Essays on Real Interest Rates, Government Debt and Monetary Policy

Srobona Mitra

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ProQuest Information and Learning Company
300 North Zeeb Road
P.O. Box 1346
Ann Arbor, MI 48106-1346

University of Washington

Graduate School

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Srobona Mitra

and have found that it is complete and satisfactory in all respects, and that any and all revisions required by the final examining committee have been made.

Chair of Supervisory Committee:

____

Charles R. Nelson

Reading Committee:

____

Charles R. Nelson

____

Richard Startz

____

Eric Zivot

Date: 8/2/02

University of Washington

Abstract

Essays on Interest Rate and Monetary Policy

Srobona Mitra

Chair of the Supervisory Committee:
Professor Charles R. Nelson
Economics

The three dissertation chapters explore various issues regarding interest rate as an instrument for monetary policy in industrial countries.

Chapter 1: "Is the Quantity of Debt a Constrain on Monetary Policy?" Monetary authorities have often voiced their concerns about a high debt level that could potentially restrain their ability to control the short-term interest rate as an instrument of monetary policy. This paper derives an augmented interest rate rule in which the response of the interest rate to expected inflation changes with the level of debt-to-GDP ratio. In particular, the interest rate response of the central bank towards an increase in expected inflation falls as debts increase beyond a certain threshold level. We estimate this threshold level for Canada and find evidence of its monetary policy having been constrained by debt during its inflation-targeting regime of the 1990s.

Chapter 2: "A Model of the Ex Ante Real Rate and the Natural Rate of Interest." (Co-authored with Charles R. Nelson and Richard Startz)

The ex ante real rate is unobserved in the United States. We model this unobserved ex ante rate using a state-space model of the ex post real rate and assume only that the inflation forecast errors are rational and that the unobserved ex ante real rate follows an autoregressive process. Finding evidence of time-varying volatility in the forecast error, we model the variances of both inflation uncertainty and monetary policy shocks. We, then, extract a measure of the natural rate as the more persistent component of the real rate.

Chapter 3: "Unknown Number of Mean Shifts and Persistence Around Mean: An Application to Short Term Real Interest Rates." (Co-authored with Arabinda Basistha)

We have provided a model for certain time-series processes that are characterized by two distinct phases: one in which the mean is stable or constant, and the other in which the mean is gradually shifting to another level. The number of these slow mean-shifts is unknown. Using an unobserved components model, the process can be divided into a 2-state Markov-switching process of the mean and a stationary cyclical component. The model is able to predict a slow shift in mean to another level without having to test for structural breaks or unit root in the data. This is particularly appealing for modeling the real interest rate of the United States, which give different results regarding presence of a unit root across various unit-root tests and sample periods.

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Acknowledgements

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Dedication

To my parents

Chapter 1: Is the Quantity of Debt a Constraint on Monetary Policy?

Chapter Abstract

Monetary authorities have often voiced their concerns about a high debt level that could potentially restrain their ability to control the short-term interest rate as an instrument of monetary policy. However, monetary policy rules in the literature do not account for such constraints. This paper derives an augmented interest rate rule in which the response of the interest rate to expected inflation changes with the level of debt-to-GDP ratio. In particular, the interest rate response of the central bank towards an increase in expected inflation falls as debts increase beyond a certain threshold level. We estimate this threshold level for Canada and find evidence of its monetary policy having been constrained by debt during its inflation-targeting regime of the 1990s.

1 Introduction

The literature on monetary policy using an interest rate rule of the type introduced in Taylor (1993) is vast and growing. Almost all of the papers talk about these rules in the context of industrialized countries, particularly the US, Canada, UK, Sweden, Australia, New Zealand among others; countries where central banks take decisions independent of those of fiscal authorities. The motivation for this paper comes from the fact that inspite of independence, monetary authorities have sometimes expressed concern about high levels of government debt. An example of such a concern can be found in the following quote from Bernanke, Mishkin et al (1999), pg.137: " *In the Bank of Canada's Annual Report, 1993 (released in March 1994), John Crow, in his last official act as Governor,* *called for the reduction of government debt in order to take pressure off interest rates and exchange rates."*

A similar concern was voiced by the US Federal Reserve Bank Chairman Alan Greenspan in 1995 when he said that he expected that "a substantial reduction in the long-term prospective deficit of the US will significantly lower very long-term inflation expectations vis-à-vis other countries" (Elmendorf and Mankiw (1999)). However, papers estimating interest rate rules do not account for "pressure" caused by high debt situations. If policy makers are responding to the state of the economy at any point in time, then the reaction function should explicitly take into account the constraint faced by policy makers regarding a high debt state.

The goal of this paper is, therefore, to answer the following question: *In an interest rate setting regime, how does the quantity of government debt affect the level of interest rate consistent with achieving the central bank's monetary policy objectives?* This paper has derived an augmented interest rate rule that explicitly takes into account the 'constraint' posed by a high debt level. Using an existing model, making modifications to it to incorporate 'pressure' due to debts, and setting up an econometric framework, we find both theoretical and empirical evidence of monetary policy being 'constrained' by a high debt level. We have used data for Canada during its inflation-targeting regime of the 1990s for empirical evidence.

To this end, we have discussed the existing literature on the links between debt and monetary policy in the next section. A formal model from which a generalized interest-rate rule can be derived is laid out in section 3. The current theoretical research on inflation targeting (as a goal for monetary policy) merges Real Business Cycle optimization models with price rigidities to set up a case for monetary policy to have real effects in the short and the medium run. We take as our starting point the results from one such model (Clarida, Gali and Gertler (1999)) and make changes in it to motivate a model for empirical estimation and testing. Data generated from the new debt-constrained interest rate rule and an empirical model with its results for Canada is discussed in section 4, while section 5 summarizes and concludes. All algebraic details of derivation are in the Appendix.

2 Existing Literature on the Links between Debt and Monetary Policy

A direct link between debt and monetary policy is seigniorage; a high debt leads to monetization of debt that in turn leads to inflation. In post war Germany, hyperinflation was used to wipe out the debt. For long-term debt, a surprise-inflation would erode away its real value; for short-term debt, the case is different (Dornbusch (1998)). Rolling over of short debt would involve higher interest rates; but if inflation expectations were perfect, this higher rate would offset the inflationary erosion. For the United States, Dornbusch has shown that a percentage point increase in the interest rate amounts to roughly $30 billion in extra deficit, or half a percentage point of GDP. There was a long period – mid-1930s to mid-1950s – where bond-yields were kept at or below 2.5 per cent; a high debt situation was accommodated by pegged, low interest rates. Thus interest rate provides a more useful link between debt and monetary policy, especially in the highly industrial countries where seigniorage accounts for a negligible portion of budgetary finance.

A different approach to the quantity-of-debt link to monetary policy has been explored by Leeper (1989), Sims (1994), Woodford (1998, 2001) et al. In this fiscal theory of price level determination, if the real primary surplus is determined exogenously, the quantity of debt could affect the price level through a wealth effect upon private consumption. A non-Ricardian tax cut – a tax cut unaccompanied by expectations of future tax increases – makes agents increase their consumption creating an excess demand in the market. This excess demand drives up prices until a fall in the real wealth – partly comprised of real debt holdings – reduces this excess demand. The maturity structure of government debt would affect monetary conditions too; in particular, shorter the maturity, larger is the inflation required to reduce the value of the public debt enough to restore equilibrium following an expansionary fiscal shock.

The above insight into the maturity structure of debt is opposite to the usual observation that a higher proportion of short-term or indexed linked debt is associated with a lower incentive to inflate (Missale and Blanchard (1994)). A government will resist the urge to inflate away debt if the rewards are small, and the cost of a lost reputation is high. Given that the rewards from unexpected inflation are increasing in the level of debt and increasing in maturity, the government will keep its non-inflation pledge credible by decreasing maturity as debt increases. However, the maturity structure of debt is not a part of our analysis.

It has been pointed out by Goodhart (1999) that the importance of the link between debt and monetary policy has been reduced to second-order level, if at all, in countries like the US and UK. However, the question of whether high and rising debt has become an important constraint for policy by making the monetary authorities accept more inflation than they otherwise would, has received little empirical evidence (Dornbusch (1998)). The European Central Bank (ECB) represents an extreme case where monetary policy is conducted independently of fiscal policies of the participating nations. Yet, high debts (beyond 60% of GDP as specified by the Stability and Growth Pact) accumulated by the member nations' governments clearly represent a risk for the ECB.

This paper is an attempt to explore this issue with the help of an empirical model in the context of an interest rate rule. We look at how the Taylor rule can be generalized to incorporate the constraint that debt imposes on the monetary authorities' ability to use the short term interest rate as an instrument for monetary policy. The model would also enable us to estimate the threshold level of debt beyond which it acts as a constraint for policy. We find evidence of such a constraint in the context of Canada.

3 Model

The model is an extension of the one used by Clarida, Gali and Gertler (1999) (henceforth, CGG). The latter had used an optimization-based closed economy framework to derive optimal interest rate rules under discretion and commitment. We adopt their two-step procedure to derive the optimal interest rate rule under discretionary policy. First, a baseline rule is derived by minimizing the central bank's loss function subject to the expectational IS and Aggregate Supply functions. Next, the rule is re-derived for the case when a debt-constraint operates – the IS equation is modified to incorporate debt in the analysis. For this purpose, we are going to distinguish between 'active' and 'passive' monetary and fiscal policies in the sense of Leeper (1991).

Monetary policy is active and fiscal policy is passive if direct lump sum (net) taxes back debt shocks entirely. The monetary authority is not constrained by current budgetary conditions; it is free to choose a decision rule that depends on past, present or future expected variables influencing its target variables. The forward-looking version of Taylor rule proposed by CGG (1999) had assumed this policy-coordination scheme. In their model, fiscal behavior supported the prevailing monetary policy by raising net taxes enough to prevent explosive debt paths. However, a reason for concern (by the central bank) about a very high debt level at certain times could be the unresponsiveness of net taxes to debt. This is when monetary authorities could 'cooperate' with fiscal authorities to bring down debt service costs – either by responding less to expected inflation or by responding to the future expected real interest rate or both. In this sense, monetary policy becomes less active and fiscal policy less passive. The interest rate rule derived in this paper is intended to capture the activeness and passiveness of monetary policy.

The objective function of central banks is characterized by a quadratic loss function increasing in, both present and future expected, inflation and output variability. The central bank minimizes this loss function over the target variables, inflation and output gap subject to the state of the economy. Under discretionary policy, the problem to be solved is the same each period; the monetary authorities make no attempt to manipulate private sector beliefs. This objective function can be written as follows:

$$\max -\frac{1}{2}[\alpha x_t^2 + \pi_t^2] \tag{1}$$

This function penalizes for deviations of (log of) output, y, from its trend, z, and for deviations of inflation, π, from its target level. Here the target for inflation is taken as zero; the analysis will not change qualitatively if, as in countries like Canada, UK and the US, the target is different from zero or is stated as a range rather than as a point. The variable $x \equiv y - z$, and represents the output gap. It should be noted that in some of the inflation-targeting countries, like Canada and New Zealand, the inflation target is jointly determined and announced by both the government and the central bank (Bernanke, Laubach, et al, 1999); however, it is often the central bank that is responsible for meeting the target. The parameter α represents the relative weight put on output in the objective function – the weight on inflation is normalized to 1. An independent central bank will not target any fiscal policy variable, as the latter is the sole concern of the fiscal authorities, the European Central Bank being an extreme case in point.

The state of the economy is summarized by the following set of equations:

$$x_t = -\varphi[i_t - E_t\pi_{t+1}] + E_t x_{t+1} + \varepsilon_t \tag{2}$$
$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t \tag{3}$$
$$\varepsilon_t = \mu\varepsilon_{t-1} + \hat{\varepsilon}_t$$
$$u_t = \rho u_{t-1} + \hat{u}_t$$
$$\hat{\varepsilon}_t \sim i.i.d.(0, \sigma_\varepsilon^2),\ \hat{u}_t \sim i.i.d(0, \sigma_u^2)$$

Equation (2) is an expectational IS curve relating the present output gap, x_t, to the expected real interest rate and future expected output gap. As with traditional Keynesian analysis, we have a negative relationship between the real interest rate and the output gap. A rise in the real interest rate causes a decline in investment and durable consumption demand; this lowers current output causing the output gap to go down. Consumption smoothing objectives also lead individuals to respond to an increase in future expected output by raising current consumption and, hence, the current output gap. This equation can be derived from a consumption euler equation resulting from a utility maximization exercise of a representative agent under standard assumptions as in CGG. The autoregressive disturbance term, ε_t, is crucial to our analysis and is a function of expected changes in government purchases relative to expected changes in potential output. Under the assumption of an active monetary policy, government purchases is assumed to be exogenous and any change in it is unpredictable.

Equation (3) is a Phillips curve assuming Calvo (1983) pricing – each of the monopolistically competitive firms chooses its nominal price to maximize profits subject to constraints on the frequency of future price adjustments; equation (3) is derived by aggregating individual firm pricing decisions. Current inflation depends both upon current and future expected output and inflation – inflation is a result of increases in marginal costs partly due to variations in excess demand and partly arising from "cost push" factors, u_t; this disturbance is also an autoregressive error term. Both $\hat{u}_t$ and $\hat{\varepsilon}_t$ are i.i.d. errors with zero-mean and constant variances.

In solving the central bank's problem, we adopt a two-step procedure. In the first step, objective function (1) is maximized at time t with respect to π_t and x_t subject to the Phillips Curve (PC) equation (3). In the second step, the first order conditions for π_t and x_t are put back into the IS equation (2) to get the optimal interest rate rule. The resulting interest rate rule for the baseline model is as follows:

$$i_t = \beta_\pi E_t \pi_{t+1} + \frac{1}{\varphi}\varepsilon_t \tag{4}$$

where, $\beta_\pi = 1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha} > 1$

The monetary policy rule above incorporates concern for both output and inflation variability and responds to both demand and supply side shocks. It is important to note that $\beta_\pi > 1$ – an increase in expected inflation calls for short term nominal interest rates to be moved up more than proportionately so that an increase in real rates influences real output and ultimately inflation. However, this rule, along with other forms of Taylor rules, does not reflect any concern posed by a high level of government debt; yet central bankers have expressed concern over high debt levels and how the latter could constrain their ability to use the nominal interest rate as an instrument to fight inflation. Recognizing that the interest rate response to higher expected inflation could be an approximation to a more general function, we generalize this rule to incorporate concerns over debt.

The government purchases term, G_t, is embedded in the error term of the IS equation as follows:

$$\varepsilon_t \equiv E_t\{\Delta z_{t+1} - \Delta g_{t+1}\} \tag{5}$$

where, $\quad g_t \equiv -\log(1 - \frac{G_t}{Y_t})$.

Y_t is the current output level. When fiscal policy is less passive, owing to a debt-GDP ratio above a certain threshold level, the expected change in government purchases will depend on the existing level and growth in debt and its servicing costs. This is because taxes (net of transfers) are no longer expected to back debt shocks. We need to take into account the government budget constraint to form expectations about future growth in government purchases. If the debt level is growing, then government purchases will be expected to fall in the future with increases in real debt service costs.

The government budget constraint, an identity, can be written as a ratio to output in the following form (see, for example, Blanchard (2000), pg.523):

$$\Delta b_t = (r_t - \hat{y})b_{t-1} + \frac{G_t}{Y_t} - \left(\frac{tx - tr}{Y_t}\right) \tag{6}$$

Here b_t is the debt-to-GDP ratio, r_t is real interest rate, $\hat{y}$ is growth rate of output, approximated by $y_t - y_{t-1}$, $\frac{G_t}{Y_t}$ is government purchases to GDP ratio and net tax (tax - transfer) expressed as ratio to GDP is given by $\frac{tx - tr}{Y_t}$.

Equation (6) relates the growth of government debt – a large part of which is marketable securities issued by the government – to its interest payment obligations on past debt and its primary deficit. The latter comprises of the excess of government purchases and transfer payments beyond total tax collections. We have not included seigniorage as a source of revenue for the government as these revenues are very small and insignificant for the major industrial countries.

Under a less passive (or active) fiscal policy, net taxes are evolving as an exogenous process such that its change is unpredictable. Government spending is assumed to evolve as follows:

$$g_{t+1} = \phi_0 - \phi_1[(r_{t+1} - \hat{y}_{t+1})b_t] + \eta_{t+1}, \quad \eta_t \sim i.i.d(0, \sigma_\eta^2) \qquad (6a)$$
$$\phi_1 > 0 \; if \; b_t > b^*$$
$$\phi_1 = 0 \; if \; b_t \leq b^*$$

Equation (6a) captures the fact that government expects spending to decrease with higher debt service costs if debt-GDP ratio is above a threshold level of b^*. This is the time when fiscal policy becomes less passive and puts pressure on monetary policy. However, when debt is below the threshold level, then monetary policy is active and there is no attention paid to the level of debt.

Substituting for $\frac{G_t}{Y_t}$ in (5) from (6a), we can make some algebraic manipulations, as is shown in the Appendix, to derive a new IS equation :

$$x_t = -(\varphi + \phi_1 b_{t-1})[i_t - E_t\pi_{t+1}] + E_t x_{t+1} + E_t[\phi_1 b_t\{i_{t+1} - \pi_{t+2}\}] - \phi_1 E_t\{\Delta x_{t+1} b_t - \Delta x_t b_{t-1}\} + \varepsilon_t^{'} \quad (7)$$

In this new IS equation, real interest elasticity parameter, φ, interacts with the past period debt-GDP ratio, b_{t-1}, to reinforce the effect of a high interest rate on the output gap, x_t. The second term on the right hand side, $E_t x_{t+1}$, is a term that lends the IS curve its expectational nature and was also present in the previous IS equation (2). There are two new terms – one related to future expected debt servicing costs and the other related to interactions between debt and growth in the output gap. The error term, $\varepsilon_t^{'} \equiv (1-\phi_1 b_t)E_t(\Delta z_{t+1}) + \phi_1 b_{t-1} E_t \Delta z_t - E_t(\Delta b_{t+1} - \Delta b_t)$, is an autoregressive (AR) process and is assumed to have the same AR coefficient as ε_t in (2).

Re-solving the problem – using the same method as is shown in the Appendix – with an unchanged objective function, equation (1), an unchanged PC, equation (3), and the newly derived IS, equation (7), we come upon a new interest rate rule which takes the following form:

$$i_t = \beta_\pi^{'} E_t \pi_{t+1} + \frac{\phi_1 b_t}{\varphi + \phi_1 b_{t-1}} r_{t+1}^{e} + \frac{\lambda}{\alpha}\left(\frac{\phi_1 b_{t-1}}{\varphi + \phi_1 b_{t-1}}\right)\pi_{t-1} + \frac{\varepsilon_t^{'}}{\varphi + \phi_1 b_{t-1}} \quad (8)$$

where,

$$\beta_\pi^{'} \equiv \left\{\beta_\pi + \left(\frac{\varphi}{\varphi + \phi_1 b_{t-1}} - 1\right)\beta_\pi + \phi_1 \frac{\varphi}{\varphi + \phi_1 b_{t-1}}\left(\frac{(\alpha\rho - \lambda)_1 b_{t-1} - \lambda(1-\rho) b_t}{\alpha\rho\varphi}\right)\right\}$$

$$\beta_\pi = 1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha} > 1$$

$$r_{t+1}^{e} \equiv E_t(i_{t+1} - \pi_{t+2})$$

There are several things to notice about this interest rate rule. First, if $\phi_1 = 0$ then $b \leq b^*$ we get back the earlier benchmark rule of CGG given by (4). Second, apart from expected inflation, policy also depends upon lagged inflation rate. This indicates that the central bank looks at past inflation beyond its role in forecasting future inflation. Third, there is a new variable that the central bank's policy responds to, independent of past or future inflation rates – the future expected short-term real interest rate, r^e_{t+1}. We can also interpret this term, with some caution, as an indicator for future debt-service costs for the government. Fourth, the coefficients of the three variables on the right hand side of equation (8) vary with the (present or past) debt level.

The first term on the right hand side of (8) is the response of short-term nominal interest rate to expected future inflation – a higher expected inflation calls for a hike in the nominal rate as a 'lean against the wind' policy (see Appendix). However, this response may not be greater than 1. In order to analyze how the policy maker's response to expected inflation alters in the case of debt above the threshold, let us assume debts are at their steady state level, b, which is assumed to exist. Imposing this condition on (8), we have the following:

$$i_t = \left\{ \beta_\pi + \left(\frac{\varphi}{\varphi + \phi_1 b} - 1 \right) \beta_\pi + \frac{\phi_1 \varphi b}{\varphi + \phi_1 b} \left(\frac{\rho(\alpha + \lambda) - 2\lambda}{\alpha \rho \varphi} \right) \right\} E_t \pi_{t+1} + \frac{\phi_1 b}{\varphi + \phi_1 b} r^e_{t+1} + \frac{\lambda}{\alpha} \left(\frac{\phi_1 b}{\varphi + \phi_1 b} \right) \pi_{t-1} + \frac{\varepsilon'_t}{\varphi + \phi_1 b} \quad (8')$$

The interest rate response to $E_t \pi_{t+1}$ is given by

$$\beta'_\pi \equiv \beta_\pi + \left(\frac{\varphi}{\varphi + \phi_1 b} - 1 \right) \beta_\pi + \frac{\phi_1 \varphi}{\varphi + \phi_1 b} \left(\frac{(\rho(\alpha + \lambda) - 2\lambda) b}{\alpha \rho \varphi} \right) \quad (8'a)$$

Whether $\beta_{\pi}^{'}$ is larger than β_{π} depends upon the relative magnitudes and signs of the second and the third terms of $(8'a)$.

We have already seen that if $b \leq b^{*}$, there is no attention paid to it by monetary authorities and the original baseline rule given by (4) applies. However, if $b > b^{*}$, then $(\frac{\varphi}{\varphi + \phi_1 b} - 1) < 0$, given $\varphi > 0$. This implies that the second term of $(8'a)$ is negative. The sign of the third term of $(8'a)$ is ambiguous unless we make assumptions about the relative magnitudes of α and λ, given $\rho < 1$. If $\alpha \leq \lambda$ -- weight given to output variations is 'low' in the central bank's objective function (1) -- then the third term is negative. If $\alpha > \lambda$, then the ambiguity remains. In other words, if the policy maker is tough (low α) towards inflation – as during a nearly pure inflation-targeting regime – then $\beta_{\pi}^{'} < \beta_{\pi}$.

Furthermore, $\beta_{\pi}^{'}$ decreases if debts, b, increase and $\alpha \leq \lambda$. To see this, we can differentiate $(8'a)$ partially with respect to b:

$$\frac{\partial}{\partial b}\beta_{\pi}^{'} \equiv \frac{\partial}{\partial b}\left\{\beta_{\pi} + \left(\frac{\varphi}{\varphi + \phi_1 b} - 1\right)\beta_{\pi} + \frac{\phi_1 \varphi}{\varphi + \phi_1 b}\left(\frac{(\rho(\alpha + \lambda) - 2\lambda)b}{\alpha\rho\varphi}\right)\right\}$$

$$= -\frac{\phi_1 \varphi \beta_{\pi}}{(\varphi + \phi_1 b)^2} + \frac{\phi_1}{\alpha\rho(\varphi + \phi_1 b)^2}[\varphi\{\rho(\alpha + \lambda) - 2\lambda\}]$$

$$< 0, \text{ if } \alpha \leq \lambda.$$

Our conclusion is ambiguous if $\alpha > \lambda$. Summarizing the central bank's response to expected inflation: in an inflation-targeting regime (low α), the policy responds less to expected inflation if debts are high and this response falls if debts increase over time. This could translate into the central bankers feeling 'pressure' when debts are growing – the coefficient of, and hence the ability to fight, inflation being constrained by high and growing debts. Monetary policy is less active during this period.

The sensitivity of the current short-term interest rate to expected future real interest rate, as is given by the second term on the right hand side of (8), is positive. Independent of whether there is a boom or a recession in the economy, if debts are high, the interest rate is moved up if future expected real interest rate, r^e_{t+1}, goes up. So high debts can pose a constraint on the monetary authorities if there are recessionary pressures in the economy calling for a fall in the interest rate. On the other hand, during a boom, although the coefficient for expected inflation is low when debts are high, that for r^e_{t+1} is high, thus 'helping' the monetary authorities to increase interest rates to some extent. However, it should be borne in mind that this is the expected future *short* real rate and might not reflect debt-service costs accurately.

The third term of equation (8) is the interest rate response to the past period inflation rate – it is a positive response when debts are high. This makes the central bank pay attention to the lagged inflation rate beside its ability to forecast future inflation. In the next section, using different parameter values for central bank's weight on output-variability, α, we find that the response to expected inflation is nearly 0.

In what follows, an empirical model is set up to estimate both the threshold level of debt and the debt-constrained policy rule (using the estimated threshold level) in the context of Canada. Then, the baseline rule (4) and the debt-constrained rule (8) are generated using actual data and estimates of IS and PC for Canada.

4 Empirical Support and Simulation

4.1 Empirical model

For the purpose of estimating and testing for an interest rate rule given by equation (8), we will be assuming that debts are above a certain level. This level is estimated by using a threshold regression model. A dummy variable approach is used in setting up the model and estimating it by generalized method of moments (GMM). One way of writing an econometric model for our purpose is as follows:

$$\begin{aligned} i_t = {} & \alpha + \beta_\pi E_t(\pi_{t+n} \mid \Omega_t) + \theta_1 (DUM) E_t(\pi_{t+n} \mid \Omega_t) + \gamma \pi_{t-1} + \theta_2 (DUM) \pi_{t-1} \\ & + \sigma r^e_{t+1} + \theta_3 (DUM) r^e_{t+1} + \mu z_t + \varepsilon_t \qquad (9) \end{aligned}$$

In this model, we have the monetary policy instrument as a short-term interest rate, i responding to expected future inflation n-period ahead, $E_t \pi_{t+n}$, future expected real interest rate (exogenously given), r^e_{t+1}, and to other variables such as exchange rates and foreign interest rates contained in the vector z. Expectations on inflation are formed given the time-t information set, Ω_t. The policy maker takes lagged inflation rate into account only when debts are high (or above a certain level); lagged inflation has, therefore, been included in (9). Concern about high debts is incorporated using the dummy variable *DUM* defined as follows:

DUM = 1, when debt-GDP ratio > estimated threshold level (b^*)

= 0, otherwise.

As has been explained in the previous section, the coefficients of (9) are expected to have the following range of values:

$$\beta_\pi > 1, \gamma = 0, \sigma = 0, \mu > 0, \theta_1 < 0, \theta_2 > 0, \theta_3 > 0.$$

In order to eliminate the unobserved expectation term, $E_t \pi_{t+n}$, we add and subtract the realized n-period ahead inflation rate, π_{t+n}, from the right hand side of (9) and write the interest rate rule in terms of the realized variables and a new error term, v_t. Here v_t is a linear function of the forecast error of inflation (adjusted for the dummy variable) and the exogenous disturbance, ε_t:

$$\begin{aligned} i_t = {} & \alpha + \beta_\pi \pi_{t+n} + \theta_1 (DUM)\pi_{t+n} + \gamma \pi_{t-1} + \theta_2 (DUM)\pi_{t-1} \\ & + \sigma r^e_{t+1} + \theta_3 (DUM) r^e_{t+1} + \mu z_t + v_t \end{aligned} \qquad (9')$$

Assuming that the actual interest rate immediately hits its target, we have allowed the error term, v_t, to be an autoregressive (AR) process of order 1, to absorb the serial correlation in the data. In particular,

$$v_t = \phi v_{t-1} + \hat{v}_t, \text{ where } |\phi| < 1, \quad \hat{v}_t \sim i.i.d(0, \sigma_v^2).$$

In actual estimation, lagged regressors were added to the set of instruments for consistent ARMA estimation.

If $w_t \in \Omega_t$ is the vector of variables within the bank's information set, that are orthogonal to $\hat{\nu}_t$, we have a set of orthogonality conditions that can be exploited to estimate model (9) by GMM. These are $E[\hat{\nu}_t \mid w_t] = 0$, which implies:

$$E[i_t - \alpha - \beta_\pi \pi_{t+n} - \theta_1 (DUM)\pi_{t+n} - \gamma\pi_{t-1} - \theta_2 (DUM)\pi_{t-1} \\ - \sigma r^e_{t+1} - \theta_3 (DUM) r^e_{t+1} - \mu z_t \mid w_t] = 0 \quad (10)$$

The parameter vector $[\alpha, \beta, \theta_1, \gamma, \theta_2, \sigma, \theta_3, \mu, \rho]$ and the threshold level of debt are then estimated with the instruments being elements of the vector w_t. There are three advantages of using GMM in our context. Firstly, it enables us to use lead variables on the right hand side of the regression equation and use an instrumental variables approach in the iterative process. Ordinary Least Squares (OLS) estimates would be biased and inconsistent since the lead inflation variable is correlated with the error term. Secondly, GMM is robust to any distributional assumption about the error term. With Maximum Likelihood Estimation (MLE), we would have to specify its distribution. Thirdly, we can choose more instruments than there are parameters to estimate; the GMM estimator will be consistent. The set of instruments is chosen such that they are potentially useful in forecasting future inflation. We are assuming that the central bank sets interest rate according to (9) taking into account all relevant information available at that time.

For the right hand side variables of (9), we have used their lagged (8 lags) values as instruments. These are discussed in more detail with the results. Since we have more instruments than there are parameters to estimate, we have used Hansen (1982)'s *J*-statistic to test for over-identifying restrictions in the system. The p-value (the minimum level of significance at which the null hypothesis of over-identifying restrictions being valid, is rejected) is reported with the estimation results.

In order to choose the threshold level of debt, we search over a grid of values for debt-GDP ratio ranging from 46% (the minimum during the sample period) to 52% (almost the maximum during this period). The step size is 0.1% point. We choose that level of the debt for which the objective function of the GMM is minimized – that is the lowest J-statistic.

4.2 Data

We have used data for Canada to see if debt does constrain monetary policy in an interest rate-setting regime. The choice of the country was given by the motivation stated in the introduction -- the central bank had expressed concern over fiscal issues, especially debt. A second reason for the choice was the ready availability of high-frequency data. Canada adopted (by making a public statement) inflation targeting in 1991, and this fact guides our choice of the sample period.

Monthly data for Canada is taken from the Bank of Canada, DRI Economics Database and International Financial Statistics, and ranges from 1991:11 to 2000:12. We have used the Overnight Rate as the short-term interest rate, i, the Bank of Canada uses as an instrument for monetary policy. The Consumer Price Index (CPI) is used to calculate the rate of inflation (at an annual rate), π, and the Gross Domestic Product (GDP) deflated by the GDP deflator is used as a measure of real output. Since data on GDP is only available quarterly, we have calculated the output gap, x, by de-trending the log of

Index of Industrial Production (IIP) by means of a quadratic trend. The linear-detrended adult unemployment rate has also been used as another measure of the output gap as a robustness check, since there are numerous ways of calculating this gap.

We have divided the monthly series for Total Interest-bearing Marketable Debt issued by the government of Canada (Federal), by GDP to derive the debt-GDP ratio – in this case we have interpolated the quarterly series for GDP into a monthly one. The debt-GDP ratio, although not used directly in actual estimation, guides our definition of the dummy variable, DUM. In Figure 1.1, DUM = 1 for the shaded region when debt-GDP ratio > threshold level.

The series for expected future short-term real interest rate is approximated by the rate on long-term indexed bonds. Unavailability of data on a short-term indexed bond prevented us from exploiting a term structure relationship to derive the future expected short-term real interest rate. Also, we had constructed a series for expected inflation by taking the difference in rates for a long-term nominal bond and the long term indexed bond – this series would have been exogenous to i. However, it failed to provide convincing results for the stance of monetary policy. This could also provide a guide to the horizon, n, that the policy maker considers while taking a decision – near-to-medium term (a quarter to about a year ahead) as opposed to long term (ten to thirty years).

The US Federal Funds Rate has been used as z. Movements in some of the variables discussed above are shown in Figure 1.1. For the short sample – 1991 to 2000 – we are considering, monthly data for inflation rate, output gap and expected real interest rate are all I(0). Estimation results are discussed in the next sub-section.

4.3 Empirical Results

The threshold level of debt is estimated to be 48% ($\pm$ 0.1% point). The series of J-statistics for the GMM estimations using a grid search over a range of debt levels is given in Figure 1.2. This corresponds to the actual debt level in Canada, when the central bank had expressed concern over a high debt level. The parameter estimates for GMM is given in Table 1.1. Column 1 has the parameter estimates for the baseline model such as those used by CGG (1998). We find that the estimate for β_π is 0.16; inspite of announcing an inflation targeting policy in 1991, the Bank of Canada was not moving interest rates aggressively enough to affect the real rate. The Bank was instead accommodating inflation. One explanation for this could be the fact that Bank of Canada was responding to movements in the Federal Funds Rate – estimate of μ is 0.78. Besides, the inflation rate was already low when the announcement about targeting inflation was made. The unemployment gap has not been used in the baseline-estimation, as it was not significant. The Bank did not directly respond to exchange rate movements; however its lags were used as instruments. This suggests that the exchange rate only helped to forecast future inflation.

The instruments used for the baseline model are the first through sixth, ninth and twelfth lags of the following variables: overnight interest rate, inflation rate, rate on indexed bond, nominal exchange rate and Fed Funds Rate. A test for over-identifying restrictions revealed a p-value for the *J*-statistic to be 0.97 – we therefore cannot reject the null hypothesis of the instruments used being valid. The horizon, n, for expected inflation is taken as 3 for the baseline case suggesting that the bank looks a quarter ahead while setting interest rates. A horizon of 12 and beyond failed to be statistically significant.

We next consider the debt-constrained case. The estimates for the debt-constrained rule are presented in column 2 of Table 1.1. The forecast horizon for inflation is again taken as 3. The estimate for β_π is 0.23 and slightly higher than that for the baseline case. The unemployment gap does seem to influence policy independent of concerns about expected inflation.

The estimates for θ_1 and θ_2 have the signs predicted in subsection 4.1. The fact that θ_1 is negative suggests that the Bank's concern about high debts do show up in its monetary reaction function. However, its size is too small to influence policy significantly. In a high-debt situation, the bank seems to rely on lagged inflation; this also seems to influence policy slightly during other times. When debts go beyond 48%, the bank moves interest rate up by nearly 29 basis points in response to lagged inflation. The total effect of inflation, past and future, $\beta_\pi + \theta_1 + \gamma + \theta_2$, is 0.64; this is less than unity.

The expected debt service cost as proxied by r^e does influence policy. However, its negative coefficient suggests that the authorities *decrease* interest rates in response to an increase in r^e when debts are high. Although the sign is opposite to that predicted by (8), this makes more intuitive sense. A high level of debt would mean higher debt service costs which is reflected in higher ex ante *long-term* real interest rates. Therefore, rate on long-term indexed bonds give a better proxy for expected debt-service costs than the expected future short real rate does.

The instrument set is almost identical to that used for the previous model. The lag structure is the same. Exchange rate is left out but unemployment gap and return on indexed bonds are included. A test for over-identifying restrictions failed to reject the null, as is given by the p-value of the J-statistic. In summary, we do find evidence of debts posing a constraint for monetary policy.

4.4 Simulation Results

In this subsection, the baseline and the debt-constrained rules are generated using actual data on i, b, π and r^e, and the four unknown parameters, φ, λ, ρ and α. The first two of these parameters are estimated directly from the IS and the PC by GMM. For this purpose, the expected real interest rate is replaced by the ex-post short-term real interest rate in the IS. The expected inflation and expected output gap, in the PC and the IS respectively, are replaced by their realized values. Eight lags (similar in structure to those used in subsection 4.3) of each of the right hand side variables are used as instruments. The GMM estimates of the parameters in the IS and PC are provided in Table 1.2. As a simplifying assumption, $\phi_2 = 1$.

Using the estimates, $\lambda = 0.03$, $\varphi = 0.04$, and different combinations of values for the free parameters α and ρ, we generate a family of baseline and debt-constrained rules. The combination of the values of these parameters for which the RMSE of the debt-constrained rule is the least, given that the RMSE for the baseline is virtually unchanged at 3.8 (within 1 place of decimal) is the following: $\alpha = 0.05$, $\rho = 0.97$. This implies that the weight given to output variability in the Bank of Canada's objective function is only 5% of that given to inflation variability. As expected in an inflation-targeting regime, the weight on inflation variation is high in the central bank's objective function. Also, the high value of the autoregressive parameter for the Phillips Curve, ρ, indicates high persistence of shocks to inflation.

The performance of the baseline rule versus the debt-constrained rule is compared in Figure 1.3, panel (a). We find that the debt-constrained rule tracks the actual overnight rate better than the baseline model and has a lower RMSE of 1.13. Furthermore, the coefficient of expected inflation, β_π, is plotted against the debt-GDP ratio in panel (b). With a correlation coefficient of –0.99, we see a strongly negative relationship between debt-GDP ratio and β_π, although the magnitude of the latter is very small. This complements our empirical observation that Canadian monetary policy reacts cautiously to expected inflation when debt-GDP ratio is high, although the magnitude of θ_1 is close to zero.

Finally, we generate β_π for the baseline case $(b = 0\%)$ and for the debt-constrained case $(b = 48\%)$ for different values of α and ρ, in Figure 1.4. In panel (a), the baseline model will generate a stable value of β_π if both α and ρ are not very small. In other words, for $\alpha = 0.05$ *and* $\rho = 0.97$, β_π will be close to 1 (within two places of decimal). In panel (b), β_π will only stabilize for (approximately) $\alpha > 0.5$ and $\rho > 0.4$. For very low α and high ρ, β_π can be very small and even negative.

5 Summary and Conclusions

The attempt of this paper was to find out if the quantity of debt was constraining monetary policy in one of the major industrial countries, in an interest rate-setting regime. There has been historical evidence of central banks expressing concern over high debt levels. To this direction, an interest rate rule for discretionary monetary policy that explicitly takes account of debt as a constraint, has been developed; this helps to pick up the times during which monetary policy has been less active and fiscal policy less passive. This debt-constrained rule gives way to the baseline (Taylor-type) rule as a special case.

It has been shown algebraically that such constraining behavior comes into effect *only* when debts are above a certain level. An empirical model has been constructed and then estimated by generalized method of moments. The threshold level of debt has been estimated and shown to correspond with the actual level of 48% when the Bank of Canada governor had expressed concern over high debts. For the case of Canada during the nineties, we have shown that although the response to expected inflation has been mildly constrained by debt, response to ex ante long-term real rate has significantly been negative. Rising debt-service costs did make the Bank more cautious. Furthermore, simulation results are provided to complement our empirical evidence.

There are important policy implications for the findings of this paper. Even though major industrialized countries have independent central banks, we have established a channel through which fiscal decisions – leading to higher debts – could constrain monetary authorities in their policy decisions. Implications for policies formulated by the newly formed European Central Bank (ECB) are especially important since the fiscal position of the member nations are going to constrain ECB decisions if the total debt is higher than a certain level. With the availability of data, one can estimate the threshold level of combined debt of the member nations that could potentially constrain ECB objectives, using the model in this paper.

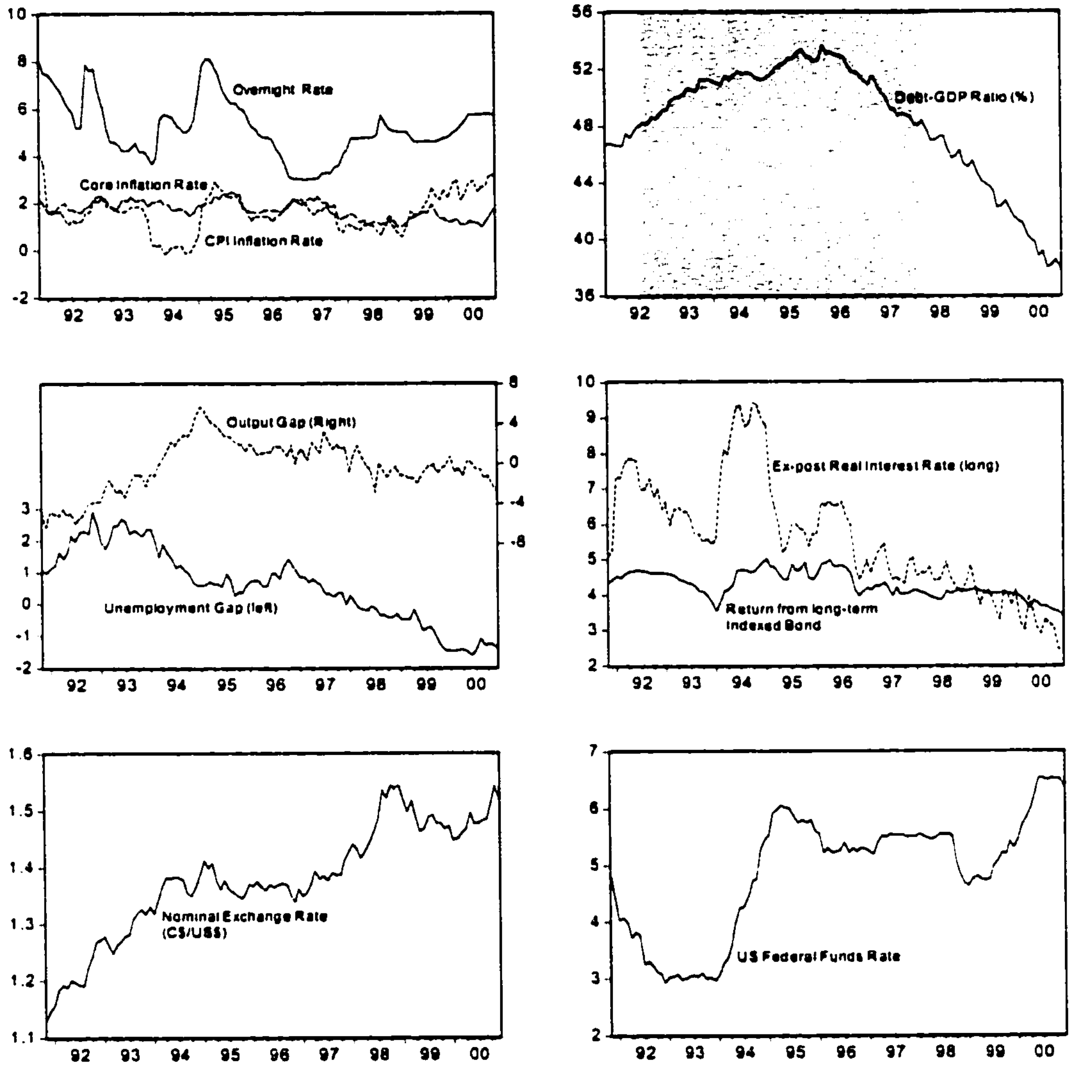


Figure 1.1: Movements in some Key Variables in Canada, Monthly Data, 1991:11 to 2000:12.

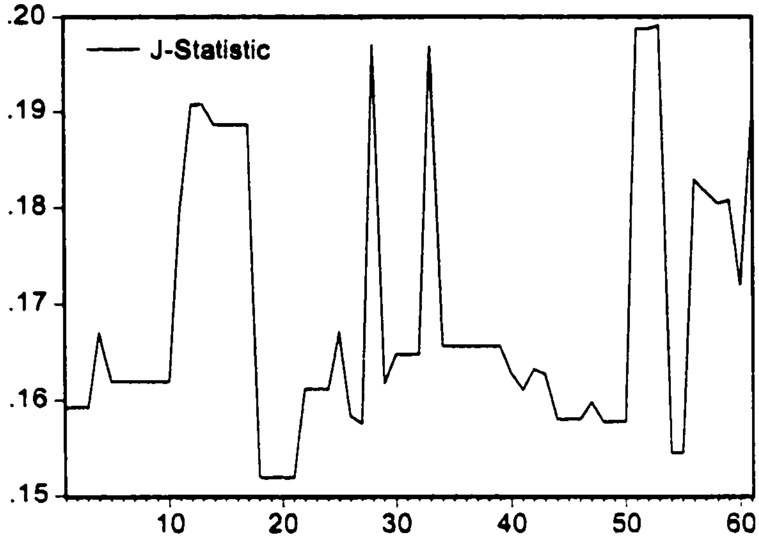
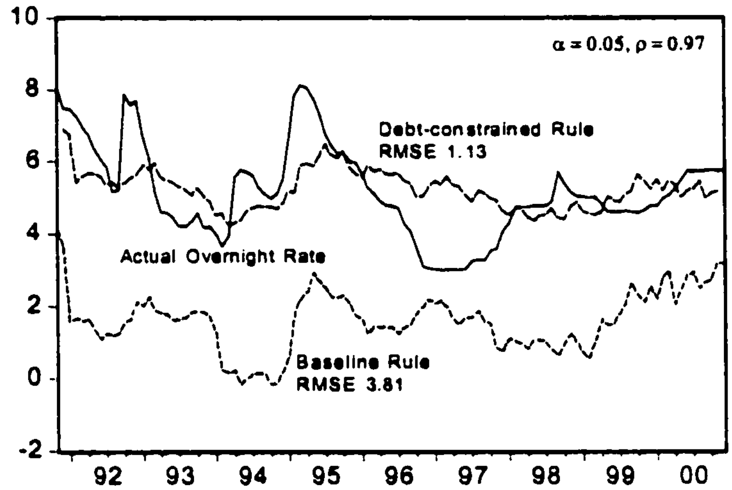
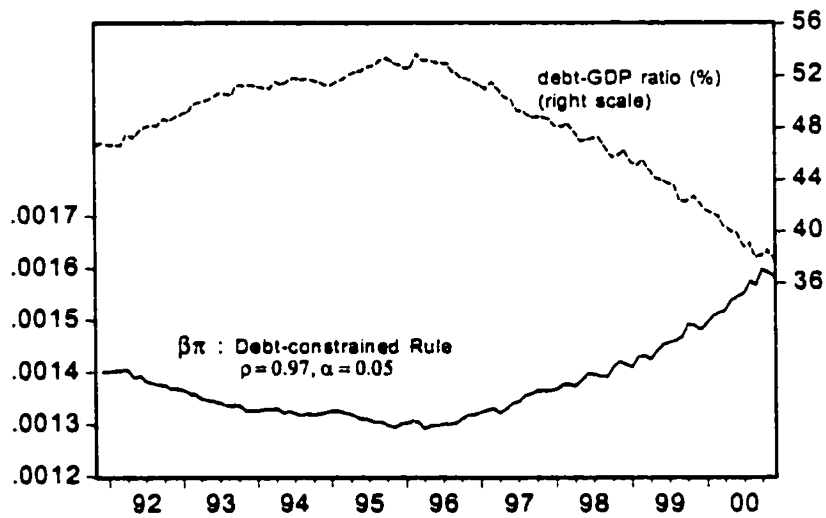


Figure 1.2: J-Statistics for Choosing the Threshold level of Debt.

Note: Each of the J-statistics is for a GMM regression of (9') for threshold levels of debt ranging from 46% to 52% of GDP. The horizontal axis gives the step-size, ten times the percentage points above 46%, for the grid search. The J-statistic is minimized at 20; this implies a threshold level of debt of 48%.

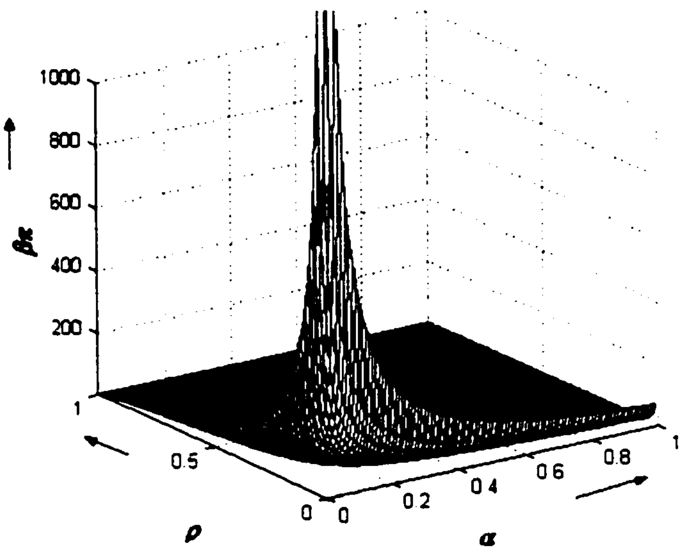


(a)

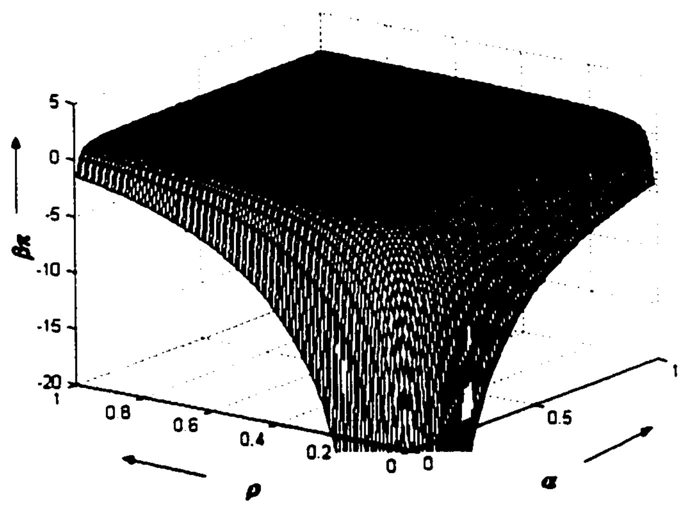


(b)

Figure 1.3: Comparison of Baseline and Debt-constrained Interest Rate Rules for Canada



(a) Baseline Model: $b = 0$, $\lambda = 0.03$, $\varphi = 0.04$



(b) Debt-constrained Model: $b = 48\%$, $\lambda = 0.03$, $\varphi = 0.04$

Figure 1.4: Simulation Results for β_π with Different Values of α and ρ.

Table 1.1: GMM Estimates of parameters of Baseline And Debt-constrained Models, Canada, Monthly Data, 1991:11 to 2000:12

Parameter Name	1 Estimates	2 Estimates
α	-0.25	-0.51
	(0.76)	(0.91)
β_π	**0.16**	**0.23**
	(0.05)	(0.06)
θ_1	--	-0.02
		(0.09)
γ	--	**0.10**
		(0.05)
θ_2	--	**0.29**
		(0.08)
σ	--	-0.10
		(0.14)
θ_3	--	**-0.11**
		(0.05)
μ	**0.78**	**0.70**
	(0.11)	(0.08)
ξ		**-0.16**
	--	(0.06)
ϕ	**0.94**	**0.93**
	(0.01)	(0.01)
p-value for J-statistic	0.97	0.95

Note: (i) Statistically significant (at 5% level) estimates are in **bold**. Standard errors of the estimates are in parenthesis.

Table 1.2: Estimates of IS and PC for Canada, Monthly Data, 1991:11- 2000:12.

Estimates of IS parameters of (2)		**Estimates of PC parameters of (3)**	
constant	0.15 (0.16)	*constant*	0.07 (0.05)
φ	0.04 (0.05)	λ	0.03 (0.01)
κ	1.06 (0.02)	β	0.97 (0.02)

Note: Estimates are by GMM. The coefficient of $E_t x_{t+1}$ in the IS equation is denoted by κ, although the actual value of the coefficient is 1.

Chapter 2: A Model of the Ex ante Real Rate and the Natural Rate of Interest[1]

Chapter Abstract

We model the unobserved ex ante real interest rate of the United States using a parsimonious state-space representation which assumes only that inflation forecast errors are rational and the unobserved ex ante rate follows an autoregressive process. Incorporating time-varying volatility in the inflation forecast errors and in the ex ante rate innovation variance, we have extracted the natural rate of interest as the more persistent component of the natural rate.

1 Introduction

The ex ante real interest rate is a crucial determinant of investment, savings and intertemporal decisions, and the ability to measure it is of intrinsic interest for monetary and fiscal policy. We model the unobserved ex ante real interest rate of the United States using a parsimonious state-space representation which assumes only that inflation forecast errors are rational and the unobserved ex ante rate follows an autoregressive process. Recognizing that results of unit root tests on the real rate are mixed (Rose (1988), Garcia and Perron (1996)), we allow for considerable flexibility by imposing neither stationarity nor constancy of the real rate (as in Fama (1975)) a priori. We have paid close attention to modeling time varying volatility of both inflation uncertainty and monetary policy shocks before making inferences about the ex ante rate.

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[1] With Charles R. Nelson and Richard Startz

In countries where the short-term interest rate is the primary policy instrument, the "natural" rate of interest provides the policy maker with a metric of overall policy stance. The natural rate of interest can be defined as the medium run, time varying, real rate consistent with actual output converging to potential output with stable inflation (see Bomfim (1997), Laubach and Williams (2001)). In forward-looking Taylor (1993) rules, the natural rate is generally assumed to be constant. After estimating the unobserved ex ante real rate, we have provided a measure of the natural rate as the more persistent component of the ex ante real rate.

Finding evidence of time-varying volatility in the inflation forecast errors we model heteroskedasticity by using a version of the stochastic volatility model (Harvey, Ruiz and Shephard (1994)). We assume that volatility varies with an unobserved stochastic variance component that itself could be affected by the inflation rate. As part of a three-step procedure, we first model the inflation forecast errors as being a product of a white-noise component scaled by an unobserved time-varying standard deviation, the squared-log of which follows an autoregressive process. The Kalman-filtered volatility component is then used as the observed volatility of the forecast errors while re-estimating the baseline model in the third step. Using this unobserved volatility model, and another to take care of heteroskedasticity in ex ante rate innovation-variance, we extract estimates of the natural rate of interest and the real rate gap.

The next section introduces the baseline model in state space form, discusses the data and estimates the model by maximum likelihood. The ex ante rate is the Kalman-filtered estimates of the unobserved component. Section 3 models deterministic and stochastic forms of time varying volatility of the inflation forecast error-variances while section 4 models time-volatility in innovation-variance of the ex ante rate. An estimate of the natural rate of interest is given in section 5, while section 6 concludes.

2 Model I: A Baseline Model

The ex post real rate, r_t, by definition, consists of the (unobserved) ex ante real rate, $\tilde{r}_t^{ea}$, plus an inflation forecast error. To avoid measurement errors, we do not make assumptions about either a model for inflation expectations (Laubach and Williams (2001)), or an information set that helps determine the ex ante rate (Huizinga and Mishkin (1986), Bonser-Neal (1990)). Instead, we only assume that inflation expectations are rational; this implies that the forecast errors are unforecastable given any information available at time t. Since they are serially uncorrelated, any autocorrelation in the ex post rate could be attributed to autocorrelations in the ex ante rate (Nelson and Schwert (1977)). The model can be written in state space form as follows:

$$r_t = \tilde{r}_t^{ea} + \varepsilon_t \qquad (1)$$
$$\tilde{r}_t^{ea} = \gamma + \rho \tilde{r}_{t-1}^{ea} + v_t \qquad (2)$$

The pure forecasting error, ε_t, and shocks, such as monetary and fiscal policy shocks, v_t, affecting the ex ante real rate, are uncorrelated. The first equation in model (1) is the measurement equation and the second one is the transition or state equation in the unobserved state variable, $\tilde{r}_t^{ea}$. The state equation has a first-order autoregressive structure.[2] The parameters, $\sigma_\varepsilon^2, \sigma_v^2, \gamma$ and ρ are estimated by maximum likelihood based on prediction error decomposition; $\tilde{r}_t^{ea}$ is then estimated by using the Kalman filter, based on information up to time t.

We have used data from January 1955 to January 2002 of the beginning of the month Federal Funds Rate as the nominal interest rate, from the DRI database. The ex post real rate is calculated by taking the actual CPI inflation rate (annualized) for the month. The nominal and the ex post real rate are shown in Figure 2.1.

The estimation results for this model are given in Table 2.1. The ex ante rate shows considerable variability, since the ex ante error variance, σ_v^2, is significantly different from zero. Furthermore, a high value of ρ suggests a high persistence in the real rate. The long run mean, $\gamma/(1-\rho)$, of the ex ante real rate is approximately 2.3%. The Kalman-filtered estimates of the ex ante real rate, shown in Figure 2.2, closely tracks the ex post rate minus the transients.

The squared difference between the ex post and the filtered ex ante rate gives an estimate of squared (inflation) forecast errors, $\hat{\varepsilon}_t^2$, that is shown in Figure 2.3. Finding evidence of time varying volatility, we next proceed to incorporate the latter in our baseline model before making any inference about the ex ante real rate.

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[2] An AR (2) specification was rejected in favor of AR (1) by the Schwarz criterion.

3 Model II: A Model Incorporating Time Varying Volatility as an Unobserved State Variable, in Variance of Inflation Forecast Errors

Modeling the specific form of time variation in volatility of inflation forecast error variance could improve our inferences about the ex ante real rate from the baseline model. However, incorporating heteroskedasticity should not radically change the parameter estimates of the baseline model – the estimate of the ex ante rate should be qualitatively similar. In order to test for homoskedasticity, we test for autoregressive conditional heteroskedasticity (ARCH) effects in the baseline model and find presence of GARCH (1,1) in the forecast errors. One approach is to treat inflation forecast volatility as an unobserved stochastic variable following a lognormal distribution following Harvey, Ruiz and Shephard (1994). Assuming that the log of this unobserved volatility is an autoregressive process, the baseline model becomes:

$$r_t = \tilde{r}_t^{ea} + \varepsilon_t \quad (1)$$

$$\tilde{r}_t^{ea} = \gamma + \rho \tilde{r}_{t-1}^{ea} + v_t \quad (2)$$

$$V(\varepsilon_t) = \exp(\alpha + \beta \tilde{h}_t) \quad (3)$$

$$V(v_t) = \sigma_v^2$$

$$\varepsilon_t = \tilde{\sigma}_t w_t, \qquad w_t \sim iidN(0,1) \quad (3.1)$$

$$h_t \equiv \ln(\tilde{\sigma}_t^2) \sim AR(1) \quad (3.2)$$

$$h_t = \delta + \phi h_{t-1} + \eta_t, \qquad \eta_t \sim NID(0, \sigma_\eta^2) \quad (3.3)$$

In model II, we postulate the inflation forecast errors to be made up of a Gaussian white noise process with unit variance, w_t, multiplied by a factor, $\tilde{\sigma}_t$, the unobserved time-varying standard deviation of the errors. Since our model has an unobserved state variable, $\tilde{r}_t^{ea}$, and an unobserved variable, $\tilde{\sigma}_t$, in the inflation forecast error, we cannot estimate both of them together by Kalman filter. The model is simplified, however, if we square both sides of equation (3.1) and take logarithms. Now the model can be estimated in three stages.

In the first stage, we estimate the baseline model (I) and take the logarithm of squared forecast errors[3], denoted y_t. In the second stage, we employ the methodology of Harvey, Ruiz and Shephard to estimate the following stochastic volatility model:

$$y_t = -1.25 + \tilde{h}_t + \ln(w_t^2) \qquad (3.1')$$
$$\tilde{h}_t = \delta + \phi\tilde{h}_{t-1} + \eta_t \qquad (3.3)$$

Equation (3.1') above is the log-squared transformation of equation (3.1), adding and subtracting 1.25 to make the errors mean-zero. Although the error in the measurement equation is not conditionally Gaussian, we treat it as though it were NID (0, 4.93), given $\ln(w_t^2) \sim \ln(\chi_1^2)$. Equations 3.1' and 3.3 are then estimated in state-space using a Quasi Maximum Likelihood (QML) method where the asymptotic standard errors should take account of the specific form of non-normality of errors. We adopt White (1982)'s variance-covariance matrix to report approximate standard errors for the QML estimates. Following evidence provided by Ball (1992), Evans (1991) and Kim (1993), that the level of inflation itself influences the volatility of inflation forecast errors, we find that log-squared annual inflation, $\ln(\pi_t^2)$, and $\tilde{h}_t$ have a sample correlation coefficient of 0.34.

[3] The forecast errors here refer to the difference between the observed ex post real rate and the filtered estimates of the unobserved ex ante rate from model I.

Therefore it is likely that forecast volatility varies with $\ln(\pi_t^2)$. We incorporate this in the second stage estimation by allowing $\tilde{h}_t$ to vary with $\ln(\pi_t^2)$ as well. Thus, instead of equation (3.3), we estimate a variant of it given by:

$$\tilde{h}_t = \delta + \phi\tilde{h}_{t-1} + \lambda \ln(\pi_t^2) + \eta_t \qquad (3.3')$$

In the third and final stage, we use the filtered estimates of $\tilde{h}_t$ in the variance specification of ε_t, and re-estimate our baseline model assuming $V(\varepsilon_t)$ is known.

The results from the first stage estimates are the same as the one for the baseline model in Table 2.1. The filtered estimates of $\tilde{h}_t$ from second stage model II are shown in Figure 2.4 and the QML estimates of the parameters are shown in Table 2.2. The standard errors have been adjusted for the non-normality of the measurement equation error by using White's QML standard errors. The unobserved volatility shows considerable fluctuation and is high during the high-inflation seventies' decade. Thus, $\tilde{h}_t$ is a stationary stochastic process with $\ln(\pi_t^2)$ as a significant explanatory variable, besides its own lag.

The third stage estimation results are given in Table 2.3 and the filtered ex ante rate from model II is compared to that from Model I (Baseline) in Figure 2.5. The ex ante rate from model II has almost the same mean and persistence, but a lower variance than the baseline one. The unobserved volatility measure, $\tilde{h}_t$, has a positive effect on the variance of inflation forecast errors: a unit increase in $\tilde{h}_t$ raises variance by almost 7.25 $(= \beta\exp(\alpha + \beta\tilde{h}_t))$ at $\tilde{h}_t = 1$ or $\tilde{\sigma}_t^2 = 2.72$. Given the significant increase in the value of the log-likelihood from Table 2.1 to Table 2.3, model II is a better specification than model I.

4 Model III: A Model with Unobserved Volatility in the Inflation Forecast Errors and Observed Volatility in Innovations of the Ex Ante Rate

The error in the state (unobserved ex ante real rate) equation is a measure of policy and technology shocks affecting the ex ante rate. A vast literature on monetary policy rules (for example, Taylor (1993), Clarida, Gali and Gertler (1998)) provides evidence that the Federal Reserve has been implicitly targeting the inflation rate using the Federal Funds Rate as the policy instrument since 1979. It reacts to an increase in inflation by increasing interest rate more than proportionally to raise the real rate. Therefore, it is likely that the variance of the ex ante rate would be influenced by the level of inflation. We incorporate this type of volatility by making the variance of the error in the ex ante rate vary with annual inflation:

$$r_t = \tilde{r}_t^{ea} + \varepsilon_t \quad (1)$$
$$\tilde{r}_t^{ea} = \gamma + \rho \tilde{r}_{t-1}^{ea} + v_t \quad (2)$$
$$V(\varepsilon_t) = \exp(\alpha + \beta \tilde{h}_t) \quad (3)$$
$$V(v_t) = \exp(\alpha' + \beta' \pi_t) \quad (4)$$

Like model II, this model is also estimated in three stages. However, unlike the previous model, (1), (2) and (4) are estimated together in the first stage. In the second stage, $\tilde{h}_t$ with specification (3.3') is estimated and its filtered estimates are used in the third stage.

The third stage results are shown in Table 2.4 and the filtered estimates of ex ante rates from the baseline and model III are compared in Figure 2.6. This specification yields a similar mean but a significantly lower persistence (0.90) of the ex ante rate; it also has a higher variance than the baseline one. An increase in the inflation rate by 1% increases the variance of the ex ante rate innovations by 0.117 when inflation is 2%, and by 0.318 when inflation is 6%. However, even though the estimates of α and β are higher for this model, the effect of a unit increase in $\tilde{h}_t$ on inflation forecast error variance is almost the same (7.04) as in model II, when $\tilde{h}_t = 1$.

Given the significant increase in the value of the log-likelihood at the inclusion of β', over model II, we are going to adopt model III to extract the more persistent component of the ex ante rate and call this component the natural rate of interest.

5 Model IV: An Estimate of the Natural Rate of Interest

Woodford (2001) revived the concept of the "Wicksellian" natural rate of interest as the real rate of interest required to keep aggregate demand equal to natural rate of output or the output consistent with flexible prices. It is the component of the real rate that is free of monetary policy shocks, but is affected by real shocks to government spending, technology, labor supply or taste parameters. Woodford has also shown that the paths of inflation, output gap and the nominal interest rate are solely determined by current and future expected 'gaps' between the ex ante real rate and the natural rate, in a sticky price New Keynesian model. Thus, if the natural rate were known to the monetary policy maker, then he or she would adjust the short term nominal rate to close the real rate gap at all times, without having to measure potential output. Therefore, the gap serves as an indicator variable for monetary policy.

Laubach and Williams (2001) define the natural rate of interest as the medium run equilibrium real interest consistent with real output reaching potential and inflation stabilizing (in the medium run). It should be a component that is free of transients induced by policy shocks. Laubach and Williams use this definition to estimate the natural rate as a low-frequency component of the real rate, using a multivariate state space model of inflation and output gap. However, their results are sensitive to assumptions regarding potential output growth and its variability, and the construction of the ex ante real rate. We take advantage of our parsimonious model to estimate the natural rate that is not sensitive to assumptions about other variables.

We adopt Laubach and Williams' definition of the natural rate by extracting the more persistent component of the unobserved ex ante real rate series from a variant of the observed and unobserved volatility model presented in the previous section. This model can be written as follows:

$$r_t = \tilde{r}_t^{ea} + \varepsilon_t \qquad (1)$$
$$\tilde{r}_t^{ea} = \tilde{r}_t^{s} + \tilde{r}_t^{p} \qquad (2.1)$$
$$\tilde{r}_t^{s} = 0.4\tilde{r}_{t-1}^{s} + v_{1t} \qquad (2.2)$$
$$\tilde{r}_t^{p} = \gamma_p + \rho_p \tilde{r}_{t-1}^{p} + v_{2t} \qquad (2.3)$$
$$V(\varepsilon_t) = exp(\alpha + \beta \tilde{h}_t) \qquad (3)$$
$$V(v_{1t}) = exp(\alpha' + \beta' \pi_t) \qquad (4)$$
$$V(v_{2t}) = \sigma_{v2}^2$$

The ex ante rate is divided into two components – less persistent, $\tilde{r}_t^{s}$, and more persistent, $\tilde{r}_t^{p}$. In order to identify the two stationary processes, we treat the persistence of $\tilde{r}_t^{s}$ as known and less than 0.5. In particular, we choose 0.4 by doing a grid search over [0.1, 0.2…., 0.5] and choosing the highest log likelihood value and the lowest Schwarz Information Criterion. Since the variance of v_{1t} is made to vary with the annual inflation rate, we can call v_{1t} monetary policy shocks. Thus the less persistent component of the real rate varies with the inflation rate only. All other shocks, including real shocks, affect the more persistent component that we call the natural rate. The difference between the ex ante real rate and the natural rate, referred to as the real rate "gap", is the less persistent component.

Model IV is estimated by the three-stage procedure outlined in the previous section. The estimation results are shown in Table 2.5 and Figure 2.7. The filtered estimates effectively separate out the transients from the more persistent component of the ex ante rate. The natural rate is very persistent (almost the same as the ex ante rate in our baseline model (I)). The real rate gap shows up as a high frequency component around its long run mean of zero for most of the sample period, except during periods of very high inflation in the seventies and early eighties.

The real rate gap could be used as an indicator of monetary policy stance. A positive gap, where the actual ex ante rate is higher than the natural rate, indicates a more restrictive or contractionary policy and a negative gap indicates an expansionary policy. In order to see how effectively the gap functions as an indicator of policy stance, we plot it against the inflation rate in Figure 2.8. The gap shows high fluctuations during the mid-seventies and early eighties, but is very low during the rest of the sample period. Policy seems to be both contractionary and expansionary during the seventies that might have contributed to ineffectiveness in decreasing inflation rates during that period. Furthermore, inflation was allowed to increase later. However, the Volcker-disinflation period (from later half of 1979) coincides with very high real rate gaps indicating a highly contractionary policy. This was also the period during which the Federal Reserve implicitly targeted the inflation rate as a policy objective. A higher current gap is also an indicator for lower inflation later. The inflation rate from the second half of the eighties behaved like the one from before the seventies, and there was no further need for a large real rate gap.

6 Conclusions

In this paper, we have estimated the ex ante real interest rate by using a very simple state-space model that only uses the assumption of rationality of inflation forecast errors. We have made inferences about this ex ante rate by modeling the time-varying volatility of the forecast error variance and the policy shocks. In particular, we have used an unobserved volatility model that allows the inflation forecast variance to vary with the log-squared inflation rate at very high rates of inflation, and with an unobserved volatility component at other times. The baseline model estimates are robust to assumptions about volatility.

Using the unobserved volatility model, we have extracted the natural rate of interest and the real rate gap as the more persistent and the less persistent components, respectively, of the unobserved ex ante real rate. The real rate gap is shown to act as a gauge of the stimulative or contractionary impetus of monetary policy.

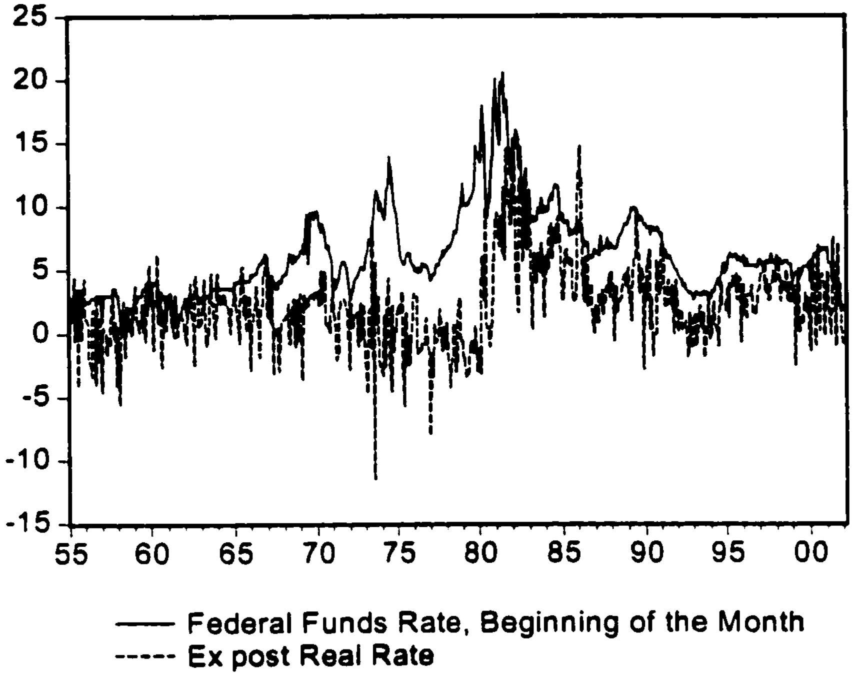


Figure 2.1: Nominal and Ex post Real Interest Rate, January 1955 to January 2002

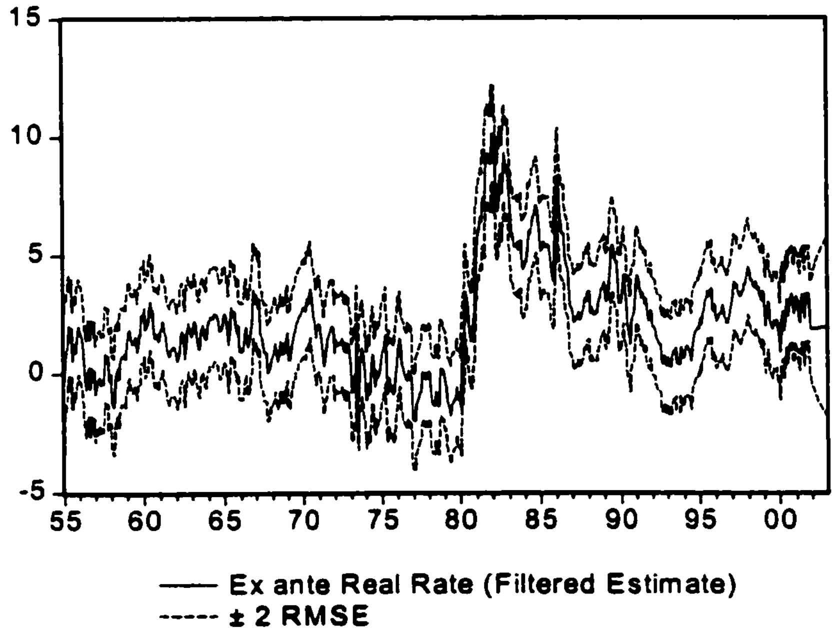


Figure 2.2: Filtered Estimate of Ex ante Real Rate from (Baseline) Model I

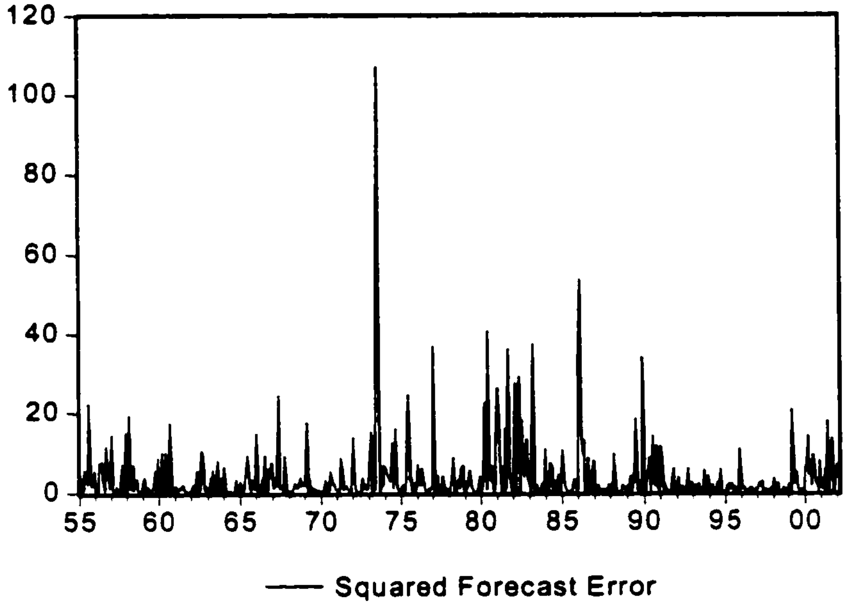


Figure 2.3: Squared Estimates of Inflation Forecast Error from (Baseline) Model I

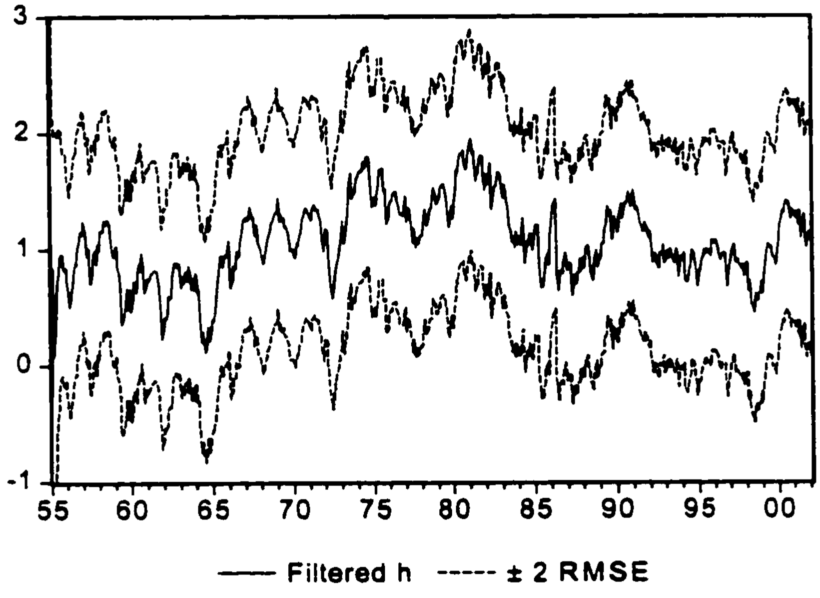


Figure 2.4: Filtered Estimates of $\tilde{h}_t$ from Second Stage (Unobserved Volatility) Model II

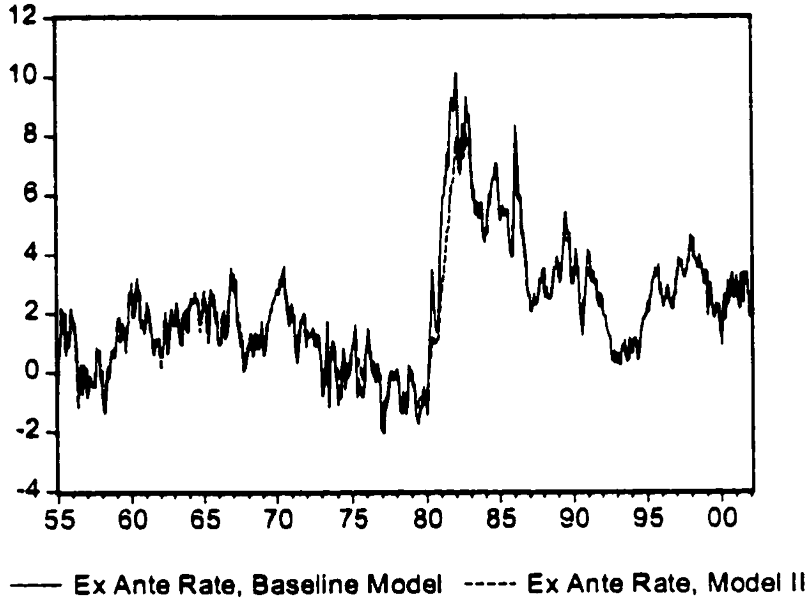


Figure 2.5: Filtered estimates of the Ex ante Real Rate from Baseline and Unobserved Volatility Model II compared

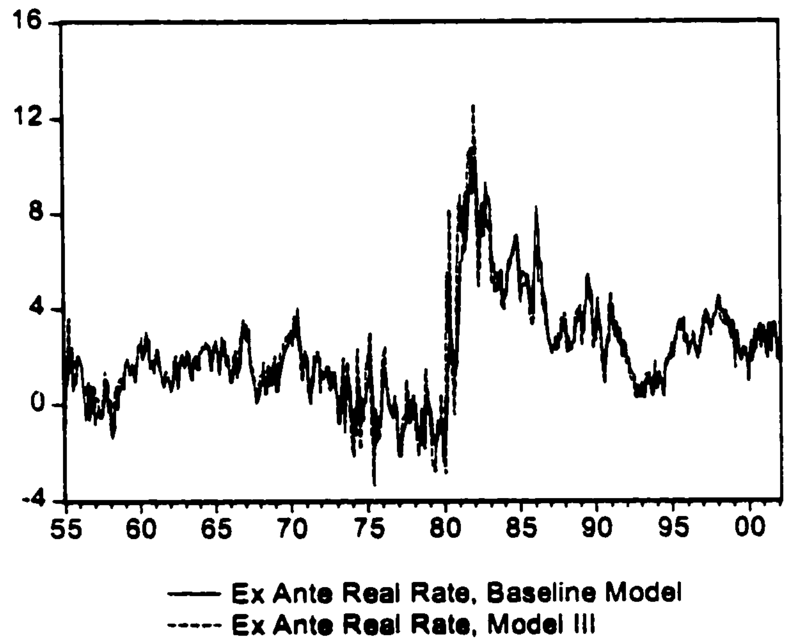
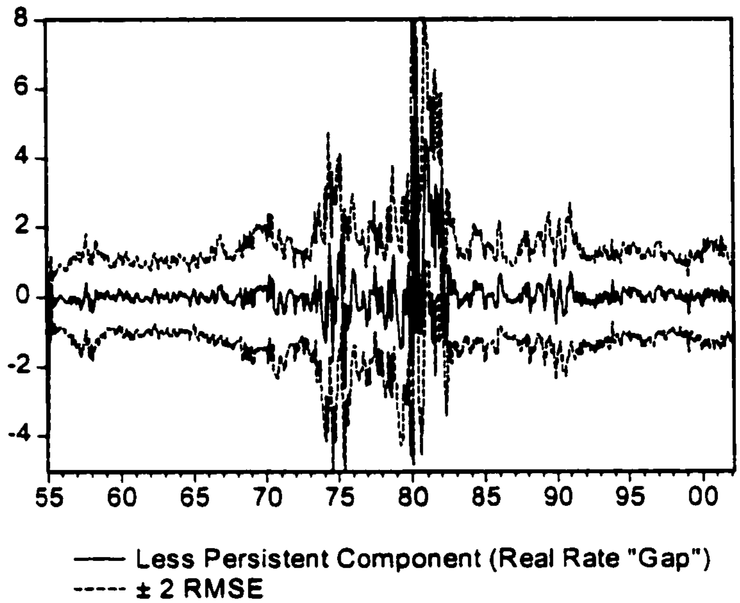


Figure 2.6: Filtered estimates of the Ex ante Real Rate from Baseline and Unobserved and Observed Volatility Model III



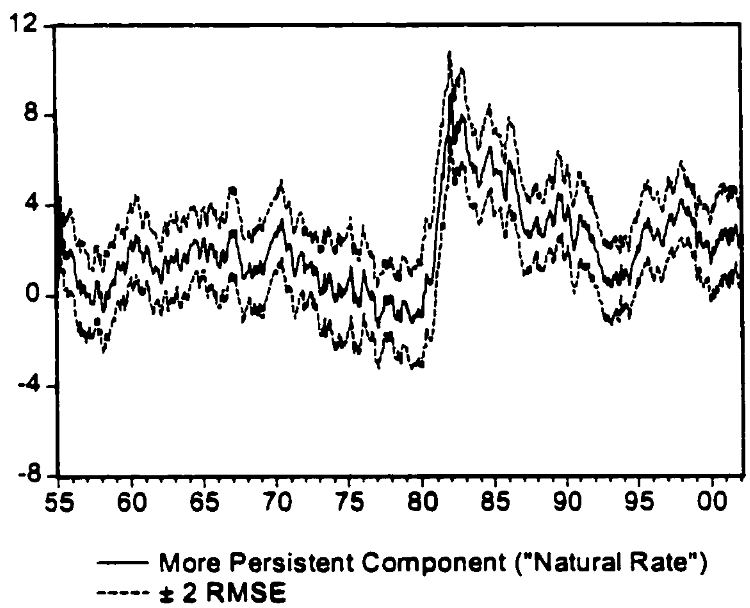


Figure 2.7: Filtered Estimates of the Ex ante Real Rate Gap and the Natural Rate From Model IV

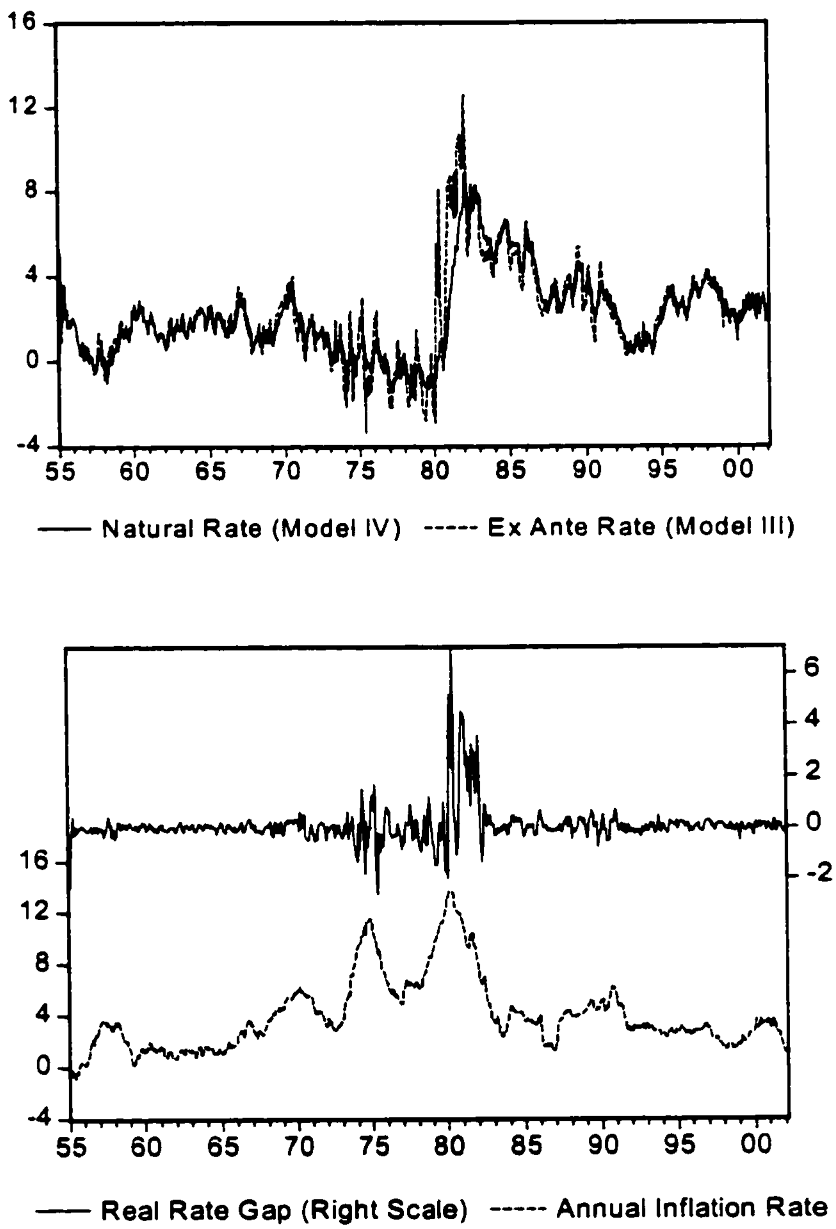


Figure 2.8: Real Rate Gap as an Indicator of Monetary Policy Stance

Table 2.1: Maximum Likelihood estimates of parameters from Baseline Model I

Parameter	Estimates	S.E.
σ_{ε}^2	4.76	0.24
σ_{v}^2	0.36	0.09
γ	0.07	0.035
ρ	0.97	0.01
Log-likelihood	-1312.1	
Schwarz Criterion	4.69	

Table 2.2: Second Stage QML Estimates from Unobserved Volatility Model II

Parameter	Estimates	S.E.*
σ_{η}^2	0.074	0.002
δ	0.096	0.0002
ϕ	0.848	0.0005
λ	0.029	0.0002
Log-Likelihood	-1235.2	
Schwarz Criterion	4.4	

** Approximate QML S.E. from White (1982)*

Table 2.3: Third Stage ML Estimates from Unobserved Volatility Model II

Parameter	Estimates	S.E.
α	-0.92	0.21
β	2.14	0.18
σ_v^2	0.24	0.07
γ	0.06	0.04
ρ	0.97	0.01
Log-Likelihood	-1258.24	
Schwarz Criterion	4.51	

Table 2.4: Third Stage ML Estimates from Unobserved and Observed Volatility Model III

Parameter	**Estimates**	**S.E.**
α	-7.62	0.33
β	7.55	0.21
α'	-1.26	0.15
β'	0.25	0.02
γ	0.20	0.04
ρ	0.90	0.02
Log-Likelihood	-1237.90	
Schwarz Criterion	4.45	

Table 2.5: Maximum Likelihood Estimates from Model IV

Parameter	**Estimates**	**S.E.**
α	-7.88	0.48
β	7.76	0.32
α'	-1.84	0.51
β'	0.30	0.05
γ_p	0.05	0.03
ρ_p	0.98	0.01
σ^2_{v2}	0.2	0.01
Log-Likelihood	-1230.1	
Schwarz Criterion	4.43	

Chapter 3: Unknown Number of Mean Shifts and Persistence Around Mean: An Application to Short Term Real Interest Rates[4]

Chapter Abstract

We have provided a model for certain time-series processes that are characterized by two distinct phases: one in which the mean is stable or constant, and the other in which the mean is gradually shifting to another level. The number of these slow mean-shifts is unknown. Using an unobserved components model, the process can be divided into a 2-state Markov-switching process of the mean and a stationary cyclical component. The model is able to predict a slow shift in mean to another level without having to test for structural breaks in the data. In addition, there is no necessity of testing for unit roots in the series prior to estimation. This is particularly appealing for modeling the real interest rate of the United States, which give different results regarding presence of a unit root across various unit-root tests and sample periods.

1 Introduction

In an influential paper, Garcia and Perron (1996) modeled the ex post real rate of the United States as a three-state Markov-switching process in its mean and variance, employing a second-order autoregressive process of the real rate. Their study suggests that, instead of having a unit root, shocks to the real rate are temporary, with a tendency to revert to the mean which itself is subject to infrequent regime shifts. The estimated ex ante rate derived as the conditional mean of the ex post rate, was constant within a regime but almost jumped to a new level when a new regime approached. The study ignored

[4] With Arabinda Basistha

various episodes when the mean of the real rate slowly shifted to a new level and became constant for some time before slowly shifting to another level. The number of shifts in the mean could be unknown.

The goal of this paper is to provide a model for a time series process that is subjected to unknown number of slow shifts in mean with a stationary cyclical component around that mean. Our benchmark model, a generalized version of Basistha and Startz (2002), has two regimes: one in which the mean of the series is steady, and the other in which the mean is gradually shifting to another level. Both the regimes have the same cyclical component around mean. The two regimes are captured by an unobserved 2-state, first-order Markov-switching state variable.

The major contribution of the paper is that one does not need to test for stationarity of the data to estimate the model. This is particularly useful for the US real interest rate series that yield different results on stationarity tests both across different types of tests and sample periods. Furthermore, opinion is divided on the number of structural changes in it. For instance, Wang and Zivot(2000) found "very strong" evidence in favor of three versus two breaks and "weak" evidence in favor of four versus three breaks, whereas Garcia and Perron(1996) modeled it assuming two trend breaks. Instead of testing for the number of breaks, the model proposed in this paper would endogenously predict them if there is any.

In what follows, we will give a description of the benchmark model, and provide estimates of persistence of a generated series drawn from different data generating processes in section 2. A version of the benchmark model is estimated for the ex post real interest rate series of the United States in section 3. Various robustness checks of our model are done in section 4 while section 5 summarizes and concludes.

2 The Benchmark Model and Measures of Persistence for Different Data Generating Processes

To take a very simple example of a data with a slow mean shift, we can think of a time series process, y_t, that is made up of two unobserved components -- a mean, x_t, and a cyclical component, z_t:

$$y_t = x_t + z_t \tag{1}$$

The cyclical component is a first-order autoregressive process, and is present during the whole sample period:

$$z_t = \rho z_{t-1} + w_t, \quad w_t \sim i.i.d.N(0,\sigma_w^2) \tag{2}$$

There are two distinct phases that the mean, x_t, passes through. In the first, the mean is constant so that

$$x_t = x_{t-1} \tag{3.1}$$

In the second, the mean slowly shifts to another (constant) level but the cyclical component remains the same in both the phases. The slowly shifting mean can be thought of as a unit-root process with first-order serial correlation in the error:

$$\begin{aligned} x_t &= x_{t-1} + u_t \\ u_t &= \theta u_{t-1} + v_t, \quad v_t \sim i.i.d.N(0,\sigma_v^2) \end{aligned} \tag{3.2}$$

The number of mean shifts is unknown as is the number or levels of constant-means. Since the data, y_t, has a tendency to persist for some time while in each regime before switching to the other, we can nest (3.1) and (3.2) with the help of a 2-state first-order Markov-switching state variable, S_t, that takes the value of 0 or 1, according to whether y_t is in the first or the second regime:

$$\begin{aligned}
&x_t = x_{t-1} + S_t u_t \\
&u_t = \theta u_{t-1} + v_t, \quad v_t \sim i.i.d.\ N(0, \sigma_v^2) \qquad\qquad (3) \\
&S_t = \{0,1\} \\
&\Pr[S_t = 0 \mid S_{t-1} = 0] = p_{00}, \quad \Pr[S_t = 1 \mid S_{t-1} = 1] = p_{11}
\end{aligned}$$

The transition probabilities, p_{00} and p_{11}, serve as indicators of persistence within each regime.

The model, given by equations (1), (2) and (3) are jointly estimated using Maximum Likelihood (ML) based on prediction error decomposition. Using the ML estimates of $p_{00}, p_{11}, \sigma_v, \sigma_w, \theta, \rho$, we then use the Kalman filter and the Hamilton (1989) filter to derive filtered estimates of the unobserved steady state probabilities of each state based on information up to time t (see Kim and Nelson (1999)).

In what follows, we will estimate the model on simulated data making four different assumptions about the data generating process (DGP). In subsection 2.1, we assume that the DGP is exactly the model we have described above. In subsection 2.2, we assume three other DGP given by a first-order autoregressive process with constant mean, a random walk and a first-order autoregressive process with 2-state Markov switch in the mean.

2.1 Model Simulation with Unknown Number of Mean-Shifts as the DGP

We generate two sets of data, each with 500 observations, following exactly the same DGP as the model given by (1), (2) and (3). The two series differ only in the average duration of the second regime. In particular, the average duration of regime 1 and 2 are assumed to be 50 each for DGP I, and to be 50 and 10, respectively, for DGP II. While generating data, the parameters are assumed to have the following values: $p_{00} = p_{11} = 0.98, \sigma_v = \sigma_w = 1$, $\theta = \rho = 0.5$ for the first DGP (DGP I) and $p_{00} = 0.98$, $p_{11} = 0.90$, $\sigma_v = \sigma_w = 1, \theta = \rho = 0.5$ for the second DGP (DGP II). The values for the transition probabilities, p_{00} and p_{11}, are consistent with steady-state probabilities of 0.50 and 0.83 for $S_t = 0$, for the two data sets, respectively. The two datasets DGP I and DGP II are shown in Figure 3.2.

Maximum Likelihood estimates of the model parameters, shown in Table 3.1, are not, except for θ, statistically different from the true values of the structural parameters. The Hamilton-filtered probabilities of the constant-mean state, shown in Figure 3.3, correctly match the phases in the two datasets where the mean seems to be a constant. It is robust to varying durations of the changing-mean state, since the filtered probability of the constant-mean state takes a value of (almost) 0 even when the changing-mean state does not last long (DGP II).

Next, in order to show that our estimation method would still yield correct measures of persistence if the true DGP is anything other than the model given by (1), (2) and (3), we consider three extreme cases where the actual DGP is alternatively given by processes other than that assumed to estimate the model.

2.2 Model Simulations with Three Different Data Generating Processes

We consider three different cases where the DGP is given by a first-order autoregressive process (AR(1)) with constant mean, a random walk and an AR(1) with 2-state Markov-switching in mean. The first is a special case of the benchmark model where the second state (that of mean-shifting) does not exist. The second is a special case where constant-mean state does not exist. The third is a variant of Garcia and Perron's specification of the ex post real rate as an AR(2) with 3-state Markov-switching in the mean and variance. In the following subsections, we are going to demonstrate that estimation of the structural model given by (1), (2) and (3) on these three types of generated data will still yield correct measures of persistence. The three sets of data are shown in Figure 3.4.

2.2.1 Data Generated by an AR(1) with Constant Mean

We postulate the true DGP to be given by

$$y_t = 1.5 + 0.5y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim i.i.d.N(0,1) \qquad (4)$$

However, we estimate (4) by assuming that the system of equations given by (1), (2) and (3) represents the true model. In effect, (4) is a special case where the second regime, that of changing-mean state, does not exist. Generating data with (4) and using it to estimate (1), (2) and (3), yields results with almost correct persistence of the process. The second column of Table 3.2 gives the estimation results. The persistence is given by estimate of ρ as 0.49 that is not significantly different from 0.50. The steady state probability of constant-mean regime is 0.99. This is consistent with the fact that regime 2 is not identifiable in (4). Once in regime 1, the probability of remaining in regime 1 (p_{00}) is near 1. The three non-precisely estimated parameters, p_{11}, σ_v, θ, have no interpretation in the context of (4) since regime 2 does not exist. The filtered probability of constant-mean state has a value of almost 1[5] throughout the sample. Thus the benchmark model estimation is robust to a DGP different from the one given by (1), (2) and (3).

2.2.2 Data Generated by a Random Walk Process

Next, we assume the true DGP to be following a random walk:

$$y_t = y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim i.i.d.N(0,1) \tag{5}$$

However, we estimate (1) – (3) on data generated by (5) that is a special case with a non-existent constant-mean regime. The estimation results are given in the third column of Table 3.2. The steady state probability of the first regime is indeed very small (0.065), and the two imprecisely estimated parameters, θ and ρ, have no interpretation in this context since the cyclical component, (2), and the serial correlation of mean innovations in (3.2) do not exist. Given the estimate of σ_v is not significantly different from 1, the estimates correctly identify the data as a random walk with standard normal errors. The filtered probability of constant-mean state, shown in Figure 3.5, is mostly near zero.

[5] The probability estimates differ only in the sixth place of decimal.

2.2.3 Data Generated by AR(1) with 2-state Markov-Switch in Mean

Next, we take data generated by an AR(1) with 2-state Markov-switching mean:

$$
\begin{aligned}
& y_t = \mu_{S_t} + 0.5 y_{t-1} + \varepsilon_t,\ \varepsilon_t \sim i.i.d.\ N(0,\ 1) \\
& \mu_{S_t} = 1.5(1 - S_t) + 3.0(S_t) \qquad (6) \\
& S_t = \{0,\ 1\} \\
& \Pr[S_t = 0 \mid S_{t-1} = 0] = p_{00},\ \Pr[S_t = 1 \mid S_{t-1} = 1] = p_{11}
\end{aligned}
$$

The state-dependent constant, μ_{S_t}, is 1.5 in a low mean state $(S_t = 0)$ and 3 in a high mean state $(S_t = 1)$. In the context of our model, a changing-mean state does not exist, since we allow for the mean to 'jump' to its new level. However, during estimation, the brief period when the level of the mean changes might be mistakenly recognized as a mean-shift phase. Estimating model (1) – (3) on data generated by (6), we indeed get a fairly high probability of constant-mean state, as is shown in the fourth column of Table 3.2. The filtered probability of constant-mean state mostly identifies the phases when the data was stationary within a high or a low mean state, but also identifies the process of mean-switches as a slow change in mean. However, the probability of persisting within the changing-mean regime is 0.87 with an average duration of around 7 (not shown); this is much lower than the duration of 87.77 of a constant mean state. Again the persistence of the cyclical component is not significantly different from its true value of 0.5.

The various experiments with different data generating processes show that our model is robust to various types of stationary and non-stationary data. One does not have to assume stationarity to estimate the model. Also, there is no need for structural break tests, since the model would predict a break if there is any. In what follows, we will apply the model to ex post real interest rate of the United States.

3 Unknown Number of Mean Shifts in the US Real Interest Rate: An Example

The ex post real interest rate of the United States yields different results when tested for the presence of a unit-root both across types of tests and sample periods (see, Rose(1988), Perron(1990)). Results are also varied regarding the number of structural breaks and regime shifts (Wang and Zivot (2000), Garcia and Perron (1996)). The generated data in Figure 3.2 resembles actual data on the ex post real rate of the United States shown in Figure 3.1. We therefore estimate our benchmark model with monthly real rate data to show that there could be numerous mean shifts and constant means in it. The model is capable of predicting these shifts endogenously.

We postulate the ex post real rate, r_t, to be made up of two independent unobserved components – a second order autoregressive[6] cyclical component, z_t, moving around a two-state first-order Markov-switching mean, x_t. The two-state Markov-switching model of the real rate with unknown number of mean shifts can be written as follows:

---

[6] The order of autoregressive process was chosen to compare our result with Garcia and Perron(1996)'s who model the ex post rate as an AR(2) with 3-state Markov-switching in the mean and variance.

$$\begin{aligned}
&r_t = x_t + z_t \\
&x_t = x_{t-1} + S_t u_t \\
&u_t = \theta u_{t-1} + v_t, \quad v_t \sim N(0, \sigma_v^2) \\
&z_t = \rho_1 z_{t-1} + \rho_2 z_{t-2} + w_t, \quad w_t \sim N(0, \sigma_w^2) \qquad (7) \\
&S_t = \{0,1\} \\
&\Pr[S_t = 0 \mid S_{t-1} = 0] = p_{00}, \ \Pr[S_t = 1 \mid S_{t-1} = 1] = p_{11}
\end{aligned}$$

The unobserved, discrete-valued, two-state Markov-switching variable, S_t, takes the value of 0 or 1 according to whether the mean, x_t, is constant ($S_t = 0$) or is in the process of shifting to a new steady state ($S_t = 1$). In the first regime of constant mean state, the only source of variation of the real rate is that of the stationary cyclical component, z_t; in the second regime where the mean itself follows a unit root process, the sources of variance in the real rate are that of the unit root component and that of the stationary cyclical component. By specification, therefore, the real rate has a higher variance in the second regime. The transition probabilities of persisting in a particular state ($\Pr[S_t = i \mid S_{t-1} = i]$, $i = 0,1$) are given by p_{00} and p_{11} respectively.

We use monthly data on 3-month Treasury Bills rate of the United States from January 1955 to May 2002 as the nominal interest rate. From the latter we subtract three-month ahead CPI inflation rate to get ex post real rate. All data is from the FRED® database of the Federal Reserve. The unknown parameters $(p_{00}, p_{11}, \sigma_v, \sigma_w, \theta, \rho_1, \rho_2)$ of model (7) are estimated by first considering the joint density of r_t, S_t and S_{t-1} conditional on information up to t-1. Then we form the log-likelihood function by taking the weighted average of conditional densities of r_t, weighted by the joint probability of

S_t and S_{t-1}. We estimate it by prediction error decomposition and Maximum Likelihood (ML) using Kalman filter to derive estimates of the unobserved state variables and the Hamilton (1989) filter to derive estimates of the unconditional probability of a particular state based on information up to time t.

Estimation results are provided in Table 3.3 and Figure 3.6. The high values of the estimated transition probabilities, p_{00} and p_{11}, imply considerable persistence within a regime. The serial correlation coefficient, θ, of the shocks to the mean in regime 2 is negative but small in absolute value. Thus, in the changing-mean state, the shocks to the mean are not very persistent in nature; this is consistent with our specification that the mean would return to another constant in a short while. In fact, the average duration of the second regime is approximately 10 months; nearly thirteen months shorter than that of the first regime. The filtered probability of regime 1, shown in Figure 3.6, separates out the constant-mean and the changing-mean states in an effective way. In particular, the 1990s decade is the only one with a high probability of being in the constant mean state. In the other decades, we see shorter periods of constant-mean interspersed with frequent mean-shifts. This is in contrast to the findings of Garcia and Perron (1996), who show that the mean was constant but different between 1961-1973, 1973-1981 and 1981-1994. Our model predicts the fairly long episode of changing-mean from 1979 to 1981.

Next, we show that this model for real rates is robust to sub-sample analysis and state-variant cyclical components.

4 Robustness Checks

In order to check the robustness of the model for real interest rate, we conduct three different robustness checks. First, we divide up the sample into the pre-1982 and the post-1982 periods to account for a change in monetary policy regime from inflation accommodating to inflation targeting. We estimate our model for each of the sub-samples. Second, we allow for two different cyclical components in the two states. We consider each of them in the next two subsections.

4.1 Sub-sample Analysis

We choose October 1982 as a break in our sample to account for a change in monetary policy targeting goal in the US. Although Paul Volcker had been targeting inflation since October 1979, we leave out from our second sub-sample the intermediate period between 1979 and 1982 when there was non-borrowed-reserves-targeting. Interest rate was used as a policy instrument to implicitly target inflation after October 1982. Estimating our model for the two sub-samples, we find, in Table 3.4 and Figure 3.7, that the filtered probabilities correctly predict the two regimes in both the sample periods. In particular, the average duration of constant-mean state is approximately 22 and 57 for the two periods. The long upward swing towards the end of the first sample period has contributed to a lower duration of the first regime. The second sub-sample results might be a little imprecise due to a smaller sample size. The persistence in the cyclical component is 0.51 and 0.56 respectively. Both are smaller than that of the whole sample

(0.6) given in Table 3.3. The imprecise estimate of θ in the first sub-sample could suggest that the changing mean follows a random walk process during this sub-sample. Thus the qualitative results are preserved even when the sample is divided into smaller sub-samples.

4.2 State-Variant Cyclical Component

Next, we assume that the cyclical component varies between regimes. Denoting the two different components as z_{1t} and z_{2t}, we consider the following variation of the model in (7):

$$\begin{aligned}
&r_t = x_t + (1-S_t)z_{1t} + S_t z_{2t} \\
&x_t = x_{t-1} + S_t u_t \\
&u_t = \theta u_{t-1} + v_t,\ v_t \sim N(0, \sigma_v^2) \\
&z_{1t} = \rho_1 z_{1t-1} + \rho_2 z_{1t-2} + w_{1t},\ w_{1t} \sim N(0,\ \sigma_{w1}^2) \quad (8) \\
&z_{2t} = \phi z_{2t-1} + w_{2t},\ w_{2t} \sim N(0,\ \sigma_{w2}^2) \\
&S_t = \{0,\ 1\} \\
&\Pr[S_t = 0 \mid S_{t-1} = 0] = p_{00},\ \Pr[S_t = 1 \mid S_{t-1} = 1] = p_{11}
\end{aligned}$$

For simplicity, we model z_{2t} as an AR(1). The results are presented in Table 3.4 and Figure 3.7. The persistence of the cyclical components are not significantly different in the two states; the absolute value of ϕ and the sum of ρ_1 and ρ_2 are higher than 0.6, the persistence of cyclical component for the benchmark case. The average duration of constant-mean state is also higher than that for the benchmark one. The estimates of error-variances of z_{1t} and z_{2t} are also significantly different from each other. However, the estimates of transition probabilities, p_{00} and p_{11}, are almost the same as that for the benchmark model.

5 Summary and Conclusions

We have provided a model for certain time-series processes that are characterized by two distinct phases: one in which the mean is stable or constant, and the other in which the mean is gradually shifting to another level. The number of these slow mean-shifts is unknown. Using an unobserved components model, the process can be divided into a 2-state Markov-switching process of the mean and a stationary cyclical component. The model is able to predict a slow shift in mean to another level without having to test for structural breaks in the data. In addition, there is no necessity of testing for unit roots in the series prior to estimation. This is particularly appealing for modeling the real interest rate of the United States, which give different results regarding presence of a unit root across various unit-root tests and sample periods.

Estimating the model on simulated data assuming various data generating processes, we have shown that our benchmark model is robust to them. When the model is applied to data on US ex post real interest rate, it is able to predict both the constant-mean states and the slowly-changing mean states fairly accurately. This model for real rates is robust to sub-sample analysis and state-variant cyclical components. It can be applied to other types of data where unit root tests could give varied results.

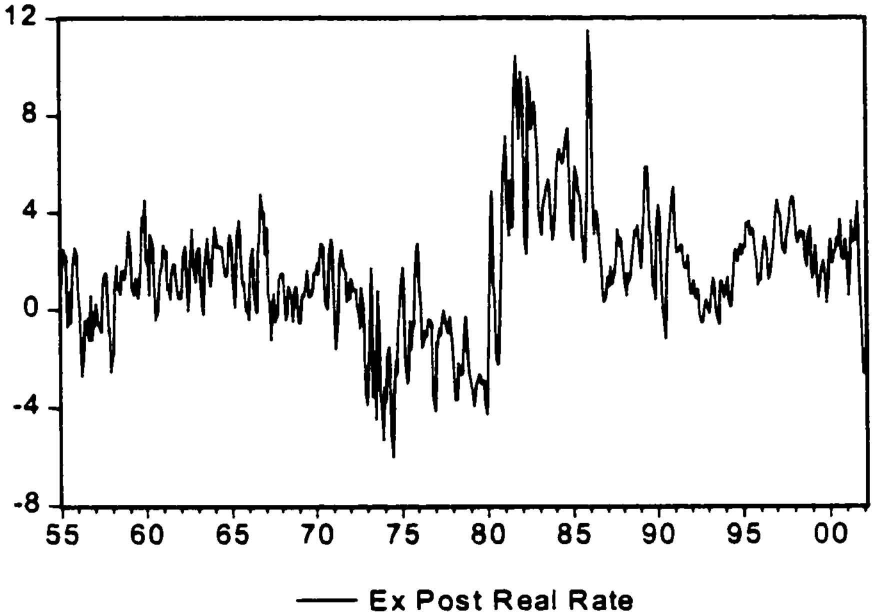


Figure 3.1: Ex Post Real Rate of the United States: 3-month Treasury Bill Rate minus actual 3-month CPI Inflation Rate.

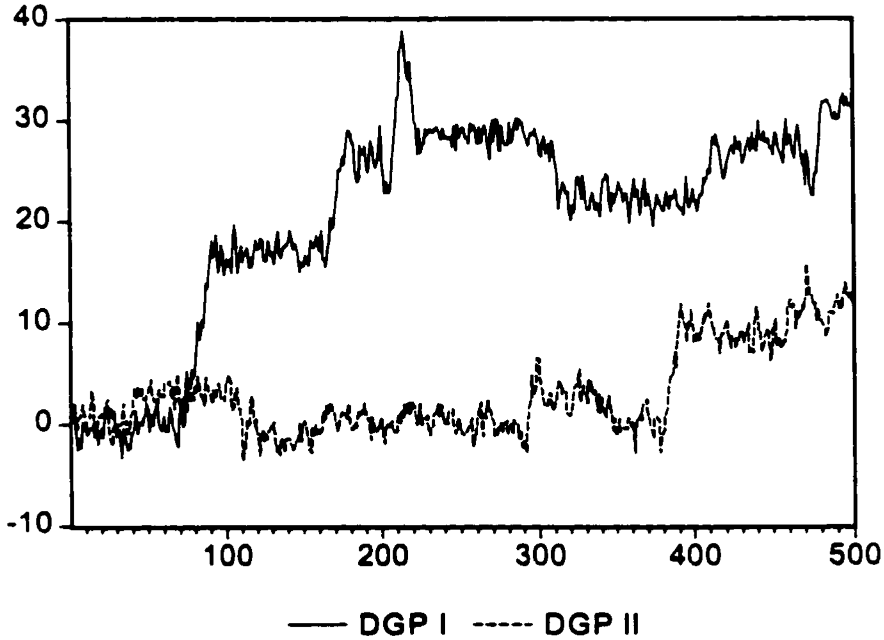
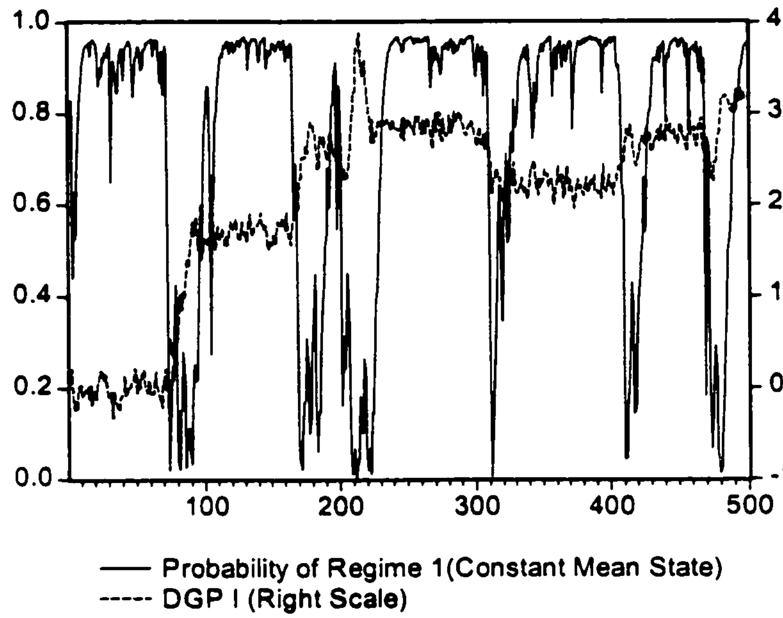


Figure 3.2: Two Sets of Generated Data Showing Unknown Number of Mean Shifts and Slowly Changing Mean



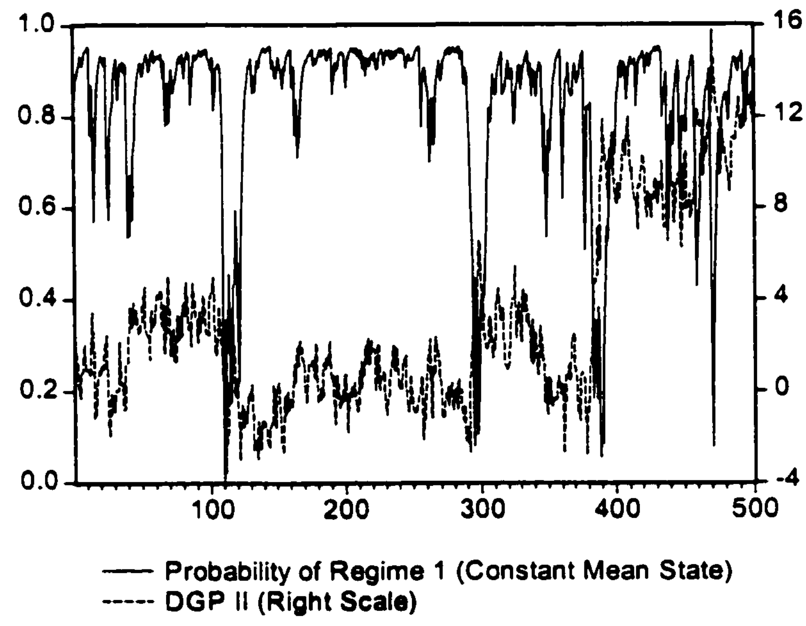


Figure 3.3: Filtered Probability of Constant-Mean State for DGP I and DGP II

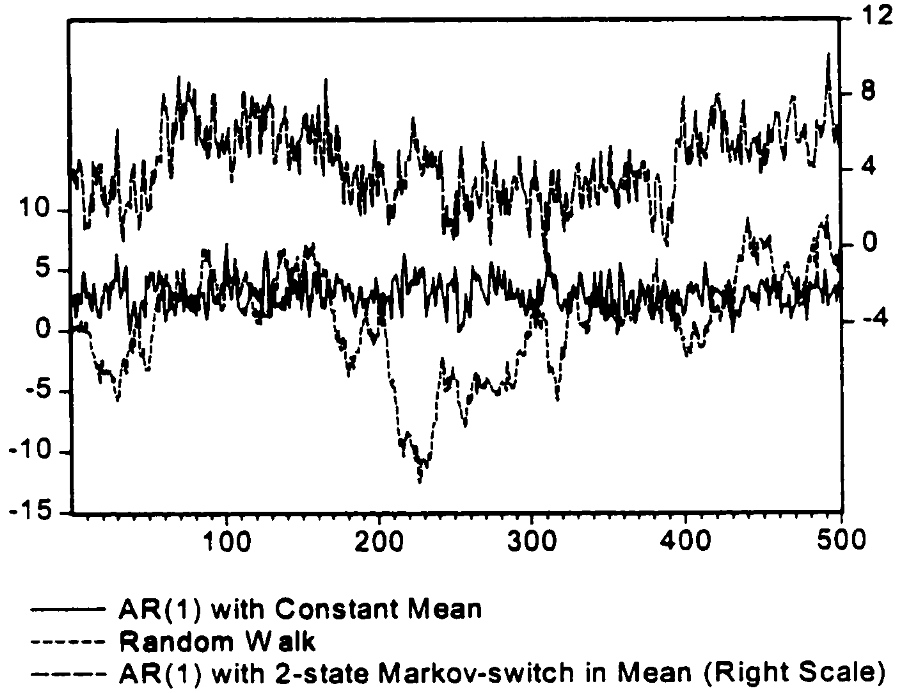


Figure 3.4: Three Alternative Data Generating Processes

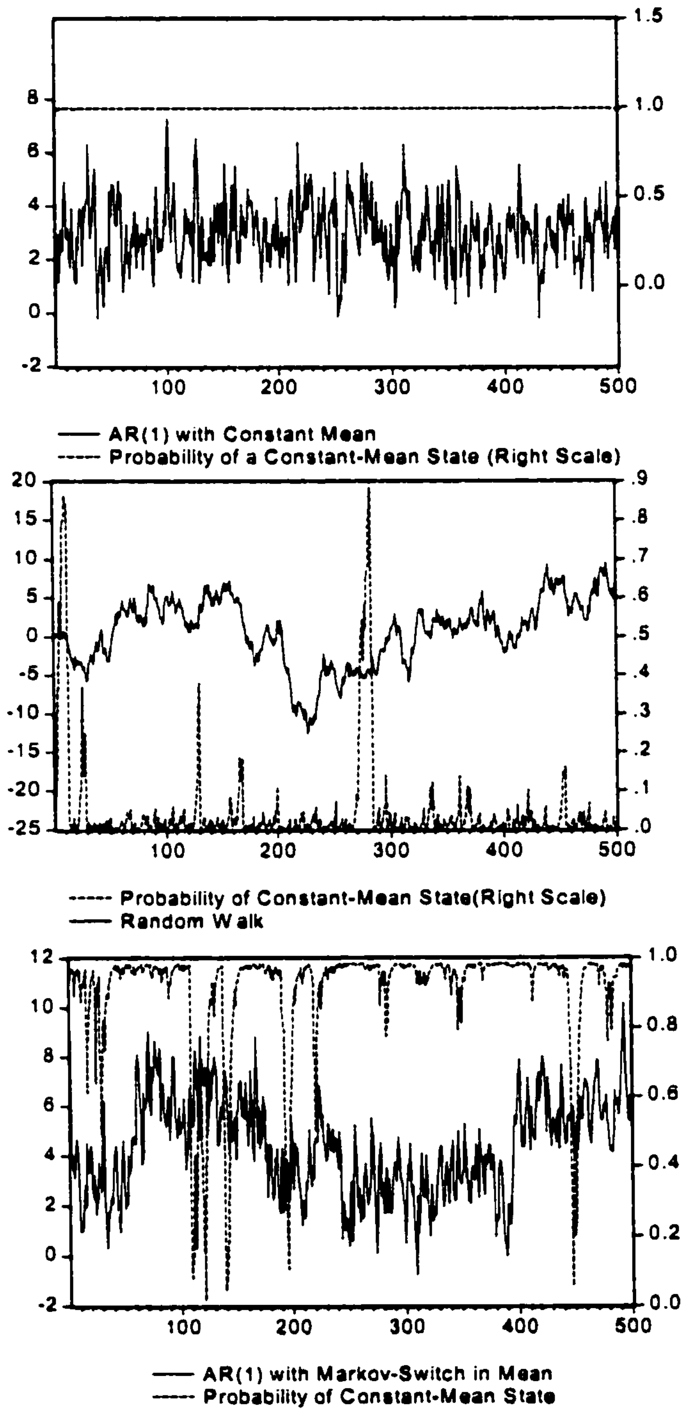


Figure 3.5: Three Alternative DGP with Filtered Probability of Constant-Mean State

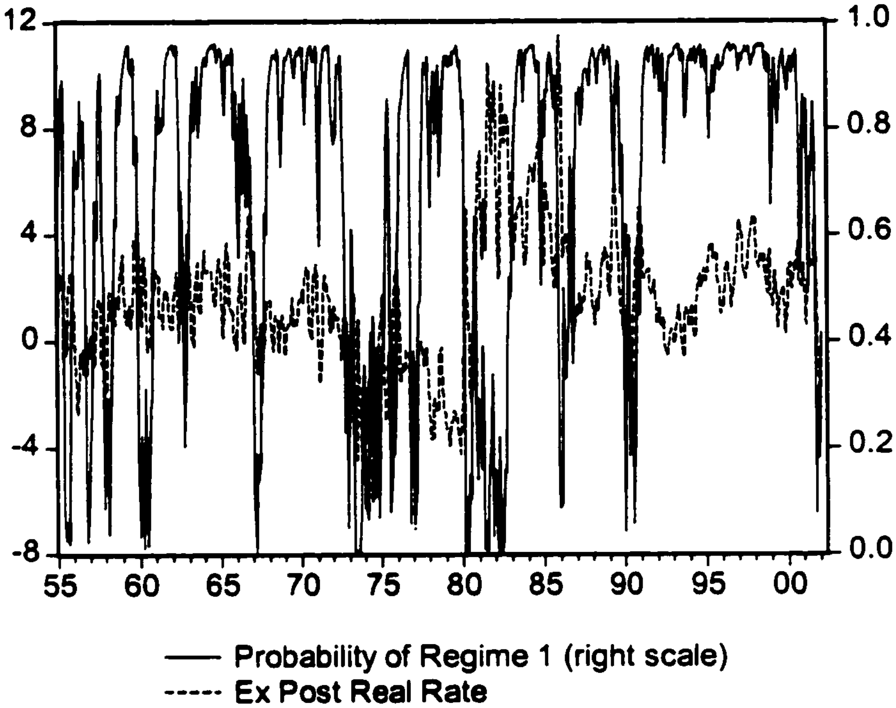
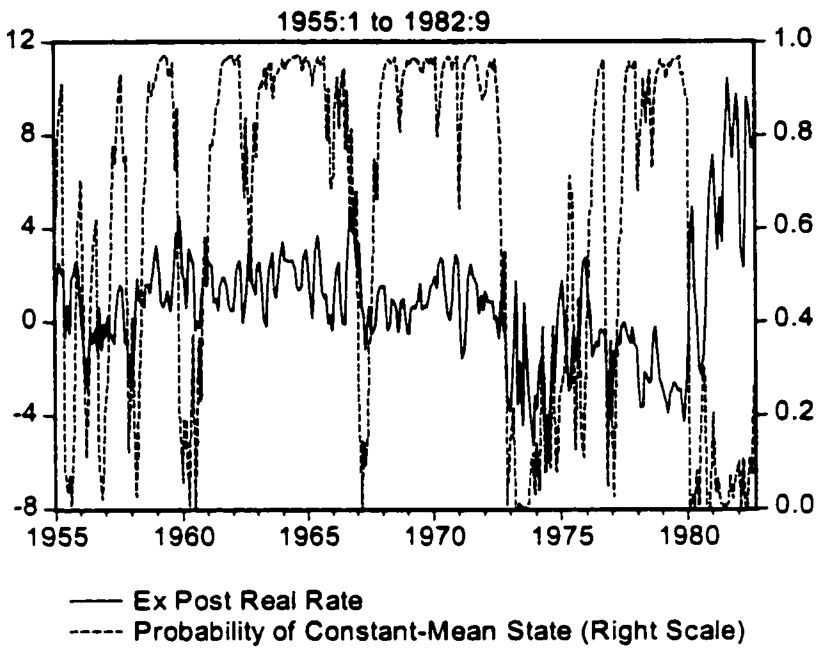


Figure 3.6: Filtered Probability of Regime 1, Benchmark Model



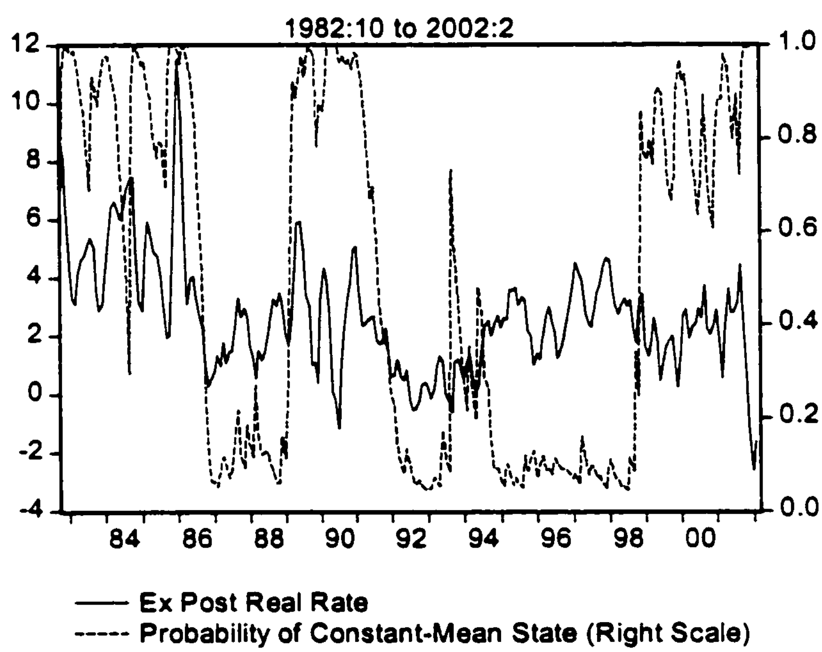


Figure 3.7: Filtered Probability of Constant-Mean State in Sub-sample Analysis

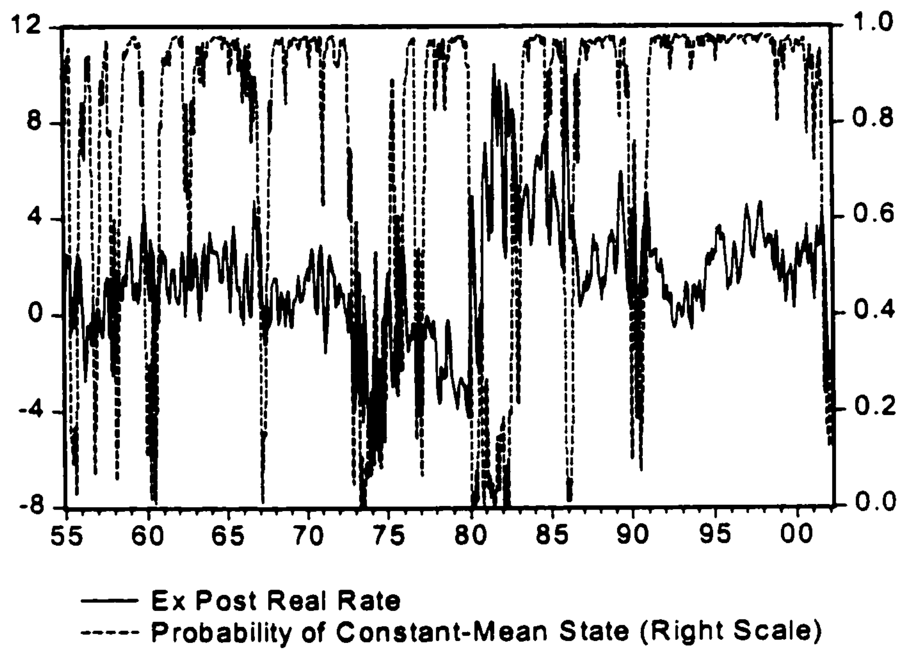


Figure 3.8: Filtered Probability of Constant Mean State with State-Variant Cyclical Component

Table 3.1: ML Estimates of Benchmark Model for DGP I and DGP II

Parameter	DGP I MLE (Standard Error)	DGP II MLE (Standard Error)
p_{00}	0.984 (0.007)	0.975 (0.011)
p_{11}	0.947 (0.025)	0.873 (0.067)
σ_w	0.999 (0.038)	0.948 (0.041)
σ_v	0.777 (0.217)	0.960 (0.286)
θ	0.663 (0.163)	0.234 (0.319)
ρ	0.424 (0.051)	0.552 (0.056)
Steady-State probability of $S_t = 0$	0.770	0.834
Average Duration of Regime 1	63	40
Log Likelihood	-341.46	-293.15

Table 3.2: MLE for Three Alternative DGP

Parameter	AR(1) with Constant Mean (Standard Error)	Random Walk (Standard Error)	AR(1) with Markov-switching Mean (Standard Error)
p_{00}	0.997 (0.539)	0.919 (0.079)	0.989 (0.007)
p_{11}	0.607 (8.009)	0.994 (0.005)	0.873 (0.081)
σ_w	1.037 (0.033)	0.386 (0.081)	0.999 (0.039)
σ_v	0.001 (0.260)	0.891 (0.095)	1.139 (0.427)
θ	-0.808 (8.668)	0.083 (0.130)	0.447 (0.331)
ρ	0.494 (0.039)	0.158 (0.249)	0.465 (0.054)
Steady-State probability of $S_t = 0$	0.990	0.065	0.918
Average Duration of Regime 1	368.60	12.49	87.77
Log Likelihood	-275.82	-256.34	-306.29

Table 3.3: MLE of Benchmark Model with Ex Post Real Interest Rate of the US January 1955 to February 2002

Parameter	Maximum Likelihood Estimate	Standard Error
p_{00}	0.9565	0.0176
p_{11}	0.9037	0.0384
σ_w	0.7078	0.0424
σ_v	1.2177	0.1736
θ	-0.3374	0.1407
ρ_1	1.1591	0.0465
ρ_2	-0.5529	0.0506
Steady-State probability of $S_t = 0$	0.6888	
Average Duration of Regime 1	22.98	
Log Likelihood	-280.7054	

Table 3.4: Robustness Checks

Parameter	Sub-Sample 1 1955:1 – 1982:9 (Standard Error)	Sub-Sample 2 1982:10 – 2002:2 (Standard Error)	State-Variant Cyclical Component (Standard Error)
p_{00}	0.955 (0.017)	0.983 (0.013)	0.970 (0.011)
p_{11}	0.932 (0.030)	0.976 (0.016)	0.907 (0.036)
σ_w	0.699 (0.062)	0.934 (0.066)	--
σ_{w1}	--	--	0.731 (0.043)
σ_{w2}	--	--	0.300 (0.105)
σ_v	1.802 (0.137)	0.560 (0.040)	1.663 (0.156)
θ	-0.026 (0.097)	0.198 (0.093)	0.222 (0.109)
ρ_1	0.987 (0.077)	1.217 (0.075)	1.119 (0.056)
ρ_2	-0.474 (0.069)	-0.660 (0.079)	-0.432 (0.055)
ϕ	--	--	-0.734 (0.132)
Steady-State probability of $S_t = 0$	0.602	0.579	0.754
Average Duration of Regime 1	22.1	57.68	32.90
Log Likelihood	-214.05	-62.93	-290.05

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Appendix: Derivation of optimal interest rate rule under discretionary policy.

Baseline Rule (CGG (1999))

Step1: Choose x_t and π_t to maximize objective (1), given inflation equation (3)

$$\text{FOC: } x_t = -\frac{\lambda}{\alpha}\pi_t \qquad (A1)$$

This is a 'lean against the wind' policy. Fighting against inflation calls for a lowering of the output gap. The extent to which this policy is going to be successful depends on the parameters λ and α. A higher λ indicates a greater response of inflation to output gap variations as is seen in the Phillips curve and the extent to which the policy maker will be helped in her task; a higher α indicates a higher relative weight put on output variability in the objective function and so this is going to 'hurt' the policy maker in her fight against inflation.

Step 2: Combining (A1) with Phillips Curve and assuming Rational Expectations and using the IS equation (2) we solve for i_t to get the baseline interest rate rule:

$$i_t = \beta_\pi E_t \pi_{t+1} + \frac{1}{\varphi}\varepsilon_t \qquad (A2)$$

where, $$\beta_\pi = 1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha} > 1$$

Debt-constrained Rule

The original IS equation (2) was derived from an euler equation from optimization framework, after substituting $Y_t - G_t \equiv C_t$, log-linearizing and writing

$y_t \equiv x_t + z_t$, where z_t is trend output and x_t is the output gap or the cyclical part of output and y_t is log (Y_t). The resulting IS equation (replicating (2)) was the following:

$$x_t = -\varphi[i_t - E_t\pi_{t+1}] + E_t x_{t+1} + \varepsilon_t \qquad (2)$$

The error term $\varepsilon_t \equiv E_t\{\Delta z_{t+1} - \Delta g_{t+1}\}$

$$g_t \equiv -\log(1 - \frac{G_t}{Y_t}) \qquad (A3)$$

where G_t is Government consumption.

Using the government spending equation, (6a) to substitute for $\frac{G_t}{Y_t}$ in (A3), we have

$$\Delta g_{t+1} = g_{t+1} - g_t$$

$$= -[\log(1-\phi_0 + \phi_1(i_{t+1} - E_{t+1}\pi_{t+2} - y_{t+1} + y_t)b_t) - \log(1-\phi_0 + \phi_1(i_t - E_t\pi_{t+1} - y_t + y_{t-1})b_{t-1})]$$

$$= -\log\left(\frac{1-\phi_0 + \phi_1(i_{t+1} - E_{t+1}\pi_{t+2} - y_{t+1} + y_t)b_t}{1-\phi_0 + \phi_1(i_t - E_t\,\pi_{t+1} - y_t + y_{t-1})b_{t-1}}\right)$$

Using approximation $\frac{1+\delta}{1+\psi} \approx 1 + \delta - \psi$, when δ and ψ are small,

$$\approx -\log[1-\phi_0 + \phi_1(i_{t+1} - E_{t+1}\pi_{t+2} - \Delta y_{t+1})b_t + \phi_0 - \phi_1(i_t - E_t\pi_{t+1} - \Delta y_t)b_{t-1}]$$

Further using approximation $log(1+a) \approx a$, for small a, and rearranging:

$$= -\phi_1\{i_{t+1} - E_{t+1}\pi_{t+2} - \Delta y_{t+1})b_t - \phi_1(i_t - E_t\pi_{t+1} - \Delta y_t)b_{t-1}\} \qquad (A4)$$

Replacing (A4) in (A3):

$$\varepsilon_t \equiv E_t\{\Delta z_{t+1} - \Delta g_{t+1}\}$$

$$= E_t[\Delta z_{t+1}] + E_t[\phi_1\{i_{t+1} - E_{t+1}\pi_{t+2} - \Delta y_{t+1}\}b_t \; - E_t[\phi_1\{i_t - E_t\pi_{t+1} - \Delta y_t\}b_{t-1}] \quad (A5)$$

Substituting for ε_t from (A5) in (2), and writing $y_t \equiv x_t + z_t$, we come upon the new IS curve given by (7), replicated here for convenience:

$$x_t = -(\varphi + \phi_1 b_{t-1})[i_t - E_t\pi_{t+1}] + E_t x_{t+1} + E_t[\phi_1 b_t\{i_{t+1} - \pi_{t+2}\}] - \phi_1 E_t\{\Delta x_{t+1} b_t - \Delta x_t b_{t-1}\} + \varepsilon_t^{'} \quad (7)$$

Here $\varepsilon_t^{'} \equiv (1 - \phi_1 b_t)E_t(\Delta z_{t+1}) + \phi_1 b_{t-1} E_t \Delta z_t$

In deriving the new interest rate rule, we use the same first order conditions as (A1), since the objective function and the Phillips Curve have not changed; substituting these conditions in the new IS equation (7), we get the new interest rate rule as in (8) which is also replicated here:

$$i_t = \left[\frac{(\lambda + \varphi\alpha\rho - \lambda\rho) + \phi_1(\alpha\rho b_{t-1} + \lambda\rho b_t - \lambda b_t - \lambda b_{t-1})}{\alpha\rho(\varphi + \phi_1 b_{t-1})}\right] E_t\pi_{t+1}$$
$$+ \left(\frac{b_t}{\varphi + b_{t-1}}\right) E_t(i_{t+1} - \pi_{t+2}) + \frac{\lambda}{\alpha}\left(\frac{\phi_1 b_{t-1}}{\varphi + \phi_1 b_{t-1}}\right)\pi_{t-1} + \frac{\varepsilon_t^{'}}{\varphi + \phi_1 b_{t-1}}$$

or,

$$i_t = \left\{\beta_\pi + \left(\frac{\varphi}{\varphi + \phi_1 b_{t-1}} - 1\right)\beta_\pi + \frac{\phi_1\varphi}{\varphi + \phi_1 b_{t-1}}\left(\frac{(\alpha\rho - \lambda)b_{t-1} - \lambda(1-\rho)b_t}{\alpha\rho\varphi}\right)\right\} E_t\pi_{t+1}$$
$$+ \frac{\phi_1 b_t}{\varphi + \phi_1 b_{t-1}} r_{t+1}^e + \frac{\lambda}{\alpha}\left(\frac{\phi_1 b_{t-1}}{\varphi + \phi_1 b_{t-1}}\right)\pi_{t-1} + \frac{\varepsilon_t^{'}}{\varphi + \phi_1 b_{t-1}}$$

where, $\beta_\pi = 1 + \dfrac{(1-\rho)\lambda}{\rho\varphi\alpha} > 1$

$r^e_{t+1} \equiv E_t(i_{t+1} - \pi_{t+2})$

Vita

Srobona Mitra was born and brought up in Calcutta, India, and went to Presidency College for her Bachelor's Degree in Economics. She moved to Delhi in 1994 to earn her Master of Arts in Economics from Delhi School of Economics, worked for a year at the Institute of Economic Growth in Delhi before moving to the United States. She earned a Doctor of Philosophy in Economics at the University of Washington in 2002.