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Yang Su

Three Essays on Household Asset Allocation

Yang Su

A dissertation
submitted in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy

University of Washington

2019

Reading Committee:

Thomas Gilbert, Chair

Stephen Turnovsky, Chair

Levis Kochin

Program Authorized to Offer Degree:
Economics

University of Washington

Abstract

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Yang Su

Co-Chairs of the Supervisory Committee:
Associate Professor Thomas Gilbert
Department of Finance & Business Economics

Ford and Louisa Van Voorhis Professor of Economics Stephen Turnovsky
Department of Economics

With high-quality household level asset holding data becoming available as well as the exponential increase in computing power, there is a growing literature that studies how households make investment decisions facing various types of uninsurable background risks. In this dissertation, I build theoretical models and conduct empirical studies to investigate different problems on household asset allocation.

In chapter 1, I build a life-cycle model of portfolio choice with endogenous labor supply and a fixed cost of labor market participation to incorporate both the extensive and intensive margins of labor supply decisions. I show that the risky asset holdings of young agents (agents younger than 45-year-old) are lower when compared to a model that only incorporates the intensive margin of labor supply. The risky asset holdings of young agents are further reduced and become hump-shaped when two additional features are included to the model: 1) endogenous Social Security accumulation and 2) a small possibility of a zero-income state. These two features increase the uncertainties faced by the agents while the fixed cost of labor market participation reduces the agents's ability to use labor supply to buffer against future income uncertainties. My model therefore reduces the gap between the empirical observa-

tions on household risky asset holdings and the predictions made by life-cycle models with endogenous labor supply.

In chapter 2, we build a three-period model to study asset allocation ("how much to invest") and location ("which account to use") consequences when an economic agent has internal habit formation utility and has access to both an illiquid but tax-favored retirement account and a taxable personal account. We show that the incentive to maintain high consumption relative to the habit level and the restriction of only having access to the personal account before retirement induces the agent to hold a safer portfolio in her personal account and a riskier portfolio in her retirement account, in accordance with empirical findings on retirement asset allocation. We also show that retirement asset allocation and location decisions are affected by bequest motives and employer match, providing policy implications for retirement plan designers.

In chapter 3, I provide updated estimations of the age profiles of stock market participation and risky share in the United States using data from the Panel Study of Income Dynamics (PSID). This chapter is motivated by the recent findings of Fagereng, Gottlieb, and Guiso (2017) on Norwegian data that the age profiles of stock market participation rate and risky share become closer to theoretical predictions when they employ more precise empirical strategies to identify the age, cohort and year effects, control for demographic variables and use a Heckman selection model to control for the endogeneity of stock market participation decision. I apply the same empirical strategies in Fagereng et al. (2017) on the U.S. data. I find that the age profile of stock market participation rate is increasing over the life cycle instead of hump-shaped. The estimated conditional risky share, after controlling for selection, is higher than the risky shares reported in previous papers and it is slightly increasing over the life cycle.

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ACKNOWLEDGMENTS

I am indebted to Professor Thomas Gilbert, for his patience, guidance and encouragements, to Professor Levis Kochin, for all the interesting discussions and all his (mental and lunch) supports. I am also very grateful for Professor Stephen Turnovsky for being the co-chair of my committee and giving me valuable advice.

It has been a long, sometimes painful, journey for me to get this PhD done. I am thankful for all the friends who were with me, gave me words of encouragements, listened to my complaints and had faith in me throughout this process. Dr. Dorothy Paun has also given me tons of encouragements and supports, and I am very grateful for her. Special thanks to Jiabin Wang and Juncheng Ke for their technical help on Chapter 1 of this dissertation.

Last but not the least, I am infinitely indebted to my parents, for their mental and financial supports, as well as their love and belief on me.

DEDICATION

To my mom and dad, Miaoyan Zhang and Xikun Su.

Chapter 1

LABOR MARKET PARTICIPATION AND LIFE-CYCLE ASSET ALLOCATION

1.1 Introduction

In this chapter, I build a life-cycle model of portfolio choice which incorporates both the extensive and intensive margins of labor supply and allows for endogenous Social Security accumulations. I show that these features lead to lower risky asset holdings than the prediction of the model in Gomes, Kotlikoff, and Viceira (2008), which does not incorporate the extensive margin of labor supply or endogenous Social Security accumulation.

One puzzle that the literature on life-cycle portfolio choice strives to solve is the discrepancy between the low and hump-shaped percentage of risky assets in wealth (henceforth *risky share*) observed in data (Ameriks and Zeldes (2004), Gomes and Michaelides (2005)), and the age profile of risky share generated by a standard calibrated life-cycle model of portfolio choice that features exogenous and uninsurable risky labor income such as the one in Cocco, Gomes, and Maenhout (2005) (henceforth CGM). CGM show that risky share decreases with age as young agents have a larger stock of human capital, an implicitly safe asset given the low correlation between income shocks and stock returns. When agents are able to adjust their labor supply as in Gomes et al. (2008), the discrepancies between model and data become even larger because agents are now able to use labor supply to buffer against shocks to risky returns, therefore they hold riskier portfolio at all ages.

However, an unrealistic assumption made in Gomes et al. (2008) is that agents are always in the labor market. Various labor economics papers document that a substantial fraction of individuals at any given date are out of the labor market. For example, Blundell, Bozio, and Laroque (2013) and Erosa, Fuster, and Kambourov (2016). Labor market participation has a strong age pattern, although the pattern depends on whether or not we control for year or cohort effects. According to French (2005), after controlling for cohort effects, labor market participation stays closed to 100% when agents are younger than 55 year-old and starts to decline sharply. When an economic agent leaves the labor market, she receives no income and faces potential destruction on her stock of human capital. With a possibility to leave the labor market at any time, an agent may choose to reduce her investment in risky assets. I therefore build a life-cycle model of portfolio choice with endogenous labor supply and a fixed cost of labor market participation to incorporate both the extensive and intensive margins of labor supply decisions.

My hypothesis is that, when we allow agents to leave the labor market, those who leave the labor market should reduce their risky share. Therefore, my model should predict lower risky share than the model with no fixed cost of labor market participation. I start with a benchmark model which simply adds a fixed utility cost of labor market participation to the life-cycle portfolio choice model of Gomes et al. (2008). The benchmark model features a small utility cost calibrated to match the observed labor market participation rate and labor supply hours, flexible labor supply, uninsurable labor income and deterministic retirement income, and a risky asset and a riskless asset. In my benchmark model, agents work less hours and start to leave the labor market at age 39, but only agents between ages 30 and 44 reduce their risky share. An unrealistically larger utility cost helps to generate much lower risky share, consistent with my conjecture that labor market participation costs reduces risky asset holdings, but it deviates too much from the observed labor supply patterns.

I then expand the benchmark model by allowing retirement income, more specifically, the Social Security income to be endogenous. Following the U.S. Social Security scheme, I extend the benchmark model by letting Social Security to be determined by the highest 35 years of income. Since agents are allowed to change work efforts and even leave the labor market in my model, allowing Social Securities to be determined endogenously better captures the risks that agents need to face: when the agent leaves the labor market, she faces the trade off of higher utility today and lower current as well as future retirement income. When the agent is subject to an income shock today, her retirement income is also affected. Endogenous Social Security accumulation further reduces the risky asset holdings for agents younger than 45, indicating that incorporating the extensive margin of labor supply does help to reduce the discrepancies between model and data when its effects are fully captured.

Next, I include an additional source of background risk that is not considered in Gomes et al. (2008): a small possibility that income temporally drops to zero, which resembles a possibility of become unemployed. When the agent face a possibility of losing her job, she would take less risk even though she could freely adjust her labor supply. With endogenous Social Security accumulation and the possibility of a zero-income state, the simulated conditional risky share becomes hump-shaped and lower than 60% before retirement, more in line with the empirical observation of risky share.

To my knowledge, only a few papers in the life-cycle model of portfolio choice allow for endogenous labor supply, and none of them incorporate the extensive margin of labor supply. Bodie, Merton, and Samuelson (1992) allow flexible labor supply and show that optimal risky asset holdings should be higher than the case when the agent cannot freely adjust labor supply, but they unrealistically assume that labor income could be fully insured. Gomes et al. (2008), as mentioned before, build a life-cycle model of portfolio choice with flexible labor supply as well as uninsurable labor income risk, and provide predictions in line with Bodie et al. (1992). Chai, Horneff, Maurer, and Mitchell (2011) allow endogenous labor supply hours, flexible retirement age as well as the purchase of annuities. Their predicted stock holdings are even larger than Gomes et al. (2008), as the agents in their model could buffer against risks in stock returns by delaying retirement as well as purchasing annuity. Polkovnichenko

(2007) considers endogenous labor supply hours in his model, but the focus of his paper are implications of habit formation utility on portfolio choice.

My contribution is to show that allowing endogenous labor supply does not necessarily imply higher risky asset holdings than the standard models if we take into account the extensive margin as well as the uncertainties that the agents face even when they can adjust their labor supply. My paper belongs to the vast literature that studies household's life-cycle asset allocation decisions. I provide a brief literature review on household finance in Appendix 3.6.

My model also builds on the vast literature that studies the implications of the extensive margin of labor supply. French (2005) studies how health status and Social Security affects the labor supply patterns, but abstracts from portfolio choices. Erosa et al. (2016) build a life-cycle model of labor supply with heterogeneous agents to study aggregate labor supply elasticities, but they also ignore portfolio choice. Attanasio, Low, and Sanchez-Marcos (2008) study how women's labor supply decisions are affected by child bearing costs and educations. Wu and Krueger (2018) build a two-earner life-cycle model and show that endogenous labor supply as well as incorporating the extensive margin of female labor supply helps to explains the consumption smoothness observed in the data[1]. My model provides implications of incorporating the extensive margin of labor supply on risky assets investments, which are not considered in the aforementioned papers.

1.2 Empirical Evidence

This section summarizes the current empirical evidence on the age profile of stock market participation, risky shares, labor market participation as well as labor supply hours. There is a well-known identification challenge when constructing the age profiles of investment and labor supply decisions. That is, the linear relationship between age, year and cohort (repre-

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[1]See Blundell, French, and Tetlow (2016) for a comprehensive summary of how the labor economics literature model labor supply decisions.

sented by birth year): Year = Birth Year + Age[2]. Year could affect risky asset investment because transaction costs may have reduced over the years (Gomes and Michaelides (2005)). As for cohort effects, generation specific experience of the stock market could affect the agent's risk preference. For example, Malmendier and Nagel (2011) show that cohorts that have experienced low stock returns when young are less likely to participate in the stock market and, conditional on participation, invest less of their financial wealth in risky assets.

Figure 1.1 depicts the conditional risky share using 1999 to 2017 waves of Panel Study of Income Dynamics (PSID) data reported by Su (2019). We can see that the age profile of conditional risky share is slightly upward sloping over the life-cycle both when cohort effects are controlled and when year effects are controlled. Stock market participation rate is increasing over the life–cycle, with the rate of increase becoming smaller after age 60. Since my model does not account for the extensive margin of risky asset investing, my analysis focuses on the risky share.

Year and cohort could also have different effects on labor supply decisions. Blundell et al. (2013) show that the labor market participation rate in the United States has increased from 1968 to 2008, while the mean labor supply hours have been reduced. Also, Blundell et al. (2013) report that the labor market participation rate for younger cohorts of women have increased. Figure 1.2 provides the age profile of labor market participation rate as well as labor supply hours, conditional on health status, reported by French (2005). French (2005) uses PSID data for the years 1968-1997. We can see that the labor market participation rate stays closed to 100% and is above 90% for for healthy agents who are younger than 58 and then quickly declines to about 30% when agents are 66 and more. The labor market participation rate starts to decline steadily for unhealthy agents at round 40.

[2] Ameriks and Zeldes (2004) contains an extensive discussion of this empirical challenge.

Using the 1999 - 2017 waves of the PSID, I also generate the age profiles of labor market participation for household heads between ages 20 to 65, as shown in Figure 1.3. I use year and cohort dummies to control for year and cohort effects and include family size as a control variable, following French (2005). Using the latest data, I find that the age profiles of labor market participation are at lower levels than French (2005). When using year dummies (which assumes that cohort effects are zero), labor market participation rate stays flat at about 80-85% and then starts to decline after age 55. When using cohort dummies (which assumes that year effects are zero), labor market participation rate is overall much lower - around 65% before the age of 55 and then starts to decline.

The reason that the age profile of labor market participation rate in this paper differs from French (2005)'s could be the follows. First, French (2005) includes only men in his calculation, while the head of households in PSID could be a woman (25% of the heads are women in the 1999 to 2017 PSID data). Overall, women has lower labor market participation rate. Second, French (2005) controls for health status, while my estimation does not take health status into account. Lastly, French (2005) uses individual fixed effects to control for cohort effects but does not explicitly control for year effects. I use dummy variables to control for cohort and year effects.

As for the average work hours, according to French (2005), labor supply hours hours per year cluster in between 2000 hours and 0. The age profile in Figure 1.2 shows that labor supply hours slowly decrease from about 2400 hours per year to about 1200 hours for healthy agent in between ages 30 to 70. Unhealthy agents have more fluctuations in labor supply hours over the life-cycle and on average work for less hours than healthy agents.

1.3 The Model

1.3.1 Preferences

Time is discrete and finite: $t = 1, 2, ..., T$. I assume that the agent starts her life at age 21 and dies at age 100, so $t = 1$ in my model corresponds to age 21 and $T = 80$ corresponds to age 100. Following Gomes et al. (2008) as well as other life-cycle models, I assume that the agent works until the age 65 (corresponding to $t = K = 45$ in my model), and retires afterwards. Modeling flexible retirement age would significantly complicate this model, hence the retirement age is exogeneous. Chai et al. (2011) allow the retirement age to be between ages 62 and 70 and their simulation shows all individuals retire by the age of 66.

In each period, the agent receives utility from consumptionand leisure, and incurs disutility from working as shown by equation (1.1):

$$U(C_t, L_t) = \frac{(C_t L_t^{\alpha})^{1-\rho}}{1-\rho} - \mathbb{1}(1 - L_t > 0) \cdot fcost \cdot (P_t^{1-\rho}). \tag{1.1}$$

C_t denotes period t consumption and L_t denotes period t leisure. Total leisure is normalized to be 1 and the agent enjoys full leisure when she retires ($L_t = 1 \forall t > K$). $\rho > 1$ is the coefficient of risk aversion. $\mathbb{1}(1 - L_t > 0)$ is an indicator function that equal to 1 if leisure is less than 1, and equal to zero otherwise. $fcost$ is the disutility when the agent works, and P_t is the permanent income in period t (defined later in this section). To facilitate comparisons, I follow Gomes et al. (2008) and use a standard CRRA utility function on the composite of consumption and leisure. The composite of consumption and leisure has a special form of a Cobb-Douglas function, in which $\alpha > 0$ denotes the relative importance of leisure.

Two properties of this utility function are worth mentioning. First, it is an old stylized fact of the United States, recently reconfirmed by Ramey and Francis (2009), that despite wages having risen since World War II, the labor supply hours have stayed relatively constant. A

Cobb-Douglas function generates such a balanced growth trend because it has an elasticity of substitution equal to one (King, Plosser, and Rebelo (1988)). Secondly, since the risk aversion coefficient ρ of the CRRA utility function is assumed to be greater than 1, leisure and consumption are assumed to be substitutes[3].

To account for labor market participation decision, I assume that the agent faces a fixed utility cost $fcost$ scaled by $P_t^{1-\rho}$ whenever she chooses to work, represented by the indicator function $\mathbb{1}(1 - L_t > 0)$ in equation 1.1. P_t is the permanent income of the period that the agent chooses to work. Assuming the utility cost of labor market participation is a portion of the permanent income helps with normalization of the problem when solving the model.

Discussion of the fixed cost in labor market participation

Conditional on working ($I(1 - L_t > 0)$), the agent incurs disutility both from the reduction in leisure in the CRRA utility and from the fixed utility cost. This makes the average utility cost per hour to first decrease then slightly increase with hours of work, because of the negative second order derivative of the CRRA utility. This property generates a threshold for labor market participation - the agent would not work unless she finds a job with some minimum number of working hours as shown in Cogan (1980). As a result, this fixed utility cost affects the labor supply decision both on the intensive and extensive margin: other things equal, the agent would either not work or work for long hours.

This simple specification of fixed utility cost to account for the labor market participation is commonly used in the labor economics literature for the study of female labor market participation. For example, Attanasio et al. (2008) and Wu and Krueger (2018). It is worth mentioning that several labor economics papers use richer model specifications than just

[3]Low (2005) provides detail discussions on how leisure and consumption being substitutes affect precautionary savings, thought he does not study asset allocation problem nor the extensive margin of labor supply.

a utility cost to capture the complexity of labor supply decision. For example, Attanasio et al. (2008) also include child care costs in their budget constraints as part of the labor market participation cost specification. Also, both French (2005) and Erosa et al. (2016) introduce non-linear labor earnings, that is, hourly wage is higher for full-time workers than part-time workers. Erosa et al. (2016) shows that non-linear earnings amplify the effects of the fixed utility cost and generate higher elasticities of labor supply on the extensive margin. I abstract from child care costs in my model and leave it for future studies. In subsection 1.6.1, I consider an extension of my model which incorporates non-linear earnings.

1.3.2 Wealth accumulations

Before retirement, the agent receives stochastic earnings represented by

$$Y_t = (1-\tau^l) \cdot (1-h(t)) \cdot \exp(f(t)) \cdot P_t \cdot \Theta_t \cdot (1-L_t), \forall t \leq K, \tag{1.2}$$

where τ^l is the tax rate on labor income [4], $h(t)$ is the per-period housing expenditures, and $exp(f(t))$ is the age-specific deterministic income. Note that housing expenditure is assumed to be exogenous here for simplicity. Several papers studies the impact of housing decisions on risky asset investment. For example, Yao and Zhang (2005) and Cocco (2005). P_t is the permanent income and has the following law of motion:

$$logP_t = logP_{t-1} + log(\kappa_t), \tag{1.3}$$

where κ_t is the permanent income shock and I assume it has log-normal distribution: $log(\kappa_t) \sim N(0, \sigma_\kappa)$. Θ_t is the transitory income shock and IT also has log-normal distribution: $\theta_t = log\Theta_t \sim N(0, \sigma_\theta)$. $1 - L_t$ represents labor supply hours. Following Gomes et al. (2008), I

[4]For simplicity, I do not separately assume a Social Security income tax, although I assume in the model that the retirement income is in the form of Social Security. This is a partial equilibrium model, therefore I do not keep track of the funding status of the overall Social Security.

assume that the agent must enjoy leisure for 1/3 of her time [5]. Therefore, we have

$$L_t \in [\underline{L}, 1], \underline{L} = 1/3. \tag{1.4}$$

When retired, the agent receives a simplified specification of Social Security. The log of retirement income equals a fixed portion λ of her average life-time deterministic income had she worked full time. I assume that retirement income is scaled by the permanent income of the year right before retirement (age = 65) because it is easier for normalization. The agent pays a lower tax rate denoted by τ^R and still has housing expenditure when retired.[6]:

$$Y_t = (1-\tau^R)\cdot(1-h(t))\cdot exp(\lambda\cdot\frac{\sum_{t=1}^{K} f(t)}{K}(1-\underline{L}))\cdot P_K, \forall t > K. \tag{1.5}$$

The agent can invest in one riskless asset with constant gross returns R_f and one risky asset with uncertain gross returns $\tilde{R}$[7]. I assume that the risky asset's return is log-normal $\tilde{R} \sim N(\mu, \sigma_{\tilde{R}})$. Let π_t denote the fraction of savings invested in risky assets. We have the per-period portfolio returns:

$$\tilde{R}_{p,t+1} = 1 + (1-\tau^c)\cdot(\pi_t\tilde{R}_{t+1} + (1-\pi_t)R_f - 1), \tag{1.6}$$

where τ^c is the tax rate on capital income. I assume that the agent is subject to short-sale and borrowing constraints, so

$$\pi_t \in [0, 1]. \tag{1.7}$$

[5]Polkovnichenko (2007) sets the lower bound of leisure to be 0.4, and argues that as long as the optimal leisure choices lie inside the interval of leisure, it does not matter what the lower bound of leisure is. In Gomes et al. (2008), they assume the agent is endowed with 100 hours per week, so the maximum hours for work are 67 hours per week.

[6]The published version of Gomes et al. (2008) does not provide a formula for the retirement income, but the working paper version has (equation (7) of the working paper version). My specification of retirement income is slightly different from the equation (7) of the working paper version of Gomes et al. (2008) by scaling the retirement income by the permanent income at age 65 while equation (7) in Gomes et al. (2008) includes the average of all working period permanent income.

[7]Chai et al. (2011) introduce a third kind of asset into their life-cycle model - annuities.

In each period, the agent observes her beginning-of-period wealth as well as her permanent and transitory income shocks, then she decides how many hours to work, her consumption, as well as the fraction of her savings invested in risky assets. Let W_{t+1} be the beginning-of-period wealth in period $t+1$. We have the law of motion of wealth to be the follows:

$$W_{t+1} = \tilde{R}_{p,t+1}(W_t + Y_t - C_t) \tag{1.8}$$

1.3.3 Recursive Form, Normalization and Solution Method

The agent chooses the optimal series of consumption, leisure and risky share ($\{C_t\}, \{L_t\}, \{\pi_t\}, t = 1, ..., 80$) in order to maximize her expected discounted life-time utility:

$$\max_{\{C_t\},\{L_t\},\{\pi_t\}} E_t[\sum_{n=0}^{T-n} \beta^t (\prod_{n=0}^{t} p_n) U(C_{t+n}, L_{t+n})], \tag{1.9}$$

subject to constraints and law of motions defined in equations (1.2) to (1.8). β_t denotes the discount rate at time t and p_t denotes the conditional survival probability at time t.

This problem can be restated recursively using the Bellman equation:

$$V_t(W_t, P_t, \Theta_t) = \max_{C_t, L_t, \pi_t} \left[U(C_t, L_t) + \delta^t E_t V_{t+1}(W_{t+1}, P_{t+1}, \Theta_{t+1})\right], \tag{1.10}$$

where δ^t denotes the discount factor adjusted by the survival probability. In each period, there are four state variables: age (t), beginning-of-period wealth (W_t), permanent income shock (P_t), transitory income shock (Θ_t). The control variables are consumption(C_t), leisure(L_t), and risky share(π_t). In this setup, we can normalize the problem using permanent income and hence reduce the number of state variables by one. Using lower case letters to denote the variables normalized by the permanent income, we can rewrite the Bellman equation and the wealth accumulation equation as the follows [8]:

[8]Note that for periods greater than K = 65, we normalize the problem and the variables by using P_K,

$$v_t(w_t, \Theta_t) = \max_{c_t, L_t, \pi_t} U(c_t, L_t) + \delta^t E_t(\frac{P_{t+1}}{P_t}^{1-\rho}) v_{t+1}(w_{t+1}, \Theta_{t+1}), \tag{1.11}$$

$$w_{t+1} = \frac{\tilde{R}_{p,t+1}}{\kappa_{t+1}} \cdot (w_t + y_t - c_t), \tag{1.12}$$

where

$$y_t = \begin{cases} (1-\tau^l) \cdot (1-h(t)) \cdot \exp(f(t)) \cdot \Theta_t \cdot (1-L_t), \forall t \leq K \\ (1-\tau^R) \cdot (1-h(t)) \cdot exp(\lambda \cdot \frac{\sum_{t=1}^{K} f(t)}{K} \cdot (1-\underline{L})), \forall t > K. \end{cases} \tag{1.13}$$

For periods $t > K$, leisure always equals one, so we have one less control variable to solve for when the agent is retired. In addition, since labor income is deterministic when the agent is retired, the transitory income shock can be dropped from the state variables in the retirement periods. The only source of uncertainty that the agent face during retirement is the uncertainty in stock returns. For periods $t > K$, this problem is therefore equivalent to the standard life-cycle model in CGM.

This problem can be solved using numerical methods. The continuous state variables as well as income and stock return shocks are first discretized. Then, I solve the model from the last period and go backward to $t = 1$ on each state-space grid point. The solution for the last period is trivial: the agent consumes all of her beginning of period wealth w_T and her last-period-income y_T. The last period value function is therefore equal to the indirect utility function:

$$v_T(w_T) = U(w_T + y_T). \tag{1.14}$$

Moving backward to period $T-1$, given that we have the value function of period T, the agent can then choose the optimal period $T-1$ consumption and risky share to maximize the sum of $T-1$ utility and discounted expected period T value function. The period T value function is retrieved using the law of motion of wealth: for a given combination of the

which is the permanent income at age 65.

control variables in period $T-1$, we can obtain period T wealth and hence the expected period T value function based on the current state (wealth, in our case) as well as the distribution of shocks. For period T wealth values that are not on the pre-specified wealth grid, I interpolate the period T value function to obtain corresponding value. This process can then be iterated backward until we reach $t = 1$.

For periods $t > K$, the agent only needs to choose consumption and risky share and is only subject to shocks in stock returns. For periods $t \leq K$, the agent needs to choose all three control variables and is subject to shocks in stock returns as well as the permanent and transitory income shocks.

I discretize the shocks in stock returns, permanent income and transitory income using Guassian Quadrature method with three node points each[9]. I also discretize the wealth space into $n = 40$ grid points with equal distance[10]. Therefore, this model is solved over a state space with 40 wealth grid points times 3 transitory income shock points for each working period. The optimal control variables are obtained using standard grid search. Specifically, for working periods, I discretize the admissible consumptions, leisure and risky shares into 1500, 80 and 100 grid points respectively, which gives me $1500 \times 80 \times 100 = 12,000,000$ grid points. Then on each grid point, I obtain the current-period value function corresponding to each of these solutions and choose the combination that gives me to largest value function.

I use the shape-preserving Piecewise Cubic Hermite Interpolating Polynomial (PCHIP) to interpolate and extrapolate the value function when the control combination gives me a next-period-wealth value that does not lie on the wealth grid. Many other papers use cubic spline interpolation, but since cubic spline is not shape-preserving, it does not perform well in my setup. For retirement periods, I use the same method. Since there is only one state

[9] Increasing Guassian Quadrature node points to five does not qualitatively change the results.

[10] I have experimented with denser wealth grids and the results are qualitatively the same.

variable, two control variable and one shock, the speed to solve for the retirement periods solutions is much faster than the speed to solve for the working periods solutions.

Overall, it takes about 8 hours to solve for the whole model on my double core computer. I use several techniques to increase the computation speed. First, I restrict the range of search for the control variables at time t to be dependent on the solution at time $t+1$. Secondly, I utilize the parallel computing toolbox of MATLAB and used parallel computation whenever it is feasible. Horneff, Maurer, and Schober (2016) present a detailed discussion of how to use parallel computation techniques to solve dynamic portfolio choice models in discrete time.

1.4 Calibration

Table 1.1 provides the values and sources of all parameters used in my benchmark model described in section 1.3. I follow the calibration of Gomes et al. (2008) closely in order to compare results. Coefficient of risk aversion (ρ) is set at 5 and discount rate (β) is set at 0.97. Both are standard values used in the literature. The standard deviations of permanent and transitory income shocks are set at 10.95% and 13.89%, respectively, which are estimates provided by CGM. Following Gomes et al. (2008), the tax rates on labor income, retirement income and capital gains are set at 30%, 15% and 20%, respectively. I use the same deterministic wage profile used in Gomes et al. (2008), which is taken from Fehr, Jokisch, and Kotlikoff (2007)[11]. I use the same housing expenditure function as in Gomes et al. (2008), which is estimated by Gomes and Michaelides (2005). The replacement rate of the Social Security income is set to 68.212%, using the estimate from CGM.

The equity premium is assumed to be modest compared to historical values - 4%, which is

[11] Specifically, I use equation (9) of Fehr et al. (2007). The deterministic wage profile is $E(a,k) = \xi(k)e^{4.47+0.33(a-20)-0.00067(a-20)^2}(1+\lambda)^{a-21}$. a refers to age. k indicates class and $k=2$ is chosen to refer to the middle-class, making $\xi(k)=1$. λ represents to the age-specific longitudinal earnings ability and is assumed to be 0. This income profile is actually weekly earnings of white males who are high school graduates but no college in 1967-75, estimated by Welch (1979).

a common value used in the literature of life-cycle models of portfolio choice [12]. A modest equity premium value helps to reflect the transaction costs of risky asset trading and helps to generate more realistic portfolio choices. The risk free rate is set at 1%. The housing expenditure profile $\{h_t\}_{t=1}^{T}$ is taken from Gomes and Michaelides (2005). The survival probabilities are taken from CGM.

Gomes et al. (2008) choose the leisure preference parameter (α) to be 0.9 and argue that this generates realistic labor supply hours, close to 2080 per year. On the other hand, Chai et al. (2011) use a higher value, 1.3, in their calibration and show that using 0.9 would generate unrealistically high work hours of 50 to 60 hours per week. Since I introduce fixed cost of labor supply in my model, the resulting labor supply hours differ from Gomes et al. (2008) and Chai et al. (2011) even when I use the same α: agents now either not work or work for longer hours. Moreover, holding the fixed cost of labor supply constant, the higher the leisure preference parameter, the less likely the agent would leave the labor market, since one additional unit of leisure now provides higher utility. As a result, when the leisure parameter increases, other things equal, the agent is more likely to stay in the labor market but would work for less hours.

I calibrate the leisure preference parameter α and the fixed cost of labor supply to match with two sets of statistics: 1) the mean labor force participation rate of household heads with ages 20-65 in the 1999 - 2017 waves of the PSID, which is 77.86%; and 2) the mean weekly hours of male in between ages 35 to 50 with a college degree, as estimated by Erosa et al. (2016)[13]. To match the low labor supply hours while matching the extensive margin, I use a higher leisure supply parameter, 1.45, and a moderate utility cost of labor supply, 8e-6.

[12]To name a few:Gomes et al. (2008), Cocco et al. (2005), Gomes et al. (2008), Fagereng et al. (2017).

[13]Admittedly, these two sets of statistics overs two different groups of the population. Future work should be devoted on getting the work hours from the PSID and re-calibrate the model.

1.5 *Results*

1.5.1 Policy Functions

Before presenting the simulation results, I plot policy functions of the agent to first show how the labor market participation cost affects the agent's consumption, labor supply, as well as investment decisions at different ages. I solve three versions of the model. First, I solve a version with no labor market participation cost, which is very similar to the model in Gomes et al. (2008), to serve as a reference. Second, I solve the benchmark model described in Section 1.3 using calibration values listed in Table 1.1. To further examine the effect of labor market participation cost on consumption, leisure and risky share choices, I solve the model again with a much higher labor market participation cost ($fcost = 1e - 2$) compared to value used in the benchmark model ($fcost = 8e - 6$).

Figure 1.4 depicts the value functions of benchmark/low-cost version of the model when the agent is at ages 65, 45 and 25. The value functions of other fixed cost levels are qualitatively similar. In general, the value function increases with wealth and transitory income shocks.

Figure 1.5a compares the optimal leisure levels in the no-cost and the two with-cost versions when the agent is at ages 65, 45 and 25. I take the average of leisure over different transitory income shock levels so that it is easier to compare across ages and wealth levels. In the no-cost version, optimal leisure never reaches 1, so the agent never leaves the labor market[14]. For the two with-cost versions, the leisure policy functions look like step functions: when wealth reaches to a certain level, the optimal leisure jumps to 1, that is, the agent leaves the

[14]Interestingly, the optimal leisure levels are lower at age 65 at extreme low and extreme high levels of wealth. In other words, if the agent is very poor or very rich at age 65, it is optimal for her to work more than she does at ages 45 and 25. This is because of the low retirement income that we assumed: knowing that the agent will get low income starting tomorrow, the agent works more to boost savings when she is very poor. When the agent is rich, the agent also works more so that the drop in "cash-on-hand (sum of wealth and income)" would not be too large.To confirm that this interpretation is correct, I solved the model with no-cost again with an extremely high retirement income, and the resulted optimal leisure at age 65 are higher than the optimal leisure at ages 45 and 25 at most levels of wealth(see Appendix 3.6).

labor market. When the labor market participation cost is high, optimal leisure jumps to 1 at smaller levels of wealth. For all three versions, leisure increases with wealth. Optimal leisure levels are higher when the agent is at age 25 than when the agent is at age 45 at all wealth levels for the no-cost and benchmark/low-cost versions, while the optimal leisure levels are very close at ages 25 and 45 in the high-cost version.

Figure 1.5b compares the optimal consumption levels in the no-cost and the two with-cost versions when the agent is at age 65, age 45 and age 25. In the no-cost version, consumption increases monotonically with wealth at all three ages which is in accordance with the literature. Since consumption and leisure are substitutes in our setting, the agent reduces her consumption at age 65 in order to maintain a balanced consumption-leisure ratio given that she will enjoy full leisure starting at age 66. Both Polkovnichenko (2007) and French (2005) have similar findings in their models.

For the benchmark/low-cost version, optimal consumption drops in medium level of wealth while optimal leisure jumps to one, again because consumption and leisure are substitutes, and consistent with findings in French (2005). After the consumption drop, the agent then increases consumption with wealth since she has more resources available. For the high-cost version, optimal consumption drops with wealth at lower levels of wealth compared to the benchmark/low-cost version because leisure jumps to 1 at low levels of wealth. Consumption levels are lower for all wealth levels and all ages as the cost of labor market participation increases - with labor market participation costs, the agent would leave the labor market and have less resources to consume.

Figure 1.5c compares the optimal risky shares in the no-cost and the two with-cost versions when the agent is at age 65, age 45 and age 25. When there is no labor market participation cost, the optimal risky share decreases with age because young agents have a larger stock of human capital compared to middle-age and old agents. In addition, the risky share decreases with wealth because the relative size of human capital, a bond-like asset, is small at high levels of wealth. Both properties are found in typical life-cycle models such as CGM.

The introduction of a fixed labor market participation cost induces some distortion of the optimal investment behavior of the agent. For example, the risky share at age 65 is higher when the fixed cost of labor market participation is higher. Also, when the fixed cost is low, the optimal risky share is actually higher at low levels of wealth than the no-cost case. This is because the agent works for longer hours and hence has higher earnings with the fixed cost. At low wealth levels, the share of "safe" earnings is large, hence the optimal risky share is larger than in the no-cost case. When the fixed cost of labor force participation is high, the agent drops out of the labor force at low levels of wealth and the risky share policy function drops quickly with wealth since the agent does not make any income when she is not working.

In summary, we can see that the introduction of the fixed labor market participation cost generates considerable distortions in the policy functions compared to the no-cost version. With the fixed labor market participation costs, the agent chooses to leave the labor market when wealth is high enough, and adjusts her consumption and investment decisions accordingly. To get a picture of how the fixed cost changes the behavior of households over the life-cycle, we move on to simulation exercises in the next section.

1.5.2 Simulation Results

Using the policy functions, we can simulate the paths of wealth, income, consumption, labor supply and risky share for a large number of households. First, I draw an initial level of wealth (relative to permanent income) from an lognormal distribution using the estimates provided by Wachter and Yogo (2010). Second, I generate the life-cycle paths of income shocks and stock return shocks for 10,000 households. In each period, the household observes their beginning-of-period wealth and income shocks, then choose their consumption, labor supply and risky share following the policy functions. Given the household's consumption, labor supply and risky share choices as well the stock return shock, the household enters the next period with a new beginning-of-period wealth. We then repeat this process for all 80 periods and for all households. I simulate several versions of the model, and each simulation has 10,000 households. First, the version with no labor market participation cost, which is a close replication of Gomes et al. (2008), to serve as a reference. Then, a simulation for the benchmark model with low labor market participation cost, and another for the high-cost version of the model.

Figure 1.6 compares the simulated life-time income, wealth and consumption in the no-cost, benchmark/low-cost, and the high-cost versions. The life paths generated by the no-cost version in the left panel is qualitatively similar to the results in Gomes et al. (2008) and is similar to the results of a typical life-cycle model: wealth peaks before retirement, at which time the agent starts to dis-save. Consumption tracks income closely over the life-cycle. Given that the agent enjoys full leisure after retirement, consumption drops upon retirement and decreases steadily. As pointed out by Chai et al. (2011), the drop in consumption upon retirement is widely observed in data[15] but is not generated by models that do not allow labor supply to be endogenous.

Wealth, income and consumption in Figure 1.6 are normalized by first year income. In the no-cost version, wealth is close to eleven-fold the first year income, consistent with the results reported in Chai et al. (2011), whose model also allows endogenous labor supply but assumes no labor market participation costs. Chai et al. (2011) add the purchase of annuity in their model but show that few individuals utilize this investment option. For the benchmark/low-cost case, the peak wealth level is higher than the no-cost case. This is because, as shown later, the simulated life-path of risky share in the benchmark/low-cost version is higher for most of the ages than the in no-cost version. Since risky asset has higher expected returns

[15] See references cited in Chai et al. (2011)

than riskless asset, investing more in risky asset helps with wealth accumulation. For the high-cost version, since a lot of agents leave the labor market and receive no labor income, wealth accumulation over the life-cycle is only about one-third of the no-cost version.

In the benchmark/low-cost version, income peaks at around age 40 and then starts to decline, while in the no-cost version, income peaks later in life, at around 45. In the benchmark/low-cost version, consumption also drops upon retirement and steadily decreases. When the fixed cost is high, income peaks at a very early stage of work life, age 23, and decline throughout the rest of the work life. The early declines in income in the two with-cost versions correspond to the fact that agents leave the labor market and reduce their labor supply hours (see later discussion).

Consumption remains low when the fixed cost is high, because agents have very small amount of wealth to consume when they leave the labor market early in life. Moreover, income counterfactually jumps up after retirement in the high-cost version, because even though the agents make little earning in their work life, they still receive a constant stream of retirement income. As a result, the agents dis-save early in their life and rely purely on the retirement income for consumption in the high-cost version.

Figure 1.7a compares the simulated labor market participation rates in the two with-cost versions. In the benchmark/low-cost version, labor market participation rate is 100% up to age 37 and gradually declines to about 60% at age 50. The decline in labor market participation rate is somewhat sharper from age 62 to 65: there is a 10% decline in labor market participation rate from age 62 to 65. A high labor market participation cost generates extreme and unrealistic results - labor market participation rate drops to below 10% when the agents are in their early 30s. As shown in the wealth path in Figure 1.6, the agents leave the labor market early in life and rely on the small amount of savings accumulated at the beginning of their work life. When the agents are close to retirement age, the available wealth becomes so small that some of them return to the labor market, which explains the increase in labor market participation rate after age 60 in the high-cost version.

The labor market participation pattern does not match the age profile provided in French (2005) (see Figure 1.2). In French (2005), labor market participation rate starts from somewhere closed to 100% and then declines to about 80% when the agents are in their early 60. The participation rate of agents up to age 55 is more persistent than the participation rate in my simulation result. In addition, French (2005) finds a much sharper decline in labor market participation rate after age 62. The reason that the agents start to leave the labor market at earlier ages in my model than in French (2005) is because I include risky asset investment while French (2005)'s model only has a riskless asset with lower expected returns. High expected returns from the risky asset helps with wealth accumulation, which then triggers the incentives for some agents in my model to leave the labor market early in the work life. As shown in my later sections, when we allow Social Securityto be accumulated endogenously, which essentially increases the uncertainties that the agents face and provides more incentives for the agents to stay in the labor force despite the participation cost, we generate a more persistent labor market participation pattern in the first half of the work life.

Figure 1.7b plots the labor supply hours conditional on participating in the labor force. We see that the conditional labor supply hours in the benchmark/low-cost version are persistently higher than in the no-cost version over the work life. As discussed before, the higher conditional labor supply hours is generated by the labor market participation cost - given the same preference on leisure, agents with labor market participation cost either not work or work more hours. When the labor market participation cost is very large, however, the average conditional labor supply hours are higher only when the agents are younger than 35. The conditional labor supply hours then decline as the agents age. We see that the conditional labor supply hours are less smooth in the high-cost version. This is likely because the percentage of the population that participates in the labor market is significantly lower in the high-cost version, hence we observe more heterogeneity on the intensive margin of labor supply.

Figure 1.7c compares the simulated life-time risky asset holdings in the no-cost, benchmark/low-cost, and the high-cost versions. The no-cost version is qualitatively similar to the result in Gomes et al. (2008): risky share is U-shaped over the life-cycle. The agents hold 100% risky assets and start to reduce the risky share after they reach age 35. This is because the relative size of human capital, an implicitly safe asset, is decreasing before retirement. Correspondingly, agents hold less risky assets as human capital depletes. Risky share drops to about 48 percent upon retirement and then starts to rise again. After retirement, as agents dis-save, the relative size of safe pension in wealth increases, and the risky share increases accordingly.

In the benchmark model, the risky share starts to decline earlier in life than the no-cost version. As a result, the simulated risky share in our benchmark model is lower for agents between 30 and 44 than the no-cost case. Since the rate of decline in the risky share in the low-cost version is slower than the rate of decline in the no-cost version, the risky asset holdings for agents older than 40 and before retirement are higher in the low-cost version than the no-cost version. The differences in the simulated risky share between the benchmark and the no-cost versions are caused by two forces. First, when agents leave the labor market and receive no labor income, they reduce their risky share. Second, agents who remain in the labor force work longer hours with the small fixed cost and hence make more "safe" earnings. Therefore, they tend to increase their risky share. The first force dominates when agents are in between 30 and 44, when agents start to leave the labor force. On the hand, the second force dominates for agents older than 44. The risky share after retirement in the low-cost version closely tracks no-cost version but are at higher levels. This is because the agents in the benchmark/low-cost version accumulates more wealth and hence are willing to take more risk after retirement than in the no-cost case.

The risky share in the high-cost version strongly differs from the other two versions: risky shares are lower than 100% starting the fifth year of work life and are considerably lower than the other two versions up to age 60. This is understandable as a large amount of agents leave the labor force in the first few year of life-cycle and reduces their risky asset investment. After age 60, the risky share starts to jump to close to 100% after retirement in the high cost version. This is because with the small wealth accumulated, the relative size of retirement income in wealth is so large that it is optimal to invest 100% of savings in stocks after retirement for the high-cost agents.

Summarizing, when I extend Gomes et al. (2008) by introducing a small fixed cost as in my benchmark model, I find that the agents in the simulation starts to leave the labor force after age 37. Since some agents are not working, they reduce their risky share. However, those who stay in the labor force work for more hours in the benchmark model than in the no-cost case, they therefore prefer to hold more risky assets. As a result, the simulated risky share in our benchmark model is lower than the no-cost case only for agents in between ages 30 and 44. The simulated risky share is much lower in the high-cost case than in the no-cost case, but the labor market participation rate is too low when the fixed cost is higher. In conclusion, I confirm my hypothesis that introducing a fixed labor market participation cost to account for the extensive margin of labor supply does reduce risky asset holdings. The effects of the fixed cost on risky asset investment seems to be small in the benchmark model, because I do not allow the labor market participation rate to be too low.

As shown in the next section, the benchmark model do not fully capture the uncertainty that the agent face. For example, when an agent leaves the labor force, her retirement income should be affected. However, in the benchmark model, retirement income is assumed to be constant. The next section shows that when incorporating these uncertainties into the benchmark model, the resulting risky share is lower than the benchmark model.

1.6 Extensions

1.6.1 Social Security

One unrealistic feature of my benchmark model is that the retirement income stays unchanged even though I allow individuals to drop out of the labor market. If the agent expects to receive a constant stream of income after age 65, she increases her risky asset investment after retirement, as shown in the simulation results in Section 1.5.2.

In this section, I experiment with two alternative specifications of Social Security. I begin with simply increasing the Social Security replacement ratio to 90% from 68.8% in the benchmark. In this case the Social Security is still deterministic and unaffected by the fact that the agent could leave the labor market in her 30s. I then extend the model by allowing Social Security income to be determined endogenously, which also corresponds to a lower realization of retirement income than the benchmark model.

In the United States, an individual's Social Security income depends mainly on two factors. First, the monthly average of the highest thirty-five years of income, adjusted by a national wage index in the year when claiming Social Security, called the Average Indexed Monthly Earnings (AIME). If an individual works for less than thirty-five years, her total income would still be divided by thirty-five. Therefore, leaving the labor market affects the accumulation of Social Security. AIME is then used to calculate the Primary Insurance Amount (PIA), which is the basis of Social Security. PIA is computed using a "bend point" system in which low AIME gets a high replacement ratio and low AIME gets a relatively higher replacement ratio. The second factor is the age of claiming the Social Security - PIA is discounted if an individual claims Social Security between ages 62 and 65 and is increased if an individual claims Social Security after age 65. Social security income is equal to PIA if an individual retires at age 65.

Note that the age of retirement and the age of claiming Social Security are not necessarily the same, but modeling this could complicates the model considerably. What further complicates the modeling of Social Security accumulation is that the bend points in the calculation of PIA are based on the national wage level of the year. For simplicity, I make the assumption that the national wage level when the agent is 65 is equal to her permanent income, and use the normalized formula of PIA provided in Catherine (2016). This assumption is typical in life-cycle models such as CGM, since they assume the the retirement income of the agent is a fraction of the permanent income at age 65. In addition, I maintain the assumption that the agent is going to retire and claim Social Security at age 65 instead of allowing the retirement age to vary between 62 and 70.

More specifically, to account for the accumulation of Social Securities endogenously, I include an additional state variable, $AIME_t$, which has the following law of motion:

$$AIME_{t+1} = \begin{cases} AIME_t + \frac{\exp(f(t))\cdot P_t\cdot\Theta_t\cdot(1-L_t)}{35}, \forall t \leq 55 \\ AIME_t + \max\{0, \frac{\exp(f(t))\cdot P_t\cdot\Theta_t\cdot(1-L_t)-AIME_t}{35}\}, \forall t > 55. \end{cases} \tag{1.15}$$

$AIME_t$ keeps track of the average of the highest thirty-five years of income for all years on or before time t. For the first thirty-five years of work life, $AIME_t$ can only increase; for the years beyond the first thirty-five years of work life, $AIME_t$ increases only if the income in the year is higher than the current $AIME_t$. At age 66 and above, the agent's income is determined by the following formula

$$\frac{PIA_t}{P_{65}} = \begin{cases} 0.9 \times AIME_t, \text{ if } AIME_t < 0.2 \\ 0.18 + 0.32 \times AIME_t, \text{ if } 0.2 \leq AIME_t < 1 \\ 0.5 + 0.15 \times AIME_t, \text{ if } 1 \leq AIME_t. \end{cases} \tag{1.16}$$

Figure 1.8 displays the resulting labor market participation rates, conditional labor supply hours as well as risky shares with different specifications of the Social Security system. When we raise the replacement ratio to 90%, agents leave the labor market at a younger age, despite the same labor market participation cost. Conditional on working, agents start to reduce their hours after age 45 compared to the benchmark model. Both are reasonable outcomes as agents are expecting a higher stream of income after retirement. A high stream of safe retirement income also induces the agents to invest more in stocks as shown in Figure 1.8c.

When we allow Social Security income to be determined endogenously, several interesting results occur. First of all, the labor market participation rate is overall higher and becomes more persistent compared to the benchmark model, as shown in Figure 1.8a. Labor market participation rate stays above 80% up to age 60 and then slowly declines to about 60%. The increase in labor market participation rate is a reasonable outcome of making Social Security accumulation endogenous. Since retirement income totally depends on the work-life income, agents have strong incentives to stay in the labor force to accumulate Social Security, despite the fixed cost of labor market participation.

As shown in Figure 1.8b, agents younger than 31 work for longer hours than the benchmark, which makes sense as making more effort today is beneficial to the retirement income. Exogenous Social Security does not provide such an incentive. In addition, working more hours when young increases retirement income more as wages are assumed to be hump-shaped over the work life.

As shown in Figure 1.8c, the agents hold less risky assets when the agents are in ages 20 to 23 and in ages 30 to 45. This is understandable because, with endogenous Social Security, agents essentially face more uncertainties - a negative shock in current income would affect both current wealth accumulation as well as retirement income. Agents hold more risky assets after age 45 than the benchmark case on average. My understanding is that as the agents get older and work for more years, the uncertainties on retirement income have reduced, so it is optimal to hold more risky assets.

My result provides an interesting contrast to the finding in CGM. In CGM, they introduce a possibility of a 25% drop in retirement income to imitate sudden health care expense during retirement, and they find young agents' investment decision is less affected by the uncertainty introduced to retirement income. This because agents in CGM only start to prepare for uncertainties in retirement income when they are close to retirement. In my model, since the uncertainties in retirement income could be reduced if the agents work more hours and work for more longer years, the agents respond by working more hours and reducing risky asset investments when young and when there are more uncertainties.

Zero Income State

Introducing endogenous Social Security pushes the simulated risky shares of agents younger than 45 to be lower but the life-cycle profile of the risky share is still inverse U-shaped. In this section, we further extend the model to include a small possibility of a zero income state. A zero income state is modeled as allowing the transitory income shock θ to become zero. For simplicity, we assume that, in each period, the agent faces an independent 0.5% possibility that income drops to zero, and there is no subsequent destruction on human capital. Since it is possible to enter a zero income state, the ability of the agent to use labor supply to adjust for shocks is further reduced.

Figure 1.9 depicts the conditional risky share when we allow endogenous Social Security accumulation and include a 0.5% possibility that the transitory income shock equals to zero. Conditional risky share is now hump-shaped before retirement - agents start out holding about 10% of their savings in risky asset, increase the risky share to about 65% by age 40, and the risky share then swings around 50% to 60% when the agents are between ages 40 and 65. The risky share before retirement is much lower than the benchmark model and when social security is endogenous, in both cases we do not allow the zero-income state.

Several other papers considered zero income state and our findings are consistent with theirs. CGM also find that including a zero income state lowers the risky shares of young agents, but they predicted higher risky shares overall than what we find when we allow endogenous Social Security and a zero income state. Polkovnichenko (2007) shows that it is important to include a zero income state when considering the risky asset investment outcomes with additive habit preference. When Polkovnichenko (2007) includes are zero income state, he also finds that younger agents hold less risky assets than the middle-aged.

Non-Linear Earnings

In this subsection, I relax the assumption that earnings are a linear function of labor supply. That is, the law of motion of earnings in Equation (1.2) now becomes

$$Y_t = (1-\tau^l)\cdot(1-h(t))\cdot\exp(f(t))\cdot P_t\cdot\Theta_t\cdot(1-L_t)^\gamma, \forall t \leq K, \tag{1.17}$$

in which the parameter $\gamma > 1$ captures the degree non-linearity of labor supply and earnings. Allowing the relationship between labor supply and earnings to be non-linear captures the empirical fact that full-time workers receive higher wages than part-time workers (Erosa et al. (2016)). Introducing non-linear earnings provides yet another incentive for the agent to either leave the labor market or work for longer hours, since now working fewer hours would result in significantly smaller earnings than working for long hours.

Following Erosa et al. (2016), I set the parameter γ to be 1.4, which makes the wages of part-time workers (who work 1000 hours per year) to be 25% less than the wages of full-time workers, and run the benchmark model. Figure 1.10 provides the simulation results when we allow non-linear earnings. Consistent with the discussion of Erosa et al. (2016) that non-linear earnings amplify the effects of the fixed utility cost, we can see that introducing non-linear earnings has similar effects on labor supply decisions as increasing the fixed cost of labor supply - labor market participation rate declines, while the conditional labor supply hours significantly increases. The conditional risky share is higher than the benchmark by about 10 percentage points for all ages.

My understanding is that non-linear earnings increase the agents' ability to use labor supply to buffer against stock market shocks, since working one more hour is more rewarding than before. Therefore, the agents increase their risky asset holdings at all ages.

In Figure 1.10, I show the results of an experiment when there is non-linear earnings and the fixed participation cost is reduced by half. Reducing the fixed participation cost should increase the agents' incentive to participate in the labor market. The resulting labor market participation rate is slightly higher than the benchmark model before age 55 and starts to decline at a slightly faster rate than the benchmark. Reducing the fixed cost by half closes the gap in labor market participation rate between the benchmark model and the non-linear earning case. This result is consistent with the observation of French (2005) that it is hard to distinguish whether the observed labor market participation pattern in the data is generated by fixed cost of labor supply or non-linear earnings.

The conditional labor supply hours when fixed cost is reduced by half remain almost unchanged. Therefore, the incentive to work for long hours given the wage difference between full-time and part-time jobs dominates the incentive to work for fewer hours when the fixed cost is lower. Given the long work hours when the fixed cost is reduced by half, the risky share is similar to the case with non-linear earnings and is much higher than the benchmark case.

1.7 Sensitivity Analysis

1.7.1 Leisure Preference Parameter

The leisure preference parameter α governs the relative importance of leisure to consumption. The higher the α, the less willing is the agent to substitute consumption with labor supply. In this subsection, I explore how the labor supply and investment decisions are affected by different values of α. Results are presented in Figure 1.11. I start with lowering α to 0.9, which is used by Gomes et al. (2008). The resultING labor supply hours are much higher when α is lower. The average labor supply hours rises to about 2800 hours per year, consistent with the result reported in Chai et al. (2011) that $\alpha = 0.9$ depicts the behavior of college professors. When α is lower, the agents also on average hold less risky assets because wealth accumulates faster with longer work hours, and the risky share reduces with wealth. The labor market participation rate with low α is lower than the benchmark case as shown in Figure 1.11a. This is understandable because with lower α, the same level of leisure gives lower utility, so the agents now have more incentives to enjoy full leisure given the same utility costs of labor supply.

1.7.2 Risk Aversion Coefficient

In this subsection, I investigate how the simulation results are sensitive to the assumed value of the risk aversion coefficient. Figure 1.12c shows that the average conditional risky share is higher when we lower the risk aversion coefficient to 3 for all ages, since the agents now less risk-adversed. When we raise the risk aversion coefficient to 7, the risky share is lower than the benchmark for agents below retirement age and higher than the benchmark after retirement. When risk averse coefficient is high, the agents prefer to hold less risky assets. As a result, they accumulate less wealth and the optimal risky share after retirement is higher than the benchmark because of the large relatively size of the deterministic retirement income.

The curvature of the utility function also affects labor supply decisions. When the risk aversion coefficient is low, the agent almost never leaves the labor market and work for a little less hours than the benchmark model. In contrast, agents with high risk aversion behave like agents with high fixed cost of labor market participation: labor market participation rate declines to below 10% before age 40 and the conditional labor supply hours are higher for younger agents and then decline as agents approach retirement.

1.8 Conclusion

In this paper, I build a life-cycle model of portfolio choice which incorporates the extensive and intensive margins of labor supply as well as endogenous Social Security accumulation and show that these features lead to lower risky asset holdings compared to the model in Gomes et al. (2008). Risky asset holdings are lower for young agents when I introduce a fixed cost of labor supply which generates an operative extensive margins of labor supply. Young agents further reduce their risky asset holdings as uncertainties introduced by the endogenous Social Security accumulations are largest when agents are young. When we allow for a zero-income state, the age profile of risky share becomes hump-shaped and significantly reduced to below 65%.

My model attempts to simultaneously match the risky asset holding and the labor supply pattern. Since the agents could quickly accumulate wealth via risky asset investments, the labor market participation rate decreases steadily starting at a younger age, which is inconsistent with the data. Simultaneously matching risky asset holding and labor supply patterns requires careful specifications of the preference parameters as well as labor market participation costs. Therefore, using the Simulated Methods of Moments (SMM) to match simulated moments with empirical moments might be a potential route to pin down the relevant parameters.

1.9 Tables

Table 1.1: Parameter Values in Benchmark Model

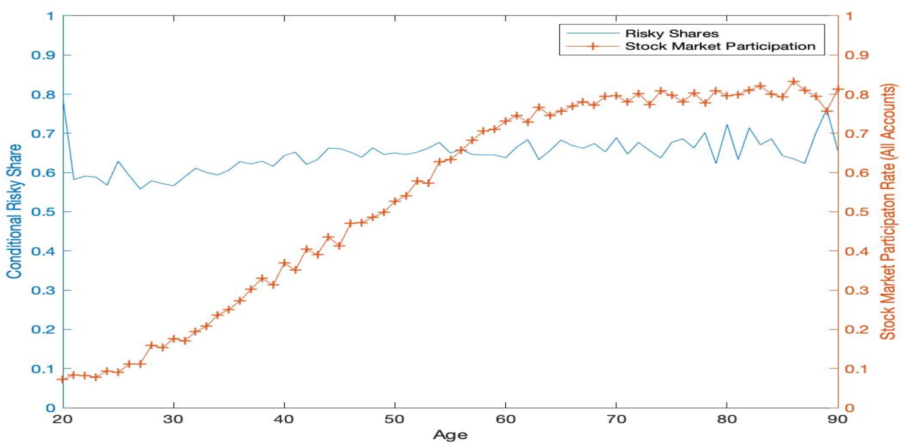
Parameter	Definition	Value	Source or Goal
ρ	Coefficient of risk aversion	5	Standard in the life-cycle literature
β	Discount rate	0.97	Gomes et al. (2008)
σ_κ	Standard deviation of permanent income shocks	0.1095	CGM
σ_θ	Standard deviation of transitory income shocks	0.1389	CGM
τ^l	Tax rate on labor income	30%	Gomes et al. (2008)
τ^R	Tax rate on retirement income	15%	Gomes et al. (2008)
τ^c	Tax rate on capital gains	20.5%	Gomes et al. (2008)
λ	Replacement rate of Social Security income	68.212%	CGM
Rf	Risk free rate	1%	Gomes et al. (2008)
μ	Equity Premium	4%	Gomes et al. (2008)
σ_θ	Standard deviation of innovations to the risky asset	20%	Gomes et al. (2008)
α	Relative importance of leisure	1.45	Data on labor supply
$fcost$	Utility cost of labor market participation	8e-6	Data on labor supply

1.10 Figures

Figure 1.1: Age Profiles of Conditional Risky Share and Stock Market Participation Rate

This figure depicts the age profiles of conditional risky share and stock market participation rate generated by a Heckman selection model using 1999-2017 waves of PSID, using two different methods to control of cohort and year effects and controlling for demographic variables. "Cohort Proxy" means cohort effects are controlled by using youth stock returns to proxy for each cohort, and "Deaton-Paxson" means year effects are controlled using the method outlined in Deaton and Paxson (1994).

(a) Cohort Proxy



(b) Deaton-Paxson

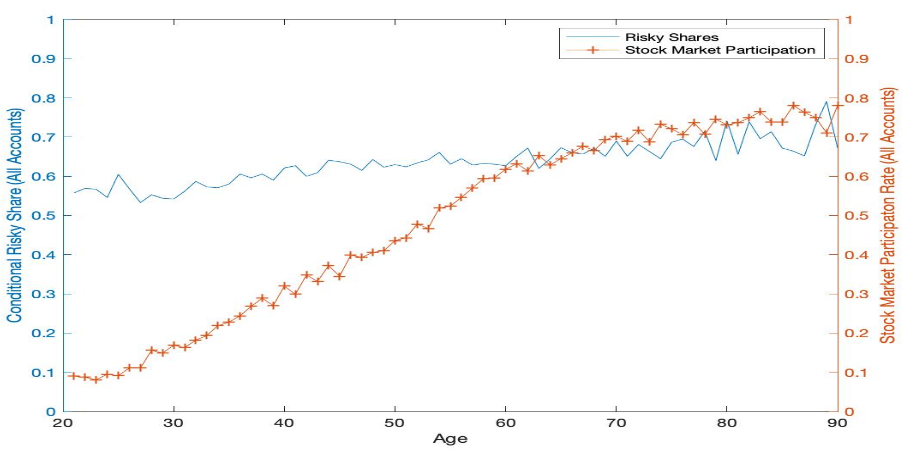


Figure 1.2: Annual Hours Worked, Labor Market Participation Rate and Asset Accumulation

Source: Figure 2 of French (2005)

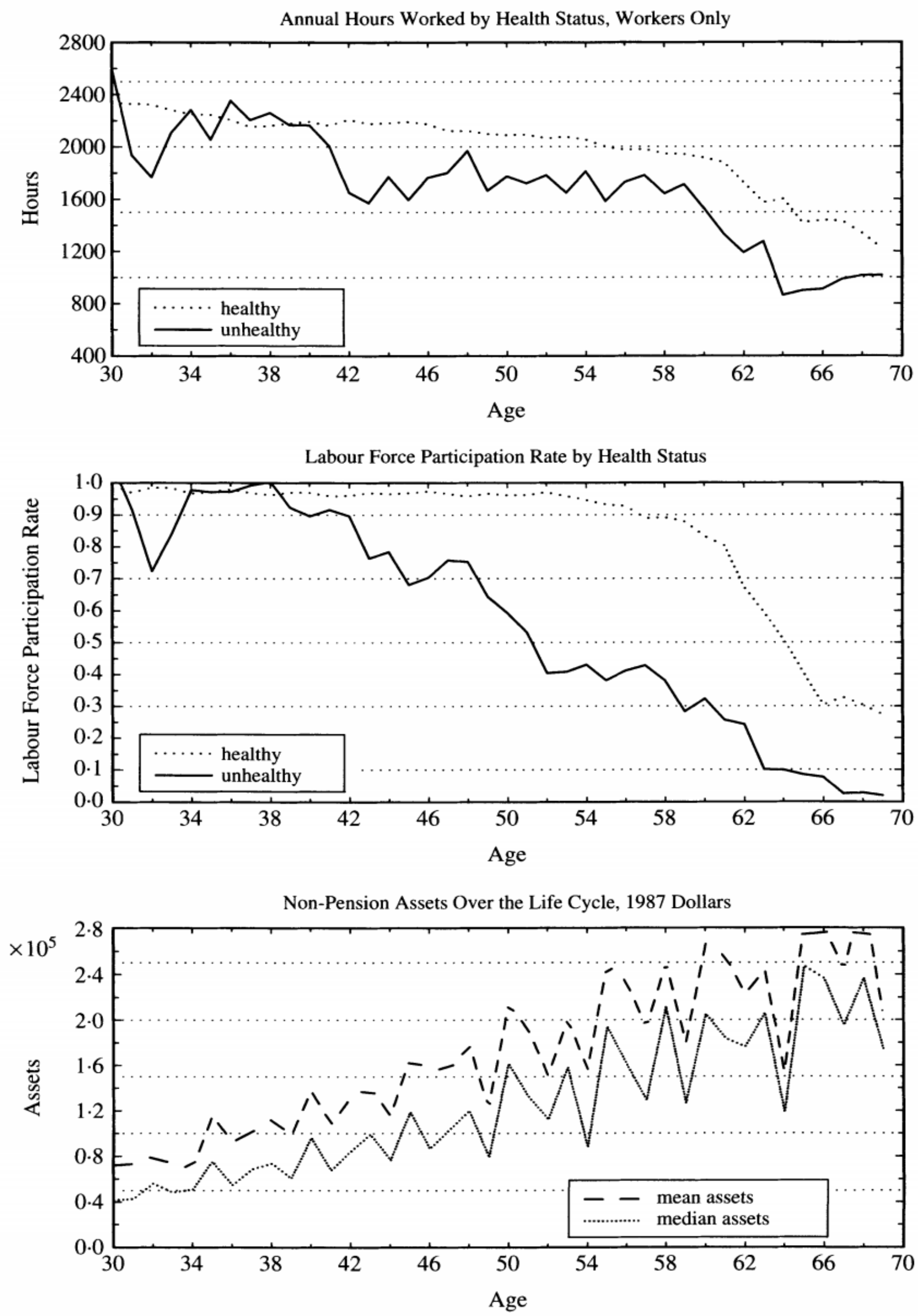


FIGURE 2

Life cycle profiles for decision variables

Figure 1.3: Labor Market Participation Rate Using 1999 - 2017 PSID Data

This figure plots the predicted labor market participation rate by age using the 1999 to 2017 waves of the PSID data. "Year Dummies" means I run a Probit regression with both age dummies and year dummies, the implicit assumption is that cohort effects are zero. "Cohort Dummies" means I run a Probit regression with both age dummies and cohort dummies, the implicit assumption is that year effects are zero.

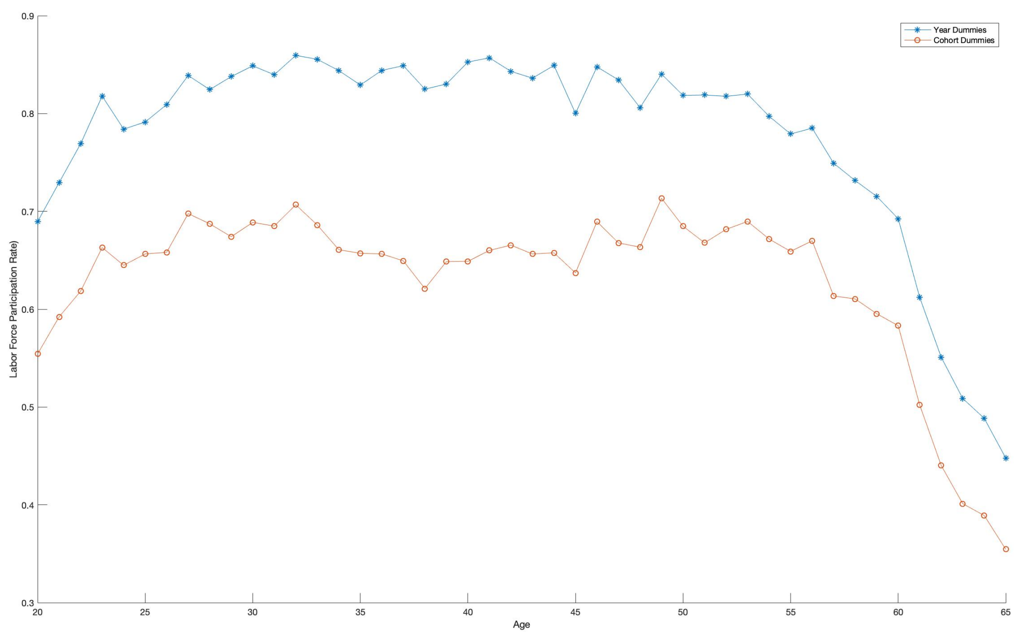


Figure 1.4: Value Functions at Different Ages

This figure provides value functions over two state variables, wealth and transitory income shocks, in the benchmark model when the agent is at ages 65, 45 and 25.

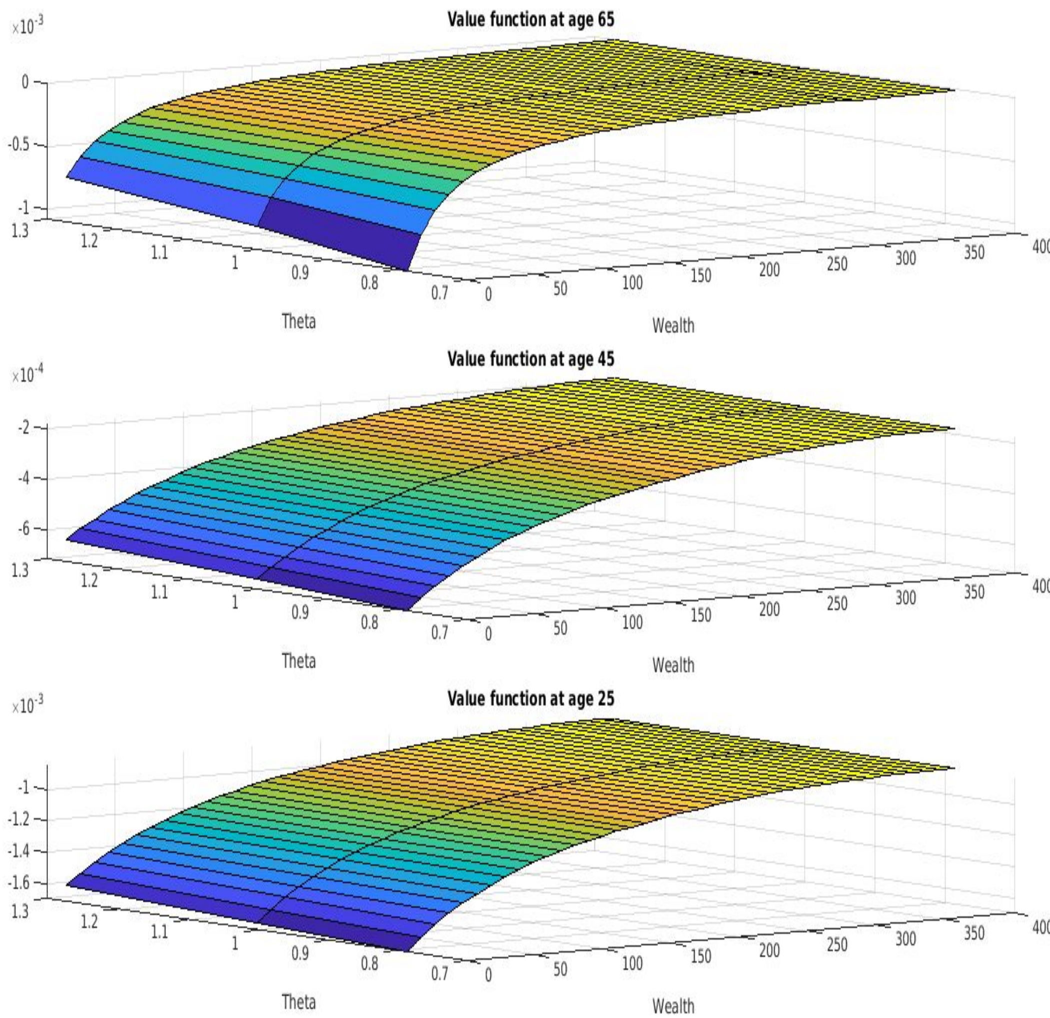
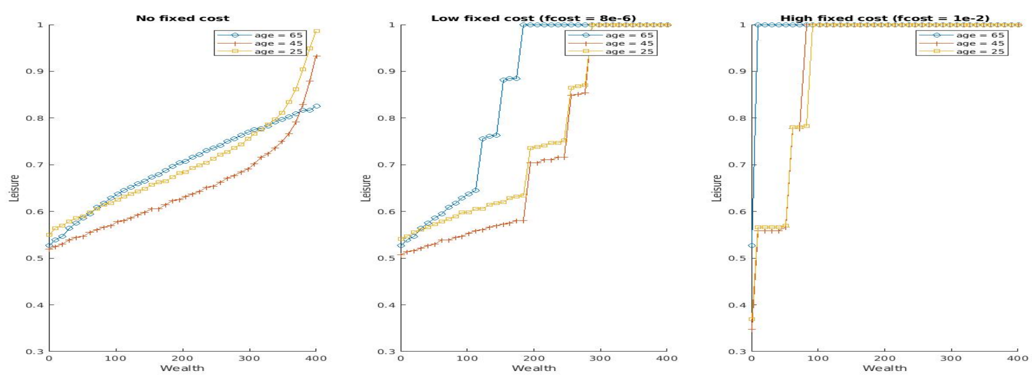


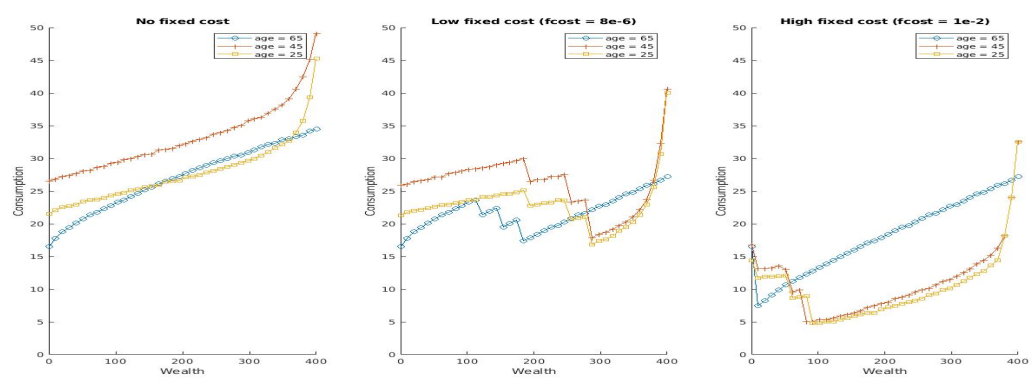
Figure 1.5: Policy Functions at Different Ages

This figure provides policy functions on leisure, consumption and risky shares against wealth when the agent is at ages 65, 45 and 25 in three different versions of the model. The left column displays the policy functions in the no-cost version, middle column displays the low-cost version, and the right column displays the high-cost version.

(a) Leisure Policy Functions



(b) Consumption Policy Functions



(c) Risky Share Policy Functions

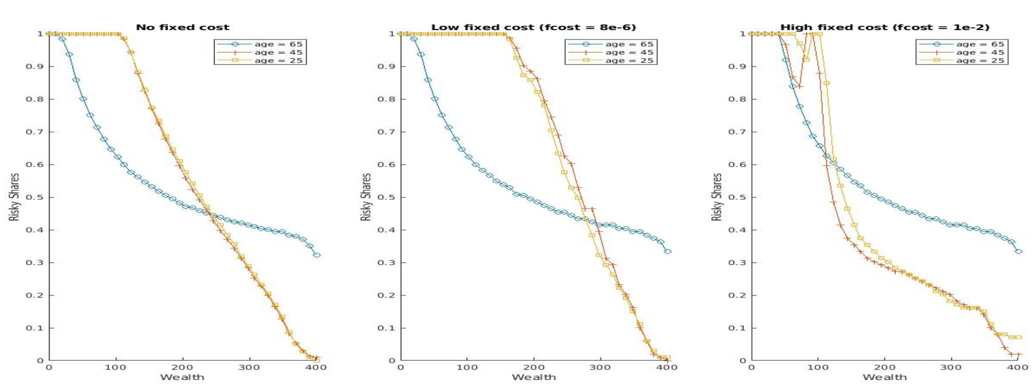


Figure 1.6: Simulated Wealth, Income and Consumption Paths

This figure provides simulated wealth, income and consumption paths, normalized by first-year income, for the no-cost (left panel), low-cost (middle panel), and high-cost (right panel) versions.

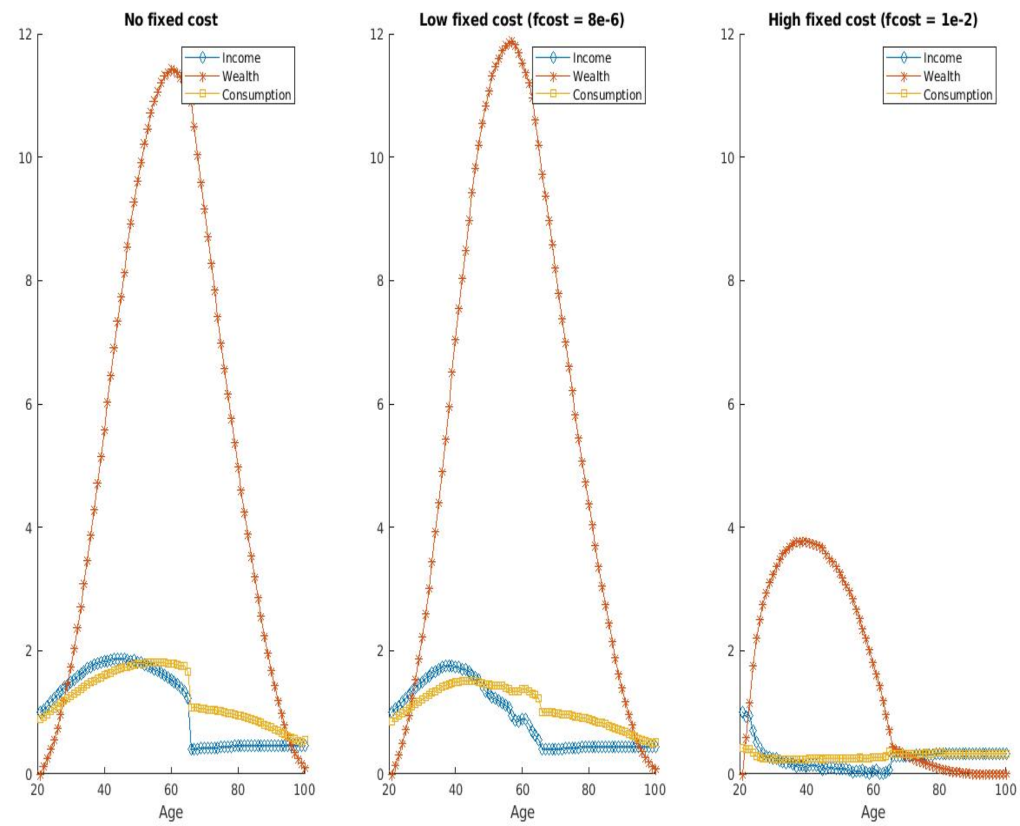
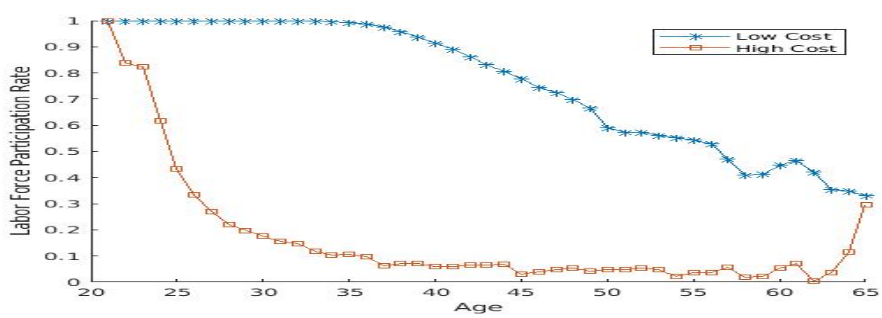


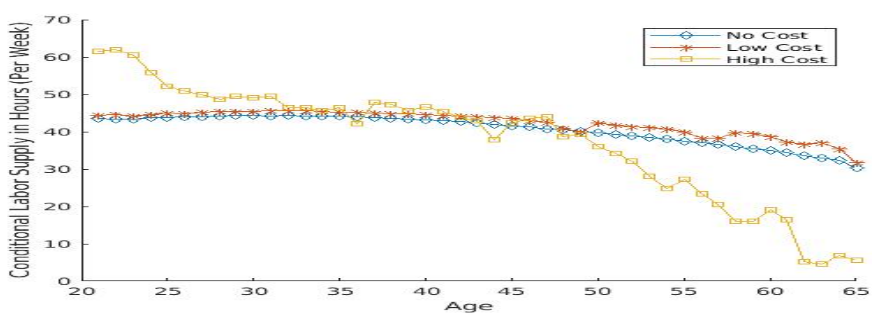
Figure 1.7: Simulated Labor Market Participation Rates, Labor Supply Hours and Risky Shares

This figure provides simulated labor market participation rates, labor supply hours, conditional labor supply hours and risky shares for the simulation exercises of three versions of the model.

(a) Labor market Participation Rates



(b) Conditional Labor Supply



(c) Conditional Risky Shares

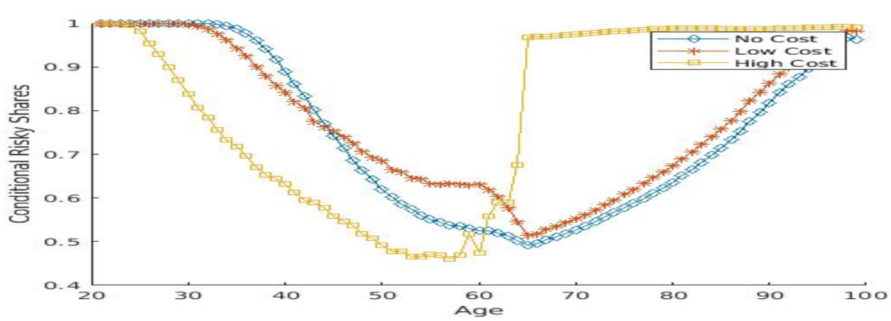
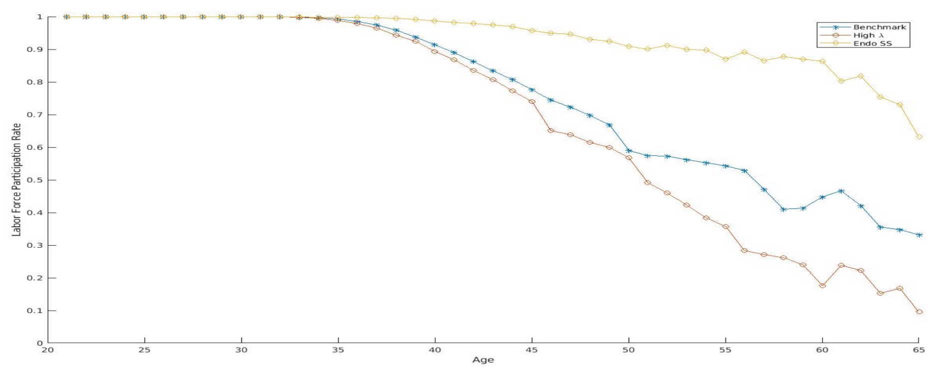


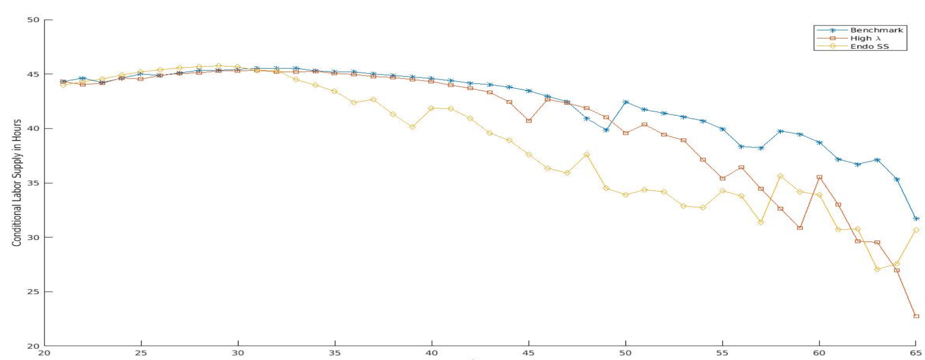
Figure 1.8: Simulation Results with Various Social Security Specifications

This figure provides simulated labor market participation rates, conditional labor supply hours and risky shares for two alternative specifications of Social Security, one with a high (90%) replacement rate, and the other with endogenous Social Security.

(a) Labor market Participation Rates



(b) Conditional Labor Supply in Hours



(c) Risky Shares

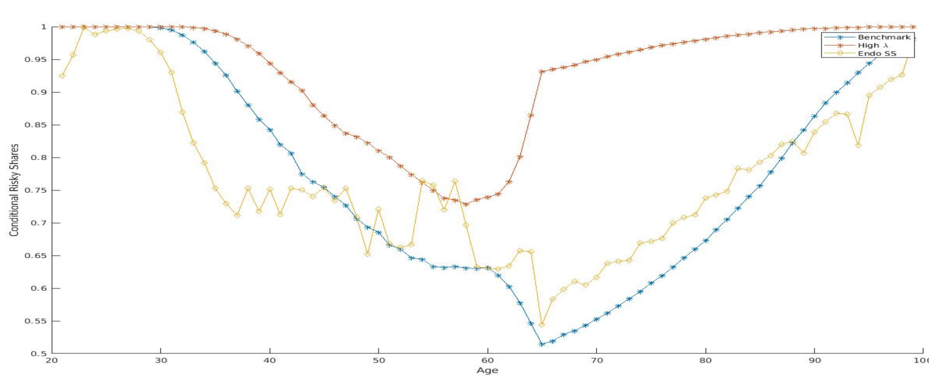


Figure 1.9: Simulation Results with Zero Income States

This figure provides simulated risky shares when we allow income to fall to zero with a small probability of 0.5%. We consider two cases: exogenous Social Securities and endogenous Social Securities.

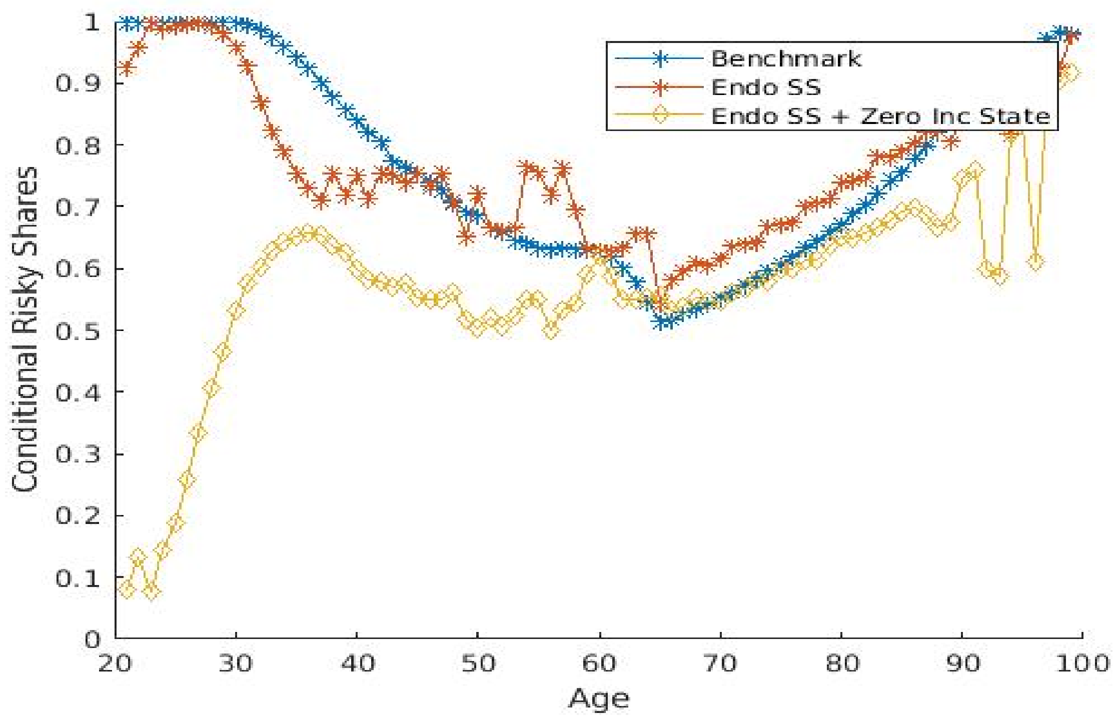
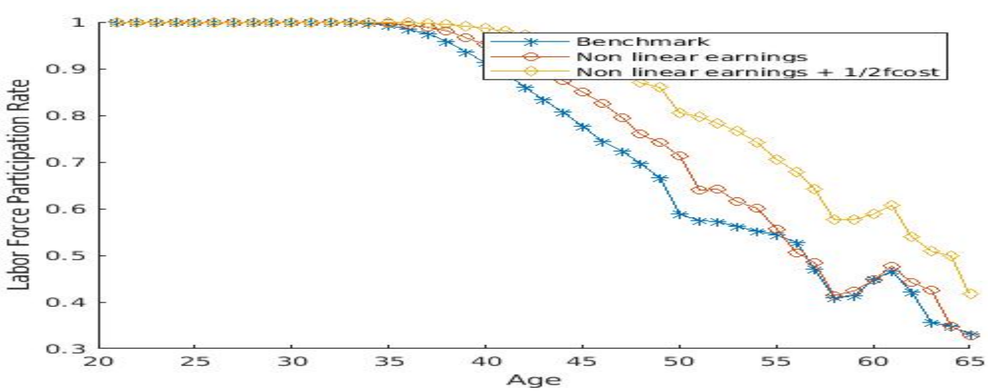


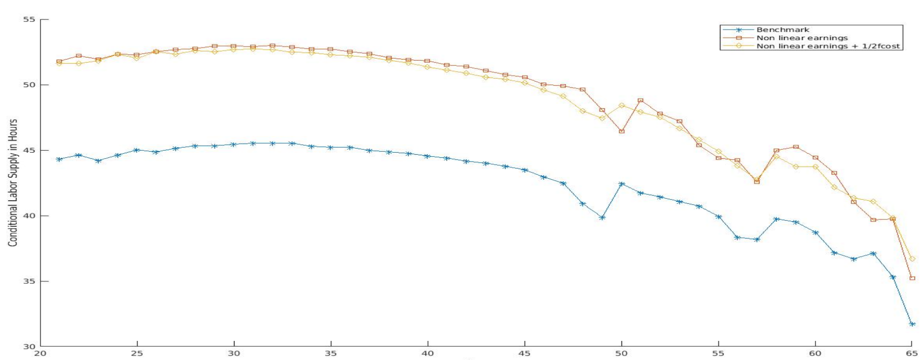
Figure 1.10: Simulation Results with Nonlinear Earnings

This figure provides simulated labor market participation rates, conditional labor supply hours and risky shares when we allow non-linearity between labor supply and earnings

(a) Labor market Participation Rates



(b) Conditional Labor Supply in Hours



(c) Risky Shares

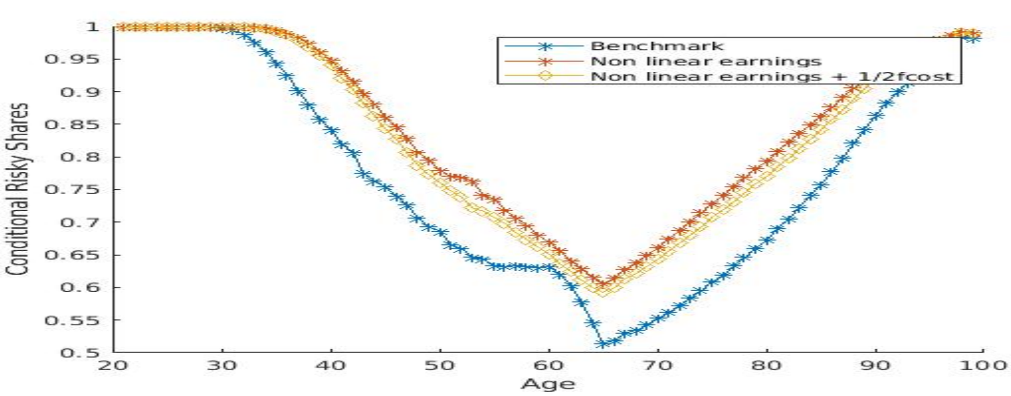
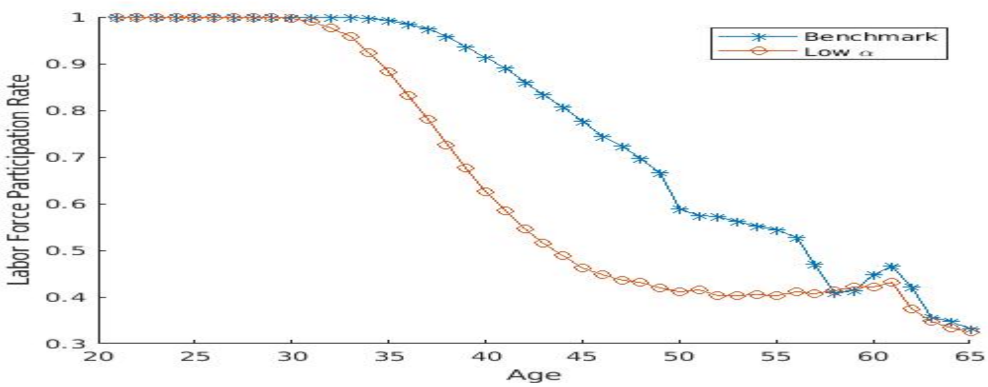


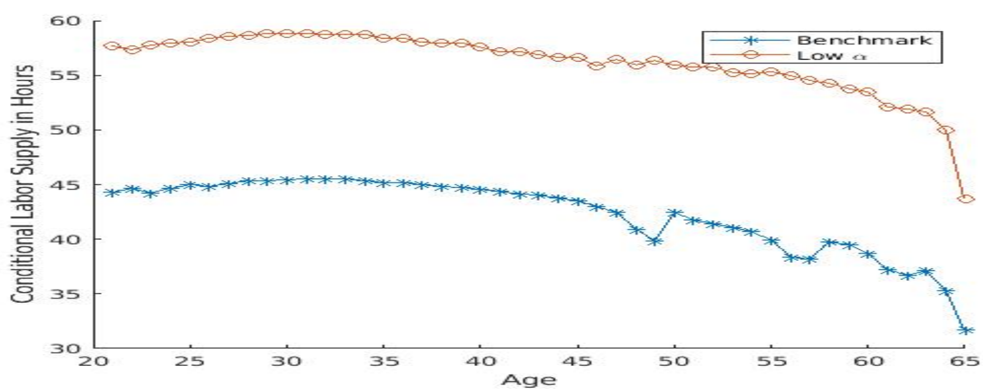
Figure 1.11: Simulation Results with Low Leisure Preference Parameters

This figure provides simulated labor market participation rates, labor supply hours and risky shares when we lower the leisure preference parameter to 0.9.

(a) Labor market Participation Rates



(b) Conditional Labor Supply in Hours



(c) Risky Shares

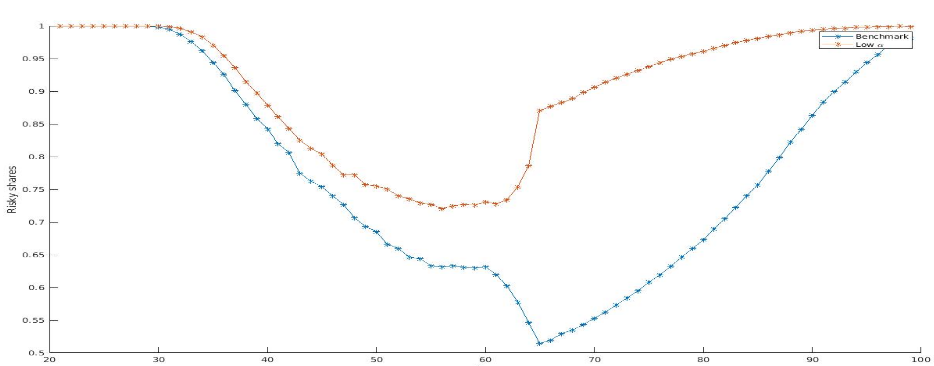
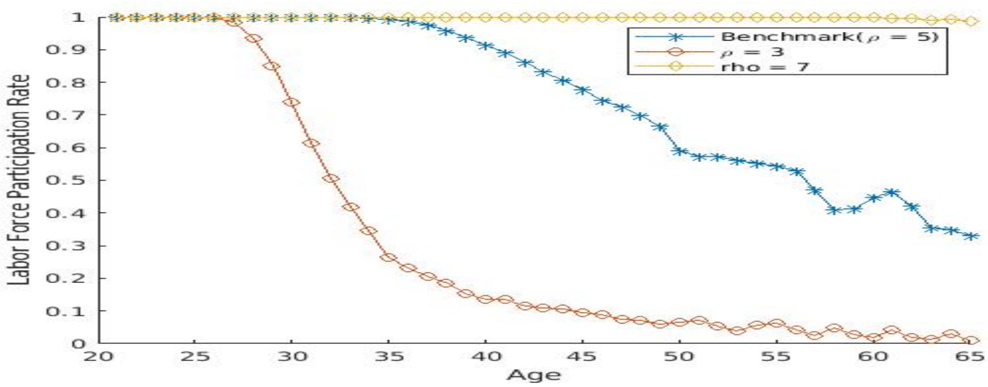


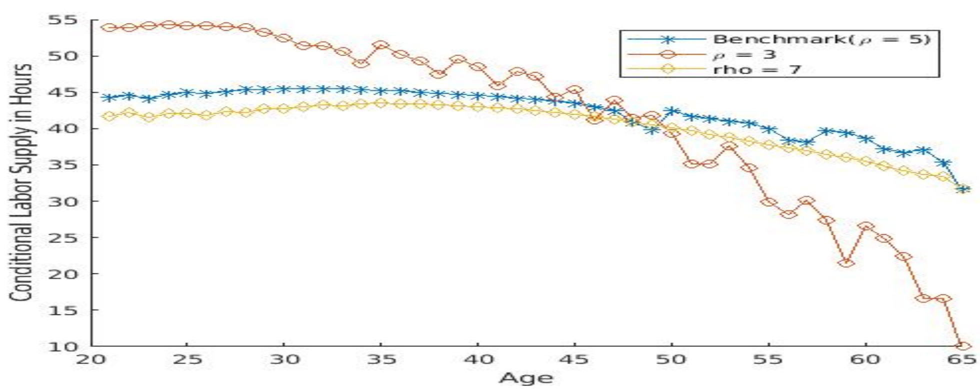
Figure 1.12: Simulation Results with Various Risk Aversion Coefficients

This figure provides simulated labor market participation rates, labor supply hours and risky shares with higher and lower risk aversion coefficients than the benchmark model.

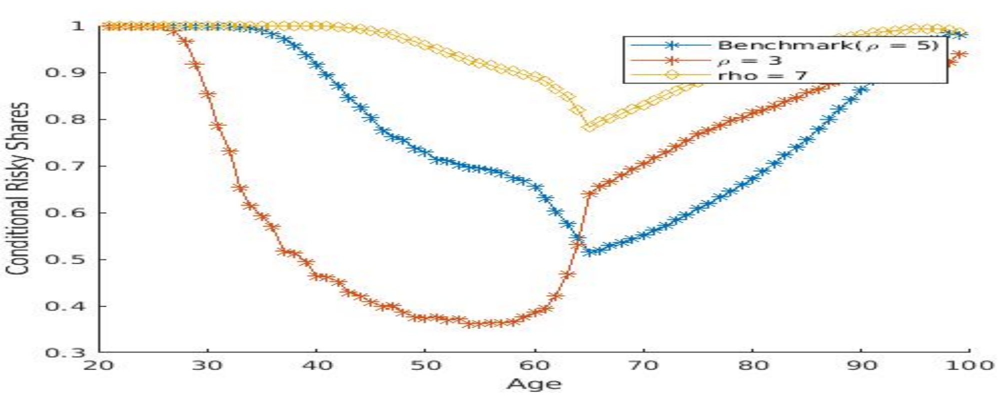
(a) Labor market Participation Rates



(b) Conditional Labor Supply in Hours



(c) Risky Shares



Chapter 2

HABIT FORMATION UTILITY AND RETIREMENT ASSET ALLOCATION – A THREE-PERIOD MODEL

2.1 Introduction

> Our Rabbis taught: 'Sufficient for his need' [implies] you are commanded to maintain him, but you are not commanded to make him rich;' in that which he wanted '[includes] even a horse to ride upon and a slave to run before him'. It was related about Hillel the Elder that he bought for a certain poor man who was of a good family a horse to ride upon and a slave to run before him. On one occasion he could not find a slave to run before him, so he himself ran before him for three miles. [Gemara, Kesubos67b][1]

More and more individuals rely on employer sponsored defined contribution plans (401(k)s or 403(b)s) and IRAs (Individual Retirement Accounts) in replacement of defined benefit plans for retirement. As of March 2018, 42 percent of the U.S. workers participate in defined contribution plans[2]. As of mid-2017, 34.8 percent of U.S. households owned at least one type of IRAs[3]. By the end of the year 2017, 27% of the retirement assets in the United States are under defined contribution plans and 33% of the retirement assets in the United States are under IRAs[4].

Assets inside defined contribution plans and IRAs (henceforth *retirement accounts*) receive favorable tax treatments but are subject to accessibility restrictions. In contrast, assets in traditional brokerage accounts or savings and checking accounts (henceforth *personal accounts*)

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[1]I thank Professor Levis Kochin for this quote and all his thoughtful inputs on this chapter.

[2]Source: Bureau of Labor Statistics https://www.bls.gov/ncs/ebs/benefits/2018/ownership/civilian/table02a.htm.

[3]Source: Investment Company Institute https://www.ici.org/pdf/per23-10a.pdf

[4]Source: Investment Company Institute https://www.ici.org/pdf/ten_facts_iras.pdf

can be accessed anytime but are not tax-favored. In addition, the size of the retirement asset depends a lot on the decisions made by the account owner since it is usually she who decides on which asset class (risky or riskless) to hold and how much of each type of asset to hold in the personal versus in the retirement accounts. Since the amount of funds in the retirement accounts determines the standard of living after retirement for individuals, it is important for individuals to make wise investment decisions for both retirement accounts and personal accounts.

In this paper, we study the asset allocation (”how much to invest”) and asset location (”which account to invest in”) consequences when an economic agent has habit formation utility and has access to both a retirement account and a personal account. Retirement account in this paper is set up to mimic the key characteristics of the employer sponsored defined contribution accounts and IRAs. The key characteristics of the retirement account include the following. First, individuals could allocate a portion of their earnings into their retirement accounts. Secondly, retirement accounts are tax favored because investment earnings are exempt from taxes. Moreover, for traditional retirement accounts, individuals could contribute pre-tax earnings to the accounts and pay income tax upon withdrawal[5]. Lastly, a tax-favored retirement account is illiquid to the extent that the funds in the retirement account could only be withdrawn after the age of $59\frac{1}{2}$; early withdrawals are subject to a 10% penalty. On the other hand, earnings (including interests, dividends and realized capital gains) in personal accounts are subject to taxes immediately, but individuals can withdraw funds from personal accounts at any time.

Our study contributes to literature on the ”asset location puzzle”. Theoretical papers[6]

[5]Given this feature, the literature often refers retirement accounts as ”Tax-Deferred Accounts (TDAs)”. ”Tax-Deferred Accounts (TDAs)” is an inaccurate name because individuals could also contribute after-tax money to Roth type accounts. In this case, there is no tax deferral.

[6]See Black (1980), Tepper (1981), Auerbach and King (1983), Dammon, Spatt, and Zhang (2004), Gomes, Michaelides, and Polkovnichenko (2009), among others.

that study retirement account investment decisions predict that individuals should follow a pecking-order strategy – hold the more heavily-taxed assets in retirement accounts and hold the less heavily-taxed assets in personal accounts. Given that bonds are more heavily taxed than stocks in the United States, these papers hence predict that retirement account is the preferred location to hold bonds while personal account is the preferred location to hold stocks. ”Mixed” allocation (holding both stock and bond in the same account), as shown by Dammon et al. (2004), only occurs when the relative size of personal account assets is too large or too small compared to the size of retirement account assets. Specifically, when the relative size of the personal account is too small (large), equity holding (bond holding) ”spills over” from personal (retirement) account to the retirement (personal) account. Therefore, the literature predicts that we should not observe mixed allocations in both the retirement and personal accounts at the same time. Holding both equities and bonds in both the retirement and personal accounts at the same time or holding only stocks in the retirement account are often labelled to be ”tax inefficient”, given the tax arbitrage one could gain if investors move the highly taxed asset (bonds) towards the retirement accounts. However, empirical findings[7] suggest that retirement accounts tilt heavily towards equity. Empirical papers also find that the asset allocation between personal and retirement accounts are often tax-inefficient (Amromin (2003)). This discrepancy is called the ”asset location puzzle” (Amromin (2003)).

Several explanations are provided for the ”asset location puzzle”, most of them focusing on reducing the tax incentives of holding bonds in retirement accounts. For example, Zhou (2009) shows that when introducing a more detailed and realistic income tax code into a life-cycle model, agents hold more risky assets in their retirement accounts because of the higher pre-tax accumulation of wealth given by the high expected stock returns. Garlappi and Huang (2006) use a two-period model and show that for risk averse agents with liquid-

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[7] See Bodie and Crane (1997), Amromin (2003) and Bergstresser and Poterba (2004), among others.

ity constraints, although tax benefits provide incentives to hold more bonds in retirement accounts, the incentive to reduce the volatility of the tax benefits over time leads to violations of the pecking-order asset location strategy. Poterba, Shoven, and Sialm (2000) argue that equity mutual funds are more heavily taxed than individual stocks, and given the high historical returns of equities, holding equity mutual funds in the retirement account helps individuals accumulate more wealth. Moreover, high-income investors could reduce their tax burden of holding bonds in personal accounts by purchasing tax-exempt bonds. More recently, Fischer and Gallmeyer (2016) show that the asymmetric tax treatment of capital gains and losses make it optimal to hold equity mutual funds in retirement accounts, in line with the conclusions in Poterba et al. (2000). However, since recently the share of ETFs (Exchange Traded Funds) has increased and ETFs hand far less capital gains to investors[8], Poterba et al. (2000) and Fischer and Gallmeyer (2016)' s argument might not apply to the current situation.

Our study offers an additional explanation of the asset location puzzle that is not yet studied in the literature – habit formation utility. Habit formation preference assumes that the agent's utility depends on her consumption relative to a reference consumption level instead of her absolute consumption level. The reference level could be internal, such as past consumption, or external, such as overall standard of living (external habit utility is also called "Catching-up with the Jones" utility). Note that the habit formation utility is also in line with the rational addiction utility defined by Becker and Murphy (1988)[9]. As for how the relative consumption is measured, we could categorize habit formation utility into additive

[8] A news report on the increase in the market share of ETFs can be found here: https://www.ipe.com/reports/special-reports/etfs-guide/how-big-can-the-etf-market-become/10021539.article. An explanation of why ETFs are more tax efficient than equity mutual funds can be found here: https://www.etf.com/etf-education-center/21017-why-are-etfs-transparent-and-tax-efficient.html

[9] Specifically, Becker and Murphy (1988) formulates the rational addiction utility as a concave function on past consumption and current consumption; past consumption affects current utility through the "learning by doing" effect similar to the effect of habit or addiction on a person's well-being.

habit, in which the relative consumption is measured as the difference between consumption and habit level, and ratio habit, in which relative consumption is measured as the ratio between consumption and habit level.

In this paper, we consider internal habit formation utility, both in the form of additive habit and ratio habit. By focusing on how habit formation utility affects asset location and allocation decisions in the two investment habitats, we concentrate on the liquidity side of the story rather than breaking the tax advantage of holding bonds in retirement account. Our hypothesis is that, with internal habit formation utility, individuals have strong incentive to smooth their "relative" consumption over time, and thus would prefer to hold less risky assets in their personal accounts to ensure sufficient funds to smooth relative consumption levels given the accessibility restrictions of the retirement accounts. The individuals would hold more risky assets in their retirement accounts for faster accumulation of tax-exempt wealth so that they could raise relative consumption after retirement. In other words, our hypothesis is that we should observe that retirement account is the preferred place for stock, or if not, we should observe tax-inefficient, "mixed", allocation in both accounts.

We test our hypothesis by building a three-period model. Each period of our model is calibrated to match with a twenty-year period so that the periods correspond to young, middle – aged and old. We assume that the agent works in the first two periods while making investment decisions in personal and retirement accounts and is retired in the third period. In each period, the agent first observes the realized stock returns and income, and then makes decisions on consumption, asset allocations in personal and retirement accounts.

We find that our hypothesis holds for the additive habit utility but not for the ratio habit utility. However, when we extend the model with a bequest motive, which provides additional incentive for the agent to save, we generate tax-inefficient asset allocation for the ratio habit case. The reason that ratio habit utility generates weaker result is that for the ratio habit utility function, the coefficient of risky aversion and the coefficient of prudent are the same as a CRRA utility, while for the additive habit utility both the coefficient of risk aversion and coefficient of prudent are higher.

For the additive habit utility case, we find the agent takes less risk in her personal account than in the retirement account when she is young. This is because the agent needs to rely on the money in the personal account to maintain her habit level when she is middle-aged. If the consumption level is lower than the habit level, the additive habit agent would get negative infinity utility, therefore the agent has strong incentive to hold some "cash reserves". Since relative consumption level matters, taking more risk (our model predicts 100% equity) in the retirement account when the agent is young helps to increase the relative consumption when the agent is retired. We also explore various habit formation strength, various tax incentives, and extend the model to allow for bequest motives and employer matches; our hypothesis stands for the additive habit utility in all extensions. In summary, our study shows that the amplified liquidity needs generated by maintaining habit does help to solve the asset location puzzle.

Only a few papers on the asset location puzzle discuss the relationship between liquidity constraints and equity holdings in retirement account and they all study economic agents with CRRA utilities instead of habit formation utilities. With CRRA utilities, these papers need to assume that investors face extremely large liquidity shocks in order to produce deviations from the pecking-order strategy. For example, Amromin (2003) assumes liquidity shocks of 0.5% probability of realizing only 1% of labor income in his three-period model. Dammon et al. (2004) also assume a strong liquidity shock: possibility of a 50% increase in consumption. We show that the liquidity needs generated by maintaining the habit stock are large enough to make agents violate the pecking-order strategy without extreme assumptions of income or consumption shocks.

Our study also contributes to the literature that incorporates habit formation utility to study asset allocation decisions. This literature abstracts from the existence of a retirement account and only studies asset allocation decisions in personal accounts throughout the life-cycle. Gomes and Michaelides (2003) show in a life-cycle model of portfolio choice that both ratio-habit preference and additive-habit preference counterfactually generate stock returns and stock market participation rates that are closed to 100% early in the life-cycle. Their explanation is that habit formation utilities make agents save a larger buffer of wealth and hence increase their incentive to invest in stocks given the high stock returns. Polkovnichenko (2007) shows that with additive-habit utility, agents hold conservative portfolio when young, which is consistent with the data on life-cycle asset holdings. Both Gomes and Michaelides (2003) and Polkovnichenko (2007) consider internal habit formation utilities. Our results show that habit formation utility does increase the saving rates of the agents, but it does not produce risky share closes to 100%, taking into account the risk exposure in both the personal and retirement accounts. Moreover, the agent reduces her risk exposure as she goes from young to middle-aged, consistent with the predictions of the life-cycle literature, but still at odds with the empirical facts that young people hold less stocks than the middle-aged and the old.

The built of the retirement system and habit formation utility

Another motivation for introducing habit formation utility to solve retirement asset allocation problem is that the built of the retirement system seems to assume people have habit formation utilities. In other words, the designers of the retirement system seems to assume that people prefer their consumption level to be similar to their past consumption level when they retire, similar to the Rabbi at the beginning of chapter who provided his service as to run before a poor man who once was from a rich family. Various policies are implemented to ensure that employees receive similar, if not more, income when they retire.

First, consider Social Security – employees are required to contribute a fixed portion of their income to Social Security, and their Social Security income is based on a formula that depends on the highest 35 years of income, which could correlate with their habit levels. As a result, the higher the income an employee has, the higher the contribution, and the higher the Social Security income she receives. Similarly, the pension formula for defined benefit plans often depends on the last five years of income during employment. Therefore, the higher the income an employee receives, with potentially higher level of habit stock, she also receives higher pension income after retirement.

For the voluntary part of the retirement system, that is, defined contributions, there are also incentives for employees to make sure their retirement income is similar to habit level. For instance, defined contribution accounts are tax favored, as mentioned in the previous part of this chapter, so that employees have the incentives to save in their defined contribution accounts. Moreover, many companies match the contribution that the employee make. After all, the matching provided by the employer could be distributed as other forms of employee benefits. In summary, even though it is usually voluntary to contribute to defined contribution plans, employees are induced to make contributions.

Since the contribution to defined contributions plans is also a fraction instead of a fixed amount of the wages, the higher an employee's income, the higher her contribution. Contributions to IRAs and 401(k)s are subject to a fixed dollar threshold, therefore employees with extremely high income might not be able to accumulate defined contributions assets that are proportion to their income. This is reasonable because individuals with extremely high income have more saving capacities.

Is the retirement income from defined contribution proportional to an employee's potential habit level? Assume a 30-year-old earns \$380,000 after-tax per year, and contributes 5%, which equals to \$19,000[10], of her after tax income to a Roth 401(k) account. This 30-year-old's company provides 1 to 1 matching on her contribution. Assume that this worker earns the same amount and contributes the same amount each year for 35 years[11], and that the rate of return of her retirement portfolio stays at 6% per year, then her retirement account balance will be at \$4,188,920 when she is 65-year-old. At age 65, a woman is expected to live for 22 more years, till age 86. Assume that the rate of return after retirement is still 6%, and the inflation rate is 0%, then she could withdraw \$328,500 per year for 22 years, each withdrawal equals to 86% of her after-tax income. If we include the Social Security income this worker receives after retirement, we could see that her retirement income is very closed to her work-life income – her potential habit level.

2.2 The Model

In this section, we present a three-period model to show how an agent with internal habit formation utility makes investment decisions, along with consumption and saving decisions, when having access to a traditional, taxable personal account and a tax-favored but illiquid retirement account. This three-period model builds on the three-period model in Gomes et al. (2009). The three-period model in Gomes et al. (2009) is used to study the implication of the introduction of a retirement account on wealth accumulation, hence they assume that there is only one safe asset for simplicity. We expand their model by introducing a risky asset and using habit formation utility.

Time is discrete, t = 1, 2, 3, and the periods correspond to young, middle-aged, and retirement periods. We assume that each of the three periods spans 20 calendar years. The agent gains utility from the ratio between consumption and a habit level. Specifically, the utility

[10] The limit of contribution to a Roth 401(k) and a traditional 401(k) is \$19,000 per year.

[11] Note that this is a conservative calculation because the limit on contribution is increased after age 50.

function is given by:

$$U_t(C_t) = \frac{(C_t/H_t{}^{\lambda})^{1-\rho}}{1-\rho}, \tag{2.1}$$

where C_t refers to period t consumption, H_t refers to period t habit level, ρ is the coefficient of risk aversion in the CRRA utility and $\lambda \in [0,1]$ is the habit strength parameter. For simplicity, we assume that the habit level equals to the previous period's consumption: $H_t = C_{t-1}$. Given this assumption, the utility function can be re-written as:

$$U_t(C_t) = \frac{(C_t^{1-\lambda} \cdot \frac{C_t}{C_{t-1}}{}^{\lambda})^{1-\rho}}{1-\rho}. \tag{2.2}$$

For λ with values strictly between 0 and 1, the utility function is determined by a weighted average of consumption and relative consumption. When $\lambda = 0$, this utility function collapses into a traditional CRRA utility function in which only current consumption matters. When $\lambda = 1$, the agent only cares about the ratio of current consumption and habit level. For ratio habit formation utility, the coefficient of relative risk aversion is constant and it equals to the CRRA risk aversion coefficient ρ. Therefore, habit strength does not affect the agent's risk aversion. However, habit strength does affect the Elasticity of Intertemporal Substitution (EIS): the agent has stronger incentive to smooth consumption intertemporally as habit strength increases, while her risk aversion stays constant. Later in this chapter, we consider an alternative functional form of internal habit formation, in which the difference of current consumption to habit matters instead of the ratio.

The problem that the agent faces is to maximize the expected utility defined over the admissible consumption sequences $\{C_t\}_{t=1}^{3}$ over the three periods. The expected utility can be written as:

$$U_1 = E_1 \sum_{t=1}^{4} \beta^{t-1}[p_t \cdot u_t + (1-p_t) \cdot b \cdot \frac{W_t^{1-\rho}}{1-\rho}], \tag{2.3}$$

where $\beta < 1$ is the discount factor, p_t denotes the conditional survival probability with $p_4 = 0$

(the agent dies in period 4), and b denotes the strength of bequest motive. In our baseline model, we assume that there is no bequest motive, therefore we set $b = 0$. As mentioned before, we assume the habit level equals to the previous period's consumption and the habit level in the first period is a constant denoted by H^1:

$$H_t = C_{t-1}, \text{and } H_1 = H^1 \tag{2.4}$$

In each period, the agent receives labor income which follows:

$$y_1 = 1, y_2 = (1+g)\tilde{\eta}, y_3 = \gamma y_2. \tag{2.5}$$

g in period 2 income denotes the income growth rate from young to middle-aged, $\tilde{\eta}$ is a random shock. For simplicity, we assume that the retirement income y_3 is a constant fraction γ of middle-aged income y_2.

We also assume that in periods 1 and 2, the agent contributes a pre-determined x percent of her income into a defined-contribution retirement account. In practice, most companies use an opt-in policy on defined contribution plans enrollment. Conditional on enrollment, contribution rate clusters at the default contribution rate, if a default is available (Choi (2015)). Therefore, we could think of the agent in our benchmark model choosing to opt in to the retirement plan provided by her company and deciding to contribute a default x percent of her income[12]. We will later allow the agent to freely change the percent of income contributed to the retirement account, that is, making the contribution rate x a choice variable in the model.

The balance in the retirement account is only available when the agent is retired (t=3)[13].

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[12] There also are institutions, for example, the University of Washington, whose contribution rates are fixed: if employees choose to enroll in the retirement plan, they cannot change the contribution rate. Source: https://hr.uw.edu/benefits/retirement-plans/uw-retirement-plan/.

[13] In this model, we do not allow early withdrawal of the retirement account.

Therefore, the agent can only use personal account savings for consumption in the first two periods. We assume that the retirement account is a Roth account. That is, contributions made to the retirement account are after-tax labor income, and the investment returns in the retirement account are tax-exempt. Roth accounts and traditional retirement accounts are equivalent when the tax rate that the agent faces at different age is unchanged, and the contribution limits stay the same.

There are two types of assets, bonds and stocks. Interest payments of bonds are taxed at τ_d and capital gains of stocks are taxed at τ_c. To reflect that fact that bonds are more heavily taxed than stocks, we have $\tau_c < \tau_d$. We assume that the stocks are not paying dividends for simplification.

The budget constraints that the agent faces are as follows. In period t = 1, it is

$$C_1 + S_1^T = (1 - x) \cdot y_1; \tag{2.6}$$

in period t = 2, it is

$$C_2 + S_2^T = (1 - x) \cdot y_2 + \tilde{R_2^T} \cdot S_1^T; \tag{2.7}$$

and in period t = 3, it is

$$C_3 = y_3 + \tilde{R_3^T} \cdot S_2^T + \tilde{R_3^R}(x \cdot y_2 + \tilde{R_2^R}(x \cdot y_1)), \tag{2.8}$$

where C_T denotes consumption in time t, S_t^T denotes savings in the personal account in time t (capital T represents that investment returns in this account are subject to taxes), $\tilde{R_t^T}$ denotes the gross portfolio return in the personal account at time t and $\tilde{R_t^R}$ denotes the gross portfolio return in the retirement account at time t. The budget constraints in the first two periods reflect the fact that the agent could only use the funds in the personal account for consumption. In period 3, the agent has access to both the personal and the retirement accounts. Since the agent dies in period 4, she will consume all her wealth in period 3.

Portfolio returns in the personal account are subject to taxes:

$$\tilde{R_t^T} = \pi_{t-1}^T \cdot (1 + (1 - \tau_c)(\tilde{R}_t - 1)) + (1 - \pi_{t-1}^T) \cdot (1 + (1 - \tau_b)(R_f - 1)) \tag{2.9}$$

where π_{t-1}^T denotes the share of period $t-1$ personal account savings invested in stocks. Portfolio returns in the retirement account are exempt from capital gains taxes and taxes on interest rates, so

$$\tilde{R_t^R} = \pi_{t-1}^R \cdot \tilde{R}_t + (1 - \pi_{t-1}^R) \cdot R_f, \tag{2.10}$$

where π_{t-1}^T denotes the share of period $t-1$ retirement account savings invested in stocks. In summary, the agent chooses first two periods' consumptions ($\{C_t\}_{t=1}^2$), or equivalently savings in the personal account$\{S_t^T\}_{t=1}^2$, risky share in personal account ($\{\pi_t^T\}_{t=1}^2$) and risky share retirement account ($\{\pi_t^R\}_{t=1}^2$) to maximize equation (2.3), subject to constraints (2.4) – (2.10).

2.3 Calibration and Solution Method

We assume each period in this model corresponds to a 20-year period and calibrate the parameters accordingly. We set the discount rate $\beta = 0.987^{20}$, risk aversion coefficient $\rho = 5$, both values commonly used in the literature. Habit strength parameter is assumed to be $\lambda = 0.8$ in our baseline case. We also consider other values of the habit strength parameters in our sensitivity analysis section. Bond return is assumed to be $R_f = (1 + 0.02)^{20}$. The survival probabilities are obtained from Cocco et al. (2005): $p_1 = 0.9582$ is the conditional probability of surviving from age 21 to age 40, $p_2 = 0.8316$ is the conditional probability of surviving from age 41 to age 60, and $p_3 = 0.3756$ is the conditional probability of surviving from age 61 to age 80. For simplicity, we assume the initial habit level $H^1 = 1$. In the case of ratio habit utility, it means that the agent enters the first period without any habit. We assume the tax rate on interest to be 25%, and the capital gains tax rate to be 20%. The agent is assumed to be forced to allocate $x = 6\%$ of her income into the retirement account[14].

There are two sources of uncertainties. First, labor income in period 2, y_2, is subject to a random shock $\tilde{\eta}$. Following Gomes et al. (2009), we assume the labor income shock has a standard deviation $\sigma_\eta = 0.15\sqrt{20}$, and $\tilde{\eta}$ takes the two values with equal possibilities: $\tilde{\eta} = 1 \pm \sigma_\eta$. Note that the income shocks assumed here is much smaller than the catastrophic income shocks assumed in Amromin (2003). The gross growth of income from period 1 to 2 is set at $g = (1 + 0.04)^{20}$. The second source of uncertainty is the gross stock returns $\tilde{R}_t$. We assume that $\tilde{R}_t$ takes two values with equal possibilities: $\tilde{R}_t = \tilde{R}_t + \sigma_R$ or $\tilde{R}_t = 1$, where $\tilde{R}_t = (1 + 0.06)^{20}$ and $\sigma_R = 0.20\sqrt{20}$. That is, stock returns over a 20-year period could be about 300% or zero. We also assume the agent is subject to borrowing and short-sale constraints, so we have

$$C_t \leq 0, \forall t = 1, 2, 3 \tag{2.11}$$

$$1 \geq \pi_t^T, \pi_t^R \geq 0, \forall t = 1, 2. \tag{2.12}$$

To solve this model, first re-arrange constraints 2.6 - 2.8 and plug into the objective function 2.3 . We have

$$\max_{C_1 S_2^T, \pi_1^T, pi_2^T, \pi_1^R, \pi_2^R} \frac{(C_1/H_1^\lambda)^{1-\rho}}{1-\rho} + \beta E \frac{(((1-x)\cdot y_2 + \tilde{R_2^T} \cdot ((1-x)\cdot y_1 - C_1) - S_2^T)/C_1^\lambda)^{1-\rho}}{1-\rho}$$
$$+ \beta^2 E \frac{\left(\frac{y_3 + \tilde{R_3^T} \cdot S_2^T + \tilde{R_3^R}(x \cdot y_2 + \tilde{R_2^R}(x \cdot y_1))}{((1-x)\cdot y_2 + \tilde{R_2^T}\cdot((1-x)\cdot y_1 - C_1) - S_2^T)^\lambda}\right)^{1-\rho}}{1-\rho}. \tag{2.13}$$

This problem is solved using grid search: we discretize the six control variables $C_1, S_2^T, \pi_1^T, pi_2^T, \pi_1^R, \pi_2^R$ into nodes on their respective admissible ranges, then plug all the potential values into the

[14] No employer matching is assumed for the base case.

objective function (2.13), and search for the combination of control variable values that maximizes equation (2.13). Due to the curse of dimensionality, six control variables produce a large grid that request a lot of computation power. We therefore use the method of adaptive grid search: start with a coarse grid and solve the problem multiple times, each time refining the search range of the control variable surrounding the new solution.

2.4 Results

2.4.1 Baseline results

We solve five versions of this model. We start with a very simple model with deterministic labor income, fixed contribution rates, standard CRRA utility, no difference in tax rates in the taxable and retirement account. Then we gradually add risky labor income, habit formation utility, tax rate differences in the two accounts, as well as flexible contribution rates to each subsequent versions and examine how each of the additional feature affects the results. Table 2.1 summarizes the features in each version of the model.

Table 2.2 presents the results from the five versions of the model using the same set of parameters. To help with interpretation, note that the income process without the random shock in our model are $y_1 = 1$, $y_2 = 3.19$ and $y_3 = 2.18$. The gross stock returns in our model could take two values, 4.1016 and 1. The risk-free rate over a 20-year period is 1.4859. In this setting, the expected returns of stocks are higher than the risk-free rate but investing in stocks could result in a return that is lower than that risk-free returns with 50% probability.

Version 1: Deterministic income

Column (1) of Table 2.2 shows the solutions for Version 1 of our model, in which labor income is deterministic and there is no habit formation utility, no tax rate differences between bonds and stocks. Contribution rate to retirement account is fixed at 6%. In both periods 1 and 2, the agent does not save in her personal account. Given that there is no uncertainty in the income process, the agent saves only to smooth consumption over the three periods. The 6% mandatory saving to the retirement account is sufficient for the agent's saving incentive, hence she does not keep extra savings in her personal account. In the retirement account, risky share equals 100% and slightly reduces to 87.6% in the second period, consistent with the age pattern of risky asset holding predicted by the life-cycle models. The risky share is high because of the high expected returns of the stock, and because the saving rate (6%) is not very high. When I experiment with higher mandatory saving rates, the risky share in the second period starts to decrease (results available upon request).

Version 2: Risky Income

Column (2) of Table 2.2 shows the solutions for Version 2 of our model, in which income becomes risky. The agent now saves 10.18% of her period 1 income in her personal account. When the agent is young, she faces income uncertainty in the next period. The model generates a stock-spill over result in the first period: the agent holds 100% stocks in personal account and 89.76% stocks in her retirement account. In both periods 1 and 2, the agent holds less risky assets compared to Version 1. This is intuitive because in Version 2, period 2 income fluctuates between 5.33 and 1.05 with equal likelihood. Given the possibility of a low-income state in period 2 and consequently, a low-income state in period 3 (note that period 3 income is assumed to be a fixed portion of period 2 income), the agent optimally chooses to hold less risky assets in the first two periods in case both stock returns and income are in their low states. In summary, when income becomes uncertain, the agent starts to accumulate precautionary savings the personal and reduces her risky asset holdings in both periods.

Version 3: Risky Income Ratio Habit Formation Utility

Column (3) of Table 2.2 shows the solutions for Version 3 of our model, in which ratio habit formation is introduced. The agent reduces her period 1 saving slightly while increasing her period 1 risky asset holding to 100% in both accounts. This result is consistent with the finding in Gomes and Michaelides (2003) that habit formation utility makes the agent take more risk in order to accumulate more precautionary wealth. In the first period, the agent holds 100% of the retirement account savings in stocks.

Version 4: Risky Income + Habit Formation Utility + Tax Incentives

Since most discussions in the asset location puzzle literature focus on tax incentives, we then introduce tax rate differences in version 4 to see whether tax incentive distorts the investment decisions in personal and retirement account for agents with habit formation utilities. As shown in Column (4) of Table 2.2, when tax incentives are introduced to the model, the results are almost the same as Version 3: the agent slightly increases her period 1 personal account saving given the lower after-taxed returns in the personal account. The risky asset positions are very similar to Version 3: the agent holds 100% risky assets in both accounts in period 1, does not save in personal account in period 2 and reduces the risky share in retirement account in period 2.

To investigate whether holding 100% risky asset in both personal account and retirement account is consistent with the pecking order strategy, we run some experiments by raising the mandatory contribution rates. When saving rates are forced to increase, the desired risk exposure reduces. We find that the agent first reduces her risky asset holding in her retirement account, consistent with the pecking order strategy that personal account is the desired habitat for risky assets. More specifically, when the mandatory contribution rate is raised to 20%, the risky share in personal account in period 1 is still 100% while the risky share in retirement account reduces to 79.25%. When mandatory contribution to retirement account increases, the relative capacity of personal account reduces, hence we observe a stock-spill-over from personal account to retirement account.

Version 5: Risky Income + Habit Formation Utility + Tax Incentives + Flexible Contribution Rates

Column (5) of Table 2.2 shows the solutions for Version 5 of our model. In version 5, we relax the requirement that the agent must contribute 6% of her income to the retirement account in periods 1 and 2. Instead, we allow the agent to decide the fraction of her income contributed to the retirement account. We assume that the agent can choose to contribute a maximum of 10% of per-period income to her retirement account, following Gomes et al. (2009). In period 1, the agent contributes 4.28% of her income to retirement account. The fact that the agent is willing to contribute some of her period 1 income to the retirement account indicates that the incentive to smooth relative consumption introduced by the ratio habit utility is strong enough for the agent to give up some liquidity in order to boost future consumption.

In period 2, the agent contributes a smaller portion, 2.50% of her period 2 income to the retirement account. The personal account savings in period 1 is slightly higher than the case with mandatory retirement account savings – the difference in the saving rates in Version 4 and 5 shows that the agent would save a little more in her personal account and contribute a little less to her retirement account given the accessibility constraint in the retirement account. Since the contribution rate is less than the default 6%, the agent allocates all her retirement account wealth to stocks in period 1, and a higher percentage, compared to Version 4, of retirement account wealth to stocks in period 2.

Summary

For an agent with ratio habit utility, the accessibility restriction of retirement account does not seem to generate additional liquidity need to hold safe assets in personal account when young. This result is consistent with the findings in Gomes and Michaelides (2003): ratio habit utility does not change the agent's risk aversion coefficient, hence the saving rates are similar in models with ratio habit and in models with CRRA utility. The risky share when young is higher with ratio habit utility because of the strong incentive to boost future relative consumption and because of the high stock returns. Young agents hold more risky assets than middle-aged agents, consistent with the predictions of the majority of the life-cycle models of portfolio choice that risky share declines with age. We conclude that in our current three-period setting the model with ratio habit utility cannot generate asset allocation strategies that contradict with the pecking-order strategy.

2.4.2 Baseline Results with Additive Habit Formation Utility

Setup

Since the ratio habit utility does not generate incentives to hold riskless assets in the personal account, we now consider the case when the agent has additive habit formation utility (following Polkovnichenko (2007) , Detemple and Zapatero (1991), Constantinides (1990), Sundaresan (1989), among others):

$$u_t(C_t) = \frac{(C_t - H_t)^{1-\rho}}{1-\rho}, \tag{2.14}$$

where $\rho > 0$ denotes the curvature of the CRRA utility function, C_t denotes the level of consumption at time t and H_t denotes the habit stock at time t. We assume that the habit level at time t is a constant fraction δ of pervious period's consumption:

$$H_t = \delta C_{t-1}, \tag{2.15}$$

where $\delta \in [0, 1]$ denotes habit strength. When $\delta = 0$, the additive habit utility collapses into CRRA utility. Note that with additive habit utility, the effective relative risk aversion coefficient now equals to

$$RRA = -(cU_{cc})/U_c = C/(C - H) > \rho. \tag{2.16}$$

The coefficient of relative prudence (Kimball (1990)) also depends on the habit level:

$$Prudence = -(cU_{ccc})/U_{cc} = (\rho + 1)C/(C - H). \tag{2.17}$$

When consumption is closed to the habit level, relative risk aversion and the coefficient of prudence become very large. When consumption is equal or less than the habit level, marginal utility as well as relative risk aversion become infinity. This is arguably an undesirable characteristic of additive habit utility. However, the property that risk aversion and prudence change as the difference between consumption and habit level changes provides more meaningful result for us to compare - the agent saves more and start to hold riskless assets in her personal account in both periods 1 and 2. We can therefore compare how the risky asset is allocated in the personal account versus in the retirement account. The results are presented in Table 2.3. The first two columns of Table 2.3 are the same as the first two columns of Table 2.2, because in Version 1 and 2 of our model, we have not introduced habit formation utility. Our discussion of the additive habit utility therefore starts with Version 3 of our model, in which income is risky and the agent has additive habit utility.

Version 3: Risky Income + Additive Habit Formation Utility

As shown in Column (3) of Table 2.3, when additive habit formation is introduced to Version 3, the agent raises her personal account saving rate significantly: saving rate is 43.40% in period 1 as compared to a 10.18% saving rate in the CRRA case. The agent also saves 12.51% of her personal account in the second period. The saving incentive with habit formation utility differs from the incentive to smooth consumption observed in the CRRA utility case of versions 1 and 2: with additive-habit formation utility, the relative level of consumption across periods becomes important – more important than the ratio habit case – the agent thus wants to increase her savings in order to increase future consumptions.

The agent takes more risk in period 1 compared to period 2, as the risky shares are higher both in the personal and in the retirement account in period 1 than in period 2. This is again consistent with the predictions of traditional life-cycle models, that young agents, who have more "bond-like" human capital stocks compared to the middle-aged and the old, should hold more risky assets, and then reduce the risky share when they age.

In period 1, we observe a reverse spill-over of stock holding: the agent invests 100% of her retirement account savings in stocks, while holding 13.79% of her personal account savings in stocks. This is consistent with our hypothesis that, since the agent could only maintain her habit level via her personal account savings, she would take less risk in her personal account than in her retirement account in period 1.

In period 2, when income uncertainty disappears and the accessibility restriction of the retirement account is going to be relaxed, the agent's allocation rule tilts towards the pecking order strategy even though we have not introduced the tax incentive feature to the model: the agent holds 13.32% stocks in her personal account while reducing the risky share of her retirement account savings significantly from 100% to 5.73%. In period 2, the agent knows that she would consume all the realized returns from her personal and retirement account investments in the next period, so she optimally adjusts the risky share to maximize expected consumption in period 3. Taking risks in personal account or in retirement account should not matter in period 2, since investments in both accounts have the same horizon and liquidity. Therefore, the lower risky share in the retirement account than in the personal account in period 2 does not contradict our hypothesis.

The result of Version 3 indicates that the agent first adjusts the risky share in retirement account before adjusting the risky share in personal account when reducing her overall risk exposure in period 2. In addition, we can see that overall the agent takes much less risk in the additive habit case than the ratio habit case, consistent with the intuition that additive habit utility increases the agent's effective risk aversion.

Version 4: Risky Income + Additive Habit Formation Utility + Tax Incentives

As shown in column (4) of Table 2.3, when tax incentives are introduced in Version 4, the expected returns of investments in the personal accounts are lower. The agent responds by slightly increasing the personal account saving rate in period 1 while increasing the personal account risky share to 16.80% and 20.00%, respectively in periods 1 and 2. The saving rate in the personal account in period 2 remains closed to version 3, the case with no tax rate difference. The agent still holds 100% risky assets in her retirement account in period 1 but reduces the risky share to 1.54% in period 2. This shows that overall, the tax rate difference in bonds and stocks does shift risky assets from the retirement account towards personal account. However, with additive habit utility, the incentive to increase relative future consumption and the inability of accessing retirement account savings before retirement is strong enough for the agent to keep holding 100% risky assets in retirement account and holding less risky assets in her personal account when young.

Version 5: Risky Income + Additive Habit Formation Utility + Tax Incentives + Flexible Contribution Rates

As shown in column (4) of Table 2.3, when flexible contribution is allowed, the agent does not contribute to the retirement account in the first period, when the tension between maintaining habit level and access to liquidity is the strongest. Instead, she keeps all her savings in her personal account. Her personal account saving rate is hence higher than all previous cases – 50%. Since personal account saving rates are higher, the agent optimally takes less risk in her personal account. In period 2, savings in the retirement account and in personal account have the same liquidity, so the agent allocates a small portion (5.00%) of period 2 saving to the retirement account and follows the tax-efficient pecking order strategy when allocating assets: all her period 2 retirement account savings are invested in bonds while 17.05% of personal account assets are invested in stocks.

Summary

In the additive-habit formation case, the agent increases her personal account saving rate significantly, consistent with the property of additive habit that the coefficient of prudence increases. In period 1, when the tension between liquidity constraints and maintaining habit through the personal account is the strongest, the agent holds 100% risky assets in her retirement account while holding much less risky assets in her personal account, in contradiction with the pecking order strategy and in line with our hypothesis. Young agents hold more risky assets than middle-aged agents, consistent with the predictions of the majority of the life-cycle models of portfolio choice. Moreover, since additive habit formation utility effectively increases the agent's risk aversion, the magnitude of risky share in personal account is closer to the data than other papers. For example, Zhou (2012) uses CRRA utility in a life-cycle model with both personal and retirement accounts, and he finds that risky shares are closed to 100% in both accounts when investors are young.

2.5 Sensitivity Analysis

2.5.1 Habit Strength

We conduct sensitivity analysis to see how the behavior of the agent is affected by the habit strength level. In all of the models that we solve above, the habit strength level is set at 0.8. Using the setting of Version 4, we re-solve the model using various habit strength levels for both the ratio habit case and additive habit case. For the additive habit case, we use habit strength parameters $\delta = [0.2, 0.4, 0.6, 0.8]$. When habit strength parameter increases, the relative risk aversion and the level of prudence of the agent increase.

Figure 2.1 shows the risky shares in personal and retirement accounts and Figure 2.2 shows the saving rates in the personal account under various habit levels for the additive habit case. We can see that savings and investment decisions are sensitive to habit strengths. As shown in Figure 2.2 , saving rate in personal account increases monotonically with habit strength in both periods as the coefficient of prudence increases with habit strength parameter. As shown in Figure 2.1 , in period 1, the agent holds 100% risky assets in retirement account (represented by the line with square nodes) for all habit strengths except for the case with the lowest habit strength ($\delta = 0.2$). Risky asset holding in personal account reduces monotonically with habit strength in period 1 as shown by the line with circle nodes. Therefore, for all the habit strength levels we consider, we observe the reverse spill over of stock from retirement account to personal account when the agent is young. In period 2, except for the case of low habit parameter, the agent's asset allocation strategy tilts towards the pecking order strategy, in which the agent holds close to 0% of risky asset in the retirement account, represented by the line with diamond nodes, and slightly higher risky assets in personal account, represented by the line with star nodes. The overall risky share also reduces with habit strength.

For the ratio habit utility case, we use habit strength parameters $\lambda = [0.2, 0.4, 0.6, 0.8, 1]$. The resulted risky shares in personal and retirement accounts are shown in Figure 2.3. We can see that with ratio habit utility, the saving rate and risky share are less affected by the habit strength parameter, which is understandable since the coefficient of prudence and risk aversion stay constant and are the same with CRRA utility. Consistent with the original Version 4 of our model, the agent holds 100% risky asset in both the personal and retirement accounts in period 1, as shown by the overlapped lines with circle and square nodes. As shown in Figure 2.4, the agent does not save in the personal account in period 2 while her personal account saving rate is decreasing with the habit strength parameters. On the other hand, the agent increases her risky asset holding in the retirement account, represented by the line with diamond nodes, slightly as habit strength increases to boost relative consumption.

2.5.2 Various Bond Tax Rates

Results of Version 4 and Version 5 of our model show that, with additive-habit formation utility, the incentive to hold safe but heavily-taxed asset (bonds) in the retirement account is dominated by the incentive to hold a safer portfolio in personal account to maintain habit level. Here, we investigate how the asset allocation decision changes when the tax rate difference between stocks and bonds increases.

To increase the tax rate gap between bonds and stocks, we increase the bond tax rate from 25% to 40%, 60% and 70%, respectively, and run Version 4 of our model with additive-habit. The higher the tax differences, the more tax-arbitrage loss for the agent to hold stocks in her retirement account.

As shown in Figure 2.4, consistent with our intuition, when the tax rate difference increases, the agent has more incentive to hold more risky assets in her personal account, so the risky share in personal account in both periods 1 and 2 increases, but only slightly: the risky share in the personal account stays almost flat in period 1 as bond tax rate increases while the increase risky share in period 2 is less than fifteen percentage points when the bond tax rate is raised to 70% from 25%. Moreover, the risky share in retirement account is still at 100% in period 1 for all tax rate differences. This shows that, when the tax rate gap becomes unrealistically large, the incentive to hold a safer portfolio in personal account to maintain habit level when the agent is young still dominates the incentive to hold heavily-taxed asset in the retirement account.

The risky share in the retirement account in period 2 drop to 0% (from 0.8%) while the risky share in the personal increasing with the bond tax rate. Again, in the second period, since there is no accessibility constraint in the retirement account in the next period, tax incentives dominates the incentive to use the personal account to retain liquidity.

As for the ratio-habit case (results not shown in the figure), difference in tax rates does not seem to be a factor that affects the agent's decision because all of the agent's savings are in retirement account. The incentive to smooth relative consumption makes the agent to still hold stocks in her retirement account, given the higher expected returns of stock.

2.6 Extensions

2.6.1 Bequest Motive

Investors with bequest motive are expected to save more. Given the stronger saving motive and the longer effective investment horizon, the differences in investment strategies in retirement and personal accounts could also be bigger. We therefore extend Version 4 of our model (which features habit formation utilities and different tax treatments on retirement account and personal accounts) to include a bequest motive. The modification we make is to allow the bequest parameter b in equation 2.3 to be positive.

When b is positive, the agent will gain utility from wealth that she leaves behind whenever she dies. We assume the motive of leaving bequest is altruistic instead of strategic (i.e., the agent leaves a bequest in order to exchange for service provided by their children). With the bequest parameter $b > 0$, the agent also needs to make saving and investment decision in period 3 since she is not willing to consume everything in period 3. We assume that capital gains of the stock from period 3 to period 4 are tax-free to take into account the fact that the cost basis of a stock is "stepped-up" when the decedent dies. However, interest payments are taxed when a bond is inherited, so bond returns are taxed from period 3 to period 4. Given such an asymmetry in tax treatment when financial asset is inherited, an agent with bequest motives might have more incentive to hold more stocks in her portfolio.

The bequest motive parameter b could be loosely interpreted as the effective extension in investment horizon generated by the bequest motive, but the value of b is hard to calibrate. Dynan, Skinner, and Zeldes (2002) use a two-period life-cycle model and show that one cannot distinguish bequest motives and precautionary savings when there are uncertainties in the model. This is because a dollar saved today serves both as a precautionary life-cycle function, for example guarding against health shocks, and a bequest function, for example when medical expenses are less than expected, the extra wealth saved becomes bequest. Cocco et al. (2005) also show in their multi-period life-cycle model that changing the value of b from 1 to 5 does not change the risky asset holdings of agents over the life cycle much until b equals to 5. Motivated by the findings of Cocco et al. (2005), we therefore set b to be 5.

The result of this extension is shown in Table 2.4 for both ratio habit utility and additive habit utility. Columns (1) and (3) display the Version 4 of the model without bequest motive, for the ratio habit and additive habit, respectively, in order to facility comparisons. Columns (2) and (4) show the results with bequest motive, for the ratio habit and additive habit, respectively.

For both functional forms of the habit utility, introducing a bequest motive changes the saving and investment behavior of the agent significantly. First, the agent increases her savings rate in personal account by a lot. Without bequest motive, the agent saves 8.22% of her period 1 income in her personal account and does not save in her personal account in period 2 with ratio habit utility. With bequest motive, the agent saves 35.07% in period 1 and 26.35% in period 2. For the additive habit utility, the agent saves even more in her personal account, 80.22% and 45.97% in periods 1 and 2 respectively, since additive habit utility increases prudence. The higher savings rates show that with longer implicit horizon, the agent with habit formation utility tends to further compress current consumption in order to increase relative future consumption.

Given the higher saving rate, the agent reduces her overall risk exposure in the two accounts in the first two periods as compared to the no bequest motive case. For the ratio habit utility case, the agent invests 15.92% of her personal account savings in stocks while holding 37.30% of her retirement account savings in stocks, confirming our hypothesis that, when the agent does save in both her personal account and retirement account, she takes less risk in her personal account in period 1 in order to maintain habit level in period 2. The agent reduces the risky shares in both accounts in period 2 by taking almost no risk (1.22%) in the retirement account and holding 11.72% of period 2 personal account saving in stocks, again consistent with the pecking order strategy. For the additive habit case, as shown in Column (4), the overall risky share is much lower than the ratio habit case, consistent with the fact that additive habit utility increases risk aversion. The agent barely takes risk in her personal account in both periods.

In period 3, the agent invests about 20% of period 3 wealth in stocks for both the ratio habit and additive habit case, higher than the pervious period's risky asset holdings because of the tax incentive for inherited capital gains. We also consider a weaker bequest motive, in which $b = 3$ and the resulted behavior is similar to the case when $b = 5$ (results available upon request). Therefore, in contrast to the result of Cocco et al. (2005), a small bequest motive (in our case $b = 3$) is enough to generate behavior differences.

2.6.2 Employer Match

Many employers provide matching to contributions to DC plans. In this section, we expand our model to consider the effect of employer match on asset location and allocation decisions in personal and retirement accounts. Other things equal, employer match, if utilized by the agent, increases the relative size of retirement account assets. The agent may then adjust her asset location and allocation strategies.

We consider two scenarios. First, an employer that provides 1 to 1 matching up to 10% of the employee's income. Secondly, an employer that provides matching up to 3% of the employee's income while the employee could choose to contribute up to 10% of her income. 3% is chosen following Gomes et al. (2009). The results are shown in Tables 2.5 and 2.6. Both Tables 2.5 and 2.6 have three columns, the first column display the result of Version 5 of our model, in which the agent can freely choose contribution rate but here is no employer match, the second column displays the results of 3% employer match and the third column displays the results of 10% employer match.

For the ratio habit case presented in Table 2.5, as shown in Column (2), a 3% employer match makes the agent contribute a similar percentage of income to the retirement account in both periods 1 and 2 while taking a little less risk in the second period in the retirement account given the increase in the size of the retirement account resulted from employer matching. A 10% employer match, as shown in Column (3), does not generate equivalent contribution in the first two periods. When liquidity need is the strongest, i.e., in period 1, the agent only contributes 4.38% of income to retirement account and hence only gets 4.38% of income from employer matching despite the 10% potential employer match. This contribution rate, however, is higher than when employer match is 3%, indicating there is some incentive for the agent to increase contribution when the matching potential is high. In period 2, given the potentially high wealth accumulation from the first period, the agent reduces her contribution to 2.50% while optimally reducing her risk exposure. The agent does not save in her personal account in both periods – all savings are in the retirement account to enjoy pre-tax investment returns.

For the additive habit case presented in Table 2.6, a 3% employer match does not generate much distortion in the agent's behavior as shown in Column (2): the agent contributes only 0.74% of period 1 income to the retirement account. In period 2, the agent contributes 2.93% her income to retirement account, closed to the 3% matching limit. Compared to the case without employer matching as shown in column (1), a 3% employer match still results in higher retirement wealth. Given the slight increase in retirement wealth and hence an increase in overall wealth, the agent also increases her personal account risky asset holding slightly in both periods.

For the 10% match case, we generate strong results as shown in Column (3): the agent contributes 10% of income to the retirement account in both periods, which greatly increases her retirement account wealth and hence her expected last period consumption. The agent thus saves less in her personal account in period 1 (28.57%, from 50% without matching) and invests 29.29% of her personal account savings to stocks, consumes all the personal account wealth in period 2, and relies only on retirement account wealth for consumption in period 3. For the additive habit agent, we observe the reverse stock spill over from retirement account to personal account the 3% match case. When there is a 10% match, since the retirement asset is large with the chosen 10% contribution rate and the employer match, the risky share in retirement account is 30.44% instead of 100% in period 1 and the personal account risky share is 29.29%. Although we don't observe a reverse spill-over of stocks, the asset holdings in both accounts are tax-inefficient in the 110% match case. Therefore, our hypothesis holds for the additive habit utility when we expand the model with employer match.

The literature has different opinions on the effects of employer matching on retirement plan participation and contributions (see Engelhardt and Kumar (2006) and references therein). Our result here shows that the matching rate matters when the agent has habit formation utilities. Agent with additive habit utility, given her strong risk aversion and prudence, does not respond much to small employer match. Agent with ratio habit utility, on the other hand, does increase her contribution rate when matching rate increases, but only when she is young.

2.7 Conclusion

In this chapter, we build a three-period model to show that internal-habit formation utility could be an answer to solve the asset allocation puzzle. An individual with habit formation utility needs to maintain a targeted habit level and can only depend on their personal accounts to do so before retirement. On the other hand, investments in retirement accounts have longer durations and higher returns for both bonds and stocks. Therefore, our hypothesis is that we should observe that the retirement account is the preferred place for stock, or if not, we should observe tax-inefficient, "mixed", allocation in both accounts.

We find that if habit is of the additive form, the contrasts between the risky asset holdings in personal and in retirement account is stronger, since additive habit utility increases the agent's risk aversion and prudence. An agent with additive habit utility optimally holds less risky asset in her personal account and takes more risk (in our model, 100% equity) in the retirement account when she is young. The tension between maintaining habit level and access to liquidity vanishes when the agent makes investment decisions in middle-age: the agent reduces the risky share in retirement account while maintaining similar risky share in her personal account. When we extend the model with a bequest motive, which provides additional incentive for the agent to save, we generate tax-inefficient asset allocation for the ratio habit case. Our finding persists for the additive habit utility when we allow the strength of habit to vary, when we introduce bequest motive and when we allow employer match.

2.8 Tables

Table 2.1: Features in Each Version of the Model

	Version 1	Version 2	Version 3	Version 4	Version 5
Risky Stock Returns	✓	✓	✓	✓	✓
Risky Labor Income		✓	✓	✓	✓
Habit Formation Utility			✓	✓	✓
Tax Rate Differences				✓	✓
Flexible Contribution Rate				✓	✓

Table 2.2: Results from Various Versions of the Ratio Habit Model Using the Same Set of Parameters

Note, contribution rate is fixed at 6% in Versions 1 to 4, and it becomes a choice variable in Version 5.

	(1) Version 1	(2) Version 2	(3) Version 3	(4) Version 4	(5) Version 5
t = 1					
Personal Account					
Saving Rate	0%	10.18%	8.37%	8.22%	9.34%
Risky Share	-	100%	100%	100%	100%
Retirement Account					
Contribution Rate	6%	6%	6%	6%	5%
Risky Share	100%	89.76%	100%	100%	100%
t = 2					
Personal Account					
Saving Rate	0%	0%	0%	0%	0%
Risky Share	–	–	–	–	–
Retirement Account					
Contribution Rate	6%	6%	6%	6%	2.5%
Risky Share	87.06%	60.43%	59.28%	60.64%	72.45%

Table 2.3: Results from Various Versions of the Additive Habit Model Using the Same Set of Parameters

	(1) Version 1	(2) Version 2	(3) Version 3	(4) Version 4	(5) Version 5
t = 1					
Personal Account					
Saving Rate	0%	10.18%	43.40%	45.49%	50.00%
Risky Share	-	100%	13.79%	16.80%	11.03%
t = 1 Retirement Account					
Contribution Rate	6%	6%	6%	6%	0%
Risky Share	100%	89.76%	100%	100%	-
t = 2					
Personal Account					
Saving Rate	0%	0%	12.51%	12.77%	18.78%
Risky Share	–	–	13.32%	20.00%	17.05%
t = 2 Retirement Account					
Contribution Rate	6%	6%	6%	6%	4.8%
Risky Share	87.06%	60.43%	5.73 %	1.54%	0%

Table 2.4: Extending Version 4 of the Baseline Model with Bequest Motive

	Ratio Habit		Additive Habit	
	(1)	(2)	(3)	(4)
	Version 4	w/ Bequest Motive	Version 4	w/ Bequest Motive
t = 1 Personal Account				
Saving Rate	8.22%	35.07%	45.49%	80.22%
Risky Share	100%	15.92%	16.80%	1.95%
t = 1 Retirement Account				
Contribution Rate	6%	6%	6%	6%
Risky Share	100%	37.30%	100%	28.12%
t = 2 Personal Account				
Saving Rate	0%	26.35%	12.77%	45.97%
Risky Share	0%	11.72%	20.00%	–
t = 2 Retirement Account				
Contribution Rate	6%	6%	6%	6%
Risky Share	60.64%	1.22%	1.54%	15.62%
t = 3				
Saving Rate		24.22%		3.47%
Risky Share		20.70%		20.31%

Table 2.5: Extending Version 5 with Employer Match in Ratio Habit Case

	(1)	(2)	(3)
	Version 5	3% Match	10% Match
t = 1 Personal Account			
Saving %	9.34%	7.14%	7.27%
Risky Share	100%	100%	100%
t = 1 Retirement Account			
Contribution Rate	5.00%	3.13%	4.38%
Risky Share	100%	100%	100%
t = 2 Personal Account			
Saving Rate	0%	0%	0%
Risky Share	-	-	-
t = 2 Retirement Account			
Contribution Rate	2.5%	3.12%	2.50%
Risky Share	72.45%	62.50%	59.72%

Table 2.6: Extending Version 5 with Employer Match in Additive Habit Case

	(1) Version 5	(2) 3% Match	(3) 10% Match
t = 1 Personal Account			
Saving %	50.00%	50.00%	28.57%
Risky Share	11.03%	12.66%	29.29%
t = 1 Retirement Account			
Contribution Rate	0%	0.74%	10%
Risky Share	-	100%	30.44%
t = 2 Personal Account			
Saving Rate	18.78%	18.58%	0%
Risky Share	17.05%	18.37%	-
t = 2 Retirement Account			
Contribution Rate	4.8%	2.93%	10%
Risky Share	0%	0%	9.56%

2.9 Figures

Figure 2.1: Risky Shares in Personal and Retirement Accounts Under various habit strength levels with Additive Habit Formation Utility

This figure plots the period 1 and period 2 risky shares in personal and retirement accounts that corresponds to habit strength parameters 0.2, 0.4, 0.6 and 0.8, using the setting of Version 4 of our model.

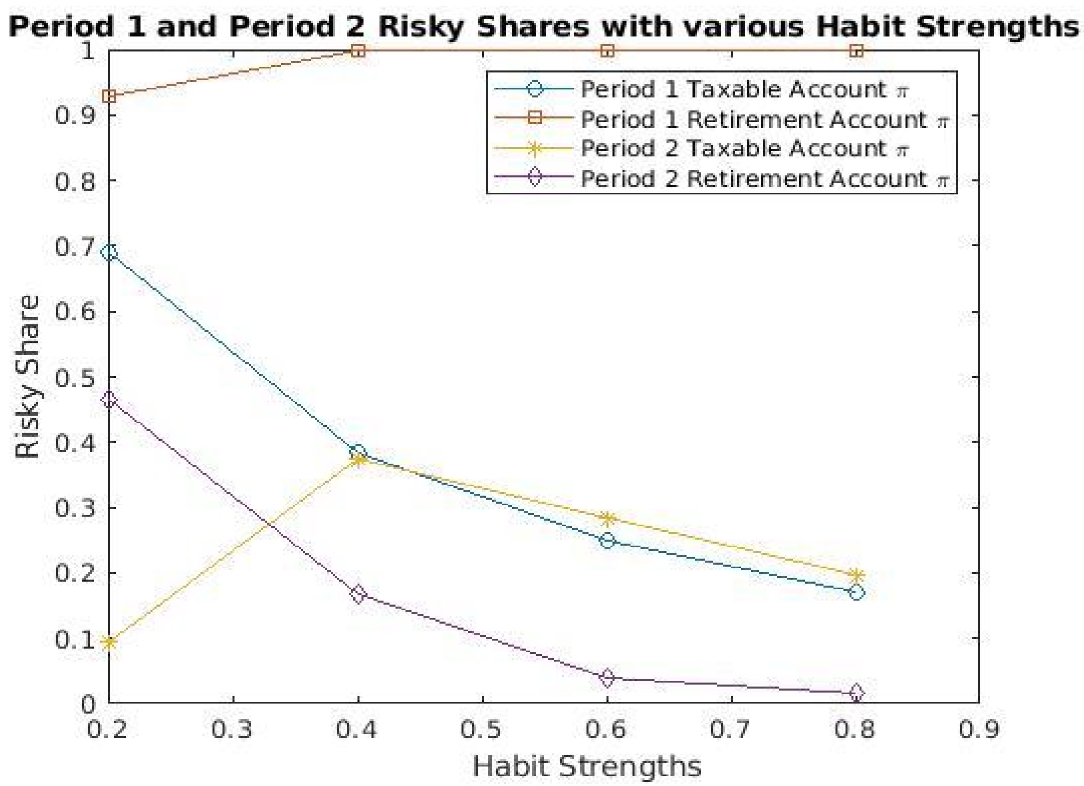


Figure 2.2: Personal Account Saving Rates Under Various Habit Strength Levels with Additive Habit Formation Utility

This figure plots the period 1 and period 2 saving rate in the personal account that corresponds to habit strength parameters 0.2, 0.4, 0.6 and 0.8, using the setting of Version 4 of our model.

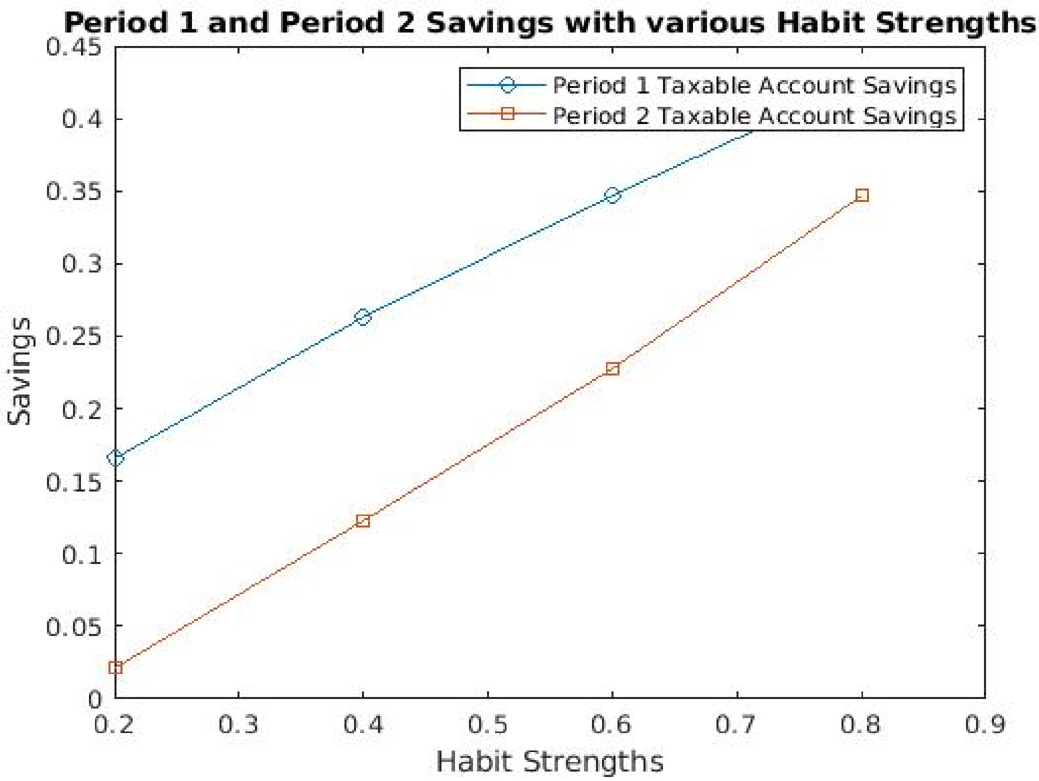


Figure 2.3: Risky Shares in Personal and Retirement Accounts Under Various Habit Strength Levels with Ratio Habit Formation Utility

This figure plots the period 1 and period 2 risky shares in personal and retirement accounts that corresponds to habit strength parameters 0.2, 0.4, 0.6, 0.8 and 1, using the setting of Version 4 of our model.

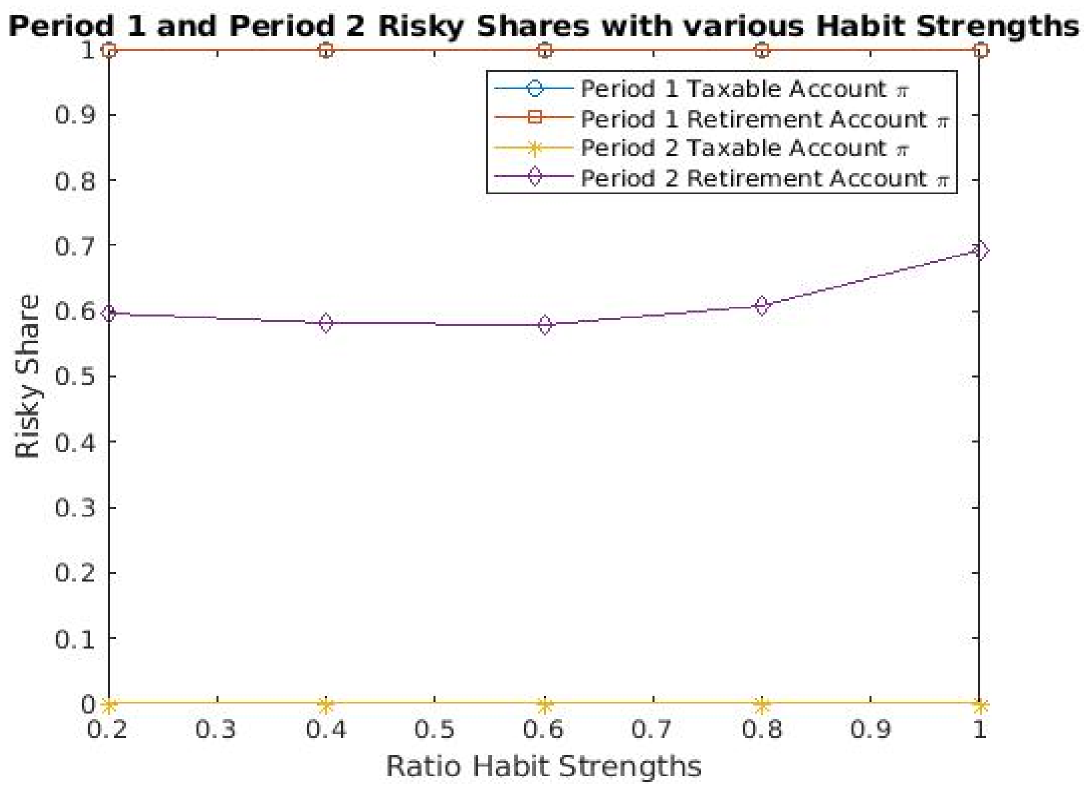


Figure 2.4: Personal Account Saving Rates under Various Habit Strength Levels with Ratio Habit Formation Utility

This figure plots the period 1 and period 2 saving rate in the personal account that corresponds to habit strength parameters 0.2, 0.4, 0.6, 0.8 and 1, using the setting of Version 4 of our model.

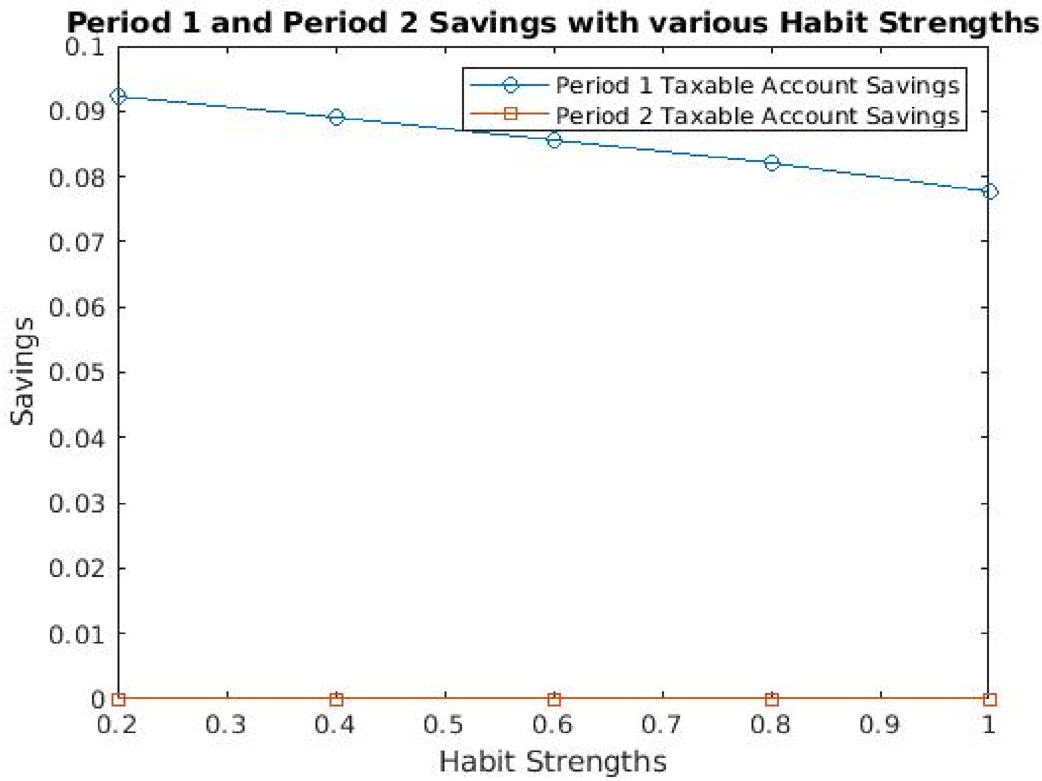
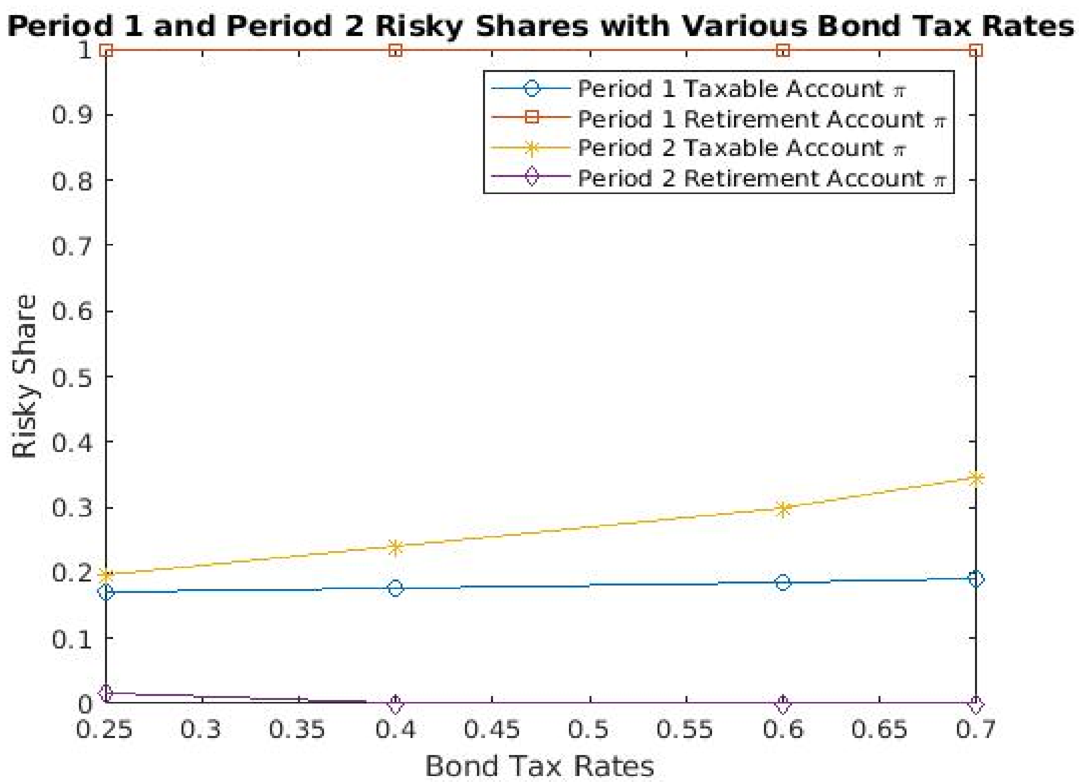


Figure 2.5: Risky Shares in Personal and Retirement Accounts under Various Bond Tax Rates

This figure plots the period 1 and period 2 risky shares in personal and retirement accounts that corresponds to bond tax rates of 25% (benchmark bond tax rate), 40%, 60% and 70%, using the setting of Version 4 of our model and additive habit utility.



Chapter 3

LIFE-CYCLE PATTERNS OF STOCK MARKET PARTICIPATION RATE AND RISKY SHARE IN THE UNITED STATES

3.1 Introduction

Life-cycle models of portfolio choice predict that agents should reduce the proportion of savings allocated to risky assets (henceforth *risky share*) as they age, both when they receive deterministic labor income as in Merton (1971) and when labor income is risky and uninsurable as in Cocco et al. (2005). The reasoning behind this prediction is that human capital, even when it is risky and uninsurable, has low correlation with stock returns and is the largest when an agent is young. The agent should therefore hold more risky asset when she is young, and as human capital depletes, reduce the risky share as she ages.

Another implicit prediction of the life-cycle models is that the agent should always hold a positive amount of stocks as long as she has some savings. That is, the optimal risky share is always positive. However, empirical findings are often at odds with these clear-cut theoretical predictions. In many countries, stock market participation rate is significantly lower than 100% – that means many people do not hold any risky asset. Stock market participation rate displays strong age pattern. It is usually found to be hump-shaped over the life cycle(Giuliano and Spilimbergo (2013)). The age-profile of the risky share, depending on the data, the way to measure risky share and the identification strategies, is either found to be flat or slightly upward sloping over the life cycle (Ameriks and Zeldes (2004)), or is found to be hump-shaped over the life-cycle (Chang, Hong, and Karabarbounis (2018), Guiso and Jappelli (2000)).

In this chapter, we provide updated estimations of the age profiles of stock market participation and risky share in the United States using a panel data set from the Panel Study of Income Dynamics (PSID). The motivation of this chapter is the recent findings in Fagereng et al. (2017). Fagereng et al. (2017) use a panel of administrative data from Norway and find that the risky share is high and stable during the early years and gradually declines over the life-cycle. This finding is much more consistent with the theoretical predictions than previous empirical findings. Fagereng et al. (2017) attribute their novel findings to the quality of data and the identification strategies that they employ - they use a Heckman selection model to account for the endogeneity of stock market participation while estimating the age profiles of conditional risky share.

Indeed, both the quality of data and identification strategies are very important to provide accurate estimations of the age profiles of stock market participation and risky share[1]. Most of the empirical studies in the United States use data from the Survey of Consumer Finances, which is a cross-sectional survey[2]. Using cross-sectional data is problematic because when we pool the data of people with the same age but are from different cohorts, we can not distinguish age and cohort effects. Panel data set tracks the same groups of people over a certain period of time and hence allows us to distinguish age and cohort effects. The administrative data used by Fagereng et al. (2017) is also better than survey data, because survey data are prone to measurement errors. In this chapter, we use a panel data set that spans an 18-year period which helps us to better distinguish age and cohort effects. The drawback of our data set is that it is not administrative data, so it could still be subject to measurement errors. Nonetheless, we think it offers new insights on the age profiles of

[1]A much more in depth discussion can be found in Ameriks and Zeldes (2004) and Giuliano and Spilimbergo (2013).

[2]Ameriks and Zeldes (2004) use a panel data set from the TIAA-CREF in their study, but their data only covers asset holdings in defined contribution accounts. In addition, the owners of the TIAA-CREF accounts are university employees, who are not representative enough for the whole U.S. population.

the stock market participation rate and conditional risky share since no previous empirical study on the United States use a national representative panel data set.

There is a well-known identification challenge in separately identifying age, cohort and year effects[3]. Using birth year to label each cohort, there is a linear relationship among age, cohort and year: age = year - cohort (birth year). Assuming that the dependent variable that we are interested in is a function of age, cohort and year ($y = f(a, c, t)$), and given the linear relationship among age, cohort and year, the data generating process of the dependent variable can also be written as $y = f(a, c, a + c) = f(a, a - t, t) = f(t - c, c, t)$. As a result, we cannot separately identify age, cohort and year effects without further assumptions on the age, cohort and year effects.

We use both the dummy variable regressions used in Ameriks and Zeldes (2004) as well as the identification method used in Fagereng et al. (2017) to break the linear relationship of the age, cohort and year effects. Ameriks and Zeldes (2004)'s empirical strategy is to assume either cohort or year effects are zero, and then estimate the remaining year or cohort effects un-restrictively using dummy variables. We start our estimation process following Ameriks and Zeldes (2004)[4]. We are interested to see how the age profiles differ when we use the same identification strategy but with a different data set.

Fagereng et al. (2017) make different assumptions than Ameriks and Zeldes (2004) to break the linear relationship among age, cohort and year. Namely, Fagereng et al. (2017) use a proxy variable (youth stock returns) instead of birth years to account for cohort effects. Their argument is that past studies, such as, Giuliano and Spilimbergo (2013) and Malmendier and Nagel (2011) show that past social economic conditions affect a person's risk preference as

[3]A much more in depth discussion on the identification challenge of the age, cohort and year effects can be found in Ameriks and Zeldes (2004)

[4]Ameriks and Zeldes (2004) is the most cited paper on the age profiles of stock market participation and risky share in the United States.

well as investment choices. To control for year effects, Fagereng et al. (2017) use the identification strategy in Deaton and Paxson (1994) and assume that year effects sum to zero when de-trended. We follow Fagereng et al. (2017)'s method to estimate age, cohort and year effects as well.

Another often ignored identification problem is the endogeneity of the decision to invest in risky assets. Since the risky share that we observe is concentrated among those people who choose to invest in risky assets, the estimated age profile of the risky share could be biased if the decision to invest in stocks was not random. Following Fagereng et al. (2017), we use a Heckman selection model to account of the selection while estimating the age profiles of conditional risky share. Fagereng et al. (2017) also include a set of demographic variables to generate age profiles conditional on these demographic variables, and we do the same. This exercise is an external validation of Fagereng et al. (2017): is the estimated age profile of conditional risky share decreasing when using data from the United States if we follow the empirical strategy in Fagereng et al. (2017)?

Our study stand out from others in two aspects. First, as mentioned before, we use a national representative panel data set that spans an 18-year period which helps to better control for age, cohorts and year effects. To my knowledge, almost all the previous studies on the age profiles of stock market participation and risky share in the United States use cross-sectional data such as the SCF or even a single-year cross-sectional data set. For example, Ameriks and Zeldes (2004) (and the reference therein) and Chang et al. (2018). Our study is also the first study that utilizes the panel structure of the PSID to study the age pattern of stock market participation and risky share[5]. Second, we employ alternative empirical strategies to

[5]Many papers use the PSID to study other aspects of household asset holdings decisions. To name a few, Chen and Stafford (2016) study how the stock market participation pattern changes as households face mortgage payment difficulties during the 2007-09 financial crisis. Zhou (2018) investigates how stock ownership changes before and after the Great Recession. Shen (2018) studies how risky shares are affected by countercyclical income risks. Love (2009) studies how marital status affect risky share.

control for age, cohort and year effects and to control for the selection into the stock market. Previous studies on the United States, such as Ameriks and Zeldes (2004) and Chang et al. (2018) use only the dummy variables to control for age, cohort and year effects and do not take into account endogeneity of selection into the stock market.

The main findings of our paper are the follows. First, when using dummy variable regressions as in Ameriks and Zeldes (2004), stock market participation rate is hump-shaped over the life-cycle. However, we find households start to gradually exit the stock market at earlier age when we use age and cohort dummies than when we use age and cohort dummies. The conditional risky share is mostly flat, only slightly increasing over the life cycle, with a high initial risky share at the very beginning of the life-cycle (age 20). Second, stock market participation rate becomes strongly increasing, from 10% at the beginning of the life-cycle, to 80% at later stages of life, when we follow the empirical strategy in Fagereng et al. (2017). In other words, the hump-shaped pattern of stock market participation rate disappears and we do not see gradual exit from the stock market as in previous papers. Third, the conditional risky share is still slightly increasing over the life cycle, but is at an overall higher level when we account for selection and use Fagereng et al. (2017)'s method to separately identify age, cohort and year effects. We also see a very high starting conditional risky share at the very beginning of the life cycle.

In summary, our results differ quite a bit from Fagereng et al. (2017), who use Norwegian data, since we do not see decreasing risky share using the U.S. data. Our result also differ from Ameriks and Zeldes (2004), who use cross-sectional U.S. data from an earlier period, when we use more complicated empirical strategies to estimate the age profiles – stock market participation rate is increasing instead of hump-shaped, and risky share is at a higher level, and is slightly increasing over the life cycle.

3.2 Data

The Panel Study of Income Dynamics (PSID) is a longitudinal household survey started in 1968. PSID contains a national representative sample of individuals and their families living in the United States as well as subsamples that focus on low-income families, immigrants and young adults. Starting in 1984, PSID collected data on wealth in five-year frequencies, and it made some major changes in the wealth survey and started to collect wealth data bi-annually in 1999. We therefore assemble our sample using the 1999 to 2017 waves of the PSID survey. Our sample spans an 18-year period and it contains 10 waves of the PSID survey. Although PSID contains individual level data, we use family level data assuming each family is one decision unit. We include only the families that are in the national representative core PSID sample and the families that have wealth data available.

3.2.1 Descriptive Results - Summary Statistics

We construct two samples and conduct our analysis on both samples. Table 3.1 presents summary statistics on the two samples. "Full sample" includes all the families who are present in any waves of the 1999 - 2017 sample period. "Balanced panel" includes only the families that are present PSID throughout the whole sample period. The number of families in the balanced panel is only 30% of the number of families in the full sample, indicating that the attrition rate of PSID during the 1999 - 2017 period is high. As shown in the first section of Table 3.1, families in the balanced panel are on average older, more educated and have slightly fewer children. There are significantlyfewer families with a female head in the balanced panel, while the family size in the balanced panel is on average slightly larger than the family size in the full sample.

The second section of Table 3.1 display the asset holding information on the two samples. To examine the asset holding patterns of the families, four different measures of wealth are presented in Table 3.1. *Liquid wealth* is constructed as the value of checking or savings accounts, money market funds, certificates of deposits, government saving bonds, treasury bills, bond funds, cash value in a life insurance policy, valuable collection for investment purposes, rights in a trust or estate, stocks in publicly held corporations, mutual funds, and investment trusts. *Financial Wealth* is liquid wealth plus the value of private annuities or IRAs. Assets included in the calculation of *Net Worth w/o Home Equity* includes financial wealth as well as value of vehicles, farm or business, and real estate other than the household's home equity. *Net Worth w/ Home Equity* includes both *Net Worth w/o Home Equity* and the value of home equity. We exclude the top and bottom 1% of the wealth distribution when calculating the summary statistics. All dollar values reported in Table 3.1 are in 1999 USD. Comparing the full sample and the balanced panel sample, families in the balanced panel are much wealthier in every measure of wealth.

PSID reports information on stock holdings in private annuities or IRAs (Individual Retirement Accounts) and value of stocks in publicly held corporations, mutual funds, and investment trusts outside of employer-based pensions or IRAs. For simplicity, we label stocks in private annuities or IRAs as *Stocks in IRA Accounts*, and stocks outside of employer-based pensions or IRAs as *Stocks in non-IRA Accounts.* Accordingly, we construct three measures of stock market participation: a family is considered a "participant" if it holds positive amount of stocks in the designated account. *All Accounts* takes into account stock market participation in both the non-IRA accounts and IRA accounts. *Non-IRA Accounts* only considers stock market participation in non-IRA accounts, and *IRA Accounts Only* considers stock market participation in IRA accounts, conditional on having an IRA account[6].

On average, the balanced panel has higher stock market participation rate than the full sample. The mean stock market participation rate in the full sample, including both the non-IRA accounts and IRA accounts, is 32%. The stock market participation rate in non-

[6]29.2% of the families in the full sample and 40.5% of the families in the balanced panel have money in private annuities or IRAs.

IRA accounts is 21% in the full sample and 29% in the balanced panel, which is consistent with the finding in Chen and Stafford (2016), who also use PSID data and report stock market participation rate in non-IRA accounts. The mean stock market participation rate in our sample is lower than the mean of around 50% reported by papers that use the Survey of Consumer Finances (SCF) as data source[7]. This is likely because of two reasons. First, the SCF over-samples high income families, who are more likely to invest in stocks. Secondly, the SCF has stock holding information in defined contribution plans, while PSID does not have information on the value of defined contribution or the asset holdings in defined contribution plans[8]. Notably, the stock market participation rate in the IRA accounts is much higher than the stock market participation rate in non-IRA accounts - 73% in the full sample and 77% in the balanced panel, suggesting that many households participate in the stock market through their retirement accounts.

PSID reports value of stocks in non-IRA accounts but it does not ask families for the specific value of the stocks in IRA accounts, private annuities or IRAs. Instead, there is a question about how the funds in private annuities or IRA are invested and the respondents are asked to choose the following three answers: A) Mostly stocks, B) Mostly interest earning, C) Split. We therefore infer the value of stocks in private annuities or IRA by assuming that those who answered A) in fact allocated 100% of their IRA funds to stocks, 0% when they answered B) and 50% when they answered C). Those who answered A) or C) are also labelled as a stock market participant through IRA accounts only. This is a common practice used in Ameriks and Zeldes (2004) and Zhou (2018). The conditional risky share in all accounts is then calculated as the value of stocks in both the non-IRA and IRA accounts divided by liquid wealth, conditional on owning stocks in either type of account.

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[7]For example, Ameriks and Zeldes (2004) and Chang et al. (2018).

[8]In fact, if we look at percent of households owning stock through directly owner ship + mutual funds + trusts (equivalent to the stock market participation rate in non-IRA accounts in this chapter), reported in Row (3) of Table 6 in Ameriks and Zeldes (2004), we can see that this number is in similar range (20 to 30%) with the stock market participation rate in non-IRA accounts reported in this chapter.

The conditional risky share in non-IRA accounts is the value of stocks in non-IRA accounts divided by liquid wealth, conditional on owning stocks in non-IRA accounts. The conditional risky share in IRA accounts is the value of stocks in IRA accounts divided by the value of private annuities and IRAs. Interestingly, all three measures of the conditional risky share of families in the balanced panel are only slightly higher than the conditional risky share of families in the full sample. The conditional risky share in IRA accounts is higher than the conditional risky share in non-IRA accounts. The differences in stock market participation rate and conditional risky share between IRA accounts and non-IRA accounts indicate that households are prefer to take more risk through private annuities or IRAs than through non-IRA accounts. All three measures of the risky share are also in line with Ameriks and Zeldes (2004).

The subsequent analysis in this chapter will focus on the full sample only. We also do the same analysis on the balanced panel, and the results are in the Appendix 3.6.

3.2.2 Age Profiles by Cohort

For an initial examination of the age profiles of stock market participation rate and risky shares, we plot two measures of stock market participation rate and their corresponding conditional risky share by age for five-year cohort groups in Figure 3.1. The cohort labelled "1920" includes families with heads born in years 1920-1924, cohort labelled "1925" includes families with heads born in years 1925-1929, ..., and so on.

The cohort view of stock market participation rate in all accounts, presented in Figure 3.1a, shows that for cohort groups between 1935 and 1970, the stock market participation rate first increases and then decreases over the 1999 to 2017 sample period. We could say that stock market participation rate is roughly hump shaped over the life-cycle for these cohorts, but it could also be that these cohorts were all affected by a common time effect. For the younger, 1975 and 1980 cohorts groups, stock market participation rate increases rapidly in the first half of the sample period then decreases. For older cohorts born before 1934, stock market participation rate decreases over the sample period. The cohort view also shows that there is a steady increase in stock market participation rate across cohort groups between 1980 and 1945, while the levels of stock market participation rate for cohorts groups between 1925 and 1940 are similar. Given the differences in stock market participation rate across cohorts, it seems that cohort effects are also influencing the stock market participation decisions of families in the sample.

The cohort view of the stock market participation rate in non-IRA accounts, shown in Figure 3.1b differs from the cohort view of all accounts in three aspects. First, the stock market participation rate in non-IRA accounts for all cohort groups are lower. This is consistent the summary statistic table that the mean stock market participation rate in non-IRA accounts is much lower than the mean stock market participation rate in the IRA accounts, and hence it is lower than the mean stock market participation rate in all accounts. Second, except for the youngest two cohorts groups, the stock market participation rate decreases over the 1999-2017 sample period, while there are a few increases during the sample period. Third, for the youngest two cohort groups, the stock market participation rate in non-IRA accounts increases at a slower rate than the stock market participation rate in all accounts. We can therefore infer that families in the 1980 and 1975 cohort groups increase their participation in the stock market mostly through their IRA accounts during the sample period. Similar to the cohort view of the stock market participation rate in all accounts, the stock market participation rates in non-IRA accounts are cohort specific - there is a steady increase across cohort groups for cohort groups younger than the 1945 cohort group, and then some small decreases across cohort groups that are older.

Figure 3.1c displays the conditional risky share in all accounts by age for five-year cohorts. For all cohorts except for the youngest and the oldest cohorts, the risky share swings around a slightly upward sloping trend over the sample periods. This could be interpreted in two ways: the risky share only slightly increases with age, or all cohorts are affected by common time effects that makes the risky share slightly increase over the years. Older cohorts on average hold more equities than younger cohorts, which contradicts to the theoretical predictions of the life-cycle models of portfolio choice that households should reduce their exposure to risky assets as they age.

The cohort view of risky share in non-IRA accounts in Figure 3.1d differs a lot from the cohort view of risky share in all accounts. Notably, the age profile of risky share for cohort groups younger than the 1950 cohort group looks like a "W" - risky share first declines in the first two or three sample periods (1999 to 2003), then sharpely increases to a level lower than the initial level during 2005 to 2007, and then it declines further to a even lower level, and then it rises again starting in 2011. Since the timing of the change in risky share matches quite well with the dot-com bubble and the great recession, it seems the younger cohorts are affected by time effects. The age profiles of the risky share for older cohorts do not show as large decline over the whole sample period as the middle aged and younger cohorts. The differences in risky share in non-IRA accounts across cohorts are more distinct than the differences in all accounts - older cohorts have higher risky shares in most of the sample periods than younger cohorts.

From the above analysis, it seems that we cannot single out the effects of age on stock market participation decision or risky share without further assumptions.

3.3 Estimation

3.3.1 Dummy Regressions

Given the linear relationship between age, year and cohort (age = year - cohort), it is impossible to separately identify each of these three effects without further assumptions. In this section, we start with simple dummy variable regressions following Ameriks and Zeldes (2004). More specifically, we regress the dependent variable of interest, which are two measures of stock market participation rate and their corresponding risky share measures, on 1) age dummies and cohort dummies and 2) age dummies and year dummies. We use Probit regressions when using stock market participation rates as dependent variable, and Ordinary Least Squares (OLS) when using risky shares as dependent variable. When we regress the dependent variable on age dummies and cohort dummies, the implicit assumption is that there are no year effects. When we regress the dependent variable on age dummies and year dummies, the implicit assumption is that there are no cohort effects.

Figure 3.2 plots the predicted stock market participation rates and risky shares against age for the year 1999, when age and year dummies are used, and for the 1960 cohort (who are 39 year-old in 1999), when age and cohort dummies are used. As shown in Figure 3.2a, the predicted stock market participation rate is hump-shaped over the life cycle, but peaks at different ages when age and year dummies are used compared to when age and cohort dummies are used. When age and year dummies are used, stock market participation rate starts at about 10% at age 20, keeps increasing util about age 62, then it decreases to about 30% at age 90. On the other hand, when age and cohort dummies are used, stock market participation rate starts at about 23% at age 20, keeps increasing until about age 30, and then decreases for the rest of the life cycle.

The stock market participation rate in non-IRA accounts becomes increasing over the life cycle when age and year dummies are used, as shown in Figure 3.2b. When age and cohort dummies are used, the age profile of stock market participation rate in the non-IRA accounts is similar to the age profile of stock market participation rate in all accounts - hump-shaped and peaking at around age 30.

According to Panels (c) and (d) of Figure 3.2, the predicted conditional risky share starts from about 70% for all accounts and 80% for non-IRA accounts at the beginning of the life-cycle, stays at a relatively high level during the first ten years of the life cycle (between 50 to 60%). The risky share then drop to about 45% to 50% after age 30, and then starts increasing. The fact that the conditional risky shares start at a high level at the beginning of the life-cycle indicates that the young families are behaving closer to the prediction of the standard life-cycle portfolio choice model, i.e. risky share should be high when agents are young, given the bond-like human capital is largest when agents are young. The predicted conditional risky share is slightly higher when age and year dummies are used than when age and cohort dummies are used.

Summarizing, the estimated age profile of stock market participation rate in this chapter looks almost the same as the age profile reported in Ameriks and Zeldes (2004), who used data from an earlier period (1989 to 1998) and from a cross-sectional survey SCF, when age and year dummies are used. When age and cohort dummies are used, the estimated age profile in this chapter is hump-shaped, while the corresponding age profile in Ameriks and Zeldes (2004) is increasing. The estimated age profile of risky share is at an overall higher level and has a higher upward slope than the age profile of risky share reported in Ameriks and Zeldes (2004), when age and year dummies are used. When age and cohort dummies are used, Ameriks and Zeldes (2004) finds their estimated age profile of risky share strongly increasing, while in this chapter the age profile is only slightly increasing. Overall, the age profiles generated by dummy variable regressions in this chapter do not differ materially from those reported in Ameriks and Zeldes (2004).

3.3.2 Heckman Section Model

We now impose more parametric assumptions and use proxies to identify age, cohort and year effects, following Fagereng et al. (2017). To control for cohort effects, Fagereng et al. (2017) use a weighted average of Norwegian and world stock returns as a proxy for cohort effects. The literature shows that the cohort-specific experiences of macroeconomic conditions and stock returns during the cohort's formative ages (18 to 25 year old) affect each cohort's view of the stock market, their likelihood to invest, and the proportion of risky assets that they hold conditional on investing. In this chapter, we use the average S&P 500 returns at age 18-25 for each cohort as a proxy for cohort effects. With cohort dummies being replaced by the cohort proxy, we can estimate year and age effects unrestrictively.

To control for time effects, Fagereng et al. (2017) adopt the Deaton and Paxson (1994) assumption that time effects sum to zero and are orthogonal to a linear time trend. The idea is that with a long enough panel from which we can distinguish trend and cycle, the time effects are cyclical and would cancel each other out once they are de-trended (Deaton (2019)). Since our sample covers an eighteen-year period, we should be able to distinguish trend and cycle and use the parametric assumption on time effects.

Following Fagereng et al. (2017), we include a set of demographic variables such as the number of children, whether the head is female, family size, health status of head, and whether the head is single while estimating the age, cohort and year effects. Using dummy variables without the demographic variables as in the previous section gives us unconditional effects of age. With demographic variables included, the estimated age effects could be interpreted as age effects conditional on the demographic variables.

In addition, we use a Heckman selection model to account for the endogeneity of the decision to participate in the stock market. As pointed out by Fagereng et al. (2017), previous papers that try to estimate the age profile of risky share ignore the fact that the decision to invest in stock market is endogenous, hence the risky share that we observe are biased by the selection into the stock market. To correct for the selection bias, we estimate a Probit model on the possibility of stock market participation and a risky share equation for those who holds stocks, taking selection into account. For the exclusion restriction of the selection into stock market, Fagereng et al. (2017) use a measure of total wealth[9], which is the sum of financial wealth and the present discounted value of human capital. The idea behind using total wealth for the exclusion restriction is the classic life-cycle model of portfolio choice introduced by Merton (1971). Namely, the absolute level of wealth, including both the value of financial wealth and the present value of human capital, does not affect the share of risky assets over financial wealth - only the relative size of risky asset to total wealth affects the decision of how much of the financial wealth to invest in the stock market. On the other hand, the value of total wealth affects the decision to participation in the stock market, assuming there is a fixed cost to participate in the stock market (Vissing-Jorgensen (2002)).

The Heckman selection model can be written as:

$$s_{iact} = \beta_a A_a + \beta_c C_c + \beta_t Y_t + \theta Z_{iact} + \theta_2 \lambda_{iact} + \epsilon_{iact}, \tag{3.1}$$

with:

$$prob(P_{iact} = 1|x) = prob(P^*_{iact} > 0|x) = prob(\delta_a A_a + \delta_c C_c + \delta_t Y_t + v Z_{iact} + v_2 W_{iact} > 0). \tag{3.2}$$

Equation (3.1) is often called the outcome equation in the Heckman selection model, where

[9]Our approach differs from Fagereng et al. (2017) as Fagereng et al. (2017) use lagged value of total wealth while we use the current value of total wealth. The reason for using current instead of lagged value in this chapter is two fold. First, since each wave of the PSID data is two years apart, it is unlikely that households make stock market participation decision based on total wealth two years ago than using their current level of total wealth. Secondly, using lagged total wealth will result in a big loss of data as the 1999 wave could not be used for the Heckman selection model. Therefore, we used current level of wealth instead of lagged total wealth.

s_{iact} is the conditional risky share for household i that we observe in year t whose age is a and who belongs to cohort c. We only observe s_{iact} for those individuals who choose to hold risky assets, hence we need some additional method to control of this selection bias. A_a, C_c, Y_t are age, cohort and time dummies, respectively. Z_{iact} are above-mentioned demographic variables for household i. λ_{iact} is the inverse Mills ratio. The significance of the coefficient of λ_{iact} tells us whether there is non-zero covariance between the decision to participate in the stock market and the decision of how much risky assets to hold. Equation (3.2) is the selection equation. The decision to participate in stock market ($P_{iact} = 1$) depends on age, cohort, year, demographic variables as well as the level of total wealth W_{iact}.

When we are controlling for cohort effect, we replace the cohort dummies in Equations (3.1) and (3.2) with the average S&P 500 returns when each cohort c is at age 18-25. When we are controlling for time effect, we follow the method outlined in Deaton (2019) and replace the year dummies in Equations (3.1) and (3.2) with a set of $T - 2$ year dummies defined as follows, from t = 3, ..., T:

$$d_t^* = d_t - [(t-1)d_2 - (t-2)d_1] \tag{3.3}$$

where t equals to 1 when the year is in 1999, equals to 2 when the year is in 2001, ..., and T equals to 10 and corresponds to the year 2017. The coefficients for d_t^* give us the year effects for the third year (2003) and forward. The coefficients for the first and second year could be recovered by the assumptions that (1) year effects sum to zero and (2) year effects are orthogonal to a linear time trend.

We run the Heckman selection model on both measures of stock market participation and their corresponding risky share - in non-IRA accounts and in all accounts. The regression coefficients for the cohort proxy (Youth Stock Return), total wealth, demographic variables and inverse Mill ratio as well as chi-squared tests results on the unrestricted year and cohort effects, are presented in Tables 3.2 and 3.3, respectively. The coefficients for the age, year and cohort dummies are available upon request. Columns (1) and (2) of the two tables contain the coefficients for the selection and outcome equations when we use cohort proxy to control for cohort effects, and columns (3) and (4) contain the coefficients for the selection and outcome equations when we use the Deaton and Paxson method to control for time effects.

Comparing Tables 3.2 and 3.3, for both measures of stock market participation, youth stock return has economically very small effects on stock market participation and risky share, and it is insignificant for stock market participation and risky share in all accounts. The level of total life time wealth has positive and significant effects on stock market participation for both measures of stock market participation and both identification strategies of the cohort and year effects, indicating that life time wealth is a strong exclusion restriction and the behavior of the households in our sample is consistent with the presence of a fixed stock market participation cost.

The coefficients of the demographic variables have similar signs and magnitudes under both identification methods of the cohort and year effects and have the same signs for both measures of stock market participation. Specifically, the number of children has negative effects on participation and positive effects on the conditional risky share. Having a female head has negative effects on both stock market participation and risky share, consistent with the literature that women tend to take less risk than men (for example, see Almenberg and Dreber (2015)). Family size has positive effects on stock market participation, likely because of the coinsurance among the family members, but has negative effects on conditional risky share. Having bad health also have negative effects on stock market participation and conditional risky share, consistent with the idea the health risk crowds out risk taking in stock market (for example, De Nardi, French, and Jones (2006)).

The coefficient of the inverse Mill ratio is significant at 1% level for both measures of stock market participation and identification strategies, indicating that it is important to take selection into the stock market into account when estimating the age profile of conditional risky share. Following Fagereng et al. (2017), we also run chi-squared tests on the unrestricted year and cohort effects, and find that year and cohort effects are significant for both measures of stock market participation and both identification strategies.

Figure 3.3 presents the estimated age profiles of stock market participation rate and risky share using the Heckman selection model. Comparing with the age profiles using only the dummy variables, the hump-shaped pattern of stock market participation rate disappears. The estimated stock market participation rate is increasing and becomes flat at later stage of life. This is true for both measures of stock market participation (non-IRA accounts and all accounts) and is true both when we use cohort proxy and when we use the Deaton-Paxson method. Moreover, the peak stock market participation rate is much higher, at about 80%[10], than when using the dummy variables. Both measures of the conditional risky share are higher after controlling for selection - the conditional risky share ranges from 55% to 80% in the Heckman selection model, while with the dummy variables the conditional risky shares ranges from 30% to 70% in all accounts and 15% to 85% in non-IRA accounts. The smaller range of the risky share over the life cycle also indicates that the age profiles of conditional risky share are much flatter after controlling for selection, controlling for demographic variables and using the more restrictive assumptions on cohort and year effects. We see a high starting risky share - at about 90% - in non-IRA accounts when we use both the cohort proxy and the Deaton-Paxson method, indicating that the young agents in our sample are behaving similar to the prediction of the life-cycle models.

In order to get a more straightforward view on the effects of selection, we plot the age pro-

[10]Except for the stock market participation rate in non-IRA accounts using the Deaton-Paxson method (Figure 3.3b), which peaks at about 70%.

files of stock market participation and conditional risky share in all accounts generated by various models *without* accounting for selection in Figure 3.4. Subfigures 3.4a and 3.4b plot age profiles of stock market participation rate and conditional risky share using age and cohort dummies, and age and year dummies, respectively, plus the demographic variables that we used in the Heckman selection model. In essence, subfigures 3.4a and 3.4b tell us the age profiles conditional on demographic variables assuming either year effects are zero or cohort effects are zero. Subfigures 3.4c and 3.4d plot the age profile of stock market participation rate and risky share using cohort proxy and the Deaton-Paxson method, respectively. Subfigures 3.4e plots the age profiles of conditional risky share using cohort proxy plus demographic variables and the Deaton-Paxson method plus the demographic variables. The only difference between the age profiles in subfigure 3.4e and the age profiles of conditional risky share in Figure 3.3 (which is also in subfigure 3.4f to facilitate comparison) is that the age profiles in subfigure 3.4e do not account for the selection into the stock market.

Comparing with the age profiles with only the dummy variables as shown in Figures 3.2a and 3.2c, there is not much difference in the age profiles of stock market participation rate when the demographic variables are included - the stock market participation rates are hump-shaped, and peaks at a later age when using age and year dummies. There is also not much difference in the age profiles of the stock market participation rate when we use the Deaton-Paxson method to restrict the year effects or when cohort proxy to control for cohort effects as shown in subfigures 3.4c and 3.4d. Therefore, it seems that the increasing age profile of stock market participation in the Heckman selection model is a result of BOTH including the demographic variables AND using the restrictions on cohort and year effects. The age profile of conditional risky share with the demographic variables and with restrictive cohort and year effects are at similar levels with the age profiles of conditional risky share using only the cohort or year dummies, but flatter. After controlling for selection, the conditional risky shares are overall higher.

3.4 Discussion and Conclusion

The main findings can be summarized as the follows. First, when using dummy variable regressions as in Ameriks and Zeldes (2004) but on the PSID data, stock market participation rate is hump-shaped over the life-cycle, with earlier peaks when we use age and cohort dummies. The conditional risky share is flat, only slightly increasing over the life cycle, with a high initial risky share at the very beginning of the life-cycle (age 20). Second, stock market participation rate becomes strongly increasing, from 10% at the beginning of the life-cycle, to 80% at later stages of life, when we follow the empirical strategies in Fagereng et al. (2017). In other words, the hump-shaped pattern of stock market participation rate disappears and we do not see gradual exit from the stock market as in previous papers. Third, the conditional risky share is still slightly increasing over the life cycle, but is at an overall higher level when we account for selection and use the more restricted empirical strategies to account for the age cohort and year effects as in Fagereng et al. (2017). We also see a very high starting conditional risky share at the very beginning of the life cycle. In summary, U.S. results differ significantly from Fagereng et al. (2017).

Overall, our estimated stock market participation rate is higher than the stock market participation rate reported in Fagereng et al. (2017). Others also report the U.S. has higher stock market participation rate than other countries (Giuliano and Spilimbergo (2013)). One reason for such a difference might be that many households in the U.S. participate in the stock market through IRAs and defined contribution accounts, while the current retirement system in Norway is dominated by defined benefit schemes[11].

The age profile of stock market participation in our chapter also differs from Ameriks and Zeldes (2004), who use cross-sectional U.S. data from an earlier period. Note that Ameriks and Zeldes (2004) provide unconditional age profiles of stock market participation rate, so

[11] Source: https://www.pensionfundsonline.co.uk/content/country-profiles/norway

when they observe a decrease in stock market participation rate after retirement, it could be that many of the households leave the stock market because of health problems, or because the head of the family became a woman (who tend to take less risk). Our estimation suggests that conditional on demographic variables such as number of children, gender of the head, family size, health status of the head, and whether there is a spouse or not, stock market participation rate is actually increasing over the life cycle, with a slower rate of increase after retirement. One potential reason for the stable or increasing stock market participation rate after retirement could be that capital gains tax is exempt in inheritance due the to "step-up" in basis once a person die. This tax treatment provides an incentive for families to keep holding stocks.

The age profile of the conditional risky share remains puzzling. The estimated risky share is higher after controlling for selection, but the flat or slightly increasing age profile is not consistent with the idea that human capital is shrinking over the life cycle and hence the optimal risky share should be decreasing. One can hypothesize why the overall estimated risky share is higher in the U.S. than in Norway, although estimating the validity of the explanations is beyond the scope of this chapter. For example, it could be that the U.S. economy has been strong while Norway's has experienced two big drops over the past twenty years, which makes U.S. households willing to hold more risky assets, conditional on participation in the stock market. But it remains a challenge to provide theoretical explanations on why the age profile of conditional risky share is increasing over the life-cycle in the U.S..

3.5 Tables

Table 3.1: Summary Statistics

This table provides summary statistics on two samples obtained from the 1999 to 2017 waves of the PSID. "Full sample" contains all the families that ever presented in these waves of the survey, and "Balanced Panel" contains only the families that stayed in the survey through the whole 1999-2017 period. Descriptions of how we construct the wealth variables and different measures of stock market participation rate and their corresponding conditional risky share can be found in section 3.2.1.

	Full Sample			***Balanced Panel***		
	Mean	Std Dev	Median	Mean	Std Dev	Median
Demographics						
Age Head	45.91	16.72	44	51.36	13.32	51
Age Spouse	44.03	14.48	42	48.93	12.52	49
Number of Children	0.76	1.13	0	0.71	1.11	0
No High School	0.15	0.35	0	0.12	0.32	0
No College	0.63	0.48	1	0.62	0.49	1
College or More	0.33	0.47	0	0.39	0.49	0
Female Head	0.25	0.44	0	0.14	0.35	0
Family Size	2.35	1.35	2	2.44	1.28	2
Asset Holdings (1999 USD)						
Liquid Wealth	38221	100997	3515	53419	117826	7526
Financial Wealth	63171	151379	4822	97354	186532	14800
Net Worth w/o Home Equity	105896	250443	14274	159894	299252	34951
Net Worth w/ Home Equity	164214	315092	42653	244275	370365	102254
Stock Market Participation Rate						
All Accounts	0.32	0.47	0.00	0.44	0.50	0.00
non-IRA Accounts	0.21	0.41	0.00	0.29	0.45	0.00
IRA Accounts only	0.73	0.44	1.00	0.77	0.42	1.00
Conditional Riksy Share						
All Accounts	0.54	0.29	0.50	0.56	0.29	0.52
non-IRA Accounts	0.57	0.32	0.62	0.59	0.32	0.65
IRA Accounts only	0.64	0.33	0.50	0.65	0.32	0.50
		N = 56548			N = 17160	

Table 3.2: Heckman Selection Model – Non-IRA Accounts

This table displays selected regression coefficients and test statistics for two estimated Heckman selection models discussed in Section 3.3.2. The dependent variable for the selection equation is stock market participation in non-IRA accounts. The dependent variable for the outcome equation is the conditional risky share in non-IRA accounts. Total wealth is the sum of financial wealth and human wealth in 1999 USD. Health Status of Head is a categorical variable that labels the general health status of the head being (1) excellent, (2) very good, (3) good, (4) fair and (5) poor. The marginal effects of age are presented in Figure 3.3. The coefficients for years and cohorts are provided in the Appendix. Standard errors are in parentheses. $*p < 0.10$, $**p < 0.05$, $***p < 0.01$.

	Cohort Proxy		*Deaton-Paxson*	
	(1)	(2)	(3)	(4)
	Participation	Share	Participation	Share
Youth Stock Return	-0.003*	0.001**		
	(0.001)	(0.001)		
Total Wealth	0.120***		0.121***	
	(0.002)		(0.002)	
Num of Minor Children	-0.264***	0.041***	-0.256***	0.040**
	(0.029)	(0.014)	(0.029)	(0.013)
Female Head	-0.103***	-0.012	-0.107***	-0.011
	(0.025)	(0.012)	(0.025)	(0.012)
Num of People in Household	0.195***	-0.034***	0.191***	-0.034***
	(0.026)	(0.012)	(0.026)	(0.012)
Health Status of Head	-0.131***	-0.003	-0.130***	-0.002
	(0.007)	(0.004)	(0.007)	(0.003)
No Spouse	-0.028	0.023**	-0.27	0.022**
	(0.024)	(0.011)	(0.024)	(0.011)
λ_{iact}		-0.066***		-0.068***
		(0.008)		(0.009)
Observations	50,288	39,450	50,288	39,450
Joint Sign. Tests				
Year $\chi^2(9)$	1033.20***	46.79***	46.19***	43.48***
Cohort $\chi^2(86)$			524.38***	157.51***

Table 3.3: Heckman Selection Model - All Accounts

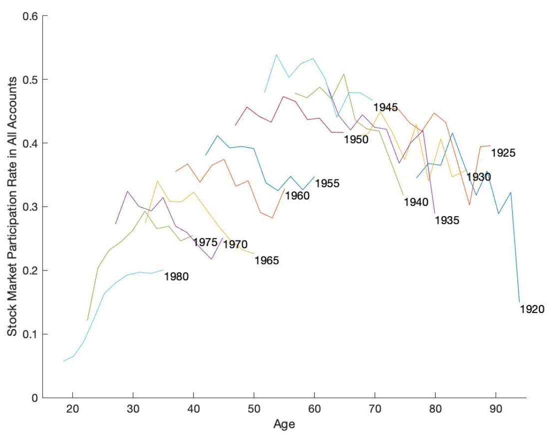
This table displays selected regression coefficients and test statistics for two estimated Heckman selection models discussed in Section 3.3.2. The dependent variable for the selection equation is stock market participation in all accounts. The dependent variable for the outcome equation is the conditional risky share in all accounts. Total wealth is the sum of financial wealth and human wealth in 1999 USD. Health Status of Head is a categorical variable that labels the general health status of the head being (1) excellent, (2) very good, (3) good, (4) fair and (5) poor. The marginal effects of age are presented in Figure 3.3. The coefficients for years and cohorts are provided in the Appendix. Standard errors are in parentheses. *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$.

	Cohort Proxy		*Deaton-Paxson*	
	(1)	(2)	(3)	(4)
	Participation	Share	Participation	Share
Youth Stock Return	-0.002	0.001		
	(0.001)	(0.000)		
Total Wealth	0.124***		0.125***	
	(0.002)		(0.002)	
Num of Minor Children	-0.370***	0.054***	-0.362***	0.056**
	(0.026)	(0.010)	(0.027)	(0.010)
Female Head	-0.077***	-0.027***	-0.079***	-0.030***
	(0.022)	(0.009)	(0.023)	(0.009)
Num of People in Household	0.293***	-0.047***	0.288***	-0.049***
	(0.024)	(0.009)	(0.024)	(0.009)
Health Status of Head	-0.178***	-0.001	-0.177**	-0.000
	(0.007)	(0.003)	(0.007)	(0.003)
No Spouse	-0.074***	0.018**	-0.70***	0.017**
	(0.021)	(0.008)	(0.021)	(0.008)
λ_{iact}		-0.071***		-0.072***
		(0.007)		(0.007)
Observations	50,292	33,573	50,292	33,573
Joint Sign. Tests				
Year $\chi^2(9)$	733.68***	51.16***	47.33***	47.60***
Cohort $\chi^2(86)$			510.46***	186.75***

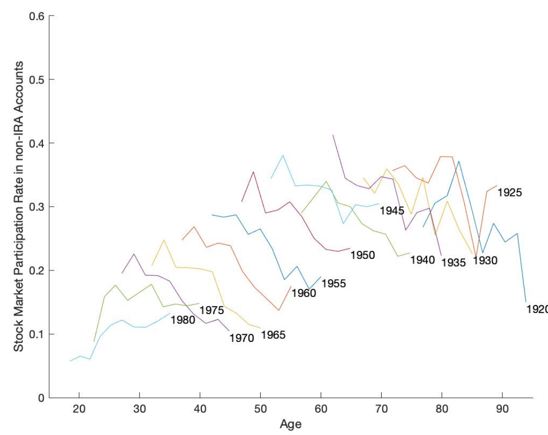
3.6 Figures

Figure 3.1: Cohort Views: Stock Market Participation Rate and Risky Share in the Full Sample

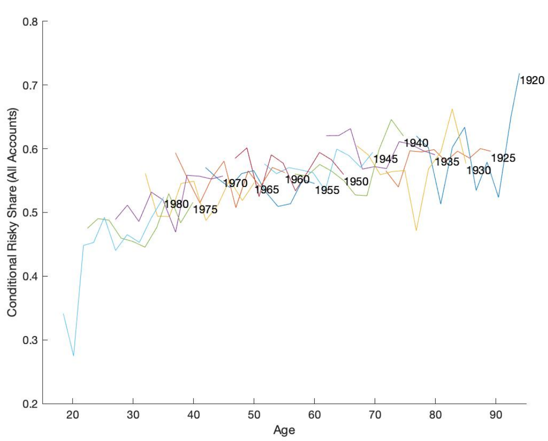
This figure plots two measures of stock market participation rate and the corresponding conditional risky share by age for five-year cohorts in the full sample. The cohort labelled "1920" includes families with heads born in years 1920-1924, cohort labelled "1925" includes families with heads born in years 1925-1929, ..., and so on.



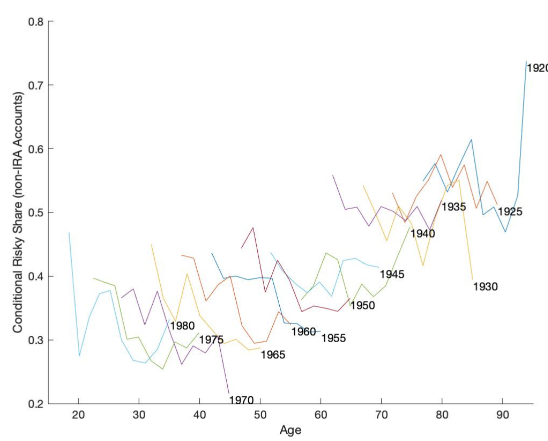
(a) Stock Market Participation Rate in All Accounts



(b) Stock Market Participation Rate in non-IRA Accounts



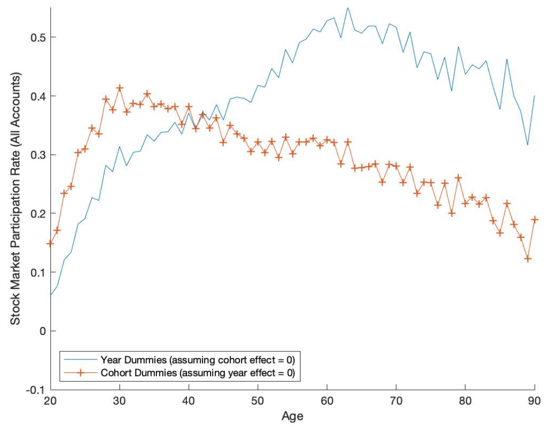
(c) Risky Share in All Accounts



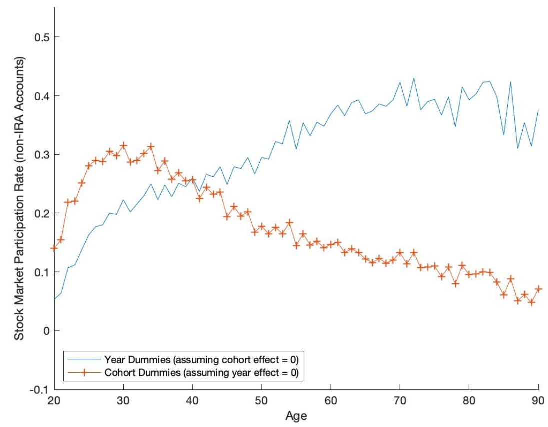
(d) Risky Share in non-IRA Accounts

Figure 3.2: Age Profiles of Stock Market Participation Using Dummy Variables: Full Sample

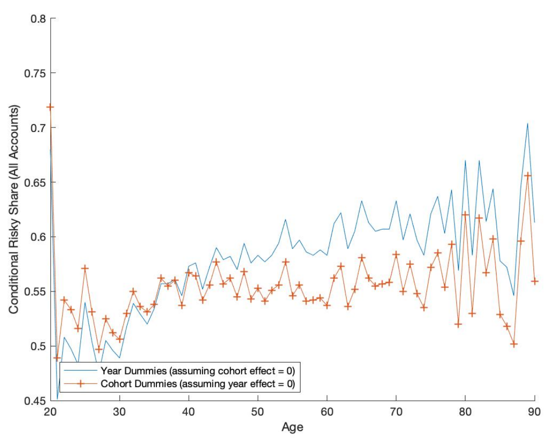
This figure presents the predicted age profiles of two measures of stock market participation rate and conditional risky share on the full sample. Panels (a) and (b) plot the predicted stock market participation rate against age in all accounts and in non-IRA accounts, respectively. Panels (c) and (d) plot the conditional risky share against age in all accounts and in non-IRA accounts, respectively.



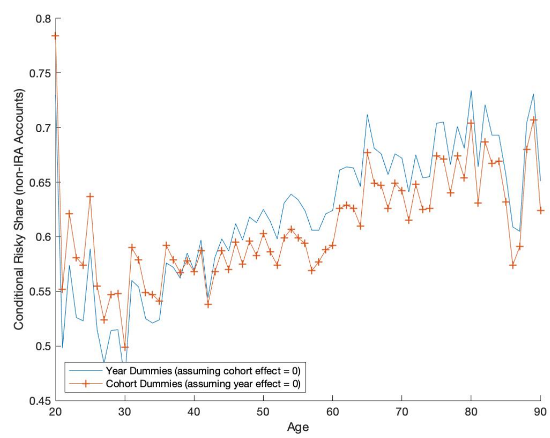
(a) Stock Market Participation Rate in All Accounts



(b) Stock Market Participation Rate in non-IRA Accounts



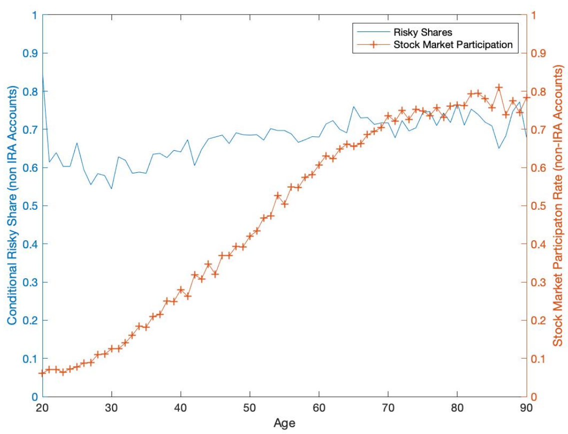
(c) Conditional Risky Share in All Accounts



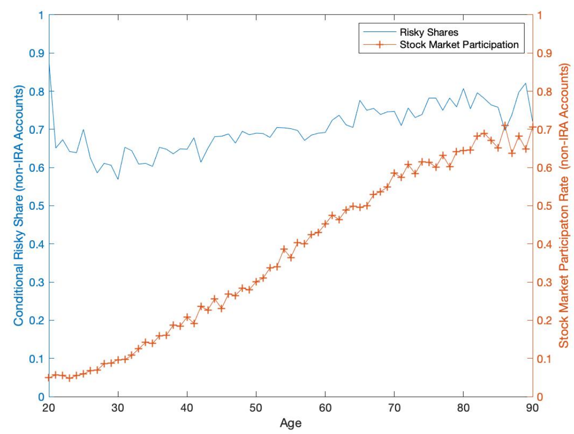
(d) Risky Share in non-IRA Accounts

Figure 3.3: Estimated Age Profiles of Stock Market Participation Rate and Risky Share Using Heckman Selection Model

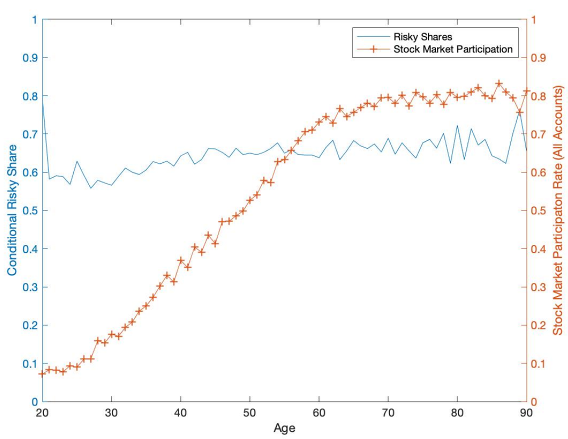
This figure presents the predicted age profiles of two measures of stock market participation rate and conditional risky share using the Heckman selection model. Panels (a) and (b) considers stock market participation rate and the corresponding conditional risky share in non-IRA accounts, while panels (c) and (d) only considers stock market participation rate and the corresponding conditional risky share in all accounts. Cohort proxy means that we use past stock return as a proxy for cohort effects and Deaton-Paxon means that we use the method in Deaton and Paxson (1994) to control for time effects.



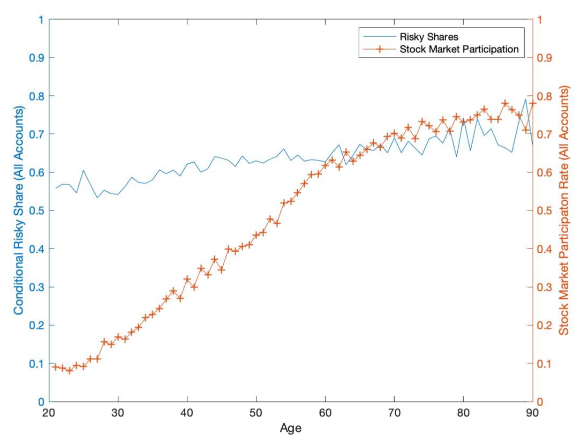
(a) Cohort Proxy - non- IRA Accounts



(b) Deaton-Paxson - non- IRA Accounts

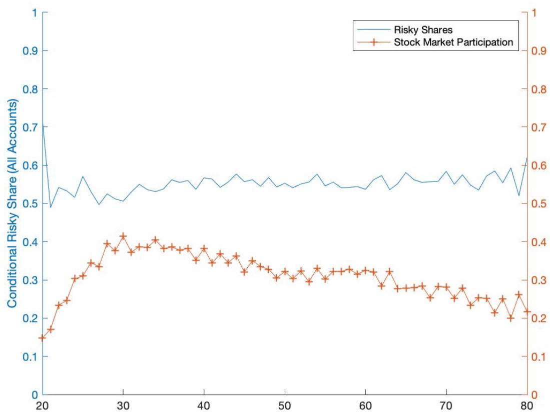


(c) Cohort Proxy - All Accounts

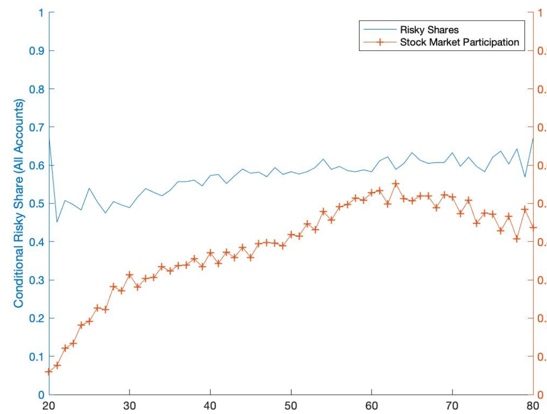


(d) Deaton-Paxson - All Accounts

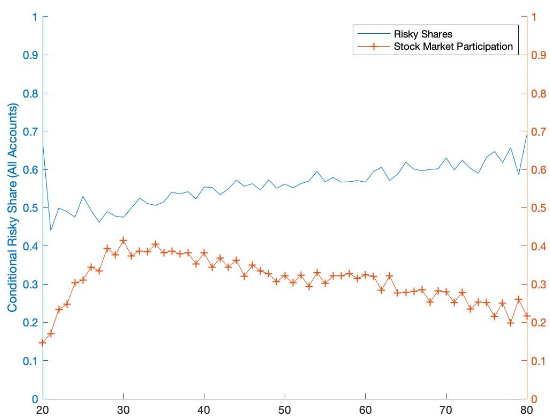
Figure 3.4: Heckman Selection Model v.s. Various Models without Selection (All Accounts)



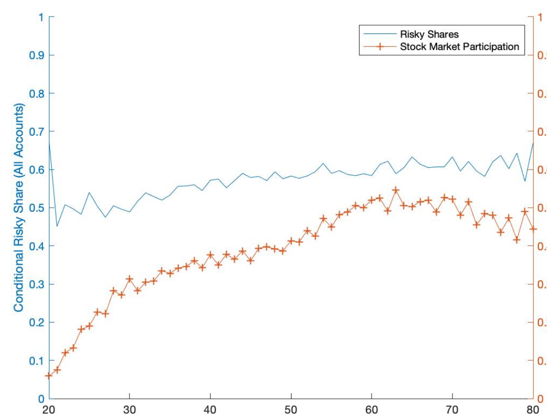
(a) Cohort Dummies with Demographic Variables



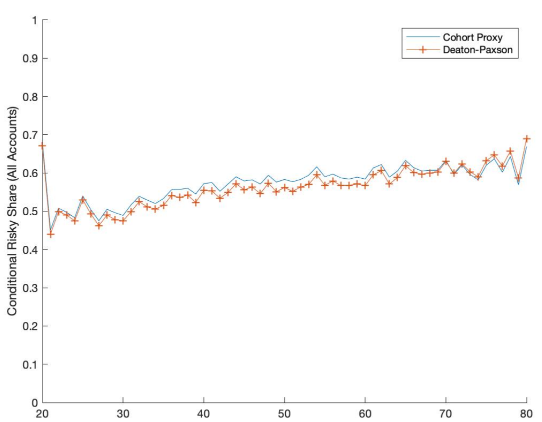
(b) Year Dummies with Demographic Variables

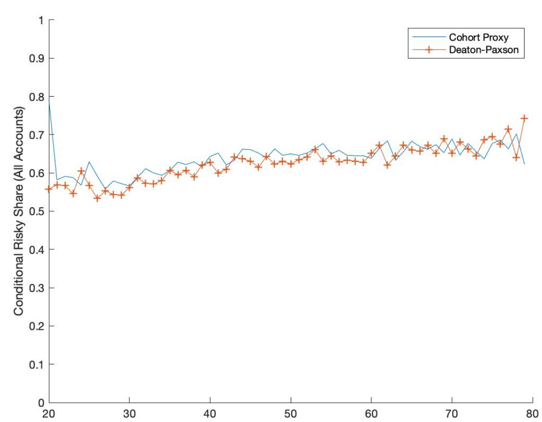


(c) Deaton-Paxson w/o Demographic Variables



(d) Cohort Proxy w/o Demographic Variables





(e) Risky Share not accounting for selection (f) Risky Share with Heckman Selection Model

References

Almenberg, J., & Dreber, A. (2015). Gender, stock market participation and financial literacy. *Economics Letters*, *137*, 140 - 142.

Ameriks, J., & Zeldes, S. P. (2004). How do household portfolio shares vary with age. *Working Paper*.

Amromin, G. (2003). Household Portfolio Choices in Taxable and Tax-Deferred Accounts: Another Puzzle? *Review of Finance*, *7*(3), 547-582.

Attanasio, O., Low, H., & Sanchez-Marcos, V. (2008). Explaining Changes in Female Labor Supply in a Life-Cycle Model. *American Economic Review*, *98*(4), 1517-1552.

Auerbach, A., & King, M. A. (1983). Taxation, portfolio choice, and debt-equity ratios: A general equilibrium model. *The Quarterly Journal of Economics*, *98*(4), 587-609. Retrieved from `https://EconPapers.repec.org/RePEc:oup:qjecon:v:98:y:1983:i:4:p:587-609`.

Becker, G. S., & Murphy, K. M. (1988). A theory of rational addiction. *Journal of Political Economy*, *96*(4), 675–700.

Bergstresser, D., & Poterba, J. (2004). Asset allocation and asset location: household evidence from the survey of consumer finances. *Journal of Public Economics*, *88*(9-10), 1893-1915.

Black, F. (1980). *The tax advantages of pension fund investments in bonds* (Working Paper No. 533). National Bureau of Economic Research.

Blundell, R., Bozio, A., & Laroque, G. (2013). Extensive and intensive margins of labour supply: Work and working hours in the us, the uk and france. *Fiscal Studies*, *34*(1), 1-29.

Blundell, R., French, E., & Tetlow, G. (2016). Chapter 8 - retirement incentives and labor supply. In J. Piggott & A. Woodland (Eds.), (Vol. 1, p. 457 - 566). North-Holland.

Bodie, Z., & Crane, D. B. (1997). Personal investing: Advice, theory, and evidence. *Financial Analysts Journal*, *53*, 13–23.

Bodie, Z., Merton, R. C., & Samuelson, W. F. (1992). Labor supply flexibility and portfolio choice in a life cycle model. *Journal of Economic Dynamics and Control*, *16*(3), 427 - 449.

Campbell, J. (2006). Household finance. *Journal of Finance*, *61*(4), 1553–1604.

Catherine, S. (2016). *Countercyclical Income Risk and Portfolio Choices over the Life-Cycle* (HEC Research Papers Series No. 1147). HEC Paris.

Chai, J., Horneff, W., Maurer, R., & Mitchell, O. S. (2011). Optimal portfolio choice over the life cycle with flexible work, endogenous retirement, and lifetime payouts". *Review of Finance*, *15*(4), 875-907.

Chang, Y., Hong, J. H., & Karabarbounis, M. (2018). Labor market uncertainty and portfolio choice puzzles. *American Economic Journal: Macroeconomics*, *10*(2), 222-62.

Chen, B., & Stafford, F. P. (2016). Stock market participation: Family responses to housing consumption commitments. *Journal of Money, Credit and Banking*, *48*(4), 635-659.

Choi, J. J. (2015). Contributions to defined contribution pension plans. *Annual Review of Financial Economics*, *7*(1), 161-178.

Cocco, J. F. (2005). Portfolio choice in the presence housing. *Review of Financial Studies*, *18*, 535-567.

Cocco, J. F., Gomes, F. J., & Maenhout, P. J. (2005). Consumption and portfolio choice over the life cycle. *The Review of Financial Studies*, *18*(2), 491-533.

Cogan, J. F. (1980). *Fixed costs and labor supply* (Working Paper No. 484). National Bureau of Economic Research.

Constantinides, G. M. (1990). Habit formation: A resolution of the equity premium puzzle. *Journal of Political Economy*, *98*(3), 519–543.

Dammon, R., Spatt, C. S., & Zhang, H. (2004, 06). Optimal asset location and allocation with taxable and tax-deferred investing. *The Journal of Finance*, *59*, 999 - 1037.

Deaton, A. (2019). *The Analysis of Household Surveys*. The World Bank.

Deaton, A., & Paxson, C. (1994). Saving, growth, and aging in taiwan. In *Studies in the economics of aging* (p. 331-362). National Bureau of Economic Research, Inc.

De Nardi, M., French, E., & Jones, J. B. (2006, October). *Differential mortality, uncertain medical expenses, and the saving of elderly singles* (Working Paper No. 12554). National Bureau of Economic Research.

Detemple, J. B., & Zapatero, F. (1991). Asset prices in an exchange economy with habit formation. *Econometrica*, *59*(6), 1633–1657.

Dynan, K. E., Skinner, J., & Zeldes, S. P. (2002). The importance of bequests and life-cycle saving in capital accumulation: A new answer. *American Economic Review*, *92*, 274-279.

Engelhardt, G. V., & Kumar, A. (2006). *Employer matching and 401(k) saving: Evidence from the health and retirement study* (Working Paper No. 12447). National Bureau of Economic Research.

Erosa, A., Fuster, L., & Kambourov, G. (2016). Towards a micro-founded theory of aggregate labour supply. *The Review of Economic Studies*, *83*(3), 1001-1039.

Fagereng, A., Gottlieb, C., & Guiso, L. (2017). Asset market participation and portfolio choice over the life-cycle. *Journal of Finance*, *72*(2), 705–750.

Fehr, H., Jokisch, S., & Kotlikoff, L. J. (2007). Will china eat our lunch or take us to dinner? simulating the transition paths of the united states, the european union, japan, and china [Book]. In *Fiscal policy and management in east asia, nber-ease, volume 16* (p. 133-193). University of Chicago Press. Retrieved from `http://www.nber.org/books/ito_07-1`

Fischer, M., & Gallmeyer, M. (2016, 09). Taxable and Tax-Deferred Investing with the Limited Use of Losses. *Review of Finance*, *21*(5), 1847-1873.

French, E. (2005). The effects of health, wealth, and wages on labour supply and retirement behaviour. *The Review of Economic Studies*, *72*(2), 395-427.

Garlappi, L., & Huang, J. (2006). Are stocks desirable in tax-deferred accounts? *Journal of Public Economics*, *90*(12), 2257 - 2283.

Giuliano, P., & Spilimbergo, A. (2013, 11). Growing up in a Recession. *The Review of Economic Studies*, *81*(2), 787-817.

Gollier, C. (2006, 04). Does ambiguity aversion reinforce risk aversion? applications to portfolio choices and asset pricing.

Gomes, F., Kotlikoff, L. J., & Viceira, L. M. (2008). Optimal Life-Cycle Investing with Flexible Labor Supply: A Welfare Analysis of Life-Cycle Funds. *American Economic Review*, *98*(2), 297-303.

Gomes, F., & Michaelides, A. (2003). Portfolio choice with internal habit formation: A life-cycle model with uninsurable labor income risk. *Review of Economic Dynamics*, *6*, 729-766.

Gomes, F., & Michaelides, A. (2005). Optimal life-cycle asset allocation: Understanding the empirical evidence. *The Journal of Finance*, *60*(2), 869-904.

Gomes, F., Michaelides, A., & Polkovnichenko, V. (2009). Optimal savings with taxable and tax-deferred accounts. *Review of Economic Dynamics*, *12*(4), 718 - 735.

Guiso, L., & Jappelli, T. (2000). *Household Portfolios in Italy* (CSEF Working Papers). Centre for Studies in Economics and Finance (CSEF), University of Naples, Italy.

Heaton, J., & Lucas, D. (2002). Portfolio choice and asset prices: The importance of entrepreneurial risk. *The Journal of Finance*, *55*, 1163 - 1198.

Horneff, V., Maurer, R., & Schober, P. (2016). Efficient parallel solution methods for dynamic portfolio choice models in discrete time [Working Paper].

Kimball, M. S. (1990). Precautionary saving in the small and in the large. *Econometrica*, *58*(1), 53–73.

King, R. G., Plosser, C. I., & Rebelo, S. T. (1988). Production, growth and business cycles: Ii. new directions. *Journal of Monetary Economics*, *21*(2), 309 - 341.

Love, D. A. (2009, 04). The Effects of Marital Status and Children on Savings and Portfolio Choice. *The Review of Financial Studies*, *23*(1), 385-432.

Low, H. W. (2005). Self-insurance in a life-cycle model of labour supply and savings. *Review of Economic Dynamics*, *8*(4), 945 - 975.

Malmendier, U., & Nagel, S. (2011). Depression Babies: Do Macroeconomic Experiences Affect Risk Taking? *The Quarterly Journal of Economics*, *126*(1), 373-416.

Merton, R. (1969). Lifetime portfolio selection under uncertainty: The continuous-time case. *The Review of Economics and Statistics*, *51*(3), 247-257.

Merton, R. (1971). Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory*, *3*(4), 373-413.

Mossin, J. (1968). Optimal multiperiod portfolio policies. *The Journal of Business*, *41*(2), 215-229.

Polkovnichenko, V. (2007). Life-cycle portfolio choice with additive habit formation preferences and uninsurable labor income risk. *The Review of Financial Studies*, *20*(1), 83-124.

Poterba, J. M., Shoven, J. B., & Sialm, C. (2000, November). *Asset location for retirement savers* (Working Paper No. 7991). National Bureau of Economic Research.

Ramey, V. A., & Francis, N. (2009, July). A century of work and leisure. *American Economic Journal: Macroeconomics*, *1*(2), 189-224.

Samuelson, P. A. (1969). Lifetime portfolio selection by dynamic stochastic programming. *The Review of Economics and Statistics*, *51*(3), 239-246.

Shen, J. (2018). Countercyclical risks and portfolio choice over the life cycle : Evidence and theory..

Stein, J. C., Hong, H. G., & Kubik, J. D. (2004). Social interaction and stock-market participation. , *59*, 137-163.

Su, Y. (2019). Life-cycle patterns of stock market participation rate and risky share in the united states. *Working Paper*.

Sundaresan, S. M. (1989). Intertemporally dependent preferences and the volatility of consumption and wealth. *Review of Financial Studies*, *2*(1), 73-89.

Tepper, I. (1981). Taxation and corporate pension policy. *The Journal of Finance*.

Vissing-Jorgensen, A. (2002). Limited asset market participation and the elasticity of intertemporal substitution. *Journal of Political Economy*, *110*(4), 825-853.

Wachter, J. A., & Yogo, M. (2010). Why do household portfolio shares rise in wealth? *The Review of Financial Studies*, *23*(11), 3929-3965.

Welch, F. (1979). Effects of cohort size on earnings: The baby boom babies' financial bust. *Journal of Political Economy*, *87*(5), S65–S97. Retrieved from http://www.jstor.org/stable/1829909

Wu, C., & Krueger, D. (2018). *How much consumption insurance in bewley models with endogenous family labor supply?* (Working Paper No. 24472). National Bureau of Economic Research.

Yao, R., & Zhang, H. H. (2005). Optimal consumption and portfolio choices with risky housing and borrowing constraints. *The Review of Financial Studies*, *18*(1), 197-239.

Zhou, J. (2009, April). The asset location puzzle: Taxes matter. *Journal of Economic Dynamics and Control*, *33*(4), 955-969.

Zhou, J. (2012). Life-cycle stock market participation in taxable and tax-deferred accounts. *Journal of Economic Dynamics and Control*, *36*(11), 1814-1829.

Zhou, J. (2018, December). *Household Stock Market Participation During the Great Financial Crisis* (Departmental Working Papers No. 2018-02). The University of Winnipeg, Department of Economics. Retrieved from https://ideas.repec.org/p/win/winwop/2018-02.html

Appendix A: Literature Review on Household Finance

My model builds on the vast literature on the life-cycle model of portfolio choice. Samuelson (1969), Mossin (1968) and Merton (1969, 1971) are among the earliest work on the life-cycle asset allocation problem in which an agent maximizes expected life-time utilities by choosing the optimal share of wealth invested in risky assets and consumption.

Samuelson (1969), Mossin (1968) and Merton (1969) show that the optimal share of wealth invested in risky assets (henceforth the "risky share"), should be independent of age and wealth. Specifically, the risky share derived by Merton (1969) is

$$\omega = \frac{Er}{\sigma\lambda},$$

where ω is the risky share, $\frac{Er}{\sigma}$ is the Sharpe ratio of individual portfolio (often assumed to be the same across individuals), and λ is the coefficient of relative risk aversion. The Samuelson- Mossin-Merton framework assumes i.i.d. asset returns, time-additive separable utilities, no labor income, frictionless and complete market.

Later in Merton (1971), Merton introduced insurable human capital to the classical models and showed that the optimal risky share should decrease with age since the value of human capital declines as people age. Over the last twenty years, empirical studies using household level datasets find discrepancies between the predictions made by the classical models and actual household asset allocation activities. First, the classical models imply that all households should invest some portion of their savings in stocks if the equity premium is greater than zero, but the percentage of population that hold stocks, directly or indirectly through retirement accounts, is very low (52.7% in the U.S. in 2007). The limited participation phenomenon is robust in all asset classes and across all wealth levels. This is the so-called "limited participation puzzle". Participation rates display a strong life-cycle patten. It is generally agreed that stock market participation is hump-shaped over the life-cycle (Giuliano and Spilimbergo (2013)).

Second, Merton (1971) predicts that with labor income, young households should invest all their savings in stocks and reduce the share of risky assets when they are older, but empirical studies find households invest less than 100% of their financial wealth to stocks across all age groups and wealth level. There are mixed results on the age-profile of risky share. Papers that use the U.S. data (e.g. Ameriks and Zeldes (2004)) usually find the risky share of the U.S. households, conditional on participating the stock market, is hump-shaped over the life-cycle (i.e. ”too low” risky share for young people). Fagereng et al. (2017)uses Norwegian data and finds household risky share is relatively high when young and middle-aged, then declines before retirement. Thirdly, there is high heterogeneity in terms of stock market participation and risky share after controlling for wealth, education, and age. Other empirical facts that are hard to explain include infrequent portfolio adjustments and under-diversified household portfolios. See Campbell (2006) and Guiso and Sodini (2013) for detailed descriptions and discussions of all these empirical facts.

As a result, there is a growing literature building models of life-cycle asset allocation trying to reconcile the discrepancies between theoretical predictions and household data. Campbell (2006) coined this field of study ”Household Finance”. Focusing on the first two discrepancies listed in the paragraph above, the literature on life-cycle models of portfolio choice has evolved into, roughly, the following directions.

First, introducing fixed or per-period stock market participation costs (e.g. Vissing-Jorgensen (2002)) to the standard model to explain the limited participation puzzle. This strand of papers show that stock market participation costs, often interpreted as the time, money and energy a person needs to learn about the stock market before investing, help to explain the low stock market participation rates observed in data.

Second, adding ”background risks”. For example, by introducing uninsurable labor income risks, Cocco et al. (2005) shows that the optimal risky share still decreases with age as in Merton (1971) because of the low correlation between labor income and stock returns. Heaton and Lucas (2002) shows that risky business income could explain the low stock market participation rates observed in the wealthy households. Yao and Zhang (2005) explicitly models housing decisions and shows that the risks and wealth pressures associated with owning and renting a house could explain the low risky share of the young households.

Third, many papers use other utility specifications such as Epstein- Zin utilities (Gomes and Michaelides (2005) and Habit formation (Gomes and Michaelides (2003) , Polkovnichenko (2007)). Last but not the least, many papers provide behavioral explanations to the observe discrepancies. For example, Gollier (2006) solve a life-cycle model of portfolio choice in which agents have ambiguity-aversions, Campbell (2006) points out that households could be making investment mistakes, Stein, Hong, and Kubik (2004) shows empirically that social interactions have positive effects on stock market participation, and Love and Phelan (2015) assumes that agents have hyperbolic discounting. Most of these papers feature more than one extensions of the classical models.

Notably, most of these papers generate results that could only partially account for observations in data. For example, fixed stock market participation costs could explain limited participation among the young households who are relatively poor, but not for the middle-aged and the old households. Habit formation could be the reason young households holding low risky shares, again because young households have relatively low wealth to maintain habit stock and hence do not have spare savings for risky investments, but it is silent about limited stock market participation and risky share held by older populations (Polkovnichenko (2007)). Plus, habit formation generates unrealistically high wealth accumulations over the life-cycle.

As pointed out by Guiso and Sodini (2013), probably all the features contributes to the explanation of the limited participation as well as the low risky share phenomenon, and "... [t]he challenge for future research is to identify when and for which investors some of the explanations are more relevant than others."

Appendix B: Leisure Policy Functions When Retirement Income Is Extremely High

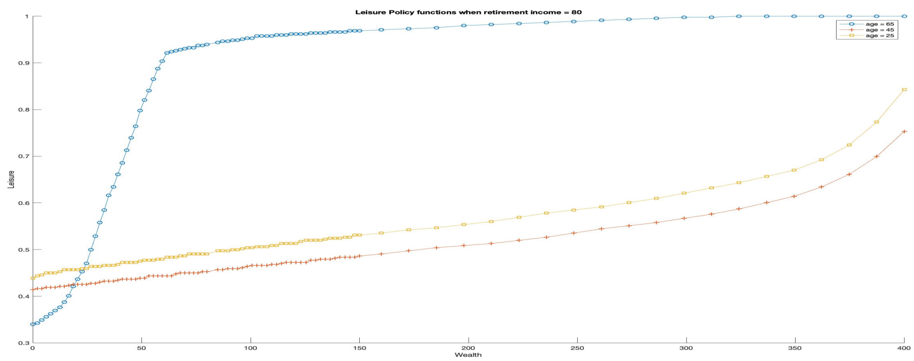


Figure A-1: Leisure Policy functions when retirement income is extremely high

I resolved the no-cost version of my model assuming retirement income is fixed at 80. Note that in my baseline setting the retirement income is at 8.7 and the optimal leisure at age 65 is lower than the optimal leisure at ages 45 and 25 at small and high levels of wealth. When retirement income is much higher, at age 65, the agent still works more than she does when she is 25 and 45 years old at low levels of wealth, but then her optimal leisure rapidly rise and becomes close to 1.

Appendix C: The U.S. Retirement System

A typical worker in the United States can gain retirement income from three sources: employer sponsored retirement plans, self-directed retirement plans or IRAs (Individual Retirement Accounts), social securities and lastly, personal savings.

Employer sponsored retirement plans have two forms: defined-benefit and defined-contribution. First established by the American Express Company in 1875, corporates provide defined-benefit plans which are guaranteed pensions for employees. The amount paid by the pension is usually determined by the length of time the employee has worked, their life-time earnings and their life-expectancy.

Defined-benefit plans have been gradually replaced by defined-contribution plans started in the 1990s. A defined-contribution plan refers to a retirement plan offered by the employer in which an employee contributes a specified portion of their income into a tax-favored retirement account and decides how to invest the funds in the account. The contributions are sometimes matched by the employer . Examples of define-contribution plans include 401(k)s, defined-contribution plans for workers in the private-sector, 403(b)s for workers in non-profits and higher educations, and 457(b)s for government employees.

The first defined contribution plans were likely offered by TIAA-CREF (Teachers Insurance and Annuity Association & College Retirement Equities Fund). TIAA was founded in 1918 by Andrew Carnegie and his Carnegie Foundation for the Advancement of Teaching. TIAA first provided fully funded pension for teachers. Started in 1952, CREF was created and it started to provide variable annuity for teachers. The participants in TIAA-CREF are mainly workers in higher educations and non-profit organizations.

A "traditional" defined-contribution plan usually has the following tax-advantages. First, employees contribute pre-tax income into the retirement account. That is, the funds allocated to their retirement account is deducted from their earnings and hence will not be taxed. Second, earnings from investments in the retirement account are only subject to income tax upon withdrawal. Employees could also choose to contribute after-tax income into their defined-contribution retirement accounts under plans called Roth 401(k) and Roth 403(b). In a Roth 401(k) or 403(b), both investment earnings and withdrawals are tax-free .

Defined contribution plans started to expand in the 1990s while defined benefits plans were shrinking. Poterba, Venti, and Wise (2009) show that given the declined in participation rate of DB plans and the increased participation rate in DC plans, the projected present value of real DB benefits at age 65 reached a historical maximum in 2003 and the average value of 401(k) assets at age 65 will surpass the average present value of DB benefits at age 65 in about 2010. They also project that the averaged (per person) 401(k) assets will be over six times larger than the historical maximum level of DB benefits at age 64, if equity returns between 2006 and 2040 are at the same historical rates.

Introduced with the enactment of the Employee Retirement Income Security Act (ERISA) in 1974, self-directed retirement plans, or IRAs (Individual Retirement Accounts/Arrangements), are retirement plans offered by financial institutions in which individuals can choose to participate. IRAs are similar to defined-contribution plans in that individuals can decide how much to contribute (subject to annual limits) and how to invest the funds inside their IRAs. Also similar to defined-contribution plans, depending on whether the participate contributes pre-tax or after-tax income to their accounts, we can divide IRAs into traditional IRAs and Roth IRAs.

Traditional retirement accounts and Roth retirement accounts provide equivalent tax advantages when a person's income falls into the same tax bracket over the time. For instance, suppose a person faces 50% income tax and decides to contribute \$100 after-tax income (or \$200 pre-tax income) to her retirement account and use the money to buy an asset that has 100% returns. For simplicity, assume there is no inflation. After two periods, if this person puts her money into a Roth account, then she would receive \$(100*2) *2 = \$400. If this person puts her money into a traditional account, then the balance in the retirement account after two periods will be \$(200*2) *2 = \$800, upon withdrawal the person would get 800*50% = \$400. Roth plans are more suitable for individuals who expect their income tax rate to be higher when they retire.

For both employer-sponsored plans and IRAs, individuals can withdraw money from their define-contribution accounts at the age of 591/2. Early withdrawals will incur a 10% penalty in addition to income tax. However, under certain circumstances defined as hardship, examples include payment of college education and purchase of primary residence, early withdrawals are exempt from the 10% penalty. Individuals are forced to withdraw funds from their employer-sponsored plans and IRAs starting at age 701/2, under a rule called Required Minimum Distributions (RMDs). Roth accounts are not subject to RMDs, there are beneficial to individuals with long planning horizons. Some 401(k) plans allow employees to take loans on the 401(k) distributions and repay the loans using after-tax earnings. Conditions of such a loan, among others, are that it is a loan of less than 5 years, and that interest payments are paid back to the defined-contributions account on time.

The third source of retirement income is social security benefits. Workers who earned enough social security credits (usually by working for sufficient years and paying the social security and Medicare tax) are qualified for social security benefits. One of the benefits provided by social security are retirement benefits in the form of monthly cash benefits. Workers can claim retirement benefits as early as 62-year-old.

The last source of retirement income is personal savings. For various reasons, incomes from pensions, employer-sponsored or self-directed retirement accounts, and social securities might not be enough for retirement consumptions, then a person could also use personal savings for retirement consumptions.

Given that investment incomes are exempt from taxes in tax-favored retirement accounts, it is instructive to review the taxes on returns of investments. In the United States, only realized net capital gains are taxed. Capital gains tax rate depends on the individual's income tax bracket and the length of time the individual holds the asset. Short-term (1 year or less) capital gains are taxed at the same rate as ordinary income, while long-term capital gains tax rate is lower than short-term capital gains tax rate. Up to $3,000 capital loss can be deducted from an individual's personal income and the unused portion of capital loss can be carried over as long-term capital loss to the following tax years.

Capital gains taxes could be avoided when a person inherits stocks from another person. Specifically, when the heir sells the stocks, the cost basis, which roughly speaking is the cost of the asset, is "stepped up (or down)" from the price at which the stock was originally purchased to the price of the stock on the date of death. For example, if a person bought a share of stock in 1978 at the price of $1 and never sold it, and when this person died the share of stock is worth $100, then the tax on the $99 capital gain is forgiven because the cost basis of this share of stock is "stepped up" to $100. The fact that capital gains are taxed only when they are realized, capital losses could be deducted from income, and that changes in cost basis when stocks are passed to the next generation give incentives for people to realize capital losses while defer capital gains.

On the other hand, dividends and interests are taxed when they are accrued. Dividends are taxed at the same rate ordinary income unless the individual has owned the stock for more than 60 days. Dividends paid by stocks that the individual owns for more than 60 days are called qualified dividends and are taxed at the same rate as capital gains. Interest income is taxed at the same rate as ordinary income.

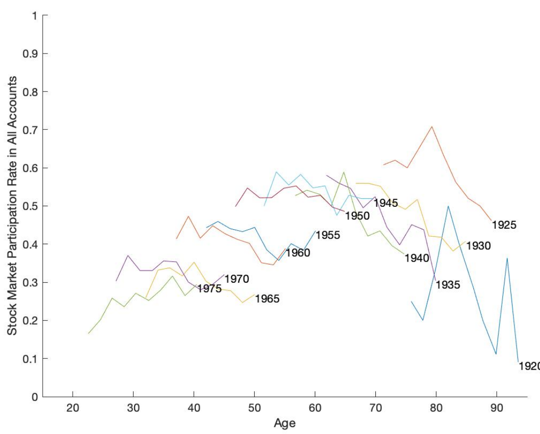
There have been various changes in the law regarding retirement accounts and tax rates on capital gains and dividends. Therefore, the observed asset allocations of individuals or households may reflect the policies when individuals first started to accumulate assets in retirement accounts. The Economic Growth and Tax Relief Reconciliation Act (EGTRRA) of 2001 made a number of changes to retirement plans. For example, it raised pre-tax contribution limits for IRA, 401(k) and 403(k) plans, created a "catch-up" provision for old workers (i.e. allow them to contribute more money to their retirement plans) and increased defined benefit compensation limits.

The Pension Protection Act of 2006 relaxed more restrictions on retirement plans by allowing automatic enrollment and increasing the cap on contribution. As for the capital gain taxes, capital gain was initially taxed at the same rate as ordinary income in 1913 (up to 7%). After that taxpayers were allowed to exclude a certain portion of the qualified capital gains from income tax and as a result the effective capital gains tax rate had been lowered than income tax rate. Dividends had been long been subject to income taxes, while under the Jobs and Growth Tax Relief Reconciliation Act of 2003 (JGTRRA), qualified dividends are taxed at the same rate as long-term capital gains. So the trends are that restrictions on retirement plans have been relaxed and that tax rates on capital gains and dividends have been reduced. The "stepped up" basis of inherited asset was once canceled in the Tax Reform Act of 1976 and was re-introduced in 1980.

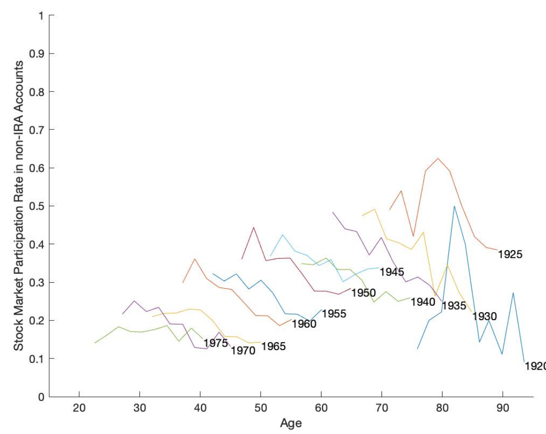
Appendix D. Results from the Balanced Panel

Figure A-2: Cohort Views: Stork Market Participation Rate and Risky Share in the Balanced Panel

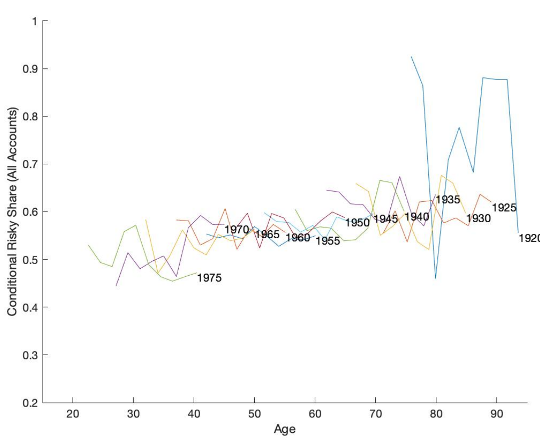
This figure plots two measures of stock market participation rate and the corresponding conditional risky share by age for five-year cohorts in the balanced panel. The cohort labelled "1920" includes families with heads born in years 1920-1924, and so on.



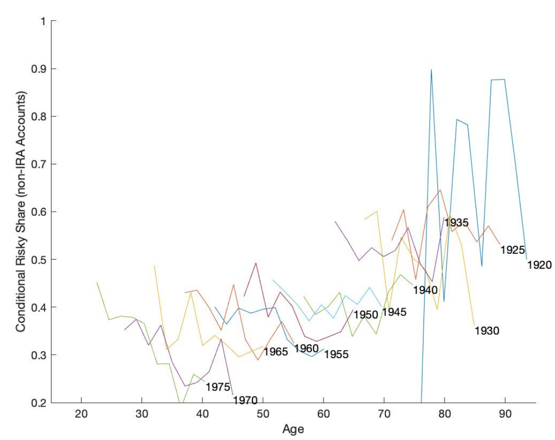
(a) Stock Market Participation Rate in All Accounts



(b) Stock Market Participation Rate in non-IRA Accounts



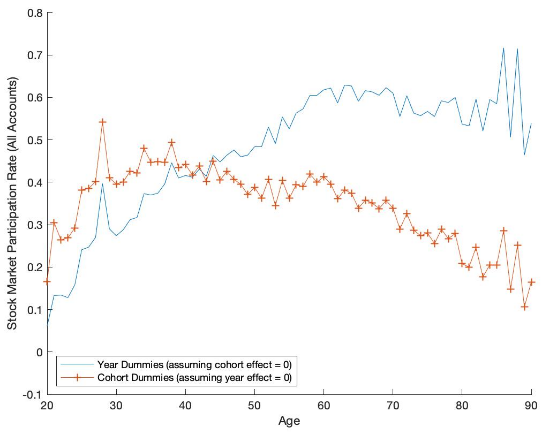
(c) Risky Share in All Accounts



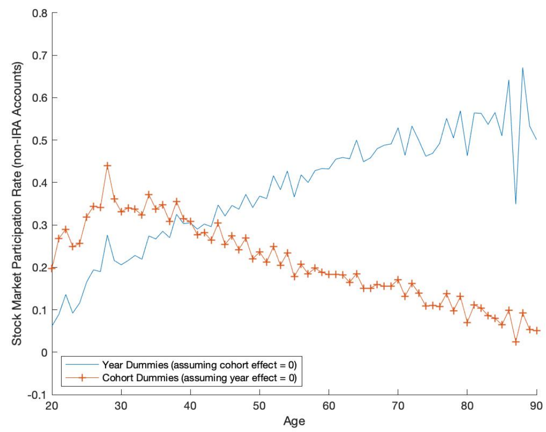
(d) Risky Share in non-IRA Accounts

Figure A-3: Age Profiles of Stock Market Participation Using Dummy Variables: Balanced Panel

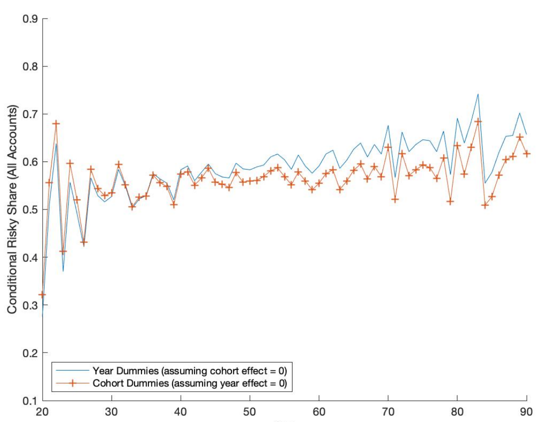
This figure presents the predicted age profiles of two measures of stock market participation rate and conditional risky share on the balanced panel. Panels (a) and (b) considers stock market participation rate and the corresponding conditional risky share in all accounts, while panels (c) and (d) only considers stock market participation rate and the corresponding conditional risky share in non-IRA accounts



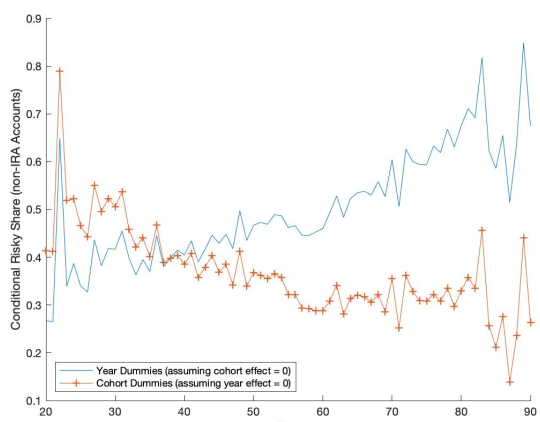
(a) Stock Market Participation Rate in All Accounts



(b) Stock Market Participation Rate in non-IRA Accounts



(c) Risky Share in All Accounts



(d) Risky Share in non-IRA Accounts