"Ms. Martin is secretly teaching us!"

High school mathematics practices of a teacher striving toward equity

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Abstract

"Ms. Martin is secretly teaching us!"
High school mathematics practices that strive toward equity

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Attention to the pursuit of equity has been a growing priority for mathematics classrooms in the last two and a half decades. While classroom discourse has become a central feature of classrooms that strive toward equity, one continuing concern for classroom researchers in mathematics education involves understanding how and when group interactions support or interfere with student learning. In this dissertation, I respond to these calls by simultaneously offering perspectives of mathematical learning from underrepresented students alongside empirical evidence of these students' transformative classroom experiences. In this study, I used ethnographic methods to analyze student and teacher classroom interactions and interviews during the first and last units in a semester one Algebra 1 class where learning was facilitated primarily by student-led whole class and student-student small-group interactions. In Paper 1, I propose a combined analytic framework for studying classroom interactions by coordinating Status and Positioning Theories. In Paper 2, I investigate the teacher's classroom structures and pedagogical practices to reveal a process for delegating mathematical authority to students. Students' perceptions reveal the classroom structures, and pedagogical practices expand what counts as mathematical smartness in this classroom. In Paper 3, I investigate students' changed perceptions of competence over the course of the study. I find that the development of specific social and sociomathematical norms mediated students' perceptions of improved competence in this classroom. Throughout this dissertation, I argue that this teacher's practice strives toward equity by expanding mathematical smartness and changing students' perceptions of competence in mathematics. I suggest that future work should analyze the systemic inequities that lead to uneven representation of any groupings of students in tracked classrooms. I also argue that future work striving toward equity in mathematics education should further investigate empirical evidence of additional processes for delegating authority to high school students, additional classroom structures and pedagogical practices that expand mathematical smartness, additional social and sociomathematical norms that can mediate students' perceptions of competence, and additional classrooms that contribute to successes for *all* students in high school mathematics.

Keywords: equity, delegating authority, Status Theory, Positioning Theory, social norms, sociomathematical norms, perceptions of competence

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ACKNOWLEDGMENTS

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intellectual challenges. I would not be the same researcher, teacher, scholar, or person without you. Thank you.

Thank you also to anyone I failed to mention and thank you in advance to anyone I will work with in the future as we pursue an understanding toward equitable teaching and learning. Thank you all for allowing me to grow this work. And I so greatly look forward to how we will all continue to support one another into the future.

DEDICATION

To all of the students who are affected by inequities in mathematics education, this work is devoted to you.

And to my partner Matt, whose support and love is inspiring.

INTRODUCTION

Attention to the pursuit of equity has been a growing priority for mathematics classrooms in the last two and a half decades (e.g., Esmonde, 2009a; 2009b; Nasir & Cobb, 2007; JRME Equity Issue, 2013; National Council of Teachers of Mathematics (NCTM), 1989, 2000). This study is a response to the ongoing need for significant qualitative documentation of what elements of equitable teaching and learning can look like in practice. In this series of papers, I respond to the call for rich empirical evidence of cooperative learning instructional practices that: a) expand mathematical smartness, b) improve students' perceptions of competence in mathematics, and c) strive toward equity.

Striving toward classroom equity is a construct I use in this study that draws from at least two foundational ideas. First, in equitable classrooms, teachers and students view everyone as competent and capable of learning high-level concepts (Cohen, 1997a). Cohen's conviction suggests smartness is not as narrow as conventionally believed; rather, she asserts *all students* are capable of high-level concepts. Second, I assert equitable classrooms demonstrate a "fair distribution of opportunities to learn" (Esmonde, 2009b, p. 1010). Esmonde's construction of equity suggests students in equitable classrooms have access to opportunities to learn. I use Cohen's and Esmonde's interpretations of equity and equitable classrooms to outline how striving toward classroom equity implies working toward access to and expectations that all students can and will learn high-level mathematical concepts.

In this study, I examined the classroom practices and student and teacher experiences of one high school Algebra 1 class during the 2011-2012 school year. Twenty-five of the 28 students formally agreed to participate in the study. Seventeen students in this class were African American or African immigrants, 5 students were Asian American, 4 students were European

American, and 2 students were Latino/a.[1] Twenty-two students were girls and 6 students were boys. I outline the class period's demographics in detail because they were not representative of the make-up of the school's student body. The school-reported demographic included about 50% boys and 50% girls. The reported racial backgrounds of the student body, compared with the classroom demographics, are available in Table 1.

Table 1

School Reported & Class Observed Student Demographics

School & Algebra Class Demographics	Approximate School-Reported Percentage	Class Number Female	Class Percent Female	Class Number Male	Class Percent Male
American Indian/ Alaskan Native	1%	0	0%	0	0%
Asian	20%	3	10.7%	2	7.1 %
Pacific Islander	1%	0	0%	0	0%
Asian/ Pacific Islander	20%	0	0%	0	0%
African American/ African Immigrant	30%	14	50%	3	10.7%
Latino/a	10%	2	7.1%	0	0%
European American	40%	3	10.7%	1	3.6%
Two or More Races	22%	0	0%	0	0%
Total Class		22	79%	6	21%

Table 1 demonstrates that when compared with the school, this Algebra 1 class had an overrepresentation of African Americans, Latinas, and girls, and an underrepresentation of Asian Americans, European Americans, and boys. The difference mirrors a school trend (and arguably also a district and national trend) in which, prior to entering high school, disproportionately more

[1] I describe the demographics on the basis of my observation. Any error or misrepresentation of how students perceived their identities is my error, alone.
[2] Demographics were not available for other Algebra 1 classes at this school during this year. That said, the teacher reported a belief that this same demographic trend was present throughout the other Algebra 1 classes at this school.
[3] Ms. Martin (all names are pseudonyms) identifies her learning and interpretation of complex instruction to the work that came out of Railside High School (Boaler & Staples, 2008).

European Americans, Asian Americans, and boys successfully complete Algebra 1, while disproportionately fewer African Americans, Latinos/as, and girls take or complete Algebra 1 in high school.[2]

Table 1 reveals an overrepresentation of African American students and girls, and especially of females of color, in this class. In particular, the proportion of African American students in this class was about 2.5 times greater than the proportion of African American students at the school as a whole. This overrepresentation of certain groups of students implies that African American students, and in particular girls of color, were underrepresented as freshman in the school's Geometry and Algebra 2 classes. The discrepancy also points to a disproportionate rate of success for African Americans and girls in mathematics prior to coming to this high school. I point out the discrepancy between the proportions of students in this class and the school as a whole because, in spite of a disproportionate representation of students in the different math tracks in this school as a whole, the students in this study worked on high-level mathematics and found themselves successful in mathematics by the end of the first Algebra 1 semester.

Educators and researchers have established cooperative learning as a means to offer students more equitable learning opportunities (Boaler & Staples, 2008; Cohen, Lotan, Scarloss, & Arellano, 1999; Esmonde, 2009b; Gutiérrez, 2002). One leading concern for classroom researchers in mathematics education involves understanding how and when group interactions support or interfere with student learning. Because cooperative learning practices such as complex instruction have been linked to equitable teaching and learning opportunities (Cohen,

[2] Demographics were not available for other Algebra 1 classes at this school during this year. That said, the teacher reported a belief that this same demographic trend was present throughout the other Algebra 1 classes at this school.

1994; Cohen & Lotan, 1997; Featherstone et al., 2011), in this paper I analyze the work of one teacher whose pedagogical practices center around the use of complex instruction as a way to position students as competent sensemakers and to offer access to high-level mathematics tasks.[3] Complex instruction is also a form of ambitious teaching, because it is a cooperative learning strategy that offers students opportunities to reason to construct their understandings of authentic mathematics problems (Lampert, 1990; Lampert & Graziani, 2009).

In Paper 1, I contribute an analytic framework for studying classroom interactions that coordinates Status and Positioning Theories. Status Theory explains how expectations of competence develop over time (Kalkhoff & Thye, 2006; Webster & Foschi, 1988). Positioning Theory explains moment-to-moment classroom interactions (Harré & Moghaddam, 2003; Harré & van Langenhove, 1999). The combined framework offers a structure for analyzing moment-to-moment classroom interactions that have resulted from status generalization and have led to or can predict status issues.

In Paper 2, I respond to a call to understand the process for transferring mathematical authority to students as a way to strive toward equity (Esmonde, 2009a; Zahner, 2011). I reveal how the teacher delegated mathematical authority using particular classroom structures, including (a) student-led presentations, (b) regular random assignment to small groups, (c) groupworthy tasks, (d) Shuffle Quizzes, (e) Participation Quizzes, and (f) group tests[4]. I also analyze the ways the teacher's pedagogical practices delegated authority, including positioning students as competent and orienting students to use one another and justify the mathematics.

[3] Ms. Martin (all names are pseudonyms) identifies her learning and interpretation of complex instruction to the work that came out of Railside High School (Boaler & Staples, 2008).
[4] Shuffle Quizzes and Participation Quizzes were developed by the teachers of Railside High School.

In Paper 3, I respond to a gap in the research, offering a description and analysis of the development of social and sociomathematical norms in a secondary mathematics classroom. In Paper 3, I also respond to the need to understand norms that facilitate mediating students' positive perceptions of competence in mathematics (Esmonde, 2009b). The social norms revealed include *everyone discusses ideas with everyone else, everyone offers and receives help,* and *groups work on the same problem.* The sociomathematical norms revealed include *mathematical work involves making sense of your own ideas, mathematical work involves making sense of others' ideas,* and *groups work to achieve mathematical consensus.*

By attending to the transfer of mathematical authority through specific classroom structures and pedagogical practices, the expansion of mathematical smartness, the development of specific social and sociomathematical norms, and the mediation of students' perceptions of competence, the combined contribution of this series of papers is an in-depth analysis of the way one teacher attended to striving toward equity in her mathematics classroom.

Abstract Paper 1

One leading concern for classroom researchers in mathematics education involves understanding how and when group interactions support or interfere with student learning. Researchers have proposed that students' perceptions of competence are mediated by the ways their ideas are treated in class. Such perceptions grow out of day-to-day classroom interactions. In this essay, I first describe Status Theory as a framework for explaining how expectations for competence develop over time. Next I describe Positioning Theory as a framework for explaining moment-to-moment classroom interactions. I then propose the coordination of Status and Positioning Theories as an analytic means for understanding how classroom interactions explain the process of status generalization. I suggest the resulting combined framework can illustrate the development of valued status characteristics and can be used to explain and analyze dynamic, moment-to-moment classroom interactions that lead to expectations for competence. I next analyze three classroom cases with the combined framework, as a way to demonstrate the development of classroom status characteristics. Each case illustrates how positioning moves can lead to expectations for competence and an understanding of which status characteristics become valued in this classroom. Case 1 is a study of student-to-student positioning moves, which can predict the development of expectations for competence and the generalization of certain status characteristics. I analyze a group interaction first using Status Theory alone, next using Positioning Theory alone, and finally using the combined Status and Positioning Theory framework. I use Case 1 to demonstrate how the combined framework can simultaneously explain student discourse moves and predict the development of expectations for competence. In Case 2, I use the combined framework to illustrate how teacher positioning moves contribute to

determining expectations for competence and the valuation of specific status characteristics. In Case 3, I use the combined framework to analyze the effect of a teacher intervention on the development of expectations for competence. I study student and teacher positioning moves prior to a teacher intervention and during a teacher intervention in order to understand how teacher positioning can interrupt the development of potential status issues. The three cases demonstrate how teacher and student discourse can position students as competent sensemakers, develop expectations for competence, and attend to status issues. I end the essay by considering the implications for using the combined framework to analyze other episodes of learning.

PAPER 1: An Analytic Framework for Coordinating Status and Positioning Theories

I describe *learning* as the process that unfolds as students develop through changing participation in the social community of the classroom (Rogoff, 2003). One leading concern for classroom researchers in mathematics education involves understanding how and when group interactions support or interfere with student learning. I use a sociocultural perspective on learning to understand learners through their social and historical contexts as they develop through their changing participation in the classroom community (Rogoff, 1995, 1997, 2003). By adopting Rogoff's Theory to frame my understanding of mathematical learning, I suggest students develop[5] through a process of changing participation in the social and cultural community of the mathematics classroom.

In this essay, I analyze the changing participation of mathematics learners in one classroom through Status and Positioning Theories, in order to show how discursive positioning moves lead mathematics teachers and students to develop expectations for individuals' competence over time. *Status* is the relative standing an individual holds in a community, a standing based on perceptions of the individual's ability to contribute to the community (Cohen, 1994a). Status Theory explains how individuals have interpreted competence to rank themselves and others (Webster & Foschi, 1988). Positioning Theory is the study of symbolically mediated interactions and conventions around the ways people speak to and act with one another. Positioning Theory can be used to analyze the dynamics of classroom interactions (Harré & van Langenhove, 1999).

I first describe Status Theory and Positioning Theory separately. I then propose a combined Status and Positioning Theory framework that explains the development of

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[5] In line with Rogoff (1997), I consider the terms *learning* and *development* to refer to the same individual processes. I therefore refer to them interchangeably.

expectations for competence through classroom discursive positioning interactions, and I consider the resulting effect on classroom learning. Although I will not use this paper to offer a complete analysis of a particular claim, I show how three discursive classroom episodes demonstrated the usefulness of the blended lens. I discuss how these discursive episodes lead to the (potential) development of valued status characteristics in this classroom. Each case demonstrates how the combined framework allows Positioning Theory to be used as a vehicle for explaining the process of status generalization.

Status Theory

Status Theory[6] is rooted in social psychology (Kalkhoff & Thye, 2006; Webster & Foschi, 1988). In this theory, individuals in problem-solving groups form a set of largely unconscious performance expectations for how likely it is believed that each group member will successfully contribute to a problem-solving task. In the mathematics classroom, performance expectations determine who will have the most influence to complete an assigned task. Unequal rates of influence among students can have negative effects on the learning of all students. Group members perceived to be the most competent to complete an assigned task tend to be the members who are the most influential, most able, and most favorably evaluated. When some students are perceived to be more competent than others, status issues arise, and student learning is threatened. Elizabeth Cohen and Rachel Lotan began using Status Theory in the late 1970s to

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[6] Status Theory has been theorized and studied under various names, including Status Generalization Theory (Webster & Foschi, 1988), Status Characteristic Theory (Cohen & Lotan, 1997), and also, named after its theory construct, Expectation States Theory (Kalkhoff & Thye, 2006). For my purposes, these theories are founded on the same principles. For simplicity, I consistently refer to this construct as *Status Theory*.

develop ways to help teachers attend to status issues in the classroom (e.g., Cohen, 1994a; Cohen & Lotan, 1997).

According to Status Theory (Cohen, 1994a), it is better to have high status than low status. In a classroom, the teacher and students negotiate what gets valued in their classroom community. The status of individuals in a community is often referred to as the result of *status ordering*. Status ordering happens when characteristics become consciously and subconsciously valued in the given community in some sort of hierarchy, and those values lead to generalized expectations for an individual's behavior. I will refer to the result of this ordering as a process of *status generalization*. In a classroom, a learner holds her or his status on the basis of the individual characteristics that are valued in that community, regardless of whether the teacher or students consciously state what they value (Cohen, 1994a). Figure 1 provides a visual representation of this process, explained below in greater detail.

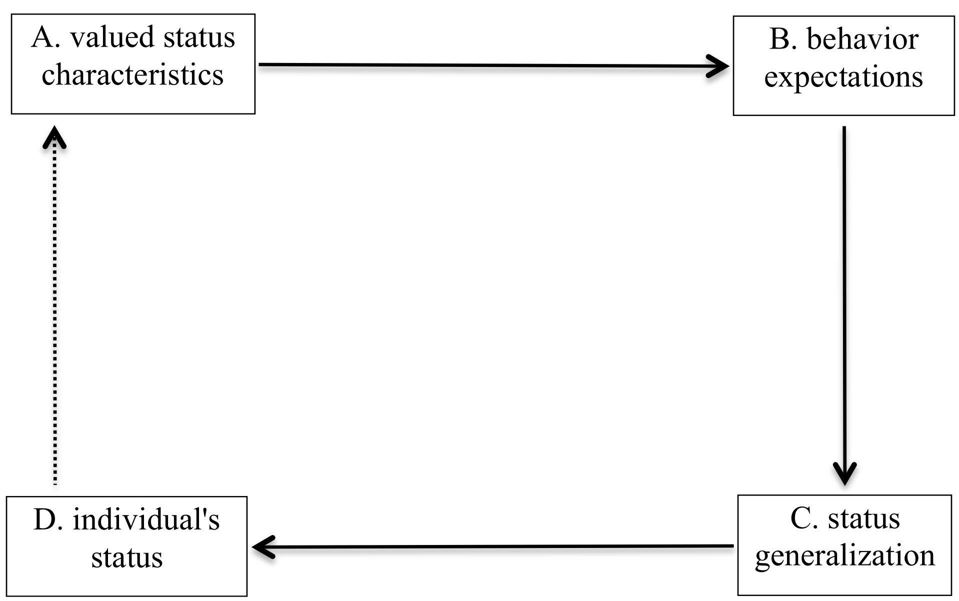

Figure 1. The process of status generalization.

Status characteristics include individual characteristics that a community may associate with different levels of competence to perform an assigned task. In a secondary mathematics classroom, the individual characteristics perceived to contribute to competence may include gender, race, language preference, speed, perceived academic competence, perceived popularity, athletic ability, work ethic, and so on. The society at large, the school community, and the developing classroom community all influence what gets valued in a mathematics classroom as a status characteristic, pictured as *valued status characteristics* in position A of Figure 1. The status characteristics that are valued by the members of a given community influence the process of status generalization for that community. For example, suppose the members of a mathematics class believe that athletes at their school tend to be highly competent in mathematics. When the students of one class demonstrate expectations for the athletes' mathematical competence by turning to them for assistance on the assigned work over others, "athlete" becomes a valued status characteristic in that classroom.

Valuing certain status characteristics over others generates differing *behavioral expectations* for students' competence to offer contributions to their group's work. Because expectations for individuals' behavior result from valuing certain status characteristics in a given community, they are pictured in the top right in position B of Figure 1. Using the athlete example from above, if students look to athletes over non-athletes to help with mathematics assignments, they establish *expectations* for those athletes' competence. When the expectations for competence become normalized in the classroom, *status generalization*, pictured in the bottom right in position C of Figure 1, has occurred. *Status generalization* can interfere with learning because it presupposes amounts of influence among community members (Berger, Cohen, & Zelditch, 1972). When certain students are predicted to have more or less influence than others, the community starts to generalize expectations that students' potential for intellectual contributions varies. Once expectations are generalized, an individual's *status* becomes exposed to the members of the community, pictured as the final part of the cycle, in the bottom left in position D of Figure 1. Although status generalization tends to depict an enduring generalization of expectations for performance, what is valued can be challenged and may change in a given community. When community values change, the cycle can start over again, as depicted in Figure 1 by the dotted line.

In American society, status characteristics like race and gender often set behavior expectations for mathematical competence. Race and gender stereotyping in mathematics classes are evidenced when, for example, students turn to Asian-American students or to the boys in their class for mathematics contributions more than the students from other groups because the former historically have been perceived to have higher mathematical competence. When people believe that students from one race or gender are more mathematically competent, expectations for behavior develop and status generalization happens, reaffirming individuals' stereotypical places in a status hierarchy.

Once status is generalized, behavior expectations continue to predict amounts of influence, power, voice, and control within a learning group. In a mathematics class, differing levels of influence in a group mean one or more students will regularly contribute significantly more, threatening opportunities for equitable learning for all students. Status generalization therefore may indicate the presence of *status issues*. For example, when a group of students acts as if one student is the most competent, the group's behavior indicates the leader's high status, the followers' lower status, and the presence of generalized performance expectations for the high-status student(s) to intellectually contribute more and the followers contribute less to the intellectual work of the group. Status issues interfere with learning opportunities for lower status students because lower status indicates perceived lower competence. Perceived competence may lead lower status students to act according to the lower amounts of influence that are (perhaps newly) expected of them. Status issues also interfere with learning for higher status students, who may lose the opportunity to learn from group members who are perceived to be less competent and therefore not consulted for their intellectual contributions.

In a classroom, attention to status issues is intended to move students toward *equal-status* interactions. Teachers can attend to status issues by publicly *assigning competence* to attempt to raise the status of students. When teachers successfully assign mathematical competence to lower status students, peers change their perceptions of these students' competence, finding them newly competent to work on high-level mathematics. When teachers successfully promote and develop a classroom community in which all learners are considered active and influential, everyone can be perceived as competent, moving the community toward *equal-status* interactions.

Positioning Theory

Positioning Theory is the study of symbolically mediated interactions and conventions around the ways people speak to and act with one another. The theory is rooted in dynamic psychological study of episodes of interpersonal interactions (Harré & Moghaddam, 2003; Harré & van Langenhove, 1999). Positioning Theory is helpful for explaining moment-to-moment classroom interactions and the dynamics of social episodes.

The concepts of *position* and *positioning* have been credited to Holloway (1984) and have also been used in the work of Harré and van Langenhove (1999) and Harré and

Moghaddam (2003). Positioning is consciously or unconsciously assigning people specific locations in a hierarchy with respect to one another. Positioning Theory offers meaning to the dynamic discursive construction of personal stories within a given context and according to the what is valued in the local community (Harré & van Langenhove, 1999).

Positioning is a way to describe the fluid *roles* individuals take up as they interact with one another (Harré & van Langenhove, 1999). A person's *position* in a given community refers to her or his role in comparison with others in the community. Sometimes positions are created intentionally, such as teacher and student in a mathematics class. Other times positions are co-constructed, such as when mathematics students position themselves and others to have a certain amount of competence while working together. Positioning oneself or being positioned by others may increase or lower an individual's status relative to other members of a group. These positioning moves influence the co-construction of an individual's and a group's learning, as students and groups develop through changing participation. For example, if students in one school position athletes to have higher mathematical competence, competence positioning will be evidenced through discursive body language and speech acts as students work together.

The dynamic nature of the positioning process can be illustrated through a positioning triad, where *individual position, discourse/positioning moves,* and *storyline* are pictured at the vertices. In Figure 2, arrows indicate the ways that interactions develop positions fluidly and continuously through discourse/positioning moves as a storyline develops over time (van Langenhove & Harré, 1999).

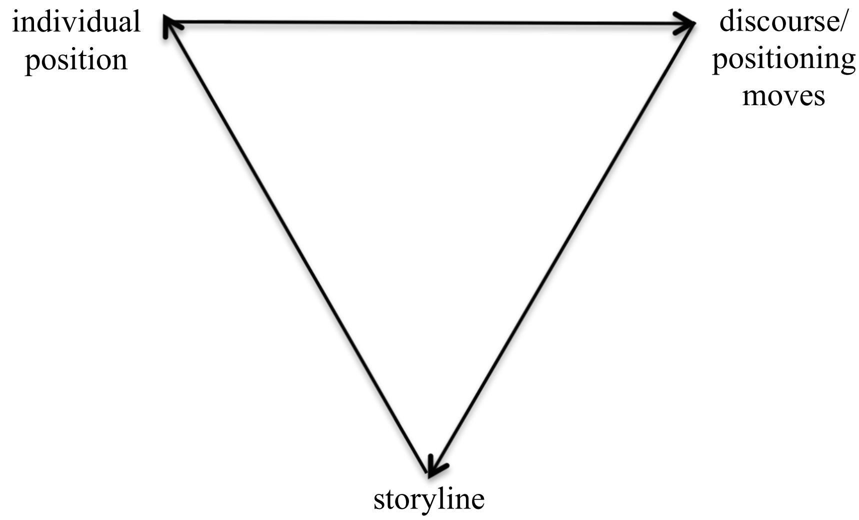

Figure 2. Position-discourse moves-storyline triad (modified from van Langenhove & Harré, 1999).

An individual's positioning cycles continuously from an individual's position, to the discursive moves in which individuals participate, to the storyline that develops, and back to the individual's position. The cycle can continue indefinitely, because continued discourse can change an individual's storyline through positioning challenges over time.

Metapositioning occurs when someone challenges the right of someone else to assign positions, or when an individual challenges the way she or he has been positioned in a certain context. Metapositioning results in either reinforcing or challenging previously understood positions. When positioning challenges are unsuccessful, the original positioning structure is reinforced and remains intact. When positioning challenges are accepted as valid, new positions are formed. An example of a successful positioning challenge occurs, for instance, when mathematics students work together in a group and a student typically positioned to have lower competence offers an answer different from a student typically positioned to have higher competence. If the group takes up the idea of the student previously positioned to have lower competence, they demonstrate a new belief that the student is competent and, consequently, the metapositioning challenge has been successful. Successful positioning challenges in mathematics classes result in redefining who can be competent to complete mathematics tasks.

When students are assigned to work together in groups, interactions are required for task completion. Positioning Theory is helpful for understanding how students and teachers perceive one another to have varying levels of competence to do intellectual work in the classroom. Below, I offer an example of a positioning challenge that occurred in one small group in a mathematics class, and I explain how status generalization may have interfered with the group's potential for correctly understanding the mathematics.

Positioning Theory as a Vehicle for Explaining Status Generalization

Table 3 illustrates a side-by-side comparison of Status and Positioning Theories. When used together, a combined Positioning and Status Theory framework can dynamically explain the process of status generalization.

Table 2

Status Theory and Positioning Theory

Status Theory	**Positioning Theory**
Explains what characteristics have been established as valued.	Explains how moment-to-moment interactions can position individuals as competent.
Explains generalized performance expectations for an individual's contributions.	Explains the dynamic process of positioning: the discursive acts that create positions during social episodes over time.
Justifies when, why, and which status characteristics have led to status issues.	Describes how participants use discourse to position one another with varying levels of competence.
Status cannot be assigned. Individuals can assign competence to one another, which can change their status.	Individuals use discursive moves to position one another as competent.
Limitation: Does not explain the language and discursive acts used for competence	Limitation: Does not explain the status characteristics students have used to

Status Theory	Positioning Theory
positioning.	perceive/position classmates as competent.

Table 2 demonstrates that whereas Status Theory explains how status generalization has been interpreted in a given community and therefore which status characteristics have led to generalized expectations for students' competence, Positioning Theory explains the moment-to-moment interactions that position individuals as competent in the given community. Status Theory offers an explanation for the ways people have formed performance expectations for individuals' contributions on the basis of the generalization of what people value in a given community (Webster & Foschi, 1988). Positioning Theory offers the opportunity to understand the dynamic process of positioning: the discursive acts that create positions during social episodes over time (Harré & van Langenhove, 1999). In a classroom community, Status Theory can help justify when, why, and which status characteristics led to status issues, and Positioning Theory can describe how participants use language to position one another as competent (or not). Although status cannot be assigned, individuals can use discursive moves to position one another as competent. Metapositioning challenges can result in changed status over time. So while Status Theory does not explain the language and discursive acts used for competence positioning, Positioning Theory does not explain the status characteristics students have used to perceive/position classmates as competent. Figure 3 postulates a combined analytic framework that explains how to coordinate the two theories.

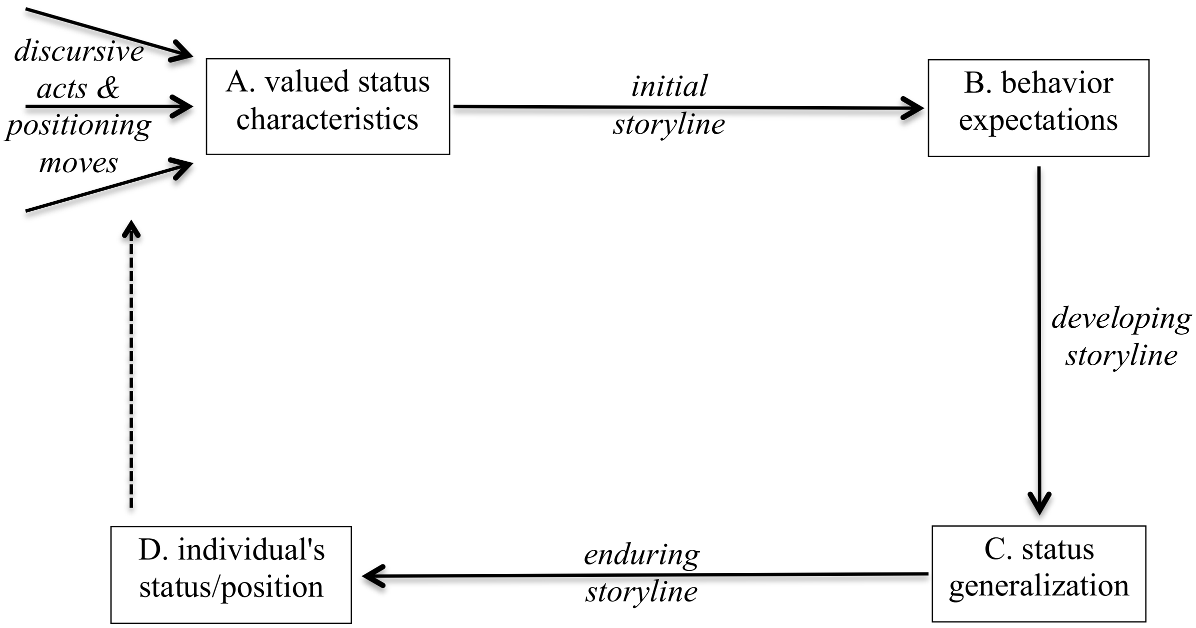

Figure 3. A combined analytic framework: Positioning Theory as a vehicle for explaining status generalization.

In Figure 3, similar to Figure 1, the process of status generalization is demonstrated by the cycling of stages pictured at nodes A, B, C, and D. The arrows indicate the ways Positioning Theory enables movement from one stage of status generalization to the next. The cycle is pictured as dynamic and fluid, meaning that positioning and repositioning challenges are ongoing for a given context. The arrows are meant to demonstrate a general progression and not an automatic movement from one node to the next. The process of status generalization is messy and can and does stay at one stage and move back and forth before moving to the next stage. The general direction represents the movement toward the development of an individual's status over time.

In the combined framework, the process of status generalization begins with individuals in the community interacting, indicated in the top left of the diagram by discursive acts and positioning moves. As individuals continue to interact, valued status characteristics are displayed in the community, depicted at stage A. An initial storyline begins to develop, evidenced by the ways community members engage in discourse and by the status characteristics they demonstrate valuing. The developing storyline is further revealed by the ways the members of the community demonstrate behavior expectations for one another, as indicated at stage B. As individuals act on the developed behavior expectations, consciously or unconsciously turning to certain members of the community over others on the basis of expectations for competence, status generalization, stage C, becomes evident. The result of status generalization is the visibility of differing expectations for competence, depicted as the enduring storyline. The enduring storyline leads to the display of an individual's status or position for the given community, pictured at stage D.

The process continues with further discursive acts and positioning moves that reify an individual's current status/position or challenge that status/position. When positioning challenges are successful, the process of status generalization begins anew, redefining the community members' positions. Although this framework shows both Status Theory and Positioning Theory cycling toward an individual's status or position with respect to others, it is Positioning Theory that is useful for describing the movement between the stages of Status Theory. The moves are cyclical and unending for a given community. As individuals continue to engage in discourse, they may either maintain or change the ways they value status characteristics, expect individuals to behave, (re)generalize status, and (re)position community members as competent (or not).

In a classroom, the teacher sets the tone for the status characteristics that are displayed by giving the students a set of explicit and implicit guidelines for behavior as a member of the classroom community. The teacher then responds to students' initial discursive acts as students take up those explicit and implicit guidelines. For example, if a teacher explicitly says, "This is a classroom where all students can learn," and a student responds, "But I'm a girl!" or displays evidence of any other societal stereotype about who can learn, the teacher's response and ongoing classroom dialogue about a relationship between gender and competence will influence whether gender becomes a valued status characteristic for the classroom community. The teacher therefore influences which status characteristics are displayed at stage A of this framework. Over time, the explicit and implicit classroom guidelines set by the teacher and the way the teacher responds to the students' taking up those guidelines (or not) will determine the community's movement toward stage B, in which valued status characteristics turn into behavior expectations. So if a teacher says, "All students are expected to share their mathematical thinking with their group members," and some students respond by explaining mathematics to their group, students may start to develop expectations for competence on the basis of who shares mathematical thinking. If students do not take up the guideline to explain mathematical thinking right away and the teacher does not reinforce this guideline, the students may not develop a classroom behavior expectation for mathematical explanation. Another example of a classroom guideline is if the teacher leads whole-class discussions and calls only on students who raise their hands. If some students raise their hand regularly while others do not and the teacher continues to call only on students who raise their hands, then the community may start to develop the expectation that students who raise their hands are competent. Over time, the guidelines the teacher indicates explicitly and implicitly and that the students take up become the valued characteristics that lead to status generalization (stage C) and a status hierarchy (stage D) for the classroom community. I next analyze specific classroom examples from this study using the combined framework.

Status Generalization Through Positioning in a Mathematics Classroom

During classroom interactions, the teacher and students make conscious and unconscious decisions about to whom they turn and how they position themselves and others to have varying levels of competence for making mathematical contributions. I describe the classroom context for this research study briefly in order to frame the nature of this classroom's interactions. The data comes from one research study I conducted, where students worked together daily in small groups of 3 to 4 students in an Algebra 1 class. In this classroom, the teacher randomly assigned students to new groups every 2 weeks. During the 26 days I collected video during the first and last units of first semester, the teacher facilitated whole-class conversation about 22% of the class time, students worked in small groups about 72% of the class time, and about 6% of the class time was largely unstructured.

The teacher in this class consistently positioned all students to be mathematically competent. The students in this class positioned themselves and were consistently positioned by the teacher and other students to have varying levels of competence at different times throughout the course of the study. Over time, students' perceptions of their own and their classmates' mathematical competence generally improved. Still, some students developed higher status, some developed lower status, and some students' status changed from group to group, on the basis of their own, their classmates', and the teacher's influence and positioning moves. As mentioned above, my purpose is not to prove claims but to analyze classroom interactions using the combined Status and Positioning Theory framework. I have chosen to analyze these particular interactions to illustrate three different ways Positioning Theory can be used as a vehicle for explaining status generalization in the mathematics classroom.

In *Case 1: Student Positioning Moves Predict Expectations for Competence*, I analyze one group interaction in which certain students contributed the majority of the mathematical ideas for a particular problem on a group test day. I analyze the first example first using Status Theory alone, then using Positioning Theory alone, and finally using the combined framework in order to illustrate the power of the combined framework. In *Case 2: Teacher Actions Develop Expectations for Competence*, I use the combined framework to demonstrate the types of discourse and positioning moves that can lead to varying expectations for competence for different group members. In *Case 3: The Effect of a Teacher Intervention,* I investigate how a teacher intervention repositioned two students as mathematically competent during an equal-status interaction. Case 3 reveals the potential for teacher interventions to interrupt the development of status issues.

Case 1: Student Positioning Moves Predict Expectations for Competence

This case demonstrates the kinds of student-student discourse and positioning moves that can lead to varying expectations for competence and the development of a status hierarchy for group members. This episode occurred during approximately the same time period, about 6 weeks into the school year, one day before the individual test on Unit 1: Linears. In this example, four students were working together in a small group. This was the second day this group had been assigned to work together and the fourth small group that students had been randomly assigned to work with in this class. The group included Naima, Phoebe, Helen, and Brayden, shown in Figure 4.

Figure 4. Student positioning moves predict expectations for competence.

On this class day, the students had been assigned to take a Participation Quiz and a group test. For a Participation Quiz, students earned a grade that represented how well they worked together as a group. In order to receive full credit on the Participation Quiz grade, Ms. Martin suggested that students interact together in specific ways, such as “Get off to a quick start,” “Point and explain,” and use “WHY?!” and “BECAUSE!” sentences. For group tests, students turned in their mathematics work together, and Ms. Martin randomly graded one student’s problem number 1, another student’s problem number 2, and so on, encouraging students to discuss and show work for each problem.[7] In this episode, the students exchanged names and then discussed the first problem on the group test:

> *An electrician started with 52 light bulbs. He installed four light bulbs every three days. Write the equation for the number of light bulbs he has left based on the number of days. Define your variables.*

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[7] For more information on Participation Quizzes and group tests, see Dunleavy (2013, dissertation Paper 2, delegating authority paper).

The group started their work together after the teacher finished giving her whole-class directions:

0 Ms. Martin: ((To the whole class.)) Alright! Out come group tests. I want to see a quick start from every group.

1 Naima: ((In their small group.)) What's your name? ((Pointing and looking at Brayden.))

2 Brayden: Brayden. B-r-a-y-d-e-n. ((Spelling his name.))

3 Phoebe: Wait, what?

4 Naima: B-r-a-y-d-e-n. What's your name? ((Pointing and looking at Phoebe.))

5 Phoebe: P-h-o-e-b-e. Is it B-r-a...?

6 Brayden: B-r-a-y-d-e-n.

7 Naima: And what's your name? ((Pointing and looking at Helen.))

8 Helen: Helen. H-e-l-e-n.

9 Phoebe: Wait, what's your name? ((Pointing and looking at Naima.))

10 Naima: Naima. N-a-i-m-a.

11 Phoebe: N-a- ? Naima: i-m-a. ((Pause.)) Okay.

12 ((Ms. Martin could be overheard talking to another group as the group looked over at her.))

13 Ms. Martin: You guys ((to the other group)) just had the best quick start I've ever seen. Thank you.

14 ((Naima looked back from the other group to whom Ms. Martin had spoken, toward her group, and then read number 1 out loud as Helen and Brayden looked up at what Ms. Martin had already written on the Participation Quiz on the overhead.))

15 Naima: ((Naima read problem 1.)) “An electrician started with 52 light bulbs. He installed 4 light bulbs every 3 days. Write the equation for the number of light bulbs he has left based on the number of days. Define your variables.”

16 ((Slight silence for a moment as everyone looked at their paper.))

17 Phoebe: Okay, ((looking up slightly at Naima)) we'll just do it and then we'll come back together.

18 Naima: Yeah, check in.

19 ((30 seconds of silence while they worked.))

20 Phoebe: ((Not looking up,)) Wait, so he started with zero days and then it will be zero to fifty two, right?

21 Naima: Mmhmm.

22 ((30 seconds of silence while they worked.))

23 Naima: So did anybody else get the equation yet?

24 Phoebe: Wait.

25 ((Another minute of a silence while they worked individually.))

26 Phoebe: ((To Naima, making eye contact)): I got rate of change 1.4.

27 Naima: I got 1.3.

28 Phoebe: If you round it, yeah.

29 Naima: But if you round it, see you're rounding down. Because it's point 5 and up.

30 Phoebe: Yeah. Oh you just put 1.3?

31 Naima: 3, yeah.

32 Phoebe: Wait, yeah. ((Erasing.)) Wait, no.... Yeah.

33 Naima: ((Looking around to the whole group at everyone's paper after a slight pause.)) So did you get your equation? ((Pointing to Brayden.))

34 Brayden: Mmhmm.

35 Naima: Did you get your equation? ((Looking at Helen.))

36 Helen: Mmhmm.

37 Naima: Did you get your equation Phoebe?

38 Phoebe: Mmhmm.

39 Naima: What did you guys get?

40 Phoebe: *y* equals 52 plus 1.3*x*.

41 Naima: ((Joining & nodding.)) Plus 1.3. Yep. Everybody got that? ((Nodding.)) Everybody got rate of change, starting point, blah blah blah?

42 Brayden: Mmhmm.

43 Naima: Find the variables. x = days, y = number of light bulbs? ((Helen nodding.))

44 Naima: Yep? Okay.

45 Namia: Number 2. “Write an equation for the line that goes through the point (5, 1) and has a rate of change of 4 over 5.”

46 ((Group becomes silent and starts working individually on the next problem.))

During this interaction, the group exchanged names (turns 1-11), read problem one out loud and worked on it (turns 12-19), came to consensus on the equation for problem one (turns 20-41), agreed they each had written the rate of change and starting point (turns 41-42), agreed the variables were x = days and y = number of light bulbs (turns 43-44), and started working on problem two (turns 45-46).

Status Theory interpretation of student positioning moves predict expectations for competence. Status Theory can inform how a status hierarchy was developing for this group. Because students were only in their second day of working together, their impressions of one another's mathematical competence were developing on the basis of work together in previous group assignments (in the first three assigned groups, only Naima and Helen had worked together, once) and on any impressions they had gathered from whole-class interactions and outside-class interactions to this point. By this class day, Naima had completed two student-led presentations, while Brayden and Phoebe had each presented once, and Helen had not presented. These students may have correlated mathematical competence with perceived ability to lead student presentations.

Status Theory suggests the number of utterances might set expectations for competence and thus be correlated to who was developing high status in the group. During this interaction, Naima spoke in 19 turns, Phoebe spoke in 11 turns, Brayden spoke in 4 turns, and Helen spoke in 2 turns. Naima initiated the conversation about everyone's name (turn 1), she read problems 1 and 2 out loud (turns 15 and 45), and she initiated the check-in conversations about problem one, "So did anybody else get the equation yet?" (turn 23) and "Did you get your equation?" to each of the group members (turns 33, 35, and 37). She also asked someone to say the equation out loud, saying: "What did you guys get?" (turn 39), and she confirmed whether everyone in the group had the same equation, "Everybody got that? Everybody got rate of change, starting point, blah blah blah?" (turn 41), and she stated the variables "x = days, y = number of light bulbs" (turn 43). Phoebe asked clarifying questions about individual's names (turns 3, 5, 9), she suggested the group members do the problem separately and "Then we'll come back together" (turn 17), she confirmed her understanding of the starting point: "Wait so he started with zero days and then it will be zero to fifty two, right?" (turn 20), she said "Wait" (turn 24) when Naima asked the group if anyone had the equation (turn 23), she initiated the first answer for the rate of change "I got rate of change 1.4" (turn 27), and she responded to Naima's request for an equation, saying "y equals 52 plus 1.3*x*" (turn 40). Brayden responded with his name, confirmed he had the equation "Mmhmm" (turn 34) and that he had the rate of change and starting point "Mmhmm" (turn 42). Helen was also relatively quiet, responding with her name and confirming she had the equation "Mmhmm" (turn 36). The varying number of utterances among group members suggests this group's storyline may produce status differences over time.

When Phoebe asked, "Wait, so he started with zero days and then it will be zero to fifty-two, right?" (turn 20), she indicated she felt comfortable asking a clarifying question. After another brief pause when Naima asked, "So did anybody else get the equation yet?" (turn 23), Phoebe was the first to assert an understanding of the equation, possibly responding quickly to be perceived as competent. Phoebe's response to Naima, "Wait" (turn 24), indicated she felt comfortable valuing time before responding. Phoebe was also the first to respond after Naima's silence, saying "I got rate of change 1.4" (turn 26).

The majority of utterances in this interaction came from Naima and Phoebe, while the minority of utterances came from Brayden and Helen. Naima and Phoebe asserted mathematical ideas during this interaction, while Helen and Brayden confirmed their group members' assertions. Status generalization can by hypothesized but not confirmed from this interaction alone. Status Theory suggests if this interaction pattern were to continue, it may have led Naima and Phoebe to develop higher status, while Brayden and Helen may have developed lower status.

Positioning Theory interpretation of student positioning moves predict expectations for competence. Using Positioning Theory to analyze this interaction suggests interpreting the group's moment-to-moment positioning moves, the group's storyline, and the individuals' resulting positions. Positioning Theory explains that Naima positioned Phoebe's idea for how to engage in groupwork as valid. Phoebe said, "We'll just do [the problem] and then we'll come back together" (turn 17), and Naima responded, "Yeah, check in" (turn 18). Phoebe first positioned Naima as mathematically competent when she accepted Naima's confirmation that the starting point was 52 (turns 20 and 21). Next Phoebe accepted Naima's explanation that rounding meant the rate of change was 1.3 instead of 1.4 (turns 26-32). Naima positioned herself both as competent and as complying with Ms. Martin's expectations for groupwork when she read the group test questions aloud (turns 15 and 45), when she asked the group to come to consensus on their equations (turns 23, 33, 35, 37, and 39), and when she confirmed rate of change and starting point (turn 41), and variables (turns 43 and 44). Brayden also positioned Phoebe and Naima as competent when Phoebe gave the equation "" (turn 40), Naima asked, "Everybody got that?" (turn 41), and he agreed (turn 42).

Positioning Theory suggests Brayden missed an opportunity for a metapositioning challenge when the group made a mathematical mistake in coming to consensus on the equation as $y = 52 + 1.3x$. The group's work, in alphabetical order of each student's pseudonym, is shared in Figures 5-8. Naima's work, shown in Figure 7, illustrates her work was graded for problem one for which the group received partial credit for not having the correct sign for the rate of change. Brayden's work, shown in Figure 5, illustrates he wrote $y = 52 – 1.3x$. If Brayden had used a metapositioning challenge to suggest the equation was $y = 52 – 1.3x$ instead of $y = 52 + 1.3x$, his ideas may have been positioned as competent, the group may have better understood rate of change, and the group may have received full credit on this problem on the group test.

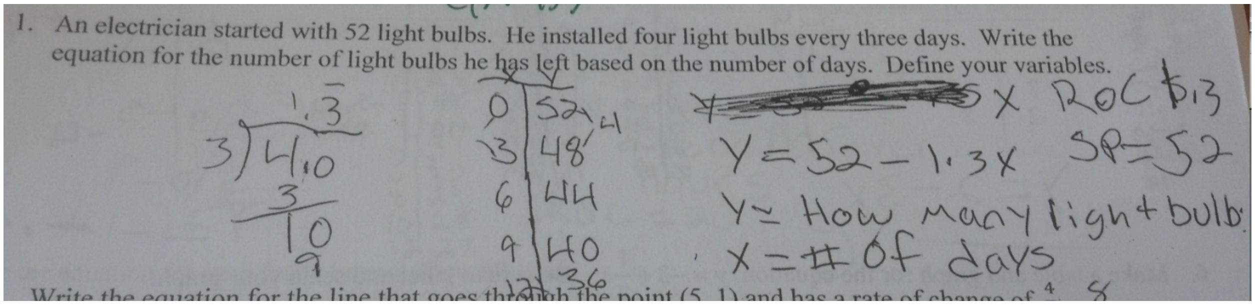

Figure 5. Brayden's problem 1.

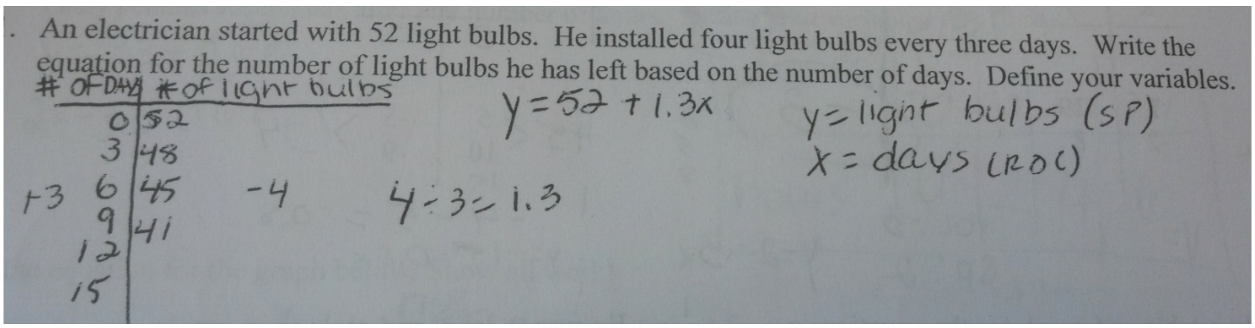

Figure 6. Helen's problem 1.

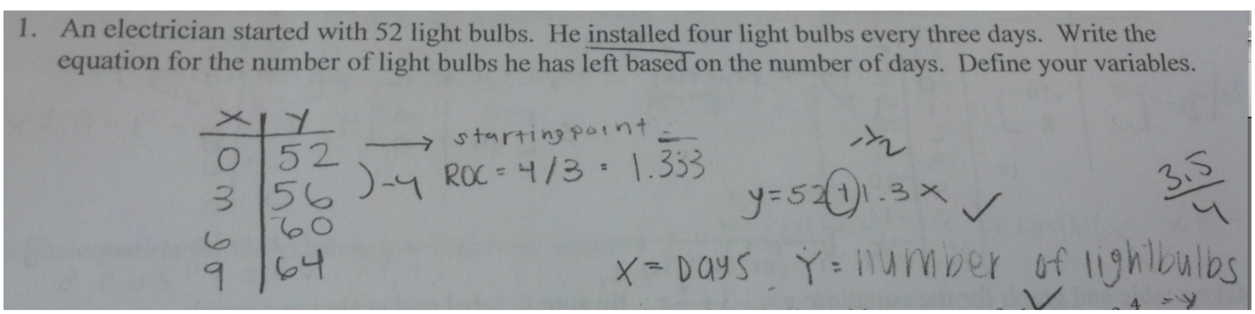

Figure 7. Naima's problem 1.

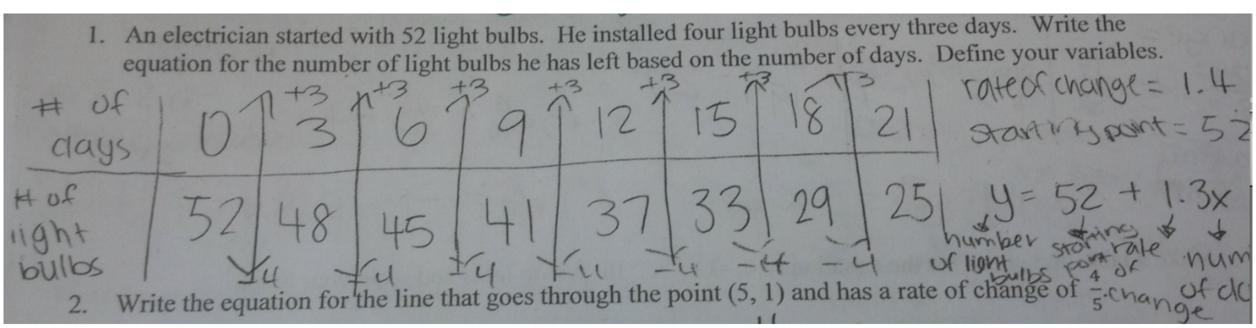

Figure 8. Phoebe's problem 1.

Combined Status and Positioning Theory interpretation of student positioning moves predict expectations for competence. The *Student Positioning Moves Predict Expectations for Competence* interaction and the students' short prior history does not show evidence of significant prior status generalization among these students. However, Status Theory suggests that if similar interaction patterns continued, the group may have developed status issues. Naima and Phoebe may have developed high status, while Brayden and Helen may have developed low status. The status issues may mean that Brayden's and Helen's mathematical ideas could not be heard as often as Naima's and Phoebe's. Because Brayden's paper showed the correct mathematical equation ($y = 52 - 1.3x$) and this different equation was not discussed before the group moved on, Positioning Theory suggests Brayden may have missed an opportunity to use a metapositioning challenge to suggest that the equation offered by Naima and Phoebe ($y = 52 + 1.3x$) had a mathematical error. However, analyzing this episode with each theory alone does not provide a complete picture of the nature of interaction and learning in this group. The combined framework offers the potential for status generalization to inform an understanding of the group's moment-to-moment interactions, suggesting Brayden's silence may have led the group to position him to have different status than either Naima or Helen.

The combined framework points to the potential development of valued status characteristics through the ways students interacted here and before this episode took place. Prior to the episode, the teacher had been fostering norm development that encouraged students to value engaging in mathematical justification.[8] During the episode, Naima and Phoebe interacted through mathematical justification. The group also verbally positioned Naima's and Phoebe's ideas as valid, such as when Naima explained why the rate of change was 1.3 instead of 1.4

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[8] See evidence of norm development in Dunleavy, 2013, dissertation Paper 3, norms and perceptions of competence.

(turns 26-32) and when Phoebe gave the starting point (turn 20) and the equation (turn 40). In this way, the group was starting to position mathematical talk as a valued status characteristic (position A in Figure 3). If and when mathematical talk became an established valued status characteristic, the group also may have started to expect students to offer mathematical explanations, thus moving along the framework from position A to position B. Brayden also had a correct mathematical equation (Figure 5) that he did not share. If he continued to remain silent in this group, his mathematical ideas would not be heard and he may have been eventually positioned to have lower perceived competence and therefore lower status. A projected analysis of the group's developing and enduring storyline could then lead to a generalization of individuals' status in this group and classroom (positions C and D). Further discursive acts and positioning moves could indicate metapositioning moves with the potential to restart the process of status generalization.[9]

Case 2: Teacher Actions Develop Expectations for Competence

During the 26 days I observed and collected video data from Units 1 and 4 of this study, about 9% of the class time was dedicated to student-led class presentations.[10] Fifteen different students presented during this time period. In one case about 5 weeks into the school year, during Unit 1: Linears, Ms. Martin asked students to present their work from problems on the warm up. Neesha,[11] the tenth student to present by that point in the school year, presented problem 1b, illustrated in Figure 9, below.

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[9] Additional analysis showed that as time went on in this class and this group, Brayden and Helen were positioned as competent by their classmates during different episodes.

[10] This 9% was a part of the 22% of whole-class time mentioned earlier.

[11] All names are pseudonyms.

1. For each problem, fill in the missing information.
b. Equation: ____________

x	*y*
-2	11
3	6

Figure 9. Problem 1b.

A transcript of Neesha's presentation of Problem 1b is included here, starting with Ms. Martin bringing the class together after small-group work time:

1	Ms. Martin:	I actually bugged some students to present.
2	Ms. Martin:	You're going to have to be willing to add to the conversation because I like your way of talking about it.
3	Ms. Martin:	Okay, 1b? Neesha!
4	Neesha:	((In front of the room, writing on the Smartboard on and near the table pictured in Figure 9.)) Okay, so the difference between negative 2 and 3 is 5. And the difference between 11 and 6 is 5, but it's negative so it would be negative 5 because that's on the *y* side, divided by 5, which equals 1... negative 1. So that would be the rate of change.
5	Neesha:	And so now I have to find the starting point, so now I just go down by negative 2, or up by 2, and it's 11, then 10, because it just goes down by 1. 9, 8, 7, 6, 5...and so this is the starting point. ((Circling 0, 9.))
6	Neesha:	And so it would be $y = 9$ minus 1*x*. ((See Figure 10.))

7 Ms. Martin: Good. Questions?

8 Ms. Martin: So I bugged [Neesha] because I really liked her extended table. I thought that was really straightforward what the starting point was.

9 Ms. Martin: I worry when I walk around and I see the zero pushed into the table. I don't know if some people really understood how they got that, or if they just got the answer from talking to some people in their group, but aren't sure about where it came from. And all of Neesha's work shows us where it comes from. Yeah. Thank you my dear.

1. For each problem, fill in the missing information.
b. Equation: ____________

x	*y*
-2	11
-1	10
0	9
1	8
2	7
3	6

Rate of change:
Equation:

Figure 10. Neesha's presentation of problem 1b.

Figure 10 represents what Neesha had written on the board by the end of her presentation.[12] As she said, "And the difference between 11 and 6 is 5, but it's negative so it would be negative 5 because that's on the y side, divided by 5, which equals 1" (turn 4), she wrote on the board "-5/5

[12] Because the video data does not clearly show Neesha's work, I created Figure 10 as a representation of what she wrote.

= -1." Neesha then explained and circled the starting point (turn 5), and then wrote the equation (turn 6).

Analysis of teacher actions develop expectations for competence. In Case 1, I first used each framework to separately analyze a group interaction. In Case 2, I use the power of the combined analytic framework to analyze what can happen when the teacher positions students as competent. The example offered in Case 2 illustrates Ms. Martin assigning competence to Neesha, positioning her as able to explain her mathematical understanding of rate of change and starting point to the class. It also illustrates Neesha accepting Ms. Martin positioning her as competent. In turn 1, Ms. Martin said, "I bugged some students to present," suggesting she had made arrangements with students in advance. In turn 2, Ms. Martin started by saying, "You're going to have to be willing to add to the conversation." Status Theory suggests Ms. Martin was setting an expectation that students who presented would be expected to share mathematical thinking. Next she said to the class (about the presenters), "…because I like your way of talking about it," positioning the students who would present to have worthwhile methods for describing how they were thinking about rate of change and starting point. Turns 1 through 3 both positioned Neesha as competent and set expectations for mathematical contributions. In turn 4, Neesha accepted this positioning and shared her method for finding that the rate of change was negative 1. In turn 5, Neesha continued presenting her mathematical ideas, sharing how she found the starting point, which she circled on her table, as illustrated in Figure 10. In turn 6, Neesha shared her equation for problem 1b. In turns 7 and 8, Ms. Martin again positioned Neesha's mathematical contributions as competent after her presentation, saying, "Good," and then telling the class what was mathematically valid about Neesha's presentation: "I really liked [Neesha's] extended table. I thought that was really straightforward what the starting point was."

Ms. Martin used the word “straightforward,” positioning Neesha’s extended-table method for finding starting point as accessible to the mathematics and to her classmates. In turn 9, Ms. Martin linked Neesha’s explanation to what may have been a misconception other students had made during the work time: “I worry when I walk around and see the zero pushed into the table. I don’t know if people really understood how they got that…And all of Neesha’s work shows us where it comes from.” Assigning competence to Neesha and her mathematical explanation for rate of change, starting point, and table representation was a deliberate positioning move Ms. Martin may have used to demonstrate Neesha was capable of publicly sharing a mathematical explanation.

Throughout the year, Ms. Martin nurtured public mathematical sensemaking as a classroom value by regularly asking students to engage in presentations and by assigning them competence for mathematical justification. This episode features Ms. Martin positioning Neesha as competent to explain her method for finding rate of change and starting point, and it demonstrates Neesha justifying her mathematical ideas to the whole class. Because the presentation took place during the fifth week of the school year, Ms. Martin may have been trying to influence and reinforce valuing people who use mathematical justification by encouraging writing representations and explaining mathematical thinking. The example illustrates how Ms. Martin positioned student-led presentations and how she potentially influenced Neesha’s self-image, her public image, and potentially which status characteristics would be valued in the classroom. Ms. Martin used presentations to position students as competent mathematical sensemakers, and she also may have hoped presentations would increase students’ status by linking students’ mathematical talk to perceived mathematical competence.

Classroom presentations are often occasions for competence positioning moves, displays of status, and valued status characteristics. Researchers can analyze teacher moves to make conjectures about which status characteristics have become or may become valued in the classroom. Other data could be analyzed using constant comparison methods in order to confirm or disconfirm the offered conjectures.

Case 3: The Effect of a Teacher Intervention

In Case 3, Ms. Martin repositioned two potentially *equal-status* students as competent when they called her over to intervene in their mathematical conversation. The teacher intervention positioned each student's ideas as competent, reinforcing their equal-status interaction to position both students' mathematical approaches as valuable. In this group, Elena and Neesha were assigned to work together. Although they had not worked together in previous groups, both students had observed at least one whole-class presentation from the other student. In the interaction that follows, Elena and Neesha were discussing solving for y in problem 6 from their assignment. Problem 6 read: $y + 5 - x = 2y + 3x - 1$. A mathematical discussion resulted from their two different methods for solving this problem.

1	Elena:	I got my answer.
2	Neesha:	What'd you get?
3	Elena:	y equals 6 minus 4x.
4	Neesha:	Well, it will be negative 4x... or…and negative 6...yeah.
5	Elena:	Not on mine.
6	Neesha:	Well, cuz you have to make… because, because the y is originally negative, so you have to make the 4.
7	Elena:	Wait, no it's not. Where? Where is it negative?

8	Neesha:	Because you're subtracting 2y from y.
9	Elena:	Oh, I subtracted y from 2y ((pointing to two spots on her paper)).
10		((Slight pause.))
11	Neesha:	But the thing is, is that, but then...
12	Elena:	((Pointing to two spots on her paper again)) I did it this way, not that way.
13	Neesha:	Because, the thing is that then like all the numbers would be on the same side. Yeah. ((Slight pause while Elena looks at her own paper.))
14	Elena:	Because I don't think we can have a negative y.
15		((Slight pause, during which Neesha calls Ms. Martin over.))
16	Neesha:	Wait, Ms. Martin? So, like, what I did was subtract 2y from y ((Ms. Martin looks at Elena's paper while Neesha talks)), but [she] subtracted y from 2y.
17	Ms. Martin:	Okay.
18	Neesha:	But we pretty much got the same answers, just the negatives...like...
19	Ms. Martin:	Okay.
20	Neesha:	Because I got negative y equals 4x minus 6 so I had to make the 4x a negative. And, yeah.
21	Ms. Martin:	So you got negative y equals 4x minus 6? Can I show the visual on [your explanation] really quick? So I can wrap my brain around it?
22		((Ms. Martin pulls out Lab Gear[13] to illustrate the problem.))
23	Ms. Martin:	So you're saying, you ended up here? Negative y ((placing the y piece in the negative area of the left-hand side of the workmat)) equals positive 4x

[13] Henri Picciotto, Wright Group/McGraw Hill – Creative Publications: https://www.wrightgroup.com/launch/wright_group.html. For more information on using Lab Gear in a mathematics classroom, see http://www.mathedpage.org/manipulatives/lab-gear.html.

((placing the $4x$s on the right-hand side of the workmat in the positive area)) and then subtract 6 ((placing the 6 ones on the right-hand side of the workmat in the negative area? Now did you guys get the same, did you get the same thing? Did you get $4x$s and 6 numbers?

24 Elena: Mmmhmm.

25 Ms. Martin: And so then you said you can’t have a negative y and so you opposite-ed it? ((moving the Lab Gear piece from the negative to the positive area of the left-hand side of the workmat.))

26 Neesha: Mmmhmm.

27 Ms. Martin: And then what happened to the other side?

28 Neesha: These have to go ((moving the $4x$s)).

29 Neesha: Wait, but do these have to go down? ((touching the 6 ones and looking at Ms. Martin.))

30 Ms. Martin: Mmhmm. Everything goes opposite.

31 Ms. Martin: Now does that match, is that okay? What do you have? Can you read it? So you have 6 minus $4x$ equals y ((reading from Elena’s paper)). Is that what you have? ((asking Neesha.))

32 Neesha: No, I have negative $4x$ and positive 6.

33 Elena: That’s what I have.

34 Ms. Martin: But she said subtract $4x$ and you said negative $4x$ plus 6.

35 Neesha: So we’re okay.

36 Ms. Martin: Yeah, we’re okay on that one.

Combined Status and Positioning Theory interpretation of the effect of a teacher intervention. For Case 3, I use the power of the combined Status and Positioning Theory framework (Figure 3) to explain the developing storyline for a group where Neesha and Elena each positioned her own ideas as mathematically valid (turns 3, 4, 6, 7, 8, 9, 12, 13, and 14). The exchange of ideas demonstrates that perceived competence to engage in mathematical sensemaking was emerging as a valued status characteristic in this group (position A in Figure 3). Other developing valued status characteristics included perceived ability to check in with group members (turns 1-2 and 33-36), perceived competence to use visuals and manipulatives (turns 9-11 and 28-29), perceived reasons for calling over the teacher (turn 15), and perceived ability to arrive at mathematical consensus (turns 1, 2, 7, 12, 18, 23, 31, 35, 36).

Because Neesha and Elena kept talking about the same problem for several exchanges, they were moving toward the shared behavior expectations that perceived competence to engage in sensemaking involved responding to group members' questions (Figure 3, position B). Initially, Elena shared a correct representation of solving for *y*, saying, "*y* equals 6 minus 4*x*" (turn 3). Neesha then explained that she thought because the *y* was negative, it would make the 4*x* negative. She did not mention what would happen to the 6 in the equation, saying, "The *y* is originally negative so you have to make the 4..." (turn 6). Elena did not initially seem to understand why Neesha thought the *y* was "originally negative," presumably because they had each worked on the problem in a different way. Elena replied, "Wait, no it's not. Where? Where is it negative?" (turn 7). Neesha explained, "Because you're subtracting 2*y* from *y*" (turn 8). At this point Elena seemed to understand their different approaches, as she replied, "Oh, I subtracted *y* from 2*y*" (turn 9) and then ((pointing to two different places on her paper)) "I did it this way, not that way" (turn 12). Neesha explained why she was concerned about subtracting *y* from 2*y*, “Then like all the numbers would be on the same side” (turn 13). Elena next explained her concern with subtracting 2*y* from *y*: “I don’t think we can have a negative *y*” (turn 14). While Neesha was correct that solving for *y* ultimately meant not have everything on one side, Elena was also correct that the problem would not be finished by leaving a negative *y.* This series of exchanges moved the group’s storyline from behavior expectations (position B) toward status generalization (position C).

Neesha next called Ms. Martin over to help (turn 15), explaining she and Elena had each subtracted *y* and 2*y* in opposite ways (turn 16). Neesha went on to explain that their answers seemed to differ on “just the negatives” (turn 18), and she went on, “I got negative *y* equals 4*x* minus 6 so I had to make the 4*x* a negative” (turn 19). Ms. Martin listened to Neesha as she talked, reinforcing a value of engaging in mathematical sensemaking (turns 17, 19, 21).

Ms. Martin next reinforced students’ competence to use visuals through pointing, explaining, and using manipulatives as another status characteristic for the students to potentially value. She asked Neesha and Elena to consider the problem using Lab Gear (turns 21, 23, 25, 28, 29). After listening to Neesha explain, Ms. Martin said: “Can I show the visual [of your explanation] so I can wrap my brain around it?” (turn 21). Ms. Martin then set up Neesha’s method using Lab Gear on a workmat facing Neesha, as illustrated in Figure 11.

Figure 11. Problem 6 using Lab Gear.

Ms. Martin repeated what Neesha said as she placed the Lab Gear pieces,

> So you're saying, you ended up here? Negative *y* ((placing the *y* piece in the negative area of the left-hand side of the workmat)) equals positive 4*x* ((placing the 4*x*'s on the right-hand side of the workmat in the positive area)) and then subtract 6? ((placing the 6 one's on the right-hand side of the workmat in the negative area)) (turn 23).

Here Ms. Martin reinforced the mathematical talk of problem 6 around the use of a visual, as a way to continue to develop mathematical sensemaking and consensus in the classroom. A worksheet representation of how Ms. Martin set up the workmat as she talked is illustrated in Figure 12.

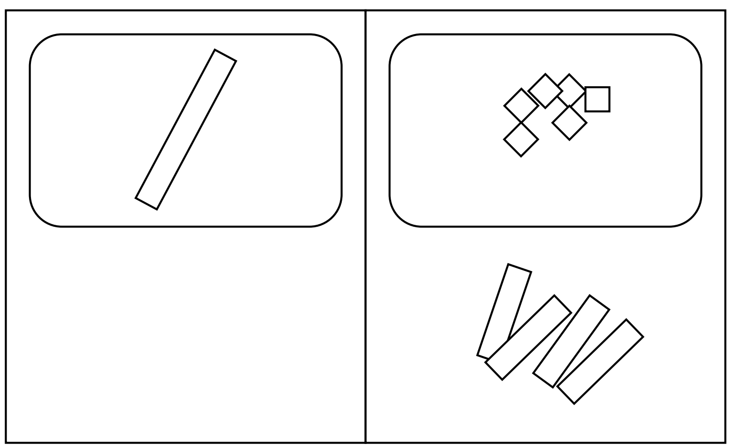

Figure 12. Lab Gear on the workmat.

As she looked at Elena and Neesha's papers, Ms. Martin next asked Neesha: "And so then you said you can't have a negative *y* and so you opposite-ed it? ((moving the *y* Lab Gear piece from the negative to the positive area of the left-hand side of the workmat)) (turn 25). Neesha confirmed that was her thinking, so Ms. Martin asked her, "And then what happened to the other side?" (turn 27). Neesha replied, "These have to go" as she moved the 4*x*s up to the negative area. Neesha next asked, "Wait, but do these have to go down?" ((touching the 6 ones and looking at

Ms. Martin)) (turn 29). Ms. Martin replied, “Mmmhmm. Everything goes opposite” (turn 30). The shifting of Lab Gear tiles where “everything goes opposite” is illustrated in Figure 13.

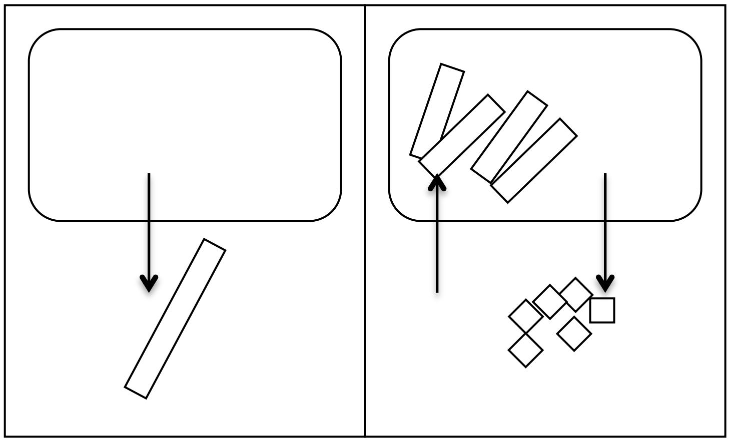

Figure 13. Everything goes opposite.

Ms. Martin next read from Elena’s paper, “you have 6 minus 4*x* equals *y*,” and the she turned back to Neesha, “Is that what you have?” (turn 31). Neesha replied, “No, I have negative 4*x* and positive 6” (turn 32), to which Elena responded, “That’s what I have” (turn 33). At this point, Neesha realized the answers were the same, saying, “So we’re okay” (turn 35). Ms. Martin validated Neesha’s ideas by listening, developing the expectation that mathematical competence can be perceived and mathematical consensus can be reached through visuals and use of the Lab Gear. Elena and Neesha’s work on this problem is shown in Figures 14 and 15.

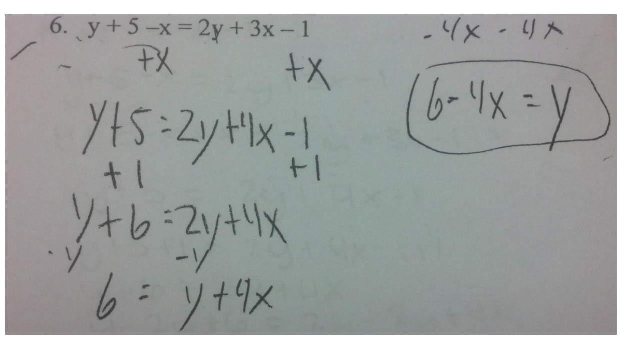

Figure 14. Elena’s problem 6.

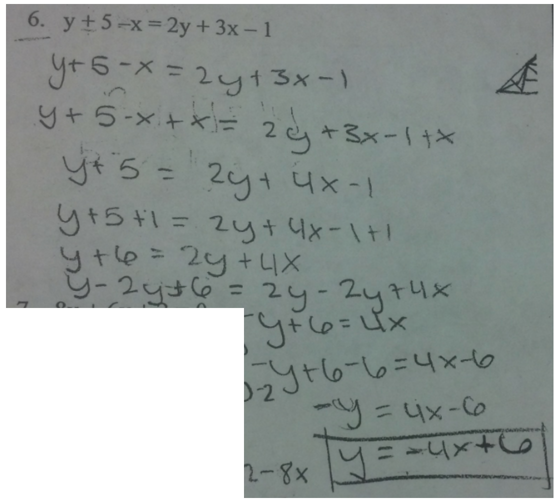

Figure 15. Neesha's problem 6.

The Case of *So We're Okay* demonstrates how Ms. Martin intervened in this group's equal-status interaction. The storyline for Elena and Neesha developed both through previous classroom interactions and through the moment-to-moment exchanges in the discussion of Problem 6. Some status characteristics may have been moving toward status generalization (position C in Figure 3), including perceived competence to engage in mathematical sensemaking and perceived ability to arrive at mathematical consensus. In this episode, Neesha and Elena had similar rates of interaction, asked a similar number of questions, and therefore demonstrated similar abilities to engage in mathematical sensemaking. These students, therefore, demonstrated approximately *equal-status* (position D). If these interaction patterns continued, this teacher positioning move could interrupt the development of any status issue. Elena and Neesha would then maintain similar rates of influence in their group, they would continue to be perceived to have mathematically valid ideas, and their learning could thrive.

Addressing Status and Positioning in the Classroom

Together, Status and Positioning Theory can explain how to attend to the complex nature of status and positioning in the classroom. The combined Status and Positioning Theory framework can be used to explain how expectations for competence are mediated by teacher and student actions in cases like *Case 1: Student Positioning Moves Predict Expectations for Competence*, *Case 2: Teacher Actions Develop Expectations for Competence*, and *Case 3: The Effect of a Teacher Intervention*. These cases demonstrate how a combined analytic framework could be used over a series of classroom episodes to predict and explain why and when students turn to one another during group interactions, how the teacher can interrupt the development of status issues, and how the students and teacher position one another to develop certain expectations for competence over time.

Whereas Status Theory allows for explanations of which expectations for competence have been generalized over time, Positioning Theory addresses the moment-to-moment dynamic roles individuals carry in their collaborative construction of personal stories (Harré & van Langenhove, 1999). Positions, discourse moves, and the ways students go about working on mathematics together drive the group's resulting storyline. A group member can challenge the right of another to assign positions, and a group member can challenge the way she or he has been positioned in a certain context. When positioning challenges are accepted as valid, the outcome can result in newly understood positions, which can result in students increasing their perceived competence. Research focused on reformed education suggests students benefit from group interactions (see Boaler & Staples, 2008; Cohen, 1997a; Esmonde, 2009). Because Status Theory does not explain how moment-to-moment interactions unfold, using Status Theory alone cannot explain how positioning moves lead to status differences. The combined Status and

Positioning Theory analytic framework suggests students can establish valued and generalized status characteristics that would lead to enduring perceptions of competence for students in this class.

I offer the combined framework to explain classroom positioning moves in action and to suggest ways that analysis of classroom interactions can inform expectations for competence, status generalization, and learning. An understanding of the complexity of group interactions and how to address them can inform what teachers need to do to combat status issues in the classroom. Elizabeth Cohen, Rachel Lotan, and a growing group of scholars have cited several strategies for teachers to address status and positioning issues in the classroom (see Cohen, 1994a; Cohen & Lotan, 1997; Featherstone et al., 2011; Turner, Dominguez, Maldonado, & Empson, 2012). These strategies include but are not limited to publicly assigning competence to students to display their mathematical abilities and increase their status, assigning open-ended tasks where multiple solution paths are possible, offering tasks that require multiple abilities and therefore multiple ways to be competent in mathematics and publicly naming these multiple abilities before students work on the task, offering opportunities for students to take on agentive problem-solving roles, and fostering norms in which students need one another's multiple unique perspectives to complete mathematics tasks better than they would on their own. Status Theory alone does not attend to how strategies like these disrupt and mitigate status issues. Addressing status and positioning issues through the lens of the combined Status and Positioning Theory framework helps explain the complexity of group interactions and the possibility for providing opportunities for all students to see the ways they are mathematically competent.

Abstract Paper 2

In pursuit of high-level opportunities for all students, mathematics education research still lacks significant qualitative documentation of equitable teaching and learning practices in secondary classrooms. In this study, I used ethnographic methods to analyze student and teacher interviews and classroom interactions during the first and last units in a semester one Algebra 1 class. Classroom observations and video analysis revealed that the teacher delegated mathematical authority through the use of specific pedagogical structures and practices. In this paper, I address how the teacher was striving toward equity through the use of classroom structures that transferred mathematical authority to students. The examined pedagogical structures include regular random small group assignments, groupworthy tasks, student-led presentations, Shuffle Quizzes, Participation Quizzes, and group tests. The examined pedagogical practices include positioning students as competent sensemakers, orienting students to use one another as mathematical resources, requiring mathematical justification, and expanding the definition of smartness in mathematics. Interviews with three classroom students reveal that the teacher's pedagogical structures and practices expanded students' perceptions of smartness in mathematics. This study offers insight into one teacher's process for transferring mathematical authority to her students through specific pedagogical structures and practices as a means to strive toward equity.

PAPER 2: *"Ms. Martin Is Secretly Teaching Us!"*

Delegating Mathematical Authority as a Means to Strive Toward Equity

Attention to the pursuit of equity has been a growing priority in mathematics education in the last two and a half decades (e.g., Esmonde, 2009a; JRME, 2013; Martin, 2003; Nasir & Cobb, 2007; NCTM, 1989, 2000). Equitable classrooms offer "a fair distribution of opportunities to learn" (Esmonde, 2009b, p. 1010), in which teachers and students view everyone as capable of learning high-level content (Cohen, 1997a). Striving toward equity is a mark of the pursuit of justice in mathematics educational learning outcomes. The field is beginning to depict the complex work entailed in classroom practices that strives toward equity. Making progress toward equitable learning opportunities in mathematics education also requires constant, purposeful work that seeks to diminish differences in access, opportunities, and outcomes (Gutiérrez, 2007). Because mathematics educators do not yet deeply understand the processes involved in teaching and learning pedagogies that effectively move toward equitable learning outcomes, the analysis in this paper is motivated by the need for more qualitative documentation of what equitable teaching and learning looks like in practice.

Educators and researchers have established cooperative learning as a means to offer students with more equitable learning opportunities (e.g., Boaler & Staples, 2008; Cohen et al., 1999; Esmonde, 2009b; Gutiérrez, 2002). Effectively implemented cooperative learning strategies (Cohen & Lotan, 1997) have also been related to positive gains in mathematics achievement, learning, and movement toward equity in heterogeneous classrooms (e.g., Boaler & Staples, 2008; Cohen, 1997b; Esmonde, 2009a). In spite of establishing cooperative learning as a central construct to the teaching and learning of mathematics that strives toward equity, the field is just beginning to analyze classroom cases that strive toward equitable opportunities and outcomes (see Boaler, 2002; Boaler & Staples, 2008; Esmonde, 2009b; Zahner, 2011).

Because cooperative learning practices such as complex instruction (Boaler & Staples, 2008; Cohen, 1994; Cohen & Lotan, 1997, Featherstone et al., 2011) have been linked to equitable teaching and learning opportunities, in this paper I analyze the work of one teacher whose pedagogical practices were centered around the use of complex instruction. The analysis attends to the elements of the teacher's practice that strive toward equity, thus filling a gap in the empirical research on equity to "uncover a range of solutions focused on what works, where, when, and why" (Martin, 2003, p. 18). Analysis of the teacher's pedagogical practices reveals that she positioned students as competent sensemakers, oriented students to use one another as mathematical resources, and required mathematical justification. In this paper, I demonstrate how these instructional strategies served to delegate mathematical authority to students, thus striving toward equitable learning opportunities and expanding the definition of smartness in a mathematics classroom.

This study reveals delegating mathematical authority as a pedagogical practice that offers equitable learning opportunities to students. Specifically, I argue that one aspect of teaching and learning that can be significant in the complex work of striving toward equity is transferring mathematical authority to students. In pursuit of instructional strategies that strive toward equity, I contribute to a need to understand the teacher's process for delegating mathematical authority to her students, as called for by Esmonde (2009a) and Zahner (2011).

Delegating Authority as a Means to Strive Toward Equity in Mathematics Classrooms

In this section, I address the voices of researchers who have contributed to the pursuit of equity in mathematics education in critically important ways. I highlight the ways in which inequity in mathematics education has created a lack of opportunities for some underrepresented students, particularly African American, Latino/a, Native American, poverty impacted students, and English learners. I address how some researchers have critiqued and (re)defined equity, demonstrating that student positioning matters and delegating mathematical authority can become a valued characteristic in striving toward equity.

Striving Toward Equity in Mathematics Classrooms

In 1989, NCTM published mathematics *Curriculum and Evaluation Standards*, stating the expectation that all students be given opportunities to learn and reason about mathematics. Now, in 2013, more than two decades after the *Curriculum and Evaluation Standards*, the field of mathematics education has yet to offer significant evidence of acceptable classroom solutions to differential experiences for underrepresented students. In particular, mathematics education researchers and educators must continue to deliberately pay attention to the school experiences and investment in learning for underrepresented students, including African American, Latino/a, Native American, students impacted by poverty, and English learners.

Researchers in mathematics education have addressed the creation of equitable classroom learning opportunities for students by arguing it matters how teachers view, attend to, and develop students' mathematical competence. Cohen (1997a) asserted that in equitable classrooms, teachers and students view each student as capable and competent in learning high-level content. When teachers and students view *each* student as competent, classrooms have increased access to work on mathematics. Gresalfi and Cobb (2006) highlighted understanding the construction of competence by studying the construction of classroom norms. They suggested that mathematics education researchers and educators focus on ways students are positioned to interact with other students, instructional materials, and associated tools, including the teacher. In particular, Gresalfi and Cobb argued that understanding students' dispositions informs how and whether students develop mathematical knowledge. Bishop's (2012) study presented a case for examining the nature of individual student interactions as a way to strive toward equity. Bishop argued that because the two students she studied developed a consistent pattern of interaction in which one student dominated the pair's conversations and mathematics decisions, they enacted inequitable identities. Turner et al. (2012) called for more study of instructional practices that support equitable participation. The work of Cohen (1997a), Gresalfi and Cobb (2006), Bishop (2012), and Turner et al. (2012) highlights a need to pay attention to the development of classroom norms and classroom interaction patterns. Taken together, these arguments reveal that whether and how all students are expected to know mathematics in a particular classroom is linked to a belief and enactment about whether and how all students are capable of learning mathematics. The ways teachers attend to these beliefs indicates the presence or absence of striving toward equity.

Delegating Mathematical Authority as a Means to Strive Toward Classroom Equity

Gresalfi and Cobb (2006) define *mathematical authority* as:

> the degree to which students are given opportunities to be involved in decision making and whether they have a say in establishing priorities in task completion, method, or pace of learning. Thus authority is not about "who's in charge" in terms of classroom management but "who's in charge" in terms of making mathematical contributions (p. 51).

Gresalfi and Cobb asserted that *who* has mathematical authority in a particular classroom depends on *who* has been given the opportunity to verify that a given mathematics contribution is reasonable. Researchers have studied the delegation of authority as a way in which teachers can make students responsible for their own and their classmates' learning (Bianchini, 1999; Lotan, 1997); support students to use one another as resources (Cohen, 1997a); increase student-student interdependence and shift checking for understanding to students (Ehrlich & Zack, 1997); and make students' life experiences, opinions, and points of view legitimate components of what is learned (Hand, 2003; Lotan, 2003).

Teachers delegate mathematical authority when they offer the opportunity for students to verify a mathematical contribution is valid. When students are required to interact with one another, work to convince their peers that their solutions make mathematical sense (Gresalfi, Martin, Hand, & Greeno, 2009), and produce individual and group products (Lotan, 1997), teachers have delegated some mathematical authority away from themselves and into the hands of their students. Delegating authority empowers students to argue, evaluate, and confirm the validity of their mathematical ideas. Students take on responsibility to explain mathematical concepts, answer questions, demonstrate multiple solution methods, and co-construct their overall conceptual understanding (Bianchini, 1999; Boaler & Staples, 2008; Gresalfi, 2009). Delegating authority is visible in part when the teacher analytically scaffolds opportunities for students to participate in sustainable mathematical discourse (Nathan & Knuth, 2003). Delegating authority can also be visible through: a) substantive student discourse, b) task cards, c) cooperative norms, and d) use of procedural roles that set clear expectations for group and individual products (Cohen et al., 1994).

I argue that delegating mathematical authority can be an equitable teaching and learning practice. This study serves to inform the research base for equitable mathematics teaching and learning by uncovering one teacher's process to delegate mathematical authority to her students. (Esmonde, 2009a and Zahner, 2011, have called for such analysis to uncover the process of transferring mathematical authority from teachers to students.) In this paper I show how an Algebra 1 teacher used the pedagogical strategy *delegating mathematical authority* as a way to strive toward equitable learning opportunities in her mathematics classroom.

Research Questions. *Striving toward equity* in mathematics education invokes constant, purposeful work that seeks to diminish differences in access, opportunities, and outcomes for mathematics students. Mathematics educators must diligently and unapologetically monitor individual and collective progress toward achieving equitable learning outcomes. In this paper, I build on and contribute to current understandings of striving toward equity in mathematics education by investigating the processes in which mathematical authority is delegated from teacher to students (Esmonde 2009a; Zahner 2011), asking

1. What structures and practices did the teacher engage to transfer mathematical authority to students?
2. How was the process of transferring mathematical authority to students linked to striving toward equity?

Theoretical Framework: A Sociocultural Perspective of Learning

I describe learning as the process that unfolds as students develop through changing participation in the social community of the classroom (Rogoff, 2003). In particular, I draw on Rogoff's characterization of learning as changing participation in the culture of the classroom (Rogoff, 1995, 1997, 2003). In the classroom I studied, students engaged with one another through small group and whole class interactions. Working together in groups of 3 to 4 students, they co-constructed their individual and group learning experiences, positioning themselves and their classmates at various levels of mathematical competence to complete mathematical tasks. In adopting Rogoff's perspective, I suggest students developed[14] through a process of changing participation as they engaged in group interactions within the sociocultural community of the classroom.

Rogoff (2003) has drawn attention to three different planes of analysis: the community plane, the interpersonal plane, and the personal plane. The learning context is situated within the community plane. The activity of learning mathematics in a small group takes place in the interpersonal plane. The individual student experience is situated in the personal plane. The context of this classroom, the larger context of school and district, and the individual students' experiences remain in the background for this analysis. I foreground the interpersonal plane for this paper in order to examine how students' learning was influenced by and could be understood within the context of a classroom community focused on cooperative learning. The local interactions among students are situated within the classroom context on the interpersonal plane.

Suppose that a classroom teacher has 8 table groups in the classroom, such that 3-4 students are randomly assigned to sit at a group every 2 weeks (see Figure 16).[15]

[14] In line with Rogoff (1997), I consider the terms *learning* and *development* to refer to the same individual processes; I therefore refer to them interchangeably.

[15] The teacher in this study randomly assigned students to groups for 2-3 weeks at a time. Her reasons will be discussed further later in this paper.

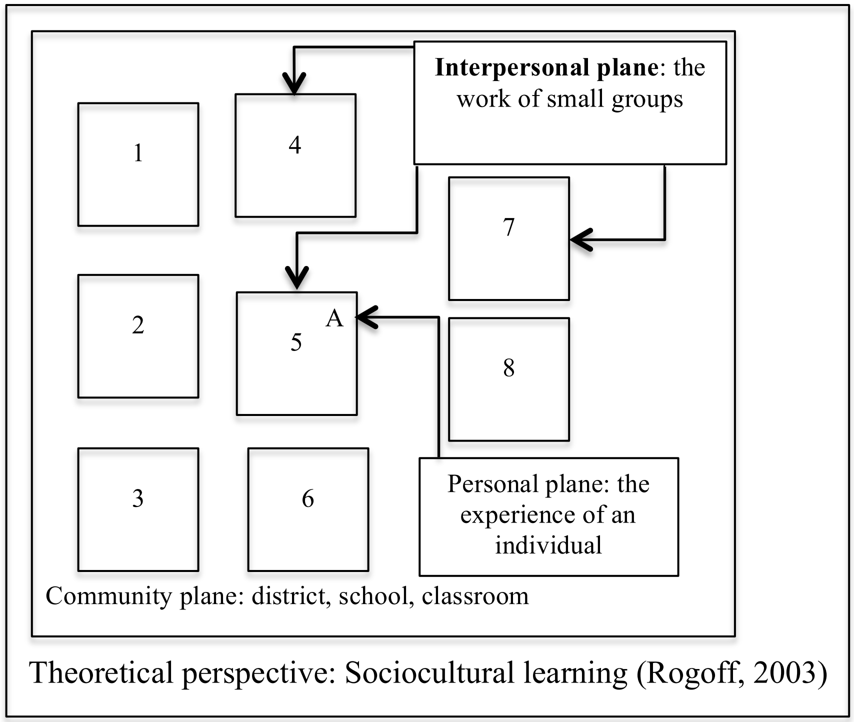

Figure 16. Learning mathematics in the interpersonal plane of small groups.

Using Rogoff's perspective on learning implies I adopt the belief that classroom teachers physically and structurally position students to have opportunities to learn together in their small groups. For example, examine the activity and interactions of a particular hypothetical small group, such as group 5, where students A, B, C, and D are assigned to sit for a 2-3 week period. The students will have opportunities to interact with one another during that time period, as shown in Figure 17.

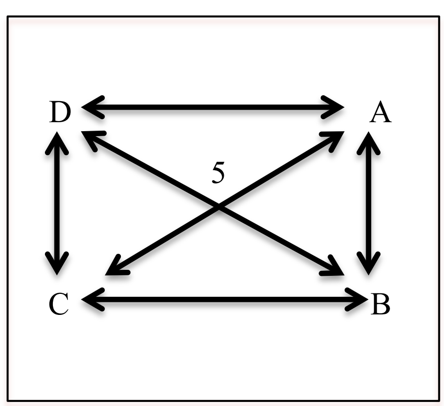

Figure 17. Learning mathematics in the interpersonal plane of one small group.

The arrows in Figure 17 are intended to show the opportunities that students have for interaction during the small group time.

Status and Positioning Theories. While the sociocultural perspective offers a frame for attending to learners' participation and interactions during groupwork, I utilize a combined Status and Positioning Theory framework to explain the dynamic nature of student interactions during groupwork. Status Theory explains which attributes have been valued in the classroom and offers an explanation for the ways students generalize their performance expectations for a student's group contributions (Kalkhoff & Thye, 2006). Positioning Theory attends to the dynamic *process* of positioning—the ways the teacher and students actively position students to be mathematically competent (or not)—and offers the opportunity to understand discourse moves that established group members' positions during social episodes in the activity of the group over time (Harré & van Langenhove, 1999). I coordinate these theories to investigate how students and the teacher generalize expectations for mathematical competence over time, as shown in Figure 18.

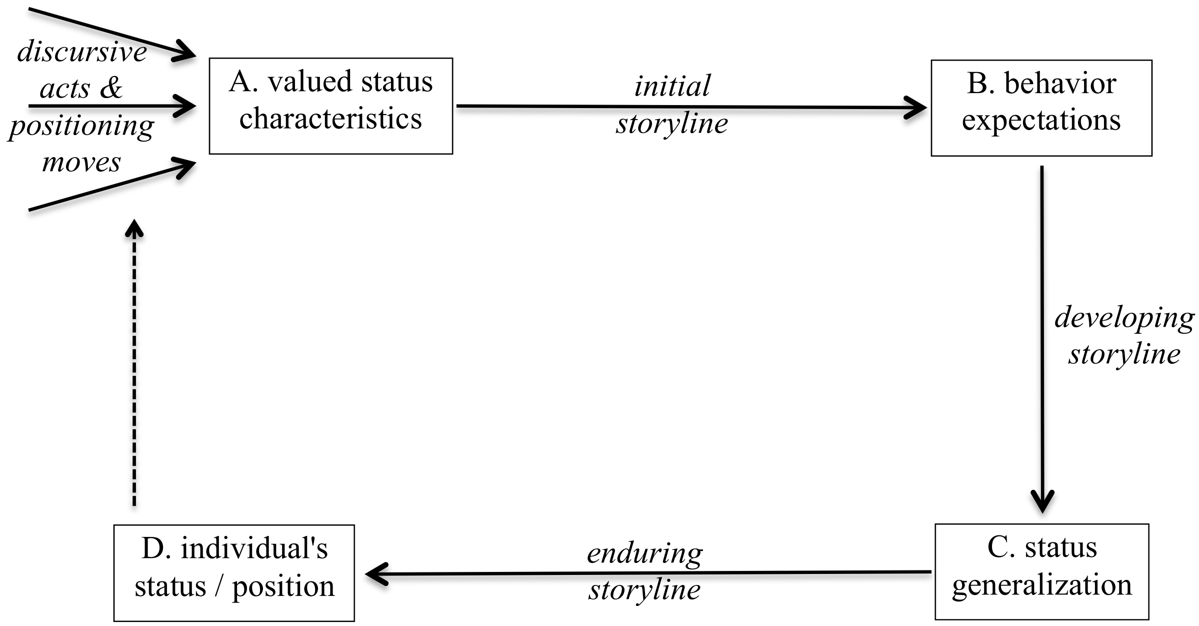

Figure 18. Positioning Theory as a vehicle for explaining status generalization.

Status Theory. I use Status Theory to understand both what characteristics get valued and the ways students' generalized expectations for competence could lead to status issues and therefore affect the overall productivity of the group (Kalkhoff & Thye, 2006; Webster & Foschi, 1988). *Status* is the relative standing an student holds in the community, on the basis of perceptions of the individual's ability to contribute to the community, regardless of whether students or the teacher consciously state what they value (Cohen, 1994a). In Status Theory, everyone believes high status is better than low status. *Status characteristics* (e.g., gender, race, language preference, perceived competence, speed) are represented on the top left-hand side of Figure 18. Status characteristics can position students to have different levels of power within a group when certain characteristics are valued by students and the teacher in a particular way (Foschi, 1989). While status differences are always present for any context, *status issues* arise when those differences interfere with a group's productivity in a collaborative environment and ultimately threaten students' opportunities to learn. Individuals in problem-solving groups form a set of largely unconscious performance expectations, illustrated in the top right of Figure 18, on the basis of what is valued in the particular setting or how likely each group member will be to successfully contribute to a task (Kalkhoff & Thye, 2006). These expectations lead to *status generalization*. For example, when a group starts to operate with one student acting as a leader and three students acting as followers, the leader is perceived as more competent, giving the leader a higher status and establishing the presence of generalized performance expectations of competence for each group member. The framework demonstrates how status characteristics dynamically and fluidly lead to behavior expectations. Once formed (or generalized), performance expectations are theorized to determine differing amounts of influence within the group, indicated by the individual's status or position on the bottom right of the framework.

Positioning Theory. While Status Theory allows me to predict which expectations for competence have been generalized in the past, I bring Positioning Theory to address *current* dynamic roles individuals bring to their construction of personal stories (Harré & van Langenhove, 1999). When looked at together, positions, discourse moves, and the ways students go about working on mathematics trace a storyline. Positions are sometimes created intentionally, such as when one person is positioned to be the teacher and a group of learners is positioned to be students in a mathematics classroom. Other times positions are co-constructed, such as when students negotiate their own and their group members' perceptions of mathematical competence. A group member can also challenge the right of another to assign positions, and a group member can challenge the way she or he has been positioned in a certain context.

Coordinating Status and Positioning Theories. Figure 18 demonstrates the combination of Status and Positioning Theories. In this combined framework, the process of status generalization begins with individuals in the community interacting, indicated in the top left of the diagram by discursive acts and positioning moves. As individuals continue to interact, valued status characteristics are displayed in the community, depicted at stage A. An initial storyline begins to develop, evidenced by the ways individuals engage in discourse and by the status characteristics they demonstrate valuing. The developing storyline is further revealed by the ways the members of the community demonstrate behavior expectations for one another, as indicated at stage B. As individuals act on the developed behavior expectations, consciously or unconsciously turning to certain members of the community over others on the basis of expectations for competence, status generalization, stage C, becomes evident. The result of status generalization is the visibility of differing expectations for competence, depicted as the enduring storyline. The enduring storyline leads to the display of an individual's status or position for the given community, pictured at stage D. The process continues with further discursive acts and positioning moves that reify an individual's current status/position or challenge that status/position. When positioning challenges are successful, status generalization starts the cycle over again, newly defining the community members' expectations and positions. Whereas Status Theory offers an explanation for the ways people have formed performance expectations for individuals' contributions on the basis of the generalization of what people value in a given setting (Webster & Foschi, 1988), Positioning Theory offers the opportunity to understand the dynamic process of positioning: the discursive acts that create positions during social episodes over time (Harré & van Langenhove, 1999). In other words, while Status Theory can explain what has already been established as valued and generalized in a community, Positioning Theory can explain the moment-to-moment interactions in a given group.

Methods

Researcher's Positioning

I share my positioning to be clear about how my identity is reflected in this study and in the ways I interpreted these data. I am a European American, middle class, female, secondary mathematics educator with a background in teaching high school mathematics. I believe all students and all people are capable of learning rigorous mathematics and are capable of highly successful outcomes, and I have a strong commitment to supporting all students to find the ways they are competent. I believe students who are not currently successful and who do not yet perceive themselves as competent in mathematics have not experienced significant opportunities to see themselves as capable.

As a teacher educator and a researcher, I believe that all teachers and students should "view each student as capable of learning both basic skills and high-level concepts" (Cohen, 1997a, p. 4). I support teachers to implement effective discourse and groupwork strategies that attend to status problems and work to increase equitable outcomes. I am interested in making a case for expanding what it means to "do mathematics" and to be competent in mathematics, so that people no longer value only speed and accuracy in mathematics. In other words, I endeavor to support change and to increase access to mathematics for all students. I was driven to conduct this study by the desire to uncover underrepresented students' perceptions of competence and to unpack teaching practices that strive toward equity.

Site and Participant Selection

As United States classrooms continue to increase in linguistic, ethnic, and socioeconomic diversity, teachers are challenged to use pedagogies that are successful with heterogeneous populations of students. As Martin (2009) has emphasized, race and racism are imperative considerations in mathematics education research that strives toward equity. As such, this study was situated in an urban school with a linguistically, ethnically, and socioeconomically diverse student population. I sought to understand how historically marginalized mathematics students (African Americans, Latinas/os, students impacted by poverty, immigrants, and girls) made sense of their mathematical learning within their classroom context.

Because I was interested in learning primarily from students who had not previously found themselves successful in mathematics, I elected to work with a high school Algebra 1 class, knowing that, at this school, many freshmen students enter prepared to take Geometry or Algebra 2. As I discuss later in this paper, it is significant that students who were taking Algebra 1 in high school were considered mathematically advanced compared to some of their peers.

I selected a collaborating teacher, Ms. Martin (all names are pseudonyms), because she intentionally made pedagogical choices to counter status issues in the classroom (Cohen, 1997b), because she cared about and intentionally developed sociomathematical norms (Kazemi & Stipek, 2001; Yackel & Cobb, 1996) that fostered interdependence during groupwork, because she regularly engaged students in mathematics discourse (Cazden, 2001), and because student-centered learning was a central part of her classroom practice. Ms. Martin and I also shared a common interest in supporting students who had previously been unsuccessful in mathematics. During the time of the study, Ms. Martin had been teaching this Algebra 1 course for 5 years.

Some mathematics educators and researchers have argued that effectively implemented groupwork is an important pedagogical tool that teachers, students, classrooms, schools, and districts can utilize to strive toward equity. For this reason, groupwork has become a consistent focus of research in teaching and learning mathematics (e.g., Boaler & Staples, 2008; Cohen, 1994a; Cohen & Lotan, 1997; Esmonde, 2009b; Webb, 1991). Effectively implemented groupwork has been shown to increase student participation, engage students more deeply in their learning, develop their academic thinking (e.g., Boaler & Staples, 2008; Herrenkohl & Guerra, 1998), promote positive mathematics identities (Hand, 2006; Jilk, 2007; Nasir & Hand, 2008), and foster classroom relational equity (Boaler, 2006).

Ms. Martin's classroom features groupwork facilitated by the principles of complex instruction (CI), a form of cooperative learning developed by Elizabeth Cohen and Rachel Lotan. CI is a form of ambitious teaching, because the effective implementation of CI offers all students opportunities to reason, understand, and use knowledge to construct their understandings of authentic problems (Lampert, 1990; Lampert & Graziani, 2009). Ms. Martin identified her learning and interpretation of CI as based on the work that came out of Railside High School (Boaler & Staples, 2008). The presumption that *all* students are capable and competent in learning high-level mathematics is the key to a faithful implementation of CI, and this presumption of competence drives CI teachers to the practice of regularly randomly assigning students to groups (Boaler & Staples, 2008), which Ms. Martin did every 2 weeks during the time of the study. CI teachers also aim to disrupt typical hierarchies that develop expectations for competence by paying attention to the role of status as students work together. They facilitate learning in small groups by delegating mathematical authority to students and by assigning tasks that require multiple abilities and multiple entry points. CI uses classwork to create opportunities for equal-status interactions in the classroom (Cohen, 1994a; Cohen & Lotan, 1997), thereby striving toward equity. As I discuss in the first finding, Ms. Martin employed all of these instructional practices in her classroom.

Data Collection

I used a qualitative case study approach in order to offer a thick and rich description (Corbin & Strauss, 2008) of the mathematical learning experience of one group of students. The data collection period took place during one period in the first semester of Ms. Martin's Algebra 1 class during the 2011-2012 school year. This Algebra 1 class had 28 students. Seventeen students were African American or African immigrants,[16] 5 students were Asian American, 4 students were European American, and 2 students were Latino/a. Twenty-two students were girls, and 6 were boys. Twenty-four of the 28 students formally agreed to participate in the study. I outline the class's demographics in detail because it did not reflect the make-up of the school's student body. This Algebra 1 class had an overrepresentation of African Americans, Latinas, and girls, and an underrepresentation of Asian Americans, European Americans, and boys. The difference mirrors a school trend (perhaps also a district and national trend) in which, prior to entering high school, disproportionately more European Americans, Asian Americans, and boys successfully completed Algebra 1, while disproportionately fewer African Americans, Latinos/as, and girls successfully completed Algebra 1.

I focused the data collection period around two units taught in Ms. Martin's Algebra 1 class during the first semester. The first focal unit (Unit 1: Linears) was taught during four weeks at the beginning of the school year in the first semester. The second focal unit (Unit 4: Lab Gear[17] and Solving Linear Equations) was taught for two weeks at the end of first semester. The

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[16] Because student demographics were not made available to me, these calculations are on the basis of my own observation, so any error or misrepresentation of how students perceived their identities is my error, alone.
[17] Henri Picciotto, Wright Group/McGraw Hill – Creative Publications: https://www.wrightgroup.com/launch/wright_group.html. For more information on using Lab Gear in a mathematics classroom, please see http://www.mathedpage.org/manipulatives/lab-gear.html.

purpose of selecting one unit at the beginning and one unit at the end of the semester was to understand the nature of the students' experiences over time. During the focal units, I attended class every day. During the middle of the semester, I attended class about twice per week, collecting fieldnotes on students' learning, and maintaining and continuing to build my relationships with the students.

Data collection for this study includes (a) fieldnotes taken during 50 classroom observations, (b) qualitative records created after each classroom observation, (c) audio and video recordings of 26 classroom sessions focused on five of seven small groups during Units 1 and 4, placing one camera near each of five of the seven small groups[18], (d) student and teacher artifacts, including but not limited to classwork, homework, projects, quizzes, and tests, gathered from observation days, (e) audio and video recordings of 22 semi-structured interviews with 12 of the 28 students, and (f) 4 semi-structured individual interviews and more than 100 informal conversations conducted with the teacher before, during, and after the study over a 3-year period between 2009 and 2012.

During the semester of the study, I became a participant observer in the class. Fieldnotes attended to students' Discourse (Gee, 1996), positioning moves, storylines, displays of status characteristics, the teacher's pedagogical choices and positioning moves, evidence of status generalization, and equity. I interacted with students as it seemed appropriate. At times I sat near groups to hear their conversations with one another, at times I responded to questions about their work, and as often as possible, per Ms. Martin's expectation, I redirected them to work with one

[18] Because 4 of the 28 classroom students elected not to participate in the study and Ms. Martin randomly assigned students to new groups every 2 weeks, we decided she would maintain random group assignment during the focal units and then shift groups that had students who elected not to participate in the study into groups that were easily off camera, leaving five groups on camera during each focal unit.

another when questions arose. As I started building relationships with students, some individuals called me over to ask questions about their mathematics work more than others. When this occurred, I redirected them, per the norm Ms. Martin hoped to develop, to use one another as mathematical resources. Redirecting students to use one another for mathematics understanding was critical, as one of Ms. Martin's focal goals for the class was to support students in finding themselves and one another as mathematically competent, often for the first time in their mathematics career. In all cases, I maintained a focus on my unit of analysis: the activity of learning mathematics in groups.

Data Analysis

Because mathematical learning and delegating authority were co-constructed between Ms. Martin and her students, my unit of analysis is the activity of delegating authority as students worked together in their groups. This group focus does not take away the importance of the community and personal planes of analysis; rather, examining perceptions of learning from the mathematics community and from individual learners provides the framing for studying student interactions in groups. Data collected on the nature of group interactions contributes to the empirical evidence of status, positioning, and equitable secondary mathematical learning practices.

Initial analyses of classroom observations and interview data took place by typing up and tagging fieldnotes and coding video sessions for teacher moves such as "assigning competence," positioning moves such as "positive self-positioning," and interactional moments such as "on-task verbal group interaction." I wrote memos related to equitable pedagogical practices and students' perceptions of competence. I read field notes, qualitative records, coding schemas of classroom sessions, and interviews, coding for common themes and interesting moments. I then used both open coding (Emerson, Fretz, & Shaw, 1995; Merriam 2009) and microanalysis (Corbin & Straus, 2008).

Once preliminary findings emerged, I questioned the data at multiple subsequent stages of analysis in order to better understand the nature of the transfer of mathematical authority in this classroom. Questioning the data also helped me get acquainted with a new piece of data: it allowed me to probe more deeply into a particular interaction in class that I was aiming to understand and helped me when I was stuck at a certain stage of analysis. For example, I used to believe that when students made eye contact with one another, it meant they were interested in what the other person was saying. This would have caused premature assignment of positioning codes. Questioning the data allowed me to determine when eye contact was a form of interest, positioning, or even dominance.

I used the constant comparison method (Merriam, 2009) to analyze segments of data alongside each other in order to seek confirming or disconfirming evidence. For example, because I argue that Ms. Martin's belief system was connected to what she did in her classroom, her potential for enacting equitable practices was related to her vision for teaching and learning. When told me she believed all of her students were capable of learning mathematics, I used constant comparison to analyze classroom observation data for evidence of assigning competence and positioning students as competent. I also used constant comparison to examine students' interviews for themes and trends. The "smartness" finding emerged from constant comparison of the ways each of three interviewed students (Neesha, Helen, and Jaelyn) discussed perceptions of competence in their interviews. I searched for similarities and differences in how these students talked about mathematical competence, and I triangulated their perceptions with specific moments from the classroom data. I used constant comparison to move my analysis from description to abstraction, to focus on more than a single case, to examine my findings in greater detail, and as a way of examining my own biases (Corbin & Strauss, 2008).

I analyzed the process of the teacher transferring mathematical authority to her students by examining her classroom structures and pedagogical practices and their effect on students. For example, I noticed that the teacher described student presentations, where students are at the front of the room presenting mathematical ideas, as a time to assign competence around thoroughly documenting the mathematics, asking questions, and providing in-depth explanations and mathematical justification.

Findings

In this section I discuss findings that emerged from investigating how and whether this teacher's pedagogy was striving toward equity by delegating mathematical authority to students. I found the teacher's vision for teaching and learning was aligned with equitable learning opportunities. Second, I found the teacher's vision was aligned both with the ways scholars have defined delegating mathematical authority and with her practice. Third, I sought to understand how the teacher's structures and practices for transferring mathematical authority were linked to striving toward equity. I found the teacher's methods strive toward equity through the structures and practices she enacted. I also found that these structures and practices expanded the definition of mathematical smartness in her classroom.

Ms. Martin's Vision for Teaching and Learning Mathematics and Equitable Classroom Opportunities

Researchers have found that teachers' beliefs and visions for teaching and learning influence what happens in the classroom (Stipek, Givvin, Salmon, & MacGyvers, 200; Zohar, Vaakin, & Degani, 2001). Stipek et al. (2001) found substantial coherence between teachers' beliefs and their practice. During interviews, I learned Ms. Martin shared a strong belief that all students are capable of learning high-level mathematics. She also believed that mathematics is a gateway subject, and problem solving is an important part of doing mathematics. Because teachers' beliefs have been connected to their practice, I use this section to describe findings about Ms. Martin's vision for teaching. The findings stem from analyzing Ms. Martin talking about her vision for teaching over the course of 4 interviews and many informal conversations and emails during the 3 years we worked together.

Vision for learning mathematics. When we talked about learning, Ms. Martin regularly shared her convictions about students and what she wanted her role to be in their journey:

> Zoom out completely and know that I come into this whole-heartedly knowing that all of my kids have something genuine to contribute. All my kids are capable of learning math. I hope that exudes from me, as a teacher, everywhere I go in that classroom. And over time, I've gotten other kids convinced that yes, they're equal contributors to this class. ... I believe that big teacher belief that all kids can learn, and learn in equally valuable ways. And I'm going to do what I can to get them there.

In this quote, Ms. Martin illustrated her belief that all her students had authentic mathematical contributions, and that part of her job was to help students discover ways they were capable of contributing to the class. Ms. Martin's perspective, that all students "have something genuine to contribute" and are "capable of learning math," aligns with Cohen's (1997a) assertion that equitable classrooms start with teachers who believe all students are "capable of learning both basic skills and high-level concepts" (p. 4) and with NCTM's (1989, 2000) expectation that all students be given opportunities to learn mathematics. By saying she convinced students they were "equal contributors" to the class, she indicated a vision for students to hold mathematical authority in a supportive classroom environment, in line with Gresalfi and Cobb's (2006) suggestion that mathematical authority was visible by "who's in charge" of the mathematical contributions.

Ms. Martin repeatedly shared her belief that mathematics provides an opening to achievement in other aspects of students' lives. The following interview excerpt is particularly representative of the way Ms. Martin related doing mathematics to an individual's overall success:

> I think [math is] a gateway… I think if you are a successful math thinker, you can be a successful problem solver in any realm of life... I think math and number sense [are] very important for anyone and everyone; I also think if you can problem solve within these set parameters, given these set of rules [dictated by mathematics], then you can extend that to any aspect of life, or any job you're in. Those critical thinking skills [are] super important.

In this passage, Ms. Martin talked about her belief in the potential for mathematics problem-solving ability to lead to success in and outside mathematics. Although her vision did not address Martin's (2003) call that historically marginalized students need to learn mathematics to critically investigate society, or Gutiérrez's (2007) call to coordinate dominant mathematics with critical mathematics, Ms. Martin acknowledged a frustration that her Algebra 1 students were not receiving the same opportunities as other students in her school: "Being in Algebra 1 as a freshman, they cannot take Calculus in high school, and they're in a system where most of their peers are a math class or two ahead of them, which is a disservice." Ms. Martin's vision for teaching and learning mathematics associates her students' potential success in mathematics with opportunities outside the mathematics classroom, which puts this part of her vision along the continuum of striving toward equity.

Description of who her students are. Ms. Martin described her Algebra 1 students as being "across the spectrum." She said that Algebra 1 in high school was for all students that did not "make it ahead":

> You get some kids that have done okay in school and in math in 7th and 8th grade, you get a good chunk of kids who haven't been successful and are still moved along in the system, kids that hung on and did fine and chose not to be accelerated or schools that didn't have [accelerated programs], and students who failed math; they're all there.

Ms. Martin's statement implied her understanding that the students in her Algebra 1 class chose not to be accelerated, had previously been unsuccessful in mathematics, or were not given opportunities to accelerate to higher level mathematics classes. She also stated that although Algebra 1 was not technically a tracked class because students entered the class having come from varying backgrounds before high school, she believed it was essentially tracked because most of the other freshmen were already taking Geometry and Algebra 2. Ms. Martin enjoyed working with students who were not the typically successful student:

> I like helping math make sense for students that aren't traditionally successful in math, or that have struggled in math in the last couple years, or decided they don't like math any more. … I prefer to teach students who traditionally struggle. … I don't ever want to give up Algebra 1.

By referring to her Algebra 1 students as “not typically successful,” Ms. Martin acknowledged that these students were at a disadvantage when compared with their peers in Geometry and Algebra 2. Further, she noted many of the students had decided they did not like mathematics and/or could not be successful in mathematics. Ms. Martin’s description of who her students were identifies her vision for helping nontraditional students discover ways they can be authentic mathematical contributors.

Instructional practices. Ms. Martin described a vision for implementing instructional strategies that required “student accountability” to the mathematics. By doing so, Ms. Martin’s classroom featured delegating mathematical authority to students across all her instructional practices. Ms. Martin described her class structure as one that offered students multiple ways to access mathematics. To accomplish this, she used complex instruction, which she learned in her teacher preparation program. When asked to describe complex instruction, Ms. Martin said:

> Complex instruction (CI) is a different perspective for looking at math and what it means to do math, know math, and be a math expert. It's opening up instructional strategies so I just don't stand and deliver. I put a lot of the learning back on the students so they're all active learners. They're all students and they're all teachers in the classroom. I think the biggest thing in a CI classroom is the community. The teacher genuinely believes that all students can learn math, and is willing to work to help all students learn math, and to value the different ways that all students learn math. And [the teacher’s belief] is apparent through verbal communication, little actions, and big actions. You can't just have the task card, a Shuffle Quiz, or a Participation Quiz; [you can’t just have] a couple instructional strategies; you really have to really have that lens of: there are equally valuable ways to do math, and, let’s try to get those going on in this classroom. I try to

make them feel safe, and I try to make it apparent that mistakes are ok, and you don't make a mistake unless you're trying. And [I try to] build a positive learning community where we're all responsible for each other. When I ask students to work in groups, I have to be aware of the social implications, meaning I have to be mindful of the sociological factors that can prevent students from participating in their groups and thus prevent them from engaging and learning. [I have to know] I cannot completely change a student's status but I can work to affect it by assigning competence and by giving tasks and problems that allow for nontraditional math smartness to be valued.

Ms. Martin described complex instruction as having assignments structured around cooperative learning and groupwork. Structuring classwork around cooperative learning meant she held individuals accountable for the material and for their participation in groups, thus delegating them mathematical authority. She said having a number of instructional strategies was necessary so that students knew there were multiple equally valuable ways of doing mathematics. Ms. Martin described using groupworthy tasks, student presentations, Shuffle Quizzes/teacher check-ins, Participation Quizzes, group tests, and small group work fostered by classroom norm development. Table 3 shows Ms. Martin's ideas about and vision for each instructional practice in the left-hand column, and a brief description of how the practice was enacted in the right column. I will refer back to these instructional practices throughout the rest of this paper.

Table 3

Ms. Martin's Classroom Structures

Classroom Structure	**Ms. Martin's Vision of the Classroom Structure** (quoting Ms. Martin)	**Description of How the Classroom Structure Was Enacted**

Classroom Structure	**Ms. Martin's Vision of the Classroom Structure** (quoting Ms. Martin)	**Description of How the Classroom Structure Was Enacted**
GWT = groupworthy tasks	Assignments are written such that students have to provide written justification of their answers, so that *whys* and *becauses* are part of the answer. Ideally the task is not scaffolded so that there is only one way to start. For example, a lot of my big linear problems ask for an equation, table, and graph, in no particular order, so students can start in different ways. Or, the task [might be] a messy word problem that takes a lot of sorting out before starting.	Tasks were written with multiple options for how to work on the mathematics. 2 task cards were available per group of 4 students. Mathematics had high-cognitive demand. Final products included visual representations like posters or graphs and always required individual accountability. See Appendix A for a sample Groupworthy Task: Trick or Treat.
SP = student presentations	I like to use student presentations to reinforce that students are also experts in the content. I also use the time to model good questions are push any expectations for documenting work/solutions and to push for thorough explanations. And I can assign competence around documenting the math, explanations, justifications, questions, and so on.	Students were asked, either in the moment or before presentation time, to volunteer to explain their mathematical solution at the front of the room. Students sometimes showed their work on the document camera or wrote out new mathematics on either the document camera or Smartboard.
SQ = Shuffle Quizzes/ teacher check-ins	I ask them to consider some questions, and then I come to each group and randomly pick a student to explain what's going on. So 3 to 4 students at a time, and I elicit what they know, and fill in blanks or gaps for them, and solidify the tools that they need to continue on or keep learning about whatever area we are working on.	Ms. Martin told students at the start of class there would be a check-in. When groups called her over, she shuffled the group's papers behind her back, which dictated the order students shared thinking on the indicated problems. She scaffolded the conversation, and if she was not satisfied, she asked them to rehearse until she returned. When she was satisfied with their answers, they received the next assignment.
PQ = Participation Quizzes	I visibly grade students and show them what they're doing well, and their group interactions, to do with the math, and how they're talking to each other. Making it public is really important, as is giving immediate feedback on the behaviors that contribute to good group work and ultimately good learning.	Ms. Martin wrote what students were doing, organized by group, reinforcing norms on a public representation. She displayed and updated the paper live during the class period. Grades for group tests and group quizzes were combined and returned in class later the same day or when students arrived the next day.

Classroom Structure	Ms. Martin's Vision of the Classroom Structure (quoting Ms. Martin)	Description of How the Classroom Structure Was Enacted
GT = group tests	One of the many strategies (along with PQs and SQs) where students put pressure on one another into understanding and knowing. A group test happens a day or two before the individual test. It is like taking a test in your small group, without the help of the teacher, but with the help of your group members, so that it's a high-stakes review. Students are exposed to the types of problems and usually the layout of the test, but they still have their groups as a resource. I hold individuals accountable by grading some problems for everyone's test and combining the points for one group grade. And, I always return graded group tests before they take the individual test so they can see mistakes and make last minute adjustments before the big assessment.	Each student received a copy of the group test. Students were told the group test was similar to the individual test. They were told to work together by staying on the same problem, discussing and justifying the mathematics, and coming to the same final answer. They were told to agree on their answers because the teacher would grade one person's number 1, another person's number 2, and so on. Grades for group tests and group quizzes were combined and returned in class later the same day or when students arrived the next day.

Ms. Martin's description of these strategies indicates not only that she intended to use a variety of intentional approaches to teaching mathematics, all of which she attributed to her interpretation of complex instruction and learning, but also that through each of these approaches she intended for her students to be positioned as competent and held accountable for doing authentic mathematics. Ms. Martin's assignments indicated what students were expected to produce, but not how students should get there. Further, the assignments included student justification: "so that the *becauses* are part of the answer," so students would have to use arrows, color coding, and/or write a paragraph to justify their work. Ms. Martin described emphasizing problem solving so "math is thinking, problem solving, and not one set way to do it." Ms. Martin used group tests, Participation Quizzes, Shuffle Quizzes, and teacher check-ins so students would put pressure on one another to learn mathematics. When successfully implemented, the presence of "pressure for learning mathematics" translated into individual and group accountability, which indicated one way she delegated mathematical authority to students.

Thus far, I have discussed Ms. Martin's vision for teaching and learning, including how she described a number of her instructional practices. Ms. Martin's vision is further revealed by how she structured the available class time. I include Table 4 to illustrate a timeline for the basic structure of the instructional practices Ms. Martin used during the course of the study:

Table 4

Classroom Structures, Algebra 1, Semester One

Days	**Unit**	**Time Period**	**Classroom Structures** classwork in small groups groupworthy tasks student presentations Shuffle Quiz/teacher check-in Participation Quiz group test unit project
First 7 days of semester	Groupworthy Tasks Unit	September 2011	7 days of assignments in the first small group 7 days of groupworthy tasks 1 Participation Quiz
Next 18 days of semester	Unit 1: Linears	September - October 2011	18 days of assignments in two new small groups 2 days of groupworthy tasks 11 student presentations in 3 class days 1 Shuffle Quiz 1 Participation Quiz alone 1 Participation Quiz + group test 1 unit project
Middle portion of semester	Units 2, 3, start of 4	October 2011 – January 2012	The same basic instructional practices were observed during weekly visits outside the focal period.
Last 8 class days of semester	Unit 4: Lab Gear and Solving	January 2012	8 days of assignments in one small group 2 days of groupworthy tasks 2 student presentations in 1 class day 1 Shuffle Quiz 2 Participation Quizzes + group tests

Table 4 shows that Ms. Martin used formal classroom structures, such as groupworthy tasks, Participation Quizzes, or student presentations, on about half of the class days during the semester. For example, in Unit 1, she used these formal instructional strategies on 8 out of the 18 days of the unit. Further analysis of the use of class time over the course of the study reveals how Ms. Martin used these instructional practices to delegate mathematical authority to her students.

Norm development. During interviews, Ms. Martin talked about how she worked to develop norms for doing mathematics. In response to what she would do, she said: “I'll kneel down, or sit down with a group, and ask questions, and put papers in the middle, and try to show them what I expect, because they don’t come in knowing how to work with each other or how to ask questions.” Here Ms. Martin reported her intention for modeling questioning strategies, habits of mind, and habits of cooperation. She also explained she would develop these norms by describing how she typically handled a conversation about expectations with her students:

> “I’m a math teacher, so I know math really well, I can do math, and I can teach you math, and that's going to be our goal of this class. We want you to know math so well that you can do math and you can explain math and you can teach math.” And so I find myself saying those types of things over and over again. I find myself modeling a conversation in a group. So I'll kneel down, or sit down with a group, and ask questions, and put papers in the middle, and try to show them what I expect, because they don't come in knowing how to work with each other or how to ask questions. And so when they say things, like “I don't get it,” I'm like, “Ah, that's totally not helping me. What do you know so far? What pieces have you picked up? Ok, so what part don't you get?” So I try to model what they need to work with each other and get information from each other.

In saying, “We want you to know math so well that you can do math and you can explain math and you can teach math,” Ms. Martin modeled students talking about what they did and did not understand. This instructional practice was her way to develop an expectation for delegating mathematical authority by expecting students to work together and talk about mathematics. This excerpt shows that when talking about how she developed norms, Ms. Martin repeatedly oriented students to doing mathematics so they were working together, explaining mathematics, and asking one another questions. She also indicated she positioned students as competent by repeatedly asking which parts they did understand. By saying she wanted students to know mathematics "so well that you can do math and you can explain math and you can teach math," Ms. Martin also indicated how students would be given the authority to *do* mathematics in her class.

Status issues. When she discussed addressing status issues arising when students worked together in groups, Ms. Martin talked about being aware of the sociological implications and challenges for students to work together:

> I know that all my kids want to be smart, and all my kids want to be good. And all my kids enjoy accolades, and enjoy being a leader. That just naturally feels good across the board. And so, whether it's just your perception, or it's a comment, students will feel not smart, not like a leader, not part of a group, based on what's going on socially, or what they have shown previously academically. And so if you're asking kids to work together, you need to address all those issues of, "I'm going to tuck away because I don't feel comfortable," or "I'm not going to participate because I don't know what's going on. I don't want anyone to know about that." So, in order to ask kids to try to work together, and genuinely work together, and help each other out, you have to be aware of how they perceive each other and how they've perceived themselves in a group setting.

This quote reflects Ms. Martin's perception that all students wanted to do well in mathematics, and she knew there would be times when students did not feel competent or comfortable on the basis of what was happening as they worked together in groups. By saying she had to speak to issues of kids "tucking away" and not participating because they did not feel comfortable, Ms. Martin addressed how she noticed and attended to status issues. She also described responding to status issues by publicly assigning competence to students whenever she could:

> In order to combat status, I try to assign competence to kids. And that only works if it's genuine and totally absolute. So there may be a day when I know a student has low status in his group, and I try really hard to get him back in the conversation, or get kids to turn him. But [maybe that day] it doesn't happen genuinely to actually bring him in. The cool part is that I haven't seen a kid where I've never been able to assign him competence. I've always been able to figure out how a kid is smart, or [find they had] some secret thing written down on their paper they didn't share with their group and [I'll be able to] make a big deal about it. Like maybe they had a perfect graph and I can use it as the example for the whole class.

In this excerpt, Ms. Martin discussed combatting status issues by assigning competence to low-status students for ways they authentically contributed to the mathematics of their group, noting that assigning competence "only works if it's genuine and totally absolute." In this way, Ms. Martin revealed her belief that this strategy was contingent on assigning competence for *genuine* mathematics contributions. She shared her belief that it was always possible to find a way that a student was competent in mathematics. When groups were not working together, she described her task in combating status issues as figuring out who had something to contribute, so that she could orient the group to work with that person:

> I glance around and see who's got stuff that I can start with, and in particular if that student isn't the typical high-status student, or the one who called me over, I will go to them first, to illustrate: this is the person you should be asking, not me. When I go to a group, I'm always trying to elicit what they know, so I'm like: "Okay, so where'd you guys get stuck?" or "What've you done so far?"

In saying, "I glance around and see who's got stuff that I can start with," Ms. Martin revealed her vision for orienting students to one another. When she described saying to students, "This is the person you should be asking, not me," she revealed one of her strategies for delegating authority to students. Ms. Martin's experiences reinforced her vision for working with status issues by always orienting students to work with one another. She could always find a way that a student was competent in mathematics, and she could elicit what students knew by delegating authority to students to talk about the mathematics they already understood. The strategies Ms. Martin turned to when addressing status issues were aligned with strategies described by Cohen (1994a), Cohen and Lotan (1997) and Featherstone et al. (2011). Because attending to status issues can increase access to mathematics for some students, Ms. Martin's attention to status as an instructional practice was a way in which she attempted to work toward equitable teaching and learning outcomes.

Summary. Through knowing who her students were as learners, by employing instructional strategies that featured students as mathematical resources, through the development of norms that oriented students to one another and required mathematical justification, and by attending to status issues, Ms. Martin's vision for teaching and learning exemplified current researchers' descriptions of delegating mathematical authority (see Gresalfi, 2009; Gresalfi & Cobb, 2006; Gresalfi et al., 2009; Lotan, 1997). Her vision also aligned with Gresalfi et al.'s (2009) assertion that delegating mathematical authority happens when students are required to work with one another to convince their peers their solutions make mathematical sense. Although Ms. Martin's vision for teaching and learning did not include the use of procedural roles typically outlined by complex instruction for groups to complete their groupwork (e.g., randomly assigning students as team captain, facilitator, recorder/reporter, resource manager), her vision for teaching is otherwise directly aligned with the principles of complex instruction, a practice for teaching that is often affiliated with equitable teaching and learning (e.g., Cohen & Lotan, 1997). Further, Ms. Martin's vision included the other requirements set forth by Cohen et al. (1994) for delegating authority to students, including (a) use of an assignment/task card that gives instructions for individual and group products, and (b) development of cooperative norms that set the expectations for students' class participation. As a result, I found that Ms. Martin's vision for teaching included key strategies for delegating mathematical authority to her students.

This section outlined the ways in which Ms. Martin's vision for teaching and learning included key instructional practices that have the potential to help teachers strive toward equity and delegate mathematical authority to students. Ms. Martin believed all of her students were capable of learning mathematics. She considered mathematics to be a gateway subject, such that achievement would help students to be successful in other aspects of life. She attended to status issues in her classroom as a way to position students as competent and increase students' access to mathematics. Furthermore, Ms. Martin believed in using instructional practices that oriented students to one another, contained problem solving with rigorous justification, and delegated mathematical authority to students. Because a teacher's vision for teaching and learning directly affects her practice (Stipek et al., 2001; Zohar et al., 2001), her perspectives affect her students' opportunities to have equitable teaching and learning experiences. In the next section, I reveal ways her vision aligned with her practice. Close examination of her practice offers insight into how the practices were enacted.

Unpacking Classroom Practice: Transferring Mathematical Authority to Students as a Vehicle to Strive Toward Equity

In the last section, I discussed how Ms. Martin envisioned her Algebra 1 class. In this section, I describe how Ms. Martin facilitated time during class and I unpack how Ms. Martin's vision for teaching and learning was subsequently related to how she delegated mathematical authority to her students during the semester. I found the nature of classroom interactions and activities could generally be linked to the ways Ms. Martin described her vision for teaching and learning. I also found Ms. Martin was consistent in using the types of instructional practices she described in her vision, including (a) complex instruction, (b) attending to status issues, (c) orienting students to use one another as mathematics resources, and (d) explaining mathematics to one another. Further, I found researchers have described the concept of transferring mathematical authority to students (see Gresalfi, 2009; Gresalfi & Cobb, 2006), and this study contributes empirical evidence for the ways Ms. Martin's specific instructional practices transferred mathematical authority to her students. Refer to Table 4 for a summary of the alignment between Ms. Martin's classroom structure and her process of transferring mathematical authority to students. I unpack the findings summarized in Table 5 throughout this section.

Table 5

Alignment of Ms. Martin's Classroom Structures, Pedagogical Practices, and the Process of Delegating Mathematical Authority to Students

Classroom Structures & Pedagogical Practices	**Classroom Structure Description**	**Alignment of Structure, Practices, & Delegating Mathematical Authority** Description of how the class structure was linked to the transfer of authority in this classroom:
A. Framing Ideas	Ms. Martin cited and used status theory to assign competence and position students as competent to make students accountable for the mathematics.	Students were in charge of validating whether mathematics methods and ideas are correct (Boaler & Staples, 2008).
B. Classroom Structures	Ms. Martin used complex instruction, groupworthy tasks, Participation Quizzes, group tests, Shuffle Quizzes, student presentations, and norm development for productive small group work.	The teacher held students accountable for: being on task, keeping groupmates on task, and producing individual and group products (Gresalfi et al., 2009; Lotan, 1997). Students talked with one another to find out what they should be doing and how to solve challenging problems, by a) task card, b) development of cooperative social and sociomathematical norms (Kazemi & Stipek, 2001; Sharan, 1999; Yackel & Cobb, 1996).
C. Orienting Teacher Moves	Ms. Martin regularly oriented students to use one another as mathematics resources during class time. This involved students asking one another questions and offering explanations.	Students were in charge of making mathematical contributions (Gresalfi & Cobb, 2006).
D. Justifying Mathematics Teacher Moves	Ms. Martin intentionally incorporated justifying the mathematics into the written and oral work of the class.	Students were given the opportunity to decide which methods they would use to solve problems (Gresalfi & Cobb, 2006).

Ms. Martin started fostering the development of norms that positioned all students as competent sensemakers on the first day of school, by randomly assigning students to groups for a 2-week groupworthy task unit.[19] When the unit was over, Ms. Martin randomly assigned students

[19] See Appendix A for a representative groupworthy task: Trick or Treat.

to new groups and oriented them to work together on the next set of assignments[20], which corresponded to the first unit of Algebra 1: Linears. Throughout the semester, Ms. Martin continued randomly assigning students to new groups every 2 weeks, telling students of her goal that they have the opportunity to work with everyone and sit everywhere in the room by the end of the year.

On all days I observed, Ms. Martin's participation structure featured student-led conversations through small group work and student-led presentations as a way to delegate mathematical authority to students to learn algebra content. By analyzing the 26 days of classroom video from Units 1 and 4 for Ms. Martin's use of class time, I found that about 72% of class time was dedicated to students working in small groups while Ms. Martin monitored, 22% of class time was spent by Ms. Martin leading a whole-class discussion, and 6% of class time was largely unstructured. The 22% of whole-class time broke down to 9% of time dedicated to student-led presentations that Ms. Martin facilitated and 13% of class time dedicated to Ms. Martin alone addressing the class as a whole. Analysis of Units 1 and 4 separately found that the use of class time was extremely consistent from Unit 1 to Unit 4. Ms. Martin's use of class time is significant to this finding because it illustrates that she dedicated about 81% of class time to students talking to one another about mathematics. This finding is evidence for the ways the participation structures used during class time delegated mathematical authority to students.

In the next part of this section, I map the findings back to the alignment between the class structure and the delegation of mathematical authority as described in Table 4. In *Case 1: Transferring mathematical authority through student presentations*, I uncover how student presentations positioned students as competent to do mathematics. In *Case 2: Transferring*

[20] See Appendix B for a representative example of a classwork assignment.

mathematical authority through early Shuffle Quizzes, I demonstrate how Ms. Martin set the expectation that students use one another to justify mathematics by leaving and returning to a group early in the year, asking them to be ready to offer further mathematical justification when she returned. In *Case 3: Transferring mathematical authority through established Shuffle Quizzes,* I analyze how justifying mathematics became normative for students as the year progressed. In *Case 4: Transferring mathematical authority through Participation Quizzes,* I reveal how Participation Quizzes introduced and reinforced interdependent self-monitoring groupwork behaviors.

Case 1: Transferring mathematical authority through student presentations. The first student presentation of the year was particularly representative of the ways that students' presentations worked and the ways Ms. Martin facilitated transferring mathematical authority through her framing ideas, participation structures, orienting moves, and justifying moves. During this student presentation, Ms. Martin asked for a volunteer to go up to the front of the room to lead the discussion on a classwork assignment. Student presentations were an opportunity for students to orient on one of their peers working at the board and justifying their mathematical process in whatever way they felt comfortable. Most often two or more students would present in a row, and each student would have different ways for justifying the presented problem. In the example featured below, students were asked to assume a fictitious student had made mistakes on their mathematics assignment. The task directions were "circle the mistake(s) and tell what the student did wrong." See Figure 19 for a representation of the graph that had mistakes students could identify.

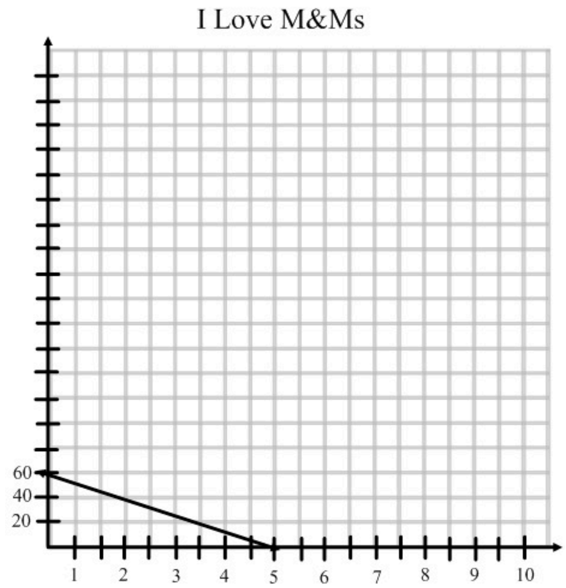

Figure 19. I Love M&Ms.

When Ms. Martin asked for a volunteer to be the first student to present for the year, Naima raised her hand. Naima's willingness to volunteer for the first presentation is salient. Naima was a sophomore student who was not successful in Algebra 1 during her freshman year. In spite of her lack of previous success, on the eighth day of this Algebra 1 class, she was willing to take a role that would place her in front of the room, justifying mathematics ideas to the class. After going to the front of the room, she said:

1 Naima: Are they s'posed to tell me?

2 Ms. Martin: Sure, you can do whatever you want. Someone tell her. Or, you, you run the class.

3 Naima: You guys, raise your hands to me.

A few students laughed at turn 3, possibly because Naima had suddenly taken a teacher role in front of the room, or possibly because this was the first time a student had presented to the class. Ms. Martin's response to Naima indicated a transfer of responsibility, "Do whatever you want;

you run the class." In this way, Ms. Martin delegated Naima the authority to decide how to handle being in front of the room justifying her mathematical thinking. Naima chose to ask students to raise their hands, which was followed by a number of students raising hands and calling out answers. When the first student raised his hand, Naima called on him and he talked about missing subtitles on the axes. Naima circled the empty spaces along the *x* and *y* axes, and wrote "no labels (subtitles)" next to the graph. The conversation continued in turn 4 (below). When the class again became quiet, Ms. Martin prompted them:

4	Ms. Martin:	What else? ((5 second pause.)) There's more. There's other big ideas too.
5		((12 second pause, during which Naima wrote in the left-hand corner: "needs to start @ 10".))
6	Ms. Martin	Why does it need to start @ 10? Tells us about that one.
7	Naima:	Because you can start this one and go from 0 to 20. ((Pointing to the *y* axis.)) This one, you go from 0 to 1.
8	Student:	It's going up by 20. ((Naima looked back at the graph, shrugged her shoulders, and erased "needs to start @ 10".))
9	Ms. Martin:	I was actually, I was thinking it was a good idea, needs to start @ 10. Or some other number?
10	Student:	It's going up by 20!
11	Student:	…doesn't start at 0.
12	Ms. Martin:	Yeah, okay, let me back that up. It's going by 20s, right? So the spacing is okay.

13 Naima: But there's stuff in between it ((Pointing to the line)). It would be easier to graph if you did it by 10s, because there's still stuff like in the 10s spot.

14 Ms. Martin: Yeah. Look how BIG that grid is. And look how much space the line takes up. ((Naima then rewrote her "needs to start @ 10.")) So if we went by 10s, or even by 5s, we could space that 60 out, we could use the whole *y*-axis. So that's, that's actually really important. You guys will – I'll ask you to make a full page graph … you want to use the whole graph to show the line, to show the whole curve. So, spacing by another number would actually help us to see the mistake up there. What else? There's another huge mistake up there.

In turn 6, Ms. Martin fostered the expectation for Naima to provide mathematical justification for "needs to start @ 10" by saying, "Tell us about that one." Naima responded in turn 7 with a mathematical reason, saying "Because you can start at this one and go from 0 to 20." Turns 6 through 14 illustrate a representative way in which Ms. Martin was able to position Naima's contribution as genuinely competent. Naima's "needs to start @ 10" was only partially mathematically valid. Simply starting the graph at 10 at the origin would be mathematically problematic because the spacing 10, 20, 40, 60 would not be even on the *y* axis. Instead of telling Naima she was wrong, Ms. Martin asked her, in turn 5, to explain what she meant. When another student suggested Naima's thinking was not valid (turn 8), Naima started erasing her suggestion. But Ms.

Martin supported Naima's idea in turn 8, saying, "I was thinking it was a good idea, needs to start @ 10" (see Figure 20).

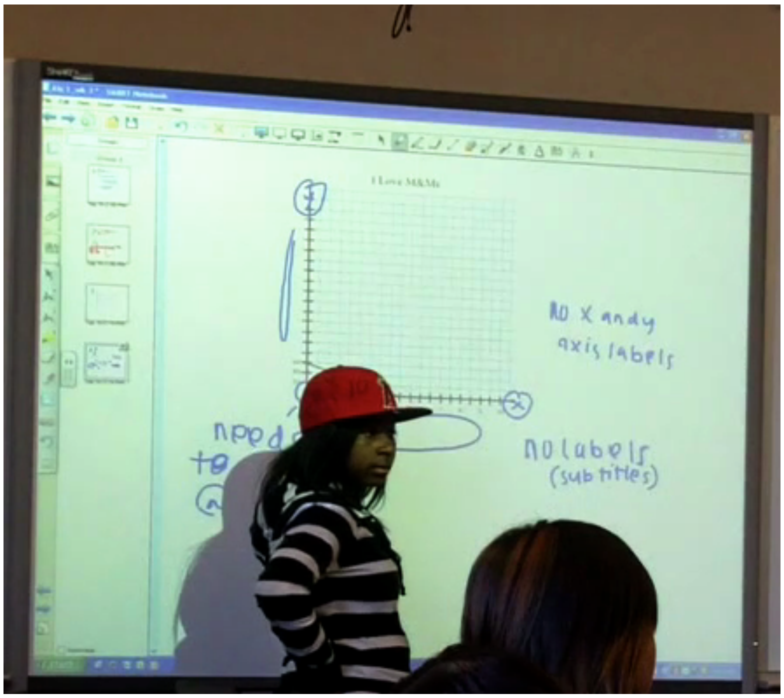

Figure 20. Needs to start @ 10.

In turn 12, Naima explained she wanted to space out the graph because "it would be easier to graph if you did it by 10s." By allowing Naima to justify her thinking, the whole class heard Naima wanted to space out the graph. In turns 8 and 13, Ms. Martin assigned Naima competence for understanding the mathematical purpose for spreading out the graph, oriented the class to Naima's justification, and attended to her pedagogical goal of discussing spacing as a common graphing mistake. Naima responded to Ms. Martin's assigning her competence by explaining her mathematical reasoning for starting the graph at 10 in order to space out the graph.

Naima's example is particularly representative of the way Ms. Martin used student presentations to transfer mathematical authority to students. She maintained a specific role of keeping the cognitive demand of the talk rigorous while orienting students to one another's mathematical ideas. In this example, Ms. Martin reframed Naima's initially problematic contribution so students could see the authentic mathematical competence the idea "needs to start @ 10" brought to the conversation. Naima responded by sharing valid mathematical thinking about how to space out the graph. During the 26 days of Unit 1 and Unit 4, 15 different students were delegated the authority to lead whole-class conversations through invited and volunteered student presentations. Ms. Martin delegated mathematical authority to students during student presentations by giving them space and time to justify mathematical thinking in their own way.

Case 2: Transferring mathematical authority through early Shuffle Quizzes. During Units 1 and 4, Ms. Martin conducted three Shuffle Quizzes.[21] The interaction between Ms. Martin and the group with Jaelyn, Tamira, Qianna, and Helen during Unit 1 was particularly representative of how mathematical authority was delegated to students through orienting and justifying norm development. During Shuffle Quizzes, Ms. Martin required students to use one another as mathematics resources. This interaction developed around the group's understanding of the assignment "Equation Time"[22] during Unit 1. After having previously walked away because she told the group they were not ready and needed to further discuss what the variables meant (mentioned in turn 5), the excerpt begins with Ms. Martin re-approaching the group for their second attempt to pass the Shuffle Quiz. Ms. Martin returned to the group, saying:

[21] Refer back to Tables 2 and 3 for descriptions and timing for Shuffle Quizzes.
[22] See Appendix C for the full "Equation Time!" assignment.

Figure 21. September Shuffle Quiz.

1 Ms. Martin: Okay, did we continue that variable conversation? ((Shuffling papers behind her back, indicating the Shuffle Quiz is about to start.))

2 Students: Sort of… Uh…

3 Ms. Martin: Sort of? Okay. I feel like there's a lot of places where you, like, you sort of have the right idea, but you're not a hundred percent on, so I'm going to bug you guys to be really detailed about it. ((Stopping the paper shuffle from behind her back and placing Jaelyn's paper on top, in front of the group.)) So, Jaelyn, part a, what do [the variables] *T* and *D* mean in this situation?

4 Jaelyn: *T* means total amount of money. *D* means number of days, or days.

5 Ms. Martin: Okay, so when I was here last, we had a discrepancy about what, what do you mean about total amount of money? Can you put more words around that?

6	Jaelyn:	No… It means total amount of money! ((Pause.)) Like, in this case, it means … would equal total amount of money…. Correct?
7	Ms. Martin:	((Pausing.)) Yeah… Well, now that you're… I'm not sure that you guys actually had this conversation. ((to Qianna)) did you guys have this conversation? ((Back to Jaelyn.)) No, no… keep going. Try again. Total amount of money. Say more about that. You're right….that. But it needs more detail.
8	Tamira:	Okay. She wants us to give more detail. Like, stretch it out!
9	Jaelyn:	Um…Okay. I'm just putting words to it…. Total amount of money he has, each week.
10		((Ms. Martin Pausing)).
11	Jaelyn:	Okay. I give up.
12		((Ms. Martin looked around at the group and then said: Okay, so I think this is a misconception. I'm not sure, but, *T* and *D* are variables, right? That means that the numbers they represent vary? They change? So what does *T* represent?))
13	Tamira:	The varying, the change.
14	Qianna:	((Inaudible.))
15	Ms. Martin:	Every, each day. It's 80 on day 0, so total amount of money left on each day.
16	Jaelyn:	What did we start with? 80 it would be at the beginning…
17	Ms. Martin:	10 is happening each day. Where is 10 in the table?....

18 ((Ms. Martin went on to tell Jaelyn that she could correct her understanding on her paper after she finished checking in. Qianna's turn was next. She explained why the equation made sense, then Tamira talked about what rate of change meant, and Helen talked about starting point. At the end of the check-in, Ms. Martin assigned them a grade and talked to the students about what to expect in future teacher check-ins/Shuffle Quizzes)):

19 Ms. Martin: Okay, so when I do check-ins, every other time except for the first time, when I walk away, I take away one point, when I walk away twice I take away 2 points, so … there were a couple hiccups here, guys, where you had about the same thing, but not exactly the same thing. So stuff like your tables are important to check, and stuff like talk about a and b, where it asks you in a and b to explain. I'm going to ask you not just to read, but to say more about it.

The excerpt indicates Ms. Martin first oriented students to develop mathematical consensus. She returned to the group after asking them to revisit justification conversations with one another about the mathematical meanings of variables. The excerpt also shows Ms. Martin held students accountable for the ways she delegated them authority. First, she required students to use one another as mathematical resources (turns 1, 7, 18). Second, she asked students to arrive at mathematical consensus (turns 1, 7). Third, she pushed students to justify their mathematical thinking (turns 3, 7, 18). When she returned, Ms. Martin completed the check-in, scaffolding students' explanations (turns 3, 5) and emphasizing her future expectations for success in check-ins (turn 19). Evidence of norm development of orienting students to use one another and develop mathematical justification was provided when Tamira told Jaelyn to give more detail in her explanation, “like, stretch it out!” (turn 8) and when Tamira supported Jaelyn in what to say after Jaelyn said, “I give up” (turns 11, 13).

This Shuffle Quiz excerpt is representative of the strategies Ms. Martin employed to normalize holding students accountable for individual and group products, mathematical consensus, and justifying mathematical thinking. Holding students accountable for being on task, for providing individual contributions, and for explaining the mathematics in their own way is consistent with transferring mathematical authority (Gresalfi et al., 2009).

Case 3: Transferring mathematical authority through established Shuffle Quizzes. The first example of a Shuffle Quiz showed how students engaged at the beginning of the study around the expectations for working together, reaching mathematical consensus, and justifying the mathematics. The next example demonstrates how a Shuffle Quiz was enacted to feature students as competent mathematical sense-makers at the end of the semester. In this later Shuffle Quiz, Jaelyn was again a member of the featured group, this time working with Irene and Phoebe. Recall in the first Shuffle Quiz, which was representative of a September teacher check-in, Ms. Martin left the group and came back, stating she was not sure all group members were on the same page. During that check-in time, she kept pressing Jaelyn for mathematical justification (turns 3 and 7). Eventually Jaelyn said, “I give up” (turn 11) and stopped trying to work on the problem. During the Shuffle Quiz featured in this next excerpt, which occurred in January, Jaelyn called Ms. Martin over, signaling her group was ready (see Figure 22).

Figure 22. January Shuffle Quiz.

Even though Ms. Martin’s expectations for participation were much higher during the January Shuffle Quiz, no one gave up, in spite of being pressed for mathematical justification. On this assignment, Ms. Martin gave the group a piece of paper showing how she solved for *y* using Lab Gear (the red page in Figure 22). The students were then meant to work out the solution on their Lab Gear mat (the orange page in Figure 22), talking to one another and preparing for the teacher check-in to explain their understanding--not just the solution--worked out on the mat. The group interaction showed Ms. Martin’s press for justification:

1 Ms. Martin: Check in...Check-in… check-in? ((Ms. Martin starts shuffling papers behind her back.))

2 Jaelyn: Well, you know I know it!

3 Ms. Martin: Oh, sure! Ladies ((to Phoebe and Irene)) how are you feeling about it?

4 Phoebe/Irene: Fine.

5 Ms. Martin: Any steps that were particularly harder?

6 Phoebe: No, just that, I'm used to it in my head.

7 Ms. Martin: But you know the mat, right?

8 Phoebe: Yeah.

9 Ms. Martin: Okay good, because I'm asking everyone both parts. Irene, how're you feeling?

10 Irene: Fine.

11 Ms. Martin: Fine? Okay, good … ((finishing shuffling and landing on Irene.)) Okay, Irene, you go first. So you're going to start in the beginning, Irene, so everyone pay attention because you have to know what she does because you're going to pick up where she left off. So Irene, what did I do first?

12 Irene: You basically set up the mat for the equation.

13 Ms. Martin: Okay. Nice!

14 Irene: And then… can I borrow that? ((Irene placed the Lab Gear in two spots on the mat.)) So, yeah.

15 Ms. Martin: Okay, so can you … you did a great job by just putting, like, the obvious words out there. You set up the mat for the equation. Can you talk about why that represents the equation? Can you talk me through it?

16 Irene: Because the equation is $2y + 4x - 10 = 0$. So basically you have $2y$, plus $4x$ ((touching the pieces)) minus 10, 10 is in the negative box. And equals zero. So, yeah.

17 Ms. Martin: People have a hard time with the equals zero, what does equals zero mean?

18 Irene: There's nothing on this side.

19 Ms. Martin: There we go. People have a hard time with the = 0, with the representation. Phoebe, you…

This interaction is representative of the ways justifying mathematics and orienting to one another became normative for members of this class. In turn 1, Ms. Martin asked, "Check in, check in, check in?" and unlike the fall, when Jaelyn said, "I give up!" Jaelyn's immediate response in January was, "You know I know it!" (turn 2). This transformation in Jaelyn's experience during Shuffle Quizzes indicates that by the end of the semester, she oriented to her classmates to prepare for Shuffle Quizzes, communicating understanding of the group's need to prepare for justifying mathematical thinking (turns 4, 6, 8, 10). Irene also displayed her confidence for explaining mathematical thinking to her peers during this Shuffle Quiz. By turns 12 and 14, when the Shuffle Quiz landed on Irene to speak first, she said, "You basically set up the mat for the equation…And then… can I borrow that?...So, yeah," (while she placed the Lab Gear in the correct spots on the workmat). When the Shuffle Quiz landed on her, Irene worked through the problem Ms. Martin asked without asking for help from anyone. Ms. Martin then pushed Irene for mathematical justification, asking, "Can you talk about why that represents the equation?" (turn 15). Irene responded without hesitation (turn 16), "Because the equation is $2y + 4x - 10 = 0$. So basically you have $2y$, plus $4x$ (touching the pieces) minus 10, 10 is in the negative box. And equals zero. So, yeah" (turn 16). Irene explained the mathematics to her peers and Ms. Martin both when she was randomly called on and when Ms. Martin pressed her for justification, demonstrating her perception of her own improved competence during the course of the semester.

This Second Shuffle Quiz Excerpt shows that by January, students took on the mathematical authority to use one another and to justify their mathematical thinking during the quiz. The group understood that they were accountable for having the authority to explain the mathematics before Ms. Martin came over (turn 2) and that they were empowered to take on that role (turns 2, 10, 14, 16, 18). Ms. Martin then held Irene accountable for mathematical justification (turns 15, 17), revealing her process of transferring mathematical authority to students (Gresalfi et al., 2009). As the interaction continued past turn 19, Ms. Martin engaged each of the students to justify her mathematics understanding of the mat for this problem.

Case 4: Transferring mathematical authority through Participation Quizzes. Ms. Martin's use of Participation Quizzes represented another instructional practice she used to transfer mathematical authority to students. The Participation Quizzes[23] displayed the ways in which students were enacting the expected behavior norms while working in groups, and they reinforced the type of interactions Ms. Martin expected to see in groups.

The Participation Quiz was a live, public record of student behaviors, thus serving to reinforce the ways Ms. Martin expected individuals to behave as students and as mathematicians for the duration of the groupwork assignment. Ms. Martin prepared the purple and red phrases in advance and displayed them at the bottom of the Participation Quiz, using them to reinforce the type of behavior she hoped students would display. When students displayed behavior she found appropriate, she dragged the phrases up to their groups on the Smartboard display, including "pointing and explaining" and "all heads in." When students did not display behavior she hoped to see, she would bring up the red phrases, such as "on different problems." She also hand wrote in quotes from students that abided by the expectation that students work together, such as, "I

[23] See Tables 2 and 3 for description and timing of Participation Quizzes in this study.

got…” for group 1 and “did you get…?” for group 2. Students who worked together and received a significant number of blue and purple behaviors and no red behaviors received full points on the Participation Quiz.

As mentioned above, Ms. Martin often paired Participation Quizzes simultaneously with group tests, as is the case for the Participation Quiz represented in Figure 23. This figure shows that “Group 1” was on target, demonstrating many of the ways groups were expected to work and talk together, including “Quick Start” and “Reading directions out loud.” The Participation Quiz also demonstrates how Ms. Martin featured other parts of student conversations, including “talking rate,” “That’s not making sense,” “showing on calculator,” and “Does it matter….?” The Participation Quiz was an instructional practice that introduced and reinforced groupwork behaviors that oriented students to use one another as resources to do mathematics. Making the Participation Quiz a public representation was one way students had the authority to monitor their own mathematical classroom orienting and justifying expectations.

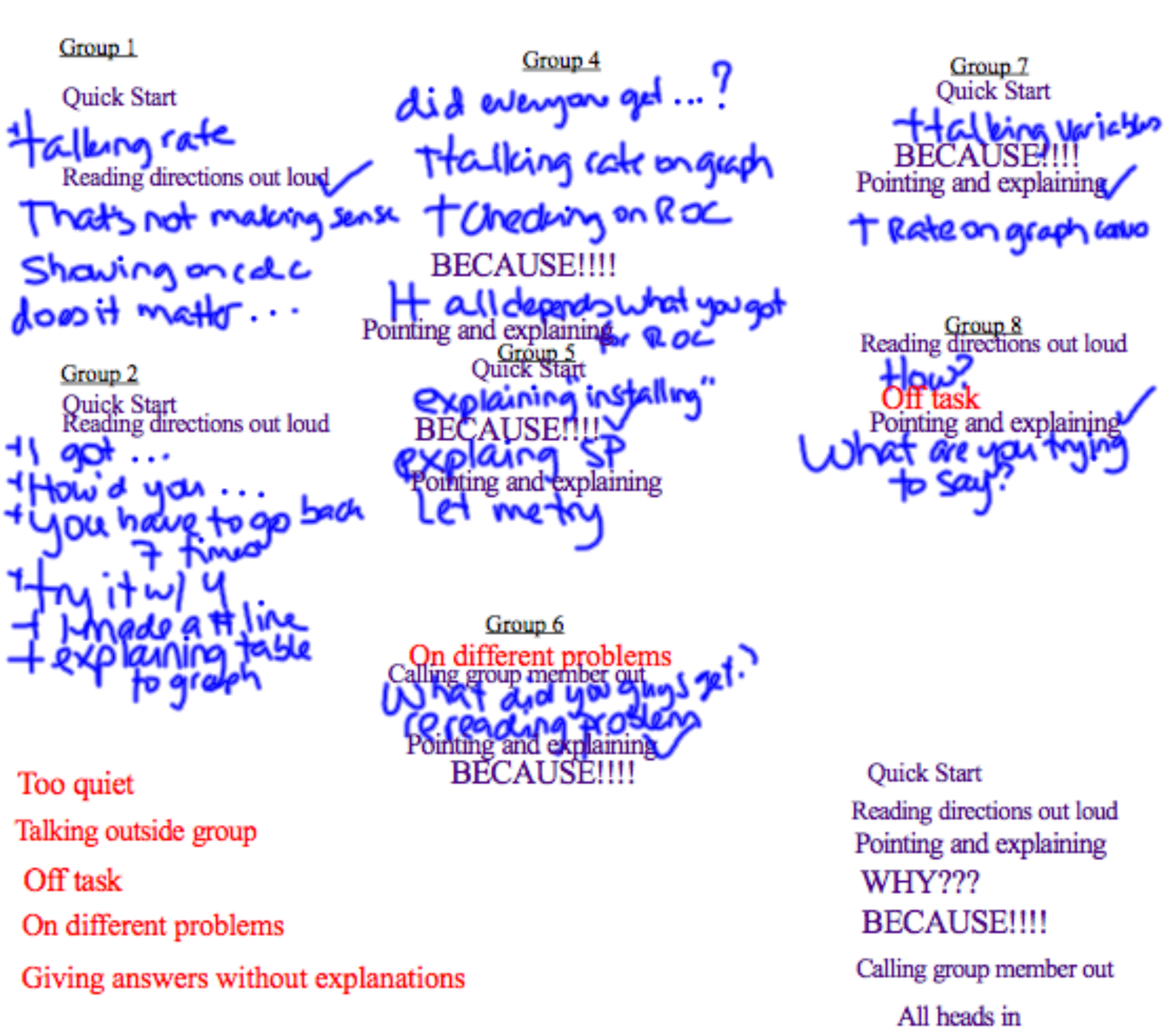

Figure 23. Using Participation Quizzes to transfer mathematical authority to students.

Summary of the process for transferring mathematical authority. The examples in this section illustrate how Ms. Martin used framing ideas and participation structures to position students as competent and attended to status issues, to orient students to work together to complete their assignments and to require students to incorporate explaining and justifying the mathematics into their classwork. Cases 1 through 4 offer evidence of the classroom processes for using these framing, participation structures, and orienting and justifying moves to transfer mathematical authority to students.

Expanding Smartness in Mathematics

The last section of this paper featured several students' class time interactions, attending to classroom participation structures that delegated mathematical authority to students. This last section will address how transferring mathematical authority to students was evidenced through students' perceptions of their *smartness* in this class. Interviews with three of the students, Neesha, Helen, and Jaelyn, illustrate how students in this class had opportunities to greatly improve their self-perceptions of mathematical competence over the course of the study.[24] Neesha, Helen, and Jaelyn each participated in three to five semi-structured interviews between November 2011 and February 2012. I feature student self-perceptions in this section as a way to understand how these students' perceptions of their classroom experiences affected their perceptions of their mathematical learning.

This *smartness* finding emerged from analyzing interview data coded around *smartness*. The typically perceived "smart mathematics student" can quickly follow an algorithm and reach a correct solution (Schoenfeld, 1988). Analyzing interview and classroom data for this class, however, revealed that students valued additional ways of being smart. First, Ms. Martin consistently told students how they were smart, assigning students competence for their unique mathematical *smartness* whenever possible. Second, Ms. Martin supported students to use mathematical authority to find their own way to solve mathematics problems. Students then discussed how they valued understanding multiple solution paths. Third, students in this class valued group members' explaining and justifying mathematical thinking. Students who justified their mathematical thinking had the mathematical authority to do so.

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[24] These are the three students for which I have complete data for this finding. However, partial analysis of other student interviews reveals these students' perspectives and backgrounds are representative of the class as a whole.

Table 6 is a reference for the voices of Neesha, Helen, and Jaelyn, illustrating each student's transformation from an initial negative self-perception of competence to an end-of-study positive self-perception of competence. I use the rest of this section to illustrate Neesha's, Helen's, and Jaelyn's transformations from negative to positive self-perceptions of mathematical competence. I also attribute this transformation to the ways Ms. Martin delegated mathematical authority by expanding the typical definition of mathematical *smartness*. *Smartness* in this class moved beyond speed and correct answers as Ms. Martin assigned competence for learning multiple solution paths, helping others, and justifying mathematics.

Table 6

Students' Self-Perceptions

Student	**Grade**	**Gender**	**Ethnicity**	**Initial Self-Perceptions**	**End-of-Study Self-Perceptions**
Neesha	9	female	African American	"I used to shut down."	"Math is one of my favorite classes."
Helen	9	female	European American	"I've never had much confidence."	"Well now I can explain it to everyone I know!"
Jaelyn	10	female	African American	"Last year I didn't like to go to class."	"I was curious to get to class and see what everybody else's answer was."

Assigning competence. Ms. Martin regularly told students in this class how they were mathematically smart, identifying their competencies and smartness through differing solution paths they found for a given mathematics problem. Neesha, Helen, and Jaelyn each discussed the different ways Ms. Martin told them they demonstrated competence in algebra throughout the semester. As a result, each student improved her self-perceptions of mathematical competence over the course of the study. In response to one interview question, Neesha described how Ms. Martin would demonstrate she knew that their method was mathematically correct:

> Ms. Martin, she'll come toward you and talk with you. And she won't give you the answer; she'll walk away. But even if she walks away, you can tell she's still listening, just by her posture. Like, she might be away, but she's kinda slightly towards you, so she can definitely still hear your group's conversation. And then, when you figure it out, she's like, "I knew you would," walking by… She just does things like that. So you can tell that she's paying attention to almost every group at the same time.

Neesha's comment indicated her belief Ms. Martin regularly paid attention to how students solved mathematics problems and subsequently told them they were smart for finding their own solution paths.

Similarly, Helen said she noticed Ms. Martin tell her ways she was smart in mathematics, not just complimenting her but assigning her authentic mathematical competence as well:

> Ms. Martin points out what I know, and it's not just like she's complimenting me all the time or something. But it's like, she's more encouraging, in that she knows that I know how to do the problem. [She knows] I know enough information to put it all together.

Helen also perceived Ms. Martin's support as assigning her competence for solving mathematics problems in her own way.

Ms. Martin assigned competence to Jaelyn by suggesting her group members use her as a mathematics resource. Ms. Martin said, "Hey [student], do you ever *bug* Jaelyn? Because I know *she* knows how to do it!" Ms. Martin deliberately asked the other group member to "bug Jaelyn" so they could learn her Lab Gear method for simplifying. In this way, Ms. Martin publicly assigned Jaelyn mathematical competence for using Lab Gear to justify mathematics. In addition to assigning Jaelyn mathematical competence, Ms. Martin was orienting the group to learn Jaelyn's method. During one interview, I showed Jaelyn this excerpt from class and asked her how watching it made her feel. Jaelyn responded, "Like the leader in the group." Ms. Martin's positioning move and Jaelyn's responsiveness is particularly powerful because Jaelyn was a sophomore student who was repeating Algebra 1. When talking about the previous year in mathematics class, Jaelyn said she would often think, "Oh my God! I have to go to mathematics class, and I don't understand anything we're doing!" But when Jaelyn was assigned competence for the mathematical knowledge she brought this year, it made her feel like she could be a leader. This example illustrates how Jaelyn's new perception of mathematical competence positioned her for increased access to do mathematics in this classroom, which gave her classmates increased access to her mathematical ideas.

These examples illustrate how Neesha, Helen, and Jaelyn, students who had not previously found themselves competent, started to learn how they were doing well in mathematics. Neesha observed Ms. Martin surveying groups and pointing out their accomplishments when they solved a particular problem in their own ways. Helen believed Ms. Martin did not give empty compliments and would point out what students understood about the mathematics. Jaelyn was assigned competence for using Lab Gear, which made her feel like a leader. Each student noticed being told they were capable of working on authentic mathematics in this classroom.

Learning multiple solution paths. When describing what it was like to be a member of this class, Neesha said students were meant to learn their own and their group members' methods for solving problems:

> So you do need to be able to explain thoroughly and [in] several different ways. So everybody can get it. Or at least, if both of you have the right answer, you both have to explain how you got it, to each other. You might not have the same way, but you need to understand both ways.

Neesha's belief that she was required to explain her own method and also listen to her partner's method illustrates her belief in a classroom norm where every student was regularly encouraged to share their own and learn their group members' ways of working on a problem. When I asked Helen to describe what it was like to be a student in this class, she noted that her experience learning mathematics this year was different than previous years. For her, it was new to be required to think for herself:

> I know [Ms. Martin] teaches us differently. I know it's more like you're thinking for yourself. And you're thinking outside the box or something. And instead of learning a formula or something, you're learning it in your way, instead of like the structured way, or whatever.

Helen's excerpt indicates her belief that learning in this class was "outside the box" because students were not learning formulas from memory or by rote procedure but were learning "in your [own] way, instead of the structured way." Similarly, Jaelyn's excerpt above, in which Ms. Martin asked her to explain simplification using Lab Gear, allowed Jaelyn to feel "like the leader in the group." These examples show the students in this class believed that *smartness* involved learning their own solution paths and the solution paths of their group members.

I also observed Ms. Martin supporting students to pursue multiple solution paths for solving an algebra problem. In this way multiple solution paths became a pedagogical practice simultaneously serving to delegate mathematical authority to students (as described above) and to expand classroom *smartness*. Ms. Martin employed this strategy starting from the first assignment, “The Garden Border” problem, a groupworthy task in which students had to determine how to tile the perimeter of a square garden of an unknown size, coming up with at least four different methods. In this way, Ms. Martin set and maintained a precedent for the semester that students value multiple solution paths. Later in the semester, the consistent value Ms. Martin placed on learning one’s own and other group members’ multiple methods was realized by emphasizing, for example, that when students used a table to determine rate of change and a linear equation for a given set of data, they could orient the table to any set of numbers they wanted. As a result, I observed that although some group members chose to make their *x*-table entries change “by 1s,” others chose to make their table entries change, “by 2s” or “by 3s”, and students were still expected to share their thinking with their group members, explaining the way they chose to look at the table. Further, when solutions were shared with the class, Ms. Martin encouraged students who thought of the problem in different ways to share their strategies. Because multiple solution paths were valued in this classroom, valuing different methods for solving problems became a classroom norm. The teacher’s moves associated with encouraging students to see problems “in their own way” expanded the concept of smartness in this classroom to include the idea that understanding multiple methods made students smarter.

Orienting students to helping one another. The examples above illustrated, in part, how Ms. Martin assigned competence to students for learning their own ways and explaining their mathematical thinking to their peers. The students also described how they valued Ms. Martin’s orienting them to help one another.

When talking about how she worked with her classmates, Neesha said, “It’s a lot more community, in this class at least. And just really, like, helpful, with everybody. Like, they wanna make sure everybody gets it." Neesha said that she believed her class was a "community" where classmates wanted to work together and wanted to ensure other classmates understood the mathematics. When I asked Neesha to describe how working with her group members was helpful, she replied:

> Talking out loud ... or thinking out loud really, it like helps you … so then you know. Cuz that's something I struggle for, because I could like, do it on paper, but then like, when I have to present like to somebody else, and then like the table group or in front of the class, it gets a lot harder. Cuz you're like, "Wait, how did I do that, in the first place?" and so, uh ... that's probably what might have happened.

Neesha reflected that "thinking out loud" was the part that was helpful and "it gets a lot harder" in working with others.

When Helen talked about her experience working in a group in this classroom, she compared her experience to previous mathematics class, noting this year was different for her because of the work in groups:

> This year, I think I'm a better math student. Because the other math environment I was in was more independent, and you had to get things on your won. This year, I think it's the [school] thing they do in the math department: We're sitting in a group. And I think that really helps me.

Helen's excerpt illustrates her belief that working in a group was helpful and meant she did not have to work alone on mathematics. This quote also illustrates Helen believed she was learning from her group members.

Jaelyn also believed Ms. Martin's orienting students to help other group members was a regular part of class. When talking about what it was like to work in groups, Jaelyn noted, "Ms.

Martin will make everyone in your group help you out and stuff." In another excerpt, Jaelyn talked excitedly about how she had received her first "A" on a mathematics test:

> Once I got the hang of it down, for the first time in years, when we took our linear equations test, I got an A on it. And then I was like, running home, and I was like telling my mom, because I never get As on math tests.

Here Jaelyn described understanding linear equations for the first time. And in response to the question, "What do you think has to happen for you to continue getting As on math tests going forward?" Jaelyn did not talk about studying or knowing mathematics, she talked about how her success could be attributed to working in a group: "As a learner, you have to like…if you don't understand it, this is where a group comes in. You should feel comfortable asking your group." Curious about how for Jaelyn getting As was related to asking her group for help, I read the previous excerpt back to her at a later interview. I then asked Jaelyn to comment further. She replied with another example of the value of helping group members:

> Even like, yesterday, I think it was yesterday, I was like, "Ms. Martin, I give up!" and then [Ms. Martin] was like, "No, no, no! You don't." And then to Qianna, [Ms. Martin] was like, "Qianna, help her. Explain!" You know. "Explain it to her!" And she was like explaining. And then I was like, "Oh, okay, I get it now!"

When I asked Jaelyn to further reflect on what she had initially said, she reported a new example, explaining that when Ms. Martin was insistent that group members help one another, she experienced understanding the mathematics.

The examples from Neesha, Helen, and Jaelyn illustrate the ways they each came to value being oriented to work with one another in this class. Neesha believed students wanted to "make sure everyone gets it." Helen reported working in a group as helpful to her understanding. And Jaelyn believed helping was expected: "Ms. Martin will make everyone in your group help you."

Justifying mathematics. Neesha's first individual interview took place in early November, by which point Ms. Martin had randomly assigned her to 4 different small groups, for a total of about 30 hours of instruction. During this first interview, I asked Neesha to describe what being a student in her class was like to that point. Interestingly, Neesha responded she perceived her classmates to be capable and committed to completing mathematics assignments:

> I really like our class…I feel like, at least all the groups that I've been in, they're really hard workers, or, if they're not hard workers, then they'll ask a lot [of questions], so they can get the answers, and so they're willing to learn. [Ms. Martin] knows that we're all talking. And I feel like that all talking thing really helps, like [she wants to] make sure everybody gets it. … I feel like our class is extremely positive. [The students] seem to get the problems, and they can go up on the board and do the problems and get the right answer.

In this excerpt, Neesha described her experiences with the mathematical activity as consistently involving students who were positive, who worked hard, who asked questions, and who justified mathematics. Neesha's comments emerged from the context of a classroom full of students taking Algebra 1 in high school while their peers were taking Geometry and Algebra 2, and many of her classmates had not been successful in mathematics prior to taking this course. Giving a response typical of how Neesha referred to her experiences in this classroom, this quote is evidence for the ways that Ms. Martin developed sociomathematical classroom norms during the first 2 months such that "doing mathematics" went beyond speed and correct answers (Yackel & Cobb, 1996). Neesha indicated she valued group members who were (a) hard working,

(b) positively oriented toward mathematics work, (c) willing to explain their mathematical thinking, and (d) able to ask questions. For Neesha, students showed they were *smart* by sharing their interest for mathematics through talking, justifying, and asking questions of their group members.

Helen also described believing justifying mathematics was helpful to developing her own understanding. About one group, she said, "We all made our statements, and it just worked out. And it helped. If [one student] said something I didn't understand, I could ask questions. And then [the other students] would say, 'oh no, it's this…'." Helen believed when groups worked together and students were able to share what they thought, other students would agree or disagree, justifying how they understood the mathematics until the group came to a consensus. When I asked Jaelyn to discuss the benefits of a Shuffle Quiz, she said,

> Yeah, there's some benefits, because if I'm in a group with somebody, and I know they don't get it, and then like, if the Shuffle Quiz comes to them, and they'll be like, oh, you know, a little like, because, I don't know, I feel like I can relate to it, because, I don't know, I was that person last year? So if I see somebody struggling I can be like, "Hey..." you know, when she walks away and she's giving us another chance she'll be like, "Okay, what exactly didn't you get? So you know, when she comes back around, and she picks you again, will you be able to explain it?"

Jaelyn described Shuffle Quizzes containing mathematical justification. Jaelyn believed she could ask her group members, "Okay, what exactly didn't you get?" so that they could be ready to justify their mathematical thinking when Ms. Martin came back to their group.

The examples here illustrate the use of mathematical justification as another way Ms. Martin expanded *smartness* in this classroom beyond traditional values of speed and accuracy.

Neesha said in her groups all the students would talk about the mathematics. Helen believed her group members were able to all make "statements" their group members would agree with or correct. Jaelyn described the value of being able to use the group to prepare for a Shuffle Quiz.

Summary of expanding *smartness*. The themes that emerged from examining the data for *smartness* in mathematics are quite different from the typically valued characteristics of speed and accuracy. The data show that mathematical *smartness* was broadened to include assigning competence for authentic mathematics contributions, learning multiple solution paths, helping one another, and justifying mathematical thinking. Table 4 displayed an alignment between the class participation structures and the ways that Ms. Martin delegated mathematical authority to her students. Table 7 maps onto Table 4 and summarizes the evidence from the finding on *expanding smartness*.

Table 7

Expanding Mathematical Smartness

Expanding *Smartness*	**B. Classroom Structures & Pedagogical Practices**	**Neesha**	**Helen**	**Jaelyn**	**Evidence of Delegating Authority**
A. <u>Framing ideas</u> Assigning Competence	Ms. Martin regularly assigns students competence.	Ms. Martin won't give you the answer, she'll listen and eventually say, "I knew you would [get it]!"	Ms. Martin points out what I know.	Ms. Martin told students to "*bug* Jaelyn" about how to use Lab Gear to explain simplifying.	Ms. Martin regularly identifies to students the ways that they are smart in mathematics.
C. <u>Orienting Moves</u> Multiple Solution Paths	e.g., the Garden Border Problem	Learn your own way and your group member's way to solve a problem.	You're thinking for yourself.	We'd put our papers in the middle.	The class values multiple ways to solve problems.

Expanding *Smartness*	B. Classroom Structures & Pedagogical Practices	Neesha	Helen	Jaelyn	Evidence of Delegating Authority
C. Orienting Moves Helping	regular random group assignment	[We] wanna make sure everyone gets it.	We're sitting in a group. And I think that really helps me.	As a learner, if you don't understand it, this is where a group comes in.	Students are expected to help one another.
D. Justifying Moves Explaining Mathematical Thinking	Development of socio-mathematical norms.	Students in this class are hard workers.	This year we're in groups and that really helps me.	Ms. Martin will make everybody in your group help you out.	Talking about your mathematics ideas is a way to show smartness.

Each of the themes that emerged around Expanding Mathematical *Smartness* is outlined in the rows of Table 7: assigning competence, valuing multiple solution paths, helping group members, and justifying mathematical thinking. Each theme is also fundamental to the pedagogical practices aligned with using complex instruction and with transferring mathematical authority to students. Assigning competence allows students to validate when mathematics contributions are correct (Boaler & Staples, 2008). Honoring students' multiple solution paths allows students to decide which solution method to use (Gresalfi & Cobb, 2006). Requiring students to help one another also empowers groups to take on the responsibility to use mathematical sensemaking to solve problems and evaluate the reasonableness of their solutions. Justifying mathematics orients students to be accountable to their group (Gresalfi et al., 2009; Sharan, 1999). Ms. Martin supported and developed these principles in particular ways in her classroom over the course of the semester. The themes contributed to cultivating a classroom environment in which being smart was more than being quick and being correct. Ms. Martin's ability to expand what it meant to be smart has important implications for students' access to smartness in her classroom. Because she broadened the definition of smartness, Ms. Martin was able to feature new ways of being smart in her classroom, thus increasing access to students' mathematical authority. Because she increased the amount of smartness available, the access to smart mathematics contributions also increased. Increasing the presence of and access to students' mathematically competent contributions indicates expanding what it means to be mathematically smart. Ms. Martin expanding smartness by delegating authority to students, moving her classroom toward equity.

Discussion

Contributing: Striving Toward Equity

This study showed that Ms. Martin was striving toward equity in her classroom through her belief system, through her process for transferring mathematical authority with specific classroom structures and pedagogical practices, and by expanding mathematical smartness. First, I found Ms. Martin's vision for teaching and learning was founded on the belief all students are mathematically competent. I then found that this belief facilitated her classroom structures and pedagogical practices, which afforded equitable learning opportunities for her students. The literature on teacher beliefs also supports the idea that teacher expectations affect opportunities in the classroom (Leonard & Evans, 2012). Second, I found Ms. Martin's process for delegating mathematical authority to students involved specific classroom structures that positioned students as competent sensemakers, oriented students to use one another as mathematical resources, and required students use mathematical justification in their work with one another. Third, I found Ms. Martin expanded the definition for mathematical smartness, which increased access to the available smartness in her classroom.

Paying attention to the teacher's vision for teaching and learning. Ms. Martin's vision for teaching and learning included a strong belief in fostering a supportive environment, where students became convinced they were capable of equally contributing to the classroom community learning high-level mathematics and participating in opportunities to reason about mathematics. If Ms. Martin's vision for equal contributors was realized year after year, and if increased participation could determine academic success, Ms. Martin's students' achievements would be in line with moving toward Gutiérrez's (2007) assertion that striving toward equity implies an inability to tell differences in achievement between different groupings of students.

In the process of transferring mathematical authority, the teacher strives toward equity. This work contributes to an ongoing conversation about the importance of striving toward equity in secondary mathematics classrooms. In this paper I outline Ms. Martin's leaning on students to do the "heavy lifting" of understanding and explaining the mathematics concepts. Ms. Martin delegated authority by featuring specific pedagogical practices that positioned students as competent, oriented students to use one another as mathematical resources, required students' mathematical justification, and expanded smartness in her classroom. A summary of Ms. Martin's enactment of her strategies to delegate authority to students was particularly revealed by a comment by Helen during an interview:

> At first I was like, *"Ms. Martin is secretly teaching us!"* At first I had to try to get used to the fact that it was about working with groups. But it's helped me a lot to improve my math skills and stuff. All of my other teachers will show you the formulas and show you the little tricks about how they will do the problem. But Ms. Martin will make us try and figure it out.

Ms. Martin's "secret teaching" offered students the mathematical authority to share their own methods for understanding mathematics problems. She created a mathematics-learning environment in which students were comfortable sharing their own thinking and asking questions whenever they did not understand. Further, she expanded what it meant to be smart in mathematics, increasing the number of smart contributions available in the room and therefore the amount of smartness students had access to while learning high-level Algebra 1 content.

Transferring authority expands smartness and increases access. Previous studies have illustrated potential benefits for effectively transferring mathematical authority to students. This study contributes an understanding of the process for transferring authority. Effectively transferred mathematical authority also expands mathematical smartness in the classroom. Expanding smartness implies more access to ways to be smart. And if there are more ways to be smart, more students will have access to smartness. The cycle of transferring authority could then theoretically continue.

Limitations

Data for this study was collected from one class period and from one teacher who had training in complex instruction. The study therefore does not attend to the scope or complexity that would be offered by simultaneously studying the transfer of authority in multiple classes, teachers, schools, and/or districts.

The data corpus indicates this classroom was a community in which groupwork was welcomed and where groupwork thrived as the preferred method for learning mathematics for all students. Still, this paper does not seek to understand the perceptions of students who already found themselves successful in mathematics and who may not have enjoyed working with other students in groups. Further research could explore the effects on students with different perspectives.

The nature of the questions I asked often uncovered students' self-reflections on what they needed to do on their own to improve their mathematics experiences. The process of interviewing students may have been positively correlated with their changed perceptions of competence and groupwork. The nature of that effect is not explored in this paper but could be a source for future understanding.

Implications

Transferring Mathematical Authority to Students Can be Powerful

This paper contributes a need for understanding the process of effectively delegating mathematical authority to students. Success with students who have typically not been successful in mathematics is an accomplishment for both teacher and students. In the context of a society struggling to reposition students who have been historically perceived as having less competence than their peers, Ms. Martin seems to have been extremely successful in meeting these significant challenges. Her work is directly aligned with the sociological approach that Elizabeth Cohen and Rachel Lotan developed through the use of complex instruction.

Call for Future Action

Future studies should attend to other pedagogical practices that delegate mathematical authority to students and explore how and whether they simultaneously lead to striving toward equity. Although most students described an incredibly positive experience in Ms. Martin's classroom, what happens to Ms. Martin's students 1, 2, 4, and 10 years after taking her class is unclear. As such, future studies should attend to the transfer and durability of students' perspectives on mathematics after participating in a mathematics classroom like Ms. Martin's. Is this experience an enduring change for students? Do students persist in this perspective when taking other mathematics classes? Does this success in mathematics open doors that were previously closed and potentially support them to successfully complete college preparatory courses? The potential effect of delegating mathematical authority could be revealed by studies that examine whether students in classrooms like Ms. Martin's later study college mathematics as often as their peers. This type of study would also serve to inform the pursuit of equitable learning opportunities. Future studies should also attend to the potential for elementary and middle school students to have decreased differences in opportunities for studying algebraic content.

As educators, we will know we have moved closer to achieving equitable outcomes in mathematics education when mathematical achievements are less and less associated with a particular group of students, whether separated by ethnicity, gender, class, religion, (dis)ability, or any other grouping.

Abstract Paper 3

In this paper, I argue that a major requirement for striving toward equity in mathematics education research involves attending to students' perceptions of mathematical competence. I respond to this call, presenting the voices of five students whose perceptions of their own mathematical competence significantly improved while taking an Algebra 1 class. I argue that the development and maintenance of specific social and sociomathematical classroom norms supported students' perceptions of their mathematical competence. The social norms that emerged from analysis include *everyone discusses ideas with everyone else, everyone offers and receives help,* and *groups work on the same problem.* The sociomathematical norms that emerged from analysis include *mathematical work involves making sense of your own ideas, mathematical work involves making sense of others' ideas,* and *groups work to achieve mathematical consensus*. I explore how the social and sociomathematical norms mediated students' changed perceptions of their competence. I argue that the development and maintenance of these norms expanded the learning opportunities in this classroom, facilitating the teacher's efforts to strive toward equity.

PAPER 3: *“Ms. Martin is Secretly Teaching Us!”* Social and Sociomathematical Norms That Mediate Students’ Perceptions of Competence

In equitable mathematics classrooms, students have access to a “fair distribution of opportunities to learn” (Esmonde, 2009b, p. 1010), and teachers and students view everyone as capable of learning high-level content (Cohen, 1997a). Increased attention to the complex work in classroom practices of teachers who strive toward equity in mathematics education has been growing in the last two and a half decades (e.g., Esmonde, 2009b; Martin, 2003; Nasir & Cobb, 2007; NCTM, 1989, 2000). In these studies, scholars created a vision and set of standards for what mathematical literacy and equity means and should look like (NCTM, 1989, 2000), proposed an equitable approach to instruction that features students’ discourse during small group work (Cohen & Lotan, 1997), argued equity discussions must be connected to the larger social and structural contexts that impact the lives of underrepresented students (Martin, 2003; Nasir & Cobb, 2007), and summarized how teachers should assist students in cooperative groups (Esmonde, 2009b). In this paper, I present the classroom norms that supported developing students’ perceptions of mathematical competence as a way to strive toward equity. I find that the development and maintenance of certain social and sociomathematical classroom norms (Kazemi & Stipek, 2001; Yackel & Cobb, 1996) can foster improving students’ perceptions of their mathematical competence. By classroom *social norms,* I refer to the ways individuals were routinely expected to behave as students in this classroom. The classroom *sociomathematical norms* refer to the ways students are routinely expected to engage around mathematical activity. In what follows, I propose students’ changed perceptions of their mathematical competence were mediated by the established social and sociomathematical classroom norms.

All mathematics teachers foster ways to participate in the classroom and ways to participate in mathematics. Over time, the routine ways students and the teacher negotiate acceptable classroom behavior as a student and a mathematician in their classroom become the classroom's social and sociomathematical norms. I therefore contend that students and teachers experience classroom practices through social and sociomathematical norms, and those experiences influence how students perceive their own levels of competence in mathematics. In the classroom featured in this study, I found the teacher's vision included fostering norms that positioned all students as competent sense-makers, thus guiding opportunities for students' equitable participation. In this paper, I argue that the teacher developed students' perceptions of mathematical competence through specific established social and sociomathematical norms. Classrooms that move toward equity have social and sociomathematical norms that promote a view of mathematics in which working together on mathematics means accessing and using mathematics reasoning to solve rigorous and relevant problems.

In this paper, I simultaneously respond to calls for qualitative research to both contribute to the development and understanding of social and sociomathematical norms and to increase empirical evidence of classroom situations in which students improved their perceptions of mathematical competence (Esmonde, 2009b). I note neither the development of social and sociomathematical norms nor the understanding of students' changed perceptions of competence has been significantly empirically studied in secondary mathematics classrooms. I further argue for studying these concepts together: I assert striving toward equity involves paying attention to classroom social and sociomathematical norms and their relationship to and influence on students' perceptions of mathematical competence. In this paper, I reveal how a teacher developed students' perceptions of competence through the use of specific social and sociomathematical norms. This paper is therefore a response to the call to document how perceptions of competence emerge in cooperative learning contexts, and how these perceptions are related to the social ecology of the classroom (Esmonde, 2009b; Esmonde, Brodie, Dookie, & Takeuchi, 2009).

Literature Review

In this section, I begin by sharing the perspective on learning that frames this study. I then highlight the ways other scholars have addressed social and sociomathematical norms and perceptions of competence in mathematics education. Next I show how my research contributes to the identified gaps in mathematics education research as I share the research questions explored for this paper.

A Sociocultural Perspective of Learning

A sociocultural perspective of learning pays attention to how students develop in their classroom community (Rogoff, 1995, 1997, 2003). In particular, I draw on Rogoff's characterization of *learning* as changing participation in the culture of the classroom (Rogoff, 1995, 1997, 2003). The collective nature of learning mathematics in schools highlights the need to pay attention to the shared processes that exist in classrooms and the ways that students perceive their mathematical learning.

In the classroom I studied, students engaged with one another and the teacher through small group and whole-class interactions. In classrooms where students are required to work together, students position themselves and their classmates to have varying levels of competence (Cohen & Lotan, 1997). Student positioning influences how students interact with one another, which impacts their learning. In adopting Rogoff's perspective of sociocultural learning, I suggest that these students developed[25] through a process of changing participation as they engaged in group interactions in the sociocultural community of the classroom. Rogoff's Theory (2003) also draws attention to three different planes of analysis: the community plane, the interpersonal plane, and the personal plane. In classrooms where students must work together to learn mathematics, the activity of learning takes place on the interpersonal plane. For this reason, I foreground the activity in the interpersonal plane in order to examine how students' individual learning was influenced by and could be understood within the context of their classroom interactions.

Previous Analyses of Classroom Social and Sociomathematical Norms

I study classroom interactions on the interpersonal plane because reform-oriented practices call for students to learn mathematics in classrooms fostering communication to support conceptual understanding (NCTM, 1989, 2000). Current research has also established that the social practices for doing mathematics are different in traditional and reform-oriented mathematics classes (e.g., Kazemi & Stipek, 2001; NCTM 1989, 2000; Wood, Williams, & McNeal, 2006; Yackel & Cobb, 1996). Yackel and Cobb (1996) explored the regular social practices students and the teacher negotiated and developed in their work together, which they called the classroom social and sociomathematical norms. Classroom *social norms* are the beliefs and values that guide the acceptable ways of participating in the classroom community; they refer to how students come to participate in classroom activities over time. Although social norms are necessary to help students understand acceptable ways of participating in classroom activities, they are insufficient to achieve content-specific conceptual thinking and learning. Classroom *sociomathematical norms* address the ways that students participate in mathematics

[25] In line with Rogoff (1997), I consider the terms *learning* and *development* to refer to the same individual processes. I therefore refer to them interchangeably.

classroom activities and regulate what counts as mathematical argumentation and participation. Sociomathematical norms also influence students' opportunities for learning mathematics (Yackel & Cobb, 1996).

Researchers have studied social and sociomathematical norms to understand ways students develop beliefs and values about what it means to be part of a mathematical learning community. Each study investigated an aspect of the relationship between classroom norms, students' learning opportunities, and their mathematics understanding. Yackel and Cobb (1996) sought to understand the role of social and sociomathematical norms in second-grade students' learning opportunities. They analyzed the teacher's role in inquiry-based second-grade mathematics classes in order to uncover how students developed dispositions toward and increased their intellectual autonomy in mathematics. Cobb, Stephan, McClain, and Gravemeijer (2001) analyzed the learning trajectory of a first-grade classroom community to understand the emergence of students' reasoning through mathematical classroom content. Kazemi and Stipek (2001) studied fourth- and fifth-grade classroom interactions that offered opportunities for *conceptual press* in students' mathematical learning. They revealed high-press opportunities, in which students linked their problem-solving strategies to mathematical reasons, were characterized by the presence of specific sociomathematical norms that promoted conceptual understanding. Hand (2003) studied the relationships between mathematical, cultural, and social practices in a high school classroom. She revealed the teacher framing and positioning norms that fostered the development of an equitable learning environment. Hufferd-Ackles, Fuson, and Sherin (2004) studied four elementary classrooms and found that in the classroom where the social norms had grown to require students to take central discourse roles, students built the most mathematical understandings. Wood et al. (2006) studied the relationship between young students' verbal social interaction patterns and children's meaning making in mathematics classroom practices. They found that social processes helped describe interaction structures created in different elementary classroom environments. Although each of these studies investigated the development of classroom norms and their relationship to students' learning opportunities, the studies did not address the ways students perceived their learning. This paper links students' perceptions of their classroom experiences to the development of social and sociomathematical classroom norms.

Analyzing Students' Perceptions of Competence, Identities, and Dispositions in Mathematics Education

Cohen and Lotan (1997) identified students' perceptions of their own and classmates' ability as one of the most important considerations in working toward equity. Boaler and Greeno (2000) and Empson (2003) found that classrooms focused on developing discourse-centered learning environments can foster students' sense of mathematical competence. A longitudinal study conducted by Boaler and Staples (2008) revealed high school students who were taught mathematics in classrooms using complex instruction changed their perceptions of who could be mathematically competent. More students in the classrooms using complex instruction reported enjoying mathematics and more of those students pursued more challenging mathematics courses after leaving their complex instruction classrooms. Students' perceptions of competence are also central to theories of motivation and achievement (Turner, Warzon, & Christensen, 2011). Jansen (2006) found that seventh-grade students with motivation to participate in mathematics were more likely to participate to demonstrate their competence. Students' perceptions of competence have also been studied to investigate discursive positioning moves that facilitated Latino/a English learners' opportunities to take on problem-solving roles for English learners

(Turner et al., 2012). The teacher and students in this study positioned other students as competent problem-solvers during mathematics discussions. The studies highlighted here found reform-oriented classes foster opportunities for students to develop broader and more complex definitions of who could be perceived as mathematically competent.

I choose to investigate students' *perceptions of competence in mathematics* in order to investigate the ways students see themselves as capable (or not) of completing mathematics work. A focus on *perceptions of competence* attends to ways students find themselves able to do mathematics work in their classroom community. A student's *perceptions of competence* in mathematics also contribute to his or her *mathematics identity* and *dispositions toward mathematics*. Mathematical *identities* and *dispositions* are reified by whom individuals recognize themselves to be and how they participate in mathematical activity.

Although classroom norms describe the routine ways that students and the teacher interact in a learning community, reform research also attends to the ways that students perceive themselves as learners. *Mathematics identity* has been defined as self-understandings about individuals' own abilities to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives (Martin, 2006). Previous scholars have also studied identity (a) during its construction (Boaler, 2002a; Boaler & Greeno, 2000; Jilk, 2007, 2011), (b) to understand students' resistance to learning mathematics (Cobb & Hodge, 2002), (c) as an interpretive scheme for analysis (Cobb & Hodge, 2007), (d) to understand the acculturation into a community (Hand, 2006; Nasir & Hand, 2008), and (e) to understand the participation of African American (Martin, 2007), Latino/a (Zavala, 2012), and English learner students in mathematics (Turner et al., 2012).

Identity-related processes are also believed to be as central to mathematics development as content learning (Esmonde, 2009b). Reform efforts in mathematics education further assert students can form positive identities as knowers and doers of mathematics (Esmonde, 2009b). Some work on students' mathematics identity development has related students' self-understandings and affiliations toward mathematics with the negotiation of their mathematics work in the cultural community of the classroom (e.g., Boaler, 2000; Boaler & Staples, 2008; Esmonde, 2009b; Jilk, 2011; Turner et al., 2012). Turner et al. (2012) revealed how positioning moves and roles individuals take up as they interact (Harré & van Langenhove, 1999) occurred during mathematics discussions. Turner et al. found positioning moves (re)structured English learners' participation in school mathematics to develop positive mathematics identities and promote all students' participation and learning.

The study of students' *mathematics dispositions* has drawn attention to the ways students engage and participate with the discipline of mathematics (Gresalfi, 2009; Gresalfi & Cobb, 2006). Work on students' dispositions toward the discipline of mathematics has related students' identification with or orientation toward mathematics in a particular classroom (Gresalfi & Cobb, 2006). Studies on dispositions toward mathematics also explore how students' dispositions shift as students engage in classroom practices over time. Sengupta-Irving (2009) studied students' dispositions toward group-based mathematical learning through peer partnerships. She found that teacher attention to partnerships afforded opportunities for students to improve their mathematical dispositions. Supporting students' discipline-specific dispositions has become a critical goal for helping students achieve an empowered relationship with mathematics (Martin, 2000), fostering positive ideas about the role of mathematics (Boaler & Greeno, 2000), and striving toward equity (Gresalfi & Cobb, 2006; Moses & Cobb, 2001).

I draw broadly on *identity* and *dispositions* research to frame how students make sense of themselves as mathematicians. I narrow my emphasis to *perceptions of competence* as a way to focus understanding on how individuals actively see themselves as capable of doing the mathematics work of the classroom. I examine how *perceptions of competence* were mediated by mathematics classroom activities.

Contribution and Research Questions

Scholars who study equitable teaching and learning, the development of classroom norms, and students' perceptions of competence have identified areas of need for future research. Gaps include a need for empirical research to grow understanding of the teaching and learning process in inquiry-oriented classrooms (Cobb, Wood, Yackel, & McNeal, 1992), a need to analyze what counts as mathematical in a classroom community (Cobb et al., 2001), a need for increased study of the instructional practices that support equitable participation (Boaler, 2002b), a need to recognize students' active roles in interpreting social norms and how students use their constructed identities when learning mathematics (Jilk, 2007), a need for empirical evidence of how social interactions and mathematical talk support mathematical learning, and a need to further document identity-related shifts and factors that position some students as more competent learners (Esmonde, 2009b, 2009c). To begin addressing these gaps, I asked the following research question: *What classroom experiences lead students to improve their perceptions of competence?* When I discovered that the development of classroom norms mediated students' change perceptions of competence, I asked: *What social and sociomathematical norms governed interactions in this classroom?* And then, *How, if at all, did the classroom negotiation of social and sociomathematical norms affect students' perceptions of competence?* I use the rest of this paper to address the ways the teacher's development and maintenance of certain social and sociomathematical norms mediated students' perceptions of their mathematical competence in this classroom.

Methods

Researcher Positioning

I share my positioning with respect to my work to be clear about how who I am is reflected in this study and in the ways I interpreted these data. I am a European American, middle class, female secondary mathematics educator with a background in teaching high school mathematics. I believe passionately that all students and all people are competent in mathematics and are capable of achieving highly successful outcomes. I believe students who are not currently successful and who do not yet perceive themselves as competent in mathematics have lacked significant opportunities to show how they can be successful. I have developed a strong commitment to supporting all students to find the ways they are competent and successful in mathematics.

As a teacher educator and a researcher, I believe that all teachers and students should "view each student as capable of learning both basic skills and high-level concepts" (Cohen, 1997a, p. 4). I support teachers to implement effective discourse and groupwork strategies that attend to ways students perceive their competence and work to increase equitable outcomes. I am interested in making a case for expanding what it means to "do mathematics" and to be competent in mathematics, so that society values more than speed and accuracy in mathematics education. In other words, I endeavor to support change and to increase access to mathematics for all students. I was driven to conduct this study by the desire to unpack teaching practices that strive toward equity and to uncover historically marginalized and underrepresented students and their perceptions of competence.

Site and Teacher Selection

Race and racism are crucial considerations in mathematics education research that strives toward equity (Martin, 2009). As American classrooms continue to increase in linguistic, ethnic, and socioeconomic diversity, districts, schools, and teachers who strive for equitable learning opportunities are obliged to use pedagogies that are successful with heterogeneous populations of students (Cohen, 1997a). As such, I situated this study during the 2011-2012 school year in an urban school with a linguistically, ethnically, and socioeconomically diverse student population. I sought to understand how historically marginalized and underrepresented mathematics students (African Americans, Latinas/os, and girls) made sense of their mathematical learning within their classroom context.

Because I was interested in learning primarily from students who had not previously found themselves successful in mathematics, I elected to work with a high school Algebra 1 class at a school where many other freshmen students enrolled in Geometry or Algebra 2. The Algebra 1 students were thus at a disadvantage because they were taking a lower mathematics class than many of their peers. In the Discussion Section, I examine the significance of how the school placed students into classes.

I selected a collaborating teacher, Hannah Martin (all names are pseudonyms), because she intentionally made pedagogical choices to counter status issues in the classroom (Cohen, 1997b), because she actively attended to the development of certain social and sociomathematical norms (Cobb et al., 1991) that fostered interdependence (Yackel & Cobb, 1996), because she regularly engaged students in mathematics discourse (Cazden, 2001), and because her classroom practice focused on student-centered learning. Hannah and I also shared a common interest in supporting students who had previously been unsuccessful in mathematics. During the time of the study, Hannah was in her fifth year of teaching Algebra 1 at this school.

I also selected Hannah because she said she employed groupwork in her classroom in order to position students as competent sensemakers. Some mathematics educators and researchers have argued that effectively implemented groupwork is an important pedagogical tool that teachers, students, classrooms, schools, and districts can implement to strive toward equity. For this reason, groupwork has become a consistent focus of equity-based research in teaching and learning mathematics (e.g., Boaler & Staples, 2008; Cohen, 1994a; Cohen & Lotan, 1997; Esmonde, 2009c; Webb, 1991). Effectively implemented groupwork has been shown to increase student participation, engage students more deeply in their learning, develop their academic thinking (e.g., Boaler & Staples, 2008; Herrenkohl & Guerra, 1998), promote positive mathematics identities (Hand, 2006; Jilk, 2007; Nasir & Hand, 2008), and foster classroom relational equity (Boaler, 2006).

I chose to work with Hannah because her classroom features groupwork facilitated by the principles of complex instruction (CI). CI is a form of cooperative learning developed by Elizabeth Cohen and Rachel Lotan. The presumption that *all* students are capable and competent in working on high-level mathematics is the key to a faithful implementation of CI (Cohen & Lotan, 1997), and this presumption of competence drives some CI teachers to regularly randomly assign students to groups (Boaler & Staples, 2008), which Hannah did every 2 weeks during the time of the study. CI teachers also aim to disrupt typical hierarchies that influence students' expectations for competence by paying attention to the role of *status*, or the relative standing a student holds on the basis of characteristics that are valued in the classroom as students work together (Cohen, 1994). CI teachers also aim to facilitate learning in small groups by delegating mathematical authority to students and by assigning tasks that require multiple abilities and multiple entry points. In all these ways, CI uses classwork to create opportunities for equal-status interactions in the classroom (Cohen, 1994a; Cohen & Lotan, 1997), thereby striving toward equity. Hannah ran her classroom aiming to employ all of these instructional strategies.

Student Selection

Twenty-eight students took this Algebra 1 class during the 2011-2012 school year. Seventeen students were African American or African immigrants, 5 students were Asian American, 4 students were European American, and 2 students were Latino/a.[26] Twenty-four of the 28 students formally agreed to participate in the study. Twenty-two students were girls, and 6 students were boys. I outline the class period's demographics in detail because they did not reflect the make-up of the school's student body. The school had about 50% boys and 50% girls. The students' reported racial backgrounds, compared with the classroom demographics, are available in Table 8. When compared with the school, this Algebra 1 class had an overrepresentation of African Americans, Latinas, and girls, and an underrepresentation of Asian Americans, European Americans, and boys. The difference mirrors a school trend (perhaps also a district and national trend) in which, prior to entering high school, disproportionately more European Americans, Asian Americans, and boys successfully complete Algebra 1, whereas disproportionately fewer African Americans, Latinos/as, and girls need to take or complete Algebra 1 in high school.

[26] I describe the demographics on the basis of my own observation. Any error or misrepresentation of how students perceived their identities is my error, alone.

Table 8

School Reported & Class Observed Student Demographics

School & Algebra Class Demographics	Approximate School-Reported Percentage	Class Number Female	Class Percent Female	Class Number Male	Class Percent Male
American Indian / Alaskan Native	1%	0	0%	0	0%
Asian	20%	3	10.7%	2	7.1 %
Pacific Islander	1%	0	0%	0	0%
Asian / Pacific Islander	20%	0	0%	0	0%
African American / African Immigrant	30%	14	50%	3	10.7%
Latino/a	10%	2	7.1%	0	0%
European American	40%	3	10.7%	1	3.6%
Two or More Races	22%	0	0%	0	0%
Total Class		22	79%	6	21%

In this paper I present findings from analyzing the interviews and classroom interactions of 5 students from this class who participated in the study: Elena, Helen, Irene, Jaelyn, and Neesha. All but 1 out of the 13 students interviewed reported positive perceptions of themselves as mathematicians by the end of the semester[27]. Although all students were invited to participate in interviews, this paper reports on the 5 students for whom I have complete data of improved self-perceptions of mathematical competence (Merriam, 2009). All 5 students featured in this analysis are girls. There are several reasons why this is true. First, there were a proportionally small number of boys in the class: Only 6 of the 28 students were boys. Second, although I interviewed 4 of the 6 boys in this class, each had individual reasons for not being picked for full analysis: 1 of these boys left the school, 2 elected not to finish the interview process, and the fourth already perceived himself competent in mathematics prior to coming to this course. Lastly,

[27] The student who did not report an improved perception of competence was an upperclassman whose background and experience in mathematics was complicated. The student also did not agree to be interviewed more than the one time.

having difficulty finding boys who significantly improved their perceptions of competence in this class can also be explained by the role gender plays in mathematical learning in this school, classroom, nation, and/or society.[28]

Although each student selected for this analysis had not found herself to be significantly successful in mathematics prior to taking Ms. Martin's Algebra 1 course, by the time of their final interviews, they each described measured improvement in their perceptions of themselves as mathematics learners in Ms. Martin's classroom. Each of the featured students participated in an initial individual interview around the middle of the semester, a final individual interview near the end of the semester, and a group interview near the end of the semester, for a total interview time of about 3 hours for each student.

Elena was a ninth-grade Latina who said in her initial interview that she was not "one of the best students," but that she would, "try to do the mathematics that we're learning." At her final interview, Elena quickly and easily shared all of the mathematics ideas she felt she had "been good at" over the course of the semester in this class: "the *y* equals method, the intercept method, substitute by zero, the elimination method, equations, and using 'the blocks' (Lab Gear[29])."

Helen was a ninth-grade European American who first described herself as lacking confidence in mathematics: "As a math learner, I've never really had much confidence in myself." At the end of the study, when asked to describe what she thought it would take for her to explain

[28] The underrepresentation of boys in this class (when compared with the school's overall 1:1 ratio of boys to girls) implies an overrepresentation of girls in this class. This will be further explored in the Discussion section of this paper.
[29] Henri Picciotto, Wright Group/McGraw Hill – Creative Publications: https://www.wrightgroup.com/launch/wright_group.html. For more information on using Lab Gear in a mathematics classroom, please see http://www.mathedpage.org/manipulatives/lab-gear.html.

her thinking more often, she said, “Oh, well that happened today. [Another student] was explaining it to me, and then I understood it and I was like, ‘Oh! Well now I can explain it to everyone I know!’ So I was explaining it to my group.”

Irene was a ninth-grade African American who described her mathematics history as uneven prior to entering high school and taking Algebra 1. She noted that she had never thought mathematics was a strong subject for her and that she had previously always found it confusing. At the end of the study, she noted that as a member of this Algebra 1 course she has found mathematics “actually kind of easier.” She notes that she is more successful and that now she “gets it.”

Jaelyn was a tenth-grade African American who was repeating Algebra 1 due to lack of success with the course during her freshman year. When she described what her learning had been like the year before, she said, “When I start to get frustrated in math, I have like this tantrum thing … I’m just gonna throw my pencil down and [say]: ‘I’m not doing this anymore because I don’t get it.’ And I did that all the time last year.” She described not wanting to go to class, skipping, and even leaving tests blank. This year was different for Jaelyn: “Now I don’t have a problem with going to class. I’ll be like, ‘Ooh. Second period. Okay.’ Cuz like, last night when I was doing my homework, I had problem two, and … I was curious to even get to class and see what everybody else’s answer was.”

Neesha was a ninth-grade African American who said that although she “used to shut down and just not talk … if I really didn’t get it, I would just stop trying,” the year of the study completely changed her perspective: “This year … I feel like I’m a really good math student and math is one of my favorite classes.” Although these students’ initial negative self-perceptions varied from shutting down to not having confidence to not liking mathematics, they all reported significant improvement in their self-perceptions of competence. For this reason, analysis of their interview data, triangulated with classroom observation fieldnotes and video, offer evidence for the relationship between sociomathematical norms and perceptions of competence.

Although in this paper I do not attempt to tell the individual storylines for each of the students who are featured, I attend to their voices as a way to identify trends in classroom norms and students' changed perceptions of competence. Table 9 is offered as a reference for the voices of Elena, Helen, Irene, Jaelyn, and Neesha.

Table 9

Student Demographics and Self-Perceptions of Competence

Student	**Grade**	**Gender**	**Ethnicity**	**Initial Self- Perceptions**	**End-of-Study Perceptions**
Elena	9	female	Latina	I'm not one of the best students in there, but I try to get things that we're learning…I'm not really good at it.	It's easy for to study and to do my homework and not struggle. I can do it all myself.
Helen	9	female	European American	I've never had much confidence.	Well now I can explain it to everyone I know!
Irene	9	female	African American	Since 7th grade math has not been my strongest subject. It is either confusing or I just don't understand it.	Now that I'm in this class, it's actually kind of easier. I don't get as much stuff wrong with math now. This year, I get it.
Jaelyn	10	female	African American	Last year I didn't like to go to class.	I was curious to get to class and see what everybody else's answer was.
Neesha	9	female	African American	I used to shut down.	Math is one of my favorite classes.

Data Collection

I used a qualitative case study approach to offer a thick and rich description (Corbin & Strauss, 2008) of the mathematics perspectives of a group of Algebra 1 students. The data collection period took place during the first semester of Hannah's Algebra 1 class during the 2011-2012 school year. I focused the data collection period around two units, Unit 1: Linears during 4 weeks at the beginning of the school year and Unit 4: Lab Gear and Solving Linear Equations for 2 weeks at the end of the semester. The purpose for in-depth study of units at the beginning and end of the semester was to understanding the nature of the students' experiences over time. During the focal units, I attended class every day. During the middle of the semester, I attended class about twice per week.

Ethnographic methods for data collection included (a) fieldnotes taken during 50 classroom observations, (b) qualitative records created after each classroom observation, (c) audio and video recordings of 26 classroom sessions focused on five of seven small groups during Units 1 and 4, placing one camera near each of five of the seven small groups,[30] (d) student and teacher artifacts, including but not limited to classwork, homework, projects, quizzes, and tests, gathered from observation days, (e) audio and video recordings of 22 semi-structured interviews with 13 of the 28 students, and (f) three semi-structured individual interviews and many informal conversations conducted with the teacher over the duration of the study.

I was a participant observer during classroom observations. Fieldnotes attended to students' discourse, positioning moves, group storylines, the teacher's pedagogical choices, displays of status characteristics and status generalization (Webster & Foshi, 1988), social and sociomathematical norms, and potentially equitable opportunities for learning. I interacted with students as Hannah and I deemed appropriate. At times I sat near groups to hear their

[30] Because 4 of the 28 classroom students elected not to participate in the study and Hannah randomly assigned students to new groups every 2 weeks, we decided she would maintain random group assignment during the focal units. She shifted groups that had students who elected not to participate into places that were easily off camera, leaving five groups on camera during each focal unit.

conversations, at times I answered questions, and as often as possible, per Hannah's expectation, I redirected them to use one another as resources. I maintained a focus on my unit of analysis: the activity of learning mathematics in groups, as displayed by social and sociomathematical norms and students' perceptions of competence.

Data Analysis

I pursued this research to contribute an understanding of classroom activities that can improve students' perceptions of mathematical competence. I started by open coding students' interviews as a way to understand how and when students described mathematical competence in this classroom (Emerson, Fretz, & Shaw, 1995; Merriam 2009). The first round of analysis lead to emergent codes like *comfort*. For example, Jaelyn said: "As a learner, if you don't understand [something], you should feel comfortable asking your group." I found students frequently used various forms of the word *comfort* to position their experiences feeling competent or not competent to discuss mathematical ideas while working in different groups throughout the semester.

I used linguistic microanalysis (Corbin & Straus, 2008) during a second phase of analysis with the *comfort* data. I studied the ways students positioned themselves and others as mathematically competent and investigated students' ideas about *comfort*, *comfortable*, and *uncomfortable* in their different groups. I differentiated microanalysis with more careful observation of data and close attention to detail. I examined commonalities and differences across students' experiences in groups. Linguistic microanalysis of *comfort* revealed a distinction between *comfortable* and *uncomfortable* groups. Whereas open coding allowed the *comfort* code to emerge, microanalysis helped me dig deeper into what *comfort* meant in this classroom, in order to break open *comfort* as a construct and consider how it was related to students' perceptions of competence.

Because perceptions of competence were co-constructed within *comfortable* and *uncomfortable* groups, I used the third phase of analysis to triangulate interview findings with the fieldnote and classroom interaction data in search of commonalities between and across *comfortable* and *uncomfortable* groups (Merriam, 2009). During the third phase of analysis, my unit of analysis was episodes of interaction where students were making sense of mathematical ideas. During whole-class interactions, I analyzed moments when students justified their mathematical ideas. During small-group interactions, I analyzed moments when students discussed mathematics problems together. This focus on group activity did not take away the importance of the community and personal planes of analysis (Rogoff, 2003), as these planes of analysis describe the community's social and sociomathematical norms and their relationship to students' perceptions of competence and therefore frame the study of student interactions in groups.

I applied the constant comparison method to analyze segments of interviews and episodes of classroom interactions and multiple classroom episodes alongside one another to seek similarities and differences (Merriam, 2009). This phase of analysis revealed *norm development* as a way of shaping students' perceptions of competence. I then categorized the different kinds of norms in the classroom, discovering the presence of *social norms* like *everyone offers and receives help* and *sociomathematical norms* like *everyone makes sense of their own mathematical ideas.* For example, when the *comfort* finding emerged, I compared the different ways students referred to a presence or lack of *comfort*, alongside the establishment or violation of the classroom's *social* and *sociomathematical norms*. I compared groups in which students felt *comfortable* to groups in which students felt *uncomfortable*. Constant comparison allowed me to uncover a relationship between *uncomfortable* groups and students' lack of interest in sharing what they did not (yet) understand. Constant comparison revealed a lack of *comfort* was related to a violation of *social* and *sociomathematical* group norms. I used constant comparison to move my analysis from description to abstraction, to focus on more than a single case, as a way of examining my own biases, and to examine my findings in greater detail (Corbin & Strauss, 2008).

I used similar codes to compile analytic memos (Corbin & Strauss, 2008; Merriam, 2009). For example, when the preliminary finding on comfort emerged, I created a memo compiling initial sources of *comfort* present in my data. I used the memo to assemble codes like *comfort, trust,* and *community* into categories and subcategories to retain and reflect on how and whether comfort held across all the data. Memos also provided a source for conceptual thought about the nature of the data corpus. Memo writing was responsive to the purpose of this work and in turn created reflective thought on the nature of norm development, the storyline of students' perceptions of mathematical competence, and the overall nature of learning in this classroom.

I questioned the data at multiple stages of analysis in order to better understand the nature of social and sociomathematical norms and students' perceptions of competence (Corbin & Straus, 2008). Questioning the data also helped me get acquainted with a new piece of data, allowed me to probe more deeply into a particular interaction in class, and helped me when I was stuck at a certain stage of analysis. For example, when I was trying to determine the role of "justifying student thinking" in the classroom, I looked at the data to determine whether this was a classroom norm. After determining that this behavior had become normative in this classroom, I questioned the data further to determine whether "explaining student thinking" was developed as a social or sociomathematical norm. Asking questions of the data allowed me to determine the role of data in this analysis.

Findings

This study was motivated by (a) a need to understand the development of social and sociomathematical classroom norms in a discourse-centered classroom, (b) a need to understand the nature of students' improving their perceptions of mathematical competence, and (c) a desire to contribute an understanding of the relationship between classroom norms and students' changed perceptions of mathematical competence. In the sections that follow, I first briefly describe the structure of Hannah's classroom. I then unpack the development and maintenance of the classroom social and sociomathematical norms that emerged from this analysis. I next explore the relationship students described between their own perceptions of competence and the feeling of comfort as they worked in their groups. I end the findings section by describing the ways the social and sociomathematical norms mediated students' changed perceptions of competence.

Social and Sociomathematical Norms That Fostered Positive Perceptions of Competence

Ms. Martin structured a large part of the learning in her classroom around students working in small groups of three to four students. During the 26 class periods recorded on video during Units 1 and 4, an average of 71% of class time was focused on students working together in small groups, 13% of class time was whole-class student presentations, and 9% was whole-class teacher talk.[31] On about one half of the 50 days observed, Ms. Martin used somewhat

[31] The remaining 7% of class was largely unstructured and included school announcements, transitions, and the start and end of the class period. These proportions were incredibly consistent from Unit 1 to Unit 4.

formally structured instructional strategies, including groupworthy tasks, teacher check-ins, Shuffle Quizzes, Participation Quizzes, group tests, and student presentations.[32] These formal instructional strategies facilitated the development and maintenance of certain social and sociomathematical norms. During the observed class time without formal instructional strategies, classroom norms were developed and sustained through teacher instructions and teacher moves that stated expectations and reinforced established or progressing norms. While students worked, Ms. Martin circulated the room, monitoring groups while they worked: She listened to group conversations, pushed papers to the center of the table to increase access to mathematical conversations, and attended to group questions most often by reorienting students to one another. On most days observed, Ms. Martin briefly set up the classwork for the day, talking for less than 5 minutes about what she wanted students to focus on during a given classwork assignment. Students then had most (about 71%) of the class time to work together in their small groups. If she wanted to highlight something in particular at the end of either the homework check or the warm up, she would ask one or more students to come up and explain their methods to the whole class. Students managed their own work, turning in and receiving back assignments from a group folder at the center of each group.

Classroom *social norms* are the ways individuals believe they are routinely expected to behave as students in the classroom (Kazemi & Stipek, 2001; Yackel & Cobb, 1996). Classroom sociomathematical norms are the ways individuals believe they are routinely expected to engage around mathematics in the classroom. A teacher has certain hopes and expectations for how students behave in the classroom, acting as students and engaging around mathematics. These teacher expectations are not norms in and of themselves. The classroom norms, rather, are the

[32] Full descriptions of these instructional strategies will be addressed in other papers (see Dunleavy, 2012, equity).

beliefs students develop as a result of negotiating with the teacher and other students about what it means to be a student and mathematician in the classroom. In this study, the norms were developed in part through specific teacher instructions, such as “same problem, same time,” through formal instructional strategies, such as assigning a group test in which students were expected to reach mathematical consensus by discussing problems and writing down any correct solution and the same final answer for each problem, and through the ways that students took up the norms. Evidence of which norms developed and were maintained emerged when students reinforced the teacher’s expectations, such as when they asked their group,, “Is everyone ready to move on?”

The social norms I uncovered for this study include *everyone discusses ideas with everyone else*, *everyone offers and receives help*, and *groups work on the same problem*. The sociomathematical norms I uncovered include *mathematical work involves making sense of your own ideas*, *mathematical work involves making sense of others’ ideas,* and *groups work to achieve mathematical consensus*. An overview of the norms I discuss in this paper can be found in Table 10. In what follows, I unpack the teacher moves and student behaviors that worked to develop, violate, negotiate, and ultimately maintain these norms.

Table 10

Social and Sociomathematical Classroom Norms

Social norms Routinized ways of engaging as a community member.	*Everyone discusses ideas with everyone else.*
	Everyone offers and receives help.
	Groups work on the same problem.
Sociomathematical Norms Routinized ways of engaging in the process of doing mathematics.	*Mathematical work involves making sense of your own ideas.*
	Mathematical work involves making sense of others’ ideas.
	Groups work to achieve mathematical consensus.

Social norm: Everyone discusses ideas with everyone else. *Everyone discusses ideas with everyone else* emerged as a classroom norm after analyzing the combined effect of the following teaching strategies: regularly randomly assigning students to groups, regularly assigning competence to students, using daily directions and individual reminders to orient students to use one another as resources, and requiring individual accountability on all assignments. In this section I show the teacher moves and the student behaviors that led to establishing the norm *everyone discusses ideas with everyone else*.

Through interviews and informal conversations, Ms. Martin repeatedly affirmed her belief that all students are competent to do mathematics. “I come into this whole-heartedly knowing that all of my kids have something genuine to contribute. All my kids are capable of learning math. I hope that exudes from me, as a teacher, everywhere I go in that classroom.”

One of the teacher moves that developed the norm *everyone discusses ideas with everyone else* was publicly, randomly assigning students to groups every 2 weeks. The seating chart was visible on the wall, and when it was time to change, Ms. Martin shuffled the cards and placed them back on the wall. At the first switch, after students had found their new seats, she addressed the change:

> We want you to sit in front, sit in back, sit with people you know, sit with people you don’t know. … So, I will be shuffling and moving you every two weeks. Sometimes you end up in the same table group again, but that’s just random chance, right? Because I just shuffle and put them up there.

After making this announcement, Jaelyn asked whether they could pick their own seats, which Ms. Martin refused. As the year progressed, Jaelyn revealed she wanted to work with other students so she could stay focused on mathematical work.

A second way Ms. Martin developed the norm *everyone discusses everyone's ideas* was through regularly assigning competence to her students. For example, in one case, students in one of Jaelyn's groups were not talking to one another but had called Ms. Martin over for help. Ms. Martin asked, "Hey [student], do you ever *bug* Jaelyn? Because I know *she* knows how to do it!" This teacher move served two purposes that developed this norm: First, it worked to assign Jaelyn mathematical competence for understanding how to simplify expressions. Second, assigning Jaelyn competence also worked to orient students to work with and value one another as resources for understanding mathematics work. Analyzing one small group and all whole-class conversations from each of the 26 class days captured on video revealed 65 moments that were coded for Ms. Martin assigning competence. Video analysis reveals each student observed Ms. Martin positioning students as competent at a rate of at least 2.5 times every class period.[33] Repeated exposure to competence positioning developed this norm.

Evidence *everyone discusses ideas with everyone else* was becoming an established classroom norm was visible through what developed during and after visible norm violations. During the second new seating chart, one student made a formal complaint about the expectation to work with everyone. In front of the class, this student said she could not work with one of the students in her newly assigned group. Ms. Martin responded by redirecting the student to stay with the group for the remainder of the class period and to discuss the situation with her after class. When they talked after class, Ms. Martin validated the student's personal feelings and discussed the potential benefits for learning how to work with others, even if there were previous conflicts. After some discussion, the student agreed to stay in the group for the duration of the 2

[33] The total number of times Ms. Martin assigned students competence throughout the class period was not evaluated but was higher than this number. This number refers to what the typical student experienced during one class period, through the combination of small-group and whole-class activities.

weeks. Because Ms. Martin convinced the student to stay in the group by pointing out the benefits of working with different people in the classroom, Ms. Martin successfully used this discussion to strengthen the development of the norm *everyone discusses ideas with everyone else*. The way Ms. Martin handled this norm violation reaffirmed and possibly strengthened the norm development itself: All students were viewed as competent to discuss ideas with every other student in the class. Another regular norm violation appeared when students talked outside their group. When this occurred, Ms. Martin redirected students to discuss ideas inside their assigned group. Over the course of the semester, visible violations of the norm decreased. I did not observe or hear of any other students refusing to work in any assigned groups, and the rate of students talking outside their group seemed to decrease as well.

Evidence the norm *everyone discusses ideas with everyone else* became established emerged through analysis of the combination of student behaviors. Students observed the seating rotation, wrote individual work on all assignments, discussed mathematics, and asked group questions. During small group time, students were on task a majority of the class time.

Further evidence *everyone discusses ideas with everyone else* was routinized arose during interviews. All the students who were interviewed referred to how they saw their classmates as competent to discuss ideas. One example occurred during Elena's first interview. When she described what learning mathematics was like in years previous to this class, she said, "We didn't learn, as in other schools did. …We had a loud class and we didn't really do the math. We had people playing around and jumping and throwing chairs. So it wasn't a very learning environment." But when Elena described what learning was like in this mathematics class, she said, "Learning how to get along with other people and work with others to figure things out." Elena's comments about her past experiences in math classes were completely different than her comments on Ms. Martin's class. That she could shift her perspective on mathematics being "not a learning environment" to "working on how to figure things out" by the middle of first semester is particularly significant. Another example of how students valued class members as competent came from Neesha, describing her initial impressions of class during her first interview:

> I really like our class. I feel like, well, at least in all the groups that I've been in, I feel like, they're really hard workers, or, if they're not hard workers, then they'll ask a lot. And so they can get answers and so they're willing to learn. I feel like our class is extremely positive. [The students] seem to get the problems, and they can go up on the board and do the problems and get the right answer.

In this excerpt, Neesha shared her belief that her classmates were capable of doing mathematics. Moreover, mentioning that students were working at the board seems to indicate she believed student presentations of the mathematics were a regular part of the class. The ways Elena and Neesha described their beliefs were representative of the ways that other students talked about classmates' abilities to be successful in this class.

Social norm: Everyone offers and receives help. The second classroom social norm that emerged was *everyone offers and receives help*. Ms. Martin developed and maintained this norm by assigning students to work together in groups, by visually and verbally communicating expectations that oriented students to use their group members as resources during every class period, and by assigning tasks that required group interdependence.

During the first 2 weeks of the school year, Ms. Martin applied a number of instructional strategies that oriented students to work together in groups. First, Ms. Martin assigned groupworthy tasks (Lotan, 2003), which she described to me as classwork assignments she and other teachers at her school designed specifically to orient students to need one another to discuss mathematics. Each groupworthy task was intended to orient students to use and need one another to do mathematics.[34] Second, Ms. Martin consistently visually displayed and orally gave the class directions that oriented students to work together. Figure 24 depicts Ms. Martin's smartboard display from the second day of school. Figure 24 also shows expectations for getting individual and group accountability: "Read entire task out loud," "Everyone records," "Group Questions Only," and "Supplies on back table." Directions used to launch students into working in small groups supported developing the norm *everyone offers and receives help* by establishing the expectation for students to read the task card together and to ask questions of their group members.

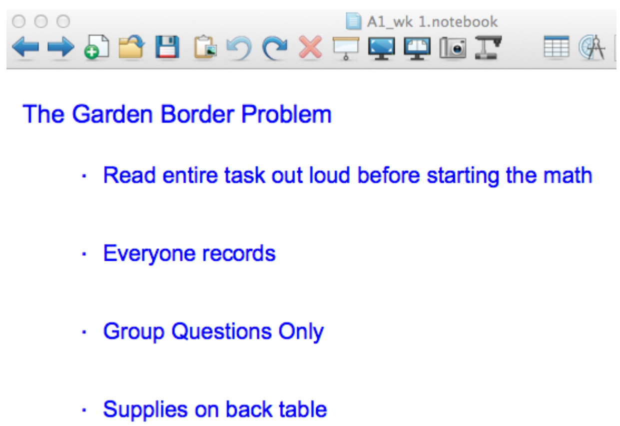

Figure 24. Group questions only.

A third way Ms. Martin contributed to developing *everyone offers and receives help* occurred through written reflection on groupwork. On the third day of school students completed an exit ticket discussing how well their group had been working together. At the end of Unit 1, students were required to discuss how well they worked together in groups as a part of their Unit 1 project. Specifically asking students to think about how they were participating as group members

[34] See Appendix D for an example of a groupworthy task from this unit: The Garden Border.

facilitated the opportunity for students to share and therefore foster group helping at the end of the first unit of the year.

Ms. Martin continued to develop and maintain this classroom norm through daily teacher moves that oriented students to work together. Some of the daily directions included telling students how to work together. For example, for homework and classwork check time, she would display and state expectations like, “Compare mistakes with group members,” “Check in with group members on the last problem!”, and “make sure you read and discuss before starting the graph.” These types of directions helped students learn how to use one another as resources. Ms. Martin also maintained the helping norm by the way she attended to groups during work time. For example, during the second week of class she approached a group and said, “Did you guys hear that question? This is worth it,” and then she walked away. This type of teacher positioning move was typical of Ms. Martin’s speech to students, orienting them to listen to one another’s contributions and questions from one another (and not her) so that they would know that conversations with group members were worthwhile. Another strategy Ms. Martin used regularly included pushing papers to the center and telling groups to ask one another questions. Each of these teacher moves oriented students to use one another to do mathematics work.

Students showed signs of taking up the norm *everyone offers and receives help* by the ways they responded to the teacher’s directives to work together and by the ways they talked about helping during interviews. Irene described how she interacted in her groups: “I volunteered to share my paper, and then I explained how I got my answer, and I put my paper in the middle. … I was supposed to put mine in the middle to share with the table. After she left, I continued with [explaining].” Irene also shared why she valued helping: “It makes it easier than just working on your own. Because then you have different opinions and you can figure out which way works best for you." Irene's statements show that she liked both helping and being helped by group members. Jaelyn cited a moment as evidence that Ms. Martin "makes everyone in the group help you out" in her first interview:

> "Even, yesterday, I was like, "Ms. Martin, I give up!" and then she was like, "No, no, no! You don't." And then [to another student in Jaelyn's group], she was like, "Help her. Explain. Explain it to her." [The other student explained it to me,] and then I was like, "Oh, okay! I get it now."

In this example, Jaelyn revealed that when she was on the brink of "giving up" on mathematics, Ms. Martin oriented a group member to help Jaelyn. She further noted that after hearing the explanation from her classmate, she understood the mathematics. This description indicates the way Jaelyn took up the norm *everyone offers and receives help.* Later in the semester, during her second interview, Jaelyn described a generalized understanding of the responsibility she felt learners in this class had to one another: "As a learner, you just have to like ... if you don't understand it, this is where a group comes in. You should feel comfortable asking your group." Ms. Martin's teacher moves facilitated opportunities for students to value helping in their groups.

Social norm: Groups stay on the same problem. The third social norm that emerged from analysis was asking groups to stay together on the same mathematics problem. Ms. Martin developed this norm by orienting students to work with one another in ways described above, by explicitly stating *groups work on the same problem* as an expectation for helping one another during certain assignments, such as by assigning Participation Quizzes in which students were assigned grades, in part, on the basis of working together.

Ms. Martin introduced norms slowly, telling me at one point, "I introduce [my expectations for student behavior] as I need them." I observed her start to introduce the expectation to stay on the same problem in the daily task assigned on the seventh day of school, as shown in Figure 25.

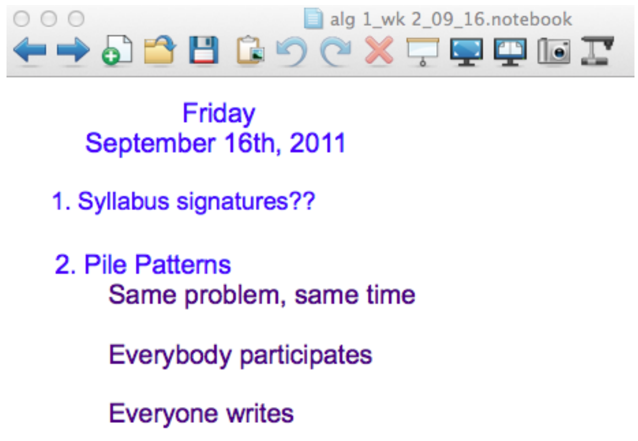

Figure 25. Same problem, same time.

Combined with the directions to turn in parent signatures from their syllabus, to complete the task "Pile Patterns" in groups in which every student participates and everyone writes work on their own papers, Figure 25 shows Ms. Martin's expectation that students worked on the "same problem" at the "same time." When she talked about it in class, she explained "same problem, same time" meant that groups worked together and helped one another, in part because "it's actually really intimidating to ask [a group member] for help when you're on the back and I'm on the front." No one moved on until the group was ready to move on. At times, Ms. Martin held students accountable for *groups work on the same problem* by assigning a Participation Quiz, as shown in Figure 26.

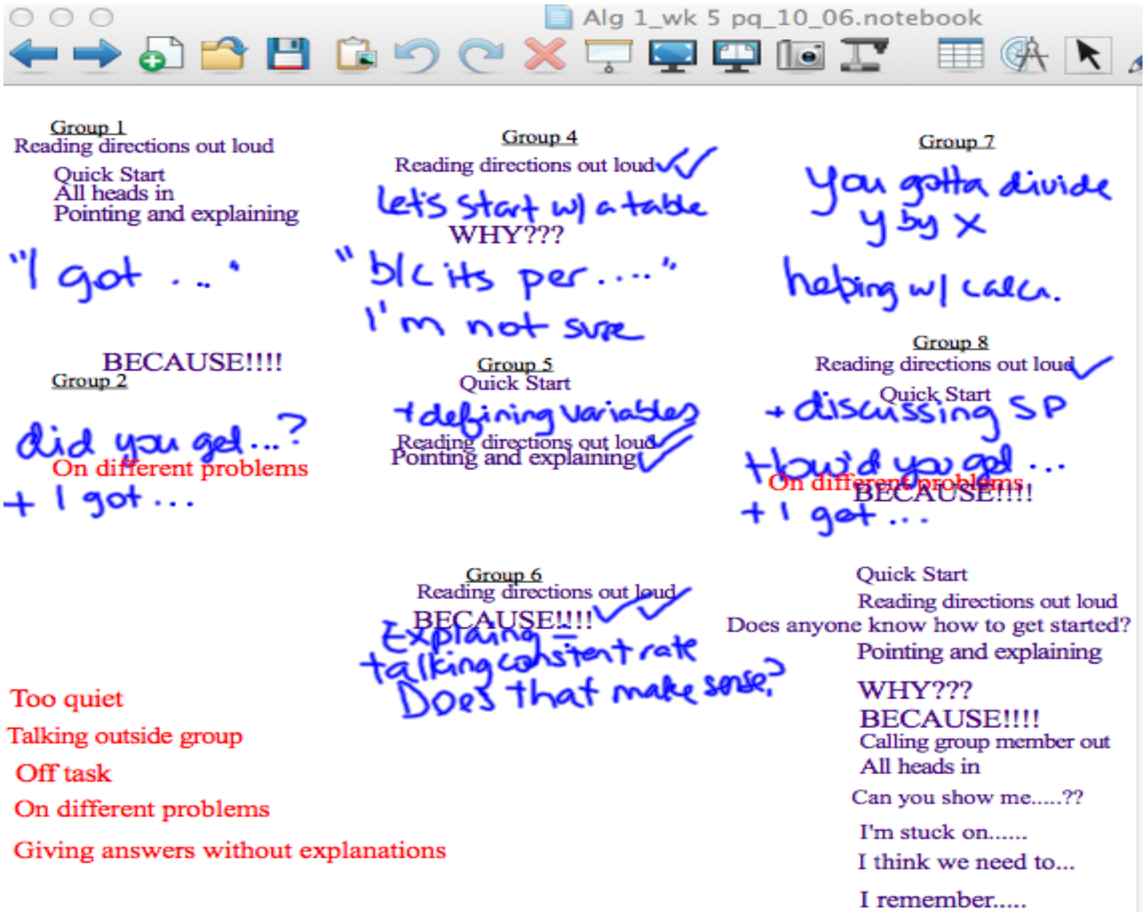

Figure 26. Participation Quiz.

In addition to the ways in which other classroom norms were developed and maintained with the Participation Quiz, the norm *groups work on the same problem* was reinforced through the phrases displayed on the Participation Quiz. The Participation Quiz was a live, public record of student behaviors, thus serving to reinforce the ways Ms. Martin expected individuals to behave as students and as mathematicians for the duration of the groupwork assignment. Ms. Martin prepared the purple and red phrases in advance, and displayed them at the bottom of the Participation Quiz, using them to reinforce the type of behavior she hoped students to display. When students displayed behavior she found appropriate, she dragged the phrases up to their groups, including "pointing and explaining" and "all heads in." When students did not display behavior she hoped to see, she would bring up the red phrases, such as "on different problems."

She also hand wrote in quotes from students who observed the expectation that students worked together, such as when she wrote “I got…” for group 1 and “did you get…?” for group 2. Students who worked together and received a significant number of blue and purple behaviors received full points on the Participation Quiz.

The Participation Quiz reinforced *groups work on the same problem* through those positive behaviors that oriented students to work together.Clearly many students observed *groups work on the same problem* because group phrases like, “Are we ready to move on?” became a normal part of classroom language. Further evidence that students took up the norm came from the ways they described staying together during interviews. Irene noted that the practice was different from what she was used to, but that she still found it helpful:

> It’s different how she does group tests, group quizzes, and how she shows us that we’re off task on a different equation, different problem. Because it’s helpful for us to be all on the same problem and then [group members] look at your paper and they see that you have the wrong answer and they show you how to get the right one. And like Ms. Martin said, you have different people giving you opinions on how to do different things, in different ways, but getting the same answer.

Irene thought staying on the same problem was useful because group members could help one another with mistakes. When I asked Helen what she thought about students staying together on the same problem, she shared a similar belief:

> I get a lot of structured stuff wrong. And I don't know; I don't have all the steps sometimes. So I think it's helpful, when someone is doing the same problem, at the same time, and not going super fast.

Students were held accountable to take up the norm *groups work on the same problem* in part by the grade assigned on Participation Quizzes. But Irene and Helen's opinions that *groups work on the same problem* was helpful suggests this social norm was taken up by some students as more than just an expectation; for these students, *groups work on the same problem* became something the students valued about their mathematical learning in this classroom.

Sociomathematical norm: Mathematical work involves making sense of your own ideas. The first sociomathematical norm to emerge was *mathematical work involves making sense of your own ideas.*

Instructional strategies fostered making sense of your own mathematical ideas. In part, Ms. Martin developed *mathematical work involves making sense of your own ideas* as a norm by using teacher moves that reinforced all of the social norms described above. By randomly assigning students to groups, offering groupworthy tasks, orienting students to work with one another, assigning competence, requiring group questions, pushing student papers to the center, assigning Participation Quizzes and group tests, and offering sentence starters that framed the ways she wanted students to talk in groups, Ms. Martin started to routinize the behaviors of *everyone discusses ideas with everyone else*, *everyone offers and receives help*, and *groups stay on the same problem*.

In addition to developing the norm *mathematical work involves making sense of your own ideas* by fostering specific social norms, this sociomathematical norm was developed by specific teacher moves that led students to take up the practice of working together to communicate mathematical understanding. Ms. Martin described how she would frame the expectation for students to participate in a mathematics conversation in their groups. One day during the third week of school at the beginning of class, she asked students to check in with one another about their homework assignment:

> I think there are some problems that are worth checking answers with your group mates. A check in might look like all of you putting your answers into the center and saying, "What did you get for the third table?" Or, a check in might look like someone saying, "Well I just didn't answer. … Can someone tell me how they got started?" So either way, can we start by checking answers? Or can we start by someone just saying, "I know I need help on this problem …?" Okay? [Students can be heard saying, "I know I need …"] Let's do that. I'm going to stamp. On your mark, get set, go!

Here Ms. Martin addressed her expectation for students to share their own answers and then compare answers with one another. She expressed an expectation the talk would be mathematical by offering students with options for what a homework check-in conversation might sound like. Sentence starters like, "What did you get for the third table …?" facilitated the expectation that homework check-ins involved not just stating answers, but making mathematical sense of them.

Another teacher move that Ms. Martin used to facilitate the expectation that students talk together about mathematics was using what she called a "teacher check-in," or "Shuffle Quiz."[35] To pass a Shuffle Quiz, Ms. Martin expected students to prepare for her to come over and randomly select one group member to explain a particular mathematics problem. Students were meant to practice explaining the mathematics with one another before she came over, thus seeming, in part, to foster the sociomathematical norm that students explained mathematics to

[35] These were referred to somewhat interchangeably in this class so for simplicity I refer to these as the same thing, and I use "Shuffle Quiz" for the rest of this article.

one another. When Ms. Martin was satisfied with the explanation of the randomly called-on student(s), the group would be allowed to move on to the next task.[36]

Classroom interactions showed students engaged in making sense of their own mathematical ideas. The social norms above displayed ways students regularly interacted with one another as students. The sociomathematical norms that follow illustrate what counted as mathematical argumentation and participation in this classroom (Yackel & Cobb, 1996). The first sociomathematical norm that emerged showed students regularly engaged in *making sense of their own mathematical ideas*. This norm is illustrated by analyzing episodes of student interaction. In one interaction near the end of the semester, Elena and Neesha began discussing the problem $3(y + 4) = -6x$. An excerpt of their interaction shows how they were *making mathematical sense* of their own ideas about solving for y:

1	Elena:	I don't know how to do this parentheses part.
2	Neesha:	So, you just make it $3y + 12$ because you're supposed to multiply the 3 by the numbers in the parentheses, so it'd be $3y + 12 = -6x$ …
3	Elena:	Mmhmm. ((Quiet, looking at her own paper.))
4	Elena:	Do I add $6x$?
5	Neesha:	Huh?
6	Elena:	Do I add $6x$?
7	Neesha:	Uh, no, you're trying to get the 12 over on the other side. So you, um, minus 12. You add a negative 12. So it would be $3y + 12$ minus 12 equals negative $6x$ minus 12.

[36] For a full example of a teacher check-in/Shuffle Quiz in which Ms. Martin presses students for more mathematical explanation, please refer to Dunleavy, 2013, delegating authority paper.

8 Neesha: And then you get rid of the 2 twelves. And then it would be 3*y* equals negative 6*x* minus 12. And then you divide by 3.

9 Elena: And that's the… Oh, and then you divide by 3. ((Silence as they work on their own papers.))

Neesha and Elena's discussion of solving for *y* in this problem illustrates that *mathematical work involves making sense of your own ideas* was an established norm for their interaction. In turn 1, Elena explained what she did not understand, saying "I don't know how to do this parentheses part" (turn 1). By saying "you just make it 3*y* + 12," Neesha explained her thinking that 3(*y* + 4) turns into 3*y* + 12 (turn 2). Elena went on to ask whether to add the -6*x* (turns 4-6). Neesha explained her belief the right next step was to move the 12: "No, you're trying to get the 12 over on the other side" (turn 7). Neesha went on, "Then you get rid of the 2 twelves … and then you divide by 3" (turn 8). Elena then seemed to agree with Neesha's explanation: "And then you divide by 3" (turn 9). The group continued to engage in mathematical sensemaking throughout their time together.

Student interviews indicated making sense of your own mathematical ideas as an established sociomathematical norm. During an interview, Neesha explained how she had come to believe that working on mathematics in groups was important:

> I feel like when you're with a group you learn how to explain it so much more. Cuz another thing that I remember [from past math classes], like would dock me off is I might get the right answer, but I wouldn't explain. I still don't really like explaining, but I learned that like, it really does help. Because if you're looking it over, you could forget how to get that answer. And during tests, that would always kinda screw me up. … I

> would see that I got the right answer, but I didn't know how I got it, and so it didn't help me with other problems.

Neesha compared her experiences in this classroom to those in other classrooms. She explained that even though she may not have liked explaining her thinking, she realized that offering a mathematical explanation that goes beyond a correct answer helped her to understand the mathematics. Helen similarly reported that she believed this class was about understanding mathematics through explaining:

> All of my other teachers will show you the formulas and show you the little tricks about how they will do the problem. And [Ms. Martin] will make us try and figure it out, rather than telling us the answer [and] how to do it right away.

Helen shared that Ms. Martin did not give away correct answers but instead expected students to work out solutions.

Sociomathematical norm: Mathematical work involves making sense of others' ideas. A second sociomathematical norm that emerged was *mathematical work involves making sense of others' ideas*.

Ms. Martin's classroom directions emphasized making sense of others' ideas. Ms. Martin often highlighted valuing different ways of thinking about mathematics when giving directions, with comments like, "People even have different ways they are thinking of [this problem], which is great." During one seating change, Ms. Martin continued to emphasize students' valuing making sense of others' mathematical ideas, saying:

> I'm excited that you're in new groups at this point because a lot of people have been working really hard for a few weeks and now that you've been working on solving you should have a good idea of what you're okay with and what you're not. Like "this part

> makes sense to me and I don't really know…" "This part doesn't make sense to me." When you're in new groups, it's great because you get to hear other voices of how they make sense of it. So things that you can be thinking about are: What are you strong at in solving for *x*? And also, what are some things that you need help on? Or what are some things where you always find yourselves getting stuck? With new voices it's always good to find another way that makes sense to you. Alright?

Ms. Martin first emphasized valuing new ways of explaining mathematics by telling students "I'm excited that you're in new groups." Next, she asked students to think about what they understood well and what they had questions about. Then she told students, "When you're in new groups, it's great because you get to hear other voices of how [other students] make sense of [mathematics]," and then, "With new voices it's always good to find another way that makes sense to you." These directions communicated the belief that hearing other students explain mathematics was helpful to understanding mathematics, thus working to establish the norm *mathematical work involves making sense of others' ideas.*

Students shared the belief that making sense of others' ideas was helpful to learning mathematics. Elena, Helen, and Irene accepted Ms. Martin's intentions for students to adopt the norm *mathematical work involves making sense of others' ideas*. During her last interview, I asked Elena why she thought that Ms. Martin regularly randomly assigned students to groups. She said: "So we can get to know everybody and so we can learn different ways people do their math." One interpretation of Elena's quote is that Elena had come to believe that Ms. Martin wanted students to learn different ways of doing mathematics.

Helen also described *mathematical work involves making sense of others' ideas* to be a helpful part of the learning process. When talking about commonalities she saw across all her groups, she talked about why students needed to be in groups:

> Well, there's always one or two kids that don't really speak at all ((raising her hand)), me, probably one of them ((We both laugh.))… We've had like 3 people and there's always been like one person who's like, "Wait, what? What's going on?" ((We both laugh again. I ask, "What do you think about that?")) I think they need to be in a group because they don't, they might not understand how much it will help them. They might not understand how much another person could help them and see a math problem in a different way. We always seem to have an idea—someone who knows how to talk to someone and explain something to them.

This excerpt attends to the way in which Helen believed groups were beneficial for a group member who did not speak as much and a group member who needed help understanding the mathematics, which provides further evidence for the social norm *groups work on the same problem* and the sociomathematical norm *mathematical work involves making sense of your own ideas.* Helen's comment "They might not understand how much another person could help them and see a math problem in a different way," also attends to the way that she had come to believe that groupwork was also good for *mathematical work involves making sense of others' ideas*.

In her interviews, Irene explained how she thought Ms. Martin wanted students to share mathematical understanding with one another:

> Like Ms. Martin said, you have different people giving you opinions on how to do different things, in different ways, but getting the same answer. … It makes it easier than just working on your own. Because you have different opinions and you can figure out

which way works best for you. And you can use that way. … The learning was probably happening through me getting different answers through my way, and then asking my tablemates if they got the same thing. If they didn't, [they would tell me] how they got their answer. So basically, after they tell me how they got their answer, I knew how to do it better. And then I knew the small things I missed. Like, putting a minus in front of different things. And then adding it with the minus.

Irene's excerpt reveals that she felt Ms. Martin expected that different members of her group give her "opinions" on how to do the mathematics, but that in the end, they all would achieve the same final answer. This excerpt reveals that Irene valued students' different opinions and solution paths as ways to participate in understanding the mathematics in this classroom.

Classroom interactions showed students engaged in making sense of others' mathematical ideas. The next sociomathematical norm that emerged suggested that in addition to making sense of their own mathematical ideas, students in this class regularly engaged in making sense of their group members' mathematical ideas as well. In one seating chart, Irene, Jaelyn, and Phoebe were members of the same group. During one group interaction, Irene leaned over and asked Phoebe how she solved for x in the following problem: $6 + x + 5x = 1 + 5x - 8$.

1 Irene: What'd you get for the last one? Negative 13?

2 Phoebe: Uh huh ((looking at Irene's paper and nodding yes.))

3 Irene: Yeah, I don't know. I did something wrong.

4 Phoebe: ((Looking over at Irene's paper.)) I don't know what you did. Wait, what'd you do?

5 Phoebe: Oh this is what I did. I put it into, you know you can add it together, to become $6x$?

6 Irene: Oh, yeah.

7 Phoebe: So then I did ... I did that.

8 Irene: Oh, yeah. I should have done that. ((Erasing and modifying her answer.))

9 Phoebe: I think it's the same way, but just longer.

10 Irene: Cuz I think if I did it right I would've got the same answer, if I did it a different way. So...

In this excerpt, Irene believed she had made a mistake and asked a group member to talk about the mathematics (turn 1). Phoebe looked at Irene's paper (turn 4) and started to try to make sense of Irene's problem, saying, "I don't know what you did. Wait, what'd you do?" Phoebe then started to explain her method (turn 5): "You know you can add it together to become 6*x*?" Irene's responded "yeah" in turns 6 and 8, showing she was starting to make sense of Phoebe's method. When Phoebe said, "I think it's the same way, but just longer" (turn 9), she started to engage in making sense of Irene's ideas and comparing them with her own, suggesting they had different but correct ways of solving the mathematics. Irene remained unconvinced that her way was correct: "I would've got the same answer" (turn 10). Irene then continued working on the problem on her own for another minute before she stopped and asked Phoebe to engage in sensemaking with her again.

11 Phoebe: ((Phoebe watching Irene's work.))

12 Irene: How'm I still getting...? I'm still getting negative 14.

13 Phoebe: Okay okay okay. Okay, let's start over ((flips over paper and starts writing on the back.))

14 Phoebe: So let's write the whole equation. ((Phoebe does all the writing.))

15	Phoebe:	So first to add this, right?
16	Irene:	Mmhmm.
17	Phoebe:	So this would be like 6 plus 6*x*…
18	Irene:	…6*x*....
19	Phoebe:	... equals 2. Wait, no that’s 5*x*. Negative 7. 1 minus 8?
20	Irene:	Oh, you did 1 minus 8? Because I just canceled 8 out and 1 out and then I did 8 minus 1 and I got negative 9.
21	Phoebe:	What?
22	Irene:	Because like, on here, I learned, that like, you’re supposed to cancel it out, so to get, end up with one regular number & one with the variable. So I canceled one out and I got. Then I took one from each side to make it even. And I got 5 over here and negative 9 over here....
23	Irene:	So what you did is you just combined variables similar to me?
24	Phoebe:	Yeah. Yes, I did because it would be easier for me. And then I... What’d I did? I minused 5 from each side. So this would be 1x, right? This would cancel out so it would be 6 plus 1x equals to negative 7. That’s what I got. I brought this down.
25	Phoebe:	And then I did... um... I minused 6 from each side. This brings me to zero...and this just brings me to 1*x*.
26	Jaelyn:	((Jaelyn looks up at this point to the conversation. She may have finished her independent work.))

27 Phoebe: And then negative 7 minus negative 6 is negative 13. Divided by 1 of *x* equals...

28 Irene: Negative 13.

29 Phoebe: Negative 13. That's how I got it.

30 Irene: Okay.

When Irene stopped Phoebe a second time to talk about the mathematics (turn 12), Phoebe flipped her paper over and suggested they start the problem over together (turns 13 and 14). Phoebe wrote the problem as originally stated at the top of the new paper and then continued to write and solve as the girls talked about the mathematics (turns 13-29). Phoebe's work during this discussion is offered in Figure 27.

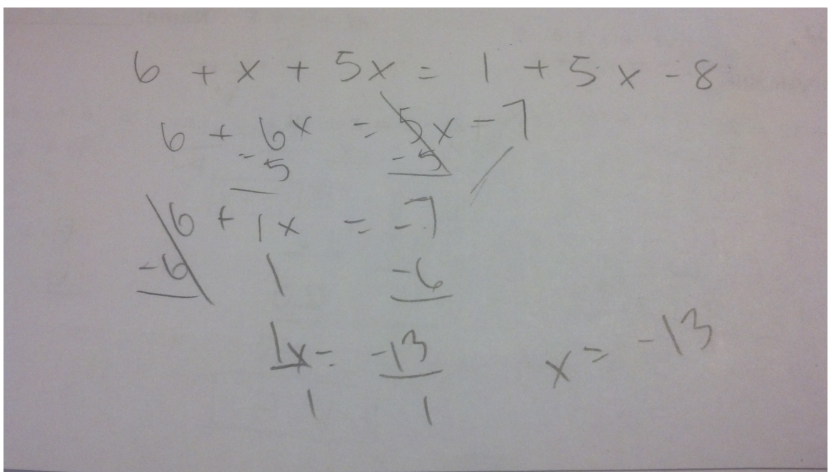

Figure 27. Mathematical work involves making sense of others' ideas: Phoebe's work.

Phoebe next said to Irene, "So first to add this, right?" and, "So this would be like 6 plus 6*x*" (turns 15 and 16), to which Irene agreed. Phoebe then made sense of the right hand side of the equation (turn 19): "Wait, no that's 5*x*. Negative 7. 1 minus 8?" Irene responded (turn 22): "I did 8 minus 1 and I got negative 9." She went on to explain her thinking, which included an understanding of solving on the algebra mat, "Because like on here, I learned you're supposed to cancel it out … so to end up with one regular number and one with the variable. … Then I took one from each side to make it even. And I got 5 over here and 9 over here." After explaining her method, Irene asked Phoebe if their mathematical thinking was similar (turn 23): "So what you did is you just combined variables similar to me?" Irene's revised work is shown in Figure 28.

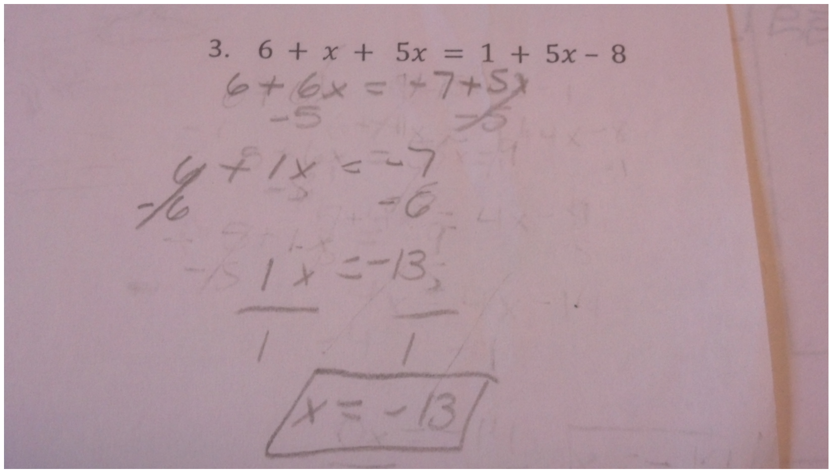

Figure 28. Mathematical work involves making sense of others' ideas: Irene's work.

The interaction continued such that Irene seemed to make sense of Phoebe's method as her own, showing that this group was working to adopt *mathematical work involves making sense of others' ideas.*

Summary of making sense of others' mathematical ideas. The data in this section includes teacher directions, student beliefs, and classroom interactions. These segments of this class point to Ms. Martin's expectation that "people have different ways of thinking of [math], which is great." The classroom result was establishing the norm *mathematical work involves making sense of others' ideas*.

Sociomathematical norm: Groups work to achieve mathematical consensus. *Groups work to achieve mathematical consensus* was fostered, in part, through formal instructional strategies like Shuffle Quizzes, group tests, and student presentations.

Shuffle quizzes fostering mathematical consensus. As mentioned above, during Shuffle Quizzes Ms. Martin asked students to agree on a way to mathematically explain a given problem before she came over to hear their mathematical explanation and allow them to move on to the next task. She asked students to rehearse in groups before she came over, discussing parts of the mathematics they did not understand. Once they reached consensus, they were expected to call Ms. Martin over for the Shuffle Quiz. When she came over, she shuffled their papers and randomly selected one student to explain the mathematics. If a student did not sufficiently justify the mathematics, which occurred on occasion earlier in the year, Ms. Martin left, instructing them to discuss the mathematics further so she could come back. Each time she had to come back, she told them their daily classwork grade was lowered by one point. These instructional moves fostered mathematical consensus by urging students to reach mathematical consensus prior to participating in the Shuffle Quiz. I observed the number of times Ms. Martin had to return to groups decrease over the course of the semester.

Group tests fostering mathematical consensus. Group tests also supported students to achieve *groups work to achieve mathematical consensus.* Although as discussed above, students were encouraged to show mathematically different solution paths, groups were required to turn in the same answers. Each group test grade was the same for every group member, and if students had different final answers, their group test grade was lowered. Group grading fostered achieving mathematical consensus.

As displayed in Figure 26 above, the Participation Quiz fostered an expectation that students explain mathematics to one another by reinforcing student behaviors like “Pointing and explaining,” discussing “WHY???,” “BECAUSE!!!!,” and “Can you show me?” Often paired with a Participation Quiz was a group test. Group tests in Ms. Martin’s class occurred on the day before the individual tests. During group tests, students were expected to work on the mathematics together and come to mathematical consensus on the answers. Although students were not required to write down the same mathematics work, they were often encouraged to write their own solutions, with the expectation that students were talking together about mathematics so as to arrive at the same final answers. On the day before the Unit 1 test, Ms. Martin assigned the first group test. In part to explain the expectations for the group test, Ms. Martin gave the following class directions:

> Today is the day before a test, and I'm going to take some time to explain what a group test is. I'm going to do [group tests] before every individual test. The group test, in the fancy world of mathematics, is a high-stakes review. It's a high-stakes review because there are a lot of points attached to it. One thing I gotta stress is that you get a group grade. So I ask you to talk about certain things but I always give you individual grades, unless it's a poster or a Shuffle Quiz. Here's the other instance where I give you a group grade. Please be mindful of how you're doing today but how your group is doing as well so you can maximize your points. Your group grade is on the math. It is just like a regular worksheet. It comes from me taking the group tests, and I will shuffle them up, and maybe I'll grade [Elena's] number 1, [Helen's] number 2, [Jaelyn's] number 3, [Neesha's] number 4. I grade random problems from everyone's test and I put those points together to make the group grade. So if you're very detailed about showing your starting point or rate of change, that is something you want to be mindful to remind your group to do, because I might grade someone else's problem besides yours. So just be aware of what else is on your test. While we work on the test, I also grade your group work. So same as last time, "same problem, same time" and "justifying the math". As

> you work, do not move on to number 2 until everyone's done with number 1. Don't work on 4b until everyone's done with 4a. Someone's got to take control of that in your group and be like, "Okay, are we ready to move on? Okay let's go…" Okay? So, same problem same time, and we're justifying the mathematics. I said this the last time I did a Participation Quiz and it was amazing what I got. So many because statements, so many why statements. People even have different ways they are thinking of it, which is great. But we're not giving away the answers, we're not just checking final answers, we're talking about what we did and why we do it that way. Okay?

In this excerpt, the group test directions emphasized students' responsibility for "same problem, same time," for justifying mathematics to one another, for arriving at common answers, for offering "why" and "because" statements, and for receiving a group grade. By saying, "Do not move on to number 2 until everyone's done with number 1," Ms. Martin reinforced the expectation that *groups work on the same problem.* By saying, "We're not giving away the answers," but we are "talking about what we did and why we do it that way," Ms. Martin asked students to explain to one another why and how they were able to achieve particular answers, reinforcing expectations that developed the norms *mathematical work to involve sensemaking your own and others' mathematical ideas*. Ms. Martin's directions, "Your group grade is on the math," "I grade random problems from everyone's test," and "If you're very detailed about showing your starting point or rate of change, that is something you want to be mindful to remind your group to do," helped develop the norm *groups work to achieve mathematical consensus.*

Later in the semester, evidence the norms *mathematical work involves making sense of your own ideas* and *groups work to achieve mathematical consensus* were established occurred in many ways. During interviews, students described good group members as students who helped, listened, and worked together on mathematics. Shuffle quizzes also became more successful as the semester progressed, so that Ms. Martin needed to leave and return less often before students passed and were allowed to move on to the next problem. Students also received a combined group test and Participation Quiz grade at the end of Units 1 and 4. See Table 11 for an illustration of this increase.

Table 11

Group Test Grades Comparison

Student Earned Scores	Unit 1 Group Test Scores Linear Equations 18 October 2011	Unit 4 Group Test Scores Solving Linear Equations 15 December 2011
Average Score	84%	92%
Median Score	89%	93%
Minimum Score	53%	85%
Maximum Score	97%	99%
Standard Deviation	4.58	2.13

Between October and December, even though the content's cognitive demand increased, the average, median, minimum, and maximum group test scores also increased. This trend indicates that over the course of the semester, students improved their abilities to work together and to make sense of mathematics. Table 11 points to the establishment of the norms *mathematical work involves sensemaking your own and others' ideas* and *groups work to achieve mathematical consensus.*

Group interactions fostered groups achieving mathematical consensus. In the interaction described above, Irene engaged in sensemaking Phoebe's ideas in order to understand how to work on the problem: $6 + x + 5x = 1 + 5x - 8$. After Irene and Phoebe finished talking, Phoebe checked in with the other group member, Jaelyn, to ask for her answer to the problem they were working on.

31 Phoebe: ((to Jaelyn)): What'd you get for number 3?

32 Jaelyn: 13.

33 Phoebe: Negative?

34 Jaelyn: Huh?

35 Phoebe: Negative? Is it negative? I got negative 13.

36 Jaelyn: Yeah. ((Looking down at her paper and seeming to add a negative in front of her answer.))

37 Irene: Oh, okay. Thank you.

This part of the excerpt shows the group members agreed on the same answer and that Jaelyn perhaps saw that her answer should be negative and changed it. See Figure 29 for Jaelyn's work.

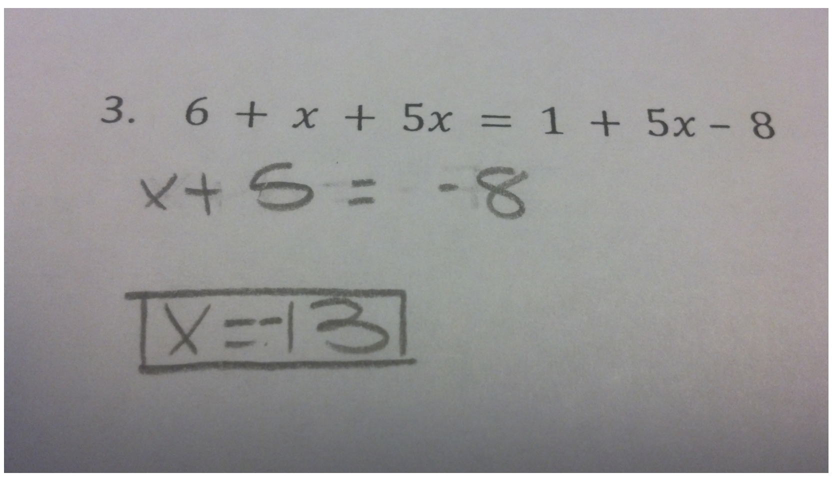

Figure 29. Groups work to achieve mathematical consensus: Jaelyn's work.

When this part of the excerpt is combined with the larger portion above, with each student's work in which the mathematics is worked slightly differently (shown in Figures 4-6), with previous experience of Jaelyn and Irene asking questions if they did not understand the mathematics, this moment illustrates the way in which *groups work to achieve mathematical consensus* was established for this group.

Social norms fostered groups achieving mathematical consensus. *Groups work to achieve mathematical consensus* was also fostered through the established social norms *everyone discusses ideas with everyone else, everyone offers and receives help, and groups work on the same problem.* Like many of the instructional strategies discussed in this paper, *groups work on the same problem* was nearly a daily group expectation. On most class days, the expectation for groups to work on the same problem was maintained informally. As the semester went on, students increasingly took up the expectation as a norm to use in their groups every day, even as they rotated into new groups every 2 weeks.

On group test days, *groups work on the same problem* was reflected in a group's shared group test grade, which also supported movement toward establishing the norm *groups work to achieve mathematical consensus*. Jaelyn talked about helping a member of her group that she wanted to get on board with understanding. She said that after a day Jaelyn was absent, the student had asked her where she was and why she hadn't been helping her:

> I just felt like she felt more comfortable asking me for help because we're friends outside of that class, rather than people that are just breezing through it. Yeah, and I was like, "Was the whole group gone?" And she was like, "No!" And I was like, "Oh," but then again, cuz we had a group test, and I felt like she was gonna like, like if Ms. Martin was gonna call on us on a question that she was gonna kinda like hold us back, because she wasn't in class the whole time she needed to be, but I mean, I didn't have a problem like going over the problems and explaining it with her. Because I've had talks with her, like, "You need to get on top of your game, because you do not want to be in this class next year."

In this excerpt, Jaelyn shared that a friend from her group who had been absent a number of days asked her for help on the mathematics. By saying, "I felt like she was gonna … kinda like hold us back, because she wasn't in class the whole time she needed to be," Jaelyn indicated that she was aware of the expectation for students to understand the mathematics for a group test, and she was willing to explain the mathematics to her friend in preparation for the test. Additionally, Jaelyn's advice to the student to "get on top of her game" was relevant, because Jaelyn herself was a sophomore, repeating the class.

When describing her last group of the semester, Irene shared that she thought it was the best group to that point in the year:

> It's going good, actually. It's going, I would say better than my other groups. Because, when I don't understand something, like when I asked about question 6, and they were reviewing question 7, Jaelyn said, "Why don't we just do 6 again?" So they explained how they did it on the mat. And that was actually really helpful, because I got to see how I was supposed to do it, how you're supposed to do it. Rather than how I did do it. So, that actually helped me. And then like, they did a lot of reviewing and checking on each other, like all of us at the table, and make sure everybody has the same, right answers. And when we didn't, we would like, go over the problem together, and see what we did wrong.

In this excerpt Irene shared a belief that her group members were so helpful they were willing to go backwards to a problem she had been working on and redo the problem with her so that everyone understood. The willingness of her group to work together on this problem showed how the group had established the norm *groups work to achieve mathematical consensus* through *groups work on the same problem*.

Summarizing the development of social and sociomathematical classroom norms. In this section, I have highlighted the social and sociomathematical norms that emerged from analysis of this classroom. As illustrated above, I found that when asked what it was like to be a student in this classroom, students mainly spoke incredibly positively about the classroom instructional strategies and teacher moves, often using words like *helpful.* I found when the teacher selected instructional strategies and employed teacher moves that oriented students to use one another as mathematics resources, these students took up sociomathematical norms that facilitated opportunities for mathematical sense-making. Table 12 is offered as a summary of the findings offered in this section.

Table 12

Summary of Emergent Social and Sociomathematical Classroom Norms

	Classroom Norms	**Teacher Moves and Instructional Strategies**	**Ways the Norms were Negotiated**
Social Norms Routinized ways of engaging as a community member.	*Everyone discusses ideas with everyone else.*	• regular random assignment to groupwork • regularly assigning mathematical competence • orienting students to one another • requiring individual accountability	• belief all students were mathematically competent • regular random assignment to groupwork • oriented group to **Jaelyn's** ideas • assigned competence on average 2.5 times per day • **Elena:** said class was about working with others • **Neesha:** said class was a positive learning place
	Everyone offers and receives help.	• regular random assignment to groupwork • assigning groupworthy tasks, which have high cognitive demand and group interdependence • orienting students to one another	• Figure 24: Group Questions Only • teacher directions: "check mistakes…" etc. • **Irene**: helped others & enjoyed being helped • **Jaelyn**: recalled when Ms. Martin made a group member help her and told

	Classroom Norms	**Teacher Moves and Instructional Strategies**	**Ways the Norms were Negotiated**
		• pushing papers to center of group	her not to give up
	Groups work on the same problem.	• regular random assignment to groupwork • teacher directions: "same problem, same time" • Participation Quizzes • Group Tests • Shuffle Quizzes	• Figure 25: Same Problem, Same Time • Students often said: "Are we ready to move on?" • **Irene**: helpful to be on the same problem for group members to catch wrong answers and help correct them • **Helen**: It's helpful because I get stuff wrong and make mistakes.
Sociomathematical Norms Routinized ways of engaging in the process of doing mathematics.	*Mathematical work involves making sense of your own ideas.*	• regular random assignment to groupwork • assigning groupworthy tasks • orienting students to one another • assigning competence • Group Questions only • pushing papers to center of group • Participation Quizzes • Group Tests • Shuffle Quizzes • sentence starters	• Ms. Martin directions: home work check & sentence starters • Shuffle Quiz • Participation Quizzes • Group Tests • Teacher Directions • **Neesha**: learn how to explain more in groups • **Helen**: class is about understanding through explaining and Ms. Martin doesn't give away correct answers. • **Elena/Neesha**: Classroom interaction
	Mathematical work involves making sense of others' ideas.	• regular random assignment to groupwork • regular teacher directions to listen to group members explain their ideas • Group Questions only • pushing papers to center of group • Participation Quizzes • Group Tests	• Ms. Martin directions: "people have different ways of thinking of it, which is great." • Shuffle Quiz • Participation Quizzes • Group Tests • **Elena**: "so we can learn different ways people do math" • **Irene**: "different people giving diff opinions and

	Classroom Norms	Teacher Moves and Instructional Strategies	Ways the Norms were Negotiated
		• Shuffle Quizzes • sentence starters	can figure out which way works best for you" • **Helen**: "other person could help see in different way" • **Irene/Jaelyn**: Classroom interaction
	Groups work to achieve mathematical consensus.	• regular random assignment to groupwork • student presentations • "same problem, same time" • Participation Quizzes • Group Tests • Shuffle Quizzes	• regular random assignment to groupwork • Shuffle Quiz decrease number of times leaving • group test scores increased • **Irene**: I got to see how I was supposed to do it, how you're supposed to do it. Rather than how I did do it. A lot of making sure everybody has the same, right answers. • **Jaelyn/Irene**: Classroom interaction

The Role of Comfort in Students' Perceptions of Competence

The previous section highlighted the social and sociomathematical norms that surfaced in this analysis. These findings emerged by analyzing interview data with the five focal students for this paper: Elena, Helen, Irene, Jaelyn, and Neesha. I use their interviews as a way to understand the student perspective of mathematical competence in this classroom. Whereas the previous section offered analysis of classroom routines and expectations, this section serves to illuminate the students' impressions of the nature of group interactions. I found that these students liked groups where they felt more comfortable, and if students felt more comfortable, they also felt more competent. I also found a relationship between students' perceptions of competence and established or violated social and sociomathematical classroom norms. I use this section to attend to the way these norms mediated students' perceptions of their mathematical learning.

Analyzing the students' interview data for trends over time showed students' positive perceptions of group interactions increased over the course of the semester. Students were randomly assigned to participate in nine different groups during the semester. I found that when students talked about the first four groups of the school year, they perceived mainly positive groups interactions for a little over half of their group experiences. When talking about the last four groups of the semester, the students reported positive perceptions of groups' functioning for about 80% of their groups and interactions.[37] This trend shows an overall increase in how students perceived their groups functioning over the course of the semester. One possible explanation for the increase in perceptions of group interactions is that as the classroom social and sociomathematical norms were increasingly accepted by students and the teacher, students simultaneously improved their perceptions of group interactions.

Analyzing the language students used to describe positive and negative perceptions of group interactions led to trends in how students characterized positive and negative group interactions. In particular, when coding interview data for perceptions of group interactions, various forms of the word "comfort" emerged prominently. In response to interview prompts such as, "Describe what it is like to be a member of this class," and, "Talk a little bit about what it was like to be a member of that group," Elena, Helen, Irene, Jaelyn, and Neesha each used the words "comfortable" and "uncomfortable" between 1 and 9 times over the course of their interviews. For example, when Elena referred to what it took for a group to be a "good" group, she said, "You feel comfortable with those people," and when Jaelyn referred to potentially not

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[37] Interestingly, only one out of the five students interviewed reported a positive perception of their work during the fifth seating chart. Including the fifth seating chart with either the first or second half of the semester still shows an overall increase in students' perceptions of their group interactions over time. Because the overall trend of changed perceptions of competence appears regardless of the placement of seating chart five, I pursue reasons for its diminished perceptions of group interactions in a future study.

being comfortable with a group, she said, “if you’re not comfortable with asking questions, then you’re not gonna ask questions, because you don’t wanna feel embarrassed.” Microanalysis (Corbin & Straus, 2008) of the ways students discussed experiencing the presence or lack of comfort revealed three themes. First, when students talked about not feeling comfortable as a member in a particular group, they also found it more difficult to ask questions of their group members. Second, when students felt comfortable as a member in a particular group, they also felt comfortable asking group questions to other group members. Third, when students described greater comfort developing over the course of time in a group, that movement resulted from influence the teacher had on the group community and on the classroom community. The presence of these three themes indicates a trajectory of comfort in this class, where sometimes groups felt uncomfortable to these students, most often students felt comfortable, and sometimes groups moved toward comfort over time.

Uncomfortable groups. Students described uncomfortable groups as having violated the developing and established social and sociomathematical norms. Groups that students described as uncomfortable had members who did not believe all group members were capable, violating *everyone discusses ideas with everyone else;* they did not feel comfortable asking one another questions, violating *everyone offers and receives help*; they worked at different paces, violating *everyone stays on the same problem;* they were more often off-task than on-task, violating *mathematical work involves sensemaking your own and others’ ideas;* and they adopted answers without discussing the mathematics, violating *groups work to achieve mathematical consensus.* Here I relate how students described these norm violations as making them feel uncomfortable.

Social norm violations in uncomfortable groups. In one group Elena did not like, she described thinking she could not trust the group with her mathematical questions:

> I didn't like that group. It's because I feel like you have to know the person and trust the person, and I feel like I can't ask them questions. I feel like they feel like they're too smart, and if I ask them questions, they'd be like, "oh, she's not smart" or something.

Here Elena said she positioned her group members as thinking they were smarter than her, which she described as preventing her from feeling comfortable asking questions. The way Elena described her lack of comfort in this group provides evidence that she thought the group violated the social norms *everyone discusses ideas with everyone else* and *everyone offers and receives help*.

In describing one group with which Jaelyn thought she did not work as well, she said, "They all knew what they were doing. I didn't feel as comfortable asking, 'Hey, how did you guys get that?' as I did in all the other groups." Jaelyn positioned the members of that group to be more competent than her, saying, "They all knew what they were doing." When Jaelyn positioned her group members as more competent, she simultaneously positioned herself as less competent, thus showing she did not believe that group honored *everyone discusses ideas with everyone else*.

Neesha described one group she did not like as "weird" for students to be working at different paces and generally to lack group cohesion:

> I didn't like that group much. I don't know. It was just weird. I felt like, [one student] was just kinda like super independent, and [that student] would be done with the whole work, and the rest of us would be catching up behind. And then me and [another group member] would always be at the same pace. And then [the fourth student], she'd get the right answers, but she was kinda lagging behind, and so, it was just everywhere. We didn't talk that much.

Neesha explained that this group was uncomfortable because not all the group members stayed at the same pace to work on mathematics together, violating the norm *groups work on the same problem*. Neesha described one group member as “super independent”, another to be working behind, and overall, she thought group did not talk to one another as much as she would have liked.

Sociomathematical norm violations in uncomfortable groups. Helen also described a group she found uncomfortable. She said the group was often off-task, and she did not feel comfortable asking this group to do mathematics:

> It seemed like they knew each other really well. It wasn’t really a good group because we were all off-topic most of the time. They all seemed like they were really good friends [and] they were kind of intimidating. I wanted to be like, “Let’s do this you guys!” but they were all talking about something else.

Helen described difficulty in focusing this group on the mathematics, showing evidence the group was violating the norm *mathematical work involves making sense of your own ideas*. Helen perceived the other group members to be good friends, and she described a resulting feeling of intimidation. Ultimately, she did not feel comfortable with the group working together, and her description of intimidation indicates how the group violated the *sensemaking* sociomathematical norm.

Irene described a group where she did not feel comfortable talking to her group members. She reported the group was quiet, group members did their own work, and she needed to go outside of her group to find mathematical help:

> And we didn't really talk. We just did our own work. And then when the teacher said we had to talk, we just talked about if we got the answers right. And then the next day I

> moved. …I wanted to be by my friend [in another group]. So I just moved there and it was kinda easier, because if I didn't get something and nobody at my table knew it, I would ask [that student behind me], and then she would explain it to me.

In this example, Irene described experiencing a group that did not work together, violating the norm *mathematical work involves making sense of your own ideas*. She described Ms. Martin's intervention as forcing the group to exchange answers for a while, but only temporarily, and that the group still did not mathematically explain their solutions to one another, thus violating the sociomathematical norm *groups work to achieve mathematical consensus*. Irene indicated that when she felt uncomfortable talking to her group, she violated these norms to turn around and sneak in a mathematical conversation with her friend in another group.

When Jaelyn described her uncomfortable group by saying, "They all knew what they were doing. I didn't feel as comfortable asking, 'Hey, how did you guys get that?' as I did in all the other groups," she reported not feeling comfortable asking her group members how they got their answers. Here Jaelyn indicated she wanted to know not just what the group's answer was, but also how they mathematically achieved their answer. Jaelyn's comment indicates she did not feel comfortable when the group violated the norm *mathematical work involves making sense of others' ideas*.

Comfortable groups. When studying what students said about their most comfortable groups, a second theme emerged around the ways students talked about their group interactions. As described in the introduction to this section, by the end of the semester, students described having more positive group experiences and more comfort in their groups than negative or uncomfortable experiences. The way they talked about these comfortable groups also offered evidence of compliance with the social and sociomathematical norms described in this study.

Social norm observances in comfortable groups. Neesha described comfortable groups as places where she perceived everyone was able to do mathematics. When describing this group, she said, “I really liked that group. I felt like we all knew what we were talking about, which was helpful.” Neesha said, “We all knew what we were talking about,” showing she believed the group observed the norm *everyone discusses ideas with everyone else*.

Helen discussed comfortable groups as ones in which she felt she could ask questions, “If [one group member] said something I didn’t understand, I could ask questions. You’re having a conversation and people show what they know, and it comes to a conclusion.” Helen described that when she felt comfortable in a group, everyone observed the norm *everyone offers and receives help*.

When Irene was talking about a group that was very comfortable, she described how they used their time to work on mathematics together:

> It was really a working environment, there were no distractions or anything. ... Most of it would be about math, and how we got different things. And it was a good environment to work in. ... It seemed like [2 of the students] got it more than me and [the other student] did. They knew what they were doing. So, I would ask them how to do stuff. So that was helpful too.

Irene described how she thought that in this “working environment” 2 of the students “got it more than her,” and that she found it “helpful.” Although “got it more than her” implies her perception of varying levels of competence among group members, the fact that she says her group members could be helpful in their working environment implies she thought the group observed both social norms *everyone discusses ideas with everyone else* and *everyone offers and receives help*.

Elena discussed groups where she was comfortable as having group members who were helpful. In one helpful group she described how a student would check her work: "[One student] gets things fast and then helps others. And it helps you because like, somebody else is there to spot your work. She wouldn't hesitate about helping. And she didn't need help. She would just help others." This quote describes how Elena believed comfort involved *offering and receiving help from group members*.

Sociomathematical norm observances in comfortable groups. Above, I shared Neesha describing how a group observed the social norm *everyone discusses ideas with everyone else*. As she continued, her comments highlighted a trend in a difference between uncomfortable and comfortable groups in this classroom. Neesha discussed how the group also observed established classroom sociomathematical norms:

> I really liked that group. I felt like we all knew what we were talking about, which was helpful. And then we all knew, like even if we did get the wrong answer, we could at least explain how we got the wrong answer, or why we thought that answer was right.

Here Neesha first expressed a belief that *everyone discusses ideas with everyone else*. She went on to discuss the group's ability to safely share wrong answers, saying, "Even if we did get the wrong answer, we could at least explain how we got the wrong answer, or why we thought the answer was right." Neesha's belief that the group regularly explained *how* and *why* answers were correct and incorrect shows this group engaged the norms *mathematical work involves making sense of your own ideas* and *mathematical work involves making sense of others' ideas*. One of the problems revealed in uncomfortable groups was not being able to ask a question or not feeling comfortable being wrong. But in this case, Neesha's group could be wrong and still feel comfortable, revealing how the group members continued to value one another as *competent* and engage in their own and their group members' *sensemaking*.

As mentioned above, Irene's talk about a group that was very comfortable included her description of a good "working environment" where "most of it would be about math and how we got different things." Thus Irene indicated that she perceived the group to observe *mathematical work involves making sense of your own ideas of their own and others' ideas*.

Helen contributed that, "If [one group member] said something I didn't understand, I could ask questions. You're having a conversation and people show what they know, and it comes to a conclusion." Above I discussed how the group took up the norm *everyone offers and receives help*. In addition to regularly asking and answering one another's mathematics questions, Helen said the group's conversations would "come to a [mathematics] conclusion," indicating this group's observance of *groups reach mathematical consensus.*

Elena described feeling comfortable in another group that was also helpful and worked together: "They helped me a lot; we all got along. It wasn't boring, and we got our work done … going through our steps and checking in and making sure everyone's on point." By saying that the group would make sure "everyone's on point," Elena described the dedication this group had to making sure the group observed *groups reach mathematical consensus*.

The analysis of comfortable groups reveals that in all cases of groups in which students described feeling comfortable, students were able to share both what they did and did not understand about the mathematics. In comfortable groups, students positioned themselves and their group members to be competent. And when students were comfortable, social and sociomathematical norms were positively guiding the group's functioning.

Groups moving toward comfort. Some groups moved toward a feeling of comfort over time. At least two possibilities are revealed for why certain groups felt more comfortable over the course of the semester. In some cases, the continued development of the social and sociomathematical classroom norms helped students feel more comfortable with certain group members as the semester progressed and the students got to know one another better. In other cases, such as when Ms. Martin successfully identified a social or sociomathematical norm violation and applied a teacher move to address it, groups moved toward feelings of greater comfort by more regularly abiding by the requested social and sociomathematical norms.

Perhaps students and groups developed greater comfort over the course of the semester as Ms. Martin influenced the development of the class's social and sociomathematical norms over time. I observed Ms. Martin regularly share her expectations for social and sociomathematical norms. Neesha shared her perception of how the classroom community was developing over time when she talked about a group from early in the year, "That group was good. We all got pretty good answers, but I think we were just shy toward each other. Like I think if we were in a group again, it wouldn't be that bad." Neesha described a feeling that as the year went on the students were getting to know one another better. She believed that the group was working toward observing the norms *mathematical work involves sensemaking your own and others' ideas,* and she predicted she would be more comfortable if she worked with those students again.

During one seating arrangement from the semester, Jaelyn and Helen were members of the same group, and both described feeling comfortable. For Jaelyn, the group was comfortable because she felt she was competent and could explain her mathematical thinking:

> That was when I was breezing through everything, and I knew what I was doing. ... Helen was like the one that was in the group, like not really understanding, and then me and [the

> other 2 students] were like, we understood it. …I felt we all explained it to Helen the most.

In this example, Jaelyn explained how she positioned herself and two other members of the group as more competent than Helen and as having the role of explaining the mathematics to Helen. Helen reported a similar competence hierarchy for this group, except that she positioned one other group member to be about her level of understanding: "I think [another student] and I are kinda on the same math page … and it kinda like evened each other out." For both Jaelyn and Helen, the levels of competence among the four group members were perceived to be different, with some students more competent than others. Helen and Jaelyn perceived Jaelyn to be able to explain the mathematics, and they thought, for that group, that dynamic worked.

About 10 weeks into the school year, Elena described one group as "okay". Elena seemed to indicate she would not have found this group to be productive at all at the beginning of the school year:

> It was okay, I think. I mean, we had to help [one student] a lot because she didn't get it. [Another student] was really good. It was okay. It was on and off. [The one student] would be like the person to help us. I would just do my work. They would ask me questions.

The way Elena described this group, she thought they were having some differences in working together, but in spite of this, the group members helped one another.

Helen noticed the effect of Ms. Martin's interventions: "When the teacher comes over and gets the group going, she asks questions. [It] helps that the teacher is there to get people on track." Although Helen may not have been consciously aware of the violation of social and sociomathematical classroom norms, she noticed that teacher intervention often led groups to reorient to one another and *offer and make sense of mathematical explanations*.

One example of the type of teacher intervention that led to increased mathematics sense-making occurred about 5 weeks into the class, in a group with Helen in which group members were “getting things so quickly” that she would tell them she got the same answer even if she had not yet finished the problem. Helen told me that in this group she often was actually working backwards to try to finish the problems the group had already completed. During class, Ms. Martin also identified norm violations in this group. On the first group test of the year, Ms. Martin noted that Helen’s group was not staying together and not offering or making mathematical sense of one another’s explanations. Ms. Martin approached this group, redirecting them to work together:

> Can I ask you guys to show off your explaining a little bit more? That's what I'm grading you on, for the participation, is justifying the math, and yesterday there was a ton of conversation, but now I'm just getting like ((pointing to the public display of the group's Participation Quiz grade)) the procedural stuff like, reading directions out loud or checking answers. [What about] “How'd I get…?” Do you know what I mean? "I got this..." I wanna hear the how's and the why's. So if you're ... maybe you start working together, right? Or check in mid-way, instead of at the end? I know all you guys here are great at communicating mathematics and I just want to be able to capture that up here for your participation grade. [Does] that work? Do you know what I mean? Yeah, okay, cool. ((Ms. Martin walked away.))

During this class moment, Ms. Martin noticed the students in Helen’s group were not working together, and she deliberately attended to this by drawing the group’s attention to the social norm of *staying on the same problem* and the sociomathematical norm of *offering and making sense of your own and others' mathematical explanations.* She also reinforced the social norm that all students were competent to make mathematical sense by asserting that they were more than capable of meeting the expectation for talking together: "I know all you guys here are great at communicating mathematics." When Ms. Martin walked away, the group increased their mathematics talk and justification. This example demonstrates the ways Ms. Martin reinforced classroom norms in order to move groups with norm violations toward increased comfort and a greater display of mathematics justifications and sense-making.

The result of the successful development of comfort in this class was also partially revealed when Jaelyn described working with Irene and one other student in different groups throughout the course of the semester: "I've gotten to know Irene and [the other student] because I've been in a group with them a couple of times … [and now] I just feel more comfortable asking questions, because I know they're not judging me or anything." Here Jaelyn revealed that over the course of time she had learned more about who Irene and [the other student] were as individuals and as learners, and as a result she felt increasingly comfortable observing the norm *everyone offers and receives help,* asking them questions and working with them so that *mathematical work involves sensemaking your own and others' ideas*.

In an excerpt cited above in the social norms section, Irene discussed a group in which *everyone offered and received help*. She then described how Ms. Martin oriented Irene's group to one another in such away that they continued talking about the mathematics after she left:

> I think Ms. Martin might have came over and asked us to start sharing about that equations. So, I volunteered to share it, and then I explained how I got my answer, and I put my thing in the middle. I mean, cuz she came over and she saw who had more, like,

> explanations. I was supposed to put mine in the middle to share with the table. And after she left, I continued with that. And then [another group member] arrived late and she was asking different questions. Then after I answered a question, [that student] started to help [other group members] more too.

In this excerpt, Irene recalled Ms. Martin intervening with the group, reorienting them to talk to one another about mathematics. Irene recalled that Ms. Martin asked her to share, and as a result she did share. During an interview at the end of the semester, Irene said that she would now sometimes feel comfortable asking her group to stay on the same problem: "In the group that I'm in right now ... I would say I would feel comfortable with saying 'Can we stay on the same problem?'" Irene's commitment to *groups stay on the same problem* indicates that her comfort level was increasing over the course of the semester.

The data from this section reveal that over the course of the study, students felt increasingly comfortable making their mathematical ideas and questions public. Increasing the public nature of learning indicates that students gradually felt more comfortable taking intellectual and social risks in front of one another. Increasing the number of intellectual and social risks may have led to more opportunities for students to learn.

Summary of the Role of Comfort in the Community of the Classroom. The analysis of the role of comfort in students' mathematical learning in this class reveals the students described comfort as changing from one group to the next, revealing an understanding that their mathematical learning was affected by what occurred on the interpersonal plane of analysis. The analysis also reveals students in this class felt comfortable in their groups more often than not, and that comfort increased over time.

The analysis also revealed a relationship between the developed social and sociomathematical norms in the classroom and students' comfort in their groups. The analysis shows that *everyone discusses ideas with everyone else, everyone offers and receives help, groups work on the same problem*, *mathematical work involves sensemaking your own and others' ideas,* and *groups reach mathematical consensus* were violated in groups that were uncomfortable, were taken up in groups that were comfortable, and were unevenly taken up in groups that were moving toward comfort. As students' norm violations decreased, comfort and perceptions of competence increased. Table 13 summarizes these findings.

Table 13

The Role of Comfort in the Community of the Classroom

	Uncomfortable Groups		**Groups That Moved Toward Comfort**		**Comfortable Groups**	
Role of comfort	**Description**	***Norms Violated***	**Description**	***Norms Developing***	**Description**	***Norm Established***
Elena	Elena felt she could not ask questions. She also thought the rest of the group thought they were smarter than her and would judge her for not being smart.	Elena described violations of the norm *everyone discusses ideas with everyone else*.	"It was okay… We had to help [one student] a lot…[Another student] was really good… I would just do my work. They would ask me questions."	Elena described the group's observance of the norm *groups work on the same problem* and *everyone offers and receives help* in spite of not taking up the norm *everyone discusses ideas with everyone else.*	Elena described group "went through their steps" together, one another, and made sure everyone was "on point."	Elena described the observance of the norms *everyone offered and received help* in this group and *mathematical work involved sensemaking your own and others' ideas*.

	Uncomfortable Groups		Groups That Moved Toward Comfort		Comfortable Groups	
Role of comfort	**Description**	***Norms Violated***	**Description**	***Norms Developing***	**Description**	***Norm Established***
Helen	The other group members were friends and talked to one another instead of working on mathematics. Even though she wanted to, Helen did not think she could say, "Let's do this you guys!"	Helen described the group not working together, violating *groups work on the same problem.* She also described the group was often off-task, violating *mathematical work involves sensemaking your own and others' ideas.*	Helen described a group where Ms. Martin was able to intervene: "When the teacher comes over and gets the group going, she asks questions. [It] helps that the teacher is there to get people on track."	Ms. Martin helped the group to start to observe the norms: *mathematical work involves sensemaking your own and others' ideas*.	"If [one group member] said something I didn't understand, I could ask questions. You're having a conversation and people show what they know, and it comes to a conclusion."	Helen described how *everyone offered and received help* in this group and how *groups reach mathematical consensus.*

	Uncomfortable Groups		Groups That Moved Toward Comfort		Comfortable Groups	
Role of comfort	**Description**	***Norms Violated***	**Description**	***Norms Developing***	**Description**	***Norm Established***
Irene	"When the teacher said we had to talk, we just talked about if we had the answers right."	Irene described violations of engaging in mathematical sensemaking, violating *mathematical work involves making sense of your own ideas* and only talking about answers, not whether answers made mathematical sense, violating *groups reach mathematical consensus.*	"Ms. Martin might have come over and asked us to start sharing about equations. So I volunteered to share it, and then I explained how I got my answer…And after she left, I continued."	Ms. Martin's intervention positioned Irene as *competent*, which helped Irene and her group start to observe the norm: *mathematical work involves making sense of your own ideas.*	We knew each other and it was a working environment.	Irene described how *mathematical work involved sensemaking everyone's mathematical ideas* in this group.

	Uncomfortable Groups		Groups That Moved Toward Comfort		Comfortable Groups	
Role of comfort	**Description**	***Norms Violated***	**Description**	***Norms Developing***	**Description**	***Norm Established***
Jaelyn	"The other group members all knew what they were doing. I didn't feel as comfortable asking, 'Hey, how did you guys get that?' "	Jaelyn believed that the other group members knew what was happening and that she couldn't ask, indicating violations of *everyone discusses ideas with everyone else* and *everyone offers and receives help.*	"I've gotten to know Irene and [the other student]…[and now] I just feel more comfortable asking questions, because I know they're not judging me or anything."	Jaelyn believed that getting to know these students indicated a developing comfort to abide by the norms *everyone offers and receives help* and *mathematical work involves making sense of your own ideas*.	"I was breezing through everything, and I knew what I was doing… Me and [the other 2 students], we understood it, and I felt like we explained it to Helen the most."	Jaelyn described how she and other group members were *competent* and how *everyone offered and received help* in this group.

	Uncomfortable Groups		**Groups That Moved Toward Comfort**		**Comfortable Groups**	
Role of comfort	**Description**	***Norms Violated***	**Description**	***Norms Developing***	**Description**	***Norm Established***
Neesha	Group members were all at different places and "we didn't talk that much."	Neesha described the group violating *groups work on the same problem* and *mathematical work involves making sense of your own ideas*.	"That group was good. We got pretty good answers, but I think we were just shy toward each other. I think if we were in a group again, it wouldn't be that bad."	Neesha described the group starting to observe *mathematical work involves sensemaking your own and others' ideas*.	"I really liked that group. I felt like we all knew what we were talking about… And then we all knew even if we did get the wrong answer, we could explain how we got the wrong answer, or why we thought the answer was right."	Neesha described how feeling comfortable being wrong was a critical part to observing the norm *mathematical work involved sensemaking your own and others' ideas* in this group.
Conclusions	**Norm violations contributed to students' general feeling of discomfort. Students' perceptions of competence were low.**		**Classroom norms were developing. Ms. Martin addressed norm violations, and groups generally responded positively, further developing the norms.**		**Norm violation decreased. Students generally observed the social and sociomathematical classroom norms. Perceptions of competence were high.**	

Table 13 illustrates the relationship between the development of the classroom soc sociomathematical norms and the role comfort played for Elena, Helen, Irene, Jaelyn, and Neesha. The left columns outline trends for these students across uncomfortable groups. I that in uncomfortable groups, social and sociomathematical norm violations impeded the g ideal functioning. Students felt less competent in uncomfortable groups. The center colum outline trends for students across groups moving toward comfort. I found that Ms. Martin worked to intervene with groups that were not functioning according to the norms she was establishing and that Ms. Martin tended to move these groups toward more comfortable le communities by reinforcing the social and sociomathematical norms described in this pape data also showed groups tended to move toward more comfortable learning communities a school year progressed, indicating the continuing development of social and sociomathem norms to support mathematical thinking and learning in this classroom. The right columns outline trends for these students across comfortable groups. I found that comfortable group observed the prescribed classroom social and sociomathematical norms. I also found that students felt more competent in comfortable groups, and that although over half the group described as comfortable when the semester started, about 80% were described as comfort by the end of the semester. The increase in comfortable groups coupled with students' inc perceptions of competence over time indicates that the social and sociomathematical norm influenced the students' level of comfort to work in groups in this classroom and the development of students' perceptions of competence in mathematics.

Discussion

Contributions

As a result of this study, I argue first that classroom social norms affected the development of sociomathematical norms. Second, social and sociomathematical norms mediated students' perceptions of competence. Third, the teacher's belief that all of her students were mathematically competent expanded their opportunities to improve their perceptions of competence. Finally, I argue that the overrepresentation of girls and students of color in this classroom indicates a larger systemic inequity present in the opportunities these students had prior to arriving in this classroom.

Social norms fostered the development of sociomathematical norms. The social norms *everyone discusses ideas with everyone else, everyone offers and receives help,* and *groups work on the same problem* fostered the development of the sociomathematical norms *mathematical work involves making sense of your own ideas, mathematical work involves making sense of others' ideas,* and *groups work to achieve mathematical consensus*. For example, I found that when Ms. Martin intervened with groups to assign competence and position group members as competent, groups started to believe they were capable of doing rigorous mathematics, creating opportunities for students to engage in mathematical *sensemaking*. When Ms. Martin positioned students as competent to help one another, groups started sensemaking their own and their group members' mathematical ideas, which positioned them to work to achieve mathematical consensus.

Social and sociomathematical norms mediated students' perceptions of competence. I found that the established social and sociomathematical norms were positively associated with students' significantly changed perceptions of competence. Social and sociomathematical norm violations indicated the absence of comfort for students to work in a given small group, whereas norm observances indicated the presence of a perceived comfortable and positive working environment. Students' perceptions of competence also indicated their capacity to see themselves as able to engage in mathematical work. Students described comfortable groups as places where they felt competent and willing to be publicly right or wrong, indicating that when the group worked together, they helped one another engage in mathematical sensemaking. Because students felt comfortable groups were safe spaces, these groups indicate students' willingness to engage in academic risk taking.

Teacher beliefs mediated students' perceptions of competence. I found that because Ms. Martin fostered the development of the norms that all students were mathematically competent, the teacher's belief system is linked to the opportunities for students to be successful in the classroom. Ms. Martin nurtured the idea that all students were competent, and each of the students highlighted significantly improved her perception of competence during the course of the study. Neesha expressed an understanding of some of Ms. Martin's teacher moves and belief in each student's ability to do the mathematics:

> Ms. Martin, she'll come toward you and talk with you. And, she won't give you the answer but she'll ... but like, not only … and then she'll walk away, but not only will she like … but even if she walks away, she's still ... like you can tell that she's still listening ... like, just by like ... her posture. Like, she might be away, but like she's kinda slightly towards you, so she can definitely still hear, like, your group's conversation. And then, like when, like you figure it out, she's like, "I knew you would!" ((motioning)) walking by, like she just does things like that. So you can tell that she's paying attention to almost every group at the same time.

By suggesting Ms. Martin regularly said, “I knew you would!” [be successful to understand a mathematics problem], Neesha attributed to Ms. Martin a consistent belief that all students were competent to do rigorous mathematics. The result was that this classroom experienced a fundamental shift in their concept of what it means to *do mathematics*. If all students are capable of engaging in high-level content, then more students in the class are available to work together and help one another in *doing mathematics*.

An overrepresentation of girls and students of color in this class indicates inequities in the system. As outlined in the methods section, this class had an overrepresentation of girls and students of color when compared with the school (and perhaps district and country). Race and racism are crucial considerations in striving toward equity in mathematics education (Martin, 2003). An overrepresentation of these groups of underrepresented students matters, because if we educators want to improve *all* students’ perceptions of competence in mathematics, we have to understand the practices that support students to improve their perceptions of competence at all levels (Gutiérrez, 2007).

Limitations

Data for this study was collected from one class period and from one teacher. This teacher had training in complex instruction and in teaching mathematics for social justice. The study therefore does not attend to the scope or complexity that simultaneously studying multiple classrooms and teachers could offer. Future research should examine the development of norms and their effects on perceptions of competence for a larger sample of classrooms.

The corpus of data indicates that this classroom was a community in which groupwork was welcomed and where groupwork thrived as the preferred method for learning mathematics. However, in this paper I have not explored the perceptions of students who may have not liked participating in groupwork activities. Further research could explore effects of students who do not prefer to work in small groups.

The process of interviewing students may have been positively correlated with their changed perceptions of competence. The nature of the questions I asked positioned students in such a way that they often realized what they needed to do on their own to improve their mathematics experiences. The nature of that effect is not explored in this paper but could be a source for future understanding.

Conclusion

Although the results of this study are specific to the field of mathematics, I propose they could be generalized to help understand how the development of social norms can impact the development of content-specific norms. I also argue the development of social and sociomathematical norms can influence students' perceptions of competence across academic content. More generally, I claim students' perceptions of competence affect their classroom learning.

Future research should attend to what happens to students' perceptions of competence as they move on to other mathematics classes and teachers. Other avenues to pursue include why these norms were successful and what other social and sociomathematical norms can mediate students' perceptions of competence and move classrooms toward equity. Questions to explore may also include: Under what conditions are changed perceptions of competence durable or not durable? Under what conditions do underrepresented students' changed perceptions of competence lead to increased numbers of students pursuing degrees and careers in mathematics? Under what conditions can the assignment of expertise be changed in a classroom?

Not all teachers believe that all students are competent to learn high-level mathematics. Future research in moving toward equity should attend to ways to positively influence teacher beliefs that all students are capable of learning high-level mathematics.

As Helen said, "If you feel like you can wave to [your group members] in the hallway, that makes me feel like I can talk to them about stuff I don't understand." If we accept Helen's desire to be able to "wave" at group members "in the hallway" as symbolic of the presence of *comfort* and positive *perceptions of competence* within a small group, her statement summarizes the idea that being comfortable with a group allows students to take risks, ask questions, and feel *competent* in academic abilities. *Comfort* allows students ultimately to expose what is not known or understood and to ask questions so that students can learn, knowing they will continue to be respected by their group members.

The findings in this paper deepen an understanding of classroom practices that move toward equity, where all students are considered capable and are given opportunities to engage in high-level content (Cohen, 1997a). In this paper I argue that by positively developing and maintaining these specific social and sociomathematical norms, Ms. Martin mediated students' *perceptions of competence* and moved this classroom toward equity.

Dissertation Conclusion

Contributions

In this dissertation, I have asserted that the pursuit of classroom equity is founded on the premise that all students are competent to complete high-level mathematical tasks and therefore should be offered rigorous opportunities to learn. Striving toward classroom equity therefore implies working toward access to and expectations that all students will and can learn high-level mathematical concepts. In this study, I offered empirical evidence for the ways one teacher used classroom practices to strive toward equity. I triangulated analysis of classroom interactions and interviews with the teacher and students as a way to capture a full picture of the secondary student and teacher mathematical learning experience. In Paper 1, I contribute an analytic framework for coordinating Status and Positioning Theories. This framework can be used to investigate the complex nature of classroom interactions and to explain positioning moves that inform expectations for competence, status generalization, and learning. In Paper 2, I contribute an understanding of the connection between a teacher's belief system and her classroom structures and pedagogical practices. I uncover her process for delegating mathematical authority through these classroom structures and its result in expanding mathematical smartness in this classroom. I argue that delegating mathematical authority and expanding mathematical smartness moved this classroom toward equitable teaching and learning opportunities for students. In Paper 3, I contribute an understanding of the development of specific social and sociomathematical classroom norms that mediated students' perceptions of mathematical competence.

Throughout this study, I deliberately feature student perspectives as a way to offer rich connections between classroom practices and students' impressions of those practices. In addition to understanding the nature of the classroom structures of a teacher who strives toward equity, I argue that developing a rich understanding of students' perspectives for learning mathematics will move mathematics research and teaching toward equitable classroom opportunities. The combined contributions of this study include understanding of the ways specific classroom structures, pedagogical practices, developed norms, and students' perspectives can inform opportunities to strive toward equity in mathematics education.

Future work in striving toward equity

Next steps. Although I have argued the teacher in this study employed equitable teaching and learning classroom structures, she did not use class time to attend to students' positions within society during class. She did not discuss with her students the possible systemic inequities that placed them disproportionately in Algebra 1 behind a majority of the school's dominant class peers. With few exceptions (see Gutstein, 2003), the field does not have significant empirical evidence of successful classroom practices that empower students to use mathematics as a way to learn about societal injustices. Next steps in striving toward equity in mathematics education include a push for teachers to utilize mathematical teaching and learning practices to enable students with knowledge of societal injustices as a way to learn mathematics.

A sociopolitical turn in mathematics education. To continue to strive toward equity, mathematics educators must continue their work. Future research must build on the conversation of equitable classroom practices and underrepresented students' empowerment to understand mathematics in society. In 1989, NCTM published mathematics *Curriculum and Evaluation Standards*, stating the expectation that all students be given opportunities to learn and reason about mathematics. NCTM renewed their efforts in 2000, outlining a vision statement for school mathematics in *Principles and Standards for School Mathematics,* suggesting that mathematics educators should make curriculum more relevant to the real world. In spite of the renewed commitment of NCTM (2000) to mathematics education, Martin (2003) highlighted the inequities and differential achievement faced by historically marginalized and underrepresented students and suggested the field use mathematics as a basis to critically investigate students' society. Martin critiqued the changes in the language of the 2000 *Principles and Standards* compared with the 1989 *Curriculum and Evaluation Standards*. He demonstrated how the 2000 *Principles and Standards* de-emphasized the inequities faced by historically marginalized students and the importance of learning mathematics to critically investigate the society in which students live by arguing that, in 2003, the field of mathematics education had yet to offer significant evidence of acceptable solutions to differential achievement for underrepresented students. Martin was calling attention to teaching and learning mathematics for social justice, such that students who had traditionally not been afforded particular mathematics opportunities might be afforded opportunities to learn mathematics by critically investigating society.

In 2007, Rochelle Gutiérrez built on Martin's findings by summarizing how the field of mathematics education might (re)define working toward equity as coordinating dominant mathematics with critical mathematics. Gutiérrez offered a working definition for monitoring progress in working toward achieving equitable outcomes in mathematics education, asserting that researchers' and educators' progress toward equity involve the intentional coordination of dominant mathematics, which she defined to be a largely Western perspective, with critical mathematics, which is mathematics that "squarely acknowledges the positioning of students as members of society rife with issues of power and domination" (p. 40). Gutiérrez suggested that educators acknowledge the presence of dominant mathematics in our classrooms so that we might take up the call to position all students to critically consider the role of power and domination in mathematics. Further, Gutiérrez asserted that progress toward equity means movement toward an inability to discern achievement outcomes as being significantly different if grouping students on the basis of any set of individual characteristics. In other words, Gutiérrez proposed mathematics researchers and educators will know there has been progress toward achieving more equitable teaching and learning outcomes when differences in achievement can no longer be seen by grouping students in any particular way, including by race, class, gender, (dis)ability, or any other grouping of students.

In 2013, the Journal for Research in Mathematics Education (JRME) published a special issue focused on the status of equity in mathematics education. Martin (2013) highlighted mathematics education as "an instantiation of White institutional space" (p. 328) where dominant-culture politics dictate what gets taught in mathematics education. Martin's claim outlines a mathematics classroom as a racialized space, and he asserted that to move toward equity, the field must pay attention to and engage the role of race and political power in mathematics education. Gutiérrez (2013) implored work in mathematics education to move beyond essentialization and victimization of historically marginalized students and toward challenging common notions of teacher quality to develop teachers' deep connection to students and political knowledge for understanding mathematical thinking and learning. Gutiérrez suggested this kind of sociopolitical turn could help deconstruct mathematics education to reveal more consciousness of mathematical learning discourses and practices.

Future research questions. In addition to corroborating the findings from this study, future studies could investigate *What other processes exist to successfully delegate mathematical authority to students? What other classroom structures can expand and therefore increase access to mathematical smartness for a given classroom community? What other social and sociomathematical norms position secondary students as competent sensemakers? What other* *classroom structures and pedagogical practices help teachers strive toward equity? What other classroom structures and practices empower students with sociopolitical knowledge as a way to learn mathematics? What happens to students' perceptions of competence when they leave classrooms where teachers strive toward equity? How do teachers and policy makers negotiate the relationship between equitable classroom practices and equitable outcomes?*

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Appendix A: Sample Groupworthy Task
Trick or Treat

Bella and Tomás are going trick or treating in the same neighborhood on Halloween. They want to meet up and have their costumes to be a surprise.

Bella starts trick or treating at the bottom of the Alder St. hill at the same time Tomás starts 935 meters away at the top of the hill. They plan on meeting somewhere in the middle of the hill.

Bella is walking up hill, so she only walks at 40 meters every minute. While Tomás is walking downhill so he walks at 70 meters every minute.

THE BIG QUESTION: WILL BELLA AND TOMÁS BE ABLE TO MEET ON ALDER HILL WITHIN 30 MINUTES???

1. Create a table for each person that shows the distance from the **bottom of the hill** based on the number of minutes. Be sure to **label** each table with the plan name and include **5 entries**.

2. Write an equation for each person. Be sure to **label** each equation with the person's name and **define the variables.**

3. Make a full-page graph for this situation that answers the BIG question. **HINT**: Remember to discuss your scaling before you get started. What number do you need to go up to for the minutes? For the meters?

   Be sure that your graph:
   - ❒ Shows the answer to the BIG question
   - ❒ Has all points accurately plotted
   - ❒ Includes all key features: title, scales, labels, and a key

4. Answer the Big Question**:** Will Bella and Tomás meet within 30 minutes? Show or explain how you know.

5. At what time will they meet on the hill? How far from the bottom of the hill will they be when they meet? Show or explain how you know.

6. If they miss each other, and Tomás keeps walking, what time will he get to the bottom of the hill? Show or explain how you know.

7. If they miss each other what time will Bella get to the top of the hill? Show or explain how you know.

Trick or Treat – Individual Report **Name________________________**

1. What are the advantages and disadvantages of using the tables to answer the questions about Bella and Tomás? Be specific.

2. What were the advantages and disadvantages of using the graph to answer questions about Bella and Tomás? Be specific!

3. What are some other ways that you might solve this problem without using the table or the graph? Be specific!

Appendix B: Sample Classwork Assignment
You Don't Need a Lot (for Linear Equations)

For each table, write the equation. Show work for credit

1.

x	y
-2	4
-4	1
-6	-2
-8	-5

Equation: __________________

2.

x	y
1	-3
3	-1
5	1
7	3

Equation: __________________

3.

x	y
4	5
-4	-1

Equation: __________________

4. Find the missing information for each. Show all work.

Starting Point: (0, 3)

Rate of Change: ______

x	y
1	5

Equation: _________________

5. Find the missing information for each. Show all work.

Starting Point: ________

Rate of Change: ___½___

Equation: _______________

x	y
2	3

6. Lucero read two points off a graph: (3, 1) and (6, 6) Use these to fill in the missing information. Show all work.

Starting Point: _________

Rate of Change: ________

Equation: ________________

7. Find the missing information for each. Show all work.

Point on the Graph: (8, -4)

Starting Point: _________

Rate of Change: ¼

Equation: ________________

8. Find the missing information for each. Show all work.

Point on the Graph: (5, 50)

Starting Point: (0, 25)

Rate of Change: ________

Equation: ________________

Appendix C
Equation Time!

1. At the beginning of each week Manuel is given $80. He spends $10 every day.

   He uses the equation below to calculate how much money he has left on any day of the week:

   T = 80 – 10D

   a. What do **T** and **D** mean in the situation? Define each variable.

   b. Explain why the equation makes sense for the situation. Be detailed!

   c. Make a table that shows how much money Manuel has each day of the week (5 entries).

   d. Manuel's rate of change: _____.

   e. **Circle and label the rate of change in the equation and table**.

   f. Manuel's starting point: ______.

   g. **Circle and label the starting point in the equation and table**.

   h. Using either your table or equation, show how long it will take until Manuel runs out of money. Show work!!

STOP HERE!!! Call the teacher over for a teacher check-in.

Everyone must be ready to show and explain the work above!

2. Delina was given $20 dollars to start a savings account at the local bank. Each week she adds $5 of her left over lunch money to her account. She uses the equation below to calculate her total money:

$$\mathbf{T = 20 + 5W}$$

a. What do T and W mean in this situation? Define each variable.

b. Make a table that shows how much money Delina has each week (5 entries).

c. Delina's rate of change: _____. Circle and label her rate of change in the equation and table.

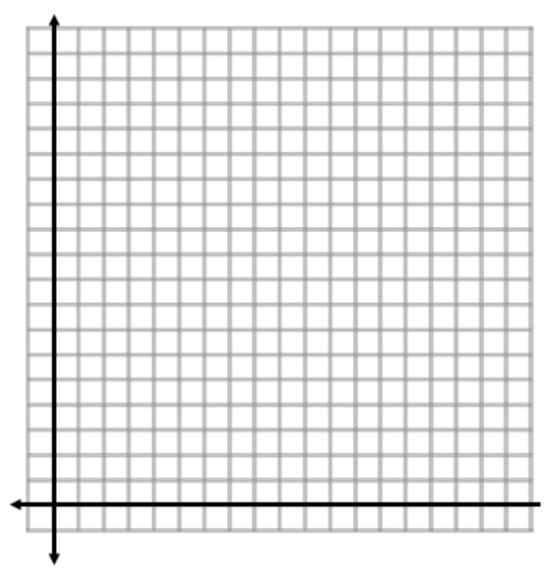
d. Delina's starting point: ______. Circle and label her starting point in the equation and table.

e. Use the equation to find out how much money will she have after 1 year (52 weeks). Show work below.

3. Kay the crazy math teacher collects a lot of things related to school. Over the years she has collected 424 staplers and she thinks she should get rid of some. She is donating 6 staplers per day to local schools. She uses the equation below to calculate how many staplers she has left after a certain number of days.

$$424 - 6D = S$$

a. What do D and S mean in the situation?



b. Make a table that shows the number of days and staplers she has left.

c. Create the graph for the situation. Make sure it fits nicely on the graph provided.

Appendix D: Sample Groupworthy Task
The Garden Border

Kiara is planning to make a garden. She wants the garden to be a square, and she wants a tile border along the inside boundary of the garden.

Other key information:

- The tiles she wants to use are each 1 foot by 1 foot.
- She only wants to use whole tiles in the border.
- The border can only be one tile wide or it will take up too much space.
- She is getting space in a local park for this project. She doesn't know how big the area will be so she needs to plan for anything.

The Task: Help Kiara figure out what happens to the number of tiles you need based on the dimensions of the garden.

Part 1:

- Draw at least 4 sample gardens. How many tiles are needed for the borders of these gardens?
- How many tiles would the border of a 100 foot by 100 foot garden have?
- Find a rule or equation for figuring out the number of tiles in the boundary of **any** square garden.

Part 2:

Although you have found at least one rule for figuring out the number of tiles in any garden, Kiara is an eccentric mathematician and insists that you figure out the number of tiles in the border of a garden by using several **different** rules. Create a diagram for *each* rule to show how you know it will calculate the number of tiles needed for any side length on the garden.

Part 3:

Organize a stand-alone summary page that explains your work. On a new sheet of paper, show your rules and use diagrams, tables or color coding to show how you know each rule works. You do not need to include your original examples unless they are helpful for showing how the rules work.

Some things to keep in mind about your summary page:

- All work should fit onto one side of a blank page
- Include a unique title
- Plan the layout so that your information is organized
- Make sure it is easy to read
- Include a diagram that shows how the rule works for the situation
- There are many correct answers to this problem so make sure another group can understand how your rules work even if they have a completely different rule.

We will do a silent presentation of each group's work. You will have a short time to look at each group's summary page and they will look at yours.

Exit Ticket

1. Write 2 math questions/comments about another group's work.
2. Describe a good and bad thing about how your group worked together on this problem.