Robust Estimation of Factor Models in Finance

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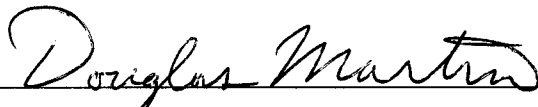
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Abstract

Robust Estimation of Factor Models in Finance

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Standard asset-pricing models entail expressions for expected returns in terms of coefficients relative to risk factors. Methods to estimate premiums of risk factors have, at its core, a single or multiple linear regression models. *Ordinary least squares* (OLS) estimation is the common choice. However, it is well established that financial returns are heavy-tailed, skewed, and vary over time. This dissertation shows that small fractions of outlying observations bias OLS estimates and inflate its variability. Outlying observations include months, firms, time periods, and gross errors. Some subset of outlying firms may have some economic value, which leads to a great fear of simply rejecting them. This dissertation uses exploratory data analysis and the robust MM-estimator to separate influential observations from the bulk of the data and to estimate risk premiums on both groups. The key results are: OLS alphas from the single-factor market model are often over-estimated due to outliers and positive asymmetry of the returns distribution. OLS betas are highly sensitive to outliers. Robust alphas and betas are superior in predicting future returns and risk, and are insensitive to the choice of returns type and returns that are dirty, e.g. not split or dividend adjusted.

The risk premium as found by Fama & French (1992) to be flat for beta and negative for size is a small size firm and seasonality effect. The risk premiums for beta (size) are positive (negative) only in January and for a tiny number of influential small size firms. Once adjusted the beta (size) risk premiums become negative (positive), confirming partial results of Knez & Ready (1997). The seasonality effect appears to be small

compared to influential firm effect, only since seasonal effects average out. The January effect is significant and spills over into February and March; in addition, size shows seasonal and book-to-market quarterly variability. Overall the MM-estimator is shown to be not only an easy-to-use alternative to the OLS estimator, unbiased towards small fractions of unusual observations, but also a tool that can be used to identify and analyze influential observations and to find trading strategies.

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Preface

This dissertation contributes to the empirical field of financial asset-pricing studies. Most theoretical asset-pricing models are based on either the principles of *no arbitrage* (market forces align prices to eliminate arbitrage opportunities) or *financial equilibrium* (mean-variance optimization), and are tested on historical financial data. A large body of empirical studies has contributed with well-refined tests, confirming, but also often contradicting each other and the theoretical models. Many of the tests engage *ordinary least squares* (OLS) regression at its core that assumes a Gaussian distribution of the underlying data. Unfortunately, classical OLS regression is highly sensitive to deviation from normality, and particularly towards extreme data points (outliers). This dissertation draws on well developed robust regression techniques with amiable statistical properties capable to cope with the predominant non-Gaussian nature of the data. Robust regression techniques are competent not only to fit models that are less sensitive to small numbers of abnormal observations, but also to identify these outliers reliably, analyze them, and possibly provide an opportunity for new trading strategies.

Acknowledgements

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1.   Introduction, Models, Methods

1.1. Introduction

The principle of financial equilibrium (mean-variance efficiency) and the principle of no arbitrage (market forces align prices to eliminate arbitrage opportunities) are the foundation of most theoretical asset pricing models. Based on the first principle, Sharpe (1964) and Linter (1965) developed the *capital asset pricing model* (CAPM). The CAPM relates the expected returns of an asset to the expected returns of the market portfolio. The relationship can be expressed by a number called beta—the asset's sensitivity to non-diversifiable risk (also known as systematic risk or market risk). The beta has come to play a fundamental role in estimating the cost of equity and in measuring the risk of an asset (Block, 1999; Graham & Harvey, 2001). Based on the second principle, Ross (1976) developed the *arbitrage pricing theory* (APT). The APT differs from the CAPM in that it is less restrictive in its assumptions. It allows for an explanatory (as opposed to statistical) model of asset returns by using the cross-sectional empirical relation of expected asset returns to the asset's attributes or other factors. Each investor can hold a unique portfolio with its own particular array of beta factors, as opposed to the identical market portfolio. The CAPM can be considered a special case of the APT in that it represents a single-factor model of the asset price, where a single beta factor is the exposure to changes in the value of the market portfolio. The CAPM can be viewed as a demand model, as it results from a maximization problem of each investor's utility function and from the resulting market equilibrium. In contrast, the APT can be seen as a supply model with its beta factors reflecting the sensitivity of the underlying asset to economic factors. Today's research is focused as much on developing and justifying theoretical models as on empirical approaches of accepting or rejecting it.

Following Ferson (2003), both the CAPM and the APT entail expressions for expected returns in terms of coefficients relative to one or more beta factors. These factor models lead to the following form for the expected return of an asset R_n at time $t+1$

$$E_t\left[R_{n,t+1}\right] = \gamma_{0,t} + \sum_{j=1}^{K} b_{nj,t}\gamma_{j,t}, \quad \forall n \tag{1.1}$$

Here, $b_{nj,t}$ are the beta factors or exposures of asset n to risk factors $f_{j,t}$. The $\gamma_{j,t}$ are the risk factor premiums (dependent of the specific asset) presenting the incremental return per unit of $b_{nj,t}$. The $\gamma_{j,t}$ coefficients are usually estimated by time series or cross-sectional regression of the asset returns on the risk factors $f_{j,t}$. If an asset R_n is uncorrelated with any of the K factors, then $\gamma_{0,t}$ would be its return. The coefficient $\gamma_{0,t}$ is also called the *zero-beta* rate or the return of a risk free asset.

Economical content in (1.1) is absent until empirical factors f_j are selected. Connor (1995) divided empirical factors into three main types: statistical, macroeconomic, and fundamental factors (all three factor models are statistical factor models in the statistical terminology).

Statistical factor models use statistical factor analytics or principal components methods. Statistical factor models are motivated by the APT, where the correct factors are the ones that capture all the risk, leaving only nonsystematic risk in the residuals (Connor & Korajczyck, 1988; Roll & Ross, 1980). Even though Burmeister & McElroy (1988) worked on interpretation relative to more intuitive economic variables, statistical factor models provide little economic intuition.

Macroeconomic factor models are used to estimate the risk premium of an asset's factor beta using time series regression. For $K=1$, an asset return R_t and benchmark return $R_{B,t}$, both in excess of the risk-free rate, $b_t = R_{B,t}$, $\alpha_t = \gamma_{0,t}$, and $\beta_t = \gamma_{1,t}$ equation (1.1) reduces to

$$E[R_t] = \alpha_t + \beta_t \cdot R_{B,t}, \quad \alpha, \beta \in \Re \qquad (1.2)$$

The estimates $\hat{\alpha}_t$ and $\hat{\beta}_t$ are obtained from an *ordinary least squares* (OLS) time series regression of excess returns of asset R_t on a benchmark $R_{B,t}$

$$R_t = \alpha + \beta \cdot R_{B,t} + \varepsilon_t, \quad t = 1,\ldots,T, \quad \alpha, \beta \in \Re \qquad (1.3)$$

The error term ε_t is assumed to be normally distributed, centered on zero, and uncorrelated with $R_{B,t}$. Under the CAPM $R_B = R_M$ (R_M the market portfolio) and $\alpha = 0$. Equation (1.3) is also called a *single-factor* model.

Fundamental factor models often estimate the risk factor premiums of the risk factors in a two-step process. In the first step, the risk factor premiums $\gamma_j(t)$ are estimated for each time period t in a cross-sectional regression of the N assets returns $R_n(t)$ on risk factors $f_{j,t}$.

$$R_i(t) = \gamma_0(t) + \sum_{j=1}^{K} \gamma_{ij}(t) f_{ij}(t) + \varepsilon_i(t), \, t = 1,\ldots,T, \, i = 1,\ldots,N, \, \gamma_0, \gamma_{ij} \in \Re \qquad (1.4)$$

In the second step, the resulting time-series of risk factor premiums is evaluated. Equation (1.4) is also called a *multi-factor* model.

Both macroeconomic and fundamental factor models use OLS regression in the estimation procedure. However, the OLS estimate is the *best linear unbiased estimate* (blue) with a convenient distribution theory for inference only when the errors are Gaussian (normally distributed). Unfortunately, this is an idealized assumption that often fails in practice. There is considerable evidence in the literature that returns are not normally distributed, but rather leptokurtotic (following some heavy-tailed distribution that generates outliers) and are positively skewed (Cable & Holland, 2000; Chou, 2002;

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Chunhachinda, Dandapani, Hamid, & Prakash, 1997; Peiró, 1999; Singleton & Windender, 1986).

Outlying returns cannot only substantially bias the values of the factor beta estimates (Hampel, Ronchetti, Rousseeuw, & Stahel, 1986; Huber, 1981; Judge, Hill, Griffiths, Lütkepohl, & Lee, 1988; Rousseeuw & Leroy, 1987), but also inflate the estimation error. The estimation error can be expressed as inflated variance or mean-squared-error and has been receiving increased attention in financial literature (Gomes, 2002; Lewellen & Shanken, 1997; Stambaugh & Pástor, 1997). However, little noticed is the fact that, while the variance of the regression estimate goes to zero like 1/n, the bias due to non-conforming outliers persists for arbitrarily large sample sizes. Therefore, it is also important to be concerned about bias caused by outliers.

Among all the robust regression methods that have been used in the empirical asset pricing research, none were obtained as the result of solving an attractive statistical optimization problem (see Section 2.1 and Section 4.1 for details). This was true until only recently, when Martin and Simin (2003) used a special maximum-likelihood type estimator (MM-estimator) to analyze stock betas. The MM-estimator has a well-developed statistical optimality theory: It minimizes the bias due to non-conforming outliers while achieving a user-specified high efficiency at the Gaussian model. The bias robust approach to regression was initiated by Martin, Yohai, & Zamar (1989), and the Gaussian efficiency-constrained bias robust solutions were obtained by Yohai & Zamar (1997) and Svarc, Yohai, & Zamar (2002). As a convenient by-product, the resulting estimator obtains high efficiencies at non-Gaussian outlier generating distributions. The optimal MM-estimator also has the property of being a consistent estimate of factor *betas* when the error distribution in (1.1) is asymmetric, which may come as a surprise to many cynics.

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Regardless of the successful developments and availability of sophisticated robust estimates in statistics, there are remarkably few published robustness papers in finance. Zaman, Rousseeuw, & Orhan (2001) found only 14 papers in a search on ECONLIT for the term "robust regression" and attribute this to the following five factors: existing belief that enough data cannot be biased; outliers can simply be detected by eye, usual residuals, or sensitivity analysis; robust regression techniques with differing strengths and weaknesses provide little guidance of proper usage; unfamiliarity with interpretation of robust results; unawareness of gains available from robust analysis of real data sets.

In addition, there is another persistent belief and controversy: robust methods are dangerous because they do not differentiate between *good* and *bad* outliers. The fear seems to be that valid data points are rejected and thereby trading opportunities missed and risk misstated. This fear even prevents from recognizing the possibility that outliers may be gross errors or isolated events that can bias estimates, inflate estimation errors, and cause trading strategies to fail. The conviction is not only caused by the failure of classical methods to dependability detect outliers (Rousseeuw & Leroy, 1987; Rousseeuw & Van Zomeren, 1990), but also by the lack of available robust alternatives.

A solution needs to be easy-to-use and able to unfailingly detect and characterize outliers so they can be classified and then rejected, included, or analyzed separately.

The MM-estimator offers just that: it reliably detects leverage and influential points, higher dimensional outliers, and even clusters of outliers. Its easy-to-use implementation in S-PLUS (2001) allows for the following two-pass procedure. The first pass flags data points identified as outliers. In the second pass, flagged data points are analyzed and either rejected, kept with the rest of the data, or treated separately. Outliers recommended for rejection would be dirty data (not adjusted for stock splits or with recording errors), or economically unmotivated (fortuitous two-dimensional stock and market return combinations that occurred independently and are not likely to recur). Outliers to be kept are data points that conform to the regression model. Outliers to be treated separately may show associations with fundamental factors and can be used to illustrate market inefficiencies and exploit arbitrage opportunities.

The remainder of Chapter 1 recalls the robust MM-estimator with its properties, and shows how it can be used to obtain robust location estimates, standard errors, and t-tests. It also introduces graphics used in Chapter 2 through Chapter 4 that may be unfamiliar to the reader. Chapter 2 is dedicated to the single-factor market model and its estimation of stock alphas and betas. The impact of the robust regression estimator is demonstrated in examples and a performance measure is introduced. Alphas and betas are then compared using the OLS and the robust estimator on discrete returns, log returns, and on returns that are not adjusted for stock splits and dividend distribution. The chapter also compares the property of the alphas and betas to predict future risk and return. Chapter 3 is based on the famous paper of Fama & MacBeth (1973) and the therein presented and widely used two-pass cross-sectional regression technique (FM). It replicates and extends their results to recent time periods. It further compares their results to a rigorous robust version of FM using the MM-estimator (RFM), and suggests various improvements. Chapter 4 extends the work of Fama & French (1992) and the robust replication approach of Knez & Ready (1997) and (Chou, Chou, & Wang, 2004) by replication and extension to recent time periods. It justifies the choice of the MM-estimator over the LTS-estimator and identifies influential observations, such as influential months, firms, and time periods. The chapter then compares the FM and RFM approach for all firms, and separately, for firms rejected.

Throughout the dissertation, acronyms are used to shorten the notation and to avoid distractions from the actual results. A list with all acronyms used can be found in the glossary.

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1.2. Robustness

"... just which robust/resistant methods you use is not important – what is important is that you use some. It is perfectly proper to use both classical and robust/resistant methods routinely, and only worry when they differ enough to matter. But when they differ, you should think hard."

J. W. Tukey (1979)

Classical *maximum likelihood estimates* (MLE) are based on idealized Gaussian or non-Gaussian distributions like exponential, Weibull, and gamma. Even though, in most of the applications, the actual data generation process follows the idealized distribution only to a certain degree, the MLEs, derived from the idealized distributions, are usually able to sufficiently summarize the data generation process.

However, often assumed-away is the well established fact that even high quality data sets can contain single data points, as well as clusters of data points that are well separated from the bulk of the data. These single data points are called outliers. Clusters of outliers can cause a distribution to be skewed or hint to multiple data generating processes. Outliers can be valid data points that are an important part of the data generating process. However, outliers can also be erroneously included data points caused by data entry errors, wrong pre-processing (such as not adjusting for stock splits or dividend distributions), or inopportune combinations in higher dimensions. Regardless if outliers are good or bad, it is important to understand that even a few can highly bias the classical MLEs and substantially increase the estimation error.

Robust statistics (Hampel, Ronchetti, Rousseeuw, & Stahel, 1986; Huber, 1981; Rousseeuw & Leroy, 1987) developed tools that help to detect and to analyze outliers, and to provide robust alternatives to classical computation of location, scale, regression, covariance, and even time series models.

This dissertation frequently draws on tools developed in robust statistics to estimate location, scale, and regression coefficients. The basic robustness concepts used are as follows.

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1.2.1. Efficiency

The efficiency of a robust estimator is a performance measure when no outliers are present. A desired property of an estimator is to have a small variability when the data is normally distributed. The classical MLE has the smallest variance, i.e., the highest efficiency of all estimators at the Gaussian model. The *relative efficiency* (RE) of the MLE to another estimator at an underlying Gaussian distribution F_{GAUSS}, is defined as

$$RE\left(\hat{\Phi}^{MLE}, \hat{\Phi}^{ROBUST}\right) = \frac{\mathrm{var}\left(\hat{\Phi}^{MLE}\right)}{\mathrm{var}\left(\hat{\Phi}^{ROBUST}\right)_{F_{GAUSS}}} \tag{1.2.1}$$

E.g., RE of the sample median is 64%, i.e., the sample mean needs roughly only 64% of the observations that the sample median needs for the same precision. The *asymptotic relative efficiency* is the limit for infinite sample size. Going forward, the efficiency of the robust estimator $\hat{\Phi}^{ROBUST}$ will be stated without mentioning $\hat{\Phi}^{MLE}$ and the underlying Gaussian distribution F_{GAUSS}.

1.2.2. Breakdown Point

The breakdown point is defined as the largest fraction of data that can be moved to infinity without completely distorting the estimate. For a regression problem this can be formally expressed as follows. Let $M = \left\{\left(x_{11}, ..., x_{1p}, y_1\right), ..., \left(x_{N1}, ..., x_{Np}, y_N\right)\right\}$ be a sample of N data points and $T : M \mapsto \Phi$ be a regression estimator. Consider all samples $\tilde{M}$ where m subsets in M are replaced by arbitrary values. Define the maximum bias caused by such m as

$$\mathrm{bias}\left(m; T, M\right) = \sup_{\tilde{M}}\left\|T\left(\tilde{M}\right) - T\left(M\right)\right\| \tag{1.2.2}$$

Then the breakdown point of T at the sample M is defined as

$$\varepsilon_n\left(T,M\right) = \min\left\{\frac{m}{N}; \text{bias}\left(m;T,M\right) \text{is infinite}\right\} \qquad (1.2.3)$$

The breakdown point for the sample mean is 0% and for the sample median is 50%. A breakpoint higher than 50% cannot be achieved.

1.3. MM-Estimator

This section begins to introduce the robust MM-regression estimator and discusses its properties. It then shows how to use the MM-estimator to compute location estimates, standard errors, and to do robust inference. It closes with a brief discussion on the efficiency of the MM-estimator.

1.3.1. Robust Linear Regression

The robust estimator of beta proposed in this paper is a special regression maximum likelihood (M) estimator based on a bounded loss function that leads to very desirable robustness properties. Because a bounded loss function results in a non-convex optimization problem, the M-estimate needs to be computed in a careful way in order to obtain a good local minimum. First, the estimator is defined and its efficacy illustrated with some simple examples. Then its attractive robustness properties are discussed and the computational method is described.

1.3.1.1. M-Estimator

Any K-factor model can be written in the general linear model form

$$
\begin{aligned}
y_i &= \phi_{0,t} + \sum_{j=1}^{K} \phi_{j,t} \cdot x_{ji} + \varepsilon_i, \quad i = 1,\dots,N \\
&= X_i^T \cdot \Phi + \varepsilon_i
\end{aligned}
\qquad (1.3.1)
$$

For example, equation (1.3) is obtained for $K=1$, $\Phi^T = \left(\alpha,\beta\right)$ and $X_i^T = \left(1, R_{B,i}\right)$. The class of regression M-estimates of Φ, introduced by Huber (1964), is defined by

$$\hat{\Phi} = arg\, min_{\Phi} \sum_{i=1}^{N} \rho\left(\frac{y_i - X_i^T\Phi}{\hat{s}}\right) \qquad (1.3.2)$$

where ρ is a symmetric robust loss function and $\hat{s}$ is a robust scale estimate for the residuals. Dividing the residuals by the scale estimate $\hat{s}$ makes the estimator $\hat{\Phi}$ invariant with respect to the scale of the error ε_i. Note that OLS and *least absolute deviations* (LAD) estimates are special M-estimates, corresponding to $\rho_{OLS}(r) = r^2$ and $\rho_{LAD}(r) = |r|$, respectively.

Although $\hat{\Phi}$ is directly estimated by minimizing (1.3.2), it is more intuitive to think of the first order condition for optimizing (1.3.2) with respect to Φ, namely

$$\sum_{i=1}^{N} X_i \psi \left(\frac{y_i - X_i^T \hat{\Phi}}{\hat{s}} \right) = 0 \qquad (1.3.3)$$

where $\psi = \rho'$.

The favorite choice of Huber (1964), ρ_{HUB}, was an unbounded convex function compromising between the OLD and LAD estimators, behaving like OLS for $|r| \le c$ and behaving like LAD for $|r| > c$, for an appropriate constant c. The convexity of ρ_{HUB} is quite attractive from a computational point of view. Unfortunately, when an unbounded loss function is used and the predictor variables other than the intercept are random, as is the present case, arbitrarily large bias of the parameter estimates can occur under normal mixture models. This was established by Martin et al. (1989), who initiated work on the use of regression M-estimators with bounded loss functions. That led to obtain bias robustness (Yohai, Stahel, & Zamar, 1991) and finally to the analytic expressions for the ρ and ψ functions

$$\rho(r;c) = \begin{cases} 3.25 \cdot c^2 & \\ c^2 \cdot \left[1.792 - 0.972 \cdot \left(\frac{r}{c} \right)^2 + 0.423 \cdot \left(\frac{r}{c} \right)^4 - 0.052 \cdot \left(\frac{r}{c} \right)^6 - 0.002 \cdot \left(\frac{r}{c} \right)^8 \right] & \\ 0.5 \cdot r^2 & \end{cases}$$

$$|r/c| > 3$$
$$2 < |r/c| \le 3$$
$$|r/c| \le 2$$

$$(1.3.4)$$

and

$$\psi(r;c) = \begin{cases} 0 \\ c \cdot \left[-1.944 \cdot \left(\frac{r}{c}\right) + 1.728 \cdot \left(\frac{r}{c}\right)^3 - 0.312 \cdot \left(\frac{r}{c}\right)^5 + 0.016 \cdot \left(\frac{r}{c}\right)^7 \right] \\ r \end{cases}$$

$$\begin{aligned} &|r/c| > 3 \\ &2 < |r/c| \le 3 \\ &|r/c| \le 2 \end{aligned}$$

(1.3.5)

The functions ρ and ψ are displayed in Figure 1 for various efficiencies.
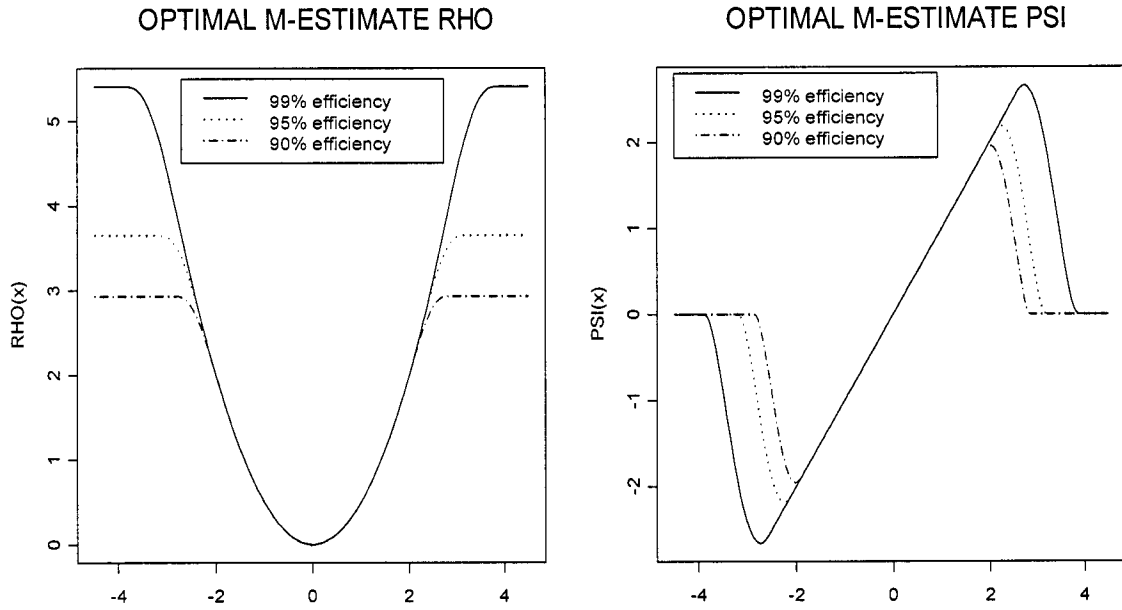
Figure 1. Examples of the Optimal ρ and ψ-Functions.
Both ρ and ψ-functions are plotted for three cutoff values c = 0.95, 1.06, and 1.29, corresponding to Gaussian efficiencies of 90%, 95%, and 99%.

The function ρ is a bounded loss function with piece-wise polynomial shapes and constant values outside a cutoff region $(-c, c)$, shown for $c = 0.95$, 1.06, and 1.29 in the right-hand panel of Figure 1. The corresponding ψ-functions are plotted in the right-hand panel of Figure 1. The three values for c yield efficiencies of 90%, 95% and 99% when the distribution of the stock returns, conditioned on market returns, are Gaussian. Note, the corresponding efficiency on the standard deviation scale is 99.5%, i.e., the standard deviations of the robust estimate is about 0.5% greater than that of the OLS estimate when the returns are Gaussian. As c increases, the Gaussian model efficiency of the M-

estimator increases, tending to 100% as c tends to infinity. In this case, the estimator becomes the fully-efficient least squares estimator $\rho_{OLS}(r) = r^2$.

The ψ functions in Figure 1 are smooth approximations to *hard- rejection* functions ψ_{HR}, with slope one on the interval $(-c, c)$ and zero outside that interval. A hard rejection function ψ_{HR} makes the untenable assumption that all observations in the interval $(-c, c)$ are perfectly good, while those outside this interval are totally bad. The discontinuity of ψ_{HR} also causes problems during optimization. The smooth transition of the ψ from its minimum and maximum toward zero avoids these undesirable features of ψ_{HR}. The region of smooth transition of ψ in Figure 1 occurs in the flanks of the distribution, where it is most difficult to decide whether an observation is good or bad.

1.3.1.2.   MM-Estimator: High Breakdown Point and High Efficiency

In order to obtain both a high breakdown point and high efficiency at the Gaussian model, a special form of robust M-estimator was introduced by Yohai (1987), called the MM-estimator. Yohai (1987) proposed the computation of the MM-estimator in three steps.

Step 1 computes an initial estimate $\hat{\Phi}_0$, which has a breakdown point of 50% but

typically an efficiency of less than 29%. The highly robust initial estimate $\hat{\Phi}_0$ is key to obtaining a good local minimum to the estimation problem (1.3.2) when using a bounded, non-convex loss function such as ρ. To compute $\hat{\Phi}_0$, Rousseeuw & Yohai (1984) proposed the S-estimate approach. The S-estimate has as its foundation an M-estimate $\hat{s}$ of an unknown scale parameter s for observations $y_1, ..., y_N$ (assumed to be robustly centered). Consider

$$\frac{1}{N-K}\sum_{i=1}^{N}\rho\left(\frac{y_i - X_i^T\Phi}{\hat{s}(\Phi)}\right) = 0.5 \qquad (1.3.6)$$

where each value of Φ corresponds to a robust scale estimate $\hat{s}(\Phi)$. The regression S-estimate is the value $\hat{\Phi}_0$ that minimizes $\hat{s}(\Phi)$, namely

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$$\hat{\Phi}_0 = arg\,min_\Phi\,\hat{s}(\Phi) \tag{1.3.7}$$

Step 2 selects the minimum value of $\hat{s}(\Phi)$, corresponding to $\hat{\Phi}_0$ in (1.3.7) as the initial

robust scale estimate $\hat{s}_0$.

Step 3 obtains the final M-estimate $\hat{\Phi}$ as the nearest local minimum of (1.3.2) to $\hat{\Phi}_0$,

using the high-efficiency and bias-robust function ρ and the robust scale estimate

$\hat{s}_0$.

Yohai (1987) showed that $\hat{\Phi}$ not only inherits the high breakdown point property

from $\hat{\Phi}_0$, but also achieves high efficiency when the data is normally distributed. The

overall computational strategy was proposed by Yohai et al. (1991) and implemented in

S-Plus (2001).

1.3.1.3.   Consistency and Asymptotic Normality.

Yohai (1987) showed that $\hat{\Phi}$, as computed in Section 1.3.1.2, is consistent and

asymptotic normal with asymptotic covariance matrix

$$C_{\hat{\Phi}} = \frac{1}{N}(X^T X)^{-1} \cdot v \tag{1.3.8}$$

where X is the (N, K)-matrix of independent variables, with the scalar v given by

$$v = \frac{s^2 \cdot E\left[\psi^2(\varepsilon/s)\right]}{E^2\left[\psi'(\varepsilon/s)\right]} \tag{1.3.9}$$

Here, ε is the error term in (1.3.1) and s the asymptotic value of the robust scale estimate

(Huber, 1981; Yohai, 1987)

Yohai (1987) established consistency of $\hat{\Phi}$ in the general linear model under the

often used assumption that the distribution of the errors is symmetric. However, this

assumption is not necessary. It can be shown that when the error term has an asymmetric

distribution and the linear regression model has an intercept, as is the case in the single

factor market model, the estimates of the slope coefficients (all coefficients except the

intercept) are consistent while the intercept will typically have some degree of asymptotic

bias.

Set $K = 1$ in model (1.3.1) and assume asymmetrically distributed errors ε. Then the conditions for a consistent unbiased solution are violated (Huber, 1981) by

$$E\left[\psi_c\left(\frac{R_t - \alpha - \beta \cdot R_{B,t}}{s}\right)\right] \neq 0 \tag{1.3.10}$$

$$E\left[R_{B,t} \cdot \psi_c\left(\frac{R_t - \alpha - \beta \cdot R_{B,t}}{s}\right)\right] \neq 0 \tag{1.3.11}$$

However, for a monotone or re-descending ψ_c with unique root and support on all real numbers, there exists a δ_0 such that (with $\varepsilon_t = R_t - \alpha - \beta \cdot R_{B,t}$)

$$E\left[\psi_c\left(\frac{\varepsilon_t - \delta_0}{s}\right)\right] = 0 \tag{1.3.12}$$

One can show that the existence of δ_0 is trivial for monotone ψ (Maronna, Martin, & Yohai, 2005). For the re-descending case see Lehman (1994) or Reeds (1985). From (1.3.12), with $\alpha^* = \alpha + \delta_0$ and $\varepsilon_t^* = \varepsilon_t - \delta_0$, it follows that

$$E\left[\psi_c\left(\frac{R_t - \alpha^* - R_{B,t} \cdot \beta}{s}\right)\right] = 0 \tag{1.3.13}$$

Further by conditioning on $R_{B,t}$, it follows directly from (1.3.12) that

$$E\left[R_{B,t} \cdot \psi_c\left(\frac{R_t - \alpha^* - R_{B,t} \cdot \beta}{s}\right)\right] = E\left[R_{B,t} \cdot \underbrace{E\left[\psi_c\left(\frac{\varepsilon_t - \delta_0}{s}\right)\right]}_{=0} \bigg| R_{B,t}\right] = 0 \tag{1.3.14}$$

This result shows that the slope estimates are consistent even when the errors have an asymmetric distribution, while the intercept estimate (alpha) can be biased. The size of the bias δ_0 is yet to be estimated and needs further research.

1.3.1.4. *Efficiency Constrained Bias Robustness*

Martin et al. (1989) obtained solutions to the problem of minimizing the maximum bias of linear model regression estimates under contaminated distributions. Within the class of M-estimator, they showed that min-max bias robust estimates minimize quantiles of the absolute residuals. Here, the absolute residuals are well-approximated by the *least*

median squares (LMS) estimate Rousseeuw (1984). Unfortunately, the LMS does not converge at the usual rate and has zero asymptotic efficiency when the data is normally distributed. This shows that it is not enough to minimize maximum bias without any other constraints.

The natural constraint to impose is the achievement of high efficiency when the data is normally distributed, thereby obtaining efficiency-constrained bias robust estimates. The first such estimate was obtained by Martin & Zamar (1993) for location estimates, but the result was not of great practical importance. Then, Yohai & Zamar (1997) obtained an efficiency-constrained robust regression estimate that minimizes the maximum bias locally for small fractions of outliers in a mixture model. This is also the estimator used and implemented in the functions ρ and ψ of Figure 1. Subsequently, Svarc, Yohai, & Zamar (2002) showed that the min-max bias property holds globally subject only to the choice of desired efficiency. For the purpose of this paper, the local property is sufficient as only a very small fraction of outliers are typically encountered.

1.3.2. Robust Location Estimate

Robust location estimates with robust standard errors (Section 1.3.3) can conveniently be obtained using the regression MM-estimator by setting $K = 0$ in equation (1.3.1). For the MM-regression, as implemented in S-Plus (2001), this is equivalent to fitting just the intercept. This will be the choice over using the median or other robust location estimates.

1.3.3. Robust Standard Errors and Covariance

Robust standard errors and t-statistics for the robust location and regression estimates can easily be derived from the asymptotic covariance matrix (1.3.8) as proposed by Yohai et al. (1991). The idea is to estimate the covariance of the estimated coefficient $C_{\hat{\phi}}$ by

$$\hat{C}_{\hat{\phi}} = \frac{1}{N} \left(\tilde{X}^T \tilde{X} \right)^{-1} \cdot \hat{v} \tag{1.3.15}$$

where v in (1.3.9) is replaced by its natural estimate $\hat{v}$ based on sample averages and X is replaced with a weighted version $\tilde{X}$ to down-weight the influence of outliers in the

independent variable. This can be done by computing the natural estimate of $\hat{v}$ in (1.3.15) as

$$\hat{v} = \frac{\hat{s}_0^2 \sum_{i=1}^{N} \psi^2\left(\tilde{r}_i\right)}{\left(\sum_{i=1}^{N} \psi'\left(\tilde{r}_i\right)\right)^2} \qquad (1.3.16)$$

where $\tilde{r}_i = \left(y_i - x_i^T \cdot \hat{\Phi}_0\right)\big/\hat{s}_0$.

Returns with very large residuals should be rejected in the estimation procedure by replacing the corresponding rows of X with vectors shrank to zero. This can be achieved by the following weighting of X in the estimation of $\hat{C}_{\hat{\Phi}}$. Define

$$\tilde{X} = \frac{1}{\sqrt{\overline{w}}} diag\left(\sqrt{w_i}\right) \cdot X, \quad i = 1, \cdots, N \qquad (1.3.17)$$

with $w_i = \psi(\tilde{r}_i)/\tilde{r}_i$ and $\overline{w} = \frac{1}{N} \sum_{i=1}^{N} w_i$. Use $\tilde{X}$ to compute

$$\hat{C}_{\hat{\Phi}} = \frac{1}{N}\left(\tilde{X}^T \tilde{X}\right)^{-1} \cdot \hat{v} \qquad (1.3.18)$$

with $\hat{v}$ as in (1.3.16).

This way, a consistent, non-parametric robust covariance matrix estimate for the estimated regression coefficients is obtained when data is Gaussian. The square roots of the diagonal elements of this matrix provide robust standard errors of the coefficient estimates.

1.3.4. Robust Inference

T-statistics lack robustness of power in the presence of outliers. To see this, let X be a sample of size N. Let $\overline{x}$ be the sample mean with sample variance s^2 and standard errors of $\overline{x}$ $s.e.(\overline{x}) = s/\sqrt{N}$. A single outlier can not only inflate $\overline{x}$ but also $s.e.(\overline{x})$, thus widening the confidence interval

$$\left[\overline{x} - t_{N-1, \alpha/2} \cdot s.e.(\overline{x}), \overline{x} + t_{N-1, \alpha/2} \cdot s.e.(\overline{x})\right]$$

While the confidence interval inflates, the coverage of the true value remains roughly constant, though at the cost of lower precision. For that reason, the t-test is robust with

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respect to a type I error but lacks power under the alternative (Adrover et al. (2004) proposed a method on how to achieve globally robust inference for the location estimate).

Robust t-tests are easily obtained by replacing the classical estimates of the mean $\bar{x}$ and the standard error $s.e.(\bar{x})$ with its robust counterpart, i.e., replacing $\bar{x}$ by the location MM-estimate (Section 1.3.2) and $s.e.(\bar{x})$ by the robust standard errors taken as the square root of the diagonal matrix $\hat{C}_{\hat{\phi}}$ (Section 1.3.3).

1.3.5. Model Selection

The goal of model selection is to optimize the trade-off between fitting error and complexity (Weisberg, 1985). Among popular methods this dissertation chose the *Akaike Information Criterion* (AIC) and the robust AIC (RAIC) to perform backward stepwise model selection to obtain OLS and robust cross-sectional regressions, respectively, in Section 4.7.3. The RAIC was proposed by Ronchetti (1985) and Yohai (1997) and implemented in S-Plus (2001).

1.3.6. Choice of Efficiency

Higher efficiency keeps the variance of the MM-estimator low compared to the OLS estimator when no outliers are present. However, it offers less protection against bias when outliers are present. Martin & Simin (2003) used an efficiency of 85%, which one might argue leads to rejection of too many data values. It came as a pleasant surprise, that efficiency of 99% provided considerable bias protection, while giving up almost nothing to the OLS estimator for normally distributed data. As shown in Chapters 2-4, even small a percentage of outliers will result in significant differences between the OLS and the proposed MM-estimates. Further, the number of outliers detected at 99% efficiency is in line with other research (Knez & Ready, 1997). Therefore, for the remainder of this paper, the efficiency is set 99%.

1.4. Graphic Displays

This section explains the graphing techniques used in the result sections. All graphic displays used are implemented in S-PLUS (2001).

1.4.1. Quantile-Quantile Plot

A quantile-quantile plot, or qqplot, plots the ordered set of quantiles from the empirical distribution against an ordered set of quantiles of the hypothesized distribution. If the scattered points cluster along a straight line, the data set likely has the hypothesized distribution. QQplots are also useful to compare two empirical distributions.

1.4.2. Box Plots

Boxplot graphics show the center, spread of a distribution, and unusually deviant data points. The box is centered at the mean of the data and the filled dot in the interior of the box is located at the median of the data. The width of the box is the interquartile distance, which is the difference between the third and first quartiles of the data. The whiskers (the lines extending from the left to right of the box) enclose about 99.3% of all data when the data has a Gaussian distribution. Data points beyond the whiskers are outliers and marked individually.

1.4.3. Time Series Plots

The time series plots in this dissertation allow showing different number of panels, with either equal scale or a scale that fits best the range of the time series. The dashed center line is set at zero. The two dashed lines above and below zero are set at plus and minus two robust standard deviations of the time series. The robust standard deviations are estimated using the *median absolute deviation about the median* (MAD).

1.4.4. Trellis Plots

Trellis graphics displays (Becker, Cleveland, & Shyu, 1996) enable to view how graphs of one or two variables change in relationship to a third variable through conditioning. The data is displayed in a series of panels, where each panel contains a subset of the first one or two variables divided into intervals of the third conditioning variable.

Trellis graphics, displaying boxplots, time series, and histograms, are utilized extensively throughout the remainder of the dissertation.

2. Single-Factor Model

2.1. Introduction

This chapter reveals the effects of outliers on the OLS regression estimates of alphas and betas as computed from the single-factor market model (1.3), with $R_B = R_M$. It shows that most OLS results are driven by a very small fraction of outliers. These outliers can be created in various ways, such as by valid data points, gross data entry errors, distribution asymmetry, the choice of return type, and even by not adjusting returns for stock splits and dividend distributions. In contrast, the MM-estimator (ROB) is shown to be capable of maintaining a low bias in presence of outliers, while keeping the variance low when no outliers are present. It also can detect and separate outliers for further analysis. In a central comparison of prediction capabilities, ROB proves to be superior. Overall, it will be shown that robust alpha and betas have sufficiently attractive performance properties to warrant routine use as a complement to, or even substitute for, the classical OLS alphas and betas.

Since alphas and betas are used for different investment purposes they are treated separately.

2.1.1. Alphas

Challenging CAPM's prediction, the $\hat{\alpha}$ estimated from the single-factor market model by OLS regression is often non-zero and plays an important role in various investment decisions. To name a few applications: plan sponsors, fund of funds, and consultants use $\hat{\alpha}$ for due diligence analysis of hedge funds and to select long-only managers; analysts often use (1.3) to compute $\hat{\alpha}$ and information ratios, $IR = \hat{\alpha}/\hat{\sigma}_\varepsilon$, where σ_ε is the residual standard error, or some tracking error of the benchmark. This is used to gauge the performance of active money managers while not rewarding managers for taking on more risk than the benchmark (Goodwin, 1998): if, $\hat{\beta} > 1$ then $\hat{\alpha}$ will be smaller than it would be if $\hat{\beta}$ was fixed at 1. This in turn will decrease the information ratio and thus punish the manager that is taking on the extra risk; and, mutual funds are

widely rated and analyzed using $\hat{\alpha}$ (Jensen, 1968; Pastor & Stambaugh, 2002). More sophisticated methods for asset pricing in a multi-factor model framework have been developed and used, e.g., to evaluate a mutual fund's performance (Lehmann & Modest, 1987). Today's established models include Fama and French's three-factor regression model (Fama & French, 1993), the four-factor regression model (Carhart, 1997), and other black-box and easy-to-use off-the-shelf products, such as Barra's Alpha Builder or FactSet's Alpha Tester.

However, alphas from the single or multi-factor models have the OLS regression estimator at its core, thus facing similar problems. The discussion of bias protection and estimation error inflation of OLS alphas in the single-factor market model will also lay out the groundwork for future research on better alphas in the multi-factor model framework.

Problems in estimating $\hat{\alpha}$ have been recognized by a number of researchers. Grinold & Kahn (2000, page 377) state: "The alphas are often unreasonable and subject to hidden biases"; further on page 378: "Implementation schemes must address two questions ... what procedures can we use to make the portfolio construction process robust in the presence of unreasonable and noisy inputs? How do you handle perfect data, and how do you handle less than perfect data..."; and finally on page 382: "Closely examine all stocks with alphas greater, in magnitude than, say, three times the scale of the alphas. A detailed analysis may show that some of these alphas depend upon questionable data and should be ignored (set to zero), while others may appear genuine. Pull in these remaining genuine alphas to three times scale in magnitude." Wermers, Kosowski, Timmermann, & White (2003, page 9) recognize that "...the empirical distribution of residuals from Jensen (and other) regressions is highly non-normal for most mutual funds in our sample." The hypothesis of normally distributed alphas as generated by various multi-factor models in their study is rejected for over 50% of funds, challenging the validity of inference tests. Using non-parametric bootstrap techniques (Efron & Tibshirani, 1993; Shao & Tu, 1995) to analyze the tails of the alpha distribution, the authors find that the right tail of the distribution of alphas, i.e., most of the positive alphas computed from

actual fund returns are substantially overestimated, compared with alphas computed from bootstrapped returns.

2.1.2. Betas

Most financial service providers report OLS estimates of $\hat{\beta}$. Martin & Simin (2003) conducted a thorough survey and pointed out that, besides Barra and Ibbotson, none of the following providers do any outliers treatment: Bloomberg, Dow Jones, Merrill Lynch, Standard and Poor's, Value Line, Vestek, and Wilshire. Some financial service providers also report an adjusted version $\tilde{\beta} = 0.35 + 0.67 \cdot \hat{\beta}$, designed to correct for the tendency of $\hat{\beta}$ to revert towards the market beta of one over time (Blume, 1975; Levy, 1971).

The financial literature proposed a number of alternatives to the OLS estimator that are robust toward outliers according to various statistical criteria. The LAD estimate is perhaps the oldest and most widely known. In the context of estimating beta, the LAD estimate was studied early on by Sharpe (1971), who considered thirty common stocks, used to compute the Dow Jones Industrial Average, and thirty mutual funds, both in the mid-to-late 1960's. Cornell and Dietrich (1978) also studied the LAD estimate using 100 companies randomly drawn from the S&P 500 from 1962 to 1975. Both studies were motivated by the knowledge that returns sometimes have influential outliers associated with non-Gaussian distributions, and that an alternative to OLS might therefore perform better. Sharpe (1971) and Cornell & Dietrich (1978) concluded that the LAD alternative did little to improve the OLS estimate of beta. These results are evidently due to the lack of influential outliers in portfolio returns of large size firms and mutual funds, and to the lack of relatively high volatility firms for these early time periods. Nevertheless, Sharpe (1971) mentioned that, on the stock level, the differences were significant.

Motivated by these findings, a number of more sophisticated robust estimators were studied. Fong (1997) used a generalized t-distribution to model skewness, as well as kurtosis to obtain robust estimates of the beta factor. He found improvements over using the normal distribution or the standard t-distribution. Connolly (1989) compared the weekend effect for different estimators, such as the OLS, a Huber-type M-estimate (Huber, 1981), and a regression-quantiles estimate proposed by Koenker & Basset

(1978). His results indicated that the weekend effect is much smaller than previously thought. Barnes & Hughes (2002) used regression-quantiles in the Fama & MacBeth (1973) method to test the CAPM and found significant beta coefficients for over-performing and under-performing stocks. Chan & Lakonishok (1992) showed that the regression-quantiles method could provide higher variance efficiency than OLS when estimating beta. Mills, Coutts, & Roberts (1996) and Mills (1999, section 6.4) also advocated using the regression quantiles approach to obtain robust estimates of the beta factor, and presented some convincing results in support of the approach. Cable & Holland (2000) studied the use of several robust estimators with a focus on applications to event studies. They found that robust estimators only increase the efficiency of the beta estimate, but do not restore normality to the residuals. However, this is an inherent strength of the robust estimator, as it makes outliers better detectable.

Martin & Simin (2003) proposed to obtain robust betas using the MM-estimator. On a moderately limited universe of US equity returns, Martin & Simin (2003) showed that OLS and robust betas differ by more than a sizeable 0.5 for about 12% of the firms. They also showed that robust betas predict future robust betas better than OLS betas, and that the existence of influential outliers is mainly a small firm effect.

Section 2.2 describes the data, data characteristics, and conditioning variables used in Trellis graphs. It also shows the percentage of outliers rejected at various efficiencies of the MM-estimator and discusses the issue of discrete and continuous returns. Section 2.3 demonstrates, with striking examples, the effects of outliers on the OLS regression. Section 2.6 and Section 2.7 present the alpha and beta results from a rigorous empirical data analysis using all U.S. equity returns from 1964 to 2003. Section 2.6.3 and Section 2.7.4 show the strength of the robust estimator on an out-of-sample prediction study. Both sections also show the effects of return type choice (discrete versus continuous returns) on alphas and betas, as well as the effects of using data that is not adjusted for stock splits and dividends (dirty data).

2.2. Data, Data Characteristics, Conditioning Variables

The study on alphas and betas uses weekly stock prices and daily returns aggregated to weekly returns for firms listed on NYSE, AMEX, and NASDAQ, and the value weighted NYSE/AMEX/NASDAQ composite as the market proxy in excess of the one-month T-Bill. The data was obtained from the *Center for Research in Security Prices* (CRSP). Weekly data was chosen as a compromise between data volume and loss of granularity. The analysis is carried out on contiguous two-year intervals from 1964 through 2003. The two-year period reflects a reasonable timeframe used by practitioners, contains sufficient number of data for inference purposes, and does not disguise time variant trends. To be included, a firm must have been listed for at least one of the entire two-year intervals. To be included in the prediction studies (Section 2.6.3 and Section 2.7.4) a firm must have been listed for at least four consecutive years.

2.2.1. Characteristics and Implications

Standard measures of the shape of the return distribution are skewness and kurtosis, which are close to zero for Gaussian data. Positive (negative) skewness means that there is a greater than normal probability of having large positive (negative) returns, i.e., skewness measures the fatness of one tail. Kurtosis measures the peaked-ness of the return distribution or the fatness of both tails. Larger kurtosis means higher chance for large positive and large negative returns.

To examine the distribution characteristics of discrete returns, the distribution of skewness parameters over time is shown in Figure 2.
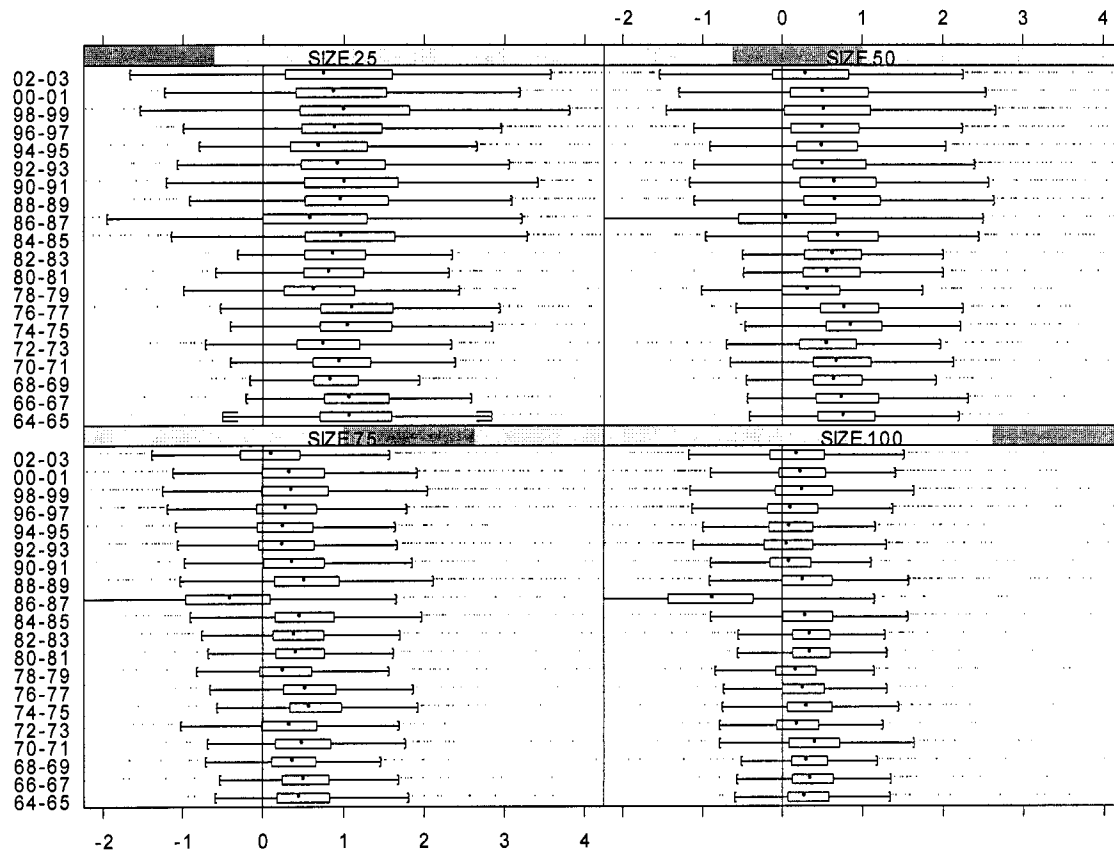
Figure 2. Skewness of Discrete Returns.
The positive asymmetry is clearly evident across all firm sizes with small size firms showing a stronger positive skewness. The irregularity in 1986-1987 is attributed to the October 1987 market crash.

With the exception of the 1986-1987 period that included the October 1987 stock market crash, more than 75% of the firms have positive skewness, many with very large positive skewness that decreases as firm size increases.

The classical skewness and kurtosis measures are based on the first moments of the data and therefore sensitive to outliers. The kurtosis measure is further restricted to symmetric distribution. These disadvantages can be overcome by using an approach that is not based on moments of the data as suggested in Brys, Hubert, & Struyf (11/2005; , 2003). This robust measure of skewness and kurtosis has a breakdown point of 25% and is illustrated in Figure 3 for discrete returns only.

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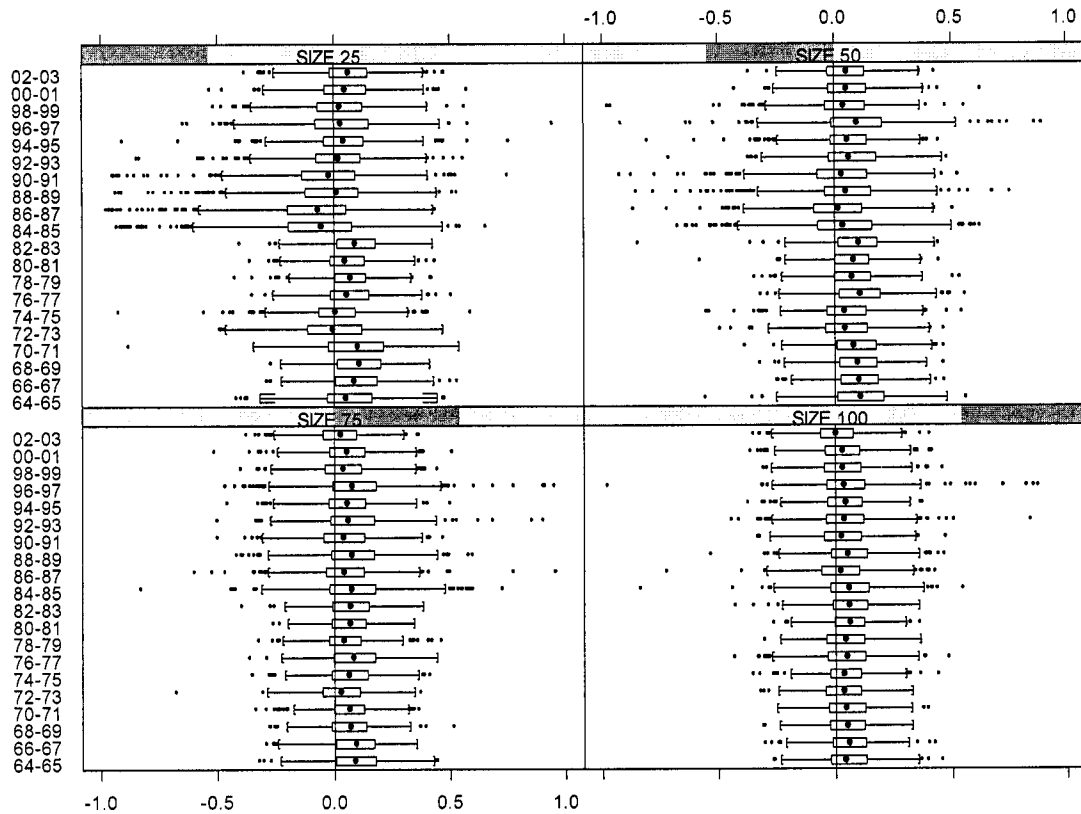
Figure 3. Robust Skewness of Discrete Returns.
The positive asymmetry is clearly evident across all firm sizes with small size firms showing a stronger positive skewness.

The range of skewness parameter is much smaller than in Figure 2. While the smallest size firms showed the most positive skewness in Figure 2, their overall skewness in Figure 3 is not much different anymore to the other size quartiles, even though they show a stronger sensitivity to market behavior than the other size quartiles. Extreme outliers, such as in October 1987 that distort the skewness in Figure 2, do not show much impact in Figure 3.

Figure 4 shows the kurtosis for discrete returns.
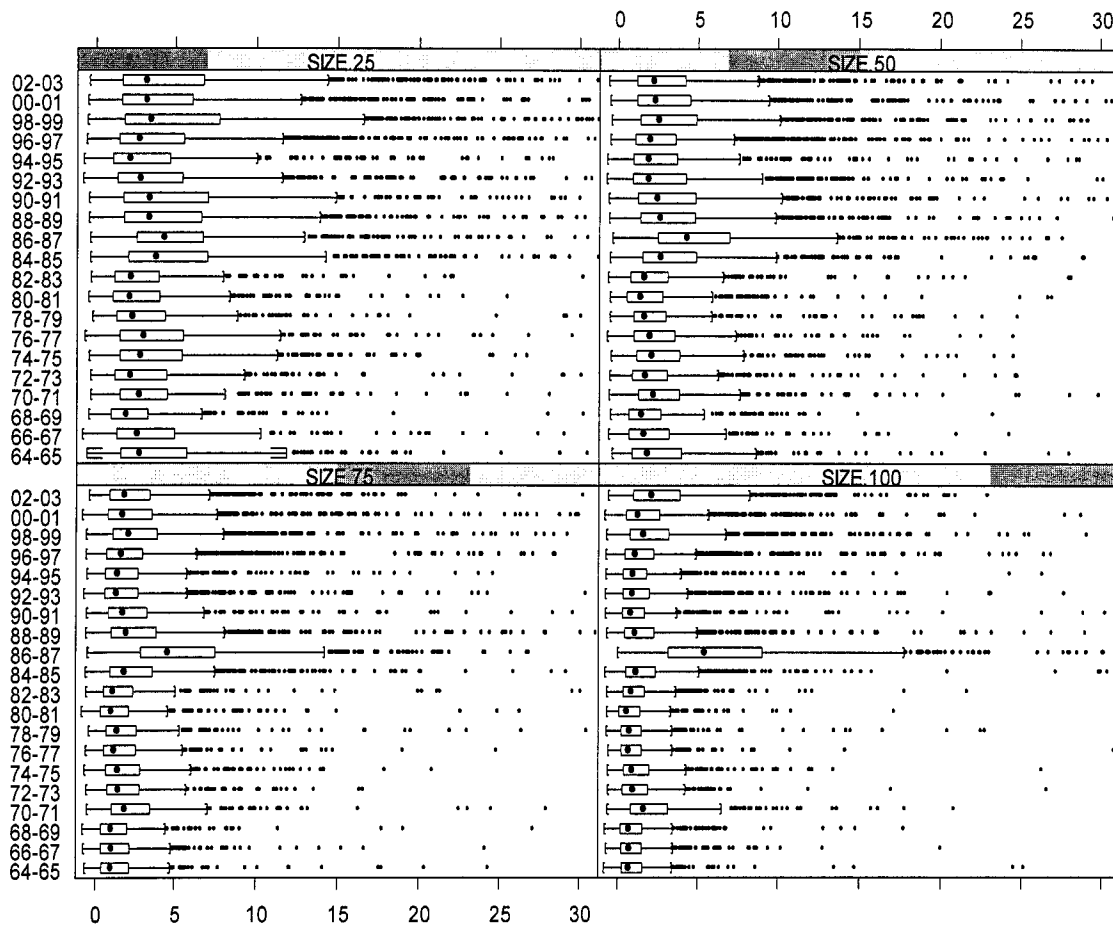
Figure 4. Kurtosis of Discrete Returns.
Kurtosis parameter is plotted over contiguous two-year time periods, sliced by firm size quartile. Across all time periods and size quartiles, firms are showing a positive median skewness.

Figure 4, shows that kurtosis is virtually always positive and often quite large. In addition, kurtosis decreases as size increases, which hardly comes as a surprise.

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Figure 5 shows pair-wise differences of the skewness parameter computed on discrete and on continuous returns.
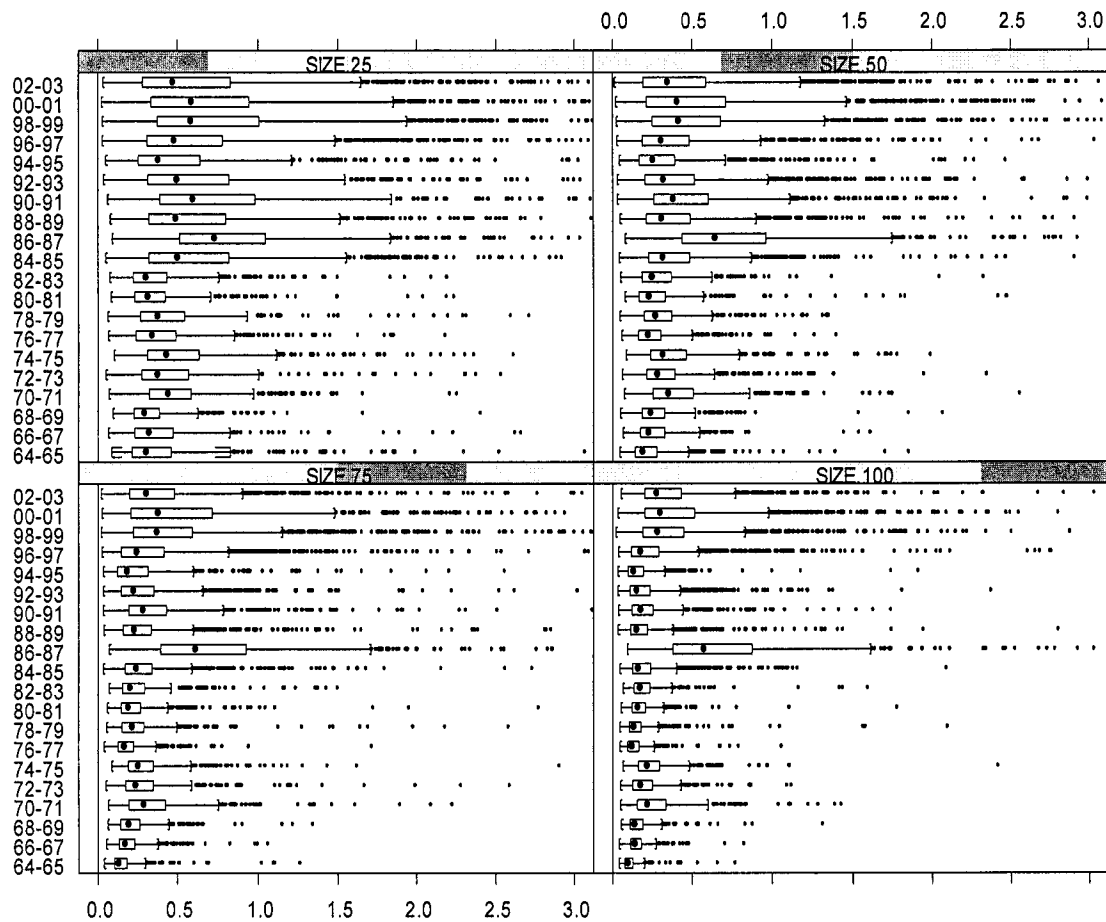
Figure 5. Skewness: Difference on Discrete and Continuous Return (pair-wise).
Continuous returns are symmetrizing, reducing the positive skewness and center the distribution closer around zero when compared to discrete returns. The irregularity in 1986-1987 is attributed to the October 1987 market crash.

The pair-wise differences are positive due to the property of the log transformation to symmetries. Returns are shifted closer to zero, resulting in reduction of skewness. As expected, the effect is strongest for the smallest size quartile.

Figure 6 shows pair-wise differences of the kurtosis parameter computed on discrete and on continuous returns, sliced by size quartiles.
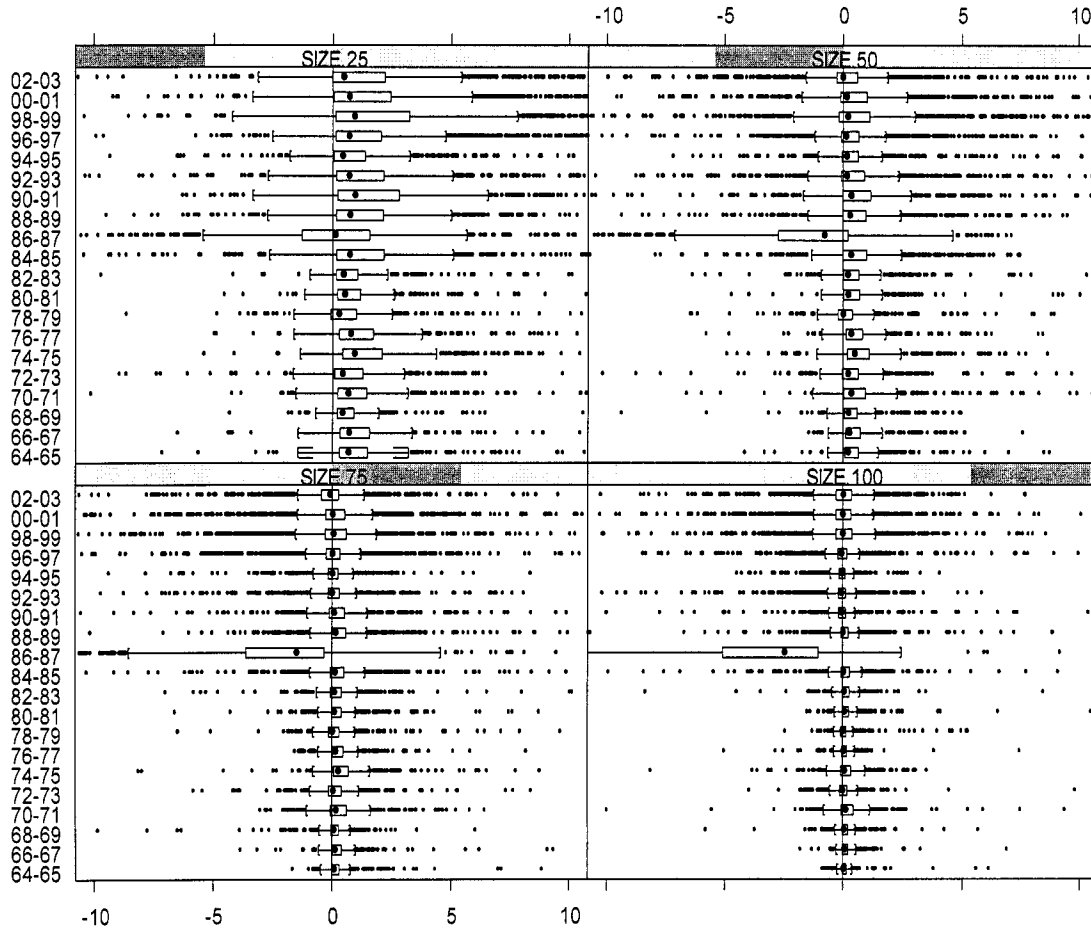
Figure 6. Kurtosis: Difference on Discrete and Continuous Return (pair-wise).
With the exception of the smallest size quartile, the kurtosis parameter is not very much affected by using continuous over discrete returns.

As can be seen from the centering of the medians around zero, kurtosis computed on continuous returns is only slightly different than that computed on discrete returns. The main result is: the median skewness almost disappears when using continuous returns, while kurtosis persists at roughly the same levels, as when using discrete returns. With respect to reducing bias in the presence of outliers, continuous returns are of little help.

2.2.2. *Conditioning Variables*

For the analysis of classical versus robust estimates of alphas and betas, it is informative to condition or *slice* on intervals of other factors. Results can then be displayed graphically in Trellis plots (Section 1.4.4).

During the analysis it turned out that slicing on robust mean returns was most effective when analyzing alpha estimates and conversely slicing on firm size (where size is the log of the market capitalization in millions) when analyzing beta estimates. For slicing by robust mean returns, firms are split in four equal sized groups, called R.25, R.50, R.75, and R.100 for the lower quartile, median, upper quartile, and highest return quartile, respectively. For slicing by size, firms are split in four sized groups, SIZE.25, SIZE.50, SIZE.75, and SIZE.100 for the lower quartile, median, upper quartile, and highest size bracket, respectively. The breakpoints for the size quartiles where chosen to reflect the current ratios of small to mid to large to super large cap firms and were recomputed for every two-year interval.

Figure 7 shows the distribution of firm's robust mean return across the robust mean return quartiles on two-year intervals from 1964 to 2003.
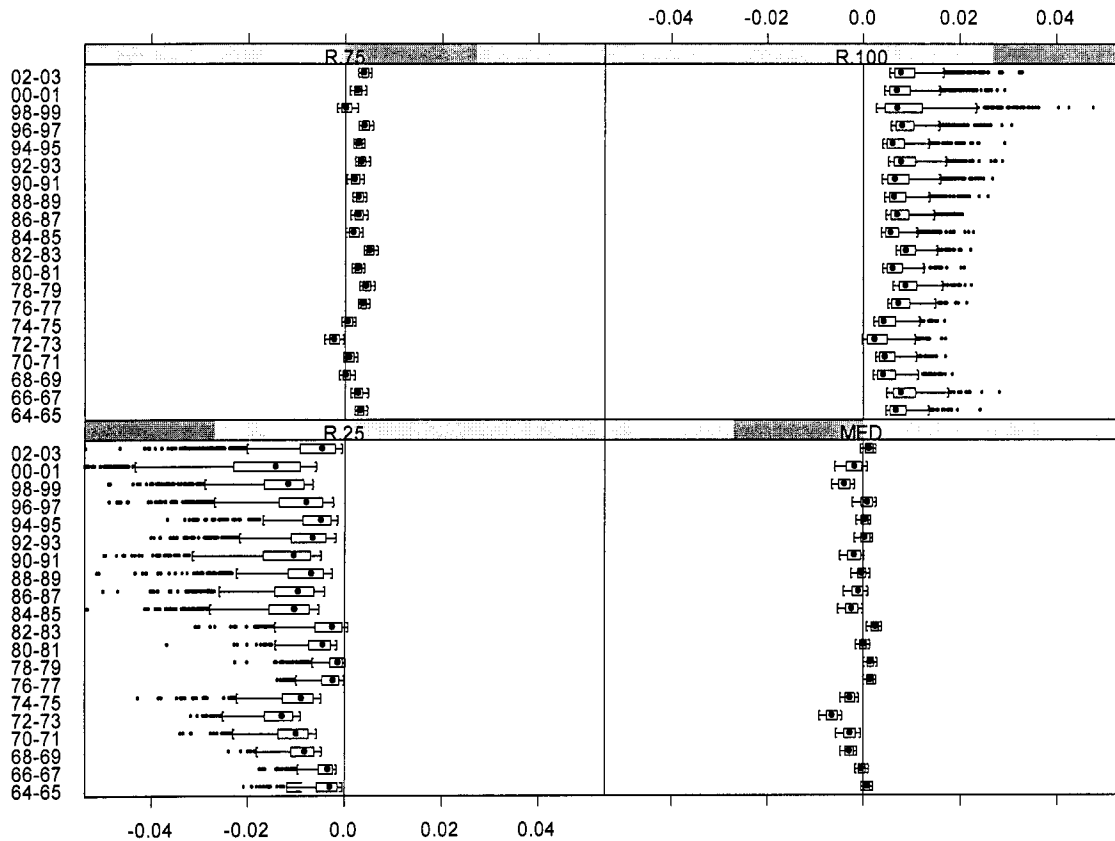
Figure 7. Distribution of Firm's Robust Mean Returns.
Firm's robust mean returns are ordered and split into quartiles with R.25 the lowest and R.100 the highest return quartile. R.25 and R.100 vary widely.

By way of construction, the two middle quartiles are fairly tight. The lowest and the highest return quartile are skewed to lower and higher returns, respectively.

Figure 8 shows the break points of the size quartiles on two-year intervals from 1964 to 2003.
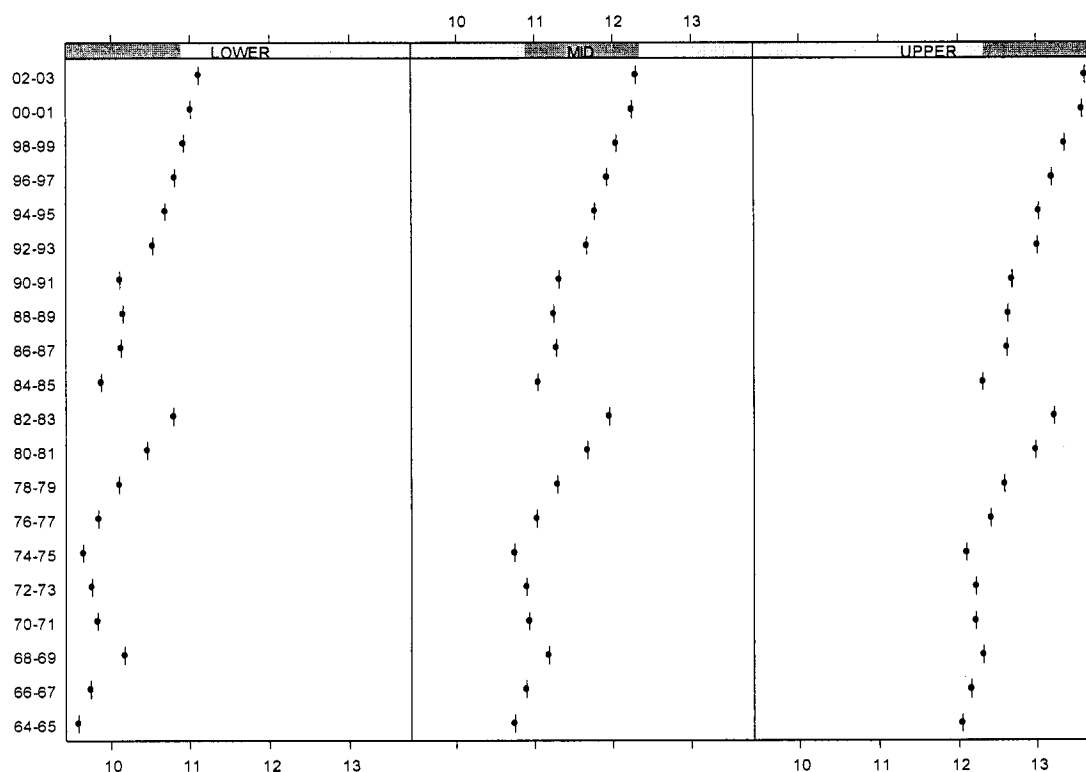
Figure 8. Size Breakpoints.
The size breakpoints in millions of dollars. The drop in 1984 corresponds to a steep increase of small size firms listed on NASDAQ.

The drop in the 1984-1985 period corresponds to a large number of new listings of small size firms on NASDAQ when listing requirements were eased. The above-average increase of size break point in the largest size quartile during 1994-2001 is peculiar and possibly due to large capitalizations of Internet firms.

Figure 9 shows the number of firms included in this study over time.
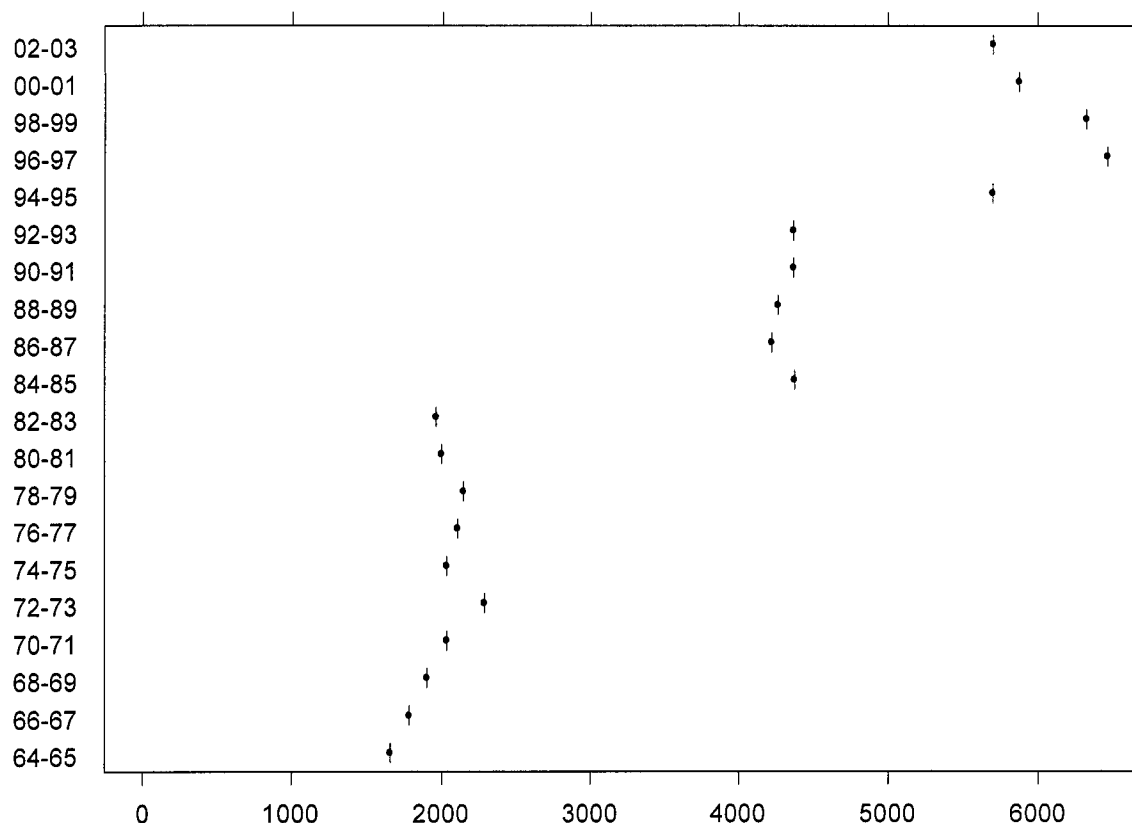
Figure 9. Number of Firms Included in Analysis.
Spikes and dips correspond to changes in regulation or market events. The number of firms jumped up when NASDAQ eased the listing barrier in 1985 and 1997 and when Internet trading was introduced in 1994. It fell during the bear market of the Seventies, aftermath of October 1987, Asia and Russian crises in 1998/1999, the 2001 technology bubble burst, and September 11.

Spikes and dips in number correspond to changes in regulations or market events. For example, the number of firms decreased from 1972 through 1983 during the bear market of the Seventies, as oil crisis, inflation, and unemployment stifled the U.S. and world economy. It further dropped in the aftermath of the October 1987 crash when the DJIA fell by 508 points (22.6%), during the Asia currency crisis in 1998, Russia's default on its debt in 1999, technology sector bubble burst, and the September 11, 2001 terrorist attacks. The number of firms included soared in the 1984-1985 period when NASDAQ eased its listing barrier, in 1994 when Internet trading was introduced, and even further in 1997 when NASDAQ eased its listing standards again.

2.2.3. Outlier Protection versus Efficiency

Computations over the full data range from 1964 to 2003 at efficiencies of 85%, 90%, 95%, and 99% resulted in median (mean) data rejection of 6.5% (7.3%), 5.3% (6.1%), 3.9% (4.7%), and 1.44% (1.78%), respectively. Table 1 shows the percentage of outliers rejected on contiguous two-year intervals at 99% efficiency, sliced by firm size.

Table 1. Percentage of Outliers deleted at 99% Efficiency.

	Median	Mean
SIZE.25	1.92	2.79
SIZE.50	1.92	1.93
SIZE.75	0.96	1.45
SIZE.100	0.96	0.98

The percentage of data rejected increases with decreasing firm size, which is not surprising since smaller size firms tend to be more non-normal. The median percentage of outliers rejected over all time periods, ranges from 0.96% for the largest size quartile to 1.92% for the smallest firm size quartile. The results in Section 2.6 and Section 2.7 will show that even the small percentage of outliers rejected at efficiency of 99% will result in significant differences between the OLS and the proposed MM-estimates.

2.2.4. Discrete versus Continuous Returns

Practitioners typically compute relative difference in price (discrete model) returns

$$r_{t+1}^{discrete} = \left(\frac{p_{t+1} - p_t}{p_t} \right)$$

where p_t is the price at time t. Inspired by the continuous time, geometric Brownian motion stock model, much of the finance literature and some practitioners use the natural logarithmic difference returns computed as

$$r_{t+1}^{log} = log\left(\frac{p_{t+1}}{p_t} \right)$$

Since split and dividend adjusted log returns are not directly available from CRSP, log returns, in this dissertation, were computed from weekly discrete returns using the relationship:

$$r_{t+1}^{log} = log\left(p_{t+1}/p_t \right) = log\left(\left(p_{t+1} - p_t \right)/p_t + 1 \right) = log\left(r_{t+1}^{discrete} + 1 \right)$$

Standard Taylor series approximation arguments are often used to show that there is little difference between discrete and continuous returns. However, this argument breaks down when returns are large, e.g., containing outliers. In this case, the difference between the two definitions can be substantial. For example, a price jump from \$5 to \$15 corresponds to a discrete return of 200%, but a continuous return of only 110%. A drop in price from \$15 to \$5 corresponds to a discrete return of -66%, but to a continuous return of -109%.

This demonstrates the fundamental problem: discrete returns are asymmetric, ranging from -100% to arbitrarily large values, while logarithmic returns are more symmetric with arbitrarily large and small values. Section 2.6.4 and Section 2.7.5 show how the choice of returns type impacts alphas and betas.

2.2.5. Clean versus Dirty Returns Data

Clean stock returns data are adjusted and checked for stock splits and dividend distributions. Access to clean data can prove to be expensive and still not offer 100% protection against gross data entrée errors. A common cause of returns to be *dirty* (besides data recording errors) is when providers fail to adjust for stock splits and dividends. Not adjusting for stock splits creates negative outliers while not adjusting for reverse splits creates positive outliers. Outliers caused by splits and reverse splits are expected to be of several orders larger than outliers caused by dividend distributions. Section 2.2.5 and Section 2.7.6 compare OLS and ROB alphas and betas for 18,316 firms, listed on at least one two-year time period. The data sets used are clean weekly CRSP returns and *dirty* weekly returns computed directly from weekly CRSP prices, unadjusted for splits and dividend distributions.

2.3. Motivation

2.3.1. Real Data Example

Figure 10 shows scatter plots of monthly returns, from 1997 through 2003, of National Healthcare Corp. (NHC) and Zenix Income Fund Inc. (ZIF) with S&P500 in excess of the Libor as the market proxy. Both panels compare the OLS (dashed) with the ROB (solid) regression line. The upper panel displays the values of the intercept alpha, the lower panel the values of the slope beta.
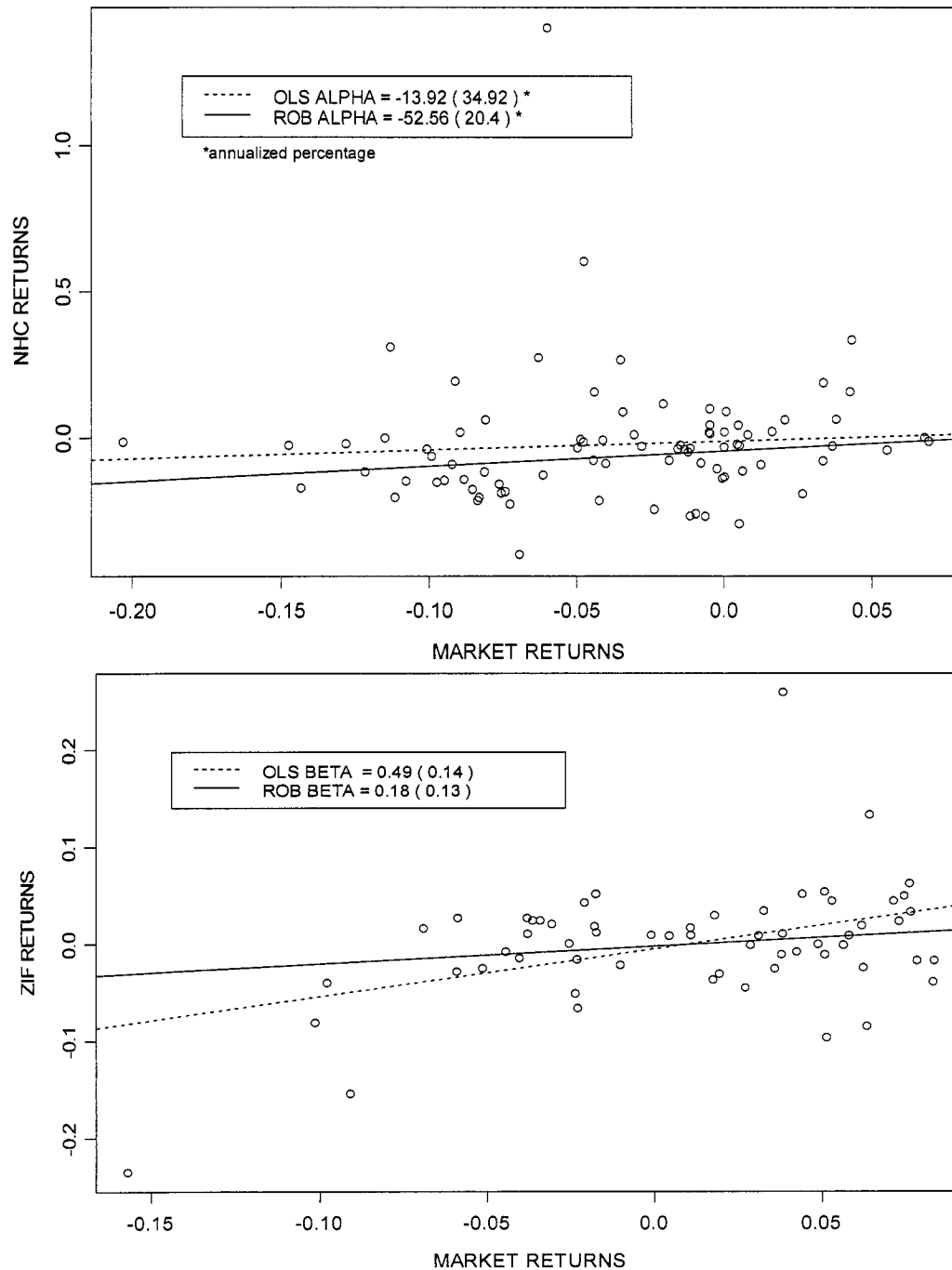
Figure 10. OLS versus Robust Estimates for Alpha and Beta.
The upper panel shows the alpha estimates for National Healthcare Corp. (NHC). ROB rejects the single outlier in the upper part of the graph causing an alpha difference of 38.6%, or 1.9 times the robust standard deviation. The lower panel shows the beta estimates for Zenix Income Fund Inc. (ZIF). ROB rejects only the large outlier in the upper right causing a beta difference of 0.31, or 2.4 times the ROB standard deviation.

In the upper panel, only the single large positive outlier is rejected. This creates a difference between the OLS and robust alphas of 38.6%, or 1.9 times the ROB standard error. In the lower panel, only the large positive outlier on the right side is rejected. This creates a difference between the OLS and ROB beta of 0.31, or more than twice the standard error of the ROB beta.

2.3.2. Monte Carlo Simulation

The Monte Carlo simulation compares the behavior of the OLS and ROB alphas and betas for normal returns versus normal mixture returns with a small *contamination* (outliers). The simulation compares 1000 replicates of OLS and ROB regression estimates computed from returns data as follows: 1000 replicates $\left(R_{B,t}, \varepsilon_t \right)_{t=1,\dots,100}$ are drawn from a bivariate normal distribution with mean zero, standard deviation 4%, and correlation zero. The replicates $\left(R_{B,t}, \varepsilon_t \right)_{t=1,\dots,100}$ are used to compute 1000 replicates $\left(R_t \right)_{t=1,\dots,100}$ from (1.3), assuming $\alpha = 0$ and $\beta = 1$. Now, $\gamma \%$ of the 100 observations in each of the 1000 replicates $\left(R_{B,t}, R_t \right)_{t=1,\dots,100}$ are randomly replaced (or contaminated) by pairs $\left(R_{B,t}, R_t \right)$ that are drawn from a bivariate normal distribution with means -12 and 16, respectively, standard deviations one, and correlation zero.

Now, (1.3) with the original uncontaminated replicates $\left(R_{B,t}, R_t \right)_{t=1,\dots,100}$ and the contaminated 1000 replicates $\left(\tilde{R}_{B,t}, \tilde{R}_t \right)_{t=1,\dots,100}$ are used to estimate OLS and ROB alphas and betas.

Table 2 shows the estimated alphas for $\gamma = 1\%$, 2%, and 3%.

Table 2. Monte Carlo Simulation Results for Alpha Estimates.

	OLS	ROB
$\gamma = 1\%$		
Mean	0.265	0.004
Standard Deviation	0.412	0.409
$\gamma = 2\%$		
Mean	0.482	0.004
Standard Deviation	0.432	0.413
$\gamma = 3\%$		
Mean	0.669	0.006
Standard Deviation	0.454	0.414

The mean of the OLS alphas shifts away from zero with rising contamination γ, while the mean of the ROB alphas are virtually not affected. The OLS standard deviation (estimation error) increases with rising γ, while the ROB estimation errors remain basically unaffected.

The mean of the OLS alphas consistently increase for increasing γ, while the ROB alphas are virtually unaltered.

Figure 11 shows histograms of the OLS and ROB alphas of 1000 Monte Carlo replicates for $\gamma = 3\%$.
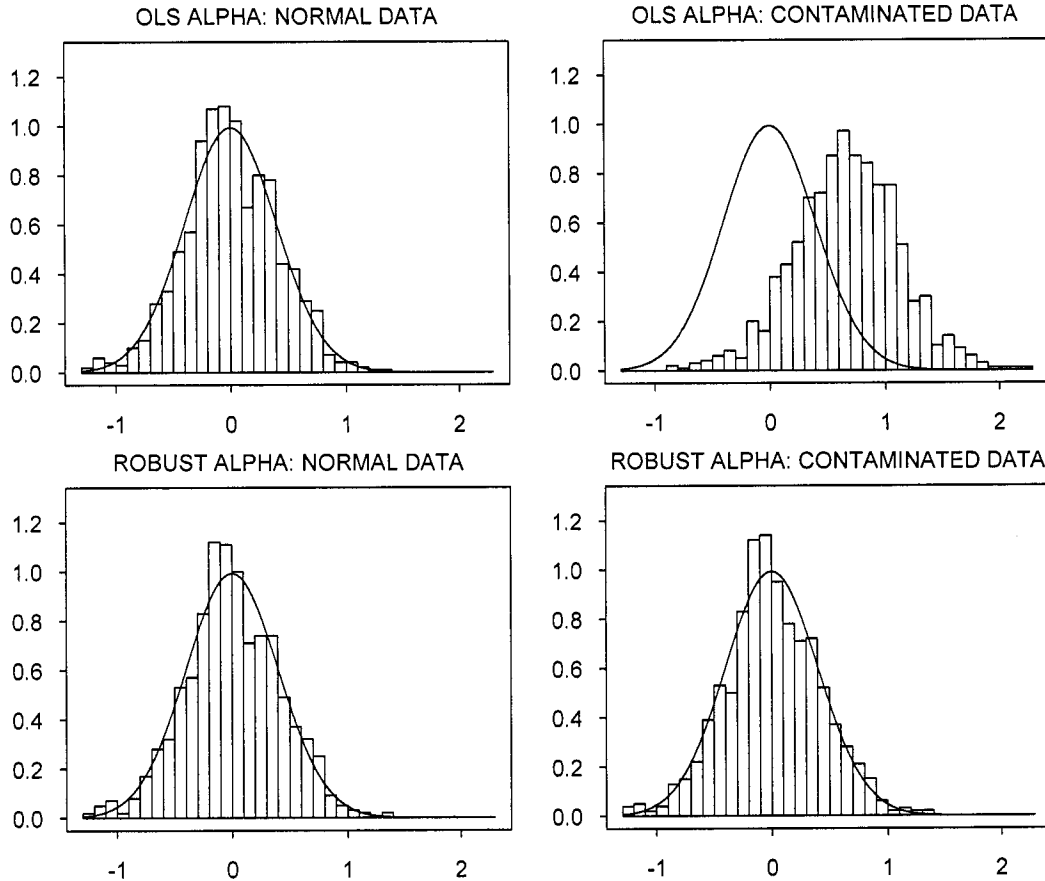
Figure 11. OLS versus ROB Alpha Estimates on Simulated Data $(\gamma = 3\%)$.

The left two panels are not contaminated and the reference distribution overlays the simulated alpha distribution. The right two panels contain outliers in the negative market and positive stock return direction.

The histograms are quite close to the theoretical normal distribution for $\gamma = 0\%$, or zero contamination. At $\gamma = 3\%$, the mean of the OLS alphas is overestimated by 0.67, with standard deviation of 0.45, resulting in bias as a percentage of the root-mean-squared-error of 83%. The location for the ROB alpha estimate is virtually not effected, at 0.006, with a bias as a percentage of the root-mean-squared-error of merely 1%.

Table 3 shows the estimated beta for $\gamma = 1\%$, 2%, and 3%.

Table 3. Monte Carlo Simulation Results for Beta Estimates.

	OLS	ROB
	$\gamma = 1\%$	
Mean	0.800	0.999
Standard Deviation	0.099	0.102
	$\gamma = 2\%$	
Mean	0.634	0.999
Standard Deviation	0.100	0.103
	$\gamma = 3\%$	
Mean	0.492	0.999
Standard Deviation	0.101	0.104

For 1%, 2%, and 3% contamination, the summary statistics for the regression beta estimates of 1000 OLS and ROB regressions are shown. The shift of the OLS betas away from one due to the presence of outliers, while the ROB betas are virtually not influenced.

The means of the OLS beta consistently decrease for increasing γ, while the ROB beta is virtually unaltered. The slightly larger standard deviation of the ROB beta estimate reflects the very small price paid in return for bias robustness toward outliers.

Figure 12 shows histograms of the OLS and ROB betas of 1000 Monte Carlo replicates for $\gamma = 3\%$.
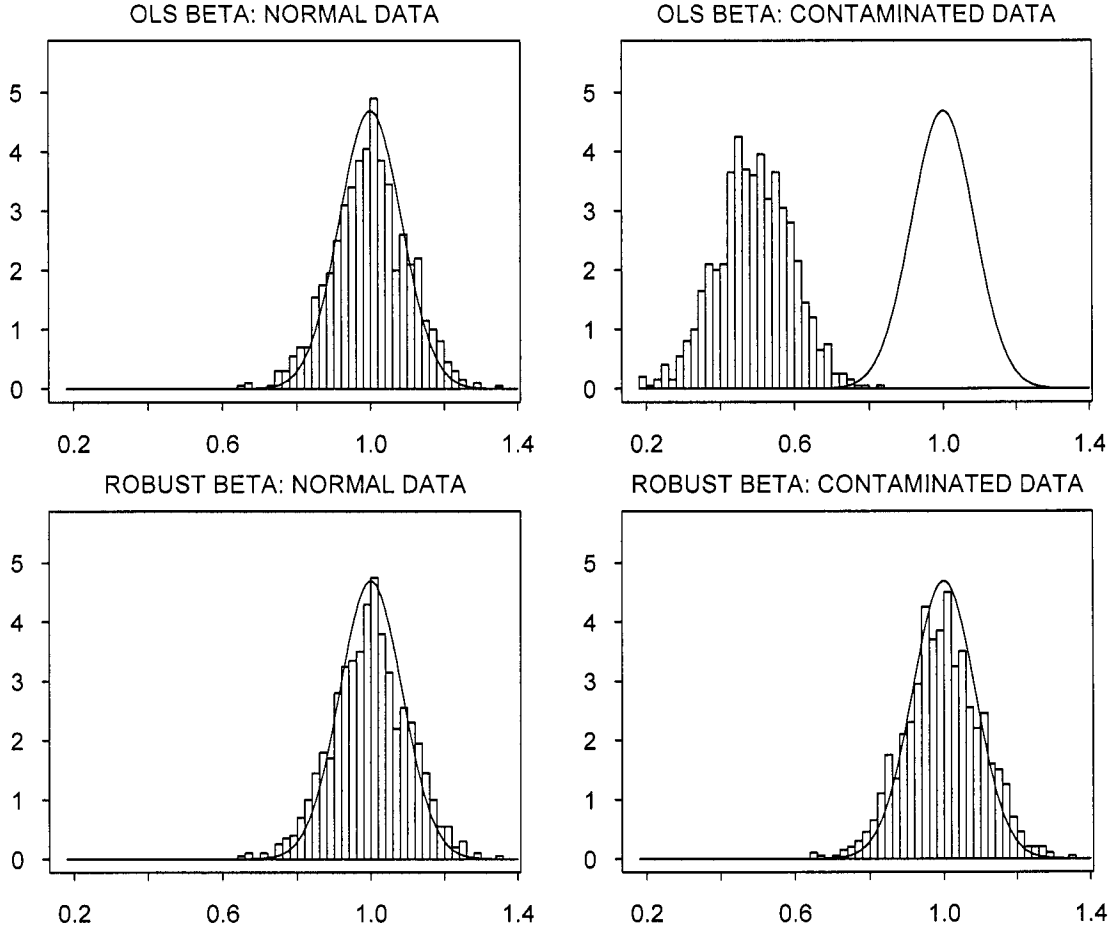
Figure 12. OLS versus ROB Beta Estimates on Simulated Data ($\gamma = 3\%$).
The left two panels are not contaminated and the reference distribution overlays the simulated alpha distribution. The right two panels contain outliers in the negative market and positive stock return direction. OLS estimators underestimate the beta coefficient, while ROB estimators remain unaffected.

The histograms are quite close to the theoretical normal distribution for $\gamma = 0\%$, or zero contamination. At $\gamma = 3\%$, the mean of the OLS betas is underestimated by 0.51 with standard deviation of 0.101. This results in bias as a percentage of the root-mean-squared-error of 98.1%. The location for the ROB beta estimate is not effected at 0.99 with a bias as a percentage of the root-mean-squared-error of merely 0.1%.

2.4. Performance Measure

The *root-mean-squared-error* (*RMSE*) is a commonly used performance measure of prediction capabilities of an estimator (Klemkosky & Martin, 1975). The *RMSE* is defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(\tilde{\Phi}_j - \hat{\Phi}_{j+k}\right)^2} \qquad (2.4.1)$$

where $\hat{\Phi}_j$ is the prediction based on data in time interval j of an estimate $\hat{\Phi}_{j+k}$ based on data in time interval k units of time in the future. That is, an interval shifted forward in time by a week or a quarter, and m is the number of time intervals used. The *RMSE* compares the predictive performance of classical versus ROB alphas and betas. It is estimated on two-year moving windows of weekly returns, using the estimate on a current window as the predictor of the estimate on windows one week and one quarter in the future. Performance is compared by computing the ratio of OLS to ROB *RMSE* denoted *RATIO*.

When no outliers are present in the prediction window, the OLS and ROB regression results are very close, and there is no advantage of replacing the OLS estimate. Therefore, the aim is to compare performance when at least one outlier is present. Here, an outlier is defined as an observation rejected by the ROB at 99% efficiency. Thus, the *conditional RATIO* (*CRATIO*), conditioned on the presence of at least one outlier in the prediction window, is computed only over those j for which the prediction window contains at least one outlier.

2.5. Intrinsic Variability of the MM-Estimator

The influence of outliers of the OLS and ROB estimate, computed in Section 2.6 and Section 2.7, can best be compared by taking pairwise differences. Yet, one might be concerned that the pairwise differences is merely intrinsic variability that is caused by the inefficiency of the ROB estimator at the Gaussian model.

The variance of the differences $\theta_{OLS} - \theta_{ROB}$ can be roughly estimated as follows.

Assume that the differences are normally distributed

$$\theta_{OLS} - \theta_{ROB} = N(0, V_{DIFF})$$
(2.5.1)

where

$$V_{DIFF} = \sigma_{OLS}^2 + \sigma_{ROB}^2 - 2 \cdot \sigma_{OLS} \sigma_{ROB} \cdot \rho$$
(2.5.2)

High efficiency is equivalent to high correlation between θ_{OLS} and θ_{ROB}, (Lehmann & Casella, 1998) i.e.,

$$\rho^2 = EFF = \frac{\sigma_{OLS}^2}{\sigma_{ROB}^2}$$
(2.5.3)

or equivalently

$$\sigma_{OLS}^2 = \sigma_{ROB}^2 \cdot EFF$$
(2.5.4)

Thus

$$
\begin{aligned}
V_{DIFF} &= \sigma_{OLS}^2 + \sigma_{ROB}^2 - 2 \cdot \sigma_{OLS} \sigma_{ROB} \cdot \rho \\
&= \sigma_{ROB}^2 \cdot EFF + \sigma_{ROB}^2 - 2 \cdot \sigma_{ROB} \cdot \sqrt{EFF} \cdot \sigma_{ROB} \cdot \sqrt{EFF} \\
&= \sigma_{ROB}^2 \cdot (1 - EFF)
\end{aligned}
$$
(2.5.5)

For the efficiency level used in this paper of $EFF = 99\%$

$$
\begin{aligned}
\hat{V}_{DIFF} &= \hat{\sigma}_{ROB}^2 \cdot (1 - 0.99) \\
&= 0.01 \cdot \hat{\sigma}_{ROB}^2
\end{aligned}
$$
(2.5.6)

The value of $\hat{V}_{DIFF}$ can be used to compute approximate t-statistics for the differences between the OLS and ROB estimates.

2.6. Alphas

Section 2.6.1 and Section 2.6.2 examine the performance of OLS and ROB alphas, as well as their standard errors. Section 2.6.3 continues with an out-off-sample test of the prediction capabilities of OLS and ROB alphas for one-week and one-quarter ahead time periods. Section 2.6.4 and Section 2.6.5 show the effects on alphas of using discrete versus continuous returns alphas and clean versus dirty returns, respectively.

2.6.1. OLS versus ROB

This section examines the performance of OLS and ROB alphas, as well as their standard errors on twenty contiguous two-year intervals from 1964 through 2003. Alphas are estimated by OLS and ROB regressions from the single-factor market model (1.3), with $R_B = R_M$, and the market proxy as described in Section 2.2.

Figure 13 shows alphas computed using OLS regression, on two-year contiguous time-intervals, sliced by their ROB mean return quartiles. All alpha estimates are displayed as annual percentages returns.
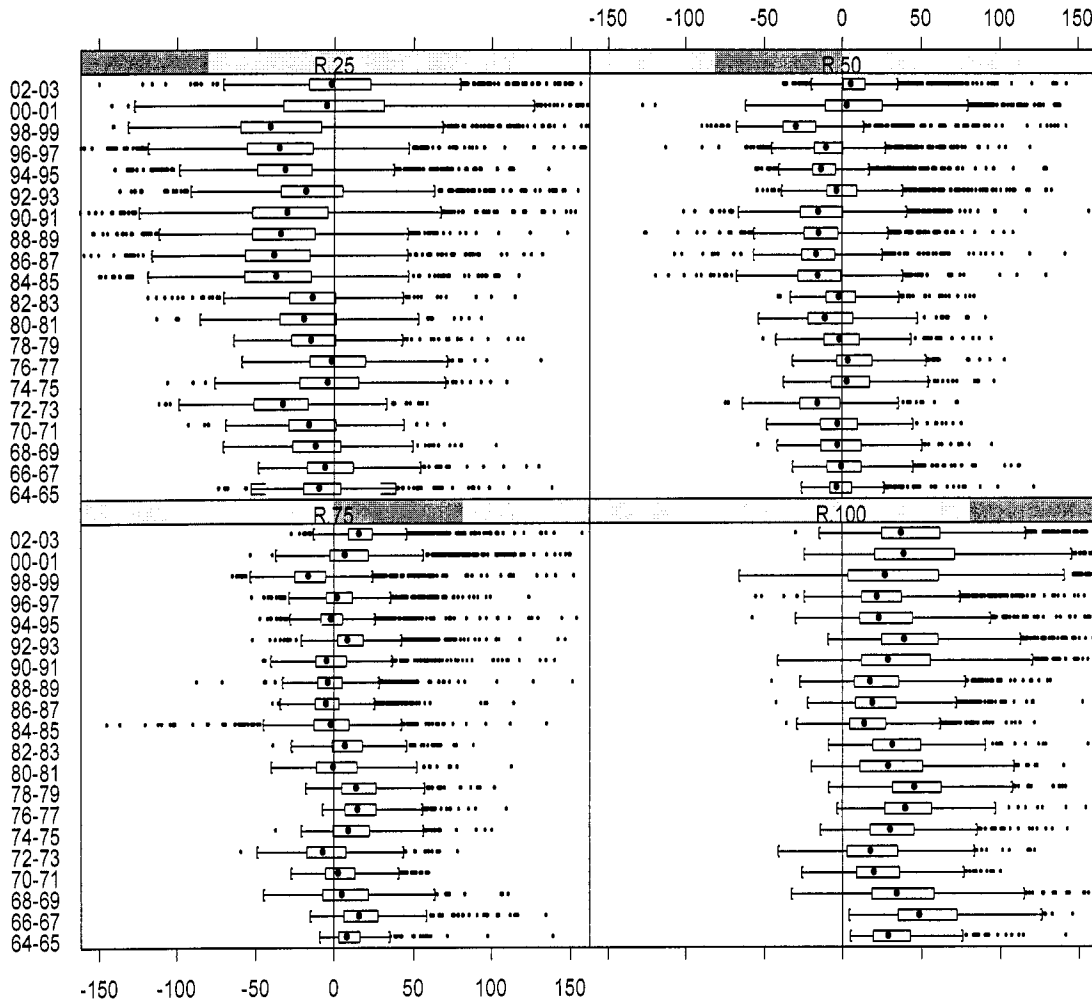
Figure 13. OLS Alphas.
Alphas tend to follow market events. This effect is strongest for the lowest return quartile. Alphas across all return quartiles tend to be skewed to the right.

Slicing OLS alphas by return quartiles shows on average that firms with lower (higher) ROB mean returns tend to have lower (higher) alphas. While the alphas are highly skewed in the lowest and highest return quartile, they have a smaller spread and are much more symmetrically in the two middle return quartiles. The positive skewness and distinct tendency of alphas to follow market trends across all return quartiles is visible. Alphas across all return quartiles are lower during down market times, such as the energy crisis and Vietnam War in 1972/73, Emerging Market crisis mid-1980s and Black Monday 1987, Asia Turmoil starting in late 1995 with its peak in 1997, and Russia's default on their debt in 1998. ROB alphas, sliced by return quartiles, give an even clearer picture, as shown in Figure 14.
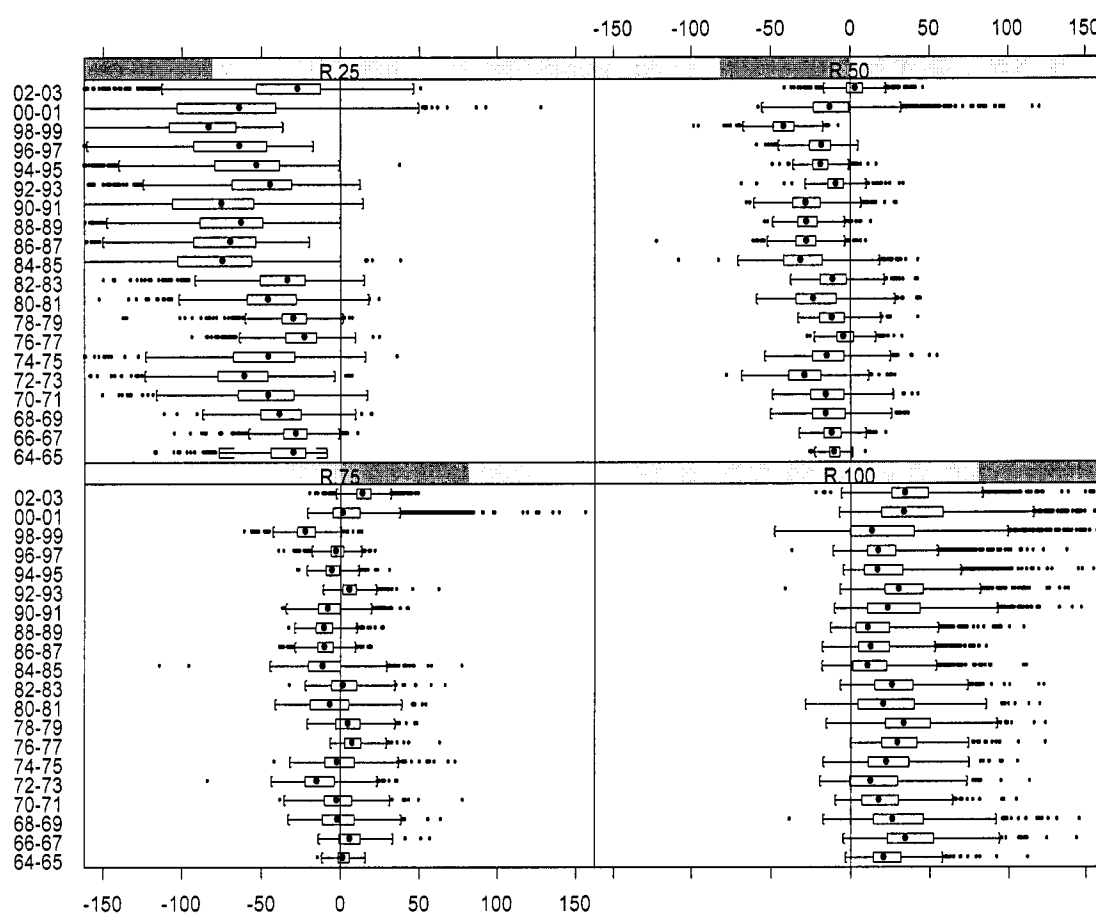
Figure 14. ROB Alphas.
Alphas tend to follow market events. This effect is strongest for the lowest return quartile. Alphas across all return quartiles tend to be skewed to the right.

ROB alphas estimated for lower return firms tend to have an asymmetric distribution skewed towards negative alpha values, while alphas of the highest returning firms have a positively skewed distribution.

Of greater interest and concern, however, is the pair-wise distribution shown in Figure 15 and summarized in Table 4.
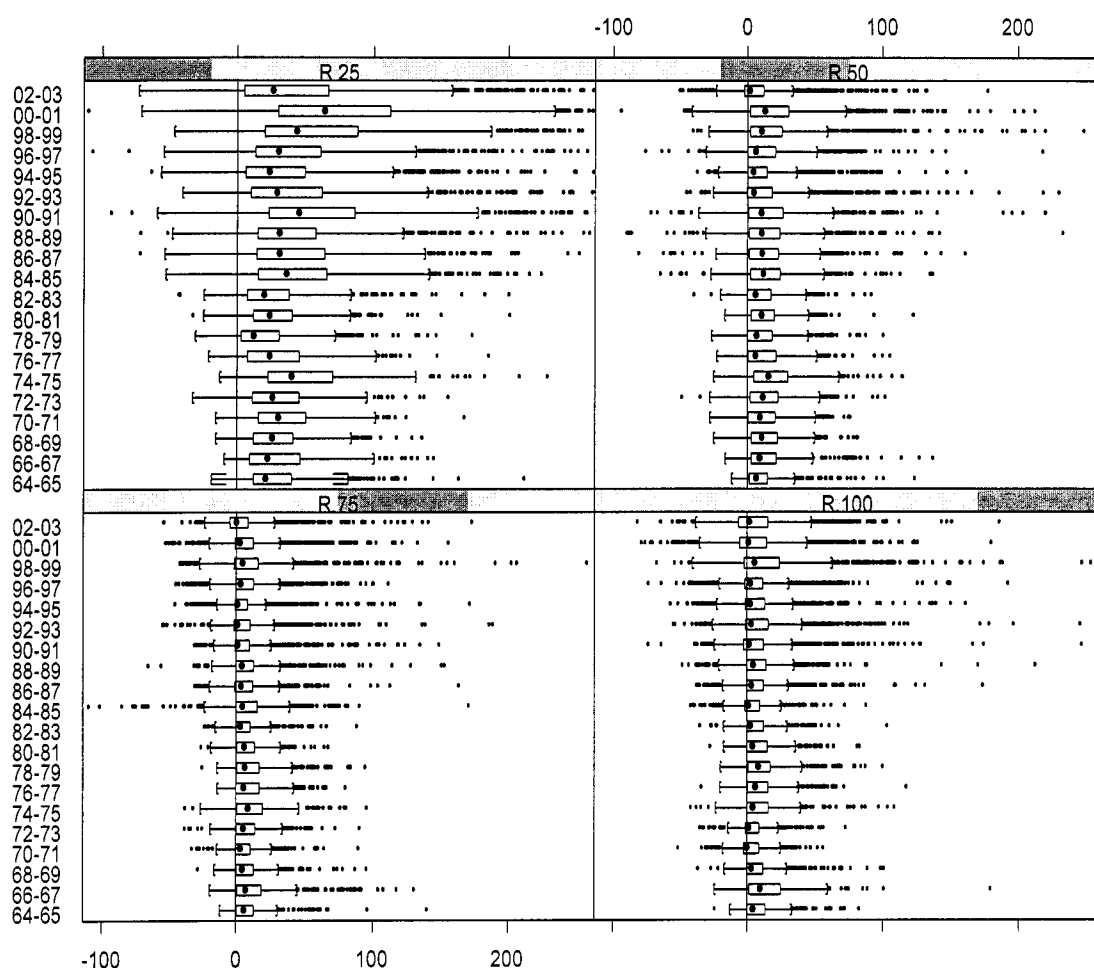
Figure 15. Pair-wise Difference of OLS and ROB Alphas.
The association with market events has almost vanished. The positive median difference across all return quartiles shows that OLS alphas tend to be significantly overestimated, in particular for lower returning firms.

Across all return quartiles, the distribution of the pair-wise differences between OLS and ROB alphas is positively skewed with positive medians. The effect is stronger for low

returning firms. Median differences for the lowest return quartile range between 10%-30% annually, with positive differences well over 100%. This compares to findings in Mamysky, Spiegel, Zhang (2003, p.2): "static OLS alphas can be off anywhere from 5 to 87 percent from a fund's time averaged alpha." This is a striking result considering that the alpha differences arise from rejecting only a small fraction of the most extreme outlying returns. The pair-wise differences in the lowest return quartile still show market effects, whereas in down and higher volatile markets the median of the pair-wise differences increases. The increase of volatility and positive skewness of the pair-wise differences is also clearly visible in recent years. This can be attributed partially to the larger numbers of small firms that are new listings on NASDAQ (Figure 9).

Table 4. Summary of Pair-wise Differences of OLS and ROB Alphas.

	Min	1st Quartile	Median	3rd Quartile	Max
R.25	-140.73	12.73	31.87	63.18	1019.76
R.50	-126.22	0.40	8.79	21.98	949.96
R.75	-109.18	-0.25	3.68	12.88	288.96
R.100	-81.35	-0.81	3.57	13.98	303.58

Differences show a positive asymmetry with median close to zero, with considerable numbers of very large differences across all size quartiles.

Figure 16 provides even greater detail. It shows the complementary empirical distribution function of the alpha differences sliced by ROB mean quartile. The size of the differences is plotted on the horizontal axis, while the vertical axis shows the percentage of the corresponding percentage of firms.
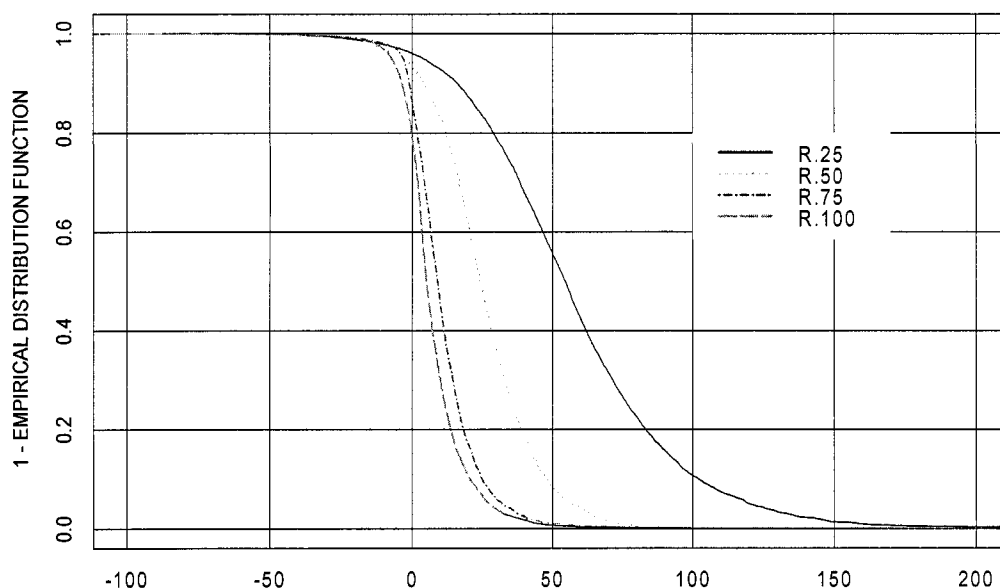
Figure 16. Distribution of Pair-wise Difference of OLS and ROB Alphas.
Well visible is the positive bias of the OLS estimator caused by a small fraction of outlying returns.

More than 80% of all pair-wise alpha differences are positive. More than 10% of the pair-wise differences in second smallest return bracket (R.50), and about 55% of the smallest return bracket (R.25) are greater than 50% on an annualized percentage basis. Again, it is important to remember that the differences are caused by rejecting only a small fraction of outlying returns.

The alpha differences from Figure 15, scaled by the estimate of the variability of the differences $\hat{V}_{DIFF}$ (Section 2.5) are shown in Figure 15.
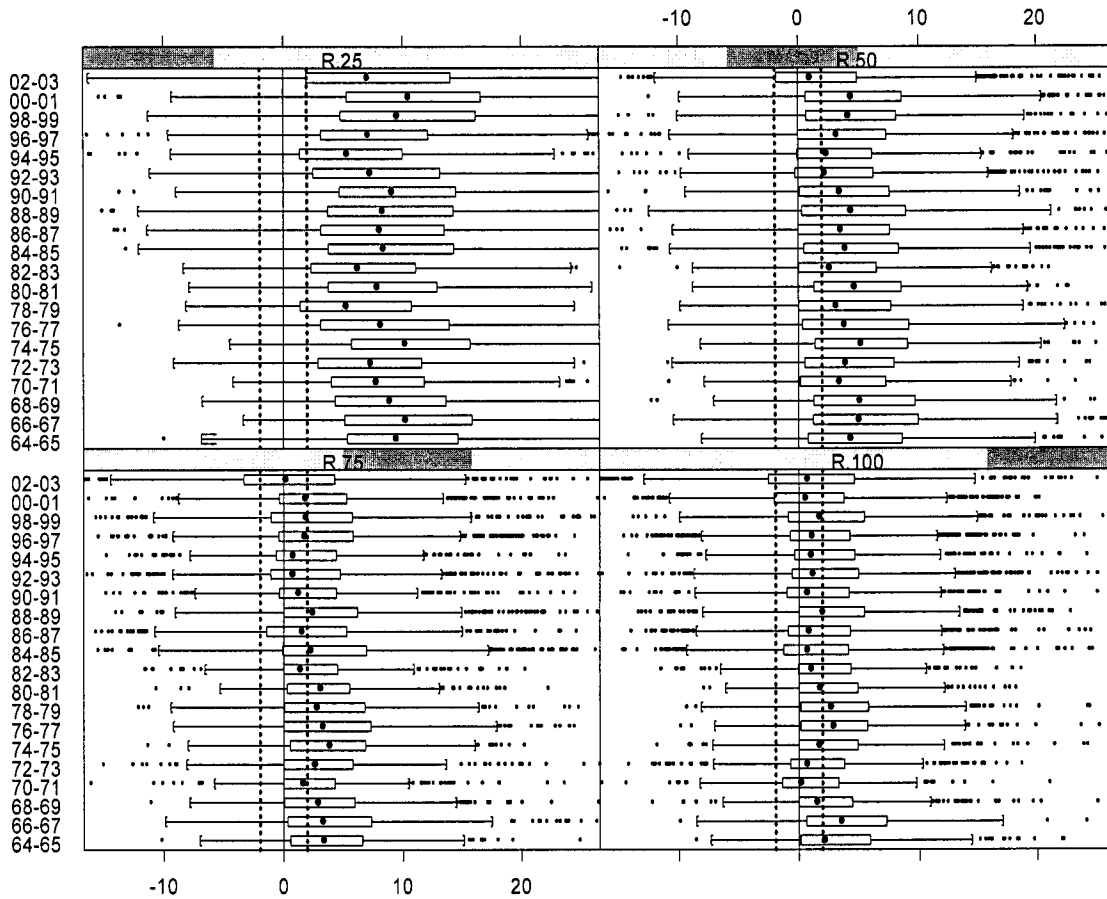
Figure 17. Scaled Pair-wise Difference of OLS and ROB Alphas.
Scaled by approximate variability of the differences at the Gaussian model. The majority of the median differences are significant, with even higher significance levels in the lower return quartiles.

The vertical dashed lines show the fraction of the t-statistics that are significant at the 5% level. This fraction is much larger than what one expects under a null hypothesis of equal OLS and ROB alpha under the Gaussian model. Thus, the majority the differences between OLS and ROB estimates are statistically significant.

2.6.2. Standard (Estimation) Errors

The MM-estimator provides standard errors that are, unlike their OLS counterpart, less inflated by outliers. Figure 18 compares the pair-wise ratios of OLS to ROB standard errors of the alpha estimates.
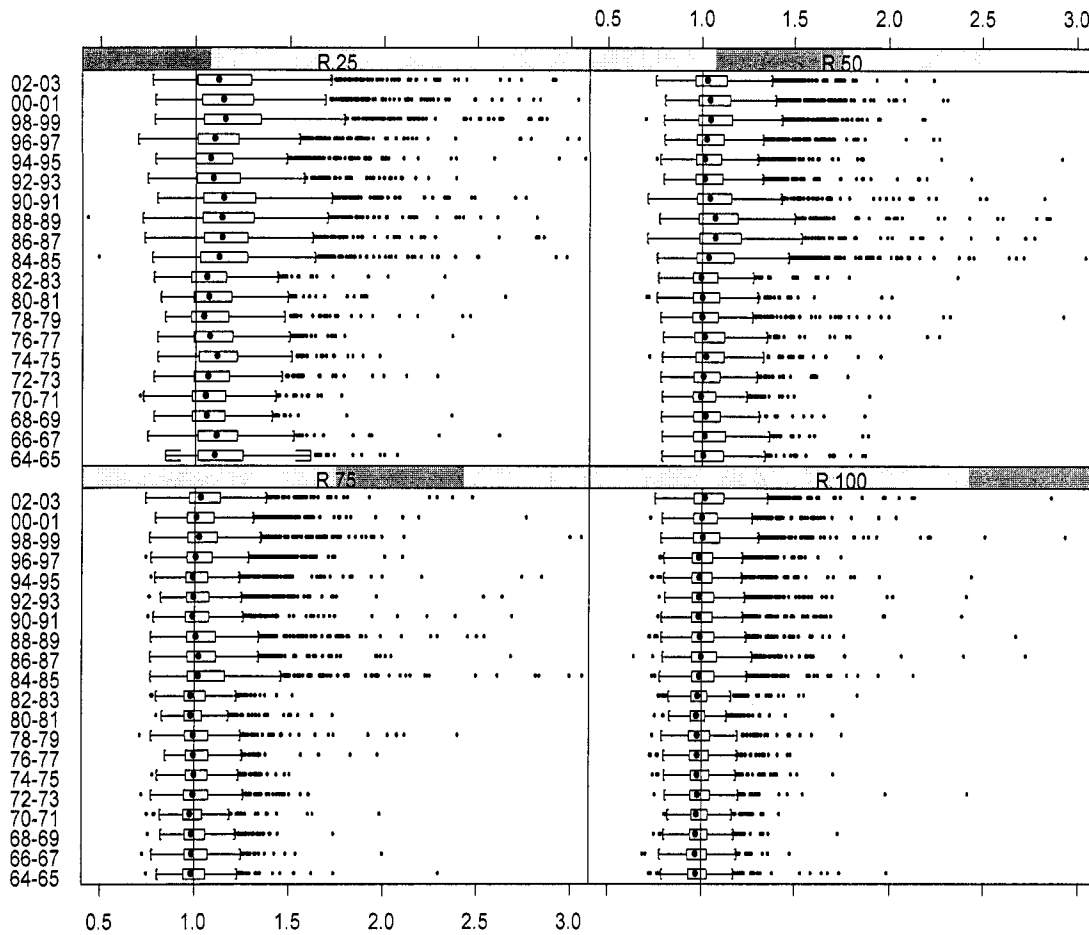
Figure 18. Ratio of OLS to ROB Alpha Standard Errors.
The cross-sectional distributions show a median ratio greater one for the two smallest size quartiles, while the largest size quartile shows a median ratio smaller than one.

The overall OLS standard errors are about 46% larger. Figure 18 shows that the median of the cross-sectional distributions for the two smallest size quartiles is greater than one, while the largest size quartile has a median slightly less than one. Responsible for the seeming contradiction are the properties of the ROB estimator, namely bias protection and efficiency loss. The bias robustness of the MM-estimator prevents the inflation of its standard errors when influential outliers are present, but inflates the standard errors only very slightly when influential outliers are not present, as is the case with most of the larger size firms.

2.6.3. Prediction

Alphas obtained from historical data must show a certain degree of predictability over future time periods to be useful in applications. This section compares the performance of the OLS and ROB estimator using the *CRATIO* (Section 2.4). For a firm to be included in this study it had to be listed for a minimum of four years consecutively. This condition left 17,402 firms in the predictive study.

Table 5 displays the mean and median *CRATIO* for the alpha estimates for both the one-week-ahead and one-quarter-ahead predictions by return quartiles.

Table 5. Conditional Ratios (*CRATIO*) of Prediction Errors (OLS to ROB).

	No. Firms	One-Week-Ahead		One-Quarter-Ahead	
		Median	Mean	Median	Mean
R.25	4349	1.183	1.279	1.097	1.217
R.50	4351	1.108	1.175	1.068	1.146
R.75	4351	1.066	1.230	1.031	1.139
R.100	4351	1.041	1.081	1.017	1.080

A *CRATIO* of 1.279 means a 27.9% increase in prediction error of the OLS estimate. The positive bias of the mean hints at a great number of positive large *CRATIO*, with much higher OLS than ROB *RMSE*.

All median *CRATIO* values are greater than one. This indicates a smaller *RMSE* and therefore a better performance of the ROB estimator. Further, the positive bias of the mean, compared to the median *CRATIO*, indicates that many *CRATIO* values are substantially greater than one. The ROB estimator performs slightly better on weekly than on quarterly predictions. For weekly alpha predictions, the gains are greatest for the smallest size quartile and minimal for the largest size quartile, while for quarterly alpha predictions the gains are small and uniform across firm size. Overall, the ROB estimator has clearly superior prediction capabilities.

2.6.4. Discrete versus Continuous Returns

Figure 19 shows the distribution of pair-wise alpha differences estimated from discrete and continuous returns for both the OLS and ROB estimator.
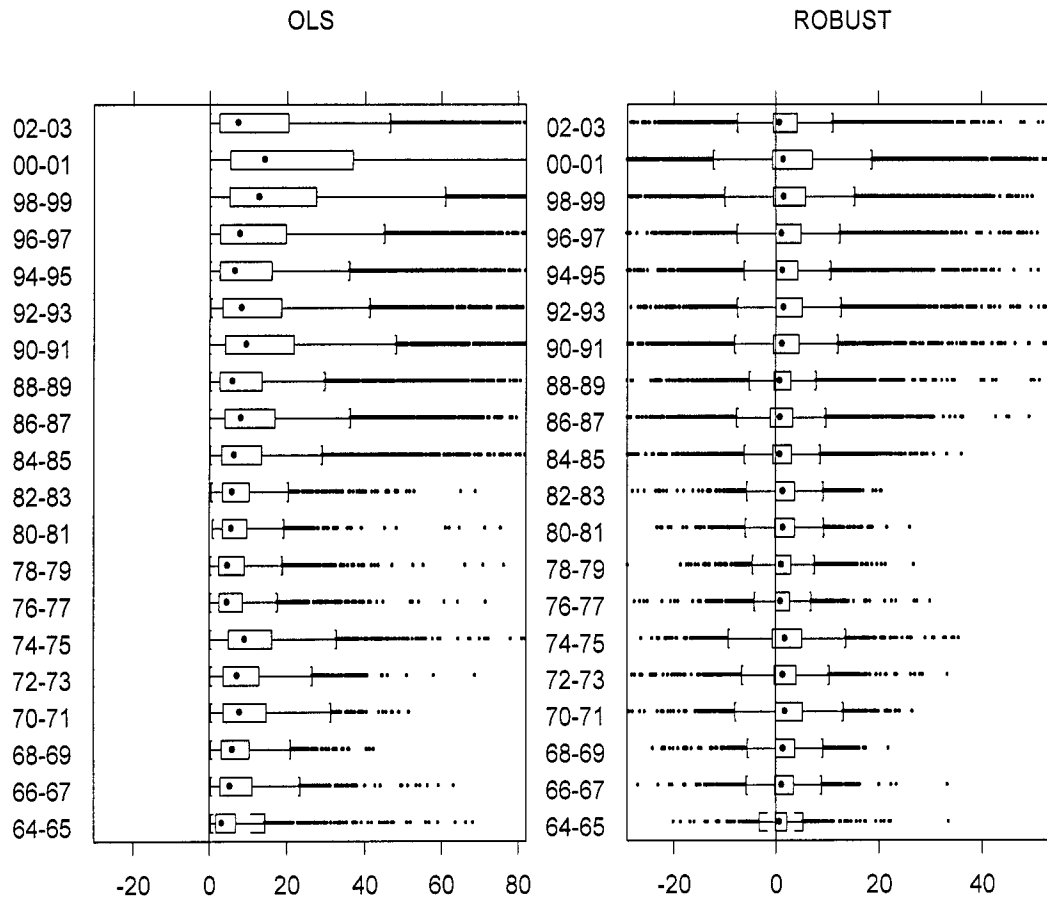
Figure 19. OLS and ROB Alpha Estimates on Discrete and Continuous Returns.
OLS alphas differences are consistently positively biased, while ROB alphas differences are more symmetrically, though still showing a positive median bias across all time periods.

The effects of choice of return-type are quite remarkable for the OLS estimator. It shows a systematic positive bias towards alphas computed from discrete returns. The median differences are as large as 10% annually. The bias has also been steadily increasing over the years with only a small decrease in the 2002-2003 time period. On the other side, ROB alphas are indifferent to large outliers in discrete returns, rejecting them, and therefore arriving almost at the same results as with continuous returns. The use of

continuous returns can therefore help to reduce bias caused by asymmetry but is of little help with respect to reducing bias.

2.6.5.   Clean versus Dirty Returns Data

The left panel of Figure 20 shows pair-wise scatterplots of OLS alphas computed on clean data (horizontal axis) and on dirty data (vertical axis). The right panel of Figure 20 shows pair-wise scatterplots of ROB alphas.
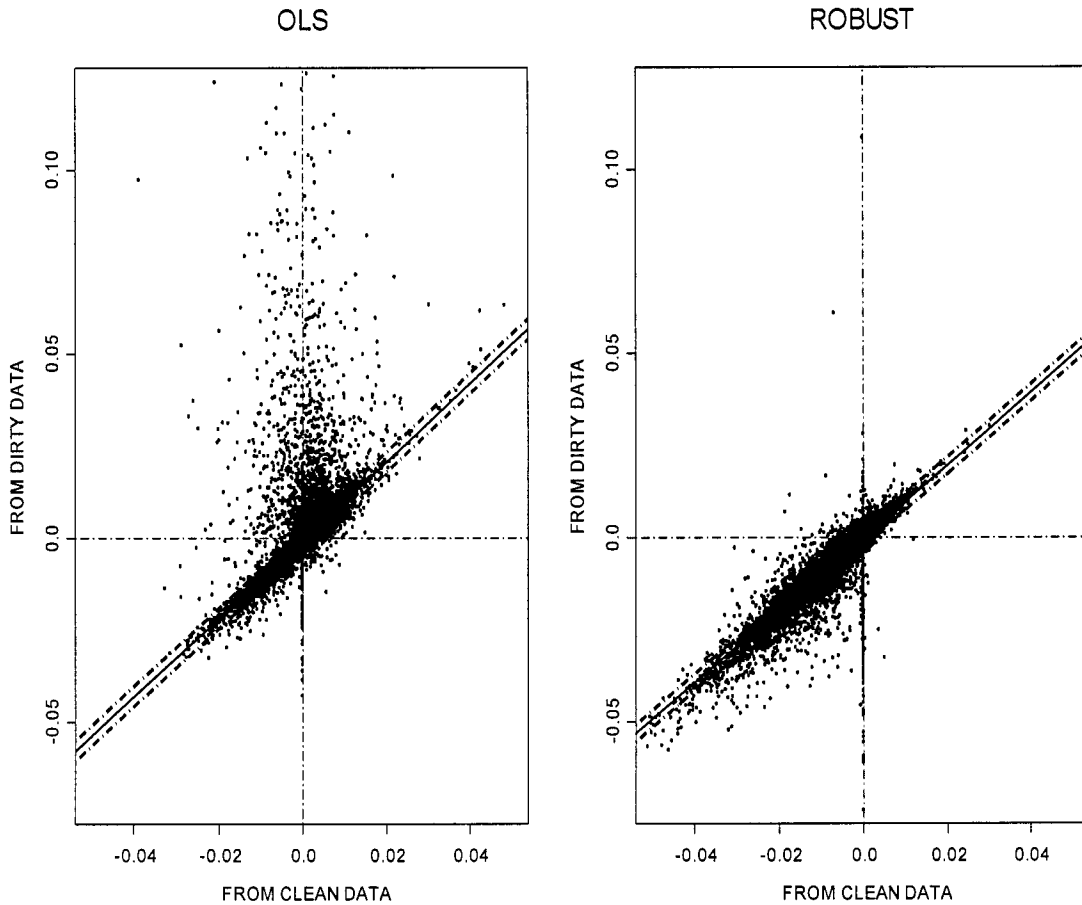
Figure 20. OLS and ROB Alpha Estimates on Clean and Dirty Returns.
The solid lines through the distribution have intercept zero and slope one. The dotted lines are plotted at ± two ROB standard deviations to the center line. The center of the distribution of OLS alphas computed on dirty data is even more positively biased than when computed on clean data. There are also a large number of positive outliers in the "dirty" data direction. The center of the ROB alpha scatterplot is shifted to the lower right.

If alphas were not influenced by the choice of returns type they would plot on a straight line through the origin with slope one. The left panel of Figure 20 shows that most OLS alphas plot close to the straight line; however, there are a considerable number of outliers. These outlying points are caused by splits and reverse splits in combination with positive and negative market returns. Overall, the OLS fit has an R^2 of only 21.4%, i.e., the variation of the clean alpha explains only 21.4% of the variation of the dirty alphas. The pair-wise scatterplot of ROB alphas, on the right panel of Figure 20, however, shows the majority of the points plot close to the straight line. The center of the scatterplot is located well into the 3rd quadrant. Further, the ROB fit of dirty on clean alphas has an R^2 of 91.7%, meaning that ROB alphas are quite reliable for returns data not adjusted for splits and dividends. The proposed ROB alpha estimator is not much influenced by outliers regardless of origin; i.e., whether outliers are true returns associated with unusually large price movements or results of erroneous data. Additionally, the center of the OLS scatterplot shifts from the upper right quadrant to the lower left quadrant for the ROB scatterplot, confirming the positive biased-ness of OLS alphas.

2.6.6. Conclusion

OLS alpha estimates are widely used by practitioners, either directly through the single-factor market model, or indirectly in most of the sophisticated multi-factor models. The results demonstrated that OLS alpha estimates cannot only be severely biased by small fractions of outlying returns or even just one single outlier, but can also have significantly inflated estimation errors. The ROB alpha, however, greatly reduced the bias due to outliers caused by non-Gaussian and asymmetric distributions, and thereby giving a better point estimate of alpha. In addition to controlling bias, ROB alphas have smaller standard errors that were not inflated by outliers. This has the important effect that the power of the t-tests increases, thereby providing a great tool to detect and further analyze outliers.

Slicing OLS and ROB alphas by return quartiles shows that firms with lower mean returns tended to have lower alphas and that alphas followed market trends, with lower alphas in times of down markets. However, the cross-sectional distribution of the pair-wise differences of OLS and ROB alphas was heavily positively skewed, showing that

OLS alphas were often overestimated. This held particularly for lower return firms with median differences between 10%-30% (annualized) and single larger differences well over 100%. In the empirical distribution of the pair-wise differences, over 80% of the differences were positive and about half of the lowest returning firms showed a difference greater than 50%. The ROB estimator rejected only a very small fraction of outlying returns and performed for all practical purposes as well as the OLS estimator in absence of outliers.

Even more important for the practitioners is the property of an estimator to predict future risk and return. Using the standard root-mean-squared-error performance measure when outliers are present (*CRATIO*), it has been shown that the median (mean) increase of the OLS alpha one-week-ahead prediction error, ranges from 4.1% (8.1%) for the highest return firm size quartile to 18.3% (27.9). For the lowest returning firm quartile and for the one-quarter-ahead prediction error, it ranges from 2.7% (8%) for the largest firm size quartile to 9.7% (21.7%) for the lowest returning firm quartile.

As a convenient by-product, it has also been shown that ROB alphas are insensitive to the choice of discrete over continuous returns, and when clean data is not available, situations where OLS alphas are highly biased.

In summary, the results strongly support the use of ROB alphas as a complement to, or even replacement of classical OLS alphas. When the two estimates agree there is usually nothing to worry about. When they disagree it is almost always because outliers or the asymmetry of returns is influencing the OLS alpha. The final decision to choose ROB over OLS estimators and to be protected against bias caused by a small fraction of outliers (and simultaneously solving the issue of choosing discrete versus continuous return types) is up to the investor. However, the question should be asked: is a large alpha estimate a good predictor of future excess returns if this alpha value is entirely caused by very small fractions of outliers, possibly only a single outlier?

2.7. Betas

Section 2.7.1 and Section 2.7.2 examine the performance of OLS and ROB betas, as well as their standard errors. Section 2.7.3 examines the idiosyncratic risk. Section 2.7.4 continues with an out-off-sample test of the prediction capabilities of OLS and ROB betas for one-week and one-quarter ahead time periods. Section 2.7.5 and Section 2.7.6 show the effects on betas of using discrete versus continuous returns betas, and clean versus dirty returns, respectively.

2.7.1. OLS versus ROB

Figure 21 shows betas computed using OLS regression for the 20 two-year time periods, sliced by firm size.
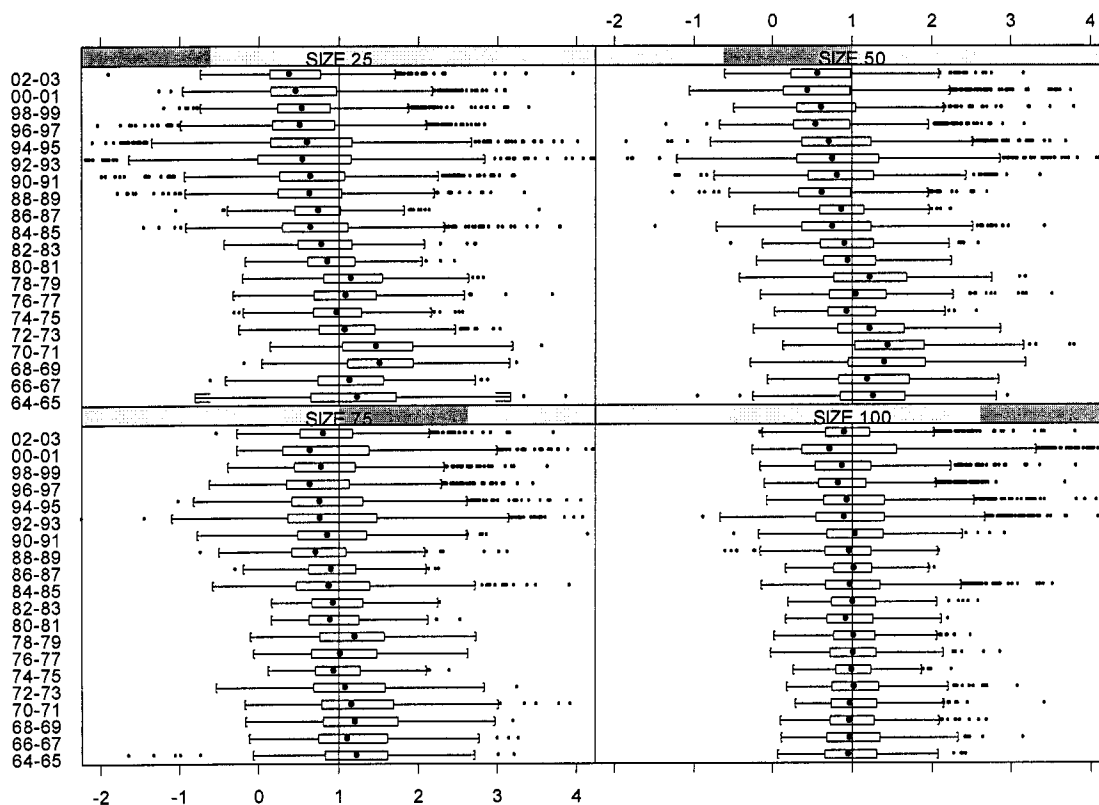
Figure 21. OLS Betas.
Median betas show little association with market events but a striking trend towards smaller values below the market beta of one, in particular for the smallest size quartile.

Figure 22 shows ROB betas for the same 20 two-year time periods, sliced by firm size.
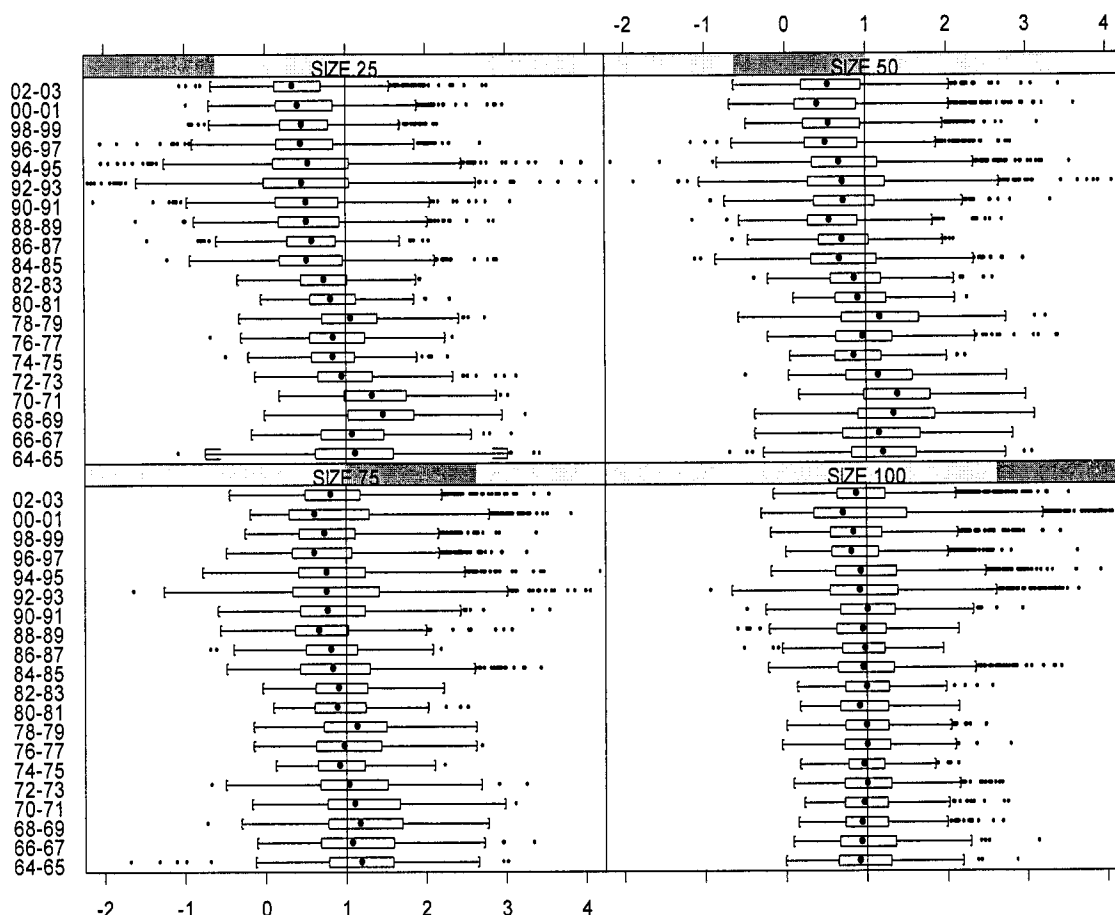
Figure 22. ROB Betas.
Median betas show little association with market events, but a striking trend towards smaller values below the market beta of one, in particular for the smallest size quartile. While more outliers start to appear in recent years, the width of the cross-sectional distribution on each of the two-year intervals is relatively constant.

Figure 21 and Figure 22 look very similar on first sight. The cross-sectional distributions of the OLS and ROB betas over time show little association with market events—only a consistent trend towards smaller beta values, with this effect being more pronounced for smaller size firms. The early oscillations of the median, up during 1968-1971 and down during 1972-1978, are most apparent for smaller sized firms, and may be related to market or political events, such as the Vietnam War and energy crisis in the early Seventies. However, after 1980 there is a consistent decrease in median betas across the

lower three size quartiles and even for the largest size quartile starting in 1986, indicating decreasing systematic risk, as seen within a CAPM framework; and, correspondingly increasing specific risk or presence of other risk factors that explain returns, see Section 2.7.3 for further discussion.

The similarity between Figure 21 and Figure 22 disappears when comparing the pair-wise differences of OLS and ROB betas, graphed in Figure 23.
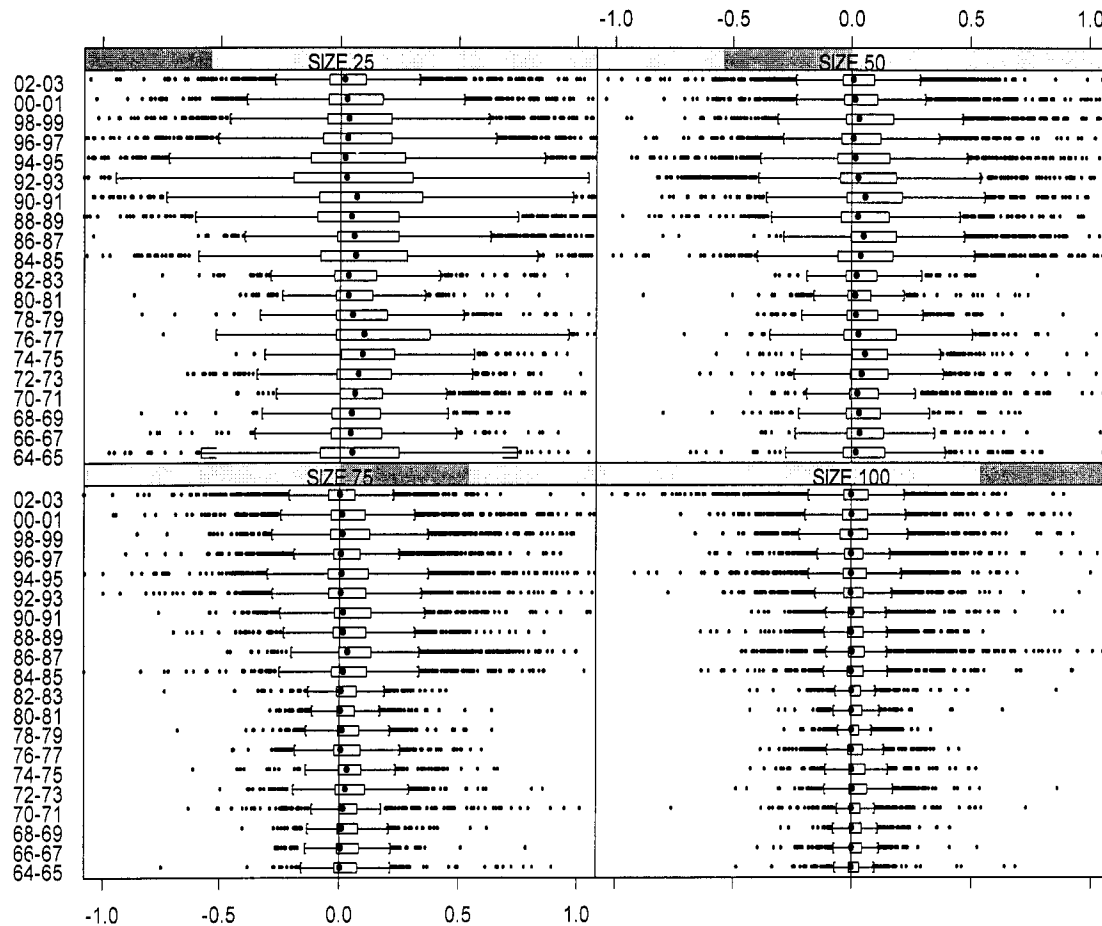
Figure 23. Pair-wise Differences of OLS and ROB Betas.
The nearly symmetrical cross-sectional distribution around zero with a positive median difference across all size quartiles indicate that OLS betas tend to over and understate the ROB betas.

The substantial difference between the OLS and ROB betas, despite that only a very small fraction of the most outlying returns are rejected, comes as a surprise. Over time, the distribution of the difference between OLS and ROB betas are fairly symmetric, with

some positive skewness about their mostly positive medians and a considerable number of very large differences across all size quartiles. There is an unclear pattern of the distribution over time, with an abruptly increased variability starting in 1984 across all firm sizes. This increase can possibly be attributed to the increasing number of firms (Figure 9).

Table 6 summarizes the pair-wise difference between OLS and ROB betas.

Table 6. Pair-wise Differences of OLS and ROB Betas.

	Min	1st Quartile	Median	3rd Quartile	Max
SIZE.25	-12.17	-0.05	0.045	0.22	6.47
SIZE.50	-2.35	-0.03	0.027	0.15	3.12
SIZE.75	-1.43	-0.023	0.014	0.101	2.66
SIZE.100	-1.01	0.018	0.0024	0.054	1.60

The differences show a positive asymmetry with median close to zero, with considerable numbers of very large differences across all size quartiles.

Figure 24 provides even more detail that shows the complementary empirical distribution function of the beta differences sliced on firm size.
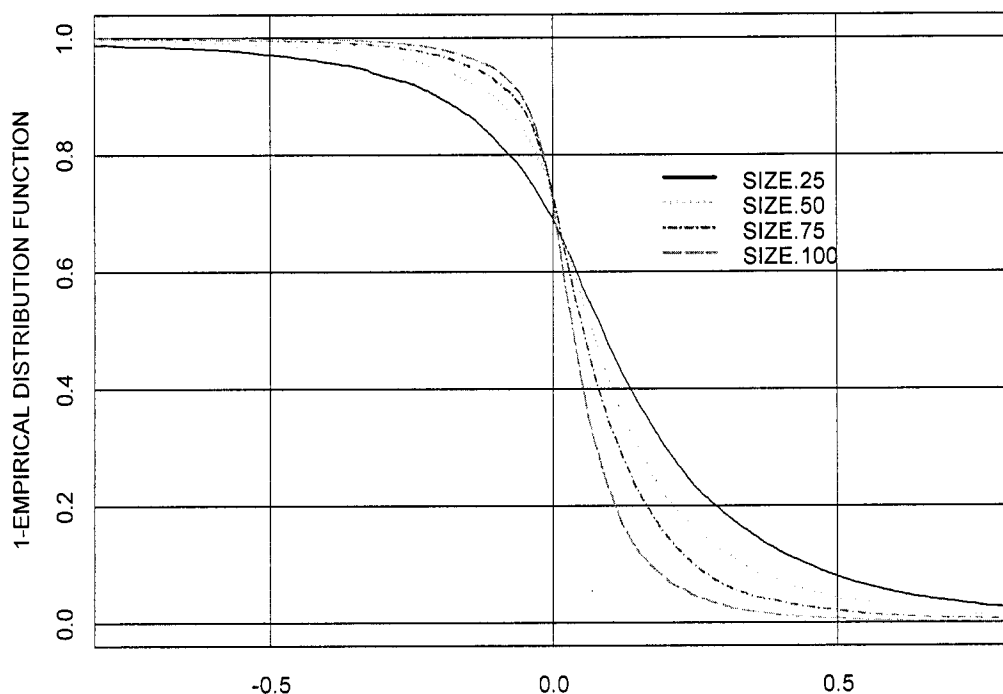
Figure 24. Distribution of Pair-wise Difference of OLS and ROB Betas.
Beta differences plotted on horizontal axis and percentage of firms on vertical axis. E.g., 20% of smallest quartile firms have a difference greater than 0.35 and about 70% of all firms have a positive beta difference.

For example, about 8.5% of the smallest sized firms have differences greater than 0.5, while about 3.5% have values less than -0.5, and together 12% have absolute differences greater than 0.5. In general, the graphs show that there are more positive differences than negative differences, and positive differences are larger than negative differences across all firm sizes. These results are reasonably consistent with Martin & Simin (2003) who studied a more limited time period of 1990 through 1997.

The beta differences from Figure 23, scaled by the estimate of the variability of the differences $\hat{V}_{DIFF}$ (Section 2.5) are shown in Figure 25.
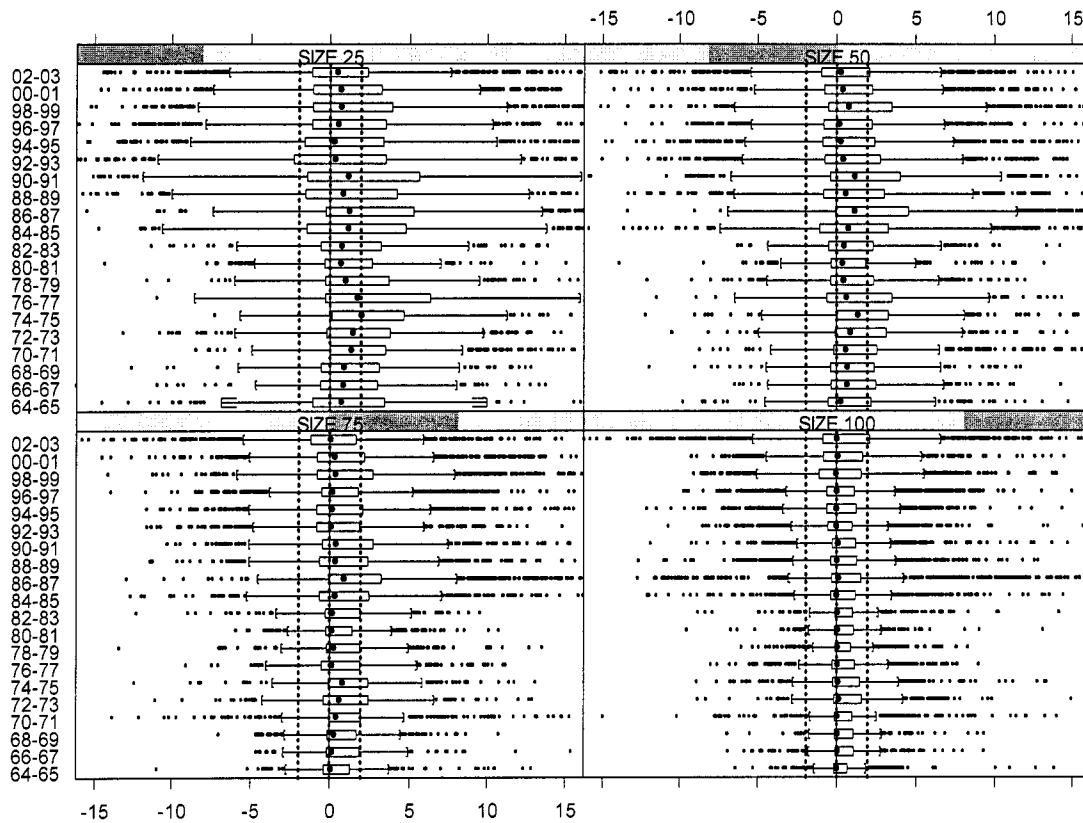
Figure 25. Scaled Pair-wise Difference of OLS and ROB Betas.
Beta differences are scaled by the crude variability of the difference of the estimators at the Gaussian model. Even though the medians are mainly within two standard deviations, a large portion of differences is significantly different from zero.

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The fraction of the significant t-statistics at the 5% level is much larger than what one expects under a null hypothesis of equal OLS and ROB beta. Thus, many of the differences between OLS and ROB estimates are statistically significant.

2.7.2. Standard Errors and Significance of Differences for Beta Estimates

As mentioned in Section 2.4, ROB estimators provide ROB beta standard errors that are not as inflated by outliers as OLS standard errors. Figure 26 compares the pair-wise ratios of OLS to ROB standard errors.
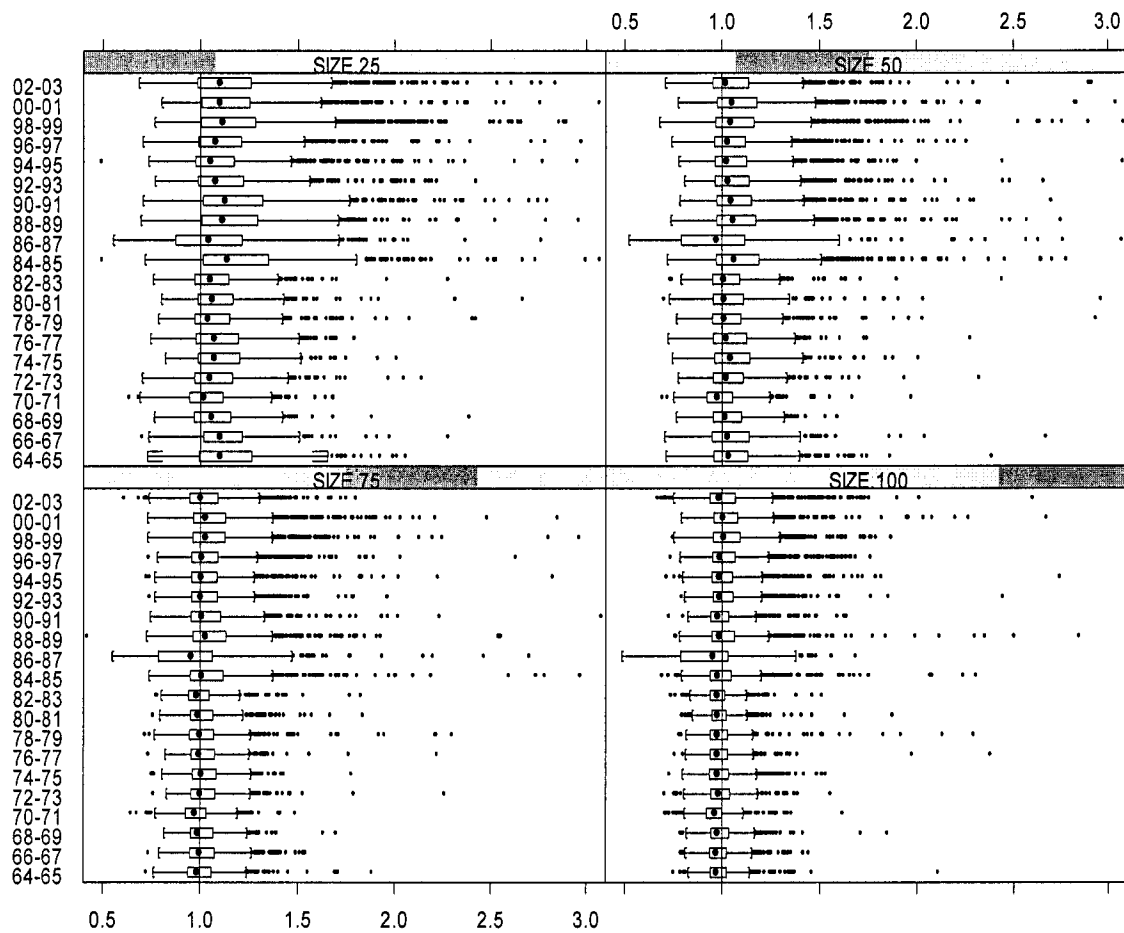
Figure 26. Ratio of OLS to ROB Betas Standard Errors.
The cross-sectional distributions show a positive median for the two smallest size quartiles. This is due to the dominant numbers of outliers in small size firms inflating the OLS standard errors, while the largest size quartile shows a slight negative median, which is due to the inefficiency of the ROB estimator when data contains no outliers.

The OLS standard error is on average about 43% larger than the ROB standard error. A closer look shows that, with the exceptions of the 1986-1987 period that included the October 1987 stock market crash, the median of the cross-sectional distributions for the two smallest size quartiles is greater than one, while the largest size quartile has a median slightly less than one. The bias robustness of the ROB betas prevents its standard errors from being inflated when influential outliers are present, while the high Gaussian efficiency of 99% of the ROB betas inflates the standard errors only very slightly when influential outliers are not present, as is the case with most of the larger size firms.

2.7.3. Idiosyncratic Risk Factors

The CAPM states that only market risk is rewarded and that the risk premium varies linearly in beta. As observed in Figure 21 and Figure 22, the distribution of OLS and ROB betas decreases over time to values well below the market beta value of one.

At the same time, the left panel of Figure 27 shows that the distributions of market returns do not show a monotonic decreasing time trend.

MARKET   INDIVIDUAL FIRMS
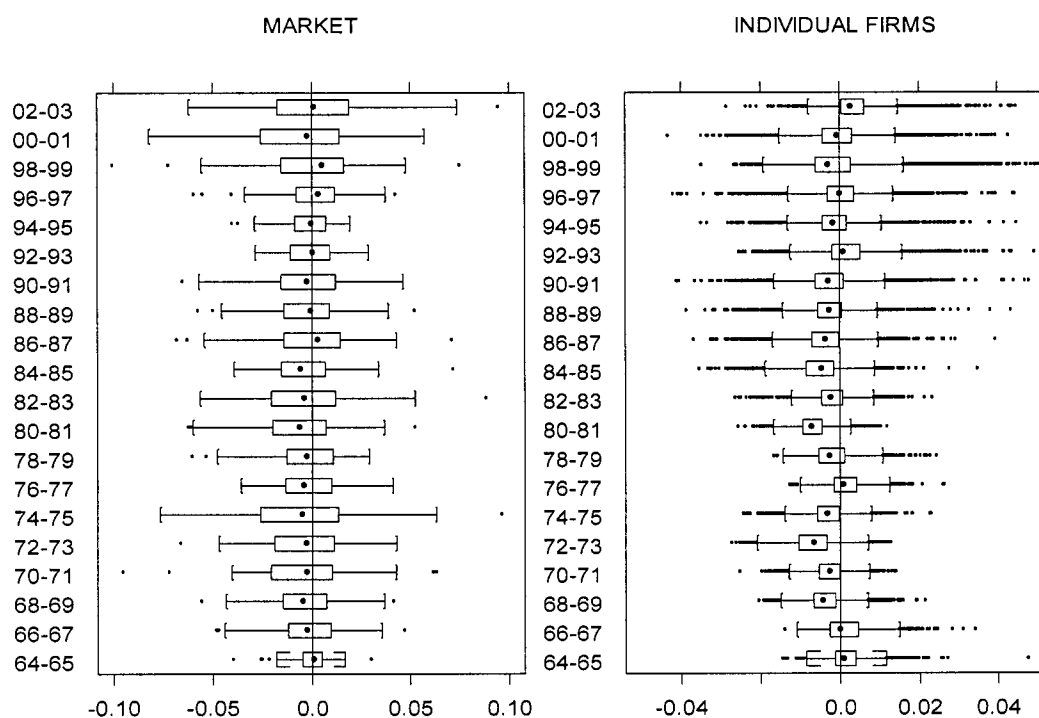
Figure 27. Market Returns and Mean Stock Returns.
The left panel summarizes market returns, while the right panel shows the distributions of mean returns. The horizontal scale of the left panel was restricted to not show the large negative outlier in the 1986-1987 time period caused by the October 1987 market crash.

According to CAPM this would imply that mean firm stock returns (right panel of Figure 27) should exhibit a monotonically decreasing expected return. However, the distributions of mean firm stock returns, in the right panel of Figure 27, reveal no such decreasing expected return pattern. This questions the validity of the CAPM, and suggests that other risk factors may be rewarded, including the possibility that idiosyncratic risk is rewarded. The median of the distribution of mean stock returns over time in the right-hand panel of Figure 27 is highly correlated with the median market returns in the left panel of Figure 27. However, unlike the market volatility, which tends to increase in down markets and decrease in up markets, firm returns show a significant increase in volatility since the mid-1980s.

Figure 28 shows boxplots of $\Delta_i = \bar{R}_i - \hat{\beta}_i \cdot \bar{R}_M$, where i is a firm index, and the mean

return estimates $\bar{R}_i$ and $\bar{R}_M$, and the beta estimates $\hat{\beta}_i$ are computed over each two-year

time interval, using both OLS and ROB betas.
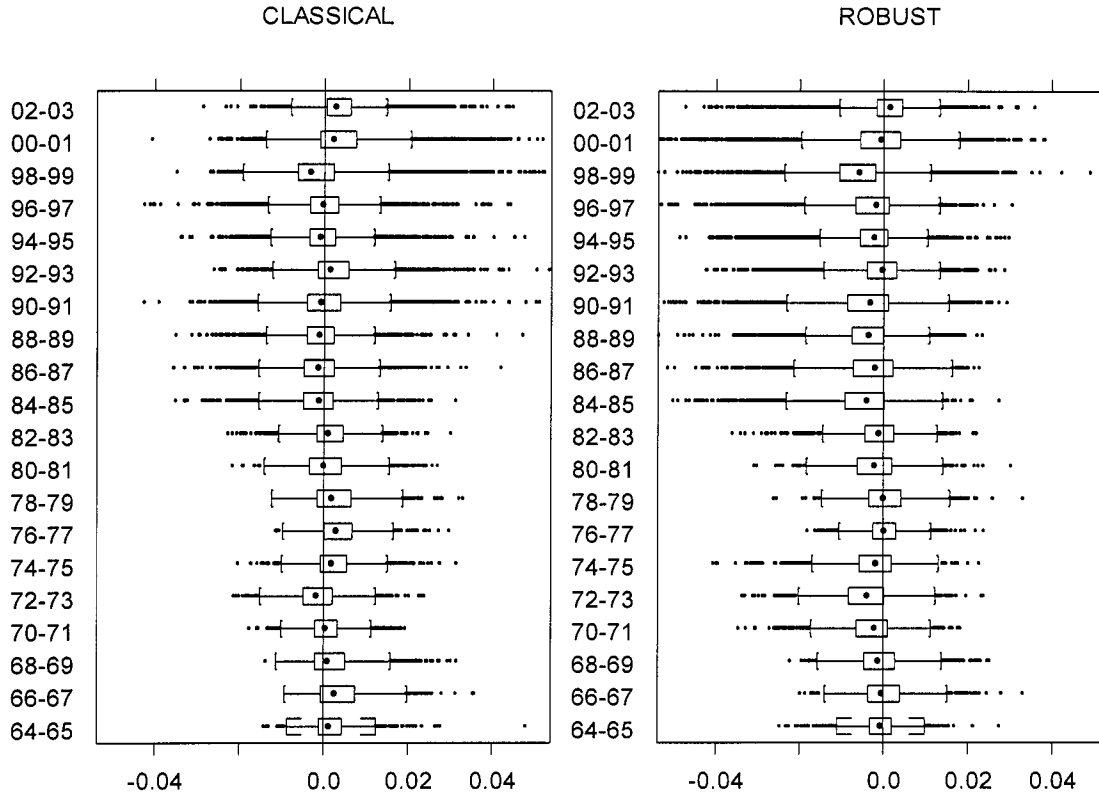
Figure 28. Difference of Firm Returns and Firm's Beta times Average Market Returns. Differences are taken for each firm and quantities are estimated on all two-year time intervals. The overall pattern is similar, though ROB median returns as well as skewness are shifted to the left.

The CAPM predicts that the average of the Δ_i should be close to zero, while the

decreasing betas of Figure 21 and Figure 22 and the results in Figure 27 predict that the

Δ_i should increase over time. However, Figure 28 does not reflect either type of

behavior, and exhibits only increasing volatility after 1982-1983 and some associated

skewness that is positive for the OLS betas and negative for the ROB betas.

This kind of contradiction has recently been observed by other researchers. E.g.,

Malkiel & Xu (1997) suggest that other idiosyncratic factors may be relevant and can

serve as risk proxies, in which case the role of beta in explaining cross-sectional returns

will decrease. Malkiel & Xu (1997) found that idiosyncratic volatility is useful in explaining cross-sectional returns. Further, Campbell, Lettau, Malkiel, & Xu, (2001) found strong evidence of a positive deterministic trend in the idiosyncratic firm-level volatility, not caused by an increase in numbers of publicly traded companies. Even after hand deletion of outliers associated with market crashes, such as October 19th, 1987 ("Black Monday"), in the variance calculations, they find that firm-level variance doubled between 1962 and 1997, with no similar trends in industry or market volatility.

This dissertation finds similar results as follows: the single factor model (1.3) with the usual assumptions results in the variance decomposition $\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{\varepsilon,i}^2$, where, on each two-year time interval, the sample standard deviation estimates $\hat{\sigma}_i$ and $\hat{\sigma}_M$ are directly computed from each firm's returns and the market returns, and the estimates $\hat{\beta}_i$ and $\hat{\sigma}_{\varepsilon,i}$ by fitting the model (1.3) via a OLS or ROB regression over the same time interval. Since the residual standard error $\sigma_{\varepsilon,i}$ represents the idiosyncratic risk, an increase of the ratio $\sigma_{\varepsilon,i}/\sigma_i$ over time would indicate a decreasing contribution of systematic risk, and an increasing presence of idiosyncratic risk.

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The systematic contribution to total risk, $\hat{\beta}_i \hat{\sigma}_M / \hat{\sigma}_i$, is displayed in the left panel, while

the ratio $\hat{\sigma}_{\varepsilon,i} / \hat{\sigma}_i$, estimated over each two-year time interval, is displayed in the right

panel of Figure 29.
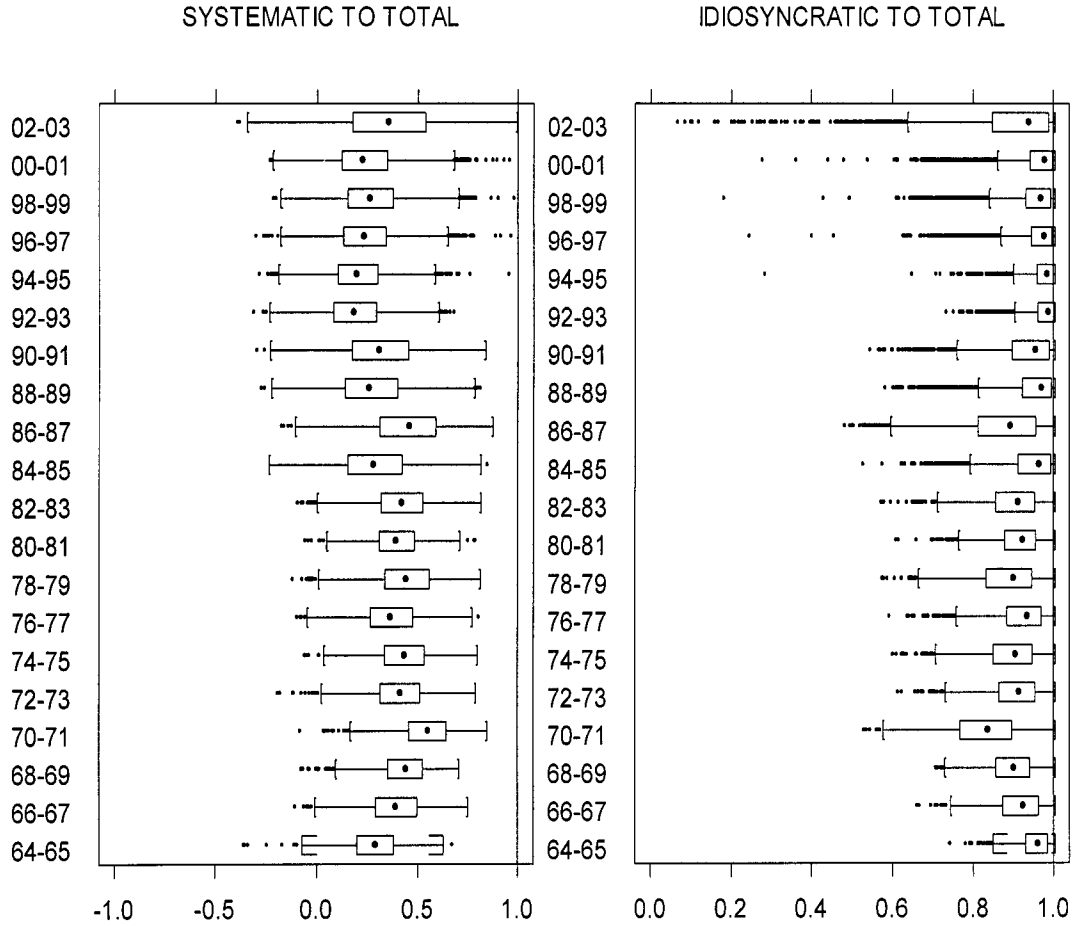
Figure 29. Ratio of Systematic and Idiosyncratic to Total Risk.
Left panel: systematic to total risk, $\hat{\beta}_i \hat{\sigma}_M / \hat{\sigma}_i$. Right panel: idiosyncratic to total Risk,
$\hat{\sigma}_{\varepsilon,i} / \hat{\sigma}_i$. The left panel shows that the systematic risk component remains fairly
constant, while the left panel shows that the dominating portion of the total risk is the
idiosyncratic risk, which increased since the 1970-1971 time period until the 2002-
2003 time period.

The right panel of Figure 29 reveals a varying pattern of location and scale (volatility)

over time. The idiosyncratic risk decreases only from 1964 to 1971 and shows, thereafter,

an overall increasing trend until 2002. It consumes almost the total risk, with a distinctive

shift to the left in 2003-2004. The left panel of Figure 29 shows, overall, a relative constant pattern in location as well as scale. A good explanation for the shift in 2002-2003 is still missing.

Of further interest is the contribution to the total variance. On the volatility scale, Figure 30 shows the differences $\hat{\sigma}_{e,i} - \hat{\beta}_i \hat{\sigma}_M$ for all firms, where the estimates are computed on two-year time intervals, using both classical OLS and ROB model fitting.

CLASSICAL   ROBUST
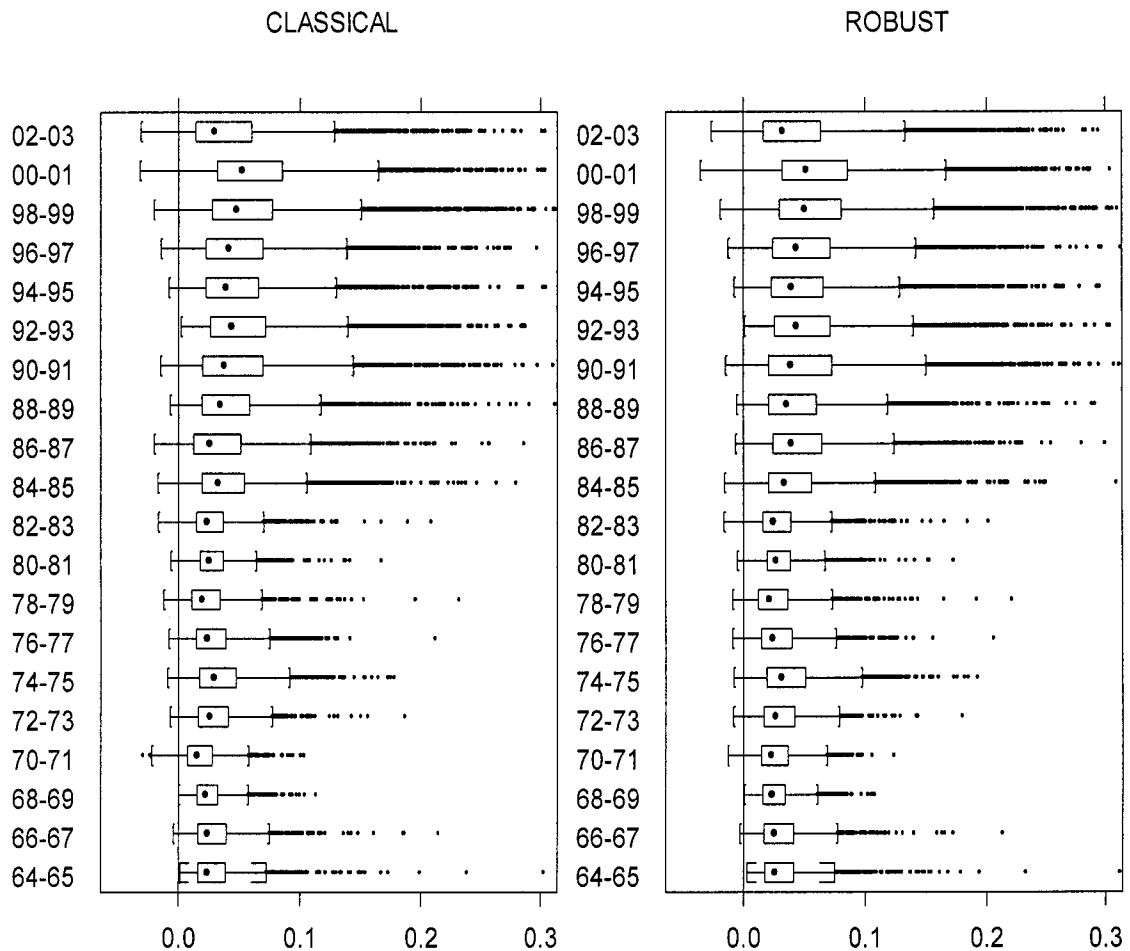
Figure 30. Residual Standard Error minus Firm's Beta times Market Volatility.
For each firm, the differences are estimated on two-year time intervals. Classical and ROB differences are very similar. The residual standard error is increasingly dominating the total volatility, i.e., the idiosyncratic risk is systematically increasing until the 2002-2003 time interval

The predominantly positive locations of the distributions show that the residual volatility is almost always the main contributor to the overall volatility. It also shows, starting in 1984-1985, that residual volatility is steadily increasing, with a steep drop in 2002-2003.

2.7.4. Prediction

Betas obtained from historical data must show a certain degree of predictability over future time periods to be useful in applications. This section compares the performance of the OLS and ROB estimator using the *RMSE* and *CRATIO* introduced in Section 2.4. For a firm to be included in this study, it had to have at least four years of consecutive listing, which left 17,402 firms in the predictive study.

Table 5 displays the mean and median increase of *CRATIO* for the beta estimates, for both the one-week-ahead and one-quarter-ahead predictions, sliced by size quartiles. The *CRATIO* values for the ROB betas increase, for both weekly and quarterly predictions across all groups indicating a significantly smaller *RMSE* of the ROB estimator. Note the positive bias of the mean compared to the median *CRATIO*, indicating great number of very large *CRATIO* values. Furthermore, the median gains are higher for weekly predictions than for quarterly predictions. For weekly beta predictions, the gains are greatest for the smallest size quartile and minimal for the largest size quartile, while for quarterly beta predictions, the gains are small and uniform across firm size.

Table 7. Conditional Ratios (*CRATIO*) of Prediction Errors.

		Beta			
		One-Week-Ahead		One-Quarter-Ahead	
	No. Firms	Mean	Median	Mean	Median
SIZE.25	4310	1.334	1.109	1.207	1.065
SIZE.50	4351	1.152	1.073	1.189	1.071
SIZE.75	4351	1.120	1.049	1.226	1.069
SIZE.100	4351	1.059	1.015	1.217	1.069

Mean and median of *CRATIO* of OLS to ROB *RMSE* for betas, sliced by firm size quartile. The positive bias of the mean hints at a great number of positive large *CRATIOs*. E.g., a *CRATIO* of 1.334 means a 33.4% increase in prediction error of the OLS estimate.

2.7.5. Discrete versus Continuous Returns

As discussed in Section 1.3.1.3, both OLS and ROB betas are consistent estimates even when the error distribution in the single factor model (1.3) is asymmetric. Thus, one would expect OLS and ROB betas estimates to be similar when compared, not between each other, but compared on different choices or return types.

Figure 31 compares the distribution of the paired beta differences between using OLS and ROB regression on discrete and continuous returns.
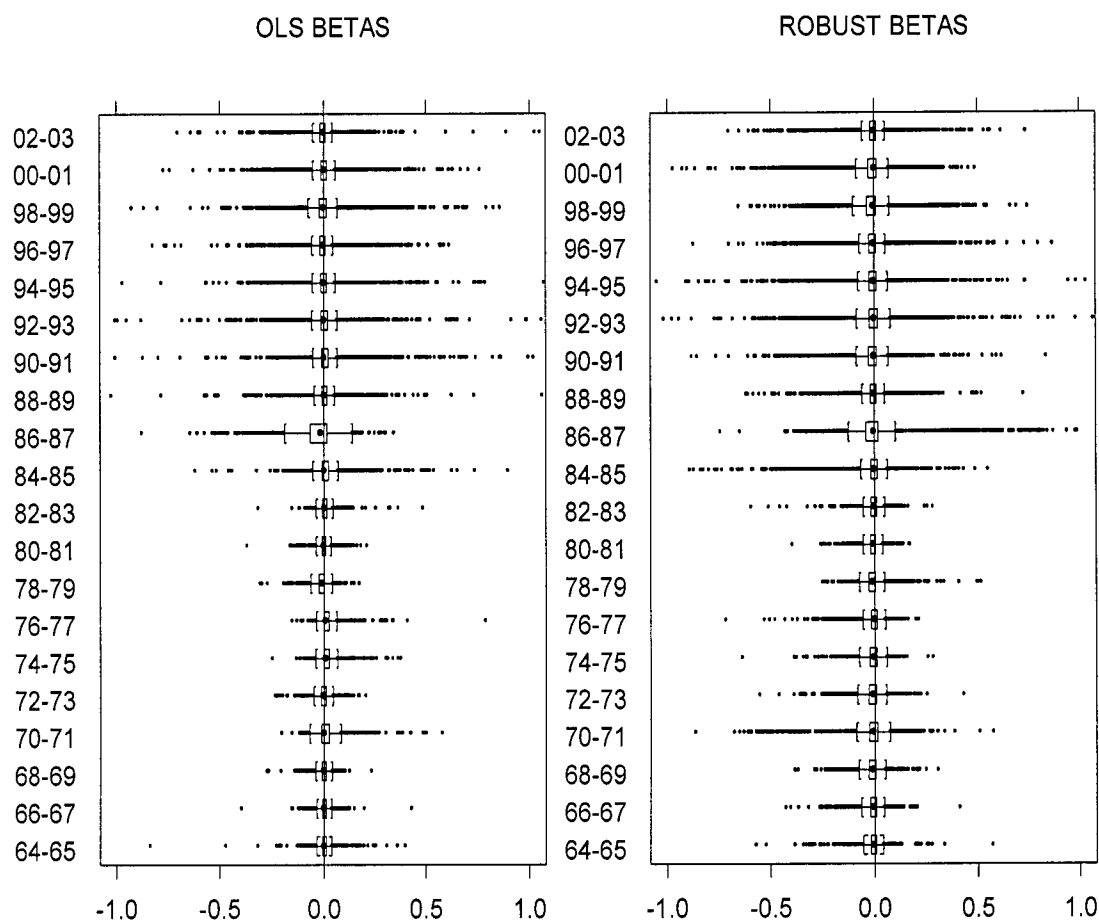
Figure 31. Betas: OLS and ROB Estimates on Discrete and Continuous Returns.
Betas are computed via OLS and ROB regression using discrete and continuous returns. Due to the properties of the slope estimate (Section 1.3.1.3), the symmetrizing log-transformation does not have strong effects. Note however, that the log-transformation does not solve the problem of kurtosis.

While there is somewhat greater variability in the values of both OLS and ROB estimates with respect to the choice of returns definition for decreasing firm size, in the majority, the differences are quite small, e.g., with respect to a market beta of one.

2.7.6. Clean versus Dirty Returns Data

The left panel of Figure 32 shows OLS betas computed on clean data on the horizontal axis and on dirty data on the vertical axis. The right panel shows the ROB beta counterpart.
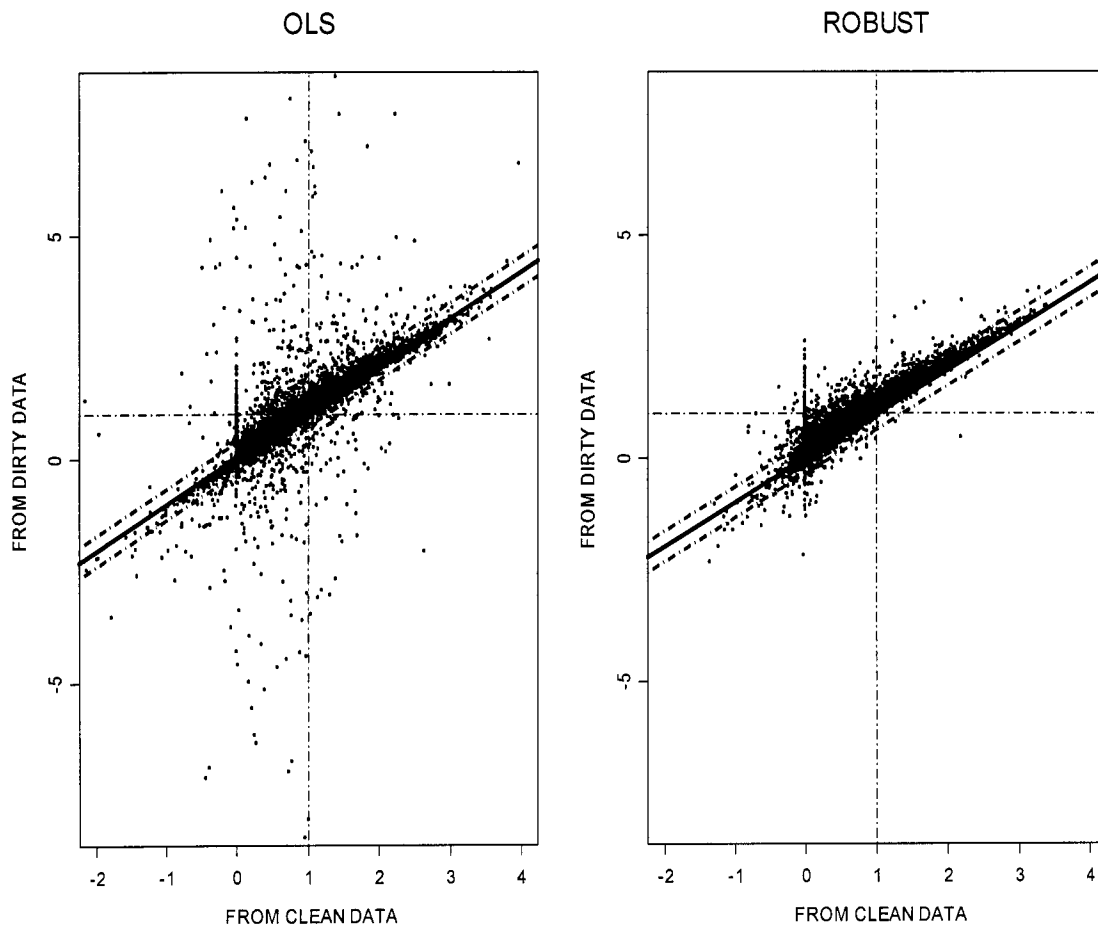
Figure 32. Betas: OLS and ROB Estimates on Clean and Dirty Returns.
BETA is computed, using OLS and ROB regression, from clean and dirty returns for 18314 firms listed at least over a two-year time period. The center straight lines have intercept zero and slope one, the dotted line are plotted at plus and minus two ROB standard deviations to the center line. OLS *BETA* a wide spread indicating many large differences due to data imputations.

If betas were not influenced by the choice of returns type, they would plot on a straight line through the origin with slope one. The left panel of Figure 32 shows that the points plot close to the straight line, about 6% at a distance more than twice the average standard error of the ROB betas on clean data, and that there are a considerable number of outlying points. These outlying points are caused by splits and reverse splits in combination with positive and negative market returns. Overall, the OLS fit has an R^2 of 21.4%, i.e., the variation of the clean beta explains only 21.4% of the variation of the dirty betas. The right panel shows that the majority of the points plot close to the straight line, with about 5.6% at a greater distance than twice the ROB standard error of the beta estimate of the clean data. The ROB fit of dirty on clean betas has an R^2 of 91.7%. This means that ROB betas quite reliable for returns data not adjusted for splits and dividends.

In summary, the ROB beta estimator has the attractive property to be relatively immune to data errors, such as not adjusting for splits and dividend distributions, thereby offering an easy solution to arrive at acceptable estimates, even if the underlying data contains small fractions of unclean or erroneous entries.

2.7.7. Conclusion

It was shown that the commonly used OLS regression method of estimating betas can be severely biased by small fractions of outliers, while the ROB betas greatly reduced the bias due to outliers. At the same time, the ROB betas achieved a high efficiency when the returns are Gaussian. In addition to controlling bias, ROB betas had smaller standard errors that are not inflated by outliers. It has also been shown that ROB betas are consistent estimates under asymmetric error distributions; and, as a consequence, the above results hold even when the returns distribution is positively skewed.

The cross-sectional distribution of the pair-wise differences of OLS and ROB betas were reasonably symmetric about a slightly positive median. The spread tended to widen in times of down markets, an effect particularly noticeable in the smallest size quartile. For the smallest firm size quartile, 75% of the firms had an OLS versus ROB beta difference that ranges in value between 0.20 and 1, though beta differences much larger than one were visible across all size quartiles and time periods. It is to be noted that these differences were obtained using a ROB estimator that rejects only 1-3% of outliers and

performs for all practical purposes, as well as OLS when the data has a Gaussian distribution. Differences of this magnitude will be financially significant to many investors, who need to be alerted to such differences, and who often will prefer the ROB beta as a better description of the vast majority of the data.

The most important property of a good beta estimate is its ability to accurately predict over future time periods. Using the standard root-mean-squared-error performance measure when outlier are present (*CRATIO*), it has been shown that the median (mean) increase of the OLS beta one-week-ahead prediction error ranges from 1.5% (5.9%) for the largest firm size quartile to 10.9% (33.4) for the smallest firm size quartile, and for the one-quarter-ahead prediction error from 6.9% (21.7%) for the largest firm size quartile to 6.5% (20.7%) for the smallest firm size quartile.

As a convenient by-product, it has also been shown that the ROB beta is useful when the data is not clean, e.g., not adjusted for stock-splits, or simply containing gross data recording or transmission errors.

In summary: the results strongly supported the use of ROB betas as a complement to or even replacement of classical OLS betas. When the two estimates agreed, there was usually nothing to worry about, but when they disagreed, it was almost always because outliers were influencing the OLS beta. The investor should be alerted by such differences to investigate the causes and timing of outliers, and may well prefer the ROB beta as a better indicator of risk and return.

3. Cross-Sectional Regression

3.1. Introduction

This chapter analyzes the multi-stage procedure of Fama & MacBeth (1973), and applies ROB statistics on various steps and evaluates its impact.

The Fama & MacBeth (1973) *cross-sectional regression technique* (FM) is not only an historically important method, but is also still one of the most widely used tools in basic empirical finance, with main contributions in empirically validating the CAPM or ATP models. The empirical FM approach is a very intuitive three-step procedure and can easily be extended to time-varying factors (Campbell, Lo, & MacKinlay, 1997; Cochrane, 2001; Elton & Gruber, 1995). The three-step procedure works as follows:

S I: Use *ordinary least squares* (OLS) regression to fit the single-factor market model (1.3), with $R_B = R_M$.

$$R_{i,t} = \alpha_i + \beta_i \cdot R_{M,t} + \varepsilon_t, \quad t = 1,...,T, \, i = 1,...,N, \quad \alpha, \beta \in \Re \qquad (3.1.1)$$

For each security N, this results in an estimate $\hat{\beta}_i$ of the CAPM measure of market risk.

S II: Given the $\hat{\beta}_i$ and residual standard errors $\hat{s}_i$ computed in S I for time period t, and $\tilde{R}_{it}$ the one-month percentage return on security i in time period t-1 to t, use the cross-sectional regression model (1.4) and subsets of risk factors

$$\tilde{R}_{it} = \tilde{\gamma}_{0t} + \tilde{\gamma}_{1t}\hat{\beta}_i + \tilde{\gamma}_{2t}\hat{\beta}_i^2 + \tilde{\gamma}_{3t}\hat{s}_i + \tilde{\eta}_{it}, \quad i = 1,...,N \qquad (3.1.2)$$

to produce time series of estimates of $\tilde{\gamma}_{jt}$, assuming that $\tilde{\eta}_{it}$ is independent of the predictor variables.

S III: Analyze the time series of the $\hat{\tilde{\gamma}}_{jt}$ estimates using averages and t-statistics to test for the main three hypotheses C1 through C3:

C1: In any efficient portfolio, the relationship between a security's expected return and risk is linear, i.e., $H_0 : E[\gamma_{2,t}] = 0$.

C2: β_i is a complete measure of the risk of security i in the efficient market

portfolio, i.e., $H_0 : E[\gamma_{3,t}] = 0$.

C3: Higher risk should be associated with higher expected excess returns, i.e.,

$H_0 : E[\gamma_{1,t}] = E[R_{M,t}] - E[R_{f,t}] > 0$, with $R_{f,t}$ the risk free rate.

FM recognized that in S III, the use of estimated $\hat{\beta}_i$ in place of true β_i introduces an

errors-in-the-variables problem, and that $\hat{\beta}_i$ averaged over portfolios are more precise

estimates of the true β_i. Therefore, FM decided to group stocks into 20 portfolios based

on ranked values of $\hat{\beta}_i$.

However, high (low) observed $\hat{\beta}_i$ tend to be above (below) the true β_i and portfolios

formed that way tend to over (under) estimate the true β_i. To avoid this serious

regression problem, FM chose to use different data to first form the $\hat{\beta}_p$-portfolios, then

estimated the initial $\hat{\beta}_p$-values, and finally to ran cross-sectional regression for the

testing period. This was done as follows:

Formation: Over a 5 or 7-year formation period, $\hat{\beta}_i$ are estimated for each stock and

initial portfolios were formed by grouping all stocks into 20 portfolios based

on their ranked $\hat{\beta}_i$.

Estimation: Over subsequent 5-year estimation periods following the formation period,

$\hat{\beta}_i$ are re-estimated for each stock and $\hat{\beta}_p$ re-computed by averaging over

each of the 20 portfolios.

Testing: Over another final subsequent 4-year testing period, $\hat{\beta}_p$ were re-averaged

monthly (without re-computing the $\hat{\beta}_i$-component) to allow for delisting of

firms, while the individual $\hat{\beta}_i$ components were annually re-computed over

the beginning of the estimation period to the end of the current year of the

testing period.

To test hypothesis C1, C2, and C3, FM used the following four cross-sectional regression models (subsets of (3.1.2)), labeled Panel A through Panel D:

$$\text{Panel A: } R_{p,t} = \gamma_{0,t} + \gamma_{1,t}\hat{\beta}_{p,t-1} + \eta_{p,t}$$

$$\text{Panel B: } R_{p,t} = \gamma_{0,t} + \gamma_{1,t}\hat{\beta}_{p,t-1} + \gamma_{2,t}\hat{\beta}^2_{p,t-1} + \eta_{p,t}$$

$$\text{Panel C: } R_{p,t} = \gamma_{0,t} + \gamma_{1,t}\hat{\beta}_{p,t-1} + \gamma_{3,t}\overline{s}_{p,t-1} + \eta_{p,t}$$

$$\text{Panel D: } R_{p,t} = \gamma_{0,t} + \gamma_{1,t}\hat{\beta}_{p,t-1} + \gamma_{2,t}\hat{\beta}^2_{p,t-1} + \gamma_{3,t}\overline{s}_{p,t-1} + \eta_{p,t}$$

(3.1.3)

where $p = 1,...,20$. FM's results are summarized in FM, Table 3.

This chapter will be restricted to FM's results of Panel A and Panel D, since Panel B and Panel C deal with the linearity of the single-factor market model and sources of measures of risk, which are also tested in Panel A and Panel D.

The center of attention will be the reproducibility of the results by providing code written in S-PLUS (2001), available upon request, and the extension of FM by applying the code to data through December 2002.

Robust methods are used when forming β-portfolios, running cross-sectional regressions, and averaging over month-to-month regression coefficients. The impact of using robust methods may be washed out on a portfolio level (Cable & Holland, 2000; Sharpe, 1971). However, the use of robust MM-estimator may show significant differences when applied in S I through S III, and may also improve the regression model assumptions. Furthermore, the introduction of the MM-estimator, within the factor model framework, is intended to lay the groundwork for its use in larger multi-factor models, as in the framework of the (Fama & French, 1992) model and the Barra-type fundamental factor models (Chapter 4).

Section 3.2 describes the data. Section 3.3 calibrates the data to the results of FM. Section 3.4 extends FM through December 2002 and analyzes the month-to-month time series averages on various time intervals and tests conditions C1 through C3. Section 3.5 shows the effects of the robust estimator. Section 3.6 provides concluding comments.

3.2. Data

The data are monthly discrete returns (Section 2.2.4), adjusted for splits and dividend distributions for all stocks traded on NYSE during the time period January 1926 through December 2002. The market proxy is the equally weighted average return of all stocks listed on NYSE in month t and the risk-free rate the 1-month Treasury bill. The data was provided by the CRSP.

3.3. Calibration to FM

Since the FM procedure may be confusing, Table 8 recalls the different time periods and lengths of formation, estimation, and testing periods used in S I and S II.

Table 8. Overview of FM S I and S II Time Periods.

Period	S I		S II
	Formation	Estimation	Testing
1	1/1926-12/1929 5 years	1/1930-12/1934 5 years	1/1935-12/1938 4 years
2	1/1927-12/1933 7 years	1/1934-12/1938 5 years	1/1939-12/1942 4 years
3	1/1931-12/1937 7 years	1/1938-12/1942 5 years	1/1943-12/1946 4 years
4	1/1935-12/1941 7 years	1/1942-12/1946 5 years	1/1947-12/1950 4 years
...	...	...	...
9	1/1955-12/1961 7 years	1/1962-12/1966 5 years	1/1967-6/1968 1.5 years

The results in S II of Table 8 are time series of the month-to-month cross-sectional regression coefficients $\hat{\beta}_{p,t}$, $\hat{\beta}^2_{p,t}$, $\bar{s}_{p,t}$, and η_{pt} over the testing periods from 1/1935 through 6/1968. S III produces averages and t-statistics from these time series over various time-periods.

The different time periods over which FM analyzes S II are summarized in Table 9.

Table 9. Time periods used in FM, S III.

Period	S III Testing	Length
1	1/1935-6/1968	33.5 years
2	1/1935-12/1945	11 years
3	1/1946-12/1955	10 years
4	1/1956-6/1968	12.5 years
5	1/1935-12/1940	6 years
6	1/1941-12/1945	5 years
7	1/1946-12/1950	5 years
8	1/1951-12/1955	5 years
9	1/1956-12/1960	5 years
10	1/1961-6/1968	7.5 years

The comparison of the replicates will be restricted to regression coefficients, corresponding t-statistics, and coefficient of determination of FM, Table 3, Panel A and D. Due to software restrictions, the notation in graphs will be g_0, g_1, g_2, g_3, and R_f in place of $\gamma_0, \gamma_1, \gamma_2, \gamma_3$, and R_f respectively. Results that are based on the data in this study are referred to as replicates.

3.3.1. FM Table 1

In order to be included in a portfolio, a security must be available for at least 4 years in the portfolio formation period, for the full 5 years in the estimation period, and additionally in the first month of the testing period.

Figure 33 shows the percentage difference between the number of securities meeting the data requirements in FM, Table 1 and the replicates.
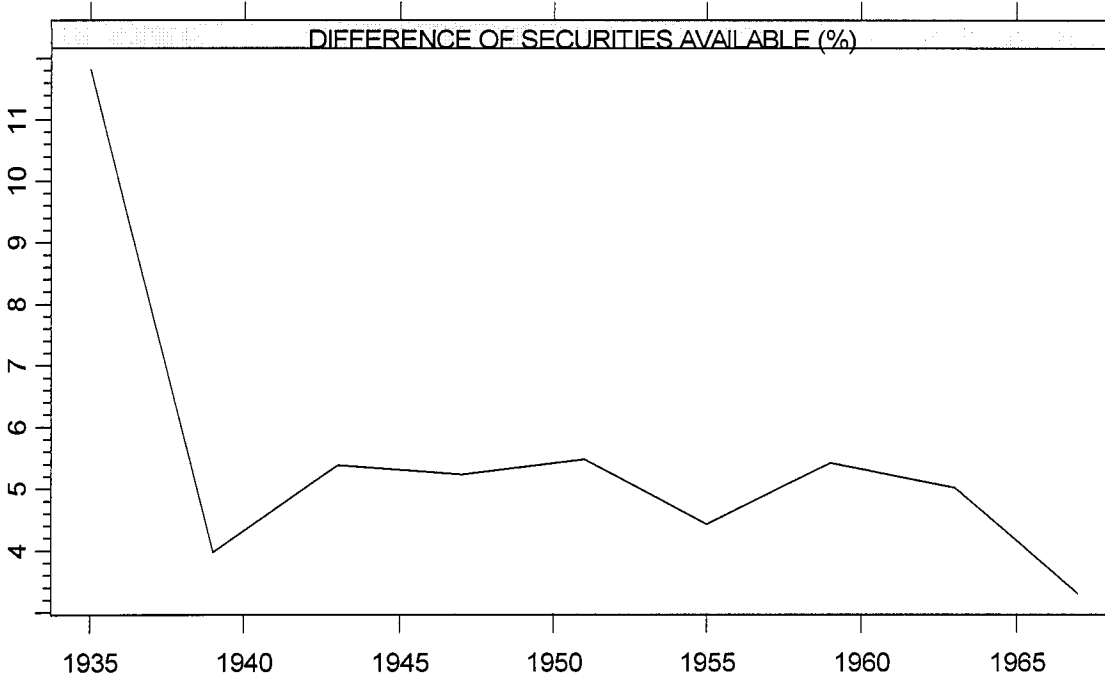
Figure 33. Percentage Difference of Available Securities in FM versus Replicates.

The number of available securities in this study is, overall, somewhat smaller than that in FM due to revisions of the CRSP database.

3.3.2. FM Table 2

FM, Table 2 provides sample statistics of the estimation periods only for the periods 2, 4, 6, and 8 of Table 8.

Figure 34 shows a qqplot (Section 1.4.1) of the values of the 20 portfolios $\hat{\beta}_{p,t-1}$ taken from FM, Table 2 versus the replicated $\hat{\beta}_{p,t-1}$ values of this study. The solid line has slope one and intercept zero. The closer the points are to the solid line, the better the match between FM, Table 2, and the replicates.

Period: 1934-1938

Period: 1942-1946

Period: 1950-1954

Period: 1958-1962
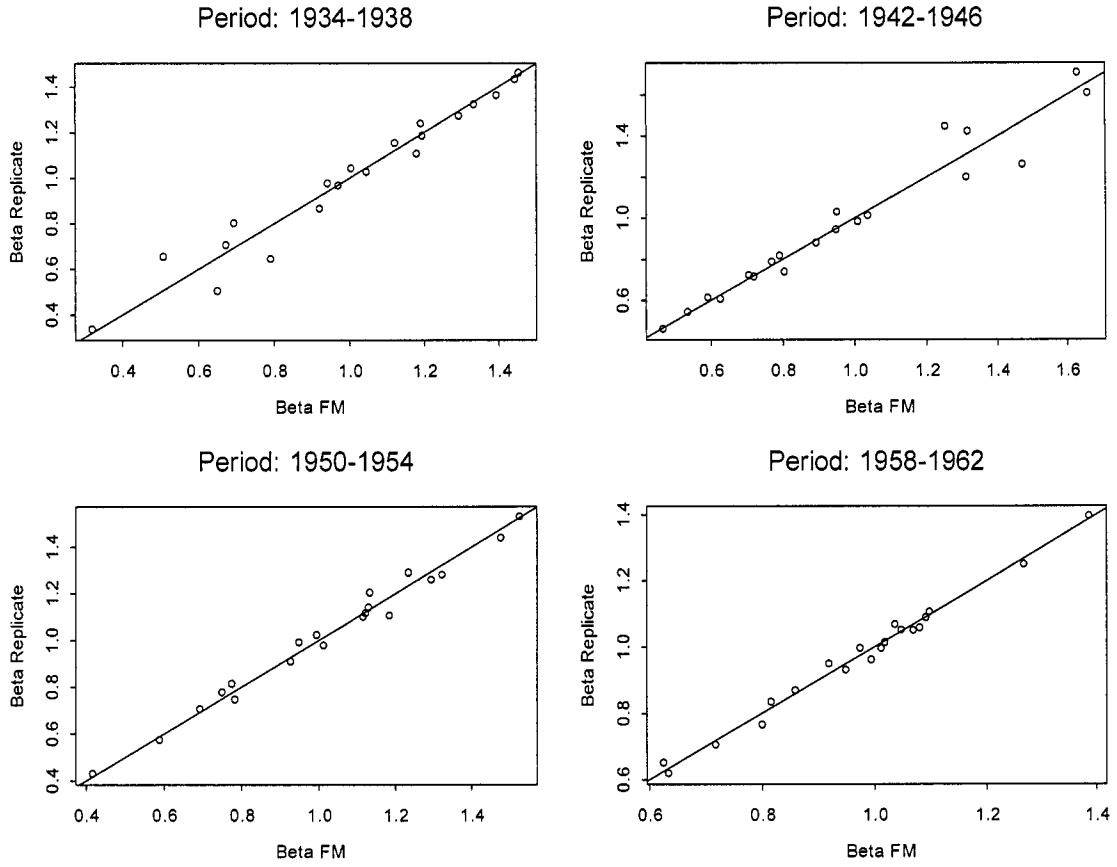
Figure 34. Comparison of $\hat{\beta}_{p,t-1}$ of FM, Table 2 versus Replicates.

The replicates match FM, Table 2 closely for all time periods.

3.3.3. FM Table 3

FM, Table 3 shows the average of the month-by-month regression coefficient

estimates $\overline{\hat{\gamma}_j}$, $\overline{R^2}$, and the corresponding t-statistics computed as

$$\overline{t(\hat{\gamma}_j)} = \frac{\overline{\hat{\gamma}_j}}{s(\hat{\gamma}_j)/\sqrt{N}} \qquad (3.1.4)$$

with $s(\gamma_j)$ the standard deviation of the time series over time periods described in Table

9.

The comparison with the replicates for FM, Table 3, Panel A on these 10 time periods

is shown in Figure 35 and Figure 36. The solid line has slope one and intercept zero.
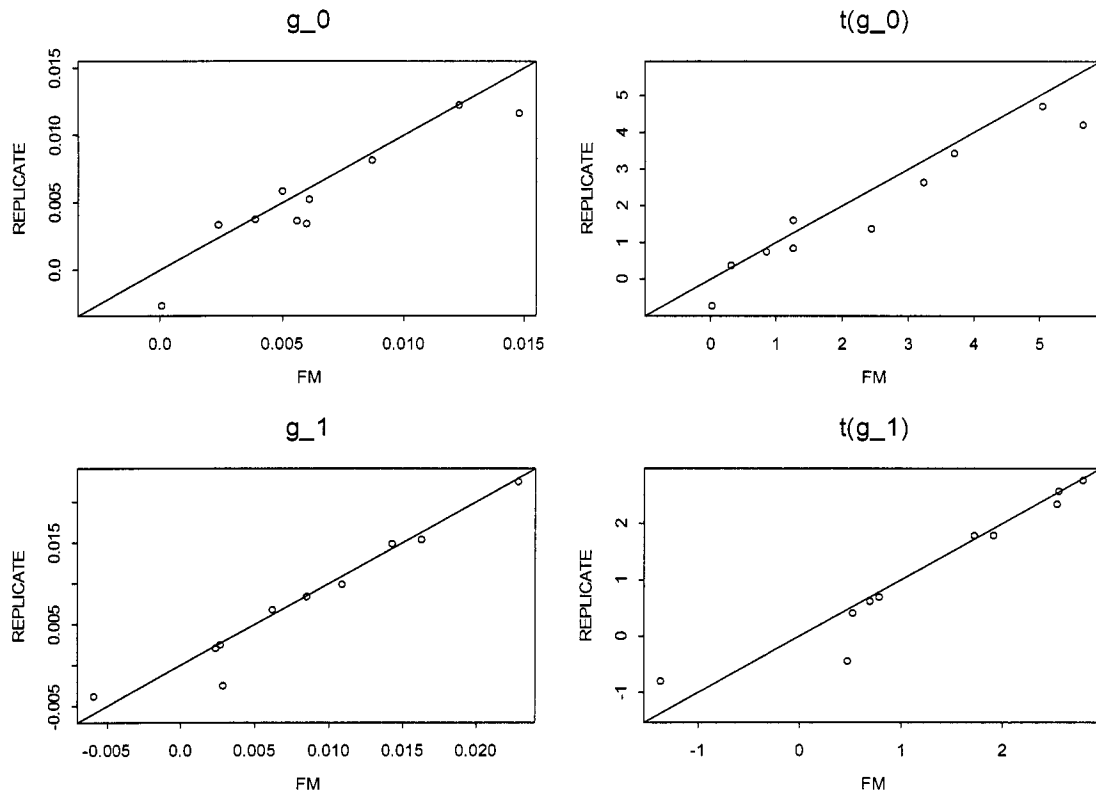
Figure 35. Estimates of FM, Table 3, Panel A versus Replicates.
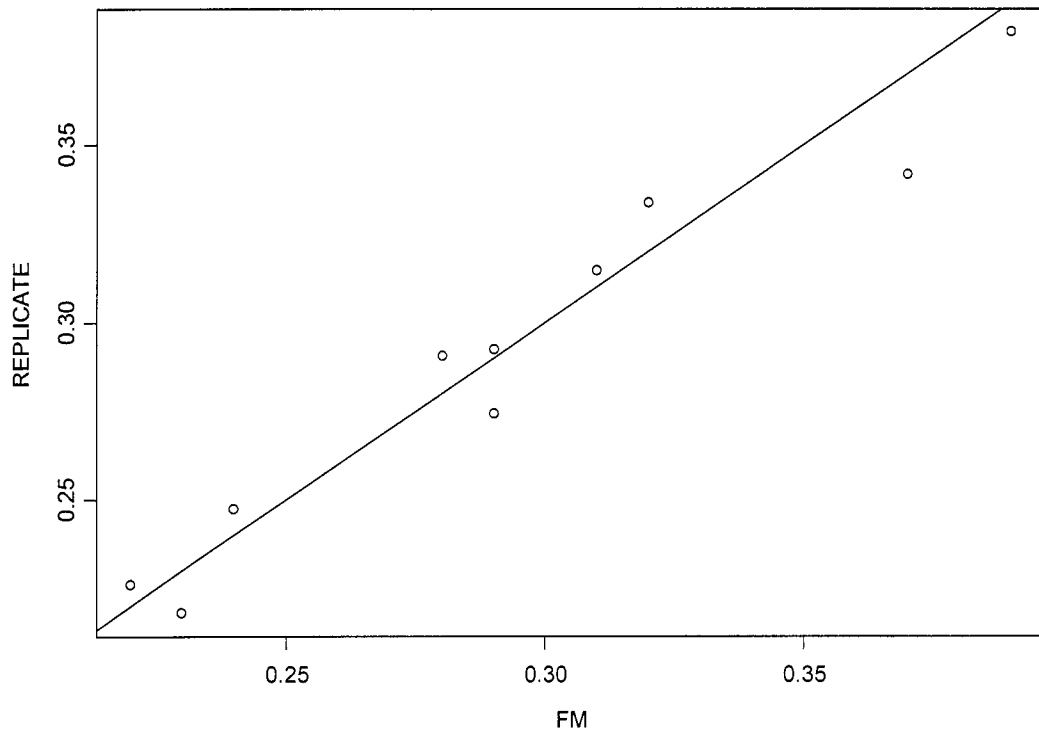
Figure 36. R^2 Estimate of FM, Table 3, Panel A versus Replicates.

The replicates in Figure 35 and Figure 36 match the coefficient estimates, corresponding t-statistics, and coefficient of determination of FM, Table 3 closely for all time periods. Figure 37 and Figure 38 show the comparison for Panel D. The estimated intercept is not shown as its values are close to zero.
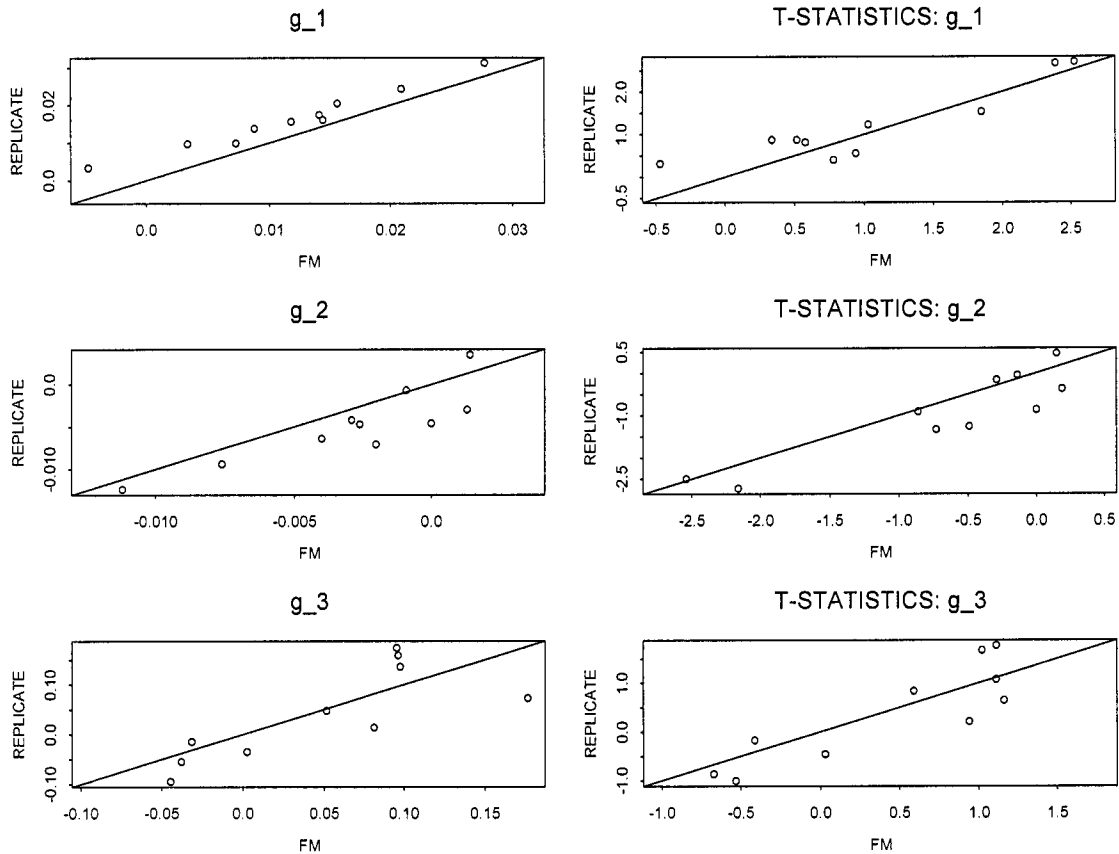
Figure 37. Estimates of FM, Table 3, Panel D versus Replicates.

The replicates in Figure 37 and Figure 38 match FM, Table 3 not as well as in Figure 35. The intercept γ_0 shows an upward bias of the replicates, while the slope γ_1 shows a downward bias. Overall, they are still relatively similar across all time periods.

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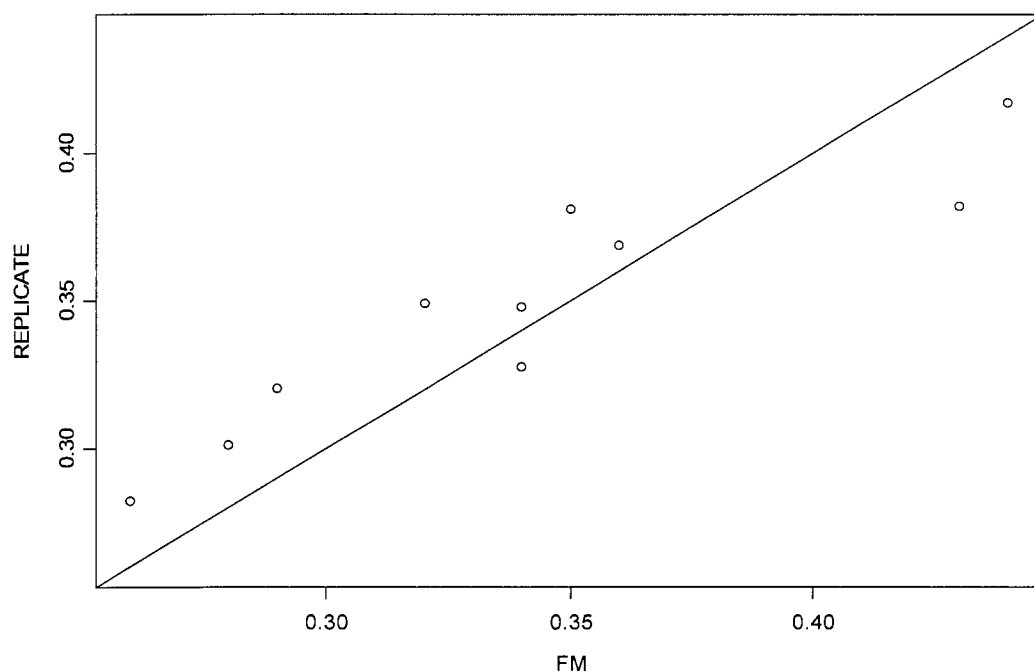
Figure 38. R^2 of FM, Table 3, Panel D versus Replicates.

3.4. Extension of FM

Table 10 gives an overview of the natural extension through 12/2002 of FM, with

respect to formation, estimation, and testing periods.

Table 10. Overview of Extended S I and S II Time Periods.

Period	S I		S II
	Formation	Estimation	Testing
1	1/1926-12/1929 5 years	1/1930-12/1934 5 years	1/1935-12/1938 4years
2	1/1927-12/1933 7 years	1/1934-12/1938 5 years	1/1939-12/1942 4 years
...	...	...	...
17	1/1987-12/1993 7 years	1/1994-12/1998 5 years	1/1999-12/2002 4 years

The time series averages and t-statistics in S III will be taken over 14 five-year and 7 ten-year periods, as well as over the full time periods from 1/1935 through 12/2002, and excluding the World War II (WW II) period, i.e., from 1/1947 through 2002.

3.4.1. FM Table 1

Figure 39 shows the number of available securities in this study for all testing periods from 1935 through 2002.
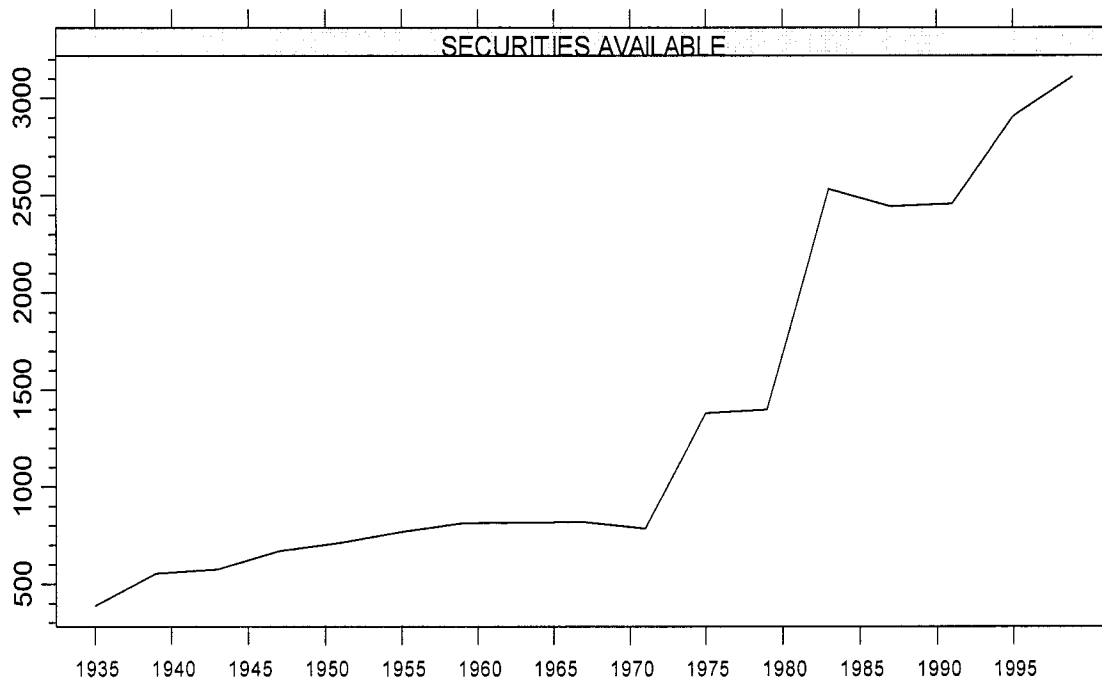
Figure 39. Number of Available Securities.

The number of firms available steeply increases mid-1970s and mid-1980s.

3.4.2. FM Table 3, Panel A, 5-year Averages

Figure 40 shows contiguous 5-year averages of the month-by-month regression coefficient estimates $\hat{\gamma}_j$ (intercept annualized) and corresponding t-statistics for the model of Panel A, while Figure 41 shows the corresponding R^2.
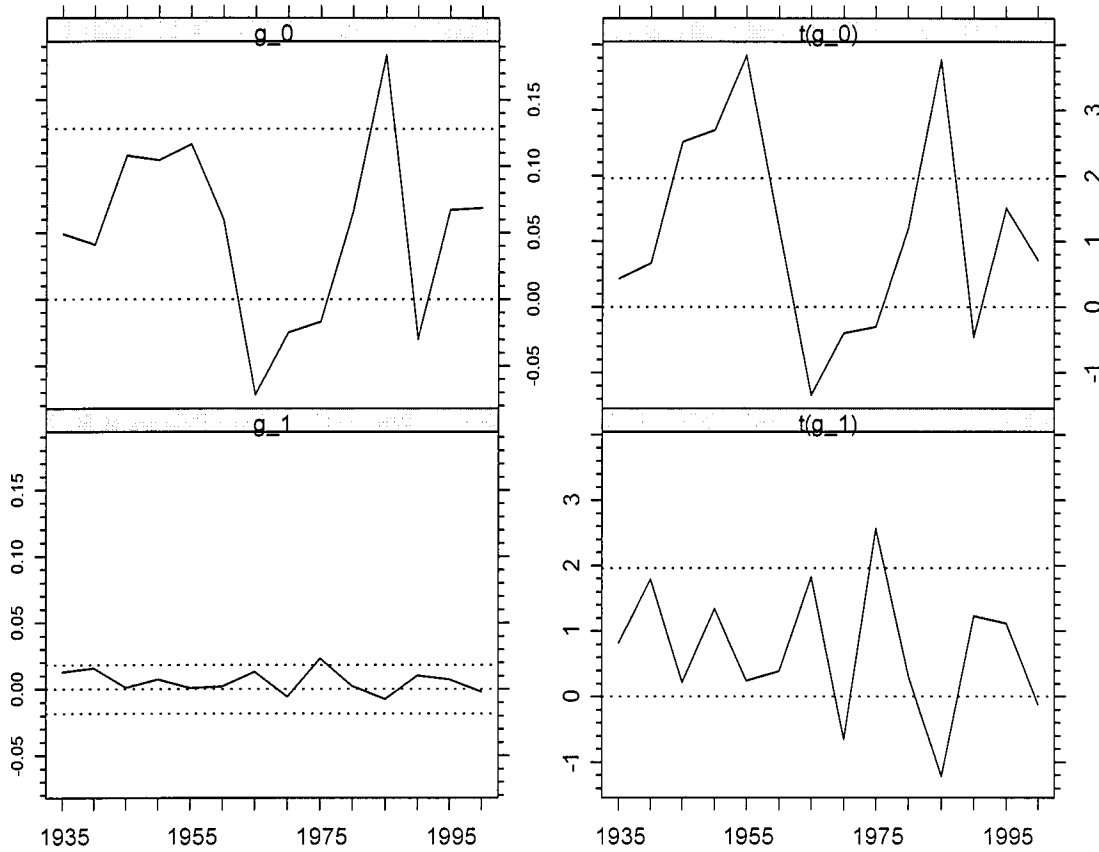
Figure 40. Panel A: 5-Year Averages of Coefficient Estimates and T-Statistics.

The intercept $\hat{\gamma}_0$ is significant for the time periods starting in 1945, 1950, 1955, and 1985. The slope $\hat{\gamma}_1$ is never significant under the null-hypothesis C3, but shows a large t-statistic in the 5-year period starting in 1975. Note that the null hypothesis in C3 can only be rejected when negative values with a large t-statistics occur. The slope coefficient $\hat{\gamma}_1$ is negative for the time periods starting in 1970, 1985, and 2000, but without significant t-statistics.
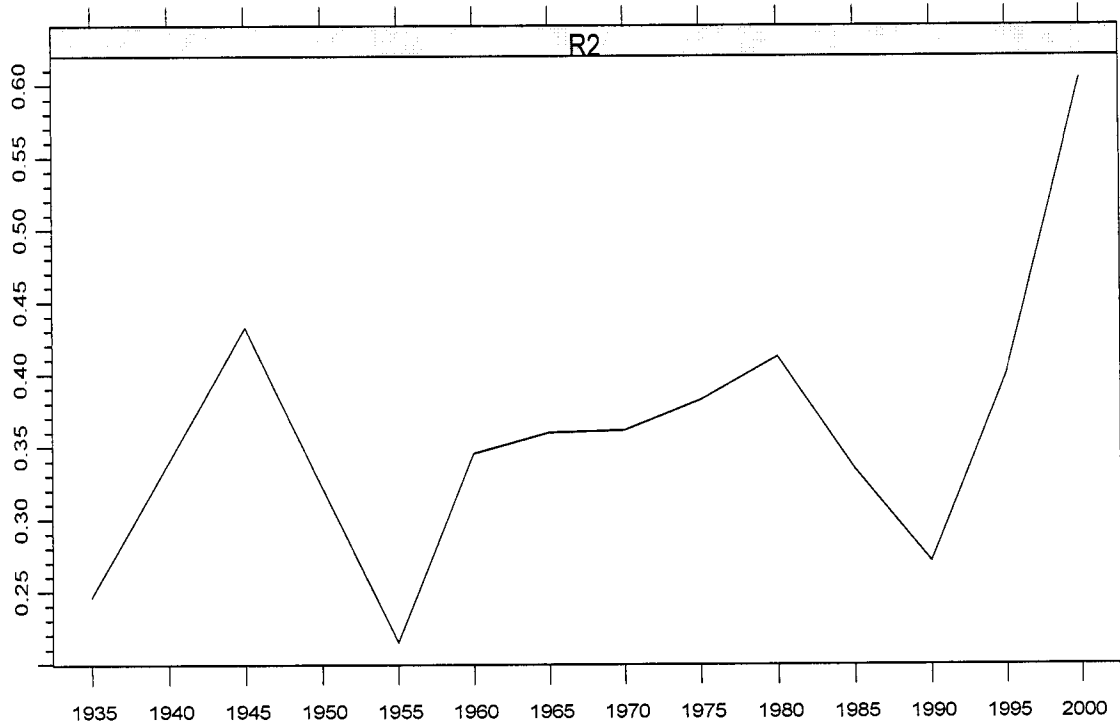
Figure 41. Panel A: 5-Year Averages of R^2.

The uptrend in the R^2, since 1955 with the exception of the 1980-1990 period, is noticeable, and could be due to market turbulences decreasing the validity of the model.

3.4.3. FM Table 3, Panel D, 5-year Averages

Figure 42 shows contiguous 5-year averages of the month-by-month regression coefficient estimates $\hat{\gamma}_j$ (intercept annualized) and corresponding t-statistics for Panel D., and Figure 43 shows the corresponding R^2 values.
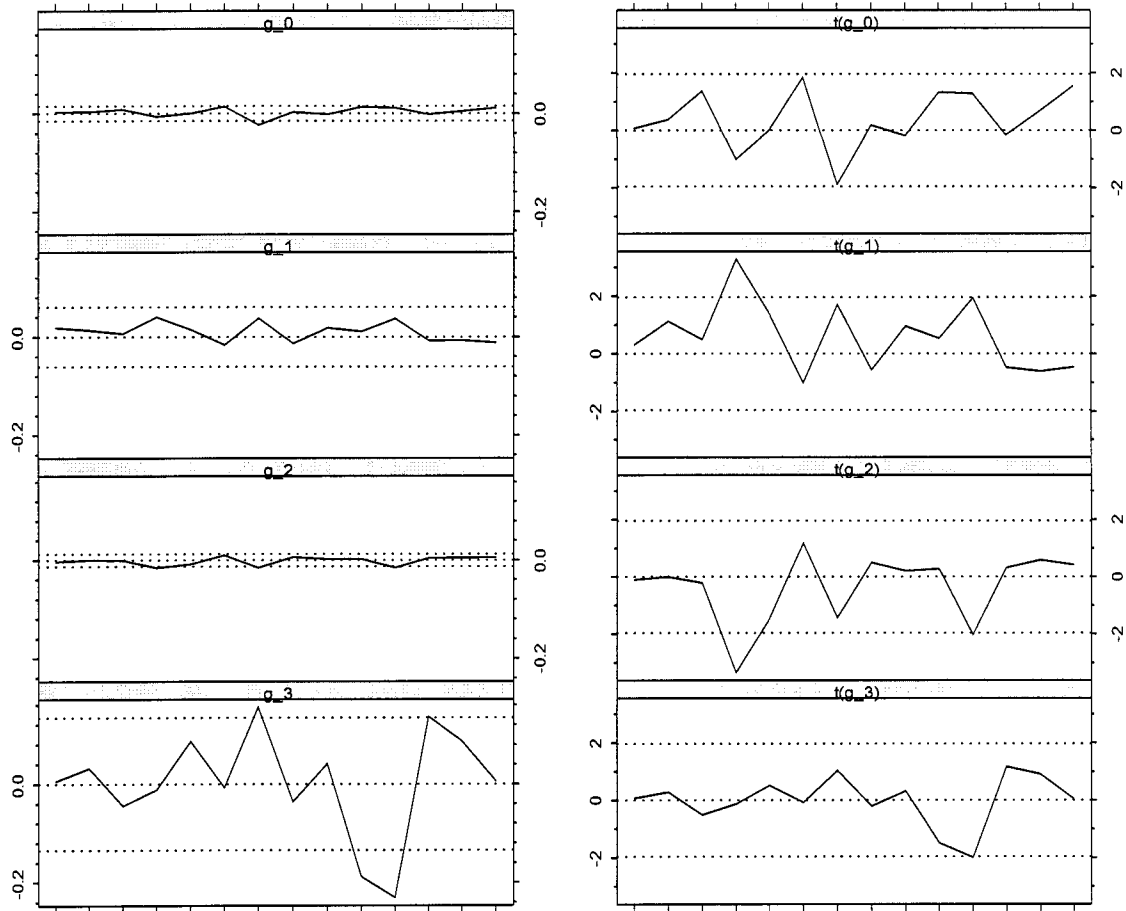
Figure 42. Panel D: 5-Year Averages of Coefficient Estimates and T-Statistics.
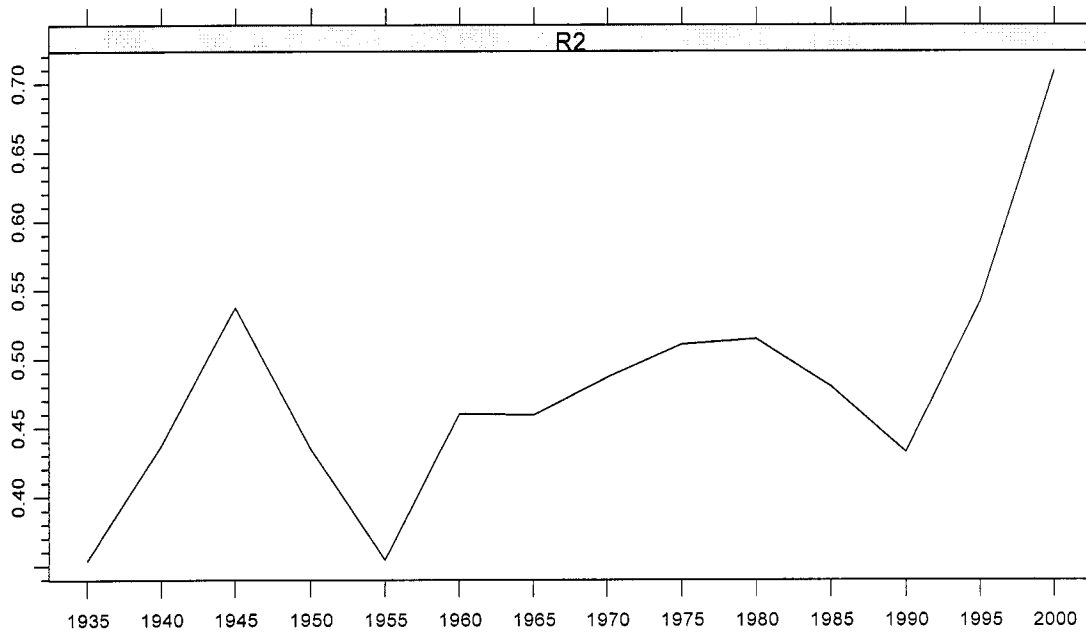
Figure 43. Panel D: 5-Year Averages R^2.

The only significant coefficient is the $\hat{\gamma}_2$ coefficient, which is significant in the 5-year time period starting in 1950. The $\hat{\gamma}_3$ coefficients fluctuate; however, it is insignificant on all 5-year periods. Similar to Figure 41, Figure 43 shows an uptrend in R^2 after 1955 with the exception of the 1980-1990 period.

3.4.4. FM Table 3, Panel A, 10-year Averages

Figure 44 shows contiguous 10-year averages of the month-by-month regression coefficient estimates $\hat{\gamma}_j$ (intercept annualized) and t-statistics for Panel A. Figure 45 shows the corresponding R^2.
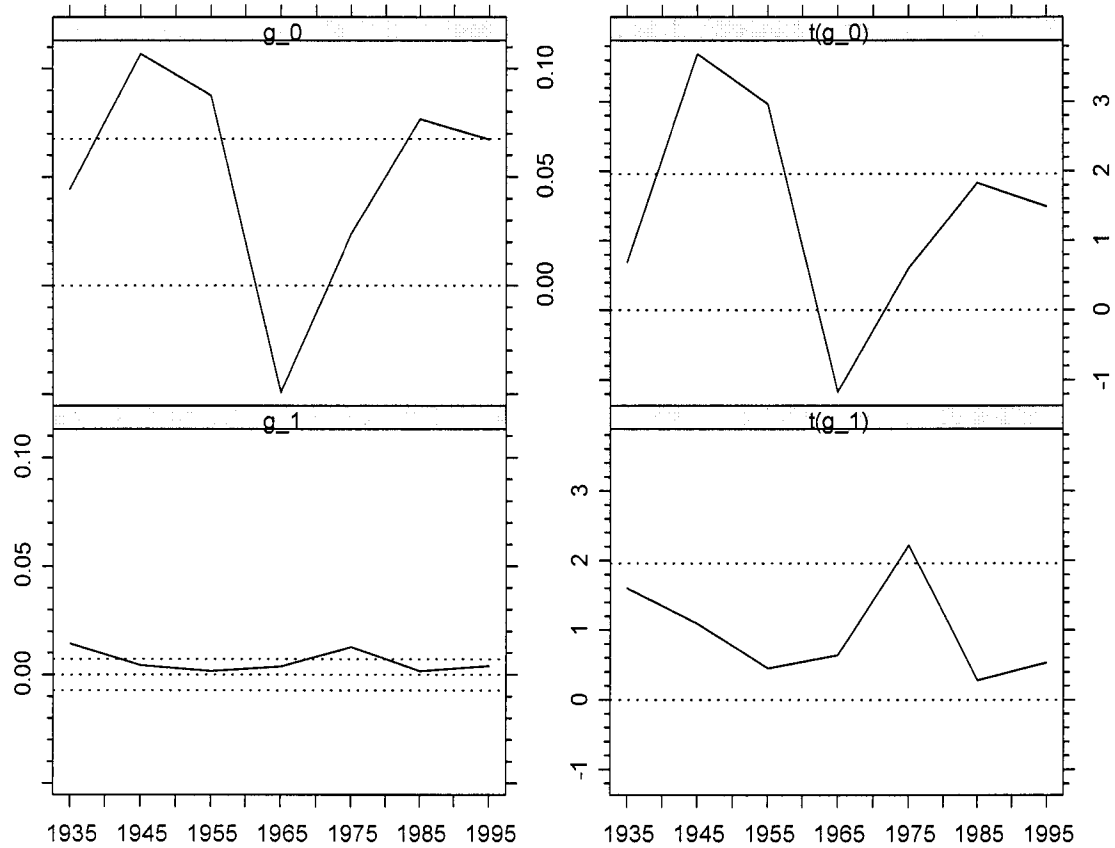
Figure 44. Panel A: 10-Year Averages Coefficient Estimates and T-Statistics.

The intercept $\hat{\gamma}_0$ is only significant in the 10-year time period that contains WW II. The slope $\hat{\gamma}_1$ is never significant, but shows a large t-statistic in the 10-year period starting in 1975. The slope coefficient $\hat{\gamma}_1$ is always positive.
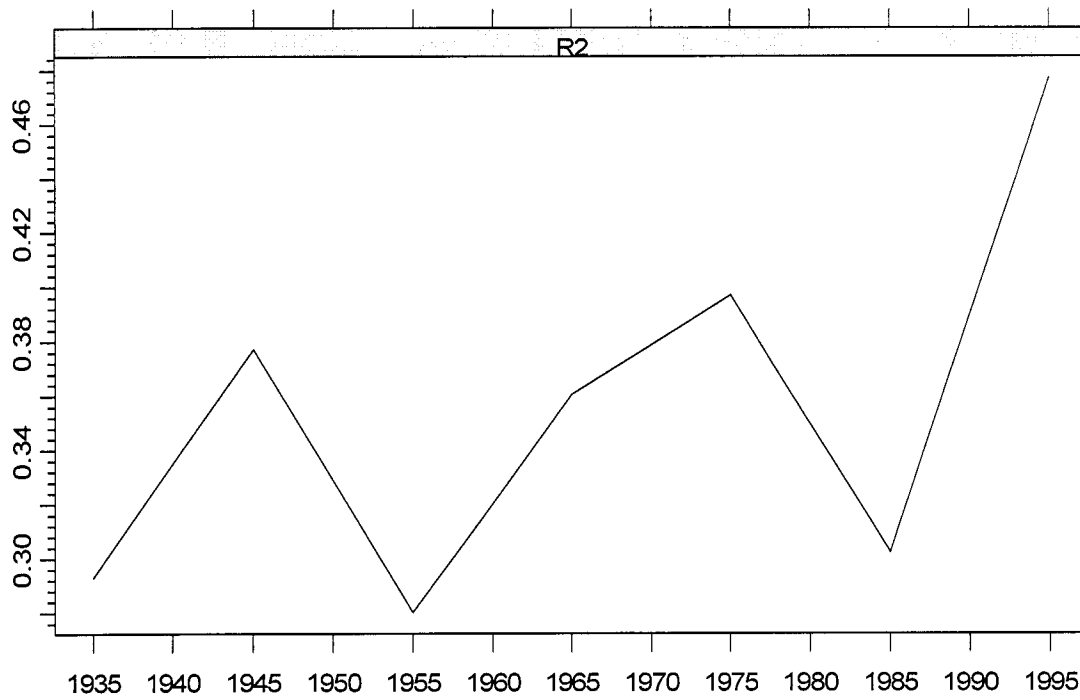
Figure 45. Panel A: 10-Year Averages R^2.

As in the 5-year windows, there is an uptrend in the R^2 since 1955, with the exception of the 1980-1990 period.

3.4.5. FM Table 3, Panel D, 10-year Averages

Figure 46 shows contiguous 10-year averages of the month-by-month regression coefficient estimates $\hat{\gamma}_j$ (intercept annualized) and t-statistics of Panel D. Figure 47 shows the corresponding R^2.
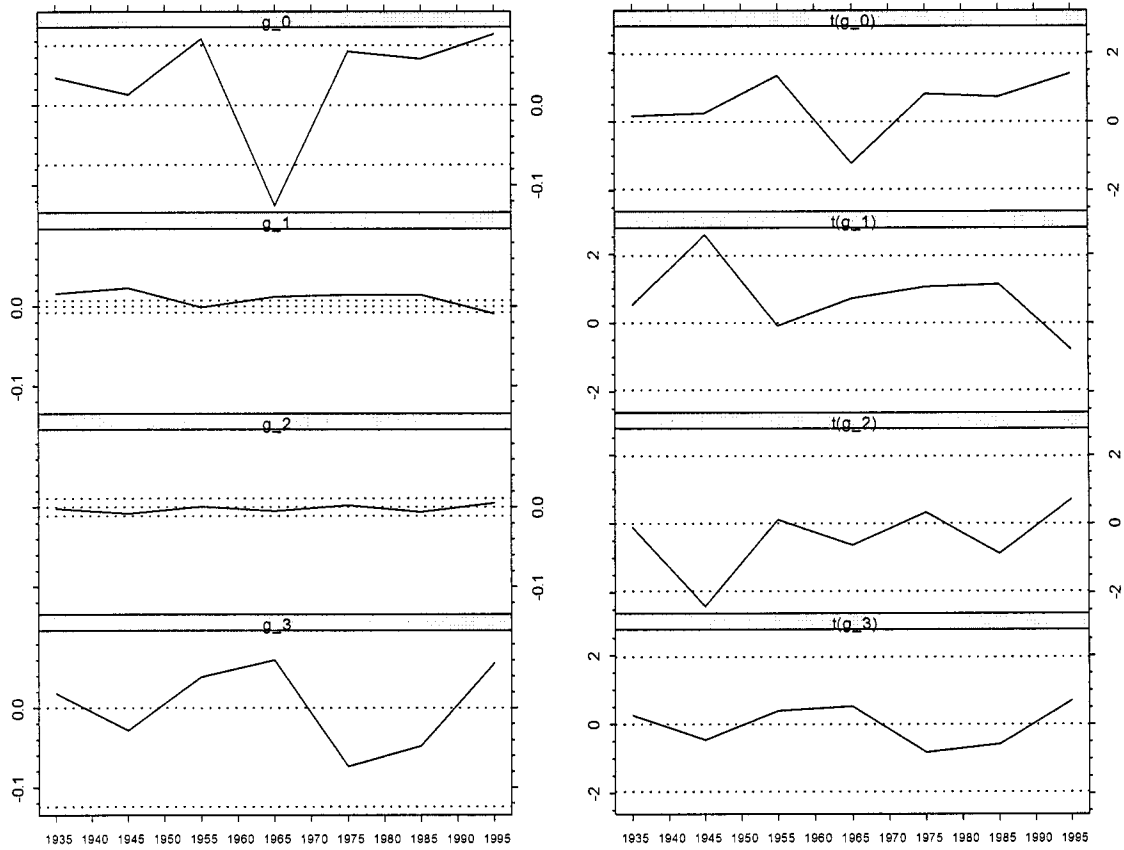
Figure 46. Panel D: 10-Year Averages of Coefficient Estimates and T-Statistics.

Only the quadratic term $\hat{\gamma}_2$ is significant on the 10-year time periods starting right after WW II in 1945. The slope coefficient $\hat{\gamma}_1$ is positive except for the time periods starting in 1955 and 1995, with a large t-statistics in the 10-year periods starting during WW II.
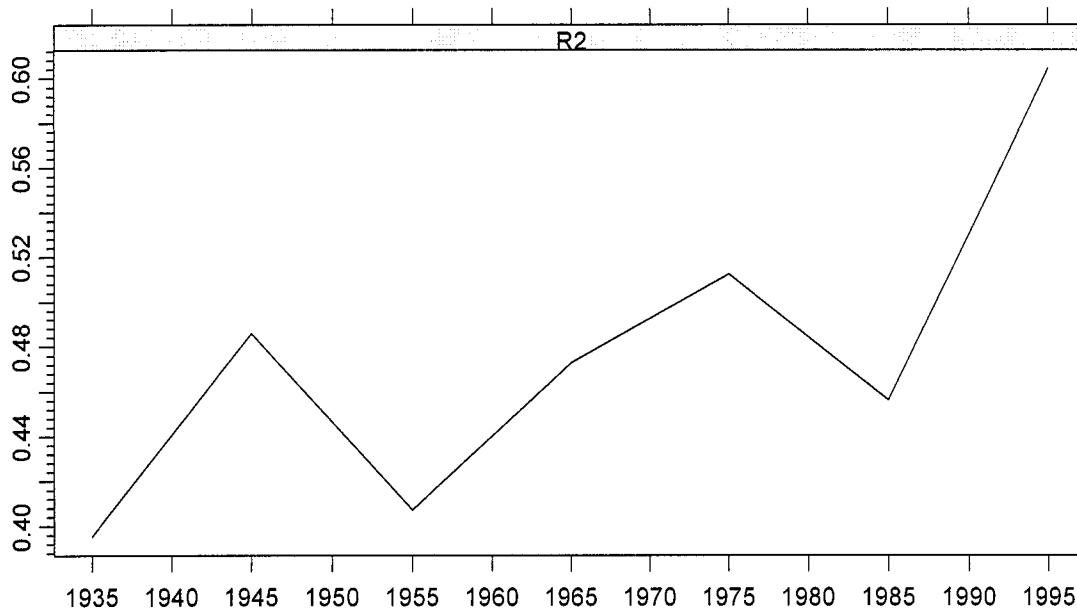
Figure 47. Panel D: 10-Year Averages of R^2.

The R^2 shows the usual uptrend since 1955, with the exception of the 1980-1990 period.

3.4.6. FM Table 3, Panel A, Other Time Periods

Figure 48 shows the time series of the coefficients γ_0 and γ_1 from the month-to-month cross-sectional regressions over the full testing period from 1/1935 through 12/2002. The time series show a large number of outliers and show different regimes of high and low, as well as increasing and decreasing volatility. A regimes, e.g., of higher volatility is the WW II period from 1935 through 1940, followed by a extremely low and then rising volatility regime until 1943. The next periods of high volatilities are in the Seventies, early Nineties, and early Millennium.
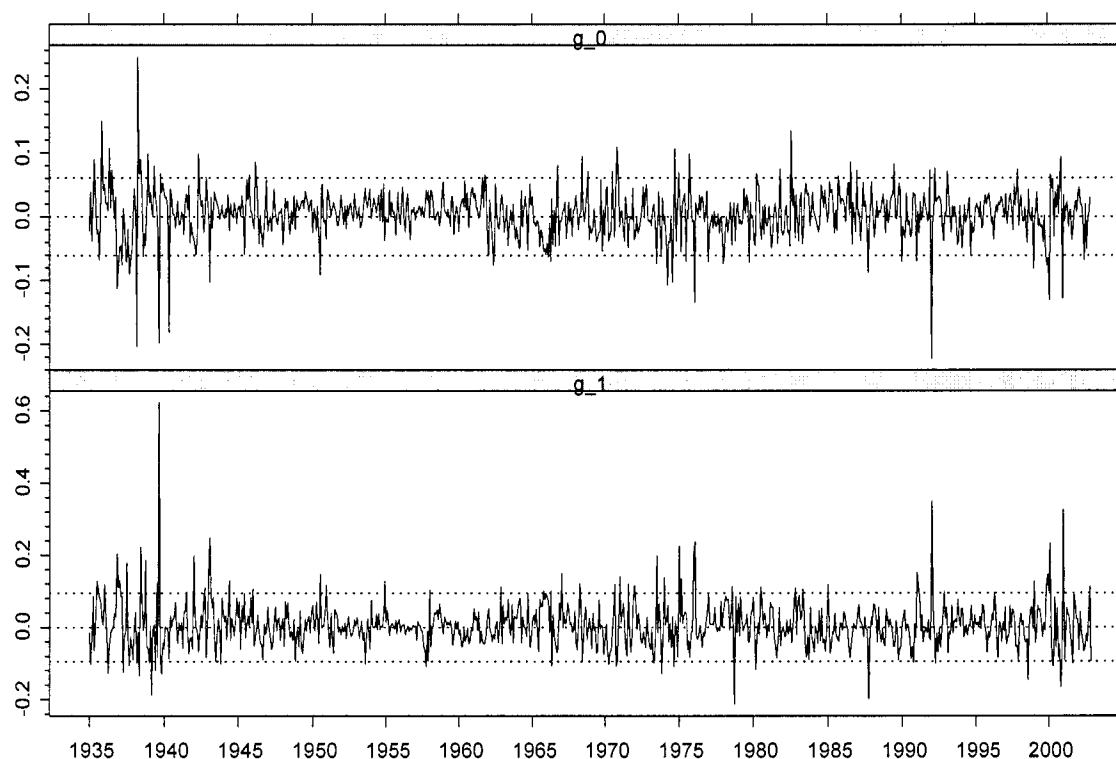
Figure 48. Time series of Coefficients Estimates γ_0 and γ_1 from S II.

Averages and t-statistics taken over sub-time periods will easily be biased by outlying values that occur more often in periods of high volatility. To make this point more clear, Table 11 shows the t-statistics for the full and sub-periods. The t-statistic of the intercept stays high when volatility time periods are excluded, but the t-statistics of the slope parameter decreases.

Table 11. T-Statistics over selected Time Periods.

	γ_0	γ_1
01/1935 – 06/1968	2.62	2.56
01/1935 – 12/2002	3.11	2.69
01/1946 – 12/2002	3.14	1.96
01/1946 – 06/1968	3.12	1.81
01/1946 – 12/1974 01/1977 – 12/2002	2.97	1.61

Even though the power of the t-test decreases in times of higher volatility, one would still think that the t-statistics of the slope parameter could be positive and large, but this does not conform to Table 12.

Table 12. T-Statistics over High Volatile Time Periods.

	γ_0	γ_1
1/1935-02/1946	1.05	1.89
1/1974-12/1976	-0.10	1.25
01/1991-03/1992	-0.05	0.48
10/1999-01/2001	-0.61	1.59

Moreover, Table 12 that shows in times of high volatility an insignificant intercept and very small values of the t-statistics of the slope. This also confirms findings of Section 4.2 and Section 4.4, emphasizing the impact of the choice of time periods on the overall results.

3.4.7. Tests of C1, C2, and C3

C1: Panel D tests if the relationship between expected returns and β is linear. With the exceptions of the 5-year and 10-year time-periods that include 1955, all t-statistics of the γ_2 coefficient are insignificant, thus C1 cannot be rejected.

C2: Panel D also tests if there is additional systematic risk besides β that affects expected returns. On 5-year and 10-year time periods, the t-statistics of the coefficient γ_3 are insignificant. Thus, condition C2 cannot be rejected.

C3: Panel A tests for a positive trade-off between expected return and risk. It shows that the slope coefficient γ_1 is negative in the 5-year time periods starting in 1970, 1985, and 2000. That is in 3 out of 14 time periods or 21.4% of the times. However, its t-statistics are never significant (under the null-hypothesis), and large only in the 5-years time period starting in 1975. On 10-year windows, Panel A shows all γ_1 to be positive; however, only the t-statistics for the 10-year time period starting in 1975 is large. Excluding time periods with high volatility, as shown in Table 12, renders all t-statistics of γ_1 to be small, and they remain small when only periods of high volatility

are considered (Table 12). All taken into consideration, there is not enough evidence to reject C3 as it only tests for significant $\gamma_1 \leq 0$.

3.5. Impact of Robust MM-Regression

It was established in Section 3.3 that the data and methodology of this study sufficiently replicate the results of FM. Section 3.4 extended FM's methods through 12/2002. In this section, FM will be compared with robust versions of FM. Robust versions of FM are obtained by applying the MM-estimator (ROB, Section 1.3.1) to various combinations of steps S I through S III. The following notation will be adapted going forward.

RS I: S I is robustified, i.e., the OLS estimator in (3.1.1) is replaced by ROB.

RS II: S I and S II are robustified, i.e., the OLS estimator in (3.1.1) and (3.1.2) is replaced by ROB.

RS II ex I: S II but not S I is robustified, i.e., only the OLS estimator in (3.1.2) is replaced by ROB, but β-portfolios are formed using the FM approach.

RS III: S I, S II, and S III are robustified, i.e., the OLS estimator in (3.1.1) and (3.1.2) is replaced by ROB, and in S III the classical averages of the time series of the $\tilde{\gamma}_{jt}$ estimates and its t-statistics are computed robustly, again using ROB.

RS III ex I&II: Only S III is robustified by replacing the classical averages of the time series of the $\tilde{\gamma}_{jt}$ estimates and its t-statistics by robust methods, while S I and S II are done the classical FM way.

Section 3.5.1 motivates the potential impacts when using robust methods. Section 3.5.2 compares FM and RS I. Section 3.5.3 compares FM and RS II. Section 3.5.4 implements the full robust approach comparing FM and RS III ex I&II and RS III. Section 3.5.5 uses RS III in Panel A and Panel D to test conditions C1, C2, and C3. Section 3.5.6 looks at the regression model assumptions.

3.5.1. EDA and Potential Impact of ROB Regression

This section points out at which steps of the FM approach robust methods may have impact. Figure 49 shows the scatter plot of the month-to-month β-portfolio returns, obtained using the FM and RS I approach over the time period starting in 1/1935 through 12/2002.
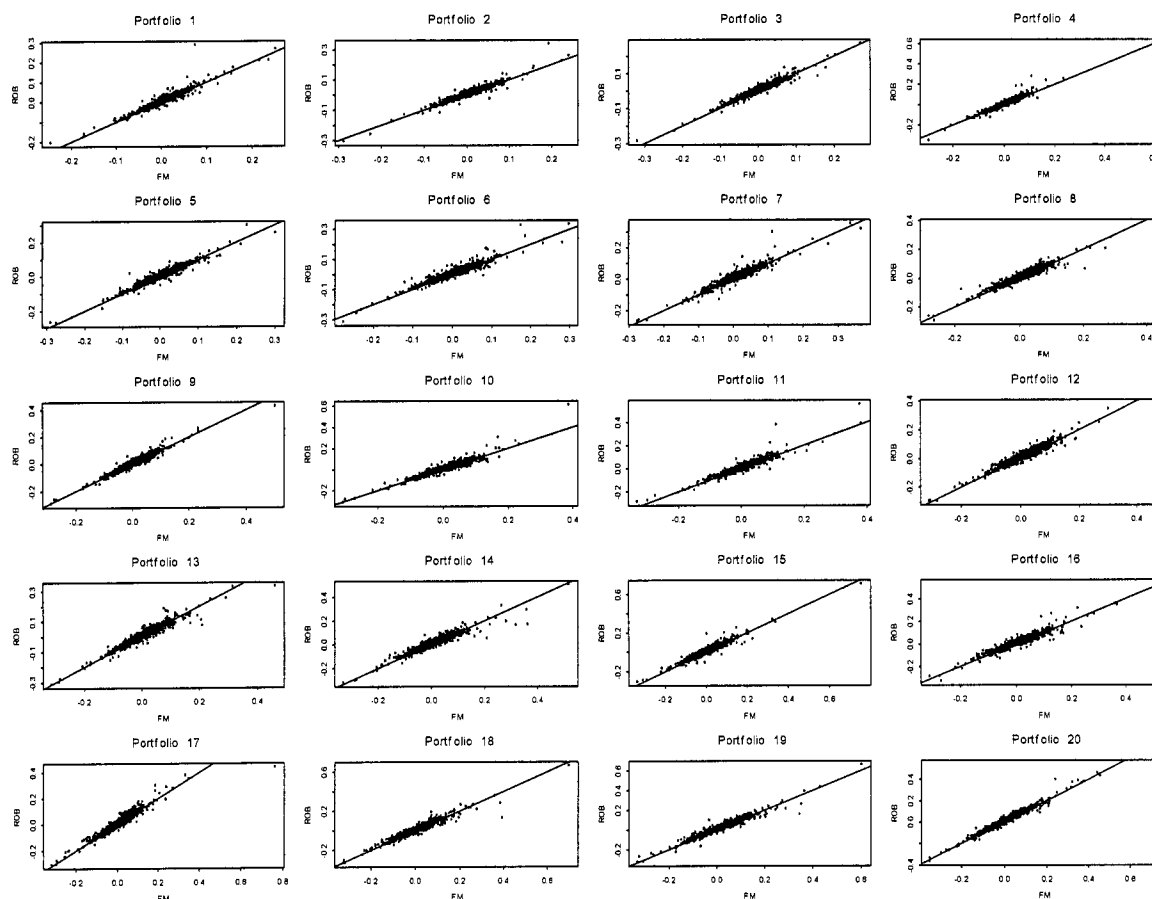
Figure 49. Monthly β-Portfolio Returns of FM versus RS I, 1/1935 - 12/2002.

Overall, the FM and RS I portfolio returns are similar, but there are a number of months with large disparities, such as the outlier in the upper right corner of Portfolio 17 and the outlier in the positive ROB direction of Portfolio 1—both corresponding to September 1939. Further, Figure 50 shows the portfolio β obtained using the S1 I and RS I approach over 1/1935 through 12/2002.
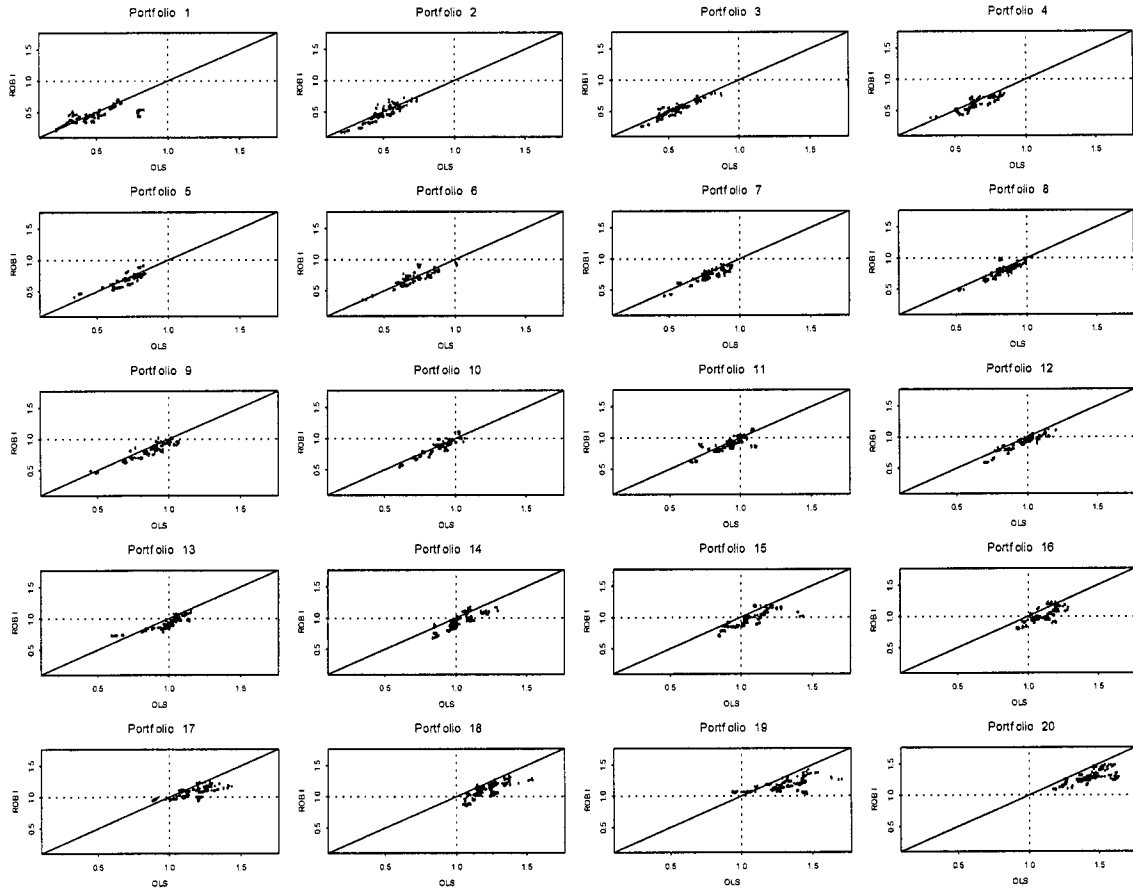
Figure 50. Monthly Portfolio Betas of S I versus RS I, 1/1935 - 12/2002.

As with the β -portfolio returns, both approaches give overall similar results, but still show outlying values. E.g., in Portfolio 1, the number of the S I β is much larger than the RS I β. It also seems that for smaller β -portfolios, S I seems downward biased, while for larger β -portfolios, S I seems upward biased. Thus, the grouping of stocks by ranked β could be affected when using ROB regression to compute the individual stocks β, thereby creating β -portfolios that are lower or higher then the S I β -portfolios.

S II computes regression estimates on the portfolio level. To understand the impact of outliers on the month-to-month cross-sectional regressions, Figure 51 shows the month-to-month portfolio returns obtained from forming β -portfolios using S I.
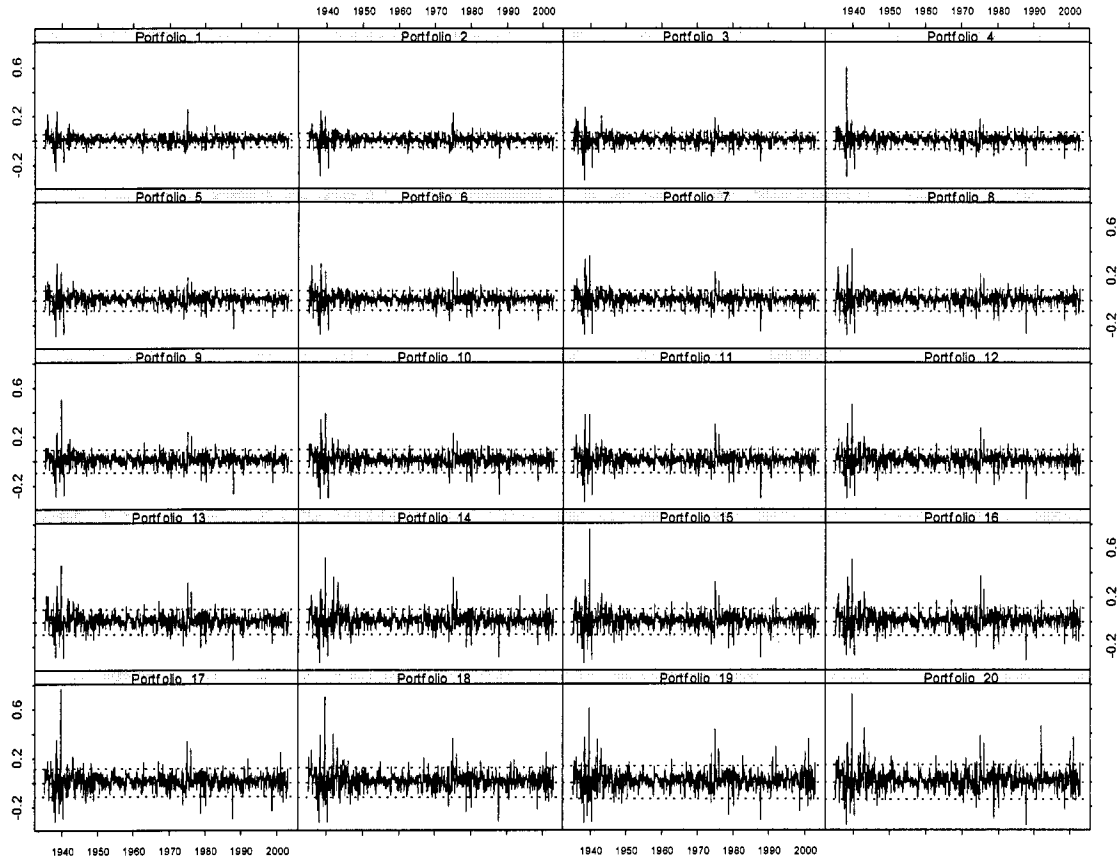
Figure 51. Time Series of Portfolio Returns using S I.

The dotted lines around the center distribution, at plus and minus twice the ROB standard deviation of the returns, clearly display a large number of outlying portfolio returns.

Figure 52 shows the number of return–β pairs that are classified by the ROB regression as two-dimensional outliers and therefore rejected.
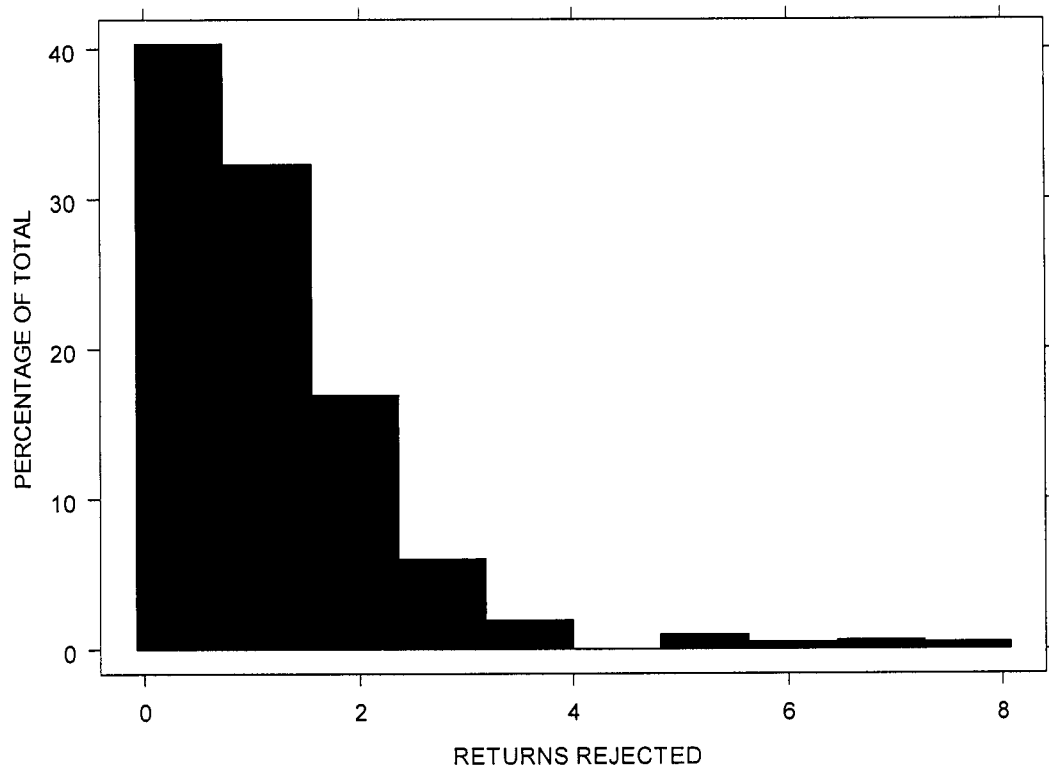
Figure 52. Panel A: Number of Outliers rejected in RS II.

While the majority of the 816 month-to-month cross-sectional regressions reject none or just one of the returns, there are a number of months where up to 8 out of the 20 month-to-month returns were rejected. Therefore, cross-sectional regression estimates can be affected when choosing ROB over OLS regression.

In S III time series are averaged and t-statistics computed. Figure 53 shows the time series of the month-to-month cross-sectional regression coefficients γ_0 and γ_1 of Panel A, comparing S II, RS II ex I, and RS II approach. The two parallel dotted lines are two times the ROB standard deviation around zero.
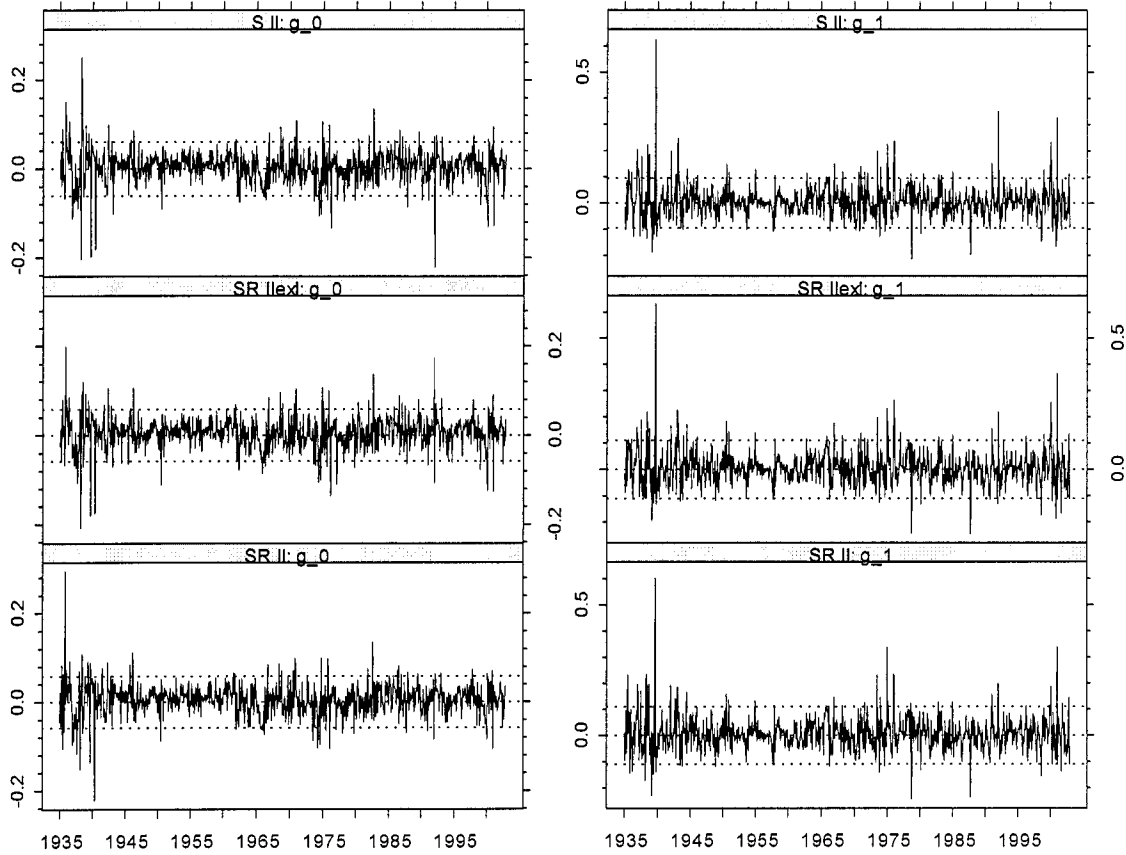
Figure 53. Panel A: Estimates from S II using S I, RS I ex II, and RS II.

Non-stationary volatility and a number of large positive and negative outliers are clearly visible. Averages taken over time periods that include large negative or positive outliers can be substantially biased. Standard deviations computed over time periods that include a large negative or positive outlier can be substantially inflated. T-tests formed with such quantities have little power. Therefore the biggest effects of using ROB will be expected in S III.

3.5.2. S I versus RS I

This section compares S I to the RS I approach on five-year contiguous time intervals from 1/1935 through 12/2002. The results are shown in Figure 54.
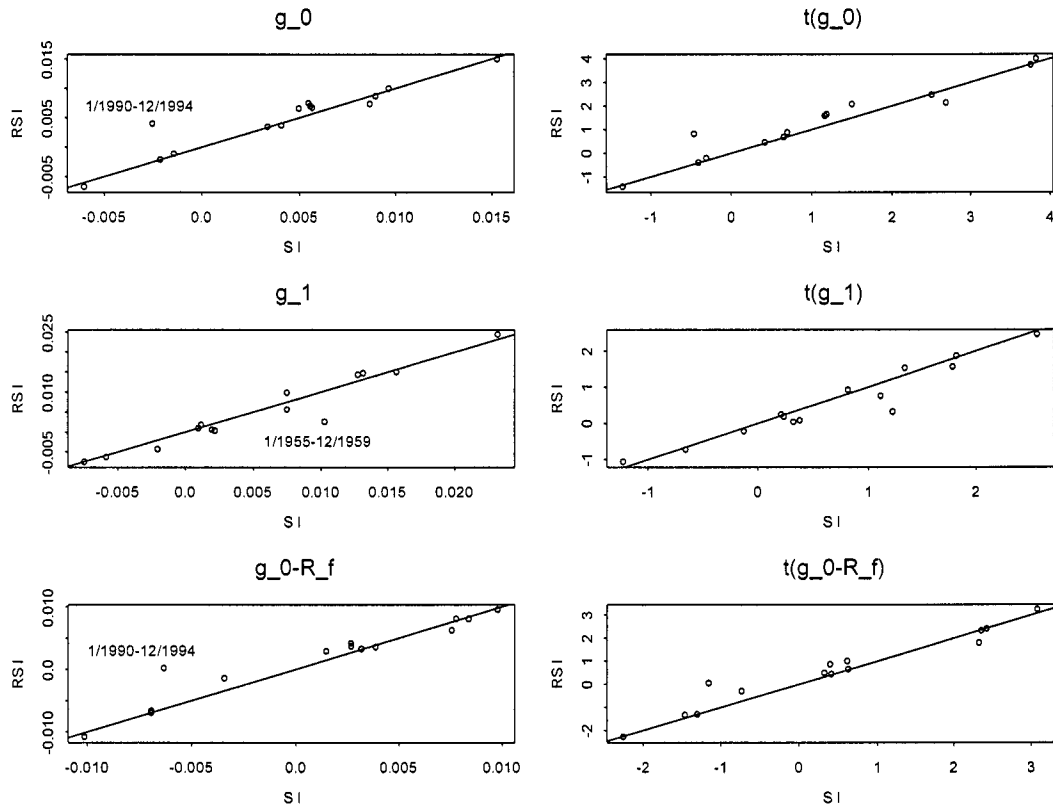
Figure 54. FM versus RS I, 5-Year Contiguous Time Periods, 1/1935 - 12/2002.

The points scatter mostly near the straight line, with the exception of a few points that correspond to the 5-year time periods starting in 1955 and 1990.

The scatter plot of R^2 in Figure 55 indicates that the use of ROB cross-sectional regression seems to increase the R^2, with the exception of the last time period.
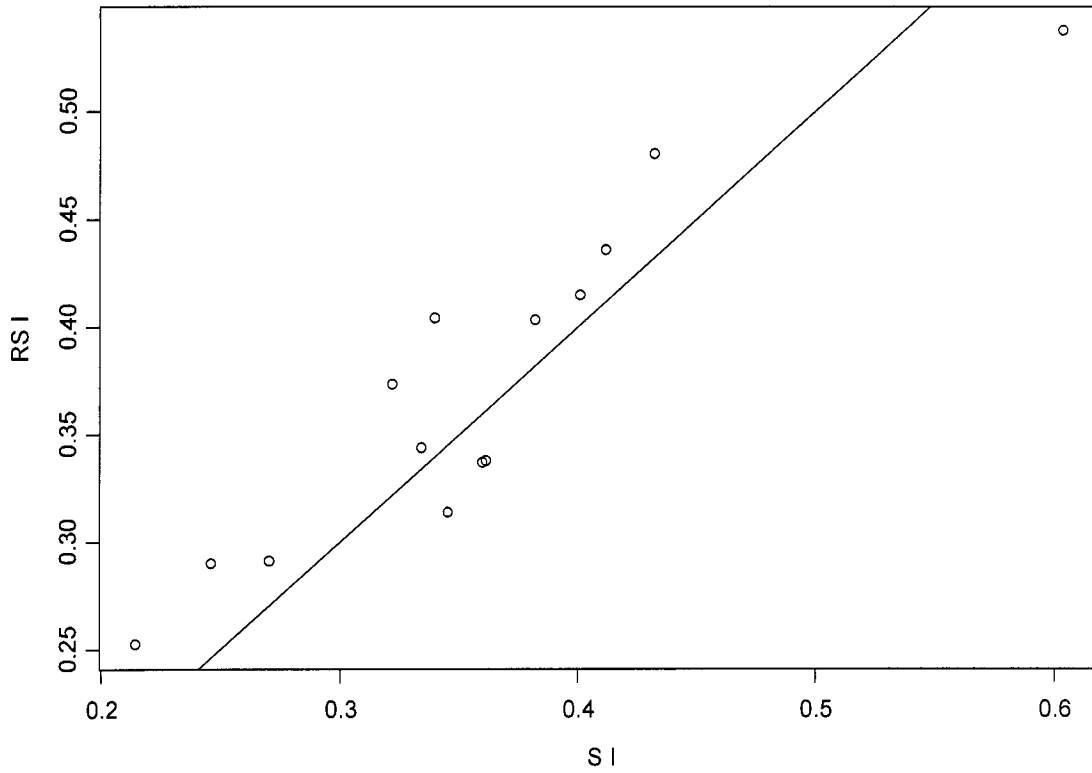
Figure 55. Corresponding R^2 of Figure 54.

3.5.3. S II versus RS II

This section compares S II to the RS II approach on five-year contiguous time intervals over 1/1935 through 12/2002. The results are shown in Figure 56.
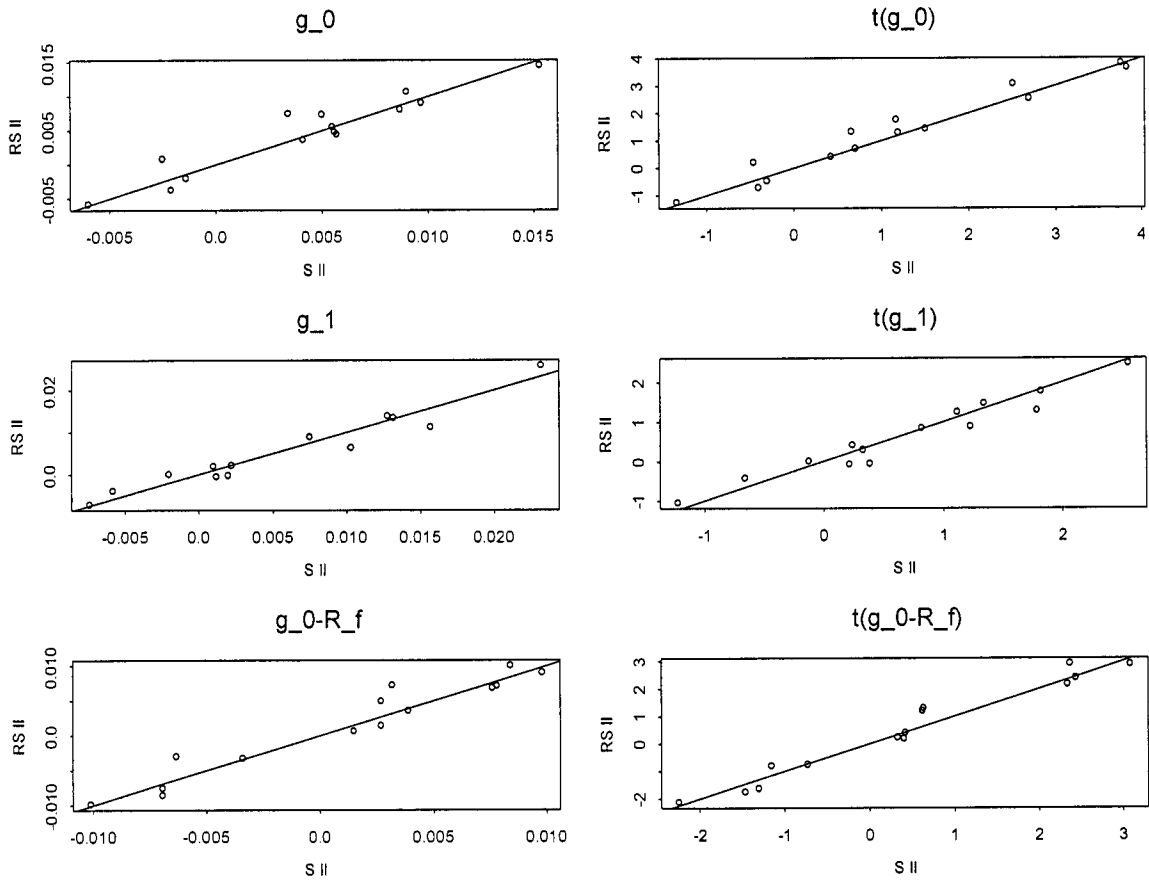
Figure 56. FM versus RS II, 5-Year Contiguous Time Periods, 1/1935 - 12/2002.

R II matches S II sufficiently well.

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Figure 57 mainly confirms FM, Table 3, Panel A, but again, a few data points are fairly different—one belonging to the WW II period and other ones to more recent time periods.
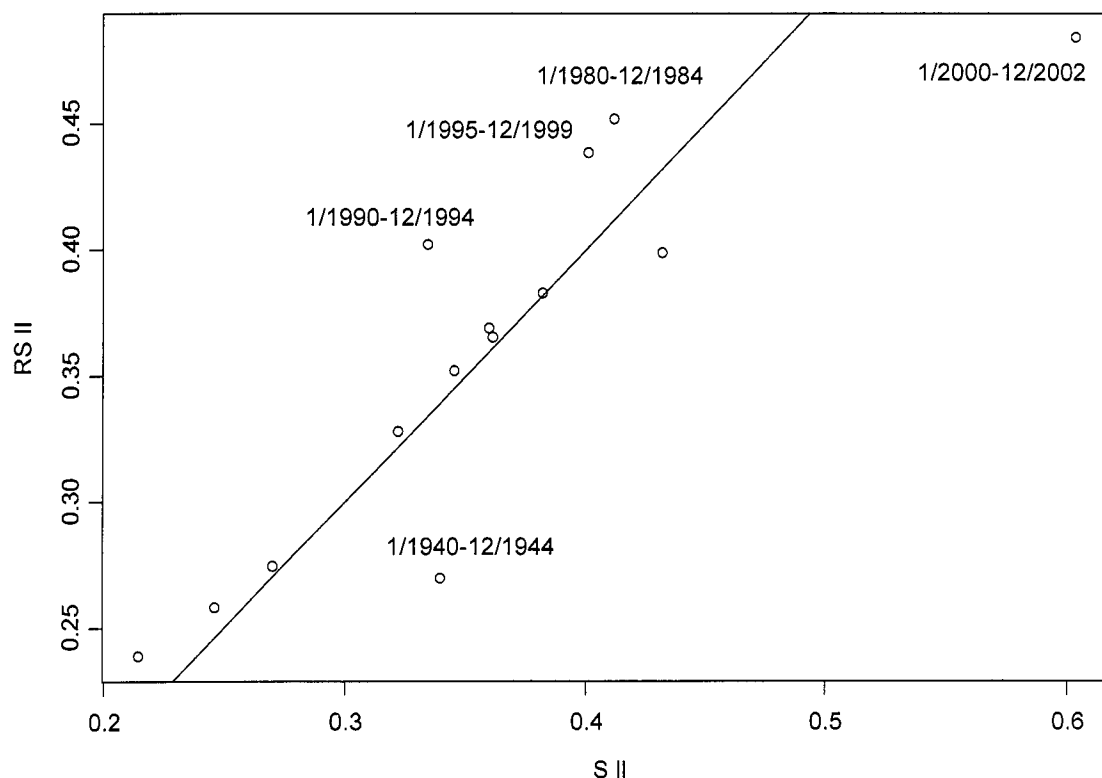
Figure 57. Corresponding R^2 of Figure 56.

3.5.4. S III versus RS III ex I&II and RS III

As described in Section 3.5.1, time series averages and t-statistics as computed in FM, Table 3 are sensitive to outliers, and the inflated standard deviations lower the power of the t-tests. This section compares the FM approach to the RS III ex I&II and RS III approach on 5-year and 10-year contiguous time intervals over 1/1935 through 12/2002. Note that RS III ex I&II means just to make step S III robust, while computing S I and S II like FM.

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The results are shown in Figure 58.
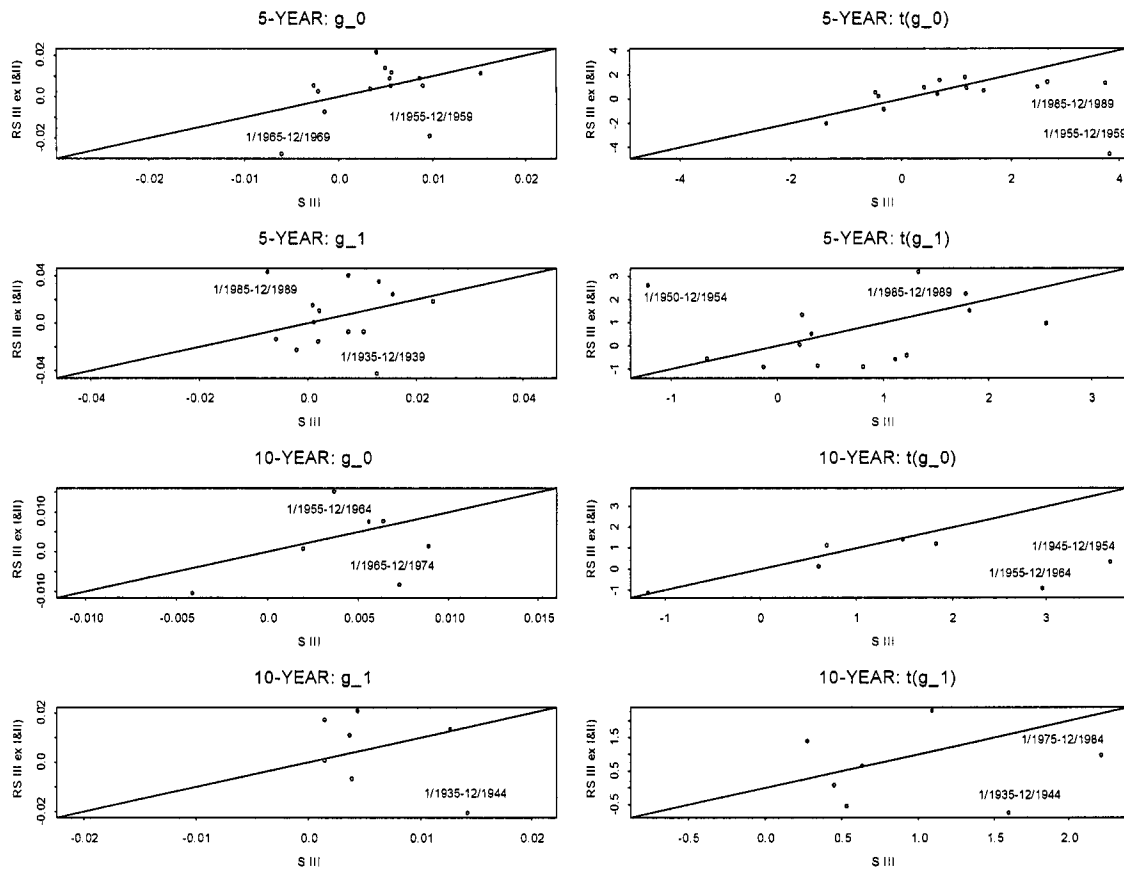
Figure 58. S III versus RS III ex I&II, 5 & 10-Year Intervals: 1/1935 - 12/2002.

Figure 58 shows that replacing just the classical approach of building time series averages and t-statistics using (3.1.4) by ROB methods already results in significant differences on both the 5-year and 10-year periods.

Figure 59 uses the full robust RS III approach over 5-year and 10-year contiguous intervals.
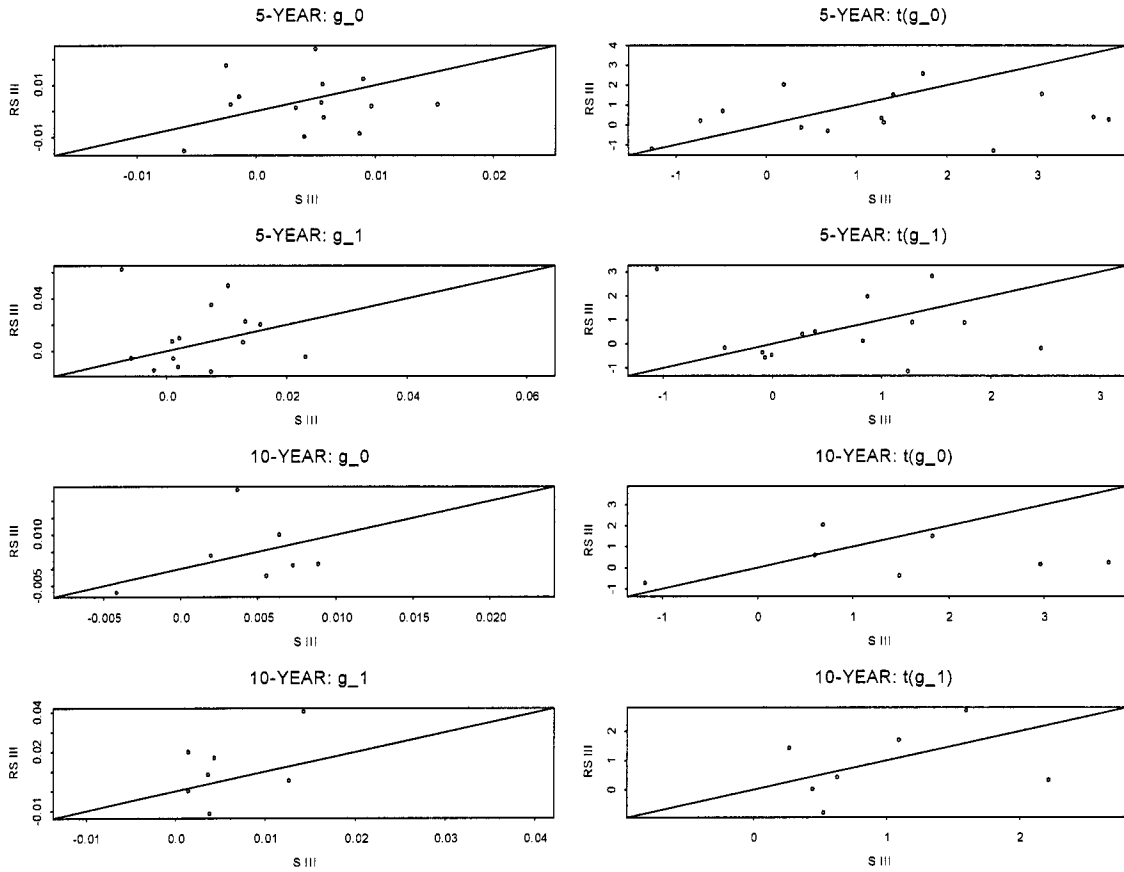
Figure 59. S III versus RS III, 5 & 10-Year Intervals: 1/1935 - 12/2002.

The differences are even greater. The results of Figure 59 are also displayed in Table 13 and Table 14.

Table 13. S III versus RS III.

5-Year	S III		RS III	
	γ_0	$t(\gamma_0)$	γ_0	$t(\gamma_0)$
1/1935-12/1939	0.004	0.43	-0.010	-0.16
1/1940-12/1944	0.003	0.66	0.001	0.09
1/1945-12/1949	0.009	2.51	0.012	1.53
1/1950-12/1954	0.009	2.69	-0.009	-1.33
1/1955-12/1959	0.010	3.84	0.002	0.36
1/1960-12/1964	0.005	1.17	0.024	2.54
1/1965-12/1969	-0.006	-1.34	-0.016	-1.23
1/1970-12/1974	-0.002	-0.39	0.002	0.17
1/1975-12/1979	-0.001	-0.30	0.005	0.66
1/1980-12/1984	0.006	1.19	0.003	0.31
1/1985-12/1989	0.015	3.77	0.002	0.24
1/1990-12/1994	-0.003	-0.46	0.017	1.99
1/1995-12/1999	0.006	1.51	0.010	1.47
1/2000-12/2002	0.006	0.71	-0.003	-0.33
5-Year	γ_1	$t(\gamma_1)$	γ_1	$t(\gamma_1)$
1/1935-12/1939	0.013	0.82	0.006	0.08
1/1940-12/1944	0.016	1.79	0.020	0.87
1/1945-12/1949	0.001	0.22	-0.006	-0.37
1/1950-12/1954	0.008	1.34	0.035	2.79
1/1955-12/1959	0.001	0.24	0.007	0.49
1/1960-12/1964	0.002	0.39	-0.013	-0.59
1/1965-12/1969	0.013	1.82	0.022	0.85
1/1970-12/1974	-0.006	-0.65	-0.006	-0.18
1/1975-12/1979	0.023	2.57	-0.005	-0.20
1/1980-12/1984	0.002	0.33	0.010	0.38
1/1985-12/1989	-0.007	-1.22	0.062	3.08
1/1990-12/1994	0.010	1.23	0.049	1.94
1/1995-12/1999	0.008	1.12	-0.016	-1.15
1/2000-12/2002	-0.002	-0.12	-0.015	-0.49

RS III lowers the percentage of significant time periods for the intercept γ_0 from 29% to 14% and raises the percentage of t-statistics greater than 1.96 of the slope parameter γ_1 from 7% to 21% on 5-year time periods; however, on 10-year periods, it lowers the significance of the γ_0 from 29% to 14%, while the percentage of large γ_1 remains even.

Table 14. S III versus RS III, 10-Year Intervals: 1935 - 12/2002.

10-Year	S III		RS III	
	γ_0	$t(\gamma_0)$	γ_0	$t(\gamma_0)$
1/1935-12/1944	0.004	0.69	0.023	2.01
1/1945-12/1954	0.009	3.68	0.001	0.22
1/1955-12/1964	0.007	2.96	0.001	0.13
1/1965-12/1974	-0.004	-1.17	-0.007	-0.74
1/1975-12/1984	0.002	0.61	0.004	0.56
1/1985-12/1994	0.006	1.83	0.010	1.47
1/1995-12/2002	0.006	1.48	-0.002	-0.41
10-Year	γ_1	$t(\gamma_1)$	γ_1	$t(\gamma_1)$
1/1935-12/1944	0.014	1.59	0.040	2.67
1/1945-12/1954	0.004	1.09	0.017	1.67
1/1955-12/1964	0.002	0.45	0.000	0.00
1/1965-12/1974	-0.004	0.63	0.008	0.40
1/1975-12/1984	-0.013	2.22	0.005	0.31
1/1985-12/1994	0.002	0.27	0.020	1.40
1/1995-12/2002	0.004	0.53	-0.012	-0.81

3.5.5. ROB Tests of C1, C2, and C3

C1: Table 15 shows the percentage of month-to-month significant t-statistics for the FM, RS II ex I, and RS III approach from 1/1935 through 12/2002 using Panel D.

Table 15. Panel D: Significant T-Statistics, 1/1935 through 12/2002.

	FM	RS II ex I	RS II
$t(\gamma_2)$	18.3%	13.4%	13.9%
$t(\gamma_3)$	14.5%	10.3%	6.9%

Figure 60 shows the 5-year time series averages and t-statistics of the month-to-month cross-sectional regression estimates obtained using RS III and Panel D.
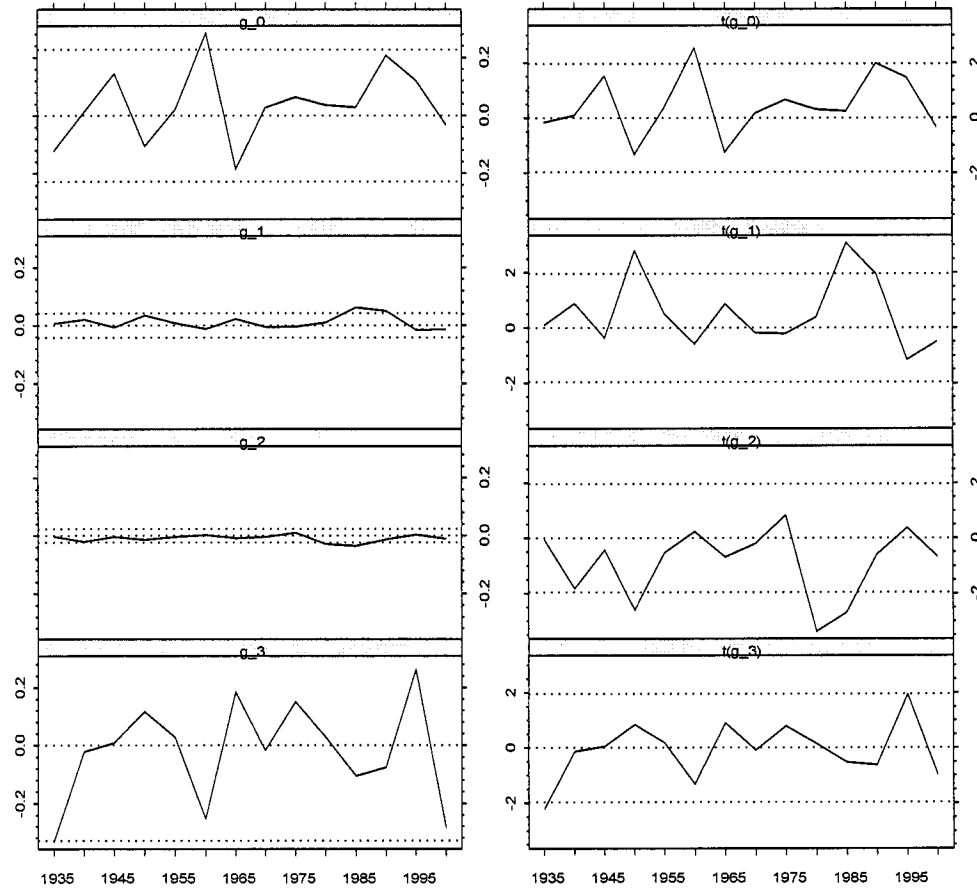
Figure 60. Panel D: Estimates using RS III.

Table 15 showed that the monthly percentages of significant t-statistics of γ_2 and γ_3 in the FM, Panel D are only 18.3% and 14.5%, respectively. It also showed the percentage of significant t-statistics for the RS II approach as 13.9% and 6.9%, respectively. Thus, the percentage of significant t-statistics for RS II ex I are right in the middle. Figure 60 shows that the γ_2 t-statistics are significant in only two out of 14 time periods, i.e., in 14.3% of the times, and values of γ_2 are close to zero. Therefore it seems plausible not to reject C1.

C2: For similar reasons as in C1, the hypothesis in C2 cannot be rejected.

C3: Panel A, Figure 61, Table 16, and Table 17 will provide some answers. Figure 61 shows the 5-year continuous averages and t-statistics using the RS III approach.
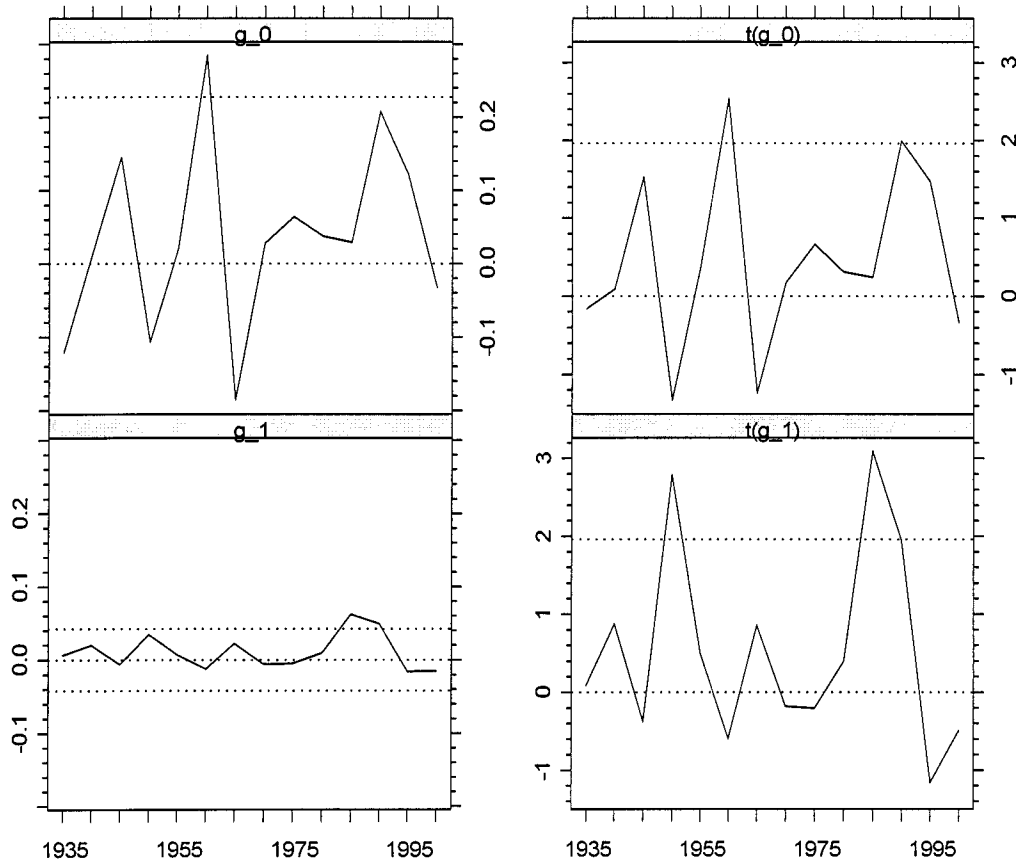
Figure 61. Panel A: 5-Year Averages and T-Statistics using RS III.

The 5-year averages of the slope parameter γ_1 are positive, but the t-statistics of C3 is never significant (under the null hypothesis), and values larger then 1.96 occur only in two out of 14 time periods, i.e. in 14.3% of the time periods.

Table 16 shows the percentage of month-to-month large t-statistics for the FM, RS II ex I, and RS III approach from 1/1935 through 12/2002.

Table 16. Panel A: T-Statistics larger than 1.96, 1/1935 - 12/2002.

	FM	RS II ex I	RS II
$t(\gamma_0)$	49.9%	40.4%	38.6%
$t(\gamma_1)$	61.9%	56.5%	54.8%
$t(\gamma_1)\&(\gamma_1 > 0)$	31.7%	28.6%	27.8%

Table 16 shows that the percentage of t-statistics larger than 1.96 to be only 31.7% and 27.8% for the FM and RS II approach, respectively. Table 17 compares the S III to the RS III for the full time period, with and without the WW II period.

Table 17. FM versus RS III, 1/1935 - 12/2002.

	S III		RS III	
	γ_0	$t(\gamma_0)$	γ_0	$t(\gamma_0)$
1/1935-6/1968	0.005	2.62	0.007	1.80
1/1947-6/1968	0.005	3.10	0.002	0.61
1/1935-12/2002	0.004	3.12	0.005	1.85
1/1947-12/2002	0.004	3.09	0.003	0.97
	γ_1	$t(\gamma_1)$	γ_1	$t(\gamma_1)$
1/1935-6/1968	0.008	2.56	0.006	0.69
1/1947-6/1968	0.006	2.09	0.009	0.98
1/1935-12/2002	0.006	2.69	0.005	0.81
1/1947-12/2002	0.005	2.08	0.006	0.98

For S III, regardless of post WW II period or not, the t-statistics for the intercept γ_0 is always significant and the slope parameter γ_1 always insignificant, in the sense of C3, with t-statistics always larger then 1.96. The t-statistics are small throughout for the RS III approach. Overall, the slope parameter is non-zero. It appears that t-statistics become insignificant when taken over shorter time periods and the ROB approach tends to show smaller t-statistics that do not so much depend on the time-frame chosen. Keeping in mind that differences in the methods are caused by a very small fraction of outliers, it suggests that the ROB approach is more accurate. While C3 cannot be rejected, there is not much evidence for a strong positive risk-return trade-off.

3.5.6. Model Check

The cross-sectional regression models (3.1.3) assume that the error terms η_{p_t} are serially uncorrelated and contemporaneously uncorrelated across assets, i.e.,

$$
\begin{aligned}
\text{cov}\left(\eta_{q_s}, \eta_{p_t}\right) &= \sigma_p^2, \quad \forall q = p, \text{ and } s = t \\
&= 0, \qquad\qquad \text{otherwise}
\end{aligned}
\tag{3.5.1}
$$

Figure 62 shows the density estimates of the lower triangular covariance matrix from (3.5.1) obtained using FM, RS I, and RS II.
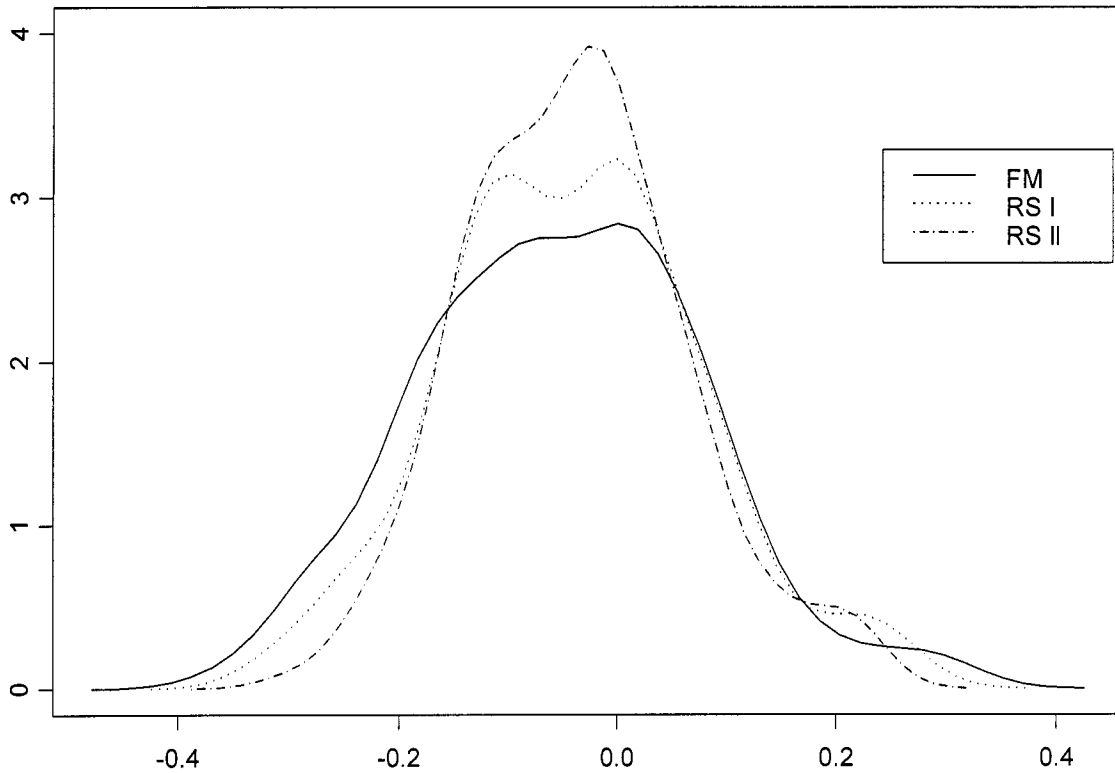
Figure 62. Panel A: Probability Densities of CSR Residuals, 1/1935-12/2002.

The model assumptions expect the density distribution to center steeply around zero. From the three methods, RS II meets the model assumptions better then the other two approaches; still, however, the off-diagonals of the covariance matrix of the residuals are far from being zero.

3.6. Conclusion

This chapter replicated the results of Fama & MacBeth (1973) (FM). It extended the results to more recent time periods, re-evaluated conclusions on various time-frames, and studied the effects of robust regression, using the MM-estimator, on various stages of FM's three-step approach. Although the data source was the same as in FM, data base revisions and consolidations made an exact data match impossible. However, a close match on FM's timeframe of 1/1935 through 6/1968 was convincingly achieved.

FM's method was extended through 12/2002 and evaluated on 5-year and 10-year contiguous time-periods, as well as on time periods excluding times with much higher then usual volatility. FM's conclusions on C1 and C2 could be confirmed and C3 not rejected; however, a strong positive trade-off between return and risk was not found on neither FM's time horizon, when excluding WW II, nor on the extended time horizon excluding small time periods of unusual volatility. This finding also holds when looking only at time periods of high volatility.

The discrepancy of the results caused by outliers was strikingly confirmed in Section 3.5, where OLS regression and classical time series averages and t-statistics were replaced by the MM-estimator. The MM-estimator was set at an efficiency of 99% rejecting only a very small fraction of outliers. Nevertheless, the slope parameter beta was mostly insignificant, regardless of time periods. The linear regression model assumption, i.e., that the error terms are serially uncorrelated across assets and time, hold better using the robust MM-estimator.

4. Multi-factor Model

4.1. Introduction

This chapter makes use of the classical Fama & MacBeth (1973) (FM) and the robust FM (RFM) techniques, developed in Chapter 3, to analyze and to extend a central result in empirical asset pricing—the findings of Fama & French (1992) (FF) on beta and other fundamental factors. It further compares and extends the ground-breaking work in robustness of FF's results by Knez & Ready (1997) (KR). The acronyms FM, RFM, FF, KR, and others will be used through this chapter, and can be found in the glossary.

The FM method has become a standard tool to test theoretical models such as the CAPM or the APT. Key results were published in FF, using returns and firm characteristic factors of all US stocks 7/1963 to 7/1970 that met certain criteria and FM to show that mainly two firm characteristics, size (natural log of market capitalization) and the beme (book equity to market equity ratio), are able to capture the cross-sectional variation in average stock returns, and that the CAPM beta (using a size-beta portfolio classification system) is insignificant.

FF's paper can be viewed as the departure of the academic research from the CAPM beta and the beginning of a focus on size and beme, as well as other factors.

The controversial results in FF also initiated research to improve FM and to make it less sensitive to outliers. KR applied, in a particularly motivating application, the *least trimmed squares* (LTS) regression estimator (Rousseeuw, 1984) to the methodology and data of FF. They showed that the negative risk premium on size, as reported in FF, is caused by observations in only 16 months (out of 330) and a fraction (less than 1%) of small sized firms. Trimming these outliers produced a positive relationship between average returns and firm size. Motivated by the success of KR, Garza-Gomez, Hodoshima, & Kunimura (2001) applied the KR method, using the same risk factors, to the Japanese stock market and confirmed their results.

Chou, Chou, & Wang (2004) extended FF and KR through 2001. Unlike KR, they also analyzed beta. Chou, Chou, & Wang (2004) were able to confirm the effects of

influential months found in KR. Additionally they found that the beta coefficient is positive and significant only in January, while in the rest of the time periods the beta coefficient is either negative or flat.

KR, Garza-Gomez, Hodoshima, & Kunimura (2001), and Chou, Chou, & Wang (2004) used the LTS estimator. The LTS has the following drawbacks: it lacks a simple formula to compute coefficient standard errors; when the fraction of contamination is greater then the fraction of trimming, then its efficiency is not clear anymore; and its breakdown point depends on the fractions of contamination and trimming (Stefanski, 1991). Furthermore, in all these papers, robustness has only been applied to the cross-sectional regression part, but not to the evaluation of the resulting time series regression coefficients.

This chapter applies robust methods to all aspects of the methodology of FF that can be influenced by outliers and also extends the results to recent time periods and additional analysis.

The next section introduces notations and describes the data and FF's specific technique to compute the explanatory variable—stock beta. It then specifies which of the models used in FF are focused on in this chapter, and recalls how the FM technique can be robustified. The section ends with a brief comparison of the robust time series test-statistics with the non-parametric Wilcox test. Section 4.3 calibrates the data to the results of FF and KR. Section 4.5 and 4.6 detect influential months, such as the January effect, and influential firms. Section 4.7 compares cross-sectional regression results using FM and RFM on various time periods and on firms accepted and rejected separately. It further compares results from FM and RFM, only on firms with significant month-to-month coefficients, and ends with a proper model selection procedure using the classical and a robust *Akaike Information Criterion* (AIC, RAIC) criterion.

4.2. Data, Notation, and Methods

The data are monthly discrete returns from 7/1970 through 6/2004 of non-financial firms listed on NYSE, AMEX, and NASDAQ, intersected with the merged COMPUSTAT annual industrial files of income-statement and balance-sheet data,

maintained by CRSP.

FF uses FM's approach (Chapter 3) to empirically test asset pricing. *Cross-Sectional regression* (CSR) is conducted by OLS regression of the cross-section of returns on variables hypothesized to explain expected returns. The resulting time series of regression coefficients are then used to evaluate the model and to test for significance of the explanatory variables. The time series of regression coefficients are evaluated using FM's *time series averages* and *t-statistics* approach (TT), detailed in Section 3.

Even though the data has the same structure and origin as in FF, the starting time period chosen was not 7/1963 but 7/1970 for three main reasons: in the 1960s and early 1970s, the US secondary market for stocks was fragmented, meaning, that orders for a given stock were handled differently from other orders (small versus large, several exchanges, OTC). This was true until the Security Act of 1975; the number of stocks available to build portfolios was insufficient for reliable inferences; before 1970, the CSR of firm size shows unusual trend from negative to positive, see Figure 63.
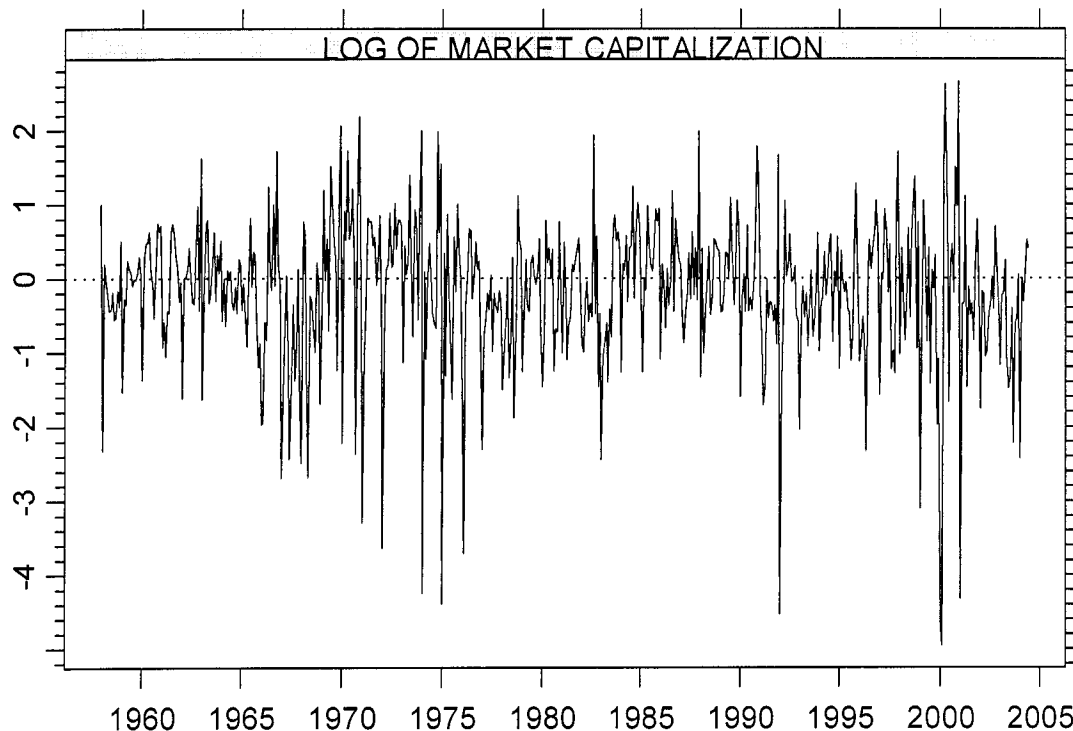

Figure 63. CSR Coefficients of Risk Factor Log of Market Capitalization.

As will be shown in Section 4.3, this truncation does not affect the results from robust methods, which justifies the truncation even more.

Explanatory variables used by FF are Earnings to Price, Stock Leverage: Asset to Book Equity, Market Leverage: Asset to Market Equity, the natural log of Book to Market Equity (*BEME*), the natural logarithm of Market Equity (*SIZE*), and the stock *BETA*. All variables but the *BETA* can be measured precisely for each firm; however, the estimate of market *BETA* is more precise for portfolios. Therefore, FF estimated *post-ranked BETA* for portfolios and then assigned the portfolio post-ranked *BETA* to each stock in the portfolio. The post-ranked *BETA* portfolios are obtained as follows.

4.2.1. SIZE-BETA *Portfolios and Post-Ranked* BETA

To allow for variation in *BETA* that is unrelated to *SIZE*, FF introduced the following procedure (see details in FF). Portfolios are formed each year *t*. In July of year *t*, all stocks are divided into *SIZE* deciles. The *SIZE* deciles are determined in June of year *t* using only NYSE stocks. The *SIZE* deciles are sub-divided into *BETA* deciles using *pre-ranked BETA* of individual stocks. The *BETA* deciles breakpoints are determined using only NYSE stocks that meet the CRSP-COMPUSTAT data requirement with 2-5 years of monthly returns history ending in June of year *t*. From July of year *t* to June of year *t* + 1, all stocks are assigned to the *SIZE* and *BETA* deciles and 100 equal-weighted monthly portfolio returns are computed. The replicate data has 408 (7/1970 through 6/2004) monthly post-ranked portfolio returns on 100 portfolios formed by *SIZE*, then by (pre-ranked) *BETA*. Finally, 100 post-ranked portfolio *BETA* is estimated using the full time series (408 months) of post-ranking portfolio returns. The post-ranking BETA are then assigned to the individual stocks according to their monthly *SIZE-BETA* portfolio membership and used as explanatory variable in the cross-sectional regression.

Note that pre-ranked and post-ranked *BETA* is the sum of the slopes from a regression of monthly firm returns on the current and prior month's market returns. Stock can move across portfolios with year-to-year changes in Stock *SIZE* and its pre-ranked *BETA* estimate for the preceding 2-5 years. Monthly portfolio averages throughout the year can also change when stocks are de-listed.

4.2.2. Models used in Cross-Sectional Regression

FF uses various combinations of explanatory variables to test asset pricing. However, the analysis of this chapter is restricted to the most discussed and promising models with the explanatory variables the post-ranked portfolio *BETA*, *SIZE*, and *BEME*. Five combinations of explanatory variables are studied:

$$RET_{i,t} = \gamma_{0,t} + \gamma_{1,t}BETA_{i,t} + \varepsilon_{i,t}$$
$$RET_{i,t} = \gamma_{0,t} + \gamma_{1,t}SIZE_{i,t} + \varepsilon_{i,t}$$
$$RET_{i,t} = \gamma_{0,t} + \gamma_{1,t}BEME_{i,t} + \varepsilon_{i,t} \tag{4.2.1}$$
$$RET_{i,t} = \gamma_{0,t} + \gamma_{1,t}SIZE_{i,t} + \gamma_{2,t}BEME_{i,t} + \varepsilon_{i,t}$$
$$RET_{i,t} = \gamma_{0,t} + \gamma_{1,t}BETA_{i,t} + \gamma_{2,t}SIZE_{i,t} + \gamma_{3,t}BEME_{i,t} + \varepsilon_{i,t}$$

where $i = 1,...,N_t$, N_t the number of firms in each month, and $RET_{i,t}$ the return of the individual stock i in month t. *SIZE* is used to distinguish between *small-cap* and *large-cap* stocks, while *BEME* is used to distinguish between *value* stocks and *growth* stocks. *SIZE* is a market measure, and *BEME* is a combination of accounting and market measures.

4.2.3. Robustness of SIZE-BETA Portfolios, CSR, and TT

The MM-estimator (ROB) is the robust estimator of choice for reasons explained in Section 1.2. It replaces the OLS estimator in the regressions and the classical location estimate. The latter is done to be consistent with the regression and to be able to take advantage of the robust standard errors provided by the MM-estimator.

FF's *SIZE-BETA* portfolio construction uses the outlier sensitive OLS estimator to compute pre-ranked and post-ranked betas. By replacing the OLS with the ROB estimator, robust *SIZE-BETA* portfolios are obtained and thus robust post-ranked *BETA*.

A *robust cross-sectional regression* (RCSR) technique is created when the OLS regression estimator in CSR is replaced by the ROB estimator.

The *robust time series analysis (*RTT*)* is obtained when the classical time series average in TT is replaced by the ROB location estimate (Section 1.3.2) and the t-statistic of the time series averages is replaced by the t-statistics of ROB.

TT is highly sensitive to the choice of time period, and its t-statistics may have little

power. The proper use of t-statistics as pointed out in Section 1.3.4 and implemented in RTT cannot be emphasized enough. KR, p. 1362, recognized that the violation of normality can cause potential problems when calculating the significance levels from the t-distributions, but dismiss the effect with: "This is because t-statistics tend to be robust to deviations from normality as a consequence of the central limit theorem." However, on p. 1373, they recognize that the t-test may lack the power to detect the size premium in certain periods.

4.2.4. Serial Correlation, T-statistics, and Non-Parametric Tests

T-statistics as well as, e.g., the nonparametric *Wilcox rank method* can break down when the data is serially correlated.

Figure 64 shows the serial correlation of the CSR coefficients for model

$$RET_{i,t} = \gamma_{0,t} + \gamma_{1,t}BETA_{i,t} + \gamma_{2,t}SIZE_{i,t} + \gamma_{3,t}BEME_{i,t} + \varepsilon_{i,t} \tag{4.2.2}$$
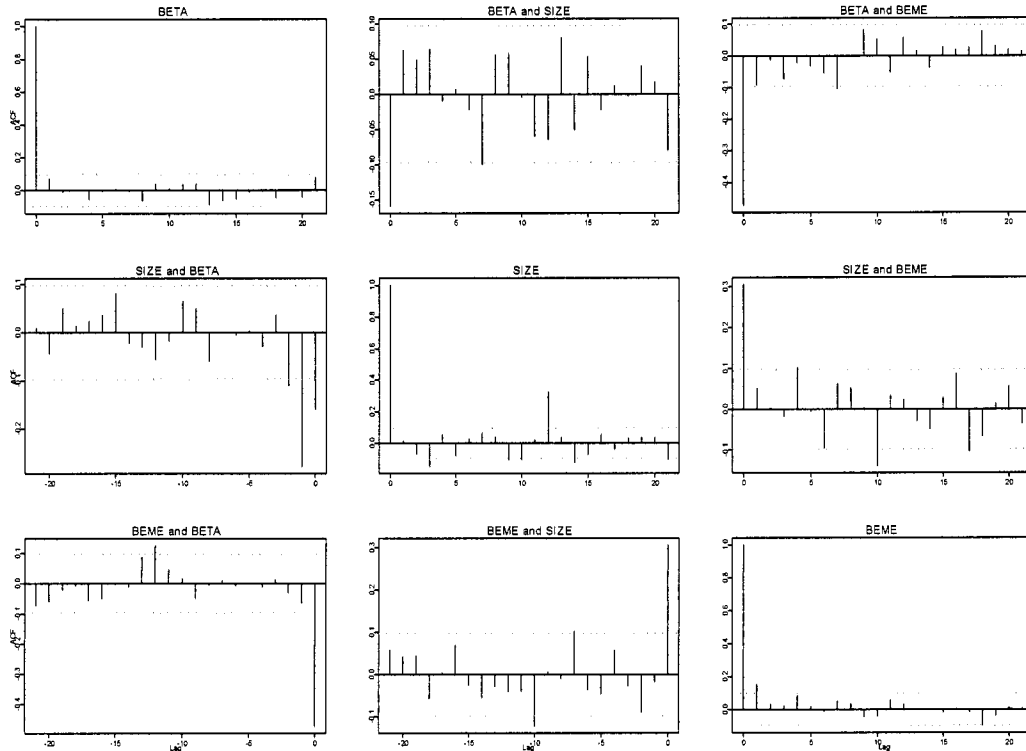
Figure 64. Autocorrelation of CSR Coefficients.

Fortunately, it only shows significant serial correlation in *SIZE* at lag 12, indicating a seasonal effect. The Wilcox test statistics, which works well for non-Gaussian data, were computed along with RTT on all time periods (not shown). For the most part, both tests match closely. This reconfirms the correctness of the ROB t-statistics.

4.3. Calibration of OLS and ROB Replicates to FF and KR

The purpose of this section is to calibrate the data and the proposed MM-estimator to data and methodologies used in FF and KR. Due to the use of different time periods (Section 4.2) and data base revisions at CRSP, results are expected to differ slightly. Table 18 compares the time series averages and t-statistics of the monthly CSR coefficients over a 1963-1990 period, as computed in FF and KR, with the replicates computed over a 1970-1990 period.

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Table 18. TT and (T-Statistics) of the Monthly CSR Coefficients.

	FF (1963-1990)	KR (1963-1990)				
	OLS	OLS	LTS, 1%	LTS, 5%	LTS, 25%	LTS, 50%
BETA	0.15	NA	NA	NA	NA	NA
	(-0.46)	NA	NA	NA	NA	NA
SIZE	-0.15	-0.12	0.14	0.33	0.37	0.30
	(-2.58)	(-2.02)	(2.63)	(6.5)	(8.87)	(7.32)
BEME	0.50	0.47	0.48	0.35	0.11	0.07
	(5.71)	(5.41)	(5.73)	(4.24)	(1.24)	(0.81)
SIZE, BEME	-0.11, 0.35	-0.08, 0.35	0.17, 0.50	0.37, 0.57	0.32, 0.32	0.34, 0.37
	(-1.99, 4.44)	(-1.41, 4.35)	(2.94, 6.39)	(7.71, 7.70)	(7.46, 3.90)	(7.98, 4.35)

REPLICATES (1970-1990)

	OLS	ROB	LTS, 1%	LTS, 2%	LTS, 3%	LTS, 5%	LTS, 25%	LTS, 50%
BETA	-0.24	-1.46	-1.03	-1.33	-1.54	-1.74	-2.23	-2.29
	(-0.87)	(-5.87)	(-4.0)	(-5.24)	(-6.54)	(-7.12)	(-9.32)	(-9.81)
SIZE	-0.08	0.26	0.15	0.17	0.27	0.32	0.38	0.35
	(-0.87)	(5.88)	(3.27)	(3.73)	(5.94)	(7.45)	(9.67)	(8.36)
BEME	0.46	0.29	0.36	0.30	0.30	0.19	-0.11	-0.24
	(5.40)	(3.47)	(4.32)	(3.58)	(3.55)	(2.29)	(-1.27)	(-2.27)
SIZE, BEME	-0.02, 0.41	0.34, 0.55	0.23, 0.50	0.32, 0.52	0.34, 0.54	0.38, 0.53	0.46, 0.41	0.40, 0.34
	(-0.30, 5.8)	(7.94, 8.20)	(5.05, 7.19)	(7.19, 7.45)	(7.92, 7.89)	(9.16, 7.47)	(12.07, 5.56)	(9.96, 4.25)

NA stands for Not Available, t-statistics are in parenthesis.

FF's and KR's OLS values match closely. The *BETA* coefficients are insignificant and *SIZE* coefficients negative and significant, except for the *SIZE* replicates. The *BEME* are remarkably similar, and the two-parameter model *SIZE, BEME* has an insignificant *SIZE* replicate.

Comparing FF with the replicates, OLS *BETA* is insignificant, though with opposite signs, and OLS *SIZE* is insignificant, but similar in values. *BEME* matches FF well. The differences in values for *BETA* and *SIZE* are caused by the truncation of the data in 1970. Interestingly enough, without the non-stationary trend as seen in Figure 63, the negative *SIZE* effect has disappeared.

In the comparison of KR, LTS with the LTS replicates: for each trimming fraction, all values and t-statistics match closely. This proves again that the truncation of the time period before 1970 does not matter for ROB, since they are fitted to the bulk of the data.

Looking at the LTS replicates, it is peculiar that the coefficients are almost monotone functions of the trimming values. The risk premium for *SIZE* becomes more and more positive for rising trimming fractions. That indicates that the core of firms have a positive *SIZE* premium.

The values of the ROB replicates (using the MM-estimator) fall right in between the values of the LTS estimator with 2% and 3% trimming fractions. This means that a trimming fraction of the LTS estimator between 2% and 3% corresponds to some rejection percentage of the MM-estimator at efficiency of 99%. Note, as shown in Figure 68, the MM-estimator rejects a percentage of observations depending on the time period, which can vary from below 1% to over 8% (during the dot.com bubble burst). On average however, the MM-estimator rejects between 1% and 3%. Most results in KR were done for a 5% trimming fraction, which is more than the MM-estimator at 99% would reject.

The calibration was very successful. Going forward, the OLS replicates will be used. Further, in view of the drawbacks of the LTS estimator mentioned in Section 4.1 and the superior properties of the MM-estimator, the MM-estimator at 99% efficiency will be used.

4.4. EDA and Preliminary Results

This section explores the raw data of the *SIZE-BETA* portfolios obtained using OLS and ROB, and the coefficients from CSR and RCSR. The hope is to be able to detect distinct features that allow to fine tune the analysis.

4.4.1. Post-Ranking BETA: *OLS versus ROB*

As described in Section 4.2.1, the 100 *SIZE-BETA* portfolios are used to compute 100 post-ranking *BETA*, which are used as explanatory variable in the cross-sectional regressions.

Figure 65 shows histograms of the 100 post-ranking OLS and ROB *BETA*.
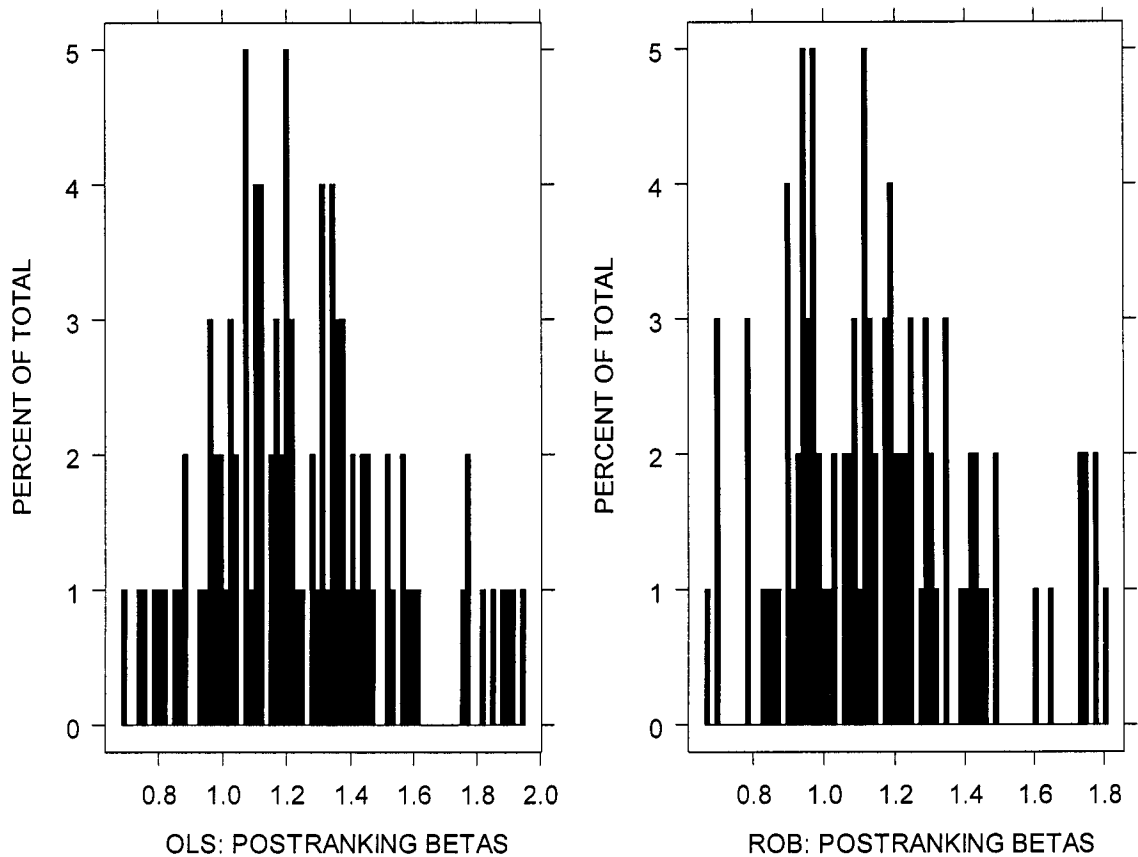
Figure 65. Histogram of Post-ranking *BETA*.

There is a notable gap between 1.6 and 1.75 for the OLS and between 1.5 and 1.7 (except for two *BETA* around 1.6). There could be an economical explanation of the gap or it

could have occurred just by chance. Nevertheless, the OLS and ROB post-ranking *BETA* are very similar.

The left panel of Figure 66 calibrates the average returns from FF, Table 1, Panel A with OLS replicates in a scatter plot. The right panel compares ROB replicates with OLS replicates.
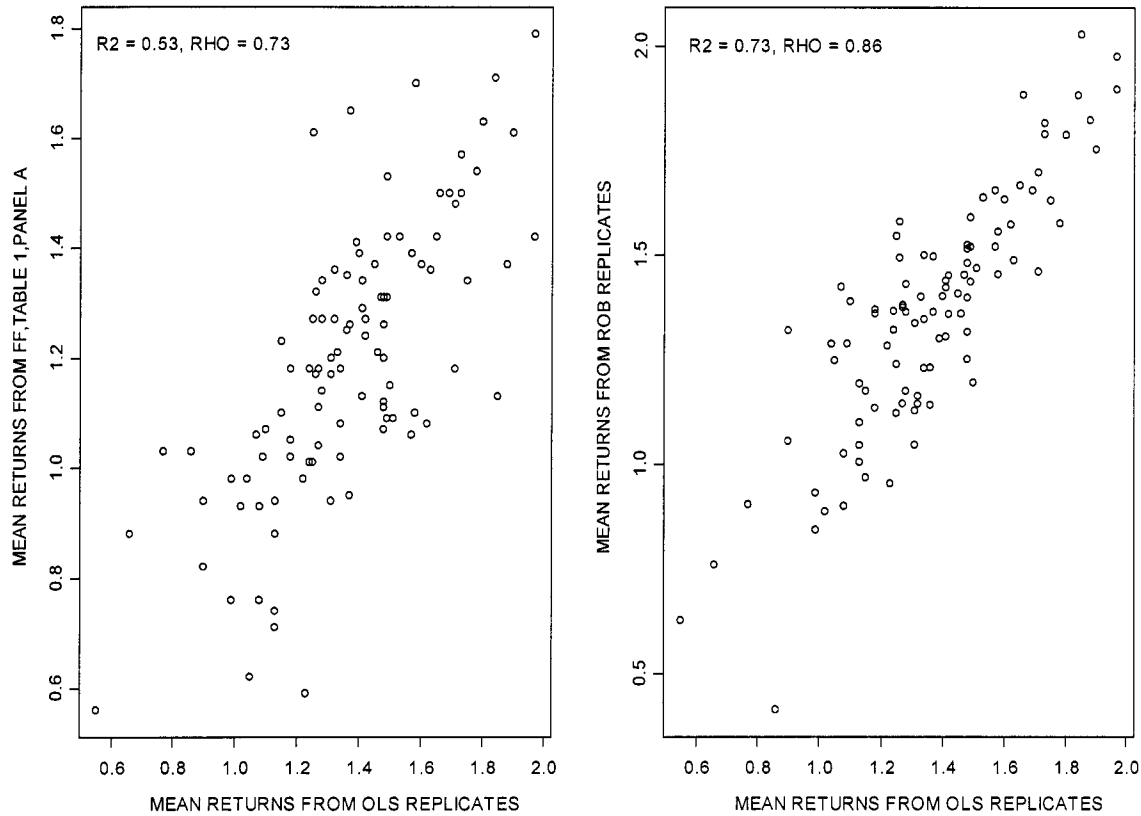
Figure 66. Average *SIZE-BETA* Portfolio Returns.

In the left panel, the averages returns from FF, Table 1, Panel A match its OLS replicates fairly well. In the right panel, the OLS and ROB replicates are very similar.

The left panel of Figure 67 calibrates the post-ranked *BETA* from FF, Table 1, Panel B with OLS replicates in a scatter plot. The right panel compares ROB replicates with OLS replicates.
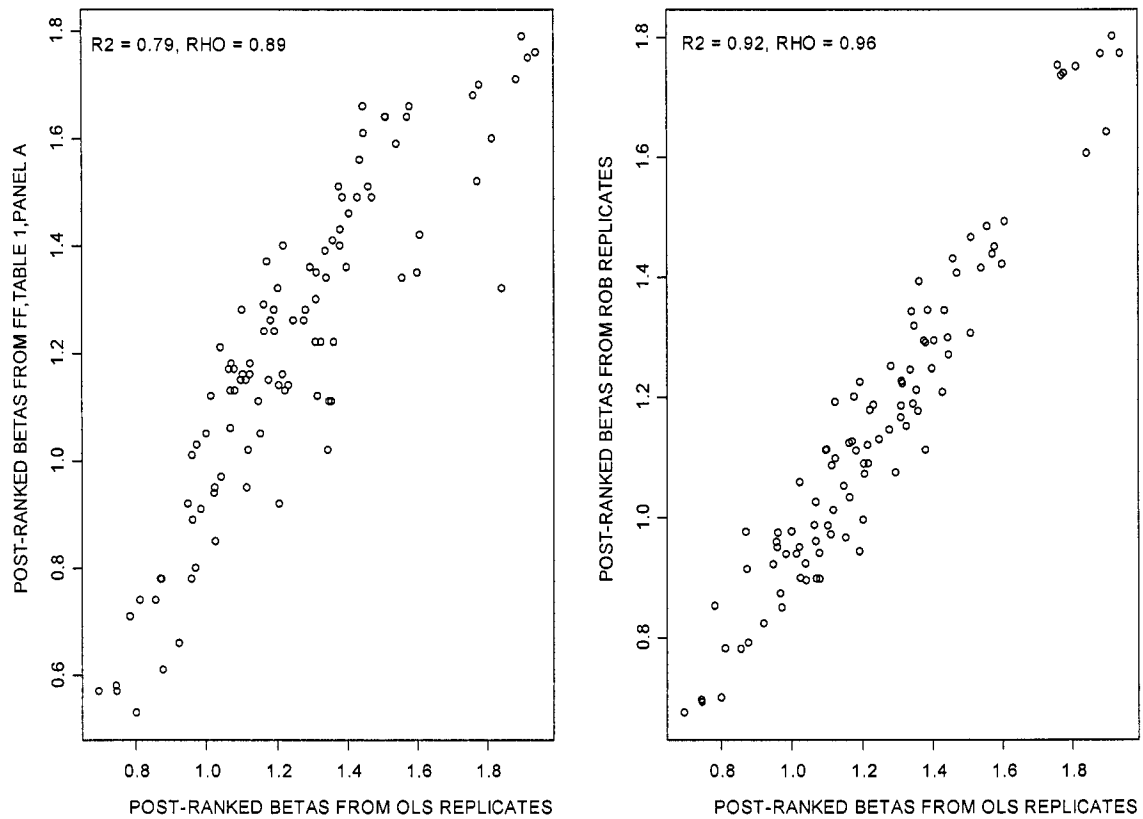
Figure 67. Post-Ranking *BETA*.

In the left panel, the post-ranking *BETA* from FF, Table 1, Panel B match its OLS replicates fairly well. In the right panel, the ROB replicates and OLS replicates are very similar, and the gap noted in Figure 65 appears again. Both Figures show, that the replicates are close to the values in FF, Table 1, and that the impact of replacing OLS with ROB is small. Therefore and to be able to better compare to the classical FM results, the replicated OLS *SIZE-BETA* portfolios are used for the remainder of the chapter.

4.4.2. Leverage Points, Influential Points, and Variation across Time

In any of the cross-sectional regression models (4.2.1), outliers can occur in the distribution of the explanatory variables (leverage points) and the independent variable (influential points). Further, since the final analysis evaluates the month-to-month regression coefficients, outliers can also occur across months.

4.4.2.1. Leverage Points

Figure 68 shows the month-to-month percentage of firms rejected (Section 1.3.1) for all models in (4.2.1), as well as the total number of firms.
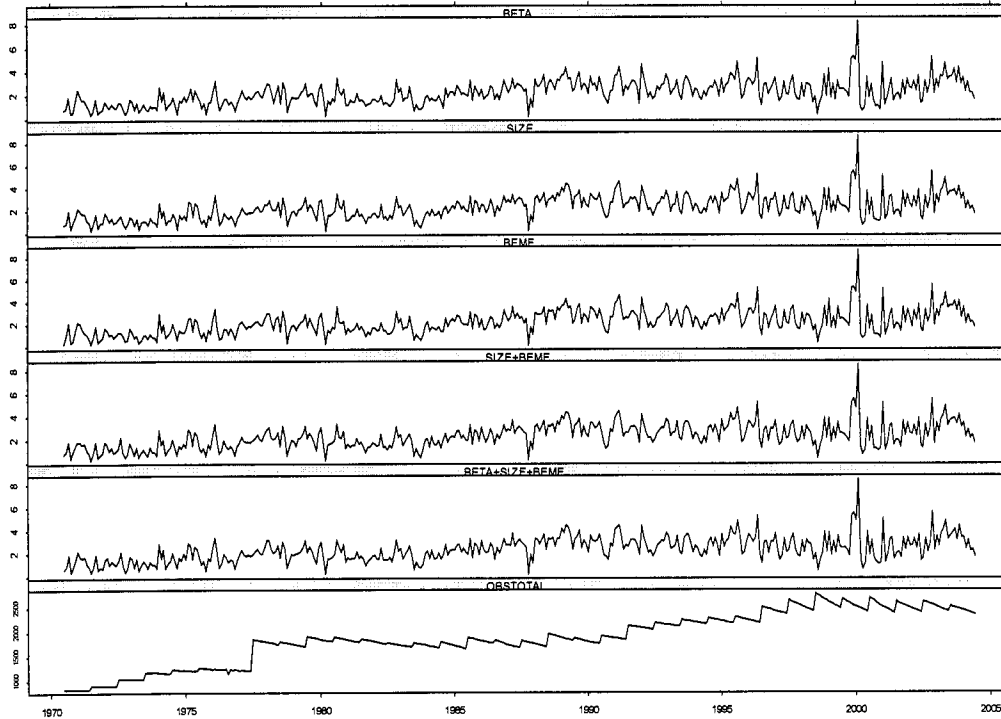
Figure 68. Percentage of Firms Rejected in the RCSR.
The percentage of firms rejected varies little from model to model, indicating that outliers are mostly rejected due to extreme returns, not due to leverage points.

Over the period from 7/1970 through 6/2004, there are 763876 observations, of which 18707 observations were rejected. The percentage of firms rejected varies little from model to model, indicating that outliers are mostly rejected due to extreme returns, not due to leverage points. The analysis will, therefore, not focus on leverage points. The lowest panel in Figure 68 shows the number of total firms per month. The falling number of firms from July of year t to Jun of year $t+1$ is caused by FF's construction of the *SIZE-BETA* portfolios (firms are assigned in July of year t and can only leave the portfolio when de-listed from the exchange within the year).

4.4.2.2. Influential Points

Influential points occur in the returns direction of firms. During the *SIZE-BETA* portfolios construction, the returns of all firms (7/1970-12/2004) were classified into portfolio *SIZE* and *BETA* deciles. This classification can be used to plot the quantiles of returns against the quantiles of a normal distribution, by *SIZE* or *BETA* deciles.

Figure 69 shows the distribution of firm returns versus the quantiles of a normal distribution for the *SIZE* deciles (recall that *SIZE* 1 is the smallest of the *SIZE* deciles).
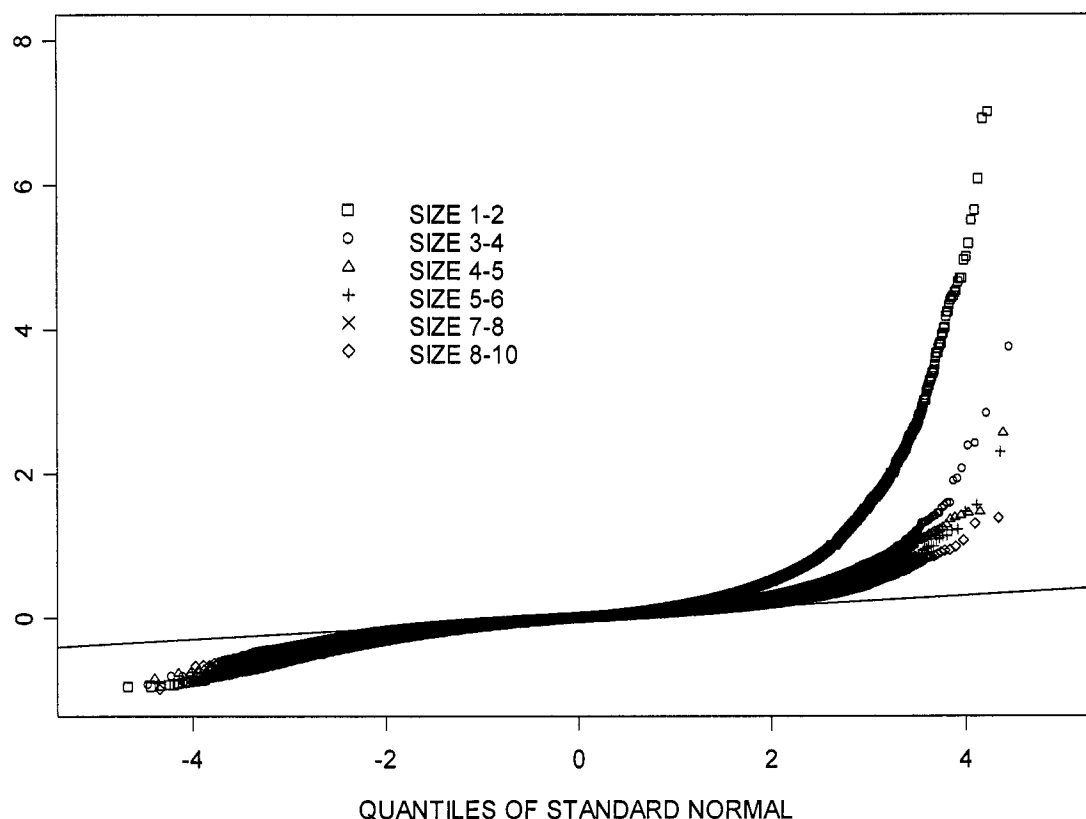
Figure 69. Quantiles of Returns versus Normal Quantiles by *SIZE*.
The smaller the *SIZE* deciles, the larger the skewness and the more extreme the departure from normality.

Figure 69 shows that the smaller the *SIZE* deciles, the larger the positive skewness and the more extreme the departure from normality. It shows also significant non-normality for larger size firms. Figure 70 shows the distribution of firm returns versus the quantiles of a normal for the *BETA* deciles (*BETA* 1 is the smallest of the *BETA* deciles).
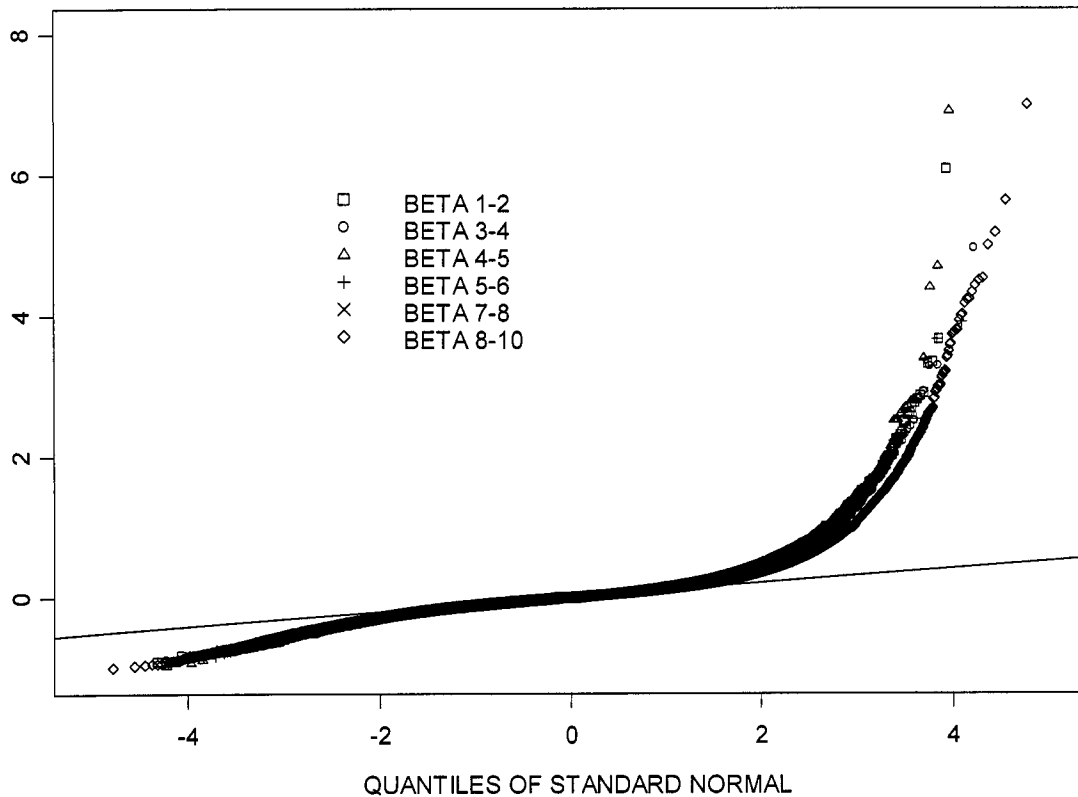
Figure 70. Quantiles of Returns versus Normal Quantiles by *BETA*.
The returns distribution is positively skewed regardless the *BETA* deciles.

Again, the returns distribution is positively skewed. There is a clear departure from non-normality across all *BETA* deciles, however, not much *BETA* effect. Influential returns are likely to have effect on the analysis.

4.4.2.3.   Variation across Time

The exploratory data analysis of the time series of cross-sectional regression coefficients will be shown only for model (4.2.2).

The raw data used in the monthly cross-sectional regressions can be shown as scatter plots revealing the typical structure. The best visualization of the characteristics would be an animation of all monthly scatterplots. Obviously, that is not possible within this dissertation document so scatter plots are shown for the year of 1997 in Figure 71 and Figure 72 for *BETA* versus *RET* and *SIZE* versus *RET,* respectively, together with an OLS

(dotted line) and ROB (solid line) regression fit. The year of 1997 was chosen since it was a year without dramatic market events in a bull market.

Figure 71. Raw data for Cross-Sectional Regressions: *BETA* versus *RET*, in 1999. The dotted line is the OLS and the solid line the ROB regression fit.

In Figure 71 the dotted and solid lines show the OLS and the ROB regression fit of *BETA* on *RET*, respectively. In January both regression fits show a positive slope with the OLS slope distinctly more positive. From February through April both regression slopes are negative. From May through September the OLS regression slope is positive while the ROB regression slope is flat. From October through December both regression slopes are negative again.

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01/1997, rejected: 90 (3.7%)   02/1997, rejected: 55 (2.3%)   03/1997, rejected: 34 (1.4%)   04/1997, rejected: 42 (1.8%)

05/1997, rejected: 78 (3.3%)   06/1997, rejected: 49 (2.1%)   07/1997, rejected: 60 (2.3%)   08/1997, rejected: 83 (3.2%)

09/1997, rejected: 95 (3.7%)   10/1997, rejected: 60 (2.3%)   11/1997, rejected: 46 (1.8%)   12/1997, rejected: 43 (1.7%)
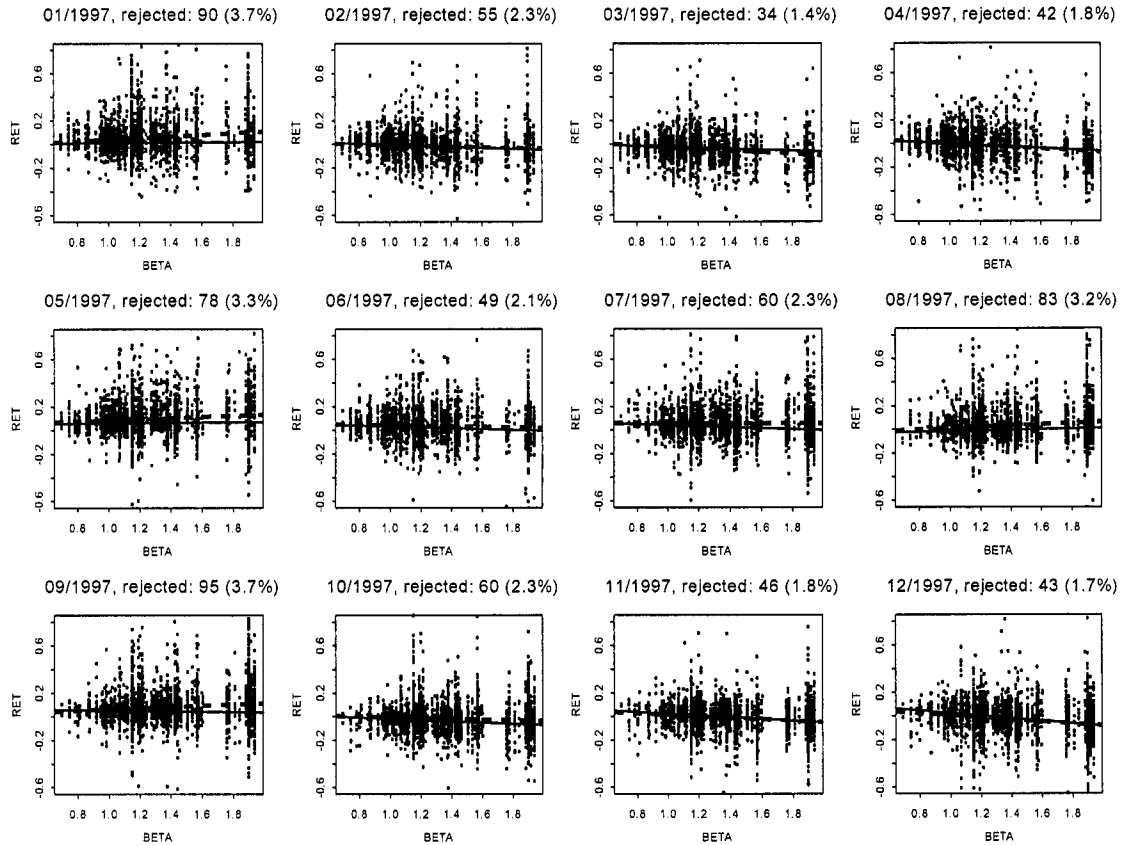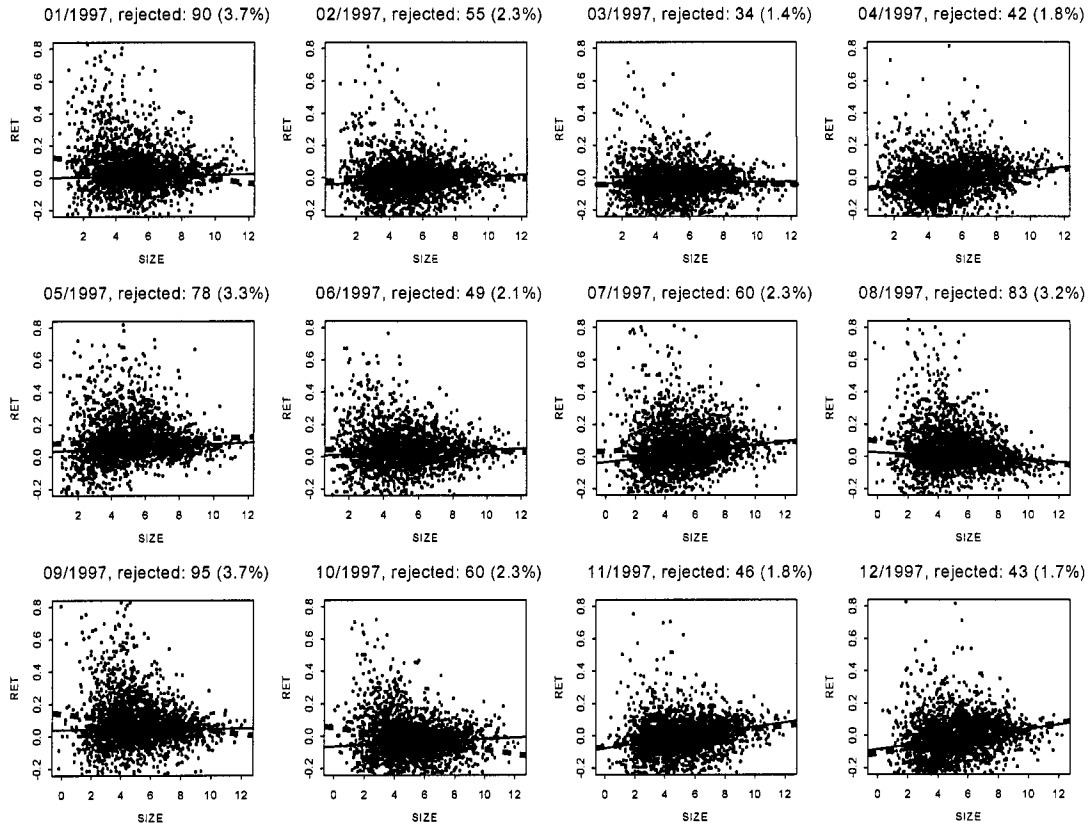
Figure 72. Raw data for Cross-Sectional Regressions: *SIZE* versus *RET*, in 1999.
The dotted line is the OLS and the solid line the ROB regression fit.

In Figure 72 the dotted and solid lines show the OLS and the ROB regression fit of *SIZE* on *RET*, respectively. In January the OLS slope is clearly negative while the ROB slope is flat. While both slopes are flat in February and March, they become positive in April through July then negative in August. In September and October the OLS slope is negative while the ROB slope is flat. In November and December both regression fits are positive again. The OLS and ROB fit disagree whenever small fractions of influential points were rejected in the ROB fit. In Figure 71 the slopes disagree from May through September where the OLS slope is positive; once the 1%-4% of influential points were removed, the ROB slope is flat. Note that outliers mostly appear for positive *BETA* and positive *RET*. In Figure 72 the slopes disagree in January, September, and October where the OLS fit is negative and the ROB fit is flat. Note that for *SIZE* most outliers have small *SIZE* and positive *RET*.

For greater detail, the following graphs show scatterplots of *BETA* and *SIZE* on *RET* for four different years for the months of January, August, and December.
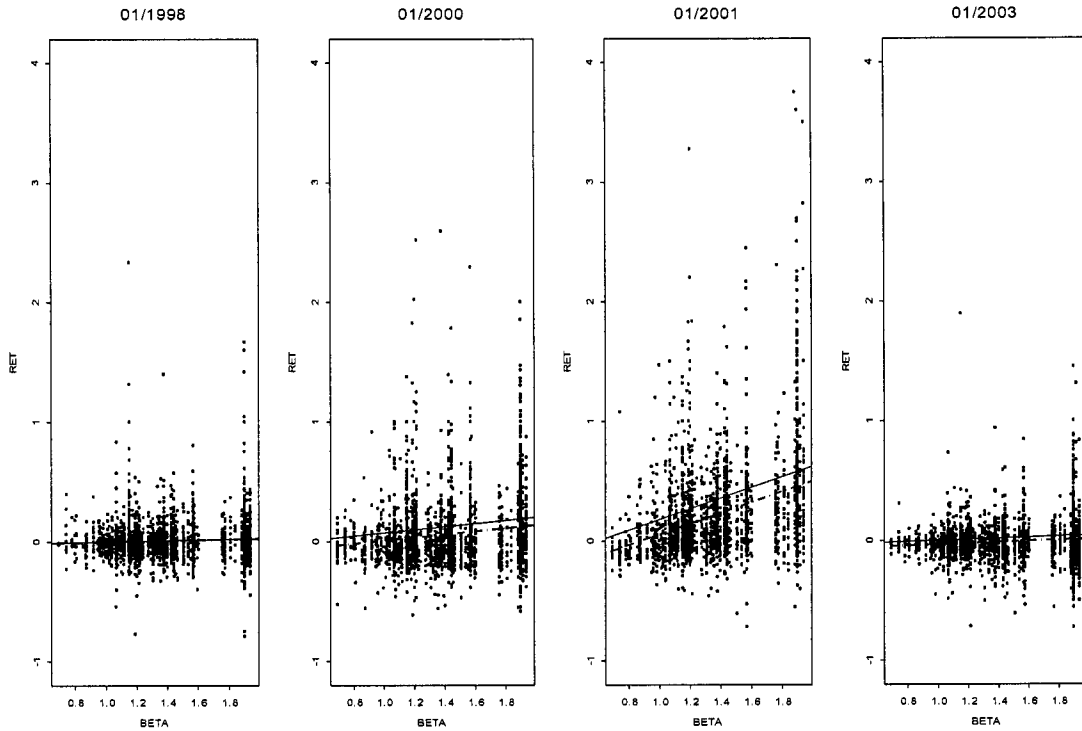
Figure 73. *BETA* versus *RET*: January 1998, 2000, 2001, 2003.
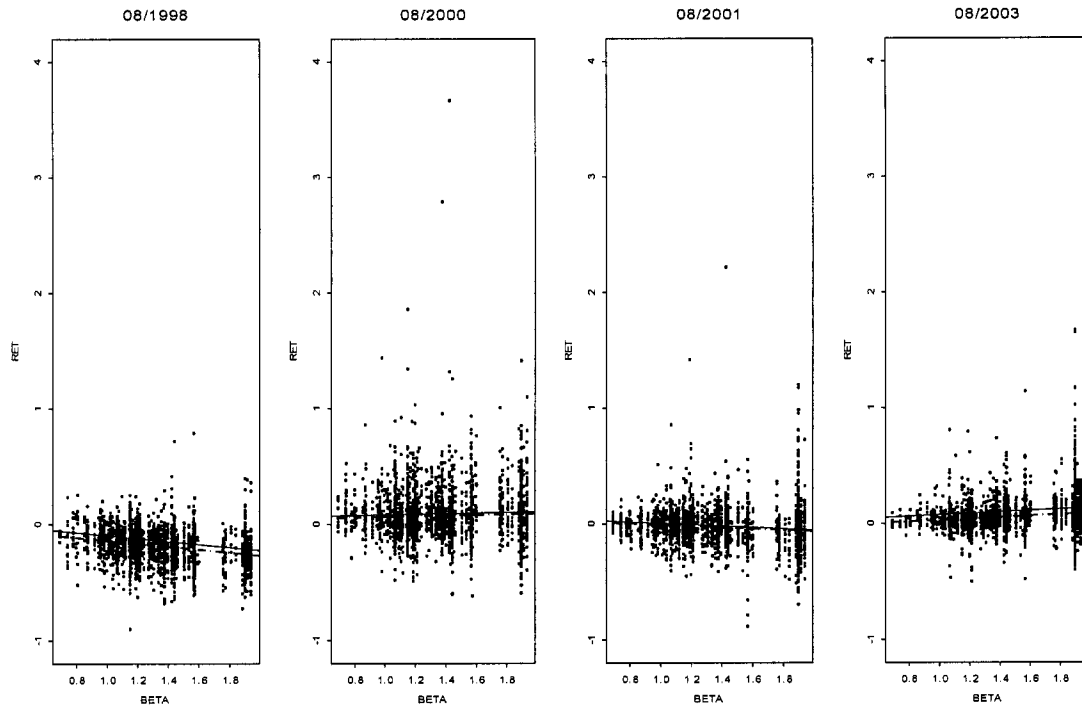
Figure 74. *BETA* versus *RET*: August 1998, 2000, 2001, 2003.
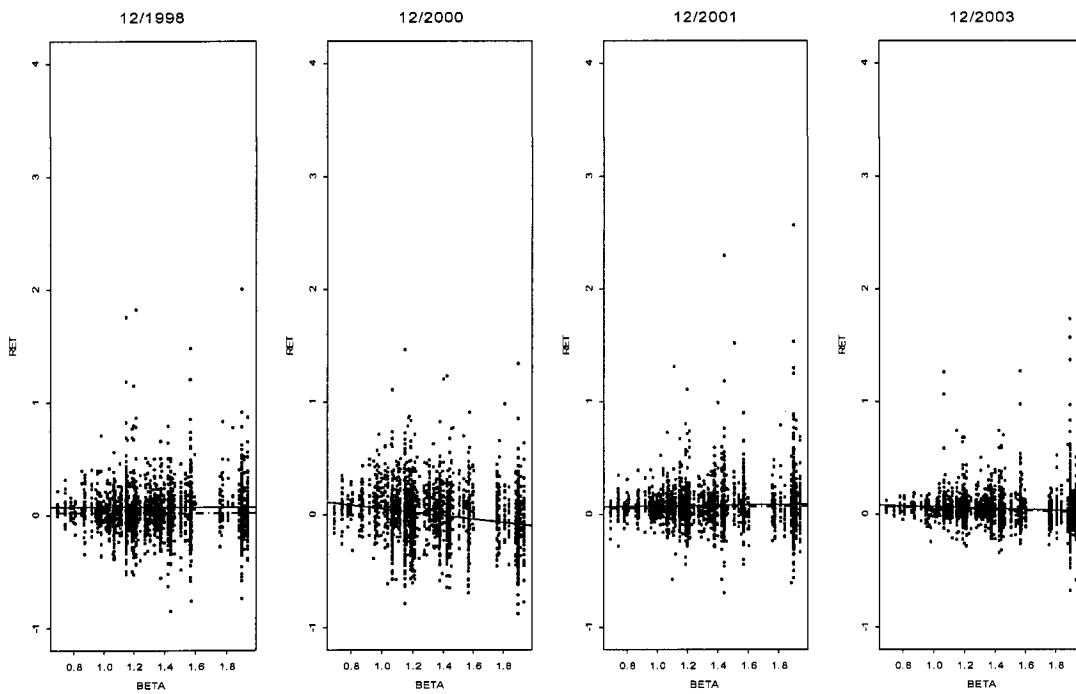
Figure 75. *BETA* versus *RET*: December 1998, 2000, 2001, 2003.

For *SIZE*:
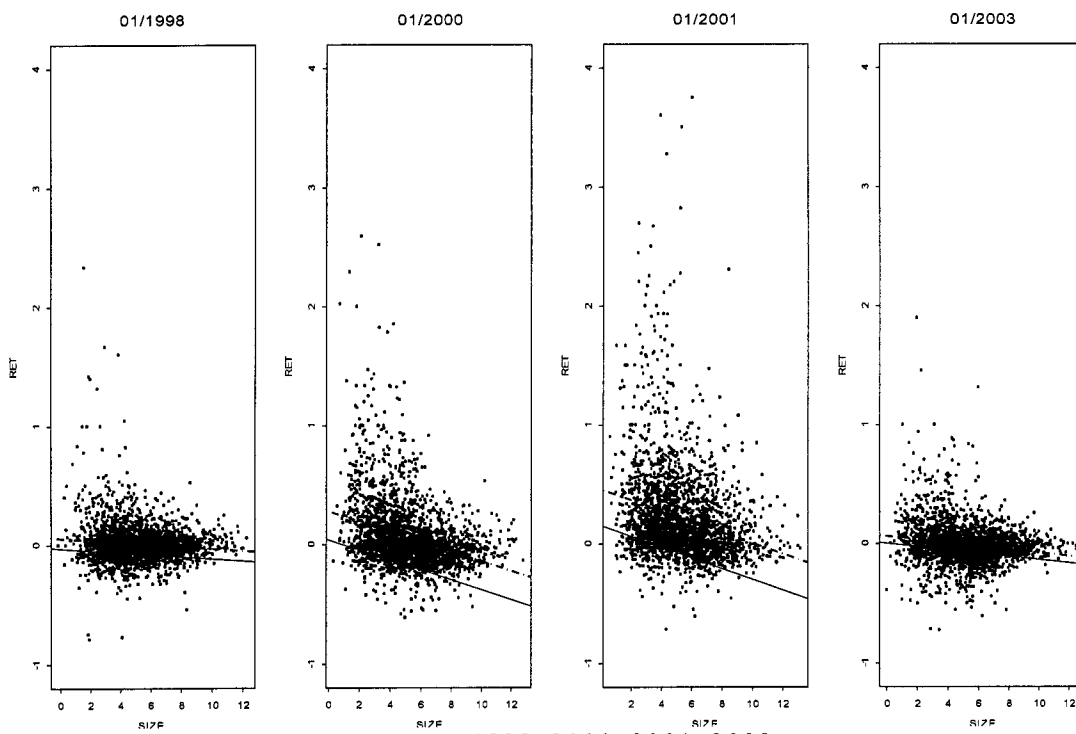
Figure 76. *SIZE* versus *RET*: January 1998, 2000, 2001, 2003.
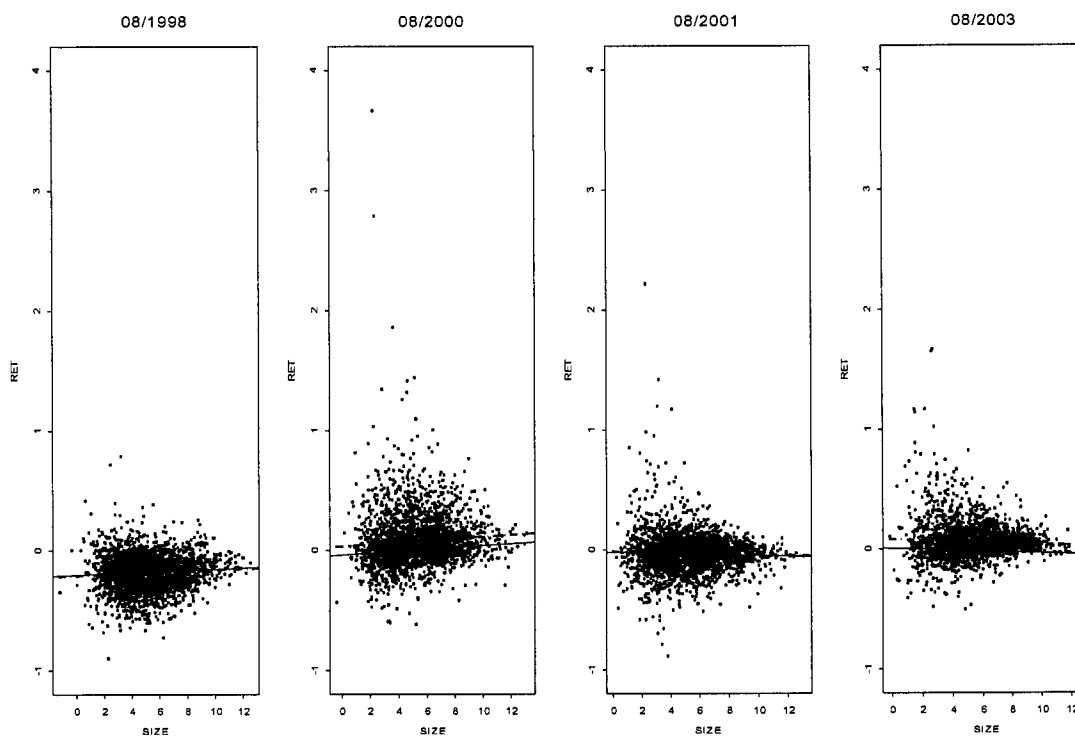
Figure 77. *SIZE* versus *RET*: August 1998, 2000, 2001, 2003.
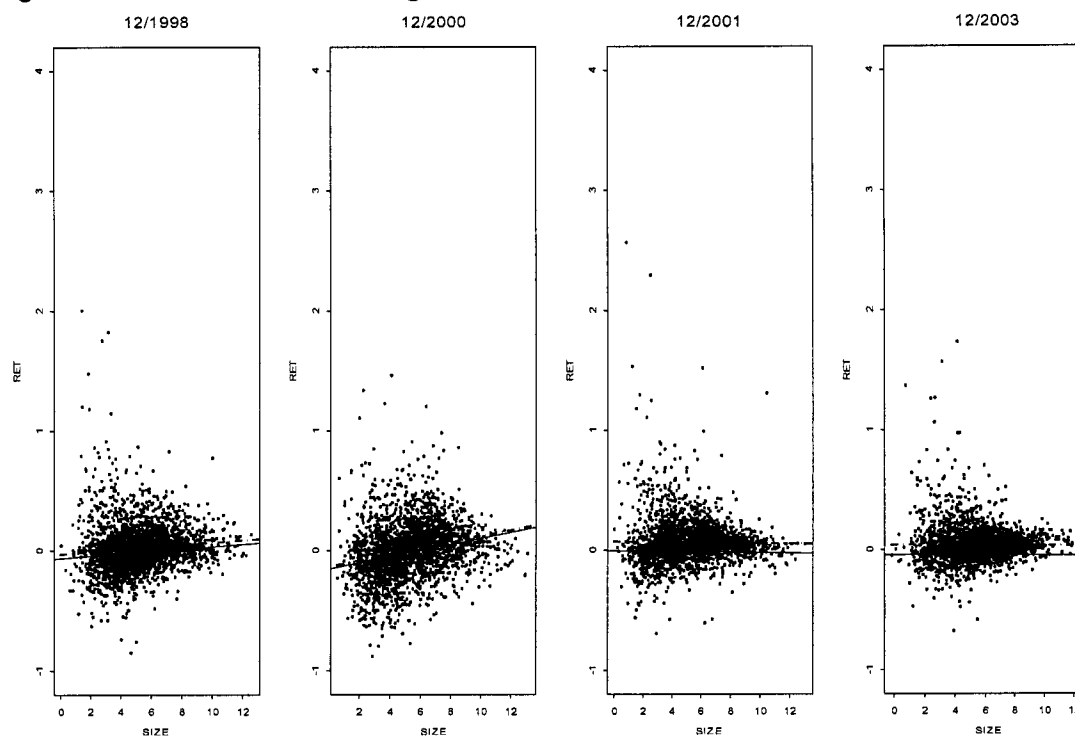
Figure 78. *SIZE* versus *RET*: December 1998, 2000, 2001, 2003.

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Figure 73 throughFigure 78 shows greater details of the seasonal effects: positive risk premium for *BETA* in January, while it is negative of flat in August and December; negative risk premium for *SIZE*, while it is positive in August and December. Furthermore, there is a tendency for outliers to occur for positive *RET* and large *BETA* and small *SIZE*. Outliers in these quadrants tend to upward bias *BETA* and downward bias *SIZE*.

The show the impact of months on the cross-sectional regression coefficients, the time series of the cross-sectional regression coefficients will be compared for: CSR with all months included, January excluded, January and February excluded, and RCSR. The reason why certain months are excluded is to show the January effect (Keim, 1983), explained in greater detail in Section 4.5.

Figure 79 compares the *BETA* coefficients.
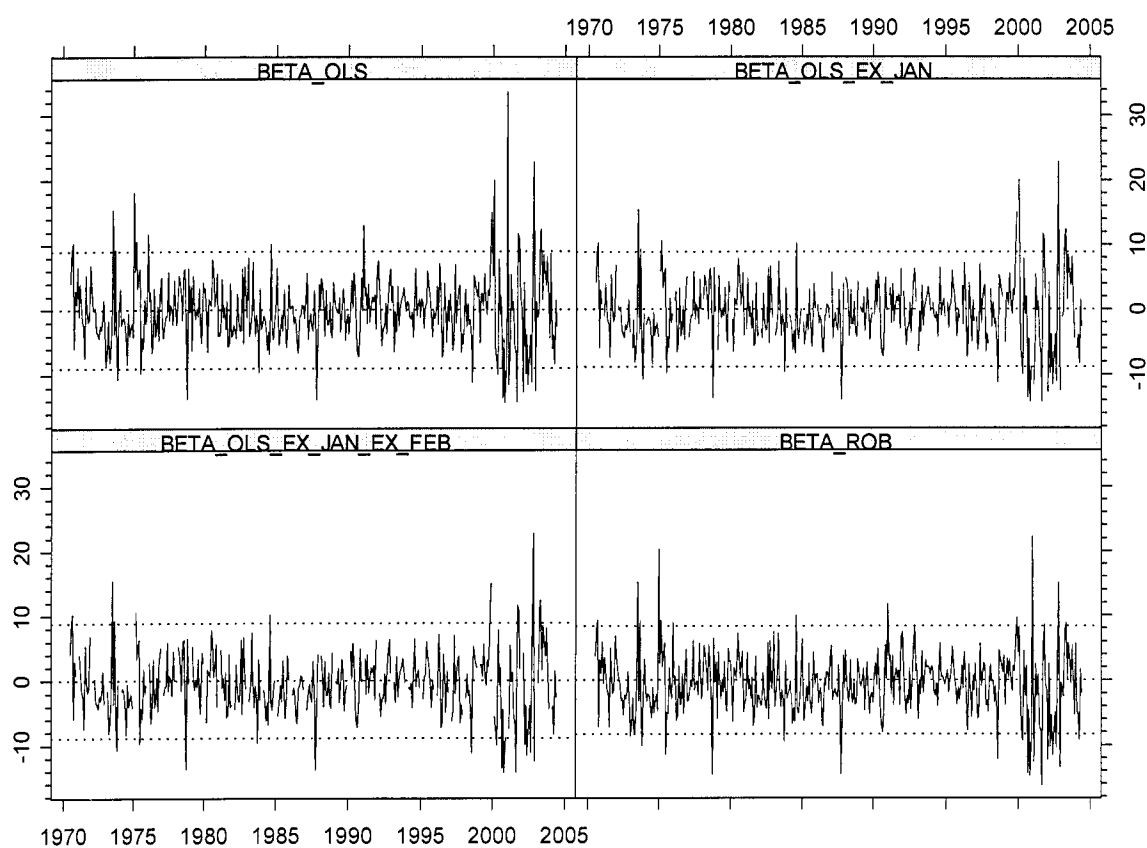
Figure 79. *BETA*: Time Series of CSR and RCSR Coefficients.

Figure 80 compares the *SIZE* coefficient, Figure 81 compares the *BEME* coefficient.
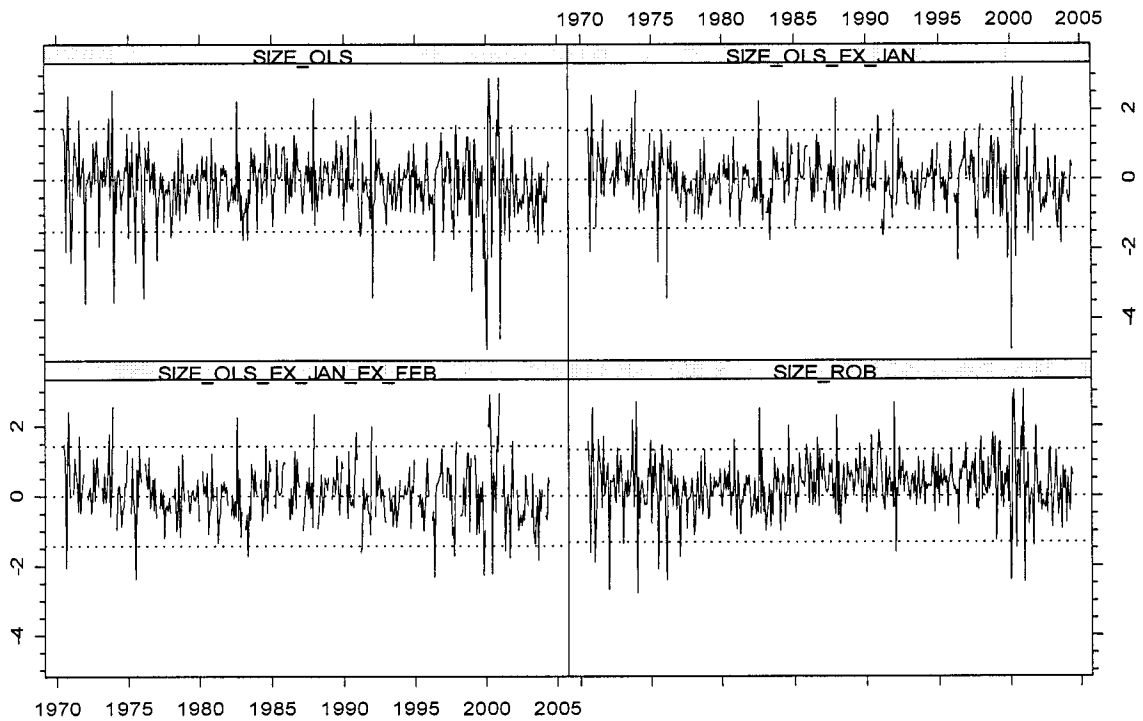
Figure 80. *SIZE*: Time Series of CSR and RCSR Coefficients.
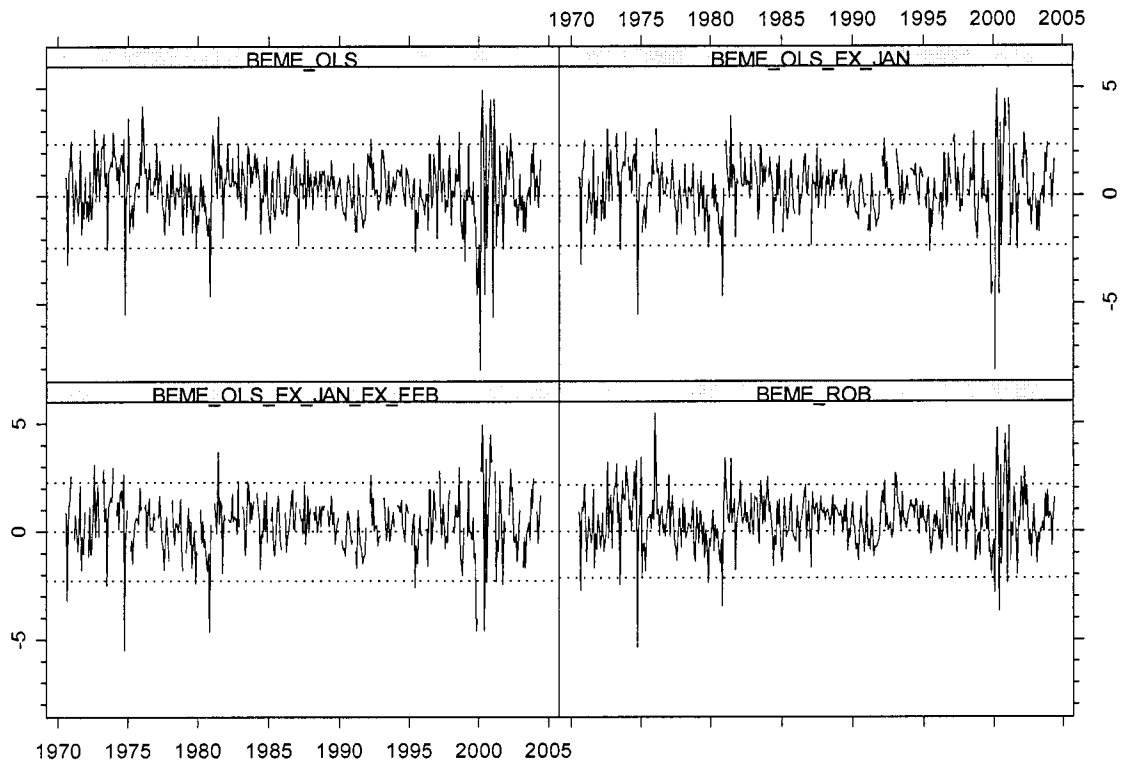
Figure 81. *BEME*: Time Series of CSR and RCSR Coefficients.

All CSR coefficients show non-stationary over longer periods of time. The OLS coefficients show higher volatility and more extreme outliers then the ROB coefficients. However, removing the months of January and then both January and February from the CSR removes the extreme outliers and make OLS and ROB look more similar. This indicates weakly the existence of a January/February effect.

Also noticeable is the frequent outliers and non-stationary volatility. There are periods of high volatility (1970-1976, 1998-2004), and one period with a low volatility (1976-1997).

4.4.3. SIZE-BETA portfolios: OLS versus ROB

The *SIZE-BETA* portfolios are used in FF, Table 1, Panels A through C as informal tests to evaluate the relationship between *SIZE*, post-ranked *BETA*, and average *SIZE-BETA* portfolio returns. The equal weighted average of the *SIZE-BETA* portfolio returns is the same as the time series average of the mean returns of each portfolio. The averages across months and across portfolios can be computed using the classical mean estimator or the MM-estimator. Note that the notion of equal weighted portfolio return does not hold anymore when using the MM-estimator; however, in the sense that an equal weighted portfolio return measures the central tendency, the ROB mean also measures a central tendency.

FF explains FF, Table 1, Panels A through C without proper tests. A simple two-way analysis can test for significance and interaction. Figure 82 shows the two-way analysis replicating FF, Table 1, Panels A, the average monthly returns in percent for the 100 *SIZE-BETA* portfolios.
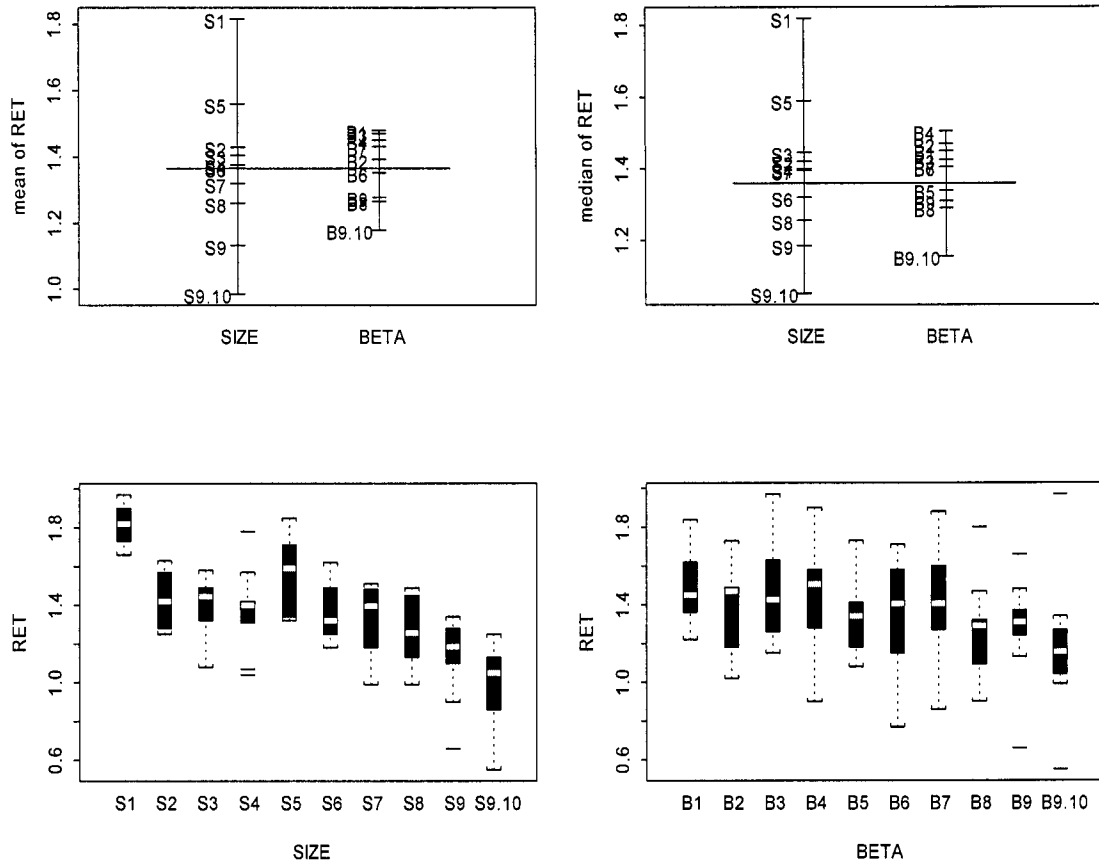
Figure 82. Two-Way Analysis: Average Returns of *SIZE-BETA* Portfolios.
SIZE explains more variability then *BETA*. Higher *RET* is associated with smaller *SIZE*. *RET* and *BETA* show a slight negative trend, however, returns in the large *BETA* buckets vary widely.

Figure 82 shows that *SIZE* explains more variability then *BETA*, and that the mean and the median two-way main effects are fairly similar. Higher *RET* is associated with smaller *SIZE*. *RET* versus *BETA* show a slight negative trend, however, the average returns in the large *BETA* buckets do not show a clear trend. The F-tests are significant for both factors ($F_{SIZE} = 21.1$, $F_{BETA} = 4.1$).

Figure 83 shows that there may be local interactions that could be tested for.
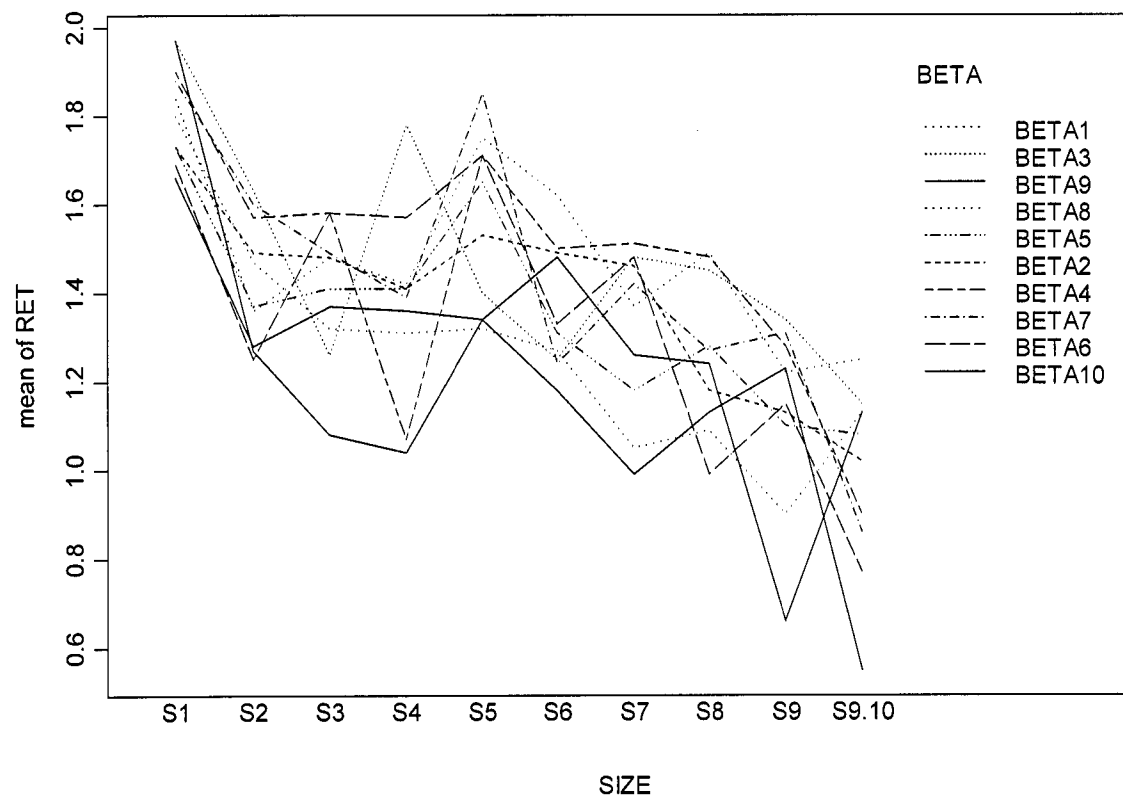
Figure 83. Two-Way Analysis: Interaction between *SIZE* and *BETA*.

In Figure 82, the time series averages and equal-weighted averages across portfolios were computed classically. In Figure 84, the equal-weighted averages across portfolios were computed using ROB.
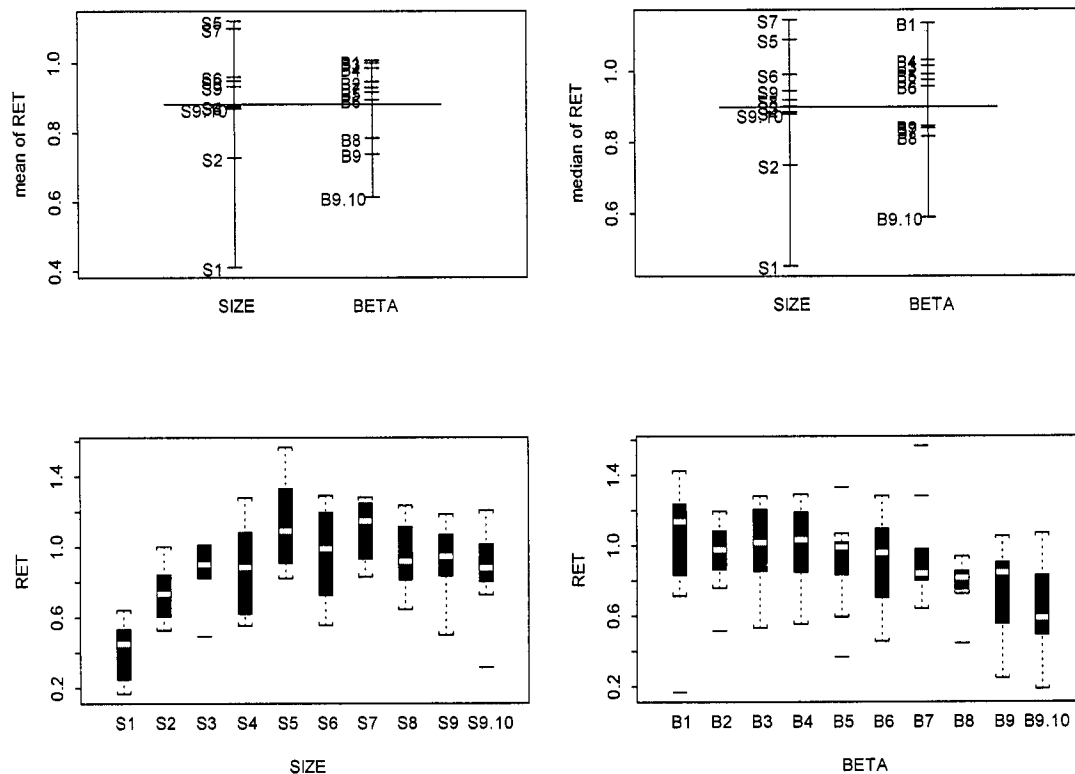
Figure 84. Two-Way Analysis: ROB Average Returns of *SIZE-BETA* Portfolios.
In comparison with Figure 82 *SIZE* still explains more variability then *BETA*. However, now, higher *RET* is associated with larger *SIZE*; and *RET* and *BETA* show a negative trend.

Compared to Figure 82, *SIZE* still explains more variability then *BETA*; but, in contrast, higher *RET* is associated with larger *SIZE* for the smallest five *SIZE* groups, while *RET* is relatively constant across the five largest *SIZE* groups. *RET* and *BETA* show a negative trend that is slight for the five smallest *BETA* and stronger for the five largest *BETA*. Again, both F-statistics are significant ($F_{SIZE} = 14.1$, $F_{BETA} = 5.8$) and negligible interaction is present (not shown). Additionally, RTT does not provide new insight (not shown).

The negative trend between *SIZE* and *BETA* in this informal analysis was already noted in FF, who recognize on p. 433 that: "average returns are flat, or show a slight tendency to decline." However, it is a new result, indicated by the ROB *SIZE-BETA* portfolios, that *RET* may be positively associated with *SIZE*.

4.5. Influential Months and the January Effect

Keim (1983) examined month-by-month, the empirical relation between abnormal returns and market value of NYSE and AMEX common stocks. He found abnormal returns and a negative SIZE effect in January, and that January explains more than 50 percent of the size effect for the period 1963 through 1979. Knez & Ready (1997) found that 20-30% of the months trimmed are in January, and also that *SIZE* is significant and negative in January alone and significant and positive for all other combined months. Chou, Chou, & Wang (2004) studied the January effect on *BETA, SIZE,* and *BEME* for various time periods between 1963 and 2001. They found, in all time periods, that in January, *BETA* is significant and positive, *SIZE* is significant and negative, and *BEME* has mixed signals.

This section extends previous work to more recent time periods, analyzes all individual months on 1971-1990, and 1971-2004, and compares on 5-year intervals and various other time intervals the effect of January when using CSR and RCSR. Keep in mind the preliminary indications of the raw cross-sectional regression data displayed for the year of 1999 in Figure 71 and Figure 72.

Figure 85 shows the monthly TT (1971-1990) of CSR and RCSR coefficients using model (4.2.2) in the upper panel, and the corresponding t-statistics in the lower panel.
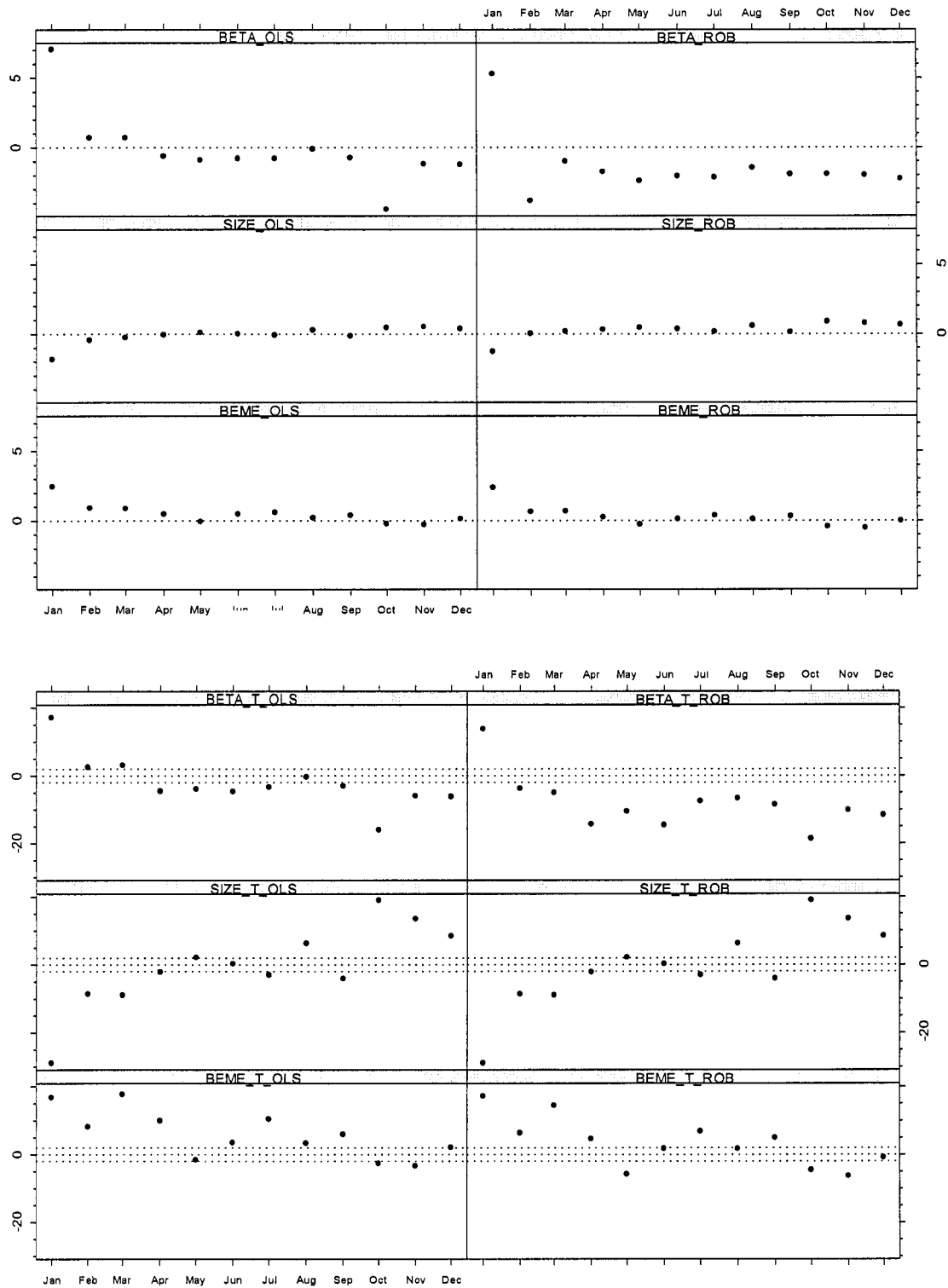
Figure 85. By Month: TT of CSR and RCSR Coefficients (1971-1990).
The January and other seasonable effects are visible in all models.

The January effect is visible for all three models. The effect is pronounced regardless of the estimator used in the cross-sectional regressions, which confirms findings of Knez & Ready (1997). Note that the January effect spills over into February, and even March.

OLS *BETA* is significant and positive in January through March. From April to December, it is significant and negative with the exception of August, where it is flat. The ROB *BETA* is always significant, however, positive only in January. OLS *SIZE* is negative and significant in January through March, then mixed through September, and thereafter positive and significant through December. The ROB *SIZE* is similar to OLS-*SIZE*, but slightly more positive. OLS *BEME* and the ROB *BEME* behave equally, positive from January through September with the exception of May, then negative from October through December. Additionally, *SIZE* and *BEME* show very distinct seasonal trends. *SIZE* is very negative and rises until the end of the summer. In September, it jumps to large positive values and decreases monotonically through the end of the year. *BEME* is positive in January and declines with up and downs through the end of the year, even to negative values. These up and down trends appear to be a quarterly trend, with peaks at the end of each fiscal quarter. This observation may not be know and needs further research.

The next four graphs compare classical averages of CSR and RCSR coefficients computed using (4.2.2) and averaged over 5-year periods from 1970 to 2004 (with only 4 years in the last period), and four other odd-size periods: 1970-1976, 1976-1998, 1998-2004, 1970-2004. The former three odd-size periods were chosen by looking at Figure 80, Figure 81, and Figure 64 and represent time periods of distinct volatilities.

Figure 86 shows the TT on CSR and RCSR using only January, and Figure 87 use all months except January.
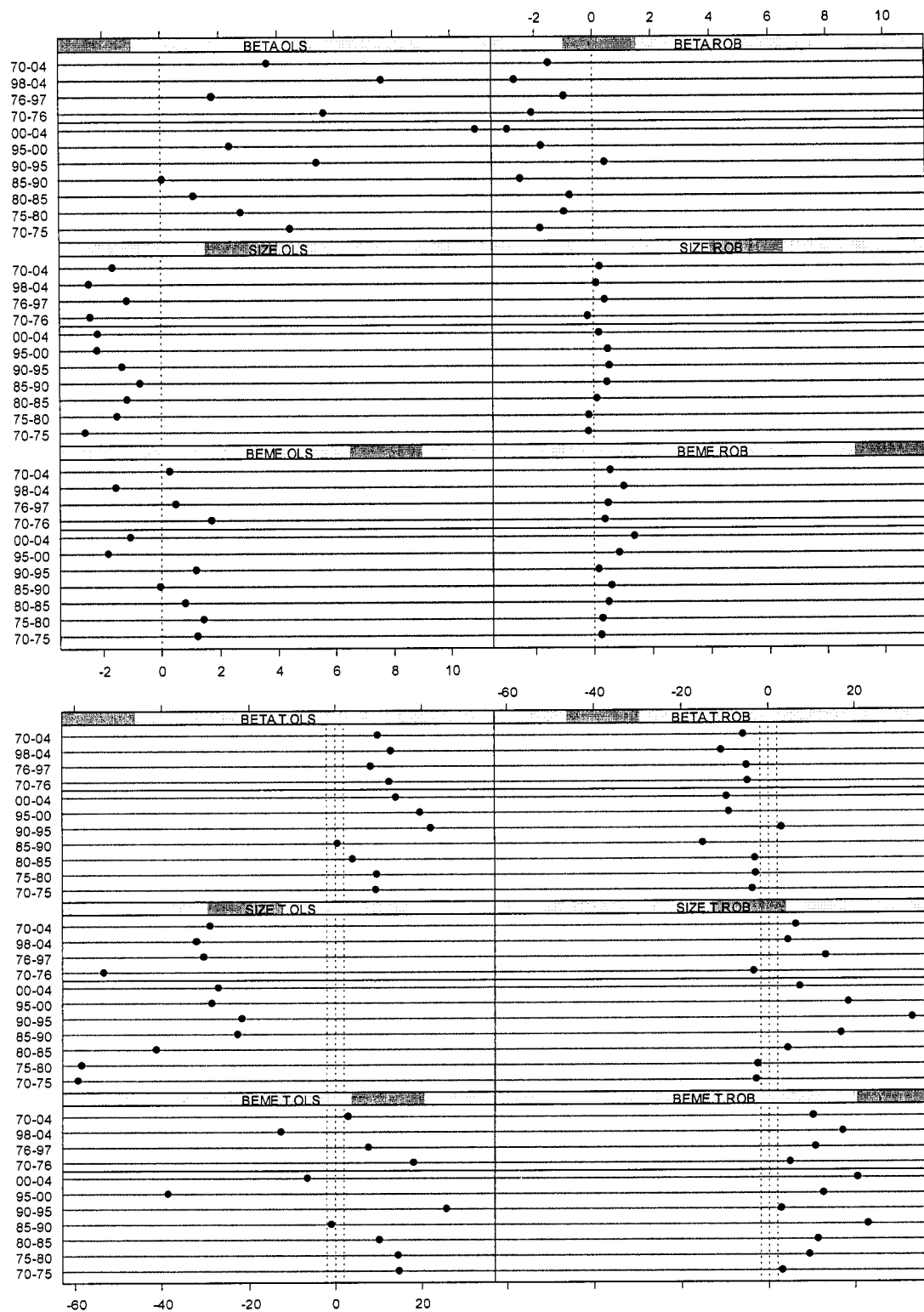
Figure 86. TT using only January.
Upper Panel: Averages, Lower Panel: T-Statistics.
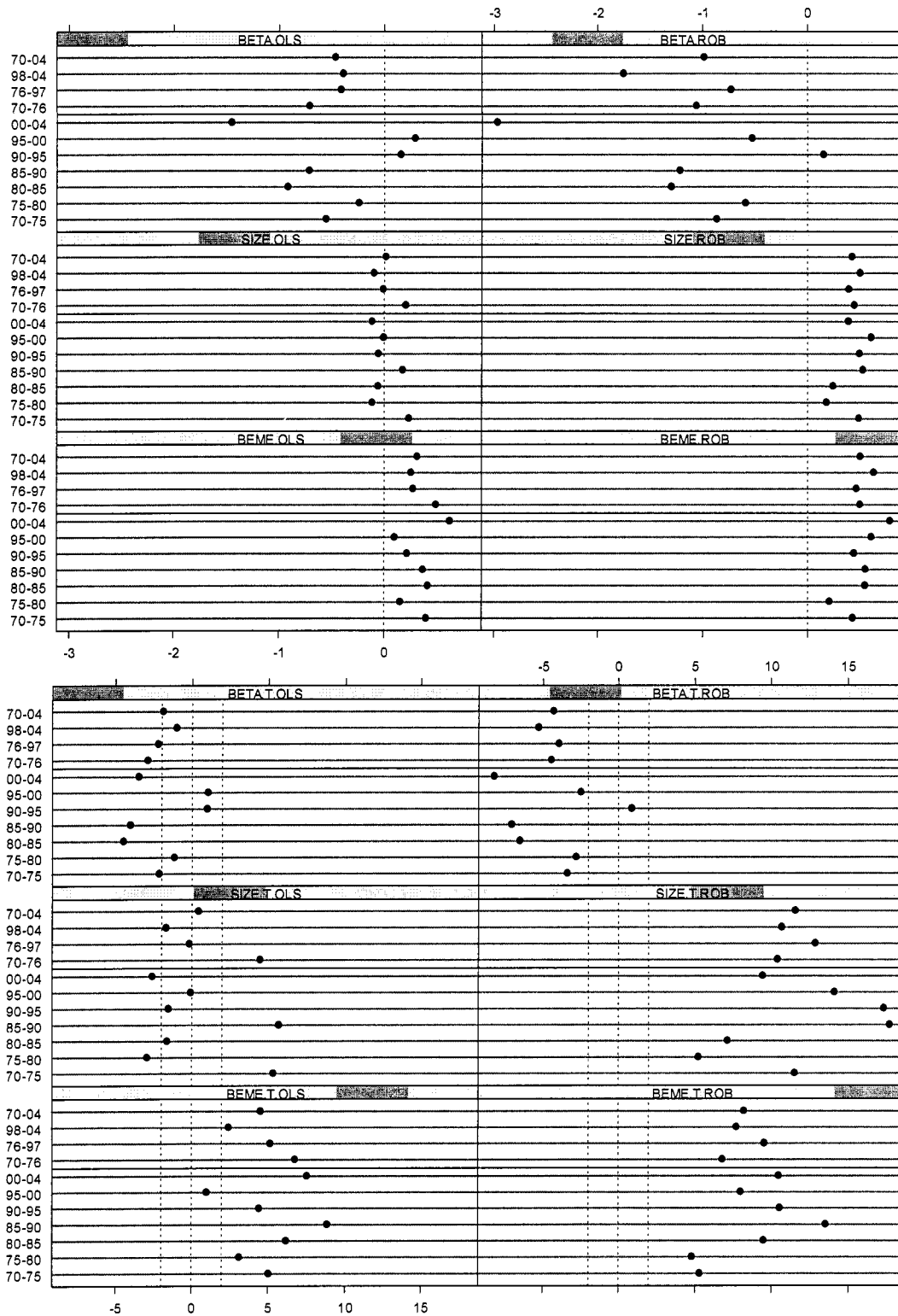
Figure 87. TT using All Months except January.
Upper Panel: Averages, Lower Panel: T-Statistics.

The analysis in Figure 86 of just the month of January shows that most risk premiums are significant. The left panel confirms the classical FF results for *BETA* and *SIZE*, however, with *BEME* negative on recent time periods. Note the extremely high positive *BETA* values, e.g., for the 2000-2004 period an average *BETA* larger then 10. For the RCSR on the right hand side, however, *BETA* is negative with the exception of the 1990-1995 period, *SIZE* is negative before 1980 and positive thereafter, and *BEME* is always positive. The odd-sized periods show similar findings.

Without the month of January, the left hand panel of Figure 87 clearly contradicts the classical FF results: *BETA*, with the exception of 1990-1995 and 1995-2000 period, is negative and *SIZE* is either close to zero and insignificant, or positive and significant. RCSR shows an even more negative and significant *BETA*, strongly positive *SIZE*, and *BEME* coefficients.

These results go beyond findings of Chou, Chou, & Wang (2004), and they confirm the general trends: when January is excluded, *BETA* get smaller, even negative; *SIZE* gets larger, mostly positive; and *BEME* is not much affected, but shows a tendency towards negative values in recent years. In general, RCSR coefficients are less affected by the influential month of January.

4.6. Influential Firms

Section 4.5 provided the insight that certain months, mainly January, are highly influential on CSR and to a much lesser degree on the RCSR coefficients. The focus of this section is to identify firms that highly influence the CSR and RCSR coefficients, and to analyze them separately from the bulk of firms. Figure 69 and Figure 70 indicated that firms with positively skewed returns tend to be small *SIZE* firms with *BETA* ranging from small to large.

The following graphs further split the rejected firms by the firm's *SIZE* and *BETA* characteristics, and the sign of its residual as measured in RCSR. Of the 18707 observations/firms rejected over 7/1970 through 6/2004, 15601 have positive residuals.

Figure 88 shows the distribution of rejected firms across *SIZE* break points as well as by positive and negative residuals. Figure 89 shows the distribution of rejected firms

across *BETA* break points as well as across positive and negative residuals.
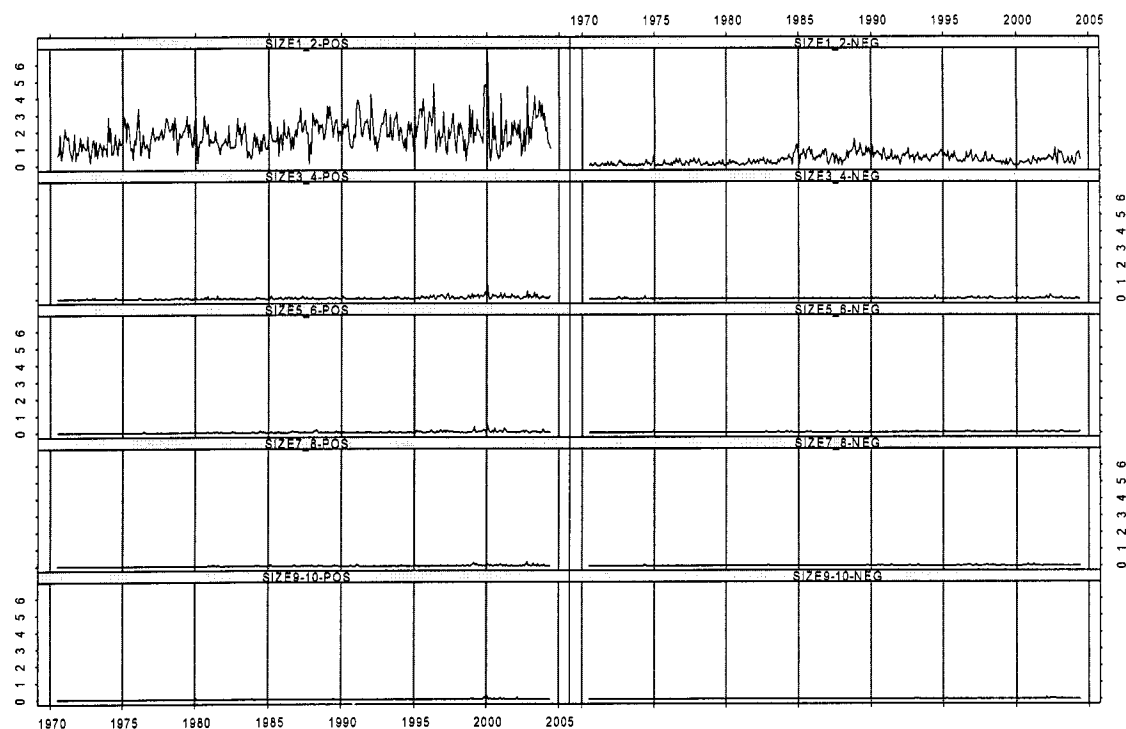
Figure 88. Rejected Firms by *SIZE* Break Point and by Residual (Percent of Total).
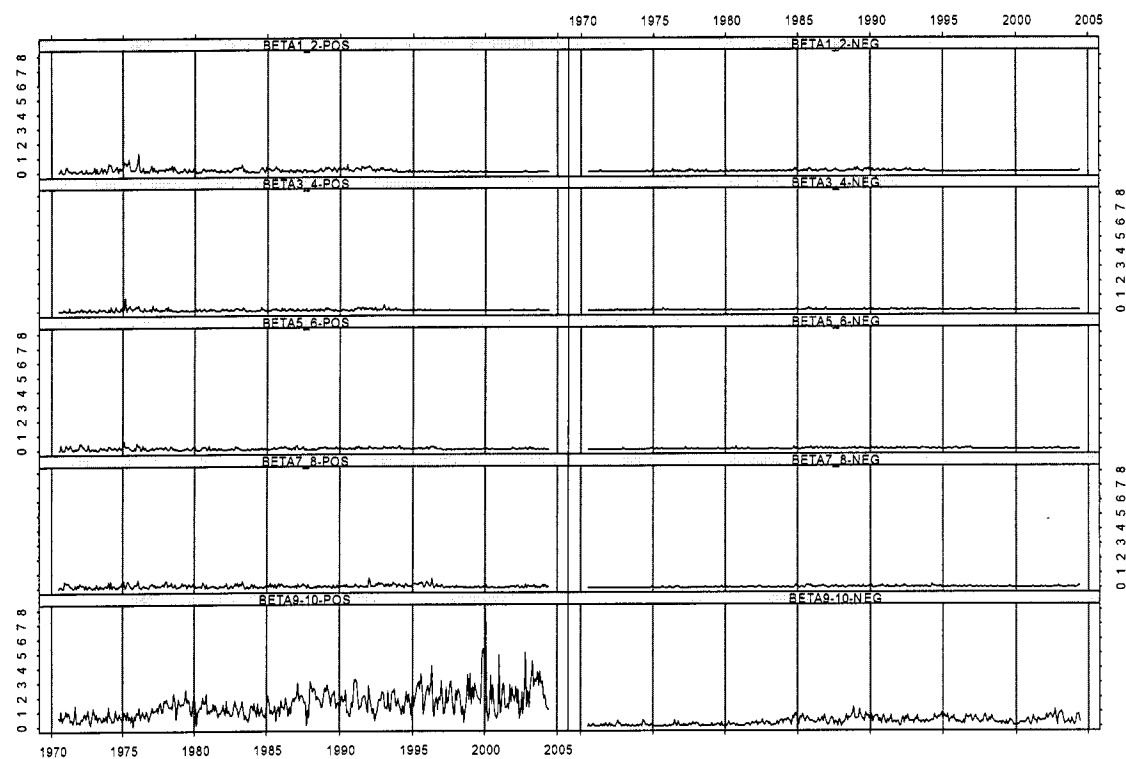
Figure 89. Rejected Firms by *BETA* Break Point and by Residual (Percent of Total).

Figure 88 shows that most rejected firms have positive residuals and are in the smallest *SIZE* bracket. Figure 89 shows that Most rejected firms have positive residuals, and belong to the largest *BETA* bracket, specifically in recent years. The CSR and RCSC performed on only rejected firms will separately be compared to CSR and RCSC on accepted firms in the next section.

4.7. Influential Time Periods

The focus of this section is the impact of chosen time periods on TT and RTT.

4.7.1. TT and RTT on various Time periods

The TT and RTT on different time periods are compared for: CSR; CSR without the months of January (no influential months); RCSR (no influential firms); and CSR on firms rejected (only influential firms) with negative and positive residuals.

Figure 90 compares boxplots of cross-sectional coefficients over the full time period 1970-2004. The coefficients were obtained from classical CSR (OLS), CSR without the months of January (EX.JAN), RCSR (ROB), and CSR on firms rejected with negative residuals (REJ.NEG) and firms rejected with positive (REJ.POS).
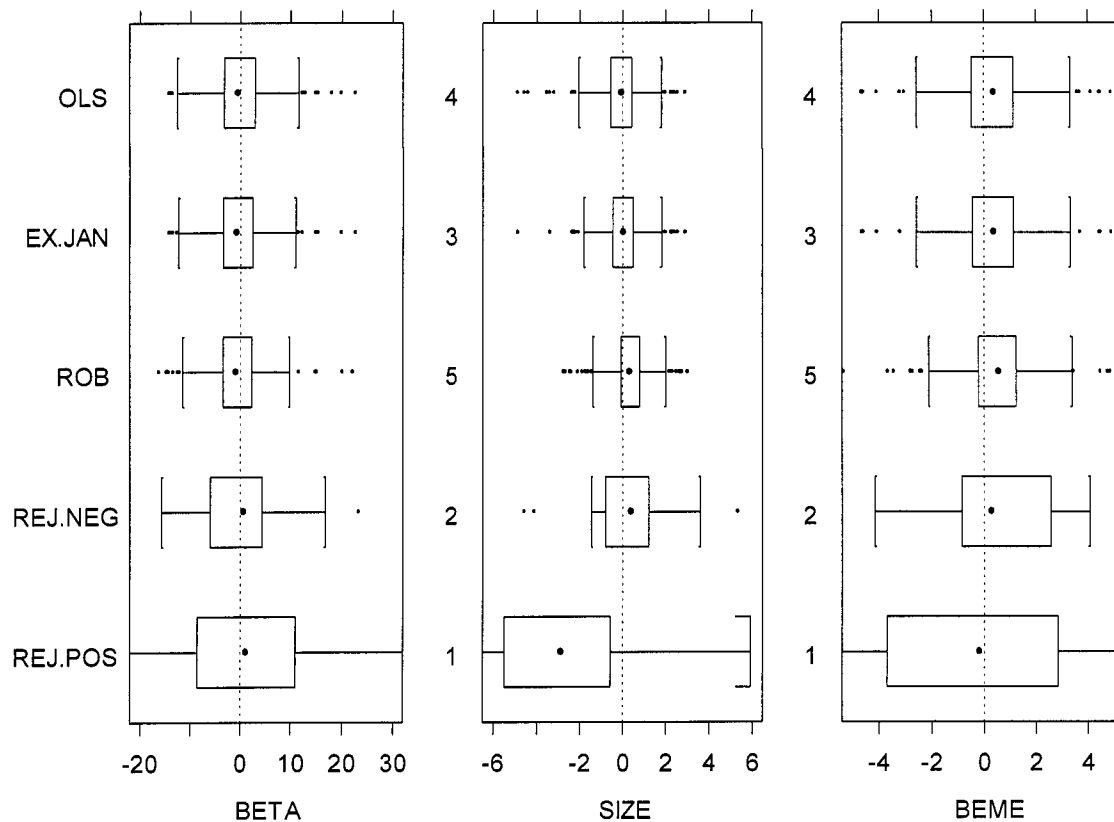
Figure 90. Cross-Sectional Regression Coefficients: 1970-2004.
Firms rejected with positive residuals are highly skewed, with *BETA* whiskers range from
-45 to 50, the *SIZE* whiskers from -16 to 6, the *BEME* whiskers from -16 to 11.
Specifically *SIZE* show negative outliers up to -40.

The median for *BETA* is similar and negative for the OLS, EX.JAN, and ROB, while the
mean for EX.JAN is closer to zero then for OLS and ROB. Rejected firms have a positive
BETA risk premium. The mean for *SIZE* is close to zero, for both OLS and EX.JAN, but
positive for ROB. While for *SIZE*, the rejected firms with negative residuals show a
positive median, the rejected firms with positive residuals (the majority of the rejected
firms) are for the most part negative. The median for *BEME* is always positive, except for
the rejected firms with positive residuals. The valuable point here is that, overall, the
influential month effect is small compared to the influential firm effect.

Regimes of different volatility (Figure 79, Figure 80, and Figure 81) suggest
analyzing TT and RTT on the following periods: five-year intervals, 7/1970-7/1976,
7/1976-12/1997, 1/1998-6/2004, and the overall period 7/1970-6/2004.

Figure 91 and Figure 92 compare TT and RTT, respectively for CSR, RCSR, and CSR for firms rejected on various periods.
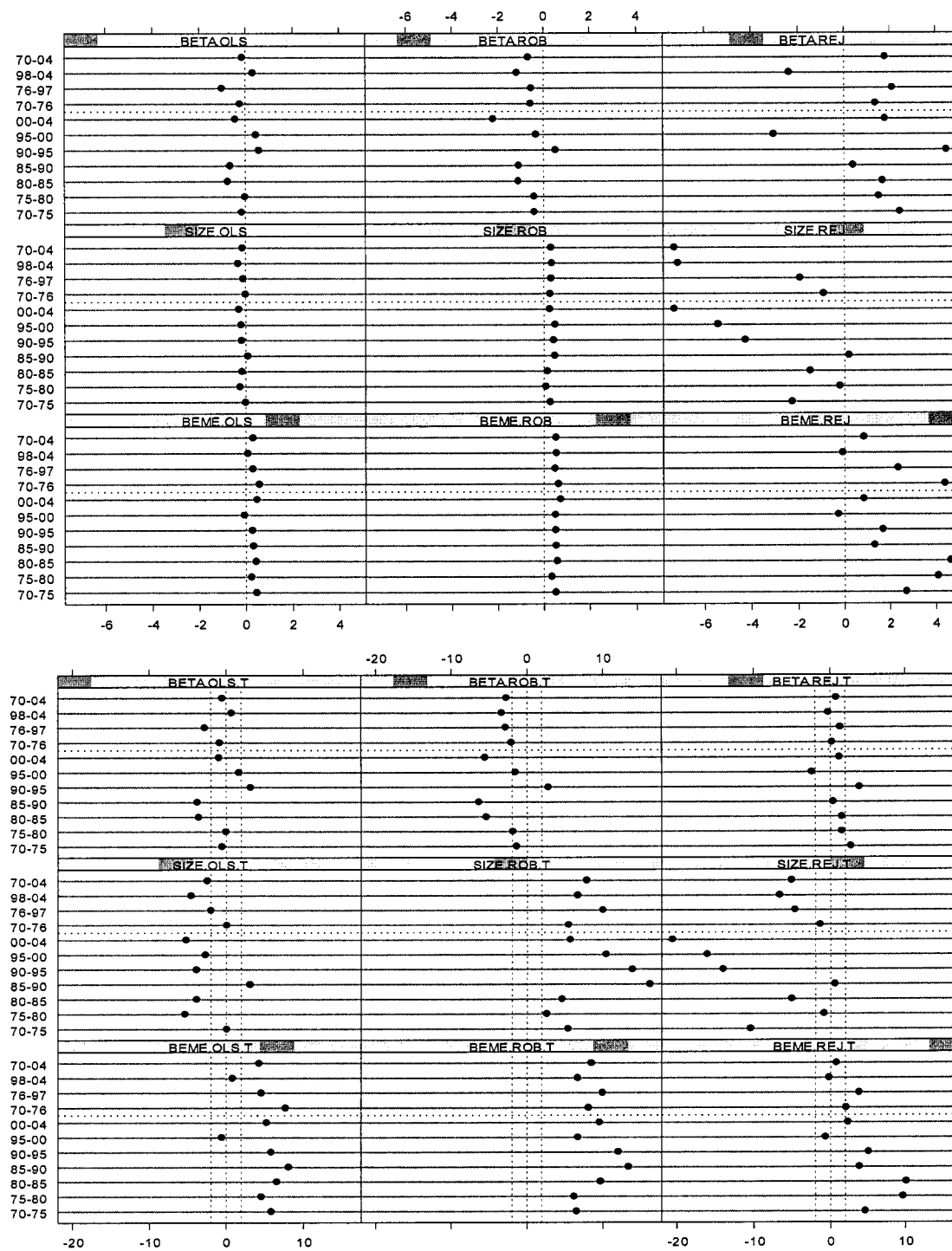
Figure 91. TT on CSR, RCSR, and CSR on Firms Rejected.
Upper Panel: Averages, Lower Panel: T-Statistics.
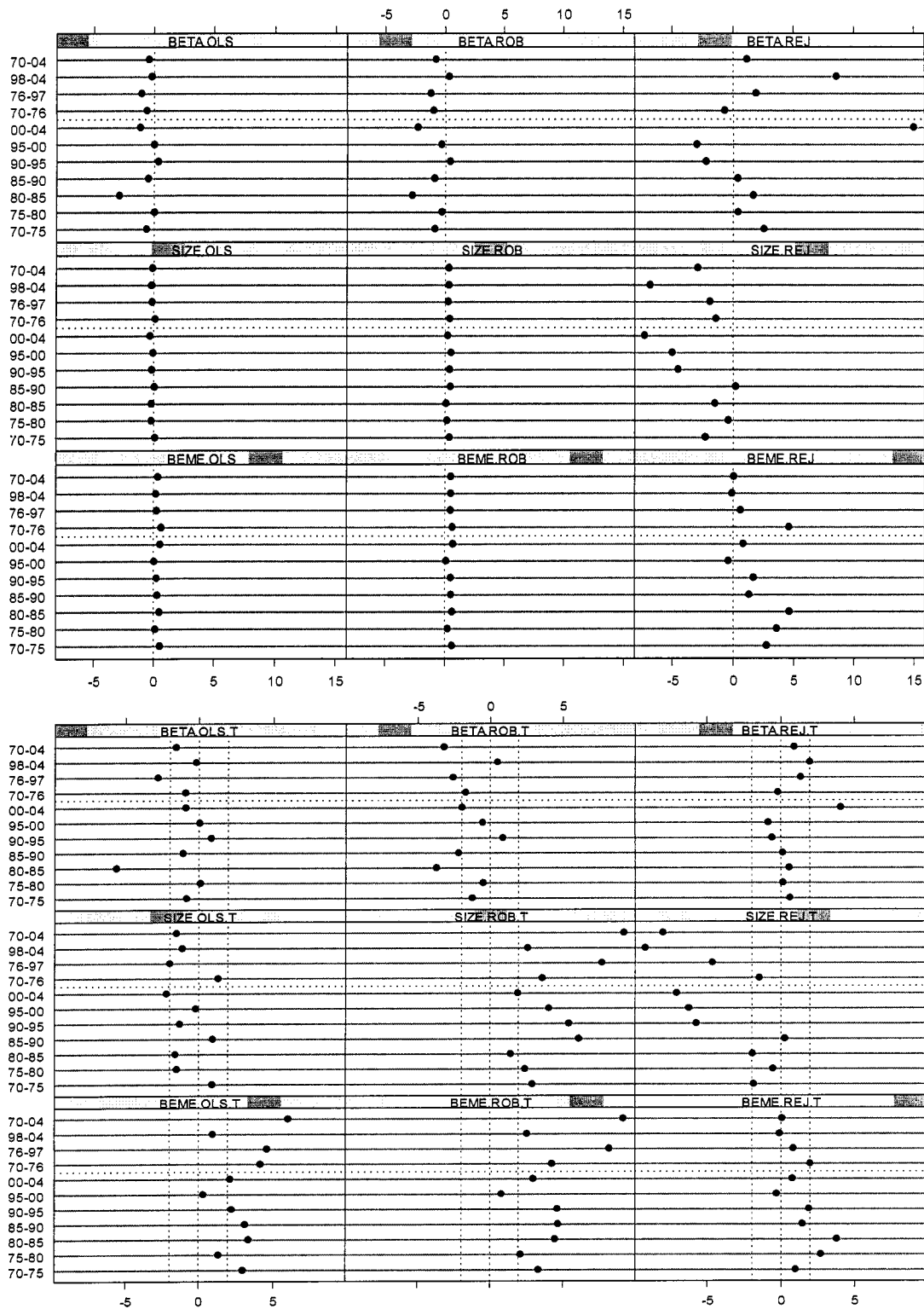
Figure 92. RTT on CSR, RCSR, and CSR on Firms Rejected.
Upper Panel: Averages, Lower Panel: T-Statistics

OLS *BETA* is either significant and negative or flat, with the exception in the 1990-1995 and 1995-2000 periods where it is significant and positive. The ROB *BETA* is only significant and positive in the 1990-1995 period, and significant and negative otherwise. The OLS *BETA* is insignificant on all other odd periods, while the ROB *BETA* is always significant and negative. Rejected firms are mostly positive in all five year periods, except the 1995-2000 period. Longer periods confirm the results on 5-year periods. RTT shows that OLS and ROB *BETA*, and *BETA* of firms rejected are mostly flat.

Most CSR show a significant and negative *SIZE* effect, except for the 1985-1990 period. RCSR is consistently significant and positive. Rejected firms however, show a significant and negative *SIZE* effect of several magnitudes larger than that of the CSR, except for the 1975-1980 and 1985-1990 period, where it is insignificant.

All methods agree more or less on a significant and positive *BEME*. The rejected firms tend to have insignificant *BEME* effects in most periods using RTT.

BETA and *SIZE* offer the conclusion that, once rejected firms are removed, the positive *BETA* effect and the negative *SIZE* effect vanish completely. Now, the *BETA* coefficient (the ROB *BETA*) is significant and negative, or insignificant, which is also partially confirmed by results in Chou et al. (2004), and the *SIZE* coefficient is significant and positive, partially confirmed by KR and Chou et al. (2004).

4.7.2. Significant Coefficients

The analysis in Section 4.7.1 considered all time series coefficients regardless of their significance. The cross-sectional regressions in this section are also based on (4.2.2), but the analysis is focused on significant coefficients only.

Table 19 shows the percentage of months in 7/1970-6/2004 for which the cross-sectional regression coefficients are significant. The five columns show CSR (OLS), CSR without the months of January (EX.JAN), RCSR (ROB), and CSR on firms rejected with negative residuals (REJ.NEG) and firms rejected with positive (REJ.POS).

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Table 19. Significant Months in 7/1970-6/2004.

%	OLS	EX.JAN	ROB	REJ.NEG	REJ.POS
BETA	62.25	56.86	70.83	0.98	5.15
SIZE	65.44	57.84	67.4	0.74	20.1
BEME	51.23	46.32	58.82	1.72	7.6

Total of 408 months.

Notably, ROB has the highest percentage of significant coefficients with 70.83%, and 20% of the rejected firms with positive residuals have a significant *SIZE* coefficient. ROB has 8%-10% higher significance than OLS. When January is removed the significance drops by 7%-8%.

Figure 93 repeats Figure 90 with only significant coefficients.
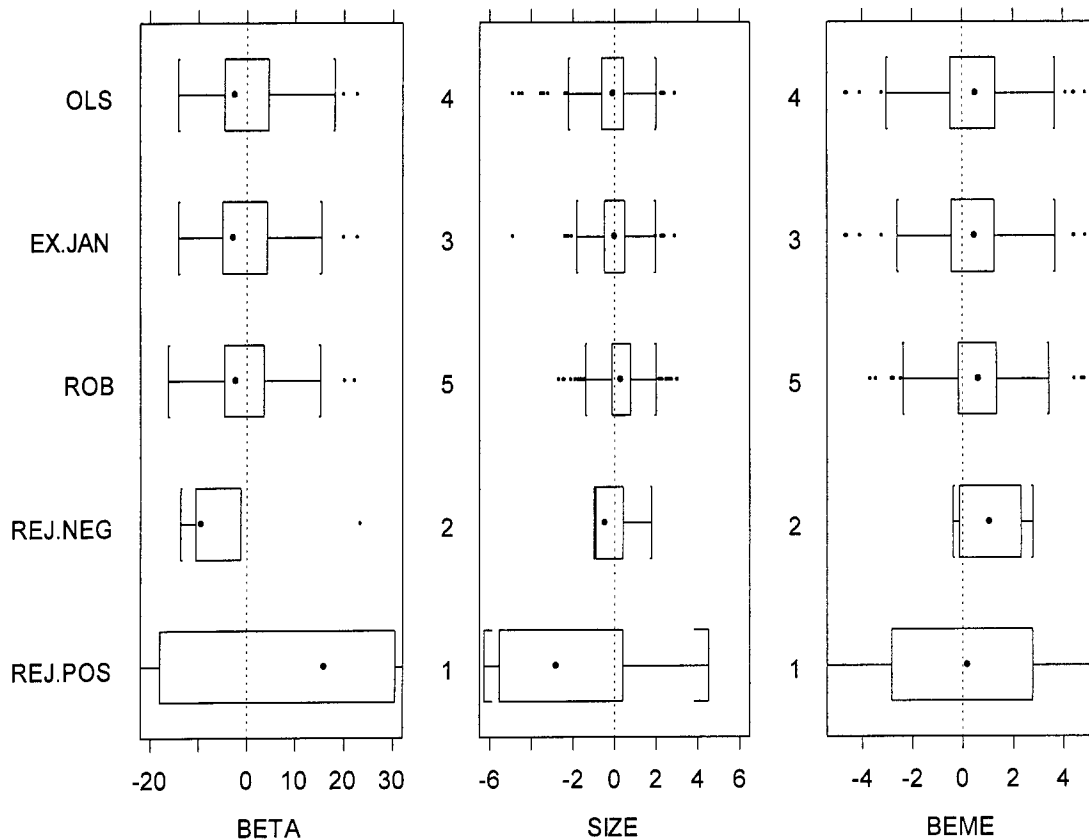
Figure 93. Significant Cross-Sectional Regression Coefficients: 1970-2004.
Firms rejected with positive residuals are highly skewed, with *BETA* whiskers range from -30 to 60, the *SIZE* whiskers from -8 to 5, the *BEME* whiskers from -9 to 9. Specifically *SIZE* shows negative outliers up to -40.

Most of the conclusions are similar to Figure 90, but the distributions have a wider inter-quartile range. Exceptions are the rejected firms: firms with negative residuals (representing only a small fraction of the observations) have now a negative *BETA* and *SIZE* premium, and *BEME* of firms with positive residuals is not positive anymore. Firms rejected with positive residuals are further highly skewed, with *BETA* whiskers ranging from -30 to 60, *SIZE* whiskers from -8 to 5, and *BEME* whiskers from -9 to 9. *SIZE* shows negative outliers up to -40. Thus, the majority of the rejected firms with significant risk premiums tend to have high positive *BETA*, large negative *SIZE*, and slightly positive *BEME*.

The two valuable points here are that the January effect is small compared with the influential firm effect (since effects average out over the year) and that only rejected firms confirm with the classical CSR results. Once the rejected firms are out of the analysis, i.e., in the RCSR, *BETA* has a negative and *SIZE* a positive risk premium.

4.7.3. Stepwise Model Selection

Table 19 shows that model (4.2.2) has a high percentage of months with insignificant coefficients. It makes sense to drop insignificant terms from the model, but only if the explanatory power can be retained. This can be achieved using model selection (Section 1.3.5).

Figure 94 shows the time series of cross-sectional regression coefficients from the CSR-AIC and RCSR-RAIC.
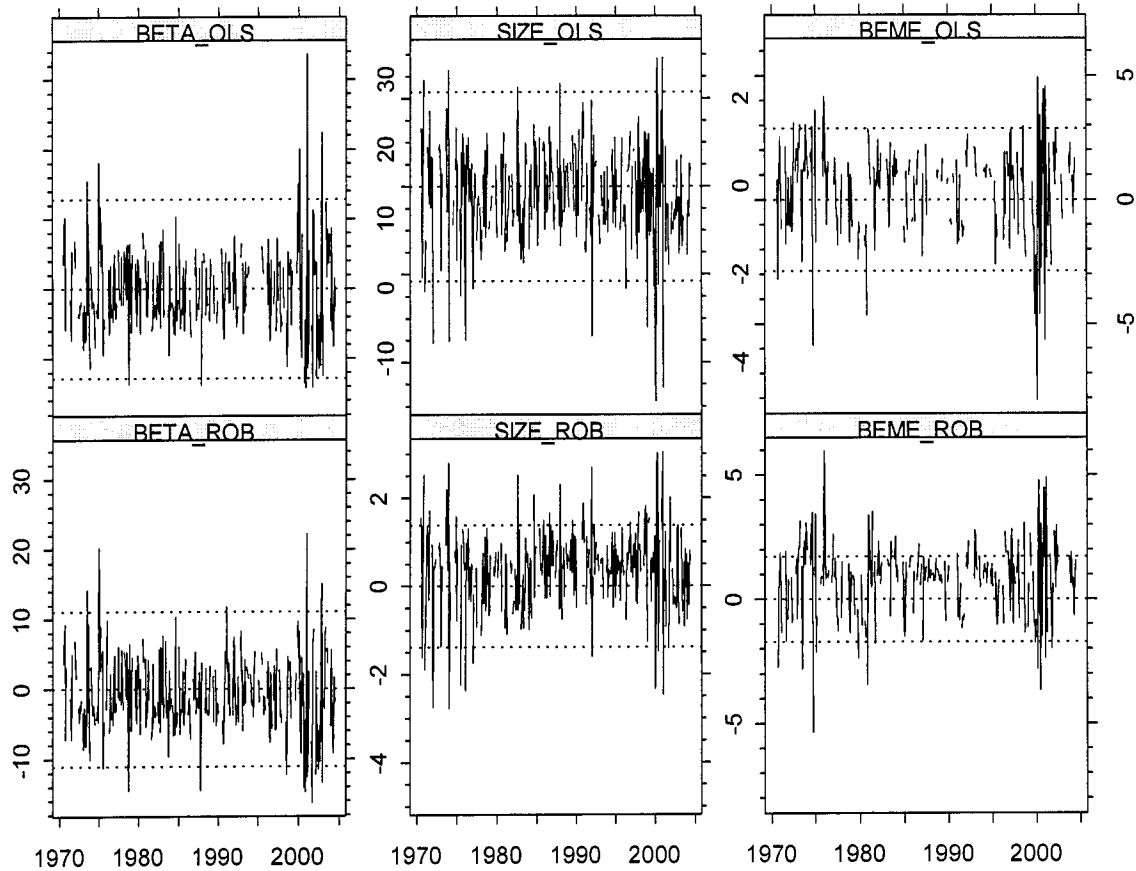
Figure 94. Time Series of CSR-AIC and RCSR-RAIC.

The time series show higher volatility in the first and last years. Averages taken over these time periods are highly sensitive to the time frame chosen. The RCSR-RAIC coefficient has less volatility than the CSR-AIC coefficients.

Figure 95 shows the pairwise differences of the CSR-AIC and RCSR-RAIC, corresponding to Figure 85.
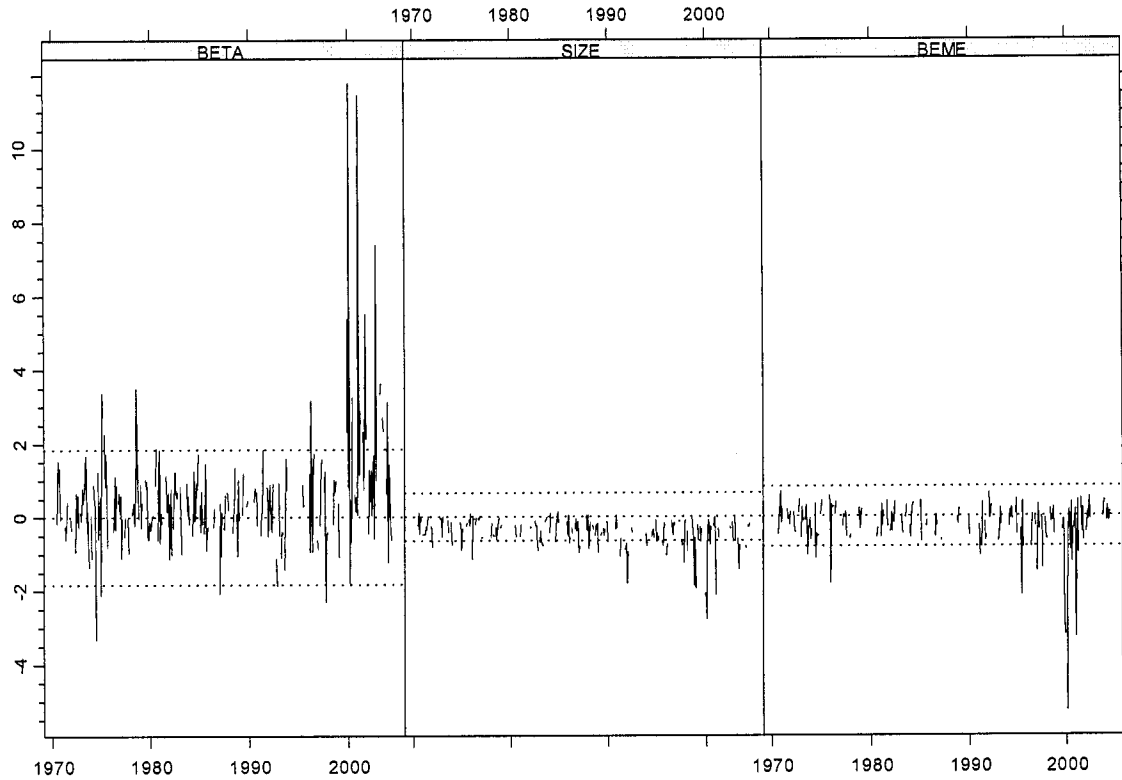
Figure 95. Pairwise Differences of CSR-AIC and RCSR-RAIC.

As expected from looking at Figure 85, the pairwise differences in Figure 95 are large, especially for *BETA* on recent time periods. Note that for *BETA,* the pairwise differences are mostly positive, while for *SIZE* and *BEME* the opposite is true. This compares with earlier findings of a more positive OLS *BETA* premium and a more negative OLS *SIZE* premium.

Table 20 gives the percentage of months in which the eight combinations of variables in model (4.2.2) are significant. The coefficients are ordered as *BETA, SIZE,* and *BEME,* and assigned the letter "F" if insignificant and "T" if significant.

Table 20. Percentage of Significant Months by Combinations of Models (4.2.2).

%	FFF	FFT	FTF	FTT	TFF	TFT	TTF	TTT
CSR	1.5	2.7	9.6	12.5	5.6	13.7	19.1	35.3
RCSR	1.2	3.7	5.6	10.3	4.9	14.7	19.1	40.4

Table 21 shows the percentage of times each variable combination was significant across the eight combinations shown in Table 20. The placeholder • stands for "F" or "T".

Table 21. Significance of Individual Combinations of Explanatory variables.

%	BETA,•,•	SIZE,•,•	BEME,•,•	BETA,SIZE,•	BETA,•,BEME	•,SIZE,BEME
CSR	73.7	76.5	64.4	54.4	49.0	47.8
RCSR	79.1	75.4	69.1	59.5	55.1	50.7

From the three explanatory variables, *BETA* is the most often included, both in CSR and RCSR. Further, it is noteworthy that RCSR-RAIC has mostly a higher percentage of significant explanatory variables than CSR-AIC. Comparing the significance of the coefficients to Table 19 shows that the stepwise model selection arrives up to 8%-10% of the times at a larger model (with significant individual explanatory variables).

4.7.4. Significant Coefficients and Stepwise Model Selection

Section 4.7.1 compared CSR, RCSR, and CSR on rejected firms using TT and RTT, regardless of the monthly coefficients' significance, and showed that a small percentage of small *SIZE* and high *BETA* firms are driving the results. Section 4.7.2 showed that a large portion of the coefficients used in TT and RTT were insignificant. Section 4.7.3 showed how to use OLS and ROB model selection to obtain CSR-AIC and RCSR-RAIC and to improve the model fit.

This section compares CSR-AIC and RCSR-RAIC evaluated using TT and RTT to Section 4.7.2. Figure 96 shows the results.
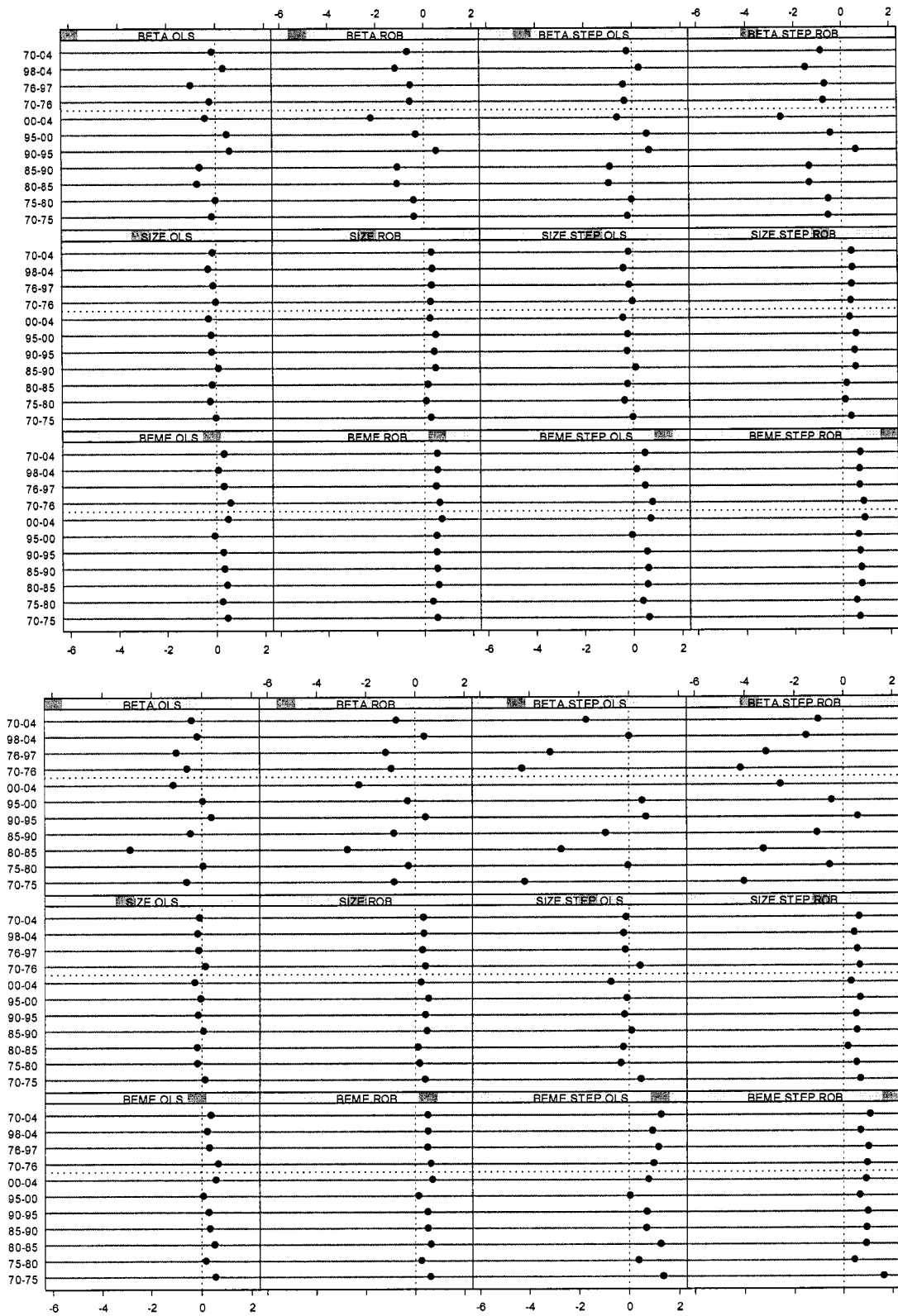
Figure 96. CSR and RSCR Coefficients: Upper Panel TT, Lower Panel RTT.

With the exception of RTT *BETA,* the CSR and CSR-AIC *BETA,* as well as RCSR and RCSR-RAIC, are very similar. OLS *BETA* is positive in 90-00, while ROB *BETA* is positive only in 90-95. OLS *SIZE* is negative, except on 90-95, where it is close to zero, while ROB *SIZE* is always positive. All *BEME* are positive or flat.

4.8. Conclusion

This chapter began by calibrating data and methods to the main results of Fama & French (1992) (FF) and the robust FF replication of Knez & Ready (1997) (KR). Results were shown for the most important risk factors: *BETA, SIZE,* and *BEME.* The chapter then extended the results to more recent time periods and applied a modern robust regression method (the robust MM-estimate, ROB) to the FF method. The efficiency of the MM-estimator was set to 99%, which corresponds roughly to a trimming fraction of the LTS-estimator of only 2%-3%. Beyond replication and extension, the chapter also added improved data analysis relative to FF, e.g., to robustly compute *SIZE-BETA* portfolios and robust time series averages, and also robust t-statistics of the cross-sectional regression coefficients. The non-parametric Wilcox tests, used as a check, confirmed the results of the robust t-test. The chapter further drew on classical AIC and robust AIC stepwise model selection to arrive at cross-sectional regression coefficients with a good model fit.

The calibration to FF and KR was done on data from 7/1970 through 12/1990 only, truncating the data periods analyzed in FF and KR (7/1963 to 7/1970). However, as it turned out, the truncation can be perfectly justified. Without the earlier time periods, there was no significant difference in the OLS beta and the OLS size premium had already vanished completely. The robust results are virtually not affected—the robust LTS replicates and KR results match almost perfectly.

Leverage points (outliers in the explanatory variable or risk factor direction) were not of concern and not a focus of the analysis. Classical and robust cross-sectional regression results were then obtained for 7/1970 through 6/2004 and analyzed and compared for influential months, firms, and time periods.

In the analysis of influential months, the January effect (Keim, 1983) was confirmed

for *BETA*, *SIZE*, and *BEME*, but interestingly, the January effect was not the only effect. *BETA* spilled into February and March then became negative through the rest of the year, with an extreme low in October. *SIZE* showed an annual trend from negative to positive and *BEME* an annual trend from positive to negative values with peaks at the end of each fiscal quarter. This asks for further research and may provide opportunities for short-term trading strategies. With respect to the influence of the month's effects on the risk premium, the analysis showed that the effects are small compared to the effects of influential firms, which is likely caused by the fact that the seasonal effects cancel out when averaged over the year.

The analysis of influential firms focused on equally treating firms rejected and accepted under the robust regression estimator. Rejected firms were further categorized by *SIZE*, *BETA*, and sign of residual return. Most rejected firms were small *SIZE* and high *BETA* firms with positive residual returns (positive outliers). The fact that mostly positive outliers were rejected excludes a potential survivorship bias. The characteristics of rejected firms did not come as a surprise and confirmed results of KR. It was also shown that those firms were mostly high *BETA* firms. This seemed to be specifically true for recent time periods.

A crucial part is the analysis of the time series averages of the cross-sectional regression coefficients, and thus, the analysis of influential time periods. Time series plots of the cross-sectional regression coefficients clearly showed periods of varying volatility and frequent outliers. This initiated the two main critics of the evaluation of time series coefficients used by FF and KR: results depend on chosen time periods, are biased by outliers, and leave t-tests with little statistical power.

For *BETA*, the classical time series analysis showed a negative (only partially significant) relationship to expected returns on most, except for the 1990-1995 period, where OLS *BETA* is significant and positive. On the other hand, ROB *BETA* is mostly negative and significant, while the rejected firms have positive *BETA* (only partially significant). Thus, once the rejected firms were separated from the bulk of the data, the remaining firms pointed towards a flat or negative *BETA*-return relationship.

For *SIZE*, the classical time series analysis showed a negative and mostly significant relationship on all sub-periods, while ROB *SIZE* was positive and significant, as expected. The rejected firms showed a large negative and significant *SIZE* relationship to expected returns.

For *BEME*, the relationship was positive and significant for OLS,ROB *BEME*, as well as for rejected firms. Using robust time series analysis, these effects were even more distinct. All results also confirmed findings by KR and Chou, Chou, & Wang (Chou, Chou, & Wang, 2004).

Overlooked, or at the minimum neglected, is the fact that a large percentage of the cross-sectional regression coefficients are insignificant in their monthly cross-sectional regressions. It was interesting to find out that RCSR produced a 2%-8% higher number of significant coefficients than CSR, with 70.1% for *BETA*, 67.4% for *SIZE*, and 58.8% for *BEME*. The analysis of influential firms repeated on only significant coefficients yielded a similar but more distinct conclusion. When classical and robust model selection was used to compute CSR-AIC and RCSR-RAIC it chose a larger model in 8%-10% of the times, compared only to CSR and RSCR with significant coefficients. The volatility CSR-AIC is higher than that of RCSR-RAIC, and the pairwise differences confirm previous results, in that OLS has a larger risk premium for BETA and a lower risk premium for SIZE compared to ROB.

The valuable points from this chapter are that tiny fractions (1%-3%) of small *SIZE* and high *BETA* firms with extraordinary (positive) period returns, as well as the choice of test period, drive the positive *BETA* and negative *SIZE* risk premium, as reported by FF. Furthermore, seasonal effects are very influential, but cancel out when averaged across the year. Once the small fraction of influential firms is rejected, the risk premium for *BETA* factor becomes negative or flat and for the *SIZE* factor positive.

5. Conclusion

The goal of this dissertation was to study key empirical financial pricing models with proper exploratory data analysis and to utilize robust statistical techniques that are appropriate for the characteristics of financial data. Within a financial data framework, the dissertation introduced the notion of robustness and summarized the properties of the robust MM-estimator. The robust MM-estimator was then used to estimate factor models and to compare the robust with the classical results. Furthermore, the robust MM-estimator was used to identify and analyze influential observations (factors, firms, months, and time periods). The estimation focused on stock alphas and betas from the single-factor market model and on risk premiums for stock beta, firm size, and book-to-market equity from a multi-factor model. For the single-factor model, the results of the robust estimator were directly compared with the results of the OLS estimator. For the multi-factor model, the OLS results were first calibrated on key papers (Fama & French, 1992; Fama & MacBeth, 1973; Knez & Ready, 1997) and then extended to recent time periods.

The surprising and consistent message with all three large data applications was that the rejection on average of 1%-3% of the most influential observations led to important differences to the classical results: classical alphas tend to be over-biased and classical betas turned out to be highly sensitive, frequently even changing signs, both robust alphas and betas are superior predictors or return and risk, respectively; the positive risk-return relationship, as found significant in Fama & MacBeth (1973), could not be confirmed. The risk premiums, as found in Fama & French (1992) to be flat for beta and negative for size, once adjusted for influential firm and seasonal effects, turned out to be negative or flat for beta and distinctly positive for size. This confirms partial results of Knez & Ready (1997) and Chou, Chou, & Wang, 2004. For a more detailed summary, the reader is referred to Sections 2.6.6, 2.7.7, 3.6, and 4.8.

It was shown that the robust estimator has the property to retain an efficiency of 99% at the Gaussian model, while protecting against bias caused not only by a small fraction

of influential returns, but also by asymmetrically distributed returns. This suggests its routine use along, or even as a replacement, of the OLS estimator.

Beyond being merely a bias protection tool that fits to the bulk of the data, the robust estimator was also shown to be helpful in identifying influential returns. Once influential returns are identified, they can be analyzed and either discarded or used to exploit market inefficiencies, i.e., to construct trading strategies. The decision to keep an influential return with the bulk of the data may be depend on the type of application, e.g., an analyst who wants to build an alpha engine may well decide not to use positively biased alphas unless she is confident that they will reoccur, while a conservative risk manager may include positively biased beta values to make sure she is covered in a worst case scenario.

Natural extension of the research in this dissertation is to analyze other factors used by practitioners, such as the five common risk factors in the returns on stocks and bonds Fama & French (1993), the three-factor model of Fama & French (1996), or to revaluate Barra-type factor models. Risk factors in up and down markets have also been treated (Davis & Desai, 1998; Grundy K. & Malkiel, 1996; Woodard & Anderson, 2003) and robust methods should be applied in this situation as well. An important application will also be in conditional asset pricing models that allow tracing time-varying expected returns and risk-factor relationships (Ferson, 2003).

The author also has successfully used robust techniques in various applications in the fund of fund / plan sponsor environment: in a portable alpha strategy using robust regression to compute the important beta exposure of a manager to the stock and bond market; in asset allocation using a robust version of Stambaugh (1997) to compute a robust covariance matrix when returns histories of the asset classes have unequal histories (Reistad & Bailer, 2005); and in risk management to compute robust conditional value-at-risk as well as spending and impairment risk.

The essence of the various applications of the MM-estimator is that it can be used to automatically protect against a fixed level of bias when influential returns are present, while paying only a small insurance premium in terms of efficiency loss (the MM-estimator yields almost the same results as the OLS estimator when no outliers are present), or it can be used to reliably recognize influential returns that can then be

exploited for trading strategies, to be included with the bulk of the data, or simply be rejected as gross data errors. Either way, the MM-estimator is an easy-to-use alternative and that provides additional opportunities over simply disregarding the presence of small fractions of unusual returns (possibly just one single return) that can distort the results.

Glossary

AIC	Akaike Information Criterion
CSR	Cross-Sectional Regressions
CSR-AIC	Cross-Sectional Model Selection using AIC
CRSP	Center for Research in Security Prices at the University of Chicago
FF	Fama & French (1992)
FM	Fama-MacBeth Procedure, Fama & MacBeth (1973)
LAD	Least Absolute Deviations
LMS	Least Median Squares
MAD	Median Absolute Deviation about the Median
KR	Knez & Ready (1997)
Replicates	Results on Data and Methods of this Dissertation
RAIC	Robust AIC
RCSR	Robust Cross-Sectional Regressions
RCSR-RAIC	Robust Cross-Sectional Model Selection using RAIC
ROB	Methods using the MM-Estimator
RFM	Robust Fama-MacBeth Procedure
RTT	Robust Time Series Averages and T-Statistics using the MM-Estimator
TT	Classical Time Series Averages and T-Statistics

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- Introduced leading-edge strategic asset allocation process with bias protection against outliers and replaced mean-variance (Markovitz) with mean-conditional-Value-at-Risk optimization, allowing a more aggressive allocation while dramatically improving the risk management process

- Put into practice an algorithm to select candidates from manager pool with unequal return histories and to simulate and rank their performance within the existing portfolio

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