the results of several thermal physics demonstrations.[103] Many students gave responses suggesting a belief that heat and cold are material substances and that temperature is a measure of the amount of those substances present in an object. In Erickson's study and later work, many students confused the ideas of heat and temperature. In particular, the extensive nature of heat and intensive nature of temperature were misunderstood by many students. Surveys of research on the understanding of heat and temperature among precollege students are reported by Tiberghien and Driver.[104,105]

Student understanding of work in mechanical contexts

Research on student understanding of the concept of mechanical work has been carried out with students at the precollege and introductory college level.

Warrington and Driver did an early study in which it was reported that many students have difficulty in applying the concept of work to simple mechanical systems.[106] Lawson and McDermott reported that many students fail to apply the work-energy and impulse-momentum theorems in simple one-dimensional situations.[107] A study by O'Brien Pride *et. al.* built upon the work of Lawson and McDermott and involved detailed investigations of instructional strategies and assessment techniques.[108]

Student understanding of the behavior of an ideal gas

Research on student understanding of the behavior of an ideal gas has been performed with students at the precollege level as well as at the introductory college level. A study by de Berg has examined understanding of pressure, volume, and mass of air in a closed syringe among 17- and 18-year old students in England.[109] Approximately one-quarter to one-third of the students made incorrect predictions with the volume, mass, and density of the air in a syringe during a compression. Rozier and Viennot performed studies with students at the introductory college level.[110] They reported that many students made incorrect predictions based on inappropriate sequential reasoning with macroscopic gas quantities. In addition to these studies with macroscopic quantities, a more thorough summary of previous research into student understanding of microscopic models for an ideal gas can be found in Kautz.[111]

Student understanding of thermodynamics

Much of the research on student understanding of thermodynamics has been performed with students who are in the final stages of secondary school or in introductory college courses. Granville and Johnstone described several 'misconceptions' in thermodynamics, many of which have to do with student understanding of physical chemistry.[112] Kesidou and Duit reported on student understanding of the first and second laws of thermodynamics, finding that many students have difficulties in applying the concept of conservation of energy to simple physical situations.[113] In particular, the idea that energy is 'used up' was quite prevalent. Van Roon *et. al.* investigated student understanding of heat and work, finding difficulties with both ideas.[114]

OVERVIEW OF PART II

Part II of the dissertation includes two chapters. The first, Chapter 8, is the draft form of a paper that has been prepared for submission to the *American Journal of Physics.* This chapter describes our work on identifying the ability of students to apply the concept of work to ideal gas processes and the underlying difficulties with mechanics that hinder student understanding. The second of the chapters, Chapter 9, describes the development and assessment of curriculum to address the difficulties described in the paper. Appendix L reproduces the current form of this curriculum. There are also two appendices that include details of student responses to several of the written problems described in Chapter 8. Appendix J describes some details of student responses to various written problems that require students to recognize the need for the application of the concept of work to gas processes. Appendix K describes details of responses to problems in which students were asked explicitly about energy quantities in general and work in particular in the context of ideal gas processes.

8. IDENTIFYING STUDENT DIFFICULTIES IN APPLYING THE CONCEPT OF WORK AND THE FIRST LAW OF THERMODYNAMICS TO GAS PROCESSES

This chapter has been prepared for submission to the *American Journal of Physics*. Additional details on student responses to several of the written problems described in this chapter can be found in Appendices J and K.

IDENTIFYING STUDENT DIFFICULTIES IN APPLYING THE CONCEPT OF WORK AND THE FIRST LAW OF THERMODYNAMICS TO GAS PROCESSES

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8.1 INTRODUCTION

The Physics Education Group at the University of Washington is engaged in an ongoing investigation of student understanding of thermal physics. Part of our goal is to develop a research base for the development of curriculum, both by our group and by others. We have been studying student ability to apply fundamental principles, such as the first and second laws of thermodynamics, in real-world situations.[115] For this paper, we focus on a part of the investigation in which we examined the extent to which students are able to: (1) recognize the relevance of the first law to a given physical situation, and (2) use the first law to predict or account for the behavior of a sample of an ideal gas.

The first law of thermodynamics relates the change in internal energy of a system (ΔU) to the energy transferred to (or from) the system in the form of work and/or heat.[116] It can be expressed algebraically as $\Delta U = Q + W$ where Q is the heat transferred *to* the system and W is the work done *on* the system.[117]

At the precollege level, heat transfer is the primary (if not only) mechanism discussed for changes in temperature. Previous research, much of it done with precollege students, has shown that the idea of heat is difficult for many students.[118] In particular, many younger students tend to confuse heat, internal energy, and temperature. We have found that college students also often have difficulties with heat, but that many students in addition have difficulty applying the concept of work to situations in thermal physics. For this discussion, the emphasis is on the ability of students in introductory-level university physics courses to apply the concept of work in the context of thermal processes.

In the courses in this study, the first and second laws of thermodynamics are typically introduced in the context of ideal gas processes. We have found that student responses to the problems we have posed to probe student understanding of the first law often also elicited conceptual difficulties with ideal gas behavior. As a result, we have also been investigating student understanding of the macroscopic quantities that characterize the state of a sample of an ideal gas (P, V, n, and T); the ideal gas law ($PV = nRT$), which relates these quantities; and the microscopic model of an ideal gas. Here we briefly describe student difficulties with these ideas that are relevant to understanding their ability to apply the first law to a sample of an ideal gas.

At the University of Washington, the investigation has involved students in the introductory algebra-based course and in a second-year course in thermal physics.[119] We will refer to these courses as "algebra-based" and "second-year," respectively. The algebra-based course includes four standard lectures a week and no recitation or tutorial sections. Approximately half the students in the algebra-based course enroll in an associated laboratory course, which until very recently had no experiments involving gases. Students enrolled in the second-year course have completed introductory calculus-based physics, which, at UW, does not cover thermal physics. About 30% of these students are physics majors, and the course is typically taught with more mathematical sophistication than the introductory courses. The second-year course has no laboratory component. The investigation has also involved students in introductory calculus-based physics at the University of Maryland.[120] We refer to this course as "calculus-based."

8.2 RESEARCH METHODS

Our research methods include analysis of student responses to questions posed in individual student interviews and on written problems. In the interviews, student volunteers are asked a series of questions pertaining to a demonstration of a simple physical phenomenon. Typically the students who volunteer for these interviews are performing at or above the mean in their respective courses. These students' responses are useful in that the better students are typically able to articulate their beliefs in detail and are often able to reflect upon what they do and do not know. The open-ended format of the interviews allows the researchers to ask additional questions to probe student understanding and clarify responses.

In addition to individual student interviews, our group commonly uses written problems to investigate student understanding. These are typically short-answer, conceptually-oriented problems. Often the questions involve ranking the values of several quantities, predicting whether a quantity will increase, decrease, or remain the same when a situation is changed, or making a prediction of the result of a physical experiment. Only rarely do these problems require students to perform a calculation or determine a numerical answer. In virtually all of these problems, students are asked to explain their reasoning. The written problems that we ask are always asked as part of a physics course, typically one that covers the topic that is covered by the problem.[121] Written questions allow testing many students at once for the prevalence of specific difficulties. In some cases these questions are given before instruction on a topic, and in other cases they come afterward.

8.3 INVESTIGATION OF STUDENT ABILITY TO APPLY THE FIRST LAW OF THERMODYNAMICS TO REAL-WORLD PROCESSES

In informal interactions with the students, we noticed that many had difficulty analyzing simple ideal gas processes. In many cases, their difficulties reflected a failure to select and apply the correct physical law or principle. In order to systematically probe student thinking, we developed a series of research tasks, including protocols for individual demonstration interviews and written problems.

8.3.1 Interview protocols

The interview protocols we have used are based on the quick compression of a bicycle pump and are referred to here as the *Bicycle Pump* protocols. In the interviews, a student is shown a pump and told that the open end will be sealed (typically by the interviewer's thumb) while the handle of the pump is rapidly pushed inward. The student is asked to predict what will happen to the temperature of the gas enclosed in the pump and to explain the reasoning behind this prediction.

The first law of thermodynamics can be used to make the prediction. A quick compression of the pump can be considered to be approximately adiabatic and therefore $Q \approx 0$.[122] In the compression, the force on the gas by the piston and the displacement of the point of application of the force are parallel, so W, the work done on the gas, is positive. Therefore, the change in internal energy is positive and the temperature will increase.[123]

We deliberately chose a situation in which the ideal gas law alone is insufficient for predicting the change in temperature of the gas. For a fixed number of moles of an ideal gas, the temperature is proportional to the product of the pressure and the volume. In the quick compression that is demonstrated in the Bicycle Pump interviews, the pressure of the gas increases as the force by the piston increases, and the volume of the gas decreases. However, without invoking the first law, it cannot be determined whether the product of the two will increase, decrease, or remain the same.[124]

In Version I of the protocol, the questions that follow the initial prediction focus on student understanding of the microscopic and macroscopic behavior of the gas. We found that student responses to Version I often did not give much information about student understanding of work and energy concepts, so we developed a revised version of the interview protocol to investigate these ideas. In Version II, the questions posed by the interviewer in response to the initial prediction are intended to lead the student to analyze the bicycle pump compression using the first law. In Version II, if a student does not spontaneously mention energy concepts, the interviewer asks whether thinking about energy could help in either making or accounting for the temperature prediction. If that line of questioning does not lead the student to use the first law, the student is asked whether thinking about work would be useful. Finally, if the previous questions have not led a student to use the first law, the equation $\Delta U = Q + W$ is provided and the student is asked whether this equation could help in analyzing the problem.

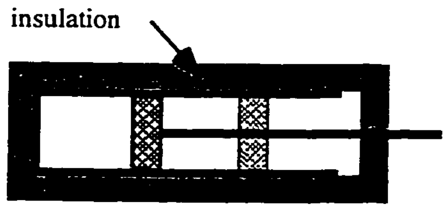
8.3.2 Written problems

To supplement the findings from the interviews we developed a series of written problems in which students must use the idea of thermodynamic work to obtain a correct answer. All of these problems involve an adiabatic compression of a sample of ideal gas.

8.3.2.1 Bicycle Pump problem

Two written versions of the Bicycle Pump problem have been used. In Version I (see Figure 8-1a), students are asked to predict what will happen to the temperature of the gas when the piston is pushed inward rapidly and to explain their prediction. In Version II, (see Figure 8-1b), the text states that the temperature of the gas increases, and students are asked to account for this change.

A cylindrical pump contains one mole of an ideal gas. The piston fits tightly so that no gas escapes, but friction is negligible between the piston and the cylinder walls.



a. Version 1	b. Version 2
The pump is thermally isolated from its surroundings. Thc piston is quickly pressed inward as shown.	The piston is quickly pressed inward as shown. A sensor in the pump records an increase in temperature.
What will happen to the temperature of the gas? Explain your reasoning.	***How can you account for the increase in internal energy of the gas? Explain your reasoning.***

Figure 8-1: Versions 1 and 2 of the Bicycle Pump problem.
a. Version 1 states that the pump is insulated and asks students to predict what will happen to the temperature.
b. Version 2 states that the temperature increases and asks students to account for the change in internal energy.

8.3.2.2 Adiabatic Compression problem

The Adiabatic Compression problem involves the same physical principles as the various Bicycle Pump tasks, but the context is slightly different. During the interviews we had seen that some students focused on the person pushing the piston, so we sought to remove this issue. We were also interested whether students would recognize that work is done in a case in which there is no 'active' agent pressing downward on the piston.[125] Therefore the compression in this problem is accomplished by adding masses to a piston (see Figure 8-2) that seals an insulated cylinder. Students are asked to predict whether the pressure, volume, and temperature of the gas will increase, decrease, or remain the same.

An ideal gas is contained in a cylinder with a tightly-fitting piston. Several small masses are on the piston.

(Neglect friction between the piston and the cylinder walls.)

The cylinder is placed in an insulating jacket. A large number of masses are added to the piston.

Tell whether the pressure, temperature, and volume of the gas will increase, decrease, or remain the same. Explain.

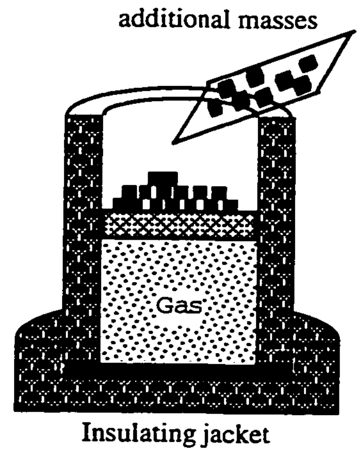


Figure 8-2 The Adiabatic Compression problem. In some versions of the question, the piston is placed over a burner in a previous part.

8.4 SUMMARY OF STUDENT RESPONSES

Among the 22 participants in the interviews using Version I of the interview protocol, fifteen were enrolled in the second-year course and seven were enrolled in the algebra-based course. Fourteen students were interviewed using Version II: five were enrolled in the second-year course and nine were enrolled in the algebra-based course. Virtually all the students had grades that were at or above the mean in their respective courses. All had completed instruction on thermal physics in their respective courses.

In both versions of the interviews, students correctly predicted a temperature increase. However, few supported their prediction with correct reasoning. Although some mentioned energy concepts, only 4 of the 16 students from the algebra-based class and 6 of the 20 from the second-year course used work to make their initial prediction. A summary of student responses is shown in Table 8-1.

Table 8-1: Students giving correct answers and reasoning for the initial temperature prediction in the Bicycle Pump interviews. Responses from Versions I and II of the protocol are similar and have been combined.

	Algebra-based Washington ($N = 16$)	Second-year Washington ($N = 20$)
Predicted temperature increase	80% (13)	75% (15)
Used work	25% (4)	30% (6)

Percentages rounded to the nearest 5%.

A summary of student responses to the written version of the Bicycle Pump problem in several different courses is shown in Table 8-2. The question was given before instruction in some cases and after standard instruction in others. As was the case with the interviews, a majority of students in each of the courses predicted that the temperature of the gas would increase. However, only a small number of students gave a correct explanation. The fraction of students giving correct reasoning was quite similar before and after instruction and the responses have been combined in the table.[126] Student responses to the Adiabatic Compression problem are summarized in Table 8-3. Very few students mentioned work or the first law of thermodynamics in their reasoning. Unlike for the Bicycle Pump problems, most of the students who failed to use work on the Adiabatic Compression problem did not correctly predict that the temperature would increase.[127]

Table 8-2: Student responses to the written versions of the Bicycle Pump problem. In version 2 of the problem, the problem text states that the temperature of the gas increases, so only student reasoning is shown.

	Algebra-based Washington Sp '98 ($N = 100$)	Calc-based Maryland 3 sections ($N = 262$)	Second-year Washington 3 sections ($N = 71$)
Version 1 or 2	2	1	1
Instruction on first law:	None	Various†	Various†
T will increase (correct)	**n/a**	**60%**	**85%**
reasoning with work (correct)	10%	~5%	25%
reasoning based on piston-molecule collisions (incomplete)	0%	0%	15%
reasoning with adiabatic equation	0%	0%	10%
incorrect reasoning	90%	55%	35%
Incorrect T predictions	**n/a**	**35%**	**15%**

Percentages rounded to the nearest 5%. Students who did not answer are not included. †Student responses were similar before and after standard instruction and have been combined.

Table 8-3: Student responses to the Adiabatic Compression problem. The second column shows only those *N'* students who answered other parts correctly (see endnote 127).

	Algebra-based Washington Wi '97	
	All students ($N = 179$)	Students predicting *P* increase, *V* decrease ($N' = 103$)
Instruction on first law:	Standard	Standard
T will increase (correct)	**10%**	**15%**
reasoning with work (correct)	~0%	~0%
incomplete reasoning	~0%	~0%
incorrect reasoning	10%	15%
Incorrect T predictions	**80%**	**80%**
Other / Blank	**10%**	**<5%**

Percentages rounded to the nearest 5%.

Both in the interviews and on the written questions, most students based their predictions on the ideal gas law or on an incorrect microscopic argument based on intermolecular collisions rather than applying the first law. Few recognized either the inadequacy of the approach they were taking or the relevance of the first law.[128] Even when prompted to do so, many students had difficulty applying the first law, especially the concept of work. Students often failed to distinguish between quantities describing a state of the gas (*e.g.*, temperature) and quantities describing a process (*e.g.*, heat transfer).

The frequency of the various incorrect forms of reasoning that we have described in student responses to the written problems described above is shown in Table 8-4. As the table shows, incorrect reasoning based on macroscopic quantities was very common in the various versions of the Bicycle Pump problem. The Adiabatic Compression problem on the other hand seemed to elicit incorrect reasoning based on the confusion between heat and internal energy and the role of insulation. Incorrect microscopic reasoning was given by only a handful of students in most versions of the written problems. Specific difficulties that we identified are discussed here. Where possible we present results from related written questions.

Table 8-4: Reasoning given by students in responses to the written problems described in this chapter. For the Adiabatic Compression problem, only those students who correctly predicted that the pressure would increase and the volume would decrease are included.

	Algebra-based Washington Winter '97 (N = 103)	Algebra-based Washington Spring '98 (N = 100)	Calc-based Maryland 3 sections (N = 262)	Second-year Washington 3 sections (N = 71)
Problem	Adiab. Comp.	Pump v2	Pump v1	Pump v1
Instruction on first law:	Standard	None	Various	Various
Correct or potentially correct reasoning	<5%	10%	<5%	50%
Incorrect reasoning with ideal gas law	10%	50%	70%	40%
Incorrect microscopic reasoning	5%	15%	5%	10%
Incorrect reasoning based on insulation	40%	0%	5%	0%
Other incorrect energy	5%	10%	5%	0%

Percentages rounded to the nearest 5%. Students who offered no reasoning are not included.

8.5 STUDENT DIFFICULTIES WITH THE IDEAL GAS

During the interviews, most of the students who relied on the ideal gas law initially referred to the change in pressure and volume of the gas. When pressed for further details, many revealed that they were thinking in terms of particles or molecules. Here we briefly describe some of the major difficulties that students had in reasoning with both macroscopic and microscopic ideal gas quantities.

8.5.1 Incorrect reasoning with macroscopic quantities

Most students argued that the changes in pressure and/or volume were responsible for the increase in temperature that they predicted. We saw two main errors in the arguments they presented. Both reflect a general difficulty in reasoning with multi-variable equations.

In written problems and interviews, some students used a two-part argument in which they sequentially isolated one variable then another in the ideal gas law. For example, several stated first that the volume decrease would cause a pressure increase (implicitly assuming constant temperature), and then that the pressure increase would cause a temperature increase (implicitly assuming constant volume). One student wrote, "decreasing the volume increases the pressure, which increases the temperature. $PV = nRT$." Similar reasoning errors have been described by other researchers.[129]

Many students gave responses suggesting that they were ignoring one variable altogether. Some students referred only to the fact that the pressure increases when the piston is pressed inward, neglecting the change in volume of the gas. For example, one student wrote, "there was a quick change in pressure. The pressure went up and so did the temperature." We also found that many students argued that the temperature of the gas would increase because the volume of the gas decreased: "in the ideal gas law, if we change the volume to a smaller amount, the temperature will rise."

8.5.2 Incorrect reasoning based on microscopic models for a gas

During the interviews, more than one third of those students who gave microscopic explanations for the temperature increase made an inappropriate connection between volume and temperature. Many appeared to assume a connection between number density and particle speed (and thus temperature). Some students made vague statements to this effect such as: "the smaller volume forces the molecules of gas to increase in speed, therefore increasing the temperature" or "the molecules of gas are closer together, making the energy higher."

A number of students made very explicit comments about the perceived connection between number density and temperature. They proposed a mechanism based on the increased number of collisions between gas particles. For example, one student said: "the molecules are getting compressed, and they have less space to move around, so they are bumping into each other a lot more, and the temperature increases." Often students suggested that the collisions would produce or release heat: "more collisions per unit time equals more heat generated."

These students had been presented in class with a microscopic model for an ideal gas that includes the assumption that gas molecules interact only via perfectly elastic collisions. In this model, the temperature of the gas is related to the average kinetic energy of the gas molecules. The microscopic mechanism for the temperature increase proposed by many students violates energy conservation, in that a change in internal energy of the gas is attributed to internal interactions of the ideal gas particles. In the interviews, we asked several of these students to consider the implications of their responses for gases in equilibrium states, and most were unable to recognize the inconsistency.

On the written problems, we also found that some students invoked collision arguments, stating, for example, that "as the molecules of gas come closer together they become more active, bouncing off each other more frequently because they are in a confined space." However, these arguments were explicitly used by less than 10% of the students on the written problems, whereas about a third had used them in interviews. In the interviews, many students started their explanations with the ideal gas law but quickly revealed the microscopic basis for their ideas after probing questions by the interviewers. As a result, we believe that some students' arguments that seem to be based on macroscopic quantities are in fact based on microscopic models.

8.6 DIFFICULTIES WITH ENERGY CONCEPTS

Although energy considerations are necessary for the analysis of the bicycle pump problem, only about half of the students that we interviewed introduced the idea of energy on their own. In Version II of the Bicycle Pump protocol, students who did not bring up the idea of energy at some point were asked whether thinking about energy would be helpful in analyzing the problem. Several stated that they could not see how energy could be helpful in this situation. Others were unsure, and when pressed by the interviewer, often turned to potential energy:

> S: I guess ... could it be that the potential energy is increasing? I mean that the air can create a force when it's let go, so the potential energy increases.

Students like this one, as well as those who talked in vague terms about the fact that energy is 'neither created nor destroyed,' were unable to analyze the Bicycle Pump problem in terms of energy considerations. In all cases, students who were asked about energy were unable to proceed further, and were subsequently asked by the interviewer to apply the idea of work to this process. When students were asked whether the idea of work could be helpful in analyzing the Bicycle Pump problem, all indicated that they recalled that work could be useful. However, few could correctly apply the concept, and a number of the students failed entirely to connect the work done on the gas to temperature:

> I: If I do work on a gas, why is that important?
>
> S: If I do work on a pencil, I don't increase its temperature. But with a gas ... it seems, well, it is related somehow.

Student responses revealed a high degree of confusion about the differences and relationships between the concepts of heat, temperature, work, and internal energy. Among those who did recognize the relevance of work, few were able to apply a general principle to determine the sign of the work done. Additional difficulties related to the absolute value of the work done were seen in both the interviews and on written problems.

8.6.1 Difficulties in distinguishing among energy quantities

Many students had difficulty distinguishing among the energy quantities heat, work, and internal energy. Some students gave responses suggesting an inability to distinguish temperature from these extensive energy quantities. The specific difficulties we observed are described in detail below.

8.6.1.1 Difficulties distinguishing heat, temperature, and internal energy

Students often related the temperature of the gas to the heat transfer rather than to the internal energy. In addition, students seemed to be unable to relate these quantities to the details of the process; few students mentioned either the speed of the compression or the insulation described in the written compression problems to make a judgment about the heat transfer.

Those students who did refer to the insulation often gave answers suggesting that the insulation would prevent a change in internal energy or temperature rather than preventing heat transfer to or from the gas. This difficulty is more than a lack of careful terminology. In the Adiabatic Compression problem, nearly half of the students stated that the temperature of the gas would not change in the compression due to the fact that the cylinder is insulated. For example, one student said that the temperature would remain the same because "an insulating jacket will hold all the heat in." Such responses suggest that students have not correctly distinguished the quantities heat, temperature, and internal energy. Researchers have reported that many precollege students confuse the concepts of heat and temperature and are confused about the role of insulating materials.[130] For example, children in some studies have said that materials like wool will make objects hot, rather than preventing heat transfer.

We have also seen responses that suggest that students are unable to distinguish the quantities associated with processes from those associated with states. In written problems as well as in the interviews, many students referred to the 'change in heat' or the 'change in work.' Some of these students expressed confusion as to why both ΔU and Q were present in the algebraic form of the first law, since both seemed to describe the 'heat in an object.' One student spontaneously wrote down the algebraic statement of the first law of thermodynamics in the interview but was then unable to reason with it:

S: I'm getting confused with how to apply the thing.

I: How are you getting confused?

S: First of all, if the heat of the system is the same as internal energy, the temperature, or if it's related to Q.

8.6.1.2 Difficulties distinguishing heat and work

Many students gave responses suggesting a failure to distinguish heat and work. Some of the students in the interviews stated that any process in which work is done must also involve a heat transfer:

S: Work and energy aren't the same thing, are they? I don't know how to distinguish the two.

I: Are they related somehow?

S: ...You're putting a parallel force onto the piston in that direction and you're making it move, so there's work being done ... you're putting a force on that pump, it has to make heat. When you rub your hand when you touch something you can feel it. I can feel my hand gets warmer.

Another student, who used work to make her initial prediction about the temperature, made a similar statement suggesting that work and heat are not independent:

I: Coming back to work and heat, is it possible to have work being done on the environment but no heat transfer?

S: Oh, well, what he was saying is that work equals $P\ dV$. So change in volume... no, there has to be heat transfer.

Several of the students in the interviews used the word 'heat' rather than 'work' to refer to the energy transfer in the process in which the piston is pressed inward quickly. In responses to written problems, students expressed many similar ideas: "the heat caused by pressing the piston inward is gained by the gas, so the temperature increased." Other students expressed similar ideas but referred instead to energy: "the rapid pushing in releases thermal energy." Some students used the term 'work' in their answers but referred to the release or conversion of energy in non-standard ways: "work = release of energy → energy = heat!"

Responses of this sort can be interpreted as being a difficulty with terminology. However, dialogues with students during the interviews suggests that at least some of these statements reflect a genuine confusion between the mechanical energy transfer that physicists describe as 'work' and the non-mechanical energy transfer that physicists describe as 'heat.' In the interviews, students who made statements of this sort also had difficulty in interpreting the algebraic statement of the first law of the thermodynamics.

8.6.2 Difficulties in determining work done in an ideal gas process

In addition to confusion about the relationship between work and temperature, students had a variety of difficulties in relating the work done to the circumstances of the process they were shown. They had been taught general definitions for work in terms of the force on the gas and the displacement of the piston and in terms of the pressure of the gas and a change in volume. Many students had difficulty relating these general definitions to the Bicycle Pump problem or various written problems that we administered. We found that several students had difficulties in determining the sign of the work and others in determining the absolute value.

8.6.2.1 Difficulties related to the sign of the work done

During the Bicycle Pump interviews, few students were able to determine the sign of the work done from the directions of the displacement of the piston and the force that it exerts on the gas. In both written problems and interviews, we found that although most students found the correct sign of the work done, their responses revealed superficial understanding and, often, incorrect reasoning. Most students were not able to interpret the sign in terms of either the definition of work or energy transfer to the gas. A description of the specific difficulties follows.

Tendency to associate the sign of work with coordinate system

Some students related the sign of the work done to the choice of coordinate system. For example, when one student referred to negative work, the interviewer asked what the student meant:

I: What's negative work?

S: I don't know. In the opposite direction.

I: Opposite to what?

S: Against your coordinate axes, for your set of motion.

Other students referred to either the force or displacement of the piston having a sign:

S: [the sign of work] is not completely arbitrary. The distance is negative, so that means ... the work done on the piston is negative. You use the distance the piston is traveling ...

Tendency to associate the sign of work with a convention

In addition to the idea that the sign of work depends on the choice of coordinate system, we found that several students believed that the sign of work was a matter of convention. Often instructors will note that some texts write the first law in terms of the work done *by* the gas and others in terms of the work done *on* the gas, and that the two statements differ in the sign of the W in the algebraic statement of the first law. It appears that many students may infer from this discussion that the sign of work is ultimately determined by convention, rather than by using the definition of work as the dot product of a force and a displacement. This confusion is illustrated by the following exchange:

I: How can you find the sign of work?

S: Isn't it a convention? It is. I think it's negative, but I can't remember why that was. If the volume is decreasing that means that there is work being done on that so the ΔV would be negative but the work will be positive.

I: What is the sign of the work done by the piston on the gas [in this process]?

S: That work is ... I don't know. I'm guessing it would be positive. I don't know the correlation between the positive and the negative part.

We have seen similar responses on written problems. For both forms of reasoning (that based on convention as well as that based on coordinate system), many of the students who used this reasoning were able to determine the correct sign of work, often based on a memorized rule. Few, however, had any understanding of the physical significance of the sign, and as a result, very few of these students were successful in applying work or the first law to gas processes.

8.6.2.2 Difficulties related to the absolute value of the work done

In addition to confusion about the sign of work done on gas samples in various processes, we found that many students had difficulties that involved the absolute value of the work done. Many used inappropriate formulas for specific cases, rather than applying a general definition.[131] Some students failed to recognize that the work done *on* the system and the work done *by* the system have the same absolute value. Others failed to recognize the path dependence of work.

Failure to recognize that work done on and by system have same value

The interview questions about the sign of the work brought out a specific difficulty in understanding work as an energy transfer. For two objects in contact, the work done on the first object by the second will be equal in absolute value to the work done on the second object by the first, but will have the opposite sign. In terms of the language typically used in the first law of thermodynamics, the work done on an object is equal and opposite to the work done by the object. Several of the students stated that nonzero work can be done *on* an object or *by* the object, but not both in the same process. Other students failed to recognize that the work done on an object and the work done by the object have the same absolute value.

Some students gave answers that seemed to be based on the belief that, for a given motion of the piston in the bicycle pump, there is one object that causes the motion, and only that object does work. Several students said that there is work done on the gas in the compression but that there would be none in an expansion, because in the expansion the *gas* would be causing the motion. For example, one student, who had spontaneously applied the idea of work to the process, went on to say that either work could be done on the gas or by the gas, but not both:

S: If the gas were to expand, the work done by the gas would be positive.

I: Would there be work done *on* the gas in that case?

S: No.

I: Why not?

S: Because the work is being done on the *piston* in that case.

Another student was quite explicit in referring to the causal relationship:

S If this [piston] suddenly just popped out like this, something would happen. I'm causing the movement, not the system, so the work is being done by me, not by this, not by the pump.

For these students, work is either done *on* the gas or *by* the gas, depending on whether they consider that the gas or the piston is the object that causes the other to move. This dichotomy seems to be connected to the idea that the object that is being moved is 'passive' and thus does no work. This belief is non-Newtonian and in fact contradicts Newton's third law.

Other students answered that work is done both on and by an object, but that the absolute values of these works are different. For example, one student said:

> S: There's some kind of force in the opposite direction. There *is* work done by the gas, but the work I do is greater. The net force is in this [inward] direction.

This student seems to be comparing the force on the gas by the piston to the force on the piston by the gas and deciding that the sum of these forces would determine whether the piston accelerated, suggesting confusion between Newton's second law, which relates the net force on a single object to the acceleration of the object, and Newton's third law, which relates the force on one object by a second object to the force on the second object by the first. Some students made statements that were even more explicit in contradicting Newton's third law. For example, a student made the following statements while attempting to determine the sign of the work done on the gas in the compression:

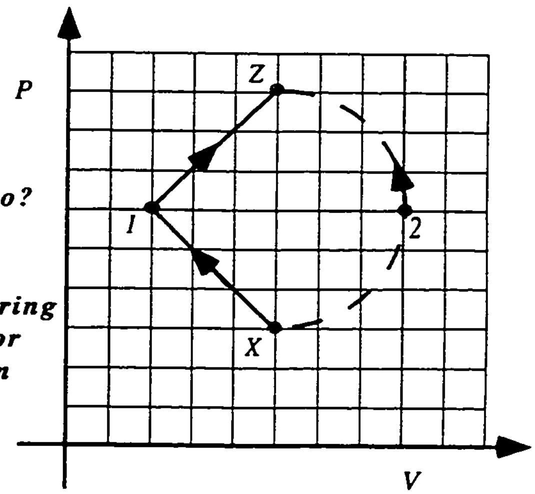
> I: So how do you decide [if work is positive or negative]?
>
> S: ... there is positive work, and there is negative work, there is also work done on the system, or work done by the system. If we are talking about work done on the system, it's positive if the volume decreases, it's negative if the volume increases. The way I thought about it in class – which I am sure the professor could blow me out of the water, if I told him this – is like in this case, you did work on the system, it kind of lost the war ... Your hand was obviously a whole lot stronger than the gas inside, you did positive work on the system. That's how I got the sign convention of it.

Failure to recognize path-dependence of work

Students are expected to be able to determine the absolute value of work done in a process given a curve representing the process on a *PV* diagram, using the fact that the area under the curve is equal to the absolute value of the work. We have seen that students often have difficulties in doing so. In particular, many students focus on the initial and final states, rather than the process, and thus fail to recognize that the absolute value of the work depends on the details of the process.

In several problems that we have posed, students were shown curves on a *PV* diagram representing two processes with the same initial and final states and asked to compare the work done on the gas in the two processes. For example, in the problem shown in Figure 8-3, students are asked to state whether the absolute value of the work done in process $X \rightarrow 2 \rightarrow Z$ is greater than, less than, or equal to the absolute value of the work done in process $X \rightarrow 1 \rightarrow Z$. To answer, students must recognize that the absolute value of the work done can be related to the area under the curve on the *PV* diagram. Since the area under the semicircular curve $X \rightarrow 2 \rightarrow Z$ is greater than that under $X \rightarrow 1 \rightarrow Z$, the work in the first process is greater. In one section of the second-year course, more than 25% of the students stated that the absolute value of the work done in the two processes is the same "because the initial and final states are the same for both processes." Several students in this class made explicit statements that "the work is independent of path taken." In a multiple-choice version of this question that was used in the calculus-based course at another university, 45% of the students chose the answer that the absolute values are equal.[132]

A student performs an experiment with an ideal gas that is confined to a cylinder with a piston. The *PV* diagram below shows the values of pressure and volume of the gas throughout two different experiments, both starting at point *X* and ending at point *Z*.



Is the work done on the gas during the process X→1→Z positive, negative, or zero? Explain.

Is the absolute value of the work done during process X→2→Z greater than, less than, or equal to that during process X→1→Z from part A? Explain your reasoning.

Figure 8-3 An example of a problem in which students are asked about the sign and absolute value of work for a process represented on a *PV* diagram.

The problem shown in Figure 8-3 proved to be particularly effective at eliciting this particular difficulty, because the initial and final volumes in the problem are the same in both processes. On a previous part of this problem, students were asked whether the work done on the gas in process *X*→*1*→*Z* is positive, negative, or zero. Students were expected to recognize that the work done on the gas in process *X*→*1* is positive and that done in *1*→*Z* is negative, but that the work done in process *1*→*Z* is greater in absolute value, so that the work done on the gas in process *X*→*1*→*Z* is negative. About 10% of the students in the second-year course answered incorrectly that the work in process *X*→*1*→*Z* is equal to zero. Approximately 45% of the students in the calculus-based course at the other university answered that the work done in the process *X*→*1*→*Z* is zero in the multiple-choice version of this problem.

Students were particularly apt to ignore the path dependence of work in cyclic processes, such as those that describe heat engines and refrigerators. In several problems we have asked students to state whether the net work done on (or by) a gas in a cyclic process is positive, negative, or zero. A significant fraction of students have answered that the net work done on the gas is zero, typically arguing that the original and final volume are the same so that the net change in volume is zero. For example, the problem shown in Figure 8-4 was given in a section of the algebra-based class. About 50% of the students said that the net work done on the gas is zero. In some cases, student responses suggested difficulties with the path-dependence of work similar to those described above. One student wrote, "No net work was done because the cycle started and ended at the same place. Since there is no displacement, then no work was done." For other students, the idea that the work done would be zero overrode correct ideas. For example, one student wrote "W = area under curve" and shaded the area corresponding to the net work done on the system, but continued and finally answered "since there was no change in volume overall there was no work done on the system."

A student performs an experiment with an ideal gas that is contained in a cylinder with a piston. The PV diagram below shows the values of pressure and volume of the gas throughout the experiment, starting at point X, continuing to points Y and Z, and returning to point X. Process $Z \rightarrow X$ is isothermal.

Is the total work done on the gas in the entire cycle (i.e., $X \rightarrow Y \rightarrow Z \rightarrow X$) positive, negative, or zero? Explain your reasoning.

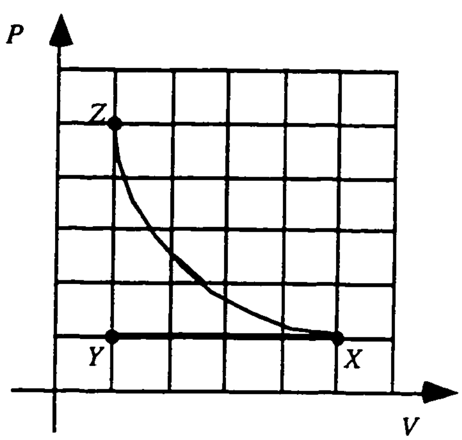


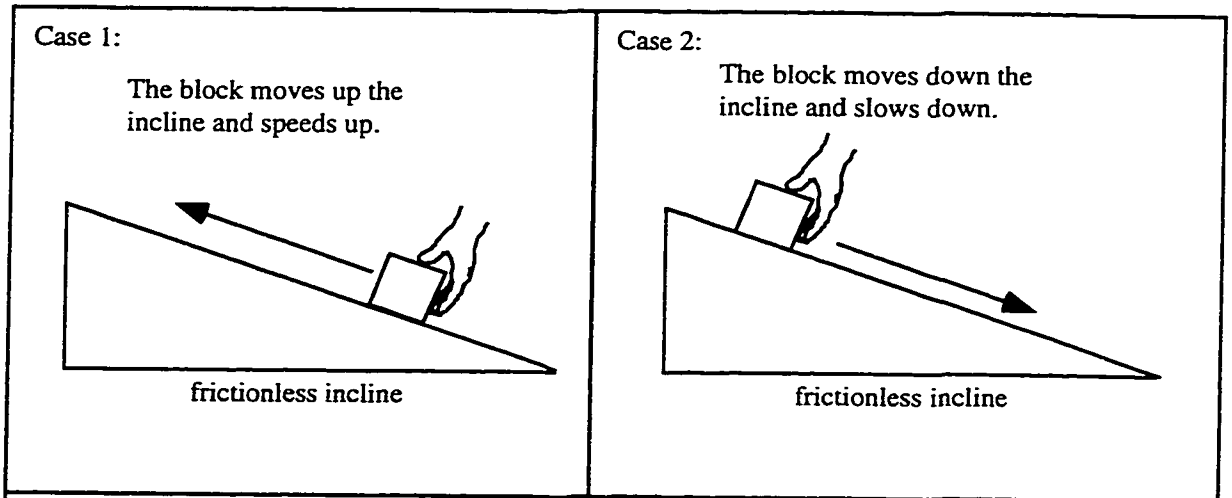
Figure 8-4 A problem in which students are asked about the work done on a gas in a cyclic process represented on a PV diagram.

These responses suggest that the difficulty with the path dependence of work is especially prevalent in processes in which the initial volume is equal to the final volume.

However, we have seen similar responses on problems in which students are asked to compare the work done in two processes for which the initial volume is different from the final volume. In one problem in which two processes had the same nonzero change in volume, nearly 20% of the students in a section of the second-year course stated that the work would be the same in the two processes despite different curves on the *PV* diagram.

8.7 STUDENT ABILITY TO APPLY CONCEPT OF WORK IN MECHANICS

It seemed that many of the difficulties that students had in applying the concept of work in thermodynamic processes were essentially difficulties with mechanics. In order to probe the ability of students to apply the concept of work in situations free from the additional complications of thermal physics, we decided to developed a short written test. We have named this test, shown in Figure 8-5, the *Mechanical Work* problem.



Tell whether the following quantities are positive, negative, or zero. Explain.

- ***the work done on the block by the hand***
- ***the work done on the block by the earth***
- ***the work done on the hand by the block (if there is no such work, state so)***

Figure 8-5 The Mechanical Work problem. In some courses, students were only asked about Case 1. In other courses, students were only asked about the work done by the hand in Case 2 and a third case was considered.

8.7.1 Written tasks

In the Mechanical Work problem, a hand exerts a force on a block that is moving on an inclined plane. The force exerted by the hand on the block is parallel to the surface of the incline. Students consider two or three different cases. In Case 1, the block moves up the incline with increasing speed while the hand pushes up. In Case 2, the hand still pushes up, but the block moves down the incline with decreasing speed. In some versions, we have asked about a third case, in which the hand pushes down the incline as the block moves down with decreasing speed (students are reminded that friction is important in this case). For each case, students are asked to determine the sign of the work done on the block by the hand and by the earth. We also ask students to determine the sign of the work done on the hand by the block, or to state explicitly if there is no such work.

To answer the questions in the Mechanical Work problem, students are expected to use the definition of work to find the sign of the work. In Case 1, the force by the hand is parallel to the displacement of the block so the work done on the block by the hand is positive. The work done on the hand by the block is negative because the force on the hand by the block is opposite to the displacement of the block. Similar reasoning can be applied to Case 2. These cases are chosen to be analogous to the processes in thermal physics that students are likely to encounter, with Case 1 similar to a compression in that the external agent (the hand) exerts a force that is in the same direction as the displacement of the point of application of the force. Case 2 is similar to an expansion in that the hand exerts a force that is opposite to the displacement.

8.7.2 Summary of student responses

Responses to the Mechanical Work problem suggest that many students have conceptual difficulties with work in mechanical situations. For example, although most students gave the correct answer for the sign of the work on the block by the hand in Case 1, most used incomplete or incorrect reasoning. Fewer than one in four referred to the definition of work or the directions of the relevant force and displacement. An additional 5% to 10% used a simplified and incomplete definition of work, referring to the fact that work is the product of force and distance and observing that the block moved through some distance but failing to refer to the vector nature of either force or distance. One such student wrote that the work done on the block by the hand is "positive, because work = (force)(distance) and since the block moves work is done." Overall, fewer than half of the students answered all parts of the Mechanical Work problem correctly.[133] A summary of the results is shown in Table 8-5.

Table 8-5: Student responses to the Mechanical Work problem.

	Algebra-based Washington Sp'97 (N = 101)	Calc-based Maryland Sp'97 (N = 184)
Sign of work on block by hand correct in Case 1	95%	90%
with complete and correct reasoning	10%	20%
with nearly complete reasoning	15%	10%
with incorrect reasoning based on coordinate system	10%	25%
with incorrect reasoning based on 'cause'	15%	5%
with other incorrect reasoning	15%	30%
All signs of work correct	25%	40%

Percentages rounded to the nearest 5%. Students who offered no reasoning are not shown.

8.7.3 Specific difficulties identified

Many students gave incorrect reasoning in responding to the questions in the Mechanical Work problem. The specific difficulties that we observed all suggested a failure to apply the definition of work. Some of the difficulties that we had seen in thermal contexts, like the idea that the sign of work is based on a sign convention, did not show up in the responses to the Mechanical Work problem. Here we discuss the specific difficulties with mechanical work that most closely mirror those found in thermodynamical contexts: (1) belief that the sign of work depends on the choice of coordinate system; and (2) difficulties relating work done on and by an object.

8.7.3.1 Tendency to associate the sign of work with coordinate system

Between 10% and 25% of the students in each class supported their answers about the sign of the work with reference to a particular coordinate system. Many students stated explicitly that the sign of work was determined by the choice of coordinate system, and that a different choice would give a different sign of work. One such student wrote, "There really is no such thing as negative work since the distance used to calculate work depends on where a person chooses to place the coordinate axis." Other students were not so explicit but described a specific coordinate system. For example, one student wrote, "the work done [on the block by the hand] is positive, because I defined my coordinate system to be positive in the up direction." Many students drew and referred to coordinate systems in their responses (see Figure 8-6). Students who used this type of reasoning based their answer on the sign of various quantities, including force, displacement, velocity, and acceleration.

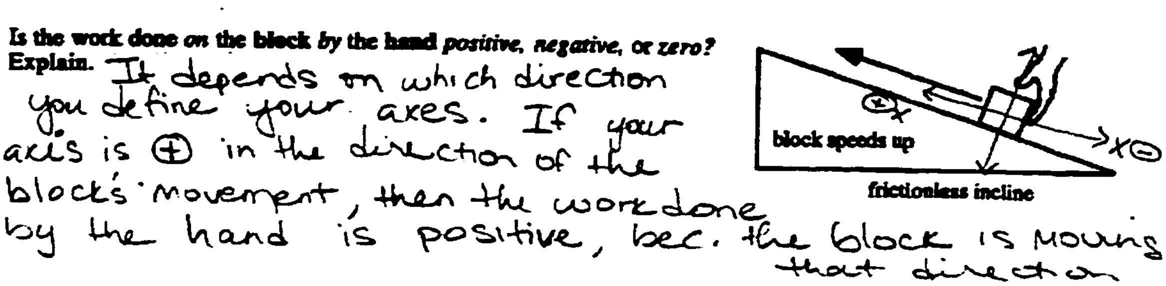


Figure 8-6: A student response to the Mechanical Work problem.

8.7.3.2 Difficulties relating work done on and by an object

When we asked students about the work done by the block on the hand, we added the statement, "if there is no such work, state so explicitly." About 20% of the students in a section of the algebra-based class did so, with another 10% stating that the work done would be zero.[134] Many incorrect responses included statements suggesting a belief that the block does no work because it is not the agent causing the motion: "the work done by the block on any object is zero because it doesn't make anything move." Another student wrote, "[the work done by the block] would be zero ... because it exerts a force but it's [sic]force is not responsible for traveling a certain distance."

As in the Bicycle Pump interviews, many student statements suggested difficulties with Newton's third law. One student wrote, "Positive... the work done on block by hand is more than vice versa." However, some students made statements that suggested an understanding of Newton's' third law and still stated that the block does no work on the hand. often based on the idea the block is not the object causing the motion. For example, for the work done on the hand by the block in case 1, one student wrote, "[there is] no such work, the block may have an equal and opposite force on hand, but the block is not actually doing any work."

8.8 STUDENT RESPONSES TO MECHANICS PROBLEMS COMPARED TO THEIR RESPONSES TO THERMAL PHYSICS PROBLEMS

We found that many of the difficulties that students have with work in thermal processes were directly related to difficulties with work in mechanical processes. In order to probe the connection between understanding of work in the context of mechanics and ability to apply work correctly in thermal physics processes, we compared the responses given by individual students on the Mechanical Work problem with the responses given by the same students on tasks involving gas processes. We have done such comparisons in several classes.

8.8.1 Tasks

In some classes we have administered a two-part ungraded quiz. On the first page are the questions from the Mechanical Work problem. On the other are those from one of the two versions of the Bicycle Pump problem. By analyzing student responses, we gain some insight into the extent that the failure to apply work in the thermal physics problem is related to student difficulties in mechanics.

8.8.2 Summary of student responses

The responses given by students in a several different courses are shown in Figure 8-7. Included are results from the algebra-based course, the calculus-based course, and the second-year course. Results from multiple sections of the same course were similar, so they have been combined. The top bar in each course represents those students who gave correct reasoning on the Mechanical Work problem. The lower bar in each group represents the students who answered incorrectly on the Mechanical Work problem. The dark portion of each bar represents the students who answered the Bicycle Pump problem correctly with correct reasoning.

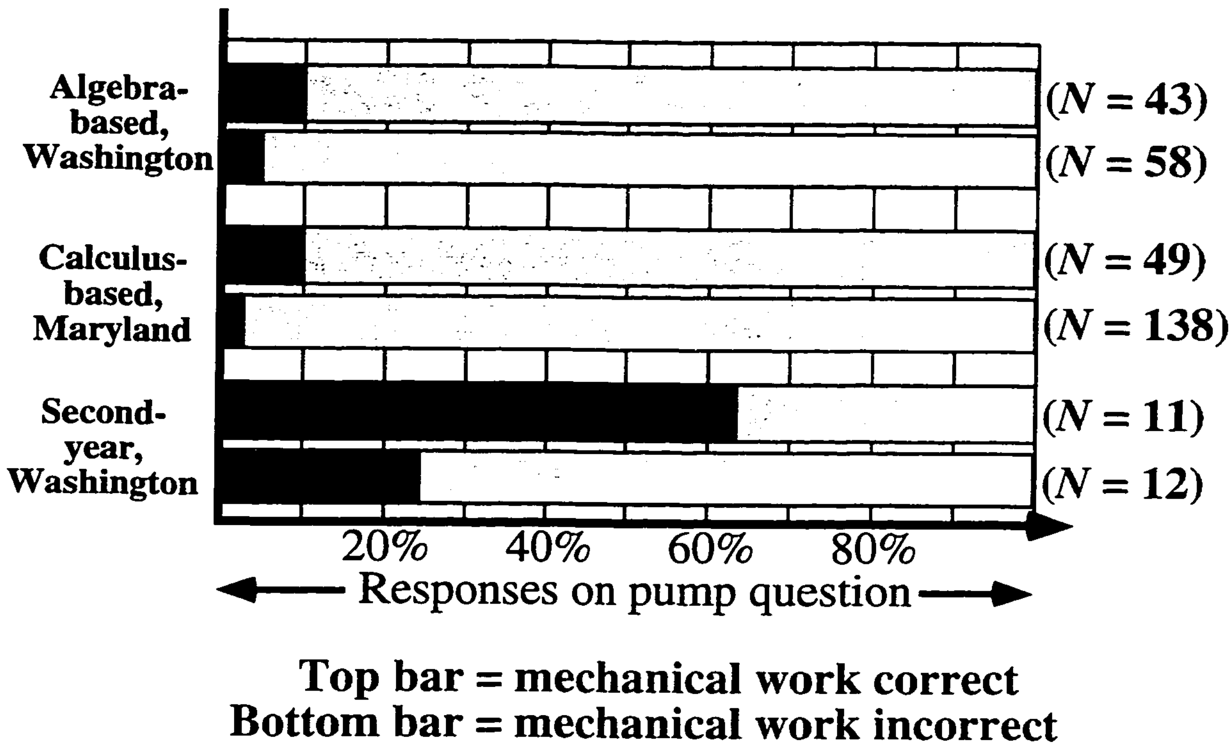


Figure 8-7: Responses on the Bicycle Pump problem compared to responses to the Mechanical Work problem. The top bar in each course represents those students who answered the Mechanical Work problem correctly. The bottom bar represents those who did not. The dark portion of each bar represents the students answering the Bicycle Pump problem correctly.

As the results in the figure show, students who answered the Mechanical Work problem based on the definition of work (the top bar in each course) were somewhat more likely to answer the pump problem using work than those who has answered based on incorrect reasoning (*e.g.*, based on a particular coordinate system or on the idea that only the object causing the motion does work).[135] However, students in both groups had a great deal of difficulty in applying work to the pump problem, even with the considerable hint offered by the page of questions on mechanical work. These results suggest that whereas an understanding of mechanical work is necessary for students to apply work correctly to the thermal physics problem, this understanding is not sufficient. Even those students with a good understanding of mechanical work have difficulty in applying this concept to the Bicycle Pump problem.

8.9 CONCLUSION

Our results suggest that for many students, a functional understanding of the first law of thermodynamics is not an outcome of an introductory course in thermal physics. We have shown that many students failed to apply the first law of thermodynamics to the problems that we have described. Most of these students used other forms of reasoning that were either based on incorrect models (particle collisions, insulation) or on incorrect application of the ideal gas law. Because these students failed to recognize that their methods were incorrect or incomplete, they were unable to see the necessity of applying the concept of work in their analysis. Even for many practicing physicists, the ideal gas law is often the first approach taken when solving the compression problem. Unlike physicists, however, many students fail to recognize when the ideal gas law alone does not allow a prediction of the final temperature. Similarly, some students believe incorrectly that a microscopic model based on collisions between gas molecules will allow them to predict the final temperature. It seems likely, then, that part of the reason that students do not apply work to these processes is that their incomplete or incorrect models allow them to make temperature predictions without the first law of thermodynamics.

When we probed student understanding, we found specific conceptual difficulties with work and the energy quantities in the first law of thermodynamics. For example, many students answered the Adiabatic Compression problem based on an incorrect understanding of the role of insulation that suggests a failure to distinguish among the quantities heat, temperature, and internal energy. Student responses in written problems and interviews suggested difficulties in recognizing the connection of work to thermal processes and difficulties in distinguishing among heat transfer, work, and changes in internal energy. Many students failed to treat heat transfer and work as independent means of changing the internal energy.

Often students gave responses suggesting a failure to recognize which quantities are determined by the state of a gas and which depend on the details of a process. For example, in problems in which students were asked to compare the absolute values of the work done in two processes, many students answered based on the initial and final states rather than on the details of the processes. In written and verbal responses to several different problems, many students made statements reflecting this confusion, referring, for example, to the 'change in heat' in a process.

In interviews, we asked students directly about the work done in the compression of the bicycle pump. Student responses indicated many conceptual difficulties with work, some of which seemed to be primarily difficulties with the definition of work and with Newton's laws. In particular, the sign of work was a confusing concept for many students. We probed student understanding of work in purely mechanical contexts, and found student responses that were parallel to many of the incorrect responses that we had seen in the context of thermal physics. Further investigation revealed that those students who lacked an understanding of the definition of work had great difficulty in applying the concept of work to gas processes. However, even those students who did have a good understanding of the definition of work had difficulty in applying the concept, suggesting that for some students the process of transferring their understanding of work to the new context of thermal physics is an additional source of difficulty.

We believe that these results have implications for instruction. In standard instruction on the first law of thermodynamics, the general definition of work is often mentioned very briefly before relationships specific to gas processes are derived. Our results suggest that such a brief treatment does not help students to connect their understanding of work to the new context of gas processes. In addition, those students who do not have an understanding of the definition of work seem to have great difficulty in understanding the physical significance of the sign of work and interpreting work as a mechanical transfer of energy. Based on these findings, we have developed a series of instructional materials that are designed to help students recognize the applicability of work to gas processes. These materials are described in the following chapter and begin with a review of the definition of mechanical work as well as exercises designed to help students apply this definition to gas processes. Our findings have also led to modifications of existing curriculum on work designed for courses in introductory mechanics.

9. ADDRESSING STUDENT DIFFICULTIES WITH WORK AND THE FIRST LAW OF THERMODYNAMICS

9.1 INTRODUCTION

Based on the research described in Chapter 8, we have developed a tutorial sequence to address student difficulties with work and the first law of thermodynamics. Our findings that many students have difficulty with the definition of work and in applying that definition to thermal processes, suggested that the tutorial should begin with a review of the definition of work in a purely mechanical context. The tutorial then guides students to apply this definition to processes in thermal physics. Additional difficulties that are particular to the context of thermal physics (discussed in Chapter 8) are also addressed.

We describe implementation of the tutorial sequence as an interactive tutorial lecture as well as a small-group tutorial and describe students' responses to post-tests after instruction using the sequence. Based on post-tests after the initial use of the curriculum, we modified the tutorial sequence somewhat. These modifications and the assessment of the revised version are described.

9.2 SUMMARY OF STUDENT DIFFICULTIES WITH WORK AND THE FIRST LAW

Research on student difficulties has guided our curriculum development in thermal physics as well as our efforts to assess the curriculum. We will briefly describe common student difficulties that the curriculum is designed to address and summarize the criteria that we have used for to assess student understanding. Not all of the difficulties discussed are explicitly tested for in the assessments that we will describe. Rather, we have chosen to focus only on a few of the most difficult and fundamental concepts.

9.2.1 Student ability to spontaneously apply work to ideal gas processes

In Chapter 8, we showed that very few students spontaneously applied work to ideal gas processes presented in interviews and written problems. The fraction of students doing so varied, ranging from 0-10% in the introductory courses to approximately 25-50% in the second-year thermal physics course.

9.2.1.1 Criteria for correct answers and reasoning

The problems that we have used to investigate student use of work often involve temperature predictions. For example, in the Adiabatic Compression problem, shown in Figure 8-2 of Chapter 8, students are asked to predict whether the pressure, volume and temperature of a gas sample will increase, decrease, or remain the same. The temperature prediction requires use of work and the first law of thermodynamics. To be considered correct, a temperature prediction should include reasoning based on the work done on or by the gas. It should be noted that many of the students who answered in this way did not specifically refer to the heat transfer or the first law of thermodynamics, suggesting that this criterion may in fact lead to an overestimate of the number of students who are reasoning correctly. Other problems will be described later in this chapter.

9.2.1.2 Incorrect reasoning used by students

After standard lecture instruction, we found that most students answered the questions discussed above without referring to work. A brief description of the incorrect forms of reasoning given by students follows. More detailed summaries can be found in Chapter 8.

Incorrect ideal gas arguments. A large number of students answered using incorrect ideal gas arguments. These arguments varied; some students responded that the pressure alone would allow prediction of temperature changes, others referred to volume alone, and some referred to both pressure and volume, often with inappropriate sequential reasoning (*e.g.*, "the volume decreases, so the pressure increases, and when the pressure increases, the temperature must increase").

Incorrect microscopic arguments. Many students answered using incorrect microscopic arguments. Most of these arguments seemed to be based on the belief that the collisions of ideal gas particles release energy and that an increase in the rate of these collisions will result in an increase in temperature. In some other cases, students did not refer to collisions but argued that the speed of gas particles is related to the particle density, such that if a fixed amount of gas is compressed, the temperature will increase as the particle density increases.

Incorrect energy arguments. Some students answered using incorrect arguments based on heat, temperature, and internal energy. The most common form of argument seemed to be based on misunderstanding of the role of insulation: many students stated that the temperature of an insulated sample of gas will not change when the volume or pressure changes, while ignoring the work done on the gas.

9.2.2 Student ability to determine the sign of work

In Chapter 8, we have shown that some students have difficulty in determining whether the sign of work is positive, negative, or zero in a given gas process. For the purpose of this section, we will restrict this discussion to problems in which students are asked to determine the sign of the work on the gas in a standard gas process that is represented by a curve on a *PV* diagram.

9.2.2.1 Criteria for correct answers and reasoning

In some of the problems discussed in Chapter 8, students are asked to determine whether the work done on a gas in a given process is positive, negative, or zero (*e.g.*, the work in an isobaric compression that takes place in a cylinder closed by a piston). To answer correctly, students must recognize that, as the gas is compressed, the force on the gas by the piston is parallel to the displacement of the piston. Applying the definition of work, the dot product of the force and the displacement of the point of application is positive. In addition to using this type of reasoning, many students referred to a specific equation for the process in question, *e.g.*, $W = - P \Delta V$ for an isobaric compression.

9.2.2.2 Incorrect reasoning used by students

We found several common incorrect forms of reasoning that students used in determining the sign of work. Although these incorrect forms of reasoning were quite common in interview responses, we saw relatively few examples in responses to written problems. These incorrect ideas are described in more detail in Chapter 8.

Work depends on coordinate system or convention. Some students in attempting to determine the sign of work, answered that the sign of work depends on the choice of coordinate system or on some other sign convention and failed to use the definition of work. Many of these students stated that the sign of work in a particular process is somewhat arbitrary and that the work could well have different sign if a different coordinate system or convention were chosen.

Work depends on object 'causing' motion. Many students made statements consistent with the idea that only the object 'causing' a motion does work in that motion. For example, students responded that in a compression of a gas in a cylinder closed by a piston, the piston does work on the gas because it causes the volume of the gas to change, but that no work is done on the piston by the gas because the gas is not causing the motion.

9.2.3 Student ability to determine the absolute value of work

In responses to several written problems, we found evidence that some students had difficulty in determining the absolute value of the work done on a gas in a process from the representation of the process on a *PV* diagram. We found such difficulties on responses to both qualitative and quantitative problems.

9.2.3.1 Criteria for correct answers and reasoning

On the qualitative problems discussed in Chapter 8 and the quantitative problems discussed in Appendix K, students were shown representations of one or more ideal gas processes on a *PV* diagram. In the qualitative problems, students were asked to rank the absolute values of the works in two processes or state whether the work done in a cycle is positive, negative, or zero. In the quantitative problems, students were asked to determine a numerical value for the work done in a process. In either case, students were expected to reason using either the integral relationship $W = -\int PdV$ or, equivalently, the area under the curve representing a process on a *PV* diagram. For the purpose of assessing the tutorial, we will focus on the qualitative tasks.

9.2.3.2 Incorrect reasoning used by students

We found several common incorrect forms of reasoning used by students in determining the absolute value of the work done in a process. These incorrect ideas and specific problems that elicit the ideas are described in more detail in Chapter 8.

Work and Newton's third law. Many students made statements that revealed difficulties with Newton's third law. Such statements typically involved the idea that in a compression the force on the gas by the piston must be greater than the force on the piston by the gas, or else the gas would not be compressed. In interviews, several students said that the absolute value of the work done on the gas in a compression would be different from that of the work done by the gas. Responses of this sort were extremely rare on written problems.

Work is path-independent and work in a cycle is zero. A common form of incorrect reasoning was based on the idea that work is path-independent. In several written problems, students have been asked to compare the work done in two processes with the same initial and final states. Many students have responded that the work is the same in the two processes because the initial and final states are the same. A special case of this reasoning involves cyclic processes, like those used for heat engines and refrigerators. Many students state that the work done in a cyclic ideal gas process is zero. Students giving this response typically state that the fact that the volume of the gas returned to its original value implies that the work is zero.

9.3 INSTRUCTION TO ADDRESS DIFFICULTIES WITH WORK IN THE FIRST LAW OF THERMODYNAMICS

In this section we will discuss a tutorial designed to address student difficulties with work in the first law: *The First Law of Thermodynamics.* The curriculum represents a series of instructional strategies that are based on the research findings described above.

9.3.1 Instructional setting

We have tested *The First Law of Thermodynamics* in two different instructional settings. At the University of Washington, we have used a version of the first law of thermodynamics in the algebra-based introductory course as part of a sequence of interactive tutorial lectures (ITLs) in thermal physics. At other institutions, including the University of Maryland, we have used the sequence as a small-group tutorial. For more details on these instructional approaches, see the Introduction.

9.3.1.1 Interactive tutorial lectures in algebra-based course

The ITLs that we have used in Physics 115 typically begin with a pretest that probes student understanding. During the ITL, students work together in their seats in the lecture hall in groups of 2 or 3. The structure is provided by worksheets that consist of a series of questions and exercises that are expressly intended to help students confront and resolve common conceptual and reasoning difficulties. There may be one or more TAs present in addition to the lecturer. During the ITL, the instructors circulate around the lecture hall and assist the students by questioning, not by telling.

At several key points, the lecturer calls the room to order. At these checkpoints, an instructor may show a demonstration or may outline some of the expected answers for the previous part of the ITL. The students are instructed to proceed from there until the next checkpoint. In this respect, ITLs differ significantly from tutorial sessions, in which students work at their own pace and there are no answers provided. The checkpoints are included because in the large lecture class it is not possible for the instructors to reach all the students. Nor is it possible to have students perform some of the simple experiments that can play an important role in helping them to confront and resolve incorrect ideas.

9.3.1.2 Tutorials in calculus-based course

We have also used *The First Law of Thermodynamics* as a tutorial in Physics 262, the calculus-based introductory physics course at the University of Maryland. In this course, the tutorial system is implemented in a fashion similar to that in the calculus-based introductory physics course at the University of Washington. For more details, see the Introduction and the references therein.

9.3.2 Preliminary version of tutorial sequence: *The First Law of Thermodynamics*

Based on the difficulties we found that students have with work and the first law, we developed a tutorial sequence designed to address student difficulties with the sign of work, Newton's third law, and the connection between mechanical work and ideal gas processes. (See also Appendix L, in which we present the complete tutorial sequence.)

9.3.2.1 Pretest

The tutorial sequence begins with a pretest (see Introduction for more details on this approach). For initial versions of the tutorial, we used a written version of the Bicycle Pump problem as a pretest. Based on our analysis of Version II of the Bicycle Pump interviews and our experiences in working with students, we discovered that application of the definition of work seemed to be difficult for students. Therefore, we later added the Mechanical Work problem to the pretest (see Figure 8-5 in Chapter 8).

9.3.2.2 Instructional sequence

Since our results have suggested that the mechanical definition of work and its connection to thermal phenomena are very difficult for students, the tutorial sequence for *The First Law of Thermodynamics* begins with an explicit review of the definition of work in a purely mechanical context. The tutorial then asks students to apply the definition to the movement of a piston in a cylinder.

Work in mechanical context. The tutorial begins by having students sketch force and displacement vectors corresponding to positive, negative, and zero work. They then generalize their responses to give a general rule for determining the sign of work. They then consider the situation from the Mechanical Work problem described in Chapter 8. In this problem, a block is pushed by a hand up a frictionless inclined plane (see Figure 9-1). As on the pretest, students are asked to determine the sign of the work done on the block by the hand, by the earth, and by the incline. They are also asked if the block in turn does work on the hand. Because of the importance of these ideas for subsequent sections, students are asked to check their answers for this section with a tutorial instructor.

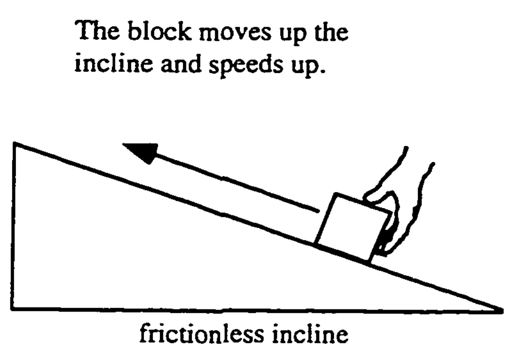


Figure 9-1: A mechanical context in which students review the definition of work in *The First Law of Thermodynamics*.

Applying the definition of work to an ideal gas process. Next, we attempted to give students help in applying the definition of work to a typical ideal gas process. In particular, students consider a sample of an ideal gas confined to a cylinder closed by a piston (see Figure 9-2). They are asked about the force on the gas by the piston and how the piston could move such that it does positive and negative work on the gas. Finally, we ask whether the gas does work on the piston in the motions they have described. These questions are designed to help students to determine the sign of the work done on the gas in various situations, and to ensure that they understand that the work done by the gas on the piston is always equal and opposite to the work done on the gas by the piston, regardless of the motion.

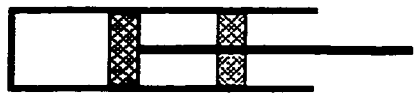


Figure 9-2: A cylinder closed by a piston. Students are asked what motion of the piston will result in positive work on the gas.

Work and changes in internal energy. The next section of the tutorial addresses student difficulties in connecting work to temperature changes. Students are first asked to make a prediction about the temperature of the gas in a cylinder during a quick compression. They then observe a similar demonstration that results in a temperature increase. After the demonstration, the tutorial introduces the concept of internal energy and a simplified form of the first law that work to the change in internal energy for a thermally insulated system. (In classes in which the demonstration is not possible, the demonstration is replaced by a statement in the tutorial that a temperature increase is observed.)

Applying the first law of thermodynamics. For the remainder of the tutorial, students apply what they have learned and extend the ideas to include heat transfer. The concept of heat transfer is introduced in the context of a temperature decrease that takes place at constant volume. In this process, no work is done, so another means of energy transfer must be invoked to account for the change in internal energy of the gas. Students are led to recognize the need for the concept of heat transfer and to differentiate heat from work. Because of the difficulties described earlier with thermal insulation, students are asked to evaluate an incorrect statement that insulation prevents a temperature change when a gas is compressed. Finally, the complete first law of thermodynamics is formulated and used in the context of an isothermal process. Students then confront another common difficulty by evaluating the incorrect statement that there is no heat transfer in an isothermal process because the temperature does not change.

9.3.2.3 Initial version of homework

In the initial version of the homework for *The First Law of Thermodynamics*, students examine isobaric and isochoric processes and determine the signs of the work done on, heat transfer to, and change in internal energy of the gas in each process. They then are asked to use the first law of thermodynamics to compare the absolute value of the heat transfer in these two processes and to use their result to compare the specific heat associated with constant-pressure processes, C_P, to the specific heat associated with constant-volume processes, C_V. In this initial version, students complete a quantitative analysis of these processes.

9.3.3 Modifications to homework

In a tutorial sequence, the homework is a place for students to apply, reflect upon, and generalize the techniques or concepts learned in the tutorial. As we show in the following section, students' ability to apply the ideas of work to processes described in terms of an experimental setup improved after the preliminary version of *The First Law of Thermodynamics*. However, students still had difficulties with processes represented on *PV* diagrams. In addition, students were still likely to confuse the concepts of work and heat. After the tutorial, many students still gave answers suggesting a failure to recognize the independence of these quantities. Therefore, we made several modifications to the homework that were designed to address these difficulties.

The revised homework includes questions designed to help students recognize how the connection between the definition of work in mechanics and the experimental processes that they had considered in the tutorial can be applied when processes are described on a *PV* diagram. For example, students are shown curves on a *PV* diagram and are asked to state whether the piston moves inward, outward, or not at all in the processes represented by the curves. For example, students are asked to consider the *PV* diagram representing a cycle shown in Figure 9-3. They are asked to compare the absolute values of the work done in processes I and III, two processes with equal volume changes but different pressures. Finally, students are asked about the sign of the work done on the gas in the complete cycle and to respond to an incorrect statement that the work done is zero because the volume returns to its original value.

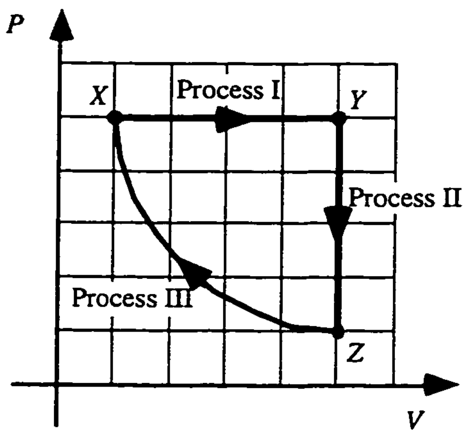


Figure 9-3: A *PV* diagram showing a cyclic process that students analyze in the revised version of the tutorial homework for *The First Law of Thermodynamics.*

We also added a series of questions to the homework designed to help students distinguish the quantities work, heat, and change in internal energy. These questions ask students to give an example of a process in which work is done on a gas but there is no heat transfer, or a process in which there is no heat transfer but the temperature changes. If the students believe that no such process is possible, they are asked to state so explicitly.

9.4 ASSESSMENT OF PRELIMINARY VERSION OF *THE FIRST LAW THERMODYNAMICS*

In this section we will examine the effectiveness of the curriculum described in the previous section. We will look at two types of written problems, both of which have been given to groups of students after standard instruction and after the use of the tutorial sequence: the Adiabatic Compression problem, and a problem based on a *PV* diagram.

9.4.1 Assessment of student ability to recognize the applicability of work: the Adiabatic Compression problem

Analysis of student responses to the interview and written problems described in Chapter 8 showed that few students spontaneously applied the first law and work to the process after standard instruction. In order to assess the effectiveness of the tutorial sequence, we administered a slightly modified version of the Adiabatic Compression problem to determine whether students were more likely to apply work and the first law of thermodynamics to this process after the tutorial.

9.4.1.1 Statement of problem and correct answer

The version of the Adiabatic Compression problem used to assess student understanding after instruction using *The First Law of Thermodynamics* was somewhat different from the question given after standard instruction. In this version, shown in Figure 9-4, the adiabatic compression followed a question involving a compression of a gas enclosed in a perfectly conducting cylinder. Although it is possible that this version of the problem called students' attention to the insulation, our experience in working with students and the difficulties that many students have with insulation suggest that highlighting the role of the insulation would not make the question significantly easier.

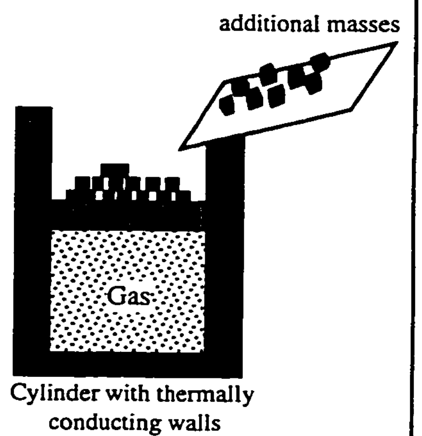
The correct answer to the various parts of the problem shown are similar to those for the problem shown in Chapter 8. In both parts of the question, the pressure of the gas will increase and the volume will decrease as masses are added to the piston. As the volume decreases, there is a force on the gas by the piston and the point of application of the force moves in the same direction as the force. Therefore, there is positive work done on the gas. For the perfectly conducting cylinder, the gas will remain in equilibrium with the room and the energy transferred to the gas in the form of work will immediately be transferred out of the gas as heat transfer to the room. Therefore, the gas will remain at room temperature. The insulating cylinder, however, prevents heat transfer to the room, so the positive work done on the gas leads to an increase in internal energy. The temperature of the gas will increase.

A cylinder constructed of perfectly thermally conducting material contains an unknown amount of an ideal gas. The cylinder is closed with a piston that is free to move but fits tightly such that no gas leaks out of the cylinder. Initially, there are a number of small masses on top of the piston. There is no friction between the piston and the cylinder walls.

A. Many additional small masses are slowly added one by one to the piston.

State whether the following quantities will increase, decrease, or remain the same. Explain.

- ***the pressure of the gas***
- ***the temperature of the gas***
- ***the volume of the gas***



B. The experiment is now repeated with the same amount of gas enclosed in a cylinder that is **perfectly insulated.** The same number of small masses are added in the same way.

State whether the following quantities will increase, decrease, or remain the same. Explain.

- ***the pressure of the gas***
- ***the temperature of the gas***
- ***the volume of the gas***

Figure 9-4: A version of the Adiabatic Compression problem.

9.4.1.2 Student responses

A summary of student responses to the Adiabatic Compression problem after the tutorial at the University of Maryland and after the ITL at the University of Washington is shown in Table 9-1. In both courses, the fraction of students who answered correctly and supported their answer with correct reasoning was substantially greater than after standard instruction at the University of Washington.[136] (There are no comparable results available from the University of Maryland after standard instruction.) Whereas after standard instruction almost no students used the first law to answer and only a few even mentioned work, after the tutorial many students explicitly referred to the first law of thermodynamics either by name or in its algebraic form. Students in this category and students who referred to work but did not explicitly refer to the first law together made up around 50% of the students in the algebra-based course and 15% of those in the calculus-based course, as compared to essentially none of the students in the section that had completed standard instruction. The frequency of the most common incorrect answer, that the temperature will would stay the same due to the presence of the insulation, was around 15% in both courses after *The First Law of Thermodynamics,* as compared to over one third in the class that had completed standard instruction. Although there are still many students who did not answer correctly, these results suggest that the instructional sequence does have a positive impact on students' ability to recognize the relevance of work and the first law of thermodynamics in this process.

There was a significant different in success rates between the students at the University of Washington and those at the University of Maryland. There are several possibilities to account for this difference. The most obvious differences between the two groups are in the level of the courses (algebra-based compared to calculus-based) and in the format of the tutorial instruction (ITLs compared to small-group tutorials). We believe that the difference in the level of the course could not account for the greater success of the students in the algebra-based course at the University of Washington, since results from other questions suggest that students in these courses perform at similar levels after standard instruction, with the calculus-based students typically performing slightly better than the algebra-based students. The difference is also probably not due to the difference in instructional format. Other studies by our group suggest that ITLs are somewhat less successful in addressing student difficulties, although those studies did not include a sequence of ITLs as long as that in this study.[137]

Two differences that are more likely to account for the difference in performance between the students in the two courses are the different appearance of the problems and the preparation of the instructors. First of all, the problems appeared in slightly different forms on the examinations in the two courses. The problem used at the University of Maryland included no diagram. This omission may have made it more difficult for students to understand the problem. Alternatively, the diagram's resemblance to those in the instructional materials may have triggered a connection for students in the course at the University of Washington, helping them to recall the association of work and the processes. Second, the details of the implementation in the two courses were quite different. The instructors in the University of Washington course were all very familiar with the student difficulties and the instructional materials, having been involved with their development. The instructors at the University of Maryland had a weekly training session, but were certainly less familiar with student difficulties than the authors of the tutorial. On results from post-tests in other topics, students in courses at pilot sites for *Tutorials in Introductory Physics* have performed at a level that is similar to that of students at the University of Washington.[138]

Table 9-1: The Adiabatic Compression problem after standard instruction and after *The First Law of Thermodynamics* in the tutorial and interactive tutorial lecture format

	Physics 115 Washington Wi '97 ($N = 179$)	Physics 115 Washington Sp '97 ($N = 194$)	Physics 262 Maryland Sp '97 ($N = 118$)
Instruction on first law:	standard	*First Law* ITL	*First Law* ttl.
Correct with correct reasoning	~0%	50%	15%
Correct without correct reasoning	10%	25%	45%
Incorrect: temperature remains the same due to insulation	35%	15%	15%
Other incorrect	50%	15%	25%

All percentages rounded to the nearest 5%.

9.4.2 Assessment of student ability to apply work to problems with *PV* diagrams

As an additional means of assessing student understanding after the tutorial, we will discuss several problems in which students were given *PV* diagrams rather than descriptions of the experimental setup. These problems showed multiple processes and asked student to find the sign of work for various processes and to compare the absolute values of the work done in two processes. We have used these problems to assess student ability to apply the definition of work to thermal processes and to use work to reason with the first law of thermodynamics. The problems involve representations of gas processes on *PV* diagrams, which was not emphasized in the tutorial (although such representations are used in the tutorial homework). Therefore, these problems often look quite different from the instructional tasks in the tutorial. We felt that such problems would be a good test of whether students could apply what they had learned in the tutorial to problems that used other representations of gas processes.

We have posed problems like those shown in Figure 9-5 on course examinations in three sections of Physics 115, one after standard instruction and two after students had completed a series of ITLs including *The First Law of Thermodynamics.* We discuss student responses to these problems in light of the criteria for student understanding that we described at the beginning of the chapter. We do not consider the first criterion (that students apply work spontaneously) because the problem asked explicitly about the work and suggested the use of the first law of thermodynamics. However, we examine the second two criteria: student ability to determine the correct sign of work in processes, and student ability to determine the absolute value of the work done in processes and distinguish work from heat and internal energy.

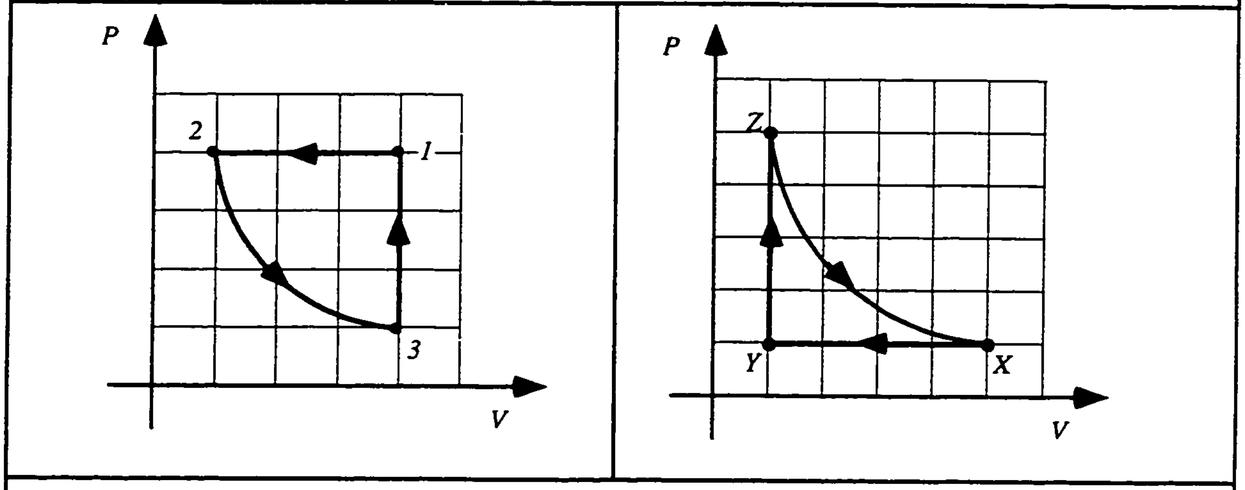
9.4.2.1 Statement of problem and correct answer

The problems that we discuss in this section all involve cyclic processes portrayed on *PV* diagrams. Two such problems are shown in Figure 9-5. Both were included on course examinations after instruction. The version using the cycle shown in Figure 9-5a was posed in a section of Physics 115 after standard instruction. The version using the cycle shown in Figure 9-5b was given to a section that had completed a series of ITLs including *The First Law of Thermodynamics.* Another section, which had completed the series of ITLs and the modified version of the tutorial homework described in Section 9.3.3, was given a similar problem with the cycle shown in Figure 9-6. The phrasing of the questions given after standard instruction were somewhat different that in Figure 9-5. The problem instead asked students to tell whether work was "done on the gas, by the gas, or no work was done" rather than asking for the sign of the work.

Below we compare student responses to the questions pertaining to the isobaric compression. In this compression, the work done on the gas is positive as the piston moves inward. Since the pressure stays the same, the temperature of the gas must decrease as the volume decreases (by the ideal gas law). Hence, the internal energy decreases in this process. Application of the first law of thermodynamics $\Delta U = Q + W$ shows that if the work on the gas is positive and the change in internal energy is negative, the heat transfer to the gas must be negative.

We have also examined student responses to questions asking about the sign of the net work done on the gas in the cycle. In the process shown in Figure 9-5a, for example, this sign can be determined by recognizing that there is positive work done on the gas in the isobaric process, negative work done on the gas in the isothermal process, and no work done in the process carried out at constant volume. Since in this case the pressure is greater in the isobaric compression than in the isothermal expansion, the absolute value of the work done in the isobaric process is greater than that in the isothermal expansion. Given the signs and relative values of these works, the work done on the gas in the entire cycle must be positive. The answers are the same for the isobaric compression in case b. In the entire cycle shown in case b, the absolute value of the work done in the isobaric compression is less than that in the isothermal expansion, so the work done on the gas in the cycle is negative.

A monatomic, ideal gas goes through a cyclic change as shown in the *PV* diagram.



State whether the following quantities are positive negative, or zero.

- ***the change in internal energy in the isobaric process***
- ***the work done on the gas in the isobaric process***
- ***the heat transfer to gas in the isobaric process***

State whether the following quantities are positive negative, or zero.

- ***the change in internal energy in the isothermal process***
- ***the work done on the gas in the isothermal process***
- ***the heat transfer to gas in the isothermal process***

Is the work done on the gas in the cycle positive, negative, or zero?

Figure 9-5: Two problems in which students are asked about energy quantities (heat, work, and change in internal energy) in processes shown on a *PV* diagram.

9.4.2.2 Student responses: sign of work

Students were asked to find the sign of work in two of the processes shown in each version of the *PV* Diagram problem. Students in the section of Physics 115 that had completed the ITL based on *The First Law of Thermodynamics* followed by the original version of the tutorial homework performed at about the same level as the class that had completed standard instruction. Approximately 60% answered correctly in each case. The students who had completed the ITL along with the modified tutorial homework performed somewhat better, with approximately 80% answering correctly. These responses are summarized in Table 9-2.

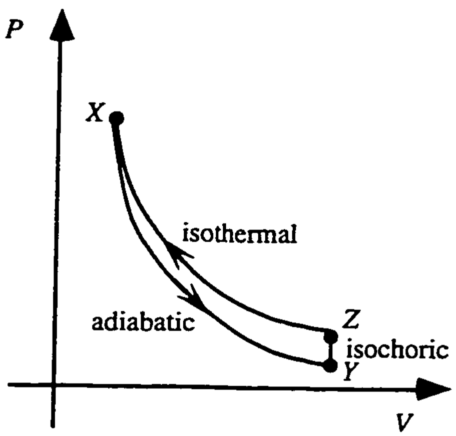


Figure 9-6: A cyclic gas process represented on a *PV* diagram. On a post-test, students were asked to determine the sign of the work done on the gas in the isothermal process, the adiabatic process, and the entire cycle. See also Figure 9-5.

The results after the modification to the homework are encouraging, as the percentage of correct responses has increased. An additional sign that the modification to the homework may have made a difference is that the reasoning given by these students was slightly different in this section. Whereas very few students used the definition of work in the Autumn '95 and Spring '97 sections, approximately 15% of the students in the Spring '98 section referred explicitly to the force and displacement in the definition of work in their responses. This result suggests that students in this section were indeed better able to connect mechanical work to work in thermal physics as represented in *PV* diagrams, and that the additional practice that these students had on such tasks in the modified version of the tutorial homework was effective.

Table 9-2: Student responses to problems requiring a determination of the sign of work in a process represented on a *PV* diagram.

	Physics 115 Washington Au'95 ($N = 111$)	Physics 115 Washington Sp'97 ($N = 201$)	Physics 115 Washington Sp'98 ($N = 122$)
Instruction	Standard	*First Law* ITL w/original HW	*First Law* ITL w/modified HW
All signs of work correct	60%	60%	80%
All signs reversed	15%	10%	10%
Other incorrect	25%	30%	10%

All percentages rounded to the nearest 5%.

9.4.2.3 Student responses: distinguishing work, heat, and internal energy

In responses to interview problems, many students were unable to distinguish the quantities work, heat, and internal energy. These difficulties also seemed to affect student responses to written problems after standard instruction. For example, in the isobaric part of the cycle problems shown in Figure 9-5, students were asked about the signs of the work done on the gas, the heat transfer to the gas, and the change in internal energy of the gas. Students often gave responses that suggested a failure to distinguish these quantities.

One common difficulty that we observed in student responses to the questions posed in the interviews was a failure to recognize the independence of heat and work. Several students made statements that whenever work is done, heat must also be transferred. In the written problem, many students gave responses to the isobaric part of the process that seemed to be consistent with this incorrect belief. These students stated that all three quantities in the first law of thermodynamics (Q, W, and ΔU) have the same sign. (Very few students said that the sign of the work and heat were the same but that the sign of the change in internal energy was different.) After standard instruction, about 30% of the students had all signs the same in the isobaric process after standard instruction. After the initial version of the *First Law of Thermodynamics* ITL, this fraction was also about 30%.

On the other hand, the ITL seemed to reduce the frequency of another common error. On the version of the cycle question given after standard instruction, we found that many students used the work done on the gas to make predictions about the internal energy, without referring to the heat transfer. These students predicted that the internal energy of the gas would increase due to the positive work done on the gas. Whereas around 40% of the class having standard instruction correctly predicted that the internal energy of the gas would decrease in the isobaric compression, nearly 60% of the class having the ITL did so. In addition, the fraction of the students predicting that the internal energy will increase in the isobaric compression, an answer consistent with a failure to consider the heat transfer, was nearly half in the class having standard instruction, as compared to approximately 25% in the class having the ITL. These results are summarized in Table 9-3. (The section of Physics 115 who completed the modified version of the tutorial homework were given the cycle shown in Figure 9-6, which does not include an isobaric process, so they did not answer a comparable question.)

Table 9-3: Student responses for the change in internal energy in the isobaric compression shown in Figure 9-5.

	Physics 115 Washington Au'95 ($N = 111$)	Physics 115 Washington Sp'97 ($N = 201$)
Instruction	Standard	*First Law* ITL
Internal energy decreases (correct)	40%	60%
Incorrect: internal energy increases because $W > 0$	50%	25%
Other incorrect	10%	15%

All percentages rounded to the nearest 5%.

9.4.2.4 Student responses: work done in a cyclic process

The preliminary version of the tutorial sequence seemed to be least successful in addressing student difficulties with the work done in a cyclic process. After the initial version of *The First Law of Thermodynamics,* with the initial homework, only about 20% of the class determined the correct sign for the net work done on the gas in the cycle. As shown in Table 9-4, this fraction is actually smaller than the fraction answering correctly in the question posed after standard instruction. However, the problem posed after standard instruction asked students to determine the numerical value of the work done in the cycle, and strongly suggested a nonzero value, asking "Is this work done on or by the system?" (See also Appendix K.) Only about 10% of the students answered that the work done in the cycle would be zero, as compared to nearly half of the students who were asked the question, "Is the work done in the cycle positive, negative, or zero?"

Regardless of whether the better performance after standard instruction was due to the wording of the question or due to the difference in instruction, we felt that we could improve student performance on this question with revisions to the tutorial sequence. This belief led in part to the modifications to the tutorial homework described in Section 9.3.3 of this chapter. In the modified homework, students are asked explicitly to compare the absolute value of the work done on the gas in two parts of a cyclic process and to determine the sign of the work done on the gas in the entire process.

The class that had completed the sequence including the ITL based on *The First Law of Thermodynamics* and the revised homework performed somewhat better in finding the sign of the work in the cycle. Approximately half of the students in this class answered correctly, with the fraction stating that the work is zero in the cycle having decreased somewhat. These results suggest that the inclusion of additional exercises in the homework can help to address some student difficulties. However, nearly half of the class still gave the common incorrect response that the work done in the cycle is zero.

Table 9-4: Student responses for the sign of work in a cyclic process.

	Physics 115 Washington Au'95 ($N = 111$)	Physics 115 Washington Sp'97 ($N = 201$)	Physics 115 Washington Sp'98 ($N = 122$)
Instruction	Standard	*First Law* ITL w/original HW	*First Law* ITL w/modified HW
Correct sign of work in cycle	40%	20%	50%
Incorrect: $W = 0$ in cycle	10%	50%	40%
Other incorrect	50%	30%	15%

All percentages rounded to the nearest 5%.

9.4.3 Summary of assessment of *The First Law of Thermodynamics*

Analysis of responses to exam questions from Physics 115 suggests that the initial version of *The First Law of Thermodynamics* was successful in many respects. Many more students used work and the first law of thermodynamics in their reasoning on the Adiabatic Compression problem. Very few students made the common error based on the idea that the insulation would prevent temperature change. Although students were more likely to spontaneously apply the concept of work to gas processes, student success in doing so in processes described on a *PV* diagram was mixed. Students were somewhat less likely to give the incorrect answers that we have attributed to a confusion between work and heat or work, heat, and internal energy. In cases in which students were asked to find the sign of work based on a *PV* diagram, either in the cyclic process or in a simple process, students did not perform significantly better after the preliminary version of the tutorial sequence.

In the initial version of the tutorial sequence, all of the examples were based on descriptions of experimental procedures. Students had little experience in the tutorial in reasoning based on the description of a process on a *PV* diagram, and no assistance in relating the absolute value of the work done in a process to the area under the curve representing that process on a *PV* diagram. Therefore, we made modifications to the homework that were designed to help students in making these connections. The performance of students after this revised tutorial sequence was an improvement over that after the initial sequence, but some difficulties seemed to persist even after these revisions. The persistence of certain difficulties led us to examine the initial knowledge of students and attempt to determine whether this initial knowledge might have been responsible for the limited effectiveness of the instructional materials.

9.5 ANALYSIS OF STUDENT RESPONSES ON MECHANICS PROBLEMS COMPARED TO RESPONSES ON THERMAL PHYSICS PROBLEMS

We have found, as shown in the previous chapter, that the level of understanding that students have about the concept of mechanical work has some impact on the ability of those students to apply work to processes in thermal physics. In classes in which we were able to do so, we have matched student responses on the Mechanical Work problem (see Chapter 8) with responses to the post-test problems described above. As shown below, in all cases, the students who entered the class with a good understanding of the definition of work (as measured by their reasoning on the Mechanical Work problem) were more likely to apply the concept of work after completing the tutorial sequence that we have described. These results suggest that the brief review of work in *The First Law of Thermodynamics* is insufficient for those students who enter the course having conceptual difficulties with work. Those students are still unlikely to apply spontaneously the concept of work in the context of thermal physics.

9.5.1 Responses on the Mechanical Work problem compared to responses on the Adiabatic Compression problem

In a section of Physics 262 at the University of Maryland, we matched students' responses on the Mechanical Work problem to their reasoning on the Adiabatic Compression problem. These students had completed the tutorial and the initial version of the tutorial homework described in this chapter and shown in Appendix L. These results are shown in Table 9-5. As the table shows, students who used the definition of work on the Mechanical Work problem were much more successful on the post-test than those who stated that the sign of work depended, for example, on the choice of coordinate system.

Table 9-5: Responses of students at the University of Maryland on the Mechanical Work problem compared to reasoning on the Adiabatic Compression problem after the tutorial *The First Law of Thermodynamics.*

	Reasoning used for sign of work by hand on Mechanical Work problem			
	Correct	Incorrect		
	Definition of work ($N = 25$)	Gravity or pot. energy ($N = 11$)	Coord. system ($N = 26$)	Other / no reas. ($N = 25$)
Correct reasoning for Adiabatic Compression	35%	25%	5%	10%
Incorrect reasoning for Adiabatic Compression	65%	70%	90%	85%

All percentages rounded to the nearest 5%.

9.5.2 Responses on the Mechanical Work problem compared to responses on the Compression Comparison problem

In a section of Physics 115 at the University of Washington, we matched students' responses on the Mechanical Work problem to their reasoning on a post-test called the *Compression Comparison* problem. The Compression Comparison problem (see Figure 9-7) is similar to the version of the Adiabatic Compression problem discussed above, but asks students to compare the results of two compressions, one with insulation and one without. A correct answer requires a recognition that positive work is done on the gas in both processes but that there is a negative heat transfer to the gas in the conducting cylinder, such that its change in internal energy is zero. Thus its temperature is less than that of the gas in the insulated cylinder. A comparison of the responses given by students to the Compression Comparison problem to the same students' responses to the Mechanical Work problem are shown in Table 9-6. As the table shows, students who had used the definition of work on the Mechanical Work problem were much more successful on the post-test than those who stated that the sign of work depended, for example, on the choice of coordinate system or on which object was 'causing' the motion. Again, those students who entered the course with a good understanding of work were more likely to refer to work in their responses after working through the tutorial.

Two cylinders, cylinder A and cylinder B, contain one mole each of ideal gas with the same initial pressure and volume. The initial temperature of each gas sample is equal to room temperature.

Cylinder A is perfectly thermally conducting, and Cylinder B is perfectly insulating. There is no friction between the piston and the cylinder walls.

Both pistons are slowly pressed inward to the same final position, as shown.

Rank the following temperatures:

- ***room temperature,***
- ***the final temperature of the gas in cylinder A, and***
- ***the final temperature of the gas in cylinder B.***

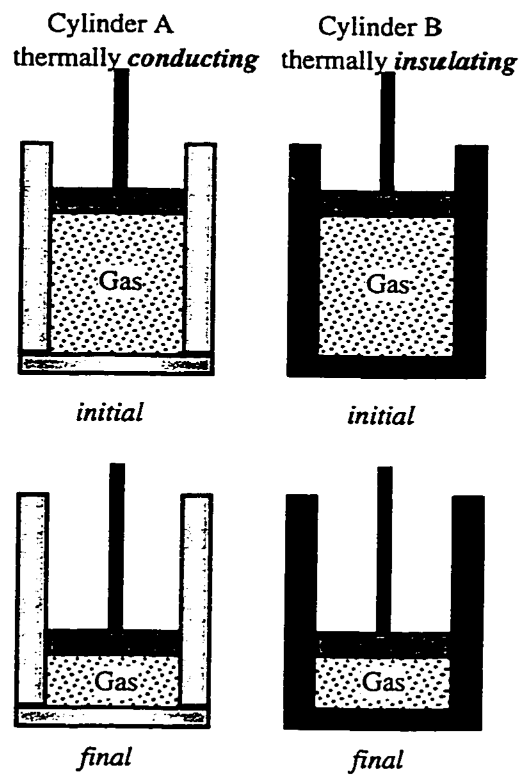


Figure 9-7: The Compression Comparison problem. This problem was used as a post-test for the ITL *The First Law of Thermodynamics.*

Table 9-6: Responses of students to the Mechanical Work problem compared to their reasoning on the Compression Comparison problem in a section of Physics 115 at the University of Washington that had completed the ITL *The First Law of Thermodynamics*.

	Reasoning used for sign of work by hand on Mechanical Work problem			
	Correct	Incorrect		
	Definition of work ($N = 27$)	Gravity or pot. energy ($N = 15$)	Coord. or cause ($N = 23$)	Other / no reas. ($N = 30$)
Correct reasoning for Compression Comparison	35%	25%	15%	15%
Incorrect reasoning for Compression Comparison	65%	75%	85%	85%

All percentages rounded to the nearest 5%.

9.5.3 Discussion

Our analysis suggests that the instructional strategies used in *The First Law of Thermodynamics* were most successful for those students who entered the course with a good understanding of mechanical work. The students who did not enter the course with this understanding performed quite poorly on the thermal physics problems even after completing the tutorial sequence. The performance of both groups is less than what we consider successful, suggesting a need for further development of the tutorial sequence.

A comparison to the results shown in **Figure 8-7** in Chapter 8 suggest that the tutorial sequence had a small positive impact on student performance. In the sections with modified instruction, approximately 10-15% of the students who entered the course without a good understanding of work applied work to the post-test problems. In comparison, almost none of the comparable group of students did so in the courses with standard instruction. In fact, the students who entered the course without a good understanding of mechanical work but completed the tutorial sequence were more successful than those who did have a good understanding of mechanical work but did had only standard instruction. However, the results suggest a need for improvement both of student preparation in mechanical work and for improvement of the tutorial sequence.

9.6 SUMMARY

Based on the results of the research discussed in Chapter 8, we developed a tutorial sequence designed to address students difficulties in applying work to gas processes. This sequence includes a set of exercises designed to provide a review of the mechanical definition of work and to help students apply this definition to gas processes. We have found that students who have completed the tutorial sequence are somewhat more likely to spontaneously apply work to gas processes, and are more successful in responding to some problems that ask explicitly about work.

There is evidence, however, that the instructional strategies that we have developed are successful for students who enter the course with the ability to apply work in mechanics contexts, but are not very successful in addressing the difficulties of other students. These results suggest that there is a need for revision of instruction on work in the introductory mechanics course. If students do not have sufficient understanding of work to apply the concept in later courses, they are unlikely to develop a meaningful understanding of topics like the first law of thermodynamics that build upon the concept of work. Although the approach taken here, that of spending some time in the thermal physics course reviewing the concept of work and building a bridge to thermodynamic work, seems like a reasonable one, it seems that this brief review of work is insufficient for many students. We have begun to use the findings of this research to modify instruction in the introductory mechanics course. In particular, a modified tutorial sequence on work is in the process of being developed and tested in the calculus-based introductory mechanics course at the University of Washington.

We believe that the results described above suggest that some review of mechanical work is useful for students before they study the first law of thermodynamics, even though such a review alone may not help those students who never developed a good understanding of the definition of work. Revision of instruction on work in the mechanics portion of the course may be a prerequisite for greater gains in student performance on thermal physics problems. Unfortunately, it is often the case that instruction on mechanical work takes place in a different quarter or semester from instruction on thermal physics. Therefore, further improvement of the instructional strategies used in *The First Law of Thermodynamics* is necessary. In addition, we have observed that after standard instruction many of the students who did not apply the concept of work instead based their answers on the ideal gas law. However the questions we have asked cannot be answered on the basis of the ideal gas law alone, despite the fact that there is insufficient information to use the ideal gas law. Additional research in student understanding of the applicability of work and in student ability to reason with the ideal gas law are in progress. In particular, we are investigating the ability of students to reason with multiple variables in the ideal gas law and to recognize when the ideal gas law is insufficient for making predictions.[139]

10. CONCLUSION AND IMPLICATIONS FOR INSTRUCTION

In this dissertation, we have described results from our investigation of student understanding of hydrostatics and thermal physics. This work is part of an ongoing project to develop a research base on the teaching and learning of physics that can serve as a resource for instructors and developers of curriculum.

We identified several specific difficulties with hydrostatic pressure, buoyancy, and the first law of thermodynamics that are commonly held by students in introductory university physics courses. Since many of these difficulties seemed to be connected to difficulties with the underlying mechanics concepts, we broadened our initial investigation to focus on student understanding of these mechanics concepts. We found that several of the conceptual difficulties identified in the contexts of liquids and gases seemed to be rooted in difficulties with purely mechanical contexts, many of which had not previously been described in the research literature. These results have been used to develop instructional materials, which we have tested, modified, and assessed in several instructional settings.

We have found that a crucial aspect of this research is the development of appropriate questions to be used as research and instructional tasks. The questions that we have found to be most useful share certain characteristics. First, these questions are almost always qualitative rather than quantitative. Although some quantitative questions elicit interesting responses from students, we have found that student responses are often very difficult to interpret. They consist primarily of mathematical manipulation that allows little insight into how students are thinking about the physical situation. Second, these problems ask students to make predictions about physical quantities, rather than to give definitions or descriptions of the significance of the quantity. We have seen students who are able to recite correct definitions or statements of physical laws but who cannot apply these ideas to simple physical systems. Finally, the most useful problems are those that ask students for explanations. We have found many situations in which students can arrive at a correct answer for a problem but then provide an explanation that is completely incorrect, casting doubt on the significance of the correct answer.

Our initial investigations revealed the existence of several specific conceptual difficulties that proved to be quite persistent. For example, we developed a written problem called the Five Blocks problem to probe student understanding of buoyancy. In response to the Five Blocks problem, the majority of students gave a single incorrect answer in which two blocks of different densities were shown at rest beneath the surface of a liquid. The percentage of students giving this answer was nearly identical in administrations of the problem before and after standard instruction. Not only were the answers given by students very similar, but the distribution of types of reasoning was also quite similar. It seems that standard instruction does not change the way most of the students think about this particular problem. Similar results have been reported on student responses to problems in other contexts as well.

Our results suggest that students are often distracted by surface features of problems. On problems requiring the ranking of pressures at various points in a static liquid, the answers given by students varied considerably depending on the shape of the container and the location of the points relative to container walls. In some cases, we observed students using different types of reasoning for pressure rankings within a single ungraded quiz. This result is consistent with previous research results suggesting that novice learners often focus on the details of problems rather than applying a consistent approach to related problems.

Students generally failed to apply the relevant physical principle to simple real-world situations. For example, we performed a series of interviews with a bicycle pump, in which we sought to investigate student ability to apply the first law of thermodynamics to the quick compression of the air inside the pump. In these interviews, almost none of the students correctly applied the first law of thermodynamics to the compression of the pump. Students instead gave answers based on the ideal gas law or on a microscopic model for a gas, even though these forms of reasoning were insufficient to allow a correct prediction.

Again, the failure of students to apply general principles is well-documented in the literature on physics education research.

Many of the incorrect ideas that students used in the contexts of hydrostatics and thermal physics seemed to be connected to incorrect understanding of principles from introductory mechanics. In each of the contexts that we have examined, there were difficulties with key ideas that seemed to be manifestations of underlying errors in mechanics. For example, students who predicted that blocks would be suspended beneath the surface of a liquid in the Five Blocks problem often supported their incorrect answer by making statements that suggested a connection between the net force on a block and its position, rather than its acceleration. Similarly, students giving incorrect pressure rankings for points in a liquid often referred to the weight of liquid above the points, while neglecting the effects of the forces by container walls on the liquid. In the bicycle pump interviews, most of the students made incorrect statements about the work done on the gas in the compression that suggested difficulties with mechanics.

We posed problems in purely mechanical contexts in order to determine whether the difficulties that we had seen were due to a failure of students to understand these ideas in the context in which they were originally learned or due to a failure to transfer an understanding of mechanics to the new contexts of gases and liquids. We found that students had many of the same difficulties in mechanical contexts. For example, even in purely mechanical situations involving springs or pulleys, students gave answers suggesting a belief that the net force on an object is related to the object's position rather than to its acceleration. Similarly, we found that there was a correspondence between several student difficulties with the concept of work that we identified in the bicycle pump problem and incorrect responses to problems that were posed in the purely mechanical context of a block on an inclined plane.

To probe the extent of the connection between mechanics concepts and difficulties in the contexts of gases and liquids, we examined the details of student responses on matched pairs of problems, one of which involved a context in hydrostatics or thermal physics, and one of which involved a mechanical context. We found that in most cases there were strong relationships between student responses on the two questions. Students who had conceptual difficulties with the mechanics problems often did very poorly on the problems in the new contexts. Students who answered the mechanics problems correctly performed at a higher level than those who did not, even though the two groups had very similar final course grades. However, some of the students whose answers indicated a good understanding of the mechanics problems had difficulty in the new context, suggesting that transferring this understanding to new contexts is also difficult for many students.

We have developed instructional materials for each of the topics that we have examined in this dissertation. These instructional materials include exercises designed to help students review and further develop their understanding of the relevant concepts from mechanics. We have tested the materials in different instructional formats including small-group tutorials, large-group interactive tutorial lectures, and, in some cases, laboratory-based courses.

Our experience has shown that an instructor's intuition is not necessarily a good guide to effective instructional strategies. In some cases, the nature and extent of student difficulties are surprising even to experienced instructors. In addition, the exercises and observations that logically lead an instructor to the correct answer may not be effective in helping students to arrive at the same result. Several of our early efforts in curriculum development proved to be very poorly matched to the needs of students, despite being informed by teaching experience and initial studies of student understanding. Some unsuccessful strategies have even received positive feedback from students. These observations highlight the need for rigorous assessment of instructional materials by testing student performance rather than by student enthusiasm or other subjective evaluations.

After students had completed instruction using the materials that we developed, we posed examination problems designed to test student conceptual understanding. By comparing student performance on these questions to that after standard instruction, we were able to assess the effectiveness of the curriculum. In several cases, we have found that student performance on these problems is significantly better in courses that used our curriculum materials than in courses in which students had completed standard instruction. In other cases, we found that some conceptual difficulties remained even after the modified instruction, suggesting the need for continuing development of the instructional materials. Subsequent research-based modifications to the curriculum have led to further improvements in student performance.

In the case of the curriculum on the first law of thermodynamics, we found that our efforts to review the mechanics ideas necessary for use of the first law were inadequate for many of the students. Most of the students who entered the course without a good understanding of mechanical work were still unable to apply the concept of work to gas processes after completing the tutorial. We conclude that a brief review of a concept as difficult as mechanical work is simply insufficient for many students to develop the understanding needed to apply this concept to the very difficult context of gas thermodynamics. This result suggests that modifications to instruction on work in the introductory mechanics course may be necessary before instruction in thermal physics can help students to achieve meaningful understanding the first law of thermodynamics. The results of this research have influenced modifications to a tutorial sequence on work that is used in the introductory calculus-based course at the University of Washington.

The research that we have conducted in the contexts of hydrostatics and thermal physics have led to new understanding of the nature of student difficulties with mechanics. We have found that the new contexts of gases and liquids often elicit student difficulties that are different from the well-known difficulties that students have with point masses. For example, we found that many students associate the force on an object with its position or displacement. For many specific difficulties, we have been able to develop problems that elicit the same types of responses in purely mechanical contexts. This result suggests that the difficulties are not particular to the new contexts of gases and liquids, but rather are general difficulties in mechanics that have not been previously described in the literature on physics education research. This study has thus been useful in expanding the research base on student understanding of mechanics as well as that on student understanding of liquids and gases.

The efforts that we have described provide an example of the three-part process that the Physics Education Group has used to improve the teaching and learning of physics based on the results of research. This process of research into student difficulties, development of curriculum to address these difficulties, and assessment of student performance after completing instruction is an iterative one, often involving several cycles of revision and assessment. By following this model, we hope to make cumulative improvements to curriculum and build upon the results of our own research as well as that of others. It is our expectation that physics instruction at all levels will benefit from the scrutiny of physics education researchers and from the systematic application of this model for research-based curriculum development.

End notes:

1 An example of a difficulty that seems to be connected to instruction can be found in the first paper in ref. 13. Students in an introductory optics course were shown a lens that was producing a real image of the filament of a light bulb on a screen. Many students predicted that covering the top half of the lens would remove half of the image, often citing the principal rays algorithm that is typically taught in the class.

2 For evidence in support of this statement, see, for example, L.C. McDermott and P.S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity, Part I: Investigation of student understanding," *Am. J. Phys.* **60,** 994–1003 (1992); Printer's erratum to Part I, *Am. J. Phys.* **61**, 81 (1993), P.S. Shaffer and L.C. McDermott, "Research as a guide for curriculum development: An example from introductory electricity, Part II: Design of instructional strategies," *Am. J. Phys.* **60**, 1003–1013 (1992), and L. C. McDermott, "Millikan Lecture 1990: What we teach and what is learned—Closing the gap," *Am. J. Phys.* **59**, 301-315 (1991).

3 See, for example, R. R. Hake, "Interactive engagement versus traditional methods: a six-thousand-student study of mechanics test data for introductory physics courses," *Am. J. Phys.* **66,** 64-74 (1998).

4 McDermott, L. C., P. S. Shaffer, and M. L. Rosenquist, *Physics by Inquiry* (John Wiley & Sons, New York, 1996).

5 L.C. McDermott, P.S. Shaffer, and the Physics Education Group, *Tutorials in Introductory Physics,* Preliminary Edition, (Upper Saddle River, NJ, Prentice Hall, 1998).

6 For descriptions of the special courses for preservice teachers developed by the Physics Education Group, see L.C. McDermott, "A perspective on teacher preparation in physics and other sciences: The need for special courses for teachers," *Am. J. Phys.* **58,** 734–742 (1990), and L.C. McDermott and L.S. DeWater, "The need for special science courses for teachers: two perspectives," to be published in *Inquiring into Inquiry in Science Learning and Teaching* (AAAS, 1999).

7 For descriptions of the use of this curriculum with underprepared students, see, for example, L.C. McDermott, L. Piternick, and M.L. Rosenquist, "Helping minority students succeed in science: I. Development of a curriculum in physics and biology; II. Implementation of a curriculum in physics and biology; III. Requirements for the operation of an academic program in physics and biology," *J. Coll. Sci. Teach.* **9**, 135–140 (January 1980), 201–205 (March 1980), 261–265 (May 1980), or L.C. McDermott, M.L. Rosenquist, and E.H. van Zee, "Instructional strategies to improve the performance of minority students in the sciences," *New Directions for Teaching and Learning* **16**, 59–72 (1983).

8 For a description of the modifications to the calculus-based introductory course at the University of Washington that led to the development of tutorials, see, for example, P.S. Shaffer, "Research as a guide for improving instruction in introductory physics,"

Ph.D. dissertation, Department of Physics, University of Washington, R. R. Harrington, "An investigation of student understanding of electric concepts in the introductory university physics course," Department of Physics, University of Washington, P. Kraus, "Promoting active learning in lecture-based courses: demonstrations, tutorials, and interactive tutorial lectures," Department of Physics, University of Washington, Ph.D. dissertation, and K. Wosilait, "Research as a guide for the development of tutorials to improve student understanding of geometrical and physical optics," Ph.D. dissertation, Department of Physics, University of Washington, as well as the papers in reference 9.

9 See K. Wosilait, P.R.L. Heron, P.S. Shaffer, and L.C. McDermott, "Development and assessment of a research-based tutorial on light and shadow," *Am. J. Phys.* **66**, 906-913 (1998), K. Wosilait, P.R.L. Heron, P.S. Shaffer, and L.C. McDermott, "Addressing student difficulties in applying a wave model to the interference and diffraction of light," accepted for publication in *Physics Education Research:* A supplement to the *American Journal of Physics,* and L.C. McDermott, P.S. Shaffer and M.D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," *Am. J. Phys.* **62**, 46–55 (1994). for more description of this approach for addressing student difficulties.

10 See ref. 6.

11 See, for example, ref. 8 and ref. 9.

12 See, for example, L.C. McDermott, "Research on conceptual understanding in mechanics," *Physics Today* **37**, 24–32 (July 1984) or L.C. McDermott and E.F. Redish, "Resource letter on Physics Education Research," to be published in *Am. J. Phys.* (1999).

13 For descriptions of the Physics Education Group's use of interviews in other contexts, see, for example, L.C. McDermott and F.M. Goldberg, "An investigation of student understanding of the real image formed by a converging lens or concave mirror," *Am. J. Phys.*, **55**, 108-119, 1987, and F.M. Goldberg and L.C. McDermott, "Student difficulties in understanding image formation by a plane mirror," *The Phys. Teach.* **11**, 472-480, 1986.

14 See, for example, B. Inhelder and J. Piaget, *The Growth of Logical Thinking from Childhood to Adolescence,* translated by A. Parsons and S. Milgram, 1958, Basic Books.

15 D.C. Giancoli, *Physics,* 5th. edition, (Prentice Hall, Upper Saddle River, NJ., 1998).

16 R. Resnick, D. Halliday, and K. S. Krane, *Physics,* 4th edition, (John Wiley & Sons, New York, 1992).

17 Since the pressure varies with depth, this argument is in general true only when the area of the side surfaces becomes infinitesimally small. In an incompressible fluid, the pressure varies linearly with depth, so the pressure at the center of the surface is equal to the average pressure over the surface. Multiplying this average pressure by the area of the surface will recover the correct magnitude for the force on the surface by the

water. In the diagram shown in Figure IA-1, we have shown the forces acting below the center of the surface, at the height of the centroid of the loading curve.

18 The fact that a vertical force and a horizontal force exerted at the same depth in a liquid on surfaces of equal area will have equal magnitudes is not trivial for many students. Further discussion of this issue can be found in A. B. Arons, *Teaching Introductory Physics* (John Wiley and Sons, New York, 1997), pp. 327-329.

19 Mercury has an extremely small vapor pressure. The 30th edition of the *Handbook of Chemistry and Physics* (Chemical Rubber Co., Cleveland, 1947) lists a vapor pressure of 0.001201 mm Hg at room temperature.

20 E. Engel Clough and R. Driver, "What do children understand about pressure in fluids?" *Res. Sci. Tech. Ed.* **3**, 133-143 (1985).

21 P. A. Giese, "Misconceptions about water pressure," in *Second International Seminar: Misconceptions and Educational Strategies in Science and Mathematics,* edited by J. D. Novak (Cornell University, Ithaca, NY, 1987), pp. 141-148.

22 P. Karitoglue and D. Psillos, "Pupils' pressure models and their implications for instruction," *Res. Sci. Tech. Ed.*, **11**, 95-108 (1993).

23 F. F. Camacho and L. G. Cazares, "Partial possible models: An approach to interpret students' physical representation," *Sci. Ed.* **82**, 15-29 (1998).

24 See, for example, A. Wilson, "The Hydrostatic Paradox," *The Physics Teacher* **33**, 538-539 (1995), and the references therein.

25 A. Arons, *Teaching Introductory Physics* (John Wiley and Sons, New York, 1997), p. 327ff.

26 We have, in some cases, asked students explicitly to explain the effect of the vapor pressure of the liquid in the barometer on the pressure measurements. We have found that very few students had a working knowledge of the term 'vapor pressure' and that few were even able to predict whether the vapor pressure would increase or decrease the height of the mercury column for a given value of atmospheric pressure.

27 Although the origin of this phrase is unknown, Texas singer-songwriter Butch Hancock played upon it in the title of his album "Firewater...Seeks its own Level," (Rainlight Records, 1980).

28 R. Resnick, D. Halliday, and K. S. Krane, *Physics,* 4th edition, (John Wiley & Sons, New York, 1992), page 381.

29 A generalization that can be made based on the results of physics education research is that novice learners, including many if not most introductory students, tend to approach problems based on the surface features of the problem rather than the general principles involved. See, for example, F. Reif and S. Allen, *Cognition and Instruction* **9**, 1-44 (1992), or J. Larkin and F. Reif, *Eur. J. Sci. Ed.*, **1**, 191-203 (1979).

30 If one is discussing absolute pressure, downward forces exerted by the atmosphere can also cause pressure to be different from that due to the weight of the liquid above a point.

31 One of the seven students said that the pressure at point X would be equal to the pressure at point Y, but that the pressure at point Z would be less. All of the others gave the ranking $P_X > P_Y > P_Z$.

32 The lecture demonstration, often referred to as the Equal-Level Tubes, involves four tubes of different shapes that are connected to a common reservoir of liquid. The tubes contain a colored liquid that is readily visible. Students are expected to observe that the free surfaces in the four tubes are all at the same level. The demonstration had just been shown and was in fact still visible in the front of the room as students answered the Different-Diameter U-tube problem. P. Kraus has cited the students' failure as evidence that many students have difficulty in correctly making observations during lecture demonstrations, P. Kraus, "Promoting Active Learning in Lecture-Based Courses: Demonstrations, Tutorials, and Interactive Tutorial Lectures," Ph.D. dissertation, Department of Physics, 1997 (unpublished).

33 Some of the students whom we have classified as answering based on the weight of the material above the points stated that the pressures at points Y and W in the Capped U-tube would be equal because there is the same amount of water above these two points.

34 Some of the students who gave this response referred only to the different area of the left and right tubes. For these students, it is not clear whether they are using 'along the tube' reasoning or simply responding based on rote use of the formula $P = F / A$.

35 P. Karitoglue and D. Psillos, "Pupils' Pressure Models and their Implications for Instruction," *Research in Sci. and Tech. Ed.* **11,** p. 95-108 (1993).

36 Approximately 40% of the students in the class did not answer this question, which was the last on a long ungraded quiz. These students have not been shown in the table.

37 L.C. McDermott, P.S. Shaffer and M.D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," *Am. J. Phys.* **62,** 46–55 (1994).

38 Such a confusion between pressure and density has been seen in the context of gases (see Kautz, C. H., "Investigation of student understanding of microscopic and macroscopic behavior of the ideal gas," Ph.D. dissertation, Department of Physics, University of Washington, 1999 (unpublished), and is surmised by Kariotoglou and Psillos in the context of liquids (see ref. 35).

39 See, for example, Halliday, Resnick, and Krane, *Physics*, p. 383: "Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel. That is, if you increase the external pressure on a fluid at one location by an amount ΔP, the same increase in pressure is experienced everywhere in the fluid."

40 E. Engel Clough and R. Driver, "What Do Children Understand About Pressure in Fluids," *Research in Sci. and Tech. Ed.* **4,** 299-309 (1985).

41 In cases where students have studied Bernoulli's equation, there is an increased potential for confusion. Bernoulli's equation is typically written

$P_0 + \rho gz + 1/2\rho gv^2 =$ constant. In this case, however, the z in the equation is measured with positive z being up, unlike the h in $P = P_0 + \rho gh$. Neither of the courses taught at the University of Washington typically cover Bernoulli's equation. However, the introductory courses at Purdue and Harvard do, and we have seen responses to hydrostatic pressure questions that suggest that students are confused by Bernoulli's equation.

42 We have also posed several problems in which students were asked to compare the pressures of points at or below the water surface to atmospheric. Students performed reasonably well on these problems. For example, on a Letter Tube problem, students in two sections of Physics 224 ($N = 70$) were asked to compare to atmospheric the pressures at two points, one at the same level as the free surface, and one below. In the two sections, 85% of the students answered correctly for the point below the free surface and 80% did so for the point at the same level as the free surface, with 70% answering both parts correctly.

43 In one of these cases the students had even taken an ungraded written quiz on which this question was included. The instructor then explicitly covered the correct reasoning for the pressure comparison and such a question was included on a midterm examination.

44 See, for example, A. Wilson, "The Hydrostatic Paradox," *The Physics Teacher* **33**, 538-539, 1995.

45 The forces on the sides should be drawn at the height of the centroid of the loading curve.

46 The force exerted by the atmosphere on the top surface of the liquid is not shown. As the problem asks specifically about the gauge pressure, it suggested that students should neglect the effects of atmospheric pressure. As is noted above, this analysis neglects the force exerted by the atmosphere on the top surface of the liquid.

47 As noted above, the solution to the problem provided by the instructor neglects forces exerted by the atmosphere. The majority of students did not include such forces, with only 7 students, or fewer than 20% of the class, mentioning a force by the atmosphere. Five of these seven either correctly took into account the forces by the container walls or neglected both the force by the atmosphere and the downward contribution of forces by the container walls, so that their consideration of the atmosphere does not affect the comparison in Table 4-1. The other two in this category correctly stated that the force on the water is greater than the weight of the water but attributed this inequality solely to the force by the atmosphere, neglecting the forces by the container walls. The asterisks in Table 4-1 indicate the responses of these students, which have been judged to include incomplete reasoning.

48 The existence of a force on the top of the oil by the stopper depends on their being a nonzero pressure at the top of the oil. The fact that the leftmost tube is filled with water to the top suggests that there is a nonzero pressure at the top of the oil. The point just below the stopper in the leftmost tube is less than the pressure at the point just below the stopper in the rightmost tube (see Chapter 1 for a similar argument applied to points

at the same level in liquids of different densities). Therefore, even if the pressure just below the leftmost stopper is zero, the pressure just beneath the rightmost stopper cannot be.

49 The average grade in the course for those students who drew correct free-body diagrams was 2.7, as compared to 3.2 for those who neglected the force exerted by the stopper on the oil and 3.0 for those who made errors based on difficulties with Newton's third law. The overall course mean grade was 2.81 and the standard deviation was 0.82.

50 See, for example, P. S. Shaffer, "Research as a guide for improving instruction in introductory physics," Ph.D. dissertation, Department of Physics, University of Washington, 1993, (unpublished). Even graduate students in physics made this error.

51 In addition, the students in these classes performed similarly on the Springs and Atwood's problems described in Chapter 6.

52 For example, in the Spring '97 section of Physics 115B, the students whose free-body diagrams showed the weight of the upper book as a force acting on the lower book had an average grade of 2.86, as compared to the course mean of 2.84.

53 In some ways this reasoning is similar to the use of a generalized coordinate system in advanced mathematical treatments of classical mechanics (e.g., Lagrangian dynamics). However, in Lagrangian dynamics, a well-established procedure exists for determining the equations of motion in terms of the generalized coordinate system. These students seem to be using this 'coordinate system' in conjunction with non-Newtonian ideas that relate the force on an object to its position or displacement.

54 D. C. Giancoli, *Physics,* 5th. ed., (Prentice Hall, Upper Saddle River, NJ., 1998).

55 McDermott, L. C., P. S. Shaffer, and M. D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," *Am. J. Phys.* **62**, 46-55 (1994).

56 This problem is also used as a pretest and an instructional task in the tutorial *Forces* in *Tutorials in Introductory Physics* (see ref. 2).

57 L.C. McDermott, P.S. Shaffer, and the Physics Education Group, *Tutorials in Introductory Physics,* Preliminary Edition, Upper Saddle River, NJ, Prentice Hall, 1998.

58 In order to have comparable groups, we have included student responses after standard instruction on a W-tube and an N-tube and not those on the Capped U-tube. The results for the Capped U-tube in Physics 224 are very similar, however, with 70% of students answering correctly.

59 One of these problems was a W-tube. The other was a tube that we have called the 'Rune Tube,' which looks like a horizontal reflection of the N-tube.

60 For other examples of uses of TA results as an goal for introductory students, see, for example, P. S. Shaffer, "Research as a guide for improving instruction in introductory

physics," Ph.D. dissertation, Department of Physics, University of Washington, 1993, (unpublished).

61 Personal communication, K. Krane.

62 If the object is in contact with multiple fluids, as in the case of an object floating in a liquid with part of its volume extending above the liquid surface, Archimedes' principle can be generalized to relate the magnitude of the buoyant force on an object to the weight of the fluids (liquids and gases) displaced by the object.

63 B. Inhelder and J. Piaget, *The Growth of Logical Thinking from Childhood to Adolescence* (Basic Books, New York, 1957).

64 J. W. Renner, *Evaluating Intellectual Development using Written Responses to Selected Science Problems,* A Report to the NSF on Grant Number EPP75-19596, "Analysis of Cognitive Processes."

65 J. McKinnon,, "The Influence of a College Inquiry-Centered Course in Science on Student Entry into the Formal Operational Stage," Ph.D. dissertation, University of Oklahoma (1971).

66 J. W. Renner and A. E. Lawson, "Promoting Intellectual Development Through Science Teaching," *Phys. Teacher* **11**, p 273-276 (1973).

67 J. McKinnon, "Earth Science, Density, and the College Freshman," *Journal of Geol. Ed.* 19, 218-220 (1971). See also ref. 65.

68 M.L. Rosenquist, "Improving preparation for college physics of minority students aspiring to science-related careers," Ph.D. dissertation, Department of Physics, University of Washington (1982), p. 74, or the papers in ref. 7.

69 R. Driver, E. Guesne and A. Tiberghien (eds.), *Children's Ideas in Science* (Open University Press, Milton Keynes, 1985).

70 Driver, R., ed., *Making Sense of Secondary Science: Research into Children's Ideas* (Routledge, London, 1994).

71 See ref. 67, p. 219.

72 E. Duckworth, "Inventing density," North Dakota Study Group on Evaluation report (Center for Teaching and Learning, University of North Dakota, 1986).

73 See, for example, the curriculum described in R. Karplus, "Physics for Beginners," *Phys. Today,* June 1972, pp. 36-46.

74 This research is described in greater detail in M.E. Loverude, "Investigation of student understanding of hydrostatics and thermal physics and of the underlying concepts from mechanics," Ph. D. dissertation, Department of Physics, University of Washington, 1999 (unpublished).

75 Although the emphasis in the second-year course described here is on thermal physics, the first two weeks are typically devoted to the study of fluid statics, with some instructors also including a brief treatment of fluid flow and Bernoulli's equation.

76 J.E. Penick, "The Mysterious Closed System," *The Science Teacher* **60** (2), 30-33 (1993).

77 The algebra-based introductory course currently uses D.C. Giancoli, *Physics,* 5th. ed., (Prentice Hall, Upper Saddle River, NJ., 1998). The second-year course uses R. Resnick, D. Halliday, and K.S. Krane, *Physics,* 4th ed. (John Wiley & Sons, Inc., New York, 1992), which is also the text for the calculus-based introductory course at the University of Washington.

78 L.C. McDermott and the Physics Education Group at the University of Washington, *Physics by Inquiry,* Vols. I and II, (John Wiley & Sons, Inc., 1995).

79 Individual demonstration interviews are described further in L.C. McDermott and F.M. Goldberg, "An investigation of student understanding of the real image formed by a converging lens or concave mirror," *Am. J. Phys.* **55**, 108-119 (1987); F.M. Goldberg and L.C. McDermott, "Student difficulties in understanding image formation by a plane mirror," *The Phys. Teach.* **11**, 472-480 (1986), as well as ref. 92. For other examples of our group's research, see Lawson, R.A. and L.C. McDermott, "Student understanding of the work-energy and impulse-momentum theorems," *Am. J. Phys.* **55**, 811-817 (1987), D.E. Trowbridge and L.C. McDermott, "Investigation of student understanding of the concept of acceleration in one dimension," *Am. J. Phys.* **49**, 242–253 (1981), D.E. Trowbridge and L.C. McDermott, "Investigation of student understanding of the concept of velocity in one dimension," *Am. J. Phys.* **48**, 1020–1028 (1980).

80 In the first version of the Five Blocks problem, the statement that block 2 is 'barely floating' was not included. Some students in this class drew block three floating at a position slightly below that of block 2, arguing that its density was greater than that of block 2 but still less than that of water.

81 If the water is assumed to be compressible, the density of the water will be greater near the bottom of the tank. In that case, the blocks may come to rest at equilibrium positions beneath the surface of the liquid. In order to avoid this issue, students are directed in the Five Blocks problem to assume that the water is incompressible. In the first version of the problem, this statement was not included. Two sections taught by the same instructor were given the Five Blocks problem as an ungraded quiz. In one version, the statement that the water was incompressible was included; in the other, it was not. The classes took the quiz at similar points in instruction. The results were virtually identical, suggesting that the density gradient in a compressible liquid is not a significant factor in student responses.

82 For other instances in which student responses were similar before and after instruction, see, for example, L. C. McDermott, "Millikan Lecture 1990: What we teach and what is learned—Closing the gap," *Am. J. Phys.* **59**, 301-315 (1991), or L. C. McDermott, "Guest comment: How we teach and how students learn—a mismatch?," *Am. J. Phys.* **61**, 295-298 (1993).

83 The failure of many students to reason successfully with the densities of the blocks is consistent with the results reported by J. McKinnon and J.W. Renner, "Are colleges

concerned with intellectual development," *Am. J. Phys.* **39**, 1047-1052 (1971), and J. McKinnon, "Earth Science, Density, and the College Freshman," *J. Geol. Ed.* **19**, 218-220, where fewer than one-third of a group of first-year college students were able to relate the density of objects to the sinking and floating behavior of the objects. McKinnon and Renner did not investigate student use of forces.

84 The 30% of students includes only those whose vectors were clearly not equal.

85 See, for example, L. Viennot, "Spontaneous reasoning in elementary dynamics," *Eur. J. Sci. Educ.* **1**, 205-221 (1979), A. Champagne, L. Klopfer, J. Anderson, "Factors influencing the learning of classical mechanics," *Am. J. Phys.* **48**, 1074-1079 (1980), and J. Clement, "Students' preconceptions in introductory mechanics," *Am. J. Phys.* **50**, 66-71. A summary of research on student understanding of mechanics can be found in L.C. McDermott, "Research on conceptual understanding in mechanics," *Phys. Today* **37** (7), 24-32 (1984).

86 R. Gunstone, *Am. J. Phys.* **55**, 691 (1987), R. F. Gunstone and R. T. White, *Sci. Ed.* **65**, 291 (1981).

87 C. Kautz, "Investigation of student understanding of the macroscopic and microscopic behavior of an ideal gas," Ph.D. dissertation, Department of Physics, University of Washington, 1999 (unpublished), L.G. Ortiz, "An investigation of student understanding of rigid-body dynamics and static equilibrium," Ph.D. dissertation, Department of Physics, University of Washington, to be completed 2000.

88 For additional evidence that students confuse the concepts of pressure and weight, see ref. 74 as well as E. Engel Clough and R. Driver, "What do Children Understand about Pressure in Fluids," *Res. Sci. Tech. Ed.* **3**, 133-143 (1985), and P. Kariotoglou and D. Psillos, "Pupils' Pressure Models and their Implications for Instruction," *Res. Sci. Tech. Ed.* **11**, 95-108 (1993).

89 In nearly all cases, we have found that the results on a particular question are essentially the same whether the question is administered as an ungraded quiz or as part of a graded course examination.

90 L.C. McDermott, P.S. Shaffer, and M.D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," *Am. J. Phys.* **62**, 46-55 (1994).

91 For example, in response to a quantitative examination problem given after traditional instruction, only slightly more than half of the students in a section of the introductory course were able to determine correctly the magnitudes of the two forces acting on a floating piece of ice when given the mass of the ice.

92 For other examples in which students seemingly apply formulas without understanding the terms in the formula, see B.S. Ambrose, P.S. Shaffer, R.N. Steinberg, and L.C. McDermott, "An investigation of student understanding of single-slit diffraction and double-slit interference," *Am. J. Phys.* 67, 146-155 (1999) and L.C. McDermott and P.S. Shaffer, "Research as a guide for curriculum development: An example from

introductory electricity, Part I: Investigation of student understanding," *Am. J. Phys.* **60,** 994–1003 (1992); Printer's erratum to Part I, *Am. J. Phys.* **61**, 81 (1993).

[93] L.C. McDermott, P.S. Shaffer, and the Physics Education Group at the University of Washington, *Tutorials in Introductory Physics,* Preliminary Edition (Prentice Hall, Upper Saddle River, NJ, 1998).

[94] P.S. Shaffer and L.C. McDermott, "Research as a guide for curriculum development: An example from introductory electricity. Part II: Design of an instructional strategy," *Am. J. Phys.* **60**, 1003-1013, (1992), as well as ref. 90.

[95] See also similar research tasks in the papers in refs. 86 and 90.

[96] These results are consistent with those reported by Gunstone in ref. 86. In a study performed with students in an introductory course, they found that, when shown an Atwood's machine constructed of a block and a bucket of sand of the same mass, many predicted the system would return to an 'equilibrium' position where the two objects of the same weight would be at rest at the same height.

[97] For related results, see the papers in ref. 83.

[98] Similar student difficulties in distinguishing mass and volume are described in M.L. Rosenquist," Improving preparation for college physics of minority students aspiring to science-related careers," Ph.D. dissertation, University of Washington, p. 74 (1982). In a course designed to prepare students for mainstream university science courses, nearly 50% of the students made incorrect predictions on a task similar to the Displaced Volume problem. See also ref. 1.

[99] Strictly speaking, this version of *Buoyancy* is not the initial version. An earlier version was used as an interactive lecture tutorial (ITL) in two sections of Physics 115 and in one section of Physics 224, the second-year course on hydrostatics and thermal physics course at the University of Washington. Student responses on post-tests after this version were no different than those after traditional instruction.

[100] For example, we have seen very similar performance on tasks from thermal physics. See C. Kautz, op. cit.

[101] This expression assumes a closed system and thus does not include a term involving, for example, the chemical potential. In some textbooks this equation is written $\Delta U = Q - W$, where W is the work done by the system. For a general discussion of the first law of thermodynamics, see, for example, F. Reif, *Fundamentals of Statistical and Thermal Physics,* (McGraw-Hill, New York, 1965).

[102] For this process, it is assumed that the piston moves at a constant speed at the instant shown. In order for the piston to move, there must be a time interval over which the net force on the piston is downward. However, if the piston then comes to rest at the end of the process, there is a time interval over which the net force on the piston is upward. By the work-kinetic energy theorem, the positive work done in the initial speeding-up interval must be equal in absolute value to the negative work done in the final slowing-down interval since the piston is at rest before and after the process.

[103] G. Erickson, "Children's conceptions of heat and temperature," *Sci. Ed.* **63**, 221-230 (1979).

[104] A. Tiberghien, "Critical review on the research aimed at elucidating the sense that notions of temperature and heat have for the students aged 10 to 16 years," in *Research on Physics Education, Proceedings of the First International Workshop, La Londe Les Maures, France*, edited by G. Delacôte, A. Tiberghien and J. Schwartz, (Éditions du CNRS, Paris, 1983), pp. 75-90.

[105] R. Driver, E. Guesne and A. Tiberghien (eds.), *Children's Ideas in Science* (Open University Press, Milton Keynes, 1985).

[106] R. Driver and L. Warrington, "Students use of the principle of energy conservation in problem situations," *Phys. Educ.* **20,** 171-176 (1985).

[107] R. A. Lawson and L. C. McDermott, "Student understanding of the work-energy and impulse-momentum theorems," *Am. J. Phys.* **55**, 811-817 (1987).

[108] T. O'Brien Pride, S. Vokos, and L.C. McDermott, "The challenge of matching learning assessments to teaching goals: An example from the work-energy and impulse-momentum theorems," *Am. J. Phys.* **66**, 147 (1998).

[109] K. C. de Berg, "Student understanding of the volume, mass, and pressure of air within a sealed syringe in different states of compression," *J. Res. Sci. Teaching* **32**, 871-884 (1995).

[110] S. Rozier and L. Viennot, "Students' reasonings in thermodynamics," *Int. J. Sci. Ed.* **13**, 159-170 (1991).

[111] C. Kautz, "Investigation of student understanding of microscopic and macroscopic behavior of the ideal gas," Ph. D. dissertation, Department of Physics, University of Washington, 1999 (unpublished).

[112] M. F. Granville, "Student misconceptions in thermodynamics," *J. Chem. Ed.* **62**, 847-848 (1985).

[113] S. Kesidou and R. Duit, "Students' conception of the second law of thermodynamics - an interpretive study," *J. Res. Sci. Teaching* **30,** 85-106 (1993).

[114] P. H. van Roon, H. F. van Sprang, and A. H. Verdonk, "'Work' and 'Heat': on a road towards thermodynamics," *Int. J. Sci. Ed.* **16**, 131-144 (1994).

[115] The research on work and the first law of thermodynamics is described in greater detail in M. E. Loverude, "An investigation of student understanding of hydrostatics and thermal physics and of the underlying concepts from mechanics," Ph.D. dissertation, Department of Physics, University of Washington, 1999 (unpublished). Additional discussion of student understanding of the ideal gas is described in C. H. Kautz, "Investigation of student understanding of microscopic and macroscopic behavior of the ideal gas," Ph.D. dissertation, Department of Physics, University of Washington, 1999 (unpublished).

[116] In the courses included in this study, students typically apply the first law of thermodynamics to closed systems, so that there are not terms including, for example,

the chemical potential. For a discussion of more general statements of the first law of thermodynamics, see, for example, F. Reif, *Fundamentals of Statistical and Thermal Physics,* (McGraw-Hill, New York, 1965).

117 In many textbooks, including those used in the algebra-based and calculus-based courses in this study (see ref. 119), the first law of thermodynamics is written $\Delta U = Q - W$, where W represents the work done *by* the system. These two formulations of the first law are equivalent. However, our experience is that the statement in terms of work done on the system is more easily understood by students. The textbook used in the second-year course uses the first law written in terms of the work done on the system.

118 For example, see G. Erickson, "Children's conceptions of heat and temperature," *Sci. Ed.* **63**, 221-230 (1979), or the review in ref. 130.

119 The algebra-based introductory course currently uses D.C. Giancoli, *Physics,* 5th. ed., (Prentice Hall, Upper Saddle River, NJ., 1998). The second-year course uses R. Resnick, D. Halliday, and K.S. Krane, *Physics,* 4th ed. (John Wiley & Sons, Inc., New York, 1992) which is also the text for the calculus-based introductory course at the University of Washington.

120 The textbook used in the calculus-based course is R. A. Serway, *Physics for Scientists and Engineers with Modern Physics,* 4th ed. (Saunders College Publishing, Philadelphia, 1996).

121 In some cases, we have posed problems on topics that are not explicitly covered in a given course. For example, we have examined student understanding of mechanics and other prerequisite concepts in the courses on hydrostatics and thermal physics described here.

122 In the interviews students were allowed to determine what approximations, if any, they should make about the details of the process. If students expressed concern about the speed with which the pump was compressed or the insulating capabilities of the pump, they would have been instructed to assume that the process would be carried out 'quickly.'

123 Students are asked to treat the gas in the pump as an ideal gas. The ideal gas law is a good approximation of the behavior of air at standard temperature and pressure. For an ideal gas, the internal energy U is proportional to the temperature of the gas and the number of moles of gas.

124 If students know how P and V depend on one another, they can predict how the product will change. In particular, for an adiabatic process, the product of P and $V^{\gamma-1}$ is a constant, where γ is a constant that depends on the gas. If students use this equation, the interviewer accepts that answer but then asks the student why the equation has the form that it does (*i.e.*, why the pressure will increase by a greater factor than the volume will decrease). None of the students in the algebra-based course and only two of the students in the second-year course applied this equation in the interviews.

125 Research in student understanding of mechanics has shown that many students have difficulty with 'passive' forces, like that exerted on a book by the table on which it is resting. See, for example, J. Minstrell, *Phys. Teach.* **20,** 10-14 (1982).

126 In addition to the correct reasoning described above, a handful of students gave other forms of reasoning that we considered to be potentially correct. For example, some students in the more advanced courses gave reasoning based on a microscopic work argument. Typically, these students argued based on the collisions of gas particles with the moving piston, concluding that the speed of the gas particles would increase as a result of these collisions and thus that the temperature would increase. This reasoning could be considered incomplete because it does not account for the presence or absence of heat transfer. For example, in an isothermal compression, gas particles also collide with a piston moving inward, yet over time the increase in molecular speed resulting from such collisions is negated by a heat transfer away from the gas. A handful of students in these courses also correctly applied the equation $PV^{\gamma-1}$ = constant for adiabatic processes to predict that the temperature would increase.

127 There are several factors that complicate analysis of this question. First of all, the adiabatic compression question as it was originally posed was part B of a problem. Part A involved heating the cylinder (without insulation). Therefore, some of the students (approximately 20% of the class) predicted that the temperature of the gas would decrease as the masses were added. About half of those who predicted a temperature decrease argued that after leaving the burner, the gas would cool to room temperature. A second complication is that some seemed to interpret the phrase "the tightly-fitting piston" to mean that the piston was not free to move (despite the additional instruction to neglect friction between the piston and cylinder). Others incorrectly associate the volume with the number of particles. Therefore, some students predicted that the volume of the gas would remain the same. All told, fifteen students responded that the volume of the gas would remain the same in both parts A and B. Even when we omitted both groups of students, we still found that almost no students applied the first law of thermodynamics.

128 Other researchers have described the failure of students to apply general principles to physics problems. In particular, many students fail to apply work and energy arguments in single-particle dynamics. See, for example, R. A. Lawson and L. C. McDermott, "Student understanding of the work-energy and impulse-momentum theorems," *Am. J. Phys.* **55**, 811-817 (1987), and T. E. O'Brien Pride, S. Vokos, and L.C. McDermott, "The challenge of matching learning assessments to teaching goals: An example from the work-energy and impulse-momentum theorems," *Am. J. Phys.* **66**, 147 (1998).

129 S. Rozier and L. Viennot, "Students' reasonings in thermodynamics," *Int. J. Sci. Ed.* **13**, 159-170 (1991).

130 A summary of this work can be found in the chapter by G. Erickson and A. Tiberghien in Driver, R., E. Guesne and A. Tiberghien (eds.), *Children's Ideas in Science* (Open University Press, Milton Keynes, 1985).

[131] For instance we have posed written problems in which students were asked to determine the value of the work done in a process or cycle given the corresponding *PV* diagram. Very few students were able to do so correctly and many applied inappropriate formulas. For example, in a process with pressure directly proportional to volume (represented by a diagonal line on the *PV* diagram), approximately 25% of the students in the calculus-based course attempted to find the work using the formula for the work done in an isothermal process. Half of students who used this formula had correctly calculated the different initial and final temperatures in an earlier part of the problem. Such responses reflect a failure to use a general definition to determine the work done as well as an indiscriminate use of formulas.

[132] This problem was posed on the final exam in Physics 113 at the University of Illinois.

[133] These students had completed standard instruction in mechanics. Typically the algebra-based introductory course at the University of Washington does not use *Tutorials in Introductory Physics.*

[134] This question was not included in this form in the version of the Mechanical Work problem given in the sections of the calculus-based course. Instead, students were asked to state whether there was work done on the block by any of the other objects in the problem. Students' responses to this question were more difficult to interpret.

[135] The responses for the students in the second-year course are sorted based on whether students answered all signs of the work correctly on the Mechanical Work problem (we have chosen a different criterion here because many students did not explain their answers, but still answered all parts correctly, suggesting that these students understood the definition of work despite their failure to explain their reasoning). If the same criterion is used for this class as for the others, i.e. reasoning used for the sign of the work by the hand on the block in the Mechanical Work problem, approximately half of those who used the definition of work (6 of 13) applied work to the Pump problem, as compared to 25% of those who used incorrect reasoning (2 of 8), suggesting that the effect does not depend on the criteria chosen.

[136] As in Chapter 8, the column showing data from the section of Physics 115 that had had standard instruction includes only those students who correctly stated that the volume of the gas decreases.

[137] P. Kraus, op. cit.

[138] Personal communication, P. R. L. Heron, P.S. Shaffer, and J.R. Thompson.

[139] C. Kautz, op. cit.

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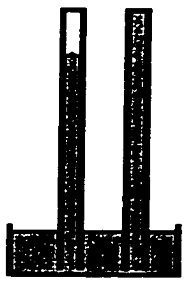
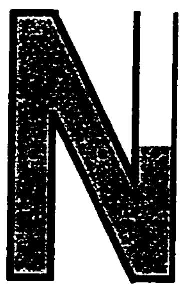
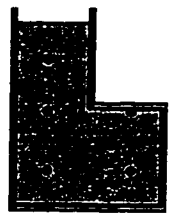
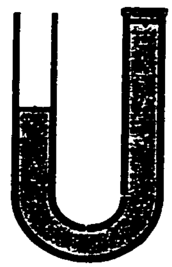
APPENDIX A STUDENT RESPONSES TO PROBLEMS REQUIRING PRESSURE COMPARISONS FOR POINTS AT THE SAME LEVEL IN THE SAME LIQUID

In Chapter 2, we described the responses of students to several written problems in which students were asked to compare the pressures at points at the same height in the same liquid. In the chapter, we showed the overall results for several populations. In this appendix, we describe some of the problems that are not shown in Chapter 1 and show the details of student responses in different sections and on different problems. Additional problems are shown at the end of the Appendix, in Figure A-2 and Figure A-3.

1. SINGLE FLUIDS

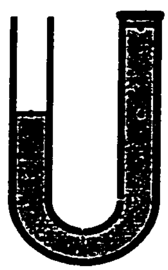
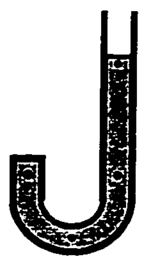
We have examined student responses to pressure comparison questions in a variety of situations. On problems in which students were asked to rank the pressures at points at the same height in a single fluid contained in various tubes, we found a consistent pattern of student responses. A summary of student responses to problems posed in different sections of Physics 115, the algebra-based introductory course at the University of Washington, is shown in Table A-1. In response to the first three problems, about two-thirds of the class correctly stated that the pressures were equal at the points at the same height. However, the Capped U-tube problem seemed to elicit quite different responses. These responses are discussed in Chapter 2. A comparison of the responses given by students to the Utah and Capped U-tube problems on a single ungraded quiz is discussed in detail in section 3 of this appendix. Table A-2 shows student responses to problems posed in two introductory courses at Harvard University: Physics 1, the algebra-based course, and Physics 11, a calculus-based course. Responses given by students in Physics 224 at the University of Washington are shown in Table A-3 and Table A-4.

Table A-1: Student responses to problems involving pressure comparisons of points at the same height in the same liquid from several sections of Physics 115 at the University of Washington.

	Physics 115 Washington Sp'98 (*N* = 64)	Physics 115 Washington Wi'97 (*N* = 201)	Physics 115 Washington Sp'98 (*N* = 109)	Physics 115 Washington Sp'98 (*N* = 109)
Type of problem	Barometers 	N-tube 	Utah-tube 	Capped U 
Instruction	Standard	Standard	Standard	Standard
No. of points	2	3	2x2	2
Correct	60% (40)	60% (127)	65% (70)	25% (26)
Not all equal	35% (24)	30% (61)	35% (39)	70% (77)
Other/Blank	0	<5% (7)	<5% (1)	5% (6)

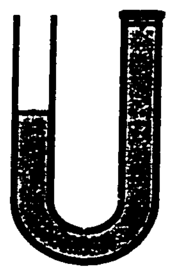
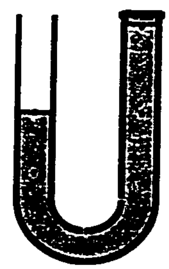
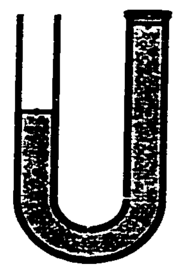
All percentages are rounded to the nearest 5%.

Table A-2: Student responses to problems involving pressure comparisons of points at the same height in the same liquid in courses at Harvard University.

	Physics 1 Harvard Au'98 (N = 120)	Physics 11 Harvard Sp'97 (N = 138)
Type of problem	Capped U 	J-tube 
Instruction	Standard	Reading only
No. of points	2	2
Correct	50% (58)	25% (35)
Not all equal	50% (62)	70% (95)
Other/Blank	0	5% (8)

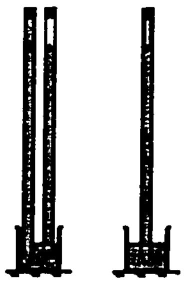
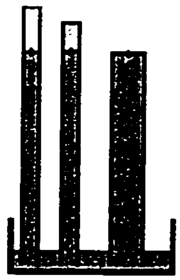
All percentages are rounded to the nearest 5%.

Table A-3: Student responses to pressure rankings for points at the same height in the same liquid in Letter Tube problems posed in Physics 224.

	Physics 224 Washington Wi'96 (*N* = 23)	Physics 224 Washington Au'97 (*N* = 50)	Physics 224 Washington Sp'97 (*N* = 22)	Physics 224 Washington Au'98 (*N* = 39)	Physics 224 Washington Su'97 (*N* = 13)
Type of problem	N-tube 	W-tube 	Capped U 	Capped U 	Capped U 
No. of points	3	3	2	2 x 2	2
Instruction	Standard	Standard	Standard	Standard	None
Correct	60% (14)	80% (39)	55% (12)	75% (30)	60% (8)
w/correct	55% (13)	70% (36)	45% (10)	75% (30)	55% (7)
Not all equal	40% (9)	20% (10)	30% (7)	25% (9)	40% (5)
Other/Blank	0	<5% (1)	15% (3)	0	0

All percentages rounded to the nearest 5%.

Table A-4 Student rankings for points at the same height in the same liquid in Barometer Problems in Physics 224. (Comparisons only include points with air or the same liquid above, i.e. the points in the left two tubes in the Au'95 problem and the left and right tubes in the Su'95 problem.)

	Physics 224 Au'95 (*N* = 39)	Physics 224 Su'95 (*N* = 17)
Type of problem	Barometer 	Barometer 
No. of points	2	2
Instruction	Standard	Standard
Correct	50% (19)	60% (10)
w/correct	40% (15)	30% (6)
Not all equal	45% (17)	45% (7)
Other/Blank	10% (3)	0

All percentages rounded to the nearest 5%.

2. RESPONSES ON PROBLEMS INCLUDING MULTIPLE FLUIDS

We have also posed several problems in which students are asked to consider points at the same height in the same fluid but with different liquids above the points. These problems seem to be more difficult than the problems in which only a single liquid is shown.

Table A-5: Student responses to problems involving pressure rankings for points at the same height but with different liquids above in several courses after standard lecture instruction.

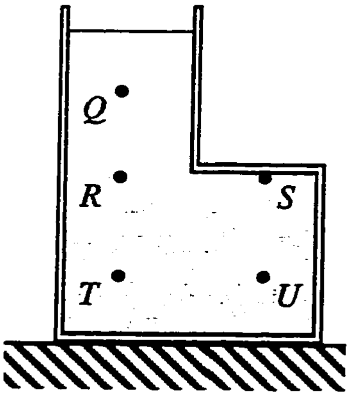
	Physics 115 Washington Au'95 (N = 120)	Physics 113 Illinois Au'98 (N = 182)	Physics 224 Washington Su'95/Au'95 (N = 56)	Phys 224 Washington Au'98 (N = 39)
Type of problem	U-tube w/diff. diam	Barometers[MC]	Barometers	Letter tube
Correct	45% (52)	65% (117)	45% (25)	65% (26)
w/correct	40% (46)	n/a	40% (21)	55% (22)
Not all equal	40% (48)	35% (65)	40% (23)	25% (09)
Other/Blank	15% (18)	n/a	15% (10)	10% (04)

All percentages rounded to the nearest 5%. [MC]Multiple-choice problem.

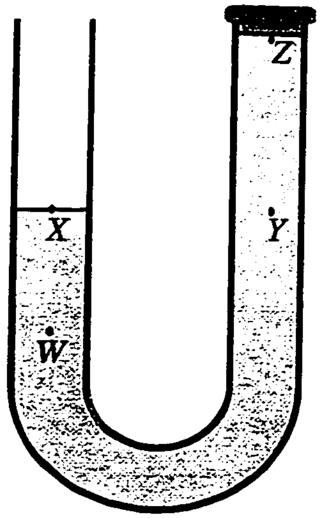
3. A COMPARISON OF STUDENT RESPONSES FOR CURVED AND CAPPED TUBES TO THOSE FOR STRAIGHT TUBES

We found that most of the problems that we used seemed to elicit similar reasoning patterns. However, curved tubes that were capped on one end seemed to elicit significantly different reasoning from other Letter Tube Problems and from Barometer Problems. We describe this reasoning in detail in Chapter 2. In this section we examine the differences between these tubes in detail. First we examine the different reasoning patterns given by students who answered the two problems on the same pretest. We then describe the difference between curved and capped tubes and other tubes as a function of course level.

In one section of Physics 115, students were given an ungraded quiz that began with a Utah-shaped tube and concluded with a Capped U-tube (see Figure A-1). A summary of student responses to the pressure comparisons is shown in Table A-6. Nearly all of the students who correctly ranked the points in the Capped U-tube also correctly ranked the points in the Utah tube (22 of 23, or 95%). For each category of incorrect response on the Capped U-tube, approximately half of the students giving this response correctly ranked the points in the Utah tube. On the other hand, only about a third of the students who correctly compared the points in the Utah tube also correctly did so in the Capped U-tube (22 of 67, or about 35%). This pattern of responses suggests that the Capped U-tube is a more strict and effective test of student understanding. Very few of the students who answered the Capped U-tube problem correctly answered the other problem incorrectly, suggesting that a 'false-positive' result (*i.e.*, a correct response given by a student whose understanding is incomplete or incorrect) is less likely on the Capped U-tube. Interestingly, not all of the students who used weight reasoning on the Capped U-tube did so on the Utah tube, suggesting that this type of response is highly context-dependent, much like the 'along the tube' reasoning described in Chapter 2.



Question 1



Question 2

1. A glass vessel is filled with water.

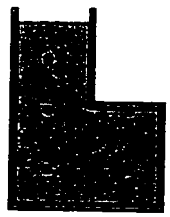
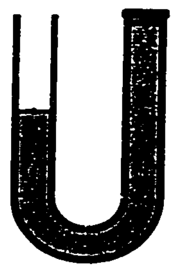
Rank the pressures at the marked points. Explain your reasoning.

2. A tube (height ~ 0.5 meters) is partly filled with water as shown. One end of the tube is closed at the top, but the other is open to the atmosphere. There is no air between the cap and the water surface on the closed side.

Rank the pressures at the marked points. Explain your reasoning.

Figure A-1: An ungraded quiz in which we asked students to rank pressures in a Utah tube and then in a Capped U-tube. We compared the responses given by individual students on the two problems.

Table A-6: Responses given by students in Physics 115 for a Curved and Capped tube compare to those for a Utah tube on the same ungraded quiz.

	Physics 115 Sp'98 ($N = 109$)	
Type of problem	Utah tube 	Capped U 
Number of points	2x2	2
Same pressure (correct)	65% (70)	25% (26)
w/correct reasoning	60% (67)	20% (23)
'Along the tube' or reverse	~0% (2)	40% (42)
Partly or fully weight-based	15% (14)	10% (10)
Points isolated from atm	5% (6)	5% (7)
Points confined	10% (9)	5% (7)

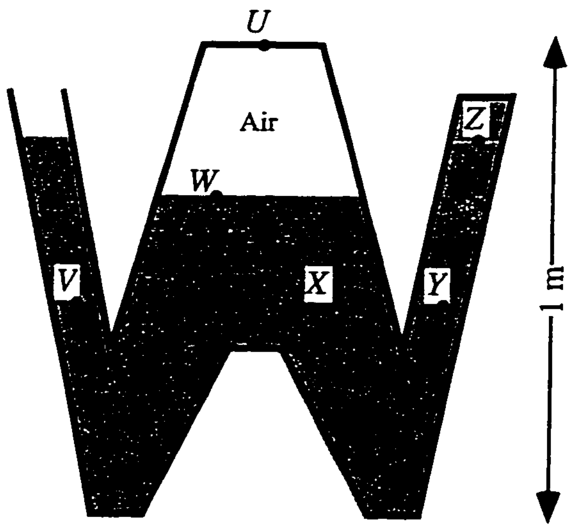
All percentages rounded to the nearest 5%.

In addition to the observation that the curved and capped tubes seem to elicit different reasoning, we have observed that the difference in correct responses between these tubes and other Letter Tube Problems seems to diminish as students grow more sophisticated. For example, this difference is large in Physics 115 and essentially non-existent among students in Physics 224. There is a small discrepancy in responses to the two types of problems among Physics Teaching Assistants at Purdue University, but responses on both types of problem were extremely good.

Table A-7: A comparison of pressure rankings on curved tube problems to those on other tube problems for students in the same course.

	Physics 115 Washington ($N = 109$)	Phys. 152 Purdue ($N > 1000$)	Physics 224 Washington ($N \sim 70$)*	TAs Purdue ($N = 18$)
Correct on curved & capped	20%	40%	70%	85%
Correct on other	65%	60%	70%	100%

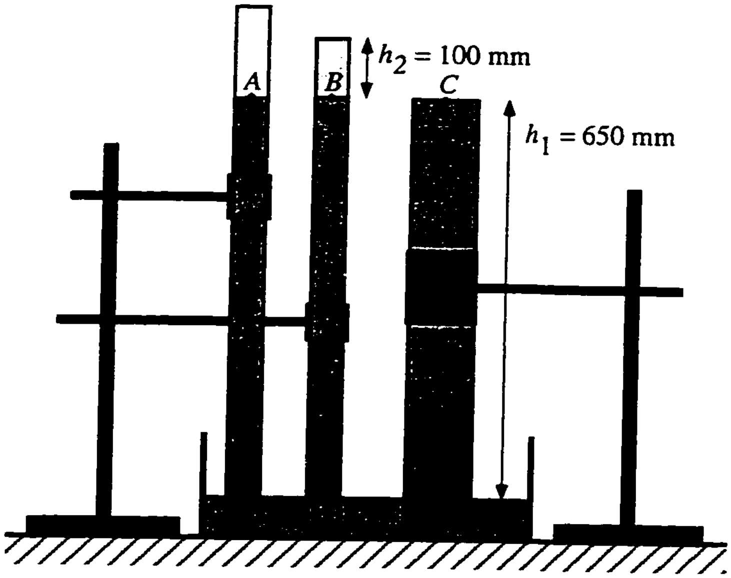
All percentages rounded to the nearest 5%. In the cases of Physics 115, Physics 152, and the Teaching Assistants, the data are taken from the same pretest. *In the case of Physics 224, average percentages over several quarters are shown.



A W-shaped glass tube is partly filled with water. The tube is open to the atmosphere on the left side only. There is air enclosed in the space above *W*. Points *V*, *X* and *Y* are at the same height above the bottom of the container. Point *Z* is at the same height as the surface of the water in the left tube.

Rank the pressures at the points *V*, *W*, *X*, *Y*, and *Z*. Explain the reasoning you used to rank the pressures.

Figure A-2: The W-tube problem.



The top of each tube is closed; the bottom end is open to a mercury reservoir. The reservoir is open at the top. In each tube the top of the column of mercury is at a height of $h1$ = 650 mm above the surface of the mercury in the reservoir. The diameters of the left two tubes are identical. The diameter of the rightmost tube is twice the diameter of any of the other tubes.

In the leftmost tube the space above the mercury column contains air at a pressure corresponding to 100 mm of mercury. In the center tube the space above the mercury column is filled with water. The height of the water column is 100 mm. In the rightmost tube there is no space between the mercury and the top of the tube.

Rank the pressures at the three points A, B and C. If any pressures are equal, state that explicitly. Explain your reasoning

Figure A-3: A Multiple-Barometer problem with multiple liquids.

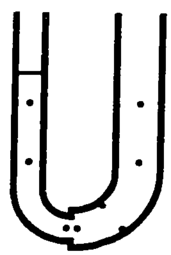
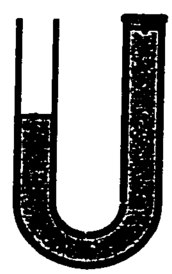
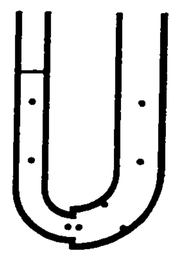
APPENDIX B ANALYSIS OF STUDENT RESPONSES TO PROBLEMS REQUIRING THE PRESSURE GRADIENT

In Chapter 3, we describe the responses of students to several written problems in which students needed to apply the idea of a pressure gradient to several different types of problems. In that chapter, we combined responses from several sections and did not include a detailed description of all of the problems that were used or the points that students were asked to consider. In this appendix, we reproduce some of the problems that are not shown in the body of the dissertation and present both the percentage and the number of students giving particular responses in various sections of the courses in the study. The structure of the appendix will be parallel to that of Chapter 3. However, we will not include descriptions of the type of reasoning that students have used or direct quotes of student responses. For this type of detail see the relevant sections of Chapter 3.

1. INCORRECT BELIEF THAT THE PRESSURE IS THE SAME AT ALL POINTS IN A STATIC LIQUID

In several different pressure problems, we posed problems in which students were asked to rank the pressures at points located at different heights in the same static liquid. As we have described in Section 3.2 of Chapter 3, these problems allow us to infer something about student understanding of the pressure gradient. We have included below a summary of the responses to such problems given by students in two different sections of Physics 115, the algebra-based introductory course at the University of Washington, one section of Physics 152, the calculus-based introductory course at Purdue University, and one section of Physics 1 at Harvard University. This summary is shown in Table B-1.

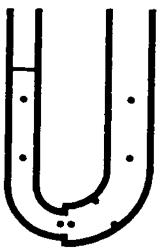
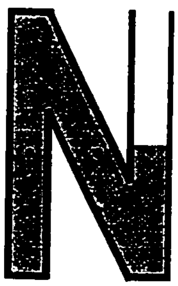
Table B-1: Responses after standard instruction in introductory courses in which students state that the pressures at all points in the same liquid are equal.

	Physics 115 Washington Wi'97 (*N* =	Physics 115 Washington Sp'95 (*N* = 110) 2 0 1)	Physics 1 Harvard Sp'98 (*N* = 120)	Physics 11 Harvard Au'96 (*N* = 138)
Problem type	N-tube 	Diff-diam U-tube 	Capped U-tube 	Diff-diam U-tube 
All pressures equal	~0% (2)	10% (10)	10% (15)	<5% (5)

All percentages rounded to the nearest 5%.

Very few students in the second-year thermal physics course make this error. Student responses to problems in four different sections of Physics 224 are shown in Table B-2.

Table B-2: Responses after standard instruction in Physics 224 at the University of Washington in which students state that the pressures at all points in the same liquid are equal.

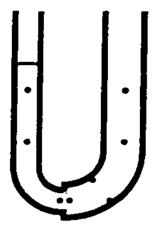
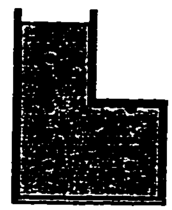
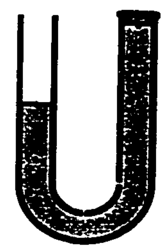
	Physics 224 Washington 3 sections (*N* = 94)	Physics 224 Washington Wi'96 (*N* = 23)
Problem type	Diff-diam U-tube 	N-tube 
All pressures equal	~0% (1)	~5% (1)

All percentages rounded to the nearest 5%.

2. BELIEF THAT PRESSURE INCREASES WITH HEIGHT

In addition to the incorrect belief that the pressures at all points in a liquid are equal, we have found that many pressure problems elicit the incorrect belief that pressure increases with height in a liquid. The percentage of student responses consistent with this belief in several introductory courses are shown in Table B-3.

Table B-3: Responses in introductory courses in which students answer as though pressure increases with height.

	Physics 115 Washington Au'95 (N = 110)	Physics 115 Washington Wi'97 (N = 201)	Physics 115 Washington Sp'98 (N = 109)	Physics 1 Harvard Sp'98 (N = 120)
Problem type	Diff-diam U-tube 	N-tube 	Utah tube 	Capped U-tube 
Instruction	Standard	Standard	Standard	Standard
Pressure *increases* with height (incorrect)	5% (8)	10% (22)	5% (4)	10% (11)

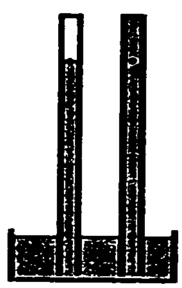
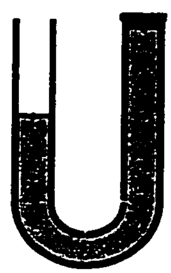
All percentages rounded to the nearest 5%.

In a series of Letter Tube Problems, only a small number of students in Physics 224 reversed the pressure gradient. In the initial version of the N-tube, no students did so. In two administrations of the W-tube, shown in Figure A-2 in Appendix A, after standard instruction, three students out of 70 reversed the pressure gradient. All of these students referred to height or to the equation $P=P_0+\rho gh$ in explaining their ranking.

3. COMPARISONS OF PRESSURE TO ATMOSPHERIC

In Chapter 3 we describe several incorrect beliefs emerging on student responses to problems requiring comparisons of the pressure at a point to atmospheric pressure. Responses for such problems in sections of Physics 115 at the University of Washington are shown in Table B-4. Responses from the second-year courses at the University of Washington and the University of Illinois are shown in Table B-5.

Table B-4: Student responses from two sections of Physics 115 for the comparison of the pressure at points above a free surface to atmospheric pressure (P_0).

	Phys 115B Washington Sp'98 ($N = 65$)	Phys 115A Washington Sp'98 ($N = 105$)
Type of Problem	Multiple-Barometer 	Capped U-tube 
Space or liquid?	gas	liquid
Less than P_0 (correct)	40% (27)	40% (41)
with correct reasoning	20% (12)	10% (13)
Equal to P_0	25% (16)	20% (21)
Greater than P_0	35% (22)	35% (38)

All percentages rounded to the nearest 5%.

Table B-5: Student responses from several sections of Physics 224 at University of Washington and one section of Physics 113 at the University of Illinois for the comparison of the pressure at points above a free surface to atmospheric pressure (P_0).

	Phys 224 Wash. Wi'95 (N = 26)	Phys 224 Wash. Sp'95 (N = 15)	Phys 224 Wash. Au'96 (N = 42)	Phys 224 Wash. Wi'97 (N = 33)	Phys 224 Wash. Au'98 (N = 39)	Phys 113 Illinois Au'98 (N = 182)
Type of Problem	Single barometer	Multiple barometer	Single barometer	Multiple barometer	Capped U-tube	Multiple barometer
Space or point?	space	space	space	point	point	gas
Less than P_0 (correct)	50% (13)	85% (13)	70% (30)	65% (21)	70% (21)	45% (82)
with correct reasoning	40% (11)	75% (11)	50% (21)	50% (17)	35% (13)	n/a^{MC}
Equal to P_0	10% (2)	5% (1)	<5% (1)	20% (7)	10% (4)	35% (67)
Greater than P_0	40% (11)	5% (1)	15% (7)	5% (2)	10% (3)	20% (33)

All percentages rounded to the nearest 5%. ^{MC}Multiple-choice problem.

4. COMPARISONS OF PRESSURES AT POINTS AT THE SAME HEIGHT IN LIQUIDS OF DIFFERENT DENSITY

In Chapter 3 we describe student responses to problems requiring comparisons of the pressures at points at the same height that are located in liquids of different densities. Responses for such problems in sections of Physics 115 at the University of Washington, Physics 224 at University of Washington, and Physics 113 at the University of Illinois are shown in Table B-6. Responses from several sections of Physics 224 in which the reasoning for this type of comparison had been explicitly shown in lecture are shown in Table B-7. As noted in Chapter 3, the percentage of correct responses in the section of Physics 115 is somewhat misleading because most of the correct answers included incorrect reasoning based on the different cross-sectional areas of the tubes containing the two points. The problem used at the University of Illinois was in a multiple-choice format.

Table B-6: Responses given by students after standard instruction in second-year courses to problems requiring a comparison of the pressures at two points at the same height in liquids of different densities.

	Phys 115 Washington Au'95 (N = 120)	Phys 224 Washington Wi'96 (N = 23)	Phys 224 Washington Wi'97 (N = 33)	Phys 224 Washington Au'98 (N = 39)	Phys. 113 Illinois Au'98 (N = 181)
Type of Problem	U-tube w/diff. diam	N-tube	Multiple-barometer	Capped U-tube	Multiple-barometer
Pressure less in denser liquid (correct)	35% (40)	10% (2)	10% (4)	15% (6)	15% (24)
with correct reasoning	~0% (2)	5% (1)	10% (3)	<5% (1)	n/a[MC]
Pressure greater in denser liquid	20% (25)	60% (14)	70% (23)	55% (22)	60% (104)
Pressures equal	30% (39)	20% (5)	10% (3)	10% (4)	30% (53)
Other / blank	15% (16)	10% (2)	10% (3)	15% (7)	n/a

All percentages rounded to the nearest 5%. [MC]Multiple-choice problem.

Table B-7: Responses given by students to problems requiring a comparison of the pressures at two points at the same height in liquids of different densities. In these sections of Physics 224, there was special emphasis in lecture on this type of reasoning.

	Phys 224 Su'95 (N = 15)	Phys 224 Au'95 (N = 34)	Phys 224 Au'98 (N = 45)
Type of Problem	Capped U-tube	Capped U-tube	multiple barometer
Pressure less in denser liquid (correct)	25% (4)	40% (14)	40% (19)
with correct reasoning	20% (3)	25% (8)	35% (16)
Pressure greater in denser liquid	25% (4)	50% (17)	40% (18)
Pressures equal	35% (5)	5% (2)	10% (5)
Other / blank	15% (2)	<5% (1)	10% (3)

All percentages rounded to the nearest 5%.

APPENDIX C CURRENT VERSION OF TUTORIAL SEQUENCE: *REVIEW OF NEWTON'S LAWS*

Contents of Appendix C:

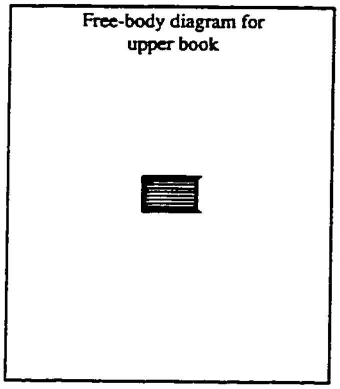
Current version of tutorial sequence: *Review of Newton's Laws*

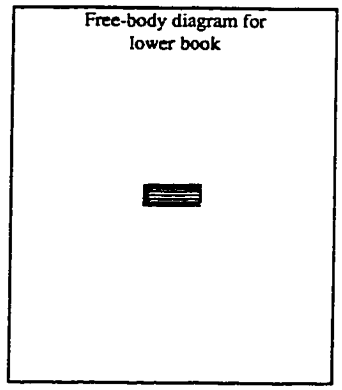
- Pretest: Two Books problem
- Tutorial: *Review of Newton's Laws*
- Homework: None

Physics 115 **PRETEST 1** Name ______________

Two books are stacked on top of each other on a level table as shown in the diagram at right. The mass of the upper book is greater than the mass of the lower book.

1. In the space provided below, draw a separate free-body diagram for each book. Recall that your diagram should include:
   - a description of each force,
   - the name of the object *on which* each force is exerted,
   - the name of the object *exerting* each force.





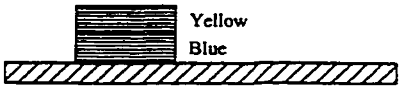
2. Rank from largest to smallest the *magnitudes* of all the forces shown in your free-body diagrams. If any of the forces have equal magnitude, then indicate that explicitly. Explain your reasoning.

Physics 115 **REVIEW OF NEWTON'S LAWS** Name ____________

In answering the following questions, you should work in groups of two to four. Neglect any forces exerted by the atmosphere unless specified otherwise.

I. A short review of Newton's Laws:

Two stacks of colored copying paper are placed on top of each other. The upper stack consists of 300 yellow sheets, while the lower stack consists of 300 blue sheets.



A. Draw a separate free-body diagram for each stack of paper. In each case:
   - draw vectors to show the forces that are exerted on the object,
   - label and describe each force (i.e., specify whether the force is a normal force, etc.),
   - identify the object on which each force is exerted and the object exerting each force.

Free-body diagram for upper stack

B. Rank the magnitudes of all the forces that you have identified in part A. If two forces are equal in magnitude, indicate that explicitly. Explain your reasoning.

Free-body diagram for lower stack

Did you apply Newton's second law in comparing the magnitudes of the forces? If so, how?

Did you apply Newton's third law? If so, how?

⧗ **Call to order: Wait for the class discussion before you continue.**

C. Consider the following dialogue between two students:

Student 1: *"There is a downward normal force on the lower stack by the upper stack."*

Student 2: *"No, a normal force is always opposite to gravity. The force you mentioned is just the weight of the upper stack."*

With which student, if either, do you agree? Explain your reasoning.

⧗ **Call to order: Wait for the class discussion before you continue.**

Physics 115

REVIEW OF NEWTON'S LAWS

II. Forces on a single sheet of paper:

A. Now consider a single sheet of paper somewhere near the center of the upper (yellow) stack.

i. At right, draw a free-body diagram for this sheet. (A complete diagram will show three forces acting on the sheet.)

ii. Rank the magnitudes of the forces that you have shown.

Free-body diagram for yellow sheet

B. Repeat the two steps in part D for a single sheet of paper somewhere near the center of the lower (blue) stack:

i. At right, draw a free-body diagram for this sheet.

ii. Rank the magnitudes of the forces that you have shown.

Free-body diagram for blue sheet

C. Is the magnitude of the upward normal force on the blue sheet (in the lower stack) *greater than, less than* or *equal to* the magnitude of the upward normal force on the yellow sheet (in the upper stack)?

D. Generalize your answer to the preceding question: How does the upward normal force exerted on a single sheet of paper change as sheets closer to the bottom of the stack are considered?

E. How does the net force on a single sheet of paper (i.e., the sum of *all* forces on the sheet of paper) change as sheets closer to the bottom of the stack are considered?

What does your answer imply about the sum of the forces on the sheet of paper by the sheets of paper above and below?

Call to order: Wait for the class discussion before you continue.

APPENDIX D INITIAL VERSION OF TUTORIAL SEQUENCE: *PRESSURE IN A LIQUID*

Contents of Appendix D:

Initial version of tutorial sequence: *Pressure in a Liquid*

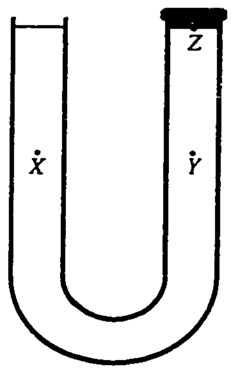
- Pretest: Capped U-tube and Different-diameter U-tube problems
- Tutorial: *Pressure in a Liquid* initial version
- Homework: None included

Physics 224 **Pretest 1** Name ____________________

I. A U-shaped glass tube (of height approximately one half meter) is filled with water. The right end is closed by a rubber stopper that touches the water. The water level is the same on both sides, and there is no air between the water in the right tube and the stopper.

Points X and Y are at the same height.

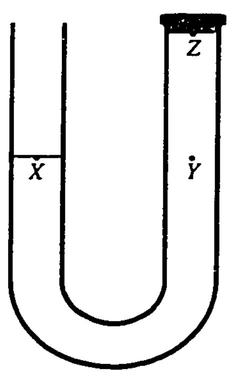
A. Is the pressure at point Y *greater than, less than,* or *equal to* the pressure at point X? Explain.



B. Is the pressure at point Z *greater than, less than,* or *equal to* atmospheric pressure? Explain.

A syringe is now used to remove some of the water from the left tube. The new water level on the left is shown in the diagram below while the water level on the right is not shown.

C. After the water has been removed, is the pressure at point Y *greater than, less than,* or *equal to* the pressure at point X? Explain your reasoning.



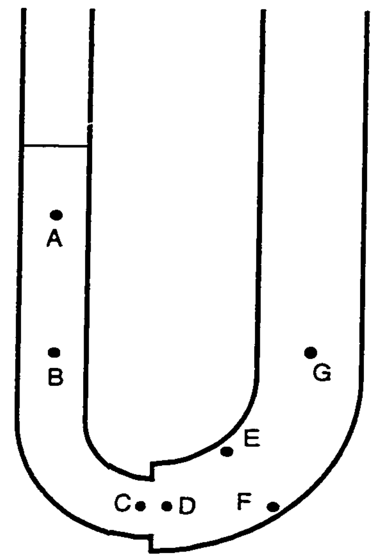
D. Will the new water level on the right be *at point Z, between points Z and Y, at point Y, or below point Y?* Explain your reasoning.

(continued on other side)

2. Consider a U-tube which is open at the top on both sides. The diameter of the right tube is one and a half times that of the left tube. Water is poured into the U-tube such that the water level on the left is as shown in the diagram.

   A. Sketch the water level on the right and explain how you determined your answer.

   B. Rank the pressures at the points indicated in the diagram. If at any point there is no water, state that explicitly. Explain your reasoning.



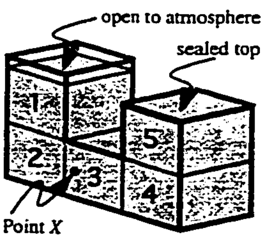
Initial version of tutorial *Pressure in a Liquid* - page 1 Appendix D

Lecture Worksheet 1
Physics 224

PRESSURE VS. DEPTH Name ______________

I. Forces in fluids

An unusually shaped piece of glassware is filled with water. A perspective view of the glassware is shown in the diagram at right. We can imagine that the water in the glassware consists of five cubes of equal size. The cubes are numbered 1 through 5 as shown.



A. Draw a free-body diagram for cube 3, i.e.
- draw vectors to show the forces that are exerted on the object,
- label and describe each force, and
- identify the object on which each force is exerted and the object exerting each force.

B. 1. Which objects, if any, exert horizontal forces on the cube of water? Explain.

Free-body diagram for cube 3

2. Imagine that a hole were drilled in the glass at point *X*. Is your answer to the previous question consistent with the behavior of the water next to the hole? Explain.

3. Compare the pressure at a given height on one side of Cube 3 to the pressure at the same height on the opposite side. (*Hint:* You may consider the horizontal forces on a very thin slice of water at this height.) Explain.

4. If necessary, modify your free-body diagram for cube 3 so that it is consistent with your answers to the preceding questions.

C. Draw a free-body diagram for cube 2.

Free-body diagram for cube 2

1. How does the pressure at the top of cube 3 compare to the pressure at the top of cube 2? (*Hint:* See part B.3)

2. What does your answer to the previous question imply about the relative magnitude of the forces exerted on the top of cube 2 and cube 3? Explain your reasoning.

3. If necessary, modify your free-body diagrams for cubes 2 and 3 so that they are consistent with one another and with your answers to the preceding questions.

D. Consider the following student statement:

"There is no water above cube 3, so there is no force pushing down on the top of cube 3."

Do you agree? Explain your reasoning.

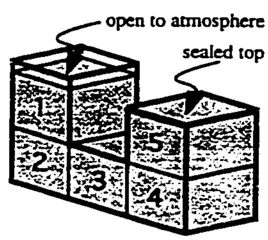
✔ Discuss your answers with a staff member before you proceed.

Lecture Worksheet 1
Physics 224

PRESSURE VS. DEPTH

II. Differences in pressure

A. 1. Use your reasoning from part II.B to identify all points at which the pressure is equal to the pressure at the top of cube 3. What do these points have in common?



2. Is the pressure at the bottom of cube 1 *greater than, less than,* or *equal to* the pressure at the bottom of cube 5? Explain.

B. Sketch free-body diagrams for cubes 1 and 5 in the spaces provided.

Free-body diagram for cube 1	**Free-body diagram for cube 5**

1. How does the force on the bottom of cube 5 compare to the force on the bottom of cube 1?

   Is your answer consistent with your statement about the pressure at the bottom of cubes 1 and 5 in part A?

2. Is there a force exerted on the top of cube 5? If so,

   - identify the object that exerts the force, and
   - compare the magnitude of this force to the magnitude of the force on the top of cube 1.

4. Rank the following quantities from largest to smallest. Explain your reasoning by referring to your free-body diagrams.

   - the pressure at the top of cube 5
   - the pressure at the bottom of cube 5
   - the pressure at the top of cube 1
   - atmospheric pressure

C. Formulate a general rule for finding the difference in pressure between any two points in the same fluid.

Does your rule depend on what is above the points that you are considering?

D. Consider the following statement made by a student:

"The pressure at a point is equal to the weight of the water above divided by the area. Therefore the pressure at the bottom of cube 3 is less than the pressure at the bottom of cube 4 because there's less water above the bottom of cube 3."

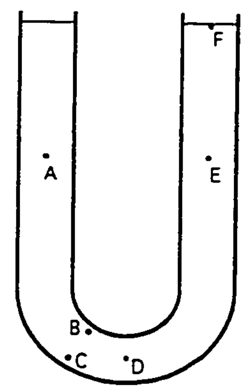
Do you agree with the statement? Explain.

Lecture Worksheet 1
Physics 224

PRESSURE VS. DEPTH

III. U-tubes

A U-shaped tube of about half a meter height with equal diameters on both sides is filled with water almost to the top as shown at right.

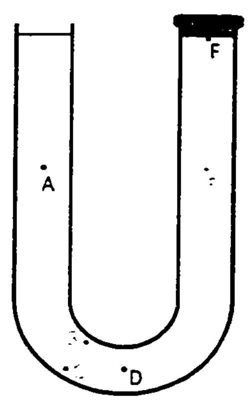


A. Rank the pressures at points *A* through *F*. Explain how you used the rule you suggested in part II.C to rank the pressures.

B. The right hand tube is now sealed at the top such that the water levels on both sides remain the same. There is no air between the rubber stopper at the top of that tube and the water.

1. Determine whether the pressures at points *A, D,* and *F* will *increase, decrease,* or *remain the same.* Explain your reasoning.

2. How does the force exerted by the rubber stopper on the water at the top of the right tube compare to the force exerted by the atmosphere on the water surface on the left?

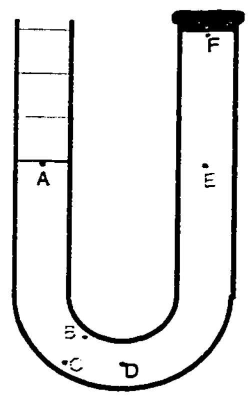


Lecture Worksheet I
Physics 224

PRESSURE VS. DEPTH

C. A syringe is now used to remove some water from the left tube.

1. As the water level on the left is lowered, how does the pressure at points *A, D, E,* and *F* change? (The water level in the right hand side has not been shown.)



2. Are there any points in the tube where the pressure is equal to atmospheric pressure? If so, identify those points. Explain.

3. What can you say about the pressure at points in the water above the points you identified in the previous question? Explain.

4. At what height, if anywhere, would you expect the pressure to be zero?

5. Use your answer to the previous questions together with the definition of pressure from part I to predict whether the water level on the right will have dropped. (*Hint:* Is there a force exerted on the top of the water in the right hand tube? If so, what object is exerting this force?)

D. Observe the demonstration at the front of the room.

What happens to the water level in the stoppered side? Is your result consistent with your prediction? If not, resolve the inconsistency.

IV. Application of pressure vs. depth: the water barometer

A very long flexible transparent tube is coiled up in a bucket of water and completely filled with water. One end of the tube is then sealed tightly and slowly raised out of the bucket. The other end of the tube remains in the bucket below the water level throughout the whole experiment.

Predict how, if at all, the level of the water in the flexible tube will change as the sealed end of the tube is raised.

✔ Observe the demonstration and check your prediction.

APPENDIX E REVISED VERSION OF TUTORIAL SEQUENCE: *PRESSURE IN A LIQUID*

Contents of Appendix E:

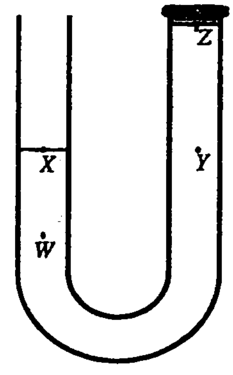
Revised version of tutorial sequence: *Pressure in a Liquid*

- Pretest: Capped U-tube and Different-diameter U-tube problems
- Tutorial: *Pressure in a Liquid*
- Homework: Capped U-tube, W-tube, and Two-Liquid problems

Pretest: Pressure Name ______________

1. A U-shaped tube (height ~ 0.5 meters) is partly filled with water as shown at right. The right end of the tube is closed at the top, but the left end is open to the atmosphere. There is no air between the stopper and the water surface on the right-hand side.

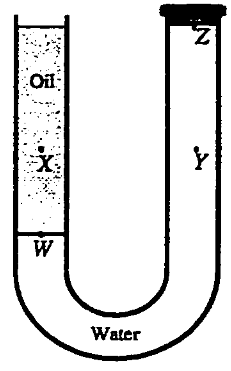
   A. Rank the pressures at points *W*, *X*, *Y*, and *Z*. Explain your reasoning.



   B. Is the pressure at point *Z* *greater than*, *less than*, or *equal to* atmospheric pressure? Explain.

A syringe is used to remove some water from the left-hand side and oil is added to the tube. The density of the oil is less than the density of water ($\rho_{oil} < \rho_{water}$).

   C. Is the pressure at point *Y* *greater than*, *less than*, or *equal to* the pressure at point *X*? Explain.



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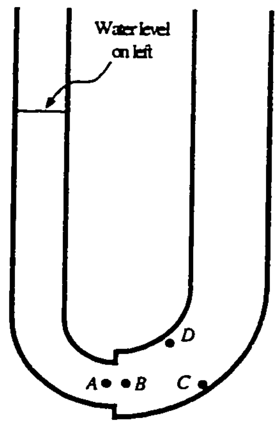
Pressure

2. Consider a U-tube that is open at the top on both sides. The diameter of the right tube is one and a half times that of the left tube. Water is poured into the U-tube such that the water level on the left is as shown in the diagram. Note that the water level on the right is not shown.

   A. Sketch the water level on the right. Explain how you determined your answer.



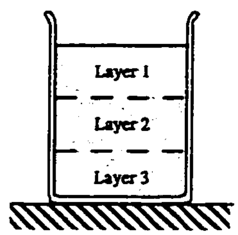
   B. Is the pressure at point *A* *greater than*, *less than*, or *equal to* the pressure at point *B*?

   C. Is the pressure at point *C* *greater than*, *less than*, or *equal to* the pressure at point *D*?

PRESSURE IN A LIQUID

I. Applying Newton's Laws to liquids

A rectangular container filled with water is at rest on a table as shown. Two imaginary boundaries that divide the water into three layers of equal volume have been drawn in the diagram. (No material barrier separates the layers.)



Free-body diagrams for

Layer 1	Layer 2	Layer 3

A. 1. For each layer, draw a free-body diagram in the space provided.

2. For each force that you have drawn, specify:

- the type of force,
- the object on which each force is exerted and the object exerting each force, and
- the surface on which each *contact* force is applied.

B. Rank the magnitudes of all the vertical forces you have drawn in the three diagrams from largest to smallest. Explain.

C. Imagine that small holes are opened in the container wall near the bottom of each layer.

1. Predict what will happen to the water near each hole. Explain.

2. Observe the demonstration and check your prediction. Describe the motion of the water near each hole. (A sketch may help.)

   What does your observation suggest about the existence of horizontal forces on the three layers of water in part A?

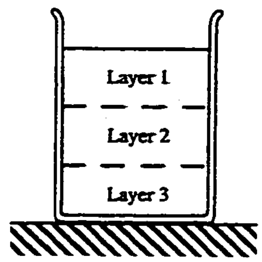
3. Based on the trajectories of the three streams of water, what can you say about the relative magnitudes of the horizontal forces on the three layers? Explain.

   If necessary, revise your free-body diagrams in part A so that they are consistent with your answers to questions 2 and 3 above.

II. Pressure

A. Recall the three layers of water from part I.

1. Which *force* would you use to determine the *pressure* at the bottom of layer 2? (There may be more than one correct answer.) Explain your reasoning. (*Hint:* Refer to your free-body diagrams from section I. Which forces are exerted at the bottom of layer 2?)



2. Which *area* would you use to determine the pressure at the bottom of layer 2? Explain.

3. Suppose that you wanted to determine the pressure at a point in the center of layer 2. For what object(s) would you draw a free-body diagram? Which force and which area would be useful in determining the pressure?

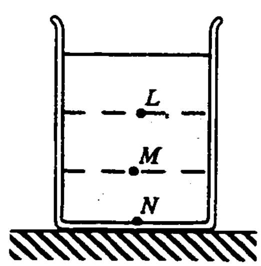
B. If you did not include forces exerted by the atmosphere in your free-body diagrams in section I, answer the following questions.

1. What additional forces would you need to include in your free-body diagrams to take forces by the atmosphere into account? (Describe each force as you did in section I.) Explain.

2. Would any of the existing forces in your free-body diagrams change in magnitude? Explain.

Three points, *L*, *M*, and *N*, are marked at the bottom of the three layers.

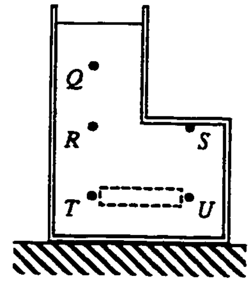
C. Rank the pressures at points *L*, *M*, and *N*. Explain how your answer is consistent with the free-body diagrams that you drew in section I.



Is your answer consistent with the equation $P = P_0 + \rho g h$ for pressure in an incompressible liquid? (*Hint:* At what point is $h = 0$? What is the pressure at that point?)

III. Pressure as a function of depth

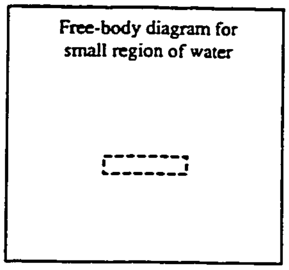
The container at right is filled with water and is at rest on a table. An imaginary boundary that outlines a small region of water has been drawn in the diagram. Treat this small region of water as a single object.



A. Draw a free-body diagram for the small region of water in the space below the figure.

B. Compare the magnitudes of all the *horizontal* forces that you have drawn.

Is your answer consistent with the motion of the small region of water? Explain.



C. Use your answer to part B to compare the pressures at points *T* and *U*. (*Hint:* How is the pressure at point *T* related to the force on the small region of water by the water to its left?)

D. Rank the pressures at points *Q*, *R*, *S*, *T*, and *U*. Explain.

E. Consider the following student dialogue:

Student 1: *"The pressure at a point is equal to the weight of the water above divided by the area. Therefore the pressure at point R is greater than the pressure at point S because there's no water above point S."*

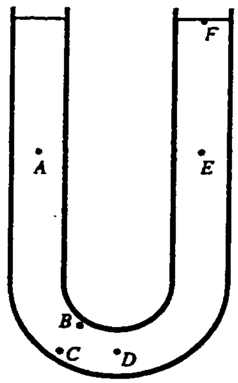
Student 2: *"I agree. The pressure is $P_o + \rho gh$, and h is zero for point S and greater than zero for point R. Therefore, the pressure at R must be greater."*

Do you agree with either student? Explain.

➪ Check your answers with a tutorial instructor before you continue.

IV. Pressure in a U-tube

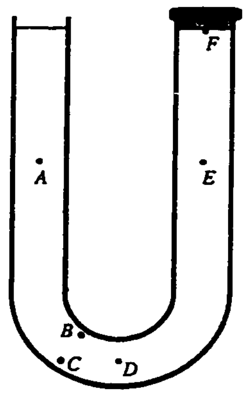
A U-shaped tube about half a meter tall is filled with water as shown.



A. Rank the pressures at points *A* through *F*. Explain.

Is your ranking consistent with the equation $P = P_o + \rho gh$? Explain.

B. The right end of the tube is now sealed with a stopper such that the water levels on both sides remain the same. There is no air between the stopper and the water surface.



1. Determine whether the pressures at points *A* and *D* will *increase, decrease,* or *remain the same.* Explain.

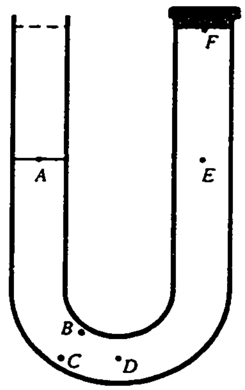
2. Is the pressure at point *E greater than, less than,* or *equal to* the pressure at point *D?*

   Does the difference in pressure ΔP_{DE} between points *D* and *E* change when the stopper is added? Explain.

3. Is the pressure at point *F greater than, less than,* or *equal to* atmospheric pressure?

4. Is the force exerted by the rubber stopper on the water at the top of the right side *greater than, less than,* or *equal to* the force exerted by the atmosphere on the water surface on the left?

C. A syringe is now used to remove some water from the left tube. (The water level in the right-hand side has not been shown.)



As the water level on the left is lowered, what do you predict will happen to the water level in the right-hand side? Explain.

Before you leave the tutorial, observe the demonstration and record your observation. This situation will be examined further in the homework.

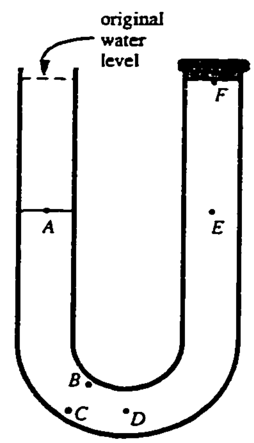
PRESSURE IN A LIQUID Name ______________

Mech HW 131

1. A U-tube filled with water is closed on one end. The tube is about one-half meter in height. As you observed in tutorial, when water is removed from the open end, the water level in the closed end does not change.

a. What is the pressure at point *F* before any water is removed?

On the basis of your answer, is there a force exerted by the stopper on the top surface of the water?



b. The water level on the left is lowered until it reaches point *A*.

Does the pressure at point *A increase, decrease,* or *remain the same?* Explain.

On the basis of your answer, does the pressure at point *D increase, decrease,* or *remain the same?* Explain. If the pressure changes, how does the change in pressure at point *A* compare to the change in pressure at point *D?*

Do the pressures at points *E* and *F increase, decrease,* or *remain the same?* How do the changes in pressure at these points compare to the change in pressure at point *A?*

Does the force exerted by the stopper on the top surface of the water *increase, decrease,* or *remain the same?* Explain.

c. Suppose that point *F* is 0.5 m above point *E*. Determine the pressure at point *F*. (*Hints:* What is the pressure at point *E?* The density of water is $\rho = 1000 \text{ kg/m}^3$, $g = 10 \text{ m/s}^2$, and $P_{atm} = 1.01 \times 10^5 \text{ N/m}^2$)

Suppose instead that the tube is much taller than 0.5 m. Calculate the distance above point *E* at which the pressure in the water would be zero (*i.e.*, find the height of water above point *E*).

d. Use your answers above and the definition of pressure to explain why the water level on the right remains at point *F* for a U-tube that is 0.5 m tall.

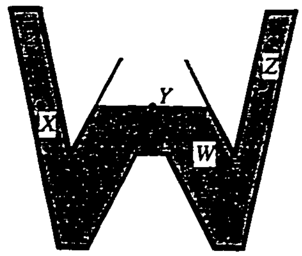
Mech HW 132

PRESSURE IN A LIQUID

2. A W-shaped piece of glassware is partially filled with water as shown. Point X is at the same height as the water level in the center of the tube.

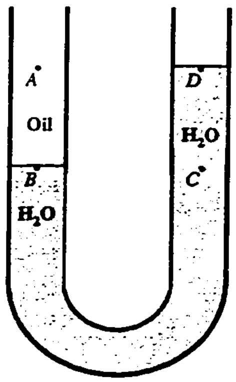
   For each of the following points, state whether the pressure is *greater than, less than,* or *equal to* atmospheric pressure. Explain.

   - point W
   - point X
   - point Y
   - point Z



3. A U-tube is partly filled with water. Oil is then poured on top of the water on one side. The final water levels on both sides are as shown. The top surface of the oil on the left is not shown.

   Point A is at the same height as point D, point B is at the same height as point C.



   a. Is the top surface of the oil *above, below,* or *at the same height as* point A? Explain.

   b. Is the pressure at point B *greater than, less than,* or *equal to* the pressure at point C? Explain.

   c. Is the difference in pressure between point B and point A *greater than, less than,* or *equal to* the difference in pressure between point C and point D? Explain how your result is consistent with the equation $P = P_0 + \rho gh$.

   Based on your answers to parts b and c above, is the pressure at point A *greater than, less than,* or *equal to* the pressure at point D?

   d. What does your answer to part c suggest about whether the top surface of the oil will be *above, below* or *at the same height as* the top surface of the water?

   If your result is inconsistent with what you predicted in part a, resolve the inconsistency.

APPENDIX F INITIAL WORK IN IDENTIFYING STUDENT DIFFICULTIES WITH ARCHIMEDES' PRINCIPLE: THE CARTESIAN DIVER INTERVIEWS

1. INTRODUCTION

In this appendix we describe an initial set of interviews designed to probe student difficulties with Archimedes' principle. We developed an exploratory interview protocol based on the Cartesian diver, a common lecture demonstration. Student responses to the questions posed in the Cartesian diver interviews suggested the existence of several difficulties with Archimedes' principle. We illustrate these difficulties with examples of student responses from the interviews.

2. CARTESIAN DIVER INTERVIEWS

In this section we will discuss an interview protocol developed to investigate student understanding of buoyancy in the context of a Cartesian diver. We will discuss the analysis of student responses. Finally, we will describe in some detail two common types of student difficulties and summarize.

A. Interview population

We interviewed seven students from Physics 224, the second-year thermal physics course at the University of Washington. The students were interviewed in the last week of the quarter and had completed all instruction on hydrostatics. As noted in the Introduction to the dissertation, the students that we interviewed all were volunteers, and most received grades above the course mean in Physics 224.

B. Interview protocol

The interview protocol centers on a Cartesian diver.[1] As previously noted, another version of the Cartesian diver had been used in lecture during the second week of class to demonstrate Archimedes' principle. A diagram of the lecture demonstration apparatus is shown in Figure F-1. A large glass cylinder is filled with water. A tube connects the column of water to a small plastic bellows that can be compressed. Inside the cylinder is a glass jar containing air and several small lead weights. The jar is capped with a plastic bellows like the one at the end of the tube.

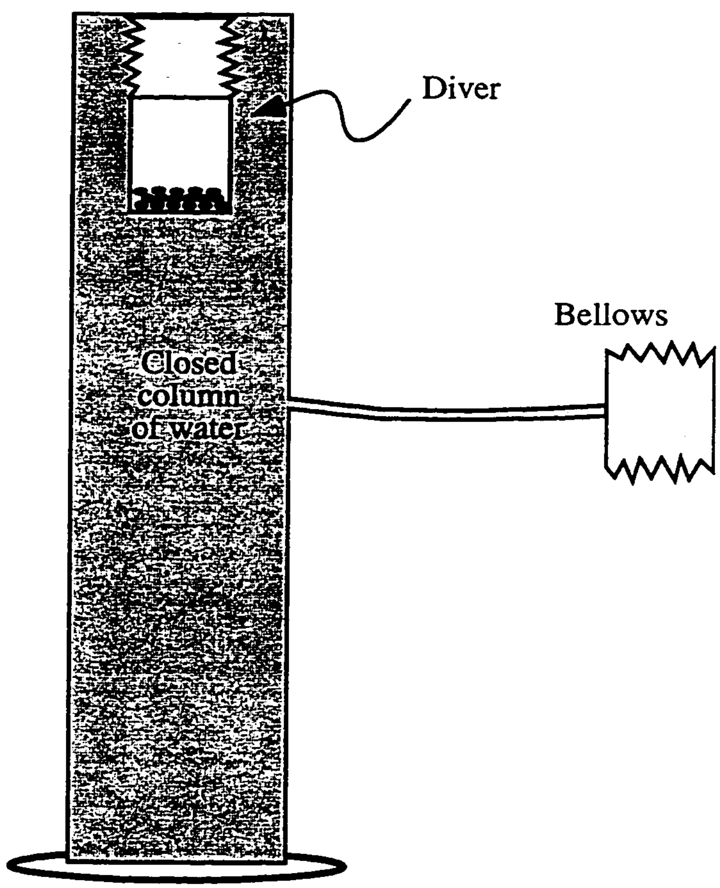


Figure F-1: An illustration of the Cartesian diver used in a lecture demonstration in Physics 224.

For the interviews, we used a smaller diver that looked very different from the one shown in the lecture demonstration. This diver was made from the bulb of a plastic dropping pipette with an attached weight, placed in a two-liter soda bottle nearly filled with water. The bottom of the bulb is open and the diver contains water and a bubble of air. The amount of air and the weight on the diver are adjusted so that the diver initially barely floats, as shown in Figure F-2. Although the two divers look very different, they operate on similar principles. When the pressure in the liquid is increased, the air trapped in each diver is compressed, so that the density of the diver increases. For the demonstration shown in Figure F-1, the volume of the bellows on top of the diver decreases as the pressure increases. For the interview apparatus, on the other hand, the pressure increase leads to a decrease in the volume of the air bubble inside the diver and water enters the open bottom of the diver.

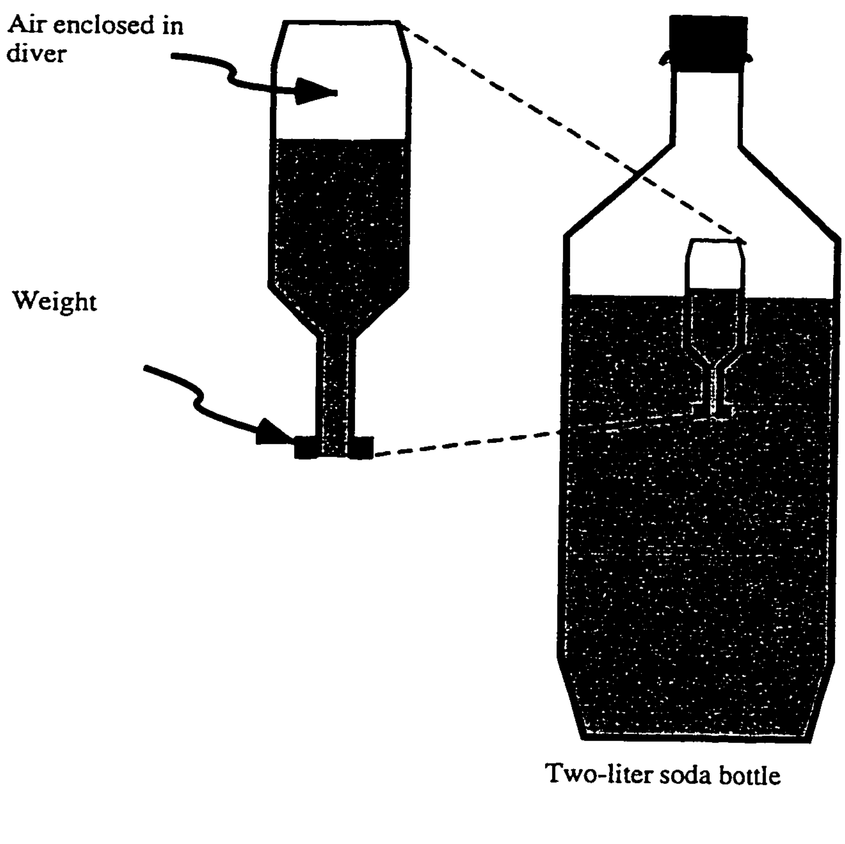


Figure F-2: The Cartesian diver used in individual demonstration interviews.

The interview begins with the Cartesian diver floating. Students are asked to describe the behavior of the diver and give a physical explanation of its behavior. Those students who do not draw a free-body diagram for the diver are asked to do so. Students are then asked what changes, if any, could be made to the apparatus in order to make the diver sink. After these preliminary questions, the interviewer asks the student to predict what will happen if the soda bottle is squeezed. After a student has made and explained a prediction, the interviewer squeezes the bottle and allows the students to observe the diver sinking. Students are then asked to account for the behavior of the diver, and to draw a free-body diagram for the diver as it is sinking.

C. Correct answers

A qualitatively correct explanation of the behavior of the diver includes a description of the gravitational force exerted on the diver by the earth and the buoyant force exerted on the diver by the surrounding liquid, as shown in Figure F-3. Since the diver is initially not accelerating, the net force on it must be zero, and the weight and the buoyant force must be of equal magnitude.

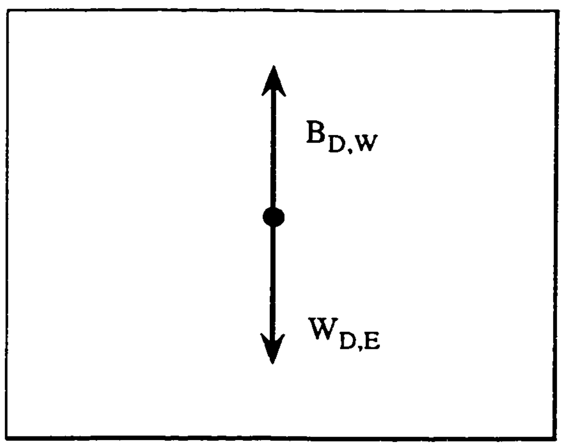


Figure F-3: A free-body diagram for the floating Cartesian diver.

When the soda bottle is squeezed, the pressure in the water and air in the bottle increases. This increase in pressure causes the air bubble inside the diver to be compressed. The mass (and thus the weight) of the diver/air/water system increases as additional water enters the diver. As the weight increases, the diver sinks lower into the water; the volume of water displaced increases until it is enough so that the buoyant force is again equal to the weight. If the weight of the diver becomes greater than the maximum value of the buoyant force, the diver then accelerates toward the bottom of the tank. In other words, the diver sinks.

3. ANALYSIS OF STUDENT RESPONSES TO CARTESIAN DIVER INTERVIEWS

All of the students were able to give a correct description of the behavior of the floating diver. Although a free-body diagram is helpful in explaining this situation, no students spontaneously drew a free-body diagram at this point. When asked, all of the students drew correct free-body diagrams. All students were able to suggest at least one method of causing the diver to sink. Most of the students proposed hanging weights on the bottom, but some suggested other methods, like replacing the liquid in the soda bottle with another liquid of smaller density.

The next part of the interview proved to be much more challenging. Despite having seen a similar demonstration in class, only two of the seven students predicted that the diver would sink when the bottle was squeezed.[2] All students correctly stated that squeezing the soda bottle would increase the pressure in the liquid. However, most of the students said that this increase in pressure would cause the diver to float higher in the liquid. For example, one student said:

> S: "At the bottom of where the [diver] is floating there was some pressure here and now it's some greater pressure. So because this pressure has increased the force that's acting up is greater than it was before. So this object is going to move up until they're equal again."

Further discussion with this student and several others revealed that many students either did not notice that the bottom of the diver was open or were not able to grasp the significance of the open bottom. Beyond this relatively minor misunderstanding of the experimental setup, responses of this type suggest a difficulty with the buoyant force. This student appears to be reasoning based on the contact force exerted by the water upward on the bottom of the diver but ignoring the contact force pushing down on the top of the diver. Although the pressure in the water increases, the pressure in the air above the water increases by the same amount, so the buoyant force, or the sum of the forces on the diver by the surrounding liquids, would not change if the diver itself did not change. As we will show, many students seemed to have difficulty in distinguishing the buoyant force from the individual contact forces on the object, and many associated the buoyant force solely with the contact force acting upward on the bottom surface of the diver.

Students who were unable to predict the behavior of the diver were shown that it sinks when the soda bottle is squeezed. After making this observation, these students should have been able to account for the behavior using their knowledge of hydrostatics. However, none of the seven students correctly explained the behavior of the diver after seeing the diver sink. Even the students who had correctly predicted that the diver would sink gave incorrect explanations for its behavior. Two common types of incorrect reasoning revealed serious misunderstanding of the physics of buoyancy. We will describe each of these common difficulties in some detail in the following sections.

A. Student difficulties distinguishing pressure and buoyant force

When asked to explain why the diver sinks, four of the seven students drew very similar incorrect free-body diagrams. These students correctly drew a weight force and a buoyant force acting on the diver, but then drew an additional force acting down on the top of the diver. The students described this force in a variety of ways. One student said:

> S: And then there's going to be another component of force down that will oppose this [indicates buoyant force] ... there's some other force pushing down on, I guess we'll just call it the weight of the water above the smaller object or the pressure exerted by the volume of fluid above it. We could call it maybe a *P dA* force, *P* being the pressure of the fluid above it acting on the surface area of the smaller object in the direction that it travels in.

This student describes the additional force alternately as "the weight of the water above the smaller object" or as "the pressure exerted by the fluid above it." In either case, his statement contradicts the description of the buoyant force as the sum of all forces on the object by the surrounding water. Another student drew the diagram shown in Figure F-4 and made the following statement:

> S: As it's accelerating, we have weight from the earth, the buoyancy from the water, and some new 'pressure force.'

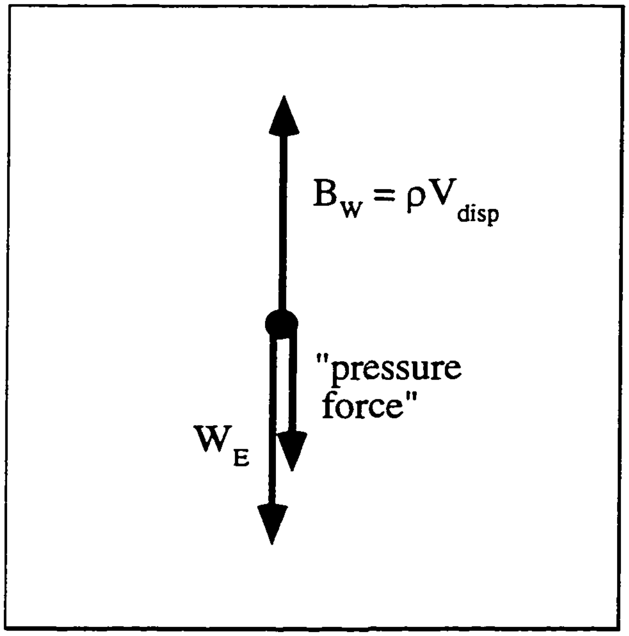


Figure F-4: An incorrect free-body diagram drawn by a student for the sinking Cartesian diver. The additional "pressure force" contradicts the definition of the buoyant force as the sum of all forces on the object by the surrounding liquid.

Again the student's description contradicts the definition of the buoyant force as the sum of all forces on the object by the surrounding liquid. More than half of the students in this interview population gave responses of this type, indicating that these students did not have a functional understanding of the definition of the buoyant force as the sum of forces on an object by the surrounding liquid(s).

The explanation given by the first student above suggests difficulty in distinguishing the buoyant force from pressure. A later statement by this student confirms that he is confusing these ideas:

> S: I might be a little ambiguous in my use of the word pressure because sometimes I get pressure and force mixed up and now I'm trying to indicate that by pressure I'm really saying that the buoyant force acting on the overall area of this object is what I'm talking about.

A second student described his struggle with the concepts of pressure and buoyant force in a similar fashion:

> S: I'm unsure right now. I've just drawn these two [forces] because at the beginning I said there was the gravitational force and there's the buoyant force and I'm still not sure if the buoyant force and what I'm calling the water pressure, are they the same thing, or are they different?

These students were very explicit about their difficulty with the concepts of force and pressure. Other students made statements that suggested that pressure is a vector quantity, like "the pressure pushes down" It is possible that these students are simply not speaking carefully; professional physicists often make similar statements, and textbooks routinely use arrows to show atmospheric pressure acting on the free surface of a liquid. However, the free-body diagrams drawn by students and the statements above suggest that the difficulty in distinguishing force and pressure in general and the buoyant force and the pressure in particular is more than a matter of carelessness. Nearly all of the students that we interviewed had difficulties of this nature, and none were able to connect the pressure in the liquid to the buoyant force in order to account for the behavior of the Cartesian diver.

B. Student difficulties with the final position of the diver

In addition to the difficulty in distinguishing force and pressure, a second major difficulty arose in the Cartesian diver interviews. In the course of probing the understanding of one of the students, an interviewer asked the student to describe the behavior of two objects with the same volume but different masses, released approximately halfway down in the liquid. The student noted that the two objects would have different densities, and the following exchange took place:

> I: How does density relate to floating?
>
> S: You have the greater weight for ... the same volume ... so the denser object is going to have a greater weight and because gravity acts on it, it's going to go down farther ... than something that is less dense.

This student went on to say that one of the objects might end up at a final position "about a third of the way down" in the water. A second student was asked a similar question, and gave a similar response:

> S: We're still keeping the volume the same, but we're increasing the mass per unit volume, so I would say that yes, it probably would rest at a different point.

The student went on to say:

> S: There'd be some equilibrium height within the fluid that would be different from the first [object].

Again, this student was referring to an equilibrium height below the surface of the liquid. Three of the seven students interviewed stated explicitly that objects of different densities will float at different depths beneath the surface of an incompressible liquid. A fourth student alluded to this belief, but the interviewer did not probe this student on the issue. It should be noted that the interviewers did not expect this particular difficulty, and that the interview protocol was not designed to test for the difficulty. Students spontaneously used this idea in attempting to explain the behavior of the Cartesian diver.

C. Summary of results

Despite having seen a similar demonstration in class, students had a great deal of difficulty accounting for the behavior of the Cartesian diver, and none gave completely correct responses. Several students drew free-body diagrams in which there were forces on the diver by the liquid in addition to the buoyant force and gave responses suggesting a confusion between pressure and buoyant force. Several students made statements suggesting that objects of different densities could be suspended at different equilibrium positions between the surface of the liquid and the bottom of the vessel. Each of these errors was made by approximately half of the seven students.

4. DISCUSSION

Analysis of student responses to the Cartesian diver interviews suggested a variety of student difficulties with buoyancy and the buoyant force. Although the small number of interviews did not allow us to make statements about the prevalence of these difficulties, these findings led to the development of several additional research tasks that will be discussed in detail in Chapter 6.

The context of the Cartesian diver seemed to be a very challenging one for the students, and indeed it seemed possible that the surprising behavior of the diver (and it was surprising to most of the students) might even suggest to students that objects can float at different levels in a liquid. In fact, many instructors, in showing the Cartesian diver demonstration, emphasize the fact that the diver can be made to rest at an equilibrium position beneath the water surface if it is adjusted such that the average density of the diver is exactly equal to the density of the water.[3] The responses given by these students suggest that this demonstration may in fact confuse students and reinforce existing incorrect ideas rather than clarifying the physical principles involved.

Notes for Appendix F:

[1] See, for example, Penick, J. E., "The mysterious closed system," *The Science Teacher* **60,** 30-33 (1993)

[2] See P. Kraus, "Promoting active learning in lecture-based courses: demonstrations, tutorials, and interactive tutorial lectures," Ph.D. dissertation, Department of Physics, University of Washington, 1997, (unpublished) for a discussion of the ability of students to recall and understand the results of the demonstrations shown in lecture.

[3] See, for example, refs. 1 and **Error! Bookmark not defined.**.

APPENDIX G DETAILED SUMMARY OF STUDENT RESPONSES TO PROBLEMS ON BUOYANCY AND ARCHIMEDES' PRINCIPLE

In Chapter 6 we have described the responses of students to several written problems probing understanding of Archimedes' principle and hydrostatic forces in the context of buoyancy. In this appendix we will include some of the details of student responses in different sections and on different problems that are omitted in Chapter 6.

1. STUDENT RESPONSES TO THE FIVE BLOCKS PROBLEM

The written version of the Five Blocks problem has been given to students in many different classes. At the introductory college level, the problem has been given to students in Physics 115, the algebra-based introductory physics course at the University of Washington, Physics 11, the calculus-based introductory physics course at Harvard University, and Physics 152, the calculus-based introductory physics course at Purdue University. In addition to the students in these introductory courses, the problem has been given to students in several sections of Physics 224, the second-year thermal physics course at the University of Washington. The students in these courses had completed various amounts of instruction at the time that they completed the problem.

Five blocks of the same size and shape but different masses are shown at right. The blocks are numbered in order of increasing mass (i.e. $m_1 < m_2 < m_3 < m_4 < m_5$).

All the blocks are held approximately halfway down in an aquarium filled with water and then released. The final positions of blocks 2 and 5 are shown.

On the diagram, sketch the final positions of blocks 1, 3, and 4. Explain your reasoning.

(Assume that the water is incompressible.)

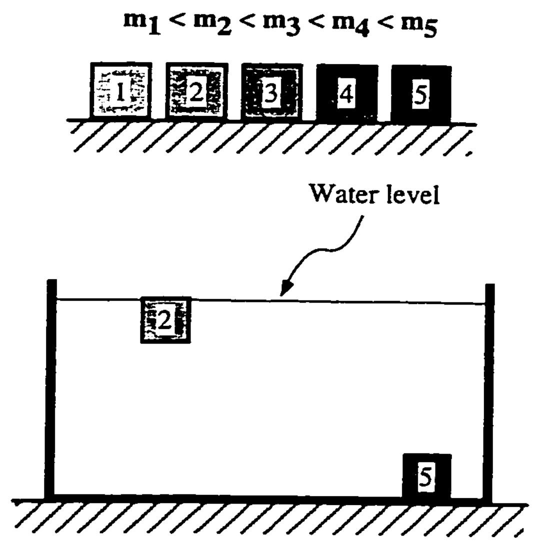
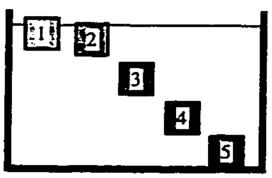


Figure G-1: The written version of the Five Blocks problem.

The written version of the Five Blocks problem has been given to four sections of Physics 115. The students in the various sections had completed different amounts of standard instruction in hydrostatics: one class had had no instruction, two classes had completed instruction on pressure vs. depth but had not studied Archimedes' principle, and one class had completed all instruction in hydrostatics. The responses given by students in these sections are summarized in Table G-1. Despite the different amounts of instruction, all the classes performed at approximately the same level.

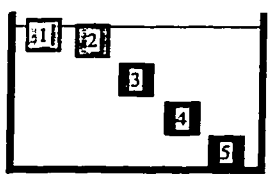
Table G-1: Student responses to the Five Blocks question in several sections of Physics 115 before and after instruction.

	Physics 115 Washington Wi '96 (*N* = 16)	Physics 115 Washington Au'95 (*N* = 110)	Physics 115 Washington Sp'98 (*N* = 109)	Physics 115 Washington Wi'96 (*N* = 136)
	Before all instruction in hydrostatics	After standard instruction on pressure only	After standard instruction on pressure only	After standard instruction on buoyancy
Correct	15%(2)	25% (25)	25% (25)	10% (16)
Incorrect: 	80% (13)	75% (80)	75% (82)	85% (119)
Other	5% (1)	5% (5)	~0% (2)	~0% (1)

All percentages rounded to the nearest 5%.

We have given the Five Blocks problem to two sections of Physics 224 at the University of Washington. In one section, it was a true pretest in that students had had no instruction in hydrostatics. In the second section, the students had completed standard lecture instruction in hydrostatics. The results from these two sections are shown in Table G-2.

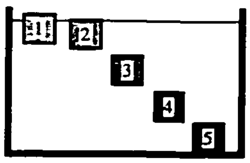
Table G-2: Student responses to the Five Blocks question in two sections of Physics 224, before and after standard instruction in hydrostatics.

	Physics 224 Washington Wi '96 (N = 13)	Physics 224 Washington Au'96 (N = 42)
	Before all instruction in hydrostatics	After standard instruction in hydrostatics
Correct	60% (8)	50% (22)
Incorrect: 	40% (5)	50% (20)

All percentages rounded to the nearest 5%.

The Five Blocks problem has also been used as a pretest for hydrostatics tutorials at Harvard University and Purdue University. The students at Harvard had not completed prior lecture instruction on buoyancy but had completed several weeks of instruction on Newtonian mechanics and had been assigned reading on Archimedes' principle. The students at Purdue had completed standard lecture instruction. The responses given by these students are displayed in Table G-3.

Table G-3: Summary of responses to the Five Blocks question for students in calculus-based courses at other universities.

	Physics 11 Harvard Au'96 (N = 138)	Physics 152 Purdue Sp'98 (N = 439)	Physics 152 Purdue Au'98 (N = 326)
	Reading only	Standard	Standard
Correct	45% (62)	45% (189)	35% (122)
Incorrect:  	55% (76)	55% (250)	65% (204)

All percentages rounded to the nearest 5%.

In addition to examining the answers, we have analyzed the reasoning used by students in responses to the Five Blocks problem before and after standard instruction. In the small section of Physics 115 that had had no previous instruction on hydrostatics, all of the students answered using either the density of the blocks or the weight of the blocks alone. In the two Physics 115 classes that had completed some or all instruction, students used a variety of different forms of reasoning. These different types of reasoning are summarized in Table G-4. Many of these students explained their answers using the formal tools of hydrostatics, like pressure and the buoyant force, but even in the class that had completed all instruction on hydrostatics, fewer than one third of the students used ideas of pressure or forces. For many of the students, it seems that standard instruction does not change the way that they think about this problem, at least to the extent that this thinking can be inferred from the written reasoning given by students.

Table G-4: Reasoning given by Physics 115 students in answering the Five Blocks question before and after standard instruction.

	Physics 115 Washington Au '95 ($N = 110$)	Physics 115 Washington Wi '96 ($N = 136$)
	After standard instruction on pressure only	After all standard instruction in hydrostatics
Forces or Displaced water	20% (22)	25% (35)
Pressure	<5% (3)	<5% (4)
Density	30% (32)	25% (32)
Mass / Weight	25% (28)	35% (48)
Other / None	25% (25)	15% (17)

All percentages rounded to the nearest 5%.

2. INSTRUCTOR RESPONSES TO THE FIVE BLOCKS PROBLEM

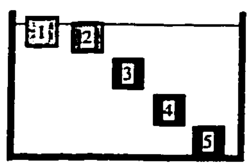
For many years, our group has taught courses for pre- and in-service elementary, middle, and high school teachers. These courses are lab-based and use *Physics by Inquiry.*[1] At the time of the development of the *Sink and Float* module in *Physics by Inquiry,* the specific conceptual difficulties described above were not known to the authors. In order to test the effectiveness of the curriculum in addressing these difficulties, we gave the Five Blocks problem to several groups of students both before and after instruction. Before instruction, we have found that pre- and in-service teachers have performed at a level that is similar to the level of students in the second-year course, with approximately half of the class giving the descending line answer.

The *Sink and Float* module of *Physics by Inquiry* does not teach the concepts of forces or pressure. Rather, students perform a series of experiments to identify the variables that influence and determine the sinking and floating behavior of objects. Students develop Archimedes' principle through observations.

After the *Sink and Float* module, we have found that approximately 75% of our students answer the Five Blocks problem correctly. Only 5% have given the 'descending line' answer. (The remaining students gave inconclusive answers to this problem.) These results are shown in, along with pretest data from several sections of pre- and in-service teachers.[2]

We have posed the Five Blocks problem to participants in workshops on *Physics by Inquiry*. Most of the participants in these workshops are college and university faculty and graduate students in sciences and education who are responsible for the preparation of teachers. The success rate for workshop participants is about 80%, with 20% giving the descending line answer. When students (including precollege teachers) perform at or above the level of workshop participants after completing research-based instruction, we consider the curriculum to be successful.

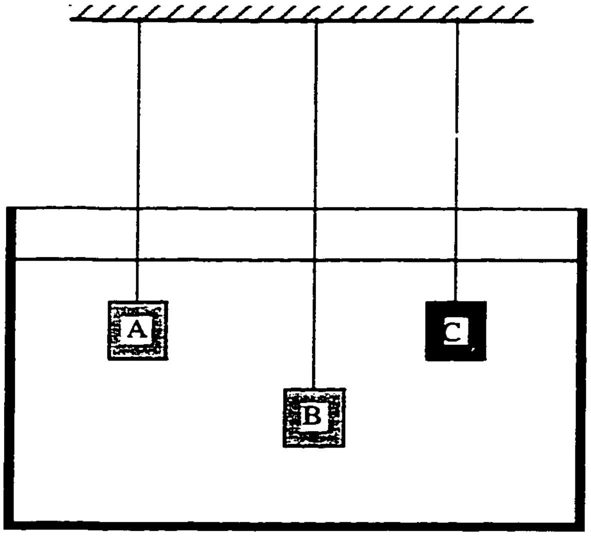
Table G-5: Responses of precollege teachers and university instructors on the Five Blocks problem.

	Inservice precollege teachers (*N* = 25)	Faculty workshop participants (*N* = 77)	Inservice precollege teachers (*N* = 40)	Tutorial instructors, Purdue (*N* = 18)
Instruction	before *Physics by Inquiry*	n/a	after *Physics by Inquiry*	before tutorial *Buoyancy*
Correct	50% (12)	80% (62)	80% (33)	70% (13)
Incorrect: 	50% (13)	20% (15)	20% (7)	30% (5)

All percentages rounded to the nearest 5%.

3. STUDENT RESPONSES TO THE BUOYANT FORCE PROBLEM

One version of the Buoyant Force problem is shown in Figure G-2. Students are asked to rank the buoyant forces on three blocks with equal volume. Two of the blocks have the same mass but are completely immersed at different depths in the liquid. A third block is at the same depth as one of the first pair but has a different mass. Thus, we have two blocks with the same volumes and masses at different depths and two blocks with the same volumes but different masses at the same depth. Students are asked to compare the magnitudes of the buoyant forces acting on the three blocks and the magnitudes of the tensions in the three strings.



Three cubical blocks of equal volume are suspended from strings. Blocks A and B have the same mass and block C has less mass. Each block is lowered into a fish tank to the depth shown in the figure below.

Rank the buoyant force acting on each block from largest to smallest. If any buoyant forces are equal indicate that explicitly. Explain.

Rank the tension in each string from largest to smallest. Explain.

Figure G-2: The Buoyant Force problem. Students are asked to compare the buoyant forces on two blocks with the same volume and mass that are located at different depths and to compare the buoyant forces on two blocks with equal volume at the same depth that have different masses. See also Figure G-3.

The version of the buoyant force problem shown in Figure G-2 was used as an ungraded posttest after lecture instruction in one section of Physics 115, as a final exam question after standard instruction in Physics 121, the algebra-based introductory physics course at the University of Maryland, and as an ungraded quiz after standard instruction in Physics 224. A very similar version of this problem, in which the blocks were suspended from spring scales (see Figure G-3), was given as an ungraded quiz after standard instruction in a section of Physics 224.

Table G-6: Student responses after standard instruction to the comparisons of the magnitudes of the buoyant forces acting on the blocks in the Buoyant Force problem (shown in Figure G-2).

	Physics 115[AB] Washington Wi'97 (*N* = 201)	Physics 121[AB] Maryland Au'97 (*N* = 126)	Physics 224[2Y] Washington Wi'96 (*N* = 22)	Physics 224[2Y] Washington Sp'97 (*N* = 15)
	Standard instruction	Standard instruction	Standard instruction	Standard instruction
Correct (all *B* equal)	10% (24)	30% (36)	25% (6)	45% (7)
B depends on mass only	30% (58)	35% (43)	20% (5)	10% (2)
B depends on depth only	10% (23)	10% (15)	20% (5)	20% (3)
B depends on mass and depth	30% (57)	25% (30)	20% (5)	20% (3)
Other / Blank	20% (39)	<5% (2)	5% (1)	0

All percentages rounded to the nearest 5%. [AB]Algebra-based. [2Y]Second-year. The Wi'96 section of Physics 224 used the version of the problem shown in Figure G-3.

Student responses to the Buoyant Force problem are summarized in Table G-7. As was suggested by student responses to the Five Blocks problem, many students answered that the buoyant forces on the blocks depended on the mass of the blocks.

Table G-7: Student responses to the comparison of the magnitudes of the buoyant forces (B) acting on blocks of equal volume but different mass that rest at the same depth. (The labels A and C refer to the blocks shown in Figure G-2)

	Physics 115[AB] Washington Wi'97 ($N = 201$)	Physics 121[AB] Maryland Au'97 ($N = 126$)	Physics 224[2Y] Washington Wi'96 ($N = 22$)	Physics 224[2Y] Washington Sp'97 ($N = 15$)
	Standard instruction	Standard instruction	Standard instruction	Standard instruction
Correct ($B_A = B_C$)	25% (48)	40% (51)	50% (11)	65% (10)
Greater B for smaller mass ($B_C > B_A$)	15% (29)	15% (19)	35% (7)	20% (3)
Greater B for greater mass ($B_A > B_C$)	45% (86)	40% (53)	15% (3)	15% (2)
Other/Blank/ Unclear	20% (38)	<5% (3)	~5% (1)	0%

All percentages rounded to the nearest 5%. [AB]Algebra-based. [2Y]Second-year. The Wi'96 section of Physics 224 used the version of the problem shown in Figure G-3.

Table G-8: Student responses to the comparison of the magnitudes of the buoyant forces (*B*) acting on identical blocks at different depths. (The labels A and B refer to the blocks shown in Figure G-2)

	Physics 115^{AB} Washington Wi'97 (*N* = 201)	Physics 121^{AB} Maryland Au'97 (*N* = 126)	Physics 224^{2Y} Washington Wi'96 (*N* = 22)	Physics 224^{2Y} Washington Sp'97 (*N* = 15)
	Standard instruction	Standard instruction	Standard instruction	Standard instruction
Correct (*B* equal)	40% (84)	65% (79)	50% (11)	55% (8)
B greater at greater depth ($B_B > B_A$)	25% (55)	25% (32)	35% (8)	25% (4)
B greater on higher block ($B_A > B_B$)	15% (26)	10% (12)	10% (2)	15% (2)
Other / Blank	20% (38)	<5% (3)	05% (1)	5% (1)

All percentages rounded to the nearest 5%. ^{AB}Algebra-based. ^{2Y}Second-year. The Wi'96 section of Physics 224 used the version of the problem shown in Figure G-3.

Student responses to the tension comparison question are shown on Table G-9. In all of the courses, the most common incorrect answer was to state that the tension in the string supporting block B is greater than in that supporting A (see Figure G-2). Most of the students who gave this answer referred to the additional water above block B.

Table G-9: Student responses to the tension comparison question in the Buoyant Force Problem after standard instruction.

	Physics 115[AB] Washington Wi'97 ($N = 201$)	Physics 121[AB] Maryland Au'97 ($N = 126$)	Physics 224[2Y] Washington Wi'96 ($N = 22$)	Physics 224[2Y] Washington Sp'97 ($N = 15$)
	Standard instruction	Standard instruction	Standard instruction	Standard instruction
Correct ($T_A = T_B > T_C$)	25% (50)	35% (43)	15% (3)	25% (4)
Incorrect ranking including $T_B > T_A$	35% (73)	45% (54)	45% (10)	45% (7)
Other incorrect	15% (31))	20% (27)	30% (6)	10% (2)
Blank	25% (47)	<5% (2)	15% (3)	10% (2)

All percentages rounded to the nearest 5%. [AB]Algebra-based. [2Y]Second-year. The Wi'96 section of Physics 224 used the version of the problem shown in Figure G-3.

There is a small difference in the response patterns in the two sections of Physics 224 is striking. Part of this difference may be due to the different versions of the problem used for these two groups. The section from Winter '96 was given a version of the problem in which the three blocks were hung from spring scales (see Figure G-3) and the Spring '97 section was given the version with strings. In the version shown in Figure G-3, students were asked about the readings on the spring scales rather than the tensions in the strings. These questions are equivalent to instructors, but may be different to students.

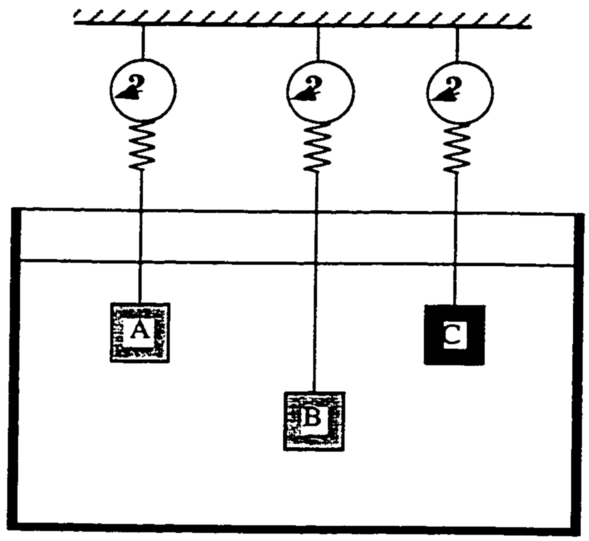


Figure G-3: A variation of the Buoyant Force problem used in one section of Physics 224. Students are asked to rank the buoyant forces and the readings on the spring scales.

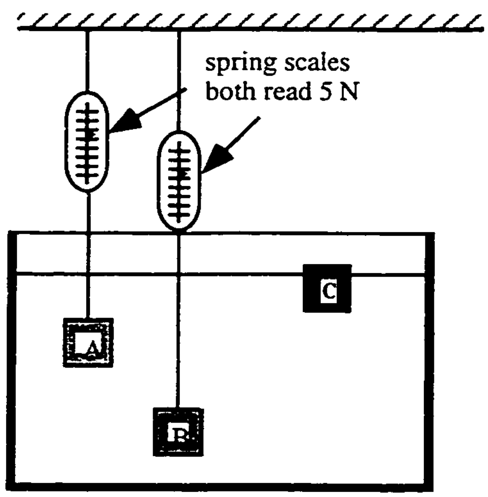
4. STUDENT RESPONSES TO SPRING SCALES PROBLEM

We have examined several problems in which students must correctly apply Newton's second law to an object at rest in a liquid. We developed the Spring Scales problem (see Figure G-4) to probe the connection between force and position that seemed to be prevalent in the Five Blocks problem described above. In Table G-10, we show student responses to the buoyant force and mass comparisons on the Spring Scales problem after standard instruction at the University of Maryland. Many students gave responses consistent with the belief that the force on an object is related to the position of the object. In the mass comparison, in particular, students said that the mass of block B is greater than that of block A because it is deeper, while the buoyant force and reading on the spring scale are the same for the two blocks: "The same two forces are acting upward on both blocks ($F_{tension}$ + $F_{buoyant}$). However, B is lower in the water. Its mg must be greater than that of block A." In some cases, students explicitly related the net force to the position. For example, one student, who had correctly found that the buoyant forces on blocks A and B are equal, wrote, "because if the buoyant forces are equal for A and B, then for B to sit lower in the tank, it must have a larger F_{net} down than A which would entail a larger mg for B."

Table G-10: Student responses to the buoyant force (*B*) and mass (*M*) comparisons on the Spring Scales problem (see Figure G-4) after standard instruction in the algebra-based introductory physics course at the University of Maryland.

	Physics 121 Maryland Au'98 (*N* = 161)
	Standard
Both parts correct (*B*, *M* equal)	15% (27)
Incorrect (*B* equal, *M* unequal)	30% (47)
Incorrect (*B* unequal)	55% (87)

All percentages rounded to the nearest 5%.



Three blocks of the same size and shape are placed in a tank of water as shown below. The masses of the blocks are unknown. Blocks A and B are suspended from springs attached to spring scales. Both spring scales read 5 N. Block C is floating as shown.

Is the magnitude of the buoyant force on block B greater than, less than, or equal to the magnitude of the buoyant force on block A? Explain.

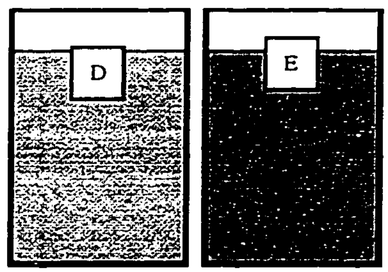
Is the mass of block A greater than, less than, or equal to the mass of block B? Explain.

Figure G-4: The Spring Scales problem.

5. FLOATING CUBES PROBLEM

In the Floating Cubes problem, shown in Figure G-5, two identical cubes are floating in liquids of different densities. Cube D is shown barely floating, while cube E rests at a greater height in its liquid. Students are asked to compare the buoyant forces in the two blocks. An answer to this problem requires use of Newton's second law. Since the blocks are at rest, the net force on each must be zero. Therefore the buoyant force on each must be equal to its weight. Since the two identical cubes have the same weight, the buoyant forces acting on the two cubes have equal magnitude. Although students were not asked to do so, they could deduce that since the floating cubes displace the same weight of liquid

(Archimedes' principle), then a smaller volume displaced must correspond to a liquid of greater density.



Two identical cubes (D and E) are placed in different liquids with different densities.

Is the buoyant force on cube D greater than, less than, or equal to the buoyant force on cube E?

Figure G-5: The Floating Cubes problem.

We have administered this problem on a course examination after standard instruction in Physics 121 at the University of Maryland. In this course, hydrostatics is covered in the last two weeks of the first semester, after instruction on mechanics. The instructor for the course reported covering Archimedes' principle, albeit briefly, but felt that the question was fair. A summary of student results is shown in Table G-11. As shown in the table, this problem proved to be very difficult for most students. A significant fraction of students responded in a fashion consistent with the belief that the force on an object is related to its position. A description of the reasoning used by students can be found in Chapter 6.

Table G-11: Student responses to the Floating Cubes problem after standard instruction in a section of the algebra-based introductory physics course at the University of Maryland.

	Phys 121 Maryland Au'98 (N = 161)
	Standard
Correct (buoyant forces equal)	20% (33)
Incorrect ($B_E > B_D$)	55% (89)
Incorrect ($B_D > B_E$)	25% (37)
Can't tell	~0% (2)

All percentages rounded to the nearest 5%.

6. STUDENT UNDERSTANDING OF MECHANICS COMPARED TO PERFORMANCE ON BUOYANCY PROBLEMS

We have examined the mechanics knowledge of students in the Winter 1999 section of Physics 115 and attempted to compare student performance on the buoyancy tasks described above to that on tasks that require application of Newton's second law in purely mechanical contexts. In particular, we analyzed student responses to the Springs problem shown in Chapter 6 and compared them to responses to the Floating Cubes problem, which is an analogous physical situation to that of the Springs problem. In both problems, identical blocks are at rest at different heights, and in both problems students tend to answer in a manner consistent with the belief that the force on an object is related to the position of the object.

We found that there was no correlation between student performance on the two problems. This lack of correlation suggests that the difficulties that students have with this problem are not solely due to a lack of understanding of Newton's second law. Rather, it seems that many students who correctly apply Newton's second law in purely mechanical problems have difficulty in recognizing how and when to apply it in situations involving objects in a liquid. For example, very few students spontaneously drew free-body diagrams in their responses to the Floating Cubes problem, despite the fact that doing so simplifies the problem solution considerably. There is considerable evidence to suggest that even students who have sound conceptual knowledge of a certain topic often fail to apply this knowledge to problems without being prompted.

Notes for Appendix G:

[1] L. C. McDermott, P. S. Shaffer, and M. L. Rosenquist, *Physics by Inquiry* (John Wiley & Sons, New York, 1996).

[2] The pre-service high school teachers that we have tested have performed at a similar level to in-service teachers in grades K-8. In only one section ($N = 6$) did teachers take both pre- and posttest, and in this case the pretest was a modified version of the problem in which the five blocks all had the same mass but had different volumes. The common incorrect answer in this case was an 'ascending line' rather than the descending line.

APPENDIX H INITIAL VERSION OF TUTORIAL SEQUENCE: *BUOYANCY*

Contents of Appendix H:

Initial version of tutorial sequence: *Buoyancy*

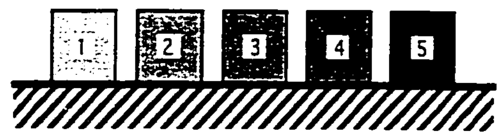
- Pretest: Five Blocks problem
- Tutorial: *Buoyancy*
- Homework: None included

Physics 115 **Pretest** Name ____________

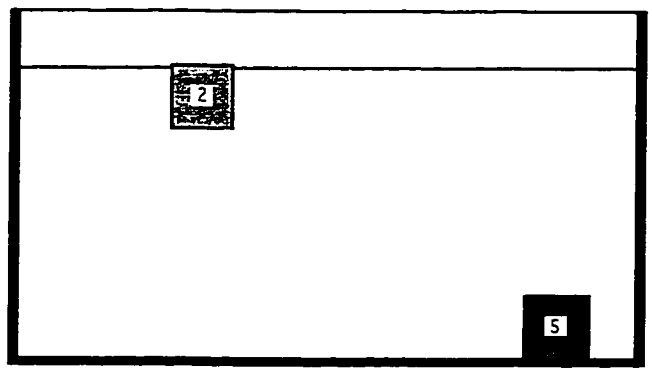
1. Five blocks of the same size and shape but different masses are shown at right. The blocks are numbered in order of increasing mass (i.e. $m_1<m_2<m_3<m_4<m_5$).

   All the blocks are held approximately halfway down in an aquarium filled with water and then released. The final positions of blocks 2 and 5 are shown.

$m_1 < m_2 < m_3 < m_4 < m_5$



A. In the diagram below, sketch the final positions of blocks 1, 3, and 4. (Assume that the water is incompressible.)



B. i. Explain why you drew block 1 where you did.

ii. Explain why you drew block 3 where you did.

iii. Explain why you drew block 4 where you did.

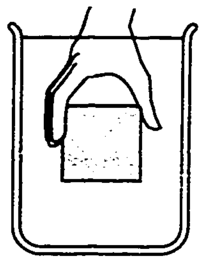
Physics 115
Section 5: Buoyancy

FLUIDS

Name ____________

VIII.Block in Water

A block is observed to float in a beaker of water. This block is then submerged near the center of the beaker as shown and released.



A. Draw a free-body diagram for the block at the instant that it is released. Draw the diagram in the box below.

B. Describe the motion of the block after it is released.

C. Rank the vertical forces you drew in your free-body diagram. If you cannot completely rank the forces, explain why you cannot.

Is your answer consistent with your answer to part A? Explain.

Free-body diagram for block at instant it is released

D. Did you draw any forces pushing down on the top of the block? Why or why not?

E. Did you draw any horizontal forces? Why or why not?

Is it possible to draw the free-body diagram differently?

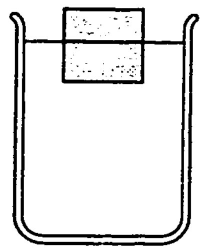
⌛ Call to order: Wait for the class discussion before you continue.

Physics 115
Section 5: Buoyancy

FLUIDS

IX. The floating block

The same block from the previous page is now floating at the top of the beaker.



A. Describe the motion of the block.

B. In the box below, draw a free-body diagram for the block.

C. Is your free-body diagram consistent with your description of the motion of the block? Explain.

D. Compare the free-body diagram you drew in part A to the free-body diagram you drew in part I.

1. Is there a force pushing down on the top of the block in each case?

2. How does the force up on the bottom of the block in part II compare to the force up on the bottom of the block in part I? How do you know?

Free-body diagram for floating block

E. Suppose you added a small piece of lead to the top of the block. Treat the piece of lead and the block as a single object.

1. Would the level at which the block floats change?

2. Would the force up on the bottom of the block change?

3. Does the force up on the bottom of the block vary with depth? Explain.

Is your answer consistent with your knowledge of pressure in a liquid? Explain.

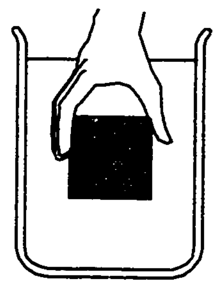
Call to order: Wait for the class discussion before you continue.

Physics 115
Section 5: Buoyancy

FLUIDS

X. Sinking blocks

A. Is it possible for a block to have the same volume and shape as the block in part I but sink when released from the center of the beaker?



Draw a free-body diagram to support your answer.

Free-body diagram for block at instant it is released

B. Compare the free-body diagram for the sinking block to the one you drew on page one. Which forces are the same and which are different? Explain.

How does the buoyant force (i.e. the sum of the forces on the block by the water) on the sinking block compare to the buoyant force on the floating block?

In general, does the buoyant force on a completely submerged object depend on the mass of the object?

C. Imagine that you were to release the block from a much greater depth.

1. Does the force up on the bottom of the block change?

2. Does the force down on the top of the block change?

3. Does the buoyant force change?

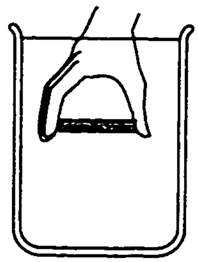
⧗ **Call to order: Wait for the class discussion before you continue.**

Physics 115
Section 5: Buoyancy

FLUIDS

XI. A smaller block

A. Consider a block made from the same material as the sinking block from part III. This block has the cross-sectional area but a much smaller height. Predict whether this block would sink or float when it is placed in the water as shown and released.



B. Draw a free-body diagram to support your answer.

Free-body diagram for block at instant it is released

C. How does the buoyant force on this block compare to the buoyant force on the larger block?

How does the weight of this block compare to the weight of the larger block?

D. In general, does the buoyant force on a *completely submerged* object depend on the *volume* of the object? Explain.

E. In general, how is the *density* of an object related to whether the object will sink or float?

⌛ **Call to order: Wait for the class discussion before you continue.**

✱ Imagine that the block were resting on the bottom of the beaker. How would the free-body diagram for the block compare to the diagram you drew in part B? Explain.

APPENDIX I REVISED VERSION OF TUTORIAL SEQUENCE: *BUOYANCY*

Contents of Appendix I:

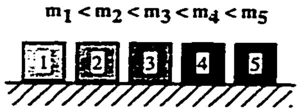
Revised version of tutorial sequence: *Buoyancy*

- Pretest: Five Blocks problem and Displaced Water problem
- Tutorial: *Buoyancy* (revised version)
- Homework: Floating Block problem and Tension problem

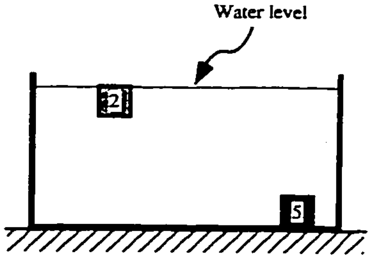
Pretest: Buoyancy Name ______________

1. Five blocks of the same size and shape but different masses are shown at right. The blocks are numbered in order of increasing mass (*i.e.*, $m_1<m_2<m_3<m_4<m_5$).

All the blocks are held approximately halfway down in an aquarium filled with water and then released. Block 2 barely floats and block 5 sinks. (The final positions of blocks 2 and 5 are shown at right.)



A. In the diagram at right, sketch the final positions of blocks 1, 3, and 4. (Assume that the water is incompressible.)



B. i. Explain why you drew block 1 where you did.

ii. Explain why you drew block 3 where you did.

iii. Explain why you drew block 4 where you did.

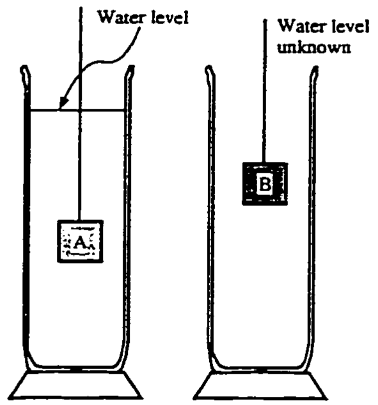
C. In the spaces provided, draw free-body diagrams for blocks 2 and 5 *at the instant they are released*. Recall that they are released at the same depth, near the center of the tank.

Buoyancy

2. An aluminum block, block A, and a brass block, block B, are attached to strings and lowered into identical graduated cylinders. The blocks are the same size and shape, but block B has a greater mass. After block A is lowered into the graduated cylinder, the water level is as shown. Block B is lowered to the level shown in the diagram below.

   The water levels in the two cylinders are the same before the blocks are added.

   A. Is the water level in the graduated cylinder containing block B *higher than*, *lower than*, or *at the same height as* the water level in the graduated cylinder containing block A? Explain. Sketch the water level in the diagram at right.



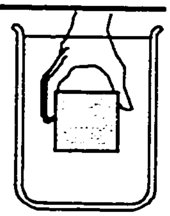
   B. Is the volume of water displaced by block B *greater than*, *less than*, or *equal to* the volume of water displaced by block A? Explain.

   C. Is the tension in the string supporting block B *greater than*, *less than*, or *equal to* the tension in the string supporting block A? Explain.

BUOYANCY

I. Hydrostatics

A. A cubical block is observed to float in a beaker of water. The block is then held near the center of the beaker as shown and released.



1. Describe the motion of the block after it is released.

2. Draw a free-body diagram for the block at the instant that it is released. Rather than drawing a single force by the water on the block, show the forces that the water exerts on each of the six surfaces of the block.

3. Rank the magnitudes of the vertical forces in your free-body diagram. If you cannot completely rank the forces, explain why you cannot.

Free-body diagram for block at instant it is released

Is your answer consistent with the motion of the block? Explain.

Is your answer consistent with how pressure varies with depth in a liquid? Explain.

4. Did you draw any forces acting downward on the top surface of the block? Explain.

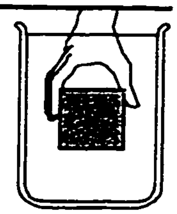
5. In the box at right, draw a vector to represent the vector sum of the forces exerted on the block by the surrounding water.

   Is this vector equal to the net force on the block? (Recall that the net force is defined as the vector sum of *all* forces acting on an object.)

   Is the magnitude of the sum of the forces exerted on the block by the water *greater than, less than,* or *equal to* the weight of the block? Explain.

➪ Check your answer with a tutorial instructor before continuing.

B. A second block has the same volume and shape as the original block.



1. Is it possible for such a block to sink when released from the center of the beaker? Explain.

   Draw a free-body diagram to support your answer. As before, draw the forces exerted on each surface of the block by the nearby water.

Free-body diagram for block at instant it is released

2. Compare the free-body diagram for the block that sinks to the one you drew in part A for the block that sinks. Which forces are the same and which are different? Explain.

3. In the box at right, draw a vector to represent the sum of the forces exerted on the block by the surrounding water.

   Compare the sum of the forces exerted on the block by the surrounding water for the block that sinks to the sum for the block that floats.

   If the object is completely submerged, does the sum of the forces on an object by the surrounding liquid depend on the mass of the object?

4. Imagine that you were to release the block from a much greater depth.

   a. Would the magnitude of the force up on the bottom surface of the block be *greater than, less than*, or *equal to* that of the force you drew above?

   b. Would the magnitude of the force down on the top surface of the block be *greater than, less than*, or *equal to* that of the force you drew above?

   c. Is the magnitude of the sum of the forces on the block by the surrounding liquid *greater than, less than*, or *equal to* the magnitude you found in part 3?

C. The sum of the forces exerted on an object by the surrounding water is called the *buoyant force*. In general, does the buoyant force on a completely submerged object depend on:

- the mass of the object?

- the distance below the surface of the water at which the object is located?

⇨ Check your answer with a tutorial instructor before continuing.

II. Archimedes' principle

Consider two blocks of the same size and shape: one made of aluminum; the other, of brass. Both blocks sink in water.

The aluminum block is placed in a graduated cylinder and the water level rises by 3 cm^3.

A. By how much does the water level rise when the brass cube is dropped in the cylinder? Assume that no water leaves the cylinder. Explain.

When an object is placed in a graduated cylinder of water, the increase in the volume reading of the water is called the *volume of water displaced* by the object.

B. Does the volume of water displaced by a *completely submerged* object depend on

- the mass of the object? Explain.

- the volume of the object? Explain.

- the shape of the object? Explain.

Archimedes' principle states that the magnitude of the buoyant force on an object is equal to the weight of a volume of water equivalent to the volume displaced by the object.

C. Consider the following statement made by a student:

> "Archimedes' principle simply means that the weight of the water displaced by an object is equal to the weight of the object itself."

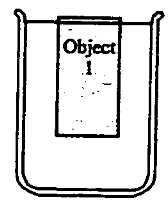
Do you agree with the student? Explain.

III. Sinking and floating

A rectangular object that floats in water, object 1, is released from rest at the center of a beaker. The object accelerates upward.

A. At the instant that object 1 is released is the buoyant force on it *greater than, less than*, or *equal to* the weight of object 1? Explain.

When object 1 reaches the surface, it is observed to float as shown. In this final position, is the buoyant force on the object *greater than, less than*, or *equal to* the weight of the object? Explain.

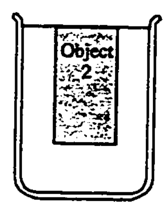


Are your answers to the questions above consistent with Archimedes' principle? Explain.

A second object of the same size and shape but slightly greater mass is released from rest at the center of the beaker. The final position of the object is shown at right.

B. How does the buoyant force on object 2 compare to the buoyant force on object 1:

- at the instant they are released?
- in their final positions?



C. A third object of the same size and shape as the first two but with slightly greater mass than object 2 is released from rest at the center of the beaker. Two students predict the subsequent motion.

Student 1: Since this object is more massive than object 2, it will not go up as high after itis released, as shown at right.

Student 2: Yes, I agree, the buoyant force is slightly less than the weight, so object 3 comes to rest just below the surface of the liquid.

Explain what is *wrong* with the statement made by each student and the diagram.

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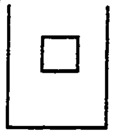
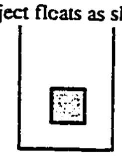
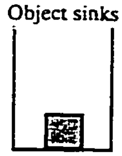
1. Three objects are at rest in three beakers of water as shown.

   a. Compare the mass, volume, and density of the objects to the mass, volume, and density of the displaced water. Explain your reasoning in each case.

Object floats on top 	Object floats as shown 	Object sinks 
Is $m_{object} \begin{pmatrix} > \\ < \\ = \end{pmatrix} m_{displaced\ water}$? Explain.	Is $m_{object} \begin{pmatrix} > \\ < \\ = \end{pmatrix} m_{displaced\ water}$? Explain.	Is $m_{object} \begin{pmatrix} > \\ < \\ = \end{pmatrix} m_{displaced\ water}$? Explain.
Is $V_{object} \begin{pmatrix} > \\ < \\ = \end{pmatrix} V_{displaced\ water}$ Explain.	Is $V_{object} \begin{pmatrix} > \\ < \\ = \end{pmatrix} V_{displaced\ water}$ Explain.	Is $V_{object} \begin{pmatrix} > \\ < \\ = \end{pmatrix} V_{displaced\ water}$ Explain.
Based on your answers above, is $\rho_{object} \begin{pmatrix} > \\ < \\ = \end{pmatrix} \rho_{displaced\ water}$? Explain.	Based on your answers above, is $\rho_{object} \begin{pmatrix} > \\ < \\ = \end{pmatrix} \rho_{displaced\ water}$? Explain.	Based on your answers above, is $\rho_{object} \begin{pmatrix} > \\ < \\ = \end{pmatrix} \rho_{displaced\ water}$? Explain.

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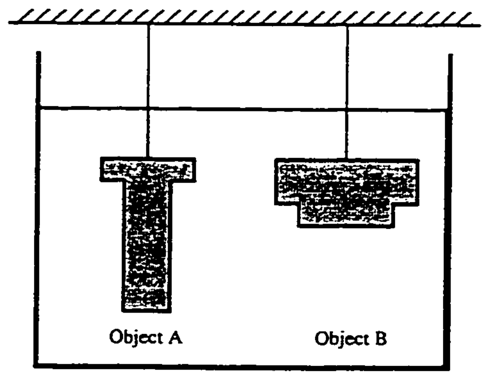
b. On the basis of your answers to part a, what must be true in order for an object to remain at rest when released in the center of an incompressible liquid?

c. Generalize your answers above to answer the following questions. Compare the relative densities of a fluid and an object for the case that (1) the object floats in the liquid and (2) the object sinks in the liquid.

2. Two objects of the same mass and volume but different shapes are suspended from strings in a tank of water as shown.



Consider the following student discussion:

Student 1: *"Both objects have the same volume, so both have the same buoyant force. Therefore the tensions in the two strings must be the same."*

Student 2: *"No, that can't be true. The bottom of object A is further down in the water where the pressure is larger. Therefore the buoyant force on object A must be greater and the tension in that string must be less."*

Student 3: *"I mostly agree with you, student 1. The buoyant force is the same on both objects. However, you forgot the force exerted down on the top of the objects by the water above. That force is larger for object B because the top surface has a greater area, so the tension in the string supporting object B must be greater."*

a. Do you agree with student 1? Explain your reasoning. If student 1 is incorrect, modify the statement so it is correct.

b. Do you agree with student 2? Explain your reasoning. If student 2 is incorrect, modify the statement so that it is correct.

c. Do you agree with student 3? Explain your reasoning. If student 3 is incorrect, modify the statement so that it is correct.

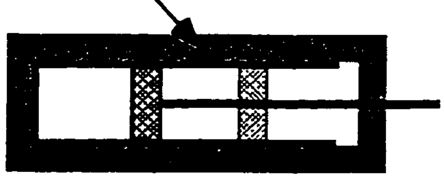
APPENDIX J DETAILED SUMMARY OF STUDENT RESPONSES TO PROBLEMS REQUIRING APPLICATION OF THE FIRST LAW OF THERMODYNAMICS

In Chapter 8, we described the responses of students to several written problems that require the application of the first law of thermodynamics to ideal gas processes. In the chapter, we showed the overall results for several populations. In this appendix, we describe some of the problems that are not shown in the chapter and show the details of student responses of students in different sections of the same course.

1. STUDENT RESPONSES TO THE WRITTEN BICYCLE PUMP PROBLEM

The two written versions of the Bicycle Pump problem are shown in Figure J-1. We have posed this problem in several different courses. At the introductory college level, the problem has been given to students in Physics 115, the second quarter of the first-year algebra-based introductory physics course at the University of Washington, Physics 262, the calculus-based introductory physics course at the University of Maryland, and Physics 224, the second-year thermal physics course at the University of Washington. We have also posed this problem in a small section of Physics 328, the third-year course in thermal and statistical physics at the University of Washington.

A cylindrical pump contains one mole of an ideal gas. The piston fits tightly so that no gas escapes, but friction is negligible between the piston and the cylinder walls.



a. Version 1	b. Version 2
The pump is thermally isolated from its surroundings. The piston is quickly pressed inward as shown.	The piston is quickly pressed inward as shown. A sensor in the pump records an increase in temperature.
What will happen to the temperature of the gas? Explain your reasoning.	***How can you account for the increase in internal energy of the gas? Explain your reasoning.***

Figure J-1: The two versions of the written Bicycle Pump problem.
a. Version 1 states that the pump is insulated and asks students to predict what will happen to the temperature.
b. Version 2 states that the temperature increases and asks students to account for the change in internal energy.

The responses given by students in Physics 115 are shown in Table J-1. As these students had completed version 2 of the problem, only the reasoning given by students to account for the increase in internal energy is described in the table. Only a few of the students gave reasoning based on work. Approximately half of the students argued based on the ideal gas law. About 10% of the students used incorrect microscopic arguments, typically based on incorrect ideas about the collisions of ideal gas particles.

Table J-1: Student responses to Version 2 of the Bicycle Pump problem in a section of Physics 115. In version 2 of the problem, the problem text states that the temperature of the gas increases, so only student reasoning is shown.

	Physics 115 Washington Sp '98 ($N = 100$)
Instruction on first law	None
Reasoning for increase in internal energy	
reasoning with work (correct)	10% (9)
reasoning based on interparticle collisions (incorrect)	10% (12)
reasoning with ideal gas law (incorrect)	50% (50)
other incorrect reasoning	20% (22)
can't account for / blank	5% (7)

All percentages are rounded to the nearest 5%.

The responses to Version 1 of the Bicycle Pump problem given by students from several sections of Physics 262 at the University of Maryland are shown in Table J-2. Although nearly two-thirds of these students were able make the correct temperature prediction, almost none used correct reasoning to make this prediction. Most students used incorrect ideal gas arguments. It is important to note that although many students gave the correct answer to this question, the correct answer alone does not mean that the student is thinking correctly. For example, one of these students wrote, "The temperature of the gas will increase." The student went on to say, "the pressure applied on the gas will increase. We know this because the force exerted by the piston is the same but the unit area gets smaller ($P = F / A$). So if the pressure gets larger and the volume and mole of gas remains the same, temperature will increase ($PV = nRT$)." This student has a number of reasoning difficulties, and his correct answer should not be interpreted as reflecting correct understanding.

Table J-2: Student responses to the Bicycle Pump problem in Physics 262 at the University of Maryland. All of these sections completed version 1 of the problem (see Figure J-1).

	Physics 262 Maryland Sp '97 ($N = 66$)	Physics 262 Maryland Au '97 ($N = 83$)	Physics 262 Maryland Sp '97 ($N = 113$)
Instruction on first law:	Standard	Standard	None
T will increase (correct)	**60% (40)**	**55% (45)**	**60% (67)**
work (correct)	~0% (1)	5% (4)	0% (0)
incomplete	0% (0)	<5% (2)	<5% (4)
ideal gas law	50% (34)	30% (24)	30% (34)
collisions	<5% (2)	10% (7)	5% (7)
T will stay the same	**20% (14)**	**30% (24)**	**25% (30)**
T will decrease	**10% (8)**	**10% (9)**	**10% (9)**
Other / Blank	**5% (4)**	**5% (5)**	**5% (6)**

All percentages are rounded to the nearest 5%.

Responses given in the various sections of Physics 224 are shown in Table J-3. In this class, a large majority of students were able to predict that the temperature of the gas would increase. Most of the students taking the test had already completed standard lecture instruction on the first law, but in one small section, students had not yet studied the first law. There is a difference in the fraction of correct answers between those students who had had instruction on the first law and those who had not.

When the reasoning used by students was considered, the number of students giving correct reasoning was quite similar before and after instruction. The major difference in the two groups seems to be that many students after instruction gave the correct answer supported by incomplete reasoning. Typically, these students made microscopic arguments based on the collisions of gas particles with the moving piston; they concluded that the speed of the gas particles would increase as a result of these collisions and thus the temperature would increase. This reasoning is considered incomplete because it does not account for the presence or absence of heat transfer. For example, in an isothermal compression, gas particles also collide with a piston moving inward, yet over time the increase in molecular speed resulting from such collisions is negated by a heat transfer away from the gas. Similarly, in an isobaric compression, such collisions take place, but in fact the temperature of the gas decreases due to heat transfer away from the gas.

In both groups of students, approximately one quarter of the students used energy arguments and work to make their prediction. In both groups, the most common type of incorrect reasoning was ideal gas reasoning; same students using this reasoning concluded that the temperature of the gas would stay the same. However, it should be noted that the group of students taking the test before instruction was extremely small.

Table J-3: Student responses to the Bicycle Pump problem in several sections of Physics 224. All of these sections completed version 1 of the problem (see Figure J-1).

	Physics 224 Washington Au '96 ($N = 34$)	Physics 224 Washington Wi '97 ($N = 26$)	Physics 224 Washington Su '97 ($N = 11$)
Instruction on first law:	Standard	Standard	None
T will increase (correct)	**90% (30)**	**90% (24)**	**65% (7)**
reasoning with work (correct)	30% (11)	15% (4)	25% (3)
incomplete reasoning	25% (9)	25% (7)	0
incorrect reasoning with ideal gas law	15% (5)	40% (11)	20% (2)
incorrect reasoning based on collisions	10% (4)	10% (2)	10% (1)
T will stay the same	**10% (3)**	**10% (2)**	**25% (3)**
T will decrease	**<5% (1)**	**0**	**10% (1)**

All percentages are rounded to the nearest 5%.

Although the Physics 328 class is very small, these students are relatively advanced in their studies, so the results are of interest as a comparison for introductory students. Although all students gave the correct answer, many of these students did not support their answer with the first law, despite their additional experience in physics. Several of the students supported their correct answer with incorrect or incomplete reasoning. It is often assumed that repeated exposures to a topic in the context of more advanced courses will help students to develop understanding of subject matter and a better appreciation of the critical ideas (like conservation of energy). For several of the students in Physics 328, that does not seem to be the case.

Table J-4: Responses to the Bicycle Pump problem in a section of Physics 328 at the University of Washington. These students completed Version 1 of the problem (see Figure J-1).

	Physics 328 Washington Sp '97 ($N = 7$)
Instruction on first law:	Standard
T will increase (correct)	100% (7)
work (correct)	1
incomplete	3
incorrect ideal gas law	2
incorrect collisions	1
Incorrect T prediction	none

All percentages are rounded to the nearest 5%.

2. STUDENT RESPONSES TO THE ADIABATIC COMPRESSION PROBLEM

The Adiabatic Compression problem (shown in Figure J-2) involves the same physical principles as the Bicycle Pump problem, but the context is slightly different. We were interested whether students would recognize that work is done in a case in which there is no 'active' agent pressing downward on the piston. Therefore the compression in this problem is accomplished by adding masses to a piston that seals an insulated cylinder. Students are asked to predict whether the pressure, volume, and temperature of the gas will increase, decrease, or remain the same.

An ideal gas is contained in a cylinder with a tightly-fitting piston. Several small masses are on the piston.

(Neglect friction between the piston and the cylinder walls.)

The cylinder is placed in an insulating jacket. A large number of masses are added to the piston.

Tell whether the pressure, temperature, and volume of the gas will increase, decrease, or remain the same. Explain.

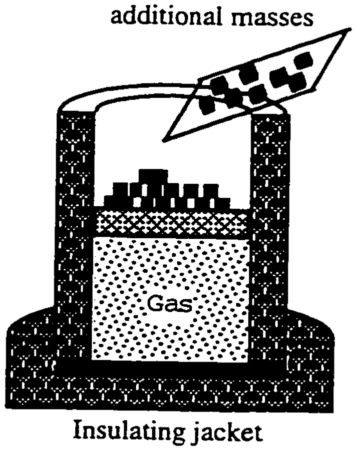


Figure J-2: The Adiabatic Compression problem.

Responses given by students in a section of Physics 115 to the Adiabatic Compression problem are summarized in Table J-5. Very few students mentioned work or the first law of thermodynamics in their reasoning. Unlike for the Bicycle Pump problems, most of the students who failed to use work on the Adiabatic Compression problem did not correctly predict that the temperature would increase.

Table J-5: Student responses to the Adiabatic Compression problem. The second column shows only those *N'* students who correctly answered the pressure and volume questions.

	Algebra-based Washington Wi '97	
	All students (*N* = 179)	Students predicting *P* increase, *V* decrease (*N'* = 103)
Instruction on first law:	Standard	Standard
T will increase (correct)	**10%** (21)	**15%** (18)
reasoning with work (correct)	~0% (2)	~0% (2)
incomplete reasoning	~0% (1)	~0% (1)
incorrect reasoning	10% (18)	15% (15)
Incorrect T predictions	**80%** (146)	**80%** (84)
Other / Blank	**10%** (14)	**<5%** (3)

Percentages rounded to the nearest 5%.

APPENDIX K STUDENT RESPONSES TO PROBLEMS SUGGESTING DIFFICULTIES IN DETERMINING THE WORK DONE IN GAS PROCESSES

In Chapter 8, we briefly described the responses of students to several written problems that require the determination of the sign or absolute value of the work done in an ideal gas process. In the chapter, we showed the overall results for several populations. In this appendix, we describe some of the problems that are not shown in the chapter and show the details of student responses in different sections and on different problems.

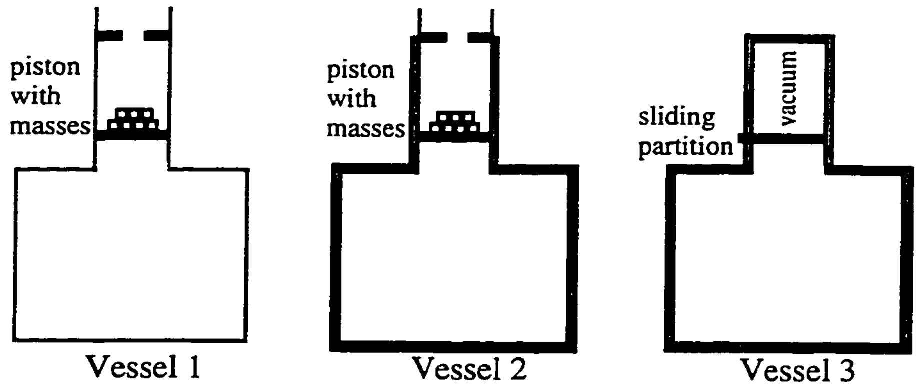
1. STUDENT DIFFICULTIES IN DETERMINING THE SIGN OF WORK

In addition to the Bicycle Pump interviews described in Chapter 8, we have also asked a series of written problems in which students are asked to state whether the work done on a sample of gas in a process is positive, negative, or zero. A summary of student responses to these problems is shown below. Nonetheless, some students' written responses included statements consistent with the difficulties we found in the interviews. A summary of these responses and examples are shown below.

A. Summary of student responses to written problems

We have posed a variety of problems in which students are asked to determine the sign of work done in a process. Some of these problems gave experimental descriptions of processes, like the problem shown in Figure K-1. Other showed curves representing processes on *PV* diagrams (see Figure K-2).

Three vessels each contain one mole of the same ideal gas at room temperature. Vessel 1 is in thermal contact with the surroundings. Vessel 2 and vessel 3 are thermally insulated. Vessels 1 and 2 are sealed with a piston on which there are several masses and vessel 3 is sealed with a partition that can slide to the left.



Each system is changed as follows: Some of the masses on the pistons sealing vessel 1 and 2 are slowly removed. The partition in vessel 3 slides to the left, allowing the gas to expand. *The final volume of the gas is the same in each case.*

A. ***Tell whether the following quantities are positive, negative, or zero. Explain your reasoning in each case.***

- ***the work done on the gas in vessel 1***
- ***the work done on the gas in vessel 2***
- ***the work done on the gas in vessel 3***

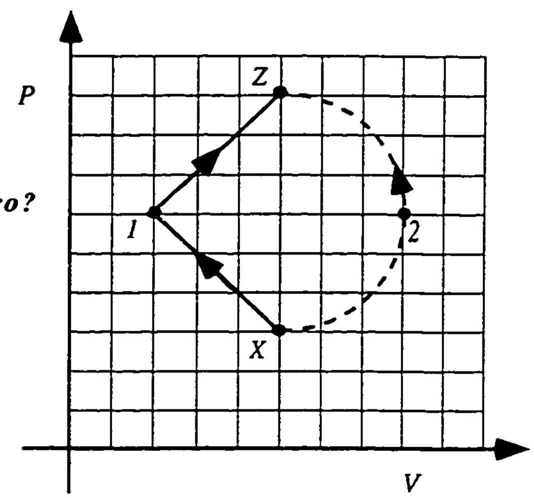
B. ***Sketch the graphs corresponding to the processes in vessel 1 and vessel 2 on a PV diagram.***

C. ***Rank the work done on the gas in the three processes, taking note of the sign (i.e., -8 < +4). Explain how you arrived at your ranking.***

Figure K-1: A problem in which students are asked to determine the sign of the work done in several gas processes.

In most cases, students were quite successful in finding the sign of the work done on the gas in various processes. However, many students gave incomplete reasoning or reasoning that seemed to be based on memorized rules. Almost no students referred to the mechanical definition of work in terms of a force and displacement. Rather, many students referred to the pressure and volume of the gas, either in the form of an integral, in the form $W = P\,\Delta V$, or in simply referring to the change in volume. In some cases, students used mnemonics that seemed to be based on cyclic processes: "$W > 0$ because it is running counterclockwise." On the problems in which standard processes were used, many students answered by writing down formulas or statements that looked like they were copied from the sheets of notes that students brought to the examination, including, "$Q = W = 0$ in free expansion," and, "isothermal: $\Delta E = 0$, $W = -Q$." For this reason, in some of the problems we specified non-standard processes like the one shown in Figure K-2. On these problems, some students still referred to standard process, often giving incorrect responses suggesting rote use of formulas (see Part 3.B of this appendix for examples).

A student performs an experiment with an ideal gas that is confined to a cylinder with a piston. The *PV* diagram below shows the values of pressure and volume of the gas throughout two different experiments, both starting at point *X* and ending at point *Z*.



Is the work done on the gas during the process X→1→Z positive, negative, or zero? Explain.

Is the absolute value of the work done during process X→2→Z greater than, less than, or equal to that during process X→1→Z from part A? Explain your reasoning.

Figure K-2: A problem specifying non-standard processes in which students are asked to determine the sign of the work and compare the absolute values of the work done in two processes.

Responses to problems requiring a determination of the sign of the work done on the gas in a process seemed to vary somewhat in the various sections of Physics 224, but with the exception of the multistep process shown in Figure K-2 the problems were answered successfully by 65% to 90% of students. Students in Physics 115 performed at a somewhat lower level. These results are summarized in Table K-1. Problems in which students were asked to consider multistep processes like the one shown in Figure K-2 proved to be quite difficult. In this process, students who applied the relationship $W = -P\,\Delta V$ indiscriminately failed to answer correctly, often concluding that the work done in the process is zero because the initial and final volumes are the same. In a multiple-choice version of this problem, students in Physics 113, the second-year introductory course in thermal physics at the University of Illinois, performed at a level similar to that of the students in Physics 224. (See Section 2 of this appendix for additional difficulties elicited by this problem.)

Table K-1: Summary of student responses to problems in which they were asked to determine the sign of the work done on a gas in a process. All results shown are after standard instruction.

Process specified by:	Phys 115 Washington	Physics 224 Washington	Physics 113 Illinois
Name (*e.g.*, adiabatic compression)		85% (30 of 36) Au '95	
Description (*e.g.*, cylinder insulated, piston pressed inward)		85%** (36 of 42) Au '97	
PV diagram (including both standard and non-standard processes)	60%* (67 of 111) Au '95	75%† (84 of 111) various	
PV diagram of multistep process with $\Delta V = 0$		45% (18 of 42) Au '98	40%MC (164 of 392) Au '98

All percentages rounded to the nearest 5%. Unless specified otherwise, each problem includes a single task with reasoning considered. MCMultiple-choice problem. Other marks indicate multiple processes: *two **three †various (several combined).

B. Tendency to connect sign of work to a convention

In the interviews, several students had difficulty in determining the sign of the work one the gas due to confusion about sign conventions. Although this particular form of reasoning is relatively rare on written problems, we have seen students give responses of this type. For example, in the Au'95 section of Physics 224 ($N = 36$), students were told that a gas undergoes an isothermal compression and asked to find the sign of the work on the gas in the process. Nearly all students (85% of the class) were able to find the sign of the work in this process correctly. However, the reasoning used by these students was often incomplete. Eight students, or nearly 25% of the class, stated explicitly that the work was positive by convention. One such student wrote, "positive, because the volume of the gas is decreasing. This is simply convention."

C. Tendency to connect sign of work to changes in pressure

Another difficulty that we observed on written responses was the connection of the sign of work with changes in pressure. For example, one student responded that the work done in a process carried out at constant volume was positive because "there is a change in pressure." A second student came to the same conclusion: "work must be done on a gas to keep it at the same volume if the pressure is increasing." Another student evidently thought carefully about her response, writing, "Yes, because $W = P\,\Delta V$, pressure increases thus work ~~increases~~ ~~occurs~~ is done." Although answers of this sort are quite uncommon (given by fewer than 5% of the students), they clearly reflect a failure to connect work in thermal processes to the definition of mechanical work.

2. STUDENT DIFFICULTIES IN DETERMINING THE ABSOLUTE VALUE OF WORK

In addition to the difficulties that we have described above with the sign of work, we found that many students had difficulty in questions in which they were asked to make statements about the absolute value of work. We have posed two types of problems, qualitative problems in which students were asked to rank the absolute values of the work done in processes, and quantitative problems in which students were asked to determine the absolute value of the work done in a process.

A. Responses to problems involving ranking tasks

We have posed a number of qualitative problems in which we have asked students to rank the absolute values of the work done in several processes. For example, in several sections of Physics 224, we asked students to draw *PV* diagrams describing three ideal gas processes and to rank the absolute values of the work done on the gas in the three processes (see Figure K-1 for an example of such a problem). Between 65% and 90% of the students gave rankings that were consistent with their *PV* diagrams. In other problems, we have given students the curves representing two or more processes on a *PV* diagram and asked the students to compare the absolute values of the works. In cases where the processes had the same nonzero ΔV, many students answered correctly, but in cases like that in Figure K-3 where the ΔV was zero in both cases, the problem seemed to be more difficult. Some common reasoning difficulties are described below.

Table K-2: Student responses to a variety of problems asking for a comparison of the absolute values of the work in two or more processes. The superscript indicates the number of processes.

	Process described by:			
	Name of process	Experimental description	PV diagram, same $\Delta V \neq 0$	PV diagram, same $\Delta V = 0$
	Physics 224 Au'94, Wi'95 ($N = 54$)	Physics 224 Au'96 ($N = 42$)	Physics 224 Sp'97 ($N = 17$)	Physics 224 Au'97 ($N = 42$)
Work ranking consistent with PV diagram	80% (44)^3	65% (27)^3	80% (14)^2	55% (23)^2

All percentages rounded to the nearest 5%.

Failure to recognize path-dependence of work

We have posed several problems in which students are asked to compare the absolute values of the work done in two processes with the same endpoints. For example, students are asked to state whether the work done in process $X \rightarrow 2 \rightarrow Z$ in Figure K-2 is greater than, less than, or equal to the work done in process $X \rightarrow 1 \rightarrow Z$. In such problems in Physics 224, we have found that a number of students argue that the work done in these processes is equal. In the problem shown, more than 25% of the students in a section of Physics 224 ($N = 42$) stated that the absolute value of the work done in process $X \rightarrow 1 \rightarrow Z$ is equal to the absolute value of the work done in process $X \rightarrow 1 \rightarrow Z$. Several of these students stated explicitly that the work done did not depend on the path: "the absolute value of the work done on the gas during process $X \rightarrow 2 \rightarrow Z$ is equal to the work done on the gas during process $X \rightarrow 1 \rightarrow Z$ because the work is independent of path taken." In a multiple-choice version of this problem given in Physics 113 at the University of Illinois, 45% of the students (179 out of 392) chose the answer that the absolute values are equal.

The problem shown in Figure K-2 is particularly effective at eliciting this particular difficulty, perhaps because the initial and final volumes in the problem are the same in both processes, so that the change in volume is zero for both processes. On this problem, about 10% of the students said that both works are equal to zero. On problems in which two processes have the same nonzero change in volume, students also make this error. For example, in another class we posed the problem shown in Figure K-3. Approximately 20% of the students stated that the work would be the same in the two processes.

A student performs an experiment with an ideal gas that is confined to a cylinder with a piston. The *PV* diagram below shows the values of pressure and volume of the gas in two different experiments, both starting at point *X* and ending at point *Z*.

Is the work done on the gas during the process X→2→Z positive, negative, or zero? Explain.

Is the absolute value of the work done during process X→2→Z greater than, less than, or equal to that during process X→1→Z from part A? Explain your reasoning.

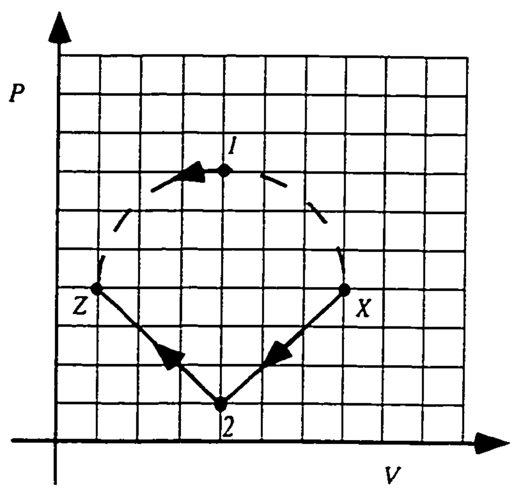


Figure K-3: A problem in which students are asked to determine the sign of work in a non-standard process and compare the absolute values of the work in two processes.

Incorrect belief that work is related to length of curve on PV *diagram*

In a handful of cases, we have seen that some students attempt to relate the work done in a process to the length of the curve on a *PV* diagram rather than the area under the curve.

For example, in one problem in which students were asked to find the net work done on a gas in a cycle, several students found the incorrect sign based on this reasoning. These students correctly recognized that the net work done in the cycle is equal to the sum of the individual works, but incorrectly related these works to the *length* of the curve. One such student wrote, "[the net work is] negative, because [process 1] does greater negative work than [process 2]'s positive work, as seen by the length of the corresponding lines."

B. Responses to quantitative problems

We have examined student responses to problems in which students are asked to determine the value of the work done given a *PV* diagram representing a process or cycle. In the problem shown in Figure K-4, we asked students in Physics 262 at the University of Maryland to determine the absolute value of the work done in the process represented by the diagonal line on the *PV* diagram. The problem shown in Figure K-5 was written by the instructor for a section of Physics 115, and asks students to determine the absolute value of the work done in a cycle similar to those used in refrigerators. The percentage of students answering correctly is shown in Table K-3. As the table shows, very few students were able to do so correctly. A description of some common errors follows.

Table K-3: Student responses to quantitative problems requiring a determination of the value of work done in a gas process. In both cases the problems were posed after standard instruction.

	Physics 115 Washington Au'95 (N = 112)	Physics 262 Maryland Sp'97 (N = 64)
Process	Cycle (See Figure K-5)	Diagonal (See Figure K-4)
Correct value of work	5% (8)	20% (12)

All percentages rounded to the nearest 5%.

Incorrect application of formulas

Many students applied formulas incorrectly. For example, in the diagonal process shown in Figure K-4, approximately 25% of the students in the section of Physics 262 attempted to find the work using the formula for the work done in an isothermal process. On a previous part of the problem, students had been asked to determine the temperatures in the initial and final states of the gas. Half of the students who used the formula for the isothermal process had calculated different temperatures for the initial and final state, suggesting that the students were not performing consistency checks.

A student performs an experiment with an ideal gas that is confined to a cylinder with a piston. A spring that is attached to the piston adjusts the pressure of the gas depending on its volume. The *PV* diagram shows the values of pressure and volume of the gas throughout the experiment.

Is the work done on the gas during the process X→Z positive, negative, or zero? Explain.

Determine the work done by the gas during the process X→Z.

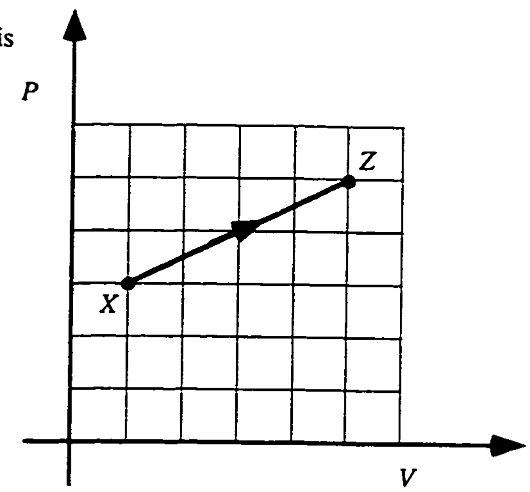


Figure K-4: A problem in which students are asked to determine the work done in a process represented on a *PV* diagram. In some courses students have been asked the qualitative question about the sign of the work; in others, the quantitative question.

Belief that work is zero in cyclic process

A related difficulty pertains to cyclic processes, like those used for heat engines or refrigerators. In a section of Physics 115, 10% of the class answered that the net work done on the gas in the cycle shown in Figure K-5 is zero. These students stated that the net work was zero despite the fact that the question statement was highly suggestive of a nonzero net work, asking "was this work done on or by the system?" when previous parts had included the possibility "no work is done."

We have posed problems that ask students to state whether the net work done on or by a gas in a cyclic process is positive, negative, or zero rather than asking for a determination of the numerical value of the work. In response to these problems, a significant fraction of students have answered that the net work done on the gas is zero, typically arguing that the original and final volume are the same so that the net change in volume is zero. On such problems posed after research-based instruction, approximately half of the students in two different sections of Physics 115 responded that the work done is zero (see Section 9.4 of Chapter 9 for details).

A monatomic, ideal gas goes through a cyclic change as shown in the PV diagram. It is initially at point #1, then is changed to point #2, then to point #3 and finally back to point #1. The path from #2 to #3 is isothermal and there are 1.5 moles of the gas. *For each case, explain your reasoning.*

A. For the part from #1 to #2, tell whether:
- the internal energy increased, decreased, or did not change.
- work was done on the system, by the system, or no work was done.
- heat was transferred to the system, from the system, or no heat was transferred.

B. For the part from #2 to #3, tell whether:
- the internal energy increased, decreased, or did not change.
- work was done on the system, by the system, or no work was done.
- heat was transferred to the system, from the system, or no heat was transferred.

C. Determine the net amount of work done in the cycle. Was this work done on or by the system?

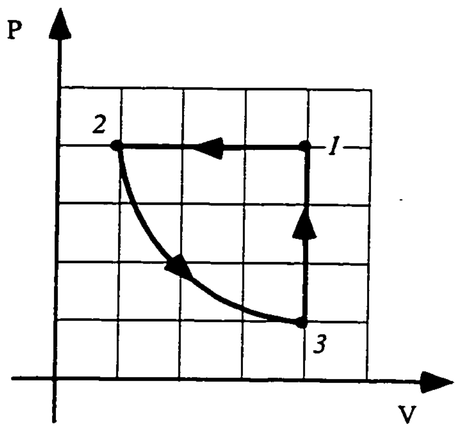


Figure K-5: A problem in which students are asked to determine the work done in a cyclic process.

In some cases, students' responses suggested difficulties with the path-dependence of work similar to those described above. One of these students wrote, "no net work was done because the cycle started and ended at the same place. Since there is no displacement, then no work was done." For other students, the idea that the work done would be zero was sufficiently compelling to override other ideas. One student wrote "W = area under curve" and shaded the correct area corresponding to the net work done by the system, but then seemingly discarded these correct ideas and answered "since there was no change in volume overall there was no work done on the system."

In some cases, students who gave this answer used a potentially correct approach, adding the work done in the processes making up the cycle, but reasoned incorrectly: "Work was positive from $X \rightarrow Y$. Work was zero from $Y \rightarrow Z$. Work was negative from $Z \rightarrow X$. The total work done on the gas was zero because the pos. & neg. work cancel out. The volume decreased & then increased by the same amount." This student has failed to consider that the different pressures in the two processes mean that the absolute value of the work in the process $X \rightarrow Y$ is not equal to that in the process $Z \rightarrow X$, perhaps due to underlying beliefs similar to those described above. We have found in other problems that some students are unable to rank the work done in two processes based on the area under the curves representing these processes on a *PV* diagram. Further difficulties of this type are described below.

Incorrect area calculations

A significant number of students approached the problem correctly, by attempting to determine the correct area from the *PV* diagram, but were unable to do so correctly. On the problem shown in Figure K-5, approximately 20% of the students in the section of Physics 115 used a conceptually sound approach and tried to find the work in the isobaric and isothermal processes and add them algebraically, but were unable to do so correctly, often because they made errors in determining one of the areas geometrically. About 10% of the students in the section of Physics 262 made similar errors on the process in Figure K-4. In addition, about 20% of the students in the section of Physics 115 found the work done in the isobaric part of the cycle alone.

APPENDIX L CURRENT VERSION OF TUTORIAL SEQUENCE: *THE FIRST LAW OF THERMODYNAMICS*

Contents of Appendix L:

Current version of tutorial sequence: *The First Law of Thermodynamics*

- Pretest: Mechanical Work and Bicycle Pump problems
- Tutorial: *The First Law of Thermodynamics*
- Initial Homework: Specific Heat problem
- Current Homework: Work in a Cycle problem

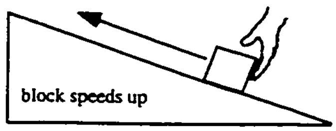
Pretest **Name** ________________

1. In each of the following situations, a block is pushed by a hand on an incline. In all cases, the hand exerts a force parallel to the incline and the initial speed of the block is v_0.

   In each of the parts below, state whether the specified quantities are *positive*, *negative*, or *zero*. Explain.

   A. The block moves up the incline and speeds up.

   - the work done *on* the **block** *by* the **hand**

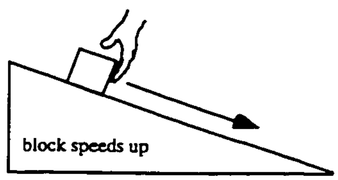


   - the work done *on* the **block** *by* the **Earth**

   - the work done *on* the **hand** *by* the **block** (if there is no such work state so explicitly).

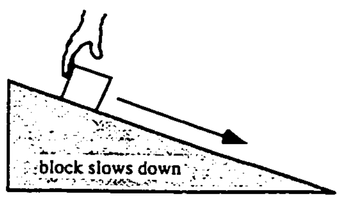
   B. The block moves down the incline and speeds up.

   - the work done *on* the **block** *by* the **hand**

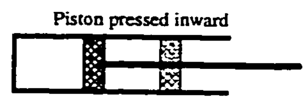


   C. The block moves down the incline and slows down. (Note that the hand is now pushing down the ramp and that the ramp has friction.)

   - the work done *on* the **block** *by* the **hand**



2. A cylindrical pump contains one mole of an ideal gas. The piston is free to move, but no gas can enter or leave the pump. Neglect friction between the piston and the cylinder walls.



The piston is quickly pressed inward as shown in the diagram. A sensor in the pump records an increase in temperature.

A. How can you account for the increase in temperature of the gas? Explain your reasoning.

B. Did the internal energy of the gas *increase, decrease*, or *remain the same* in this process? Explain.

Tutorial

FIRST LAW OF THERMODYNAMICS

Name ______________

I. Work

A. Suppose an object moves while a force is exerted on it. Recall the definition of work:

$$W = \int \vec{F} \cdot d\vec{s}$$

1. Sketch force and displacement vectors to illustrate cases in which the work done by the force is:

positive | negative | zero

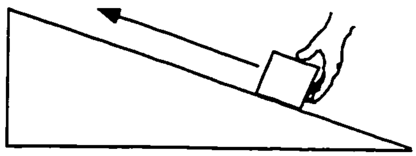
In each case state whether your sketch represents the *only* possible relative directions of the force and displacement vectors.

2. Summarize your responses above by giving a general rule for the sign of the work done by a force acting over a given displacement.

B. A block is pushed by a hand as it moves from the bottom to the top of a frictionless incline. The block is speeding up at a constant rate.

1. In the space below, draw a free-body diagram for the block. Draw an arrow in the smaller box to show the direction of the net force on the block.

net force



frictionless incline

2. Tell whether the following quantities are *positive, negative,* or *zero.* Explain.

   i. the work done on the block by the hand

   ii. the work done on the block by the earth

   iii. the work done on the block by the incline

   iv. the net work done on the block

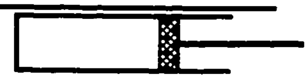
3. Is there work done on the hand by the block in this motion? If so, is this work *positive, negative,* or *zero?* Explain.

4. The *work-energy theorem* states that the change in kinetic energy of an object is equal to the net work done on it. Is your answer to part 2.iv consistent with this theorem and the motion of the block? Explain.

Tutorial

FIRST LAW OF THERMODYNAMICS

C. An ideal gas is contained in a cylindrical pump. The cylinder is closed by a piston as shown at right. There is no friction between the piston and the cylinder walls.



1. Describe the direction of the force that the piston exerts on the gas for the following cases:
   - The piston moves to the left.
   - The piston moves to the right.
   - The piston is stationary.

2. Describe how the piston could move such that it does positive work on the gas.

   Describe how the piston could move such that it does negative work on the gas.

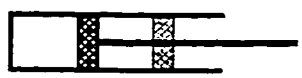
   For each of the cases above, does the gas do work on the piston? If so, how is that work related to the work done by the piston on the gas?

➪ Check your answers with a tutorial instructor.

II. Work and internal energy

A. Imagine that the pump from part I.C is thermally isolated from its surroundings by placing it in an insulating jacket. The piston is pressed inward to the position shown at right. Call this compression *Process 1.*

1. Is the work done on the gas by the piston *positive, negative,* or *zero?*



In thermal physics, we are often interested in the *internal energy (U)* of an object or in changes in the internal energy. For an ideal gas, the internal energy is proportional to the temperature and the number of moles of the gas. The internal energy can be changed in several ways. The case above is one in which the internal energy of a gas changes due to work done on the gas. When a gas is thermally insulated, the change in internal energy of the gas is equal to the work done on it:

$$\Delta U = W_{on\ gas} \qquad \text{(when a gas is thermally insulated)}$$

B. 1. Does the internal energy of the gas in an insulated cylinder *increase, decrease,* or *remain the same* when the piston is pushed inward? Explain.

2. Does the temperature of the gas in an insulated cylinder *increase, decrease,* or *remain the same* when the piston is pushed inward? Explain.

Tutorial 2

First Law of Thermodynamics

C. Two students in an introductory course are discussing *Process 1:*

> Student 1: "The volume of the gas decreases, but the pressure increases. Therefore, by the ideal gas law, the temperature must remain the same."
>
> Student 2: "No, you can't use the ideal gas law to make a prediction about the temperature. But I know the temperature goes up. The volume is less, and, therefore, the particles collide more often with one another."

Neither student is correct. Find the flaws in the reasoning of each student. Explain.

➪ Check your answers with a tutorial instructor.

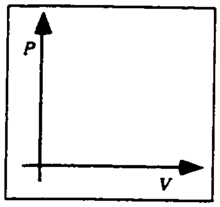
III. Heat

A. Imagine that the pump from part II is no longer thermally insulated, and the piston is locked in place. The gas is initially at room temperature.

The pump is then placed into boiling water and comes into thermal equilibrium with the water *(Process 2).*

1. In *Process 2,* do the following quantities *increase, decrease,* or *remain the same?* Explain.
   - the temperature of the gas
   - the internal energy of the gas

2. Sketch this process on the *PV* diagram at right. (*Hint:* Does the volume of the gas change? Does the pressure change?)



3. Is there work done on the gas in process 2? Explain. State how your answer is consistent with the *PV* diagram.

The energy transfer taking place in process 2 is called *heat transfer.* If the heat transferred to the gas *(Q)* is greater than zero, the internal energy of the gas tends to increase.

B. In *Process 2,* is the heat transfer to the gas *positive, negative, or zero?* Explain.

IV. Heat, Work, and Internal Energy

The *first law of thermodynamics* states that the change in internal energy of a system is equal to the sum of the work done on the system and the heat transferred to the system: $\Delta U = Q + W_{on\ gas}$. (Note that some textbooks express the first law in terms of the work done *by* the system: $\Delta U = Q - W_{by\ gas}$.)

Tutorial

First Law of Thermodynamics

A. In *Process 1* (Section II) you did not need to consider heat transfer. What feature of the experiment prevented heat transfer to the gas?

B. In *Process 2* (Section III) you did not need to consider work. What feature of the experiment prevented work from being done on the gas?

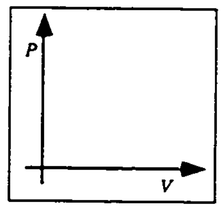
In general, a process may include energy transfer both in the form of work and in the form of heat transfer. In part C we consider such a process.

C. The pump from Sections II and III is now immersed in a mixture of ice and water and allowed to come to thermal equilibrium with the mixture.

The piston is now moved inward very slowly, such that it is always in thermal equilibrium with the ice-water mixture. Call this slow compression of the gas *Process 3.*

1. In *Process 3*, do the following quantities *increase*, *decrease*, or *remain the same?* Explain.

- the volume of the gas
- the temperature of the gas
- the internal energy of the gas



2. Sketch *Process 3* on the *PV* diagram at right.

3. Determine whether the following quantities are *positive*, *negative*, or *zero:*

- the work done on the gas in *Process 3* (Explain your reasoning by referring to a force and a displacement)
- the heat transfer to the gas in *Process 3*

4. Are your answers above consistent with the first law of thermodynamics? Explain.

D. How does the compression in *Process 3* differ from the compression in *Process 1?* Explain.

E. A student in an introductory course is considering *Process 3:*

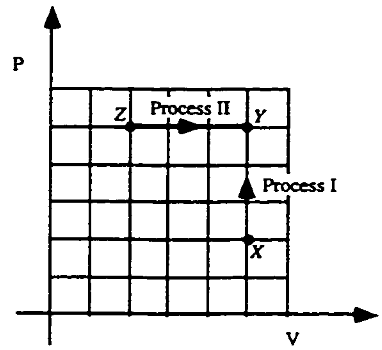
"The temperature doesn't change; it is an isothermal process. Therefore, the heat transfer must be zero."

Do you agree with this student? Explain.

Thermal Physics Homework **DUE FRIDAY, 05/01/98** Name ____
Physics 115

1. One mole of an ideal gas is confined to a container with a movable piston. The questions below refer to the processes shown on the *PV* diagram at right. Process I is a change from state *X* to state *Y* at constant volume. Process II is a change from state *Z* to state *Y* at constant pressure.

A. Rank the temperatures of states *X*, *Y*, and *Z*. If any temperatures are equal, state that explicitly. Explain.



B. Is the work done in the following processes *positive*, *negative*, or *zero?* Explain your reasoning by referring to a force and a displacement.

i. the work done **on** the gas during process I (W_I)

ii. the work done **on** the gas during process II (W_{II})

C. In process I, does the internal energy of the gas *increase*, *decrease*, or *remain the same?*

In process I, is there thermal energy transferred *to the gas*, *from the gas*, or *neither?*

In process II, does the internal energy of the gas *increase*, *decrease*, or *remain the same?*

In process II, is there thermal energy transferred *to the gas*, *from the gas*, or *neither?*

D. i. Is the change in internal energy in process I (ΔU_I) *greater than*, *less than*, or *equal to* the change in internal energy in process II (ΔU_{II})? (*Hint:* Is the absolute value of the temperature change in process I *greater than*, *less than*, or *equal to* that in process II?) Explain.

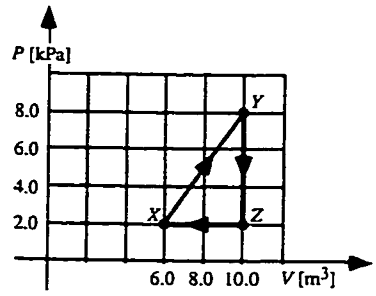
ii. Based on your answers above, is the absolute value of Q_I *greater than*, *less than*, or *equal to* that of Q_{II}? Explain your reasoning.

The *molar specific heat* of a substance is defined as the heat transfer divided by the product of the number of moles of the substance and the temperature change produced by that heat transfer ($C \equiv Q / n\Delta T$).

E. Is the molar specific heat of the gas at constant pressure (C_P) *greater than*, *less than*, or *equal to* the molar specific heat of the gas at constant volume (C_V)? (*Hint:* Use your answer to part D.ii and the definition of the molar specific heat.) Compare your answer with the values given in the table on page 404.

HOMEWORK

2. Consider the cyclical ideal gas process shown at right.



A. Tell whether the work done ***on*** the gas in each process is *positive, negative,* or *zero:*

i. process *XY* (W_{XY})

ii. process *YZ* (W_{YZ})

iii. process *ZX* (W_{ZX})

B. i. Give an interpretation of the statement that there is negative work done **on** the gas.

ii. Give an interpretation of the statement that there is negative work done **by** the gas.

C. Rank the absolute values of the works W_{XY}, W_{YZ}, and W_{ZX} from largest to smallest. If any two are equal, state that explicitly. Explain your reasoning.

D. 1. How is the work done on the gas during the complete cycle related to W_{XY}, W_{YZ}, and W_{ZX}? Write a mathematical expression.

2. Is there a region on the graph whose area is equal to the absolute value of the work done on the gas in the cycle? If so, identify that region.

3. Is the work done on the gas during the complete cycle *positive, negative,* or *zero?* Explain.

E. A student in an introductory course is considering the work done in the cycle:

"The work is given by P ΔV. Since the volume comes back to its initial value, the total work done in the process must be zero."

Do you agree with the student? Explain your reasoning.

F. 1. Calculate the work done on the gas in the cycle.

2. What is the change in internal energy ΔU in the cycle?

3. Use the first law of thermodynamics to find the thermal energy transfer Q to the gas in the cycle. Is this heat transfer *positive, negative,* or *zero?*

Revised tutorial homework for *The First Law of Thermodynamics* - Page 1 Appendix L

Homework Physics

FIRST LAW OF THERMODYNAMICS

Name ______________
Tutorial Section ______

1. For each of the following parts, state whether there exists an ideal gas process that satisfies the conditions given. If so, describe the process and give an example from tutorial if possible. If not, explain why such a process does not exist.

   a. There is heat transfer, but the temperature of the gas does not change ($Q \neq 0$, $\Delta T = 0$).

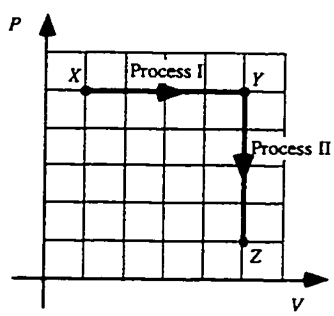
   b. There is no heat transfer, but the temperature of the gas changes ($Q = 0$, $\Delta T \neq 0$).

   c. There is no heat transfer, but work is done on the gas ($Q = 0$, $W \neq 0$).

   d. There is no work done on the gas, but there is heat transfer ($W = 0$, $Q \neq 0$).

2. One mole of an ideal gas is confined to a container with a movable piston. The questions below refer to the processes shown on the *PV* diagram at right. *Process I* is a change from state *X* to state *Y* at constant volume. *Process II* is a change from state *Y* to state *Z* at constant pressure.

   A. Rank the temperatures of states *X*, *Y*, and *Z*. If any temperatures are equal, state that explicitly. Explain.



   B. State whether the following quantities are *positive*, *negative*, or *zero*. Explain your reasoning by referring to a force and a displacement.

      i. the work done on the gas during *Process I* (W_I)

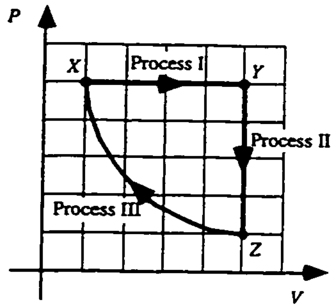
      ii. the work done on the gas during *Process II* (W_{II})

   C. Is the absolute value of the heat transfer in *Process I greater than*, *less than*, or *equal to* the absolute value of the heat transfer in *Process II*? Use the first law of thermodynamics to support your answer.

HOMEWORK

3. The processes I and II shown in part 2 are used in conjuction with an isothermal process (Process III) to form a cyclic process similar to those used in heat engines.

   A. How does the displacement of the piston in Process I compare to the displacement of the piston in Process III? Explain.



   B. Is the *absolute value* of the work done on the gas in Process I *greater than, less than,* or *equal to* the *absolute value* of the work done on the gas in Process III? Explain your reasoning. *(Hint:* How do the forces on the gas by the piston compare?)

   C. 1. How is the work done on the gas during the complete cycle (W_{cycle}) related to W_I, W_{II}, and W_{III}? Write a mathematical expression.

      2. Is there a region on the graph whose area is equal to the absolute value of the work done on the gas in the cycle? If so, identify that region.

      3. Is the work done on the gas during the complete cycle *positive, negative,* or *zero?* Explain.

   D. A student in an introductory course is considering the work done in the cycle:

      "The work is given by PΔV. Since the volume returns to its initial value, the total work done in the process must be zero."

      Do you agree with the student? Explain your reasoning.

VITA

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1999

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