Essays on Asset Pricing and International Finance

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Abstract

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Chair of the Supervisory Committee:
Chang-Jin Kim
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In this dissertation, I empirically analyze how asset prices, especially exchange rates and stock prices, are determined.

In Chapter 1[1], I develop and estimate an empirical model of exchange rates that can explain four puzzles about the relationship between exchange rates and interest rates. The delayed reaction model of Bacchetta and Van Wincoop (2021) or the latent financial shock models of Itskhoki and Mukhin (2021) and Valchev (2020) can explain various puzzles in international finance, including the Fama puzzle, the delayed overshooting puzzle, the predictability reversal puzzle, and the Engel puzzle. In this paper, we develop an empirical model of exchange rate dynamics that encompasses these models as well as the UIP model and the risk premium model. Our empirical results based on the G6 portfolio and the individual currencies suggest that most of the persistence in the expected exchange rate change stems from the delayed reaction feature. We also show that the delayed reaction and the missing premium, only when combined together, can empirically explain the above-mentioned puzzles. In particular, the missing premium plays a critical role in our explanation of the Engle puzzle. These results are robust regardless of whether we employ the interest rate differential data [1980:M1-2007:M12] or the year-on-year inflation differential data [2000:M1-2019:M12] as our explanatory variable.

[1]This chapter is based on co-authored work with Professor Chang-Jin Kim.

Chapter 2[2] extends Bansal and Yaron's (2004) long-run risks model by allowing for regime switching in the steady-state volatilities of the shocks. In addition to the persistence of the long-run component, the persistence of both inter-regime and intra-regime volatilities play important roles in our model. Additional extensions include allowing for i) serial correlation in the idiosyncratic component of the dividend growth process and ii) non-zero correlations between shocks to the common long-run and the idiosyncratic components. We first provide empirical relevances of our extensions and provide a solution to the model. We then calibrate our model based on the choice of the preference parameter values in the literature and our estimates of the model parameters. The resulting equity premium ranges between 2.36% and 5.7%, suggesting that, in order to explain the historical equity premium of 6%, a considerably higher risk aversion parameter is necessary than is assumed by Bansal and Yaron (2004).

Chapter 3 investigates the time variation of the relative importance of the news on future cash flows and news on future returns in explaining the stock return variance, by combining the VAR-based stock return decomposition in Campbell (1991) and Campbell and Ammer (1993) and the time-varying parameter VAR with stochastic volatilities (TVP-VAR-SV) in Primiceri (2005). This new approach provides two empirical findings. First, the stock market crashes in recessions because of high discount rates news, and recovery of the stock market in booms is associated with high cash flow news. Second, there is a reversal in the relative importance of CFs and DRs in explaining the stock return variance in the 1990s. Then we propose two explanations for this reversal. First, the importance of the cash flow news in stock return variance may have increased since the 1990s because of the recent developments in information technology. Second, a decline in macroeconomic volatility since the 1990s can cause less volatile DRs, if investors make inferences on the macroeconomic condition and the corresponding expected return by extracting signals from observable variables.

[2]This chapter is based on co-authored work with Professor Chang-Jin Kim.

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DEDICATION

to my dear wife, Bora

Chapter 1

THE DELAYED REACTION OF THE EXCHANGE RATE AND THE MISSING PREMIUM: AN EMPIRICAL INVESTIGATION

1.1 Introduction

Among many puzzles in international finance, this paper provides an empirical investigation of four related puzzles about the relationship between the exchange rates and the interest rates: i) the Fama puzzle,[1] which is an observation that the excess return is positively related to the interest rate differential over some short horizons, contrary to the prediction of the uncovered interest rate parity (UIP); ii) the predictability reversal puzzle, which is an observation that the relationship between excess returns and interest rate differentials is reversed at longer horizons; iii) the delayed overshooting puzzle, which is an observation that a monetary contraction that raises the interest rate leads to a period of gradual appreciation, followed by gradual depreciation;[2] and iv) the Engel puzzle,[3] which is an observation that high interest rate currencies are stronger than implied by the uncovered interest parity.

Recently, there have been developments in theoretical research on the relationship between the exchange rate and the interest rate, and several papers provide economic mechanisms that explain the four puzzles at the same time. These economic mechanisms range from the delayed portfolio adjustment of investors (Bacchetta and Van Wincoop, 2021),

[1] Bilson (1981) and Fama (1984) first document this puzzle. Refer to Engel (1996, 2014) for surveys. Recently, Engel et al. (2022) and Bussiére et al. (2022) document that the coefficient in the Fama regression is unstable over time.

[2] The delayed overshooting puzzle in the literature was first documented by Eichenbaum and Evans (1995) based on structural vector autoregressions. Subsequent papers (e.g., Scholl and Uhlig (2008), Faust and Rogers (2003), Bjornland (2009)) that employ different identifying assumptions report mixed results on its existence or the horizon for the occurrence of the maximum appreciation. As Engel (2016) notes, even though the delayed overshooting in the literature has focused on the impulse response of exchange rates to identified monetary policy shocks, it was meant to apply to any shock that leads to an increase in relative interest rates.

[3] Engle (2016) first documents that the sum of all future excess returns is lower than the equilibrium exchange rate implied by the UIP condition.

impacts of latent financial shock or the convenience yield on the interest rate differential (Valchev, 2020; Itskhoki and Mukhin, 2021), to distorted beliefs of investors (Candian and De Leo, 2023). However, empirical papers only provide indirect evidence for these mechanisms (Colacito et al., 2020; Engel et al., 2022; Bacchetta et al., 2022), or explain only subset of four puzzles above (Valchev 2020; Dahlquist and Penasse 2020). In the absence of more direct empirical evidence of the theoretical models above, it is hard to judge whether the assumed parameters in their calibrations are empirically plausible or not[4], as mentioned in Bacchetta and Van Wincoop (2021).

This paper contributes to the literature by developing and estimating an empirical model of the exchange rate that can explain all the aforementioned exchange rate puzzles. Our empirical model is built on the economic mechanism of the delayed reaction of exchange rates in Bacchetta and Van Wincoop's (2021) but at the same time depart from their model to consider the different drivers of the exchange rate. In our empirical model, the expected exchange rate change (or the expected excess currency return) is a function of a lagged exchange rate change, the interest rate differential, and the latent component, which we call the missing premium. In addition to the interest rate differential which pins down the expected exchange rate in the UIP, the lagged exchange rate change is also included because of the delayed reaction of the exchange rate. In terms of the missing premium term, it is interpreted as the financial shock in Bacchetta and Van Wincoop (2021), but it has a broader interpretation in our model. For example, it can be a function of the financial shock, the time-varying risk premium, and/or any latent variable that is related to both the exchange rate change and the interest rate differential. Unlike Bacchetta and Van Wincoop (2021) which assume that the latent financial shock does not directly impact excess currency return predictability through interest rate differential, we allow the missing premium to be correlated with the interest rate differential. We argue that, when empirically estimating Bacchetta and Van Wincoop's (2021) delayed reaction model, explicitly incorporating this missing premium is critical for consistent estimation of the effect of the delayed reaction

[4]This is true for any theoretical model, but this issue is more important in this literature because we want to explain several different but related puzzles simultaneously. For example, Bacchetta and Van Wincoop (2021) show that the first three puzzles requires the delayed reaction which is large enough, but the Engel puzzle is explained only for an intermediate range of the delayed reaction.

and the interest rate differential on the exchange rate change.

An additional contribution of this paper is to show that these puzzles can also be explained by our model for a sample that includes the post-global financial crisis period, in which Fama puzzle seems to be disappeared. Engel et al. (2022) provide an explanation for the disappearing Fama puzzle based on the fact that the interest rate differentials lose their predictive power for the excess currency return after the global financial crisis when the interest rates lost their primacy as a policy instrument and central banks relied on unconventional monetary policies. At the same time, they show that the year-on-year inflation rate differentials continue to have predictive power even after the global financial crisis, because the inflation rate is another proxy for monetary policy which is not subject to zero lower bounds. Our empirical model can explain all the puzzles above when we employ the year-on-year inflation differential instead of the interest rate differential.

Our empirical results obtained from monthly data on G6 currencies and their portfolio can be summarized as follows. First, we find that the expected exchange rate change is highly persistent after controlling for the effect of interest rate differential and the latent missing premium. This is evidence in favor of the delayed reaction hypothesis of Bacchetta and Van Wincoop (2021). Second, after controlling for the delayed reaction of the exchange rate and the missing premium, the regression coefficients of exchange rate change on the lagged interest rate differential are estimated to be ranges between 0 and 1. This result is also consistent with the prediction of Bacchetta and Van Wincoop (2021), as the expected exchange rate change is less sensitive to the current interest rate differential than in the absence of the delayed reaction. Third, the missing premium is estimated to be positively correlated with the interest rate differential and it allows us the consistent estimation of our empirical model. Fourth, our model and the estimated parameters can reproduce and explain all the aforementioned puzzles. In particular, we show that the missing premium plays a crucial role in generating the Engel puzzle. Our counterfactual analysis shows that the Engel puzzle disappears when the latent missing premium is not correlated with the interest rate differential. When we estimate our model using the year-on-year inflation rate differential instead of interest rate differential data for the period including the post-global fiancial crisis, we find the qualitatively the same parameter estimates. These estimated parameters and our model also provides an explanation for this predictive power and other related puzzles.

This paper relates to the vast literature studying the relationship between the exchange rates and the interest rates. Earlier papers have focused on providing theoretical explanations for the Fama puzzle. One strand of literature emphasizes the role of time-varying risk premium in the foreign exchange market (Verdelhan (2010), Colacito and Croce (2011), Bansal and Shaliastovich (2012), Backus et al (2013) and reference therein). In this class of models, a time-varying currency risk premium, which covaries with the interest rate differential, generates the deviation from the UIP. Another strand of literature explains the Fama puzzle and the delayed overshooting puzzle using models in which financial frictions play an important role (Gabaix and Maggiori (2015) and Bacchetta and Van Wincoop (2010)). Finally, Burnside et al. (2011), Gourinchas and Tornell (2004), and Ilut (2012) provide an explanation for the Fama puzzle based on the deviation from the rational expectation.

Since Bacchetta and Van Wincoop (2010) and Engel (2016) document the reversal of the sign of the UIP violations, recent theoretical papers build models that explain this non-monotonic pattern. Bacchetta and Van Wincoop (2021) present a model in which the delayed reaction of the exchange rate[5] to an interest rate shock generates a non-monotonic pattern of the UIP deviations that can explain not only the Fama puzzle but all the above-mentioned puzzles. Researchers like Itskhoki and Mukhin (2021) and Valchev (2020) emphasize the role of latent persistent financial shocks or convenience yield in explaining the non-monotonic pattern of the UIP deviations in the absence of the delayed reaction. Itskhoki and Mukhin (2021) show that, in segmented and incomplete financial markets, an interaction of the persistent liquidity demand shock and the foreign goods demand shock can generate this pattern. In Valchev's (2020) model, excess currency returns arise as compensation for endogenous fluctuations in bond convenience yield differentials, which is a negative function of the interest rate differential. He shows that the dynamic responses of the liquidity demands by investors can generate a the non-monotonic pattern of the UIP deviations, in the presence of a sluggish fiscal policy that follows an active monetary policy.

[5]Mitchell et al. (2006), Brunnermeier and Nagel (2008), and Bilias et al. (2009) empirically show that individual investors' asset allocation exhibit strong inertia, using micro-level data set.

Finally, Candian and De Leo (2023) show that shock misperception and over-extrapolative beliefs can generate the similar pattern of the exchange rates. This paper provide an empirical evidence in favor of the delayed reaction mechanism in Bacchetta and Van Wincoop (2021).

This paper also relates to empirical studies that investigate the sources of the excess currency return predictability and the puzzles above. Recent papers such as Colacito et al. (2020) and Engel et al. (2022) provide empirical evidence on the delayed reaction of the exchange rate to the interest rate differential, but there seems to be no empirical study that directly shows the relevance of the delayed reaction in generating the non-monotonic pattern of the UIP violations. Dahlquist and Pénasse (2021) show that incorporating the latent component in the Fama regression is also not enough to solve the four puzzles simultaneously. Within the framework of a present-value model, they provide an empirical model of real exchange rate in which the Fama regression equation is augmented with a persistent latent component. Their empirical model and the estimated parameters can generate the predictability reversal pattern observed in the data, but at the cost of the Fama puzzle getting worse. We shows that the delayed reaction of the exchange rates and the missing premium can generate the excess currency return predictability and empirically explain the puzzles above.

The rest of this paper is organized as follows. In Section 1.2, we derive an empirical model of the delayed reaction in the presence of the latent financial shock. Section 1.3 reports our empirical findings. Section 1.4 reproduces and explains the puzzles based on our model and parameter estimates. In Section 1.5 we perform a counterfactual analysis to investigate the role of the missing premium in explaining the puzzles. Section 1.6 concludes.

1.2 An Empirical Model of Delayed Reaction in the Presence of the Missing Premium

1.2.1 Derivation of an Empirical Model of Delayed Reaction in the Presence of the Missing Premium

The hypothesis on the delayed reaction of the exchange rate to the interest rate shock was first proposed by Froot and Thaler (1990), and Bacchetta and Van Wincoop (2021) construct a general-equilibrium model based on the delayed reaction, which can explain the non-monotonic pattern of the UIP violations. A key assumption in their model is that agents are subject to a quadratic portfolio adjustment cost when they maximize their lifetime utility. Because a household that is subject to the portfolio adjustment cost can not fully adjust its portfolio when the interest rate changes, the optimal portfolios of home and foreign agents are determined not only by the expected excess return on the foreign deposit but also by the lagged optimal portfolio. Thus, the model exhibits a gradual portfolio shift given the change in the interest rates and this gradual shift generates a very persistent deviation from the UIP condition.

In their model, a positive shock in the home interest rate leads to a period of continued partial appreciation of the exchange rate due to the sluggish portfolio adjustment, explaining the violation of the uncovered interest parity in the short run. At longer horizons, the exchange rate continues to appreciate until the optimal portfolio is achieved. At the same time, the interest rate gradually reverts toward its steady-state level after the shock, and this leads to a gradual depreciation of the exchange rate. If the appreciations resulting from the delayed reaction of the exchange rate dominate at short horizons and the depreciations resulting from the mean-reverting behavior of the interest rate dominate at longer horizons, their model can reproduce the delayed overshooting of the exchange rate and the predictability reversal puzzle. Bacchetta and Van Wincoop (2021) show that their model can reproduce the UIP puzzle, the delayed-overshooting puzzle, and the predictability reversal puzzle that are in the data. But they obtain these results by calibrating their model.

We consider an empirical version of Bacchetta and Van Wincoop's (2021) model, based on their model-implied equilibrium exchange rates:

$$\Delta s_{t+1} = \psi \Delta s_t + \beta(i_t - i_t^*) + f_t + \varepsilon_{t+1}, \tag{1.1}$$

$$f_{t+1} = \rho f_t + \eta_{t+1}, \tag{1.2}$$

$$i_{t+1} - i_{t+1}^* = \phi(i_t - i_t^*) + v_{t+1}, \tag{1.3}$$

$$\begin{bmatrix} \varepsilon_{t+1} \\ \eta_{t+1} \\ v_{t+1} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} & \sigma_{\varepsilon v} \\ \sigma_{\eta\varepsilon} & \sigma_\eta^2 & \sigma_{\eta v} \\ \sigma_{v\varepsilon} & \sigma_{v\eta} & \sigma_v^2 \end{bmatrix} \right), \tag{1.4}$$

where Δs_{t+1} is an exchange rate change from t to $t+1$, i_{t+1} and i^*_{t+1} are home and foreign interest rates respectively, f_{t+1} is the missing premium, and $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$ for $i, j \in \{\varepsilon, \eta, v\}$.

The main feature of the model is that the Fama regression equation is augmented by the lagged exchange rate change, and its coefficient ψ $(0 < \psi < 1)$ is an increasing function of the portfolio adjustment cost. At the same time, the Fama coefficient β $(0 < \beta < 1)$ is a decreasing function of the portfolio adjustment costIn the benchmark calibration of their model, Bacchetta and Van Wincoop (2021) set $\psi = 0.97$ and $\beta = 0.3$. . In the absence of the portfolio adjustment and the latent component, equation (1.1) reduces to the standard Fama regression, as the exchange rate reacts instantaneously to a change in the interest rate differential (i.e., $\psi = 0$ and $\beta = 1$).

Our empirical model of exchange rate departs from Bacchetta and Van Wincoop (2021) by making a different assumption and an interpretation about the latent component f_{t+1}. We name it "missing premium" in the sense that it represents a portion of the expected exchange rate change that is not explained by the delayed reaction or the interest rate differential. While the f_{t+1} term is interpreted as the financial shock in Bacchetta and Van Wincoop (2021), it has a broader interpretation in our model. For example, it can be a function of the financial shock, the time-varying risk premium, and/or any latent variable that is related to both the exchange rate change and the interest rate differential. An additional difference from Bacchetta and Van Wincoop (2021) is that the f_{t+1} term is allowed to be correlated with the interest rate differentialAllowing this additional correlation does not change the form of the equilibrium exchange rate change in equation (1.1).. In their model, the role of f_{t+1} is limited, as it does not impact the predictability of the interest rate differential, but only affects the observed exchange rate volatility. However, in exchange rate models with the latent financial shock or the convenience yield, the correlation between f_{t+1} and the interest rate differential is a key element that explains the deviation from the

UIP. Thus, it is crucial to allow the missing premium and the interest rate differential to be correlated in the estimation, to account for this possibility.

The most closely related to our paper is Dahlquist and Pénasse (2021), in which the f_{t+1} term in equation (1.1) is interpreted as the missing risk premium in the absence of the delayed reaction feature ($\psi = 0$). By casting equation (1.1) with $\psi = 0$ into the present value framework, they derive the following equation for the real exchange rate (q_t) dynamics:

$$q_t = -\frac{\beta}{1-\phi}(i_t - i_t^*) - \frac{1}{1-\rho} f_t \tag{1.5}$$

which is estimated along with equations (1.2) and (1.3). They find that their missing premium f_t is highly persistent and explains most of the variation in the real exchange rate. Even though they show that they were able to explain the predictability reversal puzzle, it was only at the cost of the Fama puzzle getting worse. This is because the correlation between the missing premium and the interest rate differential was estimated to be positive with their empirical model, which is not consistent with the prediction of theoretical models based on the risk premium or the latent financial shock.

1.2.2 The Reduced-Form Model Subject to the Long-Run Restriction

Note that our empirical model in equations (1.1)-(1.4) is not econometrically identified.[6] We thus estimate the following reduced-form model, which is derived in Appendix A.1:

[6] For identification of the model, we need at least one restriction on the variance-covariance matrix of the shocks. However, if we arbitrarily impose a restriction for identification, the estimation result would be distorted.

Reduced-form Model

$$\Delta s_{t+1} = \psi^* \Delta s_t - \psi\rho\Delta s_{t-1} + \beta(i_t - i_t^*) - \beta\rho(i_{t-1} - i_{t-1}^*) + (\gamma - \theta\delta)v_t + \delta v_{t+1} + e_{t+1} - \theta e_t,$$
$$i_{t+1} - i_{t+1}^* = \phi(i_t - i_t^*) + v_{t+1},$$
$$\begin{bmatrix} e_{t+1} \\ v_{t+1} \end{bmatrix} \sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix} \right) \tag{1.6}$$

where $\psi^* = \psi + \alpha$. Here, the γ parameter captures the contemporaneous effect of the interest rate shock on the latent financial shock, which in turn affects the expectation of the future exchange rate change. The δ parameter captures the effect of the interest rate shock on the unexpected change in the exchange rate.

In the delayed overshooting literature, Bjørnland (2009) identifies the structural vector autoregressive (SVAR) by assuming monetary policy shocks have no long-run effect on the level of the real exchange rate. He also mentions that this is a standard neutrality assumption that holds for a large class of models in the monetary policy literature. In the same vein, we incorporate and test the validity of the following long-run restriction when estimating our model:

$$\lim_{j\to\infty} \frac{\partial s_{t+j}}{\partial v_t} = \sum_{j=0}^{\infty} \frac{\partial \Delta s_{t+j}}{\partial v_t} = 0, \tag{1.7}$$

which suggests that the interest rate shock does not affect the level of the exchange rate in the long-run. From equation (1.7), we obtain the following constraint on the parameters of our model as given below:[7]

$$\delta = -\frac{1}{1-\theta}\left(\gamma + \left(\frac{1-\rho}{1-\phi}\right)\beta\right). \tag{1.8}$$

For estimation of the reduced-form model subject to the above constraint, we employ the Kalman filter and the maximum likelihood method by casting the model in (1.6) to the

[7] Refer to Appendix A.2. for a derivation of equation (1.8).

following state-space model:

Measurement Equation

$$\begin{bmatrix} \Delta s_t \\ i_t - i_t^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta s_t \\ \Delta s_{t-1} \\ i_t - i_t^* \\ i_{t-1} - i_{t-1}^* \\ e_t \\ v_t \end{bmatrix} \tag{1.9}$$

$$(\Delta \tilde{y}_t = H\xi_t)$$

Transition Equation

$$\begin{bmatrix} \Delta s_t \\ \Delta s_{t-1} \\ i_t - i_t^* \\ i_{t-1} - i_{t-1}^* \\ e_t \\ v_t \end{bmatrix} = \begin{bmatrix} \psi+\rho & -\psi\rho & \beta & -\rho\beta & -\theta & (\gamma-\psi\delta) \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta s_{t-1} \\ \Delta s_{t-2} \\ i_{t-1} - i_{t-1}^* \\ i_{t-2} - i_{t-2}^* \\ e_{t-1} \\ v_{t-1} \end{bmatrix} \begin{bmatrix} 1 & \delta \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_t \\ v_t \end{bmatrix} \tag{1.10}$$

$$(\xi_t = F\xi_{t-1} + CU_t, \qquad U_t \sim i.i.d.N(0,\Omega),)$$

where Ω is a diagonal variance-covariance matrix.

1.2.3 The Reduced-Form Model in Relation to Other Empirical Models of Exchange Rate

The model presented in the previous section encompasses various alternative models of exchange rates that include the Fama regression equation under the UIP, the risk premium model of earlier literature, the delayed reaction model, and the model with the latent financial shock. We show that the last two can generate the non-monotonic deviation of the exchange rate from the UIP (which is represented by the delayed overshooting), while the first two cannot.

The parameter restrictions on our reduced-form model, as implied by the above-mentioned four models, are summarized as follows:

UIP Model

$$\beta = 1; \quad \psi = \rho = \theta = \gamma = 0; \quad \textit{and } \delta < 0$$

Risk Premium Model of earlier literature

$$\beta = 1; \quad \psi = 0; \quad \rho = \theta; \quad \delta > 0; \textit{ and } \gamma < 0$$

Delayed Reaction Model (without financial shock)

$$0 < \beta < 1; \quad \rho = \gamma = \theta = 0; \qquad \delta < 0$$

Model with Latent Financial Shock (no delayed reaction)

$$\beta = 1; \quad \psi = 0; \quad \rho = \theta; \quad \delta < 0; \textit{ and } \gamma < 0$$

For the first two models, there exists no mechanism that can generate the delayed overshooting of the exchange rate in response to a shock in the interest rate differential[8]. Figure 1.1 depicts the response of the exchange rate over time to a shock to the interest rate differential. The parameter values we employ to generate the impulse-response coefficients in the upper panel of Figure 1.1 are as follows:[9]

[8]A fundamental mechanism of risk premium model is that a negative expected excess return is associated with an initial depreciation when the home interest rate rises, as Engel (2016) points out. With the initial depreciation, the home currency keeps appreciating until it converges to the steady-state.

[9]For all the models that we consider in this section, the parameter values are set to be consistent with the restrictions implied by each model. We also impose an additional restriction that a shock to the interest rate differential has no long-run impact on the level of the exchange rate.

<u>UIP Model</u>: $\beta = 1$; $\delta = -12.5$; $\phi = 0.92$; $\sigma_e = 2.42$; $\sigma_v = 0.07$;

<u>Risk Premium Model</u>: $\beta = 1$; $\rho = 0.9$; $\delta = 26.2$; $\gamma = -2.2$; $\phi = 0.92$; $\sigma_e = 2.42$; $\sigma_v = 0.07$; $\theta = 0.9$;

For each of the last two models, there exist sets of parameter values that can generate the delayed overshooting of the exchange rate, an important stylized fact in the data. The lower panel of Figure 1.1 depicts the delayed overshooting patterns that are generated by the following particular sets of parameters for each model:

<u>Delayed Reaction Model (without latent financial shock)</u>: $\beta = 0.3$; $\psi = 0.97$; $\delta = -3.8$; $\phi = 0.92$; $\sigma_e = 2.42$; $\sigma_v = 0.07$;

<u>Model with Latent Financial Shock (without delayed reaction)</u> [10]: $\beta = 1$; $\rho = 0.9$; $\delta = -1.4$; $\gamma = -2.2$; $\phi = 0.92$; $\sigma_e = 2.42$; $\sigma_v = 0.07$; $\theta = 0.9$;

An important note is that when we estimate the delayed reaction model (without latent financial shock) or the model with latent financial shock (without delayed reaction) using data, we cannot get parameter estimates that can generate the overshooting pattern of the kind presented the lower panel of Figure 1.1. As will be shown later, our encompassing empirical model and the corresponding parameter estimates can generate the overshooting pattern and reproduce various puzzles related to the UIP violation.

1.3 Empirical Results: Estimation of the Reduced-Form Model

Following Engel (2016), we consider data for the G7 countries: Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. In our analysis, the home country is always the United States, and all exchange rates are expressed as dollar prices of foreign currencies. We construct a portfolio of the six currencies[11] and use it for our benchmark estimation.

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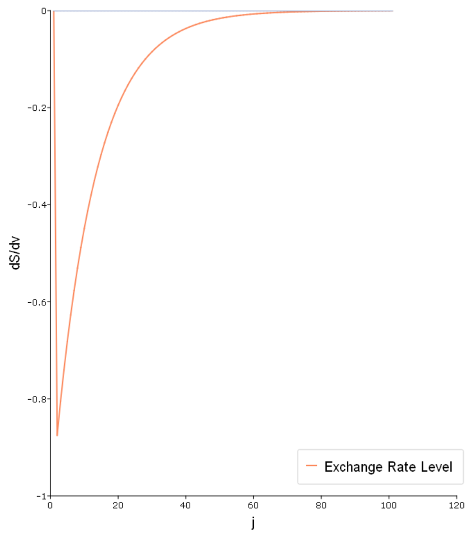
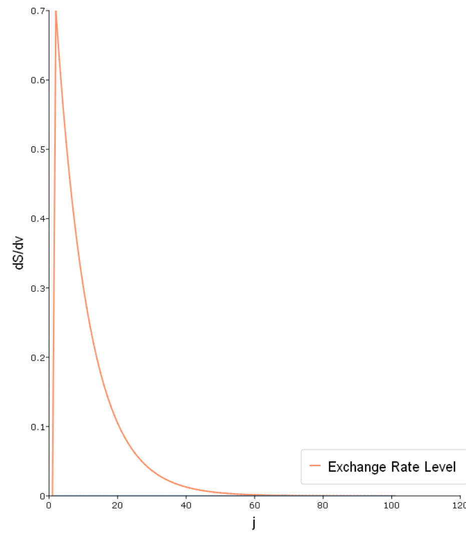
[10] Valchev (2020) shows that a higher order autoregressive process for $_t$ with complex roots can also generate the delayed overshooting battens. He shows that such dynamics can be generated by an interaction of a monetary policy a sluggish fiscal policy.

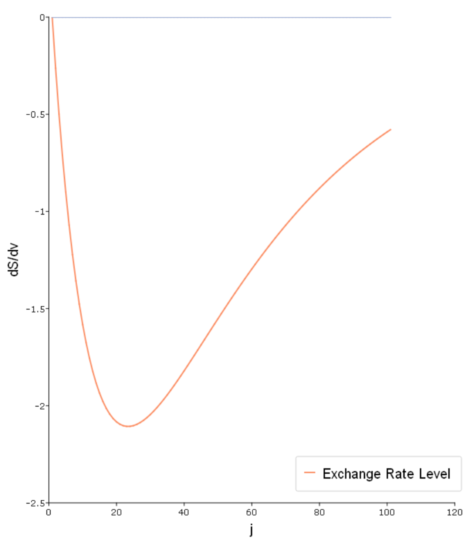
[11] Weights are measured as the value of each country's trades as a fraction of entire trades over the six countries.



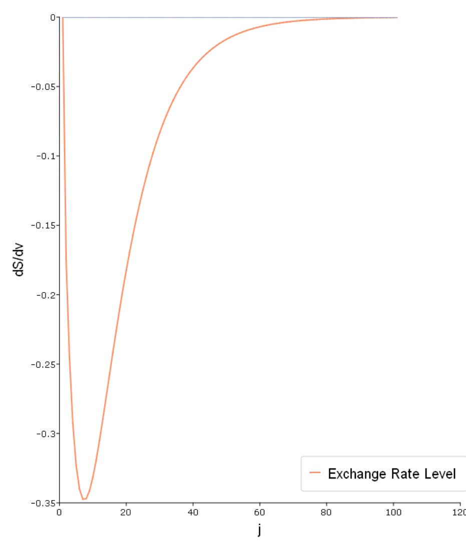
(a) Uncovered Interest Rate Parity



(b) Risk Premium Model



(c) Delayed Reaction Model



(d) Model with Latent Financial Shock

Figure 1.1: Response of the Exchange Rates to the Increase in Home Interest Rates: Various Theoretical Models with Hypothetical Parameters

Engel et al. (2022) and Bussiére et al. (2022) document that the Fama puzzle disappears in the 2000s. In particular, Engel et al. (2022) show that the evidence of interest rate differentials predicting foreign exchange returns is not stable over time and disappears altogether when interest rates are near the zero-lower bound. As they mention, interest rates lost their primacy as a policy instrument after the global financial crisis and central banks relied on unconventional monetary policies. They show that the year-on-year inflation rate differential has predictive power for the exchange rates in a sample that includes the post-global financial crisis period. This is because the inflation rate is another proxy for monetary policy, and it is not subject to zero-lower bounds.

Thus, we estimate our model using two samples. The sample we employ for estimation of the model with the interest rate differential covers the period 1980:M1-2007:M12. For a subsample (2000:M1-2019:M12) that includes the post-global financial crisis, we estimate our model by replacing the interest rate differential data with the year-on-year inflation rate differential.[12] Data on monthly exchange rates are from the Federal Reserve historic database, monthly interest rates are from Datastream, and year-on-year inflation rates are from Global Financial Data.

We first show that when the missing premium is set uncorrelated with the interest rate differential[13] as in Bacchetta and Van Wincoop (2021), key parameter estimates are not consistent with the prediction of their model. The first panel of table 1.1 presents estimates of the model parameters when γ is restricted to be zero in equation (6). The results are based on the G6 portfolio exchange rate and interest rate data for the sample that covers 1980:M1-2007:M12. The estimate of ψ coefficient is 0.23 and ρ coefficient is estimated to be negative. At the same time, β coefficient is still estimated to be negative.

The second and third panels of table 1.1 report estimates of the model parameters without the restriction that $\gamma = 0$. We find that α coefficient is still estimated to be very close to zero and statistically insignificant. Results for individual currencies were

[12] We include a dummy variable for the variance of the exchange rate shock (σ_ε^2) in order to control for the unusually high exchange rate volatility during the global financial crisis. We also estimated the model for a sample that covers the period 1987:M1-2007:M12. The results were qualitatively the same.

[13] This assumption amounts to set $\gamma = 0$ in our reduced-form model.

Table 1.1: Estimated Parameter

	Restricted Model		**Unrestricted Model**		**BW Model($\gamma = 0$)**	
ψ	0.97	(0.04)	0.93	(0.07)	0.23	(0.13)
ρ	0.04	(0.07)	0.04	(0.06)	-0.16	(0.79)
β	0.23	(0.14)	0.14	(0.17)	-0.97	(0.77)
γ	5.25	(2.22)	5.02	(2.24)	0.00	-
θ	0.94	(0.07)	0.9	(0.08)	0.00	(0.11)
ϕ	0.92	(0.02)	0.92	(0.02)	0.92	(0.02)
σ_e	2.42	(0.09)	2.42	(0.09)	2.42	(0.09)
σ_v	0.07	(0.00)	0.07	(0.00)	0.07	(0.003)
δ	-8.28	(1.97)	-8.52	(1.97)	-8.21	(1.95)
Lik	-344.58		-343.99		-344.73	

i) In the parentheses are the standard errors qualitatively the same, regardless of whether we employed the interest rate differential data or the year-on-year inflation differential data. In what follows, we thus report the empirical results based on the following reduced-form model in which α is constrained to be 0:

$$\begin{aligned} \Delta s_{t+1} &= \psi \Delta s_t + \beta x_t + \gamma v_t + \delta v_{t+1} + e_{t+1} - \theta e_t \\ x_{t+1} &= \phi x_t + v_{t+1} \\ \begin{bmatrix} e_{t+1} \\ v_{t+1} \end{bmatrix} &\sim i.i.d.N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix} \right) \end{aligned} \tag{1.11}$$

where $x_{t+1} = (i_{t+1} - i^*_{t+1})$ or $(\pi_{t+1} - \pi^*_{t+1})$

1.3.1 Results Based on the Interest Rate Differential Data [1980:M1-2007:M12]

Table 1.2 reports the results obtained based on the interest rate differential data (1980:M1-2007:M12). As large p-values for the likelihood ratio tests reported in the second last row indicate, we fail to reject the long-run restriction that the UIP holds in the long run for all countries except Japan. For Japan, the long-run restriction is rejected even at a 1% significance level. Thus, for all the countries except Japan, we report the parameter estimates obtained by imposing the long-run restriction.

Estimates of the β parameter range between 0.11-0.31. They are statistically significant at a 5% level for all the countries except Germany and Japan. Furthermore, they are all statistically different from 1. These results are consistent with the predictions of Bacchetta and Van Wincoop's (2021) delayed reaction model of the exchange rate.

Estimates of the ψ parameter, which range between 0.87-0.97 (excluding Japan), suggest that the expected exchange rate change is highly persistent. This result, combined with the estimates of the α parameter being very close to 0 and statistically insignificant, suggests that the main source of the high persistence in the expected exchange rate change is the delayed reaction of the exchange rate, with persistence in the latent financial shock playing the little role. The estimates of the ψ parameter, combined with those of the β parameter, provide evidence in support of the delayed reaction hypothesis[14].

[14] In Bacchetta and Van Wincoop's (2021) model, the ψ parameter is a proxy for the degree of the delayed

Table 1.2: Estimation of the Model (with Interest Rate Differential) [1980:1M - 2007:12M]

	G6	**UK**	**Canada**	**France**	**Germany**	**Italy**	**Japan**
ψ	0.96	0.97	0.97	0.92	0.95	0.87	0.46
	(0.04)	(0.02)	(0.03)	(0.07)	(0.07)	(0.11)	(0.37)
β	0.26	0.24	0.29	0.51	0.11	0.31	-1.80
	(0.14)	(0.11)	(0.14)	(0.31)	(0.08)	(0.18)	(1.25)
γ	5.17	1.34	-2.23	1.91	7.29	0.69	7.10
	(2.26)	(2.56)	(1.65)	(1.48)	(3.04)	(1.45)	(3.41)
θ	0.92	0.97	0.95	0.85	0.92	0.80	0.45
	(0.06)	(0.03)	(0.03)	(0.09)	(0.10)	(0.13)	(0.38)
ϕ	0.92	0.93	0.89	0.79	0.96	0.87	0.96
	(0.02)	(0.02)	(0.02)	(0.03)	(0.01)	(0.03)	(0.02)
σ_e	2.42	2.91	1.60	3.00	3.06	2.97	3.25
	(0.09)	(0.11)	(0.06)	(0.12)	(0.12)	(0.11)	(0.13)
σ_v	0.07	0.07	0.06	0.16	0.06	0.16	0.06
	(0.003)	(0.003)	(0.002)	(0.006)	(0.002)	(0.006)	(0.002)
δ	-8.29	-4.94	-0.29	-4.32	-10.13	-3.17	-9.58
	(1.97)	(2.33)	(1.45)	(1.05)	(2.74)	(1.07)	(3.13)
LR test	0.25	0.54	0.38	0.65	0.18	0.14	0.0003
β^{Fama}	-1.60	-2.42	-1.03	-0.30	-1.34	0.62	-2.73
	(0.80)	(0.86)	(0.67)	(0.64)	(0.77)	(0.52)	(0.90)

i) G6 is the weighted average of the G6 (United Kingdom, Canada, France, Germany, Italy, and Japan) exchange rates relative to the US dollar.

ii) In the parentheses are the standard errors.

iii) LR test refers to the likelihood ratio test for the long-run restriction that the UIP holds in the long run. Reported are the p-values.

Estimates of the γ parameter, which measures the effect of the interest rate shock on the missing premium, are all positive except for Japan and statistically significant for the G6 portfolio and Germany. This positive γ is consistent with the positive correlation between the interest rate differential and the latent component in the estimation results of Dahlquist and Pénasse (2021). We note that the missing premium positively correlated with the interest rate differential does not have an interpretation of the financial shock[15].

Estimates of the δ parameter, which measures the effect of the interest rate shock on the current exchange rate, are negative for all currencies. They are statically significant at a 5% level for all countries except for Canada.[16] The negative δ can be decomposed into two components. First, a contractionary monetary shock leads to a concurrent appreciation of the exchange rate. Second, the expected future depreciations due to the missing premium also lead to the current appreciation of the exchange rate.

1.3.2 Results Based on the Inflation Differential Data [2000:M1-2019:M12]

Table 1.3 reports the results obtained based on the inflation rate differential data (2000:M1-2019:M12). As the likelihood ratio test results suggest, we cannot reject the null hypothesis that the inflation shock does not have a long-run effect on the level of the exchange for any of the countries.

Estimates of the β parameter are positive and statistically significant at a 5% significance level for all countries, with an exception of Japan. The estimates of the ψ parameter range between 0.90-0.97, suggesting that the expected exchange rate change is highly persistent. These results provide evidence in favor of the delayed reaction of the exchange rate to the inflation rate.[17]

reaction, with $\psi = 0$ along with $\beta = 1$ implying no delayed reaction with the UIP holding in the short run.

[15]Itskhoki and Mukhin (2021) and Valchev (2020) explain the Fama puzzle and the predictability reversal puzzle based on a general equilibrium model that focuses on the financial shock in the absence of the delayed reaction feature. In their models, the financial shock is negatively correlated with the interest rate differential.

[16]This is consistent with Engel et al. (2022), who argue that a tighter monetary policy does not lead to an immediate appreciation for oil exporters, such as Canada and Norway, unlike other developed countries.

[17]For Germany and France, the estimates of β are higher than 1, and we cannot reject the null that $\beta = 1$ at a 5% significance level. However, we note that Bacchetta and Van Wincoop's (2021) model of delayed

Table 1.3: Estimation of the Model (with Inflation Rate Differential) [2000:1M - 2019:12M]

	G6	**UK**	**Canada**	**France**	**Germany**	**Italy**	**Japan**
ψ	0.95	0.96	0.97	0.96	0.96	0.97	0.90
	(0.04)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)	(0.09)
β	0.65	0.41	0.34	1.12	1.19	0.63	-0.07
	(0.32)	(0.22)	(0.31)	(0.50)	(0.52)	(0.32)	(0.13)
γ	-17.77	-14.86	-8.90	-21.87	-17.81	-15.63	4.04
	(5.78)	(5.59)	(6.42)	(7.05)	(6.20)	(6.35)	(5.00)
θ	0.94	0.96	0.96	0.96	0.95	0.97	0.89
	(0.05)	(0.03)	(0.04)	(0.04)	(0.04)	(0.05)	(0.10)
ϕ	0.92	0.94	0.91	0.90	0.89	0.92	0.95
	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)
σ_e	1.97	2.29	2.27	2.51	2.52	2.52	2.56
	(0.09)	(0.11)	(0.11)	(0.12)	(0.12)	(0.12)	(0.12)
σ_v	0.03	0.03	0.03	0.03	0.03	0.03	0.04
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
δ	9.44	7.76	5.07	10.95	7.18	7.62	-2.45
	(5.41)	(5.30)	(5.93)	(6.35)	(5.57)	(5.82)	(4.91)
α	0.56	0.65	0.90	0.64	0.62	0.67	0.51
	(0.24)	(0.26)	(0.29)	(0.25)	(0.25)	(0.26)	(0.23)
LR test	0.29	0.20	0.56	0.86	0.28	0.49	0.69
β^{Fama}	-5.25	-3.20	-4.66	-9.75	-7.76	-7.22	0.43
	(2.01)	(1.70)	(2.42)	(2.73)	(2.52)	(2.22)	(1.43)

i) G6 is the weighted average of the G6 (United Kingdom, Canada, France, Germany, Italy, and Japan) exchange rates relative to the US dollar.

ii) In the parentheses are the standard errors.

iii) LR test refers to the likelihood ratio test for the long-run restriction that the UIP holds in the long run. Reported are the p-values.

However, the estimates of γ and δ are significantly different from those obtained with the interest rate differential data. Estimates of the γ parameter are all negative and significant at a 5% significance level except for Japan. Thus, an increase in home inflation causes not an immediate, but an expected appreciation of the home currency. This negative γ can be explained if the actual monetary policy change occurs with delay given the change in the inflation rates, and investors react to the actual interest rate changes. Because the expected future appreciations are associated with an immediate depreciation, δ is estimated to be positive.

1.4 The Non-monotonic Pattern of the UIP violations Implied by the Model

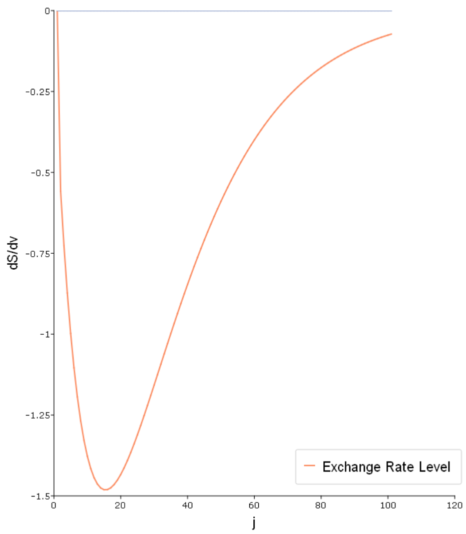
In what follows, we show that our empirical model and the parameter estimates can explain and generate the four puzzles mentioned earlier.

The Delayed Overshooting puzzle

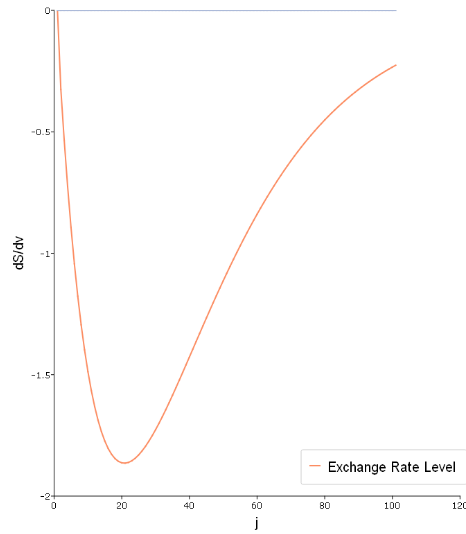
The estimates of the key parameter of our model are consistent with the predictions of the delayed response model of Bacchetta and Van Wincoop (2021) for all currencies excluding the Japanese Yen. This section exhibits how these parameter estimates generate the delayed overshooting of the exchange rate.

Figure 1.2 depicts the impulse responses of the exchange rate to a one standard error deviation increase in the home interest rate for the sample that covers the period 1980:M1-2007:M12. For all countries except Japan, our model, and the parameter estimates generate the delayed overshooting, with the maximum appreciation occurring 7-21 months after the shock. Countries with higher estimates of ψ have longer periods of appreciation at short horizons, with maximum appreciation occurring with longer lags. For Japan, the impulse-response pattern resembles that of the risk premium model in the second panel of Figure 1.1, except that the maximum depreciation occurs with a lag in response to an interest rate shock.

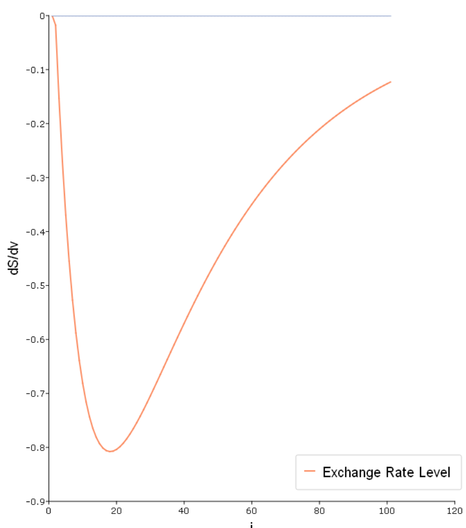
reaction bears no implication on the upper limit of the β parameter when the interest rate differential is replaced by the inflation differential.



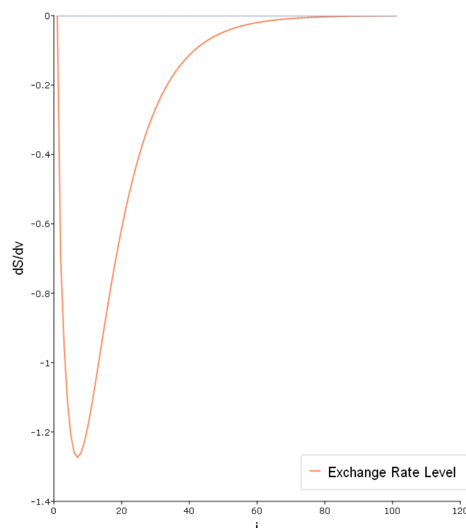
(a) G6 Portfolio



(b) United Kingdom

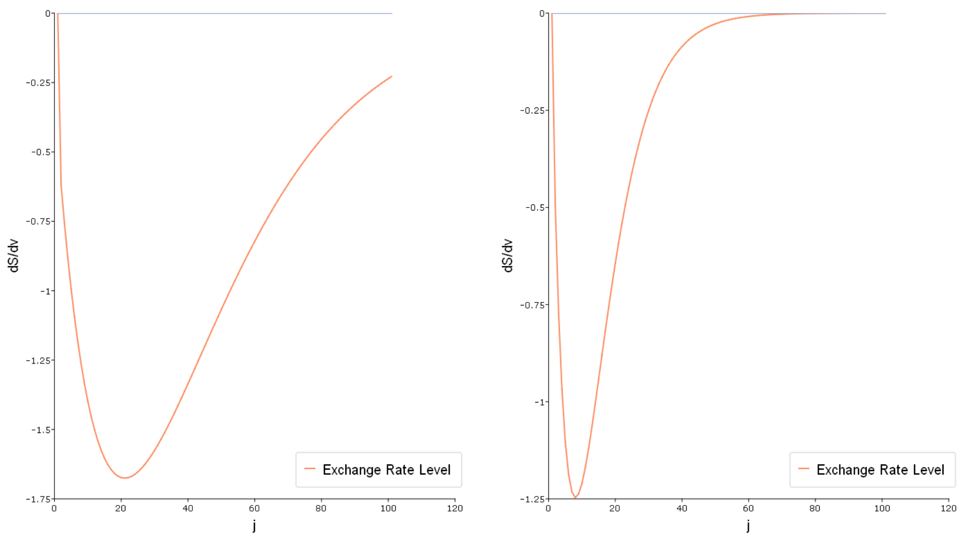


(c) Canada



(d) France

Figure 1.2: Response of the Exchange Rates to the Increase in Home Interest Rates from Our Empirical Model with Interest Rate Differential Data [Sample: 1980—2007]



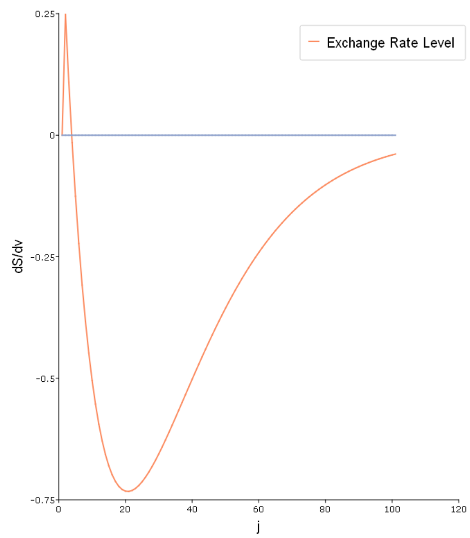
(e) Germany (f) Italy

Figure 1.2: (Continued)

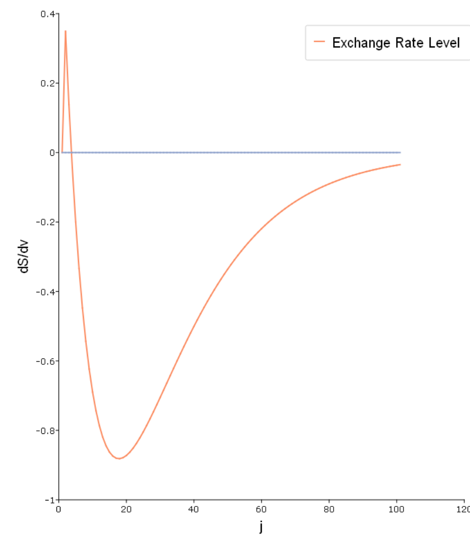
Figure 1.3 depicts the impulse responses of the exchange rate to a one standard error deviation increase in the home inflation rate for the sample that covers the period 2000:M1-2019:M12. For all currencies except for the Japanese Yen, we have delayed overshooting patterns. For these currencies, however, due to the positive δ parameter estimates, the exchange rate depreciates on impact at time t, and then gradually appreciates starting from time $t+1$. The dynamics of the exchange rate from time $t+1$ exhibit the delayed overshooting pattern, with the maximum appreciation occurring between 17-22 months after the shock. The initial depreciation is because of the positive δ, which reflects the delayed change in monetary policy given the change in the inflation rates.

The Fama Puzzle and the Predictability Reversal Puzzle

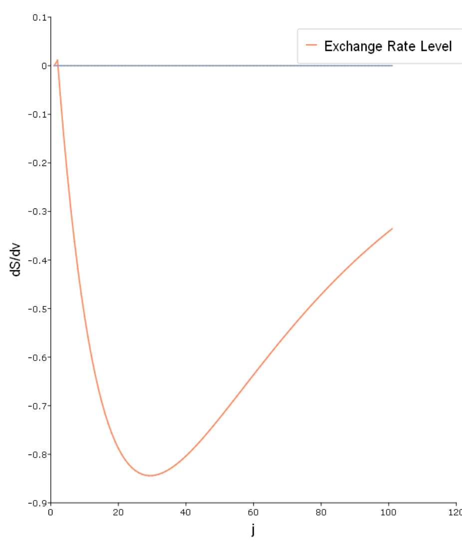
Consider the following regression:



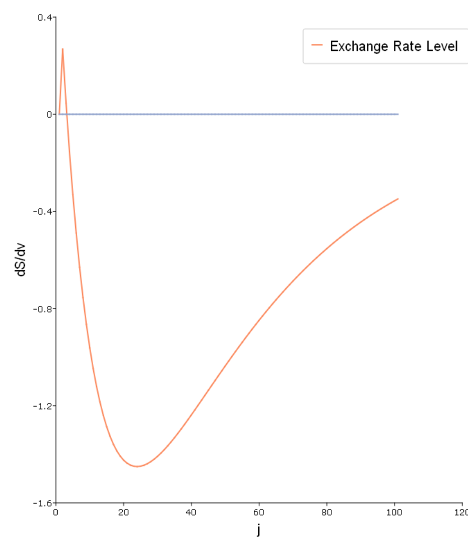
(a) G6 Portfolio



(b) United Kingdom

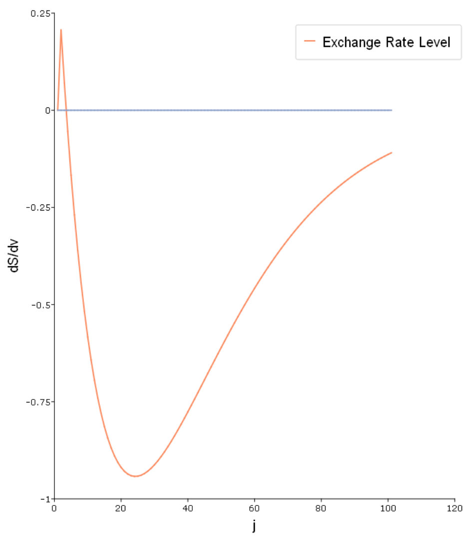


(c) Canada

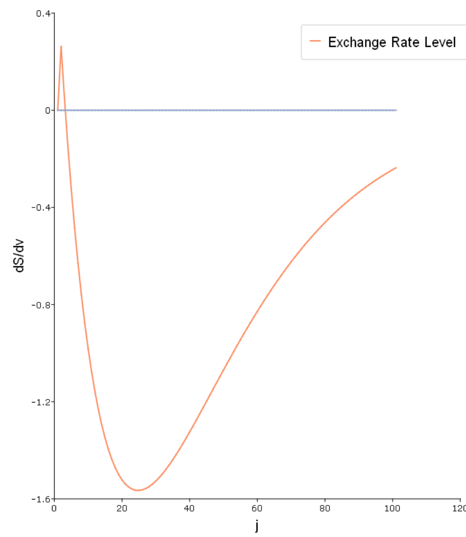


(d) France

Figure 1.3: Response of the Exchange Rates to the Increase in Home Interest Rates from Our Empirical Model with Inflation Rate Differential Data [Sample: 1987—2019]



(e) Germany



(f) Italy

Figure 1.3: (Continued)

$$\Delta s_{t+k} - (i_{t+k-1} - i^*_{t+k-1}) = \alpha_{1,k} + \alpha_{2,k}(i_t - i^*_t) + \varepsilon_{t+k}, \quad k = 1, 2, 3, \cdots , \tag{1.12}$$

where $\alpha_{2,1} = \beta^{Fama} - 1$, with β^{Fama} referring to the Fama regression β. The empirical finding that an estimate of $\alpha_{2,1}$ is negative and statistically significant, as in the Fama puzzle. Bacchetta and Van Wincoop (2010) empirically find that the $\alpha_{2,k}$ estimate is negative and statistically significant at short horizons but becomes insignificant or even positive over longer horizons. These findings are referred to as the predictability reversal puzzle.

We show that our empirical model and the parameter estimates can reproduce both puzzles by employing the following version of the excess return regression:

$$\hat{E}_t(\Delta s_{t+k} - x^\tau_{t+k}) = \alpha_{1,k} + \alpha_{2,k} x^\tau_t + \epsilon^\tau_{t+k}, \quad \tau = int, \quad or \quad inf, \\ k = 1, 2, 3, ..., \tag{1.13}$$

where x^{int}_t and x^{inf}_t are the interest rate differential and the year-on-year inflation differential, respectively. Here, $\hat{E}_t(\Delta s_{t+k} - x^{int}_{t+k-1})$ is an estimate of the ex-ante excess return on foreign deposits from $t+k-1$ to $t+k$. $\hat{E}_t(\Delta s_{t+k} - x^{inf}_{t+k-1})$ would be proportional to $\hat{E}_t(\Delta s_{t+k} - x^{int}_{t+k-1})$ in case the ex-ante real interest rates are constant or subject to a concurrent regime shift in both home and foreign countries.[18] We estimate these ex-ante excess returns from our model conditional on the parameter estimates and data.

Figures 1.4 presents estimates of $\alpha_{2,k}$, $k = 1, 2, ..., 100$, for the model with the interest rate differential (1980:M1-2007:M12). For all countries except Japan, $\alpha_{2,k}$ is negative for the first few k, but it turns positive at longer horizons. The timing of the reversal ranges from 3 to 16 months. Figure 1.5 presents the results for the model with the inflation rate differential (2000:M1-2019:M12). Similar to the regression results with the interest rate differential, the estimate of $\alpha_{2,k}$ is negative over short time horizons, but it changes the sign after 11-18 months, depending on the currency.

The Engel Puzzle

The Engel puzzle is an observation by Engel (2016) that high-interest rate currencies are stronger than implied by uncovered interest parity in levels, i.e., an observation that:

$$\hat{cov}(\sum_{j=0}^{\infty}(\Delta s_{t+j+1} - (i_{t+j} - i^*_{t+j})), i_t - i^*_t) > 0, \qquad (1.14)$$

where, $\hat{cov}(.)$ refers to the estimated covariance. Under the UIP, this measure should be close to 0. This puzzle is directly related to the predictability reversal puzzle. As Engel (2016) shows, for equation (1.14) to hold, the negative expected excess returns on the foreign currency at short horizons should be more than offset by the positive expected excess returns at longer horizons. In order to test whether our empirical model can reproduce the Engel puzzle, we estimate and run the following regression equation as in Engel (2016):

$$\hat{E}_t(\sum_{j=0}^{\infty}(\Delta s_{t+j+1} - x^{\tau}_{t+j})) = \zeta_\alpha + \beta_\alpha x^{\tau}_t + \epsilon^{\tau}_{\alpha,t}, \quad \tau = int \ \ or \ \ inf, \qquad (1.15)$$

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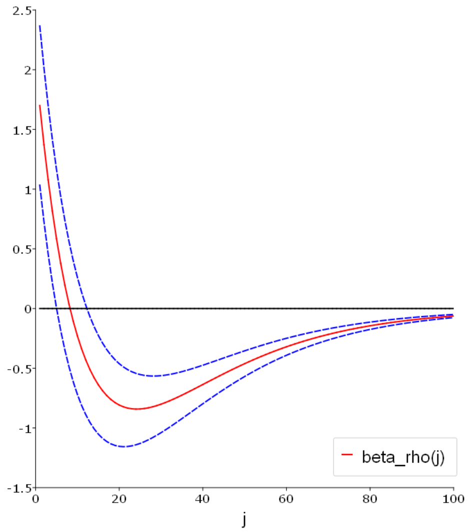
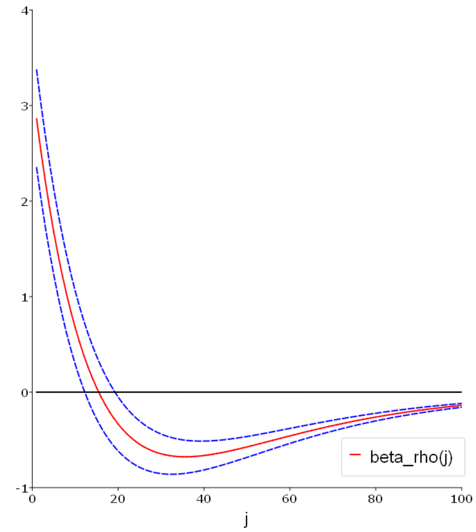
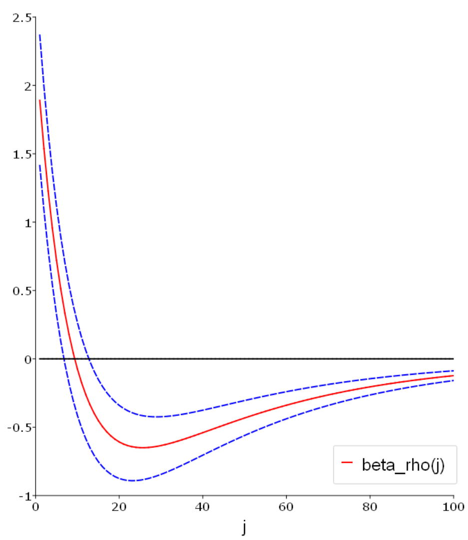
[18]Garcia and Perron (1996), for example, show that the U.S. ex-ante real interest rate (for the sample that covers the period 1961-1986) is constant subject to occasional jumps caused by important structural events.



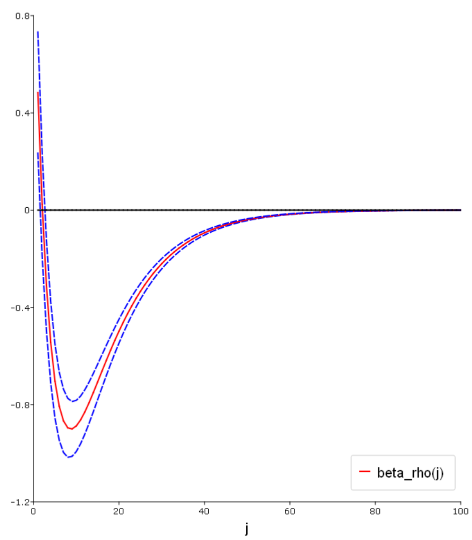
(a) G6 Portfolio



(b) United Kingdom

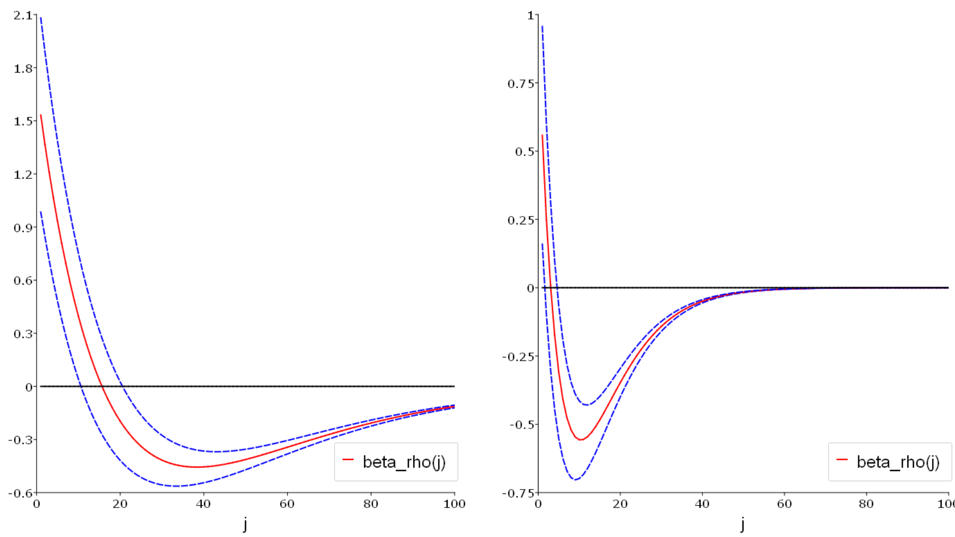


(c) Canada



(d) France

Figure 1.4: Regression of Ex-ante Excess Returns on Interest Rate Differentials [Sample: 1980—2007]



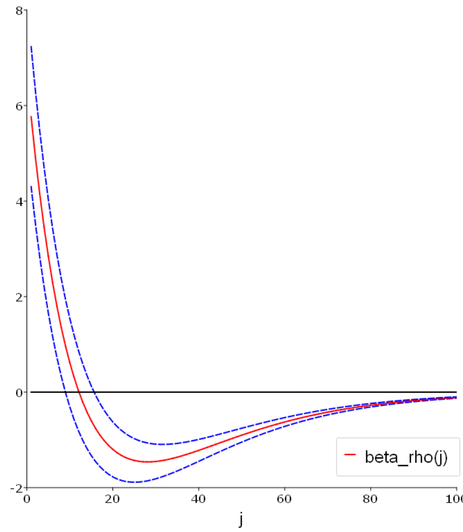
(e) Germany (f) Italy

Figure 1.4: (Continued)

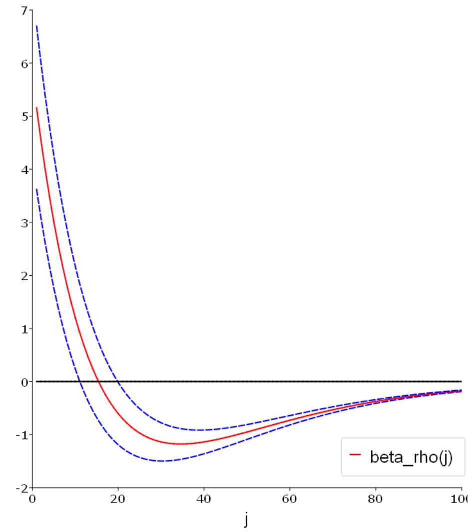
where x_t^{τ} is either the interest rate differential or the inflation rate differential; and $\hat{E}_t(\cdot)$ refers to the conditional expectation obtained based on our model and the parameter estimates.

The left panel of table 1.4 reports the results with interest rate differential for the sample that covers the period 1980:M1-2007:M12. For all countries, the estimate of β_α is positive, which means that $\hat{cov}(\sum_{j=0}^{\infty} \Delta s_{t+j} + i^*_{t+j-1} - i_{t+j-1}, i_t - i^*_t) > 0$. As in Engel (2016), the standard errors are big, and the estimates of β_α are significant at a 10% level for the G6 portfolio and two individual currencies.

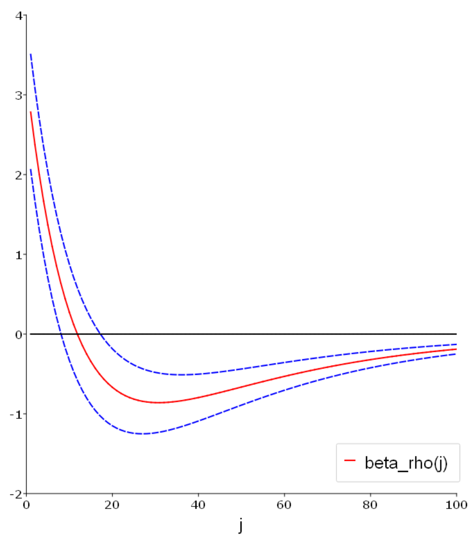
Our empirical model based on the year-on-year inflation differential data can also reproduce the Engel puzzle for a subsample that includes the post-global financial crisis period (2000:M1-2019:M12). The right panel of table 1.4 shows the results. Estimates of β_α are positive for all currencies except the Japanese Yen. They are significant, at least at a 10% level, for the G6 portfolio and three individual currencies.



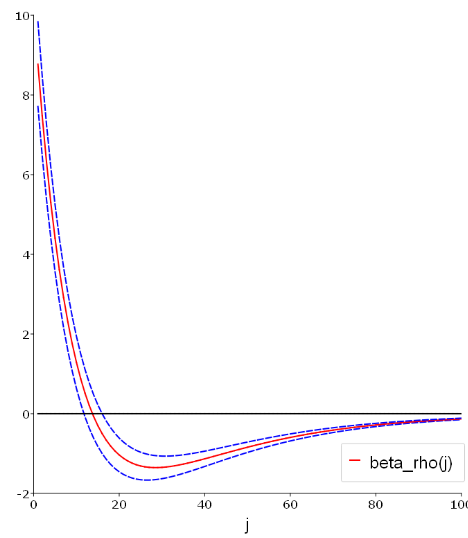
(a) G6 Portfolio



(b) United Kingdom

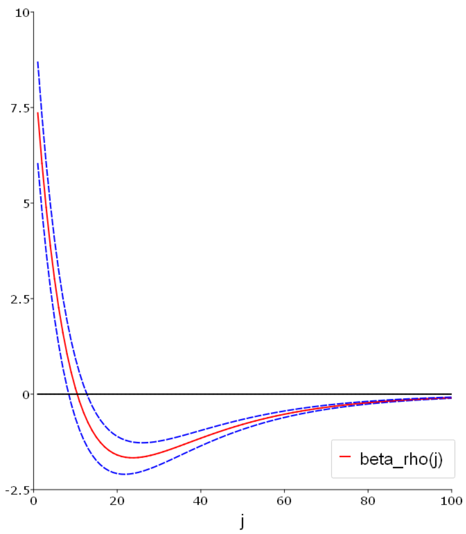


(c) Canada

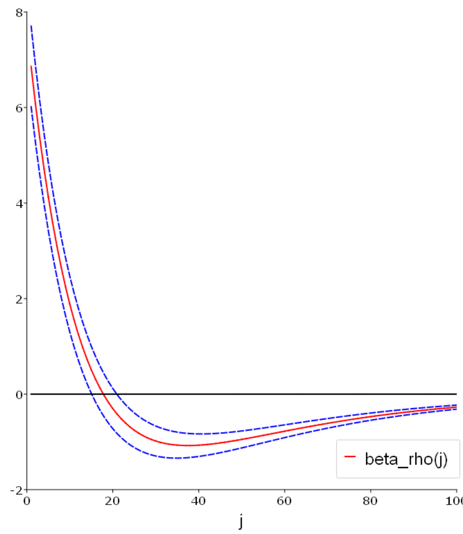


(d) France

Figure 1.5: Regression of Ex-ante Excess Returns on Inflation Differentials [Sample: 2000—2019]



(e) Germany



(f) Italy

Figure 1.5: (Continued)

Table 1.4: Test Results for the Engel Puzzle

	1980M1-2007M12 $\hat{\beta}_{\alpha}^{int}$		**2000M1-2019M12** $\hat{\beta}_{\alpha}^{inf}$	
G6	27.89**	(16.21)	39.5**	(23.31)
UK	14.41	(14.92)	37.61**	(19.72)
Canada	27.13	(33.15)	48.2***	(19.66)
France	14.65***	(4.05)	9.47	(12.46)
Germany	14.04*	(10.62)	31.89**	(17.40)
Italy	8.15***	(2.34)	20.8	(20.84)
Japan	-21.78	(0.90)	-6.39	(2.21)

i) In the parentheses are the Newey-West standard errors

ii) Levels of significance are indiciated by '***", '**', and '*' for 1%, 5%, and 10%, respectively.

1.5 The Role of the Latent Financial Shock in Explaining the Delayed Overshooting and the Engel Puzzle: Counterfactual Analysis

In this section, we evaluate the role the missing premium plays in generating four puzzles in Section 1.4. For this purpose, we perform the counterfactual analysis and provide an answer to the following question: What would happen to the overshooting pattern and the estimation results for equations (1.13) and (1.15) if it were not for the missing premium?

In the counterfactual analysis, we assume that the missing premium exists, but it is not correlated to the change in the interest rate differential[19]. This assumption implies that the indirect effect of interest rates on the expected exchange rate through the missing premium is muted. Thus, we set $\rho_{\eta v}$ in our model to be zero and obtain the impulse-response functions as well as the estimation results for equations (1.13) and (1.15) conditional on the rest of the estimated structural parameters and data. Differences between the results based on actual data and the results in the counterfactual analysis can be interpreted as an effect of the missing premium.

Because the counterfactual analysis should be based on the structural parameters, we first solve for the parameters of the structural unobserved component model that corresponds to the estimated reduced-form model parameters. As described in Morley et al. (2003), AR parameter ψ and the parameter for the interest rates β are exactly identified, because they are the same in both structural and reduced-form models. Variances and correlations of structural shocks are identified using the moments on the moving average sides of the structural model:

$$\begin{aligned}
\sigma_\varepsilon^2 + \sigma_\eta^2 &= C_1, \\
\rho_{\varepsilon v}\sigma_\varepsilon\sigma_v &= C_2, \\
\rho_{\varepsilon\eta}\sigma_\varepsilon\sigma_\eta &= C_3, \\
\rho_{\eta v}\sigma_\eta\sigma_v &= C_4,
\end{aligned} \tag{1.16}$$

[19] This is in line with the assumption about the financial shock in Bacchetta and Van Wincoop (2021), even though the missing premium in our model can not be interpreted as the financial shock. Also, it is still important to incorporate the missing premium, to match the fact that the expected exchange rate change is very persistent but the observed exchange rate change is not.

where left-hand sides of the equations are the moments implied by the structural model and C_1, C_2, C_3, and C_4 are computed from data[20], i.e. from the reduced-form parameter estimates as in the right-hand sides of equation (A.14). We find that the model is not identified, because the four moments above are not sufficient to identify five structural parameters (σ_ε, σ_η, $\rho_{\varepsilon\eta}$, $\rho_{\varepsilon v}$, and $\rho_{\eta v}$).[21] Thus, we fix one of the structural parameters $\rho_{\varepsilon\eta} = -0.95,\ -0.75, \cdots,\ -0.05$[22] and recover rest of structural parameters by solving nonlinear system of equations in (1.16). Panel A of Table 1.5 summarizes implied structural parameters of G6 Portfolio for each fixed $\rho_{\varepsilon\eta}$.

Then, for each set of structural parameters, we perform the counterfactual analysis. We first derive the counterfactual reduced-form parameters implied by $\rho_{\eta v} = 0$, by employing the following moments conditions, including the long-run restriction:

$$
\begin{aligned}
C_1 &= (\delta^2 + \gamma^2)\sigma_v^2 + (1 + \theta^2)\sigma_e^2, \\
C_2 &= \delta\sigma_v^2, \\
C_3 &= \gamma\delta\sigma_v^2 - \theta\sigma_e^2, \\
C_4 &= \gamma\sigma_v^2, \\
\delta &= -\left(\gamma + \left(\frac{\beta}{1-\phi}\right)\right),
\end{aligned}
\tag{1.17}
$$

where C_1, C_2, C_3, and C_4 are obtained from the right hand side of equation (A.12) for given structural parameters and $\rho_{\eta v} = 0$. Panel B of Table 1.5 summarizes the counterfactual reduced-form parameters. The counterfactual γ, which is zero, implies that the effects of the latent financial shock are muted. The counterfactual θ decreases when $\rho_{\varepsilon\eta}$ approaches zero, because θ is a function of $\rho_{\varepsilon\eta}$. At the same time, the counterfactual δ is much smaller than the estimated one in absolute value, to satisfy the long-run restriction in the last equation of (1.17). With these counterfactual reduced-form parameters, we generate the impulse response function and also investigate the counterfactual pattern of

[20] Refer to Appendix C for derivations of moments.

[21] σ_v is identified from the equation for the interest rate differential.

[22] Negative θ implies that $\rho_{\varepsilon\eta}$ must be negative.

the ex-ante excess return. By noting that the state vector in equation (1.10) is given by $\xi_t = [\Delta s_t,\ (i_t - i_t^*),\ (i_{t-1} - i_{t-1}^*),\ e_t,\ v_t]'$, we first obtain $E(\xi_t|I_t)$ for $t = 1, 2, \cdots, T$ by running the Kalman filter conditional on estimated parameters and data. Then the counterfactual future ex-ante excess returns are obtained as follows: $E_t^{CF}(\Delta s_{t+j+1} - (i_{t+j} - i_{t+j}^*)$ is equal to $[i_1' F^{*j} E(C_t|I_t) - i_3' F^{*j} E(C_t|I_t)]$, where i_j is a selection vector whose jth element is 1 and all other elements are all 0. F^* is the counterfactual F matrix with elements of the F matrix in equation (1.10) being replaced by the corresponding counterfactual reduced-form parameters.

Table 1.5: Implied Structural Parameters and Counterfactual Reduced-Form Parameters for Different Values of $\rho_{\varepsilon\omega}$: G6 Portfolio [1980:1M-2007:12M]

A. Implied Structural Parameters (for different values of $\rho_{\varepsilon\eta}$)					B. CF Reduced-form Parameters (when $\rho_{\eta v}$ is set to zero)			
$\rho_{\varepsilon\eta}$	σ_ε	σ_η	$\rho_{\varepsilon\eta}$	$\rho_{\eta v}$	θ	δ	σ_e	γ
-0.95	2.42	2.42	-0.23	0.13	0.73	-3.12	2.75	0.00
-0.85	2.47	2.47	-0.21	0.10	0.56	-3.12	3.05	0.00
-0.75	2.56	2.56	-0.19	0.05	0.45	-3.12	3.29	0.00
-0.65	2.46	2.46	-0.15	0.13	0.37	-3.12	3.22	0.00
-0.55	2.47	2.47	-0.13	0.09	0.30	-3.12	3.34	0.00
-0.45	2.42	2.42	-0.10	0.07	0.24	-3.12	3.32	0.00
-0.35	2.53	2.53	-0.08	0.05	0.18	-3.12	3.51	0.00
-0.25	2.46	2.46	-0.06	0.04	0.13	-3.12	3.44	0.00
-0.15	2.63	2.63	-0.04	0.04	0.08	-3.12	3.71	0.00
-0.05	2.48	2.48	-0.01	0.04	0.03	-3.12	3.50	0.00
					C. Estimated Reduced-form Parameters			
					θ	δ	σ_e	γ
					0.92	-8.29	2.42	5.17

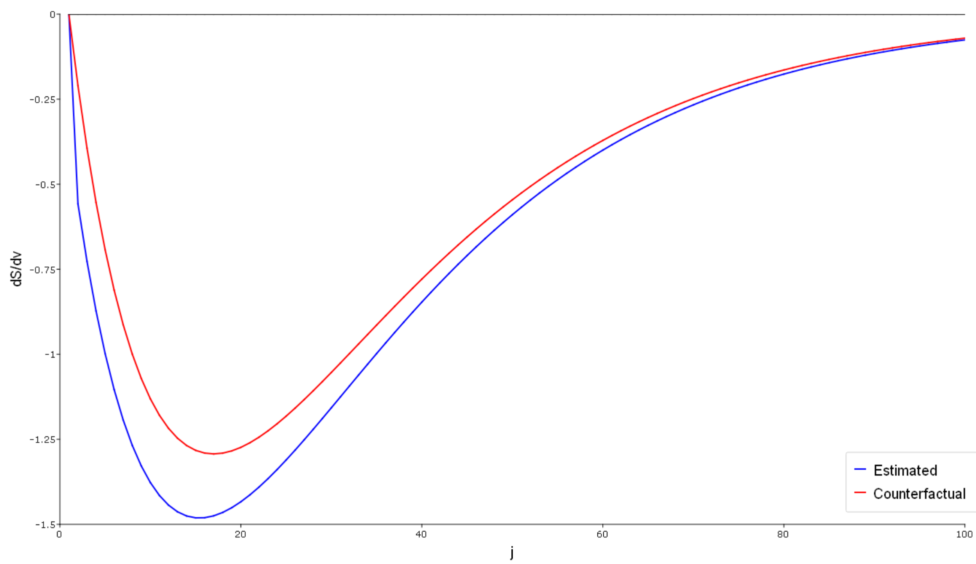


Figure 1.6: Responses of the exchange rate level to a monetary contraction

In figure 1.6, the impulse-response function with the effects of the latent financial shock muted is plotted against that conditional on all the estimated structural parameters. For the home monetary contraction, home currency initially appreciates less because the counterfactual δ is smaller. However, without the effect of the missing premium, subsequent appreciations are larger, and the timing of the maximum appreciation is later than that observed in the data.

Because of larger appreciations, the counterfactual predictability reversal pattern in Figure 1.7 shifts downward compared to that based on the estimated parameters. Without the missing premium, which is positively correlated with the interest rate differential, higher interest rate currency is expected to appreciate more in the short run and depreciate less in the longer run than those in the data.

Finally, we show that the missing premium plays a crucial role in generating the Engel puzzle. Table 1.6 summarizes the results of the counterfactual analysis for different values of $\rho_{\varepsilon\eta}$. The counterfactual $\hat{\beta}_\alpha$s are smaller than $\hat{\beta}_\alpha$ obtained from data and it turns to negative when $\rho_{\varepsilon\eta}$ is lower than -0.55. At the same time, they are insignificant for all $\rho_{\varepsilon\eta}$.

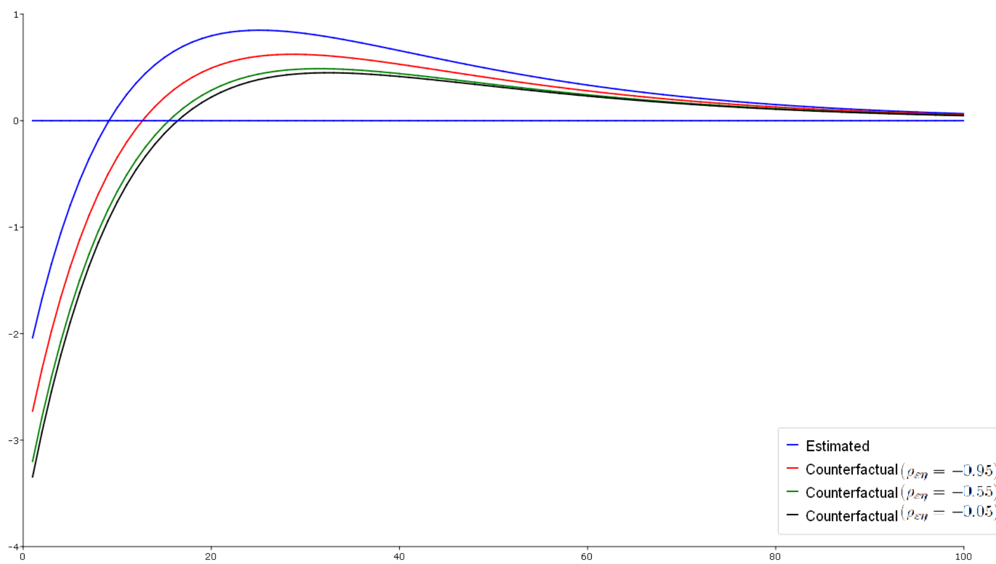


Figure 1.7: Responses to a change in the interest rate differential

These results show that even though the counterfactual ex-ante returns exhibit the pattern of reversal, expected depreciations at longer horizons are not enough to offset the short-run appreciations and generate significantly negative $\hat{\beta}_\alpha$, observed in the data.

1.6 Conclusion

This paper proposes an empirical model of the exchange rates by incorporating the delayed reaction of the exchange rate and the missing premium. Our model can empirically account for the puzzles related to the non-monotonic pattern of the UIP violations: the Fama puzzle, the predictability reversal puzzle, the delayed overshooting puzzle, and the Engel puzzle. Therefore, our model provides a comprehensive explanation of the relationship between the interest rate and the exchange rates. Both the delayed reaction of the exchange rate and the missing premium play crucial roles in explaining these puzzles.

Table 1.6: Engel Regression Results with Counterfactual Parameters: G6 Portfolio [1980:1M-2007:12M]

$\rho_{\varepsilon\eta}$	$\hat{\beta}_{\alpha}^{int}$	
-0.95	11.27	(27.22)
-0.75	2.9	(26.15)
-0.55	0.08	(27.16)
-0.45	-0.85	(27.43)
-0.25	-2.29	(27.83)
-0.05	-3.37	(28.14)
Estimated	27.59	(16.21)

i) In the parentheses are the Newey-West standard errors

Chapter 2

THE LONG-RUN RISKS MODEL REVISITED: EXTENDED MODEL AND EMPIRICAL EVIDENCE

2.1 Introduction

Since Mehra and Prescott (1985), explaining the 6% equity premium given the smooth aggregate consumption data has been one of the serious challenges to consumption-based asset pricing models. Among many attempts to solve this puzzle, the long-run risks model, first proposed by Bansal and Yaron (2004), has attracted a great deal of attention. Bansal and Yaron (2004) argued that a small, but highly persistent predictive component in consumption and dividend growth and time-varying economic volatility, in conjunction with Epstien-Zin preferences, can generate high equity premium observed in data. Voluminous subsequent works have also shown that the long-run risks can explain various salient features of asset market data. Bansal et al. (2007, 2010, 2011) show that the long-run risks model can match the key asset market facts. Croce et al (2014) generate a sizable risk premium with a downward-sloping equity term structure, using the long-run risks model with a limited information assumption. Colacito and Croce (2011) show that the long-run risks model can also explain a wide range of international finance puzzles, in addition to the equity premium puzzle in the stock market.

One strand of the long-run risks literature has focused on the empirical evaluation of the long-run risks model. This is because it is not easy to argue whether the consumption growth is serially uncorrelated as assumed in Hall's random walk hypothesis or contains a small but highly persistent component as in the long-run risks model in finite samples, even though the asset pricing implications of two models are quite different. Some papers use the simulation method to assess if the long-run risks model can explain the stylized facts about macro economy and asset market. For example, Beeler and Campbell (2012) argue that the predicting power of the log price-dividend ratio in the long-run risks model is inconsistent with the pattern observed in data. In contrast, Bansal et al. (2012) find significant empirical support for the long-run risks model using several empirical methods. Other papers investigate the existence of the long-run risks component by directly estimating the dynamics for consumption and dividend growth rates. Ma (2013) estimates a bivariate VARMA-GARCH model and argue that the model is weakly identified. Hansen et al. (2008) find strong evidence of predictable variations in consumption growth using VAR, and Bansal et al. (2016) and Schorfheide et al. (2018) identify the highly persistent long-run risks component from data, using a GMM framework with time aggregation and a Bayesian mixed frequency approach respectively. However, all of the papers above take the specification of the consumption and dividend growth in Bansal and Yaron (2004) and Bansal et al. (2012) as given in evaluating the long-run risks model. Therefore, several assumptions made in the consumption and dividend growth process in the long-run risks model has not been empirically investigated yet, even though the equity premium implied by the long-run risks model depends on these assumptions.

This paper contributes to the literature by evaluating the impact of the restrictive aspects of the long-run risks model on the model-implied equity premium. Based on recent macro and asset pricing literature, we show that several important properties of consumption and dividend growth are not considered in the long-run risks model and these these omissions can lead to a bias in the model-implied equity premium.

First, we allow the transitory component of dividend growth to be serially correlated. In the long-run risks model, the only source of the serial correlation in consumption and dividend growth rates is the long-run growth component. Therefore, the long-run risks model predicts that the consumption and dividend growth are serially uncorrelated, after controlling for the long-run risks component. However, when we regress dividend growth on consumption growth, estimated residuals from a simple regression exhibit a strong positive serial correlation, implying additional serial correlation in the transitory component of dividend growth[1]. Ignoring this independent serial correlation in dividend growth distorts some key parameter estimates and the model-implied equity premium, as shown below.

---

[1] If the transitory component of dividend growth is i.i.d, estimated residual should be serially uncorrelated. Details are provided in the section II.

Second, we allow for non-zero correlation between the long-run risk component and the idiosyncratic components of consumption and dividend growth. Beeler and Campbell (2012) point out that the empirical patterns of the variance ratios[2] are not consistent with the prediction of the long-run risks model, because variance ratios implied by the long-run risks model always have increasing pattern but the empirical variance ratios have decreasing or flat patterns in the most part of the sample. We show that it is possible for the long-run risks model to imply the decreasing or flat patterns of the variance ratio, by introducing contemporaneous correlations between the long-run and short-run shocks. To keep the model parsimonious, the long-run risk component is assumed to be uncorrelated with idiosyncratic components of consumption or dividend growth in Bansal and Yaron (2004) and others. In this case, the variance ratios always have increasing pattern because of the highly persistent long-run component. However, if the shock to the long-run risks component and the idiosyncratic components are negatively correlated, this negative correlation may offset the positive autocorrelations from the long-run risks component and generate a decreasing pattern of variance ratios. We investigate this possibility and also analyze how this extension affects the estimates of key parameters in the model.

Lastly, we introduce regime shifts in the steady-state variances of the shocks in addition to the within regime volatility variations. Lettau et al. (2008) find evidence of a discrete shift in the steady-state variance of consumption, and show that this low-frequency movement in macroeconomic volatility is strongly correlated with the low-frequency movement in stock market[3]. Econometrically, the omission of discrete shifts may cause a significant upward bias in the estimates of the persistence parameters (Diebold (1986), Mikosch and Starica (2004), Lastrapes (1989), Lamoureux and Lastrapes (1990), and Kim and Kon (1999)). Thus, ignoring regime shifts in the steady-state variance might cause a upward bias in the persistence of time-varying volatility and the model-implied equity premium in the long-run risks model.

[2] Variance ratio is commonly used in the literature to summarize the persistence of the consumption growth

[3] This economic source of risk cannot be captured in the existing long-run risks models, which assume a constant steady-state variance.

To investigate the impact of three extensions on the parameter estimates, we estimate two model specifications by maximum likelihood: the benchmark model based on Bansal and Yaron (2004) and the extended model. We find some of the key parameter estimates in the extended model are considerably different from parameter used for the calibration in Bansal and Yaron (2004) and parameters estimated in Schorfheide et al. (2018). First, the estimated serially correlation in dividend growth is significantly positive. At the same time, the impact of the long-run risk component on the dividend growth drops from 3 to 1.5 with this extension. Second, the idiosyncratic components of consumption growth and the long-run risk component are estimated to be strongly negatively correlated, and the volatility of the long-run risk component increases compared to that in the benchmark model. Finally, we find that consumption growth exhibits a sharp increase in the probability of being in a low volatility state during the 1990s. After controlling for this regime shift, The within-regime persistence of consumption volatility falls from 0.82% to 0.55% at a quarterly frequency. This confirms our presumption that ignoring structural breaks in volatility might overstate the persistence of the time-varying volatility.

We also show the model-implied equity premium considerably changes with three extensions, by providing a solution to the extended model and deriving the model-implied equity premium using simulation method. When we compare the equity premium from the benchmark model based on Bansal and Yaron (2004) and the extended model, the model-implied equity premium drops from 6% to 2.6%. More specifically, the decrease in the equity premium can be decomposed as follows. Approximately, lower persistence of the consumption volatility and smaller factor loading for dividend growth reduces the equity premium by 0.5% and 2.5%, respectively. The negative correlation between long-run and idiosyncratic components in consumption growth and higher consumption volatility associated with relaxation of the zero-correlation restriction only account for the decline of the model-implied equity premium by 0.2%.

These result suggest that the model-implied equity premium in Bansal and Yaron (2004) or Schorfheide et al (2018) might be upward biased. In other words, in order to explain the high observed equity premium in the U.S. equity market using our extended model, we need a considerably higher risk aversion than is implied by Bansal and Yaron (2004), which is around 21.

2.2 Extensions of the Existing Long-run Risks Model

Bansal and Yaron (2004), Bansal et al. (2012), and Schorfheide et al. (2018) assume the following law of motion for consumption and dividend growth.

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_{c,t}\eta_{c,t+1}, \tag{2.1}$$

$$x_{t+1} = \rho x_t + \sigma_{x,t}\eta_{x,t+1}, \tag{2.2}$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + \pi\sigma_{c,t}\eta_{c,t+1} + \sigma_{d,t}\eta_{d,t+1}, \tag{2.3}$$

$$\sigma_{i,t+1} = (1-\nu_i)\bar{\sigma}_i + \nu_i\sigma_{i,t} + \sigma_{\omega_i}\omega_{i,t+1}, \tag{2.4}$$

$$\eta_{i,t+1}, \omega_{i,t+1} \sim i.i.d.N(0,1), \quad i \in \{c, d, x\}$$

where x_t is a persistently varying component of the expected consumption growth rate (Long-run risk component) and $\sigma^2_{i,t}$ is the conditionally time-varying variance of a process i at time t. Parameter ρ and ν_x determine the persistence of the expected growth rate and the time-varying volatility respectively. $\phi > 1$ captures the higher volatility of dividend growth relative to consumption growth and the influence of the consumption shock to the dividend growth is governed by $\pi > 0$.

Important assumptions employed for the above model include: i) the sources of serial correlations in consumption and dividend growth are assumed to be the same, as the long-run growth component; ii) three shocks in the law of motions for consumption and dividend growth are mutually independent; iii) the steady-state variance of consumption growth is stable over time. However, the empirical relevance of these assumptions has not been investigated.

In this section, we extend the dynamics for consumption and dividend growth rates in the long-run risks model in three ways based on recent macroeconomics and asset pricing literature.

#1: Serial Correlation in the Idiosyncratic Component of Dividend Growth

An important implication of the law of motion for consumption and dividend growth in (2.1)-(2.4) is that the sources of serial correlations in consumption and dividend growth are the same, as the long-run growth component. In other words, transitory components of consumption and dividend growth are serially uncorrelated. However, a simple empirical analysis of the two series suggests that this might not be the case.

If the source of serial correlations in consumption and dividend growth were indeed the same, regressing dividend growth on consumption growth should give serially uncorrelated errors as follows:

$$\Delta d_t = \phi \Delta c_t + e_t, \tag{2.5}$$

where $e_t = \eta_{d,t} + \pi \eta_{c,t} - \phi \eta_{c,t}$.

However, when we regress dividend growth on consumption growth[4][5] and perform the Q-test for the residuals obtained from the regression, the null hypothesis of no serial correlation is strongly rejected, as Q-stats are all less than 0.0001 in all lags from 1 to 10. The first-order autocorrelation of the residuals e_t is estimated around 0.74, and significantly different from zero. This result implies that dividend growth is serially correlated, even after controlling for the effect of long-run risk component[6].

The omission of this serial correlation in the idiosyncratic component of dividend growth can cause an upward bias in the estimate of the factor loading ϕ in equation (2.3), as all the serial correlation in dividend growth has to be explained by the long-run risk component. This also can yield an upward-biased model-implied equity premium, given the importance of ϕ in the model-implied equity premium.

---

[4] Because Δc_t and e_t are correlated, we employ the instrumental variable approach, using Δc_{t-1} as an instrumental variable.

[5] For data description, refer to section 2.4.1.

[6] In contrast to the case of the dividend growth, when we regress earning growth on consumption growth, errors are not serially correlated.

Extension #2: Cross correlations between long-run and short-run shocks

The long-run risks model deviates from the random walk hypothesis in Hall (1978) as it assumes a persistent component in consumption growth. Beeler and Campbell (2012) cast doubt on the existence of this persistence component, based on the variance ratio test in Lo and MacKinlay (1988). The variance ratio at horizon K is defined as the variance of K-period growth rates divided by K times the variance of one-period growth rates, and it is commonly used in the literature to summarize the persistence of growth rates. For consumption growth, the variance ratio at horizon K is as follows:

$$V(K) = \frac{Var(\Delta c_{t+1} + \cdots + \Delta c_{t+K})}{KVar(\Delta c_{t+1})}. \tag{2.6}$$

If consumption follows a random walk as in Hall (1978), the variance of a K-period consumption growth rate equals K times a one-period growth rate, and therefore the $V(K) = 1$ for all K asymptotically. In contrast, if Δc_{t+1} is positively correlated because of a persistent component as assumed in Bansal and Yaron (2004), $V(K)$ must have an increasing pattern when K increases. Beeler and Campbell (2012) document that the estimated variance ratios using long-run data (from 1930 to 2008) and post-war data (from 1948 to 2008) in annual frequency are not consistent with the prediction of the long-run risks model, as the variance ratio in long-run data exhibits a decreasing pattern and the six-year variance ratio is similar to the two-year variance ratio in the post-war data[7].

We consider a way to reconcile the persistent unobserved component in the long-run risks model and the empirical pattern of the variance ratios, by allowing the long-run and short-run shocks to be contemporaneously correlated. In the presence of ρ_{cx}, the monthly variance ratio of the long-run risks model at horizon K, is:

$$V(K)^{modified} = 1 + \frac{2\sum_{j=1}^{K}(K-j)\rho^j\tilde{\sigma}_x^2}{K(\tilde{\sigma}_x^2 + \sigma_c^2)} + \frac{2\sum_{j=1}^{K}(K-j)\rho^{j-1}\sigma_{cx}}{K(\tilde{\sigma}_x^2 + \sigma_c^2)}, \tag{2.7}$$

where $\tilde{\sigma}_x^2 = \sigma_x^2/(1-\rho^2)$. The variance ratio implied by the benchmark long-run risks model

[7]When we extend the series to 2021, we find that the empirical patterns of the consumption growth variance ratios are unstable, as the variance ratios exhibit increasing pattern when the data after the global financial crisis are included.

in Bansal and Yaron (2004) does not contain the last term, as they assume that the long-run and short-run shocks are mutually independent[8]. Thus, the variance ratio in Bansal and Yaron (2004) is always greater than 1, because the second term in equation (2.7) is always positive with the persistent long-run risks component. In contrast, our extended model can reproduce a decreasing or flat pattern of the variance ratio when ρ_{cx} is negative. In table I, we consider this possibility, by comparing the model-implied variance ratios in annual frequency with various sizes of ρ_{cx}, in conjunction with the other parameters assumed in the Calibration of Bansal and Yaron (2004), Bansal, Kiku and Yaron (2012), and Schorfheide et al. (2018). The variance ratios always exhibit an increasing pattern when ρ_{cx} is zero. However, when ρ_{cx} is negative and large enough in absolute value, we can reproduce a decreasing or flat pattern of the variance ratio.

Table 2.1: Variance Ratios for Consumption Implied by the Long-run Risks Model

	ρ_{cx}	**V(2)**	**V(3)**	**V(4)**	**V(5)**	**V(6)**
BY 2004	0 (Benchmark)	1.29	1.53	1.74	1.92	2.07
	-0.5	1.17	1.32	1.45	1.56	1.65
	-0.95	1.03	1.06	1.09	1.11	1.12
BKY 2012	0 (Benchmark)	1.2	1.36	1.5	1.61	1.71
	-0.5	1.08	1.14	1.19	1.24	1.28
	-0.95	0.93	0.87	0.82	0.78	0.74
SSY 2018	0 (Benchmark)	1.31	1.59	1.85	2.08	2.28
	-0.5	1.22	1.42	1.6	1.76	1.91
	-0.95	1.11	1.21	1.31	1.39	1.47

[8]This is to keep the model parsimonious and is not an identification restriction, as the model is still identified without this zero-correlation assumption.

Extension #3: Regime Shifts in the Steady-state Consumption Volatility

Lettau et al. (2008) show that there is a strong correlation between low-frequency movements in macroeconomic volatility and low-frequency movements in stock prices. In particular, they show that there is a shift in the steady-state consumption volatility in the 1990s and they argue that the run-up in the stock price in the same period can be explained by this shift using a Markov-switching variance model of consumption. As Diebold (1986) suggests, ignoring such shifts in the low-frequency volatility may result in an upward bias in the persistent high-frequency variation in the volatility[9]. Hence, the extremely persistent time-varying volatility in Schorfheide et al (2018) may be an artifact of erroneously assuming that the steady-state variance of consumption growth is stable in their sample. In order to investigate this possibility, we estimate a univariate consumption growth process below using our full sample (1960Q1-2019Q4) and two subsamples (1960Q1-1991Q4 and 1992Q1-2019Q4):

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_{c,t}\eta_{c,t+1}, \tag{2.8}$$

$$x_{t+1} = \rho x_t + \sigma_{x,t}\eta_{x,t+1}, \tag{2.9}$$

$$\sigma_{c,t+1} = \sigma_c^2 h_{c,t}, \quad \sigma_{x,t+1} = \sigma_x^2 h_{c,t}, \tag{2.10}$$

$$h_{c,t} = 1 - \alpha_{1,c} - \alpha_{2,c} + \alpha_{1,c}\varepsilon^{*2}_{t-1|t-2} + \alpha_{2,c}h_{c,t-1}, \tag{2.11}$$

where $\alpha_{2,c}$ measures the persistence of the time-varying volatilities[10]. Two subsamples are chosen based on the identified structural break in Lettau et al. (2008). Table 2.2 summarizes the estimation results. The persistence of the long-run component in the full-sample and two subsamples are not significantly different. Even though the persistence parameter ρ in the early part of the sample is smaller than that in the later part of the sample and that in the full-sample, the difference is not significant. In contrast, the persistence of the time-varying volatility in the full-sample and two subsamples are considerably different, as

[9] Lastrapes (1989), Lamoureux and Lastrapes (1990), and Kim and Kon (1999) find for exchange rate and stock returns that the estimates of GARCH persistence decrease substantially when we allow for a deterministic shift in the unconditional variance.

[10] Detailed explanations for the equations will be provided in the next section.

$\alpha_{2,c}$ in the full sample is 0.89 but it is 0.58 and 0.59 in two subsamples respectively. This striking difference between the persistence of the long-run component and the time-varying volatility implies that the extremely high estimate of the time-varying volatility persistence is subject to the upward bias, but the estimate of the long-run component persistence is not.

Table 2.2: Maximum Likelihood Estimates for a Univariate Consumption Process

	ρ	σ_x	σ_c	α_1	α_2
1960-2019	0.82	0.2	0.3	0.11	0.89
	(0.06)	(0.04)	(0.03)	(0.05)	(0.07)
1960-1991	0.72	0.26	0.33	0.01	0.58
	(0.13)	(0.07)	(0.05)	(0.07)	(1.27)
1992-2019	0.84	0.13	0.23	0.15	0.59
	(0.07)	(0.03)	(0.03)	(0.12)	(0.27)

i) In the parentheses are the standard errors

To consider the role of regime shifts in low-frequency movements in the volatility, we incorporate the Markov-switching GARCH process for the volatility of consumption growth. This Regime-switching GARCH allows us to capture two sources of volatility persistence: persistence due to shocks within the regime and persistence due to regime shifts simultaneously. In this case, the representative agent in this economy faces additional risk, which comes from the low-frequency regime shift in steady-state volatilities, in addition to the high-frequency within regime time-varying volatilities.

2.3 Derivation of the Equity Premium from Our Extended Model

The standard solution technique for the long-run risks model is not feasible for our extended model, because of the Markov-switching volatility of consumption growth. In this section, we introduce our extended model and propose a new solution technique based on Hung (1994) and Bansal and Yaron (2004) to derive the model-implied equity premium.

We propose the following extended law of motion for consumption and dividend growth, which is based on the law of motion for consumption and dividend growth in (2.1)-(2.4), but, augmented to reflect three extensions in section 2.2:

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_{c,t}\eta_{c,t+1}, \tag{2.12}$$

$$x_{t+1} = \rho x_t + \sigma_{x,t}\eta_{x,t+1}, \tag{2.13}$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + u_{t+1}, \quad u_{t+1} = \rho_d u_t + \sigma_{d,t}\eta_{d,t+1}, \tag{2.14}$$

$$\begin{bmatrix} \eta_{x,t+1} \\ \eta_{c,t+1} \\ \eta_{d,t+1} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{cx} & 0 \\ \rho_{cx} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \tag{2.15}$$

$$\sigma^2_{c,t+1}(S_{t+1}) = (1-\nu_c)\bar{\sigma}^2_c(S_{t+1}) + \nu_c\sigma^2_{c,t}(S_t) + \sigma_{\omega_c}\omega_{c,t+1}, \tag{2.16}$$

$$\sigma^2_{x,t+1}(S_{t+1}) = \varphi^2_x\sigma^2_{c,t+1}(S_{t+1}), \tag{2.17}$$

$$\sigma^2_{d,t+1} = (1-\nu_d)\bar{\sigma}^2_d + \nu_d\sigma_{d,t} + \sigma_{\omega_d}\omega_{d,t+1}, \tag{2.18}$$

$$\eta_{i,t+1}, \omega_{i,t+1} \sim i.i.d.N(0,1), \quad i \in \{c, d, x\}$$

where S_t is a discrete regime indicator variable with the following transition probabilities:

$$Pr[S_t = 0|S_{t-1} = 0] = p_{00}, \quad Pr[S_t = 1|S_{t-1} = 1] = p_{11}. \tag{2.19}$$

First, We allow for shocks to the long-run risk component and the idiosyncratic component of consumption to be correlated[11]. This leads to a model-implied equity premium with an additional term reflecting the covariance between the long-run risk component and the idiosyncratic component of consumption growth.

Second, an additional state variable is introduced in the economy, which is the independent serial correlation in the idiosyncratic component of dividend growth. Therefore, the

[11] We still restrict other correlations to be zero, to keep the model parsimonious. Our empirical results in the next section support this restriction as two other correlations are small and insignificant.

conjectured price-dividend ratio is a linear function of the long-run risk component, time-varying volatilities, and the independent serial correlation in dividend growth. However, it does not lead to any change in our equity premium, as it does not change the stochastic discount factor.

Finally, we introduce another source of economic risk in our model, which is a Markov-switching steady-state variance of consumption growth. Thus, we have two sources of variation of time-varying volatility. Because low and high-frequency variations in volatility have different impacts on the equity premium, we combine the long-run risks model in Bansal and Yaron (2004) with an asset pricing model with Epstein-Zin preference and Markov-switching state variables in Hung (1994) and Lettau et al. (2008). The main difference between the two models lies in the way the state variables evolve over time. In the long-run risks model, all the state variables are continuously evolving over time, and the price-consumption ratio and the price-dividend ratio are linear functions of these continuous state variables. In contrast, the asset pricing model in Hung (1994) and Lettau et al. (2008) has only one state variable, which is the regime in the economy. Thus, the price-consumption ratio and the price-dividend ratio are constants within the regime and are subject to discrete shifts. We model the price-consumption ratio and the price-dividend ratio as linear functions of state variables but allow for the constants in two equations to be subject to discrete regime shifts. This is similar to Song (2017), but the way we solve the model is different from his approach, as will be shown below.

In order to incorporate the Markov-switching feature into the model, we assume that the log price-consumption ratio $pc_t(S_t)$ is linear in x_t, $\sigma^2_{x,t}(S_t)$ and a constant $A_0(S_t)$ is subject to the regime S_t:

$$pc_t(S_t) = A_0(S_t) + A_1 x_t + A_2 \sigma^2_{x,t}(S_t). \tag{2.20}$$

The log price-dividend ratio $pd_t(S_t)$ is also linear in x_t, $\sigma^2_{x,t}(S_t)$, σ^2_d, u_t and a constant $A_{0m}(S_t)$ is subject to the regime S_t:

$$pd_t(S_t) = A_{0m}(S_t) + A_{1m} x_t + A_{2xm} \sigma^2_{x,t}(S_t) + A_{2dm} \sigma^2_{d,t} + A_{1um} u_t. \tag{2.21}$$

Following the long-run risks literature, we consider a representative agent with the Epstein-Zin preference. Then the log stochastic discount factor for the economy is given by

$$m_{t+1} = \theta log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}. \tag{2.22}$$

We utilize the Campbell-Shiller (1988) decomposition for the return on the consumption claim and on the market return:

$$r_{c,t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{2.23}$$

$$r_{m,t+1} = \kappa_{0m} + \kappa_{1m} pd_{t+1} - pd_t + \Delta d_{t+1}, \tag{2.24}$$

where κ's are linearization parameters.

In order to solve the model, we need to find the unknown parameters $A_0(S_t)$, A_1, A_2, $A_{0m}(S_t)$, A_{1m}, A_{2xm}, A_{2dm}, A_{1um}, κ_0, κ_1, κ_{0m}, and κ_{1m}. Due to the nonlinearity caused by the regime S_t, $A_0(S_t)$ and $A_{0m}(S_t)$ cannot be found analytically as in Bansal and Yaron (2004), Bansal et al. (2012) and Schorfheide et al. (2018). Thus, we propose a numerical way to find these parameters, based on Hung (1994).

First, we rewrite the Euler equation for the consumption claim, considering the Markov-switching variances and transition probabilities:

$$1 = E_t[exp(m_{t+1} + r_{c,t+1})] = p_{i1} E_t[exp(C_{i1,t})] + p_{i2} E_t[exp(C_{i2,t})],\ i \in \{1, 2\}, \tag{2.25}$$

where, $E_t[exp(C_{ij,t})] = E_t[exp(m_{t+1} + r_{c,t+1}) | S_t = i, S_{t+1} = j] = exp\{E_t[C_{ij,t}] + \frac{1}{2} Var_t[C_{ij,t}]\}$, as $exp(C_{ij,t})$ follows log-normal distribution. We have a system of two nonlinear equations, as there are two states in the economy.

$$1 = p_{11} exp\{E_t[C_{11,t}] + \frac{1}{2} Var_t[C_{11,t}]\} + p_{12} exp\{E_t[C_{12,t}] + \frac{1}{2} Var_t[C_{12,t}]\}, \tag{2.26}$$

$$1 = p_{21} exp\{E_t[C_{21,t}] + \frac{1}{2} Var_t[C_{21,t}]\} + p_{22} exp\{E_t[C_{22,t}] + \frac{1}{2} Var_t[C_{22,t}]\}. \tag{2.27}$$

From (2.26) and (2.27), A_1, and A_2 are found analytically like Bansal and Yaron (2004), and $A_0(S_t)$ is found by solving the nonlinear system above numerically. Similarly, A_{1m},

A_{2xm}, A_{2dm}, A_{1um}, and $A_{0m}(S_t)$ can be found from a similar system of two nonlinear equations, which are derived from the Euler equation for the market return. Then, the market return and the risk-free rate can be derived from equation (2.24) and the Euler equation. The appendix provides more details about the solution.

2.4 The Extended Model and the Parameter Estimates

2.4.1 The Extended Model and the Estimation Procedure

To investigate how our extensions change key parameter estimates, we employ the following representation of our extended model:

$$\Delta c_{t+1} = x_t + \sigma_{c,t}\eta_{c,t+1}, \tag{2.28}$$

$$x_{t+1} = \rho x_t + \sigma_{x,t}\eta_{x,t+1}, \tag{2.29}$$

$$\Delta d_{t+1} = \phi_{S_{t+1}} x_t + u_{t+1}, \quad u_{t+1} = \rho_d u_t + \sigma_{d,t}\eta_{d,t+1}, \tag{2.30}$$

$$\begin{bmatrix} \eta_{x,t+1} \\ \eta_{c,t+1} \\ \eta_{d,t+1} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{cx} & \rho_{dx} \\ \rho_{cx} & 1 & \rho_{cd} \\ \rho_{dx} & \rho_{cd} & 1 \end{bmatrix} \right) \tag{2.31}$$

$$\sigma^2_{x,t+1} = \sigma^2_{x,S_{t+1}} h_{x,t+1},\ \sigma^2_{c,t+1} = \sigma^2_{c,S_{t+1}} h_{c,t+1},\ \sigma^2_{d,t+1} = \sigma^2_d h_{d,t+1}, \tag{2.32}$$

$$h_{i,t+1} = 1 - \alpha_{1,i} - \alpha_{2,i} + \alpha_{1,i}\epsilon^{*2}_{i,t|t-1} + \alpha_{2,i}h_{i,t},\ i \in \{x, c, d\} \tag{2.33}$$

In equation (2.32), we decompose time-varying volatility $\sigma^2_{i,t+1}$ into a steady-state variance σ^2_i and a GARCH component $h_{i,t+1}$, where $E(h_{i,t+1}) = 1$. The term $\epsilon^*_{i,t|t-1} = \epsilon_{i,t|t-1}/\sqrt{f_{i,t|t-1}}$ is the standardized prediction error, where $\epsilon_{i,t|t-1}$ is the prediction error derived from Kalman filter and $f_{i,t|t-1}$ is the variance of the prediction error (i.e., $var(\epsilon_{t+1}) = f_{t+1}$). Following the cash-flow model in section 3.2. of Schorfheide et al. (2018), we restrict $h_{x,t+1} = h_{c,t+1}$, because we have two observable data series but have three GARCH processes to be estimated.

Steady-state variances of the long-run risk component and the idiosyncratic component of consumption follow a two-state Markov-switching process: $\sigma^2_{i,S_{t+1}} = (1 - S_{t+1})\sigma^2_{i,0} + S_{t+1}\sigma^2_{i,1}$, where $i \in \{c, x\}$ and $S_{t+1} \in \{0, 1\}$. The idiosyncratic component of dividend growth is set to be serially correlated: $u_{t+1} = \rho_d u_t + \eta_{d,t+1}$, and the correlations between the long-run shock and short-run shocks are not restricted to be zero.

As Schorfheide et al. (2018) point out, the aggregate dividend growth suffers from a strong seasonality, because of non-uniform payout patterns over the calendar year. To circumvent this difficulty, we assume that in quarterly frequency, the observed consumption growth consists of true consumption growth and the seasonal component, but much of this seasonality can be removed by averaging observed dividend growth over the span of a year. Thus, we utilize the tent function for dividend growth following Schorfheide et al. (2018).

$$\Delta d_t^a = \frac{1}{4}\Delta d_t + \frac{1}{2}\Delta d_{t-1} + \frac{3}{4}\Delta d_{t-2} + \Delta d_{t-3} + \frac{3}{4}\Delta d_{t-4} + \frac{1}{2}\Delta d_{t-5} + \frac{1}{4}\Delta d_{t-6}. \tag{2.34}$$

In equation (2.34), Δd_{t-i}, $i \in \{0, 1, \cdots, 6\}$ are true quarterly dividend growth, that are different from observed dividend growth. Thus, Δd_t^a is not subject to the seasonality issue. With the tent function, equation (2.30) is transformed as follows:

$$\begin{aligned} \Delta d_{t+1}^a =& \frac{\phi}{4}x_t + \frac{\phi}{2}x_{t-1} + \frac{3\phi}{4}x_{t-2} + \phi x_{t-3} + \frac{3\phi}{4}x_{t-4} + \frac{\phi}{2}x_{t-5} + \frac{\phi}{4}x_{t-6} \\ &+ \frac{1}{4}u_{t+1} + \frac{1}{2}u_t + \frac{3}{4}u_{t-1} + u_{t-2} + \frac{3}{4}u_{t-3} + \frac{1}{2}u_{t-4} + \frac{1}{4}u_{t-5}, \end{aligned} \tag{2.35}$$

which has several MA components to control the seasonality.

Additionally, we introduce the measurement errors in dividend growth for the second half of our sample, from the first quarter of 1990. These measurement errors are motivated by the empirical findings that the dividend pay-out policies of firms have changed in the 1990s, and help us improve the model fit.

The model can be cast in a state-space form.[12] We employ the approximate maximum-likelihood estimation method of Kim (1994) to estimate the resulting state-space model with Markov-switching.

[12]For the state-space model, refer to appendix B.3.

2.4.2 Data

Consistent with the long-run risks literature, we use data on U.S. real nondurables and services consumption per capita from the Bureau of Economic Analysis and aggregate dividend data from CRSP.

The aggregate consumption data are available at annual, quarterly, and monthly frequencies. Papers before Schorfheide et al. (2018), such as Ma (2013) and Bansal et al. (2016), use the annual dataset in estimating the long-run risks model, as it covers the longest time period, from 1929. However, Schorfheide et al. (2018) find that the state-space model with stochastic volatility for consumption growth is poorly identified in annual frequency. Thus, they estimate two models for the law of motion for consumption and dividend growth. The first model is in monthly frequency from 1959M1 to 2014M12, and the second model utilizes a mixed-frequency approach, which uses annual consumption growth from 1930 to 1959 and monthly consumption growth from 1960M1 to 2014M12. The use of the state-space setup allows them to utilize datasets at different frequencies. Because of the contamination in monthly consumption data, measurement errors are included in estimating their models.

Unlike these papers, we use quarterly consumption growth from 1960Q1 to 2019Q4, assuming the representative agent decides her consumption once in a quarter for several reasons. First, because quarterly consumption data is less noisy than monthly consumption data, we do not need to include measurement errors in our model. Given the complexity of our extended model, it is better to keep the model as simple as possible. Second, Schorfheide et al. (2018) show that the estimate of the persistence parameter ρ from a univariate consumption growth model is similar in monthly and quarterly frequency, even though the confidence interval gets a little wider. One potential problem in our approach is that the degree of persistence in quarterly frequency cannot be directly compared with that in monthly frequency. Thus, instead of the absolute size of parameter estimates, we focus more on how the parameter estimates change with three extensions.

Dividend series are also smoothed out by aggregating three months' values of raw nominal dividend series and then converted to real terms using the CPI. This procedure, together with the annualization, ensures that the anomalies in data because of seasonality is removed.

2.4.3 Estimation Results

We estimate four different laws of motion for consumption and dividend growth to analyze the impact of each extension on the parameter estimates, respectively:

Benchmark model: Bansal and Yaron (2004)
Model I: Benchmark model with a serially correlated shock to dividend growth
Model II: Model I with an unrestricted covariance matrix
Model III: Model II with a Markov-switching steady-state consumption volatility

The estimation and inference results are in Table 2.3. In the benchmark model, the persistence of the long-run growth (ρ) is estimated to be 0.82, and the cube-root of ρ estimate is 0.936. It is lower than the monthly ρ estimate in Schorfheide et al. (2018), which is 0.967 and is similar to the 5% posterior quantile in Schorfheide et al. (2018). This result is consistent with the finding of Schorfheide et al. (2018), which is that the estimate of ρ from quarterly data is lower than those from monthly data and a monthly model on quarterly. In contrast, the persistence of the GARCH process for consumption growth is 0.97, which translates into 0.99 in monthly frequency. Ma (2013) estimates the bivariate VARMA-GARCH model of consumption and dividend growth using annual data from 1929 to 2008, and the annual persistence of the GARCH process in the paper is 0.9392. It is much higher than the annual persistence of the long-run risk component, which is 0.6196. Thus, the persistence of the long-run risk component declines when we use lower-frequency data, but the persistence of the GARCH process remains relatively high. As discussed above, this high persistence of the GARCH process regardless of data frequency might be because of the omission of a discrete shift in the steady-state variance.

When we allow for the idiosyncratic component of dividend growth to be serially correlated, the serial correlation parameter ρ_d is estimated to be 0.83, and it is highly significant. At the same time, the factor loading parameter ϕ declines from 3.3 to 1.61. This can be interpreted as a decline in the impact of the long-run component on dividend growth. When the independent serial correlation in the idiosyncratic component of dividend growth is not considered in the benchmark model, all the serial correlations in dividend growth have to be explained by the long-run risk component, which results in an upward bias in the factor loading parameter ϕ.

In the extended model II, we estimate the full variance-covariance matrix without any restrictions on correlation parameters. This extension delivers a strong negative correlation between a shock to the long-run component and a shock to the idiosyncratic component of consumption, which is -0.95. Two other correlations are estimated not to be significant. The estimated standard deviation of the long-run component also slightly increases with this extension, from 0.22 to 0.25. These two results are consistent with the main findings of Morley et al. (2003): shocks to trend and cycle components are highly negatively correlated, and the trend component is much more volatile when the zero-correlation restriction in their unobserved component model is relaxed.

Finally, in the extended model III, we incorporate the Markov-switching steady-state consumption variance, along with the two extensions above. In the high volatility regime, the estimated variance of the long-run growth is 0.25, and that of the idiosyncratic component of consumption growth is also 0.25. In the low volatility regime, the steady-state variance of the long-run growth is estimated to be 0.17, and that of the idiosyncratic component of consumption growth is estimated to be 0.12. This corresponds to a 32% reduction in the standard deviation of the long-run component and a 52% reduction in the standard deviation of the idiosyncratic component of consumption growth. The transition probability from a high volatility state to another high volatility state is 0.995, and the transition probability from a low volatility state to another low volatility state is 0.997. Figure 3 shows the filtered probabilities of being in a high volatility state over time. Consistent with Lettau et al. (2008), consumption exhibits a sharp increase in the probability of being in a low volatility regime around 1995 and then remains very high, almost unity, until the end of the sample. With the Markov-switching steady-state variance, the persistence of the GARCH process for the long-run risk component ($\alpha_{2,c}$) drops from 0.99 to 0.63. This confirms our conjecture that the omission of the discrete shift drives the persistence of the GARCH process to unity in the benchmark model.

Table 2.3: Maximum Likelihood Estimates

	Benchmark		**Ext. Model I**		**Ext. Model II**		**Ext. Model III**	
ρ	0.81	(0.06)	0.79	(0.07)	0.81	(0.06)	0.84	(0.05)
ϕ_0	3.60	(0.84)	1.61	(0.70)	1.23	(0.32)	1.41	(0.38)
ϕ_1	-	-	-	-	-	-	1.94	(0.40)
σ_{x0}	0.19	(0.03)	0.22	(0.04)	0.25	(0.04)	0.25	(0.05)
σ_{x1}	-	-	-	-	-	-	0.16	(0.03)
σ_{c0}	0.33	(0.04)	0.28	(0.04)	0.19	(0.03)	0.26	(0.05)
σ_{c1}	-	-	-	-	-	-	0.12	(0.02)
σ_d	2.11	(0.33)	0.99	(0.00)	1.01	(0.09)	1.01	(0.10)
ρ_d	-	-	0.83	(0.04)	0.82	(0.04)	0.83	(0.04)
ρ_{cx}	-	-	-	-	-0.95	(0.07)	-0.93	(0.06)
ρ_{dx}	-	-	-	-	0.03	(0.03)	-0.05	(0.06)
ρ_{cd}	-0.90	(0.26)	-0.21	(0.17)	-0.35	(0.20)	-0.33	(0.18)
α_{1c}	0.11	(0.04)	0.11	(0.05)	0.14	(0.05)	0.07	(0.08)
α_{2c}	0.88	(0.07)	0.88	(0.07)	0.86	(0.06)	0.52	(0.02)
α_{1d}	0.56	(0.14)	0.11	(0.04)	0.10	(0.06)	0.11	(0.07)
α_{2d}	0.41	(0.16)	0.81	(0.10)	0.82	(0.12)	0.82	(0.14)
p_{00}	-	-	-	-	-	-	0.995	(0.005)
p_{11}	-	-	-	-	-	-	0.996	(0.004)
likelihood	-484.48		-306.10		-302.49		-296.98	

i) In the parentheses are the standard errors

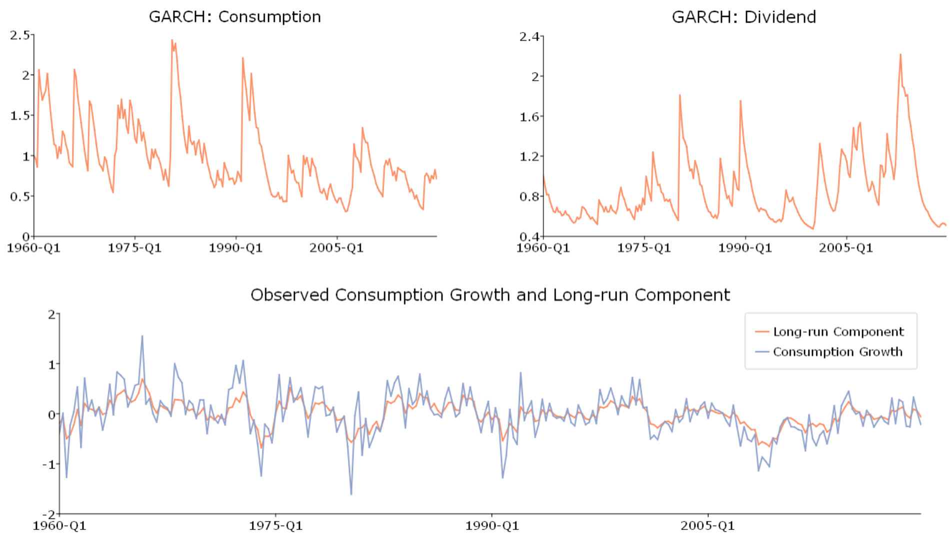


Figure 2.1: Long-run Risks Component and GARCH Series from the Long-run Risks Model without Markov-switching Variance

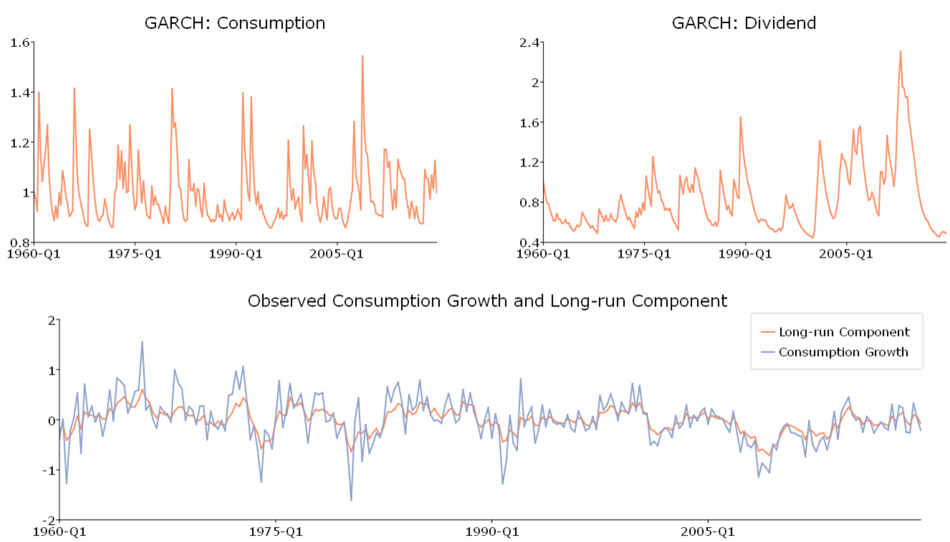


Figure 2.2: Long-run Risks Component and GARCH Series from the Long-run Risks Model with Markov-switching Variance

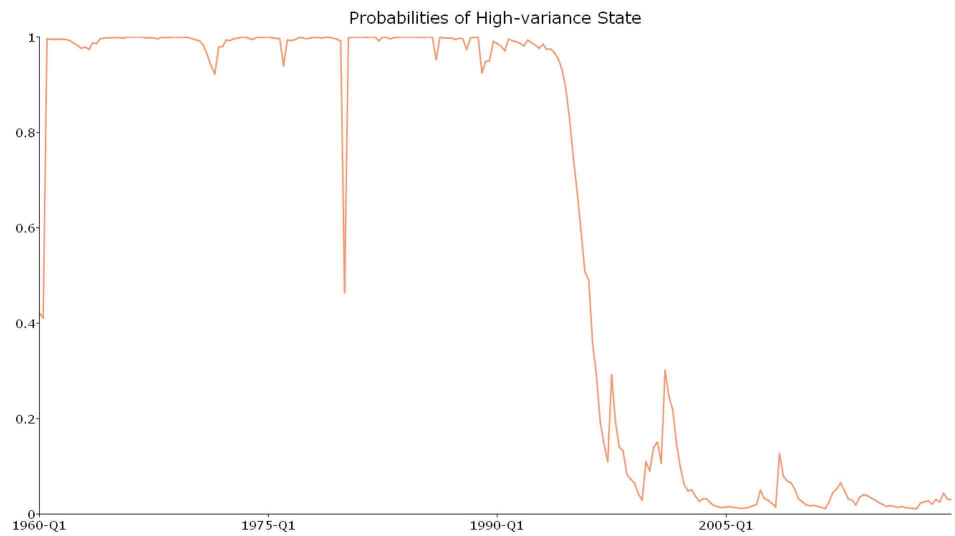


Figure 2.3: Probabilities of High-variance State

2.5 Simulation Results

2.5.1 Calibration Parameters and Simulation Method

To compare the model-implied equity premium from the extended model with that from the benchmark model based on Bansal and Yaron (2004), we assume that the decision interval of the agent is monthly, following Bansal and Yaron (2004) and Schorfheide et al. (2018). Table 2.4 provides the parameters used to calibrate the benchmark and extended models. We again simulate four models to analyze the impact of each extension on the model-implied equity premium. i) Benchmark model without any extension, ii) The extended model I with full variance-covariance matrix, iii) The extended model II, with the Markov-switching steady-state consumption variance and full variance-covariance matrix, and iv) The extended model III, in which all the extensions are applied.

Parameters in the benchmark model are chosen to generate a 6% equity premium, based on the parameters in Bansal and Yaron (2004) and Schorfheide et al. (2018). Then, parameters in the extended models are modified based on the estimation results in section 2.4. Because the estimated parameters are based on the model in quarterly frequency, we convert parameters to match frequency. Again, we focus more on the relative changes in the model-implied equity premium with extensions, than the absolute size of the model-implied equity premium.

	Benchmark	**Ext. Model I**	**Ext. Model II**	**Ext. Model III**
γ	8.89	8.89	8.89	8.89
ψ	1.97	1.97	1.97	1.97
δ	0.999	0.999	0.999	0.999
μ_c	0.0016	0.0016	0.0016	0.0016
μ_d	0.0011	0.0011	0.0011	0.0011
ρ	0.987	0.987	0.987	0.987
σ_{c0}	0.00637	0.00637	0.00637	0.00637
σ_{c1}	-	-	0.00319	0.00319
φ_x	0.0035	0.0044	0.0044	0.0044
φ_d	4.54	4.54	4.54	4.54
ϕ	3.65	3.65	3.65	1.65
ν_c	0.992	0.992	0.853	0.853
ν_d	0.969	0.969	0.969	0.969
ω_c	0.0000023	0.0000023	0.0000023	0.0000023
ω_d	0.0000023	0.0000023	0.0000023	0.0000023
ρ_{cx}	0.00	-0.95	-0.95	-0.95
p_{00}	-	-	0.9983	0.9983
p_{11}	-	-	0.9983	0.9983

Table 2.4: Calibration Parameters

We first generate the monthly series for consumption, dividend, and state variables, including the long-run components, conditional variances, and the volatility state of the economy. Log market return and the risk-free rates can be derived from those variables, following the model. Then, annual returns and risk-free rates are constructed as the sum of monthly values. Because we cannot exclude the possibility of negative conditional variance, we replace negative realizations of conditional variances with small positive numbers, as in Bansal et al. (2012) and Beeler and Campbell (2012).

A significant difference between the previous models and our extended model is that our model needs to incorporate the shift to a lower volatility state in the 1990s in simulation. We set the monthly sample size in simulation to be 760 (60 years), and introduce the shift by lowering the volatility from 421st observations. It is consistent with the estimated volatility shift identified in section 2.4. Even though the shift is deterministic, it generates additional risk in the economy, as the agent cannot predict the timing of the shift.

2.5.2 Simulation Results

In table 2.5, we display several asset pricing implications of the benchmark model and the extended model. We investigate how the model-implied equity premium changes with the extensions, by focusing on the annualized equity premium.

The model-implied equity premium from the benchmark long-run risks model with parameters in Bansal and Yaron (2004) and Schorfheide et al. (2018) is 6.0%. However, the equity premium drops to 2.4%, when it is derived from the modified asset pricing model with modified parameters reflecting the empirical findings from the estimation of the consumption and dividend growth. This decrease in the equity premium can be approximately decomposed into three parts. The decline of consumption GARCH persistence decreases the equity premium by 0.73%, which is 20% of the total decrease. A negative correlation between long-run and short-run innovations and the increase in long-run variance together bring down the equity premium by 0.27%, which is 7.4% of the total decrease. Finally, the equity premium decreases by 2.61%, as the dividend leverage ratio on the long-run growth drops. This explains 72% of the total decrease.

2.6 Conclusion

Our findings suggest some implications for the long-run risks model. Unlike Beeler and Campbell (2012) and Ma (2014), we are able to identify a long-run risk component in

	Benchmark	Ext. Model I	Ext. Model II	Ext. Model III
Consumption Mean	1.92	1.92	1.93	1.93
Consumption S.D.	2.3	2.18	1.82	1.83
Dividend Mean	1.27	1.28	1.37	1.36
Dividend S.D.	9.66	10.45	9.74	8.49
Equity Premium Mean	5.97	5.7	4.97	2.36
Equity Premium S.D.	17.72	20.28	18.79	12.18
Risk-free Rate Mean	1.09	1.99	1.99	1.98
Risk-free Rate S.D.	0.83	0.94	0.77	0.77
D/P Ratio Mean	4.26	4.15	4.44	5.09
D/P Ratio S.D.	0.25	0.31	0.33	0.17

Table 2.5: Simulation Results

consumption and dividend growth, consistent with Schorfheide et al. (2018). However, even though this long-run risk component is highly persistent, the long-run risks model is able to explain only half of the equity premium in data when restrictions in the consumption and dividend process are relaxed. In other words, in order to explain the high observed equity premium in the U.S. equity market, the risk aversion needs to be 21, which is much higher than assumed by Bansal and Yaron (2004).

Chapter 3

CHANGE IN THE IMPORTANCE OF THE DISCOUNT RATE NEWS AND THE CASH FLOW NEWS: TIME-VARYING-PARAMETER VAR WITH STOCHASTIC VOLATILITY APPROACH

3.1 Introduction

Since Campbell and Shiller (1988) show that unexpected stock returns can be decomposed into news about future discount rates (DRs) and news about future cash flows (CFs), their relative importance in explaining the variation of the stock return has been one of the fundamental topics in asset pricing. The consensus in the literature is that unexpected stock return is mainly driven by the news about future discount rates in the postwar period (Campbell (1991), Campbell and Ammer (1993), Campbell and Vuolteenaho (2004), and Binsbergen and Koijen (2009)).

Despite their importance in empirical and theoretical asset pricing, relatively less attention has been paid to the time-variation of the relative importance of CFs and DRs for the variance of the unexpected stock return. However, there are several reasons to believe their relative importance could be time-varying. First, literature on empirical asset pricing provides evidence of parameter instability in the predictive regressions for stock return (Pastor and Stambaugh (2001), Kim et al. (2005), Paye and Timmermann (2006), Dangl and Halling (2012), and reference therein). If this parameter instability reflects the time-variation of the relation between fundamentals and the stock return, the relative importance of the DRs and CFs can also be time-varying. Second, voluminous literature documents that the volatility of stock return is time-varying, and stock return is much more volatile during recessions than during booms. Thus, the relative importance of CFs and DRs could be different across regimes of the economy. Finally, previous papers such as Bernanke and Kuttner (2005) show that the relative importance of the DRs and CFs is not stable, using the subsample analysis, respectively[1].

In this paper, I directly measure the time variation of the relative importance of CFs and DRs for the unexpected stock return variance in the postwar period. For this purpose, I combine the VAR-based return decomposition in Campbell (1991) and Campbell and Ammer (1993) and the time-varying parameter VAR with stochastic volatilities (TVP-VAR-SV) in Primiceri (2005). The TVP-VAR-SV is a flexible framework that incorporates drifting VAR coefficients and multivariate stochastic volatilities, and it can be estimated using an efficient Markov Chain Monte Carlo algorithm. Given the posterior mean of drifting coefficients and stochastic volatilities, the variance of the unexpected return in each period can be decomposed into the variances of DR and CF, and their covariance using the return decomposition method.

In the benchmark TVP-VAR-SV model, the lagged stock return, the price-dividend ratio, and the relative bill rate are used as forecasting variables for the stock return, as in Campbell (1991). The model is estimated using sample 1953:M1-2019M12. When I derive time-varying variances of DRs and CFs using the return decomposition method in conjunction with the posterior mean of the drifting coefficients and the stochastic volatilities, I find two important features. First, the variance of DRs rises, and that of CFs falls in most of the recessions, but the variance of DRs falls, and that of CFs rises in expansions. This pattern implies that the stock market crashes in recessions because of high discount rates news, and recovery of the stock market in booms is associated with high cash flows news. Second, while DRs explain most of the variation in the unexpected returns from the 1950s-1980s, CFs play a larger role from 1994. In most of the period after 1994, CFs account for more than 50% of the variance of the unexpected return, and DRs account for less than 20% of the variance of the unexpected return. Even in the period of the global financial crisis, the importance of CFs is higher than that of DRs. The timing of the reversal of the relative importance of CFs and DRs coincides with the beginning of the "Great Moderation," which means a significant reduction in macroeconomic volatilities[2] (Kim and Nelson (1999),

[1]However, the subsample analysis can not capture the real-time fluctuation in variances of DRs and CFs.

[2]Several papers document that the Great Moderation affects the dynamics of the stock market. For example, Lettau et al. (2008) show that the run-up in the stock market in the 1990s is related to the

McConnell and Perez-Quiros (2000), and Stock and Watson (2002)).

To investigate the cause of the reversal in the relative importance of DRs and CFs, I conduct a counterfactual analysis. I generate an artificial set of coefficients and stochastic volatilities by fixing one of the drifting coefficients or stochastic volatilities in the forecasting VAR after 1994 as its historical average before 1994. The counterfactual path of DRs and CFs are computed using the return decomposition method with this counterfactual set of drifting coefficients and stochastic volatilities in conjunction with data. I find that a reduction in the volatility of the dividend price ratio is the primary source of the reversal. The reversal is also attributed to the reduction in the return volatility and the smaller size of the coefficient of the price-dividend ratio on the stock return. When the volatility of the price-dividend ratio and the coefficient of the price-dividend ratio on the stock return are replaced with its historical average before 1994, counterfactual DRs and CFs do not exhibit the reversal of their relative importance, as the counterfactual DRs account for most of the variations after 1994.

Finally, I provide economic mechanisms that potentially can generate the reversal. Bernanke and Kuttner (2005) argue that the larger role of CFs in the 1990s is because the stock return is not predictable from the 1990s based on R^2 of the subsample. However, I find that the MSFE of the stock return does not significantly vary across time. Thus, return predictability can not fully account for the reversal of the relative importance of DRs and CFs. Instead, I provide two alternative hypotheses, one about the CFs and another about the DRs. The first hypothesis is related to Vuolteenaho (2002) and Kothari et al. (2006). They argue that cash flow news contained in the idiosyncratic components of firm-level stocks is diversified away at the market level, which explains the significant role of DRs in explaining the variation of the market return. Based on several recent studies, I provide evidence that cash flow news may be less diversified away from 1990. Then, CFs can account for the variation of not only the individual stock return but also the aggregate market return. I also show that DRs may be less volatile since the 1990s because of the decline in macroeconomic volatility, using the noisy-information model based on Coibion

decline in macroeconomic volatility and the following reduction in the expected return. Curtis et al. (2021) find that the relation between aggregate earnings and stock return turns from negative to positive.

and Gorodnichenko (2015). I assume that investors continuously update their expectations on future stock returns based on the macroeconomic conditions, which are partially correlated with observed variables, because of noise. With these assumptions, I show that the expected return may be less updated during the Great Moderation period from the 1990s because it is harder for investors to distinguish signal from noise. This leads to less volatile DRs.

The rest of the paper is organized as follows. Section 3.2 presents the VAR-based return decomposition and the TVP-VAR-SV. Section 3.3 provides the empirical results and counterfactual analysis results. Section 3.4 discusses economic mechanisms that can explain the reversal. Section 3.5 concludes.

3.2 Methodology

3.2.1 Stock Return Variance Decomposition Based on Constant Parameter VAR

Following Campbell and Shiller (1988), Campbell (1991), and Campbell and Ammer (1993), I express the unexpected stock return in period $t+1$ to changes in rational expectations of future dividend growth, future real interest rates, and future stock returns.

$$r_{e,t+1} - E_t r_{e,t+1} = (E_{t+1} - E_t)\left[\sum_{j=0}^{\infty}\rho^j \Delta d_{t+1+j} - \sum_{j=0}^{\infty}\rho^j r_{t+1+j} - \sum_{j=0}^{\infty}\rho^j r_{e,t+1+j}\right], \quad (3.1)$$

where, $r_{e,t+1}$ is the log excess stock return relative to the risk-free rate, Δd_{t+1} is the log real dividend growth, and r_{t+1} is the risk-free rate. E_t denotes the rational expectation conditional on all the information up to period t. I can simplify the notation in equation (3.1) as follows.

$$\eta_{t+1} \equiv r_{e,t+1} - E_t r_{e,t+1} \tag{3.2}$$

$$\eta_{d,t+1} \equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} \tag{3.3}$$

$$\eta_{r,t+1} \equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r_{t+1+j} \tag{3.4}$$

$$\eta_{e,t+1} \equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r_{e,t+1+j}, \tag{3.5}$$

where, η_{t+1} is the unexpected excess return, $\eta_{d,t+1}$ represents new about future dividends, $\eta_{r,t+1}$ represents new about future interest rates, and $\eta_{e,t+1}$ represents new about future returns. Then equation (3.1) can be written as

$$\eta_{t+1} = \eta_{d,t+1} - \eta_{r,t+1} - \eta_{e,t+1}. \tag{3.6}$$

Derivation of $\eta_{d,t+1}$, $\eta_{r,t+1}$, and $\eta_{e,t+1}$ requires predictions of future stock return, interest rate, and dividend growth. Campbell (1991) and Campbell and Ammer (1993) propose a prediction method based on a reduced-form VAR model to make a forecast, where a VAR model is defined as follows:

$$Z_{t+1} = c + BZ_t + u_{t+1}, \quad u_{t+1} \sim N(0, \Omega) \tag{3.7}$$

where, Z_{t+1} is a vector which has n elements. The first element of Z_{t+1} is the excess stock return, the second element of it is the risk-free rates, and other variables are predictor variables for the stock return. Then, j-step ahead prediction of the excess stock return at period t is

$$E_t r_{e,t+1+j} = i_1' B^{j+1} Z_t, \tag{3.8}$$

where, i_1 is a n-element vector, whose first element is 1 and other elements are all 0. Then, news on future returns, which is the discounted sum of revisions in forecast return can be written as

$$\begin{aligned}\eta_{e,t+1} \equiv (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j e_{t+1+j} &= i_1'\sum_{j=0}^{\infty}\rho^j B^j u_{t+1} \\ &= i_1'\rho B(I_n - B)^{-1}u_{t+1} \\ &= \lambda_1' u_{t+1},\end{aligned} \tag{3.9}$$

where $\lambda_1' = i_1'\rho B(I_n - B)^{-1}$. In similar way, I can derive news on future interest rates.

$$\begin{aligned}\eta_{r,t+1} \equiv (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j r_{t+1+j} &= i_2'\sum_{j=0}^{\infty}\rho^j B^j u_{t+1} \\ &= i_2'\rho B(I_n - B)^{-1}u_{t+1} \\ &= \lambda_2' u_{t+1},\end{aligned} \tag{3.10}$$

where, i_2' is a n-element vector, whose second element is one and other elements are all 0, and $\lambda_2' = i_2'\rho B(I_n - B)^{-1}$. The unexpected return η_{t+1} is

$$\eta_{t+1} = i_1' u_{t+1}. \tag{3.11}$$

The news on the future dividend is derived using equations (6), (9), (10), and (11).

$$\eta_{d,t+1} = (i_1 + \lambda_1 + \lambda_2)' u_{t+1} \tag{3.12}$$

Then, equation (3) implies that the variance of excess stock return can be decomposed into the variance of the three news above and their covariances.

$$\begin{aligned}Var(\eta_{t+1}) =& Var(\eta_{d,t+1}) + Var(\eta_{r,t+1}) + Var(\eta_{e,t+1}) \\ & - 2Cov(\eta_{d,t+1}, \eta_{r,t+1}) - 2Cov(\eta_{d,t+1}, \eta_{e,t+1}) + 2Cov(\eta_{r,t+1}, \eta_{e,t+1}),\end{aligned} \tag{3.13}$$

3.2.2 Stock Return Variance Decomposition Based on TVP-VAR-SV

The VAR-based stock return variance decomposition in equation (3.13) gives the relative importance of CFs and DRs for the variance of excess stock return on average during the sample period. Because our interest is the time variation of the relative importance of CFs and DRs, I use the TVP-VAR-SV, which allows the coefficients and the covariance matrix to be time-varying during the sample period. I consider the TVP-VAR-SV version of equation (3.7):

$$Z_{t+1} = c_{t+1} + B_{t+1} Z_t + u_{t+1}, \quad u_{t+1} \sim N(0, \Omega_{t+1}). \tag{3.14}$$

Following Primiceri (2005), the reduced-form covariance matrix Ω_{t+1} can be decomposed using the triangular reduction.

$$A_{t+1} \Omega_{t+1} A_{t+1} = \Sigma_{t+1} \Sigma'_{t+1}, \tag{3.15}$$

where A_{t+1} is the lower triangular matrix

$$A_{t+1} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \alpha_{21,t+1} & 1 & \ddots & \\ \vdots & \ddots & \ddots & 0 \\ \alpha_{n1,t+1} & \cdots & \alpha_{nn-1,t+1} & 1 \end{bmatrix}, \tag{3.16}$$

and Σ_{t+1} is the diagonal matrix

$$\Sigma_{t+1} = \begin{bmatrix} \sigma_{1,t+1} & 0 & \cdots & 0 \\ 0 & \sigma_{2,t+1} & \ddots & \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{n,t+1} \end{bmatrix} \tag{3.17}$$

With the triangular reduction, equation (3.14) can be written as follows:

$$Z_{t+1} = c_{t+1} + B_{t+1} Z_t + A^{-1}_{t+1} \Sigma_{t+1} \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I_n), \tag{3.18}$$

Then, equation (3.18) can be rewritten as

$$Z_{t+1} = X'_{t+1} b_{t+1} + A^{-1}_{t+1} \Sigma_{t+1} \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I_n), \tag{3.19}$$

where, $X'_{t+1} = I_n \otimes [1,\ Z'_t]$ and $b_{t+1} = vec([c'_{t+1},\ B'_{t+1}]')$. Time-varying parameters and log of volatilities in the model are assumed to follow a random walk process, and all the innovations are assumed to be jointly normally distributed as follows:

$$b_{t+1} = b_t + \nu_{t+1}, \tag{3.20}$$

$$\alpha_{t+1} = \alpha_t + \xi_{t+1}, \tag{3.21}$$

$$log\sigma_{t+1} = log\sigma_t + \eta_{t+1}, \tag{3.22}$$

$$Var\left(\begin{bmatrix} \epsilon_{t+1} \\ \nu_{t+1} \\ \xi_{t+1} \\ \eta_{t+1} \end{bmatrix}\right) = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{bmatrix} \tag{3.23}$$

Given the complexity and nonlinearity of the model, Gibbs sampling is used to evaluate the posterior distributions of the $(B_1, B_2, \cdots, B_T)'$, $(A_1, A_2, \cdots, A_T)'$ and $(\sigma_1, \sigma_2, \cdots, \sigma_T)'$[3].

Posterior mean of the $Var_{t+1}(\eta_{t+1})$, $Var_{t+1}(\eta_{d,t+1})$, $Var_{t+1}(\eta_{e,t+1})$ and covariances of the news at time $t+1$ can be derived using the posterior mean of B_{t+1}, A_{t+1}, and $log\sigma_{t+1}$. First, the reduced form covariance matrix Ω_{t+1} is computed as

$$\Omega_{t+1} = A^{-1}_{t+1}\Sigma_{t+1}\Sigma'_{t+1}A'^{-1}_{t+1}. \tag{3.24}$$

Then, variances and covariances of the unexpected return and the news can be derived using Ω_{t+1} and $\lambda_{t+1} = i'_1\rho B_{t+1}(I_n - B_{t+1})^{-1}$.

3.3 An Application to Postwar U.S. Data

I decompose the variance of postwar US excess stock return using the TVP-VAR-SV. Three variables are used in estimating the TVP-VAR-SV: excess stock return, price-dividend ratio, and the relative bill rate. The series of excess stock returns come from Kenneth R. French's website. The price-dividend ratio is measured as the current stock price relative to total dividends paid over the previous year, where the stock price and dividends are from the

[3]See Primiceri (2005) for the details of the estimation.

Center for Research in Security Prices (CRSP) at the University of Chicago. Finally, the relative bill rate is the difference between a short-term Treasury bill rate and its one-year backward moving average. The sample runs from 1947M1 to 2019M12.

Because the short-term interest rate is not incorporated in the VAR estimation, I decompose the variance of the excess stock return into two components: news on future discount rates and news on future dividend growth and interest rates. However, because news on interest rates explains a very small portion of market returns[4], variation of the second news component is mainly attributed to the news on future dividend growth. Thus, I denote the second news component as news on future cash flows (CFs).

3.3.1 Priors

I employ Normal priors for B, A, and $log\sigma$ and inverse-Wishart distributions for Q, W, and S.

$$\begin{aligned} B_0 &\sim N(\hat{B}_{OLS}, 4V(\hat{B}_{OLS})), \\ A_0 &\sim N(\hat{A}_{OLS}, 4V(\hat{A}_{OLS})), \\ log\sigma_0 &\sim N(log\hat{\sigma}_{OLS}, I_n), \\ Q &\sim IW(k_Q^2 \cdot 60 \cdot V(\hat{B}_{OLS}), 60), \\ W &\sim IW(k_W^2 \cdot I_n, 4), \\ S_1 &\sim IW(k_Q^2 \cdot 2 \cdot V(\hat{A}_{OLS}), 2), \\ S_2 &\sim IW(k_Q^2 \cdot 3 \cdot V(\hat{A}_{OLS}), 3), \end{aligned} \tag{3.25}$$

where, $\hat{B}_{OLS}$, $V(\hat{B}_{OLS})$, $\hat{A}_{OLS}$, and $V(\hat{A}_{OLS})$ are OLS estimates using the first 60 observations (1947:M1 - 1951:M12). S_1 and S_2 denote the two blocks of S, and A_1 and A_2 are two corresponding blocks of $\hat{A}_{OLS}$. I set $k_Q^2 = 0.01$, $k_W^2 = 0.1$, and $k_W^2 = 0.01$, as in Primiceri (2005).

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[4]In most previous papers, news on future interest rates explains 1-2% of the variance of excess stock return.

3.3.2 Empirical Results

Figure 3.1 summarizes the time-variation of the variance of CFs and DRs relative to the unexpected stock return variance from 1953 to 2019. The first interesting feature is that variance of DRs rises and that of CFs falls in most recessions, but the variance of DRs falls, and that of CFs rises in most expansions. This feature is consistent with the empirical result in Gomez-Cram (2022). Using a model of stock returns in which the expected return is modeled as a latent AR(1) process and variances of innovations to expected and unexpected returns follow a Markov-switching process, he shows that the expected return is much more volatile during recessions than in expansions. This means that economic agents revise their expectations of the future stock return more during recessions. Thus, in the present-value framework, variation in the unexpected return is explained more by the news on future discount rates during recessions. One of the important exceptions is the recession of 1980, in which DRs and CFs both increase during the recession. This period coincides with the Volker disinflation. Because the tight monetary policy causes this recession, news on future interest rates included in CFs may account for a significant portion of the recession, unlike other recessions.

A more important finding is that the relative importance of CFs and DRs in explaining the variance of the unexpected stock return is reversed in the 1990s. Before 1990, more than half of the variation in the unexpected return is attributed to DRs. However, the variance of DRs starts to decrease from the 1980s, and the variance of CFs starts to increase from the early 1990s. Eventually, the relative importance of DRs and CFs is reversed in 1990. Since 1990, the variance of CFs is always higher than that of DRs, even during the Great Recession in 2008-2009. The timing of the reversal coincides with the beginning of the Great Moderation, which denotes the reduction in the macroeconomic volatility in the 1980s-1990s.

3.3.3 Counterfactual Analysis

To investigate what causes the reversal in the relative importance of CFs and DRs in explaining the variance of unexpected returns, I perform a counterfactual analysis. In the counterfactual analysis, I generate a hypothetical set of parameters, in which one of the

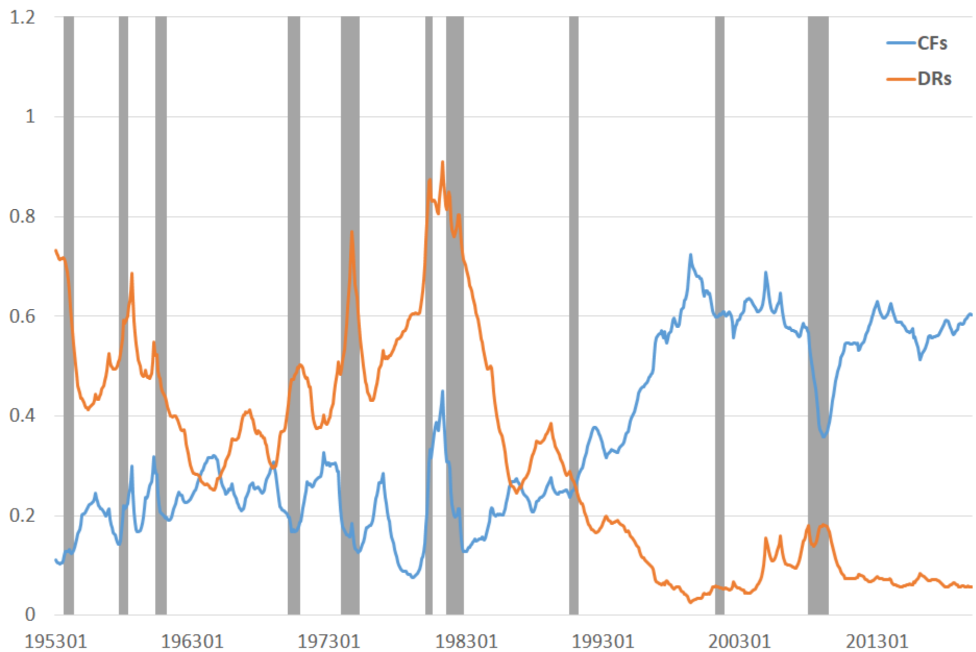
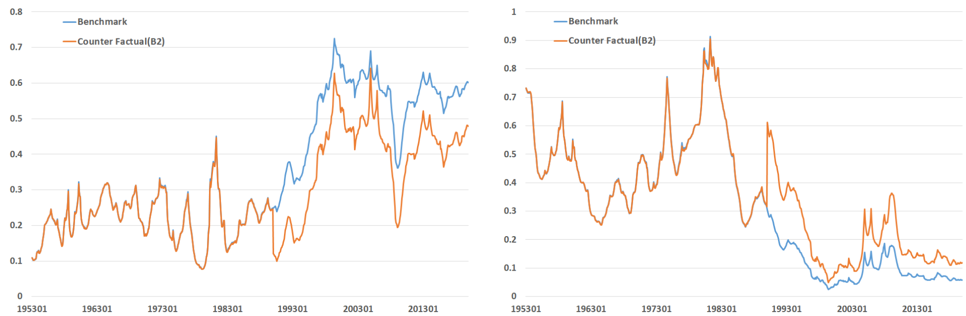


Figure 3.1: Time-varying unexpected return decomposition

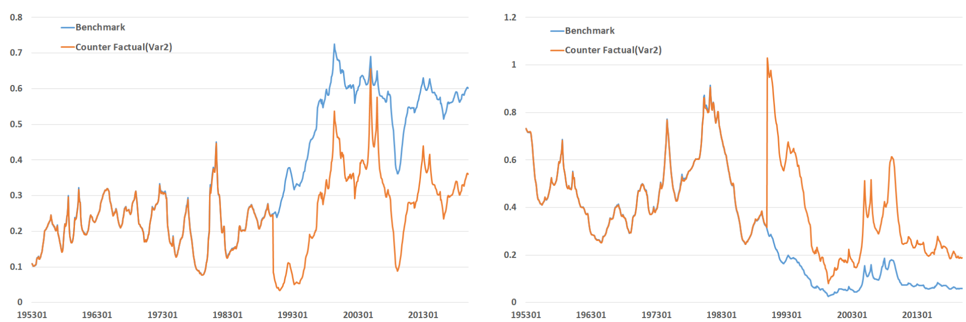
drifting coefficients or stochastic volatilities after 1990 is replaced by its historical mean before 1990. The counterfactual path of time-varying variances of DRs and CFs is derived from these counterfactual sets of parameters.

I find that the most important factor for the reversal is the reduction in the volatility of the price-dividend ratio. Figure 3.2 shows the counterfactual paths of variances of CFs and DRs relative to the variance of unexpected stock return in which the time-varying volatility of the price-dividend ratio is replaced by its historical mean before 1990. The counterfactual variance of CFs shifts downward and the counterfactual variance of DRs shifts upward. Figure 3.3 shows that another important factor for the reversal is the decrease in the drifting coefficient of the price-dividend ratio on the stock return, as the counterfactual variance of DRs increases and that of CFs decreases when the coefficient after 1990 is replaced by the historical mean before 1990. When I replace the variance of the price-dividend ratio and the coefficient of the price-dividend ratio on the stock return at the same time, DRs play a larger role in the entire sample. However, replacing other coefficients or volatilities does



(a) Counterfactual Var(CF_t)/Var(η_t) (b) Counterfactual Var(DR_t)/Var(η_t)

Figure 3.2: Counterfactual CFs and DRs when $Var_{2,t}$ is replaced



(a) Counterfactual Var(CF_t)/Var(η_t) (b) Counterfactual Var(DR_t)/Var(η_t)

Figure 3.3: Counterfactual CFs and DRs when $B_{12,t}$ is replaced

not make a significant change in the counterfactual CFs and DRs.

3.4 Economic Mechanism for the Reversal in the Relative Importances of CFs and DRs in explaining the variance of unexpected returns

In this section, I investigate economic mechanisms that can explain the increase in the variance of CFs and the decrease in the variance of DRs from the 1990s.

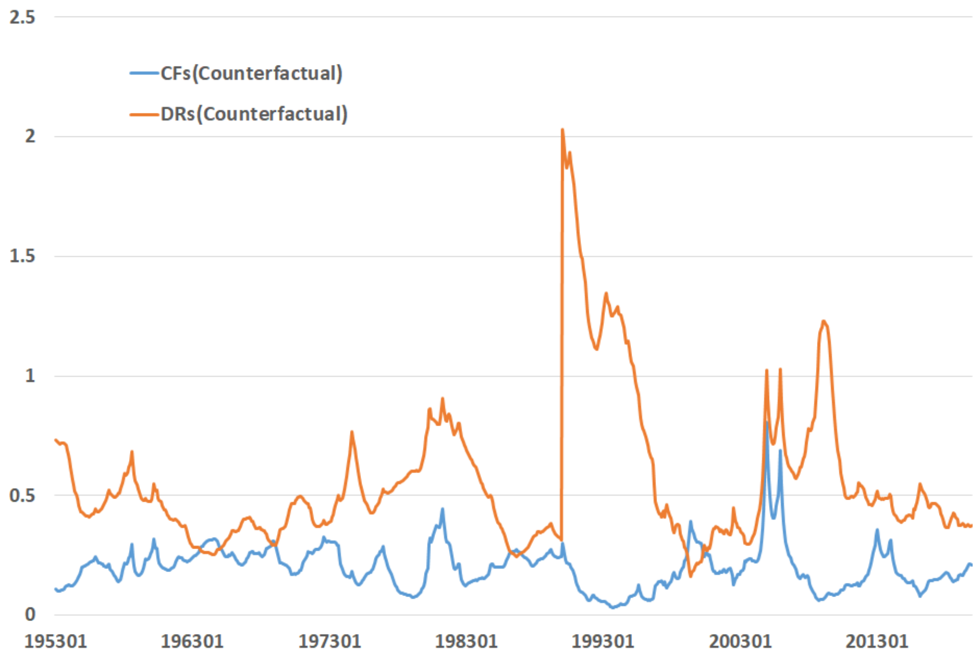


Figure 3.4: Counterfactual unexpected return decomposition

3.4.1 Predictability

Bernanke and Kuttner (2005) show that the importance of the news on cash flow increases during the 1990s, by decomposing unexpected stock returns for the 1973-2002 sample and the 1989-2002 sample. For the 1973-2002 sample, news on future discount rates accounts for 76% of the variance of unexpected returns. In contrast, news on future cash flows and news on future interest rates explain only 24.5% and 1.4% of the variance of unexpected returns respectively. However, the 1989-2002 sample yields a different result, as the importance of the news on future cash flows increases significantly. For this subsample, news on future discount rates, future cash flows, and future interest rates account for 38%, 31.9%, and 0.6% of the variance of unexpected stock return respectively. Bernanke and Kuttner explain the increase in the importance of the news on future cash flows in the 1990s by a decline in the predictability of stock returns in the later subsample, as adjusted R^2 drops from 0.04 to

-0.003[5].

Table 3.1: A Variance Decomposition of Excess Stock Return in Bernanke and Kuttner (2005)

	Var(dividends)	Var(real rate)	Var(future returns)
1973-2002	24.5%	1.4%	76.0%
1989-2002	31.9%	0.6%	38.0%

I re-examine this hypothesis of lower predictability, by comparing the mean squared forecast errors (MSFEs) before and after 1990. The point estimate of one-step-ahead forecast is computed as $\hat{r}_{e,t+1|t} = i_1'(\hat{c}_{t+1} + \hat{B}_{t+1}Z_t)$, where $\hat{c}_{t+1}$ and $\hat{B}_{t+1}$ are posterior mean of parameters. The h-step ahead forecasts are computed recursively for $h = 1, 2, \cdots, 60$. Then, MSFEs for the h-step ahead forecast is as follows:

$$MSFE_h = \sqrt{\frac{\sum_{t=1}^{T}(\hat{r}_{e,t+h|t} - r_{e,t+h})^2}{T}}. \tag{3.26}$$

Table 3.1 compares the MSFEs for the 1955-1989 period and the 1990-2019 period. I find that MSFEs for both periods are similar for $h = 1$, and MSFEs for the later period are lower for $h > 1$. Controlling for the effect of the outlier[6] does not change the result. These results imply that change in predictability can not explain the reversal in the relative importance of CFs and DRs in the 1990s.

3.4.2 An Increase in the Importance of Cash-Flow News

Several recent studies in accounting document a structural break in the relationship between the aggregate earning and aggregate stock return from negative to positive (Bailey and Lai

[5] Because the news on cash flows is treated as a residual in the VAR-based stock return decomposition method, less predictable stock return assigns more of the excess return volatility to news on future cash flows.

[6] I drop sample with the stock market crash in 1987 and 2008

Table 3.2: MSFEs of Point Forecasts

	One Month	One Year	Two Year
1955-1989	4.25	4.33	4.35
1990-2019	4.21	4.22	4.22

(2019), Chen et al. (2015), Kim et al. (2020), and Curtis et al. (2021)). The timing of the structural break in the relationship is estimated to be the early 1990s in Curtis et al. (2021).

Some papers attribute the structural break to the increase in the importance of the news on cash flows in the 1990s. Curtis et al. (2021) argue that improvements in information technology and monetary policy that triggered the Great Moderation are also responsible for the increase in the importance of the news on cash flows because news about future cash flows is reflected in a more efficient and timely fashion. Chen et al. (2015) emphasize the role of regulatory changes in the 2000s. The passage of Regulation Fair Disclosure in 2000 and the enactment of the Sarbanes-Oxley Act in 2002 have enhanced the information environment and disclosure quality, and these also allow stock returns to reflect available news in a more efficient fashion.

3.4.3 A Decrease in the Importance of Discount Rate News: Noisy-Information

As discussed in the literature, the expected returns are closely correlated with macroeconomic conditions. It means that investors continuously update their expectations of future stock returns based on new information about the stock market and the macroeconomic conditions. I show that the decline in the macroeconomic volatility can lead to less volatile expected returns and also a smaller variance of the discount rate news when investors can not fully observe the macroeconomic conditions.

Following Coibion and Gorodnichenko (2015), I assume that investors know the structure of the economy and underlying parameter values, but can not fully observe the state of the economy. Thus, they form their expectation of future stock returns based on their forecast of the state of the economy. Suppose that the state of the economy is latent and follows an AR(1) process:

$$x_t = \rho x_{t-1} + v_t, \quad v_t \sim iidN(0, \sigma_v^2), \tag{3.27}$$

where, $0 < \rho < 1$. Investors can not observe x_t but only observe a variable y_t[7], which is partially correlated with x_t:

$$y_t = x_t + \omega_t, \quad \omega_t \sim iidN(0, \sigma_\omega^2), \tag{3.28}$$

where, w_t is a noise which is uncorrelated with v_t. Finally, I assume that the expected stock return $E_t(r_{t+j})$ is a linear function of $E_t(x_{t+j})$:

$$E_t(r_{t+j}) = \gamma E_t(x_{t+j}). \tag{3.29}$$

Then, investors generate forecasts of the current state of the economy using the Kalman filter[8]:

$$\begin{aligned} E_t x_{t+j} &= \left(\frac{\sigma_v^2}{\sigma_v^2 + (1-\rho^2)\sigma_\omega^2}\right) y_t + \left(\frac{(1-\rho^2)\sigma_\omega^2}{\sigma_v^2 + (1-\rho^2)\sigma_\omega^2}\right) E_{t-1} x_{t+j} \\ &= G y_t + (1-G) E_{t-1} x_{t+j}. \end{aligned} \tag{3.30}$$

Therefore, expected returns are not fully updated with new information, but only partially updated with parameter γG.

$$E_t r_{t+j} = \gamma G y_t + (1-G) E_{t-1} r_{t+j}. \tag{3.31}$$

I can easily show that $\partial G / \partial \sigma_v^2 > 0$. Thus, the expected return is updated more when macroeconomic volatility is higher. This result is consistent with Coibion and Gorodnichenko (2015), who document that information rigidity is negatively correlated with

[7]For simplicity, I assume that investors observe only one economic variable to forecast the state of the economy. However, a model with a vector of observable economic variables delivers the same message.

[8]For simplicity, I use the steady-state Kalman gain in the derivation.

macroeconomic volatility, because a more tranquil period should be associated with greater information rigidities[9]. Because news on discount rates is a revision in future expected returns, volatility of the news on discount rates is also positively correlated with the volatility of the economy. This relationship explains why the news on discount rates is less volatile during the Great Moderation period, in which the macroeconomic volatility is small.

3.5 Conclusion

This paper investigates the time variation of the relative importance of CFs and DRs in explaining the stock return variance, by combining the VAR-based stock return decomposition in Campbell (1991) and Campbell and Ammer (1993) and the time-varying parameter VAR with stochastic volatilities (TVP-VAR-SV) in Primiceri (2005). This new approach provides two empirical findings. First, the stock market crashes in recessions because of high discount rates news, and recovery of the stock market in booms is associated with high cash flows news. Second, there is a reversal in the relative importance of CFs and DRs in explaining the stock return variance in the 1990s.

Then I propose two explanations for this reversal. First, the importance of the cash flow news in stock return variance may have increased since the 1990s because of the recent developments in information technology. Second, a decline in macroeconomic volatility since the 1990s can cause less volatile DRs, if investors make inferences on the macroeconomic condition and the corresponding expected return by extracting signals from observable variables.

[9] see also Branch et al. (2009).

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Appendix A

THE DELAYED REACTION OF THE EXCHANGE RATE AND THE LATENT FINANCIAL SHOCK: AN EMPIRICAL INVESTIGATION

A.1 Derivation of a Reduced-Form Model

This section derives a reduced-form model for our empirical model in equations (1.1)-(1.4). For this purpose, we proceed with the following steps:

Step 1: We multiply both sides of the first equation in (1.4) by $(1-\alpha L)$, where L refers to the lag operator.

$$(1-\psi L)(1-\alpha L)\Delta s_{t+1} = \beta(i_t - i_t^*) - \beta\alpha(i_{t-1} - i_{t-1}^*) + \eta_t + \varepsilon_{t+1} - \alpha\varepsilon_t \tag{A.1}$$

Step 2: We consider an orthogonal projection of η_t on v_t ($\eta_t = \gamma v_t + \tilde{\eta}_t$) and an orthogonal projection of ε_{t+1} on v_t ($\varepsilon_t = \delta v_t + \tilde{\varepsilon}_t$). This allows us to rewrite equation (A.1):

$$(1-\psi L)(1-\alpha L)\Delta s_{t+1} = \beta(i_t - i_t^*) - \beta\alpha(i_{t-1} - i_{t-1}^*) + (\gamma - \alpha\delta)v_t + \delta v_{t+1} + \tilde{\varepsilon}_{t+1} + \tilde{\eta}_t - \alpha\tilde{\varepsilon}_t \tag{A.2}$$

Step 3: Finally, we rewrite equation (A.2) as follows:

$$\Delta s_{t+1} = (\psi+\alpha)\Delta s_t - \psi\alpha\Delta s_{t-1}\beta(i_t - i_t^*) - \beta\alpha(i_{t-1} - i_{t-1}^*) + (\gamma-\alpha\delta)v_t + \delta v_{t+1} + e_{t+1} - \theta e_t. \tag{A.3}$$

Then equation (A.3) and the last equation in (1.4) form our reduced form model, where e_t and v_t are uncorrelated.

A.2 Derivation of the Long-run Restriction

The long-run restriction that the UIP holds in the long-run can be expressed as follows:

$$\sum_{j=0}^{\infty} \frac{\partial \Delta s_{t+j}}{\partial v_t} = 0. \tag{A.4}$$

To find a closed-form solution for (A.4), we first compute $\partial \Delta s_{t+j}/\partial v_t$ from our reduced-form model for $j = 0, 1, 2 \cdots$.

$$\begin{aligned}
\frac{\partial \Delta s_t}{\partial v_t} &= \delta, \\
\frac{\partial \Delta s_{t+1}}{\partial v_t} &= (\psi + \alpha)\frac{\partial \Delta s_t}{\partial v_t} + \beta \frac{\partial x_t}{\partial v_t} + (\gamma - \alpha\delta), \\
\frac{\partial \Delta s_{t+2}}{\partial v_t} &= (\psi + \alpha)\frac{\partial \Delta s_{t+1}}{\partial v_t} - \psi\alpha \frac{\partial \Delta s_t}{\partial v_t} + \beta \frac{\partial x_{t+1}}{\partial v_t} - \beta\alpha \frac{\partial x_t}{\partial v_t}, \\
\frac{\partial \Delta s_{t+3}}{\partial v_t} &= (\psi + \alpha)\frac{\partial \Delta s_{t+2}}{\partial v_t} - \psi\alpha \frac{\partial \Delta s_{t+1}}{\partial v_t} + \beta \frac{\partial x_{t+2}}{\partial v_t} - \beta\alpha \frac{\partial x_{t+1}}{\partial v_t} \\
&\vdots
\end{aligned} \tag{A.5}$$

Because $\partial x_{t+j}/\partial v_t = \phi^j$, summing up all the equations for $\partial \Delta s_{t+j}/\partial v_t$ yields

$$\sum_{j=0}^{\infty} \frac{\partial \Delta s_{t+j}}{\partial v_t} = (\psi + \alpha - \psi\alpha) \sum_{j=0}^{\infty} \frac{\partial \Delta s_{t+j}}{\partial v_t} + \delta + (\gamma - \alpha\delta) + \beta \left(\frac{1-\alpha}{1-\phi} \right), \tag{A.6}$$

$$\rightarrow \sum_{j=0}^{\infty} \frac{\partial \Delta s_{t+j}}{\partial v_t} = \frac{1}{(1-\psi)(1-\alpha)} \left[\delta + (\gamma - \alpha\delta) + \beta \left(\frac{1-\alpha}{1-\phi} \right) \right]. \tag{A.7}$$

When we substitute equation (A.7) into equation (A.4), we obtain

$$\frac{1}{(1-\psi)(1-\alpha)} \left[\delta + (\gamma - \theta\delta) + \beta \left(\frac{1-\alpha}{1-\phi} \right) \right] = 0, \tag{A.8}$$

from which we obtain the following long-run restriction:

$$\delta = -\frac{1}{1-\alpha} \left[\gamma + \beta \left(\frac{1-\alpha}{1-\phi} \right) \right]. \tag{A.9}$$

A.3 Derivation of Moments Implied by the Structural Model and the Reduced Form Model

We multiply $(1-\psi L)$ on both sides of (8)[1] to obtain

$$(1-\psi L)\Delta s_{t+1} = \beta(i_t - i_t^*) + \eta_t + \varepsilon_{t+1} \tag{A.10}$$

The MA side of the structural model is defined by combining equation (A.10) with an error term of the equation for interest rate differential:

$$U_{t+1}^s = \begin{bmatrix} u_{t+1}^s \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} \eta_t + \varepsilon_{t+1} \\ v_{t+1} \end{bmatrix} \tag{A.11}$$

Then, four moments are derived from $E(U_{t+1}^s U_{t+1}^{s'})$ and $E(U_{t+1}^s U_t^{s'})$.

$$\begin{aligned} E(u_{t+1}^{s2}) &= \sigma_\varepsilon^2 + \sigma_\eta^2 \\ E(u_{t+1}^s v_{t+1}) &= \alpha_{\varepsilon v}\sigma_\varepsilon\sigma_v \\ E(u_{t+1}^s u_t^s) &= \alpha_{\varepsilon\eta}\sigma_\varepsilon\sigma_\eta \\ E(u_{t+1}^s v_t) &= \alpha_{\eta v}\sigma_\eta\sigma_v \end{aligned} \tag{A.12}$$

Similarly, we can define the MA side of the reduced form model as follows:

$$U_{t+1}^r = \begin{bmatrix} u_{t+1}^r \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} \delta v_{t+1} + \gamma v_t + e_{t+1} - \theta e_t \\ v_{t+1} \end{bmatrix} \tag{A.13}$$

[1] We set $\alpha = 0$

Then, another four moments are derived from $E(U^r_{t+1}U^{r'}_{t+1})$ and $E(U^r_{t+1}U^{r'}_{t})$.

$$\begin{aligned} E(u^{r2}_{t+1}) &= [\delta^2 + \gamma^2]\sigma^2_v + (1+\theta^2)\sigma^2_e \\ E(u^r_{t+1}v_{t+1}) &= \delta\sigma^2_v \\ E(u^r_{t+1}u^s_t) &= \gamma\delta\sigma^2_v - \theta\sigma^2_e \\ E(u^r_{t+1}v_t) &= \gamma\sigma^2_v \end{aligned} \tag{A.14}$$

Matching structural moments in equation (A.12) and reduced-form moments in equation (A.14) yields equation (1.15) in Section 1.5.

Appendix B

THE LONG-RUN RISKS MODEL REVISITED: EXTENDED MODEL AND EMPIRICAL EVIDENCE

B.1 Solving the Benchmark Long-run Risks Model based on Bansal and Yaron (2004)

This section provides solutions for the consumption and dividend claims for the benchmark endowment process in Bansal and Yaron (2004), Bansal et al (2012), and Schorfheide et al (2018)

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_{c,t}\eta_{c,t+1}, \tag{B.1}$$

$$x_{t+1} = \rho x_t + \sigma_{x,t}\eta_{x,t+1}, \tag{B.2}$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + \pi\sigma_{c,t}\eta_{c,t+1} + \sigma_{d,t}\eta_{d,t+1}, \tag{B.3}$$

$$\sigma_{i,t+1} = (1-\nu_i)\bar{\sigma}_i + \nu_i\sigma_{i,t} + \sigma_{\omega_i}\omega_{i,t+1}, \tag{B.4}$$

$$\eta_{i,t+1}, \omega_{i,t+1} \sim i.i.d.N(0,1). \quad i \in \{c,d,x\}$$

When the representative agent has Epstein Zin (1989) preference, the Euler equation for the economy is

$$E_t[exp(m_{t+1} + r_{i,t+1})] = 1, \quad i \in \{c,m\}, \tag{B.5}$$

where

$$m_{t+1} = \theta log\delta - \frac{\theta}{\psi}\Delta c_t + (\theta-1)r_{c,t+1}, \tag{B.6}$$

is the log of the real stochastic discount factor, $r_{c,t+1}$ is the log return on consumption claim. We utilize the Campbell-Shiller (1988) decomposition for the return on the consumption claim and on the market return:

$$r_{c,t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{B.7}$$

$$r_{m,t+1} = \kappa_{0m} + \kappa_{1m} pd_{t+1} - pd_t + \Delta d_{t+1}. \tag{B.8}$$

Then, the risk premium on any asset is

$$E_t(r_{i,t+1} - r_{f,t}) + \frac{1}{2} Var_t(r_{i,t+1}) = -Cov_t(m_{t+1}, r_{i,t+1}). \tag{B.9}$$

In order to derive the dynamics of asset prices, we rely on approximate analytic solutions following the long-run risks literature. Specifically, we conjecture that the price-consumption ratio and the price-dividend ratio follow

$$pc_t = A_0 + A_1 x_t + A_{2x}\sigma^2_{x,t} + A_{2c}\sigma^2_{c,t}, \tag{B.10}$$

$$pd_t = A_{0m} + A_{1m} x_t + A_{2xm}\sigma^2_{x,t} + A_{2cm}\sigma^2_{c,t} + A_{2dm}\sigma^2_{d,t}, \tag{B.11}$$

and solve for A_0, A_1, A_2x, and A_2c using the Euler equation, the return equation, and the conjectured dynamics. The solutions for $A's$ that describe the dynamics of the price-consumption ratio are

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}$$

$$A_{2x} = \frac{\theta/2}{1 - \kappa_1 \nu_x}\{(\kappa_1 A_1)^2\}$$

$$A_{2c} = \frac{\theta/2}{1 - \kappa_1 \nu_c}\{(1 - \frac{1}{\psi})^2\}$$

The solutions for $A_m's$ that describe the dynamics of the price-dividend ratio are derived similarly and are as follows:

$$A_{1,m} = \frac{\phi - \frac{1}{\psi}}{1 - \kappa_{1m} \rho}$$

$$A_{2xm} = \frac{\frac{1}{2}\{(\theta - 1)\kappa_1 A_1 + \kappa_{1m} A_{1m}\}^2 + (\theta - 1)(\kappa_1 \nu_x - 1)A_{2x}}{1 - \kappa_{1m} \nu_x}$$

$$A_{2cm} = \frac{\frac{1}{2}\gamma^2 + (\theta - 1)A_{2c}(\kappa_1 \nu_c - 1)}{1 - \kappa_{1m} \nu_c}$$

$$A_{2dm} = \frac{\frac{1}{2}}{1 - \kappa_{1m}\nu_d}$$

Because the risk premium is determined by the covariation of the return innovation with the stochastic discount factor innovation, the risk premium is equal to the asset's exposures to systemic risks multiplied by the corresponding risk prices,

$$\begin{aligned} E_t(r_{m,t+1} - r_{f,t}) + \frac{1}{2}Var_t(r_{m,t+1}) = \\ \beta_{m,x}\lambda_{m,x}\sigma^2_{x,t} + \beta_{m,c}\lambda_{m,c}\sigma^2_{c,t} + \beta_{m,\omega_x}\lambda_{\omega_x}\sigma^2_{\omega_x} + \beta_{m,\omega_c}\lambda_{\omega_c}\sigma^2_{\omega_c}, \end{aligned} \tag{B.12}$$

where asset's βs and corresponding risk prices λs are defined as follow:

$$\lambda_x = (1 - \theta)\kappa_1 A_1$$

$$\lambda_c = \gamma$$

$$\lambda_{\omega_x} = (1 - \theta)\kappa_1 A_{2x}$$

$$\lambda_{\omega_c} = (1 - \theta)\kappa_1 A_{2c}$$

$$\beta_{m,x} = \kappa_{1m} A_{1m}$$

$$\beta_{m,c} = \pi$$

$$\beta_{m,\omega_x} = \kappa_{1m} A_{2xm}$$

$$\beta_{m,\omega_c} = \kappa_{1m} A_{2cm}$$

B.2 Solving the Extended Long-run Risks Model

B.2.1 Consumption Claim

To solve for $A_0(S_t)$, A_1, and A_2 in the extended long-run risks model, we use Euler equations, the return equation, and the conjectured dynamics. Because the dynamic properties of the economy depend on the present regime, we have two Euler equations in the extended model. When we drop the conditional expectation in the Euler equation by using the log-normal moment generator,

$$\begin{aligned} 1 &= E_t[exp(m_{t+1} + r_{c,t+1})] \\ &= \sum_{j=1}^{2} p_{ij} E_t[exp(m_{t+1} + r_{c,t+1})|S_t = i, S_{t+1} = j] \\ &= \sum_{j=1}^{2} p_{ij} exp\{E_t[C_{ij,t}] + \frac{1}{2} Var_t[C_{ij,t}]\}, \quad i \in \{1,2\}, \end{aligned} \tag{B.13}$$

where,

$$\begin{aligned} E_t[C_{ij,t}] + \frac{1}{2}Var_t[C_{ij,t}] = {} & \theta ln\delta + \theta\kappa_0 + \theta\kappa_1 A_0(j) - \theta A_0(i) + \theta\kappa_1 A_2(1-\nu_c)\sigma_{c,t}^2(j) \\ & + \theta\{1 - \frac{1}{\psi} + \kappa_1 A_1 \rho - A_1\} x_t + \theta A_2(\kappa_1\nu_c - 1)\sigma_{c,t}^2(j) \\ & + \frac{\theta^2}{2}\{(1-\frac{1}{\psi})^2 + (\kappa_1 A_1 \varphi_x)^2\}\sigma_{c,t}^2(i) + \frac{\theta^2}{2}(\kappa_1 A_2)^2 \sigma_{\omega,c}^2 \\ & + \theta^2(1-\frac{1}{\psi})\kappa_1 A_1 \varphi_x \rho_{cx} \sigma_{c,t}^2(i). \end{aligned} \tag{B.14}$$

As terms related to A_1 and A_2 are not subject to the regime, A_1 and A_2 have closed-form solutions that are equal to those in the benchmark long-run risks model.

Because $A_0(S_t)$ is subject to the regime, it does not have a closed-form solution. However, we can utilize the fact that all the terms related to the state variables are zero to satisfy the Euler equation. Thus, $A_0(1)$ and $A_0(2)$ are derived as solutions to the system of equations

$$1 = p_{11}exp(E_{11}) + p_{12}exp(E_{12}), \tag{B.15}$$

$$1 = p_{21}exp(E_{21}) + p_{22}exp(E_{22}), \tag{B.16}$$

where E_{ij}s are the part of $E_t[C_{ij,t}] + \frac{1}{2}Var_t[C_{ij,t}]$, which are not related to state variables.

$$E_{ij} = \theta ln\delta + \theta\kappa_0 + \theta\kappa_1 A_0(j) - \theta A_0(i) + \theta\kappa_1 A_{2c}(1-\nu_c)\sigma_c^2(j) + \frac{1}{2}(\theta\kappa_1 A_{\omega c})^2\sigma_{\omega,c}^2. \tag{B.17}$$

B.2.2 Market Return

We can also rewrite the Euler equation for market return as follows:

$$\begin{aligned}
1 &= E_t[exp(m_{t+1} + r_{m,t+1})] \\
&= \sum_{j=1}^{2} p_{ij} E_t[exp(m_{t+1} + r_{m,t+1})|S_t = i, S_{t+1} = j] \\
&= \sum_{j=1}^{2} p_{ij} exp\{E_t[D_{ij,t}] + \frac{1}{2}Var_t[D_{ij,t}]\}, \quad i \in \{1,2\}
\end{aligned} \tag{B.18}$$

where,

$$\begin{aligned}
&E_t[D_{ij,t}] + \frac{1}{2}Var_t[D_{ij,t}] \\
&\quad = \theta ln\delta + (\theta-1)\{\kappa_0 + \kappa_1 A_0(j) - A_0(i) + \kappa_1 A_2(1-\nu_c)\sigma_c^2(j)\} + \kappa_{0m} \\
&\qquad + \kappa_{1m}A_{0m}(j) - A_{0m}(i) + \kappa_{1m}A_{2cm}(1-\nu_c)\sigma_c^2(j) + \kappa_{1m}A_{2dm}(1-\nu_d)\sigma_d^2 \\
&\qquad + \{(1-\theta)(\kappa_1 A_1\rho - A_1 + 1) - \frac{\theta}{\psi} + \kappa_{1m}A_{1m}\rho - A_{1m} + \phi\}x_t \\
&\qquad + \{(\theta-1)A_2(\kappa_1\nu_c - 1) + A_{2cm}(\kappa_{1m}\nu_c - 1)\}\sigma_{c,t}^2(i) + A_{2dm}(\kappa_{1m}\nu_d - 1)\sigma_{d,t}^2 \\
&\qquad + \frac{1}{2}\{(\theta-1)\kappa_1 A_1 + \kappa_{1m}A_{1m}\}^2\}\varphi_x\sigma_{c,t}^2(i) + \frac{1}{2}\gamma^2\sigma_{c,t}^2(i) + \frac{1}{2}\sigma_{d,t}^2 \\
&\qquad + \frac{1}{2}\{(\theta-1)\kappa_1 A_2 + \kappa_{1m}A_{2m}\}^2\}\sigma_{\omega_c}^2 + \frac{1}{2}(\kappa_{1m}A_{2dm})^2\sigma_{\omega_d}^2 \\
&\qquad - \{(\theta-1)\kappa_1 A_1 + \kappa_{1m}A_{1m}\}\gamma\varphi_x\rho_{cx}\sigma_{c,t}^2(i).
\end{aligned} \tag{B.19}$$

Again, A_ms have closed-form solutions, except $A_{0m}(S_t)$, and are the same with the long-run risks model without Markov-switching variance.

$$A_{1m} = \frac{\phi - \frac{1}{\psi}}{1 - \kappa_{1m}\rho}$$
$$A_{2dm} = \frac{\frac{1}{2}}{1 - \kappa_{1m}\nu_d}$$
$$A_{2cm} = \frac{A_{2cm}^1 + A_{2cm}^2}{1 - \kappa_{1m}\nu_d}$$

where,

$$A_{2cm}^1 = (\theta - 1)A_2(\kappa_1\nu_c - 1)$$
$$A_{2cm}^2 = \frac{1}{2}\{(\theta-1)\kappa_1 A_1 + \kappa_{1m}A_{1m}\}^2 + \frac{1}{2}\gamma^2 - \{(\theta-1)\kappa_1 A_1 + \kappa_{1m}A_{1m}\}\gamma\varphi_x\rho_{cx}$$

Finally, $A_{0m}(1)$ and $A_{0m}(2)$ are derived as solutions to the system of equations

$$1 = p_{11}exp(F_{11}) + p_{12}exp(F_{12}), \tag{B.20}$$
$$1 = p_{21}exp(F_{21}) + p_{22}exp(F_{22}), \tag{B.21}$$

where F_{ij}s are the part of $E_t[D_{ij,t}] + \frac{1}{2}Var_t[D_{ij,t}]$, which are not related to the state variables.

$$\begin{aligned} F_{ij} =& \theta ln\delta + (\theta-1)\{\kappa_0 + \kappa_1 A_0(j) - A_0(i) + \kappa_1 A_2(1-\nu_c)\sigma_c^2(j)\} \\ &+ \kappa_{0m} + \kappa_{1m}A_{0m}(j) - A_{0m}(i) + \kappa_{1m}A_{2cm}(1-\nu_c)\sigma^2(j) \\ &+ \kappa_{1m}A_{2dm}(1-\nu_d)\sigma_d^2 + \frac{1}{2}\{(\theta-1)\kappa_1 A_2 + \kappa_{1m}A_{2cm}\}^2\sigma_{\omega_c}^2 \\ &+ \frac{1}{2}(\kappa_{1m}A_{2dm})^2\sigma_{\omega,d}^2. \end{aligned} \tag{B.22}$$

Then, the market return can be derived from the return equation and the conjectured dynamics, with As and A_ms.

B.2.3 Risk-free Rate

Finally, the model-driven equation for the risk-free rate is

$$\begin{aligned} 1/R_{f,t}(i) &= E_t[exp(m_{t+1})] \\ &= p_{i1}exp\{B_0(i1) + B_1x_t + B_2\sigma^2_{c,t}(i)\} \\ &\quad + p_{i2}exp\{B_0(i2) + B_1x_t + B_2\sigma^2_{c,t}(i)\} \end{aligned} \tag{B.23}$$

$$r_{f,t}(i) = -ln(\frac{1}{R_{f,t}(i)}) \tag{B.24}$$

where,

$$B_0(ij) = \theta ln\delta + (\theta - 1)\{\kappa_0 + \kappa_1 A_0(j) - A_0(i) + \kappa_1 A_2(1 - \nu_c)\sigma^2_c(j)\} + \frac{1}{2}\{(\theta - 1)\kappa_1 A_2\}^2\sigma^2_{\omega,c}$$

$$B_1 = -\frac{1}{\psi}$$

$$B_2 = \frac{1}{2}\{\frac{\gamma - 1}{\psi} + \gamma\} + \frac{(1 - \frac{1}{\psi})(\gamma - \frac{1}{\psi})\kappa_1^2\varphi_x^2}{2(1 - \kappa_1\rho)^2} + \frac{(1 - \gamma)(\gamma - \frac{1}{\psi})\rho_{cx}\kappa_1\varphi_x}{1 - \kappa_1\rho}$$

B.3 A State-Space Model Representation of the Empirical Model

The model that consists of (10)-(14) and (12') can be cast into the following state-space model.

Measurement Equation

$$\begin{bmatrix} \Delta c_t \\ \Delta d_t^a \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0'_{6\times1} & 1 & 0 & 0 & 0'_{5\times1} \\ 0 & \frac{\phi}{4} & B'_1 & 0 & \frac{1}{4} & \frac{1}{2} & B'_2 \end{bmatrix} \begin{bmatrix} x_t \\ x_{t-1} \\ \tilde{x}_{t-2} \\ \eta_{c,t} \\ u_t \\ u_{t-1} \\ \tilde{u}_{t-2} \end{bmatrix} \tag{B.25}$$

$$(\Delta \tilde{y}_t = H\xi_t)$$

Transition Equation

$$\begin{bmatrix} x_t \\ x_{t-1} \\ \tilde{x}_{t-2} \\ \eta_{c,t} \\ u_t \\ u_{t-1} \\ \tilde{u}_{t-2} \end{bmatrix} = \begin{bmatrix} \rho & 0 & 0'_{6\times1} & 0 & 0 & 0 & 0'_{5\times1} \\ 1 & 0 & 0'_{6\times1} & 0 & 0 & 0 & 0'_{5\times1} \\ 0'_{6\times1} & F_{x1} & F_{x2} & 0'_{6\times1} & 0'_{6\times1} & 0'_{6\times1} & 0_{6\times5} \\ 0 & 0 & 0'_{6\times1} & 0 & 0 & 0 & 0'_{5\times1} \\ 0 & 0 & 0'_{6\times1} & 0 & \rho_d & 0 & 0'_{5\times1} \\ 0 & 0 & 0'_{6\times1} & 0 & 1 & 0 & 0'_{5\times1} \\ 0_{5\times1} & 0_{5\times1} & 0_{5\times6} & 0_{5\times1} & 0_{5\times1} & F_{u1} & F_{u1} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ \tilde{x}_{t-3} \\ \eta_{c,t-1} \\ u_{t-1} \\ u_{t-2} \\ \tilde{u}_{t-3} \end{bmatrix} + \begin{bmatrix} \sigma_{x,t} & 0 & 0 \\ 0 & 0 & 0 \\ 0_{6\times1} & 0_{6\times1} & 0_{6\times1} \\ 0 & \sigma_{c,S_t} & 0 \\ 0 & 0 & \sigma_{d,t} \\ 0 & 0 & 0 \\ 0_{5\times1} & 0_{5\times1} & 0_{5\times1} \end{bmatrix} \begin{bmatrix} \eta_{x,t} \\ \eta_{c,t} \\ \eta_{d,t} \end{bmatrix} \tag{B.26}$$

$$(\xi_t = F\xi_{t-1} + R_{S_t}\eta_t, \quad \eta_t \sim i.i.d.N(0, \Omega))$$

where,

$$\tilde{x}_t = [x_t,\ x_{t-1},\ x_{t-2},\ x_{t-3},\ x_{t-4},\ x_{t-5}]'$$

$$\tilde{u}_t = [u_t,\ u_{t-1},\ u_{t-2},\ u_{t-3},\ u_{t-4}]'$$

$$B_1 = [\frac{\phi}{2},\ \frac{3\phi}{4},\ \phi, \frac{3\phi}{4},\ \frac{\phi}{2},\ \frac{\phi}{4}]', \quad B_2 = [\frac{3}{4},\ 1,\ \frac{3}{4},\ \frac{1}{2},\ \frac{1}{4}]'$$

$$F_{x1} = [1,\ 0'_{5\times 1}]', \quad F_{x2} = \begin{bmatrix} 0'_{5\times 1} & 0 \\ I_5 & 0_{5\times 1} \end{bmatrix}$$

$$F_{u1} = [1,\ 0'_{4\times 1}]', \quad F_{u2} = \begin{bmatrix} 0'_{4\times 1} & 0 \\ I_4 & 0_{4\times 1} \end{bmatrix}$$

$$\Omega = \begin{bmatrix} 1 & \rho_{cx} & 0 \\ \rho_{cx} & 0 & 1 \end{bmatrix}$$