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Experimental Assessment of Confinement and Heating on the HIT-SI3 Spheromak

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Abstract

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The presented research considers electron confinement and heating on the Steady-Inductive Helicity-Injected Torus (HIT-SI3). Specifically, the mechanisms that determine how input energy is distributed in the spheromak plasma are examined. Thomson Scattering measurements of the HIT-SI3 electron temperature and relative density are used to inform the discussion for discharges driven by steady inductive helicity injection at 15.6 kHz. These measurements are compared to simulation. They also guide the parameter choices in a 0-D energy balance model to further elucidate the confinement and heating in HIT-SI3. Results suggest that, if the heat flux is assumed to be diffusive, a Rechester-Rosenbluth stochastic field formulation for the electron thermal diffusion coefficient may be most appropriate.

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Chapter 1

INTRODUCTION

1.1 Motivation for New Energy Research

Each year, human civilization uses roughly 5.7×10^{20} J of energy, and much of this energy comes from chemical reactions that involve carbon-based reactants (combustion) [1, 2]. Though mostly formed from long-buried organisms that lived hundreds of millions of years ago, the accessibility and energy density of carbon-based fuels have made them an obvious choice for early industrial endeavors [3]. However, it is now clear that the continued use of these fuels and the consequent effusion of greenhouse and toxic gases will have consequences that may be extreme, and are likely to be both harmful and irreversible [4]. New sources of energy that are non-polluting may be necessary for maintaining human civilization on earth. Even though renewable energy resources like wind, solar, and hydroelectric power are projected to be the fastest-growing sources of electricity over the next 25 years as shown in Fig. 1.1, the current rate of emissions reduction may not be enough to avoid serious repercussions such as sea level rise due to melting polar ice [4].

Harnessing energy from nuclear fission is a proven technique that is capable of on-demand electricity generation. Fission reactors can function without the need for ancillary power production techniques or battery storage. They can be built or even transported wherever they are needed (including deep space and the deep sea), and they are also completely combustion-free, meaning their greenhouse gas emissions are negligible compared with fossil fuel sources. Further, it is likely that the fuel used by fission reactors could last far into the future, with estimates ranging from hundreds to thousands of years [5]. Others have argued that fission fuel is effectively as inexhaustible as renewable energy sources [6].

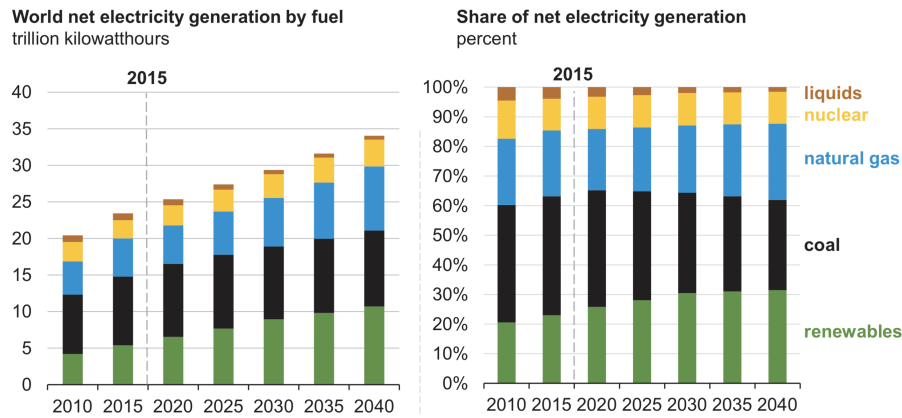


Figure 1.1: Projections from the US Energy Information Administration showing expected trends in worldwide electricity generation. Renewable and natural gas resources are projected to step up production as coal's share contracts to 31% by the year 2040. Nuclear output is expected to stay stagnant with a roughly 10% share worldwide (From Ref. [1]).

Nuclear fission has many of the desirable qualities required for the future of energy production. However, these reactors also typically suffer from unique regulatory burdens and poor public approval, both of which are due to concerns over reactor safety, nuclear waste handling, and nuclear weapons proliferation. The share of power generation taken up by nuclear fission is not expected to grow significantly in the near future.

1.2 Fusion Energy

The promise of nuclear fusion is as a means of producing power that has the benefits of a fission reactor with fewer disadvantages. Fusion is inherently safer than fission, and its fuels are likely to last many thousands of years, and possibly millions [5]. Instead of splitting large atoms as in fission, fusion relies on reactions between lower proton-number elements. This advantage is illustrated in Fig. 1.2, which shows the binding energy per nucleon due to the strong nuclear force¹. Elements near the maximum of this curve are the most tightly bound and most stable. Compared with nuclear fission reactions, whose

¹Image courtesy Wikimedia commons.

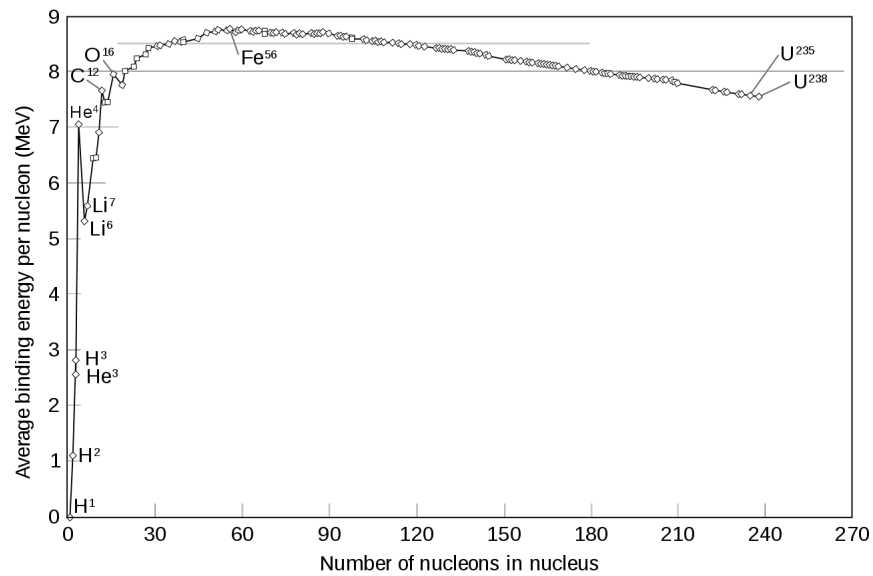


Figure 1.2: Binding energy per nucleon of elements up to U-238.

reactants begin on the right side of the curve and move toward the maximum, fusion reactions typically involve low-mass atoms on the left, and have a much larger change in binding energy between reactants and products. Therefore, per nucleon, a fusion reaction will release more energy than a fission reaction.

The likelihood of a fusion reaction occurring between two particles is described by the reaction cross section. For two atomic nuclei undergo a fusion interaction it is necessary that they pass within some critical distance of each other. This distance sets an area or “cross section” around each atom. Because the forces that bind nuclei and govern nuclear reactions act over extremely short distances, this area is on the order of the atomic cross-sectional area, and varies depending on the nuclear species involved in the reaction.

For instance, for a deuterium particle moving at some velocity v_D relative to a background of tritium particles, the rate of reactions per unit volume is equal to the product of the volume swept out by the moving particle and the number density of the tritium. Multiplying by the number density of deuterium gives us a reaction rate for a population

of deuterium streaming through the tritium:

$$\nu_{DT} = \sigma_{DT} v_D n_D n_T \quad (1.1)$$

Eq. (1.1) shows that a higher reaction cross section results in a higher reaction rate. In practice, the fusion reaction cross sections must also take into account Coulomb forces between nuclei and quantum effects such as particles tunneling through the Coulomb potential, and are determined empirically [7]. These Coulomb and quantum effects mean that the cross section in fact depends on relative velocities of the particles. In trying to assess the fusion rate of an ensemble of particles, the expected value (mean) of the product of cross section and relative velocity is used. This is referred to as the velocity-averaged cross section, $\langle \sigma v \rangle$.

Some of the fusion reactions with relatively high cross sections are listed in Fig. 1.3, together with a plot of how their velocity-averaged cross sections vary with the mean kinetic energy of the ensemble. The maximum values of $\langle \sigma v \rangle$ occur at particle energies on the order of 100 keV, which is equivalent to a temperature in excess of 1 billion Kelvin. While the particles in most small-scale magnetic confinement experiments are well below this temperature; energies between 1 eV (11600 Kelvin) and 100 eV are common. At these temperatures, atoms are prone to ionize, and thus most magnetic confinement research is based on the study of plasma², a type of highly ionized gas.

If the particles in a plasma are to provide steady-state nuclear power, the energy released must be sufficient to sustain further fusion reactions. With eq. (1.1), the energy liberated in each fusion event ξ , and for a D-T plasma in which $n_D = n_T = n/2$,

$$P_{fus} = \xi \frac{n^2}{4} \langle \sigma v \rangle_{DT} \quad (1.2)$$

For a theoretical reactor in which fusion power is the only source of heat, it is necessary to reach “breakeven,” that is P_{fus} must overcome all heat sink terms, including the power

²See Appendix A

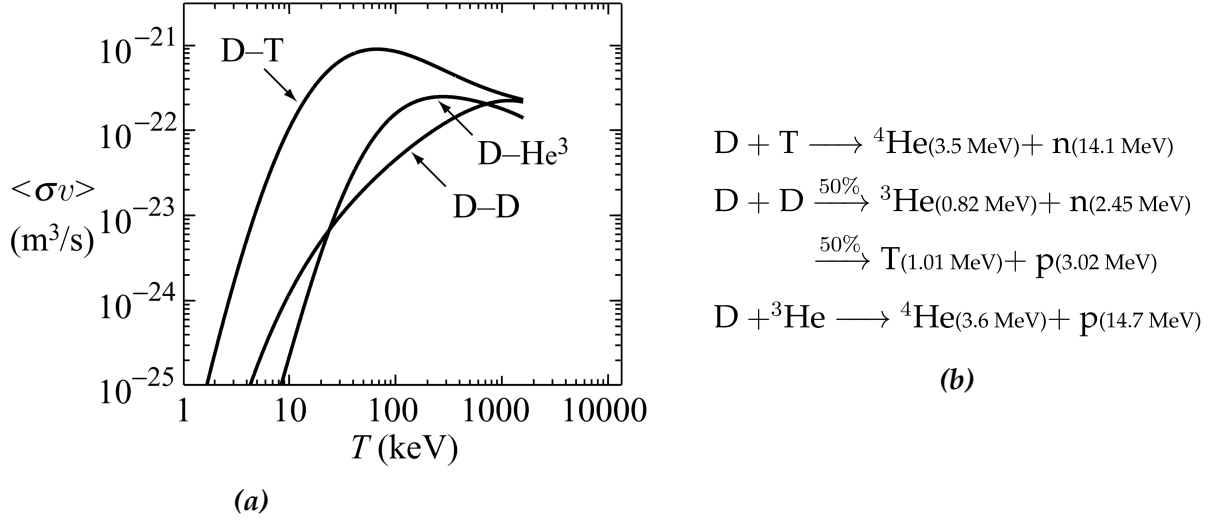


Figure 1.3: (a) Velocity-averaged cross sections for various atomic species with the relatively probable fusion reactions, as a function of the temperature of the reactants (From Ref. [7]). (b) Equations for these reactions. While deuterium can be harvested from sea water and tritium can easily be bred from fusion reactions with lithium, He-3 would need to be harvested from the moon or obtained in some other way.

loss due to thermal conduction (P_k) and loss due to bremsstrahlung radiation (P_B). This extra heat energy must be confined in a system of high enough density for enough time that it will maintain $P_{fus} \geq P_k + P_B$. This requirement is often expressed in terms of the “fusion triple product,” or the product of the density, the energy confinement time (the characteristic time for conductive heat loss), and the temperature. The specific value of $n\tau T$ required for achieving breakeven is influenced by $\langle \sigma v \rangle$, and will vary for different fuel species.

1.3 Research Goals

From the concept of the triple product it is clear that two fundamental aspects of achieving nuclear fusion power on earth are plasma confinement and plasma heating. Experimental designs for fusion reactors are thus evaluated on their effectiveness in performing these two tasks. Measurements of the plasma temperature and density are therefore key

to determining the success of fusion experiments. Additionally, the claim of any plasma physics experiment to be relevant to fusion energy must be grounded in careful and repeatable observations of the mechanisms by which that plasma is heated and confined.

The completed research had a primary goal of characterizing how the Steady-Inductive Helicity-Injected Torus (HIT-SI3) heats and confines plasma. Specifically, the phenomena that determine how input energy flows and is distributed through the HIT-SI3 plasma are examined. Using the diagnostic technique of Thomson Scattering, measurements of the HIT-SI3 temperature and relative density were made to inform the discussion. These measurements are compared to simulation of the same experiment, and they guide the development of a model that may further elucidate the mechanisms that influence confinement and heating in HIT-SI3. The evidence gathered is consistent with models of Ohmic heating as the dominant mechanism for electron heating, and suggests strong imposed magnetic fluctuations responsible for increased thermal conduction to the walls. No significant evidence of a pressure gradient was detected in the measurements recorded, which span moderate- to high-performance discharges as determined by total toroidal current and toroidal current gain.

Chapter 2

BACKGROUND

2.1 *Magnetic Confinement Fusion*

In order to confine and heat plasma and attain controlled nuclear fusion on earth, a number of methods are being researched. These investigations are often classified into two main strategies: inertial confinement, which uses various methods to induce symmetric implosions by precisely imparting momentum to the nuclear fuel, and magnetic confinement, which relies on the Lorentz force on each charged particle α to confine it to magnetic field lines:

$$\mathbf{F}_\alpha = q_\alpha (\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B}) \quad (2.1)$$

The convenient effect for magnetic confinement is that for any charge moving with some component of velocity perpendicular to an applied magnetic field, the Lorentz force has a component perpendicular to both the applied magnetic field lines and the particle velocity. This will cause the charge to move in a circular path, and the charged particles can be thought of as orbiting a field line. The radius of this orbit is called the gyroradius (or Larmor radius), and is given by $\rho_\alpha = \frac{m_\alpha v_\alpha}{q_\alpha B}$. In general, it is necessary for any magnetic confinement apparatus to have a characteristic plasma size that is larger than the typical gyroradius of particles it seeks to confine. By manipulating the magnetic field strength and structure, the behavior of the plasma can be influenced and controlled. The proposed research focuses entirely on issues related to magnetic confinement.

Another natural phenomenon that is utilized in magnetic confinement is the fact that moving charges (electric currents) produce magnetic fields in a predictable manner. This

is stated by Ampère's Law for magnetic field $\mathbf{B}$ produced by a current density $\mathbf{j}$:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \quad (2.2)$$

A consequence of this empirical law, when combined with the Lorentz force, is that electric currents flowing in the same direction tend to attract one another and compress together. This effect is fundamental to a class of magnetic confinement devices known as *pinches*, in which the driving of current tends to compress the plasma.

The tokamak is another example of a common confinement design in which plasma currents generate magnetic fields according to Ampère's Law that in turn confine plasma via the Lorentz force. Magnetic fields are produced in a torus-shaped chamber and are given closed circular trajectories. If, as each magnetic field line loops through the torus, it lies entirely on a single toroidal surface, the confinement device is said to have closed magnetic flux surfaces. Mathematically, these are surfaces of constant flux $\psi = \int_{\Sigma} \mathbf{B} \cdot d\mathbf{S}$ for which $\mathbf{B} \cdot \nabla \psi = 0$. In this case the plasma particles (which move as beads on a string along the field lines) are confined to concentric toroidal surfaces within the tokamak. An example tokamak³ structure is shown in Fig. 2.1. Additional confining fields are self-generated by the plasma via a *toroidal* current that runs along the applied magnetic field lines. This current is produced via Faraday's Law:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.3)$$

A solenoidal coil wound through the hole of the toroidal chamber produces magnetic field along the torus's vertical axis. Time-varying magnetic flux pierces the area that the plasma volume encircles, and a current in the plasma is induced.

While the confinement of the charged particles moving in a magnetic field is fundamentally due to their finite gyroradii, it is helpful to consider the macroscopic scale. The thermal pressure, $p = 2nk_B T$, of the confined plasma must be balanced by some force to

³Image courtesy of EUROfusion: www.euro-fusion.org

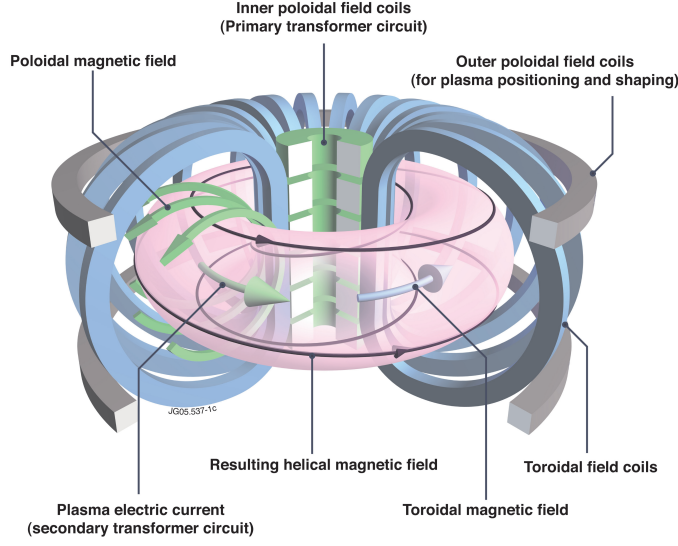


Figure 2.1: Diagram of a typical tokamak configuration.

maintain an equilibrium. Magnetic confinement uses the magnetic pressure $\frac{B^2}{2\mu_0}$ to this end.

2.1.1 Plasma Beta

The plasma equilibrium, or balance between the thermal pressure and magnetic pressure, is achieved and maintained in magnetic confinement experiments using both externally applied fields and self-generated fields. The balance of local magnetic fields against the local thermal pressure is quantified in the parameter $\beta = \frac{p}{B^2/2\mu_0}$. With this definition of β , eq. (1.2) (the power produced by fusion reactions) is rewritten as:

$$P_{fus} = \xi \frac{\beta^2 B^4}{64\mu_0^2 k_B^2 T^2} \langle \sigma v \rangle_{DT} \quad (2.4)$$

Typically the target temperature of fusion plasmas is set by the maximum of the reaction cross section for the plasma species. The fusion power output can therefore be considered simply proportional to $\beta^2 B^4$. There are thus advantages to highly magnetized confinement schemes, but producing strong magnetic fields in steady state is technologi-

cally difficult and costly. Advances in superconductor technology are promising, but the production of strong magnetic fields is not yet trivial. It may be worthwhile to focus on maximizing β in order to attain higher fusion power output without the added cost and complication of strong electromagnets.

2.2 The Spheromak

The spheromak is a relatively simple magnetic confinement design with a spherical (or “simply connected”) magnetic topology as shown in Fig. 2.2. This is in contrast to the tokamak topology in which a hole exists in the center of the torus-shaped magnetic field structure. This hole is necessitated by various physical structures that link the toroidal plasma in a tokamak, including the toroidal field coils and the transformer solenoid shown in Fig. 2.1. The spheromak plasma largely produces its own confining fields via internal currents. In general, a spheromak can be defined as a toroidal plasma-magnetic

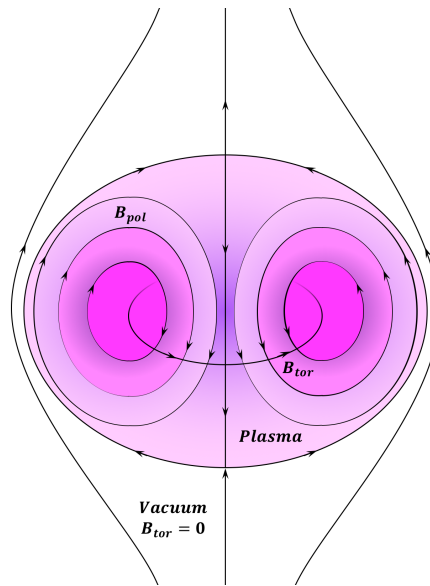


Figure 2.2: Cross-section diagram of a typical spheromak magnetic structure. The magnetic field is entirely poloidal at the separatrix and entirely toroidal at the magnetic axis (B_{tor} pierces the plane of the page).

topology having a toroidal field null at the outer separatrix and geometric axis, and a

poloidal field null at the magnetic axis. The spheromak is also generally thought of as having closed flux surfaces [8].

The spheromak has a number of advantageous characteristics. Its compact and spherical (simply-connected) geometry results in simplified engineering requirements and reduced complexity for a spheromak-based fusion power plant. The currents produced in the plasma of a spheromak reactor would be of much greater amplitude than a similar scale tokamak reactor [7]. Because plasmas are not perfectly conducting, higher currents result in greater temperatures, which facilitates fusion reactions.

2.2.1 Helicity and the MECH State

Spheromaks form spontaneously when magnetized plasmas with sufficient helicity and energy are enclosed in a conducting boundary [9, 10, 11]. Helicity is a constant of motion in magnetohydrodynamic systems (like magnetized plasmas), defined here as a volumetric quantity:

$$K = \int_{\Omega} \mathbf{A} \cdot \mathbf{B} d^3r \quad (2.5)$$

Here $\mathbf{A}$ is the magnetic vector potential and $\mathbf{B}$ is the magnetic field. Helicity is discussed in more depth in Sec. 2.3.2, but it is a global quantity that can be thought of as the linkage of magnetic flux on all scales. The tendency for spontaneous formation of spheromaks is a consequence of what is referred to as the Taylor principle, which describes the tendency of magnetohydrodynamic systems within a conducting boundary to relax to a “Taylor configuration” of minimum magnetic energy while maintaining constant total helicity [12].

Lodewijk Woltjer originally showed that a perfectly conducting magnetic fluid in a closed system has constant helicity, and that minimizing the magnetic energy of such a system results in the “force-free condition” [13]:

$$\nabla \times \mathbf{B} = \lambda \mathbf{B} \quad (2.6)$$

The force free condition implies (with Ampère’s Law) that the current and magnetic field are parallel vectors, and thus the Lorentz force (eq. (2.1)) vanishes. For a stable plasma equilibrium, this means that forces caused by a pressure gradient must also vanish, leading to $\mathbf{j} \times \mathbf{B} = \nabla p = 0$. In this case also $p \approx 0$ for plasmas with zero density at the boundaries. Thus the force-free condition also implies $\beta \approx 0$. One last implication is the concept of λ , which from the force-free condition and Ampère’s Law is $\frac{\mu_0 \mathbf{j} \cdot \mathbf{B}}{\mathbf{B} \cdot \mathbf{B}}$. This scalar quantity is often used in describing spheromaks — it quantifies the current parallel to the magnetic field, and its reciprocal is taken as the characteristic length scale of variation in the equilibrium magnetic field.

We have assumed in the above description of a Taylor configuration that the plasma is perfectly conducting and in an isolated system. All plasmas, however, have some finite resistivity through which dissipation occurs and pressure gradients are allowed. Therefore no real plasma will truly reach an exactly force-free Taylor configuration. On the other hand, even slightly resistive plasmas tend to relax toward a state of minimum energy while conserving helicity. In general this is called the Minimum Energy Constant Helicity (MECH) state. The Taylor configuration is a special case of a MECH state.

2.2.2 Spheromak Formation

The concept of the MECH state describes why spheromaks tend to form spontaneously when the right initial conditions are imposed. The relative ease with which spheromaks form means there are a number of ways to produce them experimentally. In general, an electric field is applied along magnetic field lines. When plasma is present, this results in field-aligned current drive, which is equivalent to helicity injection [8]. As the field-aligned current is driven higher, the helicity content inside the conducting boundary grows. A threshold level of current is finally reached past which magnetic reconnection events will occur to form a spheromak whose field lines are detached from the driving mechanism.

Most previously-implemented spheromaks use electrodes to inject helicity. A common approach involves plasma current being driven along field lines between two cylindrical electrodes. The Sustained Spheromak Experiment (“SSPX,” likely the highest-temperature spheromak to date) in particular is an example of a “gun-driven” spheromak, after the plasma gun originally described by John Marshall [14, 15, 16]. Gun-driven spheromaks rely on “coaxial helicity injection” (CHI), based on the coaxial geometry of the two electrodes relative to the spheromak. The SSPX configuration utilizes a capacitive spheromak formation process — the field lines along which current are driven terminate on physical electrode surfaces [17]. In order to reduce plasma-material interactions, an inductive method can be used to form the spheromak. This is the strategy used in the Steady Inductive Helicity Injected Torus (HIT-SI3).

In the HIT-SI3 experiment, helicity and energy are introduced into a topologically spherical conducting shell (confinement volume) using three inductively-driven injectors as shown in Fig. 2.3. The field lines produced in the injectors do not necessarily terminate on physical structures, and are allowed to close on themselves. As the helicity is injected, the magnetic field lines reconnect to form a spheromak object.

While the formation of spheromaks based on the DC and roughly axisymmetric CHI scheme like SSPX are easier to visualize, the formation of HIT-SI3 spheromaks are harder to picture in an intuitive way. Nonetheless, the presence of a HIT-SI3 spheromak plasma is established based on measurements of the magnetic fields produced by the plasma. These are used to verify the magnetic structure and to reconstruct the toroidal plasma current via discrete Amperian loops made up of individual $\dot{\mathbf{B}}$ probes [18].

In all cases (including inductive and coaxial helicity injection), magnetic reconnection plays a key role in spheromak formation. Turbulence quickly gives the plasma access to its minimum energy state, and reconnection occurs as a result of the turbulence [10]. Reconnection is thus fundamental to the process of forming closed spheromak flux surfaces out of originally-open field lines (called “plasma relaxation”), which is not well understood [7]. Because the charged plasma particles are confined to magnetic field lines by



Figure 2.3: (a) The bare HIT-SI3 injector and confinement volume structure. The three copper half-tori (left) inject plasma, energy, and helicity into the main chamber. The gapped midplane allows for diagnostic access. (b) “Flux” coils shown in white generate magnetic field along which current is induced by transformer action of the “voltage” coils shown in red, green, and blue.

their perpendicular velocities, and are free to move along field lines, closed flux surfaces are desirable because they approximate an infinitely long field line that confines particles.

2.3 HIT-SI3

Research on SSPX had found no clear path forward for driving a spheromak plasma-magnetic topology without causing open field lines and poor confinement. This impediment would cripple the efficiency of a spheromak fusion reactor based on the spheromak, even with the good confinement properties of a naturally decaying spheromak. HIT-SI3 is the second in a succession of inductively driven spheromaks which have investigated whether current can be driven (i.e. whether helicity can be injected) across the closed flux surfaces without destroying them and losing confinement.

As discussed in Sec. 2.2.2, HIT-SI3 injects helicity into the confinement volume inductively using AC currents. Each injector produces a confining magnetic flux and a voltage along it to inject helicity as described by the first term in eq. (2.13). The two quantities

oscillate in phase to produce constantly positive helicity. The coils that induce the voltage and “flux” magnetic field are illustrated in figs. 2.4. On the HIT-SI experiments, multi-

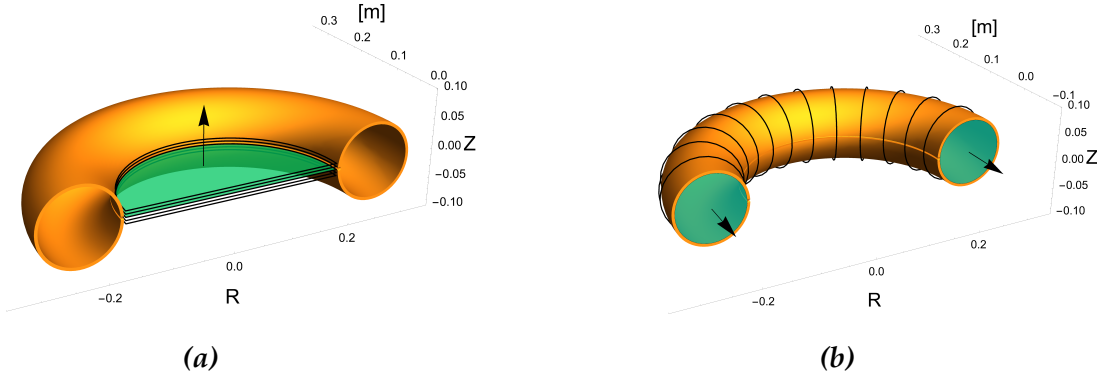


Figure 2.4: A depiction of the “voltage” (a) and “flux” (b) coils on a bare injector. Green regions with black normal arrows represent coil areas across which oscillating magnetic fields are applied by the coils in black.

ple injectors are fired with precise timing such that the rate of helicity injection can be controlled. The control of the timing between injectors is best thought of as a phase difference in their oscillatory helicity injection rates; for the HIT-SI3 configuration with three injectors,

$$\dot{K}_{inj} = 2V\Psi \left[\sin^2(\omega_{inj}t + \phi_A) + \sin^2(\omega_{inj}t + \phi_B) + \sin^2(\omega_{inj}t + \phi_C) \right] \quad (2.7)$$

The helicity injection is therefore positive and steady in spite of the AC operation of the circuits, which leads to the term Steady Inductive Helicity Injection, “SIHI” (cf. CHI from Sec. 2.2.2).

The inductively-driven injectors used on HIT-SI3 are a new area of study in spheromak sustainment research. While formation in HIT-SI3 follows the familiar Taylor relaxation and energy minimization paradigm of previous spheromak experiments, the way in which the injectors affect the spheromak is not yet completely understood. Global magnetic turbulence (see Fig. 2.5) is observed early in HIT-SI3 discharges simulated with

the NIMROD code [19], and this turbulence precedes a reconnection event that forms the spheromak [20]. After the spheromak has formed, the injectors continue to operate,

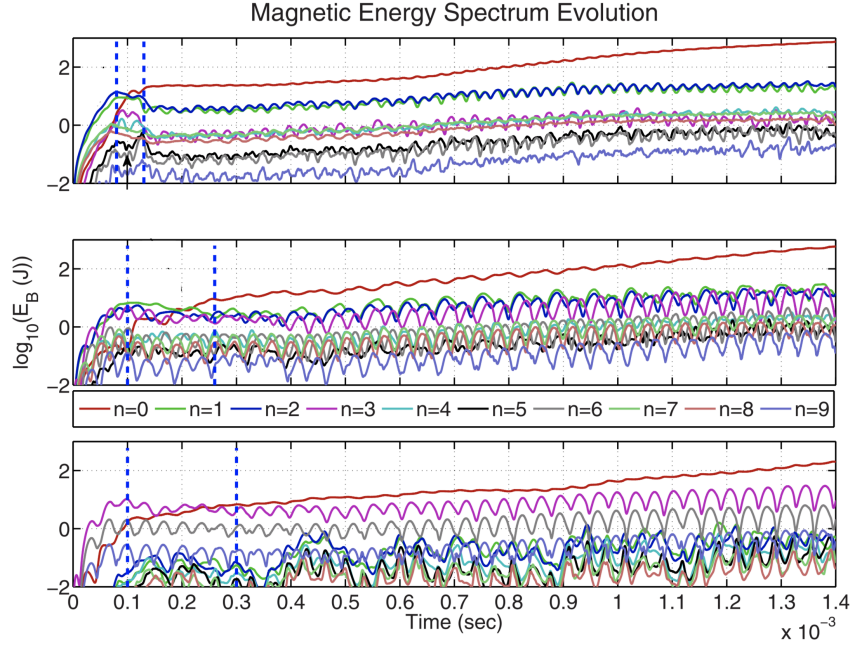


Figure 2.5: Simulated magnetic activity during typical HIT-SI3 discharges. Different colors denote different toroidal mode number as determined by Fourier analysis. The different plots are from shots with differing relative “ ϕ_A - ϕ_B - ϕ_C ” injector phasings: the upper plot was 0-120-240, the middle plot 0-60-120, and the lower plot 0-0-0. The vertical blue dashed lines mark the approximate start and end of the relaxation/formation phase of the discharge (From Ref. [20]).

expelling energy, helicity, and plasma into the confinement volume. The existence of a spheromak is supported by experimental measurements [21, 18, 22], and the injector behavior is fairly easy to measure directly, but the interface of these two systems is still unclear; exactly how the injectors interact with the spheromak plasma is still a main subject of research on the HIT-SI3 experiment.

2.3.1 Imposed Dynamo Current Drive

Imposed Dynamo Current Drive (IDCD) is a model that has been theorized to describe the mechanism by which the injectors sustain the spheromak current in HIT-SI3 plasmas. IDCD operates on the assumption that HIT-SI3 plasmas are well-described by the magnetohydrodynamic (MHD) Ohm's Law:

$$\mathbf{E} = \eta \mathbf{j} - \mathbf{v} \times \mathbf{B} + \frac{\mathbf{j} \times \mathbf{B}}{ne} \quad (2.8)$$

After Hossack, *et al.* (Ref. [23]), the pressure gradient term ∇p_e is assumed small, and the resistivity is taken to be low enough that the magnetic fields are frozen into the electron fluid. The model suggests that perturbations on current and magnetic field applied by the injectors are able to inject helicity across a closed surface of magnetic flux. The perturbations are represented in the mean dynamo theorem, in which quantities are taken to be the sum of an equilibrium (mean) value and some small fluctuating component:

$$\mathbf{j} = \mathbf{j}_0 + \delta \mathbf{j}, \quad \mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}, \quad \mathbf{v} = \mathbf{v}_0 + \delta \mathbf{v} \quad (2.9)$$

Substituting Eqs. 2.9 into 2.8 and taking the current-directed dot product of the resulting electric field gives the following:

$$\mathbf{j} \cdot \mathbf{E} = (\delta \mathbf{v} \times \delta \mathbf{B}) \cdot \mathbf{j}_0 + (\delta \mathbf{v} \times \delta \mathbf{j}) \cdot \mathbf{B}_0 + (\delta \mathbf{j} \times \delta \mathbf{B}) \cdot \mathbf{v}_0 + \eta (\mathbf{j} \cdot \mathbf{j}) \quad (2.10)$$

Eq. (2.10) demonstrates the possibility of applied perturbations counteracting the ohmic dissipation term. This would require that, for a volume surrounding a closed current loop,

$$-\oint \frac{\mathbf{j}}{|\mathbf{j}|} \cdot \mathbf{E} dl = V_{loop} = 0 \quad (2.11)$$

The model integrates the effect of the perturbations over the whole spheromak geometry, and can approximately predict the evolution of the toroidal current. Further, the model is supported by its agreement with observations of the spheromak magnetic field structure and the impedance experienced by the injectors [24].

Closed magnetic flux has not yet been observed in HIT-SI3 plasmas. While IDCD theory supports the ability to drive current without breaking the closed flux of a spheromak plasma, this has yet to be sufficiently tested in a real experiment. Simulations suggest that closed flux may develop in a decaying spheromak in a scaled-up version of the HIT-SI3 confinement volume [25], but the field lines in HIT-SI3-scale simulations remain mostly ergodic, filling the spheromak volume rather than lying on discrete surfaces.

This does not rule out the possibility of small regions of closed flux in HIT-SI3-scale plasmas; there may be effects not captured by the simulations, and in fact recent simulations of HIT-SI3 show some small and short lived regions of closed flux [26]. It is still expected that in general the degree of pressure confinement in HIT-SI3 is quite low. Localized measurements of density and temperature are used to determine to what extent HIT-SI3 heats and confines its plasma.

2.3.2 Spheromak Sustainment - Helicity, Reconnection, and Current Drive

If spheromaks are to serve as fusion plasmas they must be sustained against the dissipation that occurs due to non-zero resistivity. Both the energy and helicity of the spheromak decay through ohmic dissipation. Further, some magnetic energy is converted into kinetic energy during reconnection events that occur in relaxation, though helicity is conserved throughout reconnection.

The resistivity of the plasma causes both the conversion of current into thermal energy and the loss of helicity. The time derivative of the helicity is given by [27]:

$$\dot{K} = -2 \int_{\Omega} \mathbf{E} \cdot \mathbf{B} d^3r + 2 \int_{\Omega} \mathbf{E}_V \cdot \mathbf{B}_V d^3r \quad (2.12)$$

The first term represents the “plasma term,” or the loss of helicity through resistive dissipation. This term is often written as $\frac{K}{\tau_K}$, where τ_K is the characteristic helicity decay time, which is a constant if the system is assumed linear and time-invariant. The subscript V denotes vacuum fields, in other words the applied fields that serve as the source of

helicity. This second term is often written as $2V\Psi$, where Ψ is the magnetic flux along which the voltage V is applied. Eq. (2.12) can be rewritten as

$$\dot{K} = 2V\Psi - \frac{K}{\tau_K} \quad (2.13)$$

Injecting helicity is equivalent to driving current along magnetic field lines in a plasma. However, it cannot in general be considered to be a source of current drive within regions of closed magnetic flux. The challenge is to apply a voltage along a loop of current that exists on a closed magnetic field line.

The Sustained Spheromak Experiment (SSPX) sought to steadily replenish (without breaking the closed flux surfaces) the energy and helicity a spheromak loses through processes such as reconnection and ohmic dissipation. As discussed in Sec. 2.2.2, it used a coaxial gun to form the plasma and drive current between inner and outer electrodes. The SSPX discharge was essentially divisible into two periods or phases: the driven phase and the decaying phase. The driven (or refluxing) phase was marked by helicity injection, increased magnetic turbulence and reconnection activity, and poor confinement. The initial formation of the spheromak occurred during the driven phase, after which the the voltage applied across the electrodes was lowered and the spheromak would enter a decay phase. During the decay phase, when no helicity was being injected into the plasma volume, the confinement properties were observed to improve. In this phase the magnetic activity became more quiescent and ohmic heating raised the electron temperature up to nearly the ion temperature [28, 14].

During the decaying phases of an SSPX discharge, the high current density in the spheromak led to rapid ohmic heating. Rising temperature increases the thermal pressure and therefore the value of β in the plasma. While high β is desirable, the plasma thermal pressure can be a source of free energy for instabilities. These “pressure-driven instabilities” enforce β limits, past which the equilibrium cannot be stably sustained.

A fundamental example is the Mercier limit, which depends on how the thermal pressure gradient compares to the variation in pitch angle of the magnetic field lines of dif-

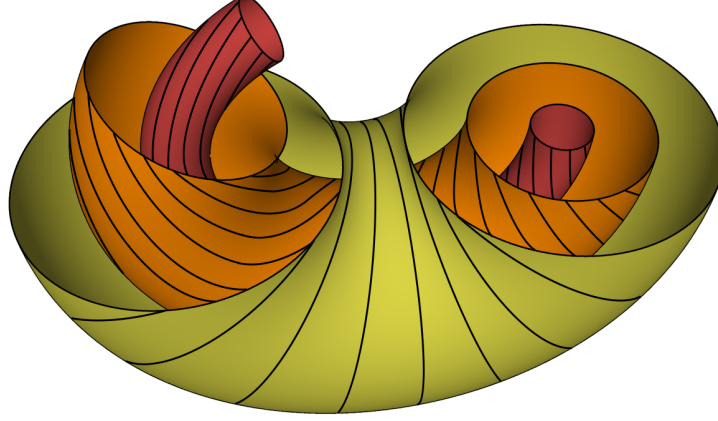


Figure 2.6: An illustration of three flux surfaces with surface magnetic field lines in black. This magnetic topology would be typical for a spheromak, with high safety factor near the magnetic axis (red surface) and low safety factor at the outer surface.

ferent flux surfaces. The pitch angle of the field lines in a toroidal configuration can be described by the parameter q , called the safety factor. In general, the safety factor can be thought of as ratio of the number of toroidal to poloidal field line transits made at a given minor radius r . In an intuitive sense q describes how tightly “twisted” the helical field lines on a given flux surface are (see Fig. 2.6). For a toroidal configuration with circular cross-section, q is [7]

$$q(r_0) = \frac{r_0 B_\phi}{R_0 B_\theta} \frac{1}{(1 - r_0^2/R_0^2)^{1/2}} \quad (2.14)$$

Here R_0 is the major radius, and toroidal and poloidal directions are denoted by ϕ and θ subscripts, respectively. The Mercier limit is described within the following criterion [29]:

$$\frac{1}{4} \left(\frac{q'(r)}{q(r)} \right)^2 + \frac{2\mu_0 p'(r)}{r B_\phi^2} (1 - q^2) > 0 \quad (2.15)$$

This criterion is a generalized version of a similar stability criterion developed by Suydam for a cylindrical geometry, and can be applied to toroidal plasmas regardless of the plasma shape, aspect ratio, or value of β [30]. The B_ϕ term is the toroidal magnetic field at the

magnetic axis. The Mercier criterion in this form makes clear that if the radial pressure gradient is negative, q has a stabilizing effect if $q^2 > 1$. The first term on the left shows the stabilizing effect of q shear.

It was shown in Ref. [31] that the Mercier β limit was reached on SSPX. This result, while reinforcing the ability of spheromaks to heat to a fundamental maximum without any auxiliary heating sources, was merely a verification of previous spheromak results [32]. The new result centered around the sustainment phase, during which magnetic reconnection events that converted toroidal magnetic flux into poloidal magnetic flux were observed. While the opening of flux surfaces during these reconnections allowed for the sustainment of toroidal current, it drastically increased the thermal conductivity to the wall, resulting in a vanishing energy confinement time [14]. The findings of the SSPX experiment did not support the idea of a steady-state spheromak-based fusion reactor.

2.4 Alternative diagnostics for measuring confinement properties

2.4.1 Far-Infrared Interferometer

The current standard for measuring electron density, n_e , on HIT-SI3 uses a Martin-Puplett interferometer operating at $184.3 \mu\text{m}$. This wavelength is chosen to allow for the electromagnetic radiation to propagate through the plasma ($\omega_{FIR} > \omega_{pe}$), to minimize diffraction effects in the plasma, and to reduce sensitivity to mechanical vibrations. As depicted in Fig. 2.7, the diagnostic involves two Far-InfraRed (FIR) lasing chambers (unfortunately dubbed “Master” and “Slave”) which are optically pumped by a $\sim 50 \text{ W}$ CO_2 laser operating at the 9R32 line, or $9.2 \mu\text{m}$. These two FIR chambers are linearly polarized orthogonally with respect to each other. The polarization difference simply allows the two beams of the interferometer to use the same optical beamline and turning mirrors, while still allowing for one beam to be selected for the plasma arm and one for the reference arm. The length of the chambers can be precisely adjusted to change their lasing frequencies and create some small $\Delta\omega = \omega_1 - \omega_2$. This is introduced so that the otherwise $\sim\text{THz}$ -frequency

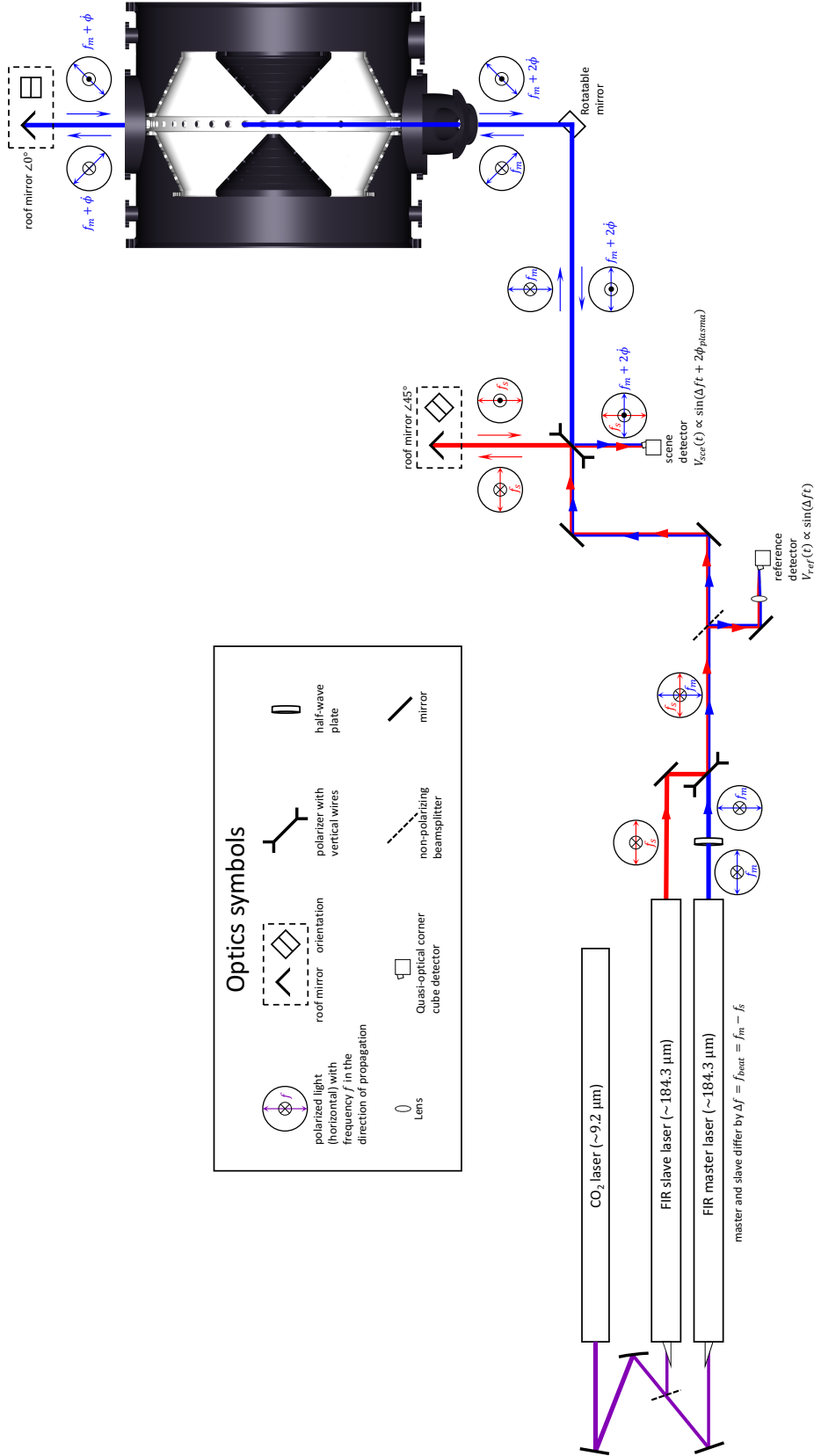


Figure 2.7: A schematic diagram of the Far-infrared (FIR) Interferometer. The beam from the master FIR chamber passes through the HIT-S13 chamber and will undergo a phase delay due to the index of refraction of the plasma.

laser radiation can be measured in the GHz range as a beat frequency between the two beams by detectors sensitive to incident power (the square of electric field, regardless of polarization). The two detectors are placed on the beamline so that one receives the beat frequency signal from the mixed beams before and incidence with the plasma, and the other receives signal from the mixed beams after one beam has traversed the plasma twice. The phase shift induced in this beam due to the plasma index of refraction is given by:

$$\Delta\phi = - \int \frac{n_e e^2 \lambda_{FIR}}{2\pi \epsilon_0 m_e c^2} dl \quad (2.16)$$

Therefore the line-integrate density is $\int n_e dl \propto \Delta\phi$, and the line-averaged density is arrived at by dividing the line-integrated density by twice the chord length.

Because the signal is phase-modulated, it is less susceptible to signal noise. This diagnostic also requires no calibration, as it is calibrated to zero plasma density during the pre-shot period when no plasma has yet broken down in the machine. Additionally, this approach to measuring electron density has high time resolution and is non-perturbative to the plasma. An example FIR density trace can be found in Fig. 2.10.

2.4.2 Langmuir Probe

A common method for measuring density and temperature in plasmas that do not approach fusion parameters is to use a Langmuir probe. The fundamental operation of Langmuir probes is based on the plasma Debye shielding and the sheath effect. In the most basic case, a single conducting probe is inserted into the plasma, and the voltage applied to it is swept to determine the steady-state electron temperature and density. The “triple-probe” method as currently implemented on HIT-SI3, the diagnostic consists of three separate but nearly collocated probes, one of which is allowed to float to the plasma potential. A fixed voltage is applied to the other two probes, and the current that flows into these is measured, allowing for the determination of electron density and temperature as functions of time in a single spatial location.

Example data from a single shot are shown in Fig. 2.10. In this figure can be seen the value of the triple probe, which is capable of time-resolved measurements only limited by the digitization rate. This is unique to the triple-probe as compared with the single-probe design, which requires a static plasma as the voltage sweep is carried out. Figs. 2.8 and 2.9 show an additional asset of Langmuir probe measurements, which is the spatial and temporal resolution available using manual scans of the probe insertion depth over a number of shots. The allows for a rough estimation of electron temperature, density, and pressure profiles through time, though this method relies on shot-to-shot repeatability in the experiment.

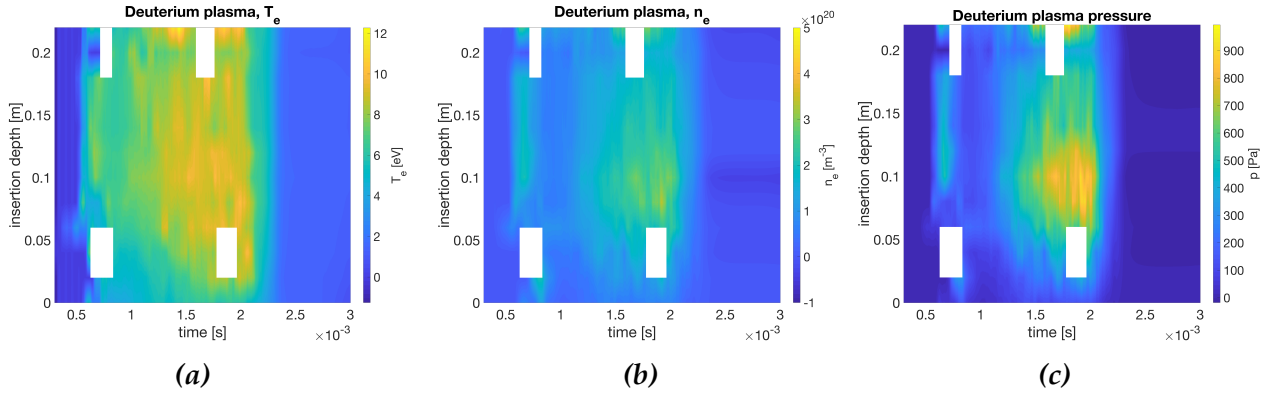


Figure 2.8: Measurements of electron temperature, density and the calculated relative pressure from a Langmuir probe radial scan of 22 cm insertion depth in Deuterium plasma.

Fig. 2.10 demonstrates some of the limitations of this diagnostic. In the strongly time-varying plasma the probe signals tend to saturate, leading to a loss of the measurement for large stretches of the shot. Additionally, as an electrical signal measurement, noise levels from the injectors are significant, leading to extremely large oscillations on the raw probe signals which must be highly smoothed to produce meaningful data. A third issue to identify is the discrepancy between the FIR and the Langmuir probe in measured density (also shown in Fig. 2.10). There is some disagreement between the two, and while the

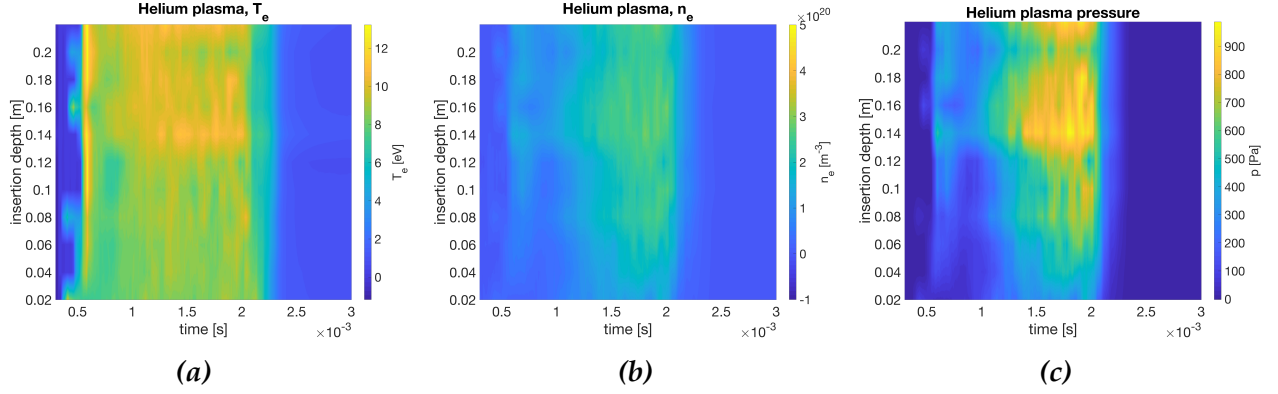


Figure 2.9: Measurements of electron temperature, density and the calculated relative pressure from a Langmuir probe radial scan of 22 cm insertion depth in helium plasma.

FIR is considered the more accurate measurement, it is chord-averaged in contrast to the single spatial point of the Langmuir probe. The effect of spatial density fluctuations in the HIT-SI3 experiment may also play a role, as the two measurements cannot be made at precisely the same location in the plasma. Finally, the Langmuir probe interacts with the plasma in ways that conceivably affect the plasma properties, including confinement. For all of these reasons, a more reliable method for measuring the temperatures and densities of HIT-SI3 plasmas, namely Thomson scattering, has been implemented.

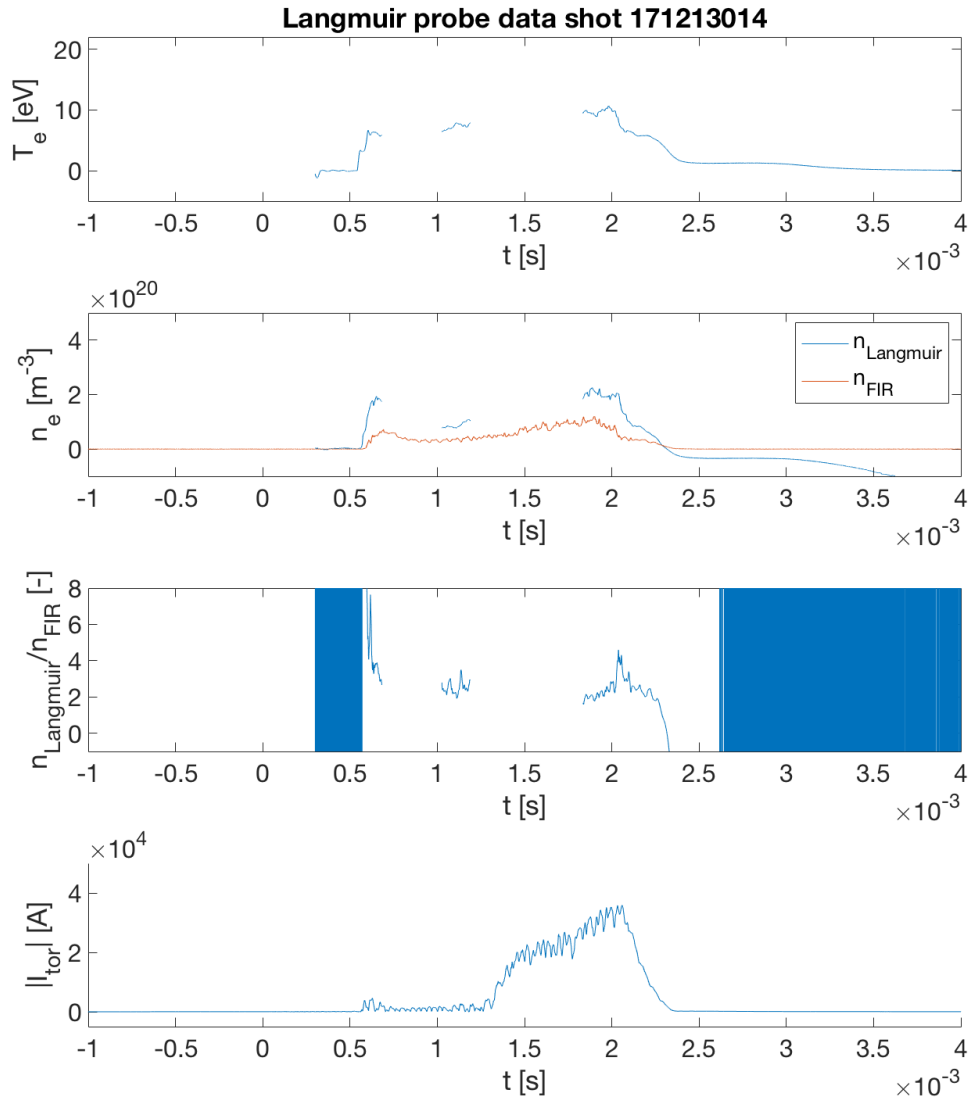


Figure 2.10: Example single-shot Langmuir probe data, including FIR density for comparison. The Langmuir probe measurement is made from the tip of the probe, and the insertion depth for this shot was as deep as possible without blocking the FIR chord, which intersects the path of the probe.

Chapter 3

THOMSON SCATTERING

3.1 *The Physics of Thomson Scattering*

The phenomenon of Thomson scattering can be used to measure the temperature and density of plasma electrons by observing how monochromatic light interacts with the plasma particles. Thomson scattering is a subset of Compton scattering — Compton scattering describes the behavior of an electromagnetic wave interacting with a free charged particle. The process involves a loss of energy in the incident photons due to inelastic scattering between the photon and the charge. In this case the particle (quantum) nature of light is clearly significant.

Thomson scattering is the classical limit of the Compton scattering phenomenon. For the Thomson scattering regime, the incident wave energy is much lower than the particle's rest energy, $\hbar\omega_i \ll mc^2$. Equivalently, the incident wavelength is much larger than the free charged particle's Compton wavelength, $\lambda_i \gg \lambda_{Compton} = \frac{h}{mc}$. The electron recoil is therefore very small and the scattering is taken to be elastic. That is, for Thomson scattering of photons with a stationary charge, $\omega_i = \omega_s$ [33]. The only alteration in the frequency of the scattered light is due to the original velocity of the charge. This fact is utilized to measure the thermal velocity of a volume of plasma electrons. The diagnostic method is to send a high-power laser pulse through the plasma and observe the spectrum of the Thomson-scattered light that results. The theory of the diagnostic is developed as follows, starting with the effect of such a laser pulse on a single plasma electron.

3.1.1 Single-Electron Scattering: Incident Beam

The electromagnetic fields associated with the input laser light are described by

$$\mathbf{E}_i = \mathbf{E}_0 \cos(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t) \quad (3.1)$$

$$\mathbf{B}_i = \mathbf{B}_0 \cos(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t)$$

where $E_0 = cB_0$, as is the case for all freely propagating electromagnetic waves. The incident wavevector is

$$\mathbf{k}_i = \frac{2\pi}{\lambda_i} \hat{\mathbf{i}} = \frac{\omega_i}{c} \hat{\mathbf{i}} \quad (3.2)$$

where $\hat{\mathbf{i}}$ is the direction of laser propagation, and λ_i and ω_i are the wavelength and angular frequency of the laser light wave, respectively. These fields produce a Lorentz force on the electron:

$$m_e \frac{d\mathbf{v}}{dt} = -e\mathbf{E}_i - e\mathbf{v} \times \mathbf{B}_i \quad (3.3)$$

To understand the relative magnitude of the second term, a conservative assumption is made that $\mathbf{v} \perp \mathbf{B}$, in which case eq. (3.3) gives the magnitude of the acceleration as

$$\frac{dv}{dt} = \frac{e}{m_e} E_i \left(1 + \frac{v}{c}\right) \approx \frac{eE_i}{m_e} \quad (3.4)$$

The contribution from the magnetic field to acceleration of the electron is negligible for non-relativistic¹ electron velocities. The acceleration induced by fields described in eqs. 3.1 is

$$\mathbf{a} = \frac{-e}{m_e} \mathbf{E}_i \quad (3.5)$$

3.1.2 Single-Electron Scattering: Scattered Radiation

The power of the radiation emitted when the electron is accelerated by the incident laser electric field is now examined following J.J. Thomson's original derivation (found in [34]).

¹For this work, non-relativistic can be taken to mean having a speed $v < 0.02c$, which corresponds to thermal velocity for electrons with energy ~ 100 eV. This is in fact a conservative boundary, as $\frac{v_{th}}{c} \sim 0.006$ at $T_e = 10$ eV, which is more typical in HIT-SI3 plasmas.

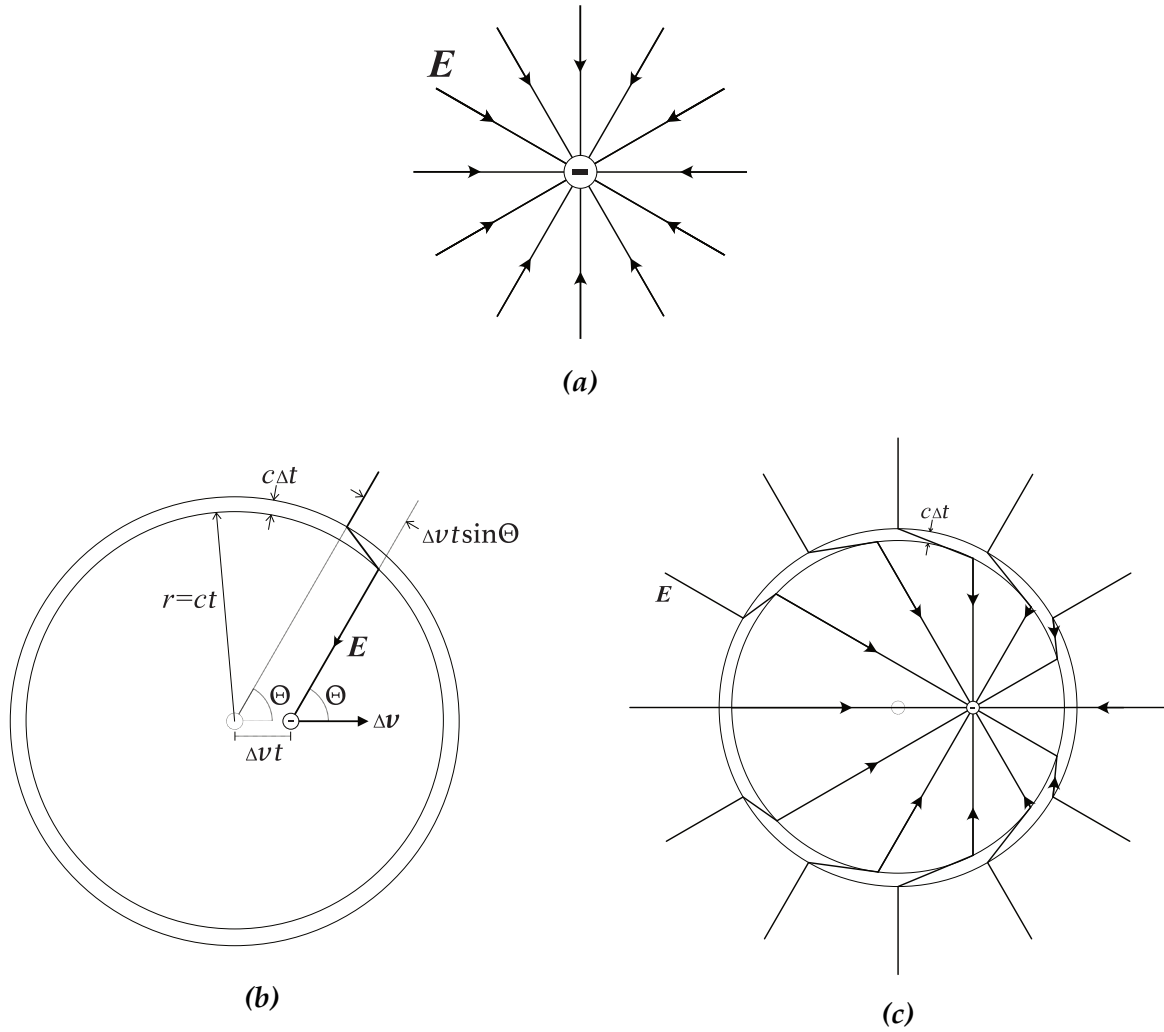


Figure 3.1: Purcell derivation of radiation from an accelerated charge. **(a)** The purely radial field of an electron before being accelerated. **(b)** During acceleration a tangential component of $\mathbf{E}$ arises. **(c)** The tangential component (which dominates far from the charge itself) is largest at $\Theta = \frac{\pi}{2}$, and vanishes in the direction parallel to the acceleration.

This derivation is more intuitive than those utilizing Liénard-Wiechert potentials in retarded time, which can be found in refs. [35, 36, 37] if an even longer story of Thomson scattering is desired.

Consider first an electron at rest. It produces a purely radial electric field (see fig. 3.1a) $\mathbf{E}$ as described by Coulomb's Law:

$$\mathbf{E} = \mathbf{E}_r = \frac{-e}{4\pi\epsilon_0} \frac{\hat{\mathbf{r}}}{r^2} \quad (3.6)$$

The charge is now accelerated to reach a velocity $\Delta v \ll c$ in a small interval of time Δt . The change this produces is encoded into the electric field as a tangential component (in the $\hat{\Theta}$ direction) and propagates away from the charge at the speed of light. Fig. 3.1b helps to demonstrate that in the spherical shell where this tangential component resides, the ratio of the tangential to radial field is:

$$\frac{|\mathbf{E}_\Theta|}{|\mathbf{E}_r|} = \frac{\Delta v t \sin\Theta}{c \Delta t} \quad (3.7)$$

With eq. (3.6), the tangential field is then

$$|\mathbf{E}_\Theta| = \frac{\Delta v}{\Delta t} \frac{t \sin\Theta}{c} \frac{e}{4\pi\epsilon_0 r^2} \quad (3.8)$$

The tangential perturbation on the radial electric field reaches a distance $r = ct$ at time t :

$$|\mathbf{E}_\Theta| = \frac{\Delta v}{\Delta t} \frac{r}{c^2} \frac{e}{4\pi\epsilon_0 r^2} \sin\Theta = \frac{|\mathbf{a}| e \sin\Theta}{4\pi\epsilon_0 c^2 r} \quad (3.9)$$

The total field scattered by the electron is then

$$\mathbf{E}_s = \frac{-e}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}} + \frac{a e \sin\Theta}{4\pi\epsilon_0 c^2 r} \hat{\Theta} \quad (3.10)$$

eq. (3.10) yields two immediate observations. First, the field is azimuthally symmetric, that is it depends only on r and Θ , with $\Theta = 0$ defined as the applied acceleration vector (in this case the linearly polarized $\mathbf{E}_i$ of the laser). Second, the radial component is inversely proportional to r^2 , such that at sufficiently large r , the $\hat{\Theta}$ component dominates

(fig. 3.1c).

Using eq. (3.5) in eq. (3.10) and assuming the tangential field dominates (in this case, it accounts for more than 99% of the magnitude of $\mathbf{E}_s$ where $r = 35$ cm) gives

$$\mathbf{E}_s = \frac{e^2 \sin \Theta}{4\pi \epsilon_0 m_e c^2 r} E_0 \cos(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t) \quad (3.11)$$

The Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{B} / \mu_0$ of an electromagnetic wave gives its power deposition per unit area (“intensity” or “radiant flux density”, in units of $\text{W}\cdot\text{m}^{-2}$):

$$|\mathbf{S}| = \frac{1}{\mu_0} |\mathbf{E}| |\mathbf{B}| = \frac{1}{c\mu_0} |\mathbf{E}|^2 = \frac{e^4 \sin^2 \Theta}{16\pi^2 \mu_0 \epsilon_0^2 m_e^2 c^5 r^2} E_0^2 \cos^2(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t) \quad (3.12)$$

Averaging over the sinusoidal variation with r and t , and noting that the classical electron radius $r_e = \frac{e^2}{4\pi \epsilon_0 m_e c^2}$, the Poynting flux of the light emitted by a single electron being accelerated by the incident laser becomes:

$$|\mathbf{S}| = \frac{dP_s}{dA} = \frac{\epsilon_0 c E_0^2}{2} \frac{r_e^2}{r^2} \sin^2 \Theta \quad (3.13)$$

The dependence on r can be removed if eq. (3.13) is rewritten in terms of the radiant intensity (units of $\text{W}\cdot\text{Sr}^{-1}$). Utilizing $dA = r^2 d\Omega$,

$$\frac{dP_s}{d\Omega} = \frac{\epsilon_0 c E_0^2}{2} r_e^2 \sin^2 \Theta = \frac{\bar{P}_i}{A_{beam}} r_e^2 \sin^2 \Theta \quad (3.14)$$

This is the power scattered by the electron into an infinitesimal solid angle, $d\Omega$. The incident laser intensity $\bar{P}_i / A_{beam}$ is the average power in the laser pulse divided by the cross-sectional area of the beam. Fig. 3.2 shows the two-dimensional cross-section and three-dimensional distribution of the scattered power.

Here it is helpful to take a closer look at the geometry of the scattering being described. To rewrite eq. (3.14) in terms of the experimental geometry, it is convenient to use a right-handed orthogonal basis of unit vectors $\{\hat{\mathbf{i}}, \hat{\mathbf{e}}_s, \hat{\mathbf{e}}_n\}$. As shown in fig. 3.3, $\hat{\mathbf{i}}$ is the propagation vector of the laser. Unit vector $\hat{\mathbf{e}}_s$ is defined to be in the plane of both $\hat{\mathbf{i}}$ and $\hat{\mathbf{s}}$ ($\hat{\mathbf{s}}$ points toward the Thomson scattering detector, so the choice of experimental

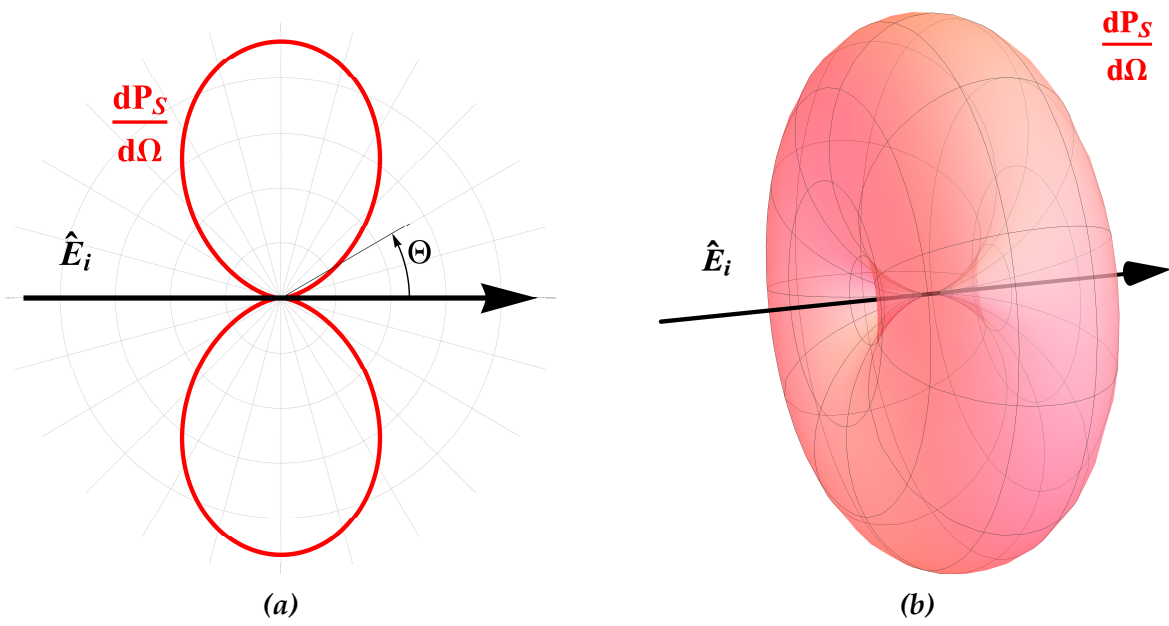


Figure 3.2: (a) 2-D cross-section of the angular distribution of radiation scattered from non-relativistic charges undergoing linear (dipole) acceleration along the electric field polarization vector of an incoming light wave. (b) 3-D distribution of (a).

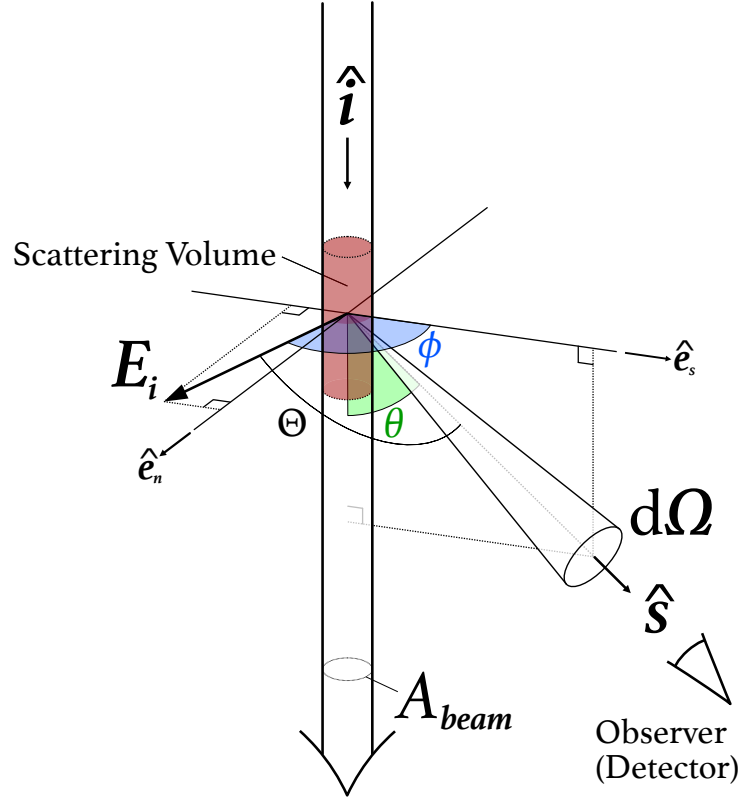


Figure 3.3: The Thomson scattering geometry for an example diagnostic setup.

setup sets $\hat{e}_s$). Finally, $\hat{e}_n \equiv \hat{\mathbf{i}} \times \hat{e}_s$.

In choosing the experimental setup, the angles θ and ϕ are set: θ is the angle between the incoming and scattered radiation, and ϕ specifies the orientation of the laser polarization in the plane perpendicular to $\hat{\mathbf{i}}$. The preceding analysis leading to eq. (3.14) can be updated by redefining unit vectors for the electric field polarization and the scattering direction with the new basis vectors:

$$\begin{aligned}\hat{\mathbf{s}} &= \cos\theta \hat{\mathbf{i}} + \sin\theta \hat{e}_s + 0 \hat{e}_n \\ \hat{\mathbf{E}}_i &= 0 \hat{\mathbf{i}} + \cos\phi \hat{e}_s + \sin\phi \hat{e}_n\end{aligned}$$

Using the definition of the dot product and a trigonometric identity,

$$\begin{aligned}\hat{\mathbf{s}} \cdot \hat{\mathbf{E}}_i &= \sin\theta\cos\phi = |\hat{\mathbf{s}}||\hat{\mathbf{E}}_i|\cos\Theta = \cos\Theta \\ \sin^2\Theta &= 1 - \cos^2\Theta = 1 - \sin^2\theta\cos^2\phi\end{aligned}\tag{3.15}$$

Then eq. (3.14) can be rewritten as:

$$\frac{dP_s}{d\Omega} = \frac{\bar{P}_i}{A_{beam}} r_e^2 (1 - \sin^2\theta\cos^2\phi)\tag{3.16}$$

Integrating over all solid angles gives the total scattered power from a single electron being irradiated by the laser²:

$$P_s = \int \int \frac{dP_s}{d\Omega} d\Omega = \int_0^{2\pi} \int_0^\pi \frac{\bar{P}_i}{A_{beam}} r_e^2 (1 - \sin^2\theta\cos^2\phi) d\theta \sin\theta d\phi = \frac{8\pi\bar{P}_i}{3A_{beam}} r_e^2\tag{3.17}$$

Eq. (3.17) demonstrates a major difficulty with Thomson scattering diagnostics. Even if it were possible to capture all of the scattered light, since $8\pi r_e^2/3 = \sigma_{TS} = 6.63 \times 10^{-29} \text{ m}^2$, scattered power levels are a very small fraction of the incident power. In practice, other factors reduce the scattered signal even further. The diagnostic therefore usually requires very high incident power from the laser source in order to achieve passable signal levels.

3.1.3 Single-Electron Scattering: Origin of the Doppler Shift

Suppose now that the electron has some non-relativistic velocity $\mathbf{v}$ prior to being irradiated by the laser. Consequentially, the frequency of the scattered radiation as observed at a stationary detector (having a wavevector $\mathbf{k}_s$ as in eq. (3.2) along $\hat{\mathbf{s}}$) will be Doppler shifted by two effects. The first is the component of the electron velocity that is parallel or

²It may be noted here that substituting the large- r approximation of the electric field magnitude (eq. (3.9)) into eq. (3.12) and integrating over all solid angles (as in eq. (3.17)) results in the more general “Larmor formula”: $P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}$. This is the total power that a non-relativistic point charge ($q = e$) radiates when accelerated at a .

anti-parallel to the laser propagation vector $\hat{\mathbf{i}}$.

$$\Delta\omega_i = \left(\frac{c - \mathbf{v} \cdot \hat{\mathbf{i}}}{c} - 1 \right) \omega_i = - \left(\mathbf{v} \cdot \hat{\mathbf{i}} \right) \frac{\omega_i}{c} \quad (3.18)$$

This captures the fact that if the electron moves against the direction of laser propagation ($\mathbf{v} \cdot \hat{\mathbf{i}} < 0$), the laser appears blue shifted (higher energy), and the apparent wavelength is higher than that in the lab frame. The second effect is the component of the electron velocity parallel or anti-parallel to $\hat{\mathbf{s}}$:

$$\Delta\omega_s = \left(\frac{c}{c - \mathbf{v} \cdot \hat{\mathbf{s}}} - 1 \right) (\omega_i + \Delta\omega_i) = \frac{c - \mathbf{v} \cdot \hat{\mathbf{i}}}{c - \mathbf{v} \cdot \hat{\mathbf{s}}} \omega_i - \Delta\omega_i - \omega_i \quad (3.19)$$

The total Doppler shift is then:

$$\Delta\omega = \Delta\omega_s + \Delta\omega_i = \frac{1 - \frac{\mathbf{v}}{c} \cdot \hat{\mathbf{i}}}{1 - \frac{\mathbf{v}}{c} \cdot \hat{\mathbf{s}}} \omega_i - \omega_i = \frac{\frac{\omega_i}{\omega_s} \mathbf{v} \cdot \mathbf{k}_s - \mathbf{v} \cdot \mathbf{k}_i}{1 - \frac{\mathbf{v}}{c} \cdot \hat{\mathbf{s}}} \quad (3.20)$$

This result may be simplified if velocity of the electron is assumed non-relativistic, and if $\omega_s \approx \omega_i$ in an elastic scattering event. The latter may seem a strange assumption to make when defining the total Doppler shift between ω_s and ω_i . However, in the context of the diagnostic as described in eq. (3.20), the geometry between wavevectors (see fig. 3.4) will play a more important role than the difference in their magnitudes. With these, the total Doppler shift may be written as

$$\Delta\omega = (\mathbf{k}_s - \mathbf{k}_i) \cdot \mathbf{v} = \mathbf{k} \cdot \mathbf{v} \quad (3.21)$$

For electron velocities perpendicular to $\mathbf{k}$, the Doppler shifts in eqs. (3.18) and (3.19) will cancel each other out. The magnitude of $\mathbf{k}$ is

$$\begin{aligned} |\mathbf{k}| &= |\mathbf{k}_s - \mathbf{k}_i| = \sqrt{\mathbf{k} \cdot \mathbf{k}} \\ |\mathbf{k}| &\approx 2|\mathbf{k}_i| \sin \frac{\theta}{2} \end{aligned} \quad (3.22)$$

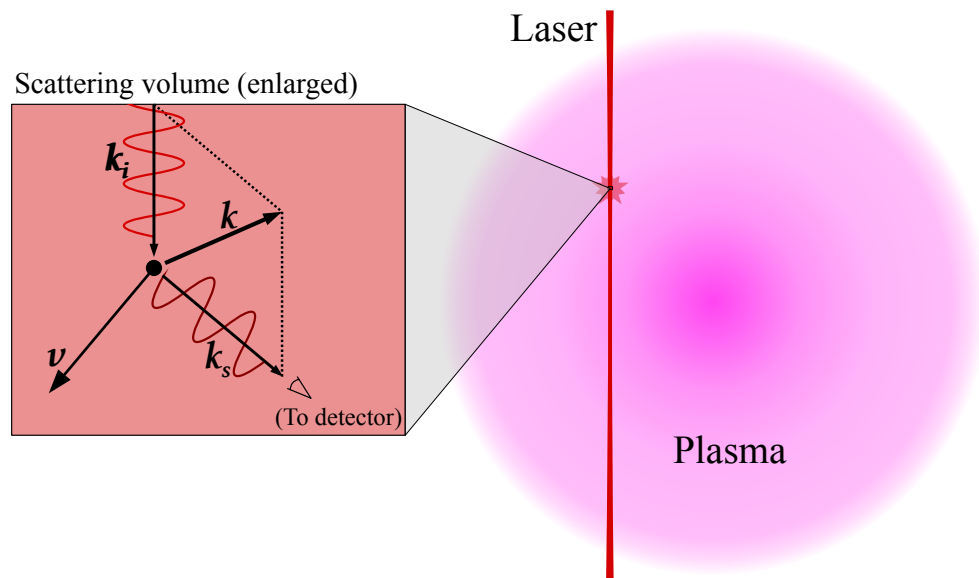


Figure 3.4: A cartoon depicting the process of Thomson scattering in a plasma diagnostic. A single electron in the scattering volume is shown whose velocity v causes a Doppler shift in the Thomson scattered radiation, eq. (3.21).

3.1.4 Plasma Scattering: Origin of the Spectrum

The analysis is now extended to the scattered power from all electrons in a specific volume of the plasma being irradiated by the laser (as in fig. 3.3). Attention is limited to a specific length segment of the beam, L , which is set by the collection optics in the particular diagnostic setup (see sec. 3.2.2). The volume of plasma in question is then $V = A_{beam}L$, and the number of electrons contributing to the scattered light depends on the number density, $N = n_e V$. For now the scattering effects of ions in the plasma are ignored, as their large mass will greatly reduce their radiated power ($P_s \propto a^2 \propto (q/m)^2$). Using the equation for scattered power from one electron (eq. (3.16)), the total power scattered through some solid angle $d\Omega$ by the electrons in volume V is then

$$\frac{dP_s}{d\Omega} = \frac{\bar{P}_i}{A_{beam}} N r_e^2 (1 - \sin^2 \theta \cos^2 \phi) = \bar{P}_i n_e L r_e^2 (1 - \sin^2 \theta \cos^2 \phi) \quad (3.23)$$

The solid angle of collection can be assumed to be small enough to be approximated by $d\Omega \sim \Delta\Omega$, rather than taking the full integral over θ and ϕ . Further, $\phi = \pi/2$ is a diagnostic design choice that maximizes the scattered power, and corresponds to the HIT-SI3 implementation (see sec. 3.2.2). These geometric simplifications lead to:

$$P_s = \bar{P}_i n_e L \Delta\Omega r_e^2 \quad (3.24)$$

It is useful to be able to describe the subset of scattered power within a certain range of frequencies between ω_s and $\omega_s + d\omega_s$. Each electron will have its own Doppler shift, directly proportional to its velocity as described in sec. 3.1.3. At present it is assumed that the electron thermal velocity dominates and thus $v = v_{th}$ in eq. (3.21). To arrive at a spectrally-resolved form of eq. (3.24), the electron velocity distribution function (EVDF), f , is used, which describes the fraction of the electron population with a given velocity:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(v_x, v_y, v_z) dv_x dv_y dv_z = 1 \quad (3.25)$$

When in thermodynamic equilibrium, the electrons in V are assumed to have a Maxwellian EVDF:

$$f(v_x, v_y, v_z) = \left(\frac{m_e}{2\pi k_B T} \right)^{3/2} \exp \left[\frac{-m_e (v_x^2 + v_y^2 + v_z^2)}{2k_B T} \right] \quad (3.26)$$

where k_B is the Boltzmann constant and T is the temperature.

It is convenient now to work in a 3-D Cartesian coordinate system with orthogonal basis vectors $\{\hat{x}, \hat{y}, \hat{k}\}$, where the unit vector $\hat{k}$ is parallel to $\mathbf{k}$ from eq. (3.21). The EVDF can therefore be redefined in these coordinates, and the velocity components v_x and v_y (which are perpendicular to $\mathbf{k}$ and thus contribute no frequency shift) can be integrated away:

$$1 = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(v_x, v_y, v_k) dv_x dv_y \right) dv_k = \int_{-\infty}^{\infty} \sqrt{\frac{m_e}{2\pi k_B T}} \exp \left[\frac{-m_e v_k^2}{2k_B T} \right] dv_k \quad (3.27)$$

Eq. (3.24) is now multiplied by 1 in the form of eq. (3.27), resulting in

$$P_s = \bar{P}_i n_e L \Delta \Omega r_e^2 \int_{-\infty}^{\infty} \sqrt{\frac{m_e}{2\pi k_B T}} \exp \left[\frac{-m_e v_k^2}{2k_B T} \right] dv_k \quad (3.28)$$

The “sifting” property³ of the Dirac delta function (which has the inverse units of its argument) can be used to select the fraction of the scattered power in the infinitesimal frequency range around a scattered wave for which $(\omega_s - \omega_i) = \Delta\omega$. First,

$$\frac{dP_s(\Delta\omega)}{d\omega} = \bar{P}_i n_e L \Delta \Omega r_e^2 \int_{-\infty}^{\infty} \sqrt{\frac{m_e}{2\pi k_B T}} \exp \left[\frac{-m_e v_k^2}{2k_B T} \right] \delta((\omega_s - \omega_i) - \Delta\omega) dv_k \quad (3.29)$$

Now another property⁴ of the delta function is noted, and this with eq. (3.21) leads to

$$\begin{aligned} \frac{dP_s(\Delta\omega)}{d\omega} &= \bar{P}_i n_e L \Delta \Omega r_e^2 \int_{-\infty}^{\infty} \sqrt{\frac{m_e}{2\pi k_B T}} \exp \left[\frac{-m_e v_k^2}{2k_B T} \right] \frac{1}{k} \delta(v_k - \frac{\Delta\omega}{k}) dv_k \\ &= \bar{P}_i n_e L \Delta \Omega r_e^2 \frac{1}{k} \sqrt{\frac{m_e}{2\pi k_B T}} \exp \left[\frac{-m_e \left(\frac{\Delta\omega}{k} \right)^2}{2k_B T} \right] \end{aligned} \quad (3.30)$$

This is the power scattered in the solid angle $\Delta\Omega$ and the infinitesimal frequency interval

³For delta functions: $\int_{-\infty}^{\infty} f(x) \delta(x - x_0) dx = f(x_0)$

⁴For some constant a : $\delta(ax) = \frac{\delta(x)}{|a|}$

$d\omega$, as a function of the Doppler shift $\Delta\omega$. It can be rewritten in terms of the wavelength as

$$\frac{dP_s(\Delta\lambda)}{d\lambda} = \frac{\bar{P}_i n_e L \Delta \Omega r_e^2}{2\lambda_i \sin(\theta/2)} c \sqrt{\frac{m_e}{2\pi k_B T}} \exp \left[\frac{-m_e \left(\frac{c\Delta\lambda}{2\lambda_i \sin(\theta/2)} \right)^2}{2k_B T} \right] \quad (3.31)$$

since $d\omega \approx 2\pi c d\lambda/\lambda_i^2$, and $\frac{\Delta\omega}{k} \approx \frac{c\Delta\lambda}{2\sin(\theta/2)\lambda_i}$ (using the approximation $\omega_s \approx \omega_i$). Fig. 3.5 shows example distributions of eq. (3.31).

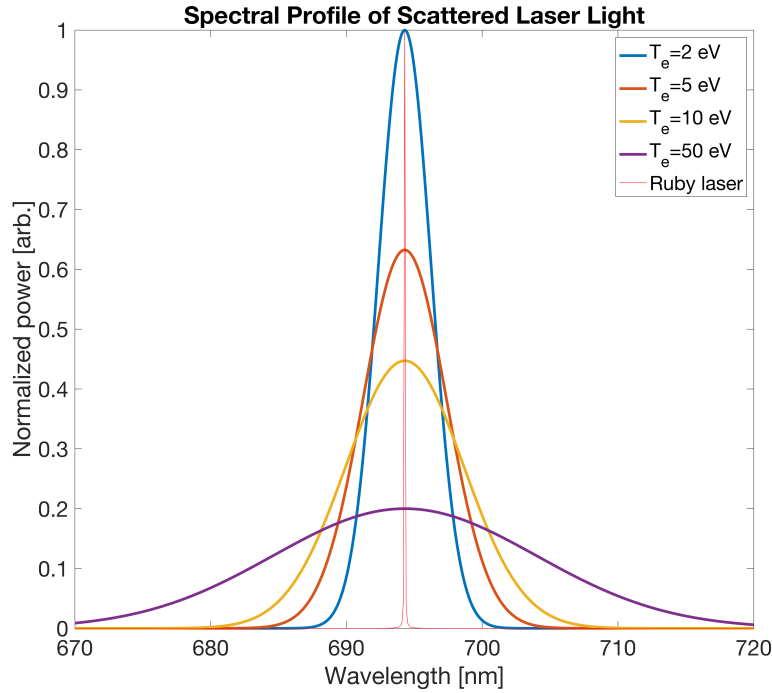


Figure 3.5: Normalized scattered power distributions based on eq. (3.31). Hotter plasmas have broader distributions, and the area under the curve scales linearly with density.

Finally it is observed that for a monochromatic light pulse with energy E and duration τ , $P = E/\tau$. Further, $E = N_{ph} \frac{hc}{\lambda_{ph}}$ for a number of photons N_{ph} at wavelength λ_{ph} (h is the Planck constant). Eq. (3.31) can finally be written in terms of number of scattered photons N_s that have been Doppler shifted from the laser wavelength λ_i to within the range $\lambda + d\lambda$,

$$\frac{dN_s(\lambda)}{d\lambda} = \frac{E_i n_e L \Delta \Omega r_e^2}{2h \sqrt{\pi} \sin(\theta/2)} \frac{1}{v_{th}} \exp \left[\frac{-c^2 (\lambda - \lambda_i)^2}{4v_{th}^2 \lambda_i^2 \sin^2(\theta/2)} \right] \quad (3.32)$$

where v_{th} is the mean thermal velocity of the the electrons in the volume $v_{th} = \sqrt{2k_B T_e / m_e}$.

3.1.5 Incoherent vs. Collective scattering

In sec. 3.1.4, it has been assumed that the scattering from a volume of plasma V is simply the sum of single-electron scattering for all electrons in the volume. The direct scattering contribution from individual ions may still be considered negligible due to their large mass, but in the quasineutral HIT-SI3 plasma, the possibility must be considered that collective effects will influence the scattering behavior. Further discussions on this topic are found in refs. [33, 37, 38].

In general, if the scattered radiation has a wavelength near the same scale as the Debye shielding length, it will interact with both the ion- and electron-shielded charge distributions surrounding its source electron. This is referred to as “collective” or “coherent” scattering, as the scattered radiation from the original charge and its shielding charges may add coherently. In this case, ion and electron shielding of electrons each contribute separately to the total scattered power, and the analysis of sec. 3.1.4 is insufficient. Salpeter (ref. [39]) is credited with the most useful early derivation of the full collective spectral distribution.

If the scattered radiation has a much shorter wavelength than the Debye length, the positions of the electrons in the shielding cloud are sufficiently stochastic on the length scale of the scattered wavelength. The contributions from the individual electrons and their shielding charge clouds are unable to add coherently. Thus the scattering is said to be “incoherent.” In this case the Doppler-broadened spectrum of scattered light from the volume is due to the sum of Doppler shifted contributions from individual electrons.

The parameter used to delineate between the two scattering regimes is called the “scattering parameter” α . Defined below for non-relativistic electrons, it is essentially

a comparison of the scattering wavelength to the Debye length (λ_D). The angle at which the scattering volume is observed plays a role as well, as it affects the magnitude of the Doppler shift, which determines the scattered wavelength.

$$\alpha = \frac{1}{k\lambda_D} = \frac{1.08 \times 10^{-14} \lambda_i}{\sin(\frac{\theta}{2})} \sqrt{\frac{n_e}{T_e}} \quad (3.33)$$

where λ_i is in nanometers, n_e is in m^{-3} , and T_e is in eV. If $\alpha \ll 1$, the scattered wavelength is much smaller than the Debye length, and the scattering is considered incoherent. This is the case for HIT-SI3 plasmas. In the conservative estimate of $n_e = 10^{20} \text{m}^{-3}$ and $T_e = 1 \text{eV}$, $\alpha = 0.1$, and it is expected to typically be at least an order of magnitude lower than this. Therefore the analysis in sec. 3.1.4 is valid and eq. (3.32) may be interpreted as an accurate representation of the scattered spectrum in HIT-SI3 plasmas.

3.2 The HIT-SI3 Thomson Scattering System

The production and installation of the Thomson scattering system involved combining equipment from two predecessor Thomson scattering experiments and customizing them to work on HIT-SI3. In most cases the design “choices” were dictated by the parameters of the equipment that was already available or installed on the machine. The design of the HIT-SI3 Thomson scattering diagnostic is detailed in the following subsections, following the general considerations for Thomson scattering as laid out in ref. [37]. Fig. 3.12 summarizes the components and layout of the diagnostic.

3.2.1 Choice of photon source

As discussed in Sec. 3.1.2, the photon source in Thomson scattering must be powerful enough to overcome the small ratio of scattered power to incident power. For HIT-SI3, a ruby laser operating at 694.3 nm is used (see inset, fig. 3.12). The laser system was manufactured by Lumonics, and was loaned to the HIT-SI3 lab from the University of Texas at Austin. For successful Thomson scattering measurements, the source radiation must

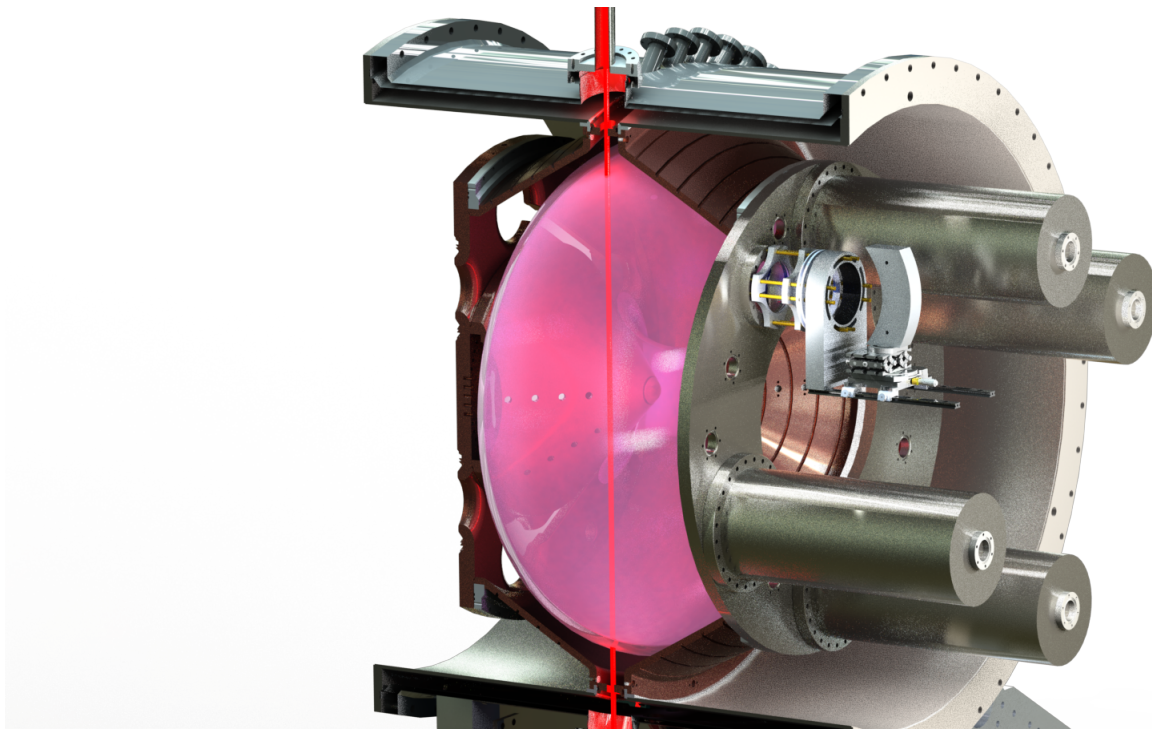


Figure 3.6: A representation of the TS system during a plasma measurement. The vacuum vessel and confinement volume have been partially cut away to view the plasma and laser beam geometry.

Description	Requirement	Typical values	
Laser Propagation	$\omega_{inc} > \omega_{pe}$	ω_{inc}	2.7×10^{15} rad/s
		ω_{pe}	4×10^{11} rad/s
Incoherent Scattering	$\alpha = \frac{1}{k\lambda_D} < 0.1$	α	0.02
Sufficient Power	$P_{inc} > 10$ MW	P_{inc}	> 100 MW (20 ns pulse, $E > 2$ J)
Low Beam Divergence	$\varnothing_{beam} \ll L_{plasma}$	$\varnothing_{beam}$	< 1 in. ($\theta_{beam} = 0.41$ mrad)
Monochromatic Light	$FWHM_{inc} \ll FWHM_{scat}$	$FWHM_{inc}$	0.04 nm
		$FWHM_{scat}$	4 nm (at $T_e = 2$ eV)

Table 3.1: Requirements for a Thomson scattering photon source and the relevant parameters of the HIT-SI3 ruby laser.

meet certain requirements based on the specific system and the plasma to be measured. Table 3.1 shows a summary of the relevant criteria and the extent to which they are satisfied.

3.2.2 Choice of scattering geometry and collection optics

The orientation of the collection optics was chosen to maximize the optical access to the laser beamline and to ensure sufficient signal from the scattering volume. The choice of scattering geometry is essentially a matter of selecting the angles in eq. (3.23). In the case of the HIT-SI3 TS system, the collection optics (and therefore the scattering vector) are oriented at 90° from both the laser propagation direction and the laser electric field vector. This allows for acceptable signal power, minimizes parasitic “stray” laser light, and ensures measurements are in the incoherent scattering regime.

The optical components that collect Thomson-scattered photons into the polychromators include two lenses to image the scattering volumes onto three bundles of optical fibers (one bundle per spatial measurement). The objective (plasma facing) lens has a diameter of 95 mm and a focal length of 500 mm, and the secondary lens has diameter 89 mm and focal length 100 mm. The mount apparatus for the lenses and fiber bundles allows for fine adjustments to be made during alignment of the collection optics.

Each fiber bundle comprises 121 individual fibers made of hard clad pure silica with

a core diameter of $200\ \mu\text{m}$, and an acceptance full cone angle of $\sim 25^\circ$ ($\text{NA} = 0.22$). They are packed into a $5\ \text{mm} \times 1.6\ \text{mm}$ bundle. Taking into account the transmission properties of each fiber and the overall packing efficiency given their cladding and jacket diameters, each bundle transmits about 45% of the incident light. Each bundle carries the scattered light (and unwanted optical noise) from the plasma volume it views to a polychromator to resolve the spectrum given by eq. (3.32).

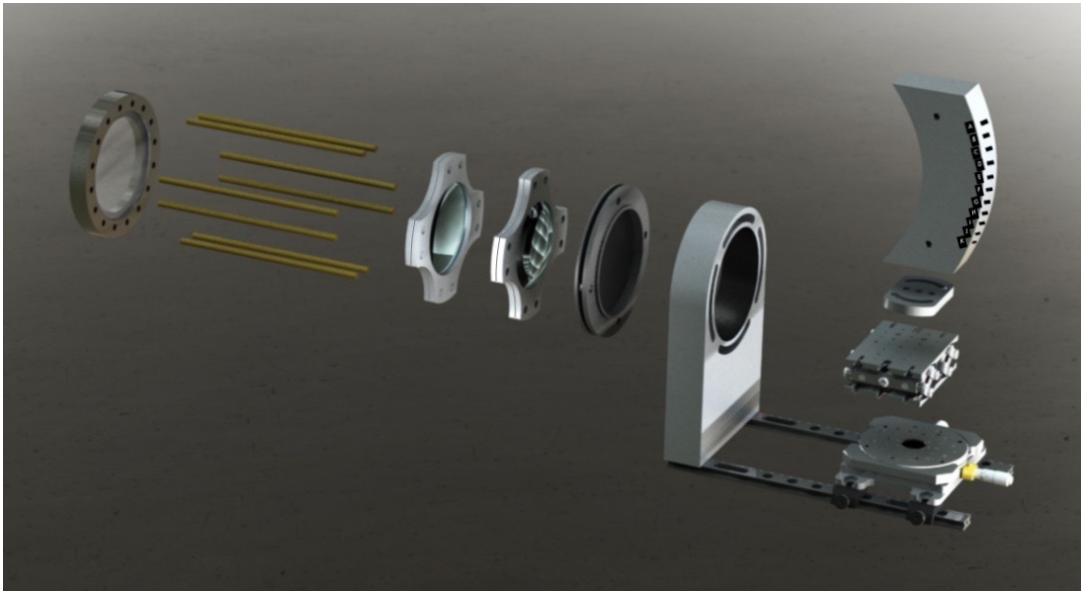


Figure 3.7: The collection elements in exploded view along the optical axis. From left to right: Conflat with anti-reflective coated window ($\varnothing 4.5$ -inch), 8 brass threaded rods, objective lens, secondary lens, polarization filter, fiber mounting hardware (exploded along the vertical axis, fibers not shown).

Figure 3.7 shows a closer exploded view of the apparatus installed to hold the collection optics. A conflat flange with 4.5-inch diameter window (anti-reflective coated for light near $694.3\ \text{nm}$) is mounted on the side of HIT-SI3 opposite from the injectors. This serves as the main TS view port. In eight of the available bolt holes, 8-inch brass $\frac{5}{16}$ -24 threaded rods are fitted that serve as optical rails for the remaining components. The two lenses are held in water-jet-cut aluminum fittings, with the objective lens situated as close as possible to the window. Score marks on the outer surfaces of the lens holders are used

to determine the distance between the lenses (3.149 inches in this case).

Next, a polarization filter aligned with the laser polarization vector is added to reduce reflected stray and unpolarized background light. Finally, the fiber mounting hardware is fitted via a heavy mount collar to which two dove-tail rails are attached. The fibers are mounted to image different points in the plasma — their faces lie tangent to the front-facing arc. In addition to the rails and the slots in the mount collar, fine control of the position and orientation of this arc is achieved with a lab jack and a translation stage with two-linear-axis and azimuthal adjustment. The front feet on which the stage is held to the rails are positioned at the 74 mm mark.

The alignment is performed by illuminating the fibers in the reverse direction to produce an image on a physical target that lies on the laser beam-line (fig. 3.8). A weighted string with demarcations at 1 cm intervals was used, as it was necessary to be able to remove the target through a small port once the machine was closed. A diode laser that is co-linear with the Thomson laser was used to ensure the target followed the correct beam path, and measurements were made with the target in place to verify and record the coordinates of the Thomson scattering collection locations.

Finally, the spots illuminated on the target by the back-lit fibers were checked for image focus, and their locations were recorded at various lab jack height settings. This was performed for every possible mount point in the fiber holder (eleven positions, labeled “A-K,” are available, see table 3.2). The alignment procedure revealed that the current TS diagnostic can measure points in the plasma spanning major radii from 22.9 cm to 44.7 cm (fig. 3.9). In practice, the small adjustments in the alignment process determine the details of the diagnostic. Lens calculations are helpful in the initial design of the collection optics, but the non-ideal nature of the real implementation requires the ability to fine tune. The actual measured scattering regions are approximately 1.5 cm in length, with diameters equal to that of the ruby laser beam, which is focused to roughly 5 mm by a $f = 3$ m converging lens “upstream” on the beamline.

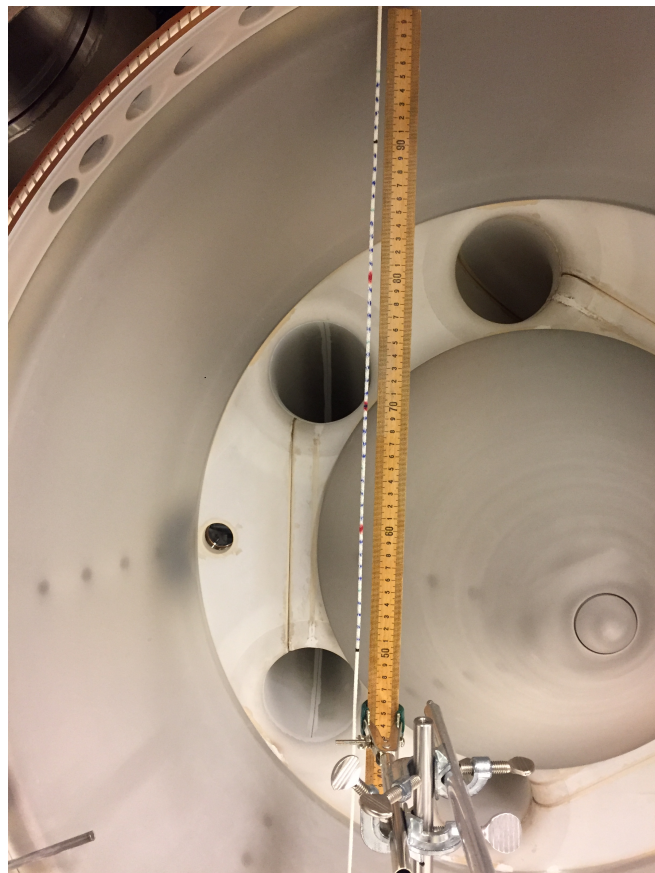
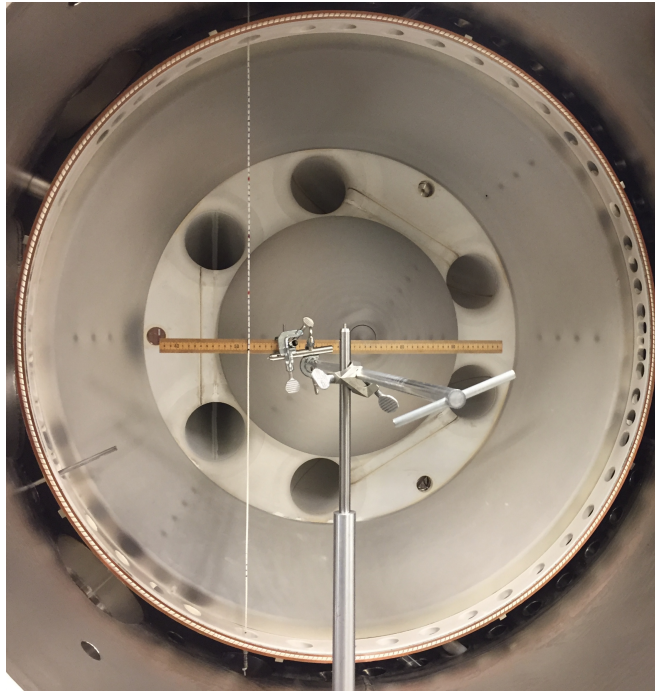


Figure 3.8: The alignment target used to verify measurement locations.

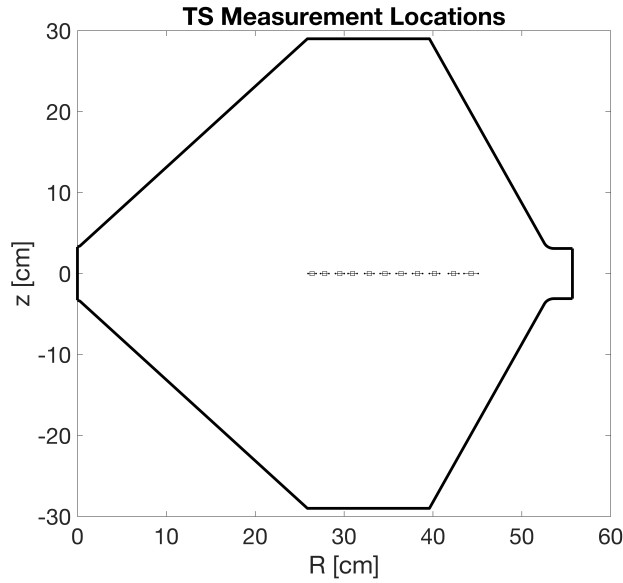


Figure 3.9: The effective radial range of the TS diagnostic, with eleven possible measurement locations. Three polychromators will be available to measure one point each concurrently.

3.2.3 Signal-to-noise ratio

The way each polychromator resolves the spectrum of light passed into it by the fiber bundle is depicted in Fig. 3.10. Incoming light is reflected from one dichroic filter to the next. A given filter only transmits a certain wavelength band of light, and reflects the rest (see Figs. 3.11 and 3.16). A silicon avalanche photodiode (APD) behind each filter serves as the detector of the in-band-radiation, and the signal from each APD is electronically filtered in an attached preamplifier circuit to boost the signal-to-noise ratio. Along with the signal of interest (the scattered and Doppler-shifted radiation from the scattering volume) a number of sources of noise contribute to unwanted light entering the optical fiber. As previously mentioned, stray (unscattered) laser radiation is to be avoided. This is an especially significant issue for some TS density calibrations, which rely on measuring the scattering signal near the laser wavelength.

One way to mitigate excess unwanted radiation is to discriminate by polarization. In

TS Measurement Major Radial Location Table (center of spot) [cm]										
fiber slot	jack height [in.]	9.270	1.000	1.125	1.250	1.375	1.500	1.625	1.750	1.875
A		26.7	26.4	25.8	25.2	24.8	24.3	23.8	23.4	22.9
B		28.2	27.8	27.2	26.7	26.1	25.7	25.1	24.6	24.1
C		29.5	29.5	28.8	28.2	27.5	27.0	26.6	26.0	25.4
D		31.3	31.0	30.4	29.7	29.2	28.5	27.8	27.3	26.9
E		32.6	32.8	32.1	31.3	30.6	30.1	29.5	28.8	28.3
F		34.8	34.6	33.8	33.2	32.5	31.7	31.1	30.6	29.9
G		36.6	36.4	35.6	34.8	34.2	33.6	32.8	32.1	31.5
H		38.5	38.3	37.7	36.8	36.0	35.2	34.6	34.0	33.2
I		40.6	40.2	39.3	38.7	37.9	37.2	36.4	35.8	35.2
J		42.5	42.3	41.5	40.2	39.9	39.0	38.5	37.7	36.6
K		44.7	44.3	43.6	42.5	41.7	40.8	40.1	39.5	38.5

Table 3.2: A list of all possible TS measurement major radial locations, as determined by observing back-lit fiber spots. Hypothetically, any value between 22.9 cm and 44.7 cm could be achieved by adjustment of the lab jack, but the exact boundaries of the spots would be less certain for jack heights not corresponding to those in this table.

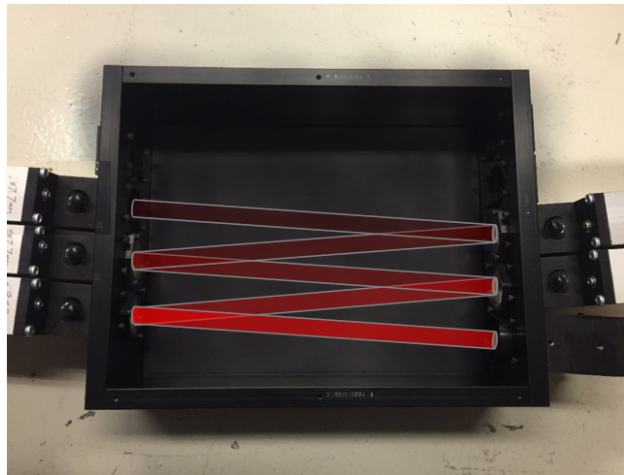


Figure 3.10: Schematic representation of the polychromator function overlaid on an image. Incoming light is collimated and subsequently reflects from one interference filter to the next. At each interference filter a certain band of the spectrum is transmitted and the out-of-band light is reflected to the next filter.

between the collection optics and the fiber mount, a polarization filter is placed to select radiation that has the same polarization as the laser. This reduces the randomly-polarized plasma line and bremsstrahlung radiation, as well as some of the reflected stray laser light that may have had its polarization altered.

Line radiation from the plasma is one of the most significant sources of noise, but can be avoided by careful selection of the filters used in the polychromators. Fig. 3.11 shows how the transmission behavior of the filters were specifically designed to avoid natural emission lines that occur in the scattering region near the ruby wavelength. The original polychromators were not designed for the HIT-SI3 plasma, and had problematic levels of line radiation in the filter transmission bands. A filter redesign completed in September of 2018 resulted in the transmission behavior exhibited in Figs. 3.11 and 3.16. In addition

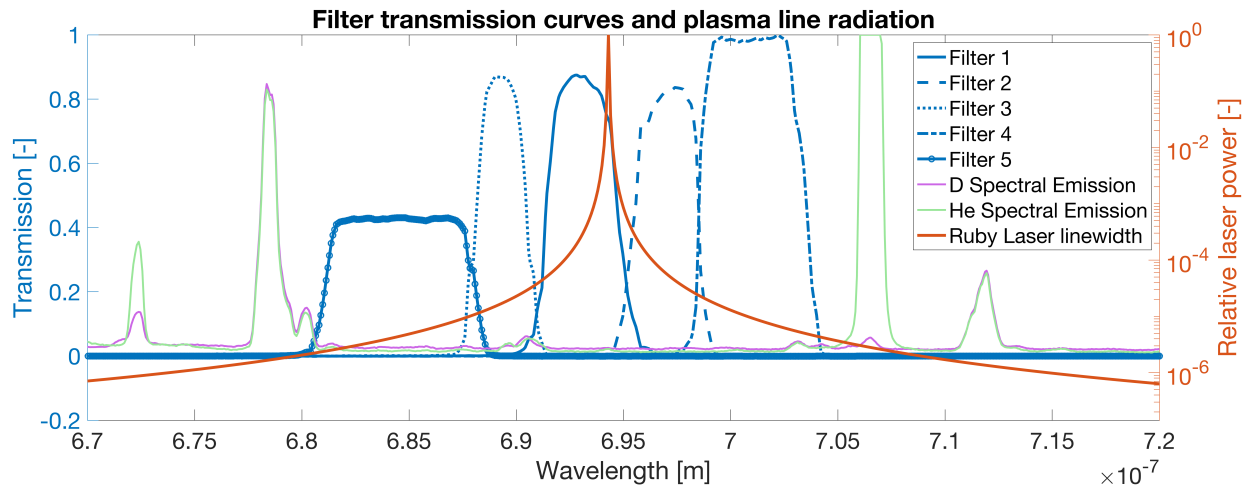


Figure 3.11: A plot of the relative spectral responses of the photodiodes behind the filters in a polychromator. Also shown are a logarithmic plot of the measured laser linewidth, and measured plasma spectral activity in the Thomson measurement wavelength region.

to the polarization filter, the main way optical noise is dealt with is built-in electronic filtering of the signal from the photodiodes. Each avalanche photodiode is connected to a pre-amplifier circuit that serves to amplify signal and reduce noise. The noise reduction is achieved using a 100 ns delay-line subtraction that removes the slowly-varying background radiation and allows through the fast-varying scattered signal that is similar in time signature to the ~ 20 ns laser pulse. This strategy (discussed further in sec. 3.3.1) is not a panacea however, as high-frequency activity in bremsstrahlung, line radiation, and recombination radiation can be caused by shocks, reconnection, and other sources of free

energy in the plasma. Averaging the photodiode signals over multiple shots can help to further reduce the noise in the measurement, but the meaning of such an average is more difficult to interpret when plasma discharges are not perfectly repeatable. Further, thermal noise and dark current noise must be considered in the process of inferring the final measurements.

3.3 Calibrations

In order to characterize the output of the polychromators in response to the various inputs (collected spectra) they receive from the experiment, three calibrations must be performed. One is for spectral resolution, one is for absolute intensity, and the third relates the two output channels of each preamplifier circuit (the “fast channel,” which makes measurements on the time scale of the laser pulse, and the “slow channel,” whose output is used in the spectral calibration). The TS laser failed before the absolute calibration could be taken, so the data collected for this dissertation has available only relative measurements of the electron density, characterized by the absolute value of the density multiplied by the unknown geometric constant c_{geom} , with units of m.

3.3.1 The Fast-Slow Calibration

This calibration is essentially a measurement of the feedback capacitance of the preamplifier circuit (fig. 3.13), and allows for the translation of signal voltages into photoelectron count. This also allows for modeling the internal circuits of the preamplifier in order to produce synthetic data, and serves to relate the fast and slow output signals from a given preamplifier circuit (fig. 3.14). The number of photoelectrons produced in the detector, which is related to the voltage of a given polychromator channel by [40, 41, 42]:

$$N_{pe} = \frac{V_{sig}}{MeG_{fast}R^{FB}} \frac{\tau}{1 - \exp\left(\frac{-\tau}{R^{FB}C^{FB}}\right)} \quad (3.34)$$

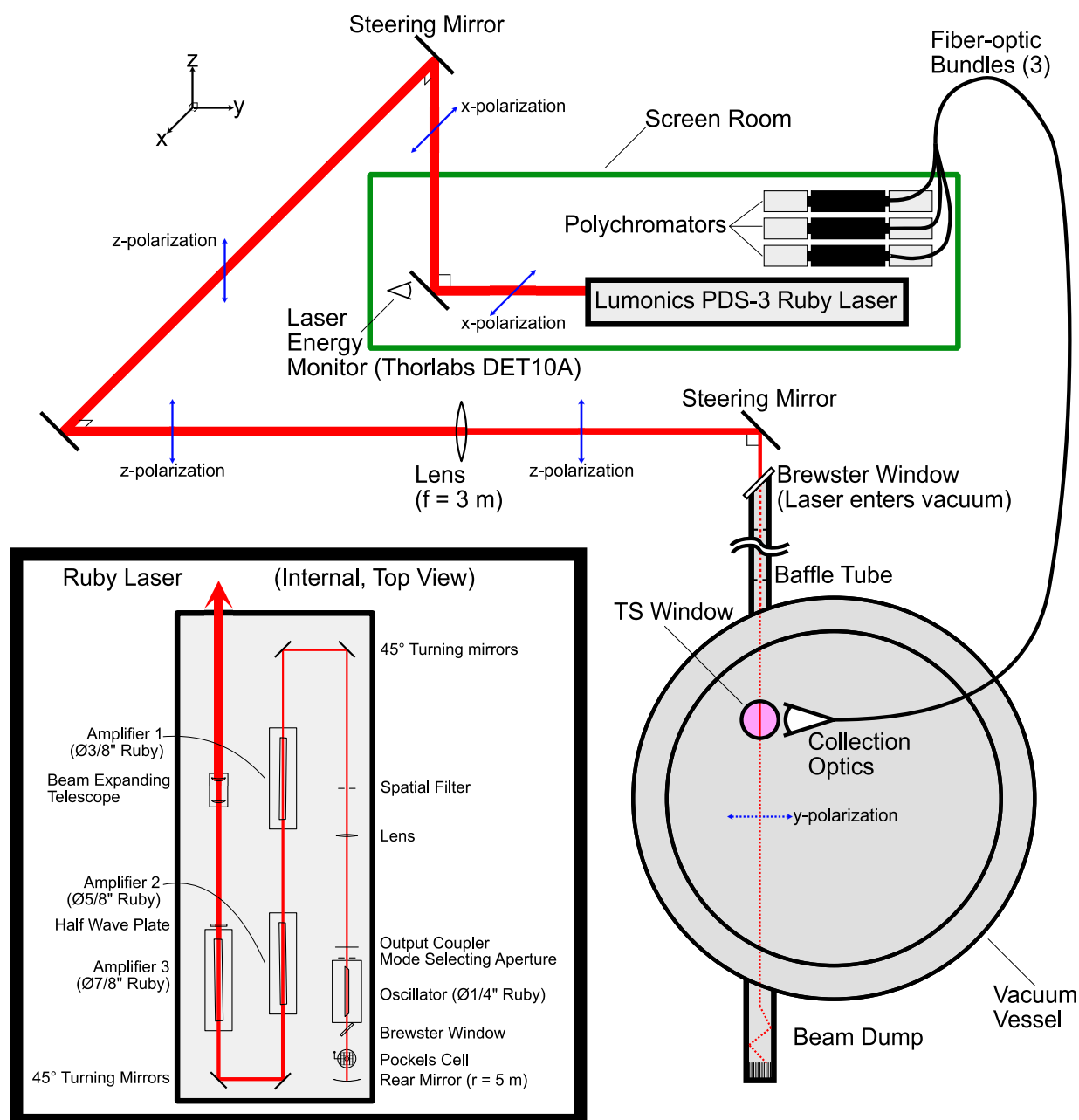


Figure 3.12: A schematic representation of the HIT-SI3 Thomson scattering system, with inset diagram of the ruby laser.

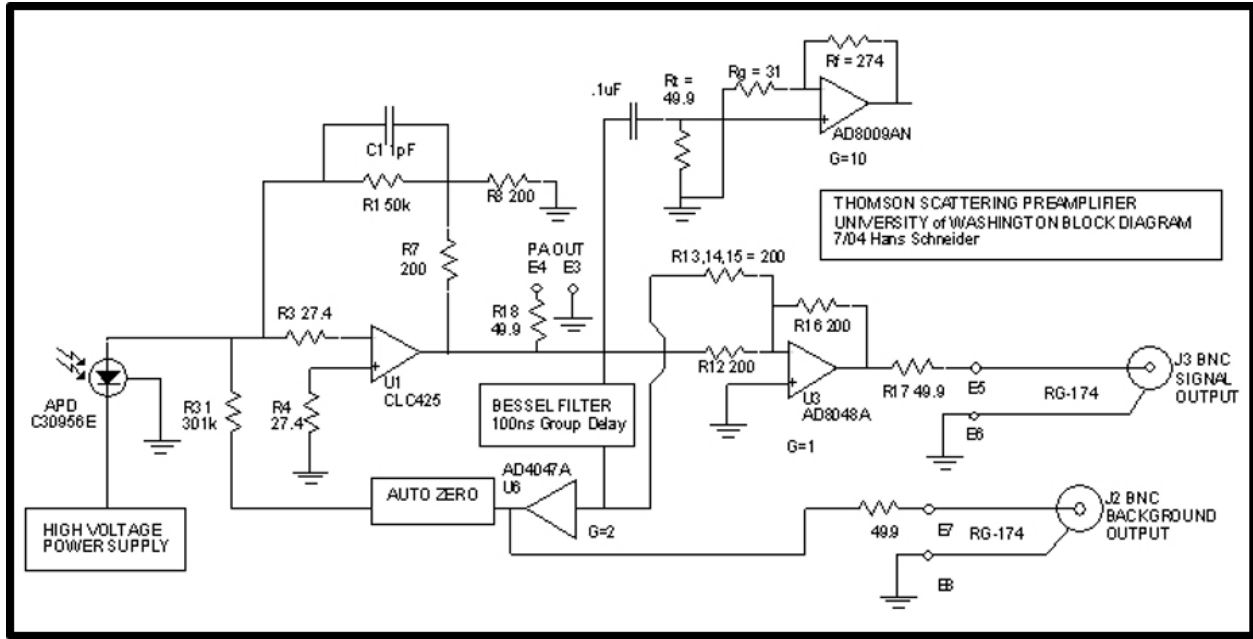


Figure 3.13: A circuit diagram of the APD/preamplifier used on each polychromator channel.

where R^{FB} and C^{FB} are the feedback resistance (50 k Ω) and capacitance of the preamplifier circuit, M is the APD gain (set at either 75 or 75/2 for the data in this dissertation), $G_{fast} = 1$ is the fast channel gain, and τ is the laser pulse period, which is defined as the duration when the signal of the first polychromator channel is above 20% of its overall peak value.

The calibration involves pulsing an LED directly incident on the Silicon APD and observing the responses of both outputs simultaneously. The original measurement of C^{FB} for the preamplifiers used in this dissertation was performed by Kiyong Lee at the Redmond Plasma Physics Laboratory. His results are shown in table 3.3.

3.3.2 The Spectral Calibration

The spectral calibration catalogs the relative response of each channel of a given polychromator over the range of wavelengths to which it is sensitive. The results of this calibration are transmission curves such as those seen in figs. 3.11 and 3.16, as well as the variances

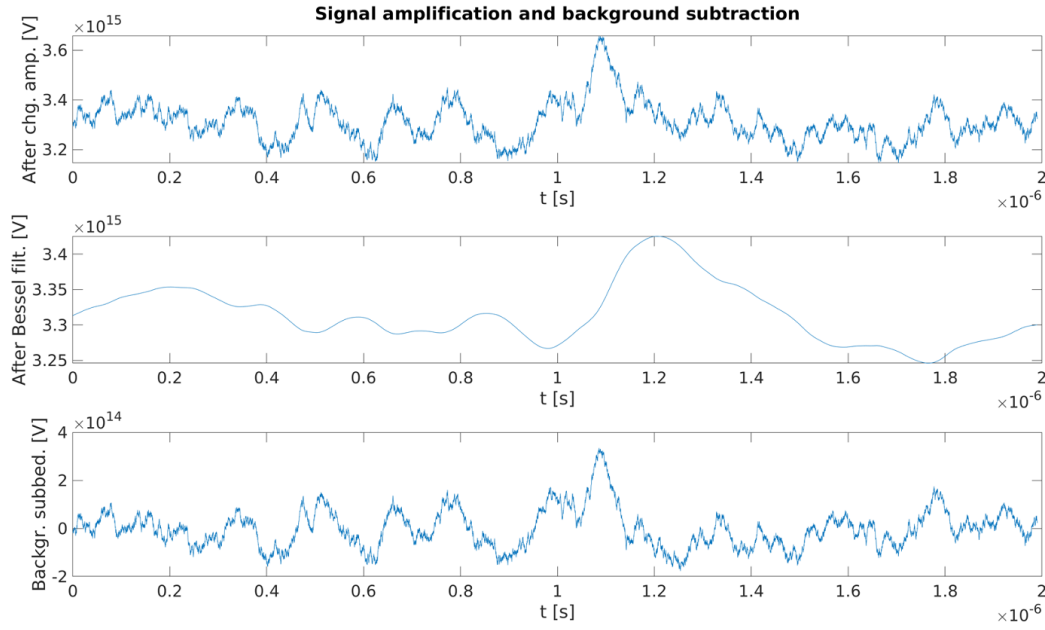


Figure 3.14: The three stages of the preamplifier circuit (synthetic data). The top trace shows the signal pulse after the first component, the charge amplifier, has operated on it. Next, the Bessel (low-pass) filter adds a 100 ns delay. The final output of the fast channel (bottom) is the middle trace subtracted from the top trace.

Preamplifier	C^{FB} [pF]	Preamplifier	C^{FB} [pF]
1_A	1.1985	3_C	1.1665
1_B	1.2131	3_D	1.1895
1_C	1.2138	4_A	1.1891
1_D	1.2433	4_B	1.1586
2_A	1.1757	4_C	1.1927
2_B	1.1999	4_D	1.2051
2_C	1.1872	5_A	1.1642
2_D	1.1974	5_B	1.2006
3_A	1.1852	5_C	1.1805
3_B	(no data)	5_D	1.1836

Table 3.3: Results of the fast-slow calibration from ref. [41].

of the transmission curves at each wavelength.

The apparatus for the spectral calibration is diagrammed in fig. 3.15. A Thorlabs stabilized white light source is focused through a mechanical chopper (run at ≈ 350 Hz) into the $50\text{ }\mu\text{m}$ entrance slit of a monochromator (Optometrics Corp., DMC1-03), which is adjustable to select wavelengths with a resolution of 0.38 nm . Upon exit, the nearly monochromatic light is refocused and split, with one leg directed toward an unfiltered APD preamplifier detector and the other directed toward the entrance of a polychromator (fiber bundle). A second optical fiber connected to a spectrometer (Ocean Optics USB2000+) is also illuminated with the second leg, and the output of this spectrometer is used to verify the monochromator-selected wavelength. The bare APD output is used to control for changes in intensity from the white light source through time. Wavelengths from 670 nm to 720 nm are scanned over manually in steps of 0.2 nm , so this calibration is a time consuming process that could be drastically sped up with automated control of a monochromator.

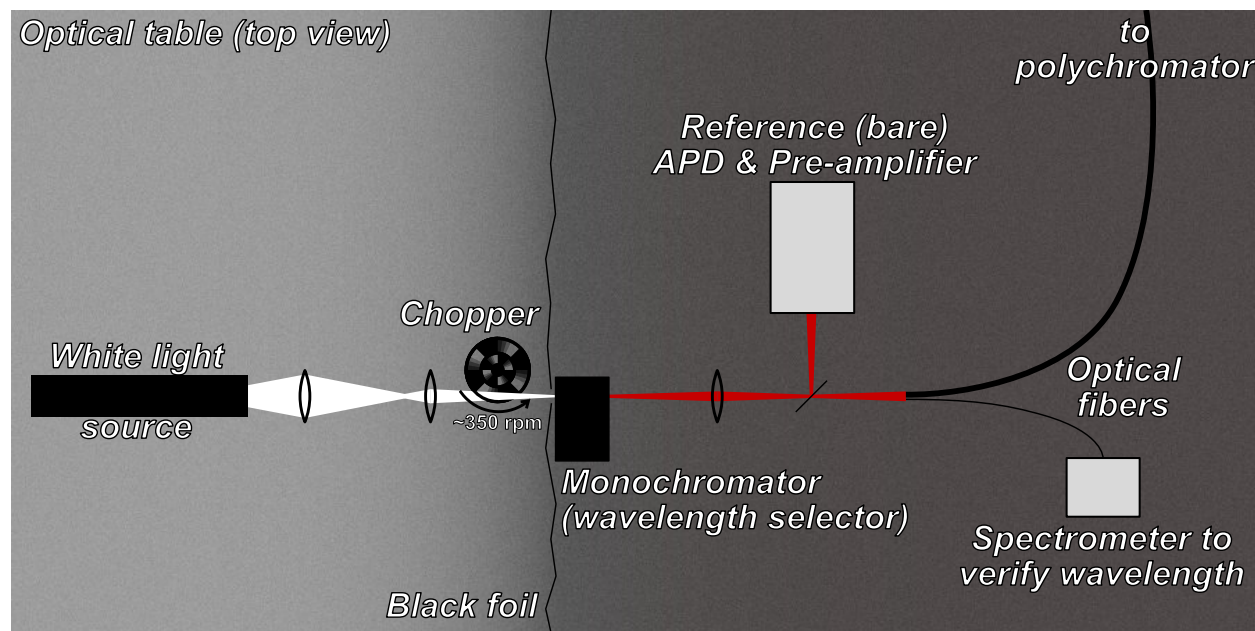


Figure 3.15: The HIT-SI3 Thomson scattering spectral calibration apparatus.

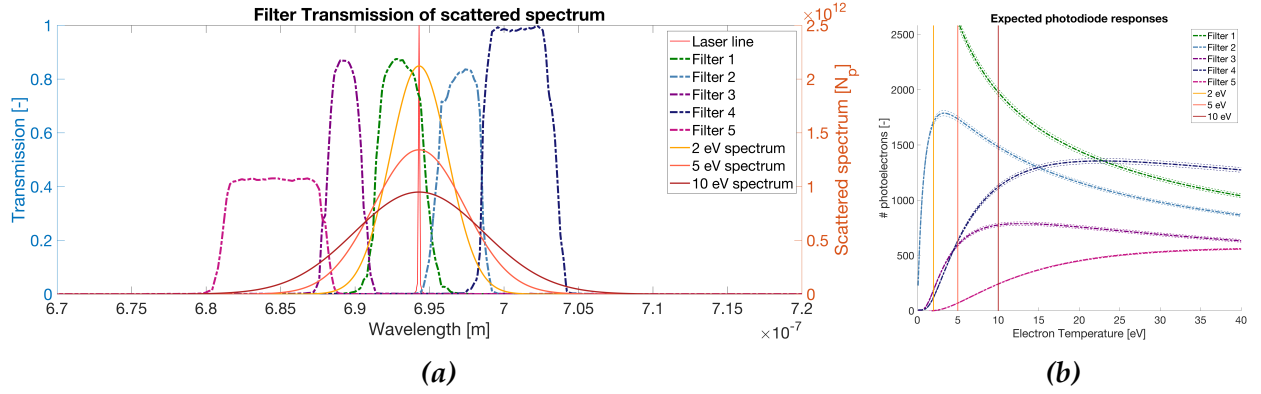


Figure 3.16: (a) Transmission traces of a polychromator system and example temperature spectra. (b) The corresponding relative photodiode response as functions of temperature.

Once the transmission profiles $\tau(\lambda)$ and their variances $\sigma(\lambda)$ have been determined, the expected number of photoelectrons measured at channel j for a given temperature and density can be calculated:

$$N_{pe,j} = \alpha \int \eta_Q(\lambda) \tau_j(\lambda) S(\lambda) d\lambda \quad (3.35)$$

Here $\alpha = \frac{r_e^2 E_{las} n_{cgeom}}{2h \sin(\frac{\theta}{2})}$ where r_e is the classical electron radius, E_{las} is the laser energy, h is the Planck constant, and θ is the detection angle for the spot being measured (fig. 3.3). The quantum efficiency η_Q of the APDs was measured by Kiyong Lee, and for the region of wavelengths under consideration is given by the equation

$$\eta_Q[\%] = (-9.167 \times 10^{-5})\lambda^2 + 0.23855\lambda - 46.722 \quad (3.36)$$

where λ is in units of nm. The spectral density function, $S(\lambda)$, is

$$S(\lambda) = \frac{1}{v_{th}} \exp \left(- \frac{\left(\frac{c(\lambda - \lambda_i)}{2\lambda_i \sin(\theta/2)} \right)^2}{v_{th}^2} \right) \quad (3.37)$$

The variance in N_{pe} for each channel can then be determined:

$$\sigma_j^2 = N_{pe,j}^2 \frac{\sigma_{E_{las}}^2}{E_{las}^2} + \alpha^2 \sigma_{\Sigma\tau,j}^2 \quad (3.38)$$

where $\sigma_{\Sigma\tau,j}^2$ is the spectrally-integrated variance of the j^{th} polychromator channel [41]:

$$\sigma_{\Sigma\tau,j}^2 = \int \eta_Q^2 \sigma_{\tau,j}^2 S^2 d\lambda \quad (3.39)$$

Additional systematic error caused by the resolution of the monochromator used in the calibration is accounted for during the inference process (sec. 4).

3.3.3 Spectral Corrections

The spectral calibration accounts for differences in how light is transmitted and detected in the fiber bundles and all components “downstream,” including all parts of the polychromators. However, before the scattered light is collected at the fiber bundles, it is transmitted through a number of components whose spectral characteristics are not included in the original spectral calibration. These are the vessel window, the two collection lenses, and a polarization filter (fig. 3.7). The plasma-facing vessel window is of particular concern, as carbon films have been noted to degrade the transmission of these windows over time, which can cause significant errors [43]. The lenses are expected to have sufficiently uniform spectral behavior to make correction unnecessary.

The relative spectral properties of these components can be determined using the same white-light source and spectrometer as used in the spectral calibration. Measuring the spectrum transmitted at various locations on the component (fig. 3.17), an approximation of the transmission can be found and factored into the spectral calibration. The average transmission behavior shown in fig. 3.18(c) is treated in the same way as the detector quantum efficiency as discussed in sec. 3.3. This procedure is performed for both the vessel window and the polarization filter. The resultant correction is determined to be quite small, residing well within the error of the measurements it affects, even in the

extreme cases shown in fig. 3.18(c). However, in future experiments this problem may grow worse, and requires consistent updating of the correction terms or a way to shield the window.

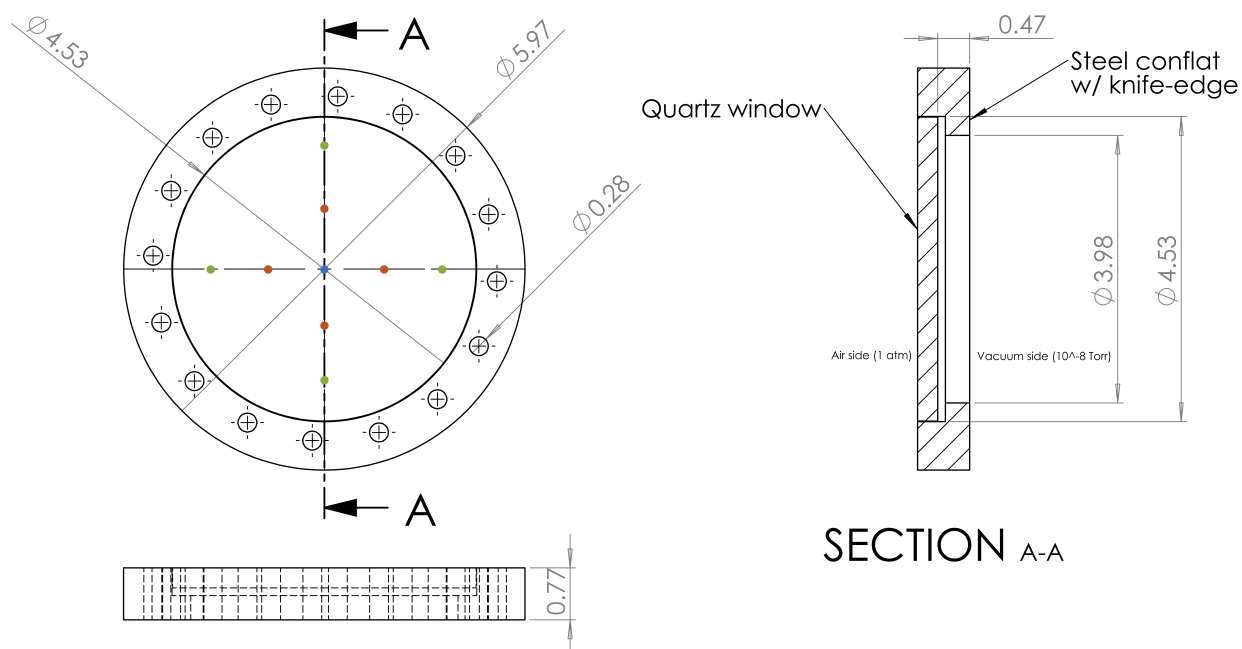
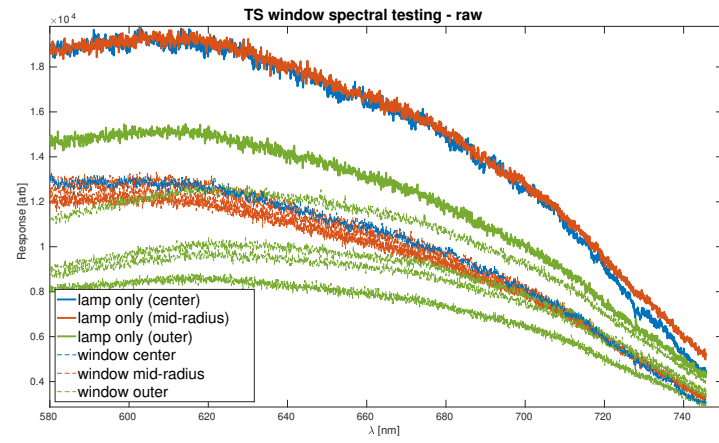
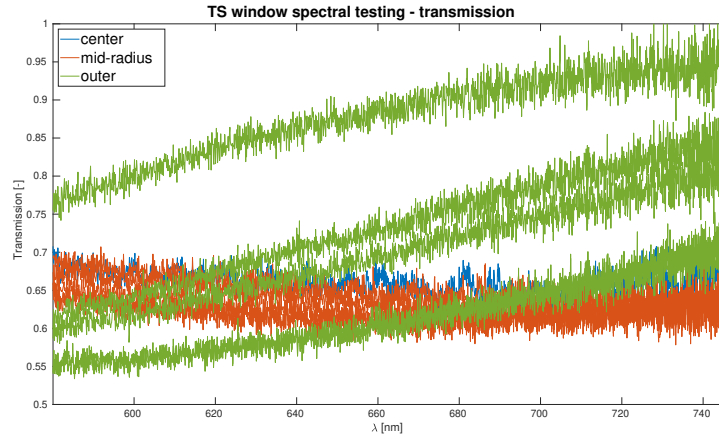


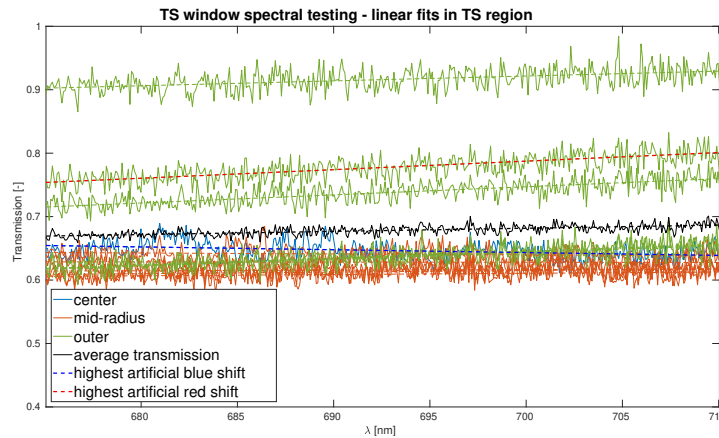
Figure 3.17: A reference drawing of the Thomson scattering window showing the approximate locations of 9 measurements of the transmitted spectrum (see fig. 3.18). Dimensions are in inches.



(a)



(b)



(c)

Figure 3.18: (a) The raw spectral behavior of the window at various location measurements (see fig. 3.17). (b) Transmissions determined from (a) normalized by their corresponding “lamp only” measurements, indicating high variation in transmission around the outer edges of the window. (c) Linear fits to the spectra at various measured locations. The average (black dashed fit) is ultimately used in the correction.

Chapter 4

INFERRING RESULTS FROM THE THOMSON SCATTERING DIAGNOSTIC

With all of the physical components of the Thomson scattering diagnostic in place, the procedure of obtaining a Thomson scattering measurement can be discussed. To understand the process of transforming the raw outputs of this (or any) diagnostic system into parameter measurements, the inverse case is considered first: if the plasma parameters were known, what sort of diagnostic response would they produce? This case is easier to start with because of its simplicity. Referred to as *forward modeling* it follows the natural flow of cause and effect, and uses deductive logic, which is less susceptible to errors and biases (fig. 4.1, adapted from ref. [44]).

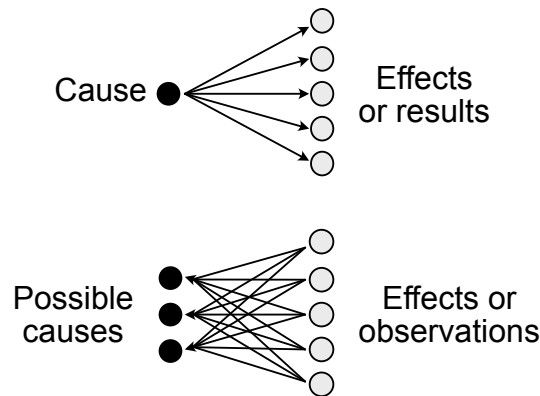


Figure 4.1: A simple representation of the difference between deductive reasoning or forward modeling (top) and inductive reasoning or inverse problems (bottom). Inductive inference only allows for conclusions in terms of relative probabilities of possible causes given certain background information, and is in general a more difficult problem.

It is also helpful to have a map to conceive of the various components in the forward modeling process (fig. 4.2). In the present case, the physical event under observation is the HIT-SI3 plasma in a small Thomson scattering measurement volume, whose electrons have some number density, n_e , and temperature, T_e . In general, perfect observation of these properties is impossible (every electron has its own thermal velocity, the density may not be uniform in the volume, the electrons may not be in thermodynamic equilibrium, etc.).

The first step in forward modeling is therefore to define a sub-model for the physical event or property under consideration. This physics model lays out the assumptions being made about what is going on behind the “veil of ignorance” in fig. 4.2. This has already been presented in sec. 3.1.4. Eq. (3.32) is the physics model. Given a chosen set of parameters, $\theta = \{n_e, T_e\}$, it gives the expected number of photons at each wavelength. The diagnostic then detects these photons and resolves the spectrum they comprise.

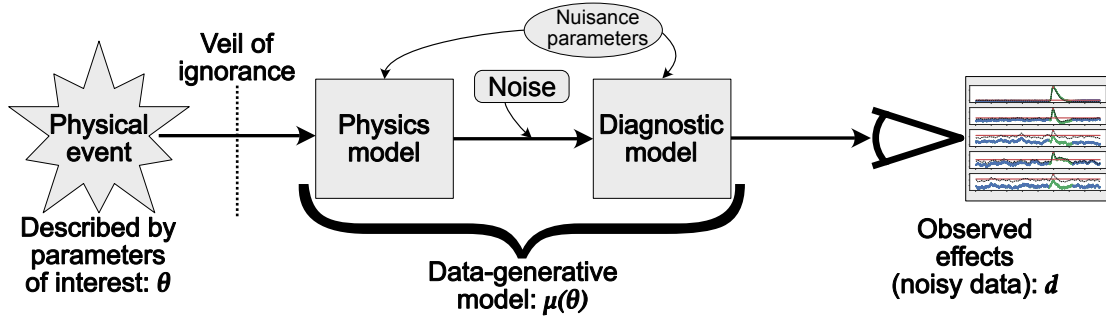


Figure 4.2: A graphical map of the forward modeling process. Starting with parameters θ , the physics and diagnostic models compose the data-generative model, which produces the theoretical diagnostic response.

4.1 Modeling the TS diagnostic

The next step in the forward modeling process is to define in detail the behavior of the diagnostic. This step must account for all parts of the diagnostic apparatus outside of

the laser-plasma interaction volume. Specifically, this entails the behavior of the laser, the collection optics and fibers, and the polychromators.

The internal details of the laser may be abstracted. Since the linewidth and wavelength have been verified to be adequate for the measurement (Table 3.1), the only important variable of the laser operation is its energy output, which varies from shot to shot. To track the laser energy, a photodiode (Thorlabs DET10A) detector is stationed behind the first turning mirror. For a number of laser pulses, a more accurate detector (Scientech AC5001S) is placed in the beam path just after the first turning mirror in order to allow simultaneous measurement. A linear regression is formed (similar to that in fig. 4.3) that allows for an energy measurement and uncertainty to be inferred for any laser pulse that is measured by the photodiode.

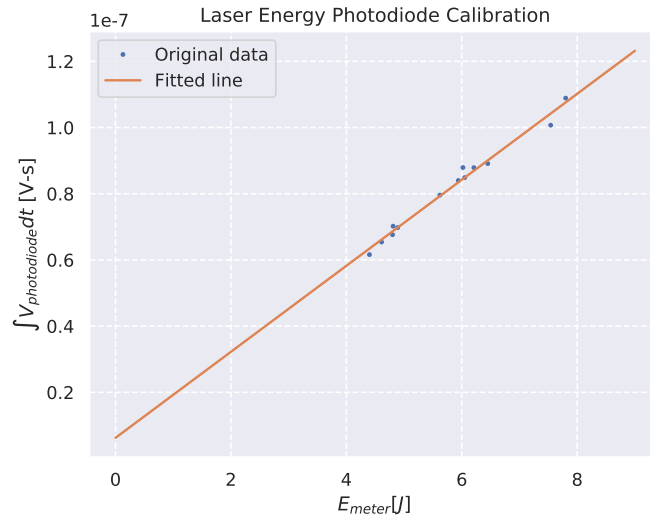


Figure 4.3: A calibration of the photodiode used to measure laser energy on each TS shot. A more precise detector is used to correlate the two signals, giving a Pearson regression value of $R \approx 0.98$.

After the laser interacts with the plasma, its spectrum is modeled by eq. (3.32). The effect of the collections optics on the intensity of the scattered light may be represented as a simple multiplicative factor η_{CO} on eq. (3.32). This factor is then combined with the

spot length, the solid angle of collection, and a factor of $1/\sqrt{\pi}$ to form c_{geom} , the geometric factor that determines the intensity of scattered spectrum at a given laser energy, changing eq. (3.32) to:

$$\frac{dN_s(\lambda)}{d\lambda} = \frac{E_i n c_{geom} r_e^2}{2h \sin(\theta/2) v_{th}} \frac{1}{v_{th}} \exp \left[\frac{-c^2 (\lambda - \lambda_i)^2}{4v_{th}^2 \lambda_i^2 \sin^2(\theta/2)} \right] \quad (4.1)$$

To be incorporated into the diagnostic model, eq. (4.1) is rewritten as eq. (3.35), which translates the physics model into an expected response from the diagnostic in photoelectron counts. The final element in the diagnostic model is then eq. (3.34), which translates between photoelectron counts and APD voltages (the raw data). There is now a fully defined path from left to right in fig. 4.2. Noise sources are the final elements to cover.

4.1.1 Noise and sources of random error

The raw voltage trace of a given photodiode is taken to be proportional to the received number of photons (eq (3.34)). The variance in the signal due to random errors must also be accounted for, in addition to the systematic errors already discussed in sec. 3.3.2. Random additive noise that affects the “fast” signal output voltage on every shot can be categorized into three sources: noise from the detector electronics, noise from natural background light intensity fluctuations in the plasma, and noise from stray laser light.

Following refs. [45, 46] via [41], the total shot noise from the j^{th} detector (including contributions from amplified dark current, signal current, and photo-induced background current) is well-modeled as:

$$\sigma_{det,j}^2 = 4N_{pe} \quad (4.2)$$

It should be noted that the units of $\sigma_{det,j}^2$ are the same as N_{pe}^2 ; eq. (4.2) is an empirical scaling. This scaling is expected to be accurate for $N_{pe} < 3.5 \times 10^4$, which is true for most HIT-SI3 TS measurements.

The background fluctuation noise for each polychromator channel is determined by sampling the raw digitized signal outside of the laser firing period. The variance of this

set of samples is taken to be the fluctuation variance,

$$\sigma_{bg}^2 = \left(\frac{\text{std}(V_{sig}|_{background})}{MeG_{fast}R^{FB}} \frac{\tau}{1 - \exp\left(\frac{-\tau}{R^{FB}C^{FB}}\right)} \right)^2 \quad (4.3)$$

Fig. 4.4 shows example raw data from a single polychromator. Occasionally background

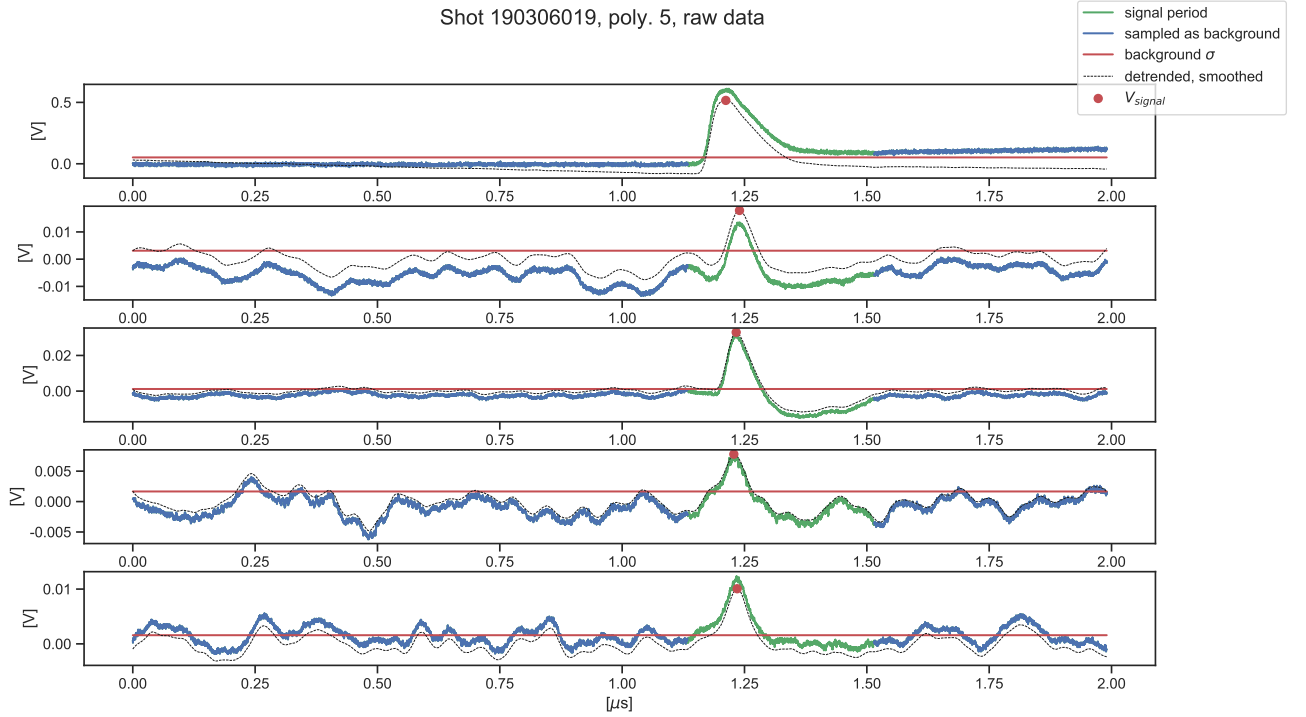


Figure 4.4: Example raw TS data. Green regions are excluded from background variance calculations.

noise causes a detector to saturate for some duration during the period it is digitized. In this case the signal from the saturated channel is not included in the measurement, which increases the uncertainty of the measurement by reducing the information available for inference. If no saturations have occurred, after the calculation of the signal is detrended and smoothed with a Savitzki-Golay filter of window ≈ 40 ns to remove high frequency noise when identifying the peak voltage. The peak voltage of any signal is taken to be its maximum during the laser pulse period.

Finally, the stray light noise is characterized by taking frequent vacuum measurements, during which no plasma is formed but the laser is still fired. The stray light can then be controlled for by subtracting the known vacuum signal from any subsequent plasma measurements. In practice the analysis code locates the four vacuum measurements taken closest in time to the shot being analyzed, averages them (after normalizing their amplitudes based on the laser energy of their respective pulses), and subtracts the average from the plasma measurement signals on the corresponding channels. The value of uncertainty in the laser energy calibration scaling (σ_{ple}^2 and σ_{vle}^2 for the plasma and average vacuum shots, respectively), as well as σ_{bg}^2 and σ_{det}^2 for all of the vacuum shots, must all be summed, giving:

$$\sigma_{stray,j}^2 = \sigma_{vac,det,j}^2 + \sigma_{vac,bg,j}^2 + N_{pe,stray}^2 \sum_{i=1}^4 \left(\frac{\sigma_{ple}^2}{E_{las,vac,i}^2} + \sigma_{vle}^2 \frac{E_{las,plas}^2}{E_{las,vac,i}^4} \right) \quad (4.4)$$

and finally:

$$\sigma_j^2 = \sigma_{det,j}^2 + \sigma_{bg,j}^2 + \sigma_{stray,j}^2 \quad (4.5)$$

Now that a model of the diagnostic has been defined, it is combined with the physics model to provide a full *data-generative*, or forward model. This model may be run forward for various reasons, including to produce synthetic test data, to characterize uncertainties in the diagnostic, and during inference when determining parameters. Fig. 4.5 shows an example polychromator sensitivity trace.

4.2 Bayesian Inference

Resolving the Thomson-scattered and thermally-broadened spectrum of laser light is a greater challenge in cooler plasmas. The signal-to-noise ratios in the raw scattering data encountered throughout this study demand a clear and careful treatment of the uncertainties in the measurement. How well a measurement parameter is known is at least as important as the value of the parameter. It is for this reason that the Bayesian probabilistic framework (sometimes referred to as integrated data analysis, or IDA) is used to analyze

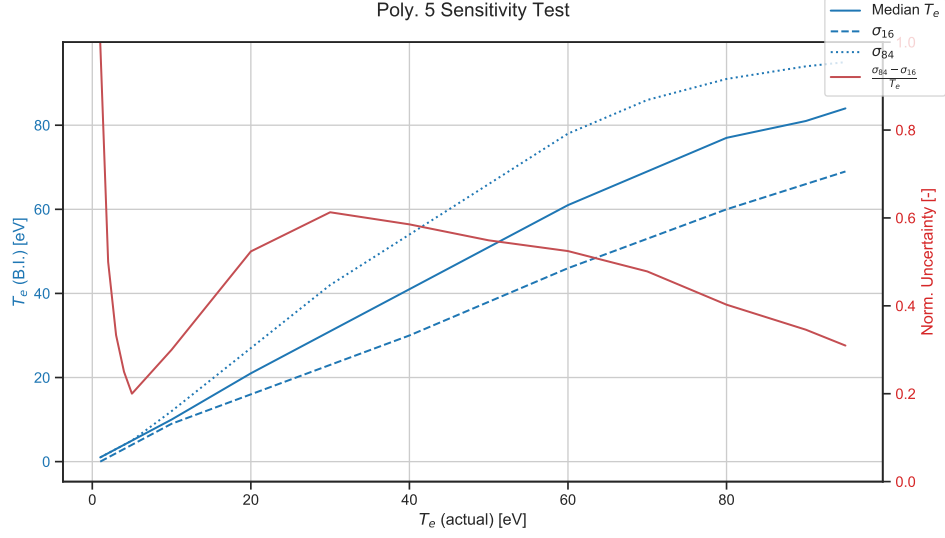


Figure 4.5: A coarse sensitivity characterization of a polychromator produced by running the diagnostic model forward (without random noise) at a range of known electron temperatures and observing the resulting uncertainty.

the data from the Thomson scattering diagnostic.

Performing the diagnostic inference using the Bayesian framework also lays the groundwork for future diagnostics to be integrated into the Bayesian model, a practice referred to as “Integrated Data Analysis” that is growing in ubiquity in plasma physics. Integrated Data Analysis (IDA) is a relatively new paradigm for processing the numerous data produced by fusion experiments and their associated diagnostics [47, 48, 49]. It utilizes Bayesian probability theory to provide a consistent framework for most of the data analysis process, including solving inverse problems to determine plasma parameters and uncertainties, combining results from heterogeneous diagnostics for increased accuracy and precision in parameter estimates, optimizing diagnostic design through forward modeling, and comparing competing physics models [50, 51].

IDA utilizes Bayesian probability theory to provide a consistent framework for data analysis, and has four main functions: (1) solving inverse problems to determine plasma

parameters and uncertainties, (2) combining results from heterogeneous diagnostics for increased accuracy and precision in parameter estimates, (3) optimization of diagnostic design through forward modeling, and (4) comparing competing physics models (hypothesis testing) [50, 51]. The first function will be of primary relevance in this research, with brief allusions to the third and fourth.

An additional advantage of the Bayesian method is the ease of dealing with nuisance parameters. *Nuisance parameters* refer to any that affect the measurement being made that are not included in the set of parameters being estimated, θ . Examples include the wavelength resolution of the monochromator used in the spectral calibration and the transmission efficiency of the components in the collection optics.

4.2.1 Solving the inverse problem: Theory

Solving inverse problems using Bayesian probability theory, or *Bayesian Inference*, requires three things: (1) data — measurements that are used to constrain (2) the data generative model, which provides a candidate explanation for how parameters produce the data, and (3) priors, or an initial expression on the probability of parameter values before inference.

In the Bayesian framework, probabilities are to be interpreted as an expression of the degree of belief in a certain proposition, quantity, or event. This can be compared with the Frequentist framework, which relies on the circular definition of *probability* as the proportion of times an event occurs in the limit of an infinite number of equally *probable* trial repetitions [50].

Bayesian inference relies on Bayes' Theorem in the following form, where $p()$ represents a probability density function (PDF), and $\mathcal{L}$ is the likelihood — an unnormalized probability density function:

$$p(\theta \mid d, \sigma, I) = \frac{\mathcal{L}(d \mid \theta, \sigma, I)p(\theta \mid I)}{p(d \mid I)} \quad (4.6)$$

In this instance, θ is some set of plasma parameters to be measured based on the observed data, d . The associated uncertainty is σ . The background information, I , is the collection of all external knowledge assumed to be true — the non-negativity of electron number density, for example.

The prior, $p(\theta \mid I)$, represents the state of knowledge before the measurement took place. Assigning the prior allows for incorporation of new information into the inference, and is also a way to define assumptions with transparency and clarity.

The likelihood, $\mathcal{L}(d \mid \theta, \sigma, I)$, describes the probability of having measured data d given a specific choice of parameters θ . The likelihood is where the forward model resides. Both physics models and diagnostic models are included in defining the likelihood.

The result sought when using Bayesian inference to estimate parameters is a PDF describing the relative likelihood at any specific choice of the parameters, θ . This is called the posterior likelihood, the left hand side of eq. (4.6). The posterior thus has the same number of dimensions as there are parameters in the vector θ .

The posterior is generally not able to be evaluated analytically, and must be sampled using techniques like Markov Chain Monte Carlo (MCMC) sampling [50]. A modern method in sampling of posteriors is nested sampling [52]. In this technique, the denominator of eq. (4.6) (the full integral of the numerator of eq. (4.6) over all parameters, referred to as the “evidence,” “model evidence,” or “marginal likelihood”) is preserved. This allows the posteriors produced by competing candidate forward models to be compared on a consistent basis. Model comparison in this framework is the Bayesian analogue of hypothesis testing in classical statistics (see sec. 4.2.3).

4.2.2 Solving the inverse problem: Example

The utility of Bayesian inference can be illustrated through the example of shot 190306019. The raw data from polychromator “5” is shown in fig. 4.4. The forward model developed earlier in this chapter is used to translate a chosen set of parameter values ($\theta =$

$\{T_e, nc_{geom}\}$) into data. The forward model represents one of three necessary components in performing Bayesian inference. The model is used to define the joint likelihood for all measurements j , where d_j and $d_{model,j}$ would be in units of photoelectron counts, determined from eqs. (3.34) and (3.35), respectively:

$$\mathcal{L}(\mathbf{d} \mid \boldsymbol{\theta}, \sigma, I) = \prod_j p_j = \prod_j \frac{1}{\sigma_j \sqrt{2\pi}} \exp \left[\frac{-(d_j - d_{model,j})^2}{\sigma_j^2} \right] \quad (4.7)$$

Here σ_j is the sum of all relevant variances for the j^{th} channel, including those discussed in secs. 3.3.2 and 4.1.1. The data, another necessary component to perform the inference, is simply d_j , the peak signal voltage (fig. 4.4) from channel j translated into N_{pe} via eq. (3.34).

The next component is the prior probabilities for the parameters. The initial assumptions in this example inform uniform priors for T_e and nc_{geom} . Previous measurements on the HIT-SI and HIT-SI3 devices suggest ion temperatures on the order of 25 eV, and lower electron temperatures, including Langmuir probe measurements near 10 eV. However, since Thomson scattering is a non-perturbative measurement and the data in this dissertation were the first measurements in HIT-SI3 plasmas being driven at low injector frequency, a uniform prior spanning 0.025 eV to 100 eV was specified. Only order-of-magnitude prior knowledge of nc_{geom} is available, so a uniform distribution ranging from $[0 \rightarrow 100] \times 10^{14} \text{m}^{-2}$ was used. These are “uninformative” priors: uniform distributions bounded only by obvious limits that give equal prior probability to all parameter values they support. To summarize, the priors are:

$$p(T_e[\text{eV}] \mid I) \sim \mathcal{U}(0.025, 100)$$

$$p(nc_{geom}[10^{14} \text{m}^{-2}] \mid I) \sim \mathcal{U}(0, 100)$$

The resulting two-dimensional posterior is sampled using two open-source sampling packages for comparison: *nestle* [53] and *PyMC3* [54]. *PyMC3* performs Markov Chain Monte Carlo sampling of the unnormalized posterior, and allows for the determination of the most likely parameter values via the numerator of eq. (4.6). *Nestle* performs nested

sampling [52, 44], which also uses MCMC methods but can in addition estimate the evidence term, allowing for model comparison (sec. 4.2.3).

These techniques search the parameter space of the posterior algorithmically in a way that involves running the forward model many times with different parameters. This facilitates the accounting for nuisance parameters like the uncertainty in the wavelength of the monochromator. In the forward model run by the sampling algorithms, a slight modification to the core physics model is made; eq. (3.37) becomes:

$$S(\lambda) = \frac{1}{v_{th}} \exp \left(- \frac{\left(\frac{c(\lambda - \lambda_i + \tilde{\lambda}_{unc})}{2\lambda_i \sin(\theta/2)} \right)^2}{v_{th}^2} \right) \quad (4.8)$$

where $\tilde{\lambda}_{unc}$ represents the uncertainty of the monochromator as a value drawn from a normal distribution with $\sigma = 0.2$ nm:

$$p \left(\tilde{\lambda}_{unc}[\text{nm}] \mid I \right) \sim \mathcal{N}(0, 0.2) \quad (4.9)$$

The two sampling methods largely agree, though in later chapters only the results from nested sampling will be used in an effort to reduce the complexity of the discussion, because it is nearly as fast as PyMC3 and provides more complete information. Figs. 4.6 and 4.7 show example results of the inference process for both sampling algorithms.

The median of the probability density function is used as the parameter value for aggregate summaries or further calculations. This is because the median is closer to the most probable parameter value in highly skewed distributions, but will still capture some of the effect of the skewness. It is also immune to residing outside the 16th and 84th percentiles, which are used as to represent an equivalent of the $\pm 1\sigma$ error bars for the skewed parameter probability distributions.

The method described above to determine T_e and n_e may be compared with the *ratio evaluation method*, in which the relative signal levels of the photodiodes in a polychromator are compared with the theoretical responses as a function of temperature. Put

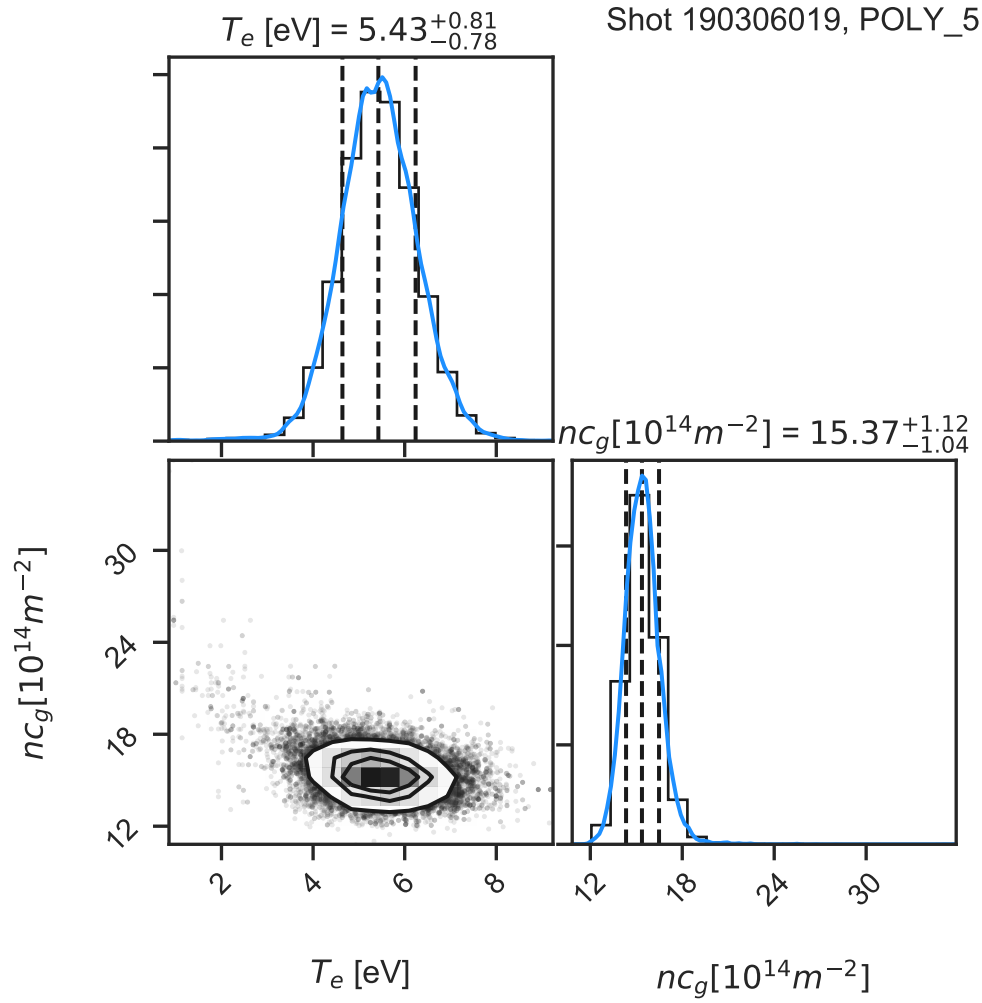


Figure 4.6: Example inference results on T_e and nc_{geom} using the Metropolis algorithm in PyMC3.

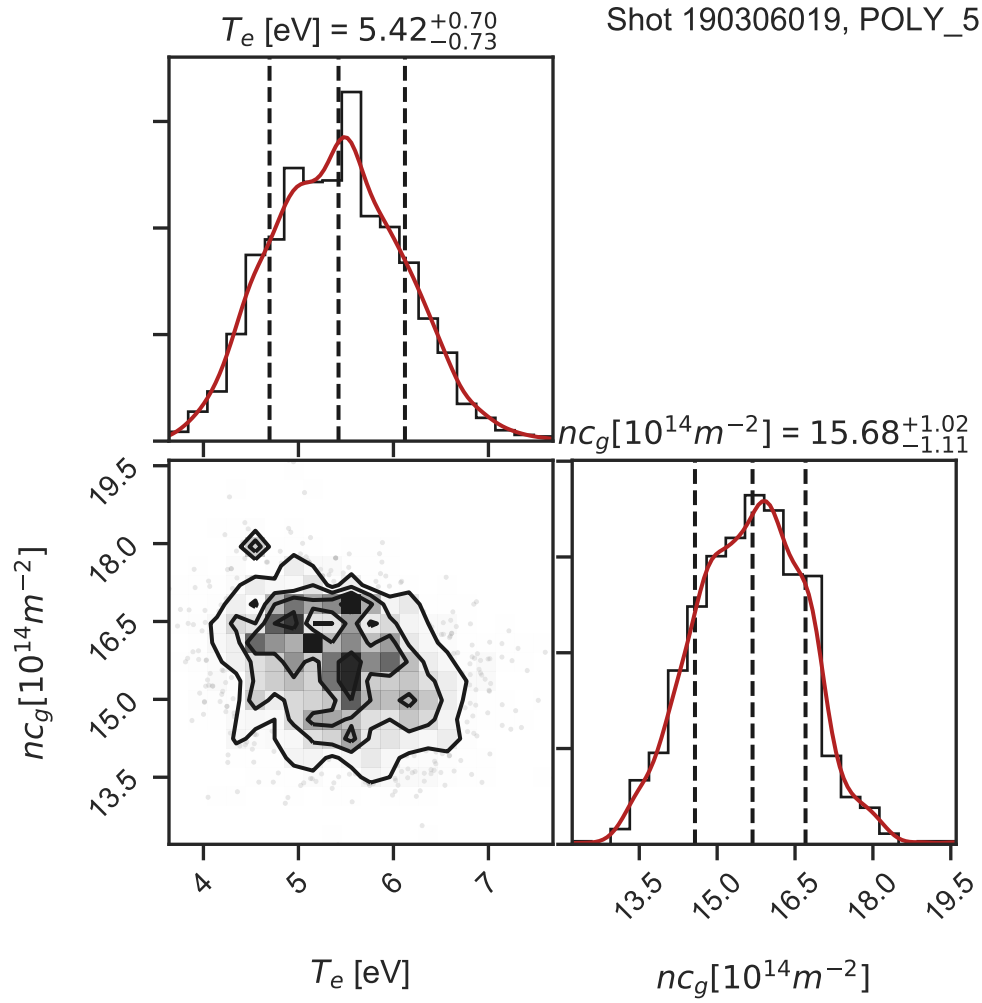


Figure 4.7: Example inference results on T_e and nc_{geom} using the nested sampling algorithm in *nestle*.

another way, a look-up table is created of the relative magnitudes of signal that each of the photodiodes in a polychromator *would* observe if the plasma were at a certain electron temperature. The mean squared error between the experimental and theoretical relative responses is minimized and a temperature measurement is arrived at (see Fig. 3.16). This method has been criticized for its tendency to oversimplify and overestimate the uncertainties of the measurement, though it is simpler and faster to compute than the Bayesian method [55].

4.2.3 Model comparison and electron velocity measurements

An additional benefit of using Bayesian inference is that it facilitates the comparison of various physics models. If the method of sampling the posterior allows for estimation of the evidence (denominator of eq. (4.6)), two competing explanations for the data with different parameters can be compared in a self-consistent way that has a built-in mechanism to avoid over-fitting [56].

Suppose there are two models to explain the generation of some data set d , and that they are *a priori* equally plausible. The two models, which in general operate on different parameters, are denoted $\mu_1(\theta_1)$ and $\mu_2(\theta_2)$. For each model, eq. (4.6) can be written as (with index i):

$$p(\theta_i | d, \mu_i, I) = \frac{p(d | \theta_i, \mu_i, I)p(\theta_i | \mu_i, I)}{p(d | \mu_i, I)} \quad (4.10)$$

where the uncertainty σ is still accounted for, but is absorbed into the “background information,” I , to simplify the equations. The specification of the model μ_i was implicit in I in eq. (4.6). Using Bayes’ Theorem again, but now to describe the conditional probability of model i , gives:

$$p(\mu_i | d, I) = \frac{p(d | \mu_i, I)p(\mu_i | I)}{p(d | I)} \quad (4.11)$$

This can be used the relative probability between two models. The ratio is known as the

K_{12}	Interpretation
1 to 3.2	Not worth more than a bare mention
3.2 to 10	Substantial
10 to 100	Strong
> 100	Decisive

Table 4.1: Suggested interpretations of the Bayes factor from ref. [57].

Bayes factor for comparing models 1 and 2, K_{12} :

$$K_{12} \equiv \frac{p(\mu_1 | d, I)}{p(\mu_2 | d, I)} = \frac{p(d | \mu_1, I)p(\mu_1 | I)}{p(d | I)} \frac{p(d | I)}{p(d | \mu_2, I)p(\mu_2 | I)} \quad (4.12)$$

Because the *prior* probability of the two models was taken to be equal ($p(\mu_1 | I) = p(\mu_2 | I)$), the Bayes factor simplifies to a ratio of the *evidence* terms of eq. (4.10):

$$K_{12} = \frac{p(d | \mu_1, I)}{p(d | \mu_2, I)} \quad (4.13)$$

Kass and Raftery suggested a now widely-referenced scale for interpretation for the Bayes factor K_{12} , (table 4.1, with $K_{12} > 1$ yielding support for model μ_1):

The above technique for model comparison was attempted with the collected Thomson data. The symmetry of measurement about the laser wavelength may allow for the inference of a bulk shift in the electron velocity distribution function. Model μ_1 was taken to be the full data-generative model described in Ch. 3, and model μ_2 is defined as nearly the same model, but with spectral density function of eq. (3.37) instead defined as:

$$S(\lambda) = \frac{1}{v_{th}} \exp \left(- \frac{\left(\frac{c(\lambda - \lambda_i)}{2\lambda_i \sin(\theta/2)} - v_d \right)^2}{v_{th}^2} \right) \quad (4.14)$$

The measured value of the streaming velocity, v_d , is taken to be the component of bulk electron flow along the direction defined by $-\mathbf{k}$, where $\mathbf{k}$ is the scattering vector discussed in sec. 3.1.3. Therefore a measurement of $v_d < 0$ implies the electrons are streaming in the direction of $+\hat{k}$.

Measurements of this electron streaming velocity have been made using Thomson scattering in the past [58, 59, 60]. It is noted that the assumption of a simple velocity shift on the background Maxwellian thermal distribution may be problematic if the electron streaming velocity is from a current that is derived from a driving electric field [61]. In this case the electron velocity distribution will distort slightly (more distortion with greater shift) from the simply shifted Maxwellian shape [62]. For simplicity the shifted Maxwellian of eq. (4.14) is nevertheless assumed given the relatively coarse spectral resolution of the TS diagnostic.

Figs. 4.8–4.11 show the results from Bayesian inference performed on the same data as in sec. 4.2.2, but with the three-parameter drifting model. Figs. 4.10 and 4.11 are more representative of the velocities observed across all measurements (fig. 4.13). A uniform prior thought to encompass plausible streaming velocities is used:

$$p(v_d[\text{km/s}] \mid I) \sim \mathcal{U}(-800, 800) \quad (4.15)$$

The concern may be raised that simply adding more parameters to any model will allow it to fit a given set of data better than a competing model with less parameters. The danger of this pitfall is reduced with Bayesian model comparison. Each evidence term involved with evaluating K_{12} is essentially the integral over all parameter space of the numerator of eq. (4.6). Since the prior PDFs are uniform and normalized, adding additional parameters to the model reduces the value of the evidence, as the prior probability for that parameter θ is $p(\theta \mid I) \propto 1/(\theta_{max} - \theta_{min})$. Thus more parameters or larger domains in the prior distributions of those parameters penalize the model they belong to when compared with more parsimonious models.

Fig. 4.12 shows the fitted distributions (using both sampling algorithms) for the two competing models: fixed and drifting Maxwellians. The value of K_{12} is 2.4, suggesting very weak support in favor of the fixed Maxwellian model.

Fig. 4.13 shows the collection of electron streaming velocity data as measured by the

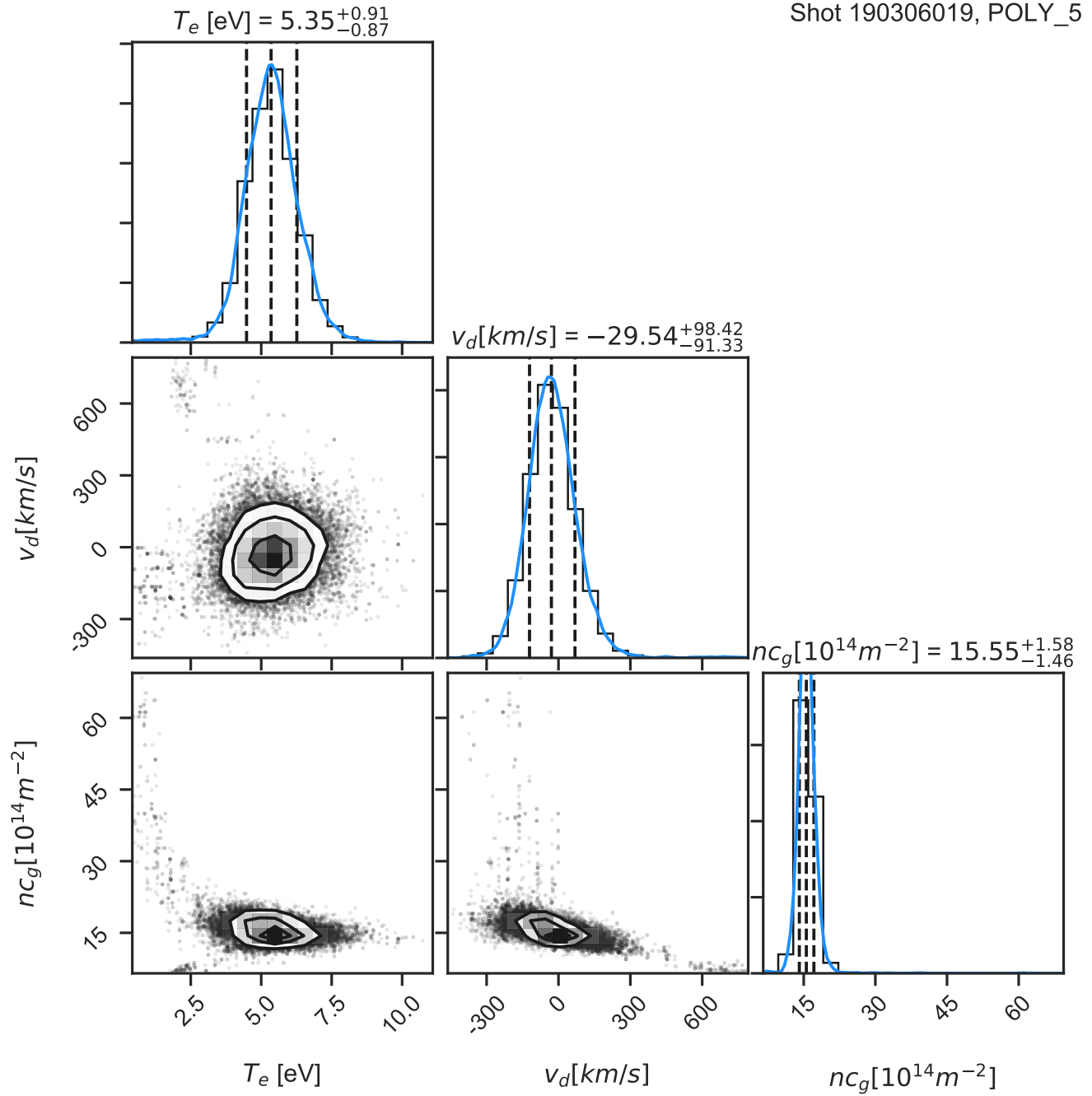


Figure 4.8: Example inference results on T_e and $n_{c_{geom}}$ using the Metropolis algorithm in PyMC3.

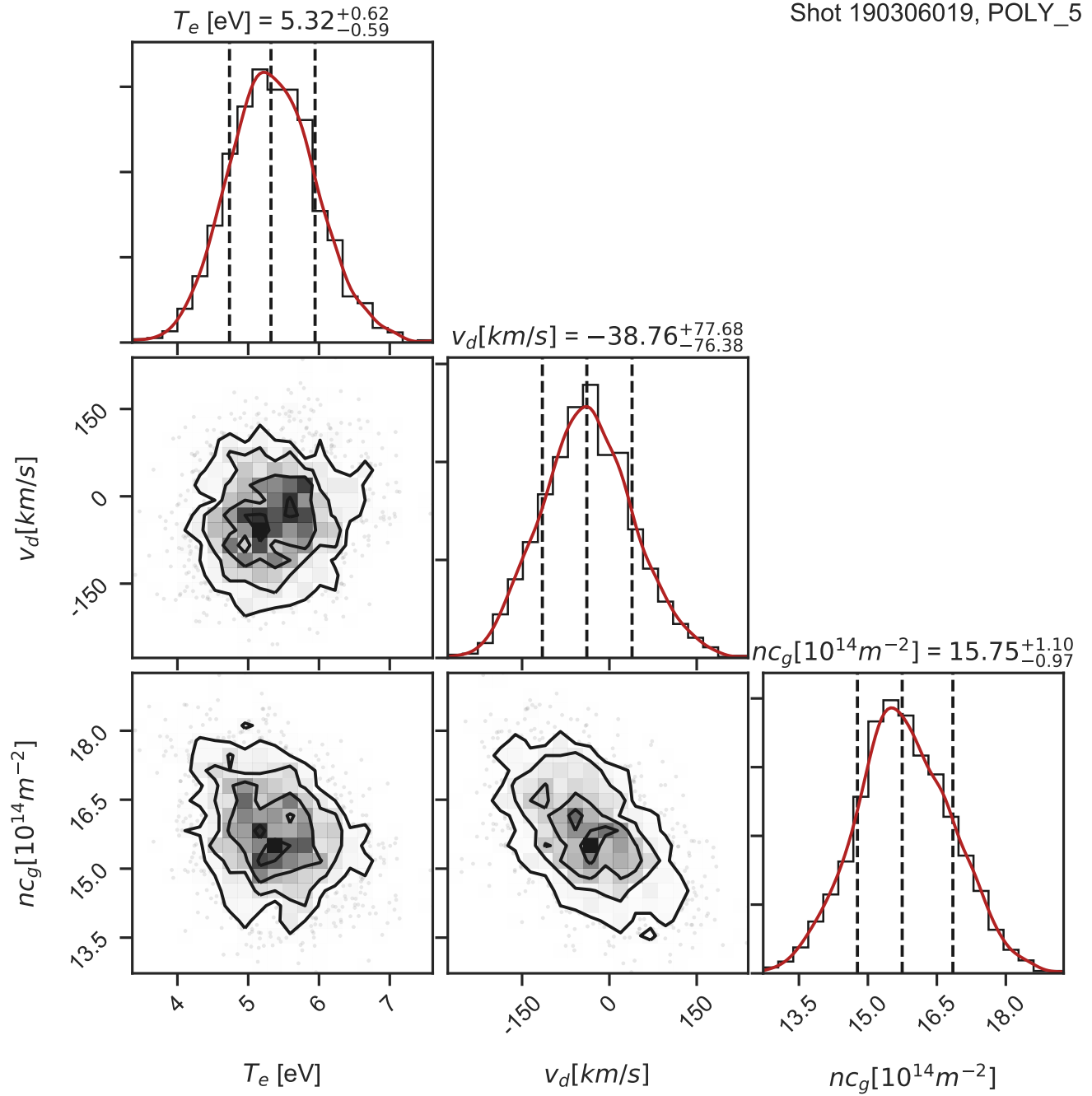


Figure 4.9: Example inference results on T_e and nc_{geom} using the nested sampling algorithm in *nestle*.

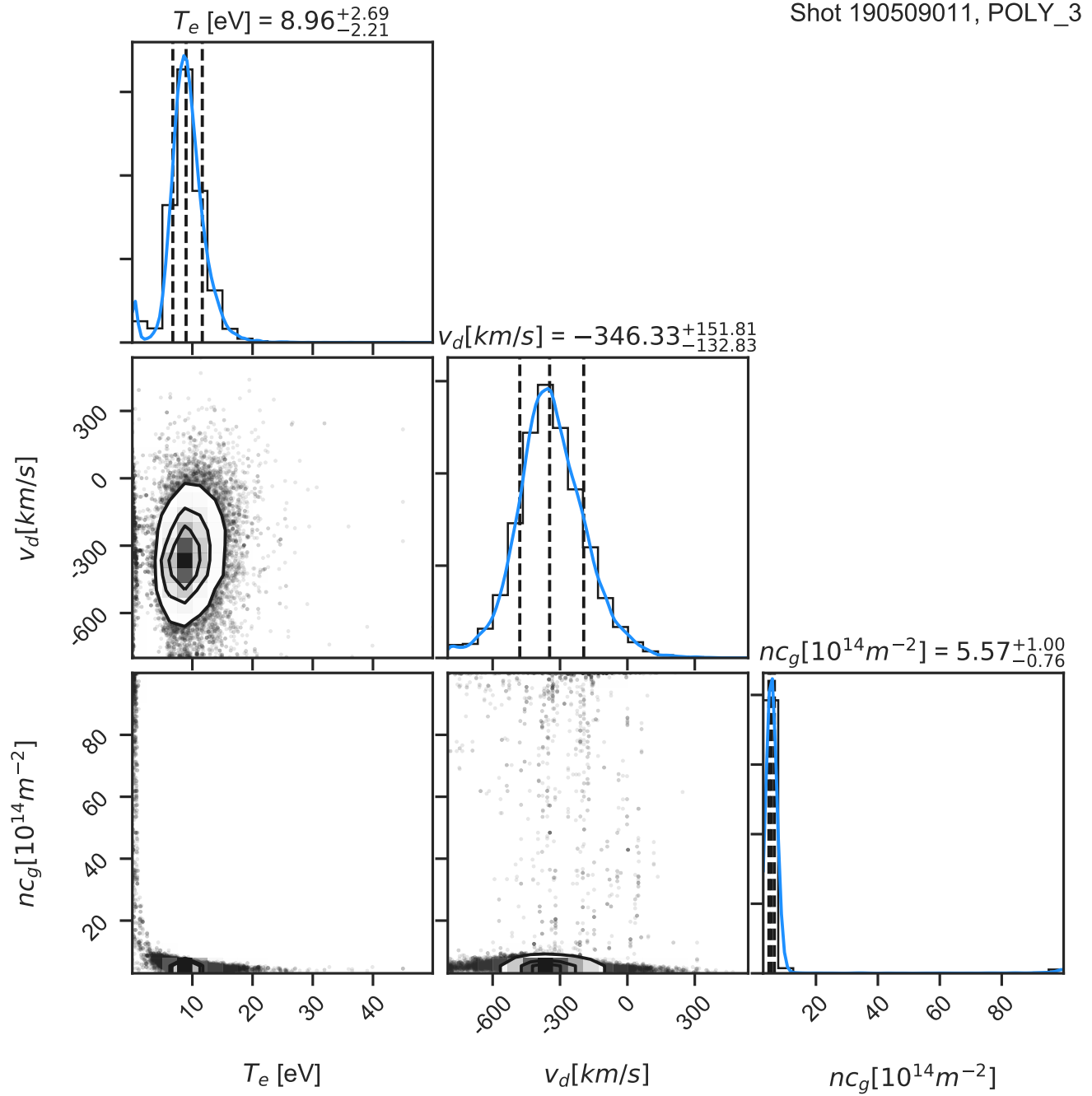


Figure 4.10: Example inference results on T_e and nc_{geom} using the Metropolis algorithm in PyMC3.

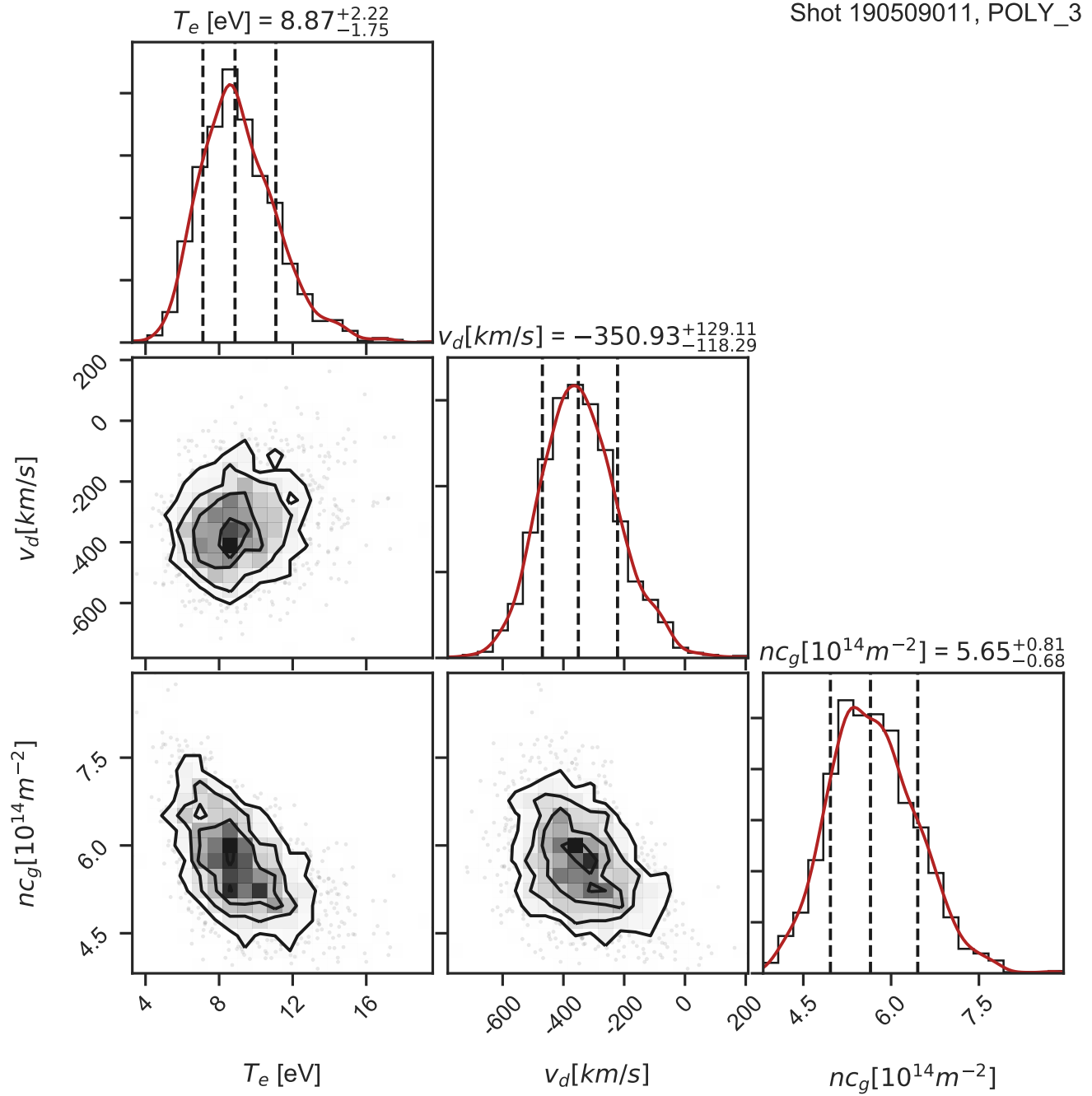


Figure 4.11: Example inference results on T_e and nc_{geom} using the nested sampling algorithm in *nestle*.

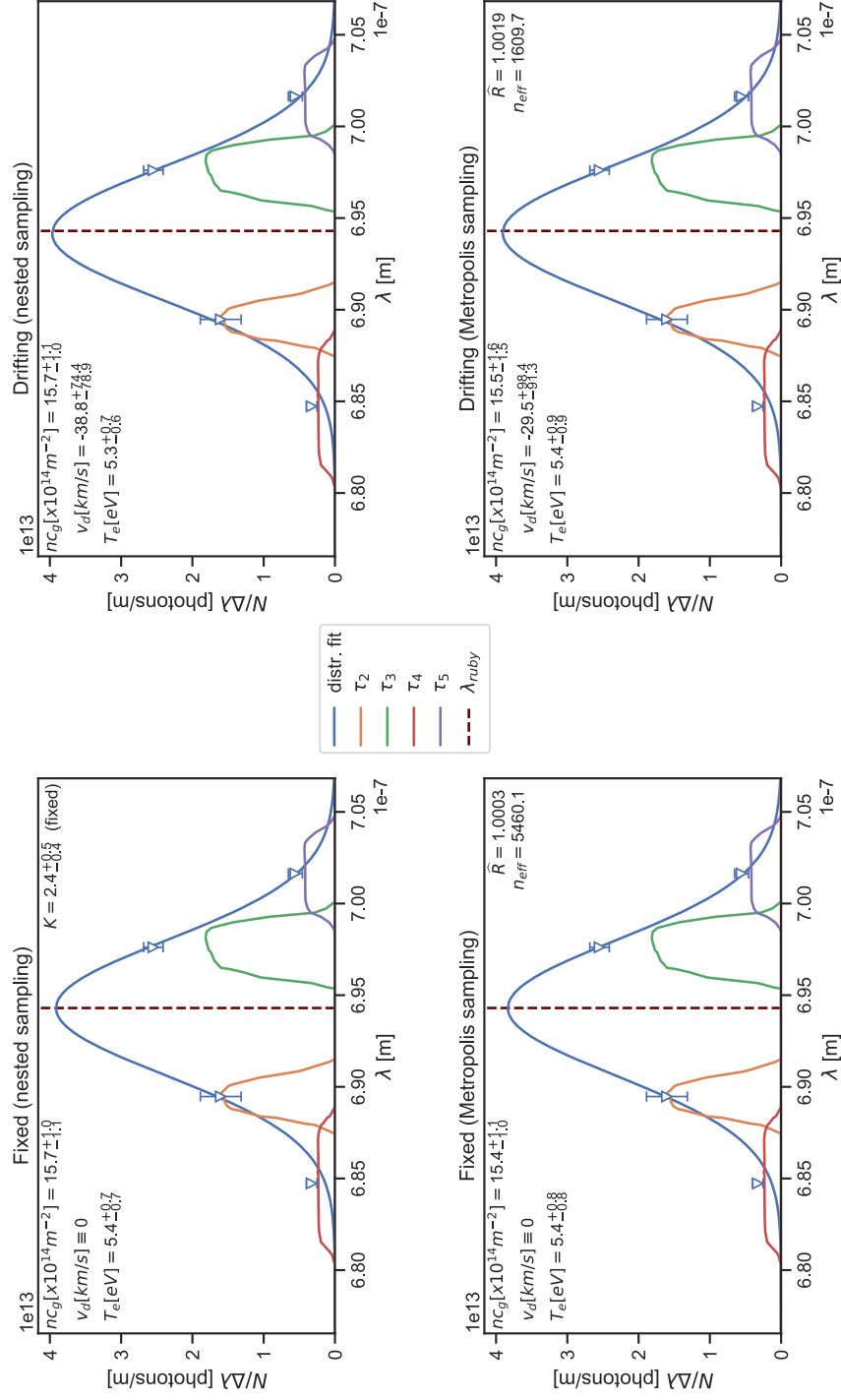


Figure 4.12: Example inference results on T_e and nc_{geom} using the Metropolis algorithm in PyMC3.

Thomson scattering system. A tendency for blue-shifted (along $+\hat{k}$) velocities is observed, as is a trend of higher magnitude shifts at higher major radii. A small positive slope in fig. 4.13 would suggest a dependence in the electron velocity on the direction of toroidal current in the spheromak. This would agree with the intuition of reversed poloidal current, though the large blue-shifted offset makes this harder to interpret, and it is plausible that these results may be from some systematic error that has not been accounted for. With the present capabilities and limitations of the diagnostic, the measurements of elec-

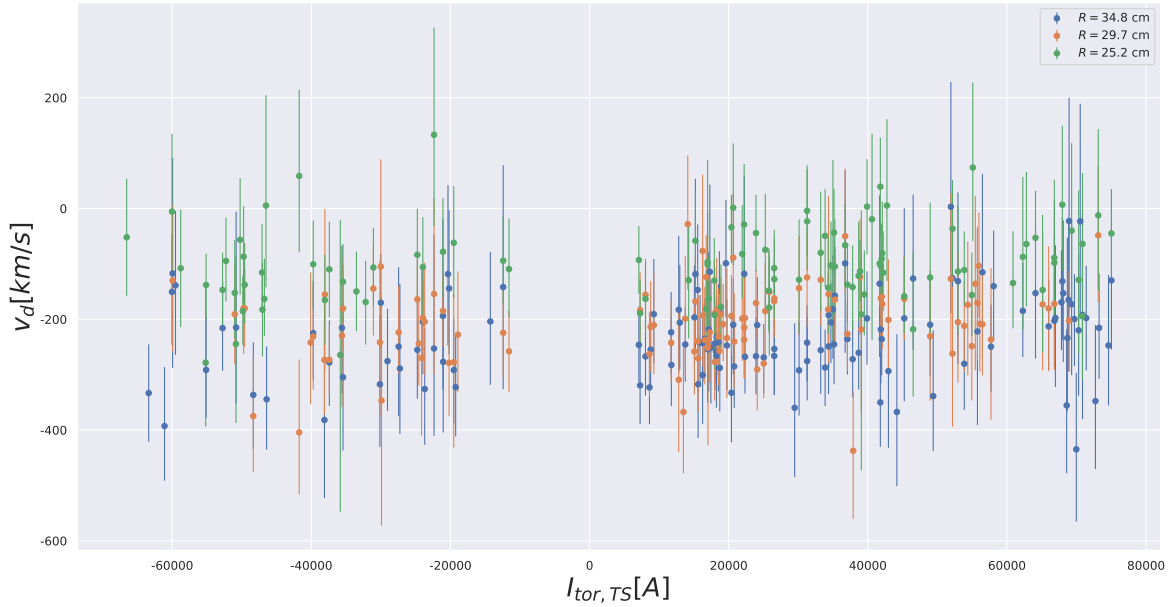


Figure 4.13: An aggregate view of electron streaming velocity measurements from HIT-SI3, plotted against the value of toroidal spheromak current at the time the measurement was taken.

tron streaming velocity are not thought to be reliable enough for significant analysis or discussion, but this avenue of measurement may prove fruitful in the future, especially if higher resolution can be achieved in the monochromator used in the spectral calibration.

Chapter 5

RESULTS AND ANALYSIS: ELECTRON CONFINEMENT IN HIT-SI3

5.1 *Thomson Measurements*

This section will attempt to organize and visualize the results from over 650 shots with Thomson data. Thomson scattering is considered the gold standard for measuring plasma electron temperature, but taken alone, its ability to fully explain any aspect of a plasma as complex as those observed in HIT-SI3 is limited. The greatest limitation of this TS diagnostic is its time resolution. The laser can only be fired once per plasma discharge, so if temporal trends are to be observed, they emerge slowly over many measurements. The total amount of data taken so far is substantial, but unlikely to circumscribe a complete picture of the behavior of the HIT-SI3 spheromak.

These are the first TS measurements on an inductively driven spheromak, and as such the data presented must be viewed as a broad initial survey of temperature and relative density over the multitude of other parameters that determine plasma behavior. Thomson scattering results of the resolution provided by this diagnostic may support or contradict more specific claims of the plasma behavior in the future, but this work represents the most complete understanding of the electron temperature and relative density to date.

5.1.1 *Pressure Profile Observations*

The first aim of the proposed research was to search for evidence of pressure confinement in the HIT-SI3 spheromak. Evidence for pressure confinement in the predecessor experiment, HIT-SI, was presented in Ref. [22]. Fig. 5.1 shows the location of the spheromak's current centroid as determined by magnetic field measurements along the flux conserver boundary. The outward radial shift observed is thought to be evidence of the Shafranov

shift, a phenomenon that occurs in pressure-confining plasma equilibria. Other possible causes of this shift, such as plasma rotation and highly peaked current distributions are shown to be unlikely. Pressure confinement as quantified by the parameter β is a funda-

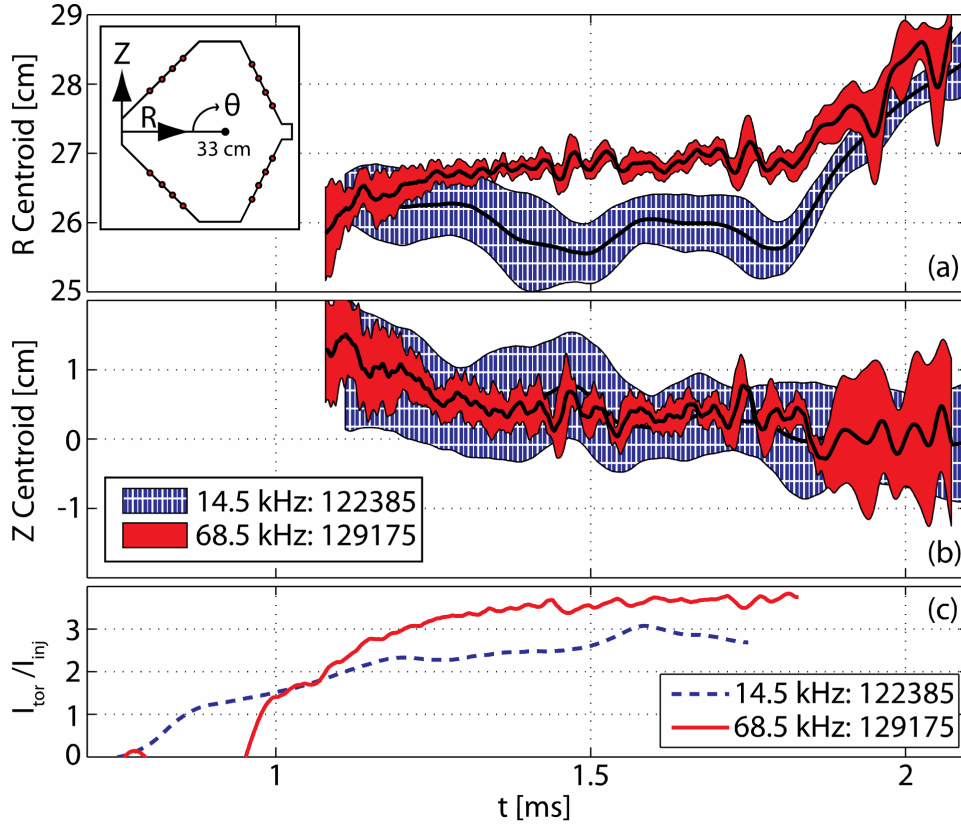


Figure 5.1: Measurements of current centroid as determined by magnetic field probes show an outward radial shift in high-frequency HIT-SI operations [22].

mental figure of merit for magnetic confinement devices. Definitive knowledge of plasma β requires concurrent and spatially-resolved measurements of the magnetic field as well as electron density and temperature. As discussed in sec. 2.3.2, the Mercier criterion balances magnetic shear against the thermal pressure gradient present in the magnetic equilibrium. A pressure profile approaching the Mercier limit, i.e. one that would affect the gross plasma equilibrium, may be observable in the measurements that have been taken.

A single shot provides temperature and relative density at up to three locations in the plasma (e.g. figs. 5.2, 5.3). The major radial locations chosen for this study span roughly

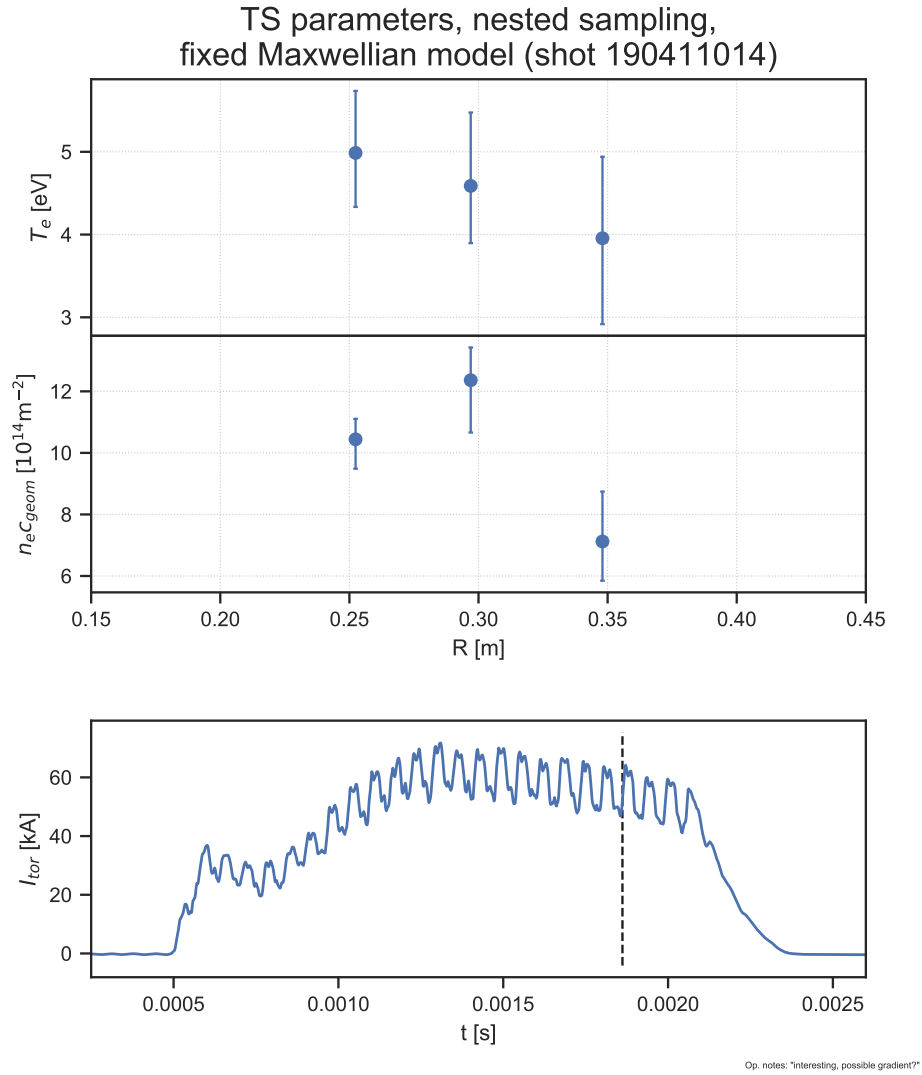


Figure 5.2: Example data of a single TS measurement

half of the radial availability of the diagnostic, and are hoped to straddle the magnetic axis of the spheromak magnetic topology. While figs. 5.2 and 5.3 demonstrate what appears to be an increase in the central spheromak electron temperature, analysis of the wider

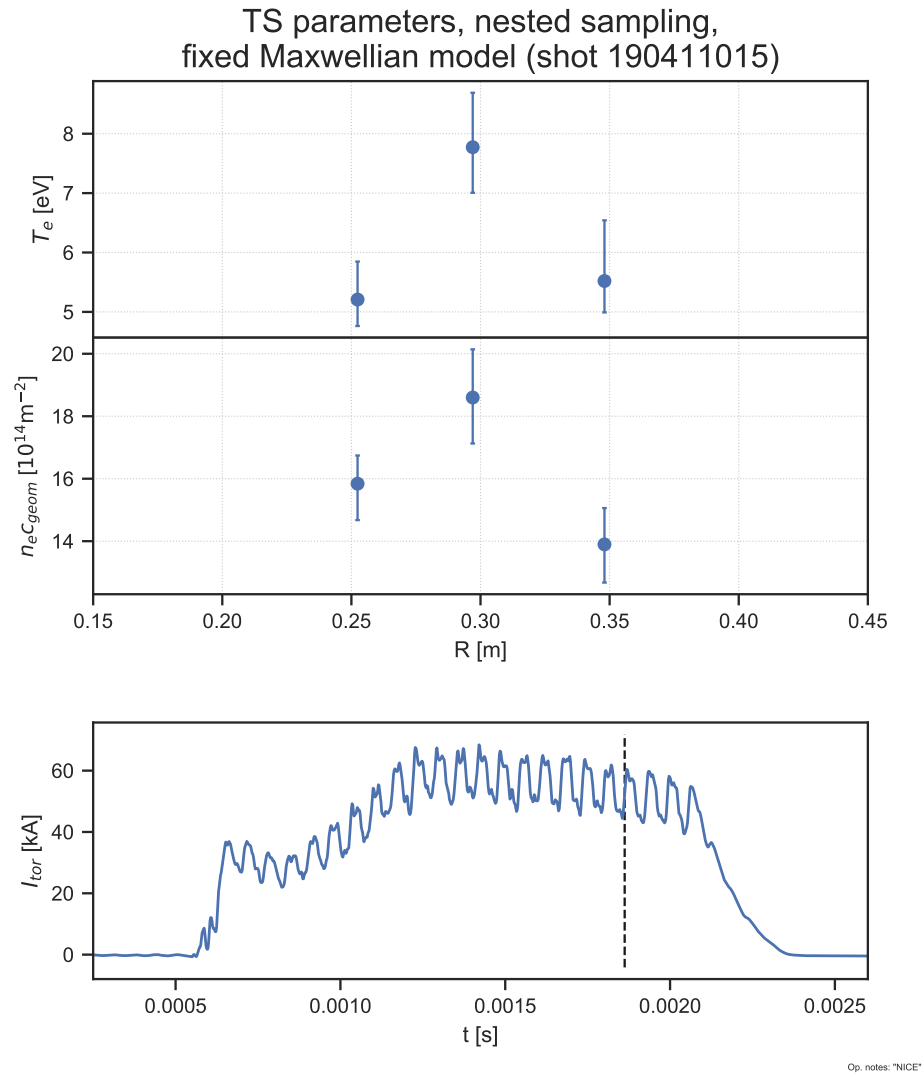


Figure 5.3: Example data of a single TS measurement

data set suggests that this is more likely a coincidence, and the radial distribution of temperature in general does not prefer a particular profile. Figs. 5.4 and 5.5 show measured profiles for every measured discharge for which the total measurement uncertainty of T_e at each of the three points was below 68%, or roughly $\pm 1\sigma$, of the measured value. This threshold is set to exclude dubious measurements that resulted from low signal-to-noise ratio (caused by low density, for instance), or sampling errors during the Bayesian inference process. Because the prior for electron temperature spans roughly 0 to 100 eV, shots with low signal-to-noise ratio on all polychromators often have posterior medians near 50 eV. This pollutes the results with apparently high temperatures. The $0.68T_e$ threshold removes these uncertain shots. It may be argued that, in light of fig. 4.5, this threshold erroneously excludes high-temperature measurements. While an exhaustive search over the eliminated measurements for high T_e and high signal-to-noise ratio in the raw data has not been carried out, only nine points are left out that would otherwise be present if the threshold were increased to 100% of the measurement, and these nine are mostly spurious. Appendix D covers explicitly the four highest-performance discharges measured by the diagnostic.

In order to fully rule out or confirm the fleeting presence of high temperature electrons (> 20 eV), more sensitive polychromators would be required, but the ubiquity of temperature measurements below 20 eV, even at high values of I_{tor} and current gain, suggests that the interpretations herein are appropriate for the vast majority of low-frequency HIT-SI3 plasmas.

The data presented here do not rule out the possibility of pressure confinement, but do not suggest the presence of a pressure gradient such as would result from a significant degree of thermal or particle confinement — due to closed magnetic flux, for instance. With relatively flat pressure profiles, a possible confinement scenario that remains plausible is a spheromak with mostly open and stochastic field lines, but with short-lived small regions of elongated field lines, perhaps approaching surfaces of closed flux.

Such “stellarator-like” phenomena have been observed in electrostatically-driven spheromak

Average radial profiles, $N = 39$, $\varepsilon_{err} < 0.68T_e$

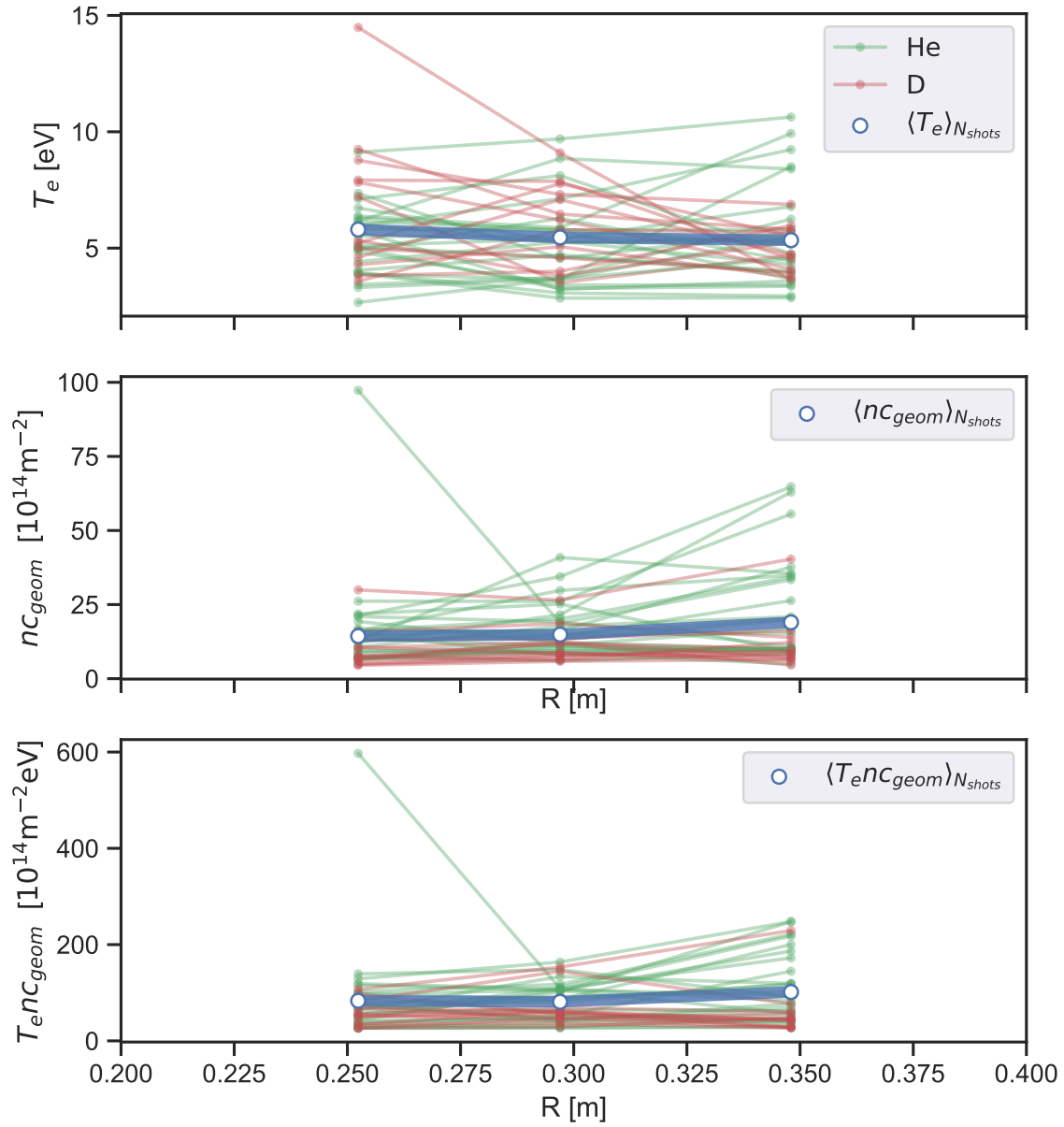


Figure 5.4: Aggregate profile measurements during drive, by fuel species

Average radial profiles, $N = 22$, $\varepsilon_{err} < 0.68T_e$

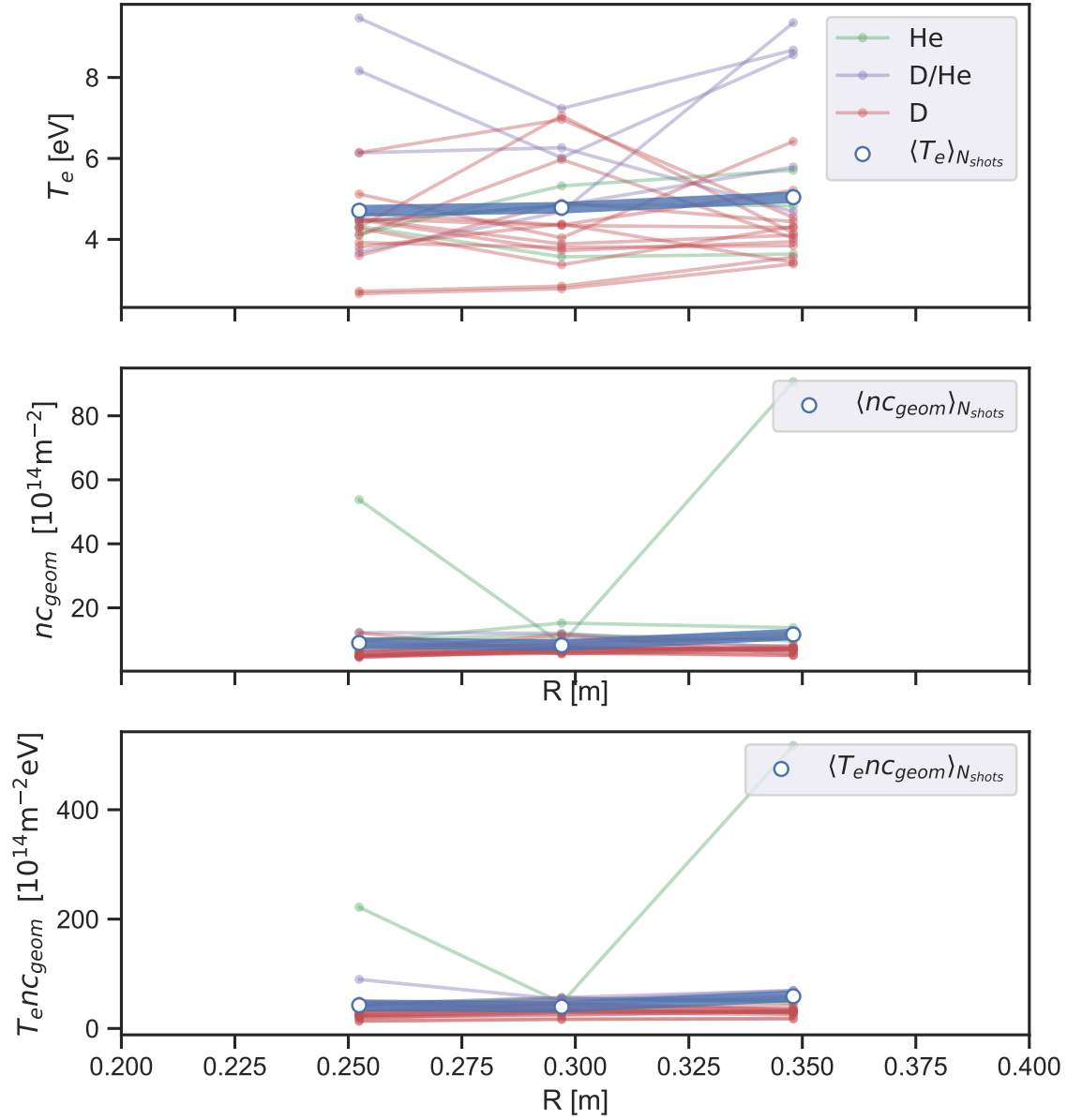


Figure 5.5: Aggregate profile measurements during decay, by fuel species

maks [14, 63, 64, 65]. The source of these regions is thought to be currents outside of the closed flux volume, with no current driven across the flux surface. As such the level of Ohmic heating observed in these volumes is expected to be negligible. Though any closed flux region may exhibit appreciable classical confinement, these volumes are observed to have high safety factor q , and neoclassical transport may be problematic for confinement. Higher q is to be expected near the magnetic axis of the spheromak. Surrounding these hypothetical regions of closed flux are observed stochastic, volume-filling magnetic fields that rapidly divert heat and particles out of the system.

It is also important here to note that the evidence for pressure confinement presented in ref. [22] suggests the centroid shifts out at higher injector frequencies relative to the centroids of low-frequency shots, which may mean that pressure confinement should only be expected for high-frequency driven HIT-SI3 spheromaks. Data in this dissertation are limited to low-frequency operations.

5.1.2 Spheromak decay

While confinement during low-frequency drive is as yet unproven, it is noted that spheromaks are known to have good confinement properties during decay [66, 27, 14]. Measurements of the decaying HIT-SI3 plasma (after the injector driving circuits have shut off) are shown in fig. 5.6 which may be compared with an ensemble of shots during drive shown in fig. 5.8.

The measurements presented in fig. 5.6 all took place during the decaying spheromak, after the injectors shut off in their respective shots. In general, the spheromak decay period provided more reliable signal. This is thought to be due to reduced line radiation during this time, but its precise cause is unknown.

The primary observation in comparing figs. 5.6 and 5.8 is that the temperatures are not seen to increase dramatically during decay; in fact they decrease monotonically throughout the decay process. This suggests that the flux surface healing and heating observed

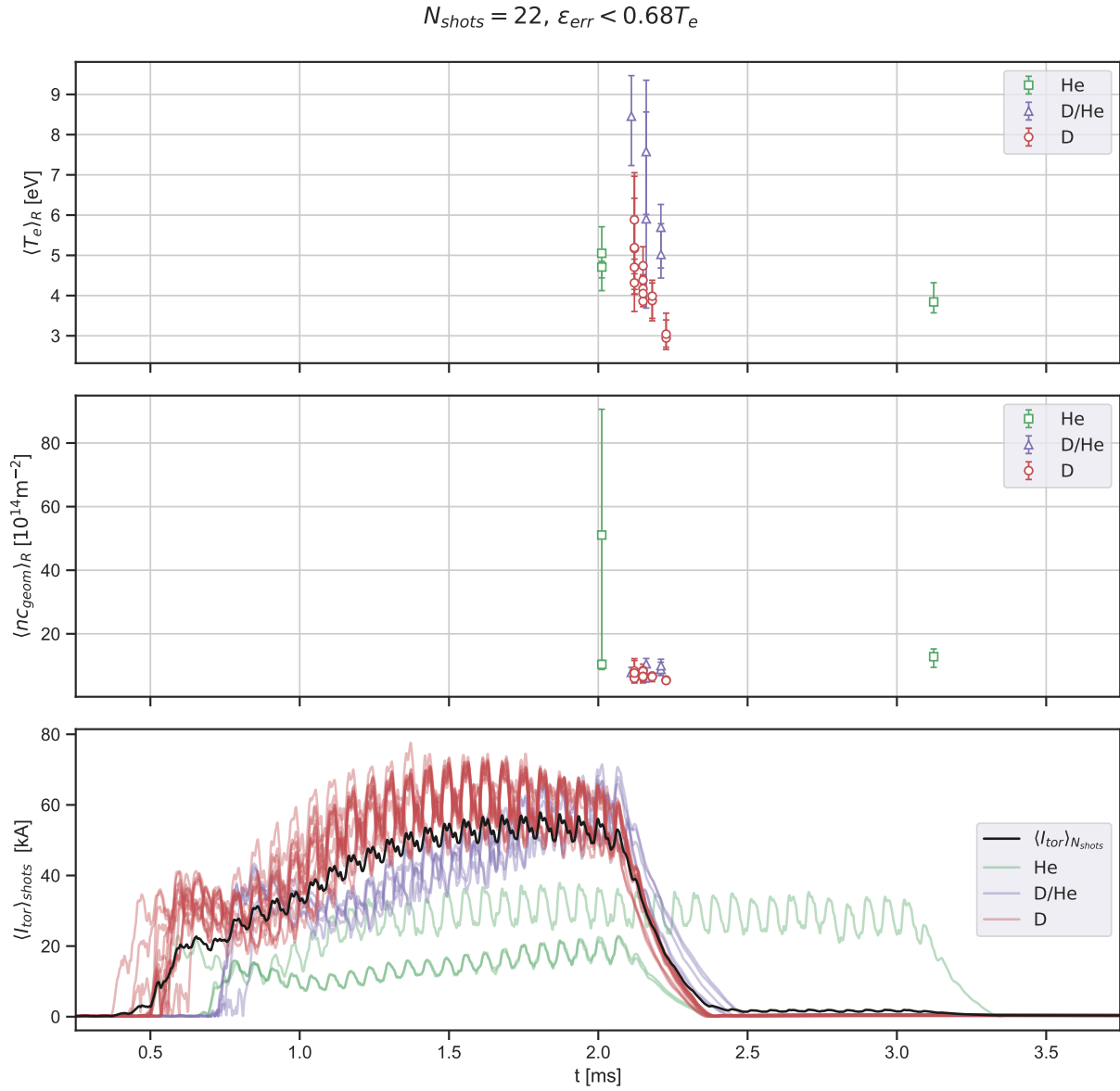


Figure 5.6: A time-resolved ensemble of TS measurements, all taken during decay (after injectors turn off). Thomson parameters are averaged over all three spatial locations, with error bars representing the highest and lowest values of T_e or nc_{geom} across the three spatial points. Shots for which any temperature measurement has $\pm\sigma$ uncertainty over the 68% threshold are excluded from this plot (only shots with all three spatial points available are included).

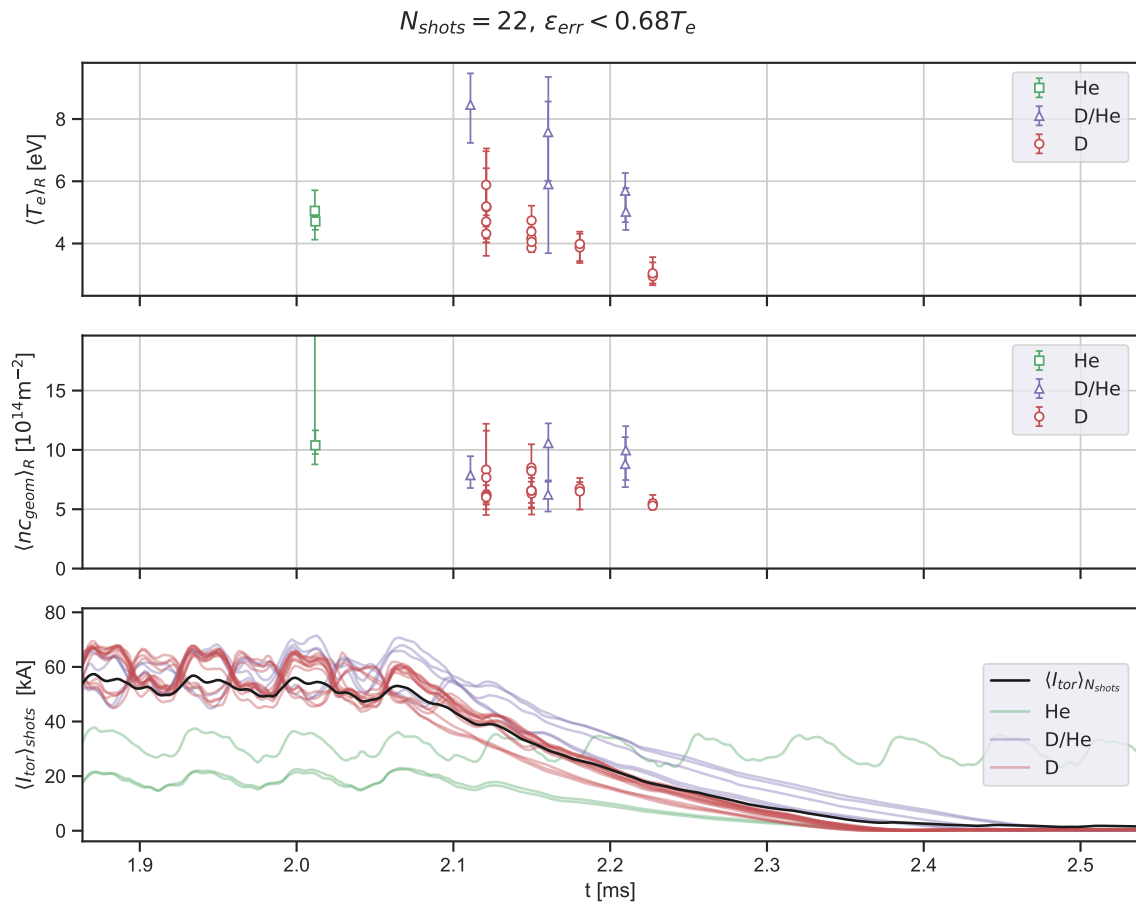


Figure 5.7: A zoomed-in view of fig. 5.6

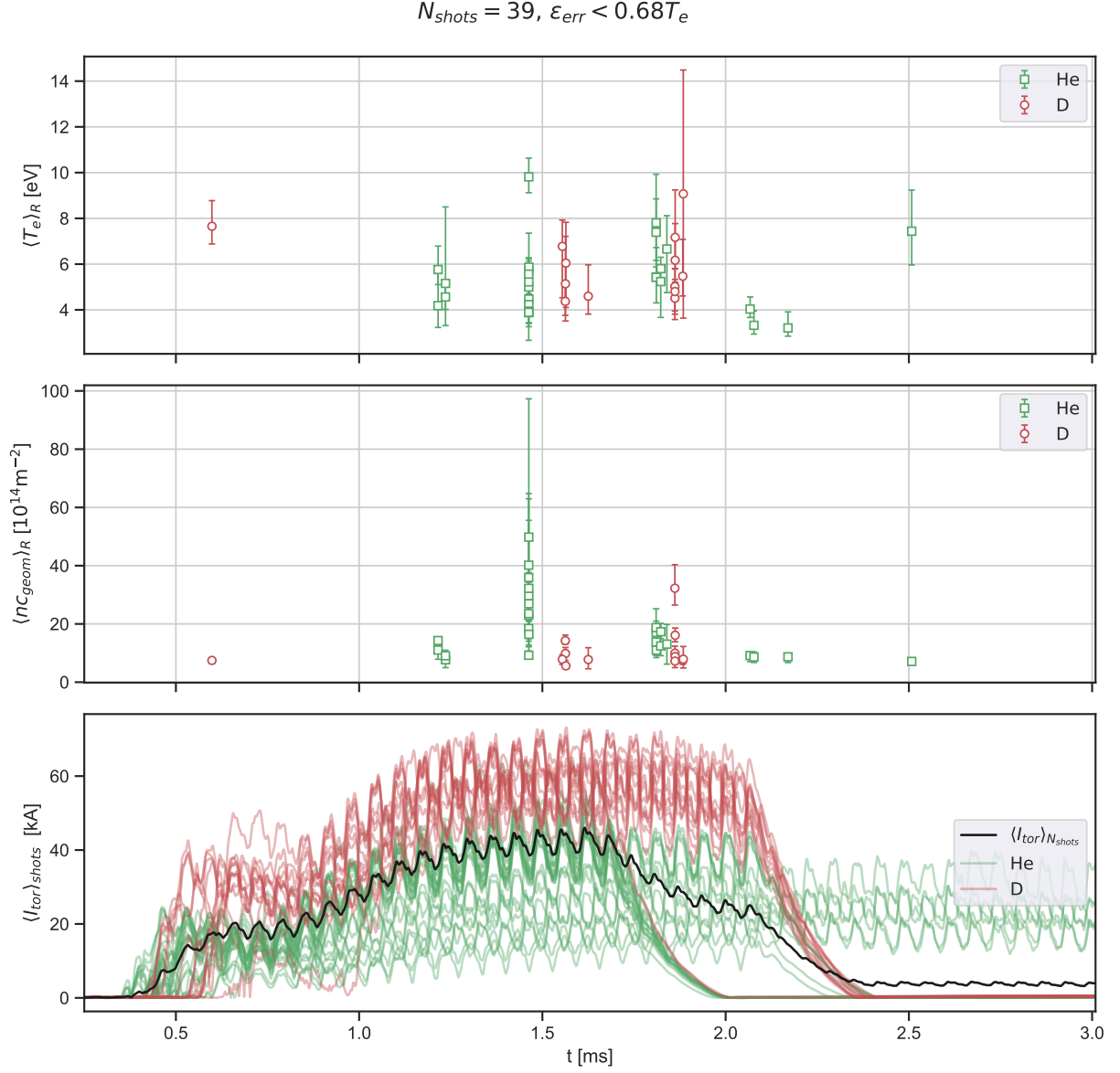


Figure 5.8: A time-resolved ensemble of TS measurements, all taken during drive (before injectors turn off). Thomson parameters are averaged over all three spatial locations, with error bars representing the highest and lowest values of T_e or nc_{geom} across the three spatial points. Shots for which any temperature measurement has $\pm\sigma$ uncertainty over the 68% threshold are excluded from this plot (only shots with all three spatial points available are included).

in other decaying spheromaks is absent in this case, perhaps because the energy confinement time does not equal the resistive decay time until late in the decay (see sec. 6.2).

5.1.3 *Summary Results*

In the interest of giving a perspective on possible relationships present in the data collected, this section presents scatter plots of temperature and density against various experimental parameters. Definite correlations are difficult to assert, but it may be observed that the highest temperatures are seen to occur at relatively low density (fig. 5.13), higher toroidal current (fig. 5.9), higher current gain (defined as the spheromak toroidal current divided by the sum of injector “toroidal” current, fig. 5.10), and lower time-averaged radial magnetic fluctuation percentage (fig. 5.15). These general trends are consistent with recent simulation results [26]. It is also noted that the highest density shots are observed to be those using helium fuel, with Deuterium-fueled shots the least dense, which agrees with other measurements made by the FIR (App. C).

5.1.4 *Comparison with 3D simulation*

Full 3D MHD simulations of HIT-SI3 are available [20, 25] and can be compared to measurements from the Thomson scattering system. One limitation of this kind of comparison is the disparity between the resolution of the simulation and that of the diagnostic. The measurements from Thomson scattering lead to insights over many single discharges, whereas simulating a large set of discharges is prohibitively expensive and time consuming. The intersection between the two methods is therefore quite small, but is nonetheless presented here in one discharge. HIT-SI3 shot 190411015 (with moderately high toroidal current and gain) was simulated in NIMROD [19]. The electron temperatures at the three spatial locations measured on the real plasma by the TS diagnostic are shown in fig. 5.16. There is significant disagreement. A reason that may account for the discrepancy in temperatures seen in fig. 5.16 is anomalous thermal conductivity. The NIMROD simulation

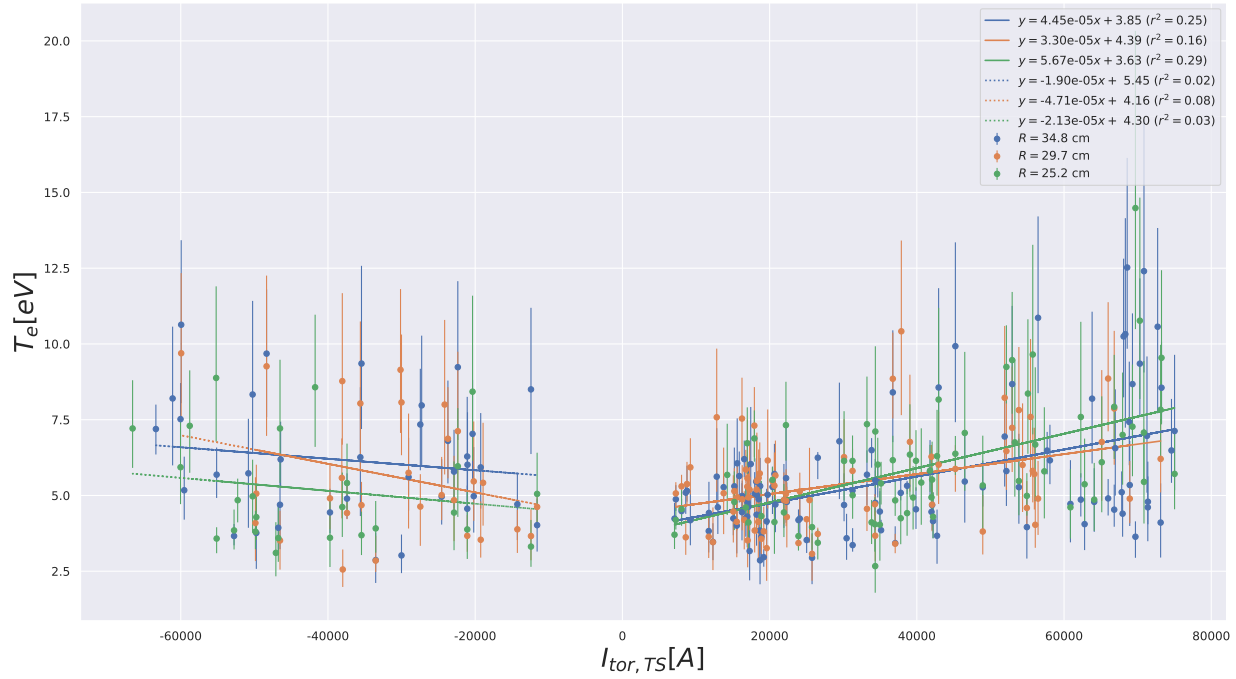


Figure 5.9: An aggregate view of electron temperature measurements from HIT-SI3, plotted against the value of toroidal spheromak current at the time the measurement was taken.

presented uses the Braginskii formulation for the electron perpendicular thermal conductivity coefficient [62]. This topic is explored in sec. 6.2. The same reason might be responsible for another notable difference between the simulation and experiment: the temperature behavior during decay. Whereas the experiment is observed to lose roughly half of its electron temperature within $100\mu\text{ s}$ (fig. 5.6), the simulation decay time is much longer, and includes brief periods of local electron heating (fig. 5.17). Late during the simulated decay, a somewhat peaked temperature profile also forms, which is not observed in the experiment (fig. 5.5).

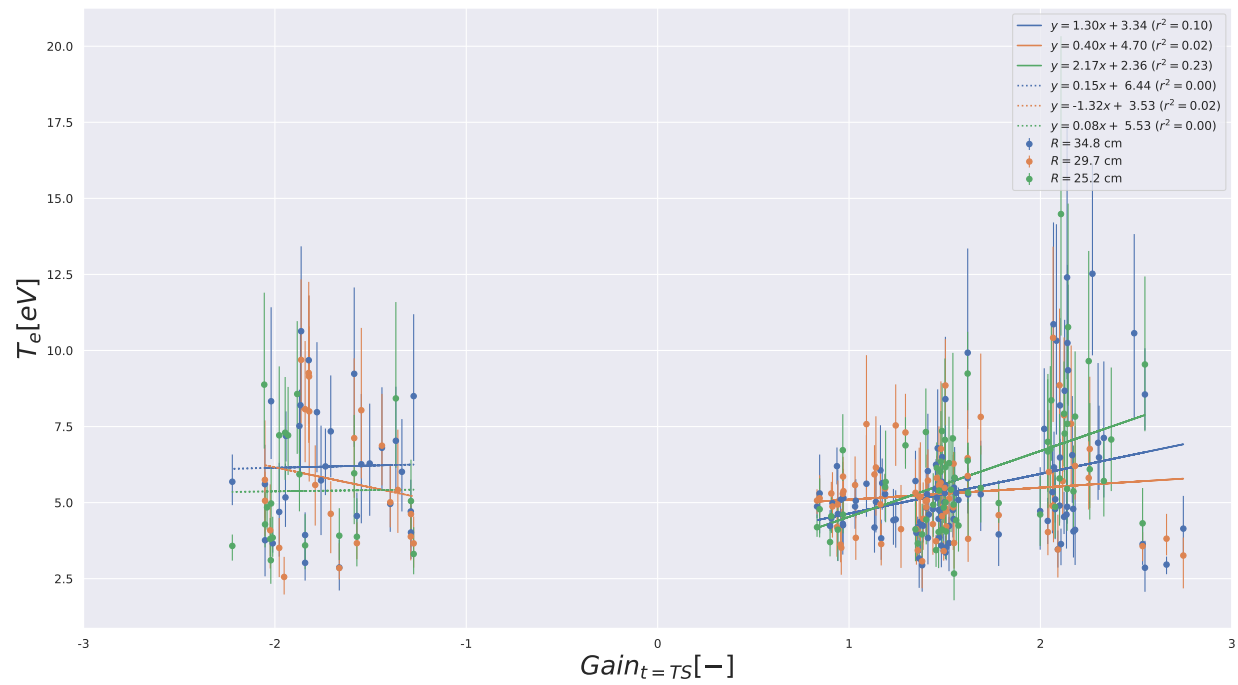


Figure 5.10: An aggregate view of electron temperature measurements from HIT-SI3, plotted against the value of toroidal current gain at the time the measurement was taken.

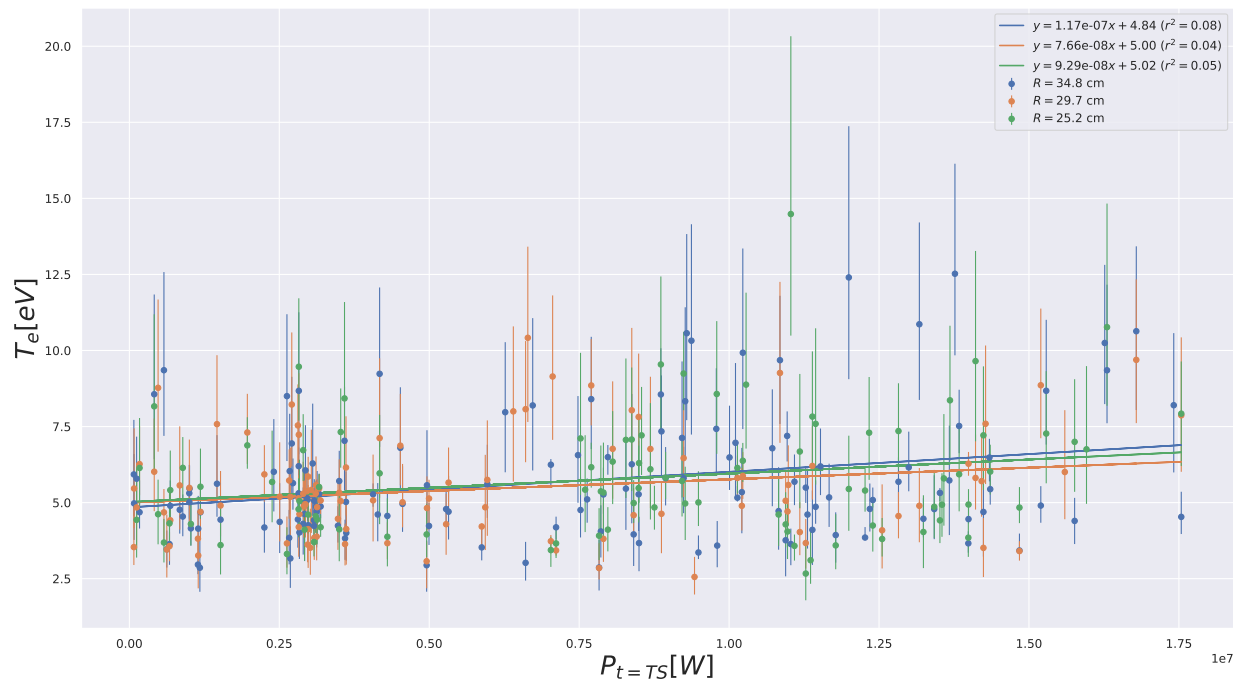


Figure 5.11: An aggregate view of electron temperature measurements from HIT-SI3, plotted against the value of total injector power at the time the measurement was taken.

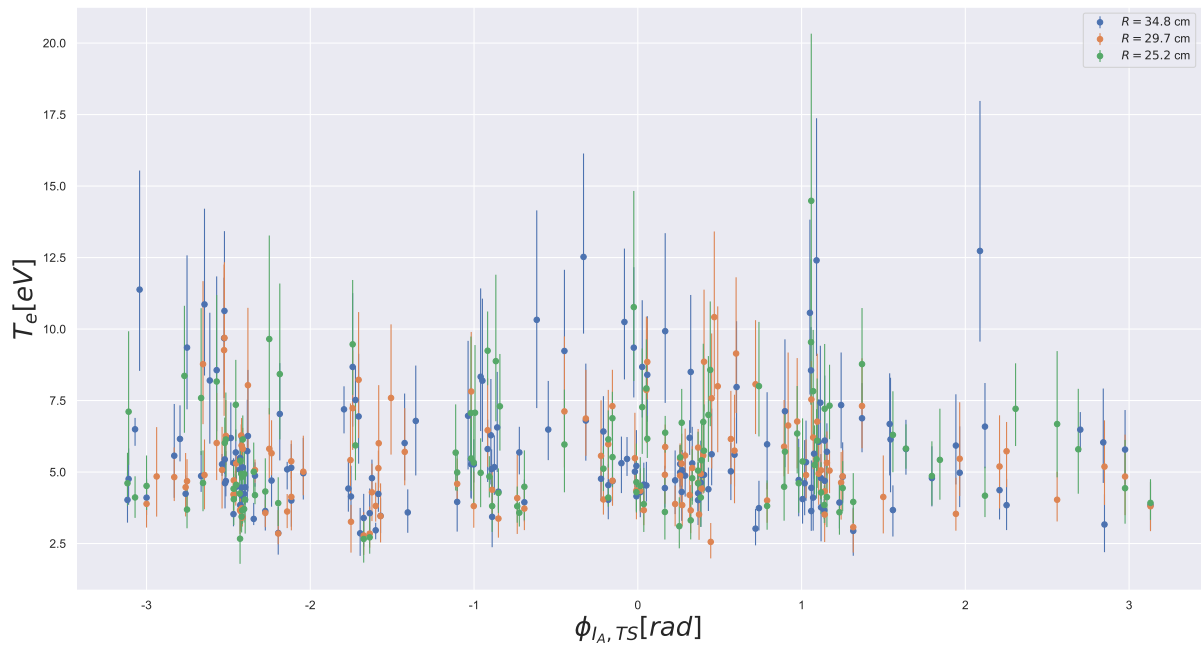


Figure 5.12: An aggregate view of electron temperature measurements from HIT-SI3, plotted against the value of injector phase (inj. A) at the time the measurement was taken.

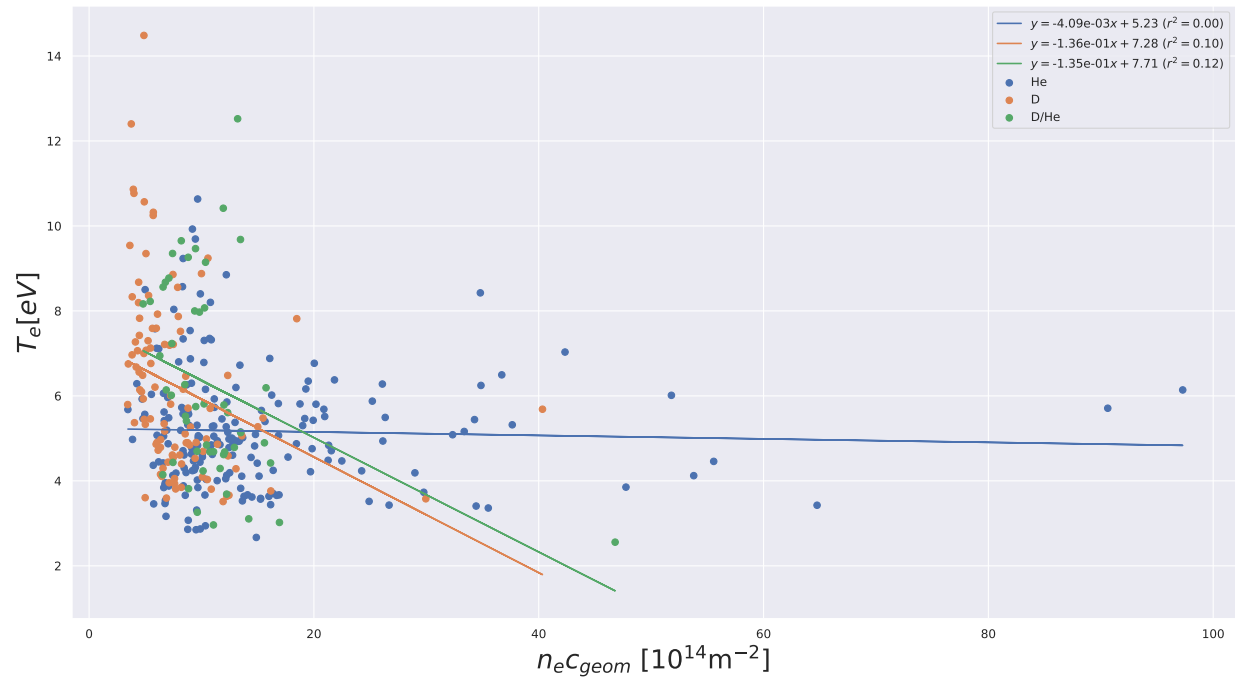


Figure 5.13: An aggregate view of electron temperature measurements from HIT-SI3, plotted against the relative density.

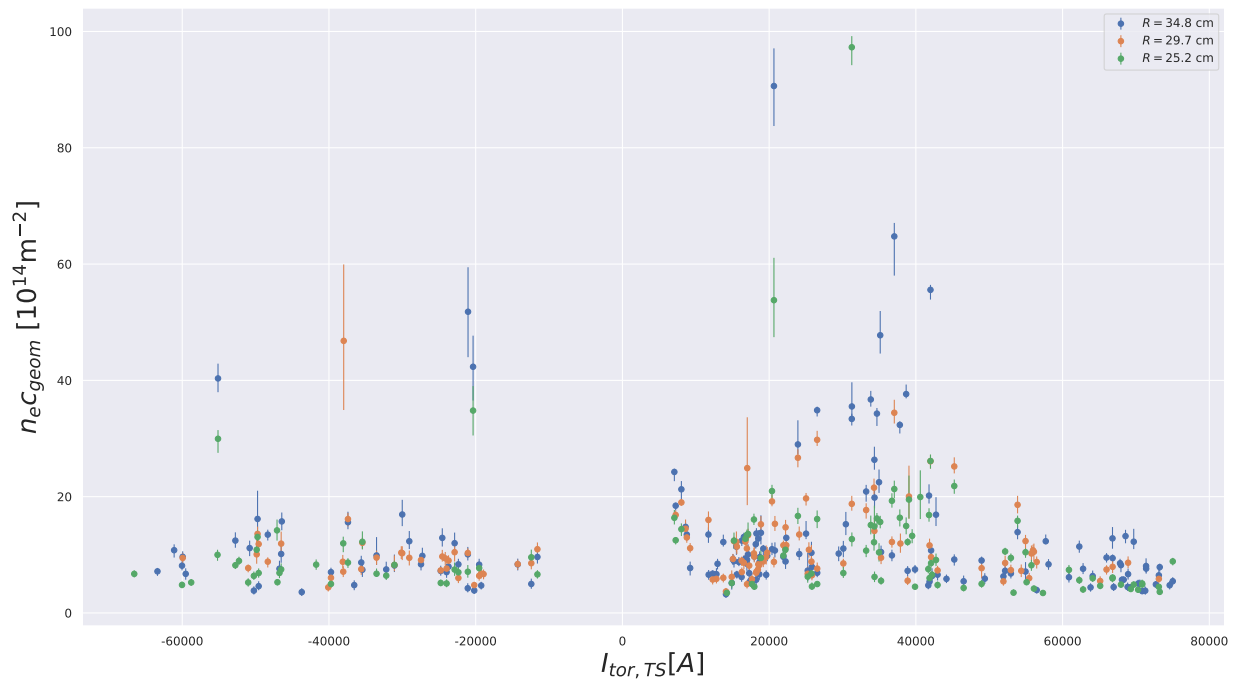


Figure 5.14: An aggregate view of relative electron density measurements from HIT-SI3, plotted against the value of toroidal spheromak current at the time the measurement was taken.

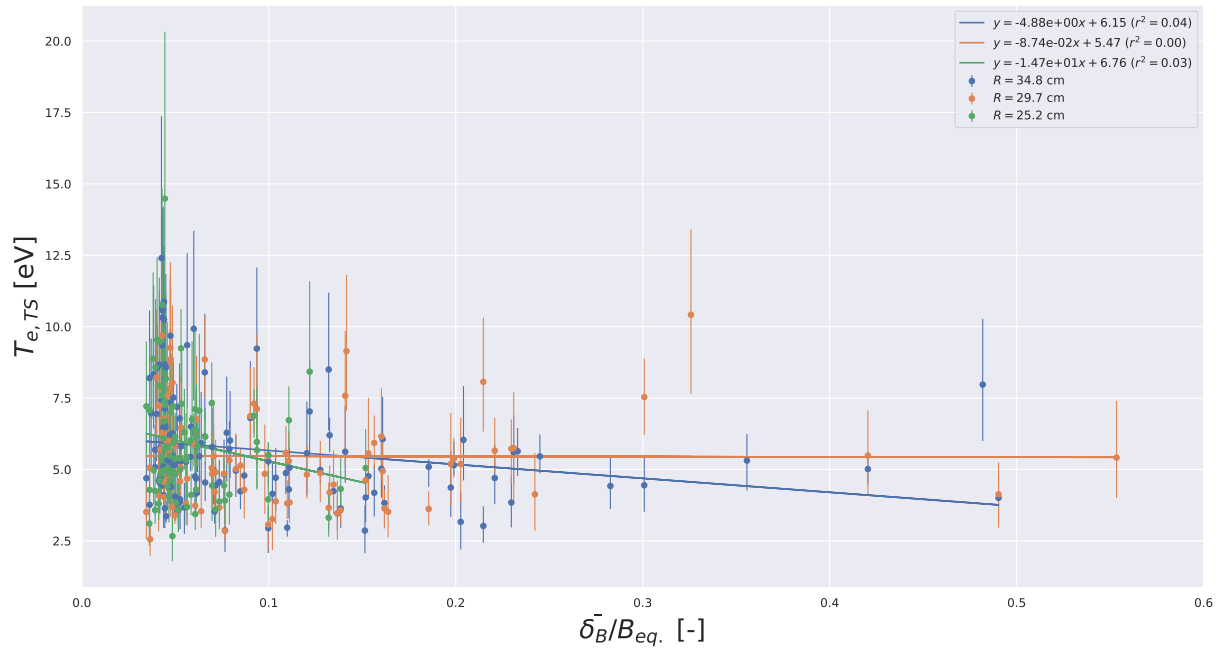


Figure 5.15: An aggregate view of electron temperature measurements from HIT-SI3, plotted against the time-averaged value of magnetic field fluctuation as measured by an internal array of probes for the shot during which the measurement was taken.

NIMROD:Thomson Comparison, shot 190411015

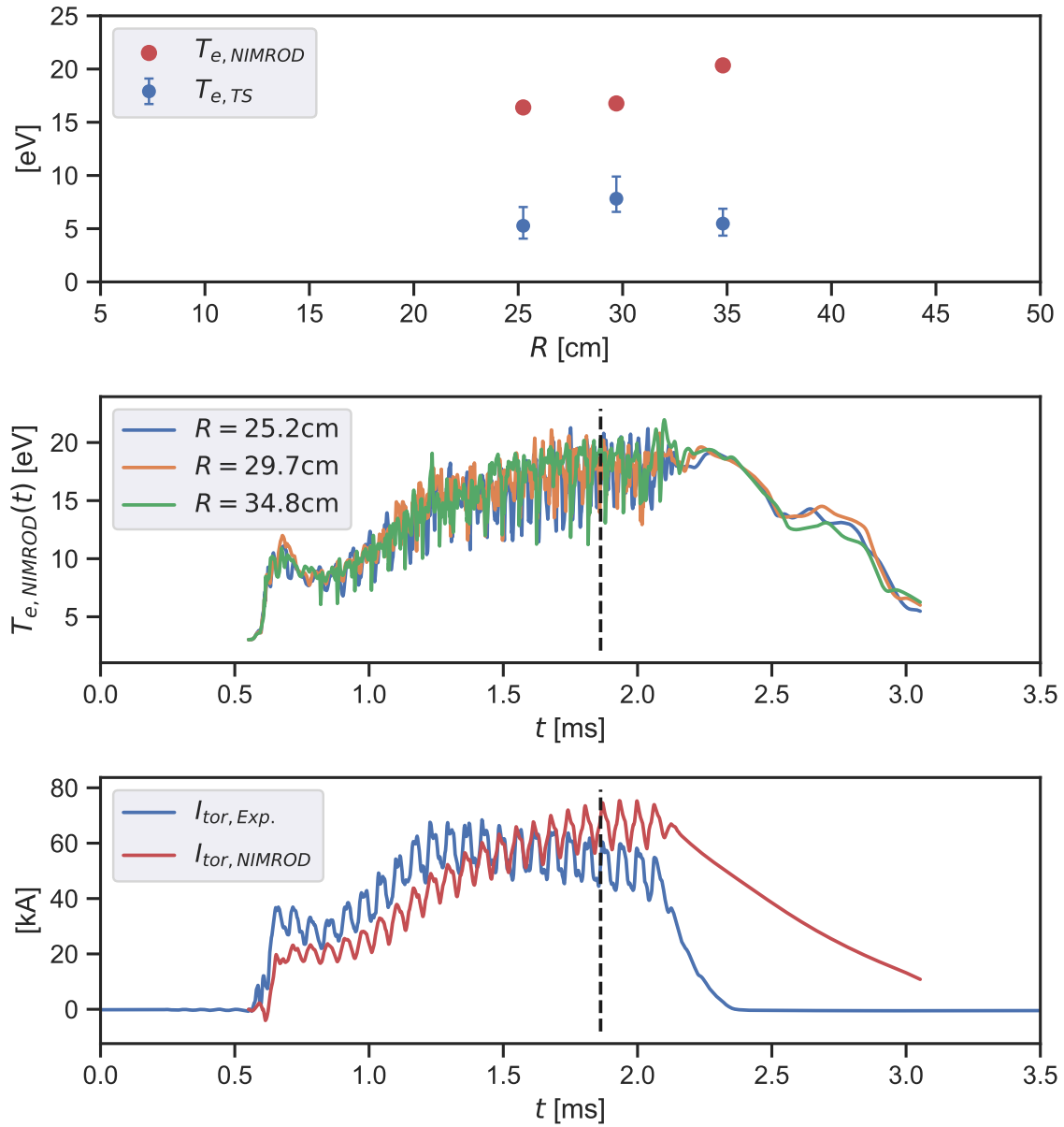


Figure 5.16: A comparison of NIMROD simulation results for electron temperature with TS measurements from the same shot.

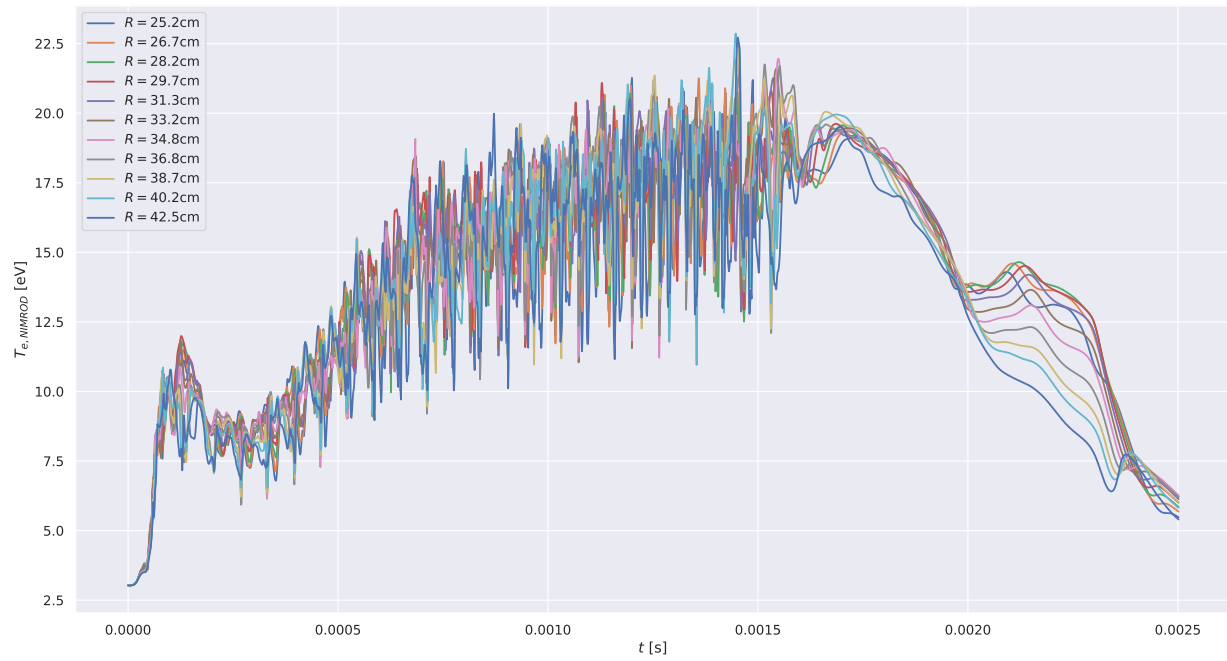


Figure 5.17: NIMROD simulation results for electron temperature at possible TS measurement locations from the shot 190411015.

Chapter 6

MODELING AND ENERGY FLOW ANALYSIS

6.1 Poynting Flux Injector Model

How can energy flow in HIT-SI3 be understood? A natural place to start the analysis is the injectors. These are the initial sources of energy and plasma in a HIT-SI3 discharge. One way to model the injectors is to use Poynting's Theorem to quantify their electromagnetic power balance. The model that results will allow for (1) the temperature of the injector plasma electrons to be estimated, (2) a complete characterization of the electromagnetic power injected into the HIT-SI3 plasma, and (3) estimation of the magnetic energy content of the injector region. In turn, this model will inform the overall map of how energy flows in the HIT-SI3 plasma. This model is determined by parameters that are measured for every discharge on the HIT-SI3 experiment.

Poynting's Theorem is

$$-\int_V \frac{\partial u}{\partial t} dV = \frac{1}{\mu_0} \oint_{\partial V} (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{A} + \int_V \mathbf{j} \cdot \mathbf{E} dV \quad (6.1)$$

This is a statement of the conservation of electromagnetic energy in a particular volume over time. The energy stored in the electric and magnetic fields of a volume V is

$$\int_V u dV = \int_V (u_E + u_B) dV = \int_V \left(\frac{B^2}{2\mu_0} + \frac{\epsilon_0 E^2}{2} \right) dV \quad (6.2)$$

Comparing the relative magnitudes of energy densities u_E and u_B using conservative estimates for the fields ($E \sim 1500$ V/m and $B \sim 1$ mT for HIT-SI3 injectors), it is observed that $u_B > 10^4 u_E$. The stored electric field energy is therefore neglected in the following analysis. It is assumed that the field energy may decrease in the volume V either by crossing the plasma volume boundary ∂V (first term on the right-hand side of eq. (6.1)),

or by performing work on the charges in the plasma within the injector (second term on the right-hand side of eq. (6.1)).

The Poynting vector, $\mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B})$, points in the direction of power flow. Since $d\mathbf{A}$ points out of the volume, the first term on the right of eq. (6.1) is positive when energy flows *out* of the volume. To help conceptualize the injectors, Poynting's Theorem can be rewritten to describe energy gained by the plasma particles in terms of the input field power (Poynting flux) and the rate at which energy is stored in the fields.

$$\int_V \mathbf{j} \cdot \mathbf{E} dV = \frac{-1}{\mu_0} \oint_{\partial V} (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{A} - \frac{1}{2\mu_0} \int_V \frac{\partial}{\partial t} (B^2) dV \quad (6.3)$$

The left-hand side of eq. (6.3) is the Ohmic power dissipation, which is a source of electron heating in the injectors. The electric field doing work on the electrons is then $\mathbf{E} = \eta \mathbf{j}$, where η is the electric resistivity of the plasma. Assuming that the plasma follows the model of parallel resistivity described by Spitzer gives a relationship between the resistivity η and the electron temperature [67].

$$\eta = \frac{\pi Z_{eff} e^2 m_e^{1/2} \ln \Lambda}{(4\pi \epsilon_0)^2 (k_B T_e)^{3/2}} \quad (6.4)$$

Other parameters are the effective ion charge Z_{eff} and the Coulomb Logarithm:

$$\ln \Lambda = \ln \left[\frac{12\pi (\epsilon_0 k_B T_e)^{3/2}}{e^3 \sqrt{n_e}} \right] \quad (6.5)$$

If both terms on the right hand side of eq. (6.3) are known, the Ohmic heating term can be used to roughly estimate the injector electron temperature. However, the injectors are not expected to have a significant level of thermal confinement. This is because the AC-driven confining magnetic fields pass through zero every injector cycle and thus likely lose much of their thermal energy to the walls. In the pursuit of explaining electron thermal energy in the HIT-SI3 spheromak, attention is subsequently focused on the terms on the right hand side of eq. (6.3). These depend on the electric and magnetic fields, both assumed to have negligible radial components: $\mathbf{B} = B_\phi \hat{\phi} + B_\theta \hat{\theta}$ and $\mathbf{E} = E_\phi \hat{\phi} + E_\theta \hat{\theta}$.

The volume being analyzed is a single HIT-SI3 injector — a half-torus with major radius $R_0 = 20.1$ cm and minor radius $a = 6.3$ cm (fig. 6.1).

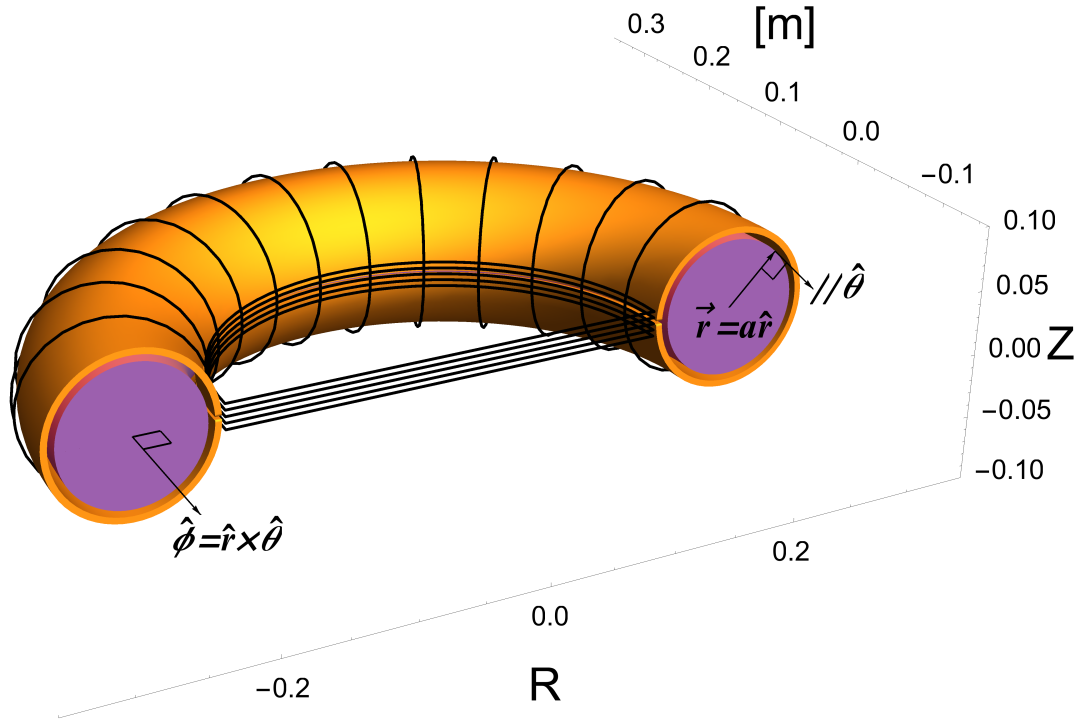


Figure 6.1: The injector-plasma volume used in the Poynting flux analysis, with $R_0 = 20.1$ cm and $a = 6.3$ cm, and the coordinate system used for integration of fields over the plasma volume V (shown in purple) and its boundary ∂V . Representative “flux” ($\mathbf{B}_\phi$) and “voltage” ($\mathbf{j}_\phi$) coils are also depicted.

6.1.1 Injector magnetic fields

Calculating the fields begins by identifying the source currents that produce them. The plasma current density in both the $\hat{\theta}$ and $\hat{\phi}$ directions, j_θ and j_ϕ , are consequent of the currents flowing in the “voltage” and “flux” circuits, respectively (fig. 2.4). The nearest HIT-SI3 measurement available to j_ϕ is given by a Rogowski loop at the injector mouth. This measures the total current flowing through the area of that mouth, I_{inj} . However, it

is possible to explore various hypothetical profiles of j_ϕ and constrain these profiles using the known value of I_{inj} :

$$I_{inj} = \int \int_{A_{mouth}} \mathbf{j}_\phi(r) \cdot d\mathbf{A} = \int_0^{2\pi} \int_0^a j_\phi(r) r dr d\theta \quad (6.6)$$

Three possible injector current profiles are assessed:

$$\begin{aligned} \mathbf{j}_\phi^h(r) &= j_{0,\phi}^h \left(\frac{r^2}{a^2} \right) \hat{\phi} = \frac{2I_{inj}}{\pi a^2} \left(\frac{r^2}{a^2} \right) \hat{\phi} && \text{(hollow)} \\ \mathbf{j}_\phi^u(r) &= j_{0,\phi}^u \hat{\phi} = \frac{I_{inj}}{\pi a^2} \hat{\phi} && \text{(uniform)} \\ \mathbf{j}_\phi^p(r) &= j_{0,\phi}^p \left(1 - \frac{r^2}{a^2} \right) \hat{\phi} = \frac{2I_{inj}}{\pi a^2} \left(1 - \frac{r^2}{a^2} \right) \hat{\phi} && \text{(peaked)} \end{aligned} \quad (6.7)$$

The azimuthal ($\hat{\phi}$) injector current is the source term for the “poloidal” ($\hat{\theta}$) component of the magnetic field, B_θ , which may be found in the following way:

$$\begin{aligned} \oint \mathbf{B} \cdot d\mathbf{l} &= \mu_0 \int \int_A \mathbf{j} \cdot d\mathbf{A} \\ \implies B_\theta^h(R, r) &= \frac{R_0}{R} \frac{\mu_0 I_{inj}}{2\pi a^4} r^3 && \text{(hollow } \mathbf{j}_\phi) \\ B_\theta^u(R, r) &= \frac{R_0}{R} \frac{\mu_0 I_{inj}}{2\pi a^2} r && \text{(uniform } \mathbf{j}_\phi) \\ B_\theta^p(R, r) &= \frac{R_0}{R} \frac{\mu_0 I_{inj}}{2\pi a^4} (2a^2 r - r^3) && \text{(peaked } \mathbf{j}_\phi) \end{aligned} \quad (6.8)$$

To determine $\mathbf{B}_\phi$, the azimuthal magnetic field, two measurements are used. The first is the current I_{flux} running through the flux circuit coil (i.e. the solenoid producing $\mathbf{B}_\phi$). The second is the measured magnetic flux from a flux loop around the injector mouth, Ψ_{inj} :

$$\Psi_{inj} = \int \int_{A_{mouth}} \mathbf{B} \cdot d\mathbf{A} = \int_0^{2\pi} \int_0^a B_\phi(r, \theta) r dr d\theta \quad (6.9)$$

Ampère’s Law gives (see fig. 6.2):

$$B_\phi(r, \theta) = \mu_0 \frac{N_{flux} I_{flux} - \int_0^\pi \int_r^a j_\theta(R_{inj} + r \cos\theta) dz d\phi}{\pi (R_{inj} + r \cos\theta)} \quad (6.10)$$

The term on the right hand side of the numerator of eq. (6.10) accounts for the contribu-

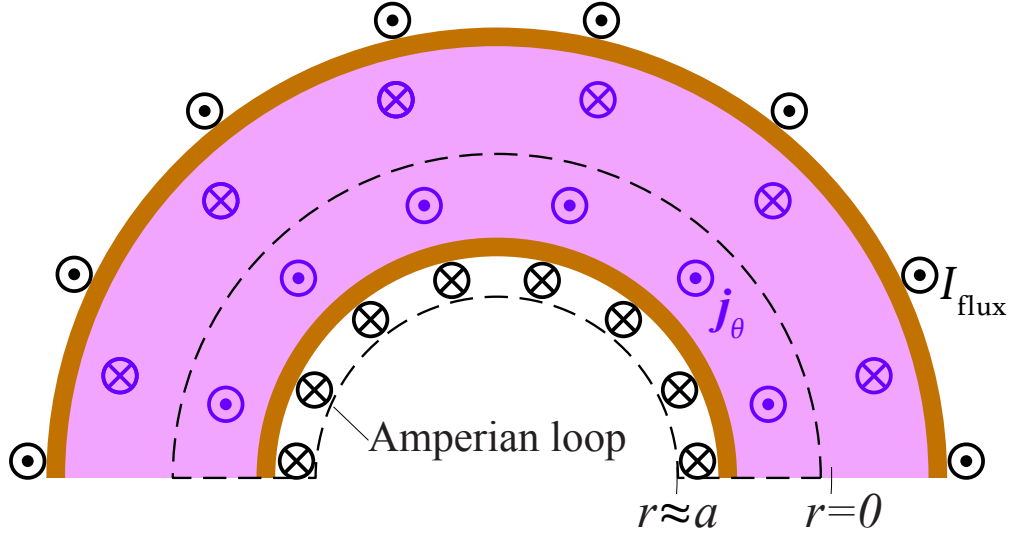


Figure 6.2: A top cutaway view of an injector showing an Amperian loop drawn to find B_ϕ resulting from j_θ and I_{flux} . The effective number of flux coil turns N_{flux} is determined by performing the analysis in the vacuum period before plasma breakdown in each shot.

tion to B_ϕ from the injector plasma current along the poloidal direction, j_θ :

$$\begin{aligned}
 j_\theta^h(r) &= j_{0,\theta}^h \left(\frac{r^2}{a^2} \right) \hat{\theta} = \frac{12I_\theta^h r^2}{\pi a^3 (4R_{inj} + 3a \cos \theta)} \hat{\theta} && \text{(hollow)} \\
 j_\theta^u(r) &= j_{0,\theta}^u \hat{\theta} = \frac{I_\theta^u}{\pi a (R_{inj} + \frac{a \cos \theta}{2})} \hat{\theta} && \text{(uniform)} \\
 j_\theta^p(r) &= j_{0,\theta}^p \left(1 - \frac{r^2}{a^2} \right) \hat{\theta} = \frac{12I_\theta^p}{\pi a (8R_{inj} + 3a \cos \theta)} \left(1 - \frac{r^2}{a^2} \right) \hat{\theta} && \text{(peaked)}
 \end{aligned} \tag{6.11}$$

The forms for $j_{0,\theta}^{h,u,p}$ are found using the definition of I_θ :

$$I_\theta^{h,u,p} = \int_0^\pi \int_0^a j_\theta^{h,u,p}(r, \theta) (R_{inj} + r \cos \theta) d\phi dr \tag{6.12}$$

There is no plasma current during the vacuum portion of the discharge before plasma breakdown, and therefore eqs. (6.9) and (6.10) can be combined to find the effective number of turns in the flux coil solenoid, N_{flux} . The same two equations can then be solved in the plasma case ($j_\theta \neq 0$) to find a relationship between the total poloidal current I_θ and

Ψ_{inj} . The equations relating I_θ and Ψ_{inj} are long when writing out the full analytical form (these are included in the injector model code repository), but can be simplified to:

$$I_\theta^h = 148.85 \left(0.01974 N_{\text{flux}} I_{\text{flux}} - \frac{\Psi_{inj}}{\mu_0} \right) \quad (\text{hollow})$$

$$I_\theta^u = 152.15 \left(0.02020 N_{\text{flux}} I_{\text{flux}} - \frac{\Psi_{inj}}{\mu_0} \right) \quad (\text{uniform}) \quad (6.13)$$

$$I_\theta^p = 170.21 \left(0.01974 N_{\text{flux}} I_{\text{flux}} - \frac{\Psi_{inj}}{\mu_0} \right) \quad (\text{peaked}) \quad (6.14)$$

With I_θ , B_ϕ can be found using eqs. (6.10) and (6.11).

6.1.2 Injector electric field and Poynting flux

To calculate the Poynting flux passing into the injector volume, the fields need only be known at the boundary. The electric field there can be approximated as follows:

$$\mathbf{E} \approx -\nabla\Phi \approx -\frac{V_{inj}}{\pi R} (-\hat{\phi}) - \frac{V_{div}}{2\pi a} \hat{\theta} \quad (6.15)$$

The two voltages that appear are measured throughout all HIT-SI3 discharges. V_{inj} is the voltage along the major-radial arc of the injector (that would drive current along the $-\hat{\phi}$ direction, as measured on HIT-SI3), and V_{div} is the voltage along the minor-radial circumference of the injector duct (that would drive current along the $\hat{\theta}$ direction).

Inspecting fig. 6.1, three surfaces define the boundary of the injector plasma volume, ∂V : the two mouths of the injector and the toroidal surface inside the copper duct. However, because $\mathbf{E}$ and $\mathbf{B}$ have components only in the $\hat{\theta}$ and $\hat{\phi}$ directions (fig. 6.1), the Poynting flux is purely radial, and energy is flowing into the injector volume:

$$\begin{aligned} \mathbf{S} &= \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} = \frac{1}{\mu_0} \left(\frac{-V_{inj} B_\theta}{\pi R} (-\hat{\phi} \times \hat{\theta}) - \frac{V_{div} B_\phi}{2\pi a} (\hat{\theta} \times \hat{\phi}) \right) \\ &= \frac{1}{\mu_0} \left(\frac{V_{inj} B_\theta}{\pi R} + \frac{V_{div} B_\phi}{2\pi a} \right) (-\hat{r}) \end{aligned} \quad (6.16)$$

Integrating $\mathbf{S}$ over the toroidal surface of ∂V gives the result that the input Poynting flux is the same for any assumed injector current profile. This is because the r dependence of

the resulting B_θ is all that distinguishes the different current profiles, and this difference vanishes at the plasma boundary, $r = a$. Eq. (6.10) also simplifies at $r = a$, where $j_\theta = 0$; with eqs. (6.8) and (6.16):

$$-\oint_{\partial V} \mathbf{S} \cdot d\mathbf{A} = -2 \int_0^\pi \int_{R_0-a}^{R_0+a} \mathbf{S} \cdot \left(dR \sqrt{\frac{a^2}{a^2 - (R - R_0)^2}} R d\phi \right) \hat{r} \quad (6.17)$$

$$= \left(\frac{R_0}{\sqrt{R_0^2 - a^2}} \right) I_{inj} V_{inj} + N_{flux} I_{flux} V_{div} \quad (6.18)$$

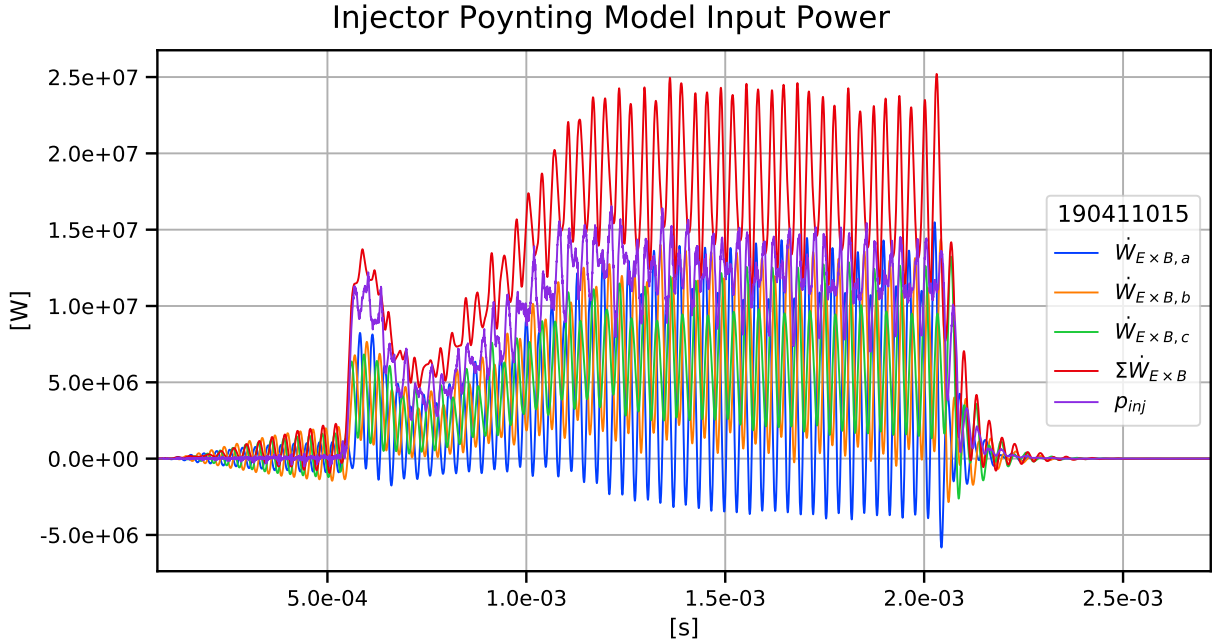


Figure 6.3: The total injected electromagnetic power in each injector (and their sum) compared to the incumbent approximation for total injected power used on the HIT-SI3 experiment “ p_{inj} ”, which accounts only for power injected by the voltage coils.

6.1.3 Rate of energy storage in the magnetic fields

The stored energy in the magnetic fields is accounted for by the final term in eq. (6.3). An analytical form is available for B_θ using eq. (6.8):

$$\frac{1}{2\mu_0} \int_V \frac{\partial}{\partial t} (B^2) dV = \frac{\partial}{\partial t} \left(\frac{1}{\mu_0} \int_0^\pi \int_{R_0-a}^{R_0+a} \int_0^{\sqrt{a^2-(R-R_0)^2}} B^2 R dZ dR d\phi \right) \quad (6.19)$$

$$\frac{1}{2\mu_0} \int_V \frac{\partial}{\partial t} (B_{\theta,i}^2) dV = l_i \mu_0 \frac{\partial}{\partial t} (I_{inj}^2) \quad (6.20)$$

where l_i is a geometric factor related to the choice of profile shape for the injector azimuthal current density j_ϕ :

$$\begin{aligned} l_h &= \frac{5a^8 R_0^2 + a^6 R_0^4 + 2a^4 R_0^6 + 8a^2 R_0^8 - 16R_0^9 \left(R_0 - \sqrt{R_0^2 - a^2} \right)}{140a^8 \sqrt{R_0^2 - a^2}} \simeq 7.71 \times 10^{-3} \text{ m} \\ l_u &= \frac{2R_0^5 - (a^2 R_0^2 + 2R_0^4) \sqrt{R_0^2 - a^2}}{12a^4} \simeq 1.54 \times 10^{-2} \text{ m} \\ l_p &= \frac{71a^8 R_0^2 + 115a^6 R_0^4 \left(31R_0 - 28\sqrt{R_0^2 - a^2} \right) - 48R_0^9 \left(R_0 - \sqrt{R_0^2 - a^2} \right) - 2a^4 R_0^5 \left(193R_0 - 140\sqrt{R_0^2 - a^2} \right)}{420a^8 \sqrt{R_0^2 - a^2}} \\ &\simeq 2.81 \times 10^{-2} \text{ m} \end{aligned} \quad (6.21)$$

In practice, the method of determining B_ϕ described in sec. 6.1.1 can be computationally cumbersome, so the following approach is used when integrating over the injector volume. The flux coils that produce B_ϕ are wound into a solenoid around the injector (sec. 2.3). The curvature of this solenoid means the magnetic field strength has an inverse dependence on the major radius, $B_\phi = \beta/R$, where β is a constant determined as follows:

$$\begin{aligned} \Psi_{inj} &= 2 \int_{R_0-a}^{R_0+a} \int_0^{\sqrt{a^2-(R-R_0)^2}} \frac{\beta}{R} dZ dR = 2\beta\pi \left(R_0 - \sqrt{R_0^2 - a^2} \right) \\ \Rightarrow B_\phi &= \frac{\Psi_{inj}}{2\pi R \left(R_0 - \sqrt{R_0^2 - a^2} \right)} \end{aligned} \quad (6.22)$$

This simpler method of calculating $B_\phi(R)$ has been found to be an adequate replacement

as the integrand in eq. (6.19) (see fig. 6.4):

$$\frac{1}{2\mu_0} \int_V \frac{\partial}{\partial t} (B_\phi^2) dV = \frac{1}{4\mu_0 (R_0 - \sqrt{R_0^2 - a^2})} \frac{\partial}{\partial t} (\Psi_{inj}^2) \quad (6.23)$$

Using eq. (6.22) in eq. (6.23) instead of eq. (6.10) saves time by allowing the flux loop

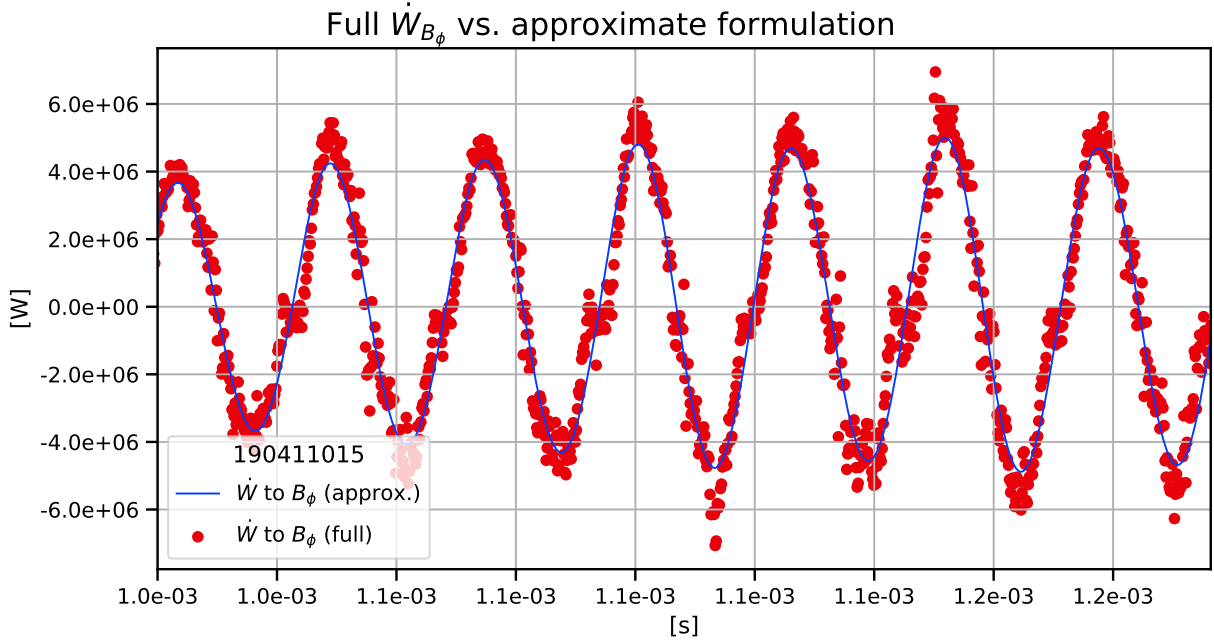


Figure 6.4: The full calculation of the total volume-integrated rate of magnetic field energy storage using eq. (6.10), compared with simpler approximation of B_ϕ from eq. (6.22).

diagnostic to perform the area integration rather than doing a 2-D numerical integral on the analytical form of B_ϕ at each time step.

6.1.4 Injector current profile selection

Choosing the current profile shape that best represents the injector behavior presents a difficulty, as the only available measurements of the injector plasma are external. One way to inform the choice is to consider the resistive diffusion time of the injector plasma,

$\tau_R = \mu_0 a^2 / \eta$, where a is the injector minor radius. This is the nominal time it takes for fields to diffuse throughout the injector volume containing a plasma with resistivity η . It is assumed that the magnetic fields (which induce and carry current) must diffuse into the volume before the driving current in the coils crosses through zero and all currents are reversed. In reality the plasma may also carry fields throughout the injector volume, but without any measurements of flow dynamics in the injector volumes, the penetration is assumed to be dominated by the resistive diffusion of fields. This requirement gives $\tau_R < \frac{\tau_{SIHL}}{2}$. Equating the two gives (through eq. (6.4)) a critical electron temperature, $T_{e,crit.}$ (fig. 6.5), marking the point at which the fields complete their diffusion in one injector half-period.

For the injector frequency (15.6 kHz) at which all data in this dissertation were taken, $T_{e,crit.} = 3.4$ eV. Measured spheromak temperatures are generally higher, so a hollow profile has been assumed. However, it should be noted that the presence of reconnection, turbulence, and other non-linear mixing events are not accounted for in this analysis, and may contribute to very different current profiles. This model is simplified to allow it to incorporate the few measurements of the injectors that are available. Additionally, outputs that are used in the rest of the work (namely the Poynting flux and stored field energy) are found to be weakly sensitive to the choice of injector current profile shape.

The above analysis allows complete calculations of injected power and injector magnetic field energy storage. It uses five measurements: the total injector current I_{inj} , the applied “flux coil” current I_{flux} , the applied injector voltage V_{inj} , the applied injector flux Ψ_{inj} , and the poloidal voltage that is induced around the injector minor-radial circumference, called V_{div} . These five quantities are routinely measured as functions of time during operation of HIT-SI3. Thus from this model are obtained important source quantities for a more complete power balance of the spheromak plasma, which is the next topic.

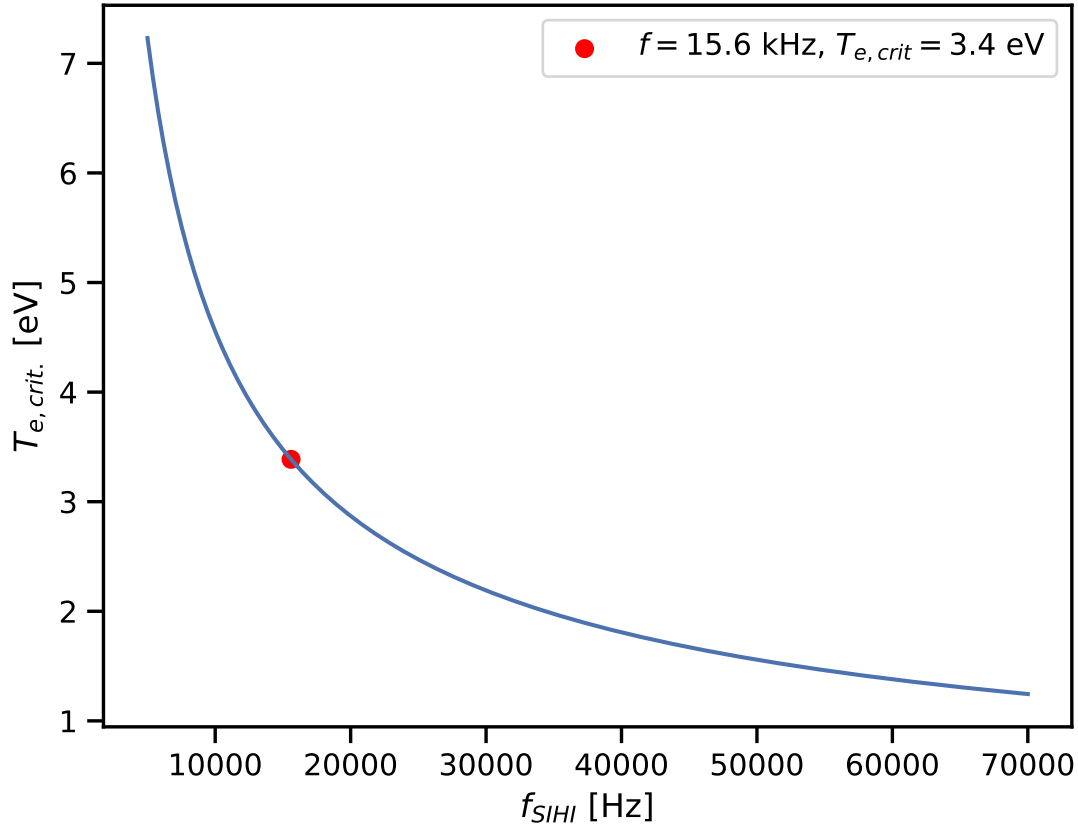


Figure 6.5: $T_{e,crit.}$ at which the fields complete their diffusion in one half of the injector period. At temperatures higher than $T_{e,crit.}$, the profile is expected to be more hollow. Below $T_{e,crit.}$, the fields permeate more quickly through the resistive plasma, and a more peaked profile is expected.

6.2 Spheromak Energy Balance Model

Having accounted for the energy that is supplied to and stored in the injector volume in sec. 6.1, analysis in this section will attempt to explain energy flows in the spheromak itself using a 0-D, time-evolved energy balance model following the work of Meyerhofer, Barnes, and others [68, 69, 70]. HIT-SI3 plasmas generally produce higher ion than electron temperatures, and in a collisional plasma this may appreciably affect the electron

temperature, so the model accounts for thermal energy of both particle species. In each case, with the simplifying assumption of zero fluid velocity, the energy balance can be written conceptually as $\frac{\partial}{\partial t}$ (internal thermal energy) = net heating power. Specifically, the following two-temperature energy conservation model is used:

$$\frac{k_B}{\gamma - 1} \int_V \left(T_e \frac{\partial n_e}{\partial t} + n_e \frac{\partial T_e}{\partial t} \right) dV = \int_V \eta j_{sph}^2 dV - P_{rad} - Q_{ei} - P_{r,e} - \oint_{\partial V} \mathbf{q}_e \cdot d\mathbf{A} \quad (6.24)$$

$$\frac{k_B}{\gamma - 1} \int_V \left(T_i \frac{\partial n_i}{\partial t} + n_i \frac{\partial T_i}{\partial t} \right) dV = P_{rec} + Q_{ei} - P_{r,i} - \oint_{\partial V} \mathbf{q}_i \cdot d\mathbf{A} \quad (6.25)$$

Here n_e is the electron number density and $n_i = n_e/Z_{eff}$ is the ion number density. k_B is the Boltzmann constant and $\gamma = 5/3$ is the heat capacity ratio. The power source and sink terms on the right hand sides of both equations are discussed in secs. 6.2.1 and 6.2.2. For the electrons these terms account for (from left to right): gains from Ohmic heating in the spheromak, radiation losses, losses from heat transfer to the ion species, losses from particle replacement, and losses from heat diffusion into the walls. The ion terms accounted for are (again from left to right): ion heating due to reconnection and other mechanisms, ion heating from electron heat transfer, losses from ion particle replacement, and heat loss to the walls.

6.2.1 Sources and losses of electron thermal energy

The main source of heating for the electrons is Ohmic heating, $\int_V \eta j_{sph}^2 dV$. Here the spheromak current density is taken to be $j_{sph} = 2I_{tor}/\pi a_{sph}^2$, where I_{tor} is the spheromak toroidal current and a_{sph} the minor radius of the spheromak, assumed to be 23 cm in keeping with ref. [71]. These definitions implicitly assume the spheromak being modeled is well-approximated by the Taylor state, force-free spheromak in which $I_{tor} \sim I_{pol}$. The resistivity, η , is assumed to follow the Spitzer parallel resistivity as in eq. (6.4).

The approximation of the spheromak as a Taylor state topology also allows for estimation of the total energy content of the magnetic field as a function of time. Using Taylor

state reconstructions, O'Neill, *et al.* integrated $B^2/2\mu_0$ over the spheromak volume for HIT-SI, the predecessor experiment to HIT-SI3 that used the same flux conserver volume [72]. The spheromak magnetic field energy is then given by:

$$W_{sph} = \frac{1.23\mu_0 I_{tor}^2}{\lambda_{sph}} \quad (6.26)$$

where $\lambda_{sph} = 10.4\text{m}^{-1}$ is the eigenvalue that solves eq. (2.6) for the HIT-SI and HIT-SI3 flux conserver. The time derivative of W_{sph} can then be added to the outputs of eqs. (6.20) and (6.9) to arrive at the full rate of change of the magnetic energy content in the experiment.

The radiated power P_{rad} is taken to be the output of an AXUV photodiode calibrated to give an estimate of the total radiated power from a plasma in the HIT-SI3 geometry, assuming a uniform distribution of radiated power per unit volume. This diagnostic is available for most shots, typically hitting peak measured powers of around 0.3–0.5 MW.

The transfer of heat from electrons to ions through collisions in the spheromak volume V is (following Braginskii [62]):

$$Q_{ei} = \frac{3V m_e n_e k_B (T_e - T_i)}{m_i \tau_{ei}} \quad (6.27)$$

After Levinton *et al.*, the rate of energy loss as a result of particle replacement is $P_{r,e} = \frac{5V_{sph} n_e k_B T_e}{2\tau_p}$ for the electrons.

The final loss term, $\oint_{\partial V} \mathbf{q}_e \cdot d\mathbf{A}$, is the diffusion of heat to the walls. It follows the Fourier law of heat conduction, $\mathbf{q}_e = -n_e \chi_e \nabla T_e$. In the simplified 0-D model, the electron heat loss to the walls is approximated as:

$$\oint_{\partial V} \mathbf{q}_e \cdot d\mathbf{A} = \oint_{\partial V} -n_e \chi_e \nabla T_e \cdot \hat{r} dA \approx n_e \chi_e A_{sph} \frac{T_e}{a_{sph}} \quad (6.28)$$

where $a_{sph} = 23\text{ cm}$ is the spheromak minor radius. The major radius of the spheromak is assumed to be 34 cm, for a volume of $V_{sph} = 2\pi^2 R_{sph} a_{sph}^2$ and surface area of $A_{sph} = 4\pi^2 R_{sph} a_{sph}$. The value of the thermal diffusion coefficient, χ_e , can be selected

from various theoretical models that describe how the electrons will conduct heat out of the system (sec. 6.2.4).

6.2.2 Sources and losses of ion thermal energy

The only direct source of thermal energy for the ions in this model is magnetic reconnection. This process, by which magnetic field lines break and merge in a plasma with finite resistivity, is known also to convert magnetic energy to thermal energy [73, 74]. Hsu *et al.* suggest 65% of the reconnected field energy is translated into ion thermal energy [75]. In the interest of keeping model assumptions simple, the reconnection power is assumed to be 100% of any remaining total injected power once the spheromak ohmic power and the rate of energy storage in the magnetic fields has been subtracted off:

$$P_{rec} = \begin{cases} P_{inj,tot} - \left(\int_V \eta j_{sph}^2 dV + \dot{W}_{mag} \right) & \text{if } P_{inj,tot} \geq \left(\int_V \eta j_{sph}^2 dV + \dot{W}_{mag} \right) \\ 0 & \text{if } P_{inj,tot} < \left(\int_V \eta j_{sph}^2 dV + \dot{W}_{mag} \right) \end{cases}$$

With this definition, the sum of the ohmic and reconnection heating may be slightly greater than the total input power (fig. 6.7), but this choice was made because it is unclear why one would expect that a *lack* of free magnetic energy would *cool* ions by demanding their thermal energy to perform reconnection. Rather than disguise this dubious loss mechanism as simply the result of “negative” reconnection power, it is avoided altogether by the piecewise definition of P_{rec} . Thus at the cost of possible slight overestimation of the ion temperature, the ion cooling mechanism that is left out is explicit.

More recent work done using full 3-D two-temperature simulations point to compressive heating as a significant driver of plasma temperature dynamics, particularly among ions in discharges driven at high-frequency [26]. The compressive term is found to take on both negative and positive values, meaning some ion cooling may be expected in regions of expansion. At low frequencies (near 15 kHz, as in the experimental discharges analyzed here), Ohmic heating is still observed to be dominant in the frequency survey performed in ref. [26].

The other source of thermal energy for the ions, Q_{ei} , is more frequently a negative quantity when modeling HIT-SI3 plasmas, and ends up acting as a loss term (the ions are often higher temperature than the electrons).

The rate of energy loss as a result of ion replacement is taken to be $P_{r,i} = \frac{5V_{sph}n_i k_B T_i}{2Z_{eff}\tau_p}$. The factor of Z_{eff} in the denominator is due to the assumed ambipolarity of diffusion to maintain a quasineutral plasma.

The loss of ion thermal energy to the wall is taken to follow the typical Braginskii perpendicular thermal conduction model [62]:

$$\kappa_{i,\perp} = n_i \chi_i = \frac{2n_i T_i}{m_i \nu_{ci}^2 \tau_i} \quad (6.29)$$

where $\nu_{ci} = Z_{eff}eB/m_i$ is the ion cyclotron frequency and τ_i is the ion-ion collision time [61]:

$$\tau_i = \frac{12(\pi k_B T_i)^{3/2} \epsilon_0^2 m_i^{1/2}}{n_i Z_{eff}^4 e^4 \ln \Lambda} \quad (6.30)$$

6.2.3 Model results, attributes, and criticisms

A strength of this model when compared with full 3D simulations is that it can be run on any HIT-SI3 shot in a matter of seconds on any modern computer. It provides an estimate of the evolution of the ion and electron temperatures, and is based on experimental measurements. Figs. 6.6 and 6.7 show the evolution of temperatures and energy flow in shot 190411015. These results are from a formulation of χ_e based on a model that accounts for radial magnetic field line fluctuations (see sec. 6.2.4).

The two least-informed parts of the model are the ways it treats particle number density, n_e , and confinement time, τ_p . Experimental FIR traces for electron density are not available for many shots, so an assumed static electron density of $5 \times 10^{19} \text{m}^{-3}$ is assumed. This is an estimation based on typical FIR measurements from shots that are available. With static density, the differential equation simplifies as the density derivative vanishes. Further, the estimate of Z_{eff} is based only on the main fuel species ($Z_{eff} = 1$ for

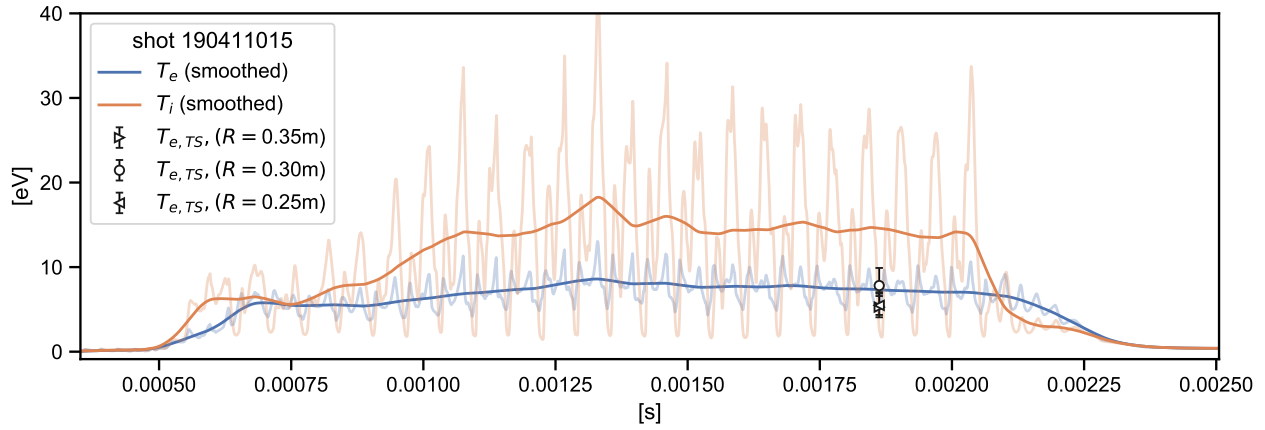


Figure 6.6: Model results of temperature in shot 190411015.

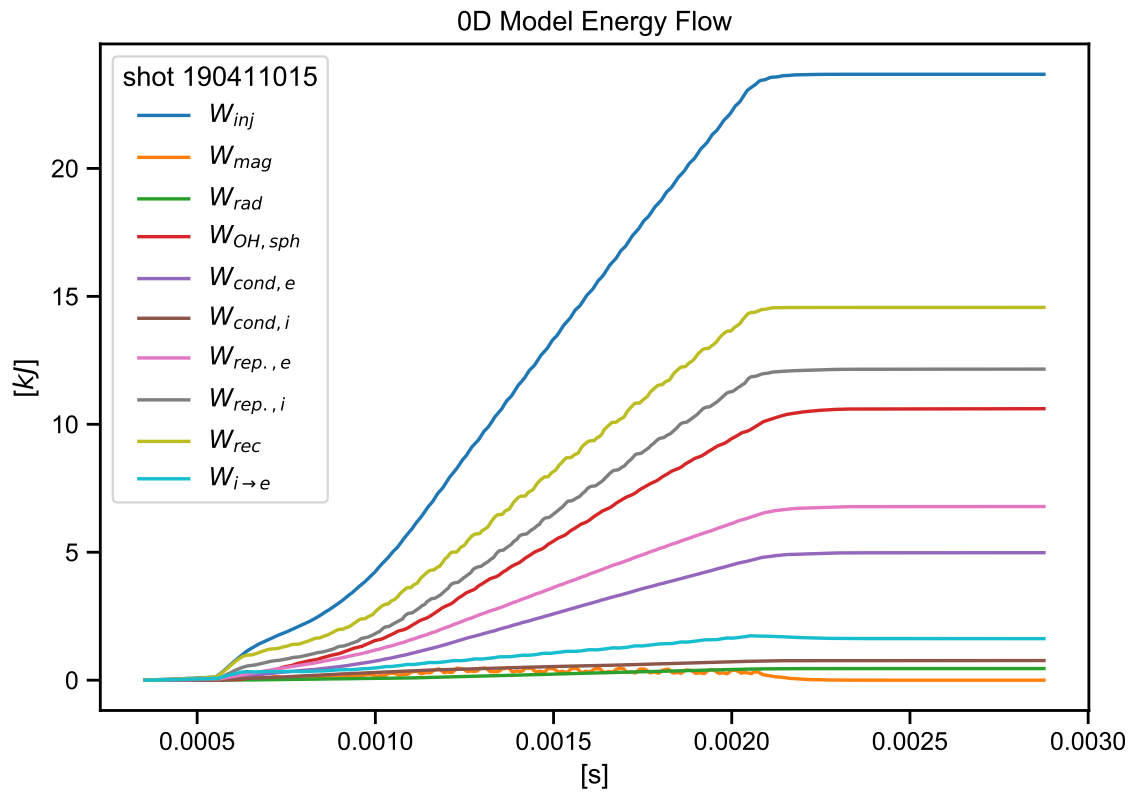


Figure 6.7: Model results of energy flow in shot 190411015.

Deuterium, $Z_{eff} = 2$ for helium), without direct insight from measurements on HIT-SI3. However, the electron temperature reported by the model is not observed to be very sensitive to the value chosen for the static density, as long as it is in the order of magnitude of densities typically observed on HIT-SI3. The particle confinement time is estimated to be equal to the energy confinement time estimated by the model:

$$\tau_E \approx \frac{a_{sph}^2}{\chi_e + \chi_i} \quad (6.31)$$

It has been noted that if τ_E is shorter than the relaxation time then sustainment may not preclude confinement [72, 76]. Various time scales as they are predicted to evolve during a HIT-SI3 discharge are shown in fig. 6.8. More accurate measurements in either τ_p or n_e would add confidence to the model.

Figs. 6.9–6.11 show other model outputs: various time scales that determine collisionality, volume averaged β , and the Lundquist number: $S_{Lund} = \mu_0 a_{sph} v_{alfven} / \eta$. The modeled values of β are in this case higher than the theoretical limit for the HIT-SI3 geometry from ideal MHD stability calculations (ref. [77]). The Lundquist number is a special case of the magnetic Reynolds dimensionless number; generally, larger, hotter plasma experiments have higher values of S_{Lund} .

6.2.4 Electron temperature comparisons for various thermal diffusivity models

Four possible cases are considered for the value of the thermal diffusivity coefficient, χ_e . These four models are expected to span the range of possible thermal confinement scenarios for HIT-SI3. Pursuant to the low temperatures and lack of observed pressure profiles in Ch. 5, other confinement regimes (such as classical diffusion or gyro-Bohm models) are left out of this discussion.

The first candidate model for the thermal diffusion is the Braginskii electron perpendicular thermal conduction (modified for $Z_{eff} = 2$) [62]:

$$\chi_{Brag.,\perp} = \frac{4.0 n_e T_e}{m_e \nu_{ce}^2 \tau_e} \quad (6.32)$$

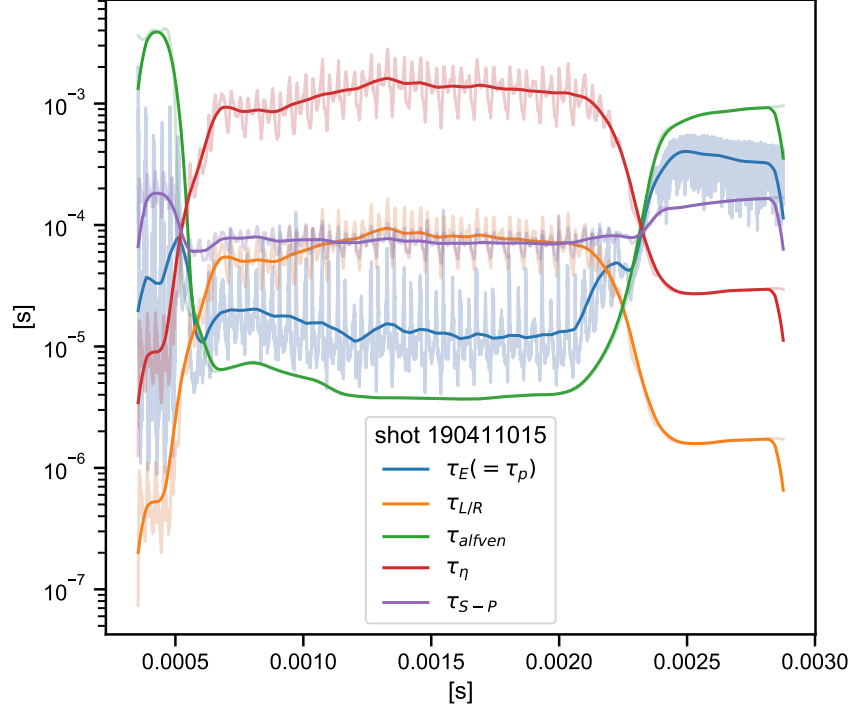


Figure 6.8: Model results of various expected physical time scales in shot 190411015, including the energy confinement time: eq. (6.31), the resistive diffusion time: $\tau_\eta = \mu_0 a_{sph}^2 / \eta$, the resistive decay time: $\tau_{L/R} = \mu_0 / (\eta \chi_{sph}^2)$, the Alfvén time: $\tau_{alfven} = a_{sph} / v_{alfven}$, and the Sweet-Parker reconnection time, $\tau_{S-P} = \sqrt{\tau_\eta \tau_{alfven}}$. $v_{alfven} = \sqrt{\frac{B_{eq}^2}{\mu_0 m_i n_i}}$.

where $\nu_{ce} = eB/m_e$ is the electron cyclotron frequency and τ_e is the electron-ion collision time [61]:

$$\tau_e = \frac{12(\pi k_B T_e)^{3/2} \epsilon_0^2 m_e^{1/2}}{\sqrt{2} n_i Z_{eff}^2 e^4 \ln \Lambda} \quad (6.33)$$

All scenarios for χ_e discussed in this model are quantifying the perpendicular (radial) heat transfer, though the notation $\chi_{Brag,\perp}$ is used to distinguish it from the commonly-used Braginskii parallel thermal conduction.

The second possible case is Bohm diffusion, an empirical scaling that suggests $\chi \propto$

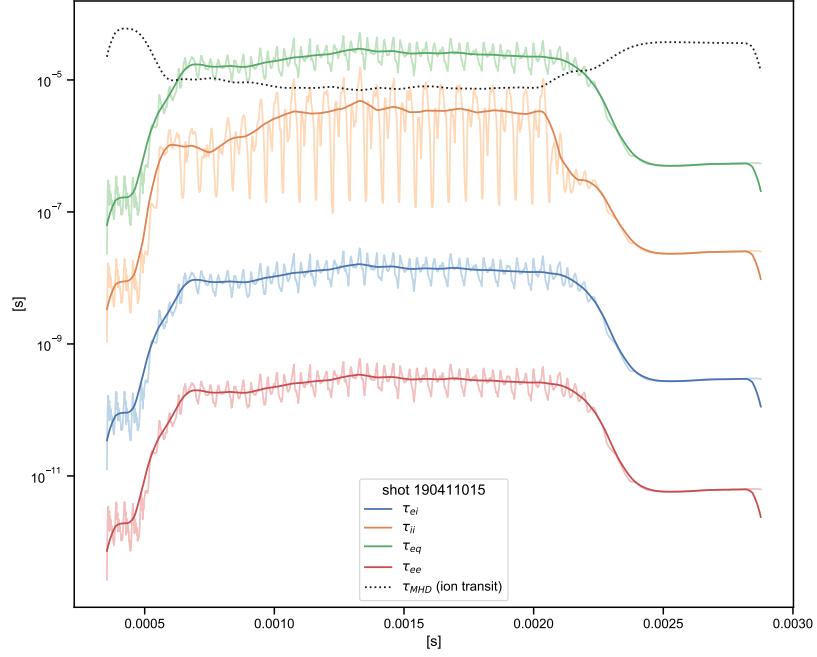


Figure 6.9: Model results of collisionality time scales in shot 190411015.

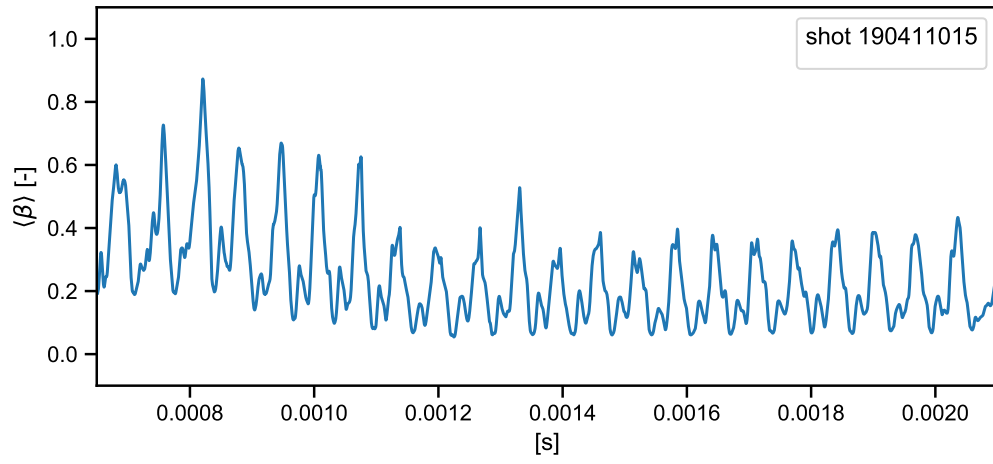


Figure 6.10: Model results of β in shot 190411015.

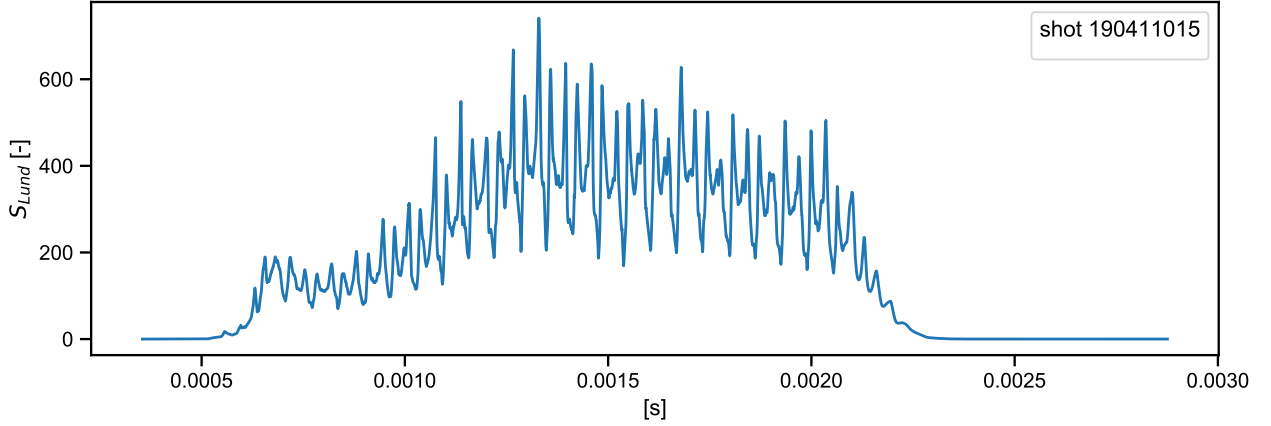


Figure 6.11: Model results of Lundquist number in shot 190411015.

T/B [78]. Specifically,

$$\chi_{\text{Bohm}} = \frac{k_B T_e}{16eB} \quad (6.34)$$

Finally, two variants of a model for χ_e in the case of field line stochasticity are tested. The following Rechester-Rosenbluth formulation is used [79]:

$$\chi_{RR} = v_{th} \pi L_{eff} \left(\frac{\delta B}{B} \right)^2 \quad (6.35)$$

where

$$L_{eff} = \left((2\pi R_{sph})^{-1} + (v_{th} \tau_e)^{-1} \right)^{-1} \quad (6.36)$$

is the effective electron autocorrelation length — here the inverse sum of the electron mean free path and the mean toroidal transit distance (after refs. [80, 81]). The normalized magnetic fluctuation in eq. (6.35) is determined from an internal magnetic probe inserted into the plasma.

The internal magnetic probe comprises three stems of boron Nitride-covered 3-axis magnetic field probes. Inserting this triplet of stems into the spheromak region allows for direct measurements of the radial, toroidal, and poloidal components of the magnetic field. The probes are spaced roughly every 2.5 cm and to cover major radial locations

from 33 cm to 53 cm. In accordance with assumption of an approximate Taylor state, the equilibrium field is defined to be the quadrature sum of the poloidal and toroidal fields (after being time-smoothed at the injector frequency). The equilibrium field traces for each probe along the middle stem are then averaged to find a mean equilibrium field.

Two possible fluctuation definitions account for the two versions of the Rechester-Rosenbluth model that were tested. The first is the radial fluctuation, δB_R , based on the original definition of the model. This is taken to be the difference in the radially-averaged measurement of the radial magnetic field before and after time-smoothing at the injector frequency. Intuitively, the radial fluctuation on a toroidal configuration counteracts the confinement by shorting the electron transport in the radial direction. Fig. 6.12 shows the equilibrium field and fluctuation measurements used for modeling a HIT-SI3 discharge.

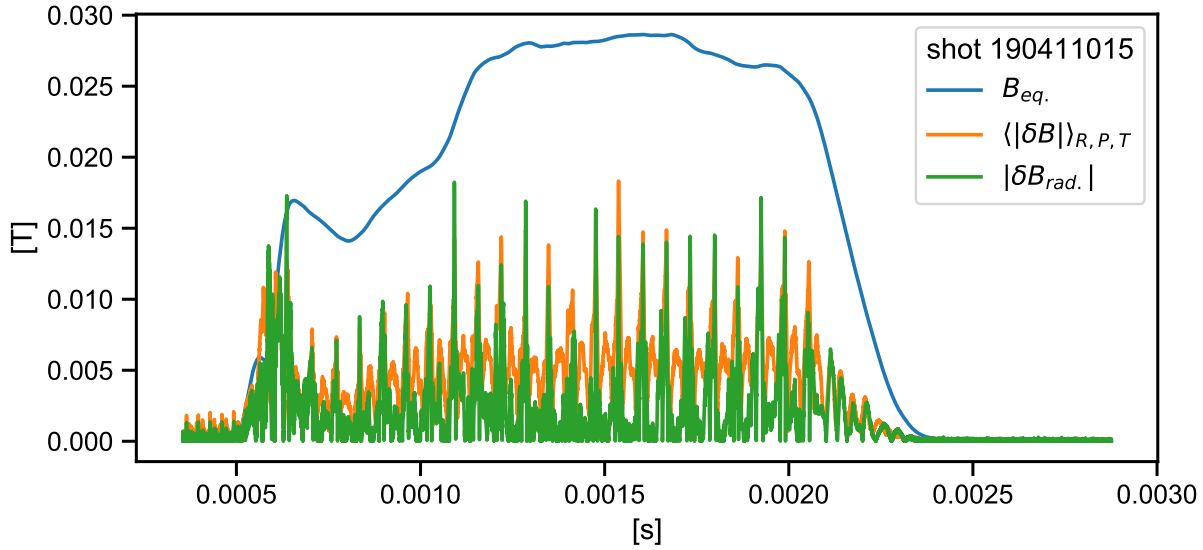


Figure 6.12: Two magnetic fluctuation definitions and the mean field. From internal magnetic probe measurements, shot 190411015.

The second magnetic fluctuation definition considered attempts to capture the possibility that the HIT-SI3 device may at times be departing from the regime of quasilinear turbulence assumed by Rechester and Rosenbluth. In the case of a highly asymmetric

equilibrium with turbulence and stochastic fields, the distinction between the various components of the magnetic field may differ in the reference frame of the plasma from that of the probe. In other words, the radial fluctuation may not be the determining quantity in thermal diffusion coefficient. In this case a simple average of the measured fluctuation magnitudes in all three directions, $\delta B_{\langle R,P,T \rangle} = (\delta B_R + \delta B_P + \delta B_T) / 3$, is used. In both cases of the Rechester-Rosenbluth models, the Braginskii perpendicular thermal conductivity is also retained, as a minimum of thermal diffusion should be expected even in a magnetized plasma without any fluctuations.

Figs. 6.13 and 6.14 compare model-predicted temperatures for the same moment as the Thomson scattering measurement for all measured shots, as well as the spatially-averaged measured temperature (excluding measurements with error above the $\pm 1\sigma$ threshold) for the same shots. The plots depict kernel density estimates of the histograms of the temperature measurements and predictions for various values of χ . Dashed vertical lines correspond to the median temperatures over the ensemble of shots measured or modeled. These plots show that, in an aggregate sense, the model that most closely aligns with measurements is the Rechester-Rosenbluth formulation that uses the larger fluctuation $\delta B_{\langle R,P,T \rangle}$. In fact, this model still appears to over-estimate the temperature. This may be caused by the simplicity of the model, which may overestimate the ion temperature thus raising the electron temperature through the Q_{ei} term, for instance. Possible physical causes include that even further deviation from the quasilinear regime may be occurring, perhaps approaching the quasisotropic stochastic magnetic field regimes described in refs. [82, 83]. Further investigation of the magnetic field structure of HIT-SI3 is suggested as future work.

6.2.5 IDCD current sustainment via magnetic fluctuations

As discussed in Sec. 2.3.1, IDCD is a model that describes how the injectors might drive a DC toroidal current in the HIT-SI3 spheromak. The evolution of toroidal current as

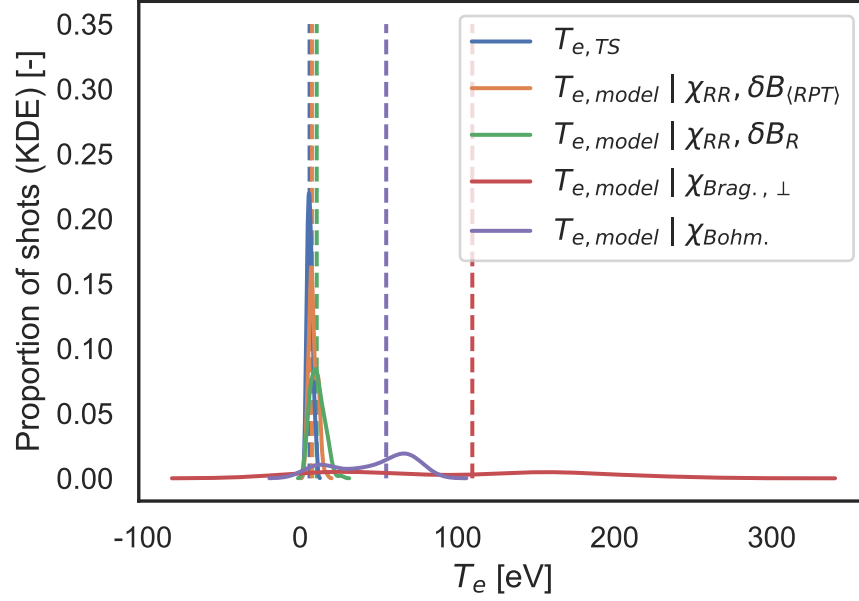


Figure 6.13: Model results for various values of χ_e , showing the full range of model-reported T_e .

predicted by IDCD is given by [23]:

$$\dot{I} = \frac{-I_{sph}}{\tau_{L/R}} + \frac{4\pi (\delta B_{\perp})^2}{\mu_0^2 n e r} \quad (6.37)$$

The outputs of the model can be used in this equation, and the result can be compared with the experimentally measured toroidal current trace (fig. 6.15a).

In the original method of estimating the IDCD-predicted toroidal current, the resistive decay time $\tau_{L/R}$ of the plasma is determined via a helicity balance model first laid out by Wrobel *et al.* [76]. The magnetic field fluctuations are taken from measurements at the edge of the plasma, and singular value decomposition is used to compare the predicted IDCD-driven current associated with different modes (i.e. δB associated with the injectors vs. δB associated with the spheromak plasma).

The measurements presented here trade the edge magnetic field measurements for internal measurements. In effect this trades toroidal (and modal) resolution for measure-

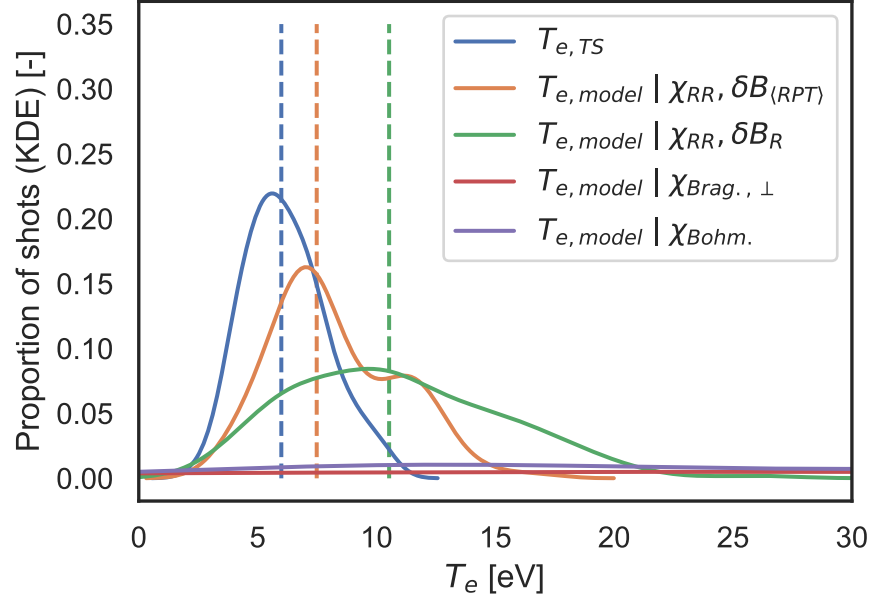
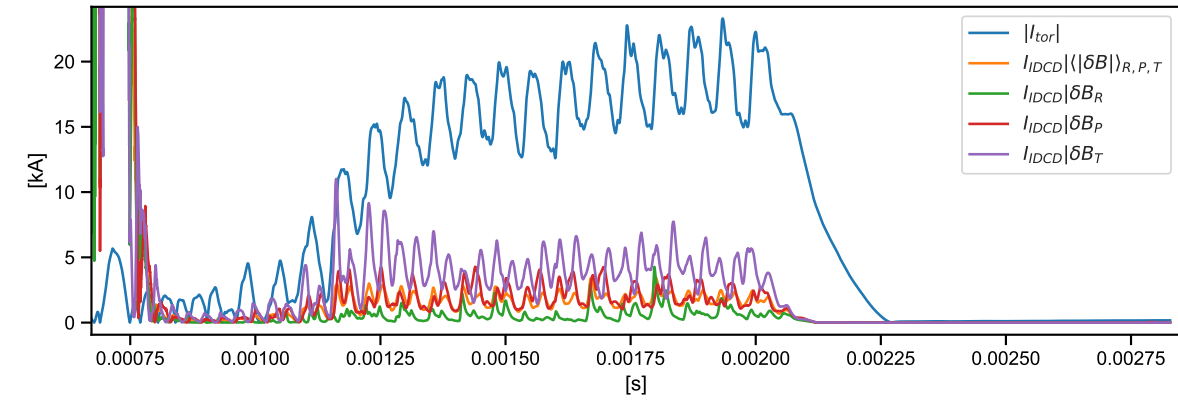


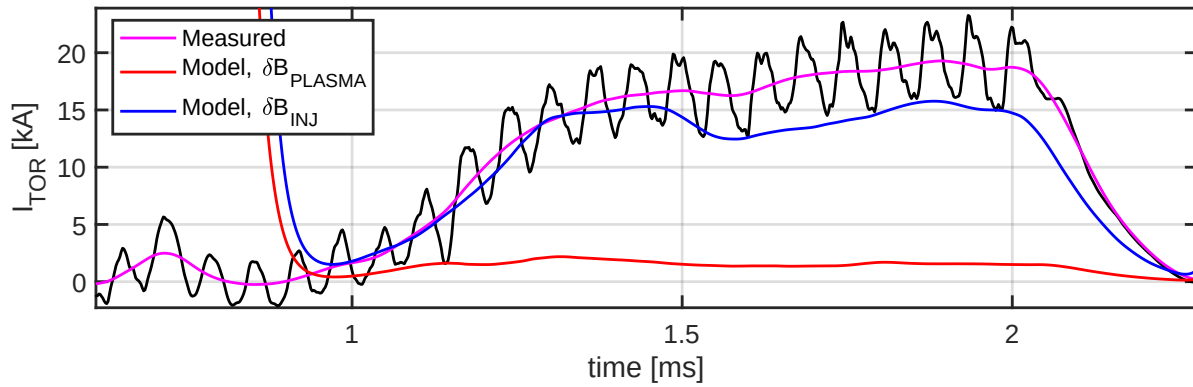
Figure 6.14: Model results for various values of χ_e , showing a closer view of the domain as measured by the TS diagnostic.

ments in the spheromak plasma itself. Further, the value of $\tau_{L/R}$ used is the 0-D energy evolution model output (as in fig. 6.8). The results from the two methods are shown in fig. 6.15.

The only terms in eq. (6.37) that differ between the two methods are $\tau_{L/R}$ and δB . The helicity balance method of determining $\tau_{L/R}$ finds it to be slightly lower, which should result in lower values of I_{tor} . The better agreement between the experimental I_{tor} and that predicted by the helicity balance model (in the case of injector-associated fluctuation) is therefore attributable to the higher fluctuation used in eq. (6.37). While the two methods of measuring the fluctuation differ slightly, this result may suggest that the stronger edge fluctuations are more capable of supplying the spheromak current if IDCD is the operant mechanism, though these strong fluctuations may present their own disadvantages to confinement as discussed in sec. 6.2.4.

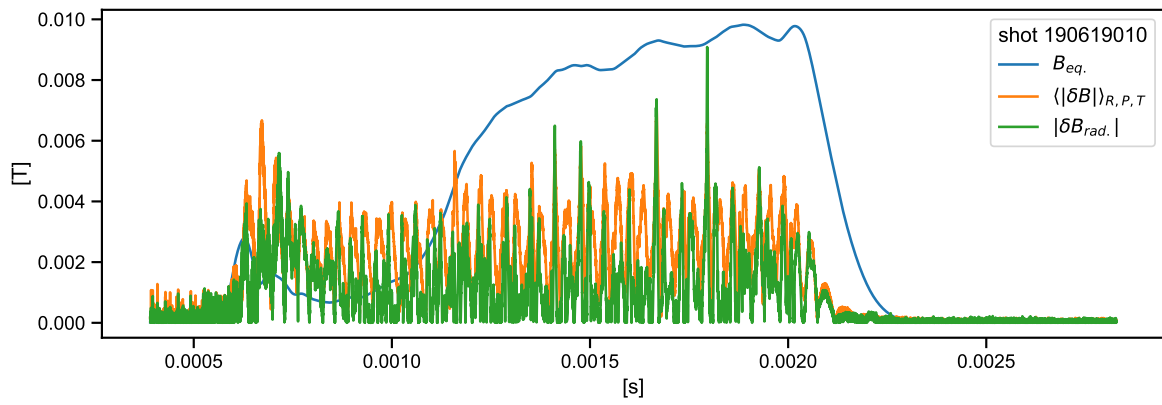


(a)

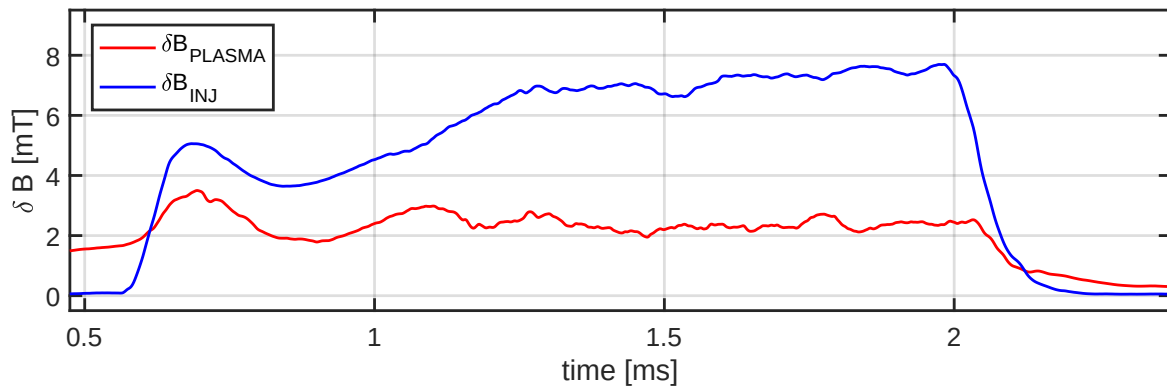


(b)

Figure 6.15: IDCD predictions of toroidal spheromak current based on 0-D energy evolution model output (a) vs. Hossack model (b).



(a)



(b)

Figure 6.16: Magnetic fluctuation measurements at the plasma edge (a) vs. interior (b).

Chapter 7

CONCLUSION AND SUGGESTIONS FOR FUTURE WORK

The questions that this dissertation sought to answer, as laid out in chapter 1, emerged naturally from the historical research context of HIT-SI3 as an inductively driven spheromak. These research questions represent a certain subset of the boundaries of understanding of the HIT-SI3 spheromak. While the spheromak concept demonstrates many advantages, a detailed analysis of energy confinement had not been performed for the HIT-SI or the HIT-SI3 device. Using the diagnostic capabilities provided by Thomson scattering, some insight into the dynamics that govern thermal confinement and the flow of energy on the HIT-SI3 spheromak has been obtained.

First, evidence of pressure confinement was evaluated. No significant indication of a gradient in pressure was found. The measurements presented were not strictly limited to the high-current or high-gain discharges, but no consistent evidence of pressure gradients was observed over the data set. Since the purported Shafranov shift effect is only seen in high injector frequency operations of HIT-SI, additional investigation of pressure behavior in high-frequency HIT-SI3 plasmas is recommended.

HIT-SI3 plasmas are often conceptually framed as having distinct injector and spheromak regions. This line of analysis is based on Taylor state calculations of the magnetic fields that describe separate minimum energy states for the injectors and the spheromak itself. Because the plasma is not a perfect conductor in an isolated system, it will never perfectly align with the Taylor states as calculated, but the concept of having separate injector and spheromak regions with distinct dynamics is to some extent supported by previous experiments [71, 23]. It is not possible to infer distinct regions of the plasma from the results presented here. This could be due to the limited resolution of the di-

agnostic, and measurements at additional spatial locations are recommended for future work.

The HIT-SI3 injectors provide all of the plasma, input energy, and current drive, as well as the perturbations that are thought to enable current drive into the spheromak region. They are the fundamental aspect of HIT-SI3 that differentiates it from previous predecessor experiments and other spheromaks. Though they are the source and driver of the experiment's dynamics, they are relatively under-diagnosed, as their construction limits the additional hardware that can be added around them. They are also strong sources of electromagnetic noise, which can confound measurement equipment nearby. Direct measurements of injector activity are recommended for future work, as they would be of particular interest to validate or set bounds on simulations, which use simplified models of the injectors as boundary conditions and therefore may not capture the full effect of the injectors on the spheromak plasma.

Second, the details of energy flow in the HIT-SI3 plasma were probed. It has been established that the plasmas formed in HIT-SI and HIT-SI3 follow the paradigm of spheromak formation as governed by Taylor relaxation to the MECH state [18, 21]. What has been less clear is how energy flows through the system after it is introduced from sources like injector coils and leaves the system through sinks like particle loss, thermal conduction to the vessel walls, and radiation. During the discharge, the energy in the system is transferred to different locations in the plasma and transformed between different types of energy including magnetic, thermal, and kinetic.

The mechanisms that cause these transformations, such as magnetic reconnection and Ohmic heating, affect the plasma equilibrium in unique ways which are not fully resolved in this study. Thomson scattering measurements and modeling have helped to some degree to distinguish the relative prevalence of these energy-transfer mechanisms and to give a better idea of how energy distributes itself in the plasma, but more work is necessary to understand exactly how the injectors and spheromak are separate, how they are linked, and how this interaction can be modeled.

Both the lack of an obvious and consistent pressure gradient and the comparison of different models for thermal conduction to the experimental results suggest a spheromak that is strongly affected by magnetic fluctuations. The estimates in Ch. 6 of the most appropriate thermal diffusion coefficient suggest a stochastic field may be present in the experiment, which would explain the discrepancy between the experiment and simulation results. The value of the thermal diffusion coefficient cannot be taken as a definitive indicator of the nature of the magnetic field structures in HIT-SI3, but instead suggest additional study of magnetic turbulence and fluctuations as plausible causes of the observations.

The discussion of magnetic fluctuations leads to the IDCD theory of current drive. Its predictions of the spheromak toroidal current evolution are found to underestimate the measured quantity when fluctuations internal to the spheromak are used. The larger fluctuations measured by the magnetic field probes at the walls of the confinement volume result in more accurate predictions. Without more measurements and evidence of closed magnetic surfaces, it is premature to assign any specific narrative to this observation. If HIT-SI3 spheromaks are in the future driven to regimes of closed flux, the effect of the injector drive on confinement will be of primary interest. For now, the suggestion of stochastic fields at low-frequency, moderate-performance operation remains an obstacle to this research.

The variability of the HIT-SI3 plasma also presents a difficulty to utilizing TS to understanding how temperature and density vary in time. The laser repetition rate is minutes long, and in the context of ms-scale plasma discharges, the measurement is limited to a single laser pulse. Thus, in order to resolve the time behavior of TS-measured quantities, extreme repeatability would be necessary in the plasma dynamics of the discharge. This level of repeatability has not been demonstrated in HIT-SI3 plasmas, so any time-resolved TS measurements have a limited interpretation.

In contrast to Thomson scattering, time resolution is an intrinsic property of the magnetic field measurements provided by the internal probe. If reliable time-resolved tem-

perature and density measurements are obtained, comparing these to magnetic field to gain an idea of how β varies in time will be possible. One possible way to extend the utility of TS measurements is to observe how consistent TS measurements are to those of a Langmuir probe positioned very close to the TS location. If the two measurements are always well-correlated, the Langmuir probe may be considered to be calibrated by the TS diagnostic, and its results are made more meaningful. Taking TS measurements on repeated shots with and without the Langmuir probe inserted would also help to quantify its perturbative effect on the plasma.

The time-resolved measurements of the magnetic field will also be useful in informing the timing of future TS measurements, as there may be repeatable events that have a certain magnetic signature, such as magnetic reconnection or instabilities. Specifically, pressure-driven instabilities observed as β limits are approached often manifest as increased small-scale magnetic turbulence [31]. Current-driven instabilities may also exhibit magnetic signatures, and the behavior of the plasma temperature and density will be of interest during these events.

If the Thomson scattering diagnostic can be absolutely calibrated, measurements should also provide a more nuanced understanding of β in HIT-SI3. Both of the values that determine β would then be directly measured, so the proportion of density and temperature could be known. In general, at a given operating value of pressure, it is more preferable for fusion devices to have a higher proportion be due to the temperature than the density. There are observed fundamental limits to the density (the Greenwald Limit, for example [7]) above which the equilibrium disrupts or becomes unstable. On the other hand, raising temperature tends to increase the plasma conductivity (see discussion of Spitzer resistivity, 6.1), thereby allowing additional current to flow, which raises the magnitude of the confining magnetic field. It is also apparent from eq. (2.4) that increases in B greatly increase fusion power output. Furthermore, the power lost to bremsstrahlung radiation increases with square of density, but with the square root of temperature, which again suggests higher temperature as the preferable parameter to limit β .

The work presented here represents a coarse foundation to understanding the behavior of electron temperature and density in HIT-SI3. Many more questions can be answered. Among these extended applications are comparison with the recently-established tomography diagnostic, using density data for analysis of fueling strategies and passive pumping behavior, and analysis of the changes that occur after the addition of higher-power driving circuits to the HIT-SI3 system. Additional information may also be derived from the system, as discussed in sec. 4.2.3, if a more precise spectral calibration setup could be implemented.

The future success of the TS diagnostic system relies on the repair of the laser. As of June, 2020, some progress has been made including replacement oscillator flash lamps and the cleaning and re-installation of its oscillator gain medium. However, the laser is still in need of alignment, and the original failure mechanism is still uncertain.

BIBLIOGRAPHY

- [1] US Energy Information Administration, "International Energy Outlook 2017," Tech. Rep. 2017, Washington, DC, USA, 2017. 1, 2
- [2] BP p.l.c., "BP Statistical Review of World Energy 2017," Tech. Rep. 66, BP (with The Centre for Energy Economics Research and Policy at Heriot-Watt University), London, UK, 2017. 1
- [3] R. A. Berner, "Fuels and Atmospheric Composition," *Nature*, vol. 426, no. November, pp. 323–326, 2003. 1
- [4] P. C. Taylor, A. M. Waple, and C. P. Weaver, "Executive Summary of the Climate Science Special Report: Fourth National Climate Assessment," tech. rep., U.S. Global Change Research Program, Washington, D.C., USA, 2017. 1
- [5] J. Ongena and G. Van Oost, "Energy for future centuries-Prospects for fusion power as a future energy source," *Fusion Sci. Technol*, vol. 53, no. 2T, pp. 3–15, 2008. 1, 2
- [6] H. D. Lightfoot, W. Manheimer, D. A. Meneley, D. Pendergast, and G. S. Stanford, "Nuclear fission fuel is inexhaustible," in *2006 IEEE EIC Climate Change Technology Conference, EICCCC 2006*, 2007. 1
- [7] J. Freidberg, *Plasma Physics and Fusion Energy*. Cambridge University Press, 2007. 4, 5, 11, 13, 20, 131
- [8] P. Bellan, *Spheromaks*. Imperial College Press, 2000. 11, 12
- [9] J. B. Taylor, "Relaxation of toroidal plasma and generation of reverse magnetic fields," *Physical Review Letters*, vol. 33, no. 19, pp. 1139–1141, 1974. 11
- [10] J. B. Taylor, "Relaxation and magnetic reconnection in plasmas," *Reviews of Modern Physics*, vol. 58, no. 3, pp. 741–763, 1986. 11, 13
- [11] A. Reiman, "Taylor relaxation in a torus of arbitrary aspect ratio and cross section," *Physics of Fluids*, vol. 24, no. 5, p. 956, 1981. 11
- [12] T. H. Jensen and M. S. Chu, "Current drive and helicity injection," *Physics of Fluids*, vol. 27, no. 1984, p. 2881, 1984. 11

- [13] L. Woltjer, "a Theorem on Force-Free Magnetic Fields," *Proceedings of the National Academy of Sciences*, vol. 44, no. 6, pp. 489–491, 1958. 11
- [14] E. B. Hooper, R. H. Bulmer, B. I. Cohen, D. N. Hill, C. T. Holcomb, B. Hudson, H. S. McLean, L. D. Pearlstein, C. A. Romero-Talamás, C. R. Sovinec, B. W. Stallard, R. D. Wood, and S. Woodruff, "Sustained Spheromak Physics Experiment (SSPX): Design and physics results," *Plasma Physics and Controlled Fusion*, vol. 54, no. 11, 2012. 13, 19, 21, 88
- [15] J. Marshall, "Performance of a Hydromagnetic Plasma Gun," *Physics of Fluids*, vol. 3, no. 1, p. 134, 1960. 13
- [16] L. C. Burkhardt and R. H. Lovberg, "Current Sheet in a Coaxial Plasma Gun," *Physics of Fluids*, vol. 5, no. 3, p. 341, 1962. 13
- [17] S. Woodruff, D. N. Hill, B. W. Stallard, R. Bulmer, B. Cohen, C. T. Holcomb, E. B. Hooper, H. S. McLean, J. Moller, and R. D. Wood, "New Mode of Operating a Magnetized Coaxial Plasma Gun for Injecting Magnetic Helicity into a Spheromak," *Physical Review Letters*, vol. 90, no. 9, p. 095001, 2003. 13
- [18] B. S. Victor, T. R. Jarboe, A. C. Hossack, D. A. Ennis, B. A. Nelson, R. J. Smith, C. Akcay, C. J. Hansen, G. J. Marklin, N. K. Hicks, and J. S. Wrobel, "Evidence for Separatrix formation and sustainment with steady inductive helicity injection," *Physical Review Letters*, vol. 107, no. 16, pp. 1–5, 2011. 13, 16, 129
- [19] C. R. Sovinec, A. H. Glasser, T. A. Gianakon, D. C. Barnes, R. A. Nebel, S. E. Kruger, D. D. Schnack, S. J. Plimpton, A. Tarditi, and M. S. Chu, "Nonlinear magnetohydrodynamics simulation using high-order finite elements," *Journal of Computational Physics*, vol. 195, no. 1, pp. 355–386, 2004. 16, 92
- [20] K. D. Morgan, T. R. Jarboe, A. C. Hossack, R. N. Chandra, and C. J. Everson, "Validation of extended magnetohydrodynamic simulations of the HIT-SI3 experiment using the NIMROD code," *Physics of Plasmas*, vol. 24, no. 12, 2017. 16, 92
- [21] T. R. Jarboe, W. T. Hamp, G. J. Marklin, B. A. Nelson, R. G. O'Neill, A. J. Redd, P. E. Sieck, R. J. Smith, and J. S. Wrobel, "Spheromak Formation by Steady Inductive Helicity Injection," *Physical Review Letters*, vol. 97, no. 11, p. 115003, 2006. 16, 129
- [22] B. S. Victor, T. R. Jarboe, C. J. Hansen, and C. Akcay, "Sustained spheromaks with ideal $n=1$ kink stability and pressure confinement," *Physics of Plasmas*, 2014. 16, 81, 82, 88

- [23] A. C. Hossack, D. A. Sutherland, T. R. Jarboe, A. C. Hossack, D. A. Sutherland, and T. R. Jarboe, "Derivation of dynamo current drive in a closed-current volume and stable current sustainment in the HIT-SI experiment," *Physics of Plasmas*, vol. 24, no. 020702, 2017. 17, 124, 128
- [24] T. Jarboe, B. Victor, B. Nelson, C. Hansen, C. Akcay, D. Ennis, N. Hicks, A. Hossack, G. Marklin, and R. Smith, "Imposed-dynamo current drive," *Nuclear Fusion*, vol. 52, no. 8, p. 083017, 2012. 17
- [25] K. Morgan, "Finite-beta simulations of HIT-SI and HIT-SI3 using the NIMROD code," 2018. 18, 92
- [26] A. A. Kaptanoglu, T. Benedett, K. Morgan, C. Hansen, and T. Jarboe, "Two-temperature effects in Hall-MHD simulations of the HIT-SI experiment," *Physics of Plasmas*, vol. 072505, no. 27, 2020. 18, 92, 115
- [27] T. R. Jarboe, "Review of spheromak research," *Plasma Physics and Controlled Fusion*, vol. 36, no. 6, pp. 945–990, 1994. 18, 88
- [28] B. Hudson, R. D. Wood, H. S. McLean, E. B. Hooper, D. N. Hill, J. Jayakumar, J. Moller, D. Montez, C. A. Romero-Talamás, T. A. Casper, J. A. Johnson, L. L. Lodestro, E. Mezonlin, and L. D. Pearlstein, "Energy confinement and magnetic field generation in the SSPX spheromak," *Physics of Plasmas*, vol. 15, no. 5, 2008. 19
- [29] V. D. Shafranov and E. I. Yurchenko, "Condition for Flute Instability of a Toroidal-Geometry Plasma," *Soviet Physics JETP*, vol. 26, no. 3, pp. 682–686, 1968. 20
- [30] G. Bateman, *MHD Instabilities*. The MIT Press, 1978. 20
- [31] S. Woodruff, E. B. Hooper, L. D. Pearlstein, R. Bulmer, D. N. Hill, C. T. Holcomb, H. S. Mclean, J. Moller, B. W. Stallard, and R. D. Wood, "Observation of a pressure limit in a gun-driven spheromak," *Physics of Plasmas*, vol. 13, no. 044506, pp. 1–5, 2006. 21, 131
- [32] F. J. Wysocki, J. C. Fernández, I. Henins, T. R. Jarboe, and G. J. Marklin, "Evidence for a pressure-driven instability in the CTX spheromak," *Physical Review Letters*, vol. 61, no. 21, pp. 2457–2460, 1988. 21
- [33] I. H. Hutchinson, *Principles of Plasma Diagnostics*. Cambridge University Press, 2 ed., 2005. 27, 40

- [34] E. M. Purcell, *Electricity and Magnetism*. Cambridge: Cambridge University Press, third edition. ed., 2013. 28
- [35] J. D. Jackson, *Classical Electrodynamics; 2nd ed.* New York, NY: Wiley, 1975. 30
- [36] D. J. Griffiths, *Introduction to Electrodynamics; 4th ed.* Boston, MA: Pearson, 2013. Republished by Cambridge University Press in 2017. 30
- [37] J. Sheffield, *Plasma Scattering of Electromagnetic Radiation*. Academic Press, 1975. 30, 40, 41
- [38] K. Warner and G. M. Hieftje, "Thomson scattering from analytical plasmas," *Spectrochimica Acta - Part B Atomic Spectroscopy*, vol. 57, no. 2, pp. 201–241, 2002. 40
- [39] E. E. Salpeter, "Electron density fluctuations in a plasma," *Phys. Rev.*, vol. 120, pp. 1528–1535, Dec 1960. 40
- [40] D. W. Johnson, B. P. LeBlanc, D. L. Long, and G. Renda, "APD detector electronics for the NSTX Thomson scattering system," *Review of Scientific Instruments*, vol. 72, no. 1 II, pp. 1129–1132, 2001. 50
- [41] K. Lee, *Development of a Thomson scattering system and its use in a rotating magnetic field driven field-reversed configurations plasma*. PhD thesis, University of Washington, ProQuest LLC, 789 East Eisenhower Parkway, Ann Arbor, MI 48106-1346, 12 2011. 50, 53, 56, 62
- [42] H. P. K.K., "TECHNICAL INFORMATION SD-37 Characteristics and use of Charge amplifier," tech. rep., 10 2001. 50
- [43] D. H. McNeill, "Effect of changes in viewing window transmission on high-temperature thomson scattering data," *Review of Scientific Instruments*, vol. 61, no. 4, pp. 1263–1266, 1990. 56
- [44] D. Sivia and J. Skilling, *Data Analysis: A Bayesian Tutorial*. Oxford University Press, 2006. 59, 69
- [45] J. McIntyre, "Multiplication Noise in Uniform Avalanche Diodes," *IEEE Transactions on Electron Devices*, vol. ED-13, no. 1, pp. 164–168, 1966. 62
- [46] P. Webb, R. McIntyre, *et al.*, "Properties of avalanche photodiodes.," 1974. 62

- [47] R. Fischer and A. Dinklage, "Integrated data analysis of fusion diagnostics by means of the Bayesian probability theory," *Review of Scientific Instruments*, vol. 75, no. 10, p. 4237, 2004. 65
- [48] L. M. Reusch *et al.*, "An integrated data analysis tool for improving measurements on the MST RFP," *Review of Scientific Instruments*, vol. 85, p. 11D844, 2014. 65
- [49] J. Svensson *et al.*, "Modeling of jet diagnostics using bayesian graphical models," *Contributions to Plasma Physics*, vol. 51, no. 2-3, pp. 152–157, 2011. 65
- [50] R. Trotta, "Bayes in the sky: Bayesian inference and model selection in cosmology," *Contemporary Physics*, vol. 49, no. 2, pp. 71–104, 2008. 65, 66, 67
- [51] R. Fischer, A. Dinklage, and E. Pasch, "Bayesian modeling of fusion diagnostics," *Plasma Physics and Controlled Fusion*, vol. 45, no. 7, pp. 1095–1111, 2003. 65, 66
- [52] J. Skilling *et al.*, "Nested sampling for general Bayesian computation," *Bayesian analysis*, vol. 1, no. 4, pp. 833–859, 2006. 67, 69
- [53] K. Barbary. <http://kbarbary.github.io/nestle>. Accessed: 11-19-2019. 68
- [54] J. Salvatier, T. V. Wiecki, and C. Fonnesbeck, "Probabilistic programming in Python using PyMC3," *PeerJ Computer Science*, vol. 2, p. e55, 2016. 68
- [55] R. Fischer, C. Wendland, A. Dinklage, S. Gori, and V. Dose, "Thomson scattering analysis with the Bayesian probability theory," *Plasma Physics and Controlled Fusion*, vol. 44, no. 8, pp. 1501–1519, 2002. 72
- [56] U. Von Toussaint, "Bayesian inference in physics," *Reviews of Modern Physics*, vol. 83, no. 3, pp. 943–999, 2011. 72
- [57] R. E. Kass and A. E. Raftery, "Bayes Factors," *Journal of the American Statistical Association*, vol. 90, no. 430, pp. 319–323, 1995. 73
- [58] S. E. Segre, "Thomson scattering from current-carrying plasma with Spitzer-Harm electron distribution function," *Plasma Physics*, vol. 20, no. 3, pp. 287–291, 1978. 74
- [59] F. Alladio and M. Martone, "Thomson scattering measurements from a current carrying plasma and the determination of local current density in a Tokamak (T.T.F.2)," *Physics Letters A*, vol. 60, no. 1, pp. 39–41, 1977. 74

- [60] I. H. Hutchinson, "Current density measurement in Tokamak-type devices by longitudinal Thompson scattering: A proposal," *Journal of Physics D: Applied Physics*, vol. 10, no. 3, 1977. 74
- [61] R. J. Goldston and P. H. Rutherford, *Introduction to plasma physics*. CRC Press, 1995. 74, 116, 119
- [62] S. Braginskii, "Transport Processes in a Plasma," 1965. 74, 93, 114, 116, 118
- [63] J. M. Finn, C. R. Sovinec, and D. Del-Castillo-Negrete, "Chaotic scattering and self-organization in spheromak sustainment," *Physical Review Letters*, vol. 85, no. 21, pp. 4538–4541, 2000. 88
- [64] C. R. Sovinec, J. M. Finn, and D. Del-Castillo-Negrete, "Formation and sustainment of electrostatically driven spheromaks in the resistive magnetohydrodynamic model," *Physics of Plasmas*, vol. 8, no. 2, pp. 475–490, 2001. 88
- [65] E. B. Hooper, C. A. Romero-Talamás, L. L. Lodestro, R. D. Wood, and H. S. McLean, "Aspect-ratio effects in the driven, flux-core spheromak," *Physics of Plasmas*, vol. 16, no. 5, 2009. 88
- [66] G. J. Wysocki, F. J. Fernandez, J. C. Henins, I. Jarboe, T. R. Marklin, "Improved Energy Confinement in Spheromaks with Reduced Field Errors," *Physical Review Letters*, vol. 65, no. 1, pp. 40–43, 1990. 88
- [67] L. Spitzer, *Physics of Fully Ionized Gases*. Interscience tracts on physics and astronomy, 3, Interscience Publishers, 1956. 103
- [68] D. D. Meyerhofer, R. A. Hulse, and E. G. Zweibel, "Zero-dimensional study of the compression of low-temperature spheromaks," *Nuclear Fusion*, vol. 26, no. 2, pp. 235–241, 1986. 112
- [69] C. W. Barnes, T. R. Jarboe, H. W. Hoida, B. L. Wright, R. A. Hulse, and D. E. Post, "Zero-dimensional energy balance modelling of the ctx spheromak experiment," *Nuclear Fusion*, vol. 25, no. 11, pp. 1657–1675, 1985. 112
- [70] F. M. Levinton, D. D. Meyerhofer, R. M. Mayo, A. C. Janos, Y. Ono, Y. Ueda, and M. Yamada, "Confinement and power balance in the S-1 spheromak," *Nuclear Fusion*, vol. 30, no. 5, pp. 871–879, 1990. 112
- [71] A. Hossack, T. Jarboe, and R. Chandra, "Plasma response to sustainment with imposed-dynamo current drive in HIT-SI and HIT-SI3," *Nuclear Fusion*, vol. 57, no. 076026, 2017. 113, 128

- [72] R. O'Neill, G. Marklin, T. Jarboe, C. Akcay, W. Hamp, B. Nelson, A. Redd, R. Smith, B. Stewart, J. Wrobel, and P. Sieck, "A fully relaxed helicity balance model for an inductively driven spheromak," *Physics of Plasmas*, vol. 14, no. 11, p. 112304, 2007. 114, 118
- [73] E. G. Zweibel and M. Yamada, "Magnetic Reconnection in Astrophysical and Laboratory Plasmas," *Annual Review of Astronomy and Astrophysics*, vol. 47, no. 1, pp. 291–332, 2009. 115
- [74] M. Yamada, J. Yoo, and C. E. Myers, "Understanding the dynamics and energetics of magnetic reconnection in a laboratory plasma: Review of recent progress on selected fronts," *Physics of Plasmas*, vol. 23, no. 5, 2016. 115
- [75] S. C. Hsu, T. A. Carter, G. Fiksel, H. Ji, R. M. Kulsrud, and M. Yamada, "Experimental study of ion heating and acceleration during magnetic reconnection," *Physics of Plasmas*, vol. 8, no. 5 II, pp. 1916–1928, 2001. 115
- [76] J. S. Wrobel, C. J. Hansen, T. R. Jarboe, R. J. Smith, A. C. Hossack, B. A. Nelson, G. J. Marklin, D. A. Ennis, C. Akcay, and B. S. Victor, "Relaxation-time measurement via a time-dependent helicity balance model," *Physics of Plasmas*, vol. 20, no. 1, 2013. 118, 124
- [77] U. Shumlak and T. R. Jarboe, "Stable high beta spheromak equilibria using concave flux conservers," *Physics of Plasmas*, vol. 7, no. 7, pp. 2959–2963, 2000. 118
- [78] E. Morse, *Nuclear fusion*. Springer International Publishing, 2018. 121
- [79] A. Rechester and M. Rosenbluth, "Electron Heat Transport in a Tokamak with Destroyed Magnetic Surfaces," *Physical Review Letters*, vol. 40, no. 1, pp. 38–41, 1978. 121
- [80] T. M. Biewer, C. B. Forest, J. K. Anderson, G. Fiksel, B. Hudson, S. C. Prager, J. S. Sarff, J. C. Wright, D. L. Brower, W. X. Ding, and S. D. Terry, "Electron Heat Transport Measured in a Stochastic Magnetic Field," *Physical Review Letters*, vol. 91, no. 4, pp. 1–4, 2003. 121
- [81] H. S. McLean, R. D. Wood, B. I. Cohen, E. B. Hooper, D. N. Hill, J. M. Moller, C. Romero-Talamas, and S. Woodruff, "Transport and fluctuations in high temperature spheromak plasmas," *Physics of Plasmas*, vol. 13, no. 5, 2006. 121
- [82] O. G. Bakunin, "Correlations and anomalous transport effects related to stochastic instability," *Plasma Physics and Controlled Fusion*, vol. 47, no. 11, pp. 1857–1876, 2005. 123

- [83] B. D. G. Chandran, S. C. Cowley, M. Ivanushkina, and R. Sydora, "Heat Transport Along an Inhomogeneous Magnetic Field. I. Periodic Magnetic Mirrors," *The Astrophysical Journal*, vol. 525, no. 2, pp. 638–650, 1999. 123
- [84] F. Chen, *Introduction to Plasma Physics and Controlled Fusion*. No. 2, Plenum Press, 1984. 142

Appendix A

DEFINITION OF A PLASMA

To use the most basic and universally familiar terms, plasma is essentially a highly ionized gas. The problem with this description is that it is too broad and too qualitative. Ionized gas is ubiquitous in the universe, so in the laboratory setting the definition of a plasma is often refined based on certain ensemble properties to refer to a subset of ionized gas. To be formally classified as a plasma, an ensemble of ionized particles must exhibit both quasineutrality and collective behavior. The term “quasineutrality” refers to a particle ensemble in which concentrations of electrons and ions are approximately equal. Collective behavior occurs in a group of charged particles that is sufficiently large and dense such that the cumulative effect of the charges produces electric forces that can compete with random particle motions (caused by collisions and thermal motion).

A concept that helps to define these requirements quantitatively is the Debye length:

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{n_e e^2}} \quad (\text{A.1})$$

This quantity is the characteristic distance at which free charges (electrons in the case of Eq. A.1) in the ionized gas will shield out electric fields caused by an opposite-sign charged particle. A related quantity is the number of particles in a sphere with radius equal to the Debye length, called the plasma parameter: $N_D = n \frac{4}{3} \pi \lambda^3$. Here n refers to the plasma number density in m^{-3} (in a quasineutral plasma the electron and ion number densities are the same, $n_e = n_i$). An ionized gas will exhibit collective behavior only if the characteristic size of the system (L) is greater than a Debye length, and there are many particles in the Debye sphere.

The qualities necessary for a collection of charged particles to be called a plasma are

then [84]:

$$n_e = \sum_{\alpha} Z_{\alpha} n_{\alpha} \quad \text{quasineutrality}$$

$$L \gg \lambda_D$$

$$N_D \gg 1 \quad \text{collective behavior}$$

$$\omega\tau > 1$$

(A.2)

Here ω refers to the characteristic angular frequency of electron inertial oscillations that occur in the plasma: $\omega_{pe} = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}}$, and τ refers to the typical time between collisions of an electron with neutral particles.

Appendix B

TS DATA ANALYSIS CODE (PYTHON)

All scripts used for data analysis and modeling in this thesis can be found at:
<https://www.dropbox.com/sh/5gmq3hnx51aim8h/AACDLm2Lys4KXH7vshXktIRla?dl=0>

Appendix C

FIR RESULTS: PUMPING CANS

The Far-infrared interferometer has been used to analyze the effect of installing four passive pumping volumes to the HIT-SI3 vessel opposite to the injector side. Many shots with different fueling parameters are compared from before and after their installation to try to determine their effect on the plasma electron density. In order to compare the discharge fueling in a consistent way, the fueling factor FF is defined:

$$FF = \Delta t_{\text{hel}} P_{\text{hel}} + \sum_i \Delta t_i P_i \quad (\text{C.1})$$

Here Δt is the gas puff duration and P is the puff pressure. The fueling factor (in units of Torr-s) just quantifies the total neutral gas inventory injected into the volume during a shot. The subscript *hel* refers to the helicon (an electrode-less plasma source used to help seed plasma breakdown), and the summation is over all three HIT-SI3 injectors.

The results pointed to the somewhat surprising behavior of lowered electron density in helium discharges, but higher density in Deuterium discharges after installation of the additional empty volumes. This may be explained by the fact that deuterium discharges are deliberately fueled at the lowest possible level to achieve higher toroidal current, so it may be that extra vacuum volume draws down the neutral pressure sufficiently quickly that additional puffing pressure is necessary to achieve breakdown. The increase in required puff pressure may then translate to higher electron densities during the shot. Helium discharges in general are fueled very conservatively (high pressure), as this is observed to reduce damage to the vessel walls caused by plasma arcing.

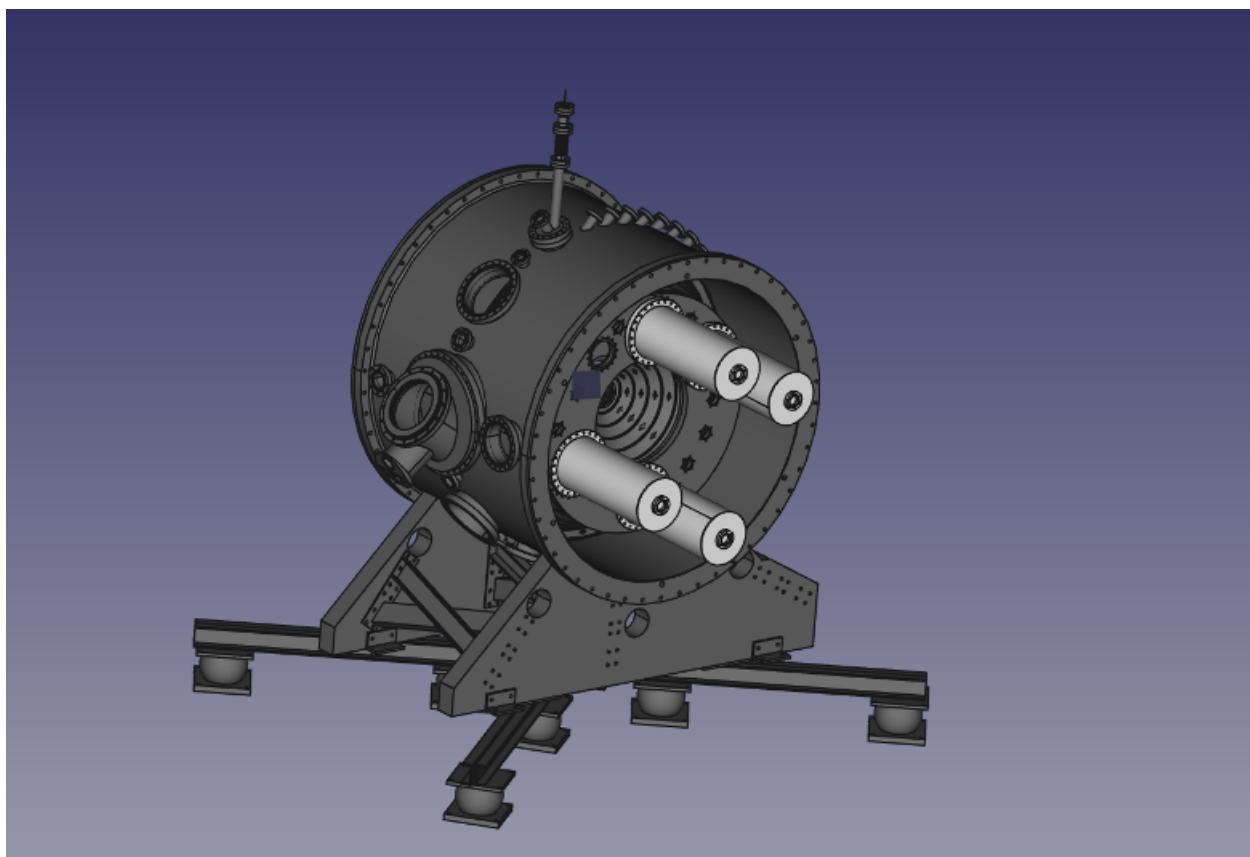


Figure C.1: Passive pumping volumes ("cans") added to the HIT-SI3 vacuum vessel before May 2, 2017.

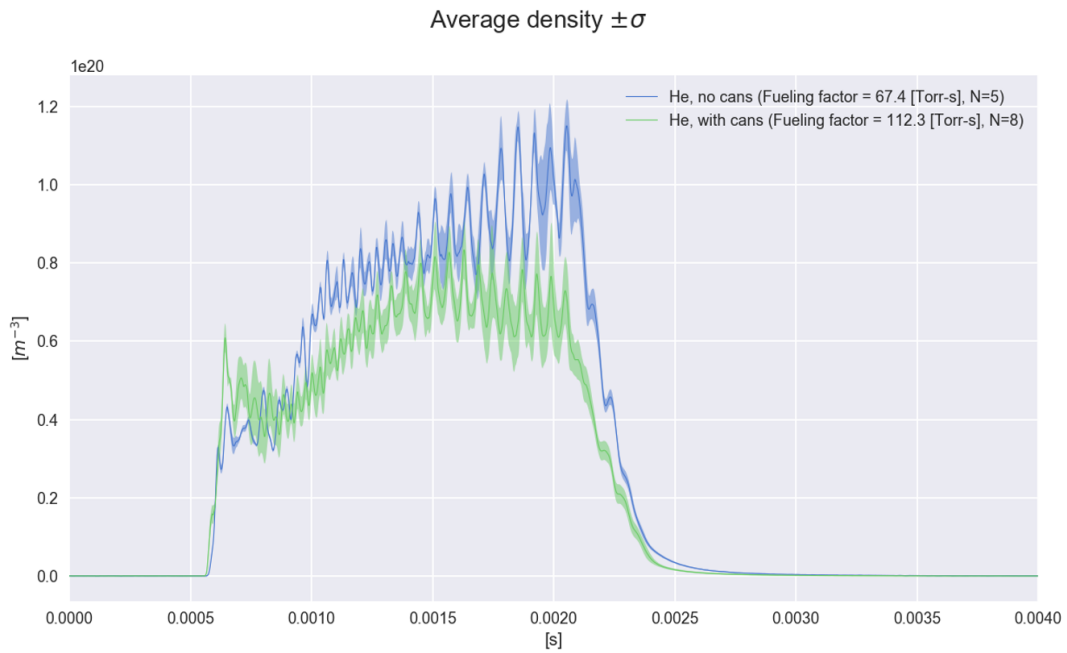


Figure C.2: Comparison of chord-integrated electron density with and without passive pumping (“cans”) in helium discharges. Even with higher fueling factor, helium discharges with passive pumping see lower densities.



Figure C.3: Comparison of chord-integrated electron density with and without passive pumping (“cans”) in deuterium discharges. In this case the opposite trend is observed.

Appendix D

INCONCLUSIVE TS MEASUREMENTS IN HIGH-PERFORMANCE PLASMAS

As discussed in Ch. 5, there is some difficulty in discriminating between high temperature measurements that result from low signal-to-noise ratio in the raw data and those that may be from high temperature plasmas. An unfortunate factor here is that discharges with the highest toroidal currents are also typically those with the lowest densities, which scales with the Thomson signal in accordance with eq. (3.32). The following are data from the best-performing shots that were measured with Thomson scattering. In general, for these measurements it is more informative to see the entire posterior distributions than to report the median and quantiles. They also show that high toroidal current discharges may still have fairly low temperatures (shot 190508008, figs. D.25–D.32). The naively interpreted TS measurements with low SNR also generally disagree with crude estimates of decaying spheromak temperature via eq. (6.4) and the following:

$$\tau_{L/R} = \mu_0 / (\eta \lambda_{sph}^2) \quad (\text{D.1})$$

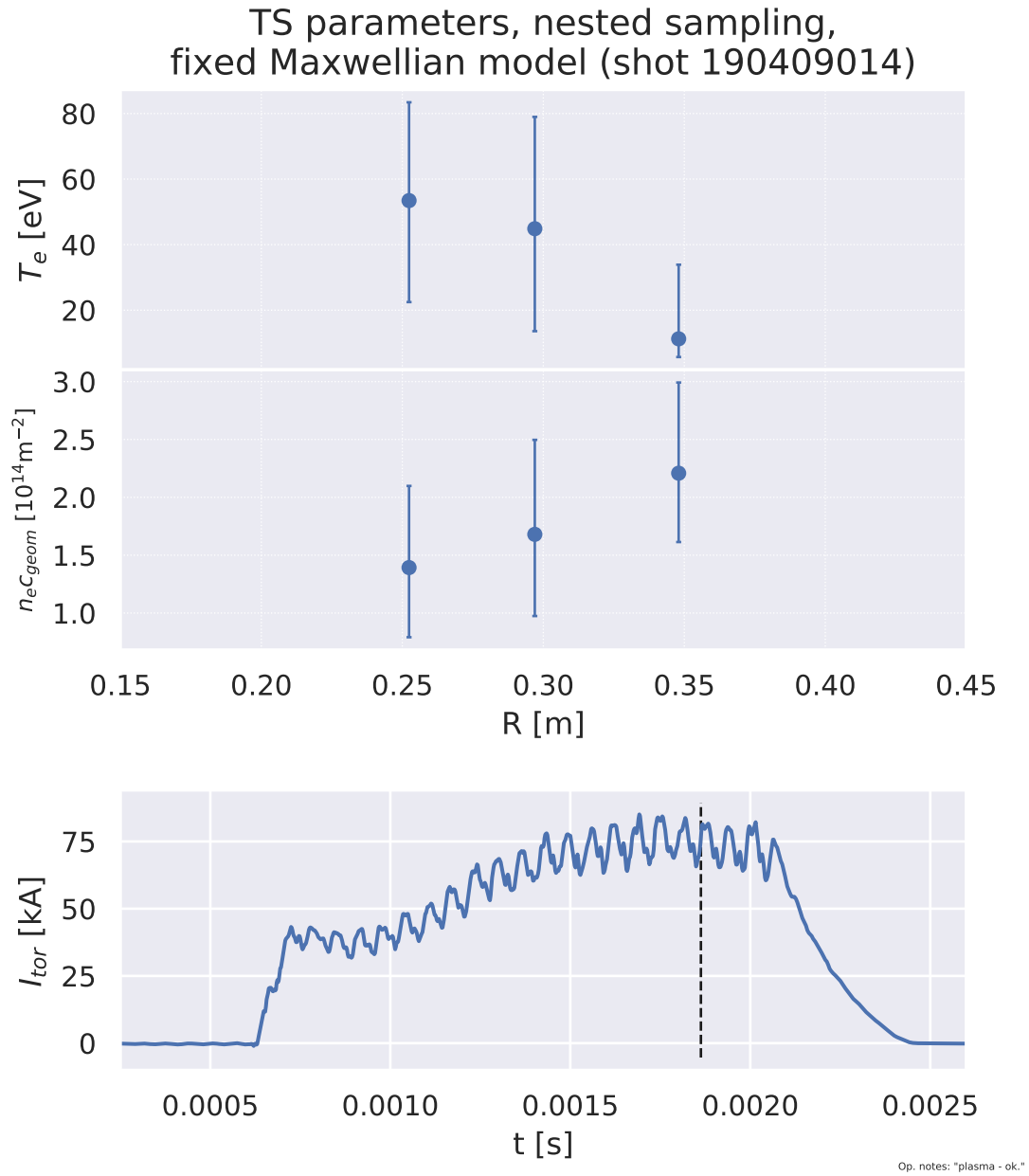


Figure D.1: TS measurements, shot 190409014.

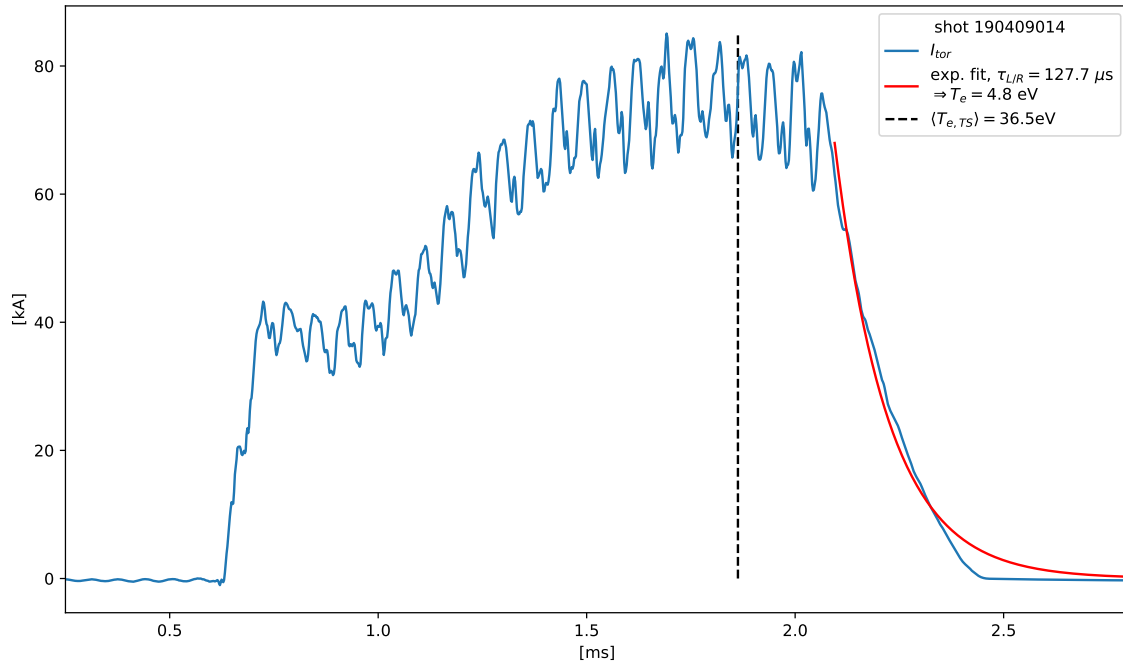


Figure D.2: TS results compared with T_e from I_{tor} decay time.

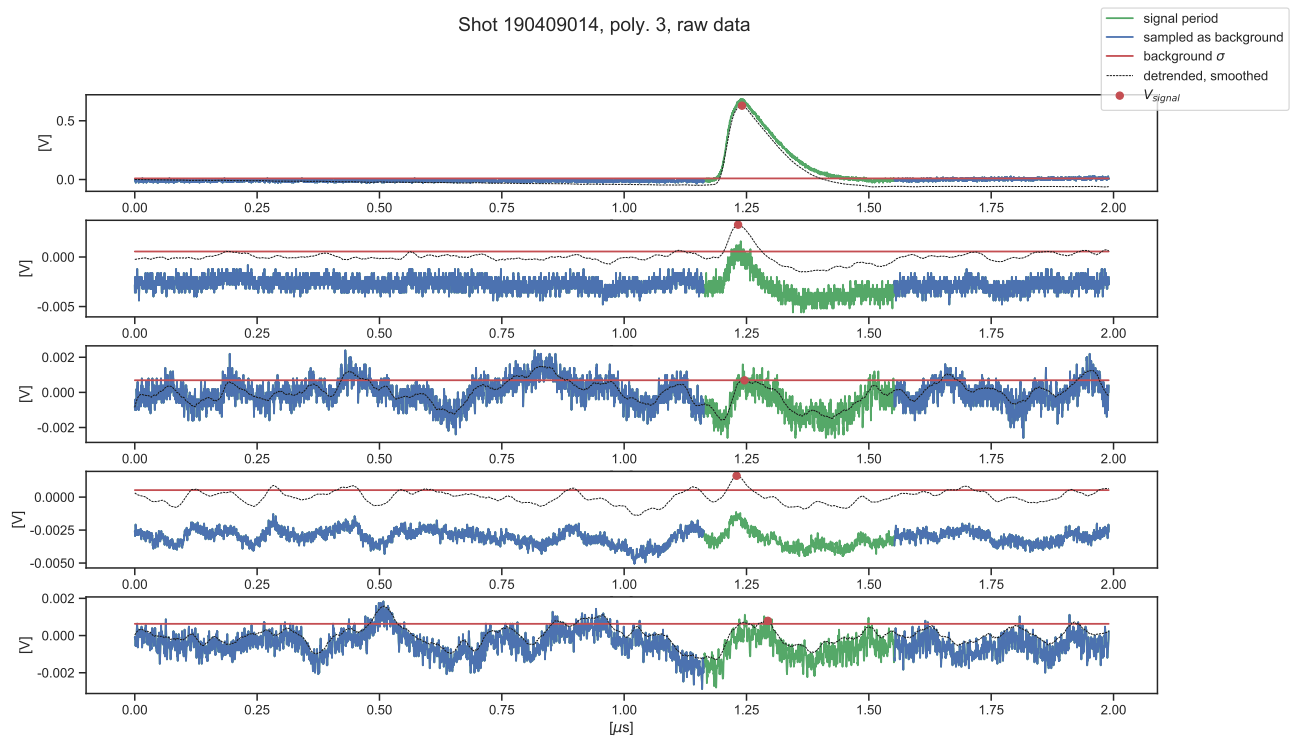


Figure D.3: Raw TS data, shot 190409014, polychromator 3.

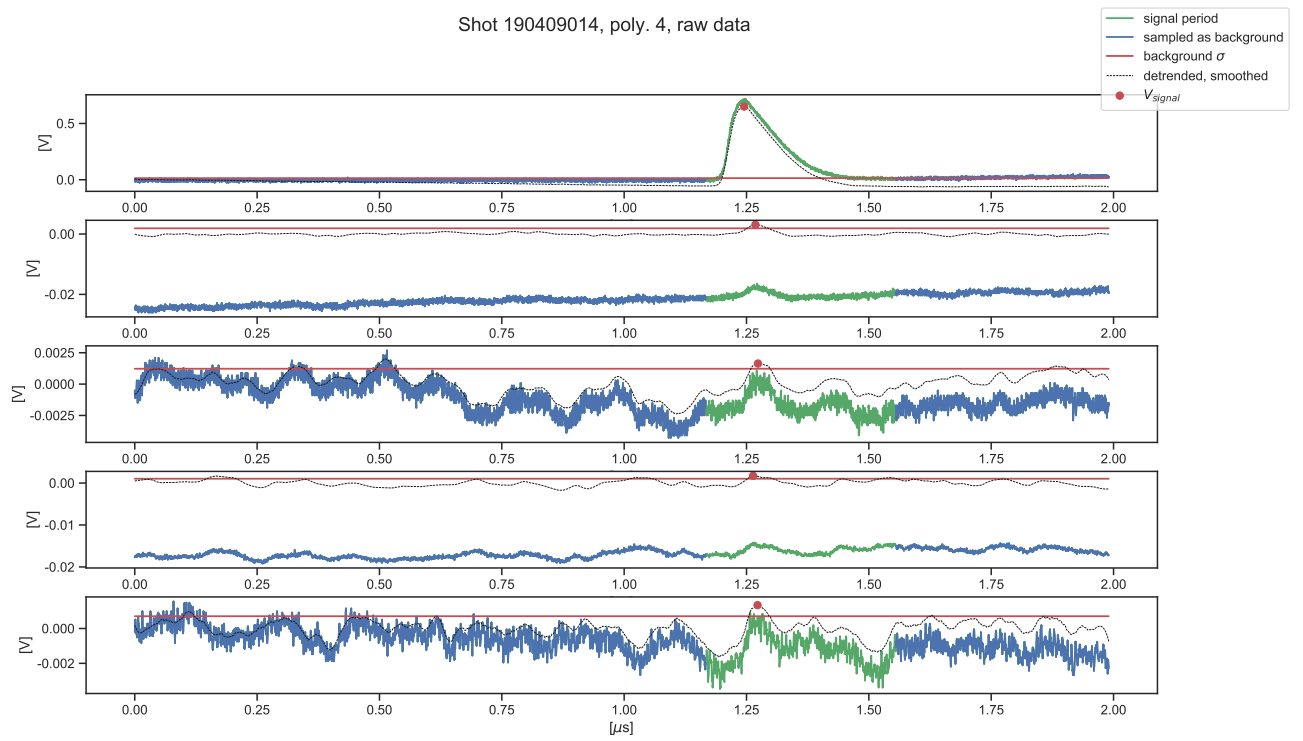


Figure D.4: Raw TS data, shot 190409014, polychromator 4.

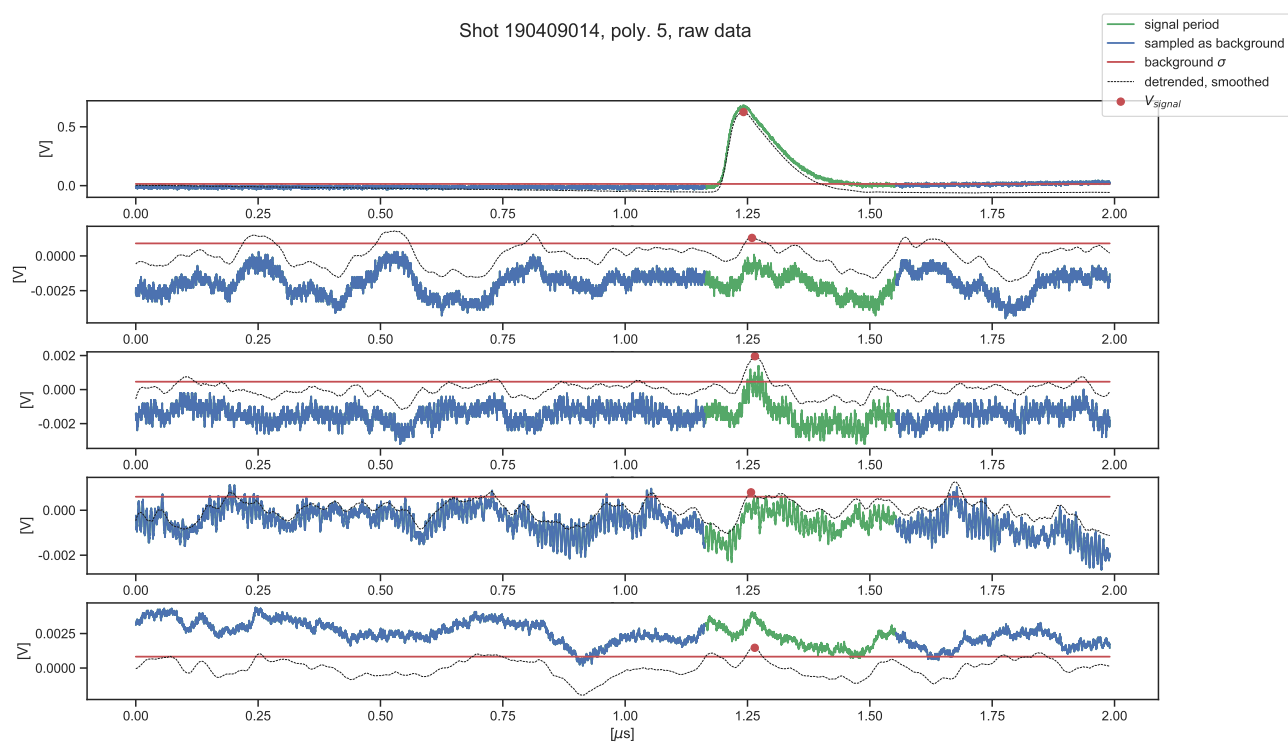


Figure D.5: Raw TS data, shot 190409014, polychromator 5.

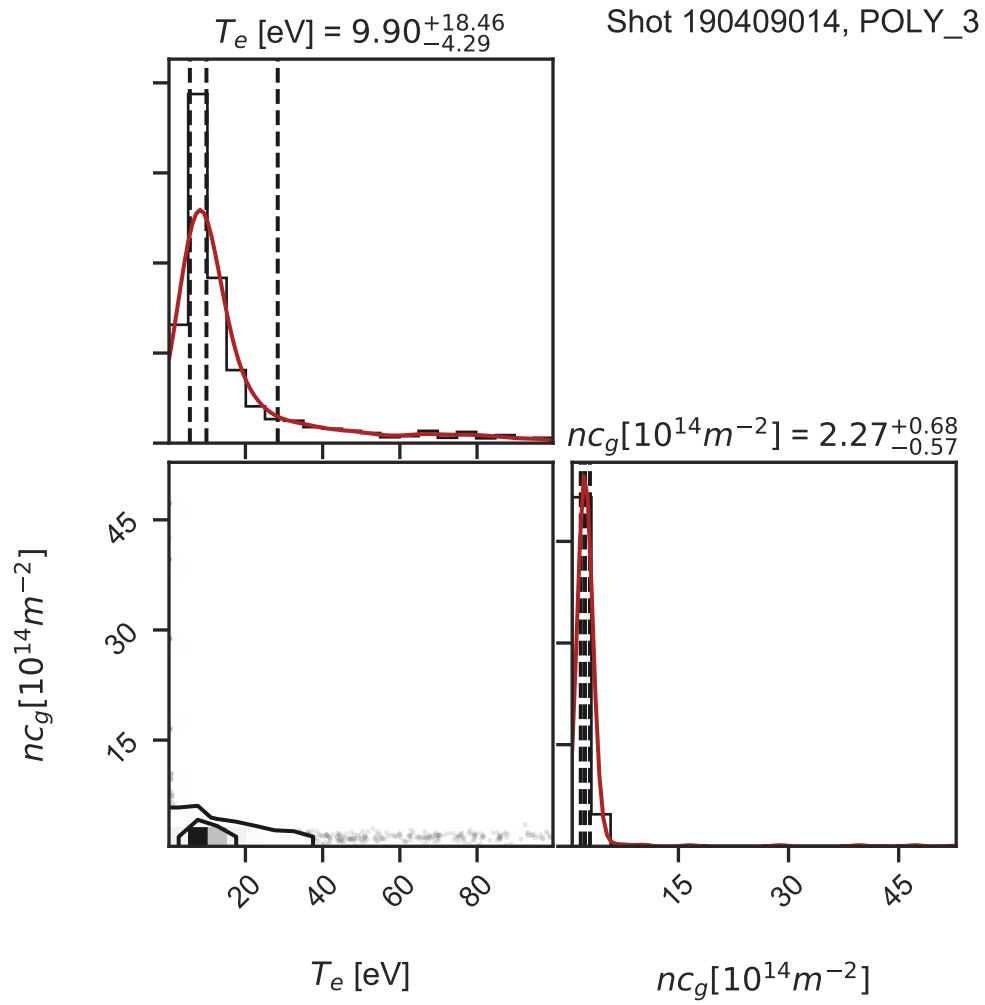


Figure D.6: Bayesian inference results on T_e and nc_{geom} for shot 190409014, polychromator 3.

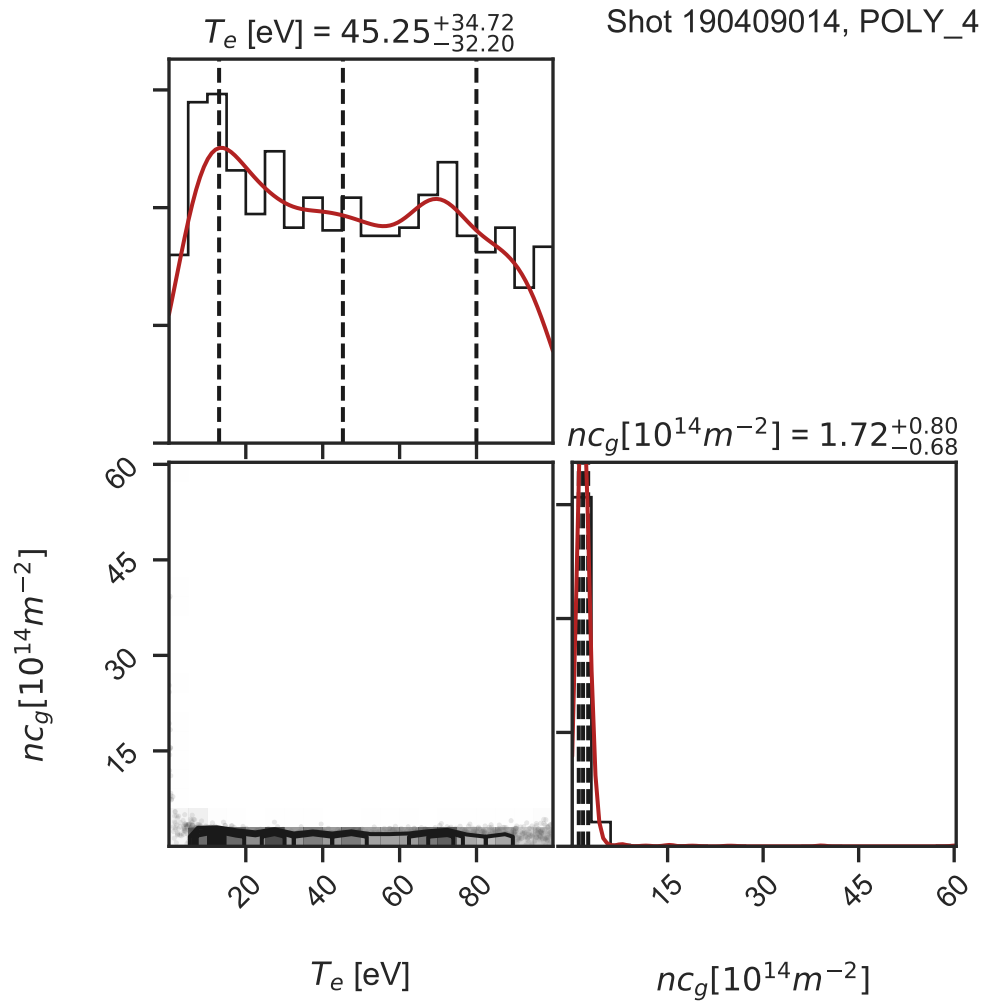


Figure D.7: Bayesian inference results on T_e and nc_{geom} for shot 190409014, polychromator 4.

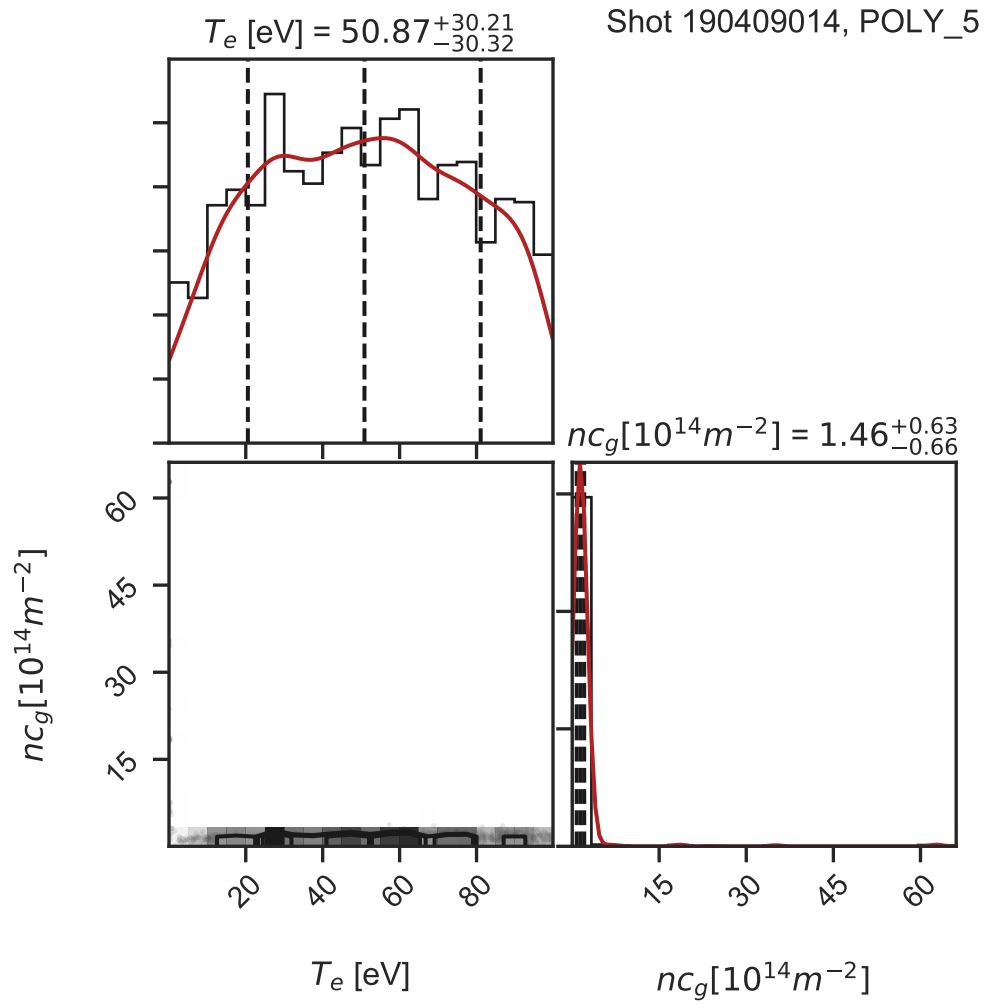


Figure D.8: Bayesian inference results on T_e and nc_{geom} for shot 190409014, polychromator 5.

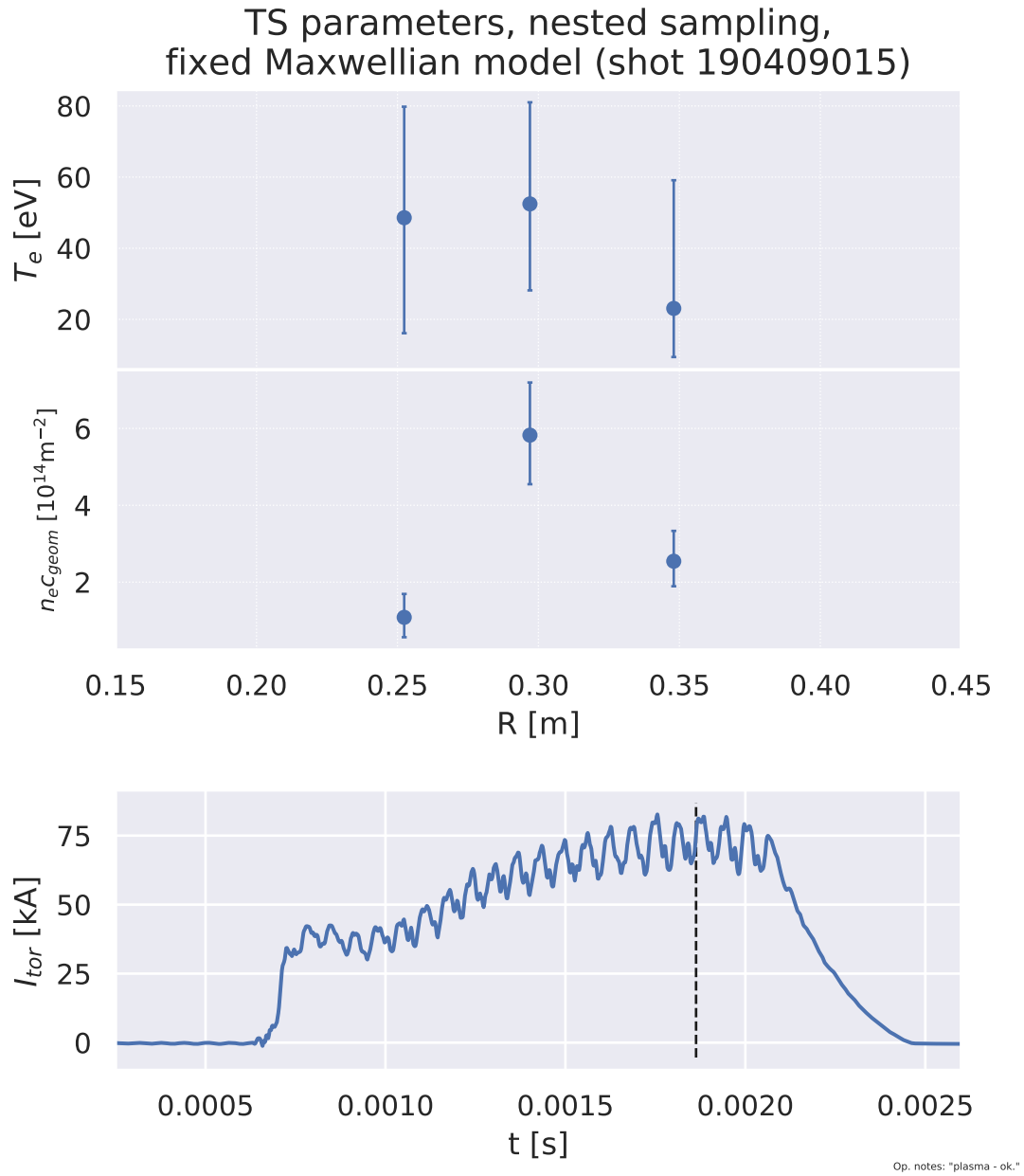


Figure D.9: TS measurements, shot 190409015.

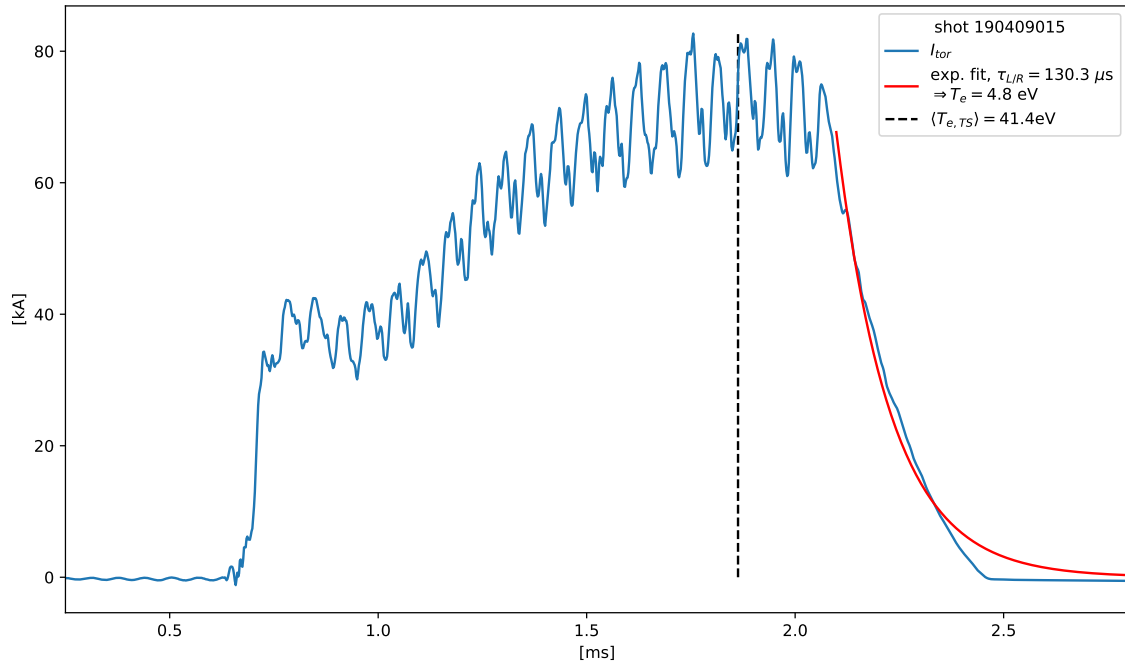


Figure D.10: TS results compared with T_e from I_{tor} decay time.

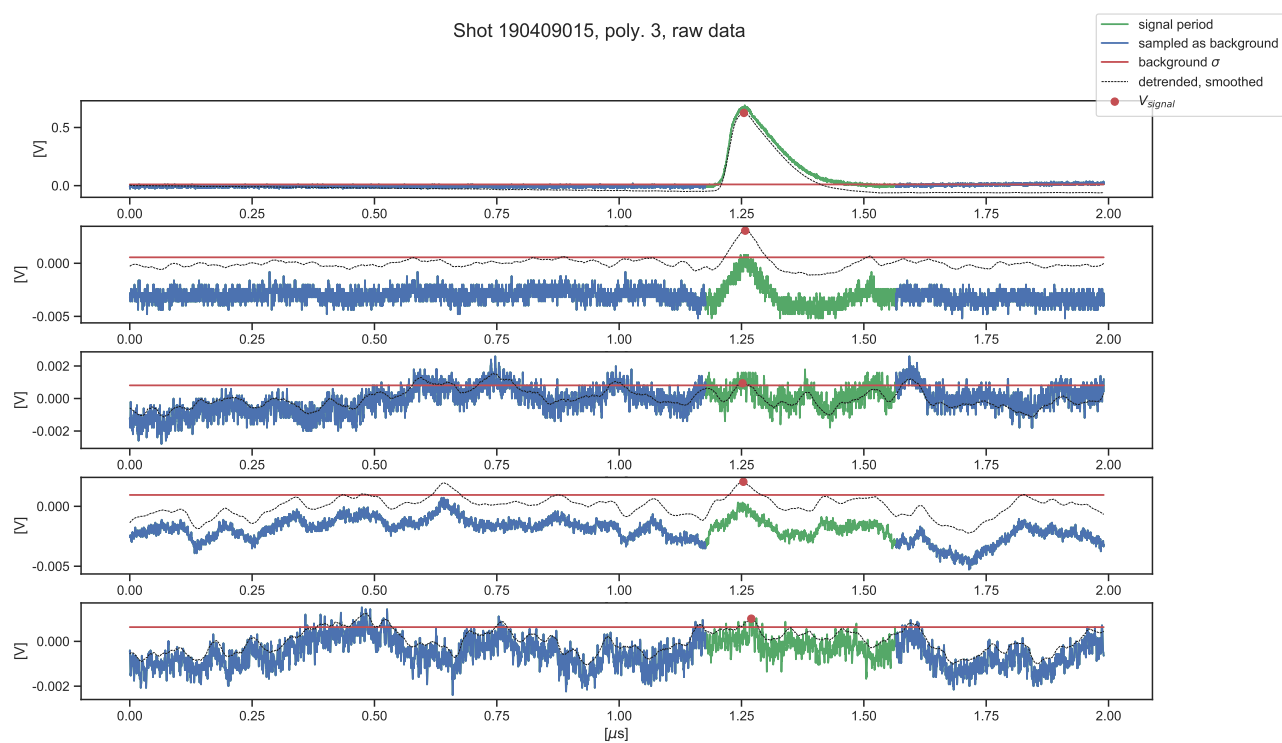


Figure D.11: Raw TS data, shot 190409015, polychromator 3.

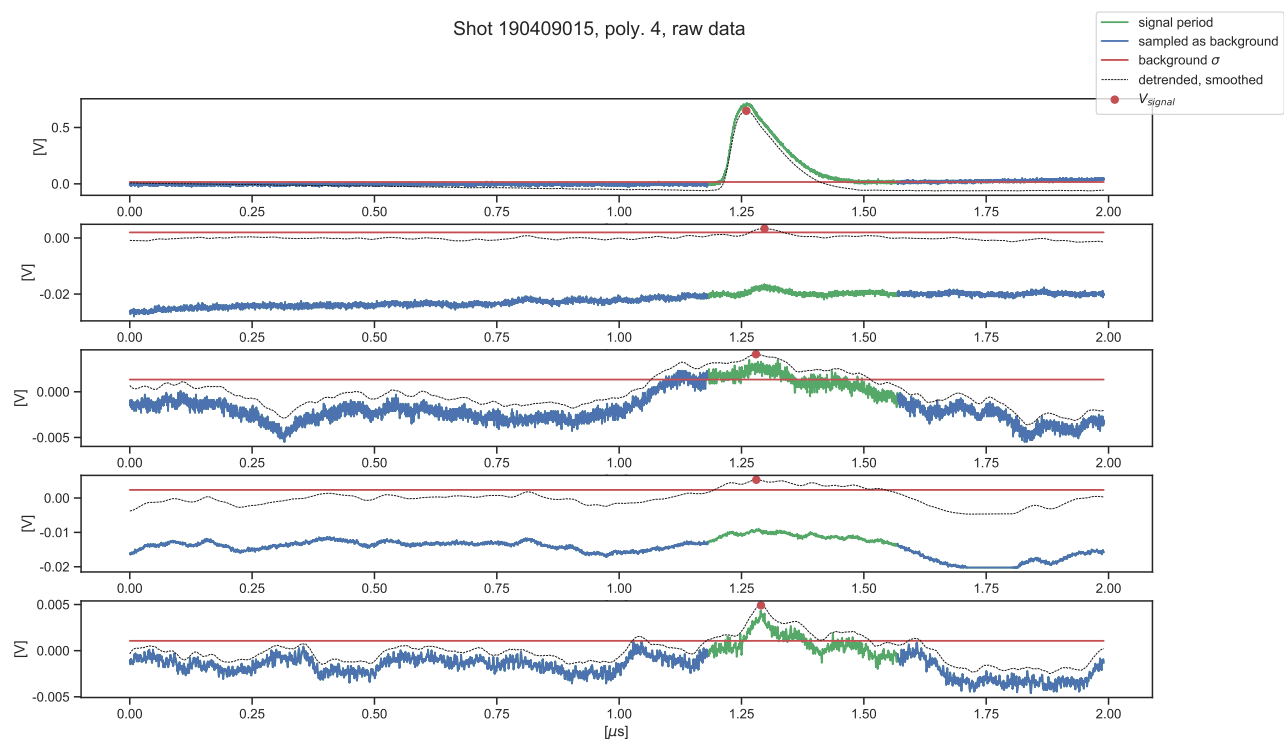


Figure D.12: Raw TS data, shot 190409015, polychromator 4.

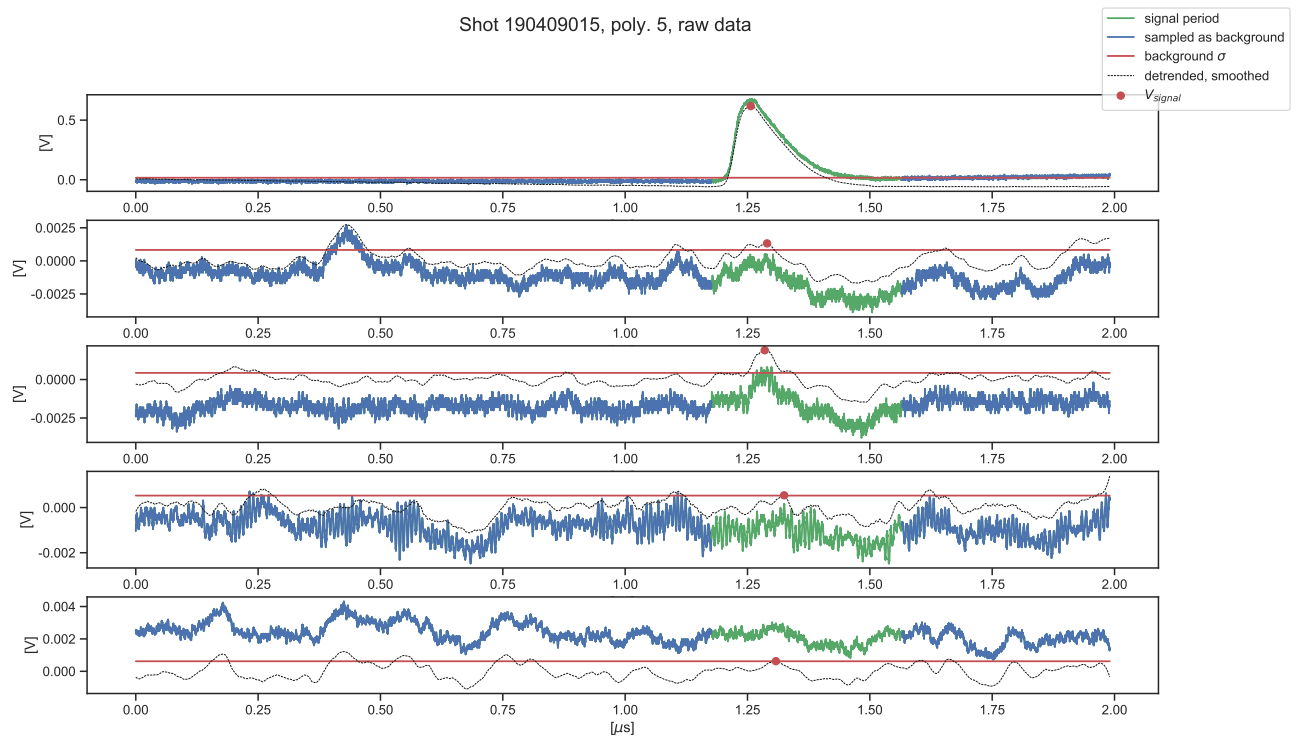


Figure D.13: Raw TS data, shot 190409015, polychromator 5.

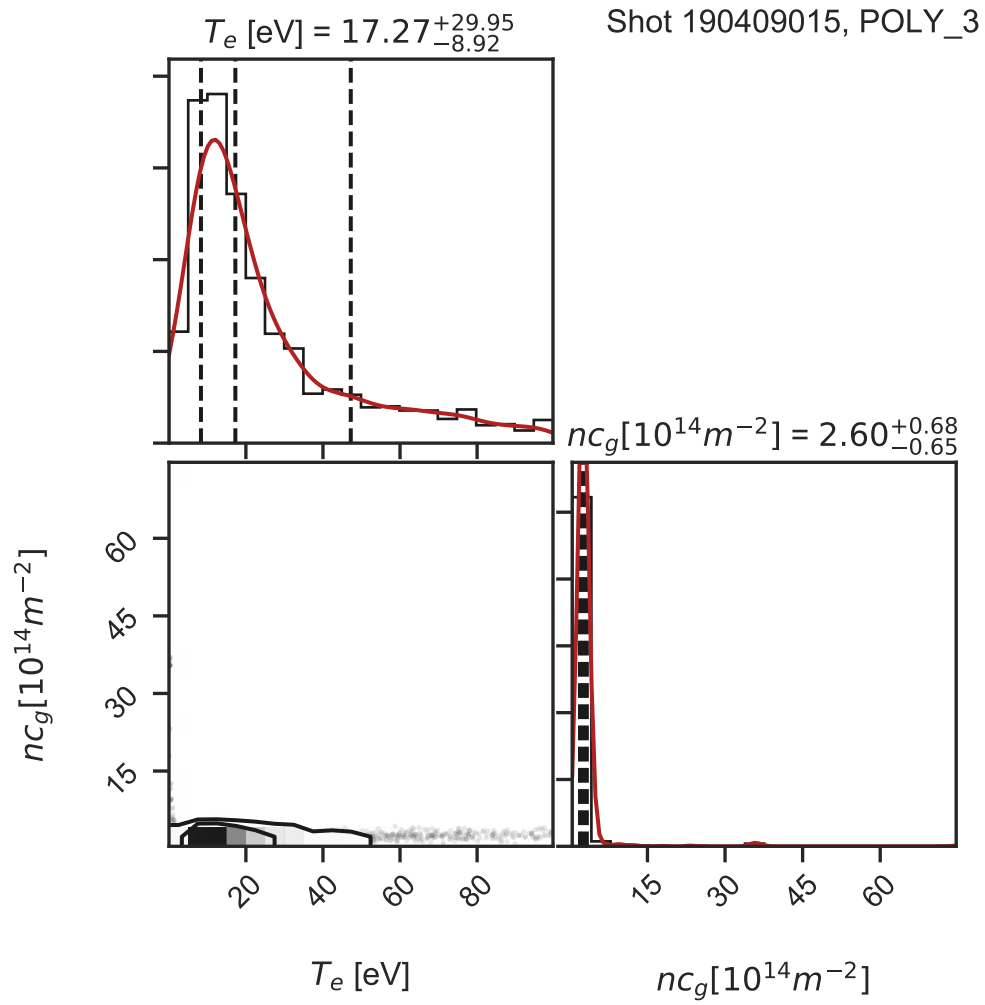


Figure D.14: Bayesian inference results on T_e and nc_{geom} for shot 190409015, polychromator 3.

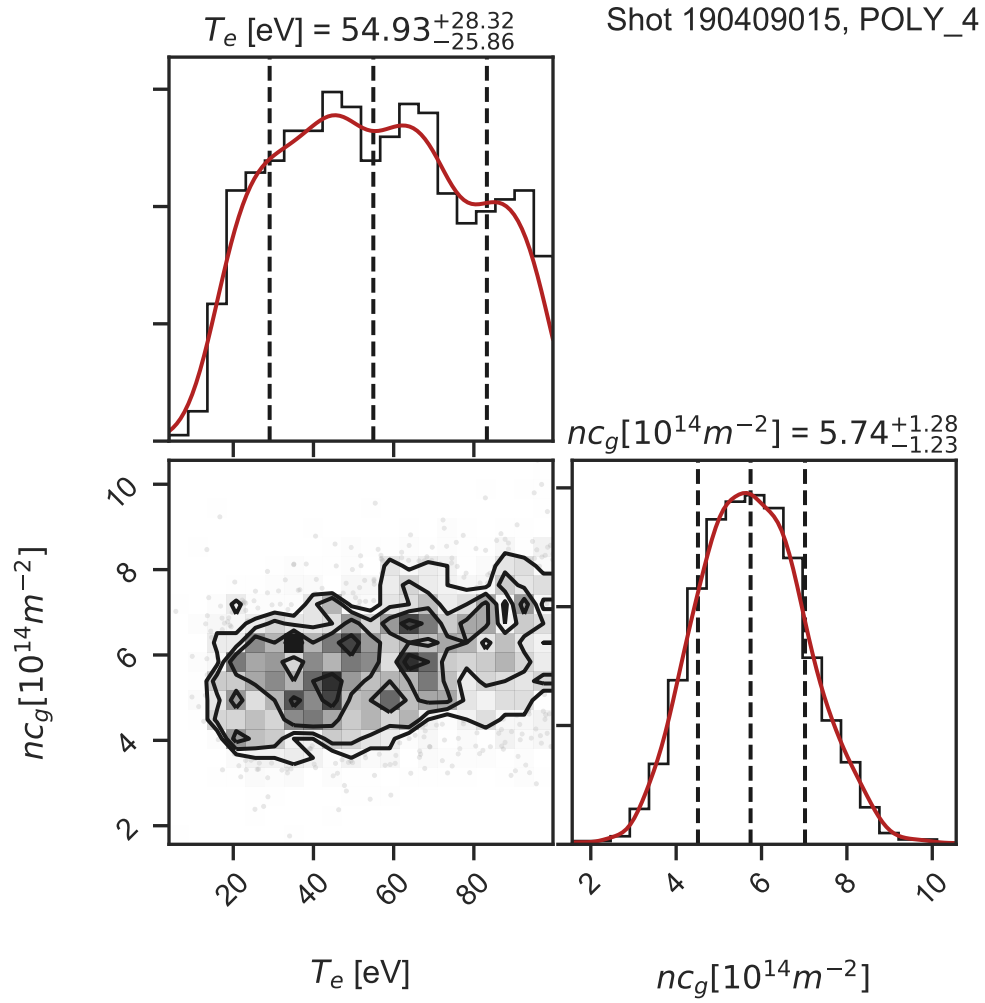


Figure D.15: Bayesian inference results on T_e and nc_{geom} for shot 190409014, polychromator 4.

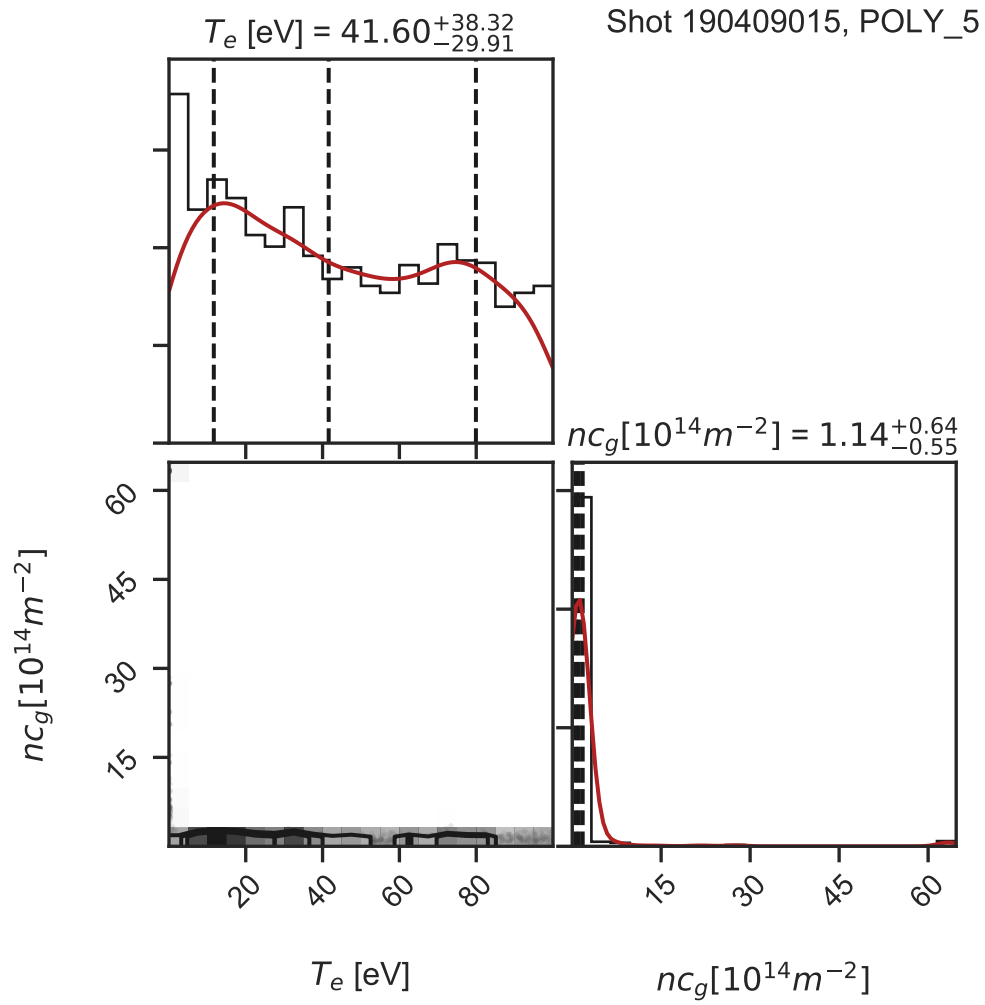


Figure D.16: Bayesian inference results on T_e and nc_{geom} for shot 190409014, polychromator 5.

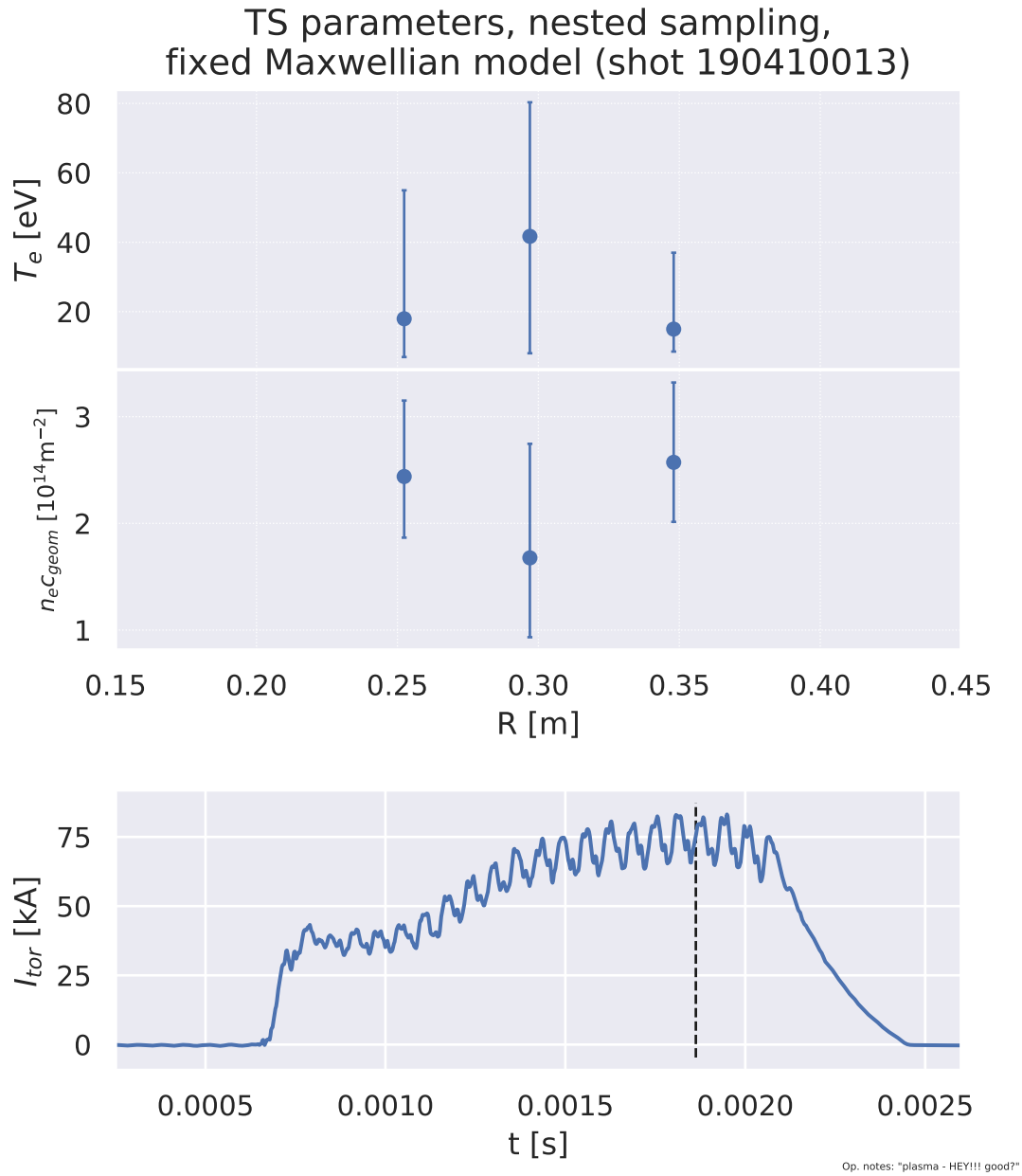


Figure D.17: TS measurements, shot 190410013.

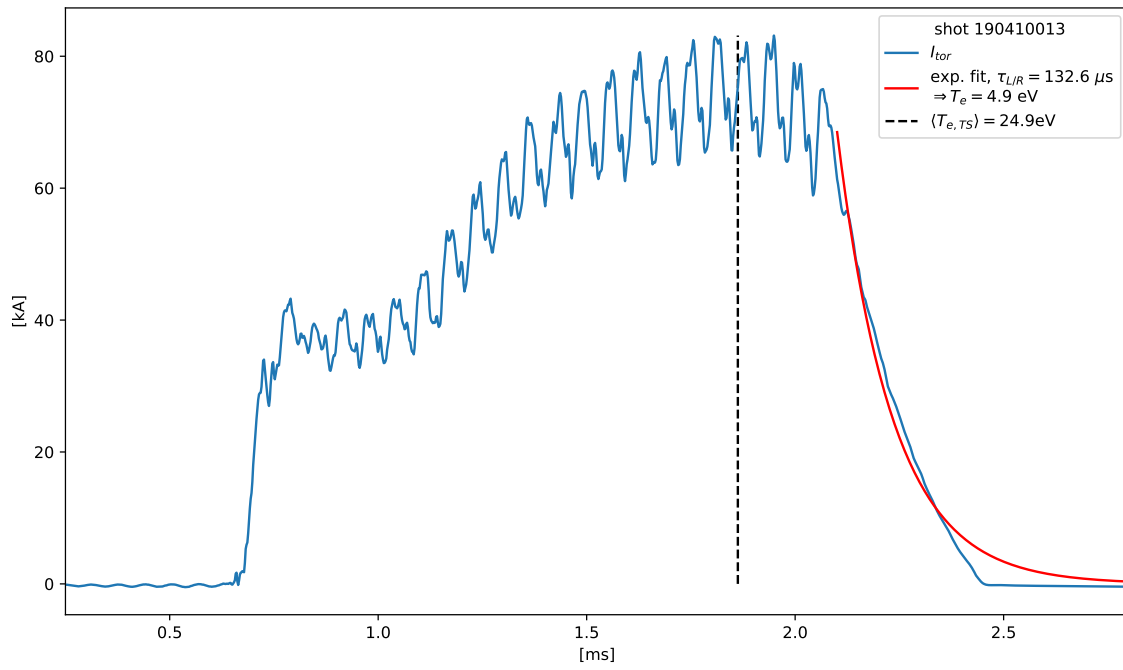


Figure D.18: TS results compared with T_e from I_{tor} decay time.

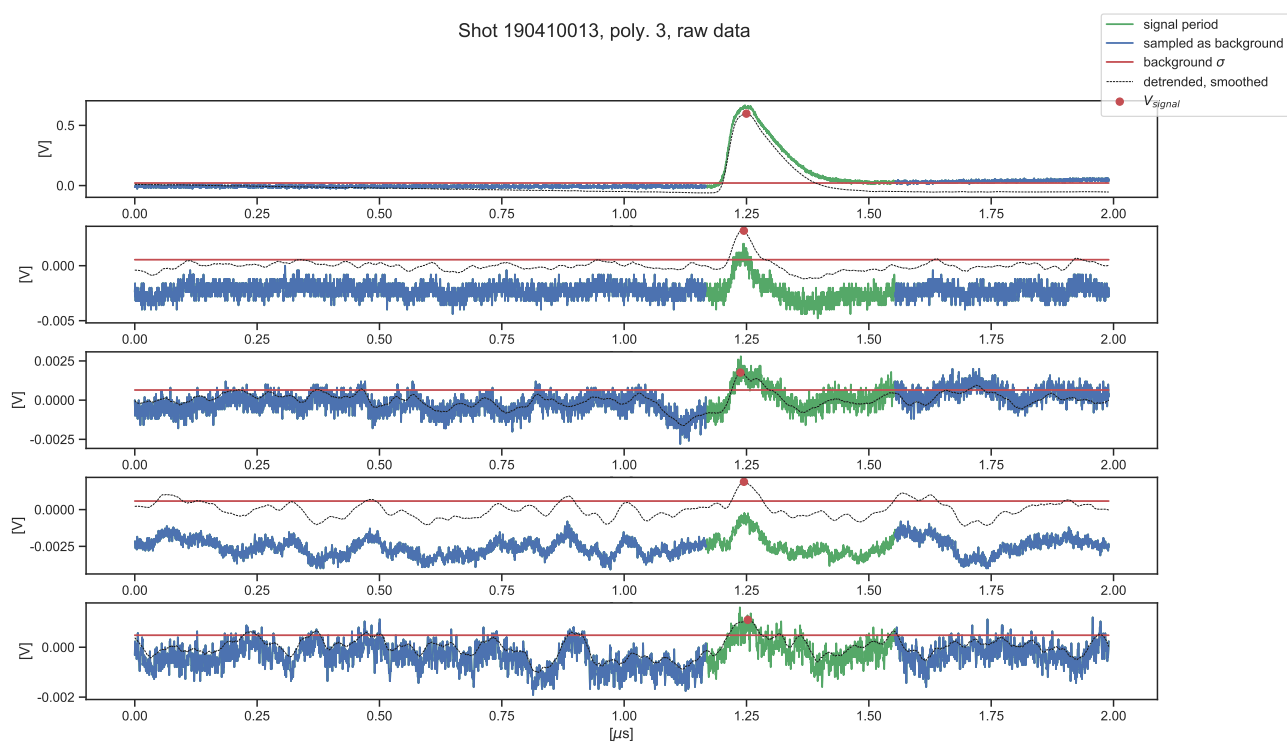


Figure D.19: Raw TS data, shot 190410013, polychromator 3.

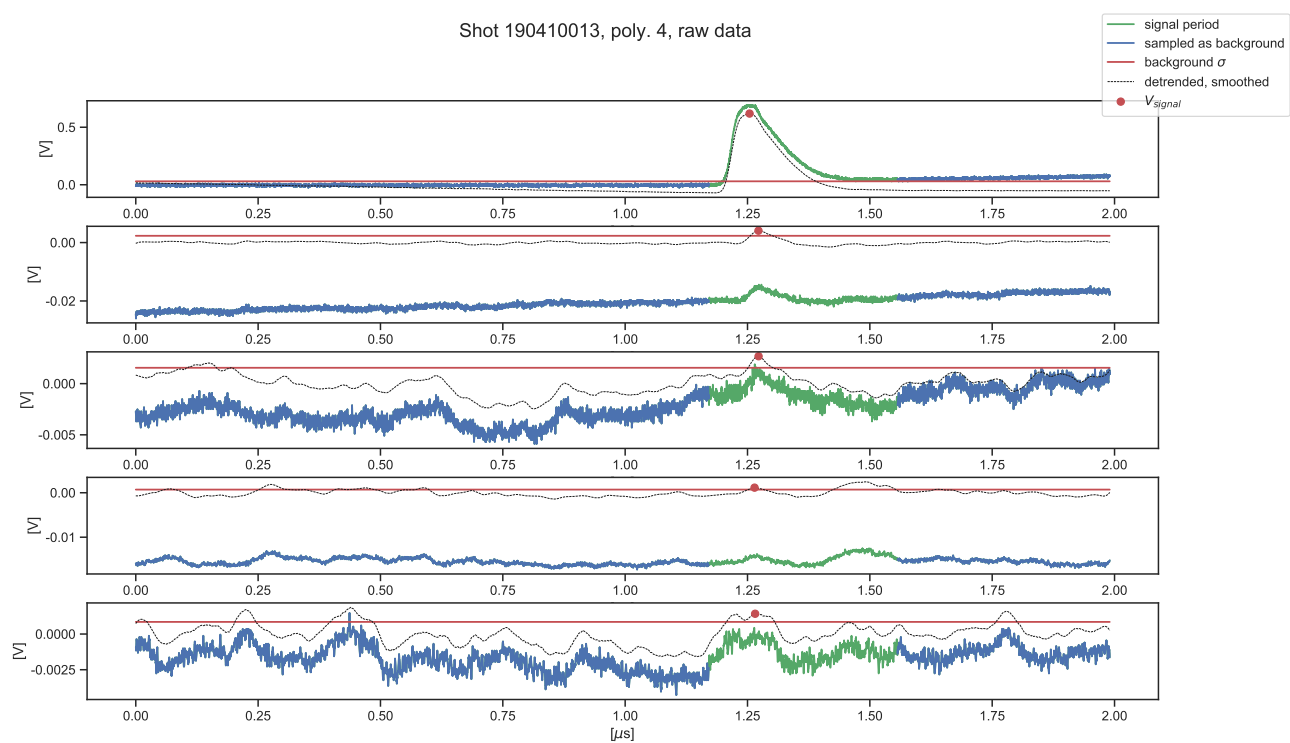


Figure D.20: Raw TS data, shot 190410013, polychromator 4.

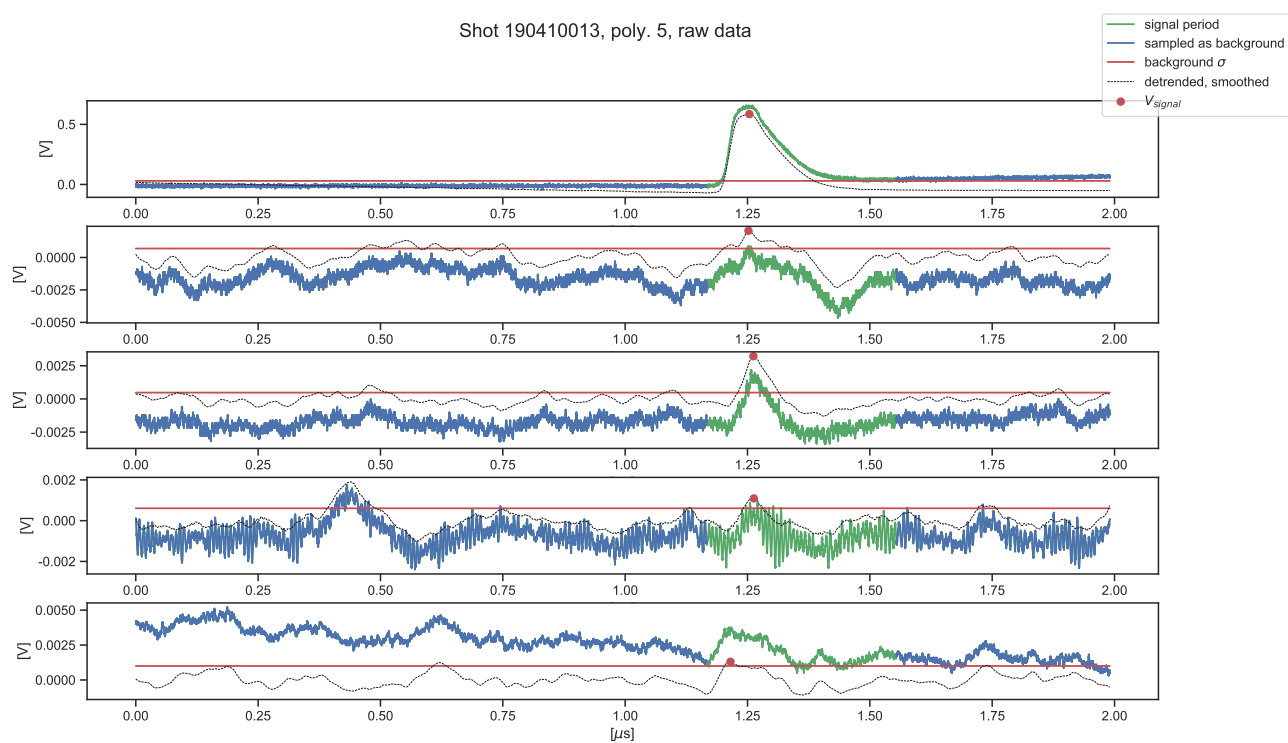


Figure D.21: Raw TS data, shot 190410013, polychromator 5.

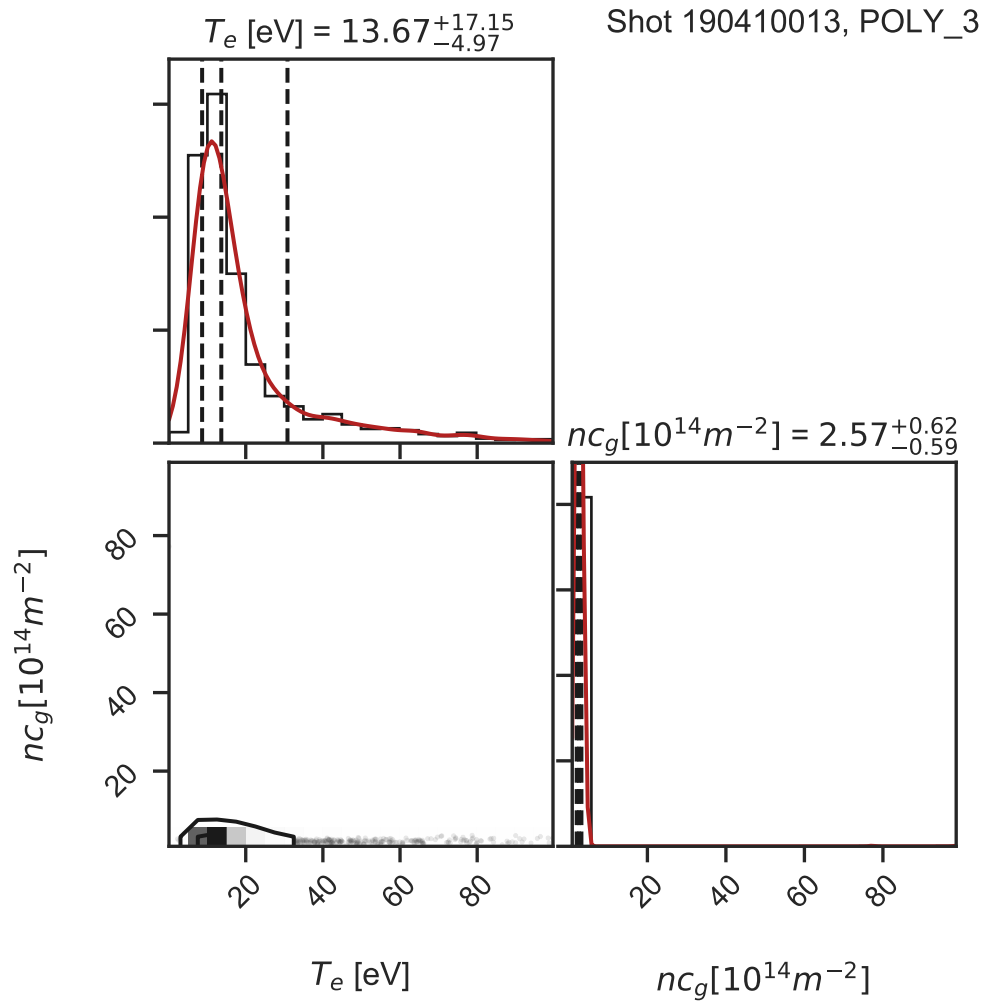


Figure D.22: Bayesian inference results on T_e and nc_{geom} for shot 190410013, polychromator 3.

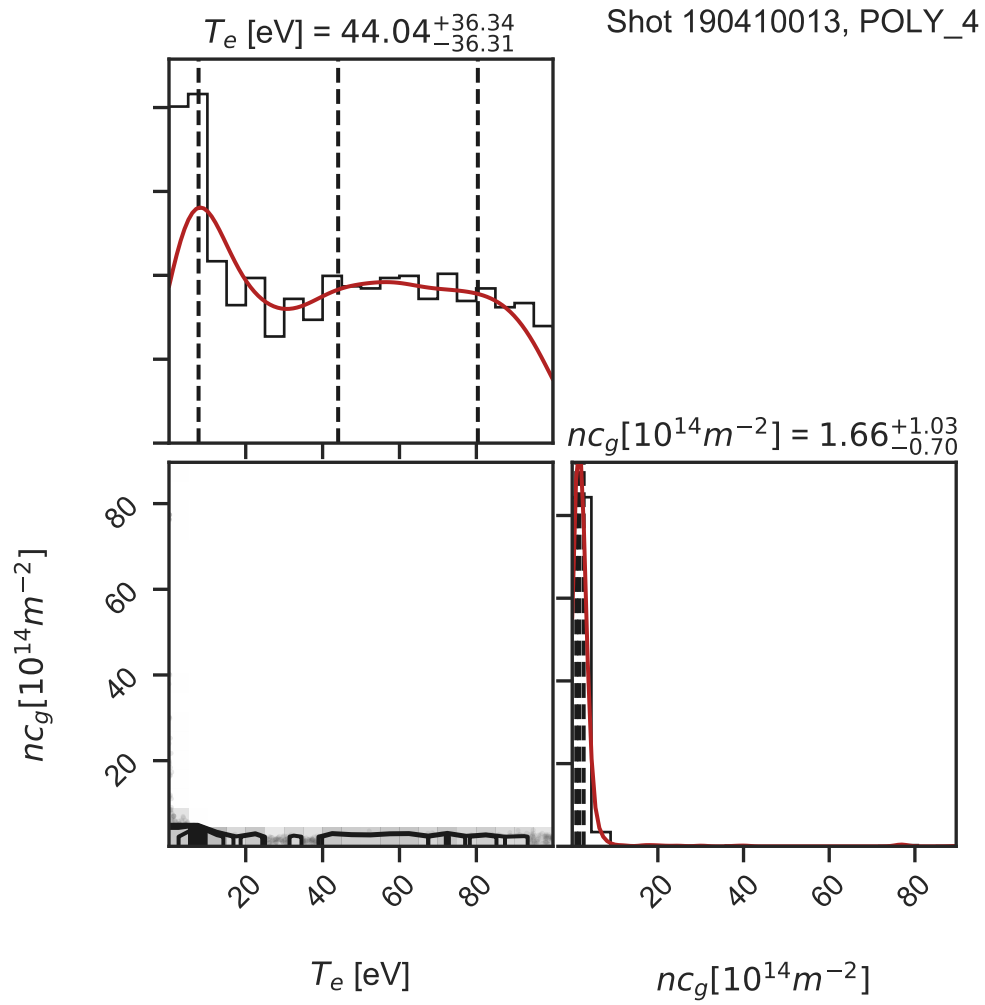


Figure D.23: Bayesian inference results on T_e and nc_{geom} for shot 190410013, polychromator 4.

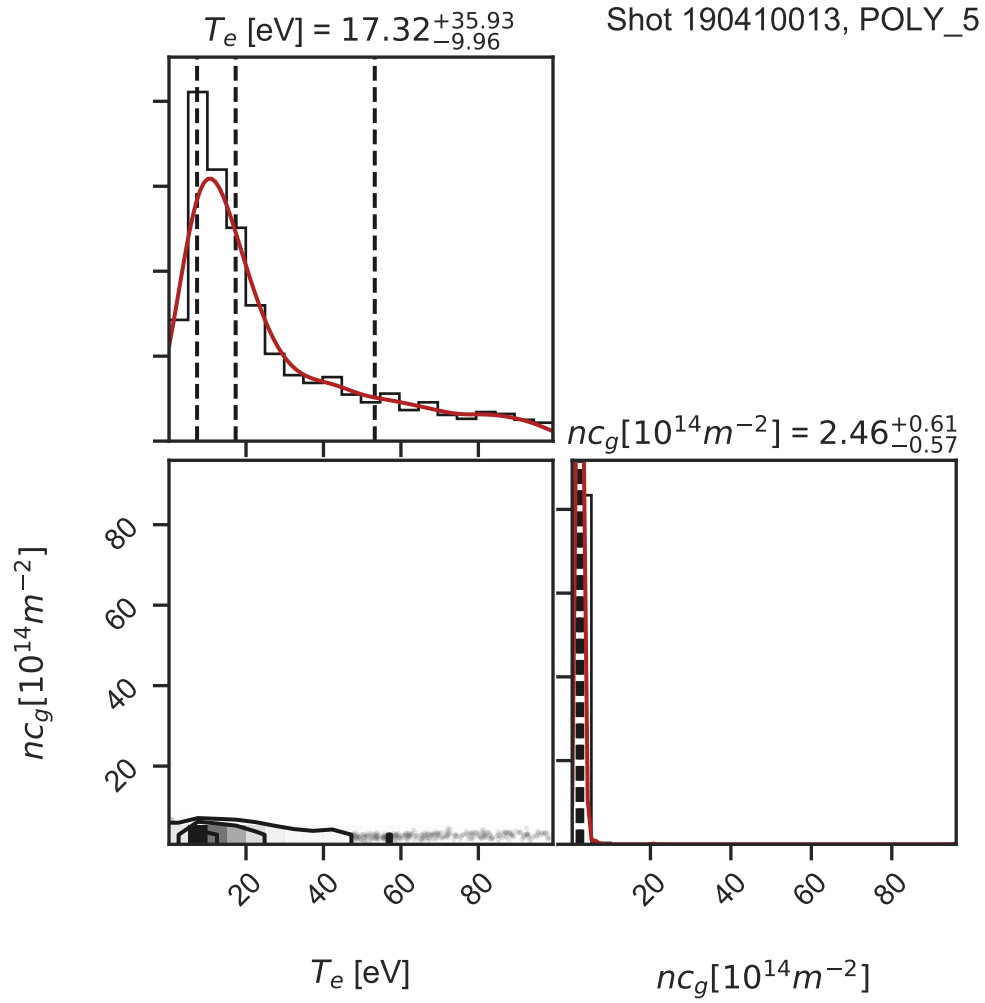


Figure D.24: Bayesian inference results on T_e and nc_{geom} for shot 190410013, polychromator 5.

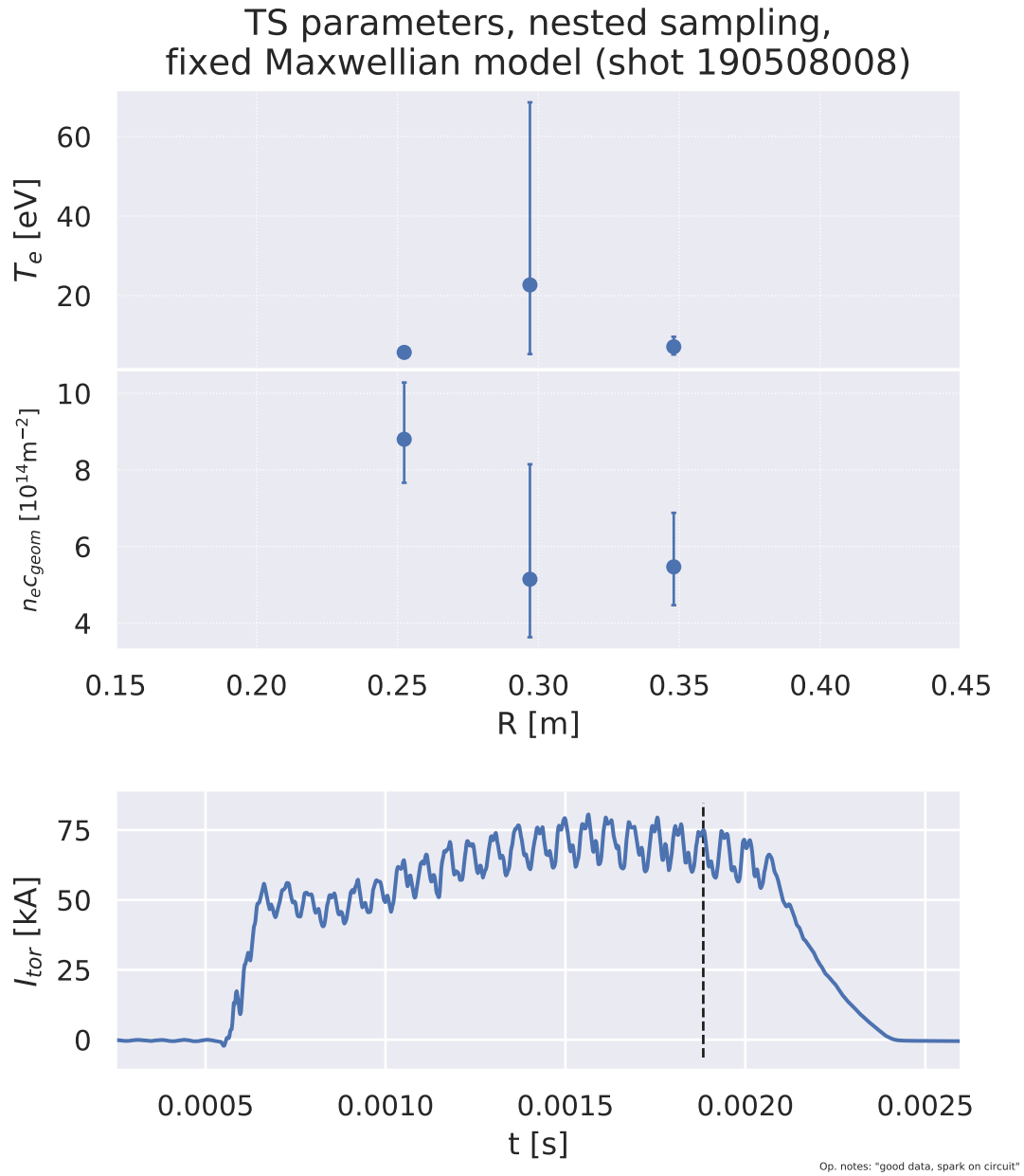


Figure D.25: TS measurements, shot 190508008.

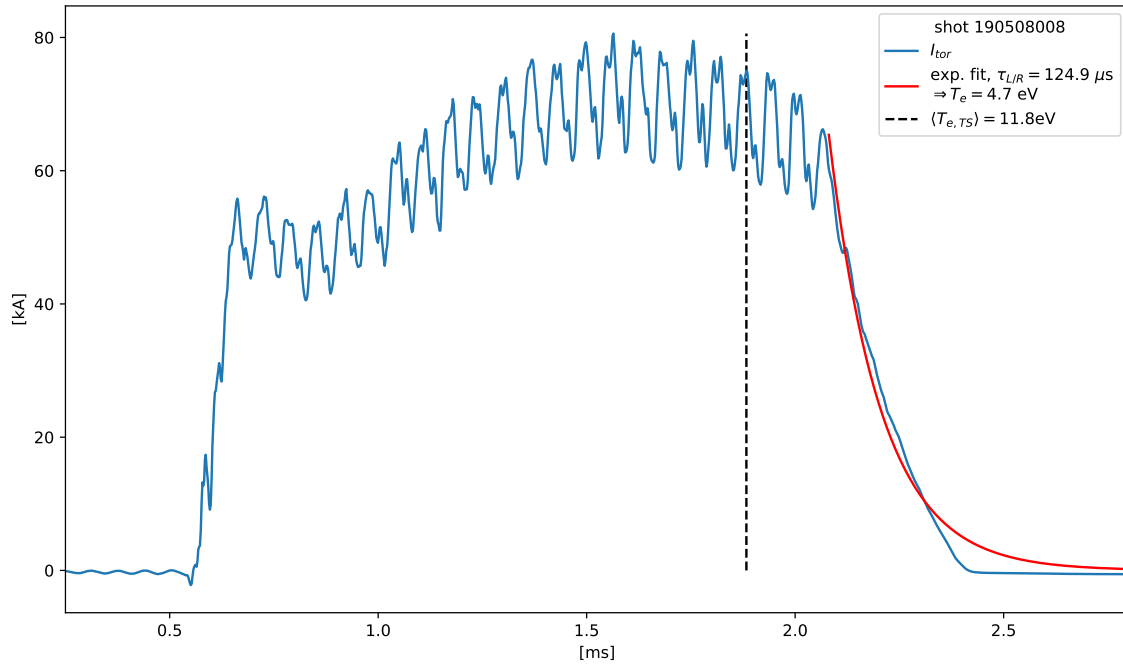


Figure D.26: TS results compared with T_e from I_{tor} decay time.

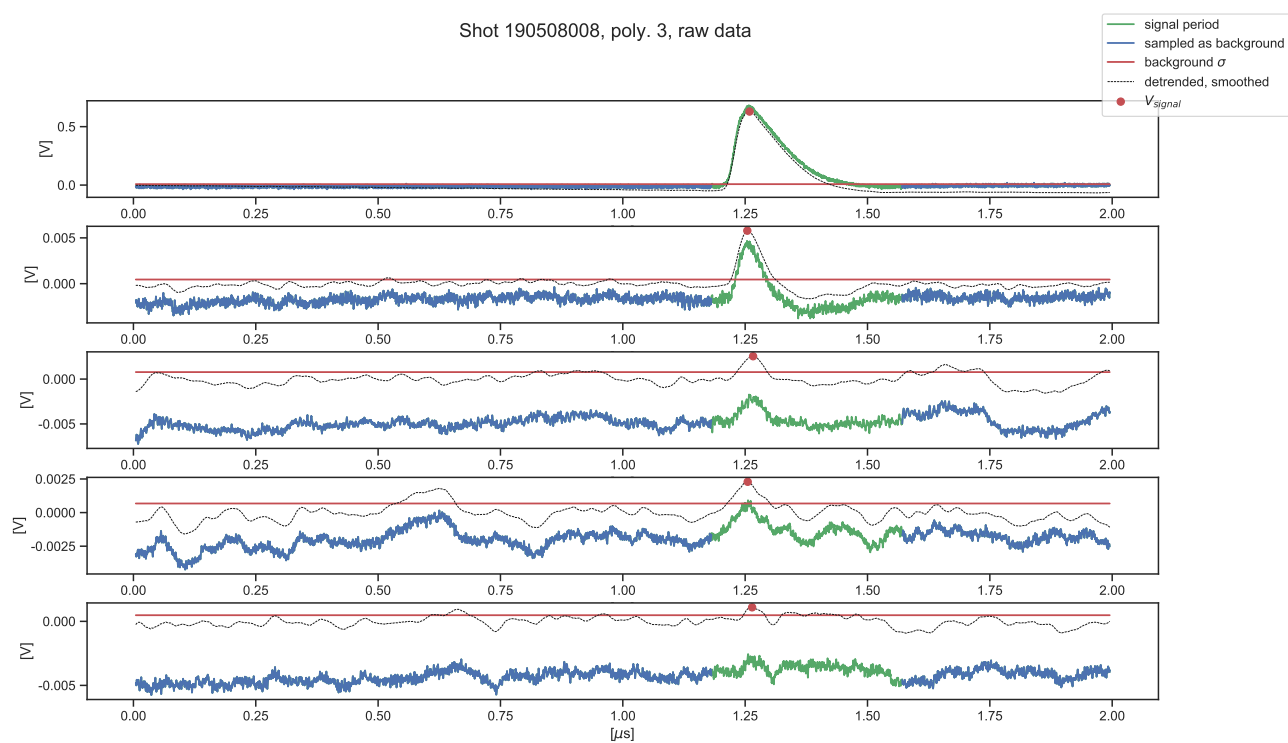


Figure D.27: Raw TS data, shot 190508008, polychromator 3.

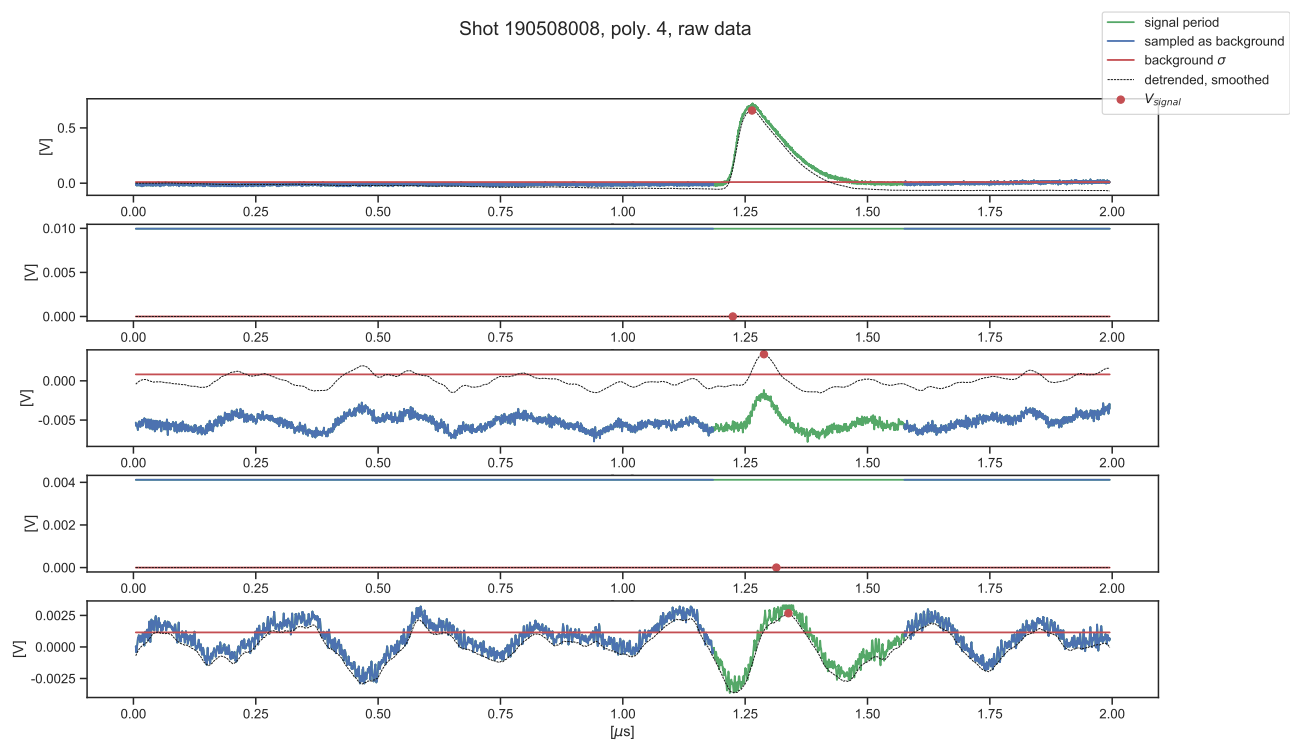


Figure D.28: Raw TS data, shot 190508008, polychromator 4.

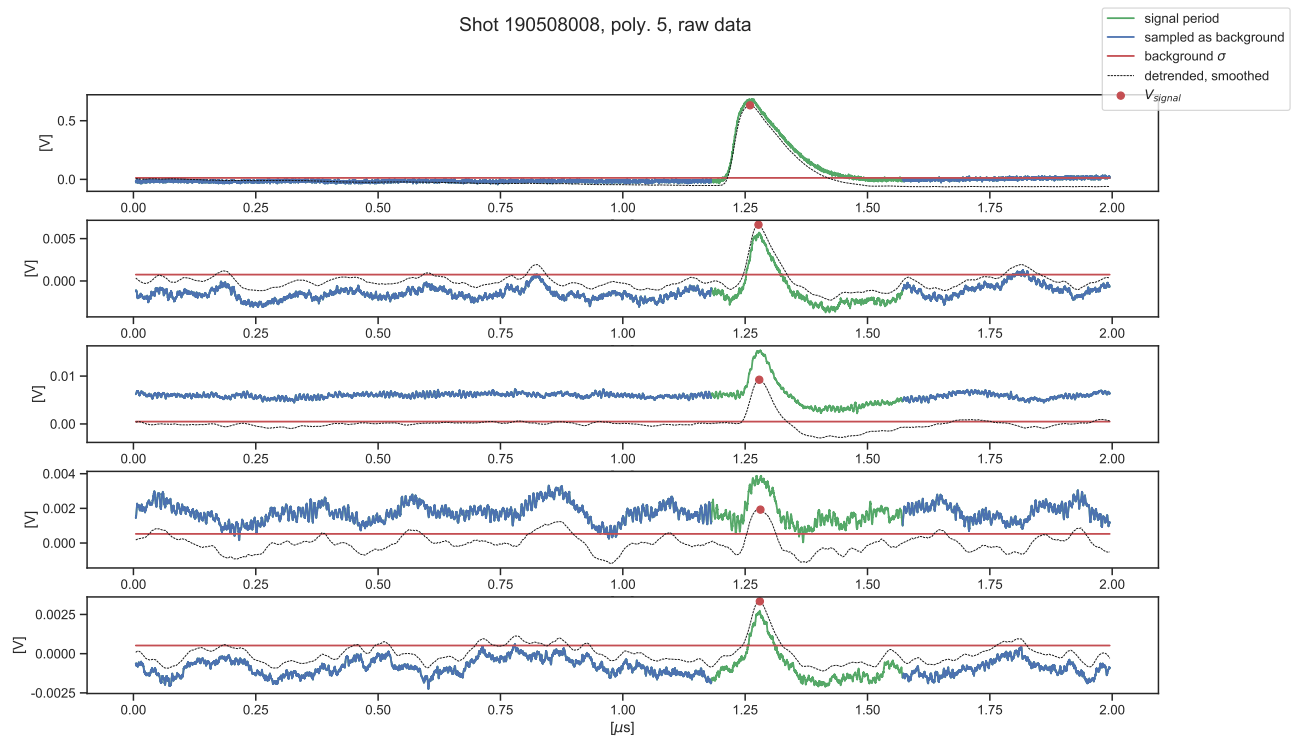


Figure D.29: Raw TS data, shot 190508008, polychromator 5.

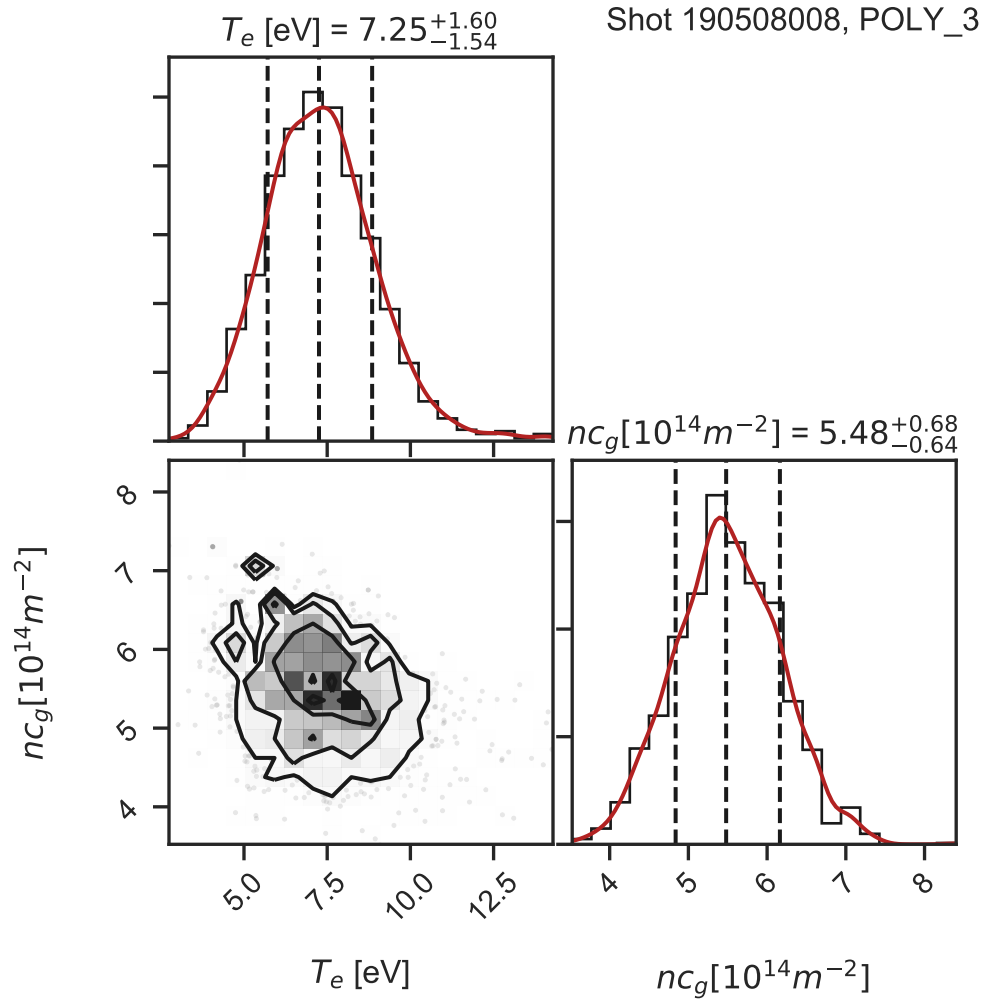


Figure D.30: Bayesian inference results on T_e and nc_{geom} for shot 190508008, polychromator 3.

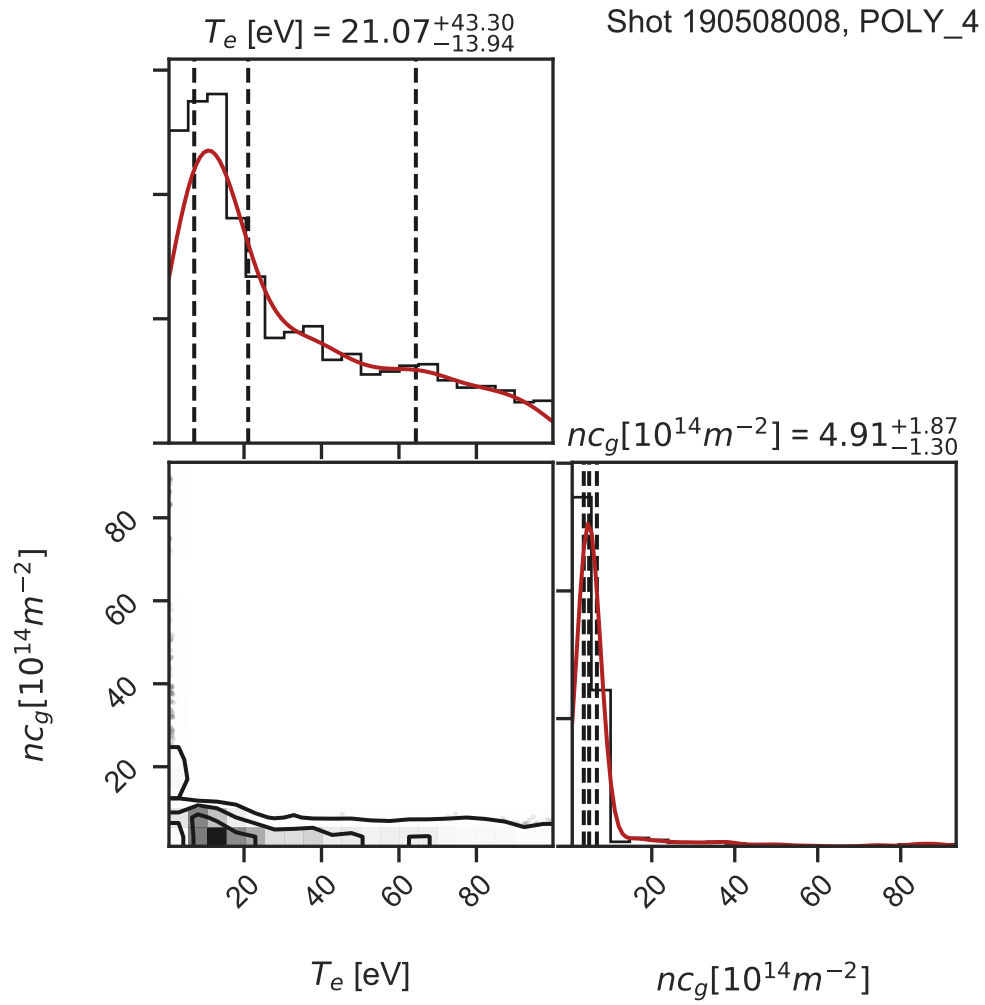


Figure D.31: Bayesian inference results on T_e and nc_{geom} for shot 190508008, polychromator 4.

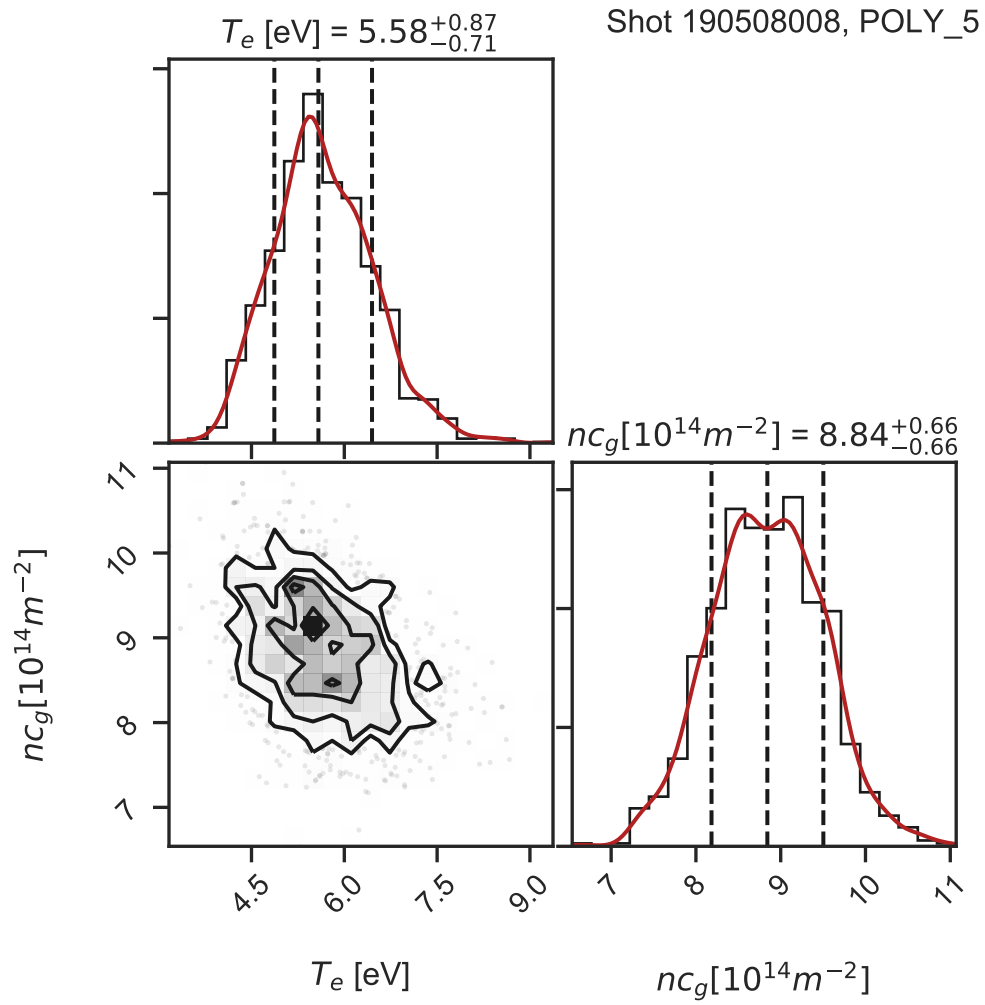


Figure D.32: Bayesian inference results on T_e and nc_{geom} for shot 190508008, polychromator 5.

Appendix E

TABULATED THOMSON SCATTERING RESULTS

1	A	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH
1	Shot	Energy	Flow	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz
154	10272009	4.065 Hz	0.348	0.029	0.254	1866.34	0.001812	14.8139	7.95156	1.607195	22.89374	16.1772	12.8031	7.06674	24.93531	3.92507	7.82412	2.51654	4.448925	25.6131	28.82525	16.96099	34.63117	5.55088	7.20532	1.87638	11.8893	15.7862	12.86205	9.812524	30.61638	26.83974	
155	10272010	4.065 Hz	0.348	0.029	0.254	-23863.46	0.001814	23.5374	11.29112	3.52567	52.47931	12.4003	8.63194	2.748802	20.09302	2.96665	2.742802	1.67846	4.182869	24.7787	21.84603	13.78675	35.43343	3.7192	3.83384	1.94891	5.584753	21.7467	17.11548	12.04799	26.83974		
156	10272011	4.065 Hz	0.348	0.029	0.254	1866.34	0.001812	14.8139	7.95156	1.607195	22.89374	16.1772	12.8031	7.06674	24.93531	3.92507	7.82412	2.51654	4.448925	25.6131	28.82525	16.96099	34.63117	5.55088	7.20532	1.87638	11.8893	15.7862	12.86205	9.812524	30.61638	26.83974	
157	10272012	4.065 Hz	0.348	0.029	0.254	-23863.46	0.001814	23.5374	11.29112	3.52567	52.47931	12.4003	8.63194	2.748802	20.09302	2.96665	2.742802	1.67846	4.182869	24.7787	21.84603	13.78675	35.43343	3.7192	3.83384	1.94891	5.584753	21.7467	17.11548	12.04799	26.83974		
158	10272013	4.065 Hz	0.348	0.029	0.254	1866.34	0.001812	14.8139	7.95156	1.607195	22.89374	16.1772	12.8031	7.06674	24.93531	3.92507	7.82412	2.51654	4.448925	25.6131	28.82525	16.96099	34.63117	5.55088	7.20532	1.87638	11.8893	15.7862	12.86205	9.812524	30.61638	26.83974	
159	10272014	4.065 Hz	0.348	0.029	0.254	-23863.46	0.001814	23.5374	11.29112	3.52567	52.47931	12.4003	8.63194	2.748802	20.09302	2.96665	2.742802	1.67846	4.182869	24.7787	21.84603	13.78675	35.43343	3.7192	3.83384	1.94891	5.584753	21.7467	17.11548	12.04799	26.83974		
160	10272015	4.065 Hz	0.348	0.029	0.254	1866.34	0.001812	14.8139	7.95156	1.607195	22.89374	16.1772	12.8031	7.06674	24.93531	3.92507	7.82412	2.51654	4.448925	25.6131	28.82525	16.96099	34.63117	5.55088	7.20532	1.87638	11.8893	15.7862	12.86205	9.812524	30.61638	26.83974	
161	10272016	4.065 Hz	0.348	0.029	0.254	-23863.46	0.001814	23.5374	11.29112	3.52567	52.47931	12.4003	8.63194	2.748802	20.09302	2.96665	2.742802	1.67846	4.182869	24.7787	21.84603	13.78675	35.43343	3.7192	3.83384	1.94891	5.584753	21.7467	17.11548	12.04799	26.83974		
162	10272017	4.257 Hz	0.348	0.029	0.254	-45458.7	0.001815	22.526	12.8847	5.61327	46.0923	14.415	11.83725	7.9667583	35.67723	8.2686	4.11542	2.60576	6.2614	19.2595	37.0629	11.77109	26.14468	5.9645	5.621925	2.63623	8.919597	19.1526	14.8928	11.20857	26.00101		
163	10272018	4.257 Hz	0.348	0.029	0.254	12.0385	0.001815	22.526	12.8847	5.61327	46.0923	14.415	11.83725	7.9667583	35.67723	8.2686	4.11542	2.60576	6.2614	19.2595	37.0629	11.77109	26.14468	5.9645	5.621925	2.63623	8.919597	19.1526	14.8928	11.20857	26.00101		
164	10272019	4.135 Hz	0.348	0.029	0.254	-43222.01	0.0004605	48.4655	50.14623	10.14296	82.3539	5.31876	3.505321	1.364075	6.7826626	13.9360	5.100047	2.182693	34.30146	10.8713	7.817896	4.972961	16.840274	54.6021	57.6723	23.74506	84.00089	3.42562	2.47938	1.145294	5.178269		
165	10272020	4.124 Hz	0.348	0.029	0.254	-43222.01	0.0004605	48.4655	50.14623	10.14296	82.3539	5.31876	3.505321	1.364075	6.7826626	13.9360	5.100047	2.182693	34.30146	10.8713	7.817896	4.972961	16.840274	54.6021	57.6723	23.74506	84.00089	3.42562	2.47938	1.145294	5.178269		
166	10272021	4.124 Hz	0.348	0.029	0.254	12.0385	0.0004605	48.4655	50.14623	10.14296	82.3539	5.31876	3.505321	1.364075	6.7826626	13.9360	5.100047	2.182693	34.30146	10.8713	7.817896	4.972961	16.840274	54.6021	57.6723	23.74506	84.00089	3.42562	2.47938	1.145294	5.178269		
167	10272026	V	0.348	0.029	0.254	-100.4882	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
168	10272027	V	0.348	0.029	0.254	-124.5497	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
169	10272028	4.568 Hz	0.348	0.029	0.254	12.0385	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
170	10272029	4.568 Hz	0.348	0.029	0.254	-124.5497	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
171	10272030	4.568 Hz	0.348	0.029	0.254	-124.5497	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
172	10272031	###	0.348	0.029	0.254	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	###	
173	10272032	4.568 Hz	0.348	0.029	0.254	12.0385	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
174	10272033	4.568 Hz	0.348	0.029	0.254	-124.5497	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
175	10272034	4.568 Hz	0.348	0.029	0.254	-124.5497	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
176	10272035	4.568 Hz	0.348	0.029	0.254	12.0385	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
177	10272036	4.568 Hz	0.348	0.029	0.254	-124.5497	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
178	10272037	4.568 Hz	0.348	0.029	0.254	12.0385	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
179	10272038	4.568 Hz	0.348	0.029	0.254	-124.5497	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
180	10272039	4.568 Hz	0.348	0.029	0.254	12.0385	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
181	10272040	4.568 Hz	0.348	0.029	0.254	-124.5497	0.0020102	23.4567	13.003	5.04057	48.87936	13.0797	10.43736	8.6009123	18.75312	41.468	38.59389	4.881082	79.72245	5.03223	3.715651	2.081164	6.4680201	45.7029	45.59767	6.620026	78.4907	3.7185	2.867909	1.394022	4.792364		
182	10272041	4.026 Hz	0.348	0.029	0.254	4068.3	0.001368	24.71	12.6892	9.39407	53.80074	16.2891	11.26727	7.6197825	25.70583	7.9914	4.67542	2.306575	3.82306	13.2522	10.5886	17.7487	34.11085	50.4879	15.2526	8.45376	21.54662	8.45376	21.54662	8.45376	21.54662		
183	10272042	4.026 Hz	0.348	0.029	0.254	-10.76639	0.001368	24.71	12.6892	9.39407	53.80074	16.2891	11.26727	7.6197825	25.70583	7.9914	4.67542	2.306575	3.82306	13.2522	10.5886	17.7487	34.11085	50.4879	15.2526	8.45376	21.54662	8.45376	21.54662	8.45376	21.54662		
184	10272043	4.026 Hz	0.348	0.029	0.254	10.76639	0.001368	24.71	12.6892	9.39407	53.80074	16.2891	11.26727	7.6197825	25.70583	7.9914	4.67542	2.															

1	A	O	E	G	H	N	O	Q	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF												
1	Shot	Energy	Flow	F2	rad/s	rad/s	rad/s	rad/s	rad/s	rad/s	rad/s	rad/s	rad/s	rad/s	rad/s	rad/s	rad/s	rad/s	rad/s	rad/s												
306	100328015	4.426	0.348	0.029	0.2524	-5213.75	0.0018128	46.3642	34.8526	17.7006	77.997	5.5584	5.245726	1.831363	6.655074	5.780	54.48063	24.1567	83.002	2.88779	2.779572	1.659915	4.038074	50.2442	40.0478	1.80077	80.2032	4.31564	4.235408	1.285472	7.776385	
307	100328016	4.272	0.348	0.029	0.2524	-5239.88	0.0018119	46.3628	34.7207	17.56578	61.77503	5.23477	5.555336	1.670005	6.168118	5.1324	51.3406	20.1409	80.0799	4.1664	4.238312	2.885245	5.931911	45.7793	41.6660	1.74508	77.7629	9.74612	5.145886	1.293679	1.336847	
308	100328017	4.461	0.348	0.029	0.2524	-5231.19	0.0018125	46.3705	34.8837	17.6965	6.9968	6.8151	5.220371	1.553283	6.424347	5.09013	50.8081	9.7423	84.8424	4.007729	1.726662	14.5778	8.50595	9.5464	24.9344	7.68062	4.31564	4.235408	1.285472	7.776385		
309	100328018	4.220	0.348	0.029	0.2524	-5235.88	0.0018122	46.3678	34.79273	17.57837	52.05978	5.23477	5.555336	1.670005	6.168118	5.1324	51.3406	20.1409	80.0799	4.1664	4.238312	2.885245	5.931911	45.7793	41.6660	1.74508	77.7629	9.74612	5.145886	1.293679	1.336847	
310	100328019	4.687	0.348	0.029	0.2524	-5239.88	0.0018122	46.3678	34.79273	17.57837	52.05978	5.23477	5.555336	1.670005	6.168118	5.1324	51.3406	20.1409	80.0799	4.1664	4.238312	2.885245	5.931911	45.7793	41.6660	1.74508	77.7629	9.74612	5.145886	1.293679	1.336847	
311	100328020	4.584	0.348	0.029	0.2524	-5239.88	0.0018122	46.3678	34.79273	17.57837	52.05978	5.23477	5.555336	1.670005	6.168118	5.1324	51.3406	20.1409	80.0799	4.1664	4.238312	2.885245	5.931911	45.7793	41.6660	1.74508	77.7629	9.74612	5.145886	1.293679	1.336847	
312	100328021	3.974	0.348	0.029	0.2524	-5239.88	0.0018122	46.3678	34.79273	17.57837	52.05978	5.23477	5.555336	1.670005	6.168118	5.1324	51.3406	20.1409	80.0799	4.1664	4.238312	2.885245	5.931911	45.7793	41.6660	1.74508	77.7629	9.74612	5.145886	1.293679	1.336847	
313	100328022	5.255	0.348	0.029	0.2524	-5239.88	0.0018122	46.3678	34.79273	17.57837	52.05978	5.23477	5.555336	1.670005	6.168118	5.1324	51.3406	20.1409	80.0799	4.1664	4.238312	2.885245	5.931911	45.7793	41.6660	1.74508	77.7629	9.74612	5.145886	1.293679	1.336847	
314	100328023	4.505	0.348	0.029	0.2524	-5239.88	0.0018122	46.3678	34.79273	17.57837	52.05978	5.23477	5.555336	1.670005	6.168118	5.1324	51.3406	20.1409	80.0799	4.1664	4.238312	2.885245	5.931911	45.7793	41.6660	1.74508	77.7629	9.74612	5.145886	1.293679	1.336847	
315	100328024		V	0.348	0.029	0.2524	-5239.88	0.0018122	46.3678	34.79273	17.57837	52.05978	5.23477	5.555336	1.670005	6.168118	5.1324	51.3406	20.1409	80.0799	4.1664	4.238312	2.885245	5.931911	45.7793	41.6660	1.74508	77.7629	9.74612	5.145886	1.293679	1.336847
316	100340007		V	0.348	0.029	0.2524	-53.4978																									
317	100340008		V	0.348	0.029	0.2524	-38.4678																									
318	100340009		V	0.348	0.029	0.2524	-38.1130																									
319	100340010		V	0.348	0.029	0.2524	-38.025																									
320	100340011	4.932	0.348	0.029	0.2524	-61045.27	0.0018634	45.2334	41.4121	24.17417	67.75656	12.1205	11.74881	8.651609	14.14205	32.9907	24.47349	6.915609	83.1973	7.60954	6.29447	4.531566	4.843145	23.5209	12.00778	2.487106	51.10384	6.96518	5.504612	1.5971824	1.1580396	
321	100340012	4.854	0.348	0.029	0.2524	-61045.27	0.0018635	45.2334	41.4121	24.17417	67.75656	12.1205	11.74881	8.651609	14.14205	32.9907	24.47349	6.915609	83.1973	7.60954	6.29447	4.531566	4.843145	23.5209	12.00778	2.487106	51.10384	6.96518	5.504612	1.5971824	1.1580396	
322	100340013	4.854	0.348	0.029	0.2524	-61045.27	0.0018635	45.2334	41.4121	24.17417	67.75656	12.1205	11.74881	8.651609	14.14205	32.9907	24.47349	6.915609	83.1973	7.60954	6.29447	4.531566	4.843145	23.5209	12.00778	2.487106	51.10384	6.96518	5.504612	1.5971824	1.1580396	
323	100340014	5.560	0.348	0.029	0.2524	-61045.27	0.0018635	45.2334	41.4121	24.17417	67.75656	12.1205	11.74881	8.651609	14.14205	32.9907	24.47349	6.915609	83.1973	7.60954	6.29447	4.531566	4.843145	23.5209	12.00778	2.487106	51.10384	6.96518	5.504612	1.5971824	1.1580396	
324	100340015	5.484	0.348	0.029	0.2524	-61045.27	0.0018635	45.2334	41.4121	24.17417	67.75656	12.1205	11.74881	8.651609	14.14205	32.9907	24.47349	6.915609	83.1973	7.60954	6.29447	4.531566	4.843145	23.5209	12.00778	2.487106	51.10384	6.96518	5.504612	1.5971824	1.1580396	
325	100340016	4.512	0.348	0.029	0.2524	-61045.27	0.0018635	45.2334	41.4121	24.17417	67.75656	12.1205	11.74881	8.651609	14.14205	32.9907	24.47349	6.915609	83.1973	7.60954	6.29447	4.531566	4.843145	23.5209	12.00778	2.487106	51.10384	6.96518	5.504612	1.5971824	1.1580396	
326	100340017	5.460	0.348	0.029	0.2524	-61045.27	0.0018635	45.2334	41.4121	24.17417	67.75656	12.1205	11.74881	8.651609	14.14205	32.9907	24.47349	6.915609	83.1973	7.60954	6.29447	4.531566	4.843145	23.5209	12.00778	2.487106	51.10384	6.96518	5.504612	1.5971824	1.1580396	
327	100340018		V	0.348	0.029	0.2524	-61.7724																									
328	100340019	5.460	0.348	0.029	0.2524	-61.7724	0.0018635	45.2334	41.4121	24.17417	67.75656	12.1205	11.74881	8.651609	14.14205	32.9907	24.47349	6.915609	83.1973	7.60954	6.29447	4.531566	4.843145	23.5209	12.00778	2.487106	51.10384	6.96518	5.504612	1.5971824	1.1580396	
329	100340020	5.460	0.348	0.029	0.2524	-61.7724	0.0018635	45.2334	41.4121	24.17417	67.75656	12.1205	11.74881	8.651609	14.14205	32.9907	24.47349	6.915609	83.1973	7.60954	6.29447	4.531566	4.843145	23.5209	12.00778	2.487106	51.10384	6.96518	5.504612	1.5971824	1.1580396	
330	100340021	4.665	0.348	0.029	0.2524	-7036.57	0.0018633	13.7666	3.89594	4.29842	19.95568	4.82652	3.14048	2.4511669	4.7952384	4.13871	42.8707	6.13009	4.16041	2.975958	5.865408	1.35448	4.62473	57.03348	28.68494	83.7767	1.25885	1.168205	0.637445	1.807312		
331	100340022	4.665	0.348	0.029	0.2524	-7036.57	0.0018633	13.7666	3.89594	4.29842	19.95568	4.82652	3.14048	2.4511669	4.7952384	4.13871	42.8707	6.13009	4.16041	2.975958	5.865408	1.35448	4.62473	57.03348	28.68494	83.7767	1.25885	1.168205	0.637445	1.807312		
332	100340023	4.665	0.348	0.029	0.2524	-7036.57	0.0018633	13.7666	3.89594	4.29842	19.95568	4.82652	3.14048	2.4511669	4.7952384	4.13871	42.8707	6.13009	4.16041	2.975958	5.865408	1.35448	4.62473	57.03348	28.68494	83.7767	1.25885	1.168205	0.637445	1.807312		
333	100340024	5.372	0.348	0.029	0.2524	-7066.81	0.0018633	13.7666	3.89594	4.29842	19.95568	4.82652	3.14048	2.4511669	4.7952384	4.13871	42.8707	6.13009	4.16041	2.975958	5.865408	1.35448	4.62473	57.03348	28.68494	83.7767	1.25885	1.168205	0.637445	1.807312		
334	100340025	5.372	0.348	0.029	0.2524	-7066.81	0.0018633	13.7666	3.89594	4.29842	19.95568	4.82652	3.14048	2.4511669	4.7952384	4.13871	42.8707	6.13009	4.16041	2.975958	5.865408	1.35448	4.62473	57.03348	28.68494	83.7767	1.25885	1.168205	0.637445	1.807312		
335	100340026	5.372	0.348	0.029	0.2524	-7066.81	0.0018633	13.7666	3.89594	4.29842	19.95568	4.82652	3.14048	2.4511669	4.7952384	4.13871	42.8707	6.13009	4.16041	2.975958	5.865408	1.35448	4.62473	57.03348	28.68494	83.7767	1.25885	1.168205	0.637445	1.807312		
336	100340027	5.372	0.348	0.029	0.2524	-7066.81	0.0018633	13.7666	3.89594	4.29842	19.95568	4.82652	3.14048	2.4511669	4.7952384	4.13871	42.8707	6.13009	4.16041	2.975958	5.865408	1.35448	4.62473	57.03348	28.68494	83.7767	1.25885	1.168205	0.637445	1.807312		
337	100340028	5.372	0.348	0.029	0.2524	-7066.81	0.0018633	13.7666	3.89594	4.29842	19.95568	4.82652	3.14048	2.4511669	4.7952384	4.13871	42.8707	6.13009	4.16041	2.975958	5.865408	1.35448	4.62473	57.03348	28.68494	83.7767	1.25885	1.168205	0.637445	1.807312		
338	100340029	5.372	0.348	0.029	0.2524	-7066.81	0.0018633	13.7666	3.89594	4.29842	19.95568	4.82652	3.14048	2.4511669	4.7952384	4.13871	42.8707	6.13009	4.16041	2.975958	5.865408	1.35448	4.62473	57.03348	28.68494	83.7767	1.25885	1.168205	0.637445	1.807312		
339	100340030	5.372	0.348	0.029	0.2524	-7066.81	0.0018633	13.7666	3.89594	4.29842	19.95568	4.82652	3.14048	2.4511669	4.7952384																	