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New Developments in Light-Front Nuclear Structure

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Abstract

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Nuclear physics is the study of many-body systems of protons and neutrons and their constituent interactions. Historically, our understanding of nuclear structure developed from low-energy scattering experiments, for which independent-particle models proved widely successful. Early $A(e, e'p)$ measurements at NIKHEF and Saclay, however, revealed that this picture is incomplete, updating our theoretical description of the nucleus toward a correlated many-body framework. In particular, these experiments—along with later high-momentum-transfer work at Jefferson Lab—highlighted the importance of nucleon-nucleon short-range correlations (SRCs), two-nucleon configurations with high relative momentum and low center-of-mass momentum. SRC phenomenology uses such configurations to explain the plateaus observed in ratios of inclusive electron-nucleus to deuterium cross sections.

With current and forthcoming high-energy electron-nucleus experiments at Jefferson Lab and the Electron-Ion Collider, nuclear structure must once again be

updated—this time into a relativistic formulation suitable for such kinematics, which has not previously been carried out. This dissertation develops nuclear structure in that direction, motivated by these future experiments. Building on existing tools from conventional nuclear physics, we reformulate them in a relativistic, light-front-quantized framework. Our light-front nuclear structure calculations, adapted from density functional theory, reproduce nuclear binding energies and shell structure well, and incorporate the physics of nucleon-nucleon SRCs through similarity renormalization group techniques. Our results indicate that a purely nucleonic description of scattering is insufficient to capture inclusive electron-nucleus data and does not fully reproduce the plateaus seen at high Bjorken- x_B , pointing to the importance of inelastic final-state interactions that current SRC phenomenology does not account for.

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DEDICATION

To my family

CONVENTIONS AND NOTATION

Throughout this thesis the following conventions are adopted unless stated otherwise.

Units. Natural units are used, with $\hbar = c = 1$. Energies, momenta, and masses are expressed in fm or MeV/GeV, as appropriate.

Light-front coordinates. Light-front coordinates follow the Lepage–Brodsky convention,

$$x^\pm \equiv x^0 \pm x^3, \quad \mathbf{x}^\perp = (x^1, x^2), \quad (1)$$

with x^+ playing the role of light-front time and x^- the longitudinal spatial coordinate. The corresponding momenta are $p^\pm = p^0 \pm p^3$, with p^- identified as the light-front energy. The on-shell condition reads $p^+p^- - (\mathbf{p}^\perp)^2 = m^2$.

All Instant Form variables will be denoted with “IF” subscript or superscript. Boldface with the subscript “IF” denotes spatial three-vectors, $\mathbf{p}_{\text{IF}} = (p^1, p^2, p^3) = (\mathbf{p}^\perp, p^3)$, while boldface symbols without subscript denote light-front three-vectors, $\mathbf{p} = (p^+, \mathbf{p}^\perp)$. The energy component is conventionally listed first in both forms:

$$p_{\text{IF}}^\mu = (p^0, \mathbf{p}_{\text{IF}}) \quad (\text{instant form}), \quad p^\mu = (p^-, \mathbf{p}) \quad (\text{front form}). \quad (2)$$

States. Plane-wave nucleon states are defined by

$$\sqrt{2p^+}\hat{a}^\dagger(\mathbf{p}, \sigma, \tau) |0\rangle = |\mathbf{p}, \sigma, \tau\rangle. \quad (3)$$

When dealing with multi-nucleon states we will take the convention that the creation operators are indexed from smallest to largest, i.e.

$$|\mathbf{p}_1, \sigma_1, \tau_1; \mathbf{p}_2, \sigma_2, \tau_2\rangle = \left(\sqrt{2p_1^+}\hat{a}^\dagger(\mathbf{p}_1, \sigma_1, \tau_1)\right) \left(\sqrt{2p_2^+}\hat{a}^\dagger(\mathbf{p}_2, \sigma_2, \tau_2)\right) |0\rangle. \quad (4)$$

Furthermore, momenta $\mathbf{p}$, spin σ , and isospin τ labels will sometimes all be packaged into a non boldfaced momentum label. For instance,

$$|\mathbf{p}, \sigma, \tau\rangle = |p\rangle,$$

$$\hat{a}(\mathbf{p}, \sigma, \tau) = \hat{a}(p), \quad (5)$$

$$|\mathbf{p}_1, \sigma_1, \tau_1; \mathbf{p}_2, \sigma_2, \tau_2\rangle = |p_1, p_2\rangle,$$

Integrals. Momentum integrals carry factors of $(2\pi)^{-3}$. The light-front Lorentz invariant phase space is

$$\int [dp] = \int \frac{dp^+ d\mathbf{p}^\perp}{(2\pi)^3 2p^+} \theta(p^+), \quad (6)$$

the $\theta(p^+)$ inside the integral, which enforces $p^+ \geq 0$, will typically be dropped and it is implicitly assumed that the domain of p^+ is positive definite, unless explicitly said so.

Metric. The mostly-minus signature, $g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$, of the Minkowski metric is used. Greek indices $\mu, \nu, \dots$ run over 0, 1, 2, 3 for instant form and $-, +, 1, 2$

for front form.

Dirac spinors. Lepage-Brodsky Dirac spinor representation is used

$$u(\mathbf{p}, \sigma) = \frac{1}{\sqrt{p^+}} (p^+ + \beta m + \boldsymbol{\alpha}^\perp \cdot \mathbf{p}^\perp) \times \begin{cases} \chi(\uparrow), & \text{for } \sigma = +1, \\ \chi(\downarrow), & \text{for } \sigma = -1, \end{cases} \quad (7)$$

$$v(\mathbf{p}, \sigma) = \frac{1}{\sqrt{p^+}} (p^+ - \beta m + \boldsymbol{\alpha}^\perp \cdot \mathbf{p}^\perp) \times \begin{cases} \chi(\downarrow), & \text{for } \sigma = +1, \\ \chi(\uparrow), & \text{for } \sigma = -1. \end{cases} \quad (8)$$

The two χ -spinors are

$$\chi(\uparrow) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \chi(\downarrow) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}. \quad (9)$$

Dirac matrices. The Dirac matrices satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

The Dirac representation is used throughout.

Chapter 1

INTRODUCTION

Nuclear physics is the study of the atomic nucleus, its constituents, and the interactions between them. The discipline is conventionally partitioned, on the basis of the energy scales involved, into the two sub-fields of low- and high-energy nuclear physics. Our understanding of nuclear structure began at low energies, with Rutherford’s gold-foil experiment published in 1911 [1]. By firing alpha particles at thin gold foil, Rutherford, Geiger, and Marsden discovered that at the center of the atom lies a small, dense, positively charged object: the nucleus. Twenty-one years later, Chadwick’s discovery of the neutron [2], together with the birth of quantum mechanics in the early twentieth century, set the stage for microscopic theoretical descriptions of nuclear structure in which protons and neutrons (nucleons) serve as the fundamental degrees of freedom. Early models—Gamow, Bohr, and Wheeler’s liquid-drop model [3] and the nuclear shell model of Mayer and Jensen [4, 5, 6]—achieved remarkable success in describing a wide range of nuclear observables, including binding energies, charge radii, and shell structure. Modern low-energy nuclear physics builds on these foundations with sophisticated *ab initio* approaches such as Density Functional Theory [7, 8] and Quantum Monte Carlo [9].

Theoretical progress has, throughout, depended on a close partnership with experiment. To study the nucleus, Rutherford relied on alpha particles emitted by naturally radioactive substances such as radium—a beam whose energy, intensity, and composition were fixed by the source and entirely outside the experimenter’s control. Recognizing this limitation, Rutherford began advocating for actively accelerating particles to interrogate the nucleus. Inspired by this vision, his students John Cockcroft and Ernest Walton constructed the first particle accelerator in 1932. By accelerating protons to 700–800 keV, they achieved the first artificial nuclear disintegration, splitting a lithium atom into two alpha particles [10, 11]. This milestone gave physicists unprecedented control over subatomic probes and established the accelerator-driven experimental program that continues to drive theoretical developments today.

The subsequent global enterprise of building ever more powerful accelerators culminated for nuclear physics in 1968 at the Stanford Linear Accelerator Center (SLAC). Deep inelastic scattering (DIS) of high-energy electrons off protons revealed the composite internal structure of the nucleon itself [12], establishing the field of high-energy nuclear physics with a new set of fundamental degrees of freedom: quarks and gluons (partons). Within five years, Quantum Chromodynamics (QCD) was formulated as the fundamental theory governing their dynamics [13, 14, 15, 16]. Understanding how nucleons and complex nuclei emerge from these underlying quark and gluon interactions has since become an overarching goal of the field. The short-distance behavior of the strong interaction is perturbatively calculable and well understood; at the larger

distances relevant to hadronic and nuclear structure, however, QCD becomes highly non-perturbative. This non-perturbative regime—the intersection of low- and high-energy nuclear physics—remains profoundly challenging and is a primary focus of contemporary research.

The earliest experimental window into this regime was opened in 1983 by the European Muon Collaboration (EMC). Performing DIS on iron and deuterium targets with high-energy muons, they observed that the ratio of per-nucleon cross-sections exhibited a distinct depletion at intermediate momentum fractions [17]. At the time, it was widely assumed that high-energy partonic observables would be insensitive to low-energy nuclear dynamics—that, at energies sufficient to resolve partons, the nucleus could be treated as a collection of free nucleons. The “EMC effect,” as this depletion came to be known, sparked intense theoretical investigation. Early approaches invoking conventional nuclear mechanisms such as Fermi motion and pionic enhancements failed to fully reproduce the data, and despite more than forty years of effort and over a thousand publications, the origin of partonic modifications in bound nucleons remains a major open question.

Experimental momentum on this question is only growing. Future measurements at the Electron–Ion Collider (EIC), which will accelerate nuclear targets to energies as high as 100 GeV/nucleon [18, 19], and current experiments at Jefferson Lab (JLab) colliding 12 GeV electrons against nuclei [20], will probe quark substructure in nuclei with unprecedented resolution. This experimental progress demands a parallel

theoretical advance: a description of the initial nuclear state suitable for high-energy processes. Nuclear structure calculations have historically been performed by solving the Schrödinger equation, but as the energy of the probe increases, relativistic effects can no longer be neglected. Without a properly relativistic treatment, purely kinematic effects can easily be misinterpreted as genuine signatures of new physics. Yet the calculation of nuclear wavefunctions within a consistent relativistic framework remains largely unexplored.

Conventional instant form quantization, using x^0 as the time variable, does not adequately meet this need because it yields frame-dependent descriptions of bound states [21]. In this formalism, a nucleus boosted to high momentum appears Lorentz contracted, and reproducing such a deformed state requires a superposition over all excited states of the system. This contraction is unphysical in the present context: it is a frame-dependent artifact rather than an intrinsic property of the nucleus. The resulting calculations are both technically cumbersome—requiring the explicit inclusion of excited nuclear states—and conceptually opaque, since frame-dependent effects become entangled with the intrinsic dynamics one aims to study. Such “fictitious dynamics” and related pathologies of instant form-based bound-state calculations have historically led to misleading conclusions, including apparent violations of the Gerasimov–Drell–Hearn sum rule in the deuteron [22] and more recent efforts to extract time-independent, three-dimensional densities for systems of relativistically moving constituents [23].

Light-front quantization, which uses $x^+ = x^0 + x^3$ as the time variable, offers a natural resolution to these difficulties: its wavefunctions are invariant under boosts, enabling a consistent, frame-independent description of nuclear structure. Despite this central advantage, relativistic light-front descriptions of finite nuclei remain largely undeveloped. Developing such a framework is essential for interpreting data from the high-energy nuclear experiments underway and forthcoming at JLab and the EIC, and is the central motivation for the work presented in this thesis.

The remainder of this thesis is organized as follows. **Chapter 2** provides an introduction into light-front quantization, with the guiding question: *why should low-energy nuclear physicists care?* Rather than offering a complete treatment of the mathematical formalism—which is extensively documented in literature—the chapter aims to build qualitative intuition for the importance of the light-front approach and to present the basic theoretical machinery needed to perform calculations

The primary observable throughout this work is the inclusive electron–nucleus scattering cross-section, and **Chapter 3** provides a theoretical overview of this process, introducing the main theoretical objects of interest.

Chapter 4 begins the calculational portion of the thesis with light-front density functional theory, the simplest realistic model of nuclear structure that provides a pathway to computing inclusive electron–nucleus cross-sections. Here, we highlight a procedure for obtaining light-front nuclear structure calculations directly from previously published instant form results.

The independent-particle picture underlying DFT, however, proves inadequate for high-energy nuclear processes. This shortcoming motivates **Chapter 5**, which introduces the similarity renormalization group (SRG) and its novel application to relativistic descriptions of nuclear structure on the light-front. We show how incorporating the SRG modifies the cross-section calculations of the previous chapter.

Chapter 6 addresses the role of final-state interactions and finally, **Chapter 7** summarizes the results of this thesis.

Chapter 2

LIGHT-FRONT QUANTIZATION

In 1949, Dirac published his foundational paper *Forms of Relativistic Dynamics*, in which he investigated distinct Hamiltonian formulations of relativistic quantum mechanics by exploring different parameterizations of spacetime [24]. His central insight was that the choice of spacetime foliation, while leaving physical observables unchanged, can offer significant computational and conceptual advantages depending on the system under investigation—much as spherical coordinates simplify spherically symmetric problems without altering their physical content. Dirac identified three classes of foliations that cannot be related by Lorentz transformations in any non-trivial way, which he termed “Forms”: the Instant Form (IF), the Front Form (FF), and the Point Form (PF). The IF, in which states are quantized on surfaces of constant time $x^0 = t$, is the most familiar and most widely adopted, aligning closely with our built-up intuition about space and time. The FF, in which states are quantized on surfaces of constant time $x^+ = x^0 + x^3$, departs from this intuition but has proven exceptionally powerful for high-energy problems and serves as the primary framework of this dissertation. The PF, in which states are quantized on a hyperboloid, has seen comparatively limited application.

This chapter aims to convey why formulating nuclear physics on the light front is essential for the accurate study of high-energy nuclear reactions, which underpins the original calculations presented later in this work. It highlights that the FF is the only relativistic Hamiltonian framework in which (i) intrinsic bound-state wavefunctions are invariant under the boosts, and (ii) the off-shell extrapolation of nucleon-level subprocesses is free of contamination from the probe. Both are necessary to compute high-energy nuclear cross sections, and only the FF satisfies both. The emphasis throughout is on qualitative motivation drawn from fundamental properties of special relativity and quantum mechanics, together with the theoretical formulation needed to support the calculations in this thesis. Dirac's FF has been independently rediscovered and rebranded across the literature as the Infinite Momentum Frame, Null-Plane Quantization, and Light-Cone Quantization; we adopt the name Light-Front (LF) Quantization here. For comprehensive pedagogical reviews, the reader is directed to Ref. [25] and references therein; for applications to nuclear physics specifically, see Refs. [26, 27, 28].

2.1 Example: Lorentz Boosting the Deuteron

To motivate the LF formulation of nuclear dynamics, consider the interplay between quantum mechanics and special relativity in a simple system under IF quantization: a deuteron at rest, with its proton and neutron aligned along the z -axis at a relative momentum of 10 MeV (Fig. 2.1). Anticipating the kinematics of the EIC, where

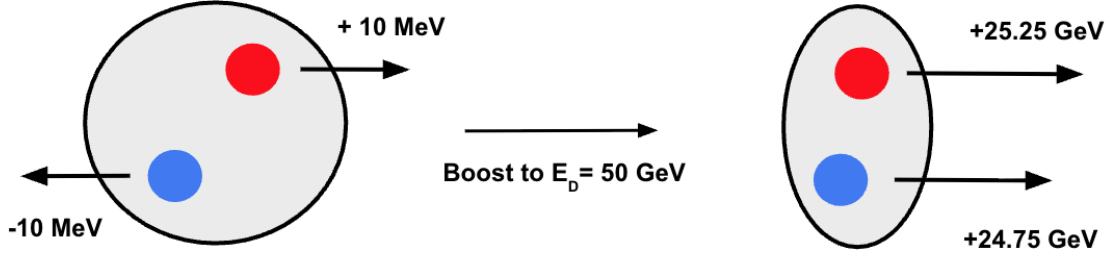


Figure 2.1: (color online) Cartoon picture of a deuteron configuration where the proton (red) and neutron (blue) have back-to-back momenta of 10 MeV along the z -axis. Boosting this configuration to an energy of 50 GeV increases both the absolute and the relative momenta of the constituents.

nuclear beams will reach hundreds of GeV per nucleon, apply a longitudinal Lorentz boost of 50 GeV. The individual nucleon momenta shift to approximately 24.75 and 25.25 GeV, and their relative momentum jumps from 10 MeV to roughly 250 MeV.

This kinematic exercise exposes a fundamental incompatibility between boosts and the traditional separation of bound-state dynamics into center-of-mass and relative motion. Non-relativistic nuclear physics—built on the Schrödinger equation—relies on this separation, yet the boost has entangled the two: the relative momentum has changed, despite us having done nothing to the internal dynamics of the system. The entanglement carries severe physical consequences. The boost-induced increase in relative momentum corresponds to a decrease in the relative distance between constituents; equivalently, the deuteron appears Lorentz-contracted along its direction

of motion. Its spatial density profile in the boosted frame is therefore not the same as in the rest frame, and no single wavefunction can describe both.

Within IF quantization, the moving state must instead be represented as a superposition of the rest-frame Hamiltonian’s eigensolutions. Although the rest-frame and boosted Hamiltonians are related by a unitary Lorentz transformation, the change in the bound-state wavefunction means that boosting effectively induces interactions. These “fictitious dynamics” are unphysical, frame-dependent artifacts rather than intrinsic nuclear properties. Historically, their entanglement with genuine dynamics in IF calculations has led to misleading conclusions, as noted in Chapter 1 (see Ref. [21] for further discussion).

Computationally, the entanglement creates a severe bottleneck: the entire nuclear wavefunction must be recalculated for every reference frame. Restricting evaluation to the nuclear rest frame does not help, since physical observables must remain frame-independent and the dynamical complexities simply migrate elsewhere in the calculation—a point we will revisit in Section 2.3, once the mechanism behind this migration has been made explicit.

2.2 General Considerations of Relativistic Hamiltonian Formulations

The difficulties of the previous section stem from the choice of coordinates used to describe the bound-state system. In the non-relativistic regime governed by the Schrödinger equation, Galilean invariance enforces the equivalence of inertial frames

and permits a clean separation of center-of-mass and relative variables. As just demonstrated, conventional IF variables are structurally unsuited for a relativistic description of bound states: they entangle overall motion with internal structure, introducing fictitious, frame-dependent dynamics. The choice of time variable lies at the heart of the puzzle. Understanding why requires examining how many-body quantum mechanics is constructed in general, and revisiting why the non-relativistic case worked in the first place.

In any canonical quantization framework—relativistic or non-relativistic—one foliates spacetime into a family of surfaces labeled by a “time” coordinate and imposes commutation relations between fields on each surface. Noether’s theorem, applied to the symmetries of the action, then produces a set of conserved charges that act as generators of unitary transformations on the Hilbert space. For non-relativistic quantum mechanics, Galilean invariance yields ten such generators: the Hamiltonian $\hat{H}$, the three momentum components $\hat{P}^i$, the three angular momenta $\hat{J}^i$, and the three Galilean boost generators $\hat{K}_G^i$. The relativistic case yields ten generators of the Poincaré group: $\hat{H}$, $\hat{P}^i$, $\hat{J}^i$, and the three Lorentz boost generators $\hat{K}^i$.

The act of foliating exposes a subtlety: a symmetry of the full four-dimensional spacetime need not act as a symmetry of any single lower-dimensional slice. A cylinder aligned with the z -axis is rotationally symmetric about z , yet only certain slices inherit that symmetry—a perpendicular cut yields a disk, still invariant under rotations about z , while a tilted cut yields an ellipse that is not. This same logic partitions

the ten generators into two classes. Those that map a foliation surface into itself are *kinematic* and contain no interactions; those that move points off the surface are *dynamical* and carry the interactions of the theory. Different foliations preserve different subgroups of the full symmetry, and the resulting split between kinematic and dynamical generators governs which transformations can be performed trivially and which require solving the dynamics.

In non-relativistic quantum mechanics, the standard choice of x^0 as the time variable happens to be optimal: of the ten Galilean generators, only the Hamiltonian is dynamical. The other nine—spatial translations, rotations, and Galilean boosts—leave equal-time surfaces invariant and are therefore kinematic. This is precisely why center-of-mass and relative coordinates separate cleanly, and why a single non-relativistic wavefunction describes the bound state in every inertial frame.

The relativistic case is more delicate. Promoting Galilean boosts to Lorentz boosts changes their character: a Lorentz boost tilts an equal-time surface rather than mapping it to itself. The three Lorentz boost generators therefore join the Hamiltonian as dynamical generators, shrinking the kinematic subgroup to six—spatial translations and rotations alone. The non-intrinsic dynamics encountered in the boosted deuteron are the direct consequence: boosting a bound state now requires evaluating an interaction-dependent operator. This is visible explicitly in physical cross sections.

Schematically,

$$\begin{aligned}
 d\sigma &\sim \sum_X \langle A, \mathbf{P} | \hat{J}^\mu(0) | X \rangle \langle X | \hat{J}^\nu(0) | A, \mathbf{P} \rangle \\
 &= \sum_X \langle A, \mathbf{P} = 0 | \hat{U}^\dagger(\Lambda) \hat{J}^\mu(0) | X \rangle \langle X | \hat{J}^\nu(0) \hat{U}(\Lambda) | A, \mathbf{P} = 0 \rangle,
 \end{aligned}
 \tag{2.1}$$

where restricting the bound state to its rest frame merely transfers the dynamical complexity into the boost operators $\hat{U}(\Lambda)$ acting on the currents. The nuclear interactions embedded in those operators are unavoidable so long as boosts are dynamical generators—they cannot be removed, only relocated. The same argument also affects cases where the final state contains bound nuclear states as well. In high-energy nuclear processes where you have moving composite objects in your initial and final states, boosts are central to the kinematics, and ensuring that bound states transform kinematically under them matters both for calculational efficiency and for physical interpretation. The FF, introduced next, does precisely this.

2.3 *Light-Front Variables*

The pathologies identified in the previous section can be traced to the use of the equal-time hyperplane $x^0 = 0$ as the quantization surface, on which the boost generators are dynamical and entangle internal motion with the overall frame. The FF, used interchangeably with LF quantization, instead quantizes on the $x^+ = 0$ hyperplane, see Fig. 2.2. The variables used in FF are called LF variables, they are defined as

$$\begin{aligned}
 x^\mu &= (x^+, x^-, x^1, x^2), \\
 x^\pm &\equiv x^0 \pm x^3.
 \end{aligned}
 \tag{2.2}$$

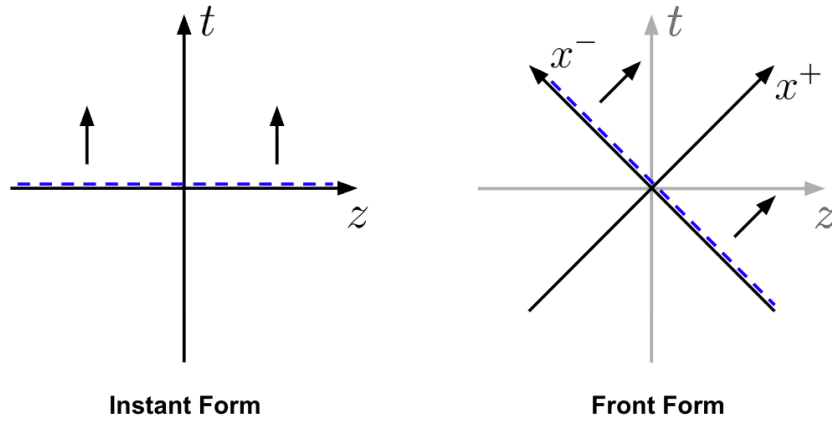


Figure 2.2: (color online) Minkowski space-time diagrams highlighting the differences between the Instant Form (IF) (left) and Front Form (FF) (right). Eigenstates of your Hamiltonian are determined at $t = \text{const}$ and $x^+ = \text{const}$ for the IF and FF respectively. The Hamiltonian generates time evolution in the respective “times” as denoted by the arrows.

The choice of time variable is a matter of convention; throughout this thesis we take x^+ as the LF time. The corresponding contraction between position and momentum is

$$x_\mu p^\mu = g_{\mu\nu} x^\nu p^\mu = \tfrac{1}{2}x^+ p^- + \tfrac{1}{2}x^- p^+ - \mathbf{x}^\perp \cdot \mathbf{p}^\perp, \quad (2.3)$$

with the LF metric tensor given in Table 2.1. From this we see that x^+ and p^- are conjugate, and we accordingly identify p^- as the LF energy. For a free massive particle, the plus component is manifestly positive,

$$p^+ = p^0 + p^z = \sqrt{m^2 + \mathbf{p}_{\text{IF}}^2} + p^z > 0, \quad (2.4)$$

and the mass-shell condition $p_\mu p^\mu = m^2$ takes the form

$$p^+ p^- - (\mathbf{p}^\perp)^2 = m^2, \quad (2.5)$$

which can be solved algebraically for the LF energy,

$$p^- = \frac{m^2 + (\mathbf{p}^\perp)^2}{p^+} > 0. \quad (2.6)$$

Unlike the equal-time dispersion relation $p^0 = \pm \sqrt{m^2 + \mathbf{p}_{\text{IF}}^2}$, the LF dispersion relation is rational in the kinematical variables $(p^+, \mathbf{p}^\perp)$ and admits no negative-energy branch.

The decisive feature of the FF, however, lies in the structure of its boost generators, which are kinematical rather than dynamical. To state these definitions, we recall that the Poincaré algebra contains three rotation generators J^i and three boost generators K^i , where J^i generates rotations about the x^i -axis and K^i generates ordinary Lorentz

boosts along the x^i -direction. The longitudinal LF boost is simply K^3 , the ordinary Lorentz boost along the z -direction. The transverse LF boosts, however, are the combinations

$$E^1 = K^1 + J^2, \quad E^2 = K^2 - J^1, \quad (2.7)$$

in which an ordinary transverse boost is paired with a rotation in such a way that the quantization surface $x^+ = 0$ is preserved. Together with the longitudinal boost, the E^i generate the full set of LF boosts; we refer to these throughout as longitudinal and transverse LF boosts. Their action on LF momenta makes their utility manifest.

Defining the rapidity ω through $e^\omega = \sqrt{(1-v)/(1+v)}$, where v is the velocity of the boost in natural units, a longitudinal LF boost acts as

$$\begin{aligned} p^+ &\rightarrow e^\omega p^+, \\ p^- &\rightarrow e^{-\omega} p^-, \\ \mathbf{p}^\perp &\rightarrow \mathbf{p}^\perp, \end{aligned} \quad (2.8)$$

so that the plus and minus components are simply rescaled and the transverse momentum is left untouched. A transverse LF boost with velocity $\mathbf{v}^\perp$ instead yields

$$\begin{aligned} p^+ &\rightarrow p^+, \\ \mathbf{p}^\perp &\rightarrow \mathbf{p}^\perp + p^+ \mathbf{v}^\perp, \\ p^- &\rightarrow p^- + 2 \mathbf{v}^\perp \cdot \mathbf{p}^\perp + p^+ (\mathbf{v}^\perp)^2, \end{aligned} \quad (2.9)$$

leaving the plus component invariant.

These transformation laws have important consequences for the identification of intrinsic variables in nuclear physics. Because longitudinal LF boosts rescale every

plus momentum by the same factor e^ω , the ratio

$$x_i = \frac{p_i^+}{P^+}, \quad (2.10)$$

with $P^+ = \sum_i p_i^+$ the total plus momentum of the bound state and the index i running over its A constituents, is invariant under both longitudinal and transverse LF boosts. Combined with the positivity of p^+ established above, this further implies $0 < x_i < 1$ for each massive constituent, so that the x_i form a natural set of boost-invariant longitudinal momentum fractions.

The transverse boost laws are equally suggestive—and reveal a remarkable feature of the FF. The transformations of p^- and $\mathbf{p}^\perp$ closely mirror the way non-relativistic kinetic energy and momentum transform under a Galilean boost,

$$\begin{aligned} [\mathbf{p} \rightarrow \mathbf{p} + m\mathbf{v}]_{\text{IF}}, \\ [E_{\text{kin}} \rightarrow E_{\text{kin}} + \mathbf{v} \cdot \mathbf{p} + \tfrac{1}{2}m\mathbf{v}^2]_{\text{IF}}, \end{aligned} \quad (2.11)$$

with the plus momentum p^+ playing the role of the mass. The subscript IF on the square brackets is there to emphasize that all variables are IF three-momenta. The very Galilean structure that broke down under Lorentz transformations re-emerges, intact, in the transverse plane. This correspondence motivates the definition of the LF relative transverse momenta

$$\mathbf{k}_i^\perp = \mathbf{p}_i^\perp - \frac{p_i^+}{P^+} \mathbf{P}^\perp = \mathbf{p}_i^\perp - x_i \mathbf{P}^\perp, \quad (2.12)$$

which takes precisely the form of their non-relativistic counterparts

$$[\mathbf{k}_i = \mathbf{p}_i - (m_i/M)\mathbf{P}]_{\text{IF}}, \quad (2.13)$$

with $M = \sum_i m_i$. By construction, the $\mathbf{k}_i^\perp$ are invariant under both longitudinal and transverse LF boosts, and satisfy the constraint $\sum_i \mathbf{k}_i^\perp = 0$.

With $(x_i, \mathbf{k}_i^\perp)$ we have thus identified a set of intrinsic variables that do not mix with the center-of-mass momentum under LF boosts. The utility of this separation is made manifest by examining the total LF kinetic energy of an A -body system, multiplied by P^+ ,

$$P^+ \sum_i p_i^- = P^+ \sum_i \frac{m_i^2 + (\mathbf{p}_i^\perp)^2}{p_i^+} = (\mathbf{P}^\perp)^2 + \sum_i \frac{m_i^2 + (\mathbf{k}_i^\perp)^2}{x_i}, \quad (2.14)$$

in which the center-of-mass and intrinsic contributions separate cleanly, in direct analogy with the non-relativistic decomposition $[T = \mathbf{P}^2/2M + \sum_i \mathbf{k}_i^2/2m_i]_{\text{IF}}$. Since the intrinsic dynamics of the bound state must be independent of its overall center-of-mass motion, any interaction term added to the Hamiltonian must depend only on the intrinsic variables $(x_i, \mathbf{k}_i^\perp)$. Hence, the bound-state wavefunctions obtained by diagonalizing such a Hamiltonian are then functions of $(x_i, \mathbf{k}_i^\perp)$ alone, and are accordingly invariant under both longitudinal and transverse LF boosts.

This is precisely the structure that was absent in the IF formulation. The fictitious dynamics that plagued the IF treatment of the boosted deuteron—the recalculation of the wavefunction in every frame, the entanglement of internal structure with center-of-mass motion, the migration of dynamical complexity into the boost operators of Eq. (2.1)—are absent here by construction. The rest-frame and boosted descriptions of the bound state are connected by a kinematical transformation acting only on P^+ and $\mathbf{P}^\perp$, while the intrinsic wavefunction $\psi(x_i, \mathbf{k}_i^\perp)$ is left invariant. The LF variables

thus provide the kinematic basis suited to relativistic bound states that the previous section called for.

2.4 *Scattering Amplitudes*

The bound-state wavefunctions identified in the previous section are one ingredient in a cross section; the other is a model of the elementary subprocess in which the probe interacts with a constituent of the bound state. We now turn to this second ingredient and show that LF quantization is equally indispensable there.

In modern quantum field theory, scattering processes are typically evaluated using the machinery of Feynman diagrams, which offer the distinct advantage of manifest Lorentz invariance for every individual graph. When considering scattering off composite targets, however, this covariant approach becomes unwieldy: the bound-state input it requires—a fully covariant wavefunction—is only available for two-body bound states, where it is given by the Bethe-Salpeter equation [29]. Even there, its practical use is hampered both by prohibitive computational costs and by well-known conceptual difficulties such as the treatment of relative time. To make progress, one reverts to a Hamiltonian framework, in which bound-state wavefunctions are rigorously defined and tractable. The cost of this shift is that scattering amplitudes must be computed using old-fashioned, time-ordered perturbation theory, which increases the number of topological graphs to be summed. The trade-off is favorable: a manageable proliferation of diagrams in exchange for access to bound-state structure.

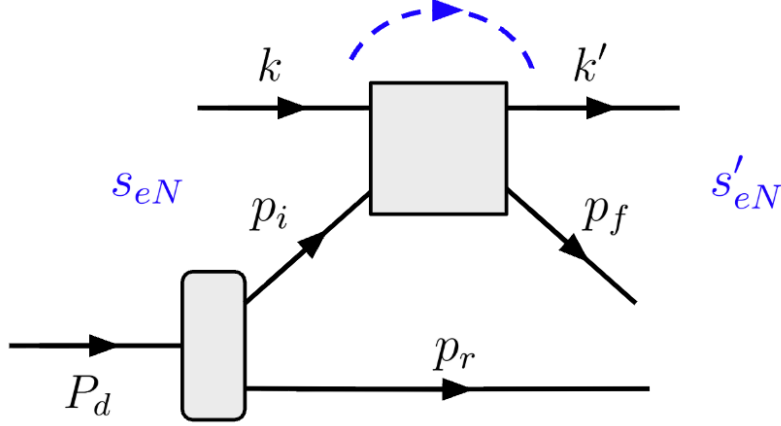


Figure 2.3: (color online) Feynman diagram for electron-deuteron scattering with an electron-nucleon subprocess. The invariant energy before and after the collision are labeled s_{eN} and s'_{eN} respectively.

For high-energy nuclear processes in which the probe resolves the individual nucleons inside the nucleus, the central theoretical challenge, aside from nuclear structure, is modeling the electron-nucleon subprocess. The standard practice is an off-shell extrapolation of the on-shell electron-nucleon amplitude: the bound nucleon, unlike a free one, does not lie on its mass shell, and the free-nucleon cross section is extended into the off-shell region using a prescription such as that of de Forest [30]. Under IF at relativistic kinematics, however, this extrapolation acquires a contamination unrelated to the nuclear target. The IF and FF, as Hamiltonian formulations, share a common feature: only three components of momentum are conserved at each interaction vertex, with intermediate particles placed on their mass shell while the remaining

momentum component absorbs the binding-induced mismatch. They differ in which three components: spatial three-momentum $\mathbf{p}_{\text{IF}}$ in IF, with energy p^0 unconserved; LF three-momentum $\mathbf{p} = (p^+, \mathbf{p}^\perp)$ in FF, with p^- unconserved. This distinction has direct consequences for the off-shellness of the struck nucleon.

Consider the Feynman diagram in Fig. 2.3, with k the incident electron, k' the scattered electron, P_d the deuteron, p_r the spectator/recoil nucleon, p_i the initial nucleon, and p_f the scattered nucleon. We quantify the off-shellness by the difference between the invariant energy of the electron-nucleon subprocess before and after the interaction, $s'_{eN} - s_{eN}$. Using overall four-momentum conservation, $k' + p_f = k + P_d - p_r$,

$$\begin{aligned} s'_{eN} - s_{eN} &= (k' + p_f)^2 - (k + p_i)^2 = (k + P_d - p_r)^2 - (k + p_i)^2 \\ &= [2k \cdot (P_d - p_r - p_i)] + [(P_d - p_r)^2 - p_i^2]. \end{aligned} \quad (2.15)$$

The square brackets separate two physically distinct contributions. The right-hand bracket, $(P_d - p_r)^2 - p_i^2$, measures the off-shellness of the struck nucleon itself: $(P_d - p_r)$ is the four-momentum that would be assigned to it if four-momentum conservation were enforced at the deuteron vertex, while p_i is the four-momentum actually assigned by the quantization scheme (on mass shell, $p_i^2 = m^2$). The two differ because only three momentum components are conserved at the vertex; the remaining component is fixed instead by the mass-shell condition. The left-hand bracket depends on the electron's four-momentum and is therefore a property of the probe rather than the target. A consistent off-shell extrapolation should depend only on the right-hand bracket; the left-hand bracket is the contamination.

Working in the deuteron rest frame with $(P_d^\mu)_{\text{IF}} = (M_d, \mathbf{0})$ and an incident electron with $k_{\text{IF}}^\mu = (\omega, 0, 0, -\omega)$ in the high-energy limit $\omega > 1$ GeV, the IF rules at the deuteron vertex give $(\mathbf{p}_i)_{\text{IF}} = -(\mathbf{p}_r)_{\text{IF}}$ and $p_i^0 = \sqrt{m^2 + (\mathbf{p}_r)_{\text{IF}}^2}$, while $p_r^0 + p_i^0 \neq M_d$ —the energy mismatch that defines the bound state. Evaluating the probe-dependent bracket to leading order in $|\mathbf{p}_r|_{\text{IF}}/m$ and neglecting the small deuteron binding energy,

$$[s'_{eN} - s_{eN}]_{\text{IF}} \approx -2\omega \frac{(\mathbf{p}_r)_{\text{IF}}^2}{m} + [(P_d - p_r)^2 - p_i^2]_{\text{IF}}. \quad (2.16)$$

The contamination grows linearly with the incident energy ω and depends explicitly on the kinematics of the probe, even though the off-shellness is meant to be an intrinsic property of the bound nucleon. At high energies the IF prescription cannot disentangle target structure from probe kinematics.

LF quantization removes the contamination. Expanding the four-vector dot product in LF components,

$$\begin{aligned} s'_{eN} - s_{eN} = & \left[k^+(P_d - p_r - p_i)^- + k^-(P_d - p_r - p_i)^+ - 2\mathbf{k}^\perp \cdot (\mathbf{P}_d - \mathbf{p}_r - \mathbf{p}_i)^\perp \right] \\ & + [(P_d - p_r)^2 - p_i^2]. \end{aligned} \quad (2.17)$$

With the chosen kinematics, $k^+ = 0$ and $\mathbf{k}^\perp = 0$, and plus-momentum conservation at the deuteron vertex enforces $(P_d - p_r - p_i)^+ = 0$. The entire probe-dependent bracket vanishes, leaving

$$s'_{eN} - s_{eN} = (P_d - p_r)^2 - p_i^2. \quad (2.18)$$

The off-shellness is determined entirely by the nuclear target: probe kinematics no

longer enter, and the extrapolation has the clean physical interpretation it was intended to have.

2.5 *Minimal Relativity and the Front Form*

The nucleon-nucleon (NN) interaction plays a central role throughout this dissertation. At present, no published work has developed phenomenological relativistic LF NN potentials capable of describing elastic NN phase shifts to high accuracy, and constructing such potentials lies outside the scope of this work. We therefore adopt an alternative prescription: we recast an existing non-relativistic NN potential—here the Argonne v_{18} (AV18) potential [31]—into relativistic form, perform a change of variables, and incorporate the effects of LF spinors to obtain an approximate LF NN potential. For further reading, the reader is directed to Refs. [26, 27, 32, 33].

The starting point is the non-relativistic Lippmann-Schwinger equation, at zero center-of-mass momenta, for a two-nucleon system in relative-momentum coordinates, neglecting spin and isospin,

$$\left[\frac{k^2}{m_N} \psi(\mathbf{k}) + \int \frac{d\mathbf{k}'}{(2\pi)^3} V(\mathbf{k}, \mathbf{k}') \psi(\mathbf{k}') = \epsilon \psi(\mathbf{k}) \right]_{\text{IF}}, \quad (2.19)$$

where again, the subscript IF on the square brackets is there to emphasize that all variables are IF three-momenta. To recast this as an effective relativistic equation, we perform the following sequence of manipulations: multiply throughout by m_N , add $m_N^2 \psi(\mathbf{k}_{\text{IF}})$ to both sides, multiply throughout by $4\sqrt{E(\mathbf{k}_{\text{IF}})}$, and insert a factor

Property	Instant Form	Front Form
Coordinates	$x^\mu = (x^0, \mathbf{x}_{\text{IF}})$	$x^\mu = (x^+, x^-, \mathbf{x}^\perp)$ $x^\pm \equiv x^0 \pm x^3$
$g_{\mu\nu}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$
Free Particle	$p^\mu = (p^0, \mathbf{p}_{\text{IF}})$ $p^\mu p_\mu = (p^0)^2 - \mathbf{p}_{\text{IF}}^2 = m^2$ $p^0 = \sqrt{m^2 + \mathbf{p}_{\text{IF}}^2} > 0$	$p^\mu = (p^-, p^+, \mathbf{p}^\perp)$ $p^\mu p_\mu = p^+ p^- - (\mathbf{p}^\perp)^2 = m^2$ $p^- = \frac{m^2 + (\mathbf{p}^\perp)^2}{p^+} > 0$ $!p^+ > 0!$
Normalization	$\langle \mathbf{p}'_{\text{IF}}, \sigma' \mathbf{p}_{\text{IF}}, \sigma \rangle =$ $(2\pi)^3 2E(\mathbf{p}_{\text{IF}}) \delta^{(3)}(\mathbf{p}'_{\text{IF}} - \mathbf{p}_{\text{IF}}) \delta_{\sigma\sigma'}$	$\langle \mathbf{p}', \sigma' \mathbf{p}, \sigma \rangle =$ $(2\pi)^3 2p^+ \delta(p'^+ - p^+) \delta^{(2)}(\mathbf{p}'^\perp - \mathbf{p}^\perp) \delta_{\sigma\sigma'}$
Phase Space	$\int d\Gamma_p = \int \frac{d\mathbf{p}_{\text{IF}}}{(2\pi)^3 2E(\mathbf{p}_{\text{IF}})}$	$\int d\Gamma_p = \int \frac{dp^+ d\mathbf{p}^\perp}{(2\pi)^3 2p^+} \theta(p^+)$

Table 2.1: Comparison of kinematic variables, the Minkowski metric, and Lorentz invariant phase space between the Instant Form and Front Form.

of $E(\mathbf{k}'_{\text{IF}})/E(\mathbf{k}'_{\text{IF}})$ inside the integral. The result is

$$\begin{aligned} \left[4(k^2 + m_N^2)\tilde{\psi}(\mathbf{k}) + \int \frac{d\mathbf{k}'}{(2\pi)^3 E(\mathbf{k}')} \tilde{V}(\mathbf{k}, \mathbf{k}') \tilde{\psi}(\mathbf{k}') = M^2 \tilde{\psi}(\mathbf{k}) \right]_{\text{IF}}, \\ \left[\tilde{\psi}(\mathbf{k}) = \sqrt{E(\mathbf{k})} \psi(\mathbf{k}) \right]_{\text{IF}}, \\ \left[\tilde{V}(\mathbf{k}, \mathbf{k}') = 4m_N \sqrt{E(\mathbf{k})} V(\mathbf{k}, \mathbf{k}') \sqrt{E(\mathbf{k}')} \right]_{\text{IF}}, \end{aligned} \quad (2.20)$$

where $M^2 = 4(m_N^2 + m_N \epsilon)$.

This procedure, known as minimal relativity, was first introduced by Brown, Jackson, and Kuo [34]. Their motivation was the following puzzle. Realistic potentials with strong repulsive cores naturally generate high-momentum components in NN wavefunctions, yet the physical validity—and the surprising success—of these components in describing high-momentum deuteron phenomena is difficult to justify when they are derived solely from the non-relativistic Schrödinger equation. To place them on firmer footing, the authors argued that such realistic potentials should instead be regarded as effective solutions of a relativistic equation—what is today understood as arising from an IF Hamiltonian in the NN rest frame. Minimal relativity then supplies the necessary relativistic kinematics without altering the phenomenologically fitted scattering phase shifts, thereby motivating the high-momentum components that emerge from realistic NN potentials.

The performance of the AV18 potential offers further support. Although it was originally fitted to reproduce nucleon-nucleon phase shifts up to a laboratory energy of 350 MeV, it nonetheless reproduces them well up to 600 MeV in several partial-wave channels (see, e.g., Fig. 2.4).

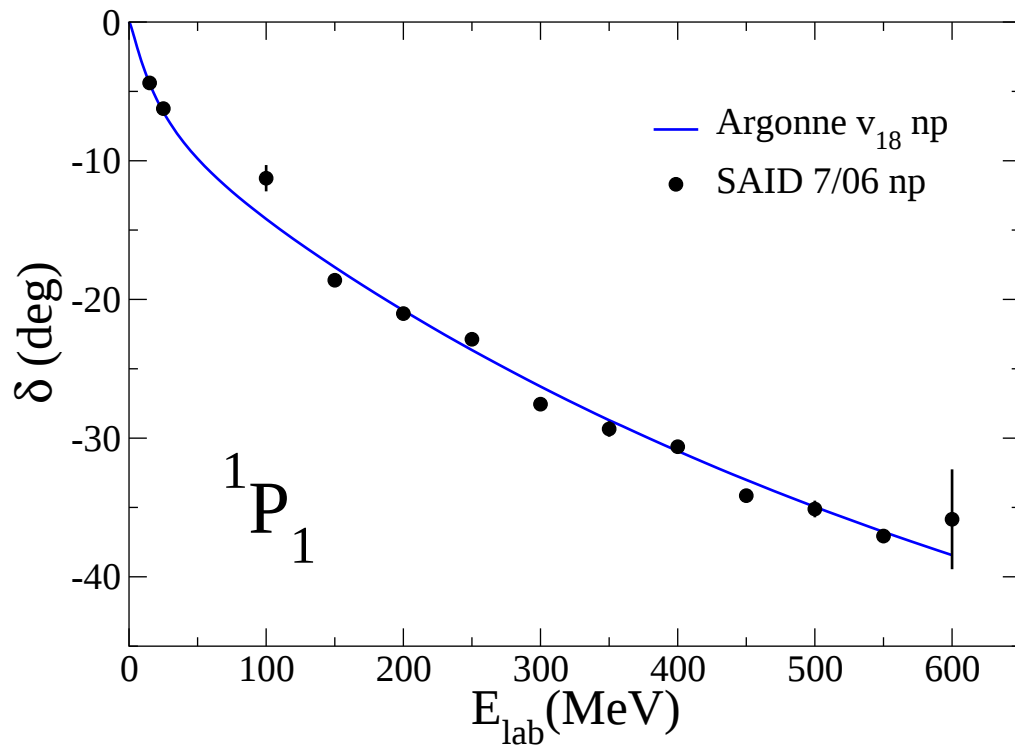


Figure 2.4: The 1P_1 nucleon-nucleon elastic phase shift, δ , calculated using the Argonne V18 potential, as a function of lab energy, E_{lab} . The solid line represents the theoretical calculation, and the black dots represent experimental data from the SAID nucleon-nucleon database in July 2006. Figure obtained from Ref. [35].

The connection to light-front quantization is established by the change of variables,

$$[k^2 + m_N^2]_{\text{IF}} = \frac{m_N^2 + (\mathbf{k}^\perp)^2}{\alpha(2 - \alpha)}, \quad (2.21)$$

where α is equal to Eq. (2.10) multiplied by the nuclear mass number A . The integration measure becomes

$$\int \frac{d\mathbf{k}'_{\text{IF}}}{(2\pi)^3 E(\mathbf{k}'_{\text{IF}})} = \int \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \alpha' (2 - \alpha')}. \quad (2.22)$$

With this substitution, Eq. (2.20) takes the form of the two-body Weinberg equation [36], where we relabel $\psi(\alpha, \mathbf{k}^\perp) \equiv \tilde{\psi}_{\text{IF}}(\mathbf{k}_{\text{IF}})$ and $V \equiv \tilde{V}_{\text{IF}}$ under the new coordinates,

$$\begin{aligned} \left(4 \frac{m_N^2 + (\mathbf{k}^\perp)^2}{\alpha(2 - \alpha)} \right) \psi(\alpha, \mathbf{k}^\perp) + \int \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \alpha' (2 - \alpha')} V(\alpha, \mathbf{k}^\perp; \alpha', \mathbf{k}'^\perp) \psi(\alpha', \mathbf{k}'^\perp) \\ = M^2 \psi(\alpha, \mathbf{k}^\perp), \end{aligned} \quad (2.23)$$

whose operator form is

$$\begin{aligned} \left[\hat{P}^+ \hat{P}^- - \hat{P}^\perp \cdot \hat{P}^\perp \right] |\Psi_{NN}, \mathbf{P}\rangle = M^2 |\Psi_{NN}, \mathbf{P}\rangle, \\ \hat{P}^- = \hat{P}_{\text{kin}}^- + \hat{V}. \end{aligned} \quad (2.24)$$

Given the operator form, one can trace back kinetic energy term in Eq. (2.23) from Eq. (2.14) and see that $V(\alpha, \mathbf{k}^\perp; \alpha', \mathbf{k}'^\perp)$ comes from $\hat{P}^+ \hat{V}$.

One subtlety remains once spin is incorporated. The manipulations above act on the IF potential—which is, in principle, a matrix element taken between IF Dirac spinors—and merely change variables to bring it into LF form, leaving the spinors themselves untouched. What remains is to transform the IF Dirac spinors into LF

Dirac spinors, which is accomplished through a Melosh rotation, a unitary relation between the two (for the original study see Ref. [37], for a detailed derivation see Ref. [33]):

$$u(\mathbf{p}, \sigma) = \sum_{\sigma'} u_{\text{IF}}(\mathbf{p}_{\text{IF}}, \sigma') U(\mathbf{p}_{\text{IF}}, \sigma', \sigma) \quad (2.25)$$

$$U(\mathbf{p}_{\text{IF}}, \sigma', \sigma) = \chi^\dagger(\sigma') \mathcal{U}(\mathbf{p}_{\text{IF}}) \chi(\sigma) \quad (2.26)$$

$$\mathcal{U}(\mathbf{p}_{\text{IF}}) \equiv \frac{p^+ + m_N + \mathbf{p}^\perp \cdot \boldsymbol{\sigma}^\perp \sigma^3}{\sqrt{2p^+} \sqrt{E(\mathbf{p}_{\text{IF}}) + m_N}}$$

$$p^+ \equiv E(\mathbf{p}_{\text{IF}}) + p^3$$

where $(\boldsymbol{\sigma}^\perp, \sigma^3)$ are the Pauli matrices and $\chi(\sigma)$ are the Pauli spinors,

$$\chi(+1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi(-1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (2.27)$$

Including spin, the bound and scattering-state wavefunction solutions of Eq. (2.23)

are therefore

$$\psi_\beta(\alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2) = \sum_{\sigma'_1 \sigma'_2} \tilde{\psi}_{\text{IF}}^\beta(\mathbf{k}_{\text{IF}}, \sigma'_1, \tau_1, \sigma'_2, \tau_2) U^*(\mathbf{k}_{\text{IF}}, \sigma'_1, \sigma_1) U^*(-\mathbf{k}_{\text{IF}}, \sigma'_2, \sigma_2), \quad (2.28)$$

where β indexes the eigenvalues of the Hamiltonian. The half-off-shell T -matrix is analogously,

$$T(\alpha_{\text{off}}, \mathbf{k}_{\text{off}}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2; \alpha, \mathbf{k}^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4) =$$

$$\sum_{\sigma'_1 \sigma'_2} \sum_{\sigma'_3 \sigma'_4} U^*(\mathbf{k}_{\text{IF}}^{\text{off}}, \sigma'_1, \sigma_1) U^*(-\mathbf{k}_{\text{IF}}^{\text{off}}, \sigma'_2, \sigma_2) U(\mathbf{k}_{\text{IF}}, \sigma'_3, \sigma_3) U(-\mathbf{k}_{\text{IF}}, \sigma'_4, \sigma_4) \quad (2.29)$$

$$\times \tilde{T}_{\text{IF}}(\mathbf{k}_{\text{IF}}^{\text{off}}, \sigma'_1, \tau_1, \sigma'_2, \tau_2; \mathbf{k}_{\text{IF}}, \sigma'_3, \tau_3, \sigma'_4, \tau_4),$$

where $\tilde{T}_{\text{IF}}(\mathbf{k}_{\text{IF}}^{\text{off}}, \mathbf{k}_{\text{IF}}) = \sqrt{E(\mathbf{k}_{\text{IF}}^{\text{off}})} T_{\text{IF}}(\mathbf{k}_{\text{IF}}^{\text{off}}, \mathbf{k}_{\text{IF}}) \sqrt{E(\mathbf{k}_{\text{IF}})}$.

Taken together, minimal relativity, the transformation to relative LF coordinates, and the inclusion of LF spinors allow us to employ well-established non-relativistic potentials within the LF quantization framework. This prescription has been widely used in high-energy nuclear physics, for instance in deuteron electrodisintegration [38] and in semi-exclusive electron-deuteron reactions in both the quasi-elastic and DIS regimes [39, 32, 33, 40]. Although the theoretical uncertainty of the prescription is difficult to quantify, these studies find that it performs well for relative momenta up to roughly 600 MeV—large enough that we can reliably describe short-range nucleon-nucleon dynamics within LF quantization. For a concrete example, Fig. 2.5 presents the LF momentum distribution, integrated over transverse momenta, of deuterium. The deuteron state is labeled as

$$|d, M_J; \mathbf{P}_d\rangle, \quad P_d^- = \frac{m_d^2 + (\mathbf{P}_d^\perp)^2}{P_d^+}, \quad (2.30)$$

the overlap with an anti-symmetric two-nucleon state being

$$\begin{aligned} &\langle \mathbf{p}_1, \sigma_1, \tau_1; \mathbf{p}_2, \sigma_2, \tau_2 | d, M_J; \mathbf{P}_d \rangle = \\ &(2\pi)^3 2P_d^+ \delta(p_1^+ + p_2^+ - P_d^+) \delta^{(2)}(\mathbf{p}_1^\perp + \mathbf{p}_2^\perp - \mathbf{P}_d^\perp) \psi_d(\alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 | M_J), \end{aligned} \quad (2.31)$$

with wavefunction normalization

$$\frac{1}{3} \sum_{M_J} \frac{1}{2} \sum_{\substack{\sigma_1 \tau_1 \\ \sigma_2 \tau_2}} \int \frac{d\alpha d\mathbf{k}^\perp}{(2\pi)^3 \alpha (2 - \alpha)} |\psi_d(\alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 | M_J)|^2 = 1. \quad (2.32)$$

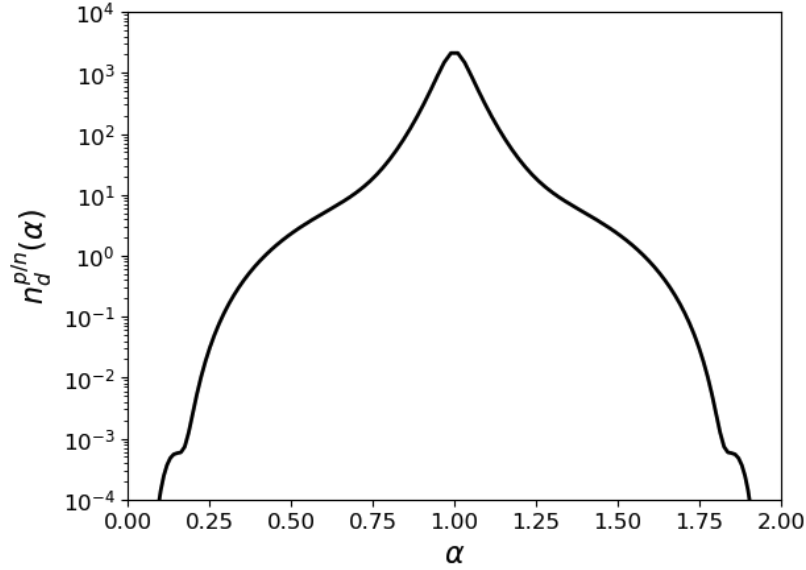


Figure 2.5: The light-front momentum distribution, integrated over transverse momenta, of deuterium as a function of the Lorentz-invariant relative variable $\alpha = 2p^+/P_d^+$.

2.6 Summary

This chapter has served to motivate the study of nuclear physics within the FF (LF quantization), the framework on which the original calculations of this dissertation rest. Its central conclusion is that, for high-energy nuclear reactions, the FF is not merely a convenient choice but a necessary one: it is the only relativistic Hamiltonian formulation in which the bound-state wavefunction admits a frame-independent treatment and in which the off-shellness of the electron-nucleon subprocess remains free of probe contamination; together, these properties permit a composite, factorized description of electron-nucleus scattering. The origin of this conclusion lies in

the choice of time variable, which dictates which Poincaré generators are kinematical and which are dynamical.

Quantization on surfaces of constant x^0 places the three Lorentz boosts in the dynamical set, entangling intrinsic structure with center-of-mass motion, so that a single wavefunction can no longer describe a bound state across different frames. The scattering cross section is itself a Lorentz-invariant quantity and may be evaluated in any frame; however, this invariance does not eliminate the dynamical complexity. Evaluating the cross section in the rest frame merely relocates that complexity into the boost operators acting on the currents, as in Eq. (2.1). Quantization on surfaces of constant x^+ instead places the longitudinal and transverse LF boosts in the kinematical set. The intrinsic variables $(x_i, \mathbf{k}_i^\perp)$ then separate cleanly from the center-of-mass coordinates, and the resulting bound-state wavefunctions are invariant under LF boosts.

The choice of quantization surface also governs the modeling of the elementary subprocess. In any Hamiltonian formulation, only three momentum components are conserved at each interaction vertex, and the off-shell extrapolation required to describe bound-nucleon kinematics depends on which component is left unconserved. In the IF, the probe-dependent contribution to the off-shellness grows linearly with the probe energy, contaminating the very target structure one seeks to isolate. In the FF, this contribution vanishes for the relevant kinematics, so that the off-shellness is determined entirely by the nuclear target. The FF is therefore the only relativistic

Hamiltonian formulation free of large, probe-induced off-shellness in the electron-nucleon subprocess.

Finally, as phenomenological LF NN potentials capable of reproducing elastic phase shifts to high accuracy are not yet available, we use the minimal-relativity prescription, along with a change of variables and the inclusion of LF spinors, to recast well established non-relativistic IF potentials into approximate LF form. Taken together, these results establish LF quantization as the Hamiltonian framework required for high-energy nuclear scattering, while the minimal-relativity prescription supplies the LF two-body NN interactions on which the calculations of the following chapters are built.

Chapter 3

INCLUSIVE ELECTRON-NUCLEUS SCATTERING

Across both low- and high-energy regimes, electron scattering serves as an indispensable tool for probing nuclear and partonic structure. For experimentalists, the electron acts as a high-resolution microscope; its low mass allows it to be readily accelerated to relativistic speeds. Furthermore, modern tracking detectors, electromagnetic calorimeters, and reconstruction techniques enable the measurement of scattered electrons with exceptional precision. From a theoretical perspective, the scattering process is governed by Quantum Electrodynamics and can therefore be modeled accurately. Consequently, electron-nucleus scattering provides a clean separation between the internal target structure and reaction mechanisms.

Inclusive electron-nucleus scattering $A(e, e')$, the main subject of this work, provides two independent kinematic “dials”—the energy transfer, ν , and the momentum transfer, q —that can be tuned to isolate specific physical regimes. The momentum transfer determines the spatial resolution of the probe, governed by the de Broglie wavelength $\lambda = \frac{\hbar}{q}$, which establishes the effective target the electron interacts with. Conversely, the energy transfer dictates the excitation energy transferred to that target. At low momentum transfers (on the order of tens of MeV), the electron probes

the nucleus as a whole; increasing ν in this regime reveals nuclear excited states and giant resonances. At intermediate momentum transfers (hundreds of MeV to a few GeV), the spatial resolution sharpens to resolve individual nucleons within the nucleus. This kinematic region is known as quasi-elastic (QE) scattering. Here, increasing ν accesses nucleon excited states, such as the Roper resonance. Finally, at momentum and energy transfers on the order of hundreds of GeV, the electron resolves the partonic structure within the nucleons and deposits enough energy to break a nucleon apart. Such processes are known as deep inelastic scattering (DIS), and they dominate the reaction cross section in this regime.

In order to develop nuclear models applicable to future high-energy nuclear physics experiments at the EIC and JLab, one must first understand QE scattering—a process that probes the nucleonic structure of the nucleus—and then build on that baseline to understand how nucleonic configurations in the nucleus interplay with their partonic dynamics. The experimental data considered in this dissertation is from JLab E08-018, an inclusive electron-nucleus scattering measurement with an electron beam energy of 3.356 GeV and a scattered-electron angle of 25° [41]. Restricting ourselves to $x_B = Q^2/(2m_N\nu) > 1$ yields $Q^2 > 1 \text{ GeV}^2$, placing us in a kinematic window with momentum transfer high enough to resolve short-distance nuclear structure, but energy transfer low enough that we need not consider the transition to quark degrees of freedom in DIS.

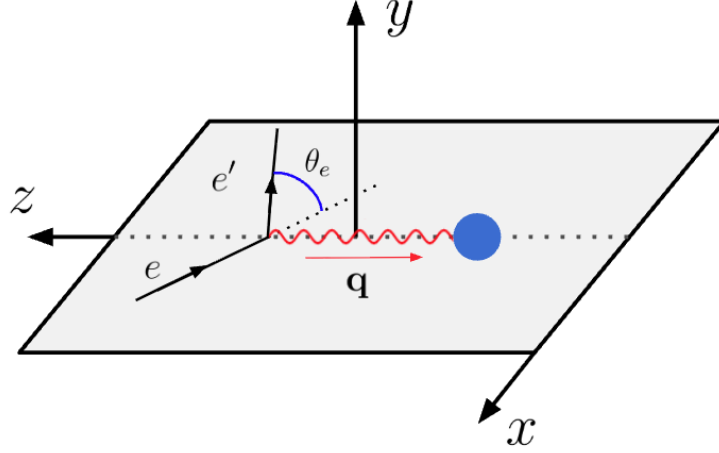


Figure 3.1: (color online) Nuclear rest frame, with the virtual photon momentum, $\mathbf{q}_{\text{IF}}$, defining the z -axis and the $x - z$ plane defined by the electron and scattered electron momentum vectors. The nucleus is represented by the filled blue circle.

3.1 Collinear Frames

Throughout this dissertation, all calculations are performed in the nucleus rest frame, with the virtual photon's IF three-momentum defining the negative z -axis (see Fig. 3.1). The nucleus rest frame belongs to a broader class of frames—known as *collinear frames*—that are related to one another by boosts along the z -axis. In any such frame, the LF momentum components of the nucleus and the virtual photon take the form

$$\begin{aligned}
 P_A^+ &> 0, & P_A^- &= \frac{m_A^2}{P_A^+}, & \mathbf{P}_A^\perp &= 0, \\
 q^+ &= -\xi_A P_A^+, & q^- &= \frac{q^2}{q^+} = \frac{Q^2}{\xi_A P_A^+}, & \mathbf{q}^\perp &= 0,
 \end{aligned}
 \tag{3.1}$$

where the parameter ξ_A is fixed by the kinematic invariant

$$2P_A \cdot q = P_A^+ q^- + P_A^- q^+ = \frac{Q^2}{x_A}. \quad (3.2)$$

Solving this constraint for ξ_A yields

$$\xi_A = \frac{2x_A}{1 \pm \sqrt{1 + 4m_A^2 x_A^2 / Q^2}}. \quad (3.3)$$

We select the positive root, known as the Nachtmann variable [42], which is what orients the virtual photon's momentum along the negative z -axis. In the scaling limit $Q^2 \gg m_A^2$ this reduces to

$$\xi_A = x_A + \mathcal{O}(m_A^2/Q^2). \quad (3.4)$$

In the nucleus rest frame, $P_A^+ = P_A^- = m_A$, and x_A is related to the Bjorken variable $x_B = Q^2/(2m_N \nu)$ by

$$x_B = \frac{m_A}{m_N} x_A. \quad (3.5)$$

3.2 Unpolarized Inclusive Electron-Nucleus Cross-Section

We start with the general parameterization of the cross-section for unpolarized inclusive electron-nucleus scattering where the final states can be anything, X . Because the electromagnetic interaction between the electron and nucleus is weak, one can model the interaction through the exchange of a single virtual photon.

$$\frac{d\sigma}{dE' d\Omega'} = \frac{E'}{E} \frac{\alpha_e^2}{Q^4} L_{\mu\nu} W_A^{\mu\nu}, \quad (3.6)$$

where q^μ is the virtual photon's four momentum and $Q^2 = -q^\mu q_\mu$, E and E' are the energies of the incident and scattered electron respectively, and α_e is the fine structure constant. Ignoring the mass of the electron, the leptonic tensor, $L_{\mu\nu}$, is given by

$$L_{\mu\nu} = \frac{1}{2} \text{Tr}[k' \gamma_\mu k \gamma_\nu] = 2(k'_\mu k_\nu + k'_\nu k_\mu - k' \cdot k g_{\mu\nu}), \quad (3.7)$$

where k^μ and k'^μ are the four momenta of the incident and scattered electrons respectively, γ^μ are the Dirac gamma matrices, and $g^{\mu\nu}$ is the Minkowski metric tensor.

The unpolarized nuclear tensor, $W_A^{\mu\nu}$, is given by

$$W_A^{\mu\nu} = \frac{1}{4\pi m_A} \frac{1}{2J+1} \sum_{M_J} \sum_X \int d\Pi_X \langle \Psi_A, \mathbf{P}_A | \hat{J}_A^\mu(0) | X \rangle \langle X | \hat{J}_A^\nu(0) | \Psi_A, \mathbf{P}_A \rangle \times (2\pi)^4 \delta^4(q^\mu + P_A^\mu - p_X^\mu), \quad (3.8)$$

where Ψ_A denotes the nuclear state which implicitly contains the mass m_A , total angular momentum J , and total angular momentum projection M_J labels. $d\Pi_X$ is the Lorentz invariant phase space measure for the final states and P_A^μ and p_X^μ are the four momenta of the nucleus and final states.

It is worthwhile to re-write Eq. (3.6) in terms of response functions, which describes the scattering of the target by a virtual photon with a given polarization. This is done by including explicit factors of the Minkowski metric in the contraction between the leptonic and nuclear tensors, $L_{\mu\nu} W_A^{\mu\nu} = L^{\rho\sigma} g_{\rho\mu} g_{\sigma\nu} W_A^{\mu\nu}$, and $\sum_\lambda \epsilon_\mu^\lambda(q) \epsilon_\nu^{*\lambda}(q) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}$. Using that the unpolarized nuclear tensor is symmetric and obeys the Ward identity, $q_\mu W_A^{\mu\nu} = 0$, we get the following decomposition of Eq. (3.6):

$$\frac{d\sigma}{dE'd\Omega'} = \left(\frac{d\sigma}{d\Omega'} \right)_{Mott} \left\{ \frac{Q^4}{|\mathbf{q}_{IF}|^4} W_A^L(x_B, Q^2) + \left(\frac{1}{2} \frac{Q^2}{|\mathbf{q}_{IF}|^2} + \tan^2 \frac{\theta_e}{2} \right) W_A^T(x_B, Q^2) \right\}, \quad (3.9)$$

$$\left(\frac{d\sigma}{d\Omega'} \right)_{Mott} = \frac{\alpha_e \cos^2 \frac{\theta_e}{2}}{4E^2 \sin^4 \frac{\theta_e}{2}}, \quad (3.10)$$

$$W_A^T = W_A^{11} + W_A^{22}, \quad (3.11)$$

$$W_A^L = \frac{|\mathbf{q}_{IF}|^2}{4Q^2} \left(\frac{Q^2}{(q^+)^2} W_A^{++} + 2W_A^{+-} + \frac{(q^+)^2}{Q^2} W_A^{--} \right), \quad (3.12)$$

where W_A^T and W_A^L are the transverse and longitudinal nuclear response functions respectively, and θ_e is the angle between the incident and scattered electron. With this, we have reduced the problem down to calculating W_A^T and W_A^L , the nuclear structure terms.

3.3 Electromagnetic Current

The following discussion closely follows Refs. [43, 44] and is presented here for pedagogical purposes; readers are directed to those references for the complete derivations.

This dissertation focuses on the QE regime, in which the momentum transfer is large enough to resolve individual nucleons inside the nucleus. In this kinematic domain the electromagnetic probe couples primarily to a single nucleon, which justifies the replacement $\hat{J}_A^\mu(0) \rightarrow \hat{J}_N^\mu(0)$ in Eq. (3.8). The operator $\hat{J}_N^\mu(0)$ is not, however,

the free-nucleon electromagnetic (EM) current, because the nucleons inside the target are bound and therefore off-shell. Modeling this *off-shell* EM current operator is the central theoretical challenge of this section.

Historically, the off-shell problem rose to prominence in the early 1980s with the advent of intermediate-energy $A(e, e'N)$ experiments at SACLAY [45, 46] and NIKHEF [47]. The most widely used prescriptions to this day are those of de Forest [30], which extrapolate the on-shell EM current to off-shell kinematics while retaining on-shell spinors for the struck nucleon. This construction is, however, fundamentally incompatible with the covariant Feynman-diagram calculations in which it is typically embedded, since a bound nucleon in such a calculation is described by a propagator rather than by on-shell spinors.

To make this incompatibility concrete, consider the covariant Feynman diagram for electron–deuteron scattering shown in Fig. 3.2. Evaluating the diagram with the standard Feynman rules gives

$$\bar{u}_{\text{IF}}(\mathbf{p}_f, \sigma_f, \tau_f) \Gamma_{\gamma^* N}^\mu \frac{\not{p}_i + m_N}{p_i^2 - m_N^2 + i\epsilon} \bar{u}_{\text{IF}}(\mathbf{p}_r, \sigma_r, \tau_r) \Gamma_{dNN}^\nu \chi_\nu(M_J), \quad (3.13)$$

where $\Gamma_{\gamma^* N}^\mu$ is the effective electromagnetic vertex between the virtual photon and bound nucleon, Γ_{dNN}^μ represents the vertex of the $d \rightarrow NN$ transition, and $\chi_\nu(M_J)$ is the polarization vector of the deuteron. Two features of this fully covariant expression are worth emphasizing. First, the EM vertex $\Gamma_{\gamma^* N}^\mu$ is never sandwiched between two on-shell spinors: the intermediate nucleon enters through a propagator rather than as an external on-shell state. Second, as discussed in Chapter 2, no nuclear wave

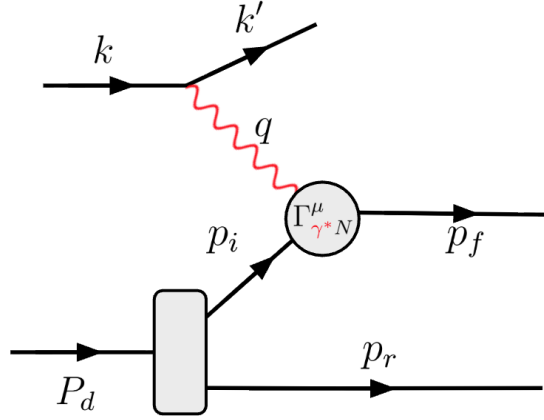


Figure 3.2: Covariant Feynman diagram describing electron–nucleon scattering inside a deuterium target.

function appears explicitly in this representation. The de Forest prescription instead finds its natural home in *noncovariant* treatments—such as the IF and FF—where intermediate nucleons are described by on-shell spinors and the nuclear wave function does appear explicitly.

The advent of high-energy eA experiments, in which electrons transfer large momenta to bound nucleons and yield final-state nucleons with momenta above a few GeV [48, 49, 50], has motivated renewed interest in formulating off-shell EM currents directly within the LF framework. References [43, 44] carry out precisely this construction. Using effective LF diagrammatic rules to sum all time-orderings of Fig. 3.2—a step required to correctly account for the LF off-shellness of the interme-

diate nucleon—they obtain an off-shell EM current operator of the form

$$\hat{J}_N(0) = \sum_{\sigma\sigma'\tau} \int [dp][dp'] \sqrt{2p'^+} \sqrt{2p^+} J_N(\mathbf{p}', \sigma', \tau; \mathbf{p}, \sigma, \tau) \hat{a}^\dagger(\mathbf{p}', \sigma', \tau) \hat{a}(\mathbf{p}, \sigma, \tau), \quad (3.14)$$

with single-particle matrix element

$$J_N(\mathbf{p}', \sigma', \tau; \mathbf{p}, \sigma, \tau) = \bar{u}(\mathbf{p}', \sigma', \tau) \Gamma_{\gamma^* N}^\mu \left(1 + \frac{\not{A}_p}{2m_N}\right) u(\mathbf{p}, \sigma, \tau), \quad (3.15)$$

where $u(\mathbf{p}, \sigma, \tau)$ is a LF spinor and the EM vertex is parametrized as

$$\Gamma_{\gamma^* N}^\mu = \gamma^\mu F_1 + i\sigma^{\mu\nu} q_\nu \frac{\kappa F_2}{2m_N} + q^\mu F_3. \quad (3.16)$$

Here $\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ is the Pauli tensor, κ is the anomalous magnetic moment of the nucleon, F_1 , F_2 , F_3 are the elastic form factors, taken to be functions of Q^2 alone, with any off-shell dependence neglected, and $q^\mu \neq p'^\mu - p^\mu$. Furthermore, the F_3 term does not contribute to the cross section by virtue of the gauge invariance of the leptonic current. The off-shell information is carried by the operator $\not{A}_p$ appearing alongside $\Gamma_{\gamma^* N}^\mu$:

$$\not{A}_p = \frac{\gamma^+}{2} (p_c^- - p^-), \quad p^- = \frac{m_N^2 + (\mathbf{p}^\perp)^2}{p^+}, \quad (3.17)$$

where p^- is the on-shell LF energy and p_c^- (subscript c for “conserved”) is the LF energy that the nucleon would carry if energy were conserved at each vertex. The difference $p_c^- - p^-$ is precisely the LF off-shellness of the bound nucleon.

Refs. [43, 44] compared their LF predictions against the widely used de Forest prescriptions. In essentially all kinematic cases examined, the LF approach predicts

smaller off-shell effects than de Forest at $Q^2 > 1 \text{ GeV}^2$. More importantly, the LF approach predicts a pronounced suppression of these effects with increasing Q^2 , which is understood intuitively as a decreasing sensitivity of the hard scattering process to the off-shellness of the target nucleon.

3.4 Inclusive Quasi-Elastic Electron–Deuteron Scattering

Equipped with the LF off-shell EM current of the preceding section, we now compute inclusive electron–deuteron scattering in the plane-wave impulse approximation (PWIA). The deuteron nuclear tensor reads

$$W_d^{\mu\nu} = \frac{1}{4\pi m_d} \cdot \frac{1}{3} \sum_{M_J} \frac{1}{2} \sum_{\substack{\sigma_1 \sigma_2 \\ \tau_1 \tau_2}} \int [dp_1][dp_2] \langle \Psi_d | \hat{J}_N^\mu(0) | p_1, p_2 \rangle \langle p_1, p_2 | \hat{J}_N^\nu(0) | \Psi_d \rangle \quad (3.18)$$

$$\times (2\pi)^4 \delta^4(q^\mu + P_d^\mu - p_1^\mu - p_2^\mu),$$

where the $\frac{1}{3} \sum_{M_J}$ averages over the three deuteron polarizations, the factor of $\frac{1}{2}$ accounts for the indistinguishability of the two-nucleon final state, and we drop the momentum label for nuclear states at rest from this point on.

The current matrix element receives two contributions, corresponding to the virtual photon striking nucleon 1 or nucleon 2 of the deuteron:

$$\begin{aligned} \langle p_1, p_2 | \hat{J}_N^\mu(0) | \Psi_d \rangle = & \sum_{\sigma_i} \left[\frac{m_d}{p_1^+ - q^+} J_N(\mathbf{p}_1, \sigma_1, \tau_1; \mathbf{p}_1 - \mathbf{q}, \sigma_i, \tau_1) \right. \\ & \times \psi_d \left(\frac{2(p_1^+ - q^+)}{P_d^+}, \mathbf{p}_1^\perp, \sigma_i, \sigma_2, \tau_1, \tau_2 \left| M_J \right. \right) \\ & - \frac{m_d}{p_2^+ - q^+} J_N(\mathbf{p}_2, \sigma_2, \tau_2; \mathbf{p}_2 - \mathbf{q}, \sigma_i, \tau_2) \\ & \left. \times \psi_d \left(\frac{2(p_2^+ - q^+)}{P_d^+}, \mathbf{p}_2^\perp, \sigma_i, \sigma_1, \tau_2, \tau_1 \left| M_J \right. \right) \right]. \end{aligned} \quad (3.19)$$

Squaring this matrix element produces interference terms between the two contributions, which are routinely neglected in calculations of this type, but are kept here.

Figure 3.3 presents the inclusive electron–deuteron cross section as a function of the Bjorken variable x_B , compared to JLab data at two kinematic settings: $E = 3.356$ GeV, $\theta_e = 25^\circ$ from JLab E08-014 [41] (left panel) and $E = 5.766$ GeV, $\theta_e = 18^\circ$ from JLab E02-019 [51] (right panel). The approximate LF deuteron wave function obtained from the procedure outlined in Chapter 2 reproduces the E02-019 data well, owing to its larger Q^2 , which provides higher spatial resolution of the high-momentum tail of the deuteron than the more modest Q^2 of E08-014. The residual discrepancy near the QE peak in the E08-014 panel reflects physics absent both from the plane-wave impulse approximation and from the AV18 interaction itself — principally two-body meson-exchange currents and the low-energy tail of the $\Delta(1232)$ excitation, neither of which is generated by a one-body current acting on a purely nucleonic NN potential without explicit meson-exchange or Δ degrees of freedom. Both contributions fall in importance at the higher Q^2 of E02-019. Overall, both datasets are well described using the AV18 NN interaction, supporting the use of the minimal-relativity AV18 potential as an approximate LF NN potential — one capable of capturing the high-momentum two-nucleon dynamics necessary for calculations developed later in this dissertation.

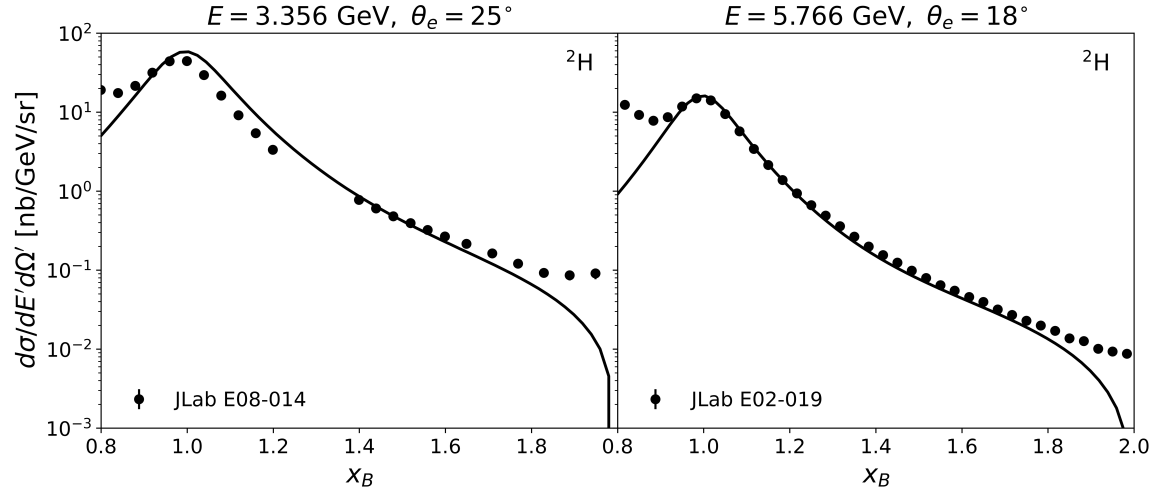


Figure 3.3: Inclusive electron–deuteron cross section as a function of the Bjorken variable x_B , at the JLab E08-014 kinematics $E = 3.356$ GeV, $\theta_e = 25^\circ$ (left) and the JLab E02-019 kinematics $E = 5.766$ GeV, $\theta_e = 18^\circ$ (right). Black points are the experimental data; the solid black curves are our light-front calculation using the AV18 NN interaction.

3.5 Summary

Benchmarking the LF nuclear structure calculation presented in this dissertation requires comparing to observables that have high enough momentum transfer to probe the short-distance nuclear structure—relevant for high-energy processes involving quarks at Jefferson Lab and the Electron-Ion Collider—but also low enough energy transfer to avoid breaking up the nucleons, probing the quarks inside. Inclusive QE electron-nucleus scattering at $Q^2 > 1 \text{ GeV}^2$, $x_B > 1$, and $0.3 < \nu < 1 \text{ GeV}$ provides exactly such an observable.

Because the electromagnetic interaction is weak, it is accurately modeled through the exchange of a single virtual photon. By utilizing the Ward identity and the symmetry of the unpolarized nuclear tensor, the cross section factorizes into transverse and longitudinal response functions, as expressed in Eq. (3.9). These response functions isolate and carry all the relevant nuclear structure information. Extracting the actual structure in the QE regime requires a theoretically consistent treatment of the electromagnetic probe interacting with a bound, off-shell nucleon. Rather than relying on legacy noncovariant extrapolations like the de Forest prescription, which fundamentally clash with covariant Feynman diagram calculations by inserting on-shell spinors, this work employs the effective LF off-shell EM current of Refs. [43, 44]. This approach correctly utilizes internal propagators and predicts a natural suppression of off-shell effects at high Q^2 .

With this off-shell current and the LF nuclear structure developed later in this work, we can accurately calculate QE cross sections within the plane-wave impulse approximation. Applying this framework to deuterium provides a critical baseline test. We explicitly retain the subtractive interference between the individual nucleon contributions when squaring the current matrix elements. Our calculation for deuterium supports that the approximate LF wave function obtained from the procedure outlined in Chapter 2, utilizing the AV18 interaction, works well. The successful reproduction of higher- Q^2 JLab data confirms that this framework captures the high-momentum two-nucleon dynamics necessary for the calculations developed in the subsequent chapters, with the expected minor deviations at lower Q^2 correctly attributable to omitted two-body meson-exchange currents and Δ -excitations.

Chapter 4

DENSITY FUNCTIONAL THEORY

Density Functional Theory (DFT) is one of the most popular ab-initio approaches towards studying many-body systems. Developed by Hohenberg and Kohn in 1964 [52], and later expanded upon by Kohn and Sham one year later [53], DFT was originally utilized to study many-electron atomic systems, but has seen great utility in many fields such as chemistry, nuclear physics, and condensed matter physics. The basic idea is that the ground-state energy of a many-body system can be represented in terms of the interacting ground-state density alone. Furthermore, under the Kohn-Sham (KS) scheme, the interacting ground-state density can be reproduced by the ground-state density of an auxiliary non-interacting system. Thus, one reduces the many-body problem from solving a coupled N-body Schrodinger equation, to solving for the spectra of a one-body Schrodinger equation. This makes DFT calculations comparatively simple to implement, often very accurate, and computationally feasible for systems involving large particle number. For a more comprehensive review and further reading, see Refs. [54, 55].

4.1 *Coulombic Density Functional Theory*

DFT rests on two fundamental theorems, proved by Hohenberg and Kohn for electromechanical many-body systems in an external potential $v_{ext}(\mathbf{r})$ [52]:

1. The ground state energy for a given external potential, E_v , is a unique functional of the electron density $\rho(\mathbf{r})$,

$$E_v[\rho] = F_{HK}[\rho] + \int d^3r v_{ext}(\mathbf{r}) \rho(\mathbf{r}),$$

where the functional F_{HK} does not depend on v_{ext} nor on the particle number of the system. It only depends on the form of the interaction between the particles, and hence is universal.

2. The electron density that minimizes the energy of the energy functional is the true electron density, corresponding to the full solution of the Schrodinger equation.

The fundamental issue facing us is that the Hohenberg-Kohn (HK) theorems only show the existence of F_{HK} , but offer no guidance in constructing the functional. This task is put into better context under the KS Scheme [53], which decomposes F_{HK} by:

$$F_{HK}[\rho] = T_s[\rho] + E_H[\rho] + E_{xc}[\rho] \tag{4.1}$$

where T_s is the kinetic energy of the non-interacting system, E_H is the Hartree energy, and in this context, is the energy corresponding to the classical Coulomb potential, and E_{xc} is the exchange-correlation energy which, by definition, contains all the many-body quantum effects not included in T_s and E_H . One can perform a variation of E_v with respect to ρ under a fixed particle number, N , constraint:

$$\frac{\delta}{\delta\rho(\mathbf{r})} \left\{ E_v[\rho] - \mu \left(\int d^3r' \rho(\mathbf{r}') - N \right) \right\} = 0, \quad (4.2)$$

where μ is the Lagrange multiplier associated to constant particle number. With Eq. (4.1) this yields,

$$\frac{\delta T_s[\rho]}{\delta\rho(\mathbf{r})} + V_{KS}(\mathbf{r}) = \mu, \quad (4.3)$$

$$V_{KS}(\mathbf{r}) = v_{ext}(\mathbf{r}) + \frac{\delta(E_H[\rho] + E_{xc}[\rho])}{\delta\rho}. \quad (4.4)$$

Consider a completely different problem: a system of non-interacting electrons moving in an external potential $V(\mathbf{r})$. If we applied the exact same variational principle to that system's energy functional, we would obtain Eq. (4.3) but with $V_{KS}(\mathbf{r}) \rightarrow V(\mathbf{r})$. Hence, Eq. (4.3) is satisfied by the density obtained from solving the single particle Schrodinger equation for electrons moving in an external potential $V_{KS}(\mathbf{r})$,

$$\left[-\frac{\nabla^2}{2m} + V_{KS}(\mathbf{r}) \right] \phi_i(\mathbf{r}) = \epsilon_i \phi_i(\mathbf{r}), \quad (4.5)$$

where

$$\rho(\mathbf{r}) = \sum_{i=1}^N |\phi_i(\mathbf{r})|^2, \quad (4.6)$$

and the N-body wavefunction is a Slater determinant of the N lowest-lying single-particle orbitals. Since $V_{KS}(\mathbf{r})$ also depends on the electron density, these equations must be evaluated self-consistently. In practice, $V_{KS}(\mathbf{r})$ is either determined from first-principles, which has been done in non-relativistic Coulombic systems, or phenomenologically with parameters optimized by fitting to a set of data. The latter is the most widely used way for DFT applications to nuclear systems.

4.2 Nuclear Density Functional Theory

For DFT applications to nuclear systems, one finds that the first HK theorem does not hold. The proof of the first theorem is based on the existence of $v_{ext}(\mathbf{r})$. However, nuclei are self-bound objects, meaning that the system is not confined due to an external potential, but due to the inter-nucleonic interactions contained in the F_{HK} functional. It has been shown that for self-bound systems, the HK theorems do hold for the intrinsic density [56, 57]. This means that nuclei that can be described using DFT all come from the same universal energy functional, F_{HK} . However, the independent, single-particle, description obtained from the KS scheme is lost because the intrinsic density depends on relative coordinates. In practice, one artificially pins the nucleus to the origin by constructing energy density functionals that depend

on absolute coordinates, breaking translational invariance. Doing so allows one to regain the independent-particle description of the system, but at the cost of losing the fundamental backing from the intrinsic density HK theorems. Regardless, such approaches to nuclear structure have seen remarkable success in describing a plethora of nuclear observables and dynamics.

4.3 Light-Front Density Functional Theory

The concepts of DFT are general and can be applied to relativistic frameworks where energy density functionals can be naturally obtained from field-theoretic Lagrangians. Historically, field-theoretic descriptions of nuclear systems began with the Walecka model [58]. Originally developed to study neutron stars, the Walecka model is a phenomenological model of nucleons interacting through the exchange of scalar and vector mesons which mimic nuclear attraction and repulsion respectively. The extension of the model to finite nuclear systems was later developed by Horowitz and Serot, called Relativistic Mean-Field Theory (RMF) [59]. Using Dyson's equations for the dressed single-particle propagator, Horowitz and Serot developed a set of self-consistent equations where nucleon densities act as current sources that generate the meson fields, which in turn influence the individual nucleon dynamics. In what follows we recast this framework in the language of Kohn–Sham DFT and extend it to the LF, with a further generalization to density-dependent couplings.

The starting point is a phenomenological field-theoretic Lagrangian, $\mathcal{L}$, describ-

ing nucleons interacting through the exchange of isoscalar-scalar σ -mesons, isoscalar-vector ω -mesons, isovector-vector ρ -mesons, and photons.

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{int}, \quad (4.7)$$

$\mathcal{L}_N$ is the Dirac Lagrangian describing free nucleons

$$\mathcal{L}_N = \bar{\psi}(i\gamma_\mu\partial^\mu - m_N)\psi, \quad (4.8)$$

$\mathcal{L}_m$ corresponds to the free meson and photon fields,

$$\begin{aligned} \mathcal{L}_m = & \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & - \frac{1}{4}\vec{R}_{\mu\nu} \cdot \vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{b}_\mu \cdot \vec{b}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \end{aligned} \quad (4.9)$$

where the field tensors are given by

$$\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu, \quad (4.10)$$

$$\vec{R}_{\mu\nu} = \partial_\mu\vec{b}_\nu - \partial_\nu\vec{b}_\mu, \quad (4.11)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (4.12)$$

The interaction terms between the meson/electromagnetic fields are encapsulated in

$\mathcal{L}_{int}$

$$\mathcal{L}_{int} = -g_\sigma\bar{\psi}\psi\sigma - g_\omega\bar{\psi}\gamma^\mu\psi\omega_\mu - \frac{1}{2}g_\rho\bar{\psi}\gamma^\mu\vec{\tau}\psi \cdot \vec{b}_\mu - e\bar{\psi}\gamma^\mu\frac{1+\tau_3}{2}\psi A_\mu, \quad (4.13)$$

In the above expressions, m_N is the nucleon mass, while m_m and g_m ($m = \sigma, \omega, \rho$) denote the meson masses and meson–nucleon coupling constants. The electromagnetic field is denoted A^μ , with coupling e , and the ρ -meson field is denoted $\vec{b}^\mu$.

RMF treats the meson mediators as classical fields. Because the meson fields are not quantized, quantum-mechanical exchange and correlation interactions are neglected here; hence, RMF is intrinsically a Hartree-level theory. Furthermore, by restricting our focus to static, spherically-symmetric nuclei, the pion fields do not contribute to the ground state due to parity conservation. These meson fields phenomenologically capture distinct aspects of the nucleon-nucleon interaction: the σ -meson simulates intermediate-range attraction, the ω -meson provides short-range repulsion, and the ρ -meson accounts for the isospin dependence. It is well known that pions play a critical role in the nuclear force, providing the long-range attraction and the short-range tensor force. Although the underlying assumptions of the RMF framework prevent the pion fields from contributing explicitly to the ground state, the specific combination of the remaining effective meson fields is designed to implicitly absorb and replicate their overall physical effects.

From Noether's Theorem, the conserved four-momenta follows from the energy momentum tensor $\mathcal{T}^{\mu\nu} = \sum_n \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \partial^\nu \phi_n - g^{\mu\nu} \mathcal{L}$, with ϕ_n running over all fields:

$$\begin{aligned} \mathcal{T}^{\mu\nu} = & \bar{\psi} i \gamma^\mu \partial^\nu \psi - \Omega^{\mu\alpha} \partial^\nu \omega_\alpha - F^{\mu\alpha} \partial^\nu A_\alpha \\ & - \vec{R}^{\mu\alpha} \cdot \partial^\nu \vec{b}_\alpha + \partial^\mu \sigma \partial^\nu \sigma - g^{\mu\nu} \mathcal{L} \\ & - \frac{1}{2} g^{\mu\nu} \left[(\partial_\alpha \omega^\alpha)^2 + (\partial_\alpha \vec{b}^\alpha)^2 + (\partial_\alpha A^\alpha)^2 \right], \end{aligned} \tag{4.14}$$

where the terms in the square brackets are all equal to zero by the equations of motion.

The conserved LF momenta on surfaces of constant x^+ are

$$P^\mu = \frac{1}{2} \int dx^- d\mathbf{x}^\perp \mathcal{T}^{+\mu}. \quad (4.15)$$

In developing LF DFT, one must carefully select a viable energy density functional. The natural choice of the light-front Hamiltonian, P^- , is fundamentally flawed in this context; given the light-front dispersion relation $P_A^- = \frac{m_A^2 + (\mathbf{P}_A^\perp)^2}{P_A^+}$, the system can trivially minimize its energy by driving the longitudinal momentum $P_A^+ \rightarrow \infty$. The linear combination $\frac{1}{2}(P^+ + P^-)$ is more promising. It avoids the unphysical $P_A^+ \rightarrow \infty$ minimization limit, and connected to the rest mass condition on the LF $P_A^+ = P_A^- = m_A$ [60]. We adopt this combination as our energy functional throughout.

At this stage, we can introduce several critical simplifications by leveraging the physical constraints of our specific approach. First, because we focus on static, ground-state nuclei, the meson and photon fields exhibit no dependence on LF time, x^+ . Second, by restricting our analysis to spherically symmetric, even-even nuclei, the perpendicular components of the meson and photon fields vanish due to the time-reversal invariance of the nuclear ground state, and the plus and minus components of the vector fields are rendered equal. Hence, we remove the Lorentz indices from the vector meson and photon fields, labeling them with subscript 0. Finally, the charged ρ meson fields do not contribute; this arises because we operate within the Hartree approximation—thereby neglecting exchange-correlation effects—and because the nucleus has a definite total charge.

On the $x^+ = 0$ hypersurface, we quantize the nucleon fields into field operators by

expanding them in terms of single-particle orbitals indexed by j ,

$$\hat{\psi}(x) = \sum_j \left[\psi_j(\mathbf{x}) \hat{a}_j + \phi_j(\mathbf{x}) \hat{b}_j^\dagger \right], \quad (4.16)$$

where $\hat{a}_j$ ($\hat{b}_j^\dagger$) is the annihilation (creation) operator for a nucleon (anti-nucleon) in state j , and $\psi_j(\mathbf{x})$ ($\phi_j(\mathbf{x})$) is the corresponding single-particle wavefunction. The creation and annihilation operators obey fermionic anti-commutation relations

$$\{\hat{a}_i, \hat{a}_j^\dagger\} = \delta_{ij} \quad \text{and} \quad \{\hat{a}_i, \hat{a}_j\} = \{\hat{a}_i^\dagger, \hat{a}_j^\dagger\} = 0, \quad (4.17)$$

and the nucleon and anti-nucleons anti-commute. The ground state of the nucleus with baryon number A is constructed by a product of the A -lowest lying nucleon single particle states, i.e. the Slater determinant

$$|\Psi_A^{MF}\rangle = \prod_{i=1}^A \hat{a}_i^\dagger |\Omega\rangle \quad \text{with} \quad \langle \Psi_A^{MF} | \Psi_A^{MF} \rangle = 1. \quad (4.18)$$

Note that our nuclear model contains no anti-nucleons and that the nuclear state is built on top of the interacting vacuum $|\Omega\rangle$. When evaluating matrix elements in the nuclear ground state, one must normal-order with respect to the trivial LF vacuum in order to remove the zero-point energy. Doing so yields two terms: one corresponding to the positive-energy nucleons and the other to contributions from the Dirac sea, the latter of which we discard [61]. Building the nucleus exclusively from the positive-energy solutions—i.e. the physical nucleons—together with this prescription for evaluating matrix elements constitutes the no-sea scheme, which explicitly omits the dynamical effects of anti-nucleons. Although it is commonly referred to in the

literature as the “no-sea approximation,” in the present context it should not be regarded as an approximation at all; it rests on no underlying physical or mathematical justification and is better understood as a foundational modeling choice [55].

Applying the simplifications above together with the quantization just introduced, the Hamiltonian density evaluated with respect to the nuclear ground state is

$$\mathcal{H}_{\text{LF}}^{RMF} = \frac{1}{2}\mathcal{T}^{+-} + \frac{1}{2}\mathcal{T}^{++} = \mathcal{H}_\psi + \mathcal{H}_\sigma + \mathcal{H}_\omega + \mathcal{H}_\rho + \mathcal{H}_A + \mathcal{H}_{\text{int}}, \quad (4.19)$$

with

$$\begin{aligned} \mathcal{H}_\psi &= \sum_{i=1}^A \bar{\psi}_i (i\boldsymbol{\gamma}^\perp \cdot \boldsymbol{\partial}^\perp + i\gamma^3 \partial^+ + m_N) \psi_i \\ \mathcal{H}_\sigma &= \frac{1}{2} [(\boldsymbol{\partial}^\perp \sigma)^2 + (\partial^+ \sigma)^2 + m_\sigma^2 \sigma^2] \\ \mathcal{H}_\omega &= -\frac{1}{2} [(\boldsymbol{\partial}^\perp \omega_0)^2 + (\partial^+ \omega_0)^2 + m_\omega^2 \omega_0^2] \\ \mathcal{H}_\rho &= -\frac{1}{2} [(\boldsymbol{\partial}^\perp b_0)^2 + (\partial^+ b_0)^2 + m_\rho^2 b_0^2] \\ \mathcal{H}_A &= -\frac{1}{2} [(\boldsymbol{\partial}^\perp A_0)^2 + (\partial^+ A_0)^2] \end{aligned} \quad (4.20)$$

$$\mathcal{H}_{\text{int}} = g_\sigma \rho_s \sigma + g_\omega \rho_B \omega_0 + g_\rho \rho_3 b_0 + e \rho_p A_0.$$

where we have introduced the scalar density ρ_s , the baryon density ρ_B , the isovector density ρ_3 , and the proton density ρ_p

$$\rho_s(\mathbf{x}) = \sum_{i=1}^A \bar{\psi}_i(\mathbf{x}) \psi_i(\mathbf{x}), \quad (4.21)$$

$$\rho_B(\mathbf{x}) = \sum_{i=1}^A \bar{\psi}_i(\mathbf{x}) \gamma^0 \psi_i(\mathbf{x}), \quad (4.22)$$

$$\rho_3(\mathbf{x}) = \sum_{i=1}^A \bar{\psi}_i(\mathbf{x}) \gamma^0 \frac{\tau_3}{2} \psi_i(\mathbf{x}), \quad (4.23)$$

$$\rho_p(\mathbf{x}) = \sum_{i=1}^A \bar{\psi}_i(\mathbf{x}) \gamma^0 \frac{1 + \tau_3}{2} \psi_i(\mathbf{x}). \quad (4.24)$$

It is important to remember that when evaluating matrix elements of the field operators with respect to the nuclear ground state, you must normal order with respect to the free vacuum. The LF energy density functional is obtained by integrating the Hamiltonian density over the LF spatial volume,

$$E_{\text{LF}}^{RMF}[\psi, \bar{\psi}, \sigma, \omega, \rho_3, A] = \frac{1}{2} \int dx^- d\mathbf{x}^\perp \mathcal{H}_{\text{LF}}^{RMF}(x^-, \mathbf{x}^\perp), \quad (4.25)$$

As discussed in Appendix A, only the $+$ projection of the nucleon field, $\psi_i^+ \equiv \Lambda^+ \psi_i$, is dynamical on the LF; the $-$ projection is a constraint solved for in terms of ψ_i^+ . We therefore impose the normalization on the $+$ components only,

$$\sum_{i=1}^A \frac{1}{2} \int dx^- d\mathbf{x}^\perp |\psi_i^+(\mathbf{x})|^2 = A. \quad (4.26)$$

Minimizing E_{LF}^{RMF} with respect to $\psi_i^\dagger$ subject to Eq. (4.26) introduces a set of Lagrange multipliers p_i^- that play the role of single-particle LF energies. The stationarity condition reads

$$\frac{\delta \mathcal{H}_{\text{LF}}^{RMF}}{\delta \psi_i^\dagger(\mathbf{x})} - p_i^- \Lambda^+ \psi_i(\mathbf{x}) = 0. \quad (4.27)$$

Performing the functional derivatives on $\mathcal{H}_{\text{LF}}^{RMF}$ yields the LF Dirac equation

$$\left[i\boldsymbol{\alpha}^\perp \cdot \boldsymbol{\partial}^\perp + i\alpha^3 \partial^+ + \gamma^0 m_N + \Sigma_H(\mathbf{x}) \right] \psi_i(\mathbf{x}) = p_i^- \Lambda^+ \psi_i(\mathbf{x}), \quad (4.28)$$

with the Hartree self-energy

$$\Sigma_H(\mathbf{x}) = g_\sigma \gamma^0 \sigma(\mathbf{x}) + g_\omega \omega_0(\mathbf{x}) + \frac{1}{2} g_\rho \tau_3 b_0(\mathbf{x}) + \frac{1}{2} e(1 + \tau_3) A_0(\mathbf{x}), \quad (4.29)$$

where $\alpha^\mu = \gamma^0 \gamma^\mu$. The meson and photon fields obey constrained Klein-Gordon equations

$$\begin{aligned} [\boldsymbol{\partial}^\perp \cdot \boldsymbol{\partial}^\perp + \partial^+ \partial^+ - m_\sigma^2] \sigma(\mathbf{x}) &= g_\sigma \rho_s(\mathbf{x}), \\ [\boldsymbol{\partial}^\perp \cdot \boldsymbol{\partial}^\perp + \partial^+ \partial^+ - m_\omega^2] \omega_0(\mathbf{x}) &= -g_\omega \rho_B(\mathbf{x}), \\ [\boldsymbol{\partial}^\perp \cdot \boldsymbol{\partial}^\perp + \partial^+ \partial^+ - m_\rho^2] b_0(\mathbf{x}) &= -g_\rho \rho_3(\mathbf{x}), \\ [\boldsymbol{\partial}^\perp \cdot \boldsymbol{\partial}^\perp + \partial^+ \partial^+] A_0(\mathbf{x}) &= -e \rho_p(\mathbf{x}). \end{aligned} \quad (4.30)$$

Equations (4.28) and (4.30) are solved numerically and self-consistently. A notable computational difficulty in the LF approach is that the Dirac single-particle Hamiltonian must be solved in cylindrical coordinates, with states labeled by their energy and the total angular momentum projection quantum number m_j . Ref. [60] carried out this calculation and verified that rotational invariance is restored by checking that the m_j states exhibit the correct degeneracy. In the same paper, along with Ref. [62], the authors further develop a procedure for recasting the LF Dirac single-particle Hamiltonian into a spherically symmetric form. However, they state the manipulations as a way to use IF DFT to 'approximate' LF DFT results. We now discuss this procedure and show that this is not an approximation.

4.4 Connecting IF and LF Formulations of DFT

To recast equation (4.28) into a form that makes the underlying spherical symmetry of the nucleus manifest, we perform two successive steps.

Step 1 — Replace x^- with the spatial z coordinate. Since we solve for the states of the Hamiltonian at $x^+ = 0$, $x^- = -2z$. In other words, the LF longitudinal direction can be mapped onto the ordinary spatial coordinate z . The LF plus-derivative becomes

$$\partial^+ \equiv 2 \frac{\partial}{\partial x^-} = -\frac{\partial}{\partial z} = \partial^3, \quad (4.31)$$

where in the last equality we used $\partial^3 = -\partial/\partial z$ in the mostly-minus metric. We further relabel the spatial argument $\mathbf{x} \rightarrow \mathbf{x}_{\text{IF}}$, as discussed in the conventions and notations in the beginning of the dissertation.

Step 2 — Phase redefinition. Writing

$$\psi_i(\mathbf{x}_{\text{IF}}) = \Psi_i(\mathbf{x}_{\text{IF}}) e^{ip_i^- z/2}, \quad (4.32)$$

the action of $i\alpha^3 \partial^3$ on ψ_i produces two terms,

$$i\alpha^3 \partial^3 \psi_i(\mathbf{x}_{\text{IF}}) = i\alpha^3 \left[\partial^3 \Psi_i(\mathbf{x}_{\text{IF}}) - i \frac{p_i^-}{2} \Psi_i(\mathbf{x}_{\text{IF}}) \right] e^{ip_i^- z/2}. \quad (4.33)$$

Moving the second term in Eq. (4.33) to the right-hand side of Eq. (4.28) collapses it to $\frac{p_i^-}{2} \Psi_i(\mathbf{x}_{\text{IF}})$.

Resulting set of equations. Putting the two steps together, the set of LF RMF equations takes the form

$$\left[i\boldsymbol{\alpha}^\perp \cdot \boldsymbol{\partial}^\perp + i\alpha^3 \partial^3 + \gamma^0 m_N + \Sigma_H(\mathbf{x}_{\text{IF}}) \right] \Psi_i(\mathbf{x}_{\text{IF}}) = \frac{p_i^-}{2} \Psi_i(\mathbf{x}_{\text{IF}}), \quad (4.34)$$

$$\begin{aligned} \left[\boldsymbol{\partial}^\perp \cdot \boldsymbol{\partial}^\perp + \partial^3 \partial^3 - m_\sigma^2 \right] \sigma(\mathbf{x}_{\text{IF}}) &= g_\sigma \rho_s(\mathbf{x}_{\text{IF}}), \\ \left[\boldsymbol{\partial}^\perp \cdot \boldsymbol{\partial}^\perp + \partial^3 \partial^3 - m_\omega^2 \right] \omega_0(\mathbf{x}_{\text{IF}}) &= -g_\omega \rho_B(\mathbf{x}_{\text{IF}}), \\ \left[\boldsymbol{\partial}^\perp \cdot \boldsymbol{\partial}^\perp + \partial^3 \partial^3 - m_\rho^2 \right] b_0(\mathbf{x}_{\text{IF}}) &= -g_\rho \rho_3(\mathbf{x}_{\text{IF}}), \\ \left[\boldsymbol{\partial}^\perp \cdot \boldsymbol{\partial}^\perp + \partial^3 \partial^3 \right] A_0(\mathbf{x}_{\text{IF}}) &= -e \rho_p(\mathbf{x}_{\text{IF}}). \end{aligned} \quad (4.35)$$

in which spherical symmetry can now be exploited directly. In fact, Eqs. (4.34) and (4.35) are the exactly same equations that are solved in the original IF DFT formulation, with $p_i^-/2$ as the single-particle energy of the nucleon [59]. Hence, IF DFT calculations can be repurposed for LF DFT.

The claim of Ref. [60] was not that they are approximate, but rather that the Fourier transform of the single-particle LF wavefunction —obtained from the IF wavefunction —has non-zero support at $p^+ = 0$, which cannot occur for massive particles. On this basis, the authors argued that the equivalence between IF and LF DFT is only approximate, attributing the discrepancy to a breaking of the LF spectral condition. However, the non-zero support at $p^+ = 0$ does not signal physics that the IF formulation fails to capture; it is instead a consequence of the independent-particle nature of DFT models. The relation $p^+ > 0$ for massive particles holds only for momentum eigenstates, and single-particle orbitals are not momentum eigenstates. The support

at $p^+ = 0$ is therefore to be expected: it would persist even if the LF–IF relation were never invoked and the entire calculation were carried out directly on the LF. More precisely, single-particle momentum distributions of the nucleus acquire support at $p^+ = 0$ because the nuclear state is not an eigenstate of LF momentum—DFT calculations of nuclear structure pin the nucleus to the origin. This interpretation is reinforced by studies of infinite nuclear matter on the light front, where translational invariance is preserved, the nucleons are momentum eigenstates, and the support at $p^+ = 0$ vanishes [63, 28]. One can correspondingly expect that properly projecting the nuclear state onto the zero-momentum eigenstate would remove the support at $p^+ = 0$, leaving a node in the wavefunction. Methods for carrying out this projection, and for incorporating it into the variational calculation, have been outlined in Refs. [64, 65]; their application here is left to future work and lies outside the scope of this dissertation.

It should be emphasized that this support is unrelated to the so-called zero modes of the quantized Dirac fields—a long-standing and still unresolved problem in light-front quantization [25, 66]. We avoid the zero modes by virtue of the no-sea scheme: the single-particle wavefunctions from which our quantized field is built were obtained without reference to anti-nucleon dynamics. Were anti-nucleons admitted into the nuclear ground state, the zero modes could no longer be avoided. More fundamentally, the equivalence between the IF and LF formulations of DFT rests on their shared underlying assumptions—static meson fields and the no-sea scheme—which lead to

identical equations.

Furthermore, Eq. (4.32) admits a natural geometric interpretation. In IF quantization, states are defined on surfaces of constant $x^0 = t$, and the Hamiltonian generates evolution from one such surface to the next; in IF DFT the Dirac fields are quantized at $t = 0$, hence the wavefunctions solved for on that surface. LF quantization proceeds analogously, but on surfaces of constant x^+ , with the Dirac fields of LF DFT quantized at $x^+ = 0$. Equation (4.32) bridges the two. It takes the IF wavefunction defined at $t = 0$ and applies a z -dependent time-evolution that transports it onto the $x^+ = 0$ hypersurface, thereby yielding the corresponding LF wavefunction (see Fig. 4.1). The relation between LF and IF wavefunctions has been examined previously in Ref. [67]. Focusing on two-body bound states, the authors showed that the IF and LF wavefunctions can each be recovered from the Bethe–Salpeter wavefunction by integrating over k^0 and k^- , respectively, but that neither can be obtained directly from the other. That a direct map exists in our case is a consequence of the single-particle nature of the problem.

From here, one can obtain the single-particle Dirac equation and the Klein-Gordon equations for the meson fields that characterize the original RMF formulation in Ref. [59]. However, despite its successes, the original RMF formulation with constant meson-nucleon couplings failed to reproduce several key properties of nuclear matter and finite nuclei. Most notably, it predicted a nuclear-matter incompressibility of roughly 500–600 MeV, far in excess of the empirical value of $K \approx 210\text{--}240$ MeV [68],

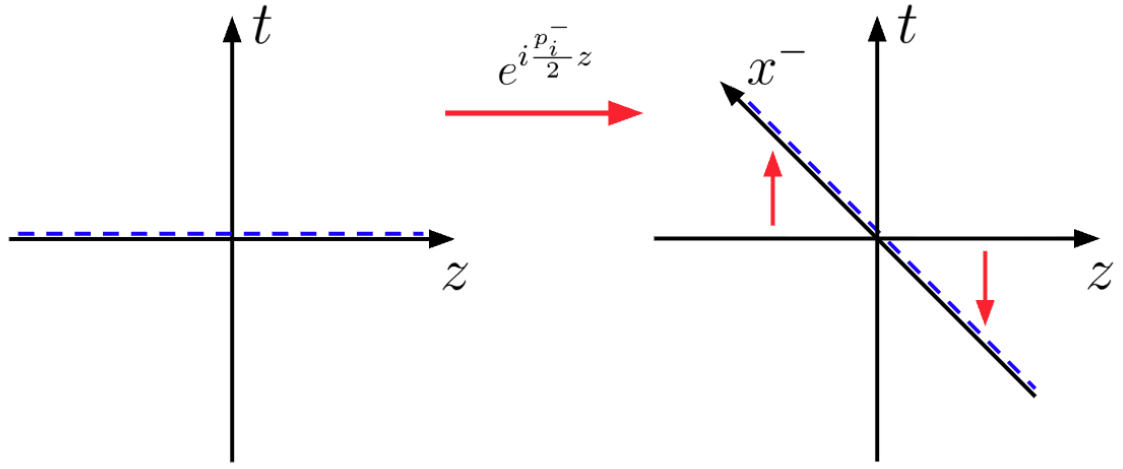


Figure 4.1: (color online) Schematic picture of converting instant form (IF) single-particle wavefunctions into light-front wavefunctions. The diagram on the left illustrates the IF wavefunction that is solved for at $t = 0$ represented by the blue dashed line. The red arrows illustrate how the z -dependent time-evolution places the wavefunction on the surface $x^+ = 0$, yielding the analogous single-particle LF wavefunction.

and yielded poor agreement with nuclear surface properties and the equation of state away from saturation. Furthermore, the original results underbind finite nuclei by roughly 2 MeV per nucleon. These deficiencies were traced to the absence of medium dependence in the effective interaction. Dirac–Brueckner–Hartree–Fock (DBHF) calculations, which resum ladder diagrams built from a realistic bare nucleon-nucleon interaction in the nuclear medium [69], demonstrated that the scalar and vector nucleon self-energies acquire a pronounced density dependence through Pauli blocking, short-range correlations, and implicit three-body effects—features that cannot be captured by fixed couplings. This microscopic insight motivated the introduction of density-dependent meson-nucleon vertices [70, 71], which phenomenologically encode the physics missing from a Hartree description.

Incorporating this density dependence into the meson coupling constants does not affect the equivalence between the IF and LF formulations. The coordinate change of Step 1 is unaffected, and the phase redefinition of Step 2 acts only on the wavefunctions, leaving the densities unchanged. The equivalence therefore persists, and density-dependent LF DFT can be solved through its IF counterpart. This equivalence allows existing IF codes to be employed directly. In this thesis we use the DIRHB package [72], a set of Fortran codes that compute the ground-state properties of even-even nuclei within relativistic self-consistent mean-field theory with density-dependent couplings. For the effective interaction we adopt the DD-ME2 functional of Ref. [73], which reproduces total nuclear binding energies for approximately 200

nuclei to within 5 MeV. Figure 4.2 presents the LF momentum distributions, integrated over longitudinal momenta, for the four doubly magic nuclei considered in this paper. To interpret these distributions qualitatively, consider the LF momentum distribution of a free proton, which would be a delta function at $\alpha = 1$. The spread in α in the nuclear case is therefore due to Fermi motion inside the nucleus, while the offset of the peak from $\alpha = 1$ reflects the effect of nuclear binding.

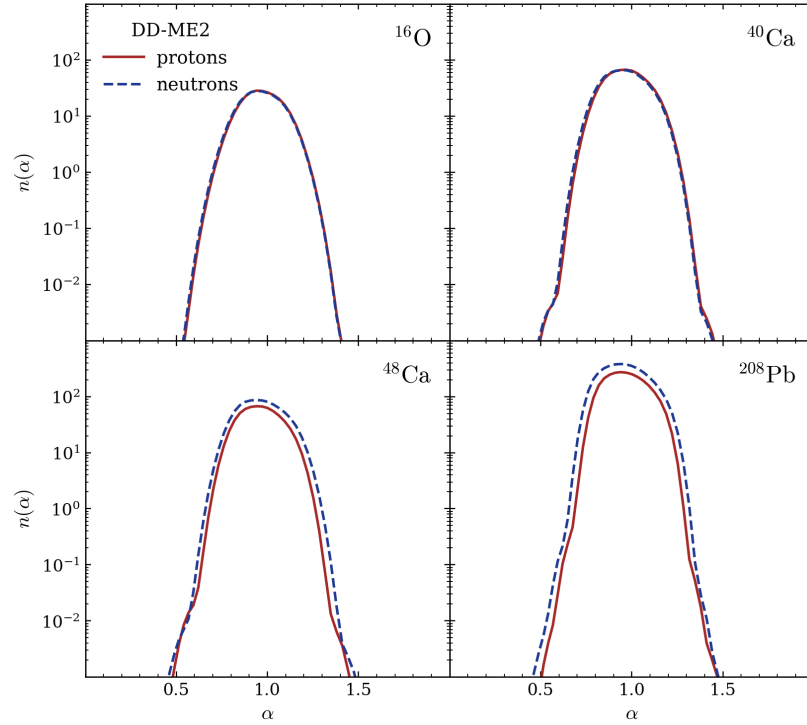


Figure 4.2: (color online) Transverse-momentum-integrated light-front momentum distributions for ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb versus $\alpha = Ap^+/P_A^+$. A is the mass number and P_A^+ is the nuclear plus momentum (m_A in the rest frame). Solid red lines denote protons; dashed blue lines denote neutrons.

4.5 *Inclusive Electron-Nucleus Cross-Sections*

Appendix B provides a detailed derivation of the inclusive electron-nucleus cross section calculated within the FF, utilizing LF DFT states to evaluate the intrinsic nuclear matrix elements. The resulting cross sections are displayed in Fig. 4.3. We find that the LF DFT framework successfully reproduces the data in the region $0.8 < x_B < 1.3$, capturing the quasi-elastic peak. However, a significant discrepancy emerges at $x_B > 1.3$, where the theoretical calculations systematically underestimate the cross section. This shortfall is directly attributable to the absence of short-distance nuclear structure—specifically, the high-momentum short-range correlations that are inherently missing from independent-particle models.

4.6 *Summary*

Mean-field approaches are among the most versatile and computationally accessible microscopic descriptions of nuclei, capable of treating systems as small as $A = 16$. Modern independent-particle models such as DFT have enjoyed remarkable success over the years: they reproduce binding energies to within roughly 3 MeV, describe charge radii, underpin nuclear fission calculations, and naturally encode nuclear shell structure. They are especially valuable for high-energy nuclear reactions, where nuclear structure must be formulated relativistically under LF quantization. In this regime *ab initio* methods are not yet viable, owing to our limited knowledge of LF NN potentials and our lack of experience solving the correlated many-body Dirac

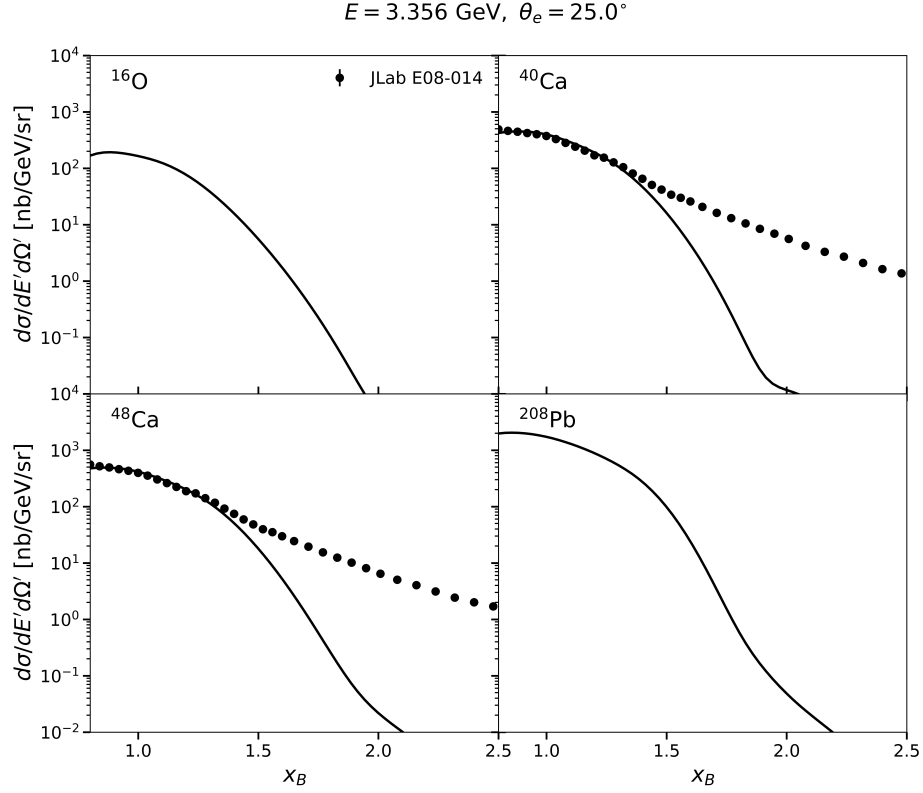


Figure 4.3: Inclusive electron–nucleus cross section at $E = 3.356 \text{ GeV}$ and $\theta_e = 25^\circ$ as a function of the Bjorken variable x_B , for ^{16}O (top left), ^{40}Ca (top right), ^{48}Ca (bottom left), and ^{208}Pb (bottom right). The solid black curves are the mean-field contribution, evaluated with relativistic mean-field single-particle wave functions. Black points are the JLab E08-014 data, available for ^{40}Ca and ^{48}Ca at these kinematics.

equation. Mean-field theories circumvent these obstacles, providing accurate descriptions of nuclei on the LF without requiring explicit knowledge of the bare NN and three-nucleon forces. The results developed here can thus help bridge the low- and high-energy nuclear physics communities.

This chapter has presented original work toward that goal. To date, only a single LF mean-field study of finite nuclei exists [60]; the major advances in IF DFT achieved since then now make a modern light-front revisit possible [55]. We find that results from IF DFT can be repurposed for LF DFT directly and naturally, without approximation. This equivalence rests on their shared underlying assumptions which lead to identical equations. Using the nuclear structure provided by LF DFT, we computed the inclusive electron-nucleus cross sections for the high-energy kinematics of Jefferson Lab experiment E08-014. We find that, while mean-field models correctly capture the quasi-elastic peak, they fail to reproduce the data for $x_B > 1.3$. This shortfall reflects the absence of high-momentum configurations in our nuclear structure calculation, and motivates the work of the next chapter.

Chapter 5

SIMILARITY RENORMALIZATION GROUP

Renormalization Group (RG) techniques originated during the development of Quantum Electrodynamics in the 1940s [74, 75]. At that time, RG methods were considered mathematical tricks used to tame the infinities arising from higher-order loop integrals in perturbative calculations. This approach was later refined and given physical interpretation by Gell-Mann and Low, who discovered that the effective charge of an electron depends on the energy scale at which it is probed [76]. In the late 1960s and early 1970s, Kenneth Wilson generalized this framework by investigating the scale dependence of Hamiltonians themselves [77, 78]. He introduced the concept of “Theory Space”—the infinite-dimensional space of all possible Hamiltonians—and mapped how these theories are mathematically connected through the flow of the RG. Ultimately, Wilson made mathematically precise the idea that scale dictates our perception of the physical world.

In the 1990s, Glazek and Wilson sought to understand the success of constituent quark models, such as the MIT bag model, by studying the RG evolution of the QCD Hamiltonian under light-front quantization [79, 80]. For this type of problem, traditional Wilsonian RG—which continuously integrates out momentum modes

above a floating cutoff—is disadvantageous because truncating the Hilbert space inherently alters the exact eigenvalue spectrum of the initial Hamiltonian. To resolve this, Glazek and Wilson, and independently Wegner for condensed matter applications [81], developed continuous unitary RG techniques, which is known as the Similarity Renormalization Group (SRG). Instead of discarding high-energy modes, these RG flow equations systematically drive the bare ultraviolet Hamiltonian toward an energy-diagonal form. Because these transformations are strictly unitary, the original physical spectrum is exactly preserved. Glazek and Wilson were unable to fully solve QCD using this technique because the multi-gluon interactions inherently induced by the RG procedure were non-negligible and could not be truncated. However, the use of unitary flow equations—particularly the form proposed by Wegner—found immense utility in low-energy nuclear physics. In the nuclear domain, induced many-body operators are found to be suppressed; as the rank of the interaction (N) increases, the corresponding N -body forces become increasingly negligible. Because this allows the interaction space to be safely truncated, SRG techniques became a critical breakthrough, directly enabling the success of *ab initio* methods like the No-Core Shell Model [82].

Recently, and of particular importance to this work, the SRG has been utilized to reconcile short-range correlation (SRC) phenomenology with independent-particle nuclear models. Historically, nuclei have been predominantly studied at low energies, where the many-body system is well described by the Schrödinger equation.

This low-energy regime has seen incredible success through the application of mean-field theories, which are often referred to as “low-resolution” pictures of the nucleus. Despite their simplifications, these models are remarkably robust; frameworks like Density Functional Theory provide some of our best microscopic descriptions, reproducing total nuclear binding energies to within an accuracy of roughly 3 MeV [83]. However, the low-resolution pictures cannot explain everything. Early work by Brueckner and collaborators [84] demonstrated that independent-particle models fail to describe high-energy, high-momentum-transfer processes, such as $(e, e'p)$ knockout reactions. They concluded that reproducing these reactions requires a nucleon momentum distribution with a significant high-momentum tail, generated by the short-range nucleon-nucleon interaction. Today, state-of-the-art numerical techniques like Quantum Monte Carlo, along with realistic two and three-body nuclear potentials such as Argonne V18 and Urbana IX, give us the ability to calculate the correlated many-body wavefunction of the nucleus [9]. These calculations confirm the existence of the high-momentum tails missing from independent-particle models, providing what is now called the “high-resolution” picture of the nucleus.

However, the success of the high-resolution picture did not definitively resolve which framework is correct, and theoretical tensions persisted. It was demonstrated that high-momentum observables could be equally well reproduced within a low-resolution framework by accounting for initial- and final-state interactions (see, e.g., early work by Greider [85], who utilized optical-model scattering to explain high-

energy observables without relying on short-range interactions). Ultimately, it is the application of the SRG that formally bridges this conceptual gap. By providing a mathematical framework to continuously transform the Hamiltonian and operators, the SRG demonstrates that both the low- and high-resolution pictures represent the exact same underlying physics evaluated at different RG scales, seamlessly reconciling the two frameworks.

The methods and applications of the SRG have been extensively documented across literature. For an excellent pedagogical introduction, complete with explicit numerical implementations, see Ref. [86]; for broader foundational reading, see Refs. [87, 88, 89, 90]. Rather than providing an exhaustive review of the formalism, this chapter focuses on establishing the minimal set of theoretical fundamentals strictly required to support the calculations in this work.

5.1 Theory Fundamentals

The basic idea of the SRG method is quite general. We wish to construct a continuous unitary transformation, $\hat{U}(s)$, which ‘simplifies’ the Hamiltonian by making it more band-diagonal when represented as a matrix [81, 86].

$$\hat{H}(s) = \hat{U}(s) \hat{H}(s=0) \hat{U}^\dagger(s). \quad (5.1)$$

By convention $\hat{H}(s=0)$ is the starting, bare Hamiltonian and s is called the flow parameter, which parameterizes $\hat{U}$. To elucidate, mathematically, if we split up the

Hamiltonian into diagonal and off-diagonal parts,

$$\hat{H}(s) = \hat{H}_d(s) + \hat{H}_{od}(s), \quad (5.2)$$

we want to construct $\hat{U}(s)$ such that,

$$\hat{H}(s) \xrightarrow{s \rightarrow \infty} \hat{H}_d(s), \quad \hat{H}_{od}(s) \xrightarrow{s \rightarrow \infty} 0. \quad (5.3)$$

To derive the differential equation for the s -dependent Hamiltonian, we take the derivative of Eq. (5.1) with respect to s on both sides.

$$\begin{aligned} \frac{d\hat{H}(s)}{ds} &= \frac{d\hat{U}(s)}{ds} \hat{H} \hat{U}^\dagger(s) + \hat{U}(s) \hat{H} \frac{d\hat{U}^\dagger(s)}{ds} \\ &= \frac{d\hat{U}(s)}{ds} \hat{U}^\dagger(s) \hat{H}(s) + \hat{H}(s) \hat{U}(s) \frac{d\hat{U}^\dagger(s)}{ds}. \end{aligned} \quad (5.4)$$

Since $\hat{U}$ is unitary we have,

$$\frac{d}{ds}(\hat{U}(s)\hat{U}^\dagger(s)) = \frac{d}{ds}(\mathbb{I}) = 0 \Rightarrow \frac{d\hat{U}(s)}{ds} \hat{U}^\dagger(s) = -\hat{U}(s) \frac{d\hat{U}^\dagger(s)}{ds}. \quad (5.5)$$

Defining the anti-Hermitian operator

$$\hat{\eta}(s) \equiv \frac{d\hat{U}(s)}{ds} \hat{U}^\dagger(s) = -\hat{\eta}^\dagger(s), \quad (5.6)$$

we can write down the SRG-flow equation as:

$$\frac{d\hat{H}(s)}{ds} = [\hat{\eta}(s), \hat{H}(s)]. \quad (5.7)$$

We call $\hat{\eta}(s)$ the dynamical generator of the SRG-transformation. Eq. (5.7) is a first-order linear differential equation that can be calculated numerically. Hence, by calculating $\hat{H}(s)$ one can build $\hat{U}(s)$ directly by using the eigenvectors of the evolved and initial Hamiltonians. There is no set procedure to determine $\hat{\eta}(s)$, which generates an s -dependent Hamiltonian that obeys Eq. (5.3). For momentum-space SRG, Wegner showed that the off-diagonal part of the Hamiltonian flows to zero, as $s \rightarrow \infty$, if we choose the generator to be

$$\hat{\eta}(s) = [\hat{T}(s=0), \hat{H}(s)], \quad (5.8)$$

where $\hat{T}(s=0)$ is the bare kinetic energy operator [81]. It is convenient to perform a change of variables, defining a new flow parameter $\lambda = s^{-1/4}$, which has units of $[\text{fm}^{-1}]$. The flow parameter λ provides a measure of the spread of the band-diagonal in the SRG-evolved Hamiltonian [86]. Under this change of variables, the SRG flow equation is written as

$$\frac{d\hat{H}(\lambda)}{d\lambda} = -\frac{4}{\lambda^5} [\hat{\eta}(\lambda), \hat{H}(\lambda)]. \quad (5.9)$$

From here on, we will drop the $\lambda = \infty$ dependency on bare operators and eigenstates.

For a concrete example, Fig. 5.1 shows the SRG transformation on the 1P_1 channel of the minimal relativity AV18 potential. $\lambda = \infty \text{ fm}^{-1}$ is the bare potential, while $\lambda = 2 \text{ fm}^{-1}$ is the SRG-evolved potential. Notice that the potential becomes more band-diagonal as λ approaches zero, meaning that the SRG flow equations under the

Wegner generator weakens the coupling between low- and high-momentum states.

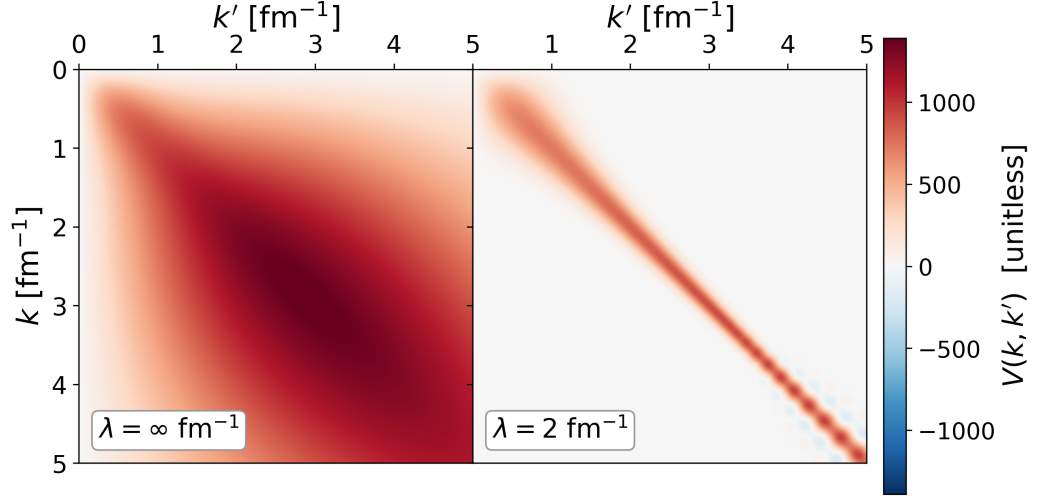


Figure 5.1: (color online) The SRG-transformed 1P_1 channel of the minimal relativity AV18 nucleon-nucleon potential.

5.2 SRG Transformation of Operators: Connecting Low- and High-Resolution Pictures

Now we discuss how the SRG can be used to bridge low- and high-resolution pictures of the nucleus. This can be immediately seen by looking at the matrix element for the momentum distribution of a single nucleon inside a nucleus,

$$\begin{aligned}
 n_A^\tau(\mathbf{p}) &= \sum_{\sigma} \langle \Psi_A | a^\dagger(\mathbf{p}, \sigma, \tau) a(\mathbf{p}, \sigma, \tau) | \Psi_A \rangle \\
 &= \sum_{\sigma} \langle \Psi_A | \hat{n}^\tau(\mathbf{p}, \sigma) | \Psi_A \rangle.
 \end{aligned}
 \tag{5.10}$$

Where σ is the spin projection, τ is the isospin projection, and $|\Psi_A\rangle$ is the nuclear many-body state. Applying $\hat{U}^\dagger(\lambda)\hat{U}(\lambda) = \mathbb{I}$ before and after the operator we find,

$$\begin{aligned}
n_A^\tau(\mathbf{p}) &= \sum_{\sigma} \langle \Psi_A | \hat{n}^\tau(\mathbf{p}, \sigma) | \Psi_A \rangle \\
&= \sum_{\sigma} \langle \Psi_A | \hat{U}^\dagger(\lambda) \hat{U}(\lambda) \hat{n}^\tau(\mathbf{p}, \sigma) \hat{U}^\dagger(\lambda) \hat{U}(\lambda) | \Psi_A \rangle \\
&= \sum_{\sigma} \langle \Psi_A(\lambda) | \hat{U}(\lambda) \hat{n}^\tau(\mathbf{p}, \sigma) \hat{U}^\dagger(\lambda) | \Psi_A(\lambda) \rangle \\
&= \sum_{\sigma} \langle \Psi_A(\lambda) | \hat{n}_\lambda^\tau(\mathbf{p}, \sigma) | \Psi_A(\lambda) \rangle.
\end{aligned} \tag{5.11}$$

Where $\hat{U}(\lambda)$ is the unitary SRG operator at flow parameter λ , $|\Psi_A(\lambda)\rangle$ is the SRG-evolved nuclear ground state, and $\hat{n}_\lambda^\tau(\mathbf{p}, \sigma)$ is the SRG-evolved momentum distribution operator for nucleon τ . Looking back at Fig. 5.1 we see that the SRG transformation decouples low- and high-momentum states. Hence, as we SRG-evolve the Hamiltonian, it's eigensolutions $|\Psi_A(\lambda)\rangle$ begin to reflect a dominantly “mean-field” or “low-resolution” description of nuclei. However, since SRG transformations are unitary, the high-resolution physics is shifted from the wavefunction to the operators. Hence, within the SRG framework, low- and high-resolution pictures of nuclei are equivalent. Both can output the same results as long as one is consistent with the SRG-scale of the reaction (operators) and structure (states).

A crucial theoretical consideration is determining, a priori, whether the chosen probing operator is truly the bare operator. Within the context of nuclear physics, the SRG transformation is driven entirely by the strong nuclear interactions governing the initial state of the target. Therefore, if the external probe—such as an electromagnetic or weak current—does not inherently contain nuclear force dynam-

ics, it constitutes the bare, high-resolution operator. For theoretical consistency, this unevolved operator must strictly be evaluated using exact high-resolution many-body eigenstates. However, as demonstrated by Eq. (5.11), when we apply the SRG transformation to evolve the high-resolution nuclear state to a low-resolution scale, the unitary transformation is formally shifted onto the operator. Consequently, the originally bare operator absorbs the short-range nuclear dynamics, naturally inducing two-body and higher-rank many-body currents which originate from nuclear forces. This has important implications for high-energy approximations in nuclear scattering processes which we will discuss shortly.

5.3 Application to Light-Front Physics

The formal connection between the low- and high-resolution pictures of nuclei provided by the SRG led Tropiano et al. to approximate high-resolution observables using shell-model wavefunctions [91]. Specifically, they substituted the exact evolved state $|\Psi_A(\lambda)\rangle$ in Eq. (5.11) with an uncorrelated mean-field nuclear wavefunction, $|\Psi_A^{MF}\rangle$. In principle, $|\Psi_A(\lambda)\rangle$ should be obtained by solving the many-body Schrödinger equation associated with the evolved Hamiltonian; however, as discussed previously, this low-resolution state reflects a dominantly independent-particle description of the nucleus. Introducing this mean-field approximation breaks the formal SRG invariance of the matrix element, inherently introducing a dependence on the resolution scale λ . This scale dependence is the theoretical tradeoff for extracting correlated ob-

servables from uncorrelated wavefunctions. Consequently, λ becomes a parameter that must be constrained. By focusing on contributions from the two-body nucleon-nucleon potential and neglecting higher-rank SRG-induced interactions, Tropiano et al. demonstrated that setting $\lambda = 1.5 \text{ fm}^{-1}$ yielded high-resolution momentum distributions that showed excellent agreement with exact high-resolution Quantum Monte Carlo calculations.

This operator-evolution approach to extracting high-resolution observables significantly simplifies computational efforts by circumventing the need to explicitly solve the correlated nuclear many-body problem. Within the domain of light-front nuclear physics, this methodology proves particularly advantageous. Given the historical scarcity of full many-body calculations performed under light-front quantization, relying on evolved operators paired with independent-particle wavefunctions provides a highly practical and effective path forward. To accomplish this, we mirror the procedure of Ref. [91]. We restrict ourselves to incorporating the effects of the two-body nucleon-nucleon interaction, we utilize the non-relativistic approximation of light-front wavefunctions developed in Chapter 3 to construct our unitary SRG transformation, and the light-front density functional theory model developed in Chapter 4 as input for a low-resolution wavefunction.

The two-body unitary light-front SRG transformation can be written as the inner product between eigenstates of the un-evolved and SRG-evolved Hamiltonians,

$$\hat{U}(\lambda) = \sum_{\beta} \int [dP] |\beta, \lambda; \mathbf{P}\rangle \langle \beta; \mathbf{P}|, \quad (5.12)$$

where β indexes the eigenvalues of the two-body light-front Hamiltonian, $\mathbf{P} = (P^+, \mathbf{P}^\perp)$ are the plus and transverse momenta of the center of mass, and λ is the SRG-scale. To be explicit,

$$\begin{aligned} \sum_{\beta} &= \sum_{\text{energy}} \sum_{JM_J} \sum_{TM_T} \sum_S, \\ \sum_{\text{energy}} &\xrightarrow{\text{bound states}} \sum_{M^2}, \\ \sum_{\text{energy}} &\xrightarrow{\text{scattering states}} \frac{1}{2} \sum_{\substack{\sigma_1 \tau_1 \\ \sigma_2 \tau_2}} \int \frac{d\alpha d\mathbf{k}^\perp}{(2\pi)^2 \alpha (2 - \alpha)}, \end{aligned} \quad (5.13)$$

where $(\alpha, \mathbf{k}^\perp)$ are the intrinsic LF relative variables. The expansion of the two-body state in terms of anti-symmetrized plane wave nucleonic states is defined by,

$$\begin{aligned} |\beta, \lambda; \mathbf{P}\rangle &= \frac{1}{2} \sum_{\substack{\sigma_1 \tau_1 \\ \sigma_2 \tau_2}} \int [dp_1][dp_2] (2\pi)^3 2P^+ \delta(p_1^+ + p_2^+ - P^+) \delta^{(2)}(\mathbf{p}_1^\perp + \mathbf{p}_2^\perp - \mathbf{P}^\perp) \\ &\quad \times \psi_\beta^\lambda(\alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2) |\mathbf{p}_1, \sigma_1, \tau_1; \mathbf{p}_2, \sigma_2, \tau_2\rangle \\ &= \frac{1}{2} \sum_{\substack{\sigma_1 \tau_1 \\ \sigma_2 \tau_2}} \int \frac{d\alpha d\mathbf{k}^\perp}{(2\pi)^2 \alpha (2 - \alpha)} \psi_\beta^\lambda(\alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2) |p_1, p_2\rangle. \end{aligned} \quad (5.14)$$

In Eq. (5.14), ψ_β^λ is the two-body light-front wavefunction for eigenstate β , SRG-evolved to λ . In the above expression, $(\alpha, \mathbf{k}^\perp)$ are defined by

$$\begin{aligned}
p_1^+ &= \frac{\alpha}{2} P^+, & p_2^+ &= \frac{2-\alpha}{2} P^+, \\
\mathbf{p}_1^\perp &= \frac{\alpha}{2} \mathbf{P}^\perp + \mathbf{k}^\perp, & \mathbf{p}_2^\perp &= \frac{2-\alpha}{2} \mathbf{P}^\perp - \mathbf{k}^\perp,
\end{aligned} \tag{5.15}$$

One important observation can explain why SRG techniques are most suitable for relativistic physics formulated using LF quantization. Notice that Eq. (5.12) has an integral over the center-of-mass momenta. If we instead attempted to use SRG techniques formulated under relativistic IF Quantization, we will find a computationally intractable problem. The IF wavefunctions are not boost invariant, and depend on center-of-mass momenta. Hence, one must re-calculate the relativistic two-body problem for every frame, making the construction of $\hat{U}(\lambda)$ extremely difficult. On the LF, however, we don't encounter this problem as our wavefunctions are boost invariant.

Using Eq. (5.14), one can write Eq. (5.12) in second quantized form

$$\begin{aligned}
\hat{U}(\lambda) &= \frac{1}{4} \sum_{\substack{\sigma_1 \sigma_2 \tau_1 \tau_2 \\ \sigma_3 \sigma_4 \tau_3 \tau_4}} \int [dP] \frac{d\alpha d\mathbf{k}^\perp}{(2\pi)^3 \sqrt{\alpha(2-\alpha)}} \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \sqrt{\alpha'(2-\alpha')}} (P^+)^2 \\
&\quad \times \langle \alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 | U(\lambda) | \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \rangle \\
&\quad \times \hat{a}^\dagger(\mathbf{p}_1, \sigma_1, \tau_1) \hat{a}^\dagger(\mathbf{p}_2, \sigma_2, \tau_2) \hat{a}(\mathbf{p}_4, \sigma_4, \tau_4) \hat{a}(\mathbf{p}_3, \sigma_3, \tau_3),
\end{aligned} \tag{5.16}$$

where p_3 and p_4 follow the same relations as p_1 and p_2 respectively, but with $\alpha \rightarrow \alpha'$ and $\mathbf{k}^\perp \rightarrow \mathbf{k}'^\perp$. The matrix element of $\hat{U}(\lambda)$ between asymmetric two-nucleon states

is

$$\begin{aligned} \langle \alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 | U(\lambda) | \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \rangle = \\ \sum_{\beta} \psi_{\beta}^{\lambda}(\alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2) \psi_{\beta}^{\dagger}(\alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4). \end{aligned} \quad (5.17)$$

5.4 High-Resolution Light-Front Momentum Distributions

It is instructive to re-write $\hat{U}(\lambda) = \hat{\mathbb{I}} + \delta\hat{U}(\lambda)$. Replacing $|\Psi_A(\lambda)\rangle$ with $|\Psi_A^{MF}\rangle$, Eq. (5.11) becomes,

$$\begin{aligned} n_A^{\tau}(\mathbf{p}) = \sum_{\sigma} \Big[& \langle \Psi_A^{MF} | \hat{n}^{\tau}(\mathbf{p}, \sigma) | \Psi_A^{MF} \rangle \\ & + \langle \Psi_A^{MF} | \delta\hat{U}(\lambda) \hat{n}^{\tau}(\mathbf{p}, \sigma) | \Psi_A^{MF} \rangle \\ & + \langle \Psi_A^{MF} | \hat{n}^{\tau}(\mathbf{p}, \sigma) \delta\hat{U}^{\dagger}(\lambda) | \Psi_A^{MF} \rangle \\ & + \langle \Psi_A^{MF} | \delta\hat{U}(\lambda) \hat{n}^{\tau}(\mathbf{p}, \sigma) \delta\hat{U}^{\dagger}(\lambda) | \Psi_A^{MF} \rangle \Big]. \end{aligned} \quad (5.18)$$

Detailed derivations of each term are presented in Appendix C. Figure 5.2 presents the approximated high-resolution light-front momentum distribution of Oxygen-16. The second and third SRG-interference terms from Eq. (5.18) are represented by the green dashed line; the left and right panels are included to illustrate the isolated behavior of these contributions. Qualitatively, these interference terms deplete probability from the low-momentum mean-field region, while the $\delta U \delta U^{\dagger}$ term subsequently redistributes it to higher momenta. This generates the high-momentum tails charac-

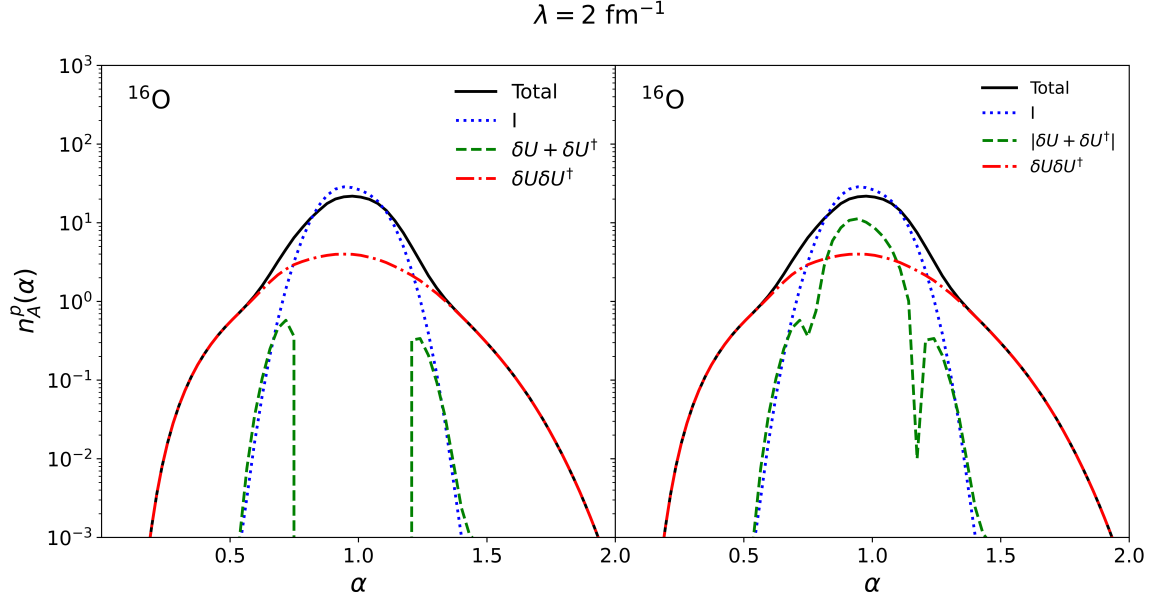


Figure 5.2: (color online) The approximated high-resolution light-front momentum distribution of Oxygen-16 using similarity renormalization group techniques. The left and right panels differ in their presentation of the green dashed line. The right figure presents the absolute value of the $\delta U + \delta U^\dagger$ term, while the left figure does not.

teristic of SRCs, consistent with both phenomenological understanding and exact non-relativistic high-resolution calculations. To our knowledge, this represents the first principled calculation of light-front momentum distributions exhibiting these high-momentum tails. Figure 5.3 presents the approximated high-resolution light-front momentum distributions for Oxygen-16, Calcium-40, Calcium-48, and Lead-208.

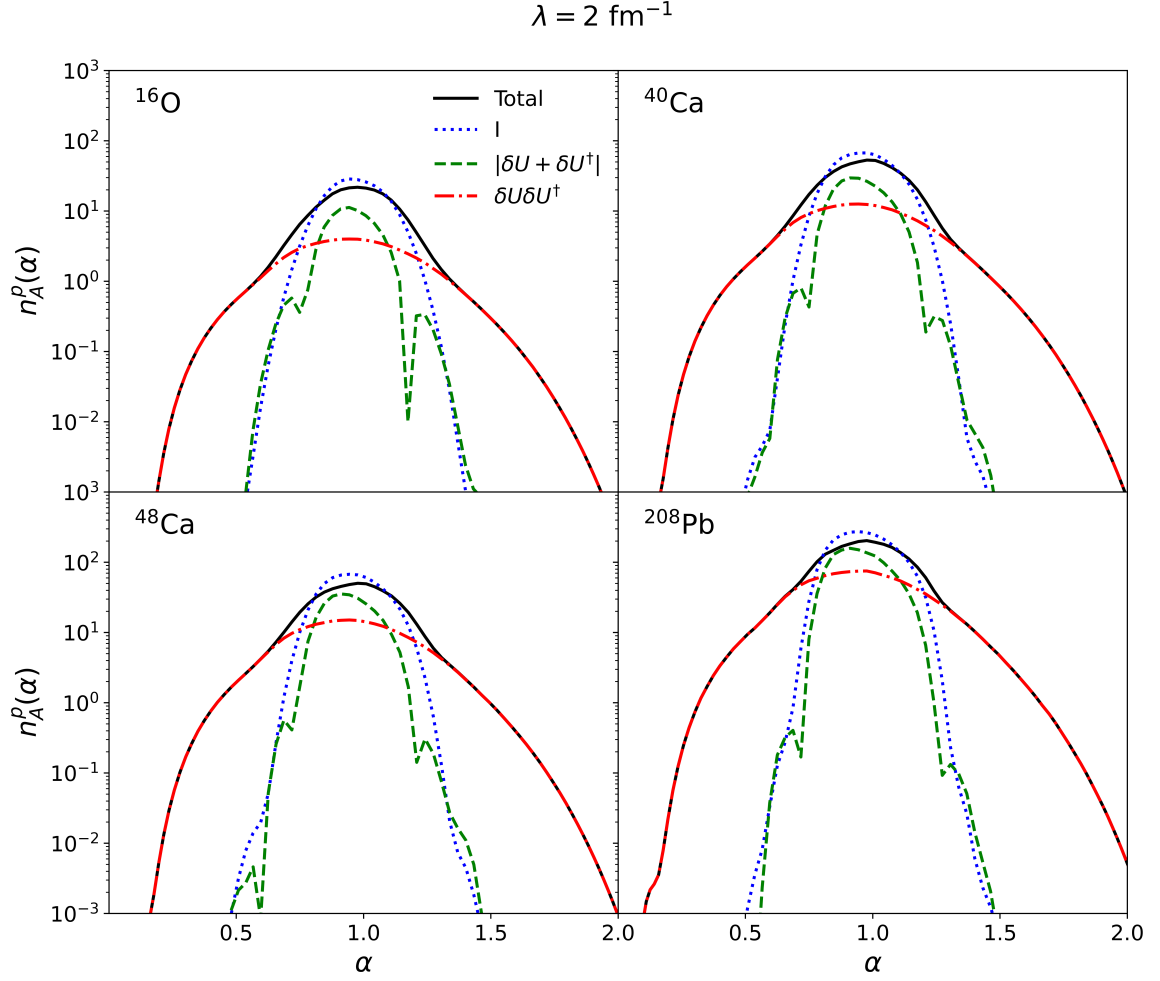


Figure 5.3: Light-front proton momentum distribution $n_A^p(\alpha)$ as a function of the longitudinal momentum fraction $\alpha = Ap^+/M_A$, for ^{16}O (top left), ^{40}Ca (top right), ^{48}Ca (bottom left), and ^{208}Pb (bottom right). The solid black curve is the total distribution $n^p = I + \delta U + \delta U^\dagger + \delta U \delta U^\dagger$; the mean-field contribution I (blue dotted), the two-body cross-term $|\delta U + \delta U^\dagger|$ (green dashed), and the two-body contribution $\delta U \delta U^\dagger$ (red dash-dot) are shown separately. SRG evolution scale $\lambda = 2 \text{ fm}^{-1}$.

5.5 *Inclusive Electron-Nucleus Scattering*

As briefly mentioned, the SRG framework carries crucial implications for our previous cross-section calculations utilizing mean-field nuclear states. Our prior analysis relied on the plane-wave impulse approximation—a high-energy approximation which assumes that the incident electron interacts with only a single nucleon inside the nucleus. In formal terms, this restricts the interaction to a bare, one-body current. Viewed through the perspective of the SRG, we now understand that employing such a simplified, high-energy approximation for the probe implicitly requires the target to be described by a high-resolution state. Consequently, it is unsurprising that uncorrelated mean-field results fail to reproduce experimental results; there is an inherent mismatch between the reaction mechanism and the nuclear structure. This discrepancy can be systematically resolved by formally applying the SRG transformation to evolve the probing operators, generating higher-rank currents that incorporate the nuclear dynamics missing from the low-resolution state.

Applying the SRG transformation systematically refines our previous mean-field calculations by depleting probability from the independent single-particle orbitals and redistributing it into short-range correlated (SRC) configurations. We implement this refinement in two steps. First, we update the one-body results of the previous chapter. Second, we incorporate 2N plus (A-2)-nuclear final-state configurations to explicitly capture the observable contributions from SRCs. Crucially, in evaluating these matrix

elements, we only apply the SRG operator terms that correspond to the initial nuclear state. By intentionally neglecting the SRG evolution of the final scattering states, we isolate the specific impact of initial-state short-range correlations on the observable cross section.

5.5.1 Updating LF DFT Results

To incorporate the initial-state short-range correlations discussed above, we first update the one-body results derived in Appendix B. This is achieved by formally applying the SRG transformation operator, $\hat{U}(\lambda)$, to the intrinsic nuclear matrix element $M_i^A(\mathbf{p}, \sigma, \tau)$. The transition from the bare matrix element to the SRG-evolved one is given by

$$M_i^A(\mathbf{p}, \sigma, \tau) = \langle \Psi_i^{MF} | \hat{a}(\mathbf{p}, \sigma, \tau) | \Psi_A^{MF} \rangle \longrightarrow \langle \Psi_i^{MF} | \hat{a}(\mathbf{p}, \sigma, \tau) \hat{U}(\lambda) | \Psi_A^{MF} \rangle. \quad (5.19)$$

By decomposing the unitary operator as $\hat{U}(\lambda) = \mathbb{I} + \delta\hat{U}(\lambda)$, we can separate the pure mean-field contribution from the SRG-induced corrections.

Figure 5.4 displays the updated inclusive cross sections evaluated at an SRG scale of $\lambda = 2 \text{ fm}^{-1}$. The total updated one-body cross section (solid black curve) is composed of the bare mean-field baseline ($\mathbb{I}$, blue dotted curve) and the linear interference terms ($\delta\hat{U} + \delta\hat{U}^\dagger$, green dashed curve). Consistent with the behavior of SRG evolution, these linear interference terms are negative in the quasi-elastic peak; they act to deplete probability from the low-momentum mean-field region. This depleted probability is precisely what gets redistributed to higher momenta via the two-nucleon

final-state configurations, which we address in the following subsection.

5.5.2 Inclusion of $2N + (A-2)$ Nuclear Final States

SRG techniques allow us to extend our previous calculations. Because the SRG-evolved current operator now contains two-body terms, the current can couple to two bound nucleons and promote them into scattering states — that is, it introduces $2N$ plus $(A-2)$ -nuclear final-state configurations into our calculations. For such configurations, the terms in Eq. (3.8) become

$$\sum_X \rightarrow \frac{1}{2} \sum_{\substack{\sigma_1 \tau_1 \\ \sigma_2 \tau_2}} \sum_f \int [dP_{A-2}] [dP'_{cm}] \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \alpha' (2 - \alpha')}, \quad (5.20)$$

$$\begin{aligned} |X\rangle \langle X| &\rightarrow |\Psi_{A-2}^f, \mathbf{P}_{A-2}\rangle \langle \Psi_{A-2}^f, \mathbf{P}_{A-2}| \otimes |p_1, p_2\rangle \langle p_1, p_2|, \\ |p_1, p_2\rangle &= |\mathbf{p}_1, \sigma_1, \tau_1; \mathbf{p}_2, \sigma_2, \tau_2\rangle, \\ \mathbf{p}_1 &= \left(\frac{\alpha'}{2} P'_{cm}, \frac{\alpha'}{2} \mathbf{P}'_{cm}^\perp + \mathbf{k}'^\perp \right), \\ \mathbf{p}_2 &= \left(\frac{2-\alpha'}{2} P'_{cm}, \frac{2-\alpha'}{2} \mathbf{P}'_{cm}^\perp - \mathbf{k}'^\perp \right), \end{aligned} \quad (5.21)$$

where f denotes all possible $(A-2)$ -nuclear final states. Utilizing the delta function in light-front momenta, we integrate over P_{A-2} and use $\mathbf{P}'_{cm} = \mathbf{P}_{cm} + \mathbf{q}$ to get

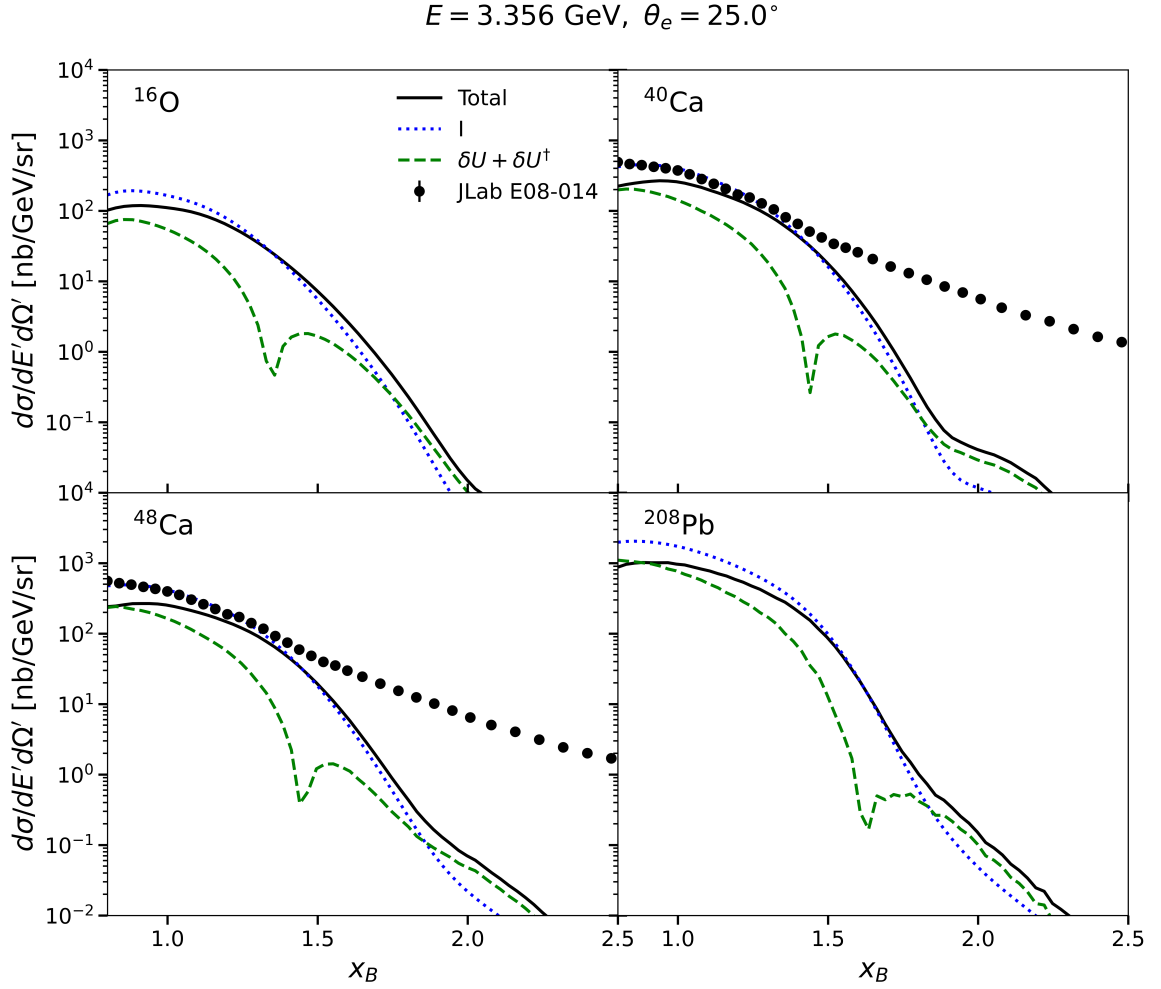


Figure 5.4: Same as Fig. 4.3, with the SRG operator corrections added. The solid black curve is the total $\mathbb{I} + \delta\hat{U} + \delta\hat{U}^\dagger$ contribution evaluated at an SRG scale of $\lambda = 2 \text{ fm}^{-1}$. The pure mean-field contribution (blue dotted) and the absolute value of the one-body cross-term $|\delta\hat{U} + \delta\hat{U}^\dagger|$ (green dashed), which depletes the cross section near the quasi-elastic peak.

$$\begin{aligned}
W_A^{\mu\nu} = & \frac{1}{2m_A} \frac{1}{2P_{A-2}^+} \frac{1}{2} \sum_{\substack{\sigma_1\tau_2 \\ \sigma_2,\tau_2}} \sum_f \int \frac{dP_{cm} d\mathbf{P}_{cm}^\perp}{(2\pi)^3} \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \alpha' (2 - \alpha')} \\
& \times \langle \Psi_A | J_A^\mu(0) | \Psi_{A-2}^f, \mathbf{P}_{A-2}; p_1, p_2 \rangle \langle \Psi_{A-2}^f, \mathbf{P}_{A-2}; p_1, p_2 | J_A^\nu(0) | \Psi_A \rangle \\
& \times \left[\frac{1}{P_{cm}^+ + q^+} \delta(q^- + P_A^- - (P_{A-2}^f)^- - P_{cm}'^-) \right],
\end{aligned} \tag{5.22}$$

$$(P_{A-2}^f)^- = \frac{M_f^2 + (\mathbf{P}_{cm}^\perp)^2}{P_A^+ - P_{cm}^+}, \quad P_{cm}'^- = \frac{4 \frac{m^2 + (\mathbf{k}'^\perp)^2}{\alpha'(2-\alpha')} + (\mathbf{P}_{cm}^\perp)^2}{P_{cm}^+ + q^+}, \tag{5.23}$$

The nuclear current matrix element is,

$$\begin{aligned}
\langle \Psi_{A-2}^f, \mathbf{P}_{A-2}; p_1, p_2 | J_A^\nu(0) | \Psi_A \rangle = & \frac{1}{2} \sum_{\substack{s_1 t_1 \\ s_2 t_2}} \sum_{\sigma \sigma' \tau} \int [dp][dp'][dq_1][dq_2] \\
& \times \sqrt{2p^+} \sqrt{2p'^+} \sqrt{2q_1^+} \sqrt{2q_2^+} \\
& \times J_N^\nu(\mathbf{p}', \sigma', \tau; \mathbf{p}, \sigma, \tau) \langle p_1, p_2 | \hat{a}^\dagger(\mathbf{p}', \sigma', \tau) \hat{a}(\mathbf{p}, \sigma, \tau) | q_1, q_2 \rangle \\
& \times \langle \Psi_{A-2}^f, \mathbf{P}_{A-2} | \hat{a}(\mathbf{q}_2, s_2, t_2) \hat{a}(\mathbf{q}_1, s_1, t_1) | \Psi_A \rangle.
\end{aligned} \tag{5.24}$$

Following the same procedure as the (A-1) calculation in Appendix B, we get

$$\begin{aligned}
\langle \Psi_{A-2}^f, \mathbf{P}_{A-2}; p_1, p_2 | J_A^\nu(0) | \Psi_A \rangle \approx & \sqrt{2P_A^+} \sqrt{2P_{A-2}^+} \\
& \times \sum_\sigma \left[\sqrt{\frac{p_2^+}{p_1^+ - q^+}} J_N^\nu(\mathbf{p}_1, \sigma_1, \tau_1; \mathbf{p}_1 - \mathbf{q}, \sigma, \tau_1) M_{ij}^A(\mathbf{p}_2, \sigma_2, \tau_2; \mathbf{p}_1 - \mathbf{q}, \sigma, \tau_1 | \lambda) \right. \\
& \left. - \sqrt{\frac{p_1^+}{p_2^+ - q^+}} J_N^\nu(\mathbf{p}_2, \sigma_2, \tau_2; \mathbf{p}_2 - \mathbf{q}, \sigma, \tau_2) M_{ij}^A(\mathbf{p}_1, \sigma_1, \tau_1; \mathbf{p}_2 - \mathbf{q}, \sigma, \tau_2 | \lambda) \right],
\end{aligned} \tag{5.25}$$

$$M_{ij}^A(\mathbf{p}_2, \sigma_2, \tau_2; \mathbf{p}_1 - \mathbf{q}, \sigma, \tau_1 | \lambda) = \langle \Psi_{ij}^{MF} | \hat{a}(\mathbf{p}_2, \sigma_2, \tau_2) \hat{a}(\mathbf{p}_1 - \mathbf{q}, \sigma, \tau_1) \delta \hat{U}^\dagger(\lambda) | \Psi_A^{MF} \rangle, \quad (5.26)$$

where we substitute the high-resolution (A-2)-nuclear states with their low-resolution mean-field counterparts, approximating them as two-hole states relative to the target A-nucleus state, i.e. $|\Psi_{ij}^{MF}\rangle = \hat{a}_j \hat{a}_i |\Psi_A^{MF}\rangle$. In doing so, we replace $\sum_f \rightarrow \frac{1}{2} \sum_{ij}$. Furthermore, note that we use $\delta \hat{U}^\dagger(\lambda)$ instead of $\hat{U}^\dagger(\lambda)$ in the above expression. This is because the $\mathbb{I}$ term is unphysical, as the unevolved current operator cannot promote two bound state nucleons into free plane-waves.

Squaring this nuclear current matrix element to compute the nuclear tensor naturally generates the quadratic $\delta \hat{U} \delta \hat{U}^\dagger$ contribution to the cross section. Physically, this term corresponds to the virtual photon striking a nucleon that belongs to a short-range correlated pair, resulting in the emission of both the struck nucleon and its correlated partner, while leaving the residual nucleus in a two-hole (A-2) spectator state.

Figure 5.5 presents the total inclusive cross section incorporating these 2N + (A-2) final states. The isolated contribution from the quadratic $\delta U \delta U^\dagger$ term is represented by the red dash-dot curve. This term acts to explicitly redistribute the probability that was depleted from the quasi-elastic peak by the linear interference terms (green dashed curve). Consequently, this missing strength is shifted to higher values of missing momentum, manifesting as an enhancement in the cross section at $x_B >$

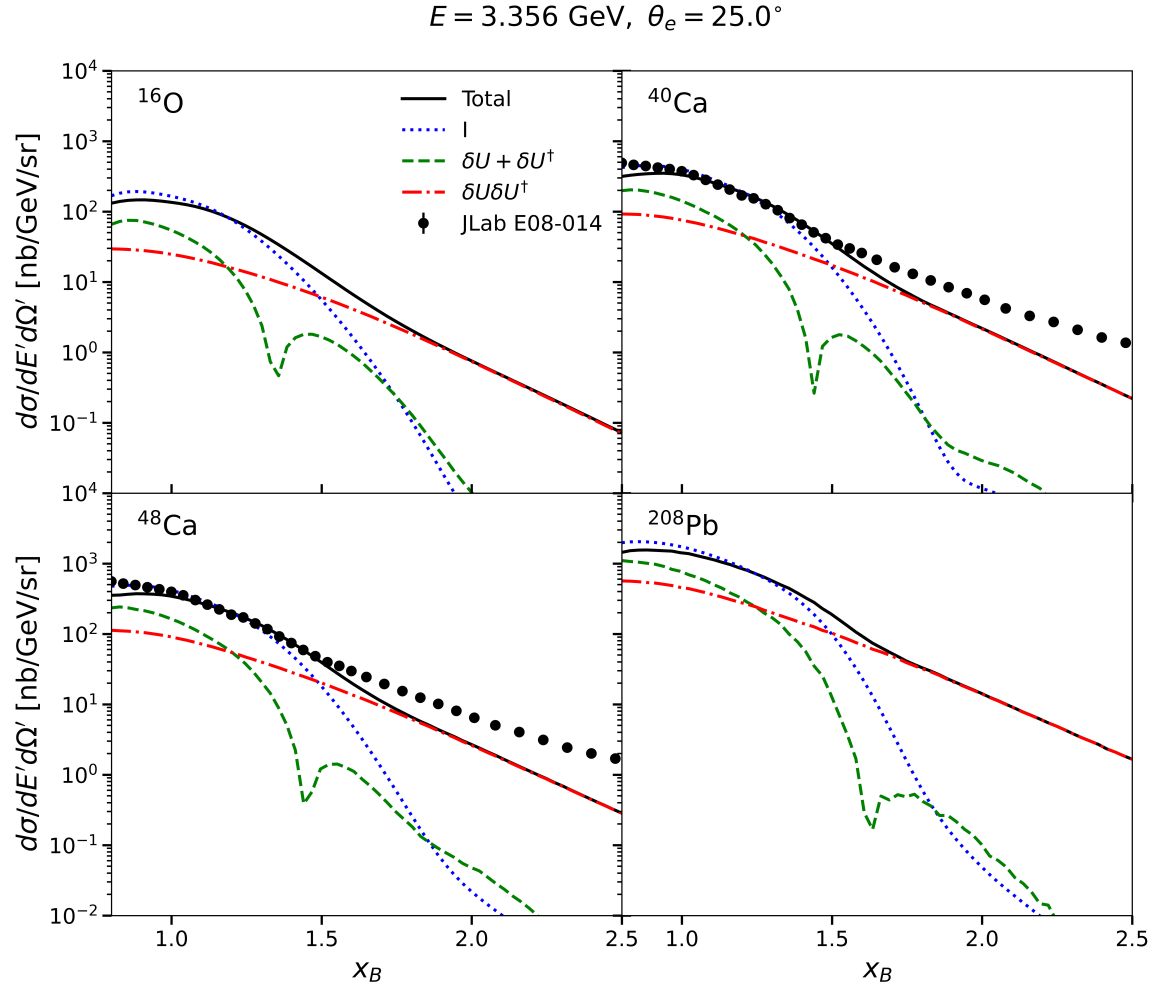


Figure 5.5: Same as Fig. 5.4, with the two-body SRG operator contribution $\delta U \delta U^\dagger$ (red dash-dot) added. The solid black curve now contains all SRG operator contributions through second order in the unitary transformation, $I + \delta U + \delta U^\dagger + \delta U \delta U^\dagger$, again at $\lambda = 2 \text{ fm}^{-1}$.

1.3. The total inclusive cross section—evaluated at $\lambda = 2 \text{ fm}^{-1}$ and incorporating all SRG operator contributions through second order in the unitary transformation $(\mathbb{I} + \delta U + \delta U^\dagger + \delta U \delta U^\dagger)$ —is shown as the solid black curve. It should be noted that the specific scale parameter $\lambda = 2 \text{ fm}^{-1}$ was not determined through a rigorous fitting procedure, but was instead chosen phenomenologically to best reproduce the cross section in the intermediate $1.3 < x_B < 1.5$ region. While this systematic inclusion of initial-state SRCs successfully generates the high-momentum tails missing from the pure mean-field results, the theoretical calculation still underestimates the experimental data in the far tail ($x_B > 1.5$). Attempting to compensate for this deficit by further decreasing λ to generate more high-momentum configurations proves unviable, as doing so leads to an overprediction of the data in the $1.3 < x_B < 1.5$ window. This residual shortfall indicates that initial-state correlations alone cannot fully account for the high-momentum reaction dynamics, confirming that the plane-wave impulse approximation remains insufficient in this regime.

5.6 Summary

This chapter demonstrated how the Similarity Renormalization Group reconciles “low-resolution” independent-particle models of the nucleus with “high-resolution” correlated many-body pictures. By utilizing a continuous unitary transformation driven by the Wegner generator, the bare Hamiltonian is systematically driven toward a band-diagonal form, decoupling low- and high-momentum modes. Because

the transformation is strictly unitary, the underlying physical spectrum remains invariant; however, the short-range nuclear dynamics are formally shifted from the highly correlated many-body state into the probing operators themselves.

This operator-evolution framework proves exceptionally powerful when formulated within LF quantization. Because LF wavefunctions are boost-invariant, the relative and center-of-mass coordinates cleanly separate—a fundamental feature that circumvents the intractable computational scaling encountered when applying SRG techniques in the IF. By pairing SRG-evolved operators with the uncorrelated wavefunctions of Chapter 4, this work achieves the first relativistic nuclear structure calculation of its kind to systematically incorporate short-range correlations into a LF mean-field framework.

The practical utility of SRG techniques was demonstrated across two primary observables. First, by evaluating the SRG-evolved one-body momentum distribution operator against a mean-field state, we demonstrated the generation of high-momentum tails characteristic of SRCs. Second, the SRG framework systematically resolves the theoretical mismatch inherent in standard plane-wave impulse approximation calculations for inclusive electron-nucleus cross sections, which incorrectly pair bare, one-body probes with low-resolution target states. By formally evolving the electromagnetic current, we naturally induced the effective two-body interactions necessary to couple the virtual photon to $2N$ plus $(A-2)$ -nuclear final-state configurations. While this systematic inclusion of short-range dynamics establishes a rigorous, computation-

ally tractable baseline for accurately modeling inclusive quasi-elastic electron-nucleus scattering at high energies, a discrepancy remains. We find that although incorporating SRCs into the LF nuclear structure successfully increases the cross-section support missing from mean-field results at $x_B > 1.5$, it still underestimates the experimental data. This residual shortfall indicates that initial-state correlations alone are insufficient, directly motivating the inclusion of final-state interactions in the next chapter.

Chapter 6

FINAL STATE INTERACTIONS

The inclusive cross section is the probability of detecting a scattered electron at given kinematics, characterized by x_B and Q^2 , or equivalently by the scattered electron's energy and angle. Experimentally, the electron interacts with the initial target and emerges into the detector, while the rest of the system is left to evolve and re-interact unobserved. The yield of detected electrons is therefore insensitive to the dynamics that occur after the initial electron-nucleus scattering: whatever rescattering takes place inside the target system cannot retroactively alter the electron that has already been measured. Theoretically, this translates into the statement that the inclusive cross section can be written without any reference to the final state at all,

$$\begin{aligned} \frac{d\sigma}{dE'd\Omega'} &\propto \sum_X \langle A | \hat{J}^\mu(0) | X \rangle \langle X | \hat{J}^\nu(0) | A \rangle \delta(P_A^- + q^- - p_X^-) \\ &= \langle A | \hat{J}^\mu(0) \delta(P_A^- + q^- - \hat{P}^-) \hat{J}^\nu(0) | A \rangle. \end{aligned} \tag{6.1}$$

The sum over final states in the first line is a book-keeping device for tabulating the contributions of individual channels — exactly the procedure followed throughout this thesis — but the second line shows that the inclusive observable is fundamentally a ground-state matrix element of an operator built from the full Hamiltonian, with

no final-state basis appearing anywhere. Why, then, do the results of the previous chapter suggest missing physics from final-state interactions (FSI)?

The answer is that the calculations performed so far have captured only a portion of the inclusive cross section within each final-state channel. The resolution lies in recognizing what is required to pass from the second line of Eq. (6.1) back to the first. The collapse of the sum over $|X\rangle$ into the operator $\delta(P_A^- + q^- - \hat{P}^-)$ is a spectral decomposition, which is valid only when the states $|X\rangle$ are eigenstates of the very Hamiltonian appearing inside the delta function. The energy-conserving delta function itself is a consequence of overall four-momentum conservation, which derives from translation invariance generated by the full Hamiltonian; it is therefore implicit at every stage that the initial and final states are eigenstates of $\hat{P}^-$. The plane-wave calculations performed so far capture only a portion of the full scattering amplitude, since the true continuum eigenstates are physical scattering states, of which plane waves are merely the asymptotic piece; the remaining T -matrix-driven contributions are generated order by order in light-front perturbation theory and have simply not yet been included. Therefore, for inclusive scattering, FSI effects as conventionally presented in the literature are not a separate dynamical mechanism that redistributes the scattered-electron yield, but the missing theoretical contribution required to restore the original inclusive observable towards its basis-independent form.

6.1 Elastic Two-Nucleon Re-scattering

In the $x_B > 1.5$ regime the dominant initial-state configurations are short-range correlated (SRC) pairs. Such pairs are characterized by a large relative momentum and a small center-of-mass momentum, so that the two correlated nucleons are close to one another while the pair as a whole is spatially separated from the residual A-2 system. Consequently, when the virtual photon strikes one member of the pair, the struck nucleon is far more likely to re-scatter from its correlated partner than from the remaining A-2 nucleons as it leaves the nucleus. We model this final-state interaction FSI by replacing the two-nucleon plane-wave final state with the scattering state generated by the LF minimal-relativity AV18 potential,

$$|p_1, p_2\rangle \langle p_1, p_2| \rightarrow |\Psi_{NN}^{(-)}, \mathbf{P}\rangle \langle \Psi_{NN}^{(-)}, \mathbf{P}|, \quad (6.2)$$

$$|\Psi_{NN}^{(-)}, \mathbf{P}\rangle = |p_1, p_2\rangle + \frac{1}{\hat{P}^+} \frac{1}{p_1^- + p_2^- - \hat{P}^- - i\epsilon} \hat{P}^+ \hat{T}^{(-)} |p_1, p_2\rangle,$$

where the $(-)$ superscript denotes the physical final-state scattering solution, $\hat{T}^{(-)}$ is the half-off-shell T -matrix, and the limit $\epsilon \rightarrow 0^+$ is understood. The factor $\hat{P}^+/\hat{P}^+$ is inserted because the T -matrix obtained from Eq. (2.23) is proportional to the matrix element of $\hat{P}^+ \hat{T}^{(-)}$.

Rather than solving for $\hat{T}^{(-)}$ directly, we first compute the half-off-shell K -matrix. The K -matrix generates standing-wave (principal-value) scattering states [92]; because it is real and satisfies an integral equation that can be rendered non-singular, it is numerically more convenient than the imaginary T -matrix. Partial-wave projec-

tion of Eq. (2.20), together with the standing-wave boundary condition, leads to the coupled integral equations for the K-matrix,

$$\left[\tilde{K}_{LL'}^{JTS}(k_{\text{off}}, p) = \tilde{V}_{LL'}^{JTS}(k_{\text{off}}, p) + \sum_{L''} \mathcal{P} \int \frac{dk' k'^2}{(2\pi)^3 E(k')} \frac{\tilde{V}_{LL''}^{JTS}(k_{\text{off}}, k') \tilde{K}_{L''L'}^{JTS}(k', p)}{4(p^2 - k'^2)} \right]_{\text{IF}}, \quad (6.3)$$

where the labels (J, T, S, L, L') specify the partial-wave channel and $\mathcal{P}$ denotes the principal value.

We solve Eq. (6.3) numerically by discretizing the momentum integral on a Gauss–Legendre grid $\{k_i, w_i\}$, which converts the integral equation into a matrix equation. The kernel is singular at $k' = p$; we regulate it by principal-value subtraction, which exploits the identity

$$\mathcal{P} \int_0^\infty \frac{dk'}{p^2 - k'^2} = 0, \quad (6.4)$$

so that

$$\mathcal{P} \int_0^\infty dk' \frac{f(k')}{p^2 - k'^2} = \int_0^\infty dk' \frac{f(k') - f(p)}{p^2 - k'^2}. \quad (6.5)$$

The subtracted integrand is finite at $k' = p$, with limiting value $-f'(p)/2p$, so the kernel of Eq. (6.3) is smooth across the grid. After subtraction and discretization, Eq. (6.3) takes the linear form $(\mathbb{I} - \tilde{V}\tilde{G})\tilde{K} = \tilde{V}$, where $\tilde{G}$ collects the (subtracted) grid weights and propagator factors; we solve this system for $\tilde{K}$ by direct matrix inversion. Finally, the half-off-shell T -matrix is recovered from $\tilde{K}$ through the (relativistic)

Heitler relation,

$$\left[\left(\tilde{T}_{LL'}^{JTS}(k_{\text{off}}, p) \right)^{(-)} = \frac{\tilde{K}_{LL'}^{JTS}(k_{\text{off}}, p)}{\hat{\mathbb{I}} - i\rho_{\text{ps}}(p)\tilde{K}_{LL'}^{JTS}(p, p)} \right]_{\text{IF}} \quad (6.6)$$

$$\left[\rho_{\text{ps}}(p) = \frac{p}{64\pi^2 E(p)} \right]_{\text{IF}}$$

where ρ_{ps} is the on-shell phase-space factor fixed by the phase-space factors in Eq. (2.20).

Finally, the LF analogue is obtained from Eq. (2.29).

Incorporating FSI requires the struck nucleon to remain off-shell even after interacting with the virtual photon. Consequently, the single-nucleon electromagnetic current matrix element introduced in Chapter 2 must be generalized to account for this final-state off-shellness:

$$J_N(\mathbf{p}', \sigma', \tau; \mathbf{p}, \sigma, \tau) = \bar{u}(\mathbf{p}', \sigma', \tau) \left(1 + \frac{\not{A}_{p'}}{2m_N} \right) \Gamma_{\gamma^* N}^\mu \left(1 + \frac{\not{A}_p}{2m_N} \right) u(\mathbf{p}, \sigma, \tau), \quad (6.7)$$

where the final-state off-shell operator, $\not{A}_{p'}$, follows the exact same structure as Eq. (3.17). We note that the recoil spectator nucleon involved in the elastic rescattering is technically also off-shell. However, we neglect its corresponding off-shell corrections here because they are kinematically suppressed. Specifically, these factors only become significant in the integration region where $\alpha_r \equiv 2p_r^+/P^+ \rightarrow 0$, with p_r the momentum of the SRC recoil nucleon. Physically, this limit describes a strongly virtual spectator nucleon emerging from the deuteron vertex and instantaneously interacting with the struck nucleon—a configuration whose contribution to the overall integral is negligible [44].

6.2 Results and Discussion for Deuterium and Calcium-40

We apply the FSI formalism developed above to the deuteron. As a pure $A = 2$ system, the deuteron admits no residual $(A - 2)$ spectators; the only final-state rescattering is the elastic re-interaction between the two nucleons modeled here. We solve the K-matrix equation (6.3) for the LF minimal-relativity AV18 potential in all S , P , and D partial-wave channels, retaining the coupled 3S_1 - 3D_1 and 3D_3 - 3G_3 channels, and construct the LF half-off-shell T -matrix via Eq. (2.29).

Figure 6.1 shows the resulting inclusive cross sections compared to JLab data at two kinematic settings: $E = 3.356$ GeV, $\theta_e = 25^\circ$ from JLab E08-014 [41] (left panel) and $E = 5.766$ GeV, $\theta_e = 18^\circ$ from JLab E02-019 [51] (right panel), together with the individual FSI contributions. The interference term includes both cross terms from the squared matrix element, $\text{PW-FSI} + \text{FSI-PW}$; we label it simply as PW-FSI . Since it removes probability from the plane-wave channel by unitarity, it is plotted as its absolute value. The quadratic term is positive and redistributes probability.

The central result is that the full cross section is nearly indistinguishable from the plane-wave result alone across the entire x_B range shown for both kinematics. The interference correction lies one to two orders of magnitude below the cross section at most kinematics, as does the quadratic correction.

We now extend the same calculation to ${}^{40}\text{Ca}$. We only apply FSI to the SRG-induced $2N + (A-2)$ final state term. The FSI is again described by the two-nucleon

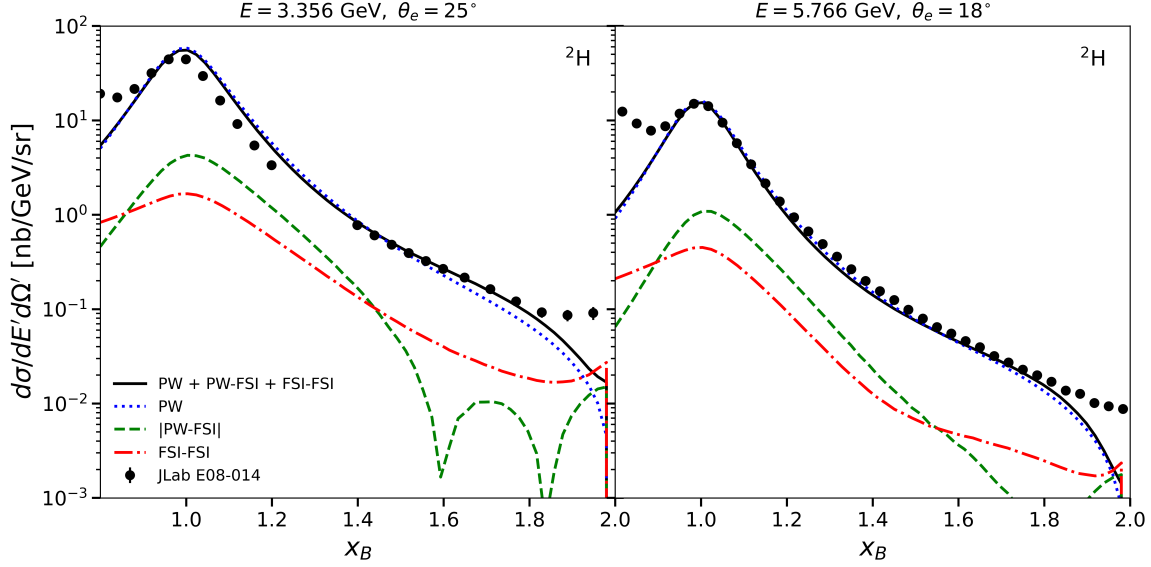


Figure 6.1: Inclusive electron–deuteron cross section as a function of the Bjorken variable x_B at the JLab E08-014 kinematics ($E = 3.356$ GeV, $\theta_e = 25^\circ$, left) and the JLab E02-019 kinematics ($E = 5.766$ GeV, $\theta_e = 18^\circ$, right), decomposed into plane-wave and FSI contributions. The solid black curve is the full theory result, $\sigma_{\text{PW}} + \sigma_{\text{PW-FSI}} + \sigma_{\text{FSI-FSI}}$; the blue dotted curve is the plane-wave piece; the green dashed curve is the PW–FSI interference $|\sigma_{\text{PW-FSI}}|$; the red dash-dot curve is the FSI quadratic contribution $\sigma_{\text{FSI-FSI}}$. Black points are the experimental data.

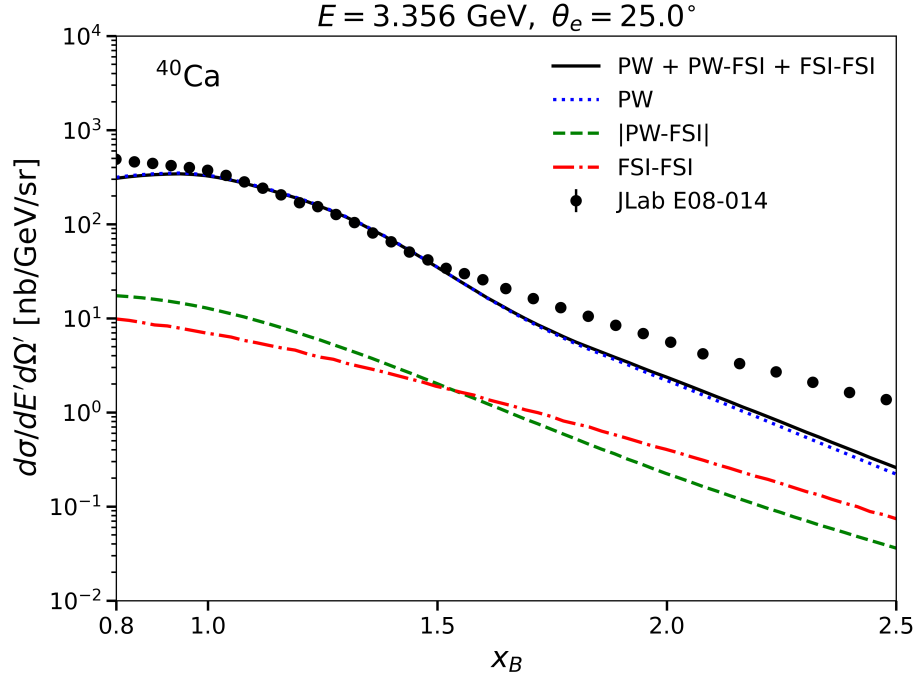


Figure 6.2: Inclusive electron- ^{40}Ca cross section as a function of x_B for JLab E08-014 kinematics. Curves as in Fig. 6.1. Data (black points) from JLab E08-014. The ‘PW’ term is the same as the ‘Total’ term labeled in Fig. 5.5.

T -matrix from the minimal-relativity AV18 potential. Results are compared to JLab E08-014 data for ^{40}Ca [41]. The results for ^{40}Ca are identical in character to those for the deuteron: the FSI corrections are negligible and the full cross section is again nearly indistinguishable from the plane-wave result alone.

Our results differ quantitatively from the similar elastic rescattering study of Ref. [93]. Both approaches employ elastic two-nucleon rescattering, so the difference is attributable to details of the nuclear wavefunction and potential, as well as to the

relativistic light-front treatment of nuclear structure in the present work, compared to the non-relativistic nuclear structure used in Ref. [93].

More broadly, the AV18 T -matrix captures only elastic NN degrees of freedom and is therefore missing the inelastic pionic channels that contribute to the physical FSI. These include virtual-photon excitation of a nucleon to a Δ -isobar, with subsequent decay $\Delta \rightarrow N\pi$, and the photon scattering off a virtual pion exchanged between nucleons and promoting it on-shell. The natural extension that incorporates these effects phenomenologically, without requiring an explicit pion-production amplitude, is to replace the AV18 T -matrix with effective amplitudes built from experimental NN cross sections, which automatically absorb all inelastic channels. This approach is the Generalized Eikonal Approximation, developed in Refs. [94, 39, 38, 44], and constitutes the natural follow-up to the present work.

These missing inelastic pionic contributions leave a clear empirical signature. While our purely nucleonic model — two-nucleon SRC configurations with elastic re-scattering — reproduces the deuterium data well for $x_B > 1$ shown in Fig. 6.1, the same model falls short of the ^{40}Ca data at large x_B (Fig. 6.2), where the cross section is dominated by the SRC contribution. The growing discrepancy with nuclear mass reflects the enhanced pionic FSI in the denser nuclear medium of heavier nuclei, which our nucleonic framework does not capture.

This has a direct consequence for the interpretation of the SRC plateau in the experimental ratio $\sigma(A)/\sigma(D)|_{x_B > 1.5}$. Because our theoretical cross section for heavy

nuclei underpredicts the data while the deuterium cross section does not, the experimental plateau exceeds the purely nucleonic ratio $\sigma_{\text{theory}}(A)/\sigma_{\text{theory}}(D)$. The standard identification of the plateau value with nucleonic SRC abundance $a_2(A)$ [48] therefore overestimates the purely nucleonic content: the experimental ratio is inflated by the pionic FSI channels that our theory is missing, and which do not cancel between the heavy nucleus and deuterium. By the same reasoning, theoretical and phenomenological extractions of the average number of SRC pairs from momentum distribution ratios [95] inherit the same sensitivity, and the pair counts extracted from experimental data should be understood as an upper bound on the purely nucleonic SRC content.

Chapter 7

CONCLUSION

The advent of high-energy experiments at Jefferson Lab and the planned Electron-Ion Collider, both of which aim to probe non-perturbative aspects of QCD using nuclear targets, places a new demand on low-energy nuclear physics: the structure models that feed these programs must be relativistic. Light-front quantization is the natural framework for this regime, but the path to relativistic many-body calculations is not obvious. Correlated *ab initio* approaches based on the many-body Dirac equation are out of reach—the requisite two- and three-body relativistic interactions have not been developed, and even the numerical machinery for solving the Dirac many-body problem is in its infancy. Furthermore, the EIC’s heavy-nuclei program compounds the difficulty, pushing well beyond what even non-relativistic *ab initio* methods can currently address.

The strategy adopted in this dissertation is to be as conservative as possible with the existing, successful machinery of IF nuclear structure, and to reformulate it over to the LF. The two key developments along this line are presented.

Light-front density functional theory

The first development, presented in Chapter 4, is the formulation of LF density functional theory and, more importantly, a demonstration of its formal equivalence to IF DFT. This equivalence is exact: no approximation is required to map one onto the other. The practical consequence is that existing and future developments in IF DFT transfers entirely to the LF. Applied to inclusive electron-nucleus scattering at high momentum transfer and low energy transfer, LF DFT reproduces the quasi-elastic peak well. It fails, however, at high Bjorken- x_B , in line with what is already established from $A(e, e'p)$ measurements: mean-field theory does not describe the high-momentum tail of the nuclear wave function.

Beyond mean field: SRG and short-range correlations

Short-range correlation phenomenology identifies the missing strength at high x_B with two-nucleon configurations of high relative momentum and low center-of-mass momentum. The second development of this program incorporates these correlations into the LF framework by using the Similarity Renormalization Group. SRCs do increase the inclusive cross section in the expected direction. They are, however, insufficient: even after including elastic nucleon-nucleon rescattering as a final-state interaction, the predicted cross section still undershoots the data. This suggests beyond purely nucleonic, elastic dynamics—toward inelastic channels as necessary ingredients in the high- x_B regime, currently missing in phenomenology.

Implications for the SRC plateau

The cross-section ratio $\sigma(A)/\sigma(D)|_{x_B>1.5}$, which exhibits a characteristic plateau, is standardly identified with the per-nucleon SRC probability $a_2(A)$ [48]. This identification rests on the implicit assumption that the dominant non-nucleonic contributions cancel between numerator and denominator. The present analysis shows this cancellation is incomplete: the experimental cross section for nuclei contain inelastic contributions, whereas the deuterium cross section is well described by the purely nucleonic model. The experimental ratio therefore exceeds the purely nucleonic ratio $\sigma_{\text{theory}}(A)/\sigma_{\text{theory}}(D)$, and the conventional $a_2(A)$ overestimates the purely nucleonic SRC content. The nucleonic theory developed here provides a lower bound on the ratio; the gap between the experimental plateau and the theoretical prediction quantifies the inelastic excess that inflates it. By the same reasoning, phenomenological extractions of SRC pair counts from ratios of nuclear momentum distributions [95] inherit the same sensitivity, and those pair counts should be understood as upper bounds on the purely nucleonic content.

Code availability

The codes developed for this dissertation are publicly available at <https://github.com/dimakim/Light-Front-Quasi-Elastic-Scattering>.

Outlook

The most immediate continuation of this work is the incorporation of inelastic FSI through the Generalized Eikonal Approximation [94, 39, 38, 44]. The GEA replaces the AV18 T -matrix with effective scattering amplitudes built from experimental nucleon-nucleon cross sections, absorbing all inelastic channels—pionic production included—without requiring an explicit pion-production amplitude. Within the LF framework developed here, the GEA is the minimal extension needed to test whether pionic FSI accounts quantitatively for the deficit observed in ^{40}Ca .

On a more fundamental level, the results of this dissertation highlight the importance of developing light-front nucleon-nucleon potentials that incorporate pion and Δ -isobar degrees of freedom explicitly. The AV18 potential used here is an instant-form potential adapted to the light-front through the minimal-relativity prescription—a procedure that does not generate the full relativistic structure of the NN interaction. A genuinely light-front NN potential with explicit pion-exchange and Δ -isobar channels would encode the same pionic physics that the GEA currently handles phenomenologically, but within a dynamically consistent relativistic framework. Such a potential would improve the FSI treatment and simultaneously provide a more faithful starting point for the SRG evolution that underlies the SRC physics of this dissertation, bringing the entire program closer to a relativistically complete description of nuclear structure for the high-energy experiments at JLab and the planned

Electron-Ion Collider.

Appendix A

QUANTIZATION OF FREE DIRAC PARTICLE

Starting from the Dirac equation, we expand the contraction between the Dirac γ -matrices and four derivative in terms of light-front coordinates,

$$(i\gamma_\mu\partial^\mu - m)\psi(x) = \left(i \left[\frac{1}{2}\gamma^+\partial^- + \frac{1}{2}\gamma^-\partial^+ - \boldsymbol{\gamma}^\perp \cdot \boldsymbol{\partial}^\perp \right] - m \right) \psi(x) = 0. \quad (\text{A.1})$$

Using light-front variables, one can develop a projection operator in Dirac space,

$$\begin{aligned} \Lambda^+ &= \frac{1}{4}\gamma^-\gamma^+ = \frac{1}{2}\gamma^0\gamma^+, \\ \Lambda^- &= \frac{1}{4}\gamma^+\gamma^- = \frac{1}{2}\gamma^0\gamma^-, \\ \Lambda^+ + \Lambda^- &= \mathbb{I}, \quad \Lambda^\pm\Lambda^\pm = \Lambda^\pm, \quad \Lambda^\pm\Lambda^\mp = 0. \end{aligned} \quad (\text{A.2})$$

Defining $\psi(x) = \psi^+(x) + \psi^-(x)$, where $\psi^+(x) = \Lambda^+\psi(x)$ and vice versa, we project the Dirac equation using Λ^+ and Λ^- ,

$$\begin{aligned} 2i\partial^-\psi^+(x) &= (-i\boldsymbol{\gamma}^\perp \cdot \boldsymbol{\partial}^\perp + m) \gamma^-\psi^-(x), \\ 2i\partial^+\psi^-(x) &= (-i\boldsymbol{\gamma}^\perp \cdot \boldsymbol{\partial}^\perp + m) \gamma^+\psi^+(x). \end{aligned} \quad (\text{A.3})$$

Notice that the second expression does not contain any LF time derivative terms, hence it is a constraint equation. In other words, the dynamical field is $\psi^+(x)$, which

can be used to solve for $\psi^-(x)$. Thus, we impose equal LF time commutation relations on the dynamical + component of the Dirac fields [96],

$$\{\hat{\psi}^+(x^+, x^-, \mathbf{x}^\perp), \hat{\psi}^{+\dagger}(y^+, y^-, \mathbf{y}^\perp)\}_{x^+=y^+} = \Lambda^+ \delta(x^- - y^-) \delta^{(2)}(\mathbf{x}^\perp - \mathbf{y}^\perp). \quad (\text{A.4})$$

The mode expansion yields,

$$\hat{\psi}(x) = \sum_{\sigma} \int \frac{dp^+ d\mathbf{p}^\perp}{(2\pi)^3 \sqrt{2p^+}} \left(\hat{a}(p) u(p) e^{-ip \cdot x} + \hat{b}^\dagger(p) v(p) e^{+ip \cdot x} \right). \quad (\text{A.5})$$

Plugging Eq. (A.5) into Eq. (A.4) yields the following commutation relations for the creation and annihilation operators

$$\{\hat{a}(p), \hat{a}^\dagger(p')\} = \{\hat{b}(p), \hat{b}^\dagger(p')\} = (2\pi)^3 \delta(p^+ - p'^+) \delta^{(2)}(\mathbf{p}^\perp - \mathbf{p}'^\perp) \delta_{\sigma\sigma'}. \quad (\text{A.6})$$

The representation of the Dirac spinors can be obtained from plugging in Eq. (A.5) into Eq. (A.3). In doing so, one finds that we have freedom in choosing what the plus projection of the spinors are, while the minus projections are constrained. For instance, the $u^-(p)$ spinor can be obtained from $u^+(p)$ through

$$u^-(p) = \frac{1}{p^+} (\boldsymbol{\alpha}^\perp \cdot \mathbf{p}^\perp + \gamma^0 m) u^+(p), \quad (\text{A.7})$$

same for $v^-(p)$ but with $-m$ in the above expression. Choosing the plus projections to be proportional to eigenstates of Λ^+ we get the Brodsky-Lepage Dirac spinor

representations [97, 25]

$$u(\mathbf{p}, \sigma) = \frac{1}{\sqrt{p^+}} (p^+ + \beta m + \boldsymbol{\alpha}^\perp \cdot \mathbf{p}^\perp) \times \begin{cases} \chi(\uparrow), & \text{for } \sigma = +1, \\ \chi(\downarrow), & \text{for } \sigma = -1, \end{cases} \quad (\text{A.8})$$

$$v(\mathbf{p}, \sigma) = \frac{1}{\sqrt{p^+}} (p^+ - \beta m + \boldsymbol{\alpha}^\perp \cdot \mathbf{p}^\perp) \times \begin{cases} \chi(\downarrow), & \text{for } \sigma = +1, \\ \chi(\uparrow), & \text{for } \sigma = -1. \end{cases} \quad (\text{A.9})$$

The two χ -spinors, which are the eigenstates of Λ^+ , are

$$\chi(\uparrow) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \chi(\downarrow) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}. \quad (\text{A.10})$$

Appendix B

INCLUSIVE ELECTRON-NUCLEUS CROSS SECTION FOR (A-1)-NUCLEAR FINAL STATES

Restricting ourselves to doubly-magic spherically symmetric ($J = 0$) nuclear initial states, with single nucleon plus (A-1)-nuclear final states, terms in Eq. (3.8) become

$$\sum_X \int d\Pi_X \rightarrow \sum_{\sigma_1 \tau_1} \sum_f \int [dP_{A-1}] [dp_1], \quad (\text{B.1})$$

$$|X\rangle \langle X| \rightarrow |\Psi_{A-1}^f, \mathbf{P}_{A-1}\rangle \langle \Psi_{A-1}^f, \mathbf{P}_{A-1}| \otimes |\mathbf{p}_1, \sigma_1, \tau_1\rangle \langle \mathbf{p}_1, \sigma_1, \tau_1|, \quad (\text{B.2})$$

where f denotes all the possible (A-1)-nuclear states. The nuclear tensor becomes

$$\begin{aligned} W_A^{\mu\nu} &= \frac{1}{4\pi m_A} \sum_{\sigma_1 \tau_1} \sum_f \int [dP_{A-1}] [dp_1] (2\pi)^4 \delta^4(q^\mu + P_A^\mu - P_{A-1}^\mu - p_1^\mu) \\ &\quad \times \langle \Psi_A | \hat{J}_A^\mu(0) | \Psi_{A-1}^f, \mathbf{P}_{A-1}; \mathbf{p}_1, \sigma_1, \tau_1 \rangle \\ &\quad \times \langle \Psi_{A-1}^f, \mathbf{P}_{A-1}; \mathbf{p}_1, \sigma_1, \tau_1 | \hat{J}_A^\nu(0) | \Psi_A \rangle \\ &= \frac{1}{2m_A} \frac{1}{2P_{A-1}^+} \sum_{\sigma_f \tau_f} \sum_f \int \frac{dp_i^+ d\mathbf{p}_i^\perp}{(2\pi)^3} \left[\frac{1}{p_i^+ + q^+} \delta(q^- + P_A^- - (P_{A-1}^f)^- - p_1^-) \right] \\ &\quad \times \langle \Psi_A | \hat{J}_A^\mu(0) | \Psi_{A-1}^f, \mathbf{P}_{A-1}; \mathbf{p}_i + \mathbf{q}, \sigma_f, \tau_f \rangle \\ &\quad \times \langle \Psi_{A-1}^f, \mathbf{P}_{A-1}; \mathbf{p}_i + \mathbf{q}, \sigma_f, \tau_f | \hat{J}_A^\nu(0) | \Psi_A \rangle, \end{aligned} \quad (\text{B.3})$$

where in the second equality we integrated over $\mathbf{P}_{A-1}$, changed variables to $\mathbf{p}_i = \mathbf{p}_1 - \mathbf{q}$, and re-labeled $(\sigma_1, \tau_1) \rightarrow (\sigma_f, \tau_f)$. Furthermore, we drop the $J = 0$ label for the initial nuclear state. Since we work in the collinear frame, where the virtual photon defines the negative z -axis,

$$(P_{A-1}^f)^- = \frac{m_f^2 + (\mathbf{p}_i^\perp)^2}{P_A^+ - p_i^+}, \quad p_1^- = \frac{m^2 + (\mathbf{p}_i^\perp)^2}{p_i^+ + q^+}. \quad (\text{B.4})$$

The nuclear current matrix element is

$$\begin{aligned} & \langle \Psi_{A-1}^f, \mathbf{P}_{A-1}; \mathbf{p}_i + \mathbf{q}, \sigma_i, \tau_i | \hat{J}_A^\nu(0) | \Psi_A \rangle \\ &= \sum_{s_1 t_1} \sum_{\sigma \sigma' \tau} \int [dp][dp'][dq_1] \sqrt{2p^+} \sqrt{2p'^+} J_N^\nu(\mathbf{p}', \sigma', \tau; \mathbf{p}, \sigma, \tau) \\ & \quad \times \langle \mathbf{p}_i + \mathbf{q}, \sigma_f, \tau_f | \hat{a}^\dagger(\mathbf{p}', \sigma', \tau) \hat{a}(\mathbf{p}, \sigma, \tau) | \mathbf{q}_1, s_1, t_1 \rangle \\ & \quad \times \langle \Psi_{A-1}^f, \mathbf{P}_{A-1}; \mathbf{q}_1, s_1, t_1 | \Psi_A \rangle \\ &= \sum_{s_1 t_1} \sum_{\sigma \sigma' \tau} \int [dp][dp'][dq_1] \sqrt{2p^+} \sqrt{2p'^+} \sqrt{2(p_i^+ + q^+)} (2q_1^+) \\ & \quad \times \langle 0 | \hat{a}(\mathbf{p}_i + \mathbf{q}, \sigma_f, \tau_f) \hat{a}^\dagger(\mathbf{p}', \sigma', \tau) \hat{a}(\mathbf{p}, \sigma, \tau) \hat{a}^\dagger(\mathbf{q}_1, s_1, t_1) | 0 \rangle \\ & \quad \times J_N^\nu(\mathbf{p}', \sigma', \tau; \mathbf{p}, \sigma, \tau) \langle \Psi_{A-1}^f, \mathbf{P}_{A-1} | \hat{a}(\mathbf{q}_1, s_1, t_1) | \Psi_A \rangle. \end{aligned} \quad (\text{B.5})$$

Here $J_N^\nu(\mathbf{p}', \sigma', \tau; \mathbf{p}, \sigma, \tau)$ is the single-nucleon electromagnetic current matrix element defined in Chapter 3.

Using Wick's Theorem to evaluate $\langle 0 | \hat{a} \hat{a}^\dagger \hat{a} \hat{a}^\dagger | 0 \rangle$ yields two delta functions which we use to integrate over q_1 and p' . This gives

$$\begin{aligned}
\langle \Psi_{A-1}^f, \mathbf{P}_{A-1}; \mathbf{p}_i + \mathbf{q}, \sigma_i, \tau_i | \hat{J}_A^\nu(0) | \Psi_A \rangle &= \sum_{\sigma} \int [dp] \sqrt{2p^+} J_N^\nu(\mathbf{p}_i + \mathbf{q}, \sigma_f, \tau_f; \mathbf{p}, \sigma, \tau_f) \\
&\times \langle \Psi_{A-1}^f, \mathbf{P}_{A-1} | \hat{a}(\mathbf{p}, \sigma, \tau_f) | \Psi_A \rangle.
\end{aligned} \tag{B.6}$$

Now we focus on the nuclear matrix element term.

$$\begin{aligned}
&\langle \Psi_{A-1}^f, \mathbf{P}_{A-1} | \hat{a}(\mathbf{p}, \sigma, \tau_f) | \Psi_A \rangle = \\
&\sqrt{2P_A^+} \sqrt{2P_{A-1}^+} (2\pi)^3 \delta(P_{A-1}^+ + p^+ - P_A^+) \delta^{(2)}(\mathbf{P}_{A-1}^\perp + \mathbf{p}^\perp) M_f^A(\mathbf{p}, \sigma, \tau_f),
\end{aligned} \tag{B.7}$$

where M_f^A is a quantity constructed from the overlap of intrinsic A- and (A-1)-nuclear light-front wavefunctions. M_f^A can be determined by looking at the matrix element of the momentum distribution operator

$$\begin{aligned}
&\langle \Psi_A, \mathbf{P}'_A | \hat{a}^\dagger(\mathbf{p}, \sigma, \tau) \hat{a}(\mathbf{p}, \sigma, \tau) | \Psi_A, \mathbf{P}_A \rangle = \\
&(2\pi)^3 2P_A^+ \delta(P^+ - P'^+) \delta^{(2)}(\mathbf{P}^\perp - \mathbf{P}'^\perp) n(\mathbf{p}, \sigma, \tau).
\end{aligned} \tag{B.8}$$

By inserting a complete set of (A-1) states in between the creation and annihilation operator and using Eq. (B.7) we find the following relation,

$$n(\mathbf{p}, \sigma, \tau) = \sum_f M_f^A(\mathbf{p}, \sigma, \tau)^* M_f^A(\mathbf{p}, \sigma, \tau). \tag{B.9}$$

Up to this point, everything has remained exact and completely independent of any specific nuclear model. The inherent model dependence is encapsulated within M_f^A ,

which we evaluate by employing mean-field theory. Within the mean-field framework, we have the following relation:

$$\begin{aligned}
 n^{MF}(\mathbf{p}, \sigma, \tau) &= \langle \Psi_A^{MF} | \hat{a}^\dagger(\mathbf{p}, \sigma, \tau) \hat{a}(\mathbf{p}, \sigma, \tau) | \Psi_A^{MF} \rangle \\
 &= \sum_i \langle \Psi_A^{MF} | \hat{a}^\dagger(\mathbf{p}, \sigma, \tau) | \Psi_i^{MF} \rangle \langle \Psi_i^{MF} | \hat{a}(\mathbf{p}, \sigma, \tau) | \Psi_A^{MF} \rangle,
 \end{aligned} \tag{B.10}$$

comparing Eq. (B.9) and Eq. (B.10) we find

$$\begin{aligned}
 \sum_f &\rightarrow \sum_i \\
 M_f^A &\rightarrow M_i^A
 \end{aligned} \tag{B.11}$$

$$M_i^A(\mathbf{p}, \sigma, \tau) = \langle \Psi_i^{MF} | \hat{a}(\mathbf{p}, \sigma, \tau) | \Psi_A^{MF} \rangle$$

where we approximate the (A-1)-nuclear states as one-hole states relative to the target A-nucleus state, i.e. $|\Psi_i^{MF}\rangle = \hat{a}_i |\Psi_A^{MF}\rangle$. The final result of the nuclear current is

$$\begin{aligned}
 &\langle \Psi_{A-1}^f, \mathbf{P}_{A-1}; \mathbf{p}_i + \mathbf{q}, \sigma_f, \tau_f | \hat{J}_A^\nu(0) | \Psi_A \rangle \approx \\
 &\sum_\sigma \frac{1}{\sqrt{2p_i^+}} J_N^\nu(\mathbf{p}_i + \mathbf{q}, \sigma_f, \tau_f; \mathbf{p}, \sigma, \tau_f) M_i^A(\mathbf{p}_i, \sigma, \tau_f)
 \end{aligned} \tag{B.12}$$

Appendix C

DERIVATION OF SIMILARITY RENORMALIZATION GROUP TERMS

In this section, we provide explicit formulas for each term presented in Eq. (5.18). To avoid redundancy, we present a detailed step-by-step derivation solely for the $\delta\hat{U}(\lambda) \hat{n}^\tau(p, \sigma) \hat{\mathbb{I}}$ interference term, as the underlying procedure straightforwardly generalizes to the remaining components, which are subsequently outlined. The corresponding non-relativistic results are documented in the appendix of Ref. [91], and the light-front adaptation presented here closely follows that study.

Firstly, the SRG unitary transformation at flow parameter λ in the plane-wave basis is given by

$$\begin{aligned}
 \hat{U}(\lambda) = \hat{\mathbb{I}} + \frac{1}{4} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma_3 \sigma_4}} \sum_{\substack{\tau_1 \tau_2 \\ \tau_3 \tau_4}} \int [dP] \frac{d\alpha d\mathbf{k}^\perp}{(2\pi)^3 \sqrt{\alpha(2-\alpha)}} \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \sqrt{\alpha'(2-\alpha')}} (P^+)^2 \\
 \times \langle \alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 | \delta U(\lambda) | \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \rangle \\
 \times \hat{a}^\dagger(p_1) \hat{a}^\dagger(p_2) \hat{a}(p_4) \hat{a}(p_3) + \dots
 \end{aligned} \tag{C.1}$$

where $(\alpha, \mathbf{k}^\perp)$ and $(\alpha', \mathbf{k}'^\perp)$ are the relative momenta and $(P^+, \mathbf{P}^\perp)$ is the center of mass momenta. $\mathbf{p}_i = (p_i^+, \mathbf{p}_i^\perp)$ for $i = 1, 2, 3, 4$ are defined as follows:

$$\begin{aligned}
\mathbf{p}_1 &= \left(\frac{\alpha}{2} P^+, \frac{\alpha}{2} \mathbf{P}^\perp + \mathbf{k}^\perp \right), & \mathbf{p}_2 &= \left(\frac{2-\alpha}{2} P^+, \frac{2-\alpha}{2} \mathbf{P}^\perp - \mathbf{k}^\perp \right), \\
\mathbf{p}_3 &= \left(\frac{\alpha'}{2} P^+, \frac{\alpha'}{2} \mathbf{P}^\perp + \mathbf{k}'^\perp \right), & \mathbf{p}_4 &= \left(\frac{2-\alpha'}{2} P^+, \frac{2-\alpha'}{2} \mathbf{P}^\perp - \mathbf{k}'^\perp \right).
\end{aligned} \tag{C.2}$$

Also, the matrix element of $\delta\hat{U}$ is antisymmetrized, i.e.

$$\begin{aligned}
&\langle \alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 | \delta U(\lambda) | \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \rangle \\
&= - \langle \alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 | \delta U(\lambda) | 2 - \alpha', -\mathbf{k}'^\perp, \sigma_4, \tau_4, \sigma_3, \tau_3 \rangle \tag{C.3} \\
&= - \langle 2 - \alpha, -\mathbf{k}^\perp, \sigma_2, \tau_2, \sigma_1, \tau_1 | \delta U(\lambda) | \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \rangle.
\end{aligned}$$

In this thesis, we focus on incorporating SRG-evolution at the two-body level only.

We will enforce this by using Wick's theorem to truncate all operators of rank $N > 2$.

For the momentum distribution operator, we have four terms to consider:

1. $\hat{\mathbb{I}} \hat{n}^\tau(\mathbf{p}, \sigma) \hat{\mathbb{I}}$
2. $\delta\hat{U}(\lambda) \hat{n}^\tau(\mathbf{p}, \sigma) \hat{\mathbb{I}}$
3. $\hat{\mathbb{I}} \hat{n}^\tau(\mathbf{p}, \sigma) \delta\hat{U}^\dagger(\lambda)$
4. $\delta\hat{U}(\lambda) \hat{n}^\tau(\mathbf{p}, \sigma) \delta\hat{U}^\dagger(\lambda)$

The first term is just the bare momentum distribution operator. To begin evaluating the second term, let us examine the string of creation and annihilation operators it involves:

$$\hat{a}^\dagger(p_1) \hat{a}^\dagger(p_2) \hat{a}(p_4) \hat{a}(p_3) \hat{a}^\dagger(\mathbf{p}, \sigma, \tau) \hat{a}(\mathbf{p}, \sigma, \tau). \quad (\text{C.4})$$

By applying Wick's theorem, we can isolate operators of rank $N < 2$,

$$\begin{aligned} & a^\dagger a^\dagger \overline{aaa}^\dagger a + a^\dagger a^\dagger \overline{aaa}^\dagger a \\ &= (2\pi)^3 \delta(p_3^+ - p^+) \delta(\mathbf{p}_3^\perp - \mathbf{p}^\perp) \delta_{\sigma_3 \sigma} \delta_{\tau_3 \tau} \\ &\quad - (2\pi)^3 \delta(p_4^+ - p^+) \delta(\mathbf{p}_4^\perp - \mathbf{p}^\perp) \delta_{\sigma_4 \sigma} \delta_{\tau_4 \tau} \\ &= \frac{2}{P^+} \left[(2\pi)^3 \delta \left(\alpha' - \frac{2p^+}{P^+} \right) \delta \left(\frac{\alpha'}{2} \mathbf{P}^\perp + \mathbf{k}'^\perp - \mathbf{p}^\perp \right) \delta_{\sigma_3 \sigma} \delta_{\tau_3 \tau} \right. \\ &\quad \left. - (2\pi)^3 \delta \left(2 - \alpha' - \frac{2p^+}{P^+} \right) \delta \left(\frac{2 - \alpha'}{2} \mathbf{P}^\perp - \mathbf{k}'^\perp - \mathbf{p}^\perp \right) \delta_{\sigma_4 \sigma} \delta_{\tau_4 \tau} \right]. \end{aligned} \quad (\text{C.5})$$

First, we focus on the first delta function term in Eq (C.5).

$$\begin{aligned} \left[\delta \hat{U}(\lambda) \hat{n}^\tau(\mathbf{p}, \sigma) \hat{\mathbb{I}} \right]_1 &= \frac{1}{4} \sum_{\substack{\sigma_1 \sigma_2 \tau_1 \tau_2 \\ \sigma_3 \sigma_4 \tau_3 \tau_4}} \int \frac{dP^+ d\mathbf{P}^\perp}{(2\pi)^3} \frac{d\alpha d\mathbf{k}^\perp}{(2\pi)^3 \sqrt{\alpha(2-\alpha)}} \frac{d\alpha' d\mathbf{k}'^\perp}{\sqrt{\alpha'(2-\alpha')}} \\ &\quad \times \langle \alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 | \delta U(\lambda) | \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \rangle \\ &\quad \times \delta \left(\alpha' - \frac{2p^+}{P^+} \right) \delta \left(\frac{\alpha'}{2} \mathbf{P}^\perp + \mathbf{k}'^\perp - \mathbf{p}^\perp \right) \delta_{\sigma_3 \sigma} \delta_{\tau_3 \tau} \\ &\quad \times \hat{a}^\dagger(p_1) \hat{a}^\dagger(p_2) \hat{a}(p_4) \hat{a}(p, \sigma, \tau) \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{4} \sum_{\sigma'_1 \sigma'_2} \sum_{\tau'_1 \tau'_2} \int \frac{dP^+ d\mathbf{P}^\perp}{(2\pi)^3} \frac{d\alpha d\mathbf{k}^\perp}{(2\pi)^3 \sqrt{\alpha(2-\alpha)}} \frac{1}{\sqrt{p^+(P^+ - p^+)}} \frac{P^+}{2} \\
&\quad \times \left\langle \alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 \left| \delta U(\lambda) \right| \frac{2p^+}{P^+}, \mathbf{p}^\perp - \frac{p^+}{P^+} \mathbf{P}^\perp, \sigma, \tau, \sigma', \tau' \right\rangle \\
&\quad \times \hat{a}^\dagger(p_1, \sigma_1, \tau_1) \hat{a}^\dagger(p_2, \sigma_2, \tau_2) \hat{a}(p_4, \sigma', \tau') \hat{a}(p, \sigma, \tau). \tag{C.6}
\end{aligned}$$

Where in the second equality, we used the delta function to integrate over $(\alpha', \mathbf{k}'^\perp)$ and re-labeled $(\sigma_4, \tau_4) \rightarrow (\sigma', \tau')$. In evaluating the second delta function term in Eq. (C.5), using the asymmetry of $\delta \hat{U}(\lambda)$ and re-labeling $(\sigma_3, \tau_3) \rightarrow (\sigma', \tau')$, we find that it is identical to Eq. (C.6). Hence the final result is,

$$\begin{aligned}
\delta \hat{U}(\lambda) \hat{n}^\tau(\mathbf{p}, \sigma) \hat{\mathbb{I}} &= \frac{1}{2} \sum_{\sigma'_1 \sigma'_2} \sum_{\tau'_1 \tau'_2} \int \frac{dP^+ d\mathbf{P}^\perp}{(2\pi)^3} \frac{d\alpha d\mathbf{k}^\perp}{(2\pi)^3 \sqrt{\alpha(2-\alpha)}} \frac{1}{\sqrt{p^+(P^+ - p^+)}} \frac{P^+}{2} \\
&\quad \times \left\langle \alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 \left| \delta U(\lambda) \right| \frac{2p^+}{P^+}, \mathbf{p}^\perp - \frac{p^+}{P^+} \mathbf{P}^\perp, \sigma, \tau, \sigma', \tau' \right\rangle \\
&\quad \times \hat{a}^\dagger(p_1) \hat{a}^\dagger(p_2) \hat{a}(p_4, \sigma', \tau') \hat{a}(p, \sigma, \tau). \tag{C.7}
\end{aligned}$$

The third term is similar

$$\begin{aligned}
\hat{\mathbb{I}} \hat{n}^\tau(\mathbf{p}, \sigma) \delta \hat{U}^\dagger(\lambda) &= \frac{1}{2} \sum_{\sigma'_3 \sigma'_4} \sum_{\tau'_3 \tau'_4} \int \frac{dP^+ d\mathbf{P}^\perp}{(2\pi)^3} \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \sqrt{\alpha'(2-\alpha')}} \frac{1}{\sqrt{p^+(P^+ - p^+)}} \frac{P^+}{2} \\
&\quad \times \left\langle \frac{2p^+}{P^+}, \mathbf{p}^\perp - \frac{p^+}{P^+} \mathbf{P}^\perp, \sigma, \tau, \sigma', \tau' \left| \delta U^\dagger(\lambda) \right| \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \right\rangle \\
&\quad \times \hat{a}^\dagger(p, \sigma, \tau) \hat{a}^\dagger(p_2, \sigma', \tau') \hat{a}(p_4) \hat{a}(p_3). \tag{C.8}
\end{aligned}$$

For the fourth term, looking at the string of creation and annihilation operators it involves, we anticipate four terms at the two-body level:

- $a^\dagger a^\dagger \overbrace{aa \ a^\dagger a \ a^\dagger a}^$
- $a^\dagger a^\dagger \overbrace{aa \ a^\dagger a \ a^\dagger a}^$
- $a^\dagger a^\dagger \overbrace{aa \ a^\dagger a \ a^\dagger a}^$
- $a^\dagger a^\dagger \overbrace{aa \ a^\dagger a \ a^\dagger a}^$

Again, utilizing the asymmetry of the δU matrix element and re-labeling spin and isospin projections we find that we only need to evaluate the first contraction above and multiply the result by four. This gives

$$\begin{aligned}
& \delta \hat{U}(\lambda) \hat{n}^\tau(\mathbf{p}, \sigma) \delta \hat{U}^\dagger(\lambda) = \\
& \frac{1}{4} \sum_{\substack{\sigma_1 \sigma_2 \sigma_3 \\ \sigma_4 \sigma \sigma'}} \sum_{\substack{\tau_1 \tau_2 \tau_3 \\ \tau_4 \tau'}} \int \frac{dP^+ d\mathbf{P}^\perp}{(2\pi)^3} \frac{d\alpha d\mathbf{k}^\perp}{(2\pi)^3 \sqrt{\alpha(2-\alpha)}} \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \sqrt{\alpha'(2-\alpha')}} \frac{1}{p^+(P^+ - p^+)} \left(\frac{P^+}{2}\right)^2 \\
& \quad \times \left\langle \alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 \left| \delta U(\lambda) \right| \frac{2p^+}{P^+}, \mathbf{p}^\perp - \frac{p^+}{P^+} \mathbf{P}^\perp, \sigma, \tau, \sigma', \tau' \right\rangle \\
& \quad \times \left\langle \frac{2p^+}{P^+}, \mathbf{p}^\perp - \frac{p^+}{P^+} \mathbf{P}^\perp, \sigma, \tau, \sigma', \tau' \left| \delta U^\dagger(\lambda) \right| \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \right\rangle \\
& \quad \times \hat{a}^\dagger(p_1) \hat{a}^\dagger(p_2) \hat{a}(p_4) \hat{a}(p_3).
\end{aligned} \tag{C.9}$$

Now we evaluate all four terms with respect to the nuclear ground state

$$|\Psi_A^{MF}\rangle = \prod_{i < F} \hat{a}_i^\dagger |\Omega\rangle, \quad (\text{C.10})$$

where $a_i^\dagger$ creates a nucleon in single-particle orbital indexed by i , $|\Omega\rangle$ is the interacting vacuum state, and F refers to the Fermi surface. We perform a change of basis, re-writing plane-wave creation and annihilation operators in terms of single-particle orbitals

$$\hat{a}(\mathbf{p}, \sigma, \tau) = \sum_i \psi_i^+(\mathbf{p}, \sigma, \tau) \hat{a}_i. \quad (\text{C.11})$$

Where ψ_i^+ is the overlap between the plus-projected single particle light-front wave-functions of nucleus A with eigenvalues i and the LF Dirac spinor.

$$\psi_i^+(\mathbf{p}, \sigma, \tau) = \frac{1}{\sqrt{2p^+}} \bar{u}(\mathbf{p}, \sigma, \tau) \gamma^+ \psi_i(\mathbf{p}) = \sqrt{\frac{2}{p^+}} u^\dagger(\mathbf{p}, \sigma, \tau) \psi_i^+(\mathbf{p}) \quad (\text{C.12})$$

Contractions of a_i with respect to the nuclear ground state are given by,

$$\overline{a_i^\dagger} a_j = \langle \Psi_A^{MF} | \hat{a}_i^\dagger \hat{a}_j | \Psi_A^{MF} \rangle = \delta_{ij} \quad (\text{C.13})$$

for $i, j < F$. Using Eqs. (C.11, C.13) we immediately get the following:

$$\langle \Psi_A^{MF} | \hat{\mathbb{I}} \hat{n}^\tau(\mathbf{p}, \sigma) \hat{\mathbb{I}} | \Psi_A^{MF} \rangle = \sum_{i < F} |\psi_i^+(\mathbf{p}, \sigma, \tau)|^2. \quad (\text{C.14})$$

The second (interference) term gives:

$$\begin{aligned}
& \langle \Psi_A^{MF} | \delta \hat{U}(\lambda) \hat{n}^\tau(\mathbf{p}, \sigma) \hat{\mathbb{I}} | \Psi_A^{MF} \rangle = \\
& \frac{1}{2} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'}} \sum_{\substack{\tau_1 \tau_2 \\ \tau'}} \sum_{ij < F} \int \frac{dP^+ d\mathbf{P}^\perp}{(2\pi)^3} \frac{d\alpha d\mathbf{k}^\perp}{(2\pi)^3 \sqrt{\alpha(2-\alpha)}} \frac{1}{\sqrt{p^+(P^+ - p^+)}} \frac{P^+}{2} \\
& \times \left\langle \alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 \left| \delta U(\lambda) \right| \frac{2p^+}{P^+}, \mathbf{p}^\perp - \frac{p^+}{P^+} \mathbf{P}^\perp, \sigma, \tau, \sigma', \tau' \right\rangle \\
& \times \psi_i^{+\dagger}(p_1) \psi_j^{+\dagger}(p_2) \left[\psi_j^+(\mathbf{p}_4, \sigma', \tau') \psi_i^+(\mathbf{p}, \sigma, \tau) - \psi_i^+(\mathbf{p}_4, \sigma', \tau') \psi_j^+(\mathbf{p}, \sigma, \tau) \right].
\end{aligned} \tag{C.15}$$

The third (conjugate interference) term gives:

$$\begin{aligned}
& \langle \Psi_A^{MF} | \hat{\mathbb{I}} \hat{n}^\tau(\mathbf{p}, \sigma) \delta \hat{U}^\dagger(\lambda) | \Psi_A^{MF} \rangle = \\
& \frac{1}{2} \sum_{\substack{\sigma_3 \sigma_4 \\ \sigma'}} \sum_{\substack{\tau_3 \tau_4 \\ \tau'}} \sum_{ij < F} \int \frac{dP^+ d\mathbf{P}^\perp}{(2\pi)^3} \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \sqrt{\alpha'(2-\alpha')}} \frac{1}{\sqrt{p^+(P^+ - p^+)}} \frac{P^+}{2} \\
& \times \left\langle \frac{2p^+}{P^+}, \mathbf{p}^\perp - \frac{p^+}{P^+} \mathbf{P}^\perp, \sigma, \tau, \sigma', \tau' \left| \delta U^\dagger(\lambda) \right| \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \right\rangle \\
& \times \psi_i^{+\dagger}(\mathbf{p}, \sigma, \tau) \psi_j^{+\dagger}(\mathbf{p}_2, \sigma', \tau') \left[\psi_j^+(p_4) \psi_i^+(p_3) - \psi_i^+(p_4) \psi_j^+(p_3) \right].
\end{aligned} \tag{C.16}$$

The fourth (quadratic) term gives:

$$\begin{aligned}
& \langle \Psi_A^{MF} | \delta \hat{U}(\lambda) \hat{n}^\tau(\mathbf{p}, \sigma) \delta \hat{U}^\dagger(\lambda) | \Psi_A^{MF} \rangle = \\
& \frac{1}{4} \sum_{\substack{\sigma_1 \sigma_2 \sigma_3 \\ \sigma_4 \sigma \sigma'}} \sum_{\substack{\tau_1 \tau_2 \tau_3 \\ \tau_4 \tau'}} \sum_{ij < F} \int \frac{dP^+ d\mathbf{P}^\perp}{(2\pi)^3} \frac{d\alpha d\mathbf{k}^\perp}{(2\pi)^3 \sqrt{\alpha(2-\alpha)}} \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \sqrt{\alpha'(2-\alpha')}} \frac{1}{p^+(P^+ - p^+)} \left(\frac{P^+}{2} \right)^2 \\
& \times \left\langle \alpha, \mathbf{k}^\perp, \sigma_1, \tau_1, \sigma_2, \tau_2 \left| \delta U(\lambda) \right| \frac{2p^+}{P^+}, \mathbf{p}^\perp - \frac{p^+}{P^+} \mathbf{P}^\perp, \sigma, \tau, \sigma', \tau' \right\rangle \\
& \times \left\langle \frac{2p^+}{P^+}, \mathbf{p}^\perp - \frac{p^+}{P^+} \mathbf{P}^\perp, \sigma, \tau, \sigma', \tau' \left| \delta U^\dagger(\lambda) \right| \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \right\rangle \\
& \times \psi_i^{+\dagger}(p_1) \psi_j^{+\dagger}(p_2) \left[\psi_j^+(p_4) \psi_i^+(p_3) - \psi_i^+(p_4) \psi_j^+(p_3) \right].
\end{aligned} \tag{C.17}$$

For calculation of inclusive electron-nucleus scattering nuclear current matrix elements, we will encounter two terms that involve the $\delta\hat{U}(\lambda)$ transformation. We write down the results here.

$$\begin{aligned}
\langle \Psi_i^{MF} | \hat{a}(\mathbf{q}_1, s_1, t_1) \delta\hat{U}^\dagger(\lambda) | \Psi_A^{MF} \rangle &= \langle \Psi_A^{MF} | \hat{a}_i^\dagger \hat{a}(\mathbf{q}_1, s_1, t_1) \delta\hat{U}^\dagger(\lambda) | \Psi_A^{MF} \rangle = \\
\frac{1}{2} \sum_{\sigma_2 \sigma_3 \sigma_4} \sum_{\tau_2 \tau_3 \tau_4} \sum_{j < F} \int \frac{dP^+ d\mathbf{P}^\perp}{(2\pi)^3} \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \sqrt{\alpha'(2-\alpha')}} \frac{1}{\sqrt{q^+(P^+ - q^+)}} \frac{P^+}{2} \\
\times \left\langle \frac{2q_1^+}{P^+}, \mathbf{q}_1^\perp - \frac{q_1^+}{P^+} \mathbf{P}^\perp, s_1, t_1, \sigma_2, \tau_2 \left| \delta U^\dagger(\lambda) \right| \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \right\rangle \\
\times \psi_j^{+\dagger}(p_2) [\psi_j^+(p_4) \psi_i^+(p_3) - \psi_i^+(p_4) \psi_j^+(p_3)].
\end{aligned} \tag{C.18}$$

$$\begin{aligned}
\langle \Psi_{ij}^{MF} | \hat{a}(\mathbf{q}_2, s_2, t_2) \hat{a}(\mathbf{q}_1, s_1, t_1) \delta\hat{U}^\dagger(\lambda) | \Psi_A^{MF} \rangle &= \\
\langle \Psi_A^{MF} | \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}(\mathbf{q}_2, s_2, t_2) \hat{a}(\mathbf{q}_1, s_1, t_1) \delta\hat{U}^\dagger(\lambda) | \Psi_A^{MF} \rangle &= \\
\frac{1}{2} \sum_{\sigma_3 \sigma_4} \sum_{\tau_3 \tau_4} \int \frac{d\alpha' d\mathbf{k}'^\perp}{(2\pi)^3 \sqrt{\alpha'(2-\alpha')}} \frac{1}{\sqrt{q_1^+ q_2^+}} \frac{q_1^+ + q_2^+}{2} \\
\times \left\langle \frac{2q_1^+}{q_1^+ + q_2^+}, \mathbf{q}_1^\perp - \frac{q_1^+}{q_1^+ + q_2^+} (\mathbf{q}_1^\perp + \mathbf{q}_2^\perp), s_1, t_1, s_2, t_2 \left| \delta U^\dagger(\lambda) \right| \alpha', \mathbf{k}'^\perp, \sigma_3, \tau_3, \sigma_4, \tau_4 \right\rangle \\
\times [\psi_j^+(p_4) \psi_i^+(p_3) - \psi_i^+(p_4) \psi_j^+(p_3)].
\end{aligned} \tag{C.19}$$

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