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**EFFECTS OF TAXATION ON  
HOUSEHOLD PORTFOLIO CHOICE  
AND RISK TAKING**

by

Joongwoo Ahn

A dissertation submitted in partial  
fulfillment of the requirements for the  
degree of

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Chairperson of Supervisory Committee

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University of Washington

Abstract

**EFFECTS OF TAXATION ON  
HOUSEHOLD PORTFOLIO CHOICE  
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by Joongwoo Ahn

Chairperson of the Supervisory Committee: Professor Neil Bruce  
Department of Economics

Tax effects on household portfolio behavior have been a controversial issue. The intuition behind this controversy is that taxes reduce not only expected return but also risk. Since expected return and risk have opposite effects on asset demand, the net tax effects would depend on the relative sizes of these conflicting effects. Therefore, this paper aims at estimating the relative sizes of these conflicting effects. This paper also attempts to estimate a direct empirical relationship between taxes and household risk taking.

First, this paper derives a system of portfolio share equations through a standard utility maximization with respect to mean and variance, utilizing a homothetic utility function. Then, the share equations are used to estimate utility parameters based on historical returns and flow of fund data for household sector. Then, using the estimated utility parameters, this paper

calculates the relative elasticities of asset demands with respect to expected return and risk, and finally tax effects on share demands.

Second, this paper develops a direct empirical model on portfolio risk in parallel to one of the benchmark models by Feldstein(1976). This paper estimates an overall measure of the household portfolio risk using the historical variance-covariances of asset returns and household-specific portfolio compositions. Then, this paper estimates tax effects on portfolio risk using the 1983 Survey of Consumer Finances (SCF) data.

- 1) The results of the estimated portfolio choice model suggest that taxation would encourage household to invest in equity assets and tax-favored assets. On the other hand, taxation would discourage household to invest in corporate bonds, federal bonds, and bank deposits. These results are consistent with the Domar and Musgrave proposition that taxation would encourage investors to invest in riskier assets.
- 2) The estimated portfolio risk model suggests that an increase in federal income taxation would encourage households to take more portfolio risk regardless of individual risk preferences. However, the sensitivity of household risk taking in response to a tax change increases as the household risk aversion increases. It implies that marginal disutility of risk is greater for risk-averse households than for risk-love households.

Overall, the empirical evidence suggests that taxation would encourage the US household to invest more in riskier assets. It also suggests that taxation would induce US households into holding riskier portfolio.

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## DEDICATION

I would like to dedicate this dissertation to my dear mother, Keum Joo Choi, my father, Soo Hwan Ahn, my lovely wife, Debbie, my children, Joonhae, Jooneil, and my yet-to-be-borns.

## Chapter 1

### Introduction

The issue of the tax effects on portfolio choice and risk taking has attracted considerable attention from economists because of its implications for macroeconomic variables such as interest rates and investment. Thus, understanding how income taxes affect household portfolio choice and risk taking is very important for evaluating tax policies and assessing the funds to be available for capital formation.

The formal discussion of this issue was initiated by an original study by Domar and Musgrave (1944). In 1944, Domar and Musgrave proposed in their seminal paper that a proportional taxation with full loss offset would encourage investment in risky assets. The intuition behind this famous Domar and Musgrave hypothesis is that, under full-loss-offset taxation, the government shares proportionately in the gains and losses to risky investment. This risk-sharing leaves the value of risk unchanged but the amount of risk borne by the investor for a given portfolio reduced. Thus, an investor is likely to

increase the share of risky assets. This rather surprising proposition has created a considerable debate since then.

Modern discussion of this proposition continued in the expected utility framework of von Neumann-Morgenstern by Tobin (1958), Hall (1960), Mossin (1968), Stiglitz (1969), and Flemming (1971). These expected utility portfolio choice models have greatly enhanced our understanding of the rather intuitive Domar and Musgrave proposition. Nevertheless, these models were not able to produce an unambiguous conclusion on the effects of taxes on portfolio choice without special assumptions on the properties of utility function or subjective probability distribution. The inherent ambiguity originates because taxation reduces not only investor's risk but also investor's expected return. A reduced risk would encourage an investment in risky assets, while a reduced expected return would discourage an investment in risky assets. Thus, the ultimate direction of tax effects on portfolio choice depends on the relative sizes of the tax effect due to risk change and the tax effect due to expected return change.

In criticism of these limitations of the expected utility models, Sandmo (1977) developed a portfolio choice theory based on more general assumptions found in a traditional consumer demand theory. Without relying on the special assumptions, he was able to analyze the effects of taxes on portfolio choice. Although his approach

provided an alternative theoretical framework, his model was not able to predict whether taxes would increase risk taking or not. His theoretical results were generally inconclusive due to two conflicting components of tax effects: a negative income effect and a positive substitution effect. The sign of tax effect on portfolio share demand depends on the relative size of income effect and substitution effect. Later, Ashan (1990) integrated a savings decision into Sandmo's pure portfolio choice model on the ground that taxes affect not only the relative attractiveness of different assets, but also the relative price of current and future consumption. Again, Ashan's results were generally ambiguous due to the conflicting effects: a negative income effect and a positive substitution effect. However, his result under the popular assumption of constant relative risk aversion contradicted the Domar and Musgrave proposition.

Although these alternative approaches further improved our understanding of the issue, they have resulted in generally ambiguous and sometimes conflicting ad hoc conclusions. That is, theoretical models have not been able to determine the direction of the tax effect on risk-taking without restrictive assumptions about the properties of investor's utility and/or subjective probability distribution. Moreover, most theoretical analyses used a proportional tax with full-loss-offset. The actual income tax is progressive and loss offset is limited<sup>1</sup>. The complex features of the actual income tax

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<sup>1</sup>The most important feature of the actual tax is the existence of heterogeneous tax rate across assets. First, capital gains are usually taxed at a lower rate than dividends, interest income, rental income. Only realized capital gains are taxed. When an asset is given away or bequeathed, future taxable gains are

system add another uncertainty to the predictability of theoretical models. Thus, the basic question of whether taxes encourage risk-taking or risky investment has been left as an empirical one.

For this reason, further understanding of the effects of taxes on portfolio behaviors requires extensive empirical analyses of the tax effects on portfolio behavior. A series of empirical studies were triggered by one of the most celebrated and original studies by Feldstein in 1976. Feldstein (1976) showed, using the 1962 Survey of Consumer Finances, that personal income tax had a very powerful effect on individual's demands for portfolio assets, after adjusting for the effects of net worth, age, sex, and the ratio of human to non-human capital. Since Feldstein (1976), few other empirical studies using individual finance data have focused on the effect on household risk-taking<sup>2</sup>, although some studies addressed the tax effect on portfolio choice as their by-product. These studies include Dicks-Mireaux and King (1982), King and Leape (1984,1987), Hubbard (1985), Agell and Edin (1990), Ioannides (1990), and Scholz (1994)<sup>3</sup>. A main

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evaluated based on the current value of the asset. This preferential capital gains will lead to a lower effective tax rate on the yield from equities and real estate. Second, the state and local government bonds are exempt from income tax. Third, the imputed income tax on personal residence is not taxed. Mortgage interest and property taxes are deductible expenses. The capital gains realized on the sale of a personal residence have also received favorable treatment. Fourth, other tax incentives for investment exist in particular type of industries such as oil and gas, pollution control facilities.

<sup>2</sup> The scarcity of empirical studies on the issue of the tax effect on risk taking has not been due to lack of interest but largely due to paucity of reliable information on individual portfolio behavior. Since a measure of individual risk preference was not available, previous studies were not able to separate the tax effect from the effects of individual preferences.

<sup>3</sup>Feldstein result was reiterated by Hubbard (1985) who considered pension wealth in portfolio choice, by Agell and Edin (1990) who used Swedish household data. On the other hand, Feldstein result was reputed

weakness of Feldstein's model and other related models, pointed out by Sandmo (1985) and Scholz (1994), was that they fail to separate the tax effect from the effects of individual preferences regarding risk that are suggested by portfolio theory to play a key role in portfolio choice. Thus, the results of these empirical models could be biased.

Scholz (1994) investigated the degree to which households manipulate their portfolios in response to tax changes, using descriptive statistics from 1983 and 1986 SCF. He concluded that taxes do affect household portfolio choice, however, with some caveats. He acknowledged that it is very difficult to separate tax effects from risk aversion that may affect household portfolio choice. Thus, he suggested that further research examining the effects of taxes on risk taking would clearly be valuable.

The weakness of the previous empirical studies can be summarized as follows. First, the previous empirical studies have mainly dealt with the lump-sum tax effects on portfolio choices that can not be identified into risk effect and expected return effect. Second, the previous studies have directly related not to the tax effects on household risk-taking behavior but to the tax effects on share demand of individual asset. Third, as pointed out by Scholz (1994), the previous studies have not explicitly dealt with the potential

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by King and Leape (1984) who tried to explain the incompleteness of household portfolio, using the 1978 Survey of Consumer Financial Decisions (SCFD). Other results in contrast to Feldsten's result were obtained by King and Leape (1987) who examined portfolio composition over the life cycle using the 1978 SCFD data, by Dicks-Mireaux and King (1982) who estimated the effects of pension wealth using Canadian individual data, and by Ioannides (1990) who tried to explain the incomplete household portfolio composition using the 1983 and 1986 SCF data.

influence of individual risk preference on portfolio choice and risk-taking. This void was primarily caused partly by the unavailability of data on individual risk preference, and partly by the paucity of utility framework models that can be empirically implemented.

However, Levy and Markowitz (1979) demonstrated that utility can be approximated by a judiciously chosen function defined over mean and variance. Such direct estimation of utility could lead to more powerful empirical results than indirect empirical results with efficient market and systemic risk. Aivazian, Callen, Krinsky, and Kwan (1983) showed that the demand for financial assets by the household sector can be modeled utilizing a mean-variance portfolio framework and a flexible functional form of utility function. They also showed that the generalized Leontief function best fits the data at least by comparison to other common flexible functional forms. Furthermore, a relatively reliable risk preference data became available by the release of the 1983 Survey of Consumer Finance in the early 1990s.

Therefore, the purpose of this study can be summarized as follows:

First, this study attempts to derive an estimable portfolio choice model from maximizing a homothetic household utility function defined over means, variances, and wealth. This paper utilizes the generalized Leontief utility function to derive a system of

portfolio share equations. Using this estimable portfolio choice model, this paper will derive and estimate the elasticities of share demand with respect to means and variances, using historical data on means and variance-covariances of each asset type. Finally, this paper will estimate each component of tax effects on asset share demand: 1) tax effects due to a change in expected returns; 2) tax effect due to a change in risk. The relative sizes of each component will determine the final direction of tax effects on investment in risky assets. This share system is quite different from the previous empirical models because an asset share demand is defined as a function of not only its own mean and variance but also the means and variance-covariances of all other assets. This paper uses the 1993 Ibbotson Associates' time series on *Stocks, Bonds, Bills, and Inflation* (SBBI).

Second, this paper develops a direct empirical model on portfolio risk in parallel to one of the benchmark models by Feldstein (1976). However, unlike Feldstein, the portfolio risk model developed in this study explicitly introduces individual risk preference into the model. This paper will estimate this portfolio risk model using the 1983 Survey of Consumer Finances (SCF), which was conducted on a sample of U.S. households<sup>4</sup>. The

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<sup>4</sup>The Survey of Consumer Finances (SCF) has been sponsored by the Board of Governors for the Federal Reserve Bank and has been conducted every third year since early 1970s. Due to an increasing awareness of privacy issues, the availability of personal finance data from the Survey has been increasingly limited over time. The 1983 SCF seems to contain the most personal finance information that is necessary for the proposed study. The 1986 SCF was conducted on the mostly same households as the 1983 SCF and validated the 1983 SCF data using the 1986 data to improve the accuracy of the 1983 SCF data. The Technical Manual and Code Book for the refined version of the 1983 SCF was published in February 15, 1990. Furthermore, the use of the most recent data is not expected to be critical for the study results.

1983 SCF data contains detailed information on household portfolio composition and household risk preference<sup>5</sup>. It also contains a great deal of information on taxable income and deductions. As a prior step toward the model estimation, this paper will also calculate fairly accurate marginal tax rates facing each household using the information available. Furthermore, the direct empirical model goes beyond the analyses of household portfolio share equations. The mean-variance models are mainly limited to the riskiness of individual asset. The portfolio risk model in this paper considers an overall measure of the household portfolio risk. Hence, the empirical results will be robust under the existence of other asset-specific characteristics such as preferential tax treatment of a particular asset.

This study will contribute to a better understanding of the tax effect on household risk taking and portfolio choice by incorporating the original features that are listed below.

First, this study explicitly introduces household risk preference to separate the tax effect from the effect of household preference. The controlling the influence of risk preference renders more accurate estimates of the tax effects. Also, if a degree of risk preference is associated with certain income groups or demographic groups, it may shed new light on socio-economic implications of tax policy.

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<sup>5</sup> The availability of this data provides a rare opportunity to test and estimate the implications of the tax effects on household risk-taking. In recognition of this opportunity, Scholz (1994) encouraged further research examining the effects of taxes on risk taking that would clearly be valuable.

Second, this study decomposes the tax effects into risk effect and expected return effect. This identification of individual factors would allow policy makers to evaluate the financial market implications of the differential taxation on different risk types of assets.

Third, this study directly relates taxes to overall household risk-taking instead of relating taxes to household risk taking indirectly through the relative share demand of risky asset.

Fourth, this study calculates household-specific federal income tax rates and uses them in the model. Thus, the tax rates can describe complex actual tax system rather than a proportional tax system.

Fifth, this study uses an established and consistent data base from one source, namely Ibbotson Associates. Thus, it eliminates a potential measurement inconsistency that sometimes leads to erratic results.

Sixth, this study combines a time series data on riskiness into a cross sectional data on household portfolio shares and other personal characteristics.

The empirical results of this study will shed a new light on the issue of tax effects on household portfolio choice and risk-taking that have been generally ambiguous in the

existing theoretical and empirical studies. Also the derivation of an estimable portfolio choice model should provide not only a method to test further theoretical development on tax effects on household portfolio behavior, but also an alternative method to empirically evaluate the implications of tax policy.

## Chapter 2

### Review of Previous Studies

The studies of the effect of taxation were first initiated by Domar and Musgrave (1944) in the wake of the sharp increase in US personal income tax rates in the 1940s. The general concern was that higher taxes would decrease the demand for high-yield common stocks and other risky investments, thus increasing the cost of business capital and distorting the pattern of investment in the economy. In contrast to the concern, they suggested that the increase in taxes would encourage the risk-taking under full-loss-offset. Their theory was that a proportional tax with full-loss-offset reduces the amount of risk borne by an individual investor, but would reduce the relative price of risk.

Since Tobin (1958) and Markowitz (1959) pioneered in applying von Neumann and Morgenstern expected utility framework to the theory of portfolio choice, many studies

on the issue have flourished.<sup>6</sup> The expected utility framework has become a modern theoretical framework for studying the portfolio choice.

The path-breaking study of Tobin (1958), based on two-asset model (risky asset and riskless asset), showed that the introduction of a proportional tax with full loss offset would induce a risk-averse investor to increase his demand for the risky asset assuming the existence of perfectly riskless asset. Tobin's model also assumed that either the return on the risky asset is normally distributed or the investors' utility function is quadratic. Arrow (1965) and Pratt (1964) showed that any quadratic utility function has an objectionable property that investors become increasingly risk-averse as their wealth increases<sup>7</sup>. The criticisms of Tobin's model include<sup>8</sup>: 1) the assumptions about the nature of preference ordering and/or subjective probabilities is too restrictive, 2) the quadratic utility assumption has an empirically objectionable implication that every risky asset is an inferior good.

The subsequent studies using Tobin's framework by Arrow (1965), Mossin (1968), Sandmo (1969), Stiglitz (1969, 1972), Merton (1969, 1971, 1972, 1973) and Cass and

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<sup>6</sup> Some of the examples are Hall (1960), Hall and Jorgenson (1969), Jorgenson and Siebert (1968), Miller (1968), Penner (1964), Richter (1960), Mossin (1968), Arrow (1965), Leland (1968), Stiglitz (1969, 1973, 1976), Cass and Stiglitz (1972), Feldstein (1969, 1972, 1976), Merton (1969), Flemming (1971), and etc.

<sup>7</sup> Levy and Markowitz (1979) showed that this objectionable properties disappears if the quadratic approximation is allowed to vary from one portfolio to another.

<sup>8</sup> See Arrow (1965, 1971), Borch (1969), and Feldstein (1969).

Stiglitz (1970, 1972) significantly improved our understanding of the portfolio behavior toward risk. However, most of them heavily depended on the restrictive assumptions such as the existence of a perfectly riskless asset, normal distribution of return, and/or quadratic utility function. In reality, the perfectly riskless asset may not exist in the presence of inflation. Feldstein (1969) demonstrated that the effects of taxes on portfolio choices are ambiguous without a perfectly riskless asset.

Hall (1960) extended Tobin's model to the case where all financial assets are risky. Hall's model suggested that a proportional tax with full-loss-offset would increase risk-taking under the assumption of increasing relative risk aversion. Although Hall's model generalized Tobin's model to the case where all assets are risky, his results were valid only under the objectionable assumption of increasing relative risk aversion (RRA). Under a more plausible assumption of decreasing RRA, Hall's model suggests that a proportional tax would decrease risk-taking. Under the assumption of constant RRA, Hall's model predicts that a proportional tax would not affect risk-taking.

Stiglitz (1973) extended these mean-variance analyses of the effects of taxation on portfolio choice to a general equilibrium framework. The general equilibrium model provides no general conclusions about the effects of taxes on portfolio behavior. Only some ad hoc conclusions can be made under special assumptions. That is, a proportional tax with full loss offset would increase the share of risky assets under one of the special

conditions: (1) increasing absolute risk aversion, (2) decreasing absolute aversion but increasing or constant relative risk aversion, (3) zero return on safe asset. However, the effect of a differential tax on portfolio behavior could not be determined even with those assumptions.

Another development of mean-variance portfolio choice theory was in the area of dynamic multiperiod models with additively separable utility functions. Leland (1968) showed that, under some restrictions on the utility function, the introduction of a tax would increase the share of the risky asset in a two-asset portfolio. The restrictions and the assumption that the asset yield in each period follows a normal distribution make this model a dynamic generalization of Tobin's original model. Merton (1971) extended the dynamic model of Leland and Samuelson (1969) to a continuous time process with continuous portfolio optimization. Merton's model implies that the relevant rates of return necessarily follow a multivariate normal distribution. Merton's optimal portfolio theory, known as mutual fund theorem, suggests that the relative proportions in which risky assets are held depend solely on the stochastic characteristics of their returns. Samuelson (1970) and Ohlson (1975) present conditions under which mean and variance are asymptotically sufficient as the length of holding period approaches zero. Flemming (1971) applied Merton's model to studying the effects of taxation and obtained Tobin's original conclusions without any restrictions on either the utility function or the distribution of returns. Although the dynamic models add another

dimension in the theory of portfolio choice, its applicability is limited by the assumption of continuous portfolio reallocation.<sup>9</sup> The assumptions in dynamic models are not sufficiently realistic to be empirically estimated. Also the value functions in the dynamic model often have complex form for general preferences to allow a tractable study of the impact of all relevant determinants of portfolio selection.

Since Tobin, although there have been many attempts to generalize the mean-variance approach, most mean-variance models critically depended on the special assumptions on the properties of utility function or probability distribution. When those restrictive assumptions were released, their ad hoc conclusions often did not hold.

In criticism of the special assumptions of expected utility models, Sandmo (1977, 1985) tried to develop a portfolio choice theory, which can be used to analyze the tax effects on risk-taking, using the general assumptions found in more traditional consumer demand theory. He developed a portfolio choice model with a very general set of assumptions that the utility functions are concave (diminishing marginal utility) and twice differentiable given the axioms ensuring the existence of the utility function. An additional assumption was that the investor's subjective probability distribution has finite first moments. Sandmo showed that these assumptions were restrictions on demand functions that are similar to those from familiar consumer demand analyses,

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<sup>9</sup>Feldstein (1976) questioned the plausibility of continuous portfolio reallocation. He also noted that the

and the expected utility theorem itself is not necessary for the derivation of portfolio behavior. He showed, with a two asset model with zero return on safe assets, that proportional tax with full loss offset will increase risk-taking. However, when he released the assumption of zero return on the safe asset, his theoretical results were generally ambiguous. He was able to make only a conditional conclusion that a proportional taxation with full loss offset is less likely to decrease the share of a risky asset as the holding period decreases or as the rate of return decreases. For example, if the holding period is reasonably short (one year), Sandmo's model suggests that a proportional taxation with full loss offset will increase the risk-taking. Sandmo also suggested the release of the restrictive assumptions such as full loss offset and two asset model would not change the fundamental ambiguity of his conclusion. In fact, Eeckhoudt and Hansen (1982) showed that a tightening of the opportunity to write off losses does not necessarily lead to less risk-taking. He also suggested that in general there is no simple connection between the properties of risk aversion functions and those of asset demand functions or that the risk functions must be defined in such a complicated way that the approach loses much of its appeal. However, his insistence on the departure of his model from the restrictive risk aversion assumptions may have limited the potential of his results. Arrow (1965, 1971), Cass and Stiglitz (1972), and Hart (1975) showed that there are not simple but empirically meaningful connection

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individual must simultaneously choose an uncertain stream of labor income and a portfolio.

between the properties of risk aversion and asset demand functions. For example, the risky asset is a normal good if and only if absolute risk aversion is decreasing.

Sandmo's model was primarily a pure portfolio choice model of an investor who is solely concerned with the level of his or her final wealth. However, taxation affects not only the relative attractiveness of different assets, but also the relative price of intertemporal choice between current and future consumption. Later, Ashan (1990) integrated a savings decision (intertemporal choice between current and future consumption) into Sandmo's pure portfolio decision model. Although the results of Ashan's portfolio model were generally ambiguous just like Sandmo's, he was able to make an unambiguous ad hoc conclusion under the assumption of constant relative aversion. Ashan (1990) suggested that, under constant relative risk aversion, a proportional taxation would decrease the risk-taking. Although this ad hoc conclusion was based on a simple two asset and two period model, his result seems to conflict with the Sandmo's results. One weakness with his model was the use of a before-tax interest rate, instead of after-tax interest rate, to compute present value of future consumption and future wealth. Thus, his results might not have fully reflected the tax-induced changes in the relative price between current and future consumption. The impacts of this weakness on his results are unknown.

In short, the theoretical models developed so far have greatly enhanced our understanding about the effect of taxation on portfolio composition. Unfortunately, the results of the theoretical analyses are generally inconclusive on the directions of tax effects on portfolio risk-taking. Thus, the basic question of the effects of taxation on risk-taking has been left unanswered.

Therefore, to further improve our understanding of the effects of taxation on the portfolio risk-taking, extensive empirical research is necessary. Despite this necessity, there has been no empirical work dealing explicitly with the tax effects on risk-taking. However, there has been some important empirical work on extensive and intensive portfolio choices that may be implicitly related to tax effects on portfolio risk-taking. They include Feldstein (1976), Dicks-Mireaux (1983), King and Leape (1984, 1986, 1987), Hubbard (1985), Agell and Edin (1990), Ioannides (1990), and Scholz (1994).

In a pioneering empirical study, Feldstein (1976) provided the first econometric study of personal taxation and portfolio composition, using data from the 1962 survey of household income and assets by the Board of Governors of the Federal Reserve System. He empirically estimated models of portfolio composition equations, and he used a constructed measure of taxes based on labor income and portfolio wealth. He concluded that the personal income tax has a very powerful effect on individuals' demands for

portfolio assets, after adjusting for the effects of net worth, age, sex, and the ratio of human to nonhuman capital.

Subsequent work dealing explicitly with tax effects on portfolio choice includes Dicks-Mireaux and King (1983), King and Leape (1984, 1987). They investigated both the determinants of households' demand for assets conditional on ownership and the factors influencing households' discrete choice of which asset to hold. Their approach attempt to account for the incomplete household portfolio composition. Their studies generally find that taxes are significantly correlated with the decision to hold a particular asset but are not significantly correlated with the level of demand for the asset, given it is held. Based on this finding, King and Leape (1984) concluded that contrary to much of the recent literature, taxes do not play a decisive role in explaining the differences in portfolio composition across households. Similar results were obtained by King and Leape (1987) who used a life cycle portfolio model of portfolio composition. These conclusions of more recent studies were in a sharp contrast to Feldstein's conclusion (1976).

Later, Hubbard (1985) presented an econometric model that anticipated social security and personal pension benefits exert a measurable influence on household portfolio allocation. Using cross-section data collected in 1979 and 1980 under the auspices of the U.S. President's Commission on Pension Policy, he also found that other things

being the same, people in higher tax brackets hold a higher proportion of their portfolios in common stock, which is quite risky, than in relatively safe assets such as money and bonds. His results suggest that omitting pension wealth variables from asset demand models can bias the estimated coefficient on the marginal tax rate to the extent that household tax situations and pension situations are correlated.

Ioannides (1992) conducted an empirical analysis based on reduced form utility comparisons. That is, he considered that the observed portfolios held by households as the outcomes of comparisons of utility households derive from different combinations of assets and their receptive quantities. Unlike King and Leape (1984, 1986, 1987), Ioannides (1992) used simple reduced form estimations by means of models that consider specific assets separately, rather than the entire portfolio. He concluded that households do indeed set their asset portfolios with the long run in mind. He also concluded that attitudes toward risk and liquidity of investments do affect the household portfolio composition.

Scholz (1994) investigated the degree to which households manipulate their portfolios in response to tax changes. He focused on explaining the relationship between the progressivity of tax burdens and the progression of statutory rates, using descriptive statistics from 1983 and 1986 SCF. He concluded that taxes do affect household portfolio choice, however, with some caveats. He acknowledged that it is very difficult

to separate tax effects from risk aversion that may affect household portfolio choice. Thus, he suggested that further research examining the effects of taxes on risk taking and measuring the magnitude of various implicit taxes would clearly be valuable.

As suggested in the introductory chapter, these previous empirical studies mainly deal with the tax effects on portfolio choices that are not explicitly related to household risk-taking behavior. So far, there has been no empirical study dealing explicitly with the tax effects on portfolio risk-taking as suggested by Scholz (1994). This void was primarily caused by the unavailability of data on individual risk preference that is critical for explaining the investor's portfolio risk-taking behavior. Also most previous empirical studies had difficulties in empirically isolating the role of taxes from a host of other non-tax factors that may affect household portfolio decisions.

In summary, theoretical results suggest that, although some ad hoc conclusion can be made under the restrictive assumptions, the effect of taxation on portfolio choice is generally ambiguous. However, under some restrictive assumptions, a majority of theoretical models seem to predict that taxation would encourage the household portfolio risk taking. Although these theoretical results are limited by the ad hoc assumptions on loss-offset provisions and utility functions<sup>10</sup>, they still provide a

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<sup>10</sup>Despite these restrictive assumptions, Young and Trent (1969), Levy and Markowitz (1979), Pulley (1981) and Kroll, Levy and Markowitz (1984) have each found mean-variance approximations to be quite accurate for a variety of utility functions and historical distributions of portfolio return.

**theoretical standard to analyze an interesting hypothesis regarding the effects of taxes on portfolio choice.**

## Chapter 3

### Econometric Model

This chapter develops two econometric models to empirically examine whether the tax system discourages or encourages households from holding portfolios with greater risk. Theoretical results are largely ambiguous. Taxes reduce the expected return, thus decreasing asset demand. On the other hand, taxes also reduce the risk and may increase a demand for risky assets. Therefore, the net effects of taxes on risky assets depend on the relative sizes of the conflicting effects. The first model aims to estimate the relative sizes of those opposing effects: the risk elasticity of asset share demand and the expected return elasticity of asset share demand. Thus, the first model is derived from mean-variance utility framework (Lancaster type). The second model directly estimates an empirical relationship between taxes and household risk-taking measured by household portfolio variances.

### 3.1 Derivation of Portfolio Choice Model

To make the model mathematically manageable without losing a focus on the main issue, this paper adopts a common assumption of a homothetic separability of household utility function. The homothetic separability assumption allows the households to make a portfolio choice independent of the overall consumption-savings decision. This kind of utility framework allows us to focus on the portfolio choice without a mathematical maze. Under these assumptions, this paper considers a portfolio share model that household investment preferences are described by a probability distribution of the household portfolio wealth and characterized by its expected portfolio wealth ( $\omega_1$ ) at the end of a period and its standard deviation ( $\sigma_\omega$ ). Then, a household utility would be a function of the expected wealth of the household portfolio and its standard deviation:

That is  $U = U(\omega_1, \sigma_\omega)$ .<sup>11</sup>

Thus, the optimization problem for the household is to choose a portfolio composition that maximizes its utility subject to its wealth and risk constraints. Given  $n$  financial assets, the household would:

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<sup>11</sup>All the standard properties of a utility function are assumed. That is, it is continuous and twice differentiable with  $U_{\omega_1} > 0$  and  $U_{\sigma_\omega} < 0$ . This assumption ensures that the household is risk-averse and that indifference curves in  $\omega_1 - \sigma_\omega$  space are upward sloping, given additional assumptions to be made later, convex from below.

Maximize:  $U = U(\omega_1, \sigma_\omega)$  with respect to  $x_i$  for all  $i = 1, 2, \dots, n$ .

$$\text{Subject to } \omega_1 = \omega_0 \left( 1 + \sum_{i=1}^n x_i \mu_i \right)$$

$$\sigma_\omega = \left( \omega_0^2 \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij} \right)^{\frac{1}{2}}$$

$$\sum_{i=1}^n x_i = 1$$

where  $x_i$  is the proportion of the household portfolio invested in financial asset  $i$ ,  $\mu_i$  is the expected rate of return on asset  $i$ ,  $\sigma_{ij}$  is the covariance of returns for  $i \neq j$  and the variance for  $i = j$ ,  $\omega_0$  is the initial household wealth invested in financial assets.

The first order conditions for the above optimization are:

$$\omega_0 U_{\omega_1} \mu_i + \omega_0^2 U_{\sigma_\omega} \sigma_\omega^{-1} \sum_{j=1}^n x_j \sigma_{ij} = \lambda \quad (i, j = 1, 2, \dots, n)$$

$$1 - \sum_{i=1}^n x_i = 0$$

where  $\lambda$  is the Lagrange multiplier,  $U_{\omega_1} = \frac{\partial U}{\partial \omega_1}$ ,  $U_{\sigma_\omega} = \frac{\partial U}{\partial \sigma_\omega}$ .

Now, in order to derive an functional form that is empirically estimable, this paper employs a generalized Box-Cox function to describe a household utility:

$$U(\delta) = \alpha_0 + \alpha_1 \omega_1(\gamma) + \alpha_2 \sigma_{\omega_1}(\gamma) + \left(\frac{1}{2}\right) \alpha_3 [\omega_1(\gamma)]^2 + \left(\frac{1}{2}\right) \alpha_4 [\sigma_{\omega_1}(\gamma)]^2 + \alpha_5 \omega_1(\gamma) \sigma_{\omega_1}(\gamma)$$

where  $U(\delta) = (U^{2\delta} - 1) / 2\delta$ ,  $\omega_1(\gamma) = (\omega_1^\gamma - 1) / \gamma$ , and  $\sigma_{\omega}(\gamma) = (\sigma_{\omega}^\gamma - 1) / \gamma$  are the Box-Cox transformations.

Differentiating the Box-Cox utility function with  $\omega_1$  and  $\sigma_{\omega}$  produces<sup>12</sup>

$$U_{\omega_1} = [\alpha_1 + \alpha_3 \omega_1(\gamma) + \alpha_5 \sigma_{\omega_1}(\gamma)] \omega_1^{\gamma-1}$$

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<sup>12</sup> The generalized Box-Cox functional form is flexible, depending on what values the parameters  $\delta$  and  $\gamma$  take on. One special case of the generalized Box-Cox functional form is the generalized Leontief

utility function where  $\delta = \gamma = \frac{1}{2}$ . This means that  $U(\delta) = (U - 1)$ ,  $\omega_1(\gamma) = 2(\omega_1^{\frac{1}{2}} - 1)$ , and

$\sigma_{\omega}(\gamma) = 2(\sigma_{\omega}^{\frac{1}{2}} - 1)$ . In this case, the marginal utility and its rate of change can be rewritten as follows:

$$U_{\omega_1} = [\alpha_1 + 2\alpha_3(\omega_1^{\frac{1}{2}} - 1) + 2\alpha_5(\sigma_{\omega}^{\frac{1}{2}} - 1)] \omega_1^{-\frac{1}{2}}$$

$$U_{\sigma_{\omega}} = [\alpha_2 + 2\alpha_4(\sigma_{\omega}^{\frac{1}{2}} - 1) + 2\alpha_5(\omega_1^{\frac{1}{2}} - 1)] \sigma_{\omega}^{-\frac{1}{2}}$$

Thus,

$$U_{\omega_1 \omega_1} = -\frac{1}{2} \alpha_1 \omega_1^{-\frac{3}{2}} + \alpha_3 \omega_1^{-\frac{3}{2}} - \alpha_5 \omega_1^{-\frac{3}{2}} (\sigma_{\omega}^{\frac{1}{2}} - 1),$$

$$U_{\sigma_{\omega} \sigma_{\omega}} = -\frac{1}{2} \alpha_2 \sigma_{\omega}^{-\frac{3}{2}} + \alpha_4 \sigma_{\omega}^{-\frac{3}{2}} - \alpha_5 \sigma_{\omega}^{-\frac{3}{2}} (\omega_1^{\frac{1}{2}} - 1),$$

$$U_{\omega_1 \sigma_{\omega}} = \alpha_5 \omega_1^{-\frac{1}{2}} \sigma_{\omega}^{-\frac{1}{2}} = U_{\sigma_{\omega} \omega_1}.$$

$$A = \frac{[\alpha_1 + 2\alpha_3(\omega_1^{\frac{1}{2}} - 1) + 2\alpha_5(\sigma_{\omega}^{\frac{1}{2}} - 1)] \omega_1^{-\frac{1}{2}}}{[\alpha_2 + 2\alpha_4(\sigma_{\omega}^{\frac{1}{2}} - 1) + 2\alpha_5(\omega_1^{\frac{1}{2}} - 1)] \omega_0 \sigma_{\omega}^{-\frac{1}{2}}}$$

$$U_{\sigma_w} = [\alpha_2 + \alpha_4 \sigma_w(\gamma) + \alpha_5 \omega_1(\gamma)] \sigma_w^{\gamma-1}$$

$$\text{Define } A = -\frac{U_{\omega_1}}{\omega_0 U_{\sigma_w} \sigma_w^{-1}} = -\frac{[\alpha_1 + \alpha_3 \omega_1(\gamma) + \alpha_5 \sigma_{\omega_1}(\gamma)] \omega_1^{\gamma-1}}{[\alpha_2 + \alpha_4 \sigma_w(\gamma) + \alpha_5 \omega_1(\gamma)] \omega_0 \sigma_w^{\gamma-2}} \quad (1)$$

Also, by subtracting the first order condition for  $x_k$  from the first order condition for  $x_i$ , we obtain:

$$U_{\omega_1} (\mu_i - \mu_k) + \omega_0 U_{\sigma_w} \sigma_w^{-1} \sum_{j=1}^n x_j (\sigma_{ij} - \sigma_{kj}) = 0 \quad (\text{for } i \neq k)$$

Rearranging the above results gives

$$\sum_{j=1}^n x_j (\sigma_{ij} - \sigma_{kj}) = -\frac{U_{\omega_1} (\mu_i - \mu_k)}{\omega_0 U_{\sigma_w} \sigma_w^{-1}} \quad (2)$$

Now using the expression (1) and (2), we can rewrite the first order condition as a system of portfolio composition equations:

$$\begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} = \mathbf{A} \begin{bmatrix} \sigma_{11} - \sigma_{21} & \cdot & \cdot & \cdot & \cdot & \sigma_{1n} - \sigma_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sigma_{11} - \sigma_{n1} & \cdot & \cdot & \cdot & \cdot & \sigma_{1n} - \sigma_{nn} \\ 1 & \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mu_1 - \mu_2 \\ \cdot \\ \cdot \\ \cdot \\ \mu_1 - \mu_n \\ 1/A \end{bmatrix}$$

In matrix notation<sup>13</sup>,

$$X = A\Omega^{-1}\Gamma$$

$$\text{where } A = -\frac{U_{\omega_1}}{\omega_0 U_{\sigma_w} \sigma_w^{-1}} = -\frac{[\alpha_1 + \alpha_3 \omega_1(\gamma) + \alpha_5 \sigma_{\omega_1}(\gamma)] \omega_1^{\gamma-1}}{[\alpha_2 + \alpha_4 \sigma_w(\gamma) + \alpha_5 \omega_1(\gamma)] \omega_0 \sigma_w^{\gamma-2}},$$

$$X = \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}, \quad \Omega = \begin{bmatrix} \sigma_{11} - \sigma_{21} & \cdot & \cdot & \cdot & \cdot & \sigma_{1n} - \sigma_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sigma_{11} - \sigma_{n1} & \cdot & \cdot & \cdot & \cdot & \sigma_{1n} - \sigma_{nn} \\ 1 & \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \mu_1 - \mu_2 \\ \cdot \\ \cdot \\ \cdot \\ \mu_1 - \mu_n \\ 1/A \end{bmatrix}$$

<sup>13</sup> Defining a element in the  $i$ th row and the  $j$ th column of  $\Omega^{-1}$  as  $d_{ij}$ , each asset demand equation can be written in a non-matrix expression:

$$x_1 = A[d_{11}(\mu_1 - \mu_2) + \cdots + d_{1(n-1)}(\mu_1 - \mu_n)] + d_{1n}$$

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$$x_{(n-1)} = A[d_{(n-1)1}(\mu_1 - \mu_2) + \cdots + d_{(n-1)(n-1)}(\mu_1 - \mu_n)] + d_{(n-1)n}$$

$$x_n = A[(\mu_1 - \mu_2) + \cdots + (\mu_1 - \mu_n)] + 1$$

where  $n$  is the number of asset types.

Again, the asset demand equation system will have  $n$  share equations and five unknown parameters  $(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$ , where  $n$  is the number of asset types. Once the household share demand system is estimated, the impacts of asset return and its variance-covariance on portfolio choice,  $\frac{\partial x_j}{\partial \mu_i}$  and  $\frac{\partial x_k}{\partial \sigma_{ij}}$ , can be calculated<sup>14</sup>.

Also, the demand elasticity of asset  $i$  with respect to expected return on asset  $j$

,  $\varepsilon(x_j, \mu_i) = \frac{\partial x_j}{\partial \mu_i} \cdot \frac{\mu_i}{x_j}$ , and the demand elasticity of asset  $i$  with respect to the variance-

covariance of returns between asset  $i$  and asset  $j$ ,  $\varepsilon(x_k, \sigma_{ij}) = \frac{\partial x_k}{\partial \sigma_{ij}} \cdot \frac{\sigma_{ij}}{x_k}$  can be estimated.

Now, the effects of taxes on portfolio choice can be broken down into a tax effect due to a change in expected return ( $\mu_i$ ) and a tax effect due to a change in riskiness ( $\sigma_{ij}$ )<sup>15</sup>.

That is,

$$\frac{\partial x_k}{\partial \tau} \cdot \frac{\tau}{x_k} = \sum_{i=1}^n \left( \frac{\partial \mu_i}{\partial \tau} \cdot \frac{\tau}{\mu_i} \right) \cdot \left( \frac{\partial x_k}{\partial \mu_i} \cdot \frac{\mu_i}{x_k} \right)$$

<sup>14</sup> Derivation of the comparative static is illustrated in APPENDIX.

<sup>15</sup> Let  $\pi_i$  be the pre-tax rate of return on asset  $i$  and  $\psi_{ij}$  be the variance-covariance of  $\pi_i$ . Also let  $\tau$  be a proportional tax rate. Then  $\mu_i = (1 - \tau)\pi_i$  and  $\sigma_{ij} = (1 - \tau)^2 \psi_{ij}$ . Thus, the tax elasticity of asset  $i$  with respect to expected return ( $\mu_i$ ) and risk ( $\sigma_{ij}$ ) can be estimated by

$$\frac{\partial \mu_i}{\partial \tau} \cdot \frac{\tau}{\mu_i} = -\frac{\pi_i}{\mu_i} \tau = -\frac{\tau}{(1 - \tau)} \quad \text{and}$$

by  $\frac{\partial \sigma_{ij}}{\partial \tau} \cdot \frac{\tau}{\sigma_{ij}} = -2(1 - \tau) \frac{\psi_{ij}}{\sigma_{ij}} \tau = -\frac{2\tau}{(1 - \tau)}$  respectively.

$$\begin{aligned}
& + \sum_{i=1}^n \left( \frac{\partial \sigma_{ii}}{\partial \tau} \cdot \frac{\tau}{\sigma_{ii}} \right) \cdot \left( \frac{\partial x_k}{\partial \sigma_{ii}} \cdot \frac{\sigma_{ii}}{x_k} \right) \\
& + \sum_{i=1}^n \sum_{\text{all } j \neq i} \left( \frac{\partial \sigma_{ij}}{\partial \tau} \cdot \frac{\tau}{\sigma_{ij}} \right) \cdot \left( \frac{\partial x_k}{\partial \sigma_{ij}} \cdot \frac{\sigma_{ij}}{x_k} \right)
\end{aligned}$$

If the effects of taxes through risk( $\sigma_{ij}$ ) reduction and through return( $\mu_i$ ) reduction have opposite signs, the final tax effect on portfolio choice would depend on the relative sizes of the opposing tax effects.

### 3.2 Derivation of Portfolio Risk Model

Feldstein's study (1976) is one of the benchmark econometric studies for estimating the effects of tax on portfolio risk. Thus, for developing a direct empirical model, this paper closely follows Feldstein's (1976) approach to develop a portfolio risk equation.

Feldstein's model assumes that the household asset demand depends primarily on its wealth and its subjective probability distribution of net asset yields. Similarly, the portfolio risk of each household is also assumed to depend on its wealth and on its perception of the probability distribution of net asset yields. The personal characteristics such as age, sex, ratio of human capital to wealth are also included in the model to

adjust the impacts of additional individual differences on portfolio risk. However, Feldstein model ignores additional differences in risk aversion among individuals. Due to lack of available information on the individual subjective risk preferences, Feldstein model assumes that these subjective risk-averse characteristics are independent of wealth and of tax situation. Since his independence assumption is not likely to hold, the exclusion of the risk preference from the model is likely to bias the coefficients of the remaining variables in his model. In short, the main shortcoming of the Feldstein's (1976) study was that it did not separate tax effects from individual risk preference that is likely to affect the household risk taking. As John K. Scholz (1994) pointed out, further empirical analyses examining the effects of taxes on risk-taking would be clearly valuable. Therefore, this paper attempts to separate the effect of risk preference from the tax effect. This paper further improves the empirical estimates of the tax effects by using the household-specific federal income taxes, instead of the aggregate income tax brackets used in Feldstein's study (1976)<sup>16</sup>.

In parallel with Feldstein approach, this paper assumes the general functional relationship of the household portfolio risk with the household wealth, the household income taxes, and the personal variables. In addition, this paper assumes that the

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<sup>16</sup> Feldstein's portfolio risk model is summarized below.

$$PORTRISK_i = \beta_0 + \sum_k \beta_k TAX_{ki} + \sum_l \gamma_l W_{li} + \sum_m \delta_m AGE_{mi} + \alpha_1 SEX_i + \alpha_2 RATIO_i + \varepsilon_i$$

household risk preferences also affect household risk-taking. Thus, the general form of the portfolio risk function is,

$$PORTRISK_i = S(TAX, PREF, PC)$$

where household tax variable is denoted by *TAX*, household risk preference by *PREF*, and other relevant personal characteristic variables by *PC*.

Thus, to incorporate the household risk preference, Feldstein portfolio risk equation for the household *i* can be revised as follows<sup>17</sup>:

$$PORTRISK_i = \alpha_0 + \alpha_1 TAX_i + \sum_k \beta_k PREF_{ki} + \delta_1 W_i + \delta_2 AGE_i + \alpha_3 SEX_i + \alpha_4 RATIO_i + \varepsilon_i$$

where the other relevant personal characteristic variables include *W* (household financial wealth), *AGE*, *SEX* (1 if female, else 0), *RATIO* (human capital to household wealth), respectively.

Portfolio risk was measured by a portfolio standard deviation that consists of asset shares and variance-covariance of asset returns<sup>18</sup>. Human capital was measured by years

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where *TAX* is tax bracket (binary) variables, *W* is financial net worth (binary) variables, and *RATIO* is the ratio of human capital (training and experience) to *W*.

<sup>17</sup> The regression coefficient represents the marginal change in portfolio risk given all the other variable remain unchanged. The mathematical implication can be seen from a Taylor expansion of the general functional form. Refer to APPENDIX 4.

<sup>18</sup> That is,  $\sigma_p = \left( \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij} \right)^{\frac{1}{2}}$

of work experience and education. The human capital and financial wealth were normalized around its respective mean. Then the normalized values were used to calculate the RATIO variable.

Singularity of the covariance matrix is avoided by omitting one group from the PREF binary variables. The individual's actual income cannot be used to define the tax situation variable.

The risk on each household's portfolio is a subjective attribute that defies precise measurement. This paper tries to measure portfolio risk by a variance estimated with historical data for the annual rates (24 observations) of returns during the 24 months before the 1983 SCF survey. The data was extracted from the SBBI 1995 yearbook. From the covariance matrix estimated with these annual values, the pretax variance of each investor's portfolio was calculated using the fractions of each type of asset as the appropriate weights in the quadratic form<sup>19</sup>.

Finally, the portfolio risk model will allow us to directly observe the empirical relationship between taxes and household portfolio risk taking. It is expected that the

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<sup>19</sup> If the risk of inflation is considered, the covariance matrix for the real yields can be calculated by first subtracting each year's rate of consumer price inflation from the nominal yields of the marketable securities, and then regarding the nominal yields on bank accounts and savings bonds as nonstochastic so that their variances and covariances reflect only the changes in the annual rate of consumer price inflation. With this inflation-adjusted covariance matrix of real yields, the variance of each individual's portfolio can again be calculated using the fractions of each type of asset as the weights in the relevant quadratic form.

**current tax system with its special provisions for the favorable tax treatment of equities would induce individuals in high tax rates to hold portfolios with greater pretax risk.**

## Chapter 4

### Empirical Results

#### 4.1 Data Descriptions

##### 4.1.1 Data for Portfolio Choice Model

The data required for estimating the portfolio choice model consist of:

- i) financial asset share holdings of US households (annual time series),
- ii) expected returns of financial assets, and the variance-covariances of their returns (annual time series).

The expected returns of financial assets and their variance-covariances were calculated using a rolling sample of the most recent 36 monthly observations of annual returns in each year. For example, the 36 monthly observations of annual returns between January 1967 and December 1969 were used to calculate the expected returns and variance-covariances for the year 1970. The 36 monthly observations of annual returns between

January 1968 and December 1970 were used to calculate the expected returns and variance-covariances for the year 1971. This rolling sampling method was applied to each year from 1964 to 1992 to obtain the annual time series data on expected returns and variance-covariances. These estimated expected returns and variance-covariances, along with portfolio share information from the Federal Reserve Flow Of Fund data, were used to estimate the portfolio choice model derived in section 3.1.

The financial assets are classified into six asset categories: Equity, Corporate Bonds, Savings Bonds, Federal Government Bonds, Municipal Bonds, and Bank Deposits. The data sources for returns and portfolio composition include:

- (1) Stocks, Bonds, Bills, and Inflation (SBBI).
- (2) Federal Reserve Bulletin 1950 - 1993
- (3) Federal Deposit Insurance Cooperation's Annual Report
- (4) Journal of Portfolio Management (Winter 1995)

The 1992 expected returns and variance-covariances, along with portfolio shares, for the six financial asset types are provided in Table 1a.

**Table 1a: 1992 Expected Returns and Shares for Six Financial Asset Types**

	x1	x2	x3	x4	x5	x6
Portfolio Share	45.11%	3.66%	1.94%	4.84%	7.16%	37.30%
Expected Return	13.20%	12.59%	12.18%	11.96%	8.86%	5.62%

Note: x1=equity, x2=corporate bond, x3=savings bond, x4=federal bond, x5=municipal bond, x6=deposit

The variance-covariances of rates of returns from six asset types are presented in Table 1b.

**Table 1b: 1992 Variance-Covariances of Returns for Six Financial Asset Types**

	x1	x2	x3	x4	x5	x6
x1	0.36051	0.0915	0.11698	0.04655	0.002739	-0.00075
x2	0.0915	0.04285	0.05579	0.02508	0.001298	-0.00048
x3	0.11698	0.05579	0.07543	0.0338	0.001387	-0.00062
x4	0.04655	0.02508	0.0338	0.01693	0.000648	-0.00029
x5	0.002739	0.001298	0.001387	0.000648	0.00067	-0.00012
x6	-0.00075	-0.00048	-0.00062	-0.00029	-0.00012	3.39E-05

Note: x1=equity, x2=corporate bond, x3=savings bond, x4=federal bond, x5=municipal bond, x6=deposit

#### 4.1.2 Data for Portfolio Risk Model

The 1983 Survey of Consumer Finances (SCF), sponsored by the Federal Reserve Board of Governors, contains interviews from a representative sample of U.S. households selected by multistage area-probability sampling methods, along with a supplemental sample of 438 high-income households. High income households were identified from the 1980 IRS tax returns using the same area probability as the main sample. The addition of a high income sample enhances the potential of having more

diverse portfolio composition because some assets are more likely to be held by high income households. In 1986, a majority of the 1983 SCF households were re-interviewed through a less detailed telephone survey. The information in the 1986 SCF was used to validate and refine the responses in the 1983 SCF. The new and improved version of the 1983 SCF data became publicly available in the early 1990s<sup>20</sup>. This study uses the households with positive equity in order to ensure non-missing portfolio risk estimation.

The 1983 SCF data provide unusually detailed information on the household portfolio composition, sources of household income, and other household characteristics such as risk preference, liquidity preference, age, education, experience, marital status, occupation, employment, pension wealth, social security (SS) benefit, home equity, debt, inheritance, and insurance. The 1983 SCF contains detailed information on the sources of income and deductible expenses of household that enable this study to simulate the fairly accurate tax rate facing each household. Details of the tax simulation model are as follows.

The 1983 SCF does not report household taxable income that is net of deductible income, but it reports the 1982 household aggregate gross income (AGI). This study first identifies the household taxable income based on the relationship between AGI and

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<sup>20</sup>The reliability of the SCF data was carefully evaluated by John Karl Scholz (1994) in comparison

taxable income found in the 1982 Statistics of Individual Income Tax Returns published by IRS. Second, since the identified taxable income is an average taxable income for a given AGI class and tax return file type, this study simulates household-specific taxable income using household tax characteristics in the SCF data and the identified average taxable income from the IRS report. This study first calibrates the marginal contributions of the household tax characteristics to the taxable income variation, controlling for the household AGI. Then this study simulates the household-specific taxable income, using the calibrated marginal contributions in the first step. Once the fairly accurate household-specific taxable income is obtained, this study applies the 1982 statutory tax rate schedule to the simulated taxable income according to tax return file types ( married and joint filing, married and separate filing, single and head of household filing, single filing) to calculate household marginal tax rate.

Lastly, the 1983 SCF reveals an interesting and valuable information on the subjective household preferences. They are:

- RISKHIGH Households who indicate that they are willing to take substantial risk.
- RISKSOME Households who indicate that they are willing to take average or above average risk.
- RISKNONE Households who indicate that they are not willing to take any risk.

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with other standard household survey data. His evaluation suggests that the SCF is fairly reliable.

The main difference in the two empirical approaches in this paper is that the portfolio risk model uses the revealed risk preference. The portfolio choice model uses the estimated risk preference obtained by estimating a representative household utility function. That is, the portfolio risk model uses a household specific cross-sectional data, while the portfolio choice model uses an aggregate time series data.

As for the household portfolio risk measure, this paper combines the variance-covariance matrix estimated from the SBBI yearbook and portfolio share information from the 1983 SCF.

Mathematically,

$$\sigma_p = \left( \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij} \right)^{\frac{1}{2}}$$

where  $\sigma_p$  is a household portfolio risk,  $\sigma_{ij}$  is a household variance-covariance between asset  $i$  and asset  $j$ ,  $x_i$  is a portfolio share of the asset type  $i$ .

The variables for estimating the portfolio risk model consists of mostly personal characteristics data including tax rates and risk preferences from the 1983 Survey of Consumer Finance. For the definitions of all the variables, refer to the section 3.2.

Descriptive Statistics of the 1983 SCF data are summarized in TABLE 2.

It should be noted that household risk preference and sex variables are binary variables.

For example, SEX=1 if household head is a female, otherwise SEX=0.

**Table 2: Descriptive Statistics for SCF Data**

Variable	No. of Obs.	Mean	Standard Deviation
EQUITY	1089	0.5150283	0.327331
CORPORATE BOND	1089	0.0135855	0.0667992
SAVINGS BOND	1089	0.0232237	0.0801263
FEDERAL BOND	1089	0.0345367	0.1374565
MUNICIPAL BOND	1089	0.0536997	0.146731
BANK DEPOSITS	1089	0.3599262	0.3277691
TAX	1089	34.8898072	14.4783299
RISKHIGH	1089	0.0743802	0.2625091
RISKSOME	1089	0.7134986	0.4523342
RISKNONE	1089	0.1910009	0.3932704
WORTH	1089	89.4277224	357.828242
AGE	1089	52.224977	15.1072112
SEX	1089	0.1303949	0.3368921
RATIO	1089	9.5341865	65.156172
PORTRISK	1089	33.8956202	17.8162104

## 4.2 Empirical Results

### 4.2.1. Portfolio Choice Model

Since the share demand equations are homogeneous of degree zero in the utility parameters, it is necessary to normalize the parameters to uniquely identify the parameters. This normalization is achieved by adding the constraint  $\sum_{i=1}^n \alpha_i = 1$  or

$\alpha_1 = 1 - \sum_{i=2}^n \alpha_i$ . This type of normalization does not affect the empirical results<sup>21</sup>. Thus,

I only need to estimate  $\alpha_2$  through  $\alpha_5$ .

Once a household parameters are empirically identified, the household utility function<sup>22</sup> and its derivatives<sup>23</sup> with respect to expected return and risk (variance-covariance) can be calculated.

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<sup>21</sup> Christensen, Jorgenson, and Lau (1975), Berndt, Darrough, and Diewert(1977), and Applebaum(1979) used the same normalization, pointing out that the results are invariant to this type of normalization.

<sup>22</sup> The Leontief utility function has the form:

$$U = 2\alpha_3\omega_1 + 2\alpha_4\sigma_w + 4\alpha_5\omega_1^{1/2}\sigma_w^{1/2} + (2\alpha_1 - 4\alpha_3 - 4\alpha_5)\omega_1^{1/2} + (2\alpha_2 - 4\alpha_4 - 4\alpha_5)\sigma_w^{1/2} + 2\alpha_3 + 2\alpha_4 + 4\alpha_5 - 2\alpha_1 - 2\alpha_2 + 1$$

<sup>23</sup> The marginal utility and its rate of change is defined as follows:

$$U_{\omega_1} = [\alpha_1 + 2\alpha_3(\omega_1^{1/2} - 1) + 2\alpha_5(\sigma_w^{1/2} - 1)]\omega_1^{-1/2}$$

$$U_{\sigma_w} = [\alpha_2 + 2\alpha_4(\sigma_w^{1/2} - 1) + 2\alpha_5(\omega_1^{1/2} - 1)]\sigma_w^{-1/2}$$

and,

$$U_{\omega_1\omega_1} = -\frac{1}{2}\alpha_1\omega_1^{-3/2} + \alpha_3\omega_1^{-3/2} - \alpha_5\omega_1^{-3/2}(\sigma_w^{1/2} - 1) ,$$

Table 3. shows the estimates of the Leontief utility function parameters.

**Table 3. Iterative Seemingly Unrelated Regression Estimates**

Parameter	Estimate	Approx.	'T'	Approx.
		Std Err	Ratio	Prob> T
$\alpha_2$	1.010177	0.0021657	466.44	0.0001
$\alpha_3$	-0.00006442	0.00001956	-3.29	0.0025
$\alpha_4$	-0.013636	0.0006818	-20.00	0.0001
$\alpha_5$	0.00010911	0.00004055	2.69	0.0115

All the estimates are statistically significant at 5% significance level.

The next step is, using the estimated utility derivatives and Hessian matrix, to calculate the elasticities of portfolio share demand with respect to expected return and risk<sup>24</sup>.

$$U_{\sigma_w \sigma_w} = -\frac{1}{2} \alpha_2 \sigma_w^{-\frac{3}{2}} + \alpha_4 \sigma_w^{-\frac{3}{2}} - \alpha_5 \sigma_w^{-\frac{3}{2}} (\omega_1^{\frac{1}{2}} - 1),$$

$$U_{\omega_1 \sigma_w} = \alpha_5 \omega_1^{-\frac{1}{2}} \sigma_w^{-\frac{3}{2}} = U_{\sigma_w \omega_1}.$$

<sup>24</sup> The demand elasticity of asset  $i$  with respect to expected return on asset  $j$  is defined as  $\varepsilon(x_j, \mu_i) = \frac{\partial x_j}{\partial \mu_i} \cdot \frac{\mu_i}{x_j}$ . The demand elasticity of asset  $i$  with respect to the variance-covariance of

returns between asset  $i$  and asset  $j$  is defined as  $\varepsilon(x_k, \sigma_{ij}) = \frac{\partial x_k}{\partial \sigma_{ij}} \cdot \frac{\sigma_{ij}}{x_k}$ . The mathematical details of

calculating the utility derivatives,  $\frac{\partial x_j}{\partial \mu_i}$  and  $\frac{\partial x_k}{\partial \sigma_{ij}}$ , are provided in the Appendix.

Table 4a shows the elasticities of portfolio share demand with respect to expected returns for the US households. As shown in Table 4a, it is important to note that a portfolio share demand will be affected not only by its own expected return but also by the expected returns of the other assets.

**Table 4a: Expected Return Elasticities for 1992 Portfolio Share**

	x1	x2	x3	x4	x5	x6
$\mu_1$	0.0025608 (0.0007083)	-0.10904 (0.05114)	0.1038 (0.03321)	-0.012794 (0.029934)	-0.085708 (0.053151)	0.020321 (0.007147)
$\mu_2$	-0.0061893 (0.0016614)	2.43852 (0.93982)	-2.84628 (0.71213)	-0.26542 (0.31542)	-0.43176 (0.12074)	0.033608 (0.017529)
$\mu_3$	-0.001065 (0.000804)	-1.46636 (0.55669)	2.80474 (0.69534)	-0.58772 (0.29664)	0.27574 (0.07881)	0.022566 (0.012284)
$\mu_4$	0.0061375 (0.0016265)	-0.32883 (0.44092)	-1.41912 (0.36547)	1.47856 (0.80302)	-0.090087 (0.095973)	-0.075824 (0.041584)
$\mu_5$	0.0007511 (0.0008816)	-0.58834 (0.2322)	0.76476 (0.15548)	-0.09707 (0.14071)	1.41171 (0.40007)	-0.24149 (0.05212)
$\mu_6$	-0.000257 (0.0007532)	0.13549 (0.24842)	0.23903 (0.18216)	-0.3092 (0.48022)	-0.83829 (0.26704)	0.17568 (0.05292)

Note1: x1=equity, x2=corporate bond, x3=savings bond, x4=federal bond, x5=municipal bond, x6=deposit  
 Note2: standard errors in parentheses are based on the estimates for the last 29 years (1964-1992)

As for the own elasticity, one percent change in expected return for equity would have little effects on its share demand. Also, as for the cross elasticities, one percent change in expected returns of non-equity assets would not have much impact on the share demand for equity asset. This may be understood that, given the volatility of the stock markets compared to fixed income securities, one percent change in expected returns is not likely to have much impacts on the share demand for equity. However, one percent

change in expected return of equity has relatively larger impacts on the share demand for the other assets.

It is interesting to observe that an increase in equity return would lead to a decrease in the portfolio shares of bond assets except savings bond. The own elasticity of the bank deposit is also small. This is not surprising because safe bank deposit may be held for non-speculative or precautionary reasons.

As for the cross elasticities, it is clear that one percent change in expected returns of other assets have little impact on the equity demand. At the same time, changes in expected equity return have relatively small impact on the demand for the other asset categories.

Table 4a also suggests that savings bonds and bank deposits are complements of equity, while government and corporate bonds are substitutes of equity. The small elasticities between bank deposits and the other assets suggest that the demand for bank deposit is affected little by changes in the expected returns of the other assets.

The variance and covariance elasticities are presented in Table 4b and 4c respectively.

**Table 4b: Variance Elasticities for 1992 Portfolio Share**

	x1	x2	x3	x4	x5	x3
$\sigma_{11}$	-0.47721 (0.50984)	-0.24931 (1.70648)	-1.82127 (0.73267)	0.5116 (0.86574)	1.7222 (1.89006)	0.29912 (1.23554)
$\sigma_{22}$	-0.0003731 (0.0025577)	-0.0001949 (0.0023815)	-0.0014239 (0.003042)	0.0004 (0.0041188)	0.0013464 (0.0026612)	0.0002339 (0.0019113)
$\sigma_{33}$	-0.0001845 (0.0051112)	-0.0000964 (0.0061746)	-0.000704 (0.0065807)	0.0001978 (0.008288)	0.0006657 (0.0049054)	0.0001156 (0.0037984)
$\sigma_{44}$	-0.0002577 (0.0031015)	-0.0001346 (0.0037443)	-0.0009835 (0.0050561)	0.0002763 (0.0034481)	0.00093 (0.0039061)	0.0001615 (0.0024466)
$\sigma_{55}$	-0.0000225 (0.0046861)	-0.0000117 (0.0035191)	-0.0000857 (0.0059634)	0.0000241 (0.0048723)	0.0000811 (0.0062644)	0.0000141 (0.0037557)
$\sigma_{66}$	-3.067E-05 (0.0006509)	-1.602E-05 (0.00092862)	-0.0001171 (0.0010257)	3.288E-05 (0.0007682)	0.0001107 (0.00104)	1.923E-05 (0.0004957)

Note1: x1=equity, x2=corporate bond, x3=savings bond, x4=federal bond, x5=municipal bond, x6=deposit

Note2: standard errors in parentheses are based on the estimates for the last 29 years (1964-1992)

As shown in Table 4a and 4b, it is important to note that a portfolio share demand will be affected not only by its own variance-covariances but also by the variance-covariances of the other assets.

In general, variance elasticities are very much smaller than expected return elasticities except own and cross elasticities of equity variance. The results suggest that one percent change in equity variance seems to have relatively large impact on the share demand of all other assets, while changes in variances of other assets have little effects on the share demand for equity. The negative share elasticity of equity with respect to its variance shows that an increase in the variance of equity return induce households to reduce equity holdings. Also, the positive share elasticities of federal and municipal bond with respect to equity variance indicate that an increase in equity riskiness would encourage

household to increase the holdings of bank deposits and municipal bonds. The results suggest that an increase in equity variance would induce households to substitute the holdings of equity and corporate bond with the holdings of federal and municipal bonds.

Table 4c (next page) reveals the covariance elasticities. The covariance elasticities indicate what percent of portfolio share will be affected by one percent change in covariance.

Most of the covariance elasticities are less than 0.01%. Thus, if risk elasticity of a portfolio share is defined as the sum of variance elasticity and covariance elasticities, the risk elasticities of most assets, except one conspicuous case of equity assets, are still much smaller than their expected return elasticities. The mostly negative share elasticities of equity with respect to its covariances show that an increase in the covariance of equity return makes households reduce equity holdings.

Also, the positive share elasticities of federal and municipal bond with respect to equity covariances indicate that an increase in equity covariance would lead households to increase the holdings of bank deposits and municipal bonds.

The results suggest that an increase in equity covariance would lead households to substitute the holdings of equity and corporate bond with the holdings of federal and municipal bonds.

**Table 4c: Covariance Elasticities for 1992 Portfolio Share**

	x1	x2	x3	x4	x5	x6
$\sigma_{12}$	-0.009822 (0.016376)	-0.005132 (0.025286)	-0.037487 (0.02008)	0.01053 (0.02158)	0.035447 (0.028288)	0.006157 (0.015134)
$\sigma_{13}$	-0.006656 (0.017645)	-0.003477 (0.040533)	-0.025401 (0.025546)	0.007135 (0.024483)	0.024019 (0.04508)	0.004172 (0.017674)
$\sigma_{14}$	-0.006606 (0.015911)	-0.003451 (0.02406)	-0.025213 (0.025895)	0.007082 (0.018404)	0.023841 (0.034652)	0.004141 (0.014406)
$\sigma_{15}$	-0.000576 (0.01124)	-0.0003008 (0.0090356)	-0.002198 (0.013447)	0.000617 (0.012937)	0.002078 (0.013074)	0.0003609 (0.0096336)
$\sigma_{16}$	0.000821 (0.0047331)	0.0004289 (0.0056181)	0.0031335 (0.006953)	-0.00088 (0.0064568)	-0.002963 (0.0052184)	-0.000515 (0.0035998)
$\sigma_{23}$	-0.0002574 (0.0026287)	-0.0001345 (0.0029041)	-0.000983 (0.0031426)	0.000276 (0.0043953)	0.000929 (0.0026983)	0.0001614 (0.0018518)
$\sigma_{34}$	-0.0002887 (0.0013652)	-0.0001508 (0.0015256)	-0.001102 (0.0023314)	0.0003095 (0.0017485)	0.0010419 (0.0019335)	0.000181 (0.0012614)
$\sigma_{25}$	-2.212E-05 (0.00051828)	-1.156E-05 (0.00046061)	-8.44E-05 (0.0006172)	2.372E-05 (0.000605)	7.984E-05 (0.0006569)	1.387E-05 (0.0004337)
$\sigma_{26}$	0.00004284 (0.00012586)	0.00002238 (0.00013912)	0.0001635 (0.0001622)	-4.59E-05 (0.0001589)	-0.000155 (0.0001941)	-2.69E-05 (0.0001002)
$\sigma_{34}$	-0.0002062 (0.002606)	-0.0001077 (0.0035795)	-0.000787 (0.0042752)	0.000221 (0.003049)	0.0007441 (0.0033546)	0.0001292 (0.0019246)
$\sigma_{35}$	-1.253E-05 (0.00093115)	-6.55E-06 (0.00072978)	-4.78E-05 (0.0011834)	0.0000134 (0.0010067)	0.0000452 (0.0012318)	7.85E-06 (0.00074)
$\sigma_{36}$	0.00002933 (0.00020733)	0.00001532 (0.000242)	0.000112 (0.0002927)	-3.15E-05 (0.0002121)	-0.000106 (0.000308)	-1.84E-05 (0.0001669)
$\sigma_{45}$	-0.0000146 (0.0012671)	-0.0000076 (0.0010171)	-5.58E-05 (0.0016289)	0.0000157 (0.0012896)	0.0000527 (0.0017609)	0.0000092 (0.0010107)
$\sigma_{46}$	0.00003431 (0.000222)	0.00001793 (0.00022028)	0.000131 (0.0003062)	-3.68E-05 (0.000223)	-0.000124 (0.0003214)	-2.15E-05 (0.0001777)
$\sigma_{56}$	0.0000211 (0.0010687)	0.00001101 (0.00092309)	0.0000804 (0.0013148)	-2.26E-05 (0.0010479)	-7.61E-05 (0.0015568)	-1.32E-05 (0.0008403)

Note1: x1=equity, x2=corporate bond, x3=savings bond, x4=federal bond, x5=municipal bond, x6=deposit

Note2: standard errors in parentheses are based on the estimates for the last 29 years (1964-1992)

An interesting exception to the above findings is that the share elasticity of equity with respect to the equity-deposit covariance is positive, and that the share elasticities of federal and municipal bond with respect to the equity-deposit covariance are negative. In this case, an increase in the equity-deposit covariance would lead households to substitute the holdings of federal and municipal bonds with the holdings of equity and corporate bonds.

In summary of Table 4a-4c, the empirical results suggest, with one notable exception, that changes in expected returns seem to have stronger effects than changes in variances. The notable exception is for the equity asset category. For equity, the changes in riskiness seem to be much stronger.

Besides the tax effects, the results here have other interesting implications on several important public finance issues: Whether a rise in government deficit financing by debt crowds out private spending will depend on the substitutability between federal debt and private securities. The estimated substitutabilities in Table 4a-4c have a direct implication on the issue. The other issues are: the impact of risk reduction regulations in the equity market on the demand for the other financial assets; the potential influence of the stock market instability on the impact of federal debt-management policies.

First, it has been suggested, by Roley (1979) and others, that a debt-financed increase in government deficits will crowd out private expenditures if federal debt and private securities are perfect substitutes and the demand for money depends on wealth. On the other hand, a complete crowding out will not occur if money and federal debt are perfect substitutes. The substitutability between federal debt and private securities may be reflected in the substitutability between federal bond (x4) and equity (x1). If the bank deposits can be considered as a proxy of a loosely defined money, the substitutability between money and federal debt may be approximated by the substitutability between bank deposits and federal bond. There are some alternative substitutability measures such as cross expected return elasticities and cross variance-covariance elasticities. Most alternative substitutability measures in this study are relatively small in their sizes. Although more interesting discussions on the crowding-out effect can be made by examining each substitutability measure one by one. However, those discussions are beyond the scope of this study.

Second, the empirical results in this paper suggest that change in risk of equity would have a substantial impact on the portfolio shares of all other assets. The policy implication of the results is that a destabilization of stock market prices by governmental policies would have substantial impact on the other financial markets. Risk reduction regulations in the equity market would also have a substantial impact on

the other financial markets. These results would be very important information for policy makers to evaluate the impacts of such policies on financial markets.

In the next step, this paper uses a proportional tax rate of 31% to calculate the tax elasticities of expected return and risk of all assets except municipal bond. Then, the calculated tax elasticities of expected return and risk are applied to the share elasticities of expected return and risk to examine the tax effects on portfolio shares. The 31% was a tax rate for a typical investor in 1992 that was estimated by Siegel and Montgomery (1995). Recall that taxation affects expected returns of all assets as well as risk (variance-covariances) of their returns<sup>25</sup>. Thus, the tax effect can be broken down into three components:

- Expected Return Factor:** Percent change in portfolio share contributed by tax-induced changes in expected returns.
- Variance Factor:** Percent change in portfolio share contributed by a tax-induced change in variance.
- Covariance Factor:** Percent change in portfolio share contributed by a tax-induced change in covariance.

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<sup>25</sup> As derived in the section 3.1, the tax effect can be broken down into three components:

$$\begin{aligned} \frac{\partial x_k}{\partial \tau} \cdot \frac{\tau}{x_k} &= \sum_{i=1}^n \left( \frac{\partial \mu_i}{\partial \tau} \cdot \frac{\tau}{\mu_i} \right) \cdot \left( \frac{\partial x_k}{\partial \mu_i} \cdot \frac{\mu_i}{x_k} \right) \\ &+ \sum_{i=1}^n \left( \frac{\partial \sigma_{ii}}{\partial \tau} \cdot \frac{\tau}{\sigma_{ii}} \right) \cdot \left( \frac{\partial x_k}{\partial \sigma_{ii}} \cdot \frac{\sigma_{ii}}{x_k} \right) \\ &+ \sum_{i=1}^n \sum_{\text{all } j \neq i} \left( \frac{\partial \sigma_{ij}}{\partial \tau} \cdot \frac{\tau}{\sigma_{ij}} \right) \cdot \left( \frac{\partial x_k}{\partial \sigma_{ij}} \cdot \frac{\sigma_{ij}}{x_k} \right) \end{aligned}$$

Each component of the tax effects is estimated separately and presented in Table 5.

**Table 5. Estimated Tax Effects on Portfolio Choice**

Portfolio Share Of	Expected Return Factor	Variance Factor	Covariance Factor	Net Effect
EQUITY	-0.000533	1.560841	0.020856	1.5811638
CORPORATE BOND	-0.300910	0.001220	0.010896	-0.2887934
SAVINGS BOND	0.502215	0.000603	0.079598	0.5824158
FEDERAL BOND	-0.136324	0.000843	-0.022359	-0.1578400
MUNICIPAL BOND	0.525699	0.000074	-0.075267	0.4505054
BANK DEPOSITS	-0.079232	0.000101	-0.013072	-0.0922038

The second column of Table 5 shows the percent change of share demand in response to one percent increase in proportional taxation through a change in expected returns. The third column of Table 5 shows the percent change of share demand in response to one percent increase in proportional taxation through a change in variances. The fourth column of Table 5 shows the percent change of share demand in response to one percent increase in proportional taxation through a change in covariances. The tax elasticities of share demand attributable to a change in covariance are mixed mainly due to complementary or substitute relationship among assets. The fifth column of Table 5 shows the net tax effect on portfolio choice. The net tax effect is the net percent change in portfolio share due to one percent increase in a proportional tax rate.

In examining the tax effects, it is important to understand that a portfolio share is affected not only by tax-induced change in its own expected return but also by tax-induced changes in the expected returns of all other assets. By the same token, a portfolio share is affected not only by tax-induced change in its own variance-covariances but also by tax-induced changes in the variance-covariances of all other assets.

According to Table 5, one percent increase in tax rate would decrease the equity share by approximately 0.0005% through a reduction in expected returns, while it would increase the equity share by approximately 1.58% through a reduction in risk. The positive risk factor dominates the negative expected return factor. Consequently, the net effect of one percent increase in a proportional tax rate would be approximately 1.581% increase in the equity share. This result is consistent with the Domar and Musgrave hypothesis that proportional taxation would encourage households to invest more in a risky asset.

As for the corporate bond asset, one percent increase in proportional tax rate would decrease its portfolio share by 0.3009% through a reduction in expected returns, while it would increase the equity share by 0.012% through a reduction in risk. In the case of corporate bond, the negative expected return factor dominates the positive risk factor.

Thus, the net effect of one percent increase in a proportional tax rate would be 0.288% decrease in its portfolio share. The results may be reflective of a relative tax disadvantage of corporate bond asset. The results suggest that a proportional taxation would discourage households from investing in the corporate bond.

The result for federal bond is similar to that of corporate bond. That is, for federal bond, excluding savings bond, the negative expected return factor dominates the positive risk factor. The results suggest that a proportional taxation would discourage households to invest in the federal bond and the municipal bond.

As for the savings bond and the municipal bond, both the expected return elasticities and the risk elasticities are positive. Thus, the net tax effects on portfolio shares are positive. The results may be explained by the fact that the returns on these assets may be sheltered from taxation or taxation can be deferred. The results suggest that a proportional taxation would encourage households to invest in the savings bond and the municipal bond.

Another interesting result in Table 5 is that a proportional taxation has a relatively small effect on the portfolio share of bank deposits. This seems reasonable given that most households may hold money in their bank accounts, at least partly, because of non-speculative motives such as liquidity and precautionary motives. Thus, one percent

taxation would have a relatively small effect on the portfolio share of their bank deposits.

Summary of the results in Table 5 is visually presented in Figure 4. Figure 4 shows the net percent change of share demand in response to one percent increase in proportional taxation.

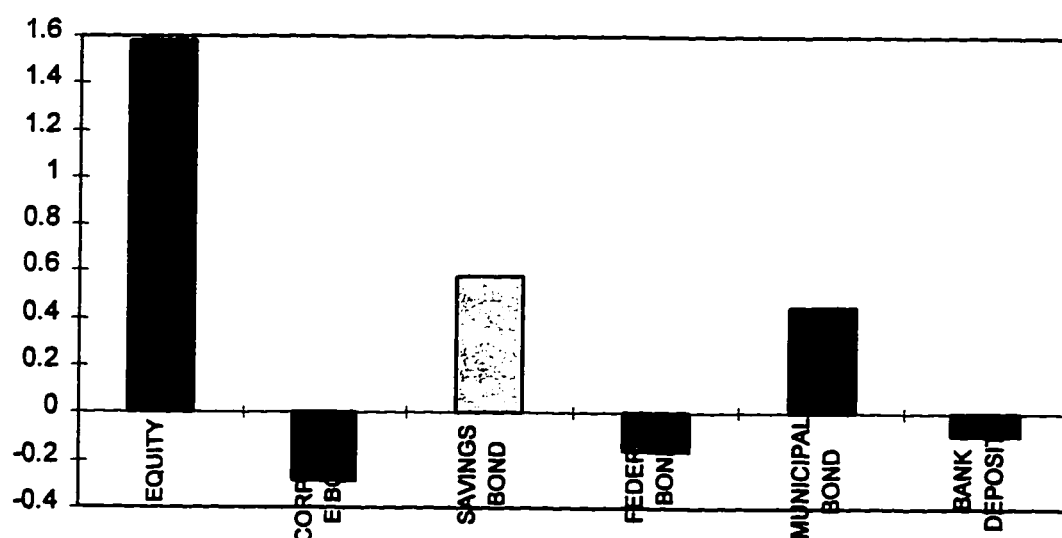


Figure 1. Net Tax Elasticity of Portfolio Choice

The results of Figure 4 suggest that an increase in taxes is likely to increase the share demand for equity, savings bond, and municipal bond. An increase in taxes is likely to decrease the share demand for corporate bond, federal bond, and bank deposits.

The discussions so far in this section focus on the tax effects on individual portfolio shares. However, the tax effect on the entire portfolio risk can not be determined by these results. Thus, in the following section, this paper presents the results of the tax effects on household portfolio risk.

#### 4.2.2 Portfolio Risk Model

The portfolio risk model was estimated with and without risk preference variables.

First, this paper estimated the model using households of all risk preference groups. The results are shown in Table 6a. The coefficient of an explanatory variable indicate a change in portfolio risk (dependent variable) due to a unit change in that variable.

According to Table 6a, the coefficient of tax variable is positive, and it is statistically significant at one percent significance level. The result implies that households with higher tax rates would take more portfolio risk than households with lower tax rates. Thus, the result is consistent with the Domar and Musgrave hypothesis that taxation would encourage household risk taking.

Also, as expected, the household risk preference variables are statistically significant. Recall that these risk preference variables are binary variables of three risk groups

(RISKNONE, RISKSOME, RISKHIGH). To avoid a perfect multi-correlation, the RISKNONE variable is excluded from the estimation. Thus, the coefficients of RISKSOME and RISKHIGH indicate the average differences in risk-taking from RISKNONE group, respectively.

**Table 6a. Parameter Estimates for Portfolio Risk Model with risk preference**

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob> T
INTERCEP	1	17.270264	2.3880379	7.232	0.0001
TAX	1	0.385323	0.0398581	9.667	0.0001
RISKHIGH	1	6.930865	2.1852291	3.172	0.0016
RISKSOME	1	2.243918	1.3014646	1.724	0.085
WORTH	1	0.005476	0.0014645	3.739	0.0002
AGE	1	0.002034	0.0342837	0.059	0.9527
SEX	1	2.152006	1.6050905	1.341	0.1803
RATIO	1	0.019765	0.0078532	2.517	0.012
Rsquare	0.1354	F Value	24.176	Prob>F	0.0001

The table shows that risk taking of RISKSOME households would be larger than that of RISKNONE households by approximately 2.24%. Also, the risk taking of RISKHIGH households would be larger than that of RISKNONE households by approximately 6.93%. The results show that household portfolio risk taking increases as household risk preference increases.

Lastly, the coefficient of the demographic variables such as AGE and SEX seems to be statistically not significant. Since SEX variable is the gender of household head, the result may reflect that household investment decisions may be made jointly between the

household head and the spouse. Also, The WORTH variable is likely to increase with the age of household head. The statistical insignificance of the AGE variable may be the result of this potential correlation between AGE and WORTH variables.

Second, the portfolio risk model was also estimated without risk preference variables to examine the role of household risk preference in explaining household risk taking behavior. The results are presented in Table 6b.

**Table 6b. Parameter Estimates for Portfolio Risk Model without risk preference**

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob> T
INTERCEP	1	19.076333	2.2124757	8.622	0.0001
TAX	1	0.404541	0.0389366	10.39	0.0001
WORTH	1	0.00542	0.0014697	3.688	0.0002
AGE	1	-0.003782	0.0341793	-0.111	0.9119
SEX	1	1.801936	1.6065346	1.122	0.2623
RATIO	1	0.019175	0.0078783	2.434	0.0151
Rsquare	0.1273	F Value	31.585	Prob>F	0.0001

The model without risk preference control also shows a positive tax coefficient. However, the size of tax coefficient seems to be larger for the regression without risk preference variables than that for the regression with risk preference variables.

The comparative results between with-preference and without-preference model are shown in Table 6c.

**TABLE 6c. Tax Effects on US Household Portfolio Risk**

	Without Risk Preference	With Risk Preference	Difference
Tax Effect	0.4045%	0.3824%	0.0211%
T-value	10.390	9.233	
Prob >  T	0.0001	0.0001	

As shown, the models without risk preference control would over-estimate the tax effects. However, both models show a positive tax effects on household portfolio risk-taking. Thus, the empirical evidence is at least consistent with the hypothesis that taxes encourage household risk taking.

This paper further estimated the portfolio risk model within each risk preference group.

**TABLE 7. Tax Effects on Household Risk-Taking By Preference Group**

	No Risk Preference Group	Some Risk Preference Group	High Risk Preference Group
Tax effect	0.296451	0.373461	0.514369
T value	2.835	7.878	3.715
Prob >  T	0.0050	0.0001	0.0004

The results show the relationship between the size of tax effect and household portfolio risk preferences. Table 7 reveals that tax effect decreases as risk preference increases. As for RISKNONE households, one percent tax increase would lead to 0.2965% increase in household portfolio risk. As for RISKSOME households, one percent tax

increase would lead to 0.3734% increase in household portfolio risk. As for RISKHIGH households, one percent tax increase would lead to 0.5144% increase in household portfolio risk.

Taxes have more impact on the risk taking of the RISKHIGH households than on the risk taking of the RISKSOME or RISKNONE households. This is consistent with our intuition that high-risk group would be more responsive to a tax-induced change in risk.

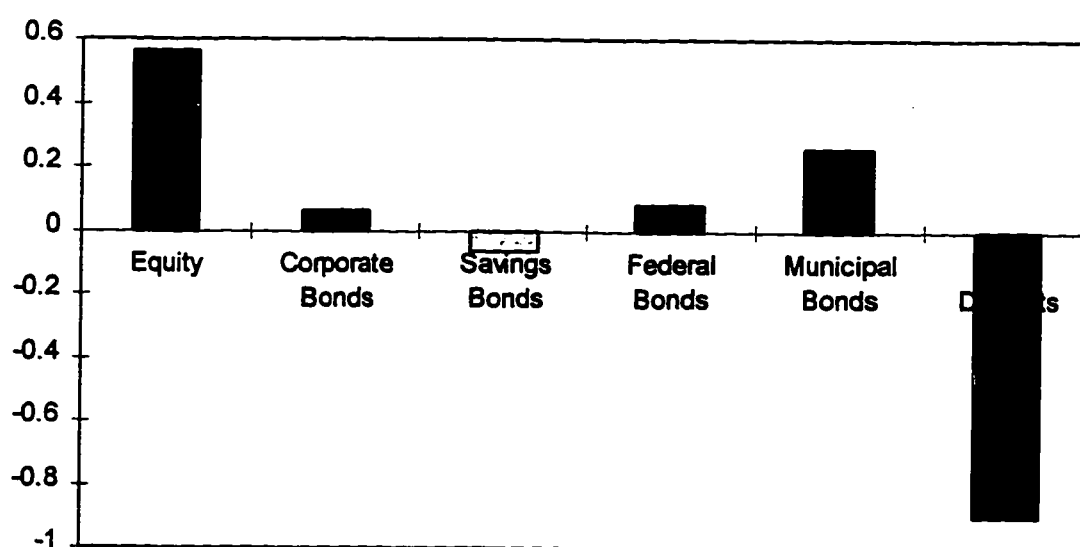


Figure 2. Tax Coefficient for Share Demand Equation

In addition to the relationship between household portfolio risk and taxes, this paper estimated the relationship between household taxes and asset share demand using the same regression model as the portfolio risk model. The only difference from the

portfolio risk model is that the dependent variable is now asset share demand. The tax coefficients of the asset share regressions are presented in Figure 2.

All the estimated tax coefficients except federal bonds are statistically significant at 1% significance level. However, these results are based on the cross-sectional data, their interpretations may be limited. For example, a positive tax coefficient of an asset share demand simply indicates that households with higher tax rates tend to hold a greater share of their portfolio wealth in that asset. However, it does not necessarily imply that an increase in tax rate would increase the share demand of that particular asset.

The positive tax coefficient for equity indicates that households with higher tax rates would hold a greater share of their portfolio wealth in equity. Although the results of asset share regression may not directly support the Domar and Musgrave proposition, they certainly do not contradict the proposition that taxation would encourage an investment in risky assets. The positive tax coefficients for municipal bonds and the negative tax coefficients for bank deposits are consistent with the results of the portfolio choice model in the previous section. The tax coefficients for corporate bonds, savings bonds, and federal bonds are small, suggesting that the share demand for those assets are relatively independent of a tax status of household.

## Chapter 5

### Conclusions

Using an estimated representative utility function (generalized Leontief function), this study was able to estimate expected return, variance, and covariance elasticities for financial assets. The estimated elasticities for financial assets were in turn utilized to calculate tax effects on portfolio choice. These elasticity estimates allowed this study to identify the relative contributions of expected returns, variances, and covariances to the final tax effect on household portfolio choice. This decomposition of the tax effects reveals the channels of taxation through which affect household portfolio choice.

This paper also estimated an empirical relationship between household portfolio risk and federal income taxes, using a portfolio risk model.

The main findings of this empirical study can be summarized as follows:

The estimated portfolio choice model shows that except equity asset, tax-induced changes in expected returns have generally a much greater impact on asset share demand than do tax-induced changes in risk (variance and covariance). In case of equity asset, tax-induced changes in risk (variance and covariance) have generally a much greater impact on asset share demand than do tax-induced changes in expected returns.

Tax-induced change in expected equity return has a relatively larger impact on the demand for all other assets. On the other hand, tax-induced changes in the expected returns of all other assets have little impact on the demand for equity. Likewise, tax-induced change in equity risk has a much larger impact on the demand for all other assets than the impacts of tax-induced changes in the risks of all other assets have on the demand for equity. The tax-induced changes in the risks of all other assets have little impacts on the demand for equity. In general, the cross effects of asset demand with respect to both expected return and risk seem to dominate the own effects.

Another interesting finding is that risk is a more important factor in share demand for equity assets, while expected return is a more important factor in determining share demand for non-equity assets.

In summary of the portfolio choice model, the estimated net tax elasticities of asset demands show that an increase in proportional taxation would increase a share demand

for equity assets and tax-favored assets such as savings bonds and municipal bonds. Also, an increase in a proportional taxation would decrease a share demand for corporate bonds, federal bonds, and bank deposits. The results imply that an increase in a proportional taxation would induce US household to shift from holding equity assets and tax-favored assets to holding non-equity assets such as corporate bonds, federal bonds, and bank deposits.

The estimated portfolio risk model shows that households with higher tax rates take more portfolio risk than households with lower tax rates regardless of individual risk preferences. However, the results suggest that individual risk preferences play a significant role in explaining the US household risk taking behavior. That is, the tax sensitivity of risk taking increases as the degree of risk preference decreases. The estimated parameters of tax and risk preference variables are statistically significant at the 5 percent significance level. The results of the household portfolio risk model is consistent with the conclusions of the portfolio choice model.

In conclusion, the empirical evidence suggests that a taxation would encourage US household to invest more in riskier assets. It also suggests that an increase in taxation would induce US households into holding riskier portfolio.

These findings contribute significantly to a better understanding of the tax effects on portfolio choice and risk taking. The empirical results of this study shed a new light on the issue of tax effects on risk-taking. These empirical revelations will be very important for policy makers to assess the potential ramifications of changing tax policies. The results also suggest that economists and policy makers should take the Domar and Musgrave proposition more seriously.

Finally, the development of an estimable portfolio choice model based on mean-variance framework would provide not only a method to test further theoretical development on the issue of tax effects on household portfolio choice and risk taking, but also an alternative method to empirically evaluate the implications of tax policies.

## BIBLIOGRAPHY

Agell, J. and Edin, P-A. 1992. "Marginal Taxes and the Asset Portfolios of Swedish Households" *Scandinavian Journal of Economics* 92(1), 47-64.

Arrow, K. J. 1965. Aspects of Theory of Risk Bearing. The Yrjo Jahnsson Lectures. Helsinki: Yrjo Jahnsson Foundation.

Ashan, S. M. 1990 "Risk-taking, Savings, and Taxation: a re-examination of theory and policy" *Canadian Journal of Economics*, XXIII, No.2 May.

Aivazian, V. A., Callen, J. L., Krinsky, I., and Kwan, C. C. Y. 1983 "Mean-Variance Utility Function and the Demand for Risky Assets: An Empirical Analysis Using Flexible Functional Forms" *Journal of Financial and Quantitative Analysis*, Vol 18. No. 4, December 1993.

Avery, R. B. 1990. 1983 Survey of Consumer Finances: Technical Manual and Codebook. Board of Governors of the Federal Reserve System. First published in April 1985 and Last revised in February 15, 1990.

Cass, D., and J. Stiglitz. 1972. "Risk Aversion and Wealth Effects on Portfolios with Many Assets." *Review of Economic Studies* 39 (July):331-354.

Domar, E.D., and R.A. Musgrave. 1944. "Proportional Income Taxation and Risk-Taking." *Quarterly Journal of Economics* 4 (August): 227-244.

Feldstein, M.S. 1969. "The Effects of Taxation on Risk-Taking." *Journal of Political Economy* 77 (September-October): 755-764.

Feldstein, M.S. 1976. "Personal Taxation and Portfolio Composition: An Econometric Analysis." *Econometrica* 44 (July): 631-650.

Flemming, J.S. 1971. "Portfolio Choice and Taxation in Continuous-Time." Mimeographed.

Friedman, M. 1959. "The demand for Money: Some Theoretical and Empirical Results." *Journal Of Political Economy* 67 (August): 327-351.

Galper, Harvey; Lucke, Robert; and Toder, Eric. 1988. "A General Equilibrium Analysis of Tax Reform." in H. Aaron, H. Galper, and J. Pechman, eds., *Uneasy Compromise* (The Brookings Institution, Washington, DC.): 59-108.

Goulder, L., J. Shoven, and J. Whalley. 1983. Domestic Tax Policy and the Foreign Sector: The Importance of Alternative Foreign Sector Formulations to Results from a General Equilibrium Tax Analysis Model, in M. Feldstein, ed., Behavioral simulation methods in tax policy analysis (University of Chicago Press for NBER, Chicago, IL): 333-364.

Hall, C. A., Jr. 1960. Fiscal Policy for Stable Growth. New York: Holt, Rinehart and Winston.

Hall, A.E, and D. W. Jorgenson. 1967. "Tax Policy and Investment Behavior." *American Economic Review* 57 (June): 391-414.

Hall, A.E, and D. W. Jorgenson. 1969. "Tax Policy and Investment Behavior: Reply and Further Results." *American Economic Review* 59 (June): 388-400.

Hall, Robert. 1987. "Intertemporal Substitution in Consumption." *Journal of Political Economy* 95 (September-October): 755-764.

Hubbard, R. Glenn. 1985. "Personal Taxation, Pension Wealth, and Portfolio Composition." *Review of Economics and Statistics* 67 (February): 53-60.

Ibbotson Associates, Inc. 1994 *Stocks, Bonds, Bills, and Inflation*.

Ioannides, Yannis M. 1990. "Dynamics of the Composition of Household Asset Portfolios and the Life Cycle." *Applied Financial Economics*, 1992, 2, 145-159.

King, M. A. and Dicks-Mireaux, L-D. L. 1982. "Asset holdings and the life cycle." *The Economic Journal*, 92, 247-67

King, M. A. and Leape, J.L. 1984. "Wealth and Portfolio Composition: Theory and Evidence" *National Bureau of Economic Research Working Paper* No. 1468. September.

King, M. A. and Leape, J.L. 1987. "Asset accumulation, information, and the life cycle, Discussion Paper No. 0014, LSE Financial Markets Group, London.

Lampman, R. 1962. *The Share of Top Wealth-Holders in National Wealth, 1922-56*. Princeton, N.J.: Princeton University Press.

Lancaster, K. *Consumer Demand: A New Approach*. New York: Columbia University Press, 1971.

Leland, H. 1968. "Dynamic Portfolio Theory." Ph.D. dissertation, Harvard University.

Levy, H., and H. M. Markowitz. "Approximating Expected Utility by a Function of Mean and Variance." *American Economic Review*, Vol. 69 (June 1979), pp. 308-317

Lintner, J. 1964. "Optimal Dividends and Corporate Growth under Uncertainty." *Quarterly Journal of Economics* 78 (February): 49-95.

Markowitz, H. 1959. *Portfolio Selection*. New York: Wiley.

Merton, R.C. 1969. "Lifetime Portfolio Selection under Uncertainty: The Continuous Time Case." *Review of Economics and Statistics* 51 (August): 247-257.

Merton, R.C. 1971. "Optimum Consumption and Portfolio Rules in a Continuous Time Model." *Journal of Economic Theory* 51 (August): 247-257.

Mossin, J. 1968. "Taxation and Risk Taking: An Expected Utility Approach." *Economics* 35 (February): 74-82.

Penner, R. G. 1964. "A Note on Portfolio Selection and Taxation." *Review of Economic Studies* 31 (January): 83-86.

Richter, M. K. 1960. "Cardinal Utility, Portfolio Selection and Taxation." *Review of Economic Studies* 27 (June): 152-166.

Roberts, Gordon S. 1975, "Lancaster's New Demand Theory: Its Application in Portfolio Analysis," *Journal of Economic Literature*, Vol. 12 (March 1975), p. 45.

Samuelson, P. A. 1969. "Lifetime Portfolio Selection by Dynamic Stochastic Programming." *Review of Economic Statistics* 51 (August): 239-246.

Sandmo, A. 1969. "Capital Risk, Consumption, and Portfolio Choice." *Econometrica* 37 (October): 586-599.

Sandmo, A. 1977. "Portfolio Theory, Asset Demand and Taxation: Comparative Statics with many assets." *Review of Economic Studies* 44, 369-379.

Sandmo, A. 1985 "The Effects of Taxation on Savings and Risk-taking" *Handbook of Public Economics*, volume 1, edited by A.J. Auerbach and M. Feldstein.

Scholz, J.K. 1994. "Tax progressivity and household portfolios: descriptive evidence from the Surveys of Consumer Finances" *Tax Progressivity and Income Inequality*, edited by Joel B. Slemrod.

Scholz, J.K. 1992. "A Direct Examination of the Dividend Clientele Hypothesis." *Journal of Public Economics* 49:261-85.

Siegel, L. B., and Montgomery, D. 1995. "Stocks, Bonds, and Bills after Taxes and Inflation", *Journal of Portfolio Management*. Winter 1995.

Stiglitz, J. E. 1972. "Portfolio Allocation with Many Risky Assets." In *Mathematical Methods in Investment and Finance*, ed. T. Szego and K. Shell. Amsterdam: North-Holland.

Stiglitz, J. E. 1969a. "Behavior toward Risk with Many Commodities." *Econometrica* 37 (October): 660-667.

Stiglitz, J. E. 1969ba. "The Effects of Income, Wealth, and Capital Gains Taxation on risk-Taking." *Quarterly Journal of Economics* 83 (May): 263-283.

Tobin, J. 1965. "The Theory of Portfolio Selection." In *The Theory of Interest Rates*, ed. F. H. Holm and F.P.R Brecking. London: Macmillan.

Tobin, J. 1958. "Liquidity Preference as Behavior towards Risk." *Review of Economic Studies* 25 (February): 65-86.

Whalley, J. 1988. "Lessons from General Equilibrium Models." In *Uneasy Compromise: Problems of a Hybrid Income-Consumption Tax*, edited by Henry J. Aaron, Harvey Galper, and Joseph A. Pechman. Washington: Brookings Inst.

## APPENDIX

### Appendix A: Comparative Statics for Portfolio Choice Model

Differentiating the first order conditions with respect to  $x_i$ 's produces the following matrix H:

$$H = \begin{bmatrix} h_{11} & \cdot & \cdot & \cdot & h_{1n} & 1 \\ \cdot & & & & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot \\ h_{n1} & & & & h_{nn} & \cdot \\ 1 & \cdot & \cdot & \cdot & 1 & 0 \end{bmatrix}$$

where 
$$h_{ij} = -\omega_0^2 U_{\omega_1 \omega_1} \mu_i \mu_j + \sigma_{\omega_1}^{-1} \omega_0^3 U_{\omega_1 \sigma_{\omega_1}} \left( \mu_i \sum_{j=1}^n x_j \sigma_{ij} + \mu_j \sum_{i=1}^n x_i \sigma_{ij} \right)$$

$$+ \omega_0^4 \left( \sigma_{\omega_1}^{-2} U_{\sigma_{\omega_1} \sigma_{\omega_1}} - \sigma_{\omega_1}^{-2} U_{\sigma_{\omega_1}} \right) \sum_{i=1}^n x_i \sigma_{ij} \sum_{j=1}^n x_j \sigma_{ij} + \omega_0^2 \sigma_{\omega_1}^{-1} U_{\sigma_{\omega_1}} \sigma_{ij}$$

and 
$$U_{\omega_1 \omega_1} = \frac{\partial^2 U_{\omega_1}}{\partial \omega_1^2}, \quad U_{\sigma_{\omega_1} \sigma_{\omega_1}} = \frac{\partial^2 U_{\sigma_{\omega_1}}}{\partial \sigma_{\omega_1}^2}, \quad U_{\omega_1 \sigma_{\omega_1}} = \frac{\partial^2 U_{\omega_1}}{\partial \sigma_{\omega_1} \partial \omega_1}, \quad U_{\sigma_{\omega_1} \omega_1} = \frac{\partial^2 U_{\sigma_{\omega_1}}}{\partial \omega_1 \partial \sigma_{\omega_1}}.$$

The second order conditions for the maximization require the principal minors of the determinant of H to alternate in sign. This second order condition ensures

the convexity of indifference curve.

Now, in order to derive the impact of a change in  $\mu_i$  on a household portfolio choice  $x_i$ 's, we differentiate the first-order conditions with respect to  $\mu_i$ .

The resulting mathematical expressions are provided below.

$$\begin{bmatrix} h_{11} & \cdot & \cdot & \cdot & h_{1n} & 1 \\ \cdot & & & & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot \\ h_{n1} & & & & h_{nn} & \cdot \\ 1 & \cdot & \cdot & \cdot & 1 & 0 \end{bmatrix} \begin{bmatrix} \partial x_1 / \partial \mu_i \\ \cdot \\ \cdot \\ \cdot \\ \partial x_n / \partial \mu_i \\ \partial \lambda / \partial \mu_i \end{bmatrix} = \begin{bmatrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_n \\ 0 \end{bmatrix}$$

where

$$z_i = -\omega_0 U_{\omega_1} \theta_{ij} - x_j [\omega_0^2 U_{\omega_1 \omega_1} \mu_i + \sigma_{\omega_1}^{-1} \omega_0^3 U_{\omega_1 \sigma_{\omega_1}} \sum_{j=1}^n x_j \sigma_{ij}], \text{ and } \theta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Solving the above equation system for  $\frac{\partial x_j}{\partial \mu_i}$  gives

$$\frac{\partial x_j}{\partial \mu_i} = -\omega_0 U_{\omega_1} \frac{c_{ij}}{|H|} - x_j [\omega_0^2 U_{\omega_1 \omega_1} \sum_{l=1}^n \mu_l \frac{c_{lj}}{|H|} + \sigma_{\omega_1}^{-1} \omega_0^3 U_{\omega_1 \sigma_{\omega_1}} \sum_{l=1}^n \sum_{j=1}^n x_j \sigma_{lj} \frac{c_{lj}}{|H|}]$$

where  $c_{ij}$  is a cofactor of  $H$ .

Analogously, in order to derive the impact of a change in  $\sigma_{ij}$  on a household portfolio choice  $x_i$ 's, differentiating the first-order conditions with respect to  $\sigma_{ij}$  gives:

$$\begin{bmatrix} h_{11} & \cdot & \cdot & \cdot & h_{1n} & 1 \\ \cdot & & & & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot \\ h_{n1} & & & & h_{nn} & \cdot \\ 1 & \cdot & \cdot & \cdot & 1 & 0 \end{bmatrix} \begin{bmatrix} \partial x_1 / \partial \sigma_{ij} \\ \cdot \\ \cdot \\ \cdot \\ \partial x_n / \partial \sigma_{ij} \\ \partial \lambda / \partial \sigma_{ij} \end{bmatrix} = \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \\ 0 \end{bmatrix}$$

where

$$y_i = -\sigma_{\omega_1}^{-1} \omega_0^2 U_{\sigma_{\sigma_{\sigma}}} (x_i + x_j) \phi_{ij} - \sigma_{\omega_1}^{-1} x_i x_j \omega_0^3 [U_{\omega_1 \sigma_{\sigma}} \mu_i + \sigma_{\omega_1}^{-1} \omega_0 (U_{\sigma_{\sigma} \sigma_{\sigma}} - U_{\sigma_{\sigma}}) \sum_{j=1}^n x_j \sigma_{ij}]$$

$$\text{and } \phi_{ij} = \begin{cases} 1/2 & \text{if } i = j \\ 1 & \text{if } i \neq j \end{cases}$$

And solving for  $\frac{\partial x_k}{\partial \sigma_{ij}}$  gives

$$\frac{\partial x_k}{\partial \sigma_{ij}} = -\sigma_{\omega_1}^{-1} \omega_0^2 U_{\sigma_{\sigma_{\sigma}}} (x_i \frac{c_{jk}}{|H|} + x_j \frac{c_{ik}}{|H|}) \phi_{ij}$$

$$- \sigma_{\omega_1}^{-1} x_i x_j \omega_0^3 [U_{\omega_1 \sigma_{\sigma}} \sum_{l=1}^n \mu_l \frac{c_{lk}}{|H|} + \sigma_{\omega_1}^{-1} \omega_0 (U_{\sigma_{\sigma} \sigma_{\sigma}} - U_{\sigma_{\sigma}}) \sum_{l=1}^n \sum_{j=1}^n x_l \sigma_{lj} \frac{c_{lk}}{|H|}]$$

## Appendix B: Generalized Leontief Utility Function

The generalized Leontief utility function is one special case of the generalized

Box-Cox functional form where  $\delta = \gamma = \frac{1}{2}$ . This means that  $U(\delta) = (U - 1)$ ,

$\omega_1(\gamma) = 2(\omega_1^{\frac{1}{2}} - 1)$ , and  $\sigma_w(\gamma) = 2(\sigma_w^{\frac{1}{2}} - 1)$ . That is,

$$U = 2\alpha_3\omega_1 + 2\alpha_4\sigma_w + 4\alpha_5\omega_1^{1/2}\sigma_w^{1/2} + (2\alpha_1 - 4\alpha_3 - 4\alpha_5)\omega_1^{1/2} + (2\alpha_2 - 4\alpha_4 - 4\alpha_5)\sigma_w^{1/2} + 2\alpha_3 + 2\alpha_4 + 4\alpha_5 - 2\alpha_1 - 2\alpha_2 + 1$$

Thus, the marginal utility and its rate of change can be written as follows:

$$U_{\omega_1} = [\alpha_1 + 2\alpha_3(\omega_1^{\frac{1}{2}} - 1) + 2\alpha_5(\sigma_w^{\frac{1}{2}} - 1)]\omega_1^{-\frac{1}{2}}$$

$$U_{\sigma_w} = [\alpha_2 + 2\alpha_4(\sigma_w^{\frac{1}{2}} - 1) + 2\alpha_5(\omega_1^{\frac{1}{2}} - 1)]\sigma_w^{-\frac{1}{2}}$$

and,

$$U_{\omega_1\omega_1} = -\frac{1}{2}\alpha_1\omega_1^{-\frac{3}{2}} + \alpha_3\omega_1^{-\frac{3}{2}} - \alpha_5\omega_1^{-\frac{3}{2}}(\sigma_w^{\frac{1}{2}} - 1),$$

$$U_{\sigma_w\sigma_w} = -\frac{1}{2}\alpha_2\sigma_w^{-\frac{3}{2}} + \alpha_4\sigma_w^{-\frac{3}{2}} - \alpha_5\sigma_w^{-\frac{3}{2}}(\omega_1^{\frac{1}{2}} - 1),$$

$$U_{\omega_1\sigma_w} = \alpha_5\omega_1^{-\frac{1}{2}}\sigma_w^{-\frac{1}{2}} = U_{\sigma_w\omega_1}.$$

Also,

$$A = \frac{[\alpha_1 + 2\alpha_3(\omega_1^{\frac{1}{2}} - 1) + 2\alpha_5(\sigma_w^{\frac{1}{2}} - 1)]\omega_1^{-\frac{1}{2}}}{[\alpha_2 + 2\alpha_4(\sigma_w^{\frac{1}{2}} - 1) + 2\alpha_5(\omega_1^{\frac{1}{2}} - 1)]\omega_1\sigma_w^{-\frac{1}{2}}}$$

## Appendix C: Mathematical Implications of Portfolio Risk Model

The actual values of explanatory variables can be viewed as a deviation from the desired values of explanatory variables.

Mathematically,

$$dS(TAX, PREF, PC) = S(TAX, PREF, PC) - S^*(TAX^*, PREF^*, PC^*) \quad (3.1.a)$$

where  $TAX^*$ ,  $PREF^*$ , and  $PC^*$  are the desired values of  $TAX$ ,  $PREF$ , and  $PC$  at the optimal portfolio risk ( $S^*$ ) derived from expected utility maximization.

Then, by total differential,

$$\begin{aligned} dS(TAX, PREF, PC) &= \frac{\partial S^*}{\partial TAX} dTAX + \frac{\partial S^*}{\partial PREF} dPREF + \frac{\partial S^*}{\partial PC} dPC \\ &= \frac{\partial S^*}{\partial TAX} (TAX - TAX^*) + \frac{\partial S^*}{\partial PREF} (PREF - PREF^*) + \frac{\partial S^*}{\partial PC} (PC - PC^*) \quad (3.1.b) \end{aligned}$$

where

$$dTAX = (TAX - TAX^*), \quad dPREF = (PREF - PREF^*), \quad \text{and} \quad dPC = (PC - PC^*).$$

Combining (3.1.a) and (3.1.b) gives

$$S(TAX, PREF, PC) - S^*(TAX^*, PREF^*, PC^*)$$

$$= \frac{\partial S^*}{\partial TAX} TAX + \frac{\partial S^*}{\partial PREF} PREF + \frac{\partial S^*}{\partial PC} PC - \frac{\partial S^*}{\partial TAX} TAX^* - \frac{\partial S^*}{\partial PREF} PREF^* - \frac{\partial S^*}{\partial PC} PC^*$$

Thus,

$$\begin{aligned} S(TAX, PREF, PC) &= \{S^*(TAX^*, PREF^*, PC^*) \\ &- \frac{\partial S^*}{\partial TAX} TAX^* - \frac{\partial S^*}{\partial PREF} PREF^* - \frac{\partial S^*}{\partial PC} PC^*\} \\ &+ \frac{\partial S^*}{\partial TAX} TAX + \frac{\partial S^*}{\partial PREF} PREF + \frac{\partial S^*}{\partial PC} PC \end{aligned}$$

Therefore, the empirical model for the portfolio risk for  $j$ th household can be written as

$$PORTRISK_j = \beta_{j0} + \beta_{j1}TAX + \beta_{j2}PREF + \beta_{j3}PC + \varepsilon_j$$

where

$$\beta_{j0} = S_j^*(TAX^*, PREF^*, PC^*) - \frac{\partial S_j^*}{\partial TAX} TAX^* - \frac{\partial S_j^*}{\partial PREF} PREF^* - \frac{\partial S_j^*}{\partial PC} PC^*,$$

$$\beta_{j1} = \frac{\partial S_j^*}{\partial TAX}, \beta_{j2} = \frac{\partial S_j^*}{\partial PREF}, \text{ and } \beta_{j3} = \frac{\partial S_j^*}{\partial PC}.$$

This paper defines the  $k$  group of risk preferences and each group is represented by a binary variable. Also, expanding the personal characteristic variables for the household  $i$  gives the specification of portfolio risk regression equation in this paper.

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