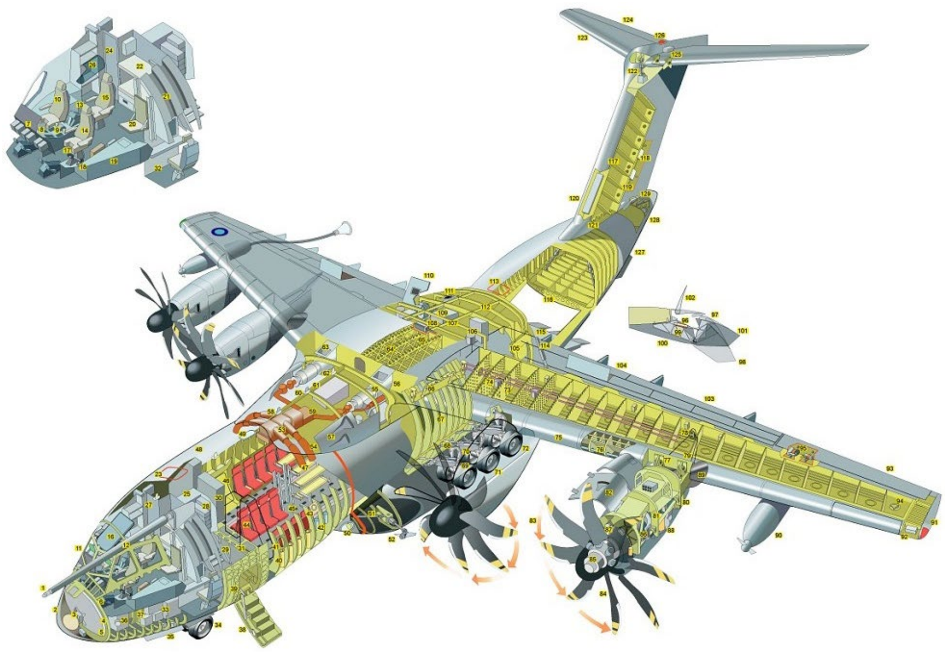
Mechanics of Solids



Muzammil Arshad

WILLIAM E. BOEING
DEPARTMENT OF AERONAUTICS & ASTRONAUTICS
UNIVERSITY of WASHINGTON

About the Author

Muzammil Arshad, William E. Boeing Department of Aeronautics & Astronautics, University of Washington, Seattle WA

Dr. Muzammil Arshad earned his PhD in Mechanical Engineering and Master of Science in Aerospace Engineering from Florida Institute of Technology. Presently he is teaching as Visiting Associate Professor in the William E. Boeing Department of Aeronautics & Astronautics at the University of Washington.

Suggested citation: Arshad, M. (2024) Mechanics of Solids. Available electronically.

Contents

Foreword....4

Statics Review....5

Chapter 1....8

Chapter 2....29

Chapter 3....39

Chapter 4....56

Chapter 5....74

Chapter 6....90

Chapter 7....110

Chapter 8....118

Chapter 9....126

Chapter 10....139

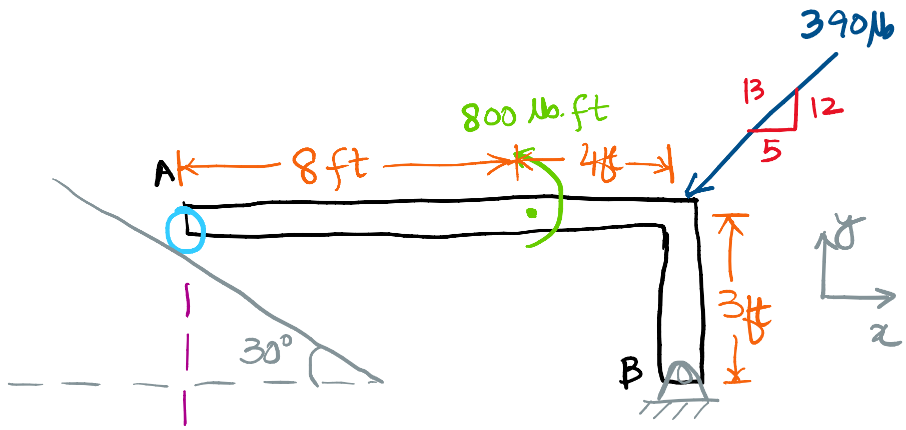
Chapter 11....148

Chapter 12....153

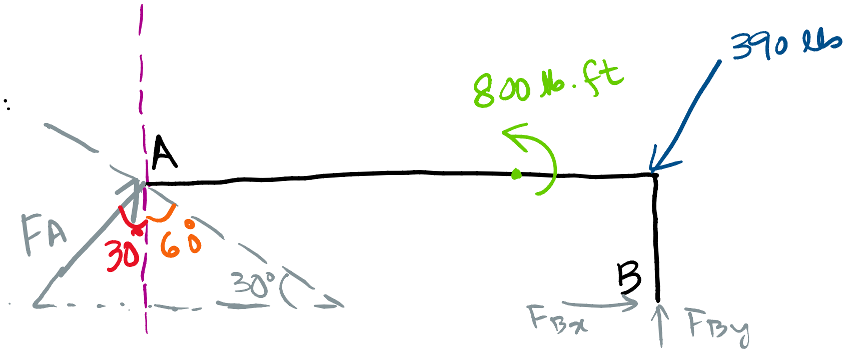
Foreword

The primary purpose of the book is to provide review material for the graduate course of AE 540 Mechanics of Solids taught at William E. Boeing Department of Aeronautics & Astronautics, University of Washington. For the students to achieve a good understanding of the advanced concepts for the graduate course in Mechanics of Solids, the entire book is designed with classroom lecture notes.

The book and lecture notes can also be used by undergraduate students for the courses of Mechanics of Materials and Solid Mechanics.



FBD:



Method:

1. Draw FBD
2. Show all reaction forces
3. Write equilibrium eqns.
4. SOLVE

$\Sigma F_x = 0;$

$$F_A \sin 30^\circ + F_{Bx} - \frac{5}{13}(390) = 0 \quad \text{—— (1)}$$

$\Sigma F_y = 0;$

$$F_A \cos 30^\circ + F_{By} - \frac{12}{13}(390) = 0 \quad \text{—— (2)}$$

$\Sigma M_B = 0;$

$$-F_A \cos 30^\circ (12) - F_A \sin 30^\circ (3) + 800 + \frac{5}{13}(390)(3) = 0 \quad \text{—— (3)}$$

Algebra: 3 unknowns, 3 eqns

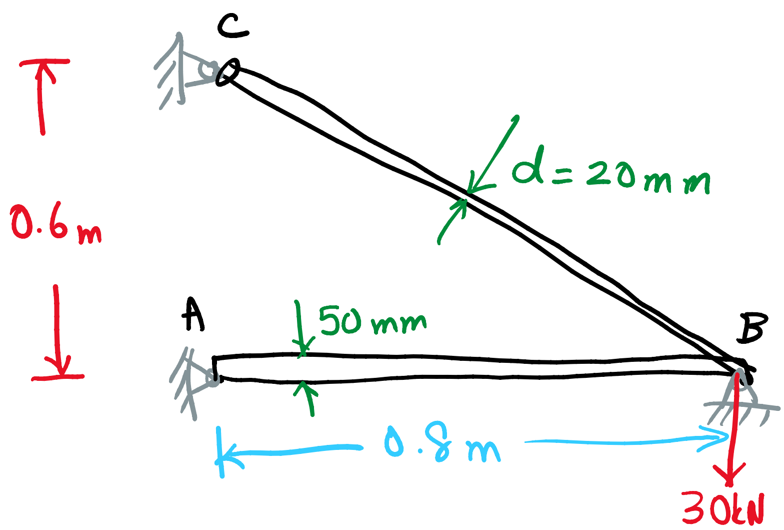
$\Rightarrow$ statically determinate

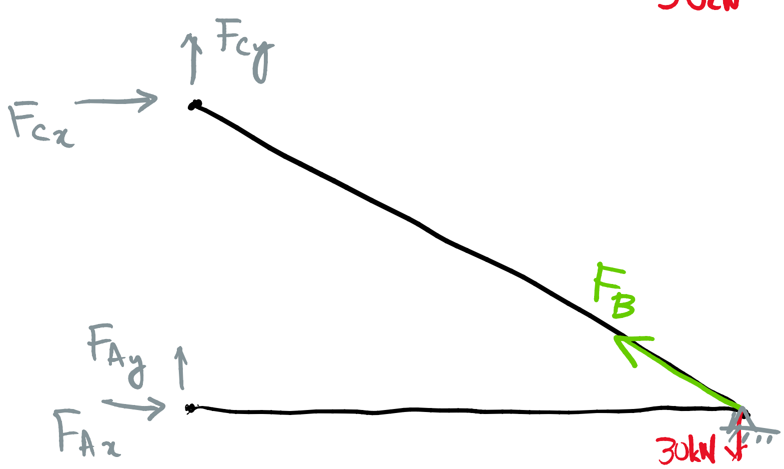
eq (3) $\Rightarrow$ $F_A = 105.11$ lb

eq (1) $\Rightarrow$ $F_{Bx} = 97.44$ lb

eq (2) $\Rightarrow$ $F_{By} = 268.97$ lb

* ——— *





$\Sigma F_x = 0;$

$$F_{Ax} + F_{Cx} = 0 \quad \text{—— (1)}$$

$\Sigma F_y = 0;$

$$F_{Ay} + F_{Cy} - 30 = 0 \quad \text{—— (2)}$$

$\Sigma M_C = 0;$

$$F_{Ax}(0.6) - 30(0.8) = 0 \quad \text{—— (3)}$$

4 unknowns
3 eqns
$\rightarrow$ indet.

$F_{Ax}(0.6) - 30(0.8) = 0$ — ③

Algebra:

eq ③ $\Rightarrow$ $F_{Ax} = 40\,kN$

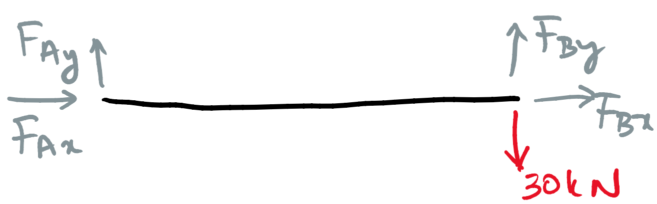
eq ① $\Rightarrow$ $F_{Cx} = -40\,kN$

$F_{Ay} = ?\,, \quad F_{Cy} = ?$

Present FBD can NOT yield any other eqn:

Consider FBD $\rightarrow$ AB

FBD2:



$\Sigma M_B = 0\,;$

$-F_{Ay}(0.8) = 0 \Rightarrow F_{Ay} = 0$

Put in eq ②

$F_{Cy} = 30\,kN$

$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = 40\,kN$

$F_C = \sqrt{F_{Cx}^2 + F_{Cy}^2} = 50\,kN$

Mechanics of Materials:

Branch of mechanics that studies the internal effects of "STRESS" and "STRAIN" in a solid body that is subjected to an external loading.

STRESS: associated with the strength of material

STRAIN: stress would cause deformation

⇒ measure of deformation of body due to loading or stress

§1.2: Equilibrium of a deformable body:

* Surface forces (F_s):
  caused by direct contact
  e.g.: car running on road (force of friction)

* Body forces (F_B):
  caused by w/o direct contact
  e.g.: gravitational force
  electromagnetic force

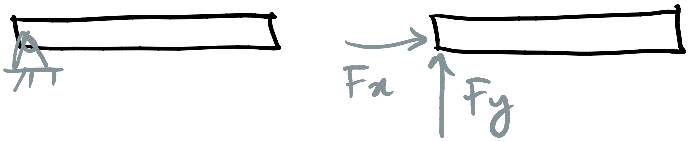
Support Reactions:

(Use Table 1-1)

Cable:

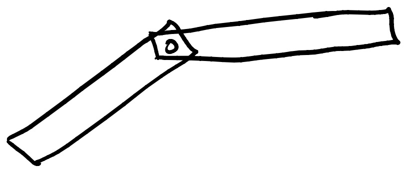
one unknown force

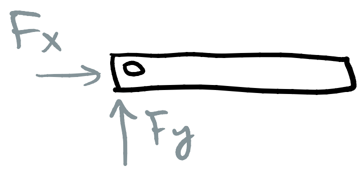
Pin (External):



two unknown forces

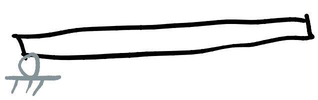
Pin (Internal):

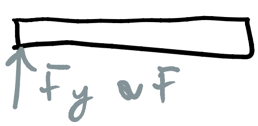




two unknown forces

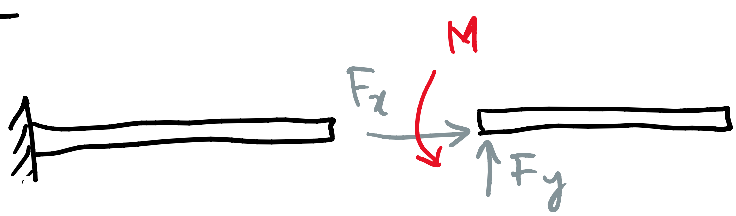
Roller:

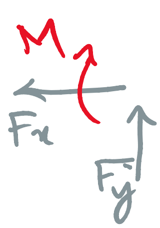




one unknown force

Fixed:



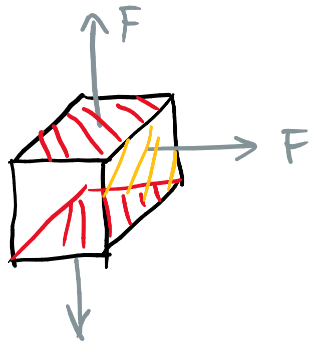


eqns of equilibrium:

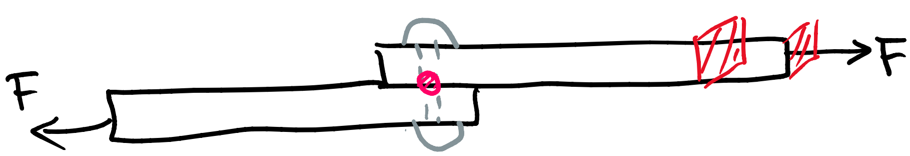
eqns of equilibrium:

$$\Sigma F_x = 0;$$
$$\Sigma F_y = 0;$$
$$\Sigma M = 0;$$

Normal force N: Force $\perp$ x-sectional area A



Shear force V: Force $\parallel$ x-sectional area A

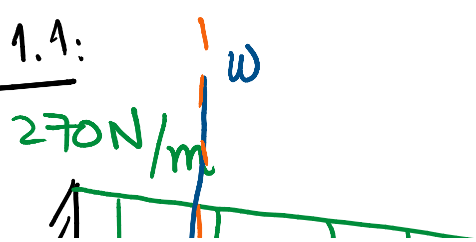


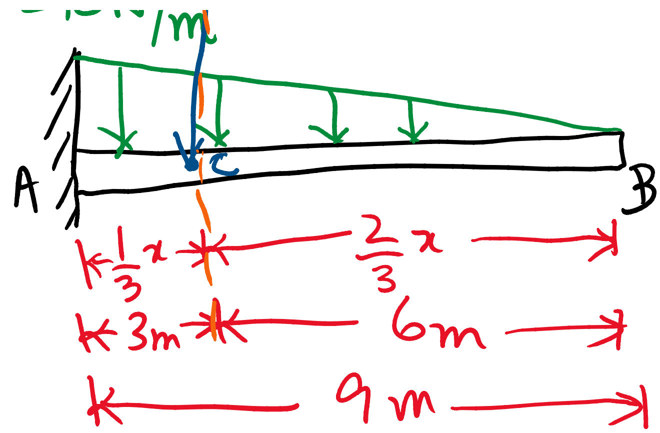
Torsional moment, Torque, T:
external load tends to twist one segment w.r.t. another segment about axis $\perp$ to area

Bending moment, M:

external load tends to bend body about an axis lying within plane of area

Ex # 1.1:





$$w = \frac{2}{3}(270) = 180 \frac{N}{m}$$

OR by similar Δs:

$$\frac{w}{6} = \frac{270}{9}$$

$$w = 180 \frac{N}{m}$$

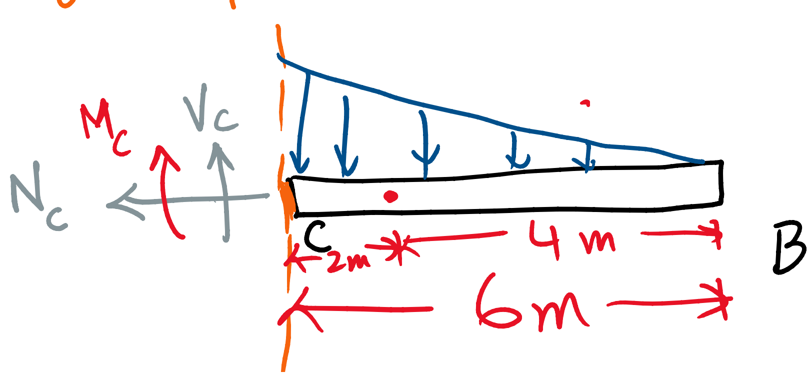
Force $F = \frac{1}{2} b x$

$$= \frac{1}{2}\left(\frac{2}{3} \times 9\right)(180)$$

$$F = 540 N$$

To find forces (N_c, V_c) and moment (M_c) @ pt. c:

<u>Cut @ c</u>



← 6m →

$\Sigma F_x = 0; \quad N_c = 0$

$\Sigma F_y = 0; \quad V_c - 540 = 0$

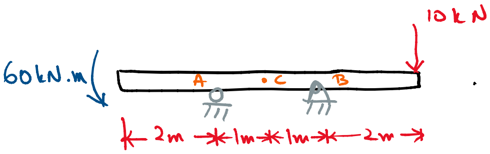
$V_c = 540N$

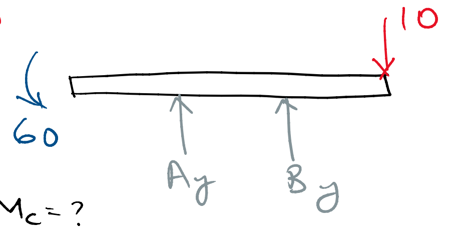
$\Sigma M_c = 0;$

$-M_c - 540(2) = 0$

$M_c = -1080 \, N.m$

Muzammil Arshad, Ph.D. | Visiting Associate Professor
William E. Boeing Department of Aeronautics & Astronautics | University of Washington





find: $N_c = ?$, $V_c = ?$ $M_c = ?$

Solution:

$$\Sigma M_B = 0;$$

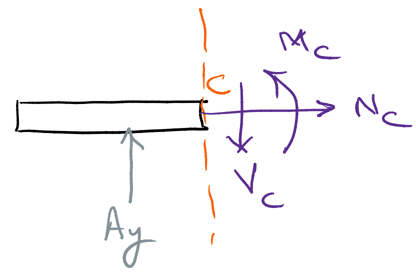
$$60 - 10(2) - A_y(2) = 0$$

$$A_y = 20\,kN$$

$\Sigma F_x = 0;$ $N_c = 0$

$\Sigma F_y = 0;$ $-V_c + 20 = 0$

$$V_c = 20\,kN$$

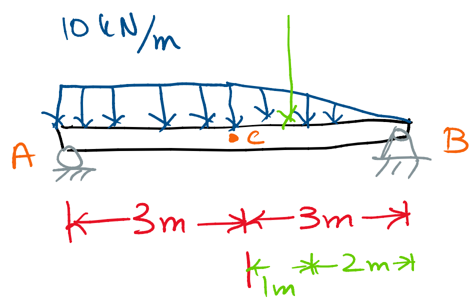


$$\Sigma M_c = 0;$$

$$M_c + 60 - 20(1) = 0$$

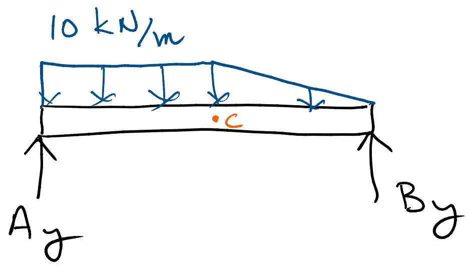
$$M_c = -40\,kN.m$$

F 1-4:



find:
$V_c = ?$ $N_c = ?$
$M_c = ?$

Solution:



$$\Sigma M_B = 0;$$

$$-A_y(6) + \underbrace{10(3)}_{\text{force}}\underbrace{(1.5+3)}_{\text{moment arm}} + \underbrace{\frac{1}{2}(10)(3)}_{\text{force}}\underbrace{(2)}_{\text{moment arm}}$$

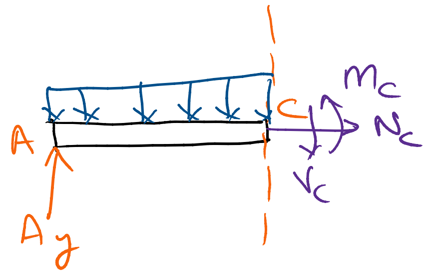
$$A_y = 27.5\,kN$$

$A_y = 27.5\,kN$

$\Sigma F_x = 0;\quad N_c = 0$

$\Sigma F_y = 0;\quad A_y - 10(3) - V_c = 0$

$V_c = -2.5\,kN$

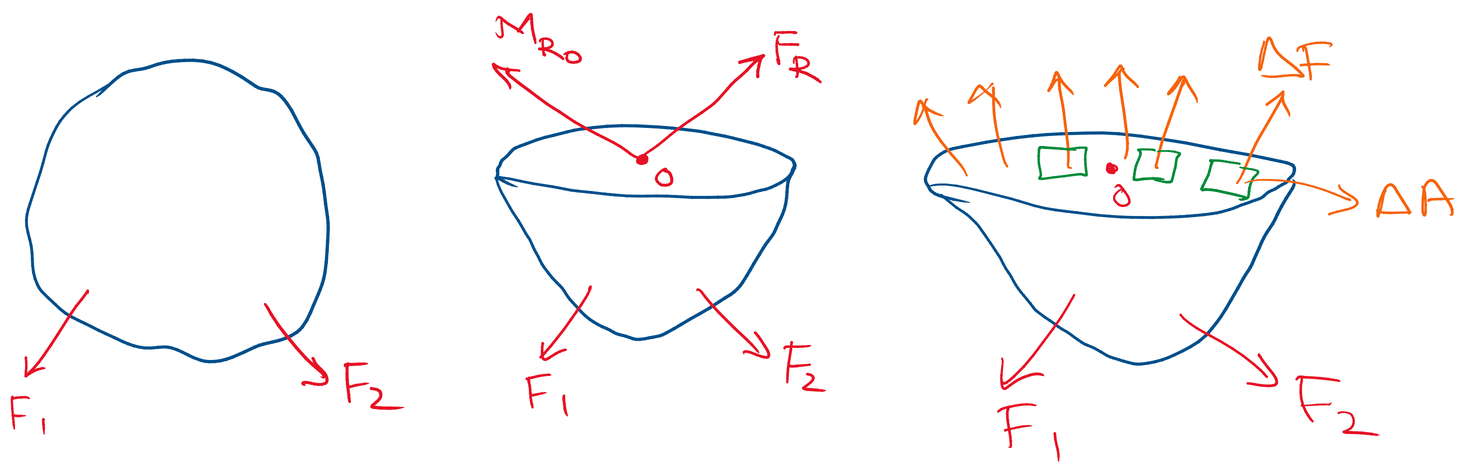


$\Sigma M_c = 0;$

$M_c - A_y(3) + 10(3)\left(\frac{3}{2}\right) = 0$

$M_c = 37.5\,kN.m$

§1.3: STRESS:



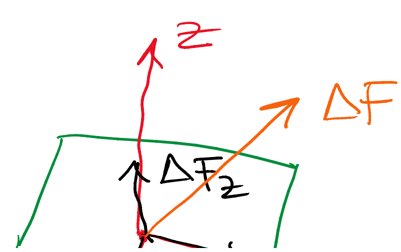
Two assumptions:

1. Mat'l: is CONTINUOUS ⇒ uniform distribution of matter with NO voids.

2. Mat'l: is COHESIVE ⇒ all portions are connected together without any breaks, cracks or separations.

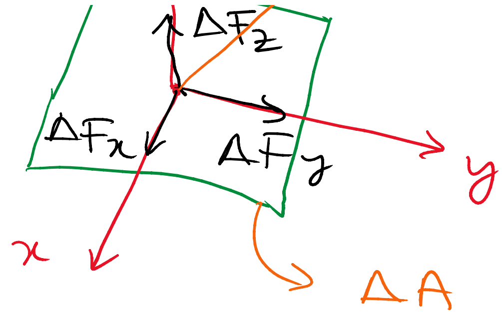
* small ΔF acting on ΔA

* components of ΔF:



* components of ΔF:

ΔF_x , ΔF_y , ΔF_z



As $\Delta A \to 0$, $\Delta F \to 0$

L'hopital's Rule:

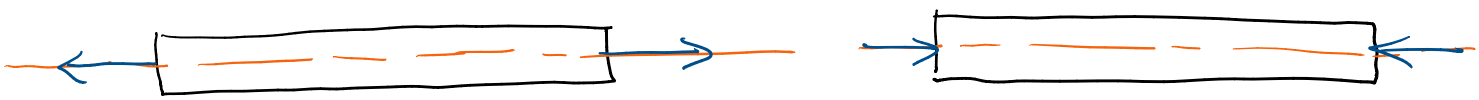
$$\lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \sigma = \delta$$

δ stress

δ displacement

Normal stress:

$$\lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A} = \delta$$



Tensile stress Compressive stress

$\underbrace{\hspace{6em}}_{\text{Axial loads}}$

Recap: $\delta = \dfrac{F}{A}$

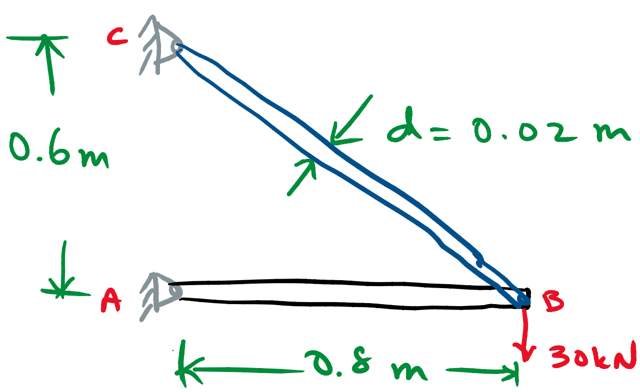
Units: <u>SI</u> $\dfrac{N}{m^2}$ = Pascal = Pa

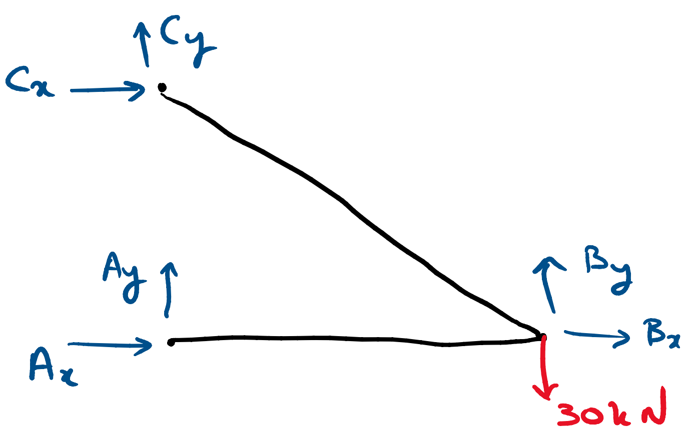
$\times 10^3$ = kPa

<u>English</u>: $\dfrac{lb}{in^2}$ = psi

$\times 10^3$ = ksi

<u>Example</u>:





<u>CASE I</u>: Assume: rod BC; made of steel

$d = 20\,mm$, $\delta_{allow} = 165\,MPa$

$F_{BC} = 50\,kN$

Question: Can rod BC safely support the load?

$$P = F_{BC} = 50\,kN$$

$$A = \frac{\pi}{4} d^2 = \pi r^2$$

$$A = \frac{\pi}{4}(0.02)^2 = 314 \times 10^{-6}\ m^2$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{50 \times 10^3}{314 \times 10^{-6}} = 159 \times 10^6\ Pa$$
$$= 159\ MPa$$

$$\sigma_{BC} < \sigma_{allow} \Rightarrow \text{SAFE}$$

CASE II: Rod BC: made of AL

$$\sigma_{allow} = 100\ MPa$$

Question: What is the safe diameter of rod BC? (Design Problem)

$$\sigma_{allow} = \frac{P}{A}$$

$$100 \times 10^6 = \frac{50 \times 10^3}{\frac{\pi}{4} d^2}$$

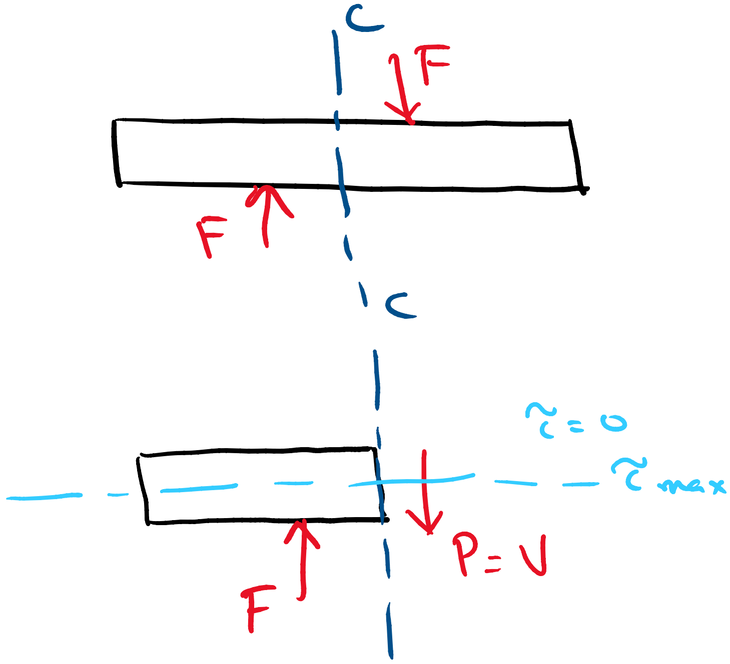
$$\frac{\pi}{4} d^2$$

$d = 25.3 \text{ mm} \approx 26 \text{ mm}$

safe dia of AL

SHEAR STRESS:

$$\tau_{avg} = \frac{P}{A} = \frac{V}{A}$$

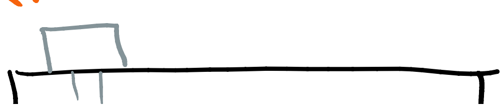


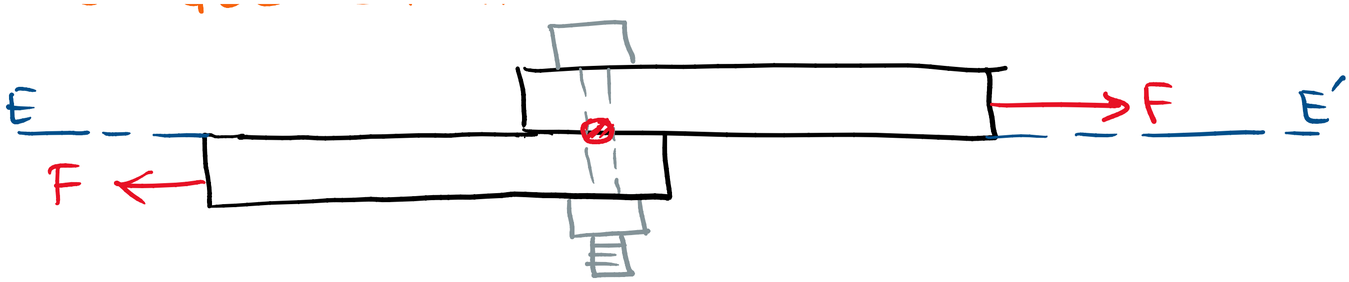
* $\tau = 0$ @ surface
* $\tau = \tau_{max}$ @ somewhere inside
* $\tau_{max} < \tau_{avg}$ (COULD BE)

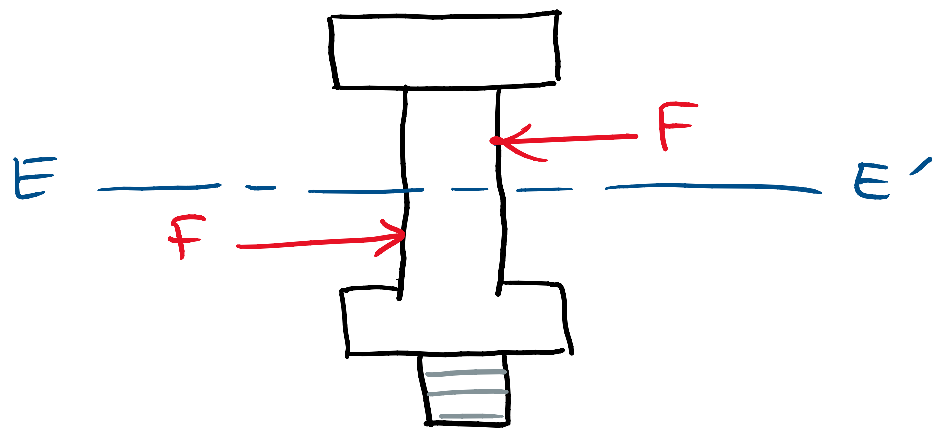
Applications:

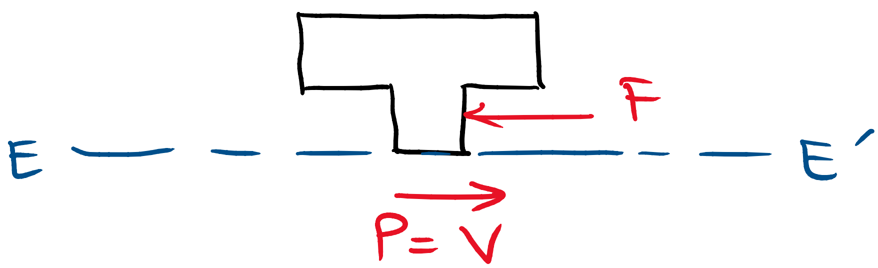
Bolts, Rivets, Pins etc.

SINGLE SHEAR:



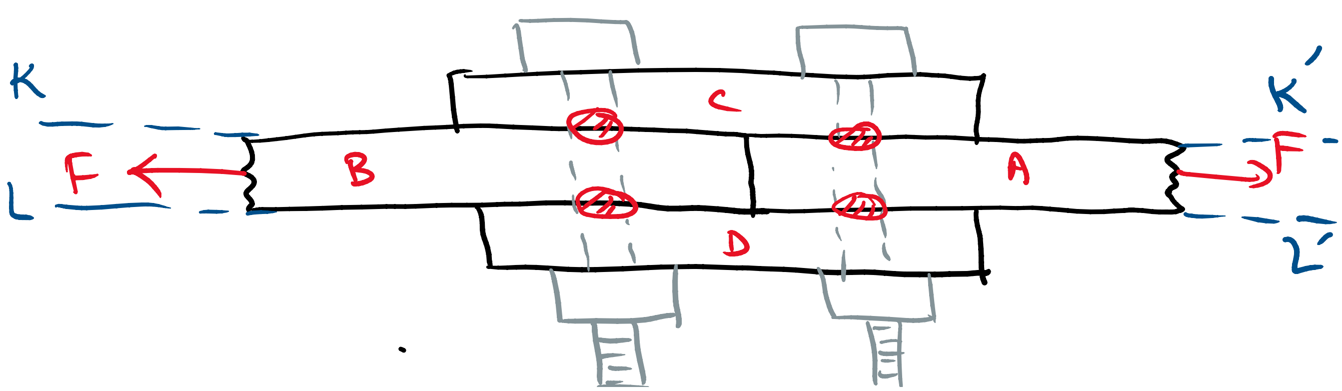


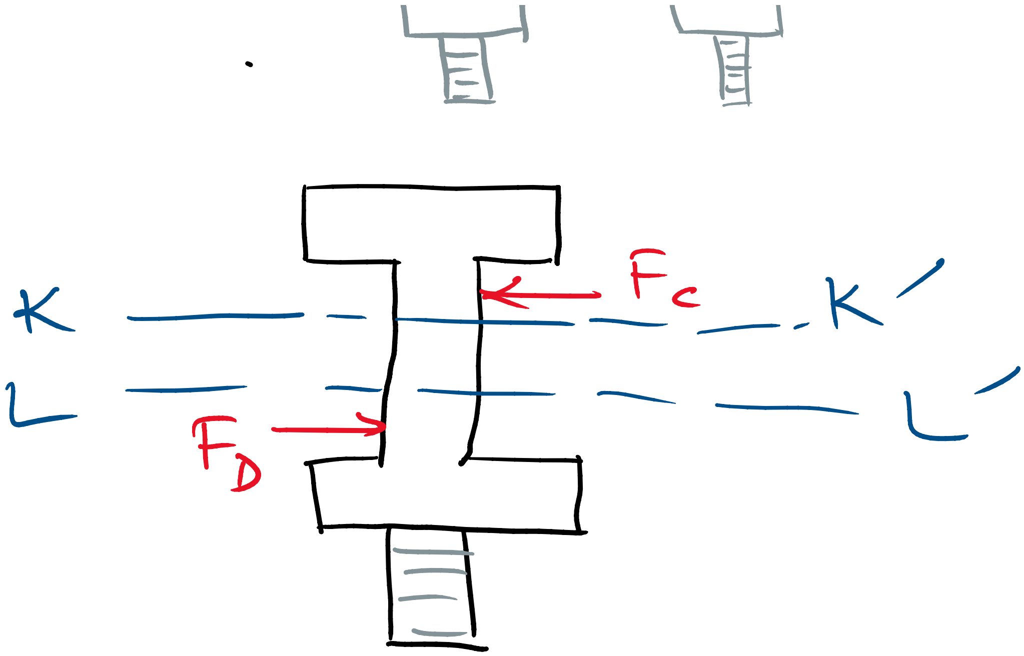


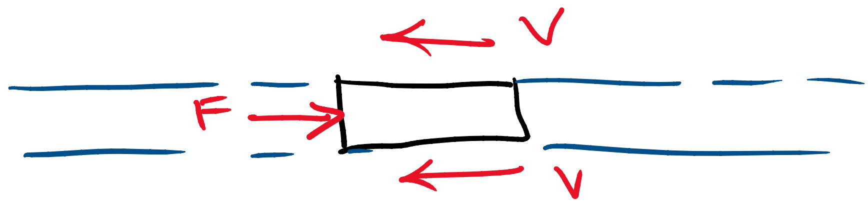


$$\tau_{avg} = \frac{V}{A}$$

DOUBLE SHEAR:

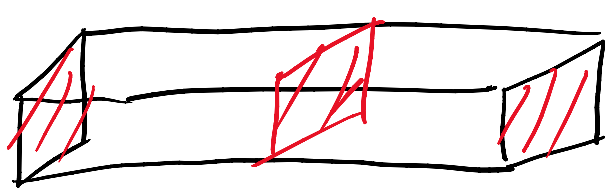


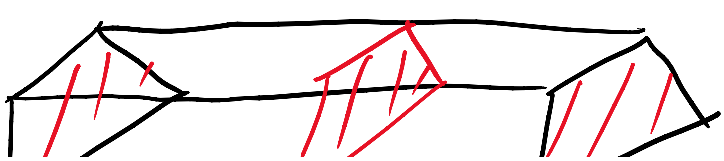




$$\tau_{avg} = \frac{V}{2A} = \frac{V/2}{A}$$

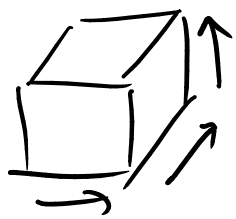
* <u>Prismatic</u> :
All x-sections are same throughout the length





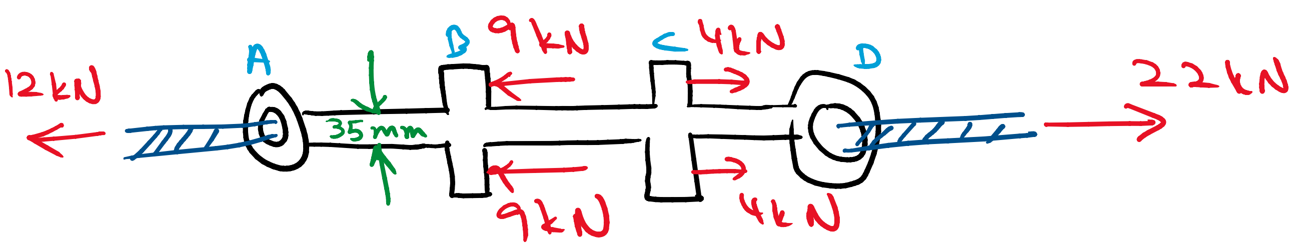
* Homogeneous mat'l : same physical (dimensions) and mechanical (material) properties

* Isotropic mat'l : same properties in all directions.

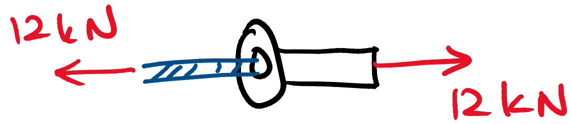


* Anisotropic mat'l : opposite to isotropic mat'l :

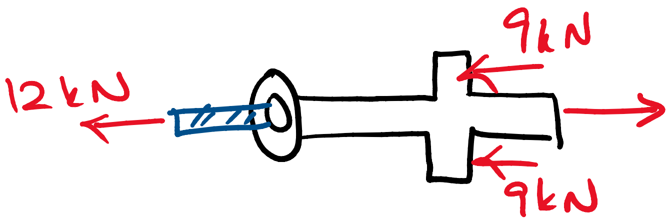
Example :



w = 35mm , t = 10 mm

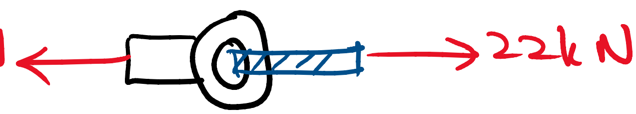


$P_{AB} = 12\text{ kN}$



$P_{BC} = 12 + 9 + 9$
$= 30\text{ kN}$

$P_{CD} = 22\text{kN}$

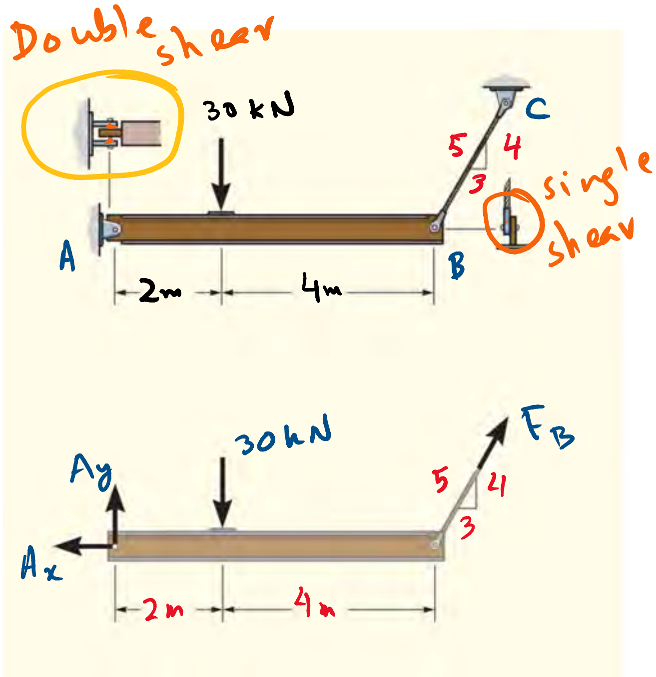


$\delta_{avg,\,max} = ?$

Largest load @ BC :

$$\delta_{BC} = \frac{P_{BC}}{A} = \frac{30 \times 10^3}{(0.035)(0.01)}$$

$$\boxed{\delta_{BC} = 85.7 \text{ MPa}}$$ Answer



Determine the average shear stress in the 20-mm-diameter pin at A and the 30-mm-diameter pin at B that support the beam

Ex # 1.9 :

Given:

$d_A = 20\,mm$

$d_B = 30\,mm$

find:

$\tau_{avg,A} = ?$

$\tau_{avg,B} = ?$

Solution :

$$\Sigma M_A = 0;$$

$$F_B\left(\frac{4}{5}\right)(6) - 30(2) = 0$$

$$\boxed{F_B = 12.5\ kN}$$

$$\Sigma F_x = 0;$$

$$12.5\left(\frac{3}{5}\right) - A_x = 0$$

$$\boxed{A_x = 7.5\ kN}$$

$$\Sigma F_y = 0;$$

$$A_y + 12.5\left(\frac{4}{5}\right) - 30 = 0 \Rightarrow \boxed{A_y = 20\,kN}$$

$$F_A = \sqrt{A_x^2 + A_y^2} = 21.36\,kN$$

At A: Double Shear :

At A: Double Shear :

$$V_A = \frac{F_A}{2} = \frac{21.36}{2} = 10.68\ kN$$

$$\tau_{avg,A} = \frac{V_A}{A_A} = \frac{10.68 \times 10^3}{\frac{\pi}{4} d_A^2}$$

$$\boxed{\tau_{avg,A} = 34\ MPa}$$ Answer

At B: Single shear

$$V_B = F_B = 12.5\ kN$$

$$\tau_{avg,B} = \frac{V_B}{A_B} = \frac{12.5 \times 10^3}{\frac{\pi}{4} d_B^2}$$

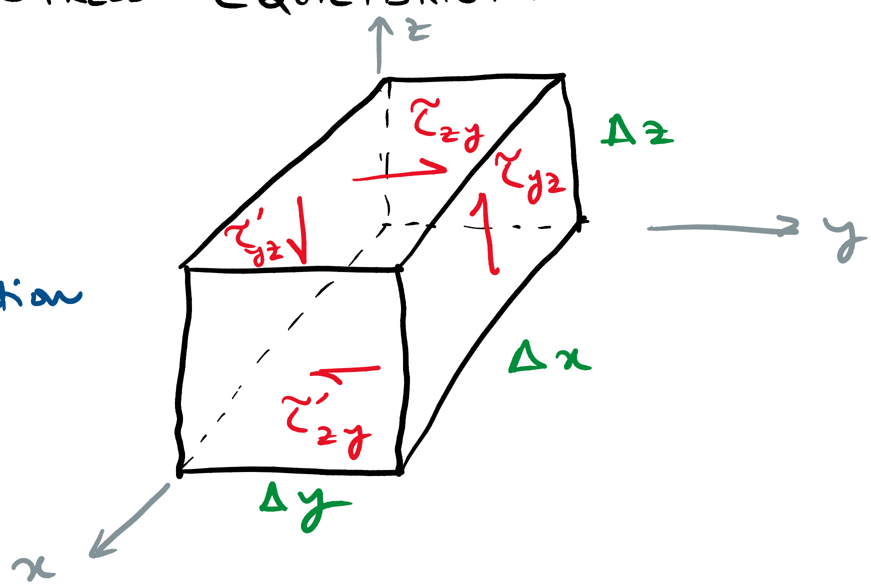
$$\boxed{\tau_{avg,B} = 17.7\ MPa}$$ Answer

* ———————————————————— *

Shear Stress Equilibrium:

$$\tau_{yz}$$

y → face z → direction



$$\Sigma F_y = 0;$$

$$\Sigma F_y = 0;$$

$$\tau_{zy}(\Delta x \Delta y) - \tau'_{zy}(\Delta x \Delta y) = 0$$

$$\boxed{\tau_{zy} = \tau'_{zy}}$$

$$\Sigma F_z = 0;$$

$$\tau_{yz}(\Delta x \Delta z) - \tau'_{yz}(\Delta x \Delta z) = 0$$

$$\boxed{\tau_{yz} = \tau'_{yz}}$$

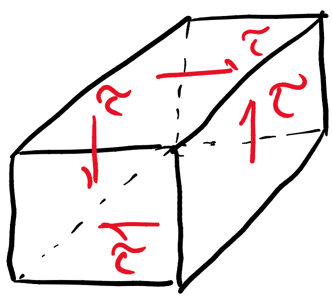
$$\Sigma M_x = 0;$$

$$-\tau_{zy}(\Delta x \Delta y)\overbrace{(\Delta z)}^{\text{moment arm}} + \tau_{yz}(\Delta x \Delta z)\overbrace{(\Delta y)}^{\text{moment arm}} = 0$$

$$\Rightarrow \boxed{\tau_{zy} = \tau_{yz}}$$

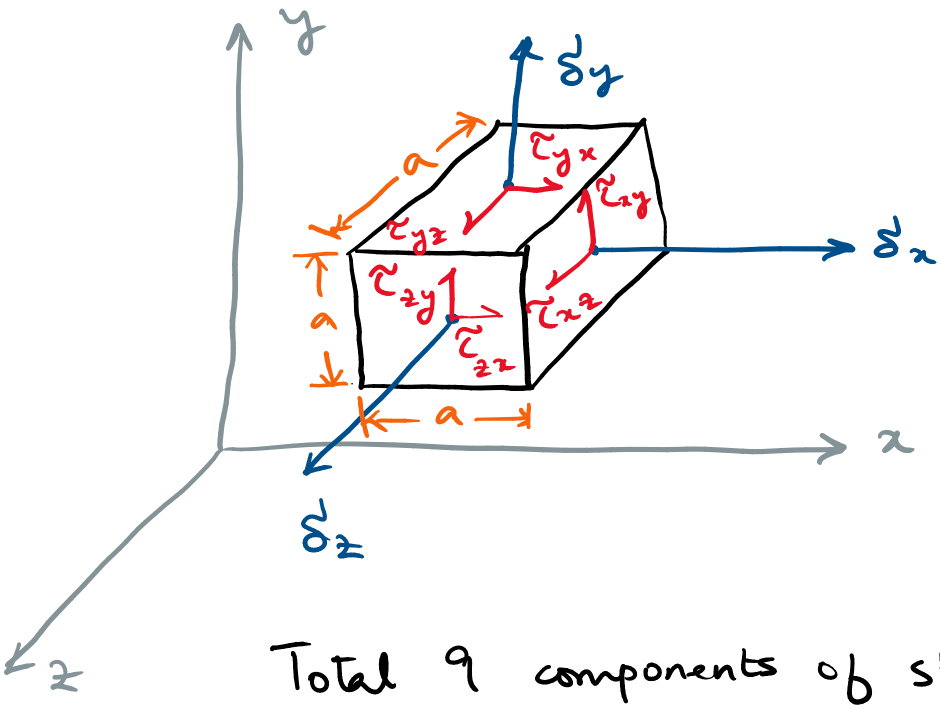
$$\Rightarrow \tau_{zy} = \tau'_{zy} = \tau_{yz} = \tau'_{yz} = \tau$$

=



Conclusion: All stresses have equal magnitude

- Uniaxial stress
- Biaxial stress
- Triaxial stress



Total 9 components of stresses :

$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yx}, \tau_{xz}, \tau_{zx}, \tau_{yz}, \tau_{zy}$

$$\Sigma F_x = 0; \quad \Sigma F_y = 0; \quad \Sigma F_z = 0;$$

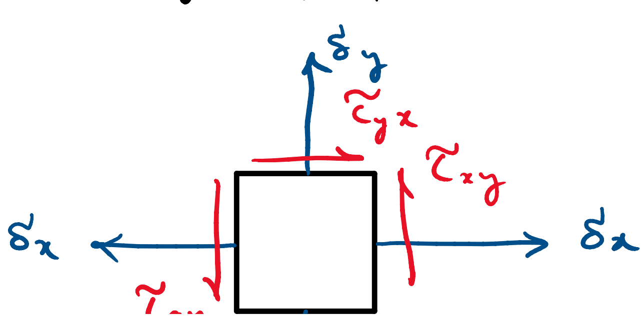
$$\Sigma M_z = 0;$$

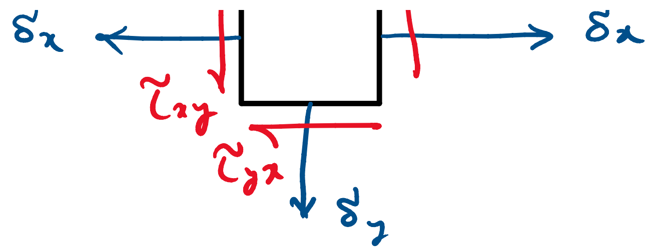
$$(\tau_{xy}\,\Delta A)\,a - (\tau_{yx}\,\Delta A)\,a = 0$$

$$\boxed{\begin{aligned} \tau_{xy} &= \tau_{yx} \\ \tau_{yz} &= \tau_{zy} \\ \tau_{zx} &= \tau_{xz} \end{aligned}}$$

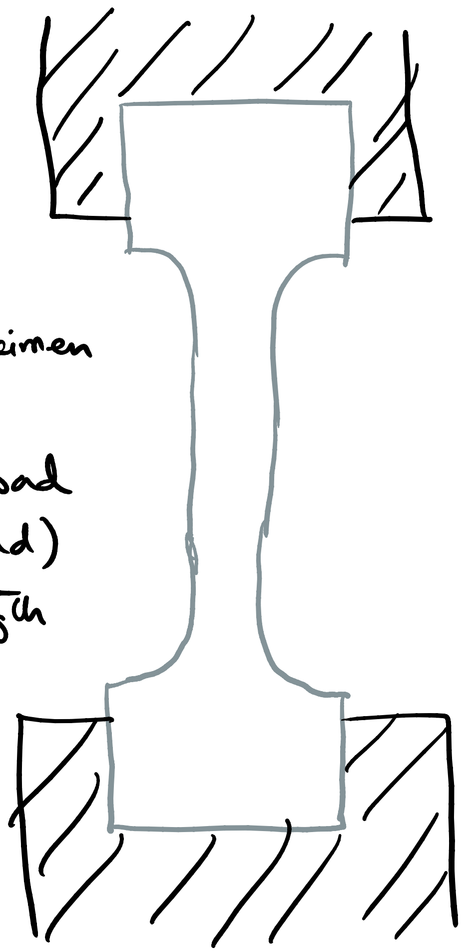
$\Rightarrow$ reduced to 6 components

$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}$





§ 1.6: Allowable Stress Design:



× Tension test:

- Largest force breaks the specimen
- Largest force
  ⇒ ultimate load P_u (axial load)
- Ultimate strength in Tension:

$$\sigma_u = \frac{P_u}{A}$$

- For ultimate strength in shear: twist a circular rod

$$\tau_u = \frac{P_u}{A_s}$$

- single shear, $A_s = A$
- double shear, $A_s = 2A$

Allowable load and Allowable stress:

Factor of Safety (FOS)

* under normal conditions, max. allowable load $< P_u$

$\Rightarrow \quad P_{allow} < P_u$

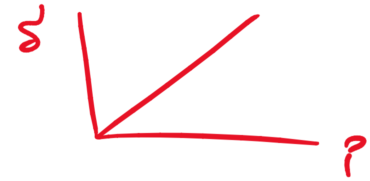
or $\quad P_{working} < P_u$

$$FOS = \frac{P_u}{P_{allow}}$$

or

$$FOS = \frac{\delta_u}{\delta_{allow}}$$

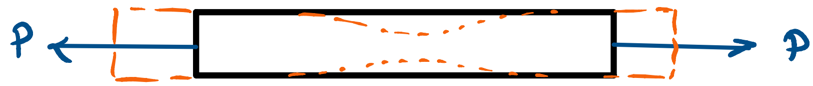
identical IF P and δ has a linear relationship

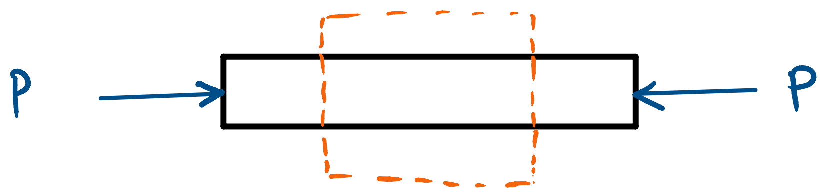


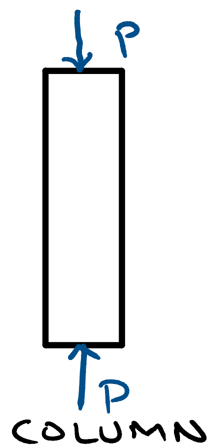
Lecture 6

CHAPTER #2:

§2.1: Deformation:





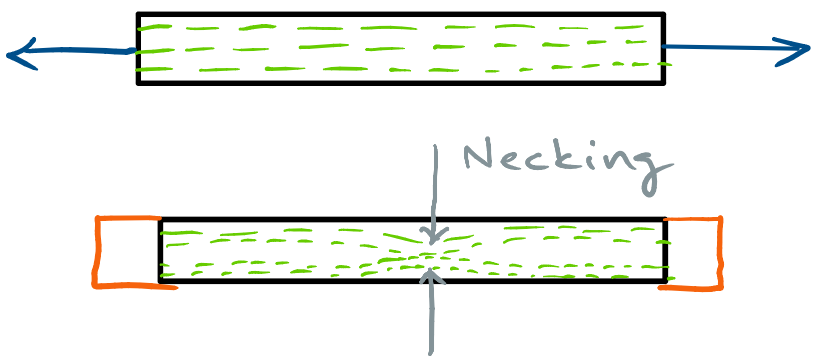


* Deformation
* Crush failure

* When force is applied ⇒ change in body's shape and size ⇒ DEFORMATION

* When temperature of body is changed ⇒ change in shape and size
  ⇒ DEFORMATION.

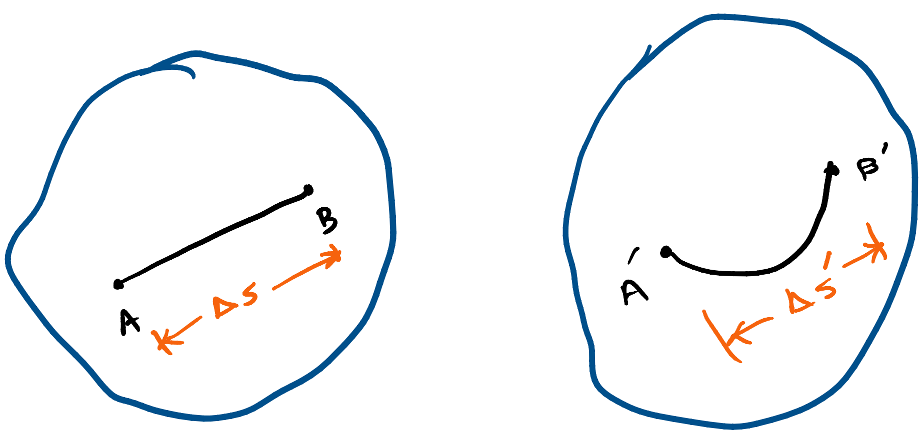
<u>e.g:</u>
* Railroad
* Bridges
* Roads
* Roofs



* change in Length of the line segments
* change in angle b/w the line segments

STRAIN:

* strain is measured by experiments
- Normal strain
- Shear strain



Before Deformation

After Deformation

change in Length $= \Delta s' - \Delta s$

Average normal strain:

$$\epsilon_{avg} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta s' - \Delta s}{\Delta s}$$

If $\Delta s \to 0 \Rightarrow \Delta s' \to 0$

If $\Delta s \to 0 \Rightarrow \Delta s' \to 0$

L'hopital's Rule: $\epsilon = \lim_{\Delta s \to 0} \dfrac{\Delta s' - \Delta s}{\Delta s}$

- ϵ +ve, IF Elongation
- ϵ -ve, IF contraction

Units: Dimensionless

Various ways to write units for strain:

- $\left(\frac{m}{m}\right)$ or $\left(\frac{mm}{mm}\right)$ or $\left(\frac{in}{in}\right)$
- % e.g: $0.001\ \frac{m}{m} = 0.1\%$

Example: $\epsilon = 480 \times 10^{-6}$

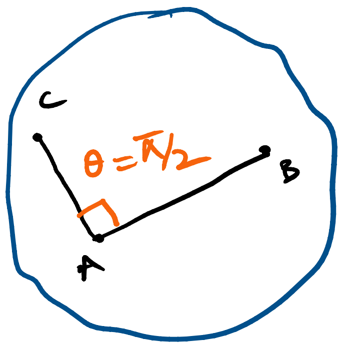
can be written as:

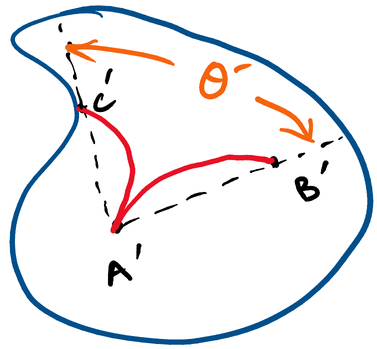
- $\epsilon = 480 \times 10^{-6} \left(\frac{m}{m}\right)$
- $\epsilon = 480 \left(\frac{\mu m}{m}\right)$
- $\epsilon = 0.048\%$
- $\epsilon = 480\mu$ (480 "micros")

SHEAR STRAIN:

- Normal strain caused by "change in length"
- Shear strain caused by "change in direction"

- Shear strain caused by "change in direction"





UNDEFORMED

- original angle b/w two line segments $= \pi/2$

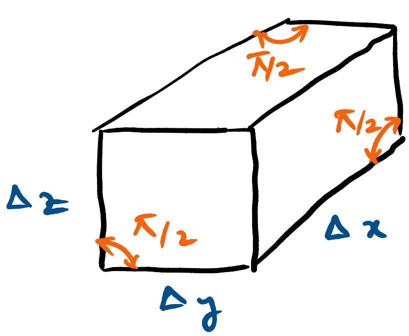
DEFORMED

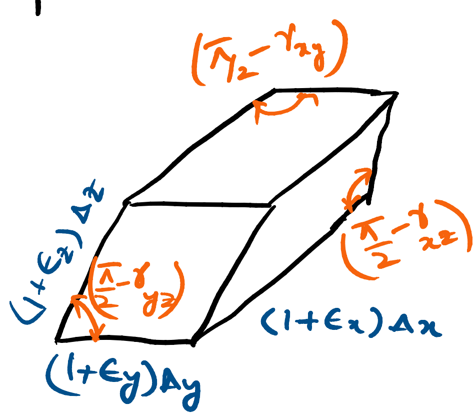
change in angle b/w line segments $= \theta'$

$$\text{Shear strain } \gamma = \frac{\pi}{2} - \lim_{\theta \to 0} \theta'$$

- If $\left(\theta' < \frac{\pi}{2}\right) \Rightarrow \gamma$ is +ve
- If $\left(\theta' > \frac{\pi}{2}\right) \Rightarrow \gamma$ is −ve

Cartesian strain components:





UNDEFORMED ELEMENT

DEFORMED ELEMENT

To find the new lengths due to deformation:

$$\epsilon_x = \frac{\Delta x' - \Delta x}{\Delta x}$$

$$\Delta x' - \Delta x = \epsilon_x \Delta x$$

$$\Delta x' = \Delta x + \epsilon_x \Delta x$$

New length: $\Delta x' = (1 + \epsilon_x) \Delta x$

<u>NOTE:</u> • Normal strain causes change in volume

• Shear strain causes change in shape

• Similar to stress, strain has 9 components that can be reduced to 6 components.

$\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}$

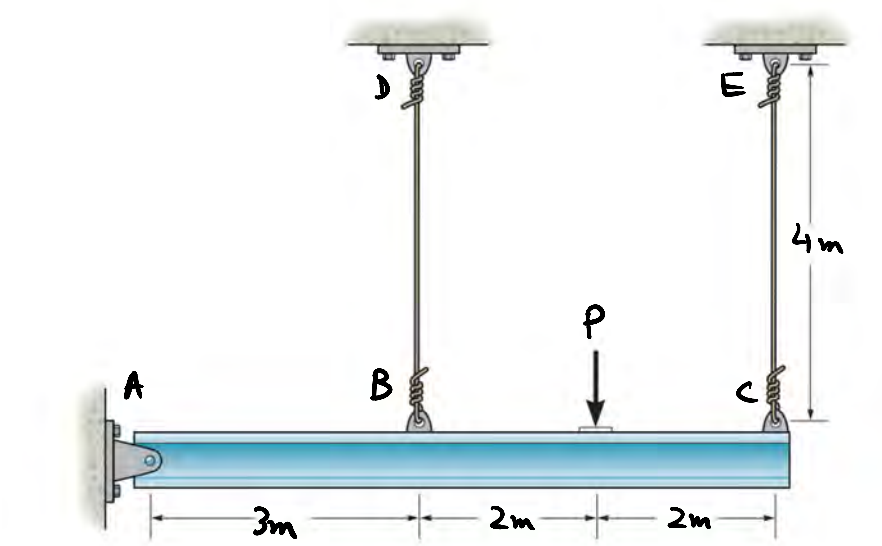
• small strain analysis:

If there is small deformation

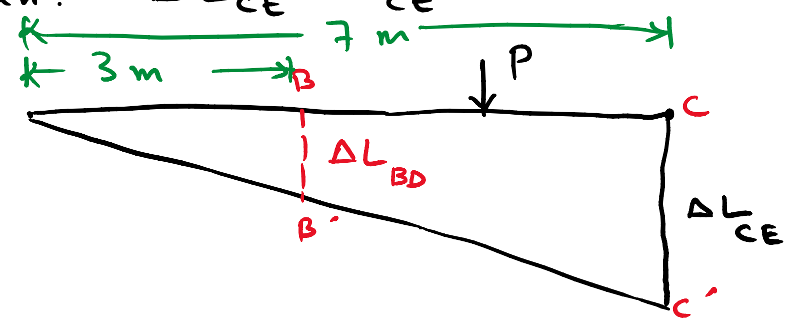
$\Rightarrow$ small strain $\Rightarrow \epsilon \ll 1$

$\Rightarrow \sin\theta = \theta, \quad \cos\theta = 1, \quad \tan\theta = \theta$

[IF θ is very small]



Given: $\Delta L_{CE} = \delta_{CE} = 10\text{ mm}$



Find: $\epsilon_{CE} = ?$, $\epsilon_{BD} = ?$

Governing Eqns:

Recall: $\epsilon_{avg} = \lim_{\Delta s \to 0} \dfrac{\Delta s' - \Delta s}{\Delta s}$

$$\Rightarrow \epsilon_{avg} = \frac{\Delta L}{L} = \frac{\delta}{L}$$

Assumptions: * Homogeneous mat'l :

<u>Solution:</u>

$$\epsilon_{avg} = \frac{\Delta L_{CE}}{L_{CE}} = \frac{\delta_{CE}}{L_{CE}}$$

$$\epsilon_{avg} = \frac{\Delta L_{CE}}{L_{CE}} = \frac{\delta_{CE}}{L_{CE}}$$

$$= \frac{10}{4000}$$

$$\boxed{\epsilon_{CE} = 0.0025 \ \frac{mm}{mm}} \quad \underline{\underline{Ans}}$$

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L_{BD}} = \frac{\delta_{BD}}{L_{BD}}$$

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{4000}$$

By similar Δs :

$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3(10)}{7}$$

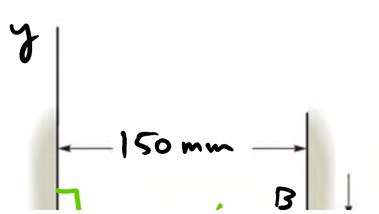
$$\Delta L_{BD} = 4.286 \ mm$$

$$\epsilon_{BD} = \frac{4.286}{4000}$$

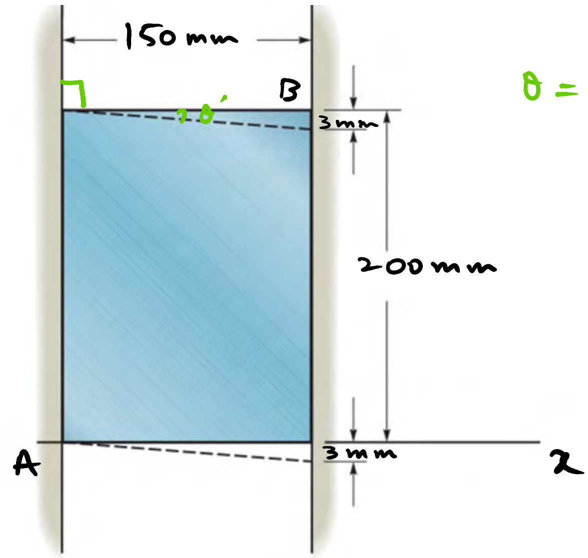
$$\boxed{\epsilon_{BD} = 0.0011 \ \frac{mm}{mm}} \quad Answer$$

<u>P#2.5</u> :

Given : rectangular plate
with given dimensions



$\theta = \frac{\pi}{} + \theta'$



$\theta = \frac{\pi}{2} + \theta'$

Find: $\gamma_{xy} = ?$

Governing Eqn:

$$\gamma = \frac{\pi}{2} - \theta$$

Assumptions:

- Homogeneous mat'l:
- uniform deformation

Solution:

$$\tan\theta' = \frac{P}{B} = \frac{3}{150}$$

$$\theta' = \tan^{-1}\left(\frac{3}{150}\right)$$

$$\theta' = 0.02 \text{ rad}$$

$$\theta = \frac{\pi}{2} + \theta'$$

$$\theta = \left(\frac{\pi}{2} + 0.02\right) \text{ rad}$$

2

$$\theta = \left(\frac{\pi}{2} + 0.02\right) \text{ rad}$$

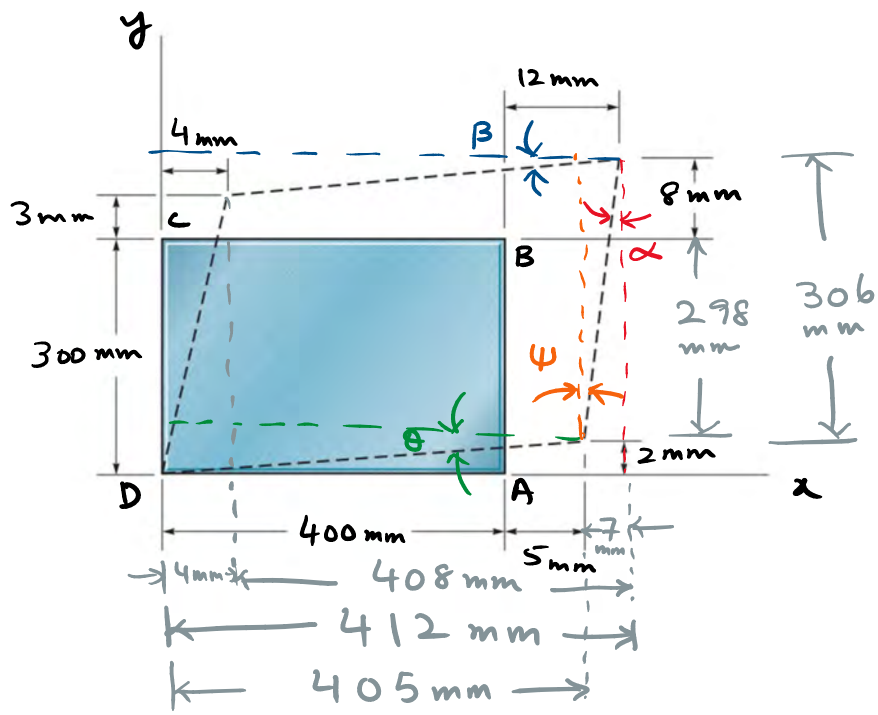
$$\gamma_{xy} = \frac{\pi}{2} - \theta$$

$$= \cancel{\frac{\pi}{2}} - \cancel{\frac{\pi}{2}} - 0.02$$

$$\boxed{\gamma_{xy} = -0.02 \text{ rad}}$$ Answer

P# 2.10:

Given: Plastic plate with given dimensions



find: γ_{xy} @ corners A and B = ?

$\Rightarrow (\gamma_A)_{xy} = ?$, $(\gamma_B)_{xy} = ?$

$\tan \alpha = \tan \psi = P/B$

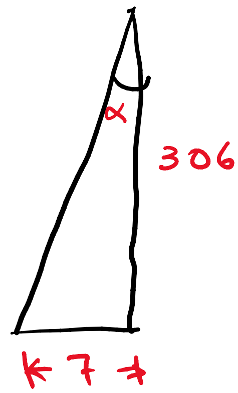
∵ small angle analysis:

$\alpha = \psi = 7$

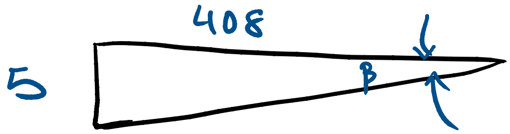
∴ small angle analysis:

$$\alpha = \psi = \frac{7}{306}$$

$$\alpha = \psi = 0.0228 \text{ rad}$$

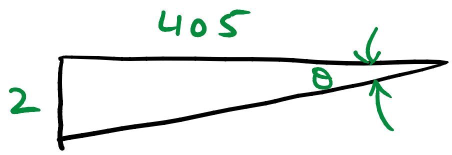


$$\beta = \frac{5}{408} = 0.01225 \text{ rad}$$



$$\theta = \frac{2}{405}$$

$$\theta = 0.005 \text{ rad}$$



$$\boxed{(\gamma_A)_{xy} = \theta + \psi = 27.8 \times 10^{-3} \text{ rad}}$$

$$\boxed{(\gamma_B)_{xy} = \alpha + \beta = 35.1 \times 10^{-3} \text{ rad}}$$

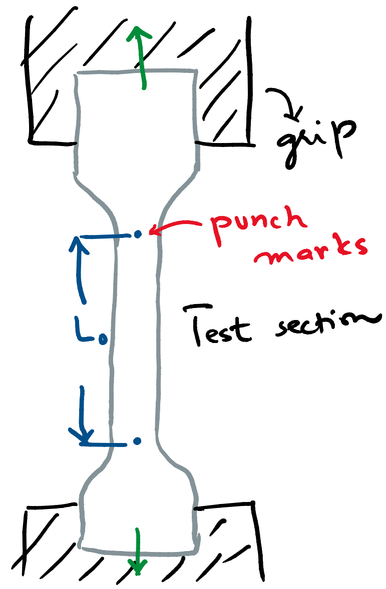
Answers.

Lecture 8

CHAPTER # 3

MECHANICAL PROPERTIES OF MATERIALS

§3.1 Tension & Compression Test:



* Measurements Taken before test:
  - $d_o \Rightarrow A_o$
  - L_o (Gage Length)
* load is applied @ slow constt: rate until failure
* Record data @ frequent intervals
  - Load, P
  - $\delta = L - L_o$
  - $\epsilon = \dfrac{\delta}{L_o}$

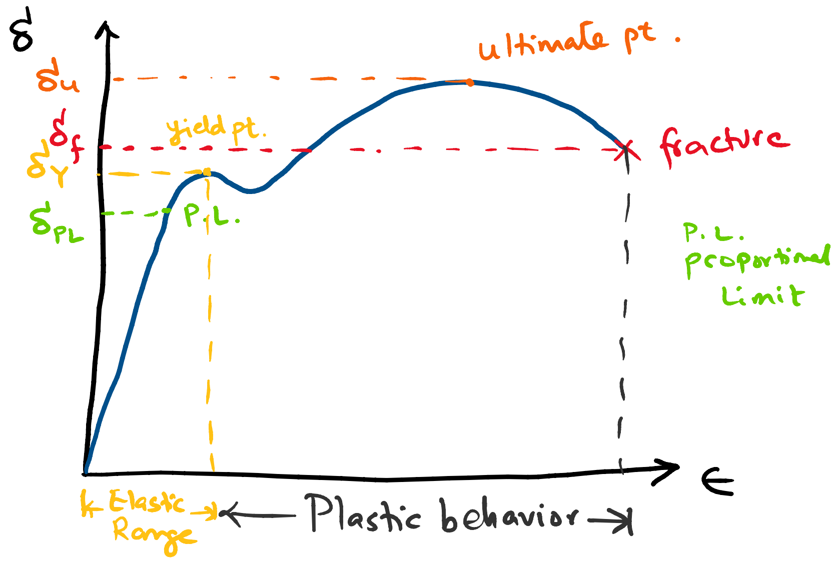
* Ability of material to sustain a load without any undue deformation or failure ⇒ strength ⇒ determined by experiments (tension or compression test) ⇒ relationship b/w σ_{avg} VS ϵ_{avg}

§3.2: Stress - Strain Diagram: σ-ϵ diagram:

- Two ways:

1. Conventional σ-ϵ diagram:
   * Calculate: $\sigma = \dfrac{P}{A}$ and $\epsilon = \dfrac{\delta}{L_o}$
   * Use original x-sectional area
   * Construct σ-ϵ diagram:
     - σ (vertical axis)

- σ (vertical axis)
- ϵ (horizontal axis)



* Highest pt. on the curve: σ_u

2. True $\sigma - \epsilon$ diagram:

   * Use ACTUAL x-sectional area (instead of original

   * Use actual specimen length @ same instant load is measured.

   $\Rightarrow$ true σ and true ϵ

   $\Rightarrow$ true $\sigma - \epsilon$ diagram

Conventional $\sigma - \epsilon$	True $\sigma - \epsilon$
• A_o is constt	• A changes with load

* Make sure: load is applied within the elastic range to avoid permanent deformation.

§ 3.3: $\sigma - \epsilon$ behavior of ductile and

§ 3.3: $\sigma - \epsilon$ behavior of ductile and brittle matls:

- Ductile matls:

  * capable of absorbing shock or energy
  * in case of overloading: large deformation before failure

* Measure of Ductility:

- % Elongation $= \left(\dfrac{L_f - L_o}{L_o}\right) \times 100$

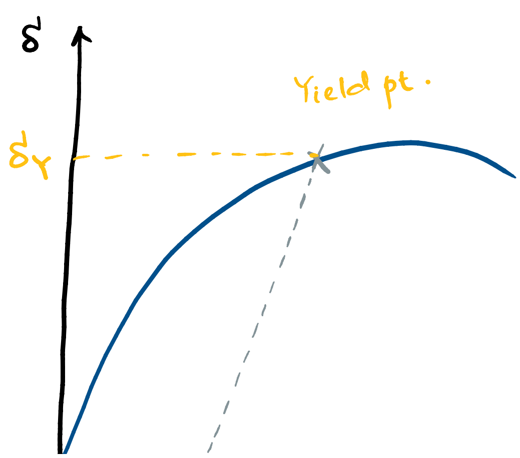
where L_o = original gage length
L_f = Length @ fracture

- % reduction of area $= \left(\dfrac{A_o - A_f}{A_o}\right) \times 100$

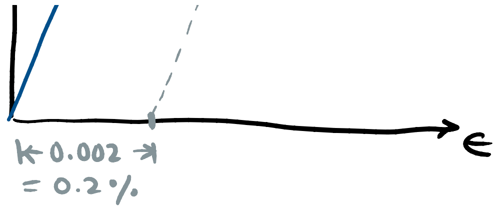
where A_o = specimen's original x-sectional area

A_f = area of neck @ fracture

* Some matls: do not exhibit constt: yielding beyond elastic range e.g. AL
⇒ Yield pt. is not very well-defined
⇒ Use OFFSET METHOD:

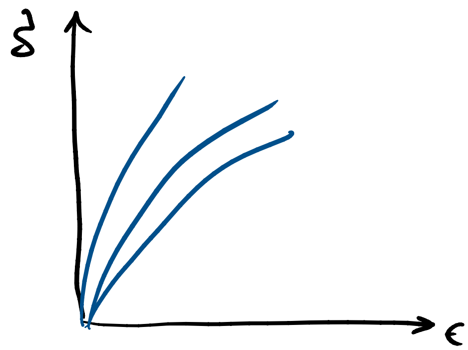


- If yield strength is not very well-defined ⇒ choose 0.002 $\left(\frac{in}{in}\right)$ strain (i.e. 0.2%) on ϵ - axis
- Draw a line ∥ to the initial straight line, until



To the initial straight line, until it intersects the δ-ϵ curve $\Rightarrow$ Pt. of intersection is yield pt.

- Brittle matls:
  - * No yielding before failure
    e.g: Gray Cast Iron



Hooke's Law:

$$\delta \propto \epsilon$$

$\Rightarrow$ δ and ϵ have a linear relationship upto "Elastic Range" (upto yield pt.)

$$\delta = E\epsilon$$

where E = modulus of elasticity or Young's modulus

$$E = \frac{\delta_{PL}}{\epsilon_{PL}}$$

Note: E represents the slope of the line

Units: same as δ

Generally; for steel:

$$E = 200\,GPa$$
$$E = 29 \times 10^3\ ksi$$

$E = 200 GPa$

$E = 29 \times 10^3$ ksi

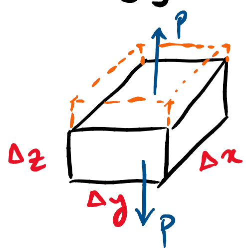
* 'E' is a mechanical property of the material
  - High E ⇒ STIFF (e.g: steel)
  - Low E ⇒ SPONGY (e.g: Rubber)

Strain hardening:

* If mat'l: is loaded beyond elastic range and load is removed, mat'l: will return to equilibrium state but not to original state

  ⇒ PERMANENT SET

§3.4: Strain Energy:



$$\sigma = \frac{\Delta F}{\Delta A}$$

$$\Delta F = \sigma \Delta A$$

$$\Delta F = \sigma (\Delta x \Delta y) \quad (1)$$

[on top & bottom face]

$$\epsilon = \frac{\Delta z' - \Delta z}{\Delta z} \Rightarrow \Delta z' - \Delta z = \underbrace{\epsilon \Delta z}_{\text{vertical displacement}}$$

Work done = Force * Displacement

$$W = F(\epsilon \Delta z)$$

Average : $F_{avg} = \dfrac{\Delta F}{2}$

$$W = \frac{\Delta F}{2}(\epsilon \Delta z) \quad (2)$$

from eq (1), Put ΔF into eq (2)

$$W = \frac{\sigma(\Delta x \Delta y)}{2}(\epsilon \Delta z)$$

$$W = \frac{\sigma \epsilon}{2} \Delta \forall$$

Work done = strain energy stored

$$\frac{\sigma \epsilon}{2} \Delta \forall = \Delta u$$

strain energy per unit volume $\dfrac{\Delta u}{\Delta \forall} = \dfrac{\sigma \epsilon}{2}$

$$u = \sigma \epsilon \quad (3)$$

per unit volume $\quad \Delta V \quad 2$

$$u = \frac{\sigma \epsilon}{2} \quad \text{—} \; ③$$

$\because$ Hooke's Law:

$$\sigma = E\epsilon$$

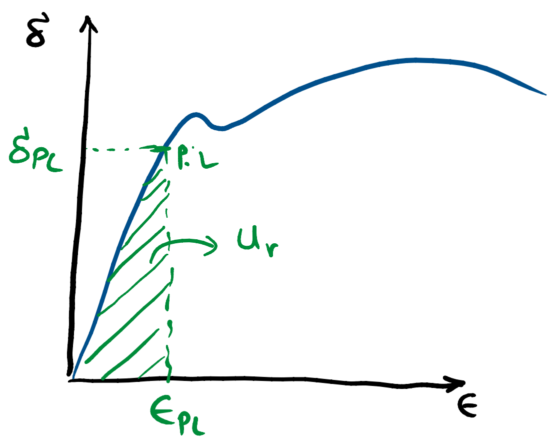
$$\epsilon = \frac{\sigma}{E} \quad \text{—} \; ④$$

Put ϵ from ④ to ③

$$u = \frac{\sigma}{2}\left(\frac{\sigma}{E}\right)$$

$$\boxed{u = \frac{\sigma^2}{2E}}$$

Modulus of Resilience (u_r):



* resilience is mat'l's ability to absorb energy w/o permanent deformation

Modulus of toughness (u_t):

* strain energy density of mat'l. just before it fractures.

Example #3.1

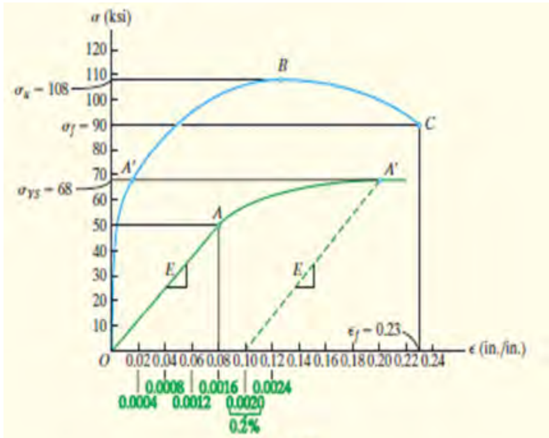


Fig. 3-18

OFFSET METHOD = 0.2%

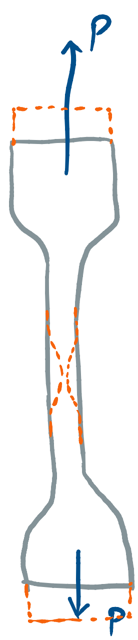
Step 1: find slope of line ⇒ E

$$E = \frac{\sigma_{PL}}{\epsilon_{PL}} = \frac{50\times10^{3}}{0.0016} = 31.2\times10^{3}\ \text{ksi}$$

from figure:

- $\sigma_Y = 68$ ksi
- $\sigma_u = 108$ ksi
- $\sigma_f = 90$ ksi

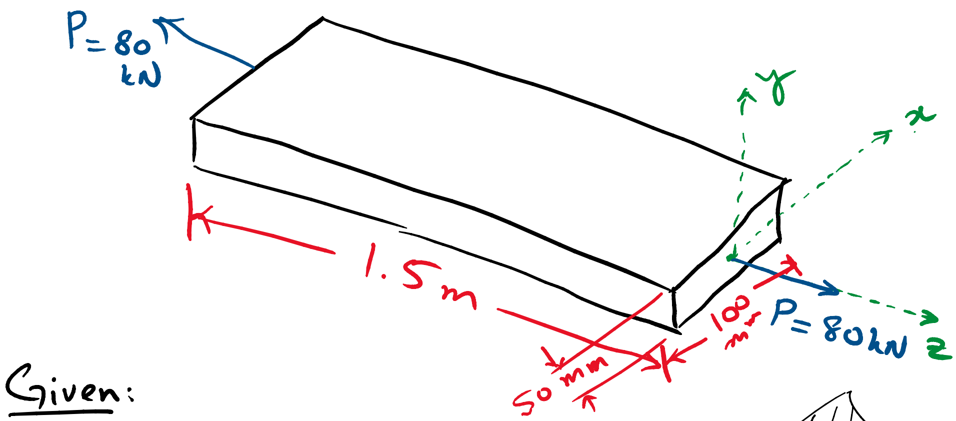
§ 3.5: Poisson's Ratio:



$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

* applicable within "Elastic range".
* ν is unique for a particular material [homogeneous and isotropic]
* -ve sign: longitudinal elongation (+ve ϵ) causes lateral contraction (-ve ϵ)
* Range: $0 \leq \nu \leq 0.5$
* Units: Dimensionless

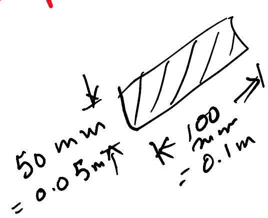
Example # 3.4 :



Given:

Mat'l : A-36 Steel

$P = 80\,kN$



Find:
- $\delta_z = ?$
- changes in dimension of x-section: $\delta_x, \delta_y = ?$

Governing Eqns:

- $\sigma = \dfrac{P}{A}$
- $\sigma = E\epsilon$
- $\nu = -\dfrac{\epsilon_{lat}}{\epsilon_{long}}$
- $\epsilon = \dfrac{\delta}{L}$

Assumptions:
- Mat'l : is homogeneous
- Hooke's Law $\Rightarrow \sigma \propto \epsilon$

Solution :

$$\sigma_z = \frac{F_z}{A} = \frac{80\times 10^3}{(0.1)(0.05)}$$

$$\sigma_z = 16\,MPa$$

Side Note : check @ table @ back

$$\sigma_z = 16\,\text{MPa}$$

Side Note: check @-table @ back cover of the book: for A-36 steel:

$$\sigma_Y = 250\,\text{MPa}$$

$$\Rightarrow \quad \sigma_z < \sigma_Y \Rightarrow \text{Hooke's law is applicable}$$

$$E_{st} = 200\,\text{GPa}$$

$$\epsilon_z = \frac{\sigma_z}{E_{st}} = \frac{16 \times 10^6}{200 \times 10^9}$$

$$\epsilon_z = 80 \times 10^{-6}\,\frac{m}{m}$$

$$\epsilon = \frac{\delta}{L} \Rightarrow \delta_z = \epsilon_z L_z$$

$$\delta_z = (80 \times 10^{-6})(1.5)$$

$$\boxed{\delta_z = 120\,\mu m} \quad \text{Answer.}$$

Go to table again:

$$\nu = 0.32$$

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z}$$

$$-\nu \epsilon_z = \epsilon_x = \epsilon_y$$

$$\epsilon_x = \epsilon_y = -0.32(80 \times 10^{-6})$$

$$\epsilon_x = \epsilon_y = -25.6\,\frac{\mu m}{m}$$

$$\epsilon = \frac{\delta}{L} \Rightarrow \delta_x = \epsilon_x L_x = (-25.6 \times 10^{-6})(0.1)$$

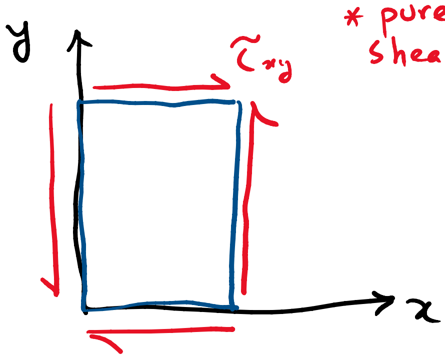
$$\boxed{\delta_x = -2.56\,\mu m} \quad \text{Answer}$$

$$\delta_y = \epsilon_y L_y = (-25.6 \times 10^{-6})(0.05)$$

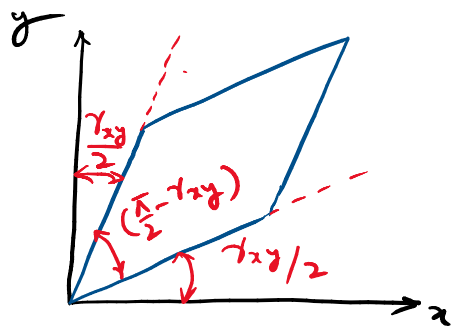
$$\boxed{\delta_y = -1.28\,\mu m} \quad \text{Answer}$$

$$\boxed{\delta_y = -1.28\ \mu m}$$ Answer

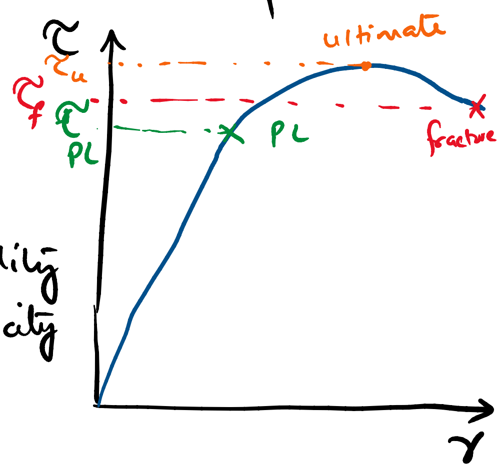
§ 3.6: $\tau - \gamma$ diagram:



* pure shear



* $\tau - \gamma$ diagram: Torsion experiment



* Hooke's Law

$$\tau \propto \gamma$$

$$\tau = G\gamma$$

where G = modulus of rigidity or shear modulus of elasticity

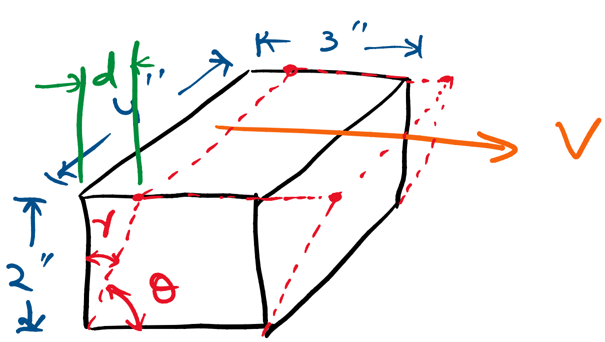
$$G = \frac{\tau_{PL}}{\gamma_{PL}}$$

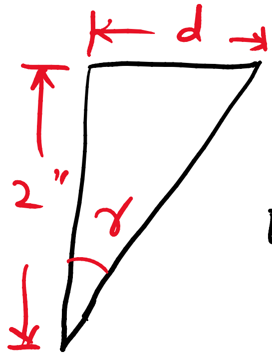
Units: same as τ

'γ' is measured in radians

Relationship between E and G

$$G = \frac{E}{2(1+\nu)}$$





$$\tan \gamma = \frac{P}{B} = \frac{d}{2}$$

$$\tan(0.008) = \frac{d}{2}$$

$$\boxed{d = 0.016''}$$

$$\tau_{PL} = \frac{V}{A}$$

$$52 \times 10^3 = \frac{V}{(3'' \times 4'')}$$

$$V = 624 \text{ kip}$$

Ex # 3.6:

Given: Mat'l: AL

$d_o = 25\text{mm}$

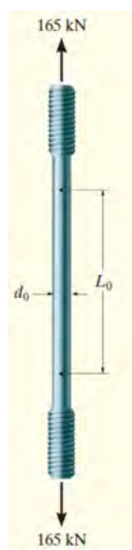
$L_o = 250 \text{ mm}$

$P = 165\text{kN}$

$\delta_L = 1.2\text{mm}$

$G_{AL} = 26 \text{ GPa}$

$\sigma_Y = 440 \text{ MPa}$



Find: $E = ?$

$\delta_d = ?$

Assumptions:

- Mat'l. is homogeneous

Governing Eqs:

- $\sigma = F/A$
- $\epsilon = \delta/L$, $\sigma = E\epsilon$
- $\nu = -\epsilon_{lat}/\epsilon_{long}$

Solution:

$$\sigma = \frac{F}{A} = \frac{165 \times 10^{3}}{\frac{\pi}{4}(0.025)^{2}}$$

$$\sigma = 336.1\,\text{MPa}$$

$\sigma < \sigma_Y \Rightarrow$ Hooke's law is applicable

$$\epsilon = \frac{\delta_L}{L_0} = \frac{1.2}{250}$$

$$\epsilon_{long} = 0.0048\ \frac{mm}{mm}$$

$$E = \frac{\sigma}{\epsilon}$$

$$= \frac{336.1 \times 10^{6}}{0.0048}$$

$$\boxed{E_{AL} = 70\,\text{GPa}}$$ Answer

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

$$G = \frac{E}{2(1+\nu)}$$

$$26 \times 10^{9} = \frac{70 \times 10^{9}}{2(1+\nu)}$$

$$26\times10 = \frac{\ldots\times10}{2(1+\nu)}$$

$$\nu = 0.347$$

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

$$0.347 = -\frac{\epsilon_{lat}}{0.0048}$$

$$\epsilon_{lat} = -0.00166\ \frac{mm}{mm}$$

$$\epsilon_d = \frac{\delta_d}{d_o}$$

$$\delta_d = \epsilon_d d_o$$

$$\delta_d = (-0.00166)(25)$$

$$\delta_d = -0.0416\ mm$$

contraction

P#3.25:

Given: matl : Plastic

$d_o = 15mm$

$P = 300N$

$L_o = 200mm$

$E_p = 2.7\ GPa$

$\nu_p = 0.4$

find: $\delta_L = ?$

$\delta_d = ?$

Solution:

$$\delta = \frac{F}{A}$$

$$\sigma = \frac{F}{A}$$

$$= \frac{300}{\frac{\pi}{4}(0.015)^2}$$

$$\sigma = 1.7 \text{ MPa}$$

$$\epsilon = \frac{\sigma}{E}$$

$$= \frac{1.7\times10^{6}}{2.7\times10^{9}}$$

$$\epsilon_{long} = 6.3\times10^{-4} \ \frac{m}{m}$$

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

$$0.4 = -\frac{\epsilon_{lat}}{6.3\times10^{-4}}$$

$$\epsilon_{lat} = -2.5\times10^{-4} \ \frac{m}{m}$$

$$\epsilon_L = \frac{\delta_L}{L_o} \ , \quad \epsilon_d = \frac{\delta_d}{d_o}$$

$$(6.3\times10^{-4}) = \frac{\delta_L}{200} \ , \quad -2.5\times10^{-4} = \frac{\delta_d}{15}$$

$$\boxed{\delta_L = 0.126\text{mm} \ , \quad \delta_d = -0.0037\text{ mm}}$$

Answers

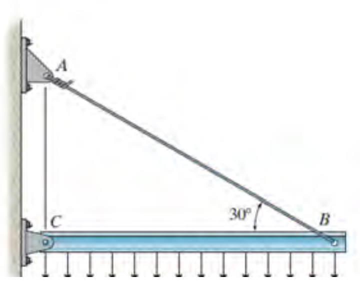
P# 3.7:

ven Matl: A-36 Steel

$d_{AB} = 0.2$ in

$w = 100$ lb/ft

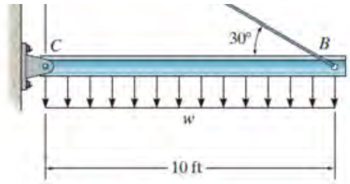
• Wire remains



/ft

- wire remains elastic

find : $\delta_{AB} = ?$



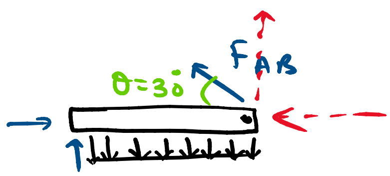
Probs. 3–7/8

Assumptions:
- Homogeneous mat'l :
- $\sigma \propto \epsilon$

Governing Eqs :
- $\sigma = F/A$
- $\sigma = E\epsilon$
- $\epsilon = \delta / L$

Solution :

$\Sigma M_c = 0$;



$F_{AB} \sin 30° (10)$

$- w(10)(5) = 0$

$\frac{\cancel{lb}}{\cancel{ft}} \times \cancel{lb}$

$F_{AB} \sin 30° (10) - (100)(10)(5) = 0$

$F_{AB} = 1000 \text{ lb} = 1 \text{ kip}$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{1}{\frac{\pi}{4}(0.2)^2}$$

$\sigma_{AB} = 31.83 \text{ ksi}$

$$\epsilon = \frac{\sigma}{E} = \frac{31.83}{29 \times 10^3}$$ → from tables

$\epsilon_{AB} = 0.0011 \text{ in/in}$

$\delta_{AB} = \epsilon_{AB} L_{AB}$

$$= 0.0011 \left(\frac{120}{\cos 30°}\right)$$

$$= \ldots \left(\frac{\ldots}{\cos 30^\circ} \right)$$

$$\boxed{\delta_{AB} = 0.152 \text{ in}} \quad \text{Answer}$$

RECAP : General Procedure :

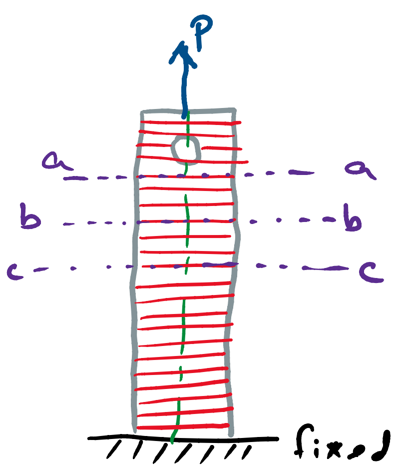
1. Find unknown forces (statics)
2. Find Stress (σ or τ)
3. Find Strain (ϵ or γ)
4. Find Deformation (δ)
5. Sketch Elongation or contraction on FBD

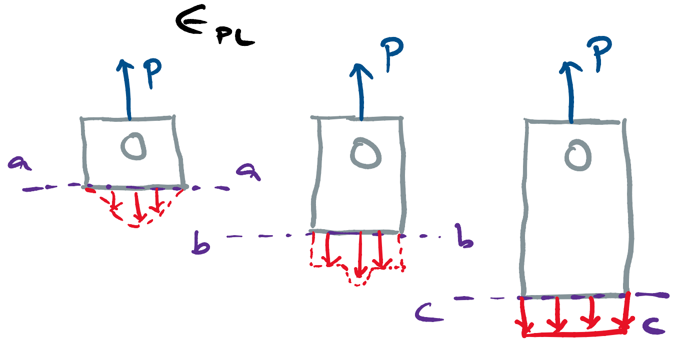
Chapter #4: AXIAL LOAD

Recall:

- Stress ⇒ measurement of force distribution within a body
- Strain ⇒ measurement of body's deformation
- Hooke's Law ⇒ $\sigma \propto \epsilon \Rightarrow \sigma = E\epsilon$

$$E = \frac{\sigma_{PL}}{\epsilon_{PL}}$$

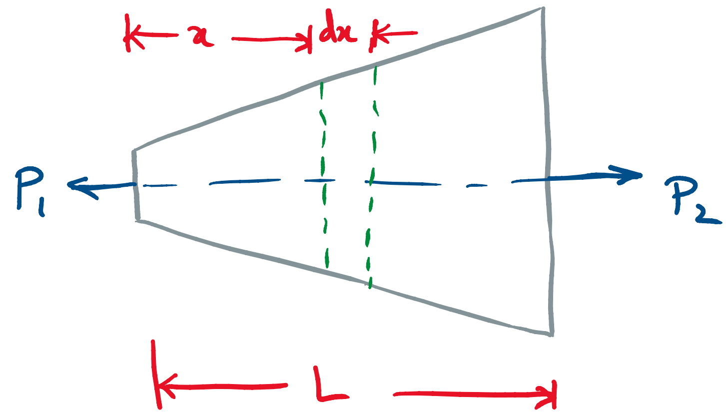


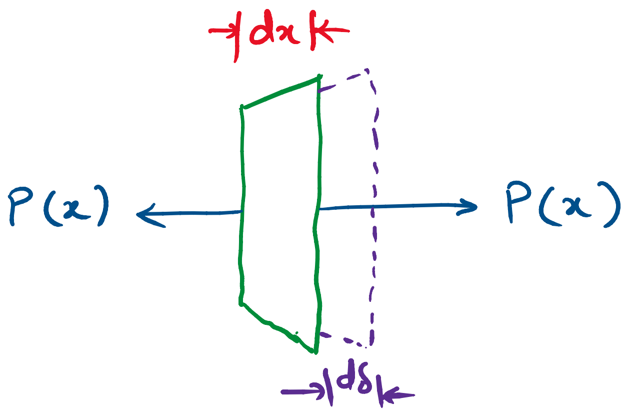


St. Venant's Principle

Elastic Deformation of an axially loaded member:

loaded member :





$$\delta = \frac{P(x)}{A(x)} \quad \text{—} \; ①$$

$$\epsilon = \frac{d\delta}{dx} \quad \text{—} \; ②$$

Elastic $\Rightarrow$ Hooke's Law $\Rightarrow$ $\delta = E\epsilon$ — ③

Plug from ① & ② into ③

$$\text{eq } ③ \Rightarrow \frac{P(x)}{A(x)} = E\left(\frac{d\delta}{dx}\right)$$

$$d\delta = \frac{P(x)\,dx}{A(x)\,E}$$

$$\int_0^{\delta} d\delta = \int_0^{L} \frac{P(x)\,dx}{A(x)\,E}$$

$$\int_0 \quad = \quad \int_0 \frac{\quad}{A(x)\,E}$$

$$\delta = \int_0^L \frac{P(x)\,dx}{A(x)\,E}$$

where: δ = displacement of one pt. on the bar relative to other pt.

L = original Length of bar

$P(x)$ = Applied force @ distance x from the end

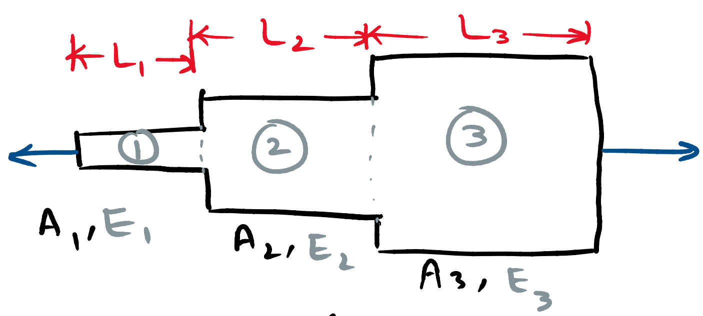
$A(x)$ = x-sectional area of bar as a $f(x)$

E = modulus of elasticity

for a constt. x-section: 

$$\delta = \frac{PL}{AE}$$

for a step-beam / bar:



$$\delta = \Sigma \left(\frac{PL}{AE} \right)$$

$$\delta = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3}$$

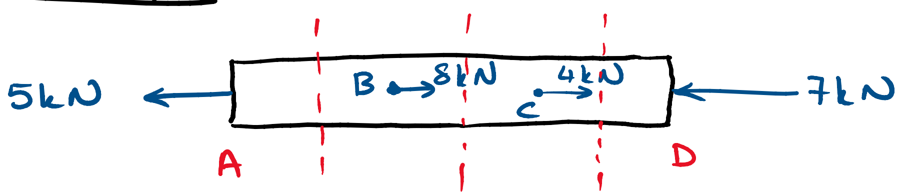
Sign Convention:

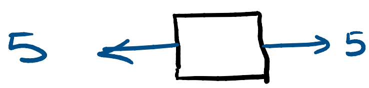
- P and δ (+ve) IF Tension or elongation

elongation

- P and δ (−ve) IF Compression or contraction

Example:





$P_{AB} = 5\,kN$

$P_{BC} = 5 - 8 = -3\,kN$

$P_{CD} = 5 - 8 - 4$

$= -7\,kN$

$$\delta = \Sigma\left(\frac{PL}{AE}\right)$$

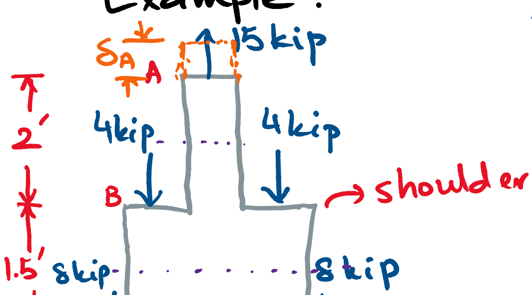
$$= \left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC} + \left(\frac{PL}{AE}\right)_{CD}$$

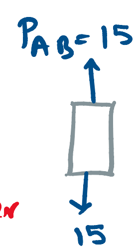
$A_{AB} = A_{BC} = A_{CD} = A$

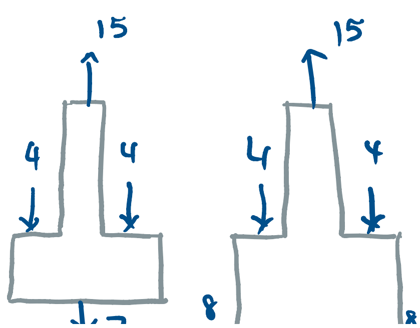
If matl. is same : $E_{AB} = E_{BC} = E_{CD} = E$

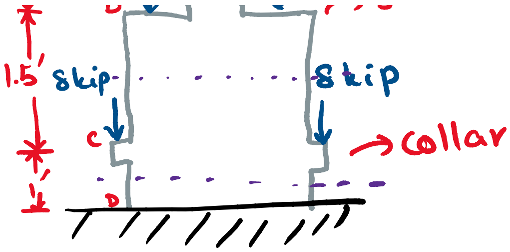
$$\delta = \frac{1}{AE}\left(P_{AB}L_{AB} + P_{BC}L_{BC} + P_{CD}L_{CD}\right)$$

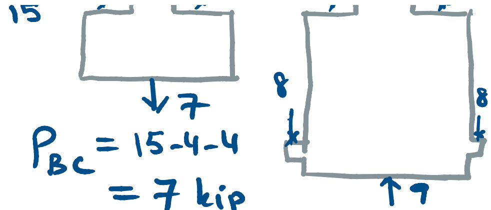
Example:











$P_{BC} = 15-4-4 = 7$ kip

$P_{CD} = 15-4-4-8-8 = -9$ kip

$A_{AB} = 1 \text{ in}^2$

$A_{BD} = 2 \text{ in}^2$

A-36 steel : $E_{st} = 29 \times 10^3$ ksi

Find: $\delta_A = ?$ $\quad \delta_{B/C} = ?$

Governing Eqn: $\delta = \Sigma \left(\frac{PL}{AE}\right)$

Solution:

$$\delta_A = \Sigma\left(\frac{PL}{AE}\right)$$

$$\delta_A = \left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC} + \left(\frac{PL}{AE}\right)_{CD}$$

$$= \frac{(15)(2 \times 12)}{(1)(29 \times 10^3)} + \frac{(7)(1.5 \times 12)}{(2)(29 \times 10^3)} + \frac{(-9)(1 \times 12)}{(2)(29 \times 10^3)}$$

$$= +0.0127 \text{ in} \Rightarrow \text{Elongation}$$

$$\delta_{B/C} = \frac{P_{BC} L_{BC}}{A_{BC} E}$$

$$= \frac{(7)(1.5 \times 12)}{(2)(29 \times 10^3)}$$

$$\delta_{B/C} = +0.00217 \text{ in} \Rightarrow \text{Elongation}$$

$$\delta_{B/D} = \Sigma\left(\frac{PL}{AE}\right) = \left(\frac{PL}{AE}\right)_{BC} + \left(\frac{PL}{AE}\right)_{CD}$$

§4.3: Principle of Superposition:

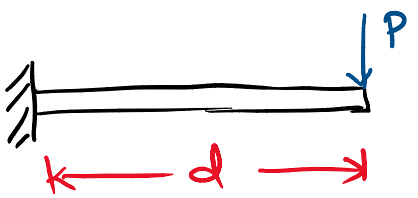
* For complicated loads

Conditions:

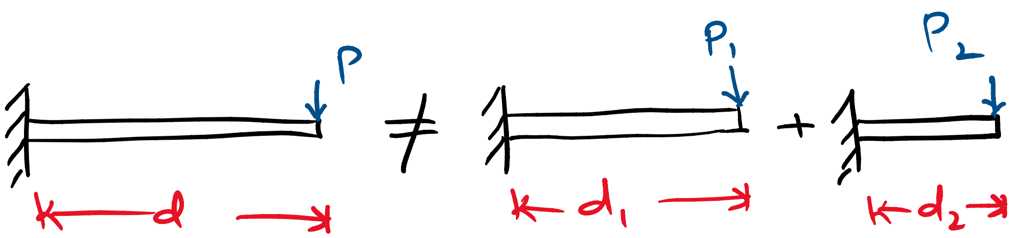
1. $\sigma \propto P$ or $\delta \propto P$

$$\Rightarrow \left[\sigma = \frac{P}{A}, \quad \delta = \frac{PL}{AE} \right] \Rightarrow \text{Hooke's Law}$$

2. original geometry must NOT change significantly.

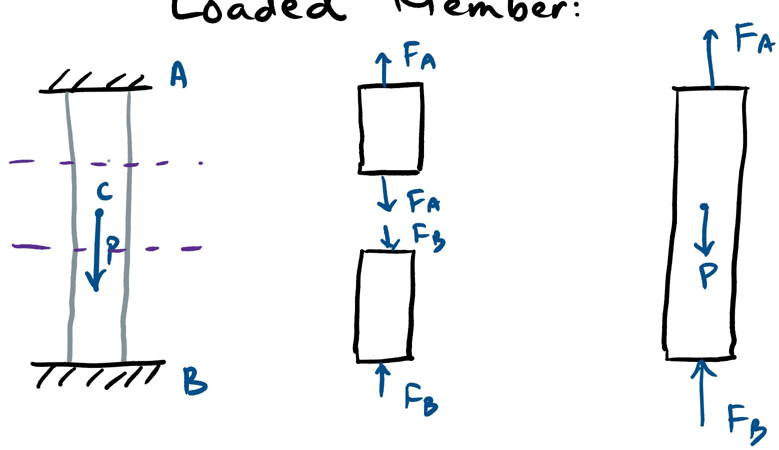


$$\boxed{P = P_1 + P_2}$$



$$Pd \neq P_1 d_1 + P_2 d_2$$

§4.4: Statically Indeterminate Axially Loaded Member:



$\Sigma F_y = 0;$

$$F_A + F_B - P = 0 \quad \text{—— (1)}$$

$\Sigma F_y = 0;$

$$F_A + F_B - P = 0 \quad \text{—— (1)}$$

$$\delta_{A/B} = 0 \quad [\text{both ends are fixed}]$$

$$\delta_{A/B} = \delta_{AC} + \delta_{BC}$$

$$0 = \frac{F_A L_{AC}}{AE} + \frac{(-F_B) L_{BC}}{AE}$$

$$\frac{F_A L_{AC}}{\cancel{AE}} = \frac{F_B L_{BC}}{\cancel{AE}}$$

$$F_A = F_B \left(\frac{L_{BC}}{L_{AC}}\right) \quad \text{—— (2)}$$

Put in eq (1):

$$F_B \left(\frac{L_{BC}}{L_{AC}}\right) + F_B = P$$

$$F_B \left(\frac{L_{BC}}{L_{AC}} + 1\right) = P$$

$$F_B \left(\frac{L_{BC} + L_{AC}}{L_{AC}}\right) = P$$

$$F_B \left(\frac{L}{L_{AC}}\right) = P$$

$$\boxed{F_B = P\left(\frac{L_{AC}}{L}\right)}$$

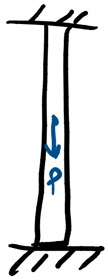
eq (2) $\Rightarrow$ $$F_A = P\left(\frac{\cancel{L_{AC}}}{L}\right)\left(\frac{L_{BC}}{\cancel{L_{AC}}}\right)$$

$$\boxed{F_A = P\left(\frac{L_{BC}}{L}\right)}$$

Hint:

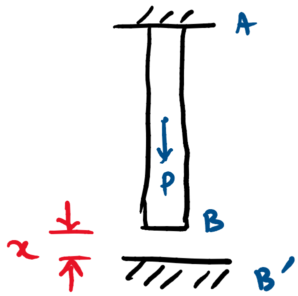
Hint:

Concept Questions / Problems:



Total deformation:

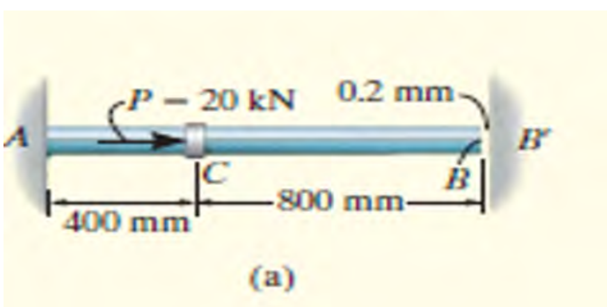
$\delta = 0$



Total deformation:

$\delta = x$

Ex # 4.5:



(a)

Steel Rod

$d = 10\,mm$

$E_{st} = 200\,GPa$

$\delta = 0.2\,mm = 0.0002\,m$

Find: $F_A = R_A = ?$

$F_{B'} = R_{B'} = ?$

Governing Eqns:

- $\Sigma F = 0$

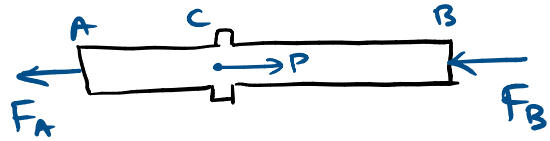
Governing Eqns:

- $\Sigma F = 0$
- $\delta = \frac{PL}{AE}$

Assumptions:

- Hooke's Law

Solution:



$\Sigma F_x = 0;$

$$F_A + F_B - P = 0 \quad \text{—— (1)}$$

$$\delta_{A/B} = \frac{F_A L_{AC}}{AE} + \frac{(-F_B) L_{BC}}{AE}$$

$$0.0002 = \frac{F_A (0.4)}{\left[\frac{\pi}{4}(0.01)^2\right]\left[200\times10^9\right]} - \frac{F_B (0.8)}{\frac{\pi}{4}(0.01)^2\left[200\times10^9\right]}$$

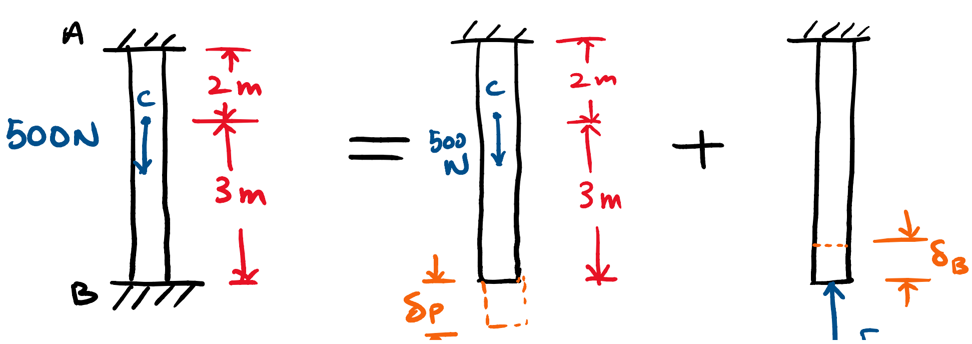
$$F_A(0.4) - F_B(0.8) = 3141.6 \text{ N.m} \quad \text{—— (2)}$$

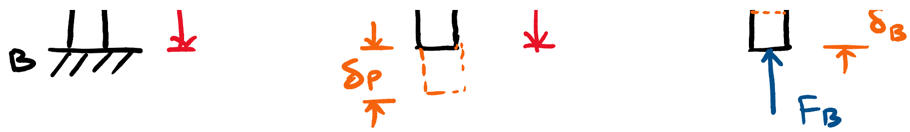
Solve eq (1) and eq (2) for F_A and F_B:

$$F_A = 16\text{ kN} \quad , \cancel{F_B} = F_B = 4\text{ kN}$$

Answers.

§4.5: The force method of Analysis for axially loaded members:





- remove the redundant support

$$0 = \delta_p - \delta_B$$

$$\delta_P = \delta_B$$

$$\frac{500(2)}{\cancel{AE}} = \frac{F_B(5)}{\cancel{AE}}$$

$$\boxed{F_B = 200N}$$

$\Sigma F_y = 0;$

$$F_B + F_A - 500 = 0$$

$$200 + F_A - 500 = 0$$

$$\boxed{F_A = 300N}$$

§4.6: THERMAL STRESS:

Recall:

- Temp: change can cause deformation
  - $T \uparrow \Rightarrow$ Expansion
  - $T \downarrow \Rightarrow$ Contraction

Conditions:

- Expansion / Contraction should be proportional to Temp:

$$\delta_T \propto T$$

- Homogeneous and istropic mat'l:

Displacement due to Temp. change:

$$\delta_T = \alpha L \Delta T$$

where: α = coefficient of thermal expansion

L = original Length of the member

ΔT = change in temp.

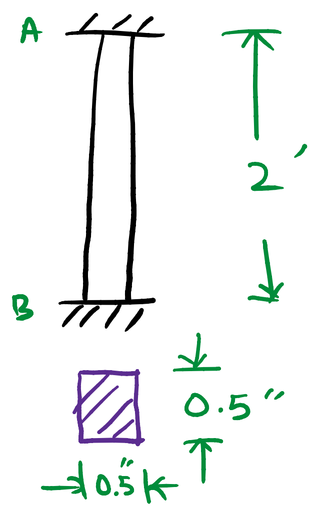
Ex # 4.10:

A-36 Steel

$T_1 = 60°F$

$T_2 = 120°F$

find: $\delta = ?$



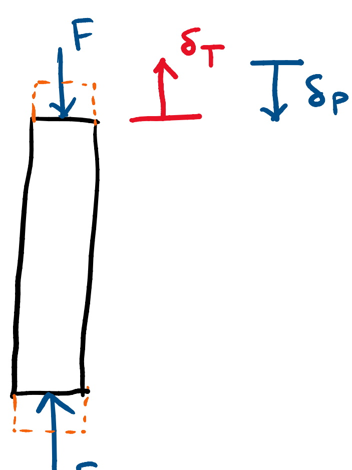
Governing Eqns:

- $\delta_T = \alpha L \Delta T$
- $\delta = F/A$, $\delta = PL/AE$

Assumptions:

- $\delta_T \propto T$
- Homogeneous mat'l.

Solution:



$$\delta_{A/B} = 0$$

$$\delta_{A/B} = \delta_T - \delta_P$$

$$0 = \alpha L \Delta T - \frac{FL}{AE}$$

$$F = \alpha \Delta T AE$$

1F

$$F = \alpha \Delta T A E$$

Go to the back cover of the book

$$\alpha = 6.6 \times 10^{-6}$$

$$E = 29 \times 10^{3} \text{ ksi}$$

[A-36]

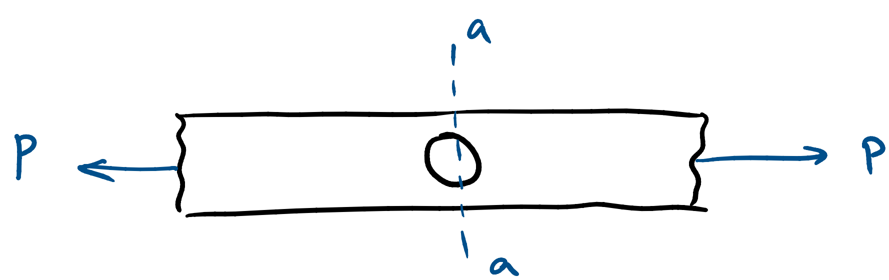
$$F = (6.6\times10^{-6})(120-60)(0.5\times0.5)(29\times10^{3})$$

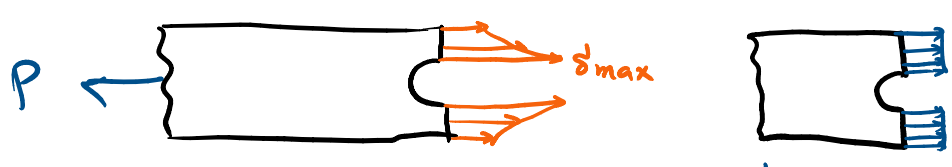
$$F = 2.8 \text{ kip}$$

$$\sigma = \frac{F}{A} = \frac{2.8}{(0.5\times0.5)}$$

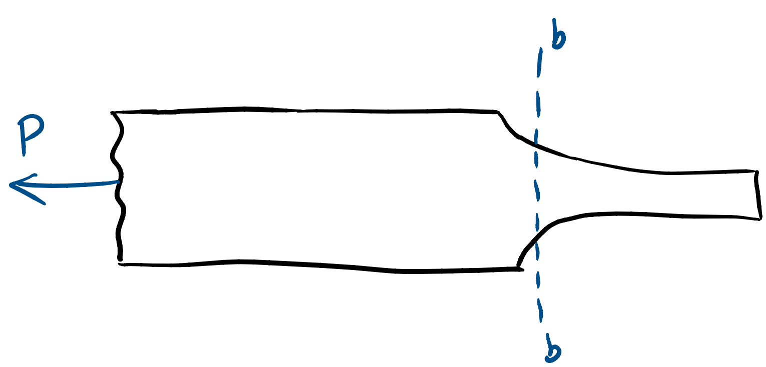
$$\boxed{\sigma = 11.2 \text{ ksi}}$$ Answer

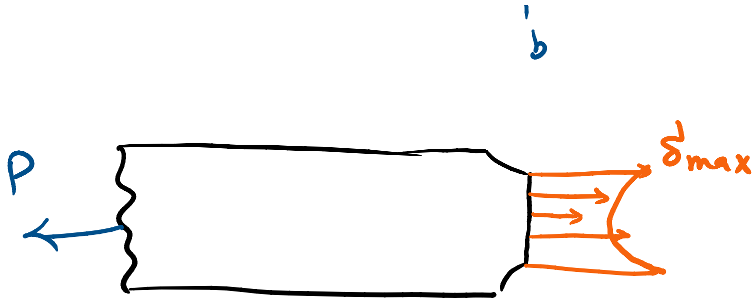
§ 4.7: Stress Concentration:

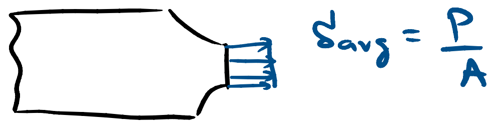




$$\sigma_{avg} = \frac{P}{A}$$







Stress Conc. Factor K:

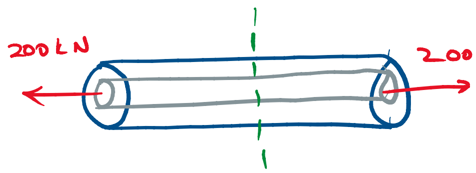
$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

Refer Fig 4-24, 4-25
find K:

$$\sigma_{max} = K \, \sigma_{avg}$$

P# 4.33:

A-36 Steel, AL



$P = 200\text{ kN}$

$E_{st} = 200\text{ GPa}$

$E_{AL} = 68.9\text{ GPa}$

$d_o = 80\text{ mm} = 0.08\text{ m}$

$d_i = 70\text{ mm} = 0.07\text{ m}$

$L = 400\text{ mm} = 0.4\text{ m}$

Find:

$\delta_{AL} = ?$

$\delta_{ST} = ?$

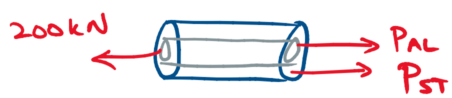
Governing Eqns:

- $\sigma = F/A$
- $\Sigma F = 0$
- $\delta = PL/AE$

Assumptions:

- Hooke's Law

Solution:



$\Sigma F_x = 0;$

$-200 + P_{AL} + P_{st} = 0$ —— ①

Compatibility:

$$\delta_{st} = \delta_{AL}$$

$$\left(\frac{PL}{AE}\right)_{st} = \left(\frac{PL}{AE}\right)_{AL}$$

$$\frac{P_{st}\cancel{L}}{A_{st}E_{st}} = \frac{P_{AL}\cancel{L}}{A_{AL}E_{AL}}$$

$$\frac{P_{st}}{\frac{\pi}{4}\left[(0.08)^2 - (0.07)^2\right](200\times10^9)} = \frac{P_{AL}}{\frac{\pi}{4}(0.07)^2(68.9\times10^9)}$$

$P_{AL} = 1.125\, P_{st}$ —— ②

Solve eq ① & eq ②

$P = 94.1\text{ kN}$ $P = 105.9\text{ kN}$

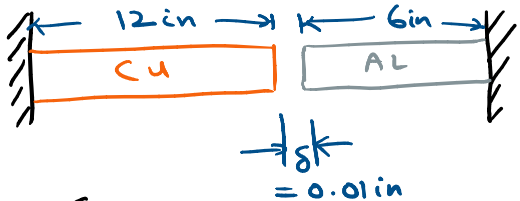
Solve eq ① & eq ②

$$P_{st} = 94.1\,kN, \quad P_{AL} = 105.9\,kN$$

$$\sigma_{AL} = \frac{P_{AL}}{A_{AL}} = 27.5\,MPa$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = 79.9\,MPa$$

<u>P#4.72</u>:



Given:- $\delta = 0.01\,in$

$T_1 = 60°F$ $\quad d_{AL} = d_{cu} = 1.25\,in$

$T_2 = 400°F$

$(\alpha)_{AL} = 13\times10^{-6}/°F \qquad (\alpha)_{cu} = 9.4\times10^{-6}/°F$

$E_{AL} = 10\times10^{3}\,ksi \qquad E_{cu} = 15\times10^{3}\,ksi$

$(\sigma_Y)_{AL} = 40\,ksi \qquad (\sigma_Y)_{cu} = 50\,ksi$

Find: (a) $\sigma_{AL} = \sigma_{cu} = ?$

(b) $L_{AL} = ?$ (New length)

Governing Equations:

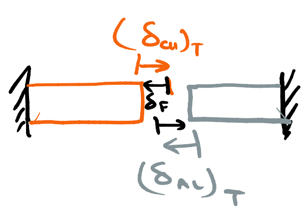
- $\delta_T = \alpha L \Delta T$
- $\delta = \dfrac{PL}{AE}$
- $\sigma = F/A$

Assumptions:

- Hooke's Law
- Homogeneous mat'l:

Solution:

Compatibility:



$$\delta = (\delta_{cu})_T - (\delta_{cu})_F + (\delta_{AL})_T - (\delta_{AL})_F$$

$$+(\delta_{AL})_T - (\delta_{AL})_F$$

$$0.01 = (\alpha L \Delta T)_{cu} - \left(\frac{PL}{AE}\right)_{cu} + (\alpha L \Delta T)_{AL} - \left(\frac{PL}{AE}\right)_{AL}$$

$$0.01 = (\alpha L \Delta T)_{cu} - \frac{P(12)}{(AE)_{cu}} + (\alpha L \Delta T)_{AL} - \frac{P(6)}{(AE)_{AL}}$$

$$P = 48.1 \text{ kip}$$

$$\delta_{AL} = \delta_{cu} = \frac{P}{A} = 39.2 \text{ ksi}$$

$$\delta_{AL} = \delta_{cu} < (\delta_Y)_{AL} \text{ and } (\delta_Y)_{cu}$$

$$\delta_{AL} = (\delta_{AL})_T - (\delta_{AL})_F$$

$$\delta_{AL} = (\alpha L \Delta T)_{AL} - \left(\frac{PL}{AE}\right)_{AL}$$

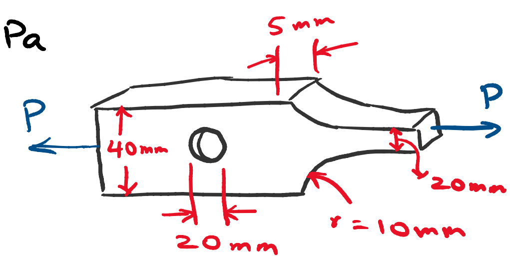
$$\delta_{AL} = 0.003 \text{ in}$$

$$L_{AL} = L_o + \delta_{AL}$$

$$\boxed{L_{AL} = 6.003 \text{ in}}$$ Answer

P # 4.88 :

$\delta_{allow} = 120 \text{ MPa}$



$w = 40 \text{ mm}$

$h = 20 \text{ mm}$

$r = 10 \text{ mm}$

find:

$P = ?$

Governing Eqn:

- $K = \dfrac{\delta_{max}}{\delta_{avg}}$
- $\delta_{avg} = \dfrac{P}{A}$

Assumptions:

- Homogeneous mat'l:

Assumptions:

- Homogeneous matl:
- St. Venant's Principle

Solution:

If failure would occur @ FILLET:

Figure 4.23 shows the nomenclature:

$$\frac{w}{h} = \frac{40}{20} = 2$$

$$\frac{r}{h} = \frac{10}{20} = 0.5$$

$$\Rightarrow K = 1.4 \quad \text{(from Fig 4-23)}$$

$$\sigma_{allow} = \sigma_{max}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

$$\sigma_{avg} = \frac{\sigma_{max}}{K}$$

$$\frac{P}{(0.02)(0.005)} = \frac{120\times 10^6}{1.4}$$

$$\boxed{P = 8.57 \text{ kN}}$$

If failure would be due to HOLE:

Fig 4.24:

$$\frac{2r}{w} = \frac{20}{40} = 0.5$$

$$K = 2.1$$

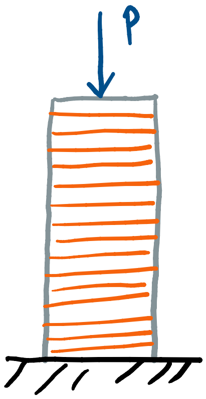
$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$120\times 10^6 = 2.1\left(\frac{P}{[0.04-0.02]0.005}\right)$$

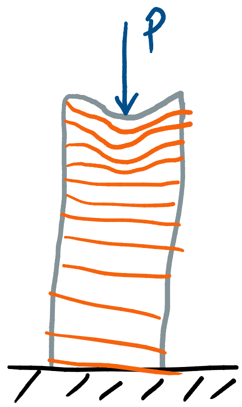
$$\boxed{P = 5.7 \text{kN}}$$ ANSWER

Chap 4:

- St. Venant's Principle



Draw deformed shape in accordance with St. Venant's Principle!

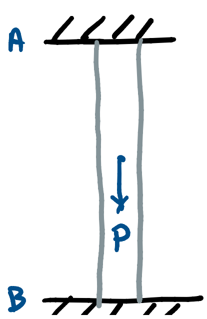


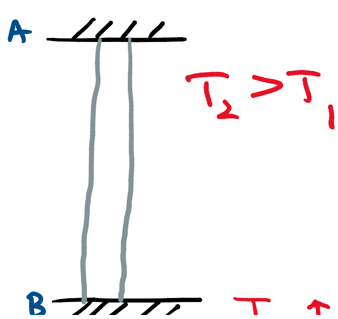
Application of Hooke's law and St. Venant's Principle:

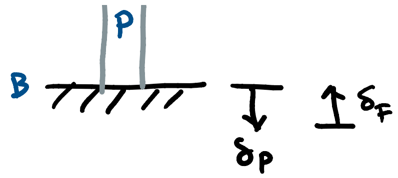
$$\delta = \frac{PL}{AE}$$

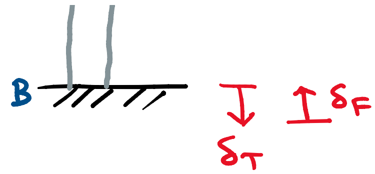
- for multiple mat'ls, x-sectional areas, lengths, loads:

$$\delta = \Sigma \left(\frac{PL}{AE} \right)$$







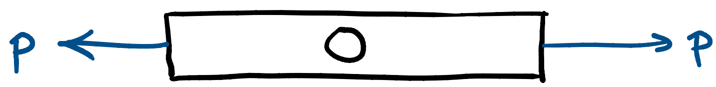


$$\delta_{A/B} = 0 = \delta_P - \delta_F \qquad \delta_{A/B} = 0 = \delta_T - \delta_F$$

- Stress Concentration Factor : K

$$K = \frac{\sigma_{max}}{\sigma_{allow}}$$

Fig 4-23, 4-24

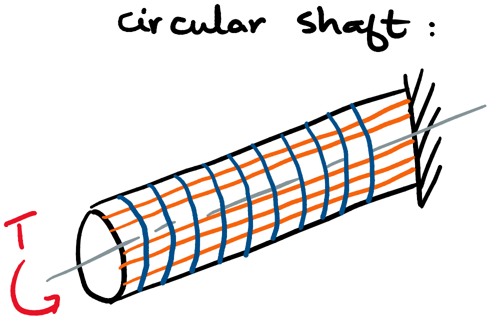


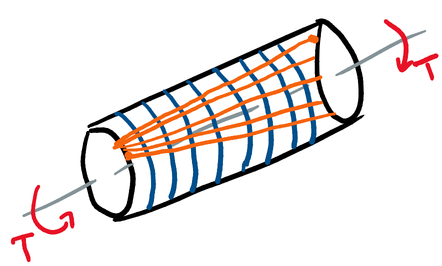
Determine the location for σ_{max}.

Answer: At the hole

Chapter # 5
Torsion

§5.1: Torsional Deformation of a circular shaft :





UNDEFORMED BODY

- shaft fixed @ one end
- Torque applied to

DEFORMED BODY

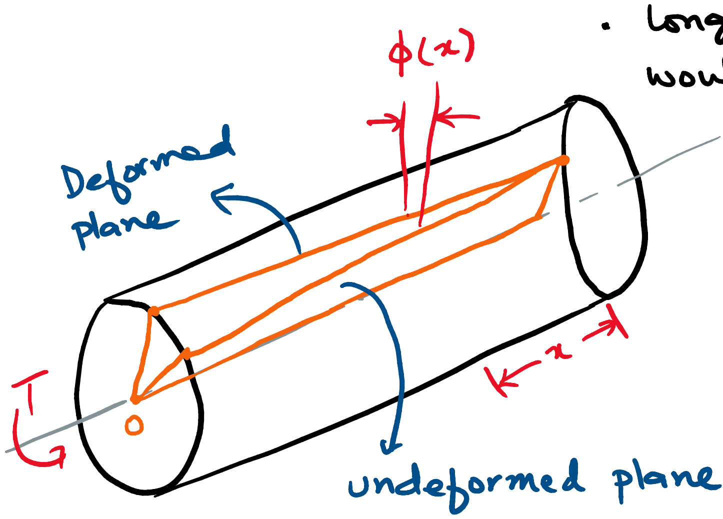
- ϕ = angle of twist
- circles would

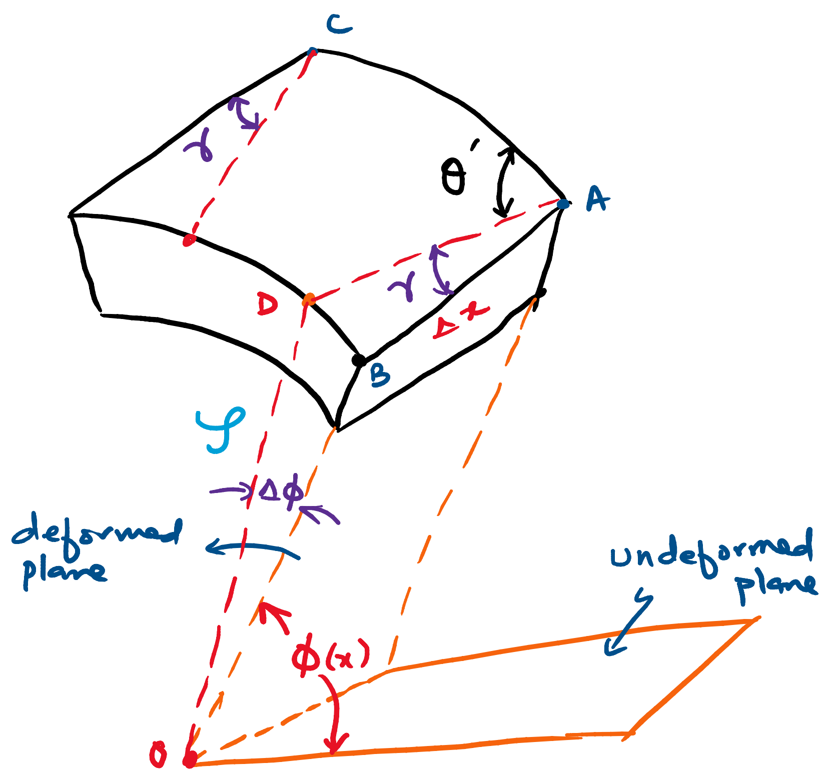
one end

- Torque applied to other end

twist

- circles would remain circle
- longitudnal lines would be twisted





Recall: shear strain : $\gamma = \frac{\pi}{2} - \theta'$

Radial distance from axis of shaft $= \rho$

Deformation of the back face $= \phi(x)$

Deformation of the front face $= \phi(x) + \Delta\phi$

shear strain causes $\Delta\phi$

Before Deformation:

shear strain causes $\Delta\phi$

Before Deformation:

Angle b/w AB and AC $= \frac{\pi}{2} = 90^\circ$

After Deformation:

Angle b/w AD and AC: $\gamma = \frac{\pi}{2} - \theta'$

Recall: Length of Arc:

$$S = r\theta$$

$$BD = \rho\,\Delta\phi = (\Delta x)\gamma$$

Recall: L'Hopital's Rule:

If $\Delta x \to dx$, $\Delta\phi \to d\phi$

$$\rho\, d\phi = \gamma\, dx$$

$$\boxed{\gamma = \rho\left(\frac{d\phi}{dx}\right)}$$

$\Rightarrow \frac{d\phi}{dx} =$ constt: over a x-section

$$\frac{\gamma}{\rho} = \frac{d\phi}{dx} = \text{constt}:$$

- γ varies only with its radial distance ρ

* $\gamma = 0$ @ shaft axis

* $\gamma = \gamma_{max}$ @ shaft surface

$$\frac{d\phi}{dx} = \frac{\gamma}{\rho} = \frac{\gamma_{max}}{c}$$

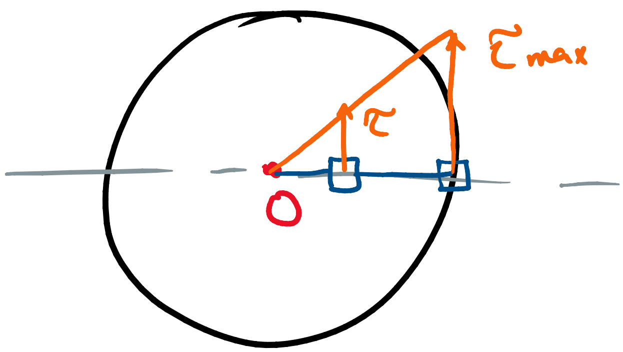
$c =$ original radial distance from

c = original radial distance from shaft axis

$$\boxed{\gamma = \gamma_{max}\left(\frac{\rho}{c}\right)} \quad (1)$$

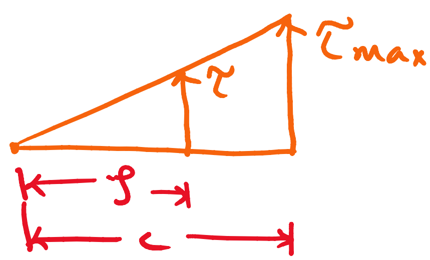
VALID for solid and hollow shafts.

§ 5.2: Torsion Formula:



If Hooke's law is satisfied ⇒ $\tau = G\gamma$ ⇒ material is linearly elastic

- $\tau = 0$ @ shaft axis
- $\tau = \tau_{max}$ @ shaft surface



By similar Δs: $\dfrac{\tau}{\rho} = \dfrac{\tau_{max}}{c}$

$$\boxed{\tau = \tau_{max}\left(\frac{\rho}{c}\right)} \quad (2)$$

- $\tau = F/A$

$$dF = \tau \, dA$$

and $dT = dF\,(\rho)$ $\qquad [T = Fr]$

$$dT = (\tau \, dA)(\rho)$$

$$T = \int_A \rho\,(\tau \, dA)$$

$$= \int_A \rho \left[\tau_{max}\left(\frac{\rho}{c}\right)\right] dA$$

$\therefore \dfrac{\gamma_{max}}{c} = \text{constt}$

Using Hooke's law: $\dfrac{\tau_{max}}{c} = \text{constt}$:

$$T = \frac{\tau_{max}}{c} \int_A \rho^2 \, dA$$

Note: ρ is a distance $\Rightarrow$ integral depends upon the shaft geometry.

$\int_A \rho^2 \, dA = J \Rightarrow$ Polar moment of inertia of shaft x-section

$$T = \frac{\tau_{max}}{c} J$$

$$\boxed{\tau_{max} = \frac{Tc}{J}} \qquad (3)$$

where: c = outer radius of shaft

T = resultant torque

$T =$ resultant torque

$\tau_{max} =$ max. shear stress in shaft

Compare eq 2 and eq 3:

$$\text{eq } 2 \Rightarrow \tau_{max} = \tau\left(\frac{c}{\rho}\right)$$

$$\text{eq } 3 \Rightarrow \tau_{max} = \frac{Tc}{J}$$

$$\tau\left(\frac{\cancel{c}}{\rho}\right) = \frac{T\cancel{c}}{J}$$

$$\boxed{\tau = \frac{T\rho}{J}} \quad 4$$

eq 3 and eq 4: TORSION FORMULA

§5.3 : POWER TRANSMISSION:

* Solid shafts } circular
* Hollow shafts } x-sections
* Power : Work performed per unit time

$$P = \frac{W}{t}$$

Units: $\frac{J}{s}$ = Watts

* $P = T\frac{d\theta}{dt}$ (θ = angle of rotation of shaft)

$\because$ Angular velocity $\omega = \frac{d\theta}{dt}$

$$\Rightarrow \boxed{P = T\omega} \longrightarrow 1$$

If frequency 'f' is given for shaft's rotation

$\Rightarrow$ measure of number of shaft rps $\Rightarrow$ Hz

1 Hz = 1 cycle/sec

1 cycle = 2π radians

$$\Rightarrow \omega = 2\pi f$$

eq 1 $\Rightarrow$ $\boxed{P = 2\pi f T} \longrightarrow 2$

$$\boxed{T = \frac{P}{2\pi f}} \longrightarrow 3$$

* When T and τ_{allow} are given [Hooke's law applicable]

$$\tau_{max} = \frac{Tc}{}$$

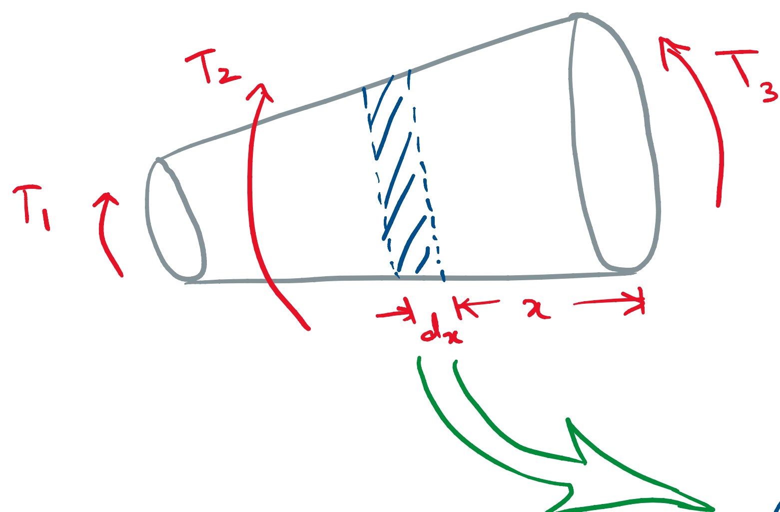
$$\tau_{max} = \frac{Tc}{J}$$

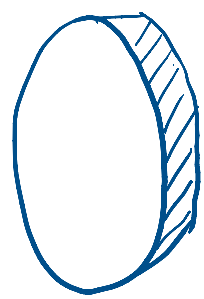
$$\Rightarrow \quad \frac{J}{c} = \frac{T}{\tau_{max}} \quad \begin{matrix} \text{known} \\ \text{known} \end{matrix}$$

$\Rightarrow$ DESIGN PROBLEM

$\Rightarrow$ Find dia or radius (c)

§ 5.4 : ANGLE OF TWIST (ϕ) :





Recall:

- $$\frac{d\phi}{dx} = \frac{\gamma}{\rho} \quad \text{—ⓐ}$$
- $$\gamma = \frac{\tau}{G} \quad \text{—ⓑ}$$
- $$\tau = \frac{T\rho}{J} \quad \text{—ⓒ}$$

eq ⓑ and ⓒ $\Rightarrow$
$$\gamma = \frac{T\rho/J}{G}$$

$$\boxed{\gamma = \frac{T\rho}{JG}} \quad \text{—④}$$

$$\frac{\quad}{JG}$$

$$\text{eq} \; (a) \Rightarrow \frac{d\phi}{dx} = \frac{T\cancel{\rho}/JG}{\cancel{\rho}} = \frac{T}{JG}$$

Integrate: $\displaystyle\int_0^{\phi} d\phi = \int_0^{L} \frac{T}{JG}\, dx$

$$\boxed{\phi = \frac{TL}{JG}} \quad \text{— (5)}$$

Side Note:
Similar to
$\delta = \dfrac{PL}{AE}$

- If shaft is subjected to several different T, L, and matls: G, J:

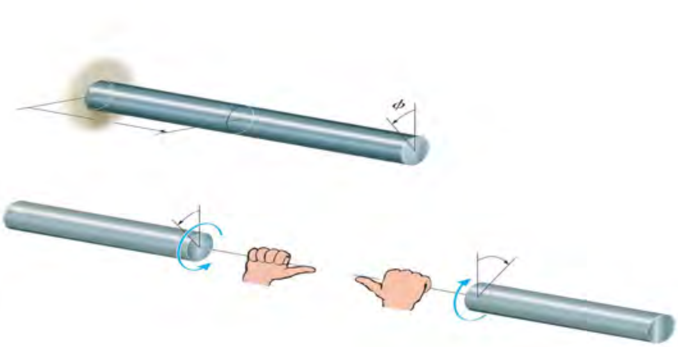
$$\boxed{\phi = \Sigma\left(\frac{TL}{JG}\right)} \quad \text{— (6)}$$

SIGN CONVENTION:

* Right Hand Rule

OR: Look from right side: CCW (+ve) ; CW (−ve)

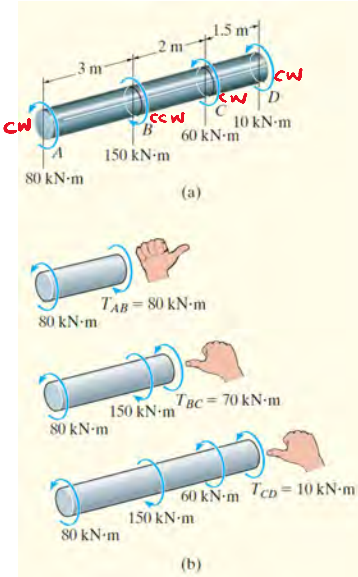
Figure 5-15 from book:



The best way to apply this equation is to use a sign convention for both the internal torque and the angle of twist of one end of the shaft with respect to the other end. To do this, we will apply the right-hand rule, whereby both the torque and angle will be positive, provided the thumb is directed outward from the shaft while the fingers curl in the direction of the torque, Fig. 5–15.

Ex # 5.5 : (Pg 211)

Look from right side



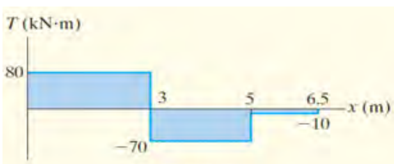
Internal Torque. Using the method of sections, the internal torques are found in each segment as shown in Fig. b. By the right-hand rule, with positive torques directed away from the sectioned end of the shaft, we have TAB = +80 kN # m, TBC = -70 kN # m, and TCD = -10 kN.m.
These results are also shown on the torque diagram, which indicates how the internal torque varies along the axis of the shaft

$$J = \frac{\pi}{2}(0.1)^4$$
$$J = 0.1571 \times 10^{-3}\ m^4$$

$$\phi_A = \Sigma \frac{TL}{JG}$$

$$= \frac{(80\times10^3)(3)}{J(75\times10^9)} + \frac{(-70\times10^3)(2)}{J(75\times10^9)} + \frac{(-10\times10^3)(1.5)}{J(75\times10^9)}$$

$$\phi_A = 7.22\times10^{-3}\ \text{rad}$$



(c)

Fig. 5–16

$$\phi_{A/C} = \Sigma\left(\frac{TL}{JG}\right)$$

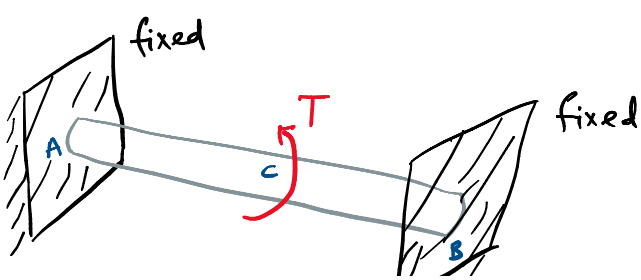
$$= \frac{(80\times10^3)(3)}{JG} + \frac{(-70\times10^3)(2)}{JG}$$

$$\phi_{A/C} = 8.49\times10^{-3}\ \text{rad}$$

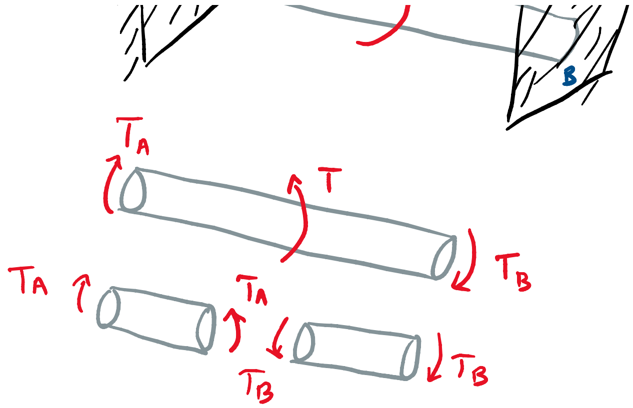
Both results are positive, which means that end A will rotate as indicated by the curl of the right-hand fingers when the thumb is directed away from the shaft.

§5.5: Statically Indeterminate Torque Members:

* If $\Sigma M_x = 0$ can NOT determine all unknown torques $\Rightarrow$ statically indet.



$$\phi_{A/B} = 0$$
$$\phi_{A/B} = \phi_{AC} + \phi_{BC}$$
$$0 = \frac{T_A L_{AC}}{JG} + \frac{(-T_B) L_{BC}}{JG}$$

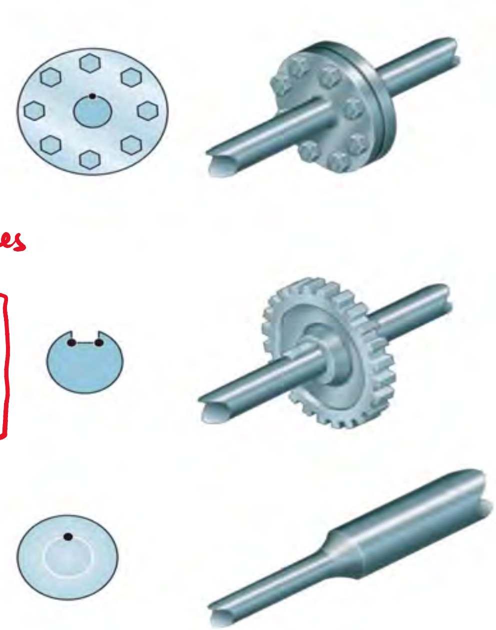


$$0 = \frac{T_A L_{AC}}{JG} + \frac{(-T_B) L_{BC}}{JG}$$

$\Rightarrow$ Find T_A and T_B

§ 5.8: Stress Concentration Factor (K):

* sudden change of x-section in all cases shown in the figure on the right $\Rightarrow$ High localized stresses



$\Rightarrow \tau_{max} = \dfrac{Tc}{J}$ [Can NOT be applied to regions with sudden change in x-section]

$$\tau_{max} = K\left(\frac{Tc}{J}\right)$$

For shoulder-fillet : Fig 5.30

- If fillet radius is increased $\Rightarrow K \downarrow \Rightarrow \tau_{max} \downarrow$
- If dia of larger shaft is reduced $\Rightarrow \dfrac{D}{d} \downarrow \Rightarrow K \downarrow \Rightarrow \tau_{max} \downarrow$

PROBLEMS.

Problem # 5. 31:

- solid shaft

$d = 25\,mm$

$P_c = 3\,kW$

$\omega = 50\ rps$

$\omega = 50(2\pi)\ rad/sec$

$P_A = 1\,kW$

$P_B = 2\,kW$

find: $(\tau_{max})_{AB} = ?$

$(\tau_{max})_{BC} = ?$

Governing Eqns:

- $\tau_{max} = \dfrac{Tc}{J}$
- $P = T\omega$

Assumptions:

- Hooke's Law applicable
- Homogeneous mat'l.

Solution:

$P_c = T_c\,\omega$

$3\times10^{3} = T_c\,[50(2\pi)]$

$T_c = 9.6\,N.m$

$P_A = T_A \omega$

$T_A = \frac{P_A}{\omega} = \frac{1\times10^3}{50(2\pi)}$

$T_A = 3.183 \text{ N.m}$

$T_B = \frac{P_B}{\omega} = \frac{2\times10^3}{100\pi}$

$T_B = 6.36 \text{ N.m}$

$P = T\omega$

$P_A = T_A \omega \;,\; P_B = T_B \omega$

$P_C = T_C \omega$

$\frac{P_A}{T_A} = \frac{P_B}{T_B} = \frac{P_C}{T_C}$

$T_A = \left(\frac{P_A}{P_C}\right) T_C$

$T_A = \left(\frac{1}{3}\right) 9.6$

$T_A = 3.18 \text{ N.m}$

$$(\tau_{AB})_{max} = \frac{T_{AC}}{J} = \frac{(3.18)(0.0125)}{\frac{\pi}{2}(0.0125)^4} = 1.04 \text{ MPa}$$

$$(\tau_{BC})_{max} = \frac{T_{BC}}{J} = 3.11 \text{ MPa}$$

P# 5.47:

- A-36 steel

$L = 60 \text{m}$

$d_o = 340 \text{mm}$

$d_i = 260 \text{mm}$

$P = 4.5 \text{MW}$

$\omega = 20 \text{ rad/s}$

find: $\tau_{max} = ?$

$\phi = ?$

Governing Eqns:

τ = T

Governing Eqns:

- $\tau_{max} = \frac{Tc}{J}$
- $P = T\omega$
- $\phi = \frac{TL}{JG}$

Assumptions:

FILL IN THE BLANKS

SOLUTION:

$$P = T\omega$$

$$4.5\times10^{6} = T\,(20)$$

$$T = 225 \text{ kN}\cdot\text{m}$$

$$\tau_{max} = \frac{Tc}{J}$$

$$= \frac{(225\times10^{3})(0.17)}{\frac{\pi}{2}\left[(0.17)^{4} - (0.13)^{4}\right]}$$

$$\tau_{max} = 44.3 \text{ MPa}$$

$$\phi = \frac{TL}{JG}$$

$$= \frac{(225\times10^{3})(60)}{J\,(75\times10^{9})}$$

$$\phi = 0.21 \text{ rad}$$

$$\phi = 11.9^{\circ}$$

P# 5.56:

Flywheel $\left\{ \begin{array}{l} s = 6\,mm \\ r = 75\,mm \end{array} \right.$

$d_i = 24\,mm$

$d_o = 32\,mm$

Find: $\tau_{max} = ?$

Governing Eqns:

- $\phi = \dfrac{TL}{JG}$
- $\tau_{max} = \dfrac{Tc}{J}$

Assumptions:

Solution:

$$s = r\theta$$

$$6 = (75)\,\phi$$

$$\phi = 0.08 \text{ rad}$$

$$\phi = \frac{TL}{JG}$$

$$0.08 = \frac{T\,(1.5)}{\frac{\pi}{2}\left[(0.016)^4 - (0.012)^4\right](75\times10^9)}$$

$T = \checkmark$

$$\tau_{max} = \frac{Tc}{J}$$

$$\tau_{max} = 64 \text{ MPa}$$

WILLIAM E. BOEING
DEPARTMENT OF AERONAUTICS & ASTRONAUTICS
UNIVERSITY of WASHINGTON

Muzammil Arshad, Ph.D. | Visiting Associate Professor
William E. Boeing Department of Aeronautics & Astronautics | University of Washington

CHAPTER # 6
BENDING

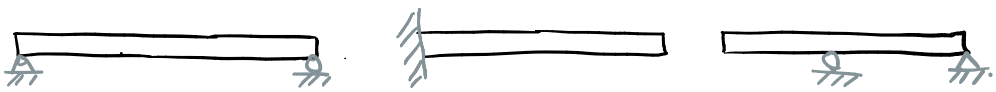
§6.1 : Shear force and moment diagrams :

Beam: slender member that supports loading ⊥ to the longitudinal axis.

* characteristic of beams:

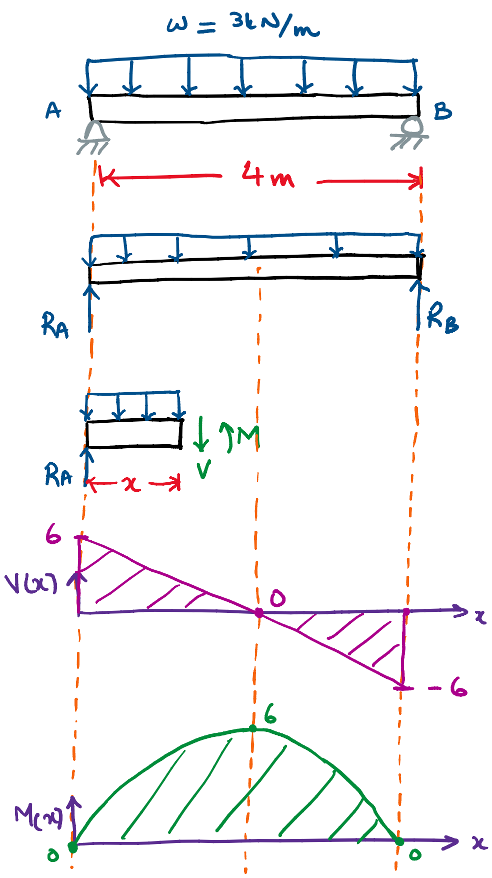
- constt : x-sectional area
- long , straight bars

Recall:



Simply supported beam — Cantilever beam — Overhanging beam

Example # 6.1 :



Step 1: Draw FBD

Step 2: Find unknowns.

$\Sigma F_y = 0;$

$R_A + R_B - 3(4) = 0$

$R_A + R_B = 12 \quad —(1)$

$\Sigma M_B = 0;$

$-R_A(4) + 3(4)\left(\frac{4}{2}\right) = 0$

$R_A = 6 \text{ kN}$

eq (1) $\Rightarrow$ $R_B = 6 \text{ kN}$

Step 3: Method of sections.

$\Sigma F_y = 0;$

$R_A - 3x - V = 0$

$V = 6 - 3x, \quad 0 \leq x \leq 4$

@ $x = 0;$ $V = 6 \text{ kN}$

@ $x = 2;$ $V = 0 \text{ kN}$

@ $x = 4;$ $V = -6 \text{ kN}$

$\Sigma M_x = 0;$

$-R_A(x) + 3x\left(\frac{x}{2}\right) + M = 0$

$M = 6x - \frac{3x^2}{2}$

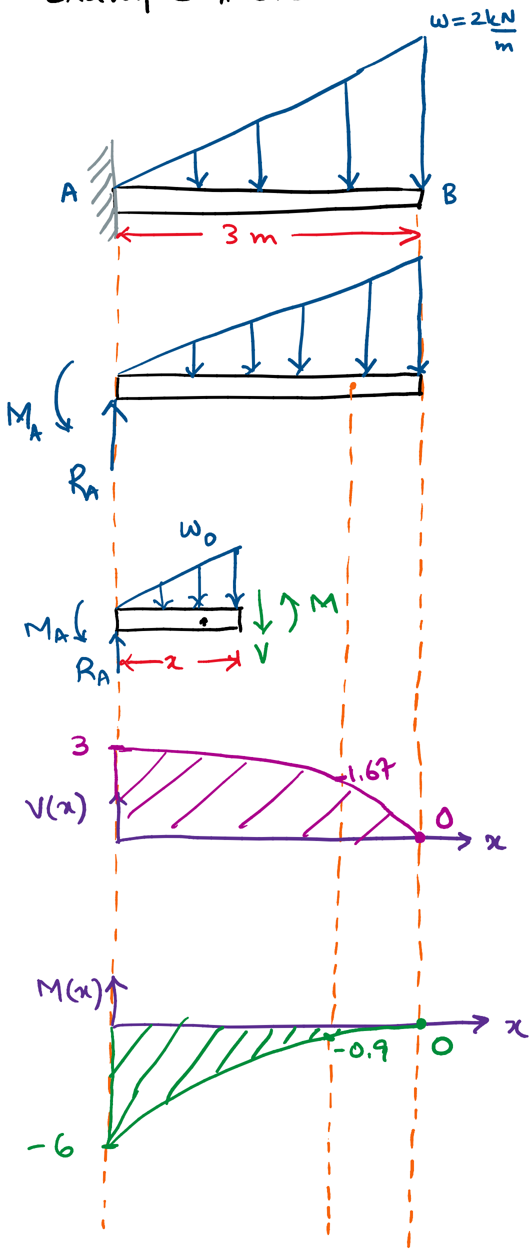
$$M = 6x - \frac{3x^2}{2}$$

$@\, x = 0;\quad M = 0$

$@\, x = 2;\quad M = 6\ \text{kN}\cdot\text{m}$

$@\, x = 4;\quad M = 0$

Example # 6.2:



$\Sigma F_y = 0;$

$$R_A - \frac{1}{2}(3)(2) = 0$$

$$R_A = 3\ \text{kN}$$

$\Sigma M_A = 0;$

$$M_A - \underbrace{\left[\frac{1}{2}(3)(2)\right]}_{\text{force}}\underbrace{\left[\frac{2}{3}(3)\right]}_{\text{moment arm}} = 0$$

$$M_A = 6\ \text{kN.m}$$

Method of sections:

$0 \leq x \leq 3;$

By similar Δs:

$$\frac{w_0}{x} = \frac{w}{L} \Rightarrow w_0 = \frac{2}{3}x$$

$\Sigma F_y = 0;$

$$R_A - V - \left[\frac{1}{2}w_0 x\right] = 0$$

$$3 - V - \left[\frac{x^2}{3}\right] = 0$$

$$V = 3 - \frac{x^2}{3}$$

$@\, x = 0;\quad V = 3\ \text{kN}$

$@\, x = 2;\quad V = \frac{5}{3} = 1.67\ \text{kN}$

$@\, x = 3;\quad V = 0\ \text{kN}$

$\Sigma M_x = 0;$

$$M_A - R_A x + \underbrace{\left[\frac{1}{2}w_0 x\right]}_{\text{force}}\underbrace{\left[\frac{1}{3}x\right]}_{\text{moment arm}} + M = 0$$

$$6 - 3x + \frac{x^3}{9} + M = 0$$

$$M = 3x - \frac{x^3}{9} - 6$$

$$M = 3x - \frac{x^3}{9} - 6$$

$@\,x=0;\quad M = -6\ \text{kN.m}$

$@\,x=2;\quad M = \cancel{6} - \frac{8}{9} - \cancel{6}$

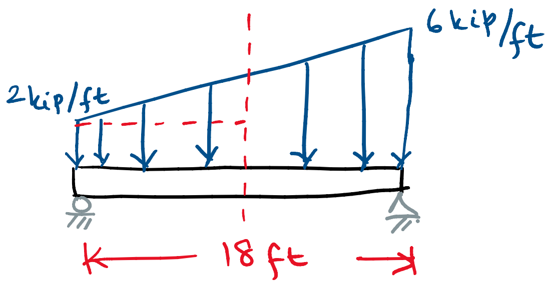
$M = -0.9\ \text{kN.m}$

$@\,x=3;\quad M = 9 - \frac{27}{9} - 6$

$= 9 - 3 - 6$

$M = 0$

Example #6.3:

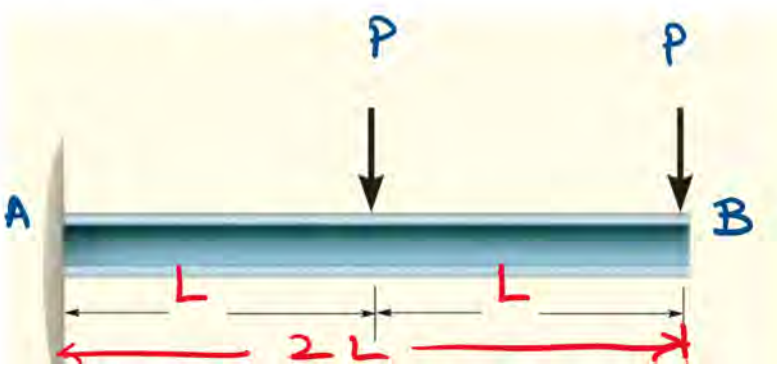


Do it yourself:

§6.2: Graphical Method for constructing shear & moment diagrams:

* simpler method
* Determine V & M as $f(x)$
* $$\underbrace{w}_{\text{dist: load intensity}} = \underbrace{\frac{dV}{dx}}_{\text{slope of shear diagram}}$$
* $$\underbrace{V}_{\text{shear @ each pt.}} = \underbrace{\frac{dM}{dx}}_{\text{slope of moment diagram @ each pt.}}$$

Ex # 6.5:

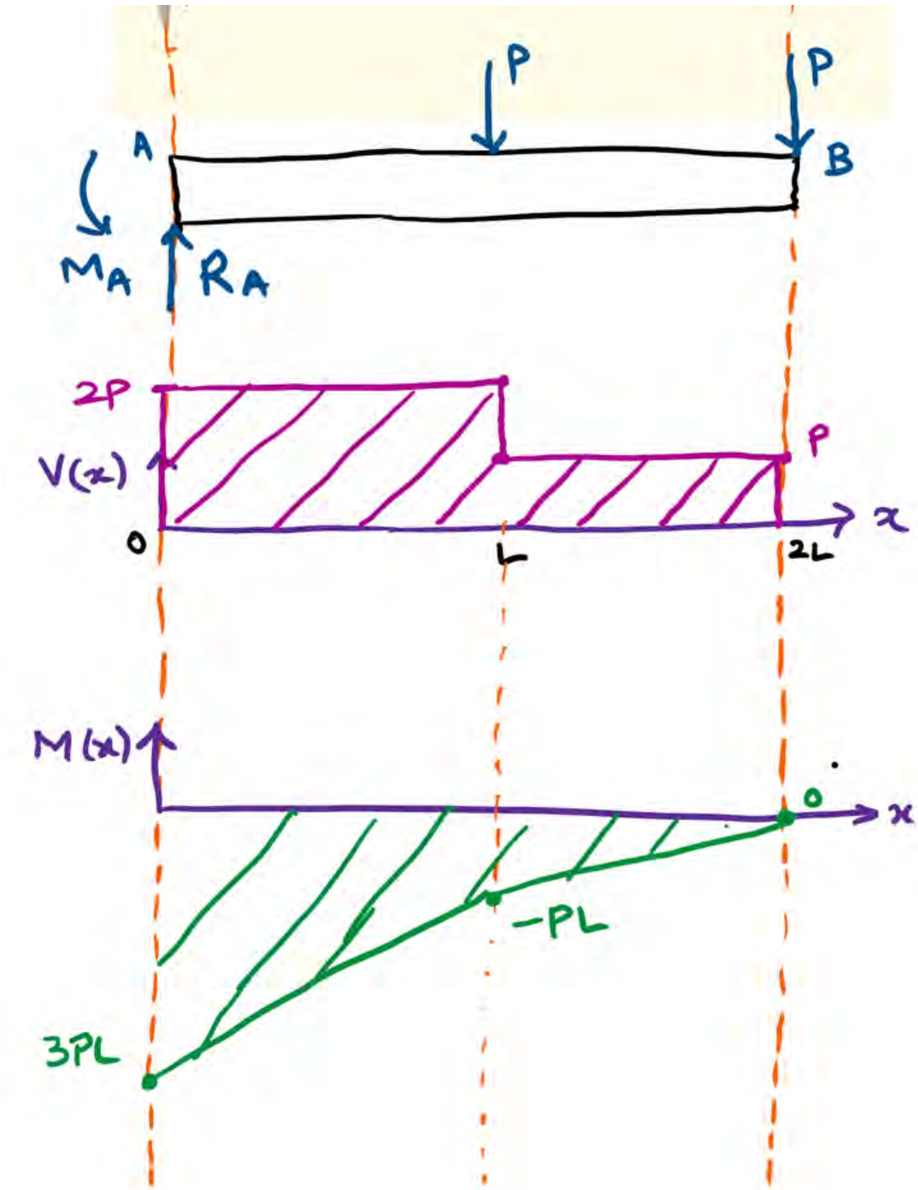


$\Sigma F_y = 0;$

$R_A - 2P = 0$

$R_A = 2P$

$\Sigma M = 0;$



$_{B} = 0,$

$$-R_A(2L) + M_A + P(L) = 0$$

$$M_A = -3PL$$

No distributed load $\Rightarrow$ slope of shear is ZERO

$$w = \frac{dV}{dx} = 0$$

$$V = \frac{dM}{dx}$$

$$M\Big|_{x=L} = M\Big|_{x=0} + \Delta M$$

$$= -3PL + \underbrace{2PL}_{\text{area of } \square \text{ of shear diagram } (0 \to L)}$$

$$M_{x=L} = -PL$$

$$M\Big|_{x=2L} = M\Big|_{x=L} + \Delta M$$

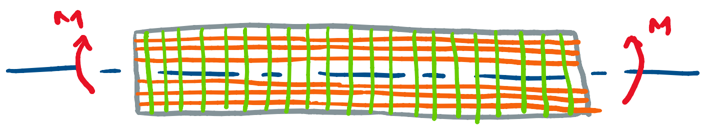
$$= -\cancel{PL} + \cancel{PL}$$

$$M_{x=2L} = 0$$

Lecture 17

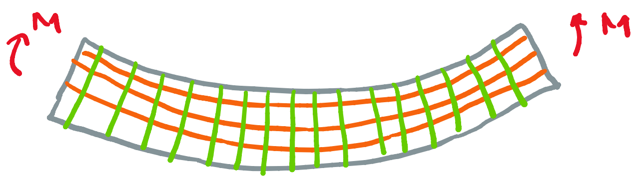
Recap: Chap 6
- Shear force diagrams
- moment diagrams

§6.3: Bending Deformation of Straight members:



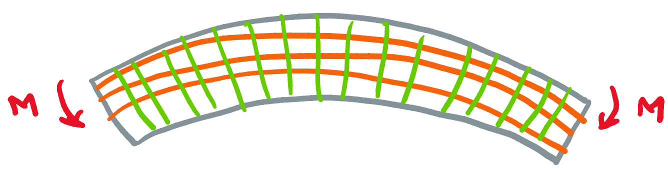
Before Deformation

Positive moment



After Deformation

Negative moment



- longitudnal lines ⇒ curve
- vertical transverse lines ⇒ remain straight but rotate
- All x-sections of the beam remain plane and ⊥ to the longitudnal axis @ every pt.
- Neutral Surface (N.S.):
  longitudnal fibres of mat'l: do NOT undergo change in length
- NEGLECT: Any deformation of x-section within its own plane.

Important Notes:

Positive moment:
- contraction $(-\epsilon)$: will occur above the Neutral Axis (N.A.) $(+y)$
- elongation $(+\epsilon)$: will occur below N.A $(-y)$

Look for figures from §6.3 in the book to understand N.S. and N.A.

$$\boxed{\epsilon = -\left(\frac{y}{c}\right)\epsilon_{max}} \quad (1)$$

⇒ normal stress in the longitudnal or x-direction but other components

$\Rightarrow$ normal stress in the longitudnal or x- direction but other components are zero $[\epsilon_y, \epsilon_z = 0]$

$\Rightarrow \sigma_y = \sigma_z = 0 \quad , \quad \tau = 0$ (in x-dir)

$\Rightarrow$ UNIAXIAL STRESS

$\Rightarrow \sigma_x$ causes ϵ_x

NEGLECT: Poisson's ratio

$$\nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}$$

§6.4: FLEXURE FORMULA:

- σ related to internal resultant bending moment, M.
- Assume: Hooke's Law $\Rightarrow \sigma \propto \epsilon$

$$\Rightarrow \sigma = E\epsilon$$

eq ① $\Rightarrow \dfrac{\sigma}{\cancel{E}} = -\left(\dfrac{y}{c}\right)\dfrac{\sigma_{max}}{\cancel{E}}$

$0 \leq \sigma \leq \sigma_{max}$

$\Downarrow$

@ distance 'c'

$$\boxed{\sigma = -\left(\frac{y}{c}\right)\sigma_{max}} \quad — ②$$

How to locate N.A.?

$$F_R = \Sigma F_x = 0$$

$$0 = \int_A dF = \int_A \sigma\, dA \qquad \left[\begin{array}{l}\text{side Note:}\\ \sigma = F/A\end{array}\right]$$

$$= \int_A \left[-\left(\frac{y}{c}\right)\sigma_{max}\right] dA$$

$$0 = -\frac{\sigma_{max}}{c}\int_A y\, dA$$

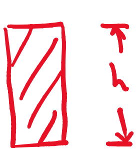
$$\because \frac{\sigma_{max}}{c} \neq 0 \quad \Rightarrow \quad \underbrace{\int_A y\, dA}_{\text{1st moment of x-section about N.A.}} = 0$$

$\Rightarrow$ Determine centroid $\Rightarrow$ N.A. is determined

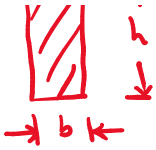
$$dM = y\, dF$$

$$(M_R)_z = \Sigma M_z = \int_A y\, dF$$

$$= \int_A y\,(\sigma\, dA)$$



$$= \int_A y\left(\frac{y}{c}\sigma_{max}\right) dA.$$



$I = \dfrac{bh^3}{12}$

$$= \int_A y \left(\frac{y}{c}\, \sigma_{max} \right) dA$$

$$= \frac{\sigma_{max}}{c} \underbrace{\int y^2 \, dA}_{\text{moment of inertia} = I}$$

$$M = \frac{\sigma_{max}}{c}\, I$$

Flexural stress

$$\boxed{\sigma_{max} = \frac{Mc}{I}} \quad \text{—— (3)}$$

FLEXURE FORMULA

σ_{max} = max. normal stress in the member (occuring @ farthest pt. from N.A.)

c = $\perp$ distance from N.A. to the farthest pt. where σ_{max} acts

M = resultant int. moment about N.A.

["method of sections" in moment diagram]

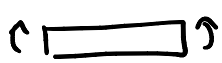
I = moment of inertia of x-section

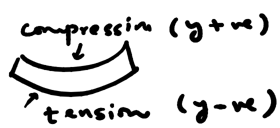
eq (2) and eq (3): $\dfrac{\sigma_{max}}{c} = -\dfrac{\sigma}{y} = \dfrac{M}{I}$

$$\boxed{\sigma = -\frac{My}{I}} \quad \text{—— (4)} \quad \text{Flexure formula}$$

$0 \leq \sigma \leq \sigma_{max}$

- If 'M' is +ve, y +ve $\Rightarrow \sigma = -$ve





Example 6.11:

Given :

$\sigma_{max} = 2$ ksi (from figure)

$c = 6''$

find: $M = ?$

(a) Flexure formula

(b) basic principles

Assumptions:

- Hooke's Law
- Homogeneous mat'l :

Gov: Eqns:

- $\sigma_{max} = \dfrac{Mc}{I}$
- $M = Fy$

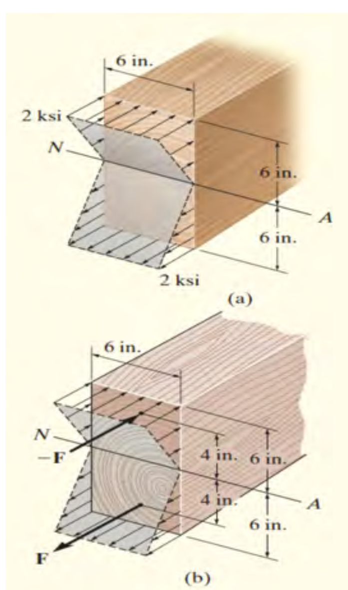


Fig. 6–25

- $\sigma_{max} = \frac{Mc}{I}$
- $M = Fy$

(b)

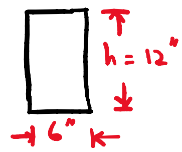
Fig. 6–25

Solution:

(a) $\sigma_{max} = \frac{Mc}{I}$

$$2 = \frac{M(6)}{\left[\frac{1}{12}(6)(12)^3\right]}$$

$$\boxed{M = 288 \text{ kip.in}}$$ Answer



(b) $F = \frac{1}{2} bh$

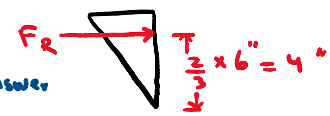
$= \frac{1}{2}(6)(6)$

$= 18 \text{ kip} \times 2 = 36 \text{ kip}$

$M = Fy$

$= (36)(4+4)$

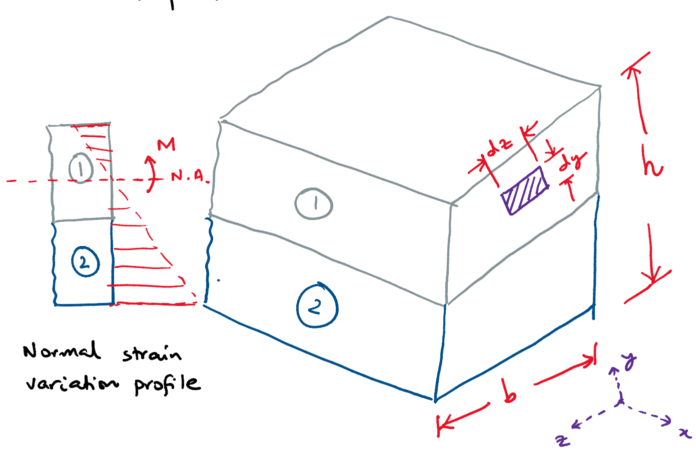
$$\boxed{M = 288 \text{ kip.in}}$$ Answer



§6.6. Composite Beams:

- Beams that are constructed of two or more matls:
  ⇒ composite beams
- Flexure formula only applicable to homogeneous matls:
  (NOT applicable to a composite beam)
- METHOD OF TRANSFORMATION:
  Transform beam into "ONE" matl.

Example:



Matl ①: stiff matl:

Matl ②: Less stiff matl:

$\Rightarrow E_1 > E_2$

Assume: $\sigma \propto \epsilon$

Matl ①: $\sigma = E_1 \epsilon$

Matl ②: $\sigma = E_2 \epsilon$

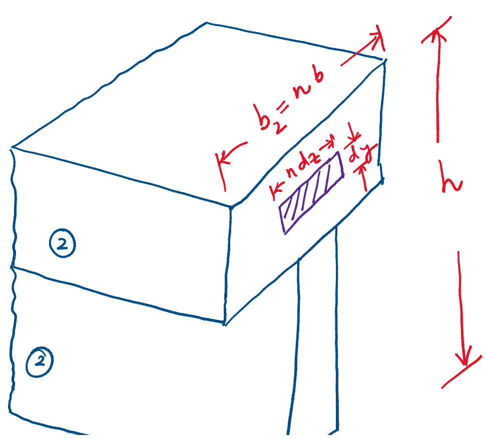
CASE 1:

If all of the beam is transformed to the less stiff matl

⇒ matl ②

Conditions:

- same height (to keep the strain distribution same)
- width would be widened to carry the load equivalent to the stiffer matl ①



* How much is the necessary width?

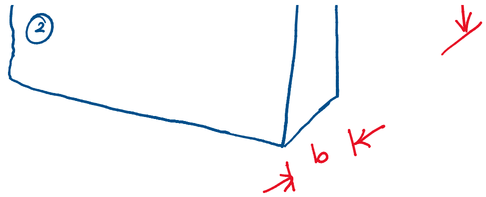
Originally: (differential element)

$dF = \sigma\, dA$

$dF = (E_1 \epsilon)(dy\, dz)$

New matl:

$dF' = \sigma' dA'$



New mat'l:

$$dF' = \delta' dA'$$

$$dF' = (E_2 \epsilon)(n\,dy\,dz)$$

$$dF = dF'$$

$$E_1 \cancel{\epsilon}\, \cancel{dy\,dz} = E_2 \cancel{\epsilon}\, n\, \cancel{dy\,dz}$$

$$\boxed{n = \frac{E_1}{E_2}}$$

where: n = transformation factor

Side Note:

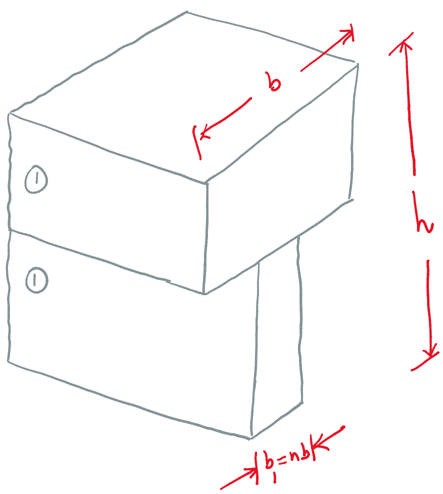
$n_1 = E_1/E_2$

$n_2 = E_2/E_2 = 1$

$\because E_1 > E_2 \Rightarrow n > 1$

CASE 2:

If less stiff mat'l 2 is transformed to mat'l 1



$$n = \frac{E_2}{E_1}$$

$$b_1 = nb$$

$$n < 1 \quad [\because E_2 < E_1]$$

* After transformation, determine the centroid ⇒ N.A.
* moment of inertia (of the transformed section)

$$dF = dF'$$

$$\delta(\cancel{dy\,dz}) = \delta'(n\,\cancel{dy\,dz})$$

$$\boxed{\delta = n\,\delta'}$$

Ex # 6.17:

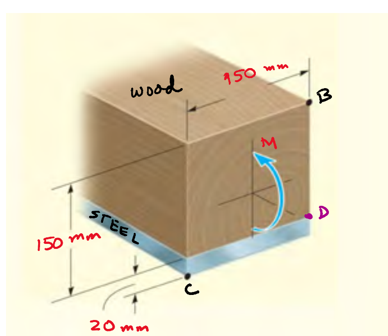
Given: $M = 2$ kN.m

$E_w = 12$ GPa

$E_{st} = 200$ GPa

Find:

$\delta_B = ?$



$\sigma_c = ?$

Gov: Eqns:

- $\sigma_{max} = \dfrac{Mc}{I}$

- $n = \dfrac{E_1}{E_2}$ or $n = \dfrac{E_2}{E_1}$

Assumptions:

- Hooke's Law
- Homogeneous matl: (after transformation)

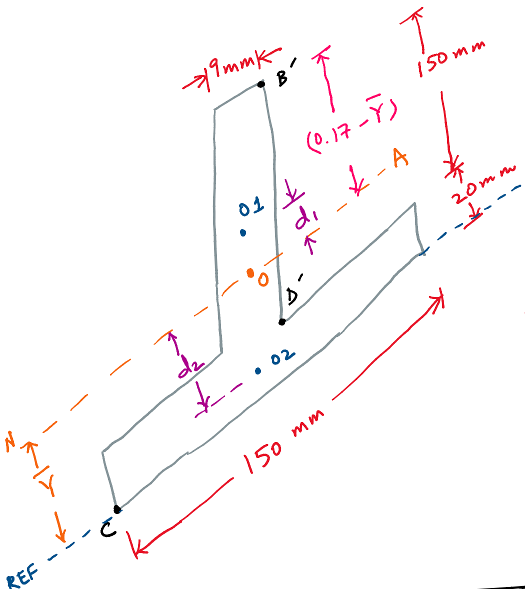
Solution:

Step 1: Transform beam to high stiff matl: ⇒ steel

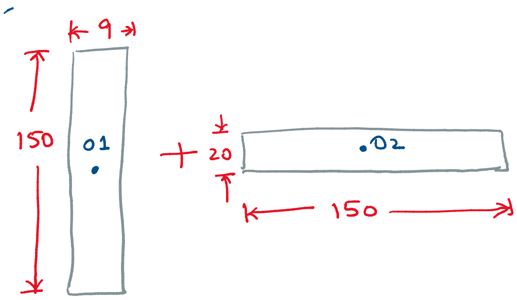
$$n = \frac{E_w}{E_{st}} = \frac{12}{200} = 0.06$$

New width: $b'_{st} = n b_w$

$= (0.06)(150) = 9mm$



Step 2: find centroid ⇒ N.A.



	1	2	Σ
AREA	(150)(9)	(150)(20)	ΣA = (150)(9) + (150)(20)
$\bar{y}$	20+75=95	10	

$$\bar{Y}\,\Sigma A = \Sigma \bar{y} A$$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$\bar{Y} = \frac{(0.095)(0.15)(0.009) + (0.01)(0.15)(0.02)}{(0.15)(0.009) + (0.15)(0.02)}$$

$$\bar{Y} = 0.03638\ m = 36.38\ mm$$

Step 3: find $I_{N.A}$:

$$I_{N.A} = I_1 + I_2$$

$$I_1 = \frac{(0.009)(0.15)^3}{12} + [(0.009)(0.15)\ d_1^2]$$

$$I_1 = \frac{(0.009)(0.15)^3}{12} + [(0.009)(0.15)(0.095 - \bar{Y})^2]$$

$$I_2 = \frac{(0.15)(0.02)^3}{12} + [(0.15)(0.02)\ d_2^2]$$

$$I_2 = \frac{(0.15)(0.02)^3}{12} + [(0.15)(0.02)(\bar{Y} - 0.01)^2]$$

$$I_{N.A} = 9.358 \times 10^{-6}\ m^4$$

Parallel Axis Theorem:

$$I = I_{xx} + Ad^2$$

$$I = \frac{bh^3}{12} + Ad^2$$

Step 4: find stresses:

$$\sigma_B' = \frac{Mc}{I} = \frac{(2\times10^3)(0.17 - \bar{Y})}{(9.358\times10^{-6})}$$

$$\sigma_B' = 28.6\ MPa$$

$$\sigma_B = n\,\sigma_B'$$

$$= (0.06)(28.6)$$

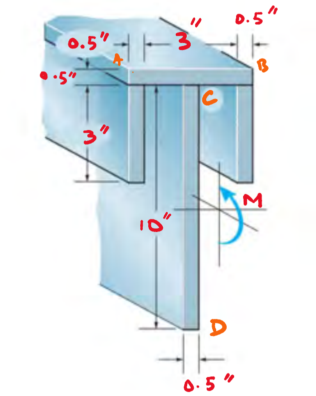
$$\boxed{\sigma_B = 1.71\ MPa}$$ Answer

$$\sigma_c = \frac{Mc}{I}$$

$$= \frac{(2\times10^3)(\bar{Y})}{9.358\times10^{-6}}$$

$$\boxed{\sigma_c = 7.78\ MPa}$$ Answer

P# 6.48:



Given:

$$\sigma_{max} = 10 \text{ ksi}$$

Find:

$$M = ?$$

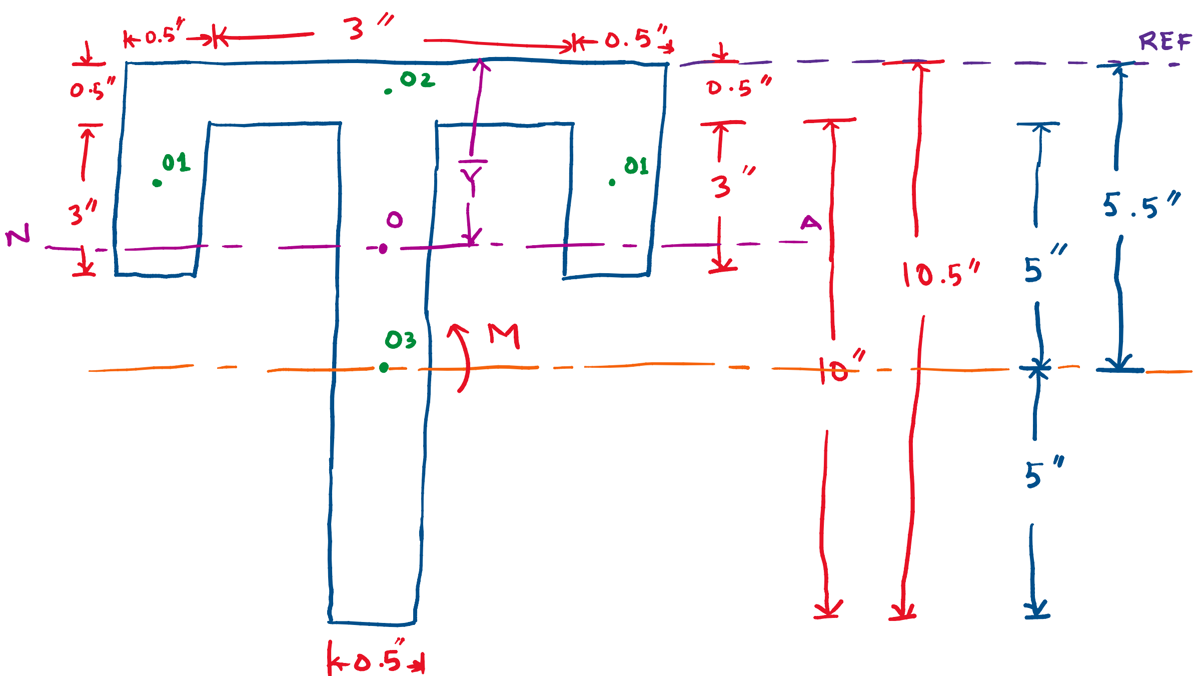
Assumptions:

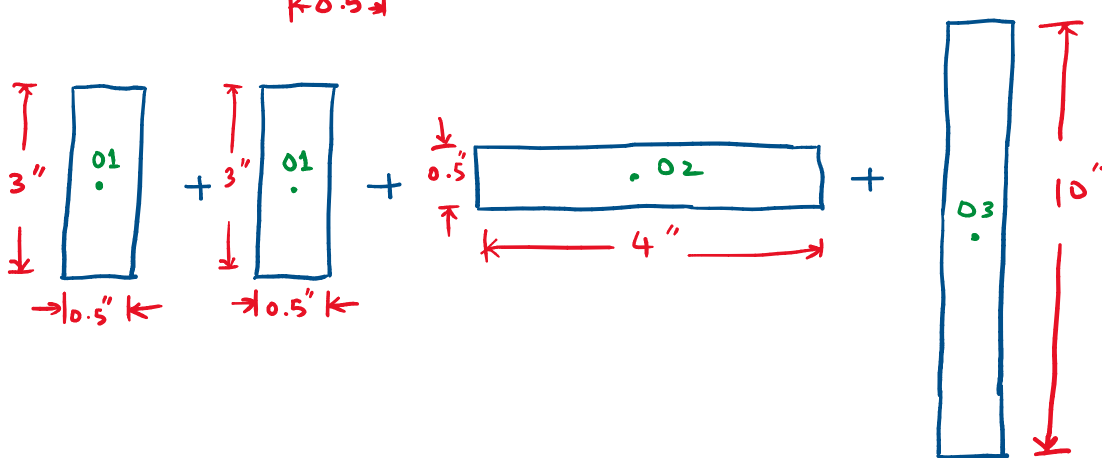
- Hooke's Law
- Homogeneous mat'l.

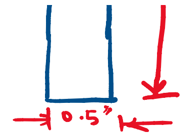
Gov. Eqns:

$$\sigma_{max} = \frac{Mc}{I}$$

Solution:







AREA	2(3)(0.5)	4(0.5)	(10)(0.5)	$\Sigma A = 2(3)(0.5) + 4(0.5) + 10(0.5)$
$\bar{y}$	2"	0.25"	5.5"	

Determine centroid: $$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$\bar{Y} = \frac{2[(2)(3)(0.5)] + 4(0.5)(0.25) + (10)(0.5)(5.5)}{2(3)(0.5) + 4(0.5) + 10(0.5)}$$

$$\bar{Y} = 3.4''$$

Determine $I_{N.A}$: $$I_{N.A.} = 2I_1 + I_2 + I_3$$

$$2I_1 = 2\left[\frac{1}{12}(0.5)(3)^3 + (0.5)(3)(\bar{Y}-2)^2\right]$$

$$I_2 = \frac{1}{12}(4)(0.5)^3 + (4)(0.5)(\bar{Y}-0.25)^2$$

$$I_3 = \frac{1}{12}(0.5)(10)^3 + (10)(0.5)(5.5-\bar{Y})^2$$

$$I_{N.A} = 91.73 \text{ in}^4$$

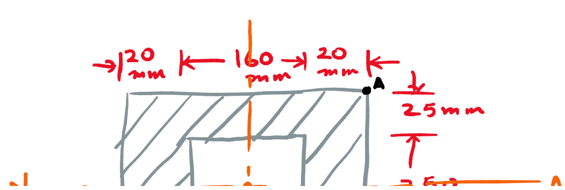
$$\sigma_{max} = \frac{Mc}{I}$$

$$10 = \frac{M(10.5-\bar{Y})}{91.73}$$

$$\boxed{M = 129.2 \text{ kip.in}}$$ Answer.

Problem:

Given: $M = 10$ kN.m

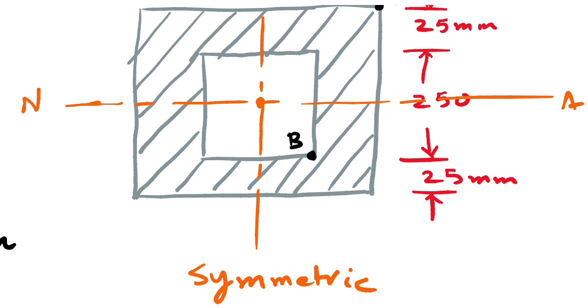


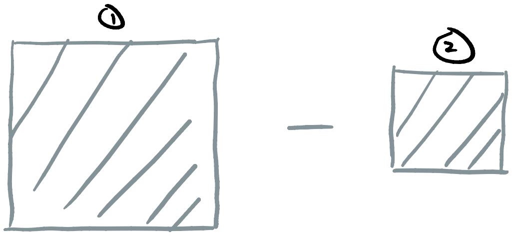
Given: $M = 10\ kN.m$

Find: $\sigma_A = ?$

$\sigma_B = ?$

Comment and sketch the scenario





$$I = \frac{bh^3}{12}$$

$$I_{N.A.} = I_1 - I_2$$

$$= \frac{1}{12}(0.2)(0.3)^3 - \frac{1}{12}(0.16)(0.25)^3$$

$$I_{N.A.} = 0.2417 \times 10^{-3}\ m^4$$

$$\sigma_A = -\frac{My_A}{I} = \frac{-(10\times10^3)(0.15)}{0.2417\times10^{-3}}$$

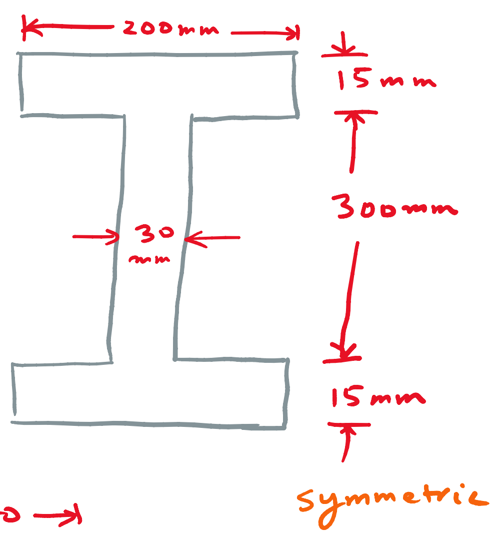
$$\boxed{\sigma_A = -6.21\ MPa}$$ Compression @ top

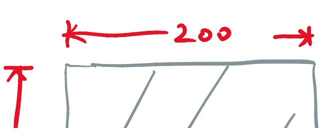
$$\sigma_B = -\frac{My_B}{I} = -\frac{(10\times10^3)(-0.125)}{0.2417\times10^{-3}}$$

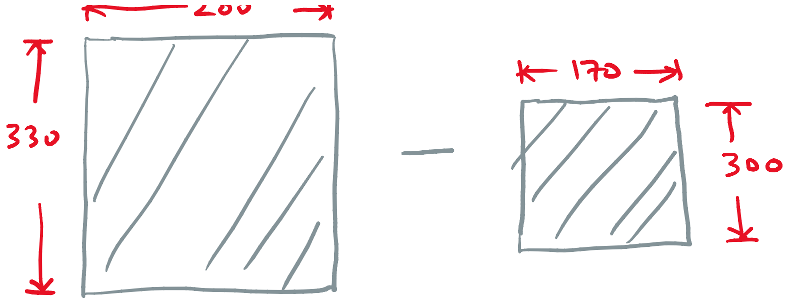
$$\boxed{\sigma_B = +5.71\ MPa}$$ Tension @ bottom

Problem:

I-beam



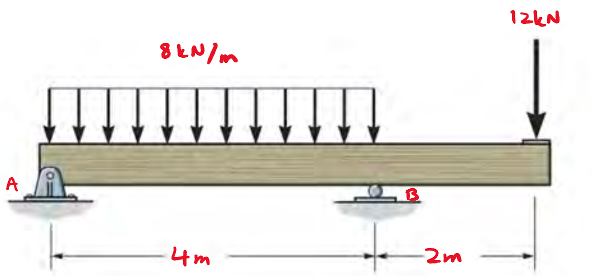


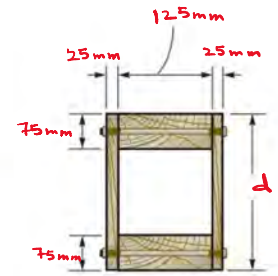


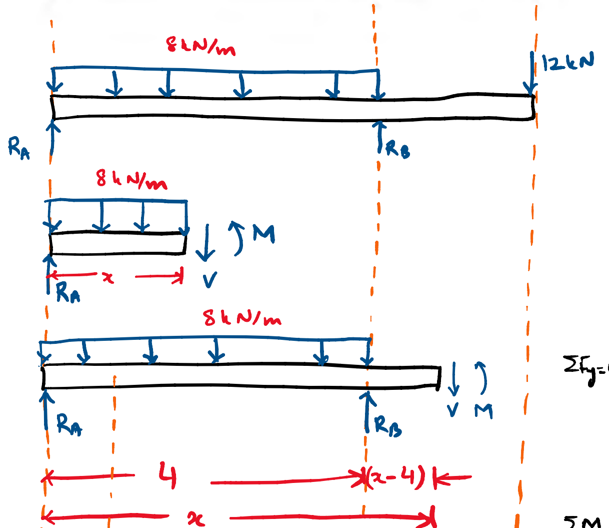
symmetric

Chapter 6 Problems

P # 6.101:







$\Sigma F_y = 0;$

$R_A + R_B - 8(4) - 12 = 0$

$R_A + R_B = 44$

$\Sigma M_A = 0;$

$R_B(4) - 12(6) - 8(4)(2) = 0$

$R_B = 34\,kN, \; R_A = 10\,kN$

$0 \leq x \leq 4$:

$\Sigma F_y = 0;\; R_A - 8x - V = 0$

$V = 10 - 8x;$

@ $x = 0;\; V = 10$

@ $x = 1.25;\; V = 0$

@ $x = 4;\; V = -22$

$\Sigma M_x = 0;\; M - R_A x + 8x\left(\frac{x}{2}\right) = 0$

$M = 10x - 4x^2$

@ $x=0;\; M = 0$; @ $x = 1.25;\; M = 6.25$

@ $x = 4;\; M = -24$

$4 \leq x \leq 6$:

$\Sigma F_y = 0;\; R_A + R_B - 8(4) - V = 0$

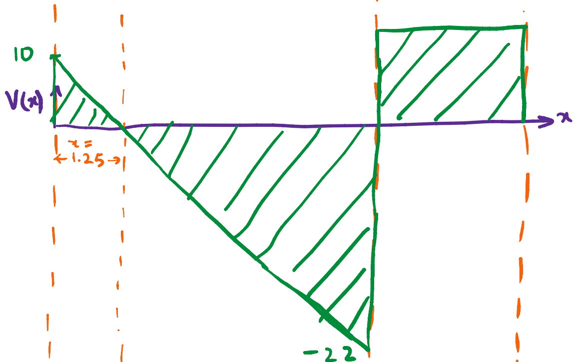
$V = 44 - 32 = 12$

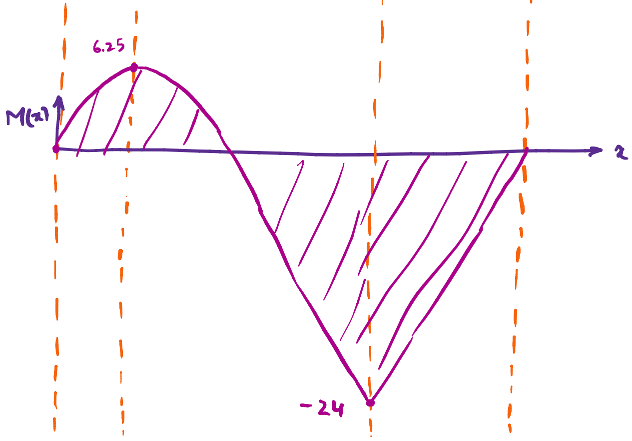
$\Sigma M_x = 0;\; -R_A x - R_B(x-4) + 8(4)(x-2) + M = 0$

$M = 10x + 34(x-4) - 32(x-2)$

@ $x = 4;\; M = -24$

@ $x = 6;\; M = 0$





$M_{max} = 24 \text{ kN.m}$

$I_{N.A.} = I_1 - I_2$

$I_{N.A.} = \frac{1}{12}(0.175)(d^3) - \frac{1}{12}(0.125)(d - 0.15)^3$

$\sigma_{max} = \frac{Mc}{I}$ where $c = \frac{d}{2}$

$$6 \times 10^6 = \frac{(24 \times 10^3)(d/2)}{\left[\frac{1}{12}(0.175)d^3 - \frac{1}{12}(0.125)(d - 0.15)^3\right]}$$

Instead for solving using Algebra: Use:

LHS = RHS

* change values of 'd' until LHS = RHS

$d = 0.409 \text{ m} = 409.4 \text{ mm}$

This is the min: dimension 'd' to bear σ_{max}. To round off the answer to nearest tenth : $\boxed{d = 410 \text{ mm}}$ Ans:

Problem:

Given: $M = 10\ kN.m$

Find:

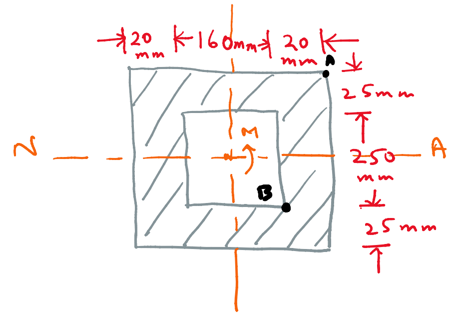
$\sigma_A = ?$

$\sigma_B = ?$

Assumptions:

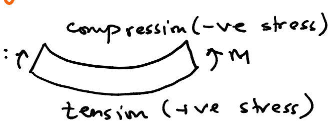
⋮

Gov: Eqns:

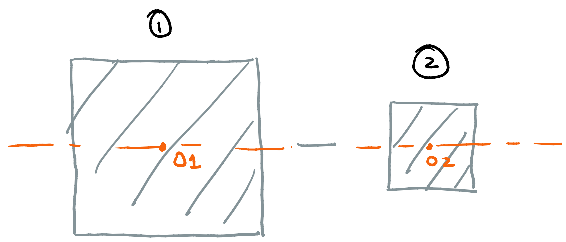


Symmetric

Recall:



Solution:



No need for ∥-axis theorem in this case

$I = \frac{bh^3}{12}$

$$I_{N.A.} = I_1 - I_2$$

$$= \frac{1}{12}(0.2)(0.3)^3 - \frac{1}{12}(0.16)(0.25)^3$$

$$I_{N.A} = 0.2417 \times 10^{-3}\ m^4$$

$$\sigma_A = -\frac{My_A}{I} = -\frac{(10\times10^3)(0.15)}{0.2417\times10^{-3}}$$

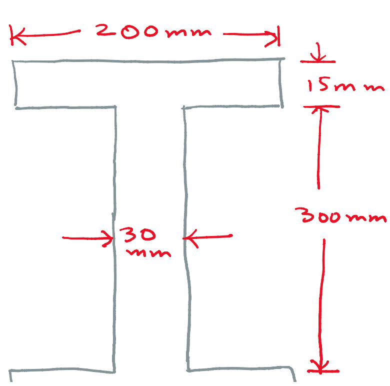
$$\boxed{\sigma_A = -6.21\ MPa}$$ Answer

$$\sigma_B = -\frac{My_B}{I} = -\frac{(10\times10^3)(-0.125)}{0.2417\times10^{-3}}$$

$$\boxed{\sigma_B = +5.71\ MPa}$$ Answer

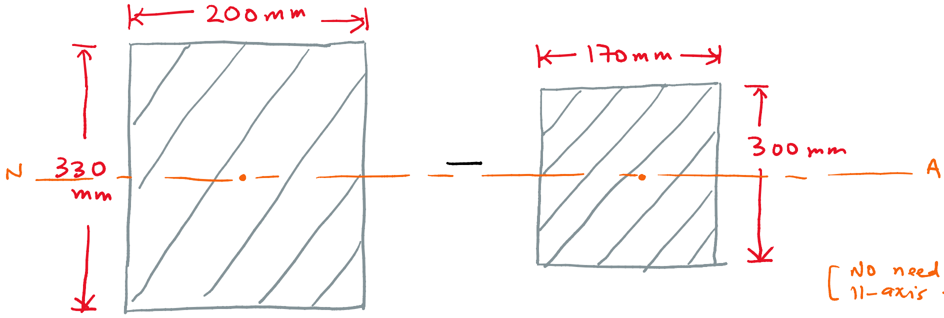
Problem:

I-beam



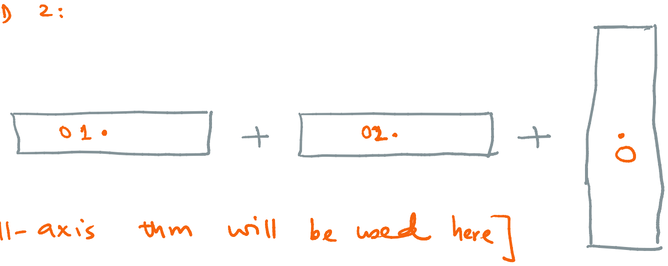
symmetric

METHOD 1:



[No need for ll-axis thm]

METHOD 2:



[ll-axis thm will be used here]

HW Problem: 6-101 (Due Wednesday)

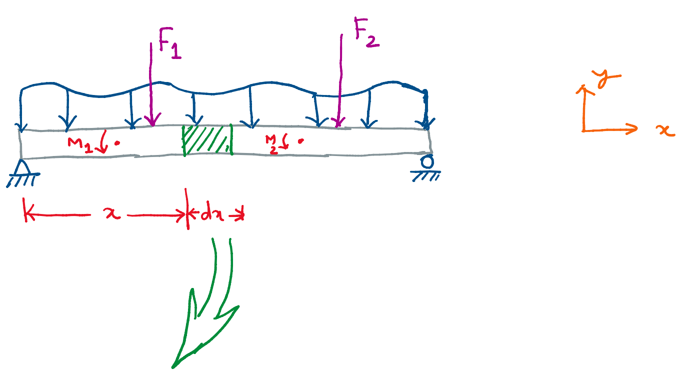
CHAPTER # 7:
TRANSVERSE SHEAR

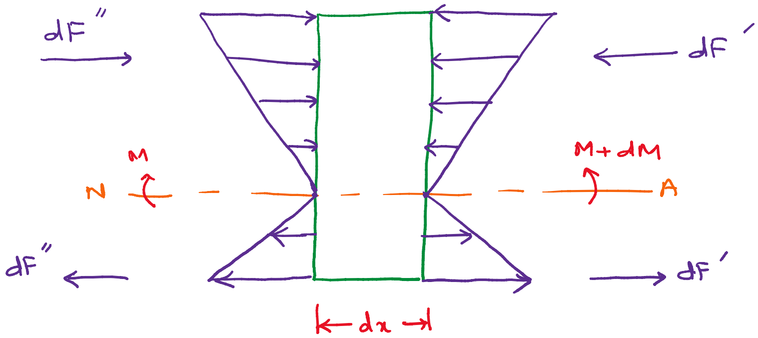
Recall: $\tau = \dfrac{V}{A}$ (fasteners)

$\downarrow V \; \curvearrowleft M \Rightarrow \sigma$ (flexure stress)

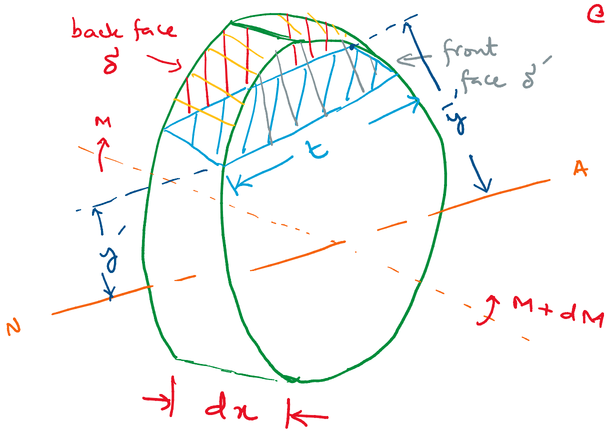
$\Downarrow \tau$ (transverse shear)

§ 7.2: THE SHEAR FORMULA:





$$\Sigma F_x = 0 \text{ (satisfy)}$$



Note: Forces in tension @ bottom & compression @ Top due to + M

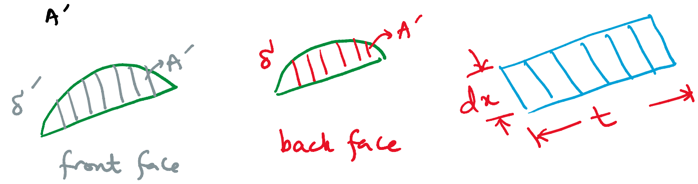
A' = area of front face as well as back face

σ = stress @ front face

σ' = stress @ back face

$$\Sigma F_x = 0$$

$$\int_{A'} \sigma' dA' - \int \sigma dA' - \tau (t\,dx) = 0$$



$$\int_{A'} \left(\frac{M+dM}{I}\right) y\,dA' - \int_{A'} \left(\frac{M}{I}\right) y\,dA' - \tau (t\,dx) = 0$$

$$\frac{dM}{I} \int_{A'} y\,dA' = \tau (t\,dx)$$

$$\tau = \frac{1}{It} \frac{dM}{dx} \int_{A'} y\,dA'$$

Recall: $V = \frac{dM}{dx}$

$$\tau = \frac{V}{It} \int_{A'} y\,dA'$$

Recall: $\bar{Y} = \Sigma y A$ $\bar{y}' = \int y\,dA'$

$It \quad A'$

Recall: $\bar{Y} = \frac{\Sigma y A}{\Sigma A}$, $\bar{y}' = \frac{\int y \, dA'}{A'}$

$$\bar{y}' A' = \int y \, dA' = Q$$

$$\boxed{\tau = \frac{VQ}{It}}$$

where: τ = shear stress in member @ distance y' from N.A.

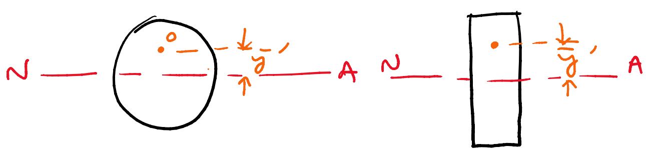
V = internal resultant shear force (method of sections)

I = moment of inertia of entire x-section

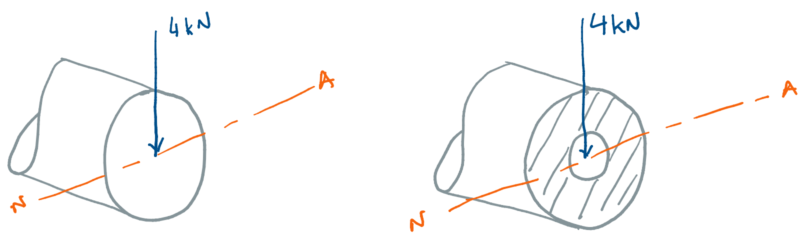
t = width of member's x-section

$Q = \bar{y}' A'$

$\bar{y}'$ = distance from N.A. to centroid



Example:



SOLID SHAFT HOLLOW SHAFT

Given: $c = 50\,mm$ $c_o = 50\,mm$, $c_i = 20\,mm$

$V = 4\,kN$

Find: $\tau = ?$

Assumptions:

- Hooke's Law
- Homogeneous mat'l:
- Symmetric member

Gov: Eqns:

- $\tau = \frac{VQ}{It}$
- $I = \frac{\pi r^4}{4}$

Solution:

$$\cdot \quad I = \frac{\pi r^4}{4}$$

Solution:

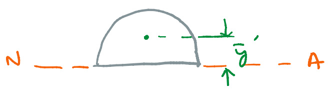
SOLID SHAFT

$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4}(0.05)^4$$

$$I = 4.9 \times 10^{-6}\ m^4$$

$$t = d = 0.1\,m$$

$$Q = \bar{y}' A'$$



$$\bar{y}' = \frac{4r}{3\pi}$$

$$Q = \left(\frac{4c}{3\pi}\right)\left(\frac{\pi c^2}{2}\right)$$

$$= \left(\frac{4(0.05)}{3\pi}\right)\left(\frac{\pi(0.05)^2}{2}\right)$$

$$Q = 83.33 \times 10^{-6}\ m^3$$

$$\tau = \frac{VQ}{It}$$

$$= \frac{(4 \times 10^3)(83.33 \times 10^{-6})}{(4.9 \times 10^{-6})(0.1)}$$

$$\boxed{\tau = 679\ kPa}$$

Answer

HOLLOW SHAFT

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = \frac{\pi}{4}[(0.05)^4 - (0.02)^4]$$

$$I = 4.78 \times 10^{-6}\ m^4$$

$$t = d_o - d_i = 0.06\,m$$

$$Q = \bar{y}' A'$$

$$Q = \left(\frac{4c_o}{3\pi}\right)\left(\frac{\pi c_o^2}{2}\right) - \left(\frac{4c_i}{3\pi}\right)\left(\frac{\pi c_i^2}{2}\right)$$

$$Q = 78 \times 10^{-6}\ m^3$$

$$\tau = \frac{VQ}{It}$$

$$= \frac{(4 \times 10^3)(78 \times 10^{-6})}{(4.78 \times 10^{-6})(0.06)}$$

$$\boxed{\tau = 1.09\ MPa}$$

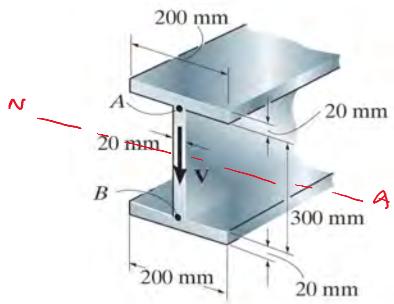
Answer

P#7.1: Given: I-Beam

- Horizontal ⇒ flanges
- vertical ⇒ web

$V = 20\,kN$

find: $\tau_{web,A} = ?$



Probs. 7–1/2/3

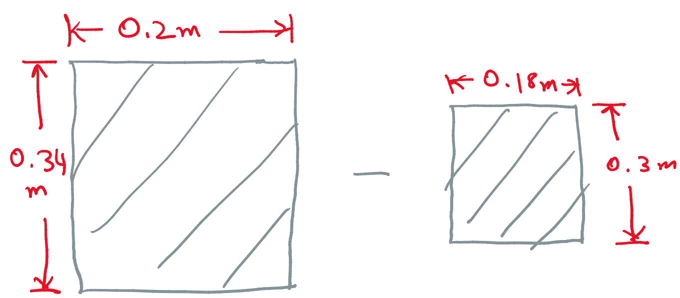
Assumptions:

- Hooke's Law
- Homogeneous mat'l.
- symmetric member

Gov: Eqns:

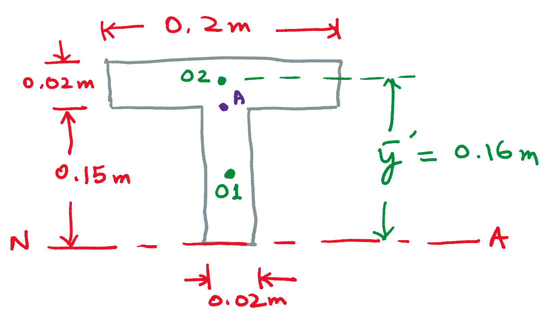
$$\tau = \frac{VQ}{It}$$

Solution:



$$I_{N.A} = \frac{1}{12}(0.2)(0.34)^3 - \frac{1}{12}(0.18)(0.3)^3$$

$$I_{N.A} = 0.25\times10^{-3}\ m^4$$



$$Q_A = \bar{y}_A' A'$$

$$= (0.16)[0.2\times0.02]$$

$$Q_A = 0.64\times10^{-3}\ m^3$$

$$\tau_A = \frac{VQ_A}{It}$$

$$= \frac{(20\times10^3)(0.64\times10^{-3})}{(0.25\times10^{-3})(0.02)}$$

$$\boxed{\tau_A = 2.56\ MPa}$$ Answer

P#7.2: same figure as P#7.1 and data

find: $\tau_{max} = ?$

P#7.2: same figure as P#7.1 and data

Find: $\tau_{max} = ?$

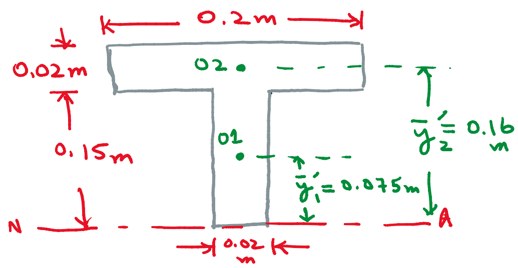
Assumptions:

Gov: Eqns:

Solution:

$$I = 0.25 \times 10^{-3}\ m^4$$

$$Q_{max} = \Sigma \bar{y}' A'$$



$$= \bar{y}_1' A_1' + \bar{y}_2' A_2'$$

$$= (0.075)\underbrace{[0.15 \times 0.02]}_{\text{area of web}}$$

$$+ (0.16)\underbrace{[0.2 \times 0.02]}_{\text{area of flange}}$$

$$Q_{max} = 0.865 \times 10^{-3}\ m^3$$

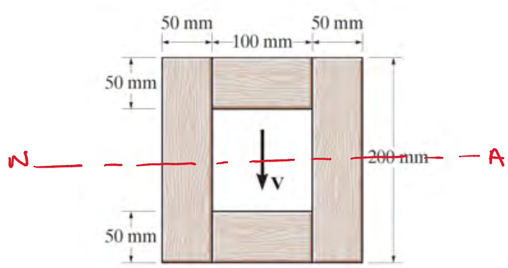
$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{(20 \times 10^3)(0.865 \times 10^{-3})}{(0.25 \times 10^{-3})(0.02)}$$

$$\boxed{\tau_{max} = 3.46\ MPa}$$ Answer

P#7.6:

Given:

$\tau_{allow} = 7\ MPa$



Prob. 7–6

Find:

$V_{max} = ?$

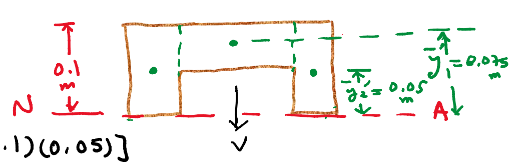
Assumptions:

- Hooke's law
- Homogeneous mat'l:
- symmetric member

Gov: Eqns:

$$\tau = \frac{VQ}{It}$$

Solution:



$$Q_{max} = \Sigma \bar{y}' A'$$

$$Q_{max} = \bar{y}_1' A_1' + 2[\bar{y}_2' A_2']$$

$$= (0.075)[0.1 \times 0.05] + 2[(0.05)(0.1)(0.05)]$$

$$Q_{max} = 4.25 \times 10^{-3}\ m^3$$

$$\tau_{max} = \frac{V_{max} Q_{max}}{It}$$

$$7 \times 10^6 = V_{max}(4.25 \times 10^{-3})$$

$$\tau_{max} = \frac{}{It}$$

$$7 \times 10^{6} = \frac{V_{max}\,(4.25 \times 10^{-3})}{(125 \times 10^{-6})(0.05+0.05)}$$

$$\boxed{V_{max} = 100 kN}$$ Answer

Lecture 21

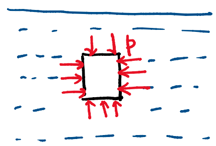
Chapter # 8

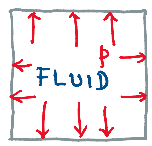
Combined Loadings

§8.1: Thin-walled Pressure vessels:

* Shapes: Cylindrical or spherical
* Usage: Boiler, Tanks, Heat Exchangers, soda cans etc.

CONCEPT 1:





$$p = \frac{F}{A}$$

We know that:

$$\sigma = \frac{F}{A}$$

• relationship b/w σ and p ?

CONCEPT 2:

$$p_g = p - p_{atm}$$

Here:

p_g = Gauge pressure

p_{atm} = Atm. pressure

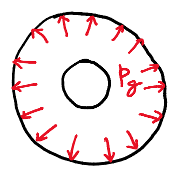
p = Absolute pressure

Example: Tire

$p_g = 32$ psi, $p_{atm} = 14.7$ psi

$32 = P - 14.7$

Abs. pressure: $p = 46.7$ psi



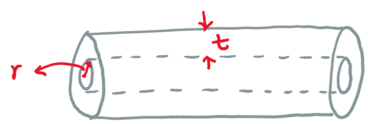
$p \perp A$

CONCEPT 3:

Assume: The pressure has a "THIN" wall.

Criteria for thin-wall:

$$\frac{r}{t} \geq 10$$



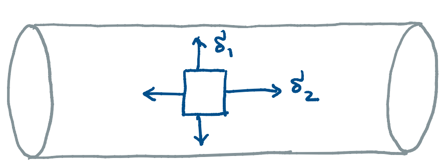
Here: r = inner radius of vessel

t = wall thickness of vessel

* For $\frac{r}{t} = 10 \Rightarrow \sigma_{predicted} = 0.04\, \sigma_{actual,max}$
  $\Rightarrow$ 4% error

* For $\frac{r}{t} > 10 \Rightarrow$ error becomes smaller

Cylindrical Pressure Vessels:

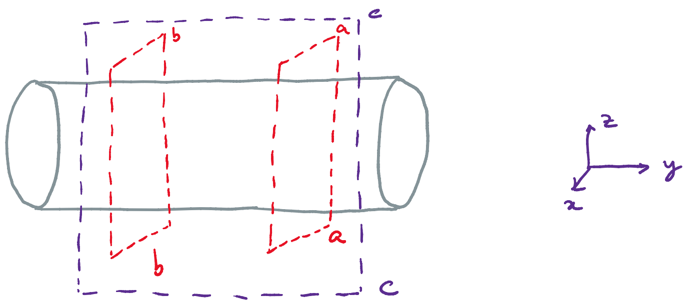


- Consider pressure vessel with wall thickness 't' and inner radius 'r', subjected to gage pressure p_g

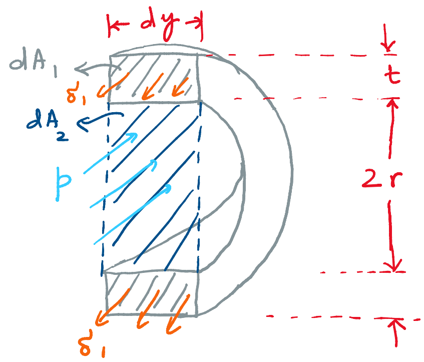
  Here: σ_1 = circumferential or hoop stress

  σ_2 = Longitudnal or axial stress

To find out relations for σ_1 and σ_2:



section c-c: cuts the cylinder longitudnally so we can see half of the cylinder



$\Sigma F_x = 0;$

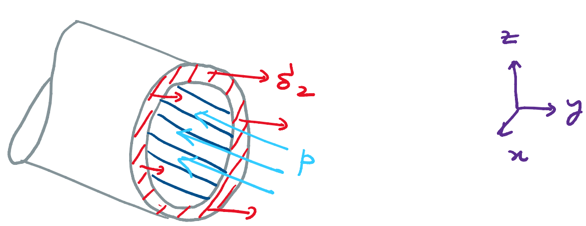
$$2(\sigma_1 dA_1) - p(dA_2) = 0$$

$$2\sigma_1(t\,dy) - p(2r\,dy) = 0$$

$$\sigma_1 = \frac{p(\cancel{2}r\cancel{dy})}{\cancel{2}(t\cancel{dy})}$$

$$\boxed{\sigma_1 = \frac{pr}{t}}$$ Circumferential or Hoop stress

To find longitudnal stress:



$\Sigma F_y = 0;$

$$\sigma_2 (dA_1) - p(dA_2) = 0$$

$$\sigma_2 (2\pi r t) - p(\pi r^2) = 0$$

$$\sigma_2 = \frac{p(\cancel{\pi} r^{\cancel{2}})}{(2\cancel{\pi}\cancel{r}t)}$$

$$\boxed{\sigma_2 = \frac{pr}{2t}}$$ longitudinal or axial stress

$$\Rightarrow \boxed{\sigma_1 = 2\sigma_2}$$

Example :

Given: Cylindrical Pressure Vessel

$d_i = 4' = 4 \times 12 = 48''$

$\Rightarrow r = 24''$

$t = \frac{1}{2}''$

$\sigma_{allow} = 20 \text{ ksi}$

Find: $p_{max} = ?$

Assumptions:

- $\frac{r}{t} = \frac{24}{1/2} = 48 > 10$

  $\Rightarrow$ Thin-walled pressure vessel

- Hooke's Law
- Homogeneous matl :

Gov: Eqns:

- $\sigma_1 = \frac{pr}{t}$
- $\sigma_2 = \frac{pr}{2t}$

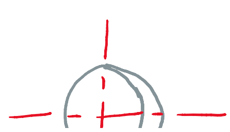
Solution:

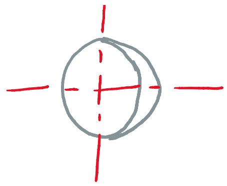
$$\sigma_{allow} = \sigma_1 = \sigma_{max} = \frac{p_{max} r}{t}$$

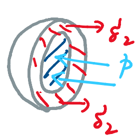
$$20 = \frac{p_{max}(24)}{0.5}$$

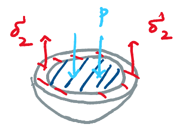
$$\boxed{p_{max} = 417 \text{ psi}}$$ Answer

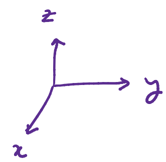
SPHERICAL PRESSURE VESSELS :











$\Sigma F_y = 0;$

$$\sigma_2(2\pi r t) - p(\pi r^2) = 0$$

$$\boxed{\sigma_2 = \frac{pr}{2t}}$$

$\Sigma F_z = 0;$

$$\sigma_2(2\pi r t) - p(\pi r^2) = 0$$

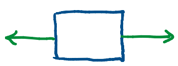
$$\boxed{\sigma_2 = \frac{pr}{2t}}$$

- For spherical pressure vessels: σ_2 remains the only stress, no matter what the orientation of hemisphere

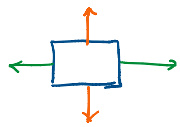
Lecture 22

Tips:
- Preparation / Practice
- Divide the time ⇒ 30 mins / Problem (Manage time)
- Always choose the easiest problem first

Recap:
- Pressure Vessels
- Thin-walled criteria: $\frac{r}{t} \geq 10$
- Cylindrical Pressure Vessels: $\sigma_1 = \frac{pr}{t}$, $\sigma_2 = \frac{pr}{2t}$
- Spherical Pressure Vessels: $\sigma_2 = \frac{pr}{2t}$

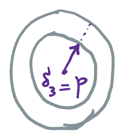


Uniaxial stress



Biaxial stress

- Third stress: Radial stress σ_3
  - * $\sigma_3 = p$ @ interior wall
  - * $\sigma_3 = 0$ @ exterior wall



- For thin-walled pressure vessels:
  IGNORE σ_3: σ_1 and $\sigma_2 > \sigma_3$

Example 8.1: (Continuation of Example from Lecture 21)

Given:

$$\sigma_{allow} = 20 \text{ ksi}$$
$$d_i = 48''$$
$$\Rightarrow r = 24''$$
$$t = \frac{1}{2}''$$

SPHERICAL PRESSURE VESSEL:

$$\sigma_{max} = \sigma_2 = \frac{pr}{2t}$$

$$20 = \frac{p_{max}(24)}{2(0.5)}$$

$$\boxed{p_{max} = 833 \text{ psi}}$$ Answer

Recall: Cylindrical P.V: $p_{max} = 417$ psi

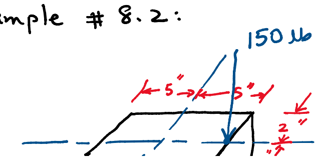
§ 8.2: STATE OF STRESS CAUSED BY COMBINED LOADINGS:

Recall: Chapter 4: Method of superposition:

* $\sigma \propto P$
* $\sigma \propto \epsilon$

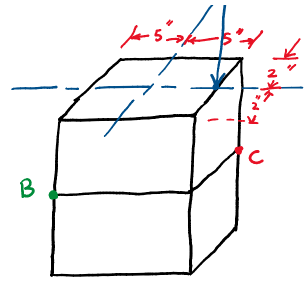
Example # 8.2:

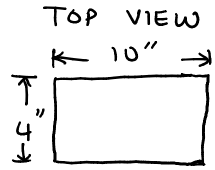
Given:



TOP VIEW

Given :





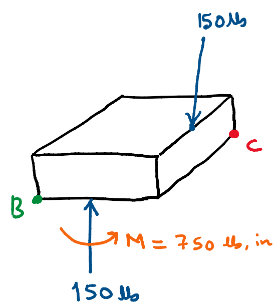
Find : $\delta_B = ?$, $\delta_C = ?$

Assumptions :

- Hooke's Law
- $\delta \propto P$
- Homogeneous matl :
- Neglect : weight

Governing Eqns :

- $\delta = \dfrac{P}{A}$
- $\delta_{max} = \dfrac{Mc}{I}$



Solution :

$$\delta = \frac{P}{A}$$

$$= \frac{150}{(10)(4)}$$

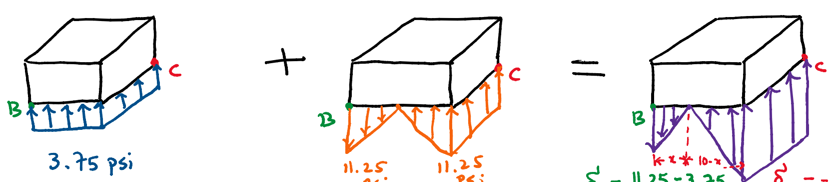
$$\delta = 3.75 \text{ psi}$$

For equilibrium : $M = 150(5) = 750$ lb. in

$$\delta_{max} = \frac{Mc}{I}$$

$$= \frac{(750)(5)}{\left[\frac{(4)(10)^3}{12}\right]}$$

$$\delta_{max} = 11.25 \text{ psi}$$



$\delta_B = 11.25 - 3.75$ $\delta_C = -11.25 - 3.75$

$\delta_B = 7.5$ psi $\delta_C = -15$ psi

To find the location of change in stress (zero stress) :

By similar Δs :

$$\frac{7.5}{x} = \frac{15}{10 - x}$$

$$\boxed{x = 3.33''}$$

P # 8.1 : Given : Spherical gas tank

$r = 1.5\,m$

$p = 300\,kPa$

Find: $t = ?$ IF $\sigma_{allow} = 12\,MPa$

Assumptions :

- Hooke's law satisfied
- Homogeneous mat'l :
- Thin-walled pressure Vessel

  If $\frac{r}{t} \geq 10$

Governing Equations:

- $\sigma_2 = \frac{pr}{2t}$

Solution :

$\sigma_2 = \sigma_{max} = \frac{pr}{2t}$

$12 \times 10^6 = \frac{(300 \times 10^3)(1.5)}{2t}$

$\boxed{t = 18.75\ mm}$ Answer

$\frac{r}{t} = \frac{1.5}{0.01875} = 80 > 10$

PROBLEM # 8.10 :

Given: A-36 Steel band

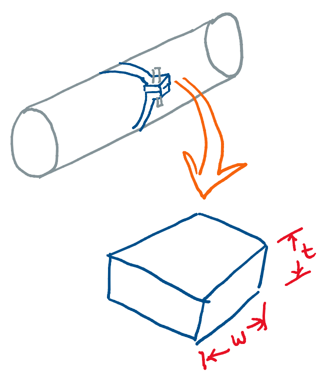
$F_{T,b} = 400\ lb$

$r = 8''$

$t = \frac{1}{8}''$

$w = 2''$

$E_{st} = 29 \times 10^3\ ksi$



Find :

$\sigma_1 = ?$

$p = ?$

$\delta = ?$

Assumptions :

- Hooke's Law
- Homogeneous mat'l :
- $\frac{r}{t} \geq 10 \Rightarrow \frac{8}{1/8} = 64 > 10 \Rightarrow$ Thin-walled P.V.

Governing Eqns:

- $\sigma = \frac{F}{A}$
- $\sigma_1 = \frac{pr}{t}$

- $\sigma = E\epsilon$
- $\epsilon = \dfrac{\delta}{L}$

Solution:

$$\sigma = \frac{F}{A} = \frac{400}{(2)\left(\frac{1}{8}\right)} = 1600 \text{ psi}$$

$$\boxed{\sigma = \sigma_1 = 1600 \text{ psi}}$$ Answer

$$\sigma_1 = \frac{pr}{t}$$

$$1600 = \frac{p\,(8)}{1/8}$$

$$\boxed{p = 25 \text{ psi}}$$ Answer

$$\sigma = E\epsilon \Rightarrow \epsilon_1 = \frac{\sigma_1}{E}$$

$$\epsilon_1 = \frac{1600}{29\times10^{6}} = 55.17\times10^{-6}$$

$$\epsilon_1 = \frac{\delta}{L} \Rightarrow \delta = \epsilon_1 L$$

$$\delta = (55.17\times10^{-6})\,\pi(r+t)$$

$$= (55.17\times10^{-6})\,\pi\left[8+\frac{1}{8}\right]$$

$$\boxed{\delta = 0.0014 \text{ in}}$$ Answer

Side Note:

circumference

$$2\pi r = \pi d$$

$$2\pi(r+t) = \pi(d+2t)$$

$$\frac{2\pi(r+t)}{2} = \frac{\pi(d+2t)}{2}$$

$$\pi(r+t) = \frac{\pi}{2}(d+2t)$$

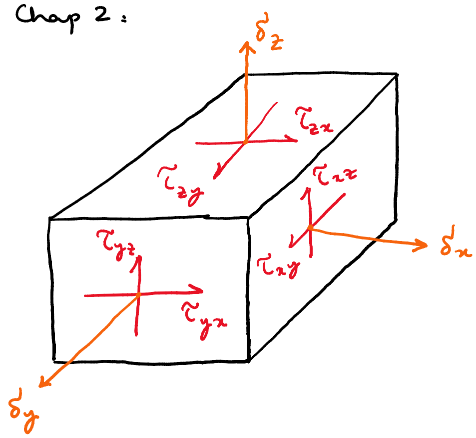
Lecture 23

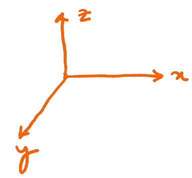
CHAPTER # 9

§9.1: Plane - Stress Transformation:

- Recall: Chap 2:

RULE:
face - direction



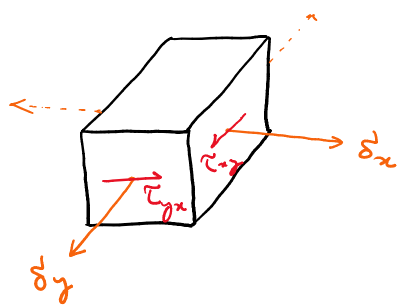


- Initially: 3 normal stresses + 6 shear stress components
- Simplify: 3 normal stresses + 3 shear stress components

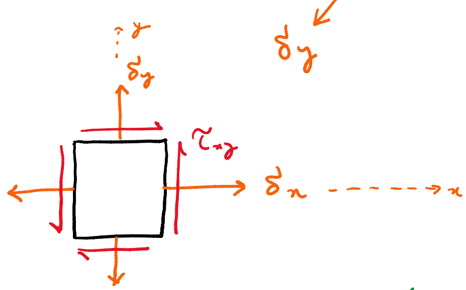
$$\Rightarrow \tau_{zy} = \tau_{yz},\ \tau_{xz} = \tau_{zx},\ \tau_{xy} = \tau_{yx}$$

- To simplify the problem mathematically:
  analyze the problem in a single-plane
  $\Rightarrow$ Plane - Stress
  e.g: IF there is no load on a surface of the body, then, σ and τ are ZERO on that face of element.

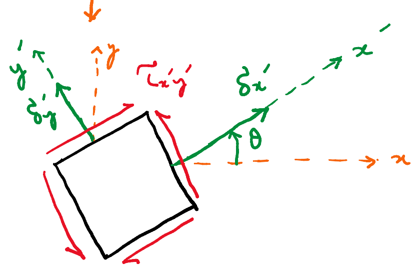
- no load on z-face:
  - $\sigma_z = 0$
  - $\tau_{xz} = 0$
  - $\tau_{yz} = 0$

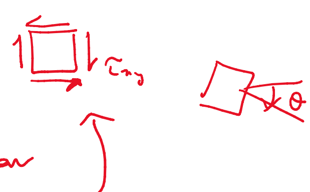


(i)



(ii)



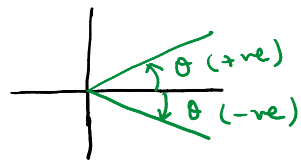


+ve shear
– ve shear

§9.2: General Eqns. of Plane-Stress Transformation:

Rules:

- $+x, +x'$ axes: outward normal
- $\sigma_x, \sigma_{x'}$ are +ve <u>WHEN</u> in $+x, +x'$ direction
- $\tau_{xy}, \tau_{x'y'}$ +ve <u>WHEN</u> in $+y, +y'$ direction
- θ is +ve $\Rightarrow$ CCW direction.

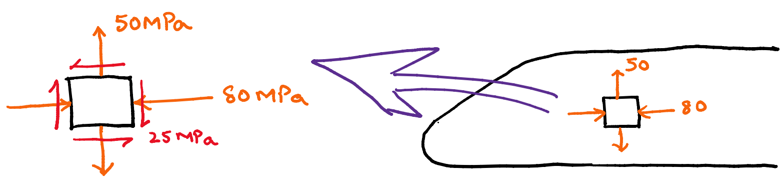


$$\sigma_{x'} = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{—— (1)}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \quad \text{—— (2)}$$

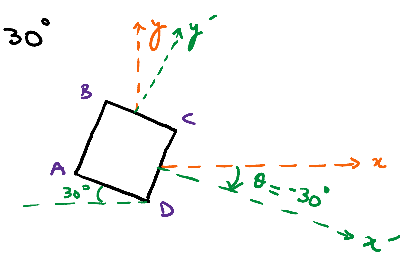
$$\sigma_{y'} = \left(\frac{\sigma_x+\sigma_y}{2}\right) - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta \quad \text{—— (3)}$$

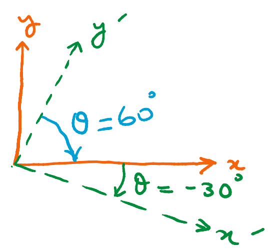
Example # 9.2 (same as Ex # 9.1)



<u>Step 1</u>: $\sigma_x = -80\text{ MPa}$, $\sigma_y = 50\text{ MPa}$, $\tau_{xy} = -25\text{ MPa}$

<u>Step 2</u>: $\theta = -30^\circ$





<u>Step 3</u>: Plane CD: $\theta = -30^\circ$:

eq (1) $\Rightarrow$ $\sigma_{x'} = \left(\frac{-80+50}{2}\right) + \left(\frac{-80+50}{2}\right)\cos 2(-30^\circ) + (-25)\sin 2(-30^\circ)$

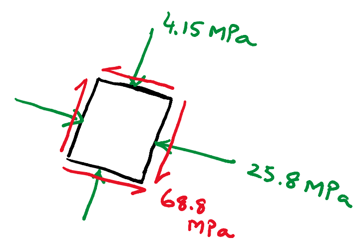
$\boxed{\sigma_{x'} = -25.8\text{ MPa}}$ Answer

eq (2) $\Rightarrow$ $\boxed{\tau_{x'y'} = -68.8\text{ MPa}}$ Answer

Plane BC: $\theta = 60^\circ$:

eq (3) $\Rightarrow$ $\boxed{\sigma_{y'} = -4.15\text{ MPa}}$ Answer

Step 4:



§9.3: Principal Stresses and Max. In-plane shear stress:

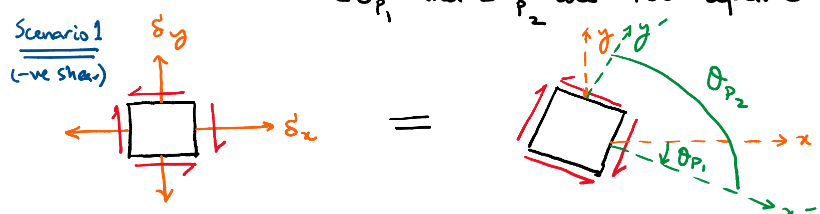
- $\sigma_{x'}$ and $\tau_{x'y'}$ depend on θ $\Rightarrow$ angle determination (orientation of the element) is very important.

If $\theta = \theta_p$:

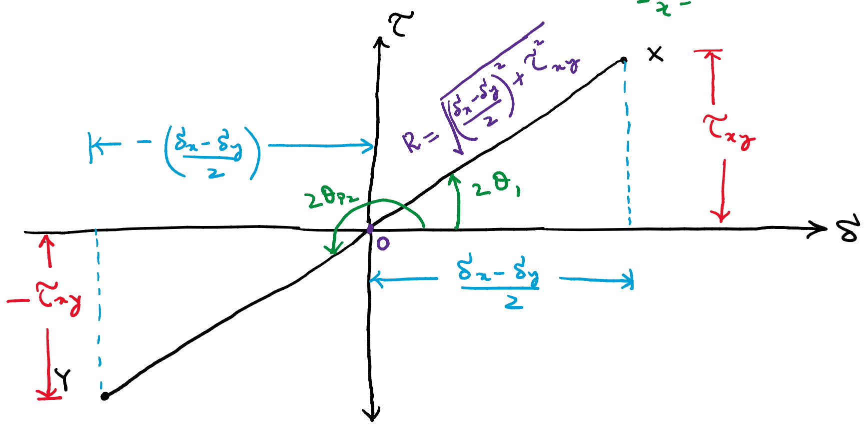
$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} \quad \text{——} \; (4)$$

- Two roots: θ_{p_1} and θ_{p_2}
- θ_{p_1} and θ_{p_2} are 90° apart
- $2\theta_{p_1}$ and $2\theta_{p_2}$ are 180° apart

Scenario 1 (-ve shear)

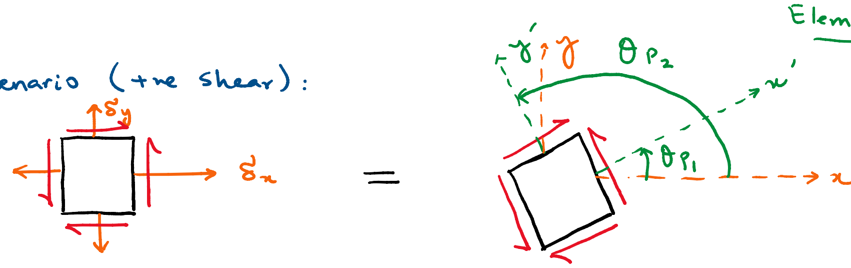


$\theta_{p_2} = \theta_{p_1} + 90^\circ$ ← Element diagram



Element diagram

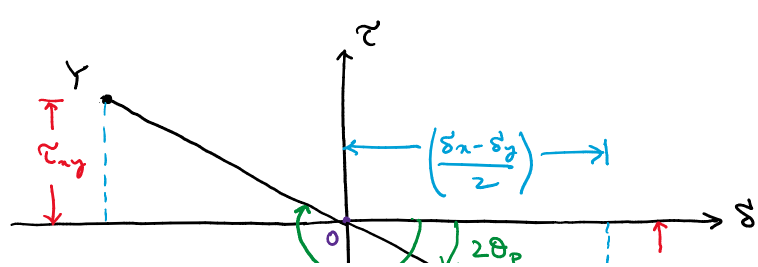
Scenario (+ve shear):

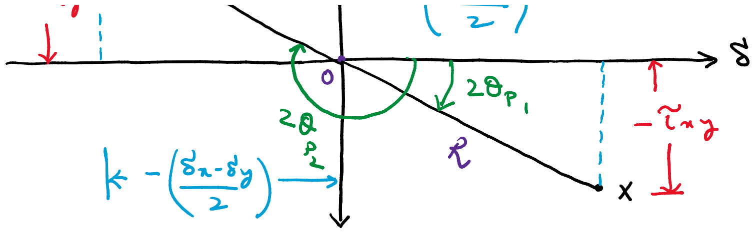


x to $x' \Rightarrow \theta_{p_1}$

x to $y' \Rightarrow \theta_{p_2}$

$\theta_{p_2} = \theta_{p_1} + 90^\circ$





NOTE : • If τ_{xy} is −ve ; X is located above δ-axis
• If τ_{xy} is +ve ; X is located below δ-axis

Lecture 24

Recap: Lecture 23: Rule about τ_{xy} being +ve or −ve

- Element diagram
- Plot $\sigma - \tau$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \quad \text{—— (5)}$$

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{—— (6)}$$

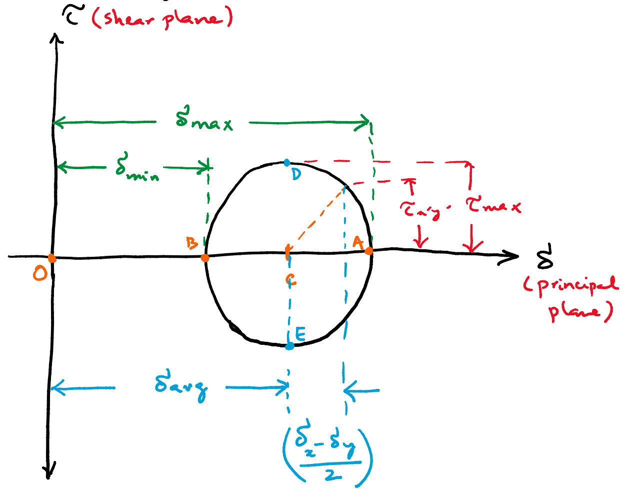
$$\left.\begin{aligned} \text{eq (1)} \Rightarrow \left(\sigma_{x'} - \sigma_{avg}\right)^2 + \left(\tau_{x'y'}\right)^2 &= R^2 \\ (x-h)^2 + (y-k)^2 &= r^2 \end{aligned}\right\} \text{Eqn: of circle}$$

$$\sigma_{max} = \sigma_{avg} + R \quad \text{—— (7)}$$

$$\sigma_{min} = \sigma_{avg} - R \quad \text{—— (8)}$$

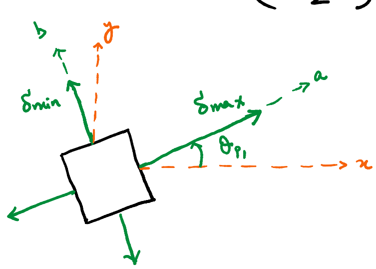
A: σ_{max} of $\sigma_{x'}$
B: σ_{min} of $\sigma_{y'}$ $\Big\}$ $\tau_{x'y'} = 0$

- No shear stress acts on the principal plane
- Pt. D and E $\Rightarrow$ largest values of $\tau_{x'y'}$ $\Rightarrow$ τ_{max}



$$\text{eq (7)} \Rightarrow \sigma_{max} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{—— (9)}$$

$$\text{eq (8)} \Rightarrow \sigma_{min} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{—— (10)}$$



NOTE: If you dont know which is σ_{max} and σ_{min}:
INSERT θ_p value in eq (1) $\Rightarrow \sigma_{x'}$
(Quick check)

<u>Shear plane</u> : $\theta = \theta_s$:

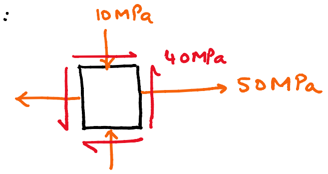
$$\tan 2\theta_s = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \quad \text{—— (11)}$$

Compare eq (4) and eq (11) :

$$\tan 2\theta_s = -\frac{1}{\tan 2\theta_p} \quad \text{—— (12)}$$

- $2\theta_s$ and $2\theta_p$ are 90° apart $\Rightarrow$ θ_s and θ_p are 45° apart
  $\Rightarrow$ Plane of max: shear stress is @ 45° to principal plane

PROBLEM:



Find: (a) Principal planes $\Rightarrow \theta_p = ?$

(b) Principal stresses $\Rightarrow \sigma_{max} = ?$, $\sigma_{min} = ?$

(c) Max: shear stress $\Rightarrow \tau_{max} = ?$

(d) Max: shear plane $\Rightarrow \theta_s = ?$

(e) Draw a comprehensive element diagram

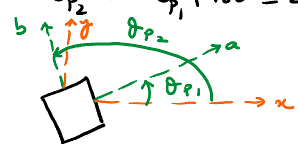
Solution:

Step 1: $\sigma_x = 50\,MPa$, $\sigma_y = -10\,MPa$, $\tau_{xy} = 40\,MPa$

(a) eq ④ $\Rightarrow$ $\tan 2\theta_p = \dfrac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$

$$2\theta_{p_1} = 53.1^\circ \Rightarrow \theta_{p_1} = 26.6^\circ$$

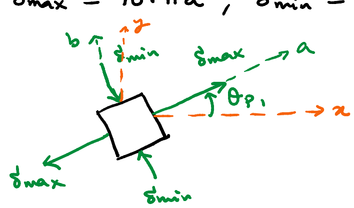
$$2\theta_{p_2} = 2\theta_{p_1} + 180^\circ = 233.1^\circ \Rightarrow \theta_{p_2} = 116.6^\circ$$



(b) eq ⑨ and eq ⑩ :

$$\sigma_{max,min} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{max} = 70\,MPa \; , \; \sigma_{min} = -30\,MPa$$



CHECK: eq ① $\Rightarrow$ $\sigma_x' = 70\,MPa = \sigma_{max}$ [Plug θ_p]

(c) eq ⑥:

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

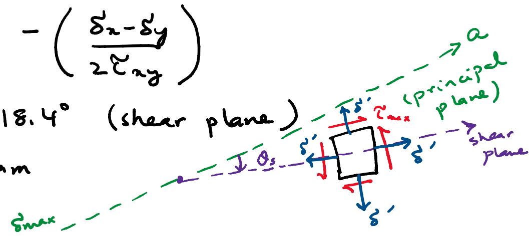
$$\tau_{max} = 50\,MPa$$

$$\sigma' = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50-10}{2} = 20\,MPa$$

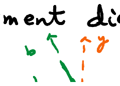
(d) eq ⑪:

$$\tan 2\theta_s = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$$

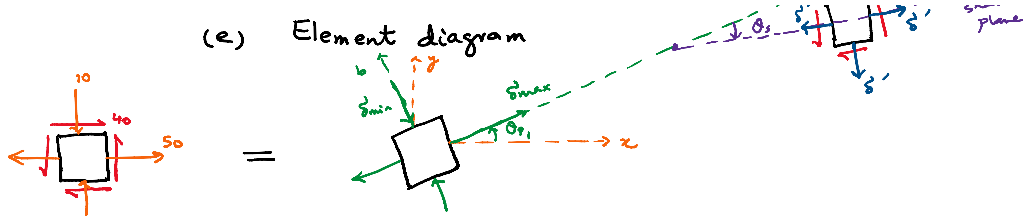
$$\theta_s = -18.4^\circ \text{ (shear plane)}$$



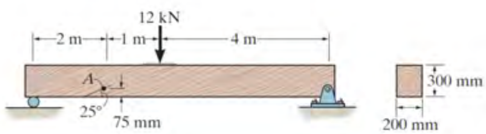
(e) Element diagram



(e) Element diagram



Problem # 9.24:



Given:

- wood beam
- symmetric

$P = 12\ kN$

Find:

- $\sigma_x = ?,\ \sigma_y = ?,\ \tau_{xy} = ?$
- $\sigma_x' = ?,\ \sigma_y' = ?,\ \tau_{x'y'} = ?$
- $\sigma_{max,min} = ?,\ \tau_{max} = ?$

Assumptions:

- Hooke's Law
- Homogeneous mat'l :

Governing Eqns:

- $\sigma = -\dfrac{My}{I}$
- $\tau = \dfrac{VQ}{It}$

- eq ① → eq ⑩ from chap 9.

Solution:

$$I = \frac{bh^3}{12} = \frac{(0.2)(0.3)^3}{12} = 0.45 \times 10^{-3}\ m^4$$

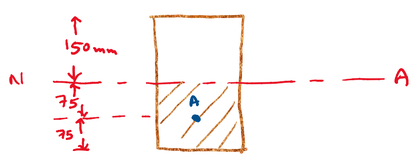
Find V_A and M_A from shear and moment diagram:

$$V_A = 6.875\ kN\ ,\quad M_A = 13.714\ kN$$

$$Q_A = \bar{y}_A' A_A$$

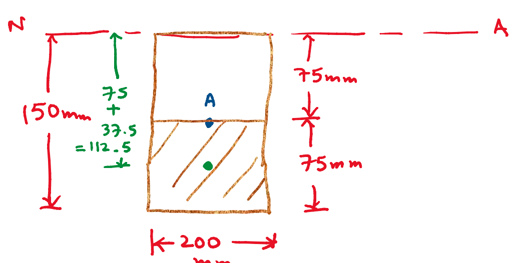
$$= (-0.1125)[(0.2)(0.075)]$$

$$Q_A = -1.6875 \times 10^{-3}\ m^3$$



$$\sigma_A = -\frac{(13.714 \times 10^3)(-0.075)}{0.45 \times 10^{-3}}$$

$$\sigma_A = 2.28\ MPa = \sigma_x$$



$$\sigma_y = 0$$

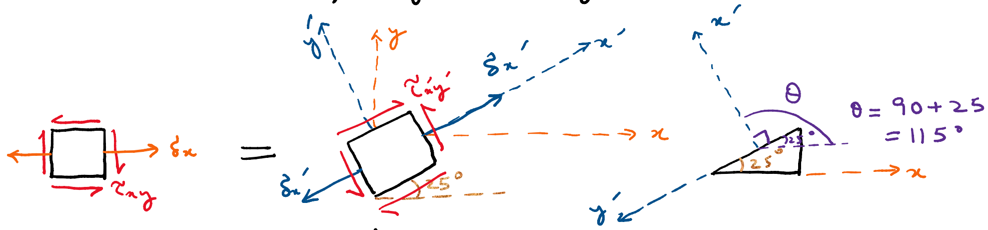
$$\tau_A = \frac{V_A Q_A}{It}$$

$$= \frac{(6.875 \times 10^3)(-1.6875 \times 10^{-3})}{(0.45 \times 10^{-3})(0.2)}$$

$$\tau_A = -0.1286\ MPa = \tau_{xy}$$

$$\sigma_x = 2.28\ MPa,\quad \sigma_y = 0,\quad \tau_{xy} = -0.1286\ MPa$$

$\sigma_x = 2.28\,MPa, \quad \sigma_y = 0, \quad \tau_{xy} = -0.1286\,MPa$



eq (1) $\Rightarrow$ $\sigma_{x'} = 0.5\,MPa$

eq (2) $\Rightarrow$ $\tau_{x'y'} = 0.95\,MPa$

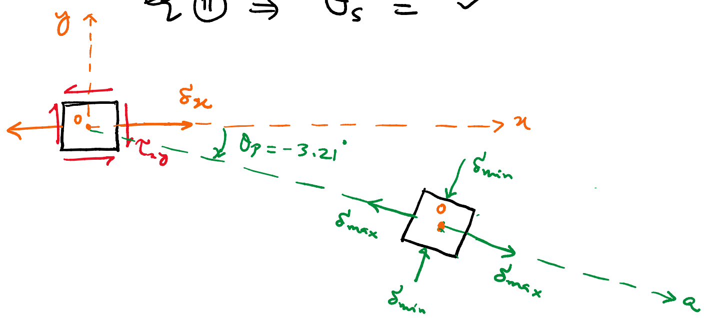
eq (9) $\Rightarrow$ $\sigma_{max} = 2.29\,MPa$

eq (10) $\Rightarrow$ $\sigma_{min} = -7.2\,MPa$

eq (4) $\Rightarrow$ $\theta_p = -3.21°$

eq (6) $\Rightarrow$ $\tau_{max} = \checkmark$, $\sigma' = \sigma_{avg} = \checkmark$

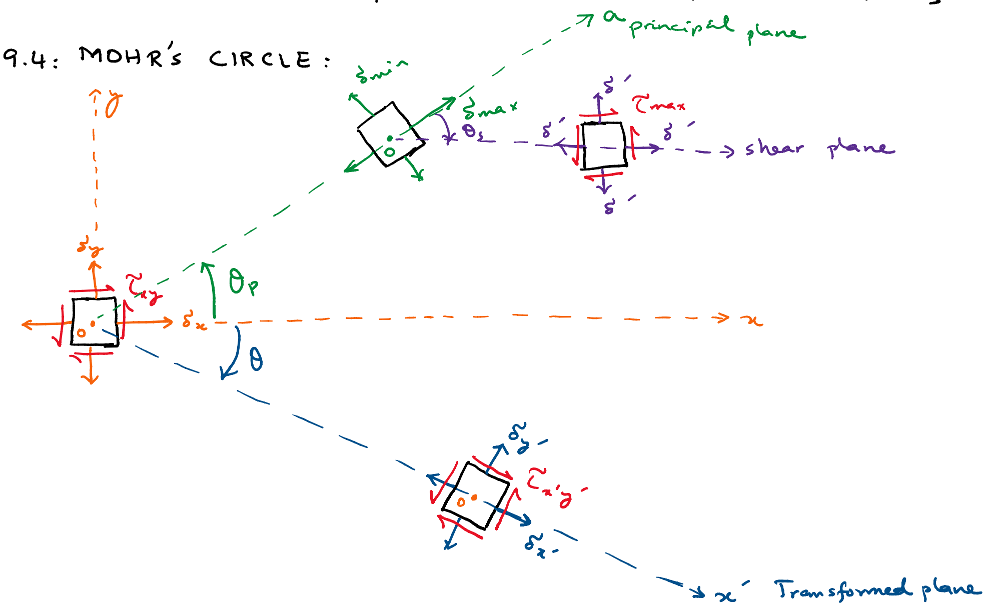
eq (11) $\Rightarrow$ $\theta_s = \checkmark$



Lecture 25

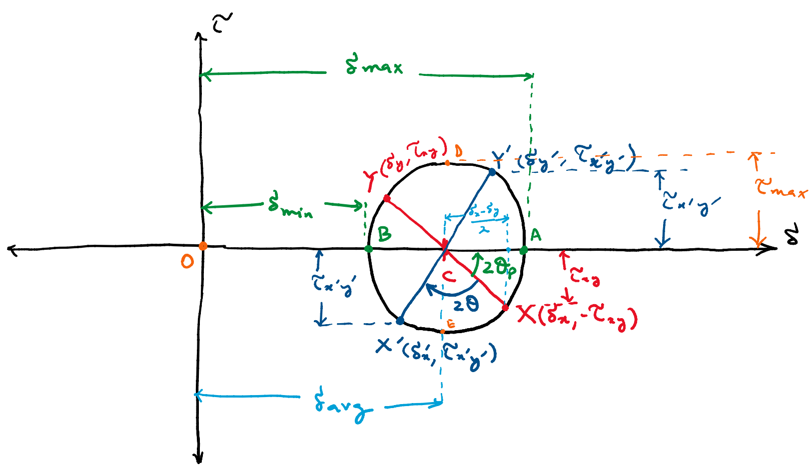
Recap:
- Transformation stresses and plane $\Rightarrow \sigma_{x'}, \sigma_{y'}, \tau_{x'y'}; \theta$
- Principal stresses and plane $\Rightarrow \sigma_{max,min}; \theta_p$
- Max. in-plane shear stress and plane $\Rightarrow \tau_{max}, \sigma'; \theta_s$

§9.4: MOHR'S CIRCLE:



To construct Mohr's circle: (Rules)

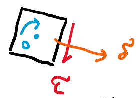
- IF τ_{xy} is +ve:
  - X is located below σ-axis
  - Y is located above σ-axis
- IF τ_{xy} is −ve:
  - X is located above σ-axis
  - Y is located below σ-axis
- Join XY
- Find center C
- Draw Mohr's Circle



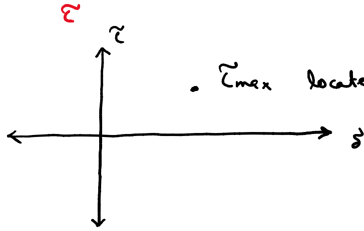
$$\tan(XCA) = \frac{P}{B} = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \tan 2\theta_p$$

$$\Rightarrow XCA = 2\theta_p$$

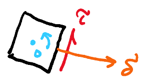
- To find max. shear on Mohr's circle :

> 

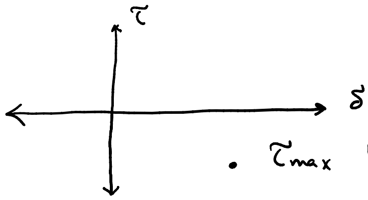
$\Rightarrow$ CW rotation



- τ_{max} located above σ- axis

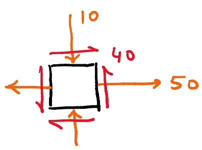
> 

$\Rightarrow$ CCW rotation



- τ_{max} located below σ-axis

Problem: Refer to previous lectures from Chap 9 for this problem



$\sigma_x = 50$ MPa , $\sigma_y = -10$ MPa , $\tau_{xy} = 40$ MPa

Step 1:

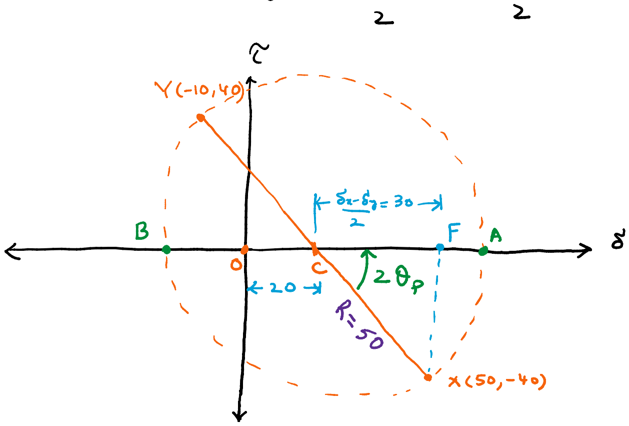
$X(\sigma_x, \tau_{xy}) = (50, -40)$

$Y(\sigma_y, \tau_{xy}) = (-10, 40)$

Step 2 :

find center C

$$OC = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50-10}{2} = 20 \text{ MPa}$$



$R = CX = \sqrt{30^2 + 40^2} = 50$ MPa

eq (7) or eq (9) : $\sigma_{max} = OA = OC + OA$

$$R = CX = \sqrt{30^2+40^2} = 50\,MPa$$

eq (7) or eq (9) : $\sigma_{max} = OA = OC + OA$
$$= \sigma_{avg} + R$$
$$= 20 + 50 = 70\,MPa$$

eq (8) or eq (10) : $\sigma_{min} = OB = OC - BC$
$$= \sigma_{avg} - R$$
$$= 20 - 50 = -30\,MPa$$

eq (4) or $\angle XCA$: $\tan 2\theta_p = \dfrac{FX}{CF} = \dfrac{40}{30}$

$$2\theta_{p_1} = 53.1^\circ \Rightarrow \theta_{p_1} = 26.6^\circ$$

P# 9-35:

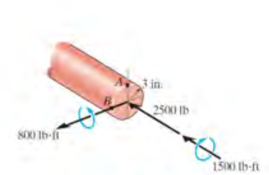
Given:

$P = 2500\ lb$

$M = 800\ lb.ft$

$T = 1500\ lb.ft$

$d = 6\ in$



Probs. 9–35/36

Find:

$\sigma_{max, min} = ?$, $\tau_{max} = ?$

Assumptions:

Gov. Eqns:

- $\sigma = \dfrac{P}{A}$
- $\sigma = \dfrac{Mc}{I}$
- $\tau = \dfrac{Tc}{J}$

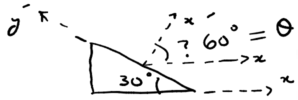
Solution:

$$A = \pi r^2 = 28.274\ in^2$$
$$J = \frac{\pi}{2} c^4 = 127.2\ in^4$$
$$I = \frac{\pi}{4} c^4 = 63.62\ in^4$$

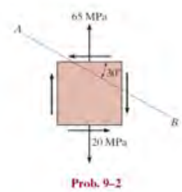
$$\sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -541\ psi$$
$$\tau_A = \frac{Tc}{J} = 424.4\ psi$$

$\sigma_x = -541\ psi$, $\sigma_y = 0$, $\tau_{xy} = 424.4\ psi$

$$\sigma_{max} = 233\ psi$$
$$\sigma_{min} = -774\ psi$$
$$\tau_{max} = 503\ psi$$



P# 9.44:



Given:

$\sigma_x = 0$, $\sigma_y = 65$ MPa, $\tau_{xy} = -20$ MPa

Find:

$\sigma_{x'} = ?$, $\sigma_{y'} = ?$, $\tau_{x'y'} = ?$ $\Rightarrow$ Using Mohr's circle

Assumptions:

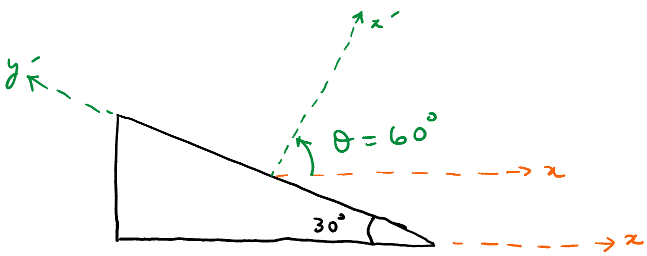
Gov: Eqns:

Solution:

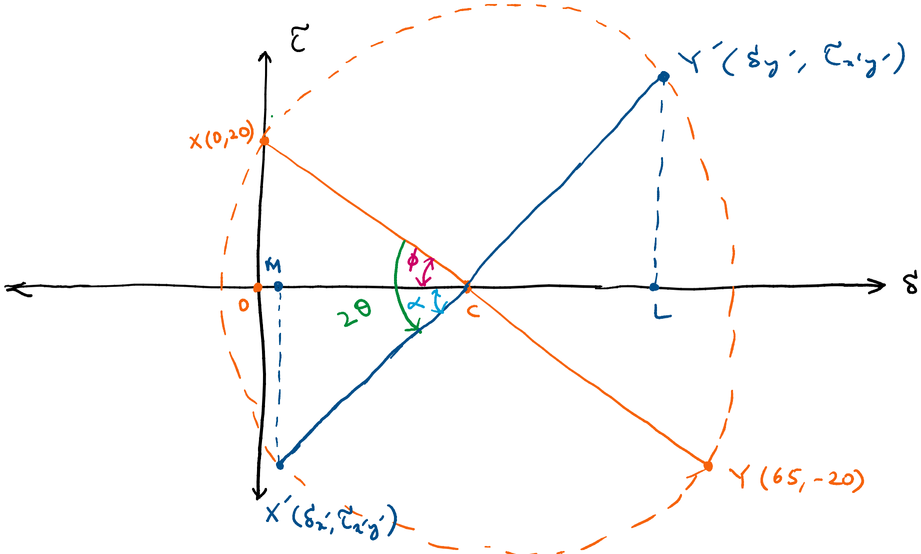
$X(0, 20)$, $Y(65, -20)$

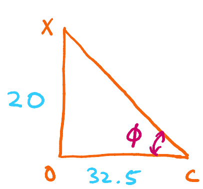
$$OC = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 32.5 \text{ MPa}$$

$$R = \sqrt{(20)^2 + (32.5)^2} = 38.16 \text{ MPa}$$



$2\theta = 120°$





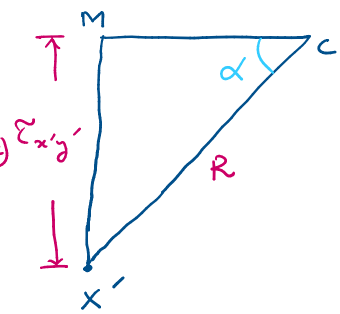
$$\tan \phi = \frac{20}{32.5}$$

$$\phi = 31.6°$$

$$2\theta = \phi + \alpha \Rightarrow \alpha = 120 - 31.6°$$

$$\alpha = 88.392°$$

$$\alpha = 88.392^\circ$$



$$\sigma_{x'} = OM = OC - CM$$

$$= 32.5 - R\cos\alpha$$

$$= 32.5 - 38.16\cos(88.392^\circ)$$

$$\sigma_{x'} = 31.4\ \text{MPa}$$

$$\sigma_{y'} = OL = OC + CL$$

$$= 32.5 + R\cos\alpha$$

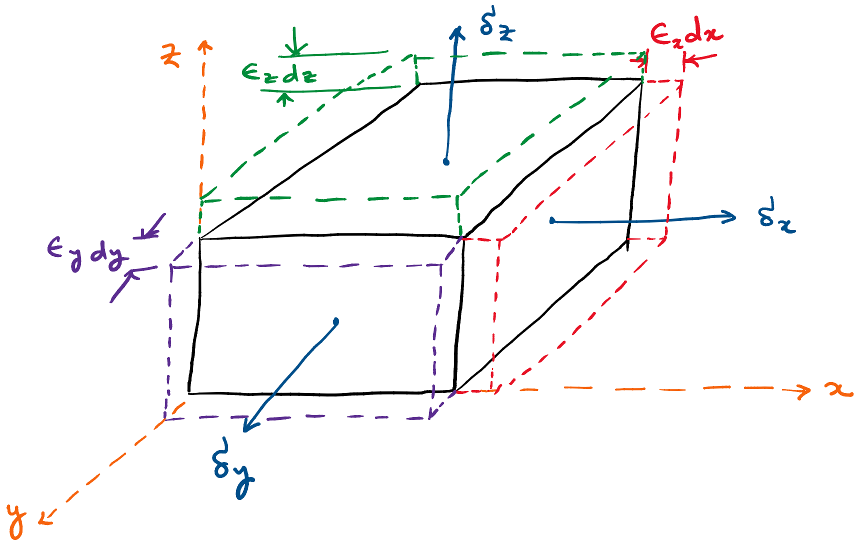
$$\sigma_{y'} = 33.6\ \text{MPa}$$

$$\tau_{x'y'} = MX' = R\sin\alpha$$

$$\tau_{x'y'} = 38.1\ \text{MPa}$$

CHAPTER # 10:
STRAIN TRANSFORMATION

- Recall: strain @ a pt: $\epsilon_x, \epsilon_y, \epsilon_z$ and $\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$
- 6 components ⇒ tend to deform [Recall: stress components]

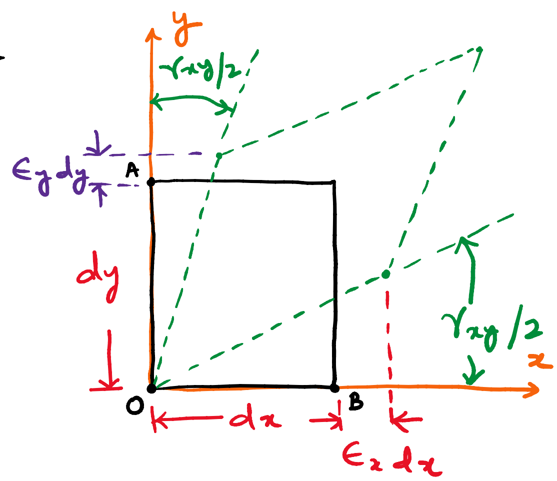


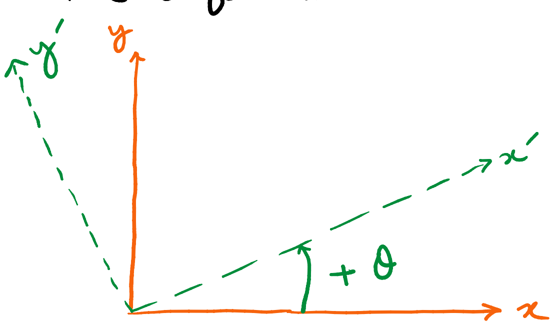
- Neglect: z-direction ⇒ $\epsilon_z, \gamma_{xz}, \gamma_{yz} = 0$

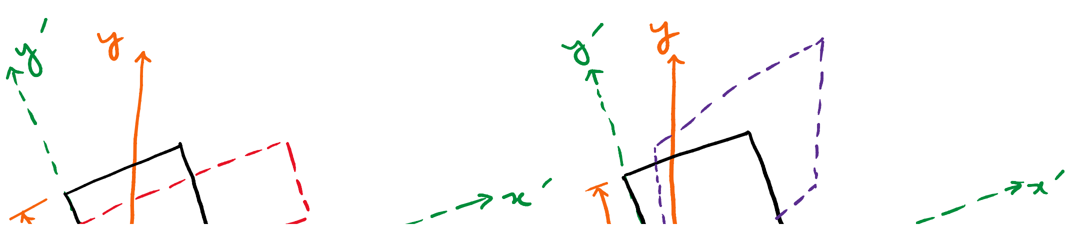
  ⇒ $\epsilon_x, \epsilon_y, \gamma_{xy}$

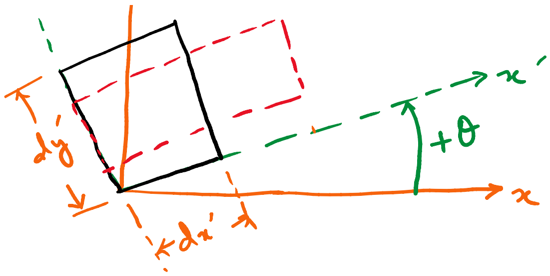
Sign Convention:

- ϵ_x, ϵ_y are +ve ⇒ Elongation
- γ_{xy} +ve ⇒ $\angle AOB < 90°$ (acute angle)
- Positive $\delta_x, \delta_y, \tau_{xy}$ cause +ve deformations



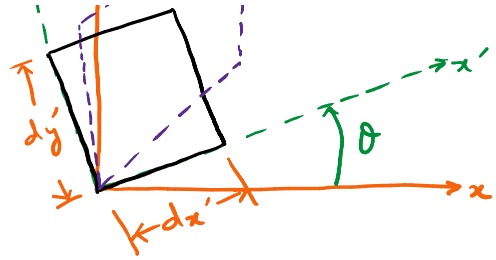






POSITIVE NORMAL STRAIN

$\epsilon_{x'}$



POSITIVE SHEAR STRAIN

$\gamma_{x'y'}$

$$\epsilon_{x'} = \left(\frac{\epsilon_x + \epsilon_y}{2}\right) + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \quad \text{— (1A)}$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta \quad \text{— (2A)}$$

$$\epsilon_{y'} = \left(\frac{\epsilon_x + \epsilon_y}{2}\right) - \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta \quad \text{— (3A)}$$

NOTE:
- $\sigma_x, \sigma_y, \sigma_{x'}, \sigma_{y'}$ correspond to $\epsilon_x, \epsilon_y, \epsilon_{x'}, \epsilon_{y'}$
- $\tau_{xy}, \tau_{x'y'}$ correspond to $\frac{\gamma_{xy}}{2}, \frac{\gamma_{x'y'}}{2}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} \quad \text{— (4A)}$$

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} \quad \text{— (5A)}$$

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \quad \text{— (6A)}$$

$$\epsilon_{max,min} = \left(\frac{\epsilon_x + \epsilon_y}{2}\right) \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \quad \text{— } \boxed{7A, 8A}$$

$$\tan 2\theta_s = -\left(\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}\right) \quad \text{— (11A)}$$

$$\epsilon_{max,min} = \epsilon_{avg} \pm R \quad \text{— } \boxed{9A, 10A}$$

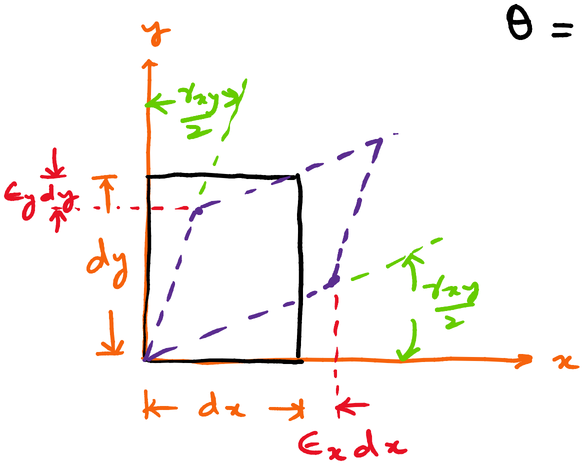
Example # 10.1:

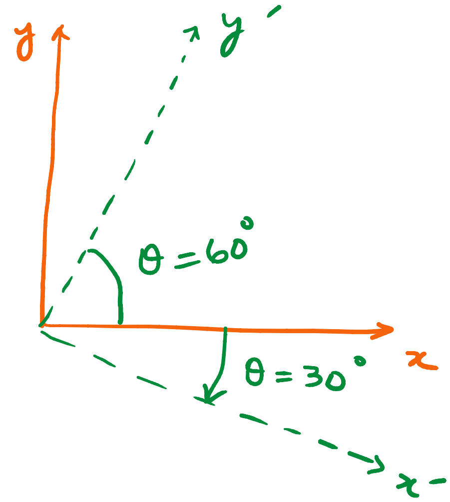
Given: $\epsilon_x = 500 \times 10^{-6}$ Find: $\epsilon_{x'} = ?$

$\epsilon_y = -300 \times 10^{-6}$ $\epsilon_{y'} = ?$

$\gamma_{xy} = 200 \times 10^{-6}$ $\gamma_{x'y'} = ?$

$\theta = -30^\circ$



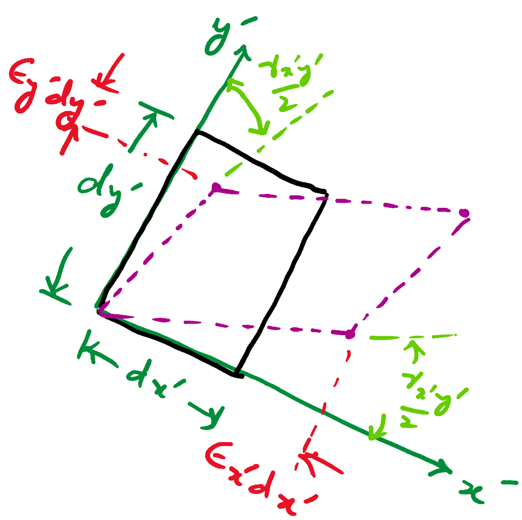


Assumptions: Gov: Eqns:

eq (1A) ⇒ $\epsilon_{x'} = 213 \times 10^{-6}$

eq (2A) ⇒ $\gamma_{x'y'} = 793 \times 10^{-6}$

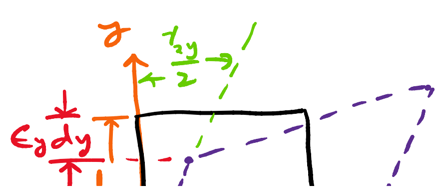
eq (3A) ⇒ $\epsilon_{y'} = -13.4 \times 10^{-6}$



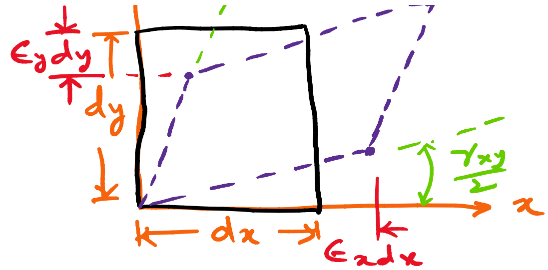
§ 10.4 MOHR'S CIRCLE:

Example # 10.4:

Given: $\epsilon_x = 250 \times 10^{-6}$



Example # 10.4:



Given: $\epsilon_x = 250 \times 10^{-6}$

$\epsilon_y = -150 \times 10^{-6}$

$\gamma_{xy} = 120 \times 10^{-6}$

Find: $\epsilon_{max} = ?$, $\epsilon_{min} = ?$, $\theta_p = ?$

Assumptions:

Gov: Eqns:

Solution:

Step 1: $X\left(\epsilon_x, \frac{\gamma_{xy}}{2}\right)$, $Y\left(\epsilon_y, \frac{\gamma_{xy}}{2}\right)$

- $X\left(250 \times 10^{-6}, -60 \times 10^{-6}\right)$
- $Y\left(-150 \times 10^{-6}, 60 \times 10^{-6}\right)$

Step 2: Locate center C

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = 50 \times 10^{-6}$$

Step 3: find radius R

$$R = \sqrt{(250-50)^2 + (60)^2}$$

$$R = 208.8 \times 10^{-6}$$

Step 4: find ϵ_{max} , ϵ_{min}:

$$\epsilon_{max} = \epsilon_{avg} + R = 259 \times 10^{-6}$$

$$\epsilon_{min} = \epsilon_{avg} - R = -159 \times 10^{-6}$$

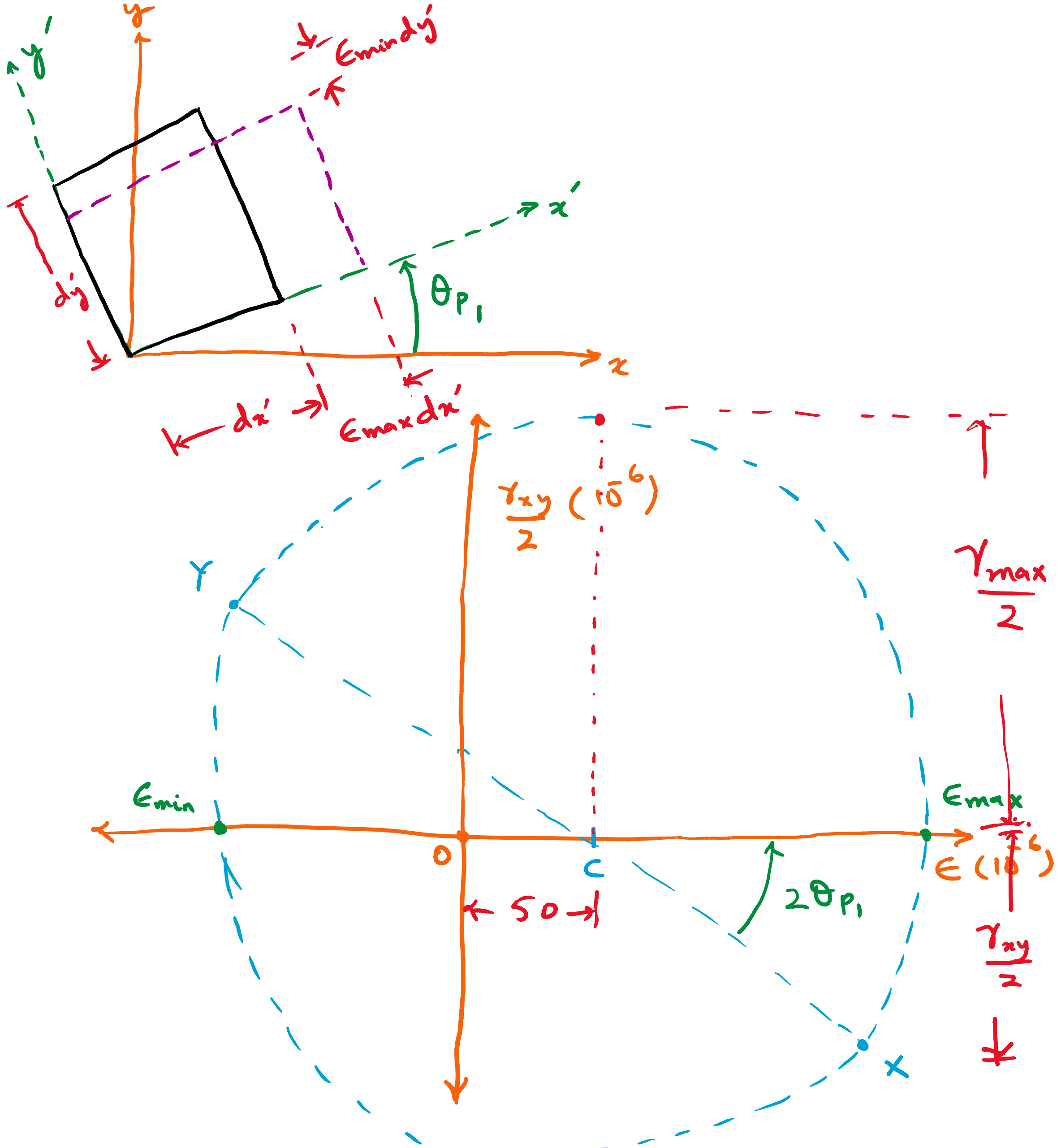
Step 5: find location of principal plane θ_p :

$$\tan 2\theta_{p_1} = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{120}{(250+150)}$$

$$2\theta_{p_1} = 16.69^\circ$$

$$\theta_{p_1} = 8.34^\circ$$

$$2\theta_{p_1} = 16.69^\circ$$
$$\theta_{p_1} = 8.34^\circ$$



P#10.2:

$$\epsilon_{x'} = \left(\frac{\epsilon_x + \epsilon_y}{2}\right) + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

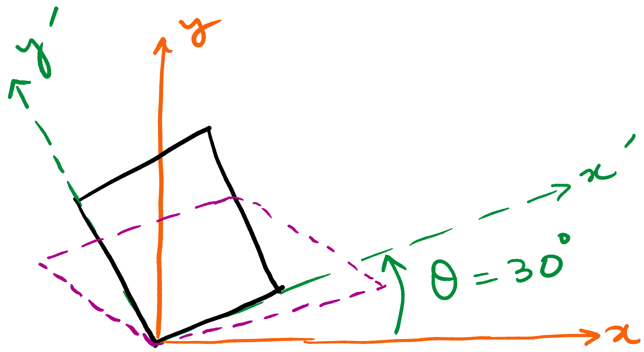
$$\epsilon_{x'} = 248 \times 10^{-6}$$

$$\epsilon_{y'} = \left(\frac{\epsilon_x+\epsilon_y}{2}\right) - \left(\frac{\epsilon_x-\epsilon_y}{2}\right)\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\epsilon_{y'} = -348 \times 10^{-6}$$

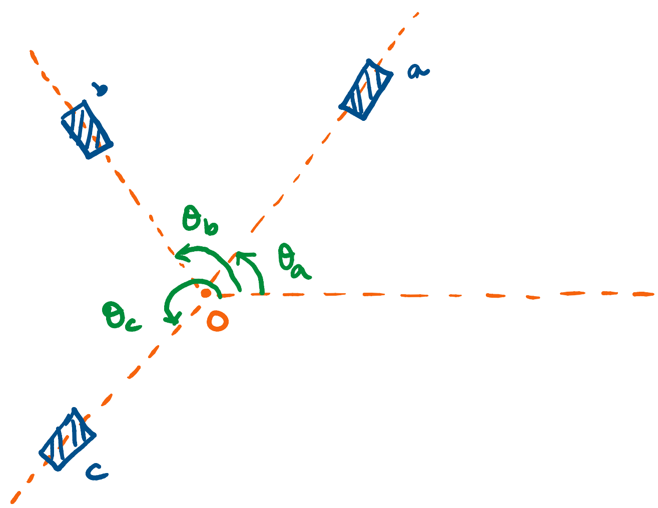
$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x-\epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = -233 \times 10^{-6}$$



§ 10.5: STRAIN ROSETTES:

- strain gauges ⇒ attached to specimen
- attached in clusters of 3 strain gauges.



Strains : ϵ_a , ϵ_b , ϵ_c

$$\epsilon_a = \epsilon_x \cos^2\theta_a + \epsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a \quad \text{——} \quad (A)$$

$$\epsilon_b = \epsilon_x \cos^2\theta_b + \epsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b \quad \text{——} \quad (B)$$

$$\epsilon_c = \epsilon_x \cos^2\theta_c + \epsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c \quad \text{——} \quad (C)$$

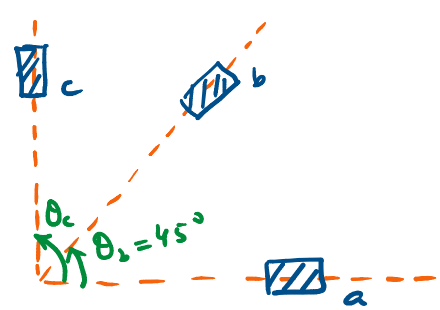
Two standard configurations: 45° and 60°

- 45° configuration: If $\theta_a = 0^\circ$, $\theta_b = 45^\circ$, $\theta_c = 90^\circ$

eq (A): $\epsilon_a = \epsilon_x$

eq (B): $2\epsilon_b - (\epsilon_a + \epsilon_c) = \gamma_{xy}$

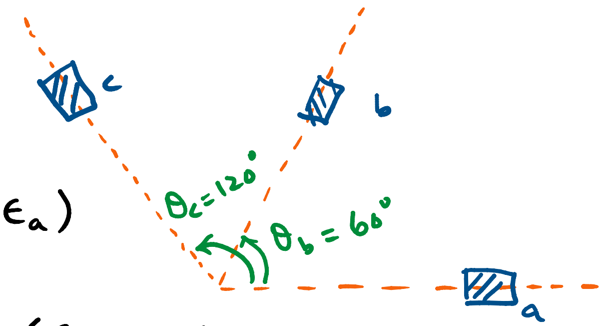
eq (C): $\epsilon_c = \epsilon_y$



- 60° configuration: If $\theta_a = 0^\circ$, $\theta_b = 60^\circ$, $\theta_c = 120^\circ$

• 60° configuration: If $\theta_a = 0^\circ$, $\theta_b = 60^\circ$, $\theta_c = 120^\circ$

eq Ⓐ : $\epsilon_a = \epsilon_x$



eq Ⓑ :

$$\epsilon_y = \frac{1}{3}(2\epsilon_b + 2\epsilon_c - \epsilon_a)$$

eq Ⓒ : $$\gamma_{xy} = \frac{2}{\sqrt{3}}(\epsilon_b - \epsilon_c)$$

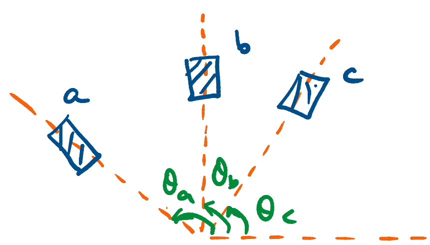
P# 10.25: Given:

configuration: 45°

$\epsilon_a = -200 \times 10^{-6}$

$\epsilon_b = 300 \times 10^{-6}$

$\epsilon_c = 250 \times 10^{-6}$



Find:

$\epsilon_{max} = ?$

$\epsilon_{min} = ?$

Assumptions:

• Hooke's Law

Gov: Eqns:

eq Ⓐ - eq Ⓒ : , ϵ_{max}, ϵ_{min}

Solution:

$\theta_a = 135^\circ$, $\theta_b = 90^\circ$, $\theta_c = 45^\circ$

eq Ⓐ : $\epsilon_x - \gamma_{xy} = -700 \times 10^{-6}$ —— ①

eq Ⓑ : $\epsilon_y = 300 \times 10^{-6}$ —— ②

eq Ⓒ : $\epsilon_x + \gamma_{xy} = 200 \times 10^{-6}$ —— Ⓒ

Solve:

$\epsilon_x = -250 \times 10^{-6}$, $\gamma_{xy} = 450 \times 10^{-6}$

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = 25\times10^{-6}$$

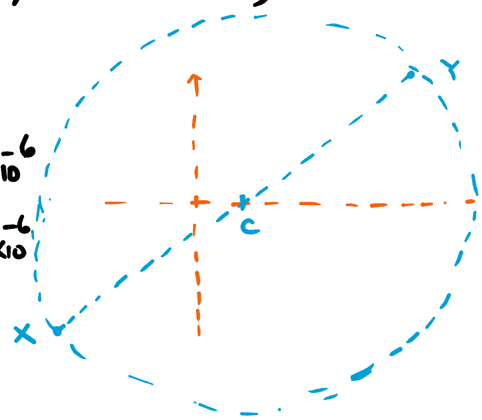
$X\ (-250\times10^{-6}, -225\times10^{-6})$

$Y\ (300\times10^{-6}, 225\times10^{-6})$

$R = 355\times10^{-6}$

$\epsilon_{max} = \epsilon_{avg} + R = 380\times10^{-6}$

$\epsilon_{min} = \epsilon_{avg} - R = -330\times10^{-6}$



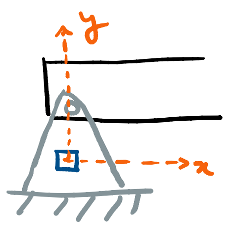
P # 10.3:

$\epsilon_x = 200\times10^{-6}$

$\epsilon_y = 180\times10^{-6}$

$\gamma_{xy} = -300\times10^{-6}$

$\theta = 60^\circ$ (CCW)



Eqns: eq 1A — 3A

find:

- $\epsilon_{x'}$, $\epsilon_{y'}$, $\gamma_{x'y'}$
- Sketch deformed element

Answers: $\epsilon_{x'} = 55.1\times10^{-6}$

$\epsilon_{y'} = 325\times10^{-6}$

$\gamma_{x'y'} = 133\times10^{-6}$

$\boxed{\text{Chap 8, 9, 10 (Exam 3): Dec 5th}}$

CHAPTER # 11

DESIGN OF BEAMS & SHAFTS

- Recall:
  - > V & M diagrams
  - > δ (flexural stress)
  - > τ

§ 11.1 : Reading Assignment

§ 11.2 : Prismatic Beam Design:

- Bending Design Criteria:
  - > Section Modulus (S)
    geometric property

$$S = \frac{I}{c}$$

Recall: $\delta_{max} = \dfrac{Mc}{I}$

$$\delta_{max} = \frac{M_{max}}{S_{reqd}} \qquad [S_{reqd} = S_{min}]$$

$$S_{reqd} = S_{min} = \frac{M_{max}}{\delta_{allow,max}}$$

> Design Criteria: $S > S_{reqd}$

> $\tau_{allow} \geq \dfrac{VQ}{It}$ $\Rightarrow$ selected mat'l: should be able to withstand the shear produced

- Read from book to get more insight.

Example # 11.1:

Given: Mat'l : steel

$\delta_{allow} = 24\ ksi$

$\tau_{allow} = 14.5\ ksi$

$\sigma_{allow} = 2\ldots$

$\tau_{allow} = 14.5$ ksi

Find:

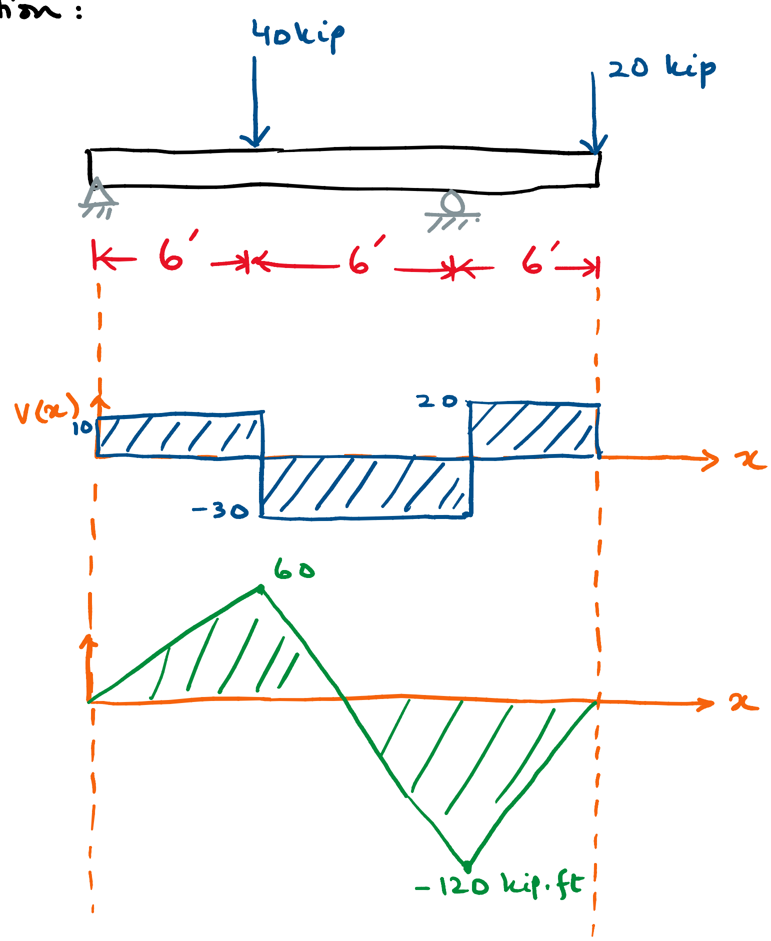
Select W-shape beam from Appendix B

Assumptions:

Governing Eqns:

- $\sigma_{max} = \dfrac{Mc}{I}$
- $\tau = \dfrac{VQ}{It}$

Solution:



$$\sigma_{allow} = \sigma_{max} = \frac{M_{max}}{S_{reqd}}$$

$$24 = \frac{(120 \times 12)}{S_{reqd}}$$

$$S_{reqd} = S_{min} = 60 \text{ in}^3$$

$$S_{reqd} = S_{min} = 60 \text{ in}^3$$

Choose a beam with $S > S_{min}$

Go to Appendix B:

List of beams

- W 12×50 — 64.7
- W 8×67 — 60.4
- W 18×40 — 68.4
- W 14×43 — 62.7

(12: in, 50: lb/ft)

$50 \frac{lb}{ft} = 0.05 \frac{kip}{ft}$

$0.05 \times 18' = 0.9$ kip

$67 \frac{lb}{ft} = 0.067$ kip/ft

$40 \frac{lb}{ft} = 0.04 \times 18$

$= 0.72$ kip

[LOWEST]

SELECTION:

$\boxed{W\ 18 \times 40}$ Answer

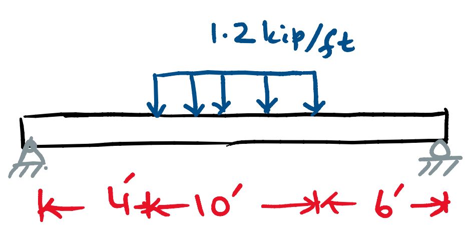
$$\tau = \underbrace{\frac{VQ}{It}}_{\text{Calculate}} < 14.5 \text{ ksi}$$

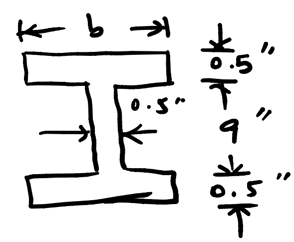
$$\tau = \frac{V_{max}}{A_{web}}$$ [approximation or estimation]

For problems in this course: Do NOT do estimation.

Problem: Given:

$\sigma_{allow} = 22$ ksi





Find:

b = ?

Assumptions:

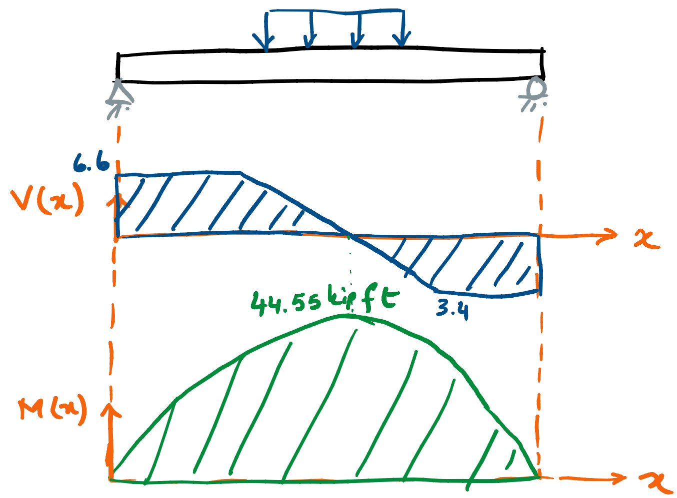
Gov. Eqns:

$S_{req} = \frac{I}{c}$

Gov: Eqns: $S_{req} = \frac{I}{c}$ ✓

$$\delta_{allow} = \frac{M}{S}$$ ✓

Solution: Choose : the beam



$$I = \frac{1}{12}(b)h^3 - \frac{1}{12}(b-0.5)(h^3)$$

$$I = 22.58\,b + 30.37$$

$$\delta_{allow} = \frac{M_{max}\,c}{I}$$

$$22 = \frac{(44.55 \times 12)(5)}{22.58b + 30.37}$$

$$b = 4.04 \text{ in}$$

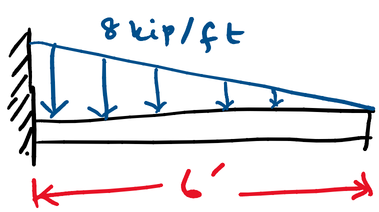
$$\boxed{b = 4.25 \text{ in}}$$ Answer

$$\tau_{allow} = ✓$$

$$\tau_{calculation} = \frac{VQ}{It}$$

P # 11.7:

Given:



shortest depth d:

$$\delta_{allow} = 24 \text{ ksi}$$

shortest depth d:

$\sigma_{allow} = 24$ ksi

$\tau_{allow} = 14$ ksi

$S_{reqd} = \dfrac{M_{max}}{\sigma_{allow}}$ → moment diagram

$= \dfrac{(48 \times 12)}{24}$

$S_{reqd} = 24 \text{ in}^3$

$\underbrace{W12\times22}_{\text{shortest depth } d}$ and W14×22

$S = 25.4 \text{ in}^3$, $d = 12.31$ in, $t_w = 0.26$ in

$\tau = \dfrac{VQ}{It} < \tau_{allow}$

$7.5 \text{ ksi} < 14 \text{ ksi}$

Use : W12×22

Lecture 31

CHAPTER # 12

DEFLECTION OF BEAMS & SHAFTS

§12.1: ELASTIC CURVE:

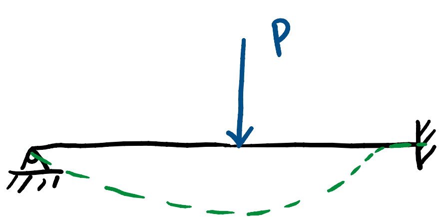
- Deflection Limit:

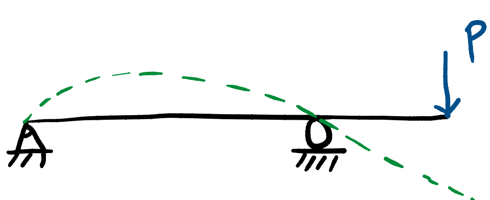
To avoid damage to the structural integrity and stability

E.g:

Electric motor: vibrating shaft will damage the bearings and other parts

- Deflections need to be determined
- Sketch deflected shape ⇒ deflection curve ⇒ elastic curve

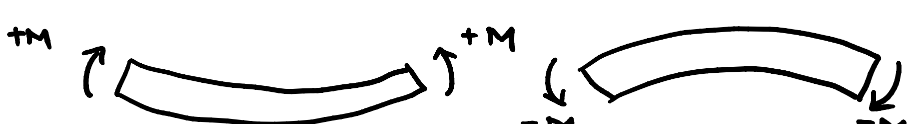


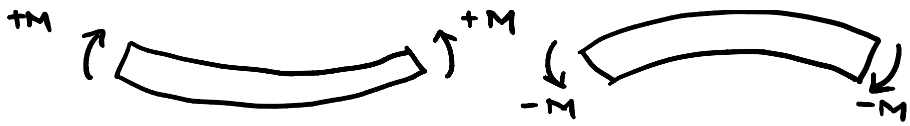


PIN SUPPORTS: resist forces ⇒ restrict displacement

FIXED WALL SUPPORT: resist moments and forces ⇒ restrict rotation as well as displacement

RECALL:

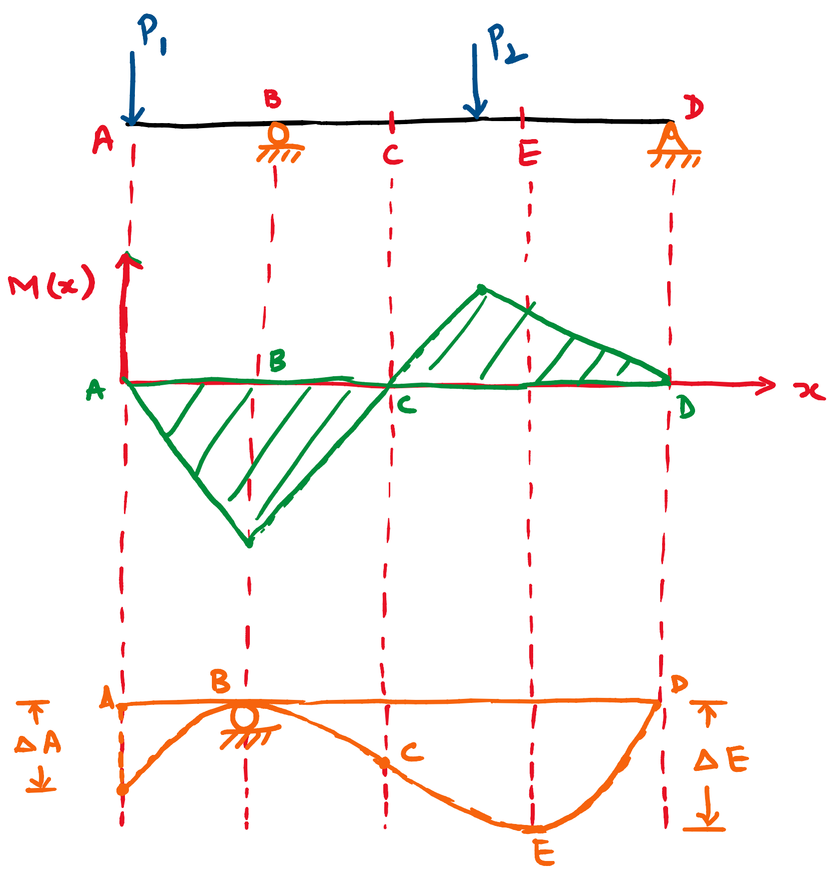




+ve internal moment $\Rightarrow$ concave upward

-ve internal moment $\Rightarrow$ concave downward

- IF elastic curve is difficult: draw the moment diagram first

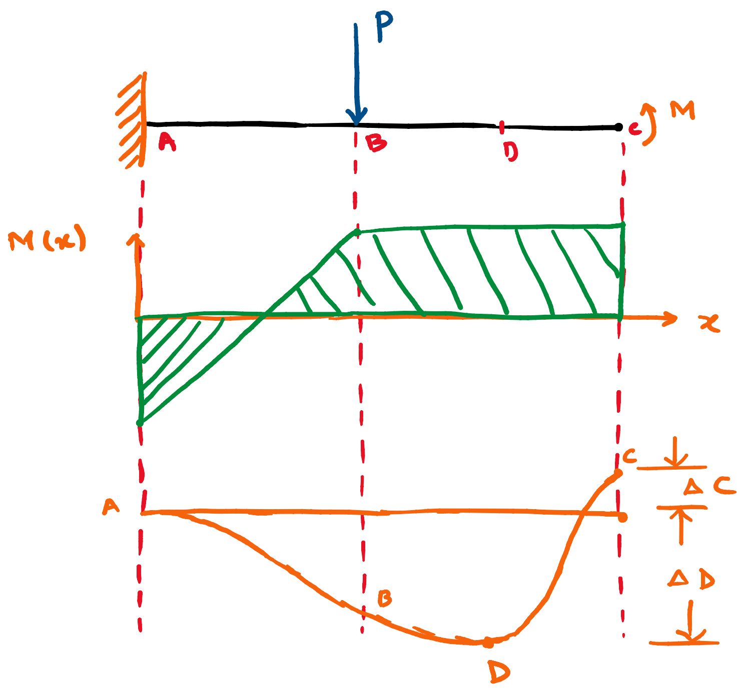


- Roller and pin supports $\Rightarrow$ ΔB and $\Delta D = 0$
- -ve moment region $\Rightarrow$ elastic curve is concave downward
- +ve moment region $\Rightarrow$ elastic curve is concave upward
- Due to the above FACT:

  inflection must exist @ pt. C
- ΔA and ΔE are displacements

inflection must exist @ pt. C

- ΔA and ΔE are displacements
- At pt. E: the slope of the curve is ZERO
  ⇒ beam deflection is maximum
- $\Delta E > \Delta A$ or $\Delta E < \Delta A$ ⇒ depends on the magnitude of P_1 & P_2 and the location of roller @ B



- At fixed support: zero displacement and zero slope @ A
- Largest displacement would occur either @ D or C

$$\frac{1}{\rho} = \frac{M}{EI} \quad \text{———} \; (1)$$

where:

ρ = radius of curvature @ a pt. on elastic curve $\left[\frac{1}{\rho} = \text{curvature}\right]$

M = internal moment in beam @ a pt.

M = internal moment in beam @ a pt.

E = Modulus of elasticity or Young's modulus

I = moment of inertia about N.A.

EI = flexural rigidity (Always +ve)

- IF M is +ve $\Rightarrow$ ρ extends above the beam
- IF M is -ve $\Rightarrow$ ρ extends below the beam
- We know:

$$\sigma = -\frac{My}{I} \quad \text{—— (2)}$$

$$-\frac{\sigma}{y} = \frac{E}{\rho} \qquad \text{[comparing eq (1) and (2)]}$$

$$\therefore \quad \frac{1}{\rho} = -\frac{\sigma}{Ey} \quad \text{—— (3)}$$

§12.2: SLOPE AND DISPLACEMENT BY INTEGRATION:

$$EI\frac{d^4v}{dx^4} = w(x) \quad \text{—— (4)}$$

$$EI\frac{d^3v}{dx^3} = V(x) \quad \text{—— (5)}$$

$$EI\frac{d^2v}{dx^2} = M(x) \quad \text{—— (6)}$$

Boundary Conditions (B.C.s) Table 12-1, Page 582

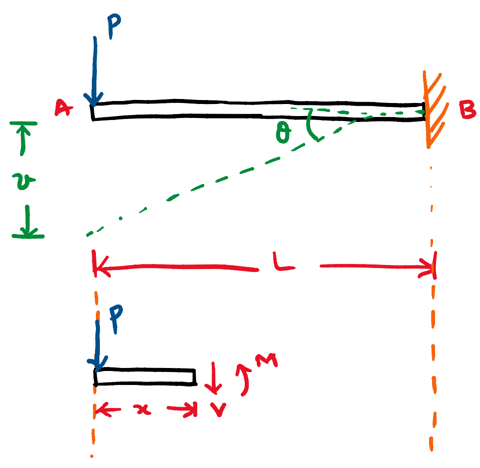
$$\underbrace{\theta = \frac{dv}{dx}}_{\text{slope}} \quad , \quad \underbrace{v = \Delta}_{\text{displacement}}$$

Example # 12.2:

find:

$\theta = ?$

$v = ?$



$\Sigma M_x = 0;$

$M + Px = 0$

$M = -Px$

eq (6) $\Rightarrow$ $$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -Px$$

Integrate twice:

$$EI\frac{dv}{dx} = -\frac{Px^2}{2} + C_1 \quad \text{—— (a)}$$

$$EI\,v = -\frac{Px^3}{6} + C_1x + C_2 \quad \text{—— (b)}$$

B.C: Table 12.1: B.C. #5 (with fixed end)

B.C: Table 12.1: B.C. #5 (with fixed end)

@ $x = L$; $\frac{dv}{dx} = \theta = 0$

@ $x = L$; $v = \Delta = 0$

eq ⓐ $\Rightarrow$ $EI(0) = -\frac{PL^2}{2} + C_1$

$$\boxed{C_1 = \frac{PL^2}{2}}$$

eq ⓑ $\Rightarrow$ $EI(0) = -\frac{PL^3}{6} + \frac{PL^3}{2} + C_2$

$$\boxed{C_2 = -\frac{PL^3}{3}}$$

eq ⓐ $\Rightarrow$ $EI\theta = -\frac{Px^2}{2} + \frac{PL^2}{2}$

$$\boxed{\theta = \frac{P}{2EI}(L^2 - x^2)} \quad \text{Ⓐ}$$

eq ⓑ $\Rightarrow$ $EIv = -\frac{Px^3}{6} + \frac{PL^2}{2}x - \frac{PL^3}{3}$

$$\boxed{v = \frac{P}{6EI}(-x^3 + 3L^2x - 2L^3)} \quad \text{Ⓑ}$$

At $x = 0$; (@ pt. A on the beam):

eq Ⓐ $\Rightarrow$ $\boxed{\theta_A = \frac{PL^2}{2EI}}$

eq Ⓑ $\Rightarrow$ $v_A = \frac{P}{6EI}(-2L^3)$

$$\boxed{v_A = -\frac{PL^3}{3EI}}$$

§12.5: METHOD OF SUPERPOSITION:

§12.5 : METHOD OF SUPERPOSITION :

$$eq\ (4) \Rightarrow w(x) = EI\frac{d^4 v}{dx^4}$$

$$w(x) \propto v(x)$$

- $w(x)$ does not significantly change the geometry of beam /shaft
- deflections due to separate loadings can be superimposed . e.g.: v_1 and $v_2 \Rightarrow v_1 + v_2 = v$

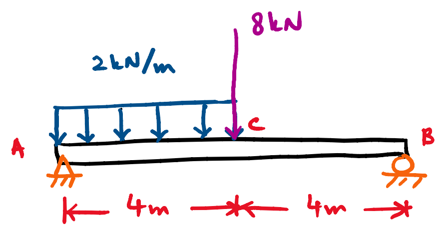
REFER and study App: C

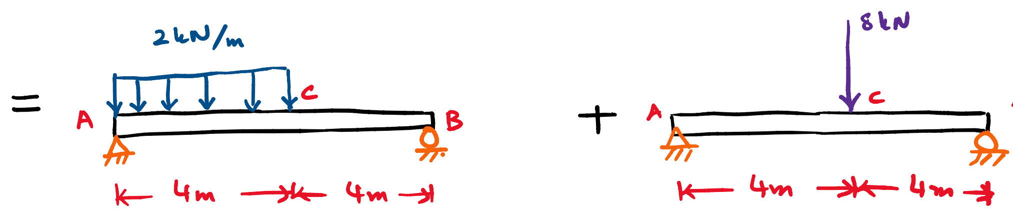
Ex # 12.13 :

find :

$\theta_A = ?$

$v_c = ?$





Appendix C : Pg 814 – 815

Distributed Load

$$\theta_{A_1} = \frac{3wL^3}{128EI}$$

$$\theta_{A_1} = \frac{3(2)(8)^3}{128\ EI} = \frac{24}{EI}\ kN.m^2$$

$$v_{c_1} = \frac{5wL^4}{768EI}$$

$$= \frac{5(2)(8)^4}{768\ EI} = \frac{53.33}{EI}\ kN.m^3$$

Concentrated Load

$$\theta_{A_2} = \frac{PL^2}{16EI}$$

$$\theta_{A_2} = \frac{8(8)^2}{16EI} = \frac{32}{EI}\ kN.m^2$$

$$v_{c_2} = \frac{PL^3}{48EI}$$

$$v_{c_2} = \frac{8(8)^3}{48\ EI} = \frac{85.33}{EI}\ kN.m^3$$

$$\theta_A = \theta_{A_1} + \theta_{A_2}$$

$$= \frac{24+32}{EI}$$

$$= \frac{24+32}{EI}$$

$$\boxed{\theta_A = \frac{56}{EI} \ kN.m^2}$$ Answer

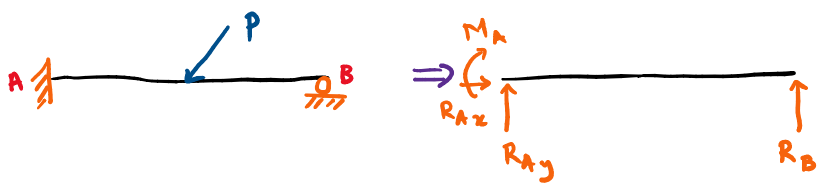
$$v_C = v_{C_1} + v_{C_2}$$

$$v_C = \frac{53.33+85.33}{EI}$$

$$\boxed{v_C = \frac{139}{EI} \ kN.m^3}$$ Answer

§12.6: Statically Indeterminate Beams and Shafts:

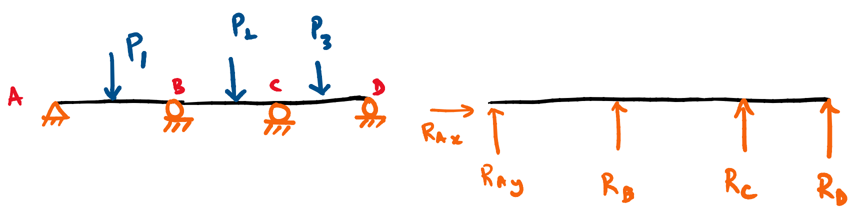
- Recall: Chapter # 4 and Chapter # 5
- If # of unknowns > # of equilibrium equations
- REDUNDANTS: additional supports that are not needed
- # of redundants ⇒ degree of indeterminacy



4 UNKNOWNS, 3 EQNS

$4-3=1$ ⇒ first degree of indeterminacy

- REDUNDANT: Could be: R_{Ay}, R_B or M_A



5 UNKNOWNS ⇒ $5-3=2$ (2nd degree indeterminacy)

- REDUNDANTS: 2 reactions: R_{Ay}, R_B, R_C or R_D

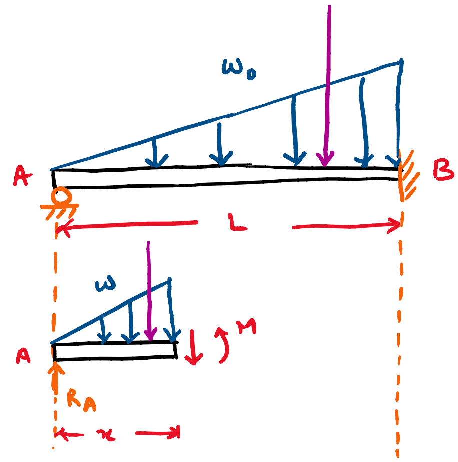
§12.7: Statically Indeterminate Beams and Shafts: METHOD OF INTEGRATION

- Recall: §12.2

$$\frac{d^2v}{dx^2} = \frac{M}{EI} \quad \text{where} \quad M = f(x)$$

Here 'M' can be expressed in terms of unknown redundants.

Example 12.17:



$F = \frac{1}{2} L w_0$

$F = \frac{1}{2} w x$

Similar Δs:

$$\frac{w_0}{L} = \frac{w}{x}$$

$$w = \frac{w_0 x}{L}$$

$$\Rightarrow F = \frac{w_0 x^2}{2L}$$

$\Sigma M_x = 0;$

$$-R_A x + M + \left(\frac{1}{2} w x\right)\left(\frac{x}{3}\right) = 0$$

$$-R_A x + M + \left[\frac{1}{2} \frac{w_0 x^2}{L}\right]\left(\frac{x}{3}\right) = 0$$

$$M = R_A x - \frac{w_0 x^3}{6L} \quad \text{—— A}$$

$$\text{eq } 6 \Rightarrow \quad EI \frac{d^2 v}{dx^2} = M(x)$$

$$EI \frac{d^2 v}{dx^2} = R_A x - \frac{w_0 x^3}{6L}$$

$$EI \frac{dv}{dx} = \frac{R_A x^2}{2} - \frac{w_0 x^4}{24L} + C_1 \quad \text{—— (i)}$$

$$EI\, v = \frac{R_A x^3}{6} - \frac{w_0 x^5}{120L} + C_1 x + C_2 \quad \text{—— (ii)}$$

Boundary conditions:

- Roller support : @ $x = 0$; $v = 0 \ (\Delta = 0)$
- fixed end : @ $x = L$; $\frac{dv}{dx} = 0 \ (\theta = 0)$, $v = 0 \ (\Delta = 0)$

B.C. 1: eq(ii) : $EI(0) = 0 - 0 + C_1(0) + C$

$$C_2 = 0$$

B.C. 2: eq (i) : $EI(0) = \frac{R_A L^2}{2} - \frac{w_0 L^4}{24L} + C_1$

$$C_1 = \frac{w_0 L^3}{24} - \frac{R_A L^2}{2} \quad \text{—— (iii)}$$

$$C_1 = \frac{w_0 L^3}{24} - \frac{R_A L^2}{2} \quad \text{—(iii)}$$

B.C. 2: eq (ii) : $EI(0) = \frac{R_A L^3}{6} - \frac{w_0 L^5}{120L} + C_1 L + C_2$

$$C_1 = \frac{w_0 L^3}{120} - \frac{R_A L^2}{6} \quad \text{—(iv)}$$

Subtract eq (iii) and eq (iv)

$$C_1 = \frac{w_0 L^3}{24} - \frac{R_A L^2}{2}$$

$$\pm C_1 = \pm \frac{w_0 L^3}{120} \mp \frac{R_A L^2}{6}$$

$$0 = \left(\frac{w_0 L^3}{24} - \frac{w_0 L^3}{120}\right) - \frac{R_A L^2}{2} + \frac{R_A L^2}{6}$$

$$\boxed{R_A = \frac{w_0 L}{10}}$$

eq (iv) : $C_1 = \frac{w_0 L^3}{120} - \left(\frac{w_0 L}{10}\right)\frac{L^2}{6}$

$$C_1 = -\frac{w_0 L^3}{120}$$

eq Ⓐ $\Rightarrow$ $\boxed{M = \frac{w_0 L}{10}x - \frac{w_0 x^3}{6L}}$ Answer

§12.9: METHOD OF SUPERPOSITION:

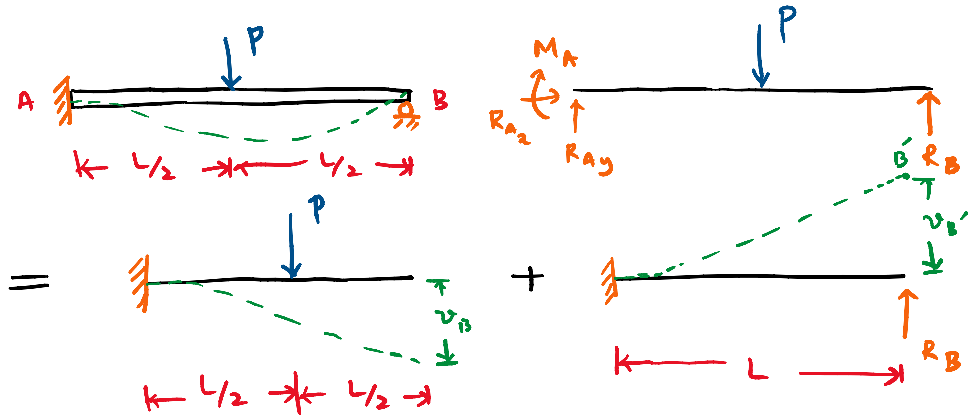
- Recall: Previously used for axially loaded bars and torsionally loaded shafts
- Apply to statically indeterminate beams/shafts

STEPS:

- 1st Step:
  - > identify redundant support reactions
  - > Remove the redundant support
- 2nd step:
  - > Load beam with redundant reaction only

FORCE METHOD:

FORCE METHOD:



Appendix C: $v_B = \frac{5PL^3}{48EI}$, $v_B' = \frac{R_B L^3}{3EI}$

$$v = 0 = -v_B + v_B' \quad \text{—— I}$$

$$0 = -\frac{5PL^3}{48EI} + \frac{R_B L^3}{3EI}$$

$$\boxed{R_B = \frac{5}{16} P}$$

$\Sigma F_x = 0$; $\quad R_{Ax} = 0$

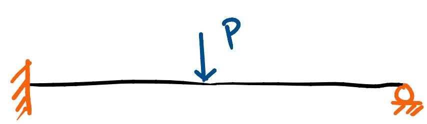
$\Sigma F_y = 0$; $\quad R_{Ay} - P + \frac{5}{16} P = 0$

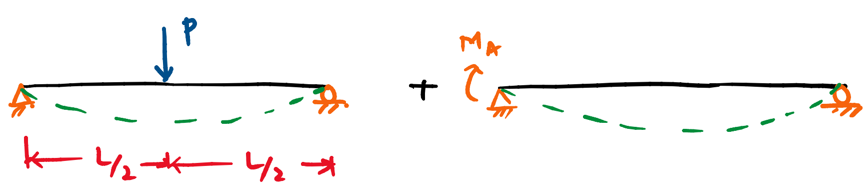
$$\boxed{R_{Ay} = \frac{11}{16} P}$$

$\Sigma M_B = 0$; $\quad M_A - P\left(\frac{L}{2}\right) + \frac{5P}{16}(L) = 0$

$$\boxed{M_A = \frac{3PL}{16}}$$

Instead of R_B: choose $M_A = 0$ (redundant)



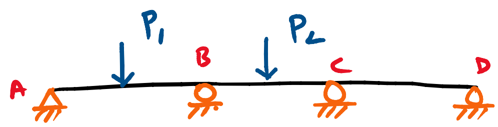


App C: $\theta_A = PL^2$, $\theta_A' = M_A L$

App C: $\theta_A = \frac{PL^2}{16EI}$, $\theta_A' = \frac{M_A L}{3EI}$

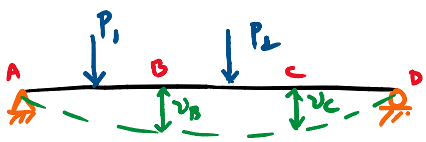
$$0 = \theta = \theta_A + \theta_A'$$

$$\Rightarrow \boxed{M_A = -\frac{3PL}{16}}$$ $\Rightarrow$ –ve sign means M acts in opp. direction to what is shown



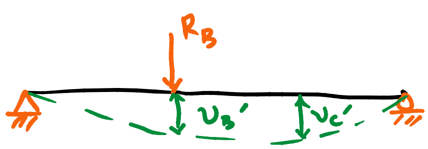
5 unknowns $\Rightarrow$ 2nd degree

- Choose R_B and R_C :

= 

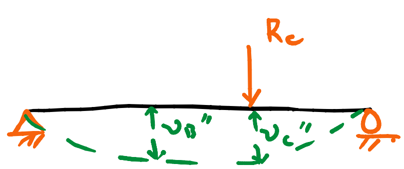
+

- Apply R_B only:



+

- Apply R_C only:



$$0 = v_B + v_B' + v_B'' \quad \text{—(1)}$$

$$0 = v_C + v_C' + v_C'' \quad \text{—(2)}$$