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The Use of Braess's Paradox for Urban Planning
A Case Study Analysis in Downtown Seattle

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Abstract

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Braess's Paradox counterintuitively states that adding capacity to roads can sometimes increase travel time. The concept provides advocates of creating better traffic performance, new urban spaces for recreation, and more effective use of resources. This thesis uses Braess's Paradox as a mathematical model to analyze the Seattle downtown road network. The Braess's Paradox demonstrates how current travel demand, can be analyzed and the results used to increase overall mobility in an urban area. The research selects road segments for analysis which have the potential to demonstrate Braess's Paradox and uses the results to inform changes that incorporate both technical data and the perspective of urban planning and policy.

Keywords: Braess's Paradox; Seattle; Urban Planning

TABLE OF CONTENTS

List of Figures	v
List of Tables	vi
Chapter 1. Introduction	1
1.1 Research Question	1
1.2 Research Objectives	3
Chapter 2. Literature Review	5
2.1 Concept of Braess's Paradox	5
2.2 The Prevalence of Braess's Paradox	7
2.3 Real-world Cases and Applications	8
Chapter 3. Methodology	12
3.1 Traffic Assignment Algorithm	12
3.2 Traffic Analysis Zone Origin Destination Demand	14
3.3 Hyperlink Establishment	15
Chapter 4. Study Area and Data Summary	18
4.1 Study Area	18
4.2 Data Format	20
4.3 Data Assumption and Summary	23

Chapter 5. Analysis and Result.....	25
5.1 Initial Scenario Analysis	25
5.2 Results of Numerical Analysis.....	29
5.3 Interactive Mapping	34
Chapter 6. Discussion and Conclusion	39
6.1 Connecting with Seattle Bike Program.....	39
6.2 Implementation Suggestion from Urban Policies Perspective.....	43
6.3 Limitations and Conclusions.....	44
Bibliography	46

LIST OF FIGURES

Figure 2.1 An Easy Braess's Paradox Example	6
Figure 3.1 Hyperlink Located in Traffic Analysis Zone 611.....	16
Figure 4.1 Study Area of the Research	19
Figure 4.2 TAZ and Hypernodes within the Model.....	21
Figure 4.3 Origin Destination Pairs from TAZ 502.....	22
Figure 5.1 Travel Time and Traffic Flow Scatter Plot under User Equilibrium.....	27
Figure 5.2 Volume over Capacity and Travel Time under User Equilibrium	28
Figure 5.3 Travel Time Assignment under User Equilibrium	29
Figure 5.4 Distribution of Total Time Change Before and After Link Closure	30
Figure 5.5 Distribution of Average Flow Change Before and After Link Closure	31
Figure 5.6 Linear Regression for Travel Time Change and Flow Change	32
Figure 5.7 Link with Negative Travel Time Variation after Closure	35
Figure 5.8 Traffic Assignment Classification under Different Scenarios.....	37
Figure 5.9 Detection of Braess's Paradox in Downtown Seattle	38
Figure 6.1 Seattle Existing Bike Network	40
Figure 6.2 Braess's Paradox Detection with Existing Bike Lanes	42

LIST OF TABLES

Table 4.1 General format of the demand table	23
Table 4.2 General format of the network table	23
Table 4.3 Parameters of the modified BPR function for different types of facilities ..	24
Table 5.1 Pearson Correlation Test.....	33
Table 5.2 Summary of the Result	34

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Chapter 1. INTRODUCTION

This section describes the reasons for choosing the research topic. It lists some possible applications of Braess's Paradox to address practical ways to improve mobility in urban environments.

1.1 RESEARCH QUESTION

Under current urban development circumstances, people need to rethink the concept of accessibility in smart cities. In particular, motorized traffic vehicles are not the only way to provide mobility. For example, use of smartphones, to access bike-sharing networks and ride-sharing services, have become more reliable mobility options than ever before.

Traditionally, the general public assumed that increasing the capacity of road networks would improve efficiency and reduce travel time. The traditional assumption was that by adding road links, vehicles will arrive at their destinations faster.

In 1968, a German mathematician, Dietrich Braess, mathematically demonstrated that adding another road could actually increase travel time for everyone. (Braess, 1968) The counterintuitive phenomenon he discovered has come to be known as Braess's Paradox (BP). Using BP as a tool in transportation planning allows planners to test how adding routes or capacity will affect traffic flow and congestion.

Modern researchers argue that banning people from driving cars in the center of cities would reduce air pollution and improve public health. In the United States, Indianapolis, Indiana, has already taken bold moves to make the city more walkable and greener by building an 18-mile pedestrian walkway and implementing both an electric car-share and bike-share program (Melanie,2015). The Indiana example provides an opportunity for a no-car zone policy to alleviate severe traffic congestion and environmental issues. Braess's Paradox is a tool that planners can use to support the public's interest in creating more walkable spaces in cities. Using Braess's Paradox, planners could analyze existing road system, and develop strategies for reducing inefficient links, thus improving traffic performance. Links not needed for motor transportation might be repurposed for more walking and recreational spaces, making the city environment more livable.

However, current studies about Braess's Paradox focus more on theoretical verification than practical urban studies. Most relevant studies are still limited to the transportation engineering field and rarely consider the road closure from the perspective of urban policy and real network assumption. Since there are few studies of the Seattle area traffic network applying Braess's Paradox, my thesis will identify examples that will prove the validity of Braess's Paradox in practice. The thesis will also focus on exploring the examples using theoretical and urban studies perspectives. The thesis raises two research questions:

- 1) How commonly is Braess's Paradox observed in the Seattle area?

2) How can planners use Braess's Paradox as an important tool in urban transportation planning?

1.2 RESEARCH OBJECTIVES

Downtown Seattle is the study area to for determining the prevalence of Braess's Paradox within the existing system of roads. Why was the city of Seattle chosen as the study area? Seattle, is a city in rapid development, devoting considerable resources to improving transportation systems and services. For example, the Commute Trip Reduction (CTR) program is a partnership among governmental organizations to guide commuters to use the transportation systems efficiently, particularly during the most congested periods of the day. (Su, 2012) The Seattle Master Bicycle Plan led by Seattle Department of Transportation, is also using urban streets to provide safe, affordable travel choices that encourage people to get out and enjoy streets on foot or by bike. As the literature documents, Braess's Paradox is prevalent in large city transportation networks. The documentation of Braess's Paradox in the Seattle area will give planners an important tool to optimize transportation networks through mathematical modeling.

Studies about Braess's Paradox, based on real-world road networks, are scarce. Many of the studies put more emphasis on engineering optimization or theoretical verification. The discovery of Braess's Paradox under real-world traffic demand is a meaningful topic. Hence, real-world cases of this issue might broaden planners' options for making the city better. Eliminating roads to improve traffic flow is a relatively new

idea, and can be further advanced professionally through an interdisciplinary approach combining transportation engineering with urban planning.

To answer the question of how to avoid Braess's Paradox, the study will explore the possibility and outcome of closing the "problematic" roads, which are the links that Braess's Paradox identifies. Transportation network planning has become a more important area for urban planners. A government may not want to suspend routes or links only because they are inefficient. The actual planning implementation should consider not only the rational, mathematical analysis but also reliable planning policy implementation, and resource constraints. Removing a road may conflict with traditional planning assumptions. This research argues that the combination of actual policy support and theoretical analysis could be a more powerful way to avoid Braess's Paradox.

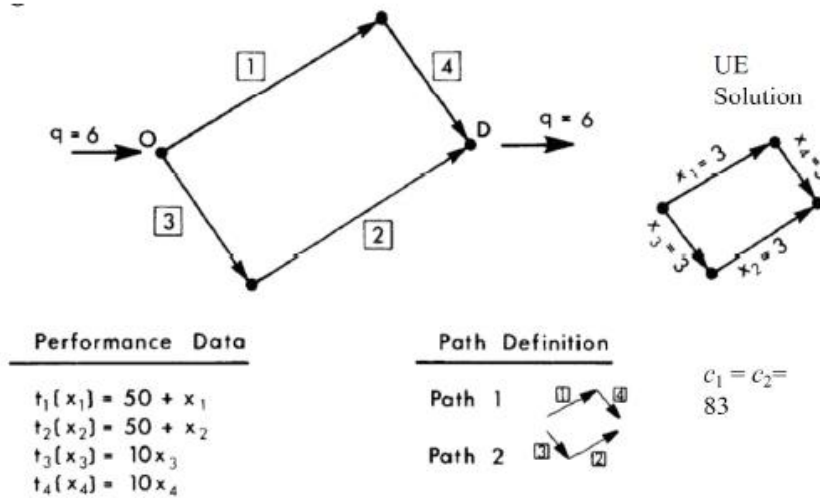
Chapter 2. LITERATURE REVIEW

This section is about the literature review of Braess's Paradox issue. It consists of three parts: the concept of the problem, the prevalence study, and some real-world cases and applications. This section proves that Braess's Paradox is not a rarely seen phenomenon but a common issue in cities. Some real-world examples are introduced in this section, and they provide the basis for choosing Downtown Seattle as the study area.

2.1 CONCEPT OF BRAESS'S PARADOX

What causes Braess's Paradox? Drivers usually tend to greedily pick out the quickest paths to their destination according to the actual road performance; this behavior is based on the user's choice, known as "user equilibrium." Another ideal traffic mode, in terms of the gross travel time balance with the aid of all users in the network, is always be considered when adding some new links. This mode assumes that drivers cooperate when selecting their routes to optimize the global road network. Some drivers may need to bear the temporal congestion in their paths, which is called by system optimum. Sometimes, adding road links or capacity will cause a gap between user equilibrium and system optimum leading to Braess's Paradox. As shown in Figure 2.1, a hypothesized simple network is created to verify the Braess's Paradox. It assumes that a total of 4 roads have their performance formula and states that the total driver flow is 6. Before adding one more link, the cost should be 83. However, after adding one more link, by obeying the principle of user equilibrium, the costs will rise to 92.

ORIGINAL NETWORK



ADD ONE MORE LINK

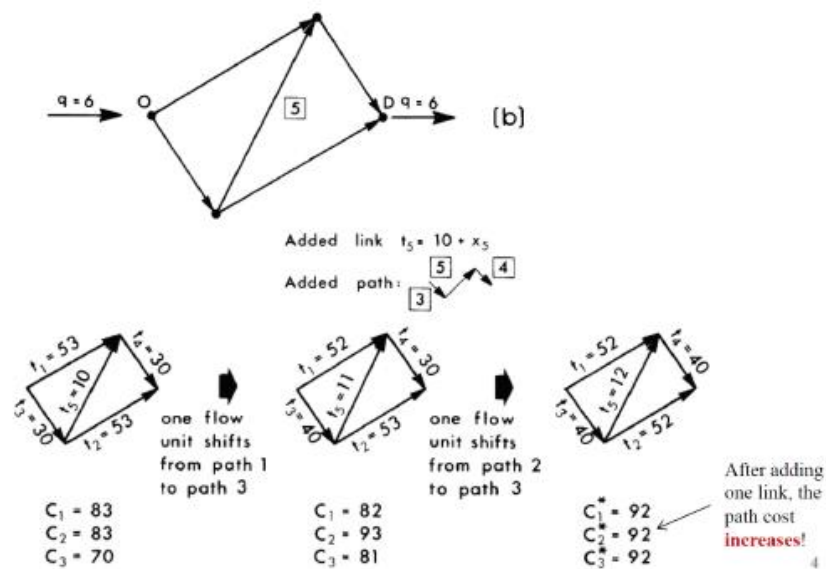


Figure 2.1 An Easy Braess's Paradox Example

Braess's Paradox happens due to users' egocentric behavior. Those who are looking for routes that could minimize their own travel cost (usually in time) might lead to more travel costs for other drivers. Drivers always tend to selfishly choose the lowest cost-consuming paths under the same travel demand. Rapoport et al. (2009) demonstrate that

a strong correlation exists between drivers' actions and the incidence of Braess's Paradox by conducting an experimental simulation based on real driving factors.

2.2 THE PREVALENCE OF BRAESS'S PARADOX

How prevalent is Braess's Paradox in the transit system? Steinberg and Zangwill (1983) prove that Braess's Paradox is not an abnormal phenomenon, but might frequently happen under reasonable assumptions. Pas (1997) examines some properties of traffic flow in using the standard network configuration used by Braess and analyzes whether the counterintuitive observation's is could be caused by some other phenomenon. From their studies, they conclude that Braess's Paradox exists extensively in most urban transportation systems. including Seattle's.

Some studies show that Braess's Paradox is a much larger factor in transportation flow than previously thought. Papadimitriou et al. (1999) introduce the idea of the price of anarchy, which means the ratio of the system's cost using user equilibrium's cost divided by the system optimum's value. The outcome is surprisingly high (2.15), which provides a convincing fact and idea for the government and transportation planners to improve the network balance by utilizing this phenomenon. Daganzo (1998) observes that it is chaotic for the time-dependent traffic assignment problem with physical queues, and it might be impossible to get initial data solved within the needed accuracy. Daganzo (1998) also makes some dependable forecasts for cumulative solved traffic flows on severely congested networks under the user equilibrium. The apparent gap between user equilibrium and system optimum provides a way to observe and test for

Braess's Paradox issue. If a road segment could be tested before and after closure, the variation of the cost consumption, such as travel time, could document the occurrence of Braess's Paradox.

There are other paradoxes similar to Braess's Paradox. Some researchers attempt to use Braess's Paradox to detect a problem discursively when testing or designing the road network. Yang et al. (1998) demonstrate that adding a new segment of road to a road network could reduce potential network capacity. Burke et al. (2012) found that adding routes and capacity to an existing network does not usually decrease the average time that individuals require to travel through the network from a source to a destination in an uncongested transportation network. In addition to transport, the Braess's Paradox phenomenon has also been observed in other fields such as computer networks, water distribution pipes, queue flow, semiconductors in electrical circuits, and crowd control (Baglee et al. 2019).

2.3 REAL-WORLD CASES AND APPLICATIONS

Braess's Paradox is not just a complicated riddle on paper that is only tested in laboratory and computer simulations, It has been documented in a variety of scenarios around the world. The following are some examples of Braess's Paradox.: Kolata (1990) reports that the temporary closure of 42nd Street in New York City for the celebration of Earth Day surprisingly improved traffic performance and decreased traffic congestion in that area. Baker (2009) identified at least six possible roads, including parts of busy streets in Boston, that when closed, could reduce congestion and delays.

Easley and Kleinberg (2010) summarize that in Seoul, the closure of a six-lane highway created space for building a new public park improved travel time within the city, even if traffic flow remained approximately the same before and after the change. Cairns et al. (1998) point out that over 70 cases of road closures or traffic restriction eventually lead to reducing traffic congestion. Sun et al. (2015) found that in San Francisco, most streets have a negligible effect on the efficiency of the road network, while the closure of a few inefficient links would counterintuitively reduce travel costs considerably. Many other examples show that Braess's Paradox in the real world is a quite common and invisible phenomenon. As a consequence, road closure can provide a meaningful improvement in real-world network performance. (Baglee et al. 2019).

The notion of Braess's Paradox has encouraged many researchers to look at transportation planning from different perspectives. Researchers try to study Braess's Paradox issues through various tools and models. Youn et al. (2008) assess the price of anarchy by analyzing the travel times in road networks of several major cities, and the results suggest that the distribution became more regular as the number of nodes increased. Baglee et al. (2019) aim to leverage the Braess's Paradox to encourage the urban policy idea of the no-car area or walkable space. The researchers formulate the Braess's Paradox Problem as a bilevel problem and develop a heuristic methodology to identify roads for closure in the center of the cities to be redeveloped as the green space or, pedestrian zones. Some government officials and planners connect the idea of Braess's Paradox with the improvement of walkability. They believe that one way to

improve downtown walkability is to use Braess's Paradox to identify under-utilized roads, that can be repurposed for pedestrian use without increasing traffic congestion (Huang, 2017; Holle, 2019). From the perspective of planning strategies, a TOD planning project in Bogota aims to make the city more walkable by reducing roads in a crowded area (Calderon-Restrepo, 2018). Fisk and Pallottino (1981) test the City of Winnipeg network data to detect the existence of Braess's Paradox and determines optimal planning strategies and future network modeling.

Although there are still insufficient large-scale urban planning practices, using Braess's Paradox to solve transportation planning problems, many scholars have conducted the experimental tests to prove the validity of Braess's Paradox. Rapoport et al. (2009) conducted two experiments to study whether the Braess's Paradox behaviorally happens in two simulated traffic networks when the networks differ from each other in their topology. Rapoport et al. (2005) report the results of a traffic network game experiment with asymmetric costs designed to test the implications of the Braess's Paradox. Bazzon (2005) experiments with the effect of giving route recommendations based on the findings of Braess's Paradox studies. In the interdisciplinary fields, some technology applications are used. Hong (2007) uses dynamic path planning for centralized vehicle navigation, using an algorithm based on the concept of Braess's Paradox. Askoura (2011) tests the optimal sub-networks based on the choice of the roads under the least impact by Braess's Paradox.

To summarize, the essential characteristics of the literature review of Braess's Paradox are as follows:

- 1) Braess's Paradox can be a tool for advocates of no-car zones.
- 2) Braess's Paradox has demonstrated validity in both theoretical experiments and practical application to specific transportation and land use problems.
- 3) Studies aiming to demonstrate how Braess's Paradox can solve problems in large-scale urban networks are rare.
- 4) Predictable or constant travel demand or travel elasticity could help to discover Braess's Paradox.
- 5) Braess's Paradox has already been successfully applied in other fields.
- 6) There are few studies about Braess's Paradox in Seattle.

In this study, I will use Braess's Paradox as the primary analytical tool to research problems in Seattle's road network. The study will implement the Braess's Paradox mathematical model that helps to identify how changes in one road can affect all links. The study will also analyze some of the roads from the perspective of urban planning and propose some ideas to redevelop the roads while avoiding Braess's Paradox.

Chapter 3. METHODOLOGY

This section discusses the method to demonstrate the presence of Braess's Paradox in selected road networks. It presents a standard method that could solve the traffic assignment problem. The sources of the demand data are presented in a matrix based on traffic analysis zones. This section introduces some hyperlinks to solve the problem of linking the location of the traffic analysis zone and to the connecting road links under study.

3.1 TRAFFIC ASSIGNMENT ALGORITHM

Experimental tests have indicated that drivers in the real world tend to find paths within transportation networks to minimize travel costs (Rapoport et al., 2005). Generally, when a road network or a list of the roads are closed, all drivers who used to use the closed links will occupy other links. Under a Nash Equilibrium, the entire system will eventually reach an equilibrium state where no driver can reduce travel time by switching routes. The travel time difference before and after a road closed is the basic principle to detect the Braess's Paradox. Then, the study goes into an optimization problem of how to optimize the road network by reducing some inefficient links. To build the user equilibrium assignment model, Sun et al. (2014) summarize a method based on Beckmann model and Wardrop's principle, in which the equilibrium flow minimizes the model function:

$$\text{Minimize } T(f_{ij}, t_{ij}) = \sum \int_0^{f_{ij}} t_{ij}(f) df$$

Where f_{ij} represents the traffic flow and t_{ij} is considered as travel time, respectively, between road intersection i and j . Link travel time t_{ij} is quantified based on the following function:

$$t_{ij}(f_{ij}) = t_0 \times \left(1 + \alpha \times \left(\frac{f_{ij}}{C_{ij}} \right)^\beta \right)$$

Where t_0 is the free-flow travel time of the road segment; C_{ij} is the capacity of the road segment; α and β are the constant parameters that defined by the empirical traffic data. f_{ij} is the flow of the link based on user equilibrium. The equilibrium flow f could be numerically calculated based on Frank-Wolfe algorithm, which is an iterative first-order optimization algorithm for constrained convex optimization.

The general step of Frank-Wolfe algorithm could be summarized as:

1) Let x_0 be any point in the compact convex set D (considered as initial input) and find the limited point (considered as equilibrium flow) s_k and $k = 0$ solving the following function:

$$\text{Minimize } s^T \nabla(x_k)$$

$$\text{Subject to } s \in D$$

2) Step size determination: set y as $\frac{2}{k+2}$, or alternatively find y that minimizes $f(x_k + y(s_k - x_k))$ subject to $0 \leq y \leq 1$

3) Update the point, let x_{k+1} as $x_k + y(s_k - x_k)$ and let k as $k+1$, then go to Step 1.

In a word, the functions listed above give the general process of how traffic demand and flow could be adequately assigned under the user equilibrium model and

what the travel cost is based on the given traffic demand and road segment data. Then, if one of the links is closed, the assigned flow and travel cost will change under the different traffic scenarios. The idea could be summarized as comparing the distribution and travel time of traffic flow under normal demand in the Seattle network by removing a road segment from the network for each road. If the road segment's removal causes trip failure, the road will be defined as the main road segment. Trip failure means the connectivity will be gone. Usually, the main road segments represent the arterials of the city. The other roads which will not cause trip failure after removal will be defined as ordinary links. The roads that possibly exist in Braess's Paradox are in the ordinary links. The general algorithm follows the principle of the Frank-Wolfe algorithm, which could be conducted by python scripts.

3.2 TRAFFIC ANALYSIS ZONE ORIGIN DESTINATION DEMAND

To obtain a more realistic result, some data was refined and manipulated to make it usable in the model. Traffic flow and travel time of each link in the study area are essential to calculate travel demand and network capacity. The study area is located in Seattle Downtown. Then some road conditions need to be analyzed, including speed limit, road length, travel demand, and capacity. Those data could be extracted from Seattle GIS Data Portal, King County GIS Data Portal, and Puget Sound Regional Council Database. The road segments include highways, arterial roads, and other links. To simplify the model, I-5 highway and other state routes would not be considered as

the downtown model, because lots of external demand from other parts of the city will influence the precision of the model.

As mentioned above, traffic demand data is one of the critical data types used in this model to identify the flow and time cost of each link. Usually, travel demand data could be generated by the home-work commuting Origin-Destination (OD) data matrix and the corresponding network. OD data estimates the information on the demand for trips from residents' original locations to their destination at a street-block level.

Puget Sound Regional Council (PSRC) estimates the whole region's travel demand by an organized 3951*3951 matrix with row index indicating the origin location and column index indicating the destination separated by the Traffic Analysis Zones (TAZs). PSRC identifies TAZs as the units of the geographic boundary system to run and report the results from its Travel Demand Forecast Model. TAZs are a set of boundaries that divide the PSRC planning region into travel demand forecast areas. The origin-destination demand matrix represents the travel demand generated among each TAZ area.

3.3 HYPERLINK ESTABLISHMENT

To document Braess's Paradox one needs to know the roads and links that exist within the study area as well as actual travel demand. However, the TAZ boundaries are different from the boundary of the study area in my research. In addition, TAZs do not show the road networks under analysis. In order to connect the TAZ to the actual road network, I created a series of hyperlinks to assign the travel demand data from each

TAZ to all of the links in that area. A "hyperlink" means that the link does not exist in the real world but is used to build up a connection between the Traffic Analysis Zone and the road segments. Firstly, the centroids of all the TAZs are identified by the related analyzed tool in ArcGIS, which could be named "hypernodes." A hypernode could then establish a hyperlink as the origin with its nearest actual road start node as the destination. Figure 3.1 makes the TAZ 611, which is shown in red lines, an example of how a hyperlink and the actual road network connect. The centroid of TAZ 611 is the hypernode, which builds up a link drawn by dash lines with its nearest node located in South Main Street. As mentioned above, travel time under user equilibrium is greatly impacted by road capacity, free-flow time, and speed limit. Therefore, for all the hyperlinks in the model, capacity and speed limit would be assumed as the maximum number and free flow time would be set as low as possible to show that the travel cost would be neglected for vehicles driving in the "hyperlinks."

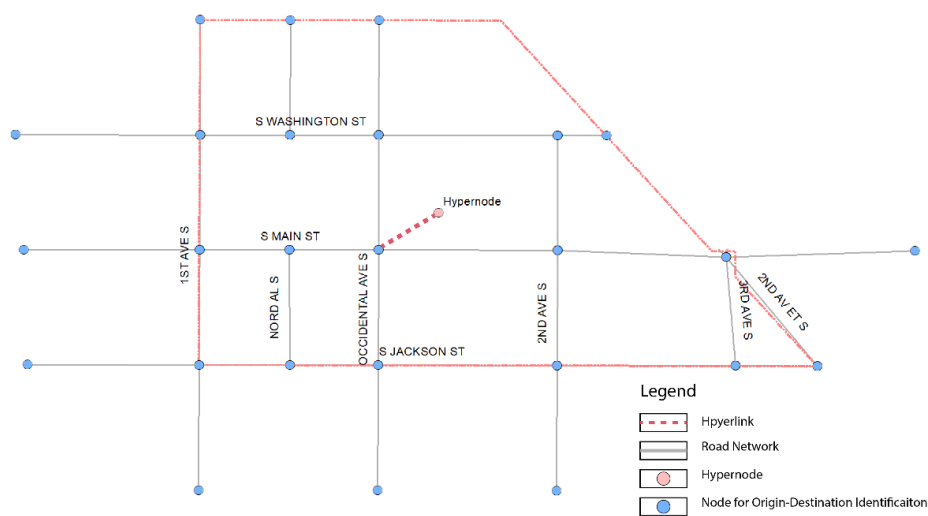


Figure 3.1 Hyperlink Located in Traffic Analysis Zone 611

Travel costs under the user equilibrium could be calculated based on the demand data and network condition data, which is comprised of hyperlinks and the network in the study area. To document the occurrence of Braess's Paradox, the tested road segment would be defined as closed before the traffic assignment algorithm runs. Then the average travel cost under user equilibrium except for the hyperlinks would compare with the road that is not evaluated under Braess's Paradox. If the difference between the two average costs is negative, Braess's Paradox might exist in the links. Python script could help to do the Frank-Wolfe iteration and automatically run the model for each link.

The quantitative analysis described above could help detect the links which display Braess's Paradox characteristics. Once links are detected, the study will pick some typical road segments and analyze possibility of road closure from the perspective of urban planning. The comprehensive plan of the area which contains the link will be discussed, focusing on the bike and walk plan in that area. The related transportation policies implemented in the study area will be considered as a critical part of future area development. If the policy or plan identifies that the link as an ordinary one (which means the government does not require the area for some other purpose), the closure of the road might allow planners to create more public spaces for residents. If not, the use of Braess's Paradox could also provide additional information that the city could consider in the area's development.

Chapter 4. STUDY AREA AND DATA SUMMARY

This section contains a discussion of the rationale for the selection of the study area and downtown network simplification. It lists the format of the data and the types of data to be used. Some parameters and virtual links to information and assumptions are also discussed in this section.

4.1 STUDY AREA

The study area is designated as Downtown Seattle, which is one of the busiest regions in King County, Washington State. In order to simplify the model, not all of the links are included in the model. Two interstate routes, the I-5 Highway and SR99 Tunnel are not included in the demand assignment because traffic demand from these two links comes from multiple external links outside of Downtown Seattle. Other links, including I-5 and SR99, that generate mostly internal travel demand were identified in the downtown network, and these links help explain the total travel demand of the network. Figure 4.1 illustrates the study area and the simplified network used in this research. The network with the grey line covered most links in the Downtown except for two interstate routes.

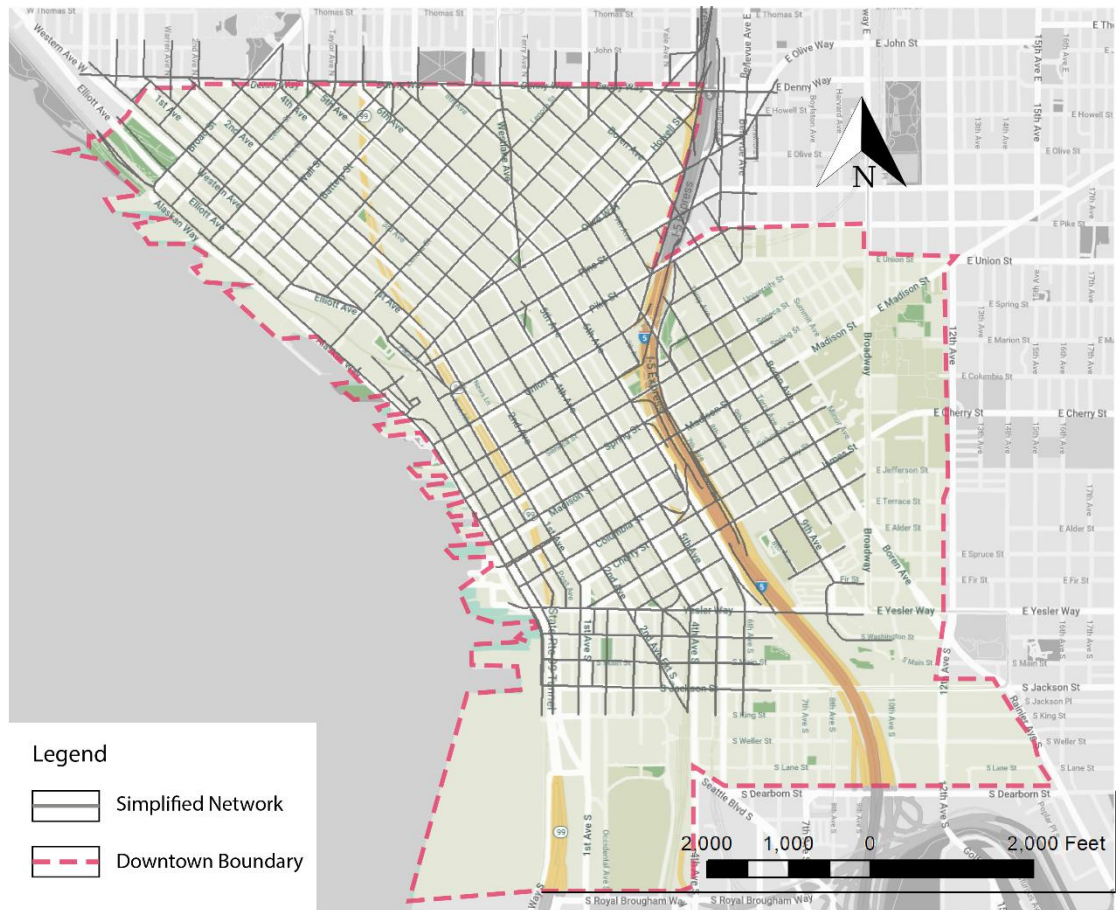


Figure 4.1 Study Area of the Research

Seattle Network Database (SND) produced by the City of Seattle provides a complete, land-based travel pathway infrastructure data, and those data can be matched with theoretical address ranges. The pathway data, consisting of a series of links or segments, is a linear graphic element representing a continuous physical travel path terminated by the path's end or physical intersection with another travel path. To sum up, 917 links will be analyzed in the model. Those 917 links are comprised of 570 start & end nodes, which form the whole Downtown Seattle Network. Capacity, speed limit and free-flow time can also be obtained from this database. The free flow time yields the equation as follow,

$$FFT = \frac{Length}{Speed}$$

Where FFT is the free flow time of the path, which could be calculated by the length of the road, divided by the speed of the vehicle. Seattle Network Database provides the speed limit of each road, and the length data can be obtained by using Calculate Geometry, an ArcGIS tool that adds geographic information to a link representing the spatial and geometric characteristics. This research assumes that capacity is relevant to the number of lanes for each link, summarized as:

$$Capacity = 900 * Lanes \left(\frac{vehicles}{hour} \right)$$

Where Lanes means the number of lanes, link capacity roughly equals 900 times the number of lanes.

4.2 DATA FORMAT

Besides the 570 actual path ends, the study area also contains 148 hypernodes from 148 TAZs, which generates the travel demand to each other based on the PSRC trip demand matrix. Centroids of the TAZs are assumed as the hypernodes that connect with the actual path ends to assign the free flow and demand on all of the links among the networks. Figure 4.2 shows that the distribution of Traffic Analysis Zones and their corresponding hypernodes.

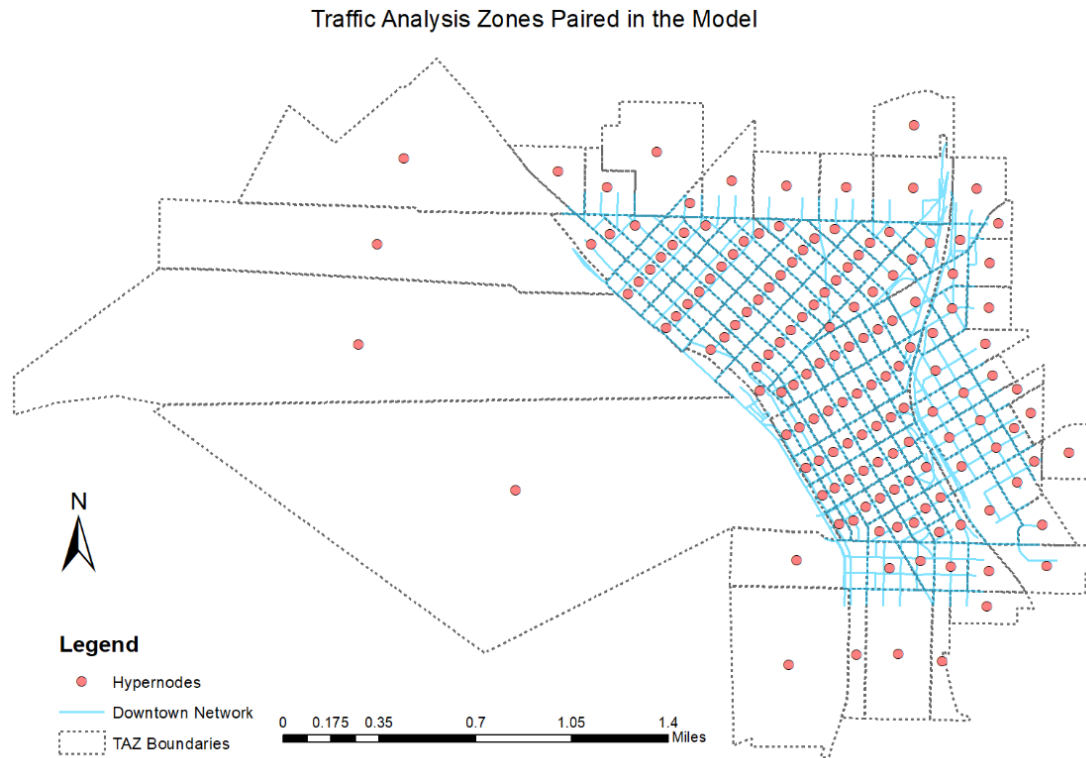


Figure 4.2 TAZ and Hypernodes within the Model

As the methodology part demonstrates, there are two data files needed to be prepared for detecting Braess's Paradox issue. Bar-Gera et al. (2016) develop a network repository for transportation research called Transportation Networks (TNTP). The general format for the data contains two tables at the list, network file; and demand table. Both data are tab text files, with each row terminated by a semicolon. First, the Demand table is constructed by an origin label and then origin node number, followed by destination node numbers and OD flow. Each hypernode has 147 OD pairs to other nodes for travel demand, requiring 147 different numbers of travel demand from the origin nodes to other nodes, respectively. Figure 4.3 illustrates the demand flow map of a node in TAZ 502 to other nodes.

Origin Destination Pairs Generated from Traffic Analysis Zone 502

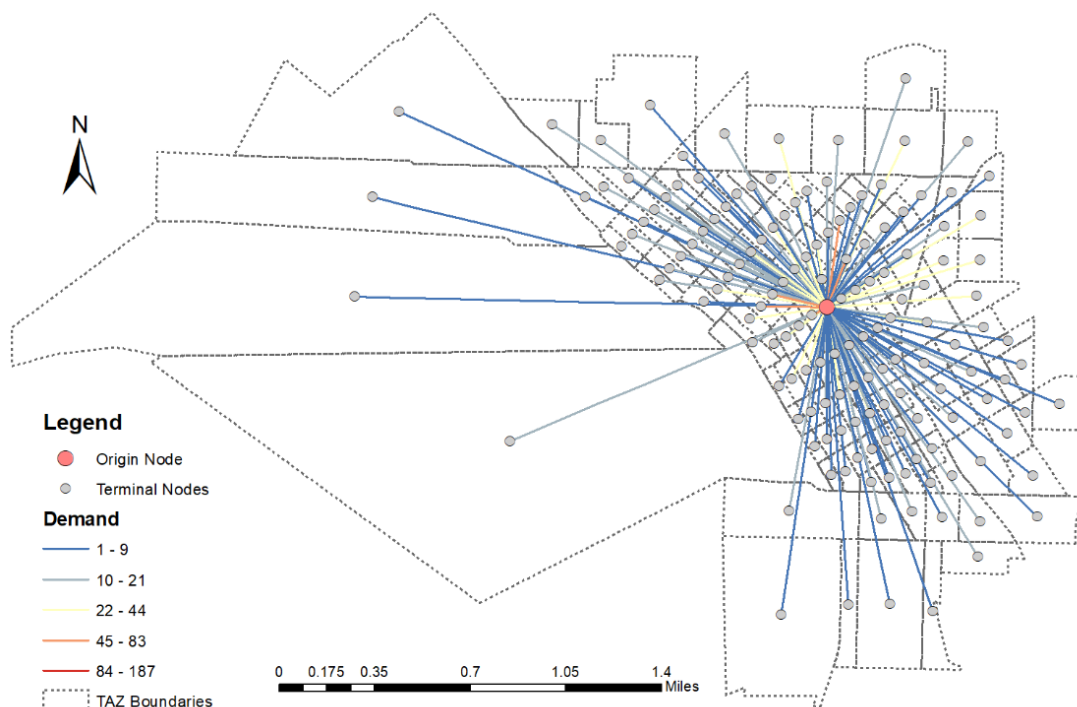


Figure 4.3 Origin Destination Pairs from TAZ 502

Meanwhile, network file includes all the links (hyperlinks and actual links), which build up the network. Links are directional, going from initial node to terminal node. Several fields should be contained in the network files, which includes Initial Node, Terminal Node, Capacity, Length, Free Flow Time, Parameter α , Power β , Speed Limit, Toll and Link Type. Kumar (2019) optimizes the format based on TNTP and develops a repository for static traffic assignment python code, which makes the demand and network tables much easier to use in the traffic assignment method. This research redevelops the downtown Seattle network based on TNTP format. Additionally, the thesis uses Kumar's (2019) python code to get the travel assignment solution using Frank-Wolfe's algorithm. Table 4.1 and Table 4.2 show the general format of the demand and network data.

Table 4.1 General format of the demand table

Origin Node	Terminal Node	Demand
1	2	60
1	3	148
...
1	148	1
2	1	79
...
148	147	1

Table 4.2 General format of the network table

Origin	Terminal	Capacity (vehicles/hour)	Length (Mile)	FFT (Min)	α	β	Speed (MPH)
1	547	100000	0.001	0.0006	0.88	6	100
2	534	100000	0.001	0.0006	0.88	6	100
...
149	150	900	0.0579	0.1738	0.71	6	20
...
718	611	900	0.0457	0.0137	0.71	6	20

4.3 DATA ASSUMPTION AND SUMMARY

The impact of hyperlinks should be as little as possible for the travel demand assignment. Therefore, the model assumes that all of hyperlinks have extremely large capacity (100000 vehicles per hour), quite short length (0.001 mile), low free flow time (0.0006 minutes) and high-speed limitation (100 MPH). Sun et al. (2015) demonstrates the parameters number. Parameter α is set based on the Table 4.3. And power parameter β is set to 6 to fit the volume of road capacity.

Table 4.3 Parameters of the modified BPR function for different types of facilities

Road Type	Speed Limit (MPH)	α
Freeway	Speed \geq 60	0.88
Major Arterials	Speed \geq 30	0.83
Collectors	Speed $<$ 30	0.71

The total zones number 148 and create 18,293 OD pairs, counting OD pairs that are larger than 0. In total, there are 718 nodes for the network, which includes 148 hypernodes and 570 actual path ends. These nodes connect with 2128 links, comprised of 147 hyperlinks and 917 road segments in two direction. The average link length is 0.0576 miles (excluding hyperlinks). Detailed data are summarized in the Table 4.4 below.

Table 4.4 Summary of the network and demand data

Summary	Hyperlinks	Actual Links	Total
Number of Nodes	148	570	718
Number of Links	147 * 2 Directions	917 * 2 Directions	2128
Number of Zones	/	/	148
Total OD Pairs	/	/	18,293
Average Link Length	0.001 miles	0.0576 miles	/
Average Free Flow Time	0.0006 minutes	0.1454 minutes	/

For the road removal test, the capacity and free flow time of the selected link are changed to ensure that no traffic flow will pass through the segment. For this model, removed road capacity will be set to 1 and its free flow time will increase to 5 minutes. Under this assumption, the travel flow and travel time solved by Frank-Wolfe's user

equilibrium will remain 0, which means nobody is willing to choose this special designated link in their routes.

Chapter 5. ANALYSIS AND RESULT

This section focuses on the identification and analysis of Braess's Paradox in Downtown Seattle. It contains the initial scenario analysis, numerical analysis of the results, and interactive mapping. In this section, two indicators are used to identify a link that produces Braess's Paradox. The first indicator is the variation of total travel time before and after a road segment is closed could help to identify the results. GIS software could help to visualize the result. The second indicator is the variation of volume over capacity before and after a road segment. Then interactive maps show which roads have a negligible effect on the efficiency of the road network, which ones are essential, the closure of which could lead to severe delays and which roads would counterintuitively reduce travel costs considerably when they are closed.

5.1 INITIAL SCENARIO ANALYSIS

As mentioned before, the Frank-Wolfe algorithm is an iterative first-order optimization algorithm for constrained convex optimization. In this research, the duality gap is set to 0.01, and the maximum iteration number is 500, which means the iteration will end when the difference between primal and dual solutions is less than 0.01 or the iteration times reach 500. Therefore, the general process for the detection of Braess's Paradox could be summarized as:

Step 0. Set the duality gap as 0.01, maximum iteration as 500; Run Frank-Wolfe traffic assignment; Summarize the result and note as the regular result. Set count = 1.

Step 1. Change one of the link's capacities and free flow time. Then run the Frank-Wolfe traffic assignment.

Step 2. Calculate the ΔT as the variation between regular travel cost and new travel cost after link removal. Mark ΔT with its origin node and terminal node in the result table. Count increase of 1.

Step 3. Return to Step 1 until the count reaches 917 (the number of tested links).

After testing the initial assignment network, which consists of 148 traffic analysis zones, 718 nodes, 2128 links, and 18,293 OD pairs, the average travel time for each actual links under user equilibrium solved by Frank-Wolfe algorithm is 0.1884 minutes (11.3302 seconds), and the average flow is 931.81 vehicles per hour. It states that drivers will feel satisfied when they spend an average of 11.3302 seconds or less traveling through the road segments. Figure 5.1 plots the correlation between travel time and traffic flow. According to the percentile summary, the research finds that only 2.7% of road segments have traffic flow, which is larger than 2000 vehicles. 12.1% of road segments have a flow less than 10 vehicles, which means such links have few traversing vehicles. The standard deviation of the flow is 603.35. The range is from 0 to 2906.302 vehicles per hour for the actual links.

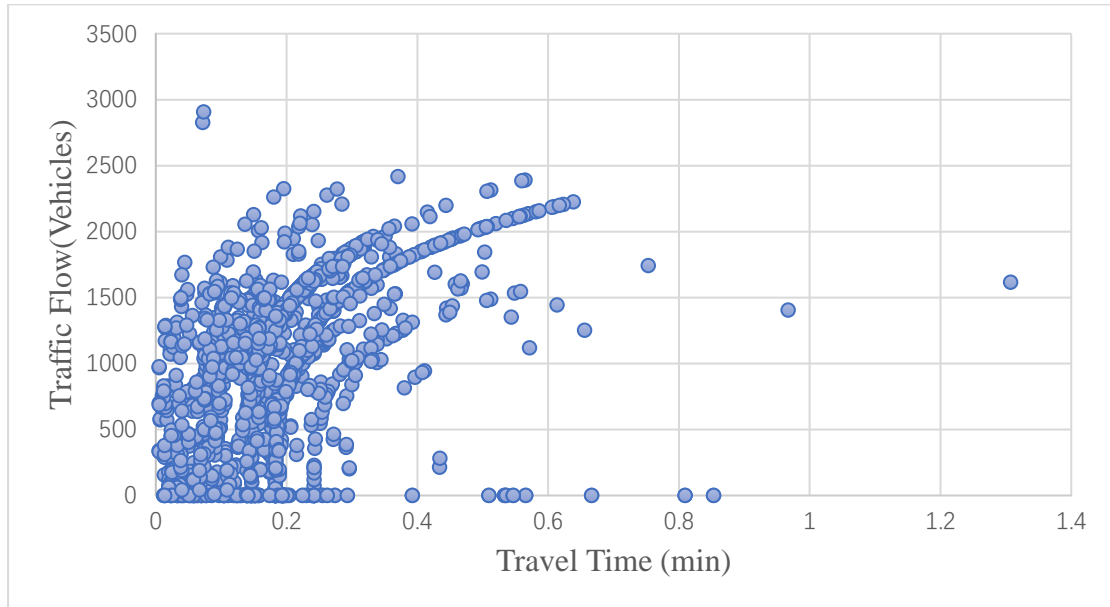


Figure 5.1 Travel Time and Traffic Flow Scatter Plot under User Equilibrium

The research includes the volume over capacity data, showing the variation between the flow for each link and its capacity. The volume over capacity (VOC) could quantify the congestion level of a road segment. It follows the equation:

$$\text{Volume over Capacity} = \text{Capacity} - \text{Traffic flow}$$

When VOC is negative, it means that the road might have traffic congestion. Figure 5.2 plots the trend of VOC under the initial scenario with travel time. The average number of VOCs is 146.272 vehicles per hour. VOC ranges from -1297.07 to 2700 vehicles per hour. Around 54.3% of the links have a negative VOC, and the standard deviation of VOC is 596.73 vehicles per hour. Based on the plot, negative VOC numbers might lead to long travel time on the links. To know the correlation between these two variables, the research does the Pearson correlation test. The coefficient of the correlation test is -0.457, which means there exists a low negative correlation between VOC and travel time under user equilibrium.

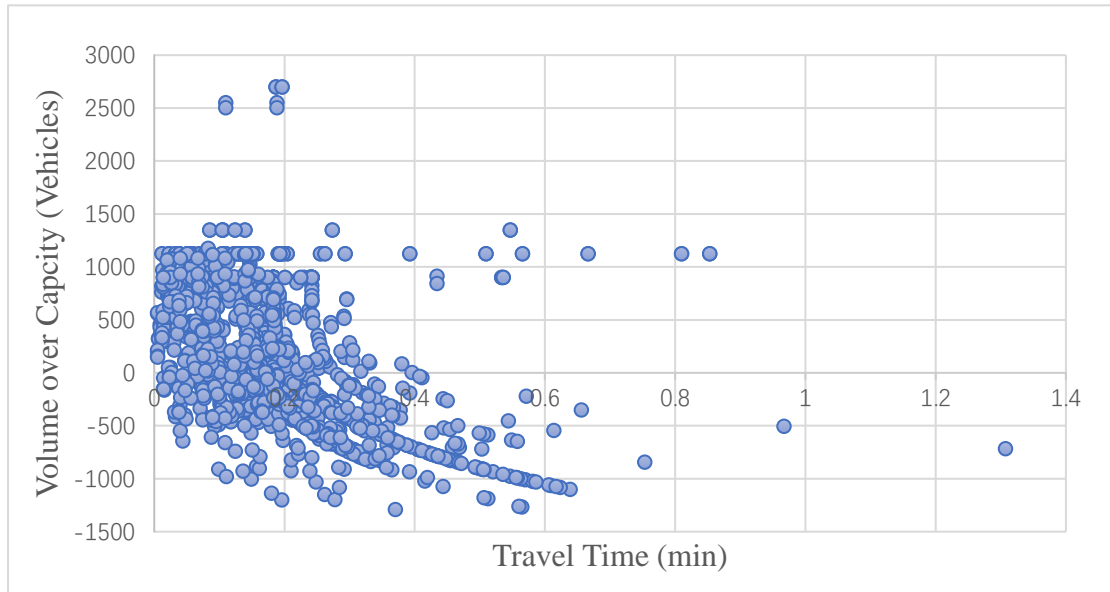


Figure 5.2 Volume over Capacity and Travel Time under User Equilibrium

The average travel time is 0.1884 minutes. Figure 5.3 illustrates the travel time of each segment by spatial visualization. The traffic assignment of each link equals the average travel time of the link in two directions under the user equilibrium. The number classification is Jenks Natural Break. From the map, some of the roads located in the center of the Downtown running in a South to North direction have the higher travel time under user equilibrium, including Elliott Ave 1st Ave, 2nd Ave, 5th Ave, and 6th Ave. The standard deviation for travel time is 0.1163.

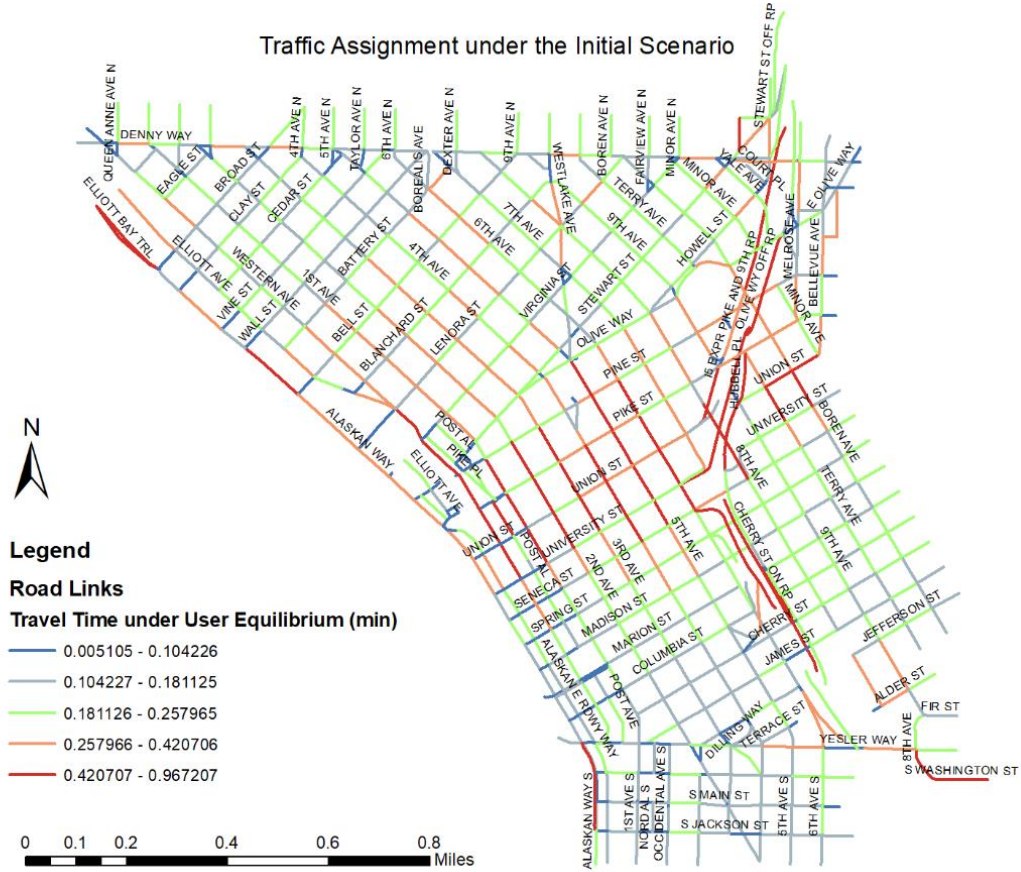


Figure 5.3 Travel Time Assignment under User Equilibrium

5.2 RESULTS OF NUMERICAL ANALYSIS

After comparing 917 different scenarios with the initial situation, ΔT is set as the index to detect the Braess's Paradox. The definition of ΔT follows the equation:

$$\Delta T_i = \sum_{j=1}^n T_{ij} - \sum_{j=1}^n T_0$$

Where $\sum_{j=1}^n T_{ij}$ represents the total travel time cost after the link i is removed.

$\sum_{j=1}^n T_0$ means the total travel time under the initial scenario ($0.1884 * 917 = 172.7628$ minutes). Hence, the index ΔT describes the travel time variation before and after the selected link change. After the testing, the range of ΔT is from -1.811 minutes to

13.196 minutes. The negative outcome shows that Braess's Paradox possibly exists and that the closure of some links, could improve the total traffic performance among the downtown links. The result shows that 34.1% of the travel time variation is lower than zero. 7.2% of the result is lower than minus 1. Figure 5.4 plots all the distribution of the variation index. The horizontal axis is the Identification number of Link, and the vertical axis is ΔT .

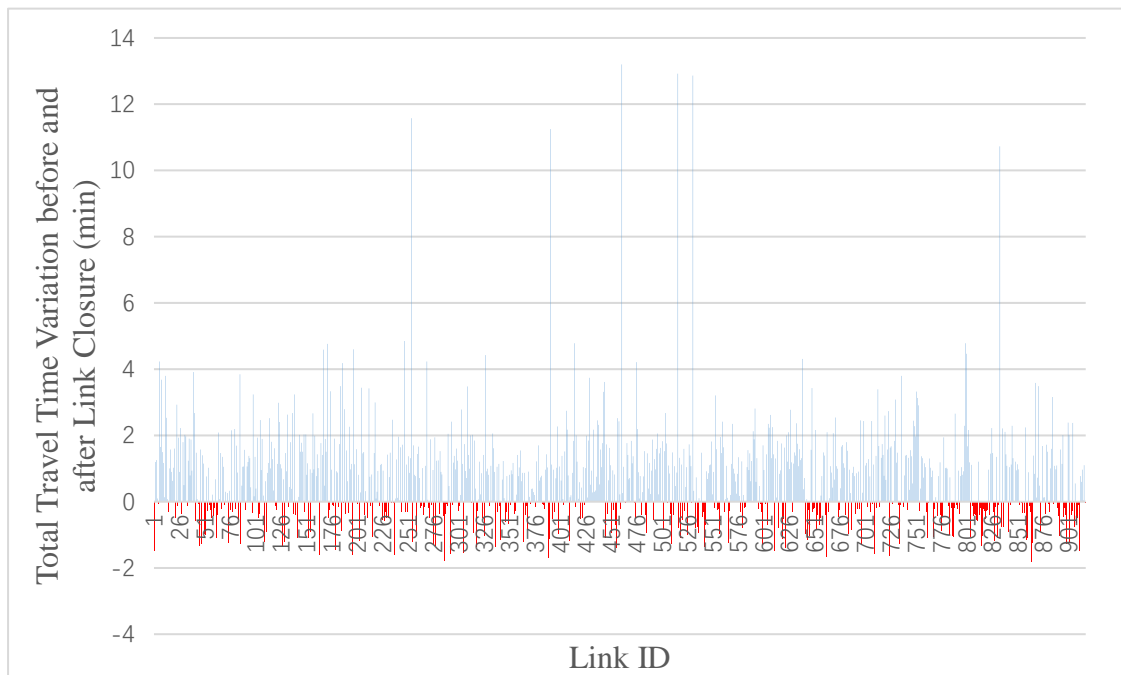


Figure 5.4 Distribution of Total Time Change Before and After Link Closure

Another index that helps measure traffic performance of the network is the traffic flow change. The higher flow might lead to more congestion to the network. The studies measure traffic flow change under 917 possibilities. The tests summarize all the difference of traffic flow under user equilibrium. The variable is defined as ΔF , which follows the equation:

$$\Delta F_i = \bar{F}_i - \bar{F}_0$$

Where \bar{F}_i is the average traffic flow over capacity after the link i is closed under user equilibrium; \bar{F}_0 represents the average traffic flow over capacity in the initial scenario, which is 146.27 vehicles per hour. Hence, ΔF describes the VOC change before and after the link closure of all the links in the network. After the testing, the range of ΔF is from -6.705 vehicles to 41.228 vehicles per hour. The result shows that 13.8% of the traffic flow variation is negative. Figure 5.5 plots all the distribution of the flow change before and after link closure. The horizontal axis is the Identification number of Link, and the vertical axis is ΔF .

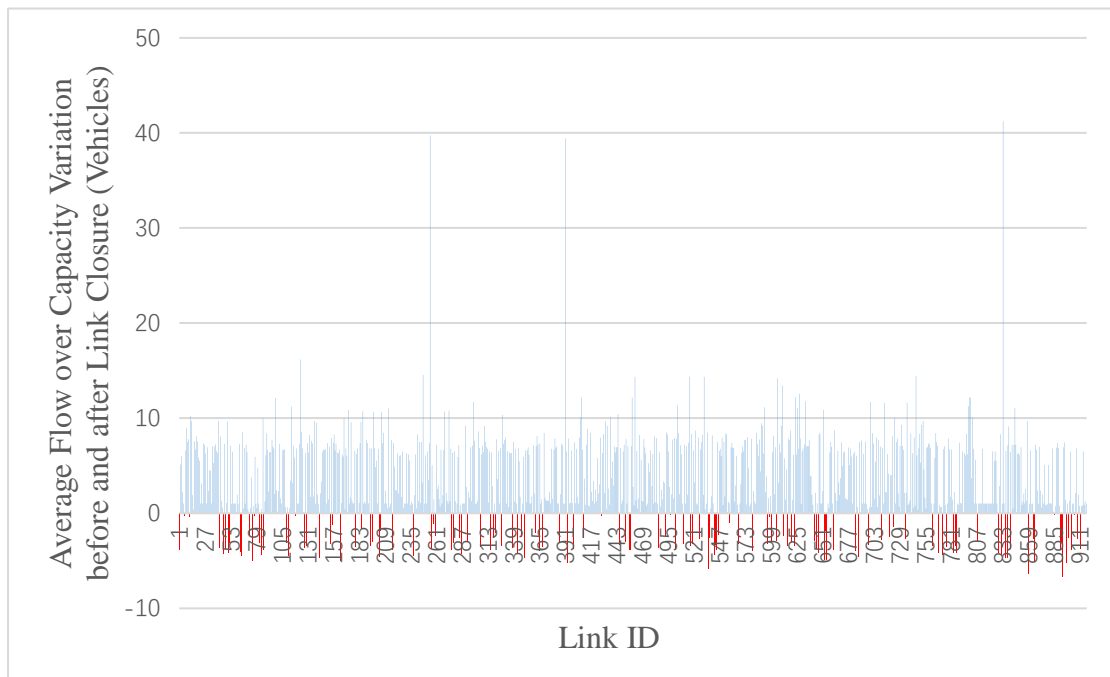


Figure 5.5 Distribution of Average Flow Change Before and After Link Closure

The flow change graph has some similarities with the travel time change graph. Then the study tests the correlation between the variables ΔT and ΔF . Based on the 917 results, linear regression is used to account for the correlation between time and flow. The average travel time change is identified as the dependent variables, and

average flow change is chosen as independent variables. The result shows that the coefficient of determination R^2 of the regression is 0.615, which means 61.5% of the data could be explained by the linear regression. The coefficient of the independent variable is 0.0082, which means each flow will increase by 0.0082 minutes in average travel time. Figure 5.6 illustrates the location of each variable and the regression curve under the linear regression model. The blue dot represents each link's closure performance, and the red line means the regression curve. Some particular points could be found in the figure, which has negative travel time change and negative flow change. It means after removing these links, the time and flow will decrease simultaneously. It indicates that the traffic performance might be better than the first network.

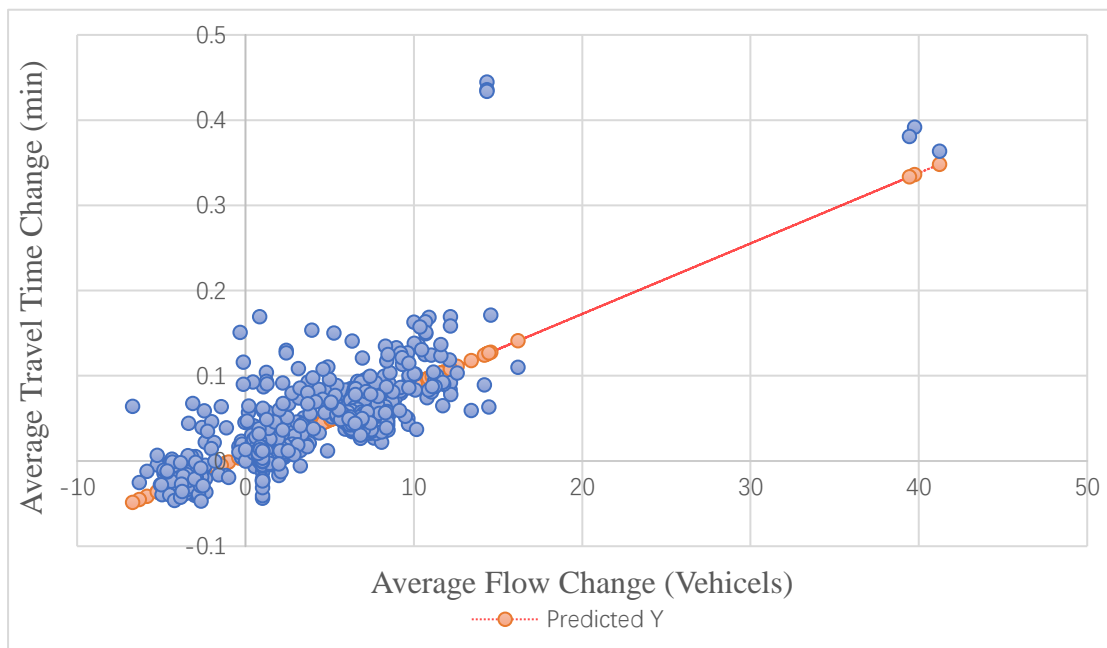


Figure 5.6 Linear Regression for Travel Time Change and Flow Change

Additionally, a Pearson Correlation Test was conducted between the results (time change, flow change) and the link's essential characteristics, which include link length,

capacity, and speed limit. These three variables decide the travel time under user equilibrium. The test helps to determine whether the speed, capacity, or length of the link could influence the entire road network system's performance. Table 5.1 states the outcome of the Pearson Correlation Test. The absolute value of all the tests is lower than 0.3, which means there is no significant correlation for the user equilibrium flow and time change before and after the link closure with the road's basic attributes, including speed limit, length and capacity.

Table 5.1 Pearson Correlation Test

Correlation Test	Link Speed Limit	Link Length	Link Capacity
VOC Change	0.095	0.056	0.095
Time Change	0.120	-0.0001	0.119

To summarize, for the total travel time change, the average of ΔT is 0.780 minutes; It ranges from -1.811 to 13.196 minutes. The standard deviation is 1.509. ΔT has no significant correlation with link basic attributes. For average flow change before and after link closure, the average of ΔF is 3.741 vehicles per hour; The range is from -6.705 to 41.228 vehicles. The standard deviation is 4.683. Same as ΔT , ΔF also has no linear correlation with the links' basic attributes (speed limit, length and capacity). The detailed summary is in Table 5.2.

Table 5.2 Summary of the Result

	Average	Max	Min	Standard Deviation	Correlation with Speed Limit/Length/Capacity
Average VOC Change (Vehicles)	3.741	41.228	-6.705	4.683	No
Total Time Change (min)	0.780	13.196	-1.811	1.509	No

5.3 INTERACTIVE MAPPING

The research model combines the results of the Braess's Paradox estimation with the original networks in ArcGIS from which maps are produced showing the roads or links that need to be optimized. Figure 5.7 illustrates the links classified by the travel time variation before and after link closure (ΔT). Figure 5.7 also categorizes all the links into 3 types. A grey line means the closure of a link will lead to an increase in travel time. A blue line represents the closure of a link that will slightly increase total travel time, and the change may be within the margin of error. The blue lines do not display Braess's Paradox. Many blue paths end along the border of the study area and are contiguous with roads that extend beyond the study area. The model does not consider the effects of external demand on the internal network. My research found that the study routes became busy as traffic exited the study area, the reason for this phenomenon is based on the information I collected I was not able to determine what cause this phenomenon. It could be intrinsic to the methodology. The red line means the links represent the top

5% greatest travel time improvement in this model indicating Braess's Paradox may exist. Surprisingly, the closure of the red link makes the travel time less than before.

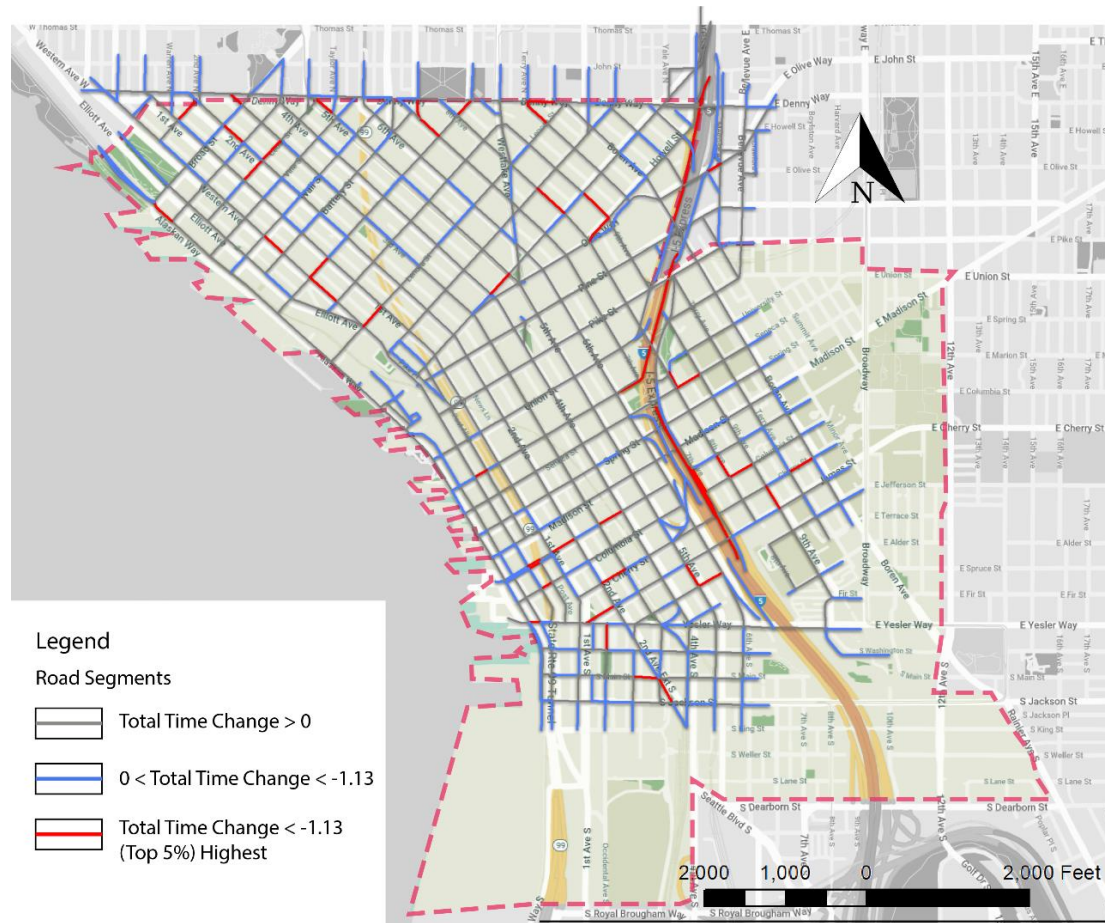


Figure 5.7 Link with Negative Travel Time Variation after Closure

Another indicator that measures traffic performance is the average change of volume over capacity (ΔF). If the change of flow over capacity is positive, it might cause some congestion in several links due to the capacity limit. The map of the study area includes the four possible scenarios based on the two indicators.

Scenario 1: $\Delta T < 0$ and $\Delta F < 0$. It means that the closure of the link could not only decrease the travel time but also lead to fewer overcapacity situations in the network. It might be the link display the Braess's Paradox.

Scenario 2: $\Delta T < 0$ and $\Delta F > 0$. It states that the closure of the link could decrease the travel time. However, it might also lead to network overload.

Scenario 3: $\Delta T > 0$ and $\Delta F > 0$. It states that the closure of the link will make the travel time longer than before and create more congestion in the network. These roads will be identified as key links that could not be removed.

Scenario 4: $\Delta T > 0$ and $\Delta F < 0$. It shows that the closure of the link will make the travel time longer than before, however, congestion situations will be less than before.

Figure 5.8 illustrates the four-scenario classification of the links. Most road segments that connect the I-5 are identified as possible links that display Braess's Paradox. It might not be entirely accurate because the ramp off I-5 has greater traffic demand and flow than shown on the current map. However, removing I-5 Highway results in a lower flow at some entrance or paths into exits to study area.

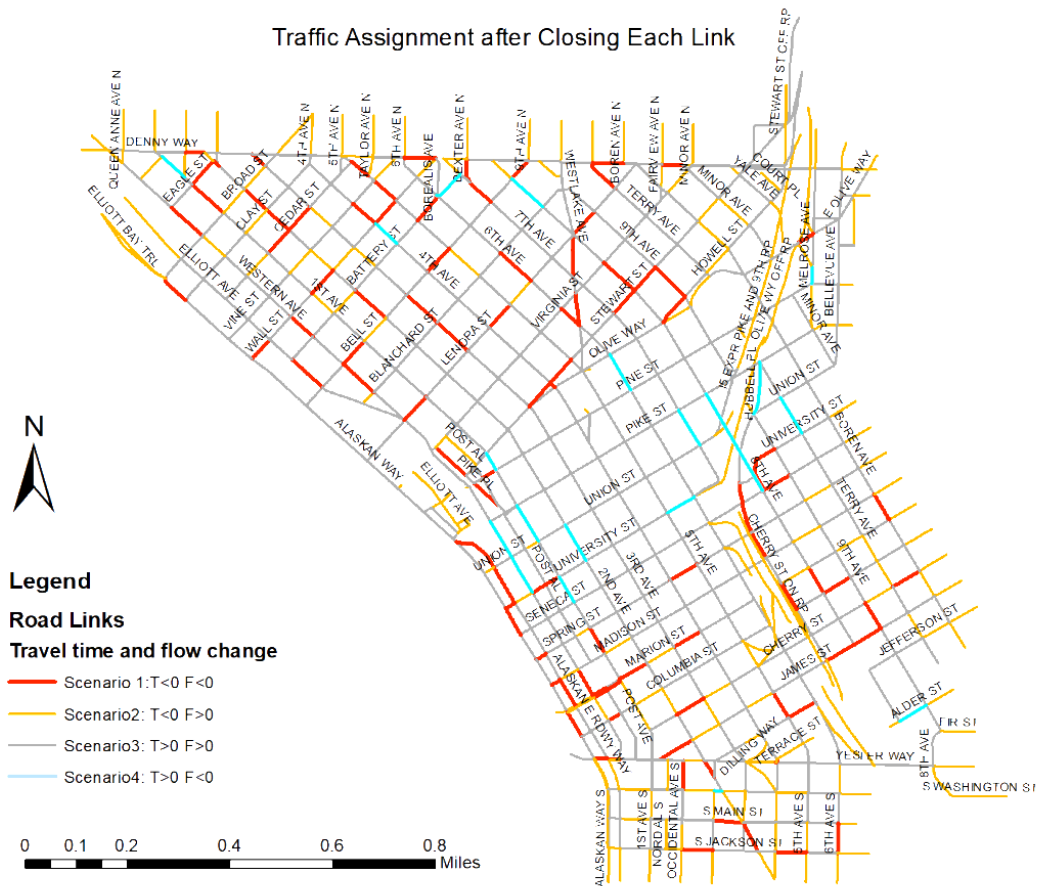


Figure 5.8 Traffic Assignment Classification under Different Scenarios

Some links classified by scenario 1 overlap with the links detected before for the red line, meanings they have the top 5% highest negative travel time change after the closure. The two finding from my research concerning links are:

- 1) if the links within the top 5% highest negative travel time change (ΔT) after the closure; and
- 2) if the links' volume over capacity change (ΔF) is negative after the closure, then Braess's Paradox is present.

Figure 5.9 shows all the location of the links in Seattle Downtown network. In general, 44 of 917 links in the network are identified as problematic link. Some streets have more than one link displaying Braess's Paradox. Examples include Marion Street, Cherry Street, and Western Ave.

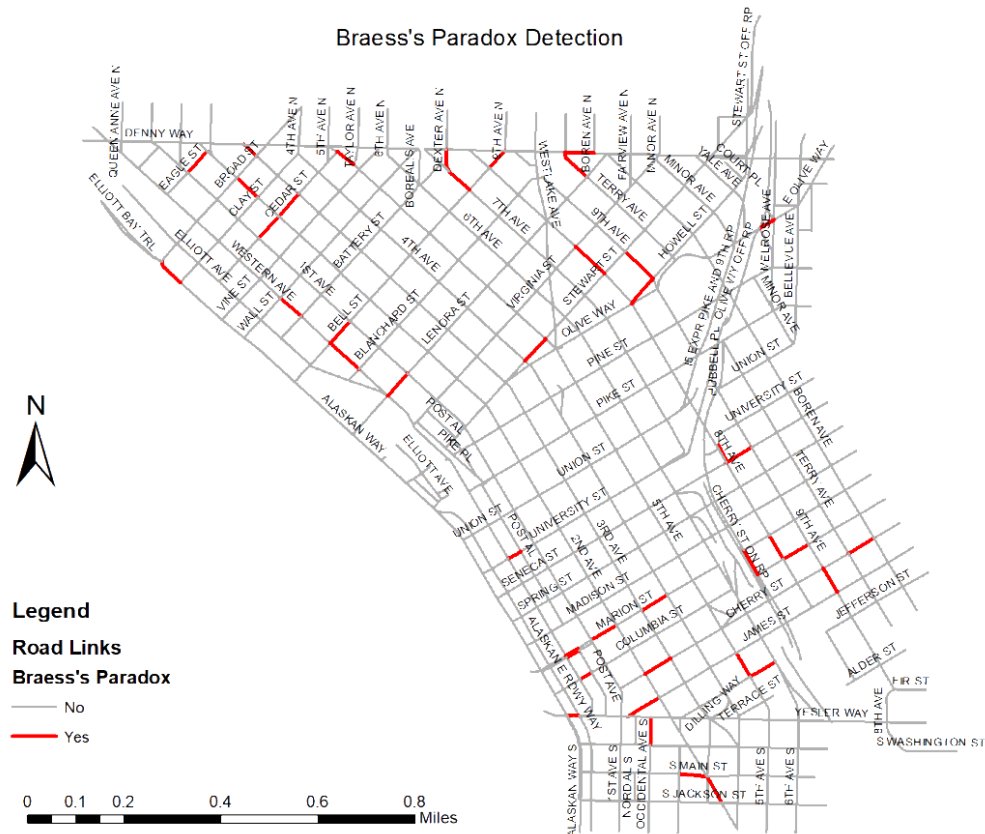


Figure 5.9 Detection of Braess's Paradox in Downtown Seattle

Chapter 6. DISCUSSION AND CONCLUSION

This section discusses the application of Braess's Paradox in the Seattle Master Bike Plan and Bridging the Gap, which is a specific 900 million dollar tax bond program voters approved several years ago to make critical improvement in transportation. The next section provides recommendations on how to connect the walking spaces in the center of the city and offers a discussion of the limitations of the methodology and finally summarizes the conclusions.

6.1 CONNECTING WITH SEATTLE BIKE PROGRAM

The Seattle Department of Transportation aims to create a robust and connected network of bike lanes and greenways to ensure that people of all ages and abilities have the choices to transport around the city. The 2019-2024 Seattle Bicycle Master plan used a quantitative methodology to rank order projects based on five factors: safety; connectivity; equity; ridership and livability. Figure 6.1 contains a map of Seattle's existing bicycle network. and identifies the bike lanes that are not connected.

While improving the connectivity of the current bike network is a focus for transportation planners, building more bike lanes might change current traffic performance. One possibility is, the closure or continued use of some links might lead to more congestion of the city network. My research demonstrates the potential for Braess's Paradox to be one of the tools transportation planners can use to improve the connectivity of bike lanes in Seattle.



Figure 6.1 Seattle Existing Bike Network

Source: 2019-2024 Seattle Bicycle Master Plan

The use of Braess's Paradox provides a practical analytical tool for planners to improve the current bike lane network. The ability to identify roads that are under-utilized or non-essential could create opportunities for establishing new bike lanes. These road closures counterintuitively improve the downtown traffic performance. Figure 6.2 connects the under-utilized roads with existing bike lanes in the study area network. Nine road segments, which were identified through the empirical analysis, that could be used to accommodate bike lanes. The streets are: Eagle Street link between 1st Ave and 2nd Ave; Cedar Street link between 1st Ave and 2nd Ave; Bell Street link between Western Ave and 1st Ave; Lenora Street between Western Ave and 1st Ave; James Street link between Yesler Way and 2nd Ave; 2nd Ave East South link between Jackson Street and South Main Street; 9th Ave link between Stewart Street and Howell Street and the two links in 8th Ave and Columbia Street between Marion Street and 9th Ave. The average length of the links is 342 feet.



Figure 6.2 Braess's Paradox Detection with Existing Bike Lanes

My research does not propose that increasing the connectivity of existing bike lanes can be solved only by using Braess's Paradox. However, Braess's Paradox is an important tool that can be useful for optimizing the city network and for improving pedestrian mobility. The margin of error produced by the Frank Wolfe algorithm running and the demand change by time might influence the outcome. However, the Braess's Paradox method provides a way for transportation planners to rethink some ignored perspectives on urban planning. Reducing car use or redesigning roads to also accommodate bikes can could optimize traffic performance for both users.

6.2 IMPLEMENTATION SUGGESTION FROM URBAN POLICIES PERSPECTIVE

What should governments or planners do when some links are identified as exhibiting Braess's Paradox under current travel demand? Road removal is not always possible for technical and political reasons. The cost of designing and building a new link is also a substantial investment and might also be inefficient. Bar-Gera et al. (2016) suggests that travel time is not the only cost users consider. Travel tolls and road type may also be factors in choosing a route. To avoid or reduce Braess's Paradox, governments may decide to increase the general travel cost by increasing tolls, changing the link type as ways to change users' behaviors that result in decreased traffic congestion.

Another strategy governments could use is temporarily closing a road link at a particular time when the Braess's Paradox is most present on some links. The closing of 42nd Street in New York in 1990 to celebrate Earth Day is an excellent example of how test traffic flow under the same demand but with a different urban network. Temporary closure could give the government and planners more time and room to optimize the road network.

Creating more walkable space by redeveloping the links exhibiting Braess's Paradox is another potential strategy. This idea is based on using the urban design method to influence the entire traffic flow. The existence of Braess's Paradox would be a useful tool for planners to use to find ways to create more walkable space in the busy

downtown area. For example, Seattle could enlarge the Seattle Walking Map, designed to help Seattle visitors choose a friendly walking route based on the traffic conditions.

6.3 LIMITATIONS AND CONCLUSIONS

The study still has some limitations for data chosen and methodology. First, compared with other large-scale networks, the size of the study area is small. Enlarging the study area would allow for more accurate traffic flow and travel time under the user equilibrium. The current test shows the links located along the border of the study area could not be well explained due to the excess demand they create as they exit the study area. However, a large-scale network requires more complex data to run the model. All the links should be connected in an orderly fashion, meaning travel demands must have a route to run from one node to the other node in the network.

Model optimization is another critical factor. The Seattle Downtown Network model takes more than 8 hours to run 917 times using the Frank-Wolfe traffic assignment. The average iteration is around 30 times. If the size of the network is quite large, the algorithm might need to run thousands of iterations to reach the accuracy of one assignment. It needs plenty of time and a powerful CPU to run the model.

Secondly, the model does not consider one-way streets in downtown Seattle. In the current study area, if some links are identified as a one-way street, some users will not find the routes to travel from one node to another node. However, the one-way street will influence the traffic assignment in all probability.

Thirdly, the Frank-Wolfe algorithm is not the only one that could solve the traffic assignment issue. Other methods like column generation (COL), method of successive average (MSA), disaggregated simplicial decomposition (DSD), and mixed stochastic user equilibrium and system optimal are also the standard way to solve the problem. It might be helpful to use a different method for the same network to detect the Braess's Paradox and find out which method is the most efficient one.

For future research, changing multiple links should also be considered to identify Braess's Paradox. The thesis only simulates the outcome of the closing a road for one link per change. However, in urban planning practices, the situation will be more complex. It is helpful to know how the travel time and flow changes when multiple links are changing.

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