A Comprehensive Guide for Learning Cello Vibrato

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Abstract

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Vibrato has become such a critical technique in today's instrumental performance that it is surprising to find it only lightly recorded in pedagogical literature, especially with respects to systematic teaching. The learning process for vibrato often comes verbally from teacher to student and is treated as the learning of a mechanism of expression. This study presents a comprehensive learning guide for vibrato on the cello. There are a number of different characteristics in vibrato technique, and based on these characteristics I demonstrate relevant exercises and excerpts for learning and practicing vibrato. I also provide a collection of cello excerpts as different applications of vibrato in performance scenarios. This study will analyze recording excerpts by renown cellists in performance environments using digital signal processing techniques to visualize and discuss the vibrato in selected passages. The goal of this study is to document a systematic method for educators to help students achieve musical expression through a thorough understanding of vibrato.

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DEDICATION

This dissertation is dedicated to my dearest husband Yilun Cui; your endless love, support, and encouragement was the key to my success for my study. You are the one who always stood by my side, the one who cheered me up and truly believed in me through my journey. Your unconditional support helped me to achieve my goals. You are not only the love of my life, you are also my mentor who has taught me so much, I can always learn new things from you. You make me become a better version of myself. To my parents, you chose the best instrument for me to learn as a little girl and gave me the opportunity to leave home and travel to Beijing, Pittsburgh and Seattle in order to pursue my studies and dream. I feel so lucky and thankful that I can make something that I love (the Cello) as my career, I could not have gotten here without your love.

INTRODUCTION

Vibrato is an expression in music performance that has the effect of varying the musical tone. The use of vibrato is intended to add depth to a note. Vibrato can produce a sound that is rich, subtle, colorful or simple. Emotional range, stylistic variety, or cultural influence are factors that affect the use of vibrato.

Nowadays, vibrato is not only an essential technique, but it is also used carefully. Fine musicians are conscious about where it is applied, and they care more about how the piece should sound according to historical performance practice, and how the piece was conceived by the composer. As musicians' skills improve and the audience's aesthetic tastes evolve, musicians have become more careful in using vibrato.

Vibrato is a twentieth-century phenomenon, in that it was used almost continuously–especially by singers and string instrumentalists–in performances. Before the nineteenth century, vibrato was introduced much more selectively than it is today (Neumann, 1991). Vibrato has slightly different definitions in different eras. Vibrato has been in use since at least the sixteenth century when it was treated as an ornamentation. Vibrato evolved from *tremolo, ondulazione, tremolio, bebung, schwebung, tremblement, tremblement serré, balancement, ondulation, flatté, close shake*, and other less common terms (Stowell, 1985).

The phenomenon of vibrato was first observed naturally occurring in the voice: all human voices can produce vibrato. String-and wind-instrument vibrato is an imitation of this aspect of singing. The Italian word vibrato means "to shake." A slight, and more or less rapid fluctuation of pitch for expressive purposes, the term is sometimes applied to fluctuation of intensity (Apel & Randel, 2003).

Vibrato in string instruments is commonly accepted as left-hand vibrato. However, the right hand controls the bow and can also produce the effect of vibrato. Drawing the bow

across the string– at first slowly and softly, then louder and gradually decreasing its pressure– the vibration will increase and decrease in intensity in proportion to both the pressure and the speed of the bow. The result of this greater or lesser amplitude in the oscillation of the string will be an undulation of the sound (Stowell, 1985), which we generally today called bow-vibrato.

Cello vibrato in particular is produced by rocking movements of the finger on the string with the help of the arm (Grove & Sadie, 1980), rather than with the wrist, as is done with the violin. Vibrato begins at the start of the note and continues to the end of the note with varying speed and width during the note (Spohr, 1833).

Generally, the development of cello technique has lagged somewhat behind that of the violin, but their developments have followed a similar path. In the nineteenth century, cello vibrato was less discussed than violin vibrato and was less systematic. Nonetheless, vibrato still achieves enough attention. Violinist Louis Sophr standardized four types of vibrato: intense and fast, slower and less intense, increasing in intensity through the note, and decreasing in intensity through the note. Dresden School cellists published cello methods and made advancements in cello technique that included the vibrato; their efforts made a large contribution to vibrato's popularity.

In the late eighteenth century and early nineteenth century, vibrato was a special means of conveying in performance the character and expression of a composition. Vibrato, sometimes indicated by the symbol but was generally freely added by the performer. It was normally employed somewhat sparingly as an expressive ornament during the period (Stowell, 1985). Information regarding the use of vibrato by eighteenth-century cellists is sparse. The terms *close shake* or *tremolo* were used in the eighteenth century to represent vibrato. Vibrato was used in very selective ways; then violinist Geminiani wrote how cellist Romberg applied vibrato in his *Treatise of Good Taste in the Art of Musick* in 1740, vibrato should apply only to the beginning of select notes (Walden, 2004). Vibrato was not accepted by all musicians in that period, some thought it made a distasteful sound.

Before the eighteenth century, there was no clear definition for vibrato. However, vibrato

was used and seriously considered an expressive effect. Leopold Mozart (1756) provides an indication of the state of vibrato in string playing at the end of the baroque period, conceding in his *Versuch einer gründlichen Violinschule* that "there are performers who tremble consistently on each note as if they had the permanent fever". He thus condemned the practice and suggested instead that vibrato should be used only on sustained notes and at the ends of phrases when used as an ornament. Therefore, the occasional use of vibrato does not mean it was used sparingly.

The Importance of Systematic Instruction

The process of learning an instrument is substantially different from the process of learning music theory or music history. Most instrumental lessons are in the form of one-on-one tutorials, where teachers give each student individual lessons. Teachers educate students verbally and through physical demonstration. They also select and recommend repertoire, whose selection is based on a student's musical understanding and physical aptitude. Teachers usually assign specific pieces to the student and teach step by step the necessary musical techniques. Teaching strategies and approaches are derived from the teacher's own background and experiences. In fact, no instrumental teacher applies the same teaching approach to each student. The teacher may use similar curricula for beginners, such as *Suzuki Method*, *Essential Element for String Instruments* by Allen et al. However, once the student's performance skills advance, the process of learning shifts to repertoire pieces and to verbal instruction in lessons. It is not easy for students and teachers to find a systematic curriculum which just focuses on learning one technique, and that also includes a comprehensive introduction to the technique, a detailed procedure for its execution, and exercises and repertoire recommendations for students from beginner to advanced levels. This type of curriculum is what I intend to investigate in my study.

Vibrato is a technique that conveys complex personal emotions. Musicians apply vibrato to express their moods, their tastes, their personalities, and their understanding of a piece of music's "soul." Vibrato is one of the hardest techniques to learn and to use appropriately. One

reason is that vibrato is not easy to execute because there are many approaches to achieve different sounds. Another more common reason is that, while learning vibrato, students do not understand when and how to apply vibrato in various scenarios. Both reasons demand that vibrato be taught in a systematic way.

A systematic instruction is the process of breaking a skill down into individual components for students and identifying the appropriate teaching method or strategy that allows for students to fully comprehend instruction in a new skill or learning objective. In this study, I break down teaching vibrato into its historical evolution, the study of its mechanics, and the characteristic analysis of repertoire that can be recommended in its teaching. Each section has a detailed description and examples, and teachers can choose any part as a single session to guide students to learn vibrato. Students can choose their matching step to enhance their vibrato or explore more exercises from this study. Teachers have the responsibility to break skills down to help students learn, regardless of what the students are struggling with. Teaching vibrato systematically also demonstrates that any student can learn vibrato. Discovering and utilizing the power of systematic instruction can ensure that educators everywhere are helping students at every grade and level (Ascherman, 2017).

Another strength of teaching vibrato systematically is time efficiency. Learning vibrato cannot be completed in just a couple of lessons; its mastery might permeate through a lifetime of lessons. Though we can define what vibrato is technically or mechanically, a few words or sentences cannot accurately define its true nature. There are countless terms to describe vibrato, this is why vibrato has been one of the most controversial and important techniques from the sixteenth century until now. Vibrato is different from other techniques, it does not have a notation on the score, and this creates a lot of ambiguity for students. Many students take a long time to work on vibrato because they only learn the mechanical motion and therefore cannot solve problems while performing it; where and when to use vibrato could become a dilemma for students. As a result, they cannot produce the sound that they desire, or they may not even notice the vibrato that they have applied is not appropriate. A systematic learning of vibrato can greatly save time on both the teaching

and learning of vibrato. This process may very well take longer than just learning how to execute the motion. Teachers can prescribe the appropriate next step by reviewing a student's prior learning history. As long as there is a systematic way to teach, teachers can rely on it as a structure and easily identify issues where students diverge from structure to rectify accordingly.

Chapter 1

CELLO VIBRATO: HISTORICAL FOUNDATIONS

1.1 Vocal Vibrato

The phenomenon of vibrato was first observed naturally occurring in the voice. All human voices can produce vibrato. String-and-wind instrument vibrato is an imitation of this aspect of singing. In 1731, later in 1740, and again in 1751, Italian violinist, composer, and music theorist Francesco Geminiani (1687-1762) wrote in his *The Art of Playing on the Violin* "consists in giving that instrument a Tone that shall in a Manner rival the most perfect human voice (Geminiani & Boyden, 1952)."

However, Wolfgang Amadeus Mozart (1756-1791), wrote a letter to his father from Paris (June 12,1778):

> Meissner, as you know, has the bad habit of purposefully pulsating the voice, marking on a long-held note all the quarters and sometimes even the eighths — and that manner of his I have never been able to tolerate. It is truly abominable and such singing runs counter to nature. The human voice vibrates by itself, but in a way and to a degree that is beautiful — this is the nature of the voice, and one imitates it not only on wind instruments, but also on strings, and even on the clavichord but as soon as one carries it too far, it ceases to be beautiful, because it is unnatural (Neumann, 1991).

Neumann (1991) remarks the importance the several points in this passage. It confirms the presence of vibrato as being "in the nature of the voice" and it stresses its beauty, which prompted its imitation on string and wind instruments. Mozart clearly approved of this imitation; it characterizes the vibrato as a natural, spontaneous component of the

voice, setting it apart from the willful, pulsating manipulation of the voice that Mozart found objectionable and outright "abominable" when done in the bad taste of Meissner's purposeful rhythmic emphases. One thing emerges clearly: Mozart desired the vocal vibrato as well as its discreet instrumental imitations.

In the schools of singing of earlier centuries, vibrato appears to be a frequently used ornament; Italian school of singing used vibrato since the third century (Grove & Sadie, 1980). In fact, vocal vibrato is a reinforcement and weakening of the note; it is a changing of the intensity of loudness of singing. However, tremolo is reiterated departure from true intonation. Therefore, vocal vibrato is similar to string players' undulating the bow, normally known as bow vibrato; but, it is vocal tremolo that produces the same effect as rocking movement of the left hand on the string that is more generally known as string vibrato. There are various descriptions of vocal vibrato or tremolo, and one of them-a wide and uneven pitch of oscillation in the voice of the elderly-is interesting. This was described by Baroque-era well known German composer and theorist Christoph Bernard (1628-1692) (Harris, 1989):

> Elderly singers feature the tremolo, but not as an artifice. Rather it creeps in by itself, as they no longer are able to hold their voices steady. If anyone would demand further evidence of the tremolo, let him listen to such an old man employing it while singing alone. Then he will be able to judge why the tremolo is not used by the most polished singers, except in ardire[1].

However, vibrato is meant to imitate the vibrations and excited quiverings of the human voice. Furthermore, the development of vibrato occurs, as it were, parallel to that of the art of singing. The expression "*tremolo*"—which was applied to violin vibrato until the middle of the last century—was, after all, originally used also for the human voice (Hauck, 1975).

True vibrato or true tremolo (i.e. vocal vibrato)-or both-, are thought to be an effect (Donington, 1992), a special means of conveying in performance the character and expres-

[1]Ardire is defined by Bernhard as a Tremolo (sic) used by a very few on the last note of a phrase.

sion of a composition (Stowell, 1985). Musicologists Thurston Dart and Robert Donington state that vibrato may have been employed as an ornament in some singing and in much instrumental playing (Zamir, 2014). When the first string instruments appeared in Western Europe in the ninth century, expressions like *appoggiatura, syncopation, mordent, portamento* were used more often. With the natural inclination to imitate the sound of human song, it was probably obvious to add to generally accepted string-instrument ornamentation the technical imitation of vocal vibrato, i.e, by vibrato or tremolo (Hauck, 1975).

1.2 Violin Vibrato

Vibrato is discussed in violin treatises earlier than it is in violoncello treatises. The violin, being the highest-pitched member of the family of string instruments, takes on a leading role and models for itself different types of techniques, including vibrato. Through the evolution of violin vibrato, we can come to a gradual comprehension of cello vibrato.

We have confirmation that vibrato was already used on an earlier form of the violin, such as lute, with the technical direction to place the fingers steeply, in contrast to the flatter positioning that is normal to fretted instruments (Hauck, 1975). In the 15th century, the lute played an important role among the stringed instruments. Later, in 1676, Thomas Mace, an English Lutenist, called vibrato a "*sting*" and described its execution in details in his *Musick's Monument*: "Strike your note, so soon as it is struck, hold your finger stopt upon the place, letting your thumb loose and wave your hand downwards and upwards several times (Mace, 1966)." The testimony for the use of the vibrato on the lute allows us to conclude that it had long since been established on this and surely also on other stringed instruments (Hauck, 1975).

In the playing of string instruments, the term "vibrato" for the oscillation of sound has not long been in use (Spohr, 1833). In 1740, Geminiani defined the term *close shake* for the violin (Geminiani & Boyden, 1952) and explained the execution of vibrato, requiring its use "as often as possible." In 1756, Leopold Mozart (1719-1787) writes about "*Tremulo*" in his *Versuch einer gründichen Violinschule*, and keeps a similar opinion to Geminiani's. Yet,

vibrato was not modish in those days (Hauck, 1975). Later, Louis Spohr (1784-1859) writes about "Tremolo" in his 1832 *Violinschule*, warning that vibrato should not be used too often but still should be taught as a means of expression to be used from time to time. Spohr described four types of vibrato that were widely used by string instruments: intense and fast, slow and less intense, increasing in intensity through the note, and decreasing in intensity (Spohr, 1833). It is remarkable that there is nothing changed in the opinions about vibrato in the next one hundred years, since Spohr was already at the apex of his art (Hauck, 1975).

Violin schools from the nineteenth century to the beginning of the twentieth century were described as having three main branches: the German, led by Joseph Joachim; the Franco-Belgian, led by Eugène Ysaÿe, and the Russian, led by Leopold Auer. The German branch firmly advocated restrained vibrato; Joachim stated that one should "use vibrato only where the expression seems to demand it." (Philip, 2004a) Although Joachim was the most influential violinist and pedagogue of his epoch, his students, such as: Elldering, Hubay, or even the generation following them, used vibrato consistently (Hauck, 1975). The Franco-Belgian leader Ysaÿe proposed a "variable vibrato" (Philip, 2004b) that became the ideal of the generation around 1900. In the Russian branch, pedagogues encouraged pupils to develop their own styles, independent of the teacher's own preferences. For example, Auer was as extremely opposed to excessive vibrato as was Joachim. Nevertheless, Auer's pupil Heifetz played with continuous vibrato. It seems that the schools' differing views on vibrato did not last very long – no more than 50 years. In fact, vibrato style did not even pass down prominently from teachers to students. The use of vibrato was more characteristic in the twentieth century; each performer developed his or her preferred way of performing vibrato. Vibrato became less limited, however, cellists generally lagged behind violinists a bit, but they followed a similar path.

1.3 Cello Vibrato

1.3.1 In the 18th Century

Before the 18th century, the cello was considered a background accompaniment instrument, which is one of the main reasons that caused cello vibrato to lack sufficient attention to warrant discussion. However, we can find the "footprint" of cello vibrato on that instrument's precursor, the viola da gamba. By the end of the 17th century, there was ample documentation concerning the French gambists' employment of vibrato. Indeed, two distinct types were recognized:

> The originally preferred one [style of vibrato] was accomplished with two fingers, the lower firmly placed on the string, the upper pressed tightly against the lower. A shaking of the hand brought the upper finger into gentle repercussive contact with the string. The result was a trill-like oscillation producing a microtone. Such a trill-type technique was possible, however, only with the lower three fingers. When the little finger was on the string, they had to resort to the shaking of the stopping finger instead. After about 1700 Marais and other players, realizing the wider color potential of the one-finger vibrato, extended its application to the other fingers as well, without abandoning the two-finger-type (Neumann, 1991).

This source points out that vibrato was created with two fingers and was treated as an ornamentation technique, like a trill. The sonic potential of vibrato was noticed by Marais and other musicians, who noticed that vibrato could change the tone color-and could be produced by each finger. Although vibrato was rarely executed in the seventeenth century, vibrato's sound was described by another gambist, Le Sieur Danoville, who noted in his gamba treatise of 1687 that vibrato "has tenderness, and fills the ear with sad and languishing sweetness (Danoville, 1972)." In the same year, the eminent gambist Jean Rousseau-who like Danoville and Marin Marais was a student of the legendary Sainte Colombe-proclaimed in his gamba treatise that one might introduce the vibrato any place ("en toutes rencontres")

where the length of the note permits it, and that it is continued for the full extent of the tone (Green, 1974).

Somehow, in the 18th century, the view of cello vibrato was still limited, it seems not all musicians found it desirable. Scottish scholar, cellist, and flutist John Gunn (1765-1824) discussed vibrato as an old-fashioned absurdity (Gunn & Boland, 1793).

In his Division-Violist, Christopher Simpson (1659) (1610-1669) calls the vibrato a "close shake, and cleverly indicates its nature by means of a trill representation that remains within the confines of a single space on the staff." *The Grove Dictionary of Music and Musicians, Macmillan Dictionary* in 1954, also mentions vibrato as "*close shake*", Based on Simpson's definition in Grove: "close-shake is that when we shake the finger as close and near the sounding note as possible may be, touching the string with the shaking finger so softly and nicely that it make no variation of the tone. This may be used where no other grace is concerned." This concept of "*close shake*" was also defined later by Bernhard Romberg, and was almost the standard English term for vibrato until the 20th century.

1.3.2 In the 19th Century

The 19th century is the turning point of cello vibrato's development. Or, more accurately, the advancement of cello techniques accelerated the development of vibrato. In the nineteenth century, the German School, especially the Dresden School, made a significant contribution towards improving the technical execution of cello vibrato, thereby increasing its popularity.

Bernhard Romberg (1767-1841) wrote of the term "*close shake*" in his *A Complete Theoretical and Practical School for Violoncello*:

> The close shake, or Tremolo, is produced by a rapid lateral motion of the finger when pressed on the string. When used with moderation, and executed with great power of bow, it gives fire and animation to the tone, but it should be made only at the beginning of the note, and ought not to be continued throughout its whole duration (Romberg, 1880).

Romberg thought that vibrato was a respectable ornamentation in art music, and that it should not be combined with any other ornamentations, such as *doppelschlag* or *mesa di voce*. Romberg expressly said that the vibrato should begin at the start of a note but end before the note duration is over. He only used the strongest finger-the second-to execute the vibrato, which lasted for the first third of long-duration notes (Kennaway, 2016). See Figure 1.2. It is unclear why he did not put emphasis on vibrato. It could be because he disliked its excessive use, but it may also be the physical consequence of his violin left-hand pronation. See Figure 1.1.

Figure 1.1: Romberg, *Violoncelloschule*, left hand

Friedrich Dotzauer (1783-1860) was credited with being the first cellist to teach vibrato. He referred to it as left-hand tremolo and used this terminology in his teaching. He remarked that, on long sustained notes, some artists (especially Italian professors) made use of a kind

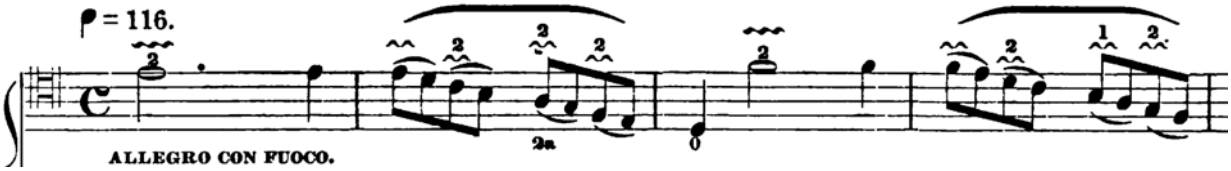


Figure 1.2: Romberg, *Violoncelloschule*, vibrato exercise, 〰 indicates vibrato

of "*trembling*" (*tremolo*) (Kennaway, 2016). Dotzauer also explains the artist can produce the same effect with a movement of the wrist that he called *ondulé*, which seems to refer to bow vibrato. In his manuscript, Dotzauer notated vibrato as made of several *son filés*[2] (Kennaway, 2016). See Figure 1.3

Figure 1.3: Dotzauer, *Violoncelloschule*, Vibrato sign

Friedrich Kummer (1797-1879) was the first cellist to write positively and in detail about vibrato. Kummer preferred to use the bow for expression, rather than vibrato. He could use any finger to vibrate, although he appears to favor the third. He uses vibrato in many places, e.g. on metrically strong beats, or on weak beats when combined with agogic lengthening. He also employs vibrato on diatonic notes much more often than chromatic notes and on longer rather than shorter notes (Kennaway, 2016). Kummer & Becker (1916) uses the same symbol as Romberg in his *Violoncelloschule* to indicate vibrato. See Figure 1.4

David Popper (1843-1913) was the turning point of vibrato for cello performance. One of his trademarks was teaching "*continuous vibrato*", with the idea that all notes should receive vibrato regardless of the duration. Despite being strongly criticized for his performance of continuous vibrato, he continued to spread this idea in his methods. His persistence played

[2] *son filé* means 'spun sound' and the term used to describe a sustained note played with the whole bow.

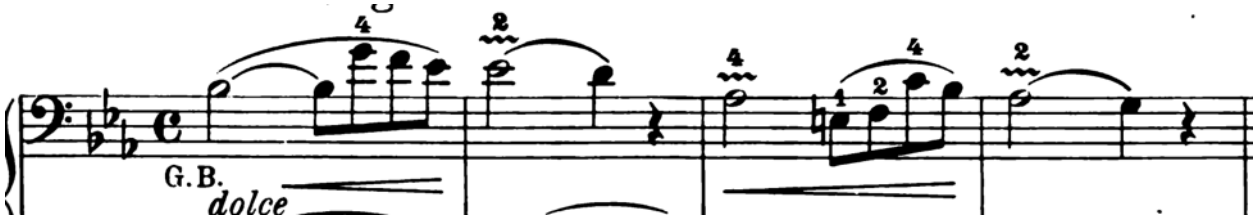


Figure 1.4: Kummer, *Violoncelloschule*, Op. 60, No. 77

a critical role in leading more cellists to accept continuous vibrato.

Vibrato started as ornamentation that needed to be used conservatively and transformed into a technique most pedagogues acknowledged, taught, and used-despite criticism-thereby giving it strong roots for further development in the 20th century. Vibrato's technical advancement and increasing popularity greatly stimulated cello's position as a solo instrument.

1.3.3 In the 20th Century

Having gained wide acceptance from cellists in the 19th century, vibrato became vogue for almost half of the 20th century. Musicians and pedagogues default to vibrato as an essential tool for enriching sound; performers and audiences started to expect performances with vibrato. Thus, vibrato became a new aesthetic. With the advancement of technology, we have greater resources (e.g. film, sound recording) by which we can observe how musicians perform and consider vibrato.

Pablo Casals (1876-1973) was one of the leading cellists in the early 20th century; his worldwide reputation for his cello playing earned him a large number of followers and fans. The way he applied vibrato thus affected many cellists. Casals said that "vibrato in itself cannot be expressive, because that depends on how it is applied. The vibrato is a means of expressing sensitivity, but it is not a proof of it." (Blum et al., 1980) Contrary to his predecessors, Casals was not against using vibrato, nor did he advocate for using it consistently. Instead, he cared more about how to apply vibrato, claiming: "... when you hear all the time a beautiful vibrato – well, you've had enough!... but the sound without vibrato is very

beautiful also, particularly in piano and pianissimo." (Blum et al., 1980) Casals absorbed vibrato as a means of helping musicians to express their interpretation of music of past ears, but he was very careful in applying it. He suggested that his students play without vibrato when they perform sonatas, because piano is an instrument that does not have vibrato. He also suggested the spare use of vibrato whenever the cellist plays an accompanying part (Blum et al., 1980).

Vibrato speed was discussed more often than its width, and Casals was probably the first cellist who considered not only vibrato width but also the direction of its oscillation. He advised cellists that the vibrating finger should move from the sharp side of pitch because he thought oscillating both sharp and flat of the in-tune note would create a vibrato that is both too wide and disagreeable to the ear (Blum et al., 1980).

The Armenian cellist Diran Alexanian (1881-1954) wrote a comprehensive work on cello vibrato in details that include its execution, duration, and emotional demands and gave clear examples to demonstrate the usage of vibrato. In his 1922 *Theoretical and Practical Treatise of the Violoncello*, he described vibrato as "one of the most active factors of the 'fullness' of tone color." Alexanian et al. (1922) demonstrated how to apply vibrato to different dynamics in both words and notation:

> For weak sounds the vibrato should be spaced and supple. For full sounds the vibrato should, on the contrary, be rapid and nervous... In a "piano" a rapid tremor would not fail to give an impression of feverishness. A "forte", on the other hand would appear weak and nerveless if played with a slow undulation (Alexanian et al., 1922).

Alexanian also thought that vibrato was discouraged in the old schools of performance because of its tendency to create poor intonation (Kennaway, 2016). He explained the direction of the vibrato thus:

> Every note attracted by another note, should be played vibrato in the interval that separates it from the note by which it is attracted.

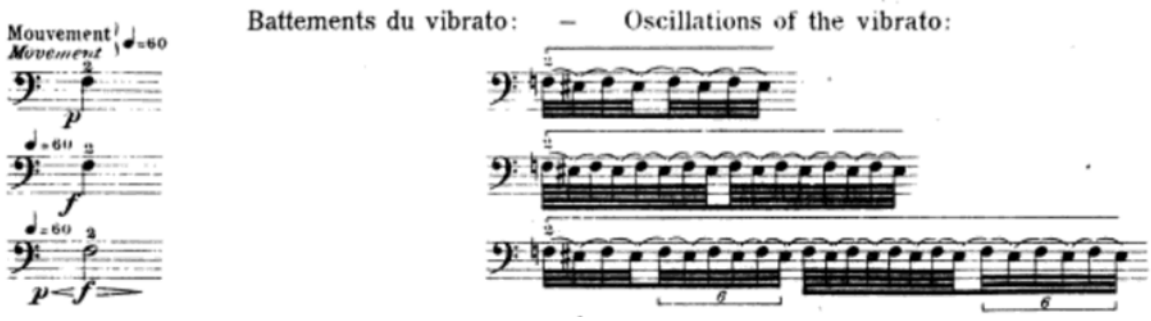


Figure 1.5: Alexanian, *Theoretical and Practical Treatise of the Violoncello*, Vibrato on different dynamics

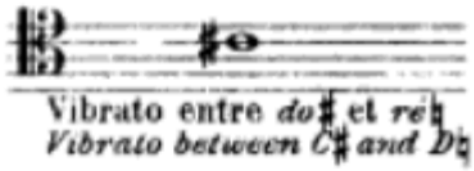


Figure 1.6: Alexanian, *Theoretical and Practical Treatise of the Violoncello*, Vibrato direction

If the vibrating note is Db, the vibrato should move from Db towards to C; this "attraction" is similar to Casals' "expressive intonation", in which leading notes and major thirds are sharpened, and minor sevenths and perfect fourths flattened, adjusting them in the direction in which they are conventionally resolved (Kennaway, 2016).

Alexanian thereby effected a smooth transition from the 19th century to the 20th century for learning of vibrato. His method brought vibrato to a deeper discussion: technically, vibrato's direction has a theoretical basis; musically, the number of vibrato oscillations directly affects both dynamics and the musical expression. Vibrato was no longer a questionable technique that discussed whether to apply or not; instead, it became more acceptable and more widely used than in the 19th century.

Since the 20th century, there are more cello pedagogues writing about the physics motion of cello vibrato. After more than 100 years, it seems that cellists are not only learning vibrato

from their teachers but also creating effective approaches for improving vibrato execution-and providing scientific bases in proof of their theories.

Later, Maurice Eisenberg (1900-1972), a student of Casals, provided further contribution on vibrato execution and application. Eisenberg thought vibrato should be taught when the student has the desire to learn, and he would be guided by the student's innate desire to beautify his or her tone (Applebaum & Applebaum, 1972). Eisenberg (1966) instructed executing the vibrato as follows: the cellist should use the entire elbow joint to the fingertip, in one unit, unbroken at the wrist, vibrating on the ball of the fingers with the motion parallel to the string. He further noted that the wrist should be on the same plane as the elbow joint. Eisenberg also talks about the thumb, which should vibrate with the other fingers during the vibrating motion so that the thumb is prepared at all times to give sympathetic support to the vibrating finger (Applebaum & Applebaum, 1972).

After establishing these fundamentals, Eisenberg let his students develop all possibilities of vibrato. However, he gave some recommendations for applying vibrato: "in a crescendo on a long note, the vibrato quickens; the intensity becomes greater until it reaches its maximum height. Also there is a greater finger pressure. But it is important that the oscillation becomes narrower." In this passage, Eisenberg (1966) includes two main elements of vibrato: speed and width. He also cites two ways to increase the dynamic or intensity, either by speeding up the vibrato or by narrowing its width. These are the most common applications of vibrato that we are still using today.

Unlike its execution in the 19th century, cello vibrato is not executed in a similar manner to violin vibrato, which is acquired by rotating the wrist. Most cellists assumed that cello vibrato involves the entire left forearm, in a motion that consists of the forearm, wrist, hand and finger. This view is also confirmed in Louis Potter, Jr's textbook-method *The Art of Cello Playing*. Potter (1980) emphasizes that the wrist (or "violinistic") vibrato should be avoided in the early development of vibrato, as this kind of vibrato might develop into an automatic and regular movement.

Potter (1980) suggested that students should start to practice vibrato in the fourth po-

sition, with the thumb at the crook of the neck of the cello, where it acts as an axis for the vibrato motion. He thinks the fourth position provides early-stage students a secure and relaxed location to start the vibrato. This is a new point that differs from any other pedagogues before. Generally speaking, cello teachers prefer to teach students vibrato from the lower positions. Most start with the first position because they think students will feel most familiar and confident in performing. However, fourth or higher positions-including thumb position would increase the difficulty of learning vibrato at the beginning.

He also encouraged students to vibrate with the thumb that it is acceptable to lift the thumb slightly off the string but not acceptable to move the thumb towards the side of the fingerboard. Potter emphasized that the left-hand shape matters while applying vibrato, and that one should try to keep all the fingers as down as is practical. He thought that if any finger were flattened or lifted too much, it would cause bad intonation. He also felt keeping fingers properly spaced would avoid unnecessary tension.

Potter points out that the emotional setting, range of the musical passage, and temperament and artistic taste of the performer are factors that influence the vibrato's speed and width (Potter, 1980). A well-developed vibrato should be performed while considering those elements. The vibrato should emerge as a personal representation of the performer's style.

Gerhard Mental's *Cello Technique: Principles and Forms of Movement* focuses on what actually happens when one plays an instrument. He uses physical and scientific theories to explain how to execute vibrato, and his theory thus attempts to help cellists to understand what happens when they play a certain passage, whether perfectly or poorly. Mental claims that cello vibrato is created by changing the pitch and is different from wind instrument vibrato, which is created by changing the volume of the tone. He proposes the cello vibrato is a double-lever movement:

> [it] is created if one shakes a match box up and down to find out whether there are any matches left. The arm is bent, but the wrist had to be kept rigid so that the shaking movement will extend all the way to the hand. The elbow moves

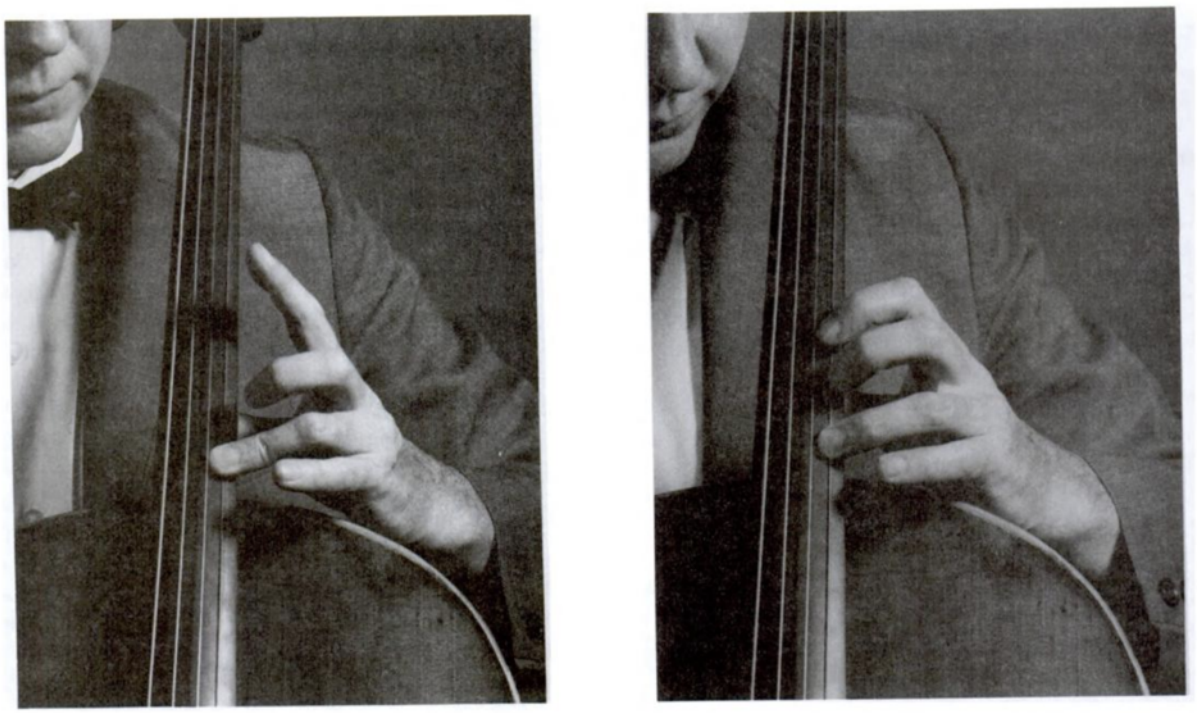


Figure 1.7: Picture from Louis Potter, Jr. *The Art of Cello Playing.* Incorrect gesture on the left, correct gesture on the right.

> in the opposite direction to the hand. This means that there is a point on the forearm, about 6 centimeters (3 inches) below the elbow, that does not move. Imagine an axis running from that point to the shoulder. On one side of the line, the upper arm and the remaining part of the forearm rotate, and on the other side the forearm and hand rotate. This movement does not run exactly parallel to the fingerboard, but at a small acute angle to it. The main direction of the hand in any case is more or less parallel to the fingerboard (Mantel & Thiem, 1995).

Mantel & Thiem (1995) asserts that the listener will hear the exact middle pitch between the extreme pitches inflections of the vibrato, Mental's opinion of perceived vibrato pitch echoes the results of Geringer's later experiment chapter 3. Both agree that vibrato should

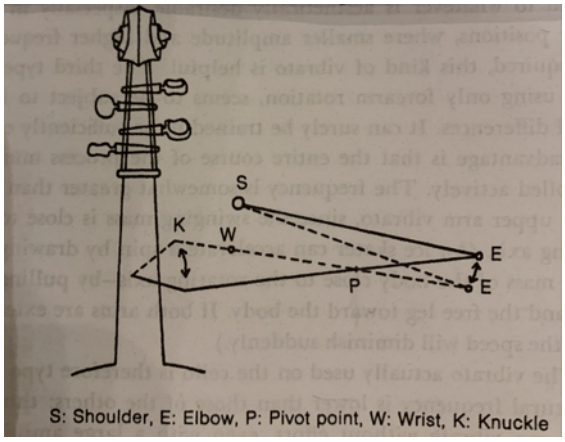


Figure 1.8: Mental, *Cello Technique: Principles and Forms of Movement*

oscillate around the in-tune note.

Some musicians might think Mental's vibrato technique is overly analytical because they prefer to relate to music and their instrument more emotionally; they do not think about how to move their joints while performing. Mental explains that he also does not care how each joint moves when he is playing, but if he needs to teach, he will want students to understand the mechanism of cello playing, rather than relying on vague analogies (Janof, 2000). Most of the learning process for an instrument is simply passed by experience from teacher to student, but Mental's description of cello vibrato helps musicians understand what they are doing. His instruction improves the quality of instrument learning and reminds music teachers to comprehend the mechanism of a technique before they teach and pay more attention to the actual facts.

1.4 Discussion of Vibrato in the 21st Century

Opinions on vibrato are countless; they exist not only between different instruments but also between periods. During the development of vibrato from the early music era to the 20th century, musicians are concerned with its execution. However, since this technique is much more mature now, musicians are more open to accept different ways of understanding vibrato.

However, people may begin to wonder whether performers still like continuous vibrato. They may wonder if it needs to be applied all the time, or if music can be performed without it.

In 2008, British conductor Roger Norrington conducted Elgar's "Pomp and Circumstance" March No. 1 of Britain's premier music festival, the Proms, without vibrato. This move shocked musicians, and columnist Stephen Pollard wrote in The Times of London that "Elgar without vibrato is the musical equivalent of dead roses." Norrington violated a "rule" of Elgar's music, which is that a fuller sound could be produced by slight vibrato. He thought that, in Elgar's day, vibrato would not have been used as much (Wakin, 2008).

Norrington's action was obviously provocative and challenged traditional classical music fans. Audiences had accepted that performances with vibrato were the norm and based on a tradition of more than two hundred years. Norrington's reversal of this norm caused extreme discomfort for them.

Later in 2010, Norrington released a controversial recording of Mahler's Ninth Symphony with Stuttgart Radio Symphony Orchestra, with the requested musicians to perform without continuous vibrato. Norrington's decision was based on an article that he had published in The New York Times in which he proposed that vibrato did not become common in European and American orchestras until 1930s. The Dallas Morning News reviewed this recording and confirmed Norrington's view of vibrato: even though some solo violinists more or less introduced continuous vibrato in orchestral performances in the early 20th century, such vibrato was not widespread until 1930s. This is evidenced by the string section of the conservative Vienna Philharmonic, which worked most closely with Mahler: until after World War II, it performed with only subtle vibrato on long notes (DallasNewsAdministrator, 2010).

Norrington's relatively vibrato-less performance was criticized by many scholars and critics. Tomoyuki Sawado listed a series of orchestra films to prove that continuous vibrato was extensively used by the New York Philharmonic in 1926, and 1931, and by the Vienna Philharmonic in 1933-1935 (Sawado, 2014).

Norrington's interpretation of Mahler's Nine Symphony is an interesting example of dis-

cussion of vibrato in the 21st century. The debate did not reach a conclusion; there is no evidence to show that Mahler's symphonic works must either be played with or without vibrato. Nevertheless, vibrato seems to garner attention no matter what the period, and people never exhaust their interest to explore and experiment with it.

I reinforce this point because I think this could bring us to think further of the application of vibrato. Vibrato seems like a fashion icon: when it first arose, people were curious about it and used it cautiously; when its technique improved, people accepted it and widely used it; when it became overused, people got tired of it and demanded less of it or even to get rid of it. There is not an absolutely "right" answer to tell musicians when and how to apply vibrato. As an educator, we should not only teach students skills, but also give students freedom to explore greater possibilities in music. As a musician, I think we should not set a boundary of excessive use of vibrato or abandon it completely. We should be sensitive about the style of music that we perform; we should understand the composer's original intention; we should combine the current audience's music tastes with our own understanding to produce a high-quality interpretation.

Beyond the consideration of whether to use vibrato, there is scientific research on vibrato that has been conducted by musical scholars and pedagogues. Beginning in 2000, John Geringer, Rebecca MacLeod, and later Michael Allen started investigations on the pitch center of vibrato. Their research brought new insights by using new technologies such as digital recording, slow-motion video, and high-resolution pitch analysis software (Ealovega, 2017) (more details will be introduced in chapter 3 on pitch center). Their research results objectively present how participants performed vibrato, in diametrical opposition to pioneer violinist Ivan Galamian's subjective conjecture.

The vital point of the newer research is not only to objectively present us the different nuances of vibrato, but also to inspire us to "take advantage" of technology. In the 21st century, high technology brings us convenience and can help us improve musical techniques. In chapter 4, I use digital recording and audio signal processing to analyze vibrato speed and width. Hopefully, this small step will also help more musicians to learn about vibrato

and apply it appropriately.

Chapter 2

VIBRATO MECHANICS AND EXERCISES

2.1 Student Development and Introduction of Vibrato Technique

Music professionals and teachers would agree that there is a proper timing for teaching vibrato. Many would agree that "learning vibrato signals that [students] have begun to move out of the beginning student category onto the path that leads to artistry." (Lamb, 1984) This is a natural course of learning in string pedagogy. Technically speaking, pupils should be able to smoothly shift into different positions, with the left hand working fluidly as a unit with the forearm. It seems like learning vibrato requires students to have these prerequisites, which help them to execute vibrato more easily.

However, some students, even if they are not yet skillful at playing an instrument, can still feel emotions from the music they hear; they observe their teachers' playing, they notice the teacher is shaking his or her hand and is making a wavering sound. They may develop strong aspirations to learn vibrato. Lack of fundamental techniques should not be a barrier for a student to express his or her true musical talent. In this study, I will provide systematic guidance by which students of different stages can learn vibrato; both teachers and students can choose the relevant exercises to practice. I will divide student progression into these three stages:

- Stage A: Students with no musical background
- Students who posesses the prerequisite skills and mechasics to start learning vibrato
- Students who know how to perform vibrato, but not appropriately

Stage A students are beginners; they observe that the teacher is oscillating with the

left hand and wonder why the teacher is making that motion. Stage B students can hear the differences in sound between vibrato and *senza* (without) vibrato; their left hand works fluidly; they intend to use vibrato to express their emotion. Stage C students know how to execute vibrato already, but their vibrato is either too slow or too fast, or they cannot perform a continuous vibrato. However, they have the ability to identify what quality vibrato sounds like. The games introduced below are premature for Stage A students, but Stage B and C students can benefit from them; Stage B students should take the exercises in order, but Stage C students can decide which games they need the most.

2.2 Left Hand

Today, most string instrumentalists consider vibrato as primarily a left-hand technique. In fact, vibrato can be achieved with both the left and the right hand by applying different techniques. As previously mentioned, left-hand vibrato is more emphasized than is right-hand vibrato, I will start by discussing aspects of the left-hand vibrato technique, starting with hand shape.

2.2.1 Hand Shape

There are two basic hand shapes on the cello, square hand shape and slanted hand shape.

Square Hand Shape

Square hand shape requires the left hand's fingers to be perpendicular to the strings. The first finger should be located about three inches below the saddle (nut) of the cello fingerboard, with the fingers equally spaced and at right angles to the strings in a relatively "square" position (Potter, 1980). (See Figure 2.1) In the nineteenth century, a "violinistic" left hand with the four fingers extremely slanted backwards was adopted by some maestros, such as Romberg (1767-1841) (See Figure 1.1), Lindley (1776-1855), and Vaslin (1794-1889) (Kennaway, 2016). Yet, the "violinistic" hand shape did not last very long because it limits hand

movement; it made executing vibrato especially difficult. Therefore, the square hand shape, as taught by Kummer, Dotzauer, and Gunn, has become widely accepted. In particular, Gunn states that the square hand shape is vital for good tone:

> [Square hand shape] is essential in order to produce a good sound to press them [the fingers] firmly on the string, and at the tip (Kennaway, 2016).

Square hand shape provides a quality tone and ensures stability of intonation. When performing vibrato with this hand shape, the left hand can rotate more, but it will have the adverse effect of making the music sound slower and wobblier. Pablo Casals, Leonardo Rose, Pierre Fournier used this hand shape.

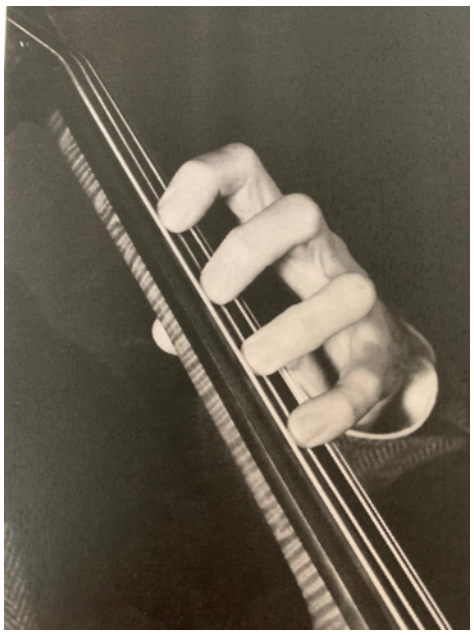


Figure 2.1: Louis Potter, Jr, *The Art of Cello Playing*, Square left hand shape

Slanted Hand Shape

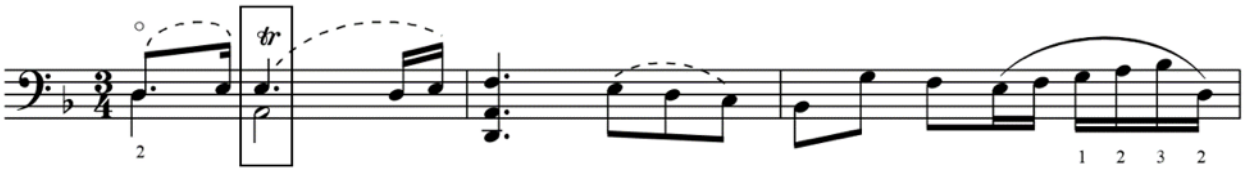
Slanted hand shape requires the left hand to be placed at an angle to the fingerboard; (See Figure 2.2) it differs from Figure 1.1 in that left hand is slanted but in a less extreme way

than in Romberg's depiction. The hand should not hold the neck so that the thumb is behind the middle finger; the palm of the hand should not be pressed in close to the neck but should be kept hollow (Kennaway, 2016). Becker wrote about the hand shape in detail:

> It should be noted that the thumb should exert an opposing pressure (Kummer speaks of a "fulcrum") in a diagonal direction; more specifically: with the fingering on the A and D strings the thumb lies more on the inner part of the neck; with the fingering on the G and C strings, however, more to the outside. The hand is correctly positioned if the channel created by the finger placement runs across the fingertips (Kennaway, 2016).

The slanted hand shape makes it easier to vibrate faster. Unlike the square hand shape, which has to change angles while playing in different registers, the slanted hand shape can be kept when shifting from lower to higher registers. With the slanted hand shape, vibrato is more of a forearm "pumping" motion parallel to the fingerboard. Piatigorsky, Starker, and Feuermann used this hand shape.

There are two exceptions while performing with slanted hand shape: performers should adjust the hand angle when playing double stops or when playing chords. Performers should make the hand shape flexible to allow for proper intonation. The examples below indicate instances in which hand shape changes to accommodate correct intonation; Excerpt 2.1 shows an excerpt from the Sarabande from Bach's Cello Suite No.2 (double stops), and Excerpt 2.2 provides an excerpt from the first movement of Elgar's Cello Concerto (chords).



Excerpt 2.1: Bach, Cello Suite No. 2, Sarabande

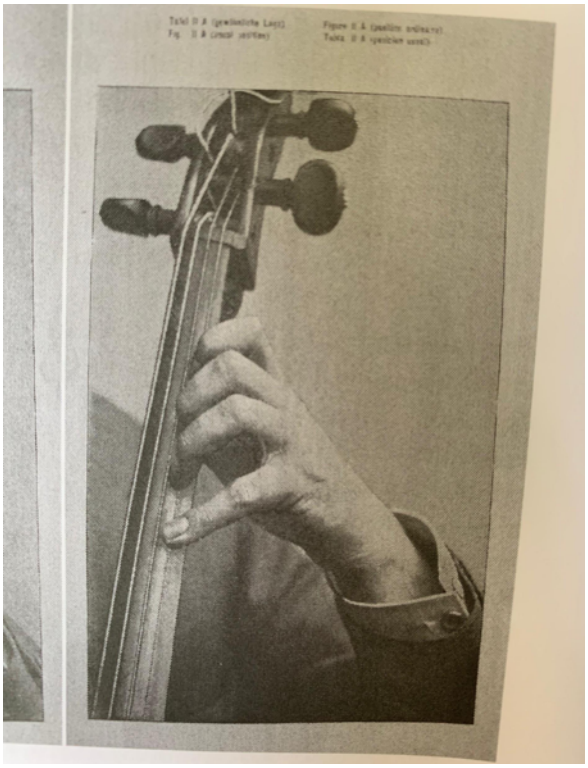
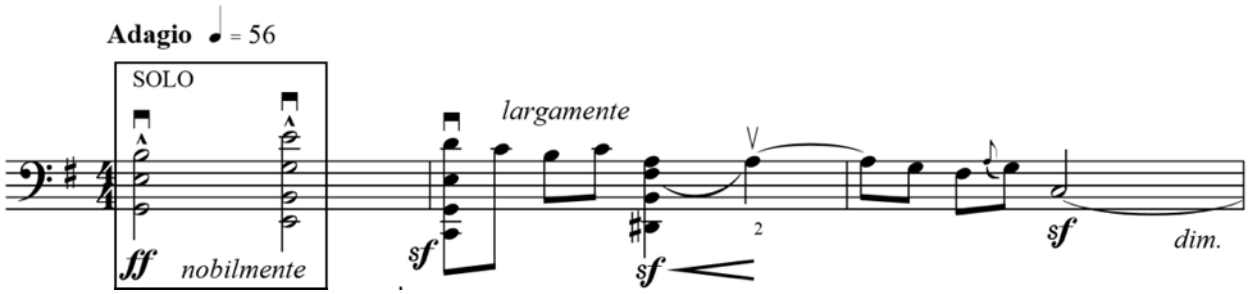


Figure 2.2: Kummer (rev. Becker), *Violoncelloschule*, Becker's slanted left hand

Applying either hand shape is feasible; each of the hand shapes has been demonstrated to be effective by well-known cellists. These two hand shapes each have their advantages and disadvantages. Stage A students are recommended to use the slanted hand shape, since they can easily execute the same angle while shifting positions on the fingerboard. Stage B and C students can keep their original hand shape, whether square or slanted; teachers should make pupils aware of that, whatever handshape the student uses, exceptions to that hand shape may be needed to preserve correct intonation. Nevertheless, a consistent hand shape is recommended because it helps maintain a consistent ease of execution.



Excerpt 2.2: Elgar, Cello Concerto in E minor, Op.85, 1st movement

2.3 Right Hand

2.3.1 Bow Vibrato

Contrary to the left-hand vibrato, bow vibrato is less discussed; however, we can still find some traces of it on the historical stage. Before the 19th century, there were various ways of creating an undulating sound; hence, bow vibrato had different names: *ondeggiamento*, *bariolage*, *coup d'archet* and *ondulé*. Accentuation was perhaps easily associated bow vibrato in the earlier nineteenth century. Generally, we commonly call bow vibrato *ondulé*; it is another form of vibrato and is a special effect as well. French composer and violinist Pierre Baillot (1771-1842) described *ondulé* is a bowing technique in which an undulation, or swell of sound, is created by fluctuation in bow pressure (Walden, 2004). In his method, Baillot et al. (1804) describes the technique as follows:

> The bowing, which is called Ondulé and which is indicated with this other sign "〰" is one composed of many swelled notes, every one of which should have its loudest part at the beginning of each half beat (Baillot et al., 1804)

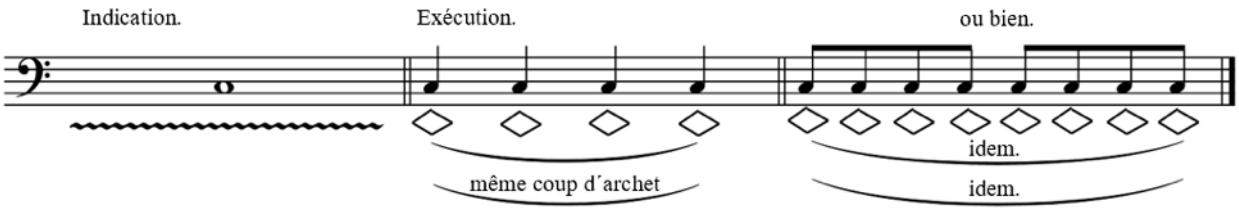


Figure 2.3: Baillot *ondulé*

Though there are very few cellists who discuss bow vibrato. Fredrick Crouch briefly mentioned bow vibrato and suggested that bow vibrato should be used at the beginning of each beat or half-beat of a long note:

> There is another species of bowing, called Undulating. It is a compound of several notes each being soft increased and diminished or to which may be given the forte, at the commencement of each beat, or half beat in the bar (Kennaway, 2016).

Both Baillot and Crouch advocate that bow vibrato should be played in a loud part of a passage and start from the beginning of each half beat.

Another cellist, Dotzauer, gives his thoughts about bow vibrato. He thinks bow vibrato should produce the same effect as left-hand vibrato but by a movement of the (right) wrist, which is indicated by this sign ⋄⋄⋄⋄. See Figure 1.3 (Kennaway, 2016).

In string playing, *ondulé* is executed by moving the bow back and forth across two or more adjacent strings. The player achieves this with an up-and-down motion of the wrist (Grove & Sadie, 1980). *Ondulé* requires a free arm and a flexible wrist; it is like a pitch tremolo across two adjacent strings. Actually, in modern practice, bow vibrato more typically occurs on one string. There are two ways one can achieve the string vibration: through increased pull or push, and through increased pressure. These two motions are based on the correct order of the transmission of impulse when moving the arm, which includes shoulder, upper arm, forearm, wrist and fingers.

2.4 Hand Techniques and Games

2.4.1 Left Hand Execution

Large to Small Motion

The path of learning vibrato progresses from larger to smaller motions. At the early stages, the student's vibrato should be wide and free, including the arm, forearm, hand and finger; the motion should not be restricted to any particular part of the left arm. In the September 1999 issue of *The Strad*, cello pedagogue Phyllis Young (2017) proposed a Michelangelo approach to vibrato:

> To create his sculptures, Michelangelo would select a large chunk of material and gradually refine it into a work of art. Although we string players start with nothing but our tools and are required to create our own building material (tone), we too need plenty of it to mould and colour. In that sense we are builders as well as sculptors (Young, 2017).

From Young's metaphor, we can infer that the "large chunk" is our left arm, which includes all the joints and parts; as we start to shake this large "chunk," it is wide and free, creating a large motion. Students of all stages can execute this. Once the arm get used to this shaking motion, we can gradually decrease the length of the sliding motion, the length of the sliding motion to perceived pitch (small motion) and change the amplitude or speed to achieve the beautiful tone that we expect. In this way, the large chunk becomes a work of art.

Young's Michelangelo approach supports the idea of learning by progressing from larger to smaller motions. This is also corresponding to another cello pedagouge-Margaret Rowell's basic principle- "*the whole before parts* (Young, 1985)." She believed that when students feel the whole range of cello, they will advance much more quickly. Both Young and Rowell's teaching principles show that learning any technique on the cello should start from a larger motion so that the students can get a main structure. Later, adding smaller details to make the learning process more effective.

Another approach that I will introduce in this study is the Orff Approach[1], which helps students learn vibrato by doing games for large motions and small motions. I believe that all concepts are learned by doing (Shamrock, 1997); in the Orff Approach, students are encouraged to learn music by experiencing, participating, imitating, and creating. Vibrato is not a simple technique that students can learn and apply to their performances after just a couple of lessons. The goal of using the Orff Approach is that teachers can create a relaxed studying atmosphere so that students of different stages and age ranges can gradually achieve proficiency at vibrato.

Large Motion Game: Ski Jumping

This game will help students to feel the left arm, forearm, wrist, hand, and the playing finger as free; all joints will be working together as one unit, and students should not perceive tension when doing the ski jump. The purpose of this game is to help students to oscillate the left hand fluently. This game works for all stages of students.

Ski jumping motion: Without using the bow, place one finger of the left hand on one string. Imagine the string is a ski jumping hill, the finger is the ski jumper, and the edge of the fingerboard (bridge-ward) is the jumping point. The finger should start at any lower position as the start point; begin to move the finger from the nut-ward to the bridge-ward, then the playing finger will pizzicato when the finger almost arrives at the edge of the fingerboard (toward the bridge).

In the ski jumping game, the left elbow will be voluntarily raised when the finger moves from the low register to the high register; the left thumb naturally lifts from the neck. This motion develops the left-hand's range of movement and freedom; the jumping hill length can be changed depending on how wide the student wants to vibrate. Students can practice on each individual finger and transform the balance from the arm to the finger without stopping

[1] *Orff-Schulwork* can be described as a model for the design of learning experiences. the goal is development of individuals who are comfortable with active music making – they can sing, move, play instruments, use speech in rhythmic and dramatic context, improvise simply in all of these areas and combine materials into original forms (Shamrock, 1997)

the motion of the left arm. For such large motion practice, students do not need to strive for a specific range of motion; the "jumping" motion could be any length, short or long. Moreover, this practice doesn't require any additional prerequisites.

Small Motion Game: Polish the String

Polish the string is a smaller-motion version of ski jumping. Ski jumping requires the finger to move from the low register to the high register, but the game of polishing the string suggests that students select a smaller length of one string. For instance, a student could move from first position to fourth position (low register), from fourth position to thumb position (low to high register), or between thumb positions (high register). The game requires students to have a sense of direction on the fingerboard. Stage A students are recommended to polish the string in the lower register; stage B and C students can be introduced to the higher positions mentioned earlier.

The motion for polishing the string is as follows: without using the bow, place one finger of the left hand on one string. Imagine that the fingertip is a cloth that has to clean the dust on the string, and repeat the motion of polishing the string. Students can try different speeds to polish (vibrate) the string; reducing the polishing range is recommended after the student feels relaxed and can naturally make the polishing motion.

Polish the string is a game that all students can easily execute because the wiping motion it employs is familiar to everyone. This game helps students relax the playing finger and thumb; if the hand is tight or very tense on the string, the wiping motion will not happen or will get difficult. After students understand the string polishing motion, they can decrease the wiping distance to make it as small as possible; then they polish the string at faster or slower speeds.

The small motion can help students achieve vibrato immediately. It not only helps stage A students, but also solve the problems that stage C students have if they do vibrato too fast or too tightly. However, the polishing motion just a simplification of the vibrato process. It does not limit the speed of the oscillation; it's just a tool for students to express their intent

for music.

2.4.2 Right Hand Execution

Bow vibrato is executed by resting the bow on one string at a normal (perpendicular) angle. As the bow moves, a very light movement is made with the right hand index finger to produce a light rotation of the frog. The bow is drawn very slowly, and the rotation is added by gradually increasing the movements of the fingers to create an "undulating" pulse in the dynamic of the note to which the vibrato is applied. This works for both down- and up-bow playing.

Right Hand Game: Teeter Totter on the String

Most people probably rode a teeter totter when they were children; normally two children play the game. When one goes up, another goes down. The playing of the game is that child A, who wants to go up first, will strongly pushes against the ground; when A releases his feet, he goes up, and child B descends down. When child B pushes against the ground, he goes up. They repeat the same motion, going up and down alternatively. We could imagine that the bow is the teeter totter, the right hand is A, and the index finger of the right hand is A's feet. The game stats from the frog; the index finger adds a little weight on the bow (A steps on the ground), and we hear a louder sound (A goes up). The index finger releases weight, and we hear a softer sound (A goes down). By repeating this motion—moving the bow from frog to the tip or the reverse—bow vibrato is executed. Listen to the sound carefully and make sure the sound is always changing between louder and softer.

On the rare occasion that string educators demonstrate bow vibrato, their pupils may observe that it is similar in effect to left-hand vibrato. Students may be curious why and when they would need this technique. One occasion for using bow vibrato is to maintain a consistent tone color in different passages. An earlier passage might use fingering on strings that are played with left hand vibrato, but a subsequent passage might require playing notes on the open string, where one cannot employ the shaking of the left hand. In this scenario, a

string musician can decide to use bow vibrato to achieve the same tone color by undulating the bow.

Chapter 3

CHARACTERISTICS OF VIBRATO AND EXERCISES

3.1 Vibrato Speed

> A mastery of vibrato is distinguished by controlling its speed and width from slow and small to fast and broad —Paul (Tortelier et al., 1976)

Vibrato speed and width are the two main elements that influence how a musician interprets the style of a musical passage. Mature, talented and well-developed musicians usually find the application of vibrato to be largely a question of instinct. In fact, applying the appropriate vibrato in a passage is based on three factors: the ability to execute vibrato at different speeds; the ability to execute vibrato of different widths; and, most vitally, the ability to choose vibrato speed and width to express musical intent. Hence, vibrato allows the audience get a better grasp of the musician's personal taste and temperament.

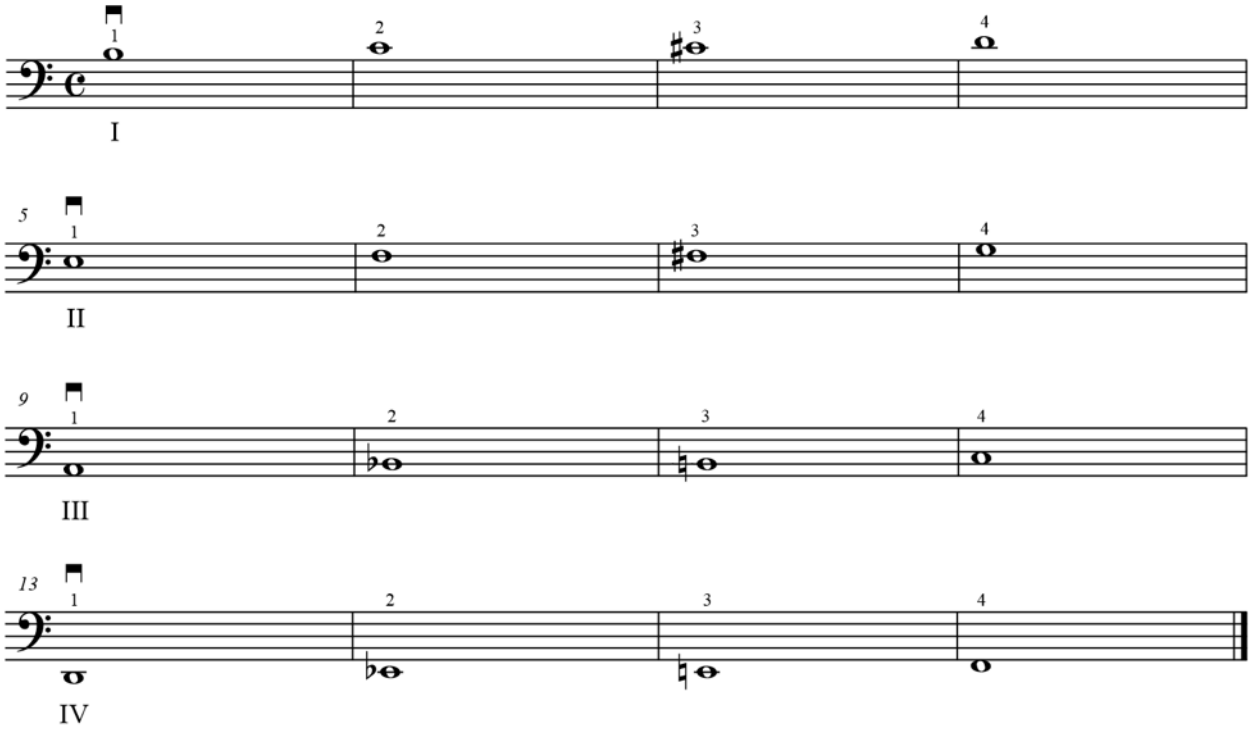
Contrary to other pedagogues, I will not suggest that students practice vibrato by using the metronome or counting oscillations of each beat in this studies below. In a live performance, the audience is not likely notice how many oscillations the performer makes. They pay attention to the music rather than the specific nuances of the technique. Vibrato is simply a tool, a performer can use vibrato to help the audience understand his or her interpretation. Vibrato is also a key with which the performer opens the door for the audience to experience the brilliant adventure that unfolds through the performance. The first step in using vibrato to add color to the sound is to control its speed.

3.1.1 Vibrato Speed Exercises

The speed of vibrato can be changed according to the dynamic, note duration, and expression of the music. Vibrato speeds can be faster or slower relative to each other; the actual speed

is less important than the relative speed by which it increases or decreases. We can practice vibrato speed through these following exercises:

It is always helpful to practice slowly, with long notes. Exercise 3.1 is aimed at establishing an equal vibrato speed, creating the vibrato with the arm, and developing the oscillation movement while playing on the cello. Fingers should always work together; every finger alternates in repeating the motion of polishing the string up and down. The student can practice this exercise in both fast and slow speeds, but he or she must ensure that the vibrato speed is consistent. Teachers should check whether the student's thumb is relaxed. The left arm will support the weight from the thumb but should not limit the movement of oscillation.



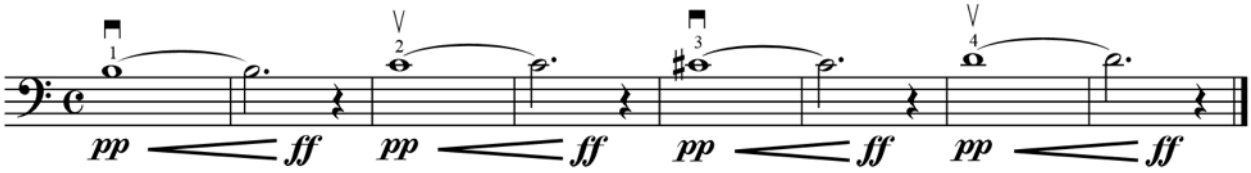
Exercise 3.1: Vibrato speed 1

There are some variations in practicing that students can explore. For instance, the student can practice with the same fingering pattern in different positions but not include shifting. Since shifting can disrupt the oscillation in the early stages, practicing vibrato in

the same position is recommended as a preliminary exercise.

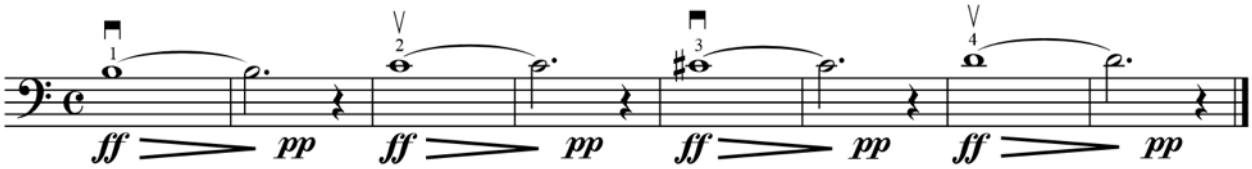
Dynamic level is one of the factors that influences how decisions on the speed of vibrato are made. Each musician has his or her preference on changing vibrato speed to match dynamic level, but for the early stage of vibrato learning, I would prefer the student not to have any practice habits. For example, the student should not limit practice to just these scenarios: vibrato speed increases when the dynamic is *forte*, speed and intensity tend to increase toward the peak of the phrase; vibrato speed decreases when the dynamic is *piano*, vibrato speed and dynamic decrease to produce a relaxed, calm tone. I would rather that students practice both fast and slow vibrato no matter dynamic so that they can develop the facility of executing variable vibrato speeds.

Exercise 3.2 gives students a process to learn how to apply a variable speed to the vibrato. There are two ways to practice Exercise 3.2: 1) gradually increase the oscillation speed from slow to fast; 2) apply a fast vibrato at the beginning and gradually slow down the oscillation. The right-hand should apply three approaches to changing dynamics are these: changing the bow speed, changing the bow pressure, and changing the contact-point. Both hands should coordinate so that the vibrato and dynamic changes are in sync with one another.



Exercise 3.2: Vibrato speed 2

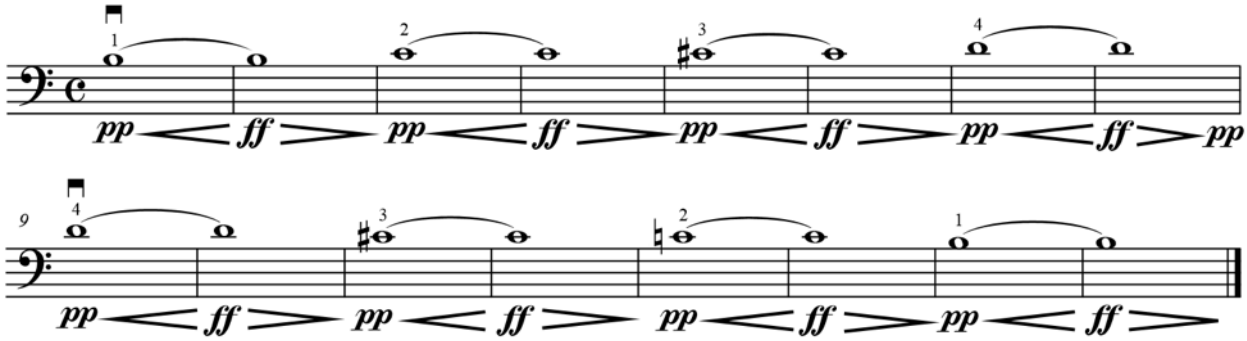
This exercise can be practiced in an alternative way as well, see Exercise 3.3. It requires students to prepare for a strong sound in the beginning and gradually decrease the volume. The dynamic starts from *fortissimo*. Students are suggested to practice in two ways: 1) apply a fast vibrato at the beginning of the note, slowing down the vibrato through the *diminuendo*; 2) still starting *fortissimo*, apply a slow vibrato at the beginning and speeding up the oscillation through the *diminuendo*.



Exercise 3.3: Vibrato speed 3

Similar to Exercise 3.2, Exercise 3.3provides students the freedom to explore a variable vibrato speed. It does not set up a standard that requires a specific vibrato speed to match a certain dynamic. Instead, there is enough room for students to explore vibrato speeds and coordinate these with the changing dynamics created by the right hand. Vibrato is never the only tool to express musician's emotions; the musician should think about the desired sound and coordinate the right-hand technique to achieve the appropriate vibrato.

After establishing a series of speed changing exercises, Exercise 3.4 integrates the earlier two exercises. Through consistent practice, students will be able to comfortably transition between slow and fast vibrato.



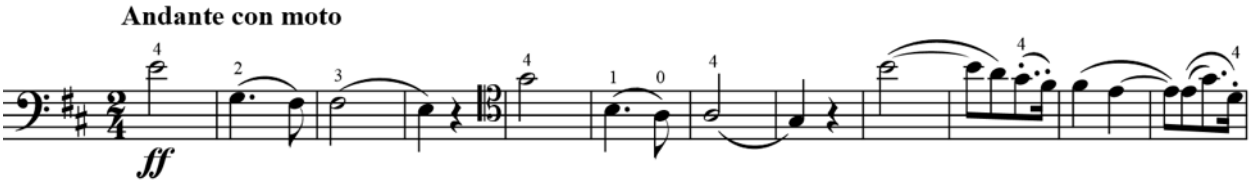
Exercise 3.4: Vibrato speed 4

Both the teacher and the student should be aware that, while playing vibrato, some students may notice tension in the left hand. The muscles of the left hand should not be tight when changing the speed of vibrato. If the student feels any tension, he or she must stop vibrating and just return to the basic "polish the string" game. This will allow the

student to relax the left hand because the polishing gesture does not require the focus on a specific pitch.

The chief factor that causes tension is that the student applies a vibrato speed that is either too fast or too slow. How do we solve the problem of students executing a vibrato that is too fast or too slow? If the vibrato sounds too fast, check whether the student is squeezing the left-hand fingers. Slightly separating out the fingers will help slow down the vibrato speed. Simultaneously check the hand position: a square hand shape may slow down the oscillation. If vibrato is too slow, make the hand more compact: vibrating with a more closed hand shape will help speed up the oscillation.

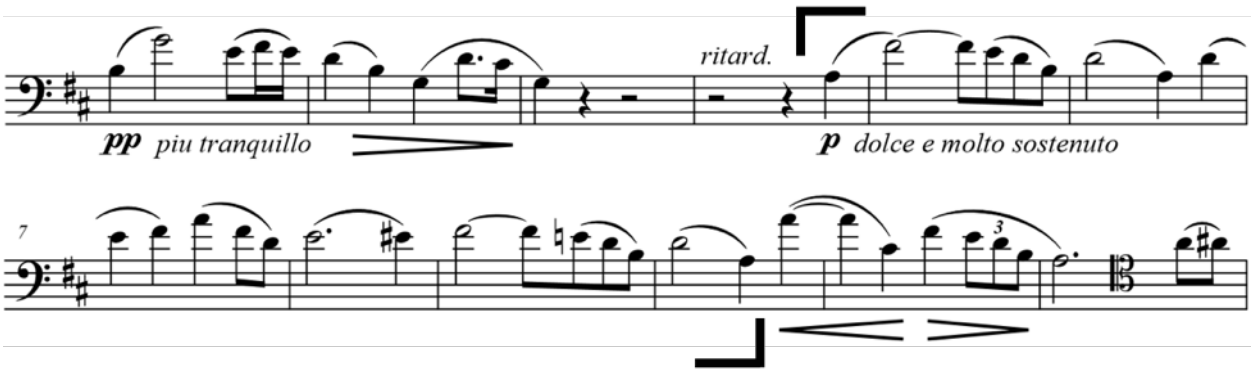
3.1.2 Vibrato Speed Examples



Excerpt 3.1: Tchaikovsky's Pezzo Capriccioso, Op. 62 for cello solo

In Excerpt 3.1, *Andante con moto* indicates the tempo of this piece is a walking speed but with motion. The dynamic is *fortissimo*, which suggests a dramatic, intense, and a powerful sound. Think about the tone quality; which kind of vibrato speed can match this intensity? A fast vibrato and an energetic left hand articulation is appropriate. To achieve an intense sound, we would not only need the left hand's fast vibrato, but also require the bow to be close to the bridge with a heavy weight on it. Yet, the bow should not make the tone sound too forced, so allow for a sensible bow speed to help play this passage beautifully.

Opposite to the previous excerpt is Dvorak's cello concerto in B minor, Op 104, 1st movement (measure 139-145). This passage emotionally demands a quiet, clean tone. (See Excerpt 3.2)



Excerpt 3.2: Dvorak Cello Concerto in B minor, Op. 104, 1st movement

The piano dynamic and "*dolce e molto sostenuto*" expressive marking together indicates a sweet, soft sound that should be played legato. These indications lead the performer to convey a warm, pure tone to audience. To achieve this effect, the vibrato should be slow and measured, so that the tone will give a peaceful and relaxing impression.

3.2 Vibrato Width

Vibrato width is a characteristic independent from vibrato speed; some feel that vibrato width is largely responsible for the expressive nature in performance. Therefore, vibrato width is the variable most frequently manipulated by the artist to create intensity and color (MacLeod, 2006). Pedagogues often mention that vibrato width can be wider or narrower relative to one another. In this chapter, we do not discuss the actual width of the vibrato; we will instead focus on the technical execution of vibrato width and the emotional variety it can convey. Generally speaking, a louder sound needs a wider vibrato, softer sound needs a narrower vibrato.

3.2.1 Difference Between Wide and Narrow Vibrato

Many students often apply different vibrato speed to a passage instead of exploring the variety of vibrato widths that could be employed. I will explain how to identify and execute

different vibrato widths.

We can first discuss the physical aspect of the hand when performing vibrato. Normally, wider fingers naturally produce a wider vibrato. This is because the finger simply covers more distance. When oscillating; a narrower finger requires compensate with the arm to achieve the same vibrato width. Students and teachers can try the following experiment:

There are three steps to understanding the width of vibrato, all of which can be explored via a single stopped note. Firstly, play the D4 with the second finger on the A string and only use the tip of the finger. There will be a short crease on the finger when it is removed from the string. Then, you use more of the pad of the finger and lower the left elbow slightly to play the note. There will be a longer crease on the finger that extends beyond the finger. Finally, turn the left-hand angle to the slanted position mentioned in chapter 2 (see Figure 2.2); there will be a diagonal crease on the finger when it is lifted from the string (Finckel, 2009).

This experiment shows following results: different contact surfaces of the finger will cause different vibrato widths. For a wider sound, use more of the pad of the finger or make a very slanted angle so that the finger has a bigger contact surface. Conversely, apply less contact surface, or use only the fingertip to achieve a narrow vibrato. Students should experiment and develop sensitivities to different vibrato widths in order to be able to decide when to utilize narrower or wider vibrato. I will later provide more excerpts to demonstrate the different scenarios where one may apply different vibrato widths.

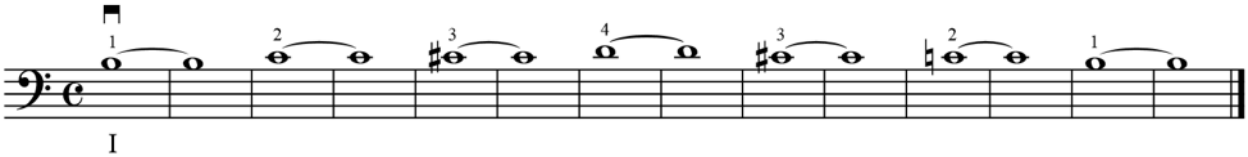
There are exercise for practicing vibrato width, but before starting such exercises, the student should be cognizant of vibrato speed and in particular the student is able to execute at a uniform speed. The student thus can focus on vibrato width and should use a consistent dynamic so that he or she can listen carefully to the change in the depth of sound that occurs as vibrato width changes.

3.2.2 Vibrato Width Exercises

There are three exercises to help students practice vibrato width; these are played in three typical positions. Unlike vibrato speed, vibrato width gets less visual attention. We can obviously observe the left-hand oscillate faster or slower; however, we should use our earns when practicing vibrato width, listening for the tone and feeling the depth of sound rather than checking left-hand movement.

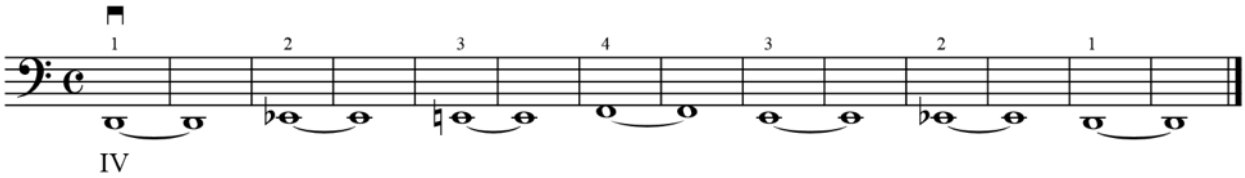
Both Exercise 3.5 and Exercise 3.6 are played in first position, where students feel it is easiest to perform. It is suggested that, for these two exercises, the student applies a wider vibrato, using more of the pad of the finger or adding more arm movement so that the finger can make more contact with the fingerboard. Both exercises reinforce practicing a deep, thick sound; the difference between them is in the tone of the string. Exercise 3.5 is played on the A-string, which has a brighter tone; Exercise 3.6 is played on the C-string, which definitely has a deeper sound.

While practicing these two exercises, we may find that a narrower vibrato on the A-string can also sound as beautiful as the wider vibrato. The difference is the difference in range: a narrow vibrato cannot produce a quality sound on the C-string because it is the thickest string and requires more weight to achieve the same effect as a higher string like the A-string. It is thus suggested that students apply a wider vibrato if the passage is on the G-string and C-strings.



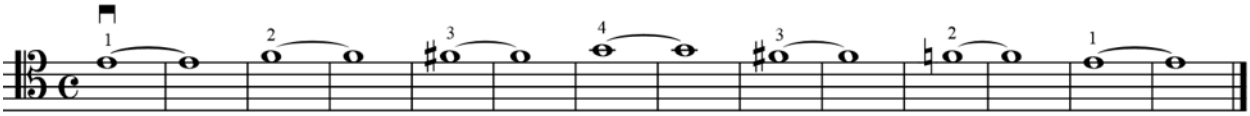
Exercise 3.5: Vibrato width 1

In the vibrato practice study, students have to identify the use of vibrato by matching the intensity of the notes to the passage's register. Fourth position is the bridge that connects the lower register to the higher register. Some students might feel awkward to perform vibrato

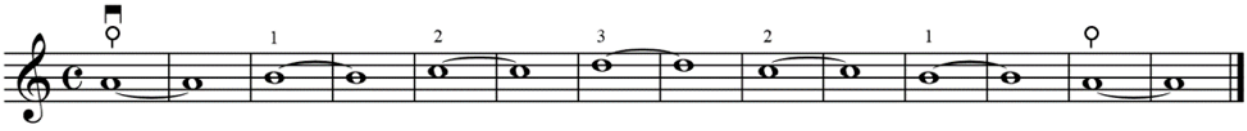


Exercise 3.6: Vibrato width 2

in the fourth position because the thumb approaches the edge of the cello's. Performing vibrato in the fourth position require a flexible thumb movement. The left elbow should be ready to be raised as well. Teachers may mention that the ski jump exercise is effective in finding the proper elbow height. Due to the special needs of the fourth position, practicing vibrato in the Exercise 3.7 will help students feel the change of balance in the left hand. Students can keep the thumb slightly off the neck, ensuring that the thumb is relaxed, and aim to achieve a clear tone; a narrower vibrato will have this effect.



Exercise 3.7: Vibrato-width, Fourth position



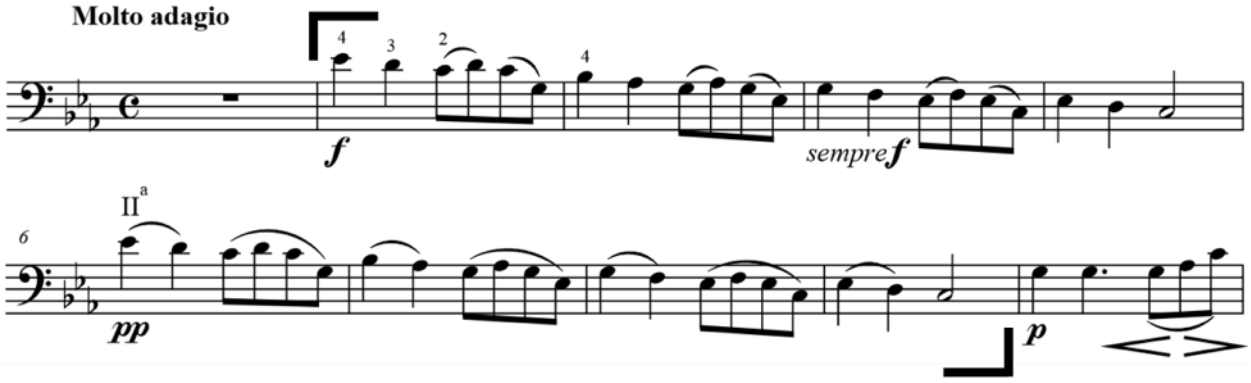
Exercise 3.8: Vibrato-width, Thumb position

The vibrato's width or speed should follow the characteristics of a passage, yet there are still some patterns we can follow to decide on the appropriate vibrato. For instance, in Exercise 3.8, this passage is in a high register, where the distances between the fingers is

smaller than they are in the lower register. A wider vibrato can cause poor intonation. Due to these limitations, performing a narrower vibrato in higher registers is recommended.

3.2.3 Vibrato Width Examples

Fauré's Elgéie for cello and piano in C minor, Op. 24, provides an excellent scenario for applying both wide and narrow vibrato. See Excerpt 3.3



Excerpt 3.3: Fauré Elgéie for cello and piano in C minor, Op. 24

To the performer, the tempo marking usually sets the most important basis for a piece's feeling. It might not be detailed, but it delivers a general direction for the composer's thoughts. *Molto adagio* means very slowly, combined with the title of the piece, this work features a sad and sombre theme. From the beginning to measure 5, the dynamic is *forte*, and it is the statement of the main theme of the piece. It requires a strong voice to make this statement, so I would suggest a wide vibrato to create the sad mood and deep sound. From measure 6 to measure 9, the dynamic suddenly changes from *forte* to *pianissimo*, but the cello has the same melody as the first phrase. Not only is the volume different, but the character also changes.

Much like telling a story, one reason for describing the same content twice to re-emphasize the theme. Here, the melancholy is emphasized again; another reason is to allude to a different feeling, besides the sad, perhaps something new? This is the performer's interpretation.

For the second phrase, we can apply a narrower vibrato to create a sense that the emotion is being held back and controlled. Since the same melody is in the high register but on the D-string, the performer has the freedom to make the color darker.

3.3 Continuous Vibrato

Vibrato is one of the techniques that string musicians use to express their understanding of the music; there are different kinds of vibrato. Musicians select the type of vibrato according to according to dynamics, registers, musical characteristics, and so on– to "draw" the picture of the music forms in their imagination. The key to make the music sound consistent is continuous vibrato. Regardless of whether the vibrato is wide, narrow, fast, or slow, continuous vibrato produces sounds that feel musically connected.

Even for advanced students, continuous vibrato might be a difficult technique to perform. Students may easily fall into the habit of vibrating on some notes, especially on long ones, and ignoring short notes. They may not even notice that they stop vibrating on certain notes. This will cause the notes that are vibrated to stand out from those that are not. The unconscious action of stopping the vibrato causes an interruption in the flow of music. The purpose of learning continuous vibrato is to help students create a consistent and connected phrase when performing.

There is a pre-exercise that is designed for students to learn continuous vibrato. They do not have to practice this pre-exercise on the cello. This is so that students can gain confidence in vibrato and worry less about intonation. Feeling the vibrato stimulus starts from the forearm and releases into the fingers. One allows the vibrato to be like an electric pulsation transferring from the forearm to the fingers. The whole left hand works as a unit; one thinks of the vibrato motion as a forearm vibrating motion instead of that of a single finger vibrating.

3.3.1 Pre-exercise for Continuous Vibrato

Practice this exercise without the cello. The vibrato will happen in the right forearm. Put the right forearm in front of your chest. Imagine the right forearm is the fingerboard, and place the left hand with a regular hand position on the right forearm. After having done the polishing exercise with all fingers down and finding the first finger go up, in order to avoid the tension that leads to the fourth finger, the pre-exercise is suggested to start from the fourth finger. Allow the fingers 4, 3, 2, 1 to play in order with vibrato in the forearm, then reverse the fingering to 1, 2, 3, 4. Repeat the process, and notice that the thumb is loose; the arm balance is changing naturally while the fingering switches. This pre-exercise is aimed at helping students vibrate with a flexible left hand and training all fingers to always vibrate together instead of individually.

There are 3 types of rhythms that students can practice in the pre-exercise; the numbers below indicates the fingering. Students can sing the fingering while practicing, and students can follow the pattern and create their own order to change the fingering:

- Two eighth notes: 4-3, 3-2, 2-1, 1-4
- Triplet: 4-3-2, 3-2-1, 2-1-4, 1-4-3
- Four sixteenth notes: 4-3-2-1, 3-2-1-4, 2-1-4-3, 1-4-3-2

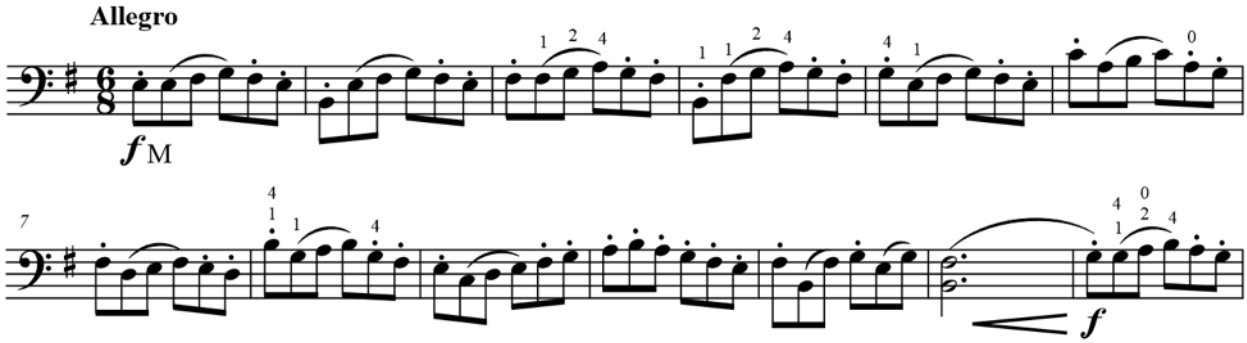
The pre-exercise can be played on the cello afterwards, and students can practice with the metronome and select different strings or positions to work on the vibrato. For example, set the metronome quarter equals 60; placing the left-hand on the A-string, play four sixteenth notes per beat for eight measures, when the left-hand is used to the vibrato motion, speed up the tempo. The tempo can range from 60 to 120 per quarter note.

Students can practice with either a wider vibrato or narrower vibrato for this exercise, but they should not mix different vibrato width in one exercise. We need to ensure that each exercise has a consistent sound. Whichever finger that vibrates should always feel that the

left-hand is alive; the switching fingering doesn't stop the vibrato motion. After practicing with different speeds and rhythms, students will be able to develop a natural continuous vibrato.

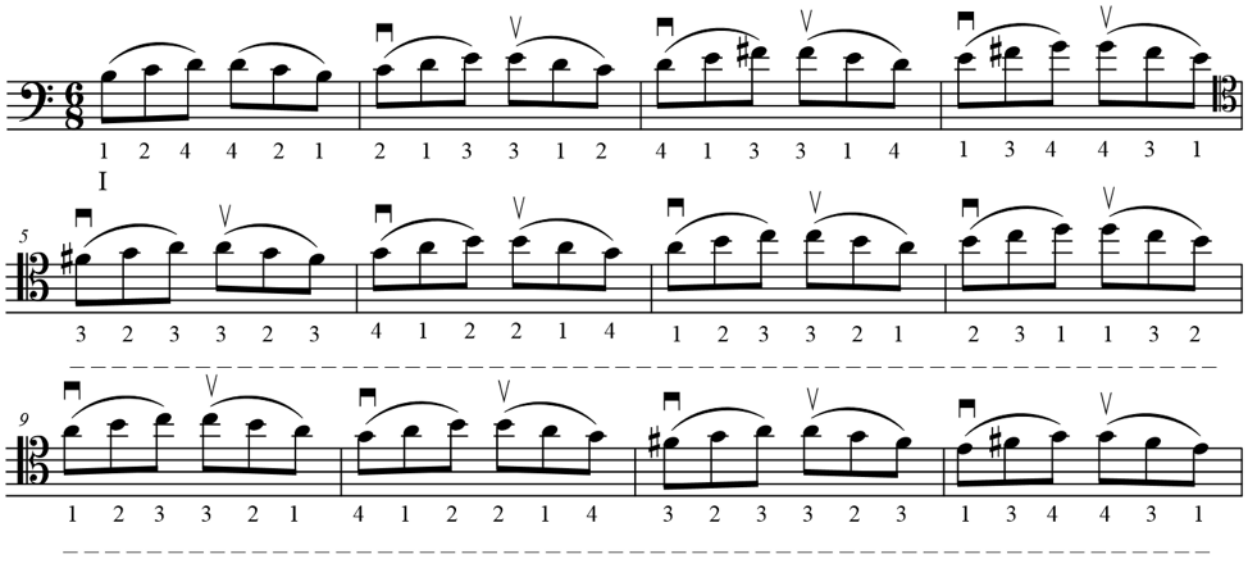
3.3.2 Continuous Vibrato Examples

After building a flexible continuous vibrato motion, we can experiment with the execution of vibrato on some etudes. For example, in Dotzauer's etude, Op. 120, No. 4 the same rhythm gives students an equal time to vibrate on different fingerings, feeling the vibrato motion from the forearm and passing the motion to each finger. Since the tempo is *Allegro*, a fast speed, we can choose a fast and narrow vibrato to perform this etude. See Excerpt 3.4



Excerpt 3.4: Dotzauer Op.120, No.4

Performing vibrato while shifting is a challenge for most students who are in the early stages of learning vibrato. Vibrato motion is easily stopped when performer's left-hand shifts from one position to another position, and this might cause an obvious gap between notes or phrases. Practicing vibrato while shifting will greatly develop the flexibility of each finger and reduce the tension from the thumb; a tight thumb will prevent smoothly shifting and affect the vibrato's consistency. Exercise 3.9 is an exercise that students can practice vibrato on every individual finger while shifting. Students will get used to transferring the weight from finger to finger by keeping their thumb free and unrestricted behind the neck and in thumb position (Jensen & Chung, 2017).

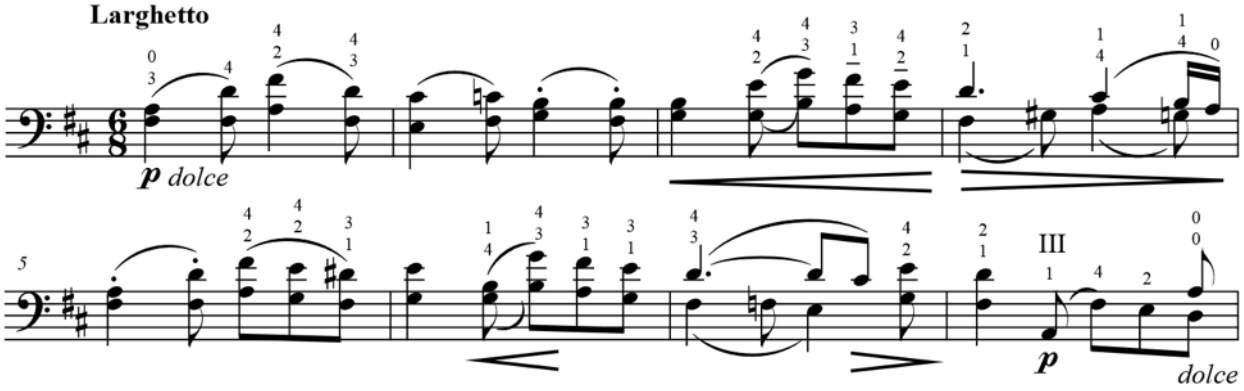


Exercise 3.9: Exercise from *CelloMind: Intonation and Technique* by Hans Jørgen Jensen/ Minna Rose Chung

Double-stop vibrato is another approach that will help students successfully achieve continuous vibrato. Based on the method that we have discussed, continuous vibrato is executed by the forearm, not any individual finger. Double-stop vibrato will help develop the ability to maintain the vibrato motion. All the double stops should have an even and stable vibrato. In general, vibrato on the first and fourth finger are the most difficult. This is because the hand shape barely leaves freedom for the hand to vibrate. One suggestion for improving quality in this scenario is to apply more arm weight on the fourth finger. It is the weakest, and it has tendency to introduce tension. In addition, arm weight allows a small amount of "sliding" on the other finger (Jensen & Chung, 2017). See Excerpt 3.5.

A connected and graceful sound is needed for this etude, so apply a relaxing vibrato is suitable. To achieve a beautiful and warm tone, loosen the hand while performing this piece; feel the drop of weight from the arm to fingers but do not squeeze the thumb.

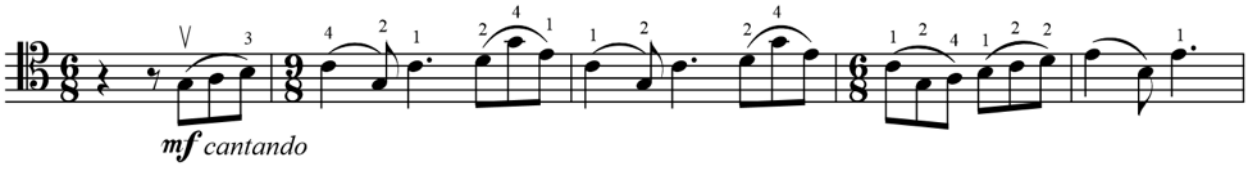
In *Cello Playing of Today*, Maurice Eisenberg (1966) discusses the influence that geographical culture had on vibrato. Many string musicians appear to reveal their national



Excerpt 3.5: Grützmacher, Op. 38, No. 5

characteristic subconsciously through the texture of the vibrato. The French, for example, are more volatile and exuberant than their neighbors to the North or East; French string musicians often use a tense, quick, and narrow finger oscillation that reflects their national temperament.

Inspired by Eisenberg's idea, I would like to consider the geographic influence on vibrato. This excerpt is from the 20th-century Russian composer Dmitry Kabalevsky's Cello Concerto No.1 in G minor. It was dedicated to fallen Russian soldiers from the World War I and II. Kabalevsky (1904-1987) was associated with the Communist Party and was called a "Soviet Composer." Eventually he became one of the most popular Soviet composers. Kabalevsky was the first composer that was educated under the Soviet system and was a member of Proletarian Collective, a group which states that "Revolutionary musical creation can only be achieved by those who grew up with the Revolution and are active participants in its development (DelosProductions, 2012)." His music works directly reflect his faith of Soviet realism. During World War II, the Russian and German governments had strict rules about the arts and what was appropriate, and they generally preferred pieces that promoted patriotism (Satz, 2006). In fact, Kabalevsky did not stray too far from traditional Russian music in his Cello Concerto. He got inspiration from post war and this concerto represented a new style of Neoclassicism, which gave Kabalevsky room to experiment.



Excerpt 3.6: Kabalevsky cello concerto No.1 Op. 49 in G minor, 1st movement

Kabalevsky put very complex emotions into the piece. The context includes war, loss, nationalism, belonging. Infused with these emotions, the first movement is *Allegro*, which is fast and lively. It requires the musician to perform with a constant energy which a fast and narrow continuous vibrato can achieve to keep this emotional tension.

Students can start practicing this excerpt without the thumb. Since it is easy to grab the neck when moving the left hand, releasing the thumb will help to move the fingers more freely to condition the hand for a better continuous vibrato.

3.4 Pitch Center

Pitch center is probably the most debated aspect of vibrato until today. Many string players are able to perform a beautiful vibrato, but they may be unable to explain how the vibrato motion is produced. The frequency of the vibrato is especially nuanced and is hard to perceive by eye or ear. Historically, pedagogues and musicians hold these three views regarding the pitch of vibrato: 1) the frequency of vibrato oscillates from the in-tune pitch and above; 2) the frequency of vibrato oscillates equally above and below the in-tune pitch; 3) the frequency of vibrato oscillate primarily from the in-tune pitch to a lower pitch.

Research by Geringer et al. (2010) found that upward from the in-tune pitch and above is little bit out of date; it's a cause for bad intonation, and after 1986, few pedagogues have advocated it. He remarked that Fischer stated that vibrating to the harp side will sound as though one is playing sharp. The other two views became more common in teaching methods for pitch center. In his 1962 *Principles of Violin Playing and Teaching*, violin pedagogue Ivan Galamian conjectured that "[t]he ear catches far more readily the highest pitch sounded,

and a vibrato that goes as much above pitch as below makes the general intonation sound too sharp."

Galamian states that vibrato should start from the in-tune pitch and oscillate downwards; this idea has influenced large number of other pedagogues like Applebaum, Fischer, Suzuki. Contrary to Galamian's viewpoint, the frequency of vibrato oscillating above and below the in-tune pitch has gotten more empirical research approval, for example from Doschek, Mantel, Young, Rolland. Mental writes that the ear chooses a medium frequency as the main pitch impression, and the listener hears what is exactly in the middle as the in-tune pitch (Mantel & Thiem, 1995). Geringer et al. (2010) himself, similarly, found that the performance of string vibrato presents a range of frequencies extending both above and below the intended pitch.

Experienced and mature string musicians may have a better sensitivity to pitch perception while performing vibrato, even when there are many other aspects that affect the intonation. Playing in tune is a challenge for all, and different styles of music may require different widths and speeds of vibrato. Different performance forms, such as solo, ensemble, orchestra, will each have their own standard for intonation. A high-quality vibrato is based on good intonation, so how can we get the perfect intonation? Compared with other fields, music has limited research resources that provide strong evidence that might help a musician to achieve the "perfect" intonation. A consistent system for learning vibrato is recommended, teachers should choose one approach to demonstrate the vibrato direction. I would recommend my students to vibrate *around* the in tune pitch.

Chapter 4

QUANTITATIVE ANALYSIS ON VIBRATO EXCERPTS

> A good vibrato is a pulsation of pitch, usually accompanied with synchronous pulsations of loudness and timbre, of such extent and rate as to give a pleasing flexibility, tenderness and richness to the tone—Carl Seashore (1937)

The appropriate use of vibrato in performance is contingent upon many aspects. An authentic performance requires the musician to consider a piece's historical period and how a piece was perceived by the period's listeners. Though the use of vibrato has changed overtime, the process of exploring how to apply appropriate vibrato gives musicians considerable room to develop vibrato width and speed.

Despite there being no perfect suggestion on how to apply a specific type of vibrato in one passage, there are some general types of vibrato that has been proposed since 1833. Spohr distinguished four types of vibrato: intense and fast, slower and less intense, increasing in intensity through the note, and decreasing in intensity. The classification of vibrato by Spohr (1833) played a significant role in improving the technique of stringed instruments in the nineteenth century. However, Spohr's recommendations placed some limitations: a fast vibrato can only be combined with intensity, and a slower vibrato can only be combined with less intensity.

Today, there are more possibilities of performing vibrato. The four major categories that are well developed and are widely accepted: fast and wide, fast and narrow, slow and wide, and slow and narrow. There are infinite ways to play vibrato, but these four categories just happen to be well known and commonly discussed.

4.1 Quantitative Analysis of Vibrato in the Cello

In this chapter, I will do a quantitative analysis on characteristics of vibrato mentioned in previous chapters by doing Fourier transformation on the audio signals of live recordings for a number of cello pieces performed by different musicians. The goal of this exercise is to objectively study patterns in the recordings and visualize vibrato so that it is easy to quantify vibrato width, and speed where it would be otherwise difficult to do by using the ear.

When we first learn a new piece, many of us listen to performances of famous cellists to get some inspiration and imitate their technique. Therefore, for each excerpt, I chose recordings from four to seven well-known musicians on which to conduct quantitative analysis with the goal of finding commonalities and differences in their vibrato technique. Even though different musicians have their own interpretations, we can still find a common understanding of the music. Through this analysis, we can make recommendations about which types of vibrato are being applied in specific scenarios.

4.1.1 What is Fourier Transform?

An audio signal is a representation of sound waves that travel through the air, consisting of compressions (dense air) and rarefactions (less dense air). The audio signal can be represented as a function of the amplitude of the compression and rarefactions over time.

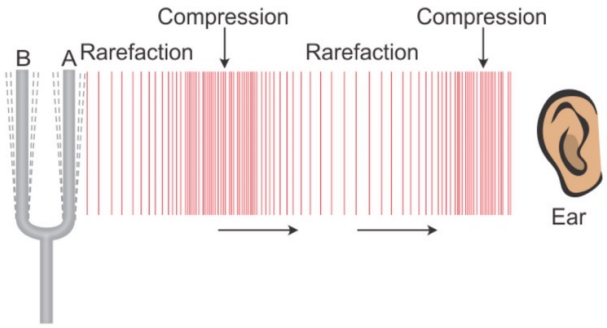


Figure 4.1: Compression and Rarefactions in sound (SeekPng)

Fourier transform is a technique that decomposes a temporal signal (function over time) into its constituent frequencies. That is, the original signal is a linear combination of sines and cosines. The mathematical properties of Fourier Transform will not be covered in the scope of this study. Müller's *Fundamentals of Music Processing: Audio, Analysis, Algorithms, Applications* provides an a comprehensive description of Fourier Transform as a foundational technique (Müller, 2015).

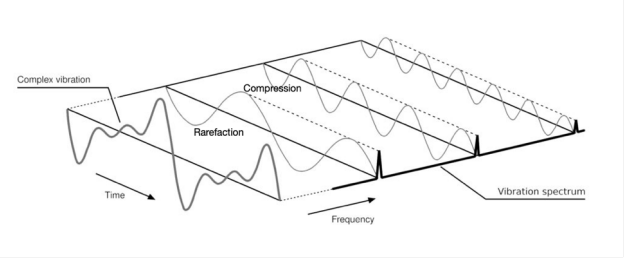


Figure 4.2: Decomposing a signal into consistuent frequencies (Fernandez, 2017)

4.1.2 Short-Time Fourier Transform and Spectrograms

Fourier transform converts a function of amplitude over time to a function of amplitude over frequency (pitch). Here the decomposition into pitches contains the pitches of the entire duration of the audio signal. However, in order to visualize the characteristics of vibrato, we need to understand how the pitch of the vibrating note is changing over time.

Short-Time Fourier Transform (STFT) breaks a long signal into a series of shorter signals using a sliding window technique. A window size W and a step size S are chosen; both are durations of time. In the discrete form of STFT, Fourier transform is conducted independently on the signal from time 0 to W, then, another one from S to S+W, then, another one from 2S to 2S+W, and so on.

Figure 4.3 shows the sliding window technique of STFT with a concrete example from an audio recording. Here Fourier transform is conducted on a window size of 1 second starting at time 0. Then the window moves by a step of 0.4 seconds where the window starts at

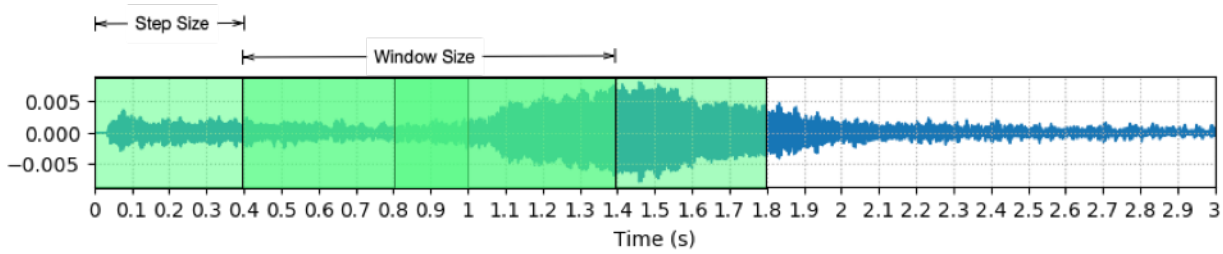


Figure 4.3: STFT sliding window

time 0.4s, and ends at 1.4s. Each window moves progressively to the right. Each Fourier transform will output the frequencies that most likely compose the sound wave within that time window.

A spectrogram is defined as an intensity plot (usually on a log scale, such as dB (Scavone, 1999b)) of a sequence of STFT (Smith, 2011) magnitude of overlapped windowed data segments, where the windows are moved forward according to the step size. It is an important representation of audio data because human hearing (Scavone, 1999a) is believed to be based on a kind of real-time spectrogram encoded by the cochlea of the inner ear (Pujol, 2007). The spectrogram has been used extensively in the field of computer music as a guide during the development of sound synthesis algorithms. When applied to an audio signal, spectrograms are sometimes called sonographs, voiceprints, or voicegrams. when the data is represented in a 3D plot, they may be called waterfalls.

4.1.3 Spectrogram Parameters

In order to see fluctuations in the pitch in a spectrogram constructed from STFT, we need to choose an adequate window size. A frame of time with a large window size may cover one or more than one oscillation of the finger during vibrato. This causes the Fourier transform to capture the variation in pitch of the oscillation as a wide indistinguishable collection of frequencies along the pitch center. On the other hand, a small window size presents a different resolution problem. The shorter the duration, the less information is available in the sample

duration to be able to distinguish the actual frequencies. The minimum frequency resolution difference that can be distinguished in a window is governed by the sampling rate of the recording (a value we consider constant, 44,100 Hz for all the recordings that we analyze) divided by the number of samples in the window. The smaller the window, the smaller the number of samples, the coarser the frequency resolution.

Hence, if we want to see both vibrato speed and width to the best of our ability through a spectrogram, we would choose the largest window size to guarantee that we can see pitch oscillation changes from one window to the next as the finger oscillates. That is to say, the window size in time duration cannot be larger than the time it takes for the finger to make one complete oscillation around the pitch center.

For the spectrogram analysis in the rest of this chapter, each recording will have a sampling rate of 44,100 Hz. We will use a window size of 185ms, or exactly 8192 frame samples. This will allow the frequency resolution to be 5.4 Hz. We will use 2.9ms, or exactly 128 frame samples as the step size.

4.1.4 Harmonics in the Spectrogram

In the spectrogram visualizations later in the study, readers might notice similar or repeating patterns over different frequency ranges. These are the harmonic frequencies from the fundamental frequency of particular notes. Harmonic frequencies are the results of standing waves formed in the instrument, in this case, the strings of a cello. Harmonic frequencies will appear at integer multiples of the fundamental frequency of the note. Figure 4.4 a spectrogram of vibrato on G3 (196.00 Hz). We can see due to harmonics there is a similar oscillation pattern on G4 (392.00 Hz).

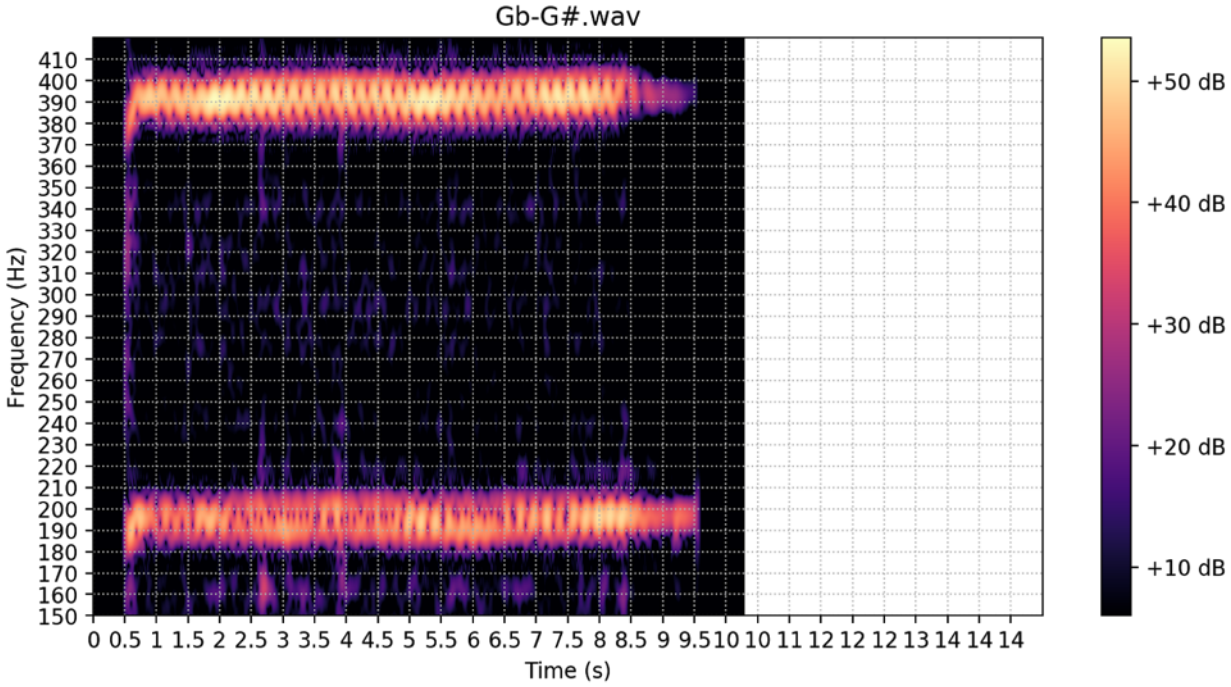
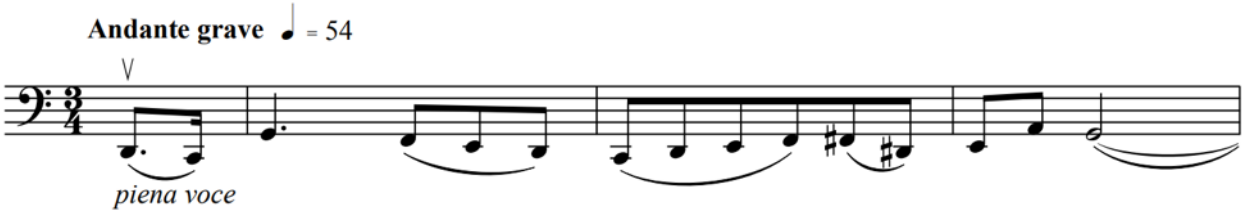


Figure 4.4: Spectrogram of vibrato on played on G3 showing harmonic frequencies for G4

4.2 Excerpts Selection

I selected 11 well known cellists' performances from the 20th and 21st centuries to analyze four excerpts. The 11 cellists are Mischa Maisky, Danill Shafran, David Finckel, Gregor Piatigorsky, Steven Isserlis, Yo-Yo Ma, Sol Gabetta, Mstislav Rostropovich, Gautier Capuçon, Leonard Rose, and Miklós Perényi.

4.2.1 Fast and wide vibrato



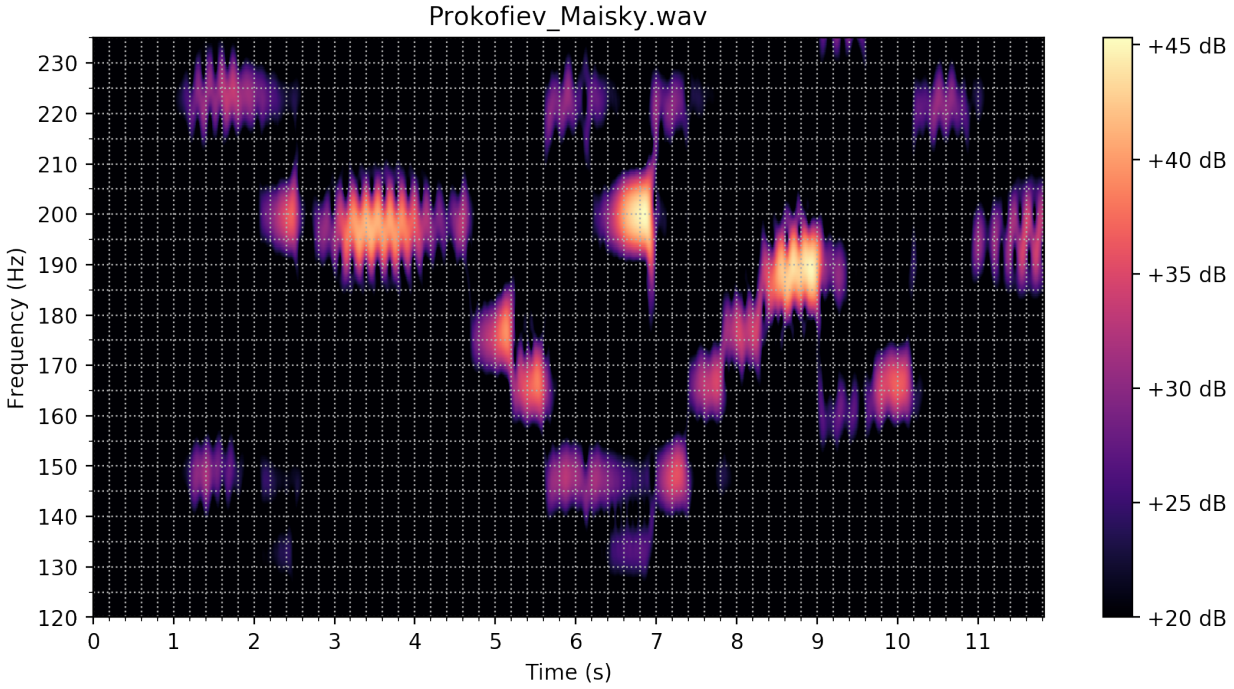
Excerpt 4.1: Prokofiev Cello Sonata in C Major, Op. 119 1st movement, measure 1-3

In this excerpt, the expression marking is *piena voce*, which means "full sound" in English. A warm and thick tone would be applied, and the dynamic should be strong. A fast vibrato could capture the intensity of the passage, if we apply a slow vibrato, the music would sound as if it were dragging. This passage is on the C-string, the lowest and thickest string on the cello, so a wide vibrato is needed.

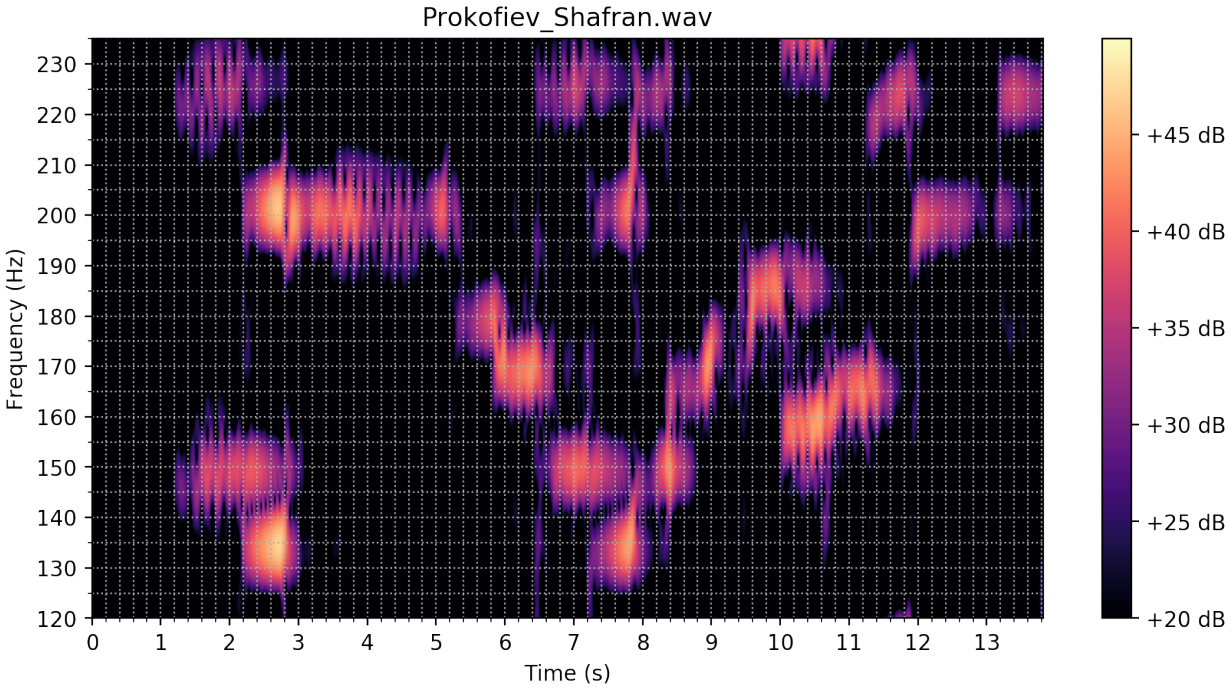
Based on standard tuning A4 (440 Hz), the frequencies of these notes in this excerpt are D2 (73.42 Hz), C2 (65.41 Hz), G2 (98.00 Hz), F2 (87.31 Hz), E2 (82.41 Hz), D2, C2, D2, E2, F2, F#2 (92.50 Hz), D#2 (77.78 Hz), E2, A2 (110.00 Hz), G2.

In the spectrogram, there are different brightness of notes which denotes their volume. In order to read the data more clearly, for the analysis, I will select pitches one octave higher than the actual pitches played. For instance, I will use D3 (146.83 Hz), C3 (130.81 Hz), G3 (196.00 Hz), F3 (174.61 Hz), E3 (164.81 Hz), D3, C3, D3, E3, F3, F#3 (185.00 Hz), D#3 (155.56 Hz),E3, A3 (220.00 Hz), G3.

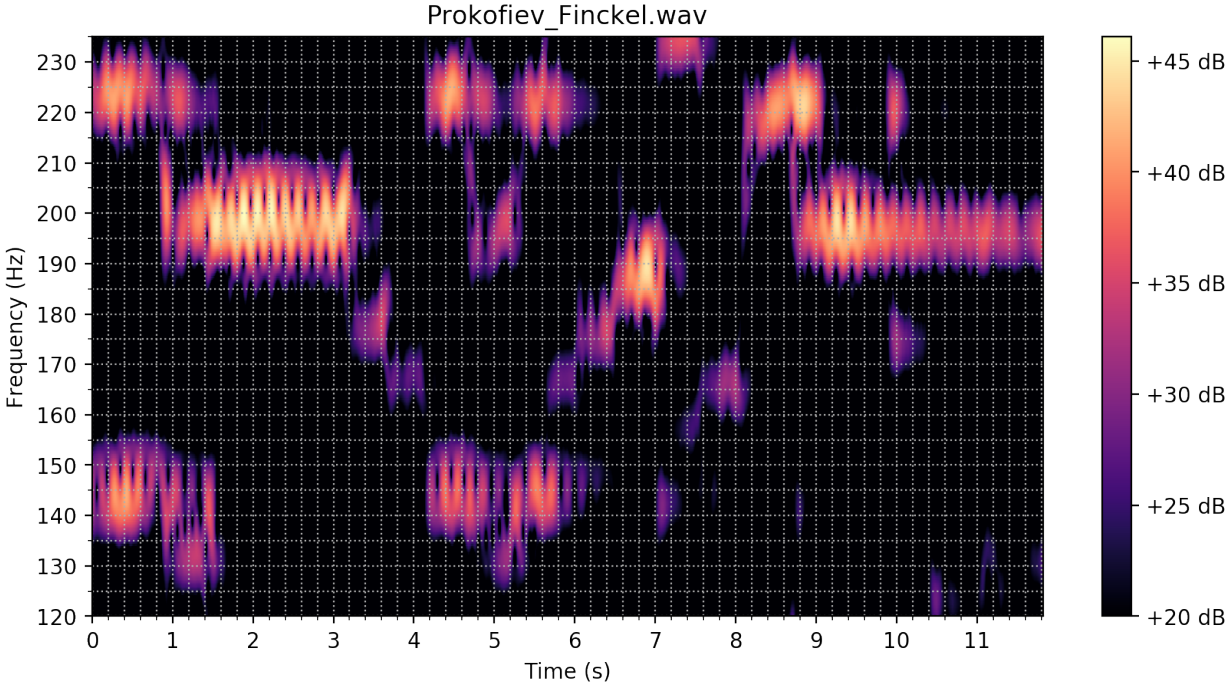
The brighter colors represent stronger dynamic; the wavy lines represent vibrato, showing the fluctuation in pitch frequency over time; the difference between the top and bottom of a wave indicate how wide of the frequency (Hz) the vibrato is; the number of oscillations per unit time is the speed of vibrato. If the same pattern shows on different frequencies, they are octave harmonics in the overtone series of the pitch being played. I count the number of peaks of fluctuation as the number of vibrato oscillations, and the frequency difference as vibrato width.



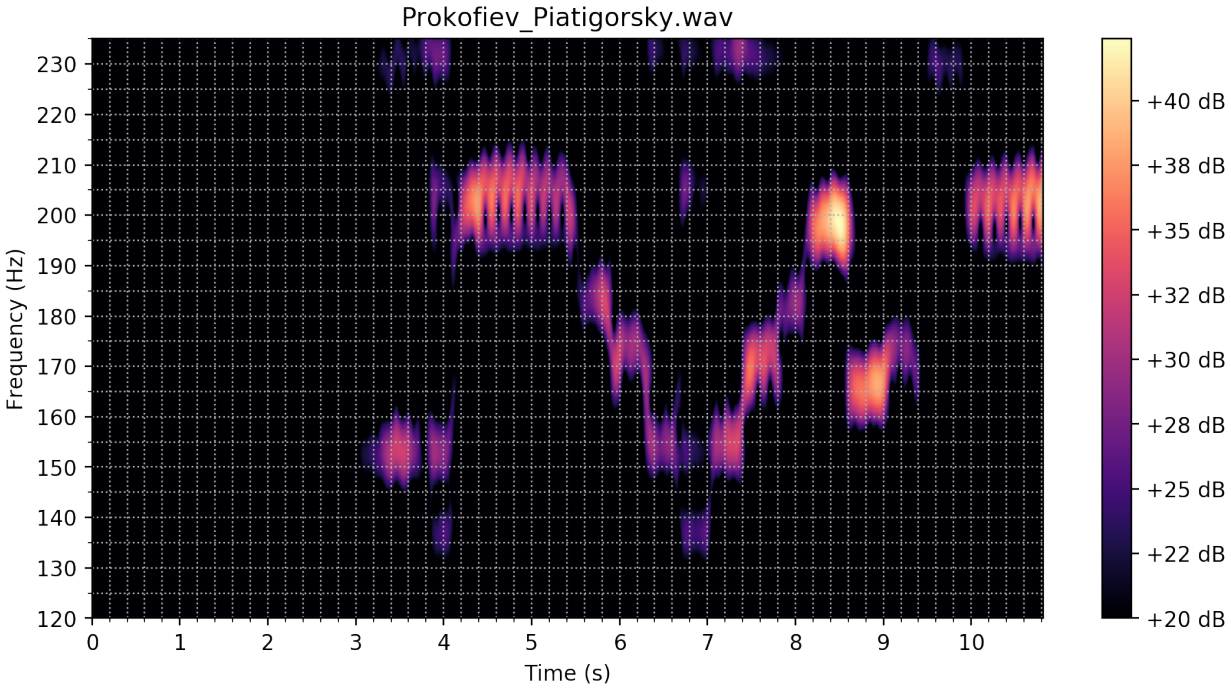
Recording 4.1: Prokofiev by Maisky (2003)



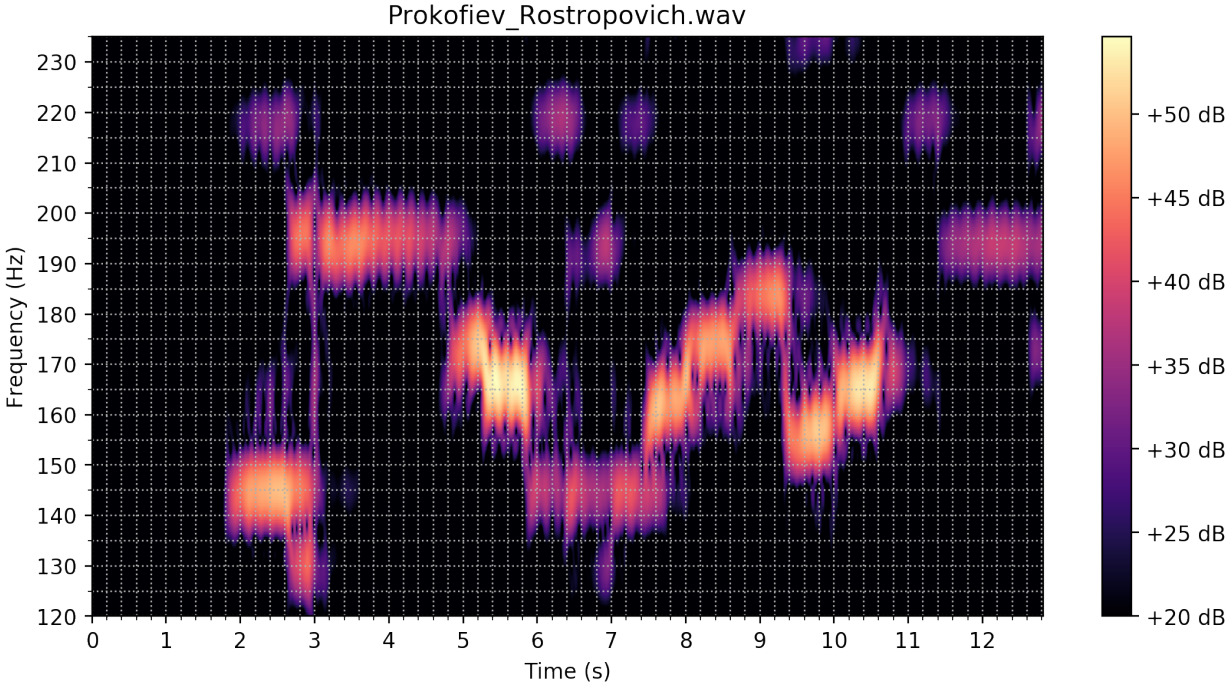
Recording 4.2: Prokofiev by Shafran (2017)



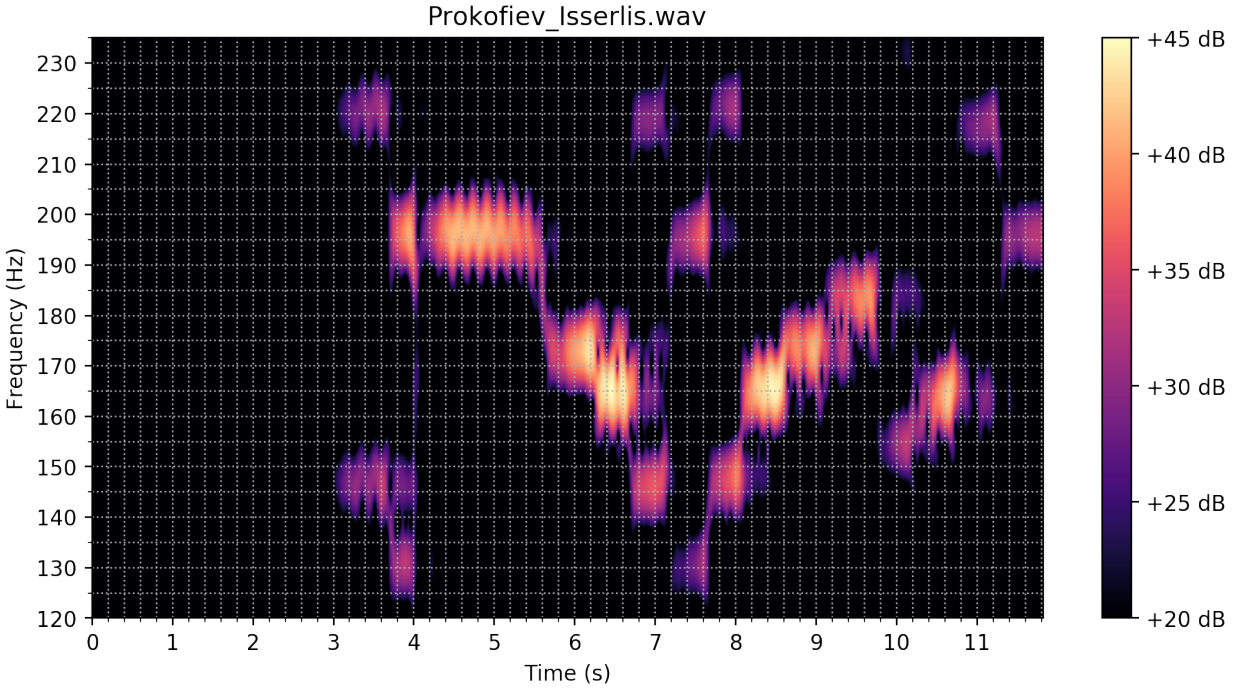
Recording 4.3: Prokofiev by Finckel (2012)



Recording 4.4: Prokofiev by Piatigorsky (1966)



Recording 4.5: Prokofiev by Rostropovich (1950)



Recording 4.6: Prokofiev by Isserlis (1996)

Prokofiev Width (Hz)	D3	C3	G3	F3	E3	D3	C3	D3	E3	F3	F#3	D#3	E3	A3	G3
Maisky	20.0	15.0	35.0	20.0	17.5	20.0	15.0	20.0	17.5	30.0	22.5	17.5	20.0	20.0	30.0
Shafran	20.0	17.0	30.0	20.0	20.0	18.0	15.0	18.0	14.0	10.0	25.0	28.0	15.0	18.0	20.0
Finckel	26.0	18.0	26.0	15.0	13.0	24.0	16.0	24.0	13.0	21.0	22.0	10.0	15.0	21.0	25.0
Piatigorsky	15.0	11.0	25.0	12.0	12.0	12.0	13.0	12.0	20.0	15.0	20.0	20.0	18.0	20.0	25.0
Rostropovich	20.0	10.0	22.0	12.0	20.0	13.0	10.0	13.0	20.0	15.0	18.0	18.0	18.0	18.0	17.0
Isserlis	17.0	18.0	30.0	20.0	20.0	15.0	19.0	15.0	17.0	17.0	20.0	14.0	20.0	18.0	15.0
Average	19.7	14.8	28.0	16.5	17.1	17.0	14.7	17.0	16.9	18.0	21.3	17.9	17.7	19.2	22.0

Table 4.1: Prokofiev Vibrato Width (Hz) - Individual Notes

Prokofiev Oscillations	D3	C3	G3	F3	E3	D3	C3	D3	E3	F3	F#3	D#3	E3	A3	G3
Maisky	7	6	18	6	5	6	7	5	3	7	9	4	8	9	6
Shafran	10	9	19	9	8	10	10	7	8	5	6	7	6	9	12
Finckel	11	8	17	6	5	7	7	7	5	5	6	1	8	8	20
Piatigorsky	6	3	10	5	2	3	5	4	2	3	4	4	3	3	6
Rostropovich	10	4	19	3	5	5	7	6	8	7	8	8	6	5	11
Isserlis	7	4	13	5	5	5	4	4	6	5	7	4	6	5	5
Average	8.5	5.7	16.0	5.7	5.0	6.0	6.7	5.5	5.3	5.3	6.7	4.7	6.2	6.5	10.0

Table 4.2: Prokofiev Vibrato Oscillations - Individual Notes

Prokofiev Speed	D3	C3	G3	F3	E3	D3	C3	D3	E3	F3	F#3	D#3	E3	A3	G3
Maisky	7.0	10.0	10.0	12.0	10.0	6.0	11.7	12.5	7.5	11.7	9.0	10.0	13.3	11.3	8.6
Shafran	10.0	11.3	7.3	11.3	8.0	7.1	12.5	8.8	10.0	8.3	5.0	8.8	7.5	11.3	7.5
Finckel	11.0	13.3	6.5	10.0	8.3	7.0	17.5	7.0	12.5	10.0	7.5	5.0	13.3	8.0	6.3
Piatigorsky	6.0	10.0	5.0	8.3	3.3	3.8	16.7	10.0	5.0	7.5	8.0	10.0	7.5	7.5	7.5
Rostropovich	7.7	6.7	7.6	4.3	5.0	4.2	35.0	6.0	10.0	7.8	6.7	11.4	5.0	8.3	7.9
Isserlis	7.0	13.3	8.1	6.3	6.3	5.6	10.0	6.7	10.0	6.3	11.7	6.7	6.7	10.0	10.0
Average	8.1	10.8	7.4	8.7	6.8	5.6	17.2	8.5	9.2	8.6	8.0	8.6	8.9	9.4	7.9

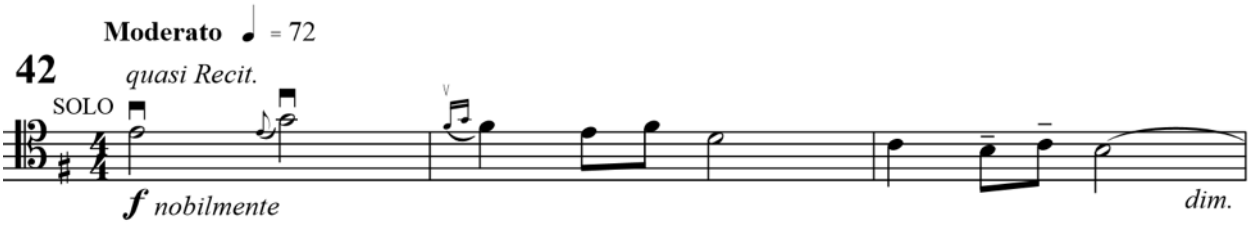
Table 4.3: Prokofiev Vibrato Speed (Oscillations/s)- Individual Notes

In Table 4.1 and similar tables in the subsequent excerpts, I calculate the difference between each note's peak and trough as the vibrato width (in terms of Hz). Although the vibrato speed is not directly shown on the spectrogram, I was able to calculate the speed by dividing the number of oscillations by the duration of time in which they occur.

Referring to on vibrato width Table 4.1 Finckel has the widest vibrato in this excerpt. Maisky also has very wide vibrato. Shafran's vibrato width is consistently wide. Piatigorsky's vibrato is the narrowest, followed by Rostropovich.

With respects to vibrato speed Table 4.3, Maisky has the fastest vibrato in this excerpt. Ten notes out of fifteen are above 10 oscillations per second. Comparatively, Finckel's vibrato is fast as well. Piatigorsky has the slowest vibrato, with only two notes above ten. Piatigorsky is consistent in numbers of oscillations on notes C3, D3, F3, E3 and A3 (see Table 4.2). Similar to Piatigorsky, Isserlis' vibrato is slow as well and has a consistent number of oscillations; perhaps this is habitual.

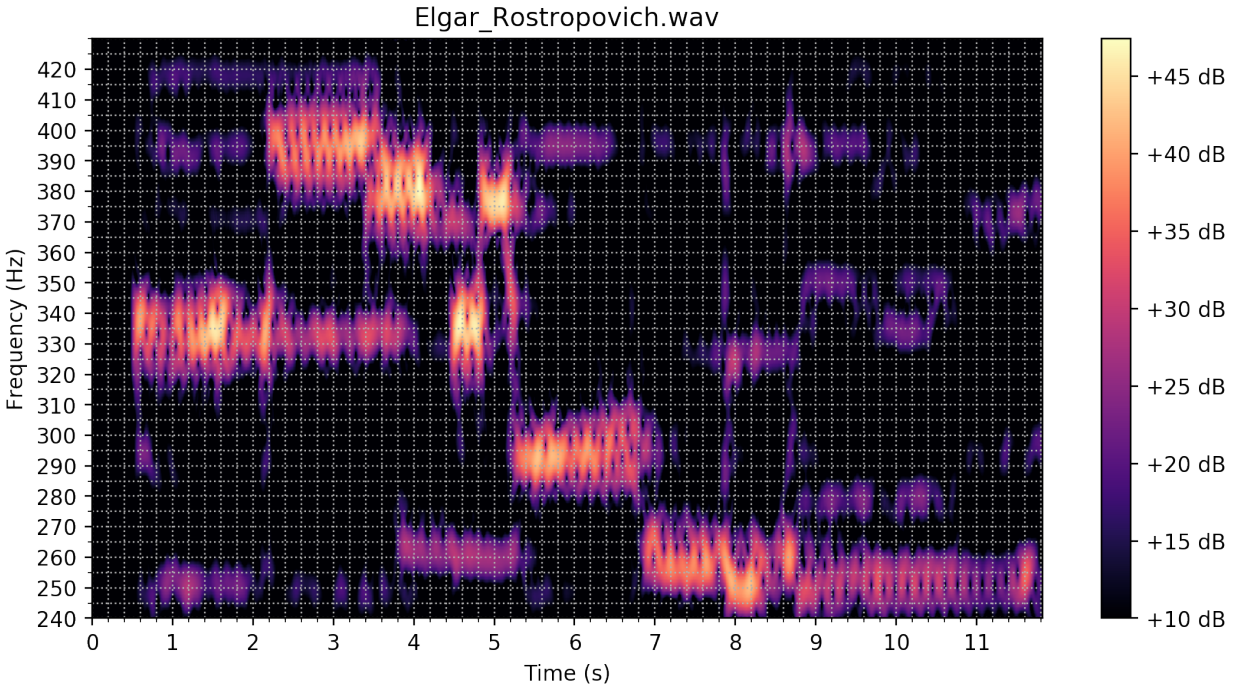
4.2.2 Fast and narrow vibrato



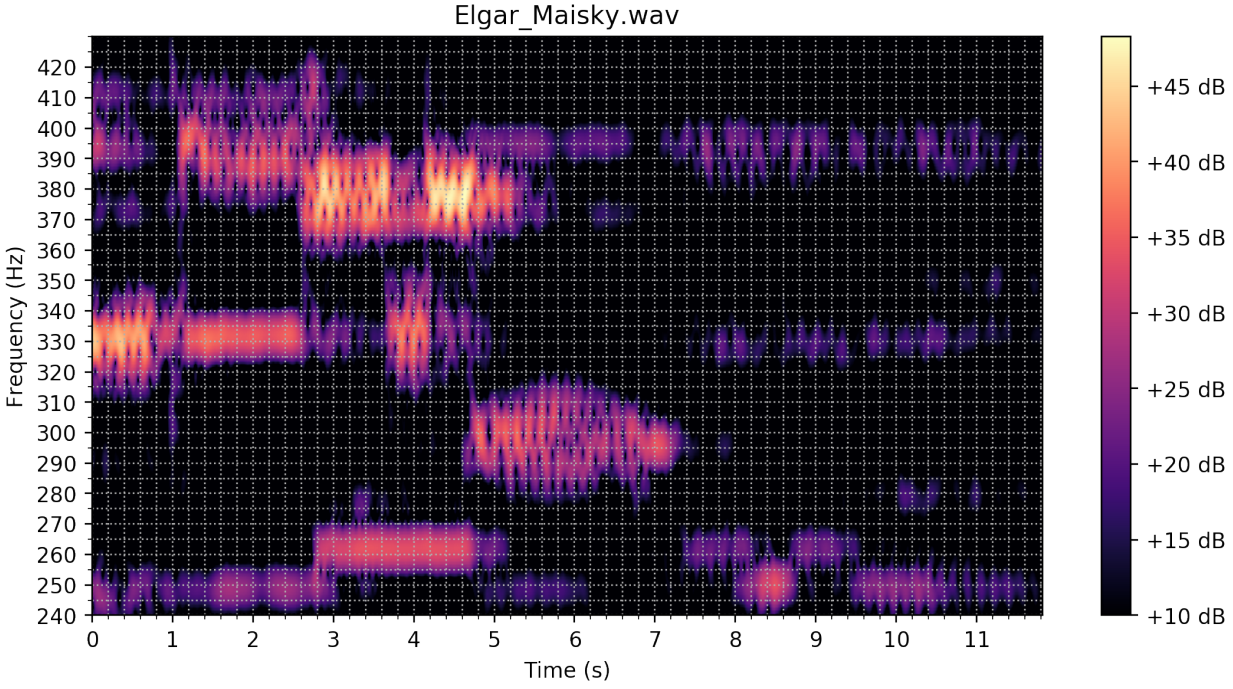
Excerpt 4.2: Elgar Cello Concerto in E Minor, Op. 85, 4th movement, measure 9-11

The dynamic of this excerpt is *forte*, *nobilmente* (this word was invented by Elgar) means nobly, in a noble style. A quicker vibrato will bring a strong and bright sound in the beginning. *Quasi Recit* means recitative-like [1], so cellists do not have to follow the ordinary rhythm. The passage can be performed a bit freely and the melody is on the A-string, a narrower vibrato will make this passage sound exquisite. The frequencies of notes from this excerpt are E4 (329.63 Hz), G4 (392.00 Hz), F#4 (369.99 Hz), E4, F#4, D4 (293.66 Hz), C#4 (277.18 Hz), B3 (246.94 Hz), C#4, B3.

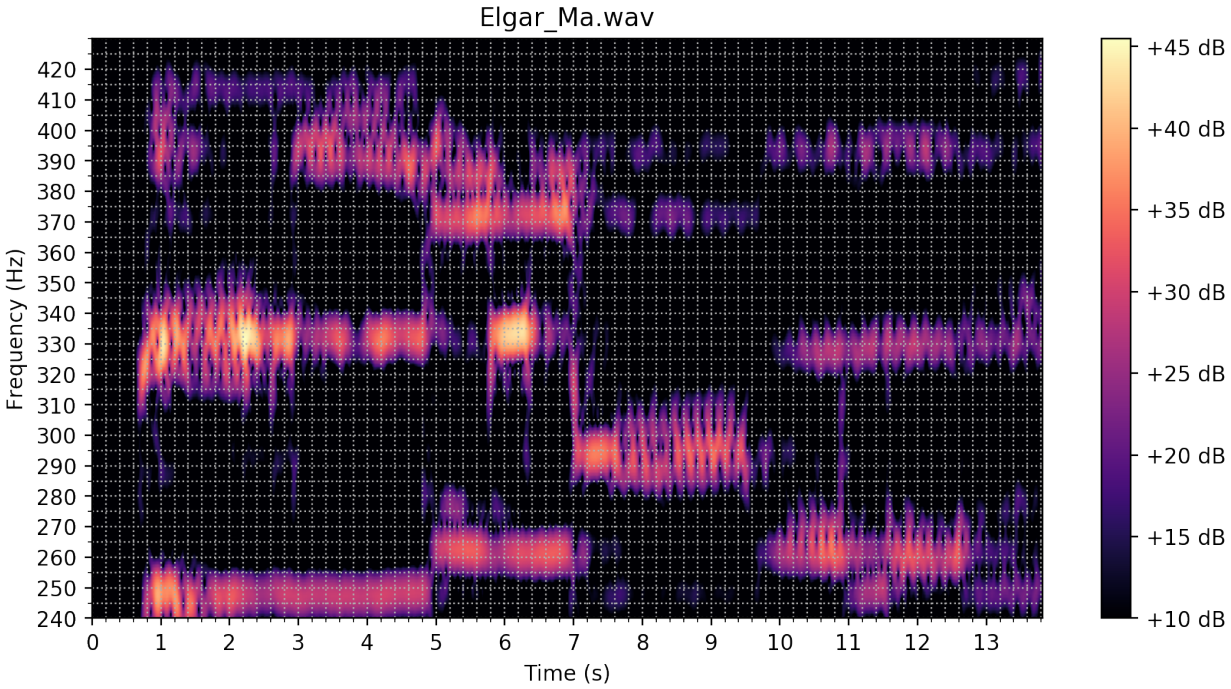
[1] **Recitative** also known by its Italian name "recitativo" is a style of delivery (much used in operas, oratorios, and cantatas) in which a singer is allowed to adopt the rhythms and delivery of ordinary speech. Recitative does not repeat lines as formally composed songs do.



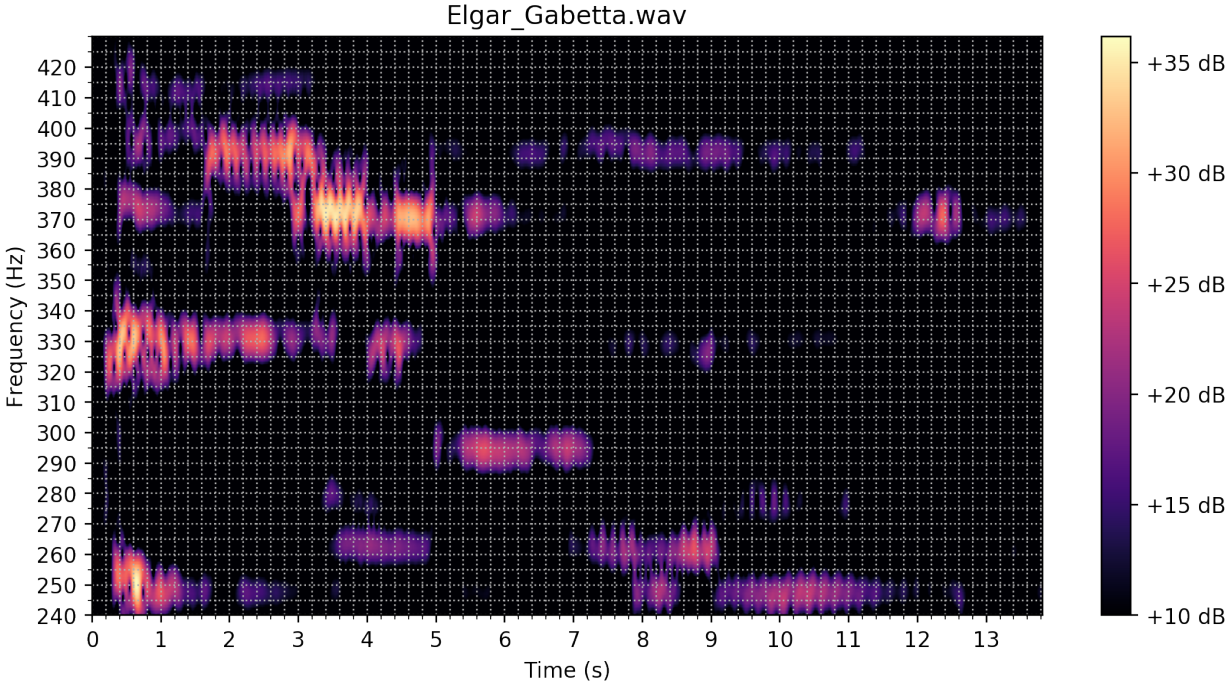
Recording 4.7: Elgar by Rostropovich (1958)



Recording 4.8: Elgar by Maisky (2014)



Recording 4.9: Elgar by Ma (2015)



Recording 4.10: Elgar by Gabetta (2016)

Elgar Width (Hz)	E4	G4	F#4	E4	F#4	D4	C#4	B3	C#4	B3
Rostropovich	30.0	30.0	30.0	40.0	25.0	27.5	20.0	25.0	20.0	15.0
Maisky	35.0	25.0	30.0	20.0	25.0	35.0	12.0	20.0	12.0	18.0
Ma	38.0	20.0	18.0	15.0	18.0	28.0	20.0	15.0	20.0	25.0
Gabetta	20.0	28.0	35.0	15.0	25.0	20.0	18.0	18.0	20.0	24.0
Average	30.8	25.8	28.3	22.5	23.3	27.6	17.5	19.5	18.0	20.5

Table 4.4: Elgar Vibrato Width (Hz) - Individual Notes

In Table 4.4 Ma shows the narrowest vibrato. Rostropovich applies a narrow vibrato as well, while Maisky's vibrato is the widest. Gabetta's vibrato width is inconsistent amongest the others and is difficult to reveal any pattern.

Elgar Oscillations	E4	G4	F#4	E4	F#4	D4	C#4	B3	C#4	B3
Rostropovich	12	11	9	3	5	13	6	5	3	14
Maisky	8	12	11	6	10	15	7	8	6	16
Ma	20	21	12	9	6	22	9	9	8	7
Gabetta	12	11	11	7	7	19	9	8	5	18
Average	13.0	13.8	10.8	6.3	7.0	17.3	7.8	7.5	5.5	13.8

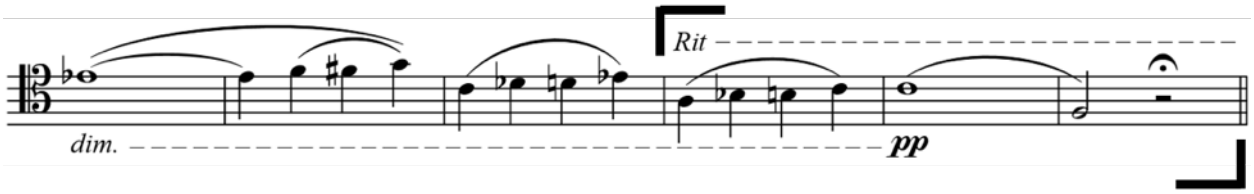
Table 4.5: Elgar Vibrato Oscillations - Individual Notes

Elgar Speed	E4	G4	F#4	E4	F#4	D4	C#4	B3	C#4	B3
Rostropovich	7.1	8.5	7.5	6.0	8.3	7.2	6.0	8.3	5.0	6.4
Maisky	6.7	8.0	9.2	10.0	8.3	5.8	8.8	10.0	7.5	7.3
Ma	8.7	10.5	6.3	9.0	8.6	8.8	6.9	15.0	7.3	6.4
Gabetta	8.6	7.9	9.2	10.0	5.0	8.6	10.0	16.0	6.3	6.0
Average	7.7	8.7	8.0	8.8	7.6	7.6	7.9	12.3	6.5	6.5

Table 4.6: Elgar Vibrato Speed (Oscillations/s) - Individual Notes

In Table 4.5 and Table 4.6, Gabetta and Ma both share incredibly fast vibrato compared to their peers. This is demonstrated in E4, G4 and D4, all of which have durations more than one second.

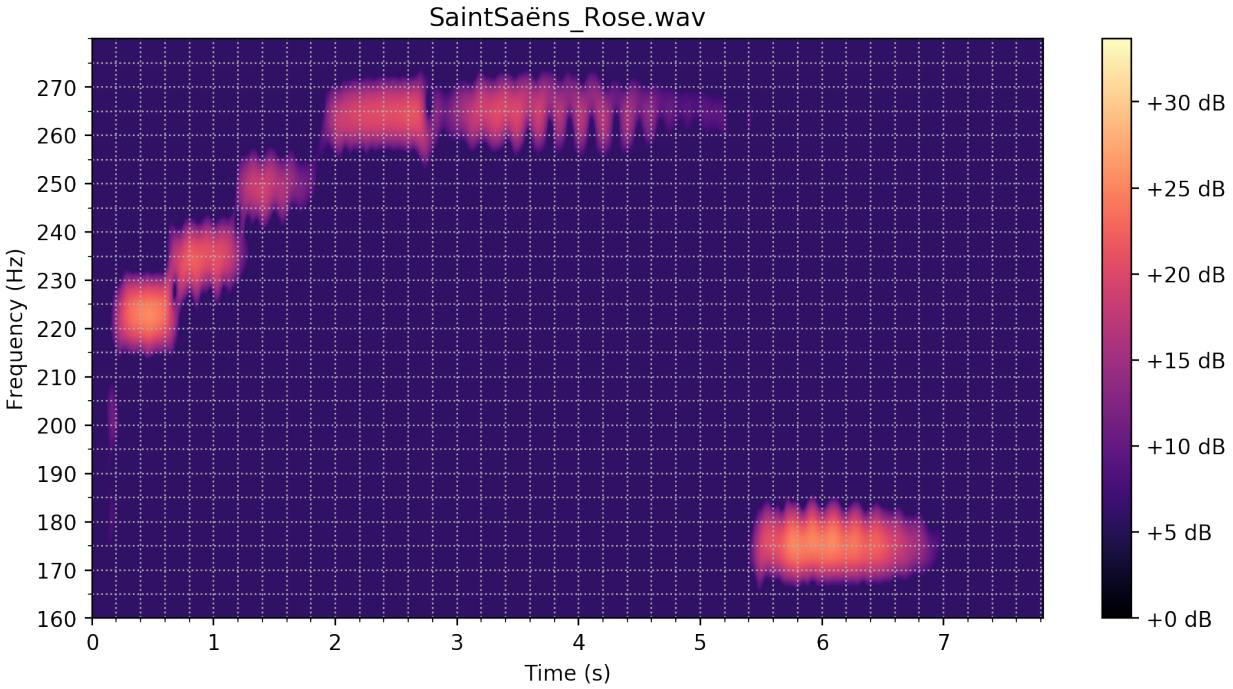
4.2.3 Slow and wide vibrato



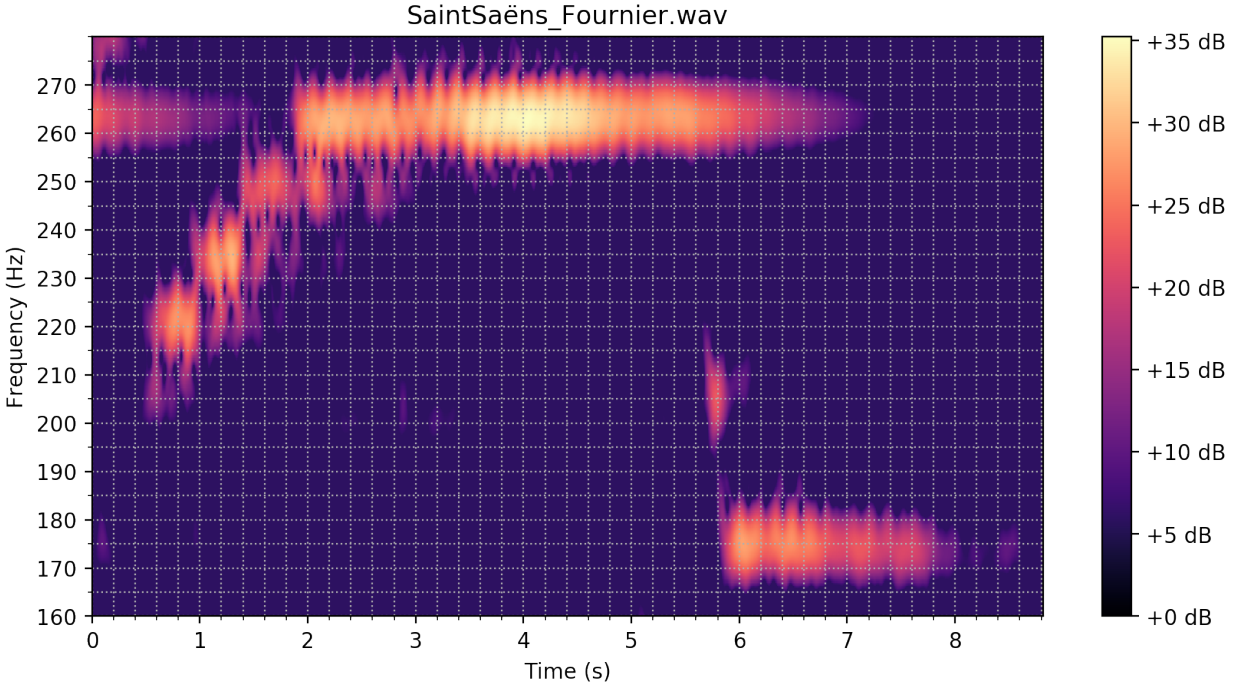
Excerpt 4.3: Saint-Saëns Cello Concerto in A Minor 1st movement, measure 205-207

The tempo of this passage, which comes at the end of the first movement, is *Allegro non troppo*, meaning "fast, but not overly so." The ending part consists of long phrases, some of which phrases are four measures long and are performed in one bow. Measures 206-207 are performed in one bow; the long phrases and *rit* is a tempo marking that means slow down the speed of this passage; *diminuendo* toward to *pianissimo* which requires the performer to decrease the volume, that can be achieved if we gradually slow down the speed of vibrato and make the width wider. This will allow the audience to clearly hear the sound as it disappears toward the end of the passage.

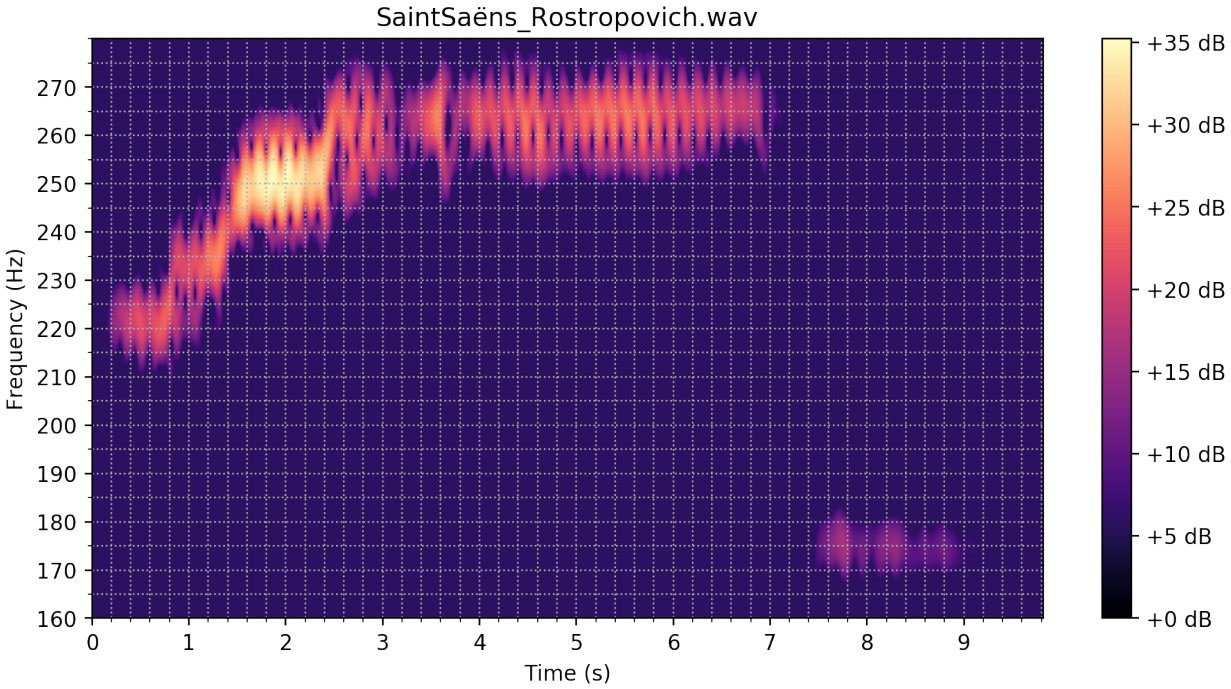
Frequencies in this excerpt are: A3 (220.00 Hz), Bb3 (233.08 Hz), B3 (246.94 Hz), C4 (261.63 Hz), C4, F3 (174.61 Hz)



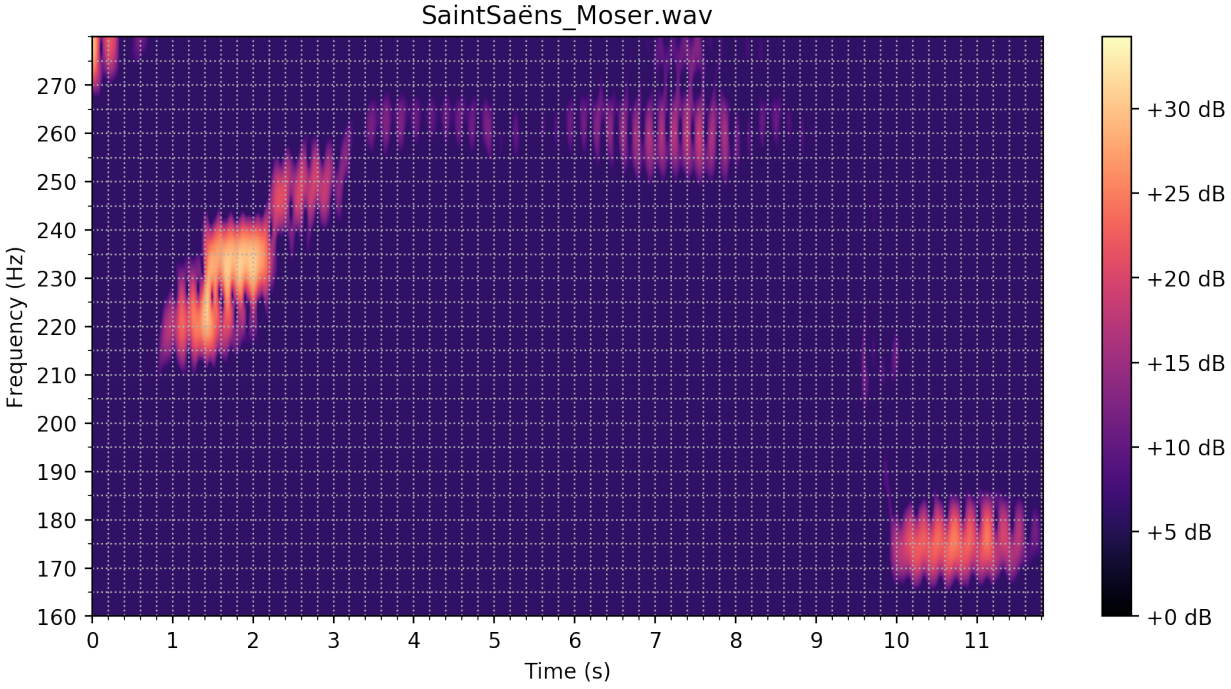
Recording 4.11: Saint-Saëns by Rose (2014)



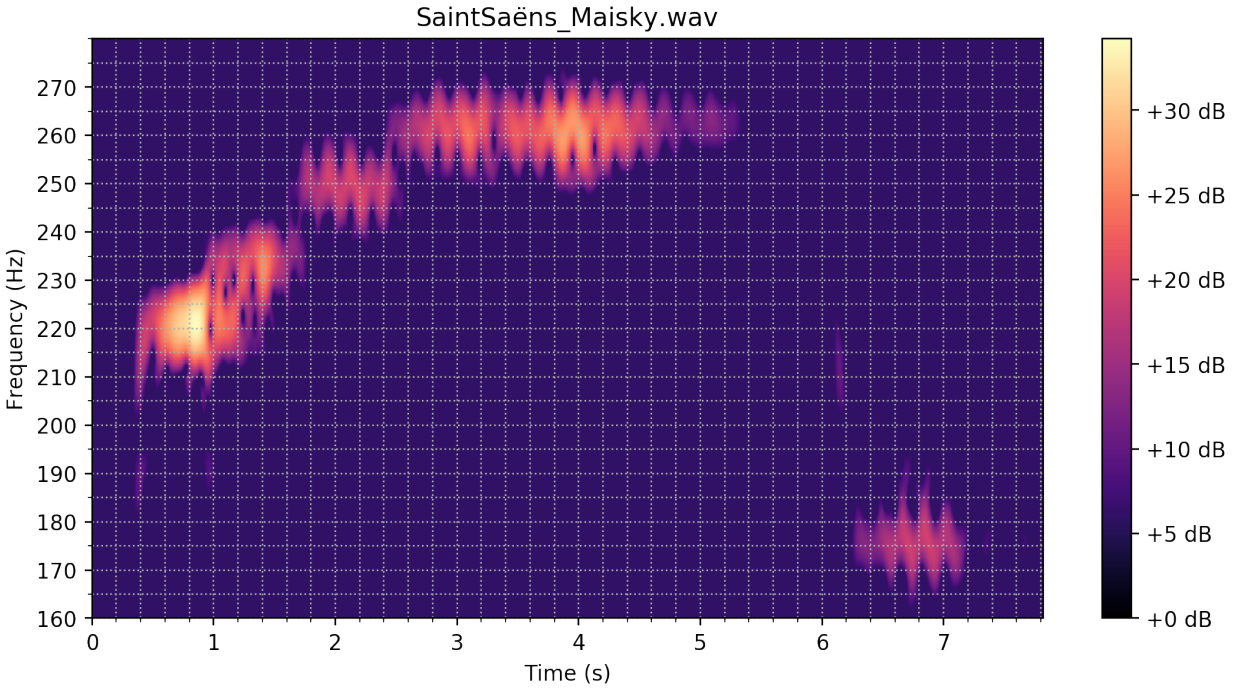
Recording 4.12: Saint-Saëns by Fournier (2012)



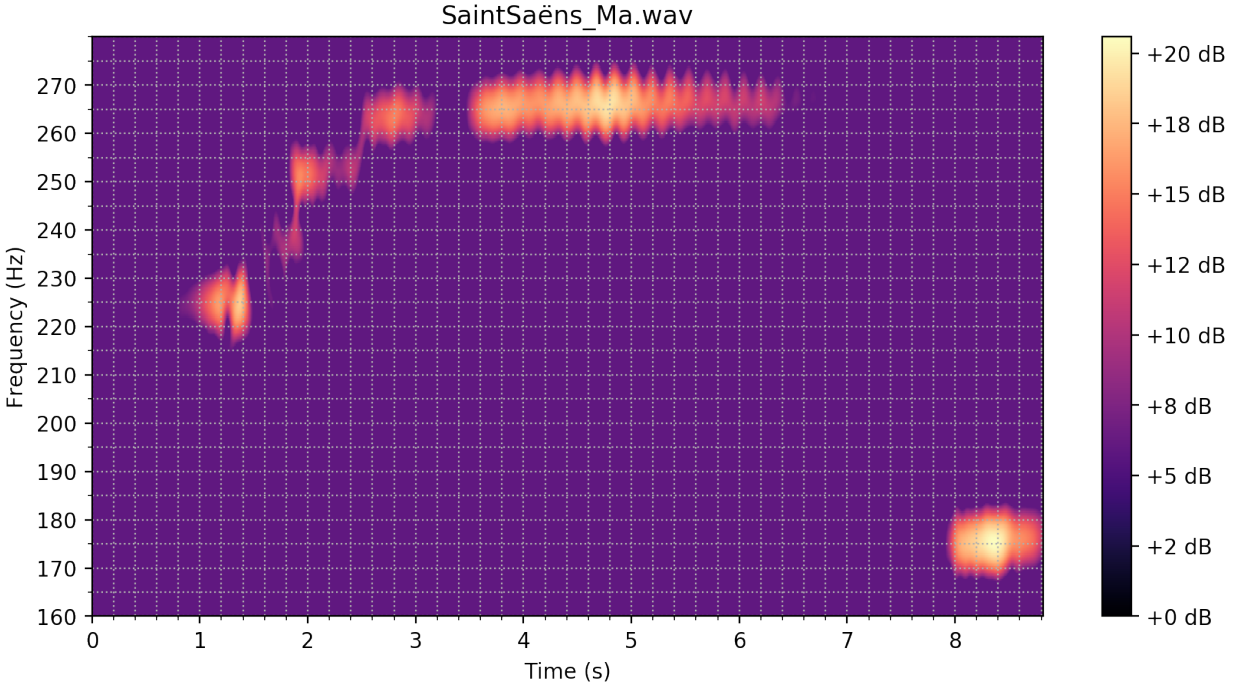
Recording 4.13: Saint-Saëns by Rostropovich (1999)



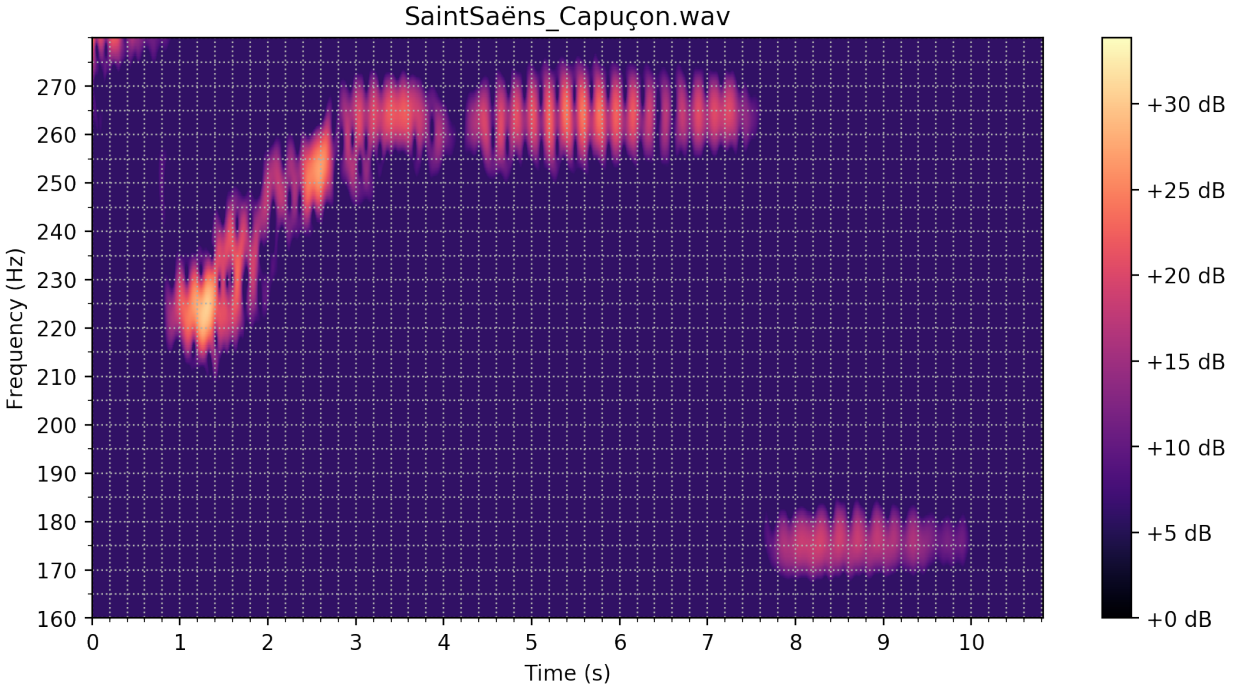
Recording 4.14: Saint-Saëns by Moser (2019)



Recording 4.15: Saint-Saëns by Maisky (2020)



Recording 4.16: Saint-Saëns by Ma (2017)



Recording 4.17: Saint-Saëns by Capuçon (2020)

Saint-Saëns Width (Hz)	A3	Bb3	B3	C4	C4	F3
Rose	17.0	18.0	21.0	9.5	11.5	17.5
Fournier	22.5	27.5	17.5	22.5	17.5	18.5
Rostropovich	17.5	20.0	25.0	22.5	25.0	11.5
Moser	20.0	19.0	20.0	12.5	20.0	20.0
Maisky	20.0	22.5	20.0	22.5	22.5	20.0
Ma	16.0	11.5	12.5	12.5	11.5	10.0
Capuçon	20.0	17.5	16.0	12.5	20.0	19.0
Averge	19.0	19.4	18.9	16.4	18.3	16.6

Table 4.7: Saint-Saëns Vibrato Width (Hz) - Individual Notes

In Table 4.7 Maisky's vibrato width is the widest, Rostropovich's vibrato width is also wide. Moser's width is consistently in the middle. Both Ma and Rose's vibrato widths are narrow in the excerpt.

Saint-Saëns Oscillation	A3	Bb3	B3	C4	C4	F3
Rose	4	6	6	10	14	8
Fournier	4	4	4	7	20	12
Rostropovich	4	6	8	6	23	13
Moser	4	7	6	6	25	10
Maisky	5	6	5	7	11	6
Ma	2	2	6	5	15	5
Capuçon	4	4	5	8	17	12
Average	3.9	5.0	5.7	7.0	17.9	9.4

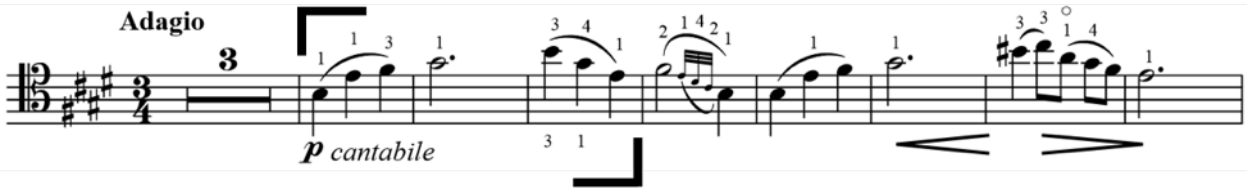
Table 4.8: Saint-Saëns Vibrato Oscillations - Individual Notes

Saint-Saëns Speed	A3	Bb3	B3	C4	C4	F3
Rose	8.0	8.6	10.0	11.1	5.8	5.3
Fournier	8.0	8.0	8.0	7.8	4.5	6.0
Rostropovich	6.7	10.0	8.0	5.0	7.0	5.4
Moser	3.3	8.8	6.0	3.0	7.4	5.0
Maisky	8.3	8.6	6.3	7.0	5.8	5.0
Ma	3.3	5.0	8.6	7.1	4.8	5.6
Capuçon	4.0	6.7	6.3	6.2	5.0	5.7
Average	6.0	7.9	7.6	6.7	5.8	5.4

Table 4.9: Saint-Saëns Vibrato Speed (Oscillations/s) - Individual Notes

Table 4.9 demonstrates that Capuçon and Moser played with the slowest vibrato in the excerpt. Ma's vibrato is almost as slow as Capuçon and Moser's. Rose and Ma both increased their vibrato speeds on A3, Bb3 and B3, which are ascending chromatic notes. The increasing of speed might show the tendency of the phrase. Rose and Ma may have wanted to give the phrase a forward moving direction toward C4. Conversely, with the exception of Rose and Maisky, all other cellists have decreased their vibrato speeds on Bb3, B3 and C4, I assume that these cellists wanted to slow down the vibrato speed to match the dynamic (*diminuendo*) and tempo (*ritardando*).

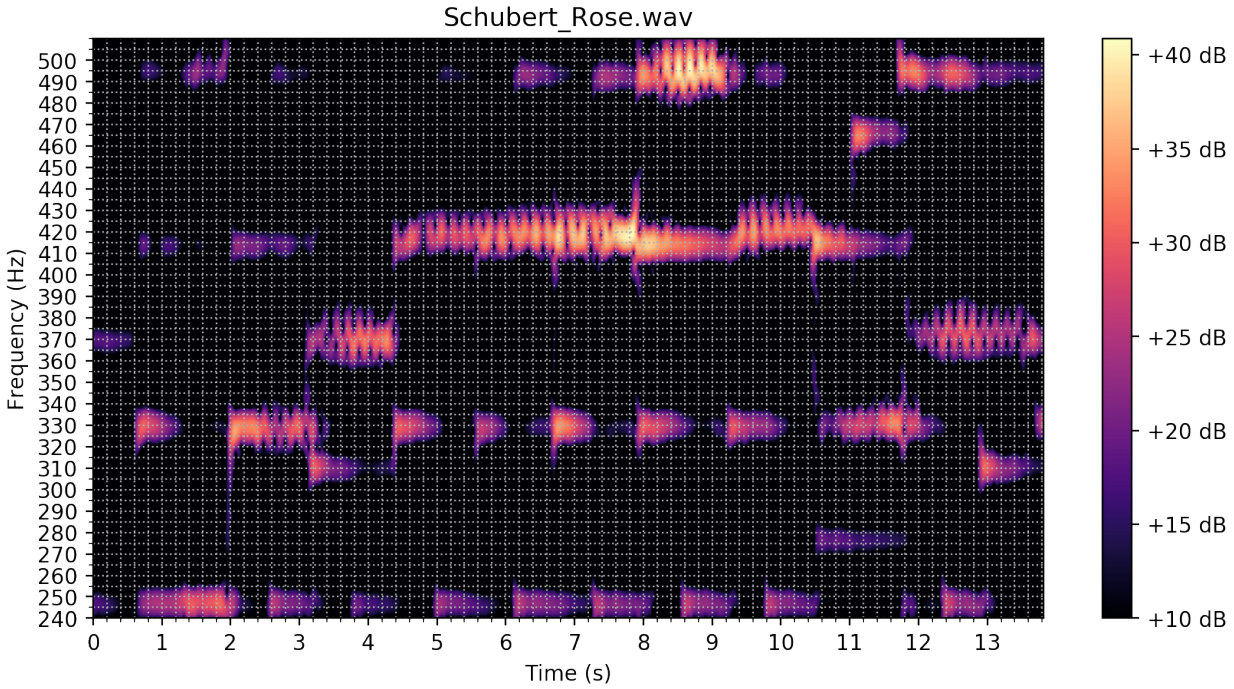
4.2.4 Slow and narrow vibrato



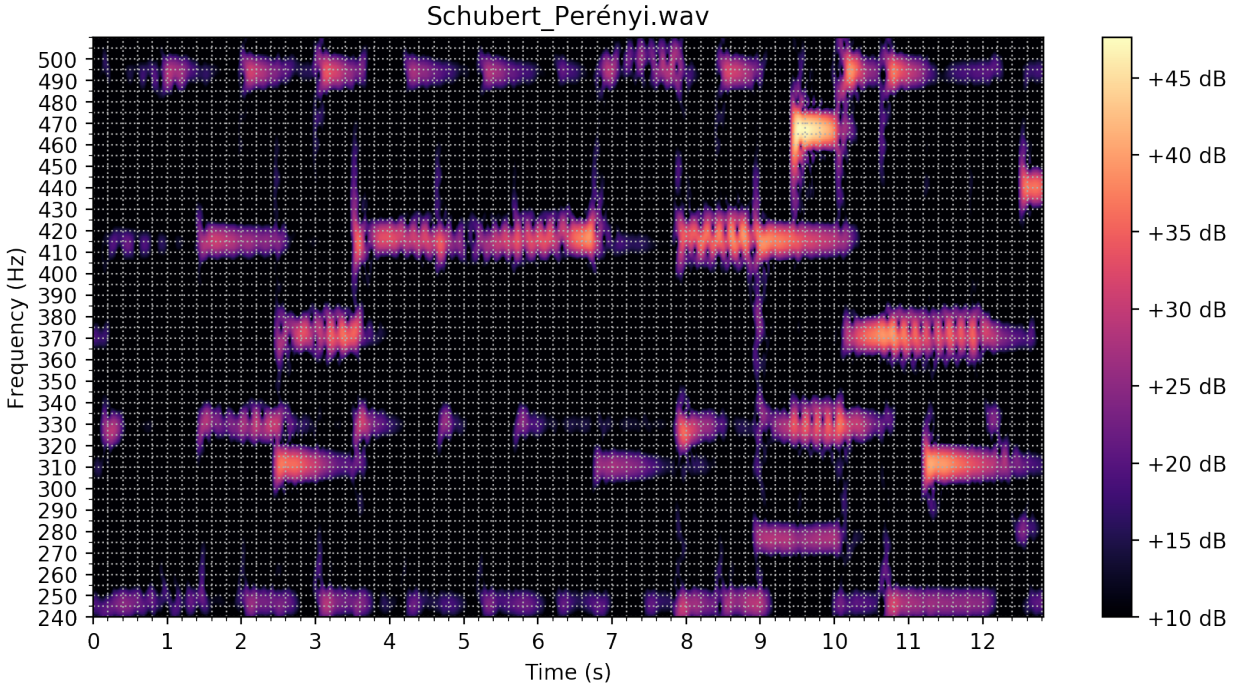
Excerpt 4.4: Schubert Arpeggione in A Minor, D 821, 2nd movement, measure 4-5

In this excerpt, the tempo is *Adagio*, which means slowing, and dynamic is *piano*. I would thus apply a slower vibrato. The melody is in a brighter string (A-string); thus, I would apply a narrower vibrato to achieve a light and clearer sound. A wider vibrato would blur the articulation.

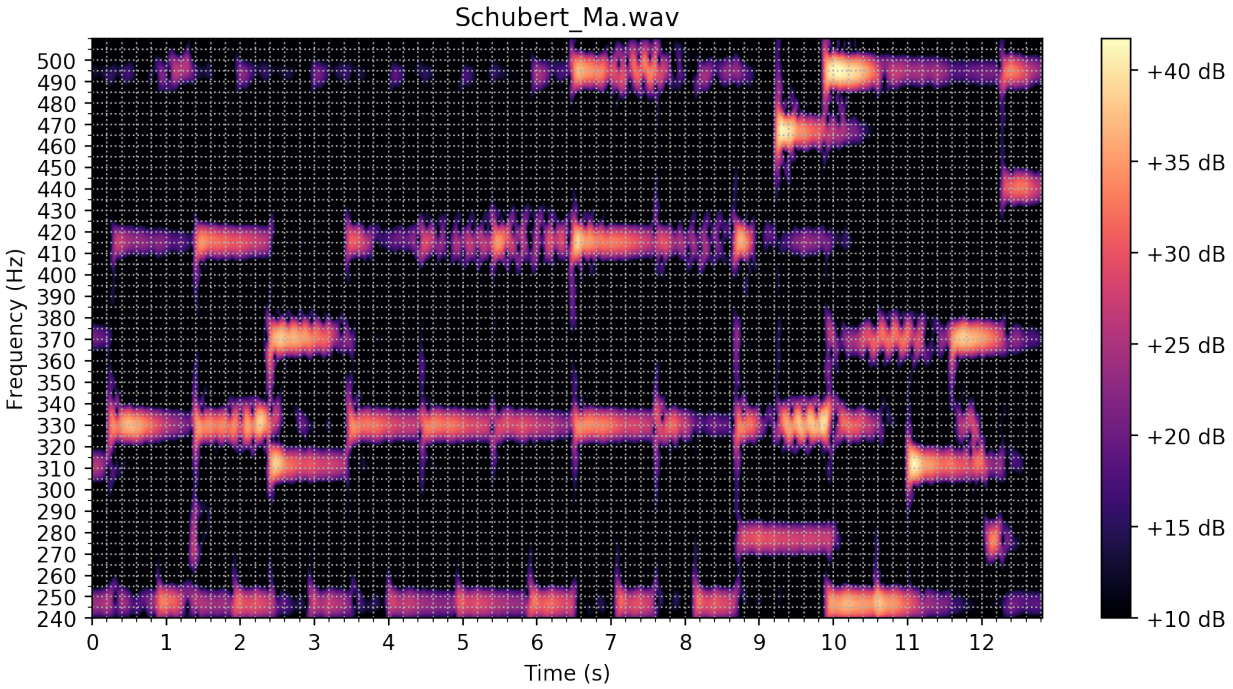
Frequencies in this excerpts are B3 (246.94 Hz), E4 (329.63 Hz), F#4 (369.99 Hz), G4 (392.00 Hz), B4 (493.88 Hz), G5, E3, F#4



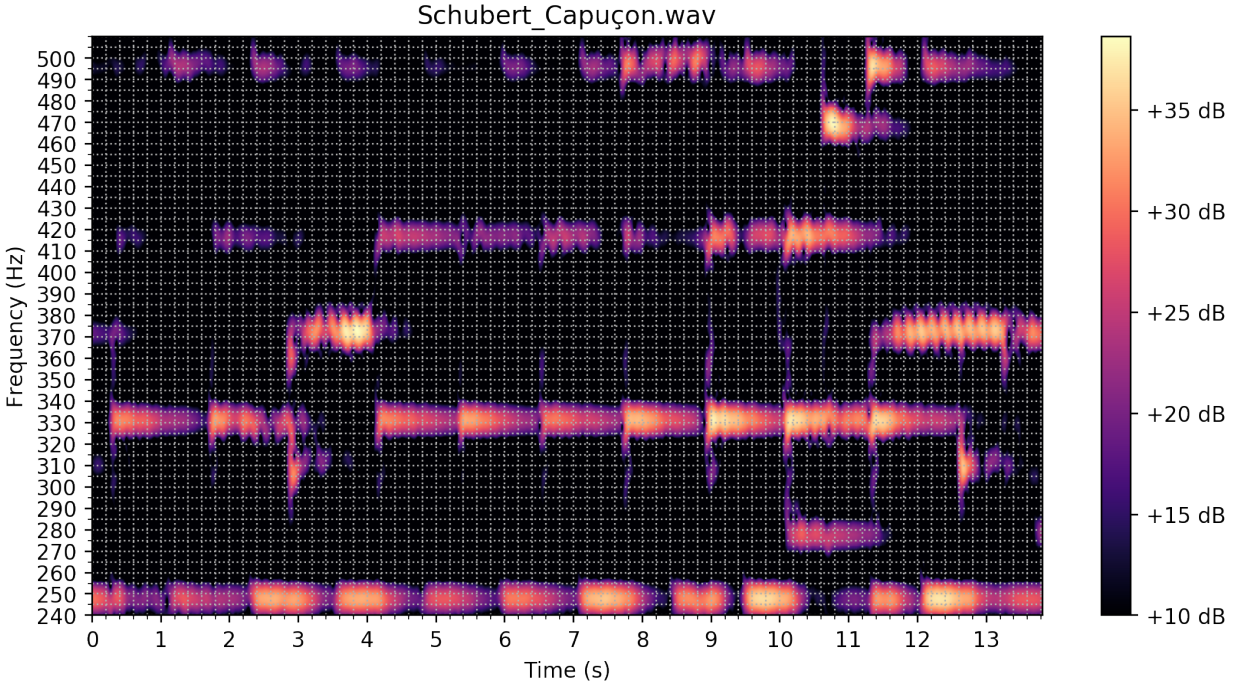
Recording 4.18: Schubert by Rose (2013)



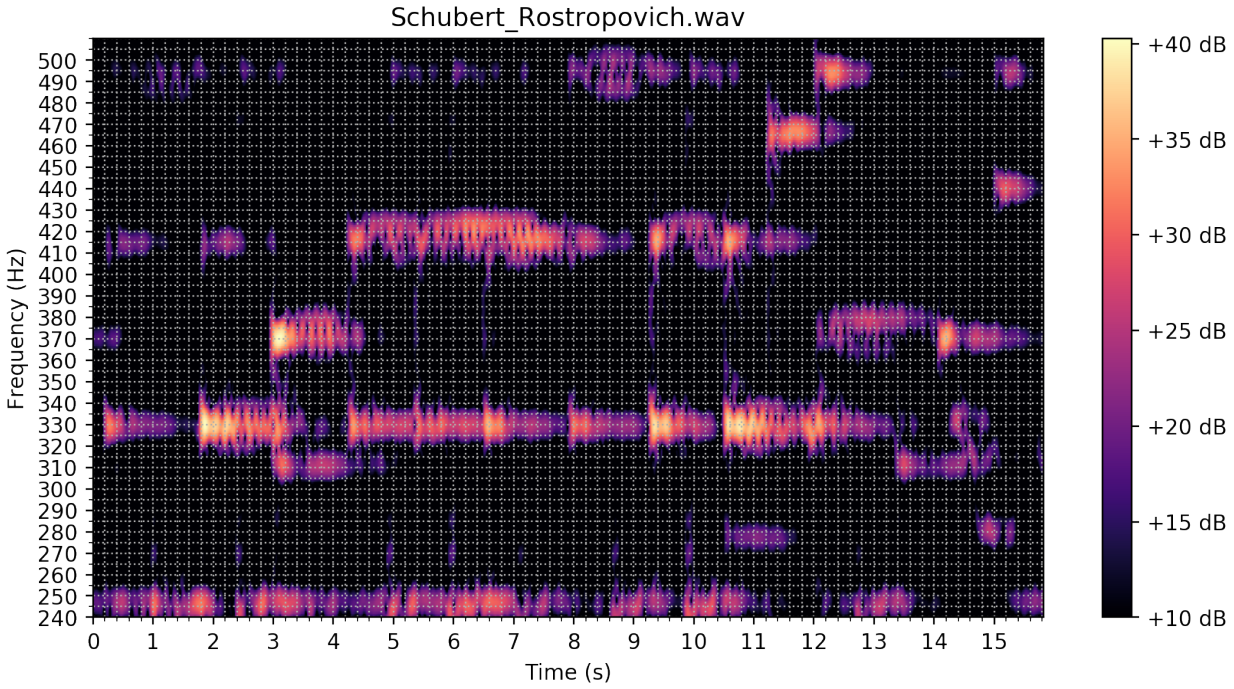
Recording 4.19: Schubert by Pérenyi (2011)



Recording 4.20: Schubert by Ma (2014)



Recording 4.21: Schubert by Capuçon (2015)



Recording 4.22: Schubert by Rostropovich (2015)

Schubert Width (Hz)	B3	E4	F#4	G#4	B4	G#4	E4	F#4
Rose	14.0	20.0	23.0	20.0	25.0	22.0	18.0	26.0
Perényi	16.0	20.0	24.0	20.0	20.0	26.0	20.0	20.0
Ma	15.0	20.0	20.0	22.0	20.0	26.0	20.0	20.0
Capuçon	15.0	20.0	24.0	18.0	20.0	18.0	18.0	20.0
Rostropovich	15.0	30.0	25.0	25.0	25.0	25.0	30.0	25.0
Average	15.0	22.0	23.2	21.0	22.0	23.4	21.2	22.2

Table 4.10: Schubert Vibrato Width (Hz) - Individual Notes

In Table 4.10, Rose's vibrato is the narrowest in the excerpt. Ma's vibrato width is narrow as well. Rostropovich's width is the widest, five out of eight notes are the widest among his peers. Perényi's vibrato width is consistently in the middle.

Schubert Oscillation	B3	E4	F#4	G#4	B4	G#4	E4	F#4
Rose	1	6	8	20	7	7	9	11
Perényi	5	6	6	21	7	6	7	9
Ma	2	4	5	20	5	8	4	6
Capuçon	4	5	6	16	12	3	7	11
Rostropovich	8	8	9	27	10	10	12	11
Average	4.0	5.8	6.8	20.8	8.2	6.8	7.8	9.6

Table 4.11: Schubert Vibrato Oscillations - Individual Notes

Schubert Speed	B3	E4	F#4	G#4	B4	G#4	E4	F#4
Rose	0.7	5.0	6.2	6.1	5.0	7.0	7.5	5.5
Perényi	3.3	5.0	5.0	6.4	5.8	6.0	5.0	3.5
Ma	1.4	3.3	4.2	6.7	4.2	8.0	2.9	2.5
Capuçon	2.9	3.3	3.8	4.8	10.0	3.0	4.7	4.4
Rostropovich	4.0	4.4	6.4	7.9	7.7	8.3	7.1	5.5
Average	2.5	4.2	5.1	6.4	6.5	6.5	5.4	4.3

Table 4.12: Schubert Vibrato Speed (Oscillations/s) - Individual Notes

In Table 4.12, Ma performed the slowest vibrato in the excerpt. Capuçon applied a very slow vibrato as well, but on B4, Capuçon is faster than everyone else. Rostropovich's vibrato is the fastest. Besides Rose, other cellists all increased their vibrato speeds on B3, E4, F#4 and G#4, which show a tendency that when notes go higher cellists intend to increase the vibrato speed; all cellists decreased their vibrato speed on G#4, E4 and F#4 when the notes are progressing to lower pitches.

4.3 Single Note Analysis

In order to observe the spectrogram better and more accurately so as to get a clear data understanding on the speed and width of the vibrato. I also chose one note from each excerpt to explain the differences among the performances; and also in the use of vibrato in different circumstances. I specifically chose notes longer than one second where the intent of vibrato is very obvious. These notes are G3 from Prokofiev, G#4 from Schubert, F3 from Saint-Saëns, and G4 from Elgar.

Prokofiev G3 Width	Low (Hz)	High (Hz)	Difference (Hz)	Interval Difference (cents)
Maisky	185.0	210.0	25.0	219.4
Shafran	185.0	215.0	30.0	260.2
Finckel	183.5	212.5	29.0	254.0
Piatigorsky	192.5	215.0	22.5	191.4
Rostropovich	180.0	202.5	22.5	203.9
Isserlis	185.0	205.0	20.0	177.7
Average			24.8	217.8

Table 4.13: Vibrato Width on G3 by Various Cellist in Prokofiev

Prokofiev G3 Speed	Oscillations	Duration (s)	Oscillations per second
Maisky	15.0	2.3	6.7
Shafran	19.0	2.5	7.6
Finckel	11.0	1.8	6.3
Piatigorsky	7.0	1.0	7.0
Rostropovich	14.0	2.0	7.0
Isserlis	10.0	1.5	6.7
Average			6.9

Table 4.14: Vibrato Speed on G3 by Various Cellist in Prokofiev

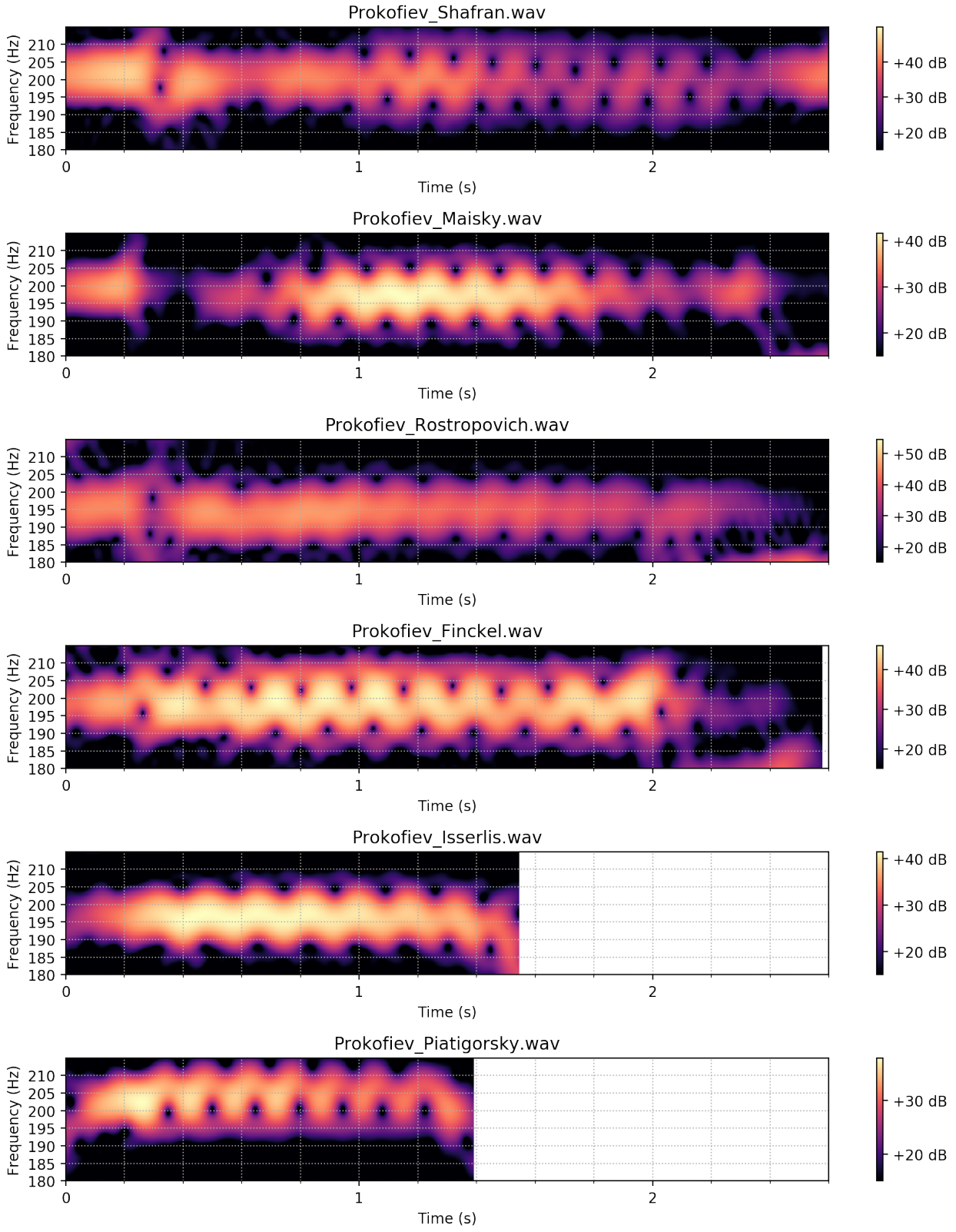


Figure 4.5: Prokofiev Excerpt G3 by Various Cellists

Elgar G4	Low (Hz)	High (Hz)	Difference (Hz)	Interval Difference (cents)
Rostropovich	380.0	410.0	30.0	131.5
Maisky	375.0	412.5	37.5	165.0
Ma	380.0	410.0	30.0	131.5
Gabetta	385.0	400.0	15.0	66.2
Average			28.1	123.6

Table 4.15: Vibrato Width on G4 by Various Cellist in Elgar

Elgar G4	Oscillations	Duration (s)	Oscillations per second
Rostropovich	11.0	1.3	8.8
Maisky	10.0	1.5	6.7
Ma	13.0	2.0	6.5
Gabetta	11.0	1.5	7.3
Average			7.3

Table 4.16: Vibrato Speed on G3 by Various Cellist in Elgar

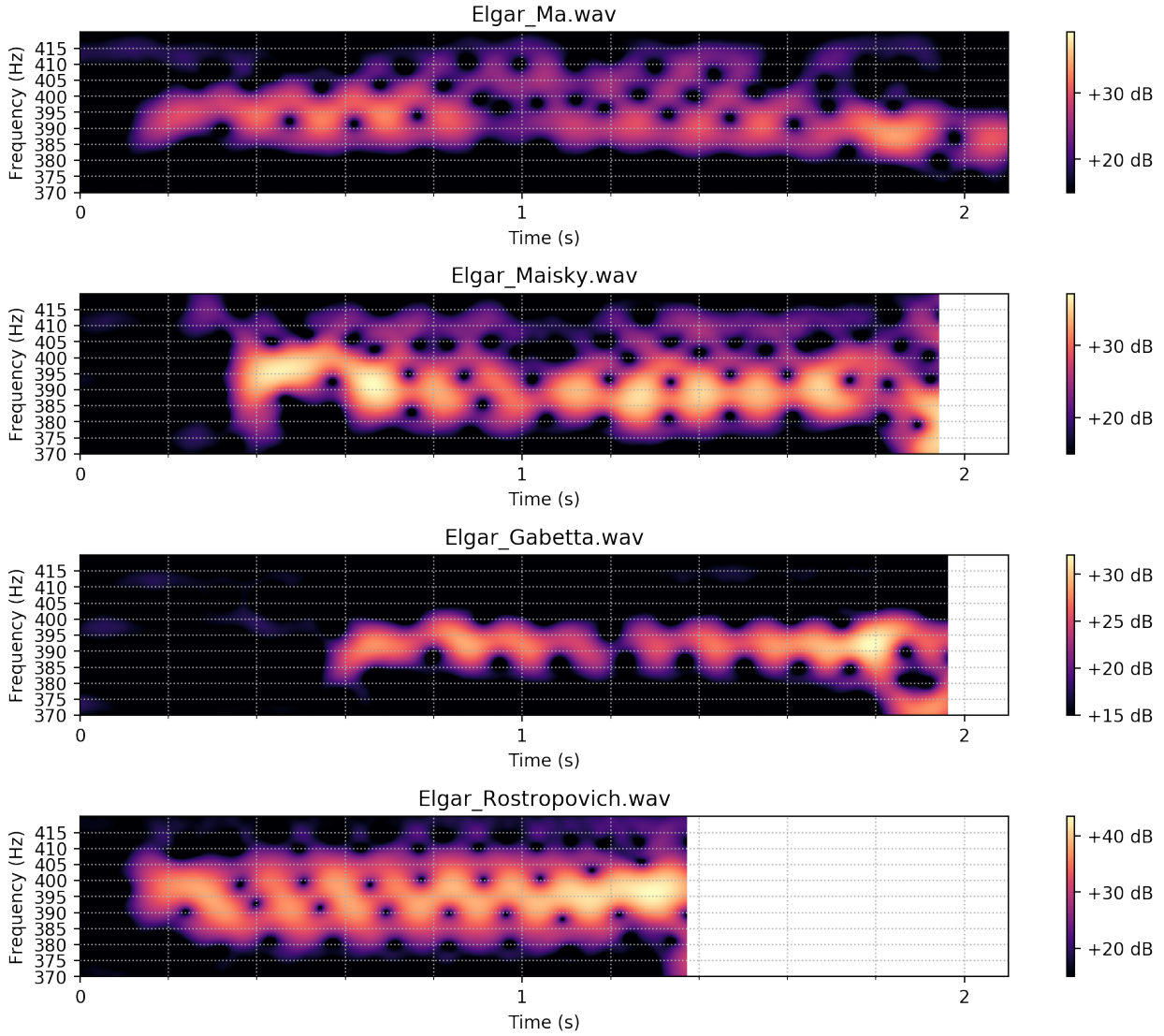


Figure 4.6: Elgar Excerpt G4 by Various Cellists

Saint-Saëns F3	Low (Hz)	High (Hz)	Difference (Hz)	Interval Difference (cents)
Rose	167.5	185.0	17.5	172.0
Capuçon	167.5	185.0	17.5	172.0
Fournier	165.0	185.0	20.0	198.1
Moser	165.0	185.0	20.0	198.1
Rostropovich	168.5	182.5	14.0	138.2
Ma	168.0	182.5	14.5	143.3
Maisky	165.0	187.5	22.5	221.3
Average			18.0	177.6

Table 4.17: Vibrato Width on F3 by Various Cellist in Saint-Saëns

Saint-Saëns F3	Oscillations	Duration (s)	Oscillations per second
Rose	8.0	1.5	5.3
Capuçon	11.0	2.1	5.2
Fournier	12.0	2.0	6.0
Moser	10.0	2.0	5.0
Rostropovich	13.0	2.4	5.4
Ma	5.0	0.9	5.6
Maisky	6.0	0.9	6.7
Average			5.7

Table 4.18: Vibrato Speed on F3 by Various Cellist in Saint-Saëns

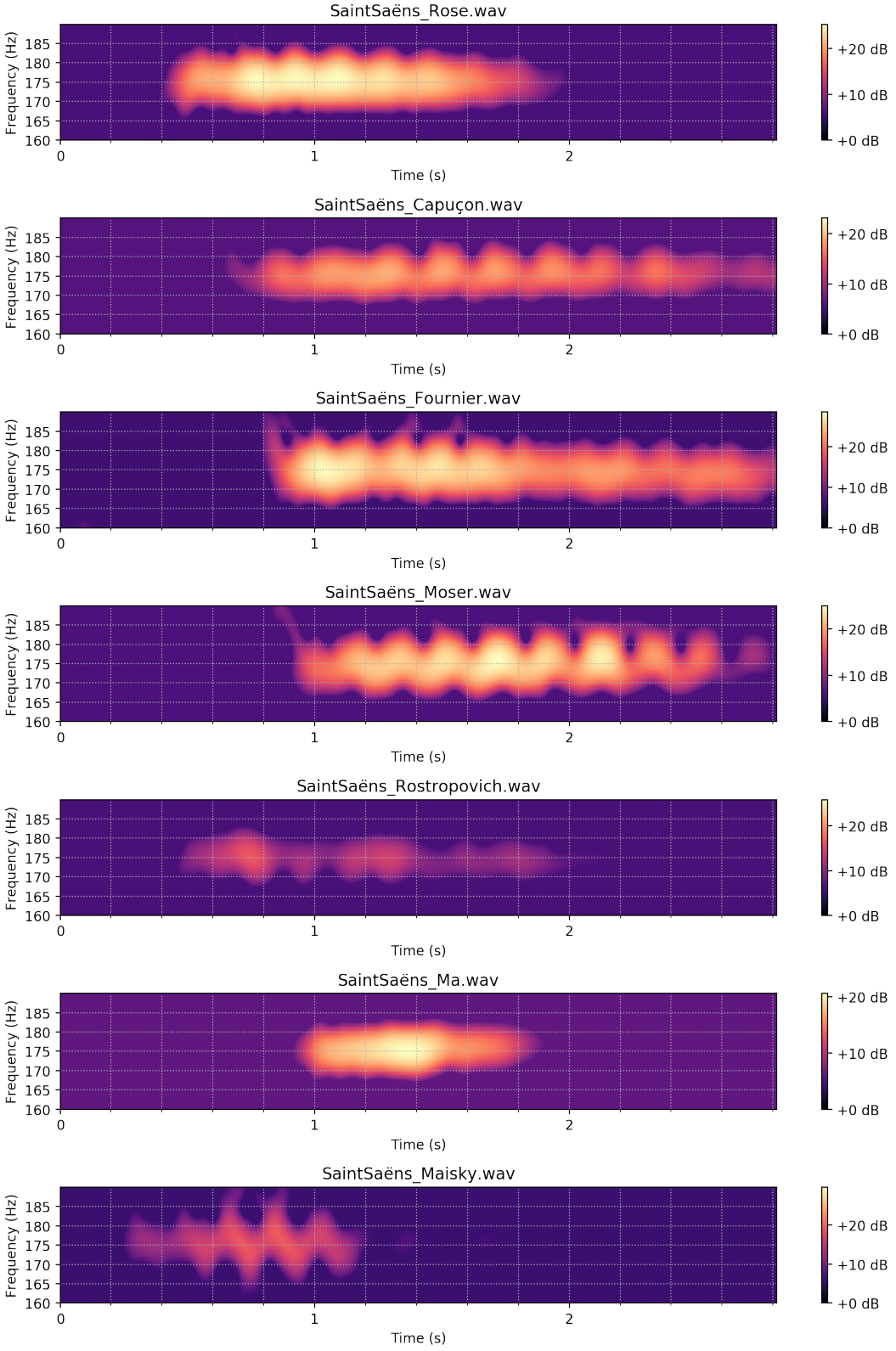


Figure 4.7: Saint-Saëns Excerpt F3 by Various Cellists

Schubert G#4	Low (Hz)	High (Hz)	Difference (Hz)	Interval Difference (cents)
Rose	410.0	430.0	20.0	82.5
Perényi	405.0	430.0	25.0	103.7
Ma	400.0	432.5	32.5	135.2
Capuçon	410.0	425.0	15.0	62.2
Rostropovich	402.5	430.0	27.5	114.4
Average			24.0	99.6

Table 4.19: Vibrato Width on G#4 by Various Cellist in Schubert

Schubert G#4	Oscillations	Duration (s)	Oscillations per second
Rose	20.0	3.3	6.2
Perényi	21.0	3.3	6.5
Ma	20.0	3.0	6.7
Capuçon	16.0	3.3	4.9
Rostropovich	27.0	3.4	7.9
Average			6.4

Table 4.20: Vibrato Speed on G#4 by Various Cellist in Schubert

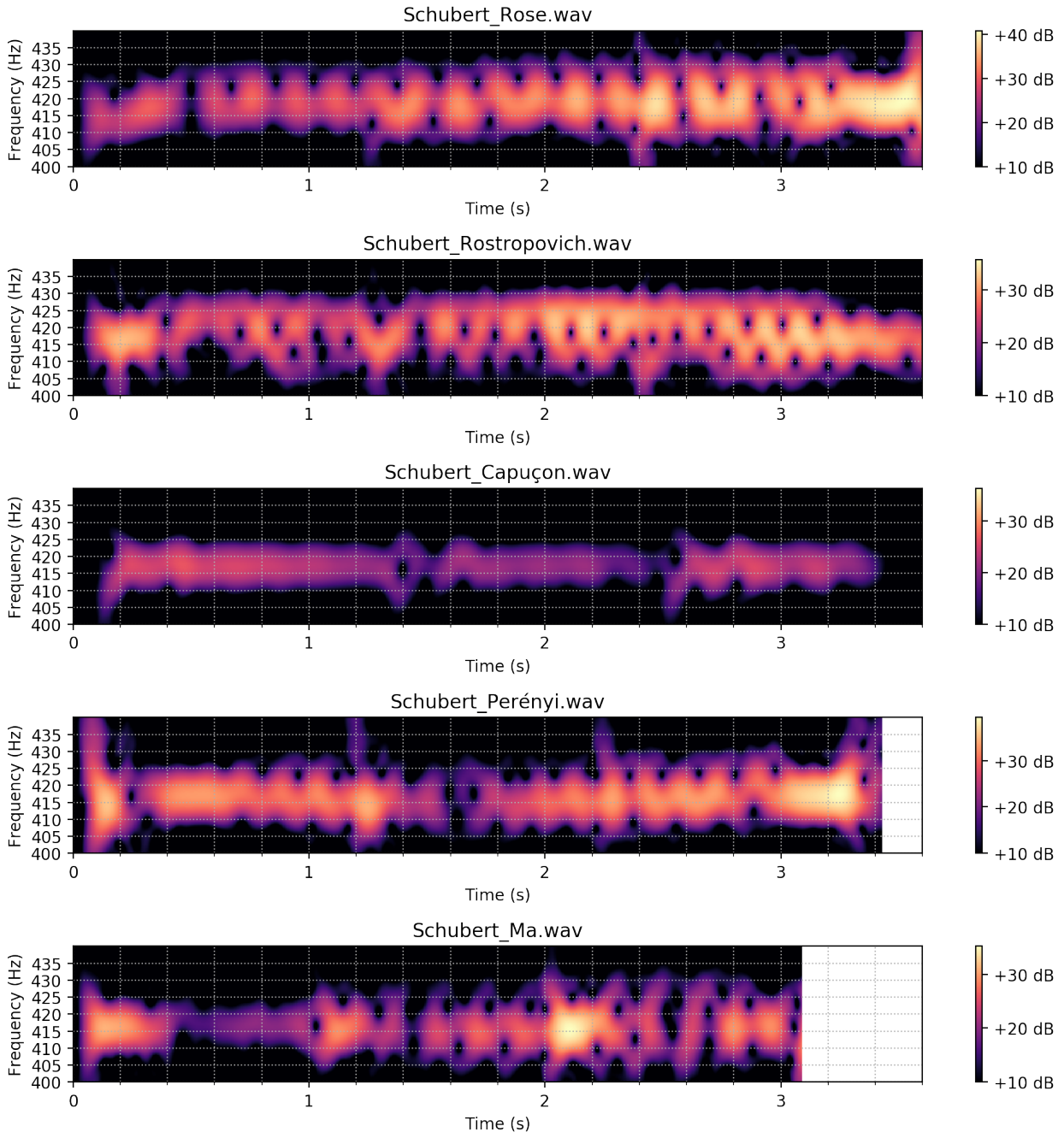


Figure 4.8: Schubert Excerpt G#4 by Various Cellists

4.4 Results and Observations

4.4.1 Vibrato Speed

I took the average speed of the musicians' vibrato by calculating oscillations per second. For pieces with faster vibrato, oscillations tend to occur closer to seven instances per second. For pieces with slower vibrato, oscillations tend to occur closer to six instances per second. This is consistent with Louis Porter Jr.'s observations (Potter, 1980). While the difference between 6 and 7 does not intuitively feel very big, however, if these were tempos, then six beats per second would be 360 bpm, and seven beats per second would be 420 bpm. From a musician's perspective, this difference would be significant. It is hard to execute vibrato faster than eight oscillations per second; such a speed may be impacted by the physical limitation of the musician.

4.4.2 Vibrato Width

A linear difference in frequency does not represent a linear difference in pitch. A constant fluctuation in frequency for different notes can sound drastically different. For example, a fluctuation of 10 Hz for C1 would be the range between F0 to E1. But on C3 the same 10 Hz fluctuation is between B2 and C#3. This is because the frequencies of notes increase exponentially rather than linearly as we go up the scale. Which is consistent with the critical bands defined for human hearing. Therefore, to accurately measure vibrato width for different notes along the scale, we need to normalize the difference in frequency with the relative frequency range of the note. A good way to normalize frequency is to use cents to denote musical interval units.

$$Difference_{interval} = 1200 \cdot log_2(\frac{f_{high}}{f_{low}}) \tag{4.1}$$

(Müller, 2015)

In Saint-Saëns, for example, the average width of F3 among the cellists is 18.6 Hz. Comparing this average width of the G4 in Elgar, 28.1 Hz, it would seem that the Saint-

Saëns' vibrato is narrower. However, when we converted into cents, the F3 vibrato width is 183.3 cents wide, whereas the G4 vibrato is 123.6 cents wide. Hence, the Saint-Saëns' vibrato is actually wider than that of Elgar.

Based on these findings, the four excerpts match our prior expectations of vibrato speed and width.

4.5 Recommendations

Apply fast and wide vibrato when a passage:

- requires strong intensity
- is lounder in volume (*forte* or *fortissimo*)
- is played on the C- or G-string
- creates constant tension in its phrase direction

Apply fast and narrow vibrato when a passage:

- is in a long phrase or melody
- is a series of short notes within one phrase
- is in higher registers
- is in an emotionally intense section

Apply slow and wide vibrato when a passage:

- is at the beginning or ending of a phrase
- is soft in dyanmic (*piano* or *pianissimo*)

- is decreasing in intensity
- is slowing down
- has intent to create a peaceful and relaxed sound

Apply slow and narrow vibrato when a passage:

- is soft in dyanmic (*piano* or *pianissimo*)
- is in a slow section
- is in higher registers
- is on the A- or D-string
- is in the beginning of a section

4.6 Discussions

This chapter verifies that a musician's perspective of a piece can be expressed in part through different types of vibrato, as reflected in the performances of famous cellists. However, each cellist has his or her own style, preference of applying vibrato, and interpretation of various types and styles of music. Despite such differences, this study reveals that well-known cellists share a common understanding of vibrato techniques, whether consciously or unconsciously.

However, there are exceptions even among famous cellists. For instance, Rostropovich's vibrato is fast. His vibrato speed is faster than that of his peers in three out of four excerpts. Rostropovich's vibrato is narrow in fast pieces and wide in slow pieces. Another example is Maisky, whose vibrato is extremely fast. In Prokofiev (Table 4.3) and Elgar (Table 4.6), he consistently reaches speeds up to ten oscillations per second.

There may also be discrepancies caused by the method used to count and measure vibrato speed and width.

When measuring vibrato speed, certain notes show abnormally fast speeds, for example C3 from Prokofiev (Table 4.3). This happens to be a note that is played on the open string where one would not expect to see any oscillation within the visualization. However, perhaps due to the instrument's own pitch fluctuation, there are many micro oscillations within the visualization that happen between a short period. We can notice similar trends with other notes where the performer intends not to vibrate; for instance in Rose's performance of A3 and the first C4 in Saint-Saëns Recording 4.11. This also occurs in shorter notes where the intent of vibrato is less clear.

The volume of each recording is different, and, while it does not affect the visualization of vibrato speed, it does make vibrato width difficult to measure. When we measure width, low volume in the recordings can cause darker regions with less visual contrast. This may lead to higher likelihood of human error or less consistency while measuring.

Moreover, the presence of other instruments, like piano and other orchestral instruments, can introduce notes that are not supposed to be measured in the visualization. We cannot remove their sound, and their presence will obscure the cello pitch from being measured reliably, especially when the other instruments perform the same note or harmonic notes at the same time.

CONCLUSION

The purpose of this thesis is to provide a thorough learning guide for vibrato to cello educators and students. It introduces how vibrato was first conceived on the cello, the selective use of vibrato in earlier periods to its wide acceptance in the 20th century, to it becoming an essential technique of all string instruments. It also brings to readers new insights through the observations of the vibrato speed and width via the signal processing technique of well-known cellists. I hope that through my method I am able to enlighten readers on the performance of various types of vibrato, through the exercises that I have provided for practice, and through the excerpts that I have quantified in order to establish a common understanding of vibrato application. Through these efforts, I am to enable students to perform in an authentic way relative to the historical aesthetic of individual musical works.

Many may wonder why the history of cello vibrato would appear in a paper that focuses on teaching how to execute the technique. While one may obtain that history from other books, the goal of mapping the history of vibrato is again aimed to allow readers to better understand performers' cognition and listeners' perspective in different periods. I wish that cellists can learn to become aware of the distinction of a piece while they perform vibrato. History not only tells us of what happened in the past, but in our study of historical practices we are also guided as to how to shape the future. It is important to recognize both historical and cultural contexts behind a musical technique, as such recognition opens one's eyes and ears to composers, performers, and listening audiences of particular times.

Today, cello has become a very popular string instrument, and there are more and more great cellists. Meanwhile, the musical audience's aesthetic tastes have also become more refined; they thus have exceedingly high expectations for the performers. A fine teacher has the responsibility of leading students to build the ability to make a transparent bridge

between composer and audience. As a part of critical thinking, young artists should accurately transfer the intent of the composer to the work; and have the potential to provide the audience with a compelling interpretation. The audience cares more about how beautifully the performer plays, while the performer has to present a fluid performance that combines technique and musicality. A spectacular concert must be based on the performer's accurate technique and the appropriate expression of his or her personality. Therefore, deeply understanding the development of cello vibrato will help students to perform with greater authenticity.

Throughout the evolution of cello practice, we can find a large number of musical scores on which can practice different techniques. Few, however, are methods books, and only a small fraction of these even provide just a mere few paragraphs of instructions on how to execute vibrato. There is an evident gap in literature on teaching vibrato technique on the cello. Cello lessons are mostly held privately, in a one-on-one setting. Teachers educate students verbally and through physical demonstration and assign pieces to practice afterwards. Students can only practice from their mental recollection of the lesson.

Textbook-like material on vibrato is incredibly rare. It is difficult to find even one source that would contain descriptions on the execution of vibrato, a series of step-by-step tutorials, possible problems that arise and how to solve them. It is meaningful to me to write this thesis so that, whether you are a cello teacher or student, you can teach or learn vibrato at any stage; when you practice any exercise that I provide above, you can find the explanation on why and how you need to work on it and what will you gain from the exercise.

Vibrato is different from other techniques. It does not have a notation on the score, and this creates a lot of ambiguity for the student. Where and when to use vibrato is thus a dilemma for students. As teachers, we should be more aware of the need to help students identify the different scenarios in which vibrato might be used. This thesis also presents in detail the vital pair of vibrato elements: vibrato speed and width. Each of these was demonstrated by fundamental exercises that work for all stages of students. These are also accompanied by example excerpts from well-known cello repertoire. I believe learning a

technique is not hard; the greater challenge is in using it appropriately. I hope the examples that I have chosen can help students understand how to apply vibrato more acceptable and persuasively. If you are a cello teacher, this work will be a friendly reminder that teaching vibrato should definitely merit more lessons to devoted acquiring that technique; you need to explain and introduce various excerpts for students. Teachers should cultivate students' sensitivity of sound so that students have an understanding of how to determine what kind of sound is desired, and then explore different types vibrato to achieve the ideal effect.

Since its earliest inception, vibrato has always been subjective; suggestions have been provided to student from the instructor's experience. This thesis tries to provide more objective recommendations via the FFT spectrogram analysis. I understand that neither performers nor audiences measure vibrato speed or width during performances, but if we want to make improvements to the quality of applying vibrato, then allowing the technique to be objectively measured through a common method is a necessity. The sample excerpts in the experiments that I have done above are far from offering an absolute objective recommendations, yet as a qualitative description, they are more objective than are our ears. The results do not fully reflect a regular pattern because the results are also affected by performers' intonation, recording quality, personal preference. Therefore, I cannot say the recommendations that I provided are definitive, but they progress from common to specific, targeted situations. Each excerpt in chapter 4 can be performed by using one type of vibrato; the analysis and data may help students to distinguish how to apply vibrato in different scenarios.

Furthermore, conducting the experiments to analyze cellists' vibrato inspires me to explore more possibilities of vibrato learning and instrumental teaching. There are several details that I would like to investigate in future endeavors. For example, all the excerpts that I analyzed were from original recordings, without using any software to normalize their tuning. If I want to figure out each cellist's pitch center, whether they oscillate from the in-tune pitch towards below or around the in-tune pitch, I can choose an open string from the same excerpt, and capture the same note from another string from which the finger is pressed. If we can set the open string as a standard, and then analyze the oscillation direc-

tion, this will help us to draw precise data on pitch center. Another development to consider for future research is whether and how to eliminate the background sound from piano or other orchestra instruments so that the spectrograms show the specific instrument of interest. This can be achieved with the application of state-of-the-art musical source separation techniques (Cano et al., 2019).

Today we live in a world where technology-assisted music performance is getting popular and even abundant in certain areas of music. However, technology-assisted music education is still in its infant stages as pertains the development of instrumentalists in their beginning and intermediate lessons. We are now just starting to break the barrier of distance learning using technology, and using electronic tuners and metronomes. I have no doubt that in the future, the marriage of interdisciplinary fields in music and technology will be able to transform the way students practice, particularly in ways that include the use of the real time Fourier Transform to analyze intonation and or the quality of vibrato. This transformation allows students to practice with more objectivity where it is needed, and to treat these aspects the same way as we tune our instruments with a tuner today. As well, this gives teachers more time and energy to instruct students on more subjective and creative elements of music such as interpretation and style.

I hope the way that I analyze vibrato may inspire students to develop music learning with any other discipline. I believe that they will create more innovative methods to improve the learning of instruments both technically and musically. As an example, in my dissertation, I measured music signals directly through a recording, but a similar exercise can be done indirectly by combining the music and health science disciplines to measure health metrics to see how a human reacts quantitatively when they listen to a piece of music. Future research here could be able to teach students how to perform music in ways that can influence the way the audience feels, physically.

Music is abstract, and we desperately love it because it has infinite room for imagination. My goal for conducting the FFT experiments is to use specific data as a basis for applying vibrato more accurately and thereby finally materializing the music we imagine. As a cellist

and educator, I encourage students to define the kinds of sounds they imagine and try to create these sounds with different types of vibrato. Combining this with what they already know about making music can help to show their musical personalities.

Listening is the key to making music, regardless of whether or not you want to learn a new technique; whether you want to explore the sound you desire or perform better with other musicians, you have to listen. Listen to great musicians' performances to get inspired; listen to your students' playing to accumulate experience about what they can avoid and what they can improve; listen to your colleagues' playing to know how to adjust your own playing in order to work together more efficiently. Listening to yourself is the most important part to becoming a better musician. We react when we listen; we think, we feel, and we come to know our own and others' demands. Such conscious and intuitive reactions are the spirit of excitement for all of us as musicians.

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