

The Importance of Coding Time in Growth Models:
Analyzing Graduation Rates in Title I Washington State High Schools 2019-2025

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Abstract

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Four-year graduation rates are a central accountability metric under the U.S. *Every Student Succeeds Act* (ESSA), yet limited research has examined how rates can evolve longitudinally across under-resourced schools, or how school-level characteristics relate to change over time. This study used multilevel growth modeling to predict graduation trajectories among $N = 93$ Title I high schools in Washington State across six academic years (2018–2019 through 2024–2025). In addition to investigating mean change over time, between-school heterogeneity, and associations with school enrollment, ESSA Section 1003 improvement funding, and Education Service District (ESD) membership. Data were drawn from publicly available administrative records from the Washington Office of Superintendent of Public Instruction. Results indicated that graduation rates increased slightly but significantly over time.

The intraclass correlation coefficient ($ICC \approx .80$) revealed that most variance in graduation outcomes was attributable to differences between schools rather than year-to-year fluctuations within schools. Enrollment size was a small but significant predictor of baseline graduation rates, while Section 1003 improvement funding was significantly associated with differences in growth trajectories—though not with baseline levels—showing that improvement funds relate more to longitudinal change than to initial standing. As well, how time points are coded can either smooth over trajectories or spotlight year-to-year differences. Taken together, findings have implications for how funding accountability systems can be designed, analyzed, and interpreted: single-time-point or smoothed-over comparisons alone risk obscuring meaningful heterogeneity whereas nuanced growth-oriented frameworks can better reflect the effects of targeted supports.

Keywords: multilevel growth modeling, Title I, graduation rates, ESSA accountability, Section 1003 funding, longitudinal analysis, time coding

The Importance of Coding Time in Growth Models:

Analyzing Graduation Rates in Title I Washington State High Schools 2019-2025

For six decades, Title I has been the U.S. government’s primary tool for addressing educational inequality associated with poverty, yet its theory of action and measure of accountability have shifted repeatedly—from remediating presumed “cultural deprivation,” to targeting low academic performance, to today’s standards-based accountability in which schools are held to measurable results. These shifts have produced durable ambiguity about what problem Title I is solving (poverty vs. performance) and who should be responsible for improvement (children, schools, or systems), making careful, historically informed interpretation of outcomes essential.

Under the U.S. *Every Student Succeeds Act* (ESSA), this ambiguity meets a results-oriented paradigm: states set standards, monitor progress, and intervene when outcomes lag; graduation rates, alongside assessment results, function as central indicators of school performance. This framework reflects a broader redefinition of the educational problem that was held after the presidential commission of the publication of *A Nation at Risk* (National Commission on Excellence in Education, 1983). Educational success is tied to economic competitiveness, schools are the locus of responsibility (rather than social policy), and success must be verified with externally validated outcomes.

A school is defined as Title I if 40 percent or more of the student population reports living in a low-income household. The Washington state educational system utilizes graduation pathways to measure accountability in Title I high schools. Thus, understanding how graduation changes over time within and across high-poverty high schools is relevant to state and local lawmakers who allocate financial support and assess progress.

Within ESSA, Title I school improvement funds (Section 1003) operate as a policy lever that varies across schools and over time as poverty concentrations shift. The policy question is not simply whether Title I high schools differ in graduation outcomes, but whether graduation trajectories—their rates of change, and the timing of meaningful shifts—systematically relate to school characteristics (Education Service District membership) and to exposure to differing amounts of Section 1003 financial supports. Answering that question requires an approach that recognizes 1) the historical evolution of Title I’s aims, 2) ESSA’s outcome-driven accountability paradigm, and 3) the cumulative, phase-based nature of four-year high school graduation.

The current study is motivated by a policy need for interpretable, decision-relevant evidence: evidence that highlights when change happens and under what conditions, not merely whether mean differences exist. By modeling graduation longitudinally and acknowledging schools’ heterogeneous contexts, the analysis speaks to stakeholders who must time and target supports, to state agencies charged with monitoring improvement, and to scholars seeking to reconcile Title I’s poverty-targeting lineage with ESSA’s outcomes emphasis.

Historical and Conceptual Background of Title I and ESSA

Title I funding began with the *Elementary and Secondary Education Act* (ESEA) and emerged from U.S. President Lyndon B. Johnson’s “War on Poverty” policies, carrying with it conceptual assumptions that have shaped and constrained federal school improvement since then (Sundquist, 1968). Through Title I, the federal government attempted to remediate the disadvantages of children living in poverty; yet the kind of disadvantage to be remediated, and who was to be held responsible, has shifted over time—from “cultural deprivation,” to low academic performance, to standards-based accountability for results (Stein, 2004). The changes

to Title I over the decades clarify why the interpretation of ESSA accountability outcomes requires caution. Over the decades, Title I has been linked to the following federal policies:

- 1) *Elementary and Secondary Education Act* (ESEA) spanning 1965 – 1981;
- 2) *Education Consolidation and Improvement Act* (ECIA) spanning 1981 – 1988;
- 3) *Hawkins Stafford ESEA Amendments* spanning 1988 – 1994;
- 4) *Improving America's Schools Act* (IASA) spanning 1994 – 2001;
- 5) *No Child Left Behind Act* (NCLB) spanning 2001 – 2015; and
- 6) *Every Student Succeeds Act* (ESSA) spanning 2015 – present.

Origins: Compensatory Aid and the “Culture of Poverty”

In the mid-1960s, legislators married the moral urgency of civil rights with the political appeal of anti-poverty action. Drawing on the then-influential “culture of poverty” scholarship (e.g., Lewis, Harrington) and the Moynihan Report, congressional debates casted Title I’s intended beneficiaries as “educationally deprived” children whose home cultures purportedly impeded school success. The legislative discourse emphasized compensating for the presumed cultural deficits of these children—often explicitly the mothers’ “cultural skills”—rather than reforming inequitable school finance or broader social structures. Schools were asked to supply the “middle-class culture” that families supposedly could not provide.

Early Ambiguity about What Counts as Need—and Who Counts as Needy

Title I intertwined poverty and performance without defining either of them precisely. Eligibility formulas rested on income thresholds; however, poverty and “educational deprivation” were often treated as the problem. This set a precedent of ambiguity: was Title I a poverty program or a low-achievement program? The answer oscillated with each

reauthorization, even as Congress steadily increased federal oversight and reporting requirements.

Reauthorizations and Rhetorical Evolution (1965–1994)

Education policy scholar Sandra J. Stein examined Title I through close analysis of congressional discourse, revealing how lawmakers' language has shaped enduring understandings of poverty, schooling, and student need (Stein, 2004). Stein's analysis of congressional floor debates showed that even though labels had shifted, deficit framings had endured. In the late 1960s, Democrats defended categorical aid for the "culturally deprived," deploying vivid images of urban "slums" and the "vicious cycle of poverty." By 1970, new categories like "predelinquent" youth appeared, linking poverty with danger and disorder; the teacher workforce in high-poverty schools was described as needing "combat pay." Through the 1970s, Congress wrestled with whether to target poverty (adopting the Orshansky index in 1974) or academic need (proposals to use criterion-referenced tests), ultimately preserving a poverty focus while mandating more evaluation and compliance.

By the 1980s, the *Education Consolidation and Improvement Act* (ECIA, 1981) pivoted from labeling these students as "educationally deprived" to "children from low-income families", signaling that administrative streamlining of Title I funding was to address disparity from high poverty. The 1988 Hawkins-Stafford amendments introduced schoolwide programs (compared to the former Title I funding being directed at individual students within schools instead of entire schools), popularizing the more elastic label "at risk," which replaced overt cultural deficit talk without resolving the poverty-versus-performance tension.

Paradigm Shift: From Compensatory Inputs to Accountability

Policy historian Jal Mehta situated Title I's later trajectory within a broader shift in education reform sparked by *A Nation at Risk* (1983), which recast schooling as a vessel of economic competitiveness (Mehta, 2013). Under this paradigm, weak academic performance was framed as a threat to national prosperity. Responsibility for improvement was assigned primarily to schools rather than to antipoverty policy, and thus, policy success was to be demonstrated through externally validated assessments. This reframing drew new political actors, particularly governors and business coalitions, into education debates, elevated education's standing on the policy agenda, and legitimized an expanded state and federal role in setting standards and measuring outcomes. Over time, these dynamics culminated in the institutionalization of standards-based reform, first at the state level and later federally through Goals 2000, the 1994 reauthorization of ESEA, and finally, the *No Child Left Behind Act* (2001).

Two features of this paradigm shift are relevant for the present study. First, it normalized outcomes (test performance, graduation) as a currency of accountability. Second, it recentered responsibility on schools and districts. These dynamics created political demand for visible gains over time and a methodological demand for models that can detect and explain growth amid nested systems.

From NCLB to ESSA: Continuity with Adjustment

The *Every Student Succeeds Act* (ESSA) of 2015 loosened some of NCLB's rigidities but retained the core paradigm Mehta describes: states must set standards, measure progress, and intervene when outcomes lag (Metra, 2013). In the Title I lineage Stein documents, ESSA represents another turn in the long oscillation between poverty targeting and performance accountability: funds still flow on the basis of poverty, but accountability systems hinge on

outcomes—now including graduation rates and other indicators beyond test scores (Stein, 2004). The historical record thus yields two cautionary implications for interpretation: 1) policy categories are constructed, not natural—they have changed repeatedly in who they include and what they mean; and 2) policy goals have shifted from compensatory inputs to standardized outcomes, without fully resolving the underlying causes of unequal opportunity. Both warrant careful, longitudinal, multilevel analysis when evaluating Title I schools under ESSA.

Implications for the Present Study

First, the justification for an outcomes-oriented lens is historically grounded: the standards-accountability paradigm makes graduation a policy-salient indicator of school performance and student opportunity. Second, the justification for a longitudinal, multilevel design is historically necessary: shifting definitions of beneficiaries, changing accountability rules, and nested governance (schools within districts/ESDs) have produced heterogeneous starting points and heterogeneous change processes that cross-sectional models cannot capture. Finally, the long arc from cultural deficit framings to at-risk and accountability vocabularies underscores the value of modeling both growth rates and timing of change—to see whether, when, and for whom policy eras align with inflection points in graduation trajectories.

Title I's historical and conceptual lineage both requires and justifies cautious, longitudinal, multilevel evaluation of graduation outcomes in high-poverty schools. The origins of compensatory aid, shaped by “culture of poverty” ideas (1960s), evolving through decades of rhetorical rebranding and compliance expansion (1970s – 1990s), and culminating in outcomes-oriented accountability paradigms (2000s – present), present a basis for meaningful evaluation to address whether differences are in fact notable as time goes on.

Prior Research on Funding and Graduation Outcomes

Research on the relationship between school funding and student outcomes has produced a substantial but uneven evidence base. Across decades of scholarship, a broad consensus has emerged that school funding matters, particularly for students experiencing poverty and other forms of structural disadvantage; however, studies diverge in their methodological approaches, outcome measures, and conclusions about magnitude, timing, and efficiency. This section reviews three strands of prior research relevant to the present study: (a) evidence linking funding to student outcomes, including graduation; (b) debates between justice-oriented and efficiency-oriented frameworks; and (c) methodological limitations of existing work that motivate a longitudinal, multilevel growth approach.

Evidence that Funding Matters—Especially for High-Need Students

A large body of school finance research demonstrates that increased education spending is associated with improved student outcomes, particularly for students from low-income backgrounds. Funding increases tied to instructional resources, support services, and staffing improvements can yield measurable gains in achievement, attainment, and engagement outcomes like attendance, especially for historically under-resourced students (Baker, 2017).

Analyses of NAEP performance alongside cumulative per-pupil expenditures show that achievement trends closely track funding changes, with notable declines following periods of fiscal contraction, such as the Great Recession (Education Next, 2018). Importantly, these declines appear with a lag, suggesting that funding effects unfold over time rather than instantaneously. This dynamic has direct implications for graduation outcomes, which reflect long-term accumulation of opportunity rather than short-term instructional inputs (see Figure 1).

While scholars largely agree that money matters, they disagree on why, how, and by how much. Justice-oriented frameworks argue that traditional adequacy and equality models are insufficient because they fail to address structural and racialized inequities embedded in education systems. Martínez (2021) explains that redistributive approaches often reproduce inequality by treating need as a technical calculation rather than a historically contingent condition shaped by racism, exclusion, and political power. Similarly, Alemán (2007) critiques human capital models for framing students primarily as economic inputs, instead arguing for approaches that center lived experience and systemic harm.

In contrast, efficiency-oriented scholars emphasize opportunity costs and the productive use of limited public resources. Levin, McEwan, and Belfield (2017) argue that education spending decisions should be evaluated through cost-effectiveness and benefit–cost frameworks, focusing on what investments generate the greatest returns per dollar. Other scholars, like Hanushek (2007), caution that adequacy studies and consultant-driven reforms may overspend without improving outcomes, advocating for reallocating resources toward interventions with strong empirical support.

Despite these tensions, all agree that high-need students require additional resources, that current finance systems frequently under-serve these populations, and that funding must be structurally targeted rather than uniformly distributed. What remains unresolved is how these investments change outcomes over time, particularly for indicators like four-year graduation.

Graduation Outcomes and Methodological Gaps in Prior Work

Although graduation rates are widely used as accountability indicators under ESSA, they have received less rigorous treatment in school finance research compared to test scores. Many studies rely on cross-sectional designs that compare districts or states at a single time point,

obscuring how funding effects accumulate across cohorts. Others use single-level models that ignore the nested nature of education systems, where students are embedded within schools, schools within districts, and districts within broader governance structures.

Moreover, much of the adequacy literature emphasizes estimating a funding “target” rather than modeling outcome change. Resource-oriented approaches such as professional judgment or evidence-based models specify the inputs required to meet standards but do not examine whether observed outcomes improve—and if so, when (Baker, Taylor, & Vedlitz, 2005). Performance-oriented approaches better link spending to outcomes, yet they often assume constant relationships over time and across contexts, neglecting heterogeneity in how schools respond to policy and resource changes.

These limitations are particularly consequential for graduation rates, which are sensitive to the delayed effects of earlier investments, policy shifts, and school capacity. Funding shocks, accountability reforms, and targeted interventions may alter graduation trajectories at different points across schools, producing nonlinear patterns of change that are poorly captured by static or pooled analyses.

Implications for the Present Study

Prior research establishes that funding matters for student outcomes, that high-need students benefit most from additional resources, and that policy effects unfold over time. At the same time, the literature reveals persistent methodological gaps: overreliance on cross-sectional designs, limited attention to growth and timing, and insufficient modeling of heterogeneity across schools and organizational contexts. This study responds directly to these gaps by examining graduation rates longitudinally, modeling change over time within schools, and accounting for between-school and between-district variation. By focusing on trajectories rather

than point estimates, the analysis aligns with both justice-oriented concerns about cumulative opportunity and efficiency-oriented demands for outcome-based evaluation under ESSA.

Why a Longitudinal, Multilevel Perspective is Needed

The structure of the data and the policy context motivating this study necessitate a longitudinal, multilevel analytic approach. Graduation outcomes among Washington Title I high schools are observed repeatedly over multiple school years, where schools differ meaningfully in both their baseline outcomes and their capacity to respond to policy support. Further, schools are embedded within organizational systems that shape implementation and support. Ignoring these differences risks incomplete or misleading inferences about graduation patterns under ESSA.

Graduation Outcomes as Longitudinal Processes

The graduation rates examined in this study correspond to four-year graduation outcomes, measured annually for each high school. Funding variables (including ESSA Section 1003 allocations) represent resources awarded and expended within a single academic year, spanning from the 2018–2019 through the 2024–2025 school year. Graduation rates are not instantaneous effects; they evolve across successive cohorts exposed to different funding and policy conditions over time.

Graduation is a cumulative outcome. It reflects students' access to instructional opportunities, support, and stability across multiple years rather than the influence of any single intervention or funding increment. The four-year graduation rate captures whether schools successfully support students through a fixed and policy-salient timeline, aligning directly with ESSA accountability requirements. A longitudinal perspective allows graduation rates to be conceptualized as trajectories, enabling the examination of both initial levels and patterns of change across time. This framing is especially important given evidence that resource effects and

policy responses do not manifest immediately, but rather unfold gradually as schools adjust practices, staffing, and supports.

Policy Disruption and Non-Uniform Change Over Time

Although ESSA was enacted in 2015, graduation and funding data suitable for analysis became systematically available beginning in the 2018–2019 school year. The observation window for this study, therefore, spans a period of substantial policy and contextual disruption, most notably the COVID-19 pandemic. During the pandemic, many Washington high schools and districts temporarily modified graduation requirements in response to unprecedented instructional and logistical challenges. These emergency adaptations produced visible disruptions in graduation rates, followed by an uneven process of returning to pre-pandemic expectations.

Such disruptions underscore the limitations of models that assume constant rates of change over time. Schools did not experience the pandemic or its recovery uniformly. Some schools exhibited sharp short-term increases in graduation due to lowered requirements, while others experienced longer-term declines or delayed recoveries as expectations were reinstated. A longitudinal framework makes it possible to distinguish these dynamics from pre-existing trends, rather than attributing abrupt shifts to cross-sectional differences between schools and Education Service Districts (ESDs).

Furthermore, ESSA Section 1003 eligibility is not static. While the eligibility criteria have remained stable since ESSA's implementation, schools move in and out of eligibility based on changes in student poverty concentrations. As a result, schools' exposure to improvement funding varies over time in ways that cannot be adequately captured by time-invariant or cross-sectional analyses. Longitudinal modeling is therefore essential for representing how graduation rates respond as schools enter, remain within, or exit eligibility for additional federal support.

Meaningful Between-School Heterogeneity Among Title I High Schools

Although all schools included in this study are designated as Title I, this classification masks substantial heterogeneity in school context and resources. Schools differ markedly in enrollment size, per-pupil expenditure, organizational capacity, and student composition, resulting in widely varying baseline graduation rates and differential responsiveness to funding and support. Two schools may both be Title I high schools yet operate under profoundly different conditions that are not fully captured by simple indicators of poverty status.

Single-level models that pool observations across schools implicitly assume that graduation trajectories are homogeneous aside from measured covariates. Such assumptions are implausible in the present context and risk attributing systematic differences in school capacity or context either to funding effects or to random noise. A multilevel approach allows baseline graduation rates and growth patterns to vary across schools and ESDs, acknowledging that schools and districts start from different positions and change at different rates.

Schools Embedded within Education Service Districts

High schools in Washington state operate within Education Service Districts (ESDs), which function as intermediary governance between districts and the state. ESDs provide specialized services, technical assistance, professional development, and compliance support that individual districts or schools may not have the capacity to deliver independently. The hierarchical structure of educational governance means that schools typically access state-level resources and guidance through their districts and ESDs.

As a result, ESDs represent more than administrative boundaries; they constitute meaningful contexts that may shape how ESSA funds are implemented, supported, and monitored. Differences among ESDs in staffing, expertise, regional conditions, and service

capacity may contribute to systematic variation in how schools respond to accountability pressures and improvement funding. Ignoring this level of nesting risks underestimating uncertainty and misattributing variation in graduation outcomes to school-level factors alone.

A multilevel framework that accounts for schools nested within ESDs allows the analysis to partition variance across organizational levels and to examine whether graduation trajectories vary systematically across regions beyond individual school characteristics.

How Coding Time Matters

A foundational decision in growth modeling is how to code “time”. As Biesanz et al. (2004) demonstrated analytically, the choice of time coding directly and predictably determines the values and standard errors of all model parameters—not just the intercept. Because the intercept represents the expected outcome when time equals zero, different time codings locate that reference point at different moments in the data. This produces different estimates of baseline levels, slopes, and their covariance, even when the underlying growth process is identical. These are not numerical reconfigurations; each coding of time answers a different substantive question.

In linear growth models, coding time so that zero falls at the first observation produces an intercept that reflects the initial status; placing zero at the final observation shifts the intercept to represent the outcomes at the end of follow-up; centering time at the midpoint locates the intercept at the middle of the observational window. Although each reparameterization is mathematically equivalent and yields an identical model fit, the associated parameter estimates, standard errors, and inferential tests differ in meaningful ways. Crucially, the precision with which a predictor can be estimated, and thus the statistical power to detect its effect, varies as a function of where time is coded (Biesanz et al., 2004). This happens because the residual

variance in a growth model parameter directly depends on how time is coded, and it is that residual variance that determines the standard error of any predictor of that parameter.

These principles apply directly to the present study. With seven academic years of graduation rate data spanning 2019 through 2025, different codings of time produce substantively different answers to the study's research questions. Coding time so that zero corresponds to the 2018–2019 baseline year, as in the primary specification here, makes the intercept interpretable as expected graduation rates at the start of the observation window and allows the slope to be read as the average annual change from that point. Biesanz et al. (2004) recommend coding time to match the substantive question, rather than adopting a default such as centering or orthogonal polynomial contrasts, which can obscure the temporal location of effects and complicate the interpretation of predictor relationships.

Purpose of the Study and Research Questions

The **methodological** purpose of this study is to demonstrate how coding time can impact the inferences drawn about mean trajectories and predictors of change over time. The **educational policy** purpose of this study is to estimate four-year graduation trajectories among Washington State Title I high schools, with attention to initial outcome levels, rates of change over time, and the timing of meaningful shifts in those trajectories. Graduation is a cumulative outcome central to ESSA accountability, yet existing research provides limited insight into how graduation patterns evolve across heterogeneous high-poverty schools and policy contexts. By modeling graduation as a longitudinal, multilevel process—situating schools within Education Service District (ESD) structures—this study seeks to move beyond cross-sectional comparisons to understand when change occurs and how contextual factors such as ESD enrollment and Section 1003 ESSA award amounts relate to that change. The research questions that follow

operationalize this purpose into a focused empirical inquiry aligned with ESSA’s outcomes orientation and the methodological demands of growth analysis.

- 1) How do graduation rates change over time across Washington Title I high schools, depending on how time is coded?
- 2) To what extent do high schools differ in baseline graduation rates and in their growth trajectories, depending on how time is coded?
- 3) Are enrollment size and ESSA 1003 funding associated with differences in:
 - a) baseline graduation rates (at the initial year of data collection: 2019) and
 - b) rates of change in graduation outcomes over time, again, depending on how time is coded?
- 4) Do graduation rate trajectories vary across Education Service Districts (ESDs), depending on how time is coded?

Methods

Sample and Setting

The analytic sample consisted of Washington State public high schools designated as Title I across the 2018–2024 academic years. Schools were included if they reported valid four-year cohort graduation rates and could be linked to enrollment, ESSA 1003 award data, and ESD membership. The final analytic sample included $N = 93$ high schools, contributing $N = 320$ school-year observations during the study period. Each school belonged to one of nine Washington State Education Service Districts (ESDs).

Data for this study were drawn from publicly available administrative sources maintained by the Washington Office of Superintendent of Public Instruction (OSPI). Graduation rate data were obtained from OSPI Graduation and Dropout Statistics / Washington State Report Card,

which reports four-year cohort graduation rates consistent with ESSA accountability definitions. School-level enrollment data were obtained from OSPI Enrollment Reports and reflect total 9th – 12th grade enrollment.

Information on ESSA Section 1003 school improvement awards was obtained from OSPI 1003 award files. Award amounts were aggregated at the school level across the 2018–2024 academic years to align with the longitudinal outcome data. All datasets included a School Identification number and Education Service District membership indicator. Files were merged using unique school identifiers. Data cleaning procedures included checks for duplicate records, inconsistent identifiers, and implausible values. Because all data were publicly available and aggregated at the school level, this study did not involve human subjects as defined by federal regulations and was not subject to IRB review.

Measures

Outcome: Graduation Rate

The primary outcome was the four-year graduation rate, defined as the percentage of students in a ninth-grade cohort who earned a regular high school diploma within four years, as reported by OSPI. Graduation rates were measured annually and scaled from 0 to 100.

Predictor: Time

Time was defined by academic year and operationalized using multiple parameterizations to examine how alternative representations of temporal change affect inferences about graduation trajectories. Because the COVID-19 pandemic represented a substantial disruption to schooling conditions and accountability contexts, different time codings were used to evaluate whether graduation trajectories reflect gradual change, a structural shift associated with the pandemic, or irregular year-to-year variation.

- 1) Linear time was coded as a continuous variable (one rate), with the baseline academic year (2018-19) coded as 0, and subsequent years increased by one unit. This specification estimates a single average rate of change over time and provides the most parsimonious representation of gradual longitudinal trends.
- 2) Two-rate linear time coded as before the pandemic (2018-19 to 2019-20) vs. after the pandemic (2020-21 to 2024-25). This specification allows the growth trajectory to shift after a major contextual disruption, capturing the possibility that COVID-19 pandemic conditions altered the rate of change in graduation outcomes.
- 3) Categorical incremental time coded by each of the six years, with 2018-19 as the baseline, with each year representing the change between consecutive academic years. This flexible specification imposes no functional form on the trajectory and allows year-specific deviations, accommodating the possibility that the effects of large-scale disruptions may emerge gradually or with delayed timing rather than immediately.

School-Level Predictors

Two continuous school-level predictors were included: **Enrollment size**, which was the student enrollment at each school aggregated across years to a mean value at the school level and then standardized into z-scores, as well as **ESSA Section 1003 dollar award amounts**, which were also aggregated and standardized. Both predictors were centered at the grand mean, such that model intercepts represent expected graduation rates for schools with mean levels of enrollment and ESSA 1003 funding.

Finally, **Education Service District (ESD) membership** was included as a factor in the model using a set of $k-1$ effect-coded indicator variables to represent k ESDs in the state. ESD 121 served as the reference category due to its larger number of Title I high schools and largest amount of ESSA 1003 funds during the study period. The set of ESD indicators were used to examine whether baseline graduation rates and/or growth trajectories differed systematically across districts.

Analytic Plan

Analyses proceeded in several stages, consistent with the study's research questions. Initial analyses examined descriptive statistics for graduation rates across years, including means and standard deviations. Descriptive statistics were also computed for enrollment and ESSA 1003 award amounts, and correlations among predictors were examined. These analyses provided context for subsequent growth modeling and informed the selection of time parameterizations.

A multilevel linear growth modeling approach was adopted as the primary analytic strategy for this study for its ease of implementation, but I note that growth modeling can also be carried out with near-identical results using structural equation modeling. As well, a linear model was deemed appropriate in this case because, although the dependent variable is a graduation rate bounded by 0 and 100, the mean observed graduation rate was 50.15% with a $SD = 25.28$, and inspection of the model residuals for each of the three mean trajectory models described next showed no skew, and only slight leptokurtosis for the simplest linear growth trajectory model ($p = .032$). All multilevel growth models were estimated using full information maximum likelihood in *R* (R Core Team, 2023) with the *lme4* package (Bates et al., 2015) and

Satterthwaite degrees of freedom-based fixed effects coefficient tests via lmerTest (Kuznetsova et al., 2017).

Coding Time

To address Research Question 1 (mean change over time), unconditional growth models were first estimated. Time was a Level 1 predictor of the graduation rates that was fixed (repeated) across schools (Level 2). I used three different time coding specifications to address Research Question 2, with the first and simplest Level 1 model as follows:

$$GraduationRate_{ij} = \beta_{0j} + \beta_{1j}(TimeLin_{ij}) + r_{ij} \quad (1)$$

and at Level 2 was:

$$\begin{aligned} \beta_{0j} &= \gamma_{00} + u_{0j} \\ \beta_{1j} &= \gamma_{10} + u_{1j} . \end{aligned} \quad (2)$$

In this model (Equations 1-2), TimeLin was coded as 0, 1, 2, 3, 4, 5, and 6 to represent the six academic year spring time points in 2019, 2020, 2021, 2022, 2023, 2024, and 2025 respectively. Significant random intercept variance indicates differences in baseline graduation rates across schools, while significant random slope variance indicates heterogeneity in growth trajectories. The variance-covariances of the Level 2 intercept and slope random effects were unconstrained for this model, meaning that variability at the intercept was free to correlate with variability in change over time (typically constraints are only placed on the random effects covariance(s) when the model complexity becomes too large for its sample size). This model is the most parsimonious with two coefficients estimated (intercept and slope over all time points) and three variance-covariance components estimated (intercept variance, slope variance, and their covariance).

In the second time coding specification, I focused on highlighting the potential change in growth rates from pre- to post-pandemic using the following model:

$$GraduationRate_{ij} = \beta_{0j} + \beta_{1j}(TimePreCovid_{ij}) + \beta_{2j}(TimePostCovid_{ij}) + r_{ij} \quad (3)$$

for Level 1, and for Level 2 was:

$$\begin{aligned} \beta_{0j} &= \gamma_{00} + u_{0j} \\ \beta_{1j} &= \gamma_{10} + u_{1j} \\ \beta_{2j} &= \gamma_{20} + u_{2j} . \end{aligned} \quad (4)$$

For this model (Equations 3-4), time for the first slope (TimePreCovid) was coded as 0, 1, 1, 1, 1, 1, and 1 (estimating the linear rate of change in graduation from spring 2019 to spring 2020) and time for the second slope (TimePostCovid) was coded 0, 0, 1, 2, 3, 4, and 5 (estimating the linear rate of change from spring 2020 to spring 2025). As in the previous specification, random intercept and slope effects were free to covary. The tradeoff between this model and the previous model is that there is added precision around the growth rates, but at the cost of a greater number of parameters estimated: three coefficients and six variance-covariance components.

Finally, the third time coding specification sliced time into equal parts to ensure that growth rates could be estimated separately across each wave of data, with Level 1 as follows:

$$\begin{aligned} GraduationRate_{ij} = \beta_{0j} + \beta_{1j}(2020_{ij}) + \beta_{2j}(2021_{ij}) + \beta_{3j}(2022_{ij}) + \\ \beta_{4j}(2023_{ij}) + \beta_{5j}(2024_{ij}) + \beta_{6j}(2025_{ij}) + r_{ij} \end{aligned} \quad (5)$$

and for Level 2 as:

$$\begin{aligned} \beta_{0j} &= \gamma_{00} + u_{0j} \\ \beta_{1j} &= \gamma_{10} + u_{1j} \\ \beta_{2j} &= \gamma_{20} + u_{2j} \\ \beta_{3j} &= \gamma_{30} + u_{3j} \end{aligned}$$

$$\begin{aligned}
\beta_{4j} &= \gamma_{40} + u_{4j} \\
\beta_{5j} &= \gamma_{50} + u_{5j} \\
\beta_{6j} &= \gamma_{60} + u_{6j} .
\end{aligned} \tag{6}$$

For this third model (Equations 5-6), time was coded into $k-1$ linear pieces for k time points coded as:

- a) spring 2019 – spring 2020 growth: 0, 1, 2, 3, 4, 5, and 6 (like the first model),
- b) spring 2020 – spring 2021 growth: 0, 0, 1, 2, 3, 4, and 5,
- c) spring 2021 – spring 2022 growth: 0, 0, 0, 1, 2, 3, and 4,
- d) spring 2022 – spring 2023 growth: 0, 0, 0, 0, 1, 2, and 3,
- e) spring 2023 – spring 2024 growth: 0, 0, 0, 0, 0, 1, and 2, and
- f) spring 2024 – spring 2025 growth: 0, 0, 0, 0, 0, 0, and 1.

In this specification, the year-to-year segments of growth could be estimated and predicted. However, due to the sheer number of growth periods, the covariances among the slopes were constrained to zero for model estimability. In total, there were seven coefficients and seven variance components estimated.

Adding Predictors

To address Research Questions 3-4, school-level characteristics and ESD indicators were added as main effects (predictors of the intercept) and as interactions (predictors of the slope(s)) for each of the three previously described model specifications (Equations 1-2, Equations 3-4, and Equations 5-6). For brevity, below I show how these Level 2 predictors were added to the first model using a mixed-model format (substituting Equation 2 into Equation 1) as follows.

$$\begin{aligned}
GraduationRate_{ij} &= \gamma_{00} + \gamma_{01}ZEnrollAgg_j + \gamma_{02}ZAwardAgg_j + \\
&\quad \gamma_{03}ESD101Eff_j + \gamma_{04}ESD105Eff_j + \cdots + \gamma_{0.10}ESD189Eff_j +
\end{aligned}$$

$$\begin{aligned}
& \gamma_{10}TimeLin_{ij} + \\
& \gamma_{11}TimeLin_{ij} \times ZEnrollAgg + \gamma_{12}TimeLin_{ij} \times ZAwardAgg_j + \\
& \gamma_{13}TimeLin_{ij} \times ESD101Eff_j + \gamma_{14}TimeLin_{ij} \times ESD105Eff_j + \dots + \\
& \gamma_{1.10}TimeLin_{ij} \times ESD189Eff_j + \\
& u_{0j} + u_{1j} + r_{ij} \tag{7}
\end{aligned}$$

This specification allowed enrollment and ESSA funding, as well as ESD membership, to predict baseline graduation rates in spring 2019 ($\gamma_{01} - \gamma_{0.10}$) and mean change over time ($\gamma_{11} - \gamma_{1.10}$).

Model Evaluation and Data Visualization

Model fit was evaluated using AIC, BIC, and likelihood ratio tests for nested comparisons. An alpha of .05, 2-tailed, was used for all statistical tests. Visualization of model-predicted values was implemented using the ggplot2 package (Wickham, 2016); interaction term predicted values were extracted with the effects package (Fox & Weisberg, 2019). Model-implied (predicted) values were used to illustrate average trends and heterogeneity across schools, funding levels, and ESDs. Example R code is provided in the Appendix.

Results

Descriptive Statistics and Correlations

Graduation rates were measured as percentages bounded between 0 and 100. Descriptive checks confirmed that all observed values fell within the admissible range. Fitted values from the linear mixed models were also examined and remained largely within the outcome bounds, supporting the appropriateness of the linear specification.

Descriptive statistics and zero-order correlations are presented in Table 1. The four-year graduation rate had a mean of $M = 50.15$ ($SD = 25.28$). The intraclass correlation coefficient (ICC) was approximately .80 across specifications, indicating that roughly 80% of the variance

in graduation rates was attributable to differences between schools rather than year-to-year variation within schools. This substantial clustering supports the use of multilevel modeling and indicates meaningful between-school heterogeneity in baseline graduation outcomes.

Variable Associations with Time (Research Question 1)

Bivariate correlations between graduation rate and linear time were small, suggesting limited uniform linear change at the descriptive level. In the categorical incremental specification, graduation rate demonstrated small positive associations with later year indicators ($r_s = .05$ to $.08$), indicating modest descriptive increases across time. Time variables were highly intercorrelated in the categorical model ($r_s = .61$ to $.99$), reflecting structural dependence among adjacent year indicators. Overall, preliminary correlations suggest relatively stable graduation rates across years with modest average change, warranting formal growth modeling to estimate longitudinal trends.

School Characteristic Associations with Graduation (Research Question 3a)

Enrollment size was moderately positively correlated with graduation rate ($r = .32$), indicating that larger schools tended to report higher graduation rates descriptively. In contrast with school size, ESSA 1003 award amounts were negatively correlated with graduation rate ($r = -.41$), representing the strongest bivariate association observed. Schools receiving higher levels of improvement funding tended to have lower graduation rates descriptively. Funding was modestly negatively correlated with enrollment ($r = -.19$).

Education Service District Membership Associations with Graduation (Research Question 4)

ESD indicators exhibited small correlations with graduation rate (generally $|r| < .20$), indicating modest cross-district differences at the descriptive level. ESD indicators were strongly intercorrelated ($r_s = .63$ to $.81$), reflecting structural coding rather than substantive overlap.

Multilevel Growth Model Results

Tables 2-4 present model fixed effects, random effects, and model fit statistics for each of the time coding specifications.

Mean Growth Trajectory (Research Question 1)

Across all specifications, the fixed linear growth term was statistically significant ($p < .001$), indicating that graduation rates changed over time on average. However, associated effect sizes were small, suggesting modest annual changes across schools. The consistency of this growth effect across linear and categorical models indicates that a steady linear trend over the study period adequately represents graduation trajectories. Figures 2-4 display the differences in the model estimates across the different time codings.

Between-School Variability (Research Question 2)

Random effects estimates demonstrated substantial variability across schools in both baseline graduation rates and growth trajectories. Random intercept variance indicated meaningful heterogeneity in baseline graduation rates. Random slope variance for linear growth indicated that schools differed in rates of change over time. The intercept–slope correlation (approximately .50–.52) suggested an association between baseline levels and subsequent growth patterns. Although within-school residual variance remained larger than between-school growth variance, significant random effects confirm that high schools differ in both initial graduation rates and longitudinal trajectories.

School-Level Predictors of Baseline and Growth (Research Question 3)

Baseline Graduation Rates (Research Question 3a). Enrollment size was significantly associated with baseline graduation rates ($p = .039$), though the effect size was small. ESSA Section 1003 funding was not significantly associated with baseline graduation rates ($p = .449$).

ESD membership indicators were not consistently significant predictors of baseline graduation rates after accounting for school-level characteristics. Figures 5-7 illustrate the mean differences among ESD effects on trajectories among the three different time codings.

Rates of Change Over Time (Research Question 3b). The interaction between time and ESSA 1003 funding was statistically significant ($p = .003$), indicating that funding levels were associated with differences in growth trajectories. Figures 8-10 display the mean effects of the ESSA award on trajectories among the three different time codings.

Enrollment size. School enrollment size did not significantly predict rates of change ($p = .560$). Interactions between ESD membership and time were not statistically significant, indicating that graduation rate trajectories did not vary systematically across districts once enrollment and funding were accounted for.

Discussion

This study was motivated by both methodology and education policy. *Methodologically*, this work shows the importance of coding time in terms of the inferences that can be drawn around mean trajectories and predictors of those trajectories. In terms of *policy*, this work surfaced a central tension in the Title I/ESSA lineage: funds are allocated on the basis of poverty, yet accountability is evaluated through outcomes like graduation rates, and how that accountability is modeled has direct consequences for what gets measured and who appears to be improving. Using longitudinal multilevel models, this study examined graduation trajectories among Washington Title I high schools, focusing on (a) average change over time, (b) heterogeneity in baseline levels and growth, and (c) associations with enrollment, Section 1003 funding, and ESD context. Two conclusions are most salient for policy interpretation and methodological design. First, Title I high schools exhibit substantial heterogeneity in graduation

outcomes. Second, average change over time is detectable but modest, and the data provided limited support for a discrete, shared phase shift during the observation window.

Salient Findings in Relation to Prior Research

Modest Average Change

Graduation rates increased slightly over time, with statistically detectable but small effects. This pattern suggests gradual movement rather than statewide inflection. Graduation is a cumulative cohort outcome, aggregating multiple years of exposure to school conditions and policy rules; as such, even substantial contextual disruptions could be seen as incremental change. The results align with literature suggesting that resource and accountability effects on long-horizon outcomes tend to produce gradual change rather than immediate change.

Substantial Between-School Heterogeneity

The intraclass correlation indicated that most variance in graduation rates was between schools rather than within schools over time. Random effects further showed variability in both baseline levels and growth trajectories.

These findings reinforce a central argument: “Title I high school” is not a homogeneous policy category. Cross-sectional or single-level approaches obscure the most policy-relevant variation, which is the differences in starting points and differences in trajectories. Under ESSA’s outcomes orientation, such heterogeneity complicates status-based accountability judgments.

Enrollment and Baseline Differences

Enrollment was positively associated with baseline graduation rates but not with growth. Size may account for some organizational capacity or access to resources, shaping initial conditions more than subsequent rates of change. This pattern is consistent with organizational

theory, suggesting that structural features influence opportunity conditions without necessarily accelerating improvement once baseline differences are accounted for.

ESSA Section 1003 Funding and Trajectories

Section 1003 funding was not associated with baseline levels in conditional models but was associated with differences in growth, albeit with small effect sizes. Descriptively, higher funding coincided with lower graduation rates, consistent with targeted allocation. In longitudinal models, however, funding is related more to trajectories than to starting points.

The magnitude of the effect warrants caution. The findings do not support strong causal claims; however, they suggest that improvement funds are associated with detectable differences in longitudinal patterns rather than cross-sectional status.

ESD Context

ESD membership showed limited predictive value once school-level characteristics were included. This does not imply that intermediary context is irrelevant; rather, membership indicators may inadequately capture variation in service provision, implementation capacity, or local labor markets. If ESDs influence improvement, those mechanisms will likely require more direct measurement.

Limitations

Several constraints hinder interpretation. First, analyses rely on school-level aggregates, limiting causal inference and precluding student-level mechanisms. Second, Section 1003 funding was operationalized in an aggregated form, potentially obscuring the timing and intensity of exposure. Third, pandemic-era policy rule changes were not explicitly modeled, complicating the interpretation of graduation metrics. Finally, the ESD context was captured through membership rather than direct measures of intermediary capacity.

Implications for Policy, Practice, and Methodology

Policy and Practice

The findings underscore that ESSA accountability operates across highly heterogeneous schools. Status comparisons alone may miss meaningful differences in trajectories. Improvement funds appear more closely associated with longitudinal change than with baseline levels, supporting evaluation frameworks that emphasize growth rather than static thresholds.

Methodological Contributions

The study demonstrates the necessity of multilevel longitudinal modeling for accountability outcomes. High between-school variance and significant random effects indicate that ignoring clustering would obscure substantively meaningful heterogeneity. The results of this study illustrate why the coding of time in growth curve models is vital for what questions get answered and how precisely they can be answered. As Biesanz et al. (2004) demonstrated, every parameterization of time produces a mathematically equivalent model — identical fit statistics, identical underlying trajectories — yet the parameter estimates, standard errors, and inferential conclusions differ systematically depending on where the origin of time is placed.

In the present study, this was not merely a theoretical concern. By coding time with the origin at the 2018–2019 academic year, the model intercept is interpretable as the expected graduation rate at baseline — the beginning of the accountability window under the *Every Student Succeeds Act* policy changes. This made it possible to directly and meaningfully test Research Questions 3a and 3b: whether enrollment size and Section 1003 funding were associated with initial graduation levels versus rates of change over the observation period. The finding that enrollment predicted baseline graduation rates but not growth, while funding predicted growth but not baseline levels, is a substantive and policy-relevant result — but it is a

result that is only clear when time is coded to locate the intercept at the time of interest. Had time been centered at the study midpoint (approximately 2021–2022) or coded using orthogonal polynomial contrasts, the intercept would no longer represent initial status in 2019, and the same fixed-effect estimates for enrollment and funding would carry fundamentally different interpretations.

The comparison of linear and categorical incremental time specifications further illustrates this principle. The categorical model codes each year-to-year transition as a separate indicator, foregoing the assumption of constant annual change. This parameterization was motivated by the policy context — the COVID-19 pandemic disrupted schooling, and OSPI modified graduation requirements beginning in 2020, raising the possibility that graduation trajectories were not adequately described by a single linear slope across the full observation window. The substantive consistency between the linear and categorical models is itself an important finding: it suggests that, within this sample of Title I high schools, graduation trajectories were sufficiently stable in their average rate of change that the simpler linear model adequately captures the trend. That conclusion, however, could not be reached without fitting both and comparing them. It required asking the question of time coding explicitly rather than assuming a functional form.

A broader implication follows for accountability research. Studies of school performance that use a single time coding — particularly one chosen for computational convenience rather than substantive alignment — risk either mislocating the intercept (and thus misinterpreting baseline-level effects) or obscuring the pattern of change across the observation window. Under ESSA's outcomes-oriented accountability framework, where schools are evaluated on both status and growth, this distinction has direct policy consequences. A school may appear unimproved

when its baseline is measured at an arbitrary centering point, but still show meaningful progress when its trajectory is evaluated from its actual starting conditions. The present study demonstrates that making time coding an explicit, theoretically justified, and reported methodological choice — rather than a default — is a precondition for drawing valid inferences about when change occurred, what predicts it, and what it means.

Conclusion

This study provides decision-relevant evidence about graduation trajectories in Washington Title I high schools while advancing a methodological argument: interpreting Title I outcomes requires models capable of representing both longitudinal change and nested heterogeneity. Average change was modest, between-school differences were substantial, and predictors related differently to baseline levels versus trajectories. By pairing multilevel growth modeling with rigorous model comparison, the analysis offers a disciplined framework for interpreting outcomes within an accountability system that evaluates performance but funds on the basis of poverty.

References

- Alemán, S. M. (2007). Critical race theory and human capital theory: Framing the discourse on the nexus of social justice and education finance. In G. M. Rodriguez & R. A. Rolle (Eds.), *To What Ends and By What Means* (pp. 35–58). Routledge.
<https://doi.org/10.4324/9780203941218>
- Aloe, A. M., & Thompson, C. G. (2013). The synthesis of partial effect sizes. *Journal of the Society for Social Work and Research*, 4(4), 390–405.
<https://doi.org/10.5243/jsswr.2013.24>
- Baker, B. D. (2017). *How Money Matters for Schools*. Learning Policy Institute.
<https://learningpolicyinstitute.org/product/how-money-matters-schools-report>
- Baker, B. D., Taylor, L., & Vedlitz, A. (2005). *Measuring Educational Adequacy in Public Schools* (Report prepared for the Texas Legislature Joint Committee on Public School Finance). The Texas School Finance Project. <https://bush.tamu.edu/wp-content/uploads/2020/08/MeasuringEducationalAdequacy.pdf>
- Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67(1), 1–48.
<https://doi.org/10.18637/jss.v067.i01>
- Biesanz, J. C., Deeb-Sossa, N., Papadakis, A. A., Bollen, K. A., & Curran, P. J. (2004). The role of coding time in estimating and interpreting growth curve models. *Psychological Methods*, 9(1), 30–52. <https://doi.org/10.1037/1082-989X.9.1.30>
- Bowden, B. (2018). Using resource and cost considerations to support educational evaluation. *American Journal of Evaluation*, 39(2), 232–249.
<https://doi.org/10.1177/1098214017735534>

- Cohen, J. (1988). *Statistical Power Analysis for the Behavioral Sciences* (2nd ed.). Lawrence Erlbaum Associates.
- Education Next. (2018). Could disappointing 2017 NAEP scores be due to the Great Recession? *Education Next*. <https://www.educationnext.org/could-disappointing-2017-naep-scores-due-to-great-recession/>
- Fox, J., & Weisberg, S. (2019). *An R Companion to Applied Regression* (3rd ed.). Sage Publications. <https://socialsciences.mcmaster.ca/jfox/Books/Companion/index.html>
- Hanushek, E. A. (2007). The confidence men: Selling adequacy, making millions. *Education Next*, 7(3), 73–78.
<https://hanushek.stanford.edu/sites/default/files/publications/Hanushek%202007%20Edn%207%283%29.pdf>
- Jimenez-Castellanos, O., & Topper, A. M. (2012). The cost of providing an adequate education to English language learners: A review of the literature. *Review of Educational Research*, 82(2), 179–232. <https://doi.org/10.3102/0034654312449876>
- Kuznetsova, A., Brockhoff, P. B., & Christensen, R. H. B. (2017). lmerTest package: Tests in linear mixed effects models. *Journal of Statistical Software*, 82(13), 1–26.
<https://doi.org/10.18637/jss.v082.i13>
- Levin, H. M., McEwan, P. J., Belfield, C. R., Bowden, A., & Shand, R. D. (2017). *Economic Evaluation in Education: Cost-Effectiveness and Benefit-Cost Analysis* (3rd ed.). Sage Publications. <https://doi.org/10.4135/9781483396514>
- Luo, W., Li, H., Baek, E., Chen, S., Lam, K. H., & Semma, B. (2021). Reporting practice in multilevel modeling: A revisit after 10 years. *Review of Educational Research*, 91(3), 311–355. <https://doi.org/10.3102/0034654321991229>

- Martínez, A. (2021). Interrogating social justice paradigms in school finance. *Educational Policy*, 35(4), 593–615. <https://doi.org/10.1177/0895904819896262>
- Mehta, J. (2013). How paradigms create politics: The transformation of American educational policy, 1980–2001. *American Educational Research Journal*, 50(2), 285–324. <https://doi.org/10.3102/0002831212471417>
- National Commission on Excellence in Education. (1983). *A Nation at Risk: The Imperative for Educational Reform* (GPO Publication No. 065-000-00177-2). U.S. Government Printing Office.
- Office of Superintendent of Public Instruction. (2023). *A Brief History of Education Funding in Washington State*. <https://ospi.k12.wa.us/sites/default/files/2023-08/edfundingchart-2.pdf>
- Office of Superintendent of Public Instruction. (2024). *Washington State Report Card*. <https://washingtonstatereportcard.ospi.k12.wa.us/>
- R Core Team. (2023). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing. <https://www.r-project.org/>
- Stein, S. J. (2004). *The Culture of Education Policy*. Teachers College Press.
- Sundquist, J. L. (1968). *Politics and Policy: The Eisenhower, Kennedy, and Johnson Years*. Brookings Institution.

Table 1*Descriptive Statistics and Correlations among Variables*

Variable	<i>M</i>	<i>(SD)</i>	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.
<i>L1 Outcome</i>																					
1. Graduation Rate	50.15	(25.28)	--																		
<i>L1 Predictors</i>																					
2. Linear Time (2019-25, Single Rate)	--	--	.05	--																	
3. Linear Time Pre-Covid (Two-Rate)	--	--	.04	--	--																
4. Linear Time Post-Covid (Two-Rate)	--	--	.04	--	.48	--															
5. Linear Time1 2019-20 (Multi-Rate)	--	--	.05	--	--	--	--														
6. Linear Time2 2020-21 (Multi-Rate)	--	--	.04	--	--	--	.99	--													
7. Linear Time3 2021-22 (Multi-Rate)	--	--	.06	--	--	--	.95	.98	--												
8. Linear Time4 2022-23 (Multi-Rate)	--	--	.07	--	--	--	.89	.92	.97	--											
9. Linear Time5 2023-24 (Multi-Rate)	--	--	.07	--	--	--	.78	.82	.88	.95	--										
10. Linear Time6 2024-25 (Multi-Rate)	--	--	.08	--	--	--	.61	.65	.70	.78	.88	--									
<i>L2 Predictors</i>																					
11. Mean Student Enrollment Count	217	(339)	.32	.00	.00	.00	.00	.00	.00	.00	.00	.00	--								
12. Mean ESSA 1003 Dollar Award	59,327	(51,614)	.32	.00	.00	.00	.00	.00	.00	.00	.00	.00	-.19	--							
13. Educ Serv Dist 101	0.10	(0.30)	.32	.00	.00	.00	.00	.00	.00	.00	.00	.00	-.16	-.07	--						
14. Educ Serv Dist 105	0.15	(0.36)	.32	.00	.00	.00	.00	.00	.00	.00	.00	.00	.06	-.11	.65	--					
15. Educ Serv Dist 112	0.11	(0.31)	.09	.00	.00	.00	.00	.00	.00	.00	.00	.00	-.15	.07	.68	.64	--				
16. Educ Serv Dist 113	0.10	(0.30)	.11	.00	.00	.00	.00	.00	.00	.00	.00	.00	-.19	-.04	.69	.65	.68	--			
17. Educ Serv Dist 114	0.02	(0.15)	.08	.00	.00	.00	.00	.00	.00	.00	.00	.00	-.16	-.07	.80	.77	.79	.80	--		
18. Educ Serv Dist 123	0.09	(0.28)	.09	.00	.00	.00	.00	.00	.00	.00	.00	.00	-.16	-.07	.70	.66	.69	.70	.81	--	
19. Educ Serv Dist 171	0.09	(0.28)	.10	.00	.00	.00	.00	.00	.00	.00	.00	.00	-.17	-.07	.70	.66	.69	.70	.81	.71	--
20. Educ Serv Dist 189	0.12	(0.32)	.07	.00	.00	.00	.00	.00	.00	.00	.00	.00	-.17	-.09	.67	.63	.66	.67	.79	.68	.68

Note. Six time points from $N = 93$ schools used. Enrollment represents the mean number of students enrolled in each school across the observed time points (standardized into z-scores), and ESSA 1003 Award is the mean Title I dollar amount a school was awarded across time points (standardized into z-scores). There were nine education service districts (ESDs) and ESD 121 served as reference category. Pearson's r reported; values in boldface indicate significance at the .05 level.

Table 2*Multilevel Model Results using Time Coded as a Single Rate*

	Model 1: Mean Growth Trajectory			Model 2: Predictors of Growth		
	<i>Coeff</i>	<i>(SE)</i>	<i>p</i>	<i>Coeff</i>	<i>(SE)</i>	<i>p</i>
<i>Fixed Effects</i>						
Conditional Mean (Spring 2019)	42.40	(3.20)	<.001	42.59	(3.45)	<.001
Mean Student Enrollment Size (Z)				10.33	(2.73)	<.001
Mean ESSA 1003 Dollar Award (Z)				-4.98	(3.21)	.124
Educ Serv Dist 101 (Effect Coded)				-1.94	(10.63)	.856
Educ Serv Dist 105 (Effect Coded)				-3.53	(7.10)	.621
Educ Serv Dist 112 (Effect Coded)				9.38	(12.43)	.452
Educ Serv Dist 113 (Effect Coded)				10.21	(8.37)	.228
Educ Serv Dist 114 (Effect Coded)				-5.77	(14.62)	.695
Educ Serv Dist 123 (Effect Coded)				0.00	(8.26)	.999
Educ Serv Dist 171 (Effect Coded)				2.50	(9.81)	.799
Educ Serv Dist 189 (Effect Coded)				-5.34	(8.06)	.510
Linear Time (2019-25, Single Rate)	2.22	(0.54)	<.001	2.41	(0.63)	<.001
Mean Student Enrollment Count (Z)				-0.74	(0.59)	.211
Mean ESSA 1003 Dollar Award (Z)				-1.14	(0.77)	.145
Educ Serv Dist 101 (Effect Coded)				4.21	(1.96)	.035
Educ Serv Dist 105 (Effect Coded)				0.59	(1.15)	.611
Educ Serv Dist 112 (Effect Coded)				-2.09	(2.74)	.446
Educ Serv Dist 113 (Effect Coded)				-1.19	(1.40)	.400
Educ Serv Dist 114 (Effect Coded)				-0.69	(2.02)	.734
Educ Serv Dist 123 (Effect Coded)				-0.46	(1.26)	.717
Educ Serv Dist 171 (Effect Coded)				0.04	(1.91)	.983
Educ Serv Dist 189 (Effect Coded)				0.51	(1.73)	.771
<i>Random Effects</i>						
Conditional Mean (Spring 2019)	653.71			458.64		
Linear Time (2019-25, Single Rate)	7.04	-.27		5.01	-.34	
Residual (Within Schools)	101.54			101.38		
<i>Model Fit</i>						
Total R^2	.87			.86		
AIC	2480.40			2479.20		
BIC	2502.40			2574.80		
Deviance (-2LL)	2468.40			2427.20		
Residual df	286			266		
LRT Chi-square test	--			41.19	.004	

Note. Six time points from $N = 93$ schools used. Enrollment represents the mean number of students enrolled in each school across the observed time points (standardized into z -scores), and ESSA 1003 Award is the mean Title I dollar amount a school was awarded across time points (standardized into z -scores). There were nine education service districts (ESDs) and ESD 121 served as reference category (effect-coded). Models were estimated with full information maximum likelihood using *R* lme4, with t -test p -values based on Satterthwaite df estimated with lmerTest. Significant effects boldfaced (alpha = .05, 2-tailed).

Table 3*Multilevel Model Results using Time Coded as Pre-Post Pandemic (Two-Rate Piecewise)*

<i>Fixed Effects</i>	Model 1: Mean Growth Trajectory			Model 2: Predictors of Growth		
	<i>Coeff</i>	<i>(SE)</i>	<i>p</i>	<i>Coeff</i>	<i>(SE)</i>	<i>p</i>
Conditional Mean (Spring 2019)	39.37	(3.74)	<.001	40.69	(4.56)	<.001
Mean Student Enrollment Count (Z)				11.49	(2.93)	<.001
Mean ESSA 1003 Dollar Award (Z)				-2.95	(6.24)	.638
Educ Serv Dist 101 (Effect Coded)				4.59	(11.55)	.693
Educ Serv Dist 105 (Effect Coded)				-5.08	(8.12)	.535
Educ Serv Dist 112 (Effect Coded)				20.67	(14.35)	.156
Educ Serv Dist 113 (Effect Coded)				10.77	(9.01)	.240
Educ Serv Dist 114 (Effect Coded)				-5.63	(15.14)	.713
Educ Serv Dist 123 (Effect Coded)				-3.57	(8.88)	.690
Educ Serv Dist 171 (Effect Coded)				3.51	(21.53)	.871
Educ Serv Dist 189 (Effect Coded)				-21.18	(10.97)	.059
Linear Rate 1 (Change 2019-2020)	6.81	(2.64)	.014	5.00	(4.06)	.222
Mean Student Enrollment Count (Z)				-2.42	(2.11)	.260
Mean ESSA 1003 Dollar Award (Z)				-3.18	(5.98)	.598
Educ Serv Dist 101 (Effect Coded)				-8.40	(9.51)	.381
Educ Serv Dist 105 (Effect Coded)				2.93	(6.14)	.635
Educ Serv Dist 112 (Effect Coded)				-18.58	(12.06)	.131
Educ Serv Dist 113 (Effect Coded)				0.46	(7.19)	.950
Educ Serv Dist 114 (Effect Coded)				0.70	(9.45)	.942
Educ Serv Dist 123 (Effect Coded)				8.07	(6.35)	.209
Educ Serv Dist 171 (Effect Coded)				-1.13	(23.26)	.961
Educ Serv Dist 189 (Effect Coded)				18.95	(9.44)	.051
Linear Rate 2 (Change 2021-25)	1.72	(0.61)	.007	2.17	(0.73)	.004
Mean Student Enrollment Count (Z)				-0.53	(0.73)	.471
Mean ESSA 1003 Dollar Award (Z)				-1.03	(0.82)	.212
Educ Serv Dist 101 (Effect Coded)				6.04	(2.37)	.012
Educ Serv Dist 105 (Effect Coded)				0.36	(1.31)	.784
Educ Serv Dist 112 (Effect Coded)				-0.30	(3.05)	.921
Educ Serv Dist 113 (Effect Coded)				-1.74	(1.81)	.340
Educ Serv Dist 114 (Effect Coded)				-1.06	(2.45)	.669
Educ Serv Dist 123 (Effect Coded)				-1.92	(1.51)	.210
Educ Serv Dist 171 (Effect Coded)				0.24	(2.05)	.908
Educ Serv Dist 189 (Effect Coded)				-0.90	(1.90)	.636
<i>Random Effects</i>	<i>Var</i>	<i>Corr</i>		<i>Var</i>	<i>Corr</i>	
Conditional Mean (Spring 2019)	762.56			446.82		
Linear Rate 1 (Change 2019-20)	78.39	-.43		30.33	-.07	
Linear Rate 2 (Change 2021-25)	8.20	-.18	.02	6.66	-.36	-.07
Residual (Within Schools)	90.27			90.03		
<i>Model Fit</i>	<i>Value</i>	<i>p</i>		<i>Value</i>	<i>p</i>	
Total R^2	.88			.87		
AIC	2482.00			2490.50		
BIC	2518.70			2637.50		
Deviance (-2LL)	2462.00			2410.50		
Residual <i>df</i>	282			252.00		
LRT Chi-square test	--			51.52		.009

Note. Six time points from $N = 93$ schools used and coded into two rates (piecewise): pre- and post-pandemic. Enrollment represents the mean number of students that were enrolled in each school across time points

(standardized into *z*-scores), and ESSA 1003 Award is the mean Title I dollar amount a school was awarded across time points (standardized into *z*-scores). There were nine education service districts (ESDs) and ESD 121 served as reference category (effect-coded). Models were estimated with full information maximum likelihood using *R* lme4, with *t*-test *p*-values based on Satterthwaite *df* estimated with lmerTest. Significant effects boldfaced (alpha = .05, 2-tailed).

Table 4*Multilevel Model Results using Time Coded by Year (Multi-Rate Piecewise)*

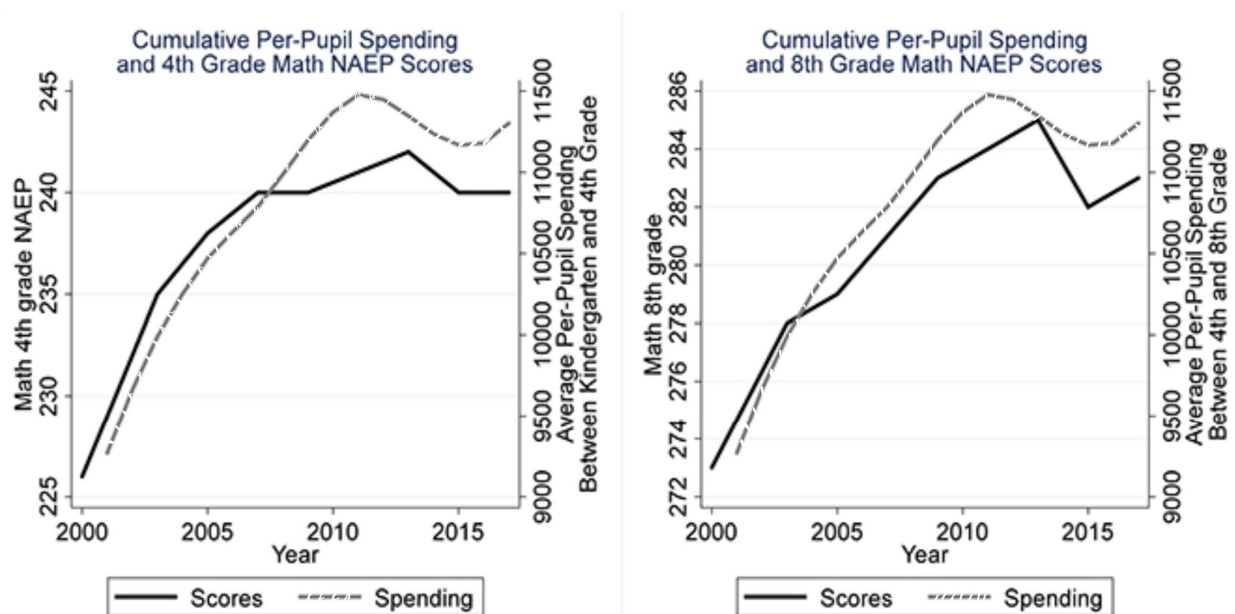
<i>Fixed Effects</i>	Model 1: Mean Growth Trajectory			Model 2: Predictors of Growth		
	<i>Coeff</i>	<i>(SE)</i>	<i>p</i>	<i>Coeff</i>	<i>(SE)</i>	<i>p</i>
Conditional Mean (Spring 2019)	40.17	(3.30)	<.001	41.18	(4.04)	<.001
Mean Student Enrollment Count (Z)				12.12	(2.60)	<.001
Mean ESSA 1003 Dollar Award (Z)				-1.88	(5.37)	.727
Educ Serv Dist 101 (Effect Coded)				4.13	(10.21)	.687
Educ Serv Dist 105 (Effect Coded)				-5.90	(7.16)	.411
Educ Serv Dist 112 (Effect Coded)				19.15	(12.59)	.130
Educ Serv Dist 113 (Effect Coded)				12.48	(8.05)	.124
Educ Serv Dist 114 (Effect Coded)				-5.58	(13.53)	.681
Educ Serv Dist 123 (Effect Coded)				-3.91	(7.89)	.621
Educ Serv Dist 171 (Effect Coded)				-19.75	(19.22)	.820
Educ Serv Dist 189 (Effect Coded)				-2.54	(9.64)	.042
<i>Linear Time1 2019-2020 (Multi-Rate)</i>	6.75	(2.57)	.009	7.13	(3.91)	.070
Educ Serv Dist 113 (Effect Coded)				34.99	(7.08)	.049
Educ Serv Dist 114 (Effect Coded)				-46.09	(10.30)	.048
<i>Linear Time2 2020-2021 (Multi-Rate)</i>	-5.70	(4.29)	.186	-8.78	(5.78)	.131
Educ Serv Dist 113 (Effect Coded)				-17.80	(12.06)	.004
Educ Serv Dist 114 (Effect Coded)				24.90	(18.43)	.013
<i>Linear Time3 2021-2022 (Multi-Rate)</i>	0.68	(3.71)	.854	5.09	(4.14)	.221
Educ Serv Dist 101 (Effect Coded)				-11.27	(12.91)	.030
Educ Serv Dist 171 (Effect Coded)				1.56	(13.29)	.023
<i>Linear Time4 2022-2023 (Multi-Rate)</i>	-0.10	(3.72)	.978	-4.30	(4.12)	.298
<i>Linear Time5 2023-2024 (Multi-Rate)</i>	-1.48	(4.09)	.718	7.16	(5.92)	.227
<i>Linear Time6 2024-2025 (Multi-Rate)</i>	4.89	(4.67)	.296	-1.20	(8.96)	.894
<i>Random Effects</i>	<i>Var</i>			<i>Var</i>		
Conditional Mean (Spring 2019)	581.96			359.90		
Linear Time1 2019-2020 (Multi-Rate)	4.92			3.26		
Linear Time2 2020-2021 (Multi-Rate)	0.00			0.00		
Linear Time3 2021-2022 (Multi-Rate)	1.12			0.00		
Linear Time4 2022-2023 (Multi-Rate)	2.86			5.43		
Linear Time5 2023-2024 (Multi-Rate)	0.00			0.00		
Linear Time6 2024-2025 (Multi-Rate)	0.00			0.00		
Residual (Within Schools)	98.71			69.56		
<i>Model Fit</i>	<i>Value</i>			<i>Value</i>	<i>p</i>	
Total R ²	.87			.90		
AIC	2494.60			2524.20		
BIC	2549.70			2836.70		
Deviance (-2LL)	2464.60			2354.20		
Residual <i>df</i>	277			207		
LRT Chi-square test	--			110.34	.002	

Note. Non-significant predictor effects on trajectory parameters omitted for brevity. Six time points from $N = 93$ schools used and coded into annual rates (piecewise). Enrollment represents the mean number of students that were enrolled in each school across time points (standardized into z-scores), and ESSA 1003 Award is the mean Title I dollar amount a school was awarded across time points (standardized into z-scores). There were nine education

service districts (ESDs) and ESD 121 served as reference category (effect-coded). Models were estimated with full information maximum likelihood using *R* lme4, with *t*-test p-values based on Satterthwaite *df* estimated with lmerTest. Significant effects boldfaced (alpha = .05, 2-tailed).

Figure 1

NAEP Scores and Cumulative Per-Pupil Spending



Note. Cumulative per-pupil expenditure (PPE) with NAEP math scores from 2000 to 2017. Both 4th- and 8th-grade scores rise as PPE increases, with a noticeable dip during the Great Recession. Economist Kirabo Jackson explains that the decline appears later than expected because the 2009 Recovery Act temporarily shielded school budgets. As a result, the first cohort exposed to sustained funding cuts was tested in 2015 after four years of reduced spending (from 2011 to 2015). Source: *Education Next*, 2018.

Figure 2

Model-Implied Graduation Rate Trajectories for Individual High Schools when Time is Coded as Linear Only

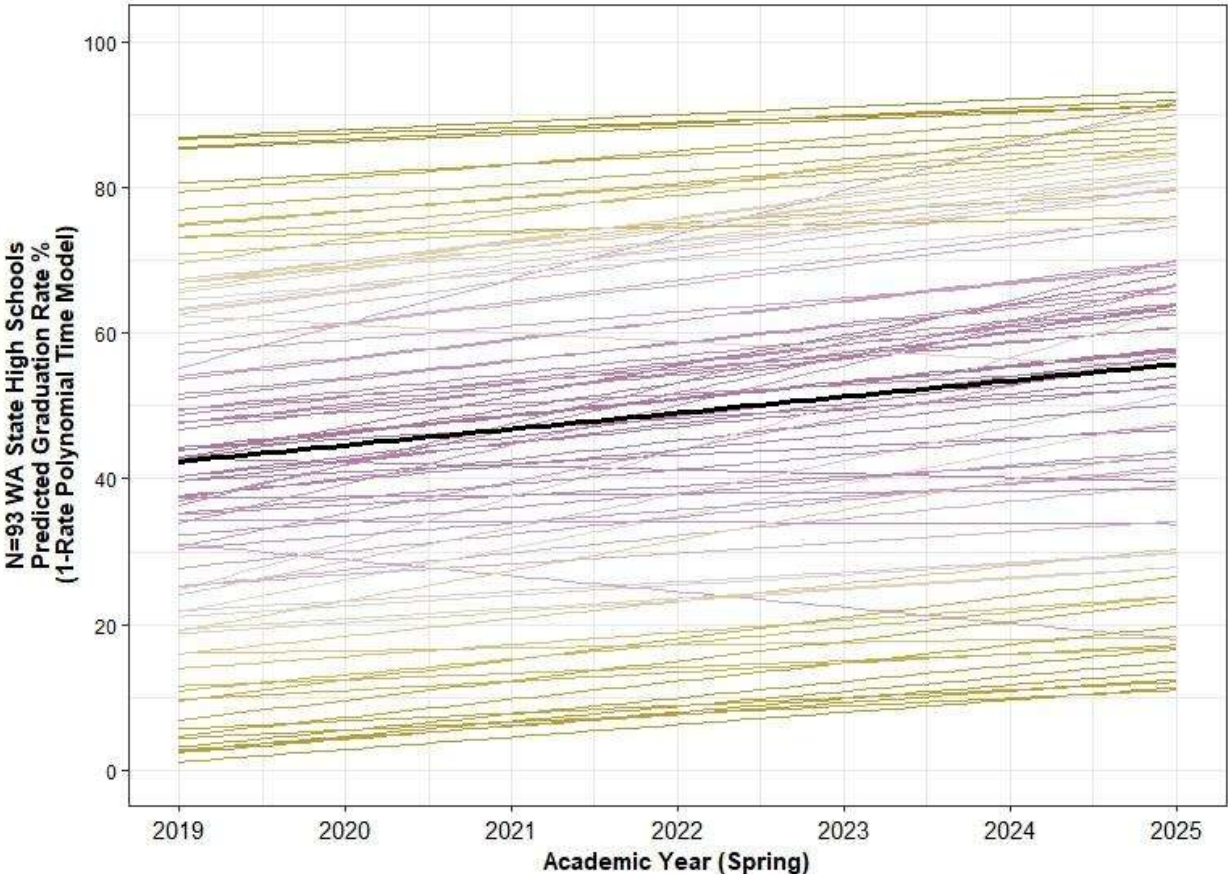


Figure 3

Model-Implied Graduation Rate Trajectories for Individual High Schools when Time is Coded as Pre-Post Pandemic (Piecewise)

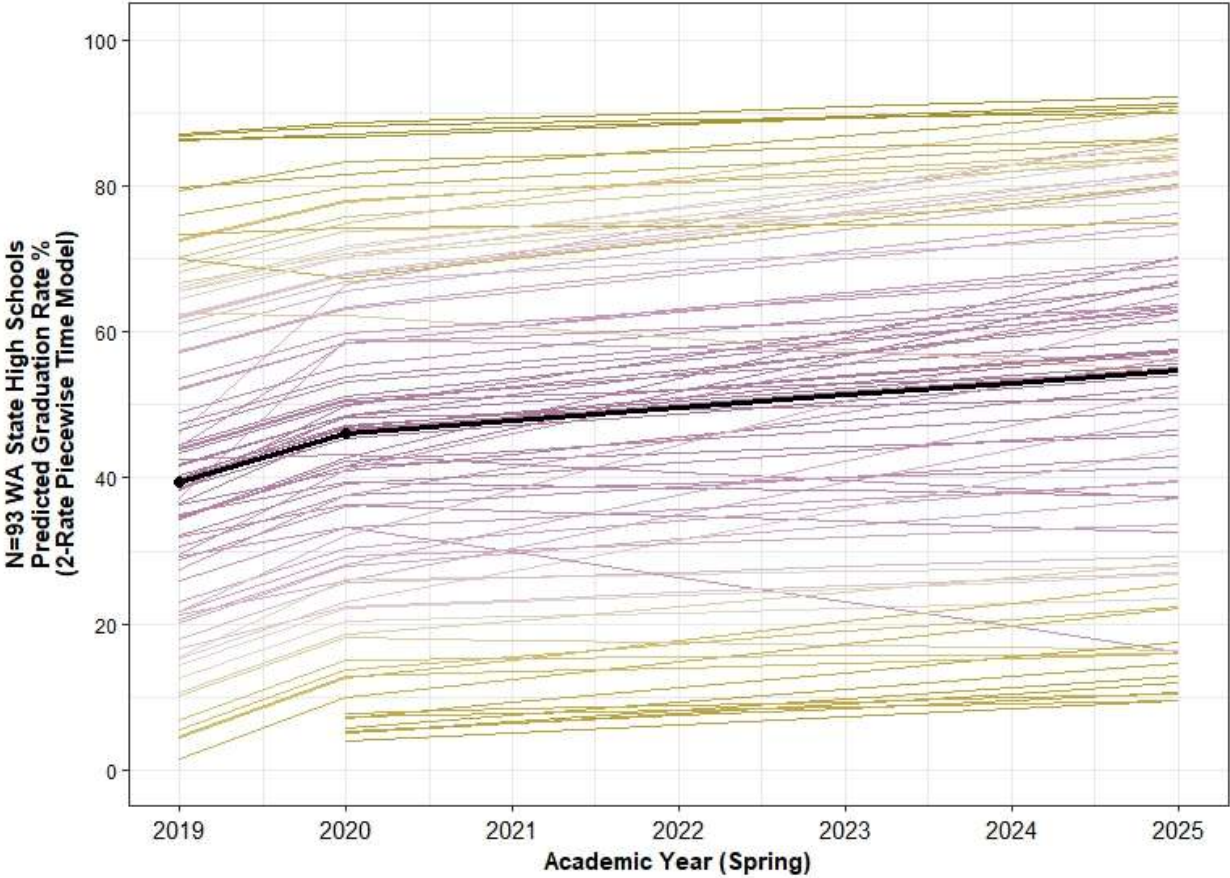


Figure 4

Model-Implied Graduation Rate Trajectories for Individual High Schools when Time is Coded Incrementally by Year (Piecewise)

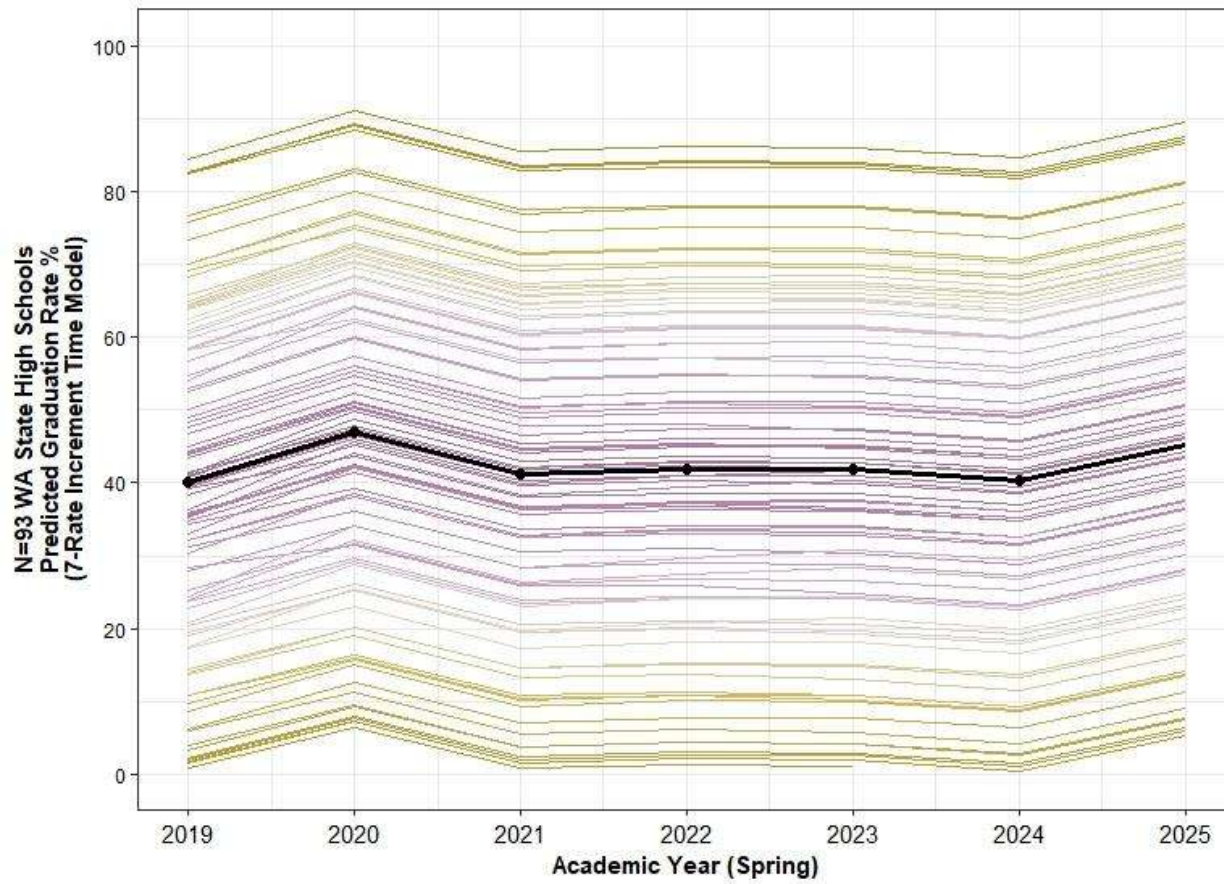


Figure 5

Model-Implied Mean Graduation Rate Trajectories for ESDs when Time is Coded as Linear Only

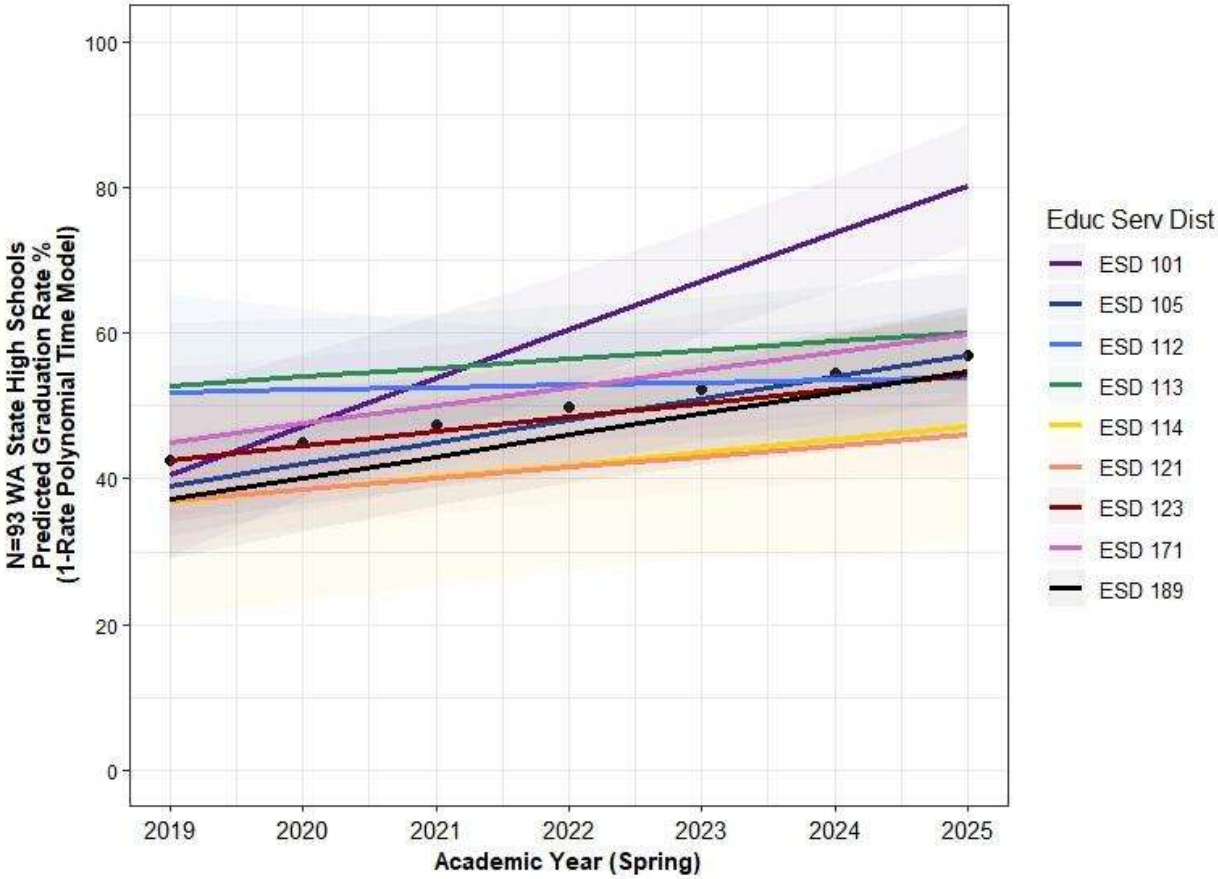


Figure 6

Model-Implied Mean Graduation Rate Trajectories for ESDs when Time is Coded as Pre-Post Pandemic (Piecewise)

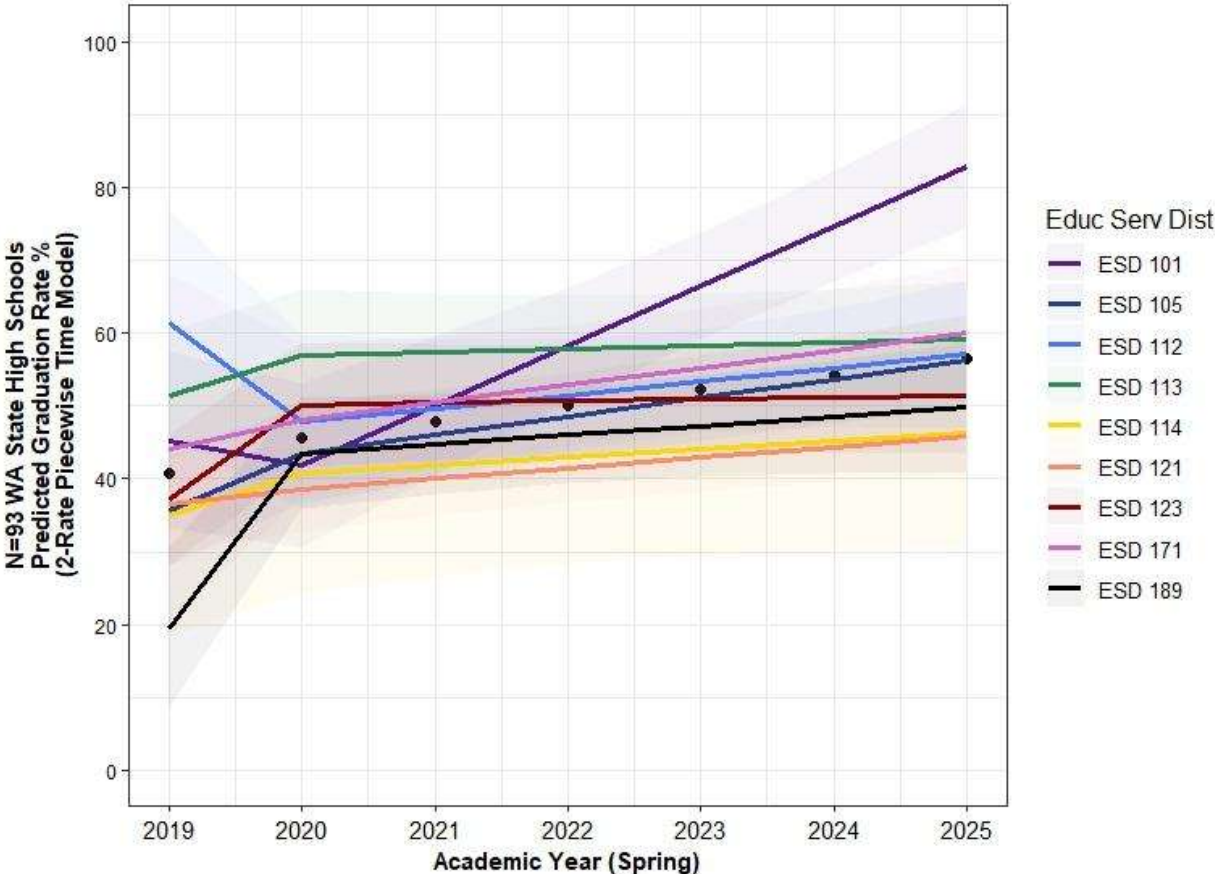


Figure 7

Model-Implied Mean Graduation Rate Trajectories for ESDs when Time is Coded Incrementally by Year (Piecewise)

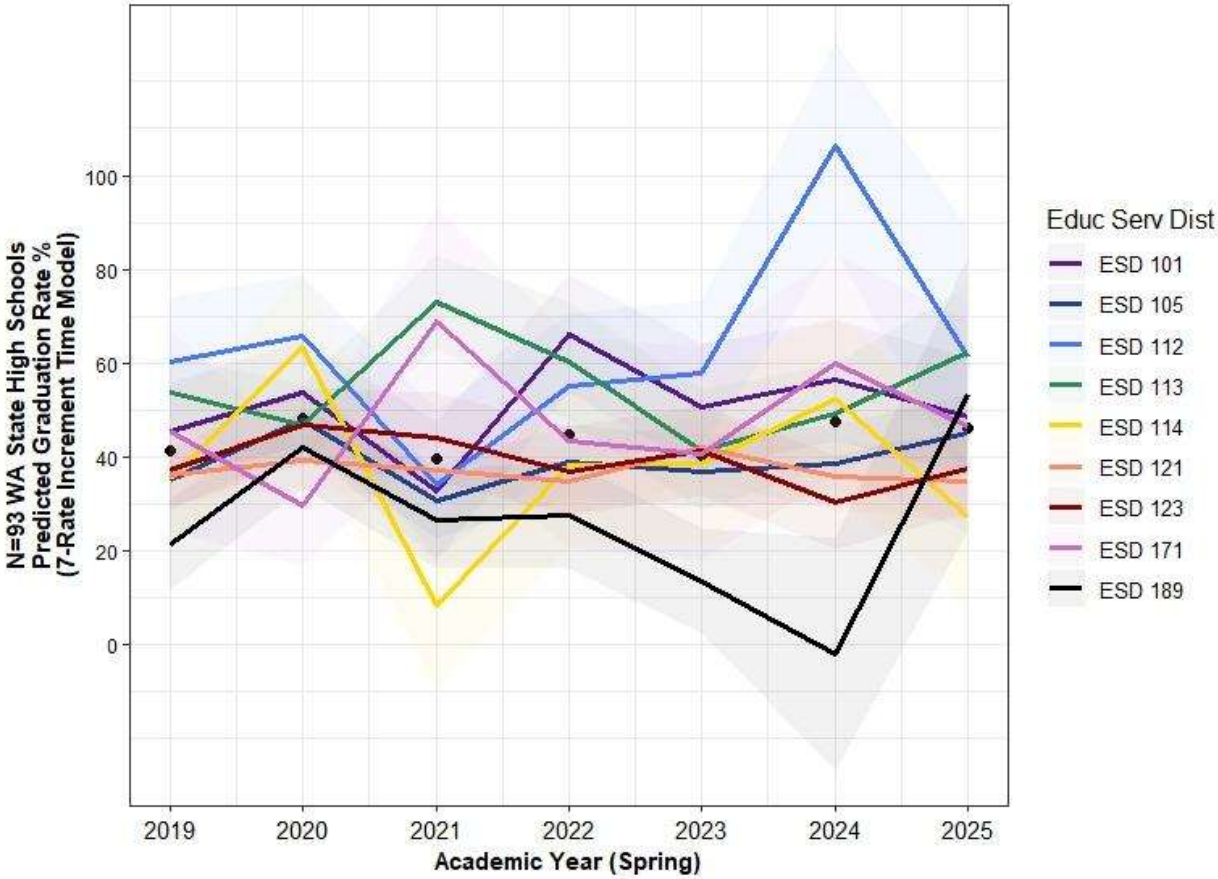


Figure 8

Model-Implied Mean Graduation Rate Trajectories by ESSA Funding Level when Time is Coded as Linear Only

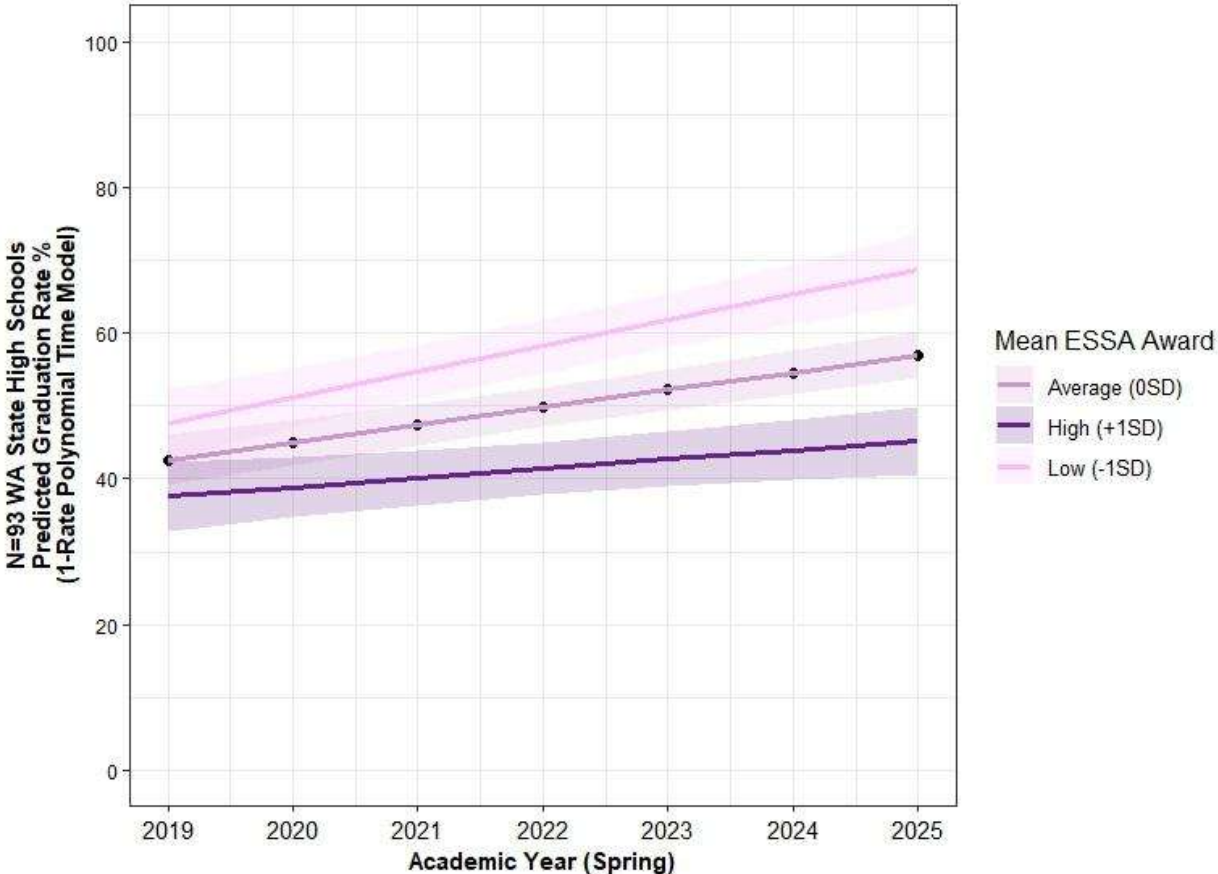


Figure 9

Model-Implied Mean Graduation Rate Trajectories by ESSA Funding Level when Time is Coded as Pre-Post Pandemic (Piecewise)

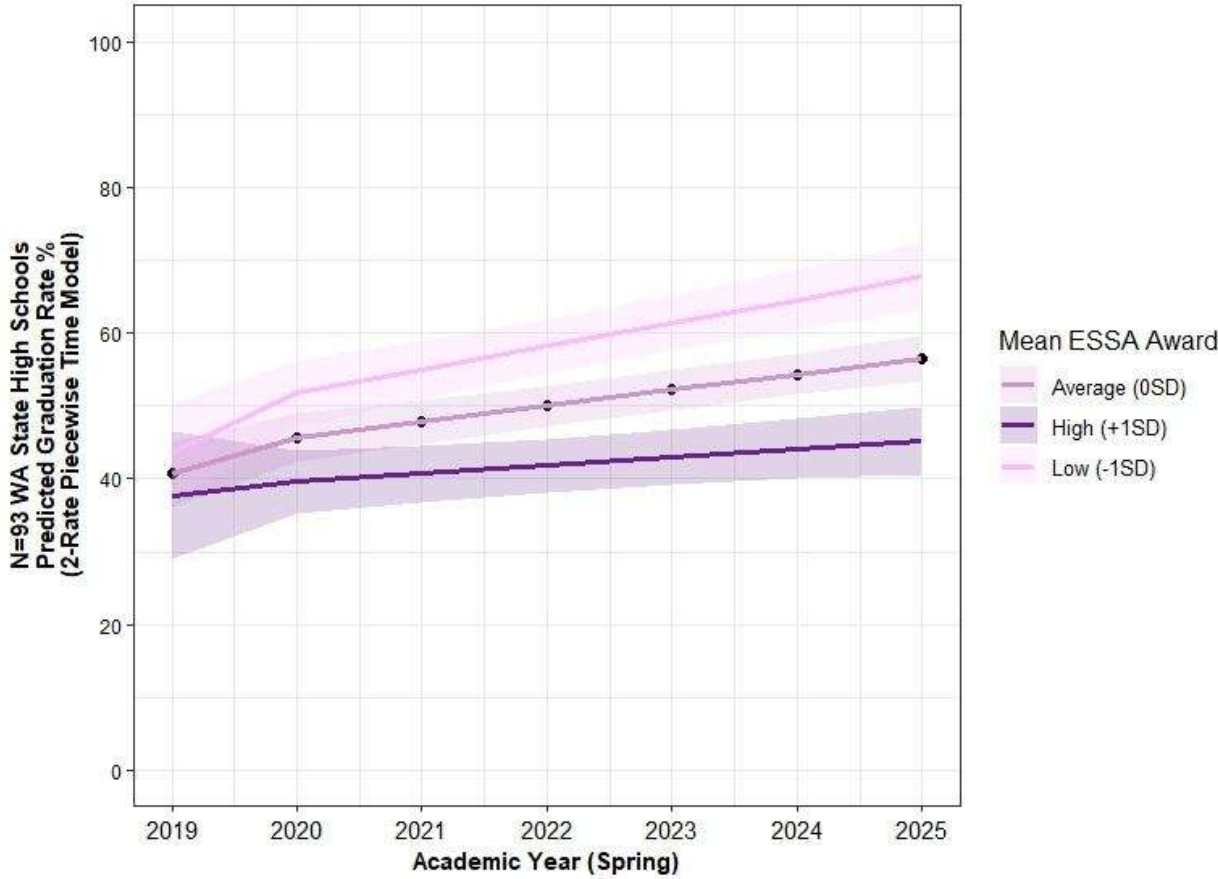
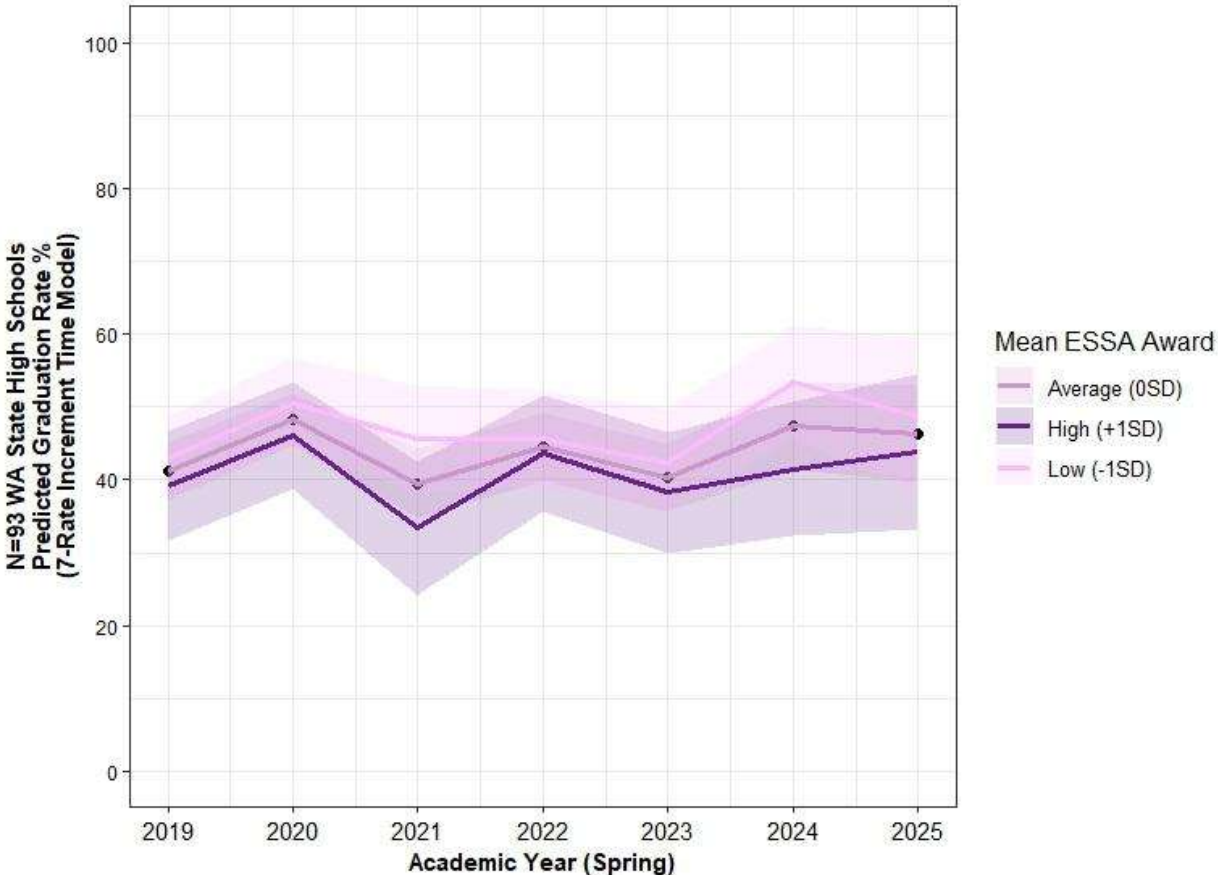


Figure 10

Model-Implied Mean Graduation Rate Trajectories by ESSA Funding Level when Time is Coded Incrementally by Year (Piecewise)



Appendix: Example R Code

```

### empty models ----
### note: we will use 2L models since little variance among districts
Grad_2L_m0 <- lmer(GraduationRate ~ (1|School.Code)
, data = alldata_long, REML = FALSE)
Grad_3L_m0 <- lmer(GraduationRate ~ (1|District.Code/School.Code)
, data = alldata_long, REML = FALSE)

summary(Grad_2L_m0)
summary(Grad_3L_m0) # very little district variance
anova(Grad_2L_m0, Grad_3L_m0, test="Chisq") #3L model not sig better than 2L

### growth models without predictors ----

#### polynomial ----
Grad_2L_poly_m1 <- lmer(GraduationRate ~ TimeLin # mean growth terms
+ (1 + TimeLin | School.Code) # variance in growth terms
, data = alldata_long, REML = FALSE, control =
lmerControl(optimizer="bobyqa"))

#### piecewise 2-rate ----
Grad_2L_pw_m1 <- lmer(GraduationRate ~ TimeRate_1
+ TimeRate_2 # mean growth terms
+ (1 + TimeRate_1 + TimeRate_2 | School.Code) # variance in
growth terms
, data = alldata_long, REML = FALSE, control = lmerControl(optimizer="bobyqa"))

#### categorical 7-rate inc/dec ----
Grad_2L_cat_m1 <- lmer(GraduationRate ~ TimeLin1 + TimeLin2 + TimeLin3
+ TimeLin4 + TimeLin5 + TimeLin6 # mean growth terms
+ (1 + TimeLin1 + TimeLin2 + TimeLin3 + TimeLin4
+ TimeLin5 + TimeLin6 || School.Code) # variance in growth terms
, data = alldata_long, REML = FALSE, control = lmerControl(optimizer="nloptwrap"))

#### model results ----
summary(Grad_2L_poly_m1) # sig linear growth
summary(Grad_2L_pw_m1) # sig rate1* and rate2** growth
summary(Grad_2L_cat_m1) # a lot of variance at time1, sig time1**

#### compare fit values ----
anova(Grad_2L_poly_m1, Grad_2L_pw_m1) # linear is better than pw
anova(Grad_2L_poly_m1, Grad_2L_cat_m1) # linear better than
categorical 7-rate inc/dec
anova(Grad_2L_pw_m1, Grad_2L_cat_m1) # pw 2-rate better than
categorical 7-rate inc/dec

### predictor models ----

#### polynomial linear ----
Grad_2L_poly_m2 <- lmer(GraduationRate ~ TimeLin # mean growth term
+ ZEnrollment_agg + ZX1003.Award_agg # L2 predictor effects on T1
+ ESD101_eff + ESD105_eff + ESD112_eff # L2 ESD predictor effects on T1
+ ESD113_eff # L2 ESD predictor effects on T1
+ ESD114_eff + ESD123_eff + ESD171_eff # L2 ESD predictor effects on T1
+ ESD189_eff # L2 ESD predictor effects on T1
+ TimeLin*ZEnrollment_agg # L2 predictor effects on growth parms
+ TimeLin*ZX1003.Award_agg # L2 predictor effects on growth parms
+ TimeLin*ESD101_eff # L2 ESD predictor effects on growth parms
+ TimeLin*ESD105_eff # L2 ESD predictor effects on growth parms
+ TimeLin*ESD112_eff # L2 ESD predictor effects on growth parms
+ TimeLin*ESD113_eff # L2 ESD predictor effects on growth parms
+ TimeLin*ESD114_eff # L2 ESD predictor effects on growth parms
+ TimeLin*ESD123_eff # L2 ESD predictor effects on growth parms
+ TimeLin*ESD171_eff # L2 ESD predictor effects on growth parms
+ TimeLin*ESD189_eff # L2 ESD predictor effects on growth parms
+ (1 + TimeLin | School.Code) # variance in growth terms
alldata_long, REML = FALSE, control = lmerControl(optimizer="bobyqa"))
, data =

```

```

#### piecewise 2-rate ----
Grad_2L_pw_m2 <- lmer(GraduationRate ~
TimeRate_1 + TimeRate_2 # mean growth terms
+ ZEnrollment_agg + ZX1003.Award_agg # L2 predictor effects on T1
+ ESD101_eff + ESD105_eff + ESD112_eff # L2 ESD predictor effects on T1
+ ESD113_eff + ESD114_eff + ESD123_eff # L2 ESD predictor effects on T1
+ ESD171_eff + ESD189_eff # L2 ESD predictor effects on T1
+ TimeRate_1*ZEnrollment_agg # L2 predictor effects on growth parms
+ TimeRate_2*ZEnrollment_agg # L2 predictor effects on growth parms
+ TimeRate_1*ZX1003.Award_agg # L2 predictor effects on growth parms
+ TimeRate_2*ZX1003.Award_agg # L2 predictor effects on growth parms
+ TimeRate_1*ESD101_eff # L2 ESD predictor effects on growth parms
+ TimeRate_2*ESD101_eff # L2 ESD predictor effects on growth parms
+ TimeRate_1*ESD105_eff # L2 ESD predictor effects on growth parms
+ TimeRate_2*ESD105_eff # L2 ESD predictor effects on growth parms
+ TimeRate_1*ESD112_eff # L2 ESD predictor effects on growth parms
+ TimeRate_2*ESD112_eff # L2 ESD predictor effects on growth parms
+ TimeRate_1*ESD113_eff # L2 ESD predictor effects on growth parms
+ TimeRate_2*ESD113_eff # L2 ESD predictor effects on growth parms
+ TimeRate_1*ESD114_eff # L2 ESD predictor effects on growth parms
+ TimeRate_2*ESD114_eff # L2 ESD predictor effects on growth parms
+ TimeRate_1*ESD123_eff # L2 ESD predictor effects on growth parms
+ TimeRate_2*ESD123_eff # L2 ESD predictor effects on growth parms
+ TimeRate_1*ESD171_eff # L2 ESD predictor effects on growth parms
+ TimeRate_2*ESD171_eff # L2 ESD predictor effects on growth parms
+ TimeRate_1*ESD189_eff # L2 ESD predictor effects on growth parms
+ TimeRate_2*ESD189_eff # L2 ESD predictor effects on growth parms
+ (1 + TimeRate_1 + TimeRate_2 | School.Code) # variance in growth terms
, data = alldata_long, REML = FALSE, control = lmerControl(optimizer =
"bobyqa"))

```

```

#### categorical 7-rate inc/dec ----
Grad_2L_cat_m2 <- lmer(GraduationRate ~ TimeLin1 + TimeLin2 + TimeLin3
+ TimeLin4 + TimeLin5 + TimeLin6 # mean growth terms
+ ZEnrollment_agg # L2 predictor effects on average across time
+ ZX1003.Award_agg # L2 predictor effects on average across time
+ ESD101_eff + ESD105_eff # L2 ESD predictor effects on average across time
+ ESD112_eff + ESD113_eff # L2 ESD predictor effects on average across time
+ ESD114_eff + ESD123_eff # L2 ESD predictor effects on average across time
+ ESD171_eff + ESD189_eff # L2 ESD predictor effects on average across time
+ TimeLin1*ZEnrollment_agg # L2 predictor effects on growth parms
+ TimeLin2*ZEnrollment_agg # L2 predictor effects on growth parms
+ TimeLin3*ZEnrollment_agg # L2 predictor effects on growth parms
+ TimeLin4*ZEnrollment_agg # L2 predictor effects on growth parms
+ TimeLin5*ZEnrollment_agg # L2 predictor effects on growth parms
+ TimeLin6*ZEnrollment_agg # L2 predictor effects on growth parms
+ TimeLin1*ZX1003.Award_agg # L2 predictor effects on growth parms
+ TimeLin2*ZX1003.Award_agg # L2 predictor effects on growth parms
+ TimeLin3*ZX1003.Award_agg # L2 predictor effects on growth parms
+ TimeLin4*ZX1003.Award_agg # L2 predictor effects on growth parms
+ TimeLin5*ZX1003.Award_agg # L2 predictor effects on growth parms
+ TimeLin6*ZX1003.Award_agg # L2 predictor effects on growth parms
+ TimeLin1*ESD101_eff # L2 ESD predictor effects on growth parms
+ TimeLin2*ESD101_eff # L2 ESD predictor effects on growth parms
+ TimeLin3*ESD101_eff # L2 ESD predictor effects on growth parms
+ TimeLin4*ESD101_eff # L2 ESD predictor effects on growth parms
+ TimeLin5*ESD101_eff # L2 ESD predictor effects on growth parms
+ TimeLin6*ESD101_eff # L2 ESD predictor effects on growth parms
+ TimeLin1*ESD105_eff # L2 ESD predictor effects on growth parms
+ TimeLin2*ESD105_eff # L2 ESD predictor effects on growth parms
+ TimeLin3*ESD105_eff # L2 ESD predictor effects on growth parms
+ TimeLin4*ESD105_eff # L2 ESD predictor effects on growth parms
+ TimeLin5*ESD105_eff # L2 ESD predictor effects on growth parms
+ TimeLin6*ESD105_eff # L2 ESD predictor effects on growth parms
+ TimeLin1*ESD112_eff # L2 ESD predictor effects on growth parms
+ TimeLin2*ESD112_eff # L2 ESD predictor effects on growth parms
+ TimeLin3*ESD112_eff # L2 ESD predictor effects on growth parms
+ TimeLin4*ESD112_eff # L2 ESD predictor effects on growth parms
+ TimeLin5*ESD112_eff # L2 ESD predictor effects on growth parms
+ TimeLin6*ESD112_eff # L2 ESD predictor effects on growth parms

```

```

+ TimeLin1*ESD113_eff      # L2 ESD predictor effects on growth parms
+ TimeLin2*ESD113_eff      # L2 ESD predictor effects on growth parms
+ TimeLin3*ESD113_eff      # L2 ESD predictor effects on growth parms
+ TimeLin4*ESD113_eff      # L2 ESD predictor effects on growth parms
+ TimeLin5*ESD113_eff      # L2 ESD predictor effects on growth parms
+ TimeLin6*ESD113_eff      # L2 ESD predictor effects on growth parms
+ TimeLin1*ESD114_eff      # L2 ESD predictor effects on growth parms
+ TimeLin2*ESD114_eff      # L2 ESD predictor effects on growth parms
+ TimeLin3*ESD114_eff      # L2 ESD predictor effects on growth parms
+ TimeLin4*ESD114_eff      # L2 ESD predictor effects on growth parms
+ TimeLin5*ESD114_eff      # L2 ESD predictor effects on growth parms
+ TimeLin6*ESD114_eff      # L2 ESD predictor effects on growth parms
+ TimeLin1*ESD123_eff      # L2 ESD predictor effects on growth parms
+ TimeLin2*ESD123_eff      # L2 ESD predictor effects on growth parms
+ TimeLin3*ESD123_eff      # L2 ESD predictor effects on growth parms
+ TimeLin4*ESD123_eff      # L2 ESD predictor effects on growth parms
+ TimeLin5*ESD123_eff      # L2 ESD predictor effects on growth parms
+ TimeLin6*ESD123_eff      # L2 ESD predictor effects on growth parms
+ TimeLin1*ESD171_eff      # L2 ESD predictor effects on growth parms
+ TimeLin2*ESD171_eff      # L2 ESD predictor effects on growth parms
+ TimeLin3*ESD171_eff      # L2 ESD predictor effects on growth parms
+ TimeLin4*ESD171_eff      # L2 ESD predictor effects on growth parms
+ TimeLin5*ESD171_eff      # L2 ESD predictor effects on growth parms
+ TimeLin6*ESD171_eff      # L2 ESD predictor effects on growth parms
+ TimeLin1*ESD189_eff      # L2 ESD predictor effects on growth parms
+ TimeLin2*ESD189_eff      # L2 ESD predictor effects on growth parms
+ TimeLin3*ESD189_eff      # L2 ESD predictor effects on growth parms
+ TimeLin4*ESD189_eff      # L2 ESD predictor effects on growth parms
+ TimeLin5*ESD189_eff      # L2 ESD predictor effects on growth parms
+ TimeLin6*ESD189_eff      # L2 ESD predictor effects on growth parms
+ (1 + TimeLin1 + TimeLin2 + TimeLin3 + TimeLin4 + TimeLin5 + TimeLin6 || School.Code) # variance
in growth terms
, data = alldata_long, REML = FALSE, control = lmerControl(optimizer =
"bobyqa"))

#### model summaries ----
summary(Grad_2L_poly_m2) #sig TimeLin*** and ZEnroll***
summary(Grad_2L_pw_m2) #sig TimeRate2** and Enroll***
summary(Grad_2L_cat_m2) #sig Enroll*** and TimeLin2:ESD113_eff**

### Model Comparisons with and without predictors
anova(Grad_2L_poly_m1,Grad_2L_poly_m2)
anova(Grad_2L_pw_m1,Grad_2L_pw_m2)
anova(Grad_2L_cat_m1,Grad_2L_cat_m2)

## Model-based data viz ----

### SCHOOL growth trajectories (m1) ----

#### (A) polynomial linear model ----

##### extract random effects ----
school_RE_data_poly <- as.data.frame(ranef(Grad_2L_poly_m1)$School.Code) # extract L2 random
effect comes after $
school_RE_data_poly$School.Code <- rownames(school_RE_data_poly) # assign IDs
colnames(school_RE_data_poly) <- c("RE_int","RE_time_lin","School.Code")

##### extract fixed effects ----
coef_int_poly <- coef(summary(Grad_2L_poly_m1))[1,1]
coef_time_lin_poly <- coef(summary(Grad_2L_poly_m1))[2,1]

##### compute schools' predicted values at each time point ----
school_RE_data_poly$time_0 <- (coef_int_poly + school_RE_data_poly$RE_int) +
(coef_time_lin_poly + school_RE_data_poly$RE_time_lin)*0
school_RE_data_poly$time_1 <- (coef_int_poly + school_RE_data_poly$RE_int) +
(coef_time_lin_poly + school_RE_data_poly$RE_time_lin)*1
school_RE_data_poly$time_2 <- (coef_int_poly + school_RE_data_poly$RE_int) +
(coef_time_lin_poly + school_RE_data_poly$RE_time_lin)*2
school_RE_data_poly$time_3 <- (coef_int_poly + school_RE_data_poly$RE_int) +

```

```

(coef_time_lin_poly + school_RE_data_poly$RE_time_lin)*3
school_RE_data_poly$time_4 <- (coef_int_poly + school_RE_data_poly$RE_int) +
(coef_time_lin_poly + school_RE_data_poly$RE_time_lin)*4
school_RE_data_poly$time_5 <- (coef_int_poly + school_RE_data_poly$RE_int) +
(coef_time_lin_poly + school_RE_data_poly$RE_time_lin)*5
school_RE_data_poly$time_6 <- (coef_int_poly + school_RE_data_poly$RE_int) +
(coef_time_lin_poly + school_RE_data_poly$RE_time_lin)*6

#### reshape predicted data wide to long ----
school_RE_data_poly_long = reshape(school_RE_data_poly,
                                   idvar = "School.Code",
                                   varying =
                                   list(c("time_0","time_1","time_2",
                                         "time_3","time_4", "time_5", "time_6")),
                                   v.names = c("Grad_Yhat"), # provide a name for DV_matrix (you
make it up)
                                   timevar = "time", # provide a name for index variable (you
make it up)
                                   direction = "long") # convert from wide to long
#### recode the 'time' variable into real time in years ----
# school_RE_data_poly_long$TimeLin <- car::recode(school_RE_data_poly_long$time, "1=0; 2=1; 3=2;
4=3; 5=4; 6=5; 7=6") # normal coding
school_RE_data_poly_long$TimeLin <- car::recode(school_RE_data_poly_long$time, "1=2019; 2=2020;
3=2021; 4=2022; 5=2023; 6=2024; 7=2025
                                   ") # for this project
#### create a categorical version of IDs for individual lines in the plot ----
school_RE_data_poly_long$school_asfactor <- as.factor(school_RE_data_poly_long$School.Code)
#### create trajectory deviation scores (can be done many ways; using the total of the random
effects, standardized) ----
school_RE_data_poly_long$sabs_Ztot_dev <- as.numeric(scale(abs(school_RE_data_poly_long$RE_int)+
abs(school_RE_data_poly_long$RE_time_lin)))

#### create predicted means dataset for mean line on top of individual trajectories ----
Grad_mean_traj_poly_0 <- c(coef_int_poly + coef_time_lin_poly*0)
Grad_mean_traj_poly_1 <- c(coef_int_poly + coef_time_lin_poly*1)
Grad_mean_traj_poly_2 <- c(coef_int_poly + coef_time_lin_poly*2)
Grad_mean_traj_poly_3 <- c(coef_int_poly + coef_time_lin_poly*3)
Grad_mean_traj_poly_4 <- c(coef_int_poly + coef_time_lin_poly*4)
Grad_mean_traj_poly_5 <- c(coef_int_poly + coef_time_lin_poly*5)
Grad_mean_traj_poly_6 <- c(coef_int_poly + coef_time_lin_poly*6)
Grad_mean_traj_poly <- as.data.frame(rbind(
  Grad_mean_traj_poly_0,
  Grad_mean_traj_poly_1,
  Grad_mean_traj_poly_2,
  Grad_mean_traj_poly_3,
  Grad_mean_traj_poly_4,
  Grad_mean_traj_poly_5,
  Grad_mean_traj_poly_6))
colnames(Grad_mean_traj_poly) <- c("Grad_Yhat") # name DV_matrix to match individual trajectory
data
# Grad_mean_traj_poly$TimeLin <- as.numeric(c(0,1,2,3,4)) # normal: add time variable to match
individual trajectory data
Grad_mean_traj_poly$TimeLin <- as.numeric(c(2019,2020,2021,2022,2023, 2024, 2025
                                   )) # for this project: add time variable to match
individual trajectory data

#### ggplot
ggplot(data=school_RE_data_poly_long, # predicted trajectory dataset
       aes(x=TimeLin, # time (always linear for x-axis)
           y=Grad_Yhat, # predicted values
           group = school_asfactor, # group must be categorical to get individual lines
           # color = abs_Ztot_dev)) + # color must be numeric when using scale_color_gradient2
below; can be categorical otherwise
       color = -abs_Ztot_dev)) + # reverse color must be numeric when using
scale_color_gradient2 below; can be categorical otherwise
       xlab("Academic Year (Spring)") + # x-axis label
       ylab("N=93 WA State High Schools\nPredicted Graduation Rate %\n(1-Rate Polynomial Time Model)")
+ # y-axis label
       geom_smooth(method = lm, formula = "y ~ x", se = FALSE, linewidth = .05, show.legend = FALSE) +
       scale_color_gradient2_tableau(palette = "Gold-Purple Diverging") + # UW

```

```

# scale_color_gradient2_tableau(palette = "Red-Black Diverging") + # WSU
# scale_color_gradient2_tableau(palette = "Green-Blue Diverging") + # Seahawks
# scale_color_gradient2_tableau(palette = "Orange-Blue Diverging") + # Summer vibes
# ?scale_color_gradient2_tableau # to see other colors in ggthemes
# MEAN LINE from means dataset
geom_smooth(method = lm, formula = "y ~ x", data = Grad_mean_traj_poly, se = FALSE,
            group = 1, linewidth = 1.25, color = "black", show.legend = FALSE) +
scale_y_continuous(limits=c(0,100), breaks=c(0,20,40,60,80,100)) +
scale_x_continuous(limits=c(2019,2025), breaks=c(2019,2020,2021,2022,2023,2024,2025)) +
theme_bw() + # this gives bordered plot with gridlines
theme(axis.text.x = element_text(color = "black", size = 10)) +
theme(axis.text.y = element_text(color = "black", size = 8)) +
theme(axis.title.x = element_text(color = "black", size=10, face="bold")) +
theme(axis.title.y = element_text(color = "black", size=10, face="bold"))
# last line of plot code cannot have a + at the end (the + is for next plot command)

#### (B) piecewise 2-rate model ----

##### extract random effects ----
school_RE_data_pw <- as.data.frame(ranef(Grad_2L_pw_m1)$School.Code) # extract L2 random effect
comes after $
school_RE_data_pw$School.Code <- rownames(school_RE_data_pw) # assign IDs
colnames(school_RE_data_pw) <- c("RE_int","RE_time_rate1","RE_time_rate2","School.Code")

##### extract fixed effects ----
coef_int_pw <- coef(summary(Grad_2L_pw_m1))[1,1]
coef_time_rate1_pw <- coef(summary(Grad_2L_pw_m1))[2,1]
coef_time_rate2_pw <- coef(summary(Grad_2L_pw_m1))[3,1]

##### compute schools' predicted values at each time point ----
##### recall how time was coded for the model:
##### alldata_long$TimeRate_1 <- as.numeric(car::recode(alldata_long$Time, "1=0; 2=1; 3=1; 4=1;
5=1; 6=1; 7=1"))
##### alldata_long$TimeRate_2 <- as.numeric(car::recode(alldata_long$Time, "1=0; 2=0; 3=1; 4=2;
5=3; 6=4; 7=5"))
school_RE_data_pw$time_0 <- (coef_int_pw + school_RE_data_pw$RE_int) +
(coef_time_rate1_pw + school_RE_data_pw$RE_time_rate1)*0 +
(coef_time_rate2_pw + school_RE_data_pw$RE_time_rate2)*0
school_RE_data_pw$time_1 <- (coef_int_pw + school_RE_data_pw$RE_int) +
(coef_time_rate1_pw + school_RE_data_pw$RE_time_rate1)*1 +
(coef_time_rate2_pw + school_RE_data_pw$RE_time_rate2)*0
school_RE_data_pw$time_2 <- (coef_int_pw + school_RE_data_pw$RE_int) +
(coef_time_rate1_pw + school_RE_data_pw$RE_time_rate1)*1 +
(coef_time_rate2_pw + school_RE_data_pw$RE_time_rate2)*1
school_RE_data_pw$time_3 <- (coef_int_pw + school_RE_data_pw$RE_int) +
(coef_time_rate1_pw + school_RE_data_pw$RE_time_rate1)*1 +
(coef_time_rate2_pw + school_RE_data_pw$RE_time_rate2)*2
school_RE_data_pw$time_4 <- (coef_int_pw + school_RE_data_pw$RE_int) +
(coef_time_rate1_pw + school_RE_data_pw$RE_time_rate1)*1 +
(coef_time_rate2_pw + school_RE_data_pw$RE_time_rate2)*3
school_RE_data_pw$time_5 <- (coef_int_pw + school_RE_data_pw$RE_int) +
(coef_time_rate1_pw + school_RE_data_pw$RE_time_rate1)*1 +
(coef_time_rate2_pw + school_RE_data_pw$RE_time_rate2)*4
school_RE_data_pw$time_6 <- (coef_int_pw + school_RE_data_pw$RE_int) +
(coef_time_rate1_pw + school_RE_data_pw$RE_time_rate1)*1 +
(coef_time_rate2_pw + school_RE_data_pw$RE_time_rate2)*5
##### reshape predicted data wide to long ----
school_RE_data_pw_long = reshape(school_RE_data_pw,
                                idvar = "School.Code",
                                varying =
list(c("time_0","time_1","time_2","time_3","time_4","time_5","time_6"
)),
                                v.names = c("Grad_Yhat"), # provide a name for DV_matrix (you
make it up)
                                timevar = "time", # provide a name for index variable (you
make it up)
                                direction = "long") # convert from wide to long
##### recode the 'time' variable into real time in years ----
# school_RE_data_pw_long$TimeLin <- car::recode(school_RE_data_pw_long$time, "1=0; 2=1; 3=2; 4=3;
5=4") # normal coding

```

```

school_RE_data_pw_long$TimeLin <- car::recode(school_RE_data_pw_long$time, "1=2019; 2=2020;
3=2021; 4=2022; 5=2023; 6=2024; 7=2025

") # for this project
##### create a categorical version of IDs for individual lines in the plot ----
school_RE_data_pw_long$school_asfactor <- as.factor(school_RE_data_pw_long$School.Code)
##### create trajectory deviation scores (can be done many ways; using the total of the random
effects, standardized) ----
school_RE_data_pw_long$abs_Ztot_dev <- as.numeric(scale(abs(school_RE_data_pw_long$RE_int)+
abs(school_RE_data_pw_long$RE_time_rate1)))

##### create predicted means dataset for mean line on top of individual trajectories ----
Grad_mean_traj_pw_0 <- c(coef_int_pw + coef_time_rate1_pw*0 + coef_time_rate2_pw*0)
Grad_mean_traj_pw_1 <- c(coef_int_pw + coef_time_rate1_pw*1 + coef_time_rate2_pw*0)
Grad_mean_traj_pw_2 <- c(coef_int_pw + coef_time_rate1_pw*1 + coef_time_rate2_pw*1)
Grad_mean_traj_pw_3 <- c(coef_int_pw + coef_time_rate1_pw*1 + coef_time_rate2_pw*2)
Grad_mean_traj_pw_4 <- c(coef_int_pw + coef_time_rate1_pw*1 + coef_time_rate2_pw*3)
Grad_mean_traj_pw_5 <- c(coef_int_pw + coef_time_rate1_pw*1 + coef_time_rate2_pw*4)
Grad_mean_traj_pw_6 <- c(coef_int_pw + coef_time_rate1_pw*1 + coef_time_rate2_pw*5)
Grad_mean_traj_pw <- as.data.frame(rbind(
  Grad_mean_traj_pw_0,
  Grad_mean_traj_pw_1,
  Grad_mean_traj_pw_2,
  Grad_mean_traj_pw_3,
  Grad_mean_traj_pw_4,
  Grad_mean_traj_pw_5,
  Grad_mean_traj_pw_6
))
colnames(Grad_mean_traj_pw) <- c("Grad_Yhat") # name DV_matrix to match individual trajectory
data
# Grad_mean_traj_pw$TimeLin <- as.numeric(c(0,1,2,3,4)) # normal: add time variable to match
individual trajectory data
Grad_mean_traj_pw$TimeLin <- as.numeric(c(2019,2020,2021,2022,2023,2024,2025
)) # for this project: add time variable to match
individual trajectory data

##### plot data ----
ggplot(data=school_RE_data_pw_long, # predicted trajectory dataset
  aes(x=TimeLin, # time (always linear for x-axis)
  y=Grad_Yhat, # predicted values
  group = school_asfactor, # group must be categorical to get individual lines
  # color = abs_Ztot_dev)) + # color must be numeric when using scale_color_gradient2
below; can be categorical otherwise
  color = -abs_Ztot_dev)) + # reverse color must be numeric when using
scale_color_gradient2 below; can be categorical otherwise
  xlab("Academic Year (Spring)") + # x-axis label
  ylab("N=93 WA State High Schools\nPredicted Graduation Rate %\n(2-Rate Piecewise Time Model)")
+ # y-axis label
  geom_line(linewidth = .05, show.legend = FALSE) + # for piecewise use geom_line() not
geom_smooth()
  scale_color_gradient2_tableau(palette = "Gold-Purple Diverging") + # UW
  # scale_color_gradient2_tableau(palette = "Red-Black Diverging") + # WSU
  # scale_color_gradient2_tableau(palette = "Green-Blue Diverging") + # Seahawks
  # scale_color_gradient2_tableau(palette = "Orange-Blue Diverging") + # Summer vibes
  # ?scale_color_gradient2_tableau # to see other colors in ggthemes
  # MEAN LINE from means dataset
  geom_line(data = Grad_mean_traj_pw, group = 1, linewidth = 1.25, color = "black", show.legend =
FALSE) + # for piecewise use geom_line() not geom_smooth()
  scale_y_continuous(limits=c(0,100), breaks=c(0,20,40,60,80,100)) +
  scale_x_continuous(limits=c(2019,2025), breaks=c(2019,2020,2021,2022,2023,2024, 2025)) +
  theme_bw() + # this gives bordered plot with gridlines
  theme(axis.text.x = element_text(color = "black", size = 10)) +
  theme(axis.text.y = element_text(color = "black", size = 8)) +
  theme(axis.title.x = element_text(color = "black", size=10, face="bold")) +
  theme(axis.title.y = element_text(color = "black", size=10, face="bold")) +
  geom_point(aes(x = 2019, y = Grad_mean_traj_pw_0), color = "black", size = 2) + # adding a dot
at breakpoints
  geom_point(aes(x = 2020, y = Grad_mean_traj_pw_1), color = "black", size = 2) # adding a dot at
breakpoints
# last line of plot code cannot have a + at the end (the + is for next plot command)

```

```

#### (C)categorical 7-rate inc/dec model ----

##### extract random effects ----
school_RE_data_cat <- as.data.frame(ranef(Grad_2L_cat_m1)$School.Code) # extract L2 random effect
comes after $
school_RE_data_cat$School.Code <- rownames(school_RE_data_cat) # assign IDs
colnames(school_RE_data_cat) <- c("RE_int", "RE_Time_Rate1", "RE_Time_Rate2",
"RE_Time_Rate3", "RE_Time_Rate4", "RE_Time_Rate5", "RE_Time_Rate6", "School.Code")

##### extract fixed effects ----
coef_int_cat <- coef(summary(Grad_2L_cat_m1))[1,1]
coef_time_rate1_cat <- coef(summary(Grad_2L_cat_m1))[2,1]
coef_time_rate2_cat <- coef(summary(Grad_2L_cat_m1))[3,1]
coef_time_rate3_cat <- coef(summary(Grad_2L_cat_m1))[4,1]
coef_time_rate4_cat <- coef(summary(Grad_2L_cat_m1))[5,1]
coef_time_rate5_cat <- coef(summary(Grad_2L_cat_m1))[6,1]
coef_time_rate6_cat <- coef(summary(Grad_2L_cat_m1))[7,1]

##### compute schools' predicted values at each time point ----
##### recall how time was coded for the model:
##### alldata_long$TimeLin1 <- car::recode(alldata_long$Time, "1=0; 2=1; 3=2; 4=3; 5=4")
##### alldata_long$TimeLin2 <- car::recode(alldata_long$Time, "1=0; 2=0; 3=1; 4=2; 5=3")
##### alldata_long$TimeLin3 <- car::recode(alldata_long$Time, "1=0; 2=0; 3=0; 4=1; 5=2")
##### alldata_long$TimeLin4 <- car::recode(alldata_long$Time, "1=0; 2=0; 3=0; 4=0; 5=1") and so
on
school_RE_data_cat$time_0 <-
  (coef_int_cat + school_RE_data_cat$RE_int)
school_RE_data_cat$time_1 <-
  school_RE_data_cat$time_0 +
  (coef_time_rate1_cat + school_RE_data_cat$RE_Time_Rate1)*(1)
school_RE_data_cat$time_2 <-
  school_RE_data_cat$time_1 +
  (coef_time_rate2_cat + school_RE_data_cat$RE_Time_Rate2)*(1)
school_RE_data_cat$time_3 <-
  school_RE_data_cat$time_2 +
  (coef_time_rate3_cat + school_RE_data_cat$RE_Time_Rate3)*(1)
school_RE_data_cat$time_4 <-
  school_RE_data_cat$time_3 +
  (coef_time_rate4_cat + school_RE_data_cat$RE_Time_Rate4)*(1)
school_RE_data_cat$time_5 <-
  school_RE_data_cat$time_4 +
  (coef_time_rate5_cat + school_RE_data_cat$RE_Time_Rate5)*(1)
school_RE_data_cat$time_6 <-
  school_RE_data_cat$time_5 +
  (coef_time_rate6_cat + school_RE_data_cat$RE_Time_Rate6)*(1)

##### reshape predicted data wide to long ----
school_RE_data_cat_long = reshape(school_RE_data_cat,
  idvar = "School.Code",
  varying = list(c("time_0", "time_1", "time_2", "time_3", "time_4",
"time_5", "time_6"
)),
  v.names = c("Grad_Yhat"), # provide a name for DV_matrix (you
make it up)
  timevar = "time", # provide a name for index variable (you
make it up)
  direction = "long") # convert from wide to long

##### recode the 'time' variable into real time in years ----
# school_RE_data_cat_long$TimeLin <- car::recode(school_RE_data_cat_long$time, "1=0; 2=1; 3=2;
4=3; 5=4") # normal coding
school_RE_data_cat_long$TimeLin <- car::recode(school_RE_data_cat_long$time, "1=2019; 2=2020;
3=2021; 4=2022; 5=2023; 6=2024; 7=2025
") # for this project

##### create a categorical version of IDs for individual lines in the plot ----
school_RE_data_cat_long$school_asfactor <- as.factor(school_RE_data_cat_long$School.Code)
##### create trajectory deviation scores (can be done many ways; using the total of the random
effects, standardized) ----
school_RE_data_cat_long$sabs_Ztot_dev <- as.numeric(scale(abs(school_RE_data_cat_long$RE_int)))

##### create predicted means dataset for mean line on top of individual trajectories ----

```

```

Grad_mean_traj_cat_0 <- c(coef_int_cat)
Grad_mean_traj_cat_1 <- c(Grad_mean_traj_cat_0 + coef_time_rate1_cat*(1))
Grad_mean_traj_cat_2 <- c(Grad_mean_traj_cat_1 + coef_time_rate2_cat*(1))
Grad_mean_traj_cat_3 <- c(Grad_mean_traj_cat_2 + coef_time_rate3_cat*(1))
Grad_mean_traj_cat_4 <- c(Grad_mean_traj_cat_3 + coef_time_rate4_cat*(1))
Grad_mean_traj_cat_5 <- c(Grad_mean_traj_cat_4 + coef_time_rate5_cat*(1))
Grad_mean_traj_cat_6 <- c(Grad_mean_traj_cat_5 + coef_time_rate6_cat*(1))

Grad_mean_traj_cat <- as.data.frame(rbind(
  Grad_mean_traj_cat_0,
  Grad_mean_traj_cat_1,
  Grad_mean_traj_cat_2,
  Grad_mean_traj_cat_3,
  Grad_mean_traj_cat_4,
  Grad_mean_traj_cat_5,
  Grad_mean_traj_cat_6
))
colnames(Grad_mean_traj_cat) <- c("Grad_Yhat") # name DV_matrix to match individual trajectory
data
# Grad_mean_traj_cat$TimeLin <- as.numeric(c(0,1,2,3,4)) # normal: add time variable to match
individual trajectory data
Grad_mean_traj_cat$TimeLin <- as.numeric(c(2019,2020,2021,2022,2023,2024,2025
)) # for this project: add time variable to match
individual trajectory data

##### plot data ----
ggplot(data=school_RE_data_cat_long, # predicted trajectory dataset
  aes(x=TimeLin, # time (always linear for x-axis)
    y=Grad_Yhat, # predicted values
    group = school_asfactor, # group must be categorical to get individual lines
    # color = abs_Ztot_dev)) + # color must be numeric when using scale_color_gradient2
below; can be categorical otherwise
  color = -abs_Ztot_dev)) + # reverse color must be numeric when using
scale_color_gradient2 below; can be categorical otherwise
  xlab("Academic Year (Spring)") + # x-axis label
  ylab("N=93 WA State High Schools\nPredicted Graduation Rate %\n(7-Rate Increment Time Model)")
+ # y-axis label
  geom_line(linewidth = .05, show.legend = FALSE) + # for categorical use geom_line() not
geom_smooth()
  scale_color_gradient2_tableau(palette = "Gold-Purple Diverging") + # UW
  # scale_color_gradient2_tableau(palette = "Red-Black Diverging") + # WSU
  # scale_color_gradient2_tableau(palette = "Green-Blue Diverging") + # Seahawks
  # scale_color_gradient2_tableau(palette = "Orange-Blue Diverging") + # Summer vibes
  # ?scale_color_gradient2_tableau # to see other colors in ggthemes
  # MEAN LINE from means dataset
  geom_line(data = Grad_mean_traj_cat, group = 1, linewidth = 1.25, color = "black", show.legend
= FALSE) + # for categorical use geom_line() not geom_smooth()
  scale_y_continuous(limits=c(0,100), breaks=c(0,20,40,60,80,100)) +
  scale_x_continuous(limits=c(2019,2025), breaks=c(2019,2020,2021,2022,2023,2024,2025
)) +
  theme_bw() + # this gives bordered plot with gridlines
  theme(axis.text.x = element_text(color = "black", size = 10)) +
  theme(axis.text.y = element_text(color = "black", size = 8)) +
  theme(axis.title.x = element_text(color = "black", size=10, face="bold")) +
  theme(axis.title.y = element_text(color = "black", size=10, face="bold")) +
  geom_point(aes(x = 2019, y = Grad_mean_traj_cat_0), color = "black", size = 2) + # adding a dot
at breakpoints
  geom_point(aes(x = 2020, y = Grad_mean_traj_cat_1), color = "black", size = 2) + # adding a dot
at breakpoints
  geom_point(aes(x = 2021, y = Grad_mean_traj_cat_2), color = "black", size = 2) + # adding a dot
at breakpoints
  geom_point(aes(x = 2022, y = Grad_mean_traj_cat_3), color = "black", size = 2) + # adding a dot
at breakpoints
  geom_point(aes(x = 2023, y = Grad_mean_traj_cat_4), color = "black", size = 2) + #adding a dot
at breakpoints
  geom_point(aes(x = 2024, y = Grad_mean_traj_cat_5), color = "black", size = 2) #+ # adding a
dot at breakpoints
  geom_point(aes(x = 2025, y = Grad_mean_traj_cat_6), color = "black", size = 2) # adding a dot
at breakpoints
# last line of plot code cannot have a + at the end (the + is for next plot command)

```

```

### ESD growth trajectories (m2) ----

#### (A) polynomial linear model ----

##### create data ----

##### define time coding for the 7 timepoints
time_points <- data.frame(
  TimeLin = c(0, 1, 2, 3, 4, 5, 6))
##### create all combinations with the 9 ESD categories
esd_combinations <- data.frame(
  ESD101_eff = c(1, 0, 0, 0, 0, 0, 0, 0, -1),
  ESD105_eff = c(0, 1, 0, 0, 0, 0, 0, 0, -1),
  ESD112_eff = c(0, 0, 1, 0, 0, 0, 0, 0, -1),
  ESD113_eff = c(0, 0, 0, 1, 0, 0, 0, 0, -1),
  ESD114_eff = c(0, 0, 0, 0, 1, 0, 0, 0, -1),
  ESD123_eff = c(0, 0, 0, 0, 0, 1, 0, 0, -1),
  ESD171_eff = c(0, 0, 0, 0, 0, 0, 1, 0, -1),
  ESD189_eff = c(0, 0, 0, 0, 0, 0, 0, 1, -1)
)
##### create the exact combinations of time x ESD
pred_grid <- expand.grid(
  timepoint = 1:7,
  esd = 1:9
)
pred_data <- data.frame(
  TimeLin = time_points$TimeLin[pred_grid$timepoint],
  ESD101_eff = esd_combinations$ESD101_eff[pred_grid$esd],
  ESD105_eff = esd_combinations$ESD105_eff[pred_grid$esd],
  ESD112_eff = esd_combinations$ESD112_eff[pred_grid$esd],
  ESD113_eff = esd_combinations$ESD113_eff[pred_grid$esd],
  ESD114_eff = esd_combinations$ESD114_eff[pred_grid$esd],
  ESD123_eff = esd_combinations$ESD123_eff[pred_grid$esd],
  ESD171_eff = esd_combinations$ESD171_eff[pred_grid$esd],
  ESD189_eff = esd_combinations$ESD189_eff[pred_grid$esd],
  ZEnrollment_agg = 0,
  ZX1003.Award_agg = 0
)
##### create model matrix
X <- model.matrix(~ TimeLin +
  ZEnrollment_agg + ZX1003.Award_agg +
  ESD101_eff + ESD105_eff + ESD112_eff + ESD113_eff +
  ESD114_eff + ESD123_eff + ESD171_eff + ESD189_eff +
  TimeLin:ZEnrollment_agg +
  TimeLin:ZX1003.Award_agg +
  TimeLin:ESD101_eff +
  TimeLin:ESD105_eff +
  TimeLin:ESD112_eff +
  TimeLin:ESD113_eff +
  TimeLin:ESD114_eff +
  TimeLin:ESD123_eff +
  TimeLin:ESD171_eff +
  TimeLin:ESD189_eff,
  data = pred_data)
##### get predictions
gamma_hat <- fixef(Grad_2L_poly_m2)
V_matrix <- vcov(Grad_2L_poly_m2)
pred_means <- X %*% gamma_hat
pred_se <- sqrt(diag(X %*% V_matrix %*% t(X)))
##### calculate Satterthwaite df using calcSatterth directly
L <- X # Our contrast matrix (45 rows)
##### use the internal function from lmerTest
##### ensure correct lmer object class
if(!inherits(Grad_2L_poly_m2, "lmerModLmerTest")) {
  Grad_2L_poly_m2 <- as(Grad_2L_poly_m2, "lmerModLmerTest")
}
##### calculate df for each row
df_satterthwaite <- numeric(nrow(X))
for(i in 1:nrow(X)) {
  # use contestMD which works better for individual contrasts

```

```

L_single <- X[i, , drop = FALSE]
# try using contest1D
tryCatch({
  # get Satterthwaite approximation
  satt_result <- contest1D(Grad_2L_poly_m2, L_single, rhs = 0, confint = TRUE, level = 0.95)
  df_satterthwaite[i] <- satt_result$df
}, error = function(e) {
  # fallback to summary method
  df_satterthwaite[i] <-<- summary(Grad_2L_poly_m2)$coefficients[1, "df"]
})
}
##### create results with Satterthwaite df
poly_m2_Yhats <- data.frame(
  timepoint = pred_grid$timepoint,
  esd = pred_grid$esd,
  TimeLin = pred_data$TimeLin,
  ESD101_eff = pred_data$ESD101_eff,
  ESD105_eff = pred_data$ESD105_eff,
  ESD112_eff = pred_data$ESD112_eff,
  ESD113_eff = pred_data$ESD113_eff,
  ESD114_eff = pred_data$ESD114_eff,
  ESD123_eff = pred_data$ESD123_eff,
  ESD171_eff = pred_data$ESD171_eff,
  ESD189_eff = pred_data$ESD189_eff,
  emmean = as.vector(pred_means),
  SE = pred_se,
  df = df_satterthwaite,
  lower.CL = as.vector(pred_means) - qt(0.975, df_satterthwaite) * pred_se,
  upper.CL = as.vector(pred_means) + qt(0.975, df_satterthwaite) * pred_se
)
View(poly_m2_Yhats) # see data
str(poly_m2_Yhats)

##### plot data ----

poly_m2_Yhats$TimeLin <- as.numeric(car::recode(poly_m2_Yhats$timepoint,
"1=2019;2=2020;3=2021;4=2022;5=2023;6=2024;7=2025
      "))
poly_m2_Yhats$ESD_asfactor <- car::recode(poly_m2_Yhats$esd, " 1='ESD 101'; 2='ESD 105'; 3='ESD
112'; 4='ESD 113';
5='ESD 114'; 6='ESD 123'; 7='ESD 171'; 8='ESD 189'; 9='ESD 121' ")
poly_m2_Yhat_2019 <- mean(poly_m2_Yhats$emmean[poly_m2_Yhats$timepoint == 1])
poly_m2_Yhat_2020 <- mean(poly_m2_Yhats$emmean[poly_m2_Yhats$timepoint == 2])
poly_m2_Yhat_2021 <- mean(poly_m2_Yhats$emmean[poly_m2_Yhats$timepoint == 3])
poly_m2_Yhat_2022 <- mean(poly_m2_Yhats$emmean[poly_m2_Yhats$timepoint == 4])
poly_m2_Yhat_2023 <- mean(poly_m2_Yhats$emmean[poly_m2_Yhats$timepoint == 5])
poly_m2_Yhat_2024 <- mean(poly_m2_Yhats$emmean[poly_m2_Yhats$timepoint == 6])
poly_m2_Yhat_2025 <- mean(poly_m2_Yhats$emmean[poly_m2_Yhats$timepoint == 7])

ggplot(data=poly_m2_Yhats,
  aes(x=TimeLin,
    y=emmean,
    fill = ESD_asfactor,
    color = ESD_asfactor)) +
  geom_point(aes(x = 2019, y = poly_m2_Yhat_2019), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2020, y = poly_m2_Yhat_2020), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2021, y = poly_m2_Yhat_2021), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2022, y = poly_m2_Yhat_2022), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2023, y = poly_m2_Yhat_2023), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2024, y = poly_m2_Yhat_2024), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2025, y = poly_m2_Yhat_2025), color = "black", size = 2, show.legend =
FALSE) +
  geom_ribbon(aes(ymin = emmean - SE,
    ymax = emmean + SE),
    alpha = 0.05, # transparency (0.2-0.3 works well)

```

```

        color = NA,                # remove border around ribbon
        show.legend = TRUE) + # usually don't need ribbon in legend
geom_line(linewidth = 1, show.legend = TRUE) +
xlab("Academic Year (Spring)") +
ylab("N=93 WA State High Schools\nPredicted Graduation Rate %\n(1-Rate Polynomial Time Model)")
+
labs(color = "Educ Serv Dist",
      fill = "Educ Serv Dist") +
scale_color_manual(values = c("purple4", "royalblue4", "royalblue1", "seagreen", "gold1",
                              "salmon1", "darkred", "orchid3", "black")) +
scale_fill_manual(values = c("purple4", "royalblue4", "royalblue1", "seagreen", "gold1",
                              "salmon1", "darkred", "orchid3", "black")) +
scale_y_continuous(limits=c(0,100), breaks=c(0,20,40,60,80,100)) +
scale_x_continuous(limits=c(2019,2025), breaks=c(2019,2020,2021,2022,2023,2024,2025)) +
theme_bw() +
theme(axis.text.x = element_text(color = "black", size = 10)) +
theme(axis.text.y = element_text(color = "black", size = 8)) +
theme(axis.title.x = element_text(color = "black", size=10, face="bold")) +
theme(axis.title.y = element_text(color = "black", size=10, face="bold"))
# last line of plot code cannot have a + at the end (the + is for next plot command)

#### (B) piecewise 2-rate model ----

##### create data ----

##### define time coding for the 7 timepoints
time_points <- data.frame(
  TimeRate_1 = c(0, 1, 1, 1, 1, 1, 1),
  TimeRate_2 = c(0, 0, 1, 2, 3, 4, 5)
)

##### create all combinations with the 9 ESD categories
esd_combinations <- data.frame(
  ESD101_eff = c(1, 0, 0, 0, 0, 0, 0, 0, -1),
  ESD105_eff = c(0, 1, 0, 0, 0, 0, 0, 0, -1),
  ESD112_eff = c(0, 0, 1, 0, 0, 0, 0, 0, -1),
  ESD113_eff = c(0, 0, 0, 1, 0, 0, 0, 0, -1),
  ESD114_eff = c(0, 0, 0, 0, 1, 0, 0, 0, -1),
  ESD123_eff = c(0, 0, 0, 0, 0, 1, 0, 0, -1),
  ESD171_eff = c(0, 0, 0, 0, 0, 0, 1, 0, -1),
  ESD189_eff = c(0, 0, 0, 0, 0, 0, 0, 1, -1)
)

##### create the exact combinations of time x ESD
pred_grid <- expand.grid(
  timepoint = 1:7,
  esd = 1:9
)

pred_data <- data.frame(
  TimeRate_1 = time_points$TimeRate_1[pred_grid$timepoint],
  TimeRate_2 = time_points$TimeRate_2[pred_grid$timepoint],
  ESD101_eff = esd_combinations$ESD101_eff[pred_grid$esd],
  ESD105_eff = esd_combinations$ESD105_eff[pred_grid$esd],
  ESD112_eff = esd_combinations$ESD112_eff[pred_grid$esd],
  ESD113_eff = esd_combinations$ESD113_eff[pred_grid$esd],
  ESD114_eff = esd_combinations$ESD114_eff[pred_grid$esd],
  ESD123_eff = esd_combinations$ESD123_eff[pred_grid$esd],
  ESD171_eff = esd_combinations$ESD171_eff[pred_grid$esd],
  ESD189_eff = esd_combinations$ESD189_eff[pred_grid$esd],
  ZEnrollment_agg = 0,
  ZX1003.Award_agg = 0
)

##### create model matrix
X <- model.matrix(~ TimeRate_1 + TimeRate_2 +
                  ZEnrollment_agg + ZX1003.Award_agg +
                  ESD101_eff + ESD105_eff + ESD112_eff + ESD113_eff +
                  ESD114_eff + ESD123_eff + ESD171_eff + ESD189_eff +
                  TimeRate_1:ZEnrollment_agg + TimeRate_2:ZEnrollment_agg +
                  TimeRate_1:ZX1003.Award_agg + TimeRate_2:ZX1003.Award_agg +
                  TimeRate_1:ESD101_eff + TimeRate_2:ESD101_eff +

```

```

TimeRate_1:ESD105_eff + TimeRate_2:ESD105_eff +
TimeRate_1:ESD112_eff + TimeRate_2:ESD112_eff +
TimeRate_1:ESD113_eff + TimeRate_2:ESD113_eff +
TimeRate_1:ESD114_eff + TimeRate_2:ESD114_eff +
TimeRate_1:ESD123_eff + TimeRate_2:ESD123_eff +
TimeRate_1:ESD171_eff + TimeRate_2:ESD171_eff +
TimeRate_1:ESD189_eff + TimeRate_2:ESD189_eff,
data = pred_data)

##### get predictions
gamma_hat <- fixef(Grad_2L_pw_m2)
V_matrix <- vcov(Grad_2L_pw_m2)
pred_means <- X %*% gamma_hat
pred_se <- sqrt(diag(X %*% V_matrix %*% t(X)))

##### calculate Satterthwaite df using calcSatterth directly
L <- X # Our contrast matrix (45 rows)
##### use the internal function from lmerTest
##### ensure correct lmer object class
if(!inherits(Grad_2L_pw_m2, "lmerModLmerTest")) {
  Grad_2L_pw_m2 <- as(Grad_2L_pw_m2, "lmerModLmerTest")
}
##### calculate df for each row
df_satterthwaite <- numeric(nrow(X))
for(i in 1:nrow(X)) {
  # use contestMD which works better for individual contrasts
  L_single <- X[i, , drop = FALSE]
  # try using contest1D
  tryCatch({
    # get Satterthwaite approximation
    satt_result <- contest1D(Grad_2L_pw_m2, L_single, rhs = 0, confint = TRUE, level = 0.95)
    df_satterthwaite[i] <- satt_result$df
  }, error = function(e) {
    # fallback to summary method
    df_satterthwaite[i] <- summary(Grad_2L_pw_m2)$coefficients[1, "df"]
  })
}
##### create results with Satterthwaite df
pw_m2_Yhats <- data.frame(
  timepoint = pred_grid$timepoint,
  esd = pred_grid$esd,
  TimeRate_1 = pred_data$TimeRate_1,
  TimeRate_2 = pred_data$TimeRate_2,
  ESD101_eff = pred_data$ESD101_eff,
  ESD105_eff = pred_data$ESD105_eff,
  ESD112_eff = pred_data$ESD112_eff,
  ESD113_eff = pred_data$ESD113_eff,
  ESD114_eff = pred_data$ESD114_eff,
  ESD123_eff = pred_data$ESD123_eff,
  ESD171_eff = pred_data$ESD171_eff,
  ESD189_eff = pred_data$ESD189_eff,
  emmean = as.vector(pred_means),
  SE = pred_se,
  df = df_satterthwaite,
  lower.CL = as.vector(pred_means) - qt(0.975, df_satterthwaite) * pred_se,
  upper.CL = as.vector(pred_means) + qt(0.975, df_satterthwaite) * pred_se
)
pw_m2_Yhats

##### plot data ----

pw_m2_Yhats$TimeLin <- as.numeric(car::recode(pw_m2_Yhats$timepoint,
"1=2019;2=2020;3=2021;4=2022;5=2023; 6=2024; 7=2025
"))
pw_m2_Yhats$ESD_asfactor <- car::recode(pw_m2_Yhats$esd, " 1='ESD 101'; 2='ESD 105'; 3='ESD 112';
4='ESD 113';
5='ESD 114'; 6='ESD 123'; 7='ESD 171'; 8='ESD 189'; 9='ESD 121' ")
pw_m2_Yhat_2019 <- mean(pw_m2_Yhats$emmean[pw_m2_Yhats$timepoint == 1])
pw_m2_Yhat_2020 <- mean(pw_m2_Yhats$emmean[pw_m2_Yhats$timepoint == 2])
pw_m2_Yhat_2021 <- mean(pw_m2_Yhats$emmean[pw_m2_Yhats$timepoint == 3])
pw_m2_Yhat_2022 <- mean(pw_m2_Yhats$emmean[pw_m2_Yhats$timepoint == 4])

```

```

pw_m2_Yhat_2023 <- mean(pw_m2_Yhats$emmean[pw_m2_Yhats$timepoint == 5])
pw_m2_Yhat_2024 <- mean(pw_m2_Yhats$emmean[pw_m2_Yhats$timepoint == 6])
pw_m2_Yhat_2025 <- mean(pw_m2_Yhats$emmean[pw_m2_Yhats$timepoint == 7])

ggplot(data=pw_m2_Yhats,
       aes(x=TimeLin,
           y=emmean,
           fill = ESD_asfactor,
           color = ESD_asfactor)) +
  geom_point(aes(x = 2019, y = pw_m2_Yhat_2019), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2020, y = pw_m2_Yhat_2020), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2021, y = pw_m2_Yhat_2021), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2022, y = pw_m2_Yhat_2022), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2023, y = pw_m2_Yhat_2023), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2024, y = pw_m2_Yhat_2024), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2025, y = pw_m2_Yhat_2025), color = "black", size = 2, show.legend = FALSE)
+
  geom_ribbon(aes(ymin = emmean - SE,
                ymax = emmean + SE),
            alpha = 0.05,
            color = NA,
            show.legend = TRUE) + # Keep ribbon in legend
  geom_line(linewidth = 1, show.legend = TRUE) +
  xlab("Academic Year (Spring)") +
  ylab("N=93 WA State High Schools\nPredicted Graduation Rate %\n(2-Rate Piecewise Time Model)")
+
  labs(color = "Educ Serv Dist",
       fill = "Educ Serv Dist") +
  scale_color_manual(values = c("purple4", "royalblue4", "royalblue1", "seagreen", "gold1",
                                "salmon1", "darkred", "orchid3", "black")) +
  scale_fill_manual(values = c("purple4", "royalblue4", "royalblue1", "seagreen", "gold1",
                                "salmon1", "darkred", "orchid3", "black")) +
  scale_y_continuous(limits=c(0,100), breaks=c(0,20,40,60,80,100)) +
  scale_x_continuous(limits=c(2019,2025), breaks=c(2019,2020,2021,2022,2023,2024,2025)) +
  theme_bw() +
  theme(axis.text.x = element_text(color = "black", size = 10)) +
  theme(axis.text.y = element_text(color = "black", size = 8)) +
  theme(axis.title.x = element_text(color = "black", size=10, face="bold")) +
  theme(axis.title.y = element_text(color = "black", size=10, face="bold"))
# last line of plot code cannot have a + at the end (the + is for next plot command)

#### (C) categorical 7-rate inc/dec model ----

##### create data ----

##### define time coding for the 7 timepoints
time_points <- data.frame(
  TimeLin1 = c(0, 1, 1, 1, 1, 1, 1),
  TimeLin2 = c(0, 0, 1, 1, 1, 1, 1),
  TimeLin3 = c(0, 0, 0, 1, 1, 1, 1),
  TimeLin4 = c(0, 0, 0, 0, 1, 1, 1),
  TimeLin5 = c(0, 0, 0, 0, 0, 1, 1),
  TimeLin6 = c(0, 0, 0, 0, 0, 0, 1)
)

##### create all combinations with the 9 ESD categories
esd_combinations <- data.frame(
  ESD101_eff = c(1, 0, 0, 0, 0, 0, 0, 0, -1),
  ESD105_eff = c(0, 1, 0, 0, 0, 0, 0, 0, -1),
  ESD112_eff = c(0, 0, 1, 0, 0, 0, 0, 0, -1),
  ESD113_eff = c(0, 0, 0, 1, 0, 0, 0, 0, -1),
  ESD114_eff = c(0, 0, 0, 0, 1, 0, 0, 0, -1),
  ESD123_eff = c(0, 0, 0, 0, 0, 1, 0, 0, -1),
  ESD171_eff = c(0, 0, 0, 0, 0, 0, 1, 0, -1),

```

```

ESD189_eff = c(0, 0, 0, 0, 0, 0, 0, 0, 1, -1)
)
##### create the exact combinations of time x ESD
pred_grid <- expand.grid(
  timepoint = 1:7,
  esd = 1:9
)
pred_data <- data.frame(
  TimeLin1 = time_points$TimeLin1[pred_grid$timepoint],
  TimeLin2 = time_points$TimeLin2[pred_grid$timepoint],
  TimeLin3 = time_points$TimeLin3[pred_grid$timepoint],
  TimeLin4 = time_points$TimeLin4[pred_grid$timepoint],
  TimeLin5 = time_points$TimeLin5[pred_grid$timepoint],
  TimeLin6 = time_points$TimeLin6[pred_grid$timepoint],
  ESD101_eff = esd_combinations$ESD101_eff[pred_grid$esd],
  ESD105_eff = esd_combinations$ESD105_eff[pred_grid$esd],
  ESD112_eff = esd_combinations$ESD112_eff[pred_grid$esd],
  ESD113_eff = esd_combinations$ESD113_eff[pred_grid$esd],
  ESD114_eff = esd_combinations$ESD114_eff[pred_grid$esd],
  ESD123_eff = esd_combinations$ESD123_eff[pred_grid$esd],
  ESD171_eff = esd_combinations$ESD171_eff[pred_grid$esd],
  ESD189_eff = esd_combinations$ESD189_eff[pred_grid$esd],
  ZEnrollment_agg = 0,
  ZX1003.Award_agg = 0
)

##### create model matrix
X <- model.matrix(~ TimeLin1 + TimeLin2 + TimeLin3 + TimeLin4 + TimeLin5 + TimeLin6 +
  ZEnrollment_agg + ZX1003.Award_agg +
  ESD101_eff + ESD105_eff + ESD112_eff + ESD113_eff +
  ESD114_eff + ESD123_eff + ESD171_eff + ESD189_eff +
  TimeLin1:ZEnrollment_agg + TimeLin2:ZEnrollment_agg +
  TimeLin3:ZEnrollment_agg + TimeLin4:ZEnrollment_agg +
  TimeLin5:ZEnrollment_agg + TimeLin6:ZEnrollment_agg +
  TimeLin1:ZX1003.Award_agg + TimeLin2:ZX1003.Award_agg +
  TimeLin3:ZX1003.Award_agg + TimeLin4:ZX1003.Award_agg +
  TimeLin5:ZX1003.Award_agg + TimeLin6:ZX1003.Award_agg +
  TimeLin1:ESD101_eff + TimeLin2:ESD101_eff + TimeLin3:ESD101_eff +
  TimeLin4:ESD101_eff + TimeLin5:ESD101_eff + TimeLin6:ESD101_eff +
  TimeLin1:ESD105_eff + TimeLin2:ESD105_eff + TimeLin3:ESD105_eff +
  TimeLin4:ESD105_eff + TimeLin5:ESD105_eff + TimeLin6:ESD105_eff +
  TimeLin1:ESD112_eff + TimeLin2:ESD112_eff + TimeLin3:ESD112_eff +
  TimeLin4:ESD112_eff + TimeLin5:ESD112_eff + TimeLin6:ESD112_eff +
  TimeLin1:ESD113_eff + TimeLin2:ESD113_eff + TimeLin3:ESD113_eff +
  TimeLin4:ESD113_eff + TimeLin5:ESD113_eff + TimeLin6:ESD113_eff +
  TimeLin1:ESD114_eff + TimeLin2:ESD114_eff + TimeLin3:ESD114_eff +
  TimeLin4:ESD114_eff + TimeLin5:ESD114_eff + TimeLin6:ESD114_eff +
  TimeLin1:ESD123_eff + TimeLin2:ESD123_eff + TimeLin3:ESD123_eff +
  TimeLin4:ESD123_eff + TimeLin5:ESD123_eff + TimeLin6:ESD123_eff +
  TimeLin1:ESD171_eff + TimeLin2:ESD171_eff + TimeLin3:ESD171_eff +
  TimeLin4:ESD171_eff + TimeLin5:ESD171_eff + TimeLin6:ESD171_eff +
  TimeLin1:ESD189_eff + TimeLin2:ESD189_eff + TimeLin3:ESD189_eff +
  TimeLin4:ESD189_eff + TimeLin5:ESD189_eff + TimeLin6:ESD189_eff
  , data = pred_data)

str(X)
##### get predictions
gamma_hat <- fixef(Grad_2L_cat_m2)
str(gamma_hat)
setdiff(colnames(X), names(gamma_hat)) # columns in X but not in gamma_hat
setdiff(names(gamma_hat), colnames(X)) # coefs in gamma_hat but not in X
V_matrix <- vcov(Grad_2L_cat_m2)
pred_means <- X %*% gamma_hat
pred_se <- sqrt(diag(X %*% V_matrix %*% t(X)))

##### calculate Satterthwaite df using calcSatterth directly
L <- X # Our contrast matrix (45 rows)
##### use the internal function from lmerTest
##### ensure correct lmer object class
if(!inherits(Grad_2L_cat_m2, "lmerModLmerTest")) {
  Grad_2L_cat_m2 <- as(Grad_2L_cat_m2, "lmerModLmerTest")
}

```

```

##### calculate df for each row
df_satterthwaite <- numeric(nrow(X))
for(i in 1:nrow(X)) {
  # use contestMD which works better for individual contrasts
  L_single <- X[i, , drop = FALSE]
  # try using contest1D
  tryCatch({
    # get Satterthwaite approximation
    satt_result <- contest1D(Grad_2L_cat_m2, L_single, rhs = 0, confint = TRUE, level = 0.95)
    df_satterthwaite[i] <- satt_result$df
  }, error = function(e) {
    # fallback to summary method
    df_satterthwaite[i] <-<- summary(Grad_2L_cat_m2)$coefficients[1, "df"]
  })
}
##### create results with Satterthwaite df
cat_m2_Yhats <- data.frame(
  timepoint = pred_grid$timepoint,
  esd = pred_grid$esd,
  TimeLin1 = pred_data$TimeLin1,
  TimeLin2 = pred_data$TimeLin2,
  TimeLin3 = pred_data$TimeLin3,
  TimeLin4 = pred_data$TimeLin4,
  TimeLin5 = pred_data$TimeLin5,
  TimeLin6 = pred_data$TimeLin6,
  ESD101_eff = pred_data$ESD101_eff,
  ESD105_eff = pred_data$ESD105_eff,
  ESD112_eff = pred_data$ESD112_eff,
  ESD113_eff = pred_data$ESD113_eff,
  ESD114_eff = pred_data$ESD114_eff,
  ESD123_eff = pred_data$ESD123_eff,
  ESD171_eff = pred_data$ESD171_eff,
  ESD189_eff = pred_data$ESD189_eff,
  emmean = as.vector(pred_means),
  SE = pred_se,
  df = df_satterthwaite,
  lower.CL = as.vector(pred_means) - qt(0.975, df_satterthwaite) * pred_se,
  upper.CL = as.vector(pred_means) + qt(0.975, df_satterthwaite) * pred_se
)
cat_m2_Yhats

##### plot data ----

cat_m2_Yhats$TimeLin <- as.numeric(car::recode(cat_m2_Yhats$timepoint,
"1=2019;2=2020;3=2021;4=2022;5=2023;6=2024;7=2025"))
cat_m2_Yhats$ESD_asfactor <- car::recode(cat_m2_Yhats$esd, " 1='ESD 101'; 2='ESD 105'; 3='ESD
112'; 4='ESD 113';
5='ESD 114'; 6='ESD 123'; 7='ESD 171'; 8='ESD 189'; 9='ESD 121' ")
cat_m2_Yhat_2019 <- mean(cat_m2_Yhats$emmean[cat_m2_Yhats$timepoint == 1])
cat_m2_Yhat_2020 <- mean(cat_m2_Yhats$emmean[cat_m2_Yhats$timepoint == 2])
cat_m2_Yhat_2021 <- mean(cat_m2_Yhats$emmean[cat_m2_Yhats$timepoint == 3])
cat_m2_Yhat_2022 <- mean(cat_m2_Yhats$emmean[cat_m2_Yhats$timepoint == 4])
cat_m2_Yhat_2023 <- mean(cat_m2_Yhats$emmean[cat_m2_Yhats$timepoint == 5])
cat_m2_Yhat_2024 <- mean(cat_m2_Yhats$emmean[cat_m2_Yhats$timepoint == 6])
cat_m2_Yhat_2025 <- mean(cat_m2_Yhats$emmean[cat_m2_Yhats$timepoint == 7])

ggplot(data=cat_m2_Yhats,
  aes(x=TimeLin,
    y=emmean,
    fill = ESD_asfactor,
    color = ESD_asfactor)) +
  geom_point(aes(x = 2019, y = cat_m2_Yhat_2019), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2020, y = cat_m2_Yhat_2020), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2021, y = cat_m2_Yhat_2021), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2022, y = cat_m2_Yhat_2022), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2023, y = cat_m2_Yhat_2023), color = "black", size = 2, show.legend = FALSE)
+

```

```

geom_point(aes(x = 2024, y = cat_m2_Yhat_2024), color = "black", size = 2, show.legend = FALSE)
+
geom_point(aes(x = 2025, y = cat_m2_Yhat_2025), color = "black", size = 2, show.legend = FALSE)
+
geom_ribbon(aes(ymin = emmean - SE,
               ymax = emmean + SE),
           alpha = 0.05, # transparency (0.2-0.3 works well)
           color = NA, # remove border around ribbon
           show.legend = TRUE) + # usually don't need ribbon in legend
geom_line(linewidth = 1, show.legend = TRUE) +
xlab("Academic Year (Spring)") +
ylab("N=93 WA State High Schools\nPredicted Graduation Rate %\n(7-Rate Increment Time Model)")
+
labs(color = "Educ Serv Dist",
     fill = "Educ Serv Dist") +
scale_color_manual(values = c("purple4", "royalblue4", "royalblue1", "seagreen", "gold1",
                              "salmon1", "darkred", "orchid3", "black")) +
scale_fill_manual(values = c("purple4", "royalblue4", "royalblue1", "seagreen", "gold1",
                              "salmon1", "darkred", "orchid3", "black")) +
scale_y_continuous(#limits=c(0,100),
                  breaks=c(0,20,40,60,80,100)) +
scale_x_continuous(limits=c(2019,2025), breaks=c(2019,2020,2021,2022,2023,2024,2025)) +
theme_bw() +
theme(axis.text.x = element_text(color = "black", size = 10)) +
theme(axis.text.y = element_text(color = "black", size = 8)) +
theme(axis.title.x = element_text(color = "black", size=10, face="bold")) +
theme(axis.title.y = element_text(color = "black", size=10, face="bold"))
# last line of plot code cannot have a + at the end (the + is for next plot command)

### ESSA effects on trajectories plots (m2) ----

#### (A) polynomial linear model ----

##### create data ----

##### define time coding for the 7 timepoints
time_points <- data.frame(
  TimeLin = c(0, 1, 2, 3, 4, 5, 6)
)

##### create three levels of ZX1003.Award_agg: -1SD, Mean (0), +1SD
award_levels <- data.frame(
  ZX1003.Award_agg = c(-1, 0, 1), # -1SD, 0SD, +1SD
  level_label = c("Low (-1SD)", "Average (0SD)", "High (+1SD)")
)

##### create all combinations of time x award level
pred_grid <- expand.grid(
  timepoint = 1:7,
  award_level = 1:3
)

pred_data <- data.frame(
  TimeLin = time_points$TimeLin[pred_grid$timepoint],
  ZX1003.Award_agg = award_levels$ZX1003.Award_agg[pred_grid$award_level],
  # Set all ESD effects to 0 (average across ESDs)
  ESD101_eff = 0,
  ESD105_eff = 0,
  ESD112_eff = 0,
  ESD113_eff = 0,
  ESD114_eff = 0,
  ESD123_eff = 0,
  ESD171_eff = 0,
  ESD189_eff = 0,
  # Set enrollment to 0 (average)
  ZEnrollment_agg = 0
)

##### create model matrix
X <- model.matrix(~ TimeLin +

```

```

ZEnrollment_agg + ZX1003.Award_agg +
ESD101_eff + ESD105_eff + ESD112_eff + ESD113_eff +
ESD114_eff + ESD123_eff + ESD171_eff + ESD189_eff +
TimeLin:ZEnrollment_agg +
TimeLin:ZX1003.Award_agg +
TimeLin:ESD101_eff +
TimeLin:ESD105_eff +
TimeLin:ESD112_eff +
TimeLin:ESD113_eff +
TimeLin:ESD114_eff +
TimeLin:ESD123_eff +
TimeLin:ESD171_eff +
TimeLin:ESD189_eff,
data = pred_data)

##### get predictions
gamma_hat <- fixef(Grad_2L_poly_m2)
V_matrix <- vcov(Grad_2L_poly_m2)
pred_means <- X %*% gamma_hat
pred_se <- sqrt(diag(X %*% V_matrix %*% t(X)))

##### calculate Satterthwaite df using calcSatterth directly
L <- X # Our contrast matrix (18 rows: 7 time points x 3 levels)
##### use the internal function from lmerTest
##### ensure correct lmer object class
if(!inherits(Grad_2L_poly_m2, "lmerModLmerTest")) {
  Grad_2L_poly_m2 <- as(Grad_2L_poly_m2, "lmerModLmerTest")
}
##### calculate df for each row
df_satterthwaite <- numeric(nrow(X))
for(i in 1:nrow(X)) {
  # use contestMD which works better for individual contrasts
  L_single <- X[i, , drop = FALSE]
  # try using contest1D
  tryCatch({
    # get Satterthwaite approximation
    satt_result <- contest1D(Grad_2L_poly_m2, L_single, rhs = 0, confint = TRUE, level = 0.95)
    df_satterthwaite[i] <- satt_result$df
  }, error = function(e) {
    # fallback to summary method
    df_satterthwaite[i] <-<- summary(Grad_2L_poly_m2)$coefficients[1, "df"]
  })
}

##### create results with Satterthwaite df
poly_m2_Award_Yhats <- data.frame(
  timepoint = pred_grid$timepoint,
  award_level = pred_grid$award_level,
  TimeLin = pred_data$TimeLin,
  ZX1003.Award_agg = pred_data$ZX1003.Award_agg,
  level_label = award_levels$level_label[pred_grid$award_level],
  emmean = as.vector(pred_means),
  SE = pred_se,
  df = df_satterthwaite,
  lower.CL = as.vector(pred_means) - qt(0.975, df_satterthwaite) * pred_se,
  upper.CL = as.vector(pred_means) + qt(0.975, df_satterthwaite) * pred_se
)
str(poly_m2_Award_Yhats)

##### plot data ----

poly_m2_Award_Yhats$TimeLin <- as.numeric(car::recode(poly_m2_Award_Yhats$timepoint,
"1=2019;2=2020;3=2021;4=2022;5=2023;6=2024;7=2025
"))
poly_m2_Award_Yhat_2019 <- mean(poly_m2_Award_Yhats$emmean[poly_m2_Award_Yhats$timepoint == 1])
poly_m2_Award_Yhat_2020 <- mean(poly_m2_Award_Yhats$emmean[poly_m2_Award_Yhats$timepoint == 2])
poly_m2_Award_Yhat_2021 <- mean(poly_m2_Award_Yhats$emmean[poly_m2_Award_Yhats$timepoint == 3])
poly_m2_Award_Yhat_2022 <- mean(poly_m2_Award_Yhats$emmean[poly_m2_Award_Yhats$timepoint == 4])
poly_m2_Award_Yhat_2023 <- mean(poly_m2_Award_Yhats$emmean[poly_m2_Award_Yhats$timepoint == 5])
poly_m2_Award_Yhat_2024 <- mean(poly_m2_Award_Yhats$emmean[poly_m2_Award_Yhats$timepoint == 6])
poly_m2_Award_Yhat_2025 <- mean(poly_m2_Award_Yhats$emmean[poly_m2_Award_Yhats$timepoint == 7])

```

```

ggplot(data=poly_m2_Award_Yhats,
       aes(x=TimeLin,
           y=emmean,
           fill = level_label,
           color = level_label)) +
  geom_point(aes(x = 2019, y = poly_m2_Award_Yhat_2019), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2020, y = poly_m2_Award_Yhat_2020), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2021, y = poly_m2_Award_Yhat_2021), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2022, y = poly_m2_Award_Yhat_2022), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2023, y = poly_m2_Award_Yhat_2023), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2024, y = poly_m2_Award_Yhat_2024), color = "black", size = 2, show.legend =
FALSE) +
  geom_point(aes(x = 2025, y = poly_m2_Award_Yhat_2025), color = "black", size = 2, show.legend =
FALSE) +
  geom_ribbon(aes(ymin = emmean - SE,
                ymax = emmean + SE),
            alpha = 0.2, # transparency (0.2-0.3 works well)
            color = NA, # remove border around ribbon
            show.legend = TRUE) + # usually don't need ribbon in legend
  geom_line(linewidth = 1, show.legend = TRUE) +
  xlab("Academic Year (Spring)") +
  ylab("N=93 WA State High Schools\nPredicted Graduation Rate %\n(1-Rate Polynomial Time Model)")
+
  labs(color = "Mean ESSA Award",
       fill = "Mean ESSA Award") +
  scale_color_manual(values = c("plum3", "darkorchid4", "plum1")) +
  scale_fill_manual(values = c("plum3", "darkorchid4", "plum1")) +
  scale_y_continuous(limits=c(0,100), breaks=c(0,20,40,60,80,100)) +
  scale_x_continuous(limits=c(2019,2025), breaks=c(2019,2020,2021,2022,2023,2024, 2025)) +
  theme_bw() +
  theme(axis.text.x = element_text(color = "black", size = 10)) +
  theme(axis.text.y = element_text(color = "black", size = 8)) +
  theme(axis.title.x = element_text(color = "black", size=10, face="bold")) +
  theme(axis.title.y = element_text(color = "black", size=10, face="bold"))
# last line of plot code cannot have a + at the end (the + is for next plot command)

#### (B) piecewise 2-rate model ----

##### create data ----

##### define time coding for the 7 timepoints
time_points <- data.frame(
  TimeRate_1 = c(0, 1, 1, 1, 1, 1, 1),
  TimeRate_2 = c(0, 0, 1, 2, 3, 4, 5)
)

##### create all combinations with the 3 ZXAward levels
zxaward_combinations <- data.frame(
  ZX1003.Award_agg = c(-1, 0, 1) # Low, Medium, High
)

##### create the exact 15 combinations of time x ZXAward
pred_grid <- expand.grid(
  timepoint = 1:7,
  zxaward_level = 1:3
)

pred_data <- data.frame(
  TimeRate_1 = time_points$TimeRate_1[pred_grid$timepoint],
  TimeRate_2 = time_points$TimeRate_2[pred_grid$timepoint],
  ZEnrollment_agg = 0,
  ZX1003.Award_agg = zxaward_combinations$ZX1003.Award_agg[pred_grid$zxaward_level],
  ESD101_eff = 0,
  ESD105_eff = 0,
  ESD112_eff = 0,

```

```

ESD113_eff = 0,
ESD114_eff = 0,
ESD123_eff = 0,
ESD171_eff = 0,
ESD189_eff = 0
)

##### create model matrix
X <- model.matrix(~ TimeRate_1 + TimeRate_2 +
  ZEnrollment_agg + ZX1003.Award_agg +
  ESD101_eff + ESD105_eff + ESD112_eff + ESD113_eff +
  ESD114_eff + ESD123_eff + ESD171_eff + ESD189_eff +
  TimeRate_1:ZEnrollment_agg + TimeRate_2:ZEnrollment_agg +
  TimeRate_1:ZX1003.Award_agg + TimeRate_2:ZX1003.Award_agg +
  TimeRate_1:ESD101_eff + TimeRate_2:ESD101_eff +
  TimeRate_1:ESD105_eff + TimeRate_2:ESD105_eff +
  TimeRate_1:ESD112_eff + TimeRate_2:ESD112_eff +
  TimeRate_1:ESD113_eff + TimeRate_2:ESD113_eff +
  TimeRate_1:ESD114_eff + TimeRate_2:ESD114_eff +
  TimeRate_1:ESD123_eff + TimeRate_2:ESD123_eff +
  TimeRate_1:ESD171_eff + TimeRate_2:ESD171_eff +
  TimeRate_1:ESD189_eff + TimeRate_2:ESD189_eff,
  data = pred_data)

##### get predictions
gamma_hat <- fixef(Grad_2L_pw_m2)
V_matrix <- vcov(Grad_2L_pw_m2)
pred_means <- X %*% gamma_hat
pred_se <- sqrt(diag(X %*% V_matrix %*% t(X)))

##### calculate Satterthwaite df using calcSatterth directly
L <- X # Our contrast matrix (15 rows)
##### use the internal function from lmerTest
##### ensure correct lmer object class
if(!inherits(Grad_2L_pw_m2, "lmerModLmerTest")) {
  Grad_2L_pw_m2 <- as(Grad_2L_pw_m2, "lmerModLmerTest")
}
##### calculate df for each row
df_satterthwaite <- numeric(nrow(X))
for(i in 1:nrow(X)) {
  # use contestMD which works better for individual contrasts
  L_single <- X[i, , drop = FALSE]
  # try using contest1D
  tryCatch({
    # get Satterthwaite approximation
    satt_result <- contest1D(Grad_2L_pw_m2, L_single, rhs = 0, confint = TRUE, level = 0.95)
    df_satterthwaite[i] <- satt_result$df
  }, error = function(e) {
    # fallback to summary method
    df_satterthwaite[i] <-<- summary(Grad_2L_pw_m2)$coefficients[1, "df"]
  })
}

##### create results with Satterthwaite df
pw_m2_Yhats_award <- data.frame(
  timepoint = pred_grid$timepoint,
  zxaward_level = pred_grid$zxaward_level,
  TimeRate_1 = pred_data$TimeRate_1,
  TimeRate_2 = pred_data$TimeRate_2,
  ZX1003.Award_agg = pred_data$ZX1003.Award_agg,
  emmean = as.vector(pred_means),
  SE = pred_se,
  df = df_satterthwaite,
  lower.CL = as.vector(pred_means) - qt(0.975, df_satterthwaite) * pred_se,
  upper.CL = as.vector(pred_means) + qt(0.975, df_satterthwaite) * pred_se
)
pw_m2_Yhats_award

##### plot data ----

pw_m2_Yhats_award$TimeLin <- as.numeric(car::recode(pw_m2_Yhats_award$timepoint,

```

```

"1=2019;2=2020;3=2021;4=2022;5=2023;6=2024;7=2025
    ")
pw_m2_Yhats_award$Award_label <- car::recode(pw_m2_Yhats_award$zxaward_level,
      "1='Low (-1SD)'; 2='Average (0SD)'; 3='High
(+1SD)'"")

# Calculate grand means for each timepoint
pw_m2_Yhat_2019 <- mean(pw_m2_Yhats_award$emmean[pw_m2_Yhats_award$timepoint == 1])
pw_m2_Yhat_2020 <- mean(pw_m2_Yhats_award$emmean[pw_m2_Yhats_award$timepoint == 2])
pw_m2_Yhat_2021 <- mean(pw_m2_Yhats_award$emmean[pw_m2_Yhats_award$timepoint == 3])
pw_m2_Yhat_2022 <- mean(pw_m2_Yhats_award$emmean[pw_m2_Yhats_award$timepoint == 4])
pw_m2_Yhat_2023 <- mean(pw_m2_Yhats_award$emmean[pw_m2_Yhats_award$timepoint == 5])
pw_m2_Yhat_2024 <- mean(pw_m2_Yhats_award$emmean[pw_m2_Yhats_award$timepoint == 6])
pw_m2_Yhat_2025 <- mean(pw_m2_Yhats_award$emmean[pw_m2_Yhats_award$timepoint == 7])

ggplot(data = pw_m2_Yhats_award,
  aes(x = TimeLin,
      y = emmean,
      fill = Award_label,
      color = Award_label)) +
  geom_point(aes(x = 2019, y = pw_m2_Yhat_2019), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2020, y = pw_m2_Yhat_2020), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2021, y = pw_m2_Yhat_2021), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2022, y = pw_m2_Yhat_2022), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2023, y = pw_m2_Yhat_2023), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2024, y = pw_m2_Yhat_2024), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2025, y = pw_m2_Yhat_2025), color = "black", size = 2, show.legend = FALSE)
+
  geom_ribbon(aes(ymin = emmean - SE,
      ymax = emmean + SE),
      alpha = 0.2,
      color = NA,
      show.legend = TRUE) +
  geom_line(linewidth = 1, show.legend = TRUE) +
  xlab("Academic Year (Spring)") +
  ylab("N=93 WA State High Schools\nPredicted Graduation Rate %\n(2-Rate Piecewise Time Model)")
+
  labs(color = "Mean ESSA Award",
      fill = "Mean ESSA Award") +
  scale_color_manual(values = c("plum3", "darkorchid4", "plum1")) +
  scale_fill_manual(values = c("plum3", "darkorchid4", "plum1")) +
  scale_y_continuous(limits = c(0, 100), breaks = c(0, 20, 40, 60, 80, 100)) +
  scale_x_continuous(limits = c(2019, 2025), breaks = c(2019, 2020, 2021, 2022, 2023, 2024,
2025)) +
  theme_bw() +
  theme(axis.text.x = element_text(color = "black", size = 10)) +
  theme(axis.text.y = element_text(color = "black", size = 8)) +
  theme(axis.title.x = element_text(color = "black", size = 10, face = "bold")) +
  theme(axis.title.y = element_text(color = "black", size = 10, face = "bold"))

#### (C) categorical 7-rate inc/dec model ----

##### create data ----

##### define time coding for the 7 timepoints
time_points <- data.frame(
  TimeLin1 = c(0, 1, 1, 1, 1, 1, 1),
  TimeLin2 = c(0, 0, 1, 1, 1, 1, 1),
  TimeLin3 = c(0, 0, 0, 1, 1, 1, 1),
  TimeLin4 = c(0, 0, 0, 0, 1, 1, 1),
  TimeLin5 = c(0, 0, 0, 0, 0, 1, 1),
  TimeLin6 = c(0, 0, 0, 0, 0, 0, 1)
)

```

```

##### create all combinations with the 3 ZXAward levels
zxaward_combinations <- data.frame(
  ZX1003.Award_agg = c(-1, 0, 1) # Low, Medium, High
)

##### create the exact 21 combinations of time x ZXAward
pred_grid <- expand.grid(
  timepoint = 1:7,
  zxaward_level = 1:3
)

pred_data <- data.frame(
  TimeLin1 = time_points$TimeLin1[pred_grid$timepoint],
  TimeLin2 = time_points$TimeLin2[pred_grid$timepoint],
  TimeLin3 = time_points$TimeLin3[pred_grid$timepoint],
  TimeLin4 = time_points$TimeLin4[pred_grid$timepoint],
  TimeLin5 = time_points$TimeLin5[pred_grid$timepoint],
  TimeLin6 = time_points$TimeLin6[pred_grid$timepoint],
  ZEnrollment_agg = 0,
  ZX1003.Award_agg = zxaward_combinations$ZX1003.Award_agg[pred_grid$zxaward_level],
  ESD101_eff = 0,
  ESD105_eff = 0,
  ESD112_eff = 0,
  ESD113_eff = 0,
  ESD114_eff = 0,
  ESD123_eff = 0,
  ESD171_eff = 0,
  ESD189_eff = 0
)

##### create model matrix
X <- model.matrix(~ TimeLin1 + TimeLin2 + TimeLin3 + TimeLin4 + TimeLin5 + TimeLin6 +
  ZEnrollment_agg + ZX1003.Award_agg +
  ESD101_eff + ESD105_eff + ESD112_eff + ESD113_eff +
  ESD114_eff + ESD123_eff + ESD171_eff + ESD189_eff +
  TimeLin1:ZEnrollment_agg + TimeLin2:ZEnrollment_agg +
  TimeLin3:ZEnrollment_agg + TimeLin4:ZEnrollment_agg +
  TimeLin5:ZEnrollment_agg + TimeLin6:ZEnrollment_agg +
  TimeLin1:ZX1003.Award_agg + TimeLin2:ZX1003.Award_agg +
  TimeLin3:ZX1003.Award_agg + TimeLin4:ZX1003.Award_agg +
  TimeLin5:ZX1003.Award_agg + TimeLin6:ZX1003.Award_agg +
  TimeLin1:ESD101_eff + TimeLin2:ESD101_eff + TimeLin3:ESD101_eff +
  TimeLin4:ESD101_eff + TimeLin5:ESD101_eff + TimeLin6:ESD101_eff +
  TimeLin1:ESD105_eff + TimeLin2:ESD105_eff + TimeLin3:ESD105_eff +
  TimeLin4:ESD105_eff + TimeLin5:ESD105_eff + TimeLin6:ESD105_eff +
  TimeLin1:ESD112_eff + TimeLin2:ESD112_eff + TimeLin3:ESD112_eff +
  TimeLin4:ESD112_eff + TimeLin5:ESD112_eff + TimeLin6:ESD112_eff +
  TimeLin1:ESD113_eff + TimeLin2:ESD113_eff + TimeLin3:ESD113_eff +
  TimeLin4:ESD113_eff + TimeLin5:ESD113_eff + TimeLin6:ESD113_eff +
  TimeLin1:ESD114_eff + TimeLin2:ESD114_eff + TimeLin3:ESD114_eff +
  TimeLin4:ESD114_eff + TimeLin5:ESD114_eff + TimeLin6:ESD114_eff +
  TimeLin1:ESD123_eff + TimeLin2:ESD123_eff + TimeLin3:ESD123_eff +
  TimeLin4:ESD123_eff + TimeLin5:ESD123_eff + TimeLin6:ESD123_eff +
  TimeLin1:ESD171_eff + TimeLin2:ESD171_eff + TimeLin3:ESD171_eff +
  TimeLin4:ESD171_eff + TimeLin5:ESD171_eff + TimeLin6:ESD171_eff +
  TimeLin1:ESD189_eff + TimeLin2:ESD189_eff + TimeLin3:ESD189_eff +
  TimeLin4:ESD189_eff + TimeLin5:ESD189_eff + TimeLin6:ESD189_eff
  , data = pred_data)

##### get predictions
gamma_hat <- fixef(Grad_2L_cat_m2)
V_matrix <- vcov(Grad_2L_cat_m2)
pred_means <- X %*% gamma_hat
pred_se <- sqrt(diag(X %*% V_matrix %*% t(X)))

##### calculate Satterthwaite df using calcSatterth directly
L <- X # Our contrast matrix (21 rows)
##### use the internal function from lmerTest
##### ensure correct lmer object class
if(!inherits(Grad_2L_cat_m2, "lmerModLmerTest")) {
  Grad_2L_cat_m2 <- as(Grad_2L_cat_m2, "lmerModLmerTest")
}

```

```

}
##### calculate df for each row
df_satterthwaite <- numeric(nrow(X))
for(i in 1:nrow(X)) {
  # use contestMD which works better for individual contrasts
  L_single <- X[i, , drop = FALSE]
  # try using contest1D
  tryCatch({
    # get Satterthwaite approximation
    satt_result <- contest1D(Grad_2L_cat_m2, L_single, rhs = 0, confint = TRUE, level = 0.95)
    df_satterthwaite[i] <- satt_result$df
  }, error = function(e) {
    # fallback to summary method
    df_satterthwaite[i] <- summary(Grad_2L_cat_m2)$coefficients[1, "df"]
  })
}

##### create results with Satterthwaite df
cat_m2_Yhats_award <- data.frame(
  timepoint = pred_grid$timepoint,
  zxaward_level = pred_grid$zxaward_level,
  TimeLin1 = pred_data$TimeLin1,
  TimeLin2 = pred_data$TimeLin2,
  TimeLin3 = pred_data$TimeLin3,
  TimeLin4 = pred_data$TimeLin4,
  TimeLin5 = pred_data$TimeLin5,
  TimeLin6 = pred_data$TimeLin6,
  ZX1003.Award_agg = pred_data$ZX1003.Award_agg,
  emmean = as.vector(pred_means),
  SE = pred_se,
  df = df_satterthwaite,
  lower.CL = as.vector(pred_means) - qt(0.975, df_satterthwaite) * pred_se,
  upper.CL = as.vector(pred_means) + qt(0.975, df_satterthwaite) * pred_se
)
cat_m2_Yhats_award

##### plot data ----

cat_m2_Yhats_award$TimeLin <- as.numeric(car::recode(cat_m2_Yhats_award$timepoint,
"1=2019;2=2020;3=2021;4=2022;5=2023;6=2024;7=2025
          "))
cat_m2_Yhats_award$Award_label <- car::recode(cat_m2_Yhats_award$zxaward_level,
"1='Low (-1SD)'; 2='Average (0SD)'; 3='High
(+1SD)'"")

# Calculate grand means for each timepoint
cat_m2_Yhat_2019 <- mean(cat_m2_Yhats_award$emmean[cat_m2_Yhats_award$timepoint == 1])
cat_m2_Yhat_2020 <- mean(cat_m2_Yhats_award$emmean[cat_m2_Yhats_award$timepoint == 2])
cat_m2_Yhat_2021 <- mean(cat_m2_Yhats_award$emmean[cat_m2_Yhats_award$timepoint == 3])
cat_m2_Yhat_2022 <- mean(cat_m2_Yhats_award$emmean[cat_m2_Yhats_award$timepoint == 4])
cat_m2_Yhat_2023 <- mean(cat_m2_Yhats_award$emmean[cat_m2_Yhats_award$timepoint == 5])
cat_m2_Yhat_2024 <- mean(cat_m2_Yhats_award$emmean[cat_m2_Yhats_award$timepoint == 6])
cat_m2_Yhat_2025 <- mean(cat_m2_Yhats_award$emmean[cat_m2_Yhats_award$timepoint == 7])

ggplot(data = cat_m2_Yhats_award,
  aes(x = TimeLin,
    y = emmean,
    fill = Award_label,
    color = Award_label)) +
  geom_point(aes(x = 2019, y = cat_m2_Yhat_2019), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2020, y = cat_m2_Yhat_2020), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2021, y = cat_m2_Yhat_2021), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2022, y = cat_m2_Yhat_2022), color = "black", size = 2, show.legend = FALSE)
+
  geom_point(aes(x = 2023, y = cat_m2_Yhat_2023), color = "black", size = 2, show.legend = FALSE)
+

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geom_point(aes(x = 2024, y = cat_m2_Yhat_2024), color = "black", size = 2, show.legend = FALSE)
+
geom_point(aes(x = 2025, y = cat_m2_Yhat_2025), color = "black", size = 2, show.legend = FALSE)
+
geom_ribbon(aes(ymin = emmean - SE,
               ymax = emmean + SE),
           alpha = 0.2,
           color = NA,
           show.legend = TRUE) +
geom_line(linewidth = 1, show.legend = TRUE) +
xlab("Academic Year (Spring)") +
ylab("N=93 WA State High Schools\nPredicted Graduation Rate %\n(7-Rate Increment Time Model)")
+
labs(color = "Mean ESSA Award",
     fill = "Mean ESSA Award") +
scale_color_manual(values = c("plum3", "darkorchid4", "plum1")) +
scale_fill_manual(values = c("plum3", "darkorchid4", "plum1")) +
scale_y_continuous(limits = c(0, 100), breaks = c(0, 20, 40, 60, 80, 100)) +
scale_x_continuous(limits = c(2019, 2025), breaks = c(2019, 2020, 2021, 2022, 2023, 2024,
2025)) +
theme_bw() +
theme(axis.text.x = element_text(color = "black", size = 10)) +
theme(axis.text.y = element_text(color = "black", size = 8)) +
theme(axis.title.x = element_text(color = "black", size = 10, face = "bold")) +
theme(axis.title.y = element_text(color = "black", size = 10, face = "bold"))

```