Insular Art Forms: Their Essence and Construction

[Revised Version]

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About This Book

This is a practical guide to understanding—by re-creating—the forms of outstanding artwork of the early Insular tradition. Its intention is to explicate principles which underlie designs that have fascinated viewers for a thousand years and more. Such treasures as the Tara Brooch and the Lindisfarne Gospels, as well as the early ringed stone crosses and Anglo-Saxon verse texts, belong to the 'Insular' tradition, evolved in Ireland and England, seventh through tenth centuries.

A lengthening series of discoveries has demonstrated the formal plans of the best pieces of Insular jewelery, 'carpet' pages, stone sculpture, and long poems to have a common source: they are all generated by a constructional cumulative geometry. Simply, compass and straight-edge are the tools to produce configurations which are conceived by rules for achieving total coherence of proportions within the forms. The designs are then executed by hand with astonishing accuracy whether on parchment, in stone, or in metal. The parchment leaves preserve direct evidence (long neglected) of the designers' tools and procedures. The designs belong to an art that is pleasing to both eye and intellect.

Demonstrations of the method of designing have appeared in separate publications over the past thirty years or so, beginning in papers on the formal divisions of long poems in Old English, on religious topics. Those divisions in the unique manuscripts had never been accounted for satisfactorily, explanations ranging through impressionistic guesses about scribal stints, duration of poetic inspiration, size of parchment leaves (or wax tablets) used in original composition or in transcribing of oral performance, pause points for reading in refectory or cell, and so on. No one had attempted to correlate any of the sectional divisions of a text with the other sectional divisions of that same text: that is, no one asked whether the sectional divisions manifested a formal scheme for the whole text in which they occurred. Once the metrical linecounts were set in full array, a web of related ratios could be recognised, and a coherent form unifying all of the 'irregular' sectional lengths became apparent. The striking thing was that the numerical ratios that linked into a comprehensive schema for a text were functions of extremely fine numerical approximations to basic geometrical ratios involving $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, along with 1, 2, 3.

The next step was fairly obvious: to examine the formal features of carpet pages of the same culture of literacy, with their squares and rectangles and interlace, to see whether there were similarities or analogous patterns. From this followed other papers on carpet pages in the principal illuminated Gospels codices. Designs of these pages embodied and interwove the same simple ratios, found when measures 1 and 2 are combined in straight and right angles, that dominate the framed illuminations: 1, 2, 3, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$. These pages—besides their extraordinary intrinsic interest—became a tool for studying the sectional divisions of texts. The graphic expression of the ratios in the carpet-pages provided a most useful analog to the verse-text forms. If 'geometrical' ratios among sectional groups in literary texts were now obvious, there was still a question of how they could have been planned and executed, especially in interlocking patterns. The graphic forms showed the way to finding an answer to that question, and particularly because construction marks in the parchment leaves took some of the guesswork out of reconstructing how the graphic forms were devised. And so the papers on divisions in poetic texts and the lengthening series of papers on page design became a dynamic enterprise, study of either one of the art forms leading to more precise and persuasive study of the other.

That was where things stood when I gathered and synthesised these parallel studies in *The Earliest Irish and English Bookarts: Visual and Poetic Forms before A.D. 1000*. Then a third art form came to my attention, the early sculptured crosses of Ireland. Slowly, one after another of these extraordinary artefacts were found answering to the same kind of formal analysis, as had the texts of English religious poems and the illuminations of the Gospels texts of Kells, Lindisfarne, Echternach, St. Gall, and others. They seem to belong to a period—in their first development, anyway—between the illuminated pages and the poetic texts. Again a series of papers followed, explicating the forms of a number of these crosses. And again, the further analyses entered into dynamic relationship with the two series that had preceded them. Other embodiments of the key ratios became apparent, along with other methods of linking them into a coherent geometry of design.

After these, at the prompting of someone who had seen some of the earlier studies, I began to examine designs in some of the fine metalwork. Its tradition precedes the objects in all three of the other art forms mentioned above. Very slowly—since there were no overt right angles outside or inside the forms to anchor the analyses—the formal schemes of two superb pieces became obvious, built from the same simple ratios and methods as were the others; papers on two famous brooches, Tara and Hunterston, have demonstrated plans more elegant than those of crosses and verse texts, and equal to the very best illuminated page designs. They enhanced still further the understanding of the forms in the other media.

It was through serendipity, perhaps, that still further objects were then discovered to have designs with the very same fundamentals: some pieces of La Tène metalwork more than a millenium earlier than the brooches, the crosses, the illuminated pages, and the poems. Then a well known British Celtic piece, the Battersea Shield, was found to follow the same principles of designing, though with some clear differences. Papers on these have begun to appear. Their contribution to understanding the tradition of designing so far has been primarily in making obvious how simply the designing of an Iron Age disc, for example, can be adapted to designing a ringed stone cross ten long-hundred years later. The Insular tradition of designing had deep indigenous taproots.

And there are artefacts such as the Cross of Cong and the St Patrick's Bell shrine to show the Insular tradition had follow-ons as well as antecedents, within a long, sweeping, smooth evolution. Papers on these have begun to appear.

So at this point, the fundamentals of early Insular designing can be made more accessible to others, I believe, if the tools, ratios, configurations, and procedures are laid out in a systematic way. When understood, they will facilitate study of design in newly discovered archives, or in a boxfull of major artefacts recovered from a fresh dig—or design in innumerable treasures already catalogued and published. The pages that follow can be thought of as a handbook for designing within this singular tradition. They do not offer interpretations of designs, divinations of their putative meanings. It has always been easy to make analyses that are incomplete, inept, over-wrought, or otherwise deserving to be dismissed. The wonderful thing about this early tradition of art, on the other hand, is that once its conceptual basis is grasped, it is hard to make mistakes in analysis of individual works, so long as analysis is rigorous and precise.

The art of designing is just as mysterious now as it ever was, but the technique and some of the aesthetic rules of Insular designing are no longer so. Students, amateurs, scholars, archaeologists—all should find this engrossing, and some may have a professional need to understand it.

Acknowledgements should begin by naming the illuminators, poets, fine metal-workers, stone carvers who created the primary forms for art in early medieval Ireland and England—and their teachers, and those who taught them, ... but nearly all the names have been lost. Even when we have one—Eadfrith, for example—we have nothing really informative about the creators of that extraordinary art.

Failing that, acknowledgements will have to begin and end by naming those who helped bring this handbook through its development and publication; they begin where *The Earliest Irish and English Bookarts* (1994) left off. It was then that my examination of Insular art forms moved beyond verse texts and book design, to find that the same principles of designing had informed standing stone crosses and the finest metalwork, particularly in Ireland; and it was after that when I began to recognise the same principles of creating the primary forms in artefacts earlier, and then still earlier than the medieval era. Patient and helpful commentary was generously given by these good people: Thomas Elwood Hart, Vincent Megaw, Brian Lacey, Dan McCarthy, B. J. Mackinder, Roger Stalley, Niamh Whitfield. Support for part-time assistants was provided by Humanities Undergraduate Research Opportunities program of the University of Washington; Annabel Castro, Miryam Minzer, Nicholas Z. Chen provided animation and video demonstrations illustrating construction of three prominent early Insular art forms. Through all of this (nearly two decades now) Dr Stacy Waters has provided the interface between my prose and ink drawings, and the materials provided to publishers of journals, which have continued to print my papers. He has also provided the interface between my old technology composition and the digital publication requirements of Witan Publishing. Finally, thanks to the Witan triad Richard Scott Nokes, Michael McNamara, Nina McNamara for venturing this publication.

1. Proem

MAGISTER: Draw me a circle.

Wonderful as it is, a circle is not an interesting design (Fig. 1.1a). It has every angle and no angle. It is not right-side-up or upside-down. It is not aslant, so it cannot be put straight. It is zero to infinity in its angles, with none being set with which to build a proportion useful in design.

MAGISTER: Draw me a straight line.

True as it may be, a line is not an interesting design either (Fig. 1.1b). It has an equal angle on either of its sides—a straight angle. There is nothing other than the-same-again, undifferentiated. It is like zero-to-infinity again, but in halves, and still not capable of yielding proportion good for design.

MAGISTER: Draw me two straight lines.

Unless they intersect, or at least touch, they will not be good for making a design either (Fig. 1.1c). But two straight lines can produce design, and in fact can hardly fail to produce design when they do intersect. At various lengths, and at various angles relative to each other, they set relations in both angles and lengths.

MAGISTER: Draw me two straight lines intersecting at right angles.

The simplest design will feature two straight lines, when they intersect and when the angles of their intersection are all equal (Fig. 1.1d). One straight line bisecting another at right angles makes a cross-shape design.

MAGISTER: Draw me a circle with its centre at the intersection of two straight lines at right angles to one another.

Not a lot by way of design lies in that, as it stands (Fig. 1.1e). But from the ratios and the proportions implicit in cross and circle can be constructed designs of brooches and discs, many of them good, a few of them excellent.

MAGISTER: Draw me a square around that circle and cross, with sides equal in measure to the diameter of the circle.

Implicit in this figure (Fig. 1.1f–g) lie ready to hand all the ratios and proportions needed for devising forms for standing 'Celtic' crosses cut from stone, forms of 'carpet-page' illuminations in early Insular Gospel manuscripts, plans of fine metal brooches, and models of the sectional divisions in manuscript texts of long religious poems in tenth-century English.

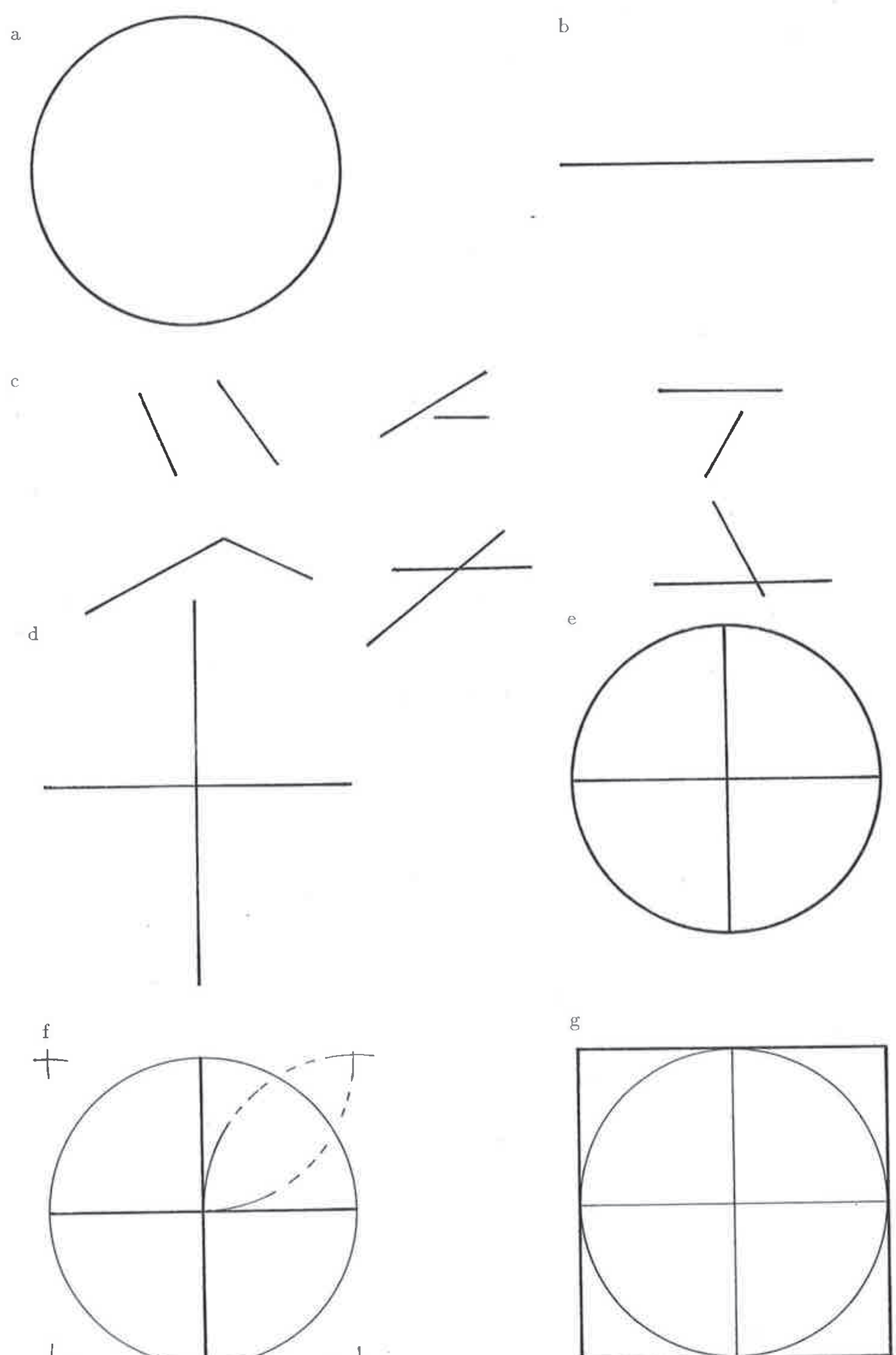


Fig. 1.1 Cross, circle, square underlying major Insular forms.

2. Inventory of Essentials

Tools

For all except verse texts, only two tools are required, a compass and a straight-edge. They work on a flat surface (a plane).

Compass, also called a **pair of compasses** Mechanically, this is nothing more than two members, usually called 'legs,' joined by a pivot, or hinge. The legs may be straight or curved or otherwise, so long as they are rigid. The joining may be at the ends of the legs, or at some point midway along their lengths. If anywhere midway, the compasses may be 'proportional,' extension of the opening at one end proportional to the extension of the opening at the other.

Functionally, the compass is used for drawing circles or parts of circles ('arcs'), and to 'take' measures. In drawing circles or portions of them, the end of one leg (its 'foot') is fixed in place, and the other end (the other 'foot') moves along a plane surface, marking its path. To take measures, the feet of the compass are placed at the points marking the distance to be measured.

A string pulled taut between two pegs will serve the same functions.

Straight-edge Mechanically, this is nothing more than a piece of flat material having one edge that is straight, that is to say, extending uniformly in a single direction.

Functionally, a straight-edge guides the path for a moving point for marking the direct (shortest) path between two points.

A taut string (a 'snapline') will also mark that path.

A lineal **measuring scale** is needed for verse texts, to convert lineal extension into numerical expression. Optimal is a straight-edge marked with ten equal measures, one of them—at one end—divided into twelfths (as fourths each divided into thirds).

(A **set square**, convenient for drawing plans incorporating right angles, seems not to have been used for carpet pages, and quite possibly was not used for the other designs either; at any rate, it is not an esssential tool.)

Configurations

The four common configurations pair up to a large extent with the materials and the contexts of the artefacts.

Circle Typically brooches and bosses in metalwork have survived.

Ringed Cross Stone sculptures or carvings most commonly survive, but the form is appropriate to bookart as well.

Rectangle Illuminated 'carpet' pages in books are many; surviving boxes made for book-shrines are few; the shape routinely is used in design of buildings as well.

Lineal Extension Narrative texts in verse are divided into sections of unequal and non-modular length, in this respect being unlike 'stanzas'.

Three of these configurations—circle, ringed cross, and rectangle—underlie two-dimensional plans laid out on a plane surface. The relative disposition of the parts of these three can be seen, making up a static visual pattern. The configuration of segmented lineal extensions is one-dimensional and cannot be grasped visually, even if the segmentations are a conspicuous visual part of the text. The configuration can be recognised nonetheless in the relations among the counts of metrical lines in the segments, which in turn have visual analogues in geometrical constructions.

Forms developed within all four configurations nearly always depend for their creation on setting lines at 90 degrees (right angles), or at 120 degrees (dividing a circle by three). That is obvious in the rectangular forms and the crosses (ringed and otherwise). In the circular designs and lineal extension plans, right angles may not appear in structural lines, yet they routinely appear in the sources of the ratios among the lengths and the distancing of the principal lines.

Ratios

The ratios which generate the finest cohesive Insular designs can be set in graphic form—without numerals, without algebraic expression—using only the tools described above, proceeding from a circle, from a concentric circle and cross, or from a concentric circle, cross, and square (see Proem). The inventory of most of these ratios comprises the relations among measures immediately implicit among 1 and 2 at either straight angle or right angle: they are the relations in basic combinations of 1, 2, $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$. These relations may be very simple in algebraic expression, such as $1 : \sqrt{2}$, or less simple, such as $1 : \frac{2}{\sqrt{5}-1}$. None of them is complicated to set, though, with the tools of the geometer.

The measure **1** is not derived. It appears as the 'given,' the measure set down prior to constructing the design, setting its size, its magnitude. All the other measures can be derived from 1.

The measure **2** in the construction of designs in Insular art is the double of 1; it is $1 + 1$ at a straight angle.

The 'geometrical' measures are illustrated in Fig. 2.1, which offers simple, typical examples of the configurations in which each of them occurs in Insular art; each will be described more fully in the next section, 'Setting the Basic Ratios'.

$\sqrt{2}$ at its simplest is the measure of $1 + 1$ at a right angle. It is routinely found among the designs in the diagonal of a square whose sides measure 1 (Fig. 2.1a), or in hypotenuse of an isoceles right triangle with sides measure 1 (Fig. 2.1b), or in the chord of a quadrant of a circle whose radius measure is 1 (Fig. 2.1c).

$\sqrt{3}$ in a common derivation is the measure of $1+\sqrt{2}$ at a right angle. It is also found typically as the longer side of a right triangle whose short side measures 1 and whose hypotenuse measures 2 (Fig. 2.1d), or as hypotenuse of a right triangle whose sides measure 1 and $\sqrt{2}$ (Fig. 2.1e), or as the long side of a right triangle inscribed in a (semi-)circle, with the short side of the triangle equal to the radius (Fig. 2.1f—same geometry as in Fig. 2.1d but from a different derivation).

($\sqrt{4}$ is the measure of $1 + \sqrt{3}$ at a right angle; in the practical geometry of Insular design, it never seems to be produced in this way, because it is also equivalent to 2, directly derived from 1, as above.)

$\sqrt{5}$ at its simplest is the measure of 1 and 2 at a right angle. It commonly occurs as the measure of the hypotenuse of a right triangle with sides measuring 1 and 2 (Fig. 2.1g) or as diagonal of two squares that share one side.

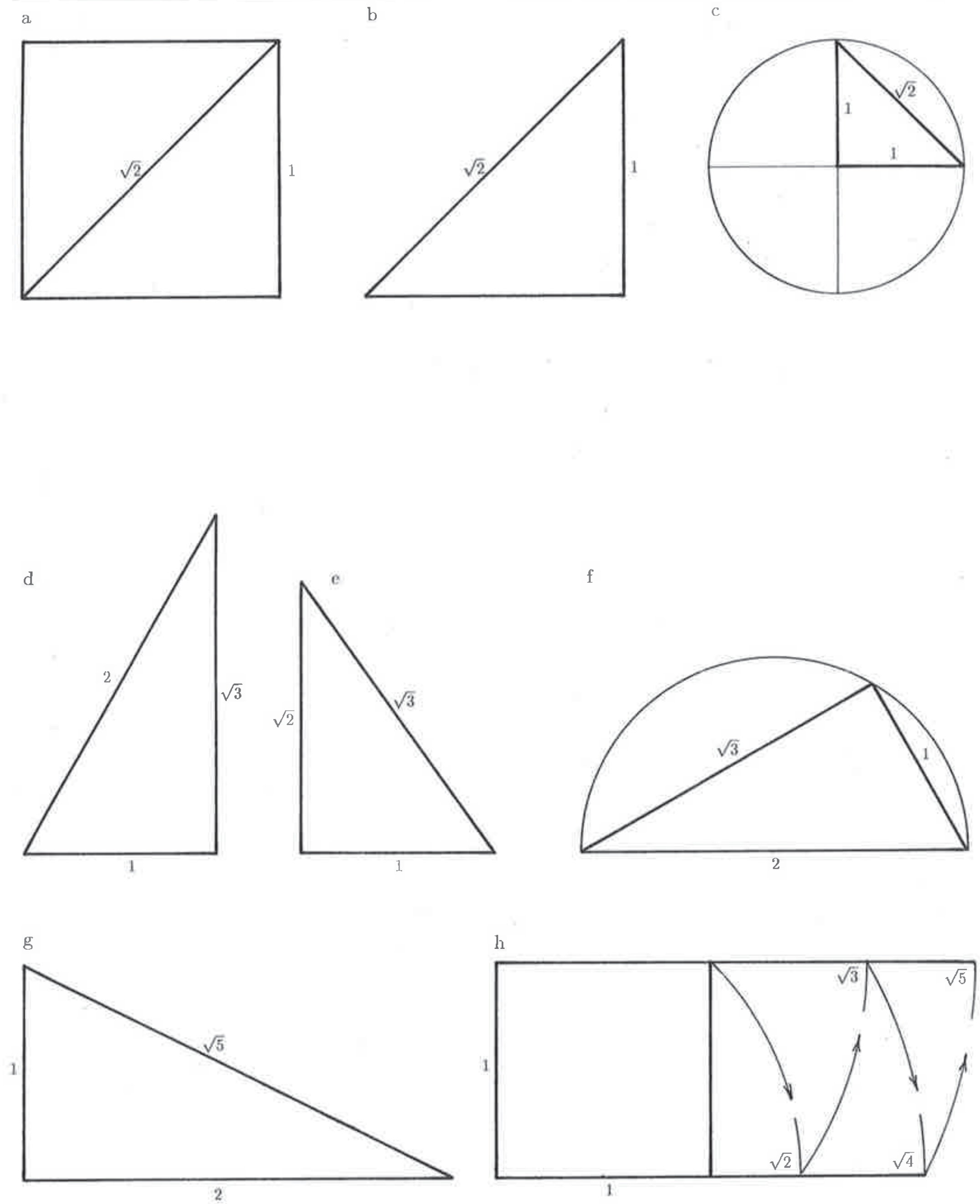


Fig. 2.1 Typical occurrences of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, employed in Insular art.

A handy summary of these measures in relation to one another is shown in Fig. 2.1h.

It will be apparent that these fundamental manifestations of 'geometrical' quantities follow the Pythagorean Theorem: that the sum of the squares of the lengths of the sides of a right triangle is equal to the square of the length of the hypotenuse of that triangle.

With other ratios incorporating integers only, measures above 1 are simply increments of that unit measure along a straight line.

On the Rôle of the Compass

That the compass was an essential tool in creating the principal symmetrical forms of Insular pages, metalwork, and high crosses needs no more evidence than the accurate circles and segments of circles embodied in so many of them. Further, when the artefact allows it, as in bone motif-pieces or vellum leaves in a codex, use of the compass is witnessed in another way, the marks left by the point of the fixed foot of the drawing tool. Some artefacts of La Tène culture, though fewer and less varied, show dependence on the compass just as plainly.

This use of the compass has been known all along, and is mentioned often in discussion of many of the artefacts. Another use needs carefully to be distinguished, however, if we are to understand the designing of typical objects of these linked traditions of art. The accurate circles of metalwork and stone sculpture and areas of ornament on pages, as well as construction marks on animal products used in them—these are products of manufacture, products of the hand in the process of making the object. Their being where they are, their rôles in the designs that underlie them, on the other hand, have not been explored as they should be. The designs were worked out by hands-on methods, a few of them leaving traces of the manufacturing process, but they were at the same time the products of the head interacting with the hand. The conceptual aspects of the designs have been studied very little. Without study of the conceptual processes along with the manual ones, the finest pieces in this artistic tradition cannot be fully understood for either their histories or their typologies.

Attention to the ideas controlling the shaping of the objects ought always to be prior to attempts to identify any 'meaning' the objects may have been created to carry, whether they are thought to have religious symbolism, or to have some bit of that grand catch-all, 'ritual significance.'

3. Setting the Basic Ratios

The ratios embodied in the forms of page illuminations, poems, brooches, ringed crosses are nearly always relations among 1, 2, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$. In a few 3 is present; 4 occurs occasionally, 5 or 7 occurs only rarely.

'Arithmetical' ratios—those involving only integers. They are developed in constructive geometry either by extension or by division.

Start with the relation of 1 and 1 as it is manifest at a straight angle (Fig. 3.1a). The effect of laying down two quantities at a straight angle is simple addition. This seems to be the normal way 2 is derived in the geometry of Insular art. The resulting ratio is 2 : 1.

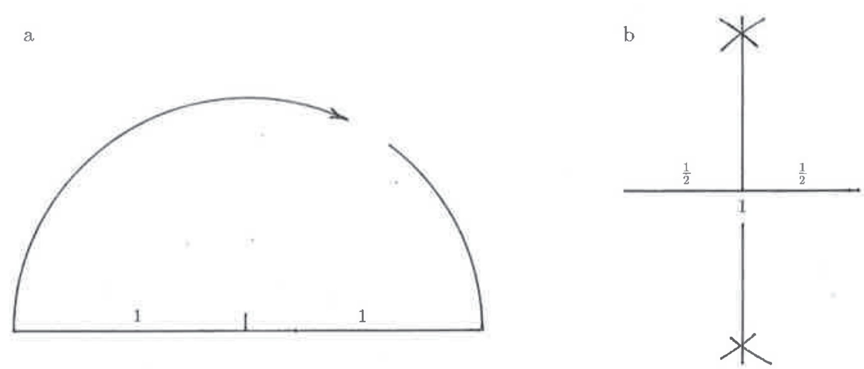


Fig. 3.1 Derivation of measure 2 (and its inverse) in relation to 1.

The inverse of 2 is derived is by a single equal division of 1. The classic method is that of bisecting a line (Fig. 3.1b). Fix one foot of the compass in turn at either end of the line representing the quantity, open it so that the other foot is at more than half the measure to be divided, describe arcs to intersect above and below, and run a straight line between the intersections: the original measure 1 is divided equally into two parts, the new measure expressed as the fraction $\frac{1}{2}$. It sets the ratio of the two measures at 1 : 2.

The straight-angle derivations seldom aggregate to more than 3 (Fig. 3.2a). In practice, it is usually produced as 2 + 1, though occasionally it seems to be 1 + 1 + 1; examples will be pointed out in the next chapter. In any case, the resulting ratios are 3 : 2 and 3 : 1.

Division of the unit measure by three is readily achieved in this way. Set up a right triangle with sides measure 3 and 1 (Fig. 3.2b). At measure 1 along the longer side, run a line at a right angle to that side: it intersects the hypotenuse of the 3 : 1 triangle at a measure $\frac{1}{3}$ above the base line (Fig. 3.2b); at measure 2 along the longer side, run a line at right angle to that side: it intersects the hypotenuse at a measure $\frac{2}{3}$ above the base line (Fig. 3.2c).

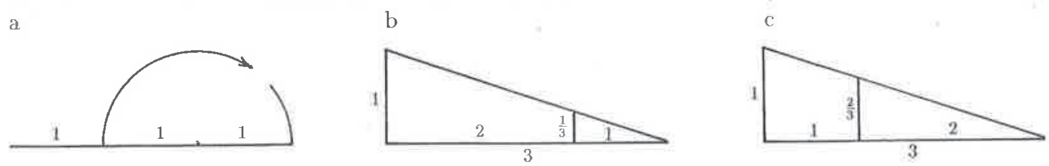


Fig. 3.2 Derivation of measure 3 (and its inverse) in relation to 1.

Within a circular form, these methods of division can be readily applied. Division of the radial measure by 2 results from this simple procedure (Fig. 3.3a): from one end of the diameter of the circle, sketch an arc (1) with radius equal to the radius of the circle; run a line (2) between its intersections with the circle, to bisect the radius. Division of the radial measure by 3 results from this further procedure (Fig. 3.3b): (1) copy half the length of the radius along the line used to bisect the radius; then run a line (2) from there to the farther end of the diameter line, cutting the centerline at one-third the length of the radius. Once again segments are in the ratios 1 : 2 and 1 : 3 and 2 : 3.

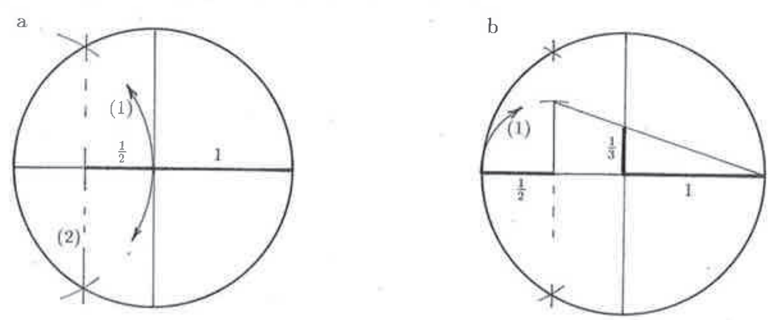


Fig. 3.3 Division by 2 and by 3 within circle.

Within a quartered square, division by 2 is as simple as marking the intersection of a diagonal of two quadrants sharing one side (Fig. 3.4a). Division by 3 can be achieved as a subsequent step (also Fig. 3.4a) resembling the procedure in Fig. 3.2c, by setting a $\frac{3}{2}$: $\frac{2}{2}$ right triangle. Another division by 3 involves the paths of two diagonals. Sketch a diagonal of two quadrants that share a side and an intersecting diagonal of one of those quadrants (Fig. 3.4b); their intersection divides each diagonal at one-third its length. When this is done on either side of the midline of a quartered square, a line through the two intersections will divide the midline at one-third its length (Fig. 3.4c). (Similarly using centerline.) Further division in thirds, if needed, is easily done by copying the $\frac{1}{3}$ measure at a straight angle.

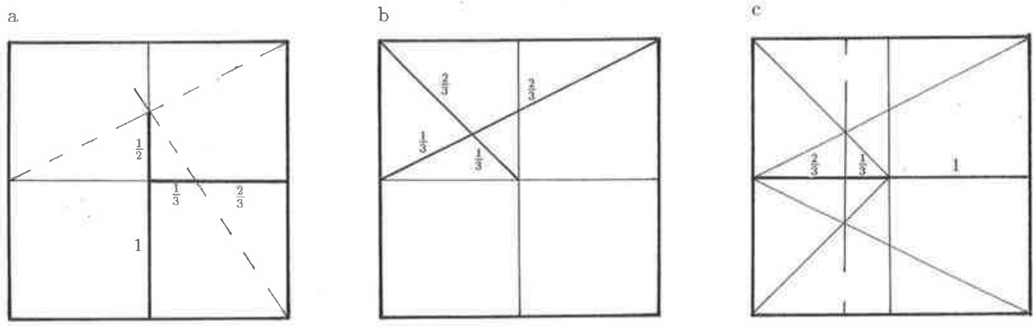


Fig. 3.4 Division by 2 and by 3 within quartered square.

The fundamental arithmetical ratios emerge as well in rectangular forms developed by extension from an underlying quartered square. A 2 : 1 rectangle is readily set out when sides are extended in one direction by a measure equal to the side of the square (Fig. 3.5a); this is an asymmetrical derivation. Alternately, the opposite sides of the square can be extended from its corners by half the measure of a side (Fig. 3.5b), or extended by full measure of a side from the midpoint of a side of the square (Fig. 3.5c).

The obviousness of these three derivations of a 2 : 1 rectangle makes them excellent examples of two key aspects of creating Insular designs. One is that there are nearly always alternate procedures ready to hand. At the same time, one or two of the alternate procedures will usually fit more naturally (so to speak) with the whole design and some will show themselves to be in fact inapposite. An asymmetric expansion, for instance, is hardly appropriate to laying out the frame for an illuminated page, especially if the frame is to be filled with design that has inverse symmetry.

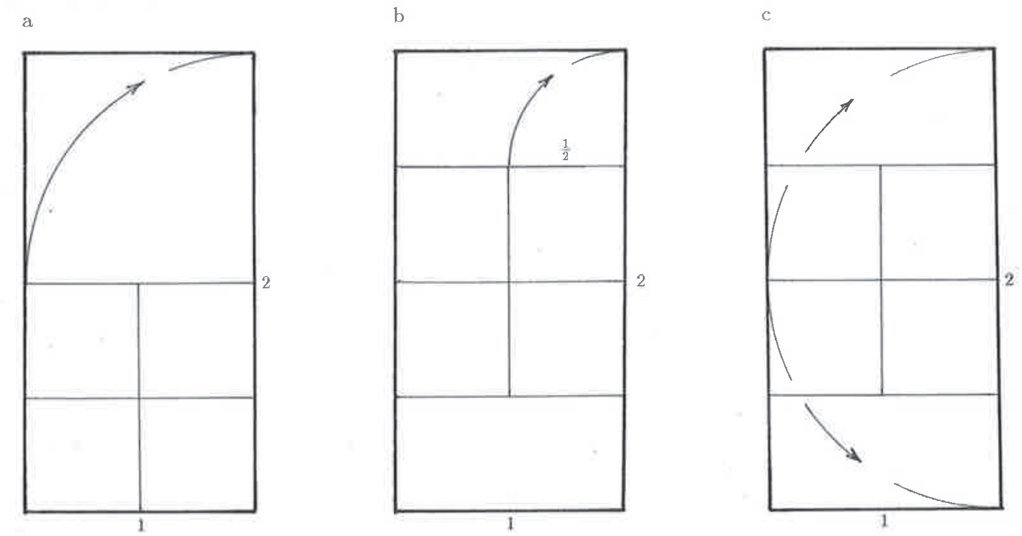


Fig. 3.5 Extension of quartered square into 2 : 1 rectangle.

Similarly, circular plans can be extended with 2 : 1 ratios of their diameters (Fig. 3.6), either asymmetrically by doubling the diameter (a, b) or symmetrically by doubling the radius at opposite ends of the diameter (c).

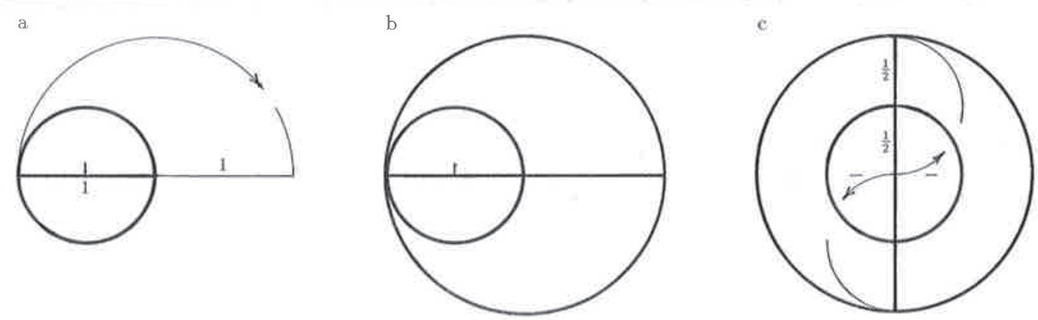


Fig. 3.6 Extension of circle into 2 : 1 circle.

Expanding a quartered square into a 3 : 2 rectangle follows procedure like those already illustrated with 2 : 1 rectangles. Fig. 3.7 illustrates an asymmetrical procedure (a) and a symmetrical procedure (b).

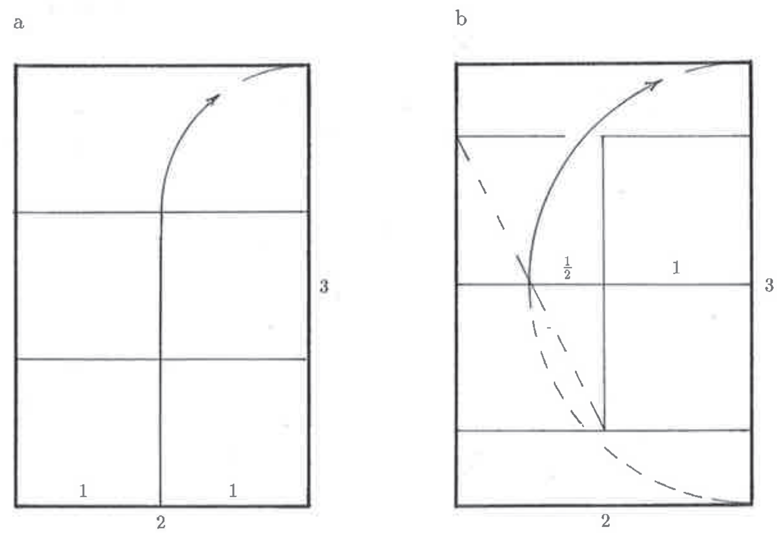


Fig. 3.7 Extension of quartered square into 3 : 2 rectangle.

The ratio 4 : 3 is not uncommon for rectangular forms. Here are three ways to construct this configuration. The first (Fig. 3.8a) proceeds from a quartered square and the division of half the midline into thirds, as in Fig. 3.4c; it then expands the square into a 4 : 3 rectangle by a procedure like the one illustrated in Figs. 3.5c and 3.7c. The second (Fig. 3.8b) is only a slight variant of the first, beginning as in Fig. 3.4a. The third procedure exploits the 3 : 4 : 5 relation among sides and hypotenuse of a right triangle, which can be assembled by any convenient method (Fig. 3.8c, for example). This ratio governs the design of such illuminations as the Lichfield Gospels Cross Page and two evangelist portraits in the St. Gall Gospels.

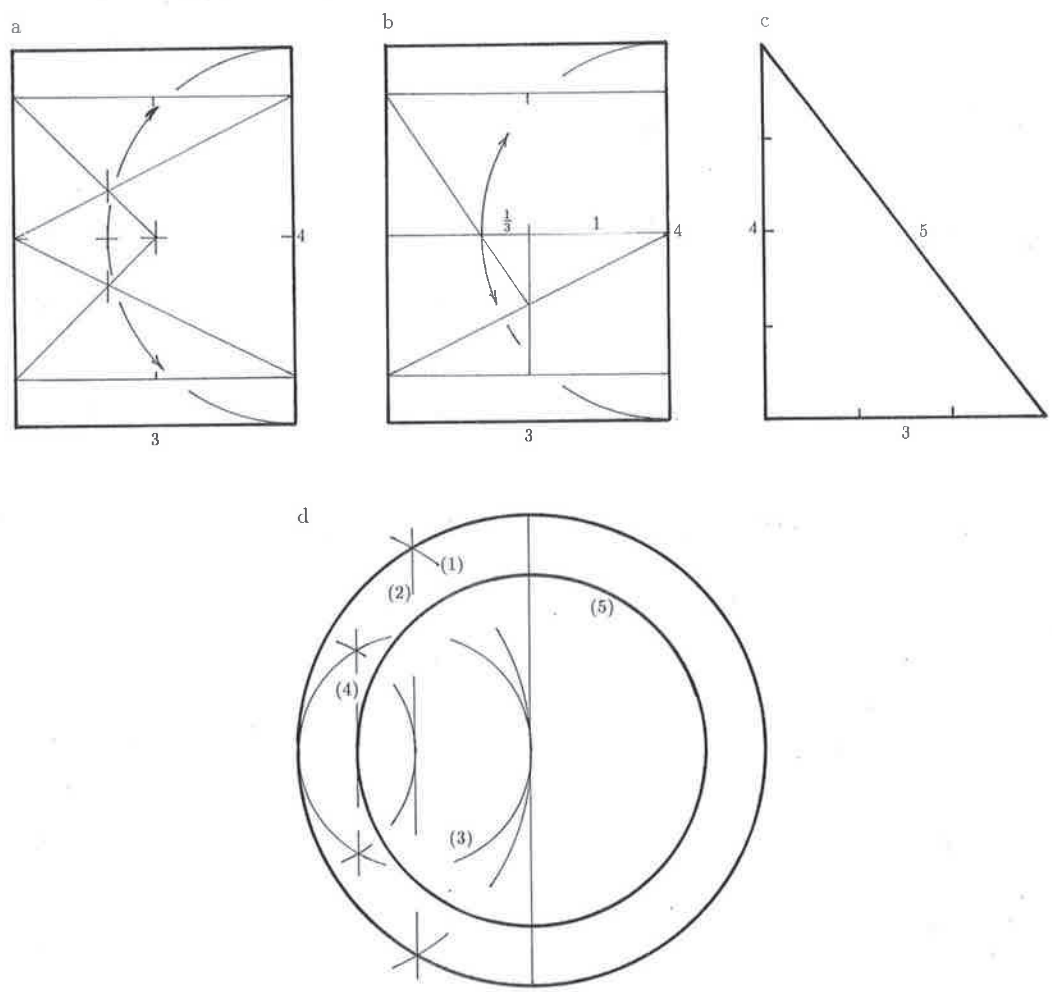


Fig. 3.8 Construction of 4 : 3 rectangle and radii.

In a circular form, a 4 : 3 relation of radii for concentric circles is readily produced by iteration of the procedure shown in Fig. 3.3a for halving a radius (Fig. 3.8d).

Beyond these, other arithmetical ratios can be set up by similar procedures, as needed. They are less frequent, and they will not exceed ratios 3 : 3, 3 : 5, and 4 : 7, in the known designs. The first (Fig. 3.9a) follows from the division of a square as 3×3, by marking one-thirds along the four sides; cf. Fig. 3.4a, where $\frac{2}{3}$ of a quadrant side is marked, which can be copied as $\frac{1}{3}$ of a side of the square. Or following on from Fig. 3.4b, c: set up corresponding divisions in each quadrant, then run lines through the intersections of the pairs of diagonals to divide the whole square as 3×3. Expansion of a 3 : 3 square into a 5 : 3 rectangle (Fig. 3.9b) follows a procedure like ones shown in Fig. 3.7. Expansion of a square into a 7 : 4 rectangle (Fig. 3.9c) is a simple variant of the one shown in Fig. 3.7c.

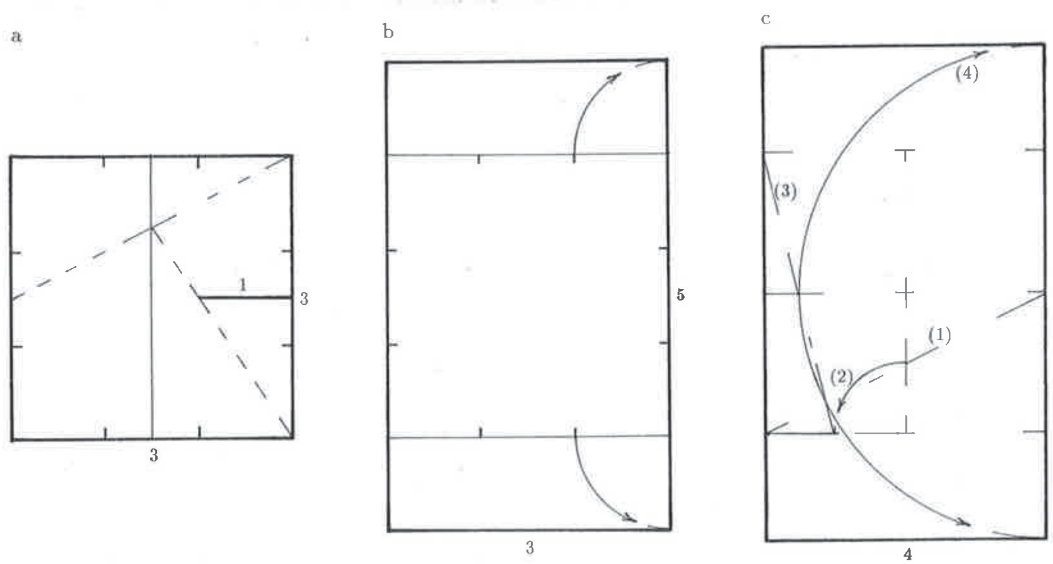


Fig. 3.9 Construction of 3 : 3, 3 : 5, and 4 : 7 rectangles.

Entr'acte

Before proceeding to 'geometrical' ratios in Insular designing, an analogy may be useful. How can one understand, say, J. S. Bach's Chaconne in G, concluding one of his suites for solo violin, without understanding the design of the instrument it was composed for? Of course, transcriptions can be made for keyboard instruments—even brass choirs—that musically may be very similar. But this movement of the partita takes a crucial part of its design from an instrument having strings tuned in fifths across a rounded bridge and played with a bow. Its shapes (so to speak) originate in the physics of the instrument on which it was composed. The physics of the instrument set certain ratios and proportions that inform principal portions of the composition. Excellence of composition is then dependent in part upon obedience to the relational rules implicit in the medium of the composition.

Correspondingly, how can one understand the formal plans of Insular art without understanding the implications of the mechanical means of shaping it? Circle-and-cross and rectangle are the simplest of regular shapes. When they are represented by human beings, they have rough approximation in free-hand construction. They will approach perfect representation with mechanical aids. In a mathematician's terms, the path of a point moving at a constant distance from a fixed point plots a circle. In an artisan's terms, a circle is something drawn on a flat surface by fixing a point, by peg or by point of compass, and guiding another point around it at the fixed length of a string or the spread of a compass. A cross is formed from two straight lines on a plane intersecting with equal angles, as in Fig. 1.1d. It is constructed by the same procedure illustrated in Fig. 3.1b that will also divide a given line into two equal segments.

So, if the sculptured stone crosses, for example, have cross-shape as the sine qua non of their design, and if many of them have circle-shape as well—the two shapes always being concentric, as in Fig. 1.1e—then how in the world can we imagine artisans creating the carefully symmetrical designs for these sculptures without mechanically plotting these basic forms?

The point, though, is not that they would have used dividers (or string) and straight-edge in laying out the designs. Rather, it is that the designs will not be understood aright without recognition that use these mechanical devices has set the proportional relations of the layout as well. We have seen their use in laying out forms with proportions expressible in relations between integers. Next to be described is their use in layout out forms with proportions incorporating irrational numbers—'geometrical' ratios.

'Geometrical' ratios—those involving irrational numbers, i.e., quantities not expressible as common fractions, which is to say, not expressible as relations among integers. These ratios are also developed by extension and by division.

The most elementary of these ratios is represented in modern notation as $1 : \sqrt{2}$. An immediate derivation, and one that is frequent in Insular art, generates $\sqrt{2}$ as the measure between 1 and 1 at a right angle (Fig. 2.1 *a*–*c*). It shows up in rectangular configurations as asymmetrical expansion of a square into a $\sqrt{2} : 1$ rectangle (Fig. 3.10a). Symmetrical derivation of the same rectangular shape is achieved by setting $\sqrt{2}$ of a quadrant measure along the midline, then copying it above and below to intersect extensions of the side of the square (Fig. 3.10b). And it can also be set readily by copying from each corner of the square the measure to an inscribed circle to intersect extensions of the sides (Fig. 3.10c).

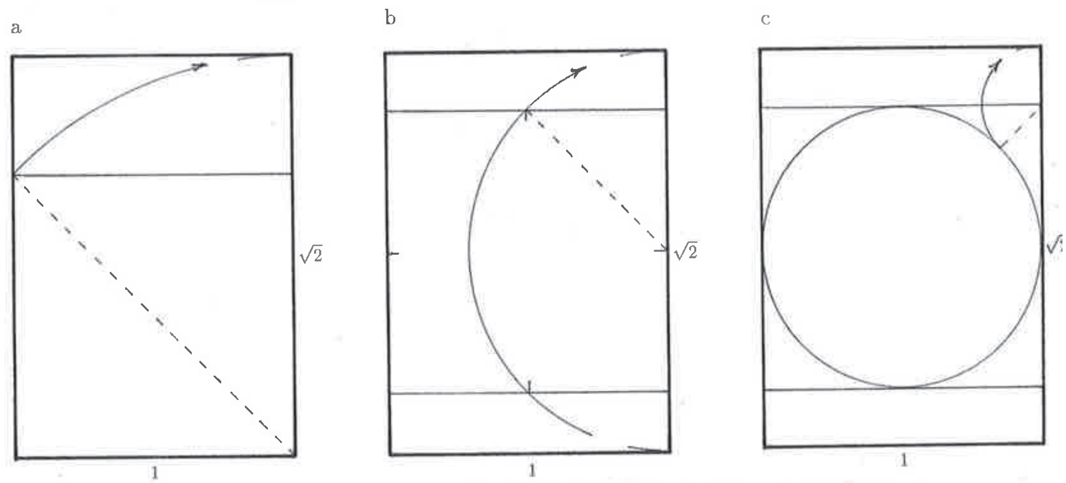


Fig. 3.10 Some derivations of $\sqrt{2}$ in relation to 1.

Within a circle or square on the given measure 1, the measure $\sqrt{2}$ is derivable in a number of ways. With diagonals of a square and an inscribed circle on the same measure, a smaller square can be plotted at $\frac{1}{\sqrt{2}}$ of the given measure (Fig. 3.11a), or a circle with diameter in the same measure can as well (Fig. 3.11b). Division of the radial measure into segments still in the ratio $1 : \sqrt{2}$ can be made at the centre of the square or circle by copying the diagonal of a quadrant onto the midline or diameter (Fig. 3.11c), where $a : b :: c : d :: 1 : \sqrt{2}$.

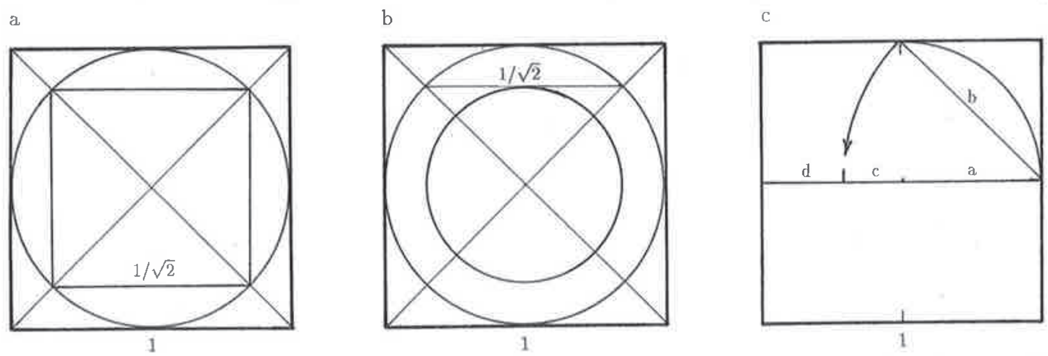


Fig. 3.11 Some derivations of the inverse of $\sqrt{2}$ in relation to 1.

Sides of a quadrant or a square can be divided easily into this same inverse ratio by any of the three procedures shown (Fig. 3.12a, b, c); the first two place the shorter segment nearer the corner and nearer the midline, respectively.

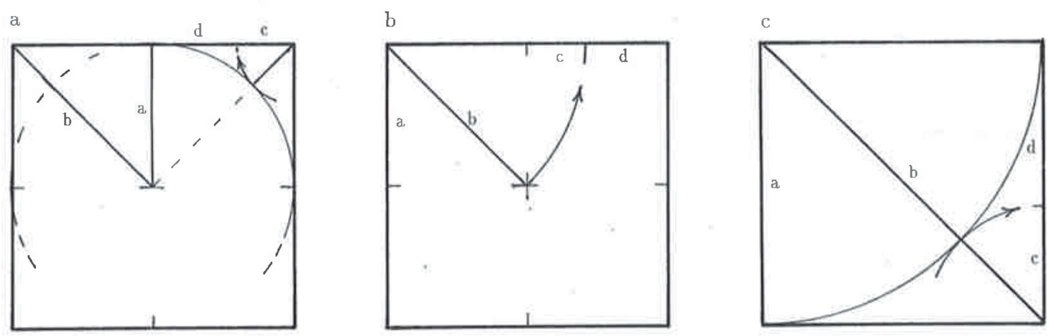


Fig. 3.12 Some further derivations of the inverse of $\sqrt{2}$ in relation to 1.

When division by $\sqrt{2}$ (as in Fig. 3.11a, b) is repeated, the resulting measure yields the ratio 1 : 2. Repeating the process yields $\frac{1}{2\sqrt{2}}$. And so on (Fig. 3.13a). Every stage yields a form at half the measure of the original square or circle. This procedure is employed as a two-stage process, producing concentric squares or circles with measures related as 1 : 2 and the mean $\frac{1}{\sqrt{2}}$: 1, but seems not to be used beyond this second stage. A $\sqrt{2}$: 1 rectangle when divided in two, as by folding a sheet of parchment to make two leaves (four pages) produces two rectangles with the same shape (at half the size). The 'folding' can be repeated with the same results each time (Fig. 3.13b). A set derived thus has coherence in the iteration of a single ratio.

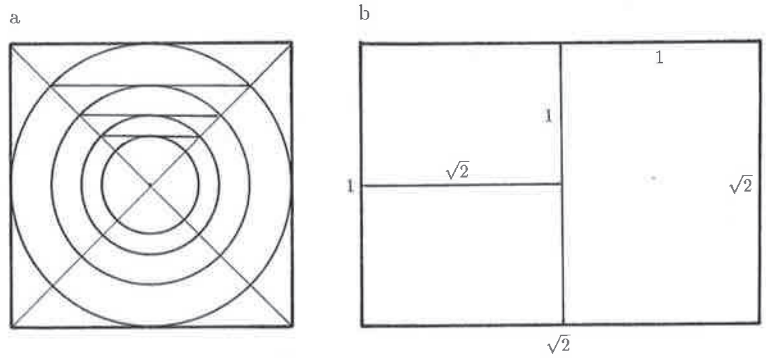


Fig. 3.13 Still further derivations of the inverse of $\sqrt{2}$ in relation to 1.

Forms governed directly by the relation between 1 and $\sqrt{2}$ are plentiful: among them, the North Cross, Castledermot, Co. Kildare, the David rex page in the Durham Cassiodorus, the Book of Mulling, p. 189, the St. Cuthbert Gospel of St. John, and Echternach Gospels, fol. 75v; or in the linecounts of sections of poetry in the Old English *Andreas* (in the Vercelli Book) and *Guthlac A* (in the Exeter Book). And then there are objects separated by more than a millenium in their date of manufacture, exceeding before and after the conventional dates of Insular art, yet devised by the same principles within a continuous tradition, such as the Cuperly Disc and the Kilfenora Cross, described in 5.1–2, below.

Direct derivation of $\sqrt{3}$ as a 'geometrical' measure is found as the hypotenuse of a right triangle with sides measuring 1 and $\sqrt{2}$ (Fig. 3.14a, cf. Fig. 2.1h). It is found as the length of a side of a right triangle with hypotenuse 2 and short side 1 (Fig. 3.14b). Measure $\sqrt{3}$ is the height of two equilateral triangles sharing one side (Fig, 3.14c); this manifestation seems not to be present in Insular art.

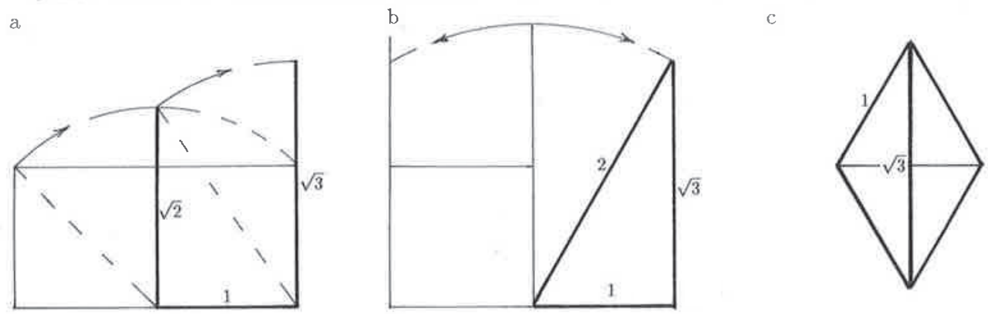


Fig. 3.14 Some common occurrences of $\sqrt{3}$ in relation to 1.

Or it appears twice in a (semi-)circle whose diameter measures 3: a perpendicular line at measure 1 intersects the circle at measure $\sqrt{3}$ from the near end of the diameter; from there to the far end of the diameter is $\sqrt{6}$, which in relation to $\sqrt{2}$ is $\sqrt{3}$ (Fig. 3.15). The interconnectedness of these primary geometrical ratios will be particularly obvious in these configurations.

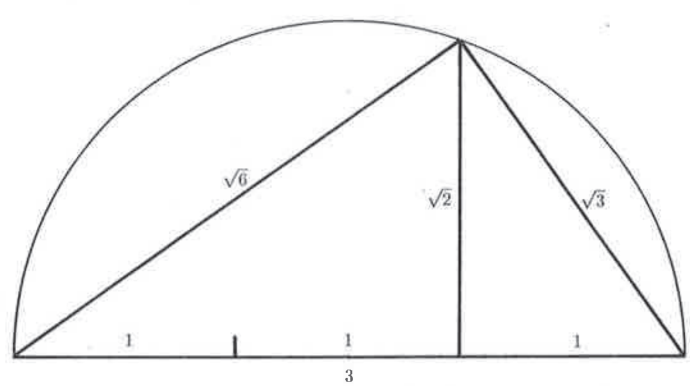


Fig. 3.15 Another occurrence of $\sqrt{3}$ in relation to 1.

The ratio represented nowadays as $\sqrt{3} : 1$ does not seem to be used directly in the main works of Insular design, because in terms of design it is 'excessive.'

Nonetheless, the measure $\sqrt{3}$ appears in less simple ratios governing the forms of a number of Insular designs, some of them among the most celebrated. The most common of these ratios is the inverse of $\sqrt{3} - 1$, that is, $\frac{1}{\sqrt{3}-1}$; a further equivalent is $\frac{\sqrt{3}+1}{2}$. This is a ratio fully appropriate to leaf-shape or illumination-shape, and can be derived readily as shown (Fig. 3.16), among other ways.

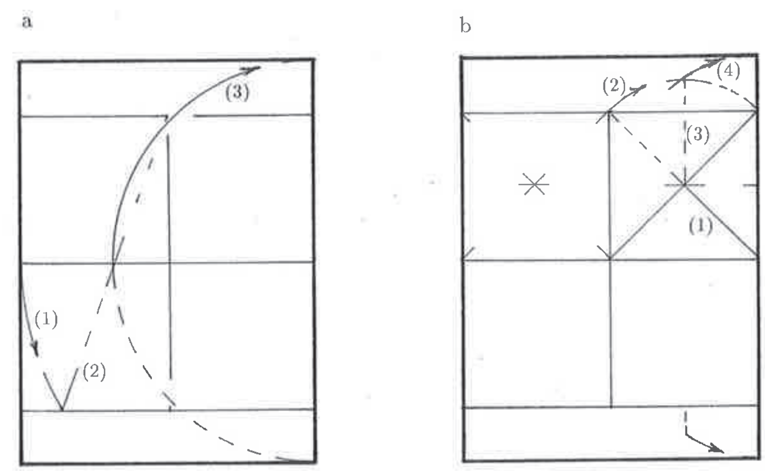


Fig. 3.16 Common occurrences of $\sqrt{3}-1$ in relation to 1.

By division of a given measure, the common ratio involving this geometrical measure is $\sqrt{3}-1$. For setting the thickness of a ring based on a quartered square there is a simple procedure (Fig. 3.17a). For setting this from a given circle the procedure is likewise simple (Fig. 3.17b). Or for setting the thickness of a rectangular frame, there are similar procedures, one of which is shown (Fig. 3.17c).

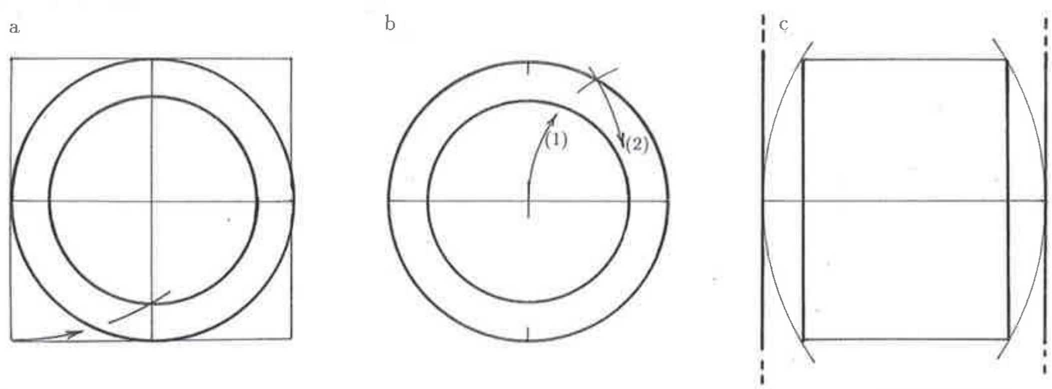


Fig. 3.17 Common occurrences of $\sqrt{3}-1$ by division.

On ratios incorporating the measure $\sqrt{3}$ are based a wide variety of designs, from a sketchy trial-piece for a brooch, to the North Cross at Ahenny, Co. Tipperary, to the spectacular St. John page in the Book of Kells.

As if to underscore the grounding of Insular design in 1 and 2 in geometrical simplicity, ratios incorporating the measure $\sqrt{5}$ seem to be even more frequent and more varied than those incorporating $\sqrt{2}$. Whereas the latter is the measure between 1 and 1 at a right angle, $\sqrt{5}$ in its simplest derivation is the measure between 1 and 2 and a right angle (Fig. 3.18a). This measure, too, is 'excessive,' being greater even than 2, by about the same amount that 2 is greater than $\sqrt{3}$.

So the measures used in these designs will be found in derivatives from 1, 2, $\sqrt{5}$. A common ratio diminishes the 'excessive' measure by 1, yielding the ratio $\sqrt{5} - 1 : 1$, the shape of two 4-by-3 cross-page frames as well as the text-space in the Lindisfarne Gospels (Fig. 3.18b), among other objects. Its reciprocal $1 : \sqrt{5} - 1$ occurs in the thickness of the ring of a stone cross, for example (Fig. 3.18c), as well as thickness of rectangular frames for illuminated pages.

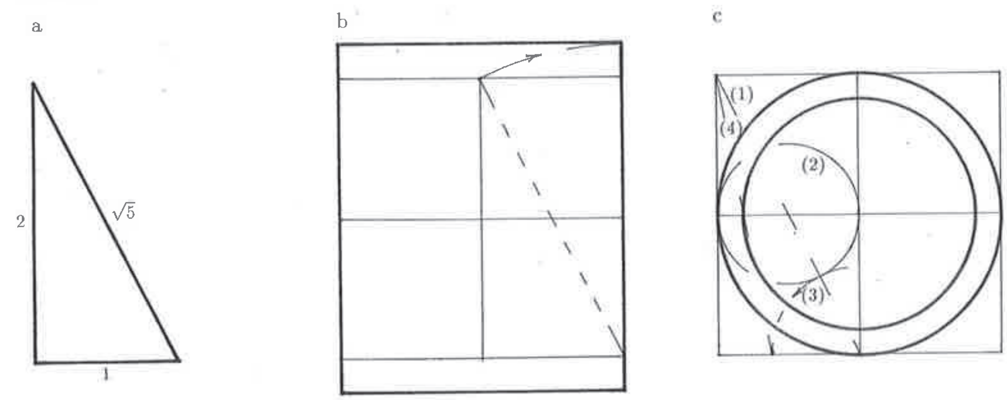


Fig. 3.18 Some derivations of $\sqrt{5}$ in relation to 1.

But it is in ratios involving the combination of these three elementary measures that are found the greatest variety and the greatest potential for excellent design: most of them grow from $2 : \sqrt{5} - 1$, which will be recognised as an expression of extreme and mean ratio, by the 'golden section' of a line. There are many procedures that will set this geometrical ratio. Some are probably not pertinent to Insular design, such as those derived from properties of a pentagon, or based on chord and arcs in a specified number of degrees of a circle: those elements do not seem to occur in the contexts that dominate Insular artworks. (They are altogether unlike the procedures—or their object-forms—described by Charles Bouleau, *The Painter's Secret Geometry*, for example, or some others recorded in the history of this aspect of mathematics.) The classic method (Fig. 3.19a) is implicit in the practical geometry of the designs, though probably seldom used in that elemental form. A derivation from a triangle with sides 2 : 1 is also implicit in the creation of forms (Fig. 3.19b), though again the usual forms of artefacts do not include this basic triangular shape. These two procedures are included only to exemplify further the underlying simplicity of the golden ratio in derivation.

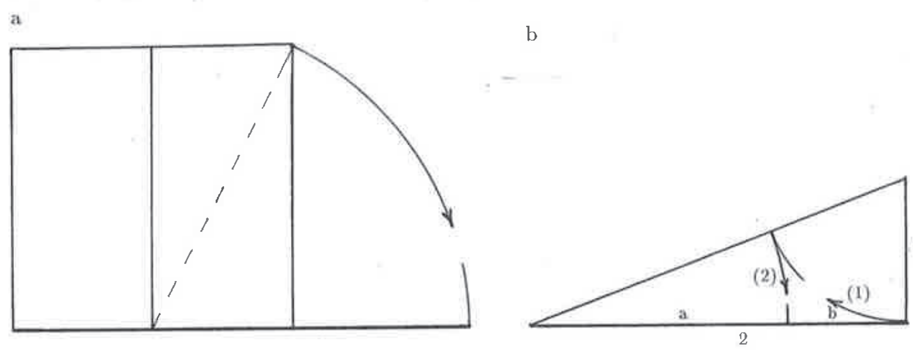


Fig. 3.19 Classic and basic derivations of $\frac{2}{\sqrt{5}-1}$, 'golden ratio'.

The two procedures for setting the golden ratio measure which seem to have been most used are illustrated next: 'most used' because the context is most often a rectangular or circular form—which includes pages, brooches, and ringed crosses, of course—and because in the medium where construction marks can be preserved, i.e., in parchment pages, the compass foot settings are sometimes still to be seen as impressions or punctures. In a quartered square an arc intersects a diagonal at $\frac{\sqrt{5}-1}{2}$ which, copied to a side of the square, divides that side into segments a and b, which manifest the golden ratio (Fig. 3.20a). The geometry underlying this is similar to that in Fig. 3.19b. In a quartered circle, the procedure is first to find the half measure of a radius (cf. Fig. 3.3a) and then to implement the diagonal of a 2 : 1 configuration—a radius and a half radius—to set the compass for an identical 'golden' cut of the radius opposite, into measures a and b, in the golden ratio. This latter procedure must have been the onset of the superb designing for the Tara Brooch.

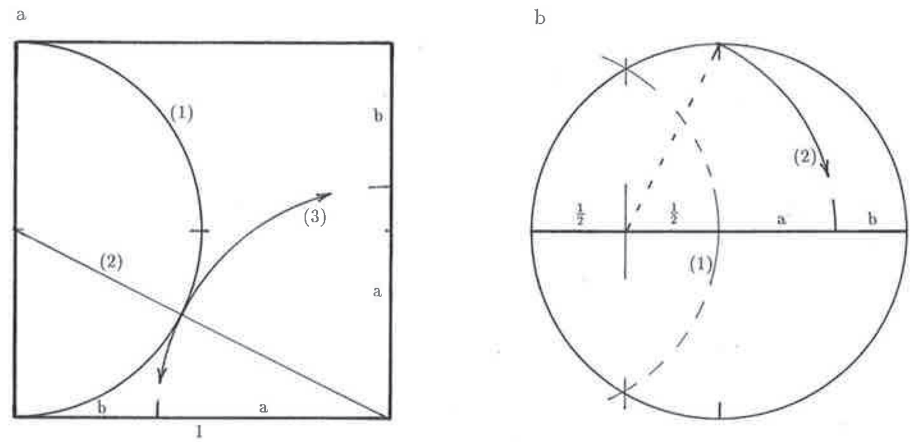


Fig. 3.20 Common derivations of 'golden ratio' $\frac{2}{\sqrt{5}-1}$ in Insular design.

There will be no need to describe one by one the many permutations involving this ratio in Insular art: the purpose of this chapter is only to provide a practical guide to setting the basic ratios. Rather, the key to understanding so many of the designs, through understanding how to create them, lies in understanding the unique internal coherence of the proportion in quantities based on this celebrated ratio. In algebraic form it is expressed this way: $b : a :: a : (b + a)$, where a given measure $(= 1)$ is divided into a and b. This is an 'extreme and mean ratio' formulation. In practical geometry it is the cutting of a linear measure such that the shorter segment is in relation to the longer segment as that longer segment is in relation to the measure before it was cut. In brief, a single division of a measure $(= 1)$ produces two additional measures a and b; together with 1 they form *two* ratios which are equivalent: short-to-long, and long to short-plus-long. More than that, there is a further equivalent ratio in $1 : (1 + a)$. There is no other ratio with the same economy of this one, or in potential for designs with simple and complete coherence.

The potential for linking does not stop there. Either a or b can be cut in the manner already shown. But further division into measures in the same ratio can be achieved more simply. A further equivalent ratio follows from the simple operation of subtracting short from long, e.g., $a - b = c$. The new measure c fits into the same chain of equal ratios, in that $c : b :: b : a :: a : 1 :: 1 : (a + 1)$. And so on repeating the procedure to set measures d and e, etc.

But algebraic notation is not needed in the procedural geometry of Insular art. All that is needed, after the first golden section of the given measure, is to subtract the shorter length from the longer, with the compass 'taking' the shorter measure and marking it from one end of the longer one (Fig. 3.21). It is in this unique capacity to link one measure to another without limit in a constant ratio that makes the golden ratio, a specific combination of $1, 2, \sqrt{5}$, such a rich source for design with full coherence of internal structure.

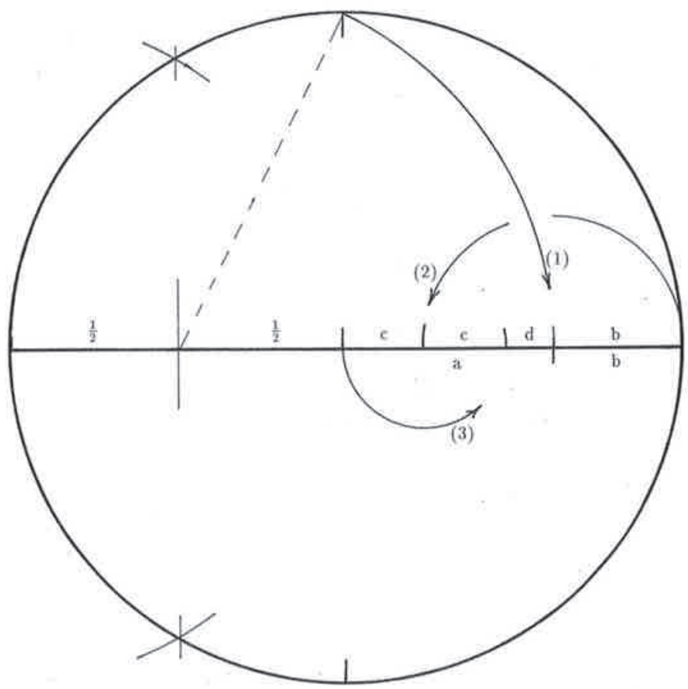


Fig. 3.21 Derivation of iterating golden ratio divisions of a given measure: $1 : a :: a : b :: b : c :: c : d \ldots$

All the procedures illustrated in the pages preceding were available to the illuminators, sculptors, poets, and fine metalworkers of early Christian Ireland and England (and their predecessors). I believe that these were the procedures they employed, and which the artists understood profoundly. Further, I cannot imagine their proceeding by any other method in creating the forms of the objects we most admire. There is not a scrap of verbal testimony that we know of from these artists or from any of their contemporaries concerning their method or rationale. But if we attend to the testimony of the few of their surviving creations, all of them seem to speak with one voice.

There are tools of computation available to us now that they did not have, whether algebra, or convenient mechanical devices such as slide rules (if anyone now remembers those logarithms on rails) and the very recent electronic calculator. These newer tools, neither available then nor needed in the creation of Insular forms, are not the tools of designing, though. That may be one reason the procedures of Insular designing became a lost art, requiring rediscovery.

The new tools can be extremely useful, on the other hand, in finding a way back into understanding the techniques and principles of Insular designing. Their principal value is in facilitating computation using three- or four-place decimal approximations, enabling one to check the accuracy of a model representing a particular form, to see whether arcs or lines or divisions of measures have been drawn with enough precision for the model to be a reliable means to understanding a form. A small error early in the construction of a model can throw off accuracy of elements developed later, enough to render the model useless or (worse) misleading. Ultimately, of course, upon the accuracy of the model depends its reliability in identifying the presence of coherent geometry in the form of an artifact, or pointing to its absence. The more accurate the models, the more can be learned about the geometrical basis of the designs: the objects, in fact, exhibit an accuracy, especially in the smaller ones, that will humble anyone working with the geometry of their designs and will stun anyone trying to replicate them by hand. For anyone using modern methods of computation, decimal approximations of the basic ratios will be useful. Only the main ones are listed here, in Table 3.1.

The internal coherence of the designs is the subject of the next section. For building it into the forms, the computation in numerals is virtually useless. Rather, the route to discovery remains in ratios of the kinds illustrated in this chapter, particularly in their expression in the small repertory of ratios assembled from 1, 2, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, with a few other numerals needed for a few particular objects.

Table 3.1 The Basic Ratios and Their Reciprocals

RATIO (: 1)	AS IN	RECIPROCAL	EXAMPLE
1.2071	$\frac{1}{2}(\sqrt{2}+1)$	0.8284	Kilfenora West Cross
1.2361	$\sqrt{5}-1$	0.8090	Lindisfarne Gospels, fol. 2v; Ahenny, South Cross
1.2679	$(2-\sqrt{3})+1$	0.7887	Soiscél Molaise bookshrine
1.2720	$\sqrt{2:\sqrt{5}-1}$	0.7862	Trier Gospels, four-symbols page
1.3090	$\frac{1}{2}(1+\frac{2}{\sqrt{5}-1})$	0.7639	Echternach Gospels, eagle page
1.3281	$3-\frac{2}{\sqrt{5}-1}$	0.7530	St Gall, Matthew & Luke pages
1.3660	$1:(\sqrt{3}-1)$	0.7321	Book of Kells, John page
1.4142	$\sqrt{2}$	0.7071	St Cuthbert Gospel of St John
1.4472	$\frac{\sqrt{5}+1}{\sqrt{5}}$	0.6910	Durham Cassiodorus, David
1.5000	$3:2$	0.6667	Book of Mulling, Mark page
1.6180	$\frac{2}{\sqrt{5}-1}$	0.6180	Tara brooch; Macdurnan Gospels, Matthew and Mark
1.7071	$1+\frac{1}{\sqrt{2}}$	0.5858	Book of Durrow, eagle page
1.7321	$\sqrt{3}$	0.5774	*Christ III*

4. Creating Coherent Geometrical Designs

Tests of Coherence

'Geometrical' is a term that is applied to any number of patterns and designs whether Byzantine or Insular, Greek or aboriginal. The term 'coherent' is joined to it here, in order to distinguish the traits of early Insular art. Within this latter tradition of designing it is quite different from key pattern, grilles, plaids and others, in which a pattern repeats and repeats. Insular coherent geometrical design, on the other hand, is in some ways like interlace, which occurs so frequently as decorative filler in Insular art, virtually as a hallmark. In interlace, intercrossing bands or ribbons pass over and under each other; they do so, however, not in the simple scheme of the warp and woof of weaving. Typically, and in all the best examples, bands in interlace patterns have no ends. That is what makes the pattern a source of fascination. The pattern presents to us a pragmatic paradox of a weaving or lace or net pattern that has been made from strands that do not have ends which could have passed over and under these strands to construct the design: there is no practical way to weave the pattern, and yet it has been woven, as we can see.

Coherent geometry is like good interlace pattern in the sense that it also links and bonds all of its elements. That provides a part of our fascination with the figures that embody it. Unlike interlace, in coherent geometrical design there is nothing palpable about it in the sense that one can trace with eye and finger the bands in an interlace pattern. The other patterns mentioned are similar, in their maze-like configurations that invite threading one's way through. Rather than bands or ribbons (some of them metamorphosed into animals), the elements linked one to another are ratios of measures. Their coherence, in turn, lies in the differing measures uniting in certain related ratios, completely and without residue. This is the trait I have also called 'commodular' linking, in a series of studies of manuscript illuminations and sculptured crosses, as well as literary compositions, in early Ireland and England.

The test for a coherent design in this sense is one that I have wrestled with a good many times. The practical lessons learned will be worth reporting with two or three distilled instances. It took a number of attempts to re-create forms of carpet pages, for example, to induce the rule that the outer rectangle of a framed illumination is the place to begin, both in analysis and in creating a form. The rule stood repeated tests; there was also the practical fact that derivations with compass and straight-edge will maintain a greater accuracy if they start from the largest element of form rather than, say, from the smaller dimensions of a cross or evangelist portrait inside the frame. The accuracy of most of the carpet pages in execution of these plans is astonishing. And then, there were built-in tests following from that. One was that there should be *continuous* derivations each determining the place of a line or an arc in the design—each derivation involving only the basic ratios and measures present in preceding manœuvers. This test cannot be stressed too strongly. It is often not very hard to 'doodle' until a mark generated by compass or straight-edge can seem to coincide—nearly enough, maybe—with a part of the form being replicated. The corrective is that if the doodling process does not fit into a continuous, consistent, economical scheme of plotting, the analysis will not be like analyses that are continuous, consistent, and economical and which do match fully the forms of the most admired artefacts in this tradition. The essence of this early art of design—and a source of its fascination—is that the forms should be rigorously rule-governed: every element must be connected to all others by a consistent and directly derived scheme of relations.

So (to continue the practical mode) finding the basis of the governing rule is crucial, obviously. More than once I took overall measures, calculated their ratio, and then proceeded in an attempt to replicate a plan. Usually it did not take long to get an 'error message' if the ratio was not the right one—usually because the decimal expression was close to one already familiar, and the expectation at the outset was that the familiar ratio was being employed once again. Any time it looks as if fudging some of the measures will be needed is a time to begin afresh.

Finally, two points about the concept of 'coherent' designing in the early Insular tradition. One is that it had been well developed in the artistic culture within which Insular designing evolved. Even as spiral pattern of long ancestry was brought into Insular art as a decorative element—on every type of artefact—so coherent structural pattern also of long ancestry was developed within the finest forms found in Insular art. This point will be developed in the section following this one, 'Some Antecedents and Follow-ons.'

The other point concerns the problem of ascertaining presence of coherent geometrical designing. A treasure-hunt for the 'golden ratio,' say, can sometimes be successful but without significance. This happens typically when two dimensions are identified which when put into numerical ratio yield an approximation of that famous ratio, without selection of the points for measuring being in an ostensible or certain relationship to the whole design. The ratio may be found without being removed from the realm of random. Presence of a ratio does not establish presence of a design. Here is an example. Mansel Spratling describes the handle of the Pegsdon Mirror (Iron Age) as measuring '81mm by 131mm, giving the proportion 0.618 : 1, a ratio known as the "golden section" ... the earliest instance of this yet recorded in Britain' ('The Iron Age Mirror Burial at Pegsdon...,' *The Antiquaries Journal*, 87 (2007), 128–30). The handle is a circular rod or tube shaped into a Y-configuration, the lower stem of the Y expanded into a loop. The meticulous measures 131 × 81 record the expanse of the handle material, irrespective of its cross-section shape and terminal embellishments. They do not report the axial dimensions of the circular metal, which may well have been primary. That is to say, the precision of the golden section ratio may be fortuitous with respect to the design. 'Found' golden ratios can be true reports of dimensions yet without relevance to the design as created.

In the instance of the Pegsdon Mirror handle, there is another pair of key measures that possibly confirm the presence of coherent geometry of the design. There is a 'loop' around the stem of the Y-element; it divides the handle-height so that the lower and upper extensions are in an accurate ratio of $\sqrt{2} : 1$. This additional accurate geometrical ratio in the simple plan may be the evidence needed to ascertain the presence of coherent designing of this artefact. Yet there remains the question of whether axial measures rather than raw overall measures are the ones to be considered.

A simpler and therefore more readily convincing example of the presence of coherent designing is the decorated bronze 'lid,' dated to first century A.D., found in Somerset, Co. Galway; NMI 1958:156. Here is the golden ratio within a design that seems both clear and planned, in the two circles of the rim of the lid and the raised boss concentric with it. The ratio and its embodiment in the piece are clearly and accurately rendered; layout could have been like that in Fig. 3.20b.

Some examples of creating forms for the various types of Insular art are given in the following pages. They range from the partial plan of a trial- or motif-piece, through examples of designs using several of the basic ratios, to the spectacular plan of the St. John page in the Book of Kells and the thoroughgoing classic elegance of the form of the Tara Brooch. Not included are plans based on numeral ratios 1 : 2 or 2 : 3 or 3 : 4; these are not unusual among dimensions of manuscript leaves or the text-space on them or even the mise-en-page of text-space or illuminations, but they are not common in designs of carpet pages and sculptured crosses, they do not occur among sectional divisions of long vernacular poems in Old English, and they are almost never found in the fine pieces of ornamental metalwork in the tradition.

A Motif-piece

There is hardly a better first example of Insular designing than a piece that preserves some of the process of creating a plan, in the form of sketchings of the kind that can eventuate in a typical Insular design. The piece described here is variously called the Dunadd trial-piece or motif-piece.

Parts of it are compass-drawn and parts are drawn free-hand. In the orientation of a brooch—which the piece prefigures—the upper portion has concentric arcs for the loop, with their centre clearly visible. In the lower portion there is only one arc that appears to be partially compass-drawn, the broad arc at the top of the lower portion. All the rest—circles, arc, and straight lines—appear to be free-hand sketches.

The free-hand portion of the plan shows the left-hand side of a tentative scheme for decoration of the lower segment of the brooch, the disposition of cells for precious stones, or interlace, or filigree, or whatever. (No need to sketch the right-hand side, which would be merely the same pattern in reverse.) This part of the design is little more than an inventory of the decoration to be worked out for this part of the brooch's surface. In this state it contains almost no information about the coherent geometry of this part of the design. As the design may have developed, of course, the elements in this area could well have been plotted according to the geometry that is present in the compass-drawn portion.

The compass-drawn portion is plotted as functions of linked ratios built around $\sqrt{3} - 1$ (see Fig. 3.17). The plan can be replicated by the procedure illustrated in Fig. 4.1.

Begin by drawing a circle. Development of the plan begins with the path of a diameter and division of the circumference by six—that is, the circle divided first by two, and then each half of the circle divided by three. The classic method is illustrated first, in Fig. 4.1a, carried out with a single setting of the compass. But much less is needed.

The short arc in the upper portion, stretching through about one-third of the upper half, has a radius $\sqrt{3} - 1$ in relation to the radius (=1). In Fig. 4.1b, one sixth of the circumference is marked successively three times starting from the top. The third mark is half way around the circle, the opposite end of the starting point, these two points standing at either end of a diameter that divides the plan for bilateral symmetry. Then the measure—length of a chord—between two of these divisions of the circumference ($= \sqrt{3}$) is copied along the central diameter (1), marking a point $\sqrt{3} - 1$ above the centre of the circle, which (2) becomes the centre of this upper arc, designated A.

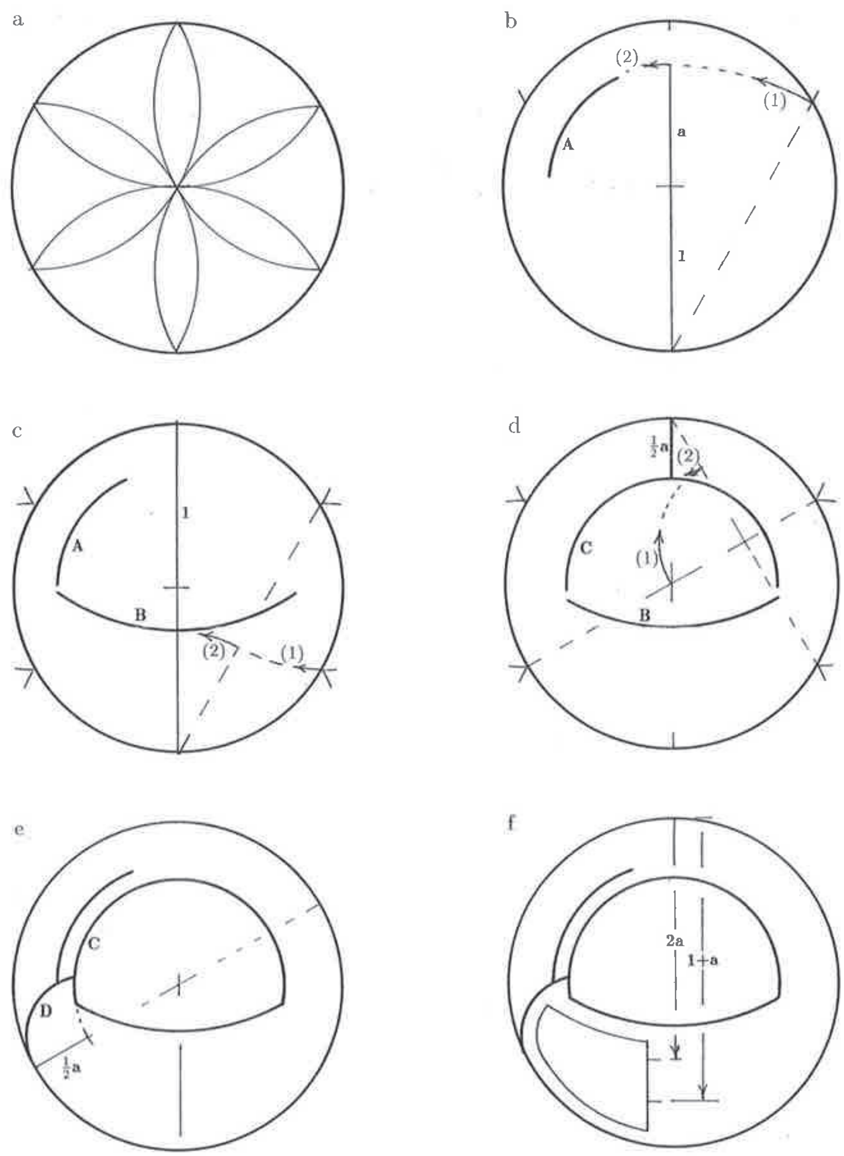


Fig. 4.1 Geometry of the compass-drawn portion of Dunadd motif-piece.

The symmetrical arc just below the centre of the plan has a radius $1+(2-\sqrt{3})$, which is to say, the diameter less $\sqrt{3}$, added to the radius to set the radius of this arc. See Fig. 4.1c. The measure $\sqrt{3}-1$ is re-computed from the circumference of the circle this time (instead of from the centre). The same straight measure $\sqrt{3}$ as the chord of two-sixths of the circle's circumference, has the radius measure of the circle subtracted from it—in the figure, it is the chord of one-sixth of the circle that is used (1). Then the remaining portion (= $\sqrt{3} - 1$ is copied (2) along the centreline. From the upper end of the diameter to this point is the radius of another arc, designated B.

The inner arc of the 'loop' in this plan has a radius $1 - \frac{1}{2}(\sqrt{3} - 1)$; in constructional rather than algebraic terms, its radius is the difference between the circle's radius (=1) and half the radius of arc A, the same as half the nearest measure between arc B and the circle, already plotted. Any convenient method of halving a measure will be appropriate; the procedure shown (Fig. 1d) places the division such that its distance from the centre of the plan sets the radius for this third arc, designated C.

In this rudimentary plan, only partially sketched and in a medium that does not facilitate precision, the traits of coherent geometry cannot be mistaken. They can be expressed in modern notation for the radii of the three arcs,

$$\mathrm{B/A} = \sqrt{3} \qquad \mathrm{C/A} = \frac{\sqrt{3}}{2} \qquad \mathrm{B/C} = 2$$

or they can be expressed in the tactile procedures which produced the primary, compass-drawn portions of the form of the Dunadd motif-piece.

One additional arc may be compass-drawn and relevant to the geometry of this plan. Where the path of arc C intersects a diameter running between the eight o'clock and two o'clock positions is the centre of arc D (Fig. 4.1e); its radius is the same measure $\frac{a}{2}$ already established. This one may be compass-drawn and then over-drawn by hand.

Two hand-drawn lines may participate in this plan as well—the nearly horizontal lines spacing the decorative cells from the centreline in the lower portion of the plan (Fig. 4.1f). The upper one seems to be $2a$ from the top, the lower one $1 + a$ from the top (same measure used in Fig. 4.1b); extending Arc A around the figure would mark its intersection with the lower part of the vertical axis. (If b is designated as the complement of a in the radius, the two horizontal lines are b and $2b$ from the bottom.)

Ringed Crosses (1)

The early standing crosses cut from stone also have outlines that follow the direct path of a straight-edge or the circular path of a moving compass point. This is obvious from direct observation. Straight-edge and compass also are employed to set the relative measures within the designs, creating the particular shapes of these crosses. This becomes obvious only by analysis. In short, these simple tools employed by geometer and artisan not only draw the structural lines but also determine where to draw them.

The next example (Fig. 4.2) embodies integration of circle and cross that is textbook-simple, evolving immediately from circle and square drawn to a single measure and divided into quadrants, as in Fig. 1.1e. It is symmetrical on both its vertical and horizontal axes. Radii of the arcs and circles used in plotting this shape are all simple functions of ratios incorporating 1, 2, $\sqrt{2}$. One procedure for drawing this cross design is the following. ('Copy' is carried out with dividers.)

Initially set concentric and commensurate circle and square ABCD divided into quadrants (cf. Fig. 1.1f–g).

For the ring, begin by sketching diagonals of the square; then beginning at any corner of the square copy (1) the measure a, as shown, to mark a point along a side of the square (measure a is diagonal of the quadrant minus radius of the circle); next find half that measure and mark it along a segment of the cross (a straight line (2) will do this); now draw an inner circle with the new radial measure.

For the cross, copy again measure a, this time (3) on either side of the underlying cross-lines where they intersect the four sides of the square. Connect these points in pairs to form the outline of a solid cross. Copy then (4) the difference in radius length between outer and inner circles, that is $\frac{1}{2}a$, to set a radius as much larger than that of the outer circle as it was larger than the radius of the inner one drawn in Step 1. This measure is used in setting the limits of the arms of the cross. Sketch part of a circle (5) above the centre with radius equal to the diagonal of a quadrant of the underlying square: this sets the limits of the top of the cross-shaft.

The 'armpits' are plotted from markings already in place, as shown.

The key measures of this plan thus are 1 = radius, and side of quadrant, $\sqrt{2}$ = diagonal of quadrant, $a = \sqrt{2} - 1$.

The second example (Fig. 4.3) shows an alternate procedure for constructing the cross-form just described, equally simple, equally adequate. It proceeds from the same source configuration (cf. Fig. 1.1f–g).

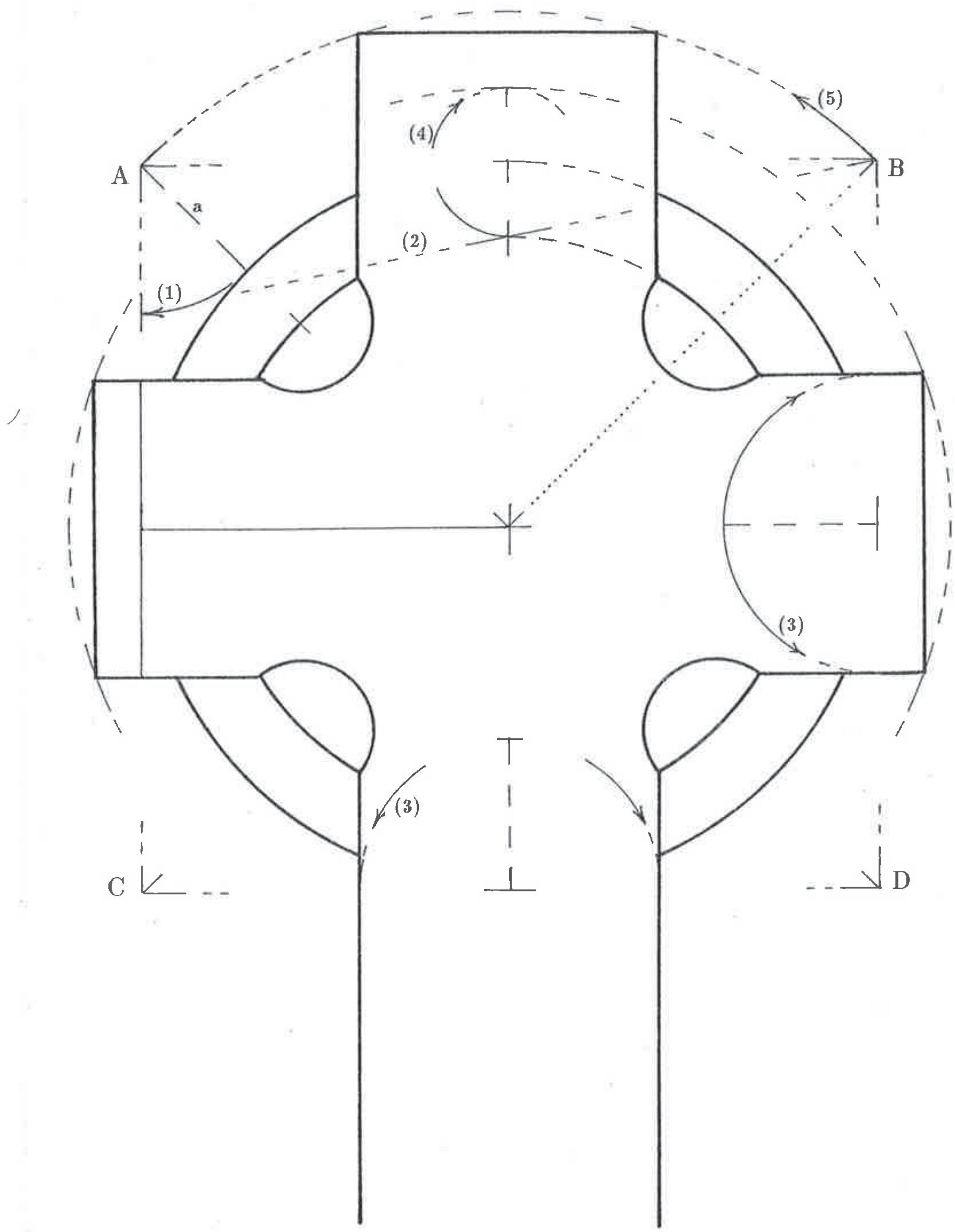


Fig. 4.2 A cross-plan based on ratios of 1, 2, $\sqrt{2}$, first method.
(Compare it to the North Cross, Castledermot, Co. Kildare)

Again, the measure of each quadrant, which is the same as that of the radius of the inscribed circle, will be the unit ($= 1$). The measure of the diagonal of a quadrant is $\sqrt{2}$ in relation to the unit. The measure a will be the difference between these other two measures (i.e., $\sqrt{2} - 1$).

For the cross, these steps:

(a) From each corner of the square in turn, (1) copy the measures of the diagonal of a quadrant to mark points on the adjacent sides of the square; these points are at measure a on either side of centerline and midline.

(b) Join pairs of these points with parallel lines to outline the shaft and the arms of a cross.

(c) Using that same diagonal measure as radius, (2) sketch an arc to intersect the upper shaft of the cross, setting its height.

(d) With radius $1 + \frac{1}{2}a$, sketch arcs centred in the plan, intersecting the arms of the cross, setting their lengths. (One way to find measure $\frac{1}{2}a$ is (3) to mark measure a along an extension of one side of the square, and then (4) run a line from there to the corner of the square nearest; it intersects a midline of the underlying square at $\frac{1}{2}a$ beyond the side of the underlying square.)

(e) The inner circle of the ring has a radius $1 - \frac{1}{2}a$.

(f) The 'armpits' are plotted directly from markings already in place, as shown.

Now compare the two procedures just described. One sets the dimensions of the ring first, then sets the dimensions of the cross, while the other follows the reverse sequence. Within this difference, one sets the radius of the inner circle of the ring first and then sets the radius of the arcs that determine lengths of the cross arms, while the other reverses the process. It will be clear from just these two differences that the design of a cross (like designs of the other objects) is distinct from any particular procedure that produces a particular form. Rather, there is a *method* of designing, which the two procedures illustrate.

There are many other alternate steps that can be used within these procedures, as well. Measure a in Fig. 4.3d could have been set by copying the length of a quadrant's diagonal from the midpoint of the bottom of the square to an extension of the bottom line (Fig. 4.4, lower left). The second design procedure then would have three steps using a single compass setting at the length of a diagonal of a quadrant: the fixed foot would be placed in turn at all four corners of the underlying square (Fig. 4.3a), at its centre (Fig. 4.3c), and at midpoint of any side (Fig. 4.4, left). This illustrates one aspect of coherent geometry in tactile terms.

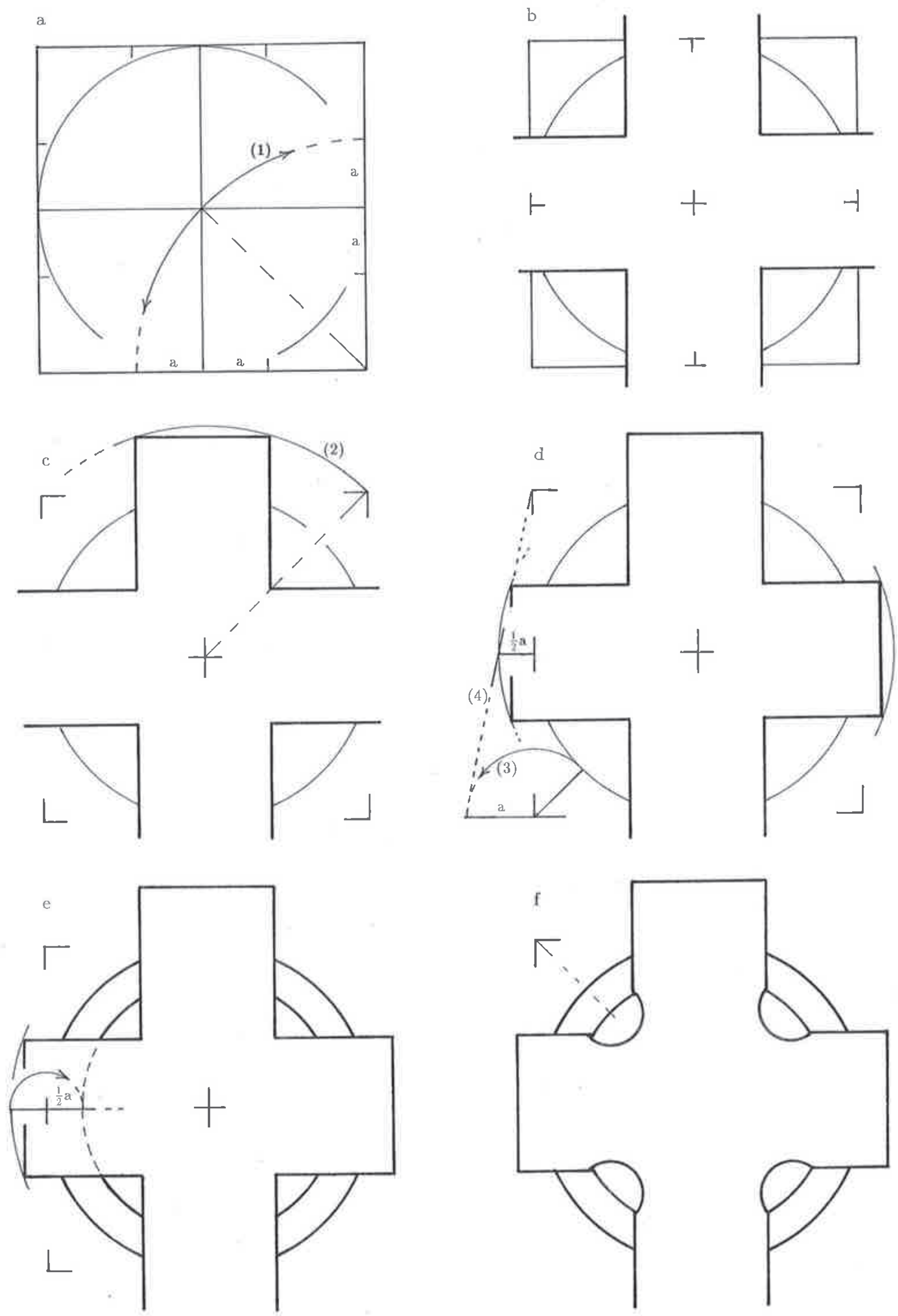


Fig. 4.3 The same cross-plan based on ratios of 1, 2, $\sqrt{2}$, second method. (Again compare it to the North Cross, Castledermot, Co. Kildare)

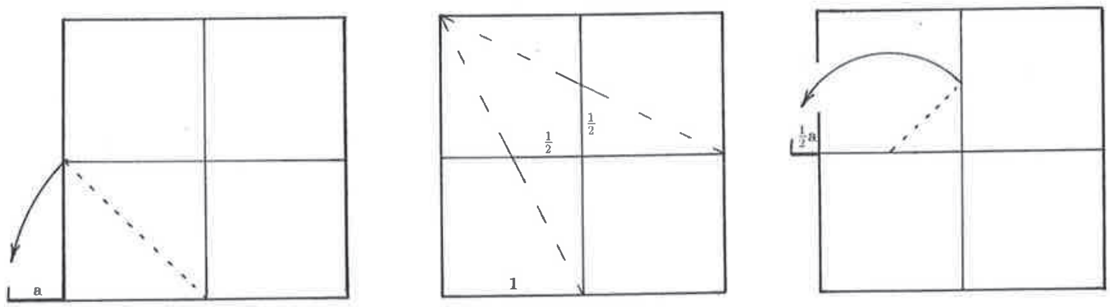


Fig. 4.4 Some alternate steps.

The measure $\frac{1}{2}a$ was set by similar procedures for halving the measure a (Fig. 4.2, steps 1–2 and Fig. 4.3d). Alternately, it could have been derived as the diagonal of a quadrant of a quadrant (Fig. 4.4, right); the halving of quadrant sides is achieved by marking the point where a diagonal intersects the side shared by two quadrants of the underlying form (Fig. 4.4, centre, cf. Fig. 3.4a and 3.7b).

Now construct the head-plan of a second standing cross. Like the first one, it will evolve immediately from commensurate circle and square ABCD divided into quadrants, as in Fig, 1.1e. See Fig. 4.5. First plot the transverse measure of the arms and shaft by copying (1) the diagonal measure of single quadrants of the square (illustrated for one quadrant). Plot then the length of the arms this time using the diagonal measure of two adjacent quadrants of the square (2). Next plot the inner radius of the ring using the difference between diagonal measure of two quadrants of the square and one of its sides (3). Plot next the height of the upper segment of the shaft by using diagonals of the underlying square (4). And finally, plot the armpits with centres at intersections of diagonals of quadrants with chords of the inner circle as shown (5), radii extending to underlying intersections of arm and shaft lines in the initial step.

The form constructed in this way is an accurate replication of the basic design of the South Cross at Castledermot, as can be seen when a drawing of this form is laid on top of an appropriate photograph on the same scale.

These two cross plans are different and they are both textbook simple. And it should be noted that they also replicate two crosses standing together in the same locale. This congeries of circumstances illustrates a key and constant aspect of Insular designing, that each form was something to be created anew, each time, and not copied. It is as if a mason had been commissioned to design a second cross for the same community. It would not be right to offer the same design

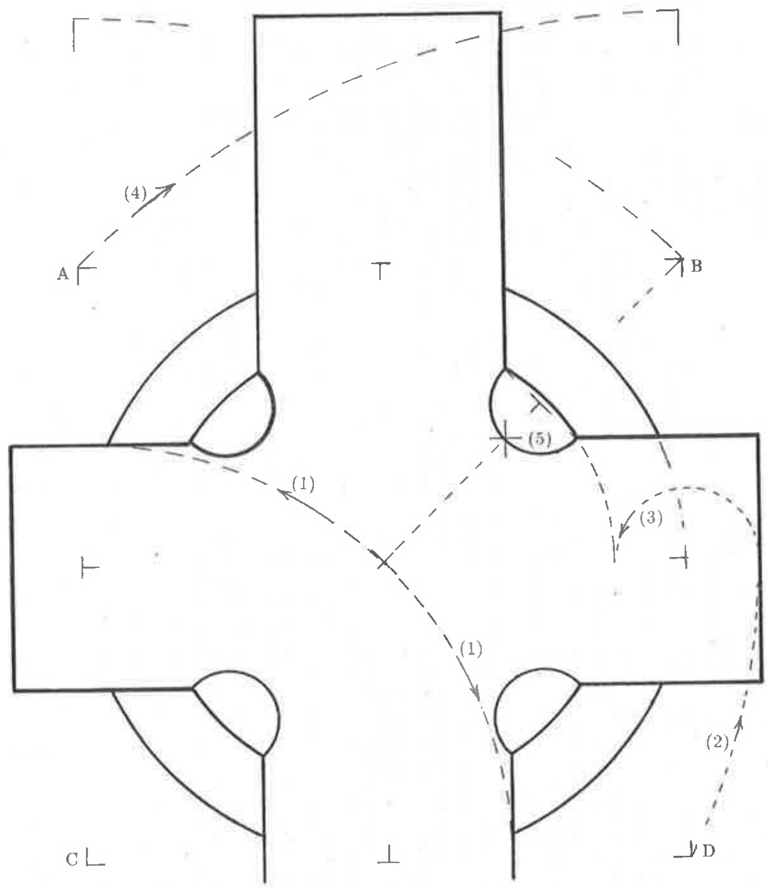


Fig. 4.5 Another cross-plan, based on 1, 2, $\sqrt{2}$, $\sqrt{5}$.
(Compare it to the South Cross, Castledermot, Co. Kildare.)

again, while it would be right that the two should be similar—alike in both size and configuration, yet different in development. In this respect the pair of crosses at Castledermot, North and South as they are called, epitomise in very simple terms the methods and the coherence of Insular designing.

Ring ratios

When the designer planned the ring for a cross, how did he proceed? Did he draw pairs of concentric circles related merely by guesses, until his eye told him he had a good design? Or did he choose among pairs of these circles with ratios he had been taught to understand and to construct, selecting the pair his eye told him was right for the whole design he was creating? The place *not* to start in deciding something as fundamental as this, is romantic or doctrinal notions already formed about *those* people, back *then*, especially notions about how they felt and what they were thinking. The place to start is in the ratios that are found. Just illustrated for a ringed cross (Figs. 4.2 and 4.3) is the ratio $1 - \frac{1}{2}(\sqrt{2} - 1) : 1$; its decimal approximation is 0.7929. In the Dunadd motif-piece (Fig. 4.1) the partial arc A was $\sqrt{3} - 1 : 1$; its decimal approximation is 0.7321. Arc C in that piece had a radius $1 - \frac{1}{2}(\sqrt{3} - 1) : 1$, decimal approximation 0.6340.

Most of the ratios fall within a narrow range: see 'Approximation' column in Table 4.1. (Reciprocals of some of these were listed in Table 3.00 at the end of Setting the Basic Ratios.) Derivations of eight ring ratios, three of them arithmetical, the other five geometrical, are illustrated in Figs. 4.6 and 4.7.

Table 4.1 The Basic Ratios in Ring Dimensions.

Figure	Ratio (: 1)	Approximation	Example
4.6a	1 : 2	0.5000	——
4.6b	2 : 3	0.6667	Terrace Cross, Tynan
4.6c	3 : 4	0.7500	SS Patrick & Columba, Kells
Cf 3.8d			
4.6d	$2(2 - \varphi)$	0.7639	South Cross, Castledermot
4.7a	$2(\sqrt{2} - 1)$	0.8284	Kilfenora
4.7b	$1 - \frac{1}{2}(2 - \varphi)$	0.8090	South Cross, Ahenny
Cf 3.18c			
4.7c	$\frac{1}{2}\sqrt{2}$	0.7071	Tower Cross, Kells
4.7d	$\sqrt{3} - 1$	0.7321	Island Cross, Tynan
Cf 3.17a–b			
4.2, 4.3	$1 - \frac{1}{2}(\sqrt{2} - 1)$	0.7929	North Cross, Castledermot

There can hardly be any surprise here, of course, in view of the material aspects of a ringed cross, as well as its perceptual aspects: the ring has to be workable in stone, has to be clearly separate from the cross which itself has to be practical in its dimensions for chiseling from a stone slab. Too narrow and the circle probably will not survive the carving process, too wide and it interferes with the clarity of the cross configuration, which is the topical shape.

Thickness of the ring of a cross, or the hoop of a brooch, normally integrates proportionally with all the other principal dimensions of an Insular design.

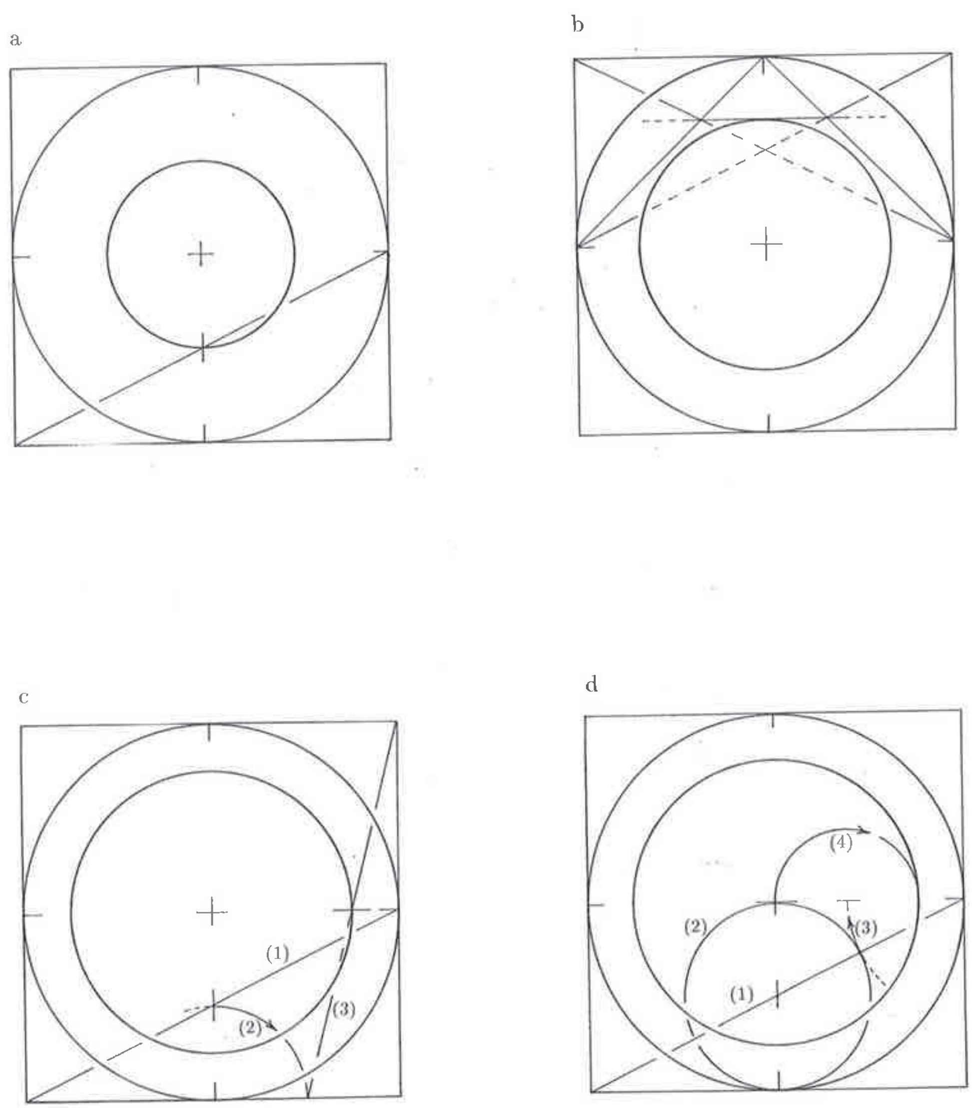


Fig. 4.6 Derivations of various ring ratios, beginning.

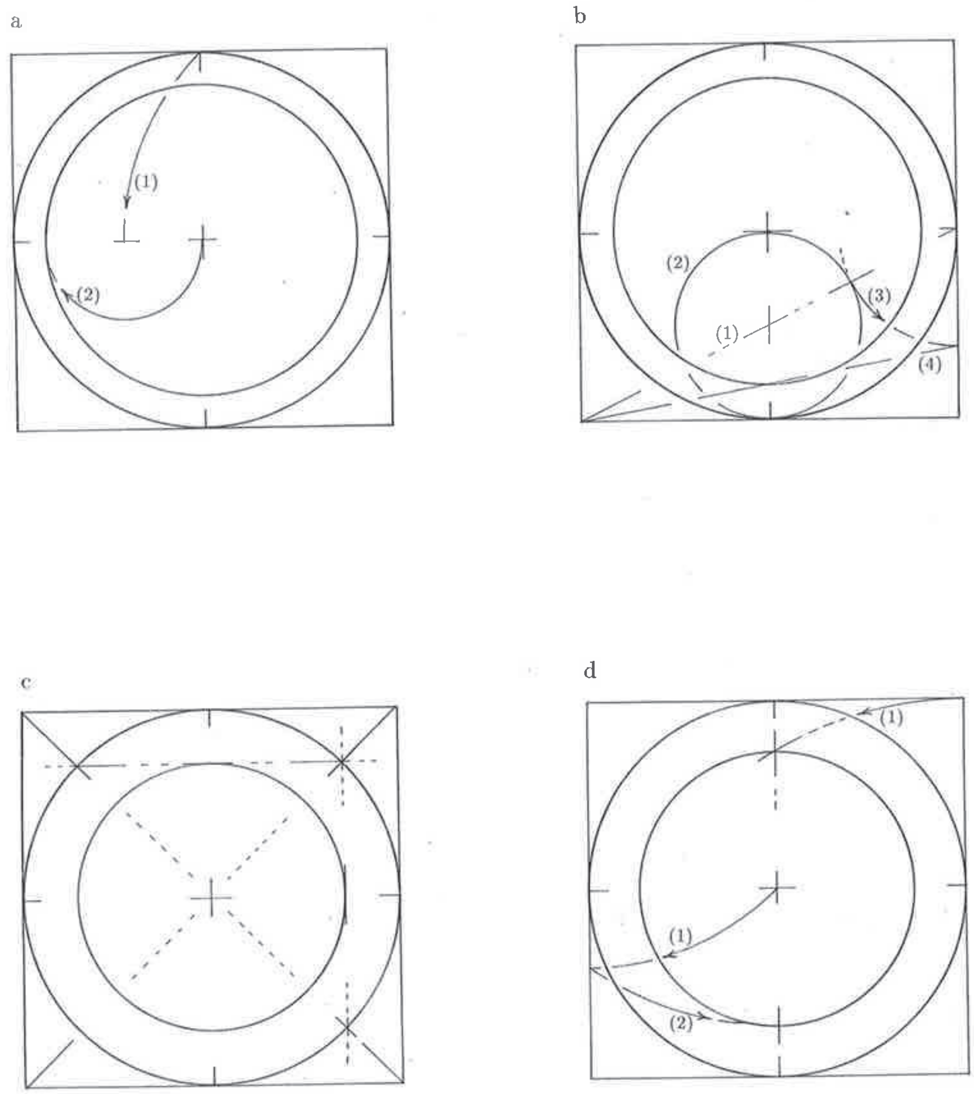


Fig. 4.7 Derivations of various ring ratios, concluding.

Ringed Crosses (2)

This example of a standing cross, the Durrow Cross, is less primitive in its geometric coherence than the two just described, though still textbook-simple. All three have the same initial setting out, or source. The key ratio, though, will be that of the golden section of a measure, or φ. Of the various ways of setting the ratio (cf. Figs. 3.19–20), probably the one similar to the procedure illustrated in 3.20b was used. One method for replicating the form is illustrated in Fig. 4.8, using the following construction steps.

Begin with concentric and commensurate circle and square divided into quadrants (cf. Fig. 1.1f–g).

(a) To plot the transverse measure of the cross members, divide an interior quadrant by golden section, as a and b; then copy b on either side of the midpoint of each side of the underlying square.

(b) For the ring, the outer circle is the original circle with diameter 2 (double the radius). For the inner circle, double the measure b from the center of the cross to set the radius. Alternatively, the diagonal measure d of two adjacent quadrants of the underlying square is marked on an extension of a midline of the square. It exceeds the original given length by the measure c (that is, $c = d - 2$); subtract the measure c from the radius of the outer circle to set radius for the inner circle.

(c) Each side of the underlying square is already divided into $b + a$. A line from the point of division to the nearest opposite corner of the square intersects the horizontal midline, giving a measure $\frac{1}{2}a$ from the side. Copy that measure to mark a point on an extension of the midline, which will be the radius for a larger circle. Intersections of this circle with the cross-arm lines sets the length of the cross arms.

(d) Use d (the diagonal of two adjacent quadrants of the underlying square) to set the overall measure of the upper extension of the shaft above the midline of the cross arms. The measure to the join of the main shaft and the capstone can then be set in either of two direct ways: from the top of the whole extension copy the transverse measure ($2b$) of the untapered shaft; or from below double the measure c above the underlying square (and the path of the outer circle of the ring).

(e) Centers of the re-entrant arcs are the points where diagonals of the square intersect the inner circle of the ring. Radii have length c.

(f) The taper—or splay—of the cross shaft is based on a widening of the lower segment of the cross. Once again use the diagonal measure d of two quadrants (dotted line), but this time mark it along extensions of the sides of the underlying square; then run a line on either side, as shown, to intersect the horizontal midline of the cross: use the resulting central measure for the transverse dimension of the lower segment of the shaft. The proportional mean between that widened measure and the original measure is the transverse dimension of the upper segment up to the join with the capstone (derivation not sketched in the figure).

The key measures are listed in Table 4.2.

Table 4.2 Measures in the Durrow Cross

FEATURE	MEASURE	RATIO (: 1)
Ring diameter, outer	2 (Given)	2
— radius, outer	1 (Given ÷2)	1
Ring radius, inner	$2b$	$2(1-1/\varphi)$ or $\frac{2}{\varphi^2}$
Transom height	$2b$	$2(1-1/\varphi)$ or $2(2-\varphi)$
Transom diagonal length	$2(1+a/2)$	$1+\frac{1}{2\varphi}$, $\frac{\varphi^2}{2}$, or $\frac{\varphi+1}{2}$
Upper shaft, above ring	$2a$	$2/\varphi$
— top segment (capstone)	$2b$	$2(1-1/\varphi)$
— bottom segment	$2c$ or $2(a-b)$	$2(\frac{1}{\varphi}-(1-\frac{1}{\varphi}))$ or $\frac{2}{\varphi^3}$
Upper shaft, above center	$2a+1$ or $2+c$	$\varphi+\frac{1}{\varphi}$
Lower shaft, below center*	$2b+3$	$2-\frac{1}{\varphi}$
— below ring*	$2b+2$	$2+2(2-\varphi)$
Lower shaft, breadth	$2/(2+c)$	$\varphi/(\varphi+2)$
Re-entrant arcs, radius	c or $a-b$	$\frac{1}{\varphi}-(1-\frac{1}{\varphi})$ or $\frac{1}{\varphi^3}$

*Length of the lower segment of the shaft is conjectured because of uncertainty about where the lower end of the shaft was intended to terminate.

See further: 'Shapes of Early Sculptured Crosses of Ireland,' *Gesta*, XXXVIII (1999), 3–21; 'The Shape of the Durrow Cross,' *Peritia*, 13 (1999), 142–53; 'The Coherent Geometry of Two Irish High Crosses,' *Peritia*, 14 (2000), 297–322; 'High Cross Design,' in *Pattern and Purpose in Insular Art*, ed. Mark Redknap, Nancy Edwards et al (Oxbow Books, 2001), 221–232; the section on the Kilfenora West Cross in 'The Ancestry of "Coherent Geometry" in Insular Art,' *JRSAI*, 134 (2004): 5–32; 'The Forms of Two Crosses on Pictish Cross-slabs, Rossie Priory, Perthshire, and Glamis no. 2,' in *Pictish Progress: New Studies on Northern Britain in the Early Middle Ages* (2011), 201–20.

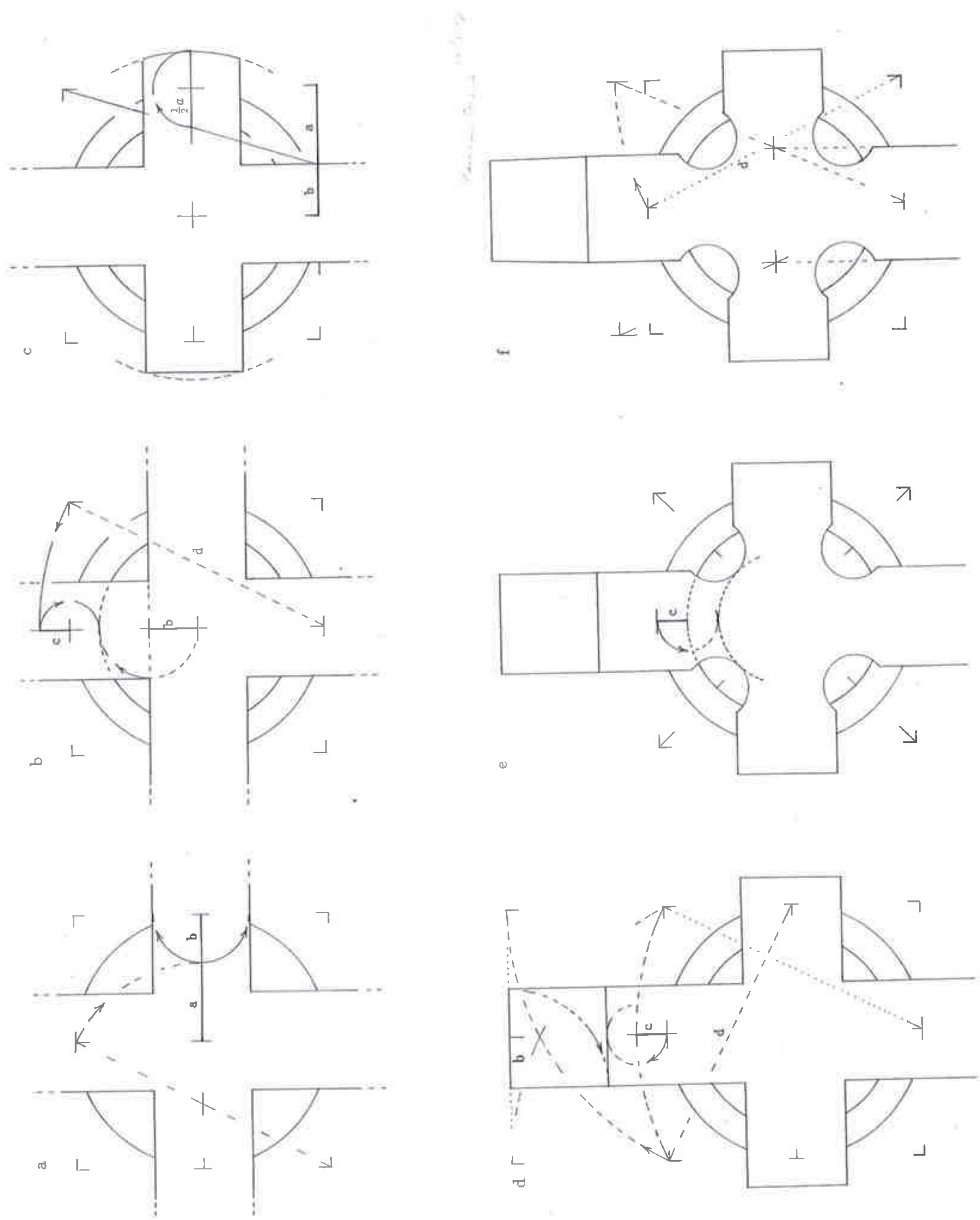


Fig. 4.8 A derivation of the form of the Durrow Cross.

Metalwork (1)

Soiscél Molaise is an eighth/ninth century isomorphic book-shrine with a front panel containing wide borders of decorative insets enclosing concentric circles at the centre of a cross. Four panels containing evangelist-symbols flank the cross to complete the layout. The basic form can be reproduced accurately by means of compass-and-straightedge construction, utilizing the simply related measures 1, 2, $\sqrt{2}$, $\sqrt{3}$ (cf Figs. 3.14–15). The plan can be drawn up as illustrated in Fig. 4.9, developed from an underlying square ABCD, quartered (as in Fig. 1.1g).

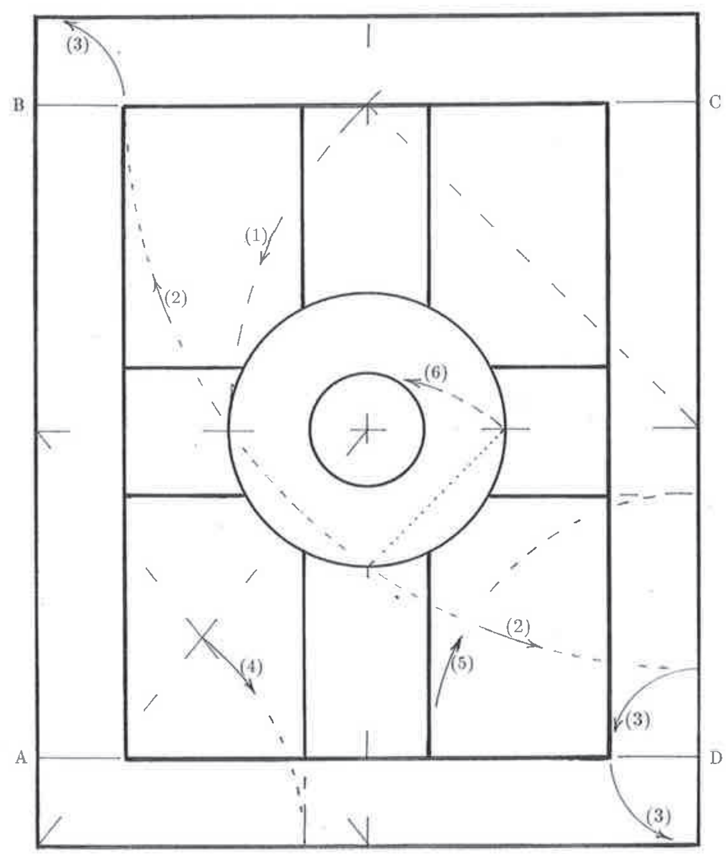


Fig. 4.9 Derivation of the primary form of Soiscél Molaise (bookshrine).

Plot the height from the width to create a rectangular panel, as illustrated in upper left quadrant (1, 2, 3). (Alternate derivation shown in lower right quadrant.) Or in narrative: In each quadrant, copy diagonal measure of a quadrant to mark a point along the midline (1), then copy the measure to that point from the farther corner of the square (on the same side) to mark a point on the horizontal side of the square (2); the first derivation produced $\sqrt{2}$ of a quadrant, the second produced $\sqrt{3}$. Copy then the measure from the corner of the square to the last-marked point to mark further points along extensions (above and below) of the sides of the square (3). Draw the lines of the enclosing rectangle using the points last marked. (Alternate procedure as shown.) Height to width is equivalent to $(3 - \sqrt{3}) : 1$.

Radius of the central circle was set in the procedure already described (1). Diameter is $\sqrt{2} - 1$ of the width (the given measure).

Plot the cross from the frame dimensions, as illustrated in lower left, then lower right quadrants. Diagonals of quadrants of the overall rectangle bisect each other: copy the half-length from each corner of the rectangle to mark points along top and bottom lines of the frame (4). Lines between corresponding pairs of points locate the upright lines of the cross. From each corner of the underlying square copy the measure to the nearest side of the cross upright to mark points along the sides of the frame (5). It is the same measure copied by (4). Lines between corresponding pairs of points locate the horizontal lines of the cross-arms. The measures involved, functions of $\sqrt{2}$ and $\sqrt{3}$, are within about two one-thousandths of functions of φ.

Plot the inner circle at the centre, as illustrated. Mark the length of a chord of the central circle along a diameter (6), setting the radius of the inner circle. (The ornamental setting mounted at that site is slightly oval in shape.) Its measure is $(\sqrt{2} - 1)^2$ of the width (the given measure).

Metalwork (2)

St. Patrick's Bell Shrine NMI R.4011 is another isomorphic box with its front panel also containing wide borders filled with decorative insets enclosing a circle at the centre of a cross. The bell-shrine is dated to about 1100 A.D., perhaps three centuries later than Soiscél Molaise. Front panels of both have configurations and layout that answer to compass-and-ruler construction, and both utilise the same repertory of relative measures, 1, 2, $\sqrt{2}$, $\sqrt{3}$. The earlier shrine is rectangular in shape, for a book, the later one trapezoidal, for a handbell. Figs. 4.10–12 illustrate a derivation of the shape of the front (and rear) panel of the bell-shrine.

Lay out first a quartered square (as in Fig. 1.1f–g) using the base-measure of the panel as its given overall dimension.

Because the front panel is to have a cross-pattern enclosing a centred circle, the ready choice is to extend the height equally above and below the underlying square, rather than extend it in one direction only. So as in Fig. 4.10 sketch the diagonals of a quadrant of underlying square ABCD and copy the distance from centre to intersection of the diagonals to mark a point along the vertical axis (1), then from an end of the midline copy the distance to the point just marked along extensions of the side of the square (2), above and below. Repeating (2) from the opposite side sets the height on the other side. Draw lines completing the underlying rectangle.

It is the inequality of the length of the top and bottom of each panel that makes them symmetrical trapezoids. In each upper corner, take the measure by which rectangle height exceeds height of the underlying square and copy it (3) to mark points along the top line: that sets the measure of the top of the panel. The trapezoid is then completed by drawing lines from lower corners of the rectangle to the points just marked.

Plot first the central circle. Its centre is at intersection of the centreline and midline of the trapezoid. Set its size as in Fig. 4.11. Begin by constructing a 2×1 right triangle on half the midline as shown (1, 2, 3). Then subtract the measure of the short side of that triangle from one end of the hypotenuse as shown (4), and copy the remaining length of the hypotenuse (5) along the horizontal axis for the circle, cutting the half-width of the frame by a golden section. This defines the radius for the circle.

For the cross, field, and frame, some lines are already mapped, and the others can be set merely by copying measures already established; see Fig. 4.12.

Inner lines of the frame (nearest the panel's borders), top and bottom, follow the horizontal lines of the underlying square.

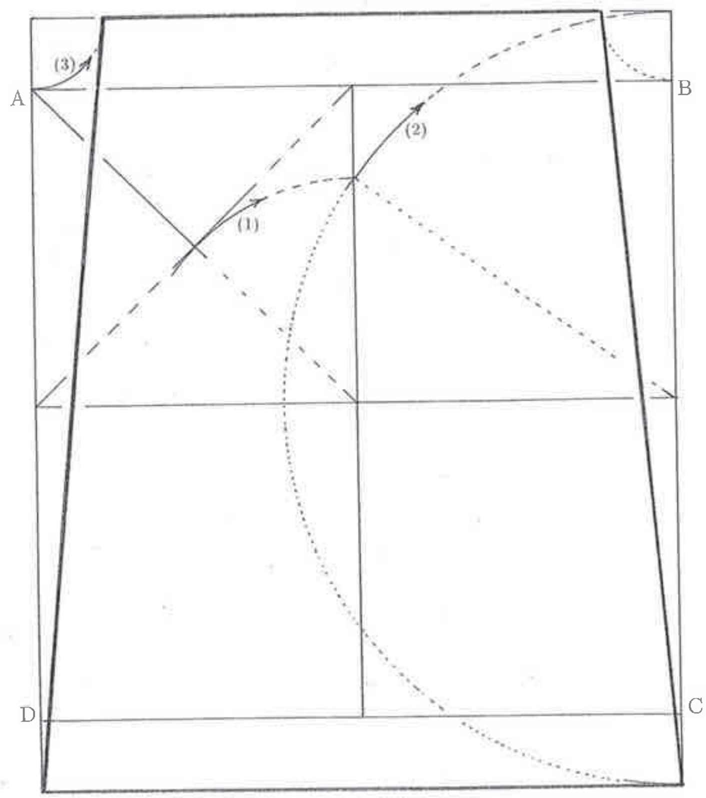


Fig. 4.10 Deriving the shape of front panel, St. Patrick's Bell Shrine (1).

The measure by which the vertical dimension of the panel exceeds the underlying square was used to set the taper of the sides: it is re-used (copied) to locate the inner lines of the tapered frame, on either side (shown only on the left side). (As a consequence, the outside width of the panel at the top is the same as the inside width of the frame at the bottom.) These *inner* lines of the frame (nearest the panel's outline) are also the *outer* lines of the panels flanking the cross, and the small bordering panels at the four extremities of the cross.

Next the outlines of cross and large quadrilaterals (Fig. 4.12). From the upper corners of the underlying rectangle, copy the measure to the midline to mark points along the upper border of the panel (1); do the same from the lower corners (1). Vertical lines connecting corresponding points (2) define the *outer* measures of the cross upright.

The upper horizontal line for the cross-arms follows a contrary derivation. From upper corners of the underlying rectangle, copy the measure to the centreline to mark points along the sides of the panel (3, 3). A horizontal line (4) connecting

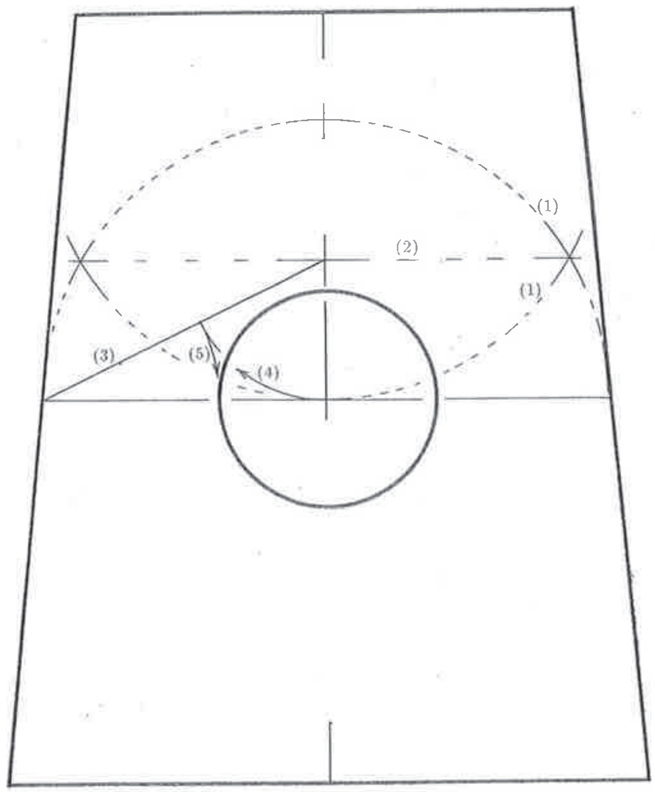


Fig. 4.11 Deriving the shape of front panel, St. Patrick's Bell Shrine (2).

corresponding points defines the *outer* lines of the band enclosing the large upper quadrilaterals (the *inner* lines of the top of the cross-arms).

(These two plottings use a strategy analogous to that used in the layout of the cross on the Harburg Gospels cross-page.)

The lower lines for the cross-arms follow from further derivatives from the basic measures. From the upper corners of the underlying rectangle, sketch lines (5) to the opposite points marked by (1) along the lower outline of the panel. From the lower corners, copy the measure to the centreline to mark points along the sides (3a), and sketch lines from those points to the lower corners opposite (6). From each lower corner then copy the measure to the intersection of the sketched lines to the side of the rectangle (7). A horizontal line (8) defines the *outer* lines of the bands enclosing the large lower quadrilaterals (*inner* lines of the lower outline of the cross arms).

To be noted is the fact that the cross-arms are *not* symmetrically placed in

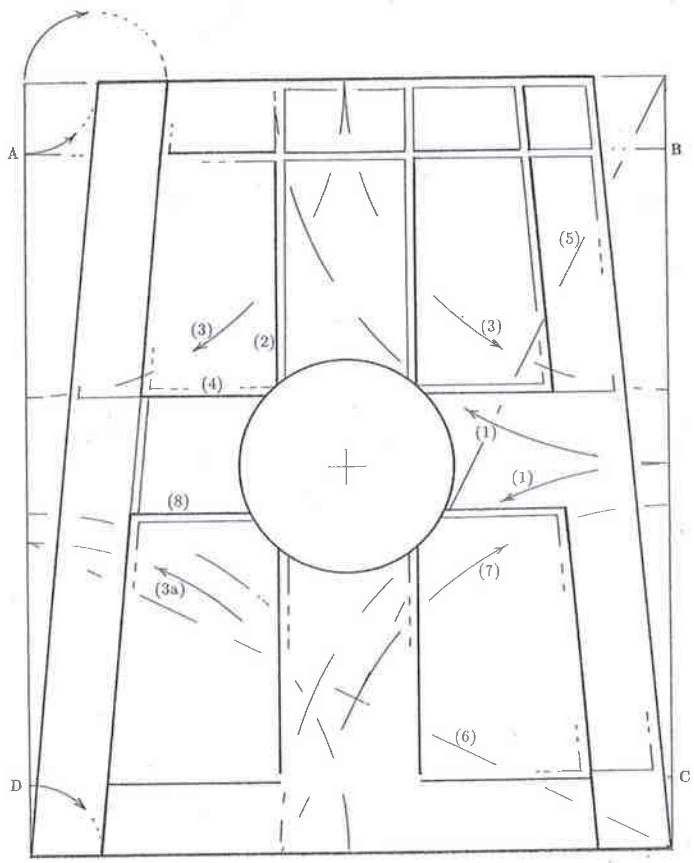


Fig. 4.12 Deriving the shape of front panel, St. Patrick's Bell Shrine (3).

relation, say, to the central circle, even though the cross upright and the large circle with its oval setting *are* symmetrically placed. The inconsistency is obvious once noticed, and it seems to be without rationale within the archetypal form of the framed-cross. It is 'off' in a way that has no immediately intuitive spatial justification. It would be 'off' as well if a simpler logic in the construction had been followed, duplicating the procedure (3, 4) in the lower half of the form: that would give the cross-arms a thickness out of balance, though, with the upright. And because the overall form is tapered, the four cells flanking the cross would in consequence appear (more) unbalanced in the areas they fill. The derivation described above (5, 6, 7) employs the other geometric constant ($\sqrt{5}$) used repeatedly elsewhere in the design to set the lower limit of the cross arms, sacrificing inverse symmetry for visual balance, without compromising the principles of Insular designing.

The operations with compass and straight-edge manifest in visual and tactile terms a coherent geometry in designing, with every structural line related to every other one by derivation direct and continuous from one given measure.

It may also useful to have in hand an analog in modern mathematical notation. Let us walk through the operations once again, this time using modern notation for the measures laid down, derived, and copied as the forms evolve. The Pythagorean Theorem is all that is needed to explicate the procedure.

The measure copied by (1) is half the diagonal of a quadrant square of the underlying square, which is $\frac{1}{2}\sqrt{2}$ of the quadrant measure, equivalent to the inverse of $\sqrt{2}$, i.e., $\frac{1}{\sqrt{2}}$. Then measure (2) picks up the hypotenuse measure of a right triangle with sides $\frac{1}{\sqrt{2}} \times 1$, and copies it for the vertical dimension of that quadrant of the panel. In terms of the given measure (underlying square), height of the panel will be $\sqrt{3} : \sqrt{2}$.

For top-and-base dimensions of the panel, use simple subtraction of the measure by which the panel height exceeds the measure of the underlying square: $1 - ((\sqrt{3} : \sqrt{2}) - 1)$.

Circle-to-width dimensions of thc panel are explained in part above. In numerical terms, Fig. 4.11 sets up a right triangle with sides 2×1, entailing a hypotenuse computed as $\sqrt{5}$. Manipulating these measures produces $(\sqrt{5} - 1) : 2$ and its complement (cf. Fig. 3.19b), so that by the internal cut in Fig. 4.11 (5), diameter of the circle is $1 - \frac{1}{\varphi}$.

The central circle's centre is at half the height of the panel.

For cross, quadrilateral complements, frame, the manœuvres (1) in Fig, 4.12 in effect set the cross-shaft width as the difference between the panel's height and its base-width (the given measure maintained in the underlying rectangle). The halves of the height exceed the halves of the base-width to mark points guiding the width of the shaft. In numerical terms, the shaft will be $(\sqrt{3} : \sqrt{2}) - 1$ in relation to the given measure. The frame has a constant breadth set as the measure by which the given measure exceeds the top of the trapezoid in either corner, that is, $\frac{1}{2}((\sqrt{3} : \sqrt{2}) - 1)$. That is, of course, also half the width of the cross upright.

·The quadrilateral complements to the cross thus have their sides already plotted except for the horizontal sides near the middle of the plan.

For the upper pair, the measure from top of the panel is a copy of half the width of the rectangle (half the given measure). For the lower pair the derivation is not an inverse copy of form the upper pair. It begins from a 1×2 right triangle, the unit of it being half the height of the panel. It then set half-height measure as the hypotenuse of a 2×1 right triangle, so that the measure of the longer side of this triangle is $2 : \sqrt{5}$ of the half-height; it is this that is copied to the sides to guide the horizontal line along the tops of this pair of quadrilaterals. Or, finally, in terms of the given measure: $\frac{Given}{2} \times (\frac{2}{\sqrt{5}} \times \frac{\sqrt{3}}{\sqrt{2}})$.

It is the difference between the measure $\frac{Given}{2}$ for the lower limit of the upper pair, and the measure just given for the upper limit of the lower pair, that puts the cross arms off-centre of the midline of the form, 'off' from symmetrical placement flanking the central circle of the plan. It is in turn this imbalance in this derivation that brings the thickness of the cross-arms into balance with the thickness of the upright, as explained above.

['St Patrick's Bell Shrine: Form and Layout of the Plates,' *JRSAI* 138 (2008), 26–46, describes the coherent geometry of all four lateral faces of this box.]

Metalwork (3)

The **Tara Brooch** has a form much more complex than that of Soiscél Molaise, somewhat more complex than that of the St. Patrick's Bell shrine, and considerably more elegant than either one. Its manufacture is eighth century, probably a bit earlier than Soiscél Molaise. Instead of evolving from relations of 1, 2, $\sqrt{2}$, $\sqrt{3}$, this piece evolves from 1, 2, φ. As different as these pieces are in shape, in function, in quantitative sets that generate their forms, all answer to—and must have been created from—the same methods of compass-and-straightedge construction. Part of the mastery of designing exhibited by the Tara Brooch is the unobtrusiveness of the complexity which underlies its form. The form of this brooch is in fact one of the finest representatives of shapes composed with a complete coherence—an 'endless knot'—of ratios among three simple measures.

The primary elements of the form, as it is manifest on the back of the piece, are the outer circle of the whole plan, together with circular arcs within that full circle. One of these arcs is concentric with the outer circle, forming with it an open ring in the upper half of the form. The other four, in the lower half of the plan, vary in their centres and radial measures: that is what gives the lower half of the brooch its distinctive form and esthetic interest. Three of these four set outlines of areas. The other one is distinctive in passing through the centres of circular (or spiral) designs.

Fig. 4.13 illustrates a method of laying out the primary form of this piece. Begin by constructing a circle, quartered (a); then divide the radius by the golden section φ (b); in this version the measures a and b are derived twice within the same figure, with a nearer the centre by one method, b nearer the centre by the other. (Compare these with Figs. 3.20b and 4.11.) A further measure c is entailed (cf. Fig. 3.21b), being either $a - b$ or $1 - 2b$ or $2a - 1$ in simplest derivation. All three are joined proportionally, in that $\frac{1}{a} = \frac{a}{b} = \frac{b}{c} = \varphi$ (golden ratio). (This being the case, it is also the case that $a = \frac{1}{\varphi}$, $b = \frac{1}{\varphi^2}$, $c = \frac{1}{\varphi^3}$.) All three are joined in their sums and differences, as well, in that $1 - a = b$ and $a - b = c$. From this generative set can be composed the primary form of the Tara Brooch—a composition in a four-tone scale, as it were. See Table 4.3. Further steps are the following.

(c) With the circle's radius divided by the golden section, double the shorter segment b to set the radius of an arc to be the inner curve of the hoop.

(c) Draw an arc with its centre at distance b above the centre and its radius measuring $b + a$. This arc is the centreline of the large pair of ornamental cells in the lower half of the brooch; it is exactly on this arc that the visible rivet has its

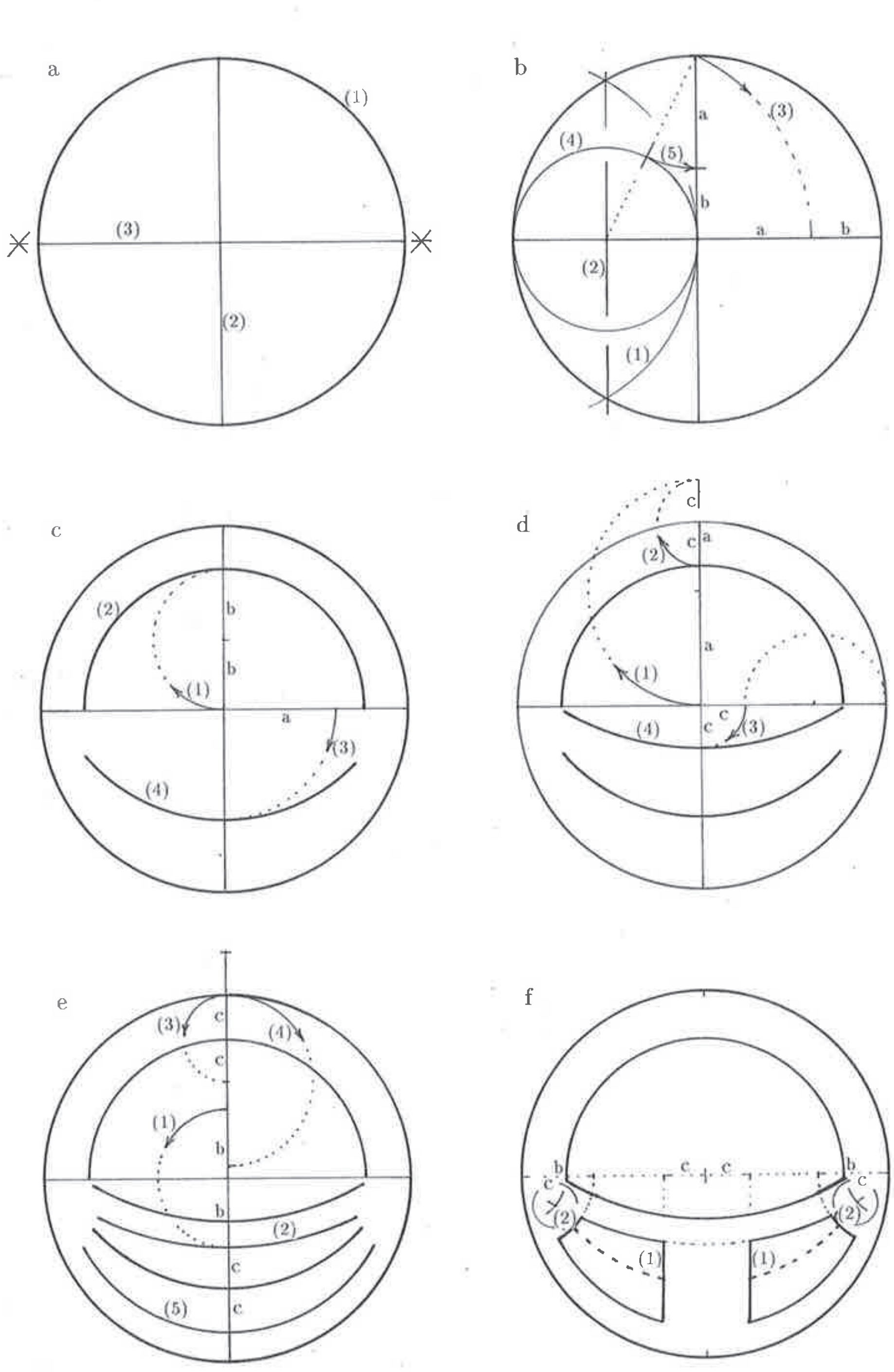


Fig. 4.13 Initial steps in constructing the plan of the Tara Brooch.

Table 4.3 Some Measures in the 'Tara' Brooch

FEATURE	MEASURE	RATIO (: 1)
Ring radius, outer	1 (Given)	1
diameter	2 (Given)	2
Ring radius, inner	$2b$	$2-\frac{2}{\varphi}=\frac{2}{\varphi^2}$
Arc 1, centre	b above centre	$\frac{1}{\varphi+1}=\frac{1}{\varphi^2}$
radius	$b+a\ (=1)$	1
Arc 2, centre	c above top	$\frac{1}{\varphi}-\frac{1}{\varphi^2}=\frac{1}{\varphi^3}$
radius	$c+1+c$	$\frac{2}{\varphi}+\frac{1}{\varphi^3}=1+\frac{2}{\varphi^3}$
Arc 3, centre	c above top	$\frac{1}{\varphi}-\frac{1}{\varphi^2}$
radius	$c+1+b$	φ
Arc 4, centre	$2(2c)$ below top	$2(\frac{2}{\varphi}-\frac{2}{\varphi^2})=\frac{4}{\varphi^3}$
radius	to $b+c+c$ below centre	$\frac{1}{\varphi}+\frac{1}{\varphi^3}=1-\frac{1}{\varphi^4}$
'Gap,' width	$2c$	$\frac{2}{\varphi^3}$
(Pin head, length	$2a$	$\frac{2}{\varphi}$)

centre.

(d) Extend the vertical diameter upward by measure c, and draw another arc from the end of that extension, with radius $c+1+c$.

(e) Draw yet another arc from the same centre as the one preceding, with radius $c+1+b$.

(e) Along the centreline, double the measure c from the top of the circle, then double it again to locate the centre of the lowest arc; draw this arc with radius from that centre to a point $b+c+c$ below the centre of the circular ring.

(f) Set the measure c on either side of the centerline to guide the parallel lines enclosing the decorative cells in the lower portion of the brooch.

Governance by the golden ratio does not end with the basic shape, but extends to the layout of areas for ornament as well. Next are construction methods for the principal ones on the back of the brooch. See Fig. 4.14.

(a) Plot the paths of arcs with centres at the midpoints of the sides and with radius c; where they intersect Arc 1 locates the centres for circular stud cells on either side.

(b) Plot the paths of arcs with centres at the midpoints of the sides and with

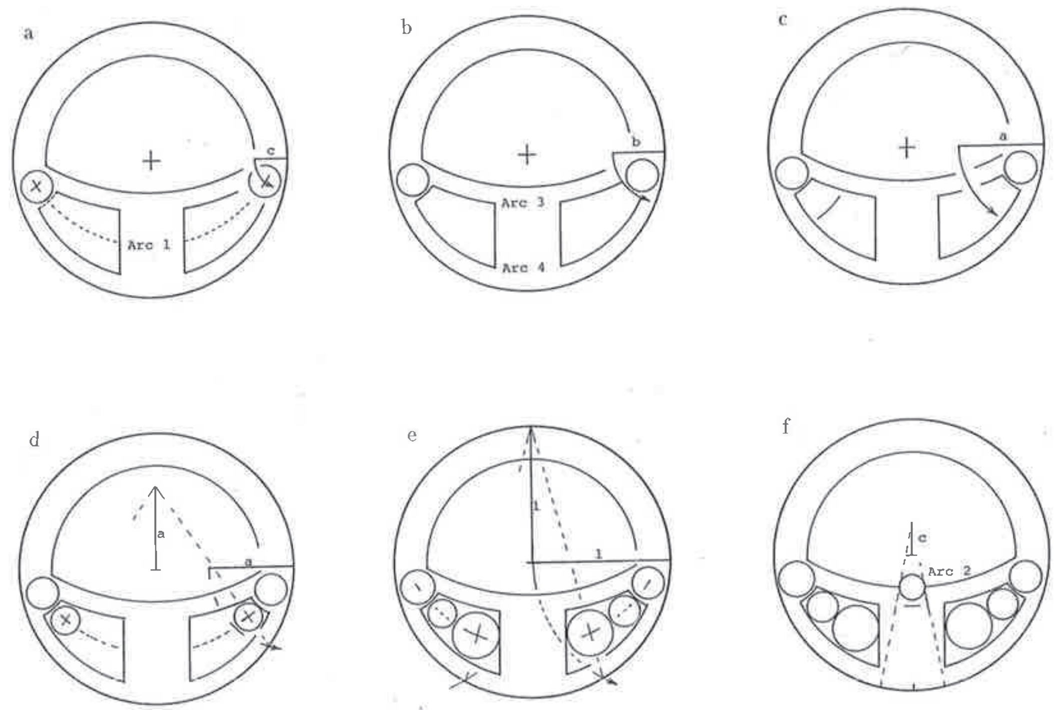


Fig. 4.14 Some further steps in constructing the plan of the Tara Brooch.

radius b; they intersect Arcs 3 and 4 to define the ends of the pair of cells in the lower part of the plan (the 'terminals').

(c) Plot the paths of arcs with centres at the midpoints of the sides and with radius a; they exactly separate the two spiral patterns in those lower cells.

(d) The centre of the smaller spiral pattern on either side is at the intersection of Arc 1 and a line between two points: the one where the arc with radius a intersects the outer circle, and the other at measure a above the centre of the outer circle.

(e) The centre of the larger spiral pattern on either side is at the intersection of Arc 1 and a line between two points: the one where the arc with radius 1 intersects the outer circle, and the other at measure 1 above the centre of the outer circle.

(f) The centre of the (now empty) circular stud cell nearest the centre of the brooch is the intersection of Arc 2 and the vertical axis; its diameter is probably defined by lines proceeding from the vertical axis at measure c above the centre of the plan to points at measure c on either side of the vertical axis at the bottom of the plan.

(The head of the pin also follows this scheme of analysis of the brooch's form in the most relevant features, its length and its outer curvature. A derivation of its form is included in 'The Form of the Tara Brooch,' *JRSAI* 128 (1998), 5–16.)

This brings up a matter in historiography, I believe, that is virtually unrecognised in art history of the early Insular tradition. The breadth of the upper loop of the brooch (that is, the difference in radii of the outer and inner arcs) is the same—in relative measures—as that of the ring of the Durrow Cross, for example; it is c, or $1 - 2b$, or $2a - 1$, in the description given. The measure $2c$ recurs in the width of the central pattern in the lower segment of the brooch (both front and back) and in the distance from the Durrow outer circle to inner reach of the armpit arcs. It also occurs in the central segments of the devices above and below the 'square' in the Lindisfarne Gospels cross-page folio 94v. Never mind that the diameter measure of the brooch is 87mm, which is less than half the horizontal measure for a typical cross-page in the Lindisfarne Gospels, and is to be compared to 99 cm diameter measure of the Durrow Cross ring (more than eleven times larger). And there are many, many more examples like these. Connections among these recurrences—these 'sames'—are as logically ascribed to a common tradition of creating forms, as they are to copying or imitation.

The form of the Tara Brooch is an exceptionally elegant construction in coherent geometry. The creator of the 'almost classical simplicity of its general outline' indeed 'seems to have been gifted with an unerring instinct for proportion whether in mass or line' (Lucas 1973, 93). 'Fine metalwork, manuscript painting and sculpture attained a perfection in early medieval Ireland (*c.* 450–1200) that has lost none of its power to delight and surprise us. All three arts are closely interlinked in their decoration, symbolism, and use of colour' (Ryan 1993, 5). To these shared characteristics can be added their formal principles, with their own power to surprise and delight us.

'Carpet' Pages The forms of two pairs of portrait carpet pages in the surviving Insular Gospels manuscripts will be described next. Then the form of a cross carpet page will be described. (Others are analyzed in *The Earliest Irish and English Bookarts.*)

David Rex and David pages, Durham Cassiodorus These two framed portrait pages precede commentary on groups of Psalms beginning with numbers 51 and 101, respectively, in Durham Cathedral Library MS. B.II.30; a fair inference is that a third portrait page preceded the first fifty. These are very early illuminated pages (eighth century), both having virtually the same height but slightly differing widths, both having forms of the simplest devising. They illustrate the variety of simple manœuvers that will create forms for carpet pages.

The David Rex page (fol. 81v) takes its shape by development of the ratio $\sqrt{2} : 1$. If the width of the plan is the given measure, the form can be constructed readily from a quartered square with that measure for its sides (cf. Fig. 1.1g). The procedure is illustrated in Fig. 4.15a. Extend the height of the square by the diagonal measure of its quadrants (1), and enclose the rectangle (2). Along upper and lower sides of the square subtract the measure by which the height of the frame exceeds the width (3) to set the inner vertical lines (4) of the frame; the inner horizontal lines lie along the top and bottom of the original square.

If the height of the plan is the given measure, the outer rectangle can be constructed readily from either a 2×1 rectangle (two squares sharing a side, cf. Fig. 3.5), or a 2×2 rectangle (a square divided into quadrants, cf. Fig.1.1g). With the first, the width is set by expansion, as in Fig. 4.15b. Diagonals of the stacked squares (1, 1) divide the midline in halves. Using that half-measure, locate midpoints of the sides of the two squares (2, 3). Then copy the diagonal of quadrants of the two squares to mark points along extensions of the upper and lower lines (4). Lines connecting corresponding pairs of these points provide the outer sides of the retangular frame. Inner frame height copies outer frame width (5–6). Inner frame width (8) is set by copying upper/lower measures (7).

With the second, beginning from a 2×2 square quartered, the width is set by narrowing, as in Fig. 4.15c. In each quadrant, locate intersection of diagonals (1, 1); copy the half-measure of diagonals to mark points along the outer horizontal lines of the square (2) to guide the outer vertical lines of the frame (3). The inner rectangle is derived from that (4 and 5, 4 continued and 6).

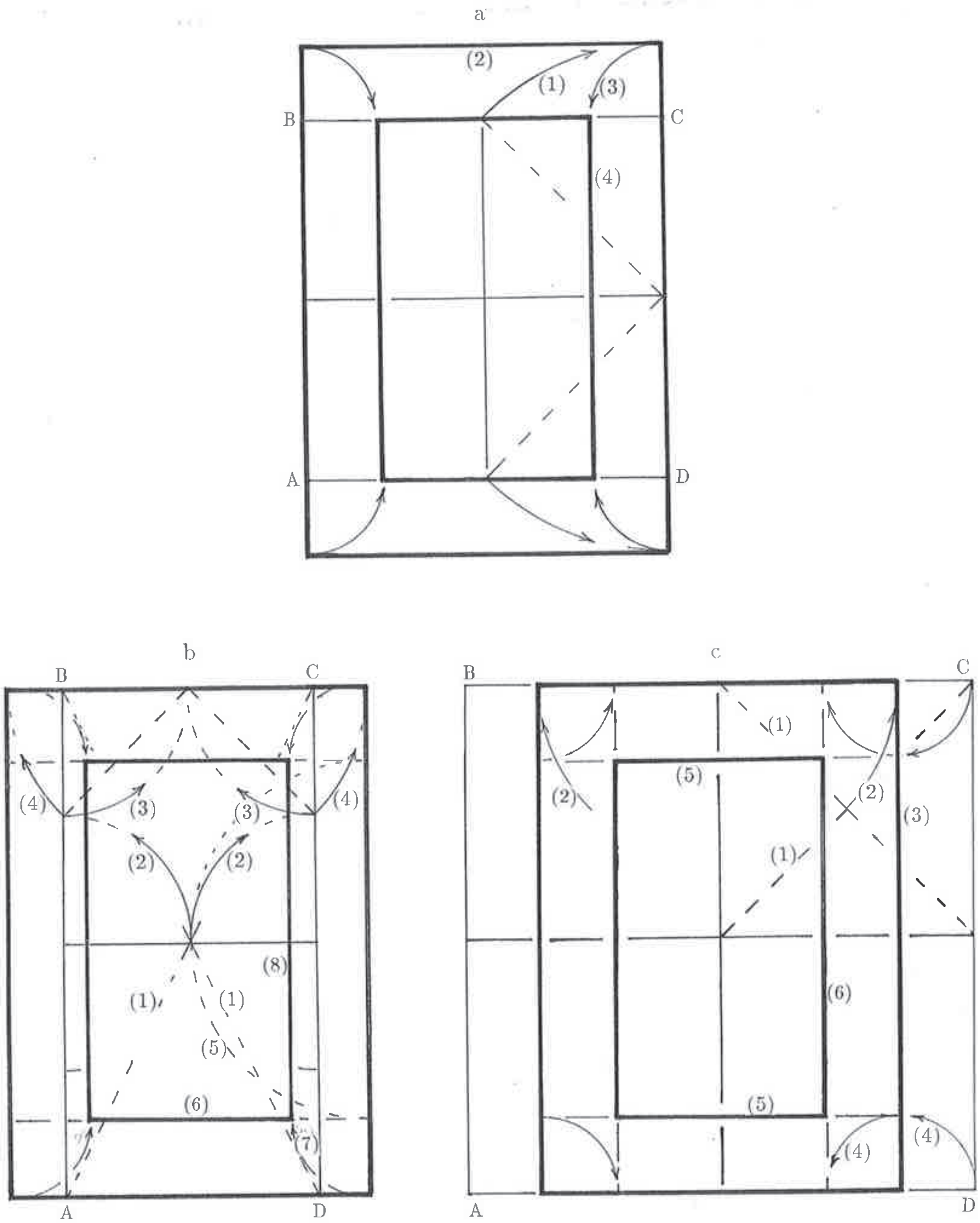


Fig. 4.15 Three constructions of David Rex page frame, Durham Cassiodorus.

The David page (fol. 172v) takes its shape form development of the ratio $\sqrt{5}:1$. If the width of the plan is the given measure, the form can be constructed readily from a quartered square with that measure for its sides (cf. Fig. 1.1g). The procedure is illustrated in Fig. 4.16a. First extend the vertical sides of the square, above and below, by the diagonal measure of two adjoined squares (1). Then sketch a line (2) from centre of the horizontal side of the square to the vertical extension opposite, marking its intersection with the midline of the quartered square. Take the measure from opposite midpoint of the side of the square to that intersection and copy it (3) to mark points above and below along extensions of the sides. Draw lines (4) connecting pairs of these points for the upper and lower sides of the frame.

For the inner rectangle of the frame, copy (5) the measure between rectangle and initial square to plot the vertical lines (6); the horizontal lines are along the upper and lower sides of the original square.

If the height of the plan is the given measure, the outer rectangle can be constructed from either a 2×1 rectangle (two squares sharing a side, cf. Fig. 3.5), or a 2×2 rectangle (a square divided into quadrants, cf. Fig. 1.1g). With the first the width is set by expansion, as in Fig. 4.16b. Begin by locating midpoints of the sides of the two squares (*a*, *b*, *c*). Divide vertical sides of the two squares by golden section (1, 2, 3). Halve the longer segment made by that section (4) marking a point along the (vertical) centreline. Along extensions of the upper and lower lines of the stacked squares copy (5) the longer segment of the centreline in each square to mark points to guide the vertical lines (6) of the frame.

With the second, from a 2×2 square quartered, the width is set by narrowing, as in Fig. 4.16c. Begin by locating the midpoint of each half of the centreline, by the intersection of the centreline with diagonals of upper and lower pairs of quadrants (1). Sketch a circle (2) centred at that midpoint, diameter equal to a quadrant measure. Copy (3) the measure shown along the outer lines of the 2×2 square, cutting each outer quadrant side by the golden section. Halve that measure along the centreline (4), and copy (5) the longer segment of the centreline to mark points along the upper and lower sides of the original square. Join the points in pairs with lines (6) that will be the outer sides of the frame.

For the inner rectangle of the frame, set the upper and lower lines (8) by copying (7) the measure between sides of the 2×2 square and the sides of the derived frame (or simply copy above and below the midline the half-width of the outer frame (7)). Then copy (7 continued) that same distance to plot the vertical lines (9) of the inner rectangle of the frame.

If there was a third David page, presumably preceding the first fifty psalms, a fair guess is that its frame was a 3 : 2 rectangle, cf. Fig. 3.7, developed like the two extant pages that followed.

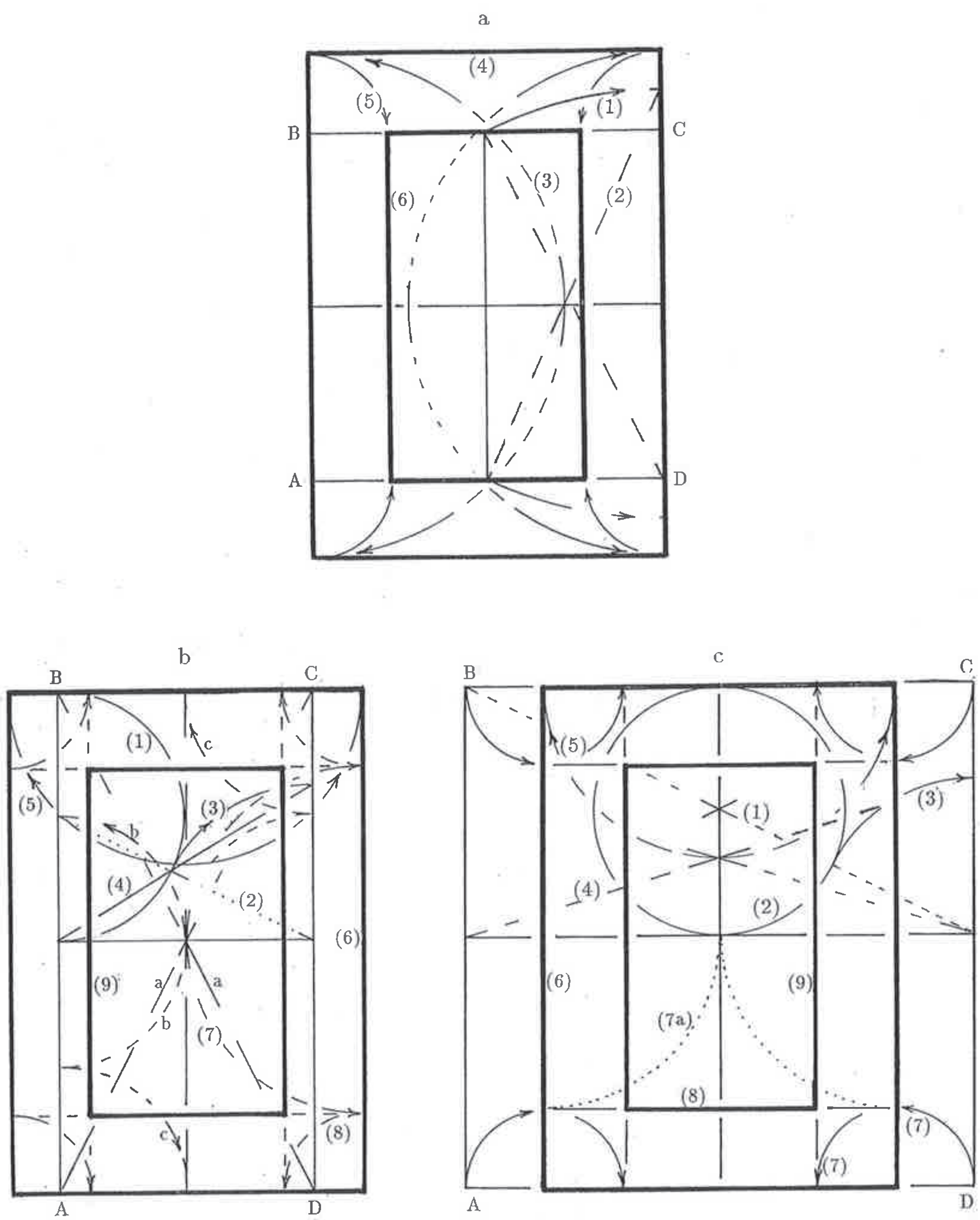


Fig. 4.16 Three constructions of David frame page, Durham Cassiodorus.

St. John Page, Book of Mulling The St. John portrait on page 193 (fol. 81v) in the Book of Mulling is a very early and elementary example of coherent geometrical design of carpet pages in Insular bookarts. The layout of the page evolves from a quartered square ABCD based on a single measure (cf. Fig. 1.1d–g) to be the width of the frame. The process of its devising is illustrated in Fig. 4.17.

(a) Set height of the rectangular frame by copying in each quadrant the diagonal measure of that quadrant along a horizontal side of the square (1), then copying (2) the remaining measure along an extension of the vertical side of the square. Draw the rectangle for a frame with dimensions now set. (The overall dimensions are in the ratio $1 : 1 + (\sqrt{2} - 1)$. Cf. Fig. 3.12b for the first step in this procedure.)

(b) Divide the width of the rectangle in fourths to set the inner horizontal dimension of the frame: intersecting diagonals of the extensions of the quartered square will guide lines defining the inner sides of the frame.

(c) Copy the measure between the inner and outer vertical lines of the frame to set the square corner cells for decoration.

(d) Using the height of the original square and its half-measure inside the frame, copy the diagonal measure of this 2×1 rectangle to mark points along the inner lines of the frame: these guide the line to be the inner side of the bottom of the frame.

(e) Intersections of diagonals of the upper extension from the original square locate the centre of the halo; a diagonal of the upper half of the frame sets, as a tangent, the radial measure of the halo.

(f) Size and placement of St. John's book proceed from dimensions set in the initial steps *a* and *b*. Width of the book is one-half the width of the inner panel (or one-fourth the width of the frame). The upper side is located by copying half the diagonal of one quadrant of the original square above the middle (1). Height of the book is copied from its width (2).

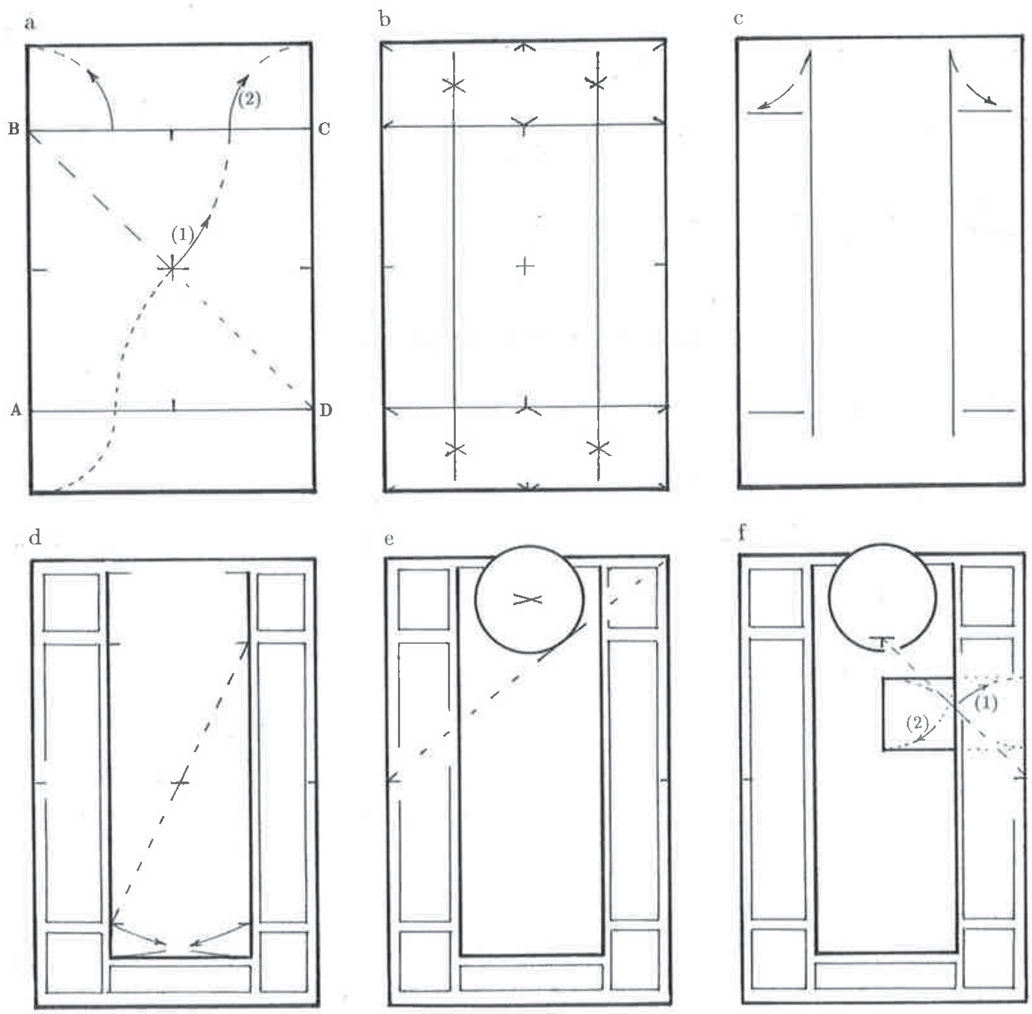


Fig. 4.17 Layout of frame, halo and book for St. John page, Book of Mulling

St. John Page, Book of Kells The St. John portrait page (fol. 291v) in the Book of Kells has to be one of the finest and most fascinating portrayals of an author and his book that we will ever encounter. It is an icon in the etymological sense—a pictorial representation. But it is more than that. It also attracts a veneration of its artistic achievement. Thankfully, it was never actively venerated for its subject and used in preparing a cure for sick cattle, as was the Book of Durrow in the 18th century, when at least one gathering from its St. John's Gospel seems to have been soaked repeatedly to obtain its curative properties.

The fantastical decoration filling the cells that join to make the frame engages and re-engages one's attention, and so does the decoration inside the halo, and so does the knotwork at the four corners of the frame. At midpoint of bottom, sides and top, the feet, hands and head outside the frame remain mysterious, though perhaps only because most of the head was lost to a binder's knife. The evangelist is not depicted as writing, as so often is the case. He holds a pen in one hand, but not a knife in the other, and he does not have an open book in front of him. Instead, he is holding a quill in one hand (and there is an inkhorn by his foot) while in the other hand he is holding a closed book. The signification in all likelihood is that his book has now been completed.

In essence, here is a page in a book, depicting a writer (with pen and inkhorn), and his completed book (it even has a fine binding). And the author is nimbed as probably no other writer has EVER been.

The rectangle of the frame for St. John's portrait, without extensions or embellishments, is approximately 266 × 195 mm.

A synopsis of a procedure that will replicate—or create—the frame surrounding the evangelist is illustrated in Fig. 4.18.

The Frame Begin with concentric and commensurate square and circle, as in Fig. 1f–g, and from it construct a frame in the ratio $\frac{\sqrt{3}+1}{2}$, as in Fig. 3.16.

(a) Sketch diagonals of this frame, and mark their intersections with the underlying circle to be the primary set of corners of the inner panel area.

(b) Another set of corners of the inner panel area is located by copying, onto the upper and lower sides, the vertical distance from the frame to the first set of corners, and then sketching the paths of parallel lines joining them in pairs.

(c) A third set of corners of the inner panel area is located by copying to the left and right sides the horizontal measures from the corners, and then sketching the paths of parallel lines joining them in pairs.

(d) Midside 'notches' in the inner frame are plotted with separate diagonals, as shown.

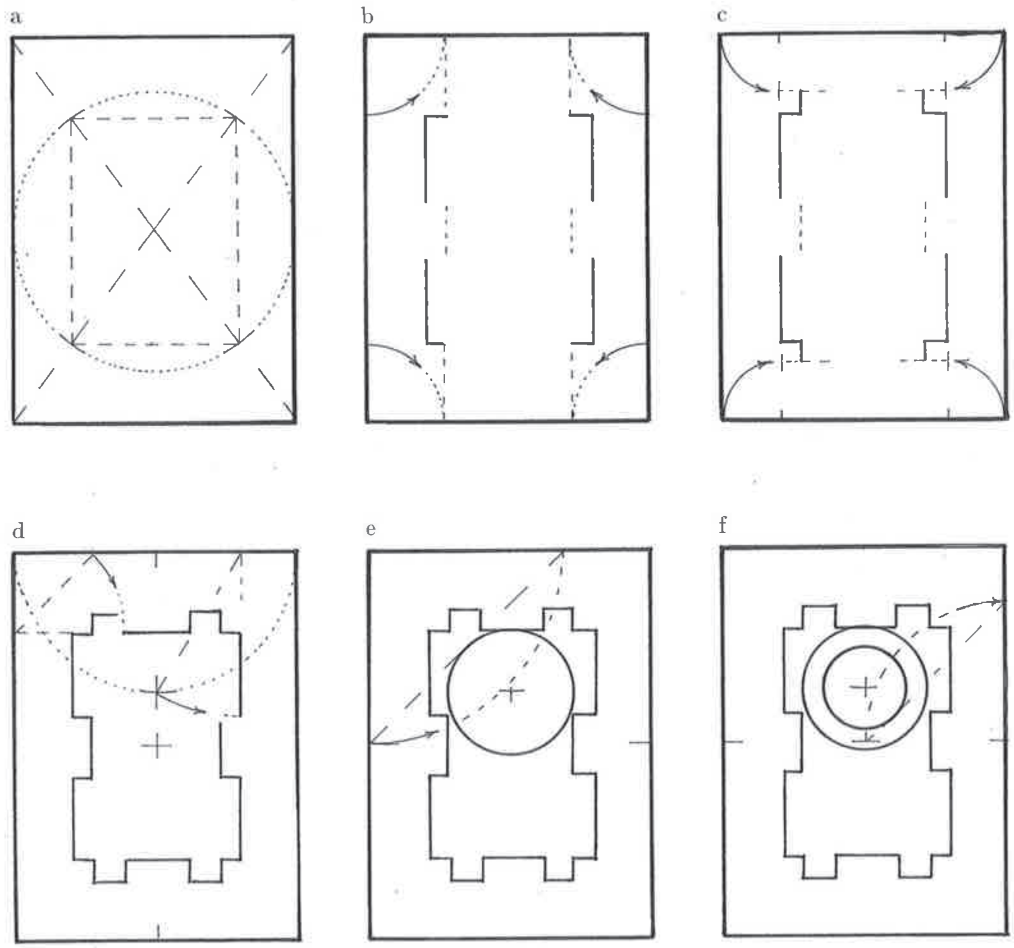


Fig. 4.18 Designing frame and halo for St. John page, Book of Kells.

The halo (e) Centre of the halo is located along the centreline a distance from the upper border equal to half the width of the frame (see preceding step). Its outer circle is drawn tangent to 45° diagonals from midpoints of the sides.

(f) The inner circle is drawn tangent to 45° diagonals from the centrepoint of the frame. The two circles are thus proportional to the frame and its underlying square. There are alternate constructions, some just as easy, all just as accurate.

It is a fair inference that the layout of the St. John page was derived by steps like the ones just illustrated, using compass and straight-edge, and needing nothing more. Proof for any method of construction, of course, is hard to come by in the absence documentation, or in the absence of the tradition that produced it continuing intact. We have proof neither from a tradition still living, nor from documentation such as instructional manuals by the artists, or reports of observers of their craft.

But we do have three kinds of evidence. One is comparative: shapes embodying the same geometrical terms and simplicity, in other media and within the same era of Insular Christian art. Another kind of evidence is deductive: how else to explain designs whose measures accurately match simple geometrical constructions again and again throughout the entire form? The third is empirical. It is not ample, and much of it is not easy to find. This is the evidence of construction marks (as they are called)—small punctures, or impressions, left by a sharp point, either a pricking knife used elsewhere to mark where the straight-edge was to be used to rule the page for text, or marks left by the fixed foot of a compass (or dividers). They also include ruled lines, typically impressed in the surface of the leaf, without ink, or curved lines similarly drawn. In the upper portion of this page construction marks are to be seen everywhere: the centres of the eyes, the innermost circle of the halo where half-circle embellishments inside the circle have their centres marked, as well as their outer limits, a horizontal ruling level with the evangelist's eyes, on both left and right of his head, and so on. There are also prickings of the parchment leaf in corners of cells of decoration that make up the frame; in profusion they guide the interlace designs.

Still, any construction marks that correspond to the ones predicted by the construction method illustrated in Fig. 4.18 for frame and halo are extremely difficult to detect. One reason is that on the portrait side of the leaf, construction marks are covered by paint. Another is that some key parts of the derivation are located along the middle of the leaf. The parchment is extra thick across the middle of the page, because that is the area of the membrane that lay along the back of the animal whose skin was turned into fol. 291. Markings will not easily show through there. And another is that pages having elegant illuminations, in this manuscript and others, seem to have been laid out with light construction markings—part of the elegance of the art. The main layout is lightly marked, and probably copied directly from a model. The decoration, on the other hand, could not be copied with a few reference points, but had to be constructed on the leaf so as to fit the specific area to be filled with embellishing devices. Thus the profusion of construction marks for ornamental details, and thus the elusiveness of marks for the overall plan.

But there is empirical evidence in the pectoral area of the evangelist depiction. In the facsimile it cannot be made out with certainty. On the manuscript leaf it is not easy to make out—in fact may not be detectable by eye when the leaf lies flat. But when the leaf is held to the light, right there in St. John's chest, the exact middle of the frame—the point where diagonals of the frame intersect—: there, exactly there, is a tiny hole letting light through from the other side. Here in the thick area of the parchment, the ridge of the animal's back, is a construction mark that does not guide interlace, or halo, or step-pattern, or boundaries of cells of the frame. It marks the point from which the mise-en-page of the illumination began, the central point of the framed image of St. John, the point of origin of the icon. It is the beginning point of the design of the St. John page in the Book of Kells.

It should not come as a surprise to learn that the book in John's hand has the same shape as the basic rectangle of the illumination itself.

Cross page, Lindisfarne Gospels fol. 94v This cross page is a masterpiece not only of knotwork, interlace, spirals, and grilles,but also a masterpiece of coherent geometry informing its structural lines and areas. It is selected for description here because its form provides a highly instructive complement to two other forms described earlier in this section, those of Durrow Cross and the Tara Brooch.

The three are complementary in several ways. They range in horizontal measure from 8.7 cm (brooch) to 18.3 cm (cross-page) to 99 cm (stone cross, ring diamater). In terms of fundamental design, the picture page uses parallel straight lines primarily with a circular pattern at the center; its frame makes the form exoskeletal, as it were. The stone cross uses parallel straight lines for the cross proper, together with circular arcs for the ring and 'armpits' of the cross, the lines and arcs contributing about equally to the form; without a frame, the cross is endoskeletal in effect. The brooch form consists primarily of circular arcs, with only minor use of parallel lines and incidental use of straight lines elsewhere; it lacks either frame or inner support.

In terms of manufacture, the first is drawing on a parchment sheet, the second is a drawing on stone with the non-enclosed areas cut away, and the third is a drawing on wax (probably), that was then carved as a model, from which a mould was made, which was then used for casting the metal base of the piece of jewelry. Accordingly, in terms of perception, the carpet page is a design drawn on a sheet of parchment, producing a figure and ground that are viewed on the flat, so to speak. The sculptured cross stands alone without a ground, being viewed in the round. The brooch also stands alone, sometimes in the round, sometimes against a background consisting of the fabric surface to which it typically is attached.

Notwithstanding these differences in size, medium, manufacture, type of structural lines, and perceptual mode, these three artifacts share the characteristics of having clear lines and careful symmetry, with their shapes defined by straight lines or straight lines and circular arcs, all emanating from the same set of relative measures, 1, 2, φ.

The primary plan of this page can be replicated by steps illustrated in Fig. 4.19. Key measures are listed in Table 4.4.

(a) Begin with the quartered square, with the upper and lower sides of its quadrants divided into extreme and mean ratio—that is, by the golden section (cf. Fig. 4.13b, top).

(b) Copy the short segment from each corner along the lateral sides of the square to plot an inner square, as shown.

(c) Within the inner square, use the diagonal of a quadrant to set (as a

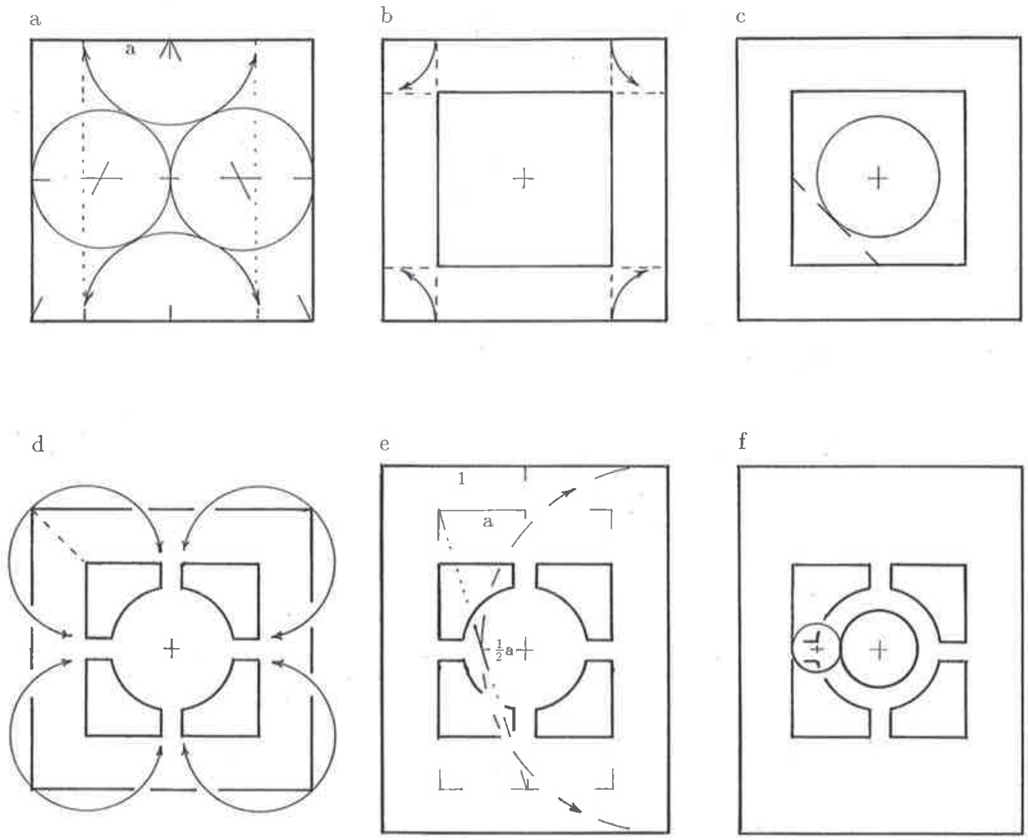


Fig. 4.19 Primary plan of Lindisfarne Gospels fol. 94v.

tangent) the centred circle.

(d) Use the diagonal measure between the two squares to plot the 'gaps' in the central device, as shown.

(e) A line from the midpoint of the lower side of the square, to the point at measure a from the centerline along a top side of an upper quadrant cuts the horizontal midline at $\frac{1}{2}a$ from the center. From the midpoint of a side of the large square the measure $1 + \frac{1}{2}a$ is then copied above and below to mark the height for the frame along extensions of its sides.

(f) Plot the inner circle as shown.

The finished plan is much more complex, of course, with its complementing panels, and with its adjustments for perceptual factors (quite apart from its decorative fillers): these matters are detailed in *The Earliest Irish and English Bookarts*, 136–41, especially Fig. 9.6. But the Lindisfarne Gospels illuminator began with the same formal principles—and same repertory of related measures

Table 4.4 Primary Measures in Lindisfarne Gospels Carpet Page folio 94v

FEATURE	MEASURE	RATIO (: 1)
Outer rectangle, width	2 (Given)	2
Outer rectangle, height	$2 + a$ as $2(1 + \frac{a}{2})$	$\frac{\varphi^2}{2}$ or $1 + \frac{1}{2\varphi}$ or $\frac{\varphi+1}{2}$
Inner 'square,' sides	$2a$	$\frac{2}{\varphi}$
Large inner circle	$\frac{\sqrt{2}}{2}(2a)$	$\frac{\sqrt{2}}{2} \times \frac{2}{\varphi}$
Small inner circle	$2a - 2(2a - \frac{\sqrt{2}}{2}(2a))$	

1, 2, φ—as did the designers of the Tara Brooch and the Durrow Cross, and developed the plan without departing from those principles.

Verse Texts Do forms of verse compositions in Old English, which are lineal (or sequential) in form, share designing concepts and methods with the two-dimensional forms of page illuminations, standing crosses, decorative metalwork?

A common response to this question is found in a review of *The Earliest Irish and English Bookarts*: 'there seems to be no practical advantage in composing poetry according to these principles' because the numerical relationships in the subdivisions of the Old English poems, 'though undoubtedly there, would not be easily apparent to the reader or listener' (*Medium Ævum* LXV.1, 126). But this response is not an answer to the question. If the quantitative patterns within poem-parts are undoubtedly there and are like those in the two-dimensional art-forms within the same culture, a better response is to pursue the question, not to dismiss it. The elemental and straightforward mathematical basis of the two-dimensional designs was not apparent to modern viewers, either, until it was pointed out a few years ago. And 'practical advantage' is hardly an historical constant.

By form in this context is meant an aspect of the verse *text* created in the late tenth century and preserved in a manuscript copy from that time. It has to do with the text as an artefact, something shaped by human workmanship. The aspect of composition relevant to Insular designing is distinct from page layout of the surviving copy. It is distinct from the metrical structure (metrical lines *not* represented by graphic lines). And it is distinct from episodes or topical segments of the 'matter' of the text. By form is meant the scheme of relations among *textual* parts. The primary divisions of the text in the manuscripts that preserve them are commonly called 'sectional divisions': they are prominent interruptions of text manifest typically in distinctive end-punctuation, blank line or line containing only a centred numeral, and resumption of text normally with one or more majuscule letters. These devices are graphically divisive of the continuous text.

The designs of two texts will be illustrated here. Both are preserved uniquely in a large manuscript anthology, The Exeter Book, written (or copied) about the year A.D. 1000.

Christ II The text generally identified now as 'Christ II' in the Exeter Book has lost one leaf, hence the original count of lines must be deduced from the narrative matter and the codicological features of the manuscript. The former gives a general notion of the length of text now missing, and the latter gives a very narrow range for the amount of lost text. Attention to formal aspects seems

Table 4.5 Lengths of the sections of *Christ II*, restored

Section	Number of lines in section groups				
[I]	77				
		144			
[II]	67[a]		228		
		151		**314**	
[III]	84[a]		237		407
		170		**330**		**495**
[IIII]	86		263		418
		179		**351**	
[V]	93		**267**		
		181			
[VI]	88				

[a]Lacuna of 67^{+} verse lines

to enable recovery of the exact length of the missing text, which works out to 67^{+} metrical lines. (All these matters discussed in detail in *The Earliest Irish and English Bookarts.*) The other side of this coin (so to speak) is that it enables us to recognise the extension of Irish and English designing methods to verse texts in late Old English.

The manuscript text of *Christ II* is interrupted every eighty-two or eighty-three lines, on average, by distinctive end-punctuation followed by a blank line, followed by prominent majuscule letters as text resumes. (None of the separate segments of texts has the average number of verse lines.) An array of the linecount for this text is given in Table 4.5.

Coherent form is hardly apparent in such a display, a quite modern method of recording these formal aspects of the text. It is essentially like that of listing measurements of key elements in the designs of page illuminations, standing crosses, and the like. Using these, though, it is possible to reconstruct the methods and particulars of the formal planning for this text. When we notice, for example, that the verse-counts 267 and 495 are in the same ratio as 144 and 267, there is motivation to explore the textual divisions further for evidence of a coherent plan. Those measures are the length of the complete text, the length of the latter three sections, and the length of the first two sections. Then by comparison to Insular form in the two-dimensional art designs, it is a short endeavor to discover the

Table 4.6 Lengths of section-groups of *Christ II*, restored

Sections	Linecount	Module = 165	
I – VI	495	3	
IV – VI	267	φ	
I – III	228	$3-\varphi$	
I – II	144	$\frac{\varphi^2}{3}$	
III – VI	351	$\sqrt{\varphi^2+(3-\varphi)^2}$	or $3-\frac{\varphi^2}{3}$
IV – V	179	$\sqrt[4]{3-\varphi}$	or $\frac{1}{\sqrt{\varphi}}\times(3-\varphi)$
I – IV	314	$\sqrt{(3-2)+\varphi^2}$	
II – V	330	2	
searo-lice	←194	$\sqrt{3-\varphi}$	

basis of the recurring ratio: multiply the ratio by three and there is the golden section ratio (φ). A best inference from that is that the key measure therefore will be one-third the total linecount, that is, 165. Let it serve as the modulus for a graphic analog.

If the golden section ratio is present, a ready model for its derivation will be two-dimensional, like those of the metal and stone artefacts. A two-by-three frame is the simplest for this (cf. Fig. 3.7), and will prove to be altogether sufficient. But first, it will be useful for analyzing the form (as distinct from constructing it) to explore the relations among 2, 3, φ that may obtain variously among the section lengths of this text. Table 4.6 tabulates them. To simplify notation, let 1 replace $3-2$ as we proceed. It will be noted that *all* the divisions of the text are expressible in relations among 1, 2, 3, φ. In addition, the location of a sentence concerning skilful literary composition fits into the scheme: **Sum m[æ]g searolice word-cwide writan** 'One is able to write formal composition ingeniously,' an item in a list of the 'Gifts of Men.'

Besides the parsimony of 1, 2, 3, φ there is linking among some of the section-groups by ratios using the same elements:

Sections II–IV and V–VI, that is, $237:181=(\varphi+1):2$, and

Sections I–VI, IV–VI, I–II, that is, $495:267=267:144$, or $3:\varphi=\varphi:\frac{\varphi}{3}$, and for good measure *searolice*, 194 lines from the end, is at the mean between the module 165 and the length of the first three sections:

$228:194=194:165$, or $3-\varphi:\sqrt{3-\varphi}=\sqrt{3-\varphi}:1$.

One final observation about these dimensions and the web of relations they manifest: the only integer counts of the inferred module (165) are 2 and 3, the the linecounts 330 and 495. Notice now the clustering of these three numerals in the array of natural, triangular, solid, and fourth-order numbers:

Natural	1	2	3	4	5	6	7	8	9	10	...	
Triangular	1	3	6	10	15	21	28	36	45	55	...	
Solid	1	4	10	20	35	56	84	120	**165**	220	...	
4th order	1	5	15	35	70	126	210	**330**	**495**	...	1,001	

Next a procedure for setting those quantities by graphic computation: the quantities of magnitude in the geometrical derivation will correspond to nearest integer quantities of multitude in the numerical display of linecounts, with no gaps, no orphans.

Begin with a 3×2 grid of squares as in Fig. 4.20, with these dimensions: the squares measure 165 along their sides, hence the grid measures length 495 (three modules), width 330 (two modules). Then introduce the golden section ratio—that is, introduce a ratio more complex to derive than arithmetical 3 : 2 (and yet of a very simple kind derivationally and perceptually). Panel **a** shows two simple methods, one in the middle one-third with solid line circle, the other in the middle and lower one-thirds with dashed lines. Either one cuts the length 495 (three modules) into segments measuring φ and $3 - \varphi$ of the modulus, matching the linecount of 267 for the latter three fitts, 228 for the first three. With this, as with the Tara Brooch plan (Metalwork 3), the first cut is where the individuating process begins, and all remaining cuts build upon this first one.

A key to formal properties of this composition, if that is what it is (found in **Sum m[æ]g searolice word-cwide writan**), can be located by a corollary of the initial and defining cut. Panel **b** shows a simple manœuvre to mark a distance 194 from the end of the text.

Panel **c** sets the two 'half' measures φ and $3-\varphi$, 267 and 228, at a right angle, and the measure 351 from end to end—the hypotenuse of the right triangle—is used to set the location for a second cut to be made complementing measure 144 from the beginning. Panel **d** sets the module measure 165 and the larger 'half' measure 267 at a right angle, and uses the measure from end to end (another hypotenuse) to compute the third cut, after line 314. Panel **e** again uses the long 'half' measure φ and the module measure 1, this time φ being the length of the hypotenuse of a right triangle, yielding measure $\sqrt{\varphi}$ for the other side; it then transfers the proportional relation they entail to the short 'half' measure $3 - \varphi$ to

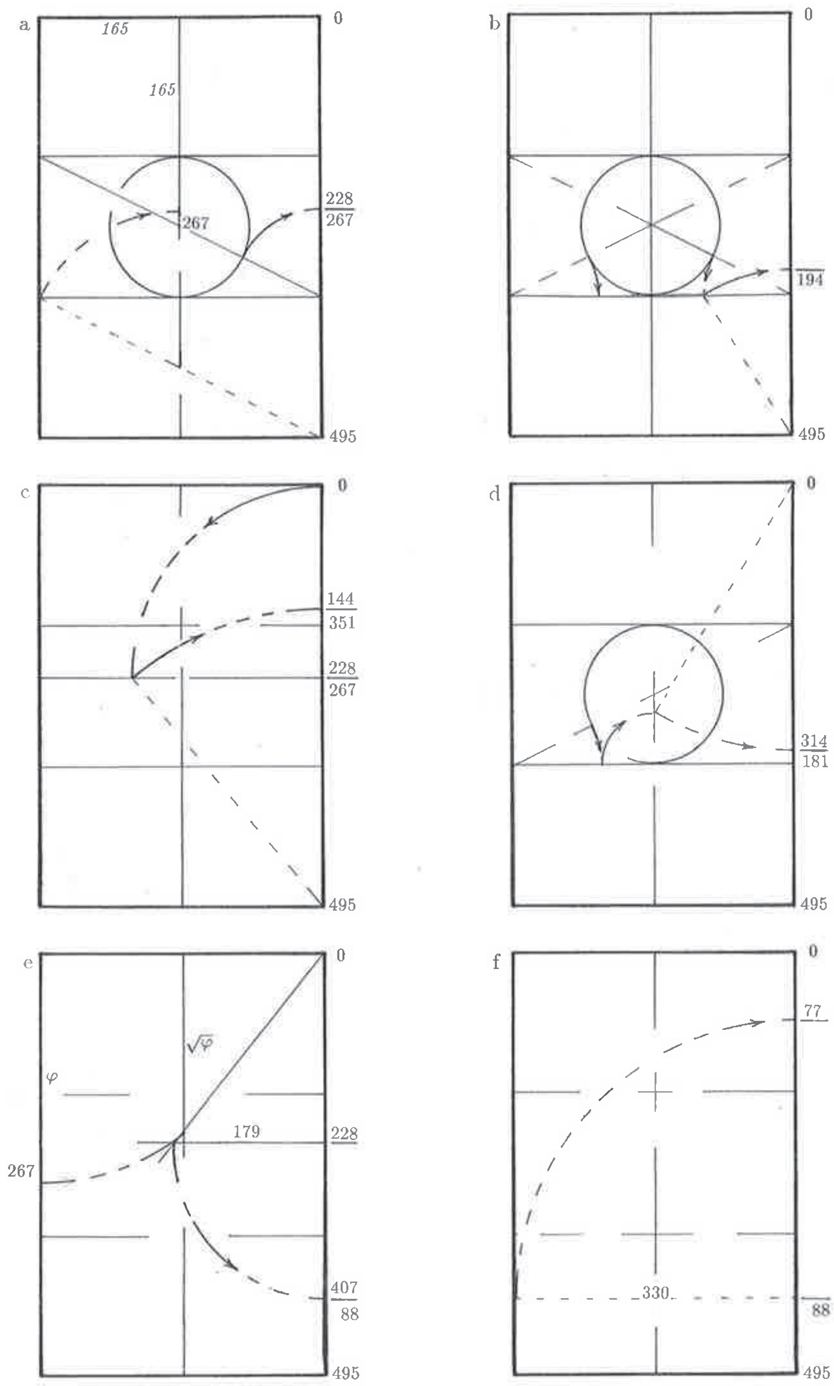


Fig. 4.20 Setting the section-lengths of *Christ II* by graphic computation.

generate 179, which is added to the earlier-set 228 for the total length 407 for the fourth cut. Panel **f** shows subtraction (graphically) of two modules, or 330, from the last-made cut, to mark a point after line 77.

With the overall length given, these five cuts divide the text into six parts with extensions accurately matching all the linecounts in the (restored) text.

The model also illustrates a point made earlier, that when main portions of this kind of formal plan are intact, a gap (the one-leaf lacuna, in this case), detected by independent analysis, can be confirmed and its linecount calculated.

One general observation should be appended. All panels except Panel **e** represent the simplest of graphic computations; the computation represented in Panel **e** is only one step more complex, yet that account 'feels' less right, even while it yields the correct measure. By its nature, $\sqrt{\varphi}$ is dependent for its derivation from φ, which in turn is dependent for its derivation on 1 and 2. Less simple in terms of manœuvres in graphic computation, but only by one step, and fully within the coherent scheme of computation, or layout method, of the rest of the form. Other Insular art plans employing the key ratio $\sqrt{\varphi}$ in their derivation are Trier Gospels tetramorph page and Echternach Gospels *imago aquile* page.

Guthlac A There are two gaps in the manuscript copy of this poem—a leaf is missing, and there is a small skip in copying: both are noted by modern editors, and the extent of missing text has been narrowly determined by best editorial procedures using codicology and comparisons to sources and parallels, where they are known. The next step, an analysis of the poem's form when the extent of the gaps is taken into account, requires attention to the lengths of the sections, both separately and in sequences. These are displayed in Table 4.7.

Table 4.7 Lengths of the sections in *Guthlac A*, reconstructed

Section	Number of lines in section groups								
[I]	92								
		169							
[II]	77		261						
		169		369					
[III]	92		277		472				
		200		380		598			
[IIII]	108[a]		303		506		690		
		211		429		598		794	
[V]	103[a]		337		521		702		891
		229		429		625		799	
[VI]	126		321		533		722		
		218		425		630			
[VII]	92[b]		322		522				
		196		419					
[VIII]	104		293						
		201							
[VIIII]	97								

[a] Lacuna of 69 lines.

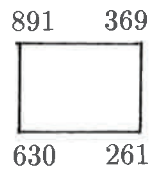
[b] Lacuna of 4 lines.

In the tabular display are four ways to represent a sample of equivalent ratios among count of metrical lines set by sectional divisions in the manuscript text of *Guthlac A*. Now develop the fourth of these models, parallel lines representing equivalence of ratios, as in Fig. 4.21 (the figure is a description, not an interpretation). Section-group lengths 891 : 794 :: 293 : 261 are shown in the central parallelogram (here a square). Graft onto this some additional pairs of equivalent

891 : 630 :: 369 : 261

$$\frac{891}{630} = \frac{369}{261} \text{ also } \frac{891}{369} = \frac{630}{261}$$

891	369
630	261



ratios of the text's section-group dimensions in linked parallelograms. Six conjoined parallelograms unite eight of the section-group dimensions. Each line-sum is linked to six other line-sums. For example:

891 : 794	is	equivalent to	293 : 261
891 : 337	"	"	690 : 261
891 : 630	"	"	369 : 261
891 : 690	"	"	337 : 261
891 : 369	"	"	630 : 261
891 : 293	"	"	794 : 261
. . .	"	"	. . .
794 : 337	"	"	690 : 293
794 : 369	"	"	630 : 293
. . .	"	"	. . .
369 : 337	"	"	690 : 630
. . .	"	"	. . .

(There is another proportional set not integrated into this model: 794 : 472 :: 722 : 429.)

Beyond this graphic linking of six pairs of equivalent ratios is another aspect of the relations among sectional lengths: all these ratios so far listed—and some others, too—have equivalences in ratios incorporating various roots of two. The second root is the easiest to derive, being found in such simple figures as the diagonal of a square with side = 1 or in the hypotenuse of an isosceles right triangle with side = 1 (as in Figs. 3.10 through 3.13). In such configurations,

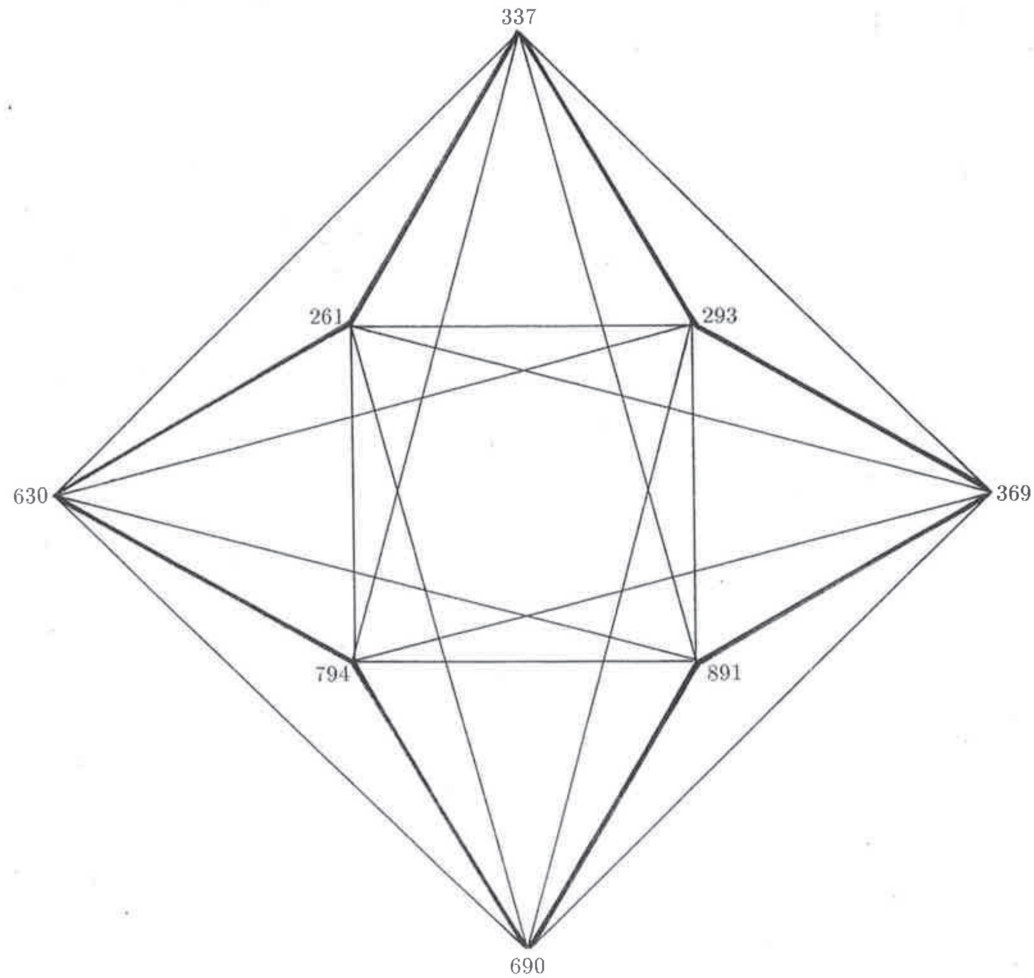


Fig. 4.21 Six parallelograms uniting eight section-group lengths in *Guthlac A*.

from the measure 630 can be derived 891 and from 261 (the complement of 630) can be derived 369; its complement in the text's line-sums is 522 (or 2 × 261). Together these measures form three equivalent ratios, which in turn are equivalent to the simplest ratio formed with $\sqrt{2}$, thus:

$$\frac{891}{630} = \frac{522}{369} = \frac{369}{261} = \sqrt{2} : 1$$

Fig. 4.22 illustrates these patterns of linecounts as graphic computation. When these are arranged differently, another dimension will fit into a related series of equivalences:

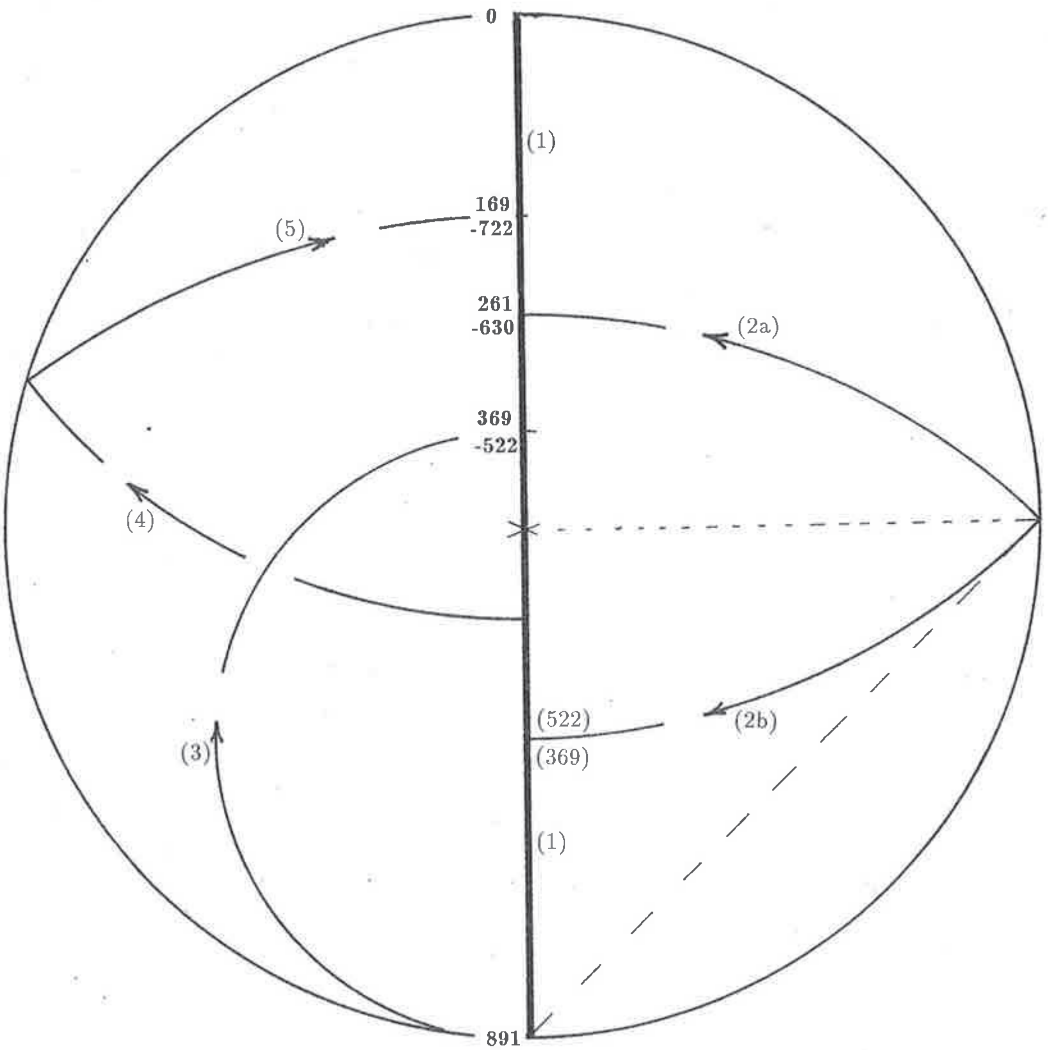


Fig. 4.22 Relations among some section-group lengths in *Guthlac A*.

$$\frac{891}{369} = \frac{630}{261} = \frac{261}{108} = (\sqrt{2} + 1) : 1$$

Other pairs of measures of the poem's sections form ratios with equivalence to ratios incorporating the fourth root of two, which can be developed by constructing $\sqrt{2}$, and then the second root of that. It is found in two pairs of section-length measures:

$$\frac{794}{472} = \frac{722}{429} = \sqrt[4]{2} : 2$$

The same root of two enters into further equivalences in these dimensions:

$$\frac{201}{169} = \frac{92}{77} \quad = \sqrt[4]{2} : 1$$

The latter set of equivalences is also illustrated in Fig. 4.22.

The second and fourth (and eighth ...) roots of two can be constructed geometrically. The second root is derived directly; the fourth root is the second root of the second root; the eighth root will be the second root of the fourth root, and so on. There is no way, however, to derive the third root of two by geometric construction, hence no way to construct the sixth root, the twelfth root, and others strictly by geometrical methods. That limitation notwithstanding, some sectional lengths of *Guthlac A* combine in ratios incorporating both third and sixth roots of two:

$$\frac{794}{630} = \frac{369}{293} = \frac{1000}{794} \quad = \sqrt[3]{2} : 1$$

$$\frac{891}{794} = \frac{293}{261} \quad = \sqrt[6]{2} : 1$$

How this unusual set of ratios was created needs exploring. If one had the value $\sqrt[3]{2}$ somehow, expressed in lineal form in relation to 1, it would be possible to construct $\sqrt[6]{2}$ from it: the second root of $\sqrt[3]{2}$ is $\sqrt[6]{2}$, and it can be found by the same procedures with which the fourth root of two can be derived from the second root. But there is no reason to suppose that the ratios were derived in that order, particularly because the ones equivalent to $\sqrt[3]{2}$ do not include the full length of the poem; for both the verse-text forms and the two-dimensionl forms, smaller dimensions normally would be derived from larger ones, and not the other way about.

Instead, these roots of two probably were developed in the plan by setting first $\sqrt[6]{2}$. When it interacted with $\sqrt{2}$, as $\sqrt{2} : \sqrt[6]{2}$, it produced the ratio $\sqrt[3]{2} : 1$, for example. A best guess seems to be that the form of *Guthlac A* began with the empirical formula $1,000 : 891$, which is a very close approximation to $\sqrt[6]{2}$ (to within one ten-thousandth). It would be set down as the given, and not derived by geometrical construction. Its mnemonics may well have been $10 \times 100 : 9 \times 99$. It proves to be a ratio from which the whole design can evolve: from the frame with these dimensions can be derived the formal plan, by geometrical construction. One attempt to replicate the method is included in *The Earliest Irish and English Bookarts*, Fig. 11-1 (p. 166); there are probably simpler methods of computation that yield the full dumensions of the verse text.

Further reason to infer that it must have been 1,000 : 891 (rather than, say 891 : 794) originally embodying the sixth root of two comes from several sources, the most obvious being the numbers related to 1,000 in the following equivalences; these numbers are the measures in linecount of fundamental divisions of the poetic text:

$$\sqrt[6]{2} = 1,000 : 891, \quad \sqrt[3]{2} = 1,000 : 794, \quad \sqrt[3]{2^2} = 1,000 : 630$$

The measure 293, which is $(1 - \frac{1}{\sqrt{2}}) \times 1,000$, is also significant for being the length of the last three sections of the poem. Probably the most persuasive confirmation lies in the complete efficiency of derivation of the design from a frame 1000×891, whereas any other ratio incorporating integers from the poem's sectional lengths either will not yield all the sectional divisions, or will not do so without very much more complicated manœuvres.

The extent to which the plan of *Guthlac A* was intended to exploit the roots of two in building commodular relations into the form is not easy to assess because there are still more equivalent ratios among the measures of the sectional groups. Many of these equivalents may be only entailed by others and, while consequent, may be only incidental to the design. For instance, some of the measures complementing ones already shown in repeating ratios will form still more repeating ratios. Thus $794 - 261 = 533$ and $891 - 293 = 598$; so

$$\frac{598}{533} = \frac{891}{794} = \frac{293}{261} = \sqrt[6]{2}$$

Again, $630 - 293 = 337$ and $794 - 369 = 425$; so

$$\frac{425}{337} = \frac{794}{630} = \frac{369}{293} = \sqrt[3]{2}$$

Even these do not complete the list. But even without any further equivalent ratios, enough have now been pointed out to put beyond question the presence and the ingenuity of an integration by proportion for the quantitative divisions in the reconstructed form of the text of *Guthlac A*, and to identify the key to it in the values of the first, second, third, fourth, and sixth roots of two. Some of the intricate, elegant pattern of this text's form is implicit in the incomplete copy that we have, and only the one reconstruction that has been presented here for the lengths of the lacunae allows such a thoroughgoing integration of proportion for any other parts of the text.

5. Some Antecedents and Follow-Ons

The essential coherent geometry of Insular designing did not come out of nowhere, nor did it vanish at an assigned date. That will be evident from analysis of a few examples, some earlier, some later than the main period of Insular art.

A La Tène Bronze Disc and an Irish High Cross

First, a pair of objects, the earlier one a bronze disc, the Cuperly Disc, Marne, early Iron Age, the later one a stone high cross from just beyond the end of the first Christian millenium, the Kilfenora west cross. The disc has a diameter of about 110 mm (c. $4\frac{1}{4}$ in.), while the Kilfenora Cross is about 450 cm (about 175 in.) high, and more if the hidden part of the high cross is considered. One may be only secular and ornamental, while the other is obviously religious and symbolic. The earlier one cannot be offered as the source of the other, or as a model, any more than the later one can be said to be a descendent or even be 'reminiscent of' the earlier. Size, material, purpose are not relevant, however, to understanding the typological relations of the two designs. Their respective venues, on the other hand, predict that they may well be related through a continuous tradition of designing. The two designs share DNA that defines their genus, so to speak, while they obviously belong to different species. Or setting aside that metaphor, they share common structural characteristics that are also present in the best of Insular designing which the one precedes, the other follows.

The plans of both can be created accurately with only compass and straightedge, without need for a measuring scale of any kind. Just how much they have in common will become evident when they are reconstructed in tandem. In fact, initial steps in drawing up models of the two pieces can be identical. Both are developed from concentric and commensurate circle and square divided into quadrants (cf. Fig. 1.1f–g).

Cuperly Disc, Marne Fig. 5.1 illustrates the construction of the disc's underlying plan.

(a) Describe a circle A (1), and run a straight line through the centrepoint of the circle (2), dividing the circle into halves. Set a second diameter perpendicular to the first one (3), dividing the circle into quadrants. Set third and fourth diameters dividing the quadrants equally (4, 5); the method used here also marks corners of a square commensurate with circle A. (This fourth manœuvre will be used for the cross design, but is not needed for the disc design.)

(b) Then copy the measure of the chord of a quadrant arc to mark a point

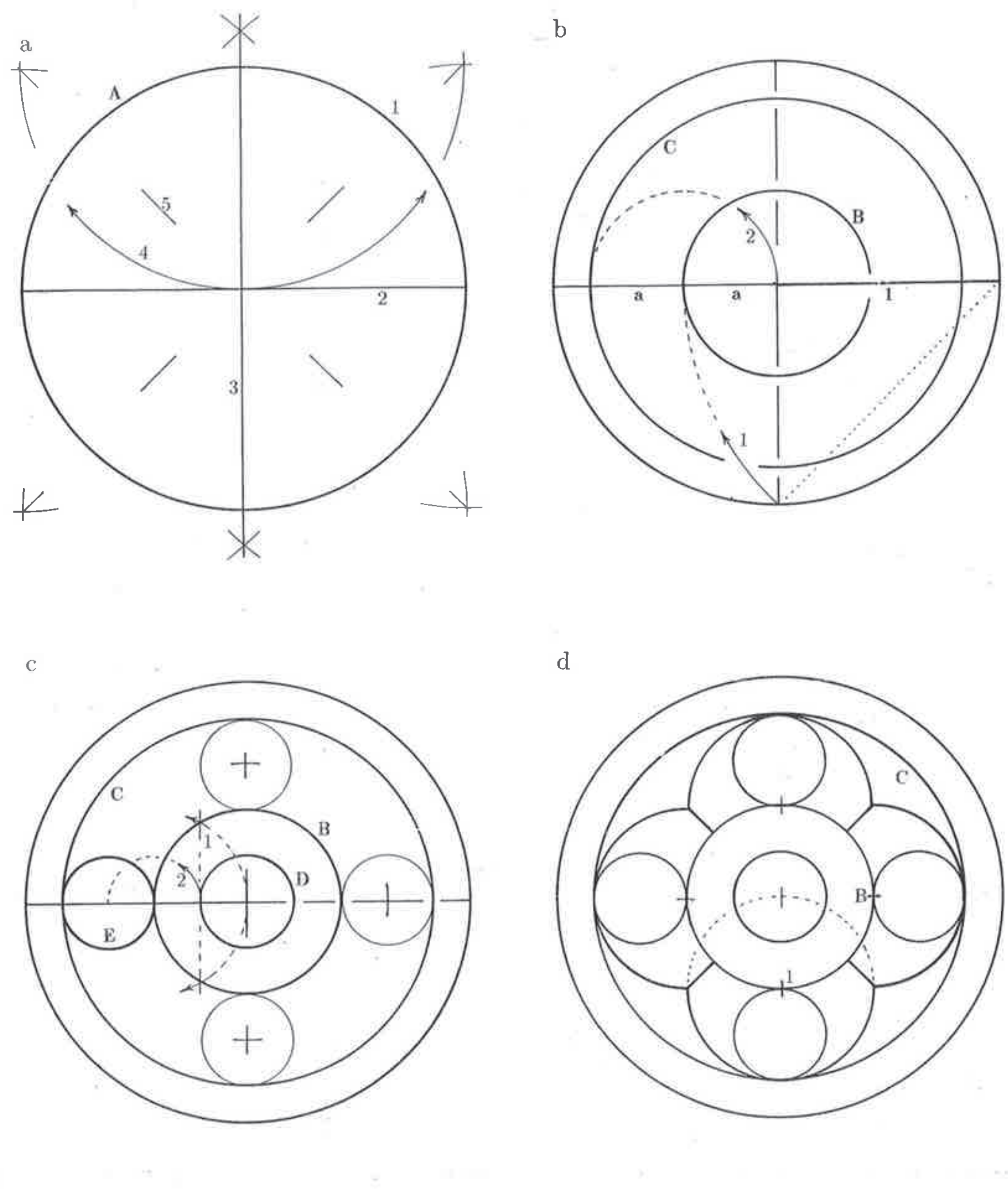


Fig. 5.1 Construction of underlying scheme of Cuperly Disc.

along a diameter (1), and describe a new circle *B* concentric to the original one, with radius to the measure just marked. Double the radius of circle *B* (2) for a new circle *C* concentric with the two others. At this point the derivations of stone cross and bronze disc will diverge.

(c) Now halve the radius of circle *B* (1), and with this new measure as radius describe circle *D* concentric with the three others. Copy that last radial measure beyond circle B along a diameter (2), and describe circle E with that same radial measure, tangent to circles B and C; repeat circle E along the vertical and horizontal axes.

(d) Next, use the intersections of circle B and the horizontal and vertical axes as centres for circles both tangent to circle C and passing through the centre of the plan (1); segments are used in the Y-shape pattern in each quadrant. (The circles could have been sketched in carrying out Step 2 in Fig. 5.1b.)

The initial circle A sets a given measure as 1 (the radius) and doubles it as 2 (the diameter). The next measure to be set (Fig. 5.1b) divides the radius unequally, with the two parts in a 'geometrical' ratio. To make the coherence of the plan easier to grasp, the shorter segment is designated as a, which is the 'geometrical' and irrational quantity $\sqrt{2}-1$. Now, ratios combining the three quantities $1, 2, a$ inform the basic layout of this disc (and the cross). They provide a three-member alphabet, as it were, in which can be written the code of this design as well as that of the Kilfenora cross. Some of the coherence of this plan's proportional structure shows clearly in Table 5.1, especially in the column giving the basic dimensions in terms of the primitive quantities 1, 2, a.

Table 5.1 Primary dimensions of the Cuperly Disc.

Radial Measure	If a stands for $\sqrt{2}-1$	Approximation
Circle A = 1	1	1.0000
Circle B = $\sqrt{2}-1$	a	0.4142
Circle C = $2(\sqrt{2}-1)$	$2a$	0.8284
Circles D, E = $\frac{\sqrt{2}-1}{2}$	$\frac{1}{2}a$	0.2071

The Cuperly Disc also has an open-work ring surrounding the central disc—Circle D—generated from crossing patterns of twenty-six arcs. This is a very interesting development within the basic plan of the disc, which will be set aside here. (How it evolves from the basic plan is shown in 'The Ancestry of "Coherent Geometry" in Insular Art,' *JRSAI*, 134 (2004)).

The Kilfenora West Cross The plan of this cross is informed by the same method, the same rules, the same initial steps, so that we can pick up after the initial stage (a) of construction as in Fig. 5.2.

(b) Copy (1) the measure of the chord of a quadrant arc to mark a point along a diameter, and describe a new circle B concentric with the original one, with radius set by the point just marked. Again, call that radial measure a. Double the radius of circle B (2) for a new circle C concentric with the two others. Halve the radius of circle A (3) for a new circle D concentric with the three others.

Now to superimpose a solid cross pattern on the circular plan:

(c) From each end of the first two diameters, (1) copy the radius length of circle B (i.e., measure a) to mark points on either side along the sides of the implicit square. Connect these points of intersection in pairs with parallel lines to set the preliminary shape of the cross. Then (2) plot the extensions of the arms and the upper portion of the shaft of the cross beyond circle A by as much as circle A exceeds circle C in radial measure.

(d) To form re-entrant arcs for this cross plan, locate points midway between circles C and A along the diagonal diameters (1); an easy way is to combine (end to end) the radii of circles B and D. Using these points as centres, describe arcs extending to the intersections of the preliminary cross with the outer circle A.

The design in this simple construction accurately matches the shape of the cross (west face) everywhere that the stonework is four-way symmetrical. Its geometric coherence in modern notation is shown in Table 5.2, with all measures given again as 1, 2, a in one combination or another.

Table 5.2 The Kilfenora West Cross.

Radial Measure	If a stands for $\sqrt{2}-1$	Approximation
Circle A $= 1$	1	1.0000
Circle B $= \sqrt{2}-1$	a	0.4142
Circle C $= 2(\sqrt{2}-1)$	$2a$	0.8284
Circles D $= \frac{1}{2}$	$\frac{1}{2}$	0.5000
Arm extension $= 2(2-\sqrt{2})$	$2(1-a)$	1.1716
(from centreline)		
Shaft width from centreline		
Top of circle $= \sqrt{2}-1$	a	0.4142
Bottom of circle $= 1-(\sqrt{2}-1)$	$1-a$	0.5858

From this preliminary—or underlying—symmetrical form, the design then introduces asymmetrical elements through taper of vertical lines of the cross.

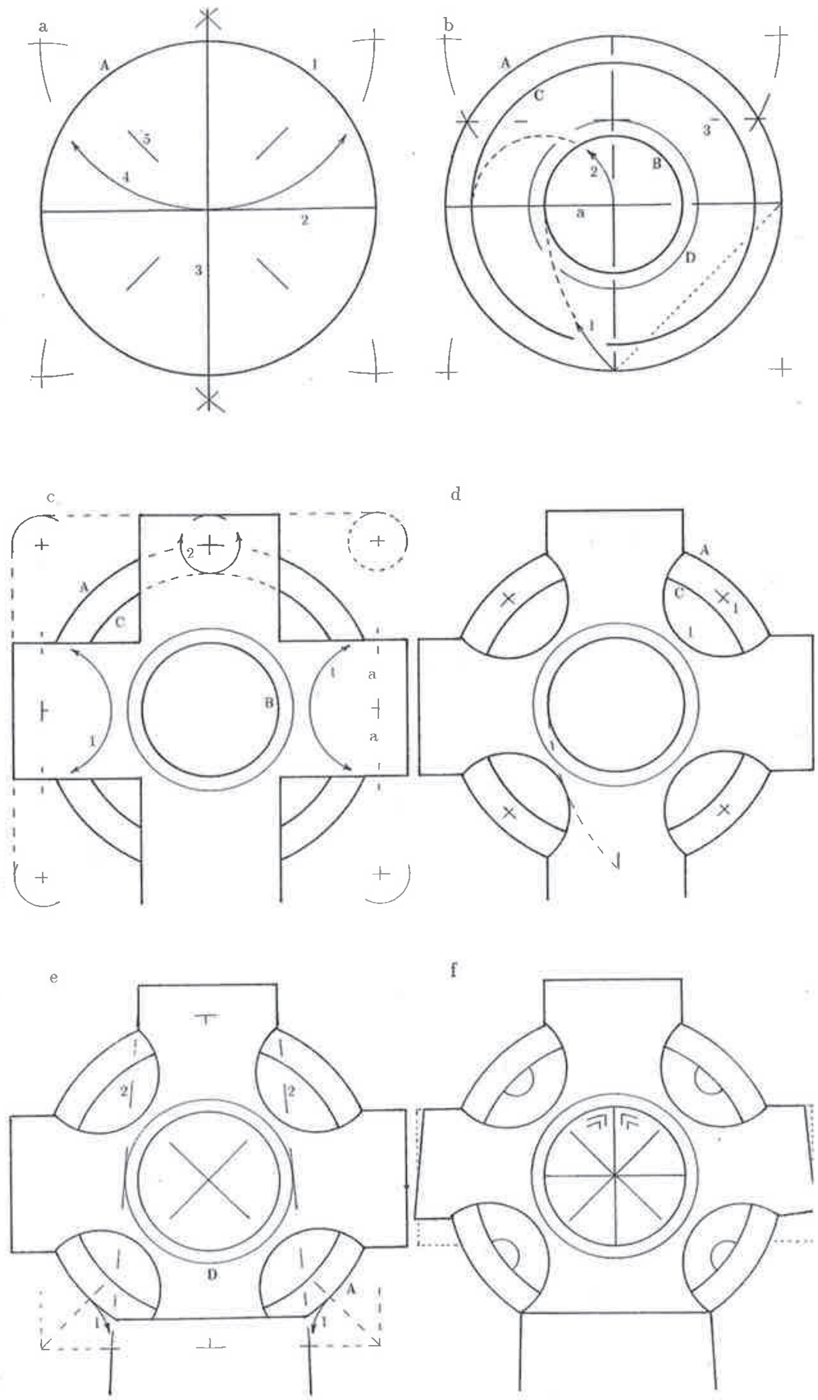


Fig. 5.2 Construction of underlying scheme of Kilfenora Cross.

It should be observed, before proceeding with reconstruction of the form, that the decoration of the shaft on which the ringed cross rests has its own formal scheme consonant with that of the key element that it supports. We cannot know whether the tall supporting pillar was designed by the same person who set out the design for the circle-and-cross that sits above it. Whoever designed it, however, must have understood the design of the ringed cross it was created to support, as the decorative scheme makes clear. The long rectangle enclosing assorted ornament in interlace represents yet another 2 : 1 relation, in its being twice the diameter measure of the ring enclosing the cross. More than that, coming down the rectangle are double layers of interlace patterns. The first two fill one-fourth of the vertical measure. The next two fill another one-fourth. Below that begins another pair, before the decorative scheme was left incomplete.

And yet, between the measure and half-measure of the ring (its diameter and radius) and the layered measures of the supporting pillar there is no meld: the top of the pillar intrudes into the ring to about the path of its inner circle. This may give added stability to the join of shaft and ring-and-cross, but there is no practical need for that. There may be aesthetic need for it, of course, if the shaft is to be both long and tapered: the path of the tapering leads to the upper extension of the cross on one side, though it misses slightly on the other. In any case, while the module for the ring design and the pillar decoration have been made the same, it is not continuous between the two elements of the cross, and its uses in the two parts are not linked in any apparent way.

Rather than reason from putative aesthetic requirements, though, it may be better to pursue the geometrical underpinning of the plan developed first in full symmetry, and see whether the inverse asymmetry may be part of that plan. It will turn out that the spread of the cross-shaft within the ring follows a simple and continuous derivation of dimensions consistent with the rest of the plan.

(e) From each of the lower two corners of the underlying square, copy the measure from there to circle A to mark points along the lower side of the square (1). Then mark the points where the path between each of these points and its corresponding point (2) crosses the lower portion of circle A. These last points will mark the top corners of the supporting pillar, matching the lower limits of the encircled portion of the cross.

(f) The top corners of the pillar also mark the lower ends of the re-entrant arcs in the lower half of the plan. Keeping the compass setting of the preliminary plan requires moving the centres of the arcs upwards around the ring; that in turn moves the upper ends of these arcs upwards, which in turn moves the lower edges of the cross-arms upward. (The upper edges remain as originally plotted.)

What occurred in the process of designing the Kilfenora west cross may have been something like this. First came decision on the scale of the cross and its style. In this instance the scale was to be quite large, at least for its loft, with a tall supporting pillar to stand as a long lower shaft of the cross. The style was to include a tapered shaft. (The tapering would be somewhat like the Killamery Cross, Co. Kilkenny, or Muirdach's Cross, Co. Louth. The massiveness and consequent need for tapering of the Kilfenora cross shaft would not be necessary, of course, as is shown by the tall crosses at Moone, Co. Kildare, and at Monasterboice, Co. Louth, both of which have slender, untapered shafts.)

The design was to proceed in full conformity with the traditions of design for sculptured crosses, which had been used as well for page illuminations in Gospels codices and for some of the finest ornamental metalwork—that is, it would evolve as an accumulation of coherent derivations carried out with compass and straightedge. It must have been undertaken in the manner of the derivation of what above was called the preliminary plan. Every aspect of the plan at this stage was linked to all others by a tactile procedure that produced simple proportional relations among all the lines shaping the ringed cross. That assumption seems to be validated by the extensive symmetry of the cross-design that has already been described.

Then came the necessity for decisions on melding the ringed cross and its tall and tapered supporting column. At this point there was a critical choice in the creation of the design: whether to make the ringed portion symmetrical inversely as well as laterally, to be set atop a tapered extension of the cross's shaft? Or to introduce taper into the ringed part of the sculpture? The choice was for the latter of these options, possibly as an attempt to introduce a primitive form of perspective, enhancing the perception of height for a cross-head atop a very tall pillar.

And with that choice came another, how to set the slant for the taper? The designer chose to follow the mathematical logic of his plan in its plainest form. The same measure by which the top of the cross extended on either side of the centreline would be the distance which the bottom of the cross came from either corner of the underlying square.

It may be useful to turn to itemize some of the coherence of the plan's proportional structure, beginning with repeating elements. The relation between the sizes of circles A and C, comprising the ring, is the same as the relation between the diameters of circles D and B. These are the pairs of concentric circles of the plan that form the rings: circle D was derived as halving half the measure of circle A, while circle C was derived as halving twice the measure of circle B. Expressed yet another way, $\frac{A}{D} = \frac{C}{B}$. That being the case, it will also be the case that $\frac{A}{C} = \frac{D}{B}$.

And what was the relation of A to C, and of D to B? Expressed in modern notation, the radius of circle B is $\sqrt{2} - 1$ in relation to the radius of circle A: the measure of the chord is $\sqrt{2}$ in relation to the radius. Now, circle C has twice the radius of circle B, which is to say, $2(\sqrt{2} - 1)$. In relation to the radius of circle A, then, the ratio between circles C and A is $2(\sqrt{2} - 1) : 1$. (The decimal approximation is 0.8284.) The relation of circles D and B, as noted, is the same.

This same relation of measures occurs as well in these elements: the width (horizontal measure) of the upper portion of the cross at the top of the ring and the width (the diameter) of circle D enclosing the central decorative scheme; and again with that latter measure and the width (horizonal measure) of the pillar at the bottom of the ring (now cut off). These recurrences of a single ratio of measures among the fundamental parts of the design witness the integrity—the coherence—of the geometry of its design. In sum, the whole design of this cross—apart from its surface decoration—proceeds in a relentless chain of geometrical derivations of a most basic kind.

Taken together, the imbalances, the distortions, and the symmetries of this cross offer a window on development of high cross design for its concepts and pragmatics together. Many other crosses have designs fully answerable to schemes of coherent geometrical construction. Mainly they are symmetrical throughout when allowance is made, where needed, for damage or erosion. The Kilfenora west cross is exceptional, its aberrations providing an additional view of designing that embraces *both* the symmetries and the aberrations in a cross form.

The creator of this cross plan proceeded in the customary way in laying out a ringed cross, to be perforated by its re-entrant arcs. He then proceeded to introduce tapering of the cross shaft *within* the encircled portion of the cross. He held to the regimen of geometrical coherence in setting the taper. That done, he had to accommodate the re-entrant arcs to the unparallel lines of the cross shaft. And that entailed moving the cylinder-like devices inside the ring away from their true diagonal positions. By *not* departing from the mathematical regimen in drawing the re-entrant arcs, he distorted the cross-arms, as explained above, along with the diagonal symmetry of the re-entrant arcs. There was no way of eliminating the distortion at this stage, and it could be reduced only by ad hoc adjustments—by fudging the disposition of the re-entrant arcs. The designer did not do this, although he altered the ends of the cross-arms by slanting them approximately parallel to the taper of the upright, perhaps as further attempt to enhance the sense of loftiness of the cross-head within the ring.

The typological relations between the designs of the Cuperly disc and the Kilfenora cross are not recondite. They lie at either end of a continuing tradition of creating designs by accumulated geometry. From the earlier to the later forms only two variant developments were needed. The first was achieved by superimposing a cross pattern on the circular plan: all it takes in this instance is copying one dimension—the radius measure of the 'geometric' inner circle—at the ends of the horizontal and vertical axes, setting the basic dimensions for the arms and the shaft of the cross. This variation, illustrated in the Kilfenora derivation, produces a plan that resembles the very early St Kevin's Cross, for example. The second is the development of re-entrant arcs that are such a common feature of high cross forms. It is achieved by reversing the direction of arcs around the centre and moving their centres to the outer ring, making them incursive rather than excursive.

The common strings of DNA in these designs have not been recognised even in mainline studies of Insular art. A review of diagnostic features of Insular art by George and Isabel Henderson, in *The Art of the Picts* (2004), omits this aspect of that tradition in art. They say: Insular art is 'essentially abstract and decorative, consisting of a set series of linear rhythmic motifs and patterns, many of them originating in the visual traditions of separate ethnic groups'—spirals, scrolls, strap- and cord-work, ... 'various zigzag motifs, formed into regular step, key and fret patterns.' It is the 'combination of a number of previously unconnected motifs' that is 'the hallmark of the style,' they say (p. 15). There can be no quarrel with any of this characterisation of Insular style, except for one crucial omission. In all the best work in the Insular tradition, the 'combination of previously unconnected motifs' takes place within plans of the kind just illustrated: plans with rigorous and thorough apportionment of areas according to a coherent scheme. This typically coherent geometry of carpet pages, high crosses, decorative metalwork is as much an essential of Insular art as is any of the decorative motifs and patterns. It has an unbroken history begun in earlier Irish art, in still earlier Celtic art, and in a lingering Irish tradition of creating form.

An Early Bronze Disc and a Late Processional Cross

Monasterevin Disc NMI W.3 is another antecedent to Insular designing, a bronze disc dated to first or second century A.D., named for its find-place in Co. Kildare. This disc is one of seven that are known.* It is large, in the range of eleven inches* in diameter, hammered into relief from the reverse side (by *repoussé*), with curvilinear ornament in the La Tène style. (It is dated conventionally by style.) The form can be replicated as in Fig. 5.3.

(a) Begin with a circle A quartered (cf. Fig. 1d–e), and plot first the 'eccentrically placed circular area.' As shown in the upper segment: sketch the chord of a quadrant **c**, copy the circle's radius measure along it (1), then copy the remaining measure a along the vertical diameter (2); double the measure a along the diameter (3). This locates the outer limit of circle B.

Then as in the lower segment of the figure, follow the same procedure inverted. This locates the centre of the eccentric circle B.

(b) Another method is simpler and also sets both circle B and circle C. As in the upper segment, copy a quadrant chord measure along the vertical diameter to set measure b (1), then double it along that diameter (2) to locate the centre for circles B and C. As in the lower segment, copy a quadrant chord measure along the vertical diameter (1), then double it along that diameter (2), setting the radial measure for circles C and B in turn.

In modern notation: if the radius of circle A is stipulated as 1 (diameter as 2), the locus of the centre is $2b$ from the top, and it is $2a$ from the bottom. The length of b is $2 - \sqrt{2}$, and the length a is $\sqrt{2} - 1$.

(c) Set the fixed point of the compass at one end of the vertical diameter (upper end in this orientation), and with the compass set equal to the radius of the circle A, mark two points on the circumference (dividing the circle into thirds), to be used in Fig. 5.3d.

Then from the centre of circles B/C copy the measure b below along the vertical diameter to locate the centre of circle D. (A derivation of measure b in place, as shown, is simple to execute, less simple to spell out.) Then copy the radius of circle B below its lowest point—i.e., double the radius below its centre—to locate a point on the circumference of circle D. Draw circle D. This

* 'Monasterevin-type' is the conventional denomination for these discs that are grouped together for design features which they all share, uniquely. This is appropriate, strictly speaking, for the five whose find-place is unrecorded, reserving 'Monasterevin discs' for the two found in the vicinity of Monasterevin, Co. Kildare. Six of these discs are now in the National Museum of Ireland, one in the British Museum. *(28 cm)

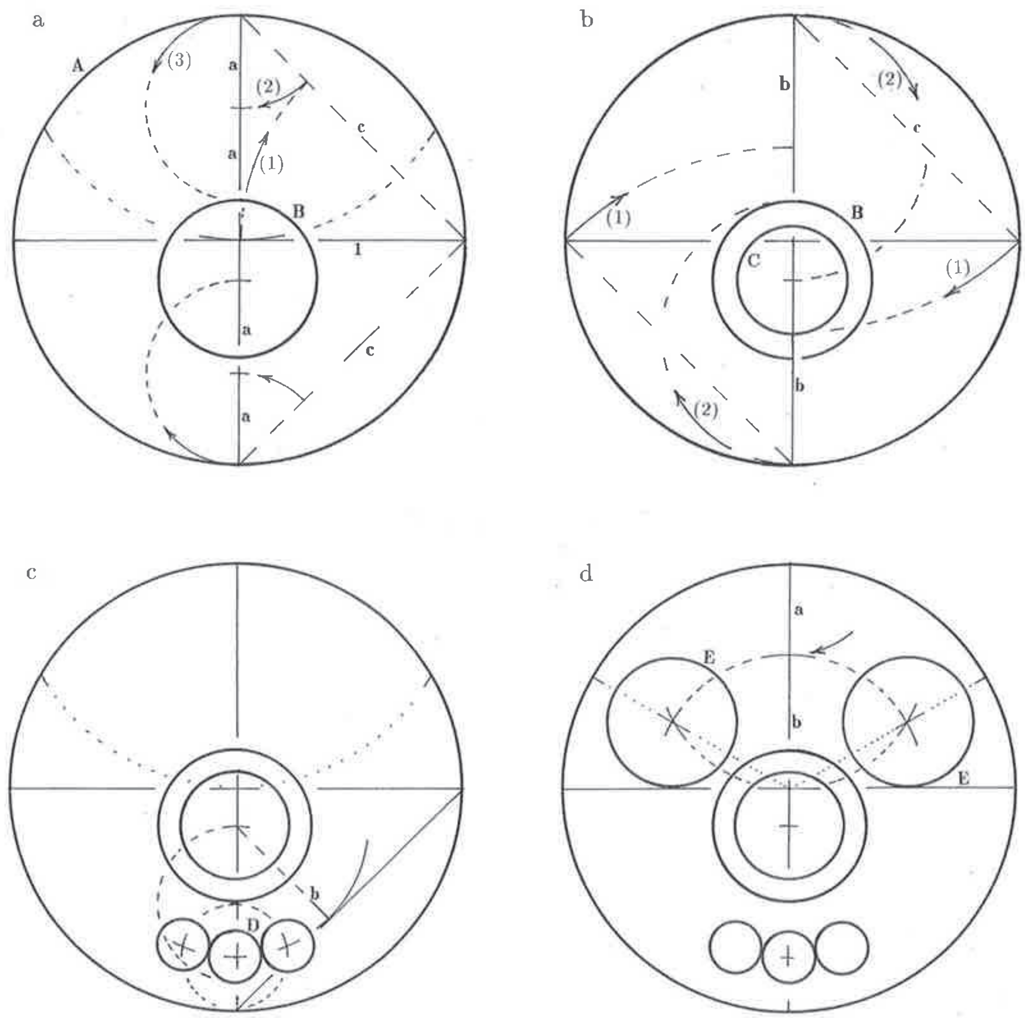


Fig. 5.3 Replicating the design of Monasterevin disc NMI W.3.

gives iteration of the diameter measure of circle *B* and the measure from its centre to the lower extent of circle *D*. Further iteration can be traced through recurrences of measure *b*, and in the listing of measures in Table 5.3.

(d) Points marked on the circle *A* dividing it into thirds were set in Fig. 5.3c. Sketch radii of circle *A* to these points just marked (10 o'clock and 2 o'clock positions): where these intersect an arc concentric with circle *A*, with radial measure *b*, will be the centres of circles *E*. (This procedure also lays down lines useful for locating the centres of the curvilinear devices just inside the perimeter of the plan.)

Table 5.3 Dimensions of NMI W.3 disc

Circle A.	Radius:	1
	Diameter:	2
Circle B.	Top:	$2a$ from top or $2b$ from bottom of circle A
	Centre:	$2a$ from bottom or $2b$ from top of circle A
Circle C.	Centre:	$2a$ from bottom or $2b$ from top of circle A
	Bottom:	c from top of circle A or b from bottom of circle A
Circle D.	Centre:	b below centre of circles B and C; b is also the measure from centres of circles B/C to chord of lower quadrant of circle A
	Bottom:	diameter of circle B below centre of circle B
Circle E.	Centre:	intersection of two arcs with radius b, one with centre at centre of circle A, the other with centre at b above centre of circle A (or, intersection of radius of $\frac{1}{3}$ of circle A with arc having centre of circle A and radius b)
	Radius:	$\frac{1}{2}b$ (its centre to (horizontal) midline of circle A)

An alternate method is this (the designer may well have used both methods). From the centre of circle A sketch an arc in the upper half having radius with measure b; from the point where that arc intersects the vertical axis, sketch another arc also having radius with measure b: where these two arcs intersect will be the centres of the pair of circles E.

Draw the pair of circles E tangent to the (horizontal) midline of the plan.

The centres of the spirals on either side of circle D are along an arc with the same radius and centre that plots the centre of circle D.

Table 5.3 lists the principal measures within the plan, all expressed in terms of 1, 2, a, b, c; these reduce to 1, 2, and $\sqrt{2}$ in their various elementary combinations: $c = \sqrt{2}$, $b = 2 - \sqrt{2}$, $a = \sqrt{2} - 1$.

The Cross of Cong, in the NMI,* is an elegant piece of early twelfth century Irish ecclesiastical art, later than the academic period 'Insular art,' yet very much a follow-on of that tradition. The form of this cross embodies another design drawn up with careful use of compass and straight-edge, and carefully conceived as interlocking measures and ratios among them in a coherent geometry. Planning of the cross and the layout of its decoration of course had to precede its complex process of manufacture in wood, metal and ornament, and both precede the aesthetic sense of admiration which the finished piece generates. Analysis of its design will be easier to track from a schematic plan for reference: see Fig. 5.4.

The cusps C mark division of the members of the cross, two segments for each arm and the upper extension, three segments for the lower extension. Then the simple arithmetical divisions of these segments appears as regular partitioning into threes, marked by studs S, as well as by the balanced interlace fillers in the areas which the studs divide. The count of these divisions is proportional to that of the divisions marked by the cusps—six for each arm and the upper member, nine for the lower member. Segments are 2 : 3, divisions 6 : 9.

To some extent—in the extremities of lower shaft and arms—the divisions within these segments appear to have equal lengths, with the overall measure of the outer segment of the arms and each of the outer two segments of the lower element also being equal. The ostensive equalities of count and proportion of measures, however, do not extend inwards along the arms, nor upwards from the cross's centre. The uppermost segment of the cross has a quite different (somewhat larger) measure. Also, the cusps at the mid-arm positions are not at half the length of the arms. Neither are the cusps of the upper segment of the upright placed at measures equal to those of the arms and lower segment of the upright. Only this much attention to detail will show that the form of this cross is anomalous with respect to its conspicuous arithmetical aspects: the numerical simplicity in counting divisions of the limbs, and even in segments of those limbs, is not present in the full dimensions of the members of the cross. In short, these ostensible regular aspects of the design of the Cross of Cong represent only a partial (and overly simple) perception of the extensional aspects of this remarkable object.

Begin by considering the most basic of formal aspects, the dimensions of the cross—the length and shapes of its members—in relational terms, the proportions and the patterns among them. The primary measures are the ones along the centreline of the upright and the midline of the arms. They belong to the cross proper, excluding the device at its base to attach it to a crosier, and they leave

* National Museum of Ireland

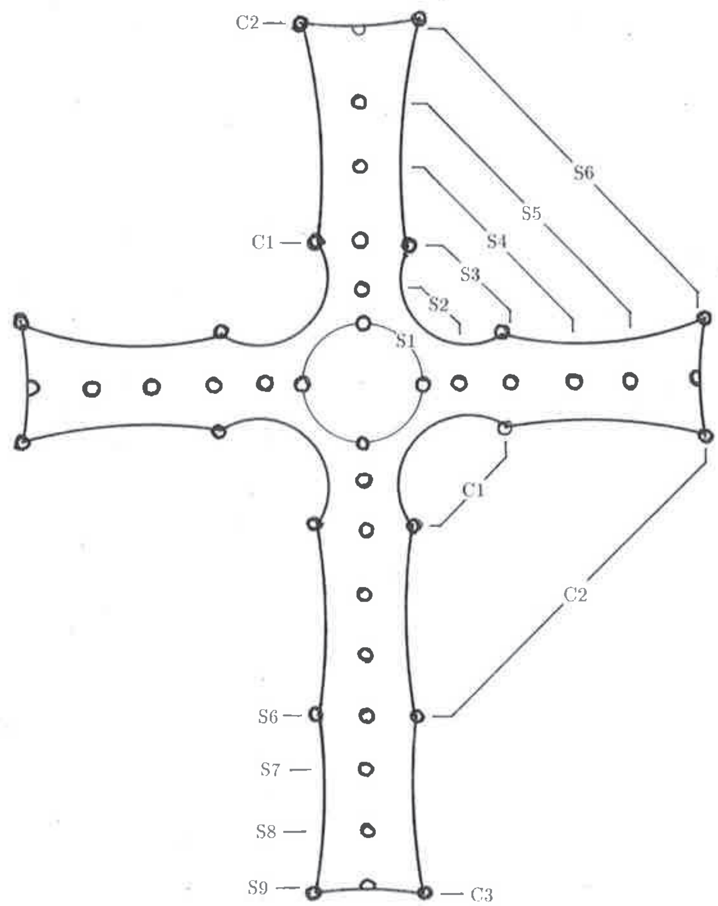


Fig. 5.4 Schematic plan of Cross of Cong.

aside the arcs at the four extremities with their attached circular ornaments. The arms are equal (or virtually so), while the shaft and upper extension of the cross are distinctly unequal, and both of them unequal to the arm extensions.

Two methods of setting these primary dimensions are illustrated in Fig. 5.5. Both use height for the cross as the given measure.

(a) As in Fig. 5.5, top, begin with a square divided into four equal squares (cf. Fig. 1f–g). Copy (1) length of diagonal of an upper quarter-square along the outer sides of the lower quadrants. Copy then (2) the complementary measure to mark points along the sides directly above these points (doubling the measure). A line (3) connecting these points will be the midline of the arms.*

* Another simple way to develop this same configuration (not illustrated) is this. The diag-

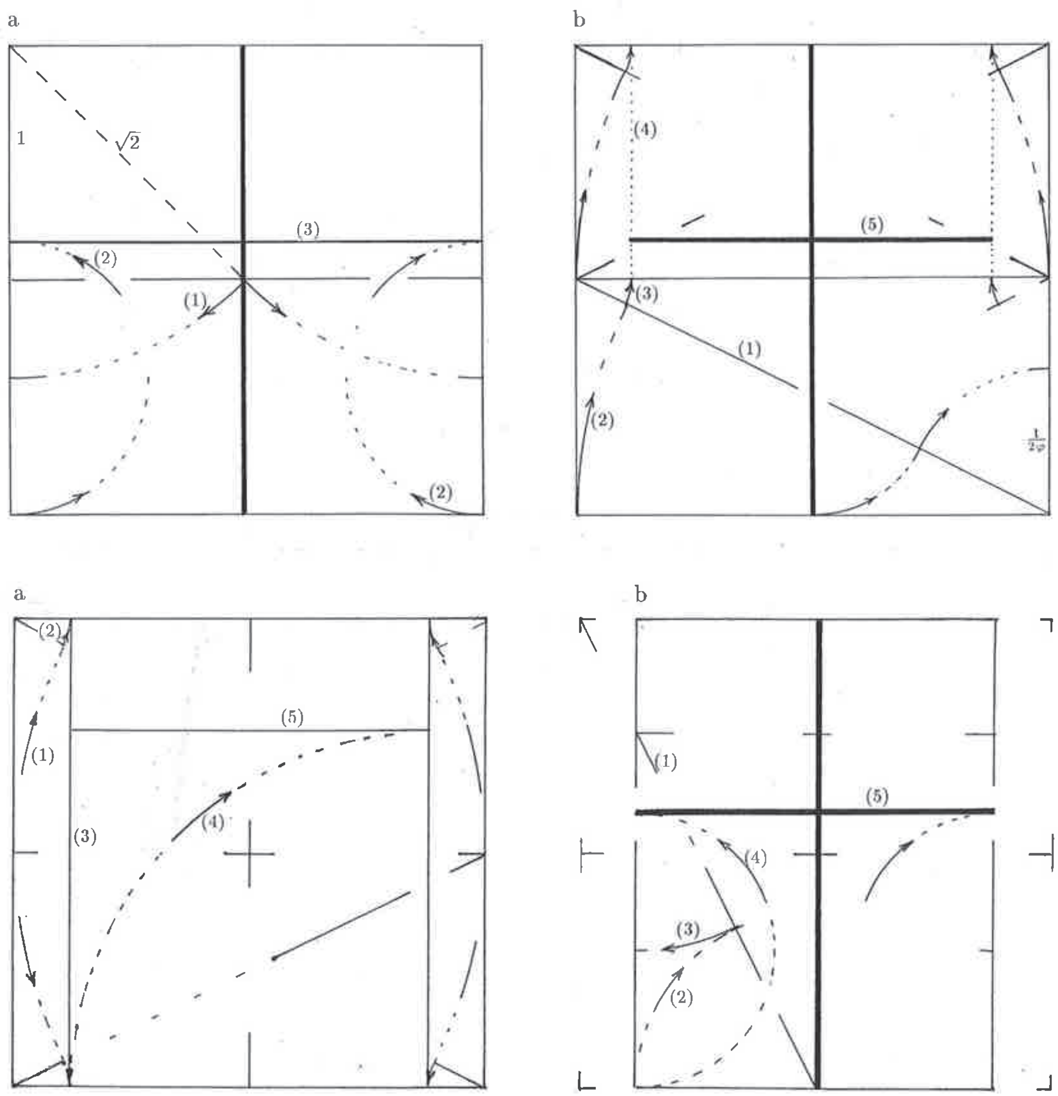


Fig. 5.5 Setting height, width, centre of Cross of Cong, two methods.

(b) Then set the horizontal dimension of the cross. Above and below, sketch a diagonal of two side-by-side squares (1); mark off from it the measure of those two squares (2); then copy the measure of the short remnant from either end along the nearer horizontal line (3), setting points to plot the length for the cross

onal of a square (1) is $\sqrt{2}$ in relation to each of its sides: sketch arcs (2) from the lower corners of the large square, radius equal to the side of the square, to intersect the diagonals. Then copy the short segments of the diagonals (3) to mark points along sides of the square. A line (4) connecting these points will become the putative midline of the cross arms.

arms (4). Draw then the midline for the arms (5).

The lateral extension could have been set before determining the vertical placement of the arms; the sequence makes no differences in the dimensions.

The method illustrated in Fig. 5.5, bottom, differs only *very* slightly in the vertical location of the horizontal axis for the arms, using again the height of the cross as the given measure of a square divided into four equal squares.

(a) Set the horizontal measure for the arms (1, 2, 3); then (4, 5) plot a square equal to the spread of the arms, from the bottom of the original square.

(b) Find the golden section of the side of the smaller square (1, 2, 3), copy the shorter segment along the side, doubling it (4). This sets the vertical measure for the midline (5) of the cross-arms.

Both constructions yield extremely close approximations to the fundamental dimensions of this cross, differing by about one-fifteenth of an inch in vertical placement of the intersection of the axes. The first of them employs $\sqrt{2}$ to determine arm height, $\sqrt{5}$ to determine arm extension and, as noted, the sequence of derivation is immaterial. The second construction employs $\sqrt{5}$ alone (of the non-integer measures) for determining both dimensions, and must proceed in the sequence described. The latter construction seems to be the more accurate of the two, though the smallness of difference and the exigencies of manufacture leave the choice in question. Here begins an enigma.

Fig. 5.6 illustrates a way to devise incursive arcs and central circle, and locate studs S3 and lateral cusps C1 and C2. Sketch a circle (1) with centre at intersection of the midline of the cross members, radius set by centre of the underlying square. Then double that radius (2) and sketch a second circle (3) concentric with the first circle. Its intersections with the midline of the cross-plan locates centres of studs S3 along the cross-arms.

Next, set lines (4) radiating from centre of the cross that divide the right angles of the cross equally (i.e., at 45 degree angles). Then, where the outer circle intersects these diagonal lines, place the fixed foot of the dividers, and set the moving foot where the inner circle intersects these lines: draw arcs (5) which will be the incursive arcs of the cross-design.

The central circle is the area which was designed to enclose and exhibit a fragment of the 'True Cross.' Its radius can be derived directly as in Fig. 5.6, inset. From the centre of S3 on either arm, copy the quadrant measure of circle 3 along the midline of the arms. Draw the inner central circle with centre and radius shown.

Centres of the small round devices at cusps C1 on either arm: they are not

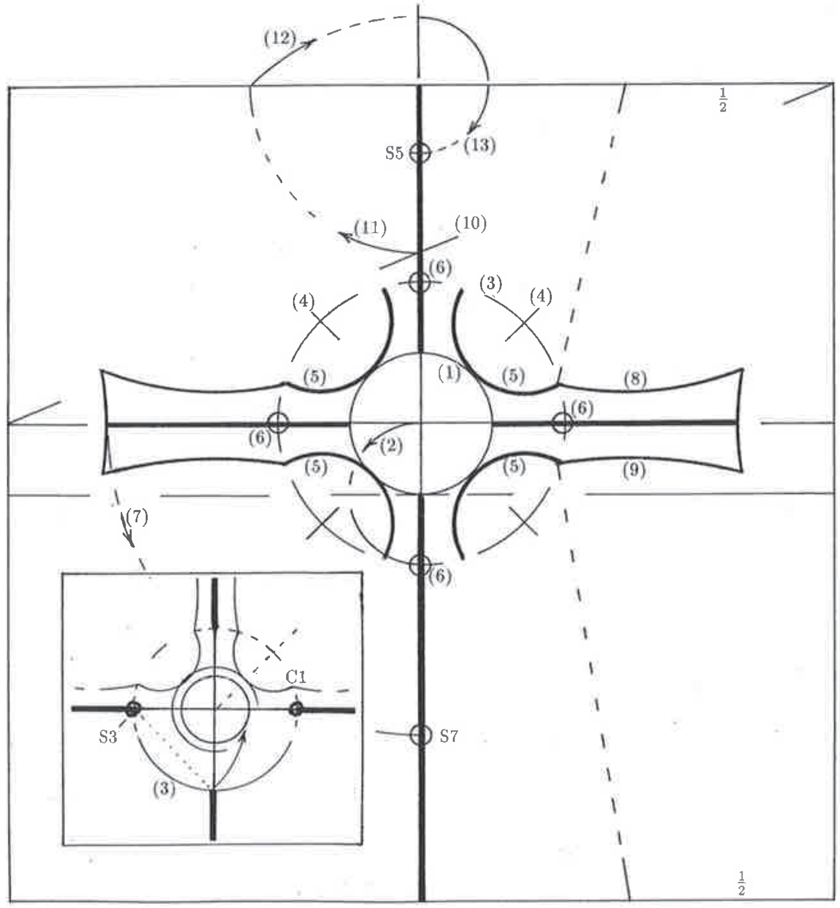


Fig. 5.6 Setting incursive arcs, central circle, S3 and C1, C2, Cross of Cong.

aligned vertically with studs S3. Corresponding devices on the shaft seem to be plotted similarly.

To plot arcs between cusps C1 and C2 on the arms, begin by finding the midpoint of the upper and lower quadrant pairs, along the original (large) square. Set the fixed point of the compass at that midpoint, the moving point to begin at C1 on the arms, and draw the curves (8, 9); they will terminate in lateral cusps C2. It should be noted that the outer curves, when drawn in this way, will not be identical, because of the different lengths of their radii. The curves in the drawing match those of the artefact in this respect.

Also in Fig. 5.6 is a possible plotting of S5 on the upper segment of the cross (10–13).

This set of graphic computations predicts the further dimension of the artefact more accurately from the first construction of the cross dimensions than from the second construction (Fig. 5.5, top, bottom). This is the case despite the second construction possibly being more accurate for the primary dimensions. The difference for measure between the pair of S3s is about one-tenth of an inch. The enigma expands.

For plotting centres of other studs S5 and S2 along the cross-arms, alternate methods will again yield virtually identical dimensions, contributing further to the enigma of plan of the Cong Cross. The derivations illustrated in Fig. 5.7 are based on squares equal to the height and the width of the cross, respectively; the smaller square can be constructed thus (not illustrated): from endpoints of the arms, copy each arm length both above and below, along the vertical lines that were used to set their length, and complete the square.

(a) As in Fig. 5.7, top left, locating the full-circle studs S5 nearest ends of the arms is simple and the result precise. Diagonals (1) of adjacent pairs of quadrants of the larger square locate centres of the halves of the midline of the original square; copying the measure shown along the midline (2) sets centres of these ornaments. In conceptual terms, these operations cut the midline of the original square by the golden section of each half. (There are alternate procedures that produce the same dimensions.)

(b) As in Fig. 5.7, top right, begin from a square that measures the reach of the cross arms. Run a diagonal of two adjacent quadrants (1), sketch a circle (2) with diameter equal to the half-measure of the square, copy the measure shown (3) to the outer edge; then with a line (4) compute half the complementary measure along the midline, as shown. The difference between these two graphic computations is hardly measurable.

As in Fig. 5.7, bottom, the studs S2 on the arms located between the mid-arm cusps and the central circle also can be plotted from either the height of the cross (the measure of the large square), or from the length of the arms; again the differences will be extremely small.

As in Fig. 5.7, bottom left, sketch the diagonal of pairs of quadrants of the large square (1). Subtract (2) the measure of a quarter-square from the diagonal, and then copy (3) the remnant measure of that diagonal along the midline. The arithmetic is $\frac{1}{2} \times \frac{Height}{\sqrt{5}}$.

As in Fig. 5.7, bottom right, sketch intersecting diagonals of each of two adjoining quadrants (1) of a square with sides equal to the horizontal extension of the cross. Then from endpoints of the midline copy the measure to the intersection

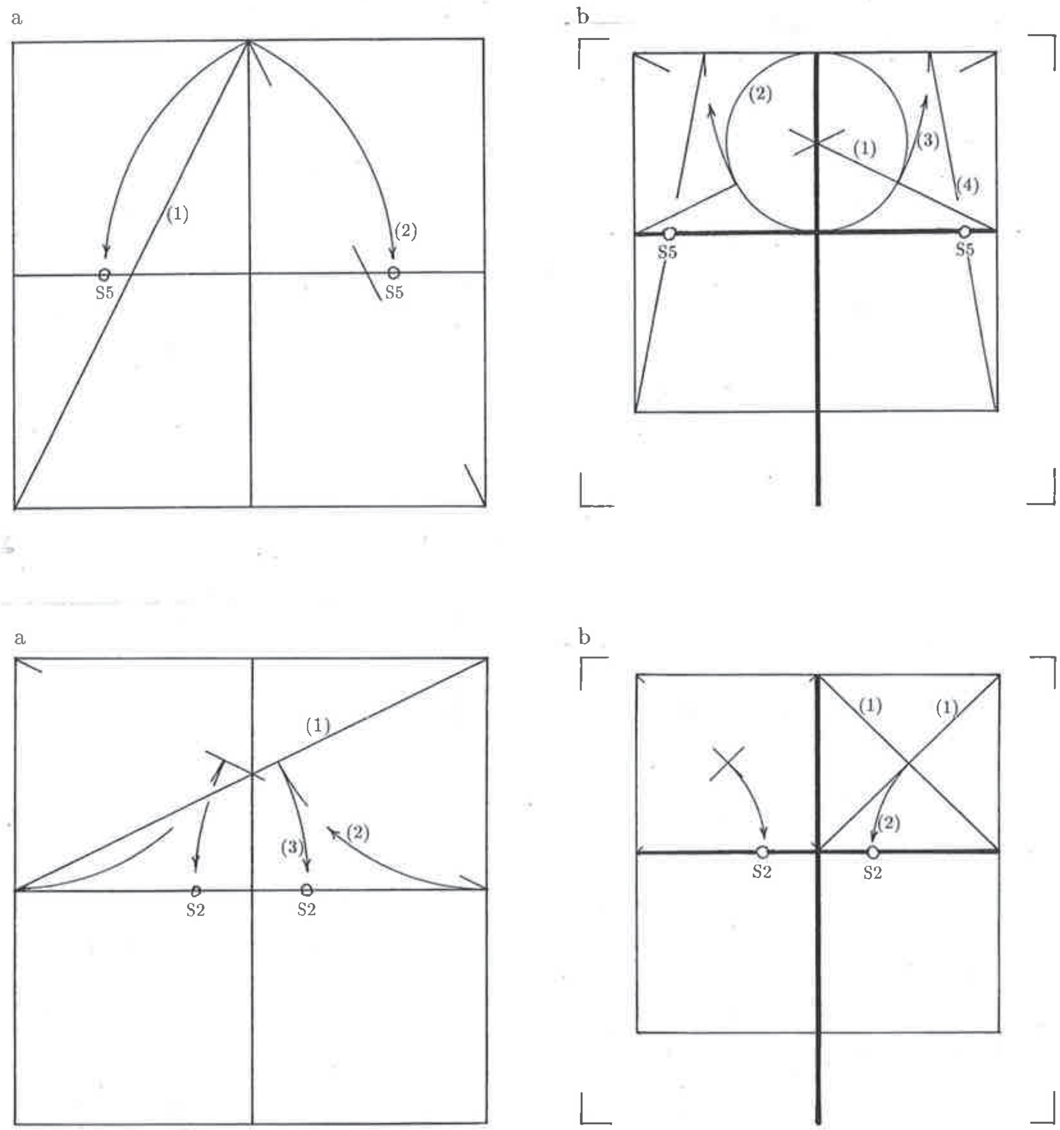


Fig. 5.7 Locating lateral S5s (top) and S2s (bottom), Cross of Cong.

along the midline (2).

The form of the Cross of Cong is a creation not merely drawn up with careful use of compass and straight-edge, but conceived as interlocked recurrences of measures and ratios among them, inherent in an arithmetic rhythm in the outlining (2 : 3 or 6 : 9), in counterpoint to a geometric web of the details of its shaping. While the finished piece presents an enigma of just which measures and manœuvres were employed, and in which order, to devise its elegant form, the more we rationalise the elements of its form, the more masterful that form appears.

POSTSCRIPT The form of the Cross of Cong is unique so far as we know from surviving artefacts. Probably it was unique when it was created, and probably its form was not copied in other pieces like it or in other media. Just how much it draws upon the tradition, though, can be illustrated with the top stone—the head—of the sculptured standing cross at Drumcliff. One object is a portable processional cross, the other is part of a massive standing sculpture. One was a cross form uncontained, the other is a cross form ringed and partially contained. Nonetheless, replication of the second form *begins exactly as described for the other cross, and illustrated without change in Fig. 5.5, either top or bottom.* (The same off-centre proportioning in the Cross of Cong and the head of the Drumcliff high cross is found elsewhere, for instance in several of the Monasterevin-type discs from a millenium earlier than the Cong Cross.) Spread of the cross-arms in the Cong Cross, in relation to its height, is the same as the outer diameter of the ring in the Drumcliff cross in relation to the height of the top stone; and entailed is the same ratio between the ring diameter and the lower segment of the cross in this top stone, exactly parallel to the Cong Cross plan. Further development of the plan will differ (which is ordinary), while the tools, techniques, and ratios will be more of the same (which is ordinary). Once the midline and centreline of the cross members are set (as in Fig. 5.5, top), dimensions of the ring can be derived simply, as in Fig. 5.8. The outer circle of the ring (1) is drawn first with its diameter being the original or underlying spread of the cross arms. A chord of that circle sets the radius for the inner circle (2). Derivations of further inner elements in place are sketched in the figure. Their linked dimensions are these:

- Transom breadth is equal to the radius of the inner circle of the ring.
- Centres of 'armpit' arcs are half the upper extension of the cross from its centre.
- Ornaments inside the 'armpits' fit within half the radius of the outer circle of the ring.

The other principal feature is the extension of the cross arms. Their corners are coextensive with a circle concentric with the ring, having diameter equal to the given measure, i.e., the height of this top stone. Aberrations from these formulae arise only from the tapering of the cross upright, a further development of the plan which is beyond the scope of this comparison.

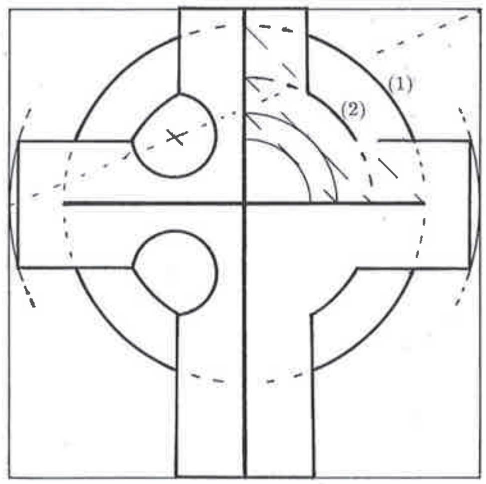


Fig. 5.8 Setting primary dimensions of Drumcliff Cross head.

Yet one more pair of objects to illustrate how long was the heredity of the coherent geometry of Insular designing.

Battersea Shield, in the British Museum, recovered from the Thames in mid nineteenth century, dates from second century B.C. The three circular ornaments have both their sizes and their placements answering to the same compass and straight-edge layout as do those of mainline 'Insular' objects of seventh to eleventh centuries, Ireland and England. See Fig. 5.9.

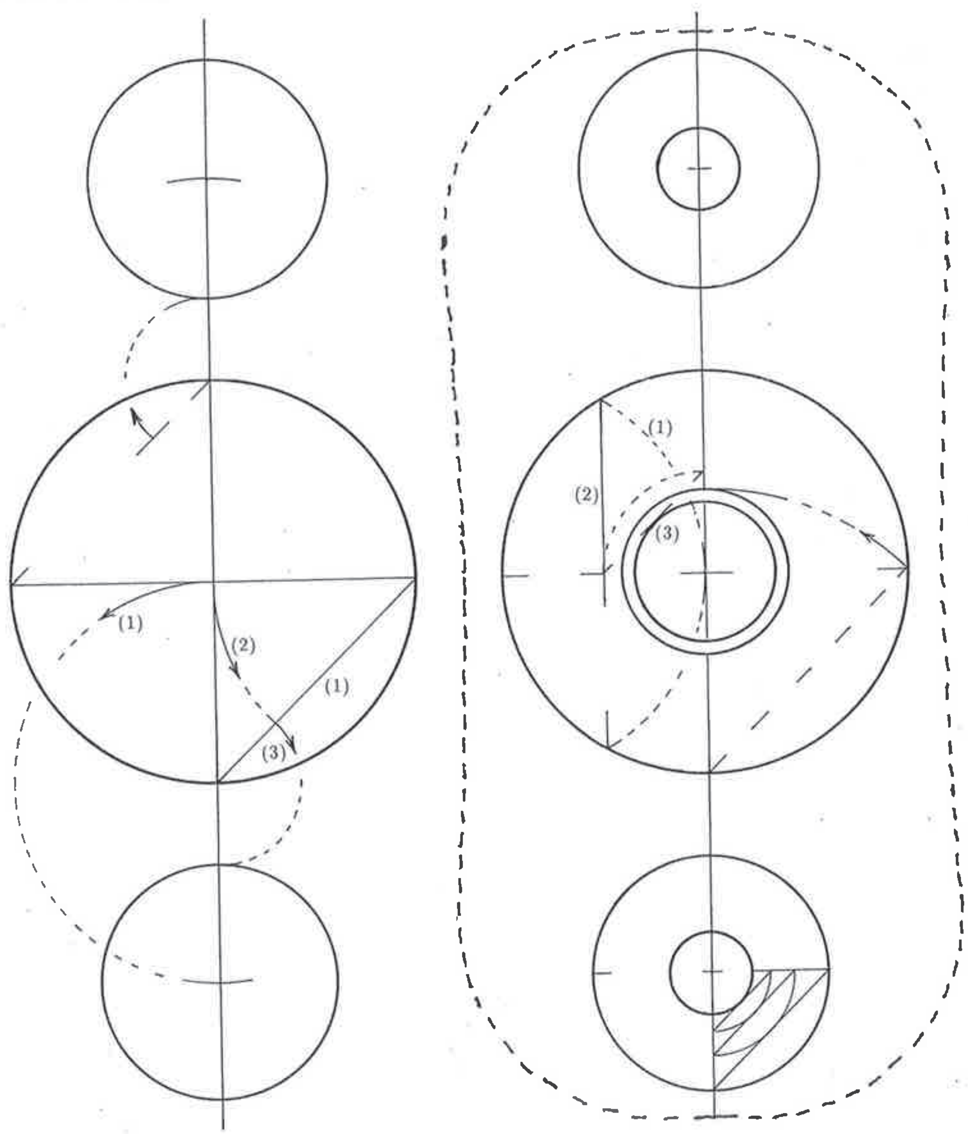


Fig. 5.9 Layout of ornament on Battersea Shield.

Gold lunula, NMI 1896.15 very much earlier—second millenium B.C.—and very much simpler than any of the other objects described in this handbook: it is a flat circular gold disc with a circular off-centre cut-out, the two circles being tangent. See Fig. 5.10.

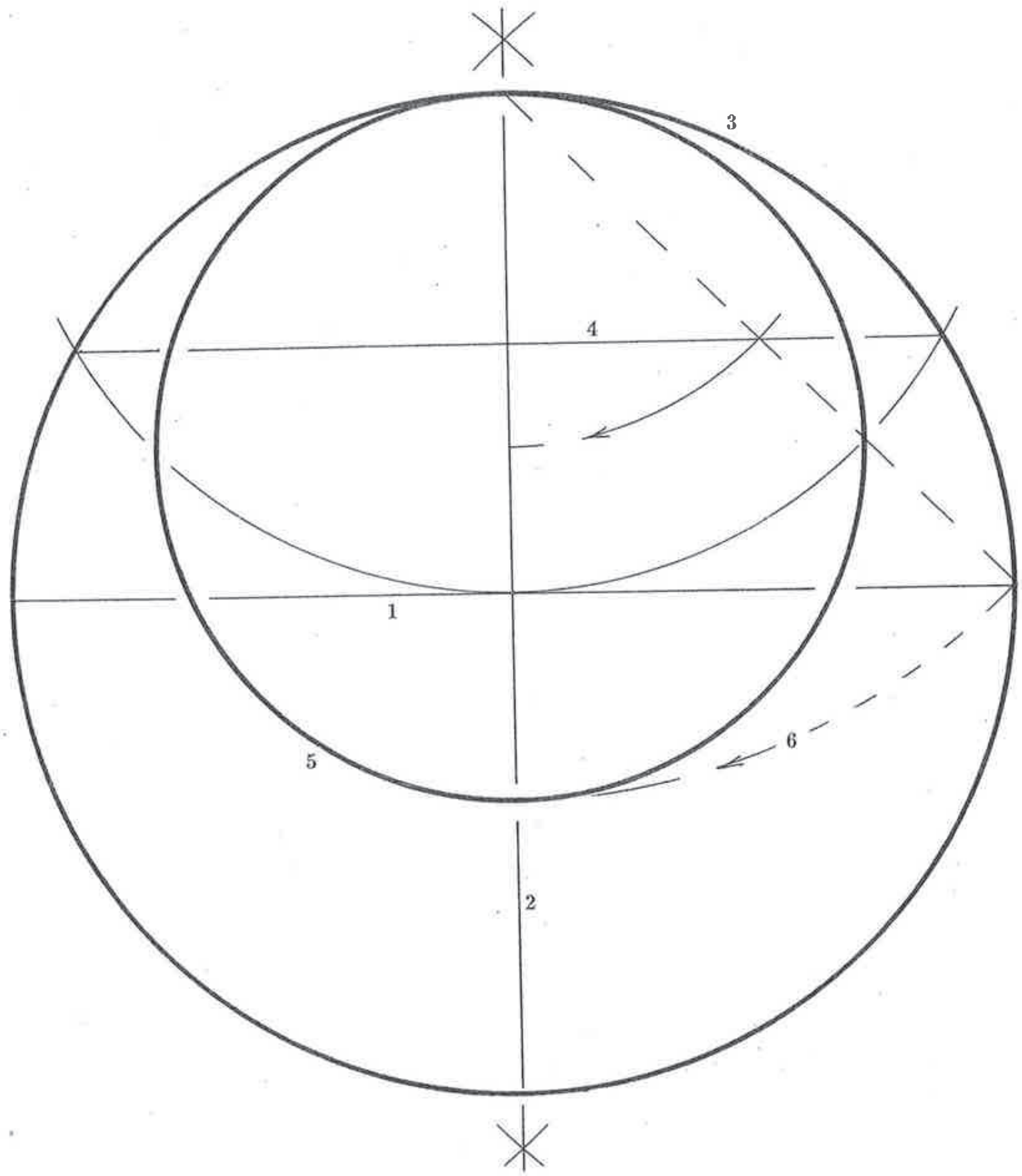


Fig. 5.10 Layout of early Irish gold lunula.

6. Some Retrospective Observations

Unit of Measure

Search for the unit of measure in various works of early Insular art—manuscript illuminations, Irish high crosses, ornamental metalwork—has not made much progress. It has seldom been undertaken in high cross design and metalwork, while in the case of manuscript illumination it has been misled by forced correlations with text-space dimensions which, again and again, can be demonstrated to be unrelated to the measures and ratios of the artwork. In the case of poem forms, the presence of a metrical unit (the 'line'), not to mention romantic notions of organic form, short-circuited any attempts to find the unit of measure which underlies whole-text forms. The search did not begin with primary forms—the shapes.

The situation in the study of this art is unlike the situation in the study of mensuration at large in the Middle Ages. For the latter there is a long, rich, and still unconcluded tradition of historical study. The metrology emerging from land survey, architecture, and trade, though, has not yet been shown to be helpful in analysis of Insular art. I think it is unlikely to be very helpful because of two major differences in the things to be measured. One is scale. The other is differences in their essential natures.

When measuring is undertaken to determine quantity in terms that can be communicated in numerals, spoken or written, a unit of measure is mandatory, and it must be recognised by anyone with an interest in knowing what that quantity is. Silver is measured in units of weight, cloth in units of length (on a bolt of constant width), grain in units of volume, land in units of area (usually square). In this kind of measuring, each quantity is determined separately from all others. Two dimensions of an object may be equals, doubles, fifths, seventeenths, cube roots—but such relational measures are not part of measuring of this kind. The 'value' of any measure, that is, consists of its relation to a single fixed measure (the unit) that exists entirely independent of any other measurement of the object being described in quantitative terms. This is the case even when terms within a system of measure may be related by geometrical or numerical ratio, either one. In short, measuring of this kind relates quantities of silver or woolen cloth or barley or plowland to an external intrinsically unrelated system of expressing quantity. The measuring serves social functions of trading, selling, taxing, tithing, and the like.

When measuring is undertaken in the creating of Insular design, nothing external to the undertaking is relevant. The overall size may have been set by reference to an external unit of measure: 'Make it 12 grains in diameter,' or 'Make it 27 palms high.' From that point on, the design need not make further use of the unit of measure unless it is conceived in modular terms of that measure (such as 13 × 18). Even so, a modular plan can make a number of equal divisions such that the resulting units do not correspond to recognised subdivisions of the external unit of measure. Most often in Insular plastic arts the overall plans are not modular, as we have seen, and the measures within the design—many of them not expressible in integers of a fixed unit in any case—are all in relation one to another and to the given measure from which they have been derived. Most will have little possibility of corresponding to conventional subdivisions of the external unit of measure. With all this, there will be no social function such as exchange or tithing served by any of the measuring that goes on in the creation of the design.

Mistaken identifications of the unit of measure have been made, for example, when regular divisions appear in the surface plan or in the construction of a design. One is the ostensible 'grid of squares' in the central rectangle of the carpet page* standing at the beginning of The Book of Durrow; another is the supposed square grid underlying the ostensibly square cross at the center of Lindisfarne Gospels fol. 138v; the grid pattern at the center of the back cover of the St. Cuthbert Gospel of St. John is yet another. In each instance, if the grid units were in fact square, the shapes of the designs would be different. The unit of measure in these three designs differs along the horizontal and the vertical axes of the grids. There is no explaining away these differences by calling them 'minor,' while trying to maintain the position that there is a single unit measure for the whole plan. Worse still is to invoke fractions. Too often the differences have not even been noticed. The fact is that the plans of these designs were not constructed on a grid based on a single unit of measure (13 × 17 or 11 × 15 or whatever), but were made instead from geometrical derivations involving irrational numbers. A rectangular area thus derived was *then* divided into integral numbers of segments to set a grid for the design: the horizontal extension of the area was divided into units of one size, the vertical extension into units of a different size. The units of measure, that is, have no independent status; they are limited in domain, resulting from dividing a portion of a particular plan that was set by different methods.

Some instances can be found of entire designs constructed according to grids whose units on both (or all) dimensions are the same, for example the Book of Durrow evangelist symbol page* for St. Matthew's gospel (7 × 4), or for the carpet cross-page preceding St. John's Gospel* in the Lindisfarne Gospels (15 × 19). But

* fol. 1v

* fol. 21v

* fol. 210v

they are relatively rare in the manuscript illuminations, at least.

The long poems in Old English illustrate a different aspect of the question of unit measure. The texts are written in metrical lines, about the identity of which there is no question or debate. Rules of the meter may still be a topic of passionate argument from time to time, but these concern the internal structure of the metrical line. The identity of the line is agreed on universally. But that is hardly the unit for constructing the forms of long poems which run into hundreds or many hundreds of lines. There is in fact no larger recurrent unit such as a stanza—four lines, seven lines, nine lines, fifteen lines—to be found in the forms of any of the poems extant. On the contrary, it is rare to find the same number of metrical lines (or even a common divisor beyond 2 and 3) in any two divisions of a single poetic text. There are several instances of the count of lines in a manuscript division of text being a prime number. So no one has ever claimed a unit of measure in the design, or large form, of English vernacular verse at this time.

The reason that a large unit of measure (aggregates of metrical lines) is not found in verse texts, and the reason that a single unit of measure is illusory in so many manuscript illuminations and other artefacts, is that the thorough forms of many of these works can be shown to have been created without them, and thus obviously without need of them. They were in fact created in a way entirely independent of them. (Ironically, modular groupings of sections of poems had also been overlooked until work began on the coherent designs in Insular bookarts—the recurrent line sums of 600 in *Andreas*, of multiples of 84 in *Phoenix*, for example.)

Why should an architect—whatever the medium of his creation—use a small-unit measure in constructing a design? The dimensions of a design may set units of interlace or grille or step pattern, or they may set the size of parts of a church, or set a frame and its parts enclosing an illumination. And so on. A fixed unit has its use in keeping consistency—in controlling the fit—among parts of a design which is constructed in multiples or modules, that is to say, made up in measures understood by counting or aggregation (bricks, tiles, panels, columns, and such). In a regular design, one that is laid out by a ruler, the fixed unit is the basis of the plan.

When it comes time to lay out the plan, the question that must be answered first is 'What size?' or 'How big?' or some other variant of the query concerning overall extension. If the question is answered with 'n units high, or long, or wide,' then the design probably will be modular—5×8, 15×21, 8×18. Commonly there will be a single module, a simple unit of measure if the units are used in all the dimensions. Public buildings and land surveys call for this kind of determination. If the question is answered with 'This big,' or 'As wide as X,' or the like, then the design does not need a ruler, and a measuring scale (a ruler) may be more hindrance than help unless 'this big' happens to coincide with unit measures of the ruler. On the small scale of carpet pages and especially of fine metalwork, modular subdivision will probably not correspond to sub-units of a customary measure, and will limit severely the ingenuity of design.

Of course, a plan can usefully be described, for comparative purposes, by its measure according to a standard scale—130 mm high, for example, 87 mm in diameter, or 1.23 m across.

Any conventional unit of measure in fact is unneeded, though, in creating a plan that begins with a given magnitude. Put another way, a unit of measure is not part of the concept when 'What size?' is expressed as magnitude extension that is determined first, which is to say, is the 'given'. It *is* the unit (or its double), and as the given measure it has no need to be defined by reference to some independent system of measuring. The given is usually the width of text-space in an elegant manuscript (which is not measured in units smaller than itself), or the overall diameter of the ring of a sculptured cross, or the diameter of a circular brooch, or the aggregate length of a verse composition. In any of these media, if the overall size is the starting point, if that size is selected as an extension to be divided, then it is beside the point to conceive the dimensions as a count of units of some independent fixed measure. There is no way to count up to one.

Binaries

Code may be simplest when it is set in binary relations of 0 and 1. This notion will be particularly attractive as a model for coding in binaries now that computer programme design is familiar—or at any time since OFF/ON became a common mode of comprehension. In computer bytes, or for that matter in the scheme of light switches, say, at top and bottom of a staircase—turn off or on at the top of the stairs or turn off or on at the bottom—binary relations can be linked and bundled with wonderful practical applications, all the while utilising what can be represented by only 0 and 1.

Also binary are 1 and 2 when they are the terms of the set. In graphic design—in the realm of extensional patterns in solid materials—where 0 and 1 do not work, the most elementary binary set is 1 and 2. An extension 1 can be doubled or halved, then cut into thirds and fourths, and so on, by manœuvres of the kinds illustrated in the 'arithmetical ratio' section of Chapter 3. It can be cut into unequal segments not answerable to integer expression by other manœuvres, illustrated in the 'geometrical ratio' section of the same chapter. The practical setting out of typical Insular designs, though, begins with 1, division into two equal parts for bilateral symmetry, and proceeds to divide those two equal parts into unequal segments of a 'geometrical' kind. The code (as it were) for design of the Tara Brooch, for example (Table 4.3), proceeds from 1 and 2 and the repeated division of 1 (into a, b, c, d) by the 'golden' ratio; that ratio itself reduces to not-terribly-elaborate relations between 1 and 2 when expressed as $\frac{2}{\sqrt{1^2+2^2}-1}$, or in linear notation, $2 : (\sqrt{1^2 + 2^2} - 1)$.

Another set is 2 and 3, which seems to be used in devising forms for verse texts primarily. The relations implicit in these two numbers, when represented as extensions at a right angle, correspond to the lengths of sections of the written texts measured by count of their metrical lines. *Andreas* has 3×600 lines, *Christ II* has 3×165 lines, etc., with sectional divisions answerable to both arithmetical and geometrical ratios involving 2 and 3. An exceptional form evolved from this binary pair is a carpet page, the only cross page surviving in the Harburg Gospels. The lines shaping the cross fall along halves and thirds (and halves and thirds ...) of the width of the frame, and at halves and thirds (...) of the length of the frame. The frame itself has dimensions in the ratio of $2 \times 2 \times 2 \times 2 \times 2 \times 2$ for width, $3 \times 3 \times 3 \times 3$ for height, or 2^6 wide, 3^4 long. The ratio is the same as for the musical interval of a major third.

Only seldom are such sets as 3×5 or 4×7 used.

Syntax and Alphabet

By syntax in this context is meant the rules for combining measures so that each measure is related to all others by rule, creating a geometrical coherence of design. Again and again the illustrations of coherent plans have been built from a sequence of continuous derivations by means of compass and straight-edge construction. In the designs of the best works, there are no elements of form not accountable to such a continuous, closed scheme of creation. Here is an explication of the syntax of coherent geometry, so to speak, as it appears in Insular art.

Recall that most designs are bilaterally symmetrical. For these there is an initial division of the given measure into $1 + 1$. In the designs evolved from geometrical ratios, the next step regularly is the division of one of the two equal measures into two segments that are unequal. With one the cut comes at $\sqrt{2} - 1$. With another it is at $\sqrt{3} - 1$. With the other it is at $\frac{1}{2}(\sqrt{5} - 1)$. The measure 1 remains the *other* of the two parts of the given dimension. The similarity holds for all three of these divisions except for the halving of the one incorporating $\sqrt{5}$, that exception being necessitated by $\sqrt{5}$ being greater than 2. Just as in the creation of the musical scale—which is encompassed by two pitches related as $2:1$ (an 'octave')—Insular designs rarely exceed the ratio $2:1$ in their relative primary measures. In fact, most of them stay within a range corresponding to the musical intervals of fifth and major third ($3:2$ and $3^4:2^6$).

In the unequal binary division of one of the two equal measures, there are further related ratios implicit, and that is a key to the coherence of the spatial schemes that we see in so many of the forms (and all of the best ones).

A very modern way to unfold that coherence is to express these divisions in alphanumeric terms, 1 for each of the two equal divisions, a and b for each of the unequal divisions of 1. These elements form an alphabet, as it were, in which can be 'written' the code for designs of the best and the characteristic pieces of Insular art.

With division of one half the overall measure at $\sqrt{3}-1$, there are two lengths, designated in Fig. 6.1 as a and b (cf. Figs. 3.17, 4.1b, and 4.6d). To follow this in numerical approximations, the values are

$\sqrt{3} = 1.73205 \qquad a = 0.73205 \qquad b = 0.26795$

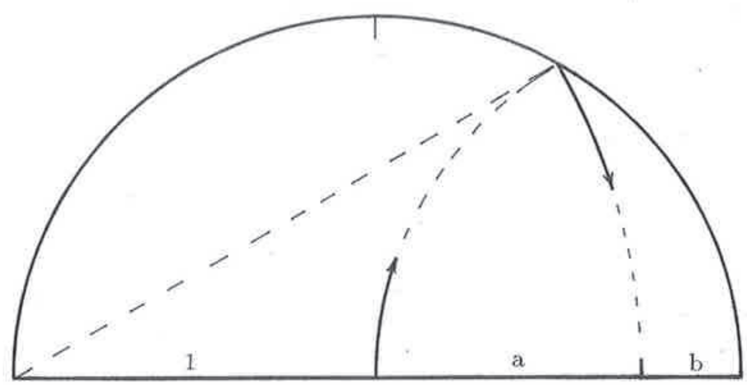


Fig. 6.1 Dividing half of a given measure at $\sqrt{3} - 1$.

Here are some of the proportional linkings of 1, 2, a, and b:

$1 + 1 = 2$

$a + b = 1$

$b/a = a/2 \quad = 1/(a+2) \quad = \frac{1-a}{1-b} \quad = \frac{1-a}{a}$

$\frac{1+a}{a} = 2 + \frac{a}{2}$

These are the most immediate combinations for equal ratios, and all that are employed in nearly all the spatial plans built around $\sqrt{3}$ geometry.

The tautology among the various formulations will be obvious in their algebraic expression above. It becomes radical when the first two equivalences are considered: $1 + 1 = 2$ and $a + b = 1$. These of course correspond to the initial dividing of the given measure equally by 2, then dividing one of the halves by using the geometrical measure $\sqrt{3}$. It will be further obvious, then, that all the relations can be reduced to 1 and a as their only terms, as is the case in several of the formulations above.

With division of one half the overall measure at $\sqrt{2}-1$, there are two lengths, designated in Fig. 6.2 as a and b (cf. Fig. 3.11c). To follow this in numerical approximations, the values are

$\sqrt{2} = 1.4142 \qquad a = 0.4142 \qquad b = 0.5858$

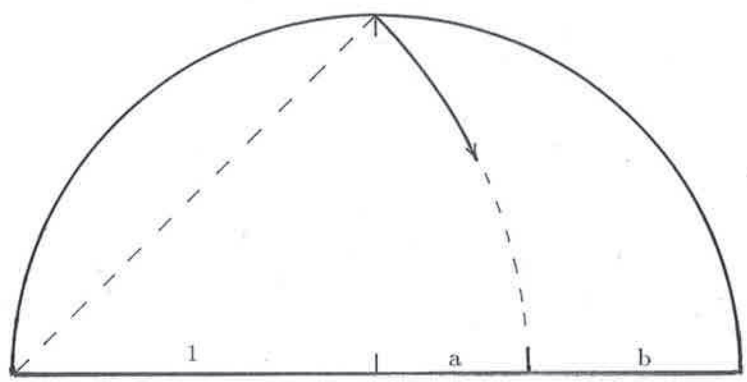


Fig. 6.2 Dividing half of a given measure at $\sqrt{2} - 1$.

Here are some of the proportional linkings of 1, 2, a, and b:

$1 + 1 = 2$

$a + b = 1$

$b/a = 1 + a \quad \text{hence } \frac{1+a}{b} = 2 + a$

$1/a = 1 + (1 + a)$

$a = \frac{1}{2+a}$

$\frac{a}{1-a} = \frac{1+a}{2}$

$\frac{1}{1+a} = \frac{1+a}{2}$, $\sqrt{2}$ (i.e., $1 + a$) being the geometric mean of 1 and 2.

These are the most immediate combinations for equal ratios, and all that are employed in many of the spatial plans built around $\sqrt{2}$ geometry. There are further divisions, such as $a/2$ and $b/2$, that enter into the schemes of several forms, as well. They merely extend the set of measures that participate in linked ratios for the coherence of forms.

The tautology among the various formulations will be obvious once more in their algebraic expression, and again it becomes radical when the first two equivalences are considered: $1 + 1 = 2$ and $a + b = 1$. And once again, all the relations can be reduced to 1 and a as their only terms.

With division of one half the overall measure at $\frac{1}{2}\sqrt{5-1}$, there are two lengths, designated in Fig. 6.3 as a and b (cf. Fig. 3.20). To follow this in numerical approximations, the values are

$$\varphi = \frac{2}{\sqrt{5}-1} = 1.61803 \qquad a = 0.61803 \qquad b = 0.38197$$

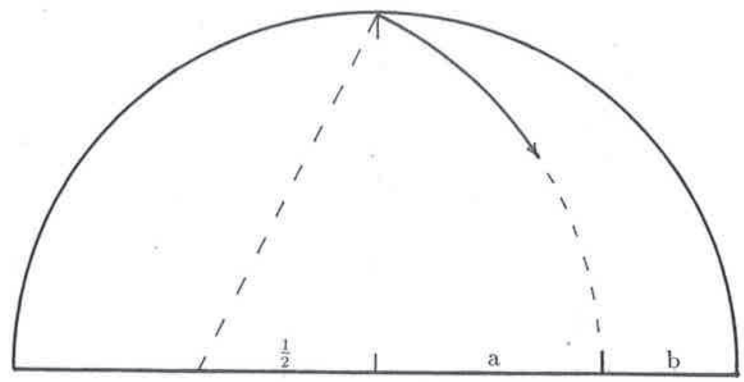


Fig. 6.3 Dividing half of a given measure at $\frac{1}{2}(\sqrt{5}-1)$.

Here are some of the proportional linkings of 1, 2, a, and b:

$$1 + 1 = 2$$
$$a + b = 1$$
$$\frac{b}{a} = \frac{a}{b+a} = \frac{1}{1+a}$$
$$a/b = 1/a = 1 + a$$

With this geometrical division into a and b there are further measures entailed that link in a chain of equivalent ratios, forming a continuous proportion—something like 'interlace':

$$\varphi = 1 + a = \frac{1}{a} = \frac{a}{b} = \frac{b}{c} = \frac{c}{d} \ldots .$$

More of the linking appears in this display:

$$a = \frac{1}{\varphi} \qquad b = \frac{1}{\varphi^2} \qquad c = \frac{1}{\varphi^3} \qquad d = \frac{1}{\varphi^4} \ldots .$$

Like the others, the whole set can be reduced to combinations of 1 and a:

$$a = \varphi - 1$$
$$b = 1 - a$$
$$c = a - b, \text{ thus } = a - (1 - a)$$
$$d = b - c, \text{ thus } = (1 - a) - (a - (1 - a))$$
$$e = c - d, \text{ thus } = (a - (1 - a)) - ((1 - a) - (a - (1 - a)))$$

. . . .

With each of these geometrical sets based on $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, together with 1 and 2, we have in effect the 'alphabets'with which can be written the code for the various coherent forms. In each alphabet the values of a and b (and any further derivations from them) are different, of course.

The purpose of the modern notation for the measures and the relations among them is to provide alternative and modern help in grasping and appreciating the radical coherence in the geometry of many of the designs. Any 'alphabet' for writing the code of these forms can be reduced to only two members, 1 and a, an ultimately simple, binary set. The syntax requires only + and − and simple nesting (the parentheses).

When all this equivalencing is done, however, in terms quite alien to the persons who created the designs of crosses, carpet pages, poems, and some metalwork, it is most important to return to the designs themselves and the practical methods of those who created them. In craftsmen's terms, the practical alphabet of Insular design was not the stark notational 1 and a that we can reduce it to, but always included the extensional correlates of 2 and b, and especially in the instance of the golden ratio it included c and sometimes d and e as well. Depending on where, say, a measure d may be called for, it can be created or it can be copied from anywhere the appropriate equivalent may lie in the design; for example:

$$d = 2a - b$$
$$d = b - c$$
$$d = 2b - a$$
$$d = 1 - (a + c)$$
$$d = a - 2c$$

The interrelations of these measures are indeed a rich resource for designs with coherent geometry. Whether the 'alphabet' is the reduced set 1 and a or whether it includes 2 and b and as needed c, d, e ..., the syntax remains the same. It works, of course, in the two dimensions of a plane, not just the one (lineal) dimension of the algebraic notation used above.

Assumptions

The assumptions that underlie this treatment of the principal forms of Insular art became clear—and in fact took shape—only in the process of articulating those forms. The process was a lengthy one, beginning in some what-ifs in trying to rationalise the 'dimensions' of some long vernacular poems on religious subjects, in Old English; proceeding to some unexpected analogs in manuscript art; extending next to some carved stone crosses, then to some fine metalwork. The evolution of the what-ifs soon showed a recognisable direction. More and more they evoked models embodying some specific mathematical relations. More and more those relations narrowed down to the most fundamental ones of arithmetic and the most elementary ones of geometry (these are the ones described in 3. 'Setting the Basic Ratios'). And more and more, the analyses of individual forms seemed to integrate all the primary dimensions of an artefact into a single, fully coherent scheme of relations, or ratios. All of this led to recognition of two assumptions that were being employed in my analysis of these forms.

> ASSUMPTION 1. Each structural plan may have an integrated (or catenate, or commodular) set of relations among its essential extensional features.

> ASSUMPTION 2. Structural lines typically can be developed without use of numerical computation, and they accordingly can be described without manipulating numbers; graphic computation is entirely sufficient and altogether fitting.

In so far as these assumptions support derivations that yield accurate representations of the structural plans of the artefacts, to that extent they are validated inductively for the study of these and other designs.

These assumptions have been employed in following two opposed approaches. One is that of creating a plan: it begins from a single quantitative given—a single magnitude in lineal measure—and emanates in a plan exhibiting coherent geometry. This first approach is made through drawings, using constructional geometry. The other is that of discovering the plan as a conceptual schema. This second approach employs algebraic notation to represent relations among constituents of the form, in the absence of examples of the mode of representation in use when the designs were created originally. We would understand this form of art much better, no doubt, if we could recover the mode of representation—the talk, the knack, the know-how—of the people who created them.

If the overall enterprise of analysis is both sound and pertinent, it may imply something about the method of the artists, in turn implying another stratum of assumptions. These could also be called inferences. They are better presented as assumptions, however: inferences tend to be regarded as conclusions—as end product—whereas assumptions are more likely to be treated as the beginning of a process of trying to understand. One of these has to do with transmission of the techniques of design, alongside transmission of techniques of metalworking, manuscript painting, devising decorative schemes, and such. Another has to do with the medium of transmission of the technique of designing. Yet another has to do with presence or absence of symbolism of the mathematical relations which inform a plan.

If these forms are code-conceived, they must have been code-transmitted. This is opposite to look-see-then-draw-a-new-one—the assumption underlying the common phrase in scholarship of one object being of 'reminiscent of' another.

The look-see-then-draw-another-one method of design may be the key to understanding the variations in decorative details—the basis, for example, of W. G. Collingwood's synthesis and synopsis in *Northumbrian Crosses of the Pre-Norman Age*. In fact, his evolutionary integration of all the examples depends on this assumption, along with two others. One is the notion that the earliest designs of a particular type are the best, the ones following inferior: 'nascent' stages are vigorous, later ones decadent. The second is that stone-cutters paid no heed to manuscript painters and metalwork designers. Otherwise we could not explain 'the taste or want of taste that hedged monumental art within such narrow lanes as we find, while all the wealth of ornament in manuscripts and metal was open to any explorer who could have got his head above the fence.' There had to be some way of transmitting types of decorative patterns. Yet there remains the problem of understanding transmission of methods for devising shapes on which and into which the decorations—including iconographic ones—had to be fitted. How to imagine that the people who designed the stone crosses neither knew nor used in their cross designs the same principles of creating form that were in use by those who were creating the forms of free-standing illuminations in the fine gospels manuscripts created in the same regions at about the same time? How to imagine their designs developing *de novo* in a culture encased in tradition?

Now, if it is true that these cross designs typically have their principal dimensions in a web of related ratios, then strict logic tells us that the design could be constructed on any of the main dimensions being selected for the given measure. There is no logic of proportional relations to gainsay that. But there are differences in practical consequences of beginning with one or another of the measures as the given quantity. The main ones are these.

Accuracy in setting relational dimensions is greater when smaller measures are derived from larger ones, rather than the other way about: small inaccuracies when reduced will detract from accuracy less than small inaccuracies when magnified. This will be of great importance, of course, in designs dependent upon symmetry—even more so in designs with linked relations in their dimensions along with the symmetry. The expanse of the cross-arms, for example, is the largest measure from the centre in typical ringed-cross design, and would be best choice for that reason. The circle is the next-to-largest of the primary measures that seem to be computed together (i.e., leaving out height of shaft), and it has only a slight practical disadvantage in that respect. It has, on the other hand, the practical advantage of being symmetrical in quadrants as it circumscribes the cross; the arms are symmetrical only bilaterally.

What about number symbolism? For many of the forms, the analyses given in earlier sections (if they are right) render dimensional relations in most full-form designs impervious to number symbolism as it is usually understood, and as it is widely documented in medieval Latin Christianity. This is the case with stone crosses, with illuminated Gospels pages, and with many other objects such as literary texts and fine metalwork. That tradition operates with numerals used to express multitude measures of quantity. The numerals are integers (one, three, seven, twelve, forty ...); or they are expressible as integer ratios such as sesquitertial, sesquialter, sesquioctave and such. A few quantities in magnitude measure in any of the forms may be numerical—twos and threes, halves and thirds, mainly. But many are not. They are the 'irrational,' which is to say, the contrary of rational numbers, for being neither integers nor quotients of integers. So while seven and four and twelve, for example, have an extensive tradition of carrying symbolic meaning, such numbers as $\sqrt{2}$ and $\sqrt{5}-1$ do not seem to have. If they had been assigned symbolic meaning within the artistic tradition, one would expect, for instance, that portrait pages of a particular evangelist would have consistent shapes, or at least shapes sharing a governing ratio utilizing the same irrational number. The fact is that they do not.

What terms did they use? No verbal analog—no talk talk—is necessary for transmitting the technique of commodular design. There doesn't need to be a tell along with the show. The independence of the technique from verbal narrative takes away any worry about how the knowledge could be spread to or through cultures whose vernaculars were disparate and lacked any koine adequate to the topic. A Latinate churchman need not have known a local vernacular to instruct an artisan in methods of layout of a plan, or an artisan need not have understood any Latin, wherever it was spoken. Yet a speaker of local Irish or English or

Franconian or Norwegian or Pictish or whatever need not have known Latin to demonstrate to a Latinate person lacking the local parlance. A guess is that the fundamentals of commodular design found in Irish and English bookarts, and fine metalwork, and stone crosses and whatever else—that these fundamentals may have arrived lively and have had a healthy heritage in Insular areas quite separately from Latin Christianity. Very recent analyses of some objects of early La Tène metalwork, Continental and Insular both, seem to demonstrate an ancestry reaching back well before Christianity.

Yet, it's hard to imagine a continuous cultural tradition employing this kind of commodular design with no one talking about it. Abbot to stonemason or book illuminator or preparer of parchment for a codex in Insular Christianity: and the other way about. Or any of these as magister instructing his underlings and successors. Surely they talked about what they were doing. And unless they never got beyond demonstrative pronouns—pointers—and devised or adopted nouns and formula labels—counters—they might as well have been talking to foreigners who had none of their own language. But at any time, surely, they must have talked in a shared language, even if only in pidgins.

Figure and Ground

Peter Harbison says of the ring of Irish high crosses that it

> must be considered as having had an important structural function.... Where the arms had constrictions, they were all the more easily liable to break at the narrowest part, and ... a support was needed to ensure that they would not snap off.... It could be argued that an almost straight line—rather than a semi-circle—would have been more natural.... [T]he rounded form may have been chosen for aesthetic reasons, and ... even though the upper half of the ring was not necessary for structural support, it was added for aesthetic reasons. [I]t is likely to have taken on very quickly ... a symbolic function as well. (*The High Crosses of Ireland*, Vol. 1, VII 'The Prototypes and Purpose of the Crosses,' 350–51).

The 'aesthetic reasons' invoked twice in the citation need not remain inscrutible.

By the analysis above, the ring—its lower half, anyway—makes sense for carving crosses from stone, for the protection which it gives to extensions of brittle material which are susceptible to fracture from impact or from changing stress (temperature, moisture, and such). That is a benefit of the form in the working of crosses in the medium of stone. It makes no accounting, though, for the arms having 'constrictions,' a characteristic element of high cross configuration. In fact, this analysis—a rationale, actually—does not consider other solutions to the structural and aesthetic issues that may have entered into Irish high cross designing; some other obvious design-types were equally practical but *not* used, such as those illustrated in Fig. 6.4. The characteristic constrictions regularly follow curves (in the 'armpits' of the crosses), some of them shallow, others deep, others in between. Yet there are the unperforated crosses (e.g., Cat. 105) similar in design but, being unperforated, without need for structure to support the horizontal members of the cross. And the circle-and-cross design without outer ring *not* supporting the horizontal arms is also found (e.g., Cat. 9). The constrictions may not be the structural problem inducting the circle into the design at all. On the contrary, the constrictions may be only a component developed from the same design concept that is more conspicuous in the circle surrounding the intersection of the cross shaft and arms. The shape of the constrictions seems always to have been derived as part of the mathematical scheme that governed all other elements of the form of ringed crosses. As that concept was developed and tested, the constrictions could increase because the physical support required for the arms was already present in the design. The notion that the upper half of the ring 'was added' lacks rationale altogether.

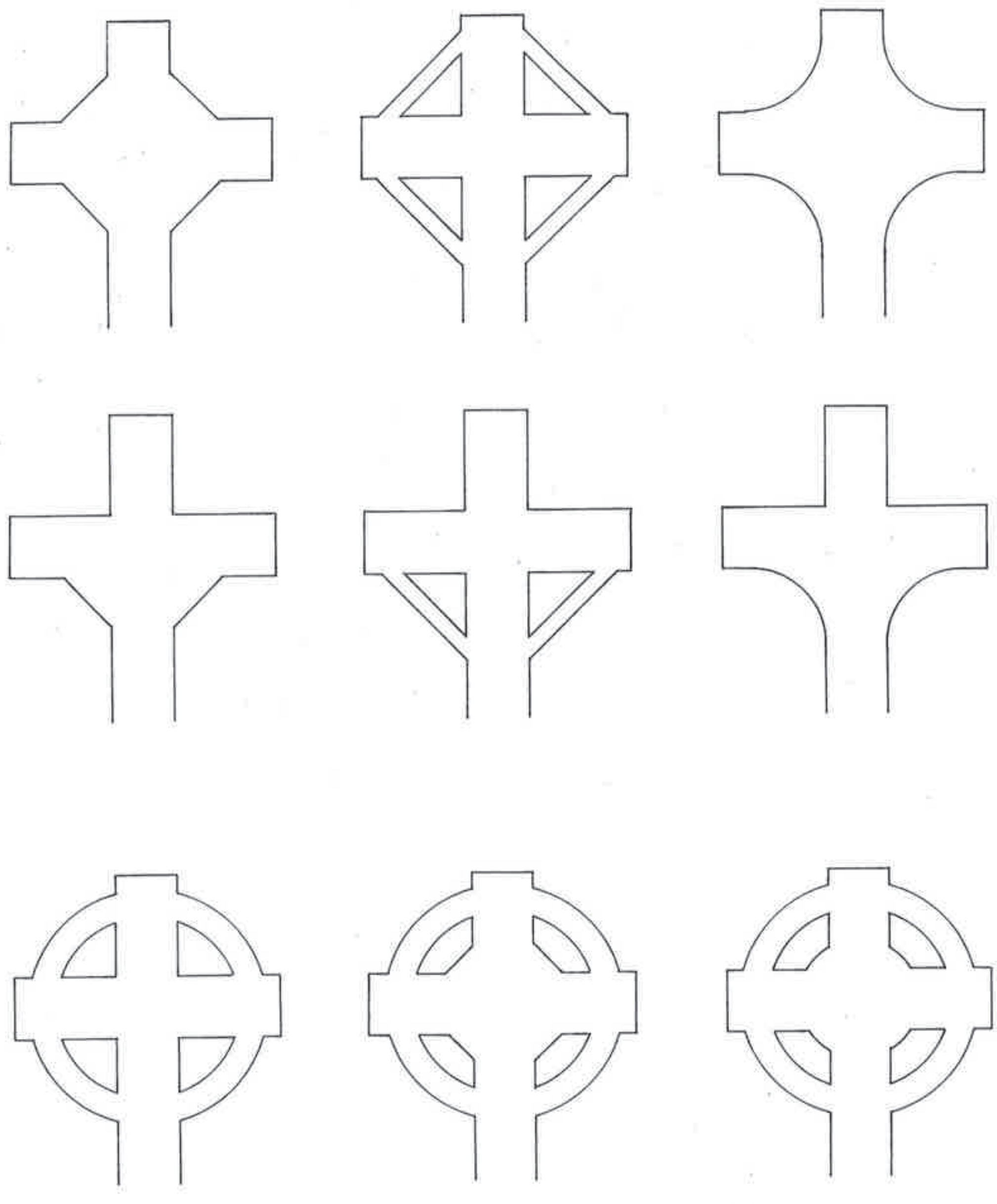


Fig. 6.4 Some practical arm supports *not* used for high crosses.

In addition, Harbison notes that 'there is also strong evidence ... that the decorated stone crosses ... were modelled on wooden crosses to which ornament of other materials were attached' (*ibid.* 345). Yet the greater the likelihood assumed for the modelling of stone crosses on wooden or wooden-and-bronze crosses, the less the evolution of the ringed cross can be rationalised: there must be some evidence that nails (rather than pegs) fastened the pieces of wooden crosses together, that bosses on those nails were taken as essential to the design in stone renderings of crosses (even though the stone sculpture used no nails),* that tubing covering edges of metal plate similarly was imitated without practical purpose in stone crosses, and so on. Otherwise, to find explanation in prior objects having 'resemblances' may be to set epiphenomenal relations in place of causal ones.

The cross symbol has been wrought in various materials. Drawn on papyrus or parchment, it is two dimensional, and is dependent on only the surface of its inscription. The surface in turn has its shape, typically that for a rectangle—or at any rate, an area with one fixed dimension and the other at right angles to it (scroll or codex). Whichever of these, the cross is the figure, the element that is drawn, and the surface is the ground. The ground is fixed in any case. Similarly with crosses in tile designs, painted on boards, carved into a surface of ivory, carved in relief on stone, and so on.

A cross not cursively drawn on a surface presents a quite different possibility for perception. A three-dimensional cross-shaped object with symbolic function typically will be portable, or it will be fixed. If portable, its size will be manageable and its material not likely to be fragile. And if portable—to be seen moving—it is a figure that has no ground, and needs none. If portable but seen mostly stationary (as on an altar, or above it), it will normally have a ground in a fixed environment. But if it stands fixed in an open place, with the viewer being the one that moves, the figure will have no ground. It will have no containing or otherwise complementing element. The crosses on carpet pages in Insular Gospels manuscripts are contained by their frames, and the frames in turn lie encompassed by the limits of the leaf. The circle in the Irish high crosses establishes containment

* When the 'bosses' on the stone crosses are said to be 'reminiscent of' knob-shaped coverings of nails that fastened the circle and cross in earlier wooden constructions of similar shape and purpose, someone with a practical eye may point out that the so-called bosses in many stone crosses are *not* centered on intersecting parts of the stone circle and cross. Any woodworker who pounded a large metal pin in some of the places we see them in the ostensibly copied stonework would have botched his work badly. In many examples, nails under the left and right bosses could never hold the circle to the cross. Otherwise we have to imagine a stone carver who saw bosses but, somehow having no conception of what they were there for, misplaced them in his imitation of a wooden cross. There is also the question whether the protuberant devices need to be understood as copies, anyway—that their presence is understandable only in terms of an isomorphic antecedent which is non-functional and meaningless in the copy. Finally, the notion of bosses somehow 'migrating' to other positions in a stone cross design is inference without method, and without anchor in the arts of craftsmen.

and complementation of the figure (the cross-symbol) that it otherwise would lack, and that it seems to need to be the more satisfying to human perception.

The frames of the cross-carpet pages not only enclose the cross figure, they are themselves complements to the figuration of the cross; at their best, the frame also is a ground that is a figure in its own right ('recursion,' as it has been called.) When the frame is drawn, the cross figure has a ground. And further, in Insular designing the relation of the correspondence is manifest also in the relations among the ratios of the respective dimensions (the lines and the areas defined by them).

With the stone crosses, the ring typically encloses without encompassing. That is to say, the ring encloses the crossing pattern of shaft and arms, surrounding the right-angle intersection with the un-angled shape of a circle, even while the head and shaft and arms of the cross typically extend beyond the circle, hence not being contained by it. This enables the cross to dominate the design, making it the figure, or subject. If on the other hand the whole of the cross pattern were contained within the circle, it would be the circle dominating the form, especially for an object standing fixed in place and with no ground of its own. The ring typically, then, provides a grounding the design would otherwise lack.

(Perforation is another aspect to consider. It makes the esthetics of the design for the high crosses unlike that of the two-dimensional page illuminations, and it makes it unlike that of relief-carving which may add depth as a third dimension but not producing a design that has both back and front, or two sides.)

The standing stone cross has only its ring to function as ground, and that may be the ultimate reason—an aesthetic reason—that it was developed to be part of high cross design. Then, to be part of the design for these prominent objects, it required integration with its subject, its ostensible figure. That integration sprang from circle and cross sharing a single centre. But as with other Insular designs (with their long prior history), its integration also demanded finding of a way to link the cross dimensions and the ring dimensions.

Forms and Fillers

For the standing high crosses, the cross carpet pages, metalwork, and similar high arts in their singular tradition, 'improvisation' should be used to characterise the art of filling areas already defined, typically with such patterns as knotwork, grille, or interlace. Improvisation does not underlie the art of creating the enclosing forms which provide the areas to be filled with images or decoration.

Improvisation may employ integer measures, but it does not employ ratios incorporating 1 and 2 with $\sqrt{2}$, or $\sqrt{3}$, or $\sqrt{5}$. Devising alone does this. 'Devise,' then, will be appropriate to signify a process of division or separation of parts; it reflects the procedures that will produce and will replicate the enclosing forms that dominate Insular designing.

An extreme illustration is provided by the North Cross at Ahenny, Co. Tipperary, Fig. 6.5. The form of the cross is devised by standard methods and executed quite well. The filler was worked out separately and clumsily improvised: the interlace devices disposed around the intersection of cross shaft and arms do not even come close to achieving symmetry which the design predicts.

There is a logic required for filling an area in the rules for interlace and similar devices, and there is a logic required for shaping an area, in the rules illustrated throughout this handbook. The two are different, one belonging to an art of improvisation, the other belonging to an art of devising, as these terms are used here. In the Insular tradition, *both* kinds of logic ask for total coherency: in good interlace, for example, it is palpable as an endless knot (as it's called), while in good page or cross or other planal forms it can be discovered in an unbroken sequence that yields full replication of the original construction.

Fig. 6.5 North Cross at Ahenny, Co. Tipperary.

Copying

If copying an Insular design meant transfering its dimensions from model to stone, or carpet-page, or mould for metalwork, all designs would have to be worked out on the scale of the completed artefact. In this circumstance the transfer could utilise an intermediary copy on material that would be portable, such as woven fabric, or wax impression, or copying measures one at a time.

Another method of copying, practical for some forms, is to employ grids. So long as the grids are identical in form, the size can vary as much as ever will be needed. That is, if lines of a grid are spaced by equal proportioning in model and artefact, the artisan then locates elements of the copied design in relation to the grid line equivalent to the relations of the corresponding elements to the grid line of the model. The grid itself marks square areas, or rectangular ones, or triangular ones, or any other kind that may be practical. Depictions of human and animal forms, or of trees or vines, for example, especially when two or more such figures belong to a single image, probably are done best by such a method. For animate forms, there is no inherent structural scheme that lends itself to copying by deriving one element of its form from another in a sequence from a given dimension to the fully executed representation. For that reason the structure of the grid merely provides reference markings for copying, with the copy being in effect a collection of mimetic representations guided by the grid. But the grid itself, an abstract, regular form, has no intrinsic relation whatsoever to the form being copied. In that lies its value.

For designs that consist of repetitive patterns, such as chains of interlace or areas of grille-pattern, use of grids (whether laid out as lines or as points where grid lines would intersect) may also be most practical, at least so long as the patterns don't embody irregular shapes. In this instance a grid is a regular pattern of repeating areas or points to guide the construction of the repeated elements on a surface, often on a different scale.

It is a different matter altogether when a design element to be copied (at whatever scale) has been created as a set of linked proportions. The relations among the parts of the plan will not be multiples of a given measure (except of course when the proportions are $1:1$, as in the last cross page in the Lindisfarne Gospels). But when the repetition involves a ratio of unequal quantities, and when one of these is an irrational number (typically $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$), copying will not be achieved by methods described above. Rather, copying requires repetition of steps mimicking those used in creating the form in the model.

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Still earlier were my papers showing the underlying rational form of sectional divisions in the manuscript texts of long religious poems of the Anglo-Saxons. These studies of page illuminations and verse texts were collected in *The Earliest Irish and English Bookarts: Visual and Poetic Forms before* A.D. *1000* (Philadelphia, 1994).