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Omar Farouk Konash

The Effect of Photovoltaic Solar Plant Participation on Different Market Structures in a Deregulated Environment without Financial Incentives in the Saudi Electricity Grid

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Abstract

The Effect of Photovoltaic Solar Plant Participation on Different Market Structures in a De-regulated Environment without Financial Incentives in the Saudi Electricity Grid

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In recent years, many nations have invested in renewable energy in an effort to improve sustainability and to reduce CO2 emissions. At present, most renewable resources are subsidized or paid a feed-in tariff. As renewable resource construction increases, subsidies will become unsustainable. Some European countries have already cut these subsidies. Hence renewables will eventually participate in energy markets the same way as conventional generators.

Saudi Arabia is in the process of introducing an electricity market, as the single Saudi Electricity Company will be divided into several competitive companies. The Saudi government is also investing heavily in renewable energy, especially after the establishment of King Abdullah City for Atomic and Renewable Energy (KACARE), as KACARE is strongly dedicated to studying and deploying new renewable energy projects in Saudi

Arabia. The Saudi national oil company (Aramco) is leading this effort and has already built two solar power sites: a 2 MW-peak PV plant site at King Abdullah University, and a 10 MW-peak PV plant site in Dhahran, Saudi Arabia.

Unlike conventional generators, renewable energy interacts differently with electricity markets because of its stochastic nature. This will be challenging, as renewables are likely to face some penalties for energy mismatch. Having a high level of renewable penetration will be a challenge for the System Operator (SO) to balance the electricity market compared with having only conventional generation. This interaction is not well understood. Therefore one of the major challenges is to understand how existing market structures will accommodate renewables when they can compete in the market, and how future market structures can be designed to incorporate renewables better. Based on KACARE's vision, the estimated solar energy capacity in Saudi Arabia will reach 41 GW by 2032 while the estimated wind energy capacity will reach 9 GW by 2032 [1] compared to a power capacity for the conventional power of 69 GW in 2013 [2]. The reason for the heavy investment in solar energy is that Saudi Arabia experiences a high value of solar irradiance throughout the year. Therefore, this research will focus principally on solar energy participation.

Energy market structures differ in details but can generally fit in two major electricity market pricing schemes: the Marginal Pricing (MP) scheme and the Pay-as-Bid (PAB) scheme. The focus of the work in this dissertation is to compare the current practice of dealing with solar power in both market schemes. Then, the more attractive scheme for solar power will be investigated thoroughly to determine existing and suggested features

that could be incorporated in the Saudi market to better motivate solar power plant construction in Saudi Arabia.

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DEDICATION

To my parents, Faikah Al-idreesi and Farouk Konash, for their affection, love, and prayers. To my amazing wife, Amal Dhaher, for her love, encouragement, and care. Without her support, this work would not have been possible. To my precious daughters, Halah and Demah. Just thinking of them keeps me going.

Chapter 1. INTRODUCTION

Many nations are investing in renewable energy resources in an effort to deal with global warming, reduce CO2 emissions, and improve sustainability. Solar power is a promising renewable energy resource that has the potential to grow rapidly in the upcoming years. In Saudi Arabia, solar power installation is very attractive as the annual solar irradiation is high, see Figure 1.1 and Figure 1.2 [3]. Recently, the USA [4], Spain [5], Germany [6], Saudi Arabia [7], the UK [8], and other countries have commissioned many solar plants in the Megawatt scale. The number of solar power projects will continue to increase in the upcoming years [9]. At present, governments are encouraging investments in solar energy by providing financial incentives such as subsidies, tax credits, and feed-in tariffs (FiT). These incentives can cause market price distortion; for example, some areas can have negative prices as renewables will bid negative prices in order to receive subsidies. FiTs do not distort the market price directly but do cause a reduction of market price by shifting the cost curve. As the number of Solar Power Plants (SPP) increases, subsidies will eventually become unaffordable. In Germany, the Photovoltaic FiT is being reduced every year as seen in Figure 1.3 [10]. Some European countries have already started cutting off renewable energy subsidies [11]. As subsidies disappear, SPPs will need to be treated similarly to conventional generators in the electricity market by bidding (or contracting) within market rules, a discussion of electricity market schemes will be discussed in this chapter in details.

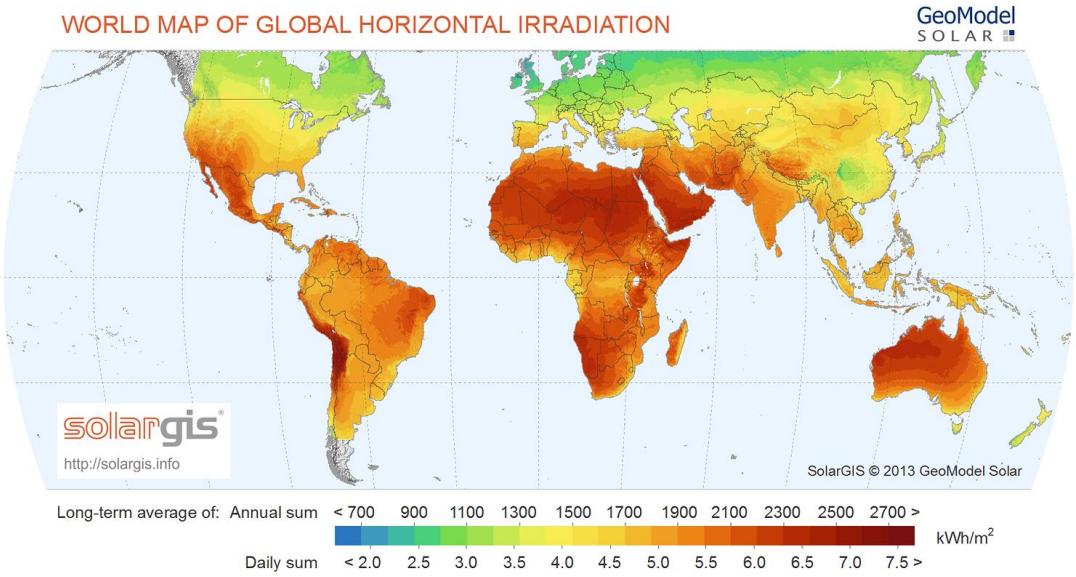

Figure 1.1 World global horizontal irradiation [3]

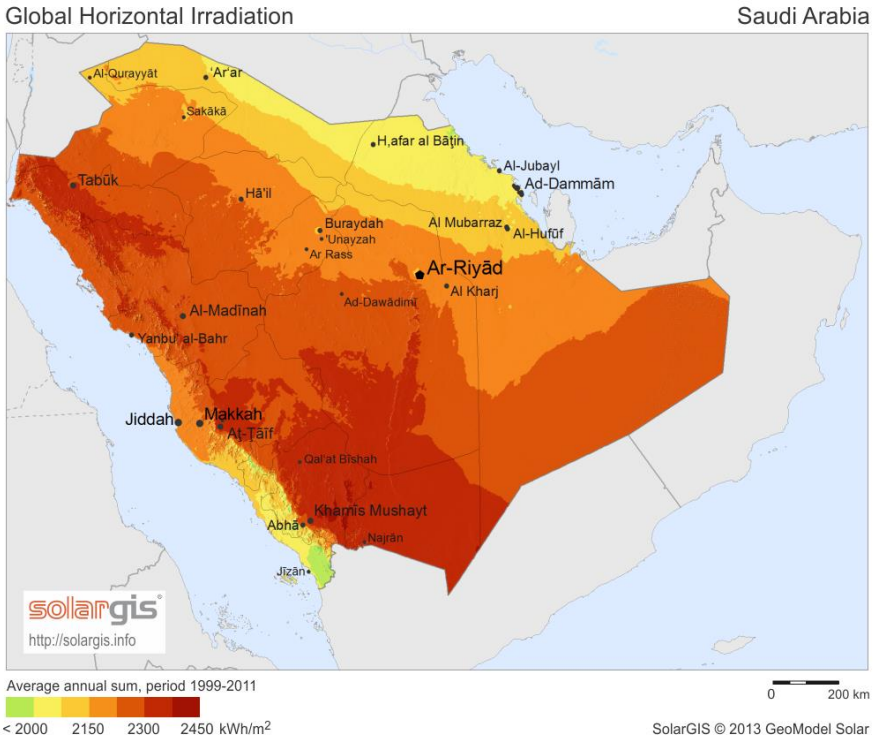

Figure 1.2 Saudi Arabia global horizontal irradiation [3]

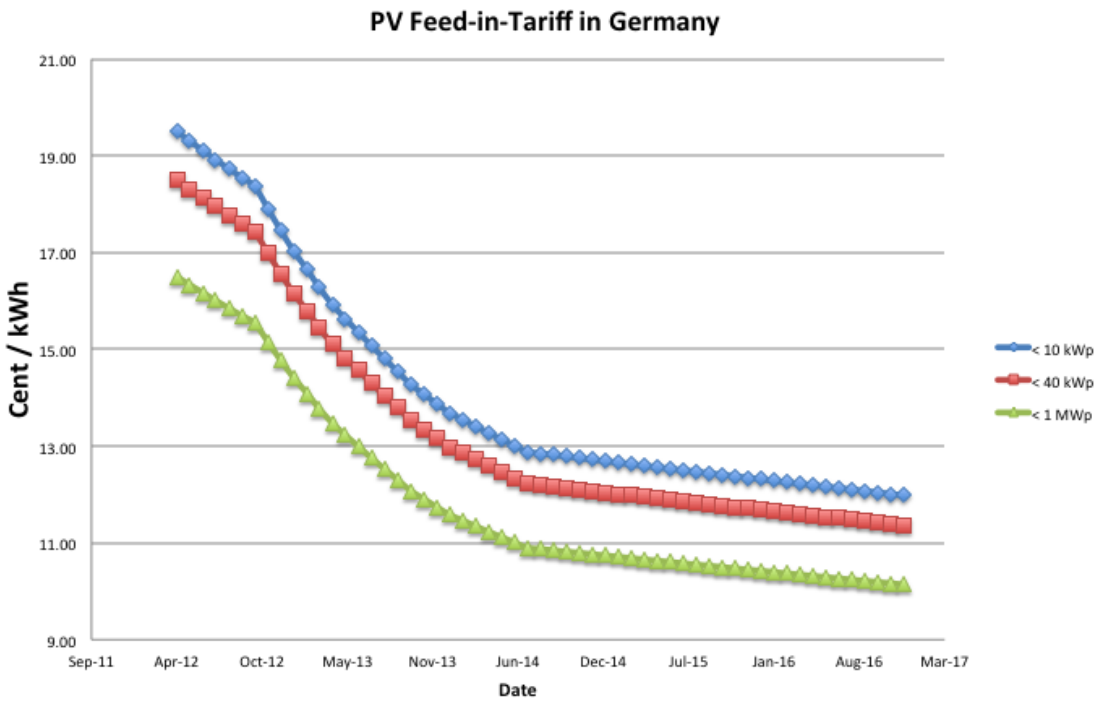

Figure 1.3: Feed-in tariff of PVs in Germany [10]

In energy markets, typical producers must bid (commit themselves) to deliver a specific amount of energy ahead of actual operation during the Day Ahead (DA) market. The DA market closure (last chance to participate in the DA market) differs between market structures. It could be one hour before Real Time (RT) delivery (e.g. UK market) or it could be up to 36 hours before RT delivery (e.g. PJM). Solar power is stochastic in nature due to changes in weather conditions and cloud movement. Because renewables are stochastic, the prediction error is significant. Hence, it is hard for renewables to predict the exact amount of energy that will be produced at any given time, which means that SPPs might not be able to meet their commitment. In markets that impose a penalty on energy mismatch (energy imbalance), this will be an extra burden on SPPs.

The most two common electricity market schemes are Marginal Price (MP) and Pay as Bid (PAB) market schemes. The focus of this research is to compare the current prac-

tice of dealing with solar power in the MP and PAB schemes. This work will investigate solar power interaction with existing market features and suggest a market structure that motivates SPPs to participate in a perfectly competitive market. The main goal of this research is to evaluate the effect of SPP installation in the Saudi Arabian Grid and to suggest the best market features on the upcoming new electricity market by achieving optimal social welfare; part of this evaluation is to explore solar power effects for different penetration levels.

1.1 Deregulation of the Electricity Market in Saudi Arabia

In Saudi Arabia, electricity was initially introduced in the 1930s by some wealthy families and some new companies for internal use. Generators were not connected to a grid, and they were either 50 Hz or 60 Hz based on the country of import (Europe, US). In the 1950s, the government introduced the electricity grid in major cities. Each city produced its own electricity. The regulatory authority was a department under the Ministry of Industry. In 1975, the Ministry of Electricity (MOE) was established, and its first roles were to unify tariffs, construct transmission lines for remote locations and to regulate electricity. In 1981, the MOE mandated all companies in each region to operate under a regional company; the Eastern, Central, Western, and Southern electricity companies, as seen in Figure 1.4. Each company had its own standards, power plants and territory of service.

In the late 1980's the first transmission line interconnection between Eastern and Central region was constructed. In 1998, the government combined the four companies into one company called the Saudi Electrical Company (SEC). The main reason for this decision was to reduce production cost and to plan better for grid expansion. In 2001 the Electricity Regulatory Authority (ERA) was established to encourage investors to construct generators. To motivate Independent Power Plant (IPP) projects to meet the rapid growth of load in the future, the role of the

authority was to set regulations for grid connection especially for independent power producers such as Aramco and Jubail and Yanbu cities independent power (MARAFIQ) and to plan the needed future generation. In 2004 the authority was renamed to Electricity & Cogeneration Regulatory Authority (ECRA) [12]. In around 2007, several debates led by ECRA concluded in favoring deregulation of the electricity market in Saudi Arabia for the purpose of having many companies compete fairly with each other. In 2008, in an effort to lay the foundation for a Saudi Electricity Market, ECRA announced that they would like to unbundle the monopoly of the government owned electricity company SEC into separate companies for Generation, Distribution and Transmission, and they would like the generation sector of SEC to be divided into at least four companies [13]. In 2012 the transmission department was split from SEC, and the new transmission company was named the "National Grid" [14]. In order to have a fair split of the Generation sector of the SEC, ACCENTURE was awarded a contract to prepare the guidelines. In order to have a free electricity market, ECRA decided to create four competitive generation companies by mid 2014 [15].[1] Having this split will lay the foundation for a free electricity market and create opportunities for investors to build new power plants to meet the annual electricity demand increase of 7.4% [16].[2]

[1] So far not implemented

[2] Although each region started with its own standard. The ministry of electricity was able to unify the electrical frequency to 60 Hz, but the voltage level of outlets depends on the city. The current plan is to phase out all 127 volt and all new facilities will be using 220 volt.

Figure 1.4 SEC jurisdiction regions [17]

1.2 Gulf Cooperation Council Interconnection Authority

The availability of the Gulf interconnection and the liberalization of the Saudi Electricity grid have the potential to create great opportunities for new investments in the power sector, especially solar energy. Some Gulf countries are already investing heavily in solar power. The Gulf interconnection will increase the accessibility of different Gulf countries to all electricity markets including the Saudi market. This interconnection will give geographical flexibility of plant site construction.

Since the 1980s the Gulf States (Saudi Arabia, Kuwait, Bahrain, Qatar, United Arab Emirates, and Oman) realized that they would benefit from having an electrical grid interconnection, as these states spread in a wide geographic area, and the demand peak differs from one state to another. These countries agreed to construct this interconnection to lay a foundation for a future healthy electricity market, and to enhance efficiency [18]. There were many challenges in this project; the main one was that the Gulf States had 50 Hz

systems while Saudi Arabia (which would be the backbone of the project) had a 60 Hz system. It was agreed to build the whole transmission line on a 50 Hz base, although this transmission line will mostly be in Saudi Arabia. Saudi Arabia will have one tapping point at the Al-Fadhili station with a HVDC back-to-back connection (See Figure 1.5 and Figure 1.6). The first phase was connecting Kuwait, Saudi Arabia, Bahrain, Qatar and UAE by a 400 kV transmission line. The second phase was to connect UAE and Oman by a 220 kV transmission line. In November 2005, construction started and by 2009 the Gulf interconnection was in operation. The headquarters and control center of the interconnection is in Saudi Arabia [19].

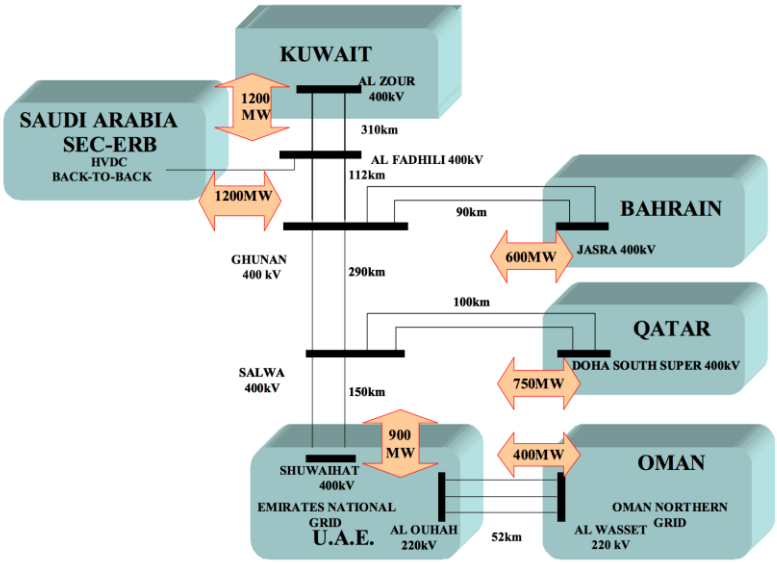

Figure 1.5 GCC interconnection single line diagram [19]

Figure 1.6 GCC interconnection layout [19]

1.3 Current Situation for Solar Power Subsidy

Currently in the USA, renewable plants receive a number of different subsidies. The most well-known subsidy is the federal Solar Investment Tax Credit (ITC), which gives around 30% tax credit on investment cost for residential and commercial facilities. This Tax Credit will be available till 2016 [20]. Also, some states and some cities have their own incentives like State tax credit and loan programs. In addition, there is a federal renewable portfolio standard (RPS) that mandates the increase of renewable energy production in the USA. The RPS differs from state to state, for example Arizona State must reach 15% of renewables by 2025, Washington State must reach 15% of renewables by 2020. California State has one of the most aggressive RPS in the USA as they must reach 33% of renewables by 2020 [21]. It is reported in California that solar power is producing more that 10% of total generation during peak hours [22]. A review in the Database of State Incentives for Renewables & Efficiency (DSIRE) website which is established by the US Department of Energy and North Carolina State University shows that

lots incentives are available in the USA that vary by state and by city [23]. In some markets, the main incentive is exempting renewable power plants from participating in the Day Ahead (DA) market, so they get the Real Time (RT) market price, and they do not face any penalties for prediction error which mean that they are guaranteed to get paid without mismatch penalty [24]. With the increase of solar power penetration, this incentive cannot continue as it will make it harder to balance the system and will distort the market. In Europe, most incentives are based on a feed-in tariff (FiT) where renewable power plants get a fixed guaranteed price [11].

Recently, some European countries have cut off renewable energy subsidies. This trend is expected to continue. Many of these subsidies are not sustainable, and eventually they will diminish, and solar plants will have to participate in the electricity market in the same way as to conventional generators. This research focuses on how solar energy will participate in the electricity market when subsidies are not provided anymore [11].

1.4 MARKET SCHEMES

There are many kinds of electricity market schemes in use around the world. However, they differ in the market details. Most markets can be categorized into one of the following: the Marginal Price (MP) scheme, or the Pay-as-Bid (PAB) scheme [25]. Each scheme has a different way to deal with participants in the market. These different market schemes will have different effects on participating solar plants.

From literature, both schemes can be categorized in having three phases: the Day Ahead (DA) phase, the Congestion Management phase, and the Real Time (RT) phase. In most of the MP scheme, congestion management is done at the same time as the DA phase (i.e. PJM, New York, ERCOT, and CAISO); also congestion management can be handled in a separate congestion management phase (i.e. Spain, Brazil). In the PAB

scheme, the UK market stands as the best example where congestion management is done in a congestion management phase only. However, this research will also propose creating a PAB scheme that will deal with congestion during the DA phase. The detail of each scheme is discussed later in this section.

1.4.1 *Marginal Price (MP)*

In the Marginal Price (MP) Scheme, there is one settlement price when the market reaches a resolution after producers and consumers submit their bids. This price is called the market clearing price (also called settlement price or Marshallian price) where the quantity of supply and demand is equal [26].

If the power transfer in a transmission line within the electricity grid reaches its capacity, then the transmission line is considered to be congested. To relieve congestion the Independent System Operator (ISO) will request generators in the uncongested area to stop increasing power production, while ISO will request generators in the congested areas to produce more power. This means that the expensive generators at congested areas will have to operate. As a result, this will cause these areas to have higher prices.

Dealing with congestion at DA (and RT) uses Locational Marginal Price (LMP). Shahidehpour defines LMP as "the marginal cost of supplying the next increment of electric energy at a specific bus considering the generation marginal cost and the physical aspects of the transmission system" [27]. In literature, there are different names for LMP like shadow price and the bus Lagrange multiplier. To simplify the above definition, it can be said that the bus LMP is the cost of re-dispatching generators in the whole grid when adding an extra 1 MW load to that bus. Generators that produce the last MW in every location of the congestion are called marginal units, as they are the units that set the

LMP at their location in the grid. As a result of congestion, there will be a split in prices in the system, as cheaper generators in some nodes cannot deliver power to the expensive nodes of the system [28].

In order to understand LMPs, it is important to understand LMP components and how to formulate the LMP equation.

Each LMP has three components, as seen equation (1.1) [29] (see Appendix A for detailed discussion):

a) System marginal cost, which is typically the marginal cost (*MC*) at the reference bus.
b) System power loss cost from reference bus, which is due to power loss in the transmission line.
c) Transmission congestion cost, which is the cost of re-dispatching due to reaching the transmission line capacity.

$$LMP = Marginal\ Price\ (MP) + Transmission\ Congestion\ Cost + Marginal\ Cost\ of\ Losses \tag{1.1}$$

Generators will achieve maximum profit by bidding their Incremental Cost (IC). The proof is discussed in details in Appendix A. Transmission Congestion Cost is the cost of re-dispatching marginal generators to relieve congestion from a transmission line as discussed earlier. Marginal Cost of Losses is the cost associated with transmission line losses, which is usually small, so they are not considered in this work [29] [30].

The MP scheme can have one of two structures. The **first structure** has two energy markets: the Day Ahead (DA) market and the Real Time (RT) market. The DA market schedules the expected amount of production, while the RT market deals with the imbalance energy between the scheduled energy amount and the actual delivery amount. These

markets clear in almost the same way. In each market supply bids (generation increases or consumption decreases) and demand bids (generation decreases or consumption increases) are settled to a LMP at each node on a determined settlement time that differs by market (i.e. some are hourly, some every half an hour, and some every 15 minutes) [31]. In the DA market settlement obligates participants to produce a specific amount of energy at the LMP. In the RT market, deviations from DA settlement are settled against appropriate bids which will pay based on the RT LMP value [30]. Typically demand is treated as a price taker as it does not set the price. The MP scheme is used in PJM, ERCOT, CAISO, NYISO, and in many countries in Europe. In this scheme, typically congestion is dealt with during both DA and RT.

The **second structure** of MP schemes consists of three phases: a DA market in which the market settles to a market price without considering congestion, a congestion Adjustment Market (AM) (also called a counter-trading market, or congestion uplift market), then a RT market that deals only with supply-demand deviations [32]. Examples of the second structure are Spain and Brazil [33].

The difference between these MP structures is that the **first** one deals with congestion in a single settlement at the time of market clearing price in which the LMPs are created, while the **second** structure will not deal with congestion at DA and will have a uniform market price, then the ISO will deal with congestion after the DA market is closed and before RT starts. In this research, both congestion management schemes are studied.

1.4.2 *Pay-as-Bid (PAB)*

In the PAB scheme, participants pay or get paid only their bid price [31]. It is believed that PAB limits producers from exercising market power [34] [35] [36]. Currently, markets that use this scheme include the UK market and Iranian market. The UK market is the most well known market and has been using the PAB scheme since 2001.

Similar to the MP scheme, PAB scheme can have DA, AM and RT markets. However, the term "gate closure" is used more often in this scheme, where gate closure is defined as "the last chance to bid or adjust a bid". Although the period before the gate closure takes place just a few hours before the actual delivery, it can be considered similar to the DA market in the MP scheme. The period between the gate closure and the actual energy delivery is the adjustment market (AM), where the ISO adjusts the scheduled generators to relieve any possible transmission system congestion. Finally, the actual delivery is similar to the RT market in the MP scheme with an adjustment market. (The existing UK model is similar to the second MP structure, in which the gate closure marks the end of the DA market).

Although there are different papers that support each scheme, one of the main claimed advantages of the PAB scheme is its ability to limit the effect of exercising market power by suppliers [35]. For example, during the DA market in a MP scheme, if a marginal unit exercises market power then this unit will set a high market price and all consumers will pay that price. Unlike the MP scheme, in a PAB scheme if a unit exercises market power, then the effect is limited as only the consumers that accepted that high bid would pay the high price. One disadvantage of the PAB scheme is that it encourages suppliers to over-bid in an attempted to maximize revenues as every supplier tries to exercise

market power by predicting the market price. Hence, some low efficiency generators might be dispatched while some high efficiency generators with a high bid price might not be dispatched. In general, in the MP scheme suppliers are motivated to bid low prices as they are guaranteed the market price, while in PAB suppliers are motivated to bid as high as they can.

In the DA market, energy is traded for a settlement period in the electricity market (half an hour in UK case). Energy is traded for the settlement period until gate closure (in the UK gate closure is currently set to an hour before the settlement period). Also, energy is traded as blocks of an electricity amount (quantity) and a price in the electricity market. When another market participant accepts a bid, only the bid price is paid, as there is no market clearing price.

At or before gate closure, participants must submit their contracted amount and price to the Independent System Operator (ISO). In the UK market, this is called the Final Physical Notification (FPN). They may also submit an **adjustment bid**, also called a supply-demand pair-bid (SDPB), which gives an adjustment price if the ISO asks the bidder to deviate from FPN volume.

Between gate closure and Real Time (RT) market, the Adjustment Market (AM) begins. During the **AM**, the ISO runs calculations to ensure that the transmission system constraints are met. If transmission line **congestion** is determined, the ISO will request producers that submitted an adjustment bid (SDPB) to relieve congestion by re-dispatching in the most economical way. The cost of relieving congestion is distributed among consumers based on their consumption (prorated) [37] [38].

In the PAB scheme, during **RT** the Independent System Operator (ISO) role is to balance power in case of any power mismatch between supply and demand. The ISO will request some generators to economically re-dispatch based on their submitted adjustment bid (SDPB) in order to achieve the power balance. Participants that deviate from their DA amount will pay the RT price (balancing price). The RT price depends on the cost needed to balance the system, which will depend on the accepted adjustment bid. The RT price will be the weighted average price of used adjustment bids. Suppliers and consumers may not intentionally deviate from FPN unless instructed by the ISO.

Current UK scheme deals with congestion at the AM and RT only. This research will introduce a **second structure** that deals with congestion at the DA market. ISO collects all bids till gate closure time then an OPF will be run to reach a market settlement solution in which no transmission line limits are hit, and all accepted bids would be paid based on their bid. The benefit is to deal with congestion ahead of time to avoid any dispatch cost in an AM.

Since UK market is the leading model for PAB, it is worth understanding how they specifically handle RT market. The UK RT market has dual pricing. A "main price" and a "reverse price" are calculated. This dual pricing mechanism is employed to encourage suppliers not to deviate from their contracted amount and made in a way that penalizes those who are contributing to system mismatch. The UK RT **main price** calculation depends on using accepted qualified adjustment bids (SDPB) that were submitted before gate closure and were used during the RT (which was discussed in a previous paragraph). The **reverse price** is the weighted average price of qualified transactions done at DA. The qualified adjustment bids must exceed a MWh threshold (in the UK market it must be

above 1 MWh). It uses only the most expensive MWh to calculate the price of accepted adjustment bids (i.e. in the UK market it is set for the most expensive 500 MWh). It can be observed that the reason for using "qualified bids" is to avoid price spikes. Also since only the most expensive adjustment bids (SDPB) are used in calculating the main price, the price will be near but not equal the to RT settlement price in MP scheme. The UK RT market has many stages to calculate the main price that makes sure price spikes will not affect the main price. For more details read [37]. The RT reverse price in PAB depends on the average price of qualified accepted bids at DA. The qualified bids are bids that are above a MWh threshold (in the UK market it is above 25 MWh), not exceeding a maximum duration (i.e. in the UK market the bid duration shouldn't exceed 4 hours), and not far away from away from gate closure (in the UK market bids between 0-20 hours before gate closure are used in price average calculation) [31] [39].

If the RT market was short (generation is less than consumption), the main price will be used for generators that are under-generating, and the reverse price will be used for generators that are over-generating. If the market was **long** (generation is more than consumption), then the main price will be used for generators that are over-generating, and the reverse price will be used for generators that are under-generating [37]. Having dual pricing that depends on whether the system was short or long is to penalize market participants that are contributing to the system mismatch.

It can be observed that since the **main price** depends only on the prices of the highest accepted adjustment bids in RT, it should have a value that is close to MP RT price. Also, the **reverse price** depends on the highest accepted bids in DA (bids that are close to gate closure), which should have a value that is close to MP DA price (quasi MP DA). This

means that both prices are expected to be close to each other. It seems like the goal is to heavily penalize those whose deviation is contributing to the power imbalance. That is why they pay the main price, while those whose deviation is helping in minimizing the imbalance will pay the reverse price (which depends on DA price) [38].

Participants who intentionally deviate from their FPN will face a special penalty by the ISO as such a practice can cause system instability. In general renewable energy deviation is not considered intentional.

For this work, the PAB scheme will use a simplified UK market scheme that retains its major features. Unlike MP where the revenue is simply the settlement price multiplied by demand (see Figure 1.7), the PAB revenue will be the area under the bidding function as seen in Figure 1.8. All bids will be considered qualified, as [39] showed that only 11 bids in a full year did not meet the MWh threshold. A perfect competition market will be assumed, i.e. no generator will try to exercise market power.

The settlement period will be set to one hour in order to have a better comparison with the MP scheme. It is expected that the price in the PAB scheme will increase as time approaches gate closure as available power supply gets less and less.

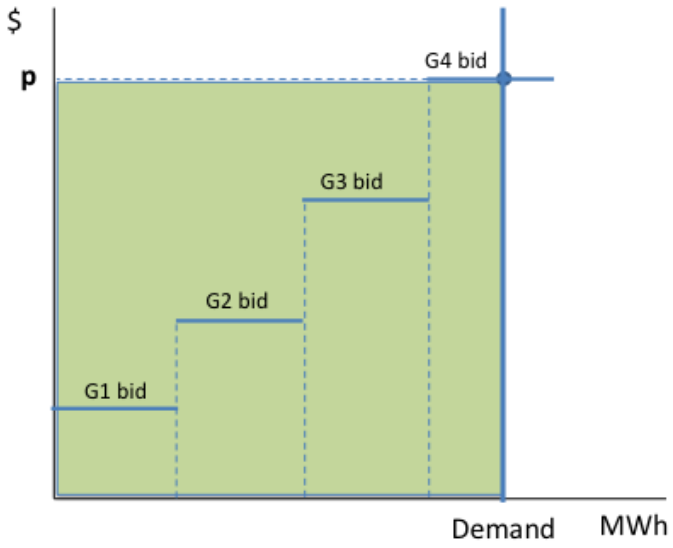

Figure 1.7 The revenue of producers in MP scheme

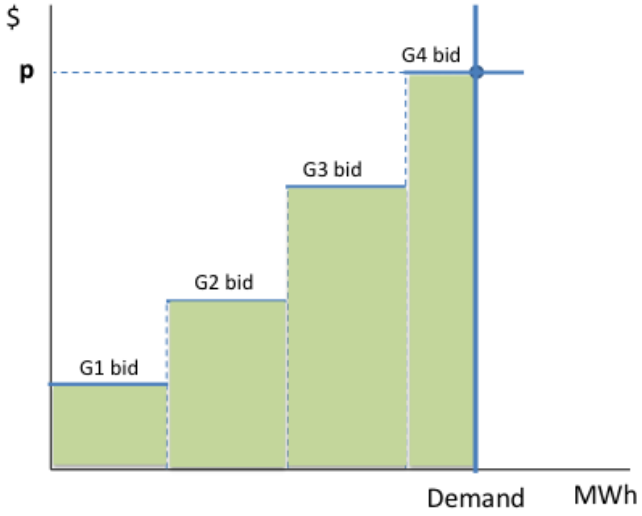

Figure 1.8 The revenue of producers in PAB scheme

1.4.3 *Hybrid scheme between MP and PAB*

There are markets that combine both the MP and PAB schemes. An example of this market is the Nordic market. It has two markets: ELSPOT represents MP, and ELBAS represents PAB. ELSPOT is the main market, which is also the DA market where "gate closure" is in the day before the actual delivery. ELBAS is open till one hour before actual

delivery [40]. There are many restrictions in ELBAS. Trading is only done at the same local transmission system operator until ELSPOT closes and publishes the available capacity of transmission lines. Then ELBAS (bilateral settlement) can accept trades outside the bidding zone until it hits the transmission capacity [41]. Thus ELBAS is a Transmission Line constrained PAB. During RT, the market price is based on MP method. Since there can be many versions of hybrid schemes, this will not be studied in this research.

1.5 Prior Work on Solar Plant Interaction with Electricity Markets

Several papers have discussed the bidding of solar power in the electricity markets. Some papers consider having storage units; others consider only MP scheme. Some of the papers have discussed bidding strategies for renewable power based on historical data without explicitly considering market impacts.

In [42] the authors discuss the new regulation in Spain where renewable power plants have the option either to get a feed-in tariff (no risk) or to bid in the DA market where they might get penalized for any energy mismatch during the RT market, but they will get a premium above the market price as an incentive. Currently, only the Sevillian solar thermal power plant participates in the Spanish electricity market. This plant uses Concentrated Solar Power (CSP) technology, which stores solar thermal energy to be converted to electricity. Thus it is essentially controllable rather than stochastic in the RT market time frame. An assumption in this paper was having the energy mismatch penalty fixed at 10% of market price, while the penalty value in the actual market varies. Another assumption in this paper is that the solar plant will not bid higher than the marginal price which means that the plant always will get the market price, as this is the main feature of having a MP scheme. This paper did not consider a PAB scheme.

This research will consider PVs without storage and will have realistic penalties (not fixed). The importance of this paper that it showed a renewable energy plant participating in the electricity market, even though it is a CSP (which is not considered in this research). Having it participating in the market may motivate other kinds of renewables to bid in the market.

In [43] the authors studied the effect of PV plants on the bidding strategy of conventional generators. They did not explore the bidding strategy of the PV plants. They explored the effect of having PV plants supplying 10% of total demand. They suggested that solar power plants simply bid (price wise) the cheapest generator's marginal cost because in a MP scheme they will get the market price. This is a good approach in the MP scheme, but it cannot be applied to a PAB scheme. The paper did consider the effect of congestion combined with PV plants on the market but it did not consider the energy imbalance issue between DA and RT, and thus did not consider PV plant penalties. The paper showed that nodes that have PVs will have a cheaper price; also that generators will take advantage and exercise market power once solar power is reduced or/and congestion appears. The importance of this paper is that it tried tackling having PVs bidding in a market although it is clear that this is not a market with perfect competition.

In [44] the authors' study is on a microgrid system that consists of 9 to 15 houses. They compared the effect of rooftop PV on a MP and PAB scheme. This paper considers the role of PV with battery storage in the bidding strategy. It considers only the effect of different game theory strategies on bidding between two generators in the microgrid. One advantage is that the paper does not consider subsidies in the market. A major outcome of the paper is that it showed that the profits under both schemes are not the same. It did not favor one scheme over another. This research will consider a large system that assumes market power will be limited.

In [45] the authors cover the issue of having renewable plants participating in the electricity market. They cover how to minimize the mismatch between the DA and RT market by using a binomial tree approach to estimate production. This paper mentioned that there are three ways that markets deal with renewables: full incentive (negative load), partial incentive (paid market clearing price in addition to subsidy) and no incentive for renewable power plants. This paper considered the third approach, which is in line with this research. However, it only considers the MP scheme. The paper proposed a way to avoid having storage by creating a secondary market where renewable plants can hedge (have an advance agreement) with typical thermal plants so that they can compensate for any energy mismatch. Hedged prices might be better than paying the mismatch penalty to the ISO but on the other hand, hedging also has its costs. This research will deal directly with energy mismatch. Hedging to balance the power mismatch will not be considered as an approach to deal with a mismatch, as hedging is like paying the penalty in advance, it will change money flow but does not change the power flow. Besides this research will compare different market schemes (i.e. MP, and PAB).

The authors in [46] studied the effect of PVs (rooftops and plants) on the Amsterdam Power Exchange (APE) which uses the MP scheme. They focused on the energy mismatch penalty. A Monte Carlo method was used to estimate the penalty cost from historical data. The first part of the paper assumed there was no energy imbalance. The second part studied energy imbalance, using previous year delivered energy and imbalance cost to produce a Gaussian relationship between imbalance amount and cost. To study the effect of energy imbalance on the bidding strategy, it simply compared varying the energy mismatch between 50% to 100% of PV capacity. The main outcome of this paper that it is better for PV cells with high kWp (Kilo Watt Peak) to participate in the APE market while PV with low kWp should get a feed-in tariff. Also,

this paper did not use any kind of solar prediction, as its focus was to see the effect of energy mismatch which was changed manually. This research will use a solar forecast for energy mismatch and will examine the effects on MP and PAB schemes.

In [47] the authors studied optimal PV bidding in a MP market. This bidding strategy (amount) of PV depends on historical and weather forecast data. This paper assumes a constant penalty factor for energy mismatch, which depends on historical penalty cost information in addition to categorizing days in the year profile with forecasted temperature and irradiance all together in one formula that maximize the expected profit. Basically, the historical data is normalized to an "ideal" clear-sky profile. The approach that the authors found to be more profitable was combining this method with a neural network classifier. The simulation was run on data from an Italian PV plant and showed higher profit using this method. The paper's bidding strategy proposes only the quantity of energy, not a price. This method will not be used in this research, as in order to get an accurate model it is important for suppliers to bid both price and quantity especially in a PAB scheme.

1.6 Difference between Solar and Wind Energy

Solar and wind energy have some common features. Both of them are considered renewable energy, both of them are stochastic, and some energy forecast methods work for both of them. Since this research is focusing on solar power, it is important to understand the difference between solar power and wind power. There are two kinds of differences; there is a physical difference, and there is a market policy difference between them.

1.6.1 *Physical*

Physically, both solar and wind power are stochastic, but wind power can peak at any time day or night, while solar power peaks during the day [48]. Wind power uncertainty is more related to air pressure change and sudden wind speed change. Although solar power it is only during daytime, it is stochastic as weather condition, and cloud movement highly affects its production. Also, high temperature and humidity reduces PV panel's efficiency. Dust accumulation on PV panels can cause no power production at all. In terms of predictability, many of the same prediction methods are used for both solar and wind power.

Prediction methods are valued for their accuracy, which is the error between actual measurement and predicted value. In order to evaluate the accuracy of prediction, there are two popular benchmarking methods that are used in literature, they are:

a) Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{1}{n}\sum_{1}^{n}(x_{predicted} - x_{actual})^2} \tag{1.2}$$

b) Mean Absolute Error (MAE)

$$MAE = \frac{1}{n}\sum_{1}^{n}\left|x_{predicted} - x_{actual}\right| \tag{1.3}$$

In literature, it can be observed that sometimes the RMSE value between prediction forecast and actual power production of renewable plants can differ by a significant amount. This difference is because some are forecasting the power produced in a single plant, and some are aggregating the forecast for a "control area" which combines forecasting different plants together [49]. It is expected that RMSE value for a control area

will be less than a single plant as having a large deviation in the entire area suddenly is rare. For example, if a cloud passes on top of PV plant then this plant's generated power will decrease, but the same cloud will not be over other plants at the same time, so the total RMSE of all these plants together will be less than this single plant. In this research the focus will be on a single plant production forecast as each plant is expected to participate in the electricity market individually. One important factor is the effects on the RMSE value of the forecast horizon time. As the forecast horizon gets further away, the RMSE gets higher.

Table 1.1 and Table 1.2 have the average RMSE values for solar and wind forecasting from different papers. It can be observed that in wind forecasting the RMSE value between hour ahead (HA) and DA usually is not large, while for solar power it varies by location. Also, it can be observed that the RMSE value for DA in solar is higher than 30% in most of the papers while for the wind it is on average around 20%. Although it might be thought that solar would be more predictable, these tables showed that wind power is the one more predictable. That is another good reason that this research will focus solar power.

Table 1.1 Solar RMSE value from different papers

Paper	Hour-Ahead	Day Ahead
[50] In humid semi-tropical environment	25%	38%
[50] In desert location	12%	11% it is lower than HA because it considers nighttime
[51] SURFRAD sites	N/A	33%
[52] US	28%	35%
[53] Germany	10.8%	N/A
[54] Germany	12%	12.8%
[55] Central Europe	N/A	40% - 60%
[55] Spain	N/A	20% - 35%
[56] Switzerland	N/A	50%
[57] US	N/A	30%
[58] Germany	N/A	40% - 60%
[58] Switzerland	N/A	40% - 50%
[58] Austria	N/A	45% - 63%
[58] Spain	N/A	20% - 40%
[50] Central Europe	22%	38%

Table 1.2 Wind RMSE value from different papers

Paper	Hour-Ahead	Day Ahead
[59] in Germany	N/A	10% - 20%
[49] in Germany	N/A	10% - 15%
[60] in Germany	7%	10% - 20%
[61] in Finland	14.5%	16.5%
[62] in Denmark (old farm)	15%	17%
[63] in Tasmania, Australia	10%	25%
[62] in Japan	15%	N/A
[62] in western Denmark	18.5%	N/A
[50] in California	14%	17%
[50] in Alberta	6% for HA and 20% for 8 hours ahead	20% - 30%

Since this work focuses on the Saudi Arabia electricity market, it is important to consider the condition of the weather in Saudi Arabia. The weather in Saudi Arabia is harsh, especially during summer. With high humidity, water vapor accumulates on the PV cells, in addition to lots of dust in the air. All can mix and form a layer that sticks on the PV cell, causing a reduction in energy production [50]. From a field visit to an experimental field in the Half Moon bay in Dhahran, Saudi Arabia, PV cells have to be cleaned regularly, cells that are left without cleaning will not produce any power [64]. The RMSE values observed in Table 1.1 are mostly in non-tropical environments. It is expected that PVs in Saudi Arabia will have higher RMSE values as dust accumulation is also unpredicted.

1.6.2 *Market Policies Related to Solar and Wind Energy*

Each market has its own policy to deal with renewables. Some treat solar and wind energy the same way and others treat them differently. It can be observed that most markets

have almost similar policies regarding dealing with Solar and Wind energy. These small differences will be discussed in this section and can easily be identified in Table 1.3.

In **PJM**, solar plants are currently treated as a negative load. Simply they do not have to participate in the DA market as they get paid the real time market price for production [65]. In PJM every wind farm has to participate in DA with its "capacity resource" based on PJM's capacity factor [24] [66]. The capacity factor is "the ratio of the total energy generated by a generating unit for a specified period to the maximum possible energy it could have generated if operated at the maximum capacity rating for the same specified period, expressed as a percent". Note that this is different than efficiency. PJM's wind capacity factor is currently set at 13%, but if a plant's wind data showed that it could not meet this value at a certain time then it is not obligated to participate in the DA market for that specific time [67] [68].

In the California ISO (**CAISO**), it is voluntary for both wind and solar plants to participate in the DA market, but they are penalized if they deviate in RT. If they do not participate in the market, then they will just simply get RT market price [48] [24]. The mechanism to participate in the market (either through an aggregator or through a scheduling coordinator) will depend on the capacity of the renewable plant [69].

In the New York ISO (**NYISO**), both wind and solar power plants are exempted from imbalance penalties between DA and RT up to 3,300 MWh/ plant [66].

In **ERCOT**, the wind farms and solar plants do not have to participate in the DA market [24] [70]. If wind generates 10% more than its DA obligation, it will be charged a penalty based on RT price [66]. Renewables in ERCOT get a state credit (including solar and wind) [71].

In the **UK**, a Feed-in-Tariff (FiT) is used for both wind and solar plants. They do not participate in the electricity market and simply receive a fixed price for every MWh [72] [73]. In **Denmark,** offshore wind farms get a fixed payment (FiT), onshore wind farms and solar plants can bid in the market and will get the market price plus a price premium [74] [75]. In **Spain,** all renewables have the option either to get a FiT or market price with premium. FiT rate depends on the renewable type [42]. In **Germany**, solar power plants get a FiT but the rate depends on whether the PV was rooftop or not, and it depends on the MW-p (Mega Watt Peak) of the PV [10]. For wind farms, the onshore FiT rate is different from offshore FiT rate [76].

Table 1.3 Market policy wind vs solar

	Wind Power	**Solar Power**
PJM	Optional to participate in DA market, but mandatory if it was "Capacity Resource"	Negative load
CAISO	Optional to participate in DA, but will be penalized for deviation	Same as wind
NYISO	Optional to participate in DA, no penalties up to 3,300 MWh	Same as wind
ERCOT	Optional to participate in DA plus state renewable energy credit	Same plus state requirement to meet high percentage of non-wind energy
UK	Feed in Tariff	Same as wind
Denmark	Off shore farms gets Feed in Tariff, On shore can bid in the market and will get market price plus premium.	Can bid in the market and will market price plus premium
Spain	Have the option either FiT or Market price plus premium	Same as wind
Germany	FiT, rate of onshore is different from offshore	FiT, rate depends on capacity and if it was roof top or not

From Table 1.3, it is clear that there are many similarities in policy between the wind and solar power. There are still few differences for example PJM treats solar as a negative load. Some FiT rates are different depending on which renewable technology is used. Also, most of US markets make it optional for renewables to participate, but as discussed in an earlier section, this is considered as an incentive that will be reduced or removed in the near future. Currently, subsidies are causing some markets to have negative prices as some renewables are bidding negative price in order to get the subsidy. Such a practice distorts the market and sends wrong signals.

1.7 SOLAR PREDICTION

The introduction of Solar Power Plants (SPPs) will have different financial effect in the MP and the PAB scheme. These effects are a function of market price, which depends on the bids of participants. Although the assumption of perfect competition means that a single participant cannot affect market price, the combined bidding of all participants does affect the market price. The amount of produced power by renewables is needed in the bidding. Therefore power prediction for renewables is important, as any mismatch between predicted energy during the DA and actual energy delivered translates into a penalty. Therefore it is important to explore available prediction methods. The following are some popular methods in solar forecasting:

1.7.1 *Persistence forecast*

This method assumes that the predicted value will be the same as previous measurement. This method gives a good estimate for short-term solar power prediction but has a high error in cloudy days. In areas that have clear sky condition, persistence forecasting might

outperform some new forecasting methods [50]. This good performance is for multi-plant forecasting as the variance get less. Currently, there are few markets that use this method for renewable energy short-term forecast production like NYISO [66], CAISO [77], and Hawaiian Electric Company [78]. In Dammam City in Saudi Arabia, between the months May to October, the average number of partial cloudy days is 3 per month, and overcast days are around 0.5 days. For the months between November and April the number of partial cloudy days are 8 per month and overcast days 3 per month [79]. Having such clear sky profile makes using persistence forecasting in Saudi Arabia attractive. The error of persistence forecasting might be higher than other more advanced forecasting methods however its simplicity makes it attractive [50] [55].

1.7.2 *Total Sky Imagery*

This method forecasts solar irradiance based on first getting a sky image of the area using a special device like the "Whole Sky Imager (WSI)". Then the captured image is analyzed to identify clouds. After that, it approximates cloud movement. Finally, based on historical solar irradiance and collected meteorological data a power prediction is generated. This method does not consider new cloud formation or dissipation. Also, it does not capture multiple cloud layers. The time horizon for this forecast is between 5 to 20 minutes [80].

1.7.3 *Satellite Cloud Motion Vector Approach*

This approach in concept is similar to the sky image method, but it uses satellite sensors instead. The main improvement is that the satellite image covers a larger area with advanced infrared image sensors. Using this technology can help in calculating accurate

cloud speed. Classical methods had poor morning prediction due to inability to collect data during the night. New models are equipped with infrared channels that function day and night. This method can predict solar irradiance up to 6 hours ahead of time [80]. [81] showed that implementing infra-red channels reduce the RMSE value by 20%.

1.7.4 *Weather-Conditioned Selective Moving Average (WCSMA) and Exponential Weighted Moving Average (EWMA)*

Both EWMA and WCSMA are typical methods used for solar irradiance prediction. Basically, EWMA uses last day data at the same time, and the previous interval with a weight factor to predict next hour production,

$$E'(i) = \propto \cdot E'(i-1) + (1-\propto) \cdot E(i) \tag{1.4}$$

Where *E(i)* represents the measurement at the end of the previous interval. *E'(i)* is the average data of previous days. $\propto$ is the weighted average. This can be problematic when sunny and cloudy days alternate as part of the prediction depends on previous day data.

The new method in this paper (WCSMA) uses sunny data for Sunny day prediction, while it will use cloudy data for cloudy day prediction. Also, it will use the average of a sliding window method.

$$E(d, n+1) = \propto \cdot E(d, n) + (1-\propto) \cdot H(d, n+1) \tag{1.5}$$

Where

$\propto$ is the weighted factor.

H(d, n+1) is the mean of past *D* days at time *n+1* which depends on the day condition (sunny, cloudy, or mix).

By using this method, the average error has dropped from 75% (EWMA) to 15% (WCSMA) in the case of alternating between sunny and cloudy days [82].

1.7.5 *Numerical Weather Prediction (NWP) / Global Meteorological Forecast*

This method is well known for its ability to forecast solar irradiance for long time horizons. It uses more than 50 parameters such as temperature, air pressure, satellite images, and more. Results of the forecast are provided in a three-dimension grid, and calculations are done between two to four times per day. Globally there are a few centers that do this forecast like the National Weather Services (NWS) in the USA, European Center for Medium Range Weather Forecast (ECMWF) in the UK, and DWDGME in Germany. These centers can predict solar irradiance up to 360 hours ahead of time [58]. [50] showed that this method is considered the best method available for DA forecasting for solar irradiance as it compared different methods in different location and NWP was shown to have the lowest RMSE value. In [53] the authors showed that in solar irradiance forecast with an extra step on ECMWF model an RMSE value of 4.6% could be achieved for DA forecast and 3.9% for intra-day forecast (this includes night time). In [57] Environment Canada's Global Environmental Multiscale (GEM) was used for solar irradiance prediction. The authors added a Kalman filter to the model. They were able to achieve a RMSE range between 6.4% to 9.2% which is around 15% lower than typical GEM value [66] [60].

1.7.6 *Neural Networks*

Neural Networks (NN) is one of the popular intelligent systems methods. It is inspired by the nerve system. There are many versions of NN, but all of them have the same concept. Like any intelligent system, it needs training before being able to give an accurate output. There could be many inputs and outputs. An NN depends on many inputs, like solar radiation, temperature, humidity, and wind velocity. This information goes to different hidden layers of "neurons" that need to be trained based on a large dataset. Depending on the

program, many inputs or all inputs can be connected to one or many neurons. Then a weight (*w*) value will evolve by training. The training data are select randomly from sample data. In solar power forecasting case, there will be one output for predicted solar power value, see Figure 1.9. An important issue with NN training is that over training can lead to "memorization" of the results which means that we will get the correct result only if we have the same input and any small difference in the input values will lead to wrong output. That is the reason that after training is done it has to be verified with none trained data.

Many papers discussed different methods of using NN in solar power prediction. In general, they showed that as the number of inputs increases (e.g. sunshine hours, air temperature, humidity, latitude, longitude, and altitude), the solar power prediction error reduces [83] [84] [85] [86].

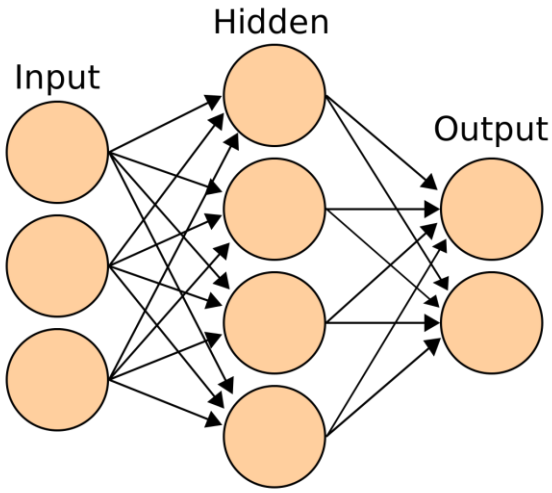

Figure 1.9 Neural network solar power forecasting model

1.8 SUMMARY

Renewable energy subsidies and incentives are not sustainable. Many nations have already reduced these incentives. In some nations, renewables are allowed to be part of the electricity market. If they participate, they have the option to either receive the market price plus a premium or they receive market price plus tax credit. In other markets, renewables are not participating in the market and just simply getting a FiT. The reason for this trend is to motivate the construction of new renewables to have a "green" and more sustainable energy in the future. These incentives cannot be sustained for long and especially that some countries have already started reducing these incentives. Renewables will eventually have to participate in a market setting in the same way as conventional generators.

The Marginal Price (MP), Pay-as-Bid (PAB), and Hybrid market schemes were discussed. Each scheme has its own unique features. In this research, the features that motivate solar power to participate in the electricity market without incentives are discussed and analyzed. This discussion lays the groundwork for analyzing and determining how different market schemes can deal with energy imbalance in the presence of high penetrations of stochastic renewables such as solar.

A perfect competition will be assumed which means that a single participant cannot affect market price, whoever the combined bidding of all participants does affect the market price. Therefore forecasting the amount produced by SPPs is needed. This research uses persistence forecasting as an initial choice due to its wide use in several electricity markets. This research is not aimed at finding a new forecasting method. Since all available information is time series data, advanced methods that use meteorology based fore-

casting cannot be used. Other non-meteorology based forecasting has an error that is not so big compared to persistence forecasting. Since this work is comparing different electricity market schemes, the same forecast error would be applied to all of them.

The focus of this research is to compare the current practice of dealing with solar power in Marginal Price (MP) and Pay as Bid (PAB) market schemes. This includes identifying and using a near optimal bidding strategy (price and quantity), which will be discussed in next chapter that maximizes solar power plant profit in all market schemes. The outcome of this work will determine the competitive market structure with rules that motivate SPPs to participate in markets without subsidies. Part of this analysis is to explore solar power economic effects for different penetration levels at different locations. The ultimate goal of this research is to evaluate the effect of SPP installation in the Saudi Grid and to suggest the best market features to be included in the upcoming new electricity market that maximizes competition, and thus social welfare.

Chapter 2. BIDDING FUNCTION FOR SOLAR POWER PLANTS IN MARGINAL PRICING AND PAY-AS-BID SCHEMES

2.1 INTRODUCTION

The introduction of Solar Power Plants (SPPs) will have different financial effect in the Marginal Price (MP) and the Pay-as-Bid (PAB) scheme. These effects are a function of market price, which depends on the bids of participants. Although the assumption of perfect competition means that a single participant cannot affect market price, the combined bidding of all participants does affect the market price. Therefore finding the optimal bidding for SPPs is needed. Knowing how producers bid in the MP and PAB market schemes will help in deciding how Solar Power Plants (SPPs) will bid in both markets. SPP do not have incremental costs like conventional generators. This is why it is important to decide what is optimal price bid in the market. This chapter will discuss how the bidding function of each market scheme is developed, and will show the adopted SPP bidding function. Since the PAB scheme is not well known, this chapter will focus more in PAB bidding.

2.2 BIDDING STRATEGY FOR GENERATORS IN MP MARKET

In the literature, it is well known that optimal bidding function in the MP scheme assuming a perfectly competitive market is to simply bid the generator's marginal cost (*MC*), which is also called Incremental Cost (IC) [28]. There are lots of methods in the literature to prove that bidding *MC* will maximize the generator's profit [28] [87] [88]. The authors of [28] proved this by

using a simple constraint optimization method. The authors of [88] proved this by using an expected value method. The authors of [87] used the Lagrange multiplier to show that having all generators operating at the same IC will minimize total cost of operation, which is achieved by having the Lagrange multiplier equal to IC.

2.3 Bidding Strategy for Solar Power Plant in the MP Scheme

From the previous section, it is well established that the optimal bidding strategy for conventional generators is to bid *IC*. Since there are no operational costs associated with solar power, therefore all SPPs will bid zero as they are guaranteed getting paid market price. If the SPP is receiving subsidies, then it will be motivated to bid a negative price in order to collect the subsidy, that is the main reason that some markets do have negative market prices. This research will assume not subsidies are available.

2.4 Bidding Strategy for Generators in PAB market

Since the Pay-as-Bid (PAB) scheme is not well known as the MP scheme, this section will spend more time in explaining bidding in PAB. As discussed in chapter one the main concept of the PAB scheme is that producers will get paid only for what they bid for, unlike the MP market where all generators will get paid market price if their bid was lower than the market clearing price. Also, it was discussed that the main claim of advocates of the PAB scheme is that it reduces the exercise of market power. In the PAB market generators need to make a good guess of market price because if their bid was high, their bid might not be accepted and the generator ends up not running. Most likely the bid will be accepted if bidding less than market price while bidding too low will lower the genera-

tor's profit. This process determines bidding strategy for PAB that maximizes the generator's profit and can be applied to Solar Power Plants (SPP).

2.4.1 *PAB Scheme Bidding Function*

Papers [34] [88] [89] discussed optimal bidding for the PAB scheme. Since the PAB bidding needs to estimate the market price, its bidding strategy depends on expected market price. This section will introduce this strategy and show a simplified proof inspired by these papers. Also, it will introduce a major improvement to the method used in these papers.

The best way to find the best bidding function for the PAB scheme is by using the expected value method. This method depends on probability and price expectation [88]. The Expected Profit (*EP*) depends on the probability of the market price and the profit function which is represented in equation (2.1). In order to find the best bidding price (x) the EP is obtained then maximized with respect to bidding price (x).

Since in this section the value used is the profit then the Expected Value (*EV*) can be called Expected Profit (*EP*). Equation (2.1) represents the expected profit when bidding value of x as bids get higher the probability of being accepted gets lower.

$$EP = \int_{x}^{\infty} f(\lambda) \cdot \pi(\lambda)\, d\lambda \tag{2.1}$$

Where

$\pi(\lambda)$ is the profit function.

$f(\lambda)$ is the pdf of market price (λ).

x is the bid price.

The profit function is represented in equation (2.2).

$$\pi = x \cdot P - IC \cdot P = (x - IC) \cdot P \tag{2.2}$$

Where

x is the bidding value. It can be any number, and it is not equal to IC.

IC is the incremental cost of the generator.

P is the amount of generated power.

With the above foundation, the next section will go through the derivations of different bidding function for PAB.

2.4.2 *PAB Scheme Bidding Function based on Uniform Distribution Function*

Papers [34] [88] [89] developed a PAB bidding function that is derived from the assumption that the pdf of market price can be represented using a uniform distribution function (see equation (2.3)). This means that the probability of having a market price (λ) between a minimum market price recorded in a rolling window period (λ_{min}) and maximum market price recorded in a rolling window period (λ_{max}) is uniform. Also, it means the probability of having a market price (λ) elsewhere is zero. The probability of having an accepted bid can be represented in the cumulative distribution function in equation (2.4). The advantage of this assumption that it is simple to use in deriving a PAB bidding function. The disadvantage, generators will not bid lower than λ_{min} or higher than λ_{max}. Also, in reality the prices are not uniformly distributed as one outlier price can offset the bidding function. Other methods will be discussed in the next section.

Uniform distribution function

$$f_u(\lambda) = \begin{cases} \dfrac{1}{\lambda_{max} - \lambda_{min}}, & \text{for } \lambda_{min} < \lambda < \lambda_{max} \\ 0, & \text{otherwise} \end{cases} \tag{2.3}$$

Cumulative Distribution Function (cdf):

$$cdf(\lambda) = \begin{cases} 0 & , \quad \lambda > \lambda_{max} \\ \dfrac{\lambda - \lambda_{min}}{\lambda_{max} - \lambda_{min}} & , \quad \lambda_{min} < \lambda < \lambda_{max} \\ 1 & , \quad \lambda < \lambda_{min} \end{cases} \tag{2.4}$$

Where

λ_{min} is the minimum market price recorded in a rolling window period.

λ_{max} is the maximum market price recorded in a rolling window period.

λ is the market price.

Profit of generator:

$$\pi = x \cdot P - \alpha \cdot P = (x - \alpha) \cdot P \tag{2.5}$$

Where

x is the bidding value, it can be any number and it is not equal to α.

α is the incremental cost of the generator.

The expected profit for $\lambda_{min} < \lambda < \lambda_{max}$ will be

$$EP = \int_{x}^{\lambda_{max}} f_u(\lambda) \cdot \pi(\lambda)\, d\lambda = \int_{x}^{\lambda_{max}} [\frac{1}{\lambda_{max} - \lambda_{min}} \cdot (x - \alpha) \cdot P]\, d\lambda$$

$$EP = \frac{P}{\lambda_{max} - \lambda_{min}} [x \cdot \lambda_{max} - \alpha \cdot \lambda_{max} - x^2 + \alpha \cdot x] \tag{2.6}$$

To find the value of (x) that maximizes expected profit, a partial derivative of EP with respect to (x) is set to zero, and solved for (x).

$$\frac{\partial(EP)}{\partial x} = \lambda_{max} - 2 \cdot x + \alpha = 0$$

$$x = \frac{\lambda_{max} - \alpha}{2} \tag{2.7}$$

Where

α is the *IC* of the generator.

Then the relationship between the bidding price and *IC* is

$$IC = 2 \cdot x - \lambda_{max} \tag{2.8}$$

To make calculation easy, it is useful to have a relationship between the maximum MP market price λ_{max}^{MP} and the maximum PAB market price λ_{max}^{PAB} in a rolling window period.

From equation (2.7)

$$Bid = \frac{\lambda_{max}^{PAB} + IC_{gen}}{2}$$

So at P_d^{max}, the IC of the generator will be IC_{gen}^{max}, which is λ_{max}^{MP}, and by definition the bid on the maximum bid value is λ_{max}^{PAB}, then

$$\lambda_{max}^{PAB} = \frac{\lambda_{max}^{PAB} + \lambda_{max}^{MP}}{2}$$

Then

$$\lambda_{max}^{PAB} = \lambda_{max}^{MP} \tag{2.9}$$

The relation between IC and λ_{min}^{PAB}, from (2.8)

$$IC = 2 \cdot \lambda_{min}^{PAB} - \lambda_{max}^{PAB} \tag{2.10}$$

Which means in order to maximize the profit of a generator that has *x* value between λ_{min}^{PAB} and λ_{max}^{PAB}, the optimal bidding value for the generator is to bid a price that is between λ_{max}^{PAB} and the generator's *IC* as shown in Figure 2.1. If the generator's *x* value was lower than λ_{min}^{PAB} then the generator will maximize its profit by simply bidding λ_{min}^{PAB} as it unlikely that its bid will not be unaccepted. If generator's *x* value was higher than λ_{max}^{PAB} then it is best for the generator not to participate in the market as bidding higher than λ_{max}^{PAB} most probably will not be accepted, and bidding λ_{max}^{PAB} means that the generator will not have its bid accepted [88]. Table 2.1 summarizes the PAB strategy.

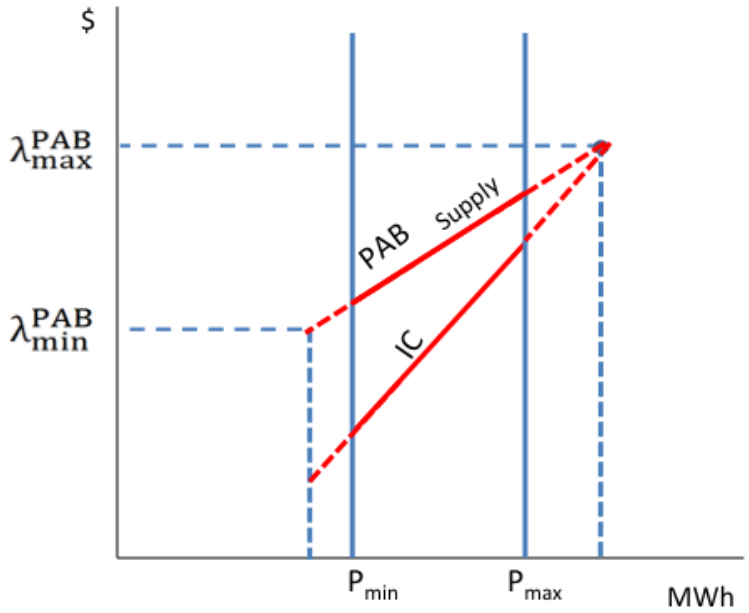

Figure 2.1 PAB bidding strategy [88]

Where

P_{min} is generator's minimum output.

P_{max} is generator's maximum output.

Table 2.1 Strategic bidding for PAB [88]

Condition	**Bid**
$IC < \lambda_{min}^{PAB}$	$Bid = \lambda_{min}^{PAB} = \frac{\lambda_{min}^{MP} + \lambda_{max}^{MP}}{2}$
$\lambda_{min}^{PAB} < IC < \lambda_{max}^{PAB}$	$Bid = \frac{\lambda_{max}^{MP} + IC_{gen}}{2}$
$IC > \lambda_{max}^{PAB}$	$Bid = \lambda_{max}^{MP}$ (Do not bid)

2.4.3 *PAB Scheme Bidding Function Based on Gaussian Distribution Function*

The work done in [34] [88] [89], assumed that the market price has a uniform distribution function. Looking at a full month of market price data from PJM and UK market (see data histogram in figures between Figure 2.2 and Figure 2.5) it can be observed that market prices do not ap-

pear to be uniform, also there are some outlier prices that will have a significant impact on the bidding function which means assuming a uniform distribution is not practical. The problem with the uniform distribution function assumption it depends on two values only the maximum and minimum recorded market prices, in which a one outlier price value can change the bidding function and results in an inaccurate outcome. In order to expand the work, the data histogram in figures between Figure 2.2 and Figure 2.5 shows that a Gaussian function will be a better choice[3]. Which actually considers the mean and standard deviation of a rolling window data will be considered. Therefore, Gaussian Distribution Function is a better representation of the market prices as it is effected by the mean value of the market prices and its standard deviation which means that the affect of outlier prices will be insignificant and it will allow predicted market price to be higher than maximum market price and lower than minimum market price. Therefore a new bidding function will be derived from the Gaussian function as follows.

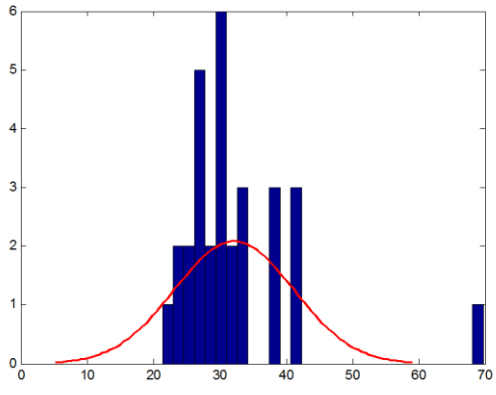

Figure 2.2 The histogram of PJM market price for the month of April, 2015 at 1 pm

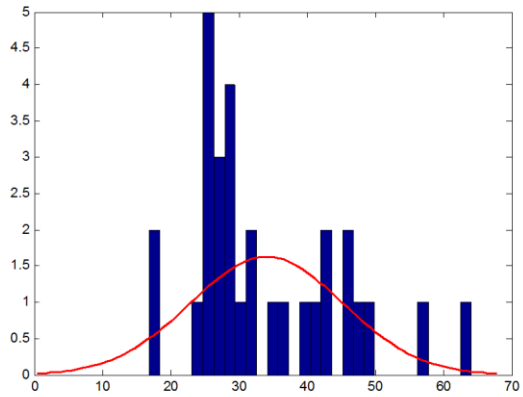

Figure 2.3 The histogram of PJM market price for the month of April, 2015 at 8 am

[3] Galiana in [88] suggested "other probability distributions such as normal are possible but less tractable analytically", but no work was actually done using this method.

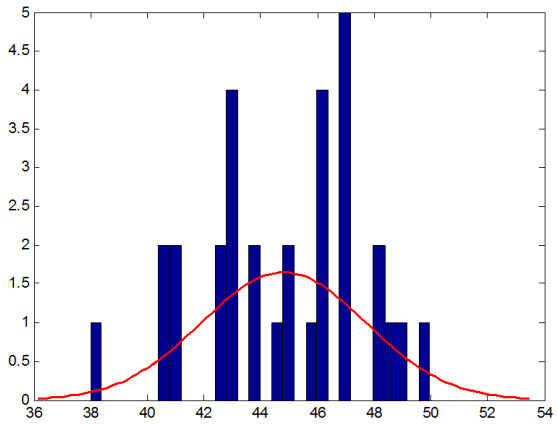

Figure 2.4 The histogram of UK market price for month before June 12, 2015 at 12 pm [90]

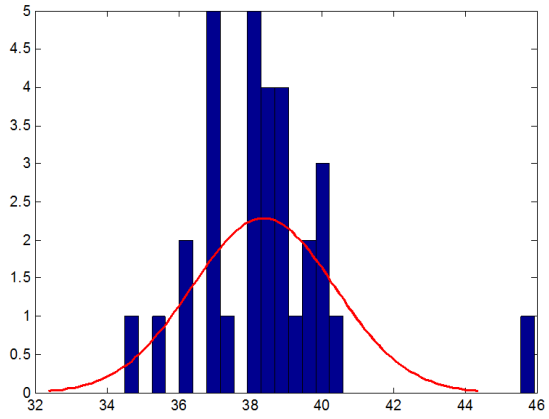

Figure 2.5 The histogram of UK market price for a month before June 12, 2015 at 2 pm [90]

The Gaussian distribution function is

$$f_g(\lambda) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{\frac{-(\lambda-\mu)^2}{2\sigma^2}} \tag{2.11}$$

Where

μ is the mean value of market price in a sliding window.

σ is the standard deviation of market price in a sliding window.

λ is market price.

The profit

$$\pi = (x - \alpha)\cdot P \tag{2.5}$$

Expected profit (EP) is

$$EP = \int_{x}^{\infty} f_g(\lambda) \cdot \pi(\lambda)\, d\lambda = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{\frac{-(\lambda-\mu)^2}{2\sigma^2}} \cdot (x-\alpha) \cdot P\, d\lambda$$

$$= \frac{P \cdot (x-\alpha)}{\sqrt{2\pi} \cdot \sigma} \int_{x}^{\infty} e^{\frac{-(\lambda-\mu)^2}{2\sigma^2}}\, d\lambda$$

$$= \frac{P \cdot (x-\alpha)}{\sqrt{2\pi} \cdot \sigma} \cdot \left[\frac{\sigma\sqrt{2\pi}}{2} \cdot erf\left[(\lambda-\mu) \cdot \sqrt{\frac{1}{2\sigma^2}} \right] \right]_{x}^{\infty}$$

$$EP = \frac{P \cdot (x-\alpha)}{2} - \frac{P \cdot (x-\alpha)}{2} \cdot erf\left[(x-\mu) \cdot \sqrt{\frac{1}{2\sigma^2}} \right] \tag{2.12}$$

To find the value of x that maximizes expected profit, a partial derivative of EP with respect to x is obtained.

$$\frac{\partial(EP)}{\partial x} = 0$$

$$\frac{P}{2} \cdot \left[1 - erf\left[(x-\mu) \cdot \sqrt{\frac{1}{2\sigma^2}} \right] \right] - \frac{P \cdot (x-\alpha)}{\sqrt{2\pi \cdot \sigma^2}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}} = 0 \tag{2.13}$$

Because of the combination of the error function (erf) and exponential quadratic function, it is hard to solve for x. The easiest way to solve it is by numerical iteration. Also, this is not possible to use in an economic dispatch solver that depends on quadratic optimization with linear constraints. Computationally tractable approximations of the Gaussian distribution may be able to rapidly provide near-optimal bid functions. Since the Gaussian distribution is already an approximation of the actual market price pdf, a computationally tractable approximation may actually be better than Gaussian. Two possible approximations are a triangle fits the Gaussian or the uniform distribution that best fits all market prices will be explored in the coming sections.

2.4.4 *PAB Scheme Bidding Function based on Gaussian Triangle fit Distribution Function*

A triangle function equivalent to Gaussian distribution (Tri-Gau) is used as an approximation fit to derive a bidding function, see Figure 2.6. The first step is to find the best value for the height and width of the triangle fit. If the width is $2 \cdot k \cdot \sigma$ then the height of the triangle is $\frac{1}{k \cdot \sigma}$ in order to have an area under the pdf equal to one.

$$Width = 2 \cdot k \cdot \sigma$$

$$Height = \frac{1}{k \cdot \sigma}$$

Triangle area

$$Area = \frac{1}{2}(Width\ X\ Height) = 1$$

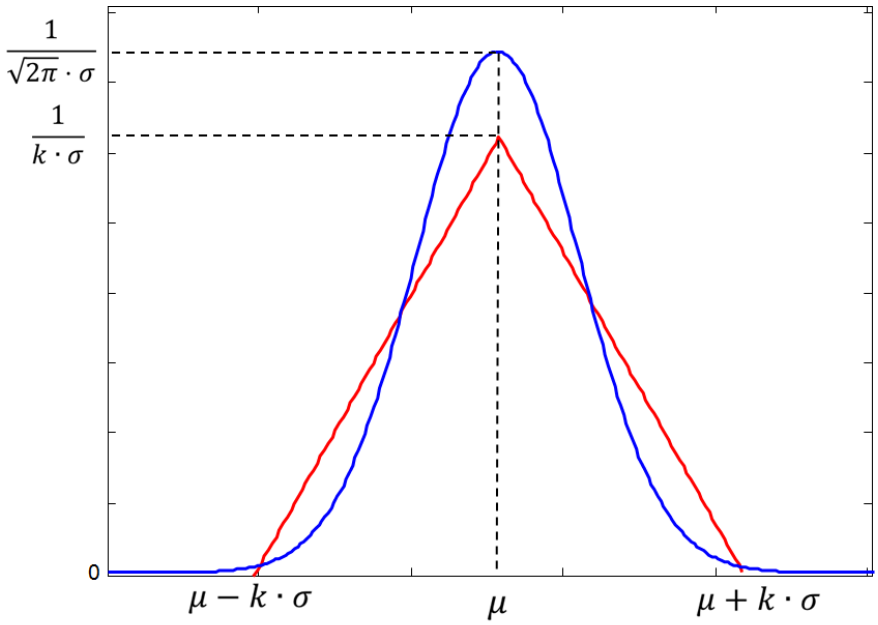

Figure 2.6 Triangle equivalent to Gaussian pdf

From these dimensions the line equations of the two sides of the triangle can be derived as two separate lines with an elbow at μ. The slope of the left side of the triangle is

$$s_1 = \frac{y_2 - y_1}{x_2 - \mathrm{x}_1} = \frac{\frac{1}{k \cdot \sigma} - 0}{\mu - (\mu - k \cdot \sigma)} = \frac{1}{(k \cdot \sigma)^2} \tag{2.14}$$

Then the line function between $(\mu - k \cdot \sigma)$ and μ will be

$$y - y_1 = s_1(x - x_1)$$

$$y = \frac{1}{(k \cdot \sigma)^2}\big(x - (\mu - k \cdot \sigma)\big) \tag{2.15}$$

For the line of the right side of the triangle between μ and $(\mu - k \cdot \sigma)$, the slope will be

$$s_2 = \frac{-1}{(k \cdot \sigma)^2}$$

The function of the line will be

$$y - y_1 = s_2(x - x_1)$$

$$y = \frac{-1}{(k \cdot \sigma)^2}(x - (\mu + k \cdot \sigma)) \tag{2.16}$$

Then the Tri-Gau pdf will be

$$f_{tg}(\lambda) = pdf(\lambda) = \begin{cases} \frac{1}{(k \cdot \sigma)^2}\big(\lambda - (\mu - k \cdot \sigma)\big) & , \quad \lambda \in (\mu - k \cdot \sigma, \mu) \\ \frac{-1}{(k \cdot \sigma)^2}\big(\lambda - (\mu + k \cdot \sigma)\big) & , \quad \lambda \in (\mu, \mu + k \cdot \sigma) \\ 0 & , \quad otherwise \end{cases}$$

Finding the best value of k:

In order to get the best triangle fit, the value of k that minimizes the Mean Square Error (MSE) value between the Triangle equivalent Gaussian and the **real** Gaussian distribution must be obtained. The first step is to normalize the pdf by assuming μ = 0. Since the pdf is symmetrical, it is enough to calculate the MSE from zero to infinity.

Mean Square Error between Gaussian and Triangle equivalent:

$$MSE = 2 \cdot \int_0^{\infty} \left[f_g(x) - f_t(x)\right]^2 dx = 2 \cdot \int_0^{k\sigma} \left[f_g(x) - f_t(x)\right]^2 dx + 2 \cdot \int_{k\sigma}^{\infty} \left[f_g(x)\right]^2 dx$$

$$= 2 \cdot \int_0^{k\sigma} \left[\frac{1}{\sqrt{2\pi} \cdot \sigma} e^{\frac{-(x)^2}{2\sigma^2}} + \frac{1}{(k \cdot \sigma)^2}(x - k \cdot \sigma) \right]^2 dx + 2$$

$$\cdot \int_{k\sigma}^{\infty} \left[\frac{1}{\sqrt{2\pi} \cdot \sigma} e^{\frac{-(x)^2}{2\sigma^2}} \right]^2 dx$$

$$MSE = 2 \cdot \frac{1}{\sigma} \left(-\frac{1}{k} erf\left(\frac{k}{\sqrt{2}}\right) + \frac{\sqrt{2}\left(1 - e^{-\frac{k^2}{2}}\right)}{k^2\sqrt{\pi}} + \frac{1}{3k} + \left[\frac{1}{4\sqrt{\pi}}\right] \right) \tag{2.17}$$

Now to find the value of k that will minimize MSE, take the partial derivative with respect to k and equal it to zero

$$\frac{\partial(MSE)}{\partial k} = 0$$

$$\frac{\partial(MSE)}{\partial k} = \frac{1}{\sigma} \left(\frac{erf\left(\frac{k}{\sqrt{2}}\right)}{k^2} + \frac{2^{\frac{3}{2}}\left(e^{\frac{-k^2}{2}} - 1\right)}{\sqrt{\pi}k^3} - \frac{1}{3k^2} \right) = 0$$

Giving a value of $\boldsymbol{k = 2.297 \approx 2.3}$

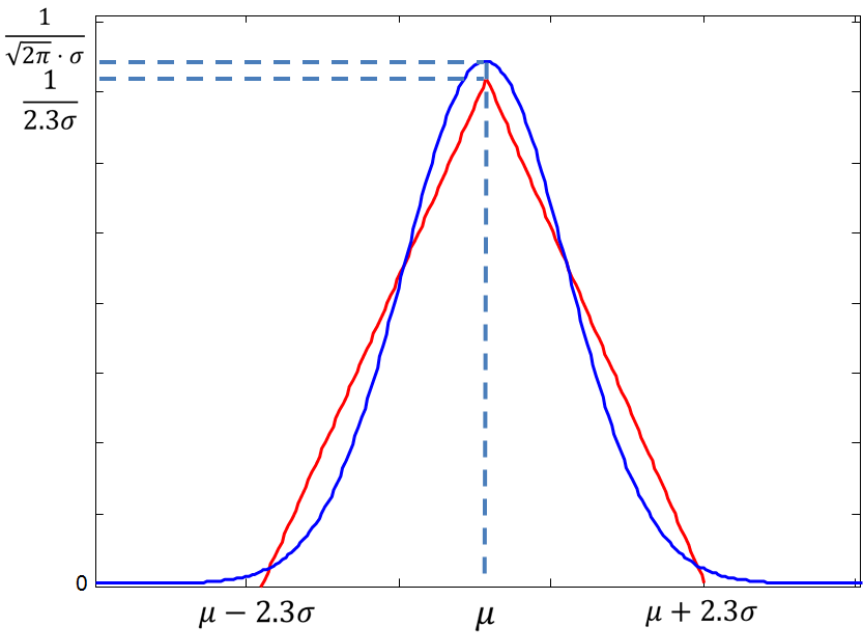

Figure 2.7 Triangle pdf equivalent to Gaussian

Figure 2.7 represents the dimensions of the triangle equivalent, which is

$$Height = \frac{1}{2.3 \cdot \sigma}$$

$$Width = 2 \cdot (2.3) \cdot \sigma$$

Then the Triangle equivalent Gaussian (Tri-Gau) pdf will be:

$$f_{tg}(\lambda) = pdf(\lambda) = \begin{cases} \frac{1}{(2.3\sigma)^2}\left(\lambda - (\mu - 2.3\sigma)\right) & , \quad \lambda \in (\mu - 2.3\sigma, \mu) \\ \frac{-1}{(2.3\sigma)^2}\left(\lambda - (\mu + 2.3\sigma)\right) & , \quad \lambda \in (\mu, \mu + 2.3\sigma) \\ 0 & , \quad otherwise \end{cases}$$

Finding the Bidding Function Using the Triangle Equivalent pdf

Since the Tri-Gau pdf has two lines, the expected profit of the right side of Tri-Gau will be different than the left side. We will start the analysis with the right side first as the result will be used in the left side

The Expected Profit (EP) of the right side of the Tri-Gau pdf when $x > \mu$

$$EP = \int_{x}^{\infty} f_{tg}(\lambda) \cdot \pi(\lambda)\, d\lambda = \int_{x}^{\mu+2.3\sigma} f_{tg}(\lambda) \cdot \pi(\lambda)\, d\lambda$$

$$= \int_{x}^{\mu+2.3\sigma} \frac{-1}{(2.3\sigma)^2}\big(\lambda - (\mu + 2.3\sigma)\big) \cdot (x - \alpha) \cdot P\, d\lambda$$

$$EP = \frac{-(x-\alpha).P}{(2.3\sigma)^2} \cdot \left[-\frac{\mu^2}{2} - (2.3 \cdot \sigma) \cdot \mu - \frac{(2.3 \cdot \sigma)^2}{2} - \frac{x^2}{2} + x \cdot (\mu + 2.3 \cdot \sigma) \right] \quad (2.18)$$

To find the value of (x) that maximizes expected profit, a partial derivative of EP with respect to (x) is obtained.

$$\frac{\partial}{\partial x}(EP) = 0$$

$$-1\left[-\frac{\mu^2}{2} - (2.3 \cdot \sigma) \cdot \mu - \frac{(2.3 \cdot \sigma)^2}{2} - \frac{x^2}{2} + x(\mu + 2.3 \cdot \sigma) \right] - (x - \alpha)$$

$$\cdot [-x + (\mu + 2.3 \cdot \sigma)] = 0$$

Solution

$$x_1 = \mu + 2.3 \cdot \sigma \quad (2.19)$$

Or

$$x_2 = \frac{2}{3}\alpha + \frac{1}{3}\mu + \frac{1}{3}(2.3\sigma) \quad (2.20)$$

Where

$x_1$is the value that minimize the expected profit.

x_2 is the value that maximize the expected profit.

Therefore x_2 will be the bidding function for $x > \mu$

$$BF(\alpha) = \frac{2}{3}\alpha + \frac{1}{3}\mu + \frac{1}{3}(2.3\sigma)$$

$$BF(P) = \frac{2}{3}(B + 2 \cdot C \cdot P) + \frac{1}{3}\mu + \frac{1}{3}(2.3\sigma) = \frac{2 \cdot B + \mu + 2.3\sigma}{3} + \frac{4 \cdot C}{3}P$$

To find the incremental cost (α) of the generator when $\boldsymbol{x = \mu}$

$$\mu = \frac{2}{3}\alpha + \frac{1}{3}\mu + \frac{1}{3}(2.3\sigma)$$

$$\frac{2}{3}\alpha = \frac{2}{3}\mu - \frac{1}{3}(2.3\sigma)$$

$$\alpha = \mu - \frac{2.3\sigma}{2} \tag{2.21}$$

Therefore the bidding function for the right side of the triangle is

$$BF(\alpha) = \frac{2}{3}\alpha + \frac{1}{3}\mu + \frac{1}{3}(2.3\sigma), \qquad where\ \alpha > \mu - \frac{2.3\sigma}{2}$$

The expected profit when $\boldsymbol{x = \mu}$ for the right side of the triangle

$$EP(\mu) = \frac{-(x-\alpha).P}{2.3^2 \cdot \sigma^2} \cdot \left[-\frac{\mu^2}{2} - (2.3 \cdot \sigma) \cdot \mu - \frac{(2.3 \cdot \sigma)^2}{2} - \frac{x^2}{2} + x \cdot (\mu + 2.3 \cdot \sigma) \right]$$

$$EP(\mu) = \frac{-(\mu-\alpha).P}{2.3^2 \cdot \sigma^2} \cdot \left[-\frac{(2.3 \cdot \sigma)^2}{2} \right] \tag{2.22}$$

The Expected Profit of the left side of the Tri-Gau pdf when $\boldsymbol{x < \mu}$

$$EP = \int_x^{\infty} f_{tg}(\lambda) \cdot \pi(\lambda)\, d\lambda = \int_x^{\mu} f_{tg}(\lambda) \cdot \pi(\lambda)\, d\lambda + \int_{\mu}^{\mu+2.3\sigma} f_{tg}(\lambda) \cdot \pi(\lambda)\, d\lambda =$$

$$= \int_x^{\mu} \frac{1}{2.3^2 \cdot \sigma^2} \left(\lambda - (\mu - 2.3\sigma)\right) . (x-\alpha).P\, d\lambda$$

$$+ \int_{\mu}^{\mu+2.3*\sigma} \frac{-1}{2.3^2 \cdot \sigma^2} \left(\lambda - (\mu + 2.3\sigma)\right) . (x-\alpha).P\, d\lambda$$

$$EP = \frac{(x-\alpha).P}{2.3^2 \cdot \sigma^2} \cdot \left[\left[-\frac{\mu^2}{2} + 2.3\sigma \cdot \mu - \frac{x^2}{2} + x \cdot (\mu - 2.3\sigma) \right] - \left[-\frac{(2.3\sigma)^2}{2} \right] \right] \tag{2.23}$$

To find the value of (x) that maximizes expected profit, a partial derivative of EP with respect to (x) is obtained.

$$\frac{\partial}{\partial x}(EP) = 0$$

$$\frac{\partial}{\partial x}(EP) = \frac{P}{2.3^2 \cdot \sigma^2} \cdot \left[\left[-\frac{\mu^2}{2} + 2.3\sigma \cdot \mu - \frac{x^2}{2} + x(\mu - 2.3\sigma) \right] + \frac{(2.3\sigma)^2}{2} \right] + \frac{(x-\alpha).P}{2.3^2 \cdot \sigma^2} \left[[-x + (\mu - 2.3\sigma)] \right] = 0$$

$$\left[\left[-\frac{\mu^2}{2} + 2.3\sigma \cdot \mu - \frac{x^2}{2} + x(\mu - 2.3\sigma) \right] + \frac{(2.3\sigma)^2}{2} \right] + (x-\alpha) \left[[-x + (\mu - 2.3\sigma)] \right] = 0$$

Now solving with respect to *x*

$$\begin{aligned} & x \\ & = \frac{1}{3}\alpha + \frac{2}{3}\mu - \frac{2}{3}(2.3\sigma) \\ & + \frac{1}{3}\sqrt{\alpha^2 - 2 \cdot \alpha \cdot \mu + 2 \cdot \alpha \cdot (2.3\sigma) + \mu^2 - 2 \cdot \mu \cdot (2.3\sigma) + 7 \cdot (2.3\sigma)^2} \end{aligned} \tag{2.24}$$

Then the BF when $x < \mu$

$$BF = \frac{1}{3}\alpha + \frac{2}{3}\mu - \frac{2}{3}(2.3\sigma) + \frac{1}{3}\sqrt{\alpha^2 - 2 \cdot \alpha \cdot \mu + 2 \cdot \alpha \cdot (2.3\sigma) + \mu^2 - 2 \cdot \mu \cdot (2.3\sigma) + 7 \cdot (2.3\sigma)^2}$$

Now find the generators incremental cost at $x = \mu$, to find the corresponding α,

$$\left[\left[-\frac{\mu^2}{2} + 2.3\sigma \cdot \mu - \frac{x^2}{2} + x(\mu - 2.3\sigma) \right] + \frac{(2.3\sigma)^2}{2} \right] + (x-\alpha) \left[[-x + (\mu - 2.3\sigma)] \right] = 0$$

Will be

$$\left[-\frac{\mu^2}{2} - (2.3\sigma) \cdot \mu - \frac{(2.3\sigma)^2}{2} - \frac{\mu^2}{2} + (\mu^2 + 2.3\sigma \cdot \mu) \right] + (\mu - \alpha) \cdot (2.3\sigma) = 0$$

$$\alpha = \mu - \frac{2.3\sigma}{2} \tag{2.25}$$

As expected equation (2.21) matches equation (2.25)

The expected profit when

$$x = \mu$$

$$EP(\mu) = \frac{(x-\alpha).P}{2.3\sigma^2 \cdot \sqrt{2\pi}} \cdot \left[\left[-\frac{\mu^2}{2} + 2.3\sigma \cdot \mu - \frac{x^2}{2} + x(\mu - 2.3\sigma) \right] + \left[\frac{(2.3\sigma)^2}{2} \right] \right]$$

$$EP(\mu) = \frac{(\mu-\alpha).P}{2.3\sigma^2 \cdot \sqrt{2\pi}} \cdot \left[\frac{(2.3\sigma)^2}{2} \right] \tag{2.26}$$

Equation (2.22) and equation (2.26) confirms that bidding a price equal to the **break point** $\boldsymbol{\mu}$ has the same expected profit. In summary the strategic bidding derived from Tri-Gau pdf is in Table 2.2. It is graphed in Figure 2.8.

Table 2.2 Strategic bidding for PAB derived from Tri-Gau

Condition	**Bid**
$IC_{gen} < \mu - \frac{2.3\sigma}{2}$	$\frac{1}{3}IC_{gen} + \frac{2}{3}\mu - \frac{2}{3}(2.3\sigma) + \frac{1}{3}\sqrt{IC_{gen}^2 - 2 \cdot IC_{gen} \cdot \mu + 2 \cdot IC_{gen} \cdot (2.3\sigma) + \mu^2 - 2 \cdot \mu \cdot (2.3\sigma) + 7(2.3\sigma)^2}$
$\mu - \frac{2.3\sigma}{2} < IC_{gen} < \mu + 2.3\sigma$	$\frac{2}{3}IC_{gen} + \frac{1}{3}\mu + \frac{1}{3}(2.3\sigma)$
$IC_{gen} > \mu + 2.3\sigma$	$\mu + 2.3\sigma$

Linearization of left side of the bidding function

It can be observed that for the condition $IC_{gen} < \mu - \frac{2.3\sigma}{2}$ is hard to fit in a bidding program, therefore it would be useful to linearize it.

From equation (2.24)

$$BF(\alpha) = \frac{1}{3}\alpha + \frac{2}{3}\mu - \frac{2}{3}(2.3\sigma) + \frac{1}{3}\sqrt{\alpha^2 - 2\cdot\alpha\cdot\mu + 2\cdot\alpha\cdot(2.3\sigma) + \mu^2 - 2\cdot\mu\cdot(2.3\sigma) + 7(2.3\sigma)^2}$$

Linearized fit as Figure 2.8

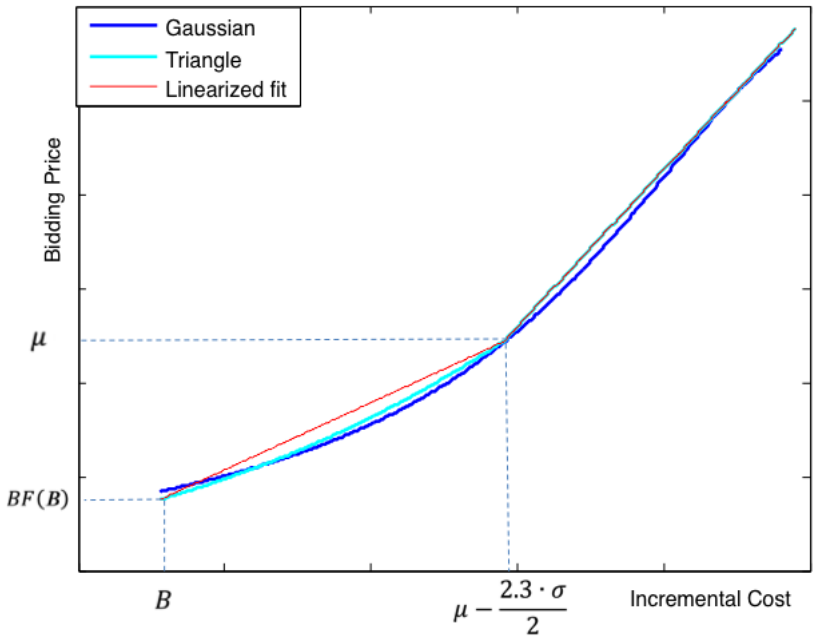

Figure 2.8 Bidding function using Gaussian, triangle equivalent, and linearized fit

From equation (2.25) the right side of this bidding function will be

$$\alpha = \mu - \frac{2.3\cdot\sigma}{2}$$

$$BF(\alpha) = \mu$$

Now the left side of this bidding function will be at point where the incremental cost of the generator at P=0

$$IC(P) = B + 2\cdot C\cdot(P)$$

$$IC(0) = B + 2\cdot C\cdot(0) = B$$

Where

B is the B coefficient of the generator's cost function.

C is the C coefficient of the generator's cost function.

Then the IC at P=0 will be

$$\alpha = B$$

From (2.24) the bidding price will be

$$BF(B) = \frac{1}{3}B + \frac{2}{3}\mu - \frac{2}{3}(2.3\sigma) + \frac{1}{3}\sqrt{B^2 - 2\cdot B\cdot \mu + 2\cdot B\cdot (2.3\sigma) + \mu^2 - 2\cdot \mu\cdot (2.3\sigma) + 7(2.3\sigma)^2}$$

Then the slope will be

$$s = \frac{\mu - BF(B)}{\mu - \frac{2.3\sigma}{2} - B} \tag{2.27}$$

Then the bidding function will be

$$BF = s\cdot \alpha + constant$$

$$\mu = (\mu - \frac{2.3\sigma}{2})\cdot s + constant$$

$$constant = Con = \mu - (\mu - \frac{2.3\sigma}{2})\cdot s \tag{2.28}$$

$$BF(\alpha) = s\cdot \alpha + Con \tag{2.29}$$

The new bidding function will be

Table 2.3 Strategic Bidding For PAB derived from Tri-Gau pdf linearized

Condition	**Bid**
$IC_{gen} < \mu - \frac{2.3\sigma}{2}$	$s\cdot IC_{gen} + Con$
$\mu - \frac{2.3\sigma}{2} < IC_{gen} < \mu + 2.3\sigma$	$\frac{2}{3}IC_{gen} + \frac{1}{3}\mu + \frac{1}{3}(2.3\sigma)$
$IC_{gen} > \mu + 2.3\sigma$	$\mu + 2.3\sigma$ (Do not bid)

2.4.5 *PAB Scheme Bidding Function Based on a Best Fit Uniform Distribution*

The Tri-Gau bidding function has a break point that splits the bidding function into two bidding functions and has high computation time. Therefore, to improve the computation time a new bidding function will be derived from a uniform distribution function using the mean (μ) and standard deviation (σ) of the market price in a sliding window instead of just minimum and maximum values of market price. This new Best Uniform distribution (B-Uni) can be represented in a rectangle fit that has a center at μ as in Figure 2.9 and it has the following features:

$$Base = 2 \cdot k \cdot \sigma$$

$$Height = \frac{1}{2 \cdot k \cdot \sigma}$$

Then the pdf of B-Uni

$$f_{bu}(\lambda) = \begin{cases} \frac{1}{2 \cdot k \cdot \sigma} & , for \quad \mu - k \cdot \sigma < \lambda < \mu + k \cdot \sigma \\ 0 & , \quad otherwise \end{cases} \tag{2.30}$$

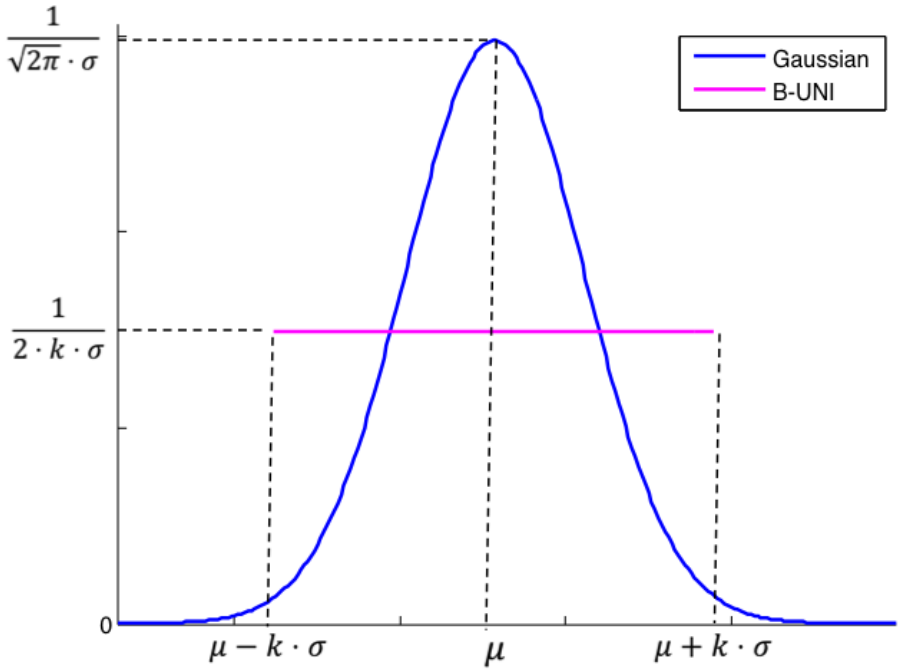

Figure 2.9 Uniform distribution equivalent to Gaussian

Finding best value of ***k***:

The optimal value of k that has the least (MSE) between the B-Uni and the real Gaussian distribution is calculated. The first step is to normalize the pdf by assuming μ = 0. Since the pdf is symmetrical it is enough to calculate from zero to infinity.

Mean Square Error between Gaussian and B-Uni:

$$MSE = \int_0^{\infty} \left[f_g(x) - f_{bu}(x)\right]^2 dx = \int_0^{k\sigma} \left[f_g(x) - f_{bu}(x)\right]^2 dx + \int_{k\sigma}^{\infty} \left[f_g(x)\right]^2 dx$$

$$= \int_0^{2k\sigma} \left[\frac{1}{\sqrt{2\pi}\cdot\sigma} e^{\frac{-(x)^2}{2\sigma^2}} - \frac{1}{2\cdot k\cdot\sigma}\right]^2 dx + \int_{k\sigma}^{\infty} \left[\frac{1}{\sqrt{2\pi}\cdot\sigma} e^{\frac{-(x)^2}{2\sigma^2}}\right]^2 dx$$

$$MSE = \frac{1}{\sigma}\left[\frac{\sqrt{\pi}}{4\pi}\cdot erf(k) - \left(\frac{1}{2} erf\left(\frac{k\sqrt{2}}{2}\right) - \frac{1}{4}\right)\cdot\frac{1}{k}\right] + \frac{\sqrt{\pi}}{4\pi\cdot\sigma}\left[1 - erf(k)\right] \qquad (2.31)$$

Now the derivative of MSE is taken with respect to k to find that value that will minimize it.

$$\frac{\partial(MSE)}{\partial k} = 0$$

$$\frac{\partial MSE}{\partial k} = \frac{1}{\sigma}\left[\left(\frac{1}{2\pi}\cdot e^{-k^2} + \frac{1}{k^2}\left(\frac{1}{2}\cdot erf\left(\frac{1}{2}k\sqrt{2}\right) - \frac{1}{4}\right) - \frac{1}{k\sqrt{2\pi}}\cdot e^{-\frac{k^2}{2}}\right) - \frac{e^{-k^2}}{2\pi}\right] = 0$$

$$\left[\left(\frac{1}{2\pi}\cdot e^{-k^2} + \frac{1}{k^2}\left(\frac{1}{2}\cdot erf\left(\frac{1}{2}k\sqrt{2}\right) - \frac{1}{4}\right) - \frac{1}{k\sqrt{2\pi}}\cdot e^{-\frac{k^2}{2}}\right) - \frac{e^{-k^2}}{2\pi}\right] = 0$$

The optimal value of *k*

$$\boldsymbol{k = 1.538}$$

Then the B-Uni can be represented as in Figure 2.10

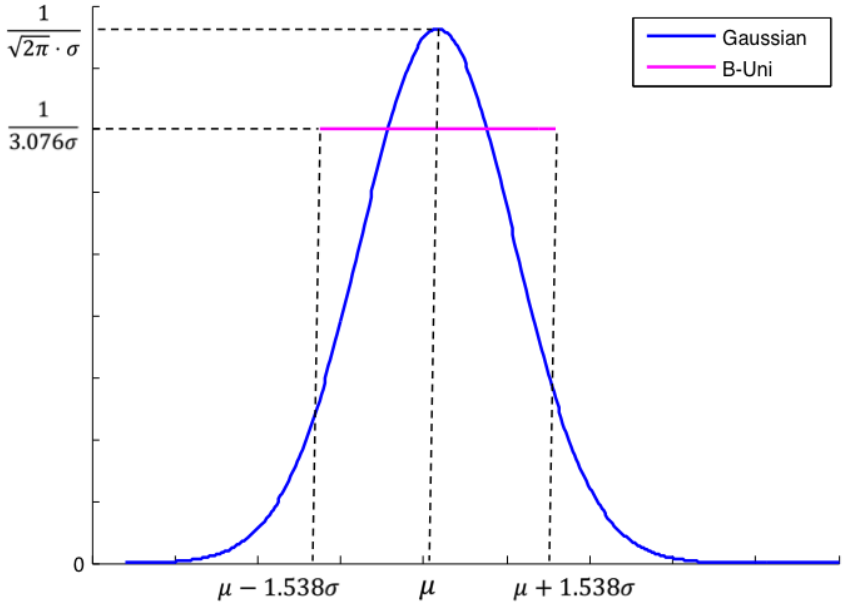

Figure 2.10 Uniform distribution equivalent to Gaussian

Expected profit for $\boldsymbol{k = 1.538}$

The expected profit for market price range between $\lambda_{min} < \lambda < \lambda_{max}$ will be

$$\lambda_{min} = \mu - 1.538\sigma$$

$$\lambda_{max} = \mu + 1.538\sigma$$

$$EP = \int_{x}^{\mu+1.538\sigma} f_{bu}(\lambda) \cdot \pi(\lambda)\, d\lambda$$

$$= \int_{x}^{\mu+1.538\sigma} \frac{1}{3.076\sigma} \cdot (x-\alpha) \cdot P\, d\lambda$$

$$EP = \frac{P}{3.076\sigma}\ [(\, x \cdot (\mu + 1.538\sigma) - \alpha \cdot (\mu + 1.538\sigma)) - (\, x^2 - \alpha \cdot x)] \qquad (2.32)$$

To find the value of (x) that maximizes expected profit, a partial derivative of EP with respect to (x) is obtained.

$$\frac{\partial(EP)}{\partial x} = 0$$

$$\frac{\partial(EP)}{\partial x} = (\mu + 1.538\sigma) - 2x + \alpha = 0$$

$$x = \frac{(\mu + 1.538\sigma) + \alpha}{2}$$

Then the bidding function will be

$$BF(\alpha) = \frac{(\mu + 1.538\sigma) + \alpha}{2}$$

Then the bidding function of PAB derived from B-Uni will be as in Table 2.4

Table 2.4 Strategic bidding for PAB derived from B-Uni

Condition	**Bid**
$IC_{gen} < \mu^{PAB} - 3 \cdot (1.538\sigma^{PAB})$	$\mu^{PAB} - 1.538\sigma^{PAB}$
$\mu^{PAB} - 3 \cdot (1.538\sigma^{PAB}) < IC_{gen} < \mu^{PAB} + 1.538\sigma$	$\frac{(\mu^{PAB} + 1.538\sigma^{PAB}) + IC_{gen}}{2}$
$IC_{gen} > \mu^{PAB} + 1.538\sigma$	$\mu^{PAB} + 1.538\sigma^{PAB}$ (Do not bid)

2.4.6 *Comapring the MSE of Proposed Distribution Functions*

MSE between Gaussian and Uniform pdf

In order to understand the improvement of using Gaussian distribution against Uniform distribution, it is worth calculating the MSE between the Uniform and Gaussian and the real Gaussian distribution. The first step is to normalize the pdf by assuming $\mu = 0$.

$$MSE = \int_0^{\infty} \left[f_g(x) - f_u(x)\right]^2 dx$$

$$= \int_{-\infty}^{\lambda_{min}} \left[f_g(x)\right]^2 dx + \int_{\lambda_{min}}^{\lambda_{max}} \left[f_g(x) - f_u(x)\right]^2 dx + \int_{\lambda_{max}}^{\infty} \left[f_g(x)\right]^2 dx$$

$$= \int_{-\infty}^{\lambda_{min}} \left(\frac{1}{\sqrt{2\pi}\cdot\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\right)^2 dx$$

$$+ \int_{\lambda_{min}}^{\lambda_{max}} \left(\frac{1}{\sqrt{2\pi}\cdot\sigma} e^{\frac{-(x)^2}{2\sigma^2}} - \frac{1}{\lambda_{max}-\lambda_{min}}\right)^2 dx$$

$$+ \int_{\lambda_{max}}^{\infty} \left(\frac{1}{\sqrt{2\pi}\cdot\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\right)^2 dx$$

$$MSE = \frac{1}{2\sqrt{\pi}\cdot\sigma}$$

$$+ \frac{1}{\lambda_{max}-\lambda_{min}}\left[erf\left(\frac{(\lambda_{max}-\mu)\sqrt{2}}{2\cdot\sigma}\right) - erf\left(\frac{(\lambda_{min}-\mu)\sqrt{2}}{2\cdot\sigma}\right) - 1\right] \quad (2.33)$$

The results show that there is a relatively high MSE between the Gaussian and uniform distribution that is highly affected by the values of λ_{max} and λ_{min}. Also, there is a fixed error that is related to the standard deviation. This MSE will be a good source of comparison for the next section.

Gaussian Triangle Fit pdf

The MSE formula was derived in equation (2.17). Then the value of $\boldsymbol{MSE}$ **at k=2.3 is**

$$MSE = \frac{0.00012}{\sigma} \tag{2.34}$$

It is clear that the MSE is small and as (σ) gets higher the triangle fit gets better.

B-Uni Distribution pdf

The MSE formula was derived in equation (2.31). Then the value of $\boldsymbol{MSE}$ **at k=1.538 is**

$$MSE = \frac{0.0188}{\sigma} \tag{2.35}$$

It can be observed that the MSE of both Gaussian Triangle fit pdf and B-Uni pdf have relatively low MSE, while the Uniform pdf is highly affected by the values of values of λ_{max} and λ_{min}. In next section, a numerical test will be done in order to adopt a PAB bidding function.

2.4.7 *Numerical Test to Compare all Proposed Bidding Functions to Gaussian Derived Bidding Function*

The proposed approximations are based on the assumption that the market prices are Gaussian but it is not clear that they actually are. Using some actual market prices in a numerical test will examine the behavior of each of the proposed approximations.

To check the accuracy of all proposed PAB bidding functions, the expected profit of Gaussian pdf is used to plot the results of several random days. Generator cost function information is obtained from [87]. PJM and UK price market were used in Figure 2.11 to ure 2.22. It can be observed that PAB bidding functions derived from Gaussian, and Tri-Gau, Tri-Gau linearized, and B-Uni are all close to each other while bidding function derived from Uniform pdf results has lower expected profit. The main reason for this the bidding function derived from Uniform pdf can easily be affected by outlier prices. Although B-Uni bidding function is simple, its expected profit is close to expected profit obtained by the bidding function derived from Gaussian pdf. Based on these results the B-Uni bidding function will be

adopted for this work as in addition to its great results as seen in Figure 2.11 to Figure 2.22, it is simple and easy to be used in solvers with less computation time.

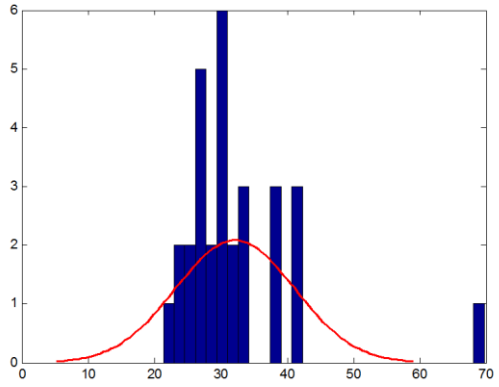

Figure 2.11 The histogram of PJM market price for the month of April, 2015 at 1 pm

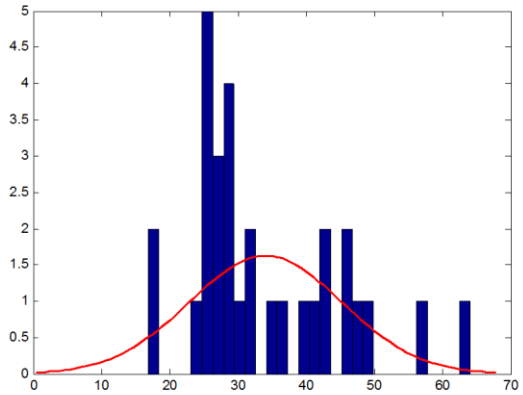

Figure 2.12 The histogram of PJM market price for the month of April, 2015 at 8 am

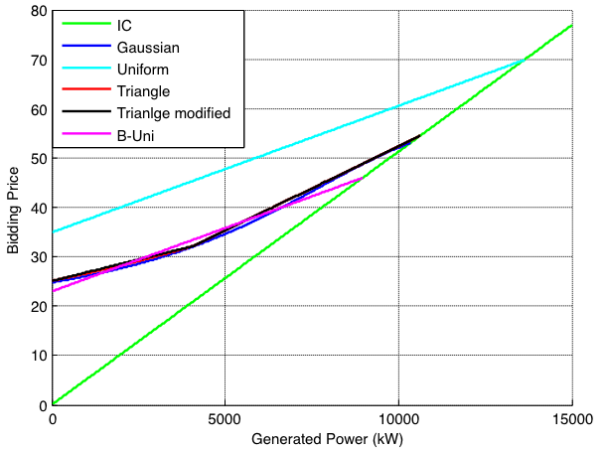

Figure 2.13 Comparing different PAB bidding functions based on PJM market price for the month of April, 2015 at 1 pm

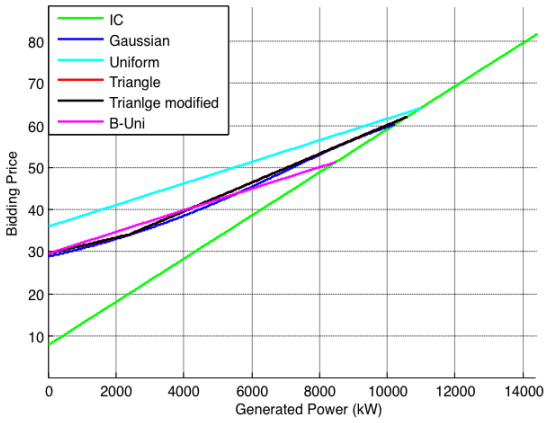

Figure 2.14 Comparing different PAB bidding functions based on PJM market price for the month of April, 2015 at 8 am

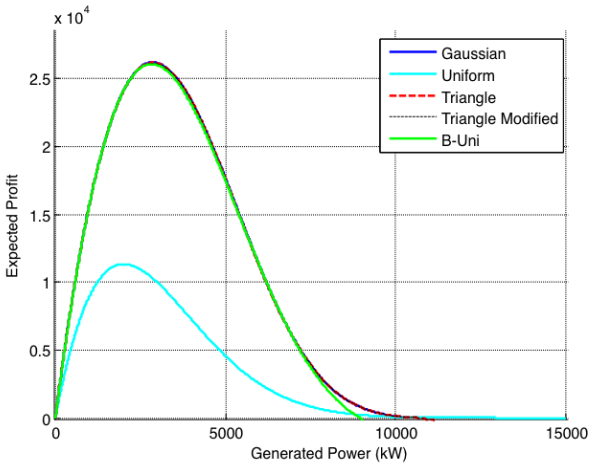

Figure 2.15 The expected profit of all proposed PAB bidding functions taking Gaussian expected profit as a reference based on PJM market price for the month of April, 2015 at 1 pm

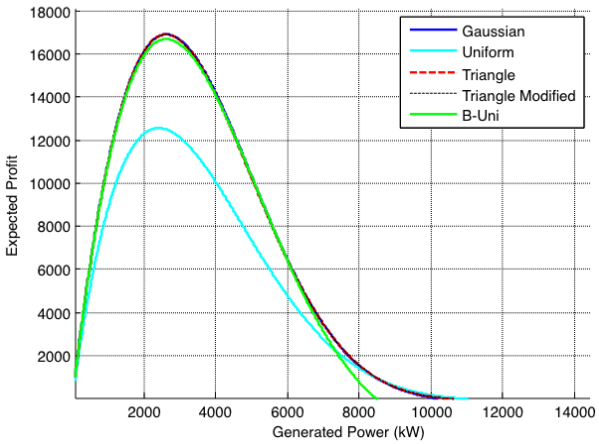

Figure 2.16 The expected profit of all proposed PAB bidding functions taking Gaussian expected profit as a reference based on PJM market price for the month of April, 2015 at 8 am

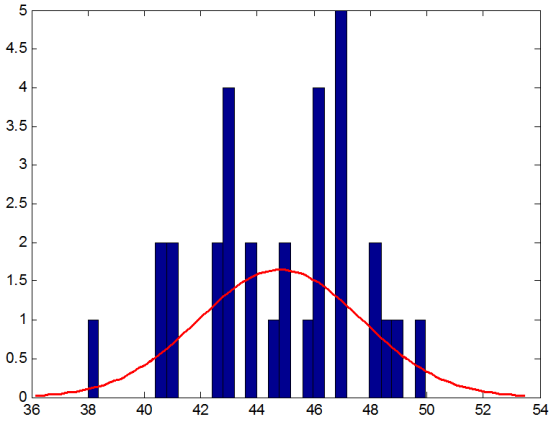

Figure 2.17 The histogram of UK market price for month before June 12, 2015 at 12 pm

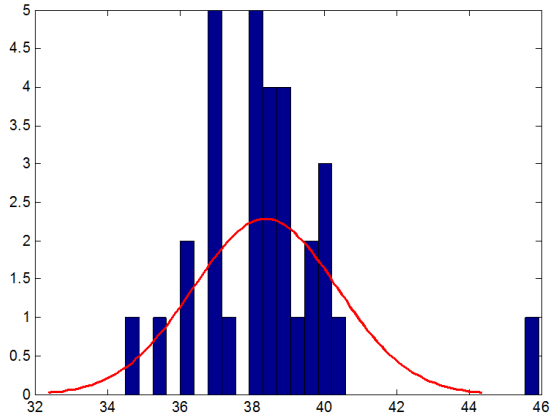

Figure 2.18 The histogram of UK market price for a month before June 12, 2015 at 2 pm

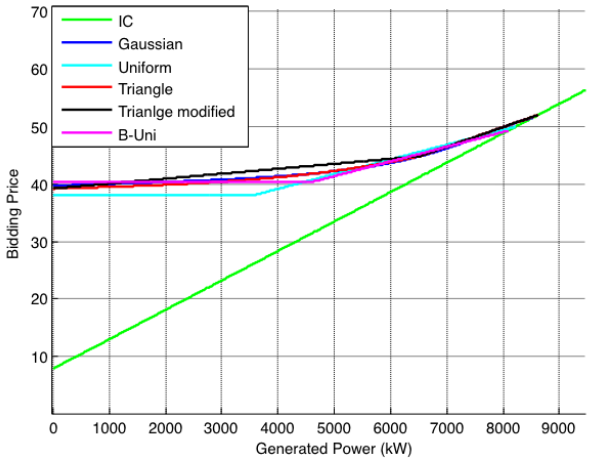

Figure 2.19 Comparing different PAB bidding functions based on UK market price for month before June 12, 2015 at 12 pm

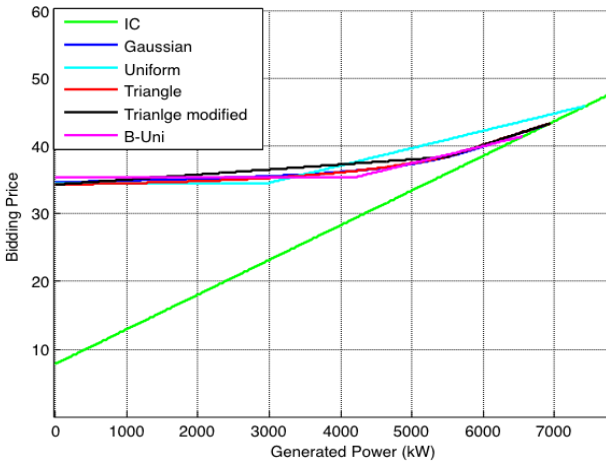

Figure 2.20 Comparing different PAB bidding functions based on UK market price for month before June 12, 2015 at 2 pm

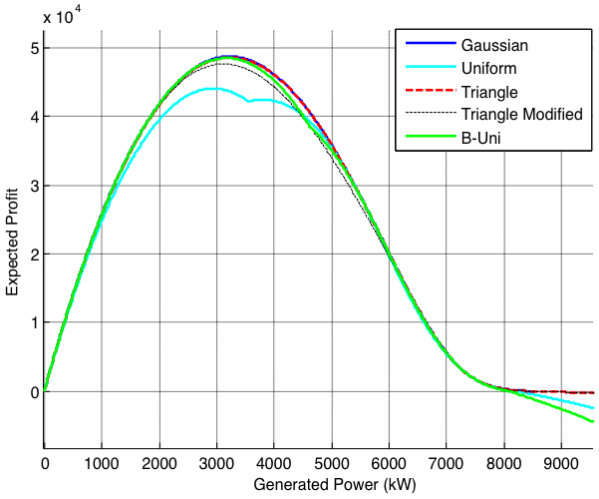

Figure 2.21 The expected profit of all proposed PAB bidding functions taking Gaussian expected profit as a reference based on UK market price for month before June 12, 2015 at 12 pm

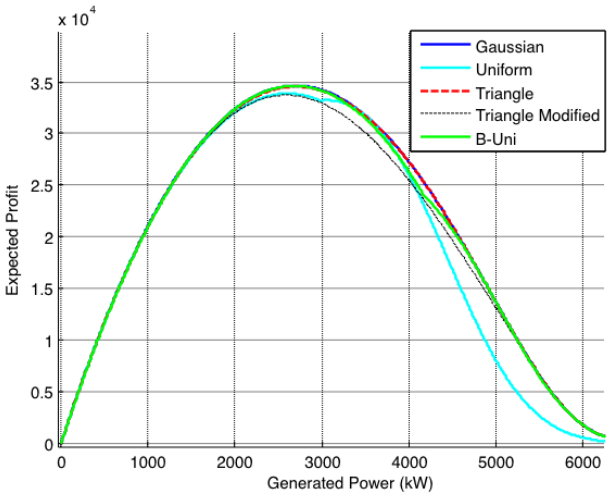

Figure 2.22 The expected profit of all proposed PAB bidding functions taking Gaussian expected profit as a reference based on UK market price for month before June 12, 2015 at 2 pm

2.4.8 *Numerical Test to Compare the Effectiveness of Uni and B-Uni Bidding Function with Respect to the Number of Units*

This section will test two of the proposed PAB bidding function. The first one is using the bidding function proposed by [88] in Table 2.1 which depends on a bidding function derived from

the Uniform distribution function. The second one will be using the B-Uni bidding function that was developed in previous as in Table 2.4.

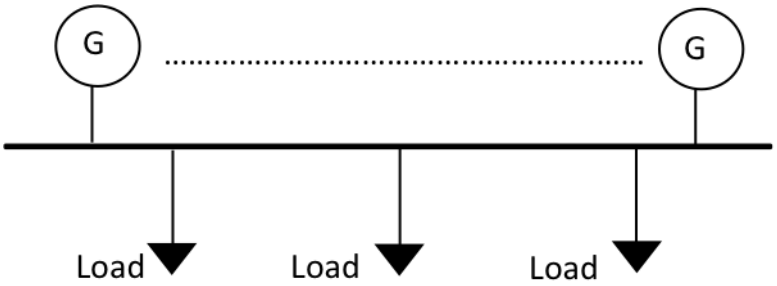

Figure 2.23 Single bus system with many generators

Table 2.5 Cost function coefficients [87]

	A	**B**	**C**
Gen (A)	78	7.80	0.002562
Gen (B)	78	7.88	0.004820
Gen (C)	100	7.97	0.004820
Gen (D)	561	8.00	0.004890
Gen (E)	50	7.85	0.001940

In order to test if the proposed bidding function will maximize generators profit as proven in the previous sections, a single bus system is assumed with many connected generators, see Figure 2.23. The cost functions of the generators are based on examples from [87] see Table 2.5. The hourly load profile data for a full year was derived from ERCOT load data [91]. A simulation is done assuming a variable number of generators comparing the difference between the real value of the bid that maximize profit of Gen (A) and the calculated value that maximizes its profit. The rest of the generators will bid optimal bidding strategy [92].

Test on PAB bidding function derived from Uniform distribution:

For the PAB bidding function derived from the Uni distribution, the bidding function based on Table 2.1 will be:

$$Bid_{PAB} = \frac{(\lambda_{max}^{MP} + IC_{gen}(P))}{2}$$

Since the incremental cost of the generators is

$$IC_{gen}(P) = B + 2 \cdot C \cdot P$$

Then

$$Bid_{PAB} = \frac{(\lambda_{max}^{MP} + (B + 2 \cdot C \cdot P))}{2}$$

$$Bid_{PAB} = \frac{\lambda_{max}^{MP} + B}{2} + C \cdot P \quad (2.36)$$

Where

λ_{max}^{MP} is the maximum recorded market price in a specific range.

λ is the market price.

B is coefficient "B" of the generator.

C is coefficient "C" of the generator.

In order to test if Gen (A) will maximize its profit using equation (2.36), equation (2.37) will be tested against a variable number of generators. The value of x will vary against Gen (A) profit. Gen (A) profit is expected to maximize when equation (2.38) is satisfied

$$Bid_{PAB} = x + C \cdot P \quad (2.37)$$

$$x = \frac{\lambda_{max}^{MP} + B}{2} \quad (2.38)$$

Test on PAB bidding function derived from B-Uni:

For the PAB bidding function derived from a B-Uni distribution, the bidding function based on Table 2.4 is

$$Bid_{PAB} = \frac{\mu + 1.538\sigma + IC_{gen}(P)}{2} \tag{2.39}$$

Since

$$IC_{gen}(P) = B + 2 \cdot C \cdot P$$

Then

$$Bid_{PAB} = \frac{\mu + 1.538\sigma + B}{2} + C \cdot P \tag{2.40}$$

Where

μ is the mean of market price for in a specific range.

B is coefficient "B" of the generator.

C is coefficient "C" of the generator.

In order to test if Gen (A) will maximize its profit using equation (2.40), equation (2.37) will be tested against a variable number of generators. The value of x will vary against Gen (A) profit. Gen (A) profit is expected to maximize when equation (2.41) is satisfied

$$Bid_{PAB} = x + C \cdot P \tag{2.37}$$

$$x = \frac{\mu + 1.538\sigma + B}{2} \tag{2.41}$$

A simulation is done on random times year round in order to check if proposed PAB bidding functions are effective. A single bus system is assumed with a **variable number of connected generators**, see Figure 2.23. The cost function of the generators are based on examples from [87], see Table 2.5. Each added generator to the single bus system after Gen (E) will repeat the generator data in Table 2.5 sequentially. The Load profile data used was derived from ERCOT load data [91].

Gen (A) cost function is

$$C_A(P_g) = A + B \cdot P_g + C \cdot P_g^2 \tag{2.42}$$

Then the bidding function should be

$$IC_A(P_g) = B + 2 \cdot C \cdot P_g \tag{2.43}$$

Six random days were chosen for the test; the results are in Figure 2.24 to Figure 2.29 where the x-axis represents increasing the number of generators based on generator data in Table C.1, while the y-axis is mismatch value between actual bid price of Gen (A) that maximize profit and proposed bidding price. It can be observed that bidding the proposed PAB bidding function has a low mismatch value in general. Having few generators in the system, bidding proposed PAB bidding does not maximize the profit of Gen (A) but it is not as bad as in the MP, while by increasing number of generators in the system it can be observed that bidding proposed PAB does not affect the mismatch a lot.

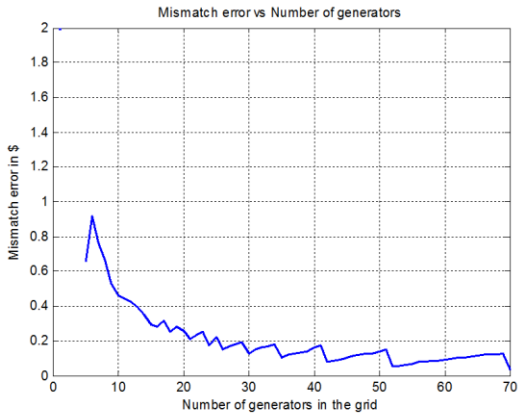

Figure 2.24 The mismatch between proposed bidding function (Uni) and actual value that maximize profit of Gen (A) in PAB market when varying the number of generators based on load at 4 am on April, 10th

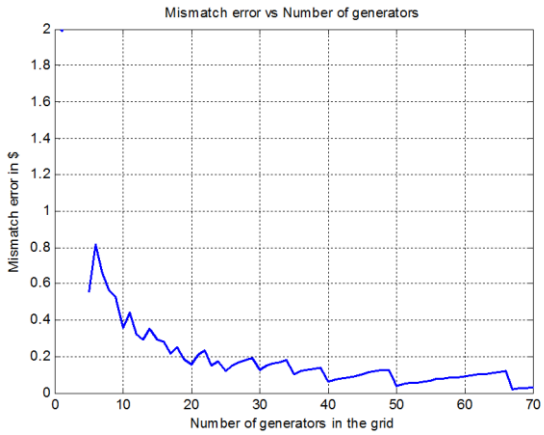

Figure 2.25 The mismatch between proposed bidding function (B-Uni) and actual value that maximize profit of Gen (A) in PAB market when varying the number of generators based on load at 4 am on April, 10th

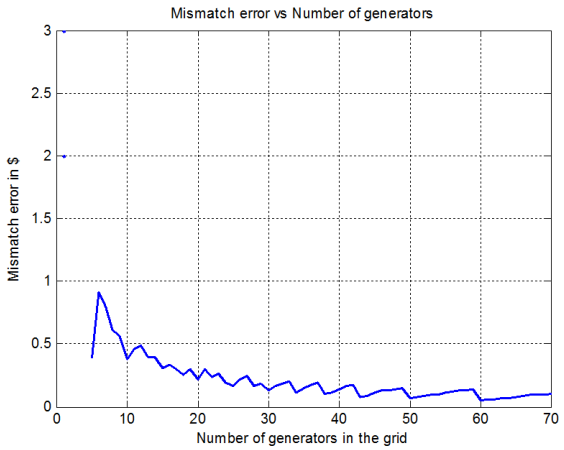

Figure 2.26 The mismatch between proposed bidding function (Uni) and actual value that maximize profit of Gen (A) in PAB market when varying the number of generators based on load at 3 am on July, 15th

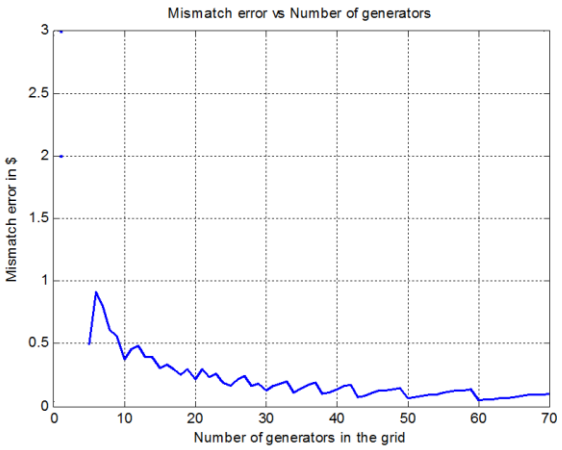

Figure 2.27 The mismatch between proposed bidding function (B-Uni) and actual value that maximize profit of Gen (A) in PAB market when varying the number of generators based on load at 3 am on July, 15th

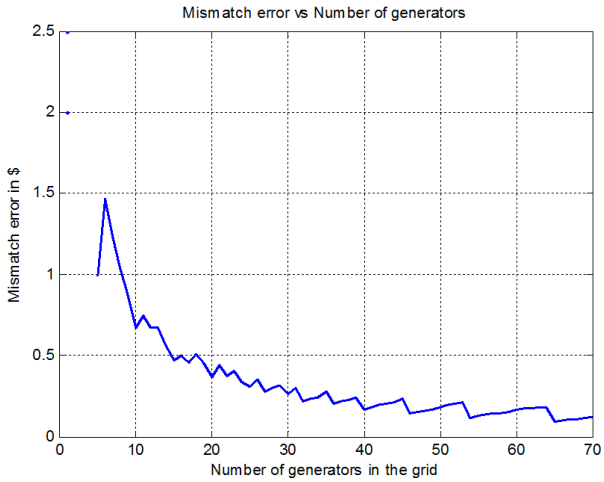

Figure 2.28 The mismatch between proposed bidding function (Uni) and actual value that maximize profit of Gen (A) in PAB market when varying the number of generators based on load at 7 pm on December, 23rd

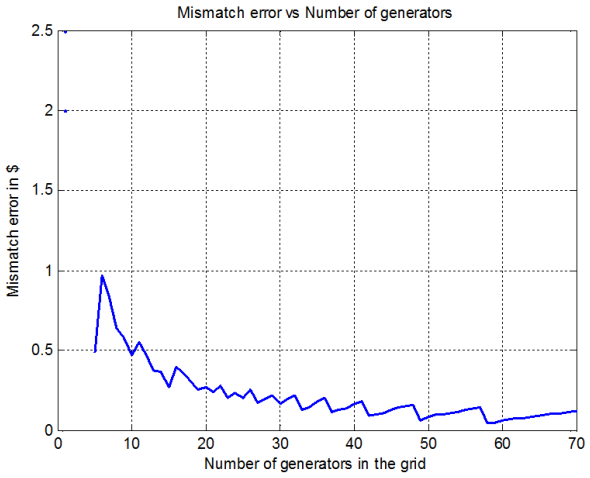

Figure 2.29 The mismatch between proposed bidding function (B-Uni) and actual value that maximize profit of Gen (A) in PAB market when varying the number of generators based on load at 7 pm on December, 23rd

This section examined a variable number of generators in a single bus model. More detailed values for a system with only 5 generators and a system with only 30 generators are explored in Appendix B.

2.4.9 *Numerical Test to Compare the Effectiveness of Uni and B-Uni Bidding Function with Respect to Other Bidding Units*

In order to compare the PAB Uni bidding function with the new PAB B-Uni bidding function, two simulations are done. The first test is having one gen bids B-Uni while the rest bid Uni. Results can be observed in Figure 2.30 where the left column is the profit of the generator if it bids B-Uni while the rest bid Uni, the right column is the generator profit if it bids Uni like the rest of the generators. It can be observed that when generator bids Uni it has less profit than when it bids B-Uni. The difference in profit is small but uniformly higher for B-Uni. The second test is having one gen bids Uni while the rest bid B-Uni. Results can be observed in Figure 2.29 where the left column is the profit of the generator if it bids Uni while the rest bid B- Uni, the right column is the generator profit if it bids B-Uni like the rest of the generators. Again this shows that when the generator bids B-Uni it has more profit when it bids Uni. Therefore this confirms the theory that bidding B-Uni is more profitable.

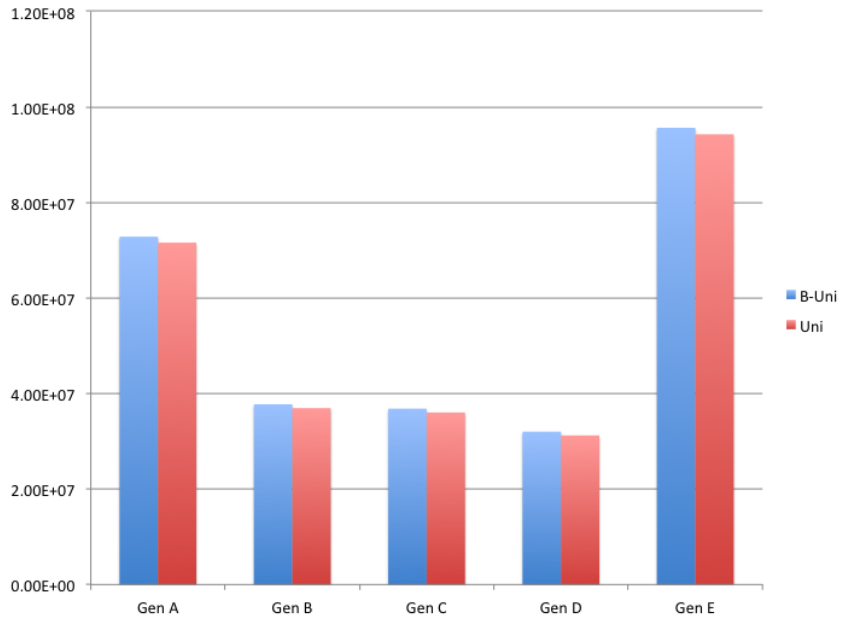

Figure 2.30 The profit of generators if one gen bids B-Uni (left column) while the rest bid Uni, the right column is the generator profit if it bids Uni

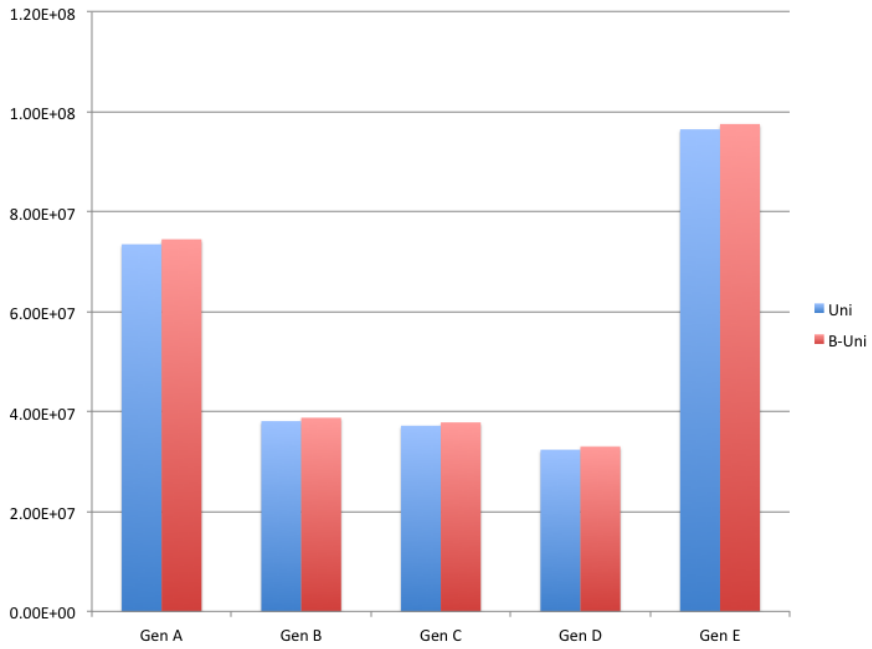

Figure 2.31 The profit of generators if one gen bids Uni (left column) while the rest bid B-Uni, the right column is the generator profit if it bids B-Uni

2.5 Bidding Strategy for Solar Power Plant in PAB Scheme

As discussed in the introduction of this chapter it is important in this research to adopt a bidding strategy for Solar Power Plants (SPP) as SPP combined bidding does affect the market price. As previously discussed the B-Uni bidding function is adopted as it is getting excellent results and easy to implement therefore it is adopted. Since there is no operational cost associated with solar power, SPPs will bid $\mu - 1.538\sigma$.

2.6 Conclusion

This chapter discussed the bidding strategy for conventional generators in the MP and the PAB schemes. Unlike MP bidding strategy which is well known, there is not much literature on bidding in the PAB scheme. That is due to the fact that there are just a few markets that use this scheme. Several PAB bidding strategies were discussed and developed

in this chapter. Based on this a bidding strategy for Solar Power Plants (SPP) was adopted for the MP and PAB market. Also, it is observed that the PAB bidding function depends on historical data in order to predict market price to maximize profit, while MP depends purely on the generator's incremental cost (*IC*). The PAB derived from B-Uni will be adopted as it has better results than the method used in [34] [88] [89] and its results are very close to results obtained from bidding function derived from the actual Gaussian distribution but the B-Uni simpler to implementing in programming.

Chapter 3. STUDYING THE EFFECT OF SOLAR POWER PLANT BIDDING ON DIFFERENT MARKET SCHEMES USING A FIVE BUS SYSTEM

3.1 INTRODUCTION

In an overview this chapter focuses on differences between Real Time (RT) performance and Day Ahead (DA) bids these differences can be due to deviation or congestion. The first part of this chapter identifies the financial effect of the MWh deviation of power producers between the DA market and the RT market in different market schemes. Solar power plant (SPP) participation affects different market structures differently principally because of the way that the market deals with deviation. The principle effect of SPP bidding on markets is a function of the way that the market resolves the bids and the way the market deals with a deviation between the DA bid amount and RT operation amount. This chapter focuses on the second. The effect of deviation on conventional generators will be explored first as it is easier to understand the effect of deviation on the generator's cost curve. Then the effect of deviation on SPP will be explored. This chapter confirms that in the MP scheme withholding some of the available energy in the DA market would increase the profit of conventional generators and Solar Power Plants (SPPs). In the PAB scheme, the financial effect of deviation is different between conventional generators and SPPs.

The second part of this chapter identifies the financial effect of SPP in the presence of congestion in different market schemes. A five-bus system is studied. The advantage of studying such a system that it is a small test system that has less computational time and the effect of transmission line congestion can be modeled. The effect of SPP bidding in a five-bus system on

the SPP's profit, generator's profit, consumer's surplus and the system social welfare is evaluated. The evaluation compares the four market structures discussed in detail in Chapter 1:

1) MP scheme that deals with congestion during the Day Ahead (DA) market, abbreviated as MP-DA.
2) MP scheme that deals with congestion in an adjustment market (AM) between DA and Real Time (RT) market, abbreviated as MP-AM.
3) PAB scheme that deals with congestion before gate closure (conceptually it is like the DA but in time it is much closer to RT), abbreviated as PAB-DA.
4) PAB scheme that deals with congestion in AM, abbreviated as PAB-AM.

In this chapter, the bidding strategy used by suppliers is the method discussed in chapter 2 assuming no subsidies or incentives are available. The PAB bidding strategy used for evaluation is the developed bidding strategy in section 2.4.5. Findings are that SPP makes more profit in the MP scheme than the PAB scheme in no congestion case, while in the presence of congestion, in the MP-DA scheme SPP makes more profit than rest of the schemes when located on the in the expensive side of the congestion. The findings of this chapter will be used as a guide for the large test system, the Saudi Grid. Table 3.1 and Table 3.2 show material covered in this chapter.

Table 3.1 Work done to study the effect of suppliers power mismatch between the DA and RT markets

	MP	PAB
Gen mismatch effect	√	√
SPP mismatch effect	√	√

Table 3.2 Work done to evaluate economic effect of SPP in different electricity market schemes

	MP-DA	MP-AM	PAB-DA	PAB-AM
Gen profit	√	√	√	√
SPP revenue	√	√	√	√
Consumer Surplus	√	√	√	√

3.2 ASSUMPTIONS

This study uses a five-bus test system with five generators obtained from PJM, with the addition of one solar power plant as seen in Figure 3.1 [29]. This research focuses on the implementation of solar power in Saudi Arabia. Hence, the solar power data was derived from the irradiation data profile of Jeddah City, Saudi Arabia obtained by NASA Remote Sensing Validation Data project, as seen in Figure 3.2. The solar irradiance data was collected with a CIMEL 8 channel sunphotometer [93].

Since there is no public daily load data available in Saudi Arabia, the load profile data was derived from an hourly load for a full year from the Electric Reliability Council of Texas (ERCOT) load database [91]. ERCOT load data was chosen because the load profile (residential, commercial, and industrial ratio) of Texas is close to Saudi Arabia load profile [94] [95]. Figure 3.3, and Figure 3.4 compare the temperature profile between the city of Dhahran in Saudi Arabia and Houston in Texas, USA. It can be observed that they are almost similar in shape but with a difference in average temperature, which might result in running more air conditioning units during summer [96] [97]. Generator data used in the five-bus system is obtained from [87] and is tabulated in Table 3.3. Although the generator data is fictional, it is close to real generator data obtained from the USA Envi-

ronmental Protection Agency [98] and should be sufficiently accurate to use for examples. More realistic generator data will be used in the next chapter.

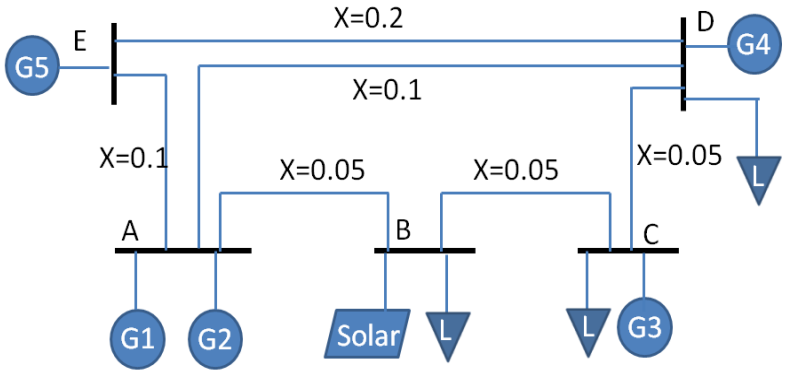

Figure 3.1 Five bus test system [29]

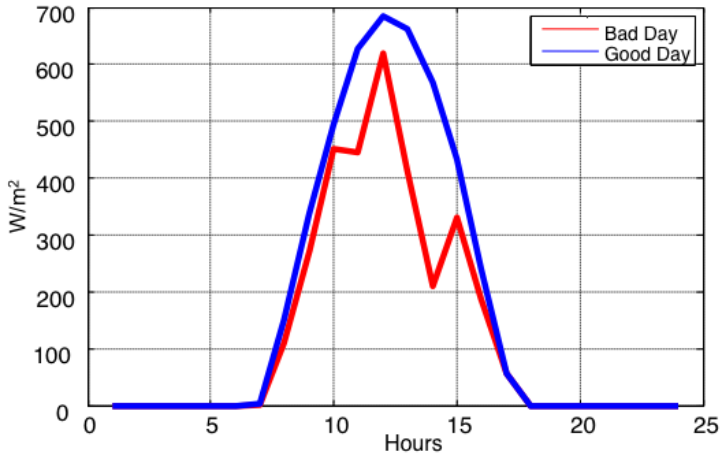

Figure 3.2 A sample of solar power profile in a good day vs bad day [93]

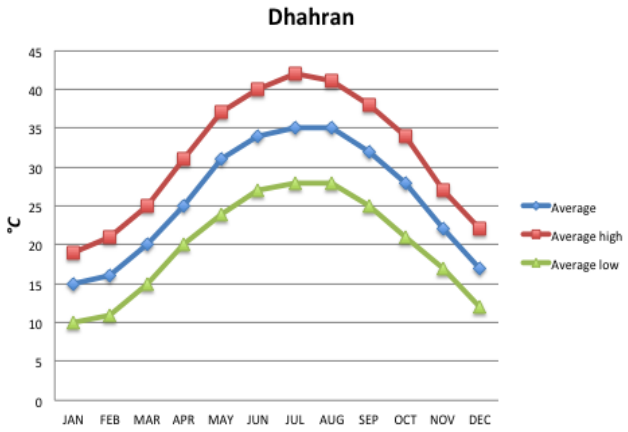

Figure 3.3 Temperature profile for Dhahran Saudi Arabia [96]

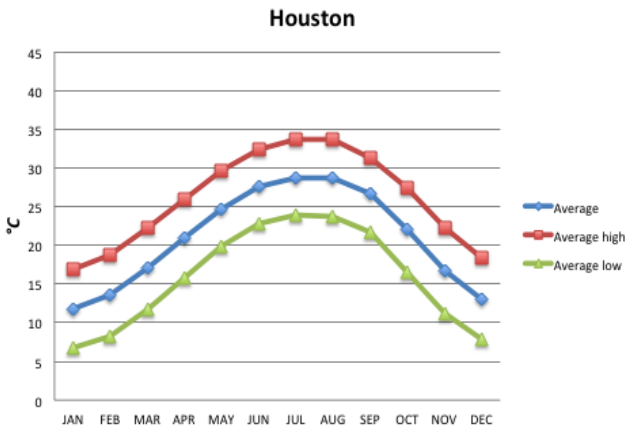

Figure 3.4 Temperature profile for Houston, TX [97]

The generators' cost functions are quadratic [87]:

$$C(P_g) = A + B \cdot P_g + C \cdot P_g^2 \tag{3.1}$$

Table 3.3 Cost function coefficients [87]

	A	B	C
Gen 1	78	7.80	0.002562
Gen 2	78	7.88	0.004820
Gen 3	100	7.97	0.004820
Gen 4	561	8.00	0.004890
Gen 5	50	7.85	0.001940

The system under study does not consider generator start up and shut down costs. Persistence forecasting is assumed for solar power prediction and load forecast. In persistence forecasting, the predicted value is the same value as the previous day at the same time. This method was discussed in section 1.7.1. It is assumed that demand has zero price elasticity because load behavior does not change in the short run in response to a change in electricity price, and no demand response is present. Generators in the MP scheme are assumed to bid their Incremental

Cost (IC) as discussed in detail in section 2.2. On the other hand, generators in the PAB scheme are assumed to bid using the optimal PAB bidding strategy discussed in section 2.4, Table 2.4.

3.3 The Effect of Power Deviation on Supplier's Profit in the MP and PAB Schemes

This section explores whether the effect of low-cost generation (i.e. solar) deviation on its profit is different from the effect of conventional generator deviation on its profit. This section assumes a power supply deviation (mismatch) between the DA and RT markets in both schemes for the five-bus test system assuming no congestion exists. As was discussed in section 1.4 there are mainly two schemes, the MP and the PAB schemes. Each main scheme is divided into two schemes depending on the way congestion is managed. However, since this section focuses only on deviation assuming no congestion, no AM is needed. Therefore, the comparison is simply between the MP and PAB schemes.

3.3.1 *The Effect of Power Deviation on Conventional Generator Profits*

This section explores the effect of MWh deviation between the DA market settlement and the actual RT market on a conventional generator's profit. Basically, there are three types of deviation: small operational deviation, outage deviation (large) and intentional deviation. Intentional deviation is not permitted by ISOs as it could lead to power system distortion and imbalance. In order to study the effect of generator deviation, it is assumed that only generator 1 (Gen 1) will have a deviation during the RT market while the rest of the generators will compensate as discussed in section 1.4. It is assumed that there is no load deviation between the DA and RT markets as the objective is to study only the effect of generator deviation on generator profit.

3.3.1.1 Theoretical Analysis of Deviation in the MP Scheme

This section covers the theoretical analysis of power mismatch between the DA and RT markets. The first step of deviation evaluation is to explore a no deviation case. Figure 3.5 shows the profit of Gen 1 assuming linear cost functions in in the event of no power deviation, where D is the total demand, Q is MWh bid in the DA market, and Q^* is the actual MWh produced in the RT market. Equation (3.2) shows the profit of Gen 1 in DA market which is the product of market clearing price at DA (p) with the amount of MWh (Q) that Gen 1 had bid in DA market subtracted from cost.

$$Profit\ in\ now\ deviation = p \cdot Q - cost \tag{3.2}$$

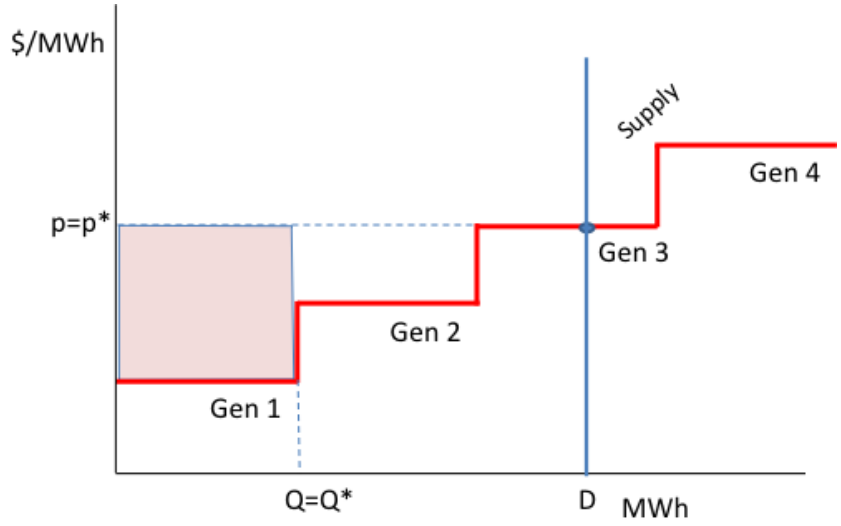

Figure 3.5. Generator's profit in MP scheme assuming no deviation between DA bid and RT production

If Gen 1 deviates from its DA bid amount Q, and produces Q^* during RT market, then the market adjusts and the new RT market settling price p^* is obtained. The profit of Gen 1 in the RT market is the amount of deviation ($Q^* - Q$) multiplied by market settling price at RT (p^*) subtracted from cost as seen in equation (3.3).

$$Profit\ in\ RT = p^* \cdot (Q^* - Q) - cost \tag{3.3}$$

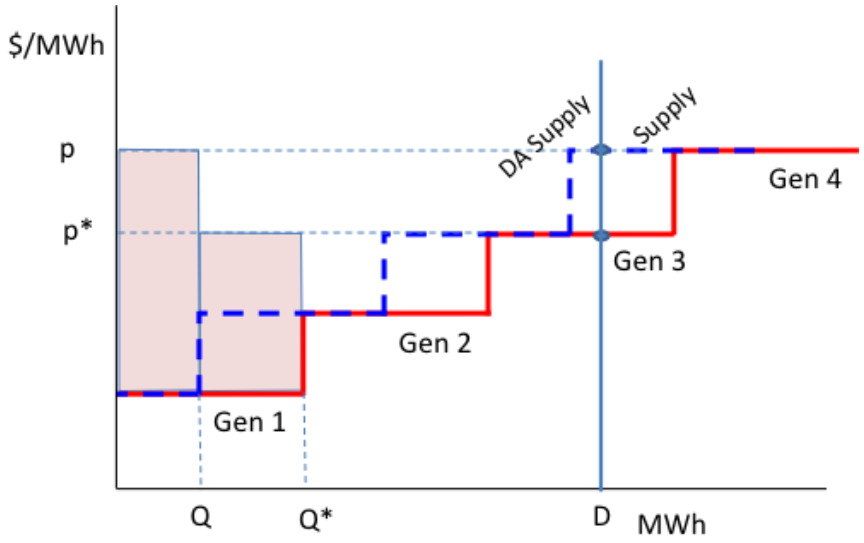

Figure 3.6. Generator's profit in MP scheme when it over-produce during the RT market. The dotted blue line is the bid in the DA market. The solid red line is the actual energy produced in the RT market

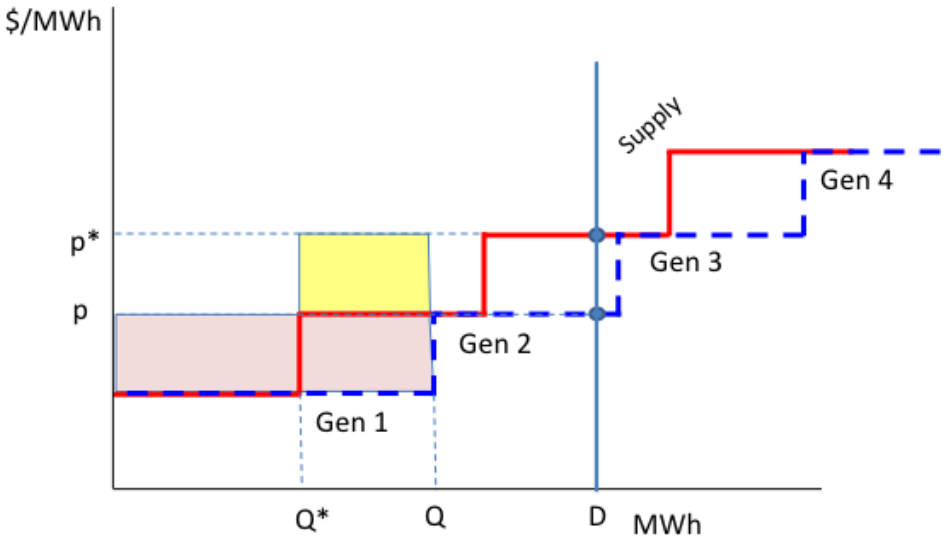

Figure 3.7. Generator's profit in MP scheme when it under-produce during the RT market. The dotted blue line is the bid in the DA market. The solid red line is the actual energy produced in the RT market

Equation (3.4) shows the total revenue of Gen 1. Figure 3.6 shows calculating Gen 1 total profit when Gen 1 over produces in the RT market. Equation (3.5) represents the profit due to

deviation in the case of a power mismatch between the DA and RT markets. It can be observed that over-producing in the RT market ($Q < Q^*$) increases Gen 1 profit. Figure 3.7 shows Gen 1 profit when Gen 1 under-produces in RT market. The yellow area in the figure represents the revenue loss. It can be observed that under-producing in the RT market ($Q^* < Q$) will decrease Gen 1 profit. Therefore generators have a financial motivation to withhold some of their MWh quantity bid during DA market. That is the reason intentional deviation is not permitted by ISOs as it cause market inefficiency and loss of social welfare.

$$Total\ Revenue = p \cdot Q + p^* \cdot (Q^* - Q) \tag{3.4}$$

$$Deviation\ Profit = (p^* - p) * Q \tag{3.5}$$

3.3.1.2 Simulation of Deviation Effects in the MP Scheme

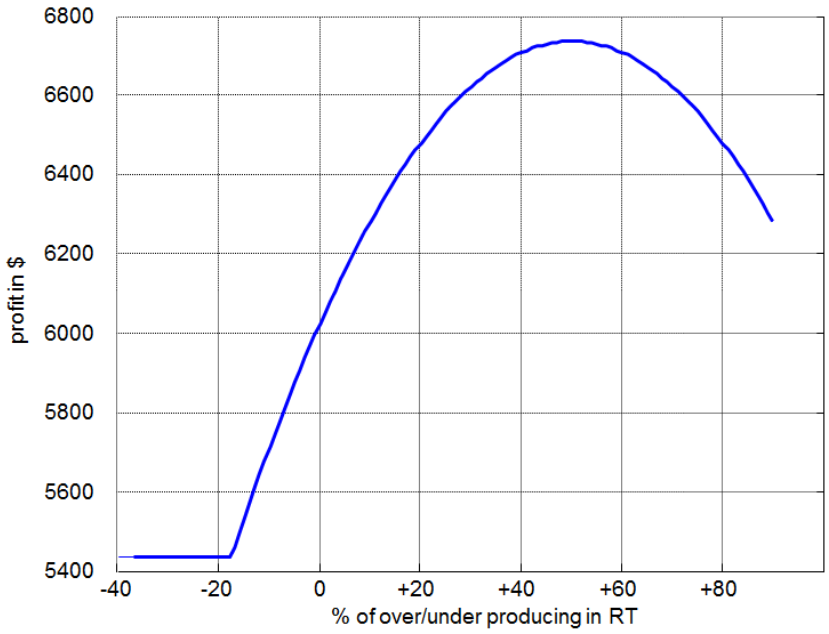

Figure 3.8 In the MP scheme Gen 1's profit when over/under producing in RT market based on ERCOT load at 6 am on September 16, 2014

To confirm the previous analysis, a simulation of deviation by Generator 1 (Gen 1) in the five bus system was performed. The assumptions in section 3.2 were used assuming a simulation for only one hour. To properly compare cases the RT market solution is held constant to study the effect on profit. Generator 1 arrives at its RT MWh value through a combination of DA bid and deviation. The extent of Generator 1 deviation is quantified as a percentage of the RT amount. Positive deviation represents over-production while negative deviation represents under-production. The results are plotted in Figure 3.8. It can be observed that over-producing in the RT market will increase profit, while under-producing will decrease profit[4]. The simulation confirms the discussion in the previous section. An observation in the figure is that the profit will be flat if generator price bid exceeds market-clearing price.

3.3.1.3 Example on Calculating the RT Market Price in the PAB Scheme

This section presents an example illustrating how the PAB scheme calculates RT market price. This is important to lay the foundation for theoretical discussion in next section on the effect of producers power mismatch between the DA market and the RT market. The RT price is the MWh weighted average price of accepted adjustment bids. If a producer deviates by over-producing or under-producing in the RT market, then it will get paid/or pays based on the weighted average price of accepted adjustment bids of all entities participating in the RT market. This price is called Average Adjustment Price (AAP), see equation (3.6). Other units will pay based on their adjustment bid [38].

$$Average\ Adjustment\ Price\ (AAP) = \frac{\sum_{n=1}^{k} AP_k \cdot \Delta P_k}{\sum_{n=1}^{k} \Delta P_k} \tag{3.6}$$

Where

[4] It can be shown mathematically that the profit is maximized at deviation of 50% in Appendix D

AP_k is the Adjustment Price for generator *k*.

ΔP_k is the MWh adjustment amount of generator *k*.

Table 3.4 Example of bids in a PAB scheme

DA market	**MWh**	**$**
Gen A accepted DA bid	50	30
Gen B accepted DA bid	50	27
Gen B increase adjustment bid	3	32
Gen B decrease adjustment bid	3	22
Gen C accepted DA bid	50	26
Gen C increase adjustment bid	2	31
Gen C decrease adjustment bid	2	21

For example, assume a PAB scheme where three generators had accepted bids in the DA market. Gen A contracted to produce 50 MWh for $30, Gen B contracted to produce 50 MWh for $27, and Gen C contracted to produce 50 MWh for $26 as in Table 3.4. Suppose that in the RT market Gen A produced only 45 MWh (5 MWh short) due to a mechanical issue. Then the system will be short 5 MWh, the ISO will accept Gen B's adjustment bid of a 3 MWh increase for $32, and will accept Gen C's adjustment bid of a 2 MWh increase for $31. Then Gen A has to pay the ISO the weighted price of $31.6 for the 5 MWh deviation. This price will be called the **Buy AAP** (buy from ISO). In the MP scheme, the RT price would've been the price of the last dispatched generator which will make the RT market price $32.

Now if during the RT market, Gen A generated an extra 5 MWh for a process reason, then the system will have an extra 5 MWh (long). The ISO will accept other generator's decrease adjustment bids. It will request Gen B to decrease its production by 3 MWh, and Gen B will re-imburse the ISO for $22 per MWh. The ISO will request Gen C to decrease its production by 2 MWh for a reimbursement price of $21 per MWh. Then Gen A will get paid a weighted average

price of only \$21.6. This price will be called the **Sell AAP** (Sell to ISO). In the MP scheme, the RT price would have been the price of the cheapest accepted bid so the RT market price would have been \$21.

3.3.1.4 Theoretical Analysis of Deviation in the PAB Scheme

This section explores the impact of power mismatch between the DA and RT markets in the PAB scheme. It is assumed that load does not change between the DA and RT market, and all generators are bidding the PAB bidding function explained earlier in section 2.4.5 in Table 2.4. Remember as explained in detail in section 1.4.2, generators in the PAB scheme get paid the price they bid. Then if Gen 1 has a mismatch between DA and RT its revenue will be one of the following cases:

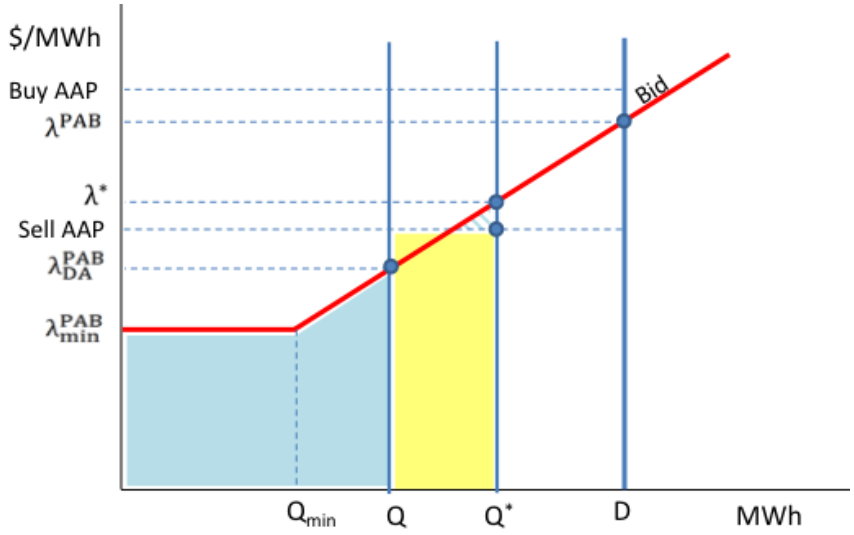

Figure 3.9. Generator revenue in the PAB scheme when it over-produces MWh in the RT market that results in a DA bid price $\lambda_{DA}^{PAB} < Sell\ AAP < \lambda^{PAB}$

a) Gen 1 over-produces MWh in the RT market (MWh contracted in DA ($\boldsymbol{Q}$) < MWh produced in RT ($\boldsymbol{Q}^*$)) that results in a DA bid price $\lambda_{DA}^{PAB} < Sell\ AAP < \lambda^{PAB}$(the price of the highest accepted bid in the DA market). Figure 3.9 is not to

scale but simplifies understanding the revenue of the generator where the DA revenue is described in equation (3.7). This is the area in the figure under the red line with the intersection of λ_{DA}^{PAB} (in light blue). The revenue during RT is the amount of deviation $(Q^* - Q)$ multiplied by Sell AAP which depends on the weighted average of adjustment bids that other generators submitted in the DA market (before gate closure) as was explained in section 1.4.2 (the area in yellow). Since this price is the weighted average, its value should be less than but close to the λ^{PAB} price. Gen 1 will have an opportunity loss as if it had bid correctly in DA then it would have got the "λ^* price" but since it deviated it is paid less. It can be observed that a large deviation might increase revenue. Note: intentional deviation is not allowed.

$$DA\ Rev = \lambda_{min}^{PAB} \cdot Q + 0.5 \cdot (Q - Q_{min}) \cdot \left(\lambda_{DA}^{PAB} - \lambda_{min}^{PAB}\right) \tag{3.7}$$

$$RT\ Rev = (Q^* - Q) \cdot Sell\ AAP \tag{3.8}$$

$$Total\ Rev = \lambda_{min}^{PAB} \cdot Q + 0.5 \cdot (Q - Q_{min}) \cdot \left(\lambda_{DA}^{PAB} - \lambda_{min}^{PAB}\right) + (Q^* - Q) \cdot Sell\ AAP \tag{3.9}$$

Where

Q is the MWh contracted at DA.

λ_{DA}^{PAB} is the DA bid price associated with generator's bidding Q MWh.

λ^{PAB} is the price of the highest accepted bid in the DA market.

D is the amount of MWh that the generator can produce at price λ^{PAB} .

λ_{min}^{PAB} is the minimum market price recorded in a rolling window period.

Q_{min} is the amount of MWh delivered when bidding $\lambda_{\min}^{\mathrm{PAB}}$.

Q^* is the actual energy delivered in the RT market.

λ^* is the DA bid price if the generator would have bid Q^* MWh in DA.

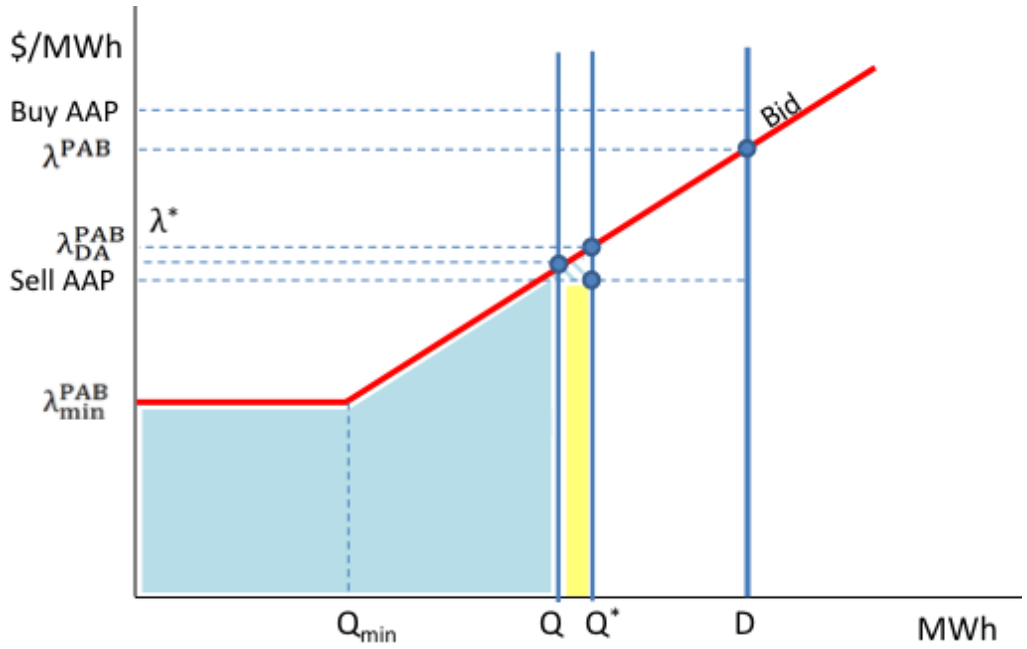

Figure 3.10. Generator revenue in the PAB scheme when it slightly over-produces MWh in the RT market that results in a $Sell\ AAP < \lambda_{DA}^{PAB} < \lambda^{PAB}$

b) Gen 1 over-produces MWh in the RT market ($Q < Q^*$) with a small deviation, which results in price $Sell\ AAP < \lambda_{DA}^{PAB} < \lambda^{PAB}$. Figure 3.10 is not to scale but simplifies understanding the revenue of the generator. The DA revenue is described in equation (3.7). This is the area in the figure under the red line with the intersection of λ_{DA}^{PAB} (in light blue). The revenue during RT is the amount of deviation ($Q^* - Q$) multiplied by Sell AAP which depends on the weighted average of adjustment bids that other generators submitted in the DA market (before gate closure) as was explained in section 1.4.2 (the area in yellow). Since this price is a weighted average, its value should be less but close to the λ^{PAB} price. Gen 1 will always have an opportunity loss as if it had bid correctly in DA then it would have got the "λ^* price" but since it deviated it is paid less.

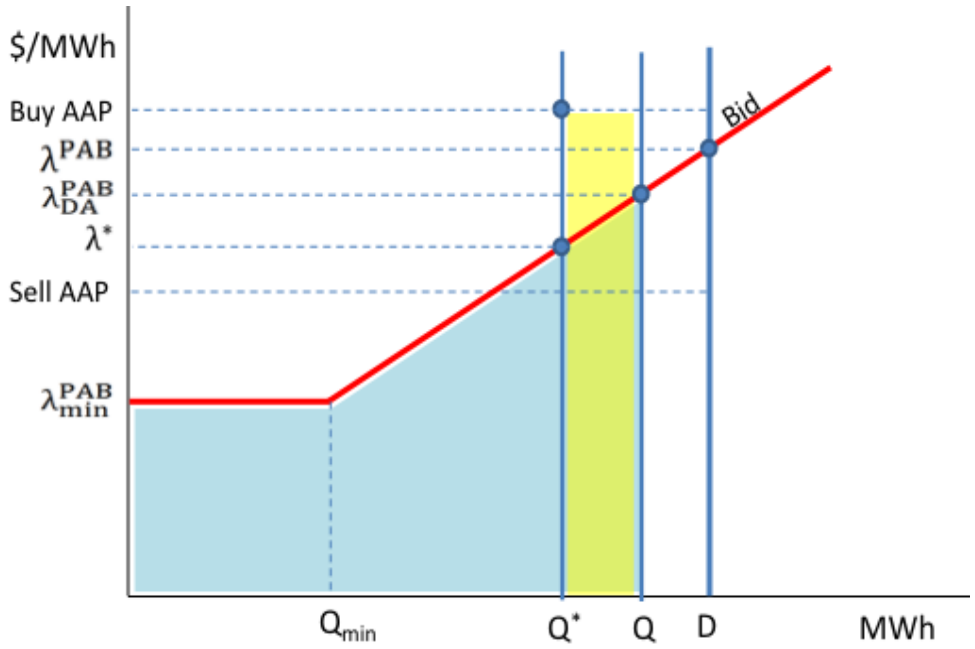

Figure 3.11. Generator revenue in the PAB scheme when it under-produce MWh in the RT market that results in a DA bid $\lambda_{DA}^{PAB} < \lambda^{PAB}$

c) Gen 1 under-produces MWh in the RT market ($Q^* < Q < D$), which results in a DA bid price $\lambda_{DA}^{PAB} < \lambda^{PAB}$. Figure 3.11 is not to scale but simplifies understanding the revenue of the generator. The DA revenue is described in equation (3.7). This is the area in the figure under the red line with the intersection of λ_{DA}^{PAB} (in light blue). The revenue (penalty) during RT is the amount of deviation ($Q^* - Q$) multiplied by the Buy AAP as in equation (3.10) (the area in yellow). Since the Buy-APP price is a weighted average, its value should be higher than but close to the λ^{PAB} price. Gen 1 will always lose in this case as if it had bid correctly in the DA market then it would have got the "λ^* price" but since it deviated it has to pay the ISO more.

$$DA\ Rev = \lambda_{min}^{PAB} \cdot Q + 0.5 \cdot (Q - Q_{min}) \cdot \left(\lambda_{DA}^{PAB} - \lambda_{\min}^{PAB}\right) \quad (3.7)$$

$$RT\ Rev = (Q^* - Q) \cdot Buy\ AAP \tag{3.10}$$

$$Total\ Rev = \lambda_{min}^{PAB} \cdot Q + 0.5 \cdot (Q - Q_{min}) \cdot \left(\lambda_{DA}^{PAB} - \lambda_{min}^{PAB}\right) + (Q^* - Q) \cdot Buy\ AAP \tag{3.11}$$

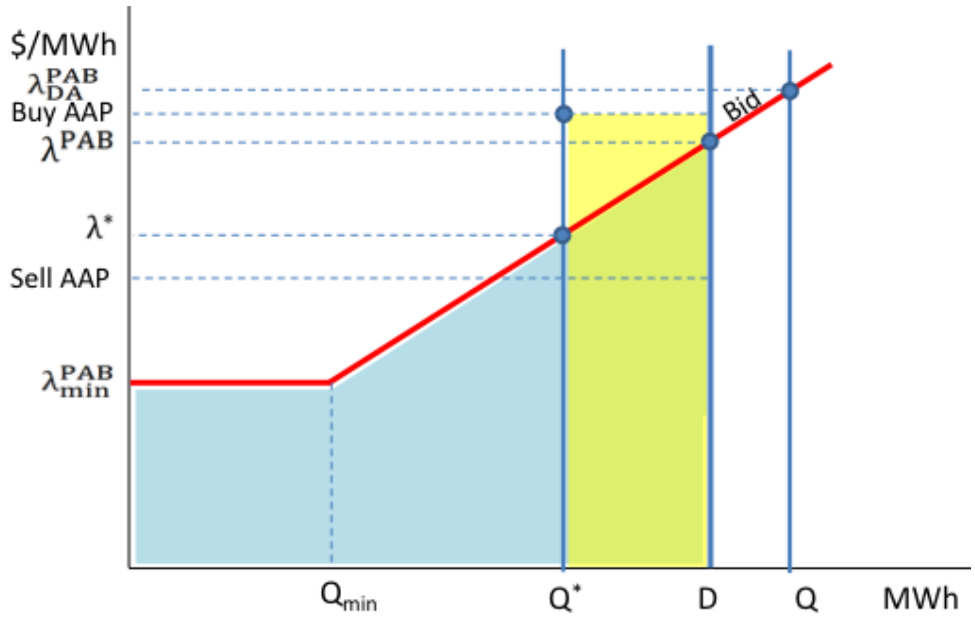

Figure 3.12. Generator revenue in the PAB scheme when it under-produces MWh in the RT market that results in a DA bid $\lambda_{DA}^{PAB} > \lambda^{PAB}$

d) Gen 1 under-produces MWh in the RT market ($Q > D > Q^*$), which results in a DA price $\lambda_{DA}^{PAB} > \lambda^{PAB}$. Since the DA bid price $\lambda_{\mathrm{DA}}^{\mathrm{PAB}}$ is higher than the highest accepted bid in DA market λ^{PAB} then this bid will not be accepted and therefore the MWh accepted bid would be D with the price of λ^{PAB}. Figure 3.12 is not to scale but simplifies understanding the revenue of the generator. The DA revenue is described in equation (3.12) which is in the area in the figure under the red line with the intersection of λ^{PAB} (in light blue). The revenue (penalty) during RT is the amount of deviation ($Q^* - D$) multiplied by the Buy AAP as in equation (3.13) (the area in yellow). Since the Buy AAP price is a weighted average,

its value should be higher than but close to the λ^{PAB} price. Gen 1 will always lose in this case as if it had bid correctly in DA then it would have got the "λ^* price" but since it deviated it has to pay the ISO more.

$$DA\ Rev = \lambda_{min}^{PAB} \cdot D + 0.5 \cdot (D - Q_{min}) \cdot \left(\lambda^{PAB} - \lambda_{min}^{PAB}\right) \tag{3.12}$$

$$RT\ Rev = (Q^* - D) \cdot Buy\ AAP \tag{3.13}$$

$$Total\ Rev = \lambda_{min}^{PAB} \cdot D + 0.5 \cdot (D - Q_{min}) \cdot \left(\lambda^{PAB} - \lambda_{min}^{PAB}\right) + (Q^* - D) \cdot Buy\ AAP \tag{3.14}$$

3.3.1.5 Simulation of Deviation Effects in the PAB Scheme

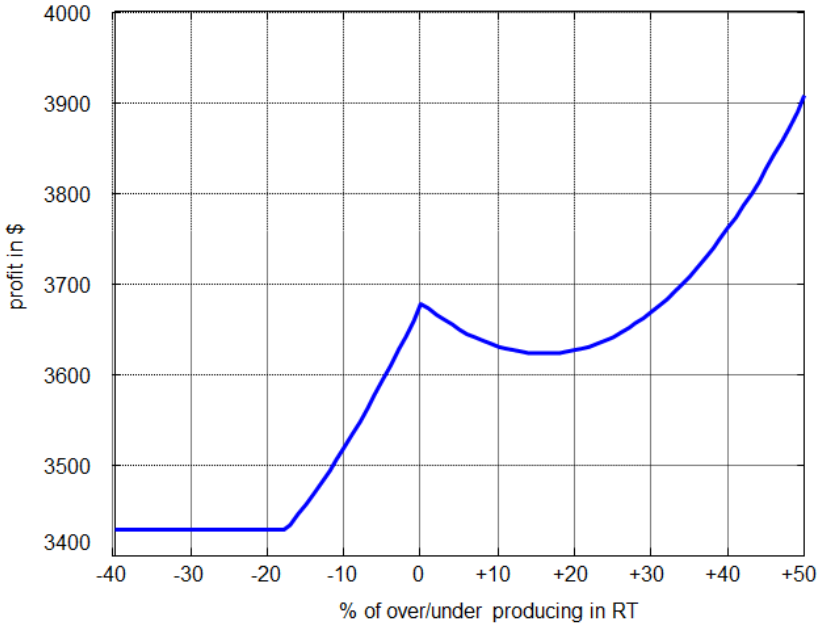

Figure 3.13. PAB conventional Gen 1's profit when over/under producing in RT market based on ERCOT load at 6 am on September 16, 2014

To confirm the previous analysis, a simulation of Gen 1 was performed. The assumptions in section 3.2 were used for one hour simulation. To properly compare cases, the RT market solution is held constant for studying the effect on profit. Gen 1 arrives at its RT MWh value

through a combination of DA bid and deviation. The extent of Gen 1 deviation is quantified as a percentage of the RT amount. Positive deviation represents over-production while negative deviation represents underproduction.

The results are plotted in Figure 3.13. This figure confirms the discussion in the previous section, where it is clear that generators in the PAB scheme are more motivated to have a large deviation between the DA market and RT market as deviation increase there profit. Current markets that utilize the PAB scheme do not allow having an intentional mismatch between the DA market and RT market. The flat line in the figure represents the portion where the generator bids in the DA market higher than PAB market price λ^{PAB}.

3.3.2 *The Effect of Power Deviation of SPP on SPP Profit*

Since solar power is stochastic, it is very likely to have a power mismatch between the DA market and the RT market. Some markets are already experiencing a high level of solar penetration (i.e. in California solar power produces more that 10% of total generation during peak hours [22]). In a system with high solar power penetration weather conditions could have a severe effect on the grid. It is important to explore the financial effect of SPP deviation between the DA and RT markets on both schemes and to compare them with of conventional generators deviation. In order to focus on the SPP deviation, it is assumed that there is no load deviation between the DA and RT market. Only the SPP will have a deviation while the rest of the generators will not deviate between the DA and RT market.

3.3.2.1 Theoretical Analysis of Deviation in the MP Scheme

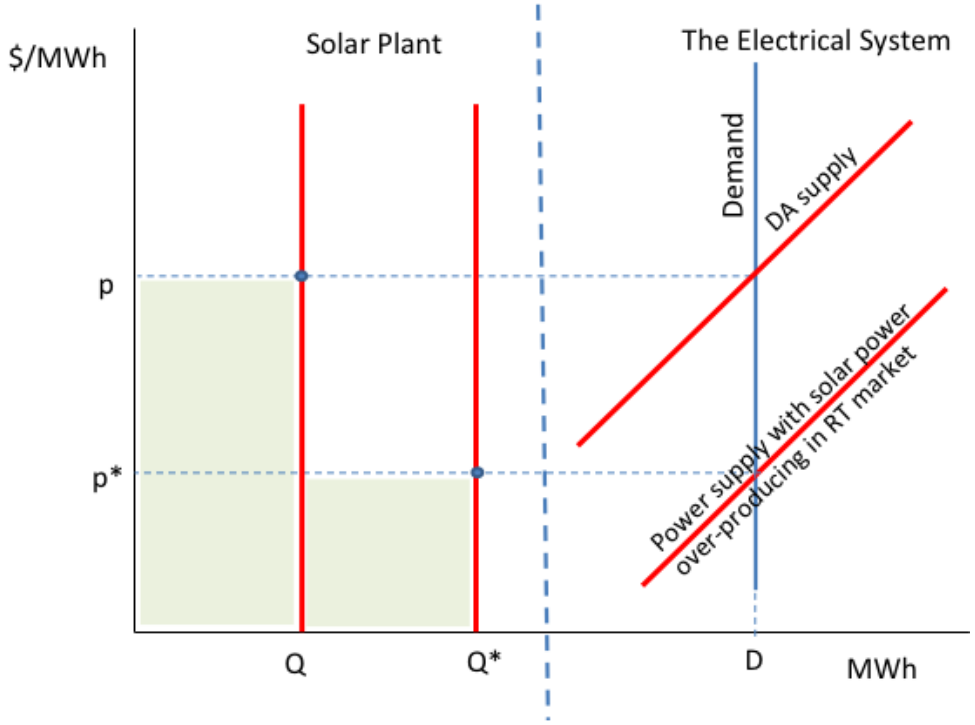

Figure 3.14 Supply-demand curves with the effect of over-producing solar power in MP scheme

In the MP scheme, in the case where the SPP contracts Q MWh in the DA market then over-produces in the RT market a total amount of Q^* MWh as in Figure 3.14, the DA market settlement price would be p. In the RT market, the SPP over-produces an amount of Q^* MWh, this will cause the supply curve to shift to the right as there is no associated operational cost with solar power which will lower the settlement price in the RT market to price p^*. Equations (3.15), (3.16), and (3.17) represent the solar revenue. It can be observed from Figure 3.14 (the area in light green) that the deviation revenue gained by SPP mismatch between the DA and RT markets can be represented in equation (3.18). So the SPP can gain more revenue in case of over-producing in the RT market.

$$DA\ SPP\ revenue = Q \cdot p \tag{3.15}$$

$$RT\ SPP\ revenue = (Q^* - Q) \cdot p^* \quad (3.16)$$

$$Total\ revenue = Q \cdot p + (Q^* - Q) \cdot p^* \quad (3.17)$$

$$Deviation\ Revenue = Q \cdot (p - p^*) \quad (3.18)$$

Where

Q is the amount of power that generator contracted at DA.

Q^* is amount of power that generator delivered at RT.

p is the price at DA.

p^* is the price at RT.

D is the total demand.

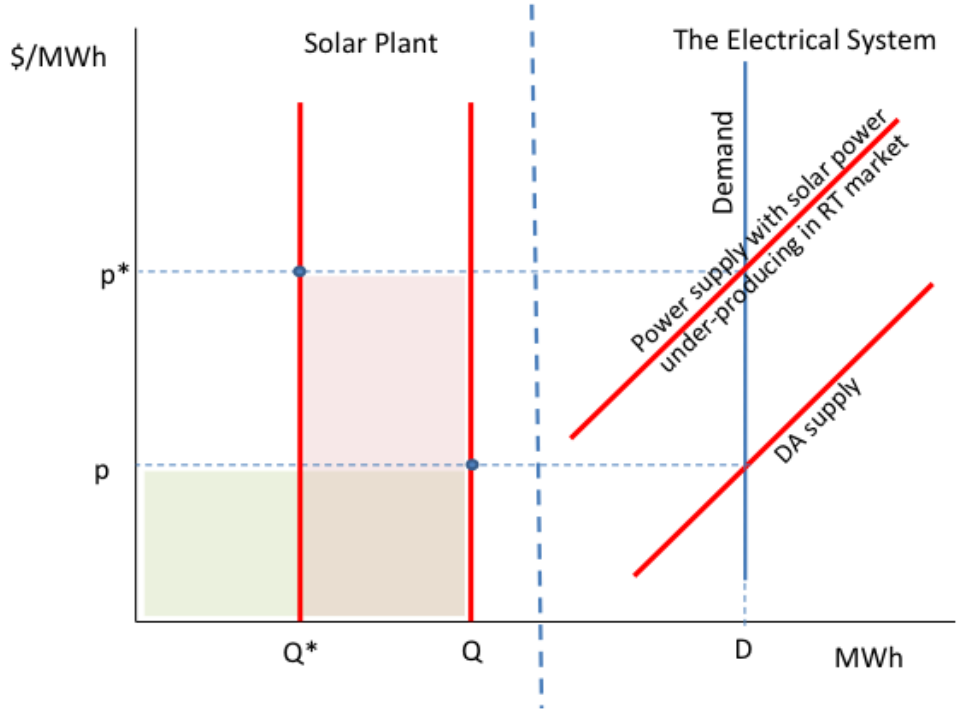

Figure 3.15 Supply-demand curves with the effect of under-producing solar power in MP scheme

In the case of SPP contracts Q MWh in the DA market and then under-produces in the RT market an amount of Q^* MWh as in Figure 3.15, the DA market settlement price would be p. In

the RT market, the SPP under-produces an amount of Q^* MWh, this will cause the supply curve to shift to the left which will raise the settlement price in the RT market to price p^*. Equations (3.19), (3.20), and (3.21) represent the solar revenue, deviation penalty, and total revenue. It can be observed from Figure 3.15, (the area in light red) that the SPP deviation penalty increases with under-production. So the SPP actually loses revenue when under-producing in the RT market.

$$DA\ SPP\ revenue = Q \cdot p \tag{3.19}$$

$$RT\ SPP\ penalty = (Q - Q^*) \cdot p^* \tag{3.20}$$

$$Total\ revenue = Q \cdot p - (Q - Q^*) \cdot p^* \tag{3.21}$$

3.3.2.2 Simulation of Financial Effects of SPP Deviation in the MP Scheme

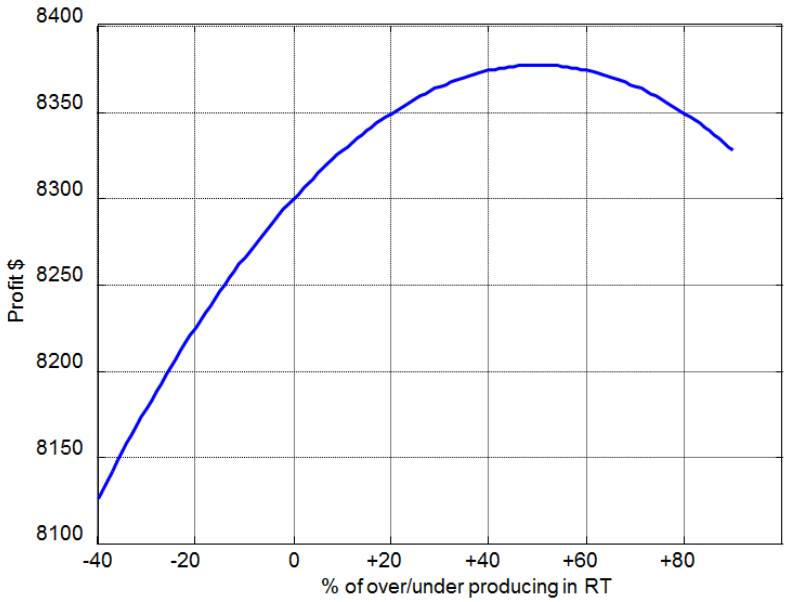

Figure 3.16 In the MP scheme, SPP profit when over/under producing in RT market based on ERCOT load at 12 pm on September 16, 2014

To confirm the previous analysis, a simulation of deviation by SPP in the five-bus system was performed. The assumptions in section 3.2 were base on one hour load. To properly compare

cases the RT market solution is held constant to study the effect on profit. SPP arrives at its RT MWh value through a combination of DA bid and deviation. The extent of SPP deviation is quantified as a percentage of the RT amount. Positive deviation represents over-production while negative deviation represents under-production. The results are plotted in Figure 3.16. It can be observed that over-producing in the RT market will increase profit, while under-producing will decrease profit. The simulation confirms the discussion in the previous section. It is clear that SPP in the MP scheme is financially motivated to underbid MWh in the DA market. It is observed that this case is similar to the conventional generator deviation case.

3.3.2.3 Theoretical Analysis of Deviation in the PAB Scheme

In the PAB scheme participants are bidding two things at the same time, the MWh amount (quantity) and the price. In case where the SPP deviates from the contracted MWh in the DA market (before gate closure), then it will get paid/buy AAP value which depends on the value of the adjustments bids. These will be different than the highest accepted bid (λ^{PAB}) in the DA market as explained in the example in section 3.3.1.3. Theoretically, there will be four cases to explore. They are:

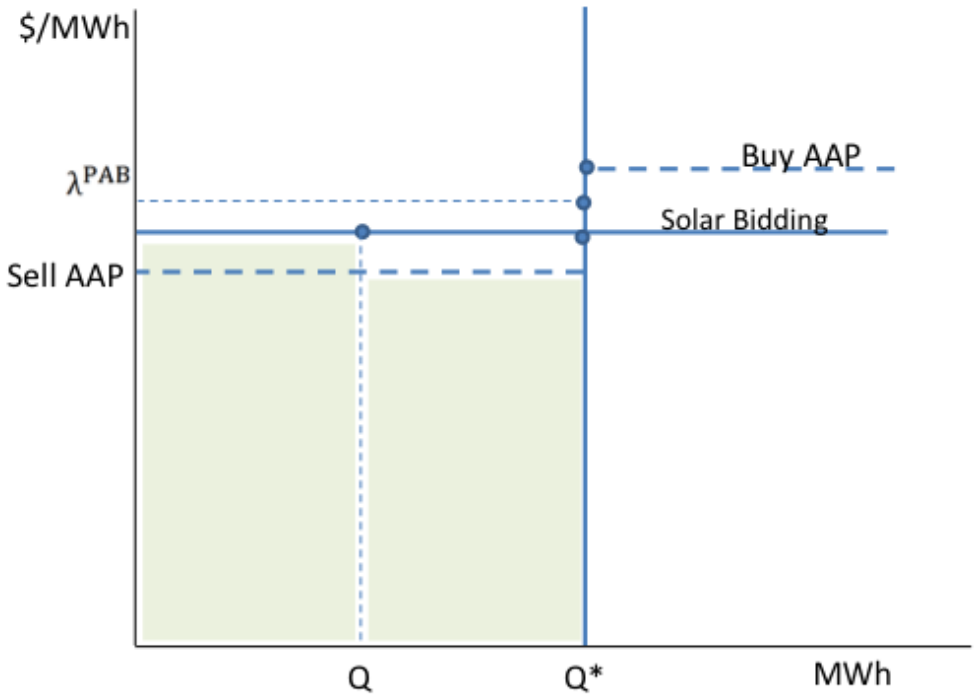

Figure 3.17. SPP revenue at PAB when and $Q < Q^*$ and $Sell\ AAP < p < \lambda^{\mathrm{PAB}}$

a) In the DA market, the SPP bids a price p that is $Sell\ AAP < p < \lambda^{PAB}$, and bids the amount of MWh Q then in the RT market it over-produces the amount of Q^*, as shown in Figure 3.17. Since $p < \lambda^{PAB}$, the SPP DA bid is accepted for the price p. The revenue in the DA market will be as in equation (3.22). In the RT market, the SPP over-produces an extra amount of $(Q^* - Q)$ MWh, so the ISO will pay a price of Sell AAP for this extra MWh, then the SPP revenue in RT market will be as in equation (3.23). The total revenue will be as in equation (3.24). Based on this the SPP will maximize its revenue by not having any de-viation between DA and RT, $(Q = Q^*)$.

$$DA\ Revenue = Q \cdot p \tag{3.22}$$

$$RT\ Revenue = (Q - Q^*) \cdot Sell\ AAP \tag{3.23}$$

$$Total\ Revenue = Q \cdot p + (Q - Q^*) \cdot Sell\ AAP \quad (3.24)$$

Where

Q is the amount of power bid in DA market (before gate closure).

Q^* is the amount of power actually delivered at RT market.

p is the bid price at DA (before gate closure).

$Sell\ AAP$ is the Average Adjustment Sell Price to ISO.

$Buy\ AAP$ is Average Adjustment buy price from ISO.

λ^{PAB} is the price of the last generator accepted bid before gate closure.

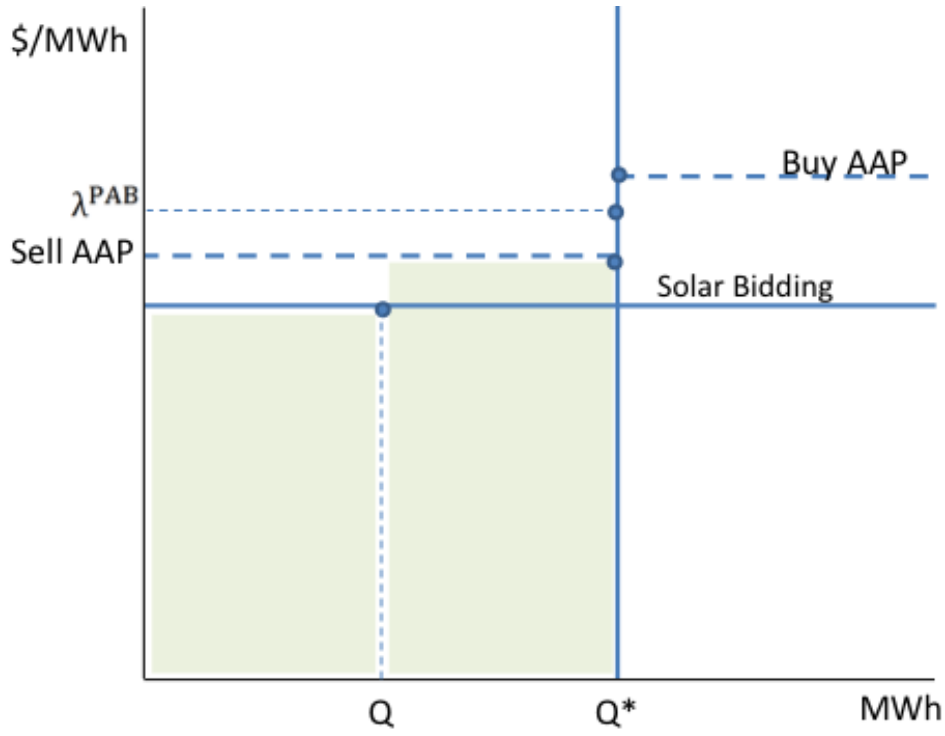

Figure 3.18. SPP revenue at PAB when $Q < Q^*$ and $p < Sell\ AAP < \lambda^{PAB}$

b) In the DA market, the SPP bids a price p that is $p < Sell\ AAP < \lambda^{PAB}$, and bids the amount of MWh Q then in the RT market it over-produces the amount of

Q^*as seen in Figure 3.18. Since $p < \lambda^{PAB}$, SPP bid will be accepted for the price p then revenue in the DA market will be as in equation (3.22). Compared to the previous scenario this DA revenue will be less as $p < Sell\ AAP$. In the RT market, the SPP over-produces an extra amount of $(Q^* - Q)$ MWh, so the ISO will pay a price of Sell AAP for this extra MWh, then the SPP revenue in RT market will be as in equation (3.23). The total revenue will be as in equation (3.24). Based on this the SPP will maximize its revenue by not bidding any MWh in the DA market, $(Q = 0)$. Of course, intentional withholding is not allowed and conflicts with the idea of having SPP participating in the market like conventional generators. So not bidding in the DA market will not be an option for solar bidding. Note: the SPP might bid in the DA market a price p that is $p > \lambda^{PAB}$ which would result in the SPP not contracting in the DA market, and then will over-produce in the RT market as discussed earlier. Such behavior is not allowed in the market, but it is discussed to know the financial effect of deviation. Also, this is an alternative form of intentional withholding. However, it might be hard to differentiate between a bad bid and intentional withholding.

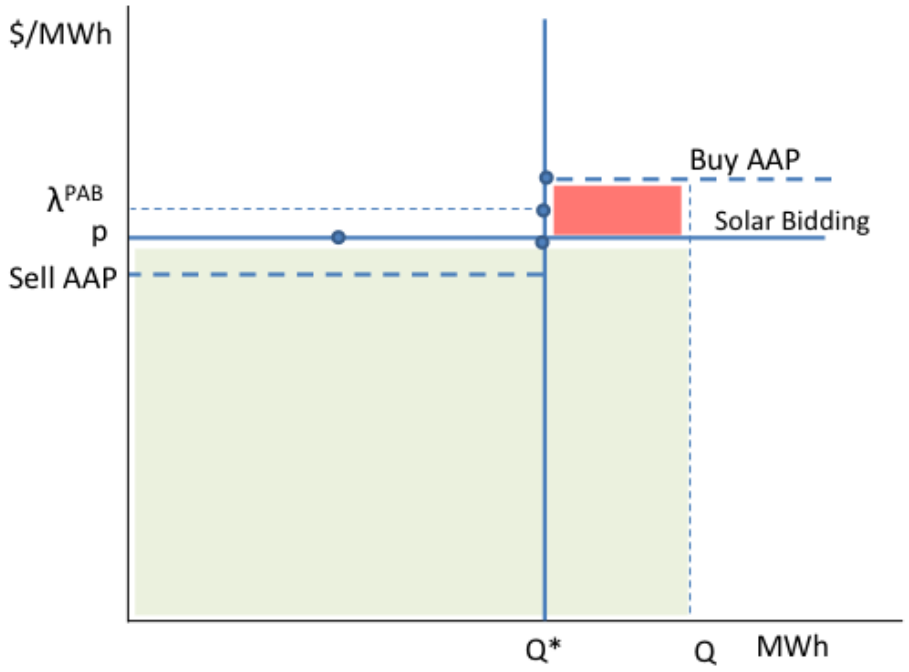

Figure 3.19. SPP revenue at PAB when and $Q > Q^*$ and $Sell\ AAP < p < \lambda^{\text{PAB}}$

c) In the DA market, the SPP bids a price p that is $Sell\ AAP < p < \lambda^{PAB}$, and bids the amount of MWh Q, then under-produces in the RT market to the amount of Q^*, as shown in Figure 3.19. Since $p < \lambda^{PAB}$, the SPP bid will be accepted for the price p. The revenue in the DA market will be as in equation (3.25). In the RT market, the SPP under-produces an amount of MWh $(Q-Q^*)$, so the ISO will be paid a price of Buy AAP for this MWh shortage, then the SPP revenue in the RT market will be as in equation (3.26). Note that it is negative revenue. The total revenue will be as in equation (3.27). The red area in the figure represents penalty cost. Based on this the SPP will maximize its revenue by not having any deviation between DA and RT, $(Q = Q^*)$.

$$DA\ Revenue = Q \cdot p \tag{3.25}$$

$$RT\ Revenue = -(Q - Q^*) \cdot Buy\ AAP \tag{3.26}$$

$$Total\ Revenue = Q \cdot p - (Q - Q^*) \cdot Buy\ AAP \tag{3.27}$$

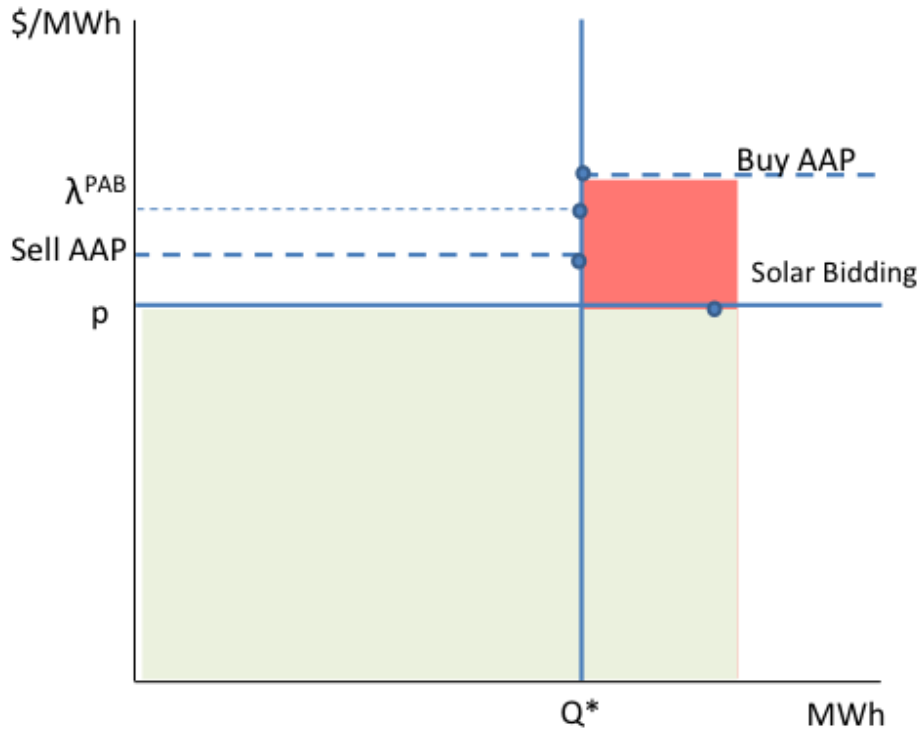

Figure 3.20. SPP revenue at PAB when and $Q > Q^*$ and $p < Sell\ AAP < \lambda^{\mathrm{PAB}}$

d) In the DA market, the SPP bids a price p that is $p < Sell\ AAP < \lambda^{PAB}$, and bids the amount of MWh Q then under-produces in the RT market to the amount of Q^*, as shown in Figure 3.20. Since $p < \lambda^{PAB}$, SPP bid will be accepted for the price p. The revenue in the DA market will be as in equation (3.25) but compared to the previous scenario this DA revenue will be less as $p < Sell\ AAP$. In the RT market, the SPP under-produces an amount of MWh $(Q - Q^*)$, so the ISO will get paid a price of a Buy AAP for this shortage MWh, then the SPP revenue in the RT market will be as in equation (3.26). Note that it is negative revenue. The total revenue will be as in equation (3.27). The red area in the figure repre-

sents penalty cost. Based on this the SPP will maximize its revenue by not having any deviation between DA and RT, ($Q = Q^*$). The only difference between this scenario and the previous one is that this scenario will have less total revenue.

3.3.2.4 Simulation of Financial Effects of SPP Deviation in the PAB Schemes

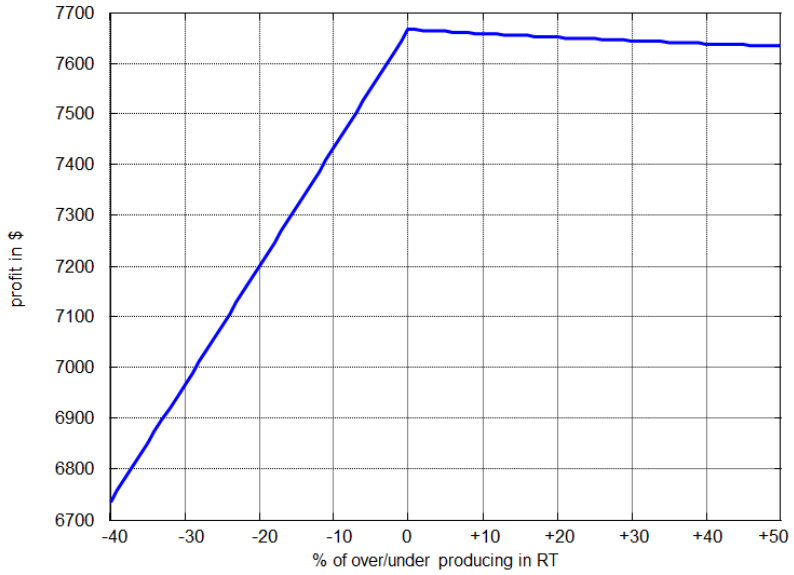

Figure 3.21 In the PAB scheme, SPP profit when over/under producing in RT market based on ERCOT load at 12 pm on September 16, 2014

To confirm the previous analysis, a simulation of deviation by SPP in the five-bus system was performed. The assumptions in section 3.2 were used base on one hour load. To properly compare cases, the RT market solution is held constant for studying the effect on revenue. The results are plotted in Figure 3.21. The figure confirms the previous theoretical discussion. It can be observed that in the PAB scheme, SPP will lose revenue in case of any mismatch, the main reason for this is the PAB scheme has the dual RT pricing that penalizes for any mismatch. Also

unlike the MP scheme, the SPP deviation financial effect is different than conventional generator deviation.

3.4 The Effect of Congestion on Social Welfare, Generators' Profit, SPP Profit, and Consumer Surplus in the MP and PAB schemes

This section will explore the financial effect of introducing SPP on the five-bus system with transmission line congestion between Bus D and Bus E. This evaluation will explore the effect on SPP revenue, consumer surplus, and generators profit. Assumptions in section 3.2 are used.

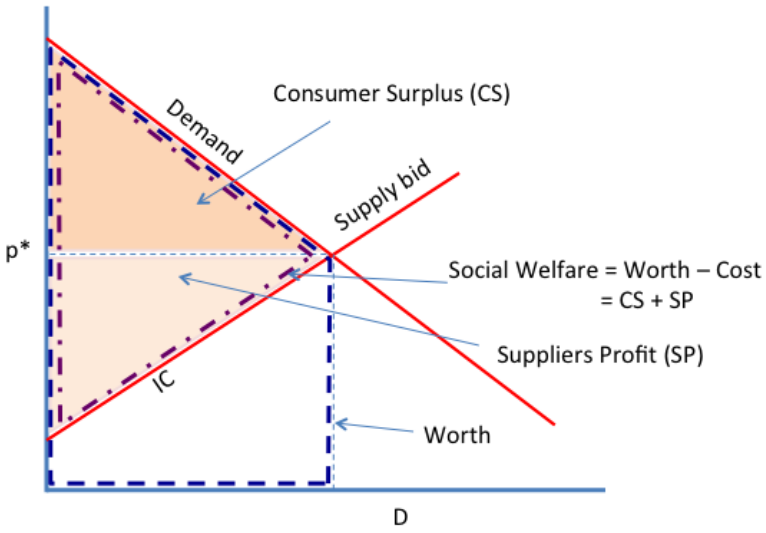

Figure 3.22 Economic social welfare

3.4.1 *Social Welfare*

It is important to clearly define **Social Welfare (SW)**, as it is the best way to measure the amount of benefit the society gains for any product or commodity. Figure 3.22 illustrates the concept of economic social welfare for an elastic supply and elastic demand (elastic demand means that the demand will change in response to supply change, inelastic demand means that

the demand will not change in response to price change). **Worth (W)** is the amount of willingness for consumers to pay for a product. The **Consumer's Surplus (CS)** is the difference between the amount that consumers were willing to pay for a quantity and actual price paid for it as in the figure**. Supplier's Profit (SP**), also called supplier's surplus, is the difference between the product market price and actual product cost. So the **Social Welfare** is the surplus gained by consumers and suppliers as in equation (3.28) and equation (3.29) [26] [99].

$$SW = CS + SP \tag{3.28}$$

or

$$SW = Worth - Cost \tag{3.29}$$

$$CS = Worth - Revenue \tag{3.30}$$

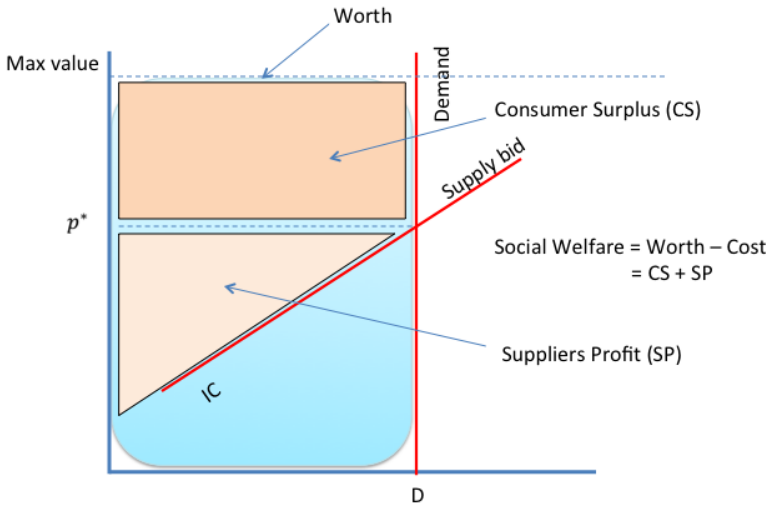

Figure 3.23 Social welfare distribution in the MP scheme

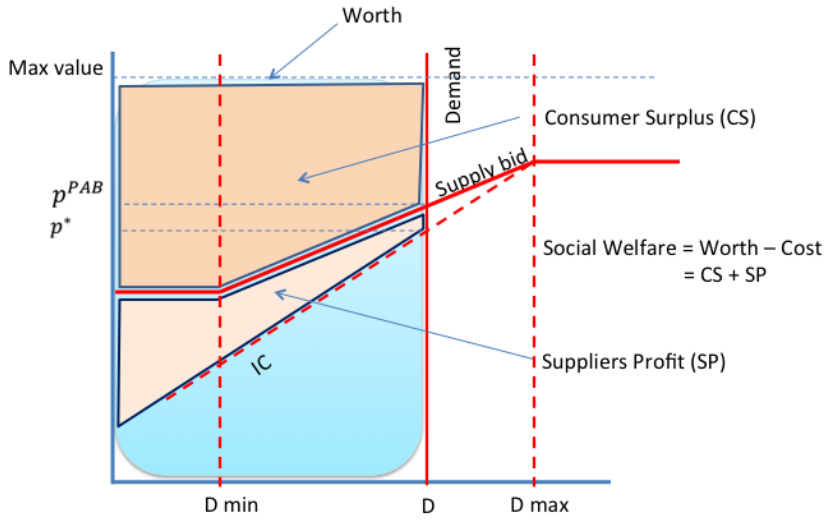

Figure 3.24 Social welfare distribution in the PAB scheme

In this study, the electricity demand is assumed to be inelastic, as in the short run consumers do not change their consuming behavior instantly in response to electricity price change. Having an inelastic demand means that the worth, consumer's surplus, and social welfare will have an infinite value. Since the objective is to find the change in social welfare, therefore a price cap can be assumed. Figure 3.23 and Figure 3.24 shows the social welfare for the MP scheme and the PAB scheme respectively. It can be observed that for the same supply and demand in both figures, the social welfare and worth will be the same but the surplus distribution will be different. Also, this shows that PAB scheme is more in favor of consumers over suppliers. Which means that in the long run there is less motivation to construct new generation plants.

This section evaluates the financial effect of introducing SPP in the presence of congestion on the SPP revenue, generators revenue, consumer surplus, and social welfare. As discussed in section 1.4, market schemes can be categorized into four schemes. All schemes are evaluated.

1) MP scheme with AM market (MP-AM).
2) MP scheme without AM market (MP-DA).
3) PAB scheme with AM market (PAB-AM).
4) PAB scheme without AM market (PAB-DA).

3.4.2 *Congestion*

In this section, the effect of congestion in a two-bus system on generator's revenue will be explored for the MP-DA and PAB-AM schemes. This is just to lay the foundation for a more complex analysis. If the power transfer in a transmission line within the electricity grid reaches its capacity, then the transmission line is considered to be congested. In a simple explanation of how the Independent System Operator (ISO) relieves congestion in a two-bus system, ISO will request generators at the cheap side of congestion to reduce power production, while ISO will request generators at the expensive side to produce more power. This means that the expensive generators at the congested side will have to operate.

A two-bus system is used as an example to compare how both market schemes deal with congestion seen in Figure 2.20. It is assumed that there are three generators. Their bid is as in Table 3.5. Load 1 consumes 150 MWh and load 2 consumes 50 MWh.

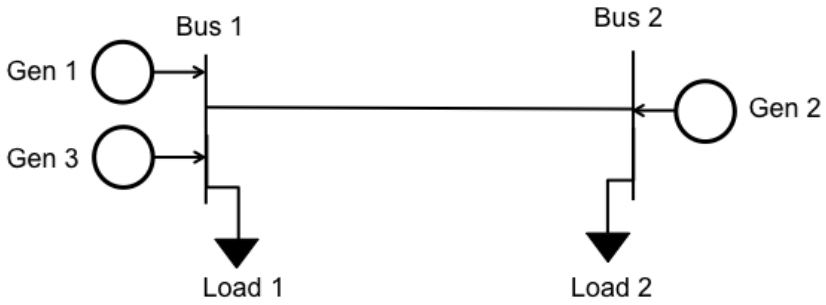

Figure 3.25 Example of a two-bus system

Table 3.5 Capacity and price of generators in the example

	MW capacity	**$/MW**
Gen 1	100	15
Gen 2	100	17
Gen 3	100	19

MP scheme with no congestion:

In the case of no congestion in the MP scheme, the total load is 100 MWh, and Gen 2 will supply the last MWh. Therefore Gen 2 will set the market price to $17, and all generators will get market price, and loads will pay market price. Total revenue is shown in Table 3.6.

Table 3.6 Revenue of generators and loads in the MP scheme assuming no congestion

	MW	**Revenue ($)**
Load 1	-150	-2550
Load 2	-50	-850
Gen 1	100	1700
Gen 2	100	1700

PAB scheme with no congestion:

In the case of no congestion in the PAB scheme, Load 1 will purchase the first 100 MW from Gen 1 for $15, then it purchases the remaining 50 MW from Gen 2 for $17. Load 2 purchases 50 MW from Gen 2 for $17. Total revenue is shown in Table 3.7.

Table 3.7 Revenue of generators and loads in the PAB scheme assuming no congestion

	MW	**Revenue ($)**
Load 1	-150	-2350
Load 2	-50	-850
Gen 1	100	1500
Gen 2	100	1700

MP scheme with congestion:

In the case of transmission line limit between bus 1 and bus 2 of 48 MW, Gen 2 will not be able to supply more than 98 MW so the line would not be congested. Then Gen 3 will supply 2 MW, which will set the LMP on bus 1 for $19, and the LMP on bus 2 for $17. Total revenue is shown in Table 3.8.

Table 3.8 Revenue of generators and loads in the MP scheme assuming congestion

	MW	**Revenue ($)**
Load 1	-150	-2850
Load 2	-50	-850
Gen 1	100	1900
Gen 2	98	1666
Gen 3	2	38

PAB scheme with congestion:

In the case of transmission line limit between bus 1 and bus 2 of 48 MW in the PAB-AM scheme, in the DA market, Load 1 will purchase the first 100 MW from Gen 1 for $15, then it purchases the remaining 50 MW from Gen 2 for $17. Load 2 purchases 50 MW from Gen 2 for $17 (similar to no congestion case). The DA market revenue is shown in Table 3.7. In the adjustment market (AM), ISO detects congestion and requests Gen 2 to reduce 2 MW and requests Gen 3 to produce 2 MW in order to relieve congestion. The AM revenue is shown in Table 3.9. The cost of relieving the congestion is $8.00 and is pro-rated among loads as in Table 3.10. The final revenue of generators and loads is shown in Table 3.11. More detailed analysis on the five bus system will be explored in the next section.

Table 3.9 Revenue of generators in the PAB scheme during adjustment market

	Adjustment MW	**Adjustment cost ($)**
Gen 2	-2	-34
Gen 3	2	38

Table 3.10 Congestion prorated cost on Loads

	MW	**Pro-rated cost ($)**
Load 1	150	3
Load 2	50	1

Table 3.11 Final revenue of generators and Loads in the PAB scheme after adjustment market

	MW	**Revenue ($)**
Load 1	-150	-2353
Load 2	-50	-851
Gen 1	100	1500
Gen 2	98	1666
Gen 3	2	38

3.4.3 *The Effect of Congestion on Supplier's Profit in All Schemes*

In order to compare the effect of congestion on suppliers profit, a full year of hourly simulation based on assumption covered in section 3.2 was done for the five-bus system in Figure 3.1. A transfer limit value was set between bus (D) and bus (E) to ensure congestion is present. The transmission limit was varied from 600 MW to 1200 MW. Low transfer line limit means higher congestion and vice versa. First, the expensive side of the congestion was identified by obtaining the annual average LMP price on all buses with respect to transmission line limit as seen in Figure 3.26. It was found that bus "D" is the expensive side of the congestion and bus (E) is the cheap side of the congestion.

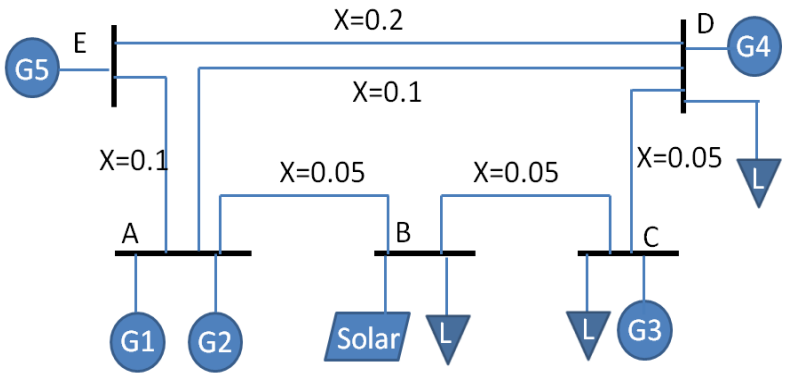

Figure 3.1 Five-bus test system [29]

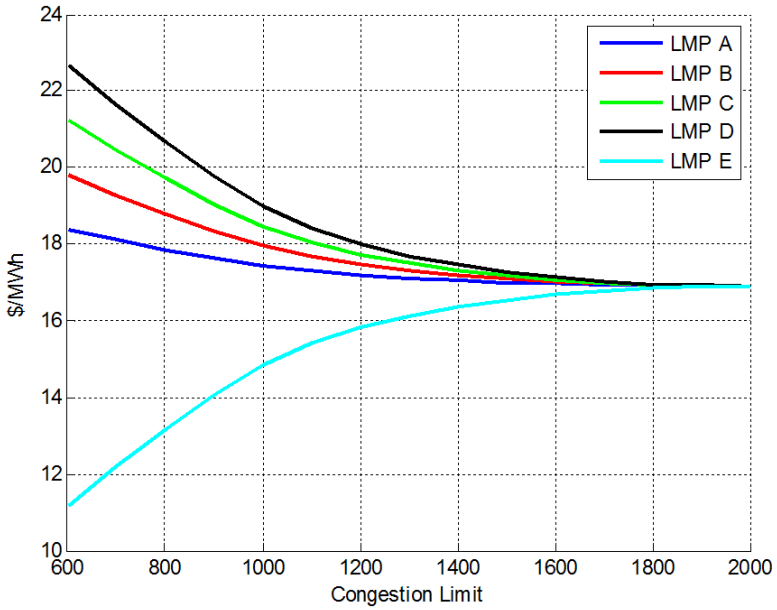

Figure 3.26 Average LMP price on all buses in full year simulation with respect to TL limit

Figure 3.27 compares the total profit in the MP scheme and the PAB scheme. For the **MP scheme**, as seen in the figure dealing with congestion either in the DA market or AM, the total profit of generators is higher with congestion. Also, it can be observed that when the SPP is on the low price bus (E), the total profit of generators is higher in both markets (**MP-DA**, and **MP-AM**). This is because if the SPP was on the expensive bus (D) then it will provide cheap power there, which will reduce the total profit of generators. In

general, because all generators on the bus are being paid the same price, this makes it more profitable for generators than the PAB scheme.

It can be observed from Figure 3.27 that with high congestion, in the PAB scheme that deals with congestion in DA market (**PAB-DA**), generators tend to make more profit than PAB scheme that deals with congestion in AM (**PAB-AM**). This is because generators that generators had contracted before gate closure, then in the AM, ISO will ask generators to adjust their production which means that they pay/get paid based on their adjustment bids unlike to the MP scheme where all generators connected to the same bus are paid the same. It is observed that in high congestion, there is some room for generators to exercise market power in the **PAB-DA**.

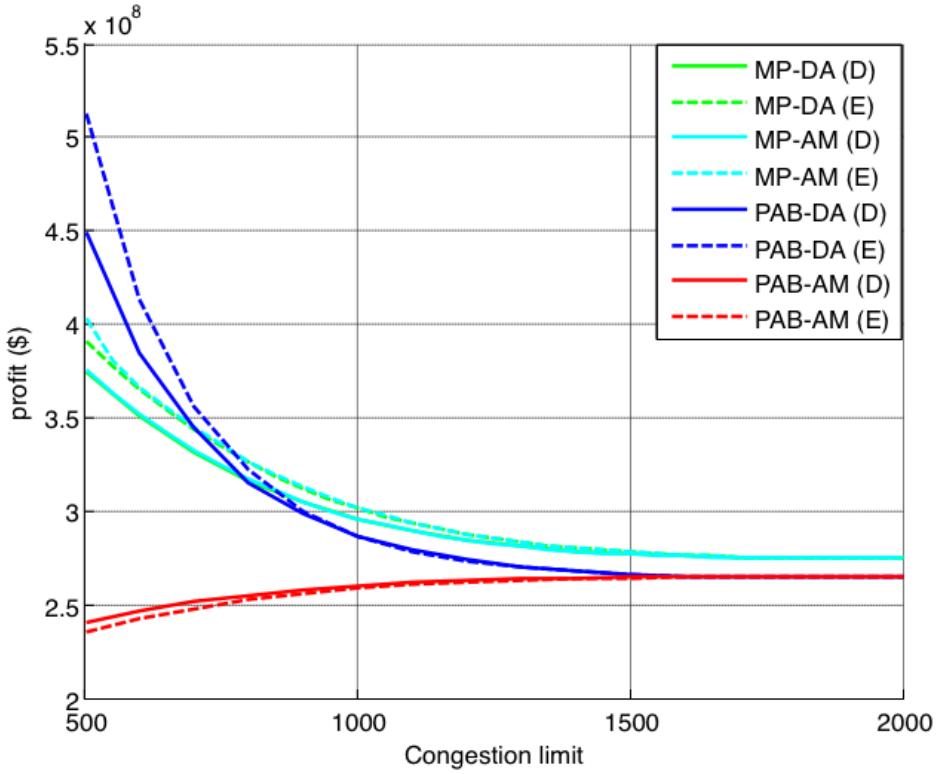

Figure 3.27 MP vs PAB scheme, total generator profit with congestion when SPP at bus (D) or (E) runs for full year.

3.4.4 *The Effect of Congestion on SPP's Revenue*

Figure 3.28 shows SPP revenue for a full year data simulation based on hourly load. For MP scheme, as it can be seen in the figure that dealing with congestion in the DA market (**MP-DA**) makes the SPP have more profit if it is on the expensive bus (D) than if it is on the cheap bus (E) because of different LMP value. In the **MP-AM** scheme, SPP will not have a change in profit with congestion, as SPP will not be able to participate in the AM. It is noted that the same behavior occurs in **PAB-AM**.

It can be observed in Figure 3.28 with high congestion, for the PAB scheme that deals with congestion in the DA market (**PAB-DA**), if SPP is on the expensive bus (D), then it tends to make more profit than it is on the cheap bus (E). In the PAB scheme that deals with congestion in AM (**PAB-AM**) similar to the **MP-AM** scheme, power is traded in the DA market before gate closure, then in the AM, the ISO will ask generators to adjust their generation (in case they submitted the optional adjustment bid) to overcome any congestion issue. Since the SPP already contracted the amount to be delivered before gate closure, it would not be required to lower its production.

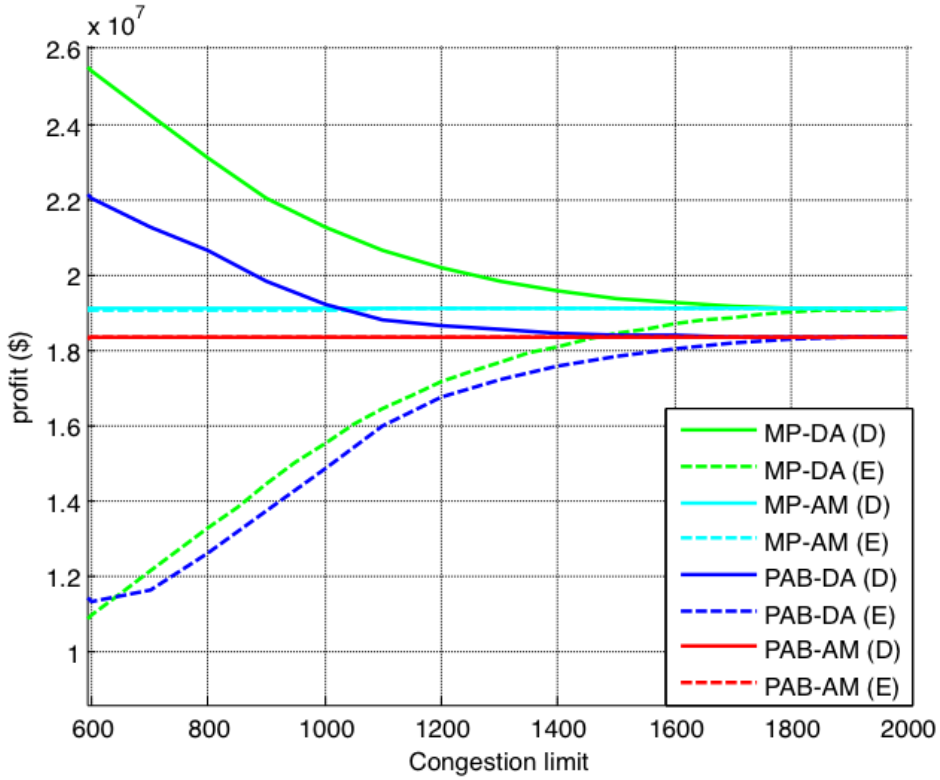

Figure 3.28 MP vs PAB scheme, Solar revenue with congestion when solar at bus (D) (expensive side) and (E) (cheap side) runs for full year

3.4.5 *The Effect of Congestion on Consumer's Surplus*

Figure 3.29 represents the consumer's surplus with respect to congestion line limit for a full year data simulation. It is observed that at low congestion, consumer surplus is much higher in the PAB scheme than in the MP scheme. This is an expected result in the PAB scheme as consumers can get low prices if they bought power in the DA market early in time.

For the MP scheme, as seen in Figure 3.29, the consumer surplus is the same in the **MP-AM** and **MP-DA** schemes. It is observed that in both schemes in the presence of high congestion the consumer surplus is bit higher when the SPP is on the expensive bus (D) because having SPP on the expensive bus (D) lowers the total generators profit which will benefit consumers.

For the PAB scheme, as seen in Figure 3.29, the consumer surplus, in general, is higher in the **PAB-AM** than the **PAB-DA** scheme. The reason for this is that the total generation profit is higher in the **PAB-DA** scheme, which causes a lower the consumer surplus. It is observed that in both schemes in the presence of high congestion the consumer surplus is bit higher when the SPP is on the expensive bus (D) because having SPP on the expensive bus (D) lowers the total generators profit that will benefit consumers.

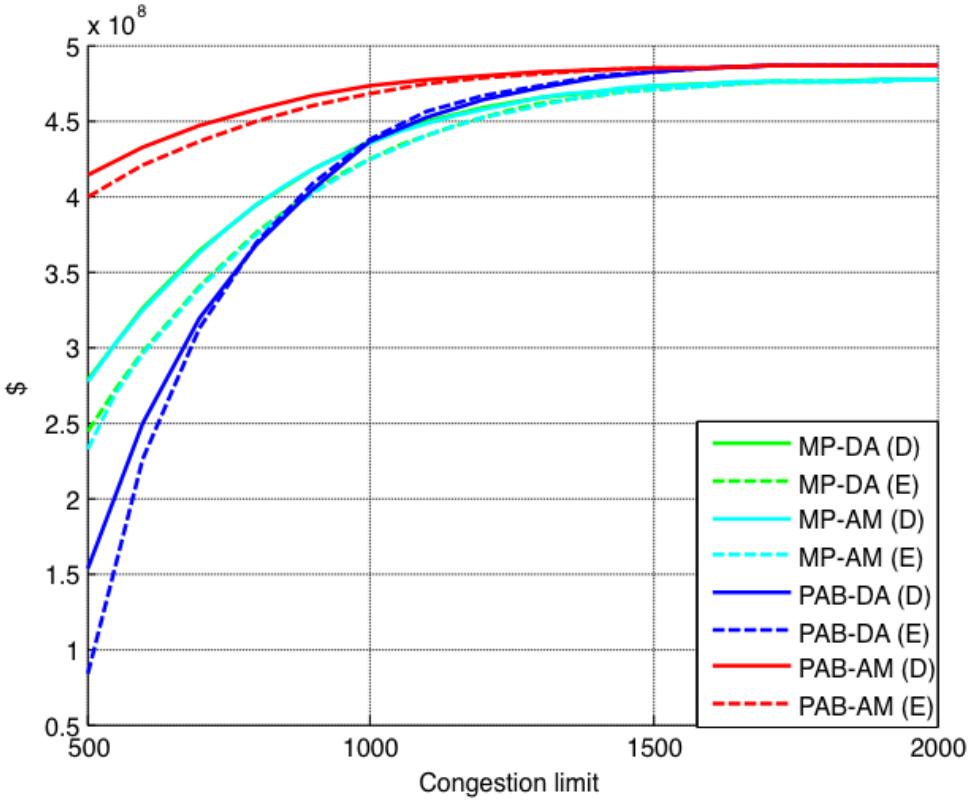

Figure 3.29. MP vs PAB scheme, consumer surplus with congestion when solar at bus (D) or (E) runs for full year

3.5 CONCLUSION

This chapter identified the financial effect of the MWh deviation of power producers between the Day Ahead (DA) market and Real Time (RT) market in different market schemes. Findings

showed that the solar power deviation in the PAB scheme is penalized more than in the MP scheme. The second part of this chapter identified the effect of SPP interaction in different electricity market pricing schemes in the presence of congestion. The schemes were examined on a five-bus system to see the financial effect of solar plant introduction in different scenarios. Findings showed that SPP would have more profit in the MP-DA scheme when treated as a regular generator; also the MP-DA scheme will encourage constructing solar plants in congested areas, which will make the market more efficient by lowering congestion. The PAB scheme is less efficient; expensive suppliers can get their bids accepted while cheap ones might not. One observation in dealing with congestion in adjustment market (MP-AM, and PAB-AM) is that SPP revenue is not affected in high congestion as the SPP already contracted the amount to be delivered in the DA market and technically it cannot adjust its production during the AM. This might change with higher SPP penetration.

Chapter 4. STUDYING THE EFFECT OF SOLAR POWER PLANT BIDDING ON DIFFERENT MARKET STRUCTURES IN THE SAUDI ARABIA GRID

4.1 INTRODUCTION

As discussed in Chapter 1, Saudi Arabia is planning on converting to an electricity market in the near future. The current electricity structure is a vertically integrated grid where Saudi Electricity Company (SEC) owns 74% of the generation in Saudi Arabia, and all transmission and distribution see (Figure 4.1). In 2008, in an effort to lay the foundation for a Saudi Electricity Market, Electricity & Cogeneration Regulatory Authority (ECRA) announced that they would like to unbundle the monopoly of the government owned electricity company SEC into separate companies for Generation, Distribution and Transmission, and they would like the generation sector of SEC to be divided into at least four companies (see Figure 4.2) [13]. In 2012 the transmission department was split from SEC and the new transmission company was named the "National Grid" [14]. During this time several Independent Power Providers (IPPs) were built with the partnership of SEC. Having the SEC split will lay the final foundation for a free electricity market like in Figure 4.3.

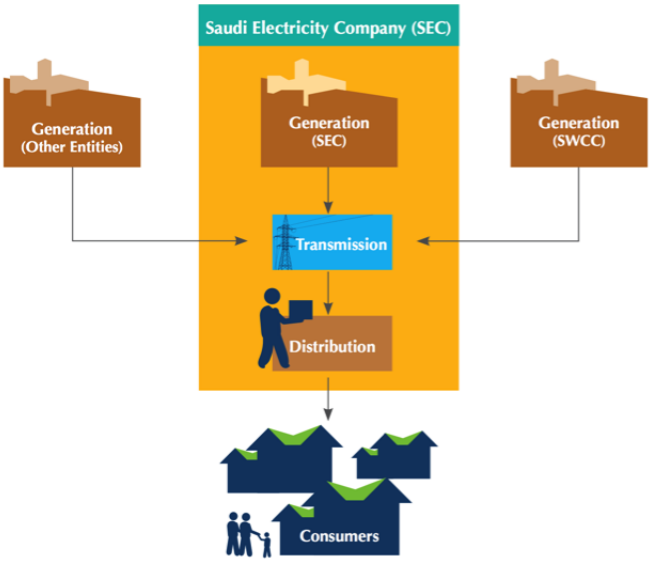

Figure 4.1 Saudi Arabia electricity structure prior deregulation [94]

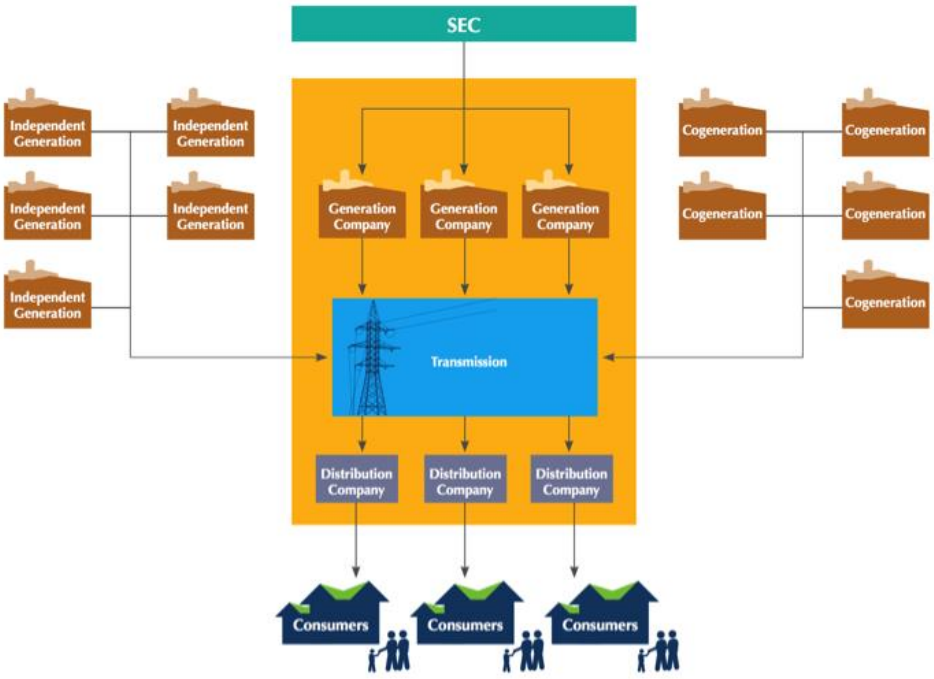

Figure 4.2 Saudi Arabia electricity structure phase-1 deregulation [94]

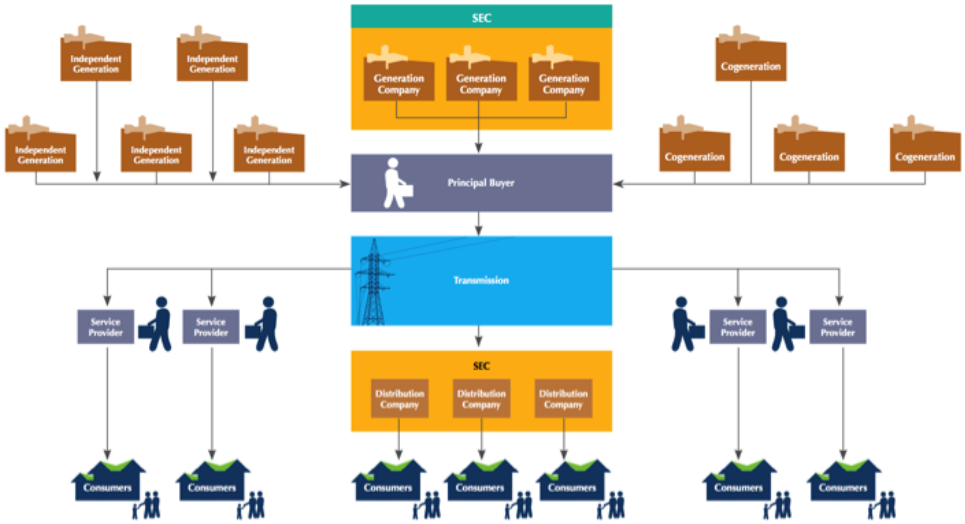

Figure 4.3 Saudi Arabia electricity structure phase-2 deregulation [94]

Once the electricity market is establish power producers will have to bid in the market depending on the market scheme adopted. Among the bidders will be the Solar Power Plants (SPPs). This chapter will evaluate the effect of SPP bidding in the Saudi grid on the SPP's profit, generators' profit, consumers' surplus, and the system social welfare. This evaluation will assume one SPP is added at different proposed locations on the Saudi grid. This simulation will compare all four-market schemes discussed in the previous chapter: MP-DA, MP-AM, PAB-DA, and PAB-AM. The PAB bidding strategy developed in this research will be used. The Saudi grid information is not publically available. An approximate model of the Saudi grid was constructed by obtaining generation data, transmission data, and load data. A considerable amount of grid data gathering was completed in collaboration with my colleague Mohammad Abahussain.

4.2 CONSTRUCTING THE SAUDI GRID MODEL

There is some published data about some parts of the Saudi grid from ECRA and SEC that can be used as a base to construct a model of the rest of the Saudi grid. Currently, there are four operation areas for SEC, the Eastern, Central, Western and Southern Operating areas [100]. See Figure 4.4. The Eastern and Central Operating areas are connected. The focus in this modeling will be the Eastern and Central region as they are interconnected and have most of Saudi Arabia generation and load.

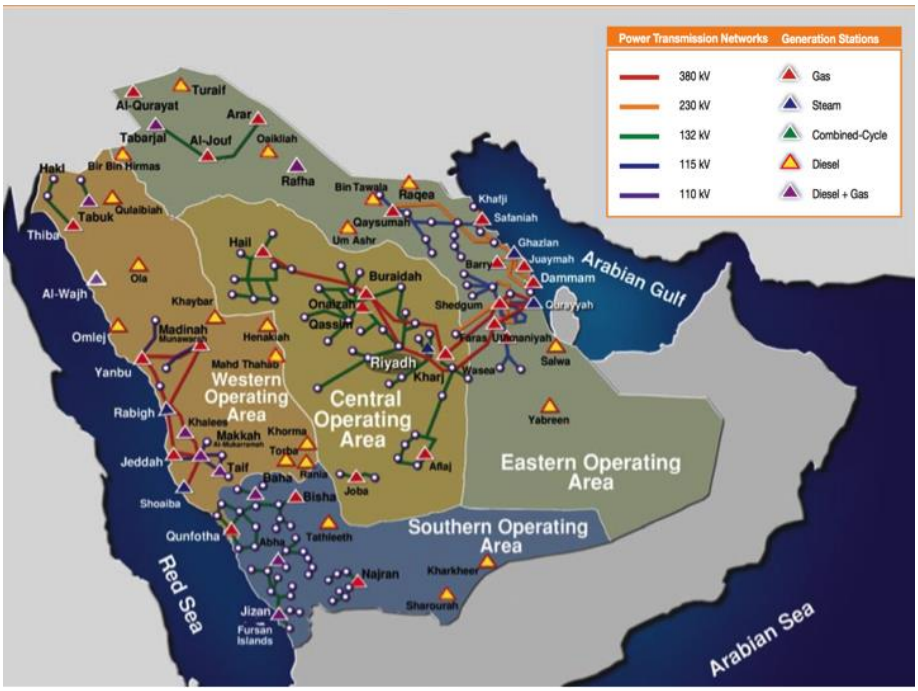

Figure 4.4 Saudi Arabia electricity grid [100]

4.2.1 *Constructing the Generation Data*

In the Saudi grid modeling the generation of each area was represented with the proper generation type mixture as in Table 4.1 [101]. The total generation capacity of Saudi Arabia is around 51 GW, with around 19 GW in the Eastern operation area and 12 GW in the Central operation area [2]. Areas that have congested transmission lines have been mod-

eled in detail. Most generators' location and capacity information are publically available from ECRA [102]. Since the generator cost curves are not publically available, cost curves of similar generators have been obtained from the United States Environmental Protection Agency (EPA), which has a massive database of many types of generators [98]. This assumption shouldn't significantly affect the accuracy of the model as most generators are manufactured by same companies and have similar characteristics with only small differences. A priority list was created to control generator status, which has must run and cheap generators in the bottom of the list and most expensive generators on the top. Based on [103] this is the current practice by SEC.

Table 4.1 Fuel type by area [101]

Area	**Fuel Type**	**Percentage (%)**
Central	Combined Cycles, natural gas	21%
	Gas turbine, natural gas	46%
	Gas turbine, crude and diesel	33%
East	Steam Cycle, natural gas	75%
	Gas turbine, natural gas	24%
	Gas turbine, diesel oil	1%

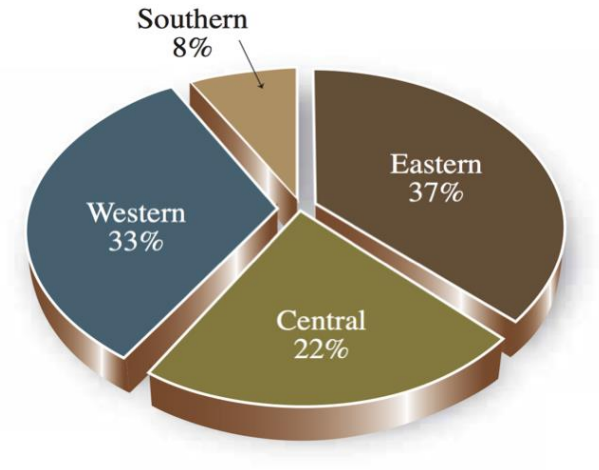

Figure 4.5 SEC generation capacity distribution [2]

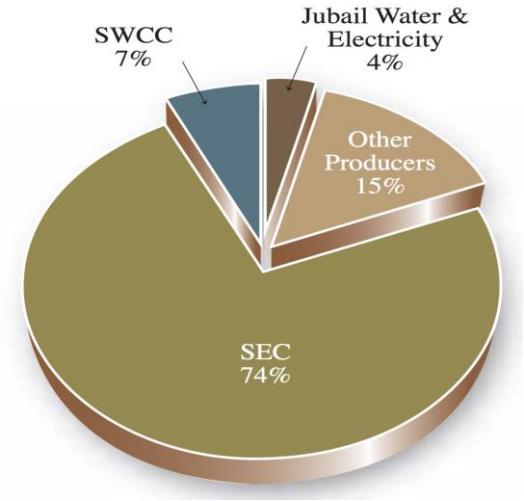

Figure 4.6 Distribution of generation capacity among producers [2]

4.2.2 *Constructing the Transmission Line Data*

In the Saudi grid modeling, the focus is on the transmission lines within Eastern region, Central region, and the interconnection between them. Figure 4.4 shows the voltage level of the main lines in the Saudi grid. The model has modeled all 380 kV, 230 kV, 132 kV and 115 kV lines. Transmission lines in lower voltage levels have been modeled in congested areas on a case-by-case basis. It is important to note that there are only five connection points (transmission lines) between the Eastern and Central area. They are in Table 4.2. Although this model does not consider low voltage, usually low voltage side effects are small on the Saudi grid. The transmission line model was based on an interview with a SEC representative [103] and manual transmission line tracing [104]. The transmission line capacity was also obtained from the SEC representative [103] as in Table 4.3. The Saudi grid model has 186 buses and 531 transmission lines. Figure 4.7 shows the constructed Saudi grid model. This modeling was done in collaboration with my colleague

Mohammad Abahussain. For confidentiality reasons most of the buses are represented in numbers.

Table 4.2 Transmission lines connecting Eastern and Central regions (publically available)

From	To	Voltage Level
Fadhili	Sudair	380 kV
Shedgum	Qurtubah	380 kV
Faras	Kharj	380 kV
Khurais	Riyadh – Sub 9019	380 kV
Wasia	Majmaah	230 kV

Table 4.3 Transmission lines capacity

Transmission line voltage	MVA capacity
380 kV	1650
230 kV	741
132 kV	212
115 kV	185
69 kV	111
33 kV	53

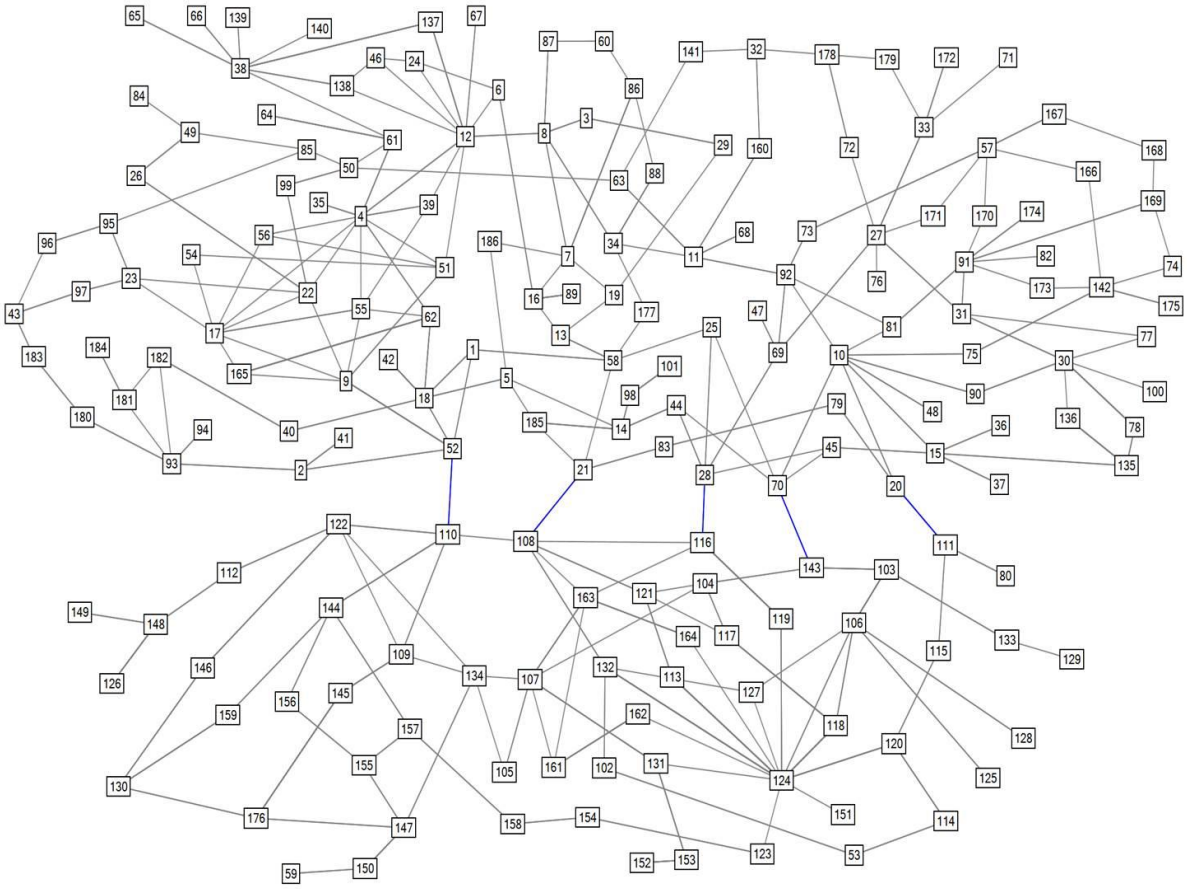

Figure 4.7 Saudi grid transmission lines topology

4.2.3 *Constructing the Load Data*

In the Saudi grid model, residential areas have been modeled as lump sum load for the main areas in each region, while large industrial loads were modeled in detail. The load distribution was obtained from ECRA and SEC annual report [2], and is shown in Figure 4.8, Figure 4.9 and Figure 4.10. The main residential areas in the Eastern region are: Dammam metropolitan, Ras Tanura, Jubail, Alhasa, Abqaiq, and Qysomah. The main residential areas in Central region are: Riyadh, Kharj, Qassim, Hail, and Aflaj. The value of load depended on several factors: in residential areas, load data for each area was derived

based on public information for the region load distributed by population in the area. In industrial areas, load data of the area was derived based on public information about total region industrial load divided based on industrial area size.

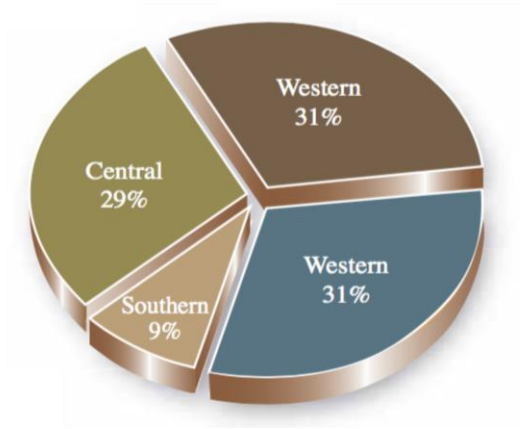

Figure 4.8 Distribution of consumption by area [2]

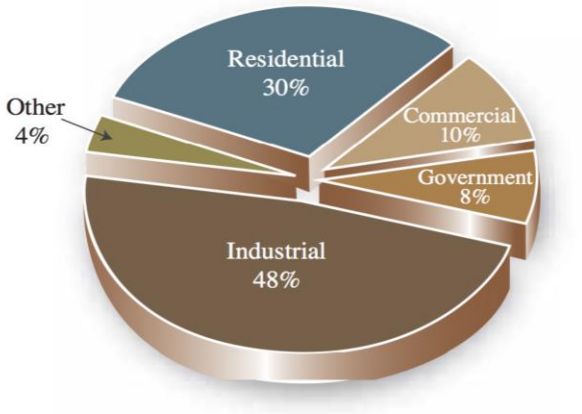

Figure 4.9 Distribution of load in Eastern region [2]

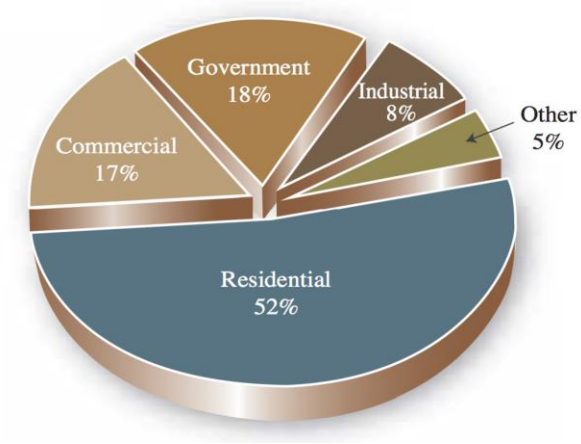

Figure 4.10 Distribution of load in Central region [2]

4.2.4 *Fuel Prices*

Fuel prices are subsidized in Saudi Arabia. Therefore in the Saudi grid model, three fuel prices are considered, old domestic fuel prices prior to December 28, 2015, new domestic fuel prices after December 28, 2015, and international fuel prices based on June 10, 2016. Comparing these different fuel prices shows how SPP reacts to different prices especially when using unsubsidized prices (international fuel prices). The domestic fuel prices that have been used were based on the published data, see Table 4.4, and Table 4.5. Since international fuel prices vary daily, prices were fixed on June 10, 2016, see Table 4.6. The old subsidized energy (MBTU) price of diesel is significantly cheaper than natural gas (16%), while crude oil energy price is slightly cheaper than natural gas. The new subsidized energy prices are higher than old prices, and the ratio between prices of different types of fuel also changed, making diesel, in particular, relatively more expensive and crude oil the cheapest energy. The international fuel price also has much higher diesel price, unlike domestic prices; natural gas price is much lower than crude oil.

Table 4.4 Old domestic fuel prices [105] [106]

Fuel Type	Fuel Price ($/BBL)	Fuel Price ($/$m^3$)	Fuel Price (SR/m^3)	Heat Content (MBTU/m^3)	Fuel Price ($/MBTU)	Fuel Price (SR/MBTU)
Natural Gas	-	-	-	42.37	0.75	2.81
Crude Oil	4.24	26.67	100.01	35.96	0.74	2.78
Diesel Oil	3.60	22.64	84.91	36.16	0.63	2.35
Heavy Fuel Oil	2.54	15.98	59.91	40.61	0.39	1.48

Table 4.5 New domestic fuel prices starting December 28, 2016 [107] [108]

Fuel Type	Fuel Price ($/BBL)	Fuel Price ($/$m^3$)	Fuel Price (SR/m^3)	Heat Content (MBTU/m^3)	Fuel Price ($/MBTU)	Fuel Price (SR/MBTU)
Natural Gas	-	-	-	42.37	1.25	4.69
Crude Oil	6.35	39.94	149.78	35.96	1.11	4.17
Diesel Oil	14	88.06	330.22	36.16	2.44	9.13
Heavy Fuel Oil	4.4	27.68	103.78	40.61	0.68	2.56

Table 4.6 International fuel prices on June 10, 2016 [107] [108]

Fuel Type	Fuel Price ($/BBL)	Fuel Price ($/$m^3$)	Fuel Price (SR/m^3)	Heat Content (MBTU/m^3)	Fuel Price ($/MBTU)	Fuel Price (SR/MBTU)
Natural Gas	-	-	-	42.37	2.62	9.83
Crude Oil	51.30	322.67	1210.00	35.96	8.97	33.65
Diesel Oil	65.10	409.47	1535.50	36.16	11.32	42.46
Heavy Fuel Oil	37.50	235.87	884.50	40.61	5.81	21.78

4.3 SIMULATING DIFFERENT MARKET SCHEMES ON THE SAUDI GRID

This research focuses on the implementation of solar power in Saudi Arabia. Hence, the solar power data was derived from the irradiation data profile of Jeddah City, Saudi Arabia obtained by NASA Remote Sensing Validation Data project. The solar irradiance data was collected with a CIMEL 8 channel sunphotometer [93]. Currently, the first commercial SPP is being constructed in Saudi Arabia with 50 MWp [109]. Since the trend in Saudi Arabia when the first Co-gen IPP was constructed is to build on the same scale [102], it is expected future SPPs will have the same capacity. Therefore this research will assume building a 50 MWp SPP.

This simulation uses persistence forecasting as an initial choice to due to its wide use in several electricity markets. This research is not aimed at finding a new forecasting method. Since all available information is time series data, advanced methods that use meteorology based forecasting cannot be used. Other non-meteorology based forecasting has an error that is not so big compared to persistence forecasting. Since this work is comparing different electricity market schemes, the same forecast error would be applied to all of them.

Since there is no public daily load data available in Saudi Arabia, the load profile data was derived from the Electric Reliability Council of Texas (ERCOT) 2011 load database [91]. ERCOT load data was chosen because the load profile (residential, commercial, and industrial ratio) of Texas is close to Saudi Arabia load profile [94] [95]. ERCOT data is normalized to fit the Saudi grid peak load information. To avoid simulation divergence when the sum of minimum generation is higher than the total load, a priority list was created that depended on turning off expensive units. This list was based on infor-

mation from SEC. This priority list took into account the must run units like Saline Water Conversion Corporation (SWCC), and MAADEN, and MARAFIQ

After constructing the Saudi grid model, the simulation used an hourly data for the full year period. The market was settled using MATLAB with the installation of a DCOPF package by MATPOWER [110]. In general, there is no congestion in the 380 kV transmission lines due to overdesign. Most congestion appears in the 115 kV and lower voltage transmission lines during high demand in summer. Congested areas were identified. Figure 4.11 shows groups of buses with similar average annual prices, where Area 1 is the most expensive area and Area 10 is the cheapest. An interesting observation is that Area 10 has negative prices due to congestion. In other words, loads in this area during high congestion are paid to increase load consumption to relieve congestion. The same observation was noted by [111].

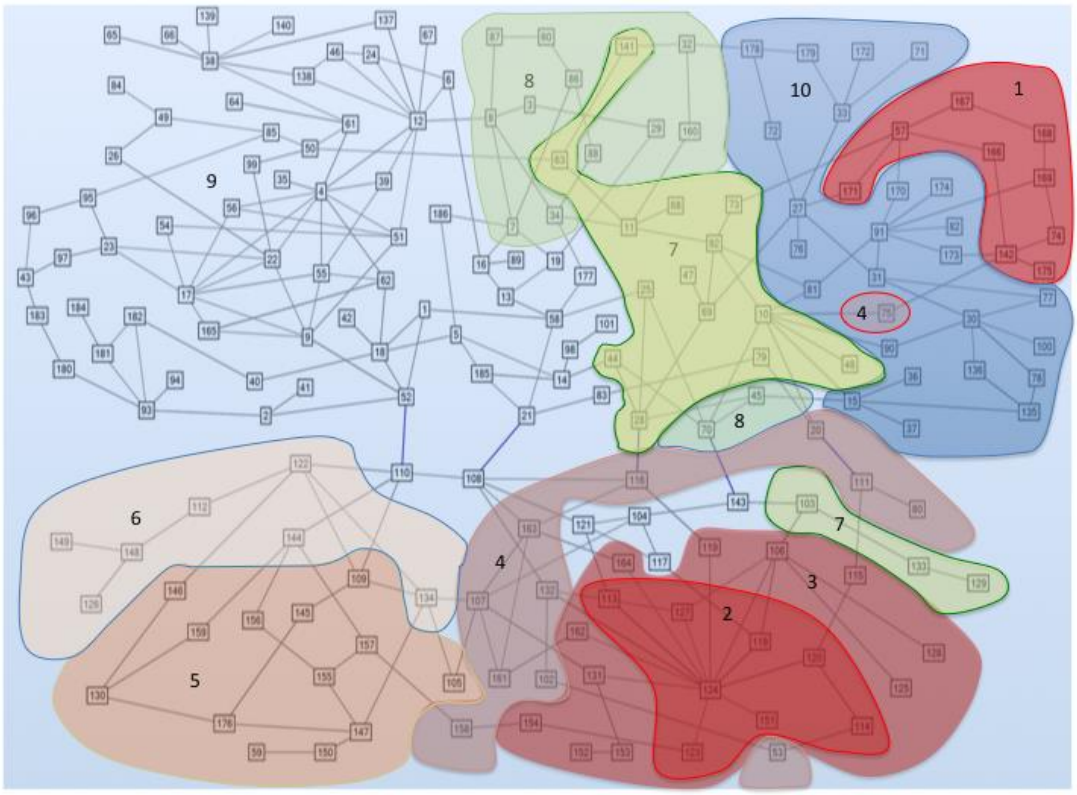

Figure 4.11 Average annual price in different areas where Area 1 is most expensive area

In order to know the economics of the SPP location, a single SPP was assumed in different areas alternately. It was assumed in the most expensive area in Eastern region (Area 1), then in the most expensive area in Central region (Area 2), the in the largest uncongested area (Area 9) and then in the area that is having negative prices (Area 10). The purpose of the test is to determine the effect of constructing SPPs in different locations using different market structures on SPP's profit, generator's profit, and consumer's surplus, also to check the effect of increasing SPP capacity in the most expensive area.

4.4 SPP Revenue

The revenue of a 50 MWp SPP using different market schemes when being installed in different areas is presented in Table 4.7, Table 4.8, and Table 4.9, and illustrated in Fig-

ure 4.12, Figure 4.13, and Figure 4.14. In general, it can be observed that SPPs make more revenue in the MP-DA scheme, and make even around 5% more than the MP-AM scheme in congested areas; this motivates SPP construction where it is needed. The only exception is Area 10; this area has negative LMP prices, so the SPP in the MP-DA scheme makes around 15% less revenue than the MP-AM scheme. Also, it is observed that in the MP-AM scheme, and the PAB-AM scheme the revenue of SPP is not very dependent on its location. The reason for this is that the SPP is getting paid in the DA market without congestion considerations, and does not participate in the AM, so its location in the DA market does not matter (no congestion). Then if there is any deviation in RT market SPP will pay a penalty price that is similar to other deviating generators.

For the PAB-DA scheme, it is observed that SPP makes more money in uncongested areas (around 1% more than PAB-AM), this encourages construction of the SPP in uncongested areas (the only exception is when the SPP is in the area that has negative prices). The reason for this is that the bidding function for the PAB scheme depends on two things: the mean value (μ) of bus price in a sliding window, and the standard deviation (σ) of bus price in a sliding window as explained in section 2.5. In congested areas, the area price fluctuation causes the standard deviation to be higher in uncongested areas. Having a high standard deviation means that the SPP price bid in this area will be low, which mean less revenue. So with MP-DA, it is more attractive for SPPs to be constructed in congested areas. Also, SPPs are more motivated to participate in a market where all generators are paying international fuel prices.

Table 4.7 Comparing SPP annual revenue in different schemes when located in different locations (generators are using old fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
Area 1	$7.91	$783,274	$818,311	$751,138	$750,275
Area 2	$7.51	$783,301	$824,240	$751,121	$758,065
Area 9	$7.22	$784,291	$805,875	$751,128	$760,894
Area 10	$4.40	$786,109	$634,372	$751,167	$718,875

Table 4.8 Comparing SPP annual revenue in different schemes when located in different locations (generators are using new fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
Area 1	$13.66	$1,301,044	$1,357,875	$1,244,717	$1,240,472
Area 2	$12.74	$1,302,260	$1,394,634	$1,244,716	$1,254,320
Area 9	$11.84	$1,303,351	$1,325,571	$1,244,727	$1,259,290
Area 10	$7.93	$1,305,264	$1,117,135	$1,244,779	$1,185,436

Table 4.9 Comparing SPP annual revenue in different schemes when located in different locations (generators are using international fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
Area 1	$33.45	$2,956,405	$3,161,475	$2,742,391	$2,696,992
Area 2	$28.32	$2,961,286	$3,113,103	$2,742,206	$2,731,257
Area 9	$26.86	$2,961,494	$2,998,530	$2,742,462	$2,768,599
Area 10	$23.68	$2,963,755	$2,607,174	$2,743,008	$2,569,515

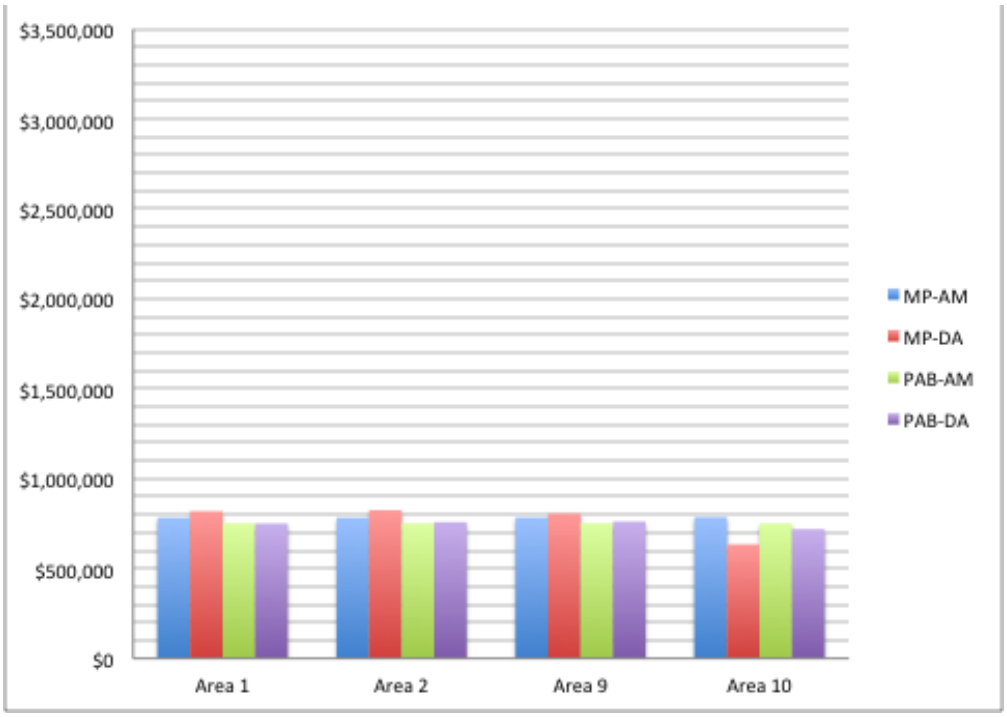

Figure 4.12 Comparing SPP revenue using different market schemes in different areas when using old fuel prices

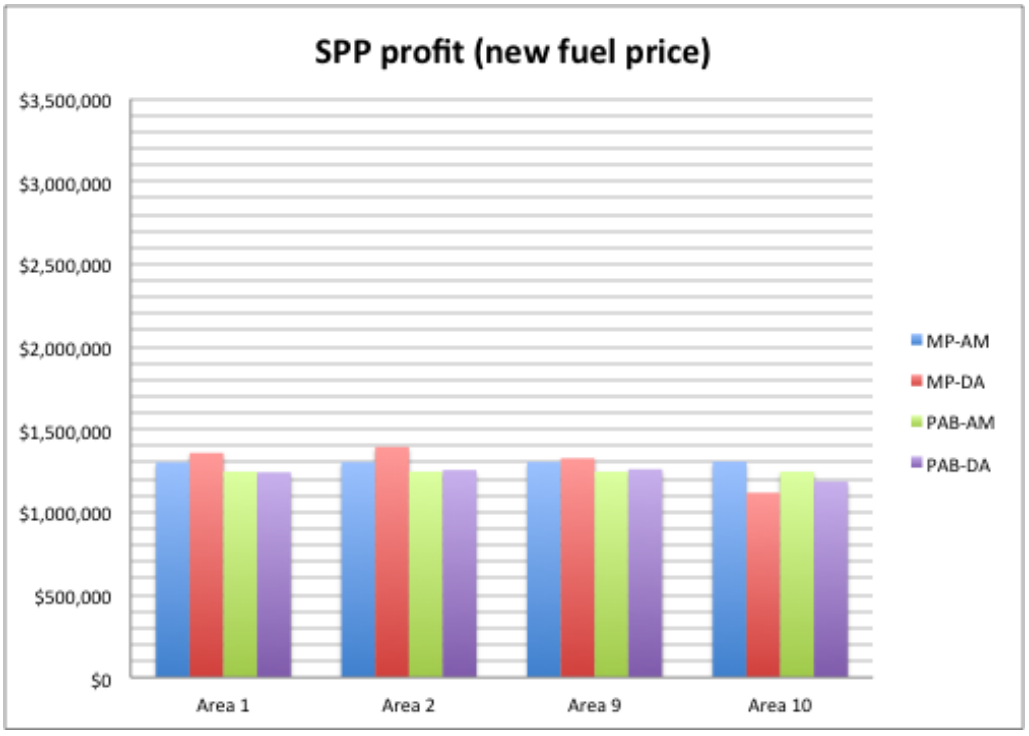

Figure 4.13 Comparing SPP revenue using different market schemes in different areas when using new fuel prices

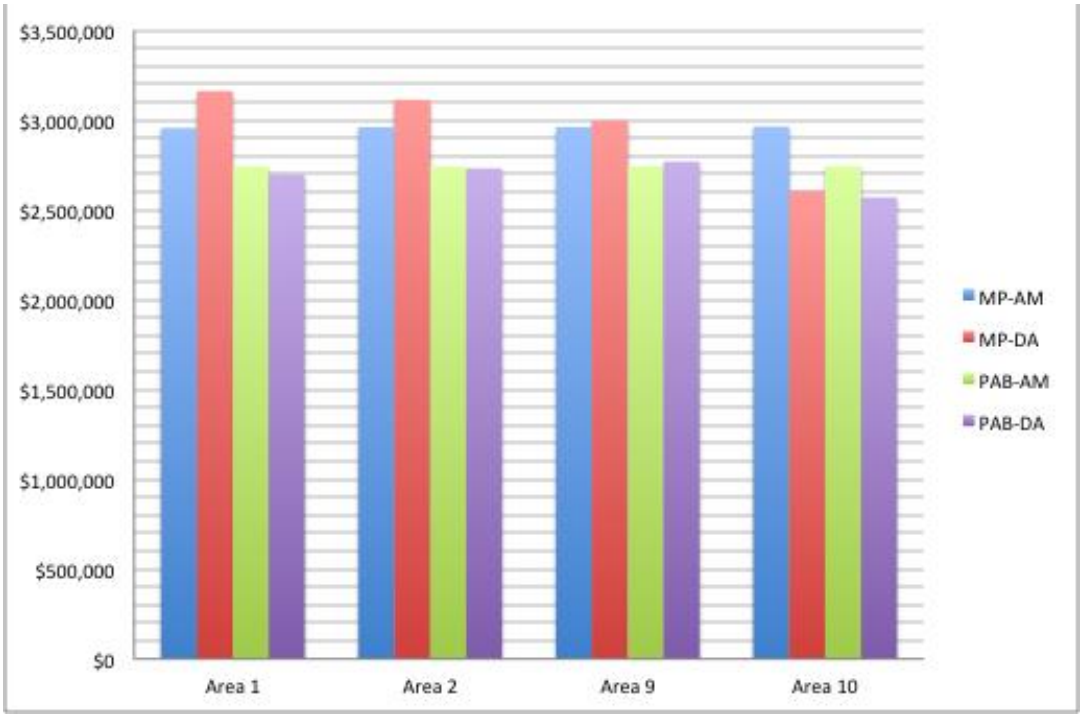

Figure 4.14 Comparing SPP revenue using different market schemes in different areas when using international fuel prices

4.5 GENERATORS' PROFIT

The total generators' profit using different market schemes without SPP and when a 50 MWp SPP is installed in different areas is presented in Table 4.10, Table 4.11, and Table 4.12, and illustrated in Figure 4.15, Figure 4.16, and Figure 4.17. It can be observed that total generators' profit in schemes that deal with congestion in the DA market (MP-DA and PAB-DA) is higher as generators will try to maximize their profit during congestion. Also, it is observed that generators in the PAB-DA scheme make more profit compared to MP-DA (around 38% when using old fuel prices, 20% when using new fuel prices, and 3% when using international fuel prices) as the generator bidding function in the PAB-

DA depends on historical data, which makes generators in congested areas continue to bid high prices.

For the total generators' profit in schemes that deal with congestion in an adjustment market (MP-AM, and PAB-AM), generators make less profit than schemes that deal with congestion in the DA market, as the MP-DA scheme makes around 80% more than the MP-AM scheme when generators are using old domestic fuel prices, around 50% more when generators are using new domestic fuel prices and around 20% more when using international fuel prices. Also the PAB-DA scheme makes around 130% more than the PAB-AM scheme when generators are using old domestic fuel prices, around 70% more when generators are using new domestic fuel prices and around 20% more when using international fuel prices. This observation supports the UK market (uses PAB-AM) claim that their market limits exercise of market power and has lower prices for consumers [35]. It is observed in schemes that deal with congestion in AM that the location of the SPP does not affect generator's profit. Also, it can be seen that when generators are using international fuel prices they make more profit. An important observation that in the MP-DA scheme in the case of using old fuel prices while having the SPP located on the area with negative prices, generators will make more profit than not installing SPP at all, as ISO will have to dispatch expensive generators to relieve the congestion.

Table 4.10 Comparing total generators' annual profit in different schemes when located in different locations (generators are using old fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
No SPP		$106,285,464	$193,598,788	$114,174,674	$267,611,812
Area 1	$7.91	$105,896,435	$191,355,558	$113,780,843	$263,036,637
Area 2	$7.51	$105,891,588	$191,051,361	$113,778,341	$262,467,842
Area 9	$7.22	$105,876,944	$192,336,011	$113,776,681	$263,455,053
Area 10	$4.40	$105,948,787	$194,715,431	$113,775,883	$267,140,277

Table 4.11 Comparing total generators' annual profit in different schemes when located in different locations (generators are using new fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
No SPP		$183,138,235	$281,150,982	$198,015,004	$342,616,843
Area 1	$13.66	$182,477,792	$276,574,125	$197,446,690	$337,547,587
Area 2	$12.74	$182,580,148	$277,328,861	$197,436,638	$336,552,211
Area 9	$11.84	$182,541,105	$279,531,982	$197,424,713	$339,746,051
Area 10	$7.93	$182,557,023	$280,986,468	$197,424,899	$341,957,098

Table 4.12 Comparing total generators' annual profit in different schemes when located in different locations (generators are using international fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
No SPP		$645,193,921	$775,900,112	$673,775,363	$796,097,621
Area 1	$33.45	$640,013,228	$770,572,378	$668,544,062	$789,709,596
Area 2	$28.32	$639,937,074	$767,010,460	$668,417,204	$783,265,572
Area 9	$26.86	$639,978,992	$770,163,548	$668,387,384	$791,425,656
Area 10	$23.68	$639,999,736	$769,981,154	$668,383,867	$792,106,433

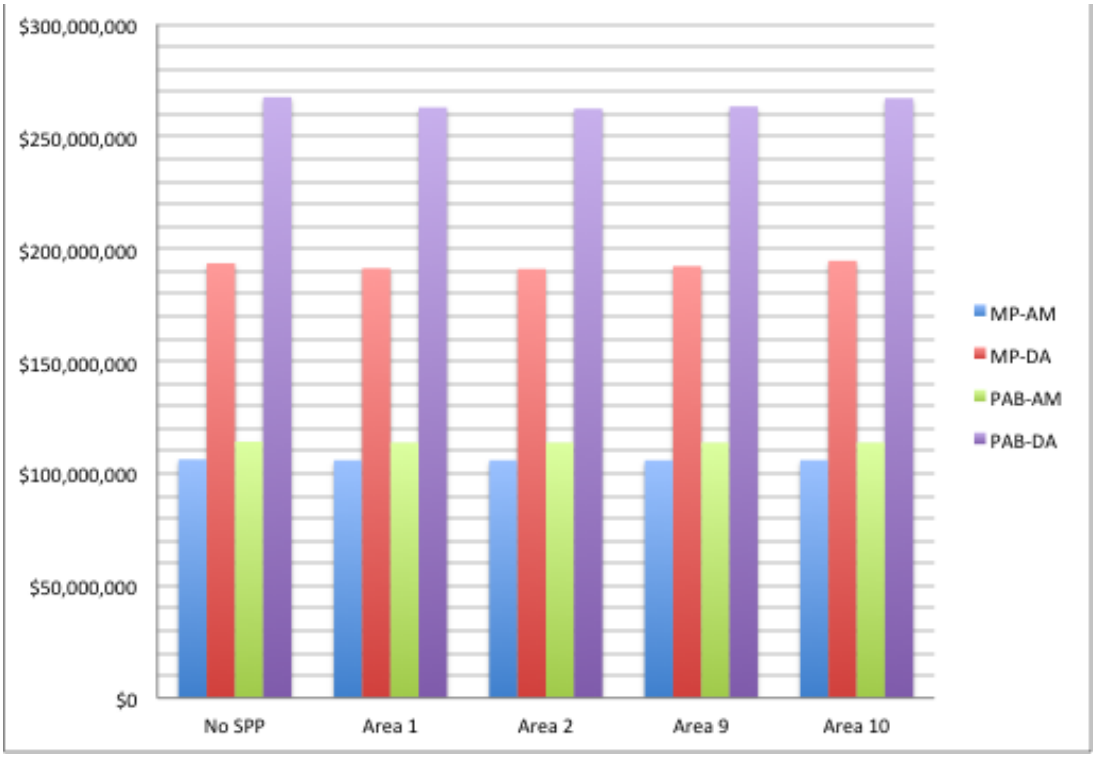

Figure 4.15 Comparing total generators profit using different market schemes in different areas when using old fuel prices

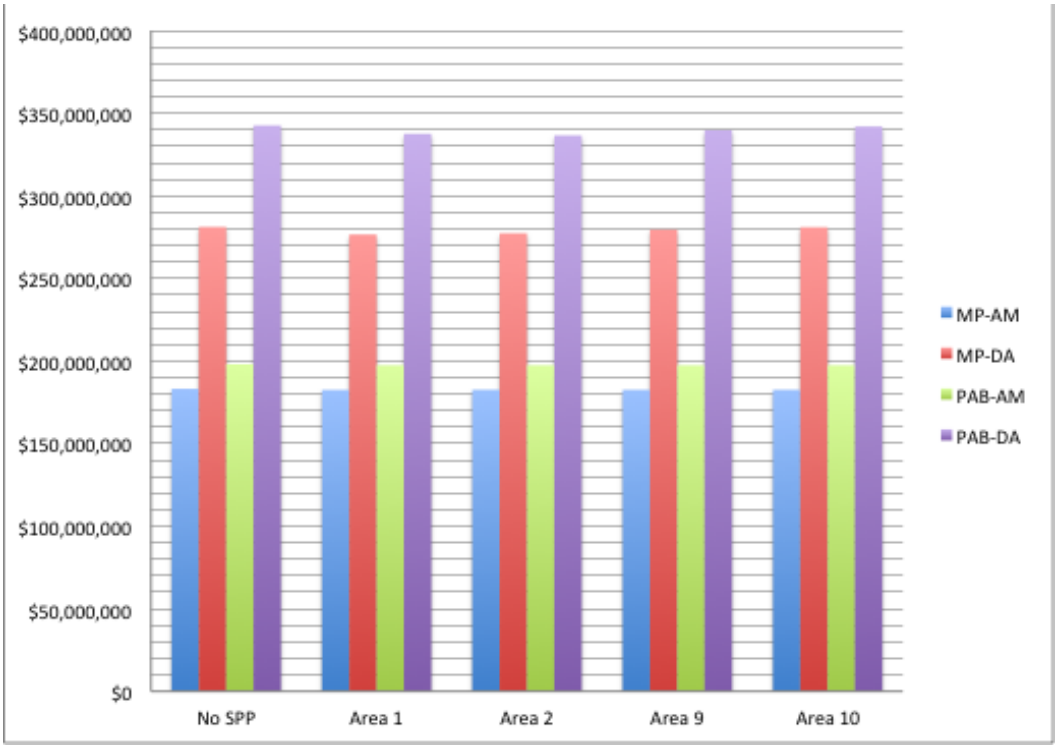

Figure 4.16 Comparing total generators profit using different market schemes in different areas when using new fuel prices

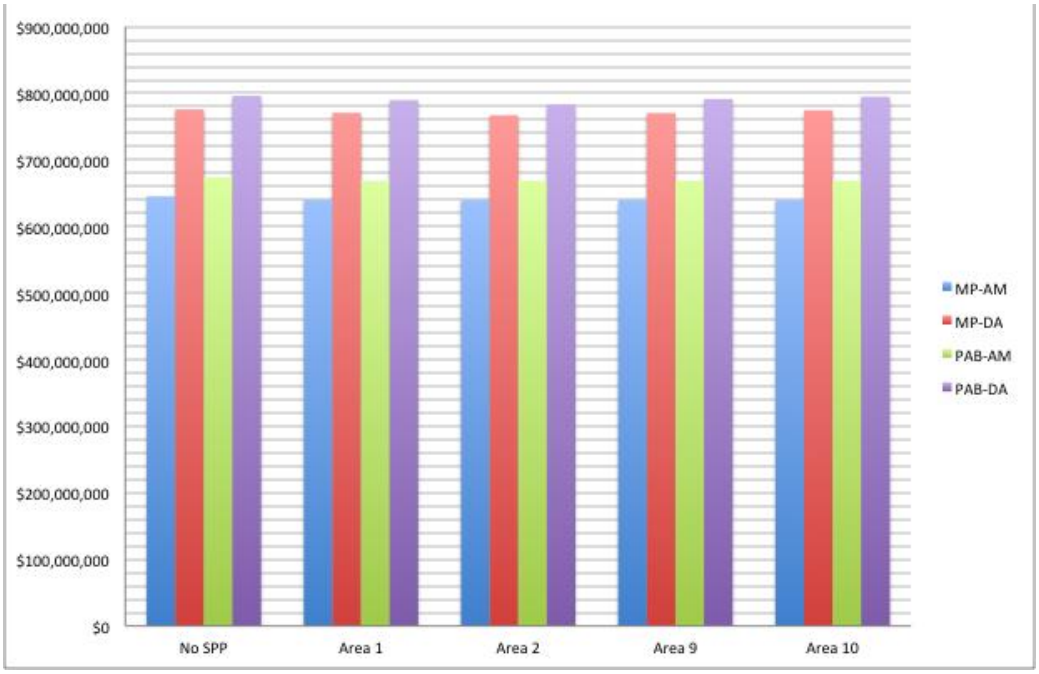

Figure 4.17 Comparing total generators profit using different market schemes in different areas when using international fuel prices

4.6 CONSUMER SURPLUS

The consumer surplus using different market schemes without SPP and when a 50 MWp SPP is installed in different areas is illustrated in Figure 4.18, Figure 4.19, and Figure 4.20. An important note: the load is assumed to be inelastic, so the theoretical consumer surplus infinite; however, differences in consumer surplus are still relevant. To see them, a cap price of $1000 was considered (see section 3.4.1). In this section, it is important to compare the difference between the consumer surplus value rather than the value itself. Therefore, only the difference of consumer surplus is presented taking the consumer surplus of the PAB-DA scheme without SPP as a reference. See Table 4.13, Table 4.14, and Table 4.15.

In general, in all the schemes the consumer surplus increases with the introduction of SPP. Schemes that deal with congestion in the AM (MP-AM, and PAB-AM) have higher consumer surplus as generators in these schemes make less revenue. Also, the location of SPP does not affect the consumer surplus. In the MP-AM scheme, the consumer surplus was higher than the PAB-AM scheme as the total generators revenue is less.

In schemes that deal with congestion in the DA market (MP-DA, and PAB-DA), the consumer surplus is higher when the SPP is installed in the congested area. Consumer surplus in the MP-DA was greater than PAB-DA consumer surplus because in the PAB-DA scheme generators make more revenue. To have a better perspective, the consumer payment is discussed in the next section.

Table 4.13 Comparing the difference of annual consumer surplus in different schemes when located in different locations taking PAB-DA with no solar as a reference (generators are using old fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
No SPP		$162,638,833	$75,325,509	$154,749,623	$0
Area 1	$7.91	$163,043,443	$77,549,282	$155,191,170	$4,700,899
Area 2	$7.51	$163,042,984	$77,842,272	$155,188,412	$5,254,528
Area 9	$7.22	$163,052,843	$76,572,193	$155,186,270	$4,248,945
Area 10	$4.40	$162,972,353	$74,357,447	$155,176,456	$527,301

Table 4.14 Comparing the difference of annual consumer surplus in different schemes when located in different locations taking PAB-DA with no solar as a reference (generators are using new fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
No SPP		$161,772,357	$63,759,610	$146,767,589	0
Area 1	$13.66	$162,477,862	$68,324,699	$147,441,899	$5,263,509
Area 2	$12.74	$162,362,891	$67,521,805	$147,430,354	$6,238,390
Area 9	$11.84	$162,371,412	$65,358,314	$147,421,212	$2,984,239
Area 10	$7.93	$162,329,169	$64,087,853	$147,399,336	$750,556

Table 4.15 Comparing the difference of annual consumer surplus in different schemes when located in different locations taking PAB-DA with no solar as a reference (generators are using international fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
No SPP		$155,524,807	$24,818,616	$125,869,168	0
Area 1	$33.45	$160,771,323	$30,007,102	$131,691,287	$6,843,933
Area 2	$28.32	$160,893,366	$33,668,164	$131,578,904	$13,472,151
Area 9	$26.86	$160,772,090	$30,550,497	$131,525,897	$4,926,012
Area 10	$23.68	$160,770,096	$31,145,259	$131,459,921	$4,320,244

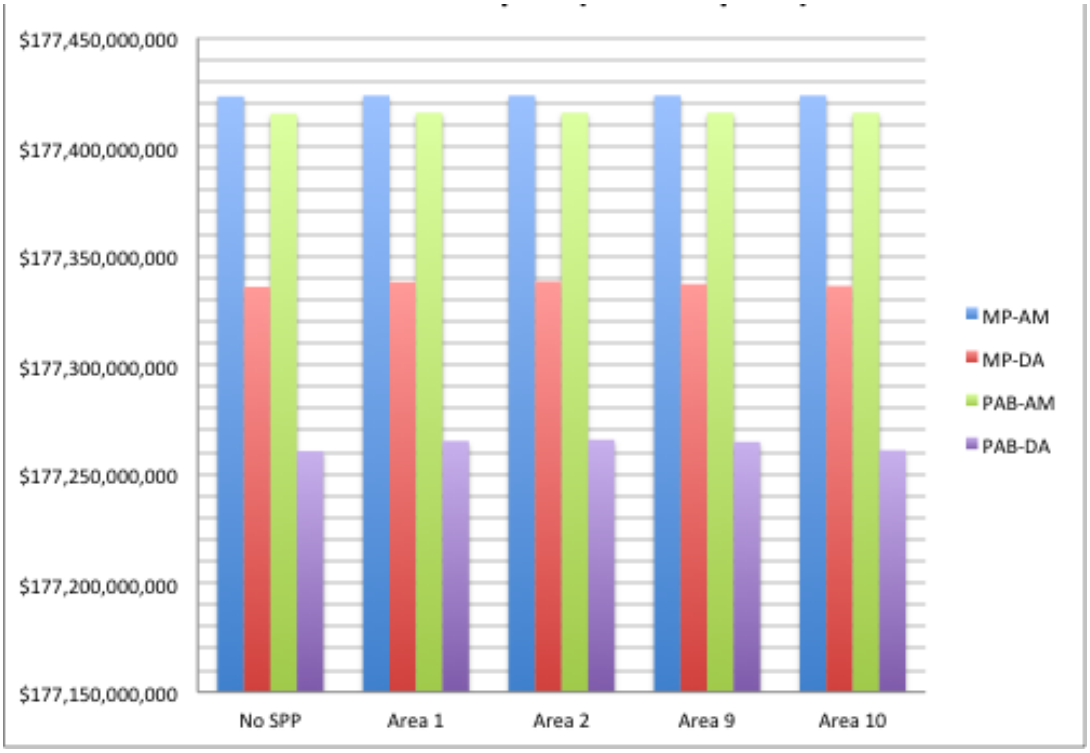

Figure 4.18 Comparing consumer surplus using different market schemes in different areas when using old fuel prices

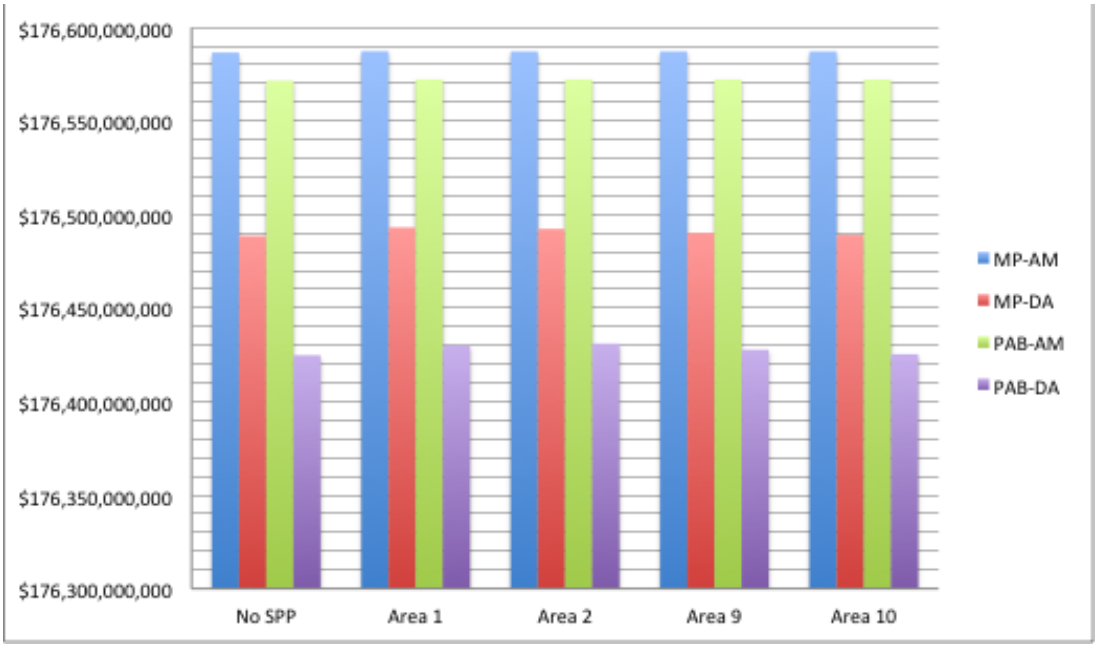

Figure 4.19 Comparing consumer surplus using different market schemes in different areas when using new fuel prices

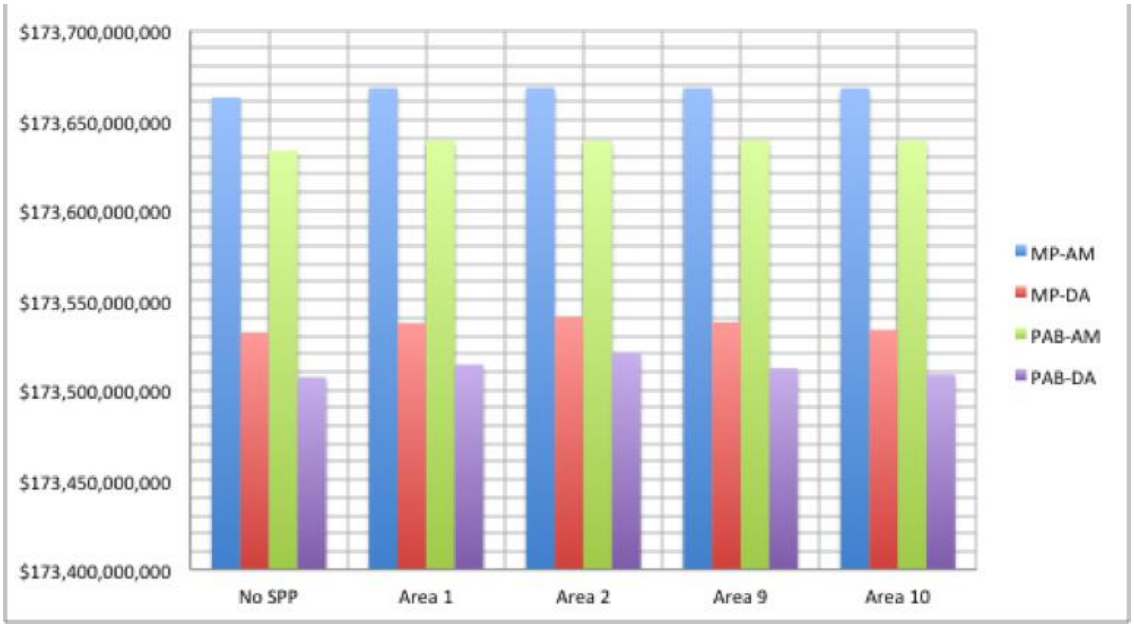

Figure 4.20 Comparing consumer surplus using different market schemes in different areas when using international fuel prices

4.7 CONSUMER PAYMENT

The consumer surplus using different market schemes without SPP and when a 50 MWp SPP is installed in different areas is presented in Table 4.16, Table 4.17, and Table 4.18, and illustrated in Figure 4.21, Figure 4.22, and Figure 4.23. As explained from consumer surplus analysis, consumers pay more in schemes that deal with congestion in the DA market (MP-DA, and PAB-DA).

In the MP scheme, consumers in the MP-DA scheme pay around 9% more than the MP-AM scheme if generators are using the old domestic fuel prices. They pay around 8% more than the MP-AM scheme if generators are using the new domestic fuel prices. And they pay around 5% more than the MP-AM scheme if generators are using the international fuel prices. For the PAB scheme, consumers in the PAB-DA scheme pay around 12% more than the PAB-AM scheme if generators are using the old domestic fuel prices. They pay around 7% more than the PAB-AM scheme if generators are using the new domestic fuel prices. And they pay around 3% more than the PAB-AM scheme if generators are using the international fuel prices. An important observation that in the MP-DA scheme in the case of using old fuel prices while having the SPP located on the area with negative prices, consumers will pay more than not installing SPP at all, as the ISO will have to dispatch expensive generators to relieve the congestion.

Very high profit for generators means less consumer surplus, and this means high prices for consumers. On the other hand, high consumer surplus means less profit for generators, which means that generators will be less motivated to participate in the market.

Table 4.16 Comparing annual total consumer payment in different schemes when SPP is located in different locations (generators are using old domestic fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
No SPP		$1,254,419,493	$1,369,381,874	$1,264,974,186	$1,424,264,535
Area 1	$7.91	$1,253,171,458	$1,365,153,861	$1,264,541,307	$1,419,402,374
Area 2	$7.51	$1,253,220,168	$1,365,744,760	$1,264,542,027	$1,418,765,102
Area 9	$7.22	$1,253,215,252	$1,367,329,867	$1,264,542,905	$1,419,843,319
Area 10	$4.40	$1,253,331,034	$1,372,292,296	$1,264,559,794	$1,423,739,637

Table 4.17 Comparing annual total consumer payment in different schemes when SPP is located in different locations (generators are using new domestic fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
No SPP		$2,091,198,208	$2,257,366,403	$2,110,522,231	$2,262,130,305
Area 1	$13.66	$2,089,101,906	$2,247,047,473	$2,109,865,180	$2,256,766,895
Area 2	$12.74	$2,089,311,045	$2,250,750,836	$2,109,867,720	$2,255,739,906
Area 9	$11.84	$2,089,289,459	$2,253,977,907	$2,109,877,339	$2,259,024,286
Area 10	$7.93	$2,089,345,143	$2,257,267,888	$2,109,904,900	$2,261,372,692

Table 4.18 Comparing annual total consumer payment in different schemes when SPP is located in different locations (generators are using international fuel prices)

SPP location	Annual average price	MP-AM	MP-DA	PAB-AM	PAB-DA
No SPP		$5,016,424,716	$5,285,474,964	$5,091,855,671	$5,223,344,419
Area 1	$33.45	$5,007,992,397	$5,266,291,824	$5,086,254,652	$5,216,578,463
Area 2	$28.32	$5,008,077,809	$5,267,601,652	$5,086,121,819	$5,209,316,349
Area 9	$26.86	$5,008,201,658	$5,275,370,697	$5,086,209,753	$5,218,424,377
Area 10	$23.68	$5,008,267,010	$5,281,213,542	$5,086,249,360	$5,218,987,387

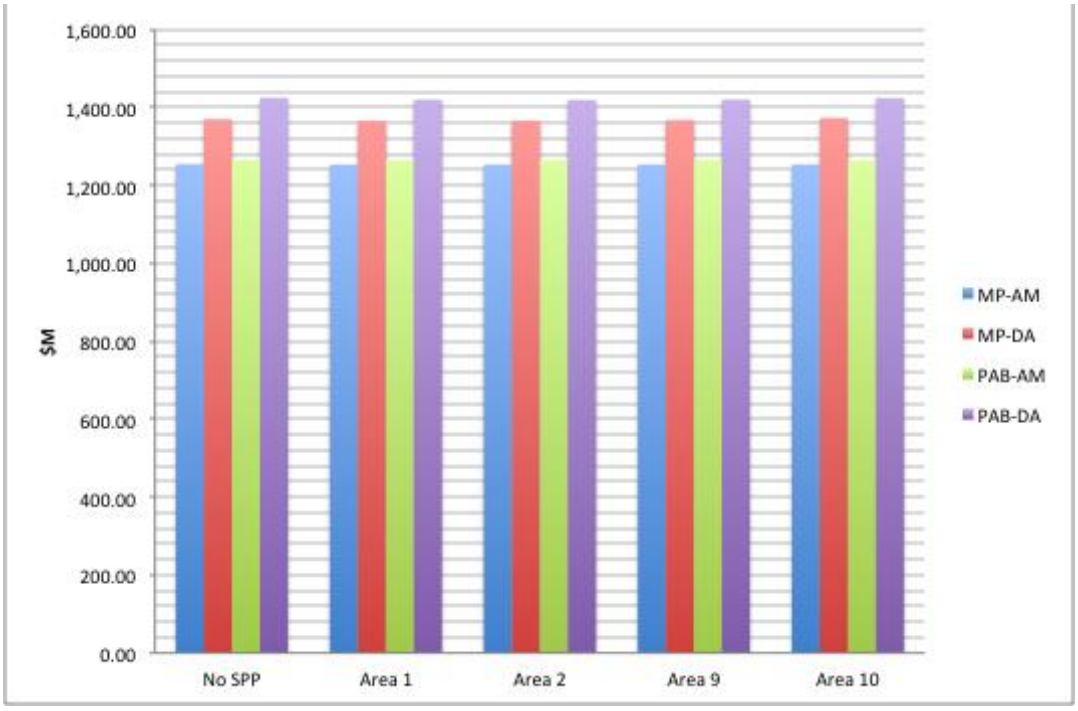

Figure 4.21 Comparing total consumer payment using different market schemes in different areas when using international fuel prices

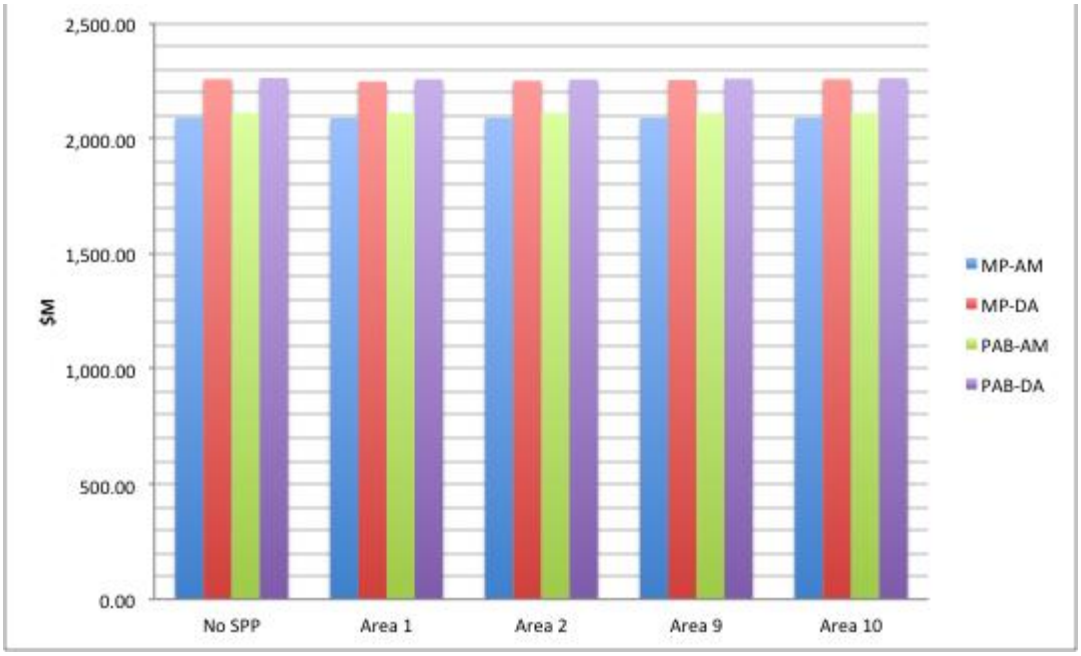

Figure 4.22 Comparing total consumer payment using different market schemes in different areas when using international fuel prices

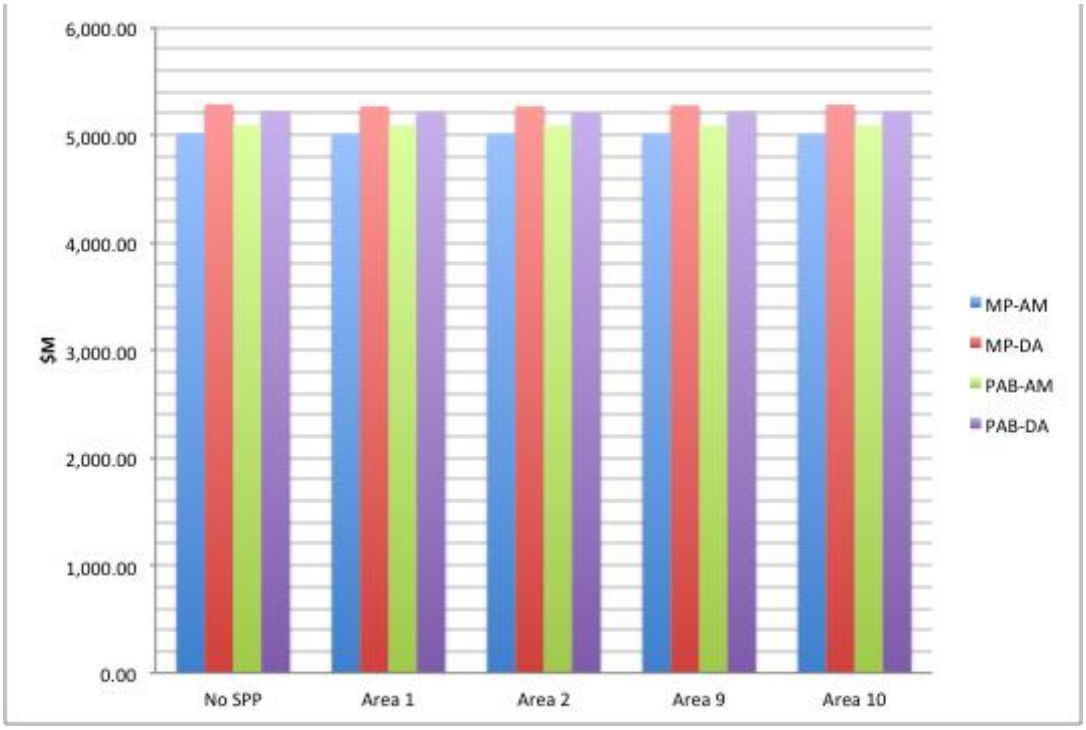

Figure 4.23 Comparing total consumer payment using different market schemes in different areas when using international fuel prices

4.8 SOCIAL WELFARE

As discussed in section 3.4.1, the social welfare for all schemes are almost the same with an inelastic demand; most of the difference will be the social welfare distribution as social welfare is the summation of generators profit and consumer surplus (or total generation cost subtracted from the worth), which was, discussed earlier. From Figure 4.24, Figure 4.25, and Figure 4.26 it can be observed that in general in the MP scheme both the MP-AM and the MP-DA scheme are the same while the PAB schemes have relatively lower social welfare value with the PAB-DA scheme as the lowest. This is because in the PAB scheme some expensive generators run while more economical generators do not because

of their unsuccessful bid. Also, it is clear that adding SPP will increase the social welfare in all schemes.

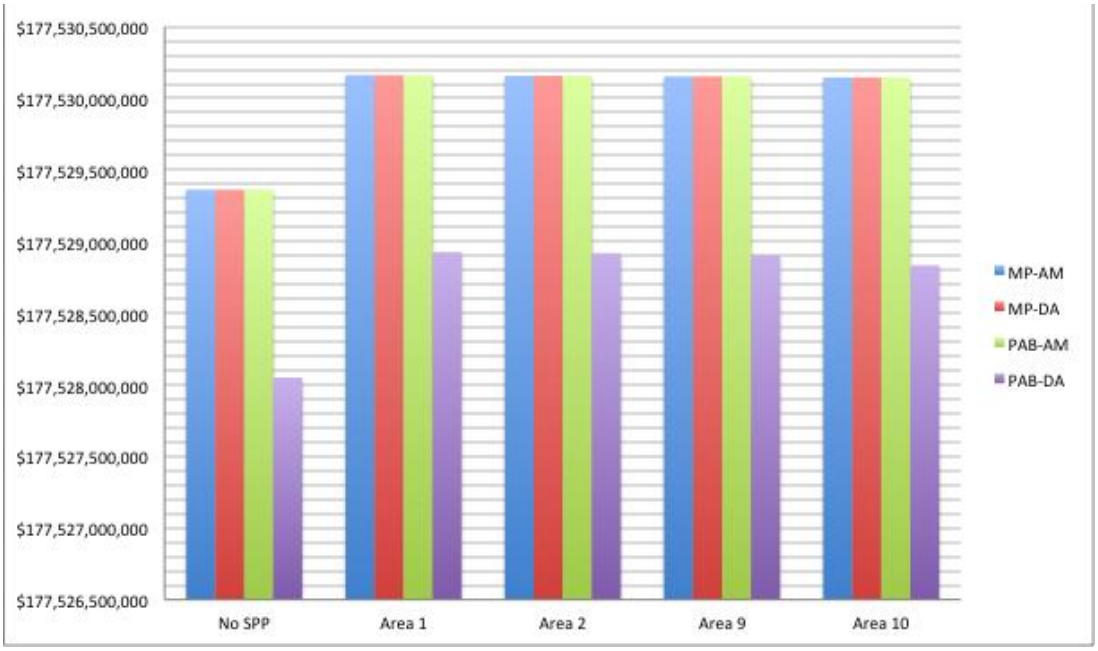

Figure 4.24 Comparing total annual social welfare using different market schemes in different areas when using international fuel prices

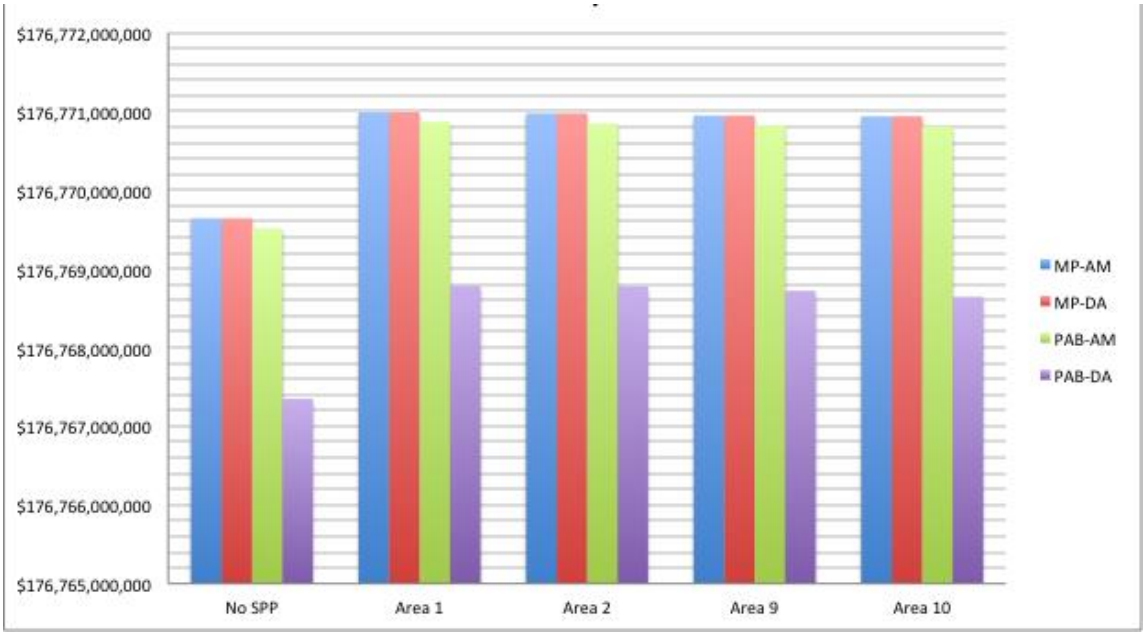

Figure 4.25 Comparing total annual social welfare using different market schemes in different areas when using international fuel prices

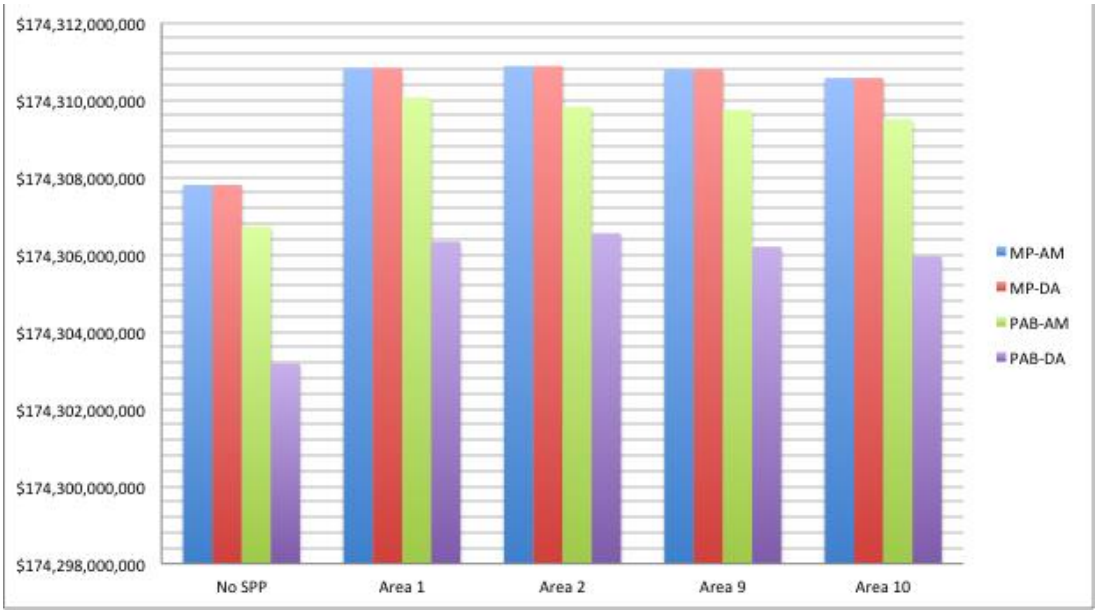

Figure 4.26 Comparing total annual social welfare using different market schemes in different areas when using international fuel prices

4.9 The Effect of Increasing Solar Penetration

For an SPP developer it is important to know how much SPP capacity can be installed on the most profitable bus (most expensive bus) before the bus price falls below the second most expensive one. Results of increasing SPP capacity in Area 1 using different fuel prices in the MP-DA scheme are illustrated in Figure 4.27, Figure 4.28, and Figure 4.29.

Looking in the figures, the market price drops as solar power capacity increases. This is expected as solar power is a cheap energy (as the rest of renewable energies), the increase of solar power capacity will cause price drop in the market. Another observation is that as SPP capacity increases on the most expensive bus, this bus price will drop to a point where it will be more profitable to install an SPP in different location (break point), with time this will encourage installing SPPs in scattered areas in the grid.

Finally, this break point changes depending on the fuel used in the market. For example it can be observed when generators are using old local fuel prices, the SPP will cause the highest bus price to drop below second highest when its capacity reaches around 140 MW. When generators are using the new fuel prices, SPP capacity can reach up 80 MW before the highest bus price drops below the second most expensive bus. While when generators are using the international fuel prices, SPP capacity can reach up to 800 MW before the highest bus price drops below the second most expensive bus. It is clear that fuel prices affect this break point. For old local fuel prices, the difference between gas and crude oil was just 0.01 $/MBTU (around 1%) while new fuel price difference between gas and crude oil is 0.13 $/MBTU (around 12%). Unlike local prices, looking at the international fuel price it can be observed that the crude oil price is much higher than the gas price.

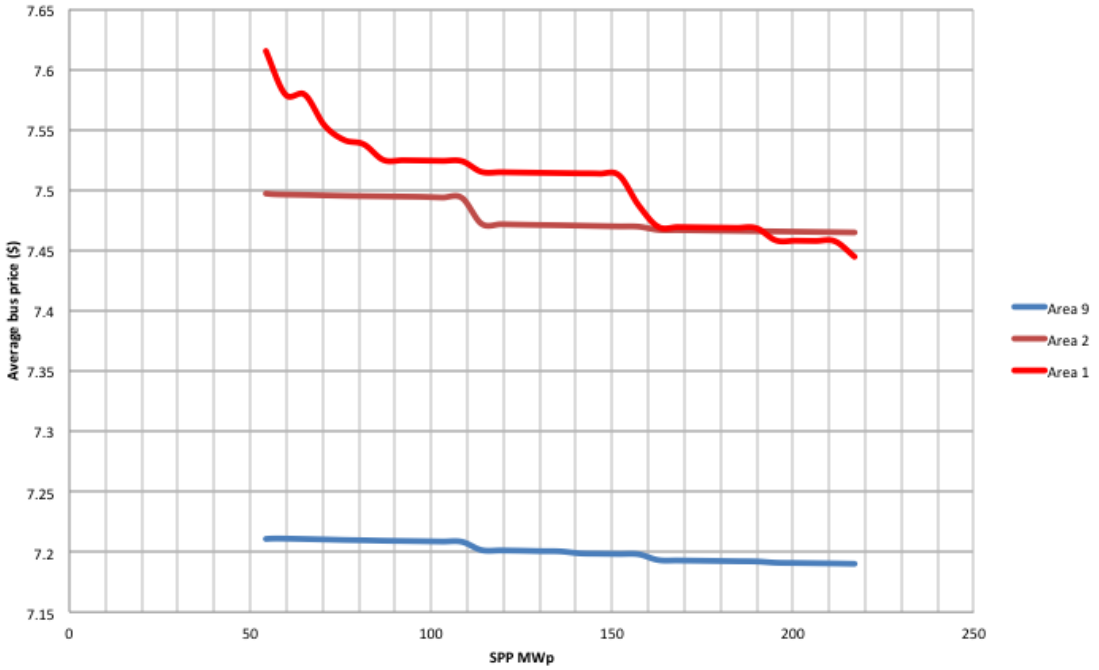

Figure 4.27 The effect of increasing SPP capacity at the most expensive bus (bus 169) on the annual average bus price with respect to second most expensive bus (bus 113). Bus 1 represents the uncongested area when using old domestic fuel prices

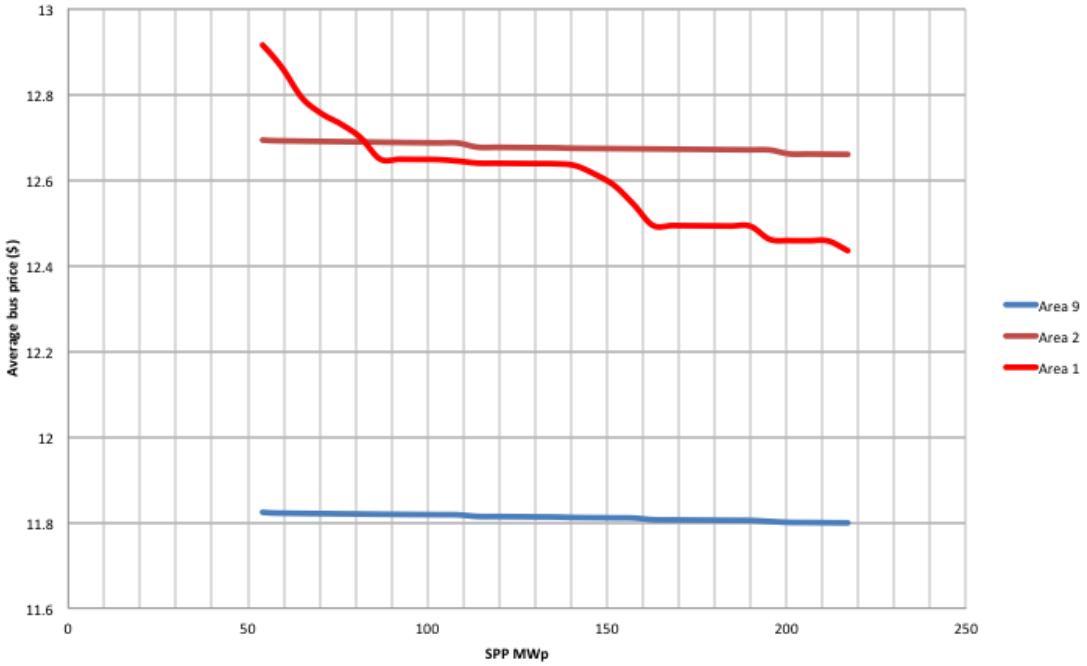

Figure 4.28 The effect of increasing SPP capacity at the most expensive bus (bus 169) on the annual average bus price with respect to second most expensive bus (bus 113). Bus 1 represents the uncongested area when using new domestic fuel prices

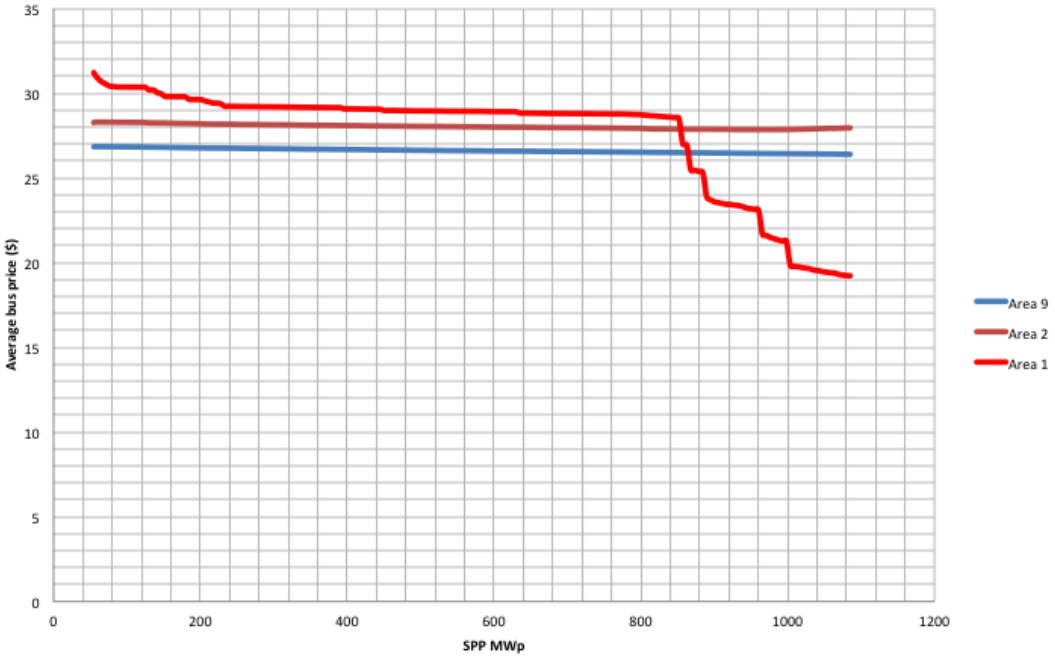

Figure 4.29 The effect of increasing SPP capacity at the most expensive bus (bus 169) on the annual average bus price with respect to second most expensive bus (bus 113). Bus 1 represents the uncongested area when using international fuel prices

4.10 Results Analysis

In this chapter the comparison of different electricity market schemes in the Saudi grid effect on SPPs was simulated. Part of the work in this chapter was constructing the Saudi grid using very limited publically available information.

Table 4.19 shows a summary comparison between all schemes. In the MP-AM scheme, the SPP location did not affect its revenue, generators profit was low, and consumer surplus was high. In the MP-DA scheme, the SPP made the highest revenue if it was located in the congested area, generators profit was high, and consumer surplus was low. In the PAB-AM scheme, the SPP revenue is low, its location does not affect its revenue, generators profit is low, and consumer surplus is high. In the PAB-DA, the SPP

revenue is low, and its revenue is higher when located in the uncongested area, the generators profit is high, and has the lowest consumer surplus.

Table 4.19 Comparing all schemes

	MP-AM	**MP-DA**	**PAB-AM**	**PAB-DA**
SPP revenue	High	Very high	Low	Low
SPP location effect on revenue	No effect	Higher in congested area	No effect	Higher in uncongested area
Total generators' profit	The lowest	High	Low	The highest
Consumer surplus	The highest	Low	High	The lowest
Social welfare	High	High	Low	The lowest

It was observed that the PAB bidding function has a disadvantage in the Saudi grid because the bidding function depends on the standard deviation value, where a few price spikes can affect the bidding function drastically.

In schemes that deal with congestion in an adjustment market, SPPs are not interested in constructing their facility in the congested area. It is clear that SPP makes the highest revenue in the MP-DA scheme. Also, the MP-DA scheme structure motivates constructing SPPs and generators in congested areas.

In addition, it was observed that the effect of increasing SPP capacity when it is in the most expensive area varies depending on the fuel price used in the market. It is clear that the break point between the most expensive bus and the next expensive bus depends on the fuel subsidy policy which will affect SPP placement.

4.11 CONCLUSION

In conclusion, based on the results in the previous section, it is clear that the MP-DA scheme is the best scheme for SPP, as the SPP makes more profit, and the scheme moti-

vates SPP construction in congested areas. In addition the MP-DA scheme has the highest social welfare. Schemes that deals with congestion in AM have higher consumer surplus with the MP-AM scheme having the highest, which is good for consumers in the short run, but will slow down constructing new power plants.

Chapter 5. CONTRIBUTIONS OF THE RESEARCH WORK AND FUTURE RESEARCH DIRECTIONS

This chapter lists all contributions of this dissertation, discusses their importance, and future research directions. The ultimate goal of this work is to identify a market scheme that motivates Solar Power Plants (SPP) and increases social welfare in the Saudi electricity grid, assuming SPPs are not getting any incentives. Part of this work was to evaluate SPP participation in different electricity market structures on a small scale (5-bus system). It was then expanded to the Saudi grid. The ultimate goal is to find the best electricity market scheme that motivates and suits SPP construction without incentives.

5.1 CONTRIBUTIONS

5.1.1 *A Comparison of Wind and Solar Energy Features Relevant to Market Performance*

Wind and solar, being stochastic renewable resources, are at first glance similar in how they will interact with markets. This work has confirmed that prediction error is the major influence on the unsubsidized performance of these resources in energy markets. The literature indicates that solar has, counter-intuitively, a significantly higher prediction error than wind, hence solar is more likely to be more sensitive to different market structures.

5.1.2 *Development of an Improved Optimal Bidding Strategy for PAB Market Scheme*

This work has developed new bidding functions for the PAB scheme, and adopted one of them. The first bidding function, which was suggested by Ren and Galiana[5] in [88], was derived di-

[5] Galiana in [88] suggested "other probability distributions such as normal are possible but less tractable analytically", but no work was actually done using this method.

rectly from the Gaussian distribution function and resulted in a theoretical optimal solution. The disadvantage of this bidding function is that it is intractable so it requires more computational effort to solve. Therefore a few computationally tractable approximations of the Gaussian distribution were obtained to provide rapidly near-optimal solution bid functions in less computational time. The first bidding function developed using an approximation was derived from a triangle approximation (Tri) probability distribution function. The developed bidding function is near optimal but difficult to implement. The next developed bidding function was based on the best fit of uniform distribution function (B-Uni) using the mean value and standard deviation of market price. Although the B-Uni bidding function is the least optimal of the three (Gaussian, Tri, and B-Uni) it is still very close but better than existing work [88] giving higher profit for SPP. Because it is easy to implement B-Uni was adopted. For simulation use where the bidding function is called many times, the computational benefits outweigh the sub optimality. This contribution was covered in detail in Chapter 2.

5.1.3 *Suggesting a New Method of Dealing with Congestion in a PAB Market Scheme*

In this work, unlike the PAB-AM scheme that deals with congestion in an Adjustment Market (AM), a PAB scheme that deals with congestion through the Day Ahead (DA) market, called PAB-DA scheme, is described, implemented and evaluated. The PAB-DA scheme has room for exercising market power because congestion separates market prices, which reduces the pool of generators that can bid in the congested areas, which creates the potential for exercise of market power. This is opposite to the general goals for adopting a PAB scheme, but on the other hand the PAB-AM does not send good price signals about generator locations to relieves congestion. Currently the well-known market that uses PAB-AM scheme is in the UK. The second market

that uses PAB is the Iranian market; but Iran's method of dealing with congestion is not well known.

5.1.4 *Identifying the Financial Effect of Power Production Deviation on Conventional Generators and Solar Power Plants in Different Market Schemes*

This research showed that having a power deviation by generators between the DA market and the RT market has different financial effects depending on the market scheme. In the MP scheme deviation will increase the generators profit in case of over-production in RT market and will reduce profit in case of under-production in RT market. On the other hand this research established that in the PAB scheme, the effect of deviation is not monotonic. Also, it established that the zero incremental cost of solar power motivates SPPs not to deviate in the PAB market unlike the MP scheme. This was discussed in Chapter 3.

5.1.5 *Identifying the Financial Effect of Congestion on the Conventional Generators and Solar Power Plants in Different Market Schemes*

The effect of installing SPP in different market schemes in the presence of congestion was studied. Findings show that with small solar power penetration, in the MP-DA, and the PAB-DA schemes, SPPs make more profit if they are installed on the expensive side of congestion. In the MP-AM, and PAB-AM schemes, SPP profit is not affected by congestion as the system deals with congestion in the adjustment market. In the presence of high congestion, in the PAB-DA scheme producers make more profit than producers in the MP-AM, PAB-AM, and PAB-DA schemes and the consumer surplus was much lower. This is discussed in detail in Chapter 3.

5.1.6 *Constructing the Saudi Grid Model*

My sponsor Saudi Aramco is the largest oil producer in the world. Like most oil companies, Aramco's new direction is to be a world leader in energy supply. Aramco is leading the installation of several Solar Power Plants in an effort to limit pollution and CO2 emission. Aramco is also involved in the electricity restructuring process. Assessing the four electricity market schemes with the introduction of unsubsidized SPPs helps to identify the best scheme for the Saudi grid. Aramco being part of SPP construction would be interested in an optimal market for SPP. This assessment included observing the SPPs' profit, generators' profit, consumers' surplus, and the system social welfare. Therefore the market structures were evaluated on a model of Saudi grid.

Very little information was available on the of the Saudi grid model from Electricity & Cogeneration Regulatory Authority (ECRA) and SEC. Using this limited information, in addition to interviews with SEC representatives, using publically available generic generation data from EPA [98], and tracing transmission lines through wikimapia [104], an approximate Saudi grid model was constructed. Currently there are four operation areas for SEC, the Eastern, Central, Western and Southern Operating areas [100]. The focus in this modeling is the Eastern and Central region as they are interconnected and have most of the Saudi Arabian generation and load. This work was completed in collaboration with my colleague Mohammad Abahussain. This was discussed in Chapter 4.

5.1.7 *Evaluation of the Effect of SPP Bidding on the Saudi Grid Using the Four Market Schemes*

The effects of SPP bidding in the Saudi grid in the four proposed market schemes were evaluated. A summary of the findings appears in Table 4.19. SPP profit was highest when using the MP-DA scheme, especially when SPP is located in a congested area. SPP profit

was low when using the PAB scheme. SPP revenue when using the MP-DA scheme was around 10% more than the PAB-DA scheme when using domestic prices and around 17% more than the MP-DA scheme when using the international fuel prices. SPP profit was not affected by location when a market scheme deals with congestion in an adjustment market (MP-AM, and PAB-AM). Total generator profits in the PAB-DA scheme were the highest. The reason for this is that generators are able to exercise market power in the DA market if they are located in a congested area. Higher consumer surplus was obtained when schemes deal with congestion in an adjustment market. One significant finding for fuel price policy in Saudi Arabia is that in an international fuel price environment, the SPP profit will be much higher, and there will be more incentive for SPP capacity to be installed. This is discussed in Chapter 4.

5.2 FUTURE WORK

5.2.1 *Using the Other PAB Bidding Function*

In this research, the adopted PAB bidding function was the "B-Uni" which reaches a near optimal solution as discussed in Chapter 2. It would be interesting do the same work using the bidding function derived from the Gaussian distribution function that achieves the theoretical optimal solution. Although the bidding function derived from the Gaussian distribution function will have a better results it would be computationally expensive to solve the function as it contains an error function (*erf*). The principle benefit of this work would be quantifying the error of the B-Unit approximation.

5.2.2 *Studying Detailed Market Rules in Each Scheme*

This work covered the simplified general form of each scheme that retains its main features, but each electricity market also has unique detailed rules. For example, many markets use the MP-DA scheme like PJM, CAISO, and ERCOT. Although they use the same scheme, each one has different detailed rules like price cap, exporting/importing electricity rules, must run units rules, congestion penalty, and renewable percentage portfolio. These additional rules may have some significant financial effects on the outcome. In addition, a hybrid scheme could be studied in which the scheme uses the MP or PAB method in the DA market then uses the opposite method in the RT market. An example of this sort of scheme is the Nordic market. In future, these detailed market rules and different combinations could be evaluated.

5.2.3 *Finding the Effect of Widespread SPP Installation in the Saudi Grid*

In this work only one SPP was installed on the grid. It would be interesting to find the effects (and/or likelihood) of widespread SPP implementation. Given the various market schemes and some model of whether SPPs will be built, it would be interesting to find the economic penetration limit. As an example, California is currently facing what is called the duck curve of load in which solar power production becomes so high during the day that as evening approaches high ramp generators are needed to compensate for solar power drop off. It is suggested to find the economic solar penetration limit. This will require gathering more data on all generators ramp rate limits with the addition of dealing with must run units.

5.2.4 *Finding the Effect of SPP Intentional Over and Under-bidding in the Saudi Grid Using all Schemes*

As discussed in Chapter 3, SPP deviation has different effects between all the schemes. This work was done on a single bus model. It would be interesting to see how much profit SPPs

would gain if they did practice this unlawful behavior in the Saudi grid. Deviation from the bid amount could result in financial benefits for SPP. In some cases this deviation is unintentional and inevitable. The challenge is to determine if the deviation was intentional of just a bad forecast.

5.3 CONCLUSION

This dissertation has explored the effects of photovoltaic solar power plant (SPP) participation in different market structures in a deregulated environment without financial incentives in the Saudi electricity grid. Although each market scheme has its own advantages and disadvantages, it is clear that the MP-DA scheme is the best environment for SPP and generators. SPP makes the best revenue in the MP-DA scheme, generators make a good profit, and the consumer surplus value is relatively low. Also, this dissertation supports the claim of PAB-AM scheme advocates that it limits exercise of market power and promotes low prices for consumers. On the other hand, this slows constructing new power plants, as investors will have less incentive for market entry. For SPPs, the PAB scheme requires that they continuously predict market price, and some times the SPP bid will not be accepted. This dissertation created a bidding function for SPP in PAB scheme that maximized its profit. Even with this strategy, SPP in the MP scheme made more revenue (as do other generators) as it can guarantee participating in the DA market to get the market price. It is important to acknowledge that market schemes studied were simplified but retained the most important features of the scheme; some simple specific rule in a scheme might have a different economic effect.

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Appendices

APPENDIX A. LMP COMPONENTS ANALYSIS

This appendix discuses in details how Locational Marginal Price (LMP) has three components: Marginal Price (MP), Power Loss Price, and Transmission Line Congestion Price. First it will go through an analysis using DCOPF in order to get the congestion price portion since this will be a DCOPF there will be no power loss portion. Then an AC system will be considered to add the power loss price portion. Then it will end with detailed example of an AC analyses of a two bus system that will support the discussion.

A.1 General Formulation of LMP

Based on [112], generators cost function can be represented as the below generators cost function:

$$C(P) = A + B \cdot P + C \cdot P^2 \tag{A.1}$$

The total cost of all generators (objective function) can be formulated as

$$\sum C(P) = \sum_{i=1}^{n} \sum_{j=0}^{2} a_{ij} \cdot P_i^j \tag{A.2}$$

Where

n is the number of generators.

The power injection on each bus is

$$P_{ik} = P_{gk} - P_{dk} \tag{A.3}$$

Where

P_{ik} is the power injection on bus (k).

P_{gk} is the power generated on bus (k).

P_{dk} is the power demand on bus (k).

The total generated power on bus (k) is

$$P_{gk} = \sum_{j=1}^{m} P_{gj} \tag{A.4}$$

Where

m is the number of generators on bus (k).

The total demand on bus (k) is

$$P_{dk} = \sum_{j=1}^{m} P_{dj} \tag{A.5}$$

Where

m is the number of loads on bus (k).

The first constraint in a **loss less** system, the total generation must equal total demand

$$\sum P_g = \sum P_d \tag{A.6}$$

Another constraint is on each bus the injected power on the bus must equal power flow leaving the bus

$$P_{im} - \sum_{n=1}^{k} F_{mn} = 0 \ , \quad P_{im} = \sum_{n=1}^{k} F_{mn} \tag{A.7}$$

Where

P_{im} is the power injection on bus (m).

F_{mn} is the power flow from bus (m) to bus (n).

k is the number of total buses.

The target is to minimize generator cost. The common way to optimize the objective function with many constraints is to use the Lagrange relaxation method. The Lagrangian

is constructed where a Lagrange multiplier is multiplied by the constraints then added to the objective function. Then partial derivative of the Lagrangian is taken with respect to all unknown variables then equaling them with zero. This will result in an equal number of equations and unknowns that can be solved.

For **example** if the objective function was

$$z = x^2 - y$$

With a constraint of

$$x + y = 1$$

Then the Lagrangian will be

$$\mathcal{L} = x^2 - y - \lambda[x + y - 1]$$

Where

λ is the Lagrange multiplier.

Then the partial derivative

$$\frac{d\mathcal{L}}{dx} = 0$$
$$\frac{d\mathcal{L}}{dy} = 0$$
$$\frac{d\mathcal{L}}{d\lambda} = 0$$

Now there are three equations and three unknown that can be solved. Applying this method to the problem will result in equation (A.8), which is a simplified Lagrangian that does not consider power line loss nor line limits.

$$\mathcal{L} = \sum C(P) - \lambda\left[\sum P_i - \sum D_i\right] \qquad \text{(A.8)}$$

Equation (A.8) has only one λ value as only one constraint is applied. A more detailed format that considers line power loss by applying the power flow constraint on each bus will result in

$$\mathcal{L} = \sum_{j=1}^{n} C_j(P_j) - \sum_{m=1}^{k} \lambda_m \left[P_{im} - \sum_{j=1}^{k} F_{mj} \right] \quad \text{(A.9)}$$

Where

n is the number of generators.

k is the number of buses.

P_{im} is the power injection on bus (m).

F_{mj} is the power flow from bus (m) to bus (j).

λ_m is the value of the Lagrange multiplier on bus (m).

Note:

$$\frac{d\mathcal{L}}{dP_{im}} = \frac{dC}{dP_{im}} - \lambda_m = 0\,, \qquad \lambda_m = \frac{dC}{dP_{im}} \quad \text{(A.10)}$$

It can be observed from (A.10) that the value of the Lagrange multiplier λ_m on bus (m) will represent the incremental cost of increasing load on the bus which is the LMP value on bus (m) by definition. The λ_m will have the cost of power loss imbedded in it, as it will be shown shortly. If there were no line loss then Lagrange multiplier in equation (A.9) should result an equal value to (A.8). Also the beauty of equation (A.9) that the result of $\lambda_m$will give the LMP value on every bus unlike solving it using (A.8). Now the more detailed Lagrangian equation that consider the power flow constraints is

$$\mathcal{L} = \sum_{m=1}^{n} C_m(P_m) - \sum_{m=1}^{k} \lambda_m \left[P_{im} - \sum_{j=1}^{k} F_{mj} \right] - \sum_{m=1}^{k} \sum_{j=1}^{k} \mu_{mj} \left[F_{mj}^{max} - F_{mj} \right] \quad \text{(A.11)}$$

Where

n is the number of generators.

k is the number of buses.

λ_m is the Lagrange multiplier of bus (m).

μ_{mj} is the Lagrange multiplier of power flow congestion between bus (m) and bus (j), if there is no power limit between bus (m) and bus (j) then μ_{mj} equals zero.

F_{mj} is the power flow between bus (m) and bus (j).

F_{mj}^{max} is the maximum power flow between bus (m) and bus (j).

To optimize the objective function, the Lagrange relaxation is applied

$$\frac{d\mathcal{L}}{dP_{im}} = \frac{dC}{dP_{im}} - \lambda_m = 0\,, \qquad \lambda_m = \frac{dC}{dP_{im}} \tag{A.12}$$

$$\frac{d\mathcal{L}}{dF_{mj}} = 0 \tag{A.13}$$

$$\frac{d\mathcal{L}}{d\lambda_m} = 0 \tag{A.14}$$

Appling the Lagrange relaxation will result in having a number of unknown variables equal to the number of equations that can be solved. It can be observed that λ_m is the incremental cost on bus (m), which by LMP definition represents the marginal cost on bus (m) or the LMP value on bus (m). The next section will prove the congestion LMP portion.

A.2 Formulation of LMP Components Using DC Power Flow

This section will show the derivation of LMP congestion component, note that DC power flow will not have a line power loss component thus no LMP line loss component.

DC power flow formula is [113]

$$F_{mj} = \frac{\theta_m - \theta_j}{X_{mj}} = y_{mj} \cdot (\theta_m - \theta_j) \tag{A.15}$$

Where

X_{mj} is the impedance between bus (m) and bus (j).

y_{mj} is the admittance between bus (m) and bus (j).

Then from equation (A.11)

$$\mathcal{L} = \sum_{m=1}^{n} C_m(P_m) - \sum_{m=1}^{k} \lambda_m \left[P_{im} - \sum_{j=1}^{k} y_{mj} \cdot (\theta_m - \theta_j) \right] - \sum_{m=1}^{k} \sum_{j=1}^{k} \mu_{mj} \left[F_{mj}^{max} - y_{mj} \cdot (\theta_m - \theta_j) \right]$$

$$= \sum_{m=1}^{n} C_m(P_m) - \sum_{m=1}^{k} \lambda_m \cdot P_{im} + \sum_{m=1}^{k} \sum_{j=1}^{k} \lambda_m . y_{mj} \cdot (\theta_m - \theta_j) - \sum_{m=1}^{k} \sum_{j=1}^{k} \mu_{mj} \left[F_{mj}^{max} \right] + \sum_{m=1}^{k} \sum_{j=1}^{k} \mu_{mj} \cdot y_{mj} \cdot (\theta_m - \theta_j)$$

$$= \sum_{m=1}^{n} C_m(P_m) - \sum_{m=1}^{k} \lambda_m \cdot P_{im} + \sum_{m=1}^{k} \sum_{j=1}^{k} \lambda_m \cdot y_{mj} \cdot \theta_m - \sum_{m=1}^{k} \sum_{j=1}^{k} \lambda_m \cdot y_{mj} \cdot \theta_j - \sum_{m=1}^{k} \sum_{j=1}^{k} \mu_{mj} \left[F_{mj}^{max} \right] + \sum_{m=1}^{k} \sum_{j=1}^{k} \mu_{mj} \cdot y_{mj} \cdot \theta_m - \sum_{m=1}^{k} \sum_{j=1}^{k} \mu_{mj} \cdot y_{mj} \cdot \theta_j$$

$$\frac{d\mathcal{L}}{d\theta_n} = \sum_{j=1}^{k} \lambda_n \cdot y_{nj} - \sum_{m=1}^{kk} \lambda_m \cdot y_{mn} + \sum_{j=1}^{k} \mu_{nj} \cdot y_{nj} - \sum_{m=1}^{k} \mu_{mn} \cdot y_{mn}$$

$$\frac{d\mathcal{L}}{d\theta_n} = \sum_{j=1}^{k} \lambda_n \cdot y_{nj} - \sum_{j=1}^{k} \lambda_j \cdot y_{jn} + \sum_{j=1}^{k} \mu_{nj} \cdot y_{nj} - \sum_{j=1}^{k} \mu_{jn} \cdot y_{jn}$$

$$= \sum_{j=1}^{k} y_{nj} (\lambda_n - \lambda_j + \mu_{nj} - \mu_{jn})$$

Note:

If μ_{nj} exists then μ_{jn} will be zero as congestion cost will be from one side.

If μ_{nj} exists it must be positive value.

Then

$$\frac{d\mathcal{L}}{dP_{im}} = \frac{C_m(P_{im})}{dP_{im}} - \lambda_m = 0 \tag{A.16}$$

$$\lambda_m = \frac{C_m(P_{im})}{dP_{im}} \tag{A.17}$$

$$\frac{d\mathcal{L}}{d\theta_m} = \sum_{j=1}^{k} \frac{1}{X_{mj}} \cdot \left(\lambda_m - \lambda_j + \mu_{mj} - \mu_{jm}\right) = 0 \tag{A.18}$$

$$\frac{d\mathcal{L}}{d\lambda_m} = P_{im} - \sum_{j=1}^{k} F_{mj} = 0 \tag{A.19}$$

$$\frac{d\mathcal{L}}{d\mu_{mj}} = F_{mj}^{max} - F_{mj} = 0 \tag{A.20}$$

Note that from (A.17), λ_m represents the LMP at bus (m) by definition.

Proof that every $\boldsymbol{\lambda_m}$ has a congestion component:

In order to formulate the congestion component in the LMP it is important to introduce the Generation Shift Factor (GSF), some texts calls it the Line Flow Sensitivity Factor [112]. GSF is the factor of change of power flow between two buses due to a change in power injection on a bus to reference bus [114] [115].

The generation shift factor by definition is

$$GSF_{m-j,u} = \frac{dF_{mj}}{dP_{iu}}$$

The change in power flow between bus (m) and (j) when change in power injection on bus (u) to reference bus.

Now back to DC power flow formula.

$$F_{mj} = \frac{\theta_m - \theta_j}{X_{mj}} = y_{mj} \cdot (\theta_m - \theta_j) \tag{A.21}$$

Admittance formula

$$y = \frac{1}{z} = g + jb \tag{A.22}$$

$$[P_i] = [B] \cdot [\theta] \tag{A.23}$$

$$[Z] = [B]^{-1} \tag{A.24}$$

Where

θ_m is the voltage angle in radian on bus (m).

$[Z]$ is the Impedance matrix.

Then

$$[\theta] = [Z] \cdot [P_i] \tag{A.25}$$

Then

$$\theta_n = \sum_{m=1}^{k} Z_{nm}.P_m \tag{A.26}$$

From (A.21)

$$F_{nj} = = y_{nj} \cdot (\theta_n - \theta_j) = y_{nj} \cdot \left(\sum_{m=1}^{k} Z_{nm} \cdot P_{im} - \sum_{m=1}^{k} Z_{jm} \cdot P_{im} \right) \tag{A.27}$$

Then

$$GSF_{n-j,u} = \frac{dF_{nj}}{d\mathrm{P_{iu}}} = y_{nj} \cdot (Z_{nu} - Z_{ju}) \tag{A.28}$$

Which means that $GSF_{n-j,u}$ is the factor of power flow change between bus (n) and bus (j) will change when power injection on bus (u) changes

Then from [114]

$$F_{nj} = \sum_{u=1}^{k} GSF_{n-j,u} \cdot P_{iu} \tag{A.29}$$

Now with adding line constraint to (A.8), the Lagrangian will be

$$\mathcal{L} = \sum_{m=1}^{n} C_m(P_m) - \lambda \sum_{m=1}^{k} \left[P_{gm} - P_{dm}\right] - \sum_{m=1}^{k}\sum_{j=1}^{k} \mu_{mj} \left[F_{mj}^{max} - F_{mj}\right]$$

$$\mathcal{L} = \sum_{m=1}^{n} C_m(P_m) - \lambda \sum_{m=1}^{k} \left[P_{gm} - P_{dm}\right] - \sum_{m=1}^{k}\sum_{j=1}^{k} \mu_{mj} \left[F_{mj}^{max} - \sum_{u=1}^{k} GSF_{(m-j)u} \cdot P_{iu}\right] \quad \text{(A.30)}$$

By definition $GSF_{(m-j)u}$ is zero for reference bus. After taking partial derivative with respect to power injection on reference bus will result

$$\frac{d\mathcal{L}}{dP_{ir}} = \frac{dC_r(P_{ir})}{dP_{ir}} - \lambda = 0 \quad \text{(A.31)}$$

$$\frac{dC_r(P_{ir})}{dP_{ir}} = \lambda \quad \text{(A.32)}$$

Partial derivative of the Lagrangian with respect to power injection on rest of buses will result

$$\frac{d\mathcal{L}}{dP_{\mathrm{in}}} = \frac{dC_n(P_{in})}{dP_{in}} - \lambda + \sum_{m=1}^{k}\sum_{j=1}^{k} \mu_{mj}\, GSF_{(m-j)n} = 0 \quad \text{(A.33)}$$

Then LMP on bus (n)

$$\frac{dC_n(P_{in})}{dP_{in}} = \lambda - \sum_{m=1}^{k}\sum_{j=1}^{k} \mu_{mj}\, GSF_{(m-j)n} \quad \text{(A.34)}$$

Then

$$\frac{dC_n(P_{in})}{dP_{in}} = \frac{dC_r(P_{ir})}{dP_{ir}} - \sum_{m=1}^{k}\sum_{j=1}^{k} \mu_{mj}\, GSF_{(m-j)n} \quad \text{(A.35)}$$

Or from (A.17) it can be written

$$\lambda_n = \lambda_r - \sum_{m=1}^{k}\sum_{j=1}^{k} \mu_{mj}\, GSF_{(m-j)n} \tag{A.36}$$

Where

λ_r is the Lagrange multiplier at reference bus.

Then

$$LMP_n = LMP_r + LMP_{Cong-n} \tag{A.37}$$

$$LMP_r = \frac{dC_r(P_{ir})}{dP_{ir}} = \lambda \tag{A.38}$$

$$LMP_{cong-n} = -\sum_{m=1}^{k}\sum_{j=1}^{k} \mu_{mj}\, GSF_{(m-j)n} \tag{A.39}$$

μ_{mj} = 0, if there is no connection nor congestion between bus (m) and bus (j).

A.3 Formulation of LMP Components Considering Line Losses

In an AC system power line losses are considered, the following will show how to get the value of the congestion and line power loss components of LMP.

$$\mathcal{L} = \sum_{m=1}^{n} C_m(P_m) - \lambda \sum_{m=1}^{k} \left[P_{gm} - P_{dm} - P_{loss}\right] - \sum_{m=1}^{k}\sum_{j=1}^{k} \mu_{mj} \left[F_{mj}^{max} - F_{mj}\right] \tag{A.40}$$

$$\begin{aligned} \mathcal{L} = \sum_{m=1}^{n} C_m(P_m) - \lambda \sum_{\mathrm{m}=1}^{k} \left[P_{gm} - P_{dm} - P_{loss}\right] \\ - \sum_{m=1}^{k}\sum_{j=1}^{k} \mu_{mj} \left[F_{mj}^{max} - \sum_{u=1}^{k} GSF_{(m-j)u} \cdot P_{iu}\right] \end{aligned} \tag{A.41}$$

At reference bus

$$\frac{d\mathcal{L}}{dP_{ir}} = \frac{dC_n(P_{in})}{dP_{ir}} - \lambda = 0 \tag{A.42}$$

At bus (*n*)

$$\frac{d\mathcal{L}}{dP_{in}} = \frac{dC_n(P_{in})}{dP_{in}} - \lambda[1 - \frac{dP_{loss}}{dP_{in}}] + \sum_{m=1}^{k}\sum_{j=1}^{k} \mu_{mj}\, GSF_{(m-j)n} = 0 \quad \text{(A.43)}$$

$$\text{LM}P_n = \frac{dC_n(P_{in})}{dP_{in}} = \lambda - \lambda \cdot \frac{dP_{loss}}{dP_{in}} - \sum_{m=1}^{k}\sum_{j=1}^{k} \mu_{mj}\, GSF_{(m-j)n} \quad \text{(A.44)}$$

Although F_{mj} has a power loss factor in it, [112] showed that this is an excellent approximation, especially when comparing it with the DC power model.

Then

$$LMP_m = LMP_{reference} + LMP_{loss-m} + LMP_{cong-m} \quad \text{(A.45)}$$

$$LMP_{reference} = \lambda \quad \text{(A.46)}$$

$$LMP_{loss-m} = -\lambda \cdot \frac{dP_{loss}}{dP_{im}} \quad \text{(A.47)}$$

$$LMP_{cong-m} = -\sum_{n=1}^{k}\sum_{j=1}^{k} \mu_{nj}\, GSF_{(n-j)m} \quad \text{(A.48)}$$

Note that μ_{nj} equales zero if bus (n) and bus (j) are not connecting or there is no congestion.

$\frac{dP_{loss}}{dP_{im}}$ also called power Loss Factor (LF).

Now to solve LF an excellent approximation is discussed in [116] as follows

Since it is assumed that Voltage is 1 p.u

$$P = V \cdot I$$

Then

$$I_{nm} = \frac{F_{nm}}{V} = F_{nm} \quad \text{(A.49)}$$

$$P_{loss} = \sum_{m=1}^{k} \sum_{\substack{j=1 \\ j \neq m}}^{k} F_{mj}^2 \cdot R_{mj} \qquad ,for \quad F_{mj} > 0 \tag{A.50}$$

From (A.29)

$$F_{mn} = \sum_{u=1}^{k} GSF_{m-n,u} \cdot P_{iu} \tag{A.51}$$

Then

$$\frac{dP_{loss}}{dP_{im}} = \frac{d}{dP_{im}} \left(\sum_{b=1}^{k} \sum_{\substack{j=1 \\ j \neq m}}^{k} F_{bn}^2 \cdot R_{bn} \right) = \sum_{b=1}^{k} \sum_{\substack{j=1 \\ j \neq b}}^{k} \left(2 \cdot F_{bn} \cdot R_{bn} \cdot \frac{dF_{bn}}{dP_{im}} \right)$$

$$= \sum_{b=1}^{k} \sum_{\substack{j=1 \\ j \neq b}}^{k} \left(2 \cdot \left(\sum_{u=1}^{k} GSF_{b-j,\mathrm{u}} \cdot P_{iu} \right) \cdot R_{bn} \cdot (GSF_{\mathrm{b}-j,m}) \right)$$

$$LF = \frac{dP_{loss}}{dP_{im}} = \sum_{b=1}^{k} \sum_{\substack{j=1 \\ j \neq b}}^{k} \left(2 \cdot R_{bn} \cdot \left(GSF_{b-j,m} \right) \cdot \left(\sum_{u=1}^{k} GSF_{b-j,u} \cdot P_{iu} \right) \right) \tag{A.52}$$

$$for \; F_{mj} > 0$$

A.4 Formulation of LMP Components Using AC Power Flow in a 2 Bus System

This section will show that there is a LMP power loss component in a addition to the congestion LMP component for a 2 bus system. This section considers line losses. For simplicity a two-bus system is assumed as the objective is to show the components of LMP. The model in [114] is used.

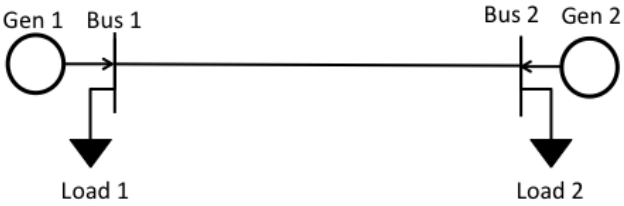

Figure A.1 Example of a two-bus system

For simplicity the generator cost function is assumed to be linear which is very common in gas turbine generators based on data obtained in [98].

$$C_n(P_n) = B_n \cdot P_n \tag{A.53}$$

Constraints are

- Line loss
- Line limit
- Gen maximum and minimum

$$P_{im} = \sum_{j=1}^{n} |V_m| \cdot |V_j| (g_{mj} \cdot cos(\theta_{mj}) + b_{mj} \cdot sin(\theta_{mj}) \tag{A.54}$$

Where

P_{im} is power injection at bus (m).

In a two-bus system it will be

At bus 1

$$P_{g1} - P_{d1} = |V_1|^2 + |V_1| \cdot |V_2| \cdot [g_{12} \cdot cos(\theta_{12}) + b_{12} \cdot sin(\theta_{12})] \tag{A.55}$$

At bus 2

$$P_{g2} - P_{d2} = |V_2|^2 + |V_2| \cdot |V_1| \cdot [g_{21} \cdot cos(\theta_{21}) + b_{21} \cdot sin(\theta_{21})] \tag{A.56}$$

The power loss will be

$$
\begin{aligned}
P_{loss} &= F_{12} + F_{21} \\
&= |V_1|^2 \cdot g_{11} + |V_1| \cdot |V_2| \cdot [g_{12} \cdot cos(\theta_{12}) + \ b_{12} \cdot sin(\theta_{12})] + |V_2|^2 \cdot g_{22} \\
&+ |V_2| \cdot |V_1| \cdot [g_{21} \cdot cos(\theta_{21}) + \ b_{21} \cdot sin(\theta_{21})] \\
&= |V_1|^2 \cdot g_{11} \ + |V_2|^2 \cdot g_{22} + |V_1| \cdot |V_2| \\
&\cdot [g_{12} \cdot cos(\theta_{12}) + b_{12} \cdot sin(\theta_{12}) + g_{21} \cdot cos(\theta_{21}) + \ b_{21} \cdot sin(\theta_{21})] \\
&= \ |V_1|^2 \ \cdot g_{11} + |V_2|^2 \cdot g_{22} + |V_1| \cdot |V_2| \\
&\cdot [g_{12} \cdot cos(\theta_{12}) + b_{12} \cdot sin(\theta_{12}) + g_{21} \cdot cos(\theta_{12}) - \ b_{21} \cdot sin(\theta_{12})] \\
&= \ |V_1|^2 \ \cdot g_{11} + |V_2|^2 \cdot g_{22} + 2 \cdot |V_1| \cdot |V_2| \cdot g_{12} \cdot cos(\theta_{12})
\end{aligned}
$$

Where

F_{12} is power flow from bus (1) to (2).

Assuming voltage is 1 p.u

$$P_{loss} = g_{11} + g_{22} + \ 2 \cdot g_{12} \cdot cos(\theta_{12}) \tag{A.57}$$

Inequality constraint for line limit

$$P_{12}^{min} \leq |V_1|^2 \cdot g_{11} + |V_1| \cdot |V_2| \cdot [g_{12} \cdot cos(\theta_{12}) + \ b_{12} \cdot sin(\theta_{12})] \leq P_{12}^{max} \tag{A.58}$$

$$P_{21}^{min} \leq |V_2|^2 \cdot g_{22} + |V_2| \cdot |V_1| \cdot [g_{21} \cdot cos(\theta_{21}) + \ b_{21} \cdot sin(\theta_{21})] \leq P_{21}^{max} \tag{A.59}$$

Assuming voltage is 1 p.u then

$$P_{12}^{min} \leq g_{11} + g_{12} \cdot cos(\theta_{12}) + \ b_{12} \cdot sin(\theta_{12}) \leq P_{12}^{max} \tag{A.60}$$

$$P_{21}^{min} \leq g_{22} + g_{21} \cdot cos(\theta_{21}) + \ b_{21} \cdot sin(\theta_{21}) \leq P_{21}^{max} \tag{A.61}$$

Then the line limit can be rewritten

$$-F_{12}^{max} \leq g_{11} + g_{12} \cdot cos(\theta_{12}) + \ b_{12} \cdot sin(\theta_{12}) \leq F_{12}^{max} \tag{A.62}$$

$$-F_{21}^{max} \leq g_{22} + g_{21} \cdot cos(\theta_{21}) + \ b_{21} \cdot sin(\theta_{21}) \leq F_{21}^{max} \tag{A.63}$$

Generator constraints

$$0 \leq P_{g1} \leq P_{g1}^{max}$$

$$0 \leq P_{g2} \leq P_{g2}^{max}$$

Using energy conservation law

At bus 1

$$P_{g1} - P_{d1} - g_{11} - g_{12} \cdot cos(\theta_{12}) - b_{12} \cdot sin(\theta_{12}) = 0 \quad \text{(A.64)}$$

At bust 2

$$P_{g2} - P_{d2} - g_{22} - g_{21} \cdot cos(\theta_{21}) - b_{21} \cdot sin(\theta_{21}) = 0 \quad \text{(A.65)}$$

Now the Lagrangian will be

$$\begin{aligned} \mathcal{L} = {} & C_1(P_{g1}) + C_2(P_{g2}) \\ & - \lambda_1\left[P_{g1} - P_{d1} - g_{11} - g_{12} \cdot cos(\theta_{12}) - b_{12} \cdot sin(\theta_{12})\right] \\ & - \lambda_2\left[P_{g2} - P_{d2} - g_{22} - g_{21} \cdot cos(\theta_{21}) - b_{21} \cdot sin(\theta_{21})\right] \\ & - \mu_{12}[F_{12}^{max} - g_{11} - g_{12} \cdot cos(\theta_{12}) - b_{12} \cdot sin(\theta_{12})] \\ & - \mu_{21}[F_{21}^{max} - g_{22} - g_{21} \cdot cos(\theta_{21}) - b_{21} \cdot sin(\theta_{21})] \\ & - \rho_1(P_{g1}^{max} - P_{g1}) - \rho_2(P_{g2}^{max} - P_{g2}) - \upsilon_1(P_{g1} - P_{g1}^{min}) \\ & - \upsilon_2(P_{g2} - P_{g2}^{min}) \end{aligned} \quad \text{(A.66)}$$

Where

λ_m is the Lagrange multiplier (coefficient) for power cost change on bus (m), which is the marginal price on that bus.

μ_{nm} is the Lagrange multiplier (coefficient) for the cost of congestion from bus (n) to bus (m) after reaching.

A.5 ACOPF NO CONGESTION CASE FOR A TWO BUS SYSTEM:

In this section only line power loss is considered with no congestion limit for a two-bus system [114].

Assume

$$C_1(P_{g1}) = B_1 \cdot P_{g1}$$

$$C_2(P_{g2}) = B_2 \cdot P_{g2}$$

$$B_1 < B_2$$

$$P_{g1} \geq 0$$

$$P_{g2} \geq 0$$

Then in an uncongested two bus system only P_{g1} will be on and P_{g2}will be off since $B_1 < B_2$ The Lagrangian will be

$$\begin{aligned}\mathcal{L} = B_1 \cdot P_{g1} + B_2 \cdot P_{g2} \\ - \lambda_1\left[P_{g1} - P_{d1} - g_{11} - g_{12} \cdot cos(\theta_{12}) - \ b_{12} \cdot sin(\theta_{12})\right] \\ - \lambda_2\left[P_{g2} - P_{d2} - g_{22} - g_{21} \cdot cos(\theta_{21}) - \ b_{21} \cdot sin(\theta_{21})\right]\end{aligned} \quad \text{(A.67)}$$

Let bus one be the reference bus, then

$$\theta_1 = 0$$

$$\begin{aligned}\mathcal{L} = B_1 \cdot P_{g1} + B_2 \cdot P_{g2} - \lambda_1\left[P_{g1} - P_{d1} - g_{11} - g_{12} \cdot cos(\theta_2) + \ b_{12} \cdot sin(\theta_2)\right] \\ - \lambda_2[-P_{d2} - g_{22} - g_{21} \cdot cos(\theta_2) - \ b_{21} \cdot sin(\theta_2)]\end{aligned} \quad \text{(A.68)}$$

$$\frac{d\mathcal{L}}{dP_{g1}} = B_1 - \lambda_1 = 0 \quad \text{(A.69)}$$

Then

$$\lambda_1 = \ B_1 = IC_{g1} \quad \text{(A.70)}$$

$$\frac{d\mathcal{L}}{d\lambda_1} = P_{g1} - P_{d1} - g_{11} - g_{12} \cdot cos(\theta_2) + \ b_{12} \cdot sin(\theta_2) = 0 \quad \text{(A.71)}$$

$$\frac{d\mathcal{L}}{d\lambda_2} = -P_{d2} - g_{22} - g_{21} \cdot cos(\theta_2) - \ b_{21} \cdot sin(\theta_2) = 0 \quad \text{(A.72)}$$

In (A.72) θ_2 is the only unknown, then θ_2 can be calculated.

$$\frac{d\mathcal{L}}{d\theta_2} = -\lambda_1[g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)] - \lambda_2[g_{21} \cdot sin(\theta_2) - b_{21} \cdot cos(\theta_2)] = 0 \quad \text{(A.73)}$$

Then

$$\lambda_2 = -\lambda_1 \frac{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)}{g_{21} \cdot sin(\theta_2) - b_{21} \cdot cos(\theta_2)} \quad \text{(A.74)}$$

From equation (A.71)

$$P_{g1} = P_{d1} + g_{11} + g_{12} \cdot cos(\theta_2) - b_{12} \cdot sin(\theta_2) \quad \text{(A.75)}$$

From equation (A.57)

$$P_{loss} = g_{11} + g_{22} + 2 \cdot g_{12} \cdot cos(\theta_{12})$$

Proof of LMP power loss component:

The LMP value has the Marginal cost and Marginal power loss elements in it. It can be observed that power loss cost component is imbedded within λ value.

$$LMP_k = \text{LMP}_k^{ref} + LMP_k^{loss} \quad \text{(A.76)}$$

From (A.10) the LMP at bus 1

$$LMP_1 = \lambda_1 = B_1 = IC_{g1} \quad \text{(A.77)}$$

LMP at bus 2

$$LMP_2 = \lambda_2 \quad \text{(A.78)}$$

To find LMP_k^{loss}

$$LMP_k^{loss} = \lambda_2^{loss}$$

Equation (A.76) can be rewritten to

$$\lambda_2 = \lambda_1 + \lambda_2^{loss} \quad \text{(A.79)}$$

$$\lambda_2^{loss} = \lambda_2 - \lambda_1 = \lambda_2 + \lambda_2 \cdot \frac{[g_{21} \cdot sin(\theta_2) - b_{21} \cdot cos(\theta_2)]}{[g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)]}$$

$$\lambda_2^{loss} = \lambda_2 . \frac{2 \cdot g_{12} \cdot sin(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)} \quad \text{(A.80)}$$

To verify that (A.80) value of λ_2^{loss} equation is the power loss incremental cost, by definition λ_2^{loss} is the incremental power loss cost on bus (2) as the below equation.

$$\lambda_2^{loss} = \lambda_2 \cdot \frac{dP_{loss(1-2)}}{dF_{12}} \quad \text{(A.81)}$$

Solve for

$$\frac{dP_{loss(1-2)}}{dF_{12}}$$

Since

$$\frac{dP_{loss(1-2)}}{d\theta_2} = \frac{dP_{loss(1-2)}}{dF_{12}} \cdot \frac{dF_{12}}{d\theta_2} \quad \text{(A.82)}$$

From (A.57)

$$P_{loss(1-2)} = g_{11} + g_{22} + 2 \cdot g_{12} \cdot cos(\theta_{12})$$

Then

$$\frac{dP_{loss(1-2)}}{d\theta_2} = 2 . g_{12} \cdot sin(\theta_{12}) \quad \text{(A.83)}$$

From (A.55)

$$F_{12} = g_{11} + g_{12} \cdot cos(\theta_{12}) + b_{12} \cdot sin(\theta_{12})$$

$$\frac{dF_{12}}{d\theta_2} = g_{12} \cdot sin(\theta_{12}) - b_{12} \cdot cos(\theta_{12}) \quad \text{(A.84)}$$

From (A.82)

$$\frac{dP_{loss(1-2)}}{dF_{12}} = \frac{2 \cdot g_{12} \cdot sin(\theta_{12})}{g_{12} \cdot sin(\theta_{12}) - b_{12} \cdot cos(\theta_{12})} \quad \text{(A.85)}$$

From (A.81)

$$\lambda_2^{loss} = \lambda_2 \cdot \frac{dP_{loss(1-2)}}{dF_{12}} = \lambda_2 \cdot \frac{2 \cdot g_{12} \cdot sin(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)}$$

$$\lambda_2^{loss} = \lambda_2 \cdot \frac{2 \cdot g_{12} \cdot sin(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)} \quad \text{(A.86)}$$

It can be observed that (A.80) equals (A.86), this proofs that LMP has the λ_m^{loss} component.

Example 1

The following is based on an example from [114], assume the following:

$$C_1(P_{g1}) = 10 \cdot P_{g1}$$

$$C_2(P_{g2}) = 20. P_{g2}$$

$$P_{L1} = 6\ MW$$

$$P_{L2} = 8\ MW$$

$$y_{12} = 4.0 - j20.0\ p.u$$

$$g_{12} = g_{21} = -4$$

$$g_{11} = g_{22} = 4$$

$$b_{12} = b_{21} = 20$$

$$b_{11} = b_{22} = -20$$

Since the generator cost function is linear and there are only two generators, then the only generator that will be dispatched is Gen 1 as it is the cheapest one, while Gen 2 will be off as there is no transmission line limit.

From (A.67) the Lagrangian will be

$$\mathcal{L} = B_1 \cdot P_{g1} + B_2 \cdot P_{g2} - \lambda_1\left[P_{g1} - P_{d1} - g_{11} - g_{12} \cdot cos(\theta_2) + b_{12} \cdot sin(\theta_2)\right] - \lambda_2[-P_{d2} - g_{22} - g_{21} \cdot cos(\theta_2) - b_{21} \cdot sin(\theta_2)]$$

From (A.69) and (A.70)

$$\frac{d\mathcal{L}}{dP_{g1}} = B_1 - \lambda_1 = 0$$

Then

$$\lambda_1 = B_1 = 10$$

From (A.72)

$$\frac{d\mathcal{L}}{d\lambda_2} = -P_{d2} - g_{22} - g_{21} \cdot cos(\theta_2) - b_{21} \cdot sin(\theta_2) = 0$$

Solve for θ_2 as it is the only unknown, for the example

$$-8 - 4 + 4 \cdot cos(\theta_2) - 20 \cdot sin(\theta_2) = 0$$

$$\theta_2 = -24.72996°$$

From (A.74)

$$\lambda_2 = -\lambda_1 \cdot \frac{-4.sin(\theta_2) + 20.cos(\theta_2)}{-4.sin(\theta_2) - 20.cos(\theta_2)} = -\lambda_1 \cdot \frac{19.83915}{-16.49242}$$

$$\lambda_2 = -\lambda_1 \cdot (-1.2029) = 10 \cdot 1.2029 = 12.029$$

$$\lambda_2 = 12.029$$

From (A.75)

$$P_{g1} = P_{d1} + g_{11} + g_{12} \cdot cos(\theta_2) - b_{12} \cdot sin(\theta_2)$$

$$P_{g1} = 6 + 4 - 4 \cdot cos(\theta_2) - 20 \cdot sin(\theta_2)$$

$$P_{g1} = 14.7337$$

From (A.57)

$$P_{loss} = g_{11} + g_{22} + 2 \cdot g_{12} \cdot cos(\theta_{12})$$

$$P_{loss} = 4 + 4 + 2 \cdot (-4) \cdot cos(-\theta_2) = 0.7337$$

To verify

$$P_{loss} = P_{g1} - P_{d1} - P_{d2} = 14.7337 - 6 - 8 = 0.7337$$

From (A.79)

$$LMP_2^{loss} = \lambda_2^{loss} = 12.0292 - 10.00 = 2.0292$$

Also this can be verified by (A.80) and (A.86)

$$\lambda_2^{loss} = \lambda_2 \cdot \frac{2 \cdot g_{12} \cdot sin(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)} = 12.029 \cdot 0.16869 = 2.0292$$

Which means that each 1 MW delivered from bus (1) to bus (2) a 0.16869 MW will be lost

Example 2

All information is similar to example 1 except

$$P_{L2} = 10\ MW$$

From (A.72)

$$-10 - 4 + 4 \cdot cos(\theta_2) - 20 \cdot sin(\theta_2) = 0$$

$$\theta_2 = -32.03639°$$

From (A.74)

$$\lambda_2 = -\lambda_1 \frac{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)}{g_{21} \cdot sin(\theta_2) - b_{21} \cdot cos(\theta_2)}$$

$$= 12.861$$

From (A.79)

$$LMP_2^{loss} = \lambda_2^{loss} = 12.861 - 10 = 2.861$$

Also this can be verified by (A.80) and (A.86)

$$\lambda_2^{loss} = \lambda_2 \cdot \frac{2 \cdot g_{12} \cdot sin(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)} = 12.86107 \cdot 0.22246 = 2.861$$

A.6 ACOPF CONGESTION CASE FOR A TWO BUS SYSTEM

This section shows calculating the LMP congestion portion [114].

From (A.66)

$$\mathcal{L} = C_1(P_{g1}) + C_2(P_{g2}) - \lambda_1\left[P_{g1} - P_{d1} - g_{11} - g_{12}.cos(\theta_{12}) - b_{12}.sin(\theta_{12})\right] - \lambda_2\left[P_{g2} - P_{d2} - g_{22} - g_{21}.cos(\theta_{12}) - b_{21}.sin(\theta_{21})\right] - \mu_{12}[F_{12}^{max} - g_{11} - g_{12}.cos(\theta_{12}) - b_{12}.sin(\theta_{12})] - \mu_{21}[F_{21}^{max} - g_{22} - g_{21}.cos(\theta_{12}) - b_{21}.sin(\theta_{21})] - \rho_1\left(P_{g1}^{max} - P_{g1}\right) - \rho_2\left(P_{g2}^{max} - P_{g2}\right) - \upsilon_1\left(P_{g1} - P_{g1}^{min}\right) - \upsilon_2\left(P_{g2} - P_{g2}^{min}\right)$$

Since in this case the line will be congested, both generators will run. Since generator 1 is cheaper then it is expected that the power flow will be from bus 1 to bus 2.

Then following coefficients will be zero

$$\mu_{21} = \rho_1 = \rho_2 = \upsilon_1 = \upsilon_2 = 0$$

$$\mathcal{L} = B_1 \cdot P_{g1} + B_2 \cdot P_{g2} - \lambda_1\left[P_{g1} - P_{d1} - g_{11} - g_{12} \cdot cos(\theta_2) + b_{12} \cdot sin(\theta_2)\right] - \lambda_2\left[P_{g2} - P_{d2} - g_{22} - g_{21} \cdot cos(\theta_2) - b_{21} \cdot sin(\theta_2)\right] - \mu_{12}[F_{12}^{max} - g_{11} - g_{12} \cdot cos(\theta_2) + b_{12} \cdot sin(\theta_2)] \quad \text{(A.87)}$$

Solve the Lagrangian

$$\frac{d\mathcal{L}}{dP_{g1}} = B_1 - \lambda_1 = 0 \quad \text{(A.88)}$$

$$\lambda_1 = B_1 = IC_{g1} \quad \text{(A.89)}$$

$$\frac{d\mathcal{L}}{dP_{g2}} = B_2 - \lambda_2 = 0 \quad \text{(A.90)}$$

$$\lambda_2 = B_2 = IC_{g2} \quad \text{(A.91)}$$

$$\frac{d\mathcal{L}}{d\mu_{12}} = F_{12}^{max} - g_{11} - g_{12} \cdot cos(\theta_2) + b_{12} \cdot sin(\theta_2) = 0 \quad \text{(A.92)}$$

Solve (A.92) for θ_2 as it is the only unknown

$$\frac{d\mathcal{L}}{d\lambda_1} = P_{g1} - P_{d1} - g_{11} - g_{12} \cdot cos(\theta_2) + b_{12} \cdot sin(\theta_2) = 0 \quad \text{(A.93)}$$

Then

$$P_{g1} = P_{d1} + g_{11} + g_{12} \cdot cos(\theta_2) - b_{12} \cdot sin(\theta_2) \tag{A.94}$$

$$\frac{d\mathcal{L}}{d\lambda_2} = P_{g2} - P_{d2} - g_{22} - g_{21} \cdot cos(\theta_2) - b_{21} \cdot sin(\theta_2) = 0 \tag{A.95}$$

Then

$$P_{g2} = P_{d2} + g_{22} + g_{21} \cdot cos(\theta_2) + b_{21} \cdot sin(\theta_2) \tag{A.96}$$

$$\begin{aligned} \frac{d\mathcal{L}}{d\theta_2} = &-\lambda_1[-g_{12} \cdot sin(\theta_{12}) + b_{12} \cdot cos(\theta_{12})] \\ &- \lambda_2[g_{21} \cdot sin(\theta_{21}) - b_{21} \cdot cos(\theta_{21})] \\ &- \mu_{12}[-g_{12} \cdot sin(\theta_{12}) + b_{12} \cdot cos(\theta_{12})] = 0 \end{aligned} \tag{A.97}$$

Solve for μ_{12}

$$\begin{gathered} -\lambda_1[-g_{12} \cdot sin(\theta_{12}) + b_{12} \cdot cos(\theta_{12})] - \lambda_2[g_{21} \cdot sin(\theta_{21}) - b_{21} \cdot cos(\theta_{21})] \\ - \mu_{12}[-g_{12} \cdot sin(\theta_{12}) + b_{12} \cdot cos(\theta_{12})] = 0 \end{gathered}$$

$$\begin{gathered} \lambda_1[-g_{12} \cdot sin(\theta_{12}) + b_{12} \cdot cos(\theta_{12})] + \lambda_2[g_{21} \cdot sin(\theta_{21}) - b_{21} \cdot cos(\theta_{21})] \\ + \mu_{12}[-g_{12} \cdot sin(\theta_{12}) + b_{12} \cdot cos(\theta_{12})] = 0 \end{gathered}$$

$$\begin{gathered} \lambda_1 \cdot g_{12} \cdot sin(\theta_2) + \lambda_1 \cdot b_{12} \cdot cos(\theta_2) + \lambda_2 \cdot g_{21} \cdot sin(\theta_2) - \lambda_2 \cdot b_{21} \cdot cos(\theta_2) \\ + \mu_{12}[g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)] = 0 \end{gathered}$$

Then μ_{12}

$$\mu_{12} = -\frac{\lambda_1 \cdot g_{12} \cdot sin(\theta_2) + \lambda_1 \cdot b_{12} \cdot cos(\theta_2) + \lambda_2 \cdot g_{21} \cdot sin(\theta_2) - \lambda_2 \cdot b_{21} \cdot cos(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)}$$

$$\mu_{12} = -\frac{g_{12} \cdot sin(\theta_2) \cdot [\lambda_1 + \lambda_2] + b_{12} \cdot cos(\theta_2) \cdot [\lambda_1 - \lambda_2]}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)} \tag{A.98}$$

Where

μ_{12} is the incremental congestion price.

Proof that μ_{12} is the LMP congestion component in two bus system [my work]:

By definition the incremental cost of congestion is dC/dF_{12}^{max}

Since

$$\frac{dC}{d\theta_2} = \frac{dC}{dF_{12}^{max}} \cdot \frac{dF_{12}^{max}}{d\theta_2} \tag{A.99}$$

$$\begin{aligned} C = B_1 \cdot P_{g1} + B_2 \cdot P_{g2} &= B_1 \cdot (P_{12} + P_{d1}) + B_2 \cdot (P_{21} + P_{d2}) \\ &= B_1 \cdot (g_{11} + g_{12} \cdot cos(\theta_{12}) + b_{12} \cdot sin(\theta_{12}) + P_{d1}) + B_2 \cdot (g_{22} + g_{21} \\ &\cdot cos(\theta_{21}) + b_{21} \cdot sin(\theta_{21}) + P_{d2}) \end{aligned}$$

Taking partial derivative

$$\begin{aligned} \frac{dC}{d\theta_2} &= B_1 \cdot (g_{12} \cdot sin(\theta_{12}) - b_{12} \cdot cos(\theta_{12})) + B_2 \cdot (-g_{21} \cdot sin(\theta_{21}) + b_{21} \cdot cos(\theta_{21})) \\ &= B_1 \cdot (-g_{12} \cdot sin(\theta_2) - b_{12} \cdot cos(\theta_2)) + B_2 \\ &\cdot (-g_{21} \cdot sin(\theta_2) + b_{21} \cdot cos(\theta_2)) \\ &= -g_{12} \cdot sin(\theta_2) \cdot [B_1 + B_2] - b_{12} \cdot cos(\theta_2) \cdot [B_1 - B_2] \end{aligned}$$

$$\frac{dC}{d\theta_2} = -g_{12} \cdot sin(\theta_2) \cdot [B_1 + B_2] - b_{12} \cdot cos(\theta_2) \cdot [B_1 - B_2] \tag{A.100}$$

From transmission line limit

$$F_{12}^{max} = g_{11} + g_{12} \cdot cos(\theta_{12}) + b_{12} \cdot sin(\theta_{12})$$

$$\frac{dF_{12}^{max}}{d\theta_2} = g_{12} \cdot sin(\theta_{12}) - b_{12} \cdot cos(\theta_{12}) = -g_{12} \cdot sin(\theta_2) - b_{12} \cdot cos(\theta_2) \tag{A.101}$$

Then based on (A.99)

$$\begin{aligned} \frac{dC}{dF_{12}^{max}} = \frac{\frac{dC}{d\theta_2}}{\frac{dF_{12}^{max}}{d\theta_2}} &= \frac{-g_{12} \cdot sin(\theta_2) \cdot [B_1 + B_2] - b_{12} \cdot cos(\theta_2) \cdot [B_1 - B_2]}{-g_{12} \cdot sin(\theta_2) - b_{12} \cdot cos(\theta_2)} \\ &= \frac{g_{12} \cdot sin(\theta_2) \cdot [B_1 + B_2] + b_{12} \cdot cos(\theta_2) \cdot [B_1 - B_2]}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)} \end{aligned}$$

$$\frac{dC}{dF_{12}^{max}} = \frac{g_{12} \cdot sin(\theta_2) \cdot [\lambda_1 + \lambda_2] + b_{12} \cdot cos(\theta_2) \cdot [\lambda_1 - \lambda_2]}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)} = \mu_{12} \quad \text{(A.102)}$$

So this proofs that μ_{12} is the congestion cost

Proof of the that λ_2^{loss} is the LMP power loss component in two bus system [my work]:

Since the price of λ_2 composes of

$$\lambda_2 = \lambda_1 + \lambda_2^{loss} + \mu_{12} \quad \text{(A.103)}$$

To find λ_2^{loss}

$$\lambda_2^{loss} = \lambda_2 - \lambda_1 - \mu_{12}$$
$$= \lambda_2 - \lambda_1$$
$$+ \frac{\lambda_1 \cdot g_{12} \cdot sin(\theta_2) + \lambda_1 \cdot b_{12} \cdot cos(\theta_2) + \lambda_2 \cdot g_{21} \cdot sin(\theta_2) - \lambda_2 \cdot b_{21} \cdot cos(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)}$$
$$= \frac{\lambda_2 \cdot g_{12} \cdot sin(\theta_2) + \lambda_2 \cdot b_{12} \cdot cos(\theta_2) - \lambda_1 \cdot g_{12} \cdot sin(\theta_2) - \lambda_1 \cdot b_{12} \cdot cos(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)}$$
$$+ \frac{\lambda_1 \cdot g_{12} \cdot sin(\theta_2) + \lambda_1 \cdot b_{12} \cdot cos(\theta_2) + \lambda_2 \cdot g_{21} \cdot sin(\theta_2) - \lambda_2 \cdot b_{21} \cdot cos(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)}$$
$$= \frac{\lambda_2 \cdot g_{12} \cdot sin(\theta_2) + \lambda_2 \cdot g_{21} \cdot sin(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)} = \lambda_2 \cdot \frac{2 \cdot g_{12} \cdot sin(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)}$$

$$\lambda_2^{loss} = \lambda_2 \cdot \frac{2 \cdot g_{12} \cdot sin(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)} \quad \text{(A.104)}$$

Referring to (A.86) which was derived in the non-congested section, based on the definition of incremental power loss.

$$\frac{dP_{loss(1-2)}}{d\theta_2} = \frac{dP_{loss(1-2)}}{dF_{12}} \cdot \frac{dF_{12}}{d\theta_2} \quad \text{(A.105)}$$

From (A.81)

$$\lambda_2^{loss} = \lambda_2 \cdot \frac{dP_{loss(1-2)}}{dF_{12}}$$

$$\lambda_2^{loss} = \lambda_2 \cdot \frac{2.\, g_{12} \cdot sin(\theta_2)}{g_{12}.\, sin(\theta_2) + b_{12} \cdot cos(\theta_2)} \quad \text{(A.106)}$$

So this proofs that λ_2^{loss} is LMP power loss component.

Example 3:

Assume same problem as in example 1 but with line limit of 5 MW between bus (1) and bus (2).

Then

$$\mu_{21} = \rho_1 = \rho_2 = \upsilon_1 = \upsilon_2 = 0$$

From (A.87)

$$\begin{aligned}\mathcal{L} = B_1 \cdot P_{g1} + B_2 \cdot P_{g2} - \lambda_1\left[P_{g1} - P_{d1} - g_{11} - g_{12} \cdot cos(\theta_2) + b_{12} \cdot sin(\theta_2)\right] \\ - \lambda_2\left[P_{g2} - P_{d2} - g_{22} - g_{21} \cdot cos(\theta_2) - b_{21} \cdot sin(\theta_2)\right] \\ - \mu_{12}[F_{12}^{max} - g_{11} - g_{12} \cdot cos(\theta_2) + b_{12} \cdot sin(\theta_2)]\end{aligned}$$

From (A.89)

$$\lambda_1 = B_1 = 10$$

From (A.91)

$$\lambda_2 = B_2 = 20$$

From (A.92)

$$4 - 4 \cdot cos(\theta_2) - 20 \cdot sin(\theta_2) - 5 = 0$$

$$\theta_2 = -14.12022$$

From (A.94)

$$P_{g1} = P_{d1} + g_{11} + g_{12} \cdot cos(\theta_2) - b_{12} \cdot sin(\theta_2)$$

$$P_{g1} = 6 + 4 - 4 \cdot cos(\theta_2) - 20 \cdot sin(\theta_2) = 11\ MW$$

From (A.96)

$$P_{g2} = P_{d2} + g_{22} + g_{21} \cdot cos(\theta_2) + b_{21} \cdot sin(\theta_2) =$$

$$P_{g2} = 8 + 4 - 4 \cdot cos(\theta_2) + 20 \cdot sin(\theta_2) = 3.2417\ MW$$

From (A.57)

$$P_{loss} = g_{11} + g_{22} + 2 \cdot g_{12} \cdot cos(\theta_{12})$$

$$P_{loss} = 4 + 4 + 2 \cdot (-4) \cdot cos(\theta_2) = 0.2417\ MW$$

From (A.98) and (A.102)

$$\mu_{12} = -\frac{g_{12} \cdot sin(\theta_2).[\lambda_1 + \lambda_2] + b_{12} \cdot cos(\theta_2) \cdot [\lambda_1 - \lambda_2]}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)}$$

$$\mu_{12} = -\frac{-4.sin(\theta_2) \cdot [10 + 20] + 20 \cdot cos(\theta_2) \cdot [10 - 20]}{-4 \cdot sin(\theta_2) + 20 \cdot cos(\theta_2)} = 8.083937$$

$$\lambda_2 = \lambda_1 + \lambda_2^{loss} + \mu_{12}$$

$$\lambda_2^{loss} = \lambda_2 - \lambda_1 - \mu_{12} = 20 - 10 - 8.08393 = 1.916$$

To double check use equation (A.86)

$$\lambda_2^{loss} = \lambda_2 \cdot \frac{dP_{loss(1-2)}}{dP_{12}}$$

$$\lambda_2^{loss} = \lambda_2 \cdot \frac{2.g_{12} \cdot sin(\theta_2)}{g_{12} \cdot sin(\theta_2) + b_{12} \cdot cos(\theta_2)} = (20).\frac{2 \cdot (-4) \cdot sin(\theta_2)}{(-4) \cdot sin(\theta_2) + (20) \cdot cos(\theta_2)}$$

$$\lambda_2^{loss} = 20 \cdot 0.0958 = 1.916$$

Based on the discussion in this section, it was proven that LMP has three components Marginal cost at reference bus, line loss component and congestion cost.

APPENDIX B. EXAMPLES OF THE EFFECT OF INCREASING GENERATORS ON PAB MARKETS

This appendix is an extension to the work done in section 2.4.8. The extra work in this section has more detailed information to check if the proposed PAB bidding function will maximize profit when applying the bidding function. A single bus system with 5 generators and a single bus with 30 generators will be explored.

B.1 EXAMPLE ON A SINGLE BUS MODEL WITH 5 GENERATORS

A simulation is done on random times year round in order to check if proposed PAB bidding functions are effective. A single bus system is assumed with **5 connected generators**, see Figure B.1. The cost function of the generators are based on examples from [87], see Table B.1. The Load profile data used was derived from ERCOT load data [91].

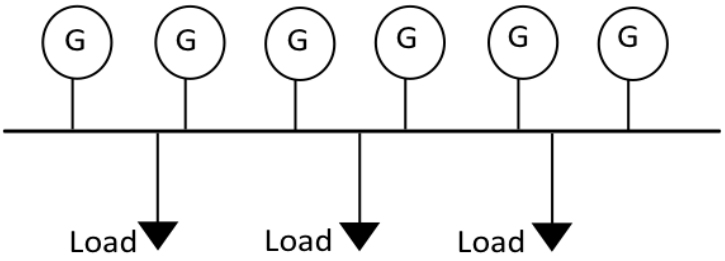

Figure B.1 Single bus system with 5 generators

Table B.1 Cost function coefficients [87]

	A	**B**	**C**
Gen (A)	78	7.80	0.002562
Gen (B)	78	7.88	0.004820
Gen (C)	100	7.97	0.004820
Gen (D)	561	8.00	0.004890
Gen (E)	50	7.85	0.001940

Gen (A) cost function is

$$C_A(P_g) = A + B \cdot P_g + C \cdot P_g^2 \tag{B.1}$$

Then the Incremental Cost (*IC*) will be

$$IC_A(P_g) = B + 2 \cdot C \cdot P_g \tag{B.2}$$

From section 2.4 of this research, the bidding function for PAB Uni is as in equation (B.3) and the bidding function for PAB B-Uni is as in equation (B.4). In order to test if Gen (A) will maximize its profit, equation (B.5) will be tested. The value of x will vary against Gen (A) profit. Gen (A) profit is expected to maximize when equation (B.6) is satisfied for PAB Uni and equation (B.7) for PAB B-Uni.

$$Bid_{PAB}^{Uni} = \frac{\lambda_{max}^{MP} + B}{2} + C \cdot P \tag{B.3}$$

$$Bid_{PAB}^{B-Uni} = \frac{\mu + 1.538\sigma + B}{2} + C \cdot P \tag{B.4}$$

$$Bid_{PAB} = x + C \cdot P \tag{B.5}$$

The point that maximize the profit of Gen (A) when bidding PAB Uni

$$x_{Uni} = \frac{\lambda_{max}^{MP} + B}{2} \tag{B.6}$$

The point that maximize the profit of Gen (A) when bidding PAB B-Uni

$$x_{B-uni} = \frac{\mu + 1.538\sigma + B}{2} \tag{B.7}$$

For this test, results of using PAB Uni are collected Table B.2, and results of using PAB B-Uni are collected in Table B.3. Looking in both tables it can be observed that the expected and actual value of variable (x) that maximizes profit of Gen (A) are very close to each other. This small difference is due to that expected value depends on the λ_{max} which depends on a historical data of a specified window. It can be observed that unlike MP market, Gen (A) cannot increase its profit by deviating for BF in a system with small number of generators, which means that the effect of exercising market power by Gen (A) is limited. Also it can be observed that using PAB B-Uni has a better result. The reason for this that PAB Uni depends on the value λ_{max} which could be an outlier price that will make effect the bidding function.

Table B.2 Values of (x) that maximizes Gen (A) profit in a 5 Gen system using PAB bidding function derived from Uniform distribution function

Time	**Day**	**Month**	**Expected value of "x" at maximum profit**	**Actual value of "x" at maximum profit**	**Error**	**Figure**
4:00	10	8	12.7	13	0.3	Figure B.2
4:00	10	4	11.2	11.5	0.3	Figure B.3
8:00	10	10	12.5	12	0.5	Figure B.4
3:00	15	7	12.8	13	0.2	Figure B.5
22:00	20	9	15.1	14.5	0.6	Figure B.6
19:00	23	12	13.3	13	0.3	Figure B.7

Table B.3 Values of (x) that Maximizes Gen (A) profit in a 5 Gen system using PAB bidding function derived from best-fit-uniform distribution function (B-Uni)

Time	**Day**	**Month**	**Expected value of "x" at maximum profit**	**Actual value of "x" at maximum profit**	**Error**	**Figure**
4:00	10	8	13.3	13.1	0.2	Figure B.8
4:00	10	4	11.6	11.6	0	Figure B.9
8:00	10	10	12.5	12.3	0.2	Figure B.10
3:00	15	7	13.2	13.2	0	Figure B.11
22:00	20	9	14.7	15	0.3	Figure B.12
19:00	23	12	12.8	13.1	0.3	Figure B.13

The figures from Figure B.2 to Figure B.7 are for bids derived PAB Uni

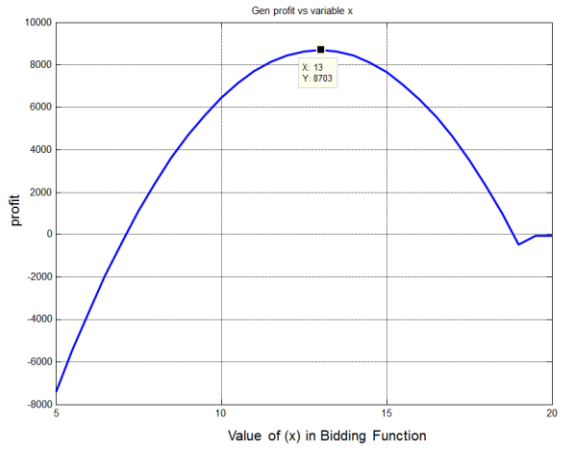

Figure B.2 The effect on profit of Gen (A) when varying the value of x of its bidding function at 4 am on Aug, 10th

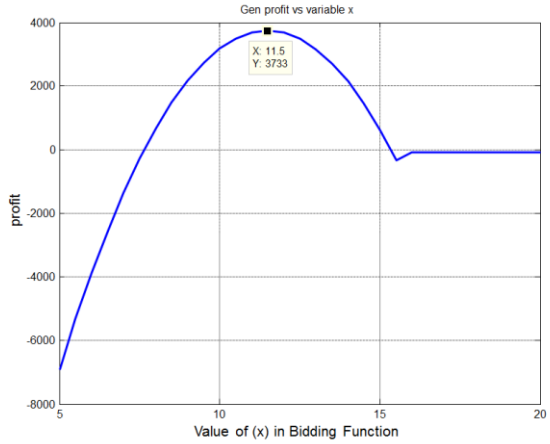

Figure B.3 The effect on profit of Gen (A) when varying the value of x of its bidding function at 4 am on April, 10th

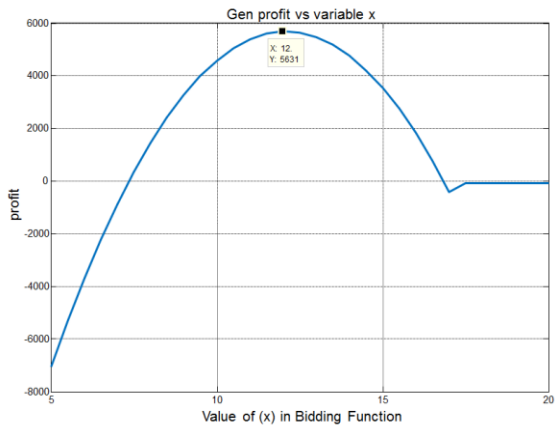

Figure B.4 The effect on profit of Gen (A) when varying the value of x of its bidding function at 8 am on Oct, 10th

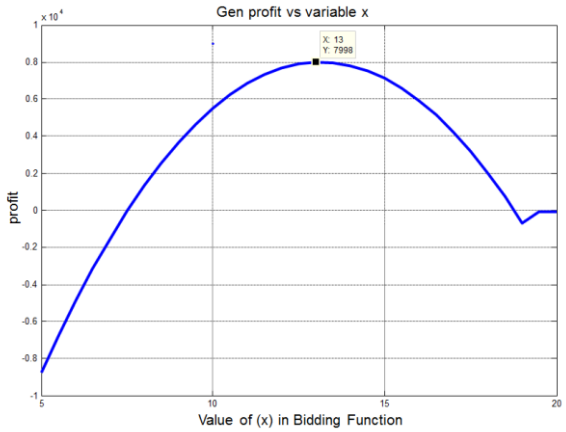

Figure B.5 The effect on profit of Gen (A) when varying the value of x of its bidding function at 3 am on July, 15th

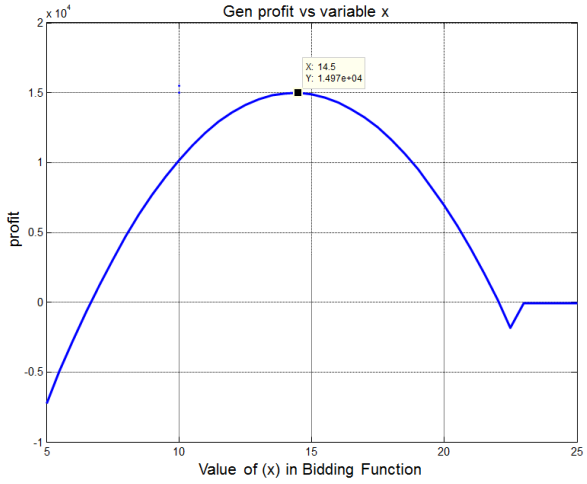

Figure B.6 The effect on profit of Gen (A) when varying the value of x of its bidding function at 10 pm on Sept, 20th

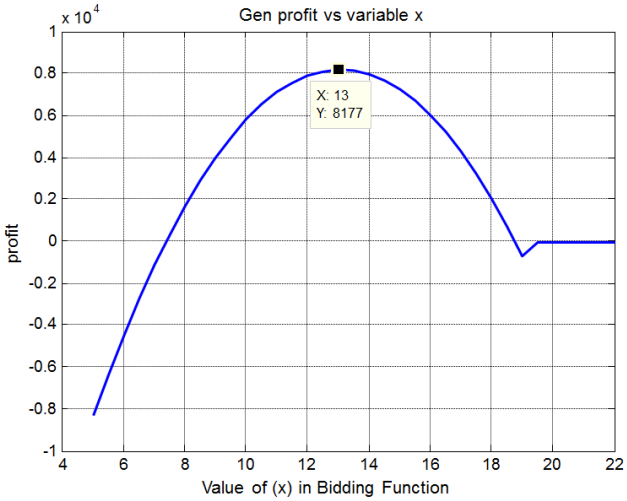

Figure B.7 The effect on profit of Gen (A) when varying the value of x of its bidding function at 7 pm on Dec, 23rd

The figures from Figure B.8 to Figure B.13 are for bids derived from PAB B-Uni

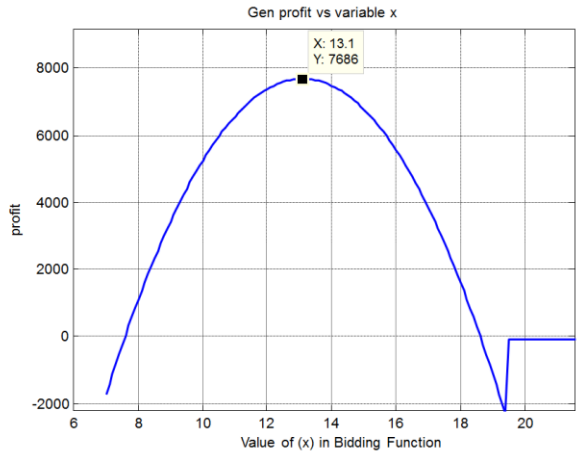

Figure B.8 The effect on profit of Gen (A) when varying the value of x of its bidding function at 4 am on Aug, 10th

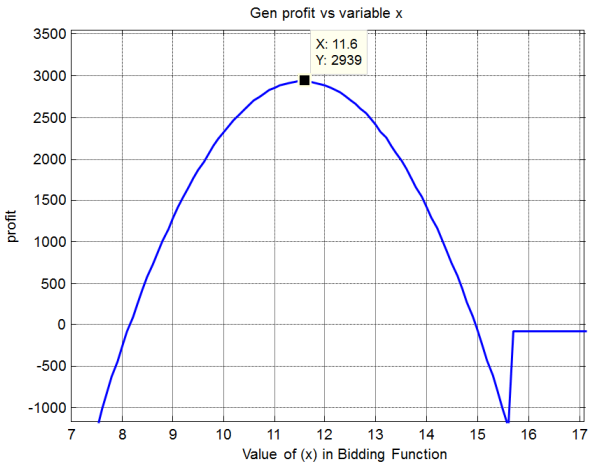

Figure B.9 The effect on profit of Gen (A) when varying the value of x of its bidding function at 4 am on April, 10th

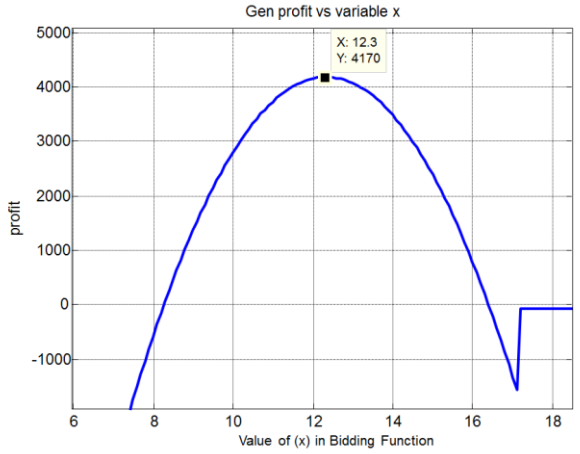

Figure B.10 The effect on profit of Gen (A) when varying the value of x of its bidding function at 8 am on Oct, 10th

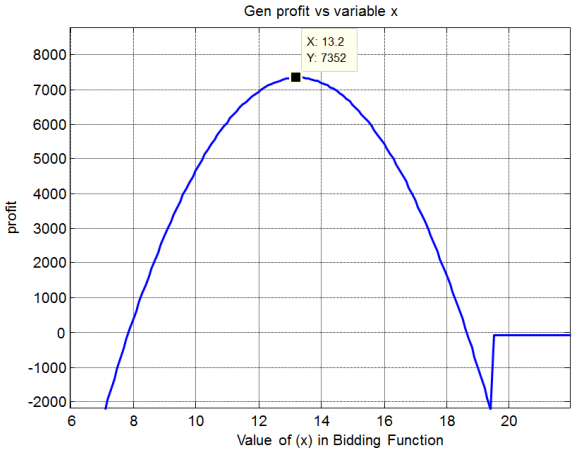

Figure B.11 The effect on profit of Gen (A) when varying the value of x of its bidding function at 3 am on July, 15th

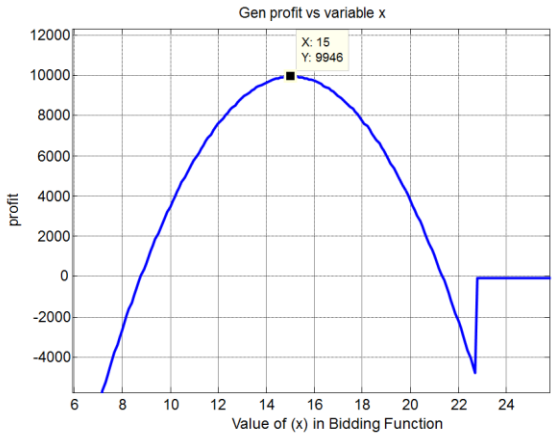

Figure B.12 The effect on profit of Gen (A) when varying the value of x of its bidding function at 10 pm on Sept, 20th

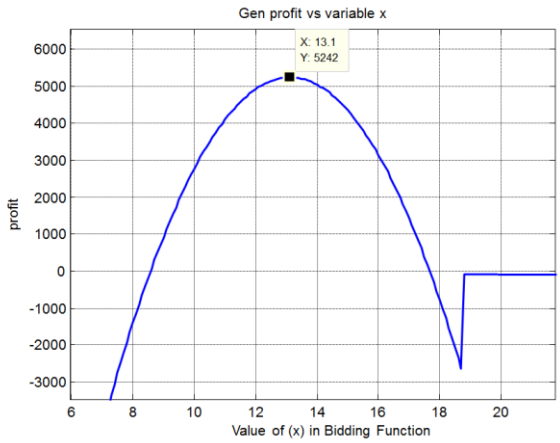

Figure B.13 The effect on profit of Gen (A) when varying the value of x of its bidding function at 7 pm on Dec, 23rd

B.2 Example on a Single Bus Model with 30 Generators

A simulation is done on random times year round on a **single bus system with 30 generators** (6 sets of the previous example) see Figure B.14. For this test, results of using PAB Uni are collected Table B.4, and results of using PAB B-Uni are collected in Table B.5. Looking in both tables it can be observed that the expected and actual value of variable (x) that maximizes profit of Gen (A) are very close to each other. This small difference is due to that expected value depends on the λ_{max} or μ which depends on a historical data of a specified window. It can be observed that unlike MP market, Gen (A) cannot increase its profit by deviating for BF in a system with small number of generators, which means that the effect of exercising market power by Gen (A) is limited. Also it can be observed that using PAB B-Uni has a better result. The reason for this that PAB-Uni depends on the value λ_{max} which could be an outlier price that will make effect the bidding function.

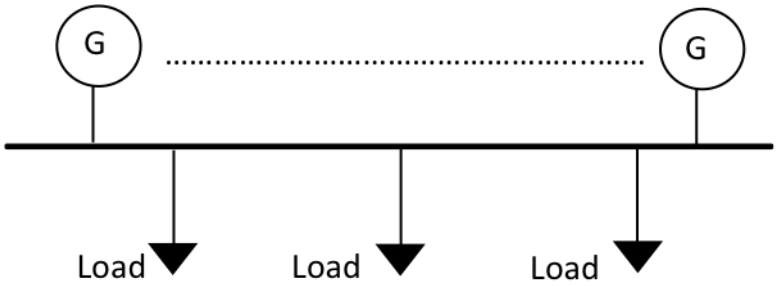

Figure B.14 Single bus system with 30 generators

Unlike MP market, it can be observed that number of generators in the system does not affect PAB bidding strategy, which supports the claim of PAB advocates that PAB limits the ability of generators to manipulate the market [35]. Therefore the PAB function in Table 2.1 can be used with confidence

Table B.4 Values of (x) that maximizes Gen (A) profit in a 30-gen system using PAB bidding function drived from Uniform distribution function

Time	**Day**	**Month**	**Expected value of "x" at maximum profit**	**Actual value of "x" at maximum profit**	**Error**	**Figure**
4:00	10	8	8.7	8.9	0.2	Figure B.15
4:00	10	4	8.4	8.6	0.2	Figure B.16
8:00	10	10	8.6	8.7	0.1	Figure B.17
3:00	15	7	8.7	8.9	0.2	Figure B.18
22:00	20	9	9.0	9.2	0.2	Figure B.19
19:00	23	12	8.7	8.9	0.2	Figure B.20

Table B.5 Values of (x) that maximizes Gen (A) profit in a 30-gen system using PAB bidding function drived from B-Uni distribution function

Time	Day	Month	Expected value of "x" at maximum profit	Actual value of "x" at maximum profit	Error	Figure
4:00	10	8	8.7	8.9	0.2	Figure B.21
4:00	10	4	8.5	8.6	0.1	Figure B.22
8:00	10	10	8.6	8.7	0.1	Figure B.23
3:00	15	7	8.8	8.9	0.1	Figure B.24
22:00	20	9	9.1	9.3	0.2	Figure B.25
19:00	23	12	8.7	8.9	0.2	Figure B.26

Below figures are assuming a single bus system with 30 generators bidding PAB derived from uniform distribution function:

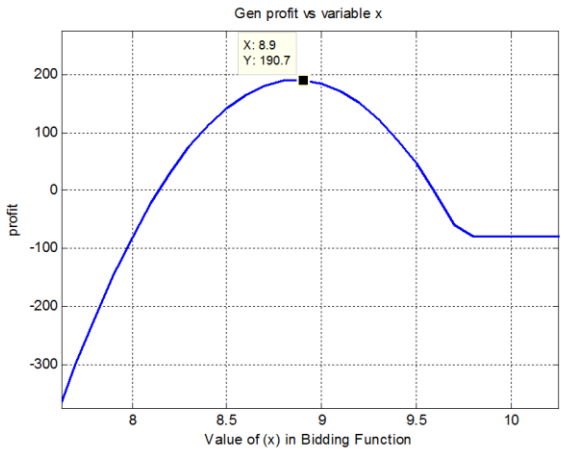

Figure B.15 The effect on profit of Gen (A) when varying the value of x of its bidding function at 4 am on Aug, 10th

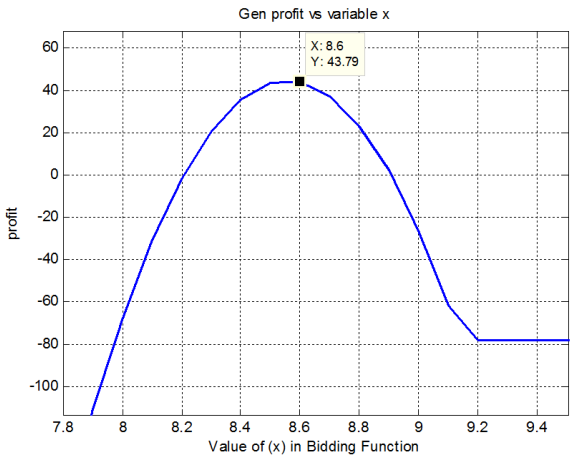

Figure B.16 The effect on profit of Gen (A) when varying the value of x of its bidding function at 4 am on April, 10th

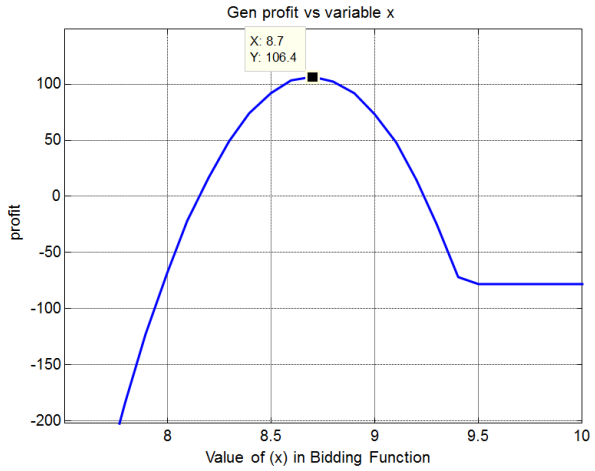

Figure B.17 The effect on profit of Gen (A) when varying the value of x of its bidding function at 8 am on Oct, 10th

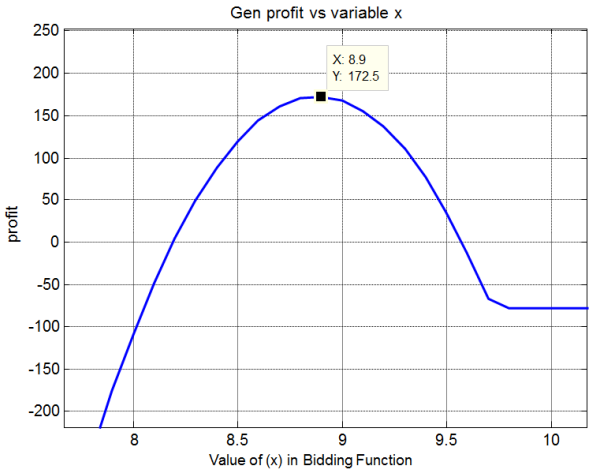

Figure B.18 The effect on profit of Gen (A) when varying the value of x of its bidding function at 3 am on July, 15th

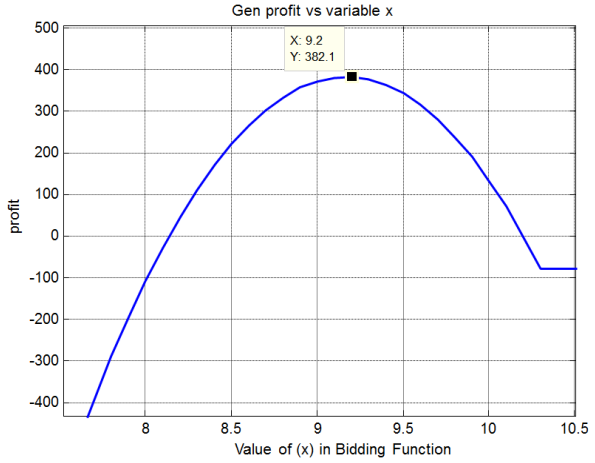

Figure B.19 The effect on profit of Gen (A) when varying the value of x of its bidding function at 10 pm on Sept, 20th

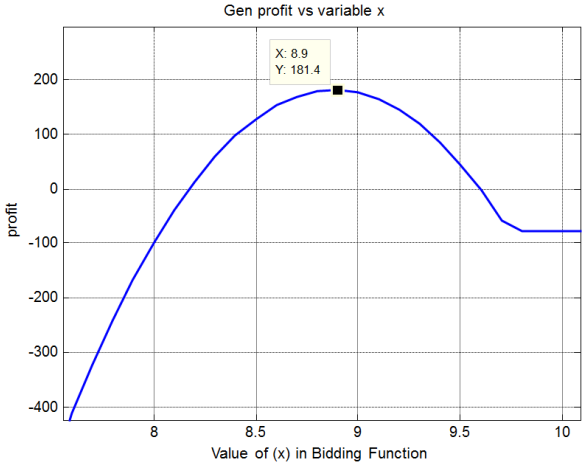

Figure B.20 The effect on profit of Gen (A) when varying the value of x of its bidding function at 7 pm on Dec, 23rd

Below figures are assuming a single bus system with 30 generators budding PAB derived from B-Uni distribution function:

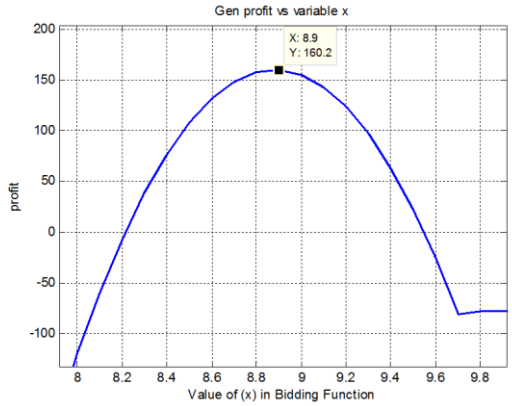

Figure B.21 The effect on profit of Gen (A) when varying the value of x of its bidding function at 4 am on Aug, 10th

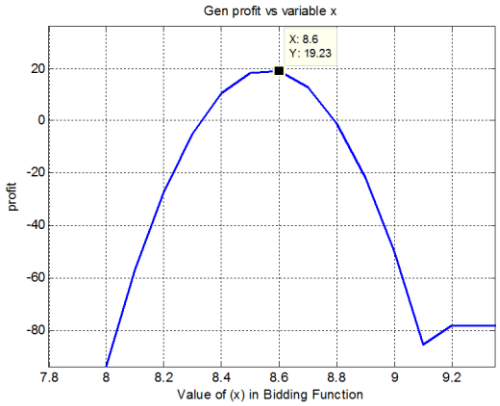

Figure B.22 The effect on profit of Gen (A) when varying the value of x of its bidding function at 4 am on April, 10th

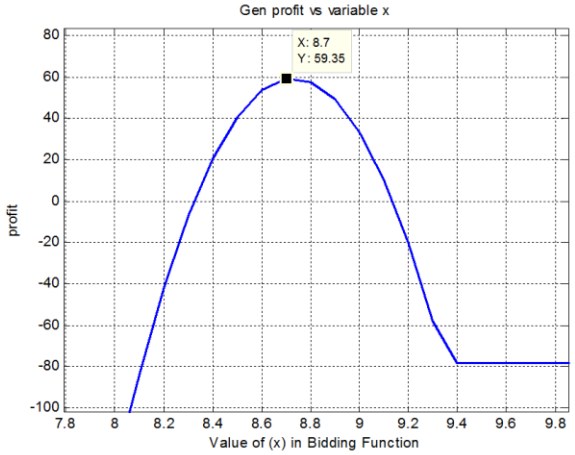

Figure B.23 The effect on profit of Gen (A) when varying the value of x of its bidding function at 8 am on Oct, 10th

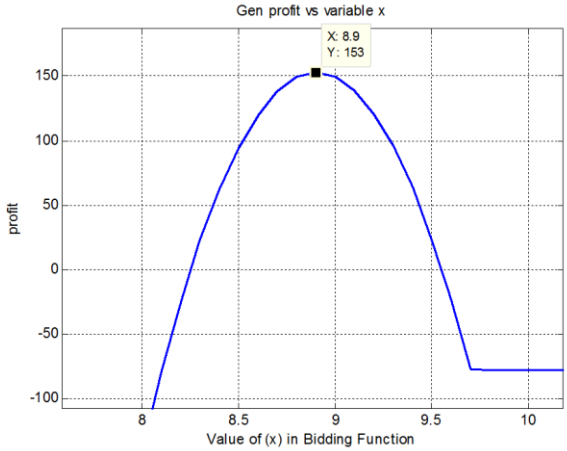

Figure B.24 The effect on profit of Gen (A) when varying the value of x of its bidding function at 3 am on July, 15th

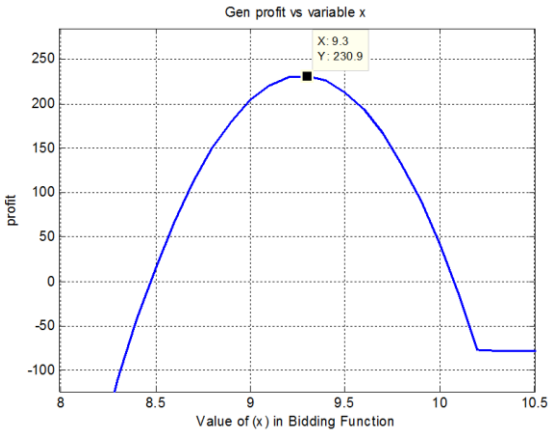

Figure B.25 The effect on profit of Gen (A) when varying the value of x of its bidding function at 10 pm on Sept, 20th

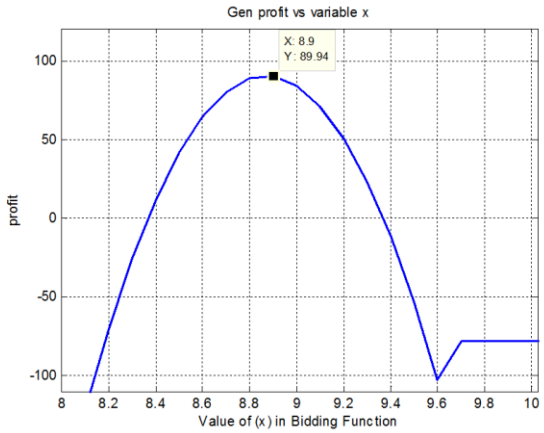

Figure B.26 The effect on profit of Gen (A) when varying the value of x of its bidding function at 7 pm on Dec, 23rd

APPENDIX C. VALIDATING THAT MP SCHEME BIDDING STRATEGY IS EFFECTED BY NUMBER OF PARTICIPANTS

In a perfectly competitive market all participants are **price takers**. This means that the bid of a single generator does not affect the market price, and it is able to sell all of its offered power for market price [117]. An example will be explored in this section that will show that in order to have bidding IC maximize generators' profit, many marginal units must be participating in the MP market.

In order to test if bidding the IC will maximize generators profit, a single bus system is assumed with many connected generators, see Figure C.1 .The cost function of the generators are based on examples from [87], see Table C.1. The load profile data used was derived from ERCOT load data [91]. A simulation is done to examine difference in value between theoretical and proposed bidding value with respect of different number of.

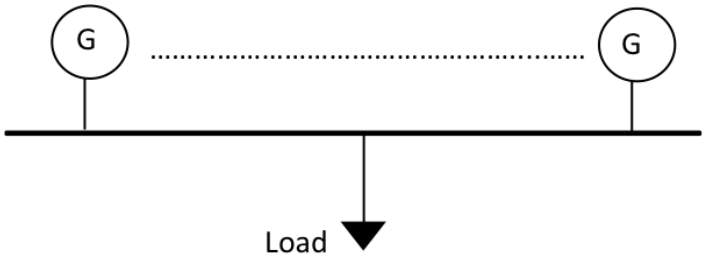

Figure C.1 Single bus system with many generators

Table C.1 Cost function coefficients [87]

	A	**B**	**C**
Gen (A)	78	7.80	0.002562
Gen (B)	78	7.88	0.004820
Gen (C)	100	7.97	0.004820
Gen (D)	561	8.00	0.004890
Gen (E)	50	7.85	0.001940

Gen (A) cost function is

$$C_A(P_g) = A_g + B_g \cdot P_g + C_g \cdot P_g^2 \tag{C.1}$$

Then the bidding function should be

$$IC_A(P_g) = B_g + 2 \cdot C_g \cdot P_g \tag{C.2}$$

In a single bus system the market price (λ) will equal the generators' IC, as discussed in detail in Appendix A.

Then the profit (π) of Gen A will be

$$\pi_A = IC_A(P_g) \cdot P_g - C_A(P_g) \tag{C.3}$$

Random days were chosen for the example. The results are in Figure C.2, Figure C.3, and Figure C.4. The *x*-axis represents increasing the number of generators based on generator data in Table C.1 by adding a new generator based on the sequence in the table then repeating them over. While the left y-axis is the mismatch value between the actual bid price of Gen (A) that maximizes profit and its IC in dollars and the right y-axis represents the percentage of **marginal units** (MU) (units that are bidding the last MW). It can be observed that with few generators in the system, bidding IC does not maximizes the profit of Gen (A), while by increasing number of generators in the system it can be observed that bidding IC will maximize the profit and the mismatch will be zero. Also it can be observed that Gen (A) maximizes its profit when all run-

ning units are marginal units. This confirms that bidding IC maximizes profit, which is achieved in a perfect competition market or near perfect competition market. The reason for this is that with few generators in the grid, many generators will hit their maximum limit which means that they are not MU anymore which gives room for Gen (A) to practice market power. The same conclusion was observed by [92].

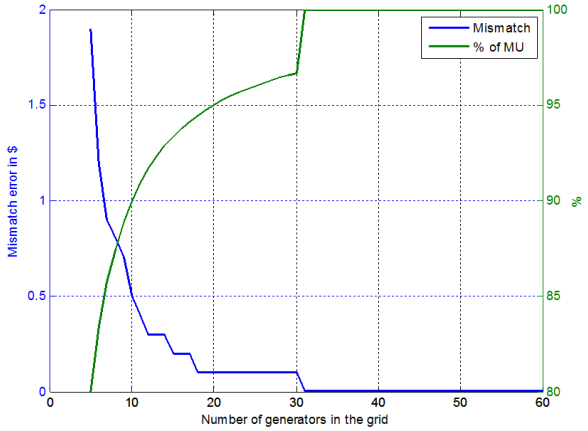

Figure C.2 The mismatch between proposed bidding function and actual value that maximize profit of Gen (A) in MP market when varying the number of generators based on load at 4 am on April, 10th

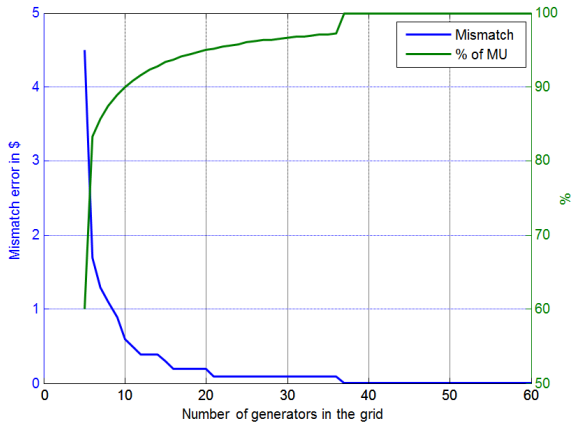

Figure C.3 The mismatch between proposed bidding function and actual value that maximize profit of Gen (A) in MP market when varying the number of generators based on load at 3 am on July, 15th .

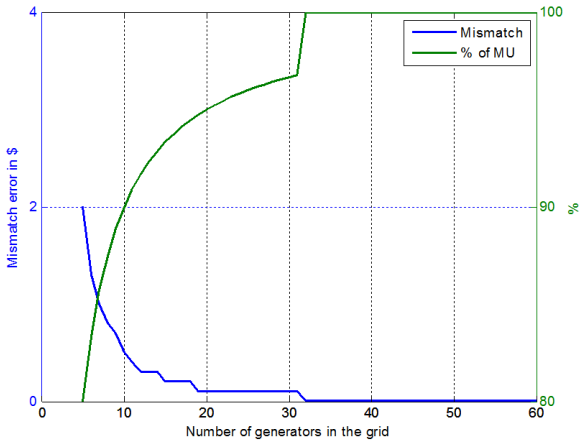

Figure C.4 The mismatch between proposed bidding function and actual value that maximize profit of Gen (A) in MP market when varying the number of generators based on load at 7 pm on December, 23rd .

APPENDIX D. FINDING THE POINT OF PROFIT MAXIMIZATION BY QUANTITY DEVIATION IN THE MP SCHEME

There is not a lot of literature about quantity bidding as most markets require all generators to submit a bid for all of their available capacity which makes no room for generators to maximize using this approach [118]. That is why generators seek to maximize their profit by price bidding. With the increase of renewable energy use, which is stochastic in nature, SPP power production deviation is inevitable. Tenorio showed that generators in the MP scheme are more motivated to withhold some of their quantity bid in the DA market to increase their profit. His work focused more on hedging in game theory in a system with a few generators [119]. Bialek analyzed "bidding of quantities" using conjectured supply function. In his analysis generators do bid IC but his analysis is limited only to 3 generators system [120].

As discussed in section 3.3.1, in the MP scheme producers will increase their profit by withholding some of their capacity in the DA market. The following will mathematically drive the percentage DA withholding that will maximize profit.

Assume that Generator 1 is called Gen 1, and the rest of the generations on the grid are lumped in a generator called Gen (2).

From equation (3.1) the cost function of Gen 1 is

$$C(P_1) = A_1 + B_1 \cdot P_1 + C_1 \cdot P_1^2 \tag{D.1}$$

Gen (2) is

$$C(P_2) = A_2 + B_2 \cdot P_2 + C_2 \cdot P_2^2 \tag{D.2}$$

Where

A, B, and C are the cost function coefficients.

P_1 is the produced power by Gen 1.

P_2 is the produced power by Gen (2).

The total profit (π) of Gen 1 will be

$$\pi = DA\ revenue + R\mathrm{T}\ revenue - Cost$$

$$\pi = x \cdot P_1 \cdot \lambda_{DA} + (1 - x) \cdot P_1 \cdot \lambda_{RT} - C(P_1) \quad \text{(D.3)}$$

Where x is the percentage of Gen 1 quantity withholding in the DA market with respect to final amount of generation (P_1) in the RT market. The profit of Gen 1 will be

$$\pi = x \cdot P_1 \cdot \lambda_{DA} + (1 - x) \cdot P_1 \cdot \lambda_{RT} - (A_1 + B_1 \cdot P_1 + C_1 \cdot P_1^2)$$

Where

λ_{DA} is the market price at DA market.

λ_{RT} is the market price at RT market.

Since Gen 1 is withholding some of its generation capacity in the DA market, and Gen (2) represents the lump sum of the rest of the grid, then the incremental cost of Gen (2) will set the market price. Then profit of Gen 1 will be

$$\begin{aligned}
\pi = x \cdot P_1 \cdot \lambda_{DA} + (1 - x) \cdot P_1 \cdot \lambda_{RT} - (A_1 + B_1 \cdot P_1 + C_1 \cdot P_1^2) \\
= x \cdot P_1 \cdot [B_2 + 2 \cdot C_2 \cdot (D - x \cdot P_1)] + (1 - x) \cdot P_1 \\
\cdot [B_2 + 2 \cdot C_2 \cdot (D - P_1)] - (A_1 + B_1 \cdot P_1 + C_1 \cdot P_1^2) \\
= [x \cdot B_2 \cdot P_1 + 2 \cdot x \cdot P_1 \cdot C_2 \cdot (D - x \cdot P_1)] + (1 - x) \\
\cdot [B_2 \cdot P_1 + 2 \cdot C_2 \cdot (D - P_1) \cdot P_1] - (A_1 + B_1 \cdot P_1 + C_1 \cdot P_1^2) = \\
= [x \cdot B_2 \cdot P_1 + 2 \cdot D \cdot x \cdot C_2 \cdot P_1 - 2 \cdot C_2 \cdot x^2 \cdot P_1^2] \\
+ [B_2 \cdot P_1 + 2 \cdot C_2 \cdot D \cdot P_1 - 2 \cdot C_2 \cdot P_1^2 - x \cdot B_2 \cdot P_1 - 2 \cdot x \cdot C_2 \cdot D \cdot P_1 + 2 \\
\cdot x \cdot C_2 \cdot P_1^2] - (A_1 + B_1 \cdot P_1 + C_1 \cdot P_1^2)
\end{aligned}$$

To get the value of (x) that maximize Gen 1 profit (π) the partial derivative of the profit with respect to (x) will result in

$$\frac{\partial \pi}{\partial x} = B_2 \cdot P_1 + 2 \cdot D \cdot C_2 \cdot P_1 - 4 \cdot C_2 \cdot P_1^2 \cdot x - B_2 \cdot P_1 - 2 \cdot C_2 \cdot D \cdot P_1 + 2 \cdot C_2 \cdot P_1^2 = 0$$

$$-4 \cdot C_2 \cdot P_1^2 \cdot x + 2 \cdot C_2 \cdot P_1^2 = 0$$

$$C_2 \cdot P_1^2 \cdot (-4 \cdot x + 2) = 0$$

$$x = \frac{1}{2} = 50\%$$

So withholding ***50%*** of the generation capacity in the DA market will maximize Gen 1 profit.

VITA

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