

The Latent Time-to-Criterion Model: Research Design Considerations  
for Optimizing Parameter Estimate Accuracy and Precision

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**ABSTRACT**

The Latent Time-to-Criterion Model: Research Design Considerations  
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The latent time-to-criterion (T2C) model is arguably a more policy action-oriented approach to modeling longitudinal data than the traditional intercept-slope growth model. However, despite their mathematical equivalence (in terms of model parameters), the traditional intercept-slope latent growth model and the latent time-to-criterion (T2C) latent models were hypothesized to differ in their *parameter estimation performance* due in part to typical attrition-type missingness in longitudinal studies that would uniquely affect the time-to-criterion ( $\tau$ ) parameter stability. To investigate this phenomenon, this dissertation used a Monte Carlo simulation that systematically varied attrition-type missingness, along with sample size, number of time points data is collected, pre-defined criterion levels, measure reliability, and growth rate variability. Simulation results showed that, in smaller sample sizes of  $N = 100$ , estimates of the time-to

criterion (tau) factor and its predictor effect can be biased and suffer from lower power when missingness and growth rate variability are high. Further, these effects were exacerbated by poor scale reliability (i.e.,  $\alpha \leq .4$ ). Not surprisingly, sample size was also found to be the key predictor of bias in both parameters of interest: a sample size of at least  $N = 250$  yielded parameter estimates with minimal to no relative bias or standard error bias as well as power close to or at 100%. In the real data analysis, which used a subsample of publicly available data from the NCES Early Childhood Longitudinal Study 2010-2011 kindergarten cohort, both the traditional latent growth model and the latent T2C model were demonstrated as a concrete example of a scenario with a large sample size and modest attrition – a scenario in which the parameter estimates would not be expected to be biased. Concrete research design recommendations for applied researchers wishing to use the T2C model, as well as future research directions, are discussed.

*Keywords:* latent time-to-criterion, latent growth model, longitudinal, attrition, missingness, measurement time points, heterogeneity, reliability

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## TABLE OF CONTENTS

<b>LIST OF TABLES</b> .....	i
<b>LIST OF FIGURES</b> .....	v
<b>CHAPTER 1</b> .....	1
Longitudinal Data Analyses and the Time-to-Criterion Model .....	1
Motivation and Educational Research Context.....	2
Context 1: Emergent Bilingual K-12 Students: Time-to-Reclassification.....	3
Context 2. First-Generation College Students: Time-to-Degree Completion.....	5
Context 3. Individualized Learning: Time-to-Mastery .....	7
The T2C Model.....	8
Traditional Latent Intercept-Slope Growth Model.....	9
Present Study.....	14
<b>CHAPTER 2</b> .....	19
Monte Carlo Simulation Study .....	19
Method .....	19
Data Generation.....	19
Analysis Plan.....	20
Results.....	21
<b>CHAPTER 3</b> .....	38
Real Data Application Example using ECLS 2010 Subsample.....	38
Method .....	38
Data Source .....	38
Results.....	39
<b>CHAPTER 4</b> .....	46
Discussion .....	46
<b>REFERENCES</b> .....	51
<b>APPENDIX A – Statistical Code</b> .....	63
<b>APPENDIX B – Figures</b> .....	66
<b>APPENDIX C – Tables</b> .....	102

## LIST OF TABLES

<b>Table 1</b> .....	22
<i>Traditional Growth Model: Relative Bias Results on Selected Conditions</i>	
<b>Table 2</b> .....	23
<i>Traditional Growth Model: Standard Error Bias Results on Selected Conditions</i>	
<b>Table 3</b> .....	24
<i>Traditional Growth Model: Power Results on Selected Conditions</i>	
<b>Table 4</b> .....	25
<i>T2C Model: Relative Bias Results on Selected Conditions</i>	
<b>Table 5</b> .....	26
<i>T2C Model: Standard Error Bias Results on Selected Conditions</i>	
<b>Table 6</b> .....	27
<i>T2C Model: Power Results on Selected Conditions</i>	
<b>Table C1</b> .....	102
<i>3-time point Models with Measure Reliability 0.8: Relative Bias Results on Selected Conditions</i>	
<b>Table C2</b> .....	103
<i>5-time point Models with Measure Reliability 0.8: Relative Bias Results on Selected Conditions</i>	
<b>Table C3</b> .....	104
<i>50-time point Models with Measure Reliability 0.8: Relative Bias Results on Selected Conditions</i>	
<b>Table C4</b> .....	105
<i>3-time point Models with Measure Reliability 0.6: Relative Bias Results on Selected Conditions</i>	

<b>Table C5</b> .....	106
<i>5-time point Models with Measure Reliability 0.6: Relative Bias Results on Selected Conditions</i>	
<b>Table C6</b> .....	107
<i>50-time point Models with Measure Reliability 0.6: Relative Bias Results on Selected Conditions</i>	
<b>Table C7</b> .....	108
<i>3-time point Models with Measure Reliability 0.4: Relative Bias Results on Selected Conditions</i>	
<b>Table C8</b> .....	109
<i>5-time point Models with Measure Reliability 0.4: Relative Bias Results on Selected Conditions</i>	
<b>Table C9</b> .....	110
<i>50-time point Models with Measure Reliability 0.4: Relative Bias Results on Selected Conditions</i>	
<b>Table C10</b> .....	111
<i>3-time point Models with Measure Reliability 0.8: Power Results on Selected Conditions</i>	
<b>Table C11</b> .....	112
<i>5-time point Models with Measure Reliability 0.8: Power Results on Selected Conditions</i>	
<b>Table C12</b> .....	113
<i>50-time point Models with Measure Reliability 0.8: Power Results on Selected Conditions</i>	
<b>Table C13</b> .....	114
<i>3-time point Models with Measure Reliability 0.6: Power Results on Selected Conditions</i>	
<b>Table C14</b> .....	115
<i>5-time point Models with Measure Reliability 0.6: Power Results on Selected Conditions</i>	

<b>Table C15</b> .....	116
<i>50-time point Models with Measure Reliability 0.6: Power Results on Selected Conditions</i>	
<b>Table C16</b> .....	117
<i>3-time point Models with Measure Reliability 0.4: Power Results on Selected Conditions</i>	
<b>Table C17</b> .....	118
<i>5-time point Models with Measure Reliability 0.4: Power Results on Selected Conditions</i>	
<b>Table C18</b> .....	119
<i>50-time point Models with Measure Reliability 0.4: Power Results on Selected Conditions</i>	
<b>Table C19</b> .....	120
<i>3-time point Models with Measure Reliability 0.8: Standard Error Bias Results on Selected Conditions</i>	
<b>Table C20</b> .....	121
<i>5-time point Models with Measure Reliability 0.8: Standard Error Bias Results on Selected Conditions</i>	
<b>Table C21</b> .....	122
<i>50-time point Models with Measure Reliability 0.8: Standard Error Bias Results on Selected Conditions</i>	
<b>Table C22</b> .....	123
<i>3-time point Models with Measure Reliability 0.6: Standard Error Bias Results on Selected Conditions</i>	
<b>Table C23</b> .....	124
<i>5-time point Models with Measure Reliability 0.6: Standard Error Bias Results on Selected Conditions</i>	
<b>Table C24</b> .....	125
<i>50-time point Models with Measure Reliability 0.6: Standard Error Bias Results on Selected Conditions</i>	
<b>Table C25</b> .....	126
<i>3-time point Models with Measure Reliability 0.4: Standard Error Bias Results on Selected Conditions</i>	

**Table C26** ..... 127  
*5-time point Models with Measure Reliability 0.4: Standard Error Bias Results on Selected Conditions*

**Table C27** ..... 128  
*50-time point Models with Measure Reliability 0.4: Standard Error Bias Results on Selected Conditions*

## LIST OF FIGURES

<b>Figure 1</b> .....	10
<i>Path Diagram of a General 2-Factor Latent Variable Model</i>	
<b>Figure 2</b> .....	12
<i>Path Diagram of Hypothetical 2-Factor Latent (Traditional) Intercept-Slope Growth Model</i>	
<b>Figure 3</b> .....	14
<i>Path Diagram of Hypothetical Latent T2C Model</i>	
<b>Figure 4</b> .....	32
<i>Relative Bias for Time-to-Criterion Factor by Indicator Level</i>	
<b>Figure 5</b> .....	33
<i>Relative Bias for Predictor Effect on Time-to-Criterion Factor by Indicator Level</i>	
<b>Figure 6</b> .....	34
<i>Power to Detect Time-to-Criterion Factor by Indicator Level</i>	
<b>Figure 7</b> .....	35
<i>Power to Detect Predictor Effect on Time-to-Criterion Factor by Indicator Level</i>	
<b>Figure 8</b> .....	36
<i>Standard Error Bias for Time-to-Criterion Factor by Indicator Level</i>	
<b>Figure 9</b> .....	37
<i>Standard Error Bias for Predictor Effect on Time-to-Criterion Factor by Indicator Level</i>	
<b>Figure 10</b> .....	41
<i>Model-Implied Individual Growth Trajectories for ECLS EBRS</i>	

<b>Figure 11</b> .....	43
<i>Model-Implied Aggregate Growth Trajectories by preLAS Level for ECLS EBRS</i>	
<b>Figure 12</b> .....	43
<i>Path Diagram of Results for the Traditional Growth Model for ECLS EBRS</i>	
<b>Figure 13</b> .....	44
<i>Relationship between Baseline/Fall preLAS Scores and Model-Implied Time-to-Criterion</i>	
<b>Figure 14</b> .....	45
<i>Path Diagram of Linear T2C Model Results for ECLS EBRS</i>	
<b>Figure B1</b> .....	66
<i>3 Indicators: Relative Bias for Latent Time-to-Criterion Factor</i>	
<b>Figure B2</b> .....	67
<i>3 Indicators: Relative Bias for Predictor Effect on Latent Time-to-Criterion Factor</i>	
<b>Figure B4</b> .....	69
<i>3 Indicators: Relative Bias for Predictor Effect on Latent Slope Factor</i>	
<b>Figure B5</b> .....	70
<i>3 Indicators: Power for Detecting Latent Time-to-Criterion Factor</i>	
<b>Figure B6</b> .....	71
<i>3 Indicators: Power for Detecting Predictor Effect on Time-to-Criterion Factor</i>	

<b>Figure B7</b> .....	72
<i>3 Indicators: Power for Detecting Latent Slope Factor</i>	
<b>Figure B8</b> .....	73
<i>3 Indicators: Power for Detecting Predictor Effect on Latent Slope Factor</i>	
<b>Figure B9</b> .....	74
<i>3 Indicators: Standard Error Bias for Latent Time-to-Criterion Factor</i>	
<b>Figure B10</b> .....	75
<i>3 Indicators: Standard Error Bias for Predictor Effect on Latent Time-to-Criterion Factor</i>	
<b>Figure B11</b> .....	76
<i>3 Indicators: Standard Error Bias for Latent Slope Factor</i>	
<b>Figure B12</b> .....	77
<i>3 Indicators: Standard Error Bias for Predictor Effect on Latent Slope Factor</i>	
<b>Figure B13</b> .....	78
<i>5 Indicators: Relative Bias for Latent Time-to-Criterion Factor</i>	
<b>Figure B14</b> .....	79
<i>5 Indicators: Relative Bias for Predictor Effect on Latent Time-to-Criterion Factor</i>	
<b>Figure B15</b> .....	80
<i>5 Indicators: Relative Bias for Latent Slope Factor</i>	
<b>Figure B16</b> .....	81
<i>5 Indicators: Relative Bias for Predictor Effect on Latent Slope Factor</i>	

<b>Figure B17</b> .....	82
<i>5 Indicators: Power for Detecting Latent Time-to-Criterion Factor</i>	
<b>Figure B18</b> .....	83
<i>5 Indicators: Power for Detecting Predictor Effect on Time-to-Criterion Factor</i>	
<b>Figure B19</b> .....	84
<i>5 Indicators: Power for Detecting Latent Slope Factor</i>	
<b>Figure B20</b> .....	85
<i>5 Indicators: Power for Detecting Predictor Effect on Latent Slope Factor</i>	
<b>Figure B21</b> .....	86
<i>5 Indicators: Standard Error Bias for Latent Time-to-Criterion Factor</i>	
<b>Figure B22</b> .....	87
<i>5 Indicators: Standard Error Bias for Predictor Effect on Latent Time-to-Criterion Factor</i>	
<b>Figure B23</b> .....	88
<i>5 Indicators: Standard Error Bias for Latent Slope Factor</i>	
<b>Figure B24</b> .....	89
<i>5 Indicators: Standard Error Bias for Predictor Effect on Latent Slope Factor</i>	
<b>Figure B25</b> .....	90
<i>50 Indicators: Relative Bias for Latent Time-to-Criterion Factor</i>	
<b>Figure B26</b> .....	91
<i>50 Indicators: Relative Bias for Predictor Effect on Latent Time-to-Criterion Factor</i>	

<b>Figure B27</b> .....	92
<i>50 Indicators: Relative Bias for Latent Slope Factor</i>	
<b>Figure B28</b> .....	93
<i>50 Indicators: Relative Bias for Predictor Effect on Latent Slope Factor</i>	
<b>Figure B29</b> .....	94
<i>50 Indicators: Power for Detecting Latent Time-to-Criterion Factor</i>	
<b>Figure B30</b> .....	95
<i>50 Indicators: Power for Detecting Predictor Effect on Time-to-Criterion Factor</i>	
<b>Figure B31</b> .....	96
<i>50 Indicators: Power for Detecting Latent Slope Factor</i>	
<b>Figure B32</b> .....	97
<i>50 Indicators: Power for Detecting Predictor Effect on Latent Slope Factor</i>	
<b>Figure B33</b> .....	98
<i>50 Indicators: Standard Error Bias for Latent Time-to-Criterion Factor</i>	
<b>Figure B34</b> .....	99
<i>50 Indicators: Standard Error Bias for Predictor Effect on Latent Time-to-Criterion Factor</i>	
<b>Figure B35</b> .....	100
<i>50 Indicators: Standard Error Bias for Latent Slope Factor</i>	
<b>Figure B36</b> .....	101
<i>50 Indicators: Standard Error Bias for Predictor Effect on Latent Slope Factor</i>	

**CHAPTER 1.**

**Longitudinal Data Analyses and the Time-to-Criterion Model**

Longitudinal data can be analyzed a number of ways, but generally there are three approaches: the “traditional” intercept-slope growth model, the “time-to-criterion” survival model, and data-intensive “time-series” models that utilize cyclical/seasonal data for forecasting. The first type of analysis is often used in the social sciences to estimate mean trajectories of developmental processes, or the mean change in an outcome during or after an intervention period. The second is often used in the health sciences for the purpose of estimating the length of time required to reach a particular “event,” such as disease onset or recovery. The third is most often found in the natural sciences and finance market studies of data with numerous time points to estimate average trends, while taking regular cycles of outcome fluctuations into account. The present study is focused on investigating the second analytic approach because: 1) a time-to-criterion (T2C) model may be more useful for policy-oriented social science research than the first approach; 2) intensive longitudinal data collection involving human subjects is more costly than natural phenomena (like financial market gains and losses), rendering the third approach less amenable for large-scale social science research; and 3) longitudinal studies involving people, especially vulnerable populations, are more prone to missingness due to attrition over time, which could have negative effects on T2C model estimates.

More specifically, missingness over time due to natural attrition would not affect the intercept factor in a traditional intercept-slope growth model (assuming it is set to the first time point of data collection), but it would impact the estimate of the tau factor (time length estimate) in the T2C model. Thus, the present study seeks to help researchers plan their studies by investigating the optimal conditions under which the T2C model can provide accurate and

## CHAPTER 1: LONGITUDINAL DATA ANALYSES AND THE T2C MODEL

precise estimates for different levels of attrition-related missingness, total sample size, growth variability, number of time points of data collection, and measurement reliability. I assess this primarily using a Monte Carlo simulation (i.e., simulated data where the true values are known), and then also demonstrate the use of both the “traditional” intercept-slope and the T2C models with a subsample of child data from the publicly available U.S. Department of Education NCES *Early Childhood Longitudinal Study, Kindergarten Class of 2010–11* dataset (Tourangeau et al., 2015).

### **Motivation and Educational Research Context**

To understand the extent to which the T2C model is in use in current social science research, I searched within the Academic Complete Search database for peer-reviewed journal articles focused on growth modeling. The specific search criteria were:

- Publication dates: between January 2018 and December 2022 (i.e., the previous five years)
- Must be peer-reviewed
- Publication type: Academic journal
- Document type: Full-text article
- Language: English
- Search terms: “growth model” or “growth curve model” or “longitudinal”
- Once the total number of articles, I also further filtered the search to just those mentioning “time-to-criterion” OR “time to criterion”

The search found 240,432 articles with any of the first set of search terms within the full text of the articles. Of these, only one paper mentioned or alluded to the T2C model – it is the article by Johnson and Hancock (2019), which provided inspiration for this dissertation study.

## CHAPTER 1: LONGITUDINAL DATA ANALYSES AND THE T2C MODEL

Thus, the T2C model is not currently being utilized the way it might be. To provide some context for the relevance of its use within education research, in the following sections I provide three possible areas that might benefit from utilizing the T2C as an analysis tool. I begin by discussing emergent bilingual student reclassification policy, followed by first-generation college students' degree attainment, and ending with mastery-based learning.

### **Context 1: Emergent Bilingual K-12 Students: Time-to-Reclassification**

Children whose home language is not English, also known as emergent bilingual (EB) students, often face more hurdles learning academic material in U.S. public schools. (Although I use the term “emergent bilingual” as an asset-oriented approach, other terms that have been used to characterize this group of learners includes English learners (ELs), English language learners (ELLs), and English as a second language students (ESLs).) While states can vary in their exact policies and procedures for serving EB students, the typical process is that students who score in a particular range on an English language proficiency test are placed into language support services that can range from language teaching services provided during certain periods within the mainstream classroom (less common) or by placing students in “EL-only” classrooms (Mavrogordato & White, 2017). Those who score above a certain threshold typically receive no services, and yet others may never be tested at all because parents can opt out of testing altogether. In any case, the language test itself is “high stakes” in that students may or may not receive language support services based on the score ranges used to make district/state policy (Cardenas, 2018).

Importantly, students who are designated to receive language services are also typically tested annually to assess whether they are ready to “exit” services, a practice known as “reclassification” (Mavrogordato & White, 2017). By and large, the criteria for exiting EL

## CHAPTER 1: LONGITUDINAL DATA ANALYSES AND THE T2C MODEL

services depends on scoring in the “advanced” range on the annual test score (Pilger Suhr & Alonzo, 2021), although other factors such as teacher or parent advocacy can be considered (Pompa & Villegas, 2017). Nevertheless, there is a lack of standardized reclassification criteria across the U.S., likely due to states’ freedom in designing their own language assessments as part of the Every Student Succeeds Act (ESSA) signed into law in 2015 (Cardenas, 2018).

Putting aside the inconsistencies across state assessments, Kim and Herman (2010) argued that there is a difference between reclassification and *effective* reclassification, where *effective* reclassification is shown by post-reclassification evidence of positive outcomes on school retention, state reading and mathematics tests, and passing assessments required for high school. In their comparative study of four school districts (two in New York and two in Canada), reclassified students can thrive in regular classrooms, but students who exit too early may fall behind peers in later grades (Kim & Herman, 2010; Robinson, 2011).

In another study, Estrada and Wang (2018) analyzed data from two districts in California and found that a substantial number of EB students had met standardized test criteria for exiting the EL program but were inexplicably *not* reclassified. Such unfounded decisions to fail to reclassify students can lead to diminished opportunities to take more advanced coursework required for 4-year college access (Callahan, 2005; Harklau, 2002; Robinson, 2011; U.S Department of Education, 2016). To that point, Johnson (2020) examined how reclassification at the middle-to-high school transition point affected students’ on-track status to high school graduation for a large California school district. One of the key findings was that reclassification by the eighth grade increased students’ chances of being on track to graduate high school (Johnson, 2020).

## CHAPTER 1: LONGITUDINAL DATA ANALYSES AND THE T2C MODEL

As Kim and Herman (2010) pointed out, there is tension between assuring that EB students have sufficient English development support to thrive in mainstream classrooms, and avoiding the potential negative consequences of spending too much time on English supports and therefore receiving reduced access to the rigorous subject content in mainstream. *Hence, studying the optimal time-to-reclassification using the T2C model could help to contribute meaningfully to solving this dilemma. Knowing the best timeframes for this reclassification (with confidence intervals) would provide educators and policymakers with some understanding of how quickly EB students can be reasonably expected to enter mainstream classrooms (Hakuta, Butler, & Witt, 2000; Motamedi et al., 2016) as well as help identify EB students who may need additional supports prior to reclassification (Kieffer & Parker, 2016; Motamedi et al., 2016; Parrish et al., 2006).*

### **Context 2. First-Generation College Students: Time-to-Degree Completion**

Another substantive area where T2C models may make more sense than traditional intercept-slope models include first-generation college student success. Research has shown that first-generation (Gen-1) college students face more barriers to college access and success compared to continuing-generation (Gen-C) peers (Nuñez & Cuccaro-Alamin, 1998). To be clear, Gen-1 status may be defined as students for whom both parents never attended college, although there are competing definitions, such as students for whom both parents may have attended college but never completed a degree (Lundberg et al., 2007), as well as students for whom both parents may have completed a two-year Associate's degree, but not a four-year Bachelor's degree (Garriot, 2020). Irrespective of the definition, the major (intrinsic) difference between Gen-1 and Gen-C college students is parental education level, and the financial and social resources associated with education.

## CHAPTER 1: LONGITUDINAL DATA ANALYSES AND THE T2C MODEL

Even when Gen-1 students are accepted into more selective colleges, they typically choose to attend less selective institutions because of the price tag, whereas Gen-C students are less likely to worry about the cost of a college education since they may have more financial resources to begin with, or have accepted the idea of student loan debt if their parents successfully navigated such loans (Berkner & Chavez, 1997; Chen, 2005; Choy, 2001; Fentress & Collopy, 2011; Inman & Mayes, 1999; Lohfink & Paulsen, 2005; Nuñez & Cuccaro-Alamin, 1998; Pascarella et al., 2004; Terenzini et al., 1996; Somers et al., 2004; Volle & Federico, 1997; Warburton et al., 2001).

Once a college is chosen, Gen-1 students also tend to face barriers in participating in campus activities and attending their classes due to unmet financial needs (Engle, 2007; Inman & Mayes, 1999; Lundberg et al., 2007; Mcdossi et al., 2022; Mehta et al., 2011; Nuñez & Cuccaro-Alamin, 1998; Paulsen & St. John, 2002; Penrose, 2002; U.S. Department of Education, 2001) as well as the multiple roles that Gen-1 students may have at home such as caring for their parents or being parents themselves (Kim et al., 2010; Zwerling, 1992), especially for students from historically marginalized backgrounds (Oseguera et al., 2009). To compound matters, because Gen-1 students may have graduated from under-resourced secondary schools and/or may also be linguistic minorities, they may be required to take added coursework to meet college class prerequisites, which in turn creates increased expenses and greater time to degree completion compared to Gen-C peers (e.g., Harrell & Forney, 2003). Indeed, Engle and Tinto (2008) found that Gen-1 students attending two-year colleges were five times *less* likely than Gen-C peers to transfer to four-year institutions and attain a Bachelor's degree within a six-year timeframe (Engle and Tinto, 2008).

## CHAPTER 1: LONGITUDINAL DATA ANALYSES AND THE T2C MODEL

*Within this context, a T2C data analysis approach may be more useful than typical growth models if regular degree progress indicators, such as credits earned as well as students' self-efficacy perceptions of their own academic progress, could be used to estimate the length of time needed for students to successfully complete their degree, or perhaps the time it takes for feeling a strong sense of efficacy that they are moving forward in their education. Such information could then be applied to monitor future students, especially Gen-1 students, for identifying those who may need additional tiers of support to stay on track for completing their degree on time.*

### **Context 3. Individualized Learning: Time-to-Mastery**

A third example of the potential for T2C to be more useful than traditional growth models is in the context of focusing on learning for mastery, rather than to be rank-ordered in achievement tests. Currently, the K-12 education system in the U.S. predominantly follows a traditional, fixed time-based curriculum – meaning students are expected to spend a fixed amount of time in each grade before advancing to the next grade (Ofgang, 2021). However, even in the 1960s researchers such as Bloom (1968) and Carroll (1963) noted that traditional instructional formats would not fully allow for individual-based academic progress. Indeed, the 1960s also saw the emergence of the idea of mastery-based learning, which is also known as “personalized learning” and “competency-based learning” (Lee, 2014). Although No Child Left Behind 2002 is no longer in effect, this federal initiative and related educational reforms in states and school districts in the early 2000s have partially powered the resurgence of the idea of tending to the learning needs of individual students (Twyman, 2014). Indeed, in the past two decades, the importance of mastery-based learning has been gaining more recognition for

## CHAPTER 1: LONGITUDINAL DATA ANALYSES AND THE T2C MODEL

adapting education curricula to diverse learners (Collins & Halverson, 2009a, b; Friedman, 2006; McCombs, 2008; Miliband, 2006; Reigeluth & Karnopp, 2013).

It should be noted the focus of mastery-based learning is on the student's individual progress and whether they have mastered a certain competency, not how they perform compared to their peers (Mammadov & Hertzog, 2021). As such, within the mastery-based learning framework, a criterion-referenced assessment (CRA) gauging student progress on a given competency or skill would be used to evaluate learning progress (Lee, 2014; Thorndike and Thorndike-Christ, 2010). *Here again is a situation where a time-to-criterion analysis would be useful for evaluating the mean length of time needed to reach particular competencies, rather than simply the growth in overall knowledge. Such information could inform reasonable expectations around individual differences in the time needed for concept mastery, as well as student characteristics that are predictive of the time length needed, in order to successfully plan individualized instruction.*

### **The T2C Model**

Though survival analysis methods have enjoyed a long history of use in health sciences research, the typical survival analysis model does not account for variation in individual trajectories and also does not consider continuous outcomes such as rating scales and total scores. Quite recently, Johnson and Hancock (2019) filled this gap by proposing a *latent* T2C model that marries the typical survival analysis model with the “traditional” intercept-slope growth model involving continuous outcomes, while also taking measurement error into account. Below I review the traditional latent intercept-slope growth model, and then turn to the newer latent T2C model.

### Traditional Latent Intercept-Slope Growth Model

In the structural equation modeling (SEM) framework, a traditional latent intercept-slope growth model can be specified as follows.

$$\mathbf{y}_j = \mathbf{v} + \mathbf{\Lambda}\boldsymbol{\eta}_j + \boldsymbol{\varepsilon}_j \quad (1)$$

In the model above, the vector of outcomes  $\mathbf{y}$  for person  $j$  measured at  $t$  time points ( $i = 0, 1, 2, 3 \dots t-1$ ) is a function of a vector of intercepts  $\mathbf{v}$  ( $t \times 1$  vector often fixed to  $\mathbf{0}$  to identify the intercept and growth factor means<sup>1</sup>); a  $t \times P$  matrix of factor loadings  $\mathbf{\Lambda}$ , where  $P$  = the number of growth factors (usually constrained to a design matrix layout within which the first column is fixed to  $\mathbf{1}$  and the next column is fixed to  $\mathbf{t}$  (if the first time point is serving as the reference for the intercept,  $\mathbf{t}$  may be coded  $\mathbf{t-1}$ ); a  $p \times 1$  vector of weights  $\boldsymbol{\eta}_j = \alpha_i$  and  $\beta_j$  with distribution  $\boldsymbol{\eta}_j \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ ; and a  $t \times 1$  vector of time point-specific errors  $\boldsymbol{\varepsilon}_j$  with distribution  $\boldsymbol{\varepsilon}_j \sim N(\mathbf{0}, \boldsymbol{\Theta})$ . The  $t \times t$  covariance matrix  $\boldsymbol{\Theta}$  is typically fixed to assume local independence for person  $j$  across  $t$  time points, conditional on the latent factors  $\boldsymbol{\eta}_j$  as follows:

$$\boldsymbol{\Theta} = \begin{bmatrix} \theta_0 & & \\ 0 & \dots & \\ 0 & 0 & \theta_{t-1} \end{bmatrix}.$$

Perhaps a more intuitive understanding of latent models can be seen using path diagrams. For a general SEM model, the path diagram in Figure 1 is an example with two factors and six indicators, with the focal interest on estimating the factor loadings (and not on the factor means). The measurement scenario in Figure 1 can be customized to other contexts; these two factors can represent any chosen latent constructs, the number of indicators can vary, and different

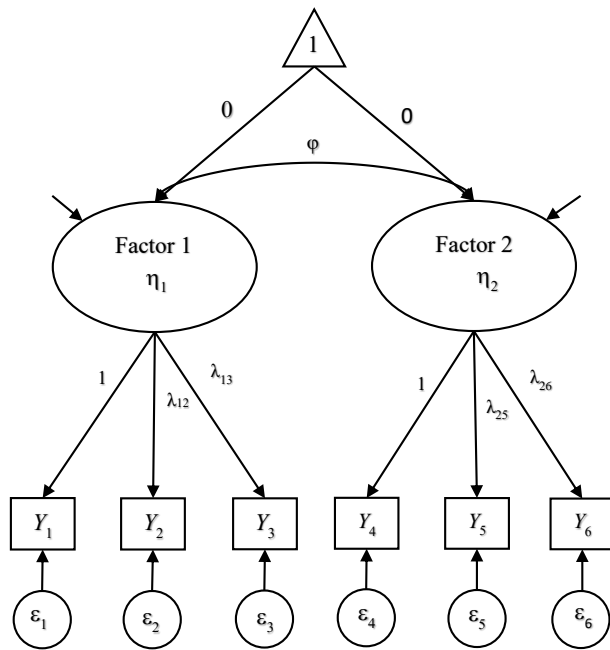
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<sup>1</sup> The intercept factor mean is the estimated mean value of  $Y$  for the time point coded 0, which is often (but not always) the first time point at which  $Y$  was measured; the growth factor mean, if assumed linear, is defined as the estimated mean change in  $Y$  for each one-unit increase in time. The difference between the SEM and univariate random effects (multilevel) approaches is that the SEM approach takes the measurement error of  $Y$  at each time point into account, rather than assuming the error variance is homogenous across time points.

constraints can be placed on the model. Figure 2, a specific case of how the general SEM model can be customized, is adapted from Preacher et al. (2010) and illustrates a latent linear growth model for a scenario with five time points. In the diagram, both the intercept (estimated value of the outcome when  $t = 0$ ) and the slope (estimated change in the outcome for each unit increase in time) are symbolized in ovals because they are latent, or unobserved, while the outcomes ( $y_i$ ) are our observed data symbolized as boxes. The five observed outcomes are assumed to be caused by both the intercept and the slope, as indicated by the arrows or “paths” from each factor to each observed variable. The error term is unspecified as it is free to vary (i.e., be estimated separately) for each time point, which essentially provides us with structural model estimates (the means, variances, and covariances among the latent variables) that are separate from observed variable measurement error (the residual error variance at each time point).

**Figure 1**

*Path Diagram of a General 2-Factor Latent Variable Model*

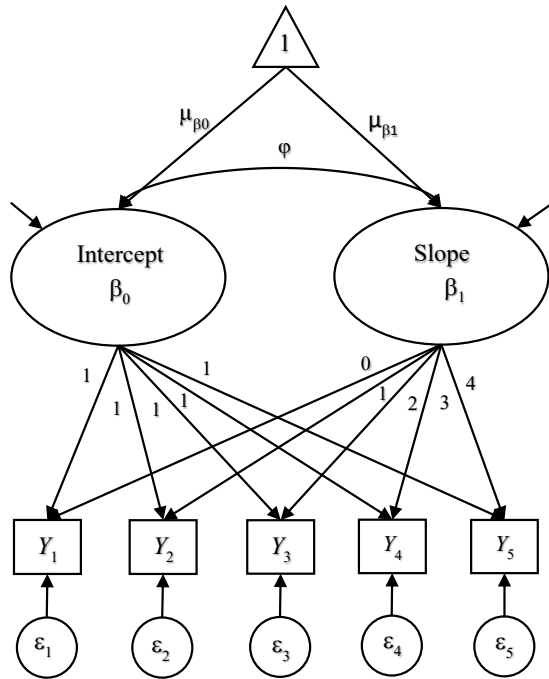


## CHAPTER 1: LONGITUDINAL DATA ANALYSES AND THE T2C MODEL

Importantly, in all structural equation models, the scales of the latent variables depend on how we relate the observed variables to them, as well as our assumptions about the latent variables themselves (i.e., a latent variable need not be a normal distribution). In a confirmatory factor analysis SEM, for example, we might assume the latent variables are unit normal and only be interested in how the observed variables relate to the latent variables via their loadings (path coefficients), or the relationships (covariation) among the latent variables themselves. In SEM growth modeling, however, we are interested in estimating the latent means and variances, and to accomplish this, we must put in place model constraints. Compared to regular latent variable models in which path coefficients are estimated, in latent growth models, the paths are (typically) constrained or “fixed” to reflect the structure of the data. In Figure 2, the paths from the intercept to each observed variable are constrained to 1, and the paths from the slope to each observed variable reflect the time point and distance of the measurements. This specification forces the estimation of the mean of the intercept to be the estimated mean at the first time point (because it has no relationship with the slope per our specification), and the slope mean to be the estimated change in the outcomes per one year increase in time.

**Figure 2**

*Path Diagram of Hypothetical 2-Factor Latent (Traditional) Intercept-Slope Growth Model*



Lastly, I wish to make a few other points of clarification. For brevity throughout this paper, the term “linear” is used because it is assumed that the effect of time on the outcome is linear/additive, and not some other polynomial or piecewise form (such models would incorporate additional latent factor(s) to represent the functional growth form assumed). Also, again for brevity, for the remainder of this paper, the focus is limited to observed variables assumed to be (conditionally) multivariate normally distributed, and equidistantly measured. Such assumptions can be relaxed in more complex model specifications.

**Latent Time-to-Criterion (T2C) Growth Model**

Recently, Johnson & Hancock (2019) mathematically reparameterized the traditional latent intercept-slope model as a latent time-to-criterion model (T2C) by sacrificing the intercept<sup>2</sup> ( $\alpha_j$ ) with the expression:  $(c - \beta_j \tau_j)$ , where  $c$  is a predefined (known) criterion of the outcome ( $y_{ij}$ ). To implement the model, the elements of the  $\mathbf{v}$  and  $\mathbf{\Lambda}$  matrices in (Eq. 1) are altered to accommodate the estimation of  $\tau$  rather than  $\alpha$ . The resulting model equation is as follows.

$$\begin{bmatrix} y_{0j} \\ \dots \\ y_{(t-1)j} \end{bmatrix} = \begin{bmatrix} c - \mu_\beta \mu_\tau \\ \dots \\ c - \mu_\beta \mu_\tau \end{bmatrix} + \begin{bmatrix} 0 - \mu_\tau & -\mu_\beta \\ \dots & \dots \\ T_{(t-1)-\mu_\tau} & -\mu_\beta \end{bmatrix} \begin{bmatrix} \beta_j \\ \tau_j \end{bmatrix} + \begin{bmatrix} \varepsilon_{0j} \\ \dots \\ \varepsilon_{(t-1)j} \end{bmatrix} \quad (2)$$

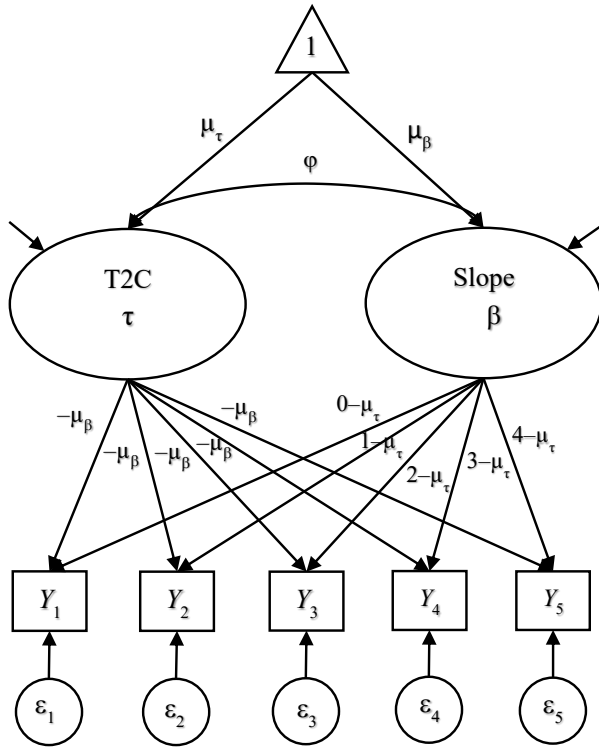
As can be seen in Eq. 2 and Figure 3, to make the model identifiable, constraints on the factor loadings and item intercepts have to be updated such that 1) the factor loadings of  $\tau$  must be fixed to  $-\mu_\beta$ , which is the negative value of the estimated growth rate factor mean, and 2) the factor loadings of  $\beta$  must be fixed to  $t_i - \mu_\tau$ , which is the difference between time point  $i$  minus 1 ( $t_i - 1$ ) and the estimated tau factor mean ( $\mu_\tau$ ). Lastly, the item intercepts must be fixed to a constant value of  $c - \mu_\beta \mu_\tau$  (not shown in the figure)<sup>3</sup>.

<sup>2</sup> Note that an alternative formulation could also be to sacrifice the growth rate to estimate the intercept instead, if the intercept is of more interest; see Johnson and Hancock (2019, p. 694) for further details.

<sup>3</sup> Again, this assumes the intercept is set at the first time point. Other specifications exist.

**Figure 3**

*Path Diagram of Hypothetical Latent T2C Model*



**Present Study**

The present study is focused on investigating realistic scenarios when a latent T2C model is being planned for use in longitudinal research designs, with an eye toward optimizing parameter estimation precision. Specifically, my research question is: What are the effects of different design facets on the accuracy (point estimate relative bias) and precision (standard error empirical bias) of latent growth and time-to-criterion factor means, as well as a predictor effect on the latent factors? The key ingredients for consideration include missingness due to attrition, sample size, number of time points measured, slope variability, and measurement reliability. Below I describe my rationale for studying the effects of each condition on T2C model estimates.

**Missingness due to attrition.** In any model that employs full information maximum likelihood (FIML) model estimation, which includes most SEMs with normally distributed indicators, parameters are estimated using the observed variance-covariance matrix based on all cases, rather than the listwise/casewise deletion used in ordinary least squares (OLS) and restricted maximum likelihood (REML) estimation. The full information maximum likelihood (FIML) approach is preferred over the OLS/REML approach in terms of better statistical power/precision; however, both methods yield unbiased parameter estimates if the data can be assumed to be missing completely at random (MCAR) or just missing at random (MAR) (e.g., Enders & Bandalos, 2001; Kenward & Molenberghs, 1998)<sup>4</sup>. MCAR refers to situations where missingness on a given outcome,  $y$ , is not systematically related to values of the  $y$  variable (e.g., individuals with higher skill deficits do not have more missing data on that skill), nor any other known variable, observed or unobserved (e.g., individuals classified as part of a particular subgroup,  $x$ , do not have more missing data on the focal skill,  $y$ ). MAR, on the other hand, allows for the latter situation, so long as the external variable related to missingness is measured and included in the model.

Both MCAR and MAR can plausibly occur in a longitudinal study. For example, MCAR can happen when a study allows new individuals to enter or leave the study at any time point, particularly if a study is focused on change over time for a specific physical space like a particular school or clinic where people tend to come and go. However, MAR is more likely to occur when only one cohort of individuals is followed over time; the longer the study goes on, the more attrition is likely. In the case of missingness due to time (i.e., attrition), as long as “time” is a variable included in the model (i.e., as it is in growth models), MAR can be assumed.

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<sup>4</sup> Missingness is considered *not* at random (MNAR) when the missingness mechanism was not measured or included in the model; in such cases, model parameter estimate biases can occur (Rubin, 1976).

Despite the advantage of using FIML estimation over casewise/listwise deletion, any presence of missing data naturally adds uncertainty to model parameter estimates because the quality of the variance-covariance information is weighted by the sample size of the matrix elements. In turn, any missingness can yield larger parameter standard errors than would occur if complete data on all cases were available. In short, power and precision for parameter estimates can still be reduced, even using FIML (Enders & Bandalos, 2001). *The present study seeks to understand how much missingness due to attrition can be tolerated in the T2C model for estimating the time-to-criterion factor  $\tau$  as well as predictors of tau, since its estimation relies on the estimated slope value, which in turn relies on available information across time points.*

**Criterion placement.** The selection of the criterion level (which must be set a priori to estimate the time it takes to achieve that level) will depend on the intensity of the study. For a typical norm-referenced assessment with  $M = 100$  and  $SD = 15$ , a small “effect” criterion would be approximately a 5-point increase (1/3 of a standard deviation gain from the starting point), whereas a large effect might be a 15-point increase (1 standard deviation from the starting point). It was surmised that the placement of the criterion will likely be a key factor for researchers planning their studies. *Thus, the present study seeks to understand how the criterion placement may interact with other design considerations for optimal parameter estimation accuracy and precision for the T2C model.*

**Sample size at study onset.** In multivariate models, sample size (number of individuals measured) tends to matter more than in univariate models because of the relatively larger number of parameters being estimated. Rules of thumb have recommended anywhere from greater than  $N = 100$  up to  $N = 1000$  (e.g., Comrey & Lee, 1992; Gorsuch, 1983; Guilford, 1954; Kline, 1979), Others focus more on the ratio of individuals per observed variable in the model, with

recommendations ranging from 2 individuals per variable up to 20 individuals per variable (Pituck & Stevens, 2016, p. 347). *The present study seeks to understand how the total sample size at the onset of a study may interact with other design considerations for optimal parameter estimation accuracy and precision for the T2C model.*

**Number of time points data is collected.** Because growth modeling involves multiple time points, the number of time points is an integral part of the “sample size” consideration. Specifically, the variance-covariance matrix used in estimating the model treats time points as observed variables that are indicators of the latent factors. For model identifiability, the minimum number of variables per factor in any SEM is three; as such, three is the minimum number of time points needing to be measured for estimating any two-factor growth model (whether it be the traditional intercept-slope or the slope-tau T2C model). Further, the more correlated measurements across time are with each other (i.e., the greater the correlation between scores of any two randomly selected time points for a given individual), the more time points are needed for estimation precision (Hecht & Zitzmann, 2020). *As such, the present study seeks to understand how the T2C model parameter estimates behave for different numbers of time points measured, in conjunction with missingness and sample size considerations.*

**Growth rate variability.** In growth modeling, variance in individual rates of change (i.e., the slope variance) can range from little to none when everyone has nearly the same rate of change, to quite variable in the case of response to intervention for a heterogeneous sample. Growth modeling flexibly allows for the inclusion of predictors of growth for testing the extent to which differences in growth rates are related to key variables. In the T2C model, researchers will likely not only wish to test predictor effects on growth rates, but also predictor effects on the time-to-criterion factor,  $\tau$ . *The present study therefore seeks to understand how growth rate*

*(slope) variability interacts with missingness, sample size, and number of time points on the T2C model parameter estimates.*

**Measure reliability.** Last but not least, some applied researchers may not be aware that the reliability of observed measures contributes directly to the precision of model estimates: the more reliable a measure is, the more precise the estimates of the model parameters. In other words, less reliable measures may require much larger sample sizes and time points than more reliable measures to achieve optimal parameter estimate precision. *Thus, the present study seeks to understand how measurement reliability interacts with missingness, sample size, number of time points, and growth variability on the T2C model parameter estimates.*

## CHAPTER 2.

## Monte Carlo Simulation Study

## Method

## Data Generation

Data was generated using a traditional latent growth model using *Mplus 8* (Muthén & Muthén, 1998/2017) within the *MplusAutomation* package in *R* (Hallquist & Wiley, 2018). Other assumptions included equidistant time points, normally distributed with  $M = 100$  ( $SD = 15$ ) to approximate a typical norm-referenced assessment. The true average growth rate,  $\beta_j$ , was assumed linear at  $M = 2$  points per year, and the true effect of one predictor (normally distributed with  $M = 0$ ,  $SD = 1$ ) was set to have a 1 point-per-point increase on the intercept and slope. The following conditions were varied:

1. **Missingness:** Four levels included: 0% missing, 10% per year, 20% per year, and 30% per year (accumulating over time).
2. **T2C tau factor:** Four levels of the time-to-criterion factor ( $\tau_j$ ) included: 2.5, 5.0, 7.5, and 10.0 units of time from study onset. In our context, this respectively translates to a criterion:
  - $c = 105$  points (0.33 standard deviation of increase between intercept and the criterion;  $(105 - 100) \div 15 = 5 \div 15$ )
  - $c = 110$  points (0.67 standard deviation of increase between intercept and criterion;  $(110 - 100) \div 15 = 10 \div 15$ )
  - $c = 115$  points (1 standard deviation of increase between intercept and criterion;  $(115 - 100) \div 15 = 15 \div 15$ )

## CHAPTER 2: MONTE CARLO SIMULATION STUDY

- $c = 120$  points (1.33 standard deviations of increase between intercept and criterion;  $(120 - 100) \div 15 = 20 \div 15$ )
3. **Sample size at study onset:** Five levels included:  $N = 100, 250, 500, 1000,$  and  $5000$ , chosen to reflect small- and relatively large-scale policy research.
  4. **Number of time points:** Three levels included:  $t = 3, 5,$  and  $50$  indicators chosen to reflect minimum, typical, and time-intensive types of data collection used in longitudinal studies.
  5. **Slope heterogeneity:** Four levels included:  $Var = 0, 1, 4, 9$ , chosen to reflect  $SD = 0, 1, 2,$  and  $3$  points relative to the mean slope  $= 2$  points, in order to capture different ratios of the mean growth rate relative to growth rate heterogeneity (i.e., 2:0, 2:1, 2:2, and 2:3, respectively).
  6. **Measure reliability:** Three levels included: Cronbach's alpha  $= .4, .6,$  and  $.8$  reliability, induced by setting the residual error variance to  $(1 - \text{reliability}) * (\sigma^2 = 225)$  to reflect poor, modest, and better reliability levels, respectively.

### Analysis Plan

For each of the  $4 \times 4 \times 5 \times 3 \times 4 \times 3 = 2880$  conditions, 1,000 replications were generated<sup>5</sup>. Each of these datasets were analyzed with a traditional latent intercept-slope growth model and the latent T2C model. The results focus on the relative bias and empirical bias in the point and standard error estimates for T2C model parameters (the means for the intercept and tau factors, as well as the predictor effects on each). Descriptive statistics and data visualization are reported for use in evaluating the research questions. Parameter point estimate bias exceeding

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<sup>5</sup> For the 50-indicator model, 200 replications, rather than 1,000, were used due to the required computing memory size and length of time needed to analyze the datasets. I do not believe there will be any change in the substantive patterns, but for publication I will increase this amount to 1,000 for better precision of simulation estimates.

$\pm 5\%$  and standard error estimate bias exceeding  $\pm 10\%$  are considered cause for concern (Hoogland & Boomsma, 1998).

### Results

As a check on the simulation process, parameter bias, standard error bias, and power were tabulated for the growth factors and their predictors for the traditional latent growth model (the data generating function) by missingness and selected sample size (collapsed across other conditions). Those results are given in Tables 1, 2, and 3 on the following pages. As can be seen, as sample size increased there was no bias in any of the parameter or standard error estimates, and power grew higher (as would be expected).

For comparison against the traditional growth data generating model, the average parameter point estimate bias, standard error bias, and power by missingness for selected sample sizes (again, results are collapsed across all levels of other design conditions like criterion, time points, slope heterogeneity, and measure reliability) for the T2C model are provided in Tables 4, 5, and 6. As will readily be seen, the growth estimates are identical to those for the traditional growth model. More interestingly, there were marked effects of missingness on the T2C-specific parameters compared to the sacrificed intercept parameter of the traditional growth model. Before examining each of the design facets, an analysis of variance was conducted on the T2C results with the design conditions serving as main effects (and all two-way interactions).

Design conditions with large effect sizes (i.e.,  $\eta_p^2 \geq 0.14$ ) were as follows. For tau, the main drivers of tau parameter bias were sample size ( $\eta_p^2 = 0.80$ ), number of indicators ( $\eta_p^2 = 0.50$ ), missingness ( $\eta_p^2 = 0.50$ ), slope heterogeneity ( $\eta_p^2 = 0.33$ ), and measure reliability ( $\eta_p^2 = 0.24$ ), whereas only sample size ( $\eta_p^2 = 0.42$ ) and number of indicators ( $\eta_p^2 = 0.20$ ) predicted parameter standard error bias. Power was similarly predicted by sample size ( $\eta_p^2 = 0.69$ ), but also

CHAPTER 2: MONTE CARLO SIMULATION STUDY

by criterion level ( $\eta_p^2 = 0.36$ ). For the predictor of tau, the main predictors of bias were sample size ( $\eta_p^2 = 0.65$ ), missingness ( $\eta_p^2 = 0.28$ ), and number of indicators ( $\eta_p^2 = 0.18$ ). Standard error bias was similarly predicted by sample size ( $\eta_p^2 = 0.54$ ) and number of indicators ( $\eta_p^2 = 0.32$ ). Comparatively, sample size was also the chief driver for power ( $\eta_p^2 = 0.90$ ), but number of indicators ( $\eta_p^2 = 0.41$ ) as well as missingness ( $\eta_p^2 = 0.33$ ) also had a large effect on this parameter estimate.

**Table 1**

*Traditional Growth Model: Relative Bias Results on Selected Conditions*

Parameter	Sample Size		
	<i>n</i> = 100	<i>n</i> = 250	<i>n</i> = 5000
Mean Intercept			
Missing Rate = 0%	0.02	0.00	0.00
Missing Rate = 10%	0.08	0.03	0.00
Missing Rate = 20%	0.09	0.03	0.00
Missing Rate = 30%	0.08	0.04	0.01
Mean Growth Rate			
Missing Rate = 0%	-0.05	-0.22	0.00
Missing Rate = 10%	-0.94	-0.60	-0.02
Missing Rate = 20%	-1.21	-0.13	-0.09
Missing Rate = 30%	-1.63	-0.85	-0.05
Predictor Effect on Intercept			
Missing Rate = 0%	6.61	3.24	-0.38
Missing Rate = 10%	12.03	5.19	-0.53
Missing Rate = 20%	11.15	1.44	-0.11
Missing Rate = 30%	12.14	5.41	-0.56
Predictor Effect on Growth Rate			
Missing Rate = 0%	0.29	-0.49	-0.05
Missing Rate = 10%	0.97	-0.23	-0.05
Missing Rate = 20%	0.85	0.40	0.06
Missing Rate = 30%	1.97	0.57	-0.16

*Note.* Results are irrespective of indicators, slope heterogeneity, and measure reliability. Relative bias is presented as percentages (%). 3- and 5-indicator models had 1000 replications, but 50-indicator models had 200 replications.

**Table 2***Traditional Growth Model: Standard Error Bias Results on Selected Conditions*

Parameter	Sample Size		
	$n = 100$	$n = 250$	$n = 5000$
Mean Intercept			
Missing Rate = 0%	0.96	1.01	1.00
Missing Rate = 10%	1.00	1.00	0.99
Missing Rate = 20%	0.99	1.00	1.01
Missing Rate = 30%	1.00	1.00	0.99
Mean Growth Rate			
Missing Rate = 0%	0.98	0.99	1.00
Missing Rate = 10%	0.97	0.99	1.02
Missing Rate = 20%	0.98	1.00	0.98
Missing Rate = 30%	0.93	0.99	1.01
Predictor Effect on Intercept			
Missing Rate = 0%	0.98	0.99	0.99
Missing Rate = 10%	0.99	0.96	1.00
Missing Rate = 20%	1.00	0.99	1.03
Missing Rate = 30%	1.00	0.97	1.01
Predictor Effect on Growth Rate			
Missing Rate = 0%	0.97	0.99	1.01
Missing Rate = 10%	0.98	0.99	1.02
Missing Rate = 20%	0.95	0.97	1.02
Missing Rate = 30%	0.96	0.99	0.99

*Note.* Results are irrespective of indicators, slope heterogeneity, and measure reliability. Standard error bias is presented as percentages (%). 3- and 5-indicator models had 1000 replications, but 50-indicator models had 200 replications.

**Table 3***Traditional Growth Model: Power Results on Selected Conditions*

Parameter	Sample Size		
	$n = 100$	$n = 250$	$n = 5000$
Mean Intercept			
Missing Rate = 0%	1.00	1.00	1.00
Missing Rate = 10%	1.00	1.00	1.00
Missing Rate = 20%	1.00	1.00	1.00
Missing Rate = 30%	1.00	1.00	1.00
Mean Growth Rate			
Missing Rate = 0%	1.00	1.00	1.00
Missing Rate = 10%	0.99	1.00	1.00
Missing Rate = 20%	0.97	1.00	1.00
Missing Rate = 30%	0.91	1.00	1.00
Predictor Effect on Intercept			
Missing Rate = 0%	0.11	0.18	0.98
Missing Rate = 10%	0.11	0.18	0.99
Missing Rate = 20%	0.12	0.18	0.99
Missing Rate = 30%	0.11	0.18	0.99
Predictor Effect on Growth Rate			
Missing Rate = 0%	0.84	0.98	1.00
Missing Rate = 10%	0.77	0.95	1.00
Missing Rate = 20%	0.67	0.90	1.00
Missing Rate = 30%	0.59	0.83	1.00

*Note.* Results are irrespective of indicators, slope heterogeneity, and measure reliability. 3- and 5-indicator models had 1000 replications, but 50-indicator model had 200 replications.

**Table 4***T2C Model: Relative Bias Results on Selected Conditions*

Parameter	Sample Size		
	$n = 100$	$n = 250$	$n = 5000$
<b>Mean Tau</b>			
Missing Rate = 0%	2.08	1.09	0.05
Missing Rate = 10%	3.63	1.47	0.02
Missing Rate = 20%	5.77	1.67	0.14
Missing Rate = 30%	10.93	3.55	0.11
<b>Mean Growth Rate</b>			
Missing Rate = 0%	-0.04	-0.22	0.00
Missing Rate = 10%	-0.92	-0.61	-0.02
Missing Rate = 20%	-1.09	-0.13	-0.09
Missing Rate = 30%	-1.21	-0.85	-0.06
<b>Predictor Effect on Tau</b>			
Missing Rate = 0%	8.67	3.17	0.02
Missing Rate = 10%	17.83	5.42	0.02
Missing Rate = 20%	28.83	6.70	0.43
Missing Rate = 30%	55.84	13.20	0.18
<b>Predictor Effect on Growth Rate</b>			
Missing Rate = 0%	0.29	-0.49	-0.05
Missing Rate = 10%	0.96	-0.23	-0.05
Missing Rate = 20%	0.81	0.40	0.06
Missing Rate = 30%	1.92	0.58	-0.16

*Note.* Results are irrespective of indicators, criterion, slope heterogeneity, and measure reliability. Relative bias is presented as percentages (%). 3- and 5-indicator models had 1000 replications, but 50-indicator models had 200 replications.

**Table 5***T2C Model: Standard Error Bias Results on Selected Conditions*

Parameter	Sample Size		
	$n = 100$	$n = 250$	$n = 5000$
<b>Mean Tau</b>			
Missing Rate = 0%	0.96	0.99	1.00
Missing Rate = 10%	0.96	0.98	0.99
Missing Rate = 20%	0.91	0.96	0.99
Missing Rate = 30%	0.91	0.94	0.98
<b>Mean Growth Rate</b>			
Missing Rate = 0%	0.98	0.99	1.00
Missing Rate = 10%	0.97	0.99	1.02
Missing Rate = 20%	0.99	1.00	0.98
Missing Rate = 30%	0.95	0.99	1.01
<b>Predictor Effect on Tau</b>			
Missing Rate = 0%	0.95	0.98	1.01
Missing Rate = 10%	0.87	0.98	1.01
Missing Rate = 20%	0.79	0.94	0.98
Missing Rate = 30%	0.79	0.89	0.98
<b>Predictor Effect on Growth Rate</b>			
Missing Rate = 0%	0.97	0.99	1.01
Missing Rate = 10%	0.98	0.99	1.02
Missing Rate = 20%	0.95	0.97	1.02
Missing Rate = 30%	0.96	0.99	0.99

*Note.* Results are irrespective of indicators, criterion, slope heterogeneity, and measure reliability. Standard error bias is presented as percentages (%). 3- and 5-indicator models had 1000 replications, but 50-indicator models had 200 replications.

**Table 6***T2C Model: Power Results on Selected Conditions*

Parameter	Sample Size		
	$n = 100$	$n = 250$	$n = 5000$
<b>Mean Tau</b>			
Missing Rate = 0%	0.97	1.00	1.00
Missing Rate = 10%	0.96	1.00	1.00
Missing Rate = 20%	0.93	1.00	1.00
Missing Rate = 30%	0.88	1.00	1.00
<b>Mean Growth Rate</b>			
Missing Rate = 0%	1.00	1.00	1.00
Missing Rate = 10%	0.99	1.00	1.00
Missing Rate = 20%	0.97	1.00	1.00
Missing Rate = 30%	0.92	1.00	1.00
<b>Predictor Effect on Tau</b>			
Missing Rate = 0%	0.62	0.95	1.00
Missing Rate = 10%	0.50	0.91	1.00
Missing Rate = 20%	0.39	0.82	1.00
Missing Rate = 30%	0.31	0.68	1.00
<b>Predictor Effect on Growth Rate</b>			
Missing Rate = 0%	0.84	0.98	1.00
Missing Rate = 10%	0.77	0.95	1.00
Missing Rate = 20%	0.67	0.90	1.00
Missing Rate = 30%	0.59	0.83	1.00

*Note.* Results are irrespective of indicators, criterion, slope heterogeneity, and measure reliability. 3- and 5-indicator models had 1000 replications, but 50-indicator models had 200 replications.

## CHAPTER 2: MONTE CARLO SIMULATION STUDY

There was no bias for the slope factor nor the predictor effect on the slope factor. The results thus focus on the relative bias and empirical bias in the point and standard error estimates for T2C model parameters (the means for the intercept and tau factors, as well as the predictor effects on each). Although the forthcoming results focus on outcomes for tau and its predictor effect, figures and tables for results on all parameter estimates, including growth, can be found in the appendices. Below, the effect of each design condition on parameter estimates for tau and its predictor effect is discussed in detail.

**Missingness.** Missingness due to attrition was a key driver for parameter estimate bias and power. Considering that missingness directly affects sample size at each time point, which is also a key driver of parameter estimate bias and power, it was no surprise that missingness explained 50% of the variance in the relative bias for tau as well as 33% and 28% of the variance in the power to detect and relative bias for the predictor effect on tau, respectively. As missingness increased, especially with high growth variability and smaller sample sizes, tau and its predictor effect are vulnerable to overestimation with its standard error prone to slight underestimation, and power to detect those parameters drops severely as low as 20%.

**Criterion placement.** Criterion placement was the only key predictor for the power to detect tau, explaining 36% of the variance in this parameter estimate. When the criterion is placed earlier in time (i.e.,  $T2C = +0.33SD$ ), especially with a small sample size (i.e.,  $N = 100$ ), power dropped below 80%. This pattern may appear to be unexpected given that relative bias results showed that tau tends to be overestimated in smaller sample sizes (i.e., power would be expected to increase if the estimated tau is larger than it should be); however, the standard error bias for tau, especially when the criterion is placed earlier in time, was also biased. Further,

## CHAPTER 2: MONTE CARLO SIMULATION STUDY

smaller effect sizes, all other things being equal, always suffer from lower power. As expected, for higher criterion levels and sample sizes, power was close to or at 100%.

**Sample size at study onset.** Sample size was the key predictor of all three simulation outcomes (parameter and standard error bias, as well as power). Compared to other design facets, sample size explained the most variance, ranging from 42% to 90%, in the parameter estimates. For the predictor effect on tau, sample size explained an overwhelming 90% of the variance in power, followed by 80% in the relative bias for tau, 69% in power to detect tau, 65% in relative bias to detect predictor effect on tau, and 54% and 42% in the standard error bias for the predictor effect on tau and the tau factor, respectively. Results showed that the T2C model parameter estimates can become unstable – yielding positive relative bias and negative standard error bias as well as lower power – in the small sample size condition especially (i.e.,  $N = 100$ ), holding all other design conditions constant. This instability was further exacerbated in the small sample size condition when there is a high level of missingness/attrition.

**Number of time points.** The number of time points that data is collected (i.e., number of indicators of a factor analysis) explained 18% to 50% (i.e.,  $\eta_p^2 = 0.18$  to  $\eta_p^2 = 0.50$ ) of the variance in bias and power estimates. More specifically, this design condition explained the following amounts of variance: 50% in the relative bias for tau, 41% in the power to detect the predictor effect on tau, 32% in the standard error bias for the predictor effect on tau, 20% in the standard error bias for tau, and 18% in the relative bias for the predictor effect on tau. T2C models with fewer time points (i.e., 3 or 5 relative to 50), especially in conjunction with a small sample size  $N = 100$ , tended to have positive relative bias, slightly negative standard error bias, and lower power. However, as expected, with greater numbers of time points (coupled with a

larger sample sizes), there was no relative bias or standard error bias, and power was close to or at 100%.

**Growth rate variability.** Growth heterogeneity was the key driver only for the relative bias for tau, explaining 33% of the variance. Tau estimates were most sensitive to high growth rate variability when there was high missingness and a small sample size (i.e.,  $N = 100$ ). As sample size increased (e.g.,  $N = 250$ ), tau and its predictor effect were only prone to overestimation when there was high missingness and poor measurement reliability. As expected, as sample size increased, growth variability impacts on bias and power diminished.

**Measurement reliability.** Similar to growth rate variability, this design facet was a key driver only for the relative bias for tau, explaining 24% of the variance. Results suggested that poor measurement reliability, along with small sample sizes, high missingness, and high growth variability, contribute to an overestimation of tau and its predictor effect, an underestimation of their standard errors, and a lower power to detect them. However, larger sample sizes remedy all conditions with poor measurement reliability. That is, as sample size increases, and holding all else constant, conditions with poor measurement reliability (i.e.,  $\alpha \leq .4$ ) still yielded excellent accuracy and precision for the parameter estimates of tau and its predictor effect.

### **Other Results**

Tables C1 – C9 in Appendix C show the mean relative bias for select design conditions across the 3-, 5-, and 50-time point T2C models. For estimating the time-to-criterion factor and its predictor effect in the small sample size condition ( $N = 100$ ) across the three T2C models, relative bias increased as missingness and growth variability increased, coupled with decreasing measure reliability, especially when the criterion is placed farther out in time.

## CHAPTER 2: MONTE CARLO SIMULATION STUDY

Tables C10 – C18 in Appendix C display power results for select design conditions across the 3-, 5-, and 50-time point T2C models. As expected, power decreased for higher amounts of missingness, but this decline was only observed for these parameter estimates:

- (a) the time-to-criterion factor for the small sample size condition ( $N = 100$ ) with low measure reliability, early criterion (i.e., 105), high missingness, and high growth variability (see Appendix B); and
- (b) the predictor effect on the time-to-criterion factor for the smaller sample size conditions ( $N = 100$  and  $N = 250$ ), especially for higher amounts of missingness (see Appendix B).

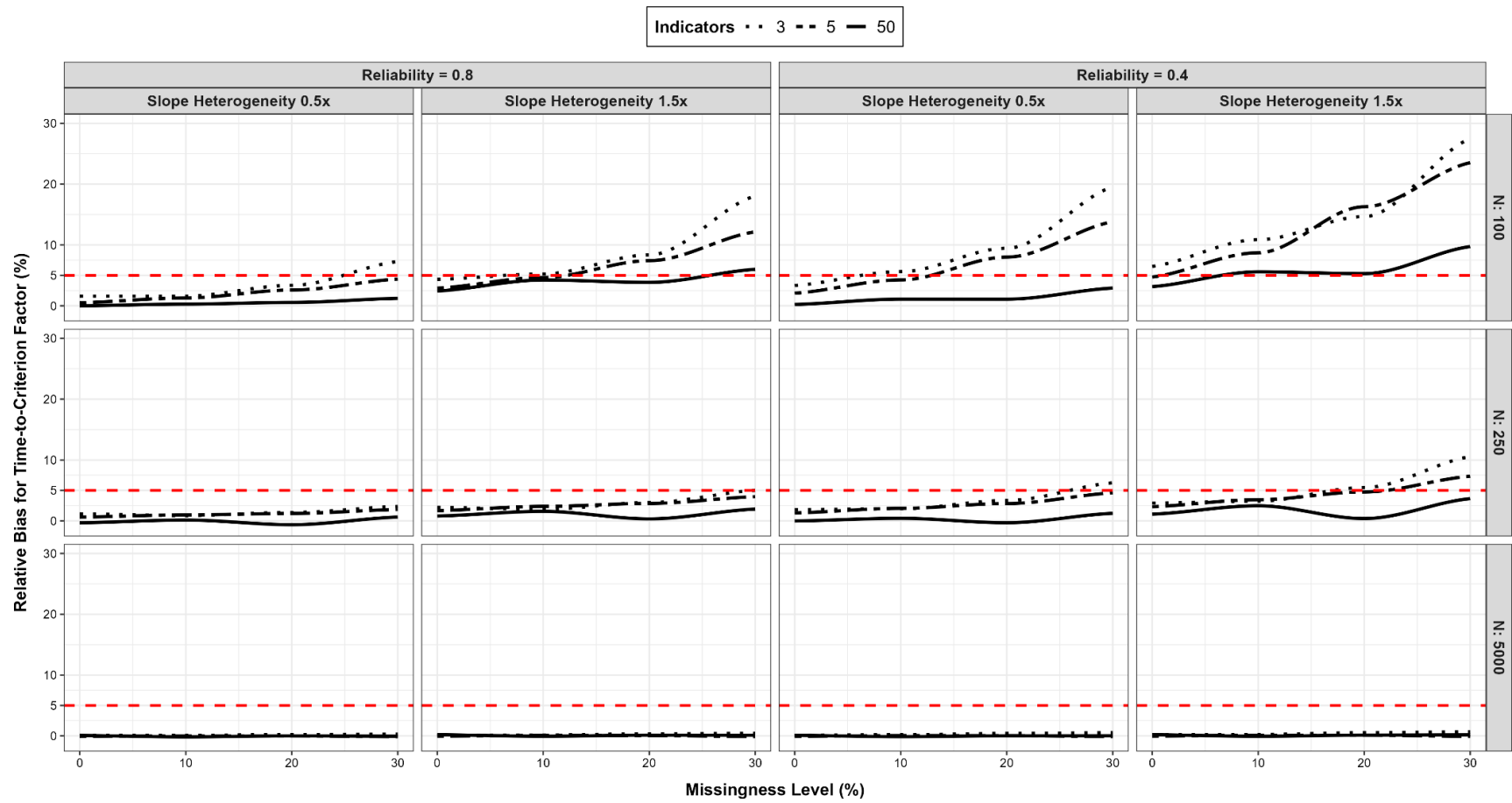
Tables C19 – C27 in Appendix C provide standard error bias results for select design conditions across the 3-, 5-, and 50-time point T2C models. Across those three T2C models, the standard error bias of the time-to-criterion factor and its predictor effect in the small sample size condition ( $N = 100$ ) was slightly underestimated with low measure reliability and/or high growth variability, especially when the criterion is placed closer in time.

However, taken altogether, with greater numbers of indicators, relative bias and standard error bias decreased and power increased for the time-to-criterion factor and predictor effect on the time-to-criterion factor, holding all else constant (see Figures 4 – 9).

CHAPTER 2: MONTE CARLO SIMULATION STUDY

Figure 4

Relative Bias for Time-to-Criterion Factor by Indicator Level



**Figure 5**

*Relative Bias for Predictor Effect on Time-to-Criterion Factor by Indicator Level*

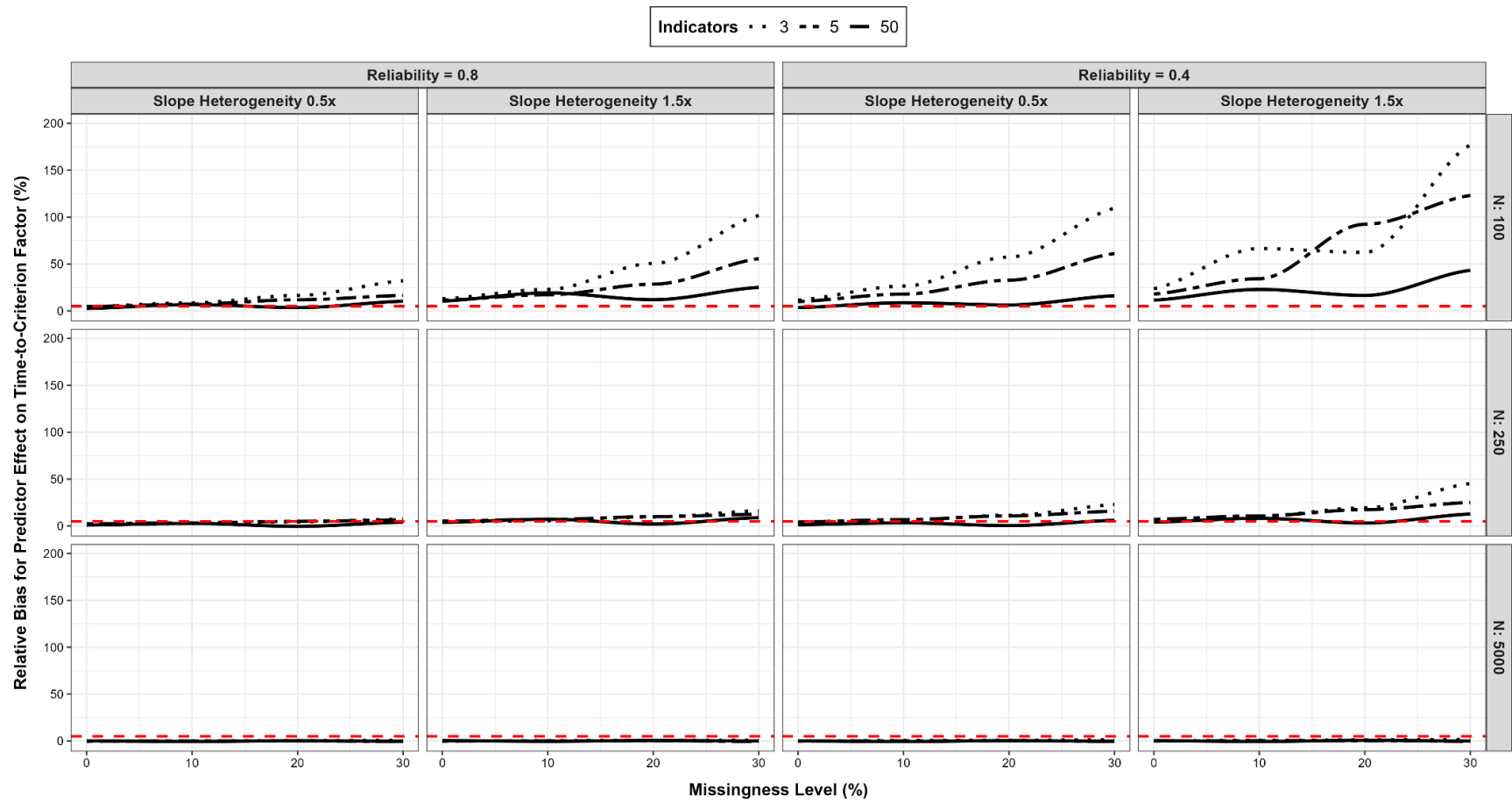
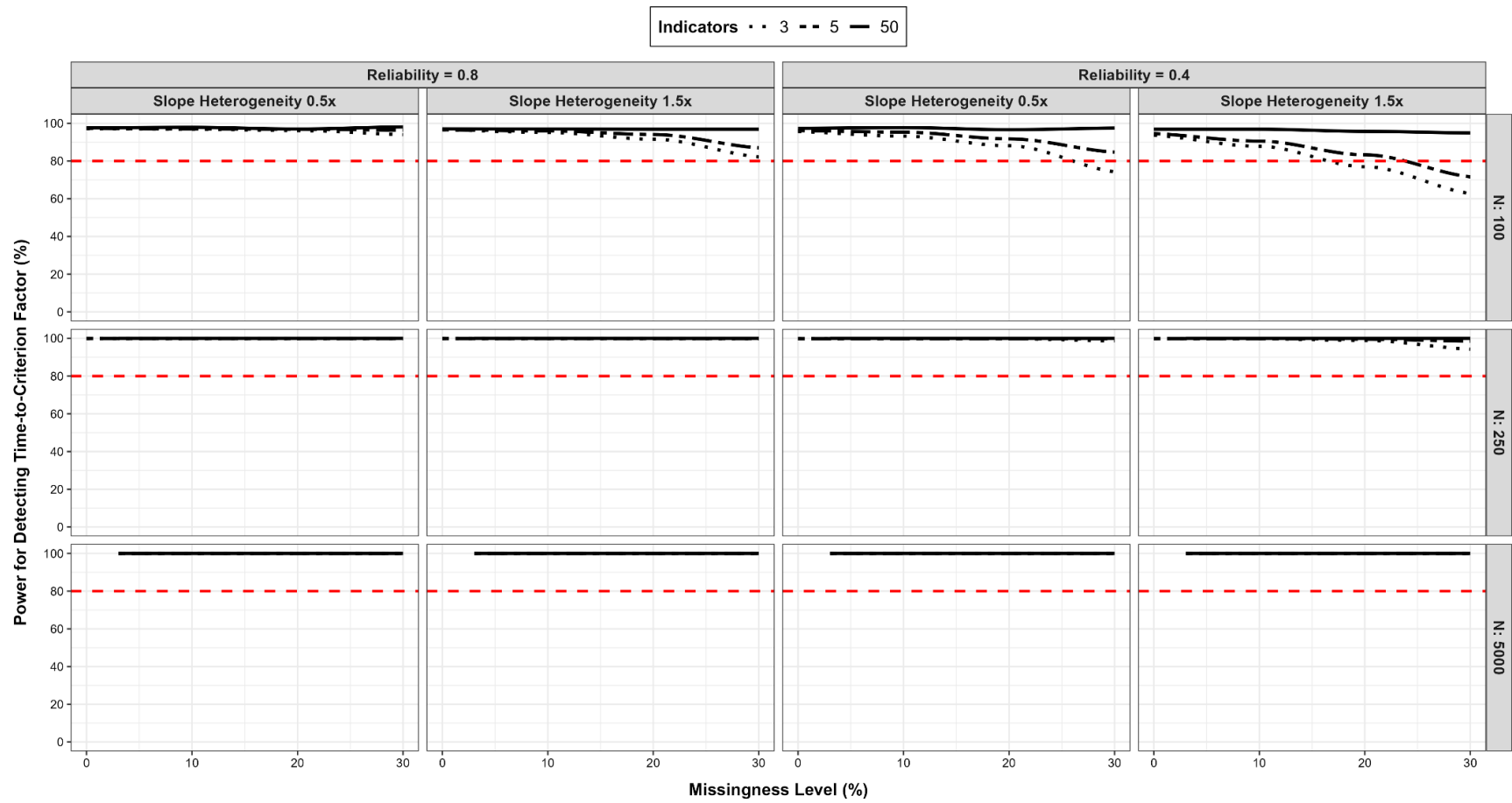


Figure 6

*Power to Detect Time-to-Criterion Factor by Indicator Level*



**Figure 7**

*Power to Detect Predictor Effect on Time-to-Criterion Factor by Indicator Level*

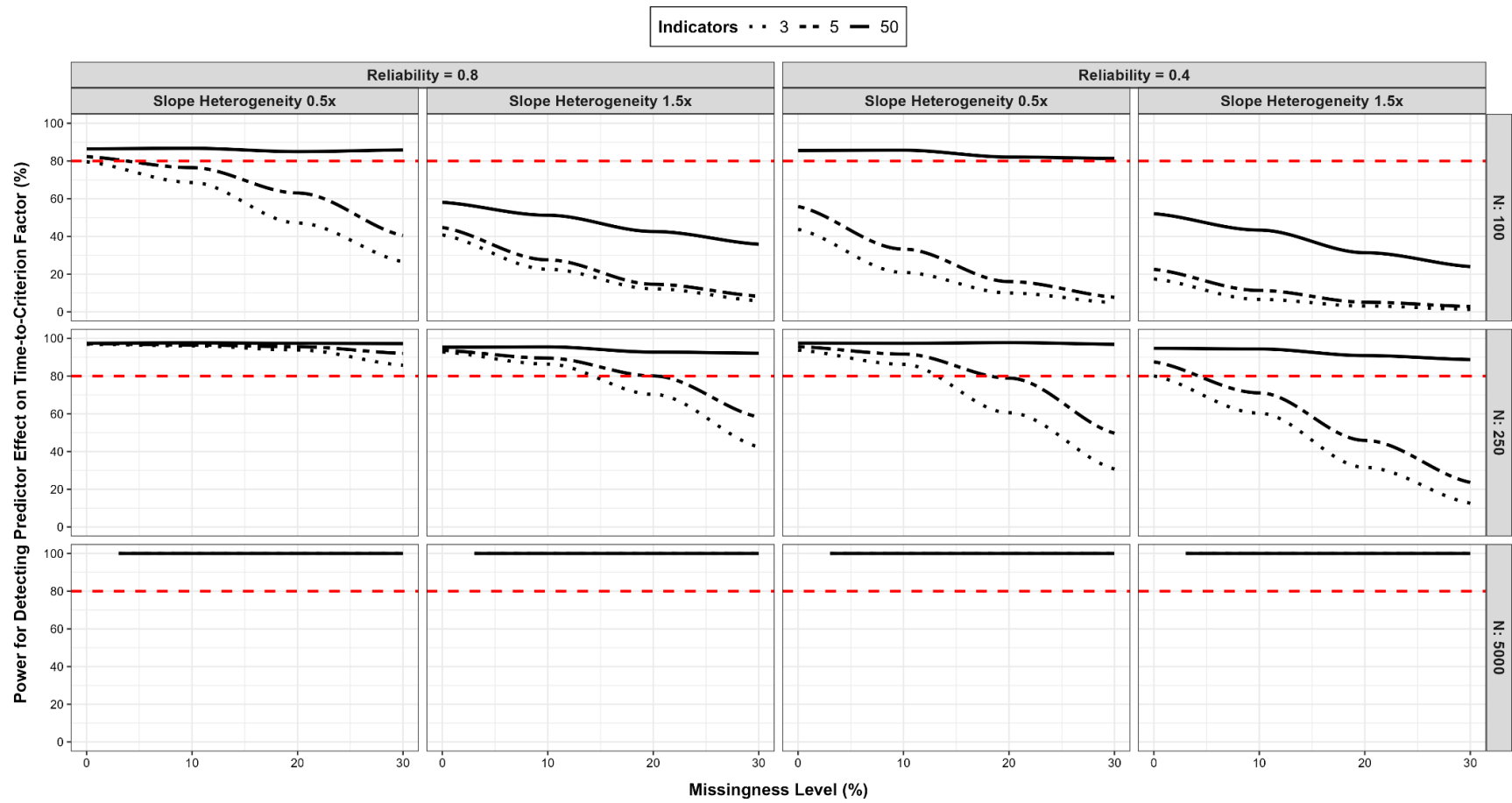
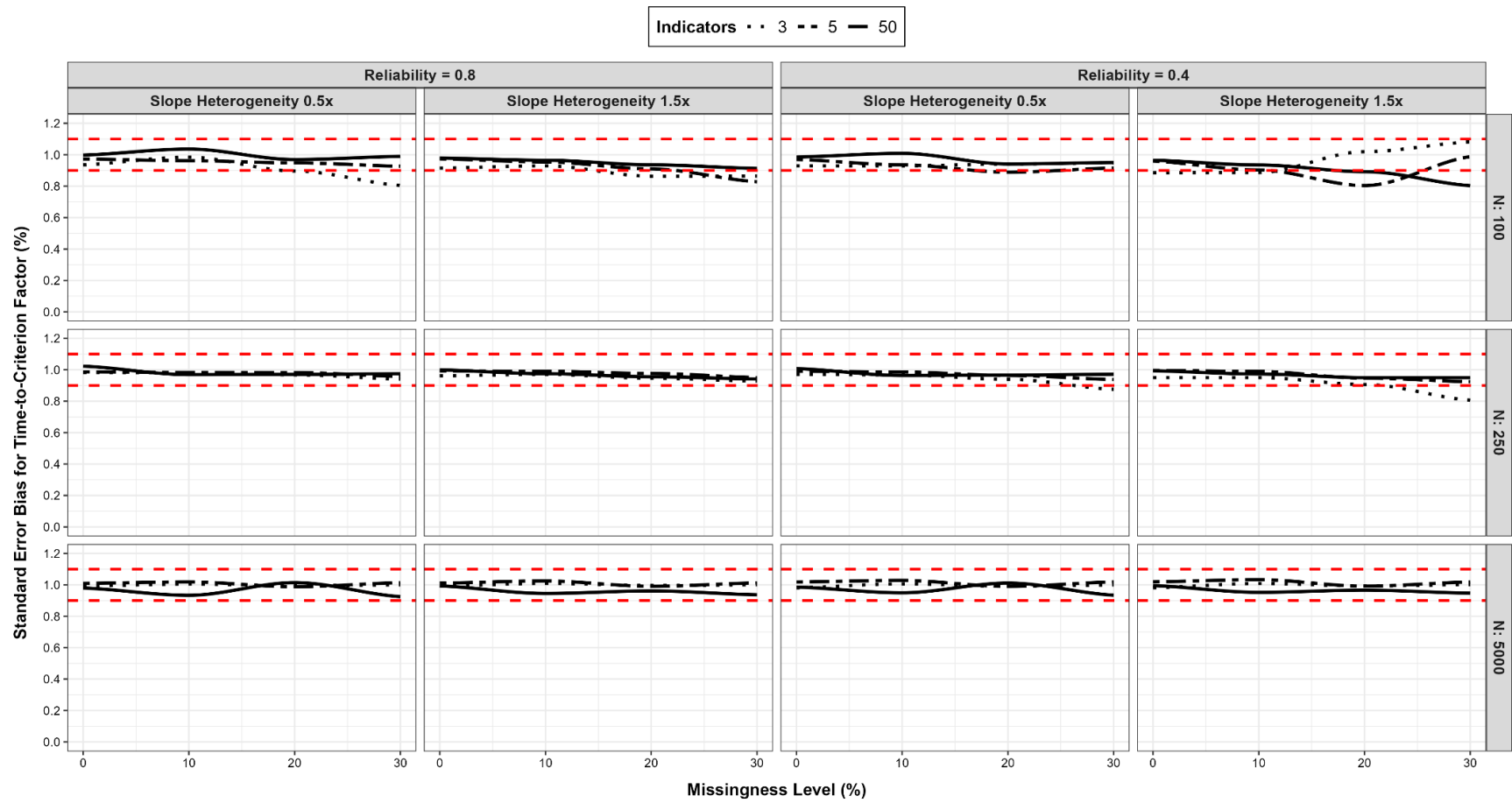


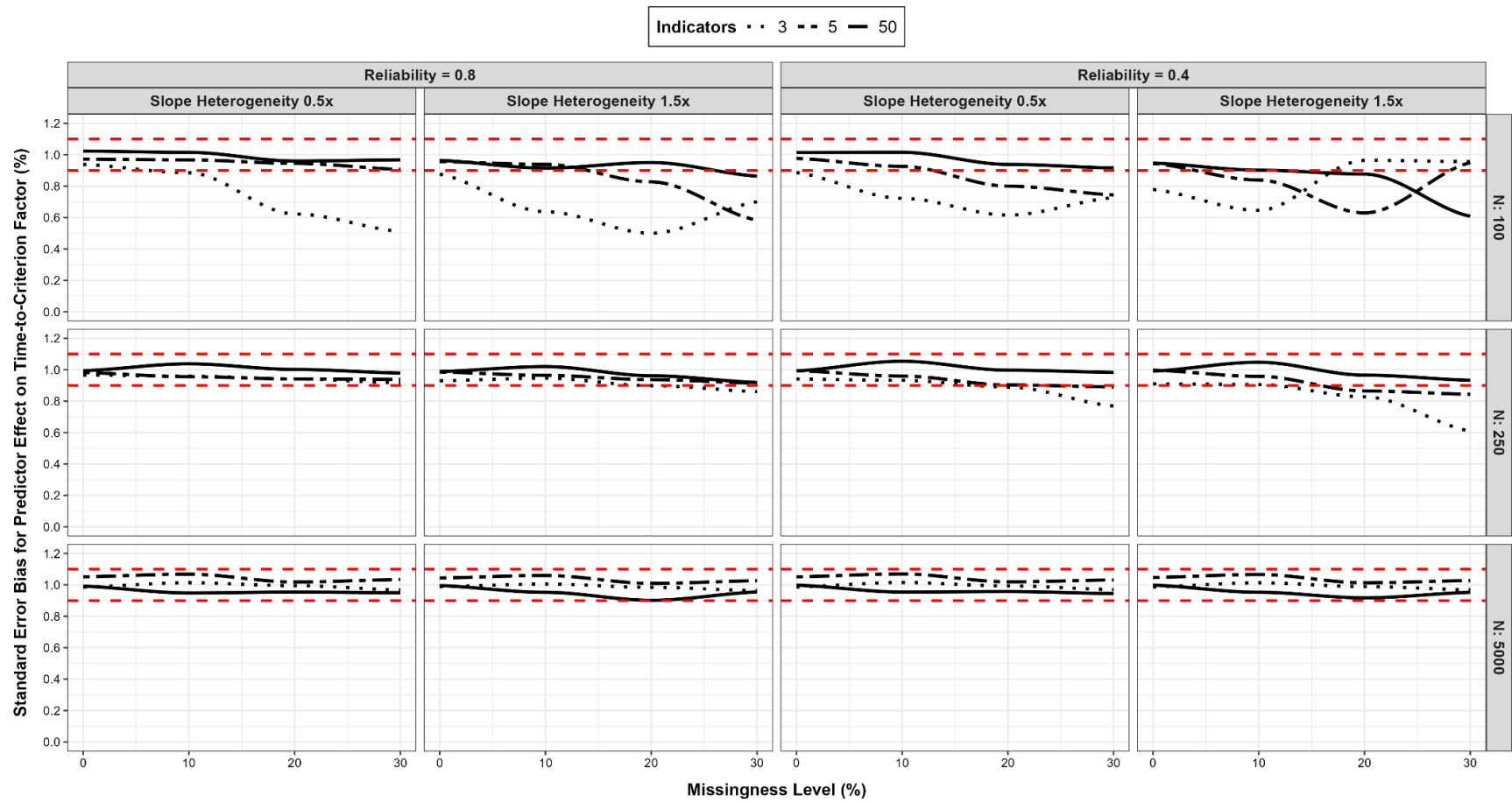
Figure 8

Standard Error Bias for Time-to-Criterion Factor by Indicator Level



**Figure 9**

*Standard Error Bias for Predictor Effect on Time-to-Criterion Factor by Indicator Level*



## CHAPTER 3.

### Real Data Application Example using ECLS 2010 Subsample

To demonstrate how inferences from the two methods – the traditional latent intercept-slope model and the latent T2C model – can be similar as well as diverge, I present an applied analysis using extant data from the U.S. Department of Education NCES *Early Childhood Longitudinal Study, Kindergarten Class of 2010–11* dataset (Tourangeau et al., 2015). I focus on analyzing the language scores of a subsample of children whose home language was not English using both methods.

### Method

#### Data Source

**Overview.** The Early Childhood Longitudinal Study (ECLS) followed a nationally representative sample of children through their elementary school years, starting in the 2010-2011 school year (kindergarten) and ending in the 2015-2016 school year (when most students were in fifth grade). The goal of ECLS is to collect multi-source data (from study participants, parents, teachers, school administrators, etc.) to provide educational researchers and policymakers with data to address questions regarding elementary school children's psychological and cognitive development as well as their social experiences in school. One of the myriad early childhood assessments ECLS collected was students' performance on the English Basic Reading Skills (EBRS) assessment, scored out of 20 points. EBRS is the focal outcome of this applied analysis and children were assessed at four time points: Fall 2010 (kindergarten), Spring 2011 (kindergarten), Fall 2011 (first grade), and Spring 2012 (first grade). To be routed to the EBRS assessment, it is a requirement that students pass the preLAS (English Language Proficiency Assessment for Early Learners) screener with a score of at least 16 points.

**Sample Description.** The sample for the present study included children who communicate in a language other than English in the home and who had available data for the preLAS and EBRS assessments at baseline/Fall 2010, and who were *not* administered the Spanish early reading skills (SERS) assessment. However, excluded from the analytic sample were children whose home language is Spanish and did not pass the preLAS and were therefore, according to ECLS' assessment routing path (Tourangeau et al., 2015; see Exhibit 2-2 on page 2-5) administered the SERS assessment (instead of the EBRS assessment). Although the ECLS dataset includes survey weights, that information was not incorporated into the applied analysis because the focus is on a specific subgroup (i.e., not to generalize results to the broader student population).

### Results

Table 1 displays results for the linear LGM and linear T2C models. Each model's growth depended on students' standardized scores on the preLAS (English Language Proficiency Assessment for Early Learners), which serves as a screener test before students can take EBRS assessments. Given the trajectory patterns found in data visualization, a quadratic model was specified and globally found to be a better fit to the observed data than a linear model. Further, it was unclear when exactly the assessments of children's EBRS took place, so different time point interval structures were explored by freeing time interval estimates for the second and third time points iteratively and selecting the model with the best fit indices (RMSEA, CFI, TLI, and SRMR) to determine the final time point spacing. Model fitting also indicated that the quadratic factor had near-zero variability (i.e., Heywood cases were found), so only the quadratic factor mean was estimated (variance was constrained to zero, and as a result, it could not covary with the intercept and linear growth factors).

**Table 7***Comparison of Latent Traditional Growth and T2C Model Results for ECLS EBRS*

Parameters	Model	
	Linear LGM	Linear T2C (c = 16)
<b>Factor means (<math>\mu</math>)</b>		
Intercept factor ( $\alpha$ )	12.34* (0.09)	—
Slope factor ( $\beta_1$ )	8.92* (0.22)	8.92* (0.22)
Quadratic factor ( $\beta_2$ )	-5.15* (0.15)	-5.15* (0.15)
T2C ( $\tau$ )	—	0.97* (0.01)
<b>Factor variances/covariances (<math>\Phi</math>)</b>		
Intercept factor ( $\alpha$ )	7.54* (0.56)	—
Slope factor ( $\beta_1$ )	2.85* (0.46)	2.85* (0.46)
T2C ( $\tau$ )	—	0.15* (0.011)
Cov ( $\alpha$ , $\beta_1$ )	-4.19* (0.47)	—
Cov ( $\alpha$ , $\tau$ )	—	—
Cov ( $\beta_1$ , $\tau$ )	—	0.38 (0.04)
<b>Factor loadings (<math>\Lambda</math>)</b>		
Intercept factor ( $\alpha$ )		
EBRS 1	1	—
EBRS 2	1	—
EBRS 3	1	—
EBRS 4	1	—
Slope factor ( $\beta_1$ )		
EBRS 1	0.00	-0.97* (0.01)
EBRS 2	0.70	-0.27* (0.01)
EBRS 3	1.10	0.13* (0.01)
EBRS 4	1.30	0.33* (0.01)
Quadratic factor ( $\beta_2$ )		
EBRS 1	0.00	-0.97* (0.01)
EBRS 2	0.49	-0.48* (0.01)
EBRS 3	1.21	0.24* (0.01)
EBRS 4	1.69	0.72* (0.01)
T2C ( $\tau$ )		
EBRS 1	—	-3.77* (0.09)
EBRS 2	—	-3.77* (0.09)
EBRS 3	—	-3.77* (0.09)
EBRS 4	—	-3.77* (0.09)
<b>Predictors of growth (<math>\pi</math>)</b>		
Intercept factor on preLAS	2.32* (0.08)	—
Slope factor on preLAS	-1.38* (0.07)	-1.38* (0.07)
T2C on preLAS	—	-0.26* (0.01)
<b>Residual variances (<math>\Theta</math>)</b>		
EBRS 1	8.41* (0.57)	8.41* (0.57)
EBRS 2	5.43* (0.22)	5.43* (0.22)
EBRS 3	4.40* (0.28)	4.40* (0.28)
EBRS 4	2.23* (0.19)	2.23* (0.19)

*Note.*  $N = 2,114$  children who speak a language other than English in the home, and who began the study in kindergarten in fall of 2010-2011 school year.

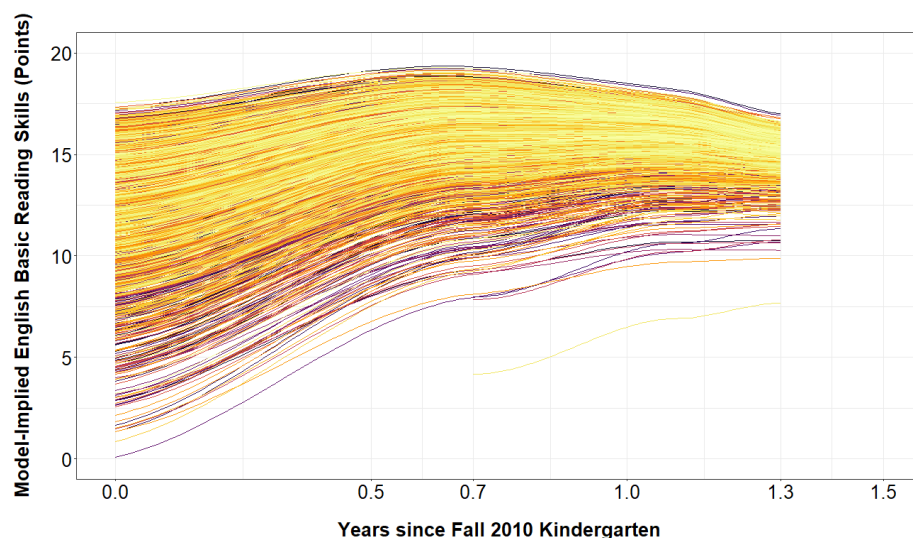
In other words, there is no individual variability in deceleration; every student decelerates in EBRS growth at the same rate. Regardless, overall model fit statistics for these data may not be useful since growth models are constrained and the focus is not to model the data perfectly.

However, it is worth noting that, as proof of their mathematical equivalence and correct model specification, the linear LGM and linear T2C models share the same model fit statistics ( $RMSEA = 0.16$ , 90% CI = [0.15, 0.18];  $CFI = 0.83$ ;  $TLI = 0.72$ ;  $SRMR = 0.11$ ;  $\chi^2(6, N = 2,114) = 336.11$ ,  $p < .001$ ).

**Linear LGM.** Table 1 (first set of columns) reports the findings for the traditional growth model. The intercept factor (fall 2010 kindergarten EBRS) was estimated at  $M = 12.34$  points at the start of kindergarten ( $SE = 0.09$ ), the linear factor (annual growth in EBRS) was estimated at  $M = 8.92$  points per year ( $SE = 0.22$ ), and the quadratic factor (deceleration in EBRS growth) was estimated at  $M = -5.15$  points deceleration per year ( $SE = 0.15$ ). Figure 10 uses a heat plot to illustrate the model-implied (predicted) individual trajectories, with yellow colors indicating greater numbers of students and purple colors indicating fewer numbers of students.

**Figure 10**

*Model-Implied Individual Growth Trajectories for ECLS EBRS*



The latent factor covariance between the intercept ( $\alpha$ ) and linear growth factors ( $\beta$ ) were estimated at -4.19 points per point ( $SE = 0.47$ ). This negative relationship indicates that students who performed better on the EBRS assessment at baseline/fall 2010 experienced less growth in EBRS over the four assessments.

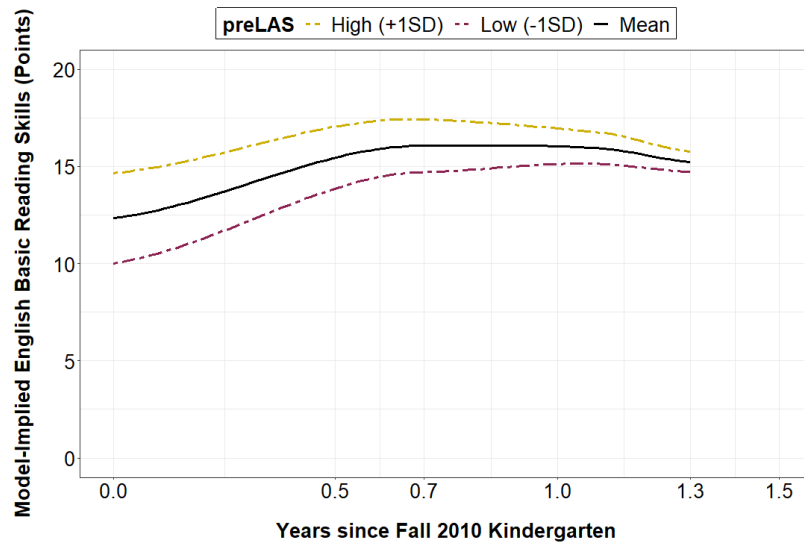
Results also indicated that the fall language preLAS assessment uniquely predicted the intercept and linear growth factors. Specifically, for every standard deviation increase in preLAS points, students' baseline EBRS scores were predicted to be higher by 2.32 points ( $SE = 0.08$ ) but simultaneously, these students were also predicted to experience 1.38 less points per year growth ( $SE = 0.07$ ), holding all else constant. That is, the better that students performed on the preLAS, the less growth they experienced over time (see Figure 11 for aggregate trajectories).

**Linear T2C.** While the slope estimate was identical to that of the traditional linear growth model, in the T2C model, focus is shifted to the estimated time-to-criterion parameter,  $\hat{\tau}$ . The mean time to reach the criterion of  $c = 16$  points was estimated at  $\hat{\tau} = 0.97$  years ( $SE = 0.01$ ), which indicates that the model predicted students to take approximately eleven months from baseline/fall 2010 to reach a threshold score of 16 points on the EBRS assessment. The model also estimated that the covariance between the growth and tau factors was positive:  $Cov(\beta, \tau) = 0.38$  ( $SE = 0.04$ ). This indicates that students who experienced more linear growth on the EBRS were predicted to take longer to reach the criterion. Given what was previously found for the covariance between the intercept and linear growth factors in the linear LGM, it seems clear that although students who started out lower were predicted to have more growth, the significance of their lower scores at baseline/fall 2010 necessitated a longer period to reach the criterion.

Last but not least, the model results showed that the preLAS was a negative predictor of mean time-to-criterion (see Figure 13).

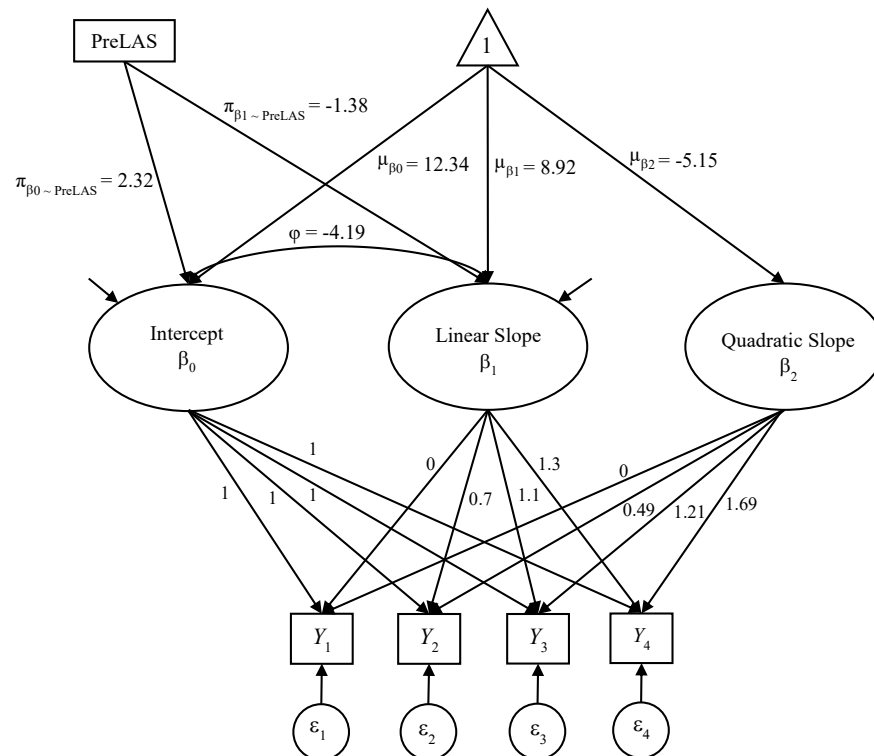
**Figure 11**

*Model-Implied Aggregate Growth Trajectories by preLAS Level for ECLS EBRS*

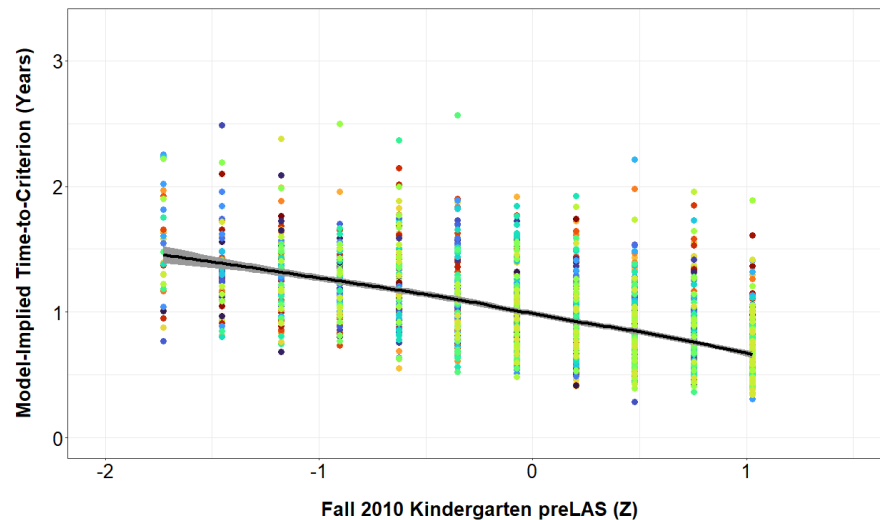


**Figure 12**

*Path Diagram of Results for the Traditional Growth Model for ECLS EBRS*



Note. All paths shown are statistically significant at the .05, 2-tailed level.

**Figure 13***Relationship between Baseline/Fall preLAS Scores and Model-Implied Time-to-Criterion*

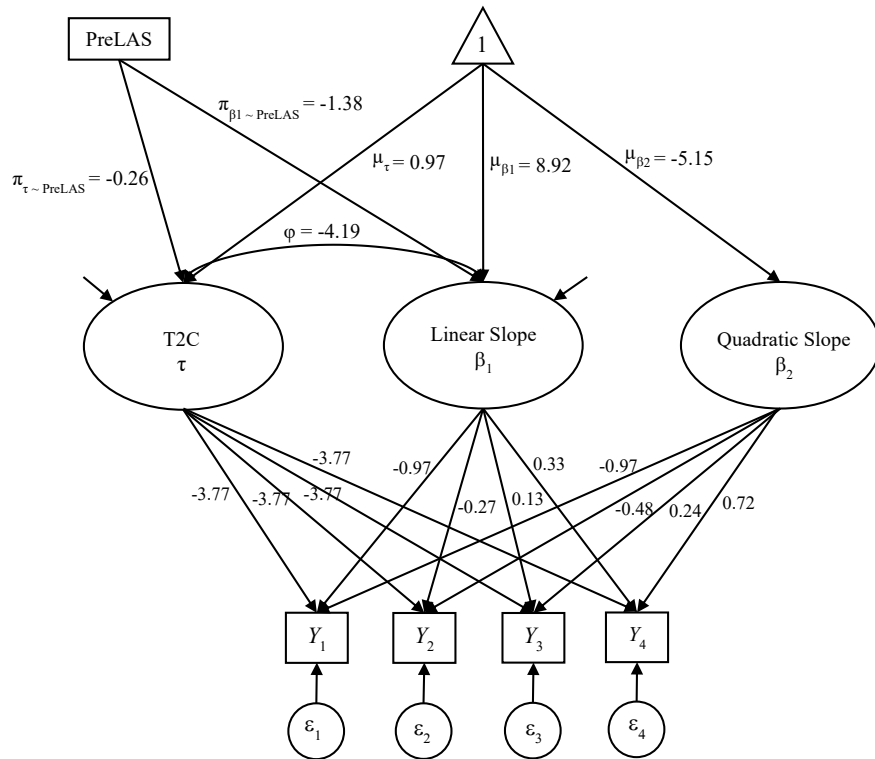
*Note.* Dots represent individual students in the analytic sample. Black trendline represents the averaged relationship (with standard errors in gray shading) between preLAS scores and Time-to-Criterion.

Specifically, the cumulative effect of higher preLAS scores was estimated at approximately 3 months *less* time to reach the criterion. In other words, the better the students performed on the preLAS (a screener test for EBRS assessments), the less time it took them to reach the criterion on the EBRS.

Last but not least, to ensure the linear T2C model was estimable, the four EBRS indicators were specified with constraints on the relationships with the time-to-criterion, linear slope, and quadratic slope factors (see Figure 14). The factor loadings of the time-to-criterion factor (which has replaced the intercept factor of the traditional model) were fixed at the negative value of the linear slope minus the quadratic slope, and the linear slope factor loadings were fixed at the time of each EBRS measurement occasion minus the mean time to reach the criterion. Lastly, the quadratic slope factor loadings were fixed at the squared value of the real time EBRS measurement occasion minus the mean time to reach the criterion.

**Figure 14**

*Path Diagram of Linear T2C Model Results for ECLS EBRS*



*Note.* All paths shown are statistically significant at the .05 level, 2-tailed.

## CHAPTER 4.

### Discussion

The present study investigated the parameter estimate performance of the recently derived latent time-to-criterion (T2C) model (Johnson & Hancock, 2019) with simulation methods under a variety of realistic conditions that would be encountered in longitudinal education research, and then demonstrated the model's use with real data. In the simulation, design facets included attrition, sample size, number of time points collected, measurement reliability, and growth variability. Results showed that relatively high attrition, high growth variability, lower measure reliability, in conjunction with small sample sizes (i.e.,  $N \leq 250$ ) and a criterion placed farther out in time, can lead to overestimation of the tau factor, inferring the time it takes to reach a criterion will take longer than it actually does.

The applied example utilized a large sample size of  $N = 2,114$  children measured on their early basic reading skills (EBRS) at four time points: fall 2010, spring 2010, fall 2011, and spring 2011. Missingness for the four time points included 3% at the second time point, 65% at the third time point, and only 35% at the last measurement occasion. The continuous predictor used in the analysis was baseline preLAS score (measured at the first time point in fall 2010), which had no missingness. The growth rate variability was approximately  $SD = 10.12$  points (i.e., the slope factor's standard error of 0.22 multiplied by the square root of the sample size  $\sqrt{2114}$ ). According to Najarian et al. (2018), the reliability for the EBRS assessment was estimated to be .87 or higher at each of the four measurement occasions (reliabilities were: .87 fall 2010, .97 spring 2010, .94 fall 2011, and .99 spring 2011). According to the simulation results, because the sample size of the applied analysis was so large, all parameter estimates would be expected to be unbiased, since sample sizes of 250 or greater yielded estimates with

## CHAPTER 4: DISCUSSION

negligible bias; as well, power levels for each parameter estimate would be expected to be greater than 99%.

### **Research Design Recommendations**

Within the context of the simulation design conditions used in this study, there are three fundamental education research scenarios to consider for planning a study in order to use the T2C model. In one scenario, an intervention program may be of interest for a study with a relatively small cohort (i.e.,  $N \leq 100$ ) of rare or vulnerable participants, which might also have a relatively high attrition rate over time. According to the simulation results, small sample sizes and/or higher attrition led to overestimation of the time-to-criterion factor as well as its predictor effect. Thus, researchers could mitigate this situation, assuming a missingness level of 20 to 30% (per measurement occasion), by selecting *very reliable measures* (e.g.,  $\alpha \geq .6$ ) to track participants over time, and if possible, *greater numbers of time points*.

In a second scenario, a just-developed, researcher-designed curriculum-based measure may be what is being used to track participants over time, and thus may have relatively low reliability. The results of the simulation showed that modest to poor measure reliability (i.e.,  $\alpha \leq 0.6$ ) led to overestimation of the time-to-criterion factor and its predictor effect, especially in smaller sample sizes of 100. However, simulation results also showed that a sample size of  $N \geq 250$  or having at least five measurement occasions reduced bias and increased power to detect key parameters. Thus, in circumstances using a potentially unreliable scale to follow participants over time, it would be best to boost the sample size or number of measurements taken.

Finally, a third research scenario would be when there is high growth variability among individuals due to differential response to intervention or due to a very diverse population being studied. The simulation results showed that high growth variability was only problematic for

## CHAPTER 4: DISCUSSION

very small sample sizes or very high attrition rates. As such, when the goal is to study response-to-intervention or a very mixed population, obtaining a large sample and committing resources to tracking participants to avoid missingness over time is recommended.

### **Limitations and Future Directions**

The limitations in the simulation study results are directly related to the design conditions. First, this study only simulated linear growth, not higher order polynomial or nonlinear growth (although a quadratic factor was demonstrated in the applied analysis). It would be especially interesting in the future to investigate how the T2C model parameter estimates would perform with a binary outcome (either with a continuous predictor or binary predictor). Second, one continuous, time-invariant predictor was used, with a relatively small effect size for the predictor on the growth and T2C factors. Because the predictor effect on the growth and T2C factors was not zero for all conditions, there were no conditions to evaluate Type I error. Future directions could consider including time-variant predictors as well as conditions evaluating Type I error for the predictor effect on latent factors. Third, the covariance between the intercept and slope was constrained to 0 for simplicity in the data generating model. In the real world, this situation is rare, as starting points typically determine growth rates; however, there is little reason to expect that relaxing this constraint would change the pattern of findings in the present study.

Fourth, the levels of missingness were relatively modest (i.e., the highest level of missingness was 30%), as was the growth rate (i.e., 2 points increase per measurement occasion). Some longitudinal studies can have much higher missingness and/or higher growth rates. Relatedly, FIML was used to handle missingness; multiple imputation may have been a better approach. Fifth, among the 3-, 5-, and 50-time point models, time intervals were assumed equidistant. In applied research, it may more realistic or practical that study participants are not

## CHAPTER 4: DISCUSSION

measured at equal intervals due to factors like program funding or the nature of how the measurement instrument is administered. Future directions could delve deeper into how uneven time intervals could affect the T2C model parameter estimation for better or worse.

Lastly, this study followed a frequentist approach, and it is possible that a Bayesian approach with informative priors might yield better T2C model estimates. In sum, future work should investigate the effects of the following conditions on parameter estimation: (a) non-linearity, (b) null effect of the predictor on the growth parameters, (c) effects of differing levels of intercept-slope covariance, (d) higher levels of missingness, and (e) Bayesian framework with informative priors.

### **Conclusion**

Compared with “traditional” latent intercept-slope types of growth modeling, the latent time-to-criterion (T2C) model may be an ideal – but underutilized – analytic tool for evaluating longitudinal data with an eye toward more actionable education policy decision-making. This said, despite their mathematical *model* equivalence, the traditional and T2C growth models differ in their parameter estimation performance under different realistic longitudinal research situations. This is because the tau factor (and its predictors) will naturally depend on the slope factor more than the intercept of a traditional model, and moreover, when there is attrition, the amount of information available to estimate the T2C model parameters is lower than that of a traditional intercept-slope growth model.

Results from this dissertation’s simulation study provide direct insight in helping researchers plan their T2C study designs to avoid bias and maximize power, and the applied analysis then demonstrates the T2C analysis with publicly available data. It is hoped that this

## CHAPTER 4: DISCUSSION

model can be employed in work that not only advances theory, but also improves and supports equitable and just decisions that can affect the lives of students and their communities.

## REFERENCES

### REFERENCES

- Anthony, C. J., & Ogg, J. (2020). Executive function, learning-related behaviors, and science growth from kindergarten to fourth grade. *Journal of Educational Psychology, 112*(8), 1563–1581. <https://doi.org/10.1037/edu0000447>
- Berkner, L. and L. (1997). Chavez. *Access to Postsecondary Education for 1992 High School Graduates*. Washington, DC: National Center for Education Statistics. Available on the World Wide Web: <https://nces.ed.gov/pubs98/98105.pdf>
- Biesanz, J. C., Deeb-Sossa, N., Papadakis, A. A., Bollen, K. A., & Curran, P. J. (2004). The role of coding time in estimating and interpreting growth curve models. *Psychological methods, 9*(1), 30–52. <https://doi.org/10.1037/1082-989X.9.1.30>
- Bloom, N. S. (1968). Learning for mastery. *Instruction and Curriculum, 1*(2), 1-10. Available on the World Wide Web: <https://files.eric.ed.gov/fulltext/ED053419.pdf>
- Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (2000). *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academy Press. Available on the World Wide Web: <http://www.csun.edu/~SB4310/How%20People%20Learn.pdf>
- Bollen, K. A., & Curran, P. J. (2006). *Latent curve models: A structural equation perspective*. Hoboken, NJ: Wiley
- Callahan, R. M. (2005). Tracking and high school English learners: Limiting opportunity to learn. *American Educational Research Journal, 42*, 305–328. <https://doi.org/10.3102/00028312042002305>
- Cardenas, A. (2018). Every English Learner Succeeds: THE NEED FOR UNIFORM ENTRY AND EXIT REQUIREMENTS. *Brooklyn Law Review, 83*(2), 755. Available on the

## REFERENCES

- World Wide Web:  
<https://brooklynworks.brooklaw.edu/cgi/viewcontent.cgi?article=2138&context=blr>
- Carroll, J. B. (1963). A model of school learning. *Teachers College Record*, 64(8), 723-733.  
<https://doi.org/10.1177/016146816306400801>
- Chen, X. (2005). First-Generation Students in Postsecondary Education: A Look at their College Transcripts. Washington, DC: National Center for Education Statistics. Available on the World Wide Web: <https://files.eric.ed.gov/fulltext/ED485756.pdf>
- Choy, S. (2001). *Students whose parents did not go to college: Postsecondary access, persistence and attainment* (NCES 2001-126). Washington, DC: U.S. Department of Education. Available on the World Wide Web:  
<https://files.eric.ed.gov/fulltext/ED460660.pdf>
- Coley, R. L., Kruzik, C., & Votruba-Drzal, E. (2020). Do family investments explain growing socioeconomic disparities in children's reading, math, and science achievement during school versus summer months?. *Journal of Educational Psychology*, 112(6), 1183–1196.  
<https://doi.org/10.1037/edu0000427>
- Collier, V. (1987). Age and rate of acquisition of second language for academic purposes. *TESOL Quarterly*, 21 : 6 17-64 1. <https://doi.org/10.2307/3586986>
- Collier, V. (1995). Acquiring a second language for school. *Directions in Language and Education*, 1 :4. Washington, DC: The National Clearinghouse for Bilingual Education. Available on the World Wide Web:  
<http://www.ncbe.gwu.edu/nchepubs/directions/O4.htm>
- Collina, A., & Halverson, R. (2009). *Rethinking education in the age of technology: The digital revolution and schooling in America*. New York: Teachers College Press.

## REFERENCES

Comrey, A. L., & Lee, H. B. (1992). *A First Course in Factor Analysis*. Hillsdale, NJ: Erlbaum.

Cook, G., Boals, T., & Lundberg, T. (2011). Academic achievement for English learners: What can we reasonably expect? *Phi Delta Kappa*, 93, 66–69.

<https://doi.org/10.1177/0031721711109300316>

Cross, J. R., & Schroth, S. (2016). *Mastery-based learning: is it good for gifted learners?* NAGC Conceptual Foundations Network. Available on the World Wide Web:

<https://www.nagc.org/sites/default/files/Publication%20PHP/Mastery%20Based%20Learning-Riedl-Schroth-December%202016.pdf>

Dabach, D. B. (2014). “I am not a shelter!” Stigma and social boundaries in teachers’ accounts of students’ experience in separate “sheltered” English learner classrooms. *Journal of Education for Students Placed at Risk*, 19, 98–124.

<https://doi.org/10.1080/10824669.2014.954044>

Enders, C. K., & Bandalos, D. L. (2001). The relative performance of full information maximum likelihood estimation for missing data in structural equation models. *Structural equation modeling*, 8(3), 430–457. [https://doi.org/10.1207/S15328007SEM0803\\_5](https://doi.org/10.1207/S15328007SEM0803_5)

Engle, J., & Tinto, V. (2008). Moving beyond access: College success for low-income, first-generation students. Pell Institute for the Study of Opportunity in Higher Education.

Available on the World Wide Web: <https://files.eric.ed.gov/fulltext/ED504448.pdf>

Estrada, P., & Wang, H. (2018). Making English Learner Reclassification to Fluent English Proficient Attainable or Elusive: When Meeting Criteria Is and Is Not Enough. *American Educational Research Journal*, 55(2), 207-242.

<https://doi.org/10.3102/0002831217733543>

Fentress J. C., and Collopy R. M. B. (2011). Promoting Resiliency Among First-Generation

## REFERENCES

- College Students. *Teacher Education Faculty Publications*, 14. Available on the World Wide Web: [https://ecommons.udayton.edu/cgi/viewcontent.cgi?article=1013&context=edt\\_fac\\_pub](https://ecommons.udayton.edu/cgi/viewcontent.cgi?article=1013&context=edt_fac_pub)
- Friedman, T. L. (2006). *The world is flat: A brief history of the twenty-first century*. New York: Farrar, Straus, and Giroux.
- Garriott. (2020). A Critical Cultural Wealth Model of First-Generation and Economically Marginalized College Students' Academic and Career Development. *Journal of Career Development*, 47(1), 80–95. <https://doi.org/10.1177/0894845319826266>
- Goldenberg, C., Rueda, R. S., & August, D. (2006). Sociocultural influences on the literacy attainment of language minority children and youth. In D. August & T. Shanahan (Ed.), *Developing literacy in second language learners: Report of the National Literacy Panel on Language Minority Children and Youth* (pp. 249–267). Mahwah, NJ: Erlbaum.
- Gorsuch, R. L. (1983). *Factor Analysis*, 2nd Ed. Hillsdale, NJ: Erlbaum.
- Guglielmi, R. S. (2012). Math and science achievement in English Language Learners: Multivariate latent growth modeling of predictors, mediators, and moderators. *Journal of Educational Psychology*, 104, 580–602. <https://doi.org/10.1037/a0027378>
- Guglielmi, R. S., & Brekke, N. (2018). A latent growth moderated mediation model of math achievement and postsecondary attainment: Focusing on context-invariant predictors. *Journal of Educational Psychology*, 110(5), 683–708. <https://doi.org/10.1037/edu0000238>
- Guilford, J. P. (1954). *Psychometric Methods* (2nd Ed.). New York, NY: McGraw-Hill.
- Hakuta, K., Butler, Y. G., & Witt, D. (2000). *How long does it take English learners to attain proficiency?* Davis, CA: The University of California Linguistic Minority Research

## REFERENCES

- Institute. Available on the World Wide Web:  
<https://files.eric.ed.gov/fulltext/ED443275.pdf>
- Hamaker, E. L., & Muthén, B. O. (2020). The fixed versus random effects debate and how it relates to centering in multilevel modeling. *Psychological methods, 25*(3), 365–379.  
<https://doi.org/10.1037/met0000239>
- Harklau, L. (2002). ESL versus mainstream classes: Contrasting L2 learning environments. In V. Zamel & R. Spack (Eds.). *Enriching ESOL pedagogy: Readings and activities for engagement, reflection, and inquiry*. Mahwah, NJ: Lawrence Erlbaum Associates.  
<https://doi.org/10.2307/3587433>
- Hecht, M., & Zitzmann, S. (2020). Sample size recommendations for continuous-time models: Compensating shorter time series with larger numbers of persons and vice versa. *Structural Equation Modeling: A Multidisciplinary Journal, 1*–8.  
<https://doi.org/10.1080/10705511.2020.1779069>
- Hoogland, J. J., & Boomsma, A. (1998). Robustness studies in covariance structure modeling: An overview and a meta-analysis. *Sociological Methods & Research, 26*(3), 329-367.  
<https://doi.org/10.1177/0049124198026003003>
- Inman, W. E., & Mayes, L. (1999). The importance of being first: Unique characteristics of first-generation community college students. *Community College Review, 26*, 3–23.  
<https://doi.org/10.1177/009155219902600402>
- Johnson, A. (2020). The Impact of English Learner Reclassification on High School Reading and Academic Progress. *Educational Evaluation and Policy Analysis, 42*(1), 46-65.  
<https://doi.org/10.3102/0162373719877197>

## REFERENCES

- Johnson, T. L., & Hancock, G. R. (2019). Time to criterion latent growth models. *Psychological methods, 24*(6), 690-707. <https://doi.org/10.1037/met0000214>
- Kenward, M. G., & Molenberghs, G. (1998). Likelihood based frequentist inference when data are missing at random. *Statistical Science, 13*(3), 236–247. <https://doi.org/10.1214/ss/1028905886>
- Kieffer, M. J., & Parker, C. E. (2016). *Patterns of English learner student reclassification in New York City public schools* (REL 2017–200). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Northeast & Islands. Retrieved from <http://ies.ed.gov/ncee/edlabs>.
- Kim, A-S., Choi, S., & Park, S. (2020). Heterogeneity in first-generation college students influencing academic success and adjustment to higher education. *The Social Science Journal, 57*(3), 288-304. <https://doi.org/10.1016/j.soscij.2018.12.002>
- Kim, J., & Herman, J. L. (2010). *When to exit ELL students: monitoring subsequent success and failure in mainstream classrooms after ELLs' reclassification*. (CRESST Report 779). Los Angeles, CA: University of California, National Center for Research on Evaluation, Standards, and Student Testing (CRESST). Available on the World Wide Web: <https://files.eric.ed.gov/fulltext/ED520430.pdf>
- Kline, P. (1979). *Psychometrics and Psychology*. London: Academic Press.
- Kopcha, T. J., Ding, L., Neumann, K. L., & Choi, I. (2016). Teaching Technology Integration to K-12 Educators: A “Gamified” Approach. *TechTrends, 60*(1), 62–69. <https://doi.org/10.1007/s11528-015-0018-z>

## REFERENCES

- Lee, D. (2014). How to Personalize Learning in K-12 Schools: Five Essential Design Features. *Educational Technology*, 54(3), 12–17.
- Lohfink, M.M. and M B. Paulsen. “Comparing the Determinants of Persistence for First-generation and Continuing-generation Students.” *Journal of College Student Development*, 46(4): 409-428. 2005. <https://doi.org/10.1353/csd.2005.0040>
- Lundberg, S. L. A., Hovaguimian, K., & Slavin Miller, S. (2007). First-Generation Status and Student Race/Ethnicity as Distinct Predictors of Student Involvement and Learning. *NASPA Journal*, 44(1). <https://doi.org/10.2202/0027-6014.1755>
- Matta, T., & Soland, J. (2019). Predicting Time to Reclassification for English Learners: A Joint Modeling Approach. *Journal of Educational and Behavioral Statistics*, 44(1), 78-102. <https://doi.org/10.3102/1076998618791259>
- Mammadov, S., & Hertzog, N. B. (2021). Changes in students’ achievement goals in advanced learning environment: a multivariate multilevel model. *Educational Psychology*, 41(9), 1097–1116. <https://doi.org/10.1080/01443410.2021.1975654>
- Matthews D. J., and Foster J. F. (2005). Mystery to mastery: Shifting paradigms in gifted education, *Roeper Review*, 28:2, 64-69. <https://doi.org/10.1080/02783190609554340>
- Mavrogordato, M., & White, R. S. (2017). Reclassification variation: How policy implementation guides the process of exiting students from English learner status. *Educational Evaluation and Policy Analysis*, 39, 281–310. <https://doi.org/10.3102/0162373716687075>
- McCoach, D. B., Rambo, K. E., & Welsh, M. (2013). Assessing the growth of gifted students. *Gifted Child Quarterly*, 57(1), 56–67. <https://doi.org/10.1177/0016986212463873>

## REFERENCES

- McCombs, B. (2008). From one-size-fits-all to personalized learner-centered learning: The evidence. *The FM Duffy Reports*, 13(2), 1-12.
- Mcdossi, O., Wright, A. L., McDaniel, A., & Roscigno, V. J. (2022). First-generation inequality and college integration. *Social Science Research*, 105, 102698-102698.  
<https://doi.org/10.1016/j.ssresearch.2022.102698>
- Mehta, S. S., Newbold, J. J., & O'Rourke, M. A. (2011). Why do first-generation students fail? *College Student Journal*, 45(1), 20-35.
- Miliband, D. (2006). Choice and voice in personalised learning. In OECD (Ed.), *Schooling for tomorrow: Personalising education* (pp. 21-30). Brussels: OECD Publishing.
- Motamedi, J. G., Singh, M., & Thompson, K. D. (2016). *English learner student characteristics and time to reclassification: An example from Washington state. REL 2016-128*. Washington, DC: Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Northwest. Available on the World Wide Web:  
<https://files.eric.ed.gov/fulltext/ED565624.pdf>
- Najarian, M., Tourangeau, K., Nord, C., and Wallner-Allen, K. (2018). Early Childhood Longitudinal Study, Kindergarten Class of 2010–11 (ECLS-K:2011), First- and Second-Grade Psychometric Report (NCES 2018-183). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education. Washington, DC. Retrieved from <https://nces.ed.gov/pubsearch>.
- Núñez, A. M., & Cuccaro-Alamin, S. (1998). First-generation students: Undergraduates whose parents never enrolled in postsecondary education (Report No. NCES 98–082). Washington, DC: National Center for Education Statistics.

## REFERENCES

- Ofgang, E. (2021). Educators Moving Away From Seat Time For Mastery-Based Education: Some districts are replacing traditional views of seat time with mastery-based education. *Technology & Learning*, 28.
- Oseguera, L., Locks, A. M., & Vega, I. I. (2009). Increasing Latina/o Students' Baccalaureate Attainment. *Journal of Hispanic Higher Education*, 8(1), 23-53.  
<https://doi.org/10.1177/1538192708326997>
- Parrish, T. B., Perez, M., Merickel, A., & Linqanti, R. (2006). *Effects of the implementation of Proposition 227 on the education of English learners, K-12: Findings from a five-year evaluation*. Palo Alto, CA: American Institutes for Research and WestEd.
- Pascarella, E. T., Pierson, C. T., Wolniak, G. C., & Terenzini, P. T. (2004). First-generation college students: Additional evidence on college experiences and outcomes. *The Journal of Higher Education*, 75(3), 249-284. <https://doi.org/10.1080/00221546.2004.11772256>
- Paulsen, M. B., & St. John, E. P. (2002). Social class and college costs: Examining the financial nexus between college choice and persistence. *Journal of Higher Education*, 73, 189-236.  
<https://doi.org/10.1080/00221546.2002.11777141>
- Penrose, A.M. (2002) "Academic Literacy Perceptions and Performance: Comparing First-generation and Continuing Generation College Students." *Research in the Teaching of English*, 36(4): 437-461.
- Pilger Suhr, M., Nese, J., & Alonzo, J. (2021). Parallel reading and mathematics growth for English learners: Does timing of reclassification matter? *Journal of School Psychology*, 85, 94-112. <https://doi.org/10.1016/j.jsp.2021.02.003>
- Pituch, K. A., & Stevens, J. P. (2016). *Applied Multivariate Statistics for the Social Sciences*, 6<sup>th</sup> Ed. New York, NY: Routledge.

## REFERENCES

- Pompa, D., & Villegas, L. (2017). Analyzing state ESSA plans for English learner accountability: A framework for community stakeholders. Washington, DC: Migration Policy Institute. Available on the World Wide Web:  
<https://www.migrationpolicy.org/sites/default/files/publications/ESSA-Framework-FINAL.pdf>
- Preacher, K. J., Zyphur, M. J., & Zhang, Z. (2010). A general multilevel SEM framework for assessing multilevel mediation. *Psychological methods, 15*(3), 209-233.  
<https://doi.org/10.1037/a0020141>
- Reigeluth, C. M., & Karnopp, J. R. (2013). *Reinventing schools: It's time to break the mold*. Lanham, MD: Rowman & Littlefield.
- Rubin, D. B. (1976). Inference and missing data. *Biometrika, 63*(3), 581-592.  
<https://doi.org/10.1093/biomet/63.3.581>
- Robinson, J. P. (2011). Evaluating criteria for English learner reclassification: A causal-effects approach using a binding-score regression discontinuity design with instrumental variables. *Educational Evaluation and Policy Analysis, 33*, 267-292.  
<https://doi.org/10.3102/0162373711407912>
- Shonkoff, J. P., & Phillips, D. A. (2000). *From neurons to neighborhoods: The science of early childhood development*. Washington, DC: National Academy Press.
- Sturgis, C., & Patrick, S. (2010). When success is the only option: Designing competency-based pathways for next generation learning. Quincy, MA: Nellie Mae Education Foundation.
- Somers, P., Woodhouse, S., & Cofer, J. E. (2004). Pushing the boulder uphill: The persistence of first-generation college students. *NASPA Journal, 41*(3), 418-435.  
<https://doi.org/10.2202/1949-6605.1353>

## REFERENCES

- Terenzini, P. T., Springer, L., Yaeger, P. M., Pascarella, E. T., & Nora, A. (1996). "First-generation college students: Characteristics, experiences, and cognitive development." *Research in Higher Education*, 37(1), 1-22. <https://doi.org/10.1007/BF01680039>
- Thorndike, R. M., & Thorndike-Christ, T. (2010). *Measurement and Evaluation in Psychology and Education (8th Ed.)*. Boston, MA: Pearson.
- Tourangeau, K., Nord, C., Lê, T., Sorongon, A. G., Hagedorn, M. C., Daly, P., Najarian, M., & Mulligan, G. (2015). *User's Manual for the ECLS-K:2011 Kindergarten Data File and Electronic Codebook, Public Version*. National Center for Education Statistics 2015-074. Available on the World Wide Web: <https://files.eric.ed.gov/fulltext/ED566378.pdf>
- Twyman, J. S. (2014). Competency-based Education: Supporting Personalized Learning. Center on Innovations in Learning, Temple University, Philadelphia, PA, 1-10.
- U.S Department of Education, National Center for Educational Statistics (2001). *Bridging the gap: Academic preparation and postsecondary success of first-generation students* (NCES Publication No 2001153). Washington, DC: U.S. Government Printing Office.
- U.S. Department of Education. (2010). *Transforming American education : Learning powered by technology*. Washington, DC: Office of Educational Technology.
- U.S. Department of Education. (2016). *Non-Regulatory Guidance: English Learners and Title III of the Elementary and Secondary Education Act (ESEA), as amended by the Every Student Succeeds Act (ESSA)*. Retrieved from <https://www2.ed.gov/policy/elsec/leg/essa/essatitleiii guidenglishlearners92016.pdf>
- Volle, K., & Federico, A. (1997). *Missed Opportunities: A New Look at Disadvantaged College Aspirants*. The Education Resources Institute and The Institute for Higher Education Policy.

## REFERENCES

Warburton, E., Bugarin, R., & Nuñez, A. (2001). *Bridging the gap: Academic preparation and postsecondary success of first-generation students* (NCES 2001-153) Washington, DC:

National Center for Education Statistics, U.S. Government Printing Office.

Zwerling, L. S (1992). First-generation adult students: In search of safe havens. In L. S. Zwerling and H. B. London (Eds.), *First Generation College Students: Confronting the Cultural Issues*. San Francisco, CA: Jossey-Bass Publishers.

## APPENDICES

### APPENDIX A

#### Data Generation and T2C Analysis R Code for 5 Time Points, 30% Missing

```
## LIBRARIES -----

library(MplusAutomation)
library(plyr)
library(tidyr)

setwd("C:/trad/ind05/miss30")

## SIMULATION CONDITIONS ----

# define sample sizes
size <- c(100, 250, 500, 1000, 5000)
# define levels of criterion (related to latent tau)
crit <- c(105, 110, 115, 120)
# define levels of latent growth slope variance
slpvar <- c(0, 1, 4, 9)
# new define levels of resid error based on reliab
# for reliab = .40: 1-.40 = .60 * var=225 = 135
# for reliab = .60: 1-.60 = .40 * var=225 = 90
# for reliab = .80: 1-.80 = .20 * var=225 = 45
reserr <- c(135, 90, 45)
# number of replications
nreps = 1000
# level of missingness
miss <- c(30)

## DATA GENERATION ----
for (a in 1:length(size)){
  A <- size[a]
  for (b in 1:length(miss)){
    B <- miss[b]
    for (c in 1:length(crit)){
      C <- crit[c]
      for (d in 1:length(slpvar)){
        D <- slpvar[d]
        for (e in 1:length(reserr)){
          E <- reserr[e]

          ### Mplus INPUT FILE TO GENERATE DATA ----
          Title <- paste0("TITLE: Trad_N",A,"_Miss",B,"_Crit",C,"_SlpVar",D,"_Res",E,";")
          Montecarlo <- paste0("MONTECARLO: NAMES ARE y1-y5 x;")
          Nobservations <- paste0("NOBSERVATIONS = ",A,";")
          Nreps <- paste0("NREPS = ",nreps,";")
          Seed <- paste0("SEED = 293117;")
          Patmiss <- paste0("PATMISS = y1(0) y2(0) y3(0) y4(0) y5(0) |
y1(0) y2(1) y3(1) y4(1) y5(1) |
y1(0) y2(0) y3(1) y4(1) y5(1) |
y1(0) y2(0) y3(0) y4(1) y5(1) |
y1(0) y2(0) y3(0) y4(0) y5(1);")
          Patprobs <- paste0("PATPROBS = .24|.30|.21|.15|.10;")
          Results <- paste0("RESULTS =
Trad_N",A,"_Miss",B,"_Crit",C,"_SlpVar",D,"_Res",E,".dat;")
          Repsave <- paste0("REPSAVE = all;")
          Save <- paste0("SAVE = Trad_N",A,"_Miss",B,"_Crit",C,"_SlpVar",D,"_Res",E,"_rep*.dat;")
          ModelPopulation <- paste0(
            "MODEL POPULATION:
i b | y1@0 y2@1 y3@2 y4@3 y5@4;
[i*100];
[b*2];
i*225;
b*",D,";
i with b*0;
y1-y5*",E,";
[x*0];
x*1;
```

## APPENDICES

```

i ON x*1;
b ON x*1;"
Analysis <- paste0(
  "ANALYSIS: ESTIMATOR = ML;COVERAGE = .05;ITERATIONS = 10000;")
Model <- paste0(
  "MODEL:
i b | y1@0 y2@1 y3@2 y4@3 y5@4;
[i*100];
[b*2];
i*225;
b*",D,";
i with b*0;
y1-y5*",E,";
i ON x*1;
b ON x*1;"
inpnm <- paste0("Trad_N",A,"_Miss",B,"_Crit",C,"_SlpVar",D,"_Res",E,".inp")
sink(file = inpnm)

cat(Title,Montecarlo,Nobservations,Nreps,Seed,Patmiss,Patprobs,Results,Repsave,Save,Model
Population,Analysis,Model,file= inpnm,sep = "\n")
sink()
}
}
}
}

### RUN Mplus MODELS TO GET TRAD MODEL RESULTS AND GENERATE DATA FOR T2C ANALYSIS ----
runModels()

## DATA ANALYSIS USING T2C ----
### reset to a new working directory
setwd("C:/t2c/ind05/miss30")

for (a in 1:length(size)){
  A <- size[a]
  for (b in 1:length(miss)){
    B <- miss[b]
    for (c in 1:length(crit)){
      C <- crit[c]
      for (d in 1:length(slpvar)){
        D <- slpvar[d]
        for (e in 1:length(reserr)){
          E <- reserr[e]

          ### Mplus INPUT FILE TO ANALYZE DATA WITH T2C ----
          ##### use prior data generated from traditional growth model
          Title <- paste0("TITLE: T2C_N",A,"_Miss",B,"_Crit",C,"_SlpVar",D,"_Res",E,";")
          Data <- paste0("DATA:
FILE=C:\\trad\\ind05\\miss30\\Trad_N",A,"_Miss",B,"_Crit",C,"_SlpVar",D,"_Res",E,"_replst.dat;
TYPE = MONTECARLO;")
          Variable <- paste0("VARIABLE: NAMES = y1-y5 x; USEVARIABLES = y1-y5 x;
MISSING ARE ALL(999.000000);")
          Analysis <- paste0(
            "ANALYSIS: ESTIMATOR=ML;COVERAGE=.05;ITERATIONS=10000; SDITERATIONS=100;")
          Model <- paste0(
            "MODEL:
tj BY y1-y5* (t);
bj BY y1-y5* (b1-b5);
[tj*5] (mt);
[bj*2] (mb);
tj*225;
bj*",D,";
tj with bj*;
[y1-y5*105] (n);
y1-y5*",E,";
bj ON x*1;
tj ON x*1;"
          ModelConstraint <- paste0(

```

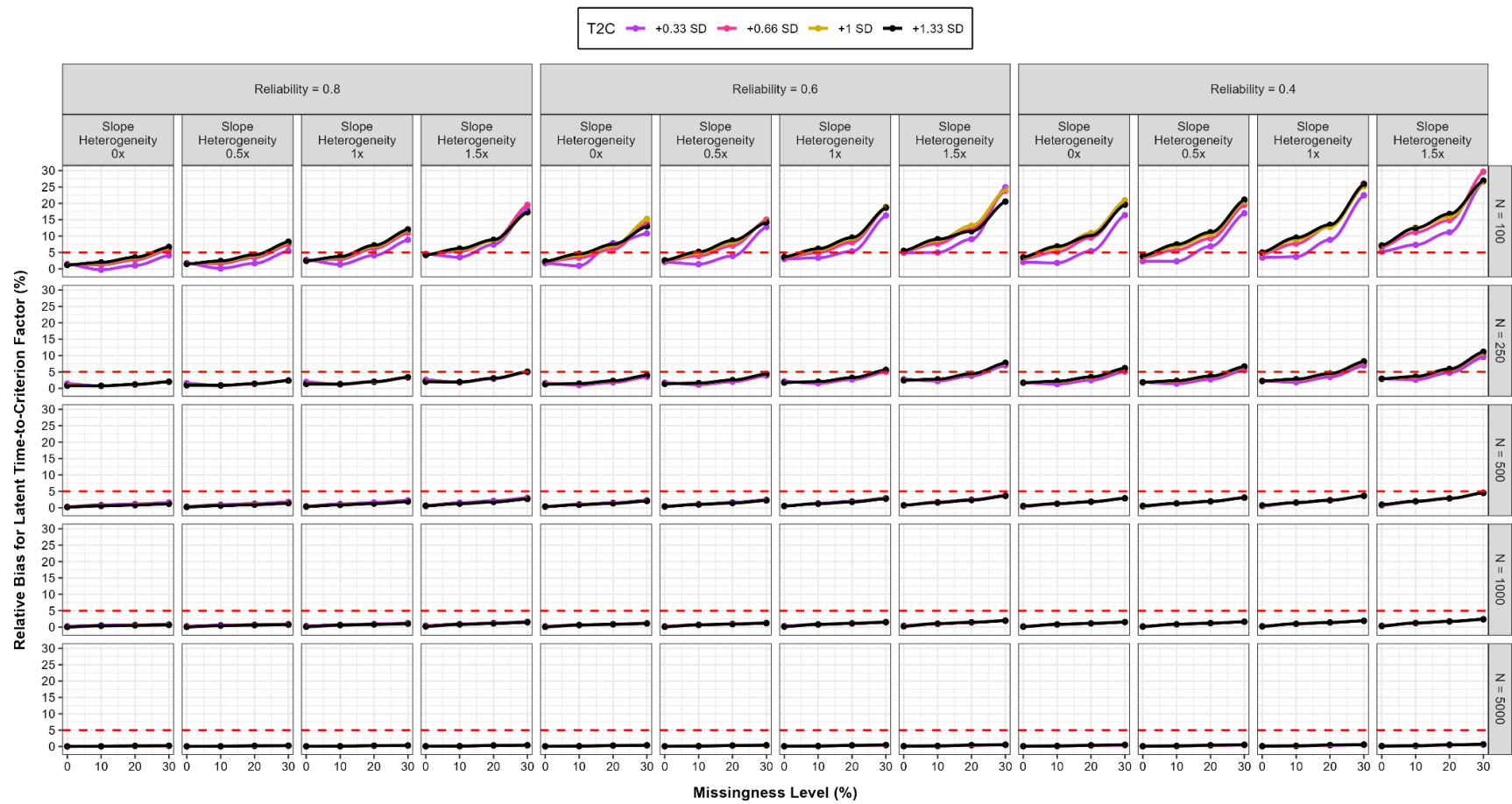


APPENDIX B

Full Results Figures

Figure B1

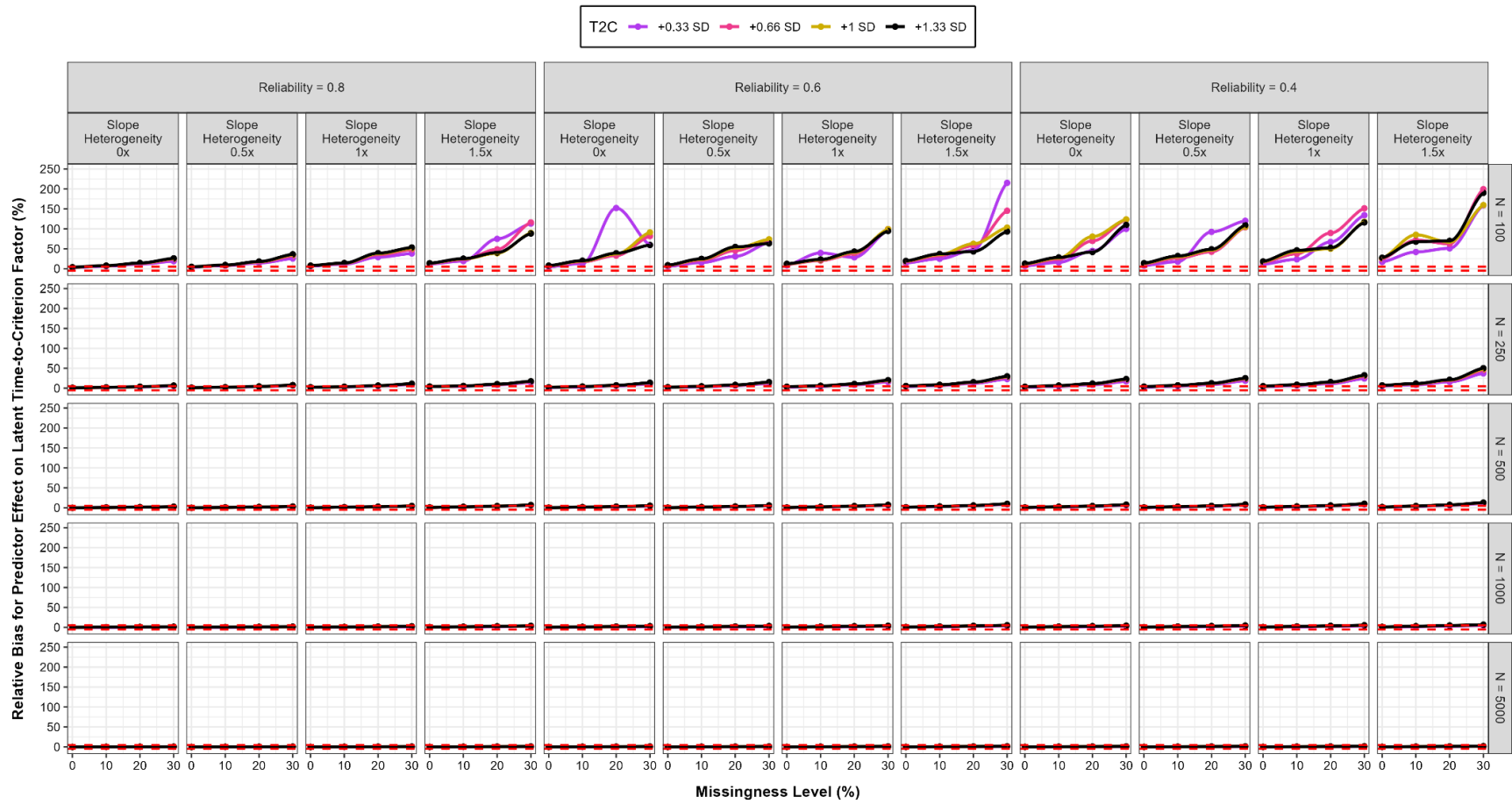
3 Indicators: Relative Bias for Latent Time-to-Criterion Factor



APPENDICES

**Figure B2.**

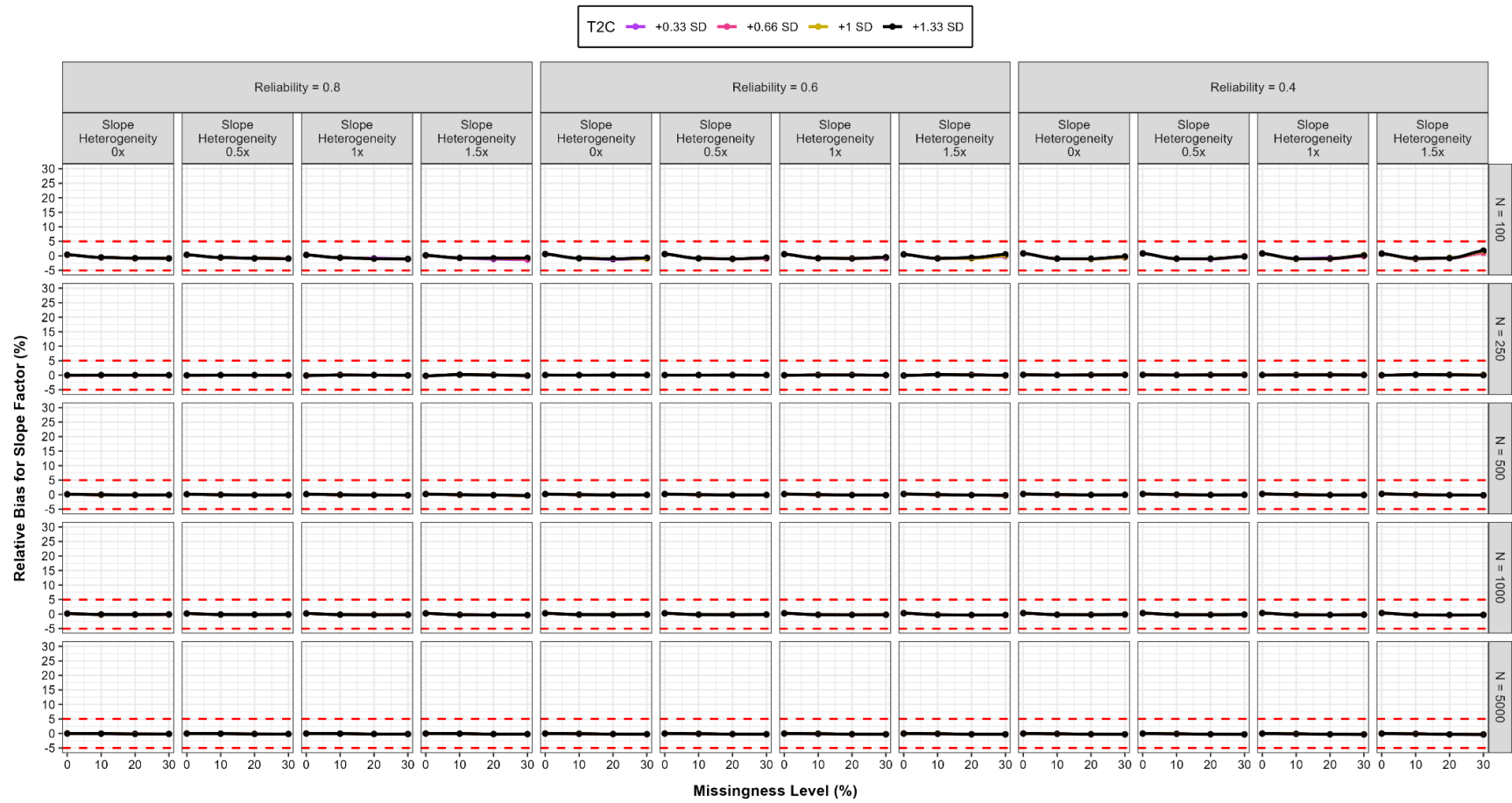
*3 Indicators: Relative Bias for Predictor Effect on Latent Time-to-Criterion Factor*



APPENDICES

**Figure B3**

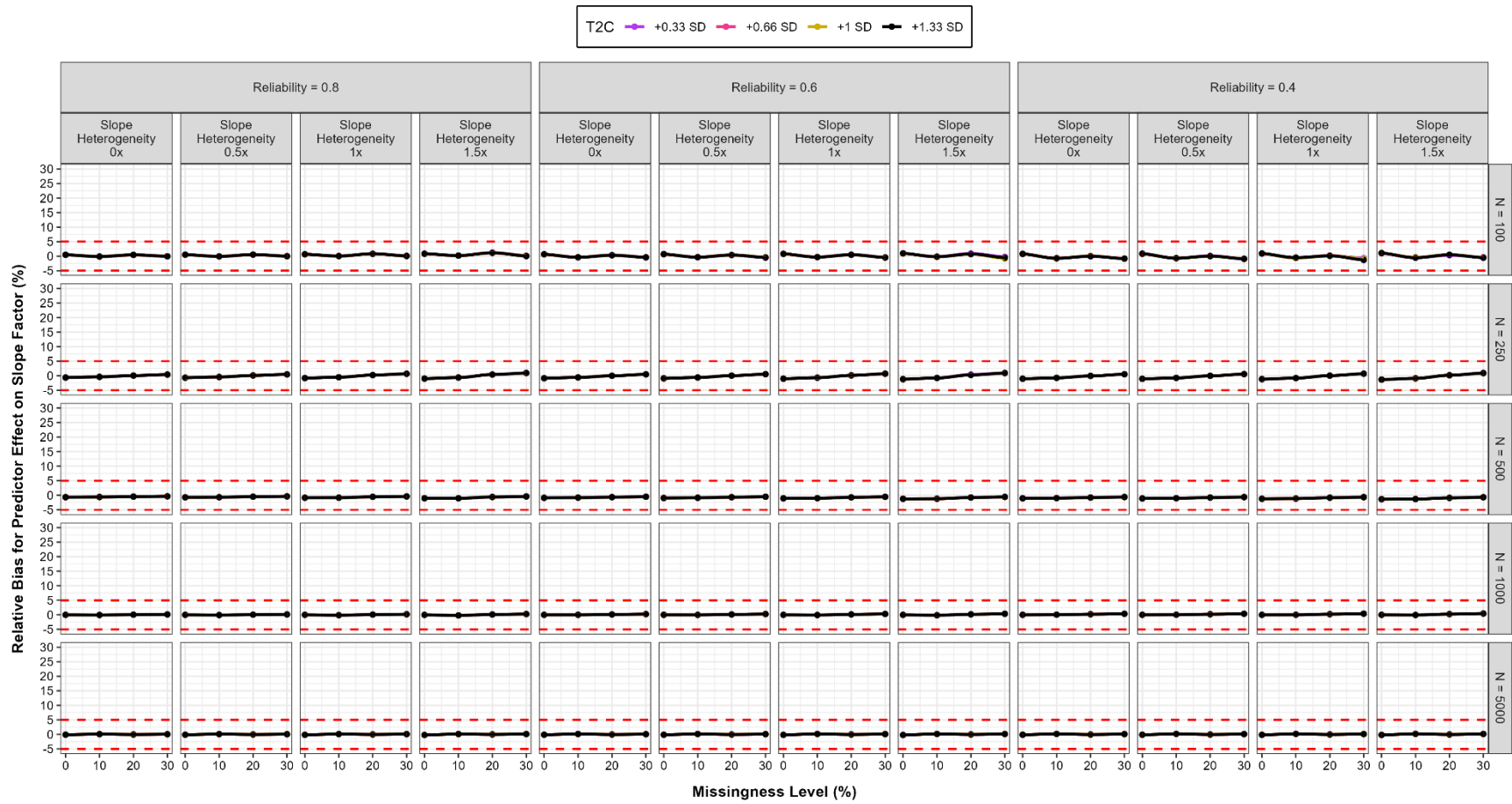
*3 Indicators: Relative Bias for Latent Slope Factor*



APPENDICES

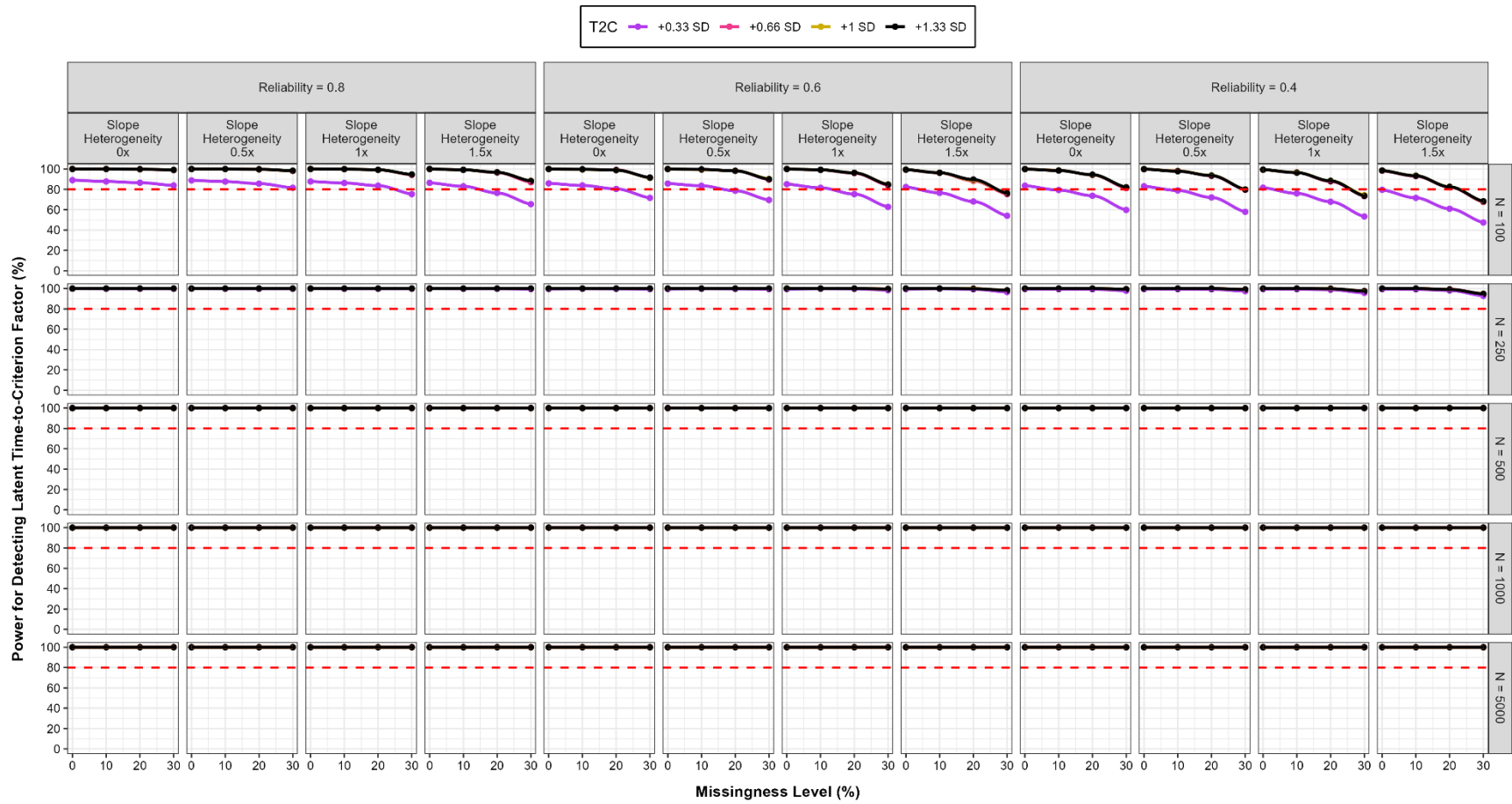
Figure B4

3 Indicators: Relative Bias for Predictor Effect on Latent Slope Factor



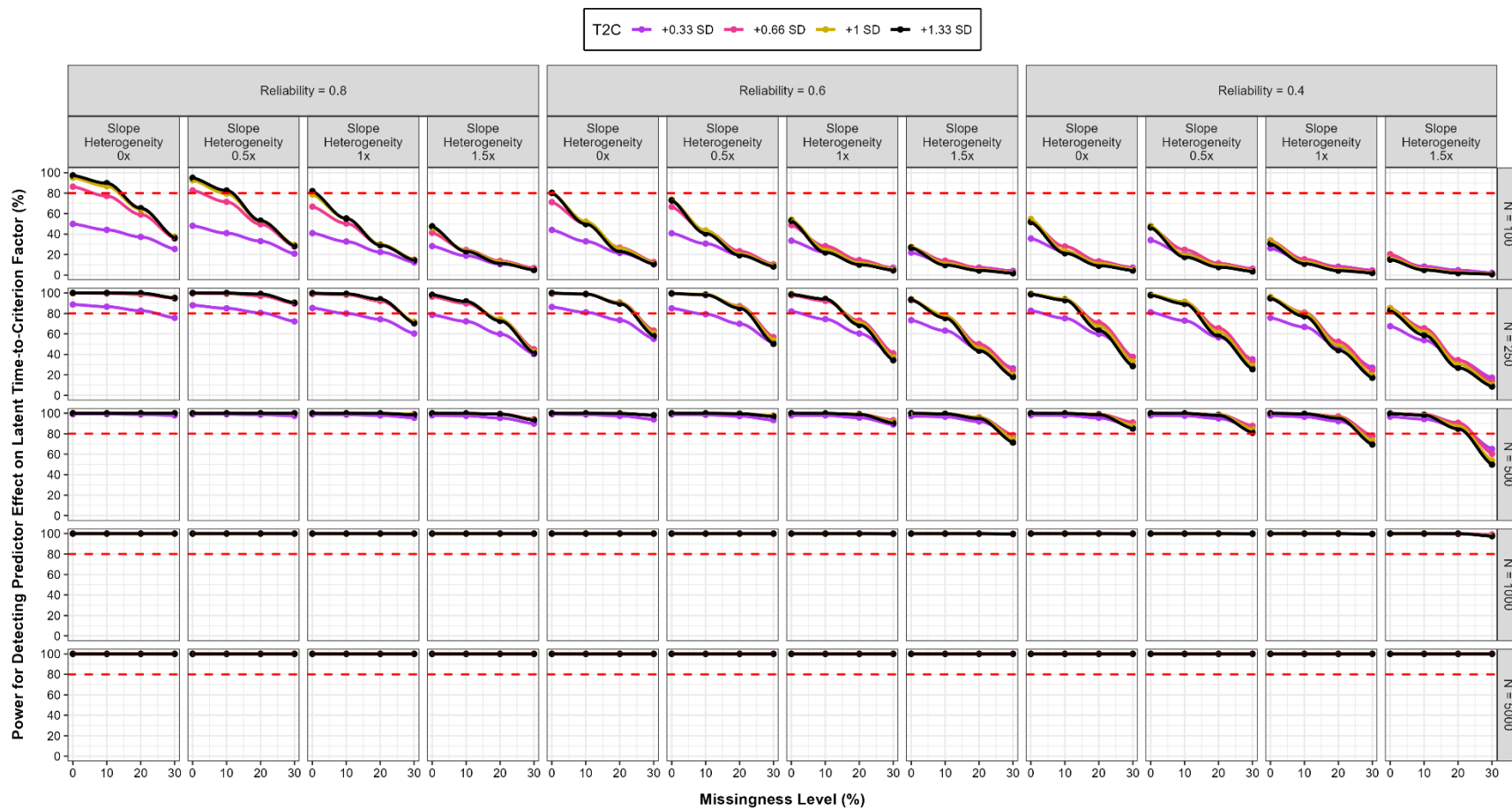
**Figure B5**

*3 Indicators: Power for Detecting Latent Time-to-Criterion Factor*



**Figure B6**

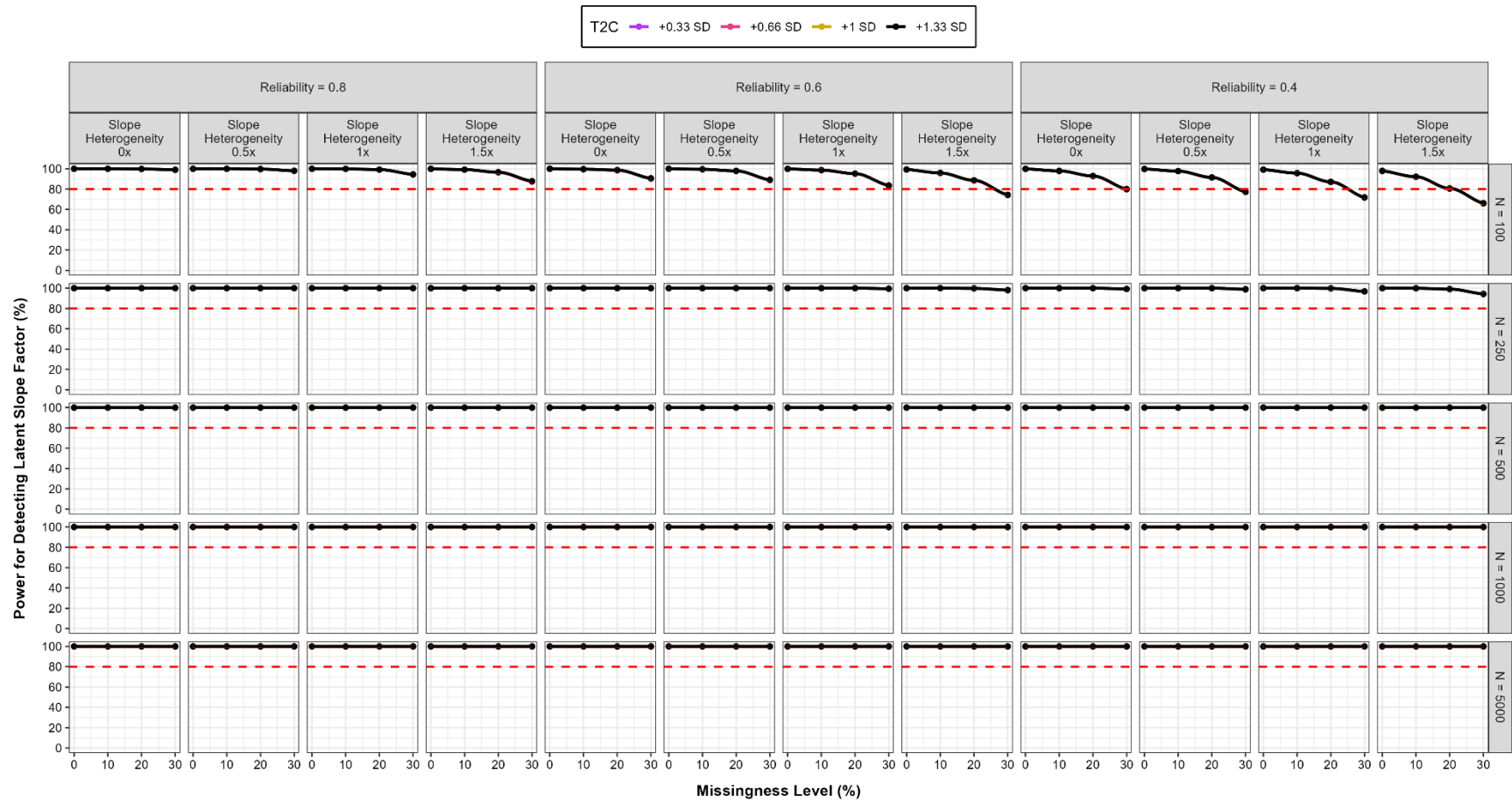
*3 Indicators: Power for Detecting Predictor Effect on Time-to-Criterion Factor*



APPENDICES

**Figure B7**

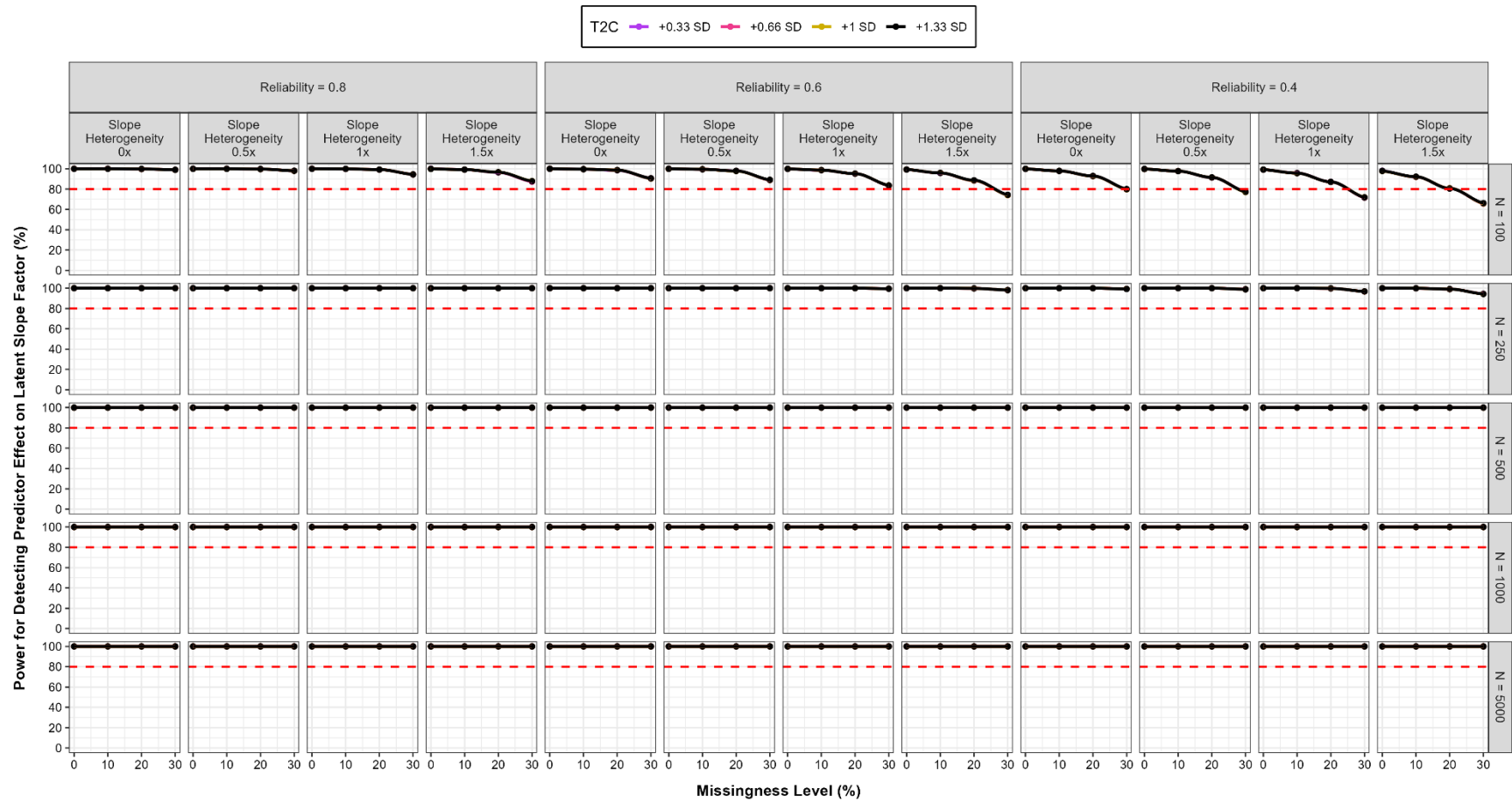
*3 Indicators: Power for Detecting Latent Slope Factor*



APPENDICES

**Figure B8**

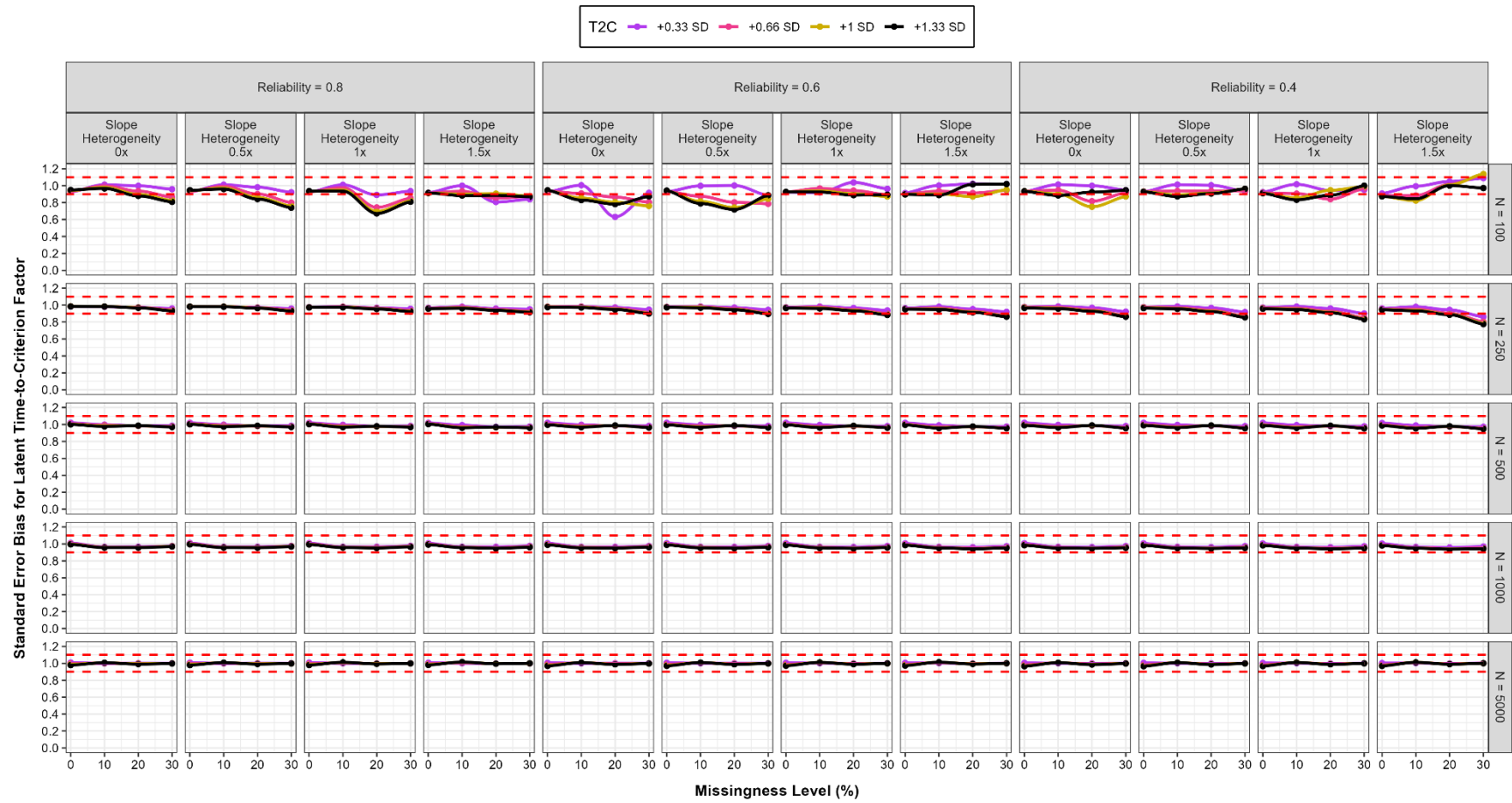
*3 Indicators: Power for Detecting Predictor Effect on Latent Slope Factor*



APPENDICES

**Figure B9**

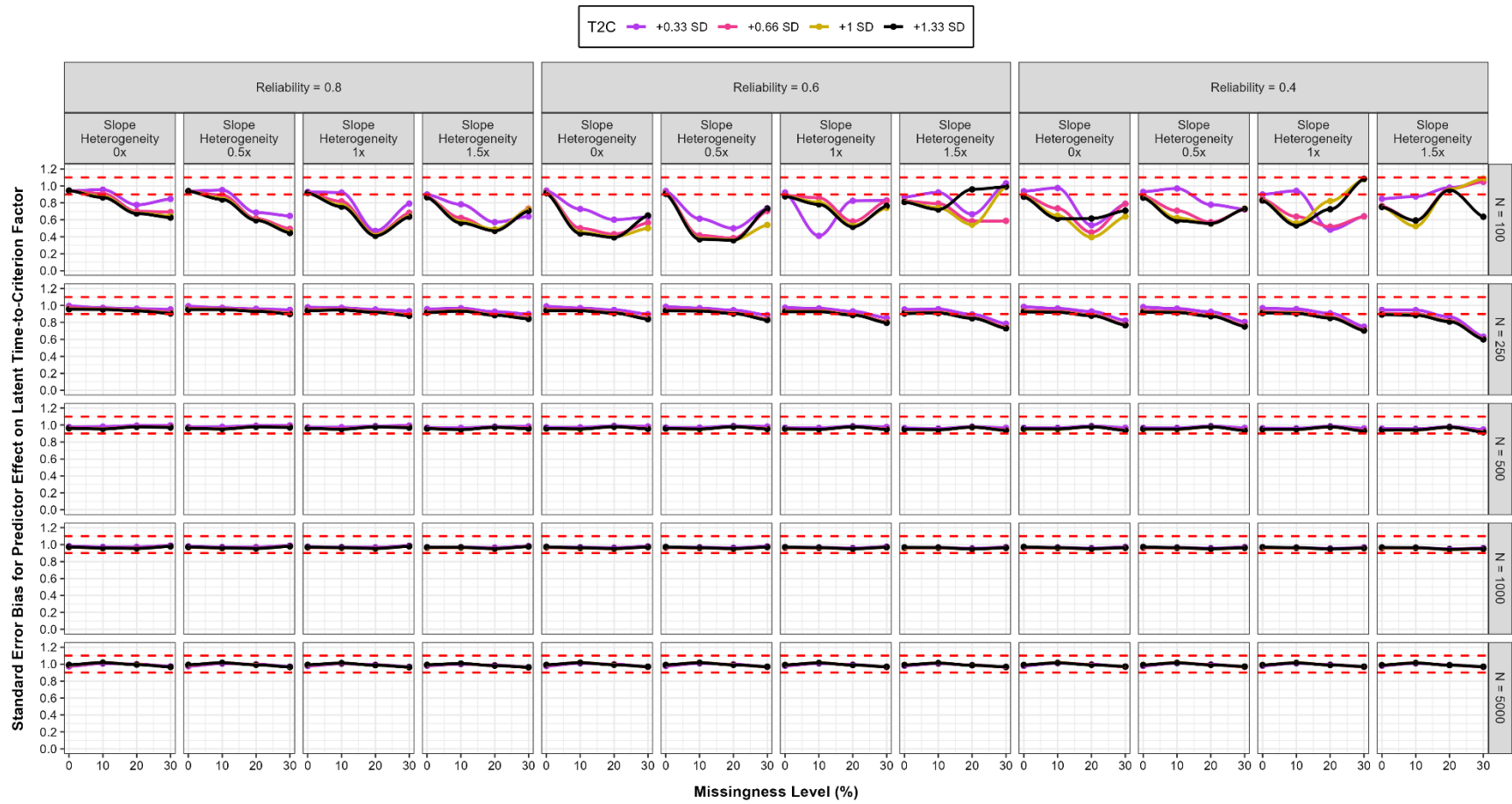
*3 Indicators: Standard Error Bias for Latent Time-to-Criterion Factor*



APPENDICES

**Figure B10**

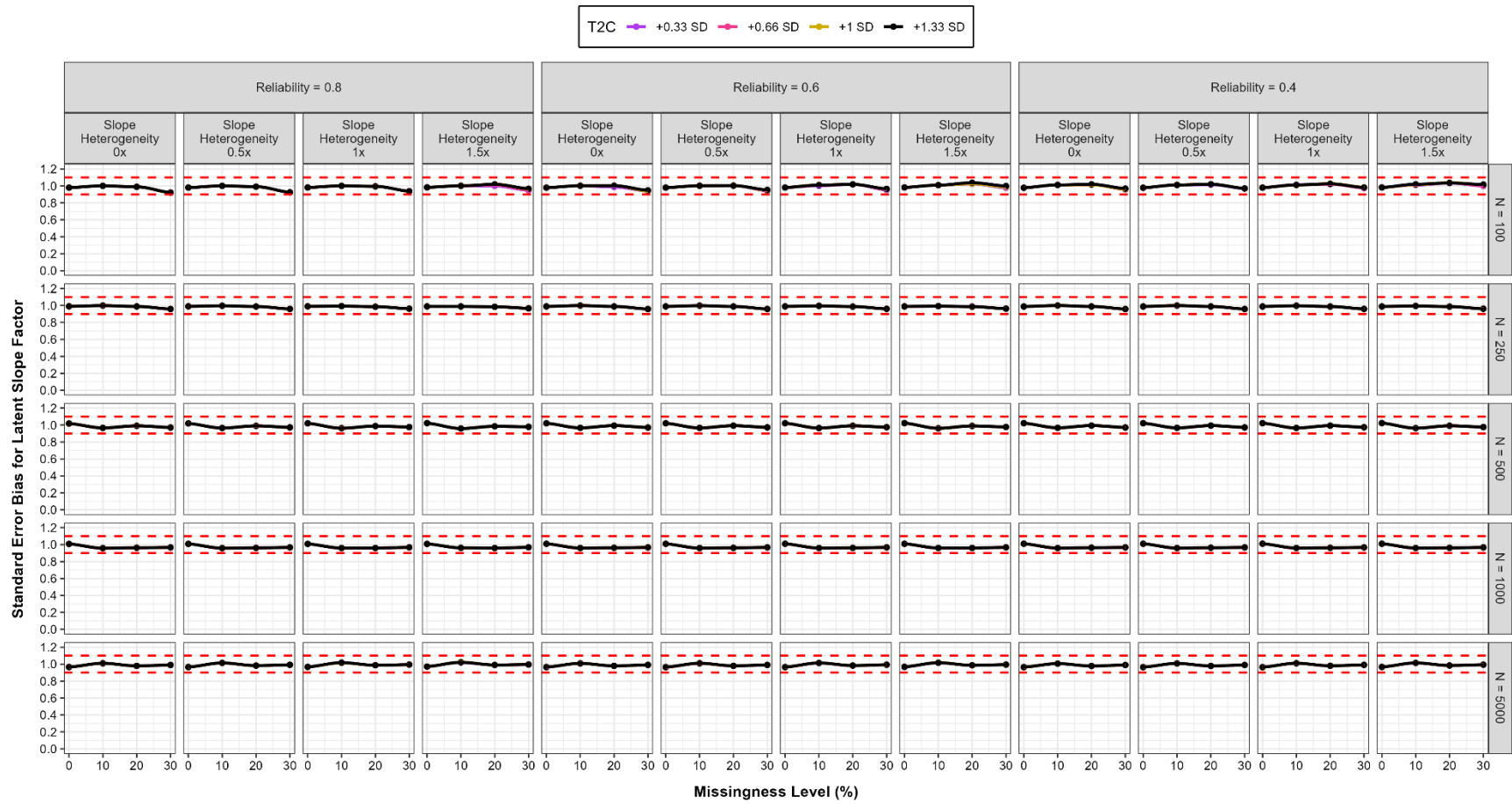
*3 Indicators: Standard Error Bias for Predictor Effect on Latent Time-to-Criterion Factor*



APPENDICES

**Figure B11**

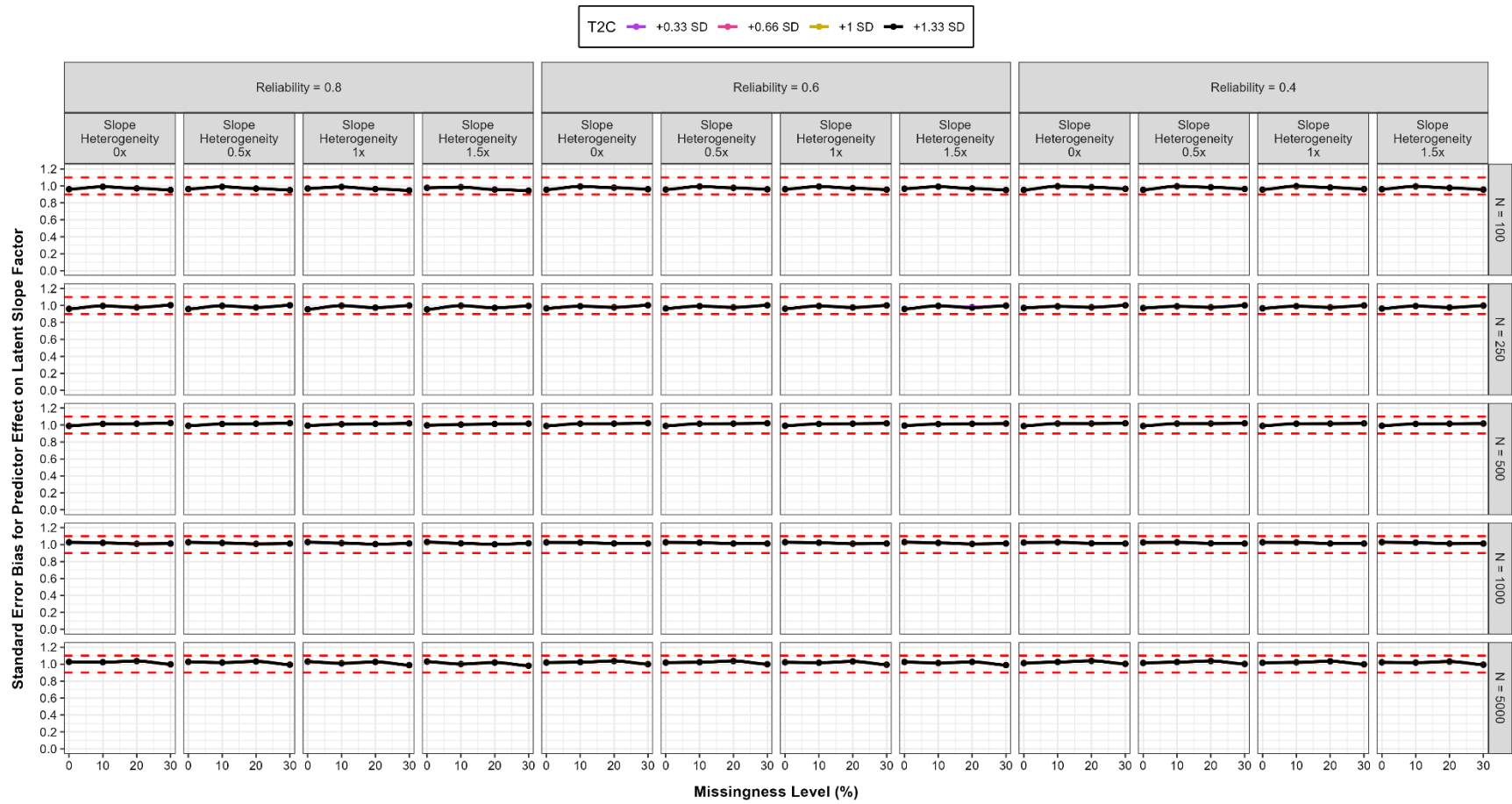
*3 Indicators: Standard Error Bias for Latent Slope Factor*



APPENDICES

**Figure B12**

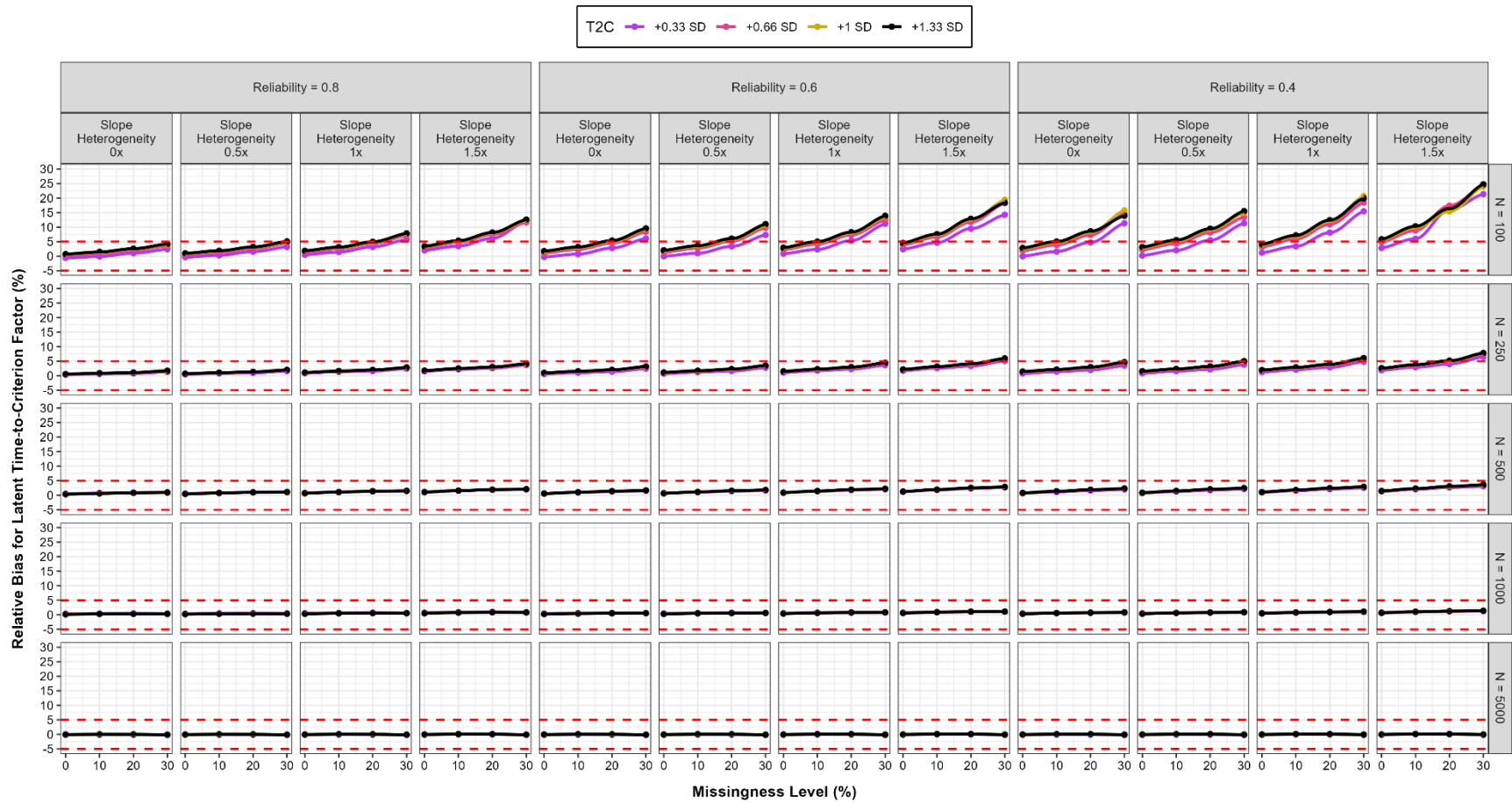
*3 Indicators: Standard Error Bias for Predictor Effect on Latent Slope Factor*



APPENDICES

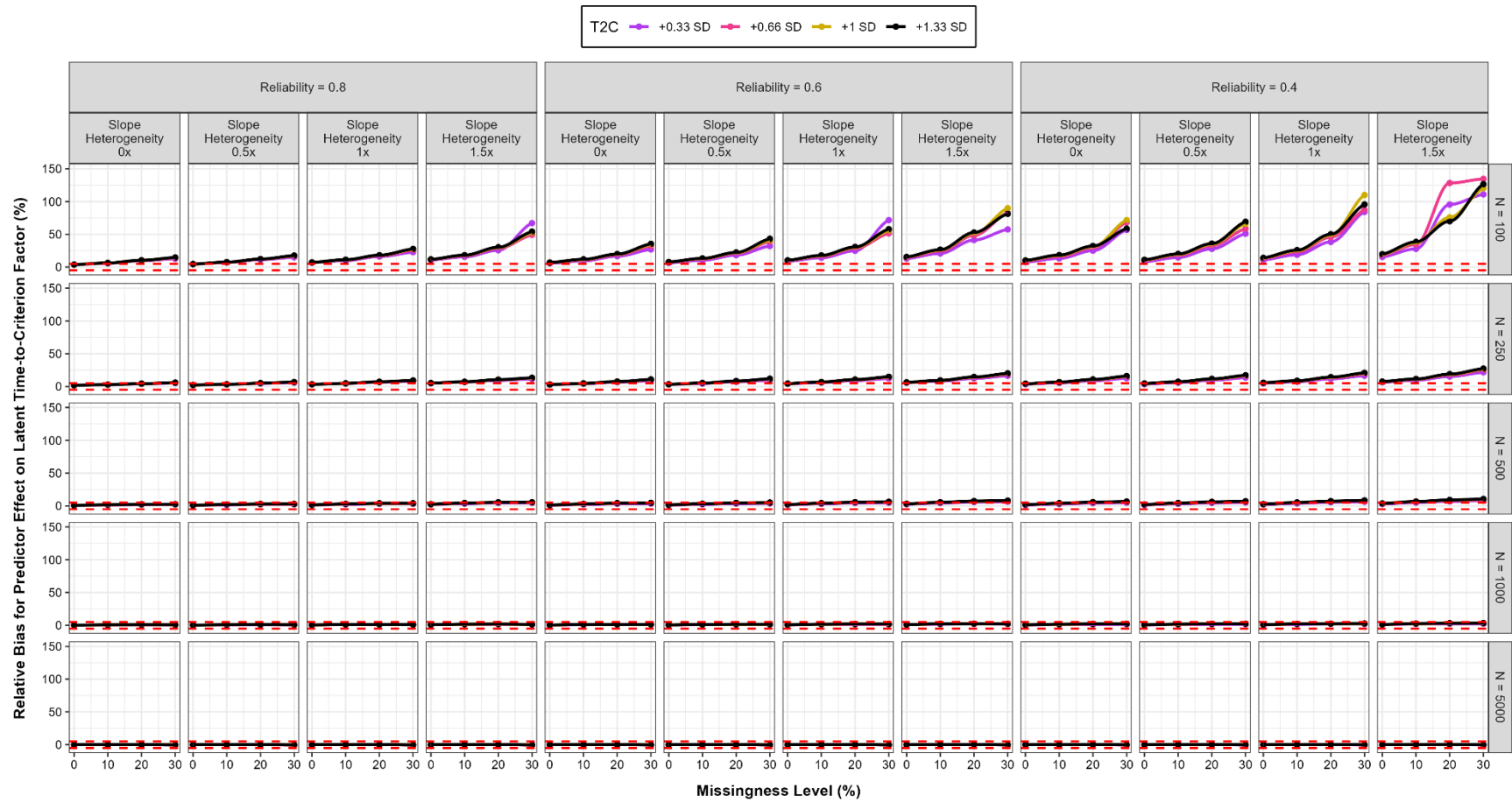
Figure B13

5 Indicators: Relative Bias for Latent Time-to-Criterion Factor



**Figure B14**

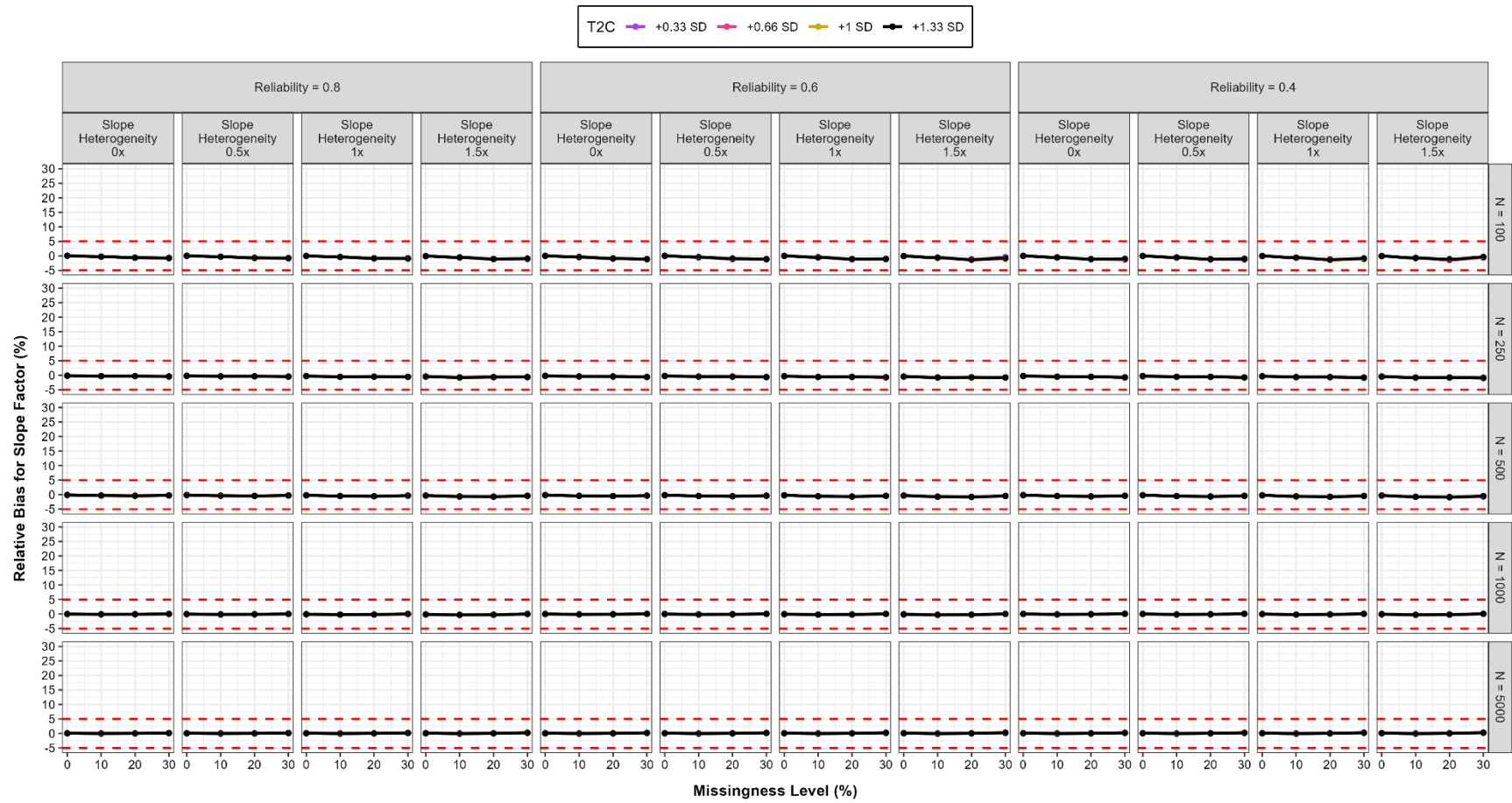
*5 Indicators: Relative Bias for Predictor Effect on Latent Time-to-Criterion Factor*



APPENDICES

**Figure B15**

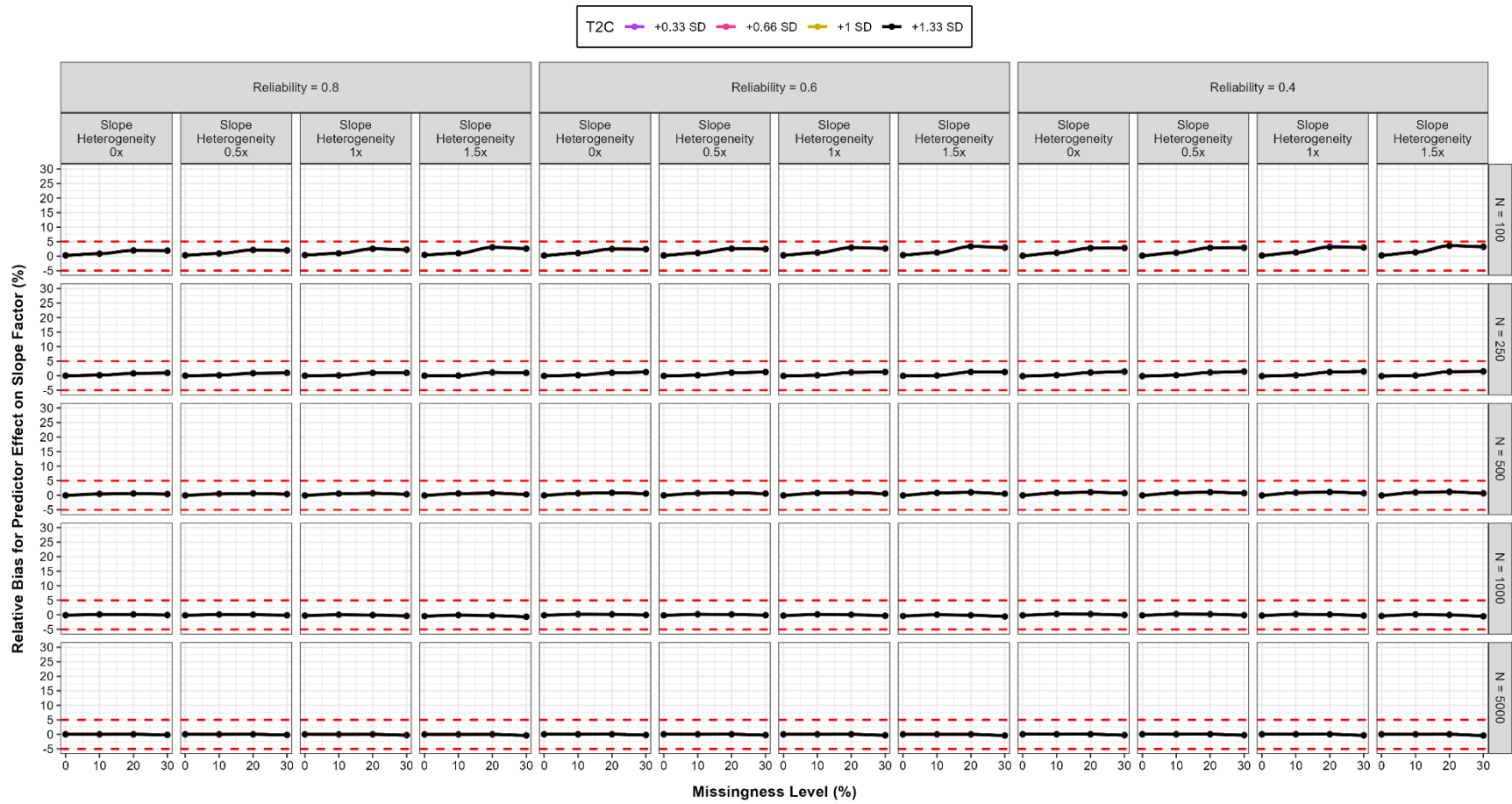
*5 Indicators: Relative Bias for Latent Slope Factor*



APPENDICES

**Figure B16**

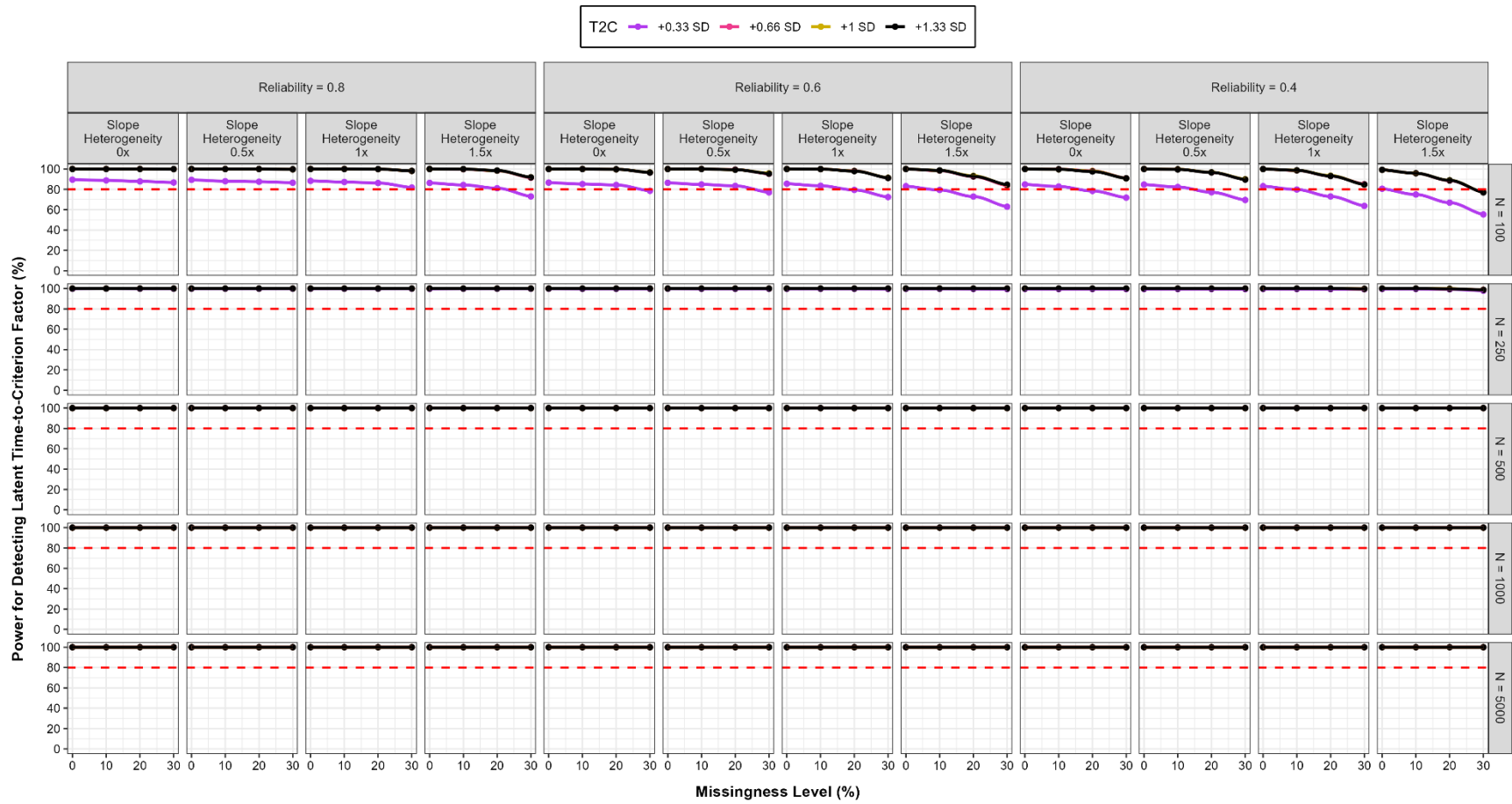
*5 Indicators: Relative Bias for Predictor Effect on Latent Slope Factor*



APPENDICES

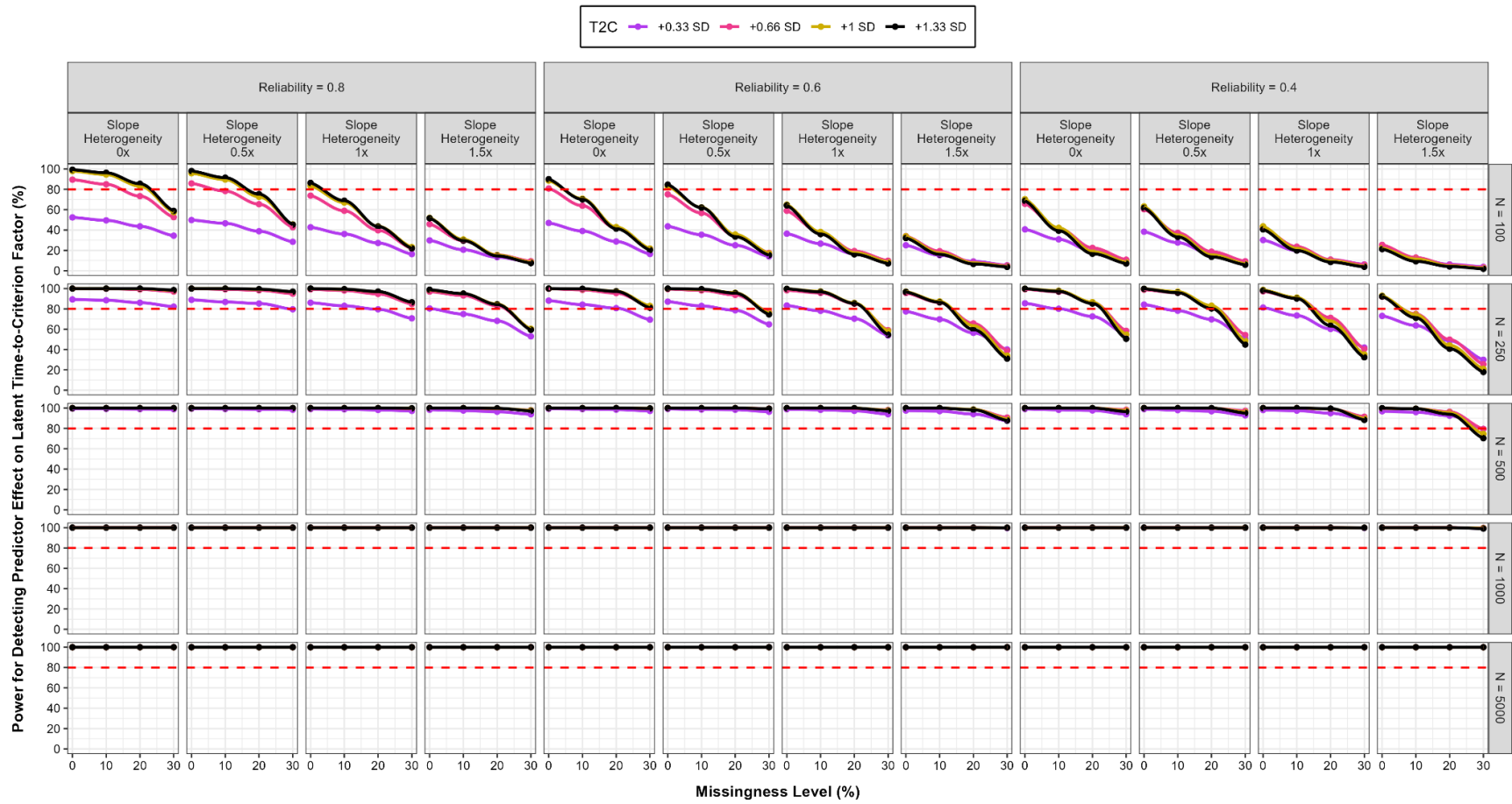
**Figure B17**

*5 Indicators: Power for Detecting Latent Time-to-Criterion Factor*



**Figure B18**

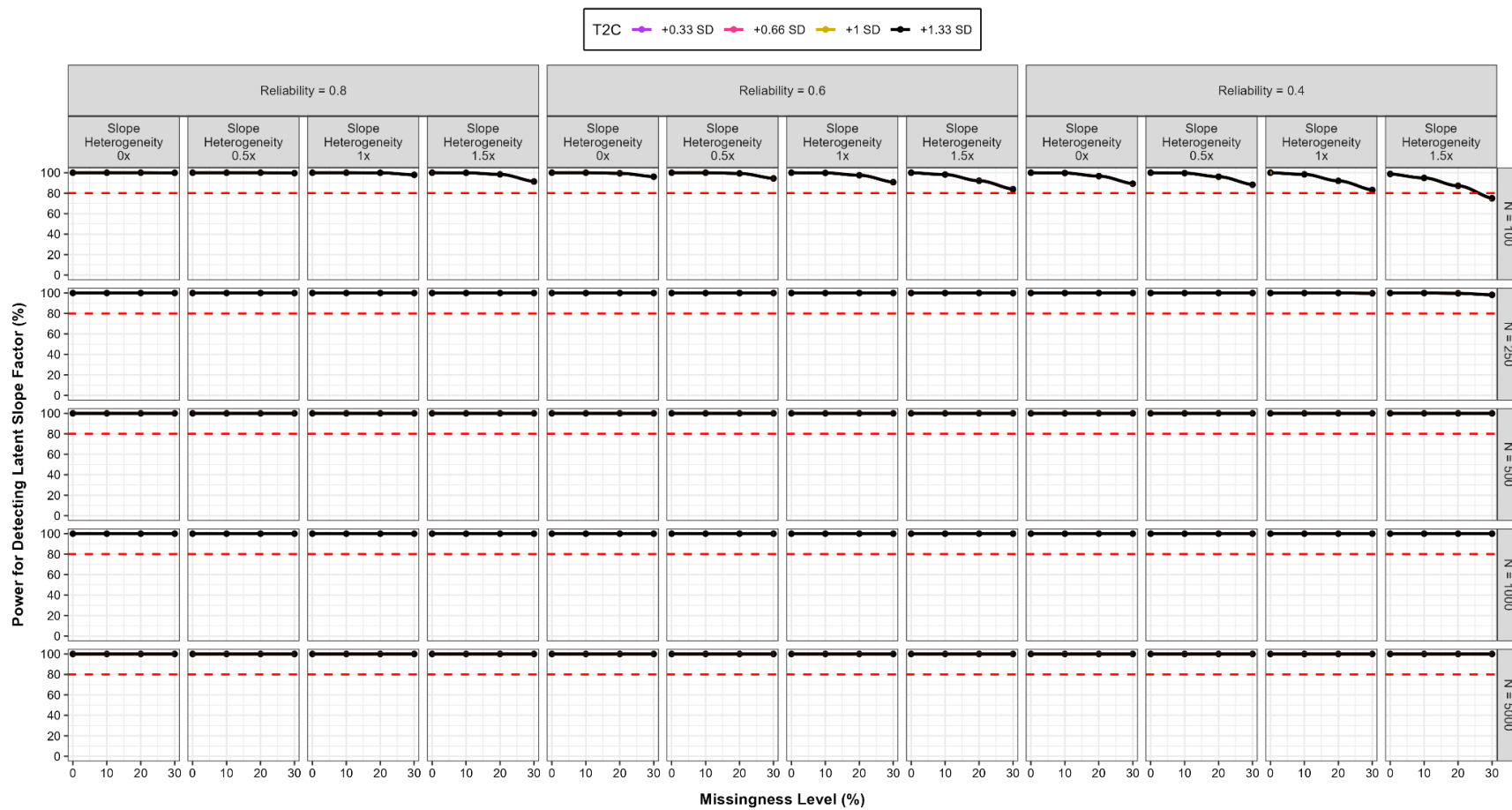
*5 Indicators: Power for Detecting Predictor Effect on Time-to-Criterion Factor*



APPENDICES

Figure B19

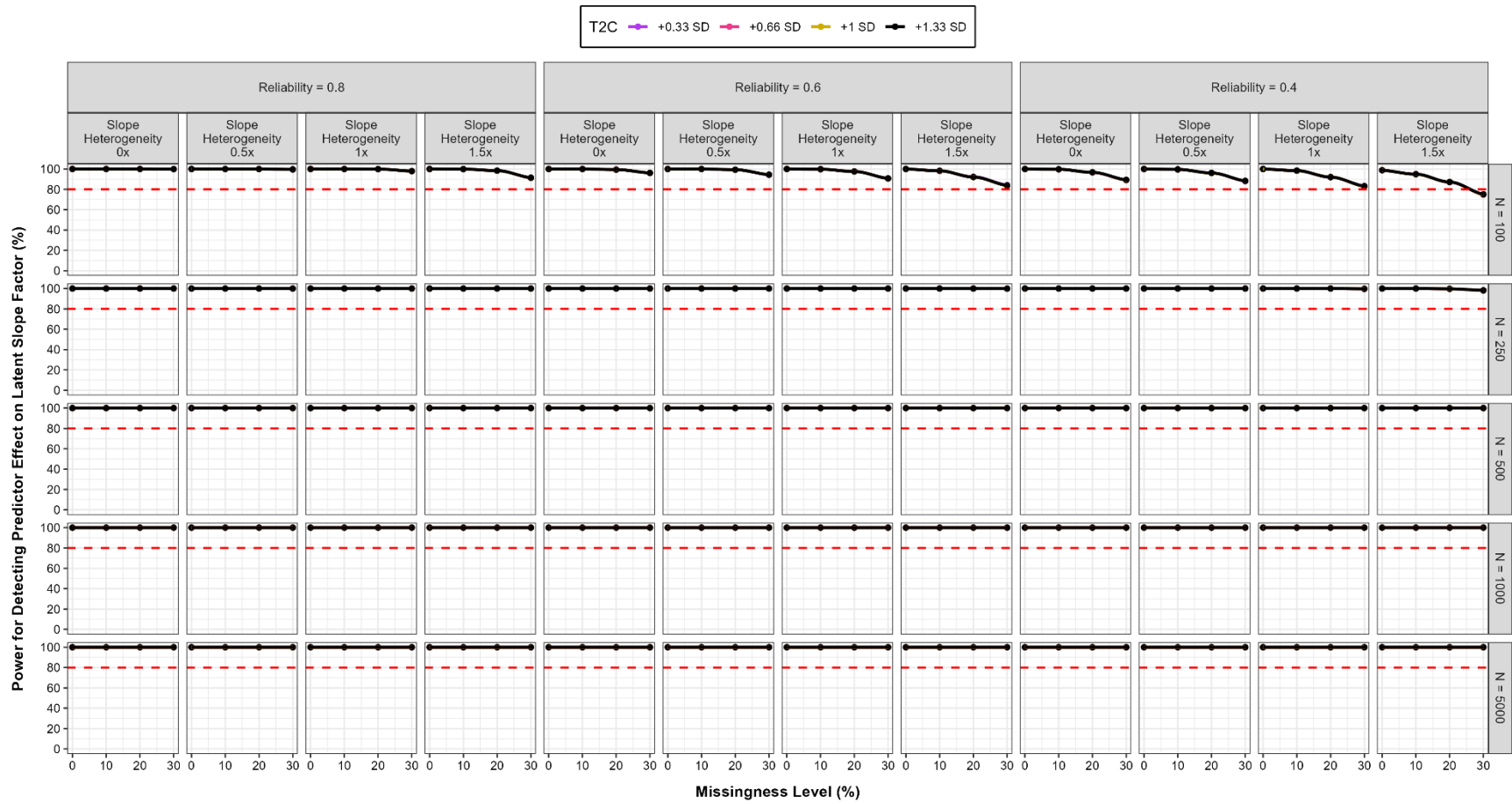
5 Indicators: Power for Detecting Latent Slope Factor



APPENDICES

**Figure B20**

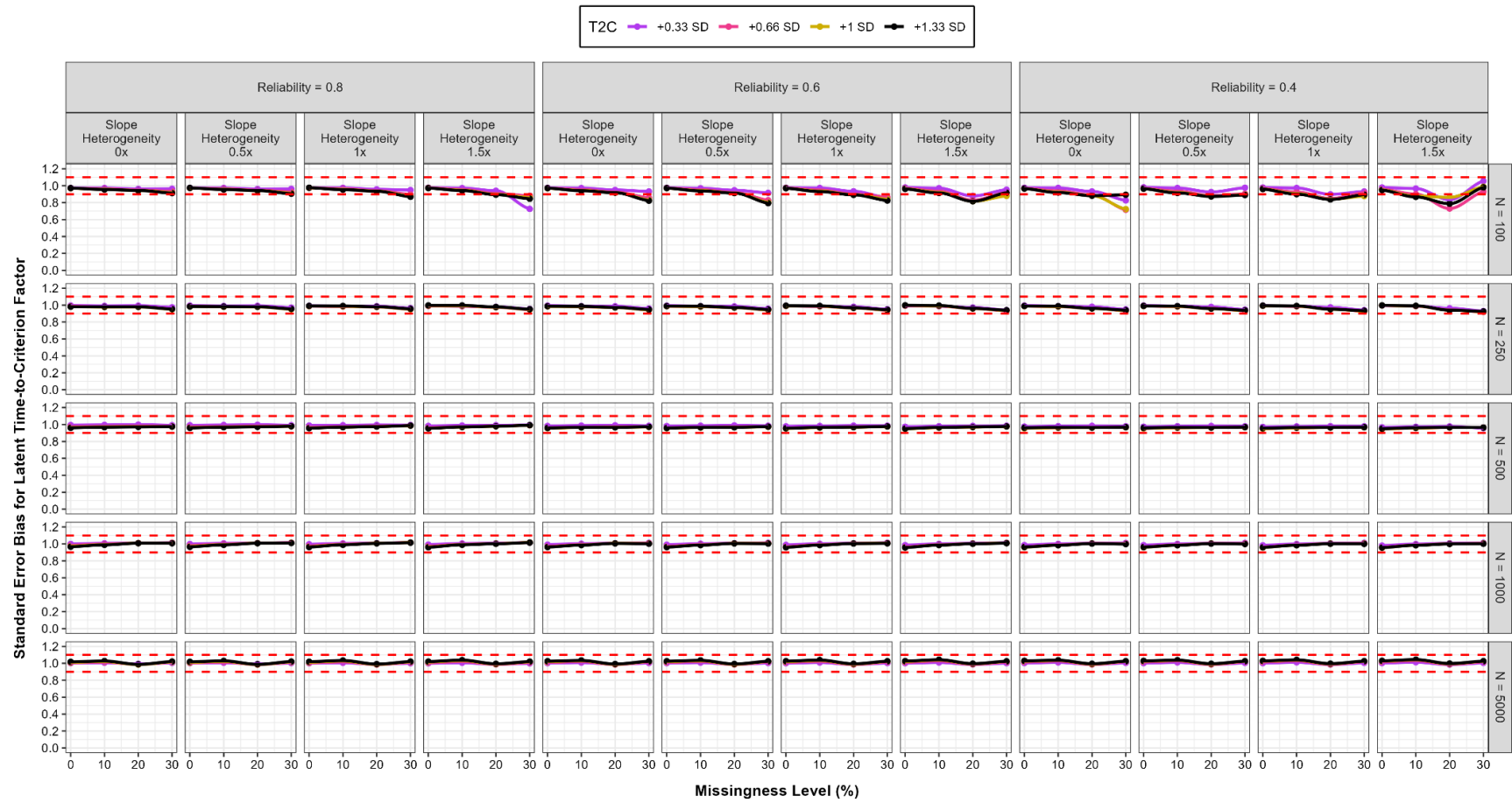
*5 Indicators: Power for Detecting Predictor Effect on Latent Slope Factor*



APPENDICES

Figure B21

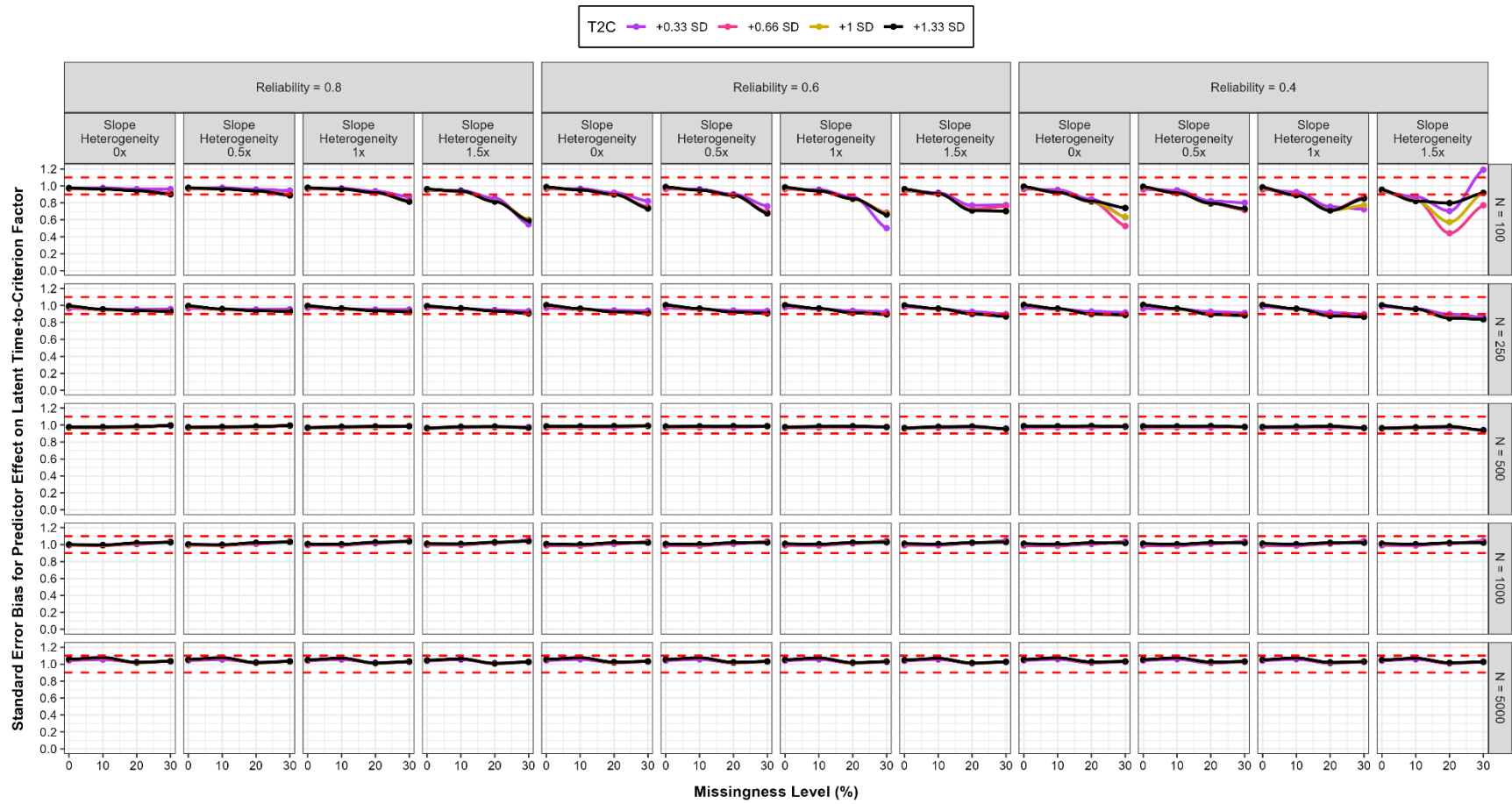
5 Indicators: Standard Error Bias for Latent Time-to-Criterion Factor



APPENDICES

Figure B22

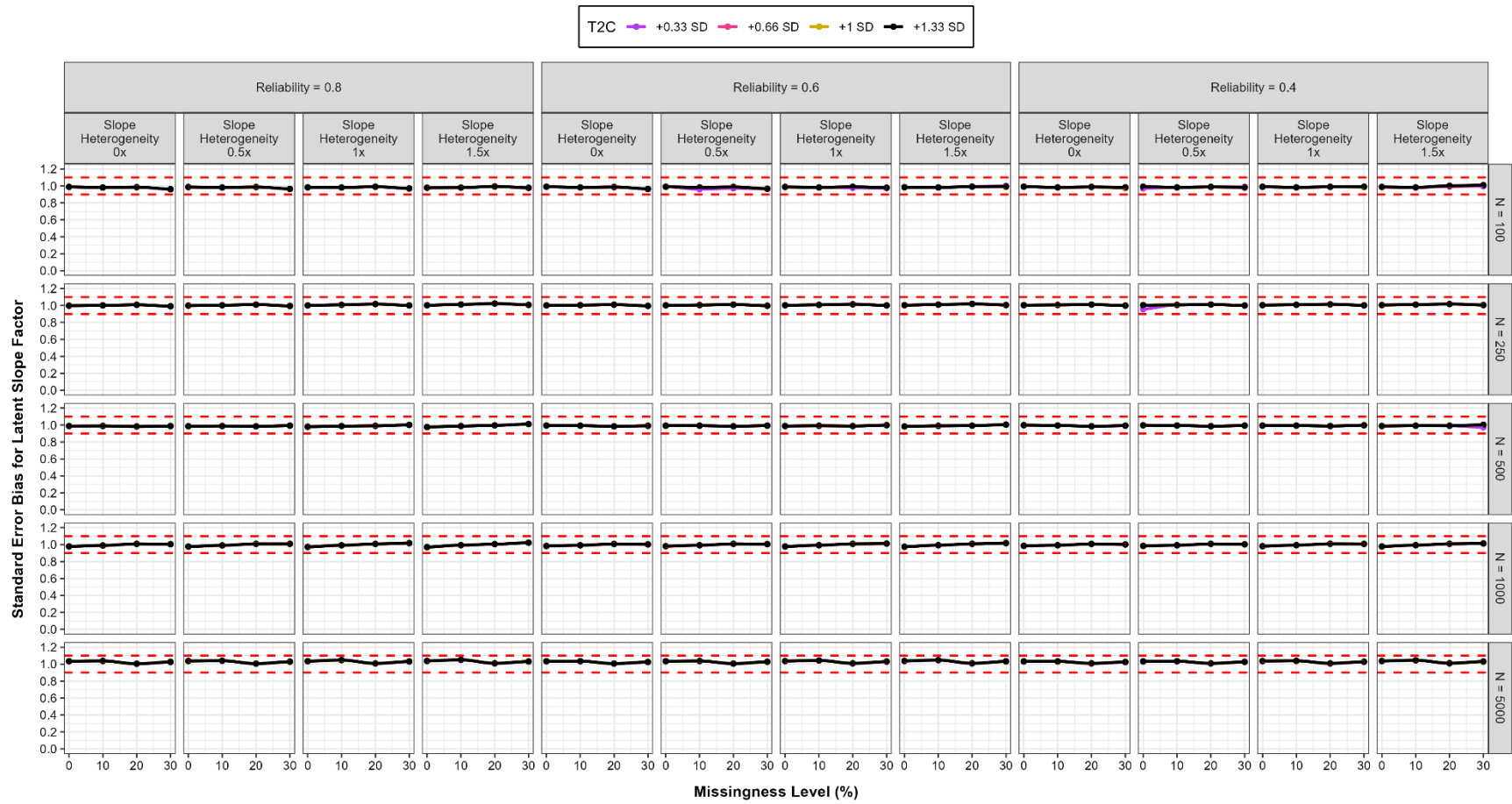
5 Indicators: Standard Error Bias for Predictor Effect on Latent Time-to-Criterion Factor



APPENDICES

Figure B23

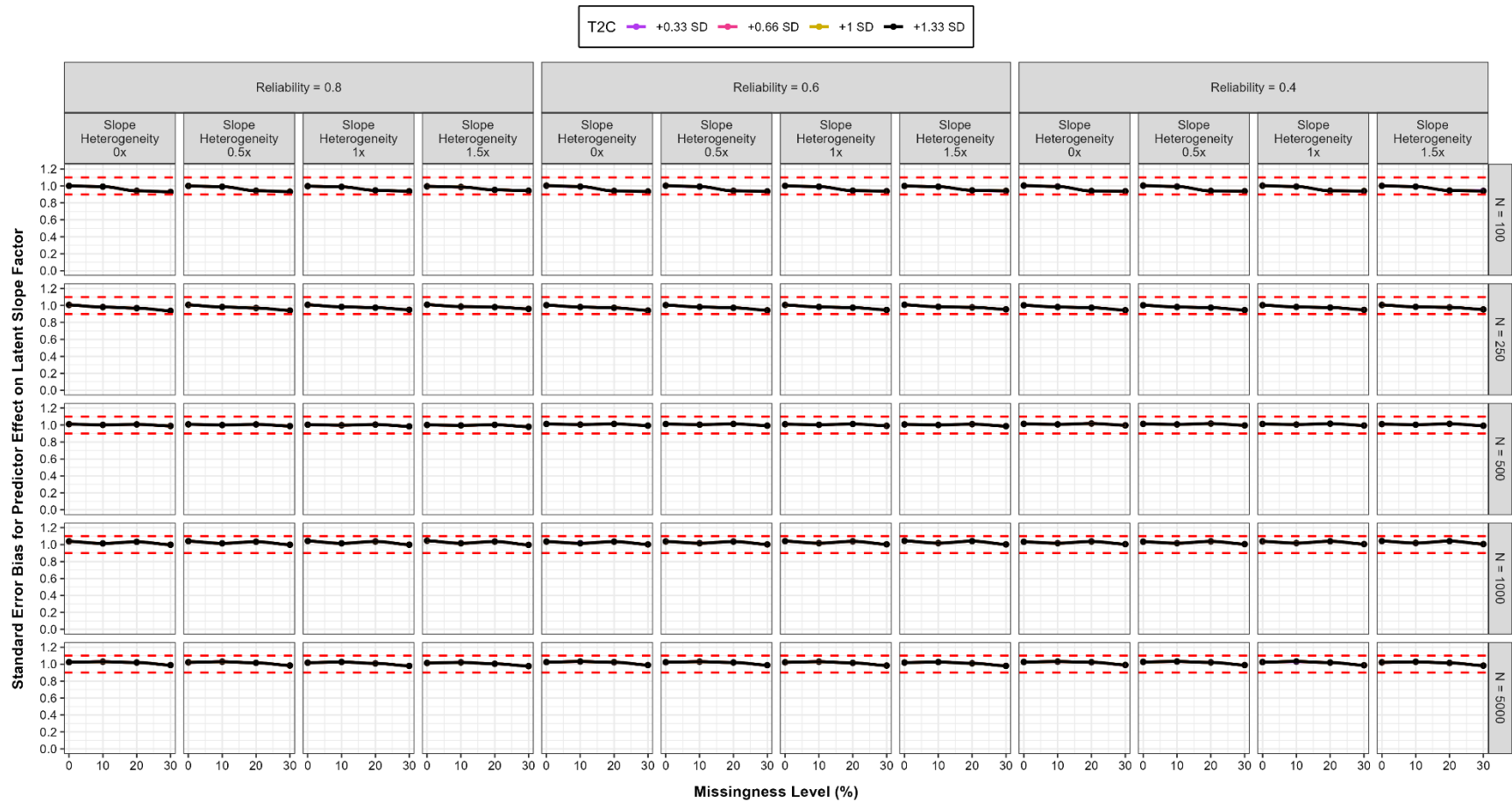
5 Indicators: Standard Error Bias for Latent Slope Factor



APPENDICES

**Figure B24**

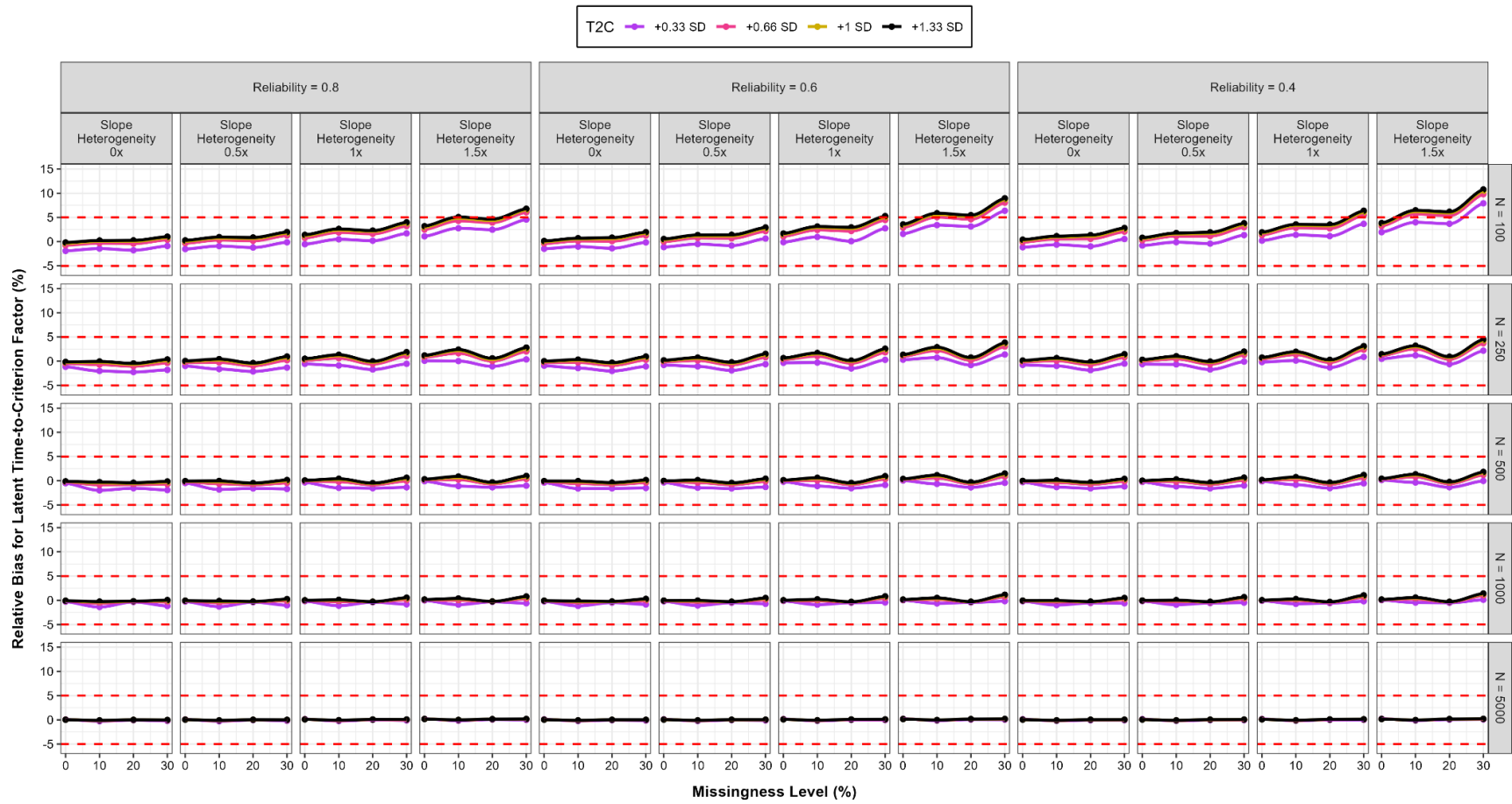
*5 Indicators: Standard Error Bias for Predictor Effect on Latent Slope Factor*



APPENDICES

Figure B25

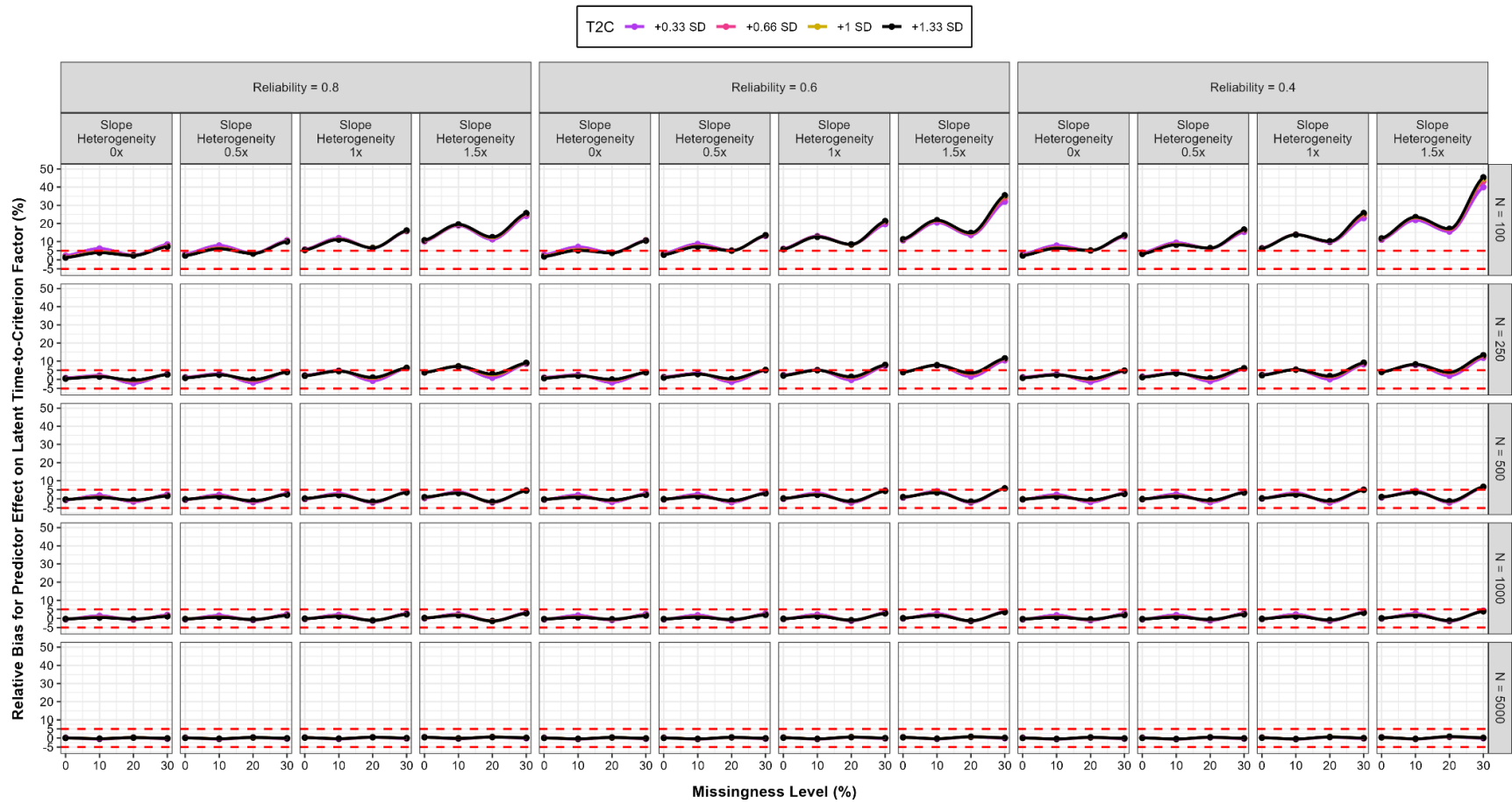
50 Indicators: Relative Bias for Latent Time-to-Criterion Factor



APPENDICES

Figure B26

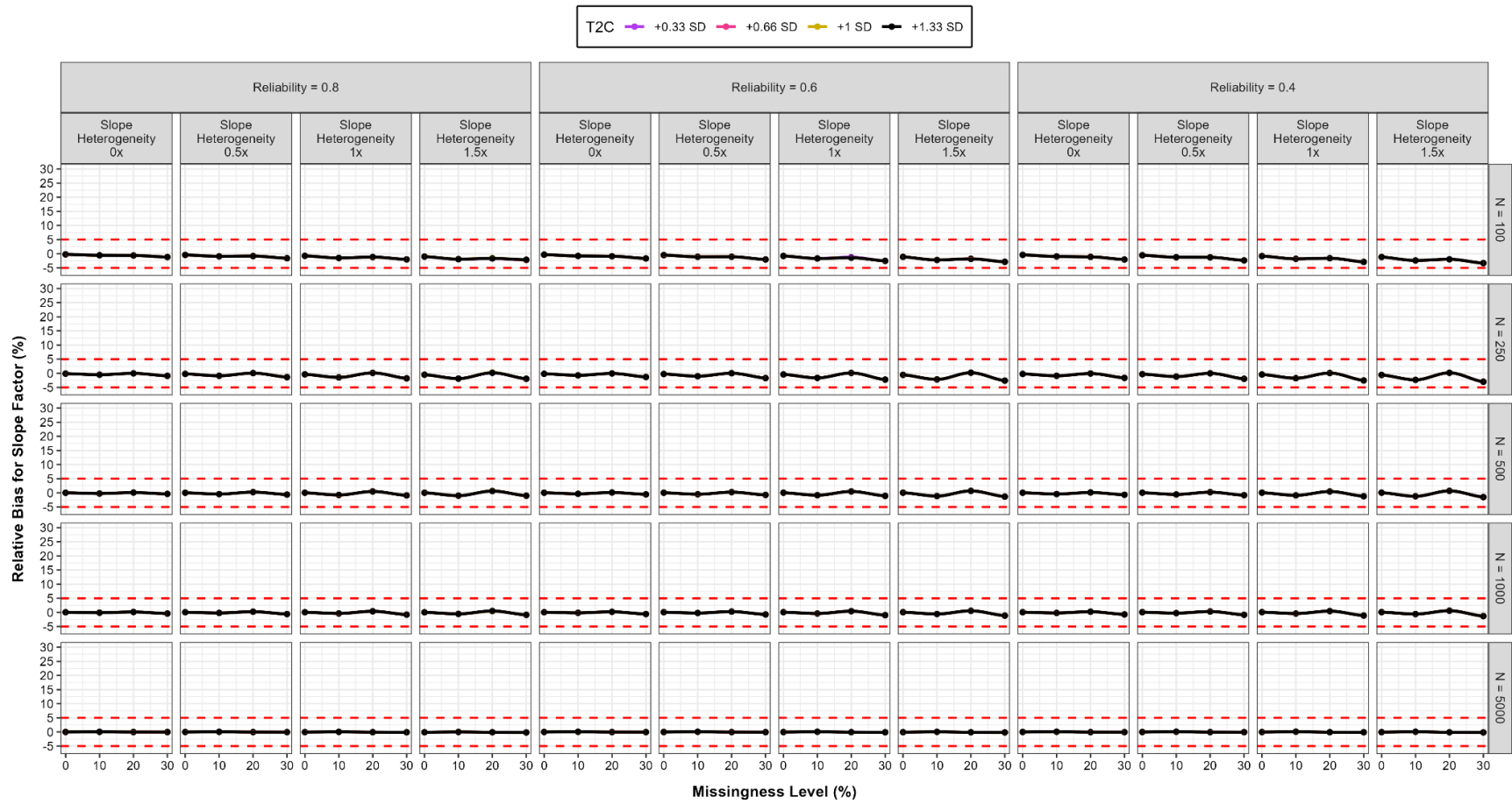
50 Indicators: Relative Bias for Predictor Effect on Latent Time-to-Criterion Factor



APPENDICES

Figure B27

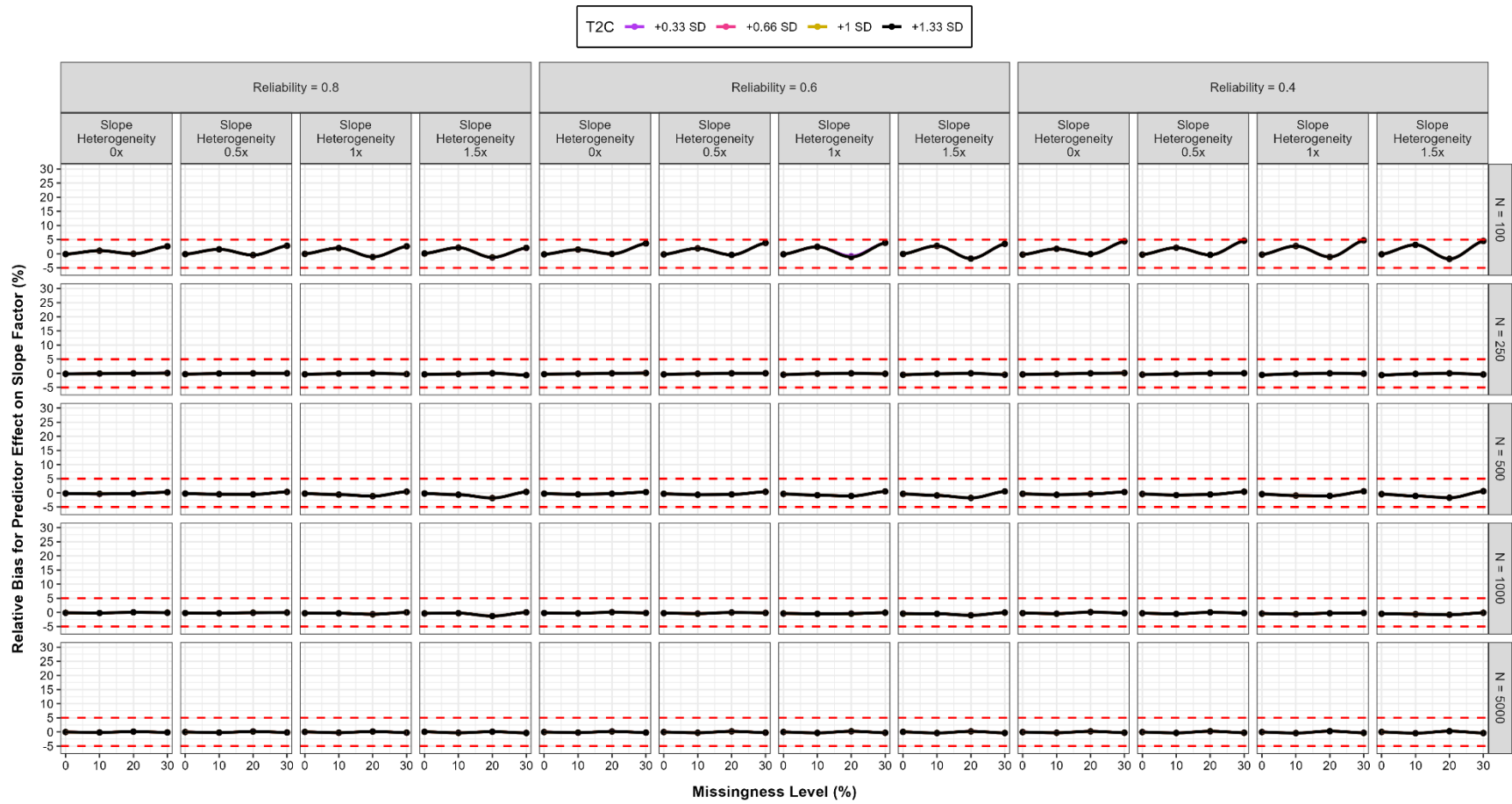
50 Indicators: Relative Bias for Latent Slope Factor



APPENDICES

Figure B28

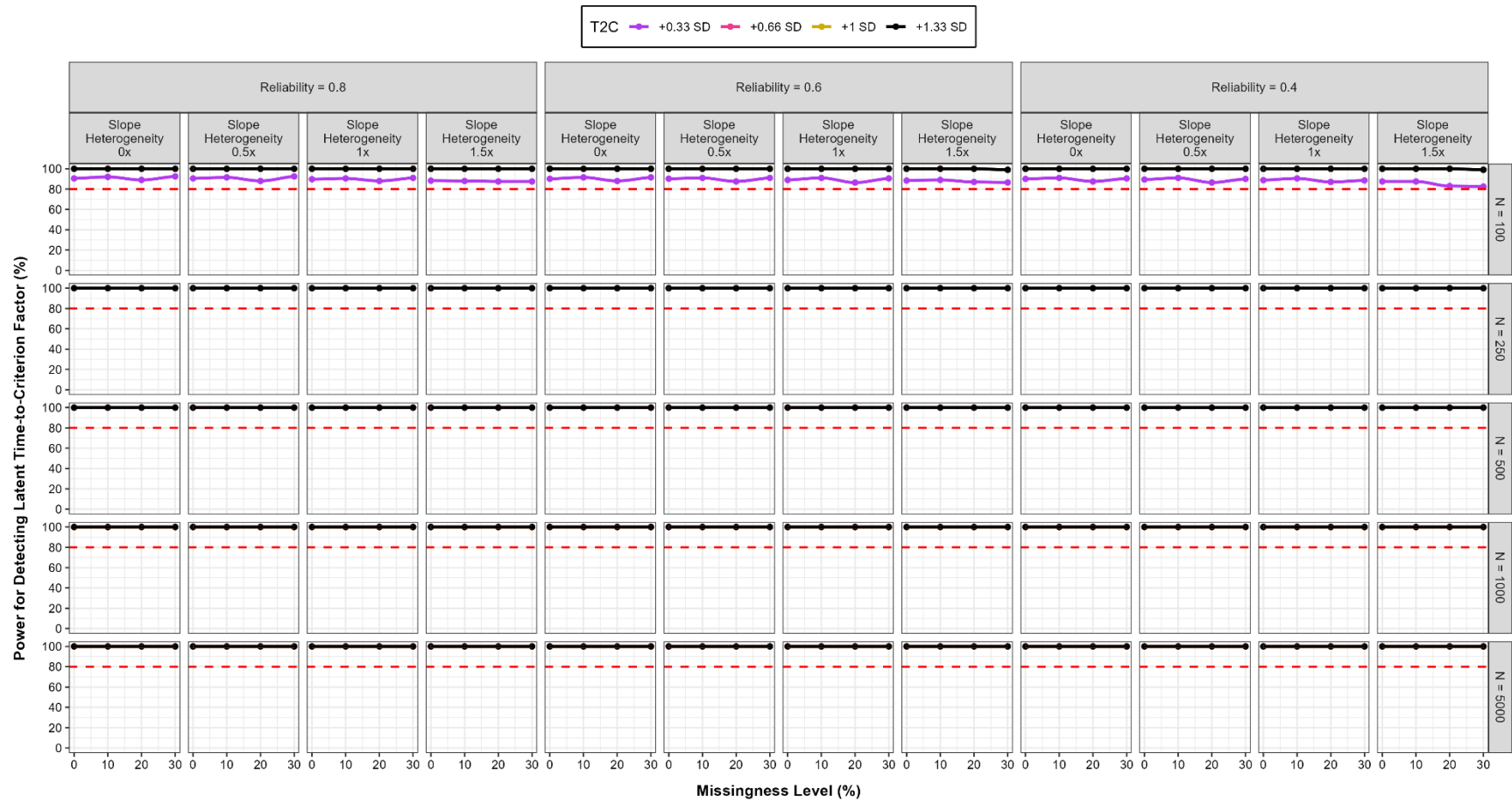
50 Indicators: Relative Bias for Predictor Effect on Latent Slope Factor



APPENDICES

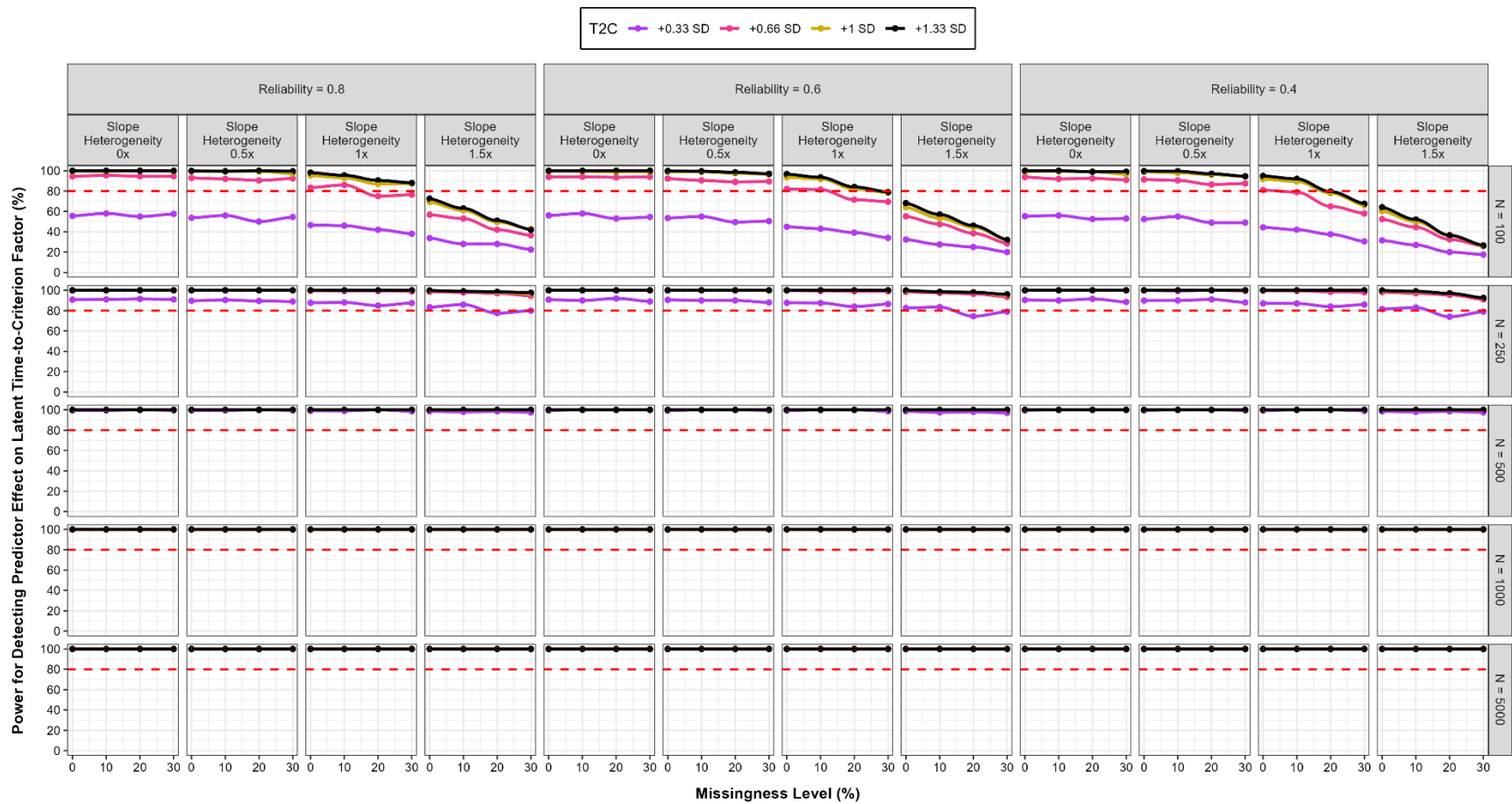
Figure B29

50 Indicators: Power for Detecting Latent Time-to-Criterion Factor



**Figure B30**

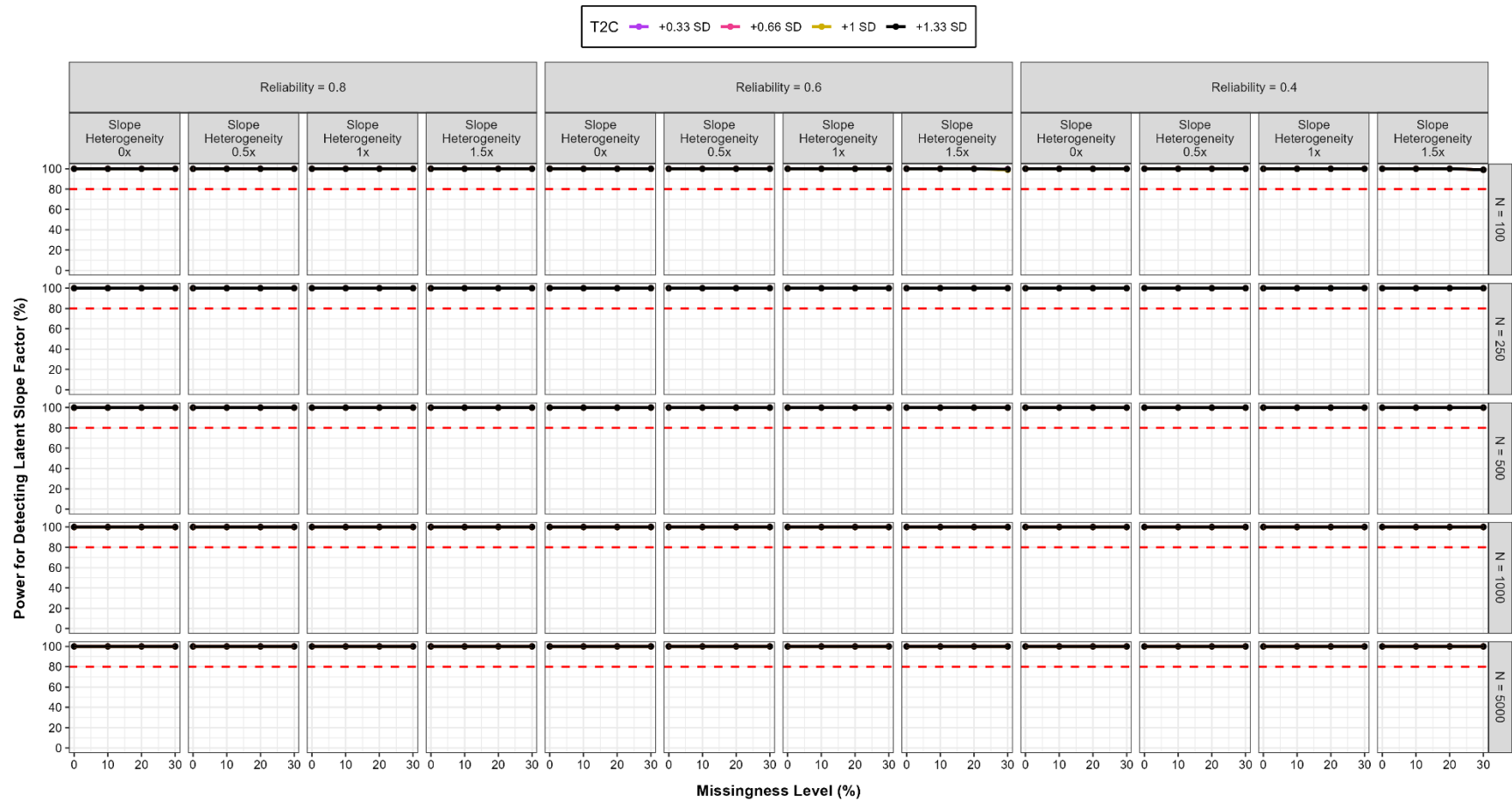
*50 Indicators: Power for Detecting Predictor Effect on Time-to-Criterion Factor*



APPENDICES

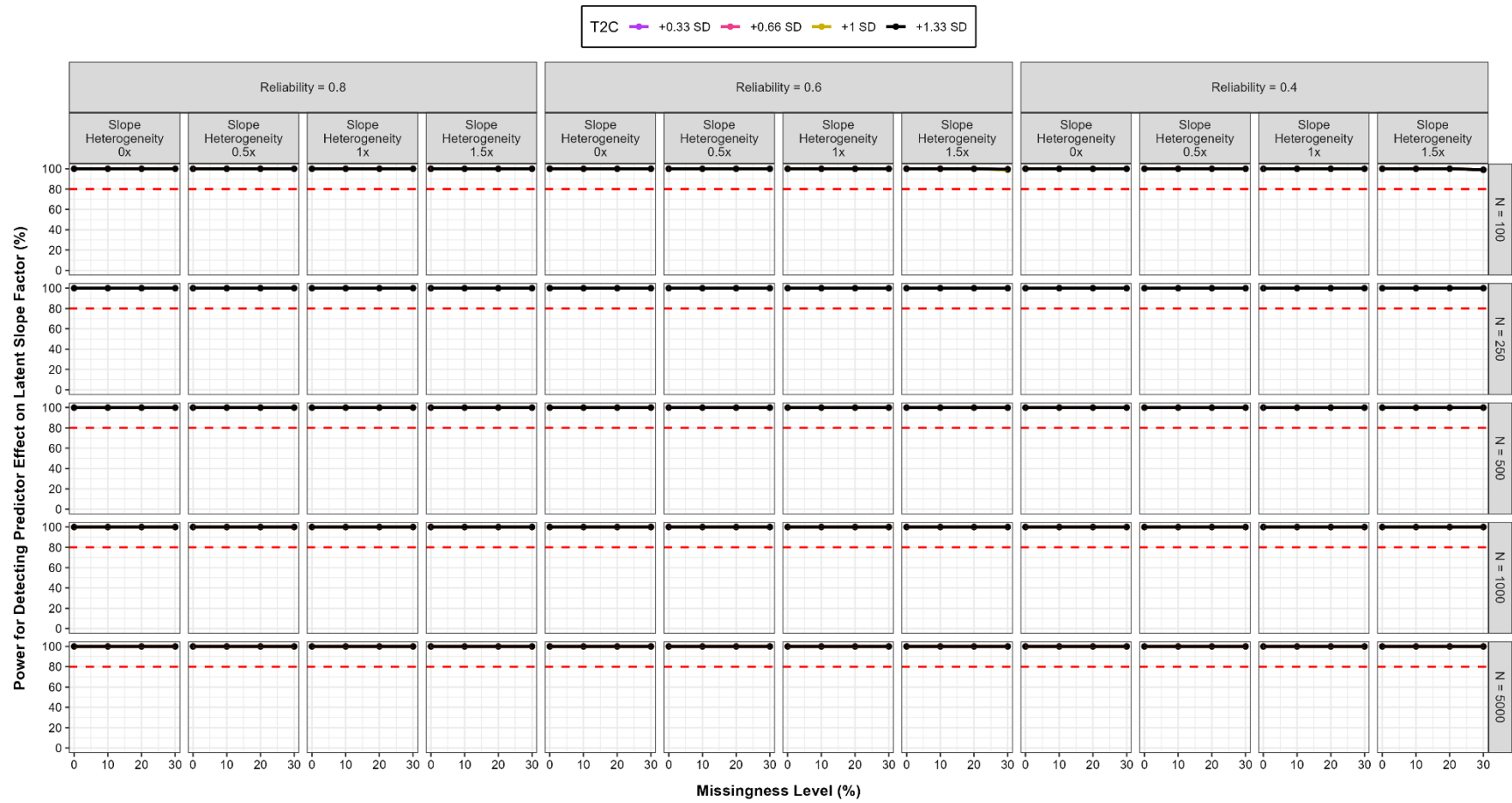
Figure B31

50 Indicators: Power for Detecting Latent Slope Factor



**Figure B32**

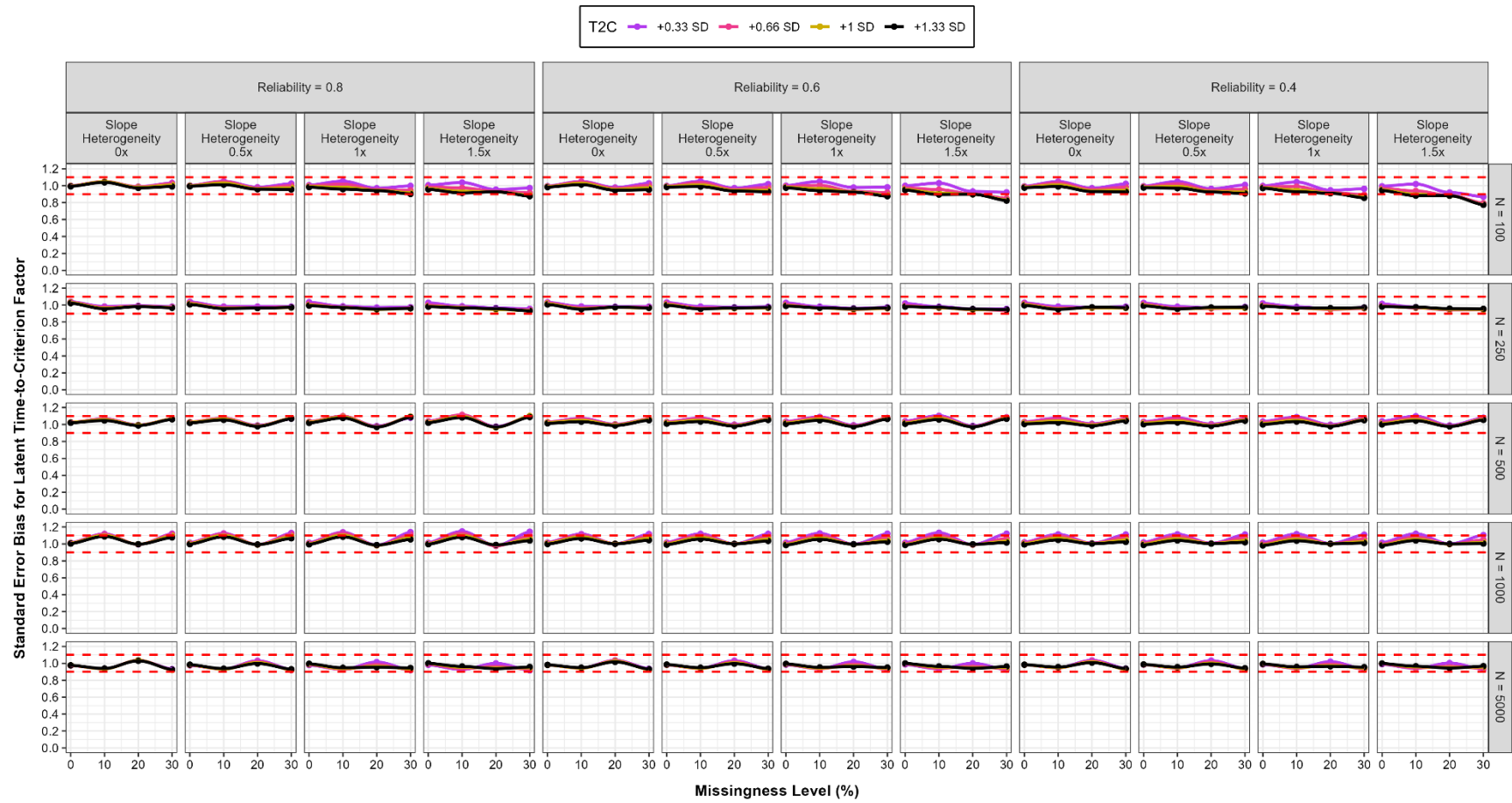
*50 Indicators: Power for Detecting Predictor Effect on Latent Slope Factor*



APPENDICES

**Figure B33**

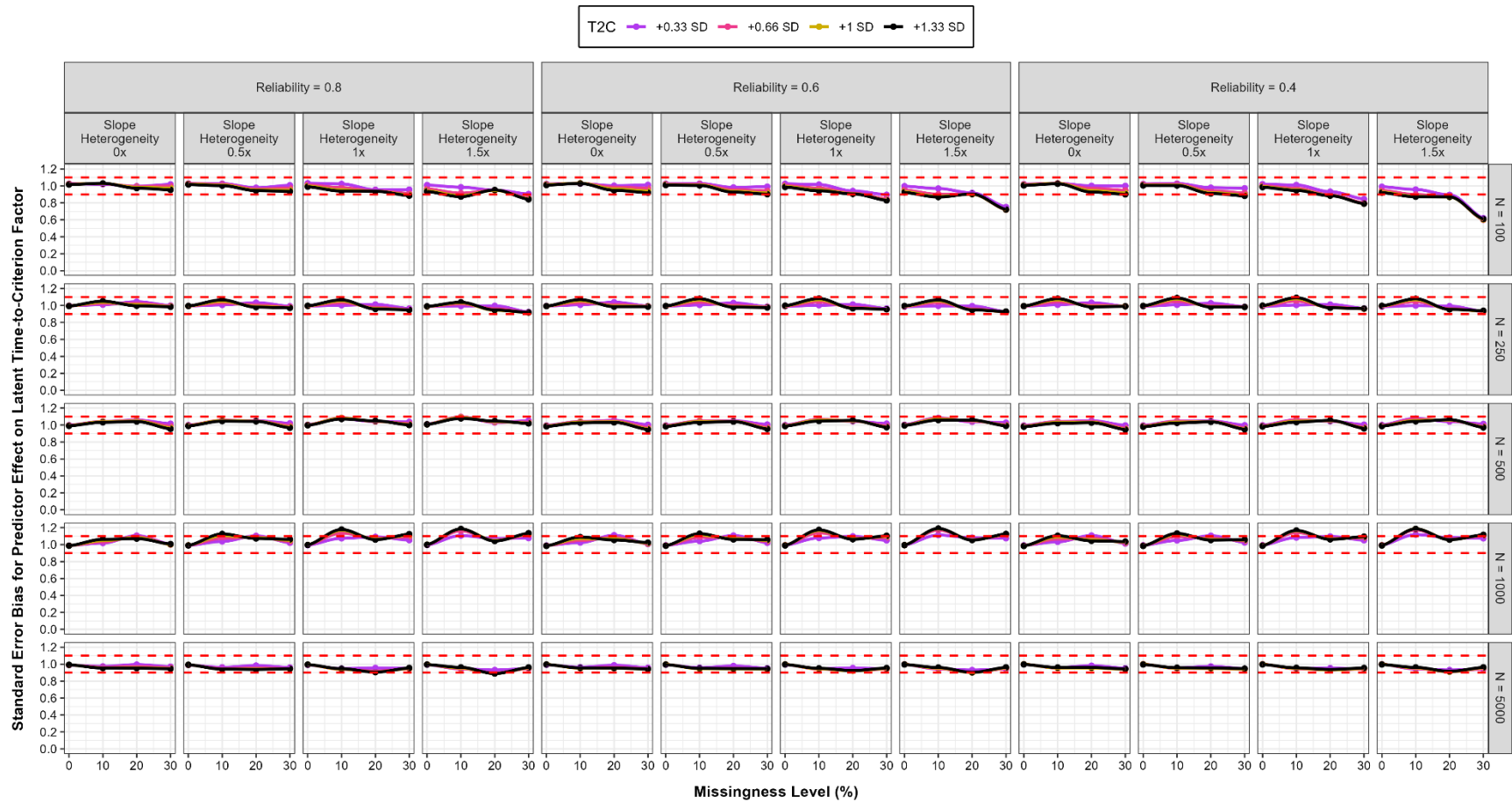
*50 Indicators: Standard Error Bias for Latent Time-to-Criterion Factor*



APPENDICES

Figure B34

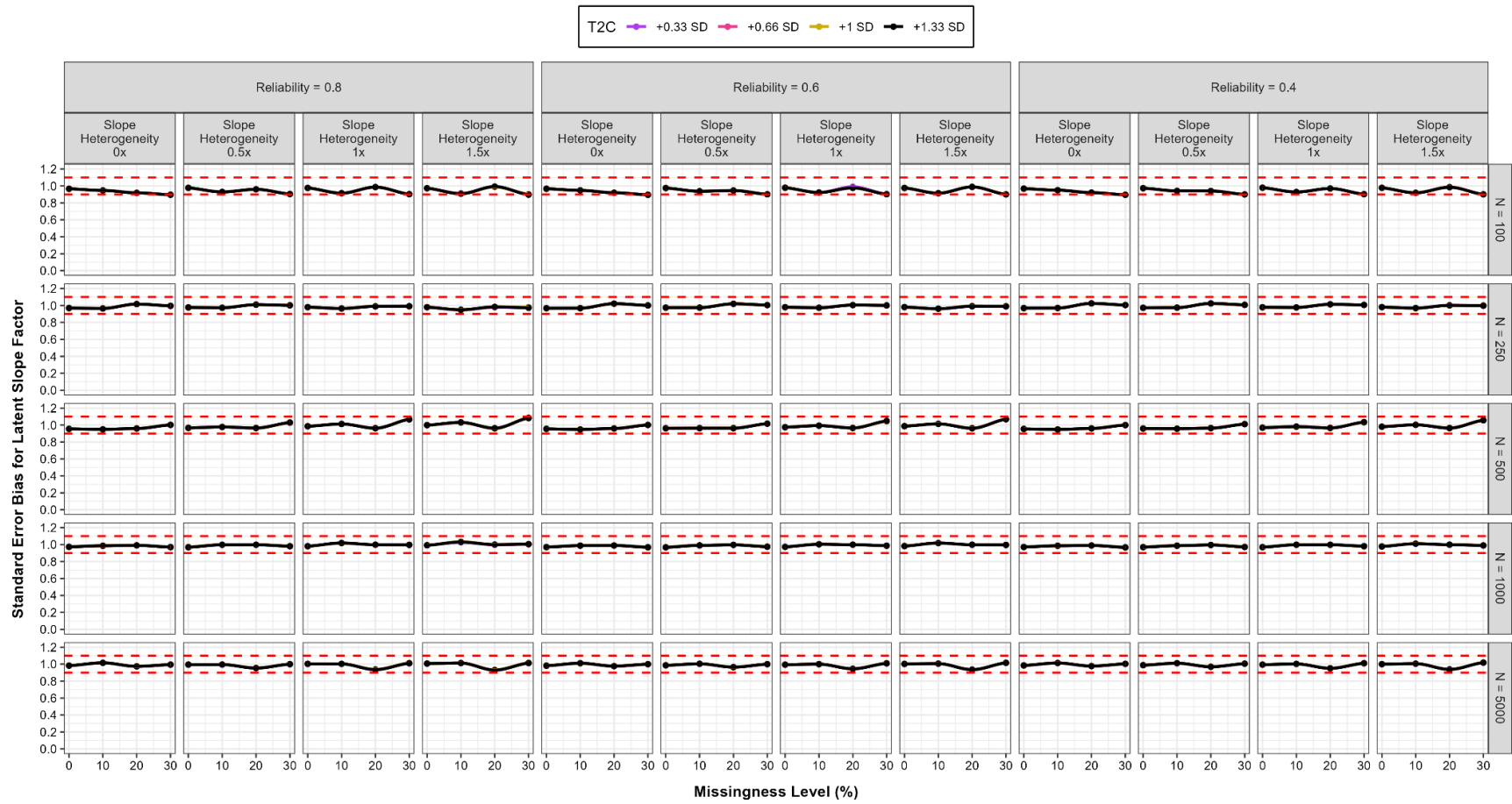
50 Indicators: Standard Error Bias for Predictor Effect on Latent Time-to-Criterion Factor



APPENDICES

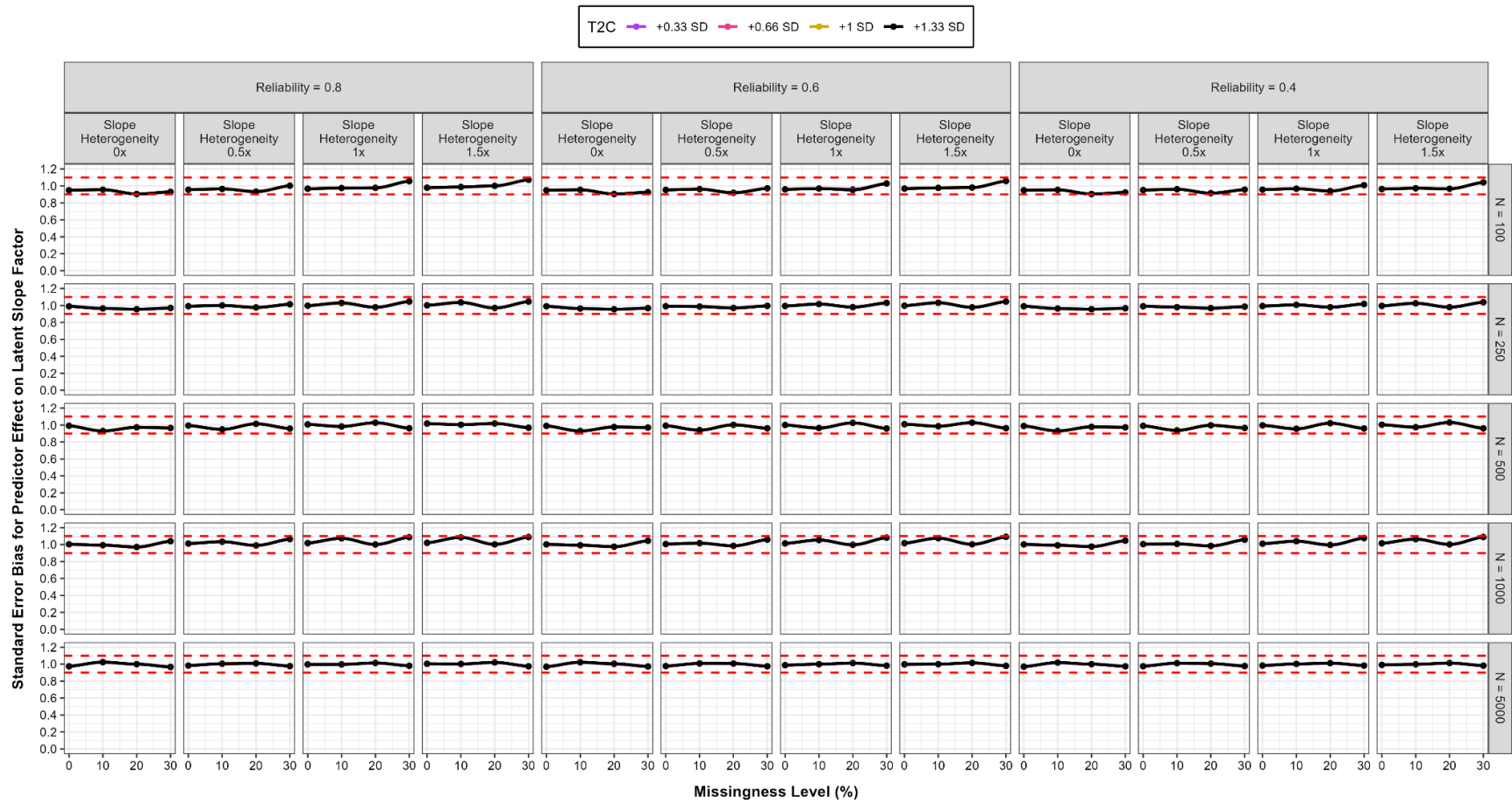
Figure B35

50 Indicators: Standard Error Bias for Latent Slope Factor



**Figure B36**

*50 Indicators: Standard Error Bias for Predictor Effect on Latent Slope Factor*



## APPENDIX C

## Results Tables

Table C1

*3-time point Models with Measure Reliability 0.8: Relative Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
Mean Intercept/Tau									
Missing Rate = 0%	-0.08	2.63	2.34	-0.06	1.88	1.26	0.00	0.09	0.07
Missing Rate = 10%	0.08	1.19	3.58	-0.02	1.13	1.22	0.00	0.07	0.11
Missing Rate = 20%	0.08	3.55	5.98	-0.02	1.83	1.91	0.00	0.24	0.27
Missing Rate = 30%	0.07	9.14	11.08	-0.03	3.13	3.21	0.00	0.30	0.33
Mean Growth Rate									
Missing Rate = 0%	0.31	0.31	0.31	-0.08	-0.09	-0.08	-0.02	-0.02	-0.02
Missing Rate = 10%	-0.63	-0.63	-0.63	0.10	0.10	0.10	-0.07	-0.07	-0.07
Missing Rate = 20%	-0.94	-0.91	-0.87	0.06	0.06	0.06	-0.18	-0.19	-0.18
Missing Rate = 30%	-1.14	-1.05	-0.93	-0.03	-0.03	-0.03	-0.20	-0.21	-0.20
Predictor Effect on Intercept/Tau									
Missing Rate = 0%	-0.62	6.06	7.46	0.14	2.20	2.35	-1.07	-0.25	-0.05
Missing Rate = 10%	9.39	11.07	14.10	2.09	3.18	3.58	-0.59	0.10	0.31
Missing Rate = 20%	10.14	32.34	27.92	2.55	5.45	6.32	-0.50	0.34	0.54
Missing Rate = 30%	9.81	48.89	50.90	2.41	8.99	10.83	-0.52	0.56	0.79
Predictor Effect on Growth Rate									
Missing Rate = 0%	0.62	0.62	0.62	-0.79	-0.79	-0.79	-0.15	-0.15	-0.15
Missing Rate = 10%	0.00	0.01	0.00	-0.50	-0.50	-0.50	0.09	0.09	0.09
Missing Rate = 20%	0.75	0.75	0.74	0.19	0.19	0.19	-0.03	-0.03	-0.03
Missing Rate = 30%	0.05	0.06	0.01	0.62	0.62	0.62	0.07	0.07	0.07

*Note.* Relative bias is presented as percentages (%).  $N = 1000$  replications per cell.

APPENDICES

**Table C2**

*5-time point Models with Measure Reliability 0.8: Relative Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.05	0.37	1.73	0.00	0.80	1.02	0.00	-0.10	-0.04
Missing Rate = 10%	0.06	1.26	2.93	0.00	1.25	1.50	0.00	-0.02	0.04
Missing Rate = 20%	0.05	2.98	4.71	0.01	1.49	1.86	0.00	-0.04	0.02
Missing Rate = 30%	0.06	5.85	7.42	0.01	2.25	2.67	0.00	-0.19	-0.10
<b>Mean Growth Rate</b>									
Missing Rate = 0%	-0.02	-0.02	-0.02	-0.29	-0.29	-0.29	0.06	0.06	0.06
Missing Rate = 10%	-0.39	-0.38	-0.39	-0.49	-0.49	-0.49	-0.01	-0.01	-0.01
Missing Rate = 20%	-0.83	-0.82	-0.83	-0.45	-0.45	-0.45	0.03	0.03	0.03
Missing Rate = 30%	-0.89	-0.85	-0.87	-0.52	-0.52	-0.52	0.18	0.17	0.18
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	8.89	6.49	6.78	3.93	3.23	3.10	-0.21	-0.14	-0.06
Missing Rate = 10%	7.95	9.51	10.84	3.31	4.35	4.53	-0.21	0.00	0.11
Missing Rate = 20%	9.71	15.67	17.78	4.13	6.15	6.75	-0.15	0.02	0.13
Missing Rate = 30%	9.18	29.35	28.49	3.73	7.65	8.80	-0.19	-0.40	-0.35
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.35	0.35	0.35	0.00	0.00	0.00	0.01	0.01	0.01
Missing Rate = 10%	0.94	0.93	0.94	0.12	0.12	0.12	0.00	0.00	0.00
Missing Rate = 20%	2.41	2.41	2.41	0.95	0.95	0.95	0.02	0.02	0.02
Missing Rate = 30%	2.15	2.17	2.16	0.98	0.99	0.98	-0.25	-0.25	-0.25

*Note.* Relative bias is presented as percentages (%).  $N = 1000$  replications per cell.

APPENDICES

**Table C3**

*50-time point Models with Measure Reliability 0.8: Relative Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.11	-0.72	1.17	0.07	-0.66	0.40	0.00	0.13	0.08
Missing Rate = 10%	0.11	0.22	2.22	0.14	-1.13	1.02	0.01	-0.25	-0.07
Missing Rate = 20%	0.13	-0.08	2.02	0.12	-1.79	-0.07	0.01	-0.01	0.07
Missing Rate = 30%	0.12	1.31	3.45	0.15	-0.83	1.50	0.01	-0.14	0.06
<b>Mean Growth Rate</b>									
Missing Rate = 0%	-0.60	-0.61	-0.61	-0.31	-0.31	-0.31	-0.05	-0.05	-0.05
Missing Rate = 10%	-1.22	-1.22	-1.22	-1.18	-1.18	-1.18	0.03	0.03	0.03
Missing Rate = 20%	-1.07	-1.08	-1.07	0.09	0.09	0.08	-0.07	-0.07	-0.07
Missing Rate = 30%	-1.75	-1.76	-1.72	-1.51	-1.51	-1.50	-0.10	-0.10	-0.10
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	11.52	5.31	4.92	5.47	1.99	1.74	0.24	0.25	0.19
Missing Rate = 10%	18.81	11.22	10.14	10.91	4.29	3.91	-0.66	-0.51	-0.32
Missing Rate = 20%	11.13	6.02	6.20	-3.00	-0.97	0.84	0.44	0.36	0.36
Missing Rate = 30%	17.63	14.78	14.77	10.73	5.47	5.49	-0.78	-0.34	-0.07
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	-0.06	-0.06	-0.06	-0.31	-0.31	-0.31	0.00	0.00	0.00
Missing Rate = 10%	1.70	1.70	1.70	-0.13	-0.13	-0.13	-0.24	-0.24	-0.24
Missing Rate = 20%	-0.73	-0.73	-0.73	-0.01	-0.01	-0.01	0.13	0.13	0.13
Missing Rate = 30%	2.54	2.54	2.54	-0.21	-0.21	-0.22	-0.24	-0.24	-0.24

*Note.* Relative bias is presented as percentages (%).  $N = 1000$  replications per cell.

APPENDICES

**Table C4**

*3-time point Models with Measure Reliability 0.6: Relative Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	-0.09	2.93	3.51	-0.07	2.03	1.69	0.00	0.10	0.11
Missing Rate = 10%	0.09	2.68	6.24	-0.02	1.45	1.95	0.00	0.11	0.17
Missing Rate = 20%	0.09	6.53	9.29	-0.03	2.56	3.11	0.00	0.32	0.37
Missing Rate = 30%	0.07	16.20	16.59	-0.03	4.83	5.44	0.00	0.41	0.47
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.62	0.59	0.58	0.06	0.01	0.01	-0.02	-0.03	-0.03
Missing Rate = 10%	-0.87	-0.86	-0.86	0.07	0.10	0.11	-0.10	-0.10	-0.10
Missing Rate = 20%	-1.25	-1.01	-0.90	0.09	0.10	0.10	-0.22	-0.24	-0.24
Missing Rate = 30%	-1.35	-0.71	-0.33	0.07	0.04	0.04	-0.26	-0.28	-0.27
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	-0.68	8.21	12.06	0.13	2.77	3.63	-1.15	-0.23	0.02
Missing Rate = 10%	10.26	23.11	26.22	2.20	4.66	6.01	-0.63	0.22	0.50
Missing Rate = 20%	11.06	64.47	44.87	2.84	8.33	10.52	-0.51	0.53	0.80
Missing Rate = 30%	10.64	109.11	77.67	2.65	15.42	19.88	-0.53	0.86	1.18
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.70	0.78	0.78	-0.92	-1.01	-1.01	-0.13	-0.16	-0.16
Missing Rate = 10%	-0.36	-0.32	-0.32	-0.62	-0.68	-0.68	0.13	0.14	0.14
Missing Rate = 20%	0.44	0.56	0.48	0.03	0.14	0.10	-0.04	-0.04	-0.04
Missing Rate = 30%	-0.28	-0.41	-0.47	0.54	0.67	0.66	0.07	0.09	0.08

*Note.* Relative bias is presented as percentages (%). N = 1000 replications per cell.

APPENDICES

**Table C5**

*5-time point Models with Measure Reliability 0.6: Relative Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.04	0.73	2.80	0.00	1.01	1.45	0.00	-0.11	-0.04
Missing Rate = 10%	0.06	2.15	4.83	0.00	1.65	2.15	0.00	-0.01	0.07
Missing Rate = 20%	0.05	5.24	8.12	0.00	2.15	2.84	0.00	-0.02	0.06
Missing Rate = 30%	0.06	9.69	13.20	0.01	3.48	4.27	0.01	-0.20	-0.09
<b>Mean Growth Rate</b>									
Missing Rate = 0%	-0.03	0.00	-0.03	-0.32	-0.32	-0.32	0.08	0.07	0.07
Missing Rate = 10%	-0.52	-0.49	-0.52	-0.56	-0.57	-0.57	-0.01	-0.02	-0.02
Missing Rate = 20%	-1.08	-1.11	-1.08	-0.55	-0.55	-0.55	0.03	0.03	0.03
Missing Rate = 30%	-1.23	-0.97	-1.04	-0.68	-0.69	-0.68	0.21	0.20	0.21
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	9.15	8.43	10.09	4.03	3.96	4.29	-0.25	-0.14	-0.02
Missing Rate = 10%	7.92	13.67	17.36	3.22	5.71	6.55	-0.23	0.05	0.19
Missing Rate = 20%	10.39	25.22	31.39	4.39	8.71	10.25	-0.16	0.10	0.23
Missing Rate = 30%	9.77	47.04	54.39	3.78	11.75	14.32	-0.23	-0.41	-0.32
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.29	0.33	0.29	-0.07	-0.07	-0.07	0.02	0.02	0.02
Missing Rate = 10%	1.12	1.14	1.12	0.14	0.14	0.14	0.02	0.02	0.02
Missing Rate = 20%	2.84	2.85	2.84	1.12	1.12	1.12	0.03	0.03	0.03
Missing Rate = 30%	2.58	2.63	2.62	1.26	1.26	1.26	-0.29	-0.29	-0.29

*Note.* Relative bias is presented as percentages (%). N = 1000 replications per cell.

APPENDICES

**Table C6**

*50-time point Models with Measure Reliability 0.6: Relative Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.10	-0.28	1.48	0.06	-0.44	0.54	0.00	0.16	0.09
Missing Rate = 10%	0.11	0.71	2.76	0.12	-0.52	1.41	0.01	-0.21	-0.08
Missing Rate = 20%	0.13	0.25	2.66	0.11	-1.56	0.09	0.01	-0.02	0.08
Missing Rate = 30%	0.12	2.40	4.78	0.14	-0.01	2.22	0.01	-0.07	0.09
<b>Mean Growth Rate</b>									
Missing Rate = 0%	-0.69	-0.69	-0.68	-0.35	-0.36	-0.35	-0.04	-0.04	-0.04
Missing Rate = 10%	-1.44	-1.44	-1.44	-1.39	-1.39	-1.39	0.06	0.06	0.06
Missing Rate = 20%	-1.30	-1.26	-1.30	0.07	0.06	0.07	-0.08	-0.08	-0.08
Missing Rate = 30%	-2.27	-2.29	-2.27	-1.96	-1.96	-1.96	-0.10	-0.10	-0.10
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	11.44	5.78	5.42	5.58	2.19	1.91	0.26	0.24	0.17
Missing Rate = 10%	17.89	12.35	11.71	10.07	4.68	4.43	-0.74	-0.57	-0.42
Missing Rate = 20%	12.28	7.78	8.02	-2.86	-0.48	1.28	0.36	0.40	0.45
Missing Rate = 30%	16.03	18.89	20.22	9.72	6.71	7.12	-0.92	-0.34	-0.07
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	-0.18	-0.18	-0.18	-0.43	-0.43	-0.43	-0.02	-0.02	-0.02
Missing Rate = 10%	2.14	2.14	2.15	-0.16	-0.16	-0.16	-0.31	-0.31	-0.31
Missing Rate = 20%	-0.84	-0.76	-0.84	-0.01	-0.01	-0.01	0.20	0.20	0.20
Missing Rate = 30%	3.71	3.71	3.71	-0.15	-0.15	-0.15	-0.28	-0.28	-0.28

*Note.* Relative bias is presented as percentages (%). N = 1000 replications per cell.

APPENDICES

**Table C7**

*3-time point Models with Measure Reliability 0.4: Relative Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	-0.10	3.27	4.90	-0.08	2.18	2.15	0.00	0.11	0.13
Missing Rate = 10%	0.10	3.78	9.07	-0.02	1.77	2.71	0.00	0.14	0.23
Missing Rate = 20%	0.09	8.07	12.89	-0.03	3.33	4.39	0.00	0.39	0.46
Missing Rate = 30%	0.08	20.63	23.37	-0.04	6.78	8.04	0.00	0.51	0.59
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.81	0.85	0.81	0.10	0.10	0.10	-0.03	-0.03	-0.03
Missing Rate = 10%	-1.13	-1.01	-0.97	0.12	0.12	0.12	-0.12	-0.13	-0.12
Missing Rate = 20%	-1.61	-0.94	-0.93	0.13	0.12	0.13	-0.28	-0.29	-0.28
Missing Rate = 30%	-1.75	0.05	0.36	0.11	0.11	0.10	-0.32	-0.33	-0.32
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	-0.81	10.66	18.07	0.17	3.43	5.09	-1.21	-0.19	0.10
Missing Rate = 10%	11.01	24.68	43.47	2.32	6.23	8.67	-0.67	0.33	0.67
Missing Rate = 20%	11.88	63.68	53.14	3.08	11.40	15.32	-0.52	0.70	1.02
Missing Rate = 30%	11.34	128.15	131.23	2.84	24.38	32.84	-0.55	1.12	1.53
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.91	0.91	0.91	-1.16	-1.16	-1.16	-0.16	-0.16	-0.16
Missing Rate = 10%	-0.56	-0.63	-0.61	-0.82	-0.82	-0.82	0.17	0.17	0.17
Missing Rate = 20%	0.43	0.13	0.14	0.02	0.02	0.02	-0.04	-0.04	-0.04
Missing Rate = 30%	-0.50	-0.71	-0.90	0.68	0.69	0.68	0.11	0.11	0.11

*Note.* Relative bias is presented as percentages (%). N = 1000 replications per cell.

APPENDICES

**Table C8**

*5-time point Models with Measure Reliability 0.4: Relative Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.04	1.05	3.95	-0.01	1.17	1.87	0.00	-0.12	-0.03
Missing Rate = 10%	0.06	3.24	6.99	0.00	1.99	2.78	0.00	0.00	0.09
Missing Rate = 20%	0.05	8.57	11.68	0.00	2.76	3.82	0.00	-0.01	0.09
Missing Rate = 30%	0.06	14.87	18.51	0.01	4.65	5.89	0.01	-0.21	-0.06
<b>Mean Growth Rate</b>									
Missing Rate = 0%	-0.04	-0.04	-0.05	-0.34	-0.37	-0.34	0.08	0.08	0.08
Missing Rate = 10%	-0.63	-0.62	-0.63	-0.62	-0.62	-0.62	-0.02	-0.03	-0.02
Missing Rate = 20%	-1.28	-1.24	-1.21	-0.62	-0.63	-0.62	0.04	0.02	0.03
Missing Rate = 30%	-1.51	-1.01	-0.86	-0.81	-0.82	-0.81	0.24	0.23	0.23
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	9.40	10.40	13.74	4.12	4.60	5.48	-0.28	-0.13	0.02
Missing Rate = 10%	7.97	18.58	25.65	3.18	6.99	8.54	-0.26	0.10	0.28
Missing Rate = 20%	10.93	46.65	47.02	4.60	11.25	13.88	-0.17	0.17	0.34
Missing Rate = 30%	10.27	75.71	87.43	3.86	16.02	20.24	-0.26	-0.39	-0.25
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.20	0.22	0.20	-0.13	-0.13	-0.13	0.03	0.03	0.03
Missing Rate = 10%	1.22	1.26	1.21	0.14	0.14	0.14	0.03	0.03	0.03
Missing Rate = 20%	3.08	3.13	3.09	1.22	1.22	1.22	0.04	0.04	0.04
Missing Rate = 30%	2.85	2.92	3.01	1.44	1.44	1.44	-0.32	-0.32	-0.32

*Note.* Relative bias is presented as percentages (%). N = 1000 replications per cell.

APPENDICES

**Table C9**

*50-time point Models with Measure Reliability 0.4: Relative Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.09	0.04	1.74	0.06	-0.27	0.66	-0.01	0.17	0.09
Missing Rate = 10%	0.10	1.16	3.23	0.11	-0.08	1.70	0.01	-0.18	-0.08
Missing Rate = 20%	0.13	0.86	3.24	0.10	-1.37	0.24	0.01	-0.03	0.09
Missing Rate = 30%	0.12	3.37	5.94	0.13	0.61	2.76	0.01	-0.02	0.11
<b>Mean Growth Rate</b>									
Missing Rate = 0%	-0.74	-0.74	-0.74	-0.39	-0.39	-0.39	-0.03	-0.03	-0.03
Missing Rate = 10%	-1.60	-1.60	-1.60	-1.53	-1.53	-1.53	0.08	0.08	0.08
Missing Rate = 20%	-1.49	-1.49	-1.49	0.04	0.04	0.04	-0.08	-0.09	-0.09
Missing Rate = 30%	-2.65	-2.66	-2.65	-2.27	-2.28	-2.27	-0.11	-0.11	-0.11
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	11.26	6.12	5.88	5.60	2.34	2.07	0.26	0.25	0.15
Missing Rate = 10%	17.04	13.25	12.98	9.41	4.94	4.81	-0.78	-0.61	-0.48
Missing Rate = 20%	12.88	9.16	9.71	-2.76	-0.08	1.68	0.27	0.42	0.51
Missing Rate = 30%	14.64	22.79	25.32	8.95	7.67	8.34	-1.00	-0.33	-0.07
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	-0.28	-0.28	-0.28	-0.53	-0.52	-0.52	-0.04	-0.04	-0.04
Missing Rate = 10%	2.43	2.43	2.43	-0.21	-0.20	-0.20	-0.36	-0.36	-0.36
Missing Rate = 20%	-0.87	-0.87	-0.87	0.00	0.00	0.00	0.25	0.25	0.25
Missing Rate = 30%	4.56	4.56	4.56	-0.10	-0.10	-0.10	-0.32	-0.32	-0.32

*Note.* Relative bias is presented as percentages (%).  $N = 200$  replications per cell.

APPENDICES

**Table C10**

*3-time point Models with Measure Reliability 0.8: Power Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	1.00	0.88	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	0.86	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	0.83	0.99	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	0.77	0.95	1.00	1.00	1.00	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	0.95	0.95	0.95	1.00	1.00	1.00	1.00	1.00	1.00
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.10	0.42	0.80	0.17	0.85	1.00	0.99	1.00	1.00
Missing Rate = 10%	0.12	0.34	0.63	0.18	0.81	0.98	1.00	1.00	1.00
Missing Rate = 20%	0.11	0.26	0.40	0.18	0.74	0.91	0.99	1.00	1.00
Missing Rate = 30%	0.11	0.16	0.21	0.17	0.62	0.74	0.99	1.00	1.00
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.89	0.89	0.89	0.99	0.99	0.99	1.00	1.00	1.00
Missing Rate = 10%	0.81	0.81	0.81	0.98	0.98	0.98	1.00	1.00	1.00
Missing Rate = 20%	0.68	0.68	0.68	0.94	0.94	0.94	1.00	1.00	1.00
Missing Rate = 30%	0.52	0.52	0.52	0.86	0.86	0.86	1.00	1.00	1.00

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C11**

*5-time point Models with Measure Reliability 0.8: Power Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	1.00	0.88	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	0.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	0.86	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	0.82	0.97	1.00	1.00	1.00	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	0.97	0.97	0.97	1.00	1.00	1.00	1.00	1.00	1.00
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.12	0.44	0.84	0.20	0.86	1.00	1.00	1.00	1.00
Missing Rate = 10%	0.11	0.38	0.72	0.20	0.83	0.99	0.99	1.00	1.00
Missing Rate = 20%	0.11	0.31	0.55	0.20	0.80	0.95	1.00	1.00	1.00
Missing Rate = 30%	0.11	0.22	0.33	0.20	0.71	0.85	1.00	1.00	1.00
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.92	0.92	0.92	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	0.85	0.85	0.85	0.99	0.99	0.99	1.00	1.00	1.00
Missing Rate = 20%	0.76	0.76	0.76	0.97	0.97	0.97	1.00	1.00	1.00
Missing Rate = 30%	0.62	0.62	0.62	0.91	0.91	0.91	1.00	1.00	1.00

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C12**

*50-time point Models with Measure Reliability 0.8: Power Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	1.00	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	0.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	0.88	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	0.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.11	0.47	0.93	0.20	0.88	1.00	1.00	1.00	1.00
Missing Rate = 10%	0.13	0.47	0.90	0.20	0.89	1.00	1.00	1.00	1.00
Missing Rate = 20%	0.13	0.44	0.85	0.19	0.86	1.00	1.00	1.00	1.00
Missing Rate = 30%	0.13	0.43	0.82	0.19	0.87	0.99	1.00	1.00	1.00
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.97	0.97	0.97	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	0.96	0.96	0.96	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	0.94	0.94	0.94	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	0.93	0.93	0.93	1.00	1.00	1.00	1.00	1.00	1.00

*Note.* N = 200 replications per cell.

APPENDICES

**Table C13**

*3-time point Models with Measure Reliability 0.6: Power Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	1.00	0.85	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	0.81	0.99	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	0.76	0.96	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	0.64	0.86	1.00	0.98	0.99	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	0.98	0.98	0.98	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	0.95	0.95	0.95	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	0.84	0.84	0.84	0.99	0.99	0.99	1.00	1.00	1.00
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.10	0.35	0.58	0.15	0.82	0.98	0.97	1.00	1.00
Missing Rate = 10%	0.11	0.25	0.31	0.17	0.75	0.92	0.98	1.00	1.00
Missing Rate = 20%	0.11	0.15	0.14	0.16	0.62	0.72	0.98	1.00	1.00
Missing Rate = 30%	0.10	0.08	0.06	0.16	0.43	0.40	0.98	1.00	1.00
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.76	0.76	0.76	0.97	0.97	0.97	1.00	1.00	1.00
Missing Rate = 10%	0.61	0.61	0.61	0.92	0.92	0.92	1.00	1.00	1.00
Missing Rate = 20%	0.48	0.48	0.48	0.83	0.83	0.83	1.00	1.00	1.00
Missing Rate = 30%	0.36	0.36	0.36	0.69	0.69	0.69	1.00	1.00	1.00

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C14**

*5-time point Models with Measure Reliability 0.6: Power Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	1.00	0.85	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	0.83	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	0.80	0.97	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	0.73	0.92	1.00	1.00	1.00	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	0.97	0.97	0.97	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	0.91	0.91	0.91	1.00	1.00	1.00	1.00	1.00	1.00
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.12	0.38	0.68	0.18	0.84	0.99	0.99	1.00	1.00
Missing Rate = 10%	0.11	0.29	0.46	0.18	0.79	0.96	0.99	1.00	1.00
Missing Rate = 20%	0.11	0.20	0.24	0.18	0.72	0.84	0.99	1.00	1.00
Missing Rate = 30%	0.11	0.11	0.12	0.19	0.57	0.60	0.99	1.00	1.00
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.81	0.81	0.81	0.99	0.99	0.99	1.00	1.00	1.00
Missing Rate = 10%	0.69	0.69	0.69	0.96	0.96	0.96	1.00	1.00	1.00
Missing Rate = 20%	0.58	0.58	0.58	0.89	0.89	0.89	1.00	1.00	1.00
Missing Rate = 30%	0.44	0.45	0.45	0.78	0.78	0.78	1.00	1.00	1.00

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C15**

*50-time point Models with Measure Reliability 0.6: Power Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	1.00	0.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	0.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	0.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.11	0.47	0.91	0.20	0.88	1.00	1.00	1.00	1.00
Missing Rate = 10%	0.12	0.46	0.88	0.19	0.88	1.00	1.00	1.00	1.00
Missing Rate = 20%	0.13	0.42	0.82	0.19	0.85	1.00	1.00	1.00	1.00
Missing Rate = 30%	0.13	0.40	0.77	0.19	0.86	0.99	1.00	1.00	1.00
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.96	0.96	0.96	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	0.95	0.95	0.95	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	0.91	0.91	0.91	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	0.90	0.90	0.90	0.99	0.99	0.99	1.00	1.00	1.00

*Note.* N = 200 replications per cell.

APPENDICES

**Table C16**

*3-time point Models with Measure Reliability 0.4: Power Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	1.00	0.82	0.99	1.00	0.99	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	0.76	0.97	1.00	0.99	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	0.69	0.90	1.00	0.99	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	0.55	0.76	1.00	0.96	0.98	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	0.96	0.96	0.96	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	0.88	0.88	0.88	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	0.72	0.74	0.74	0.97	0.97	0.97	1.00	1.00	1.00
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.09	0.28	0.36	0.14	0.77	0.93	0.95	1.00	1.00
Missing Rate = 10%	0.11	0.17	0.14	0.16	0.67	0.80	0.97	1.00	1.00
Missing Rate = 20%	0.11	0.09	0.06	0.15	0.50	0.48	0.97	1.00	1.00
Missing Rate 30%	0.10	0.05	0.03	0.15	0.29	0.20	0.97	1.00	1.00
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.62	0.62	0.62	0.92	0.92	0.92	1.00	1.00	1.00
Missing Rate = 10%	0.49	0.49	0.49	0.84	0.84	0.84	1.00	1.00	1.00
Missing Rate = 20%	0.36	0.36	0.36	0.72	0.72	0.72	1.00	1.00	1.00
Missing Rate = 30%	0.28	0.28	0.28	0.55	0.55	0.55	1.00	1.00	1.00

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C17**

*5-time point Models with Measure Reliability 0.4: Power Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	1.00	0.83	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	0.80	0.98	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	0.74	0.94	1.00	0.99	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	0.65	0.86	1.00	0.99	1.00	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	0.98	0.98	0.98	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	0.93	0.93	0.93	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	0.83	0.84	0.84	0.99	0.99	0.99	1.00	1.00	1.00
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.11	0.33	0.48	0.17	0.81	0.97	0.98	1.00	1.00
Missing Rate = 10%	0.11	0.23	0.25	0.17	0.74	0.89	0.97	1.00	1.00
Missing Rate = 20%	0.10	0.13	0.11	0.17	0.63	0.68	0.98	1.00	1.00
Missing Rate 30%	0.10	0.07	0.05	0.16	0.44	0.36	0.97	1.00	1.00
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.69	0.69	0.69	0.96	0.96	0.96	1.00	1.00	1.00
Missing Rate = 10%	0.57	0.57	0.57	0.90	0.90	0.90	1.00	1.00	1.00
Missing Rate = 20%	0.45	0.45	0.45	0.80	0.80	0.80	1.00	1.00	1.00
Missing Rate = 30%	0.36	0.36	0.36	0.66	0.66	0.66	1.00	1.00	1.00

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C18**

*50-time point Models with Measure Reliability 0.4: Power Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	1.00	0.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	0.86	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	0.88	1.00	1.00	1.00	1.00	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 30%	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.11	0.46	0.90	0.20	0.87	1.00	0.99	1.00	1.00
Missing Rate = 10%	0.13	0.45	0.86	0.19	0.88	1.00	1.00	1.00	1.00
Missing Rate = 20%	0.12	0.40	0.78	0.18	0.85	0.99	1.00	1.00	1.00
Missing Rate = 30%	0.12	0.38	0.72	0.18	0.85	0.98	1.00	1.00	1.00
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.96	0.96	0.96	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 10%	0.95	0.95	0.95	1.00	1.00	1.00	1.00	1.00	1.00
Missing Rate = 20%	0.89	0.89	0.89	0.99	0.99	0.99	1.00	1.00	1.00
Missing Rate = 30%	0.87	0.87	0.87	0.99	0.99	0.99	1.00	1.00	1.00

*Note.* N = 200 replications per cell.

APPENDICES

**Table C19**

*3-time point Models with Measure Reliability 0.8: Standard Error Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.93	0.92	0.94	0.98	0.98	0.97	1.01	1.01	0.98
Missing Rate = 10%	1.01	1.01	0.94	0.98	0.98	0.97	1.00	1.00	1.01
Missing Rate = 20%	1.01	0.92	0.82	0.97	0.97	0.96	0.99	1.00	0.99
Missing Rate = 30%	1.01	0.91	0.81	0.97	0.96	0.92	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.98	0.98	0.98	0.99	0.99	0.99	0.97	0.97	0.97
Missing Rate = 10%	1.00	1.00	1.00	0.99	0.99	0.99	1.02	1.02	1.02
Missing Rate = 20%	0.99	0.99	1.00	0.99	0.99	0.99	0.99	0.99	0.99
Missing Rate = 30%	0.93	0.93	0.94	0.96	0.96	0.96	0.99	0.99	0.99
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.97	0.93	0.92	1.03	0.98	0.94	0.95	0.98	0.99
Missing Rate = 10%	0.99	0.90	0.76	0.99	0.97	0.95	1.00	1.00	1.01
Missing Rate = 20%	1.00	0.63	0.54	0.99	0.95	0.92	1.00	1.00	0.99
Missing Rate 30%	1.00	0.73	0.60	1.00	0.93	0.88	1.00	0.97	0.97
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.97	0.97	0.97	0.96	0.96	0.96	1.03	1.03	1.03
Missing Rate = 10%	0.99	0.99	0.99	1.00	1.00	1.00	1.01	1.01	1.01
Missing Rate = 20%	0.97	0.97	0.97	0.98	0.98	0.98	1.03	1.03	1.03
Missing Rate = 30%	0.95	0.95	0.95	1.00	1.00	1.00	0.99	0.99	0.99

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C20**

*5-time point Models with Measure Reliability 0.8: Standard Error Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.97	0.98	0.97	1.01	0.99	0.99	1.01	1.00	1.02
Missing Rate = 10%	0.97	0.97	0.95	1.01	0.99	0.99	1.01	1.01	1.03
Missing Rate = 20%	0.98	0.96	0.93	1.01	0.99	0.98	1.01	0.99	0.99
Missing Rate = 30%	0.97	0.90	0.88	1.02	0.97	0.95	1.00	1.00	1.02
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.98	0.98	0.98	1.00	1.00	1.00	1.04	1.04	1.04
Missing Rate = 10%	0.98	0.98	0.98	1.01	1.01	1.01	1.04	1.04	1.04
Missing Rate = 20%	0.99	0.99	0.99	1.01	1.01	1.01	1.01	1.01	1.01
Missing Rate = 30%	0.97	0.97	0.97	1.00	1.00	1.00	1.03	1.03	1.03
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.98	0.97	0.97	0.94	0.97	0.99	1.04	1.04	1.05
Missing Rate = 10%	0.98	0.97	0.96	0.94	0.96	0.96	1.03	1.05	1.07
Missing Rate = 20%	0.98	0.93	0.91	0.95	0.95	0.94	1.04	1.02	1.02
Missing Rate 30%	0.98	0.83	0.80	0.95	0.95	0.92	1.04	1.03	1.03
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	1.00	1.00	1.00	1.01	1.01	1.01	1.02	1.02	1.02
Missing Rate = 10%	0.99	0.99	0.99	0.98	0.98	0.98	1.03	1.03	1.03
Missing Rate = 20%	0.95	0.95	0.95	0.97	0.97	0.97	1.01	1.01	1.01
Missing Rate = 30%	0.94	0.93	0.94	0.95	0.95	0.95	0.98	0.98	0.98

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C21**

*50-time point Models with Measure Reliability 0.8: Standard Error Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.99	1.00	0.98	1.04	1.04	1.00	0.97	0.98	0.99
Missing Rate = 10%	1.03	1.05	0.98	1.00	0.99	0.96	0.95	0.93	0.95
Missing Rate = 20%	1.00	0.97	0.95	1.01	0.98	0.96	1.03	1.02	0.98
Missing Rate = 30%	1.03	1.01	0.93	0.99	0.97	0.96	0.95	0.92	0.94
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.97	0.97	0.97	0.98	0.98	0.98	1.00	1.00	1.00
Missing Rate = 10%	0.92	0.92	0.92	0.96	0.96	0.96	1.01	1.01	1.01
Missing Rate = 20%	0.96	0.96	0.96	1.00	1.00	1.00	0.95	0.95	0.95
Missing Rate = 30%	0.90	0.90	0.90	0.99	0.99	0.99	1.01	1.01	1.00
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	1.01	1.02	0.99	0.99	0.99	0.99	0.97	0.99	0.99
Missing Rate = 10%	1.01	1.01	0.96	0.96	1.00	1.06	0.99	0.96	0.95
Missing Rate = 20%	1.01	0.97	0.95	1.03	1.02	0.97	1.03	0.97	0.92
Missing Rate = 30%	1.01	0.97	0.90	0.96	0.97	0.96	0.99	0.96	0.95
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.96	0.96	0.96	1.00	1.00	1.00	0.99	0.99	0.99
Missing Rate = 10%	0.97	0.97	0.97	1.01	1.01	1.01	1.01	1.01	1.01
Missing Rate = 20%	0.95	0.95	0.95	0.97	0.97	0.97	1.01	1.01	1.01
Missing Rate = 30%	1.02	1.02	1.02	1.02	1.02	1.02	0.97	0.97	0.97

*Note.* N = 200 replications per cell.

APPENDICES

**Table C22**

*3-time point Models with Measure Reliability 0.6: Standard Error Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.93	0.92	0.93	0.99	0.97	0.97	1.01	1.01	0.97
Missing Rate = 10%	1.01	0.98	0.86	0.98	0.98	0.96	0.99	1.00	1.01
Missing Rate = 20%	1.01	0.92	0.85	0.97	0.97	0.94	0.99	1.00	0.99
Missing Rate = 30%	1.00	0.95	0.92	0.97	0.94	0.89	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.98	0.98	0.98	0.99	0.99	0.99	0.97	0.96	0.97
Missing Rate = 10%	1.00	1.00	1.01	1.00	1.00	1.00	1.01	1.01	1.01
Missing Rate = 20%	0.99	1.01	1.02	0.99	0.99	0.99	0.98	0.98	0.98
Missing Rate = 30%	0.93	0.95	0.97	0.96	0.96	0.96	0.99	0.99	0.99
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.97	0.92	0.88	1.03	0.98	0.93	0.95	0.98	0.99
Missing Rate = 10%	0.99	0.67	0.58	0.98	0.97	0.93	1.00	1.01	1.02
Missing Rate = 20%	0.99	0.65	0.56	0.99	0.93	0.89	1.00	1.00	0.99
Missing Rate = 30%	1.00	0.81	0.78	1.00	0.86	0.80	1.00	0.97	0.97
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.96	0.96	0.96	0.96	0.96	0.96	1.02	1.02	1.02
Missing Rate = 10%	0.99	0.99	0.99	0.99	0.99	0.99	1.02	1.02	1.02
Missing Rate = 20%	0.98	0.98	0.98	0.98	0.98	0.98	1.03	1.03	1.03
Missing Rate = 30%	0.96	0.96	0.96	1.00	1.00	1.00	1.00	1.00	1.00

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C23**

*5-time point Models with Measure Reliability 0.6: Standard Error Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.97	0.98	0.97	1.01	0.99	0.99	1.01	1.00	1.03
Missing Rate = 10%	0.97	0.97	0.93	1.01	0.99	0.99	1.01	1.01	1.04
Missing Rate = 20%	0.98	0.93	0.88	1.01	0.98	0.97	1.01	0.99	0.99
Missing Rate = 30%	0.97	0.91	0.84	1.02	0.95	0.95	1.00	1.00	1.02
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.99	0.99	0.99	1.00	1.00	1.00	1.04	1.04	1.04
Missing Rate = 10%	0.98	0.98	0.98	1.01	1.01	1.01	1.04	1.04	1.04
Missing Rate = 20%	0.99	0.98	0.99	1.01	1.01	1.01	1.01	1.01	1.01
Missing Rate = 30%	0.97	0.98	0.98	1.00	1.00	1.00	1.03	1.03	1.03
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.97	0.96	0.98	0.94	0.98	1.00	1.04	1.04	1.05
Missing Rate = 10%	0.97	0.95	0.94	0.94	0.96	0.96	1.03	1.06	1.07
Missing Rate = 20%	0.97	0.87	0.84	0.95	0.94	0.91	1.04	1.01	1.02
Missing Rate 30%	0.98	0.71	0.69	0.95	0.92	0.90	1.04	1.03	1.03
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	1.00	1.00	1.00	1.01	1.01	1.01	1.02	1.02	1.02
Missing Rate = 10%	0.99	0.99	0.99	0.98	0.98	0.98	1.03	1.03	1.03
Missing Rate = 20%	0.94	0.94	0.94	0.97	0.97	0.97	1.02	1.02	1.02
Missing Rate = 30%	0.94	0.94	0.94	0.95	0.95	0.95	0.99	0.99	0.99

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C24**

*50-time point Models with Measure Reliability 0.6: Standard Error Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.99	1.00	0.97	1.04	1.03	0.99	0.98	0.98	0.99
Missing Rate = 10%	1.04	1.05	0.96	1.01	0.98	0.96	0.96	0.94	0.95
Missing Rate = 20%	0.99	0.97	0.93	1.01	0.97	0.96	1.03	1.02	0.98
Missing Rate = 30%	1.03	0.99	0.89	1.00	0.97	0.96	0.96	0.93	0.95
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.97	0.97	0.97	0.98	0.98	0.98	0.99	0.99	0.99
Missing Rate = 10%	0.93	0.93	0.93	0.97	0.97	0.97	1.01	1.01	1.01
Missing Rate = 20%	0.96	0.96	0.96	1.01	1.01	1.01	0.96	0.96	0.96
Missing Rate = 30%	0.90	0.90	0.90	1.00	1.00	1.00	1.01	1.01	1.01
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	1.01	1.02	0.98	0.99	0.99	0.99	0.98	0.99	1.00
Missing Rate = 10%	1.01	1.01	0.96	0.96	1.00	1.07	0.98	0.95	0.95
Missing Rate = 20%	1.02	0.96	0.92	1.03	1.02	0.97	1.04	0.96	0.93
Missing Rate 30%	1.02	0.91	0.84	0.96	0.97	0.96	0.98	0.95	0.95
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.96	0.96	0.96	0.99	0.99	0.99	0.98	0.98	0.98
Missing Rate = 10%	0.97	0.97	0.97	1.00	1.00	1.00	1.01	1.01	1.01
Missing Rate = 20%	0.94	0.94	0.94	0.97	0.97	0.97	1.01	1.01	1.01
Missing Rate = 30%	1.00	1.00	1.00	1.01	1.01	1.01	0.98	0.98	0.98

*Note.* N = 200 replications per cell.

APPENDICES

**Table C25**

*3-time point Models with Measure Reliability 0.4: Standard Error Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.93	0.92	0.91	0.99	0.97	0.96	1.01	1.00	0.97
Missing Rate = 10%	1.01	1.01	0.86	0.98	0.98	0.95	0.99	1.00	1.01
Missing Rate = 20%	1.01	1.00	0.93	0.97	0.96	0.91	0.99	1.00	0.99
Missing Rate = 30%	1.00	0.98	0.97	0.97	0.90	0.83	1.00	1.00	1.00
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.98	0.98	0.98	0.99	0.99	0.99	0.97	0.97	0.97
Missing Rate = 10%	1.00	1.01	1.01	1.00	1.00	1.00	1.01	1.01	1.01
Missing Rate = 20%	0.99	1.02	1.02	0.99	0.99	0.99	0.98	0.98	0.98
Missing Rate = 30%	0.93	0.98	0.98	0.96	0.96	0.96	0.99	0.99	0.99
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.97	0.90	0.83	1.03	0.97	0.91	0.95	0.98	0.99
Missing Rate = 10%	0.99	0.94	0.58	0.98	0.96	0.91	1.00	1.01	1.01
Missing Rate = 20%	0.99	0.69	0.71	0.99	0.91	0.85	1.00	1.00	0.99
Missing Rate = 30%	1.00	0.80	0.79	1.00	0.75	0.71	1.00	0.97	0.97
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.96	0.96	0.96	0.97	0.97	0.97	1.02	1.02	1.02
Missing Rate = 10%	0.99	0.99	1.00	0.99	0.99	0.99	1.02	1.02	1.02
Missing Rate = 20%	0.98	0.98	0.98	0.98	0.98	0.98	1.04	1.04	1.04
Missing Rate = 30%	0.96	0.96	0.96	1.00	1.00	1.00	1.00	1.00	1.00

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C26**

*5-time point Models with Measure Reliability 0.4: Standard Error Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.97	0.98	0.96	1.01	1.00	0.99	1.01	1.00	1.03
Missing Rate = 10%	0.97	0.97	0.90	1.00	0.99	0.99	1.01	1.01	1.04
Missing Rate = 20%	0.97	0.90	0.84	1.01	0.97	0.95	1.01	0.99	1.00
Missing Rate = 30%	0.97	0.95	0.91	1.02	0.94	0.93	1.00	1.01	1.03
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.99	0.99	0.99	1.00	0.99	1.00	1.03	1.03	1.03
Missing Rate = 10%	0.98	0.98	0.98	1.01	1.01	1.01	1.04	1.04	1.04
Missing Rate = 20%	0.99	0.99	0.99	1.01	1.01	1.01	1.01	1.01	1.01
Missing Rate = 30%	0.97	0.98	0.99	1.00	1.00	1.00	1.03	1.03	1.03
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	0.97	0.95	0.98	0.94	0.98	1.01	1.03	1.04	1.05
Missing Rate = 10%	0.97	0.93	0.89	0.94	0.96	0.96	1.03	1.06	1.07
Missing Rate = 20%	0.97	0.78	0.78	0.95	0.92	0.88	1.04	1.01	1.02
Missing Rate 30%	0.98	0.84	0.81	0.95	0.90	0.87	1.04	1.03	1.03
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.02	1.02
Missing Rate = 10%	0.99	0.99	0.99	0.98	0.98	0.98	1.03	1.03	1.03
Missing Rate = 20%	0.94	0.94	0.94	0.98	0.98	0.98	1.02	1.02	1.02
Missing Rate = 30%	0.94	0.94	0.94	0.95	0.95	0.95	0.99	0.99	0.99

*Note.* N = 1000 replications per cell.

APPENDICES

**Table C27**

*50-time point Models with Measure Reliability 0.4: Standard Error Bias Results on Selected Conditions*

Parameter	Sample size $n = 100$			Sample size $n = 250$			Sample size $n = 5000$		
	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD	Trad	T2C, +0.33 SD	T2C, +1.33 SD
<b>Mean Intercept/Tau</b>									
Missing Rate = 0%	0.99	0.99	0.97	1.03	1.02	0.99	0.98	0.99	0.99
Missing Rate = 10%	1.04	1.04	0.94	1.01	0.98	0.96	0.97	0.95	0.95
Missing Rate = 20%	0.99	0.95	0.91	1.00	0.96	0.97	1.03	1.02	0.98
Missing Rate = 30%	1.03	0.97	0.87	1.01	0.97	0.97	0.97	0.93	0.93
<b>Mean Growth Rate</b>									
Missing Rate = 0%	0.97	0.97	0.97	0.98	0.98	0.98	0.99	0.99	0.99
Missing Rate = 10%	0.93	0.93	0.93	0.97	0.97	0.97	1.01	1.01	1.01
Missing Rate = 20%	0.95	0.95	0.95	1.02	1.02	1.02	0.96	0.96	0.96
Missing Rate = 30%	0.90	0.90	0.90	1.00	1.00	1.00	1.01	1.01	1.01
<b>Predictor Effect on Intercept/Tau</b>									
Missing Rate = 0%	1.01	1.01	0.98	1.00	0.99	1.00	0.98	1.00	1.00
Missing Rate = 10%	1.02	1.01	0.96	0.96	1.01	1.08	0.97	0.95	0.95
Missing Rate = 20%	1.03	0.95	0.9	1.03	1.02	0.97	1.04	0.96	0.94
Missing Rate = 30%	1.02	0.86	0.80	0.95	0.97	0.97	0.97	0.94	0.94
<b>Predictor Effect on Growth Rate</b>									
Missing Rate = 0%	0.96	0.96	0.96	0.99	0.99	0.99	0.98	0.98	0.98
Missing Rate = 10%	0.96	0.96	0.96	0.99	0.99	0.99	1.01	1.01	1.01
Missing Rate = 20%	0.93	0.93	0.93	0.97	0.97	0.97	1.01	1.01	1.01
Missing Rate = 30%	0.98	0.98	0.98	1.00	1.00	1.00	0.98	0.98	0.98

*Note.* N = 200 replications per cell.