An Investigation of Student Understanding of Classical Ideas Related to Quantum Mechanics: Potential Energy Diagrams and Spatial Probability Density

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Abstract

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This dissertation describes the results of two related investigations into introductory student understanding of ideas from classical physics that are key elements of quantum mechanics. One investigation probes the extent to which students are able to interpret and apply potential energy diagrams (i.e., graphs of potential energy versus position). The other probes the extent to which students are able to reason classically about probability and spatial probability density. The results of these investigations revealed significant conceptual and reasoning difficulties that students encounter with these topics. The findings guided the design of instructional materials to address the major problems. Results from post-instructional assessments are presented that illustrate the impact of the curricula on student learning.

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DEDICATION

To Anya, Jamie, and the Little One on the way

Chapter 1

INTRODUCTION: OVERVIEW OF THE RESEARCH

This dissertation describes an investigation into student understanding of two topics in physics that are taught at both the introductory and advanced levels: potential energy diagrams (i.e., graphs of potential energy *vs.* position) and spatial probability density (i.e., the probability per unit length or volume of locating an object in a certain region). The focus of this dissertation is on identifying the extent to which students are able to interpret and apply the relevant concepts, on the difficulties they encounter in doing so, and on the design and assessment of instructional strategies that can help promote the development of a functional understanding.

Some of the motivation for this investigation came from observations made by others and ourselves that sophomore and junior level students in courses on modern physics or quantum mechanics encounter significant difficulty in relating potential energy diagrams and probability densities to real world systems and motions. Since instructors in those courses may expect a basic facility with these ideas for *classical* systems, we have focused our research efforts in this context. Furthermore, since many students are first introduced to these ideas in *introductory* courses, much of our research is concentrated at this level.

We were also motivated by instructional, intellectual, and practical reasons. Potential energy diagrams can combine several important ideas and concepts from an introductory course, including kinetic energy, potential energy, conservation of energy, kinematic and dynamic quantities (e.g., acceleration and force), and graphical representations. This requires students to relate different ideas that may have been introduced at different times in their course. Probing student understanding of potential energy diagrams can provide information about how well students synthesize various concepts, as well as give a more complete picture of student understanding of the individual ideas.

Spatial probability density is essential to understanding some of the formalism and the predictions of quantum mechanics. One of the underlying concepts, probability, is sometimes first introduced by instructors within the context of quantum mechanics. This means students must study two new, abstract concepts at the same time. Introducing probability density classically avoids this issue. Having studied probability and probability density for classical systems also allows students to contrast the predictions of classical and quantum mechanics. Furthermore, understanding the ways in which students think about classical probability density might give insights into how they may struggle in quantum mechanical contexts.

Additionally, spatial probability density is one of many forms of *spatial density.* Others that are typically included in an introductory physics course are mass and charge densities. Probing how students understand spatial densities in multiple contexts may provide a more encompassing view as to how students view this concept.

1.1 Goals and practices of the research

There are three primary goals of the research presented in this dissertation. One goal is to investigate the extent to which students understand particular topics (potential energy diagrams and spatial probability density) within their introductory physics courses. We do this mainly by developing questions that target specific physics concepts and steps in reasoning. These are administered to students in long-answer or in multiple-choice format and are followed by a prompt for students to explain their reasoning. We then examine responses to these 'pretests' in detail, looking for patterns that suggest particular ways of thinking about the relevant ideas. When reporting student understanding, we use the term 'difficulties' to describe incorrect or unproductive pathways in reasoning. In conducting the research, the pretest questions are typically administered after relevant lecture instruction, reading assignments, and laboratory sections. We often administer a given pretest in many forms and contexts to probe student ideas in detail and to ensure that the difficulties are not associated with a particular phrasing of the question or representation. Pretests are also frequently revised on the basis of examining student responses. An overarching objective is to identify areas in which students may need additional help in developing a functional understanding of a given area of physics.

The second goal of this investigation is to use our findings to develop curricula that target the difficulties that we previously identified through examining pretest responses. In the large lecture format of the courses in this investigation, these curricula typically take the form of paper-based worksheets called 'tutorials' that students complete in small group sections with the help of teaching assistants. The two tutorials we have developed are called *Potential energy diagrams* and *Probability in classical and quantum mechanics*. They are to become part of a larger body of work called *Tutorials in Introductory Physics* that emphasizes addressing conceptual issues related to the learning of physics [1]. This publication is used in physics courses both nationally and internationally to help address continuing student difficulties that are present even after lecture instruction.

The third goal is to assess the effectiveness of the curricula. We do this by developing additional questions related to the topics. These ‘posttest’ questions are incorporated into students’ course exams. We typically avoid administering the same question to a given group of students as both a pretest and posttest. Findings from analyzing student responses on posttests inform the direction of future research, such as developing new questions or revising the curricula.

1.2 Organization of dissertation

This dissertation is composed of four parts. Each is described below.

Part I provides an extended overview and background to our research and is composed of two chapters. Chapter 2 provides a description of the methods used, an overview of the student populations, and the environment in which the investigation was conducted. Chapter 3 discusses prior research that is relevant to our investigation.

Part II describes our research into student understanding of potential energy diagrams. The four chapters that comprise this part are: our initial research into student ideas and difficulties (Chapter 4); the development of a curriculum to address the student difficulties (Chapter 5); the assessment of, and modifications to, the curriculum (Chapter 6); and the development and assessment of an additional curricular component that is not typically part of *Tutorials in Introductory Physics* (Chapter 7).

Part III describes our research into student understanding of spatial probability density. The three chapters that comprise this part (Chapters 8–10) are organized in a similar fashion to the first three chapters of part II.

Throughout both Parts II and III, abbreviated descriptions of the questions and curricula we administered are provided. Unabbreviated versions of this material, however, are provided in **Part IV** of the dissertation. This part is composed of Appendices A and B, which contain the questions and curricula for potential energy diagrams and probability density, respectively.

Finally, there are two **indices** at the end of this dissertation. One index is for the material associated with the tutorial *Potential energy diagrams.* The other is for the material associated with tutorial *Probability in classical and quantum mechanics.* The indices may be used to find pretests, posttests, and course material in the appendices for a particular course section.

Part I

OVERVIEW AND BACKGROUND OF RESEARCH

Chapter 2

CONTEXT AND METHODS: RESEARCH, CURRICULUM DEVELOPMENT, AND INSTRUCTION

This chapter presents an overview of the learning and research environment at the University of Washington (UW) where the present research took place. We begin by describing the student populations, the courses, and the instructional environment (§2.1 and §2.2). We then discuss the research methodology and statistical practices used throughout this dissertation (§2.3 and §2.4).

2.1 Overview of student populations and courses

The University of Washington (UW) is a public RU/VH [2] research university that enrolls over 40 000 students. Slightly more than half of the undergraduate applicants are accepted to the university. Most of the students have done well in their high school classes. In autumn 2014, for example, the middle 50% of entering freshmen had a high school GPA between 3.64 and 3.93 [3].

The research presented in this dissertation involves several undergraduate physics courses at the UW. Each is described later in this section. However, since the majority of the research was embedded within the introductory calculus-based physics sequence, we first provide a detailed description of the sequence here.

The three courses that comprise the introductory calculus-based sequence are Phys 121 (mechanics), Phys 122 (electricity, circuits, and magnetism), and Phys 123 (waves, optics, modern physics). The term Phys 12X is used to refer to the three-course sequence as a whole. In any given quarter there are between one and four lecturers teaching different sections of each course in the sequence. About 70% of the 1300 students[1] enrolled in Phys 12X at any given time are pre-engineering majors. The fraction of students who major in physics is relatively small. (There are about 100 physics majors who graduate each year.) Most students are sophomores when they complete the three-quarter sequence.

There are three separate components associated with each Phys 12X course: lecture, laboratory, and tutorial (quiz) sections. With regard to the first of these, students attend three 50-minute lectures each week. All lecturers are encouraged to use a real-time response system (also known as clickers) in class, but both the number of clicker questions and the amount of student interactivity that occurs varies greatly from one lecturer to another. Homework assigned by the lecturer is completed once per week by students using an online system [4] in which students typically supply numerical or symbolic answers and receive immediate feedback with regard to its correctness.

[1] At the start of autumn quarter in 2014, a total of 1369 students were enrolled in one of the three Phys 12X courses.

In preparation for most lectures, students view videos and answer conceptual questions using a different online system [5]. Lecturers have the ability to view student performance in order to adjust their upcoming lecture, though in practice whether or not this occurs varies from lecturer to lecturer.

The second component to each Phys 12X course is the laboratory. Students attend their laboratory section for 110 minutes each week. During the period relevant to our investigation, the labs have been fairly traditional in that students often verify laws discussed in lecture (e.g., Newton's second law) or use in-lab measurements to determine some unknown quantity (e.g., moment of inertia of an irregular object). Pre-labs and post-labs are completed by students using the same online system as the lecture homework.

The third component to each Phys 12X course is the tutorial (quiz) section. This is the primary context for the present research. Students attend these 50-minute sections once per week and work in groups from three to five through worksheets from *Tutorials in Introductory Physics* with the help of their classmates and two teaching assistants (TAs). (An in-depth discussion of these tutorials takes place below.) These tutorials are in place of traditional recitation sections. 'Traditional' here means those recitation sections in which teaching assistants might hold class discussions, demonstrate the solving of problems, or assign typical textbook problems for students to work through in their small groups.

All sections of a given course (e.g., all sections of Phys 121) share a common schedule and common midterm exams, of which there are three. The final exams are section-specific.

Several physics courses other than the calculus-based introductory sequence were also involved in the present research. A brief overview of each, including the 12X sequence for completeness, is provided below for future reference.

Phys 114: This is the first of three courses in the introductory algebra-based sequence. It covers classical mechanics and is typically populated by pre-medical students or others not requiring calculus-based physics for their degrees. There are no small group sections associated with the course. Although enrollment in the laboratory sections are not officially required by the physics department, most students do enroll in them to fulfill requirements by their majors. The algebra-based labs are very similar to the calculus-based versions.

PHYS 12X (Described above): This is the collective name for the three-quarter introductory calculus-based sequence (see next three courses).

- PHYS 121: During the period of this research, classical mechanics has been covered in a fairly traditional order: kinematics, forces, energy & momentum, and rotational dynamics. There is also a brief treatment of universal gravitation and fluids.
- PHYS 122: Electrostatics, basic circuits, and magnetism are covered.
- PHYS 123: Waves, optics, and selected modern physics topics are covered. The amount of modern physics topics has varied greatly over the past decade. During the present investigation, basic quantum mechanics and atomic and nuclear physics were included in the syllabus.

PHYS 225: This sophomore-level course on introductory quantum mechanics is mainly populated by physics majors. Over the past decade the content has shifted from general modern physics (e.g., blackbody radiation, photoelectric effect, special relativity, atoms, basic applications of the Schrödinger equation, etc.) to primarily two-state quantum systems.

PHYS 50X: This refers to PHYS 501, PHYS 502, or PHYS 503, which act as preparation courses for graduate and undergraduate TAs who are teaching *Tutorials in Introductory Physics.* A typical class session consists of taking a pretest, working through the corresponding tutorial, and examining introductory student responses to the same pretest. This course is described in more detail in the next section.

2.2 Overview of *Tutorials in Introductory Physics* and its implementation

Tutorials in Introductory Physics is a published set of worksheets and other paper-based materials designed to supplement traditional lecture instruction in an introductory physics course [1]. Each set of worksheets (henceforth, tutorial) focuses on a single topic; examples include *Conservation of energy, Gauss' law,* and *Single-slit diffraction.* This section illustrates the implementation at the University of Washington, the instructional strategies of the tutorials, and the preparation for tutorial instructors.

2.2.1 Instructional sequence and the student experience at UW

For a given topic within *Tutorials in Introductory Physics* there are four distinct components that students typically work through: a pretest, an in-class worksheet/tutorial, a homework, and a posttest. The order here matches that experienced by students. Each component is described below, including its instructional implementation at the University of Washington.[2] (Note that aspects of some of the following components are also associated with research. These aspects are discussed in §2.3.)

2.2.1-a Pretests

'Pretests' are short quizzes administered over the weekend prior to students working the in-class tutorial. Student are given credit for completing them. One goal of the pretests is to probe student understanding of basic ideas relevant to the tutorial. Another is to provide students the opportunity to assess their own knowledge and learning. Both of these aspects of the pretests are elaborated in §2.3.

The pretests are administered through a UW-produced online 'Catalyst' survey tool [7].[3] A time limit of 15 minutes is set to encourage students to answer using only their own

[2] Some other institutions that have adopted *Tutorials in Introductory Physics* have published a description of their own implementations. See, for example, Ref. [6].

[3] Some pretests are administered on paper due to technical limitations of Catalyst (e.g., lack of functionality to draw graphs).

knowledge. Students are informed that pretests are graded for completion, not correctness.

The pretests are typically administered after lecture instruction since most of the tutorials assume some amount of background knowledge. This approach is consistent with the idea that *Tutorials in Introductory Physics* are a supplement to lecture instruction, not a replacement.

After completing the pretest over the weekend, students attend their assigned tutorial section during the upcoming week.

2.2.1-b Tutorials/In-class worksheets

The in-class tutorial worksheets are composed of questions about a concept or situation that research has shown to be somewhat challenging for students. Each tutorial is based on prior or ongoing research. Through questions embedded in the worksheets and questions posed by TAs, students practice applying the theory they have learned in lecture to new situations. They come to learn how their prior ideas do or do not fit within that theory. Examples of instructional strategies employed by the worksheets are discussed in §2.2.2.

At the UW, students work through the tutorials during their designated 50-minute quiz section, each of which can accommodate up to about 24 students. Students work in groups of between three and five. Two TAs are assigned to teach each tutorial section. They are most often physics graduate students, although the Physics Department hires some undergraduates and out-of-department graduate students to help meet demand.

During the tutorial sections, students are encouraged to work collaboratively in completing the worksheet. Should the need arise, students can ask TAs for assistance. The TAs are instructed to assist students by asking guiding questions rather than giving answers directly. This allows students to go through the requisite reasoning themselves.[4] In addition to addressing students' questions, TAs listen to groups' conversations (contributing as needed), encourage collaborative work, and pose additional questions that are extensions to those in the tutorial.

[4]Teaching through questioning is known as the *semi-Socratic* method [8].

If students do not complete the tutorial during the allotted 50 minutes, they are encouraged to seek help from their peers or TAs prior to attempting the associated homework.

2.2.1-c Homework

The tutorial homework is paper-based. For the most part, it consists of questions for which students need to apply the concepts discussed in tutorial. In some cases, the homework guides students through *new* concepts not explicitly developed in the tutorial by using a similar questioning strategy. For full credit, students must explain their reasoning.

Homework is due the week after the associated tutorial. Only a subset of the questions that students complete is graded. TAs who grade are instructed to respond to incorrect or incomplete responses by writing comments or questions rather than giving answers. Solutions are not available to students, so they are encouraged to discuss their homework with TAs.

Homework for the last tutorial of the quarter is neither collected nor graded since it occurs during the last week of classes. This situation has bearing on the present research since the tutorials that were developed are typically the last ones in their respective courses.

2.2.1-d Posttest/Exam

'Posttests' refer to questions administered after tutorial instruction on a given topic. They are almost always administered on students' course exams. In the Phys 12X sequence there are three midterm exams and one final exam. During any given quarter about half of the tutorial topics have associated posttest questions. (Not all tutorials are post-tested each quarter due to the limited length of each exam.) Twenty percent of each exam's points are composed of tutorial-based questions.

Students must explain their reasoning to obtain full points on the tutorial-based midterm questions. The final exams are by default multiple choice due to, in part, a lack of TA hours. However, explanations may be required on final exams if the particular tutorial topic is being actively researched, the instructor of the course permits it, and a grader can be obtained.

2.2.2 Instructional strategies of the tutorials

The in-class tutorial worksheets employ a variety of instructional strategies to help improve student understanding. During some tutorials, students develop a conceptual model from the ground up to explain complex phenomena. (See, for example Ref. [9].) Many tutorials use an instructional strategy of providing fictitious student statements and asking students to identify any mistakes or incorrect assumptions. Other tutorials have students use hypothetico-deductive reasoning to distinguish between two competing theories that could account for the outcome of an experiment.

Another fairly common strategy is known as *elicit, confront, resolve.* In this strategy, a question is first presented to students that research has shown to *elicit* incorrect ideas or assumptions. Next, students are *confronted* with the fact that their original answer is incorrect. This may be accomplished through a series of guiding questions. Finally, the students are guided to *resolve* the discrepancy through further questioning.

The Physics Education Group has found the elicit-confront-resolve method to be effective in helping students engage and invest in their own learning. (An example of this instructional method as applied to electric circuits, and the resulting gain in student understanding, can be seen in Ref. [10].) Theoretically, this strategy has footing within an educational theory called constructivism [11], which posits that students generate their own knowledge and understanding by active participation. The elicit-confront-resolve method is consistent with this viewpoint in that students are encouraged to become active thinkers.

2.2.3 Instructor preparation for the tutorials

All first-year physics graduate students at UW who have teaching assignments are required to enroll in Phys 50X. The primary objective of this course is to prepare tutorial instructors to teach the upcoming tutorial topic. (Undergraduate tutorial TAs also attend this course or similar preparation sessions.) Broader issues related to teaching are also discussed.

During a given preparation session, TAs first work through a paper-based pretest that is similar to the online version administered to the introductory students. These pretests give the TAs an opportunity to assess their own knowledge of the material and to reflect on the knowledge that students are expected to have for the upcoming tutorial. The TA pretests also provide data on the extent to which UW physics graduate students are able to apply knowledge from their introductory courses. Often, their performance on the pretests are compared to introductory student performance on *posttests.* The Physics Education Group has found that it is often a practical goal to bring introductory undergraduate students up to the level of incoming graduate students, who may not have thought about their introductory material in many years. This measure is often used as a gauge as to whether or not a given tutorial should continue to be modified and improved.

After completing the pretest, the TAs work in small groups through the same tutorial that the introductory students will work through later in the week. At least one member of each group is an experienced TA, Most often he or she is a graduate student or postdoctoral researcher with the Physics Education Group. While the TAs work together to answer the tutorial questions, the experienced TA facilitates group discussion. He or she also helps them through the material, if necessary, by asking guiding questions. This models the type of interactions that we expect to occur during the actual tutorial sessions with introductory students. The experienced TA also provides insights into effective questioning strategies on particular parts of the tutorial. Additional questions are also posed that are not part of the tutorial. This helps deepen the knowledge of the tutorial TAs and provides them with questions that they themselves can pose to the introductory students.

After working through the tutorial, the tutorial TAs in Phys 50X examine actual introductory student responses to the online pretest. The experienced TA facilitates group discussion about the difficulties that the students encountered. A class-wide discussion then takes place during which each group share their findings. These practices provide the TAs a sense of the level of expertise and common difficulties that the introductory students bring with them into the tutorial sessions.

2.3 Research methodology and its relation to curriculum development and instruction

One important part of the research performed by the Physics Education Group, and presented in this dissertation, is gaining insight into student thinking by probing and documenting how physics students respond to qualitative and conceptual questions. The students' answers and explanations are used to provide insight into what they do or do not understand about the relevant physics. Examining these responses often leads to the development of *different* questions that are designed, for example, to elicit more clearly an underlying student difficulty or to probe whether or not student responses are an artifact of some feature of the question. The results from detailed analysis are published in peer-reviewed journals for both other physics education researchers and physics instructors. An example of this process is described by Shaffer and McDermott [12].

In addition to examining student understanding, the Physics Education Group develops curricula, including *Tutorials in Introductory Physics*. These materials are designed to guide students toward developing and understanding sound physical models and to address underlying student difficulties that were identified through research. Since these curricula are administered at the UW (and at other institutions that serve as pilot sites), the PEG is also very active in areas related to effective instruction, including teacher preparation. These three components—research, curriculum development, and instruction—are intricately linked together. For example, while observing students who are working through a tutorial (instruction), a researcher may observe students misapplying an equation from lecture to solve a problem (e.g., $W_{\text{net,ext}} = \Delta K$) and wonder whether an additional question within the tutorial could help students realize their mistake (research). The researcher might then modify the in-class worksheet, asking students what each subscript in the variable $W_{\text{net,ext}}$ means (curriculum development). The tutorial TAs might also be informed that students could have difficulties in this section (instruction). After students work through the modified materials (instruction), a posttest is administered to help determine how and to what extent the change affected student performance (research), which would then inform whether the

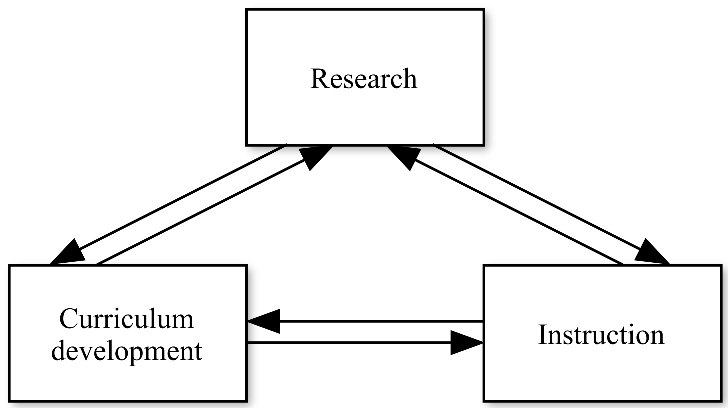


Figure 2.1: Connections between research, curriculum development, and instruction.

modification should carry over into the next quarter (curriculum development).

As the example above illustrates, performing research and taking an active role in instruction enables the Physics Education Group to develop effective curricula. The intricate connection among the three components is often illustrated using the diagram in Figure 2.1.

It should be emphasized again that the collection of data on student understanding is central to the present research. As such, there are a variety of methods that are used to probe student understanding. For reference, a list is provided below. Readers may refer to this as needed.

Interviews: One-on-one interviews between a student and a researcher are typically conducted during exploratory research, in which the difficulties that students might have on a certain topic or the way in which students approach a problem are not already known. During these 45-minute audio-recorded sessions, the researcher will pose questions of various degrees of difficulty to the student and ask the student to think aloud while attempting to solve them. The researcher must, in real time, use the student's spoken and written responses to decide on the next course of action. These actions may range from testing whether a particular sequence of questions is helpful (if the student is stuck on the problem) to asking an entirely different and unplanned series of questions (if the student's behavior indicates an interesting or incorrect underlying idea). Thus, the interviews can be described as semi-structured rather than scripted. A more detailed discussion of this data collection method is provided by Trowbridge and McDermott [13].

Online pretest quizzes: Online pretest quizzes are typically administered using WebQ, one of the Catalyst online tools developed at UW [7]. (The term *Catalyst* is used throughout this dissertation to refer to this question format.) Catalyst does not provide a way for students to draw pictures when answering a question. Therefore, questions in which students must determine the correct graph, vector, or image must be presented in multiple-choice format. This is relevant to the present research because spatial probability density and potential energy diagrams involve graphs.

As previously mentioned, the purpose of the pretests is two-fold. First, instructors and researchers can use the results of a class' pretests to gauge students' understanding. Results from the pretests are often compared to results from 'posttests' to judge the effectiveness of the tutorial in addressing a particular difficulty.

A second purpose of the pretests is student-centered. Students can use their experience in the tutorial to reflect on their own knowledge and learning. To this end, large posters that summarize the pretest questions are hung in the tutorial rooms. Students are encouraged to discuss the answers and explanations with their classmates and TAs and thus consider how their ideas have changed (metacognition).

Written pretest quizzes: Sometimes pretests are administered using a written format instead of the Catalyst system described above. This is often done when the answer to a question is graphical in nature. Results from the written questions often inform the answers choices on corresponding Catalyst questions. This is especially true if the answers are graphical in nature.

Written posttest questions: Hand-written posttest questions are most often administered in an exam setting. Answers and explanations provided by students can be compared to performance to similar pretests questions to investigate the effect of the tutorial on student understanding. (Such comparisons are the subject of §2.4.2.)

Multiple-choice posttest questions: Multiple-choice posttest questions (without explanations) are typically only administered on final exams, which are entirely multiple choice. The lack of student explanations is often not an issue in more established research (as opposed to exploratory research) because the types of student explanations are already known. Thus, the question and answer choices can be written to elicit common incorrect ideas student may have.

In-class observations: Authors of a new or newly-modified curriculum typically attend or teach in the tutorial sections in which the curriculum is used. This provides an opportunity to observe how students interact with it. These observations are useful in determining which portions of the tutorial are unclear, too difficult, or otherwise need modifying, all of which might be difficult to detect using only posttest data. Observations of the interactions between the TAs and students can also be useful. Both effective and ineffective guiding questions from TAs can be disseminated to future TAs in their preparatory course (Phys 50X).

2.4 Measures and statistics of student performance

This section describes how student performance is translated into quantitative data, and how different data are compared in order to determine the effectiveness of curricula.

2.4.1 Reporting student performance

Student performance is usually reported as percentages (e.g., percent of the students who give correct responses, correct responses with correct reasoning, incorrect types of explanation, etc.). This allows for easy comparison across groups of students of different sizes.

Additionally, we typically round all pretest and posttest percentages to the nearest 5%. One of our goals is to give a general sense of the ways in which students respond to and struggle with physics problems. Since we also use our results to develop curricula, we are sensitive to what may be considered 'instructionally significant' rather than simply 'statistically significant' (see our discussion in §2.4.2 for more information). Rounding to the nearest 5% avoids the fine-grained details that are often unnecessary for this purpose.

2.4.2 Comparing student performances

Throughout this dissertation, performance by students on a given question is compared to performance on a similar one. This is especially true when comparing pretest to posttest performance to infer whether the intervening tutorial instruction had an effect. Such comparisons are sometimes done using *different* groups of students. That is, the students taking the pretest and those taking the posttest may be enrolled in different sections of the same course, or take the course in different quarters. One of two methods may be used to establish whether or not any difference in performance between pretest and posttest is due to chance.

One method involves using the actual variation or range in performances on the questions that we have administered as part of this study. Doing so requires that the question be administered several times so that a range can be established. As an example, suppose that on six different administrations of a particular pretest question, the percentage correct always falls between 30% and 45%. Suppose further that on a similar number of posttest administrations between 50% and 60% of the students answer correctly. Since the ranges of performances in pretests and posttests do not overlap, the inference will be made that the difference in performances is likely due to tutorial instruction.

The second method for comparing performance across groups of students involves using the historical variations that the Physics Education Group has observed on other questions. In a study spanning 15 years' worth of introductory students in classes taught by a variety of instructors, with and without clicker questions, and with different textbooks, Heron [14] found that student performance on any given pretest question typically varies between 5 and 10 percentage points. For example, if the average performance on a large number of administrations of a particular question is 50%, individual administrations will typically vary at most between 40% and 60%. In this dissertation, if performance on a pretest and posttest question differs by at least 20 percentage points (e.g., 50% *vs.* 70%), then the inference will be made that the difference is likely not due to chance alone, but rather due to tutorial instruction. A difference of 15 percentage point is considered borderline especially if the questions were administered only to a single section of students.

It is our opinion that this division can also be used by some instructors in deciding what is 'instructionally significant.' Although a difference of 10 percentage points might, in some cases, be found to be statistically significant, we recognize that such a relatively 'small' change may not be worth an increase in instructional time.

Occasionally in this dissertation, statistical claims are made using chi-square tests [15, p. 637]. This is done when students in one section or course are randomized in some way and we are comparing across these different randomized groups.

2.4.3 Abbreviation used for course sections

Throughout this dissertation, student responses are summarized using tables. These tables often abbreviate course sections. For example, the abbreviation **121A112** should be read as **121A-11-2**, which is short for Physics **121,** section **A,** year 20**11,** and quarter number

2 (i.e., spring). Winter is considered quarter 1, spring is quarter 2, summer is quarter 3, and autumn is quarter 4 (which is the beginning of the academic year at the University of Washington). This code and the indices at the back of this dissertation can be used to view the questions and curricula administered to each section.

Chapter 3

PRIOR RESEARCH

As we described in §1.1, our research focuses on the difficulties that students encounter when reasoning about potential energy diagrams and spatial probability density (both for classical systems). This chapter provides an overview of existing relevant research related to these topics.

In addition to the information provided in this chapter, specific results from prior research are emphasized throughout the dissertation as relevant (e.g., if work by others influenced our own, or if our results are similar to those of other studies).

3.1 Overview of prior research related to potential energy diagrams

Potential energy diagrams can be viewed as the combination of two distinct concepts: a graphical way to represent continuous variables and the abstract concept of energy. We begin this section by describing relevant research on these two topics separately. We then discuss research that specifically focuses on graphical representations of energy. Finally, we briefly discuss existing curricula that are designed to help students understand such graphical representations.

Relevant research on graphs

There is a variety of research on student understanding related to graphs. Some of the research focuses purely on visual aspects. For example, Shah and Hoeffner [16] provides an overview of the general characteristics of graphs that can help students to interpret them correctly, including aspect ratio and the use of color. Of greater relevance to our research, though, are the findings of researchers who characterize the *difficulties* that students encounter when reasoning about graphs.

For example, McDermott et al. [17] described the tendency of some students to focus on incorrect features of a graph to answer kinematics question (e.g., confusing the slope and height of a graph). Other such 'wrong feature' tendencies are summarized by Clement [18]. Of a more general nature, some researchers have found that students' prior experiences affect their ability to interpret graphs [19, 20]. For example, Planinic et al. [20] found that first-year university students struggled more with graphical interpretations (area under curve) on questions in a kinematics context than they did on analogous questions in a purely mathematical context. This suggests that some graphical difficulties may be context-dependent.

We note, though, that some of the difficulties that have been identified in the context of graphs are also elicited in questions that do not use graphs [21]. This suggests that some difficulties in interpreting physics graphs are physics-related, not necessarily graph-related.

Relevant research on energy

There is a large body of research focusing on issues related to student understanding of *energy.* Much of this research, however, is seated at the K–12 level. For example, Lee and Liu [22] found that some characteristics of energy (conservation) are more easily generalizable by students across disciplines. Megalakaki and Thibaut [23] studied the extent to which students differentiated between the concepts of force and energy for both animate and inanimate objects. Although such studies are in some way related to our research, the research we describe below is more directly relevant to our research.

In a broad sense, some researchers and educators distinguish between two main ways in which energy can be conceptualized: either (1) as a substance-like quantity that can be thought of as existing within or as part of the object or system of interest, or (2) as an abstract concept with which a 'convenient' number can be associated [24–26]. There appear to be advantages and disadvantages to both. For example, it can be argued that treating energy as having substance-like properties more naturally encompasses the conservation of energy. Some researchers, including Scherr et al. [25], have used this view in an educational setting (e.g., using small blocks to represent units of energy) with great success.

However, one disadvantage of treating energy as a substance is that it does not readily accommodate systems with negative potential or total energy. Negative energies are a common feature of many systems (e.g., universal gravitation and attractive Coulomb interactions). Some of the difficulties students encountered when using a substance-like metaphor have been documented [27, 28]. For example, Dreyfus et al. [28] described that some students made misleading statements such as claiming that 'all' of the available energy can be in one form or another. This idea is inconsistent with the fact that, for negative potential energies, the value of kinetic energy can be greater than that of the total energy.

Outside of these general theoretical considerations, some researchers have identified difficulties that students encounter when reasoning about energy and energy-related concepts. For example, Lindsey et al. [29, 30] studied student ability to infer the sign of the change in potential or total energy and the extent to which students understand the importance of systems. She found that many students failed to consider how different choices of systems affected the analysis of work and energy. Loverude [31] has found that the motion of objects in gravitational fields is a factor in how students rank gravitational potential energies. For example, if two objects are of the same mass and at the same height, but one of them is moving upward, some students believed that the object moving upward has more potential energy. This is inconsistent with the idea that potential energy for a given object/system is a function only of position.

Relevant research on graphical representations of potential energy

There are many ways in which energy can be represented graphically. Brewe [24] and van Heuvelen and Zou [32] provide an overview of the various graphical representations that are used in the context of energy. One particularly common representation is an energy bar chart [33]. This representation has the advantage of easily accommodating negative energy unlike, for example, pie charts. Since our primary concern is with graphs of potential energy *vs.* position, below we emphasize research related to potential energy diagrams.

One of the earlier bodies of research on student understanding of potential energy diagrams is by Bao [27].[1] He performed an investigation into third-semester introductory student understanding of these graphs in classical contexts. His primary motivation, like others who have studied potential energy diagrams at more advanced levels [35, 36], was to understand the background of students prior to their studies of quantum mechanics. His main findings with regard to classical potential energy diagrams were as follows:

- *Students struggled to understand that the total energy in a system could be negative.* These students sometimes stated that negative energy "does not make sense," at least in classical contexts. Bao interpreted these responses as indicating a belief that energy is a 'physical' quantity, such as mass. This belief is consistent with treating energy as a substance-like quantity. We recognize that this belief is not necessarily tied to graphical representations.

[1] See also Bao and Redish [34].

- *Students tended to associate a one-dimensional potential well with a two-dimensional gravitational well.* In other words, they ascribed a gravitational context to an otherwise context-free potential energy diagram. Bao's evidence included student statements such as "bumping back and forth in the well" and objects being more likely to be found at the "bottom" of potential wells. Although this tendency can be helpful in some situations, it could indicate a belief that potential energy diagrams can *only* describe gravitational systems.

Since Bao's work, research into student understanding of potential energy diagrams has been rather limited. Only recently has more work been published. A previous publication by the present author describes a number of different student difficulties [37]. (We do not discuss these findings here since the current dissertation expands and refines these initial findings.) Dreyfus et al. [38] has described student difficulties with negative potential energies, similar to that described by others [27, 37]. His motivation was to understanding the extent to which students understand chemical bonds and reactions.

Examples of existing curricula on potential energy diagrams

Some educational materials use graphical representations of energy as a way to further student ability to reason about energy. For example, van Heuvelen [39] describes an instructional strategy using energy bar charts.[2] However, we focus here only on those materials that directly use potential energy diagrams.

A number of the early attempts in recent history to introduce students to potential energy diagrams rely on laboratory-based experiments [40–43]. However, none of these appeared to be based on research of student difficulties. The experience of the Physics Education Group has been that research-based curricula can be effective at addressing specific student difficulties.

To our knowledge, research-based development of curricula focusing on potential energy diagrams began with Bao [27]. The worksheet-based tutorial he developed (which itself

[2] See also Ref. [32].

was initially created by Jolly et al. [43]) was based in part on his research into the difficulties students encountered with potential energy diagrams. This curriculum also used a laboratory-based approach to introduce potential energy diagrams. The overarching goal of that tutorial was to allow students to understand *probabilistic interpretations* of classical systems.

A curriculum designed by Dreyfus et al. [38] uses potential energy diagrams in an interdisciplinary setting. The extent to which this material was designed to address difficulties specifically with potential energy diagrams is limited. Rather, its purpose was as a tool with which to connect multiple ideas in physics and chemistry and to improve understanding of chemical reactions.

Commentary

The extent to which the current literature explores the difficulties that students encounter with potential energy diagrams is limited. The research that does exist tends to view the diagrams as a secondary tool with which to understand other difficulties (e.g., probability and chemical reactions). We are not aware of any research in which student understanding of potential energy diagrams is the primary focus. A similar situation exists with *curriculum* related to potential energy diagrams. Typically, these diagrams take on a "secondary" role.

We recognize that potential energy diagrams are themselves a tool with which to characterize and understand analyze systems. However, as will be shown in Part II of this dissertation, there are a large number of difficulties that students encounter when reasoning about potential energy diagrams. (The situation is not unlike the use of a graphical electromagnetic plane wave representation to illustrate the propagation of light.) Many of these difficulties we identified appear not to be restricted to any particular system or context. Instead, they are inherent in the graphical representation itself. Other difficulties appear to be based on issues related to energy and not on graphical representations. This dissertation contributes to the literature (1) a detailed understanding of the way in which graphs of potential energy *vs.* position are and are not utilized correctly by students, and (2) a description of a research-based and research-validated curriculum for introductory mechanics students that addresses many of the difficulties they encounter. Some of the results have implications for other graphs of physical quantities, especially for graphs in which the horizontal axis represents a temporally-varying quantity other than time.

3.2 Overview of prior research related to spatial probability density

As the name implies, *spatial probability density* combines the concepts of both probability and density. We begin this section by describing some relevant research into student understanding of density. We then discuss relevant research into student understanding of probability and probability density.

Relevant research on spatial density

There is a large body of research spanning several decades that explore student difficulties with the concept of *density* [44–46]. Most of these investigations have occurred at the K–12 level and almost exclusively focus on mass density. In all of these, a recurring finding is that students tend to confuse the concepts of mass and mass density (i.e., they treat mass density as if it were mass and vice versa).

Of particular importance to our research are the findings of Kanim [47], who studied university undergraduate understanding of charge density and mass density. One of his findings was that students performed more poorly on questions about charge density than they did on analogous questions about mass density. This result can be interpreted as a failure to generalize knowledge about density in one context (mass) to another context (charge). Moreover, other researchers have found that the 'transfer' of knowledge between topics in science and math is not always straightforward for students [20, 48]. Another relevant finding of Kanim was that many students failed to distinguish between charge and charge density. This is consistent with the prior research in the context of mass described above.

Relevant research on probability and probability density

Findings from statistics education research

A great deal of research exists on student understanding of probability and statistics [49–51]. Much of this work has been conducted by statistics education researchers. The range of this research is quite broad, and includes identifying difficulties students encounter with basic probability. To provide one specific example, some researchers describe the belief by individuals of the "gambler's fallacy" [52]. This incorrect notion posits that events that have occurred in the recent past are less likely to occur in the future, even though it is known that the results are *independent* from one another (e.g., the flipping of a coin). Konold [53] notes, though, that students may answer two problems that are equivalent in contradictory ways. This finding is consistent with the results of physics education research that show student do not necessarily hold a coherent conceptual model [54]. We note that other areas of statistics education research include probing student ideas about sampling distribution and statistical inference [55, 56].

Findings from physics education research at advanced levels

Of greater relevance to our research, however, are the difficulties students encounter when reasoning about probability and probability density in the context of physical systems. Such investigations are primarily seated within physics education research. With the exception of a few studies in statistical mechanics (e.g., Ref. [57]), most of these investigations have been conducted within advanced quantum mechanics courses [35, 36, 58, 59]. The findings that are relevant to our research include the following:

- *The failure of students to distinguish between closely related concepts.* This includes distinguishing between probability and probability density, as well as between probability and expectation value [36, 60].

- *The failure of students to distinguish between classical and quantum regimes.* Some students applied classical reasoning in quantum contexts and vice versa [35, 36].

- *The belief that objects 'prefer' to be in regions with lower potential energy.* Some students claimed it was 'easier,' and thus more likely, to find objects in regions of lower potential energy. They failed to associate the speed of an object in a region to the amount of time spent in the region [27, 35].

We note that many of these researchers who focus on quantum mechanics also extend their investigation to classical systems. (Many researchers believe it is beneficial for students to have an understanding of basic probabilistic concepts in classical contexts prior to studying quantum mechanics [34, 61]. Success in this regard in advanced courses has been documented by Crouse [36].)

Findings from physics education research at the introductory level

Research into probability and probability density at the introductory level is limited. The most extensive research we are aware of is that by Bao [27], whose work on student understanding within classical physics focused mainly on probability rather than probability density.[3] Some of his findings in an introductory (third semester) course mirror those in the advanced courses (discussed above). For example, some students failed to distinguish between the predictions of classical and quantum mechanics. Other students stated that objects tend to *prefer* lower potential energies. We note that many physics education researchers have found that in many cases both introductory and advanced students encounter some of the same difficulties [62–64].

We are unaware of any research that focuses on student understanding of probability *density* in classical mechanics at the introductory calculus-based level.

Overview of curricula on classical probability density

Here we provide an overview of curricula on classical probability density by briefly characterizing three existing ones. Each is intended for a different population of students.

One of the curricula was developed by Wittmann et al. [65] and is intended for use in a non-science-major course. Many of these students enroll in the course to fulfill graduation requirements. The curriculum relies heavily on laboratory experiments or demonstrations. Students begin with an introduction to discreet probability by flipping coins. They then

[3] See also Bao and Redish [34].

transition over to thinking about (continuous) probability density through the use of histograms.

Another curriculum was initially developed by Jolly et al. [43] and then modified by Bao and Redish [34]. The modifications were based on his research into student difficulties with probability[4] and potential energy diagrams. This curriculum is intended for third-semester calculus-based students (analogous to same population that we discuss in Part III of this dissertation). However, it also relies heavily on laboratory experiments. Students use, for example, force probes to construct force *vs.* position graphs. The curriculum leverages computer-based measurements to construct probability density graphs for elastic systems and infinite square wells. An overarching goal of the curriculum was to guide students toward an understanding of probability and probability density in quantum mechanics.

The last curriculum we discuss was developed by Crouse [36]. It was initially based on the previous work of Bao and Redish, though Crouse heavily modified it on the basis of his own research. Unlike the previous two, it is not experimental-based and is intended for use with junior-level quantum mechanics students. Correspondingly, it tends to be somewhat mathematical and conceptually more difficult. For example, students distinguish between a "time-averaged" probability distribution (for which the probability density at a point is inversely proportional to the speed of the object) and an "instantaneous" probability distribution (for which the probability distribution can be modeled by a Dirac delta function). More recently, this curriculum has undergone revisions based on the work of Emigh [66].

Commentary

Much research exists on student understanding of classical probability from statistics education researchers and physics education researchers. Relatively little research, though, exists on student understanding of classical probability density. Furthermore, existing curricula on probability and probability density are either experimental-based or are intended for ad-

[4] Although the curriculum covers classical probability density, the associated classical research appears to be based only on *probability.*

vanced students. These features make them somewhat inappropriate for use in a quiz-section setting for introductory students, which is the primary context for our investigation.

This dissertation contributes to the literature (1) a detailed study of introductory student understanding of classical probability and probability density, with an emphasis the extent to which students conceptualize probability density as being distinct from probability, and (2) a research-based and research-validated worksheet-based curriculum intended to address many of the difficulties students encounter.

Part II

IDENTIFYING AND ADDRESSING STUDENT DIFFICULTIES WITH POTENTIAL ENERGY DIAGRAMS

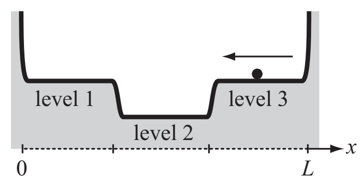
In this part, we discuss our research into identifying and addressing student difficulties related to potential energy diagrams.

Motivation for research

We initially became interested in potential energy diagrams through our research into student understanding of classical probability density, which is the topic of Part III. As part of that research we developed a new tutorial called *Probability in classical and quantum mechanics* (see Chapter 9) that was intended for students in PHYS 123, the third-quarter introductory sequence at the University of Washington. An excerpt from the tutorial is shown below:

II. Qualitative reasoning about classical probability

A ball moves back and forth forever on a frictionless track with very steep sides. The track has three horizontal levels that are of equal length. Levels 1 and 3 are of equal height. Assume that the ball is able to make it up and down the ramps and never leaves the surface of the track.

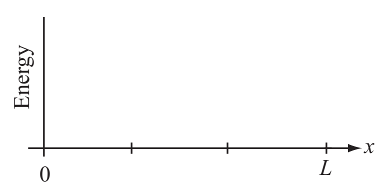


A. Suppose the position of the ball were measured by taking a photograph of the system at a random time. *Predict* the ranking, from greatest to least, of the likelihoods of the photograph showing the ball on levels 1, 2, and 3. Explain your reasoning.

B. Consider the system consisting of the ball, track, and Earth.

1. On the axes below, sketch a qualitatively correct graph of potential energy vs. position of the ball. Draw a dashed line to represent the *total* energy of the system.

   How are the following reflected in your graph?
   (i) The total energy of the system is constant.
   (ii) The ball makes it up the ramps from level 2.



Students consider a ball-track-Earth system. The track is composed of three horizontal levels, two of which are at the same height. The middle level is the lowest of the three. Students are asked to draw a potential energy diagram for the ball-track-Earth system. They are also asked to draw a line representing total energy and to explain how it is shown in their potential energy diagram that the ball is able to make it into the upper levels.

Although previous research has revealed that some students struggle in interpreting potential energy diagrams,[5] we were surprised at the prevalence and variety of the difficulties we observed during the tutorial sessions. One of the difficulties occurred in the exercises labeled part B.1 of section II (see the excerpt). While attempting to sketch a graph of potential energy *vs.* position for the system, some students were unsure where they should place the zero of potential energy on their diagram. Many wanted the minimum value of potential energy to be zero, but were unsure whether this was correct given that the bottom of the lowest level isn't at the lowest vertical position on the illustration (i.e., the lowest level itself has 'some height'). Other students became aware of this 'contradiction' only after comparing their diagram to those of their partners'. Many could not resolve the issue without the help of the TAs in the room. Our observations indicated to us that the arbitrary nature of potential energy is not apparent to some students.

Another difficulty we observed was students not incorporating the effects of the steep sides into their diagrams. Nearly all students drew their potential energy curves as three horizontal lines without vertical portions at $x = 0$ and $x = L$. This feature of the potential energy diagram is important since its presence ensures the existence of turn-around points at specific locations. This observation suggested to us that students may need specific guidance in understanding the role of turn-around points and how they are represented on a potential energy diagram.

Students were also asked in B.1 to draw on their diagram a line representing total energy. The placement of the horizontal line proved to be difficult. Some students indicated that the value of total energy was equal to the potential energy on levels 1 and 3, since, to paraphrase, "on those levels all of the energy is potential." Other students were unsure where to draw the total energy line. A related question asked students what feature of their diagram shows that the ball is able to make it up from the lower level to the upper level. Even those students who correctly drew the total energy line above the potential energy in the upper levels struggled to answer this question. The presence of these difficulties suggested to us

[5] See §3.1 for a discussion of prior research.

that the necessary relation $E > U$ between total energy E and potential energy U required for classical motion may be unfamiliar to many students.

Our observations indicated that despite two previous quarters of introductory physics, many Phys 123 students struggle with basic aspects of drawing and interpreting potential energy diagrams. Based on these observations, we decided to explore the extent to which *first-quarter* introductory mechanics students (Phys 121) understand potential energy diagrams. The mechanics course is a natural place to probe student ideas on this topic and possibly to develop targeted curriculum since it is the first place students are expected to bring together the concepts of kinematics, dynamics, and energy. We ultimately decided to create a new tutorial for students as part of the curriculum *Tutorials in Introductory Physics*. This new tutorial is called *Potential energy diagrams*.

Organization of Part II

We begin this part by describing our research into the difficulties students encountered when reasoning about potential energy diagrams (Chapter 4). Next, we describe how this research informed the development of the tutorial topic *Potential energy diagrams* (Chapter 5). We then provide a comparison of student performance before and after tutorial instruction (Chapter 6). Finally, we describe the motivation, development, and assessment of an additional component to the tutorial *Potential energy diagrams* called the *online interactive practice homework* (Chapter 7).

Chapter 4

IDENTIFYING STUDENT DIFFICULTIES WITH POTENTIAL ENERGY DIAGRAMS AFTER LECTURE INSTRUCTION

This chapter documents research into student difficulties with potential energy diagrams in classical contexts. Unless otherwise noted, all data were collected from questions administered to Phys 121 students. (See §2.1 for a description of this course.) Since the questions we asked were administered prior to any tutorial instruction, they are considered pretests, even if they were administered on students' exams. (We described the role of pretests within the larger context of tutorial instruction in §2.2.1.)

We begin this chapter by describing the difficulties that students encountered when determining kinematic and dynamic quantities (§4.1). We then discuss the extent to which students are able to use the equation $E_{\text{tot}} = K + U$ at specific points on a potential energy diagram to argue about kinetic energy, total energy, and turn-around points (§4.2). Next, we describe the difficulties associated with drawing or choosing potential energy diagrams based on descriptions of systems or motions (§4.3). Finally, we describe the failure of many students to account for the arbitrary nature of potential and total energies (§4.4).

We remind the reader of the abbreviated scheme to designate all course sections. See page 23 for a description.

4.1 Student ability to relate particle kinematics & dynamics to potential energy diagrams

In this section we discuss how students use potential energy diagrams in attempting to determine kinematic or dynamic properties associated with a particle. Our primary focus is on incorrect reasoning used by students.

We begin by describing the questions we have administered to students and providing a brief overview of student performance. We then describe the difficulties that we have found students encounter. When appropriate, examples of these difficulties from each of the question types are provided. Finally, we briefly discuss findings from variations of questions that give us additional insight into student understanding.

4.1.1 Description and development of questions

We administered four types of questions to probe the ability of students to use potential energy diagrams to determine each of the following:

- the direction of the acceleration of the particle (Figure 4.1(a)).
- the direction of the force on the particle (Figure 4.1(b)).
- how the speed of the particle is changing (Figure 4.1(d)).
- the direction of motion of the particle (Figure 4.1(c)).

In each question, students were informed either directly or indirectly that the total mechanical energy $E_{\text{tot}} = K + U$ is constant. They were also told that the one-dimensional translational kinetic and potential energy are the only forms of energy present in the system under consideration.

The reasoning needed to answer each of the questions is discussed below. The questions in Figure 4.1 are examples of the ones used in this study. Other questions are discussed as needed in the various subsections.

- To answer the **acceleration** question correctly (Figure 4.1(a)), students can note first that at the point of interest, $x = 2\,\text{cm}$, the slope is negative ($dU/dx < 0$). The relation between potential energy and force in one dimension, $F = -dU/dx$, and between force and acceleration, $F = ma$, imply that the sign of the acceleration is opposite that of the slope of the potential energy graph: $\operatorname{sgn}(a) = -\operatorname{sgn}(dU/dx)$. Thus, at $x = 2\,\text{cm}$, the acceleration is in the positive direction. Another method of determining the direction of acceleration involves finding the direction of acceleration in each possible direction of motion:

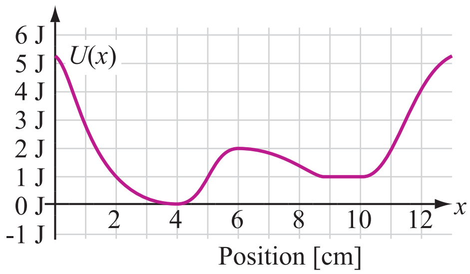
– If the particle were moving in the positive x-direction near $x = 2\,\text{cm}$, the graph of $U(x)$ in Figure 4.1(a) shows the potential energy would be decreasing. Since the total energy $K+U$ is constant, the kinetic energy K would therefore be increasing, implying that the particle speeds up. From kinematics, a particle moving in one dimension that is speeding up has its acceleration and velocity pointing in the same direction. Thus, the acceleration would be in the positive x-direction.

– If instead one assumes that the particle were moving in the *negative* x-direction, the increase in potential energy would correspond to a decreasing speed. A decreasing speed implies acceleration and velocity are in opposite directions; thus, the acceleration would again be in the positive x-direction.

With this latter method, one arrives at the same answer for the direction of acceleration regardless of which direction of motion is assumed.

- To answer the **force** question correctly (Figure 4.1(b)), students can use the force-potential energy relationship $F = -dU/dx$ to conclude that the sign of the force on the particle is opposite that of the slope of the potential energy graph. Thus, at $x = 2\,\text{cm}$, the force is in the positive direction. Alternatively, students can first determine the direction of acceleration (as described above) and use Newton's second law.

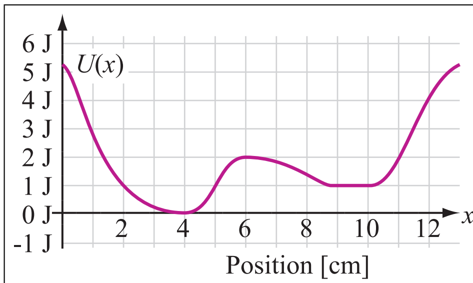
- To answer the **direction of motion** question correctly (Figure 4.1(c)), students can realize that graphs of $U(x)$ contain no inherent information about velocity. Acceleration



What is the direction of the **acceleration** of the particle at 2 cm?

[Possible answers: *positive x-direction, negative x-direction, not moving,* or *not enough information.*]

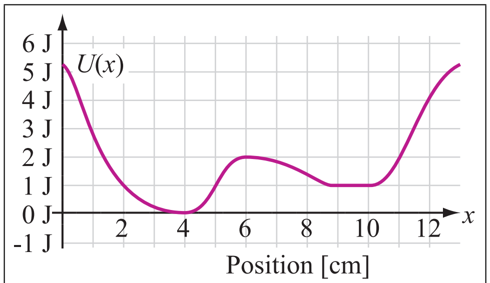
(a) Example of acceleration question.



What is the direction of the **force** on the particle at 2 cm?

[Possible answers: *positive x-direction, negative x-direction, not moving,* or *not enough information.*]

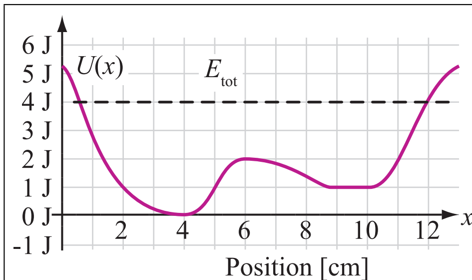
(b) Example of force question.



What is the **direction of motion** of the particle at 5 cm?

[Possible answers: *positive x-direction, negative x-direction, not moving,* or *not enough information.*]

(c) Example of direction of motion question.



How is the **speed** of the particle changing at 11 cm?

[Possible answers: *speeding up, slowing down, not changing,* or *not enough information.*]

(d) Example of speed question.

Figure 4.1: Abbreviated examples of questions used to probe student ability to relate potential energy diagrams to motion. Each of the correct answers is underlined.

and changes in potential energy can be determined, but not the direction of motion. Thus, the correct answer is *not enough information.*

- To answer the **speed** question correctly (Figure 4.1(d)), students can note that the speed of the particle will change in different ways depending on what direction the particle is moving.[1] Thus, there is *not enough information* to answer.

The answer choices for the direction of motion questions and the speed questions sometimes varied. For example, the correct answer for the speed question was sometimes phrased *either speeding up or slowing down* instead of *not enough information.* There was no noticeable effect on student performance, so all results are combined.

We now briefly discuss how we came to develop each of these four question types. We were originally interested in the extent to which students are able to connect the newer (more recently introduced) concept of potential energy to more familiar kinematics and dynamics ideas. The questions about the direction of force and the direction of acceleration were thus the first two to be administered to students in the present investigation. In analyzing student responses to the direction of acceleration question, we noticed a tendency of some students to assume that the particle was moving in the positive x-direction, which can lead student to arriving at a correct answer.[2] The other two types of questions—rate-of-change of speed and direction of motion—were developed specifically with this tendency in mind: students who believe that the particle *only* moves in the positive x-direction would answer these two additional questions incorrectly.

4.1.2 Brief overview of performance

Here we provide a summary of student performance across the four of the types of questions in Figure 4.1. We discuss incorrect answers only briefly since the major categories of incorrect reasoning are elaborated upon in §4.1.3. Much of the discussion in this section is

[1] This statement is true for points at which $dU/dx \neq 0$.

[2] This tendency is discussed at length in §4.1.3-a.

Table 4.1: Student performance on the ‘acceleration’ question. The sign of dU/dx at the point of interest was either positive or negative (i.e., the correct answer was not zero). The highlighted rows indicate correct reasoning.

Answer	N=581[a]	Reasoning	N=398[b]
Correct	65%	Left-to-right	25%
		F or $-dU/dx$	15%
		Minimize U	10%
		Concavity	5%
Neg. of correct	20%	Slope	10%
Zero	5%	Concavity	5%
Not enough info.	10%	No dir. info.	≤5%

[a] Sections 121B112, 121B142, 121A143, and 121D144.
[b] Sections 121B142, 121A143, and 121D144.

summarized in the tables presented in §4.1.2-a–4.1.2-d. We note that for all questions, we have administered versions in which the slope at the point of interest is both positive and negative. Performance on both versions was similar unless otherwise noted.

4.1.2-a Brief summary: Direction of acceleration questions

A summary of student performance on the acceleration questions is shown in Table 4.1. About two-thirds of the students answer correctly. The most common incorrect answer is the negative of the correct answer (i.e., if the correct answer is *positive x-direction*, the most common incorrect answer is *negative x-direction*).

Also categorized in the table are the most popular types of reasoning for each answer choice.[3] Most students who answer correctly use reasoning that we consider correct. Correct and incorrect lines of reasoning are elaborated upon in §4.1.3.

[3] The N values for the answer and reasoning columns are different because some administrations were multiple-choice and did not ask for an explanation.

Table 4.2: Student performance on the 'force' question. The sign of dU/dx at the point of interest was either positive or negative (i.e., the correct answer was not zero). The highlighted rows indicate correct reasoning.

Answer	N=1131[a]	Reasoning	N=1131[a]
Correct	55%	Left-to-right	20%
		F or $-dU/dx$	15%
		Minimize U	5%
		Slope	5%
Neg. of correct	25%	Slope	20%
Zero	5%	—	—
Not enough info.	10%	No dir. info.	≤5%

[a] Sections 121A132, 121A133, 121A134, 121C134, 121D134, 121A141, 121B141, and 121A142.

4.1.2-b Brief summary: Direction of force questions

A summary of student performance on the direction of force questions is shown in Table 4.2. Just over half of the students answer correctly. Most of these students use reasoning that we consider correct. The most common incorrect answer, provided by one-quarter of the students, is the negative of the correct answer (i.e., if the correct answer is *the positive x-direction*, the most common incorrect answer is *the negative x-direction*).

It is notable that student responses on direction of force questions were very similar to those on direction of acceleration questions. This is perhaps not surprising given the fundamental connection between acceleration and force.

It is worth noting that we have observed large variability on the percentage of students using "F or $-dU/dx$" reasoning. In all but one administration, the percentage was between 5% and 15%; in spring 2014, though, 40% of the students used this reasoning. This large variability may be due to the different amounts of emphasis that lecturers place on the relationship between force and potential energy. Table 4.2 combines all data.

4.1.2-c Brief summary: Direction of motion questions

A summary of student performance on direction of motion questions is shown in Table 4.3. We have asked questions for points at which the slope is positive, negative, and zero. In all three cases, 25–30% of the students answered correctly that the direction cannot be determined, and only 5–10% of the students used correct reasoning. The most common incorrect answer when $dU/dx \neq 0$ is *the positive x-direction* (Tables 4.3(a) & (b)). When $dU/dx = 0$, the most common incorrect answer is *the particle is not moving* (Table 4.3(c)).

In comparing the relative proportions of answers for questions in Tables 4.3(a) & (b), the relative proportion of answers provided by students was largely insensitive as to whether the slope of the potential energy diagram (dU/dx) was positive or negative. However, there may be a small correlation on the sign of dU/dx as to the type of reasoning students are more likely to use. When $dU/dx > 0$, about 10% used some type of "left-to-right" reasoning to argue that the particle moves in the positive x-direction, whereas 25% did so when $dU/dx < 0$. Furthermore, in comparing Tables 4.3(b) & (c), students may be more likely to identify the direction of motion with the sign of dU/dx when $dU/dx = 0$ as compared to when $dU/dx < 0$.

Some questions we have administered provided students with a horizontal line on the graph representing the total energy. Others did not. When $dU/dx \neq 0$ at the point of interest, we have seen no difference in how students perform. However, we have only asked the direction of motion question with $dU/dx = 0$ without a total energy line. It is possible student performance would be different if these students were provided with such a line. Further research is needed in order to better understand how the total energy line does or does not affect student performance when $dU/dx = 0$.

4.1.2-d Brief summary: Speed questions

A summary of student performance on the speed questions is shown in Table 4.4. Only about 10% of the students answered correctly that the rate of change of speed cannot be determined. The most common incorrect answer, provided by 70% of the students, is to

Table 4.3: Student performance on the 'direction of motion' question. The correct reasonings are highlighted.

(a) $dU/dx > 0$ at the point of interest.

Answer	N=555[a]	Reasoning	N=372
Not enough info (corr.)	30%	Correct	10%
		Left-to-right	10%
Pos. x-dir	50%	Slope	20%
		Left-to-right	10%
Neg. x-dir	15%	Slope	5%
		Minimize U	5%
$v = 0$/Neither	5%	—	—

[a] Sections 121B112, 121A132, 121A113, and 121D134.

(b) $dU/dx < 0$ at the point of interest.

Answer	N=362[a]	Reasoning	N=198
Not enough info (corr.)	30%	Correct	5%
Pos. x-dir	55%	Slope	5%
		Left-to-right	25%
Neg. x-dir	15%	Slope	15%
		Minimize U	—
$v = 0$/Neither	≤5%	—	—

[a] Sections 121A112, 121A143, and 121D144.

(c) $dU/dx = 0$ and $d^2U/dx^2 < 0$ at the point of interest.

Answer	N=180	Reasoning	N=180
Not enough info (corr.)	25%	Correct	10%
Pos. x-dir	25%	Slope	5%
		Left-to-right	5%
Neg. x-dir	10%	—	—
$v = 0$/Neither	45%	Slope	30%
		F or $-dU/dx$	5%

[a] Section 121A152.

Table 4.4: Student performance on the ‘speed’ question. The sign of dU/dx at the point of interest was either positive or negative (i.e., the correct answer was not zero). The highlighted row indicates the correct answer.

Answer	N=934[a]
Not enough information	10%
Consistent with left-to-right reasoning*	70%
Opposite of above	15%
Neither/$v = 0$	5%

[a] Sections 121B132, 121A134, 121C134, 121A141, 121B141, and 121D144.
* For example, students state an object slows down if $dU/dx > 0$ at the point of interest.

state that the particle slows down if $dU/dx > 0$ at the point of interest (or that it speeds up if $dU/dx < 0$).

4.1.3 Identification of student difficulties

Here, we provide detail on the lines of reasoning that students use to answer the four types of kinematic and dynamic questions.[4] In some cases, the lines of reasoning that we have grouped together manifest themselves in different ways. This is especially true for the “left-to-right” reasoning. For this reason, we often provide multiple student quotes that exemplify a given line of reasoning.

4.1.3-a Tendency to read graphs from left to right

Here we discuss student tendencies to interpret potential energy diagrams as being read from left to right. This particular tendency takes multiple forms. We first provide a brief overview.

[4]These questions are shown in Figure 4.1 on page 44.

Brief overview

Depending on which of the four types of questions were used, between one-quarter and three-fifths of Phys 121 students give reasoning that is designated here as 'left-to-right' reasoning. We have found that this reasoning is composed of two closely related tendencies:

1. Some students assume that the motion of the particle is in the positive x-direction. This is denoted here as a "positive-x-assumption." For example, when reasoning about how a particle's kinetic energy is changing at a point where $dU/dx > 0$, some students will state that the particle moves in the positive direction and conclude that the increasing potential energy causes a decreasing kinetic energy. They fail to recognize that the particle's kinetic energy could be increasing if it were moving in the negative x-direction.
2. Other students treat graphs of $U(x)$ as if they were graphs of $U(t)$. This is denoted here as a "time-horizontal-axis" reasoning. This is similar to the positive-x-assumption tendency above in that students would claim that the kinetic energy decreases at a point where the slope of the potential energy diagram is positive. However, when students with time-horizontal-axis reasoning are asked about the direction of motion of the particle, they claim correctly that the direction cannot be determined. Claiming that the kinetic energy is decreasing but that the direction of the motion is unknown is consistent with treating potential energy diagrams as if they plotted time on the horizontal axis. This is in contrast to students with positive-x-assumption reasoning who claim that the particle moves in the positive x-direction.

As described in detail later, it is often difficult to distinguish between these two related lines of reasonings based only on a response to a single question, even on questions about the directions of acceleration or force. For this reason, most of the discussion within this subsection will use the generic phrase *left-to-right reasoning* to describe the two related tendencies. An effort to distinguish between these two lines of reasoning is described on page 57.

Table 4.5: Percentage of students whose explanations indicate that they were using 'left-to-right' reasoning. Ranges indicate typical averages on each administration, which usually includes 100–200 students.

Question type	Left-to-right reasoning	N^{a}
Direction of acceleration	25%	398
Direction of force	20%	1131
Direction of motion	20–35%*	750
Rate of change of speed	60%	788

[a] See Tables 4.1–4.4. * The percentage varies depending on whether $dU/dx = 0$ or $dU/dx \neq 0$ at the point of interest.

Table 4.5 shows the percentage of students using left-to-right reasoning on each of the four question types. Acceleration and force questions each elicit percentages of around 25%. About 60% of the students use this reasoning on the speed questions. On the direction of motion questions, up to 55% of the students answer that the particle moves in the positive x-direction.

Below, examples of left-to-right reasoning for each of the question types are provided. The acceleration and force questions produce nearly identical reasonings, so the discussion of these two types of questions are combined. The direction of motion and speed rate-of-change questions are each discussed separately.

Examples from acceleration and force questions

Figures 4.1(a) and 4.1(b) on page 44 can be used as the context for the following student quotes. Note that $dU/dx < 0$ at the point of interest.

Students who use left-to-right reasoning to determine the direction of the acceleration or the force argue that the slope of the graph of potential energy can inform how the speed is changing, and then connect the change in speed to the direction of acceleration or force. The following quote is typical of these students

> *"[The acceleration is in the positive direction.] Potential energy is decreasing so translational kinetic energy should be increasing."*
> (Phys 121B, spring 2014)

These statements are consistent with assuming the particle moves in the positive x-direction. Students with this reasoning appear to have not considered whether the direction of the acceleration or force would be the same if the particle were moving from right to left on the graph. However, the method correctly gives the direction of acceleration and force. Thus, while not complete, we interpret these explanations as correct.

Parts of these statements are also consistent with interpreting the horizontal axis as time. It is important to note, though, that if a student were treating the horizontal axis as time in a *consistent* manner, then it should not be possible to determine the direction of acceleration; a particle that is speeding up implies only that acceleration (or force) and velocity are in the same direction. Thus, the fact that these students *do* decide on a particular direction of the acceleration might also indicate an underlying kinematic difficulty.

To probe this possibility, an additional pretest question was administered in autumn 2014 and winter 2015. The question asked students to describe the connection between the direction of acceleration and the rate of change of speed.[5] Between 25% and 40% of the students answered incorrectly that an acceleration in the positive direction corresponds to an object that is speeding up. Thus, the difficulties that students encounter in answering kinematic questions based on potential energy diagrams may result, in part, from difficulties on basic kinematic concepts. Furthermore, these results explain how even those students who are treating the horizontal axis as time might state that the direction of acceleration can be determined.

It should be mentioned that we categorized answers and explanations as left-to-right reasoning only if students discussed how the *kinetic energy* or *speed* changes if the graph is read from left to right. Students who answer correctly but describe only how the potential energy changes (e.g., "the potential energy is increasing so acceleration is in the negative

[5] These questions can be seen in the appendix in Figures A.19 and A.25 on pages 373 and 390, respectively.

direction") are not included in this reasoning category.

Examples from direction of motion questions

We now discuss examples of student explanations who argue that the particle moves in the positive x-direction (i.e., from left to right on the graph). The graph in Figure 4.1(c) on page 44 can be used as context for the following student quotes. Note that $dU/dx > 0$ at the point of interest.

As indicated in Table 4.3 on page 49, about 50% of the students answer incorrectly that the direction of motion is positive. There are several methods students use to justify this answer, but only some of them can be categorized as possibly indicating an *intrinsic* belief that the particle moves in the positive x-direction.[6] One such method was used by students who discussed how the speed or kinetic energy changes, even though they were not asked to do so. For example, at a point where $dU/dx > 0$:

> *"The particle is moving in the positive x direction but slowing down because it is losing kinetic energy and gaining potential energy."*
> (Phys 121A, spring 2013)

This student and others with similar explanations appear to assume that the particle moves in the positive x-direction (or that they interpret the horizontal axis as if it were time), and then use that assumption in explaining how the kinetic energy changes.

Another type of reasoning that could indicate an intrinsic belief in a positive direction of motion is describing that the graph or the horizontal axis itself indicates the direction of motion:

[6] Equating the sign of the slope of the graph to the direction of motion is an example of reasoning method that does not indicate such an intrinsic belief.

> *"Since the graph is increasing in x-position, it must be moving in the positive x direction."*
>
> (Phys 121A, spring 2013)

> *"Position is increaing [sic] so particle is moving in positive x direction."*
>
> (Phys 121D, autumn 2014)

> *"That is how [I] define my coordinate system, where increasing x is in the positive x direction."*
>
> (Phys 121D, autumn 2013)

One interpretation of these explanations is that the students (1) are reading the graph from left to right, (2) are noticing that the x-coordinates on the horizontal axis increases as they do this, and (3) are then ascribing this increase in x-coordinate of the axis to an increase in the x-coordinate of the particle.

We note that even students who answer correctly that the particle could be moving in either direction sometimes provide explanations that can be categorized as left-to-right reasoning. We elaborate upon this point on page 57.

Examples from speed questions

We now describe how students answer speed questions using left-to-right reasoning. The graph in Figure 4.1(d) can be used as context for the following student quotes. Note that $dU/dx > 0$ at the point of interest.

Many student explanations are similar those on direction of acceleration and force questions:

> *"Since the potential energy is increasing, the kinetic energy is decreasing which means it is slowing down. "*
>
> (Phys 121B, spring 2013)

This explanations is representative of about 50% of all Phys 121 students. They articulate that an increase (or decrease) of potential energy implies a decrease (or increase) of kinetic energy.[7]

An additional 10% of the students cited only how the kinetic energy is changing, but do not mention how it was determined. It is likely that these students had similar reasoning to the 50% mentioned above since they offer information about kinetic *energy* even though the question only asks about speed.

These explanations suggest either that students believe the particle only moves in the positive x-direction, or that they are interpreting the graph of $U(x)$ as $U(t)$ without regard to direction of motion. It is difficult to categorize a given explanation as belonging to one or the other since the same explanation can be consistent with both.

The total of 60% cited here is noticeably higher than the corresponding 25% in the context of acceleration and force. This difference can be explained by noting that for acceleration and force questions, an assumption about the direction of motion does not need to be made in order to arrive at an answer. That is, students may believe motion is in the positive direction, but they do not necessarily need to use this assumption to answer correctly about the direction of the acceleration or force.

[7] This percentage does not include the roughly 5% of the students who cited only the slope of $U(x)$ in their explanation. While it is possible these students had similar reasoning to the 50% of students mentioned above, no such claim is made.

Distinguishing between the two 'left-to-right' lines of reasoning

As previously discussed on page 51, it is difficult to distinguish between students using a positive-x-assumption reasoning and those using a time-horizontal-axis reasoning. We were initially motivated to explore these two separate lines of reasoning by noting statements made by some of the students who gave correct answers on the direction of motion question. About 10% of the students answered correctly that the direction of motion cannot be determined, but in their explanation provided unnecessary and incorrect information about how the speed of the particle is changing.[8] The following student quotes illustrates our observation (note that $dU/dx > 0$ at the point of interest):

> *"[Not enough information to determine the direction of motion.] Its kinetic energy is decreasing but kinetic energy does not depend on direction."*
> (Phys 121D, autumn 2013)

This quote and others like it suggested to us that some students were treating the graph as if time were plotted on the horizontal axis rather than position. These observations led us to pair both a direction of motion question and a speed question on a *single* pretest in autumn 2014. (The questions are labeled Questions 7 and 9 in Figure A.22, which begins on page 385.) We hypothesized that students interpreting the diagrams as graphs of $U(t)$ (time-horizontal-axis reasoning) would answer (note that $dU/dx < 0$ at the point of interest):

- it is *not* possible to determine the direction of motion.
- it *is* possible to determine how the speed is changing (the speed is increasing).

On the other hand, students correctly interpreting the graphs as $U(x)$, but assuming the particle moves in the positive x-direction (positive-x-assumption reasoning) would answer:

- it *is* possible to determine the direction of motion (positive x-direction)
- it *is* possible to determine how the speed of the particle is changing (the speed is increasing)

[8] The question itself did not ask students to provide information about speed; these students did so voluntarily.

Table 4.6: Percentage of students that used one of the two 'left-to-right' reasonings.

Type of reasoning	N=144[a]
Positive-x-assumption	40%
Time-horizontal-axis	15%
Not left-to-right reasoning	45%

[a] Section 121D144.

It should also be mentioned that we specifically asked students to consider a point with negative slope ($dU/dx < 0$). Doing so excludes from analysis those students who equate the direction of motion of the particle to the sign of dU/dx (about 15% of all students).[9]

Table 4.6 shows the percentage of students falling into each category. About 40% seemed to assume that the particle moves in the positive x-direction, whereas 15% treated the diagram as if it were a graph of $U(t)$ instead of $U(x)$.

In addition to providing a sense of the various difficulties students encounter, these percentages can inform the development of curricula designed to help students. For example, it may be more natural to spend a greater part of a tutorial on addressing the belief that the particle only moves in the positive x-direction.

Finally, we stress that the percentages in Table 4.6 come from questions in which students are provided a potential energy diagram. We have also asked questions in which students must draw a potential energy diagram. These questions tended to elicit similar difficulties but in different proportions.[10]

We now move on from left-to-right reasoning and discuss other tendencies we have observed with students answering kinematic and dynamic questions.

[9] This category of reasoning is discussed in §4.1.3-c.

[10] For a discussion of these questions, see §4.3.

Table 4.7: Percentage of PHYS 121 students that use the second derivative (concavity) of the potential energy diagram to answer kinematic and dynamic questions.

Question type	Concavity reasoning	Approx. N[a]
Direction of acceleration	10%	398
Direction of force	$\sim 0\%$	1131
Direction of motion	$\sim 0\%$	750
Rate of change of speed	$\sim 0\%$	788

[a] See Tables 4.1–4.4.

4.1.3-b Tendency to equate the second derivative of a potential energy diagram to acceleration

About 10% of the students appeared to equate the sign of the second derivative of $U(x)$ with the sign of the acceleration, as shown in Table 4.7. For example, a student attempting to answer the acceleration question shown in Figure 4.1(a) on page 44 stated:

> *"The graph is concave up [at $x = 2$ cm], so its acceleration (2nd deriv.) is positive."*
>
> (PHYS 121B, SPRING 2014)

These students may be overgeneralizing their prior knowledge about position *vs.* time graphs (in which the second derivative *would* yield acceleration) to reason how acceleration and graphs of potential energy *vs.* position are related.

Essentially no students use concavity of potential energy diagrams in answering the other three types of questions (direction of force, direction of motion, and rate of change of speed). This is in sharp contrast to the presence of left-to-right reasoning on all question types, as was discussed in §4.1.3-a.

Although the underlying physics of this type of reasoning is incorrect, the student quote given above shows that it may result in a correct answer if dU/dx and d^2U/dx^2 have opposite signs. This has implications for designing accurate multiple-choice questions.

Table 4.8: Percentage of PHYS 121 students that incorrectly use the slope of $U(x)$ to answer kinematic and dynamic questions. which usually includes 100–200 students.

Question type	Slope reasoning	Approx. N[a]
Direction of acceleration	10%	398
Direction of force	20%	1131
Direction of motion	15–30%*	750
Rate of change of speed	10%	788

[a] See Tables 4.1–4.4. * Percentage may be sensitive to the sign of dU/dx at the point of interest.

4.1.3-c Tendency to equate the slope dU/dx incorrectly to kinematic and dynamic quantities

Here we describe ways in which students incorrectly equate the slope dU/dx of potential energy diagrams to kinematic and dynamic properties. We designate this as "slope" reasoning. Table 4.8 compares the use of this reasoning on different questions. It is referenced throughout this discussion.

Examples from direction of acceleration and direction of force questions

About 10% of PHYS 121 students equated the sign of the slope of $U(x)$ to the sign of the acceleration of the particle, as indicated in Table 4.8. About 20% of the students equated the sign of the slope to the sign of the force on the particle. This method of reasoning always leads to an incorrect answer (except if $dU/dx = 0$) since the signs of both the acceleration and force are proportional to $-dU/dx$.

Explanations for the acceleration and force questions were very similar. For this reason we only discuss acceleration questions here. Much of our commentary on acceleration questions can also be applied to force questions.

There are two similar but slightly different types of explanation that we categorized as "slope" reasoning. Some students used the term *derivative* or *slope* explicitly:

> *"The derivative at $x = 2.0$ cm is negative, so the acceleration is in the negative x direction."*
> (Phys 121B, spring 2014)

Other students referred only to an *increase* or *decrease* in potential energy and did not use slope or derivative:

> *"The potential energy is decreasing at the position 2 cm, so the direction should be negative."*
> (Phys 121B, spring 2014)

Superficially, a "negative derivative" and a "decreasing value" could be seen as equivalent. However, we distinguish between these explanations because the latter statement may indicate a tendency to think about a *temporal* change in potential energy (i.e., how $U(t_o)$ and $U(t_o + \delta t)$ compare) rather than only a statement about a spatial change (i.e., how $U(x_o)$ and $U(x_o + \delta x)$ compare). In other words, students with the latter type of reasoning may be more inclined to think of the horizontal axis as representing time. Nonetheless, we have categorized both of these explanations as "slope" reasoning due to their similarity.

The question arises as to *why* these students are equating the sign of the *slope* of the potential energy diagram (or, equivalently, the left-to-right increase or decrease) to the direction of acceleration. It is likely that there are multiple reasons students equated the slope with the acceleration. One possibility is that students are recalling from lecture that the derivative of $U(x)$ is relevant in determining acceleration, but are forgetting that the *negative* of the slope yields the correct sign, rather than the slope itself. If this were the case, one might expect similar reasoning for questions about the direction of the *force.* This is indeed the case for some students, as we describe below. Another possibility is that some students are overgeneralizing their knowledge of graphs of velocity *vs.* time, for which the slope *does* yield the acceleration. If this were the case, one might expect some students to overgeneralize their knowledge of graphs of *position vs.* time and equate the second derivative of potential energy diagrams with the acceleration. This is indeed the case for a small percentage of the students, as described in §4.1.3-b.

Examples from direction of motion questions

Between 15% and 30% of the students equated the sign of the slope of $U(x)$ to the direction of motion of the particle. For example:

> *"Since the slope of U(x) graph is positive at x=11.0 cm, the particle is moving in the positive x-direction."*
>
> (Phys 121A, spring 2013)

> *"The particle is gaining potential energy. [So the particle is moving in the positive x-direction.]"*
>
> (Phys 121A, spring 2013)

Most students in this category do not describe why the slope of the graph determines the direction of motion of the particle. One possibility is that students are overgeneralizing their knowledge of position *vs.* time graphs to equate velocity to the derivative. Indeed, some students do appear to use this reasoning explicitly:

> *" Slope at x=6 cm is positive. Since slope of position is velocity, then velocity is postive value. "*
>
> (Phys 123C, spring 2011)

Only a handful of the students were this explicit. Given that about 5% of all students used the second derivative d^2U/dx^2 to determine the acceleration (as described in §4.1.3-b, roughly the same number of students may have overgeneralized in this manner.

A related possibility for students equating the slope and the direction of motion is that some students are recalling that the slope dU/dx is relevant for interpreting potential energy diagrams (as in $F = -dU/dx$), but are misapplying their knowledge to equate dU/dx to the velocity. (In other words, these students may be forgetting both the negative sign in $-dU/dx$ as well as that dU/dx is related to force and acceleration, not velocity.) If this were the case, we might expect that some students *would* recall the relative negative sign and equate the *negative* of dU/dx to the direction of motion. (In other words, these students do not forget about the negative sign, but do forget that dU/dx is related to force and acceleration, not velocity.) We have indeed observed some students giving reasoning consistent with this interpretation:

> *"The potential energy is growing which means [the particle] must be moving in the negative x-direction."*
> (Phys 121D, autumn 2013)

However, the fraction of students who used this line of reasoning is quite low; at most 5% on a given administration. We note that these students might also be attempting to describe the tendency for a particle to *accelerate* toward lower potential energy.

When we have asked direction of motion questions with $dU/dx = 0$ and $d^2U/dx^2 < 0$ at the point of interest, about 5% of the students stated that either the potential energy is increasing or decreasing (most say decreasing). In effect, they appeared to be describing the slope of the potential energy diagram at a point just to the right of the point of interest. These students may be using a type of left-to-right reasoning and describing how the potential energy *will* change.

We now move away from describing the tendency to equate the sign of dU/dx with kinematic quantities and describe additional difficulties that students encounter.

4.1.3-d Tendency to map generic potential energy diagrams to specific contexts

Some responses indicate that a small percentage of students (on the order of 5% or less) may be interpreting potential energy diagrams as always relating to gravitational interactions. For example, on the direction of motion question, some students incorporate the expression mgy into their answer:

> *"[The particle is moving in the negative x-direction.] U(x) = mgh, so when it's going down the particle is going down and when it's going up the particle is going up. At x = 7, it is going down, so velocity is downwards."*
> (Phys 121A, summer 2014)

Other students describe the direction of motion in terms of "upward" or "downward" without appealing explicitly to a gravitational expression. For example, one student stated for a point that had positive slope:

> *"[Not enough information to determine the direction of motion.] The particle could be moving in either direction [but] we know that at that point, it is moving up."*
> (Phys 121D, autumn 2014)

Despite the correct answer, this student appears to map a positive slope to an "upward" motion.

This tendency to map potential energy diagrams to specific contexts, especially gravitational ones, has been documented by others. For example, Bao [27] interpreted similar responses by students as indicating a tendency to interpret a one-dimensional potential well as a two-dimensional gravitational well.

4.1.3-e Tendency to overgeneralize the idea of minimizing potential energy

One *correct* method that some students use to determine the direction of the acceleration (and force) on the particle is to use the idea that conservative systems "tend to minimize" their potential energy. Between 5% and 20% of the students used this type of "minimize-U" reasoning in answering direction of acceleration questions. Nearly all of these students arrived at a correct answer. The following student quote is typical (note that $dU/dx < 0$ at the point of interest):

> *"A particle always aims to lower its potential energy [so the acceleration at $x = 2$ cm is in the positive x-direction.]"*
> (Phys 121B, spring 2014)

Also included in this category are students who make analogies to ramps or roller coasters. These students also tend to arrive at the correct answer:

> *"Imagining that the curve is a roller coaster, the particle moving along will be accelerating in the positive x-direction."*
> (Phys 121B, spring 2014)

This latter type of explanations that make analogies to only physical objects are less common (a few percent) than those that state explicitly that potential energy tends to be minimized, but are included in this category since some students discuss both. We emphasize that we consider this *correct* reasoning for the acceleration question.

Some students, however, appeared to overgeneralize this idea of minimization and arrived at incorrect or misleading statements that suggest they tend to treat motion as always being toward lower potential energy. The following statement is from a student who answered correctly that the acceleration is in the positive x-direction:

> *"[At $x = 2$ cm the] particle will move to minimize potential energy which is at x=4.0cm. Therefore particle must move in the positive x-direction."*
>
> (PHYS 121B, SPRING 2014)

Furthermore, in answering questions about the direction of motion, up to 5% of the students explicitly used a minimize-U reasoning and arrived at an incorrect answer. The following student quote is illustrative (note that $dU/dx > 0$ at the point of interest):

> *"[The particle moves in the negative x-direction.] The particle is being drawn to the left because it is trying to reach the lowest possible potential energy."*
>
> (PHYS 121A, SPRING 2013)

We note that students in other studies have exhibited similar tendencies. For example, Bao [27] described that some students treated the particle as, to paraphrase, preferring the bottom of the well. He interpreted such responses, though, as a tendency of some students to interpret a one-dimensional well as a two-dimensional gravitational well (see §4.1.3-d). Ambrose [35] found that some junior-level students believed that probability density is higher in regions of lower potential energy. However, this was in the context of quantum mechanics and wave functions.

4.1.4 Additional findings: Variations of questions

In this subsection we present additional findings into our study of how students use potential energy diagrams to answer kinematics questions. The data here were gathered from questions that do not fit naturally into our subsections above.

4.1.4-a Direction of motion: Two-question version

Below we discuss how student performance on the direction of motion question differed when the question is asked in a slightly different format.

In most versions of the 'direction of motion' question, students are asked to use a potential energy diagram to determine, if possible, the direction of motion of the particle at a particular location. (For an example, see Figure 4.1(c) on page 44.) In answering, students provide a *single* answer (e.g., "not enough information"). In autumn 2014, we administered a slightly different version of the question by asking a *pair* of questions on the same pretest:

- Is it possible the particle at $x = x_o$ is moving in the positive x-direction?

- Is it possible the particle at $x = x_o$ is moving in the negative x-direction?

This pair of questions was designed to allow students a more explicit opportunity to consider whether a particle *could* be moving in the negative x-direction.

Pairs of answers to the two-question version can be mapped directly to a single answer on the single-question version. For example, students who answer *yes* to the first one and *no* to the second are mapped to the answer *the particle moves in the positive x-direction* on the single-question version.

A comparison of answers to the one-question and two-question versions are shown in Table 4.9. The percentage of students who answered incorrectly that the particle moves in the positive direction is lower by 15 percentage points on the two-question version. Correspondingly, the percentage of students answering correctly increased by about the same amount. These results suggest that allowing students a more explicit opportunity to consider whether the particle can move in the negative x-direction results in a higher level of performance. However, despite the higher performance for the two-question version, it is still noteworthy that only half of the students answered correctly and that up to a third of students may have used some type of 'left-to-right' reasoning.

4.1.4-b Rate of change of speed

Speed question that we have typically administered involves using a potential energy diagram to determine, if possible, how the speed of the particle changes at a particular location (see, for example, Figure 4.1(d) on page 44). The correct answer is "not enough information" because the direction of motion is unknown. In autumn 2014 we administered a different version of the question in which students are asked to select which graphs, if either, could

Table 4.9: Comparison of performance on two versions of the 'direction of motion' question. The correct answer is highlighted.

	Single-question version N=917[a]	**Two-question version** N=168[b]
Either	30%	50%
Pos x-dir only	50%	35%
Neg x-dir only	15%	15%
Neither/$v = 0$	5%	5%

[a] See Table 4.3 on page 49. [b] Section 121C144.

Table 4.10: Phys 121 student answers about how the speed of the particle is changing. Not all answers are shown.

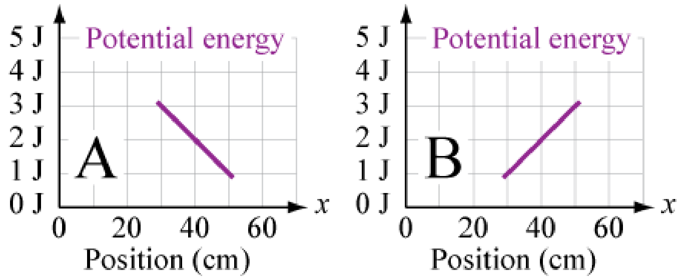
Answer	**New version:** Select graph(s) N=181[a]	**Traditional version:** Describe speed N=934[b]
Correct	25%	10%
Consistent w/left-to-right	60%	70%
Other	15%	20%

[a] Section 121A144. [b] See Table 4.4 on page 50.

correspond to a particle speeding up. An excerpt of the question is shown in Figure 4.2. The correct answer is *both graphs A and B* since the particle could be moving in either direction.

Student performance is shown in Table Table 4.10. A slightly greater percentage of students answered correctly on the new version (25%) as compared to the traditional one (10%). In both versions, the most popular answer is consistent with reading the graph from left to right. The vast majority of the students in both versions provided explanations that were consistent with this reasoning.

These results show that, when reasoning about speed, students exhibit similar difficulties whether they are given a diagram and asked to describe the speed, or are given a description of the speed and asked to select the corresponding diagram(s). We present a more focused



Question 7.
For which of the graphs, if either, **could** the particle be **speeding up** when it is located at ***x* = 40 cm?**

- ○ Neither graph
- ○ Only graph A
- ○ Only graph B
- ◉ Both graphs A and B

Figure 4.2: Additional version of the 'speed' question.

study of how students draw or choose potential energy diagrams to match descriptions of physical systems in §4.3.

4.1.5 Summary

We have identified a variety of difficulties that students encountered when reasoning about kinematic or dynamic properties based on potential energy diagrams. Of those we discussed, the most prevalent one was the tendency to interpret the graph from left to right (e.g., stating an object speeds up at a point on the graph where the slope is negative). Most of the other difficulties are associated with using incorrect features of the potential energy diagram (e.g., using the second derivative of the graph in cases where the first derivative is relevant). This tendency to use incorrect features of a graph is consistent with the findings of other researchers in different contexts [17, 18].

4.2 Student ability to relate kinetic and total energy to potential energy diagrams

In this section, we discuss the extent to which students are able to use potential energy diagrams to relate kinetic, total, and potential energies. We begin by describing many of the questions we have administered to students. Included in this description is a discussion of how we came to refine some of the potential energy diagrams. We then discuss some of the difficulties that students encountered in answering these questions. We also provide a commentary on the effect of lecture instruction on student performance.

4.2.1 Description of questions

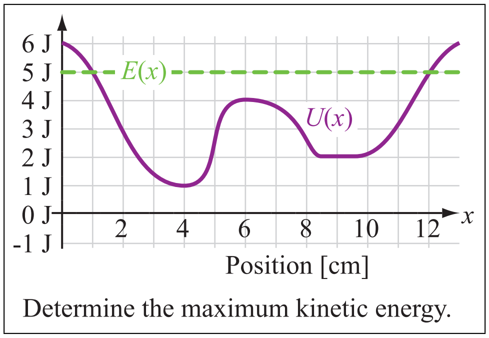
The questions we have administered require students to apply the relationship $E_{\text{tot}} = K + U$ to determine the maximum kinetic energy, the total energy, or the location of turn-around points. Below we discuss each of the questions. The ones that ask specifically about the maximum kinetic energy are described first. A description of the other questions follows. After, we briefly discuss the development of some of the questions and provide an overview of the results.

4.2.1-a Questions about the maximum kinetic energy

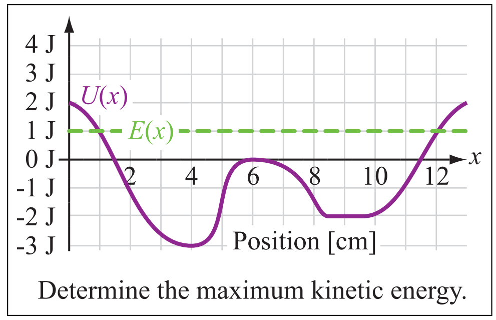
Four of the questions we have administered ask students to use a potential energy diagram to determine the maximum kinetic energy attained by the particle of interest.

In two of the questions, students are provided with a line representing total energy. These are shown in Figures 4.3(a) and (b). We call these questions the 'K_{max} from E_{tot}' and 'K_{max} with neg. PE' questions, respectively. To answer correctly, students can use the relation $E_{\text{tot}} = K + U$ to determine that the maximum kinetic energy K_{max} occurs where the value of potential energy is at a minimum (U_{min}). The numerical value of the maximum kinetic energy can then be found using $K_{\text{max}} = E_{\text{tot}} - U_{\text{min}}$. Thus, the correct answer for both questions in the figure is $K_{\text{max}} = 4\,\text{J}$.

We have also administered two questions in which students are not provided with the total



(a) The 'K_{max} from E_{tot}' question.



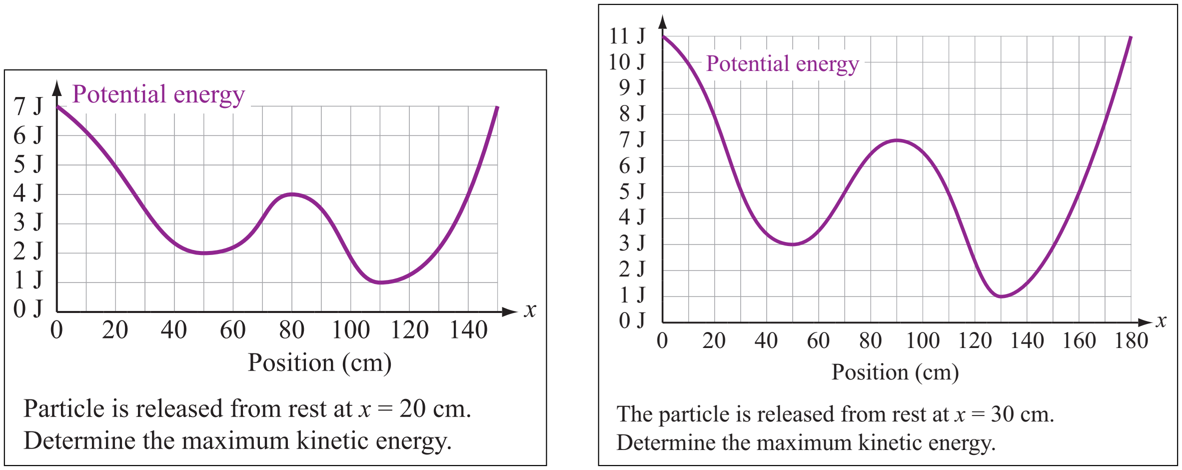
(b) The 'K_{max} with neg. PE' question.

Figure 4.3: Abbreviated versions of the 'K_{max} from E_{tot}' and 'K_{max} with neg. PE' questions. The correct answer to both questions is 4 J.

energy line, but are instead provided with the position at which the particle is released from rest. One of these questions is shown in Figure 4.4(a). We call this question the 'K_{max} from x_{rel}' question. Students are told that the particle is released from $x_{\text{rel}} = 20\,\text{cm}$. To determine the maximum kinetic energy, students must first calculate the total energy.[11] This can be done by noting the kinetic energy is zero at the point of release, so $E_{\text{tot}} = U(20\,\text{cm}) = 5\,\text{J}$. The maximum kinetic energy can then be determined in the same manner as the 'K_{max} from E_{tot}' question. Thus, the correct answer for the diagram in Figure 4.4(a) is $K_{\text{max}} = 4\,\text{J}$.

The other question that provides students with a point of release is shown in Figure 4.4(b). In this 'K_{max} w/turn-around' question, students are told that the particle is released from rest at $x = 30\,\text{cm}$ and are asked to determine the maximum kinetic energy. This question is different than the others we have discussed thus far because there are two disjointed regions that satisfy $U(x) \leq E_{\text{tot}}$: $30\,\text{cm} \leq x \leq 70\,\text{cm}$ and $110\,\text{cm} \leq x \leq 160\,\text{cm}$. To answer correctly, students should realize that the particle cannot move past $x = 70\,\text{cm}$. Therefore, the maximum kinetic energy occurs at the local minimum located at $x = 50\,\text{cm}$, and not the global minimum at $x = 130\,\text{cm}$. The correct answer is $K_{\text{max}} = 2\,\text{J}$.

[11] We note that there are methods to determine the maximum kinetic energy that do not *explicitly* require students to calculate E_{tot}. For example, students can draw or imagine a horizontal line on the graph from the point of release and then calculate the vertical distance between the horizontal line and U_{min}. However, implicit in this method is the extra step of drawing the horizontal line, which represents E_{tot}.



(a) The 'K_{max} from x_{rel}' question. The correct answer is 4 J.

(b) The 'K_{max} w/turn-around' question. The correct answer is 2 J.

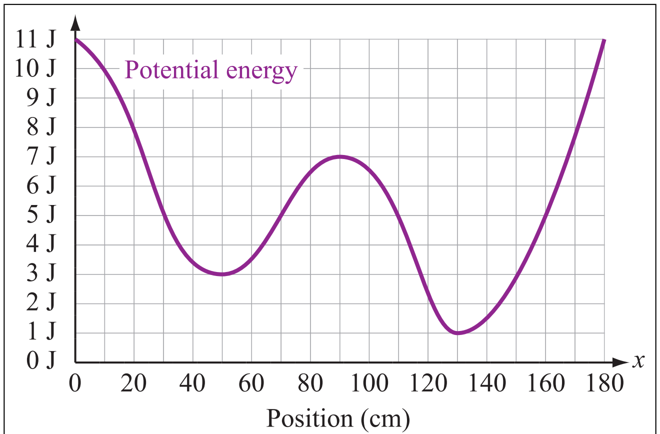
Figure 4.4: Abbreviated versions of the 'K_{max} from x_{rel}' and 'K_{max} w/turn-around' questions.

4.2.1-b Additional questions

We have also administered three other questions that *do not* ask students to determine the maximum kinetic energy.

One such question is shown in Figure 4.5. Students are provided with the point of release ($x = 70\,\text{cm}$) and are asked to determine the total energy. We call this question the 'E_{tot} from x_{rel}' question. To answer correctly, students can note that the kinetic energy is zero at the point of release, so the maximum kinetic energy is $K_{\text{max}} = U(70\,\text{cm}) = 5\,\text{J}$. Note that this question uses the same diagram as that in Figure 4.4(b). It probes the first step in the reasoning required to answer the 'K_{max} from x_{rel}' question.

A different question we have asked is shown in Figure 4.6. Students are provided with the point of release ($x = 30\,\text{cm}$) and are asked to determine the largest and smallest x-values attained by the particle. We call this question the 'determine turn-around' question. To answer correctly, students can use the same reasoning as the 'K_{max} w/turn-around' question to recognize that the particle cannot move past $x = 70\,\text{cm}$. The choices provided to students on this question were based on our research in this chapter and included positions at which



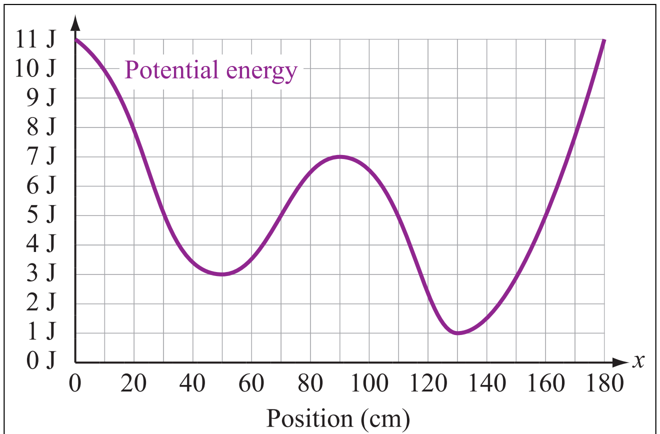
The particle is released from rest at $x = 70$ cm.
Determine the total energy of the system.

Figure 4.5: Abbreviated version of the 'E_{tot} from x_{rel}' question. The correct answer is 5 J.

$dU/dx = 0$ (50 cm, 90 cm, and 130 cm), positions at which $U(x) = E_{\text{tot}}$ (30 cm, 70 cm, 110 cm, and 160 cm, which include the correct answers), as well as the smallest and largest x-values on the graph (0 cm and 180 cm). We note that this question uses the same diagram and point of release as the 'K_{max} w/turn-around' question in Figure 4.4(b). It probes some of the reasoning required to answer that question.

Another question we have asked is shown in Figure 4.7. This question is different from the others in that students were not provided with a potential energy diagram. In this question, students are told that the kinetic energy of a particle in 'region A' is twice that in 'region B.' They are asked to compare the corresponding potential energies in the two regions. Five different choices are provided to students (see the figure). We call this question the 'PE from KE' question. The correct answer is that the potential energy is region A is less than that in region B, but not necessarily half because the total energy is not given. (We developed this question as a result of another question we administered to Phys 123 students that did use a potential energy diagram. See our discussion in §4.2.2-c.)

Finally, we briefly note that we have conducted one-on-one interviews with students enrolled in the third quarter of the introductory sequence (Phys 123). These students had



The particle is released from rest at $x = 30$ cm. Determine the smallest and largest x-values attained by the particle.

Figure 4.6: Abbreviated version of the 'determine turn-around' question. The correct answers are 30 cm and 70 cm.

Suppose it is known that the kinetic energy of the particle in region A is twice that in region B.

Question 12.
Which of the following is a true statement regarding the potential energy of the system in regions A and B?

- ◯ The potential energies in regions A and B are **the same.**
- ◯ The potential energy in region A is **half that** in region B.
- ◉ The potential energy in region A is **less than** that in region B, but not necessarily half.
- ◯ The potential energy in region A is **twice that** in region B.
- ◯ The potential energy in region A is **more than** that in region B, but not necessarily twice.

Figure 4.7: Abbreviated version of question administered designed to elicit the tendency to believe kinetic and potential energies are inversely proportional. The correct answer is selected.

completed all instruction in Phys 121 and Phys 122. We describe the questions we asked in this setting as needed.

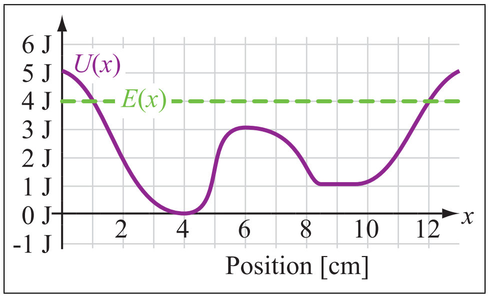
This concludes the description of the questions we have administered. Next we describe the development of 'appropriate' potential energy diagrams that lead to unambiguous answers.

4.2.1-c Commentary on the development of appropriate potential energy diagrams

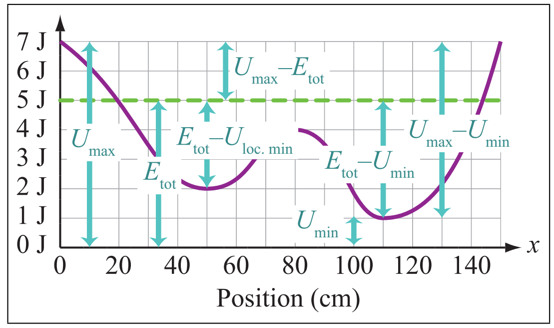
In administering several versions of the 'K_{max} from E_{tot}' question shown in Figure 4.3(a), we found that there are a variety of methods that students used in attempting to determine the maximum kinetic energy. However, not all of the potential energy diagrams we used were sufficient in disambiguating these different methods. An early version of one of the diagrams we administered is shown in Figure 4.8(a). We found that some students arrived at the correct answer of $K_{\mathrm{max}} = 4\,\mathrm{J}$ by using incorrect reasoning. For example, some students seemed to believe that the maximum kinetic energy is equal to the total energy as a general rule (i.e., $K_{\mathrm{max}} = U_{\mathrm{max}}$), without considering the minimum value of potential energy. Moreover, there were different *incorrect* methods students appeared to use that led to the same incorrect answer. For example, some students seemed to believe that the maximum kinetic energy is equal to the maximum value of potential energy (i.e., $K_{\mathrm{max}} = U_{\mathrm{max}}$). Other students explicitly found the difference between the maximum and minimum values of potential energy from the diagram (i.e., $K_{\mathrm{max}} = U_{\mathrm{max}} - U_{\mathrm{min}}$). Both of these incorrect methods led to the same answer of $5\,\mathrm{J}$.

In designing a more appropriate potential energy diagram, we enumerated the various methods students used (both correct and incorrect) to determine the maximum kinetic energy. These methods are illustrated in Figure 4.8(b). On the revised diagrams that we used on subsequent questions, we ensured that each method would lead to a different answer. (The correct method is indicated by the vertical line segment labeled $E_{\mathrm{tot}} - U_{\mathrm{min}}$.)

Finally, we note that the results we discuss in this section were obtained from these unambiguous potential energy diagrams. (An exception to this rule is if the combined percentages



(a) A PED that was used early in our investigation.



(b) Demonstration of the various ways in which students attempted to determine the kinetic energy.

Figure 4.8: Development of unambiguous potential energy diagrams to probe how students determine the maximum kinetic energy.

of two ambiguous methods were relatively low or if student explanations can easily be used to distinguish the different lines of reasoning.)

Below, we briefly summarize student performance on the questions that ask students to determine the maximum kinetic energy.

4.2.1-d Brief summary of performance on maximum kinetic energy questions

Given that we have administered four types of questions that ask students to determine the maximum kinetic energy, we find it convenient here to collectively summarize the performances. (Results from other questions are discussed throughout §4.2.2.) Table 4.11 groups the questions together according to the information that was provided to students (either the total energy or the point of release). The various rows indicate the methods students used to determine the maximum kinetic energy. (Many of these methods were shown graphically in Figure 4.8(b).) The percentages we report do not include students who provided explanations that are inconsistent with the expected method. (Doing so only changed the percentages by at most 5%, and in most cases less.) The reported N values, though, include all students.

Table 4.11: Student performance on determining the maximum kinetic energy. Blank entries indicate percentages of less than 5%. The correct answer is highlighted.

	Total energy given		Point of release given	
Method	K_{max} from E_{tot} N=135[a]	K_{max} with neg. PE N=135[b]	K_{max} from x_{rel} N=146[c]	K_{max} w/turn-around N=188[d]
Correct	55%–60%	55%–65%	20%	10%
$U_{max} - U_{min}$	25%	10%	10%	40%
E_{tot}	5%		35%	15%
U_{max}	5%	5%	5%	10%
$U_{max} - E_{tot}$			10%	5%
$E_{tot} - U_{local\ min}$ *			10%	N/A*
$E_{tot} - U_{global\ min}$ **	N/A**	N/A**	N/A**	10%
$\|U_{min}\|$		10%		

[a] Section 121D134. [b] Section 121C134. [c] Section 121D144. [d] Section 121A144.
* This row is for *incorrect* responses. The method '$E_{tot} - U_{local\ min}$' is correct for the 'K_{max} w/turn-around' question. ** This row is for *incorrect* responses. The method '$E_{tot} - U_{global\ min}$' is correct for the '$K_{max}$ from E_{tot}' and 'K_{max} from x_{rel}' questions.

On the questions in which students were provided with the total energy line, slightly more than half used the correct method (up to 65%[12]). A markedly smaller fraction of students answered correctly on the questions in which students were provided with the point of release (up to 20%). This is perhaps not surprising given that the latter version involves an extra step: Students must first recognize that the total energy must be calculated, and its value correctly determined before K_{max} can be found.

In the next subsection, we begin our discussion of the difficulties we have identified that students encounter in relating kinetic, potential, and total energies.

[12]The *range* of values we report in the table reflect that some of the responses suggest students might be providing the *location* of where the particle attains its maximum kinetic energy, rather than the value of the maximum kinetic energy. Both the correct answer and this location had numerical values of 4. Later questions that we developed avoided this issue.

4.2.2 Identification of student difficulties

We found that many of the ways in which students reason about kinetic and total energy could be categorized into two primary difficulties: (1) the tendency to treat the bounds of a potential energy diagram as the bounds of the allowed region, and (2) the tendency to treat the minimum value of potential energy as being zero. These difficulties are discussed in §4.2.2-a and 4.2.2-b, respectively. Separately from these, we found that some students treated kinetic energy as being inversely proportional to potential energy. This is discussed in §4.2.2-c. Additional difficulties that were somewhat less common are discussed in §4.2.2-d–§4.2.2-g.

4.2.2-a Belief that the bounds of the potential energy curve are the bounds of the allowed region

Some students appeared to believe that the particle reaches all positions for which the potential energy curve is defined. In other words, they treated the horizontal bounds of the potential energy curve as the bounds of the allowed region of the particle (though students did not phrase it in quite this way).[13] Below we provide examples that illustrate this difficulty using several different questions.

Evidence from one-on-one student interviews

One of the clearest examples of this tendency comes from our one-on-one semi-structured interviews that we conducted in spring 2011. We presented students with a potential energy diagram (without a line representing E_{tot}) and described the diagram as follows:

INTERVIEWER: These graphs are supposed to represent the potential energy of the system as a function of the position of the particle.

STUDENT: Okay

[13] This belief might also be interpreted as a failure to account for turn-around points.

INTERVIEWER: For example, if the particle were at $x = 25\,\text{m}$, the system would have roughly 7 J of potential.

A short time later, we ask the student to describe, if possible, the motion of the particle:

INTERVIEWER: If someone presented you this graph and this is the only information about this system you had, could you say anything about the motion of that point particle that's confined to this system? For example, is it moving? If so, what direction? Is it stationary?

STUDENT: I think it would probably have to be moving because the amount of potential energy is changing. And if you just got a particle sitting still somewhere, unless you're changing the laws of what's happening around said particle, I don't think you'd be able to change its potential energy unless it's doing something.
[...]

STUDENT: Well I guess if it's going to be at different positions it'd have to be moving because that's the definition of position. So the different positions make the different potential energies.

INTERVIEWER: Is there anywhere where we could place the particle and it could not be moving? That it would be stationary for all time?

STUDENT: Well, because we have the graph going along position, doesn't that have to mean it's moving?

This student appeared to interpret the different positions on the horizontal axis as representing actual positions that the particle reaches. (See the second to last statement made by the student.)

Evidence from the maximum kinetic energy questions

On the maximum kinetic energy questions, some students appeared to treat the maximum value of potential energy on the diagram as if it were the highest value of potential energy attained by the system. This is consistent with treating the particle as if it traveled to the smallest and largest x-values on the graph. For example, in the 'K_{max} from E_{tot}' question in Figure 4.3(a) on page 71, students were asked to determine the maximum kinetic energy given a potential energy diagram and the total energy line. Only 55% of the students answered correctly (see the first data column in Table 4.11). The most common incorrect method, provided by about 25% of the students, was to subtract the maximum and minimum values of potential energy (i.e., $K_{\mathrm{max}} = U_{\mathrm{max}} - U_{\mathrm{min}}$). These students did not incorporate the total energy into their calculation. The following quote is typical of the explanations provided by these students:

> *"[$K_{max} = 5\,\mathrm{J}$.] The object starts with six joules of potential energy and drops to 1 joule of potential energy, meaning that 5 of those joules must have been translated into kinetic energy."*
> (Phys 121D, autumn 2013)

Students with this reasoning failed to recognize that the locations on the graph where $U(x) = 6\,\mathrm{J}$ represent what the value of potential energy would be *if* the particle *were* located there. Although these students did correctly incorporate the minimum value of potential energy, they did not take into account the line representing total energy.

A small fraction of the students on the same question (about 5%) appeared to equate the maximum kinetic energy with only the maximum value of potential energy (i.e., $K_{\mathrm{max}} = U_{\mathrm{max}}$). Although similar to the previous set of students, these students did not incorporate the minimum value of potential energy U_{min}.[14] The following quote is typical of students with this answer:

[14] We elaborate on another interpretation of this method in §4.2.2-b.

Table 4.12: Student performance on the 'E_{tot} from x_{rel}' question. The correct answer is highlighted.

Method	N=226[a]
$E_{\text{tot}} = U_{\text{rel}}$	45%
$E_{\text{tot}} = U_{\text{max}}$	45%
Other	15%

[a] Sections 121B152 and 121C152.

> *"[$K_{max} = 6\,\text{J}$.] The maximum energy it reached on the graph was 6 joules. Since the energy is not lost, the max kinetic energy would be also max of 6 J."*
>
> (Phys 121D, autumn 2013)

We note that the combined percentage of students using either one of these methods that treat U_{max} as the highest value of potential energy attained by the system is 30%.

The other maximum kinetic energy questions also elicited these methods, though in different proportions (see Table 4.11).

Evidence from the 'E_{tot} from x_{rel}' question

Our observation that some students were using the maximum value of potential energy on the diagram motivated us to develop the 'E_{tot} from x_{rel}' question (see Figure 4.5 on page 73). In this question, students are given the point of release of the particle and are asked to determine the total energy of the system. Only about 45% of the students answered correctly (see Table 4.12). We note that this percentage is higher than the percentage of students correctly determining the maximum kinetic energy on the point-of-release version (20%). Of importance here, however, is that 45% of the students gave an answer consistent with treating E_{tot} as being equivalent to U_{max}. These students treated the total energy as if it were the maximum value of potential energy. They failed to account for the point at which the particle is released.

Evidence from the 'determine turn-around' question

We have also observed the tendency to treat the bounds of the potential energy diagram as the bounds of the allowed region on the 'determine turn-around' question (see Figure 4.6 on page 74). In this question, students are provided with the point of release of the particle. They are asked to determine the smallest and largest x-coordinates that the particle reaches. To answer correctly, students must consider the turn-around point at $x = 70\,\text{cm}$. The correct answer is $(x_{\text{smallest}}, x_{\text{largest}}) = (30\,\text{cm}, 70\,\text{cm})$.

Student performance is shown in Table 4.13. About 20% of students answered $(30\,\text{cm}, 180\,\text{cm})$. Explanations by students are sometimes difficult to interpret, but many are explicit in using the right-most point on the potential energy curve:

> *"[(30 cm, 180 cm).] If the particle is released at 30 and continues moving towards the right as shown by the graph, then 30 is the smallest, also, the graph ends at 180 and the [p]article is shown to be there thus 180 is the furthest right."*
>
> (PHYS 121A, SPRING 2015)

An additional 10% of the students answered $(0\,\text{cm}, 180\,\text{cm})$. The following quote is typical of these students' explanations:

> *"[(0 cm, 180 cm).] Its the range of the graph"*
>
> (PHYS 121A, SPRING 2015)

In all, about 30% of the students identified at least one of the bounds of the potential energy curve as bounding the motion of the particle.

We briefly note that about 30% of the students identified either $x = 50\,\text{cm}$ or $x = 90\,\text{cm}$ as one of the turn-around points (not shown in the table). The potential energy curve at these two points has a slope of zero. The answers provided by these students are consistent with treating the slope as indicating the direction of motion. See our discussion in §4.1.3-c.

Table 4.13: Student performance on the 'determine turn-around' question. The correct response is highlighted.

Response ($x_{\text{smallest}}, x_{\text{largest}}$)	N=178[a]
30 cm, 70 cm	35%
30 cm, 160 cm	5%
30 cm, 180 cm	20%
0 cm, 180 cm	10%

[a] Section 121A152.

Summary

Many students appeared to believe that the particle is able to 'reach' all positions for which the potential energy curve is defined. The variety of questions in which this tendency is elicited suggests that addressing the underlying difficulty could improve student performance on a large range of questions.

4.2.2-b Tendency to treat the minimum value of potential energy as being zero

Some student responses are consistent with the belief that the minimum value of potential energy of a system is zero. These students failed to recognize that the minimum value on the potential energy diagram we provided was non-zero.

Below we provide examples of student responses that illustrate this tendency from several types of questions.

Evidence from the 'K_{max} from x_{rel}' question

On the 'K_{max} from x_{rel}' question, students were asked to determine the maximum kinetic energy using the point of release of the particle (see Figure 4.4(a) on page 72). In answering, about 35% of the students appeared to equate the maximum kinetic energy to the total energy (i.e., $K_{\text{max}} = E_{\text{tot}}$; see the third data column of Table 4.11 on page 77). The following student quote is illustrative of this reasoning:

> *"[$K_{max} = 5$ J.] The particle had 5 joules at its released [sic] so it can only gain up to 5 joules."*
>
> (PHYS 121D, AUTUMN 2014)

Other students are more explicit that they are treating the minimum potential energy as zero. For example:

> *"[$K_{max} = 5$ J.] Since the PE of the ball at that point [x=20 cm] is 5 J, when it is released that is its maximum energy, and since maximum energy equals PE + KE, the most KE can be is when PE is zero, and since total energy is 5 J, that is what the maximum KE equals."*
>
> (PHYS 121D, AUTUMN 2014)

Another method that is consistent with the belief that the minimum potential energy is zero is '$K_{\text{max}} = U_{\text{max}}$.' About 5% of the students used this method on the 'K_{max} from x_{rel}' question. These students did not incorporate into their method the minimum value of potential energy U_{min} on the diagram. Additionally, they appeared to also be misinterpreting the maximum potential energy U_{max} as the total energy.[15] About 40% of students used either the '$K_{\text{max}} = E_{\text{tot}}$' or the '$K_{\text{max}} = U_{\text{max}}$' method.

We mention briefly that we have observed this reasoning on the other maximum kinetic energy questions, though slightly less often. (See Table 4.11.)

Commentary on possible underlying causes

Up to about one-third of students seemed to treat the minimum value of potential energy as being zero, despite that the potential energy diagram had a minimum value that was non-zero. In trying to understand the underlying cause of this tendency, we note that in our experience it is fairy common for both students and instructors to set the minimum potential energy to zero when solving typical textbook problems. For example, if solving for the speed of the ball after it has fallen a distance h, it is natural to associate the values mgh for the initial potential energy and 0 for the minimum value.[16] Furthermore, nearly

[15] We discussed this difficulty in §4.2.2-a.

[16] This is in contrast to simply setting the *change* in potential energy equal to $-mgh$.

all textbooks write the expression for elastic potential energy (for objects that obey Hooke's law) as $U(x) = \frac{1}{2}kx^2$.[17] This is equivalent to setting the minimum value of potential energy equal to zero. It is possible that these practices, which are commonplace in the physics classroom, contribute to the difficulty we have described here.

4.2.2-c Tendency to treat kinetic and potential energies as being inversely proportional

A difficulty we have identified which is different from those described above is the tendency to treat kinetic and potential energies as being inversely proportional (i.e., $K \times U = \text{const}$). We have only observed students exhibiting this tendency on a specific type of question.

We first identified this tendency in our research into student understanding of probability and probability density. For example, we administered a question to Phys 123 students that used the potential energy diagram shown in Figure 4.9. Note that the total energy line is not given. Students were asked to rank, if possible, the probabilities of finding the object of interest in the two regions shown (labeled A and B).[18] In attempting to answer, about 10% of students explicitly stated that either the speed or the kinetic energy in region B was twice that in region A. (Students had not been asked explicitly to give a quantitative comparison of the kinetic energies. They were only asked to rank the probabilities in the two regions, which can be done through a qualitative comparison of kinetic energies.) Explanations typically relied on the fact that the potential energy in region B is half that in region A. These students failed to realize that the value of *total energy* influences the values of kinetic energy in the two regions.

We were interested in documenting whether students in Phys 121 have the same tendency. We also hypothesized that the underlying difficulty was not restricted to *graphical* representations of potential energy. In spring 2015, we administered the 'PE from KE' question shown in Figure 4.7 on page 74. In this question, students are told that the *kinetic energy*

[17]We discuss student difficulties with the arbitrary nature of potential energy in the context of springs in §4.4.2-b.

[18]For more information on this question, see §8.2.3.

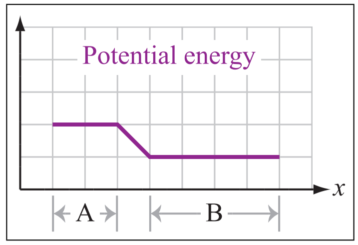


Figure 4.9: About 10% of PHYS 123 students claimed $K_B = 2K_A$ or $v_B = 2v_A$, even though they were only asked to rank the probabilities of finding an object in the two regions.

in region A is twice that in region B. They are also informed that the total energy in the system is constant. Students are asked to compare the kinetic energies in the two regions. The five different options provided to students are shown in the figure.

Student performance is shown in Table 4.14. Just over half of students answered incorrectly that $P_A = \frac{1}{2}P_B$. Most of these students cited either the conservation of energy or the constancy of energy in their explanations. The following quotes are typical of these students' explanations:

> "[$U_A = \frac{1}{2}U_B$.] *Because the total energy is a constant and since KA = 2KB PB = 2PA which leads to PA = 0.5PB*"
>
> (PHYS 121A, SPRING 2015)

> "[$U_A = \frac{1}{2}U_B$.] *Since energy is conserved in this system, the potential energy in region A must be half that in region B since the kinetic energy is double in region A what it is in region B.*"
>
> (PHYS 121A, SPRING 2015)

These students treated kinetic and potential energies as being inversely proportional and failed to apply the relation $E_{\text{tot}} = K + U$ correctly.

4.2.2-d Tendency to reason locally rather than globally

Some students appeared to use a *local* minimum of potential energy to reason about maximum potential energy instead of the global minimum.

Table 4.14: Student performance on the 'PE from KE' question. The correct answer is highlighted.

Answer	N=174[a]
$U_A < U_B$, but not necessarily half	20%
$U_A = \frac{1}{2}U_B$	55%
$U_A = 2U_B$	15%
Other	10%

[a] Section 121A152.

For example, on the 'K_{max} from x_{rel}' question, students are asked to determine the maximum potential energy by using the point of release of the particle (see Figure 4.4(a) on page 72). About 10% of the students appeared to find the maximum potential energy by using the *local* minimum of potential energy located at $x = 50\,\text{cm}$ (i.e., $K_{\text{max}} = E_{\text{tot}} - U_{\text{local min}}$). This is shown in the third data column of Table 4.11 on page 77. The following student quote is illustrative of this tendency:

> *"[$K_{\text{max}} = 3$ J.] Potential and kinetic energy must add to 5, and because the smallest potential energy is 2 J, the kinetic energy can be 3 J at max."*
> (Phys 121D, autumn 2014)

In attempting to understand why students would use the local minimum of potential energy rather than the global minimum, we note that the point of release of the particle at $x = 20\,\text{cm}$ is spatially closer to the local minimum at $x = 50\,\text{cm}$ than it is to the global minimum at $x = 110\,\text{cm}$. One possible explanation of this tendency, then, is that students may be more likely to look at only the closest minima near the point of release. In some sense, the local minimum may be more 'salient' than the global one [67]. More research is needed to understand this tendency.

Finally, we note that using this incorrect 'local' method would lead to a correct answer on the 'K_{max} w/turn-around' question (see Figure 4.4(b) on page 72). That is, some of the 10% of students who provided correct responses to this question might actually have used incorrect reasoning. (Given that few students discuss the turn-around point on this question, it seems likely that at least some students are reasoning in this manner.)

4.2.2-e Tendency to equate K_{max} with $|U_{min}|$

In attempting to determine the maximum value of kinetic energy, some students seemed to equate the maximum value of kinetic energy with the absolute value of the minimum of the potential energy. We have only observed a sizable fraction of students exhibiting this tendency on questions with potential energy diagrams that include *negative* values. For example, on the '$K_{\max}$ with neg. PE' question (see Figure 4.3(b) on page 71), students are presented with a potential energy diagram and a line representing total energy. They are asked to determine the maximum kinetic energy of the particle. About 10% of the students used the $K_{\max} = |U_{\min}|$ method (see the second data column of in Table 4.11 on page 77). The following two quotes illustrate this tendency:

> "[$K_{max} = 3\,\mathrm{J}$.] *Since potential energy is the inverse of kinetic energy, we can say that 3 joules of kinetic energy by the particle.*"
> (Phys 121C, autumn 2013)

> "[$K_{max} = 3\,\mathrm{J}$.] *The greatest kinetic energy will be when the potential energy is lowest. This is because the total energy is a constant and equals the sum of the potential and kinetic energy of the system. Therefore, when the potential energy of the system is lowest, there we will find when the system has the most kinetic energy, which would be when the potential is at 3 J.*"
> (Phys 121C, autumn 2013)

Although these students successfully identified the point on the graph at which kinetic energy takes on its maximum value, they failed to incorporate the total energy into their method.

4.2.2-f Tendency to treat the total energy as if it were the minimum value of potential energy

On the maximum kinetic energy questions in which the point of release of the particle is provided, some students appeared to treat the potential energy at the point of release (i.e., the total energy) as if it were the minimum value of potential energy of the system (i.e., they equated K_{max} with $U_{\text{max}} - E_{\text{tot}}$). On the '$K_{\text{max}}$ from x_{rel}' question (see Figure 4.4(a) on page 72), about 10% of the students used this method to determine the kinetic energy. The following student quote is illustrative of this method:

> *"[$K_{max} = 2$ J.] since you originally had 7 joules of potential energy and then five at that point, so two was converted to kinetic energy"*
> (Phys 121D, autumn 2014)

Students who used this method (1) failed to recognize that the maximum value of potential energy on the diagram is not attainable by the system, and (2) treated the potential energy of the release point of the particle as the lowest value of potential energy attained by the system. The first of these two difficulties is related to the tendency we discussed in §4.2.2-a. The second is unique to this method.

It is notable that we have not seen this difficulty arise with any regularity on the questions in which the total energy line is provided to students.

4.2.2-g Tendency to treat the particle as if it 'tunneled' through barriers

The responses of some students are consistent with treating the particle as if it were able to 'tunnel' through potential barriers. For example, on the 'K_{max} w/turn-around' question, students are provided with the point of release of a particle and are asked to determine the maximum kinetic energy (see Figure 4.4(b) on page 72). To answer correctly, students must account for the turn-around point at 70 cm to realize that the particle *does not* reach the global minimum of potential energy. However, about 10% of the students used the '$E_{\text{tot}} - U_{\text{global min}}$' method (see the last column of Table 4.11 on page 77). These students failed to recognize that the region $70\,\text{cm} < x < 110\,\text{cm}$ is inaccessible to the particle.

Partially due to this observation, we developed the 'determine turn-around' question shown in Figure 4.6 on page 74. This question uses the same potential energy diagram as the 'K_{max} w/turn-around' but asks students to determine the smallest and largest x-values that the particle attains. About 5% of the students incorrectly answered (30 cm, 160 cm). These coordinates coincide with the leftmost and rightmost positions x on the graph at which $U(x) = E_{\text{tot}}$.[19] Many of these students provided an explanation that is consistent with this description. For example:

> *"[(30 cm, 160 cm).] [...] The largest x-position that the particle can reach is 160 cm because that is where the particle will have the same amount of potential energy that is had when it started."*
>
> (PHYS 121A, SPRING 2015)

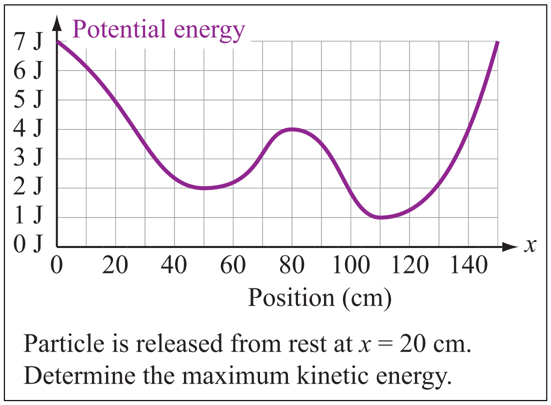
These students correctly identify two places where $U(x) = E_{\text{tot}}$, but may not recognize that the particle cannot move past the *first* location where $U(x) = E_{\text{tot}}$.

Given that one of the distinguishing characteristics of quantum mechanics is the ability of particles to tunnel, it is important for students in a classical setting to be able to account for these turn-around points.

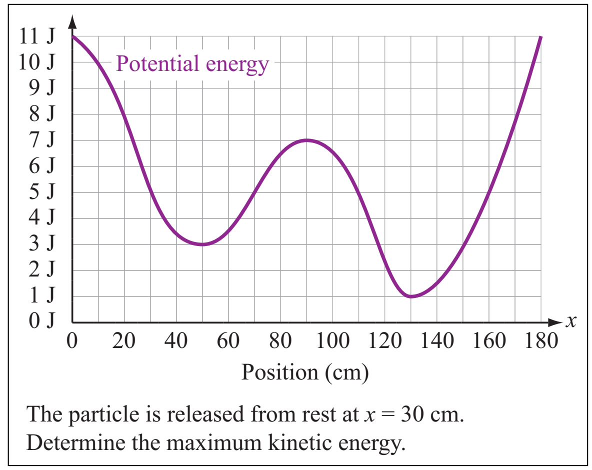
4.2.3 Commentary on the effect of lecture instruction

In general, the findings from physics education research reveal that student performance on qualitative questions is largely insensitive to lecture instruction. This has also been our experience with the questions based on potential energy diagrams that we have administered as part of the present study. However, we did find the results varied substantially on two related questions that were administered in sections with notably different amounts of lecture instruction. In this case, the percentage of students who answered correctly was similar, but the percentage of students choosing various incorrect answers seemed to differ.

[19] We note that many more students identified the largest x-value attained as being beyond $x = 70$ cm. The students we discuss here are only those who used essentially correct reasoning but failed to account for the turn-around point.



(a) The '$K_{\max}$ from x_{rel}' question. The correct answer is 4 J.



(b) The '$K_{\max}$ w/turn-around' question. The correct answer is 2 J.

Figure 4.10: Abbreviated versions of the '$K_{\max}$ from x_{rel}' and '$K_{\max}$ w/turn-around' questions.

The '$K_{\max}$ from x_{rel}' and '$K_{\max}$ w/turn-around' questions discussed earlier are similar to one another. (The questions are discussed in §4.2.1-a and the figures are repeated in Figure 4.10.) Both questions provide students with the point of release of a particle and ask them to determine the maximum kinetic energy. The primary difference between these questions is that on the latter one, students must take into account the turn-around point at $x = 70\,\text{cm}$. Thus, they must use the local minimum of kinetic energy rather than the global minimum (which is the case for the former question).

The instructors of the sections in which we administered these two questions spent considerably different amounts of time in class discussing potential energy diagrams. The instructor of the section to which we administered the '$K_{\max}$ from x_{rel}' question described potential energy diagrams in depth. In soliciting details from the instructor, he responded:

> *"Yes, I introduced PEDs [...] I did discuss motion in both directions, and made the ball-in-a-bowl analogy. I copied the picture from the book to discuss total [energy] as horizontal lines, and [kinetic energy] as the vertical distance between [total energy] and [potential energy]."*
>
> (Instructor from the 'K_{MAX} from x_{REL}' section)

In contrast, the instructor of the section to which we administered the 'K_{max} w/turn-around' question did not discuss these diagrams:

> *"I did nothing at all about energy diagrams, focusing instead on definitions and drawing pictures of the physical situation to guide the writing of equations."*
>
> (Instructor from the 'K_{MAX} w/turn-around' section)

On the 'K_{max} from x_{rel}' question, 20% of the students (who had more lecture instruction) answered correctly. (See Table 4.15.) To help compare this to performance on the other question, we include the percentage of students who incorrectly used the local minimum to calculate the maximum kinetic energy rather than the global minimum; this method is *correct* on the other question. Doing so yields about 30% of students who use essentially correct reasoning, but with possibly the wrong minimum of potential energy. On the 'K_{max} w/turn-around' question, we can include students who incorrectly used the global minimum rather than the local minimum; this method is correct on the other question. Doing so yields about 20% of students who used essentially correct reasoning.

These percentages of students who use essentially correct reasoning are not markedly different (30% *vs.* 20%), despite the notable differences in lecture instruction. This result suggests that lecture instruction is not necessarily sufficient in helping students to overcome some of the difficulties associated with potential energy diagrams. (We have also not noticed any large differences in the other questions that we administered to the section whose instructor provided extra instruction.)

However, there is a large difference between the two sections on some of the *incorrect* methods used by students. For example, only 10% of the students on the 'K_{max} from x_{rel}' question subtracted the maximum and minimum values of potential energy on the diagram

Table 4.15: Student performance on the 'K_{max} from x_{rel}' and 'K_{max} w/turn-around' questions. Blank entries indicate percentages of less than 5%. The correct answer is highlighted.

Method	K_{max} from x_{rel} Had lecture instr. N=146[a]	K_{max} w/turn-around No specific lec. instr. N=188[a]
Correct	20%	10%
$U_{max} - U_{min}$	10%	40%
E_{tot}	35%	15%
U_{max}	5%	10%
$U_{max} - E_{tot}$	10%	5%
$E_{tot} - U_{local\ min}$ *	10%	N/A*
$E_{tot} - U_{global\ min}$ **	N/A**	10%
$\|U_{min}\|$		

[a] See Table 4.11 on page 77. * This row is for *incorrect* responses. The method '$E_{tot} - U_{local\ min}$' is correct for the 'K_{max} w/turn-around' question. ** This row is for *incorrect* responses. The method '$E_{tot} - U_{global\ min}$' is correct for the '$K_{max}$ from E_{tot}' and 'K_{max} from x_{rel}' questions.

(i.e., $K_{max} = U_{max} - U_{min}$). On the '$K_{max}$ w/turn-around' question, 40% of the students used this method. This difference is also accompanied by a difference in the percentage of students who equated the maximum kinetic enregy to the total energy of the system (i.e., $K_{max} = E_{tot}$); about 35% of the students on the 'K_{max} from x_{rel}' question did so, compared to about 15% of the students on the 'K_{max} w/turn-around' question.

We hypothesize that lecture instruction is the primary cause of these differences. We do not believe that the differences between these two questions would be responsible for the difference in the percentages of students using these incorrect methods. Further research, however, is needed to understand the effect of lecture instruction on the ability of students to relate kinetic and total energies at specific points on potential energy diagrams.

4.2.4 Summary

We have found that students struggled in a variety of ways when reasoning about the kinetic energy, the total energy, or the turn-around points. Some students treated the horizontal bounds of the potential energy diagram as if they were the bounds of the allowed region of the particle. Other students treated the minimum value of the potential energy as if it were zero, despite that the students were provided with potential energy diagrams in which $U_{\mathrm{min}} \neq 0$. (These two tendencies were not mutually exclusive.) Students also struggled in understanding the role that total energy plays in determining the precise values of kinetic energy. Of the other difficulties that we identified, the most notable to us is the tendency to treat the particle as if it could tunnel through potential barriers. A correct understanding of the role that turn-around points play in determining the allowed region is crucial to being able to understand how some aspects of quantum mechanics are different than those of classical mechanics.

4.3 Ability to draw or choose graphs of potential energy vs. position

In this section we discuss the extent to which students are able to use visual, written, and mathematical descriptions of systems and motions to determine the associated graph of potential energy *vs.* position. We begin in §4.3.1 by providing an example of a typical question that we used to probe student understanding. In §4.3.2 we describe the main difficulties that students encountered in determining potential energy diagrams. As will be shown, many of the difficulties arise from a failure to recognize that the potential energy diagram is a function of position only and is independent of the motion of the object of interest. Then, in §4.3.3, we describe the extent to which modifying the questions we used to probe student understanding does or does not affect student performance.

4.3.1 Example of questions and overview of student performance

We have administered a variety of questions to the calculus-based mechanics courses (Phys 121) as well as a sophomore-level introductory quantum mechanics course (Phys 225). Our questions used either a near-Earth gravitational context ($U(y) = mgy + U_o$) or a linear spring context ($U(x) = \frac{1}{2}kx^2 + U_o$).[20] The majority of questions are in the context of near-Earth gravitation, so below we only illustrate a typical question using this context. Questions that include springs are described throughout this section as needed.

4.3.1-a Example of questions

One of the questions we have administered to students is shown in Figure 4.11 (the 'ball-drop' question). Students are told a ball is dropped from $y = H$. The positive y-direction is defined to be upward, which is the standard for textbooks and is used in all of our questions. Students are asked to select which of the five graphs best represents a potential energy

[20] We chose not to explore the universal gravitational context ($U(r) = -GMm/r + U_o$) primarily because the near-Earth context is mathematically simpler and more familiar to students. Additionally, our pretests on potential energy diagrams are most often administered concurrently with lecture instruction on universal gravitation.

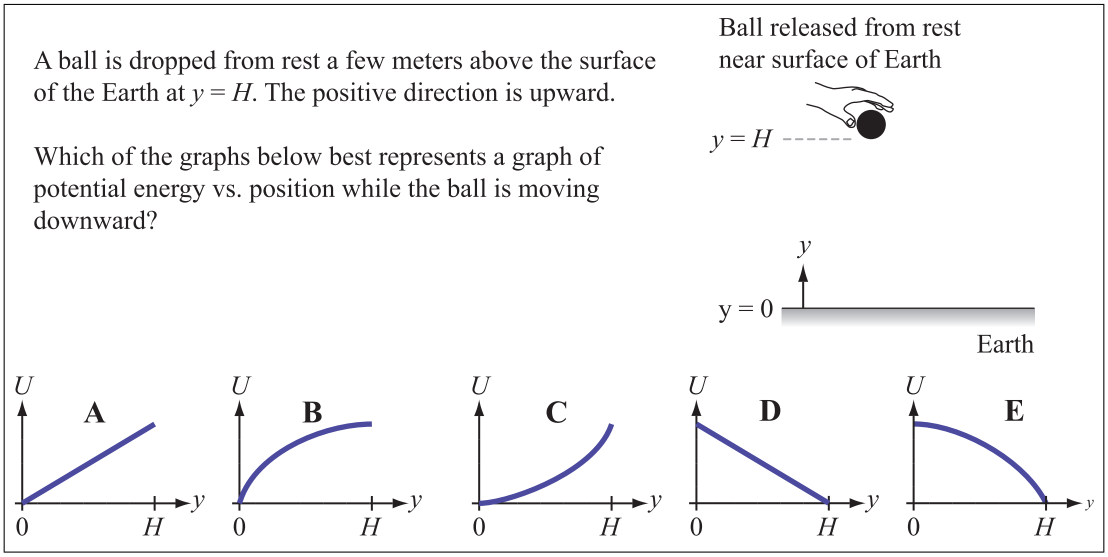
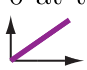


Figure 4.11: Abbreviated version the 'ball-drop' question. The correct answer is graph A.

diagram of the ball-Earth system during the interval when the ball is moving downward. It is important to note here that the horizontal axis on all of the potential energy diagrams were labeled with $y = 0$ at the origin (vertical axis-crossing) and with $y = H$ on the right. The correct answer is  (graph A) since a uniform gravitational force in the negative direction ($F = -mg$) implies a potential energy that is linear in height. (Note that the direction of motion is irrelevant to answering the question. See §4.3.3 for further discussion on this point.)

The answer choices that we provided to students were based on hand-drawn potential energy diagrams by Phys 225 students who answered a similar question on a final exam. The five choices represented the most popular graphs.[21] We hypothesized that Phys 121 students would be inclined to draw the same graphs if provided such an opportunity on a free-response question.[22] We found this to be the case on subsequent free-response final

[21]The question administered to Phys 225 students can be seen in Figure A.1 on page 321. Some examples of the hand-drawn graphs can be seen in our previous publication [37].

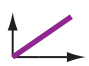
[22]This is consistent with findings from physics education research that show even advanced students can struggle in the same ways as introductory students. See our note on page 34.

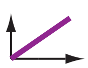
exams. Thus, the answer choices we provide to students represent diagrams that students would draw themselves.

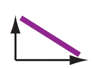
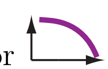
We have administered many different versions of this question. These versions differ in the motion of the ball (e.g., the ball is thrown upward, or no motion is described at all) or providing a mathematical expression for potential energy (i.e., $U = mgy$). We discuss the effect that these modifications have on student performance in §4.3.3.

4.3.1-b Brief summary of performance

The ball-drop question described above was administered to both Phys 121 and Phys 225 students. Student performance from both classes is provided in Table 4.16. Only one-fifth of Phys 121 answered correctly. Most of the explanations by these students cited either the expression for potential energy or the linearity of potential energy. The following explanations are typical of these students:

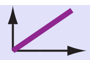
> *"[The answer is .] When H is large, potential energy is large because of the formula mgh."*
>
> (Phys 121A, autumn 2014)

> *"[The answer is .] The potential energy is proportional to the distance the ball is from the surface."*
>
> (Phys 121A, autumn 2014)

About 70% of the Phys 121 students answered either  or . A discussion the reasoning students use to arrive at these graphs is provided in §4.3.2.

A small percentage of students (on the order of 5%) appeared to use universal gravitation instead of near-Earth gravitation. Although not incorrect, our intention was for students to consider the simpler $U = mgy + U_o$ expression. We discuss modifications made to the question to address this tendency in §4.3.2-c.

Table 4.16: Student performance on the 'ball-drop' question. The correct answer is highlighted.

Answer	PHYS 121 Multiple-choice N=190[a]	PHYS 225 Hand-drawn N=68[b]
 (Graph A)	20%	60%
 (Graph B)	5%	15%
 (Graph C)	5%	5%
 (Graph D)	35%	10%
 (Graph E)	35%	5%

[a] Section 121A144. [b] Section 225A101.

A greater percentage of the PHYS 225 students answered correctly (60%). However, both correct and incorrect explanations provided by these sophomore students were similar in nature to those of the PHYS 121 students. This is consistent with other findings that show even more advanced students can struggle with the same concepts that introductory students do, albeit in different proportions [62–64].

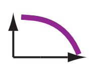
4.3.2 Identification of student difficulties

Here we summarize some of the difficulties that students encounter in choosing or drawing potential energy diagrams. Most of the discussion is based on the 'ball-drop' question shown in Figure 4.11. These difficulties seem to arise from a failure to recognize that the potential energy diagram is a function of position only and is independent of the motion of the object of interest. Additionally, some of the difficulties appear to be related to those previously discussed in §4.1. We discuss these commonalities when appropriate.

4.3.2-a Tendency to associate accelerated motion with curved potential energy diagrams

Answers and explanations provided by some students suggest that they were associating accelerated motion with curved potential energy diagrams. For the ball-drop question shown in Figure 4.11, about 40% of the students incorrectly chose curved graphs.[23] Of these students, 70% mentioned in their explanations that the ball is accelerating; only 5% of the students who drew straight graphs mentioned acceleration. The following explanations are typical of this reasoning:

> *"[The answer is .] Since the ball is released from height of H, the potential energy is going to be the greatest at that height, it has concave down curve because the ball doesn't accelerate as fast initially as it does towards the end."*
>
> (Phys 121A, autumn 2014)

> *"[The answer is .] [...] it [can't] be linear because there is the acceleration of gravity."*
>
> (Phys 121A, autumn 2014)

These students incorrectly ascribed accelerated motion to a curved graph of potential energy *vs.* position. One way to account for this observation is that they are overgeneralizing their knowledge of kinematic graphs: Since potential energy is connected with position, and the kinematic graphs encountered in a mechanics course typically plot time on the horizontal axis, theses students may be thinking of how curved position *vs.* time graphs reflect accelerated motion.

Commentary: Relation to other difficulties

We note that there is a similarity between the difficulty we described above and one of the difficulties we identified in our investigation of kinematic quantities. For example, in §4.1.3-

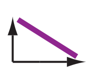
[23] This 40% is different than simply adding up the appropriate percentages in Table 4.16 due to rounding to the nearest 5%.

b, we found that about 10% of the students associated an inflection point of a potential energy diagram with zero acceleration. This reasoning is consistent with associating *curved* potential energy diagrams with a non-zero acceleration. It is interesting, though, that the proportion of students exhibiting this tendency on the direction of acceleration questions (about 10%) is quite different than on the ball-drop question (about 40%). More research is needed to understand the extent to which different questions can elicit the same underlying difficulties in very different proportions within the context potential energy diagrams.

We also note that this tendency is somewhat related to a finding by Loverude [31]. He found that some students believed the motion of an object influences the value of potential energy. These students claimed, for example, that if two objects of the same mass and at the same height, but one of them is moving upward, the object that is moving upward has more potential energy.

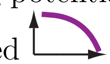
4.3.2-b Tendency to interpret the graph from left to right

Answers and explanations provided by some students suggest that they were interpreting the graph from left to right (e.g., they were treating the horizontal axis as if it represented time instead of position). On the ball-drop question shown in Figure 4.11, about 70% of the students incorrectly chose graphs with negative slopes. The vast majority of these students mentioned the decreasing potential energy. Of the students who chose positive-sloped graphs, relatively few mentioned the decreasing potential energy. The following explanations are typical of this reasoning:

> *"[The answer is*  *.] The potential energy decreases as the heigh[t] decreases"*
>
> (Phys 121A, autumn 2014)

> *"[The answer is .] y decrease, P decrease"*
> (Phys 121A, autumn 2014)

These students appear to have mapped a decreasing potential energy to a 'decreasing' graph (if that graph is read from left to right).

It was fairly common for students to discuss both the decreasing potential energy and accelerated nature of the motion. Nearly all of these students answered , as the following quote illustrates:

> *"[The answer is .] The ball accelerates toward the earth so the potential energy decreases exponentially"*
> (Phys 121A, autumn 2014)

Commentary: Relation to other difficulties

We note that there is a similarity between the difficulty we described above and some of the difficulties we identified in our investigation of kinematic quantities. In §4.1.3-a we found that up to 60% of the students interpreted graphs from left to right when arguing about how the speed of a particle is changing (these students assumed that the potential energy decreases if the slope is negative, which would imply an increase in speed). Additionally, in §4.1.3-c we found that between 15% and 30% of students associated the slope of the potential energy diagram with the direction of motion.

The responses of these students indicate a failure to recognize the horizontal axis of potential energy diagrams plots *increasing* values of *position* on the horizontal axis.

Additional question: Periodic spring motion

We have observed students exhibiting a similar tendency to interpret the graph from left to right on a written pretest in the context of a spring. The question is shown in Figure 4.12. Students are asked to draw and label a potential energy diagram for a spring-block system that begins its motion at $x_{\max}$, turns around at $x_{\min} < x_{\max}$, then ends its motion again at

A block is attached to an ideal spring of spring constant k on a level, frictionless surface, as shown. Let system SB be composed of the spring and block.

The block is initially held in place at $x = x_{max}$. It is then released. It starts to move to the left toward the equilibrium position x_o, reaches the turn-around point x_{min}, moves right toward x_o, and finally returns to x_{max}.

In the space at right, **sketch a graph** of the potential energy U *vs*. position x for system SB.. Use the convention that $U = 0$ at the equilibrium position x_o. **Label the horizontal axis** with x_{min}, x_o, and x_{max}.

No explanation is necessary.

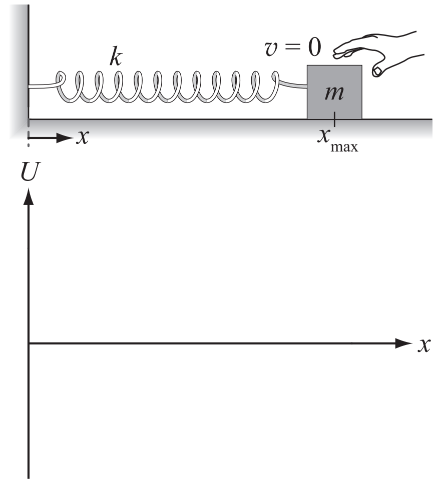
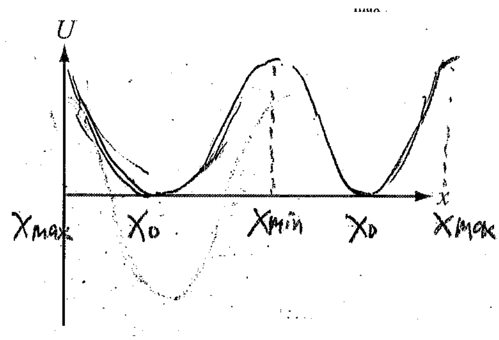


Figure 4.12: Written pretest asking students to draw a potential energy diagram for a spring-block system that undergoes periodic motion. Administered to 33 Phys 121 students in autumn 2014. The correct shape is quadratic with vertex at $x = x_o$.

x_{max}. The equilibrium position x_o (between x_{min} and x_{max}) is specified to have a potential energy of zero. We considered a drawing correct if it had $U(x = x_o) = 0$ and increasing values toward both x_{min} and x_{max}.

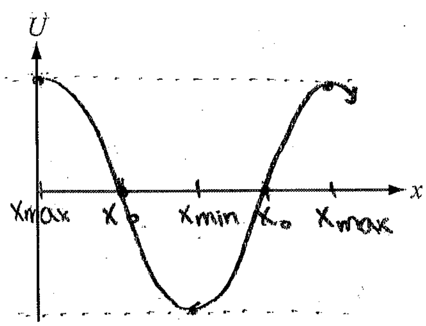
About one-third of the 33 students to whom we administered the question drew or labeled graphs consistent with interpreting the graph from left to right. These students either labeled the horizontal axis in the incorrect order (x_{max} to the left of x_{min}) or drew a periodic-like potential energy curve. Students who drew periodic-like curves typically labeled several instances of x_o and x_{max} on the horizontal axis, as the following student-drawn graph illustrates:



(Phys 121C, autumn 2014)

These students appear to be treating the graph as being 'created' from left to right and labeling the position at each moment in time. They fail to realize both that each position x should be labeled only once on the horizontal axis and that the order of the x-coordinates on the horizontal axis should be increasing from left to right.

We also note that 14 of the 33 students drew potential energy curves with at least some negative values. Many of these students incorrectly indicated that $U(x) < 0$ for $x_{\min} < x < x_{\max}$, as illustrated in the following drawing:

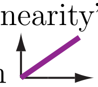
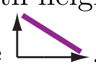


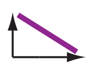
(Phys 121CK, autumn 2014)

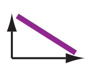
These students failed to realize that the minimum value of potential energy for a spring occurs at the equilibrium position x_o.

This pretest was administered during the first few minutes of students' tutorial sections. The question is identical to the first exercise of the in-class worksheet through which students subsequently worked. This allowed us to ask students follow up questions. We found that many students who drew incorrect graphs could reproduce the formula $U = \frac{1}{2}ks^2$ from memory. However, they did not spontaneously consider using this expression to help draw the potential energy curve. This observation may be related to the tendency to ignore or misuse mathematical expressions, which is discussed below in §4.3.2-c.

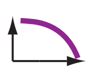
4.3.2-c Tendency to ignore or misuse mathematical expressions for potential energy

Answers and explanations provided by some students suggest that they were unable to connect a correct mathematical expression or description for potential energy with the shape of the potential energy diagram. On the ball-drop question shown in Figure 4.11, about 55% of the students explicitly wrote some form of $U = mgy$, or discussed the "linearity" of potential energy with height. Of these students, only 30% chose the correct graph . Most of the rest chose . The following explanations are typical of these latter students:

> *"[The answer is*  *.] Potential energy = Mass x gravity x height. Therefore, as the height goes down, potential energy will go down."*
>
> (Phys 121A, spring 2014)

> *"[The answer is*  *.] Within a few meters of the surface of the earth, the change in the gravitational field is negligible so the change in potential energy is pretty much linear with respect to height."*
>
> (Phys 121A, spring 2014)

About 10% of all students correctly described the mathematical relation as above, but chose a *curved* graph. The following student quote is illustrative:

> *"[The answer is*  *.]* $U = mgh$. *h is decreasing quadratically so U must as well"*
>
> (Phys 121A, spring 2014)

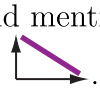
These student failed to connect their mathematical knowledge of potential energy with the correct graph of potential energy. They instead mapped the temporal decrease in the expression $U\left(y(t)\right) = mgy(t)$ to the "graphical" decrease of the potential energy curve.

4.3.3 Effects of modifications to the 'ball-drop' question on student performance

To probe student thinking in greater detail, we made several modifications to the 'ball-drop' question in Figure 4.11 on page 96. Below we describe these modifications and the extent to which they did or did not affect student performance.

4.3.3-a Effect of explicitly providing the expression for potential energy

As we described briefly in §4.3.1-b, a small percentage of students appeared to use universal gravitational potential energy when answering the 'ball-drop' question. These students tended to select curved graphs, which would be correct for $U(r) = -\frac{GMm}{r} + U_o$. To address this, and to provide a greater opportunity for students to answer correctly, we administered several questions in which we explicitly included the equation "$U = mgy$" in the question. This equation appeared in both the text of the question as well as the accompanying diagram.[24]

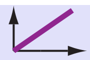
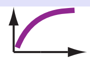
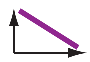
We had anticipated that fewer students would choose curved graphs when provided the expression mgy because, as described in §4.3.2-c, students who were *not* provided the expression but did mention it or the linearity of potential energy in their explanations tended to select graph .

The results of the 'ball-drop' question with and without the expression mgy present is shown in Table 4.17. Surprisingly, performance on the two versions is essentially the same. About 35% of the students chose curved graphs when provided with the expression, compared to 40% without. Furthermore, the number of students answering correctly (25%) with the expression is similar to that without (20%).

The failure of students to incorporate correctly the given mathematical expression of potential energy into their graphs emphasizes the tendency of students to focus on aspects of the balls' motion.

[24] The complete question can be seen in the appendix in Figure A.21 on page 381.

Table 4.17: Performance on the 'ball-drop' question with and without the inclusion of the expression $U = mgy$. The correct answer is highlighted.

Answer	With expression N=171[a]	Without expression N=190[b]
	25%	20%
	5%	5%
	∼ 0%	5%
	40%	35%
	30%	35%

[a] Section 121C144. [b] Section 121A144.

4.3.3-b Effect of modifying or removing the described motion

In the original 'ball-drop' question, students were told that a ball is dropped from rest a distance H above the ground. The responses by many students indicated they were incorrectly focusing on the downward and accelerated motion of the ball (§4.3.2). To explore this tendency further, we asked several additional versions with different descriptions of the motion. In one version, which we call the 'up-down' version, students are told that a ball is thrown upward from $y = 0$, turns around at $y = +H$, and returns to $y = 0$. Students are then asked to choose a potential energy diagram for the upward portion of the motion and separately for the downward portion.[25] Students were not provided with the relation $U = mgy$.

In another version, which we call the 'no-motion' version, we did not describe any motion, nor did we provide a diagram of the ball or ground. Students were asked to choose the potential energy diagram that corresponds to a ball-Earth system. Since on other versions many students seemed to incorporate temporal aspects into their graphs, we provided the

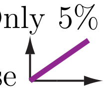
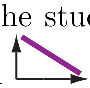
[25] The complete question can be seen in the appendix. It is labeled Questions 9 and 11 in Figure A.15, which begins on page 360.

additional answer choice of "not enough information" to these students. We provided the expression $U = mgy$ to all of these students.[26]

Based on our past observations, we expected students to perform better on the no-motion version as compared to the original ball-drop version in Figure 4.11 since students may be less likely to prescribe motion to the ball. We had no expectations of how students would perform on the up-down version as compared to the ball-drop version.

Student performance on all three versions is shown in Table 4.18(a). As expected, students performed markedly better on the no-motion version than on the original ball-drop version (65% compared to 20%). However, 35% of the students still answered incorrectly, even though they were provided the expression for potential energy. These students tended to ascribe a downward motion to the ball in their explanations, despite the fact that the question did not explicitly provide any description of motion.

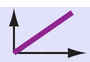
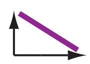
Performance for the downward motion of the up-down version is 10 percentage points lower than the original ball-drop question. We typically exclude a 10 percentage point difference between two administrations. However, we note that performance on the downward portion of the up-down question is the lowest performance of all of the versions of this question we have asked (including an additional version that we discuss below in §4.3.3-c). More research is needed to understand whether the small difference we observed can be reproduced with a larger population.

Popular answer combinations for the up-down version are shown in Figure 4.18(b). Despite there being 36 possible answer combinations (including "not enough information," which only 2 students chose), the vast majority of students fall within only one of a few combinations. Only 5% of the students answered both questions correctly. About 35% of the students chose  and . These students failed to recognize that potential energy diagram is independent of the motion of the ball. (Despite the incorrect sign of the slope, about two-thirds of answers discuss the linearity of gravitational potential energy in either

[26]Students were informed that the ball is always within a few meters of the surface of the Earth so that the equation $U = mgy$ could be motivated. The complete question can be seen in the appendix. It is labeled Question 2 in Figure A.26 on page 391.

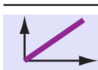
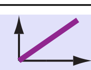
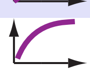
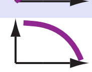
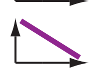
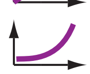
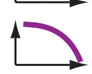
Table 4.18: Student performance on different versions of the 'ball-drop' question. The correct answers are highlighted.

(a) All versions.

	Up-down N=204[a]		**Ball-drop** N=190[b]	**No-motion** N=127[c]
Answer	Upward	Downward	Downward	N/A
	40%	10%	20%	65%
	40%	5%	5%	5%
	15%	5%	5%	5%
	∼ 0%	35%	35%	10%
	5%	45%	35%	15%

[a] Section 121B142. [b] Section 121A144. [c] Sections 121A151 and 121B151.

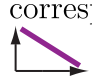
(b) Answer combinations for the 'up-down' version.

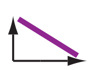
Upward	Downward	N=204
		5%
		35%
		35%
		10%
Other combinations		15%

their upward or downward explanation, compared with one-third of all other answer pairs.) In addition, about 35% of the students chose  and  for the upward motion and downward motions, respectively. (These graphical features precisely mimic those of a graph of $U(t)$.)

4.3.3-c Effect of first asking students to choose a U vs. t graph

Given the tendency of many students to incorporate temporal aspects of motion into their potential energy diagrams, we developed an additional version of the ball-drop question in which students are asked to choose a diagram representing potential energy *vs.* time prior to choosing a potential energy diagram. Our goal was to determine whether or not this change would result in fewer students incorporating temporal aspects in their potential energy diagram. We anticipated that providing students an opportunity to consider both U *vs.* t and U *vs.* x curves might help them to distinguish between spatial and temporal graphs. The choice of curves for the U *vs.* t graph was the same as that for the potential energy diagram.[27]

Student performance is shown in Table 4.19. About 75% of the students chose the correct graph for U *vs.* t. We categorized most of the corresponding explanations as correct or partially correct. About 20% of the students chose . Many of these cited either the equation $U = mgy$ or the constant gravitational acceleration. For example:

> *"[The answer for U vs. t is* *.] $U = MGH$, so this is a linear relationship. As the ball moving downward, its height decreases relative to earth, so the potential energy should decrease."*
>
> (Phys 121B/C, spring 2015)

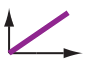
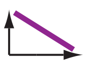
This student and others with similar explanations appear to have overgeneralized the linearity of the relationship between potential energy and position.

Somewhat surprisingly, the percentage of students who chose the correct graph for U *vs.*

[27] The complete question can be seen in Questions 11 and 13 of Figure A.28, which begins on page 404.

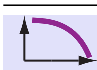
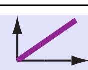
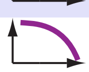
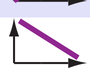
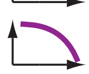
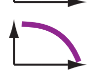
Table 4.19: Effect of asking students to choose a U *vs.* t graph prior to choosing a U *vs.* x graph for the 'ball-drop' question. The correct answers are highlighted.

(a) Comparison to original ball-drop question.

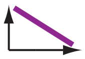
	With additional question N=228[a]		**Without additional question** N=190[b]
Answer	U *vs.* t	U *vs.* x	U *vs.* x
	∼ 0%	20%	20%
	∼ 0%	10%	5%
	5%	5%	5%
	20%	45%	35%
	75%	20%	35%

[a] Sections 121B152 and 121C152. [b] Section 121A144.

(b) Common answer combinations for U *vs.* t and U *vs.* x.

U *vs.* t	U *vs.* x	N=228
		10%
		40%
		15%
Other combinations		35%

x did not differ substantially from that for the original ball-drop question.

Common answer combinations are shown in Table 4.19(b). Only 10% of the students answered both questions correctly. About 40% of the students chose  for U *vs.* t and  for U *vs.* x. These students distinguished between the two graphs (i.e., they did not choose the same graph for both questions) but they incorrectly associated a temporal decrease in potential energy with a left-to-right decrease of the potential energy diagram. Explanations for these were similar to those that we discussed in §4.3.2-b.

About 15% of the students chose  for both graphs. Most of the explanations on the two questions were identical to each other or very similar. Some students even stated "same reasoning" when answering the second question. These students correctly identified the U *vs.* t graph but failed to distinguish it from the U *vs.* x graph. They did not realize that $U = mgy$ implies a linear potential energy diagram.

4.3.4 Summary

In choosing or drawing potential energy diagrams, many students failed to recognize that the diagram is independent of the motion of the ball. Even when no explicit discussion of motion was provided, many students ascribed a direction of motion to the object of interest. In considering the motion, these students attempted to incorporate temporal features of the system into their diagram, such as associating a temporally decreasing potential energy with a negative slope of potential energy or an accelerated motion with a curved potential energy diagram. These tendencies are similar to some of the difficulties we identified in §4.1.3-a and §4.1.3-b.

4.4 Student ability to reason about the arbitrary nature of energy

In this section we describe student difficulties related to the arbitrary nature of potential energy. The difficulties described here seem to arise from a failure to recognize that only *changes* in potential energy are physically relevant.

Other researchers have found similar results to some of those we describe below (see, for example Bao [27]). These prior findings, however, tend to be qualitative and anecdotal in nature. Our approach contributes a quantitative aspect to the literature and uses questions specifically designed to elicit student ideas related to the arbitrary nature of energy.

We begin by describing the questions we administered as part of this investigation. We then discuss the student difficulties we have identified.

4.4.1 Description of questions

We have asked a variety of questions that probe the extent to which students understand that both the potential energy and the total energy for a given system are arbitrary up to an additive constant. Many of these questions explore the idea that a given physical system or experimental setup can be represented by more than one potential energy diagram. Other questions ask only if a given diagram could represent a possible physical system.

The 'shifted diagrams' questions

There are three closely-related questions we have asked that involve comparing several potential energy diagrams (which include a total energy line) to an 'original' system. One such question is the 'four shifted diagrams' question shown in Figure 4.13. Students are shown four different potential energy diagrams. The diagrams differ from the original one in that their potential energy curves are vertically shifted upward (positively) or downward (negatively). Two of the diagrams (A and C) also had their total energy lines shifted in the same manner as the potential energy. Students are asked to select which of the four diagrams could represent the *same experimental setup as the original system with a particle*

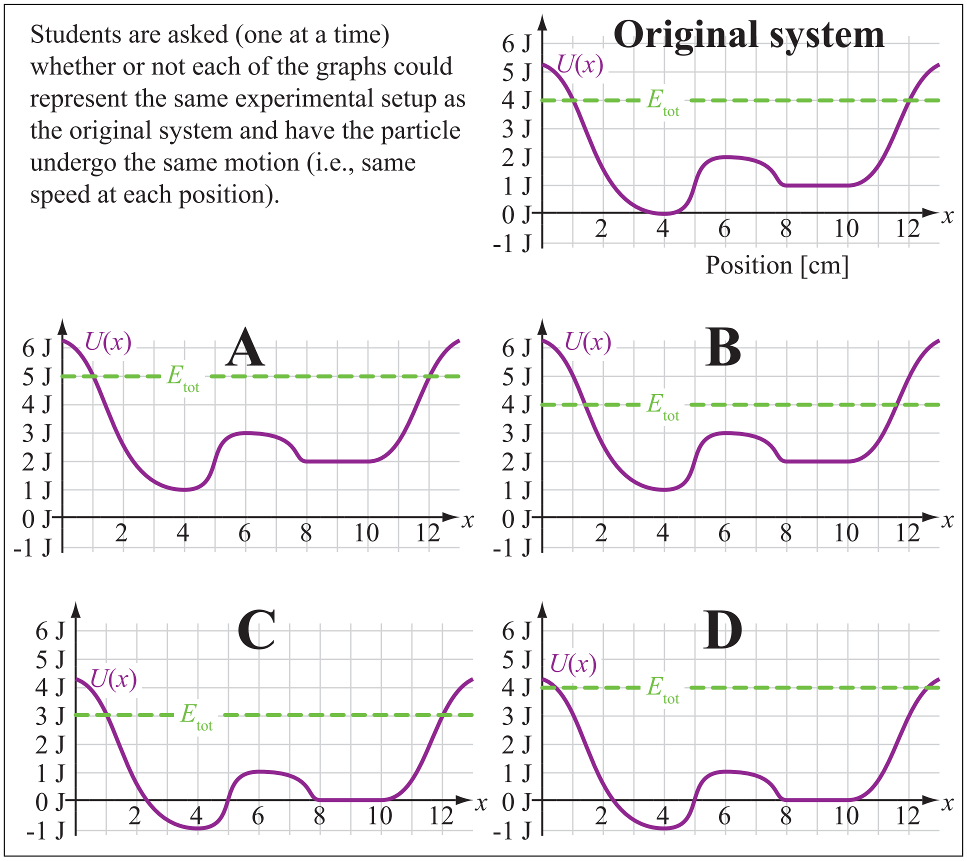


Figure 4.13: Abbreviated version of the 'four shifted diagrams' question. The correct answers are only graphs A and C.

that undergoes the same motion. The question describes the phrase *same motion* as meaning that the particle has the same speed at every position.

To answer correctly, students can note that only changes in potential energy are relevant, so none of the four diagrams shown can be ruled out on the basis of potential energy alone. However, only systems A and C have the same kinetic energy $K = E_{\text{tot}} - U$ at every position since the total energy for these graphs is shifted by the same amount as the potential energy curve. Thus, the correct answer is *Yes* to systems A and C, but *No* to systems B and D. (We typically abbreviate a given student's answers for systems A and B together, and for systems C, and D together. For example, we abbreviate the correct answer as *Yes-No, Yes-No,* which are the responses to graphs A, B, C, and D, respectively.)

Two other questions we have administered are called the 'three shifted diagrams' and 'two shifted diagrams' questions. These are shown in Figures 4.14 and 4.15, respectively. They are similar to the 'four shifted diagrams' questions but with fewer graphs from which to choose. The reasoning required to arrive at the correct answers are the same as that on the 'four shifted diagrams' question. Thus, the correct answer on the 'three shifted diagrams' is both graphs II and III. The correct answer on the 'two shifted diagrams' question is only graph II.

Note that some of the graphs on the 'four shifted diagrams' and 'three shifted diagrams' questions include negative potential energy; the graphs on the 'two shifted diagrams' question do not.

The 'positive spring shift' and 'negative spring shift' questions

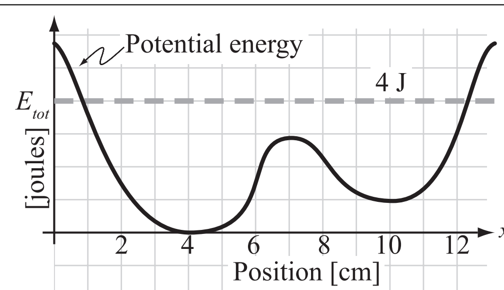
The previous three questions used generic potential energy diagrams that were not associated with a specific context. Two questions we have administered that are associated with a specific context are the 'positive spring shift' and 'negative spring shift' questions. These are shown in Figure 4.16. Students are presented with a potential energy diagram of the form $U(x) = Ax^2$ for a spring that obeys Hooke's law. Students are asked whether a new potential energy diagram that has been vertically shifted (either upward or downward) could represent the potential energy of the same spring. Each student was presented with only one of these two questions (either an upward or a downward shift). In both cases, the correct answer is that the new diagram could represent the same spring.

The 'possible system' question

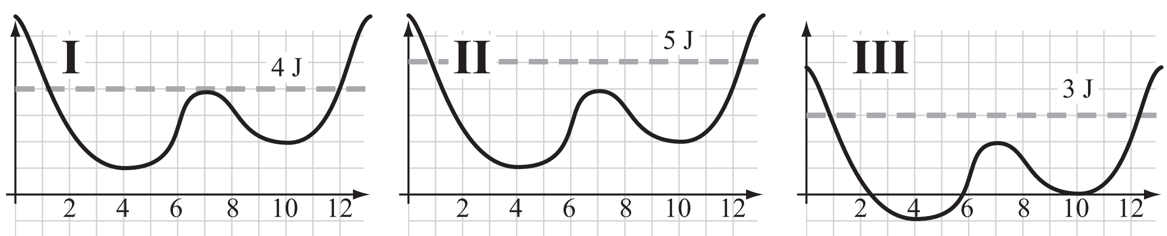
The questions we have described above involve asking students whether a given setup or system could be represented by different potential energy diagrams. In contrast, the 'possible system' question shown in Figure 4.17 does not. In this question, students are shown four potential energy diagrams that are identical except for their vertical shifting. Two of the potential energy diagrams are non-negative, and two include negative values. Students are

A particle is part of a one-dimensional system. The two forms of energy in the system are translational kinetic energy of the particle and potential energy of the system. All internal forces are conservative (*i.e.*, there is no energy loss in the system).

The graph at right shows the potential energy of the system as a function of position of the particle. The dashed line shows the total energy (kinetic plus potential) of the system.



Which of the three graphs below could represent the <u>same physical system</u> with an identical particle undergoing the <u>same motion</u>. (*Note:* Same motion means the particle has the same speed at every position.)



A. Only I

B. Only II

C. Both I and II

D. Both II and III

E. None of the graphs

Figure 4.14: The 'three shifted diagrams' question. The correct answer is choice D (both II and III).

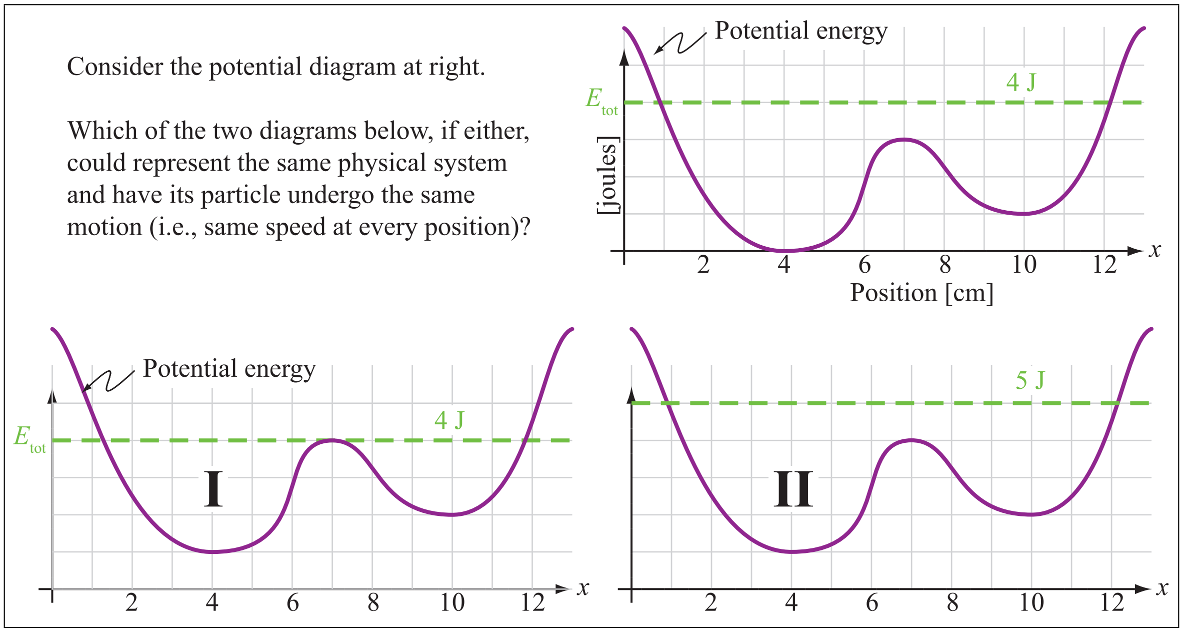
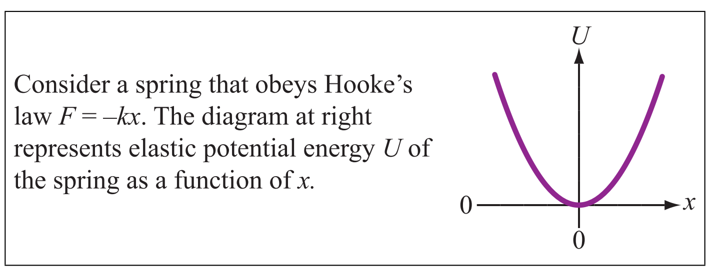
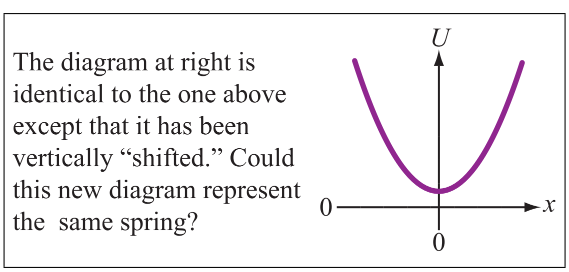


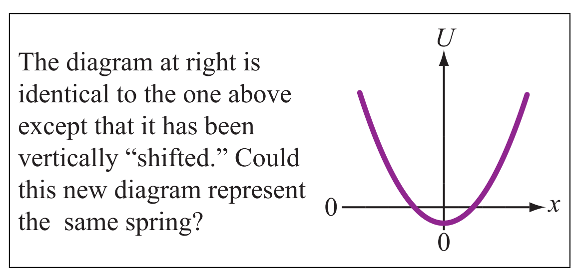
Figure 4.15: Abbreviated version of the 'two shifted diagrams' question. The correct answer is only graph II.



(a) Setup for the questions.



(b) The 'positive spring shift' question.



(c) The 'negative spring shift' question.

Figure 4.16: Abbreviated versions of the 'positive spring shift' and 'negative spring shift' questions. The correct answer is *Yes* to both versions.

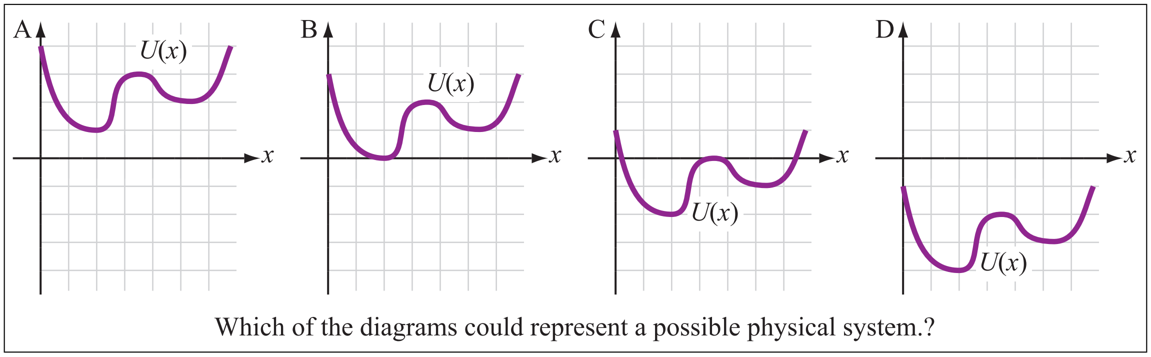


Figure 4.17: The 'possible system' question. The correct answer is *all of them*.

asked to indicate whether or not each diagram could correspond to a *possible* physical system. The correct answer is that all of the diagrams could correspond to a possible physical system.

4.4.2 Identification of student difficulties related to the arbitrary nature of potential energy

There are three difficulties we have identified that are related to the arbitrary nature of potential energy: the belief that potential energy cannot be negative; the belief that potential energy is not arbitrary; and the tendency to characterize a system by its total energy. The responses by some students suggest that these difficulties are not independent of each other. Below we provide examples of student responses that illustrate these difficulties.

4.4.2-a Belief that potential energy cannot be negative

The responses of some students indicate that they believed potential energy cannot be negative (but that potential energy is otherwise arbitrary). Other researchers have found that students struggled in interpreting negative *total* energies [27]. We extend this research to *potential* energy. Below we provide examples of student responses from several of the questions that we discussed in §4.4.1.

Evidence from the 'four shifted diagrams' question

On the four shifted diagrams question shown in Figure 4.13, students are asked which of the four graphs could represent the same experimental setup. The results are shown in Table 4.20. About 25% of the students answered something other than *No-No* to the pair of questions for systems A and B (indicating that at least one of the positive-shifted diagrams is possible) but answered *No-No* for the pair of questions for systems C and D (indicating that neither of the negative-shifted diagrams is possible). Compare Tables 4.20(b) and (c).) Explanations provided by the vast majority of these students suggest that they do not believe potential energy can be negative. For example, some students answered correctly for systems A and B (*Yes-No*), but answered *No-No* for systems C and D. The following student quotes are illustrative:

> *"[Yes-No, No-No.] The object can't have negative potential energy. "*
> (Phys 121A, spring 2014)

> *"[Yes-No, No-No.] Theres no such thing as negative potential energy "*
> (Phys 121A, spring 2014)

Some students stated (incorrectly) that both systems A and B could represent the original system (*Yes-Yes*), but believed neither C nor D could:

> *"[Yes-Yes, No-No.] No, there is no such thing as negative potential energy so the graph can not go below 0 so this would not work."*
> (Phys 121A, spring 2014)

These students believed some of the positively-shifted graphs are possible, but that none of the negatively-shifted graphs are.

Table 4.20: Student performance on the four shifted diagrams question. The correct answers are highlighted.

(a) Responses by question.

System	Description	Answer (Same setup?)	$N{\approx}200$[a]
A	U and E_{tot} shifted up	Yes	75%
B	U shifted up	No	60%
C	U and E_{tot} shifted down	Yes	50%
D	U shifted down	No	75%

[a] Section 121A142.

(b) Answer combinations for systems A and B.

Answer combination		$N{\approx}200$
A	B	
Yes	No	50%
Yes	Yes	20%
No	Yes	20%
No	No	10%

(c) Answer combinations for systems C and D.

Answer combination		$N{\approx}200$
C	D	
Yes	No	40%
Yes	Yes	10%
No	Yes	15%
No	No	35%

(d) Answer combinations for students that answer in the same way on A & C as on B & D.

Answer combination		$N{\approx}125$
A & C	B & D	
Yes	No	55%
Yes	Yes	10%
No	Yes	20%
No	No	15%

(e) Answer combinations for all systems

Answer combination				$N{\approx}200$
A	B	C	D	
Yes	No	Yes	No	35%
Yes	Yes	Yes	Yes	10%
Yes	Yes	No	No	10%
Yes	No	No	No	10%
No	Yes	No	Yes	10%
No	No	No	No	10%
Other combinations				10%

Table 4.21: Student performance on the 'three-' and 'two shifted diagrams' questions. The correct answers are highlighted.

(a) The 'three shifted diagrams' question.

Answer	N=72[a]
Only I	20%
Only II	15%
Both I and II	10%
Both II and III	45%
None of the graphs	10%

[a] Section 121A113.

(b) The 'two shifted diagrams' question.

Answer	N=165[a]
Neither	10%
Only graph I	10%
Only graph II	70%
Both graphs	10%

[a] Section 121A112.

Evidence from the 'three shifted diagrams' and 'two shifted diagrams' questions

The 'four shifted diagrams' above was administered before all lecture instruction on universal gravitation was completed. We recognize that exposure to this topic may help students realize that negative potential energy is possible (since the associated potential energy is usually written as $U(r) = -G\frac{Mm}{r}$). However, we administered both the 'three shifted diagrams' and the 'two shifted diagrams' questions to a different section's final exam after all lecture instruction. (See Figures 4.14 and 4.15, respectively.) In these question, students are asked to select which of three (or two) new graphs could represent the same system as an original one (see the figures). We found that the same tendency was elicited on these questions.

Student performance on the 'three shifted diagrams' question is shown in Table 4.21(a). About 15% of the students incorrectly chose only graph II. This answer is consistent with the belief that potential energy and total energy are arbitrary, but that any shifts must not result in negative potential energy. A similar percentage (10%) of students chose the analogous question on the 'two shifted diagrams' question, as shown in Figure 4.21(b).

Evidence from the 'possible system' question

The 'possible system' question also elicited the belief that potential energy cannot be negative. (See Figure 4.17). In this question, students are asked which of the four diagrams could represent a possible physical systems.

About 50% of the students (N=566)[28] answered that the two positive graphs are possible but that the two negative graphs are not. These results suggest that many students did not believe that potential energy can be negative.

We note that the percentage of students exhibiting this incorrect idea is higher on the 'possible system' question than the previous ones. One possible cause is that this question removes the extra step of connecting a *given* system to multiple potential energy diagrams. Additional research is needed to identify the underlying cause of the difference.

Additional finding: Stating that potential energy cannot be negative does not imply that the underlying difficulty is related to negative potential energy

We have collected evidence that suggests on certain types of questions, some students may state that negative potential energy is not possible, but that the underlying difficulty is not associated with negative potential energy. This tendency was elicited on the 'negative spring shift' and 'positive spring shift' questions (see Figure 4.16). In this question, students are shown a potential energy diagram of the form $U(x) = Ax^2$ for a spring that obeys Hooke's law. They are asked whether a different diagram that is shifted upward (positive) or downward (negative) could represent the potential energy of the same spring.

On the 'negative' version, about 65% of the students (N=189)[29] stated incorrectly that the shifted diagram could not represent the same spring. Most of these students stated that either negative potential energy is not possible or that springs in particular cannot have negative potential energy. The following student quotes are illustrative of this reasoning:

---

[28]This question was administered to sections 121A132, 121B132, 121A133, and 121A134.

[29]Section 121A144.

> *"There can't be negative energy in a system, so the new graph could not represent the U of the system."*
> (Phys 121A, autumn 2014)

> *"you cannot have negative energy stored in the spring"*
> (Phys 121A, autumn 2014)

If taken alone, these result might suggest that a large fraction of students do not believe negative potential energy is possible for the case of a spring. However, we also asked the 'positive spring shift' question to a different section in the same quarter. This question is identical to the negative version except that the diagram is shifted upward. About 60% of the students (N=170)[30] answered that the shifted diagram could not represent the original spring. This is roughly the same percentage as that on the 'negative spring shift' question. (Explanations that accompany these answers are discussed in §4.4.2-b.)

The fact that these two percentages are essentially the same suggests that some students may state that potential energy cannot be negative for a given situation, but that the underlying reason is related to the arbitrary nature of potential energy. The results can also give insights to researchers into designing valid questions that probe the belief that potential energy cannot be negative.

We note that in the 'shifted' questions we analyzed prior to our discussion of the spring questions, we always included graphs with positive shifts. This was done to isolate those students who believe potential energy cannot be negative from those students who may be struggling with the more fundamental idea of the arbitrary nature of potential energy.

Summary

Our results suggest that many students did not believe that potential energy can be negative. These students failed to realize that the arbitrary nature of potential energy implies that the potential energy associated with any given position may be set to any negative (or

[30] Section 121C144.

positive) arbitrary value, so long as the differences in potential energies between two points is preserved.

4.4.2-b Belief that potential energy is not arbitrary

The responses of some students indicate that they believed potential energy for a given system is not arbitrary (i.e., that it cannot be 'shifted' by a constant amount). Note that this is distinct from the belief that potential energy cannot be negative. Below we provide examples of student reasoning from many of the questions we described in §4.4.1.

Evidence from the 'four shifted diagrams' question

On the 'four shifted diagrams' question shown in Figure 4.13, students were asked which of the four diagrams presented could represent the same experimental setup as the original one. About 10% of the students answered that none of the shifted potential energy diagrams could represent the same experimental setup as the original one (i.e., their four answers were all *No*). The following two quotes are typical of student explanations:

> *"[No-No, No-No,] in a 1-dimensional system, the particle cannot have more potential energy at a certain point if it has the same translational kinetic energy."*
>
> (Phys 121A, spring 2014)

> *"[No-No, No-No.] No because the overall amount of potential energy has increased compared to the original setup."*
>
> (Phys 121A, spring 2014)

Some students explained which aspect of the *physical setup* would need to change in order for the new diagrams to represent the same system:

> *"[No-No, No-No.] The object would be at a greater height."*
>
> (Phys 121A, spring 2014)

(We note that this last response is consistent with the tendency of students to ascribe a gravitational interpretation to potential energy diagrams, which we discussed in §4.1.3-d.)

We interpret these responses as indicating the belief that the potential energy for a given system is not arbitrary. These students failed to realize that the potential energy U at any position x can be set to any value, provided that the differences of potential energies at any two positions is consistent with the given system.

Evidence from the 'three shifted diagrams' and 'two shifted diagrams' questions

We note briefly that both the 'three shifted diagrams' and 'two shifted diagrams' questions (see Figures 4.14 and 4.15, respectively) elicited the belief that potential energy is not arbitrary. These questions are variations of the 'four shifted diagrams' but with fewer graphs to consider. On both of these questions, about 10% of the students stated that none of the diagrams could represent the same physical system as the original one. (See Table 4.21.)

Evidence from the 'positive spring shift' question.

The percentage of students who exhibited the belief that potential energy is not arbitrary is markedly higher on the 'positive spring shift' question (see Figure 4.16(b)) than on any of the other questions we have administered. In this question, students are presented with a potential energy diagram of the form $U(x) = Ax^2$ for a spring that obeys Hooke's law. Students are asked whether a different potential energy diagram that has been vertically shifted upward could represent the potential energy of the same spring.

Only 40% of the students ($N{\approx}170$) answered this question correctly. However, most of the explanations that accompanied these correct answers were incomplete, were unclear, or were incorrect. For example, some students simply stated the vertical shift would need to be explained in some way or would need to be 'accounted for.' The following student quote is illustrative:

> *"[Yes, it could represent the same spring,] but the part responsible for the vertical shift would have to be accounted for."*
> (Phys 121C, autumn 2014)

Some students made correct general statements about potential energy, but then attempted to account for the increased potential energy through gravitational interactions:

> *"[Yes, it could represent the same spring.] You can define U to be zero wherever you want as long as the change in U is the same. Maybe the spring is held above the ground, and the second graph is including some potential energy due to gravity."*
> (Phys 121C, autumn 2014)

Other students made general statements about reference or starting points, but do not elaborate:

> *"[Yes, it could represent the same spring.] Yes it depends where you choose the starting point."*
> (Phys 121C, autumn 2014)

Of the 60% of the students who answered incorrectly, most cited that $U(x = 0)$ is no longer zero, or that $U(x) \neq 0$ for any x. The following student quotes are illustrative:

> *"[No, it could not represent the same spring.] Springs must store zero potential energy at their equilibrium positions, thus this could not be the same spring."*
> (Phys 121C, autumn 2014)

> *"[No, it could not represent the same spring.] Because if the graph below were used it would imply that the spring never reaches a state where there is no potential energy, which cannot happen."*
> (Phys 121C, autumn 2014)

These students failed to realize that the potential energy of a spring is typically *chosen* to be zero at the equilibrium position.

Although somewhat less common, some students supported their answer by citing that $U(x)$ for the two graphs are simply different, and did not discuss the equilibrium position. For example:

> *"[No, it could not represent the same spring.] There is always more potential energy in graph 2 at any point x than there is in graph 1"*
> (Phys 121C, autumn 2014)

Other students incorrectly use Hooke's law to explain their answer:

> *"[No, it could not represent the same spring] because Hooke's law is without constant. [W]hen the spiring is nether compressed or stretched, the potential energy will be 0."*
> (Phys 121C, autumn 2014)

This student failed to relate correctly the force law to the associated potential energy.

As we previously, mentioned, the percentage of students who claimed that the shifted diagram on the 'positive spring shift' question is higher than the corresponding percentages on the other questions we have administered. One possible explanation is that students are less apt to believe that an expression other than commonly used "$\frac{1}{2}kx^2$" could represent the potential energy of a spring. That is, the fact that this question was in a specific context may have contributed to the higher observed percentages. Another possible explanation is that removing the total energy line causes more students to believe that a given system cannot be represented by more than one potential energy diagram. Further research is needed to explore the underlying causes of our observation.

Summary

On some questions, only a small fraction of students (about 10%) appeared to believe that potential energy is not arbitrary. These questions used generic potential energy diagrams without relating them to any specific context. We found, though, that this belief was more strongly elicited in the specific context of a spring. Our results suggest that using questions with context-specific systems may elicit this tendency more strongly. We note, though, that we have not yet asked similar questions in the context of near-Earth gravitation (i.e., $U(y) = mgy + U_o$), in which it is common to exploit the arbitrary nature of potential energy.

4.4.2-c Tendency to characterize a system by its total energy

The responses of some students indicate that they believed a system is characterized by its total energy (i.e., that the total energy of a system is an inherent property of a system and is not arbitrary). Below we provide examples of student responses that illustrate this tendency.

Evidence from the 'four shifted diagrams' question

In the 'four shifted diagrams' question shown in Figure 4.13, students are asked to select which of the four diagrams could represent the same system as an original one. About 20% of the students answered that graph A (for which U and E_{tot} are shifted upward) could not represent the same setup, but that graph B (for which only U is shifted up) could. (See Table 4.20(b).) This answer combination of *No-Yes* for graphs A and B is consistent with the belief that a shifted potential energy diagram is possible (i.e., potential energy diagrams can be arbitrary), but that the amount of total energy for a given system is not. The following are typical explanations:

> *"[No-Yes.] System A has 5J of total energy and the original system has 4J of total energy so how could they be the same? "*
> (Phys 121A, spring 2014)

> *"[No-Yes.] The difference is that one setup has more total energy so although the graph looks the same for each experiment, the fact that one has more energy makes it not the same."*
> (Phys 121A, spring 2014)

These responses suggest that some students treated the total energy as a characteristic of a given system.

Evidence from the 'three shifted diagrams' and 'two shifted diagrams' questions

We note briefly that both the 'three shifted diagrams' and 'two shifted diagrams' questions (see Figures 4.14 and 4.15, respectively) elicited the belief that the total energy of a system is not arbitrary. These questions are variations of the 'four shifted diagrams' but with fewer graphs to consider. On the 'three shifted diagrams' question, about 20% of the students incorrectly stated that only the graph with the shifted potential energy but the same total energy (graph I) could represent the same system. On the 'two shifted diagrams' question, about 10% of the students selected the analogous choice (graph I).

These answers are consistent with the belief that a given system has a non-arbitrary value of energy but an arbitrary value of potential energy.

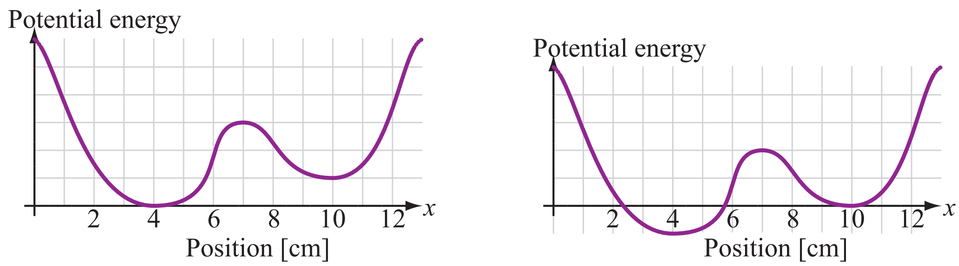
Related difficulty: Misapplication of the conservation of energy

We extended the current investigation beyond Phys 121 and into Phys 123 (the third-quarter of the introductory sequence). In administering a question designed to probe the arbitrary nature of potential and total energies, we observed that some students used a 'conservation of energy' argument to justify why the total energy of a system is not arbitrary. We did not observe this reasoning on the different questions we administered to students in Phys 121.

The question is shown in Figure 4.18 and is a variation of those that we have already discussed. Students are shown a potential energy diagram with $U_{\text{min}} = 0$ that does *not* have a line representing total energy. Then they are shown another diagram that is identical to the first but shifted downward by a constant amount so that $U_{\text{min}} < 0$. Students are asked if it is possible that the new graph represents the same physical system as the original one and, if so, they are asked how the total energy would need to compare to the original total energy. The options available for the total energy were *the same as the original graph* and *less than the original graph.* We call this question the 'determine shifted energy' question.

About 40% of the Phys 123 students (N=146)[31] answered correctly that the new diagram

[31] Section 123C112.



The graph at leftrepresents the potential energy of a system.

The graph at right is identical but has been shifted downward. Could this graph represent the same physical system? If so, how would the total energy need to compare if the particle is to undergo the same motion (i.e., have the same speed at every position)?

Figure 4.18: Abbreviated version of the 'shift determine energy' question. The correct answers are *yes* and *less than.*

could represent the original one provided that the new total energy is less than the original energy. In their explanations, students tended to state only that less potential energy implies less total energy, but provided no further elaboration.

About 35% of the students incorrectly answered that the new diagram could not represent the original system. These students could be struggling with either arbitrary shifts of potential energy or with negative potential energy. This question does not distinguish between these two difficulties, and most student explanations could be interpreted either way. (See our commentary near the end of §4.4.2-a on page 121.)

Of primary interest here, however, is that about 25% of the students incorrectly stated that the new diagram could represent the original one provided that the total energy is *the same.* Most explanations are incomplete and describe only why the negative shift of potential energy is valid. However, many of the explanations that do mention total energy support our earlier interpretation that the value of total energy is a characteristic of a system. The following student quote is illustrative:

> *"[Yes, if E_{tot} is the same.] Different reference for potential energy. Total energy remains the same since it is independent of the reference energy."*
> (PHYS 123C, SPRING 2011)

Interestingly, some students explain why the total energy must be the same by citing conservation of energy:

> *"[Yes, if E_{tot} is the same.] Yes it could, because potential energy is relative. [...] Because energy is conserved, it would have to be identical to the previous graph."*
> (PHYS 123C, SPRING 2011)

> *"[Yes, if E_{tot} is the same.] by the law of concervation [sic] of energy, all energy is conserved. No energy is lost because the graph is exactly the same just shifted downward a little bit."*
> (PHYS 123C, SPRING 2011)

These students appeared to misapply the concept of conservation of energy. They may have overgeneralized the constancy of energy within a system to the equality of total energy across identical systems.

We do not know the cause of why we have observed 'conservation of energy' reasoning only on this administration. It may be due to the different population of students (PHYS 123 *vs.* PHYS 121) or it may be due to subtle differences in the question itself. (We have not yet administered this question to students in PHYS 121.)

4.4.3 Summary

Many students struggled in recognizing that the values of potential and total energies are arbitrary. For some students, the difficulty was rooted in a failure to recognize that potential energy can be negative. For other students, it lay in not recognizing that potential energy for a given system is arbitrary up to a given constant. Other students struggled in not realizing that the total energy in a system is not a characteristic of a system.

We have also found that some difficulties appear to be more strongly elicited when a specific context is used rather than a generic ‘system’ (e.g., a spring system for which the minimum value for potential energy is not zero). More research is needed to understand the extent to which this finding can be elicited in other specific contexts (e.g., near-Earth gravitation).

Chapter 5

RESEARCH-INFORMED DEVELOPMENT OF A NEW TUTORIAL ON POTENTIAL ENERGY DIAGRAMS

In this chapter we provide an overview of the curricular material that we developed to improve student understanding of graphical representations of potential energy. This material is called *Potential energy diagrams* and will become one of the many tutorial topics in *Tutorials in Introductory Physics* (see §2.2) [1].

We first describe the initial development of the in-class worksheet for *Potential energy diagrams* and the accompanying homework. Throughout this discussion, we provide examples of how we incorporated the results from our research into student difficulties (see Chapter 4). We then briefly describe how more recent iterations of the in-class worksheet and homework have changed since their initial development based on ongoing research.

(A discussion of the effects of these curricular materials on student understanding may be found in Chapter 6. A separate component of the tutorial, the *online interactive practice homework*, is discussed in Chapter 7.)

5.1 Initial development of curricular materials for the tutorial Potential energy diagrams

Below we discuss the initial versions of the in-class worksheet and homework for *Potential energy diagrams*.

5.1.1 Initial development of the in-class worksheet

The first iteration of the tutorial *Potential energy diagrams* is shown in the appendix (see Figure A.29 on page 412). It is composed of four sections. Each section is described below.

Section I covers basic interpretations of potential energy diagrams, including the fact that a particle can move both from left to right and from right to left on the graph. As described in §4.1.3-a, many students have a tendency to 'read' potential energy diagrams from left to right (i.e., they assume that the particle moves in the positive x-direction and/or they treat the horizontal axis as if it represented time). Students are guided toward a correct understanding by considering a spring-block system; they are asked whether the direction of motion or the rate of change of speed can be determined solely from the block's location. Our intention was that students would generalize this example to other potential energy diagrams. This section of the tutorial also guides students in developing a graphical interpretation of kinetic energy (i.e., the vertical distance between the total energy line and the potential energy curve).

Section II of the tutorial continues using the spring-block context to guide students toward an understanding of allowed regions, forbidden regions, and turn-around points. Students then generalize these ideas to other potential energy diagrams. We were motivated to include these ideas based on our research that many students seemed to interpret the bounds of the potential energy curve as the bounds of the allowed region (§4.2.2-a).

Section III provides students practice using and interpreting potential energy diagrms in the context of universal gravitation. We were motivated to include this context primarily because our research indicated that some students did not believe that negative potential energy is possible (see §4.4.2-a). Universal gravitation naturally incorporates negative energy with its choice of $U_{\max} = U(r \to \infty) = 0$.

Finally, section IV covers acceleration and force. Students use their earlier graphical representation of kinetic energy to determine the direction of acceleration at various points. They then use Newton's second law to relate acceleration to force. In this way, students develop the connection between potential energy and the quantities acceleration and force prior to seeing the force-potential energy relationship $F = -dU/dx$. This method of first developing concepts and connections prior to introducing formality is consistent with prior work of others [1, 68, 69].

An overview of more recent iterations of the in-class worksheet are described in §5.2.

Next, we discuss the initial development of the written homework for *Potential energy diagrams*.

5.1.2 Initial development of the homework

The first iteration of the written homework for *Potential energy diagrams* is shown in the appendix (see Figure A.37 on page 456). It is composed of four problems. Each problem is described below.

Problem 1 guides students toward an understanding of the arbitrary nature of potential energy. Students consider two potential energy diagrams that are identical except that one is shifted downward so that $U_{\min} < 0$ instead of $U_{\min} = 0$. They discover that measurable quantities (e.g., kinetic energy) are the same for both diagrams. We included this problem based on our research that showed some students did not realize that potential energy is arbitrary and/or that potential energy can be negative (see §4.4).

Problem 2 provides students with a graph of force *vs.* position and asks them to draw the corresponding potential energy diagram. We included this problem to give students practice in understanding the construction of potential energy diagrams; most of the tutorial and the rest of the homework ask students to *use* a given potential energy diagram. Afterward, students consider allowed and forbidden regions. This provides practice in applying these ideas, which were only briefly discussed in the in-class worksheet

Problem 3 provides students practice in thinking about kinematic and dynamic quantities. It includes a question that students often answer incorrectly based on our research (a rate of change of speed question, see §4.1.2-d). Also included is a fictitious student statement that students must critique. The statement deals with the mistaken belief that when potential energy is at a minimum, the kinetic energy is equal to the total energy (see §4.2.2-b).

Finally, problem 4 addresses how potential energy diagrams change if the mass of the object of interest were to be increased. Students are guided to reach the conclusion that it depends on the form of the potential energy. Diagrams whose corresponding interactions depend on mass will change (e.g., gravitational interactions), while those that do not depend on mass will not change (e.g., springs). Our intention was to give students practice in using the expression for potential energy to guide their decision making process. As we discussed in §4.3.3-a, many students did not spontaneously connect the expression for potential energy with the corresponding potential energy diagram.

5.2 Summary of modifications made to the tutorial since its initial development

In this section we briefly describe how more recent versions of the in-class worksheet for *Potential energy diagrams* differ from the initial one described above.[1]

We modified the in-class worksheet in almost every quarter that it was used during the present research. Some of the changes were motivated by our observations during tutorial sessions. These changes tended to be relatively minor (e.g., clarifying wording, additional helper questions, etc.) Other changes were motivated by posttest results (which is the focus of Chapter 6). These changes tended to be larger in scale, such as the addition of new sections or the use of new instructional approaches. Below, we describe only a few of the more significant changes and reference relevant research from Chapter 6 as appropriate. The most recent iteration of the tutorial is shown in the appendix (see Figure A.36 on page 449).

Section I of the tutorial now places more emphasis on the motion of the block-spring system and asks students to predict the shape of the potential energy diagram. Students are then led toward constructing a correct graph of potential energy diagram by explicitly mapping specific points in the motion of the block to the graph. This decision was based on (1) the posttest results described in §6.3.1, in which we saw little to no improvement in student ability to draw potential energy diagrams for ball-Earth systems, and on (2) the Physics Education Group's experience that students are able to learn from their prior incorrect ideas if they are confronted with those ideas and are guided to resolving any inconsistencies.[2] We describe this modification to the worksheet and the accompanying research in more detail in §6.3.2-a.

Section II of the tutorial now places greater emphasis on guiding students toward an understanding that, although the direction of motion of a particle cannot be determined from a potential energy diagram, the direction of the acceleration *can* be determined. This change

[1] Modifications made to the written homework were relatively minor. This was due partially to the fact that the homework is typically not collected since it is assigned during the last week of class. Thus, it is difficult for us to gather student responses. We do not discuss changes made to the homework here.

[2] See §2.2.2 for a discussion of this instructional strategy.

The following questions can be used as a guide to check your answers to part C.

1. Suppose that the block were moving in the positive *x*-direction at $x = x_1$ (see the top diagram at right).

   Dir of motion

   - Based on the potential energy diagram, would the block be *speeding up* or *slowing down?* Circle your answer at right.

   Speeding up?
   Slowing down?

   - Use your answer to determine the direction of the acceleration. Draw an arrow to indicate the direction at right. Explain briefly.

   Dir of acceleration

   Suppose instead that the block were moving in the negative *x*-direction at $x = x_1$ (see the top diagram at right).

   Dir of motion

   - Based on the potential energy diagram, would the block be *speeding up* or *slowing down?* Circle your answer at right.

   Speeding up?
   Slowing down?

   - Use your answer to determine the direction of the acceleration. Draw an arrow to indicate the direction at right. Explain briefly.

   Dir of acceleration

Figure 5.1: Excerpt from recent iteration of the *Potential energy diagrams* tutorial that focuses on acceleration.

was motivated by the posttest results we describe in §6.1.3-a in which students showed a greater tendency after tutorial to answer that the direction of acceleration cannot be determined. Students first predict whether or not the acceleration can be determined. (During our observations of the in-class tutorial, many students predicted incorrectly.) Students are then led to the correct answer using the questions shown in Figure 5.1 in which they discover that the direction of acceleration is the same regardless of the direction of motion of the particle.

Another modification we made is the use of generic, non-context-specific potential energy diagram earlier in the tutorial (compare the autumn 2014 version, which begins on page 438, with the winter 2015 version, which begins on page 449).[3] We noticed during our observations

[3]Generic potential energy diagrams were removed from early iterations of the tutorial. What we discuss here is the re-addition of such diagrams.

of tutorial sessions that many students were relying not on the potential energy diagram to make kinematic arguments but on their experience with springs. For example, some students would use the memorized response that an object attached to a spring will accelerate toward the "relaxed" position. Our intention was to provide students the opportunity to understand how potential energy diagrams alone can be used to make kinematic arguments.

The last modification we discuss here is the removal of the section on universal gravitation. This was done primarily to keep the length of the tutorial short enough so that it can be completed by most students within the allotted 50 minutes. It is important to note that with the removal of this section, students no longer have an in-class opportunity to consider negative potential energy. We discuss possible issues related to this in §6.4.1.

Chapter 6

RESEARCH-BASED ASSESSMENT OF NEW TUTORIAL ON POTENTIAL ENERGY DIAGRAMS

In this chapter we compare student performance on questions related to potential energy diagrams prior to tutorial instruction (‘pretests’) to performance after tutorial instruction (‘posttests’). This chapter is organized according to the main categories of questions that were presented in Chapter 4. These questions probed: the ability to reason about kinematic and dynamic quantities; the ability to relate the kinetic and total energies to potential energy diagrams; the ability to choose or draw potential energy diagrams based on descriptions of motion; and the ability to argue about the arbitrary nature of potential energy. (Results from the online interactive practice homework are discussed in Chapter 7).)

Although not always possible, we avoid administering the same type of question to a given section of students as both a pretest and a posttest. A discussion of our method of comparing pretest and posttest performances is discussed in §2.4.2.

Finally, we remind the reader of the abbreviated scheme used to designate all course sections. See page 23 for a description.

6.1 Effect of tutorial instruction on student ability to reason about kinematic and dynamic quantities based on potential energy diagrams

In this section we compare student ability to reason about kinematic and dynamic properties using potential energy diagrams before and after tutorial instruction. Student reasoning prior to tutorial instruction was discussed in §4.1.

Each subsection below is devoted to one of the four question types that we have administered, which were determining (1) the direction of motion, (2) the sign of the rate of change of speed, (3) the direction of the acceleration, and (4) the direction of the force. Examples of each of these questions are shown in Figure 6.1.

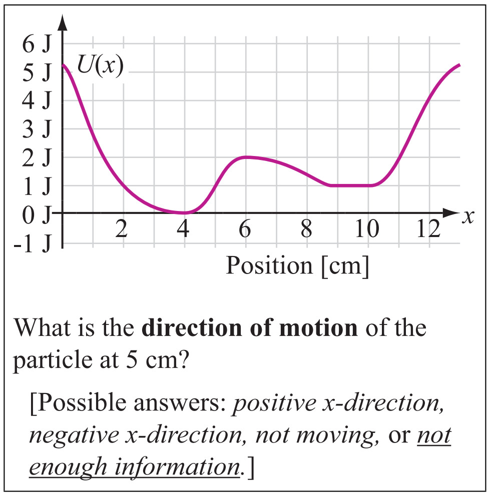
6.1.1 Effect of tutorial instruction on student ability to reason about the direction of motion

On the 'direction of motion' questions, students are presented with a potential energy diagram and are asked to determine, if possible, the direction of motion (or the direction of the velocity) of the particle. See Figure 6.1(a) for an example. The correct response to this question is that there is *not enough information* to answer since the diagrams show only what the potential energy U would be *if* the particle were located at position x; the particle could move either in the positive or negative x-directions.

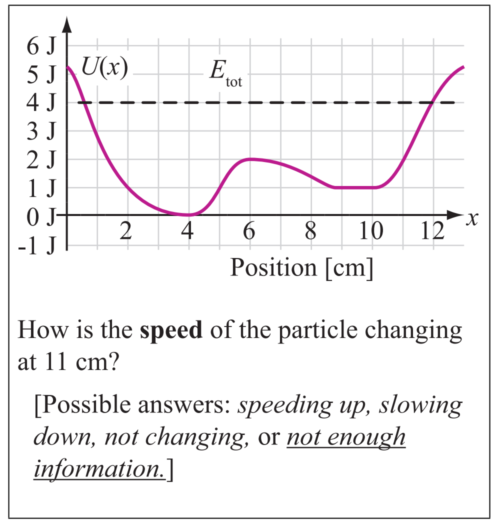
Table 6.1(a) compares pretest and posttest results. (Only results from points at which $dU/dx > 0$ or $dU/dx < 0$ are shown. We have not administered posttest questions for which $dU/dx = 0$.) The fraction of students who answered correctly increased from 30% before tutorial instruction to 70% after. This increase coincided with a decrease in the fraction of students who incorrectly answered that the particle moves in the positive x-direction (from 50% to 20%), which was the most popular answer prior to tutorial instruction.

Not all students who answered correctly provided reasoning that we categorized as correct.[1] Table 6.1(b) shows the most common student answers and explanations for the cases in which the slope of the potential energy diagram at the point of interest is positive (i.e.,

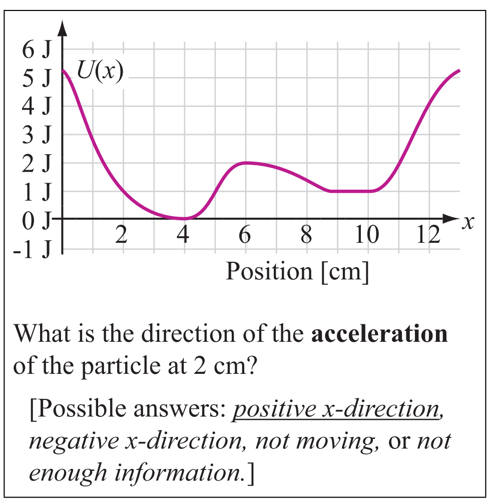
[1]See §4.1 for a discussion of our criteria for correct reasoning.



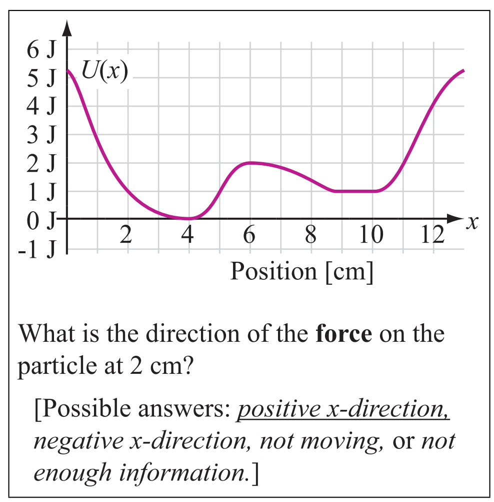
(a) Example of direction of motion question.



(b) Example of speed question.



(c) Example of acceleration question.



(d) Example of force question.

Figure 6.1: Abbreviated examples of questions used to probe student ability to relate potential energy diagrams to motion. Each of the correct answers is underlined.

Table 6.1: Pretest to posttest comparison of student performance on the 'direction of motion' question. The correct answers or explanations are highlighted.

(a) Answers for points at which $dU/dx > 0$ or $dU/dx < 0$.

Answer	Pretest N=917[a]	Posttest N=854[b]
Not enough information	30%	70%
Positive x-direction	50%	20%
Negative x–direction	15%	10%
Other/$v = 0$	5%	<5%

[a] Sections 121A112, 121B112, 121A132, 121A133, 121D134, 121A143, and 121D144.
[b] Sections 121A134, 121A141, 121B141, and 121C144.

(b) Answers and explanations for points at which $dU/dx > 0$.

Answer / Explanation	Pretest N=372[a]	Posttest N=426[b]
Not enough information	**30%**	**70%**
w/correct reasoning	10%	45%
claims particle slows	10%	5%
other	10%	20%
Positive x-direction	**50%**	**20%**
uses "slope" reasoning	20%	5%
claims particle slows	5%	5%
other	25%	10%

[a] Sections 121A132, 121A133, and 121D134.
[b] Sections 121A134 and 121C144.

$dU/dx > 0$).[2] On pretests, only 10% of the students provided both a correct answer and a correct explanation, whereas on the posttests 45% of the students did so. We also note that the percentage of students with correct answers and explanations on posttests exceeds the percentage of students with only correct answers on pretests.

Also shown in Table 6.1(b) are students who gave correct answers but indicated somehow in their reasoning that the particle slows down. (This reasoning is consistent with treating the potential energy diagram as if it were a graph of potential energy *vs.* time rather than *vs.* position.) The percentage of students in this category is roughly the same on pretests and posttests: 10% and 5%, respectively. Thus, the increase in the percentage of students who answered the 'direction of motion' questions correctly is not a result of more students using this particular incorrect reasoning.

On pretests, 20% of the students stated incorrectly that the particle moves in the positive direction because the slope dU/dx is positive. (This was the most common type of explanation students gave for this answer when $dU/dx > 0$.) On posttests, this percentage droped to 5%.

We interpret the results above as an indication of the effectiveness of the tutorial *Potential energy diagrams.*

6.1.2 Effect of tutorial instruction on student ability to reason about the rate of change of speed

We now describe the effect of tutorial instruction on the ability of students to reason about how the speed of a particle is changing at a particular location on a potential energy diagram. On these 'speed' questions, students are asked to determine, if possible, how the speed of the particle is changing. See Figure 6.1(b) for an example. On all the questions we administered, the slope at the point of interest was either positive or negative (i.e., non-zero). Thus, the correct response is that there is *not enough information* to answer since the particle could

[2] We have only administered 'direction of motion' questions that ask students for *explanations* for points at which $dU/dx > 0$. We note briefly, though, that performance on questions for which $dU/dx < 0$ increased from 30% with correct answers on pretests (N=362) to 75% with correct answers on posttests (N=195).

Table 6.2: Pretest to posttest comparison of student performance on the 'speed' question. The correct answer is highlighted.

	Pretest N=934[a]	Posttest N≈1200[b]
Not enough information	10%	55%
Consistent with "left-to-right" reasoning	75%	35%
Opposite of above	15%	5%
Other	5%	5%

[a] See Table 4.4 on page 50. [b] Sections 121A132, 121A133, 121D134, 121A143, 121C144, 121B151, and 121B152.

be moving either in the positive or negative x-direction. Note that this question is, in some respect, more difficult than the 'direction of motion' question discussed above since students must first recognize that the direction of motion is not known.

Student performance on the speed question improved from 10% correct on pretests to 55% on posttests, as shown in Table 6.2. Nearly all of the students with correct answers also provided correct reasoning.

The increase in the percentage of students who answered correctly from pretest to posttest coincided with a decrease in the percentage of students who claimed that the particle slows down if $dU/dx > 0$ (or speeds up if $dU/dx < 0$): from 75% on pretests to 35% on posttests. On both pretests and posttests, the vast majority of students who answered in this way used 'left-to-right' reasoning (i.e., either believing the particle moves in the positive x-direction or treating the graph as a function of time rather than as a function of position). See §4.1.3-a for a discussion of this reasoning.

We have also administered the 'speed' question to tutorial TAs as a pretest as part of their preparation course.[3] Of the 31 TAs, 17 answered correctly (about 55%). The rest typically used 'left-to-right' reasoning. This performance is similar to that for the introductory students. Many of these TAs were themselves undergraduates, but for the

[3]See §2.2.3 for a discussion of this practice.

data we have, there is no noticeable difference between undergraduate and graduate TA performance on this question. This result suggests that after working through the tutorial, introductory students perform as well on the 'speed' questions as our tutorial TAs perform on pretests.

Despite the improvement in introductory student performance, it is important to note that only about half answered correctly on the 'speed' question after tutorial instruction. One-third of students argued that particles slow down where $dU/dx > 0$, which is consistent with 'left-to-right' reasoning. The tendency to read graphs from left to right seemed to persist even with modifications to the tutorial that specifically address this tendency (see §5.2 for a discussion of such modifications).

Additional question: Disambiguating two related 'left-to-right' reasonings

In §4.1.3-a on page 51, we described two related 'left-to-right' lines of reasoning that some students employed to argue about the direction of motion and the sign of the rate of change of speed. We labeled one of these lines of reasoning as 'positive-x-assumption.' Students with this line of reasoning answered that the particle moves in the positive x-direction and that it would slow down if $dU/dx > 0$ (or speed up if $dU/dx < 0$). This answer is consistent with interpreting the particle as moving from left to right on the potential energy diagram. The other line of reasoning we labeled 'time-horizontal-axis.' Students with this line of reasoning answered correctly that the direction of motion cannot be determined, but answered incorrectly that the particle would slow down at a location where $dU/dx > 0$. This answer is consistent with interpreting the horizontal axis as if it were time because they answered that the rate of change of speed can be determined, but that the direction of motion is unknown. On page 57, we described our attempt to distinguish between these two lines of reasoning by administering both a speed question and a direction of motion question on the same quiz.

We administered the same pair of questions as a posttest. Table 6.3 compares pretest and posttest student performance. On pretests, about 55% of the students used one of these two lines of reasoning, compared to 25% on posttests. Most of this decrease appears to be

Table 6.3: Pretest to posttest comparison of student tendency to use one of the two 'left-to-right' lines of reasoning.

Type of 'left-to-right' reasoning	Pretest N=144[a]	Posttest N=224[b]
Positive-x-assumption	40%	15%
Time-horizontal-axis	15%	10%
Not 'left-to-right' reasoning	45%	75%

[a] Section 121D144. [b] Section 121C144.

attributed to fewer students using the positive-x-assumption reasoning (from 40% to 15%).

As the table shows, after tutorial instruction roughly equal proportions of students (15% and 10%) are using the two separate 'left-to-right' lines of reasoning to argue about speed and direction of motion. This result can inform future curriculum development: Further attempts to significantly increase student performance may need to address both of these related lines of reasoning.

6.1.3 Effect of tutorial instruction on student ability to reason about the direction of acceleration

On the 'acceleration' questions, students are presented with a potential energy diagram and asked to determine, if possible, the direction of the acceleration of the particle at a particular position x. See Figure 6.1(c) on page 143 for an example. To arrive at a correct answer, students can use the one-dimensional force-potential energy relationship $F = -dU/dx$ along with Newton's second law and conclude that, for example, a positive slope on the potential energy diagram corresponds to an acceleration in the negative direction (and vice versa). Alternatively, students can (1) assume a direction of motion (e.g., in the positive x-direction), (2) argue about how the potential energy and kinetic energy change, and then (3) use the direction of motion and the change in speed to determine the direction of the acceleration.

As we describe below, posttest performance with an early version of the tutorial were disappointing. They suggested that some students might have been overgeneralizing their

Table 6.4: Pretest to posttest comparison of student performance on the 'acceleration' question after an early version of the tutorial. The correct answer is highlighted. Note the increase in the percentage of students choosing *not enough information*.

Direction of acceleration	Pretest N=183[a]	Posttest N=232[b]
Negative x-direction	70%	60%
Positive x-direction	15%	5%
Zero	5%	5%
Not enough information	10%	35%

[a] Section 121B112. [b] Section 121B141.

ideas about the direction of motion in determining the direction of the acceleration. This motivated us to modify the tutorial, and subsequently reassess the effect that the tutorial has on student understanding of the direction of acceleration. Our discussion below first describes student performance on the early version of the tutorial. We then describe performance on the modified version.

6.1.3-a Posttest performance after an early version of the tutorial

Student performance before and after an early version of the tutorial is shown in Table 6.4.[4] The percentage of students answering correctly is essentially the same. The 10 percentage point difference is within typical fluctuations in performance. Surprisingly, however, the percentage of students answering *incorrectly* that there is not enough information increased from 10% on the pretest to 35% on the posttest.

A possible explanation for this result is that students were overgeneralizing the *correct* idea that one cannot determine the direction of the motion of the particle. Their reasoning may have been that, since one cannot know the direction of motion, the direction of acceleration is likewise unknown. (The data presented in the table were administered on a multiple-choice exam. No explanations were gathered. However, subsequent free-response

[4] We only show data that we had collected as of winter 2014.

questions on the direction of acceleration revealed that many students used this line of reasoning.) We note that the in-class worksheet (1) emphasizes early on that the direction of motion cannot be determined and (2) addresses the direction of the acceleration only on the last page of the tutorial. Moreover, a survey in another quarter indicated that 70% of the students did not reach or finish the last page of the tutorial.

In response, we modified the in-class worksheet to incorporate more guiding questions that ask students to consider the direction of acceleration. These modifications were described in §5.2. Below we describe student performance on the modified tutorial.[5]

6.1.3-b Posttest performance after modifications made to the tutorial

Table 6.5(a) compares pretest performance to posttest performance on determining the direction of the acceleration after working through the modified version of the tutorial. The modifications were discussed in §5.2. (Note that we do not compare posttest performance after the pre-modified tutorial to posttest performance after the post-modified tutorial here. See page 152 for more information.) The percentage of students who answered correctly was roughly the same before and after tutorial instruction (about two-thirds of the students). Furthermore, about 20% of the students used 'left-to-right' reasoning to arrive at a correct answer (e.g., they describe the particle as slowing down if $dU/dx > 0$, and use that as a basis to argue why the acceleration is in the negative x-direction). This is roughly the same percentage as observed on pretests. Although we categorized this reasoning as correct on acceleration questions, our results from §6.1.2 indicate that many students incorrectly apply this reasoning to argue about how the speed changes.

The most common incorrect answer on the posttest, which was chosen by 15% of the students, was *not enough information*. Some students in their explanations cited that since the direction of motion is unknown, the direction of acceleration is also unknown. These

[5]We also developed an additional component to the tutorial curriculum partially due to this same observation on the 'direction of acceleration' questions. This component was only used for a single quarter and was administered in a quarter prior to modifying the tutorial. See Chapter 7 for a discussion of this *online interactive practice homework*.

Table 6.5: Pretest to posttest comparison of student performance on the 'acceleration' question after working through a modified version of the tutorial. The correct answers/explanations are highlighted. Blank entries indicate percentages of less than 5%.

(a) Answers.

	Pretest N=581[a]	Posttest N=489[b]
Correct answer	65%	70%
Opposite of above	20%	10%
Zero	5%	5%
Not enough information	10%	15%

[a] Sections 121B112, 121B142, 121A143, and 121D144.
[b] Sections 121A143, 121D144, and 121B151.

(b) Explanations.

	Pretest N=398	Posttest N=181
Left-to-right	25%	20%
Both directions		20%
F or $-dU/dx$	20%	15%
Slope	10%	5%
Concavity	5%	
Dir. v is unknown		5%

[a] Sections 121B142, 121A133, and 121D144. [b] Section 121D144.

students appeared to be overgeneralizing their knowledge about the direction of motion, despite specific modifications made to the tutorial to address this line of reasoning. We note that even after modifying the pretest in-class worksheet, the tutorial curriculum appears not to be effective with regard to this aspect of student understanding of potential energy diagrams.

However, it is worth noting that on posttests, about 20% of the students used 'both directions' reasoning in which they described that the particle could either be slowing down or speeding up depending on its direction of motion. (See Table 6.5(b).) Almost no students used this correct line of reasoning on pretests. We attribute this increase to modifications made to the tutorial that were described in §5.2.

Commentary on comparing pre-modified data to post-modified tutorial data

It is natural to compare the pre-modification posttest data in Table 6.4 to our post-modification posttest data in Table 6.5. However, we do not do this for the following reason. We have argued that some students appeared to overgeneralize their knowledge about the direction of motion when answering a direction of acceleration question. It is possible that the presence of a 'direction of motion' question immediately prior to an 'acceleration' question could influence the likelihood that a given student might incorrectly answer *not enough information.*[6] The pre-modified data presented in Table 6.4 were gathered from a quiz that also included a direction of motion question. However, only a small fraction of the students that are represented in the post-modified data in Table 6.5 were presented with a direction of motion question on their quiz.[7] Thus, we have not collected sufficient data to compare our pre-modified posttest data to our post-modified posttest data.

[6] We note that we have not seen this phenomenon present in our pretest data.

[7] Only one summer section (N=76) had a direction of motion question paired with a direction of acceleration question.

6.1.4 Effect of tutorial instruction on student ability to reason about the direction of force

We now compare student performance before and after tutorial instruction on 'force' questions in which students are asked to determine, if possible, the direction of the force at a specific position. See Figure 6.1(d) on page 143 for an example. One way to answer these questions correctly is to use the one-dimensional force-potential energy relationship $F = -dU/dx$. Using this relationship, a positive slope on a potential energy curve corresponds to a force in the negative x-direction (and a negative slope to a force in the positive x-direction).

Table 6.6(a) compares pretest and posttest performances. There is a 15 percentage point increase in the fraction of students who answered correctly after tutorial instruction (55% before tutorial *vs.* 70% after). Had these data been gathered from single administrations, we would not make a claim about the effect of the tutorial.[8] However, our data include multiple sections for both pretests and posttests. The percentage of students who answered correctly on pretests on each administration ranges from 45% to 55%,[9] whereas the percentage correct on our posttests ranges from 60% to 75%. Thus, the tutorial appears to have an effect, although small, on the ability of students to answer force questions correctly.

Table 6.6(b) compares the lines of reasoning used by students on pretests and posttests. There are two notable differences. One is a decrease in the percentage of students using 'slope' reasoning after tutorial instruction (i.e., equating the sign of the slope to the direction of the force). This decrease is desirable since students should be equating the *negative* sign of the slope to the force. The second difference is an increase in the percentage of students who claimed that, since the direction of motion is unknown, the direction of the force is also unknown. This increase is not desirable. These students appear to have overgeneralized their knowledge about the direction of motion in much the same fashion as we saw on direction

[8] See our discussion in §2.4.2 for more information.

[9] In reporting this data we exclude one section in which a noticeably high percentage of students used 'F or $-dU/dx$' reasoning. See our discussion in §4.1.2-b on page 47 for more information.

Table 6.6: Pretest to posttest comparison of student performance on the 'force' question. Blank entries indicate percentages of less than 5%. The correct answer and various correct explanations are highlighted.

(a) Answers.

Direction of force	Pretest $N{\approx}1150$[a]	Posttest $N{=}530$[b]
Correct answer	55%	70%
Opposite of correct	30%	10%
Zero	5%	5%
Not enough information	10%	15%

[a] Sections 121A132, 121A133, 121A134, 121C134, 121D134, 121A141, 121B141, and 121A142. [b] Sections 121A144, 121B152, and 121C152.

(b) Explanations.

Reasoning	Pretest $N{\approx}1150$[a]	Posttest $N{=}234$[b]
Left-to-right	20%	15%
Both directions		5%
F or $-dU/dx$	15%	20%
Minimize U		15%
Slope	20%	5%
Dir. of v is unknown		15%

[a] See table above for a list of sections.
[b] Section 121A144. Only 60% of these students answered correctly, which is the lowest of our posttest administrations.

of acceleration questions in §6.1.3.

Also notable is the increase in the percentage of students who used a 'minimize U' reasoning. These students argued that particles or systems tend to "prefer" or "end up" in regions of lower potential energy. We consider this a correct line of reasoning for force questions. Very few students used this line of reasoning on pretests, but 15% of the students did on this posttest. The tutorial does not discuss this line of reasoning. Additional data is needed to understand whether or not this observation is specific to the particular section of Phys 121 to which we administered this question.[10]

6.1.5 Summary

We have observed some improvement in some of the kinematic/dynamic questions we have administered, and little to no improvement in others. Student performed better on both the 'direction of motion' and 'speed' questions after tutorial instruction. Fewer students treated the particle as moving from left to right on the graph, though a large fraction of students do still use this line of reasoning after tutorial instruction. Performance on the 'acceleration' question is roughly the same after tutorial instruction. However, we have observed more students using the 'both directions' line of reasoning on posttests, which is emphasized throughout tutorial instruction. Performance on the 'force' is only marginally greater after tutorial instruction, which may be a result of the relatively minor role force plays in the in-class worksheet.

[10] We have only administered one time the 'force' question as a posttest that asked students to provide explanations. The rest of our data come from multiple-choice exams.

6.2 Effect of tutorial instruction on student ability to relate kinetic and total energies to potential energy

In this section we compare pretest and posttest results on questions that probe student ability to relate kinetic and total energies to potential energy. These questions typically require students to use the relation $E_{\rm tot} = U + K$ at one or more points on a potential energy diagram in some way (e.g., determining the maximum kinetic energy or the turn-around points). In §4.2, we described a number of difficulties that students encountered in answering these questions. Here we focus only on the 'major' difficulties with which a large fraction of students struggled. These difficulties were: the belief that the bounds of the potential energy curve are the bounds of the allowed region (§4.2.2-a); the tendency to treat the minimum value of potential energy as being zero (§4.2.2-b); the tendency to treat kinetic and potential energies as being inversely proportional (§4.2.2-c); and the tendency to treat the particle as if it 'tunneled' through barriers (§4.2.2-g).

6.2.1 Effect of tutorial instruction on the belief that the bounds of the potential energy curve are the bounds of the allowed region

In §4.2.2-a, we discussed the belief by some students that the bounds of the potential energy curve are the bounds of the allowed region (i.e., treating the particle as if it were able to reach all positions for which the potential energy curve is defined). This belief was elicited by a number of different types of questions. Two of the questions were those in which students were asked to determine the maximum kinetic energy. We discuss posttest performance on these two questions first. We then discuss posttest performance on three additional questions that do not ask for the maximum kinetic energy.

Results from the 'maximum kinetic energy' questions

In the '$K_{\rm max}$ from $E_{\rm tot}$' question, students are presented with a potential energy diagram and a line representing the total energy. They are asked to determine the maximum kinetic energy that the particle attains. To answer correctly, students can use the relation $E_{\rm tot} = K + U$

and conclude that the maximum kinetic energy can be found by subtracting the minimum potential energy from the total energy (i.e., $K_{\text{max}} = E_{\text{tot}} - U_{\text{min}}$). (An example of this question can be seen in Figure 4.3(a) on page 71.) A different question, the 'K_{max} from x_{rel}' question, is similar to the previous one except that students are provided with the point of release of the particle instead of the total energy. To determine the maximum kinetic energy, students must first realize that the total energy is equal to the potential energy at the point of release (i.e., $E_{\text{tot}} = U_{\text{rel}}$). The maximum kinetic energy can then be found in the same manner as in the previous question. (An example of this question can be seen in Figure 4.4(a) on page 72.)

On the 'K_{max} from E_{tot}' question, about 30% of the students on pretests used the maximum value of potential energy U_{max} on the graph instead of the total energy to determine the kinetic energy (i.e., they used either the '$K_{\text{max}} = U_{\text{max}} - U_{\text{min}}$' or the '$K_{\text{max}} = U_{\text{max}}$' method). This result is shown in Table 6.7(a). On the 'K_{max} from x_{rel}' question, about 15% of the students on pretests used one of these two methods. (See Table 6.7(b)). These methods are consistent with the tendency to treat the particle as if it reaches the smallest and largest positions for which the potential energy curve is defined, which are outside of the allowed region. After tutorial instruction, only 5% of the students used this line of reasoning on each of these questions.

Furthermore, the percentage of students who answered correctly improved after tutorial instruction. On the 'K_{max} from E_{tot}' question performance increased from 55% correct to 85%. On the 'K_{max} from x_{rel}' question performance increased from 20% correct to 70%.

We attribute this improved performance to the tutorial *Potential energy diagrams* in which students graphically interpret kinetic energy as the vertical line between the total energy line and the potential energy curve.

We have also administered the 'K_{max} from E_{tot}' question to tutorial TAs as a pretest as part of their preparation course.[11] All of the 21 TAs answered correctly. Many of these TAs were themselves undergraduates. This result could suggest that after working through

[11]See §2.2.3 for a discussion of this practice.

Table 6.7: Pretest to posttest comparison of student performance on two types of 'maximum kinetic energy' questions. Blank entries indicate percentages of less than 5%. The correct answers are highlighted.

(a) The 'K_{max} from E_{tot}' question.

Method	Pretest N=135[a]	Posttest N=218[b]
$E_{\text{tot}} - U_{\text{min}}$	55%	85%
$U_{\text{max}} - U_{\text{min}}$	25%	5%
E_{tot}	5%	5%
U_{max}	5%	

[a] Section 121D134.
[b] Section 121B132.

(b) The 'K_{max} from x_{rel}' question.

Method	Pretest N=146[a]	Posttest N=228[b]
$E_{\text{tot}} - U_{\text{min}}$	20%	70%
$U_{\text{max}} - U_{\text{min}}$	10%	5%
E_{tot}	35%	10%
U_{max}	5%	
$U_{\text{max}} - E_{\text{tot}}$	10%	5%
$E_{\text{tot}} - U_{\text{local min}}$	10%	

[a] Section 121D134.
[b] Sections 121C144 and 121C152.

the tutorial, introductory students perform almost as well as our tutorial TAs perform on pretests. More data for both introductory student posttest performance and tutorial TA pretest performance is needed, however, to make a stronger claim.

Results from the 'E_{tot} from x_{rel}' question

Another question that elicited the belief that the bounds of the potential energy curve are the bounds of the allowed region is the 'E_{tot} from x_{rel}' question. This question is similar to the 'K_{max} from x_{rel}' question in that students are provided with the location of the point of

Table 6.8: Pretest to posttest comparison of student ability to determine the total energy when the point of release is provided. The correct method is highlighted.

Method	Pretest N=226[a]	Posttest N=224[b]
$E_{\text{tot}} = U_{\text{release}}$	45%	65%
$E_{\text{tot}} = U_{\text{max}}$	45%	25%

[a] Sections 121B152 and 121C152.
[b] Section 121A152.

release of the particle. However, fewer steps in reasoning are required. Students are asked only to determine the total energy of the system (not the maximum kinetic energy). (This question is shown in Figure 4.5 on page 73.) To answer correctly, students can note that the kinetic energy is zero at the point of release, so the total energy is equal to the potential energy at that location (i.e., $E_{\text{tot}} = U_{\text{rel}}$).

On the pretests, almost half of students (45%) incorrectly answered that the total energy is equal to the highest value of potential energy on the diagram (see Table 6.8). This is consistent with treating the bounds of the diagram as the bounds of the allowed region as described above. After tutorial instruction, a smaller percentage of students used this method (25%).

We note that the decrease in the percentage of students who equated the total energy with the maximum potential energy on the 'E_{tot} from x_{rel}' is consistent with the decrease in the percentage of students who use either the '$U_{\text{max}} - U_{\text{min}}$' or '$U_{\text{max}}$' method on the two kinetic energy questions we described above.

Results from the 'determine turn-around' question

Another question we have administered that elicited the tendency to treat the bounds of the diagram as the bounds of the allowed region is the 'determine turn-around' question. In this question, students are provided with a potential energy diagram and the point of release of the particle. They are then asked to determine the smallest and largest x-coordinates that

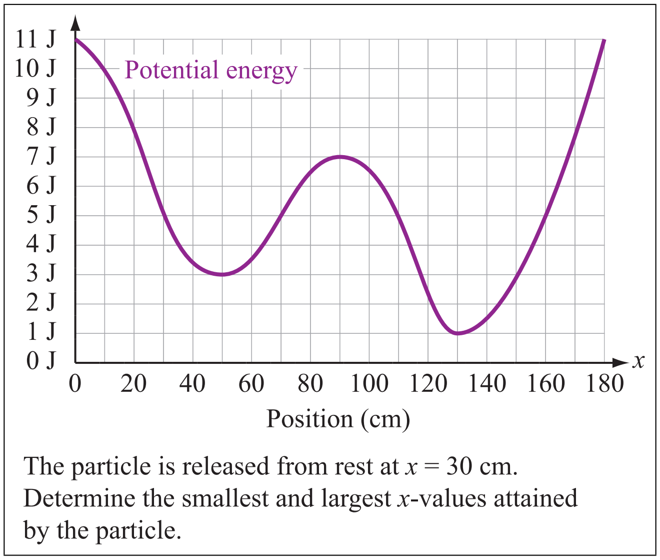


Figure 6.2: Abbreviated version of the 'determine turn-around' question. The correct answers are 30 cm and 70 cm.

the particle reaches. (This question is repeated in Figure 6.2.) The potential energy diagram that we used has two disjoint regions in which $U(x) < E_{\rm tot}$, so to answer correctly students must consider the turn-around point that is closest to the point of release. The smallest and largest x-values attained by the particle are 30 cm and 70 cm, respectively (see the figure). The answer choices provided to students include the smallest and largest x-values on the diagram (0 cm and 180 cm), locations at which $dU/dx = 0$ (50 cm, 90 cm, and 130 cm), and locations at which $U(x) = E_{\rm tot}$ (30 cm, 70 cm, 110 cm, and 160 cm, which include the correct answers).

Table 6.9 compares pretest and posttest student performance. A greater percentage of students answered correctly after tutorial instruction (65%) than before tutorial instruction (35%). On the pretest, 30% of the students stated that 180 cm is the largest x-coordinate reached. This answer is consistent with the belief that the particle 'reaches' the end of the graph, which is outside the allowed region. On the posttest, only 10% of the students answered in this way.

We note, though, that approximately the same percentage of students selected the com-

Table 6.9: Pretest to posttest comparison of student ability to determine the smallest and largest x-values reached by a particle. The correct answer is highlighted.

Response ($x_{\text{smallest}}, x_{\text{largest}}$)	Pretest N=178[a]	Posttest N=203[b]
30 cm, 70 cm	35%	65%
30 cm, 160 cm	5%	15%
30 cm, 180 cm	20%	
0 cm, 180 cm	10%	10%
Other	30%	10%

[a] Section 121A152. [b] Section 121A151.

bination 0 cm and 180 cm for the smallest and largest positions, respectively. This might suggest that it is easier to address the incorrect belief that the particle reaches the 'end' of the graph than it is to address the incorrect belief that the particle starts at the 'beginning' of the graph. More research is needed to understand how each student changes his or her answer from pretest to posttest.

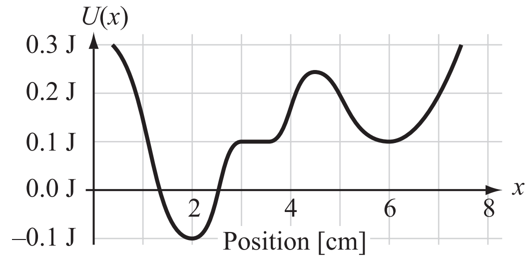
Additional question: Particle located at particular position

We have administered an additional question on a posttest that we did not administer on any pretest. Although we cannot compare performance before and after the tutorial, the results provide some insight into student understanding after tutorial instruction. The question is shown in Figure 6.3. Students are provided with a potential energy diagram without a total energy line and are asked to select one of the three statements that must be true, if any (see the figure). All three statements are false, so the correct answer is *none of these statements is true.*

About 35% of the students answered that the statement *There is an instant when the particle is located at* $x = 7$ cm was correct (N=65). This result suggests that even after tutorial instruction, a large fraction of students believed the bounds of the potential energy curve are the bounds of the allowed region.

It is important to note, though, that students might have believed that more than one of

[4 pts] Which of the following statements **must be true** regarding the potential energy diagram at right?



A. This potential energy diagram is not possible because $U < 0$ in some regions.

B. There is an instant when the particle is located at $x = 7$ cm.

C. The particle will reach $x = 2$ cm and immediately stop.

D. None of these statements must be true.

Figure 6.3: Posttest question administered in spring 2015 to PHYS 121C.

the statements was true. We did not provide a "more than one" option to students. Thus, the 35% of the students who stated the particle must have been located at $x = 7\,\text{cm}$ should only be used as a rough guide of student understanding after tutorial instruction.

6.2.2 Effect of tutorial instruction on the tendency to treat the minimum value of potential energy as being zero

In §4.2.2-b, we described the tendency of students to treat the minimum value of potential energy as if it were zero, despite the fact that we provided students with a potential energy diagram with a non-zero minimum value of potential energy. For example, on the 'K_{max} from x_{rel}' question, students are asked to determine the maximum kinetic energy given the point of release of the particle (see Figure 4.4(a) on page 72). On the pretest, about 40% of the students used either the '$K_{\text{max}} = E_{\text{tot}}$' or '$K_{\text{max}} = U_{\text{max}}$' methods to determine the maximum kinetic energy. These students failed to incorporate the minimum value of potential energy U_{min} on the curve. This failure is consistent with treating the minimum value of potential energy as being zero. (Some of these students explicitly stated in their explanations that the minimum value of potential energy *is* zero, despite the fact hat the potential energy diagram had a non-zero minimum value in the allowed region.) On the posttest, only 10% of the students used this reasoning (see Table 6.7(b) on page 158).

Although the percentage of students exhibiting this tendency on this particular question is smaller after tutorial instruction, we note that we have seen a large range in the percentages of students exhibiting this tendency on other *pretest* questions. This issue is discussed in §4.2.3 on page 90. We note, though, that none of our posttest administrations have resulted in a measurable *increase* in the percentage of students exhibiting this tendency.

6.2.3 Effect of tutorial instruction on the tendency to treat kinetic and potential energies as being inversely proportional

In §4.2.2-c we described the tendency of some students to treat kinetic and potential energy as if they were inversely proportional. This difficulty was elicited on the 'PE from KE' question in which students are told that the kinetic energy in 'region A' is twice that in 'region B.' Students are asked to select which of several statements about the relative potential energies in the two regions is correct. No potential energy diagrams or illustrations of the physical situation are provided to students. (The question and the answer choices are shown in Figure 4.7 on page 74.) The correct statement is that the potential energy in region A *is less than that in region B, but not necessarily half,* because the value of total energy E_{tot} is unknown and influences the value of kinetic energy $K = E_{\text{tot}} - U$.

Table 6.10 compares student performance before and after tutorial instruction. On the pretests, only 20% of the students answered correctly. On the posttests, 45% of the students answered correctly. Likewise, the percentage of students who incorrectly answered that the potential energy in region A is half that in region B decreased after tutorial instruction, from 55% on pretests to 35% on posttests.

However, despite the improved performance on posttests, it is important to note that roughly one-third of students still incorrectly stated that the potential energy in region A is half that in a region B after tutorial instruction. The tutorial curriculum we developed does not directly address this tendency. Instead, students are guided to construct graphical representation of kinetic energy at a given position as the vertical distance between total energy and potential energy on a potential energy diagram. Future iterations of the tutorial

Table 6.10: Pretest to posttest comparison of student tendency to treat kinetic and potential energy as being inversely proportional.

Answer	Pretest N=174[a]	Posttest N=230[b]
$U_A < U_B$, but not necessarily half	20%	45%
$U_A = \frac{1}{2}U_B$	55%	35%
$U_A = 2U_B$	15%	10%
Other	10%	5%

[a] Section 121A152. [b] Section 121B152.

and/or homework may incorporate specific questions to target this difficulty.

6.2.4 Effect of tutorial instruction on the tendency to treat a particle as if it 'tunneled' through barriers

In §4.2.2-g we described the tendency of some students to treat the particle as if it 'tunneled' through forbidden regions. Two questions elicited this tendency: the 'determine turn-around' question and the 'K_{max} w/turn-around' question.

Results from the 'determine turn-around' question

On the 'determine turn-around' question, students are provided a potential energy diagram and the point of release of a particle at $x = 30\,\text{cm}$ (see Figure 4.6 on page 74). They are asked to determine the smallest and largest x-positions that the particle reaches. The diagram has two disjoint regions that satisfy $U(x) \leq E_{\text{tot}}$: $30\,\text{cm} \leq x \leq 70\,\text{cm}$ and $110\,\text{cm} \leq x \leq 160\,\text{cm}$ (see the figure). So, to answer correctly, students must note that the particle does not move past the turn-around point located at $x = 70\,\text{cm}$. Thus, the correct answer is $(x_{\text{smallest}}, x_{\text{largest}}) = (30\,\text{cm}, 70\,\text{cm})$.

The percentage of students who answered correctly increased from 35% on the pretest to 65% on the posttest. (See Table 6.9 on page 161.) Of central importance here, though, is comparing incorrect lines of reasoning. On the pretest, about 5% of the students answered

(30 cm, 160 cm), which are the leftmost and rightmost points on the graph satisfying $U(x) = E_{\text{tot}}$. These students failed to account for the forbidden region in the interval between 70 cm and 110 cm. On the posttest, about 15% of the students answered in this manner.

Although the increase in the observed percentage is within typical fluctuations, we note that an increase could be indicative of tutorial instruction helping students to understand the relevance of the points that satisfy $U(x) = E_{\text{tot}}$ in determining the locations of turn-around points. That is, some students who would not have used either of the correct turn-around points may have attempted to used some points that satisfy $U(x) = E_{\text{tot}}$, but in an incorrect manner. More research is needed to understand a given student changes his or her response to this question after tutorial instruction.

Results from the 'K_{max} w/turn-around' question

The 'K_{max} w/turn-around' question uses the same potential energy diagram as the 'determine turn-around' question above, but asks students to determine the maximum kinetic energy of the particle (see Figure 4.4(b) on page 72). To answer correctly, students must note that the particle cannot reach $x = 130$ cm that corresponds to the global minimum of the potential energy of the system. Thus, the correct answer is $K_{\text{max}} = E_{\text{tot}} - U_{\text{local min}} = 2$ J.

Student performance before and after tutorial instruction is shown in Table 6.11.[12] A greater percentage of students answered correctly on the posttest than on the pretest (40% *vs.* 10%), suggesting that the tutorial helps students account for turn-around points when determining maximum kinetic energy. However, about 20% of the students used the '$E_{\text{tot}} - U_{\text{global min}}$' method on the pretest, compared to 10% on the pretest. This method is consistent with the belief that the particle is able to tunnel through the potential barrier in the region 70 cm $< x <$ 110 cm. As with the 'determine turn-around' question, the percentage of

[12]We most often administered the point-of-release version posttest questions in multiple-choice form. The grading form allows only a maximum of five different answer choices. As shown in Table 4.11 on page 77, there is a large variety in the answers provided by students on the 'K_{max} w/turn-around' question. Thus, we have varied the possible answers on the different posttest administrations of the point-of-release version. The percentages in the table show observed percentages when the corresponding answer choice was available to students.

Table 6.11: Pretest to posttest comparison of student ability to determine maximum kinetic energy when the turn-around point must be considered. The correct method is highlighted.

Method	Pretest N=188[a]	Posttest N≈640[b]
$E_{\text{tot}} - U_{\text{local min}}$	10%	40%
$E_{\text{tot}} - U_{\text{global min}}$	10%	20%
$U_{\text{max}} - U_{\text{global min}}$	40%	10%
E_{tot}	15%	5–15%
Not enough information	N/A*	5%

[a] Section 121A144.
[b] Sections 121D144, 121B151, and 121B152. We do not include data from 121A134 because students worked through an early iteration of the tutorial that was significantly different than later ones. * This answer choice was not provided to students.

students who used ‘tunneling’ reasoning is larger on posttests, but the percentage is within typical fluctuations.

6.2.5 Summary

The posttest results suggest that the tutorial is effective at helping some students relate the total and kinetic energies to the potential energy for certain types of questions. For example, students were better able to use correctly the total energy and the minimum value of potential energy to determine the maximum kinetic energy. We note, though, that even after tutorial instruction a large fraction of students still struggled with many ideas, including (1) using the point of release of the particle, (2) accounting for turn-around points, and (3) understanding how the relative values of kinetic energies inform the relative values of potential energies. Some of these continuing difficulties may result from the tutorial not addressing the some of the underlying tendencies. Future versions of the tutorial may guide students toward a correct understanding of these ideas.

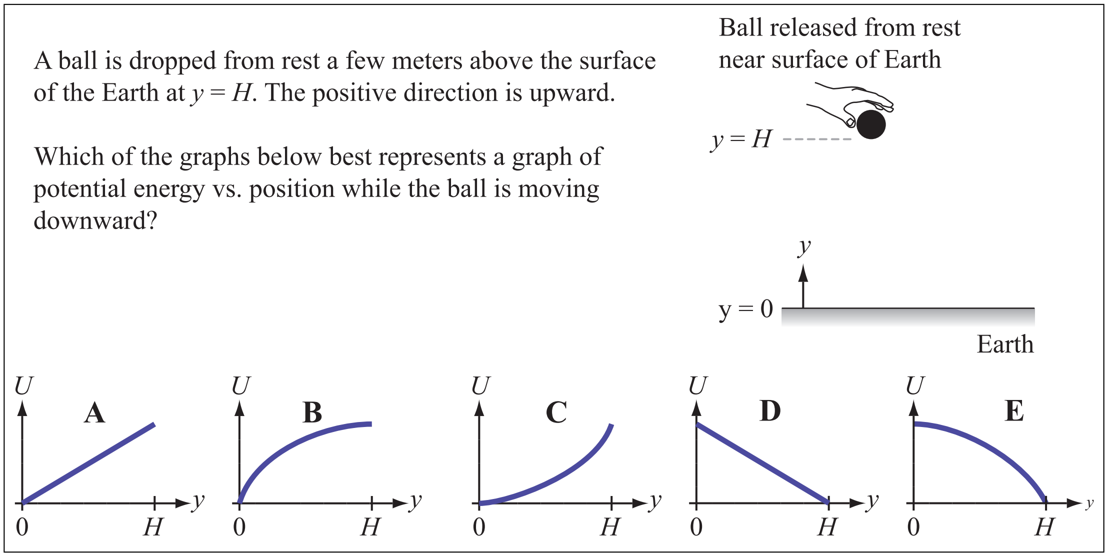
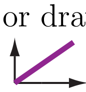


Figure 6.4: Abbreviated version the 'ball-drop' question. The correct answer is graph A.

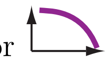
6.3 *Effect of tutorial instruction on student ability to choose or draw potential energy diagrams*

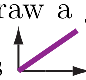
Here we describe the extent to which working through the tutorial described in Chapter 5 affects student ability to choose or draw potential energy diagrams. Student performance and common difficulties prior to tutorial instruction are discussed in §4.3.

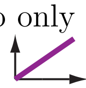
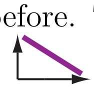
All of the questions we discuss in this section involve variations of the question shown in Figure 6.4. Each is in the context of near-Earth gravitation with the choice $U(y = 0) = 0$ (i.e., $U = mgy$). Students are asked to choose or draw the correct potential energy diagram for a ball-Earth system. The correct answer is .

We begin this section by comparing pretest and posttest results. As will be described, unlike many of the results shown in §6.1 and §6.2, performance on these questions was essentially unchanged after tutorial instruction (with the exception of one particular question). This was the case even after specific modifications to the in-class worksheet and the creation of 'teaching pretests.'

6.3.1 Comparison of performance with and without tutorial instruction

One question we have administered to students is the 'ball-drop' question in which students are asked to draw or choose a potential energy diagram for a ball that moves downward near the surface of the Earth. (See Figure 6.4.) Student performance before and after tutorial instruction is shown in Figure 6.12(a).[13] As the table shows, student performance is essentially the same before and after tutorial instruction. Only 25% of the students correctly chose . More than half of the students chose either  or . As described in §4.3.2, these students appeared to be incorrectly incorporating information about the rate of change of the potential energy and/or information about the accelerated nature of the motion into their graphs.

A related question we have administered is the 'up-down' question in which a ball moves upward, turns around, and falls downward. Students choose or draw a graph for both the upward and downward part of the motion.[14] The correct answer is  for both motions.

Performance before and after tutorial instruction on this question is shown in Table 6.12(b). Unlike the 'ball-drop' question, we have observed an increase in student performance after tutorial instruction. About 30% of the students answered the pair of questions correctly after tutorial instruction compared to only 5% before. The largest change in incorrect responses occurred for the pair of graphs  and  for the upward and downward motions, respectively. Only 15% of the students chose this pair on posttests compared to 35% on the pretest.

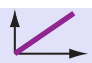
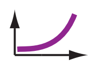
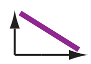
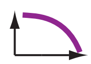
One measure of student understanding is choosing the same graph for both the upward and downward motions. Such responses are consistent with recognizing that potential energy diagrams are independent of the motion of the object of interest. On the 'up-down' question, only 10% of students on the pretest chose the same graph for both motions, compared to

[13]We have found on both pretests and posttests that student performance is unaffected by whether or not the question provides the explicit equation $U = mgy$, so we combine these administrations in reporting posttest results. See §4.3.3-a for a discussion on this matter.

[14]The complete question can be seen in the appendix. It is labeled Questions 9 and 11 in Figure A.15, which begins on page 360.

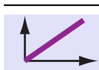
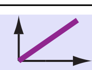
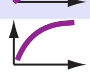
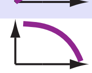
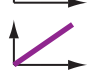
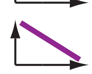
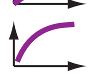
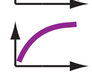
Table 6.12: Pretest to posttest comparison of student performance on the 'ball-drop' and 'up-down' questions. The correct answers are highlighted.

(a) The 'ball-drop' question.

Response	Pretest N=361[a]	Posttest N=320[b]
	25%	25%
	5%	5%
		10%
	35%	25%
	30%	35%

[a] Sections 121A142 and 121C142. [b] Sections 121D144 and 121A152.

(b) The 'up-down' question.

Response		Pretest	Posttest
Upward	Downward	N=204[a]	N=443[b]
		5%	30%
		35%	20%
		35%	15%
			10%
Other combinations		25%	25%
Same answer on both		10%	50%

[a] Section 121B142. [b] Sections 121A142 and 121A151.

about 50% on the posttest. Much of this difference is due to the correct answer. However, some is attributed to the pair of responses  . Almost no students chose this pair on the pretest, where as 10% did on the posttests.

We are unsure of how to account for the increase in performance on the up-down version but no difference in performance on the ball-drop version. It is possible that considering both motions triggered more students who have completed the tutorial into the correct line of reasoning than students who have not completed the tutorial, in a way that the ball-drop question did not. More research is needed to understand these results.

In the following subsection we discuss our attempts to improve student performance. Although none of these attempts have produced any measurable effect on the percentage of students answering correctly, it provides insight into our method for modifying and creating curricula and into the persistence of some of the specific difficulties.

6.3.2 Attempts to improve student performance

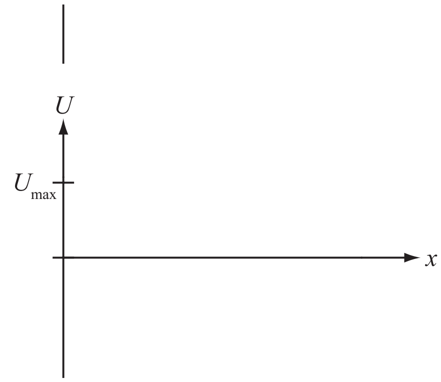
Here we describe two different methods we have used to improve student performance on the questions described in §6.3.1. One of these methods was a modification made to the in-class worksheet of the tutorial *Potential energy diagrams.* The other was the creation of a ‘teaching pretest’ that guides students toward interpreting graphs that plot position (not time) on the horizontal axis.

6.3.2-a New section of tutorial: Constructing a potential energy diagram

In early versions of the tutorial, the first section asked students to write an expression for the potential energy of a spring-block system ($U(x) = \frac{1}{2}kx^2$) and to use their expression to draw the corresponding potential energy diagram. (The first iteration of the tutorial can be seen in the appendix in Figure A.29 on page 412.) Our intention was that students would realize that the expression for $U(x)$ determines the shape of the potential energy diagram for the spring-block context and that they would generalize this to other contexts. As described in §6.3.1, the results of the posttests indicated that students failed to make this generalization

B. In this part, you will check your answer above.

1. On the horizontal axis at right, label the positions x_{min}, x_o, and x_{max}.
2. Are your labels consistent with the relation $x_{min} < x_o < x_{max}$? Resolve any inconsistencies.
3. On the diagram, draw and label *five dots* to indicate when the block: (i) is initially released from x_{max}; (ii) passes through x_o for the first time; (iii) reaches x_{min}; (iv) passes through x_o again; and (v) returns to x_{max}.



4. Connect your five dots by a smooth curve.
5. Recall from lecture that the potential energy $U(x)$ for the spring above is $\frac{1}{2}k(x-x_o)^2$. Is your curve consistent with this expression? If not, resolve the inconsistency.

Figure 6.5: A portion of section I of the tutorial that was used in spring 2015.

in a near-Earth gravitational context.

After we noticed students continuing to struggle in this manner, we began to modify the first section of the in-class worksheet. We attempted to incorporate questions that were designed to address the tendency of students to incorporate the rate of change of the potential energy into their diagrams.[15] In the new modified versions, students first consider a spring-block system that starts moving at $x = x_{\max}$, moves in the negative x-direction, turns around at $x_{\min}$, and eventually returns to its original position $x_{\max}$. Students *predict* the shape of the potential energy diagram by sketching on a blank set of axes. Immediately after predicting, students work through the questions shown in Figure 6.5. These questions ask students to draw dots on a blank set of axes to represent five different instants in the motion of the block. Some of these dots end up being drawn "on top" of each other. Students then check that their dots are consistent with the relation $x_{\min} < x_o < x_{\max}$ (x_o represents the equilibrium position). Students connect their dots by a smooth line, and then later check that their shape is consistent with the expression $U(x) = \frac{1}{2}kx^2$.

[15]We briefly described this modification to the tutorial in §5.2.

As part of this investigation, we observed the tutorial sessions as students worked through this new section of the worksheet. As expected, many students incorporated the rate of change of potential energy into their prediction for the potential energy diagram. Their graphs tended to exhibit an oscillatory-like behavior. (See the discussion of the spring question that begins on page 101). We anticipated that students would learn from their incorrect spring-block drawings and correctly generalize to the ball-Earth context.

Subsequent posttests revealed this was not the case, however. Comparing posttest results from before and after the modification to the first section reveals no measurable gains in student performance.

We have not since modified the in-class tutorial to address this continuing difficulty, although we plan to do so in the future.

In the next subsection we briefly describe one additional method we have developed in an attempt to improve student ability to choose or draw the correct potential energy diagram for a ball-Earth system.

6.3.2-b Teaching pretest: Constructing a speed vs. position graph

In winter 2015, we created a 'teaching pretest' to help students consider how oscillatory motion is or is not represented on graphs that plot position on the horizontal axis. The format of this teaching pretest is very similar to the online interactive practice homework that we describe later in Chapter 7. Most questions have built-in 'logic.' Students' answers determine the next page of the pretest that they encounter. In this way, we are able to tailor student experience to particular incorrect answers or reasoning that our research suggests students will make or use.

On the pretest, students view a simulated video of the same oscillatory motion of the block-spring system that we described in §6.3.2-a. They then predict the shape of a speed

Now consider the first instant when the block is located at x_{equil} and moving in the negative x-direction.

Which choice at right best represents where this point should be plotted on the axes? (The point you have already considered is shown in dark blue.)

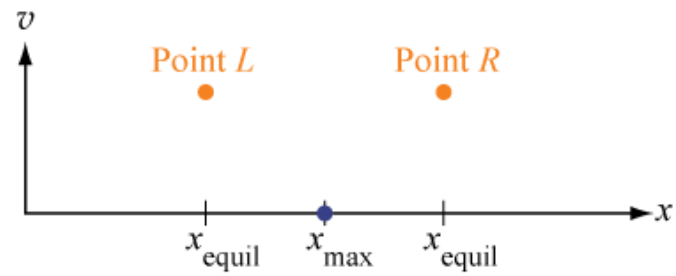


Figure 6.6: Excerpt from a teaching pretest used in winter 2015.

vs. position graph.[16] The possible graphs from which students could choose were based on our experience with potential energy diagrams. After choosing a graph, students are guided through one of two methods that they can use to check their answer. For brevity, we discuss only one of the methods here.

An excerpt from the teaching pretest is shown in Figure 6.6. Students construct the correct v *vs.* x graph one point at a time. In this way, students explicitly see that the graph is initially "created" from right to left and can "backtrack" on itself. Our intention was that many students would be "surprised" by the fact that the points on the graph appear from right to left (since our research suggests that many students treat the motion on potential energy diagrams as being from left to right). The activity would thus make an impact on student thinking and help them generalize this multi-directional idea from a speed *vs.* position graph to a potential energy *vs.* position graph.

This instructional approach is similar to the modification to the tutorial we described in §6.3.2-a. One important difference, however, is that we administered a ball-drop 'posttest' question immediately after students completed the teaching portion of the pretest. (The

[16] We chose to ask about speed rather than potential energy because an earlier pretest question we administered indicated many students were ascribing negative potential energy to a compressed spring but a positive potential energy to a stretched springs. These signs are inconsistent with each other since the *minimum* value of potential energy occurs at the equilibrium position. Using speed eliminates this issue and allows us to probe more easily the oscillatory nature of the motion.

ball-drop question was part of the teaching pretest and before tutorial instruction.) We did this to provide us with immediate feedback about the effectiveness of this method.

Despite these changes, performance on the ball-drop question was essentially the same after the teaching pretest as it was without the teaching pretest. However, during the subsequent in-class tutorial sessions, we did observe fewer students drawing oscillatory potential energy diagrams for the spring-block system. Additionally, we overheard some students correcting others on their incorrect prediction by referencing the teaching pretest. Thus, although the teaching pretest did not have the effect we anticipated on the ‘ball-drop’ question, our in-class observations suggest that teaching pretests can have some effect on student thinking.

6.3.3 Summary

The curriculum *Potential energy diagrams* appears to affect student performance on the ‘up-down’ question. A greater fraction of students were able to answer correctly. Correspondingly, a greater fraction of students recognized that the potential energy diagram is independent of the motion of the object of interest. However, our results also suggest that some ideas are deeply held by students and are resistant to change. Even after targeted instruction, many students treat potential energy diagrams as being created from left to right. Future versions of the curriculum may incorporate different instructional strategies to address the underlying difficulties.

6.4 Effect of tutorial instruction on student ability to reason about the arbitrary nature of energy

In this section we compare pretest and posttest results on student ability to reason about the arbitrary nature of potential energy. A description of the types of questions we administered and the results from the pretests are described in §4.4.

We have had only a limited number of opportunities to probe the effect of the tutorial on student ideas related to this topic. Our efforts have been primarily directed toward the reasoning required to answer the questions described in §6.1–§6.3. Consequently, the discussion in this section is brief. Further research on the effect of our tutorial curriculum is needed.

We begin this section by exploring the effect of tutorial instruction on the belief that potential energy cannot be negative. We then move on to discuss some more general difficulties about the arbitrary nature of potential and total energy.

6.4.1 Effect of tutorial instruction on the belief that potential energy cannot be negative

As described in §4.4.2-a, some students appeared to believe that potential energy could not be negative. One of the questions that elicited this belief was the 'possible system' question. In it, four potential energy diagrams that were identical except for their vertical offset. (A diagram of the question is shown in Figure 4.17 on page 117.) Two of the diagrams had non-negative potential energy values and two had negative values. Students are asked which, if any, of the diagrams could represent a possible physical system.

Table 6.13 compares student performance on this question before and after lecture instruction. On the pretests, 30% of the students answered correctly that all of the diagrams were possible, compared to 75% of the students on the posttest. Correspondingly, the number of students who answered incorrectly that only the positive diagrams were possible decreased from 50% on pretests to 15% on posttests.

There are two important notes that we give pertaining to the posttest data. First, we

Table 6.13: Pretest to posttest comparison of the belief that negative potential energy is not possible.

Answer	Pretest N=566[a]	Posttest N=76[b]
All diagrams are possible	30%	75%
Only positive diagrams are possible	50%	15%
Other	20%	10%

[a] Sections 121A132, 121B132, 121A133, and 121A134.
[b] Section 121A143.

have collected posttest data from only 76 students. More data are needed to corroborate the results we present here. Second, the in-class worksheet through which these students worked included a section on universal gravitation. In this section, students considered negative potential energy and negative total energy. We attribute at least some of the large increase in performance on this question to the inclusion of this section.[17] In fact, during the tutorial sessions, we overheard many students verbalize their surprise that negative potential energy was possible, suggesting that the tutorial provided an opportunity for students to address their incorrect beliefs. However, more recent iterations of the tutorial do not include the universal gravitation section. It was moved to the homework partially due to the fact that not all students were able to complete the entire tutorial during the allotted class time. We have not subsequently administered this posttest question. Thus, it is possible that the current version of the tutorial may be less effective at helping students reason about negative potential energy than the data presented here would suggest.

We have also administered the 'possible system' question to tutorial TAs as a pretest as part of their preparation course.[18] All but one of the 22 TAs answered correctly. It is worth noting that many of these TAs were undergraduates rather than graduate students. Although the number of TAs to whom we have administered this question is somewhat low,

[17]Some of the increase might also be attributed to the homework that addresses negative potential energy.

[18]See §2.2.3 for a discussion of this practice.

[4 pts] Which of the following statements **must be true** regarding the potential energy diagram at right?

A. This potential energy diagram is not possible because $U < 0$ in some regions.

B. There is an instant when the particle is located at $x = 7$ cm.

C. The particle will reach $x = 2$ cm and immediately stop.

D. None of these statements must be true.

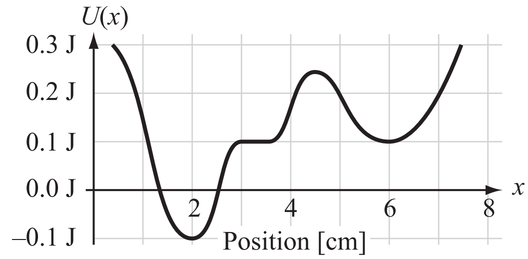


Figure 6.7: Posttest question administered in spring 2015 to Phys 121C.

this result could indicate that introductory undergraduate students do not perform as well after tutorial instruction as tutorial TAs. More data on TA pretest performance is needed.

We have also administered a different question to introductory students that is designed, in part, to elicit the belief that potential energy cannot be negative. The question is shown in Figure 6.7. It was administered *after* the in-class worksheet section on universal gravitation was removed. (We have not administered a similar question as a pretest. Although we cannot compare performance before and after the tutorial, the results provide some insight into student understanding after tutorial instruction.) Students are provided with a potential energy diagram without a total energy line and are asked to select one of the three statements that must be true, if any (see the diagram). All three statements are false, so the correct answer is *none of these statements is true.*

About 10% of the students (N=65) selected choice A that states *This potential energy diagram is not possible because* $U < 0$ *in some regions.* These students failed to recognize that potential energy energy can be negative. It is important to note, though, that students might have believed that more than one of the statements was true. We did not provide a "more than one" option to students. Student responses may be different if the answer choices were refined.

6.4.2 *Effect of tutorial instruction on student understanding of the arbitrary nature of potential and total energy*

As described in §4.4, some students appeared to struggle in understanding the arbitrary nature of potential and total energies. The difficulties students encountered were elicited by many different questions (see, for example, §4.4.1). Below we compare student performance on pretest and posttest questions. As described above, we have only administered a limited number of these questions as posttests, so some of our results are inconclusive. More research is needed to understand the effect of tutorial instruction with regard to the arbitrary nature of potential and total energies.

Results from the 'two shifted diagrams' question

In §4.4.2-b we described the belief by some students that potential energy is not arbitrary (i.e., that two potential energy diagrams, which are different only by their vertical 'shifts,' could not represent the same system). Also, in §4.4.2-c we described the related tendency of some students to characterize a system by the total energy (i.e., that the energy of a system has a definite value and is not arbitrary). This latter idea is incorrect since an arbitrary value of potential energy U implies an arbitrary value of total energy $E_{\text{tot}} = K + U$. Both of these difficulties were elicited, for example, by the 'two shifted diagrams' question. In this question, which is repeated in Figure 6.8, students are provided with a potential energy diagram with a line representing total energy. They are asked which of two different diagrams could represent the same physical system. The correct answer is *only graph II* because both the potential and total energies were shifted by the same amount.

Table 6.14 compares student performance on this question before and after tutorial instruction. We found no significant difference after tutorial instruction. Thus, on the basis of this question, tutorial instruction appears to have no impact on student ability to reason about the arbitrary nature of potential energy or the tendency to characterize a system by its total energy. We emphasize, though, that the percentage of students exhibiting incorrect

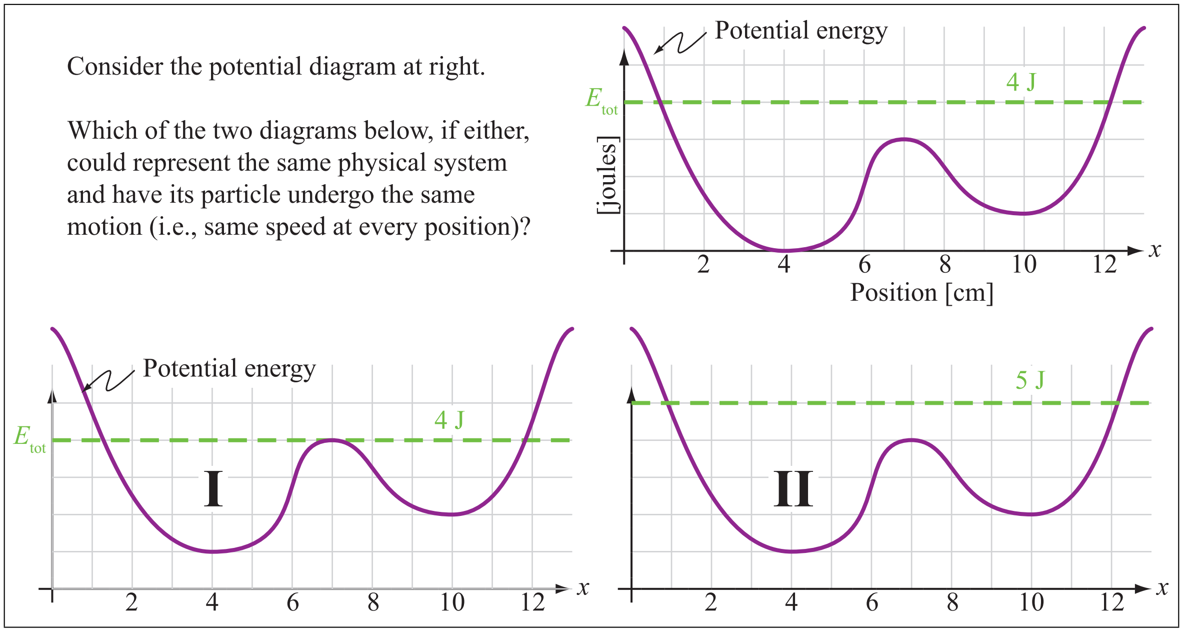


Figure 6.8: Abbreviated version of the 'two shifted diagrams' question. The correct answer is only graph II.

ideas did not *increase* as a result of tutorial instruction.

Results from the 'negative spring shift' question

In §4.4.2-a on page 121 we described the performance on the 'negative spring shift' question. This question is repeated in Figure 6.9. Students are presented with a potential energy diagram for a spring (with $U_{\text{min}} = 0$) and are asked if a different diagram (with $U_{\text{min}} < 0$) could represent the same spring. The correct answer is that it could represent the same spring since only changes in potential energy are physically significant.

Student performance is shown in Table 6.15. Before tutorial instruction, only 35% of the students answered correctly. (As we described on page 121, we interpreted the results as primarily indicating difficulty with arbitrary potential energy for the case of a spring, and not necessarily difficulty associated with negative potential energy.) After tutorial instruction, 50% of the students answered correctly. The difference in percentages is at the borderline between what we would consider significant for a single administration and what we would

Table 6.14: Pretest to posttest comparison of student performance on the 'two shifted diagrams' question. The correct answer is highlighted.

Response	Pretest N=165	Posttest N=195
Only diagram II	70%	80%
Only diagram I	10%	5%
Both diagrams	10%	5%
Neither diagram	10%	5%

[a] Section 121A112.
[b] Section 121A141.

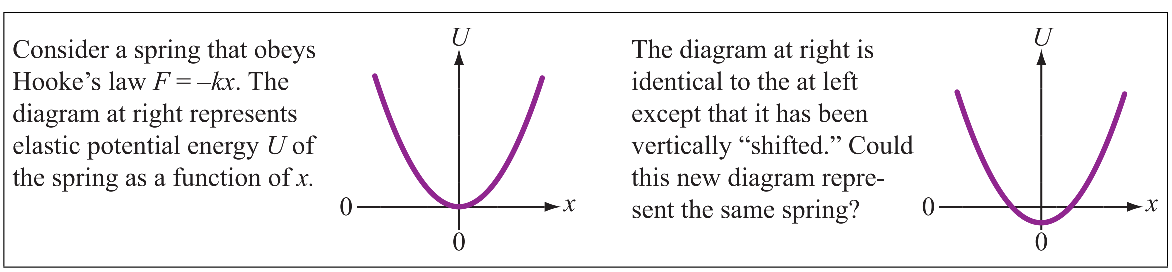


Figure 6.9: Abbreviated version of the 'negative spring shift' question. The correct answer is *yes.*

Table 6.15: Pretest to posttest comparison of student performance on the 'negative spring shift' question. The correct answer is highlighted.

Response	Pretest N=189[a]	Posttest N=181[b]
Could represent same spring	35%	50%
Could not represent same spring	65%	50%

[a] Section 121A144. [b] Section 121D144.

consider not significant. Even if the results are indicative of an actual effect of the tutorial curriculum, we stress that half of the students still struggled in understanding that potential energy is arbitrary.

In interpreting these results, we note that, although the written homework does address the arbitrary nature of potential energy, it does so for the case of a generic, non-context-specific potential energy diagram. Future versions of the homework or in-class tutorial may incorporate arbitrary changes to non-generic potential energy diagrams.

6.5 Commentary on posttest results after tutorial instruction

Throughout this chapter, we provided examples that illustrate the effect of tutorial instruction on student ability to interpret, to use, and to draw potential energy diagrams. Student performance on many of the questions improved. Correspondingly, many of the difficulties students encountered were exhibited by a smaller proportion of students after tutorial instruction. A greater fraction of students were able to reason correctly about how the speed and direction of motion at particular locations, avoiding the 'left-to-right' line of reasoning that was prevalent prior to tutorial instruction. Students were better able to relate the kinetic and total energy to the maximum kinetic energy, understanding the role that the minimum and maximum values of potential energy on the diagram do and do not play, respectively, with regard to the maximum kinetic energy.

Despite these improvements, some of the tendencies that students encountered prior to tutorial instruction continue to be present even after targeted instruction. For example, many students still used 'left-to-right' reasoning when drawing graphs of potential energy. In some cases, a large fraction of students still did not reason correctly about the arbitrary nature of potential energy. Some of the tendencies appear to be deeply held and resistant to change. Different instructional strategies may need to be developed to address the underlying difficulties.

Chapter 7

DEVELOPMENT AND ASSESSMENT OF A NEW TUTORIAL COMPONENT: AN ONLINE INTERACTIVE PRACTICE HOMEWORK

In this chapter we discuss a component to the tutorial curriculum that we have called an *online interactive practice homework* (OIPH). This component is new and not a part of other topics within *Tutorials in Introductory Physics*. It was developed specifically to supplement the tutorial curriculum *Potential energy diagrams*, though the insights gained could inform the development of additional OIPHs.

We begin this chapter by describing the motivation for the OIPH, the development, and the content in detail. Afterward, we discuss the effect of the OIPH on two particular student difficulties that we identified in Chapter 4.

Finally, we remind the reader of the abbreviated scheme to designate all course sections. See page 23 for a description.

7.1 Development of the online interactive practice homework (OIPH)

7.1.1 Motivation for development

We were initially motivated to develop supplementary exercises for students based on posttest results from early iterations of the tutorial *Potential energy diagrams*. We found that after working through the tutorial: (1) a significant portion of students still answered 'speed' questions using 'left-to-right' reasoning (see, for example, §6.1.2); (2) there was an increase in the percentage of students who incorrectly stated that the direction of acceleration of the particle could not be determined (see §6.1.3-a); and (3) many students did not take into account the turn-around points when determining the maximum kinetic energy (see §6.2.4).

The typical response by the Physics Education Group when encountering continued student difficulty after tutorial instruction has been to modify the in-class worksheet and/or the accompanying written homework. A complicating factor that led us to initially prefer developing an additional component rather than begin modifying the existing curriculum was that the homework for *Potential energy diagrams* is assigned during the last week of the quarter and is not collected. A survey administered on an exam in the spring of 2013 indicated that 25% of the students did not attempt the last homework. Another 45% completed only a portion. Even for the 30% of students who completed the homework, there was no formal opportunity for feedback from TAs because the last assigned tutorial homework of the quarter is not collected. To help address these results, we developed and administered a new online curricular component in spring 2014 that supplements the in-class worksheet and written homework.[1]

The 'online interactive practice homework,' or OIPH, was designed to be completed by students after tutorial instruction but before the written homework is attempted. The primary function is to provide interactive feedback to students in a way that addresses the ongoing difficulties we described above.

[1] Later, we attempted to address the three difficulties discussed above by modifying the tutorial. See our discussion in §6.1.3.

A broader aim of this new component, though, is to develop a model that might be used with other topics within *Tutorials in Introductory Physics*. Since this is a new component of the tutorials, we describe the structure of the OIPH in the next subsection.

7.1.2 General structure of online interactive practice homework

This subsection describes the overarching structure of the OIPH. The specific exercises and content are described in §7.1.3.

The defining characteristics of the OIPH are as follows:

- *Online-based:* We used the same online system that students use for their pretests.
- *Interactive:* Students are provided with feedback that varies depending on which answer they choose. This was intended to echo the interaction between students and TAs during tutorial sessions in a way that traditional written homework cannot.
- *Practice homework:* Students are informed that they will receive a grade for the OIPH, but that the written homework should still be attempted.

Each of the questions on the OIPH is multiple choice. Some ask for an answer to a specific question (e.g., determining the rate of change of kinetic energy with respect to time). Others ask students to select the best explanation for their previous (correct) answer. The flowchart in Figure 7.1 illustrates the interactive nature of the OIPH. A description of the interactive sequence for a given question follows (beginning at the top of the figure):

- On their first attempt of a given question, students read the question and select one of the answer choices. The number of answer choices varies from three to five. The full text of the six questions is shown in Appendix A.2.3-b that begins on page 483.
- Students are allowed a maximum of three attempts per question.
- If students answer *correctly* on any of the three attempts, they are taken to a "confirmation page" that states their choice is correct. The full text of confirmation pages is shown in Appendix A.2.3-e that begins on page 518. After the confirmation page, students move on to the next question.

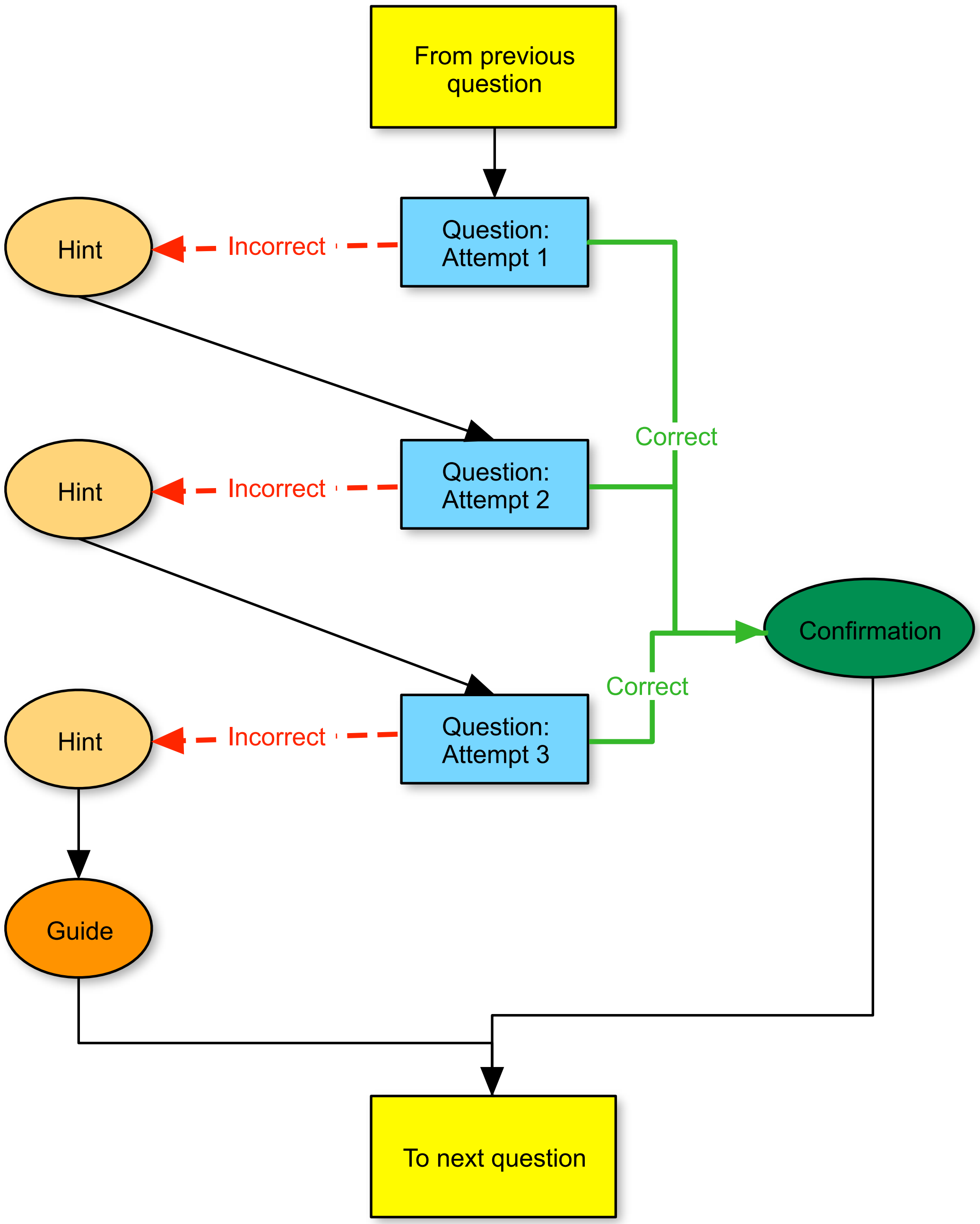


Figure 7.1: Flowchart for a typical question in the OIPH.

- If instead students answer *incorrectly* on any of the three attempts, they are taken to a page that reiterates their incorrect answer and provides a possible reason they may have had for making the incorrect choice. These reasons are based on the research described in Chapter 4. Students are then given a suggestion or 'hint' that attempts to bridge the gap between the particular incorrect answer and the correct reasoning. These hints are based on reasoning provided by students in long-answer pretest and posttest questions, as well as the instructional tutorial experience of the author. The full text of the 'hint' pages is shown in Appendix A.2.3-c that begins on page 490.
- After viewing the hint page following an incorrect answer, students re-attempt the question. The answer choices provided during the second and third attempts are identical to those on the first, including the previously-selected incorrect choice.
- If students provide three incorrect attempts they first move on to the standard hint page described above. Afterward they are taken to a special 'guide' page that describes the correct reasoning. The full text of these guides is shown in Appendix A.2.3-d that begins on page 511. In cases where more than one possible method could be used to arrive at a solution, only one is discussed in detail; the other method is only briefly mentioned.

In the next subsection we describe the content of the OIPH.

7.1.3 Description of content

The OIPH consists of three parts, each of which consists of two questions.[2] These are summarized in Table 7.1. In each question, students are given a potential energy diagram and are asked about some kinematic aspect of the particle's motion.

Part I asks how the kinetic energy is changing at a particular location. Part II asks for the direction of acceleration at a particular location. Part III asks for the location of

[2]Also included in the OIPH are preliminary questions asking students how much of the accompanying tutorial they finished as well as a free-response question at the end in which students can leave comments about the OIPH itself.

Table 7.1: Description of the six questions on the OIPH.

Part	Ques.	Description of question	Figure
I	1st	How is kinetic energy changing?	7.2(a)
	2nd	Explain reasoning to previous question.	7.2(b)
II	1st	Direction of acceleration?	7.4(a)
	2nd	Explain reasoning to previous question.	7.4(b)
III	1st	At what location is the speed first zero?	7.6(a)
	2nd	Determine boundaries of particle's motion.	7.6(b)

turn-around points and the allowed region.

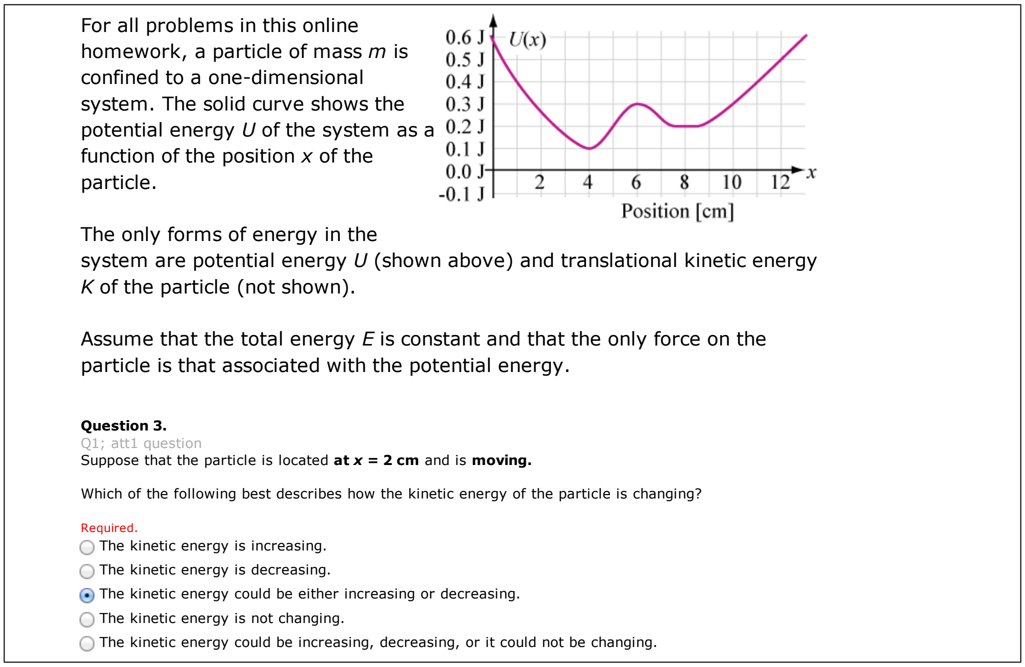
These first two parts are designed to complement each other: In part I, students first recognize that it is not possible to decide how the kinetic energy is changing. Then, in part II, they find that the direction of the acceleration *can* be determined. By allowing students to contrast these results, we anticipated that students would develop a more complete picture of what can and cannot be known from a potential energy diagram.

Each of the six questions, along with the hints that accompany the more common incorrect answer choices, are described in detail below. Relevant research that informed the development of these elements is also discussed. Results from the OIPH are discussed in §7.2.1.

7.1.3-a Part I of the OIPH: Change in kinetic energy

The first question of part I, shown in Figure 7.2(a), presents students with a graph of $U(x)$ and asks them to determine how the kinetic energy is changing at $x = 2$ cm. Students are not told whether the particle is moving in the positive or negative x-direction. To answer correctly, students can note that the slope of $U(x)$ at the specified point is non-zero. Thus, the correct answer is that *the kinetic energy could be either increasing or decreasing,* depending on the direction of motion.

This question (or analogous ones about speed) have been administered many times on



For all problems in this online homework, a particle of mass *m* is confined to a one-dimensional system. The solid curve shows the potential energy *U* of the system as a function of the position *x* of the particle.

The only forms of energy in the system are potential energy *U* (shown above) and translational kinetic energy *K* of the particle (not shown).

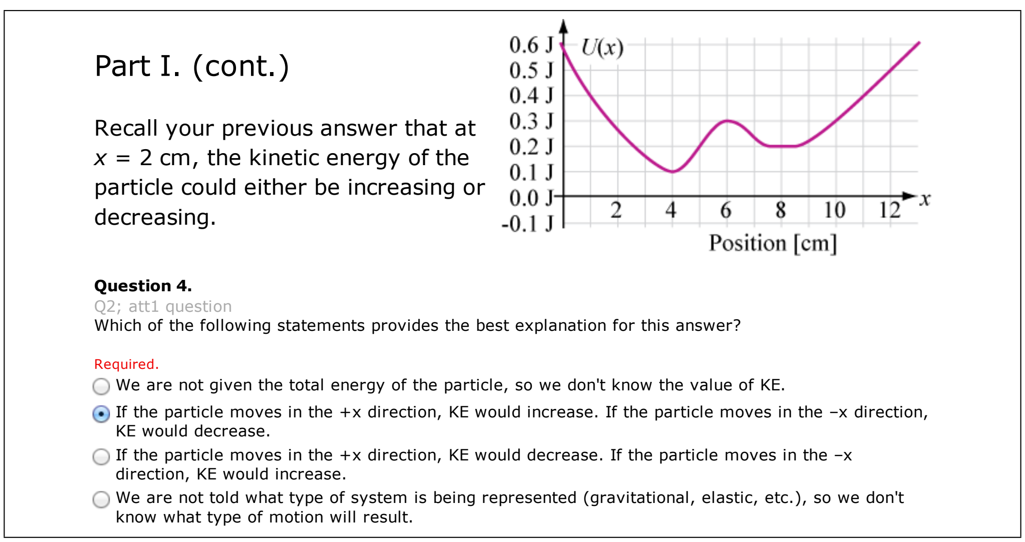
Assume that the total energy *E* is constant and that the only force on the particle is that associated with the potential energy.

Question 3.
Q1; att1 question
Suppose that the particle is located **at *x* = 2 cm** and is **moving.**

Which of the following best describes how the kinetic energy of the particle is changing?

Required.
- ○ The kinetic energy is increasing.
- ○ The kinetic energy is decreasing.
- ◉ The kinetic energy could be either increasing or decreasing.
- ○ The kinetic energy is not changing.
- ○ The kinetic energy could be increasing, decreasing, or it could not be changing.

(a) First question of part I: How is kinetic energy changing at $x = 2$ cm?



Part I. (cont.)

Recall your previous answer that at *x* = 2 cm, the kinetic energy of the particle could either be increasing or decreasing.

Question 4.
Q2; att1 question
Which of the following statements provides the best explanation for this answer?

Required.
- ○ We are not given the total energy of the particle, so we don't know the value of KE.
- ◉ If the particle moves in the +x direction, KE would increase. If the particle moves in the −x direction, KE would decrease.
- ○ If the particle moves in the +x direction, KE would decrease. If the particle moves in the −x direction, KE would increase.
- ○ We are not told what type of system is being represented (gravitational, elastic, etc.), so we don't know what type of motion will result.

(b) Second question of part II: Which statement provides the best explanation for the prior question?

Figure 7.2: Part I of the OIPH is concerned with the direction of motion at a particular location. The selected buttons correspond to the correct answers.

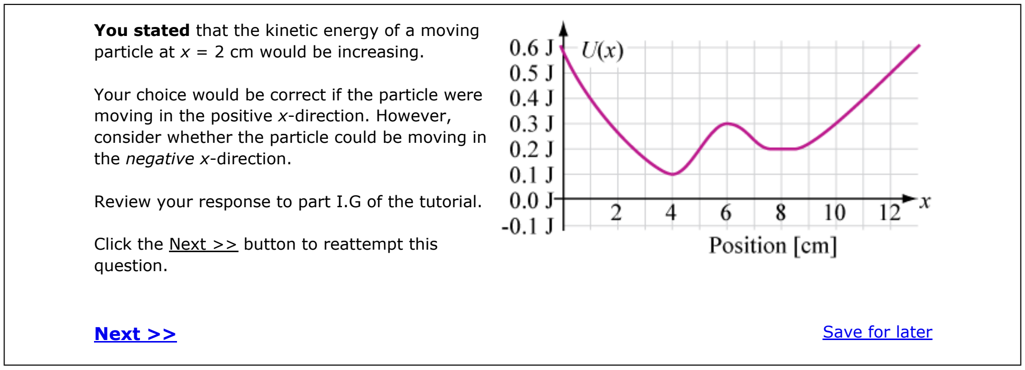


Figure 7.3: Hint shown in question 1 if students answer incorrectly that the kinetic energy is increasing at $x = 2\,\text{cm}$.

pretests and posttests. The most common incorrect method used by students in answering is to treat the particle as if it moves in the positive x-direction or to treat the graph as if time were plotted on the horizontal axis (see §4.1.3-a). Noting that the slope at the point of interest is negative, students who use this reasoning would arrive at the incorrect answer that the potential energy is decreasing and *the kinetic energy is increasing.* The hint that is presented to students if they choose this answer is shown in Figure 7.3. Students are informed that this answer would be correct *if* the particle were moving in the positive x-direction. It is then suggested that they consider whether the particle could move in the negative x-direction. In this way, the hint attempts to address one of the underlying issues that we identified through our research.

Hints for the other incorrect answers are provided in the appendix. See the caption on page 484.

After completing the first question of part I, students move on to the second question (see Figure 7.2(b)). This question is a follow up to the first one. It asks students to provide an explanation for why the kinetic energy could either be increasing or decreasing at $x = 2\,\text{cm}$ (i.e., it asks students to explain the correct answer to the previous question). The correct explanation is the selected option in the figure, which states:

> If the particle moves in the $+x$ direction, KE would increase. If the particle moves in the $-x$ direction, KE would decrease.

The first and fourth answer choices for this question (shown in the figure) were included because they contain some true statements, encouraging students to think critically about why the sign of dK/dt cannot be determined. The similarity of the second and third choices requires students to connect correctly the sign of the change in the given quantity to the sign of the change in the desired quantity.

Hints for the other incorrect answers are provided in the appendix. See the caption on page 485.

7.1.3-b Part II of OIPH: Direction of acceleration

Part II of the OIPH focuses on the direction of the acceleration of the particle. The first question of this part, shown in Figure 7.4(a), asks students to determine the direction of the acceleration at $x = 5$ cm. One method to arrive at the correct answer is to use the force-potential energy relationship in one dimension, $F = -dU/dx$, and Newton's second law $F = ma$. Combining these two relationships results in $\text{sign}(a) = -\text{sign}(dU/dx)$. Thus, since $dU/dx > 0$, the acceleration is in the negative x-direction.

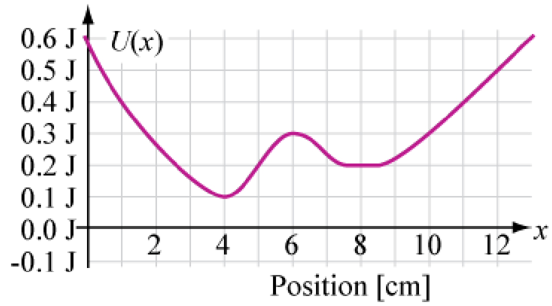
Based on other post-tutorial assessments, it was expected that the answer choice *either in the positive or negative x-directions* would be the most popular incorrect choice. The accompanying hint, shown in Figure 7.5, asks students to determine the direction of the acceleration if the particle were moving in the positive x-direction, and again if it were moving in the negative x-direction. Students are then in a position to understand that the direction of the acceleration is independent of the direction of motion.

Hints for the other two incorrect answers are provided in the appendix. See the caption on page 486.

After completing the first question of part II, students move on to the second question,

Part II.

Consider again the potential energy diagram shown at right.



Question 5.
Q3; att1 question
Suppose now that the particle were located at ***x* = 5 cm** and is **moving.** (Note that this is a different location than the previous questions.)

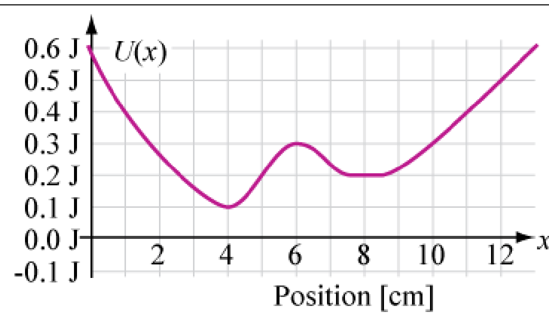
Which of the following statements best describes the **direction of the acceleration** of the particle?

Required.
- ○ The acceleration is in the +x-direction.
- ◉ The acceleration is in the −x-direction.
- ○ The acceleration could be in either the +x-direction or −x-direction.
- ○ The acceleration is zero.

(a) Question 3: What is the direction of the acceleration at $x = 5$ cm?

Part II. (cont.)

Recall your previous answer that at *x* = 5 cm, the acceleration is in the negative *x*-direction.



Question 6.
Q4; att1 question
Which of the following statements provides the best explanation for this answer?

Required.
- ○ Any time a particle slows down, the acceleration must be negative.
- ○ The particle is slowing down, which is opposite its direction of motion. This corresponds to a negative acceleration.
- ◉ A particle moving in the positive direction would be slowing down, while a particle moving in the negative direction would be speeding up. Both correspond to a negative acceleration.

(b) Question 4: Which statement provides the best explanation for question 2?

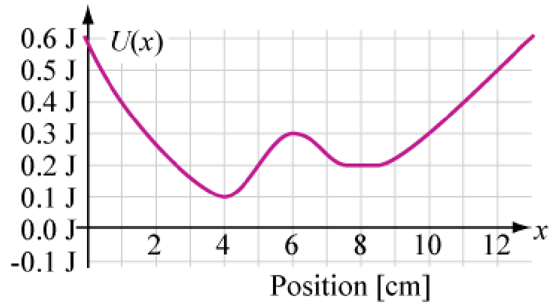
Figure 7.4: Part II of the OIPH is concerned with the direction of acceleration at a particular location. The selected buttons correspond to the correct answers.

You stated that the acceleration of a moving particle at *x* = 5 cm could be in either the +x-direction or the −x-direction.

You may have chosen this answer because the direction of motion of the particle cannot be determined. (Recall Part I of this online homework.) However, it may still be possible to know the direction of acceleration even without the direction of motion.

Consider what the direction of the acceleration *would* be if:

- the particle were moving in the +x-direction
- the particle were moving in the −x-direction



It may also help to review part IV of the tutorial that discusses acceleration and potential energy diagrams.

Figure 7.5: Hint shown if students answer incorrectly that the direction of acceleration is unknown.

which is shown in Figure 7.4(b). This question asks students to select the statement that correctly explains why the direction of the acceleration is in the negative x-direction. The correct answer is selected in the figure and discusses both directions of motion. Both of the other two answer choices assume that the particle moves in the positive x-direction, and are typical of student explanations on pretests (see our discussion in §4.1.3-a).

Hints for the two incorrect answers are provided in the appendix. See the caption on page 487.

7.1.3-c Part III of OIPH: Turn-around points

Part III of the OIPH is designed to help students account for turn-around points. The first question, shown in Figure 7.6(a), uses a different potential energy diagram than the previous two parts. The question asks students to determine the location at which the particle first attains a speed of zero after it is released at $x = 2\,\text{cm}$. To answer correctly, students can note first that the particle would initially accelerate and move in the positive x-direction. The kinetic energy, and hence the speed, is zero when the potential energy U is equal to the $E_{\text{tot}} = U_{\text{release}} = 2\,\text{J}$. This first occurs at $x = 5\,\text{cm}$. The other choices correspond to locations at which $U(x) = E_{\text{tot}}$ or $dU/dx = 0$.

Hints for all of the answer choices are shown in the appendix. See the caption on page 488.

The second question of part III, shown in Figure 7.6(b), is a follow up to the first one. Students are asked to determine the region(s) in which the particle will oscillate back and forth. To answer correctly, students should recognize that when the particle first attains a speed of zero (at $x = 5\,\text{cm}$ from the first question) the acceleration is in the negative x-direction back toward $x = 2\,\text{cm}$. Thus, the correct answer is the first one in the figure (i.e., only between 2 cm and 5 cm).

The incorrect answers to the second question of part III each use the positions x that satisfy $U(x) = E_{\text{tot}}$ in various ways.[3] In particular, the second option in the figure lists both of the regions that satisfy $E_{\text{tot}} > U(x)$. This choice is consistent with students treating the particle as if it could "tunnel' through the forbidden region (see §4.2.2-g). The hint for this incorrect answer, shown in Figure 7.7, suggests to students that they consider the direction of the acceleration at $x = 5\,\text{cm}$.

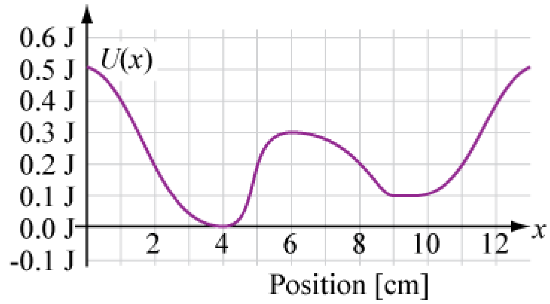
Hints for all of the answer choices are shown in the appendix. See the caption on page 489.

The three parts that we have described comprise the main body of the OIPH. Next we discuss results from the OIPH.

[3] Note that we did not provide students the option of selecting the rightmost x-coordinate on the graph. See §4.2.2-a for a discussion of why this option ideally should have been available to students.

Part III.

Consider now the different potential energy diagram shown at right.



Question 7.

Q5; att1 question

Suppose that the particle were **released from rest at *x* = 2 cm.**

After being released, at what location will the particle **first** attain a speed of zero?

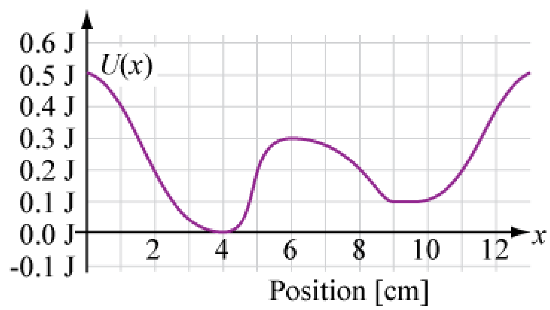
Required.

- ○ *x* = 4 cm
- ◉ *x* = 5 cm
- ○ *x* = 6 cm
- ○ *x* = 8 cm
- ○ *x* = 11 cm

(a) Question 5: Determine first location where speed is zero.

Part III. (cont.)

Consider again the potential energy diagram at right.



Question 8.

Q6; att1 question

Suppose that the particle were again **released from rest at *x* = 2 cm.**

Between which locations will the particle oscillate back and forth?

Required.

- ◉ Between 2 cm and 5 cm only
- ○ Between 2 cm and 5 cm, as well as between 8 cm and 11 cm
- ○ Between 2 cm and 8 cm only
- ○ Between 2 cm and 11 cm only
- ○ Between 8 cm and 11 cm only

(b) Question 6: Between what locations will the particle oscillate?

Figure 7.6: Part III of the OIPH is concerned with the allowed regions when the particle is released from rest from $x = 2$ cm. The selected buttons correspond to the correct answers.

You stated that the particle, if released from rest at x = 2 cm, would oscillate between 2 cm and 5 cm, and between 8 cm and 11 cm.

You may have chosen this answer because $E > U$ in both regions.

Consider, however, the value of the particle's speed at x = 5 cm, as well as the direction of the acceleration at this location. You may find part IV of the tutorial helpful.

Click the Next >> button to reattempt this question.

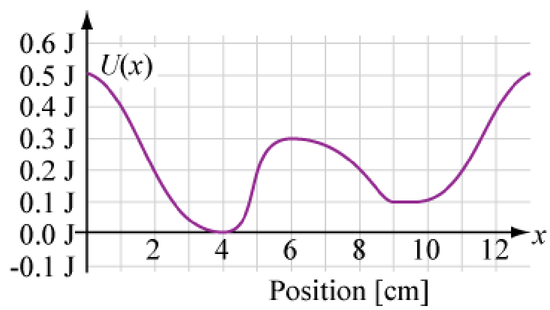


Figure 7.7: Hint shown if students answer incorrectly to the second question of part III that the particle oscillates between 2 cm and 5 cm as well as between 8 cm and 11 cm.

7.2 Results from the online interactive practice homework

In this section we describe results from the online interactive practice homework (OIPH) that was administered to all sections of Phys 121 in spring 2014. We begin by describing performance on the OIPH itself. We then present results from posttests that were administered after students worked through the OIPH.

7.2.1 Student performance on the online interactive practice homework

Nearly all of the students enrolled in Phys 121 completed the OIPH during the quarter when it was used (478 out of 488). On their first attempt, only 30% of the students answered all six questions correctly. This observation suggested to us that the OIPH was addressing a need for the majority of students.

Student performance on each of the six questions is summarized in Table 7.2. Only the correct responses and the most common incorrect answers are shown. The first column of the results shows the percentage of students who chose each answer on their first attempt. The next column gives the percentage of students who selected each answer on *any* attempt. (Thus, for any given question response, the percentages in this column can sum to more than 100%.) Note that nearly all students answered each question correctly by the time they had completed the OIPH. Table 7.2 is used as the basis for the analysis of student responses below.

It is interesting to note that on parts I and II of the OIPH (the kinetic energy and acceleration questions, respectively), student performance on the first attempt was essentially the same as we had previously obtained on corresponding questions administered on exams after tutorial instruction. (See Table 6.2 on page 146 and Table 6.4 on page 149.) This might be expected since both groups of students had completed similar versions of the tutorial and, as discussed above, relatively few students complete the final homework of the quarter. (Note, there is no corresponding exam question to serve as a benchmark for part III of the OIPH, the turn-around question.)

Table 7.2: Distribution of student responses on OIPH in spring 2014. Highlighted rows indicate correct answer.

	Question	Answer choice	On first attempt[a]	On any attempt[b]
Part I	Kinetic energy	Either incr or decr	60%	95%
		Increasing	40%	40%
	Explain previous	Correct	85%	95%
		System is unknown	5%	10%
Part II	Direction of acceleration	$-x$	55%	90%
		Either $+x$ or $-x$	40%	45%
		$+x$	5%	20%
		Zero	5%	15%
	Explain previous	Correct	80%	>95%
		$\vec{a}$ is opp. $\vec{v}$	15%	20%
		Slowing, so $a < 0$	5%	10%
Part III	Turn-around point	5 cm	75%	95%
		4 cm	15%	20%
		6 cm	10%	15%
	Range of motion	2–5 cm only	75%	95%
		2–5 cm and 8–11 cm	20%	20%

[a] Column indicates the percentage of students who select the indicated answer on their first attempt.

[b] Column indicates the percentage of students who select the indicated answer on any attempt. The percentages for a given question may add up to more than 100% since a single student may answer incorrectly before answering correctly, if at all.

On the kinetic energy question (the first question of part I), essentially all of the students who answered incorrectly on their first attempt (40%) stated that the kinetic energy was increasing. This answer is consistent with the 'left-to-right' line of reasoning described in §4.1.3-a. Of these students, 85% answered correctly on their second attempt (not shown in the table). This suggests that the hint we developed was effective in helping students realize their mistake.

On the acceleration question (the first question of part II), about 45% of students answered incorrectly on their first attempt. Nearly all of them (about 40% of all students) stated that the acceleration is *either in the positive or negative x-directions*. These students may have overgeneralized the idea that the direction of motion cannot be determined to conclude that the direction of the acceleration cannot be determined. Of these students, only 60% answered correctly on their second attempt; about 25% answered that the acceleration is in the *positive x-direction*. This percentage of students who chose the correct answer on their second attempt is lower than the percentage who did so on the kinetic energy question. This might suggest that it is more difficult for students to reason about acceleration and/or that the hint we developed is not as effective in helping students realize their mistake.

Only 35% of the students answered both the kinetic energy and the acceleration question correctly on their first attempt. The most common incorrect answer combination, given by 20% of the students, was *increasing* on the kinetic energy question and *either positive or negative* on the acceleration question, both of which are incorrect. This answer combination is consistent with left-to-right reasoning on the kinetic energy question (i.e., the particle moves to the right), but then an overgeneralization of the fact that the direction of motion cannot be determined when answering the acceleration question.

Of those students who answered correctly on the kinetic energy question on their first attempt (the first question of part I), 65% went on to answer the acceleration question correctly on their first attempt. Of those students who answered *incorrectly* on the kinetic energy question on their first attempt, only 35% went on to answer the acceleration question correctly on their first attempt. This suggests that the OIPH was not effective in bringing those students who answered incorrectly on the kinetic energy question up to the level of the students who answered it correctly.

On part III, only 65% of the students answered both questions correctly on their first attempt. The most common incorrect answer combination, chosen by 10% of the students, was *5 cm* on the turn-around question (correct) and *2–5 cm and 8–11 cm* on the allowed region question (incorrect). These students failed to account for the motion of the particle after it comes to a stop. This result suggests that for many students, only knowing the locations where $v = 0$ is not sufficient for them to determine the allowed regions.

7.2.2 Effect of online interactive practice homework on posttest scores

After students had completed the OIPH (and had worked through the in-class worksheet), we administered two questions on their final exam that tested material from the OIPH. One of the questions asks students to determine the direction of the acceleration at a particular point. This question is identical to the first question of part II of the OIPH. We chose to administer an identical question to provide an upper bound on the extent to which the OIPH could improve student learning. The other question asks students to determine the maximum kinetic energy of a particle given its point of release. This question is an extension of the turn-around and allowed region problems of part III of the OIPH. To answer correctly, students must account for the turn-around point and realize that the particle does not reach the global minimum of $U(x)$.

7.2.2-a Determining the direction of acceleration

A comparison of student performance on questions regarding the direction of acceleration is shown in Table 7.3. Pretest data are shown in the leftmost column. Posttest data in the rightmost two columns differ as to whether or not students worked through the OIPH. Those who did not work through the OIPH had worked through a modified version of the tutorial that we specifically developed to help students think about the direction of acceleration.[4]

[4] See §5.2 for a discussion of these modifications.

Table 7.3: Student performance on the 'acceleration' question with and without the OIPH.

	Pretest	Posttest	
		without OIPH[b]	with OIPH
Dir. of accel.	N=581[a]	N=489[a]	N=239[c]
Correct	65%	70%	75%
Not enough info	10%	15%	15%
Other	25%	15%	10%

[a] See Table 6.5(a) on page 151.
[b] Data are from sections that used the post-modified tutorials that specifically addressed direction of acceleration questions as described in §6.1.3-a. c Section 121A142.

Student performance on the acceleration question after working through the OIPH is roughly the same as it was on the pretest (i.e., without tutorial or OIPH instruction). We were initially surprised that more students who had worked through the OIPH did not answer correctly given that almost all had successfully answered the acceleration question on one of their three attempts. One explanation is that simply *reading* hints and correct explanations in the OIPH is insufficient for a large portion of the class.

Furthermore, student performance after working through the OIPH is identical to performance without the OIPH but after working through a modified version of the tutorial that was designed specifically to address 'left-to-right' reasoning on acceleration questions (see the footer in Table 7.3). This result suggests that the modified tutorial used by the latter group of students was no better than the combination of the unmodified tutorial and the OIPH in developing student ability to reason about the direction of acceleration from a potential energy diagram.[5]

[5]We do not compare post-OIPH data with posttest data that were gathered prior to the development of the OIPH. See the end of §6.1.3-a for more information.

Table 7.4: Comparison of student performance on maximum kinetic energy posttest questions with and without the OIPH. The correct method is highlighted.

	Without OIPH		**With OIPH**
Method to find K_{max}	Early version of tut. N=196[a]	Later versions of tut. N=640[b]	Early version of tut. N=239[c]
$E_{\text{tot}} - U_{\text{local min}}$	25%	40%	65%
$E_{\text{tot}} - U_{\text{global min}}$	40%	20%	10%
Other	35%	40%	25%

[a] Section 121A144. [b] See Table 6.11 on page 166. [c] Section 121B142.

7.2.2-b Accounting for turn-around points

A comparison of student performance on maximum kinetic energy questions *on posttests* with and without the OIPH is shown in Table 7.4. Students performed better on posttests with the OIPH (65% correct) than they did on both pretests (10%) and on posttests without the OIPH (40%). In addition, the percentage of students who incorrectly determined the maximum kinetic energy K_{max} by subtracting E_{tot} and $U_{\text{global min}}$ has decreased since early iterations of the tutorial (10% compared to 40%). These positive results are expected: The OIPH is the only part of the sequence of materials within *Potential energy diagrams* that specifically helps students account for turn-around points when there are multiple regions satisfying $U(x) \leq E_{\text{tot}}$. These results suggest that the guiding students toward an understanding of turn-around points and allowed regions can be effective in helping students use those skills in determining the maximum kinetic energy.

7.3 Summary

The relatively low percentage of students who answered all six questions of the OIPH correctly on their first attempt suggests that the OIPH was addressing a need for the vast majority of students. Student performance on the posttests, though, was mixed when compared to pretest results. A larger percentage of students were able to account for turn-around points when reasoning about the maximum kinetic energy, but about the same percentage of students were able to reason correctly about the direction of the acceleration. We have not yet used our insights gained from the OIPH to modify the in-class worksheet or homework.

A broader aim of the OIPH was to develop a model that might be used with other topics within *Tutorials in Introductory Physics*. The insights we provided into the development of this component may inform the development of additional OIPHs for other topics within *Tutorials in Introductory Physics*.

Part III

IDENTIFYING AND ADDRESSING STUDENT DIFFICULTIES WITH SPATIAL PROBABILITY DENSITY

In this part, we discuss our research into identifying and addressing student difficulties related to probability and spatial probability density.

Motivation for research

Our motivation for investigating these topics arose from a decision by the Physics Department at the University of Washington in spring 2008 to include several new physics topics to Phys 123, the third and final course in the introductory physics sequence. (For a discussion of this sequence, see §2.1.) These new topics included basic quantum mechanics, nuclear interactions, and atomic structure. Additionally, several topics that were previously part of Phys 122 (e.g., inductance and AC circuits) were moved into Phys 123.

In considering these changes to the course, we noticed that there was relatively little instructional time devoted to these additional topics. Previous research by others has revealed that introductory students who study quantum mechanics struggled with some of the basic conceptual ideas, including probability [27]. Studies have also found that some of the difficulties extended to physics majors enrolled in upper-division courses [35, 36, 58, 59]. (See §3.2 for a discussion of prior research.) We were motivated by these factors to identify and address any difficulties that students encountered in their study of probability and spatial probability density.

Rather than focus on probability from a quantum mechanical point of view, we chose mainly to investigate the extent to which students understand *classical* aspects of probability. The tutorial topic that we developed, *Probability in classical and quantum mechanics*, also emphasizes classical reasoning. We anticipated that being introduced probability and probability density in a classical context could help students in a quantum mechanical setting, for example by allowing them to distinguish between classical and quantum mechanical models. Other researchers have used similar methods in a junior-level course with documented success [36].

Organization of Part III

We begin this part by describing student understanding of probability and spatial probability density prior to tutorial instruction (Chapter 8). We then describe the development of a new tutorial called *Probability in classical and quantum mechanics* that is designed to address the difficulties we identified through our research (Chapter 9). Finally, we describe the effect of tutorial instruction on student understanding (Chapter 10).

Chapter 8

IDENTIFYING STUDENT DIFFICULTIES WITH PROBABILITY AND PROBABILITY DENSITY

In this chapter we document our research into student difficulties with probability and spatial probability density in classical contexts. The data we present are from tutorial pretests administered to students enrolled in Phys 123 at the University of Washington. (See Chapter 2 for a discussion of pretests and courses.) Some of the pretests were administered before relevant lecture instruction on this topic, while others were administered after.[1] However, student performance was similar with respect to this timing, so all results are combined.

We begin this chapter in §8.1 by describing many of the pretest questions from which our data were gathered. We then describe the results of the administration of these questions in §8.2 and §8.3, which focus on probability and probability density, respectively.

We remind the reader of the abbreviated scheme to designate all course sections. See page 23 for a description.

[1]We note that probability and probability density in *classical* contexts is not a required part of the course. However, without a basic understanding of the classical counterpart, students are not in a position to understand how the behavior of quantum systems differ from their classical counterparts.

8.1 Description of questions used

Much of the data presented in this chapter are based on student responses to one of several types of questions. In this section we describe these questions and the corresponding correct answers. Other questions that we have administered only a limited number of times sometimes provide additional insight into student thinking. These are described as appropriate throughout this chapter.

As discussed in the introduction to Part III, the questions we developed focus on probability in the context of *classical* physics. Below, we categorize the questions according to whether or not they involve explicit descriptions of the motion of an object (e.g., a ball bouncing back and forth on a track or being dropped).

8.1.1 Questions involving explicit descriptions of motion

Some of the questions we have administered involve explicit descriptions of motion of an object. Given the deterministic nature of classical mechanics, we motivate the need for probability in our questions by asking students to suppose that the position of an object is measured at a random time. We note that other researchers have asked similar questions to probe student understanding of probability and related ideas. See, for example, Refs. [27, 35, 36, 65].

8.1.1-a The 'equal-length track' and 'equal-length drop' questions

One of the questions we have administered is called the 'equal-length track' question. In it, students are told that a track is composed of two horizontal levels of equal size that are connected by a short ramp. (See Figure 8.1(a).) A ball rolls back and forth and is able to make it up to the upper level. (The track is described as frictionless and the collisions with the walls as elastic. Sometimes the question states that the ball moves back and forth *forever*.) Students are told to suppose that a photograph is taken at a random time and are asked to compare the probabilities that it shows the ball in the upper and lower levels.

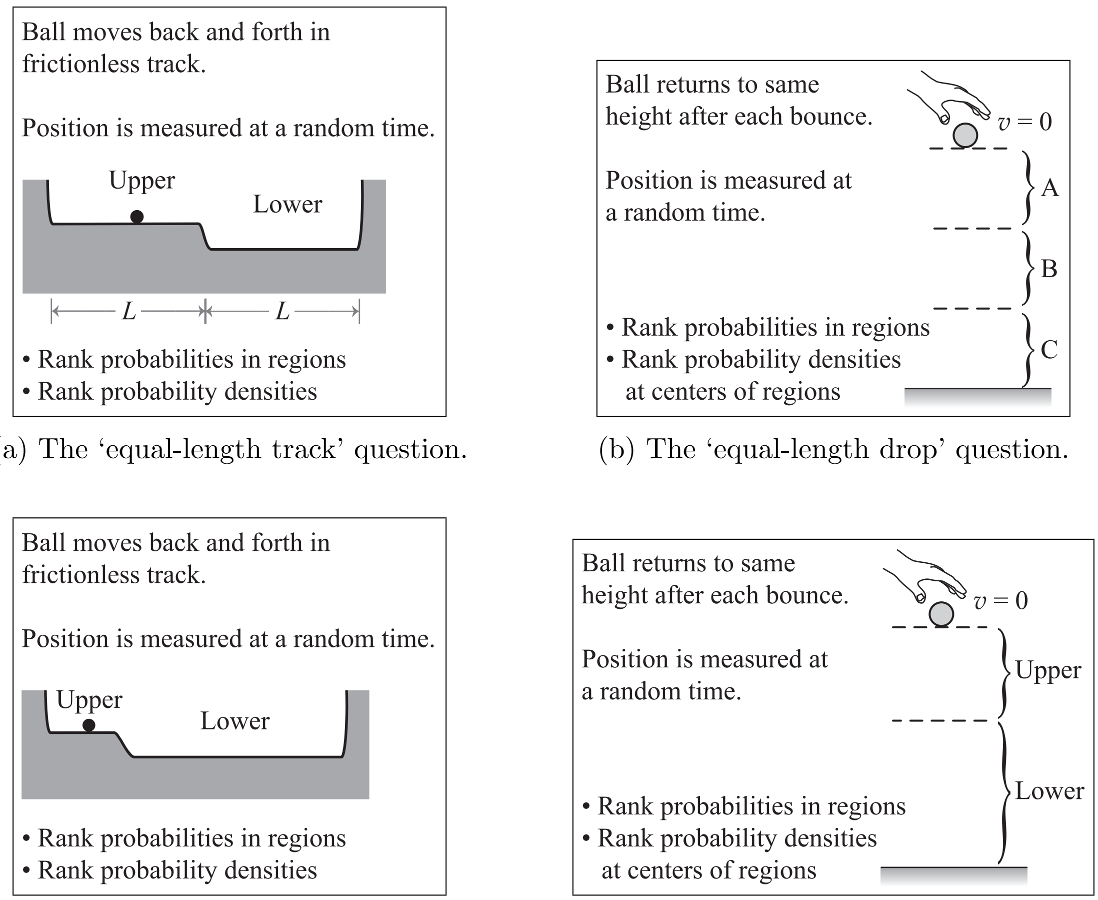
To answer correctly, students can note that the ball moves more slowly on the upper level than on the lower level. Since the levels are of equal size, the ball will spend a longer time on the upper level. Therefore, the photograph will be more likely to show the ball on the upper level ($P_{\text{upper}} > P_{\text{lower}}$).

On some administrations of the question, students were also asked to rank the probability *densities* on the two levels. One way to answer correctly is to consider the probabilities in infinitesimal-sized regions of width dx on each level. Since the ball moves more slowly through the infinitesimal region on the upper level, the probability dP is greater in the upper level. Thus, the probability density $\rho = dP/dx$ is greater in the upper level than in the lower level ($\rho_{\text{upper}} > \rho_{\text{lower}}$).

An alternative way to answer the density question is to first note that the probability density is uniform on each level (due to the constant speed of the ball), so the probability densities may be calculated using $\rho = P/L$. By noting $P_{\text{upper}} > P_{\text{lower}}$ and that the lengths of the levels are the same, the correct answer is that the density in the upper level is greater than that in the lower level.

A similar question we have administered is called the 'equal-length drop' question, shown in Figure 8.1(b). Students consider a ball that is dropped from rest. The ball bounces off the ground and always returns to the same height after each bounce. Students are asked to rank the probabilities in three equal-sized regions, labeled A, B, and C (see the figure). To answer correctly, students can use similar reasoning to the 'equal-length track' question and conclude that the ball spends more time in the upper regions than the lower regions. Thus, the correct ranking for the 'equal-length drop' question is $P_A > P_B > P_C$.

In all administrations of this question, students were also asked to rank the probability densities at the centers of the three regions. To answer correctly, students can use the same infinitesimal argument as the 'equal-length track' question above. The correct answer is $\rho_A > \rho_B > \rho_C$.



(a) The 'equal-length track' question.

(b) The 'equal-length drop' question.

(c) The 'unequal-length track' question.

(d) The 'unequal-length drop' question.

Figure 8.1: Abbreviated versions of two 'track' and two 'drop' questions.

8.1.1-b The 'unequal-length track' and 'unequal-length drop' questions

We have also administered two questions called the 'unequal-length track' and 'unequal-length drop' questions, shown in Figures 8.1(c) and (d), respectively. They are similar to the 'equal-length' versions except that the upper level is smaller than the lower level.

In attempting to rank the probabilities on these two questions, students can note that the ball moves slower in the upper region and faster in the lower region, so the relative amounts of time that the ball spends in the two regions *cannot* be compared. Thus, the correct answer is that there is *not enough information* to rank the probabilities. (This answer choice was available in each of the four questions shown in Figure 8.1.)

To rank the probability densities in these two regions, students can use the infinitesimal-region argument described above. The correct ranking is still $\rho_{\text{upper}} > \rho_{\text{lower}}$.

We emphasize here that although the *probabilities* on the 'unequal-length' versions cannot be ranked, the probability *densities* can be ranked.

8.1.1-c The 'infinite square well' question

Another question we have asked in the context of motion is the 'infinite square well' question, which is shown in Figure 8.2. In this question, students are told that a ball moves back and forth on a level, frictionless track, and that it bounces elastically from the walls. An imaginary line divides the track into two unequal-sized regions, A and B, with $L_A < L_B$. There are four sub-questions. Questions 1 and 2 ask students to rank the probabilities and probability densities in the two unequal-sized regions, respectively. The correct answers are $P_A < P_B$ and $\rho_A = \rho_B$. Questions 3 and 4 ask students to write expressions for the probability and probability density in region A, respectively. The correct answers are $P_A = \frac{L_A}{L_A+L_B}$ and $\rho_A = \frac{P_A}{L_A} = \frac{1}{L_A+L_B}$.[2]

[2] In some administrations of this question, a partition was placed between regions A and B while the marble was still moving. The track was then physically separated. We found no significant difference in student performances on these two versions, so we combine results.

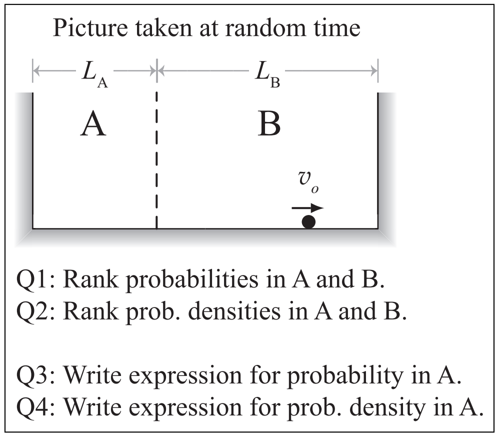


Figure 8.2: Abbreviated version of the 'infinite square well' question.

8.1.2 Questions not involving descriptions of motion

In addition to the questions described above, some of the questions we have administered do not explicitly describe the motion of an object. Each of these pretests were administered only in the context of probability density (not probability). These are described below.

8.1.2-a The 'density from probability' question

One question we administered is the 'density from probability' question shown in Figure 8.3. In this question, students are told that the probability of finding the ball in region A of a track is greater than that in region B. No illustration of the track is given, but students are told explicitly that the relative lengths of the two regions are unknown. Students are then asked to compare, if possible, the probability densities in the two regions. (They are told that the density in each region is uniform.) The correct answer is that there is *not enough information* to rank the probability densities.[3]

[3] In another version of the question (not shown), the track context is removed and students are told only that the probability of finding an object in a given region is greater than that in a different region. The difference in student performance on these versions was not significant, so we combine results throughout this dissertation.

A ball is confined to a frictionless track that is composed of two regions, A and B. It is known that the probability of finding the ball in region A is *greater than* the probability of finding the ball in region B.

Question 11.
Is the *probability density* in region A *greater than, less than,* or *equal to* the probability density in region B? Assume that the probability density in each region is constant.

not enough information

Figure 8.3: Abbreviated version of the 'density from probability' question. The correct answer is selected in the drop-down box.

8.1.2-b The 'broken block' question

Another question we have administered is the 'broken block' question, which is shown in Figure 8.4. In this question, students are informed that a small marble is placed at a random location in a block of material. There are two tasks. The 'expression task' asks students to write an expression for the probability density associated with finding the marble in the block. To answer correctly, students can note that the probability density is the probability of finding the marble somewhere in the block divided by the volume of the block. Thus, the correct answer to the expression task is $\rho = 1/HWD$. Next, the 'ranking task' informs students that the block of material is broken into two unequal sized pieces, with piece A smaller than piece B. Students are asked to rank the probability densities in piece A, piece B, and the original block. To answer correctly, students can note that breaking the block does not alter the probability that the marble is in any given region. Thus, the correct answer to the ranking task is $\rho_{\text{orig}} = \rho_B = \rho_A$.

The broken block question was modeled after a similar set of questions developed by Kanim [47] in the context of mass and charge. We describe his questions and provide a summary of some of his results in §8.3.5-a.

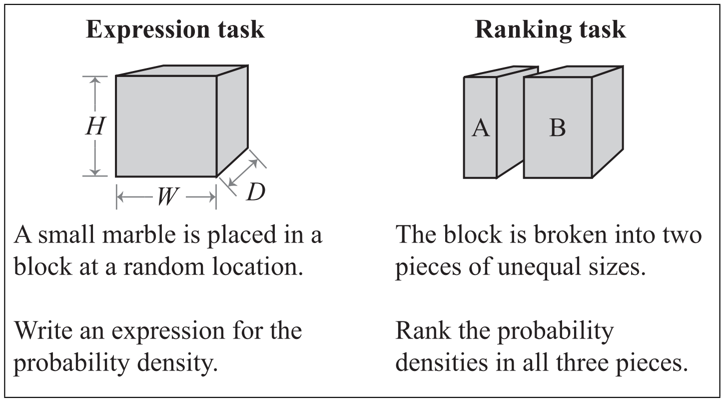


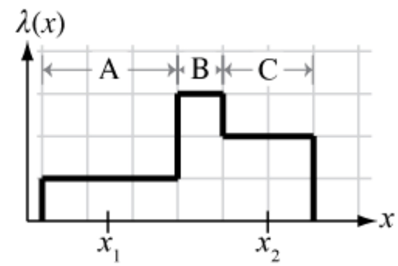
Figure 8.4: Abbreviated version of the 'broken block' question. The correct answers are $1/HWD$ and $\rho_{\text{orig}} = \rho_A = \rho_B$.

8.1.2-c The 'density graph' question

Another set of questions we have administered are the 'density graph' questions shown in Figure 8.5. Students are shown a graph of either "probability per unit length" *vs.* position or "charge per unit length" *vs.* position. (The graphs are identical.) They are asked to rank the probabilities or charges in the indicated regions. To answer correctly, students can compare the integrals of the given density functions in the three regions. The correct answer for both questions is $C > A = B$.

In the following section we begin our discussion of student difficulties, starting with *probability.*

A one-dimensional rod contains a positive charge. A graph of the amount of charge per unit length λ versus position *x* is shown at right.



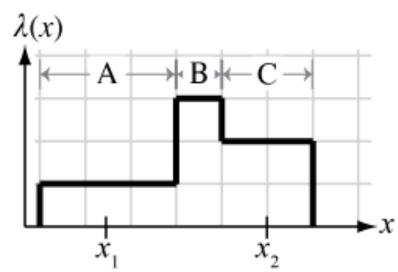
Question 6.

Rank, from **greatest to least,** the amount of charge in regions A, B, and C. (Use only the symbols **A**, **B**, **C**, **>**, and **=**. Do not use spaces or the < symbol.)

C>A=B

(a) Charge density version.

Many photographs are taken of a pebble confined to some unknown frictionless track. A graph of the probability of a photograph showing the pebble per unit length λ versus position *x* is shown at right.



Question 8.

Rank, from largest to smallest, the probability of a photograph showing the pebble in regions A, B, and C. (Use only the symbols **A**, **B**, **C**, **>** and/or **=**. Do not use spaces or the < symbol.)

C>A=B

(b) Probability density version.

Figure 8.5: Abbreviated versions of the 'density graph' questions.

8.2 Student difficulties with probability

The concept of probability is intricately tied to the concept of probability density. It can be argued that the former is more basic. We therefore begin by describing student difficulties with probability that we have identified through our research. (Difficulties that relate to probability *density* are discussed later in §8.3.)

We identified several difficulties related to probability. Some of these were associated with the failure to incorporate (or recognize the need to consider) the speed of the ball and/or the lengths of the regions (§8.2.1 and §8.2.2). Other difficulties involved using the correct rankings of speed and lengths in an incorrect manner (§8.2.3) or a failure to recognize that changes made to one region can affect the probability in another (§8.2.4). Additional difficulties are discussed in later subsections (§8.2.5 and §8.2.6). Prior research related to probability was discussed in §3.2.

8.2.1 Tendency to rank probabilities solely according to the sizes of the regions

In this subsection we describe a common tendency of students to rank probabilities solely according to the sizes of the regions (i.e., larger regions have larger probabilities). In §8.2.1-a we present examples of student reasoning from a few different types of questions. We then describe in §8.2.1-b how this tendency was affected by asking an additional precursory question or by slightly varying the context of the question.

8.2.1-a Examples from ‘track’ and ‘drop’ questions

The tendency to rank probabilities solely according to the sizes of the regions was elicited in both the two ‘track’ and two ‘drop’ questions (see Figure 8.1 on page 212). We provide examples from these and other questions below.

Examples from the two 'track' questions

In §8.1.1-a we described the 'equal-length track' question in which a ball moves back and forth on a track that is composed of two equal-length levels that are at different heights (see Figure 8.1(a) on page 212). Only 30% of the students answered correctly, as shown in Table 8.1(a). Almost half of the students incorrectly answered that the probability in the two levels are equal. Most of these students cited only the relative lengths of the levels and did not mention the relative speeds of the ball. The following response is illustrative of this reasoning:

> *"[$P_{\text{upper}} = P_{\text{lower}}$.] They have the same relative lengths, so the pebble should be equally likely to be on one side of the track vs. the other."*
> (Phys 123C, spring 2008)

As this response shows, many students did not explicitly discuss the *times* spent in the two regions. Considering the relative amounts of time is crucial to understanding how to rank the probabilities. However, some students who ranked the probabilities as being equal did discuss the relative amounts of time. The following student quote is illustrative:

> *"The regions A and B are equal size, so the ball spends the same amount of time in each one through one cycle of motion. Therefore the probability that the pebble will be in one of the regions is equal."*
> (Phys 121A, spring 2008)

Although somewhat rare, some of these students did discuss the speed of the ball in the two regions, but incorrectly stated that the speeds were the same. This might indicate that some students did not correctly consider the effect of the ramp on the speed of the ball, or failed to recognize its relevance.

It is interesting to note that many of the students who answered *correctly* did not mention the relative lengths of the two levels. This could indicate that they focused *only* on relative speeds.[4] We were motivated partially by this observation to ask the 'unequal-length track'

[4] We discuss this student tendency explicitly in §8.2.2.

Table 8.1: Student performance on 'equal-length track' and 'unequal-length track' questions. The correct answers are highlighted.

(a) 'Equal-length track' question.

Answer	N=275[a]
$P_{\text{upper}} > P_{\text{lower}}$	30%
$P_{\text{upper}} = P_{\text{lower}}$ (by size)	45%
$P_{\text{upper}} < P_{\text{lower}}$	25%
Not enough information	<5%

[a] Sections 123A082, 123C082, 123A083, 123A084, and 123A091.

(b) 'Unequal-length track' question.

Answer	N=253[a]
Not enough information	10%
$P_{\text{upper}} < P_{\text{lower}}$ (by size)	75%
$P_{\text{upper}} = P_{\text{lower}}$	10%
$P_{\text{upper}} > P_{\text{lower}}$	10%

[a] Sections 123A092 and 123C092.

question (see Figure 8.1(c) on page 212). In this question, the upper level of the track has a smaller length than the lower level, so the relative amounts of time that the ball spends in the two levels cannot be compared. Therefore, the correct answer is that there is *not enough information* to compare the probabilities.

Student performance on the 'unequal-length track' question is shown in Table 8.1(b). Only 10% of the students answered correctly, a reduction from the 30% who answered the 'equal-length track' question correctly. The vast majority of the students (about 75%) answered incorrectly that the probability in the upper, smaller region is less than that in the lower, larger region. These students tended to cite only the relative lengths of the two levels in their explanations. The following student quote is typical:

> *"[The probability in the upper level is] less, since upper level has less area for the ball to be."*
>
> (Phys 123A/C, spring 2009)

Some of these students are more explicit about why the probability is greater in the larger region. The following student quote is one such example:

> *"[$P_{\text{upper}} < P_{\text{lower}}$.] The probability that the ball will be at any given point is equal throughout the track: however, the lower level has more track and thus more possible points."*
>
> (Phys 123A/C, spring 2009)

This statement and other similar ones are consistent with the belief that a measurement of position at a random time is equivalent to choosing a random position on the track. These students failed to recognize that the speed of the ball in each region is important.

It is notable that a smaller fraction of students answered correctly on the unequal-length track question than on the equal-length track question (10% compared to 30%). This observation suggests that answering the equal-length track question correctly does not necessarily imply a robust understanding of probability in these contexts. It suggests further that the unequal-length track question may be better at identifying students who have a more complete understanding of probability.

We also note that a greater fraction of students on the 'unequal-length track' question ranked the probabilities solely according to the size of the region (75% compared to 45%). This could indicate that the relative lengths of the levels of a track may have had an affect on the reasoning that students used to rank probabilities.

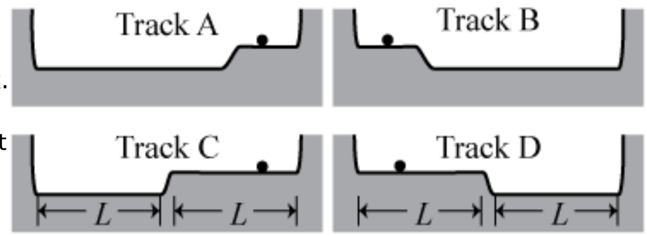
Additional question: The 'choose track' question

In both the 'equal-length track' and the 'unequal-length track' questions, students are asked to use a given track to rank probabilities. A different type of question we have asked is the 'choose track' question, which is shown in Figure 8.6. In this question, students are shown four different two-level tracks and are asked for which track(s) could the probability of measuring a ball to be in the left level be greater than that in the right level (see the figure). The correct choices are tracks *A, B*, and *D* because the ball is either moving slower in the left level or the left level is of greater length.

Student performance is shown in Table 8.2. The upper portion of the table represents the most common answer combinations. The lower portion represents the percentage of students who selected a given track as *part* of their answer.

Four balls are confined to move back and forth in the four frictionless tracks shown. The balls are able to make it up to the upper level in each track.

Suppose a photograph of each track were taken at a random time.



Question 13.
For which of the tracks shown could the probability of the photograph showing the ball in the left region be greater than that in the right region (*i.e.,* for which track could Prob(left) > Prob(right) be true)? Select all that apply.

- [x] Prob(left) > Prob(right) could be true for track A
- [x] Prob(left) > Prob(right) could be true for track B
- [ ] Prob(left) > Prob(right) could be true for track C
- [x] Prob(left) > Prob(right) could be true for track D
- [ ] P(left)>P(right) could not be true for any of the shown tracks

Figure 8.6: The 'choose track' question. The correct answers are selected.

Table 8.2: Student performance on the 'choose track' question. The correct answers are highlighted.

Response	N=249[a]
Only tracks ABD are consistent	5%
Only track A is consistent	50%
Only tracks AC are consistent	10%
Only tracks AD are consistent	5%
Only tracks BD are consistent	<5%
Only track D is consistent	<5%
No tracks are consistent	10%
Track A is consistent	75%
Track B is consistent	20%
Track C is consistent	25%
Track D is consistent	20%

[a] Sections 123A092 and 123C092.

Only 5% of the students answered this question correctly. The most common answer combination, chosen by 50% of the students, was that *only track A* could have a greater probability on the left level than that on the right. We note that track A is the only track whose left region is larger than the right. Thus, it seems that up to half of students chose tracks based only on their lengths. They failed to account for the speed of the ball in the two levels. This tendency was also the most common incorrect method used by students on the two 'track' questions in which students ranked probabilities in the two levels of a given track.

Examples from the two 'drop' questions

We note briefly that many students also ranked probabilities solely according to the sizes of the regions on the 'equal-length drop' and 'unequal-length drop' questions.[5] Student performance for these questions are shown in Tables 8.3(a) and 8.3(b), respectively. About half of students (50%) answered the equal-length question correctly. Slightly fewer students (40%) answered the unequal-length question correctly. Nearly all of the students who answered the more difficult unequal-length question correctly also answered the equal-length question correctly.[6]

On both questions, the most common incorrect answer was to rank the probabilities according to the sizes of the regions (i.e., $P_A = P_B = P_C$ on the equal-length question, and $P_{\text{upper}} < P_{\text{lower}}$ on the unequal-length question). The percentages were 30% and 40% for the equal-length and unequal-length questions, respectively. Explanations provided by the vast majority of these students are similar to those on the 'equal-length track' and 'unequal-length track' questions, so we do not provide examples of student responses here.

Finally, we note that a comparison of student performance on the two 'track' questions to performance on the two 'drop' questions is made difficult due to circumstances surrounding their administrations. We discuss these circumstances below.

[5]For a description of these questions, see Figure 8.1 on page 212.

[6]Students answered both questions as part of the same pretest.

Table 8.3: Student performance on the two 'drop' questions. The correct answers are highlighted.

(a) 'Equal-length drop' question.

Answer	N=404[a]
Correct	50%
All probabilities equal	30%
Opposite ranking of correct	5%
Not enough information	<5%

[a] Sections 123A102, 123C102, and 123A114.

(b) 'Unequal-length drop' question.

Answer	N=404[a]
Not enough information	40%
$P_{\text{upper}} < P_{\text{lower}}$	40%
$P_{\text{upper}} = P_{\text{lower}}$	15%
$P_{\text{upper}} > P_{\text{lower}}$	5%

[a] Sections 123A102, 123C102, and 123A114.

Commentary on comparing track and ball-drop performance

A direct comparison of student performance on the track questions and ball-drop questions (Tables 8.1 and 8.3) is difficult due to a difference in how the questions were administered:

- We always administered both the 'equal-length drop' and the 'unequal-length drop' questions on the *same* pretest (i.e., students answered both questions).
- The 'equal-length track' and 'unequal-length track' questions were never administered on the same pretest.

To probe the possible effects of these different circumstances, we asked student enrolled in Phys 121[7] one of two versions of a pretest:

- One version had both the equal-length drop and unequal-length drop questions.
- The other version had only the unequal-length drop question.

This combination of pretests allows us to probe whether the presence of the 'equal-length drop' question affects performance on the 'unequal-length drop' question.

On the version with both questions, student performance on the 'unequal-length drop' question was similar to Phys 123 student performance: about 40% (N=165) of students

[7] Sections include 121A152 and 121B152. We have not yet performed this study with Phys 123 students.

answered correctly. However, on the version with only the unequal-length drop question, only 20% (N=180) answered correctly.

This result suggests that asking both versions of the 'drop' question can improve student performance on the unequal-length drop question. For this reason, we do not make any direct comparisons between performance on the 'track' and 'drop' questions.

Despite the possibility that Phys 123 student performance that we reported in Table 8.3(b) is 'artificially' high due to the presence of the 'equal-length drop' question, we emphasize that only 40% of the students were able to answer correctly. About the same fraction of students ranked the probabilities solely according to the sizes of the regions ($P_{\text{upper}} < P_{\text{lower}}$).

8.2.1-b Examination of the effect of asking student to consider speed or potential energy

The results above (§8.2.1-a) indicate that a large fraction of students considered only the relative sizes of regions when ranking probabilities. These students failed to incorporate the speed of the object. To explore this tendency further, we administered variations of some of the track questions. In one of the variations, we administered a precursory question in which students are asked to rank the speed of the ball in the two levels. In the other variation, we provided explicit information about the *potential energy* associated with the two levels without the precursory speed question. (The original questions do not include any mention of potential energy.) We begin by discussing results from the precursory speed questions.

Effect of providing a precursory speed question

The precursory speed question was provided as part of the 'equal-length track' question. (The unmodified equal-length track question is shown in Figure 8.1(a) on page 212.) The results with and without the speed question are shown in Table 8.4. About 75% of the students answered the equal-track question correctly with the speed question present, compared to only 30% without. Only 15% of the students incorrectly ranked the probabilities solely according to the sizes of the regions with the speed question, compared to 45% without. This result suggests that more students correctly incorporated speed into their answer after

Table 8.4: Effect of precursory speed question on student performance on the 'equal-length track' qusetion. The correct answer is highlighted.

Answer	Without speed question N=275[a]	With precursory speed question N=159[b]
$P_{\text{upper}} > P_{\text{lower}}$	30%	75%
$P_{\text{upper}} = P_{\text{lower}}$ (by size)	45%	15%
$P_{\text{upper}} < P_{\text{lower}}$	25%	10%
Not enough information	<5%	<5%

[a] See Table 8.1(a) on page 220.
[b] Sections 123A074 (version W2A) and 123B091.

being explicitly asked about speed.

We have also asked a precursory speed question on the 'choose track' question described on page 221. In this question, students are asked to select the track(s) for which the probability in the left level could be greater than that in the right level. (The four tracks from which students could choose were shown in Figure 8.6.) The results are shown in Table 8.5. The percentage of students answering correctly is roughly the same (15% correct with the speed question compared to 5% without). However, the percentage of other answers changed in notable ways. With the precursory speed question, only 15% of the students chose *only* track A (which is incorrect) compared to 50% without. Furthermore, when the speed question was present, 60% of the students selected track D as one of the possible tracks (which is correct) compared to only 20% without. An increase in the percentage of students including track D is expected since the precursory speed question appeared to produce a sizable increase in the percentage of students who correctly answered the equal-length track question, which was based on track D. This suggests that fewer students are using the size of the region alone to choose tracks based on probabilistic information.

We note also that the percentage of students who included track B in their answer (which is correct) increased only marginally with the speed question present, from 20% to 35%. This is at the threshold of what we consider significant. This suggests that students struggled in

Table 8.5: Student performance on the 'choose track' question with and without a precursory speed question. The correct answers are highlighted.

Response	Without speed question N=249[a]	With speed question N=115[b]
Only tracks ABD are consistent	5%	15%
Only track A is consistent	50%	15%
Only tracks AC are consistent	10%	10%
Only tracks AD are consistent	5%	15%
Only tracks BD are consistent	<5%	10%
Only track D is consistent	<5%	15%
No tracks are consistent	10%	5%
Track A is consistent	75%	60%
Track B is consistent	20%	35%
Track C is consistent	25%	15%
Track D is consistent	20%	60%

[a] See Table 8.2 on page 222. [b] Section 123B091.

ranking probabilities when both the speed and length must be considered, even when the precursory speed question is present. Interestingly, though, the percentage of students who included track A as one of the possible tracks (which is correct) decreased slightly from 75% to 60%. Further research is needed to understand these results.

Effect of providing information about potential energy

In one quarter we administered two different versions of the 'unequal-length track' question: the 'PED' version and the 'track and PE' version. In the 'PED' version, students are provided with a potential energy diagram instead of an illustration of a track. The diagram was generic and did not pertain to a ball-Earth-track system. In the 'track and PE' version, students are provided with a figure of a track and a statement that the two regions have "different potential energies." (This information was not present in the original version.) Each student answered only one of these versions, which was randomized based on his or her University-assigned student identification number.

Table 8.6: Comparison of student performance on different versions of the 'unequal-length track' question. The correct answer is highlighted.

	Version		
Response	Original N=253[a]	PED N=150[b]	Track and PE N=133[b]
Not enough information	10%	20%	40%
$P_{\text{upper}} < P_{\text{lower}}$	75%	45%	40%
$P_{\text{upper}} = P_{\text{lower}}$	10%	25%	10%
$P_{\text{upper}} > P_{\text{lower}}$	10%	10%	5%

[a] See Table 8.1(b) on page 220.
[b] Sections 123A152 and 123C152.

The results are shown in Table 8.6. Students performed better on the 'track and PE' version than they did on both the original and 'PED' versions (40% correct compared to 10% and 20% correct, respectively). Furthermore, less than half of the students on both of the modified versions ranked the probabilities according to the sizes of the regions, whereas 75% of the students did so on the original question. These results are similar to those from the precursory speed question.

8.2.1-c Summary

We have found that a large fraction of students ranked probabilities solely according to the sizes of the regions (i.e., larger regions have larger associated probabilities). These students failed to account for the speed of the ball and/or failed to recognize that the speed of the ball is relevant in comparing probabilities. The percentage of students doing this was relatively large for versions of questions in which students were required, on their own, to recognize the role of speed in the ranking of the probability. We found, however, that either asking students a precursory speed question or providing students with information about the potential energy could increase the fraction of students who took speed into account. This suggests that associating potential energy with a physical system can reduce the tendency to use only the sizes of regions to rank probabilities. This result has implications for curriculum development; it may be relatively straightforward to guide students toward understanding the role that kinematic quantities play in ranking probabilities on a given track.

8.2.2 Tendency to rank probabilities solely according to speed

The responses of some students to the pretest questions indicate that when ranking probabilities they considered only the relative speeds of the object of interest in the different regions. For example, on page 227 we described a variation of the 'unequal-length track' question in which students are provided information regarding the potential energy. On this question, about 5% of the students answered that the probability in the upper, smaller region is less than that in the lower, larger region (i.e., $P_{\text{upper}} > P_{\text{lower}}$) and mentioned only the relative speeds in their explanation. (See Table 8.6.) The following student quote is illustrative of this tendency (note that "side A" is the upper level):

> *"[$P_{\text{upper}} > P_{\text{lower}}$.] Side A has more potential energy than side B which means it has less kinetic energy meaning it has less speed and is in side A for more time"*
>
> (PHYS 123A/C, SPRING 2015)

This student and others with similar explanations failed to account for the length of the regions of interest when ranking probabilities.

We note that essentially no students used this line of reasoning on the original unequal-length track question in which students were not prompted to consider potential energy. This is not surprising given that very few students incorporate the speed of the ball in any fashion on the original track questions.

8.2.3 Tendency to use compensation-based reasoning

The responses of some students to the pretest questions we have administered indicate that they used 'compensation-based' reasoning when ranking probabilities. For example, on the 'unequal-length track' question described in Figure 8.1(c) on page 212, students were asked to rank the probabilities of finding a ball on two unequal-sized horizontal levels of a track. About 10% of the students answered incorrectly that the probability on the smaller, upper level is equal to that on the larger, lower level (see Table 8.1(b) on page 220). The following student quote is illustrative of this reasoning:

> *"[The probabilities are equal.] Although the section is smaller, the ball moves slower in it than in [the other section] so they would even out."*
> (Phys 123C, spring 2010)

We have observed similar reasoning on the 'unequal-length drop' question in which a ball is dropped and students are asked to rank the probabilities in two unequal-sized regions. On this question, 15% of the students answered that the probabilities were equal. The reasoning provided by these students is similar to those on the 'unequal-length track' question.

Unlike the students discussed in §8.2.1 who considered only the lengths of the regions, or the students discussed in §8.2.2 who considered only the relative speeds, students who used compensation-based reasoning attempted to incorporate both the speed of the ball and the lengths of the regions to arrive at an answer. However, they incorrectly stated that these two effects compensate and effectively 'cancel.'

This tendency to use compensation-based reasoning was also evident on questions that are more quantitative in nature. In §8.2.1-b we described a variation of the unequal-track question in which we provided students a potential energy diagram. The length of region A was half as long as that of region B, but the potential energy was twice as much.

About 10% of the students attempted to use the ratios of the potential energies in the two regions to explicitly claim how the speeds or kinetic energies compare. Many of these students went on to conclude that the relative amounts of times spent in the two regions are the same.[8] The following student quote is illustrative of this line of reasoning:

> *"[$P_A = P_B$.] Because the potential energy at B is half that of potential energy at A, the object moves twice as fast in B. However, B is twice as long as A, so the object spends equal amounts of time in both areas. Thus, the probabilities are equal."*
>
> (Phys 123A/C, spring 2015)

An additional 5% of the students stated that the probabilities in the two regions are equal but did not explicitly describe the ratios of the speeds or the kinetic energies. Thus, up to 15% of the students may have been using compensation-based reasoning.

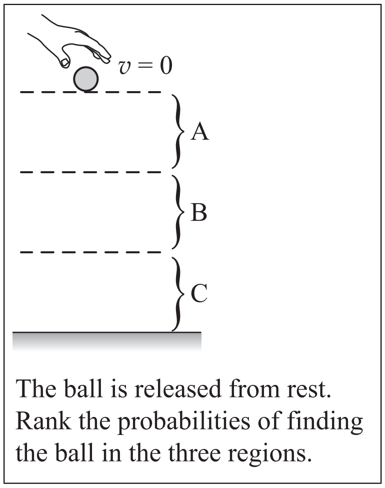
Commentary on the tendency to use compensation-based reasoning in other areas of physics

The belief by students that two competing causes exactly cancel to produce the same effect has been documented in several other areas of physics. For example, Lawson and McDermott [70] administered a question in which two pucks of different masses $m_1 > m_2$ and different speeds $v_2 > v_1$ had different impulses applied. They found that some students argued on the basis of mass and speed to conclude that the pucks have the same momentum mv. Kautz et al. [71] found that students use similar reasoning in arguing about speed and volume in the context of a microscopic model for ideal gases.

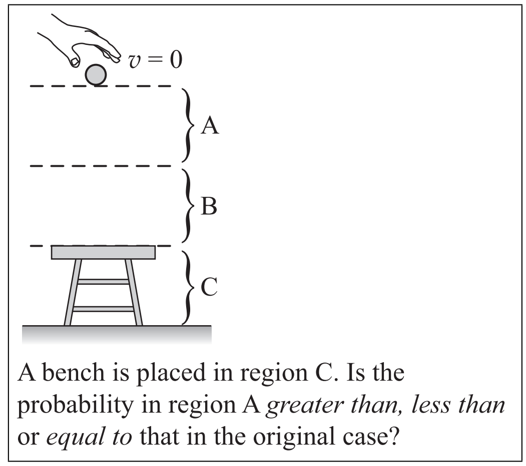
8.2.4 Tendency to reason locally about probability

Some questions we have administered ask students to predict how a change made to one region affects the probability of finding the ball in another region. The response of some students indicate that they believed that the probability of finding an object in each region

[8]This observation led us to ask a related question as part of our study of student understanding of potential energy diagrams. See §4.2.2-c.



(a) The 'equal-length drop' question. The correct answer is $P_A > P_B > P_C$.



(b) Follow-up question. The correct answer is *greater than.*

Figure 8.7: Abbreviated versions of the 'equal-length drop' question and follow-up question.

is independent of the other regions. For example, in the 'equal-length drop' question, a ball is dropped from rest and students are asked to rank the probabilities of finding the ball in three equal-sized regions, labeled A, B, and C (see Figure 8.7(a)). In some administrations of this question, students were asked a follow-up question in which they are told that a bench is placed in region C (the lowest region). Thus, the ball is no longer able to reach region C. Students are asked how the probability in region A (the upper region) after the bench is added compares to that before the bench is added (see Figure 8.7(b)). To answer correctly, students can note that the amount of time that the ball spends in region A is the same but the total amount of time to complete a bounce is less. Thus, since the *fraction* of time the ball spends in region A is greater, the probability of finding it there is also greater.

On this question, about 80% of the students (N=117)[9] answered correctly. Most students had reasoning that we considered correct or partially correct (e.g., stating that the ball can no longer reach region C). However, about 15% answered incorrectly that the probability in region A does not change. Many of these students explained either that the amount of time or the speed of the ball in region A does not change. The following response is typical of this line of reasoning:

> *"[The probability in region A is the same after the bench is added.] It will still spend the exact same amount of time in region A as it did before the bench was added."*
>
> (Phys 123A, spring 2010)

This student and others with similar responses failed to recognize that the *fraction* of time spent in a region determines the probability, not the *absolute* amount of time. Essentially, these students treated the probability in the regions as being independent of one another.

8.2.5 Failure to determine an expression for probability

The results presented in §8.2.1–§8.2.4 above indicate that many students failed to identify and/or incorporate, qualitatively, the factors that determine the relative probability of finding a ball in two regions of a track (i.e., the speed, the length, or both). We were interested in whether or not students also would struggle if the question posed is to find the probability in a more quantitatively manner.

A question we developed as part of this investigation uses a track composed of only a single level (see Figure 8.8). As such, the question does not require students to consider relative speed. Since a significant fraction of students struggled to incorporate the speed of the ball correctly (see §8.2.1), we decided to focus first on the 'simpler' spatial aspects of probability. Future questions may incorporate the speed of the ball.

The question we developed is called the 'infinite square well' question. An imaginary line divides the track into two unequal-sized regions, labeled A and B, with $L_A < L_B$ (see

[9] Section Phys 123A, spring 2010.

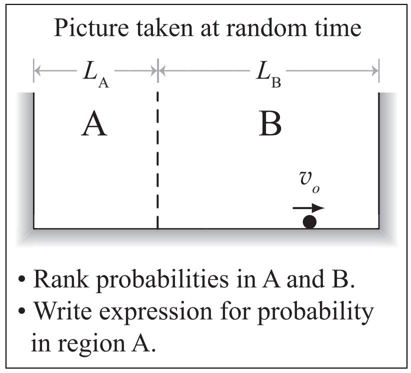


Figure 8.8: Abbreviated version of the 'infinite square well' question. The correct answers to the two questions shown are $P_A < P_B$ and $P_A = \frac{L_A}{L_A+L_B}$.

the figure). Students are first asked to rank the probabilities of finding the ball in the two regions. To answer correctly, students can note that the speed of the ball is the same in the two regions, so the probability will be greater in the larger region (i.e., $P_A < P_B$). They are then asked to write an expression for the probability of finding the ball in region A. The correct answer is $P_A = \frac{L_A}{L_A+L_B}$ because, in this case, the probability is proportional to the length of the region (since the speed is constant) and the two probabilities must sum to 1.[10]

Student performance is shown in Table 8.7. About 90% of the students answered the 'ranking task' correctly. This relatively high performance is not surprising given that a large fraction of students appear to reason about probability solely based on the lengths of the regions (see §8.2.1), which in this case leads to the correct answer.

Despite this high performance on the ranking task, only 60% answered correctly on the 'expression task.' Explanations typically did not go beyond describing the mathematical expression in words, though given the relative simplicity of the physical system (as opposed to tracks with different speeds in different regions), we categorize these as correct.

The most common incorrect answer on the expression task (given by about 10% of the students) was *not enough information.* These students typically stated that the exact values

[10]Students were also asked to rank the probability densities in the two regions and to write an expression for the probability density in region A. We discuss performance on these questions throughout §8.3.

Table 8.7: Student performance on the two probability questions of the 'infinite square well' question. The correct answers are highlighted.

Question	Respnose	N=401[a]
Rank probabilities	$P_A < P_B$	90%
	$P_A > P_B$	5%
	$P_A = P_B$	5%
Write probability in reg. A	$\frac{L_A}{L_A+L_B}$	60%
	Not enough info	10%

[a] Sections 123A102, 123C102, 123A152, and 123C152.

of the lengths are needed, despite that the symbols L_A and L_B are provided in the question text. Expressions provided by many of the other students varied greatly and are difficult to categorize. For example, a small fraction of the responses had non-dimensionless units (e.g., "$v_o L_a$"). Other students appeared to incorporate ideas from quantum mechanics, either from lecture or their textbook (e.g., "$\sqrt{2/L_A}\sin(\pi x/L_A)$"). This tendency to incorporate quantum mechanics in classical situations or vice versa has been identified by other researchers, including Bao [27], Ambrose [35], and Crouse [36].

The lower performance on the expression task compared to the ranking task (see the table) suggests that the ability to rank probabilities in simple situations does not necessarily imply an ability to reason symbolically.

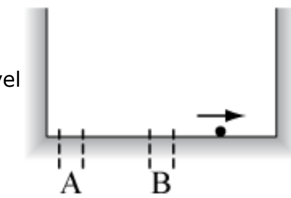
8.2.6 Additional difficulties

In addition to the difficulties described above, there were others that were either less common or different in nature, but still seemed significant to document. In this subsection we describe these additional difficulties.

8.2.6-a Belief that object passes through middle region twice, but edge regions only once

One of the questions we administered is the 'middle twice' question, shown in Figure 8.9. In this question, students are presented with a single-level track and two small, equal-sized

A pebble of mass m = 5 g moves back and forth in a one-dimensional region with rigid walls, as shown. Assume that the pebble rebounds from the walls elastically (*i.e.,* without loss of energy) and that the surface of the region is level and frictionless. Regions A and B are of equal width, as shown.



Suppose a photograph of the pebble were taken at a random time.

Question 6.
Is the likelihood that the photograph shows the pebble in region A *greater than, less than,* or *equal to* the likelihood that it shows the pebble in region B?

likelihood in region A is equal to likelihood in region B

Figure 8.9: The 'middle twice' question. The correct answer is that the probabilities are equal.

regions labeled A and B. Region A is near the edge of the track and region B is near the middle. Students are asked to rank the probabilities of finding the ball in the two regions. The correct answer is that the two probabilities are equal to each other since the ball moves through each equal-sized region with the same speed.

We were motivated to ask this question based on the research by others who studied upper-division physics courses. On a similar question, Ambrose [35] found that 30% of students believed the particle was more likely to be found in regions near the center of the track than in regions near the edge. The reasoning that students who answered in this manner used was either quantum mechanical in nature (e.g., failing to distinguish between classical motion and the ground state of the quantum mechanical infinite square well [35]) or incorrect classical reasoning (e.g., the belief that a classical object passes through the middle twice per period but 'reaches' each edge only once [36]). We wished to determine to what degree introductory students exhibited similar lines of reasoning.

On the 'middle twice' question, only about 5% of the students gave answers and explanations consistent with the belief that the ball moves more often through the middle (i.e., a classical reasoning.) The following student quote is illustrative of this tendency:

> *"[$P_B > P_A$.] if the pebble rolls back and forth, it will roll over B 2 times more often as it rolls through the middle to reach the sides of the region."*
> (Phys 123C, spring 2008, version U1b.)

We note that even though the introductory students had had some lecture instruction on quantum mechanics at the time of the pretest, essentially no students used quantum mechanical reasoning on this question as the junior-level students did. We comment on this result at the end of this section (§8.2.7).

We have observed a similar percentage of student exhibiting this belief on the 'equal-length drop' question that is described in §8.1.1-a (see Figure 8.1(b) on page 212). In this question, a ball bounces under the influence of gravity. Students are asked to rank the probabilities in three equal-sized regions. The following response illustrates the belief that the ball passes through the middle region twice:

> *"[$P_B > P_A = P_C$.] Since B is in the middle it will have the highest probability since one cycle of the ball takes it through B twice, but A and C only once each."*
> (Phys 123C, spring 2010)

8.2.6-b Belief that the probability at a point is non-zero

Other researchers have found that some students incorrectly believed that the probability at a point is non-zero. For example, Crouse [36] found that only 35% of junior-level students correctly identified the probability at a point to be zero. Many of the other students attempted to relate the relative probability densities two regions to the probabilities at a point within each region.

We were interested in probing whether introductory students would exhibit similar reasoning. A question that we designed to probe this tendency is shown in Figure 8.10. We call this question the 'probability at a point' question. Students are presented with a potential energy diagram resembling that for a spring. Two particular x-coordinates, F and G, were labeled on the diagram. Students were asked to rank the probabilities of finding the center

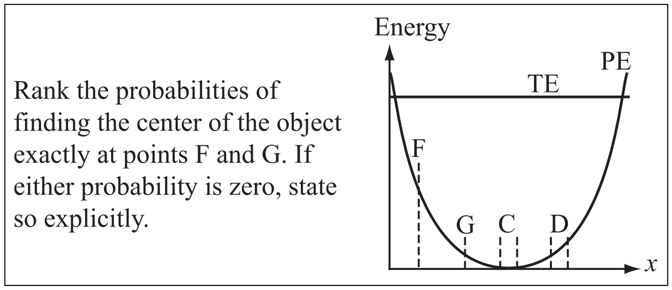


Figure 8.10: Abbreviated version of 'probability at a point' question. The correct answer is $P_F = P_G = 0$.

of the object exactly at points F and G, and to compare the values of each probability to zero. The correct answer is $P_F = P_G = 0$ since the widths of the regions of interest are zero. We note that the students had not had any instruction in quantum mechanics at the time this question was administered.

Only 10% of the students answered correctly (N=147). About 75% of the students answered either $P_F > P_G > 0$, $P_F = P_G > 0$, or $P_G > P_F > 0$, all of which reveal a belief that the probability at a point is non-zero.

Most of the explanations by these latter students focused only on the relative rankings of P_F and P_G. They did not tend to explain why they thought the probability at a point was non-zero. The following student quote is illustrative:

> *"[$P_G > P_F > 0$.] The object is traveling slower at G."*
> (Phys 123B, winter 2008)

Students who did explain why the probabilities are greater than zero tended to answer $P_F = P_G > 0$, and emphasized only that F and G are *points*. The following two responses are typical of this line of reasoning:

> *"[$P_F = P_G > 0$.] They are both points so there is equal likelihood for all points to contain the object."*
> (Phys 123B, winter 2008)

> *"[$P_F = P_G > 0$.] because the particle will be exactly at each point only once in a cycle so there would only be one point of time for each"*
> (Phys 123B, winter 2008)

The results from this question indicate that a large fraction of introductory students ascribe a non-zero probability to finding the particle at a point.

8.2.6-c Tendency to associate probability with the area under a potential energy curve

In §8.2.1-b we described a variation of the 'unequal-length track' question in which students are presented with a potential energy diagram and are asked to rank the probabilities of finding an object in two unequal-sized regions. On the diagram that we provided, the smaller region has a length of 2 units and the value of the potential energy is 2 units. The larger region has a length of 4 units and the value of the potential energy is 1 unit.

About 5% of the students provided answers and explanations consistent with using the area under the graph of $U(x)$ in the two regions to compare the probabilities. The following quote is illustrative:

> *"[The probabilities in the two regions are equal.] The area under the curve for each section is 4 [units] so they are the same"*
> (Phys 123A/C, spring 2015)

These students appeared to be overgeneralizing the procedure for calculating probability from a graph of probability density. This suggests that they did not have a robust understanding of the connection between probability, potential energy, and probability density.

8.2.7 Summary

We have found that a large fraction of students failed to incorporate correctly the effects of the speed of an object and the sizes of the regions in ranking probabilities. These students either incorporated only one of these quantities (i.e., either speed or size) or believed incorrectly that the two effects compensate (§8.2.1–§8.2.3). An important finding is that students were more likely to reason correctly if asked to consider the speed of the particle or if they were provided with information about the potential energy (§8.2.1-b). Of the other difficulties we have identified, the two most prominent ones are the failure to write an expression for probability (despite their ability to rank probabilities on the same question) and the incorrect belief that the probability at a point is non-zero (§8.2.5 and §8.2.6-b).

Our results also suggest that introductory students may not confuse the predictions of classical and quantum mechanics as readily as readily as junior-level students (§8.2.6-a). One possible reason is that introductory student spend much less time on quantum mechanics. They may not understand the predictions of quantum mechanics at a basic level, which could prevent them from confusing the two regimes.

8.3 Student difficulties with probability density

In this section we describe the difficulties that we have identified as a result of administering questions that probe the extent to which students can reason about spatial probability density. The questions were described in §8.1, though we briefly summarize them as needed.

The difficulties we have identified are varied in nature. We begin by describing the failure of students to distinguish between probability, probability density, and other closely related concepts (§8.3.1). Next, we describe the tendency to associate smaller regions with larger probability densities (§8.3.2) and the tendency to use compensation-based reasoning (§8.3.3). (These three difficulties can also be interpreted as a failure to reason using multi-variable equations.) Finally, we discuss two difficulties that we characterize as being more 'general' than the others. One of these is the observation that the phrase 'probability density' may itself have contributed to some of the difficulties that have been identified (§8.3.4). The other is the failure to generalize knowledge of more familiar forms of spatial density, such as mass density or charge density (§8.3.5). Prior research related to probability density was discussed in §3.2.

8.3.1 Failure to distinguish between probability density and closely related concepts

The results of many of the pretest questions we have administered suggest that students failed to distinguish between probability, probability density, and other closely related concepts. In some cases, the students answered questions about probability density as if they were describing probability. In other cases, they answered questions about probability as if they were describing probability density.

It is important to note, though, that this failure may have resulted from one of several different underlying difficulties. For example, some students may believe probability and probability density are two different names for the same quantity. Other students may have trouble with multivariable equations, focusing only on changes to the numerator of a ratio (e.g., $\rho = P/L$) even though the denominator changes as well.[11] In many cases, these two underlying difficulties would result in the same responses to the pretests we have administered. For this reason, we characterize these difficulties using the generic description 'a failure to distinguish between closely related concepts.' Below we provide results from several different types of questions that illustrate this failure.

8.3.1-a Examples from 'density graph' question

In §8.1.2-c on page 216 we described the 'density graph' question in which students are asked to use a graph of probability per unit length *vs.* position to rank probabilities in three regions. To answer correctly, students can compare the areas underneath the probability density curve in each region. Student performance is shown in Table 8.8. About 20% of the students ranked the probabilities solely according to the values of the probability densities (i.e., $B > C > A$). This was the most common incorrect answer. Most of these students emphasized in their explanations the height of the graphs. The following student quote is illustrative of this line of reasoning:

> *"[B > C > A.] We can tell this because the graph is the highest at B, then C, and then A."*
>
> (Phys 123A/C, spring 2008)

Some students, though, were not as explicit as the student above. For example:

> *"[B > C > A.] This is the distribution of probability starting with the greatest at B."*
>
> (Phys 123A/C, spring 2008)

We infer that this student and others with similar explanations are ranking according to the height of the graph from the fact that the ranking $B > C > A$ is most readily obtained by comparing heights. That these students are using the values of probability density to

[11] Student difficulties with multivariable equations has been identified by other researchers in different contexts, including thermodynamics and physical waves [72–74].

Table 8.8: Student performance on the 'density graph' question. The correct answer is highlighted.

Response	Consistent with	N=149[a]
$C > A = B$	Rank by area	60%
$B > C > A$	Rank by density	20%
$A > C > B$	Rank by length	10%
Other	Other	5%

[a] Sections 121A082 and 121C082, version U1B.

rank probabilities suggests that they did not distinguish between these two closely-related concepts.

8.3.1-b Examples from 'density from probability' question

In §8.1.2-a on page 214 we described the 'density from probability' question in which students are told that the probability of finding a ball in region A of a track is greater than that in region B. No illustration of the track is given, but students are told explicitly that the relative lengths of the two regions are unknown. Students are then asked to compare, if possible, the probability densities in the two regions. They are told that the density in each region is uniform. The correct answer is *not enough information.*

Student performance on this question is shown in Table 8.9. Only 30% of the students answered correctly. Most of these students provided explanations that we consider correct or partially correct (e.g., stating that the relative lengths are unknown). About half of students, though, answered incorrectly that the probability density in region A (which has the higher probability) is greater than that in region B. These students typically stated that since the probability is greater, the corresponding density is greater as well. The following student responses illustrate this tendency:

Table 8.9: Student performance on the 'density from probability' question. The correct answer is highlighted.

Response	N=413[a]
Not enough information	30%
$\rho_A > \rho_B$	50%
$\rho_A = \rho_B$	5%
$\rho_A < \rho_B$	10%

[a] Sections 123A083, 123A084, 123A091, 123A142, and 123C142.

> *"If it is in A more often then the probability density is higher."*
> (Phys 123A, autumn 2008)

> *"There seems to be a correlation here. A larger probability means a more dense area therefore the density should be larger"*
> (Phys 123A, autumn 2008)

These responses also suggest that some students did not distinguish between probability and probability density.

8.3.1-c Examples from the 'broken block' question

In §8.1.2-b on page 215 we described the 'broken block' question in which a marble is placed at a random location in a block of material. There are two tasks associated with this question, the 'expression task' and the 'ranking task.' The expression task asks students to write an expression for the probability density associated with finding a small marble in a large block of material at a random location. The correct answer to the expression task is $1/HWD$. Students are then told that the block of material is broken into two unequal-sized pieces. The ranking task asks students to rank the probability densities for all three pieces. The correct answer to the ranking task is $\rho_{\text{orig}} = \rho_{\text{small}} = \rho_{\text{large}}$.

Student performance on this question is shown in Table 8.10. Only 25% of the students correctly answered the expression task. The two most common incorrect answers were $\rho = HWD$ and $\rho = 1$,[12] which were provided by about 35% of the students. Explanations for these incorrect answers tended to emphasize the total probability or the total volume of the region in which the marble could be found. The following student quotes are illustrative:

> *"[The probability density is 1.] the marble is in the block, so there is a 100% chance of finding it in the box."*
> (PHYS 123A/C, SPRING 2009)

> *"[The probability density is HWD.] This gives the volume of the box, and the marble is somewhere in that volume."*
> (PHYS 123A/C SPRING 2009)

These responses are consistent with not distinguishing among probability, probability density, and volume.[13]

About 10% of the students attempted to incorporate some aspect of the *marble* into their expression for probability density, such as mass or volume, despite the fact that we provided no variables to represent either of these. Students who incorporated the volume of the marble typically wrote '$V_{\text{marble}}/V_{\text{block}}$' or a similar expression. This expression is equal to the *probability* of a randomly chosen point within the block also lying within the marble. Given that the expression represents a probability, and not a probability density, it can be interpreted as indicating a failure to distinguish between probability and probability density. Furthermore, some students attempted to incorporate the mass of the marble into their expression (e.g., using 'm_{marble}' in the numerator of their expression). These students appear to have failed to distinguish between different forms of spatial density.

[12] Some students gave an answer of '$HWD = 1$.' These were classified as '1.'

[13] Kanim [47] found similar confusion with volume in the context of charge density. See §8.3.5-a.

Table 8.10: Student performance on the 'broken block' question. The correct answers are highlighted.

Task	Answer	N=412[a]
Write expression	$\frac{1}{HWD}$	25%
	HWD	20%
	1	15%
	$\frac{\text{"marble"}}{HWD}$	10%
	Other	30%
Rank density	$\rho_{\text{orig}} = \rho_{\text{small}} = \rho_{\text{large}}$	10%
	$\rho_{\text{small}} > \rho_{\text{large}} > \rho_{\text{orig}}$ (by size)	25%
	$\rho_{\text{orig}} > \rho_{\text{large}} > \rho_{\text{small}}$ (by inverse size)	50%
	Other	15%
Combinations	$\frac{1}{HWD}$ & $\rho_{\text{orig}} = \rho_{\text{small}} = \rho_{\text{large}}$	≤5%
	$\frac{1}{HWD}$ & $\rho_{\text{small}} > \rho_{\text{large}} > \rho_{\text{orig}}$	10%
	$\frac{1}{HWD}$ & $\rho_{\text{orig}} > \rho_{\text{large}} > \rho_{\text{small}}$	10%
	HWD & $\rho_{\text{orig}} > \rho_{\text{large}} > \rho_{\text{small}}$	10%
	1 & $\rho_{\text{orig}} > \rho_{\text{large}} > \rho_{\text{small}}$	10%

[a] Sections 123A083, 123B091, 123A092, and 123C092.

On the ranking task, only 10% of the students answered correctly. Half of the students incorrectly ranked the densities according to volume ($\rho_{\text{orig}} > \rho_{\text{large}} > \rho_{\text{small}}$) and tended to support their answer by describing the relative probabilities. The following student quote illustrates this line of reasoning:

> *"[$\rho_O > \rho_B > \rho_A$.] The original block is bigger followed by B then A as the smallest. The marble would have a greater chance in appearing in the block that is the biggest."*
>
> (Phys 123A/C, spring 2009)

This explanation and others like it are consistent with the failure to distinguish between probability and probability density.

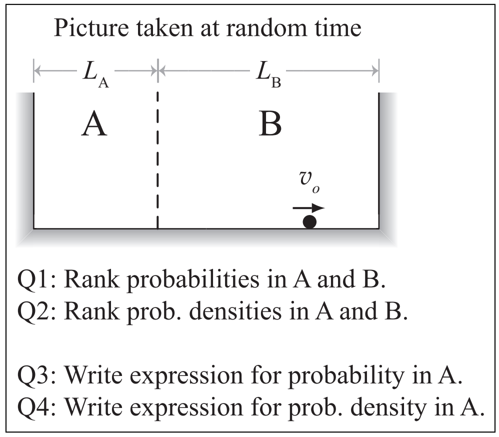


Figure 8.11: Abbreviated version of the 'infinite square well' question.

8.3.1-d Examples from the 'infinite square well' question

On the 'infinite square well' question, which is repeated in Figure 8.11, students are told that a ball moves back and forth on a single-level track. An imaginary line divides the track into two unequal-sized regions, labeled A and B, with $L_A < L_B$. There are four sub-questions. Questions 1 and 2 (see the figure) ask students to rank the probabilities and probability densities in the two unequal-sized regions. The correct answers are $P_A < P_B$ and $\rho_A = \rho_B$, respectively. Questions 3 and 4 ask students to write expressions for the probability and probability density in region A. The correct answers are $P_A = \frac{L_A}{L_A+L_B}$ and $\rho_A = \frac{P_A}{L_A} = \frac{1}{L_A+L_B}$, respectively.

Student performance on the 'infinite square well' question is shown in Table 8.11. Only 15% of the students answered all four questions correctly. Students performed better on the probability questions (Questions 1 and 3) than they did on the corresponding density questions (Questions 2 and 4).

On the probability density ranking task (Question 2), 40% of the students stated incorrectly that the probability density in region A is less than that in region B. (Nearly all of these students answered correctly that the probability in region A was less than that in region B.) In their explanations, students typically discussed either the relative probabilities or relative sizes. The following two student quotes are illustrative:

> *"[$\rho_A < \rho_B$.] Since A has a smaller area, its probability density is smaller than B."*
>
> (PHYS 123A, SPRING 2010)

> *"[$\rho_A < \rho_B$.] since it is less likely to be in A than B, its probability will be less."*
>
> (PHYS 123A, SPRING 2010)

We note that this latter student, like many others, uses the term 'probability' when 'probability density' would be more appropriate.

The responses of these students and others with similar explanations are consistent the failure to distinguish between probability and probability density.

Only 15% of the students correctly answered the density expression task (Question 4), despite that 60% of the students answered the probability expression task (Question 3) correctly. Most of the incorrect answers are quite varied and difficult to categorize. However, the most common incorrect expression, provided by 10–15% of the students, was $\rho = \frac{L_A}{L_A+L_B}$. Nearly all of these students provided the same expression on the probability task, which in that case was correct. That these students provided the same expression for probability and probability density strongly suggests that they do not distinguish between these two concepts.

Table 8.11: Student performance on the 'infinite square well' question. The correct answers are highlighted.

Question	Response	N=401[a]
Q1: Rank prob.	$P_A < P_B$	90%
	$P_A > P_B$	5%
	$P_A = P_B$	5%
Q2: Rank dens.	$\rho_A = \rho_B$	45%
	$\rho_A < \rho_B$	40%
	$\rho_A > \rho_B$	10%
Q3: Prob. in A	$\frac{L_A}{L_A+L_B}$	60%
	Not enough info	10%
Q4: Density in A	$\frac{1}{L_A+L_B}$	15%
	Consistent*	<5%
	$\frac{L_A}{L_A+L_B}$	10%
	Not enough info	20%

[a] Sections 123A102, 123C102, 123A152, and 123C152.

* Incorrect but consistent: Students divide their incorrect expression for probability by length.

8.3.1-e Additional evidence for the failure to distinguish between probability and probability density

In §8.1.1-b on page 213 we described the 'unequal-length track' question in which a ball moves back and forth on a track composed of two, unequal-length levels that lie at different heights. In some administrations of the question, students were asked to rank both the probabilities and the probability densities for the two levels. About 40% of the students (N=253) answered that both the probability and the probability density were smaller in the upper, smaller region. (We discussed our interpretation of ranking probabilities according to size in §8.2.1.) Explanations for the density ranking were similar to those on the infinite square well question in §8.3.1-d above.

8.3.1-f Summary

The evidence presented in this section suggests that some students failed to distinguish between probability density and closely related concepts. Some seemed to equate probability density and probability, or probability density and volume. Our results suggest that learning to distinguish between these related concepts could significantly improve overall performance on some questions related to probability density.

It is interesting to note that the percentage of students who exhibited this difficulty varied widely from question to question: from about 10% of the students on the 'infinite square well' expression task to about 50% on the 'density from probability' question. More research is needed to understand the subtleties involved in eliciting this tendency.

8.3.2 Tendency to associate smaller regions with larger densities

The responses of some students on the tutorial pretest questions suggest that they had the tendency to associate smaller regions with larger probability densities (and vice versa). Below we provide responses from several of the pretest questions we have administered that illustrate this tendency.

8.3.2-a Example from 'infinite square well' question

One of the questions in which this tendency was elicited is the 'infinite square well' question, which we described above in §8.3.1-d. In this question, students consider a ball moving back and forth in a single-level track. An imaginary line divides the track into two unequal-sized regions, labeled A and B, with region A smaller than region B. As part of this question, students are asked to rank both the probability and probability densities in the two regions.

Student performance is shown in Table 8.11 on page 249. About 10% of the students stated *correctly* that the *probability* in region A is less than that in region B ($P_A < P_B$) and stated *incorrectly* that the probability density in region A is greater than that in region B ($\rho_A > \rho_B$). In explaining their ranking for probability density, many students stated only that smaller areas have higher probability densities. The following student quotes are illustrative of this reasoning:

> *"[$P_A < P_B$ because length] B is greater than Length A so probability is higher in B. [...] [$\rho_A > \rho_B$] since the volume in section A is smaller."*
> (Phys 123A, spring 2010)

> *"[$P_A < P_B$.] [...] [$\rho_A > \rho_B$] because section A is smaller in length than section B is, indicating that it's a more 'dense' area."*
> (Phys 123A, spring 2010)

These students seem to have focused only on the denominator of the relation $\rho = P/L$ to argue that a smaller region implies a larger density.

8.3.2-b Example from the 'unequal-length track' question

We have also observed a tendency to rank probabilities by the inverse sizes of the regions on the 'unequal-length track' question. In this question, students consider a ball that moves back and forth on a track composed of two, unequal-length levels. The length of the upper level is smaller than that of the lower level ($L_{\text{upper}} < L_{\text{lower}}$). On the administration we describe below, students are asked to rank both the probabilities and the probability densities on each level. The correct answer for the probability question is that there is *not enough information* to answer because the ball moves more slowly in the upper region but the upper region is smaller. The correct answer for the probability density question is $\rho_{\text{upper}} > \rho_{\text{lower}}$ because the ball moves more slowly in the upper region, so the ball spends more time per unit length there.

Student performance is shown in Table 8.12. About 15% of the students answered incorrectly that the probability in the upper, smaller region was less than that in the lower, larger region ($P_{\text{upper}} < P_{\text{lower}}$) but answered correctly that the density in the upper, smaller region was greater than that in the lower, larger region ($\rho_{\text{upper}} > \rho_{\text{lower}}$).[14] In explaining their ranking for probability density, many students cited only that the upper region is smaller than the lower region. However, we have seen additional patterns of reasoning that help elucidate this particular tendency. For example, one student provided the following answer and explanation for the probability and probability density questions, respectively:

> *"[$P_{\text{upper}} < P_{\text{lower}}$ because the] lower step has a greater area so there's a greater possibility of find it on the lower step. [...] [$\rho_{\text{upper}} > \rho_{\text{lower}}$ because the] upper step has a smaller area so if the ball is on the upper step, there's a smaller area [for it to] go through."*
>
> (Phys 123A/C, spring 2009)

This student seemed to imagine that the ball was on the upper level when arguing about the probability density in the upper level, and then separately imagined that the ball was on the lower level when arguing about that in the lower level. Essentially, this student treated the two levels independently. Mathematically, this is analogous to writing '100%/L' for the probability density in each level separately.

We note that this student and others with responses appeared to disregard their ranking for the probability ($P_{\text{upper}} < P_{\text{lower}}$). It is unclear whether students who did not explicitly use this reasoning did so implicitly to argue $P_{\text{upper}} < P_{\text{lower}}$ and $\rho_{\text{upper}} > \rho_{\text{lower}}$.

[14] Although this ranking for probability density is correct, the answer *and* reasoning that these students provide is inconsistent with their previous answer of $P_{\text{upper}} < P_{\text{lower}}$.

Table 8.12: Student performance on the 'unequal-length track' and 'unequal-length drop' questions. The correct answers are highlighted.

Question	Response	Unequal-length track N=253[b]	Unequal-length drop N=404[a]
Rank probabilities	Not enough info	10%	40%
	$P_{\text{upper}} < P_{\text{lower}}$ (by size)	75%	40%
	$P_{\text{upper}} = P_{\text{lower}}$	10%	15%
	$P_{\text{upper}} > P_{\text{lower}}$	10%	5%
Rank densities	$\rho_{\text{upper}} > \rho_{\text{lower}}$	25%	25%
	$\rho_{\text{upper}} = \rho_{\text{lower}}$	20%	30%
	$\rho_{\text{upper}} < \rho_{\text{lower}}$ (by size)	45%	20%
	Not enough info	10%	25%
Common combos.	Both correct	<5%	15%
	P by size, ρ equal	15%	20%
	P by size, ρ by size	40%	15%
	P by size, ρ correct	15%	5%
	Both not enough info	5%	25%

[a] Sections 123A102, 123C102, and 123A114. [b] Sections 123A092 and 123C092.

8.3.2-c Example from the 'broken block' question

The tendency to associate smaller regions with larger densities was also elicited on the 'broken block' question, shown in Figure 8.4 on page 216. In this question, students are asked to write an expression for the probability density associated with finding a marble in a block of material. Next, the block is broken into two unequal-sized pieces and students are asked to rank the probability densities in all three pieces.

About 25% of the students answered incorrectly that the probability density in the smaller piece is greater than that in the larger pieces. (See Table 8.10 on page 246.) These students tended to state only that the volume of the smaller block is smaller. (We briefly note that about 10% of all students answered correctly that the probability density is $1/HWD$, but answered the ranking task incorrectly as indicated above.)

8.3.2-d Summary

Up to 25% of students appeared to rank probability densities by inverse size. These students failed to account for how the *probability* changes. One explanation is that these students are relying on a memorized rule that for a *given* (fixed) amount of a 'numerator,' the ratio is inversely proportional to the 'denominator.'

8.3.3 Tendency to use compensation-based reasoning

Responses by a small fraction of students indicate that they use 'compensation-based reasoning' when ranking probability densities. That is, some students use their ranking of probability in two regions and the relative lengths of the regions to argue that the probability density in the two regions is the same.

This tendency was elicited on the 'unequal-length drop' question (see Figure 8.1(d) on page 212). In this question, students consider a ball bouncing under the influence of gravity. They are asked to rank the probabilities in two unequal-sized regions and to rank the probability densities at the centers of those regions. About 5% of the students provided answers and explanations consistent with compensation-based reasoning. They stated that the probability in the upper, smaller region was less than that in the lower, larger region ($P_{\text{upper}} < P_{\text{lower}}$) but that the probability densities associated with the regions were the same ($\rho_{\text{upper}} = \rho_{\text{lower}}$).[15] These students typically cited in their explanation the competing effects of probability and length. The following student quote is illustrative:

> *"[The probability densities are the same.] There is a smaller probability in [the upper region], but also a smaller area, so the probability density is the same in both regions."*
>
> (Phys 123A, spring 2010)

About the same percentage of students used compensation-based reasoning on the 'unequal-length track' question.

[15] About 15% of the students who answered $P_{\text{upper}} < P_{\text{lower}}$ and $\rho_{\text{upper}} = \rho_{\text{lower}}$ argued that the densities were the same because any point or region is equally probable as any other. See Table 8.12.

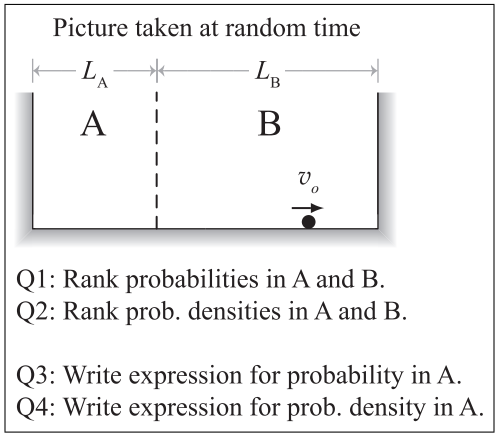


Figure 8.12: Abbreviated 'original' version of the 'infinite square well' question.

Although compensation-based reasoning was somewhat rare in the context of probability density, we recognize that the failure of students to distinguish between probability and probability density (§8.3.1) may play a role in masking this difficulty.

8.3.4 Difficulty in interpreting the phrase 'probability density'

As described below, we have gathered evidence that suggests that some of the difficulties students encountered with probability density is due to the phrase 'probability density' itself.

In the 'infinite square well' question, students consider a ball moving back-and-forth in a single-level track. (The question is reproduced in Figure 8.12.) An imaginary line divides the track into two unequal-sized regions, labeled A and B, with region A smaller than region B (see the figure). Students are asked first to rank both the probability and probability densities in the two regions (Questions 1 and 2). Students are then asked to write expressions for both the probability and probability density in region A (Questions 3 and 4). We call this version of the infinite square well question the 'original' version.

We also administered two additional versions of this question. One version uses the phrase *probability per unit length* instead of *probability density.* We call this version the 'per unit length' version. The other version uses both of the phrases *probability density* and *probability per unit length* as well as provides students with an operational definition for probability density (divide probability by length). We call this version the 'definition' version. The four questions themselves were otherwise the same across all of the three versions.

Table 8.13 shows student performance on the modified versions as well as the original version. Performance on the two probability questions (Questions 1 and 3) was essentially the same. This is not surprising given that we did not modify these questions.

Performance on the two probability density questions (Questions 2 and 4), however, was greater for both the 'per unit length' and the 'definition' versions. Between 75% and 80% of the students on the modified versions correctly ranked the densities (Question 2) compared to only 45% on the original version. Likewise, between 35% and 40% of the students correctly wrote an expression for probability density (Question 4) compared to only 15% on the original version. Students performed equally well on the per unit length and definition versions.

The greater student performance on the 'per unit length' version compared to the original version suggests to us that the phrase *probability density* may contribute to some of the student difficulties that we and other researchers have identified. We discuss curricular implications of this result at the end of this section (§8.3.6).

About 10% of the students on the 'definition' version wrote an incorrect but consistent expression for probability density (Q4) compared to essentially no students on either of the other two versions. However, a difference of 10% percentage points may not be instructionally significant. This result suggests that simply telling students how to calculate probability density effects very little change, if any, in student ability to use their previous expression for probability to arrive at a correct expression for probability density.

It is important to note that even on the modified versions on which students performed better, more than half of the students failed to answer all four questions correctly. Furthermore, that only up to 65% of the students wrote a correct expression for *probability* suggests that a significant hurdle in improving student understanding of probability density is improving their understanding of probability.

Table 8.13: Student performance on multiple versions of the 'infinite square well' question. The correct answers are highlighted.

		Original[a]	Modified	
Question	Response	N=401[d]	Per-unit-length[b] N=123[e]	Definition[c] N=151[f]
Q1: Rank prob.	$P_A < P_B$	90%	90%	85%
	$P_A > P_B$	5%	5%	10%
	$P_A = P_B$	5%	5%	<5%
Q2: Rank dens.	$\rho_A = \rho_B$	45%	75%	80%
	$\rho_A < \rho_B$	40%	20%	10%
	$\rho_A > \rho_B$	10%	5%	10%
Q3: Prob. in A	$\frac{L_A}{L_A+L_B}$	60%	65%	60%
	Not enough info	10%	5%	10%
Q4: Density in A	$\frac{1}{L_A+L_B}$	15%	35%	40%
	Consistent*	<5%	<5%	10%
	$\frac{L_A}{L_A+L_B}$	10%	10%	<5%
	Not enough info	20%	10%	10%

[a] Question uses "probability density."
[b] Question uses "probability per unit length."
[c] Question uses both "probability density" and "probability per unit length" and provides an operational definition (divide probability by length).
[d] Sections 123A102, 123C102, 123A152, and 123C152. [e] Sections 123A152 and 123C152. [f] Section 123A114. * Students are incorrect but consistent: They divide their incorrect expression for probability by length.

8.3.5 Failure to generalize knowledge of familiar forms of density

The results from some of the pretest questions we have administered suggest that many students who can reason about familiar forms of density (e.g., mass) failed to use the same reasoning on analogous questions in the context of probability. Below we present some evidence from two different sets of questions that were designed to probe student thinking about densities in different contexts. The specific difficulties that arose from questions about probability density were discussed above.

8.3.5-a Example from 'broken block' question

In §8.1.2-b on page 215 we described the 'broken block' question in which a small marble is placed at a random location in a block of material. Students are asked first to write an expression for the probability density associated with finding the marble in the block. The block is then broken into two pieces, and students are asked to rank the probability densities of the original block and the two broken pieces.

This question was based on analogous questions developed by Kanim [47] for students enrolled in Phys 122 (the second-quarter introductory course that covers electricity and magnetism). In his question, a block with charge uniformly distributed throughout its volume is broken into two smaller pieces. Students were first asked to write an expression for the charge density of the original piece, and were then asked to rank the charge densities of all three pieces. Kanim also asked an analogous question in the context mass with the intent to compare student performance in the two contexts.

Kanim's results are reproduced in Table 8.14. He found that the vast majority of the students (about 85%) were able to rank the mass densities correctly. However, only 55% of the students were able to rank the charge densities, even after all lecture and tutorial instruction. (We note briefly that we have administered these same charge and mass questions to a larger population of Phys 123 students and found similar results.)

Kanim's results can be interpreted as indicating that not all students are able to generalize

Table 8.14: Student performance on mass and charge density question administered by Kanim [47] to Phys 122 students.

Task	Response[a]	Context: Mass Winter 1997 N=108	Context: Charge Summer 1996 N=63
Write expression	$\rho = Q/hwt$	85%	80%
	$\rho = hwt/Q$	5%	0%
	$\rho = hwtQ$	<5%	5%
Rank densities	$\rho_o = \rho_A = \rho_B$	85%	55%
	Rank by size[b]	10%	15%
	Rank by inverse size[c]	5%	25%

[a] Responses use the symbol 'Q' to represent either charge or mass.
[b] Students ranked the largest piece (the original block) as having the largest density, and the smallest piece as having the smallest density.
[c] Students rank the smallest piece as having the largest density, and the largest piece as having the smallest density.

their prior knowledge of mass density in answering questions about charge density. (He also found that the disparity in performance on the charge and mass questions was greater prior to lecture instruction on Gauss' law. The data that we reproduced represents the greatest *parity* in performance between the two questions he found without tutorial instruction.)

We were motivated by Kanim's results to probe the parity of student performance on the 'broken block' question in the context of probability density to that in the context of mass density and charge density. Our results from the probability version are shown in Table 8.15.[16] Only 25% of the students correctly wrote an expression for probability density, compared to 85% and 80% of the students on the mass and charge contexts, respectively. Likewise, on the ranking task, only 10% of the students gave a correct ranking, compared to 85% and 55% of the students on the mass and charge contexts, respectively.

These results suggest that students who may have otherwise answered the 'broken block'

[16]We have also administered a similar question in which a specific point (x_o, y_o, z_o) is chosen at random in the block rather placing a small marble at a random location. We found largely similar results on both questions, but we only report results from the marble version here.

Table 8.15: Student performance on 'broken block' question for probability density. The correct answers are highlighted.

Task	Answer	N=412[a]
Write expression	$\frac{1}{HWD}$	25%
	HWD	20%
	1	15%
	$\frac{\text{"marble"}}{HWD}$	10%
	Other	30%
Rank density	$\rho_{\text{orig}} = \rho_{\text{small}} = \rho_{\text{large}}$	10%
	$\rho_{\text{small}} > \rho_{\text{large}} > \rho_{\text{orig}}$ (by inverse size)	25%
	$\rho_{\text{orig}} > \rho_{\text{large}} > \rho_{\text{small}}$ (by size)	50%
	Other	15%
Combinations	$\frac{1}{HWD}$ & $\rho_{\text{orig}} = \rho_{\text{small}} = \rho_{\text{large}}$	≤5%
	$\frac{1}{HWD}$ & $\rho_{\text{small}} > \rho_{\text{large}} > \rho_{\text{orig}}$	10%
	$\frac{1}{HWD}$ & $\rho_{\text{orig}} > \rho_{\text{large}} > \rho_{\text{small}}$	10%
	HWD & $\rho_{\text{orig}} > \rho_{\text{large}} > \rho_{\text{small}}$	10%
	1 & $\rho_{\text{orig}} > \rho_{\text{large}} > \rho_{\text{small}}$	10%

[a] Sections 123A083, 123B091, 123A092, and 123C092.

Table 8.16: Comparison of student performance on the 'density graph' question in the context of charge and probability. The correct answer is highlighted.

		Context	
Response	Consistent with	Charge[a] N=115	Probability[b] N=149
$C > A = B$	Rank by area	75%	60%
$B > C > A$	Rank by density	15%	20%
$A > C > B$	Rank by length	$< 5\%$	10%
Other	Other	5%	5%

[a] Sections 123A082 and 123C082, version U1A.
[b] Sections 123A082 and 123C082, version U1B.

question correctly in the context of mass or charge may not have done so in the context of probability. That is, students may not be able to generalize their knowledge of density in one context to another.

8.3.5-b Example from 'density graph' questions

In §8.1.2-c on page 216 we described the 'density graph' question in which students are asked to rank either the probabilities or the charges in three different regions based on a graph of probability per unit length *vs.* position or a graph of charge per unit length *vs.* position. The question was administered in Phys 123 prior to any lecture instruction on probability but after all instruction on charge. As shown in Table 8.16, students performed somewhat better in the context of charge than in the context of probability (75% compared to 60%)[17] despite the similarity in wording on the two versions.

This result is consistent with our result from the broken block question described above: Student may not be able to generalize their knowledge of density in one context to another context.

[17]The version of the pretest that students took was randomized. A χ^2 test comparing the number of students answering correctly yields $\chi^2 = 5.23, p < 0.023$.

8.3.5-c Summary

In both the broken block and the density graph questions, students performed more poorly in the context of probability than they did in the context of charge (and, for the broken block question, in the context of mass). Student ability to argue about density appears to be context-dependent. We interpret these results as indicating that students do not necessarily generalize their knowledge of more familiar forms of density when reasoning about probability density.

8.3.6 Summary of difficulties associated with probability density

Many of the difficulties students encountered appear to result from an underlying difficulty with multivariable equations or, relatedly, a failure to distinguish between closely related concepts (§8.3.1–§8.3.3). For example, some students may only consider changes to the numerator in the relation $\rho = P/L$. We have also shown that the phrase "probability density" itself may have contributed to the prevalence of many of these difficulties because students performed better when this phrase was replaced by "probability per unit length" (§8.3.4). This result suggests that students may benefit from a curriculum that guides students toward making connections between the concept of density and the idea of "so much of the numerator per unit length." We also demonstrated that many students may not generalize their knowledge of familiar forms of density (e.g., mass density or charge density) when arguing about probability density (§8.3.5).

This concludes our discussion of student difficulties with probability and probability density. In the next chapter, we discuss the development of curricula that is designed to target many of these difficulties.

Chapter 9

RESEARCH-INFORMED DEVELOPMENT OF A NEW TUTORIAL ON SPATIAL PROBABILITY DENSITY

In this chapter we give a broad overview of the curricular material we developed to improve student understanding of spatial probability density. The development of this material was directly informed by the research discussed in Chapter 8.

We first describe the initial development of the in-class worksheet for *Probability in classical and quantum mechanics* and the accompanying homework (§9.1). The discussion illustrates how the exercises within the curriculum relate to the findings described in Chapter 8. We then briefly describe how the tutorial has changed since its initial development based on ongoing research (§9.2).

(The impact of the curriculum on student understanding is discussed in Chapter 10. The development and assessment of an alternative curricular form of *Probability in classical and quantum mechanics*, called an *interactive tutorial-lecture*, is discussed in §10.4.)

9.1 Initial development of curricular materials for Probability in classical and quantum mechanics

The development of the in-class worksheet and the written homework are discussed separately below.

9.1.1 Initial development of the in-class worksheet

The first iteration of the in-class worksheet for *Probability in classical and quantum mechanics* is shown in the appendix (see Figure B.19 on page 608). It is composed of four sections. Each is described below.

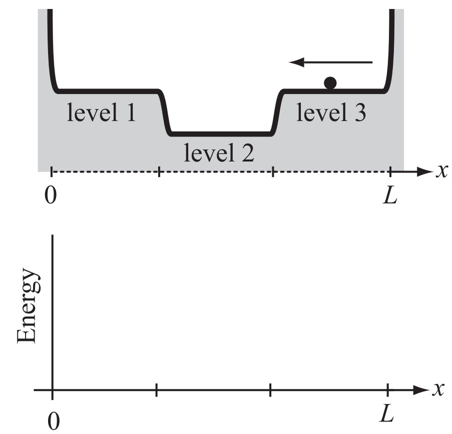
Section I of the in-class worksheet provides a review of linear density in the context of charge. Students consider a one-dimensional rod with total charge $+Q$. Several of the questions in this section are designed to guide students toward a basic conceptual understanding of density, including how charge density is different than, but related to, charge. For example, one of the exercises asks students to use the given charge and length of the rod to draw and label a graph of linear charge density *vs.* position. Students are then asked to describe what geometric feature of the graph represents the amount of charge in a particular region (i.e., the area under the curve).

We were motivated to include this section in part by our research and the research by others that showed student understanding of density is context dependent (see §8.3.5 and Ref. [47]). Our intention was to provide students a review of relevant concepts with a familiar form of density. Later sections that focus on *probability* density then build upon these ideas. Students are guided to realize that many of the same ideas from charge density can apply to other contexts.

Section II guides students toward an understanding of how the speed of an object in a region influences the probability of measuring the object to be in that region. Students do this by considering a ball that moves back and forth on a three-level track (see Figure 9.1). To help students think about speed, they are asked to draw a potential energy diagram for

II. Qualitative arguments of classical probability

A ball moves back and forth forever on a frictionless track with very steep sides. The three levels of the track are of equal length and are joined by steep ramps. Levels 1 and 3 are of equal height. Assume that the time spent on the steep portions is negligible and that the ball never leaves the surface of the track.



In the following questions, consider the system consisting of the ball, track, and Earth.

A. On the axes at right, sketch a qualitatively correct graph of potential energy versus position of the ball.

On the same axes, draw a line that could represent the *total* energy of the system. How is the fact that the total energy of the system is conserved reflected in your graph?

Figure 9.1: Excerpt from section II of *Probability in classical and quantum mechanics.*

the ball-Earth system. They also label a line representing total energy.[1]

We included this section in part because many students on pretests did not consider the speed of the ball when ranking probabilities in different regions. Instead, they considered only the relative lengths of the regions (see §8.2.1).

Section III guides students toward an understanding of probability density. We designed the tutorial to be an appropriate *introduction* to probability density (i.e., it assumes no prior knowledge on the topic). Many of the tutorials within *Tutorials in Introductory Physics* do not introduce topics; they assume some prior knowledge from lecture. We deemed this an inappropriate choice for probability density due in part to the relatively little amount of time in Phys 123 that is devoted to probability density, and that very little of that, if any, focuses on probability density applied to classical physics contexts. This section begins by using the same track as in section II. Students are provided with the probability of finding the ball within a 2 cm region on one of the levels. They are then asked to use this information

[1]We note that our observations of students struggling to draw potential energy diagrams were part of our motivation to begin studying student understanding of potential energy diagrams. See page 38 for more information on our observations.

to calculate the probability of finding the ball within a 1 cm region. Later, they use that number to calculate the probability of finding the ball on the entire level.

After calculating these probabilities, students are asked to consider how the probability they calculated for the 1 cm region can be considered a type of 'density.' They are then guided to consider their work on charge density in section I. Finally, students also draw a graph of probability density *vs.* position and are guided to consider normalization (i.e., $\int \rho(x)\, dx = 1$).

One of the goals of this section is to help students distinguish between probability and probability density. Our research and the research of others have shown that many students fail to distinguish between these and other closely related concepts (see §8.3.1 and Refs [27, 36]). This section attempts to address this difficulty by having students go back and forth between probability and probability density multiple times.

Finally, section IV of the in-class worksheet contrasts the predictions of classical and quantum mechanics. Students first predict the shape of a graph of probability density *vs.* position for (classical) ball confined to an infinite square well; they arrive at a horizontal graph with value $\rho = 1/L$. They are then told than an *electron* is confined to a similar potential and are provided a handout of a graph of $\rho(x)$ corresponding to one of the energy eigenstates. In this way, students discover that the ideas of classical mechanics fail to predict all phenomena related to probability. However, they also discover that the relation between probability and probability density is applicable to quantum mechanical systems.

This section is included in the in-class tutorial based on our research and the research of others that suggest some students have difficulty in distinguishing between the predictions of classical and quantum mechanics (see page 235 and Refs. [27, 35, 36]).

9.1.2 Initial development of the homework

The first version of the homework for *Probability in classical and quantum mechanics* is shown in the appendix (see Figure B.29 on page 656). It is composed of four problems. Each is described below.

Problem 1 revisits the infinite square well scenario from section IV of the in-class worksheet. In our initial development, we were unsure whether the length of the in-class worksheet was appropriate. Revisiting the situation from the last section of the tutorial provides those students who do not finish the worksheet with an opportunity to contrast classical and quantum mechanical predictions.

The other problems in the homework are within the context of classical mechanics. Problem 2 asks students to construct a mathematical relation between the probability in a region and (1) the speed of an object, (2) the length of a region, (3) and the total time spent in a track. Students later construct a mathematical relation for probability density using the same variables. This problem gives students additional practice in connecting time and probability as well as distinguishing between probability and probability density. Both of these topics were difficult for students on the pretests we administered (see §8.2.1 and §8.3.1).

Problems 3 and 4 provide students with a graph of probability density *vs.* position and use it as a basis to ask students questions about probability and the corresponding physical system. Up until this point, both the in-class worksheet and homework have asked students to use the opposite reasoning (i.e., they have asked students to go from a physical system or probabilistic information to probability density). These two questions provided students an opportunity to go from probability density to more familiar quantities.

9.2 Examples of modifications made to the tutorial since its initial development

During the course of our research, we made several significant changes to the in-class worksheet for *Probability in classical and quantum mechanics*. Below we briefly discuss two important changes: (1) the removal of the first section that focused on charge density and (2) the addition of a more conceptual guide to probability density.[2] These changes were motivated by examining student responses to posttests, which we discuss later in §10.3.

A recent version of the in-class worksheet is shown in the appendix in Figure B.26 on page 642.

9.2.1 The removal of the first section on linear charge density

The most significant change to the tutorial was the removal of section I of the in-class worksheet. This section provided students with a review of linear density using the context of a charged rod. Our original intention with this section was to provide students a basis for thinking about linear density in a familiar context. Students could then connect these concepts to the more unfamiliar context of probability density.

As described in §10.3.1, we found no measurable effect of the first section. Student performed equally well on specifically-chosen posttest questions with and without it. The current version of the in-class worksheet now begins with what was formerly labeled section II in which students consider how speed affects probability.

9.2.2 The addition of a more conceptual guide to probability density

In early versions of the in-class worksheet, students were introduced to probability density by calculating and using a value for the probability of finding a ball in a 1 cm region. They later interpreted this quantity as a probability density. We realized, though, that by only having students calculate numerical values for probability density for specific situations, they were

[2] Although we have modified the accompanying homework as well, we do not discuss these changes.

not asked to think qualitatively about the role that speed plays in influencing probability density. Subsequent posttests that require students to consider the role of speed in ranking probability densities revealed that relatively few students were acquiring this skill.

After several different modifications to the tutorial to address the continuing difficulty (which we describe along with the accompanying research in §10.3.2), we arrived at the current sequence of exercises regarding probability density. Students now consider probability density in a qualitative manner for most of the relevant section. For example, they are guided to think about how the speed of the ball in small, equal-sized regions can be used to rank probability densities. Only after working through these qualitative problems do students encounter a numerical aspects of probability density.

Chapter 10

RESEARCH-BASED ASSESSMENT OF NEW TUTORIAL ON SPATIAL PROBABILITY DENSITY

In this chapter we discuss student performance on questions related to probability and probability density after tutorial instruction with *Probability in classical and quantum mechanics*.

Many of the questions we have administered as posttests (i.e., post-tutorial assessments) are similar to the pretest questions that we discussed in §8.1. A discussion of our method of comparing pretest and posttest performances is discussed in §2.4.2.

This chapter begins by examining student responses on questions about probability (§10.1). Next, student understanding of probability density is discussed (§10.2). Finally, we provide a discussion of other issues related to probability and probability density, including the research that guided us in our modification of the in-class worksheet (§10.3)[1] and the development and assessment of an *interactive tutorial-lecture,* which is an alternative instructional strategy to the tutorial *Probability in classical and quantum mechanics* (§10.4).

We remind the reader of the abbreviated scheme used to designate all course sections. See page 23 for a description.

[1]We discussed these modifications in §9.2.

10.1 Effect of tutorial instruction on student understanding of probability

In this section we describe the effect of the tutorial *Probability in classical and quantum mechanics* on student ability to rank the probabilities of finding an object in different regions. An overview of student performance after tutorial instruction is provided in §10.1.1. We then briefly discuss in §10.1.2–§10.1.3 the impact of the tutorial on student understanding through the lens of some of the difficulties we identified in §8.2.[2]

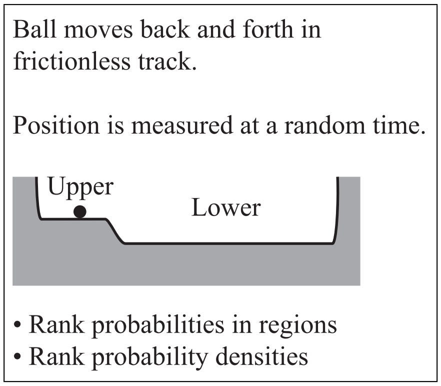
10.1.1 Overview of student performance on probability questions after tutorial instruction

In an effort to probe the effect of tutorial instruction on student understanding of probability, we administered the 'unequal-length track' question (see Figure 10.1(a)). In this question, students are told that a ball moves back and forth on a track composed of two levels that are of unequal size. The upper level is smaller than the lower level. Students are asked to rank, if possible, the probabilities of finding the ball in the two regions. To answer correctly, students can note that the ball moves more slowly in the smaller level. Thus, the correct answer is that there is not enough information to rank the probabilities.

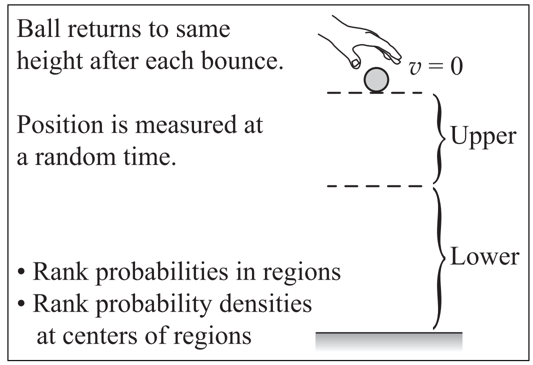
Student performance on the 'unequal-length track' question before and after tutorial instruction is shown in Table 10.1(a). On pretests (prior to tutorial instruction), only 10% of the students answered correctly. On posttests (after tutorial instruction), 80% of the students answered correctly. The vast majority of these students provided correct reasoning. This increase in performance correlates with fewer students on posttests answering that the probability in the smaller, upper level is less than that in the lower, larger level (75% on pretests compared to 10% on posttests). The percentage of students who selected the other two answers remains somewhat unchanged after tutorial instruction (20% on pretests compared to 10% on posttests).

It is worth noting that on the in-class worksheet, students explicitly consider the unequal-

[2] We do not analyze posttests for the difficulties we identified in §8.2.5 and §8.2.6. Further research is needed to probe the effectiveness of the tutorial on these difficulties.



(a) The 'unequal-length track' question.



(b) The 'unequal-length drop' question.

Figure 10.1: Abbreviated versions of two 'track' and two 'drop' questions.

Table 10.1: Pretest to posttest comparison of student performance the 'unequal-length track' and 'unequal-length drop' probability questions. The correct answers are highlighted.

(a) The 'unequal-length track' question.

Response	Pretest N=253[a]	Posttest N=350[b]
Not enough info.	10%	80%
$P_{\text{upper}} < P_{\text{lower}}$	75%	10%
$P_{\text{upper}} = P_{\text{lower}}$	10%	5%
$P_{\text{upper}} > P_{\text{lower}}$	10%	5%

[a] See Table 8.1(b) on page 220.
[b] Sections 123A082 and 123B091.

(b) The 'unequal-length drop' question.

Response	Pretest N=404[a]	Posttest N=367[b]
Not enough info.	40%	75%
$P_{\text{upper}} < P_{\text{lower}}$	40%	10%
$P_{\text{upper}} = P_{\text{lower}}$	15%	10%
$P_{\text{upper}} > P_{\text{lower}}$	5%	5%

[a] See Table 8.3(b) on page 224.
[b] Sections 123C102 and 123A152.

length track situation. In an attempt to address the possible concern that students are simply recalling the arguments used in tutorial without any robust understanding, we also administered the 'unequal-length drop' question to students as a posttest. Students did not see this situation in the in-class worksheet or the accompanying homework.

The 'unequal-length drop' question is shown in Figure 10.1(b). Students are told that ball is dropped from rest. They are are asked to rank the probabilities of finding the ball in two unequal-sized regions. As was the case for the track question, the correct answer is that there is not enough information.

Performance on this question is shown in Table 10.1(b). On pretests, only 40% of the students answered correctly, compared to about 75% on posttests (see Table 10.1(b)). This improvement coincides with fewer students selecting that the probability in the smaller, upper region is less than that in the larger, lower region (40% on pretests compared to 10% on posttests). The percentage of students who selected other answers is fairly unchanged after tutorial instruction, as shown in the table.

The improvement in the ability of introductory students to rank probabilities on both the 'unequal-length drop' and 'unequal-length track' questions suggests that not only were students able to recall their experience in tutorial, but also able to generalize it to other situations.[3] We attribute this increase in performance to the portion of the in-class worksheet of the tutorial in which students are guided to consider how both the speed of the ball and the length of the regions influence the probability of finding an object in a particular region (see Chapter 9 for a description of the tutorial).

We have also administered both the 'unequal-length track' and 'unequal-length drop' probability questions to tutorial TAs as a pretest as part of their preparation course.[4] The number of TAs for whom we have data is low. Nonetheless, we report their findings for

[3]We have not administered the 'equal-length track' or the 'equal-length drop' questions as posttests because we found the unequal-sized versions more strongly elicited student difficulties. However, we do note that students performed better on both unequal-sized questions on posttests than they did on the equal-sized questions on pretests.

[4]See §2.2.3 for a discussion of this practice.

documentation purposes. Of the 10 graduate TAs who answered the 'unequal-length track' question, nine did so correctly. (Compare this result to 80% of introductory students on posttests.) All of the 15 graduate TAs who answered the 'unequal-length drop' question did so correctly. (Compare this to 75% of introductory students on posttests.) Although these results could indicate that introductory students did not perform as well as graduate TAs did on pretests, we recognize that more data is needed on TA performance to make a stronger claim.

Commentary: Precursory speed questions

As we described in §8.2.1-b, on some versions of the pretest questions we asked a precursory speed question. In the presence of such a 'helper' question, student performance improved significantly. None of the posttests we administered included such a precursory speed question. It is telling, then, that students performed as well on the posttest 'unequal-length track' question (80% correct) as they did on the pretest 'equal-length track' question that included a precursory speed helper question (75% correct).[5]

10.1.2 Effect of tutorial instruction on tendency to rank probabilities solely based on size

As described in §8.2.1, prior to tutorial instruction a large fraction of students ranked probabilities in different regions solely based on the sizes of the regions. For example, on the 'unequal-length track' question in Figure 10.1(a), about 75% of the students answered incorrectly that the probability in the smaller, upper region was less than that in the larger, lower region. Most of these students cited only relative lengths in their explanations. They failed to realize that length is not the only relevant quantity in ranking probabilities in different regions.

On posttests (after tutorial instruction), only 10% of the students ranked the probabilities on the 'unequal-length track' question in this manner, as shown in Table 10.1(a). We see a

[5] See Table 8.4 on page 226.

similar decrease on the 'unequal-length drop' question (see Table 10.1(b)).

We attribute this increase in performance to the portion of the in-class worksheet of the tutorial *Probability in classical and quantum mechanics* in which students are guided to consider how both the speed of the ball and the length of the regions influence the probability of finding a ball in a particular region (see Chapter 9 for a description of the tutorial).

10.1.3 Effect of tutorial instruction on tendency to rank probabilities solely based on speed

In §8.2.2 we described a tendency of some students to rank probabilities based only on the relative speeds of an object. For example, on a modified version of the 'unequal-length track' question shown in Figure 10.1(a), roughly 5% of the students argued incorrectly that the probability is larger in the smaller level because the ball moves more slowly through that level. These students did not account for the fact that the ball moves more slowly in a *smaller* level (i.e., they did not account for the relative sizes of the levels).

On the posttests, about the same percentage of the students (5%) argued that the probability is higher in the region with slower speed (see Table 8.1(b)). Of relevance here is that we did not observed an *increase* in the fraction of students who used this incorrect line of reasoning. Thus, although we would not expect a measurable decrease in the proportion of students exhibiting this line of reasoning to decrease, tutorial instruction appears not to *increase* the likelihood of students using it.

10.1.4 Effect of tutorial instruction on tendency to use compensation-based reasoning

In §8.2.3 we described a tendency of some students to use compensation-based reasoning when comparing probabilities in two regions. For example, on the 'unequal-length track' and 'unequal-length drop' questions (see Figure 10.1), between 10% and 15% of the students stated that the probability of finding the ball in the upper region is equal to that in the lower region ($P_{\text{upper}} = P_{\text{lower}}$). Most of these students stated that since the ball moves more slowly in the upper region, which is of smaller in size, the ball will spend an equal amount of time on the two levels.

On posttests, about the same percentage of students answered incorrectly that $P_{\text{upper}} = P_{\text{lower}}$ (5%–10%) as they did on the pretests (10%–15%). Given that only a small fraction of students used this reasoning on pretests, it is perhaps not surprising that there is no significant change from pretest to posttest. We emphasize, though, that tutorial instruction appears not to have *increased* the likelihood that students use this line of reasoning.

10.1.5 Summary

After working through the tutorial curriculum *Probability in classical and quantum mechanics*, significantly more students are able to argue correctly about relative probabilities. Much of this improvement coincides with a decrease in the number of students who rank probabilities solely according to size, which was the most prevalent difficulty as measured by the pretests. We attribute this gain to the sections of the tutorial that guide students toward an understanding of the role that speed and time play in influencing relative probabilities. The percentage of students who used other incorrect lines of reasoning remained relatively low after tutorial instruction.

10.2 Effect of tutorial instruction on student understanding of probability density

In this section we describe the impact of tutorial instruction on student understanding of spatial probability density. In §10.2.1, a comparison of pretest and posttest student performance on different types of questions is provided. Then, in §10.2.2–§10.2.5, we use the data presented in §10.2.1 to discuss the impact of the tutorial on many of the specific student difficulties that were identified in §8.3.

10.2.1 Overview of student performance after tutorial instruction

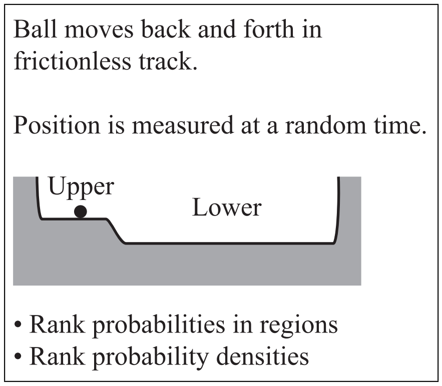
In general, student performance on probability density questions improved after tutorial instruction. Below, we compare pretest and posttest performance on several different questions. These questions are:

- The 'unequal-length track' question
- The 'unequal-length drop' question
- The 'infinite square well' question
- The 'density from probability' question
- The 'broken block' question

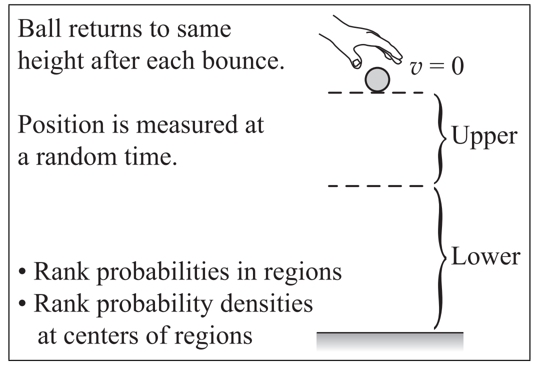
The first three of these questions are in the context of the motion of an object. The last two probe understanding of probability density more generally. (The effect of tutorial instruction on the *difficulties* are discussed later in §10.2.2–§10.2.5.)

10.2.1-a The 'unequal-length track' and 'unequal-length drop' questions

We administered the 'unequal-length track' question as both a pretest and posttest. In this question, students are told that a ball moves back and forth on a track composed of two unequal-sized levels at different heights (see Figure 10.2(a)). They are asked to rank the probability density in the two levels. The correct answer is that the probability density in



(a) The 'unequal-length track' question.



(b) The 'unequal-length drop' question.

Figure 10.2: Abbreviated versions of two 'track' and two 'drop' questions.

the upper region is greater than that in the lower region because the ball spends more time per unit length in the upper region.

Student performance on this question before and after tutorial instruction is shown in Table 10.2. On pretests (prior to tutorial instruction), only 25% of the students answered correctly, and only 5% of all students provided correct or partially correct reasoning. On posttests (after tutorial instruction), 70% of the students answered correctly, and about 45% (of all students) provided correct or partially correct reasoning. We note that the percentage of students on posttests who provided correct answers and explanations is greater than that of students who only provided correct *answers* on pretests. This improvement in student performance coincides with a decrease in the percentage of students who answered incorrectly that the probability density in the smaller, upper region is less than that in the larger, lower region (45% on pretests compared to 5% on posttests). (As discussed in §10.2.2, this incorrect answer is consistent with the failure to distinguish between probability and probability density.) The percentage of students who selected other answers are essentially unchanged (30% on pretests compared to 25% on posttests).

It is important to note that on the in-class worksheet, students are guided to rank the probability densities on the unequal-length track situation. To address the possible con-

Table 10.2: Pretest to posttest comparison of student performance on the 'unequal-length track' and 'unequal-length drop' density questions. Correct reasoning is highlighted.

	Unequal-length track		**Unequal-length drop**	
Rank prob. densities	Pretest N=253[a]	Posttest N=196[b]	Pretest N=404[a]	Posttest N=367[c]
$\rho_{\text{upper}} > \rho_{\text{lower}}$	25%	70%	25%	75%
w/partial or correct reasoning	≤5%	45%	15%	55%
$\rho_{\text{upper}} = \rho_{\text{lower}}$	20%	10%	30%	10%
$\rho_{\text{upper}} < \rho_{\text{lower}}$	45%	5%	20%	5%
Not enough information	10%	15%	25%	5%

[a] See Table 8.12 on page 253. [b] Section 123C102. [c] See Table 10.1(b) on page 273.

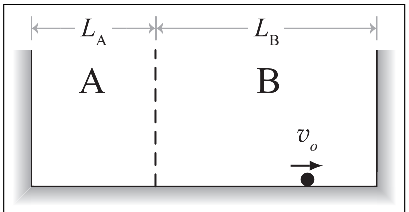
cern that students may simply be recalling the argument used in the tutorial without any deeper understanding, we also administered the 'unequal-length drop' question to students on posttests. Students did not see the ball-drop context in the in-class worksheet or the accompanying homework.

Performance on the 'unequal-length drop' question is also shown in Table 10.2. We see a similar result on this question as we do on the track question. On pretests, only 15% of the students answered correctly and provided correct or partially correct reasoning, compared to about 55% on posttests. This suggests that not only were students able to recall their experience in tutorial, but also able to generalize it to other situations.

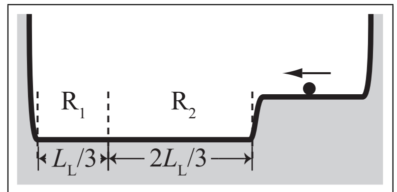
We attribute the improvement in student performance on the 'unequal-length track' and 'unequal-length drop' questions to the portion of the in-class worksheet of the tutorial *Probability in classical and quantum mechanics* in which students consider how the probability density can be ranked by comparing the probabilities in small, equal-sized regions.

We have also administered both the 'unequal-length track' and 'unequal-length drop' density questions to tutorial TAs as a pretest as part of their preparation course.[6] The number of TAs for whom we have data is low. Nonetheless, we report their findings for

[6]See §2.2.3 for a discussion of this practice.



(a) Original pretest version.



(b) Modified posttest version.

Figure 10.3: Abbreviated versions of the 'infinite square well' questions.

documentation purposes. Of the 10 graduate TAs who answered the 'unequal-length track' question, eight did so correctly. (Compare this result to 70% of introductory students on posttests.) Of the 15 graduate TAs who answered the 'unequal-length drop' question, all did so correctly. (Compare this to 75% of introductory students on posttests.) Although these results could indicate that introductory students did not perform as well as graduate TAs performed on pretests, we recognize that more data is needed on TA performance to make a stronger claim.

10.2.1-b The modified 'infinite square well' question

On pretests, we asked the 'infinite square well' question that is shown in Figure 10.3(a). In this 'original' version, students are asked to rank the probability densities in the two regions that are part of a single-level track (see the figure). The correct answer is that the densities are equal because the speed of the ball is constant.

On posttests, we asked a modified version of the 'infinite square well' question. In this version, students are shown a *two*-level track instead of a one-level track as on the pretest (see Figure 10.3(b)). However, the question asks students to rank the probability densities in two regions within a single level.[7] The correct answer is that the densities are equal.

[7] The addition of the second level to the track on the posttest version facilitated us asking another question, which we do not discuss here.

Table 10.3: Pretest to posttest comparison of student performance on the 'infinite square well' question. The correct answer is highlighted.

Answer	Pretest N=401[a]	Posttest N=481[b]
$\rho_{\text{small}} = \rho_{\text{large}}$	45%	70%
$\rho_{\text{small}} < \rho_{\text{large}}$	40%	20%
$\rho_{\text{small}} > \rho_{\text{large}}$	10%	10%

[a] See Table 8.13 on page 257.
[b] Sections 123A091, 123A102, and 123A114.

Student performance before and after tutorial instruction is shown in Table 10.3. On pretests, about 45% of the students ranked the probability densities correctly, compared to 70% on posttests. This improvement in student performance coincides with a decrease in the percentage of students who stated that the probability density in the smaller region is less than that in the larger region (40% on pretests compared to 20% on posttests).

10.2.1-c The 'density from probability' question

On the 'density from probability' question, students are told that the probability of finding a ball in 'region A' of a track is greater than that in 'region B.' Students are told that no other information about the track is known, including the relative heights and lengths of the regions. They are asked to rank, if possible, the probability densities in the two regions. The correct answer is that there is not enough information. (For an example of this question, see Figure 8.3 on page 215.)

Student performance before and after tutorial instruction is shown in Table 10.4.[8] On pretests, 30% of the students answered correctly, compared to 65% on posttests. This improvement coincides with a smaller percentage of students after tutorial instruction stating

[8] We exclude several posttest administrations from this analysis. Some sections also answered an 'unequal-length track' question as part of their posttest. These students performed better than those students who did not (80% correct, 15% $\rho_A > \rho_B$).

Table 10.4: Pretest to posttest comparison of student performance on the 'density from probability' question. The correct answer is highlighted.

Responses	Pretest N=413[a]	Posttest N=796[b]
Not enough information	30%	65%
$\rho_A > \rho_B$	50%	35%
Other	15%	5%

[a] See Table 8.9 on page 244. [b] Sections 123C082, 123A091, 123C092, 123A102, and 123A114.

incorrectly that the probability density in the region with larger probability is greater than that in the region with smaller probability (50% on pretests compared to 35% on posttests). We have administered these questions to several sections and there is essentially no overlap in the percentages on pretest and posttest administrations. Thus, we consider this 15 percentage point decrease to be a result of working through the tutorial.[9]

10.2.1-d The 'broken block' question

On pretests, we administered the 'broken block' question (see Figure 8.4 on page 216). In this question, students are informed that a small marble is placed at a random location in a block of material. There are two tasks. The 'expression task' asks students to write an expression for the probability density associated with finding the marble in the block. To answer correctly, students can note that the probability density is the probability of finding the marble somewhere in the block divided by the volume of the block. Thus, the correct answer to the expression task is $\rho = 1/HWD$. Next, the 'ranking task' informs students that the block of material is broken into two unequal sized pieces, with piece A smaller than piece B. Students are asked to rank the probability densities in piece A, piece B, and the original block. To answer correctly, students can note that breaking the block does not

[9] See §2.4.2.

alter the probability that the marble is in any given region. Thus, the correct answer to the ranking task is $\rho_{\text{orig}} = \rho_B = \rho_A$.

We asked a similar question as a posttest but with two modifications: (1) the block was two-dimensional rather than three-dimensional; and (2) students were asked to rank the probability densities of only the two 'broken' pieces rather than the two broken pieces and the original block. We recognize the argument could be made that the two-dimensional posttest question is more straightforward to answer than a three-dimensional pretest question. To address these concerns, we note the following:

1. On the three-dimensional pretest, of those students who incorporated the dimensions H, W, and D into their expression for probability density, very few used them in a manner that would not generalize to two dimensions (e.g., HWD and $H+W+D$ easily generalize to two dimensions, but H/WD does not). In other words, students' use of these dimensions were 'symmetric' with respect to interchanging any two dimensions.

2. On the three-dimensional pretest, about 15% of the students provided a ranking for probability density that was uninterpretable (e.g., $\rho_{\text{small}} < \rho_{\text{large}} > \rho_{\text{large}}$), incomplete (e.g., $\rho_{\text{small}} > \rho_{\text{large}} > \rho_{\text{large}}$), or otherwise inconsistent with the relative sizes of the three pieces (e.g., $\rho_{\text{small}} > \rho_{\text{large}} = \rho_{\text{orig}}$). For the purposes of the pretest-posttest comparison in this section, we remove these 15% of the students from the analysis.

With these notes, we regard the three-dimensional and and two-dimensional versions of the 'broken block' questions as appropriate for comparison.

Student performance before and after tutorial instruction is shown in Table 10.5. Performance on the expression task improved from 25% to 60%. This improvement in performance coincides with a decrease in the percentage of students who equated the probability density to either the volume of the original block (e.g., '$\rho = HWD$') or the probability associated with the entire block (e.g., '$\rho = 1$').

Performance on the ranking task of the 'broken block' question also improved, from 10% correct on pretests to 70% on posttests). The largest corresponding decrease for *incorrect*

Table 10.5: Pretest to posttest comparison of student performance on the 'broken block' question. Only a subset of pretest data is included. See the text for more information. The correct answers are highlighted.

Task	Answer	Pretest 3-dim N=341[a]	Posttest 2-dim N=262[b]
Write expression	$\frac{1}{HWD}$	25%	60%
	HWD or LW (volume)	20%	5%
	1	15%	10%
	$\frac{\text{"marble"}}{HWD}$	5%	5%
	Other	35%	20%
Rank density	$\rho_{\text{small}} = \rho_{\text{large}}$	10%	70%
	$\rho_{\text{small}} > \rho_{\text{large}}$	30%	10%
	$\rho_{\text{large}} > \rho_{\text{small}}$	60%	25%
Combinations	$\frac{1}{HWD}$ & $\rho_{\text{small}} = \rho_{\text{large}}$	5%	50%
	$\frac{1}{HWD}$ & $\rho_{\text{small}} > \rho_{\text{large}}$	15%	5%
	$\frac{1}{HWD}$ & $\rho_{\text{large}} > \rho_{\text{small}}$	10%	5%
	HWD & $\rho_{\text{large}} > \rho_{\text{small}}$	15%	5%
	1 & $\rho_{\text{large}} > \rho_{\text{small}}$	15%	5%

[a] See Table 8.15 on page 260. [b] Sections 123A083 and 123A092.

answers occurred for the answer that the probability density is larger in the larger-sized piece (from 60% to 25%).

Finally, we note that the percentage of students who answered both questions correctly increased from 5% on pretests to 50% on posttests.

10.2.2 Effect of tutorial instruction on failure to distinguish between probability density and closely related concepts

In §8.3.1 we demonstrated that many students did not distinguish between probability density and related concepts, including probability and volume. In general, after tutorial instruction, the ability of students to distinguish between probability density and related concepts improved. Below, we provide examples of improvement from a few types of questions. We also provide examples of ways in which this tendency manifested that we have not observed on pretests.

10.2.2-a Examples from specific questions

Example from the 'broken block' question

One of the questions that illustrated this difficulty is the 'broken block' question, which is described above in §10.2.1-d. As shown in Table 10.5 on page 285, about 15% of the students on pretests answered $\rho = 1$ on the expression task. This answer is consistent with an inability to distinguish probability and probability density. Another 20% of students answered on pretests that $\rho = HWD$. In all, about 35% of the students had difficulty distinguishing among probability, probability density, and volume.

On posttests, only 10% of the students answered $\rho = 1$ and only 5% answered $\rho = LW$. The former percentage is essentially unchanged from pretests (from 15% to 10%), and the latter percentage is somewhat improved (from 20% to 5%).[10] When combined, only about 15% of the students on posttests failed to distinguish among probability, probability density,

[10] The percentage on posttests on each administration has been 10% or lower; the percentage on pretests for each administration has been 15% or greater.

and/or volume, compared to 35% on pretests. We interpret this decrease in the percentage of students failing to distinguish between closely related concepts a result of tutorial instruction.

Example from the 'density from probability' question

Another question in which students struggled to distinguish between probability density and related concepts is the 'density from probability' question. In this question, students are told that the probability of finding the ball in region A of a track is greater than that in region B. No illustration of the track is given, but students are told explicitly that the relative lengths of the two regions are unknown. Students are then asked to compare, if possible, the probability densities in the two regions. (They are told that the density in each region is uniform.) The correct answer is that there is *not enough information* to rank the probability densities.

Table 10.4 on page 283 compares student performance before and after tutorial instruction. On pretests, 50% of the students answered $\rho_A > \rho_B$. We interpret this response as indicating a failure to distinguish between probability and probability density. On posttests, this percentage drops to 35%, which we consider to be an effect of the tutorial (see page 283).

Example from the 'infinite square well' question

The last question that we discuss in this subsection is the 'infinite square well' question. The original version used on pretests is shown in Figure 10.3(a) on page 281. The modified version used on posttests is shown in §10.3(b). In both questions, students are asked to rank the probability densities in two unequal-sized regions that are on the same horizontal level. The correct answer is that the densities are equal because the speed of the ball is constant.

Student performance before and after tutorial instruction is shown in Table 10.3 on page 282. On pretests, 40% of the students incorrectly stated that the probability density in the smaller region is less than that in the larger region. (Nearly all of these students also answered that the probability in region A was less than that in region B.) We interpret this response as indicating a failure to distinguish between probability and probability density.

On posttests, only 20% of the students answered that the probability density in the smaller region is less than that in the larger region.

Summary

We illustrated above that a smaller percentage of students failed to distinguish between probability, probability density, and other closely related concepts. We interpret this improvement to the tutorial that guides students in distinguishing probability and probability density multiple times.

Despite this improvement, it is important to note that up to 35% of students after tutorial instruction still give answers suggesting a failure to distinguish between these closely related concepts. The questions seem sufficiently basic that we believe it is reasonable to expect a larger fraction of students to answer correctly at the end of an introductory calculus-based course. More research is needed to address the underlying causes of this difficulty.

10.2.2-b New manifestations of the failure to distinguish between closely related concepts

On posttests, we have observed additional ways in which students failed to distinguish between closely related concepts that we did not observe on pretests. This may be a result of tutorial instruction. However, it may also be the case that the specific types of questions we asked as posttests (not all of which we asked as pretests) enabled us to probe different aspects of student thinking that we had not probed previously. Below, we provide two examples of new manifestations.

Summing probability densities

We previously described the modified 'infinite square well' question that we administered as a posttest. In addition to answering the 'ranking task' on this question, students answered another task that we call the 'expression task.' (See Figure 10.4.) Students are told that the probability in the upper and lower levels are P_U and P_L, respectively, and that the lengths of the upper and lower levels are L_U and L_L, respectively (see the figure). They are then

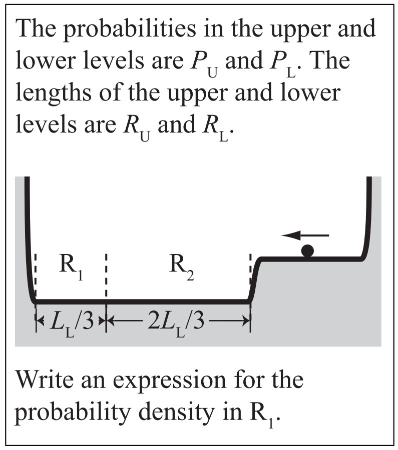
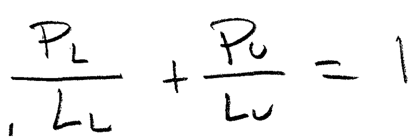


Figure 10.4: Question that elicited the tendency to treat probability density as an extensive quantity.

asked to write an expression for the probability in region R_1, which is the leftmost one-third of the lower region. To answer correctly, students can note that the probability density in the entire lower region is uniform, so the probability density in it, and thus in R_1, is just $\rho_1 = \rho_L = P_L/L_L$.

In attempting to arrive at an answer, about 5% of the students first *summed one or more probability densities together,* and used the resulting expression to solve for the probability density in region R_1. The following student response is illustrative of this addition of probability densities:

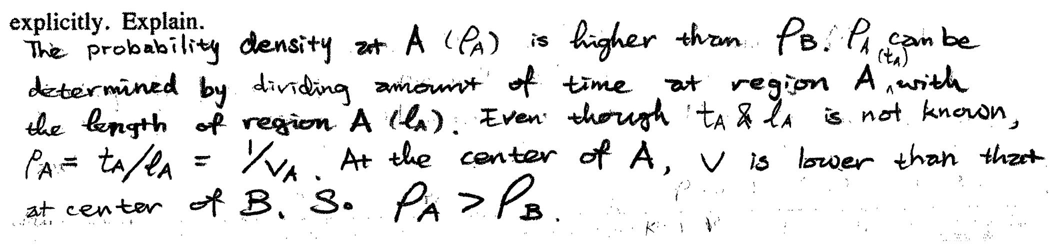


(Phys 123A, winter 2009)

This student set the sum $\frac{P_L}{L_L} + \frac{P_U}{L_U}$ equal to 1. This student and others with similar responses may be attempting to "normalize" probability density, but in doing so failed to distinguish between probability and probability density.

Treating time and probability as equivalent

Another new manifestation of the failure to distinguish between closely related concepts involved treating the *probability* in a region as if it were equivalent to the amount of *time* spent in that region. This tendency arose in questions in which students are asked to compare probability densities. For example, on the 'unequal-length drop' density question, students are asked to rank probability densities at the centers of two regions, labeled regions A and B. (See Figure 10.2(b) on page 279.) In arriving at an answer, some students divided the time the region by the length of the region to arrive at probability density. The following student response is illustrative:



explicitly. Explain.
The probability density at A (ρ_A) is higher than ρ_B. ρ_A (t_A) can be determined by dividing amount of time at region A with the length of region A (l_A). Even though t_A & l_A is not known, $\rho_A = t_A/l_A = 1/V_A$. At the center of A, V is lower than that at center of B. So $\rho_A > \rho_B$.

(PHYS 123C, SPRING 2010)

This student and others with similar responses equated the probability density to the quantity $\Delta t/L$. They effectively replaced probability in a region with the amount of time spent in that region in writing an expression for probability density.

We note that we do not include in this category students who simply *discuss* the time spent in equal-sized regions on the two levels. For example, we categorized the following explanation as correct:

> xplicitly. Explain.
> As previously mentioned, the speed at which the ball travels through A is less than the speed at which it travels through B. Therefore, the ball spends more time per length in section A, so there is a greater probability density at the center of region A than at the center of region B.
>
> (Phys 123C, spring 2010)

This student does not *explicitly* equate time and probability.

Summary of new manifestations

As illustrated, some students failed to distinguish between closely related concepts in a manner that we did not observe on pretests. This may be due to a combination of the specific types of questions we asked as posttests (not all of which we asked as pretests) and/or working through the tutorial, which emphasizes using the amount of time spent in a region to rank the probabilities. More research is needed to understanding the underlying reason(s).

10.2.3 Effect of tutorial instruction on tendency to associate smaller regions with larger densities

In §8.3.2 we described a tendency of some students to associate smaller regions with larger densities, regardless of the speed of the particle. We interpreted these responses as students overgeneralizing the rule that *for a given (fixed)* probability (or mass, charge, etc.), the corresponding density is inversely proportional to the size of the region. Below, we compare pretest and posttest results for two different questions.

Results from the 'broken block' question

One of the questions that elicited this tendency was the 'broken block' question, which is described above in §10.2.1-d. In this question, students consider a small marble that is located somewhere in a block of material. The block is broken into unequal-sized pieces, and students are asked to rank the probability densities associated with finding the marble in the different sized pieces. On pretests, about 30% of the students stated incorrectly that the probability in the smaller region was greater than that in the larger region. On posttests, only 10% of the students answered in this manner. This improvement suggests that after tutorial instruction, students were better able to avoid associating smaller regions with larger probabilities.

Results from the 'infinite square well' question

Another question that elicited the tendency to associate smaller regions with larger probability densities was the 'infinite square well' question, which we described above in §10.2.1-b. In this question, students are asked to rank the probability densities in two unequal-sized regions that are on the same horizontal level of a track. On pretests, about 10% of the students stated that $\rho_{\text{small}} > \rho_{\text{large}}$. In their explanations, these students typically stated that a smaller size implies a large density.

Given that only about 10% of the students used this reasoning on pretests, we would not expect to see any measurable *decrease* in the percentage of students using this reasoning on posttests. Indeed, as shown in Table 10.3 on page 282, the same percentage of students on posttests rank densities by inverse-size. Most of the explanations are consistent with our previous interpretations.

10.2.4 Effect of tutorial instruction on tendency to use compensation-based reasoning

In §8.3.3 we described a tendency of some students (about 5% on the 'unequal-length track' question) to argue that large regions with large probabilities have probability densities that are equal to those of small regions with small probabilities. We characterized this tendency as compensation-based reasoning (i.e., they believed the larger probability and larger sized exactly compensated in the expression P/L). On posttests, about the same percentage of students used compensation-based reasoning.

10.2.5 Effect of tutorial instruction on failure to generalize knowledge of familiar forms of density

In §8.3.5 we illustrated that many students failed to generalize their knowledge of familiar forms of density (e.g., charge density or mass density) to reason about probability density. In other words, reasoning about density appeared to be context-dependent.[11]

One of the questions that elicited this difficulty is the 'broken block' question, which was just described above in §10.2.1-d. Of particular relevance here is comparing student performance on this question in the context of probability to that in other contexts. Student performance before and after tutorial instruction in the context of probability is shown in Table 10.5 on page 285. About 60% and 70% of the students answered the expression task and ranking task correctly, respectively. In the context of *mass,* about 85% of the students answered each of the tasks correctly (see Table 8.14 on page 259).[12] In comparing performance in the context of probability and mass, we emphasize that (1) students who answered the mass questions did not have tutorial instruction in mass density, and that (2) many iterations of the in-class worksheet for *Probability in classical and quantum mechanics* included a page focusing on density in a different context with the intention of helping students generalize their knowledge from familiar forms of density.

Commentary on results

These results suggest that even with targeted instruction, student performance on density questions can be context-dependent. That students performed more poorly on the 'broken block' question in the context of probability, even with targeted instruction, that they did in the context of mass without targeted instruction. We note that the improvements in the 'broken block' question we have observed cannot necessarily be attributed to more students generalizing their knowledge of other forms of density (e.g., by using what they learned from

[11] Kanim [47] found similar results when comparing mass and charge density.

[12] This data was collected by Kanim [47], though we have found similar results when we administered this question in the context of mass.

section I of the tutorial). Students may simply have learned the answer in the context of probability density. Indeed, we have evidence that this may be the case, as we discuss later in §10.3.1.

10.2.6 Summary

After working through the tutorial curriculum *Probability in classical and quantum mechanics*, student performance on many of the posttest questions significantly improved. A greater fraction of students were able to relate the speed of an object with the probability density. The frequency with which students failed to distinguish between probability, probability density, and other related concepts decreased. The percentage of students who exhibited the other difficulties we investigated (the tendency to associate smaller regions with larger densities and the tendency to use compensation based reasoning) either decreased slightly or remained relatively low.

A continuing difficulty is the ability to generalize knowledge of more familiar forms of density. Although more students were able to do so after tutorial instruction, students did not perform as well on probability density questions as they did on mass density or charge density questions.

10.3 Research that led to modifications of the tutorial

In §9.2, we described two ways in which we modified the in-class worksheet for *Probability in classical and quantum mechanics* after its initial development: the removal of the first section that provided a review of linear *charge* density and the addition of a conceptual guide to probability density. In this section, we describe the research results that motivated these changes.

10.3.1 Testing the effect of the first page of the in-class worksheet

Early versions of the tutorial *Probability in classical and quantum mechanics* included a section on linear charge density. It was used for several quarters. The intention of this section was to allow students to review spatial density in a familiar context so that they could refer to it when they subsequently encounter spatial probability density (see §9.1.1). In particular, we believed it would help students distinguish between probability and probability density. Figure 10.5 shows this section as it was used in spring 2010.

To investigate whether or not this section had the desired effect, we removed the first section of the in-class worksheet in autumn 2011. We subsequently administered the two questions shown in Figure 10.6 as posttests. (We already had posttest data from these questions when the section was present.) We first describe these two questions separately, then present results from them simultaneously.

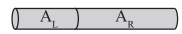
The question in Figure 10.6(a) is a modified 'infinite square well' question. In it, students are asked to rank the probability densities in two unequal-sized regions that are part of the same horizontal level. The correct answer is that the probability densities are equal because the speed of the particle is uniform. As we described in §10.2.2, we interpret the incorrect answer that the probability density in the smaller region is less than that in the larger region ($R_1 < R_2$) as indicating a failure to distinguish probability and probability.

The other posttest question we administered as part of this investigation is the 'density from probability' question. (See Figure 10.6(b).) In it, students are told that the probability

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS — ST 1

I. Review of linear density

A. A thin insulating rod has positive charge evenly distributed over its length. A line divides the rod into two pieces (A_L and A_R), as shown. Treat the rod as one-dimensional.



Rank the (i) charge and (ii) charge densities associated with pieces A_L and A_R. Explain.

B. A different rod is composed of two pieces that have *equal* amounts of charge. The charge is uniformly distributed on each piece.

Rank the charge densities associated with pieces B_L and B_R. Explain.

C. Consider the following student statements.

Student 1: "The charge density at a point is the amount of charge at that point."

Student 2: "The charge density at a point is the charge per unit length near that point. You calculate it with Q_{total} / L_{total}."

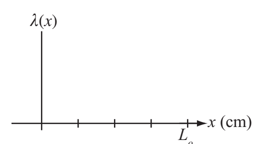
Do you agree with either of the students? Are there situations in which the student(s) with whom you agreed might not be correct?

D. The rod from part B has *total* positive charge Q_o and *total* length L_o. Piece B_L has length $L_o/4$.

Determine the charge densities of the two pieces, λ_L and λ_R. Show your work.

Draw a graph of the linear charge density λ *vs*. position x. Take $x = 0$ to be the left edge of the rod.

What geometric feature of your graph indicates that the charges on the two pieces are the same?



What would you expect for the quantity $\int_{\text{all } x} \lambda(x)\, dx$,

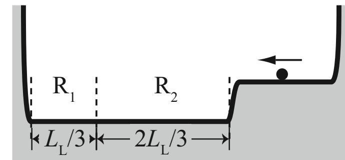
where $\lambda(x)$ is the charge density λ at position x?

➪ Discuss your answers to this section with a tutorial instructor.

Figure 10.5: First page of the in-class worksheet that reviewed linear charge density. This version was used in spring 2010.

IX. [25 point total] This page consists of two unrelated parts, A and B.

A. A classical (*i.e.*, non-quantum mechanical) object moves back and forth forever on the frictionless track at right. The two levels are horizontal and the object is able to make it up the ramp to the upper level. An imaginary line divides the lower level into two regions, R_1 and R_2, as shown.



i. [6 pts] Is the probability density in region R_1 *greater than, less than,* or *equal to* that in region R_2? If there is not enough information, state so explicitly. Explain.

(a) Modified 'infinite square well' question.

B. A classical object is confined to a frictionless track that is composed of two horizontal levels (C and D) joined by a short ramp. The heights and lengths of the two levels are unknown.

For each of the following cases, use the given information to determine if the probability density on level C is *greater than, less than,* or *equal to* the probability density on level D. If there is not enough information, state so explicitly. *Consider each case separately and explain your reasoning.*

i. [6 pts] Out of a very large number of photographs taken at random times, more than half show the object on level C.

(b) 'Density from probability' question.

Figure 10.6: Exam questions designed to probe the effect of the first section of *Probability in classical and quantum mechanics.*

in 'region C' of a track is greater than that in 'region D.' No diagram of the track is given, and students are told that the heights and lengths of the regions are unknown. They are asked to rank, if possible, the probability densities in the two regions. The correct answer is that there is not enough information to rank the probability densities. As described in §8.3.1-b, some students gave the incorrect answer that the probability density is greater in the region with greater probability density (i.e., $\rho_C > \rho_D$). We interpreted this incorrect answer as indicating a failure to distinguish between probability and probability density.

Table 10.6 compares student posttest performance on these two questions with and without the section on charge density.[13] The percentage of students who answered in a manner

[13]We include data only for those sections who answered both of the questions on the same exam.

Table 10.6: Student performance with and without the first section of the in-class tutorial. The correct answers are highlighted.

Question	Response	With first section N=279[a]	Without first section N=202[b]
Modified infinite square well	$\rho_1 = \rho_2$	65%	75%
	$\rho_1 < \rho_2$	20%	15%
Density from probability	Not enough info.	60%	70%
	$\rho_C > \rho_D$	35%	30%

[a] Sections 123A091 and 123A102. [b] Section 123A114.

consistent with the failure to distinguish between probability and probability density is within the typical range of fluctuations. Based on the results of this investigation, we chose not to replace the first section of the tutorial that covers charge density since first removing it in autumn 2011. This allows students to spend more time on the sections of the in-class worksheet for which we have documented evidence of a positive effect on student understanding.

Commentary

We were initially surprised that there was no measurable increase in the percentage of students failing to distinguish between probability and probability density after removing the charge density section of the in-class worksheet. One possible explanation is that this section is simply ineffective, and that the gains we observed from pretest to posttest on these two questions (see Tables 10.4 and 10.3) are a result of the later sections of the worksheet that focus on probability density. Another possibility is that the charge density section is somewhat effective, but that the later sections on probability density are *sufficient* in helping students answer the posttest questions that we administered.

10.3.2 The addition of a conceptually-based guide to probability density

One of the goals of the tutorial *Probability in classical and quantum mechanics* is to help students understand the role that the speed of an object plays in influencing the probability density associated with finding the particle in a particular region. As we described in §8.3 and §10.2.1, few students were able to apply this idea prior to tutorial instruction without prompting that directed their attention to any differences in speed.

To gauge the effectiveness of the tutorial in guiding students toward a correct understanding, we administered the 'unequal-length track' question as a posttest. In this question, students are asked to rank the probability densities in two unequal-sized levels of a track on which a ball is moving back and forth. The correct answer is that the probability density in the upper level is greater than that in the lower level ($\rho_{\text{upper}} > \rho_{\text{lower}}$) because the ball moves slower on the upper level, so it spends more time in a given equal-sized region there than it does on the lower level.

Performance after an early version of the tutorial

Performance on this question on *pretests* is shown in the leftmost column of results in Table 10.7. Also shown in the table is performance by students who worked through an early version of the tutorial (see the column labeled 'Before spring 2010'). The percentage of students who answered correctly improved from 25% on pretests to 45% on posttests. The percentage of students who gave the correct answer and correct or partially correct explanation improved from up to 5% to 35%.

Despite this improvement in student performance, we were interested in attempting to raise the percentage of students who give correct answers and explanations to above 35%. Subsequently, we administered the 'unequal-length track' question to graduate TAs prior to them working through the tutorial. Eight out of ten TAs answered this question correctly with correct reasoning. That TAs on pretests outperformed introductory students on posttests is consistent with us aiming for improved student performance.

First attempt to modify the tutorial: No change in performance

When we obtained the posttest results from the early version of the tutorial, we decided to review the sequence of exercises in the in-class worksheet. In this early version, students are

Table 10.7: Student performance on the 'unequal-length track' question before and after tutorial modifications. Correct explanations are highlighted.

	Pretest	**Posttest**	
Response	N=253[a]	Before spring 2010 N=542[b]	Spring 2010 N=196[c]
$\rho_{\text{upper}} > \rho_{\text{lower}}$ (corr.)	25%	45%	70%
w/partial or correct reasoning	≤5%	35%	45%
$\rho_{\text{upper}} = \rho_{\text{lower}}$	20%	5%	10%
$\rho_{\text{upper}} < \rho_{\text{lower}}$	45%	5%	5%
Not enough information	10%	40%	15%

[a] See Table 10.2 on page 280. [b] Sections 123A082, 123B091, and 123A092.
[c] Section 123C102.

introduced to probability density by being asked to calculate a value for the probability of finding a ball in a 1 cm region. They are then asked to interpret this quantity as a probability density. We realized, though, that by only having students calculate numerical values for probability density for specific situations, they were not asked to think qualitatively about the role that speed plays in influencing probability density. In response, we added some conceptual questions to the in-class worksheet in which students are guided to consider small, equal-length segments as a way to compare probability densities. The intention was to guide students to realize that, although the probabilities cannot be ranked in the two unequal-sized levels of the track, the probability densities can be ranked. We placed these additional exercises *after* the numerical ones. (Compare section III of the tutorial in the appendix in Figure B.19 that begins on page 608 to that in Figure B.23 that begins on page 628.)

On subsequent posttest administrations of the 'unequal-length track' question, however, we found that student performance did not improve. Roughly the same percentage of students provided correct answers and explanations. (This data is combined into the column of Table 10.7 labeled 'Before spring 2010.') The most common incorrect answer was *not enough information.* These students tended to argue that since the probabilities in the two levels cannot be ranked (which is correct), the probability densities also cannot be ranked (which is incorrect).

Second attempt to modify the tutorial: Improved performance on answer

Given the lack of gain in student performance and the relatively high percentage of students answering 'not enough information' on the 'unequal-length track' probability density question, we hypothesized that (1) students were relying too much on the *given* probabilistic information in the worksheet, and that (2) since the conceptual questions occur *after* the numerical ones, some students may have been using the conceptual exercises as *verification* of their prior numerical responses rather than viewing it as a way in which one can argue qualitatively about probability density. In response, we decided in spring 2010 to remove most of the numerical exercises from the worksheet and to replace them with conceptual ones. The few numerical questions that were left were placed after the conceptual ones. The version of the tutorial we used in spring 2010 is shown in the appendix in Figure B.24 on page 633.

As shown in Table 10.7, the percentage of students who answered correctly increased from 45% on the early posttests to 70% in spring 2010. Interestingly, however, the percentage of students who provided correct or partially correct *explanations* remained essentially unchanged (from 35% to 45%, which is within typical fluctuations). From the table, it appears that most of the increase in correct answers resulted from a decrease in the percentage of students answering *not enough information.* In spring 2010, the roughly 25% of the students with correct answer but incorrect reasoning used a variety of methods to arrive at their answers, including stating that the combination of a smaller length and slower speed results in a larger probability density. (We consider this incorrect reasoning because the length of the upper region plays no role in ranking the densities.)

Commentary

We interpret the results from spring 2010 as indicating that the new version of the tutorial can help more students realize or recall that the probability densities *can* be ranked, but that it was not markedly effective in helping these students understand the underlying reason. We have not since made attempts to significantly modify the tutorial to address the still relatively low percentage of students who can provide correct answers and explanations on the unequal-length track question.

10.4 Adaptation of tutorial for use as an interactive tutorial-lecture

The previous sections in this chapter described the performance of students after working through the tutorial *Probability in classical and quantum mechanics.* In this section we discuss an alternative instructional form of the tutorial that we call an *interactive tutorial-lecture,* or *ITL* for short. This is part of a larger effort by the Physics Education Group to develop research-based material for large-lecture settings. (See, for example, Emigh [66].)

Below we describe the ITL and assess its effect on student understanding of probability density.

10.4.1 Initial motivation and description

Due to scheduling conflicts in autumn 2010, we were unable to hold the tutorial sessions for *Probability in classical and quantum mechanics.* We inquired and were given special permission by the instructor of the course to spend 30 minutes of a lecture covering classical probability density. Instead of using the *Probability in classical and quantum mechanics* in its then-current form, we modified the tutorial for use in a large-lecture environment to account for reduced instructional time. Briefly, the changes we made included the following:

Incorporating class discussions Students are asked in some places to stop working and to wait for a class discussion. This is intended to echo the interactions between tutorial TAs and students that are typical in the small-group tutorial sections.

Administering clicker questions Students use a real-time response system to answer carefully selected questions based on our research. Clickers are already integrated into the introductory physics courses at the University of Washington.

Reordering and removing material This allows the tutorial to be finished in the allotted time.

Providing additional questions for students who finish early We recognized that some students may finish the last section early. An ‘optional’ section provides these students with additional practice.

The interactive tutorial-lecture (ITL) is shown in Appendix B.2.2 on page 646. To help facilitate group discussion, several TAs and a postdoc who were familiar with *Tutorials in Introductory Physics* were present. There were four or five instructional staff in total for the 120 students who were present. No homework was administered.

10.4.2 Results from clicker questions

As part of the ITL, we asked several clicker questions. Two of the clicker questions were the 'unequal-length track' density question. (We previously described this question in §10.2.1-a, though we also briefly summarize it below.) These two questions were identical to each other but administered at different times. One was administered just after the section on probability but before the section on probability density. We call this administration the 'pre-question' to distinguish it from a 'traditional' tutorial pretest. The other was administered after the section on probability density. We call this administration the 'post-question' to distinguish it from a traditional tutorial posttest.

The 'unequal-length track' density question is shown in Figure 10.2(a) on page 279. In it, students are asked to rank the probability density on two unequal-sized levels of a track. The correct answer is that the probability density in the (smaller) upper level is greater than that in the (larger) lower level.

Comparison of pre-question and post-question results

Results from the pre-question and post-question are shown in the rightmost two columns of Table 10.8. On the pre-question, only 30% of the students answered correctly. Most of the rest either answered that the probability density in the two levels were the same (25%) or that they cannot be ranked (30%). On the post-question, 85% of the students answered correctly. Most of the rest answered that the probability in the two levels were the same (15%). Very few students answered that the probability densities cannot be ranked.

We attribute the increase in performance to the portion of the worksheet in which students are explicitly guided to consider the probabilities of finding the ball in two small equal-sized

Table 10.8: Student performance on the 'unequal-length track' question during the ITL. Blank entries indicate percentages of less than 5%. The correct answer is highlighted.

	Tutorial		**Tutorial-Lecture**	
Answer	Pretest N=253[a]	Posttest N=196[a]	Pre-question* N=122[b]	Post-question* N=121[b]
$\rho_{\text{upper}} > \rho_{\text{lower}}$	25%	70%	30%	85%
$\rho_{\text{upper}} = \rho_{\text{lower}}$	20%	10%	25%	15%
$\rho_{\text{upper}} < \rho_{\text{lower}}$	45%	5%	10%	
Not enough information	10%	15%	30%	

* These questions were administered during the ITL using clicker devices.
[a] See Table 10.2 on page 280. [b] Section 123A104.

regions on the two levels.

Comparison of ITL pre-question and 'traditional' pretest results

In addition to the results from the ITL, Table 10.8 shows pretest results from quarters in which the traditional tutorial was used. For the pretest and pre-question, roughly the same percentage of students answered correctly (25% and 30%, respectively). This might be expected since neither group of students had targeted instruction in probability density at the time the questions were administered.

Interestingly, though, a greater fraction of the students on the pre-question answered that the probability densities in the two levels cannot be compared (30% on the pre-question compared to 10% on the pretest). One possible explanation for this difference is that students who answered the pre-question had already worked through the probability portion of the ITL. They may have been better able to understand that the *probabilities* in the two unequal-sized levels cannot be compared (see §10.1). This, coupled with a failure to distinguish between probability and probability density (see §8.3.1 and §10.4.3), could have led to a greater fraction of students on the pre-question answering that the probability densities cannot be compared.

We briefly note that performance on the post-question was slightly greater than performance on the traditional posttest question. (The differences are only 15%, however.) We recognize, though, that a direct comparison of post-question and posttest may not be possible since the former was administered directly after the relevant section of the ITL, while the latter was administered in an exam setting roughly a week after instruction.

10.4.3 Assessment after interactive tutorial-lecture

In addition to the clicker questions during the interactive tutorial-lecture, we also administered posttests on students exams as a follow up to the ITL. (This is the typical manner in which we assess traditional tutorial instruction.) In assessing the effectiveness of the ITL, we were primarily interested in changes to basic conceptual issues surrounding probability density. To this end, we administered the 'density from probability' question as a posttest. In this question, students are told that an object is more likely to be found in 'region A' of a track than in 'region B'. They are told that no other information about the track is known. Students are asked to compare, if possible, the probability densities in the two regions. The correct answer is that there is not enough information.

Of interest here is a comparison of posttest results from the ITL to posttest results after traditional tutorial instruction (as well as to pretest results prior to tutorial instruction). Student performance is shown in Table 10.9. On pretests, about 30% of the students answered correctly. The most common incorrect answer, provided by 50% of the students, is that the probability density is greater in the region with higher probability (i.e., $\rho_A > \rho_B$). We interpret this incorrect answer as an inability to distinguish between probability density and probability.

As we previously described in §10.2.2-a, student performance is greater on *traditional* tutorial posttests as compared to pretests: After traditional tutorial instruction, a greater percentage of students provided correct answers (65% as compared to 30%) and a smaller percentage of students provided the incorrect answer $\rho_A > \rho_B$ (35% as compared to 50%).[14] However, there was not a similar improvement in student performance after instruction with

[14]See page 287 for a discussion of why we consider this 15% difference significant.

Table 10.9: Comparison of pretest and posttest student performance on the 'density from probability' question (interactive tutorial-lecture. The correct answer is highlighted.

	Pretest	**Posttest**	
		Tutorial	Tutorial-Lecture
Responses	N=413[a]	N=796[a]	N=167[b]
Not enough information	30%	65%	40%
$\rho_A > \rho_B$	50%	35%	50%
Other	15%	5%	10%

[a] See Table 10.4 on page 283. [b] Section 123A104.

the interactive tutorial-lecture. As the table shows, about 40% of the students answered correctly on the posttest question after working through the ITL, compared to 30% on pretests. This small difference is within typical fluctuations in performance. Furthermore, the percentage of students who provided the incorrect answer $\rho_A > \rho_B$ was approximately the same (50% on both pretest and posttest).

These results suggest that the ITL was not effective in helping students distinguish between probability and probability density as measured by the 'density from probability' question.

10.4.4 Commentary

The results presented in this section suggest that the effect of the interactive tutorial-lecture on student performance is mixed. Some students were able to apply lessons that are explicitly addressed in the ITL (i.e., ranking probability densities for situations in which the probabilities cannot be ranked), but were not able to generalize other lessons (i.e., realizing that probability density cannot be ranked for certain situations in which the ranking of probability is known). Furthermore, the results from the posttest question we administered suggests that the ITL was not as effective as traditional tutorial instruction as measured by our posttest question. More research is needed to characterize and compare the effect of adapting existing tutorial materials for use in a lecture setting.

10.5 Commentary on posttest results after tutorial instruction

Throughout this chapter, we provided examples that illustrate the effect of targeted instruction on student ability to reason about probability and spatial probability density. In general, student performance on questions about probability improved. Prior to tutorial instruction, many students only argued about how the length of a region influenced the probability. After tutorial instruction, more students were able to understand the role that speed plays in influencing the probability.

Likewise, student performance on many questions about probability density improved. A greater proportion of students were able to relate the speed of an object to the probability density. A smaller proportion of students failed to distinguish among probability, probability density, and volume. The percentage of students who exhibited the other difficulties we investigated either decreased slightly or remained relatively low.

We also showed that modifying an existing tutorial for use in a large lecture setting does not necessarily result in the same gains in student understanding that the original tutorial does. More research in this area.

A continuing difficulty after tutorial instruction is the ability to generalize knowledge of more familiar forms of density. Additionally, for some questions, many students did not understand that knowledge of only relative probabilities is insufficient for determining relative probability densities. Further modifications to the tutorial curriculum may be needed to address these difficulties.

Chapter 11

SUMMARY OF INVESTIGATIONS INTO IDENTIFYING AND ADDRESSING STUDENT UNDERSTANDING OF POTENTIAL ENERGY DIAGRAMS AND SPATIAL PROBABILITY DENSITY

Quantum mechanics is a topic that is gradually being introduced earlier in university (and even precollege) instruction. There is thus a need for research to identify what students do and do not understand about this topic and to design research-based and research-validated instructional materials that can help promote a functional understanding. The research discussed in this dissertation provides a foundation. The focus has been on student ability to interpret and apply potential energy diagrams and spatial probability density as they relate to classical systems. A basic premise is that in order for students to be able to recognize the difference between the predictions of classical and quantum physics, they must understand the classical counterparts.

The research spanned more than six years and involved dozens of classes taught by different instructors at the University of Washington. We found that after traditional instruction through lecture and textbook, many students have basic difficulties with potential energy diagrams and spatial probability density. Some of these seem to be based on a lack of fundamental skills, such as interpreting graphs and distinguishing between closely related concepts. Others are more closely tied to the underlying physical concepts and principles such as conservation of energy and interpretations of densities in general.

Using our insight into student conceptions and reasoning, we created two sets of instructional materials to improve student understanding and to address the underlying difficulties. These are to become part of a larger body of work that focuses on conceptual and qualitative reasoning in introductory physics (*Tutorials in Introductory Physics* [1]). Many of the problems that students encounter with these topics, however, appear to be deeply held and resistant to change. Moreover, many appear to be context dependent. In some cases, what seemed to be subtle differences in wording and question order greatly affected student response. We made good progress in the design of the materials, but found that for some difficulties, the time allotted (about 50 minutes of time in tutorial sections plus associated homework) was not sufficient. We therefore worked on developing other components to supplement instruction and improve student learning (e.g., an online interactive practice homework and teaching pretests).

We have observed that the materials seem to be effective at addressing some, but not all of the key difficulties that students encounter. Posttest performance on questions administered on course examinations is often much improved. However, in some cases, students struggle in the same way, and in roughly the same proportions as they did prior to working through the materials. Even successive revisions based on additional research was not effective at addressing some of the ongoing difficulties. We interpret this result as indicating that some of the ideas are strongly held and resistant to change. Simply providing more time to cover a particular topic or helping students to recognize contradictions in their reasoning is not sufficient to help them overcome some patterns of reasoning.

There are several avenues for additional research. Some of the results are tentative given the number of students and courses involved. Additional data are needed to refine and corroborate the results. (See, for example, §6.4.) The nature of some of the tendencies we identified suggest that the underlying difficulty may extend to contexts other than those we have probed (such as that described in §4.3.2-b). New instructional strategies may need to be developed if some of the ongoing student difficulties are to be addressed. (See, for example, §6.3.) The present research not only provides a contribution to the literature on identifying and addressing student understanding, but also can serve as a basis for future research.

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Part IV

APPENDICES: UNABBREVIATED QUESTIONS AND CURRICULAR MATERIAL

In this part, we provide unabbreviated versions of all of the pretest and posttest questions administered as part of our research. We also provide all of the curricular material developed. Appendix A (beginning on the following page) is devoted to the tutorial *Potential energy diagrams.* Appendix B (beginning on page 549) provides the material related to the tutorial *Probability in classical and quantum mechanics.*

Appendix A

QUESTIONS AND CURRICULAR MATERIAL FOR THE NEW TUTORIAL ON POTENTIAL ENERGY DIAGRAMS

In this portion of the Appendix we provide all pretests, in-class worksheets, homework, posttests, and other curricular material for the tutorial *Potential energy diagrams.*

Contents of appendix

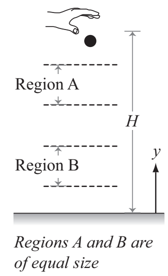
A.1 Pretests related to Potential energy diagrams

In this section we provide written and online pretests related to the tutorial *Potential energy diagrams*.

Name ____________________________________ **Student ID** ______________ **Score**________

last *first*

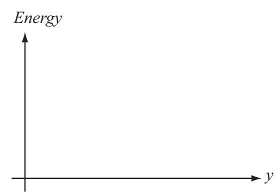
[12 points total] A ball is dropped from rest at a height *H* (a couple of meters) above the surface of the earth, as shown. While the ball is falling, its position is measured by taking a photograph at a random time. Regions A and B are of equal size.



Treat the ball classically (*i.e.*, do not use quantum mechanics in answering the following questions). The following questions are intended to be answered qualitatively; no involved calculations are expected.

A. [4 pts] On the axes below at right, sketch for the Earth-ball system both (i) a qualitative graph of the *potential energy* vs *height* of the ball and (ii) a qualitative graph of the total energy E_{tot} vs *height* of the same system. Take $y = 0$ to be the ground. Clearly label each graph.

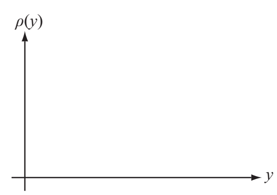
i. Explanation:



ii. Explanation:

B. [4 pts] Is the likelihood that the photograph shows the ball in region A *greater than*, *less than*, or *equal to* the likelihood that the photograph shows the ball region B? Explain.

C. [4 pts] On the axes at right, sketch a qualitative graph of the probability density $\rho(y)$ associated with measuring the position y of the ball at a random time. Take $y = 0$ to be the ground. Explain.



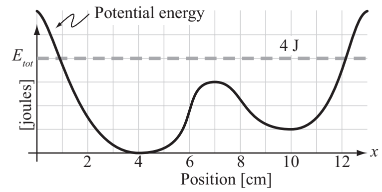
Physics 225A, Winter 2010 Final Exam QM-UWA225A101L-EF(PCQ).doc

Figure A.1: Written final exam administered in winter 2010 to Phys 225A.

Name ___________________________________ **Student ID** _______________ **Score** ___________
last *first*

Questions 16 and 17 involve a point particle that is part of a one-dimensional system. The only forms of energy of the system are potential energy of the system and translational kinetic energy of the particle. The total energy of the system is conserved. The *type* of interaction(s) between the particle and the rest of the system (*e.g.*, gravitational, magnetic, other, etc.) is not given, nor is it necessary for this question.

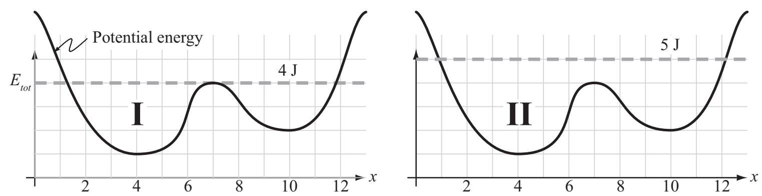
16. The graph of potential energy of the system versus position of the particle is shown at right. The total energy of the system is 4 joules, as indicated.



Is it possible to determine the direction of the **velocity** of the particle when it is located at $x = 8$ cm?

A. Yes. It is moving in the $+x$ direction.
B. Yes. It is moving in the $-x$ direction.
C. Yes. The velocity is zero.
D. No, it is not possible. More information is needed.

17. The graphs below differ from the graph above in that the potential energy and/or the total energy curves have been shifted up or down by some amount.



Which of the graphs above could represent **the same physical system** as the original system from question 16 **and** have the particle undergo **the same motion?** [*Note:* Having the *same motion* means the particle has the same speed at every position that it had in the original system.]

A. None of the graphs
B. Only I
C. Only II
D. Both I and II

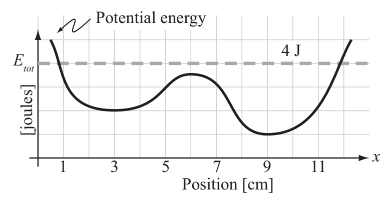
Physics 121A, Spring 2011 Final Exam ME-UWA121A112L-EF(PED)mc.doc

Figure A.2: Multiple-choice final exam administered in spring 2011 to Phys 121A. Identical to posttest administered in winter 2014 to Phys 121A (see page 532).

Name ___________________________________ **Student ID** _______________ **Score** ___________
last *first*

Questions 16 and 17 involve a point particle that is part of a one-dimensional system. The only forms of energy of the system are potential energy of the system and translational kinetic energy of the particle. The total energy of the system is conserved. The *type* of interaction(s) between the particle and the rest of the system (*e.g.*, gravitational, magnetic, other, etc.) is not given, nor is it necessary for this question.

The graph of potential energy of the system versus position of the particle is shown at right. The total energy of the system is 4 joules, as indicated.



16. Is it possible to determine the direction of the **velocity** of the particle when it is located at $x = 5$ cm?
    - A. Yes. It is moving in the $+x$ direction.
    - B. Yes. It is moving in the $-x$ direction.
    - C. Yes. The velocity is zero.
    - D. No, it is not possible. More information is needed.

17. Is it possible to determine the direction of the **acceleration** of the particle when it is located at $x = 5$ cm?
    - A. Yes. The acceleration is in the $+x$ direction.
    - B. Yes. The acceleration is in the $-x$ direction.
    - C. Yes. The acceleration is zero.
    - D. No, it is not possible. More information is needed.

Physics 121B, Spring 2011 Final Exam ME-UWA121B112L-EF(PED)mc.doc

Figure A.3: Multiple-choice final exam administered in spring 2011 to Phys 121B. Identical to a posttest administered in winter 2014 to Phys 121B (see page 533).

Catalyst WebQ https://catalyst.uw.edu/webq/build/tipwo/134811

Question 1.
In Amy's reference frame, the flash above Smith Tower occurs:

- ◯ Before the flash above the Space Needle occurs.
- ◯ After the flash above the Space Needle occurs.
- ◯ At the same time as the flash above the Space Needle occurs.
- ◯ There is not enough information provided to answer this question.

Question 2.
Explain the reasoning you used to answer the question above.

Question 3.
In Bob's reference frame, the flash above Smith Tower occurs:

- ◯ Before the flash above the Space Needle occurs.
- ◯ After the flash above the Space Needle occurs.
- ◯ At the same time as the flash above the Space Needle occurs.
- ◯ There is not enough information provided to answer this question.

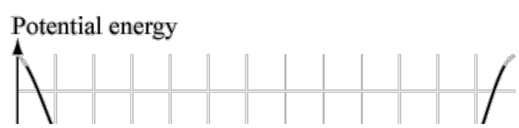
Question 4.
Explain the reasoning you used to answer the question above.

Question 5.
In Chuck's reference frame, the flash above Smith Tower occurs:

- ◯ Before the flash above the Space Needle occurs.
- ◯ After the flash above the Space Needle occurs.
- ◯ At the same time as the flash above the Space Needle occurs.
- ◯ There is not enough information provided to answer this question.

Question 6.
Explain the reasoning you used to answer the question above.

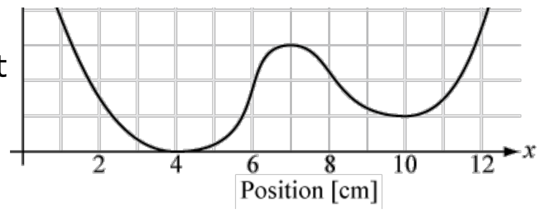
Note: The



2 of 5 5/23/11 12:49 PM

Figure A.4: Catalyst pretest version (SIM,PED)U2D administered in spring 2011 to PHYS 123C. Top portion of first page covers material related to a different tutorial. (Three pages.)

following questions may not be related to next week's tutorial.



A point particle is part of a one-dimensional system. The only forms of energy of the system are translational kinetic energy and potential energy. The total energy of the system is conserved.

The graph of potential energy of the system versus position of the particle is shown.

At some time, the particle is observed to be at x = 6 cm.

Question 7.
What is the direction of the **velocity** of the particle when it is located at x = 6 cm?

- ○ The velocity of the particle is in the positive-x direction at x = 6 cm.
- ○ The velocity of the particle is in the negative-x direction at x = 6 cm.
- ○ The velocity of the particle is zero at x = 6 cm.
- ○ There is not enough information to answer.

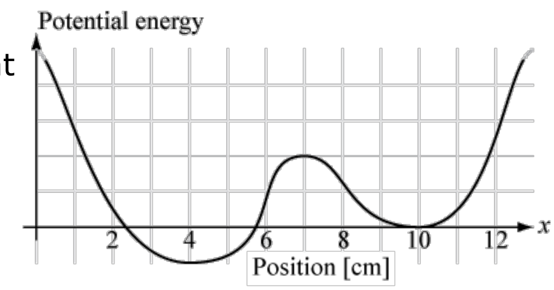
Question 8.
Explain the reasoning you used to answer the question above.

Question 9.
What is the direction of the **acceleration** of the particle when it is located at x = 6 cm?

- ○ The acceleration of the particle is in the positive-x direction at x = 6 cm.
- ○ The acceleration of the particle is in the negative-x direction at x = 6 cm.
- ○ The acceleration of the particle is zero at x = 6 cm.

- ◯ There is not enough information to answer.

Question 10.
Explain the reasoning you used to answer the question above.



The graph at right of potential energy versus position is identical to the graph above except that all values have been decreased by some amount.

Question 11.
Is it possible that this new graph represents the **same physical system** as that above? If so, how would the **total energy** of the system have to compare to that above if the particle is to undergo the same motion (*i.e.,* have the same velocity at every position)?

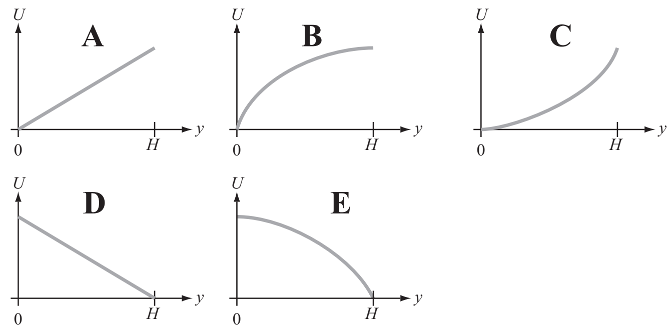
- ◯ Yes it could, and the total energy would have to be identical to that above for the particle to undergo the same motion.
- ◯ Yes it could, but the total energy would have to be less than that above for the particle to undergo the same motion.
- ◯ No it could not represent the same physical system.

Question 12.
Explain the reasoning you used to answer the question above.

Name ____________________________________ **Student ID** _______________ **Score** ___________
last *first*

7. [5 pts] Suppose that your classmate were to drop a ball from rest at a height *H* above the surface of Earth. You wish to sketch a graph of the potential energy *U* of the ball-Earth system versus the height *y* of the ball while the ball is falling. (The ground is at $y = 0$. Let "up" be the positive *y*-direction.)

   Which of the following sketches most accurately depicts the graph $U(y)$ while the ball is falling?



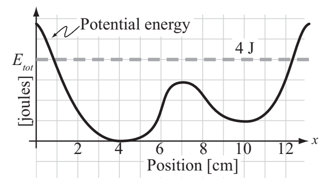
Physics 114A, Summer 2011 Final Exam ME-UWA114A113T-EF(PED)mc.doc

Figure A.5: Multiple-choice final exam administered in summer 2011 to Phys 114A.

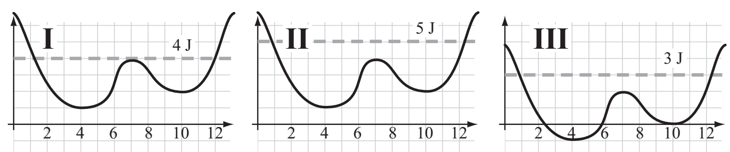
Name ___________________________________ **Student ID** _______________ **Score** ___________
last *first*

12. A particle is part of a one-dimensional system. The two forms of energy in the system are translational kinetic energy of the particle and potential energy of the system. All internal forces are conservative (*i.e.*, there is no energy loss in the system).

The graph at right shows the potential energy of the system as a function of position of the particle. The dashed line shows the total energy (kinetic plus potential) of the system.



Which of the three graphs below could represent the <u>same physical system</u> with an identical particle undergoing the <u>same motion</u>. (*Note:* Same motion means the particle has the same speed at every position.)



A. Only I

B. Only II

C. Both I and II

D. Both II and III

E. None of the graphs

Physics 121A, Summer 2011 Final Exam ME-UWA121A113T-EF(PED)mc.doc

Figure A.6: Multiple-choice final exam administered in summer 2011 to Phys 121A.

Time remaining: 0:14:20

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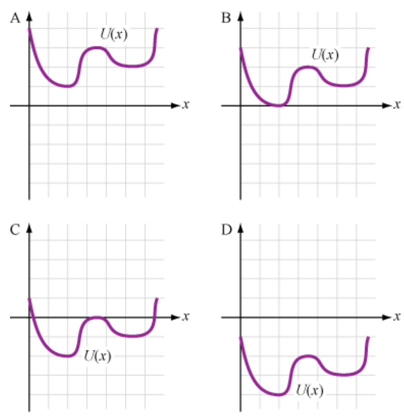
(PED)U1a
University of Washington

Page **1** of 3

Each of the four diagrams at right plots potential energy $U(x)$ of a one-dimensional system. The coordinate x represents the position of some particle within that system.



Question 1.
For each of the diagrams A–D, indicate whether or not it corresponds to a possible physical system.

	Possible	Not possible
Diagram A	○	○
Diagram B	○	○
Diagram C	○	○
Diagram D	○	○

Question 2.
Explain the reasoning you used to answer the previous question.

Next >>

Questions or Comments?
Contact the 121 tutorial pretest coordinator at uwttl121@u.washington.edu

Figure A.7: Catalyst pretest version (PED)U1a administered in spring 2013 to Phys 121A and in summer 2013 to Phys 121A. (Five pages.)

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(PED)U1a
University of Washington

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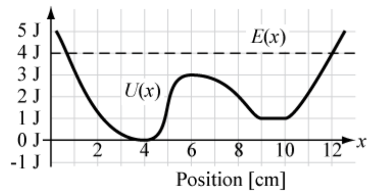


A particle of mass *m* is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle. The dashed line represents the total energy *E* of the system.

The only forms of energy in the system are potential energy (shown above) and translational kinetic energy of the particle.

Question 3.
What is the greatest kinetic energy obtained by the particle?

Select one...

Question 4.
Explain the reasoning you used to answer the previous question.

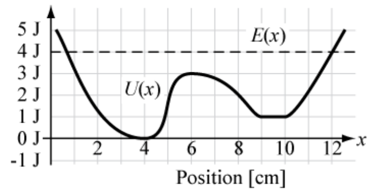
Question 5.
Suppose that the particle were located at x = 11.0 cm.

Which of the following best describes the direction of motion of the particle?

Select one...

Question 6.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



Question 7.
At what location is the magnitude of the force on the particle the largest?

Select one...

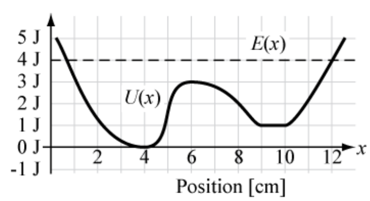
Question 8.
Explain the reasoning you used to answer the previous question.

Question 9.
At which of the following locations, if any **Time remaining: 0:12:56** equal to zero? (Check all that apply.) If there is not enough information to answer, state so explicitly in your explanation.

- ☐ x = 0.7 cm
- ☐ x = 4.0 cm
- ☐ x = 6.0 cm
- ☐ x = 9.5 cm
- ☐ x = 12.0 cm

Question 10.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



Question 11.
Which of the following best describes the direction of the force on the particle at x = 8.0 cm?

Select one...

Question 12.
Explain the reasoning you used to answer the previous question.

Question 13.
Describe the motion of the particle in the interval $9.0 < x < 10.0$ cm. If there is not enough information to answer, state so explicitly. You will have an opportunity to explain later.

Question 14.
Explain the reasoning you used to answer the previous question.

Next >>

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Time remaining: 0:12:19

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(PED)U1a

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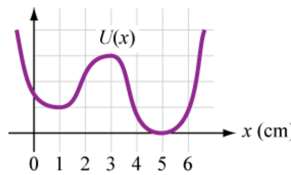
Page **3** of 3

A particle of mass *m* is part of a one-dimensional system with potential energy. The potential energy as a function of position of the particle is shown at right.



Question 15.
At which of the following locations, if any, could the particle be released from rest and remain stationary? (Check all that apply.) If there is not enough information to answer, state so explicitly in your explanation.

- ☐ x = 0 cm
- ☐ x = 1.0 cm
- ☐ x = 2.0 cm
- ☐ x = 3.0 cm
- ☐ x = 4.0 cm
- ☐ x = 5.0 cm
- ☐ x = 6.0 cm

Question 16.
Explain the reasoning you used to answer the previous question.

Submit responses

Questions or Comments?
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Time remaining: 0:14:43

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(PED)U1b

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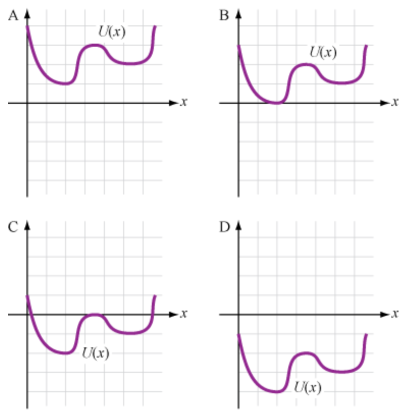
Page **1** of 3

Each of the four diagrams at right plots potential energy $U(x)$ of a one-dimensional system. The coordinate x represents the position of some particle within that system.



Question 1.
For each of the diagrams A–D, indicate whether or not it corresponds to a possible physical system.

	Possible	Not possible
Diagram A	○	○
Diagram B	○	○
Diagram C	○	○
Diagram D	○	○

Question 2.
Explain the reasoning you used to answer the previous question.

Next >>

Questions or Comments?
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Figure A.8: Catalyst pretest version (PED)U1b administered in spring 2013 to Phys 121B. (Five pages.)

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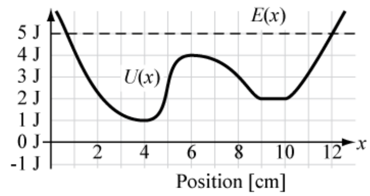
(PED)U1b
University of Washington

Page **2** of 3

A particle of mass *m* is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle. The dashed line represents the total energy *E* of the system.



The only forms of energy in the system are potential energy (shown above) and translational kinetic energy of the particle.

Question 3.
What is the greatest kinetic energy obtained by the particle?

Select one...

Question 4.
Explain the reasoning you used to answer the previous question.

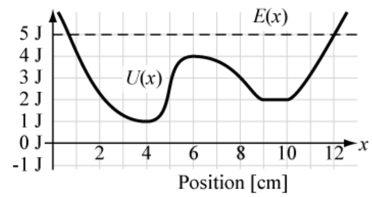
Question 5.
Suppose that the particle were located at x = 11.0 cm.

Which of the following best describes how the speed of the particle is changing?

Select one...

Question 6.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



Question 7.
At what location is the magnitude of the force on the particle the largest?

Select one...

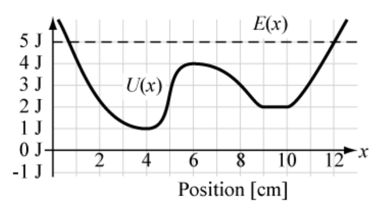
Question 8.
Explain the reasoning you used to answer the previous question.

Question 9.
At which of the following locations, if any **Time remaining: 0:11:21** equal to zero? (Check all that apply.) If there is not enough information y in your explanation.

- ☐ x = 0.7 cm
- ☐ x = 4.0 cm
- ☐ x = 6.0 cm
- ☐ x = 9.5 cm
- ☐ x = 12.0 cm

Question 10.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



Question 11.
Which of the following best describes the direction of the force on the particle at x = 8.0 cm?

Select one...

Question 12.
Explain the reasoning you used to answer the previous question.

Question 13.
Describe the motion of the particle in the interval $9.0 < x < 10.0$ cm. If there is not enough information to answer, state so explicitly. You will have an opportunity to explain later.

Question 14.
Explain the reasoning you used to answer the previous question.

Next >>

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Time remaining: 0:10:28

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(PED)U1b

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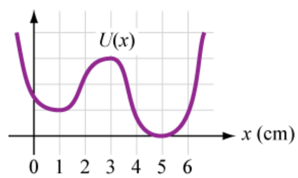
Page **3** of 3

A particle of mass *m* is part of a one-dimensional system with potential energy. The potential energy as a function of position of the particle is shown at right.



Question 15.
At which of the following locations, if any, could the particle be released from rest and remain stationary? (Check all that apply.) If there is not enough information to answer, state so explicitly in your explanation.

- ☐ x = 0 cm
- ☐ x = 1.0 cm
- ☐ x = 2.0 cm
- ☐ x = 3.0 cm
- ☐ x = 4.0 cm
- ☐ x = 5.0 cm
- ☐ x = 6.0 cm

Question 16.
Explain the reasoning you used to answer the previous question.

Submit responses

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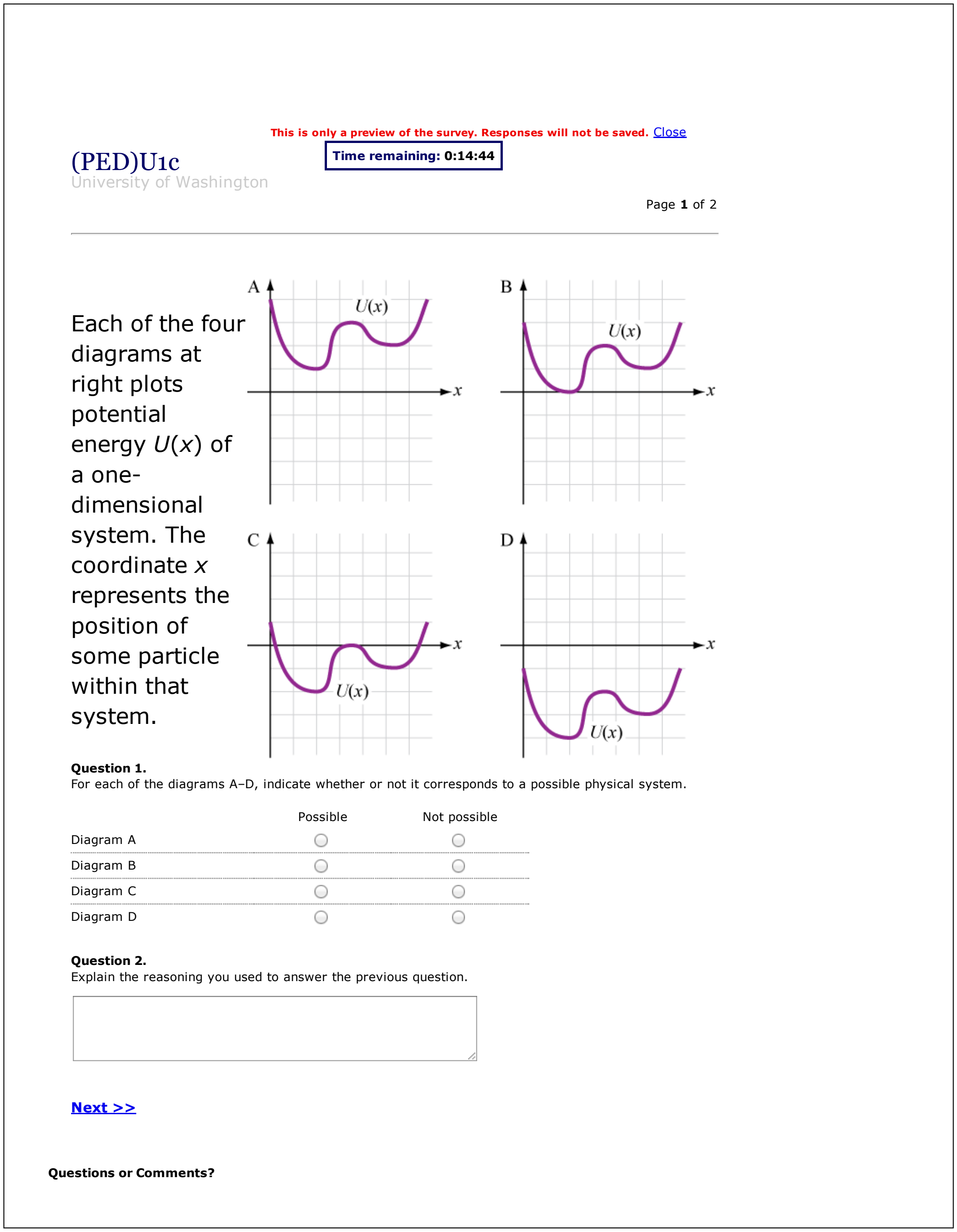

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Time remaining: 0:14:44

(PED)U1c

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Page **1** of 2

Each of the four diagrams at right plots potential energy *U*(*x*) of a one-dimensional system. The coordinate *x* represents the position of some particle within that system.

Question 1.
For each of the diagrams A–D, indicate whether or not it corresponds to a possible physical system.

	Possible	Not possible
Diagram A	○	○
Diagram B	○	○
Diagram C	○	○
Diagram D	○	○

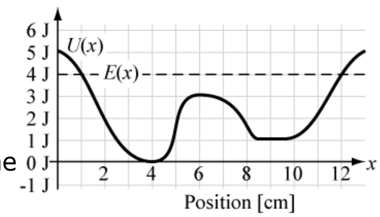
Question 2.
Explain the reasoning you used to answer the previous question.

Next >>

Questions or Comments?

Figure A.9: Catalyst pretest version (PED)U1c administered in autumn 2013 to Phys 121A. Some questions are similar to pretest version (PED)U1c_CNE (see page 353). (Four pages.)

(PED)U1c
University of Washington



A particle of mass *m* is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle. The dashed line represents the total energy *E* of the system.

The only forms of energy in the system are potential energy (shown above) and translational kinetic energy of the particle.

Question 3.
What is the greatest kinetic energy obtained by the particle?

Select one...

Question 4.
Explain the reasoning you used to answer the previous question.

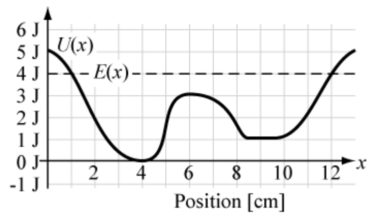
Question 5.
Suppose it is known that the particle were located at x = 11.0 cm.

When the particle is at this position, which of the following best describes how the speed of the particle is changing?

Select one...

Question 6.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



Question 7.
At what location is the magnitude of the force on the particle the largest?

Select one...

Question 8.
Explain the reasoning you used to answer the previous question.

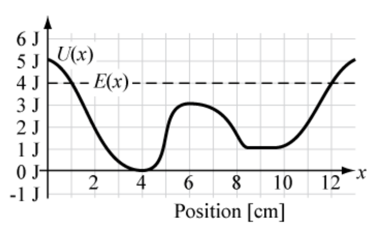
Question 9.
For each of the following locations, determine whether the force on the particle is *zero, non-zero,* or if *there is not enough information.*

	Zero	Non-zero	Not enough information
1.0 cm	○	○	○
1.5 cm	○	○	○
4.0 cm	○	○	○
6.0 cm	○		○
9.0 cm	○	○	○
11.5 cm	○	○	○
12.0 cm	○	○	○

Time remaining: 0:13:27

Question 10.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



Question 11.
Which of the following best describes the direction of the force on the particle at x = 8.0 cm?

Select one...

Question 12.
Explain the reasoning you used to answer the previous question.

Question 13.
Describe the motion of the particle in the interval 8.5 < x < 9.5 cm. If there is not enough information to answer, state so explicitly. You will have an opportunity to explain later.

Question 14.
Explain the reasoning you used to answer the previous question.

Submit responses

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(PED)U1e

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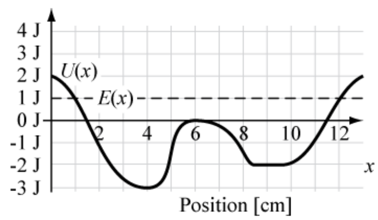
Page **1** of 2



A particle of mass *m* is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle. The dashed line represents the total energy *E* of the system.

The only forms of energy in the system are potential energy (shown above) and translational kinetic energy of the particle.

Question 1.
What is the greatest kinetic energy obtained by the particle?

Select one...

Question 2.
Explain the reasoning you used to answer the previous question.

Question 3.
Suppose it is known that the particle were located at *x* = 11.0 cm.

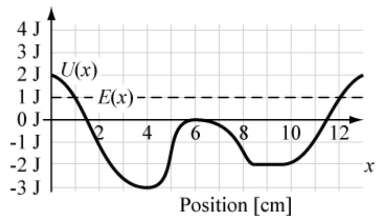
When the particle is at this position, which of the following best describes how the speed of the particle is changing?

Select one...

Question 4.
Explain the reasoning you used to answer the previous question.

Figure A.10: Catalyst pretest version (PED)U1e administered in autumn 2013 to Phys 121B (honors) and Phys 121C. (Four pages.)

The figure is reproduced at right for convenience.



Question 5.
At what location is the magnitude of the force on the particle the largest?

Select one...

Question 6.
Explain the reasoning you used to answer the previous question.

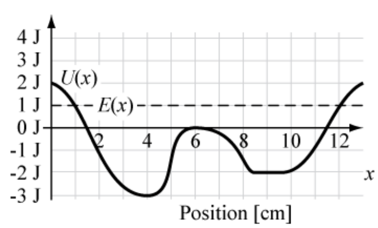
Question 7.
For each of the following locations, determine whether the force on the particle is *zero, non-zero,* or if *there is not enough information.*

	Zero	Non-zero	Not enough information
1.0 cm			
1.5 cm			
4.0 cm			
6.0 cm			
9.0 cm			
11.5 cm			
12.0 cm			

Time remaining: 0:14:44

Question 8.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



Question 9.
Which of the following best describes the direction of the force on the particle at x = 8.0 cm?

Select one...

Question 10.
Explain the reasoning you used to answer the previous question.

Question 11.
Describe the motion of the particle in the interval 8.5 < x < 9.5 cm. If there is not enough information to answer, state so explicitly. You will have an opportunity to explain later.

Question 12.
Explain the reasoning you used to answer the previous question.

Next >>

Questions or Comments?
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Time remaining: 0:13:47

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(PED)U1e

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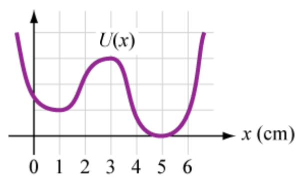
Page **2** of 2

A particle of mass *m* is part of a one-dimensional system with potential energy. The potential energy as a function of position of the particle is shown at right.



Assume that the only force on the particle is that associated with the potential energy $U(x)$ shown.

Question 13.
Suppose that the particle were released from rest at each of the following locations.

Determine whether the particle would remain stationary after being released. If there is not enough information to answer, state so explicitly in your explanation.

	Would remain stationary	Would *not* remain stationary	Not enough information
0 cm			
1.0 cm			
2.0 cm			
3.0 cm			
4.0 cm			
5.0 cm			
6.0 cm			

Question 14.
Explain the reasoning you used to answer the previous question.

Submit responses

Questions or Comments?
Contact the 121 tutorial pretest coordinator at uwttl121@u.washington.edu

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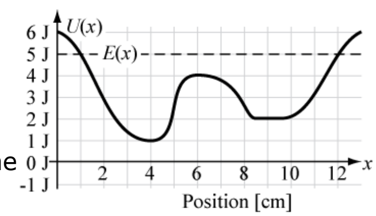
Page **1** of 2



A particle of mass *m* is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle. The dashed line represents the total energy *E* of the system.

The only forms of energy in the system are potential energy (shown above) and translational kinetic energy of the particle.

Question 1.
What is the greatest kinetic energy obtained by the particle?

Select one...

Question 2.
Explain the reasoning you used to answer the previous question.

Question 3.
Suppose it is known that the particle were located at x = 11.0 cm.

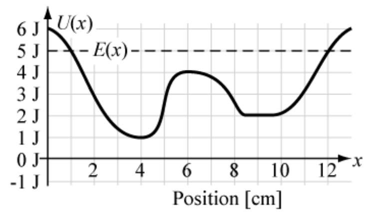
When the particle is at this position, which of the following best describes the *direction of motion* of the particle?

Select one...

Question 4.
Explain the reasoning you used to answer the previous question.

Figure A.11: Catalyst pretest version (PED)U1d administered in autumn 2013 to Phys 121D. (Four pages.)

The figure is reproduced at right for convenience.



Question 5.
At what location is the magnitude of the force on the particle the largest?

Select one...

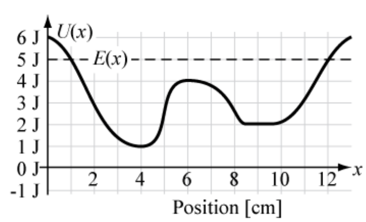
Question 6.
Explain the reasoning you used to answer the previous question.

Question 7.
For each of the following locations, determine whether the force on the particle is *zero, non-zero,* or if *there is not enough information.*

	Zero	Non-zero	Not enough information
1.0 cm	○	○	○
1.5 cm	○	○	○
4.0 cm	○	○	○
6.0 cm	○	○	○
9.0 cm	○	○	○
11.5 cm	○	○	○
12.0 cm	○	○	○

Question 8.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



Question 9.

Which of the following best describes the direction of the force on the particle at x = 8.0 cm?

Select one...

Question 10.
Explain the reasoning you used to answer the previous question.

Question 11.
Describe the motion of the particle in the interval $8.5 < x < 9.5$ cm. If there is not enough information to answer, state so explicitly. You will have an opportunity to explain later.

Question 12.
Explain the reasoning you used to answer the previous question.

Next >>

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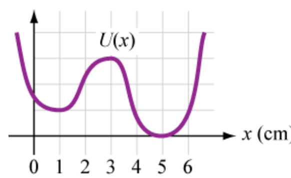
Page **2** of 2

A particle of mass *m* is part of a one-dimensional system with potential energy. The potential energy as a function of position of the particle is shown at right. Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.



Question 13.
Suppose that the particle were released from rest at each of the following locations.

Determine whether the particle would remain stationary after being released. If there is not enough information to answer, state so explicitly in your explanation.

	Would remain stationary	Would *not* remain stationary	Not enough information
0 cm	○	○	○
1.0 cm	○	○	○
2.0 cm	○	○	○
3.0 cm	○	○	○
4.0 cm	○	○	○
5.0 cm	○	○	○
6.0 cm	○	○	○

Question 14.
Explain the reasoning you used to answer the previous question.

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PRETEST 11

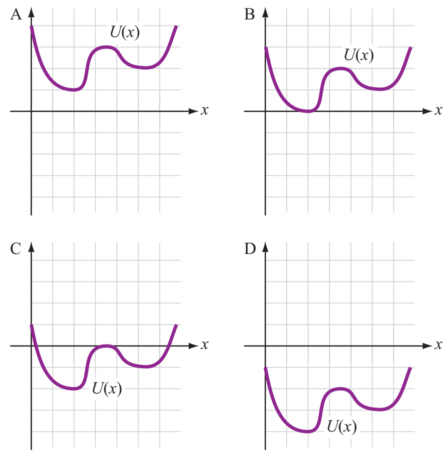
❐ Grad ❐ Undergrad ❐ Faculty ❐ Other
❐ I have done this pretest/tutorial before

1. Each of the four diagrams at right plots potential energy $U(x)$ of a one-dimensional system. The coordinate x represents the position of some particle within that system.

   a. For each diagram A–D, indicate whether it could correspond to a possible physical system. If it could, circle "Yes." If it could not, circle "No."

      - Diagram A: Yes / No
      - Diagram B: Yes / No
      - Diagram C: Yes / No
      - Diagram D: Yes / No

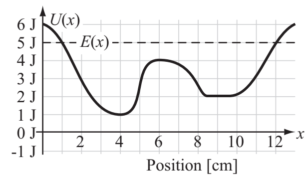
      Explain the reasoning you used in answering the question above.



ME-UWA501X134T-PB(PED) Autumn 2013

Figure A.12: Written pretest administered in autumn 2013 to Phys 501A (mainly graduate TAs). (Two pages.)

2. A particle of mass m is confined to a one-dimensional system. The solid curve at right shows the potential energy U of the system as a function of the position x of the particle. The dashed line represents the total energy E of the system. The only forms of energy in the system are potential energy (shown at right) and translational kinetic energy of the particle.



   a. What is the greatest kinetic energy obtained by the particle? Express your answer in joules. If there is not enough information to answer, state so explicitly. Explain.

   b. Suppose that the particle were located at $x = 11.0$ cm.

      Would the particle be *speeding up, slowing down,* or would the particle *not be moving?* If there is there not enough information to answer, state so explicitly. Explain.

   c. At what location(s) is the magnitude of the force on the particle the largest? If there is not enough information to answer, state so explicitly. Explain.

   d. At what location(s), if any, is the force on the particle equal to zero? If there is not enough information to answer, state so explicitly. Explain.

   e. What is the direction of the force on the particle at $x = 8.0$ cm? If there is not enough information to answer, state so explicitly. Explain.

   f. Describe the motion of the particle in the interval $9.0 \text{ cm} < x < 10.0$ cm. If there is not enough information to answer, state so explicitly. Explain.

Time remaining: 0:14:51

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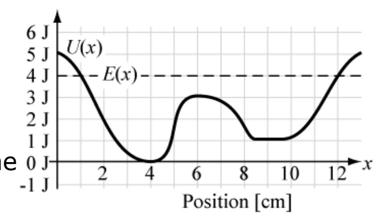
Page **1** of 2

A particle of mass m is confined to a one-dimensional system. The solid curve at right shows the potential energy U of the system as a function of the position x of the particle. The dashed line represents the total energy E of the system.



The only forms of energy in the system are potential energy (shown above) and translational kinetic energy of the particle.

Question 1.
What is the greatest kinetic energy obtained by the particle?

Select one...

Question 2.
Explain the reasoning you used to answer the previous question.

Question 3.
Suppose it is known that the particle were located at x = 11.0 cm.

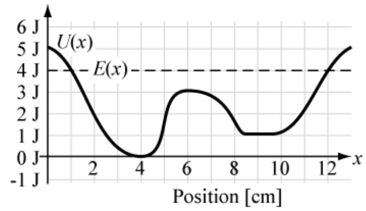
When the particle is at this position, which of the following best describes how the speed of the particle is changing?

Select one...

Question 4.
Explain the reasoning you used to answer the previous question.

Figure A.13: Catalyst pretest version (PED)U1c_CNE administered in winter 2014 to Phys 121A and 121B. Not shown are questions from the final pages of the pretest that covered topics from a different tutorial. Questions shown are similar to pretest version (PED)U1c (see page 339). (Three pages.)

The figure is reproduced at right for convenience.



Question 5.
At what location is the magnitude of the force on the particle the largest?

Select one...

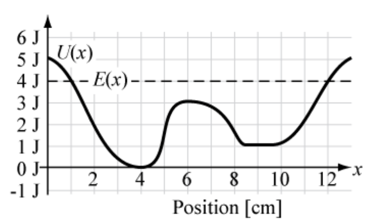
Question 6.
Explain the reasoning you used to answer the previous question.

Question 7.
For each of the following locations, determine whether the force on the particle is *zero, non-zero,* or if *there is not enough information.*

	Zero	Non-zero	Not enough information
1.0 cm	○	○	○
1.5 cm	○	○	○
4.0 cm	○	○	○
6.0 cm	○	○	○
9.0 cm	○	○	○
11.5 cm	○	○	○
12.0 cm	○	○	○

Question 8.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



Question 9.

Which of the following best describes the direction of the force on the particle at $x = 8.0$ cm?

Select one...

Question 10.
Explain the reasoning you used to answer the previous question.

Question 11.
Describe the motion of the particle in the interval $8.5 < x < 9.5$ cm. If there is not enough information to answer, state so explicitly. You will have an opportunity to explain later.

Question 12.
Explain the reasoning you used to answer the previous question.

Next >>

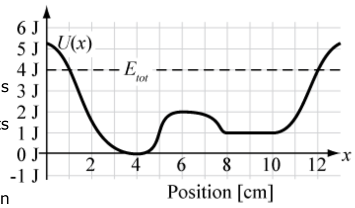
Questions or Comments?
Contact the 121 tutorial pretest coordinator at uwttl121@u.washington.edu

Print view of '(PED)U2a'

Print this page

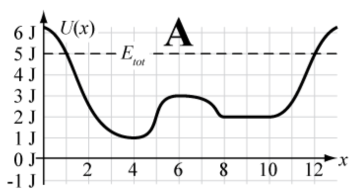
Part I.

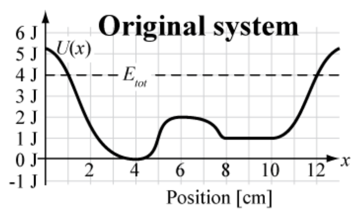
A particle of mass *m* is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle. The dashed line represents the total energy of the system.



The only forms of energy in the system are potential energy (shown above) and translational kinetic energy of the particle (not shown).

Question 1.
Could the diagram below labeled A represent the **same experimental setup** as the original system with a particle that undergoes the **same motion?** (*Note:* Having the same motion means the particle has the same speed at every position that it did in the original situation.)

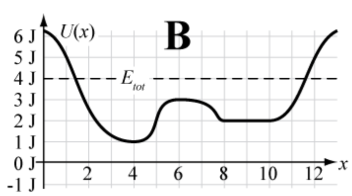


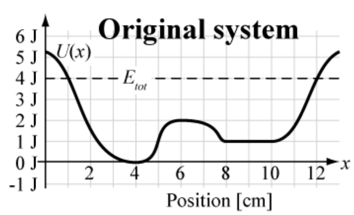


- Yes, diagram A could represent the same setup.
- No, diagram A could NOT represent the same setup.

Question 2.
Explain the reasoning you used to answer the previous question.

Question 3.
Could the diagram below labeled B represent the **same experimental setup** as the original system with a particle that undergoes the **same motion?** (*Note:* Having the same motion means the particle has the same speed at every position that it did in the original situation.)





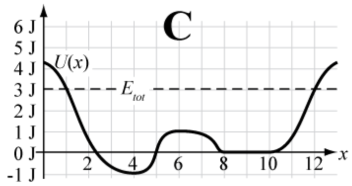
- Yes, diagram B could represent the same setup.
- No, diagram B could NOT represent the same setup.

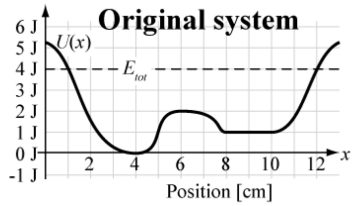
Question 4.

Figure A.14: Catalyst pretest version (PED)U2a administered in spring 2014 to Phys 121A. (Four pages.)

Explain the reasoning you used to answer the previous question.

Question 5.
Could the diagram below labeled C represent the **same experimental setup** as the original system with a particle that undergoes the **same motion?** (*Note:* Having the same motion means the particle has the same speed at every position that it did in the original situation.)

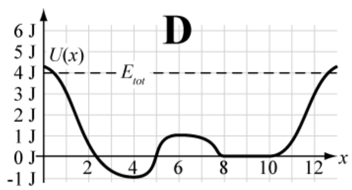


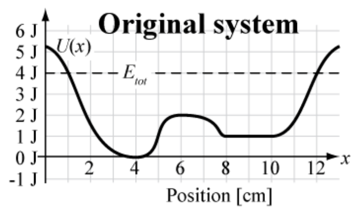


- ○ Yes, diagram C could represent the same setup.
- ○ No, diagram C could NOT represent the same setup.

Question 6.
Explain the reasoning you used to answer the previous question.

Question 7.
Could the diagram below labeled D represent the **same experimental setup** as the original system with a particle that undergoes the **same motion?** (*Note:* Having the same motion means the particle has the same speed at every position that it did in the original situation.)

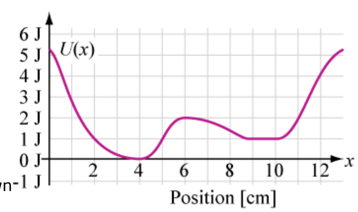




- ○ Yes, diagram D could represent the same setup.
- ○ No, diagram D could NOT represent the same setup.

Question 8.
Explain the reasoning you used to answer the previous question.

Part II.



A particle is confined to a one-dimensional system. The solid curve at right shows the potential energy U of the system as a function of the position x of the particle.

The only forms of energy in the system are potential energy (shown above) and translational kinetic energy of the particle (not shown). Assume that the only force on the particle is that associated with the potential energy $U(x)$ shown.

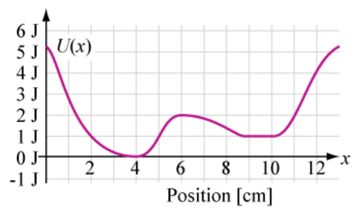
Question 9.
Suppose the particle is located at x = 2.0 cm.

What is the direction of the *force* on the particle?

- Positive x-direction
- Negative x-direction
- The force is zero.
- Not enough information to answer.

Question 10.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



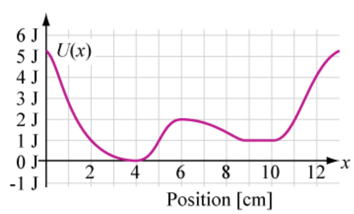
Question 11.
Suppose the particle is located at x = 8.0 cm.

What is the direction of the *force* on the particle?

- Positive x-direction
- Negative x-direction
- The force is zero.
- Not enough information to answer.

Question 12.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



Question 13.
Suppose it is known that at a particular time the particle is located at x = 5.0 cm and moving in the positive x-direction with a kinetic energy of 2 joules.

At what position x will the particle first obtain a speed of zero?

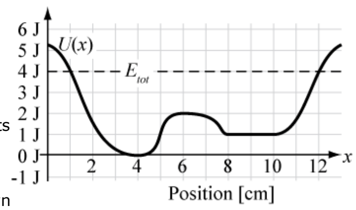
- At exactly 6.0 cm
- Between 8.0 and 9.0 cm
- Between 10.0 and 11.0 cm
- At exactly 11.0 cm
- Between 11.0 cm and 12.0 cm

- At exactly 12.0 cm
- At a position greater than 12.0 cm
- The particle does not turn around and change directions.
- Not enough information to answer
- Other:

Question 14.
Explain the reasoning you used to answer the previous question.

Part III.

A particle is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle. The dashed line represents the total energy of the system.



The only forms of energy in the system are potential energy (shown above) and translational kinetic energy of the particle (not shown).

Question 15.
Suppose that it takes the particle 3.0 seconds to travel from x = 1.0 cm to x = 12.0 cm.

Now suppose that the particle were replaced with a less massive particle ($m_{new} < m_{orig}$), while the total and potential energies remained the same.

Which of the options below best describes the time it would take the *new* particle to traverse the same path?

- Greater than 3.0 seconds
- Exactly 3.0 seconds
- Less than 3.0 seconds
- Not enough information to answer.

Question 16.
Explain the reasoning you used to answer the previous question.

Time remaining: 0:14:04

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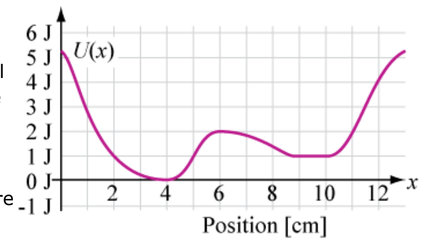
Page **1** of 3

Part I.



A particle is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle.

The only forms of energy in the system are potential energy (shown above) and translational kinetic energy of the particle (not shown). Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.

Question 1.
Suppose the particle is located at *x* = 2.0 cm.

What is the direction of the *acceleration* of the particle?

- ◯ Positive *x*-direction
- ◯ Negative *x*-direction
- ◯ The acceleration is zero.
- ◯ Not enough information to answer.

Question 2.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.

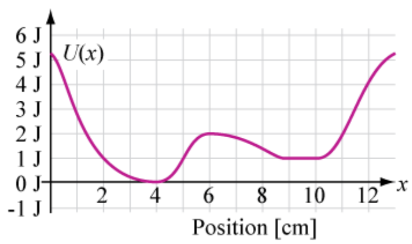


Figure A.15: Catalyst pretest version (PED)U2B administered in spring 2014 to PHYS 121B. (Five pages.)

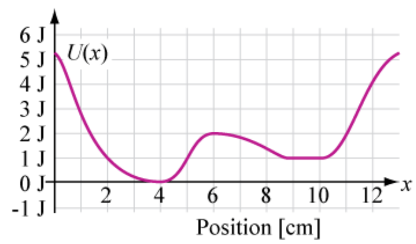
Question 3.
Suppose the particle is located at x = 8.0 cm.

What is the direction of the *acceleration* of the particle?

- Positive x-direction
- Negative x-direction
- The acceleration is zero.
- Not enough information to answer.

Question 4.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



Question 5.
Suppose that the particle is released from rest at x = 1.0 cm.

What is the *kinetic energy* of the particle when it is located at x = 5.0 cm?

- KE = 0 J
- KE = 1.0 J
- KE = 2.0 J
- KE = 3.0 J
- KE = 4.0 J
- KE = 5.0 J
- KE is more than 5.0 J
- Not enough information to answer.
- Other:

Question 6.
Explain the reasoning you used to answer the previous question.

Next >>

Time remaining: 0:13:17

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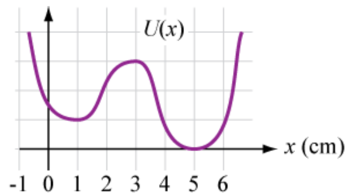
Page **2** of 3

Part II.

A particle of mass *m* is part of a one-dimensional system with potential energy. The potential energy as a function of position of the particle is shown at right. Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.



Question 7.
Suppose that the particle were released from rest at each of the following locations.

Determine whether the particle would remain stationary after being released. If there is not enough information to answer, state so explicitly in your explanation.

	Would remain stationary	Would *not* remain stationary	Not enough information
1.0 cm	○	○	○
2.0 cm	○	○	○
3.0 cm	○	○	○
4.0 cm	○	○	○
5.0 cm	○	○	○
6.0 cm	○	○	○

Question 8.
Explain the reasoning you used to answer the previous question.

Next >>

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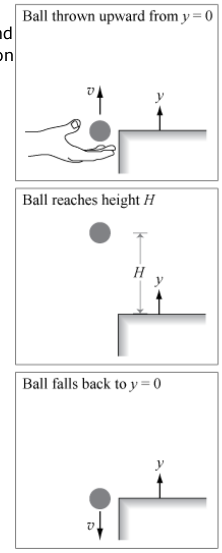
Page **3** of 3

Part III.

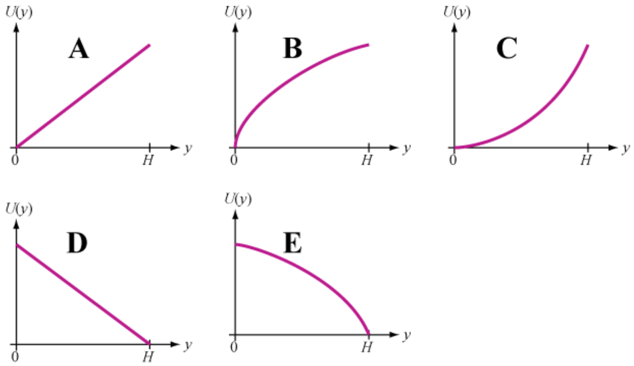
A ball is thrown upward, as shown at right. It leaves the hand at $y = 0$ (the surface of the table). Let the positive y-direction be upward.

The ball reaches $y = H$ (a few meters) before falling back down to $y = 0$.



Question 9.
Consider the interval when the ball is **moving upward.**

Which of the diagrams below best represents a graph of the potential energy U of the ball-Earth system as a function of the position y of the ball?

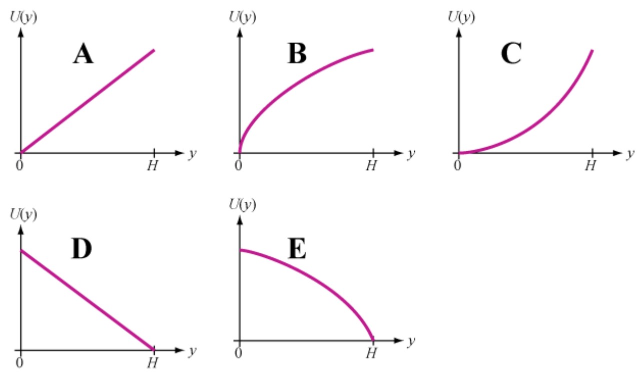


- Diagram A
- Diagram B
- Diagram C
- Diagram D
- Diagram E

Question 10.
Explain the reasoning you used to answer the previous question.

Question 11.
Consider the interval when the ball is **moving downward.**

Which of the diagrams below best represents a graph of the potential energy *U* of the ball-Earth system as a function of the position *y* of the ball?



- Diagram A
- Diagram B
- Diagram C
- Diagram D
- Diagram E

Question 12.
Explain the reasoning you used to answer the previous question.

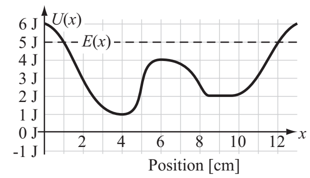
Submit responses

Questions or Comments?
Contact the 121 tutorial pretest coordinator at uwttl121@u.washington.edu

PRETEST 10

❐ Grad ❐ Undergrad ❐ Faculty ❐ Other
❐ I have done this pretest/tutorial before

1. A particle of mass *m* is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle. The dashed line represents the total energy *E* of the system. The only forms of energy in the system are potential energy (shown at right) and translational kinetic energy of the particle.



   a. What is the greatest kinetic energy obtained by the particle? Express your answer in joules. If there is not enough information to answer, state so explicitly. Explain.

   b. Suppose that the particle were located at $x = 11.0$ cm.

      Would the particle be *speeding up*, *slowing down*, or would the particle *not be moving?* If there is there not enough information to answer, state so explicitly. Explain.

   c. At what location(s) is the magnitude of the force on the particle the largest? If there is not enough information to answer, state so explicitly. Explain.

   d. At what location(s), if any, is the force on the particle equal to zero? If there is not enough information to answer, state so explicitly. Explain.

   e. What is the direction of the force on the particle at $x = 8.0$ cm? If there is not enough information to answer, state so explicitly. Explain.

   f. Describe the motion of the particle in the interval $9.0 \text{ cm} < x < 10.0$ cm. If there is not enough information to answer, state so explicitly. Explain.

Figure A.16: Written pretest administered in autumn 2013 to PHYS 501A (mainly graduate TAs).

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(DRB)U8c_Video_PED

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Part II

Note: This part concerns material that you will consider in a later week.

A block is on a level, frictionless surface and attached to a spring. The other end of the spring is fixed to a wall.

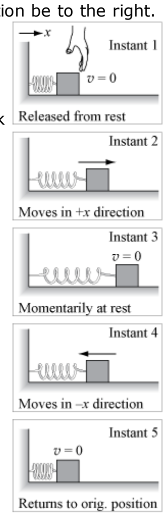
Assume that the spring obeys Hooke's law. Let the positive *x*-direction be to the right.

The block is initially held in place by a hand with the spring compressed (*i.e.,* the spring is shorter than its equilibrium length).

The hand now releases the block. Consider the motion of the block for an *entire* period, which includes:

- The instant that the block is released from rest (with the spring compressed).
- The interval while the block is moving to the right (in the positive *x*-direction).
- The instant that the block turns around (with the spring stretched).
- The interval while the block is moving to the left (in the negative *x*-direction).
- The instant that the block returns to its originial position (with the spring compressed).

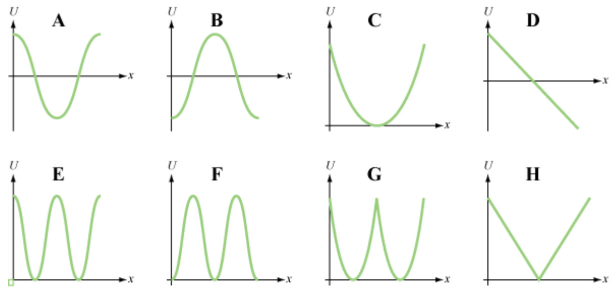
The diagrams at right show the position and direction of motion of the block during a few instants.



Question 7.
Consider system SB consisting of the spring and block for the motion described above (*i.e.,* one period).

Which diagram below best represents a possible **potential energy** *U* of system SB as a function of the **position** *x* of the block (*i.e., U*(*x*))?

Figure A.17: Catalyst pretest version (DRB)U8C_VIDEO_PED administered in summer 2014 to PHYS 121A. This pretest was appended to an already existing pretest on another topic; questions from this other topic are not shown. Students this quarter had an additional pretest that covering only PED-related material(see page 368). (Two pages.)



- Diagram A
- Diagram B
- Diagram C
- Diagram D
- Diagram E
- Diagram F
- Diagram G
- Diagram H

Question 8.
Explain your reasoning for your previous response.

Submit responses

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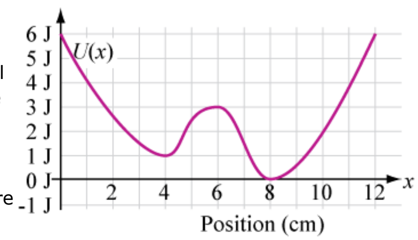
Page **1** of 2

Part I.

A particle is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle.



The only forms of energy in the system are potential energy (shown above) and translational kinetic energy of the particle (not shown). Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.

Question 1.
Consider an instant when the particle is located at $x = 7.0$ cm.

What is the direction of the *velocity* of the particle?

- ◯ Positive *x*-direction
- ◯ Negative *x*-direction
- ◯ The velocity is zero.
- ◯ Not enough information to answer.

Question 2.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.

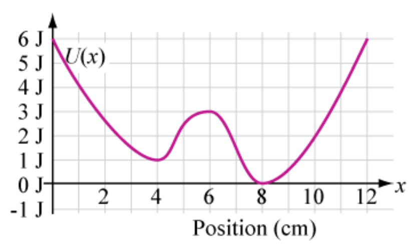


Figure A.18: Catalyst pretest version (PED)U3A administered in summer 2014 to PHYS 121A. Students this quarter had a shorter, earlier pretest (see page 366). (Five pages.)

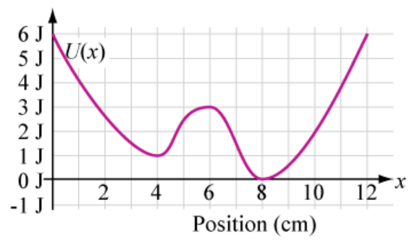
Consider an instant when the particle is located at x = 7.0 cm.

What is the direction of the *acceleration* of the particle?

- ◯ Positive x-direction
- ◯ Negative x-direction
- ◯ The acceleration is zero.
- ◯ Not enough information to answer.

Question 4.
Explain the reasoning you used to answer the previous question.

The figure is reproduced at right for convenience.



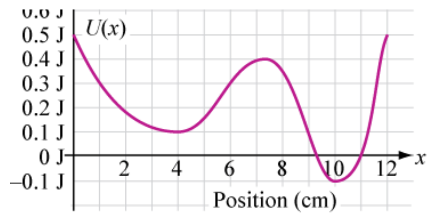
Question 5.
Suppose that the particle is released from rest at x = 1.0 cm.

What is the *kinetic energy* of the particle when it is located at x = 6.0 cm?

- ◯ K = 0 J
- ◯ K = 1.0 J
- ◯ K = 2.0 J
- ◯ K = 3.0 J
- ◯ K = 4.0 J
- ◯ K = 5.0 J
- ◯ K > 5.0 J
- ◯ Not enough information to answer.
- ◯ Other:

Question 6.
Explain the reasoning you used to answer the previous question.

potential energy *vs.* position shown at right.



Question 7.
Suppose that the particle is released from rest at x = 2.0 cm.

What is the *maximum kinetic energy* that the particle attains?

- K_{max} = 0 J (particle doesn't move)
- K_{max} = 0.1 J
- K_{max} = 0.2 J
- K_{max} = 0.3 J
- K_{max} = 0.4 J
- K_{max} = 0.5 J
- Not enough information

Question 8.
Explain the reasoning you used to answer the previous question.

Next >>

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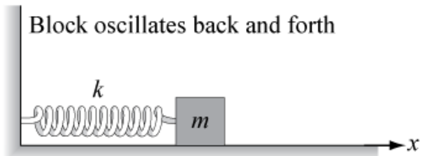
Page **2** of 2

Part II.

A block on a level, frictionless surface is attached to a spring of spring constant *k*. The other end of the spring is attached to an immovable wall.

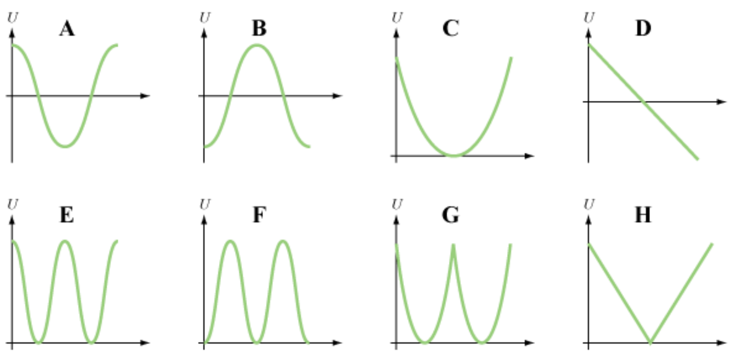


The block is initially held in place by a hand so that the spring is compressed. The hand then releases the block. The block moves back and forth.

During **one period** of its motion, the block start from its position closest to the wall, moves to the right (positive *x*-direction) to its position farthest from the wall, then moves to the left (negative *x*-direction) to its originial position.

Let system SB consist of the spring and block.

The eight diagrams below represent possible graphs for potential energy of system SB as a function of **time** (*i.e.,* $U(t)$) and potential energy of system SB as a function of **position** (*i.e.,* $U(x)$) for one entire period as described above.



Question 9.
Which diagram above best represents a possible graph for $U(t)$ (*i.e.,* potential energy *vs.* time)?

	A	B	C	D	E	F	G	H
Which best represents $U(t)$?	○	○	○	○	○	○	○	○

Question 10.
Explain the reasoning you used to answer the previous question.

Question 11.
Which diagram above best represents a possible graph for $U(x)$ (*i.e.,* potential energy *vs.* position)?

	A	B	C	D	E	F	G	H
Which best represents $U(x)$?	○	○	○	○	○	○	○	○

Question 12.
Explain the reasoning you used to answer the previous question.

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Preliminary Part (cont.)

Suppose that an object is moving in one dimension and that its acceleration is positive.

Question 2.
Which of the following best describes how the speed of the particle is changing?

- ○ The speed is increasing.
- ○ The speed is decreasing.
- ○ The speed is not changing.
- ◉ There is not enough information to answer.

Question 3.
Explain the reasoning you used to answer the previous question.

Next >>

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Figure A.19: Catalyst pretest version (DRB)U8c_Video_Research_PED_A1D administered in autumn 2014 to Phys 121A, C, and D as part of a pretest for a different tutorial near the end of the quarter. Students had all lecture, tutorial, and lab instruction on the relevant material. Students were presented with one of four questions based on the last digit of their student ID number. (Four pages.)

Time remaining: 0:13:13

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(DRB)U8c_Video_Research_PED_A1D

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Preliminary Part (cont.)

Suppose that an object is moving in one dimension and that its acceleration is in the positive direction.

Question 2.
Which of the following best describes how the speed of the particle is changing?

- ○ The speed is increasing.
- ○ The speed is decreasing.
- ○ The speed is not changing.
- ● There is not enough information to answer.

Question 3.
Explain the reasoning you used to answer the previous question.

Next >>

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Preliminary Part (cont.)

Suppose that an object is moving in one dimension and is speeding up.

Question 2.
Which of the following best describes the direction of the acceleration?

- ○ The acceleration is in the positive direction.
- ○ The acceleration is in the negative direction.
- ○ The acceleration is zero.
- ◉ There is not enough information to answer.

Question 3.
Explain the reasoning you used to answer the previous question.

Next >>

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Preliminary Part (cont.)

Suppose that an object is moving in one dimension and is speeding up.

Question 2.
Which of the following best describes the acceleration?

- ○ The acceleration is positive.
- ○ The acceleration is in negative.
- ○ The acceleration is zero.
- ◉ There is not enough information to answer.

Question 3.
Explain the reasoning you used to answer the previous question.

Next >>

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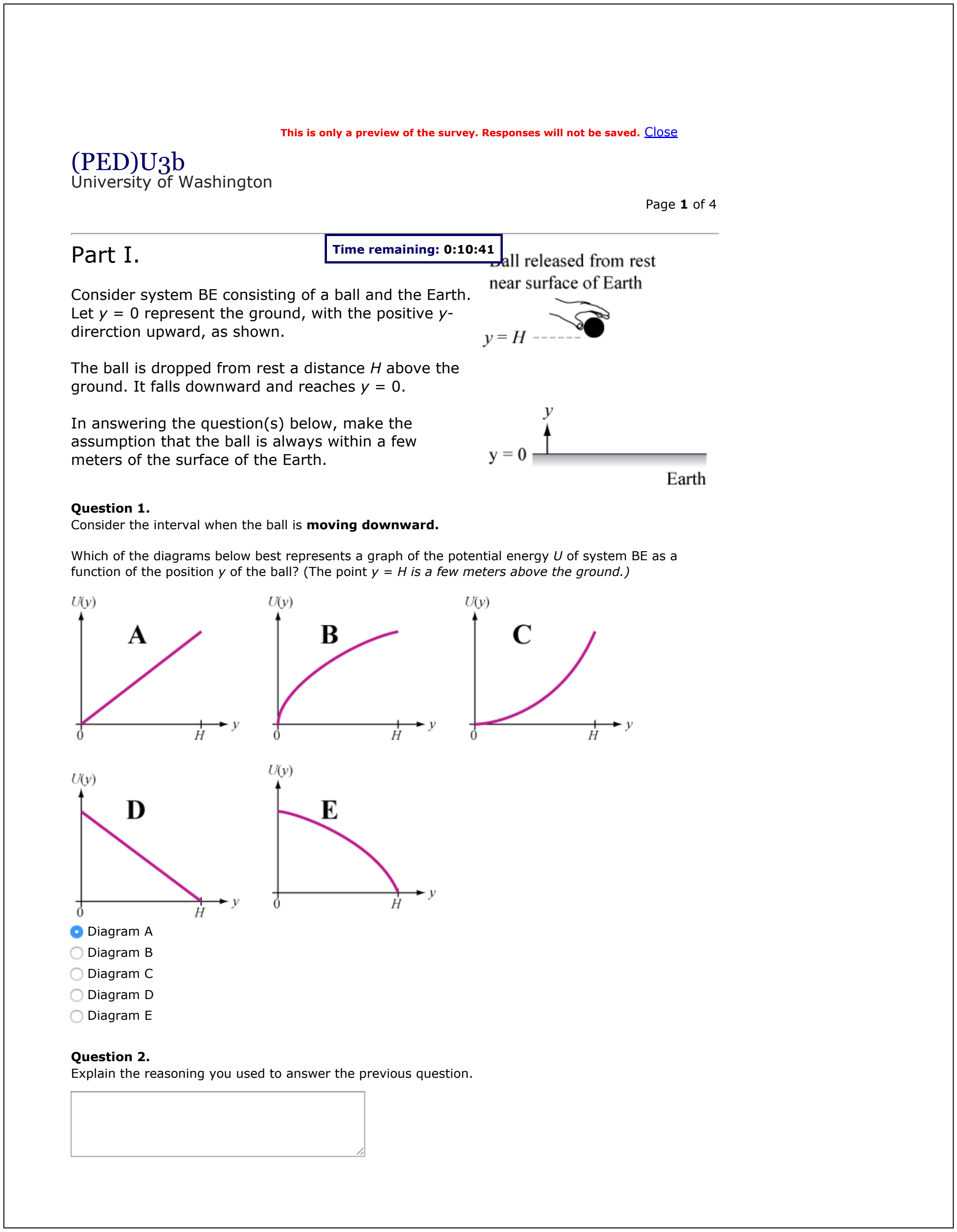

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Part I.

Consider system BE consisting of a ball and the Earth. Let *y* = 0 represent the ground, with the positive *y*-direrction upward, as shown.

The ball is dropped from rest a distance *H* above the ground. It falls downward and reaches *y* = 0.

In answering the question(s) below, make the assumption that the ball is always within a few meters of the surface of the Earth.

Question 1.
Consider the interval when the ball is **moving downward.**

Which of the diagrams below best represents a graph of the potential energy *U* of system BE as a function of the position *y* of the ball? (The point *y* = *H is a few meters above the ground.)*

- Diagram A
- Diagram B
- Diagram C
- Diagram D
- Diagram E

Question 2.
Explain the reasoning you used to answer the previous question.

Figure A.20: Catalyst pretest version (PED)U3B administered in autumn 2014 to PHYS 121A. (Four pages.)

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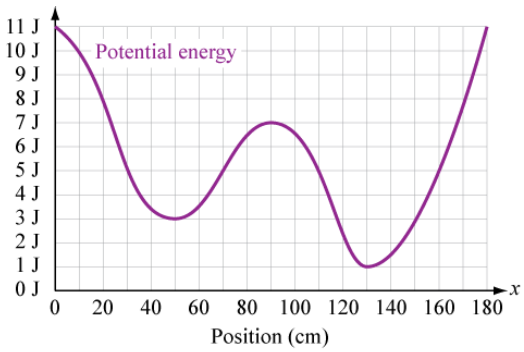
Part II.

A particle is confined to a one-dimensional system. The solid curve below shows the potential energy *U* of the system as a function of the position *x* of the particle.

The only forms of energy in the system are potential energy and translational kinetic energy of the particle. Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.



Question 3.
Suppose the particle were released from rest at x = 30 cm.

What is the *maximum kinetic energy* in joules that the particle attains after being released?

2 J

Question 4.
Explain the reasoning you used to answer the previous question.

Next >>

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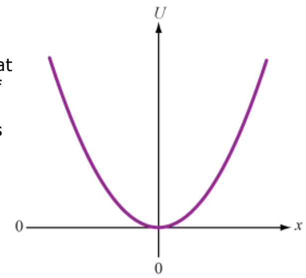
Part III.

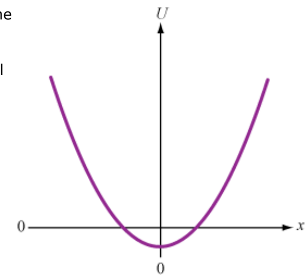
Consider a spring that obeys Hooke's law. (Recall that Hooke's law is $F=-kx$, where x is the displacement of the spring from equilibrium.) The diagram at right represents elastic potential energy U of the spring as a function of x.



Question 5.

Consider now the diagram at right. This graph is identical to the one above except that it has been vertically "shifted."

Could this new diagram also be used to represent the potential energy of the same spring?



- (●) Yes, this new graph could be used to represent the potential energy of the spring instead of the original graph above.
- ( ) No, this new graph could NOT be used to represent the potential energy of the spring instead of the original graph above.

Question 6.
Explain the reasoning you used to answer the previous question.

Next >>

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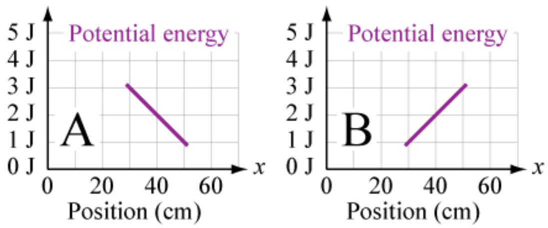
Part IV.

Each graph below represents the potential energy of a one dimensional system. The horizontal axis corresponds to the position of a particle in that system. The only forms of energy in each system are potential energy and translational kinetic energy of the particle. Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.



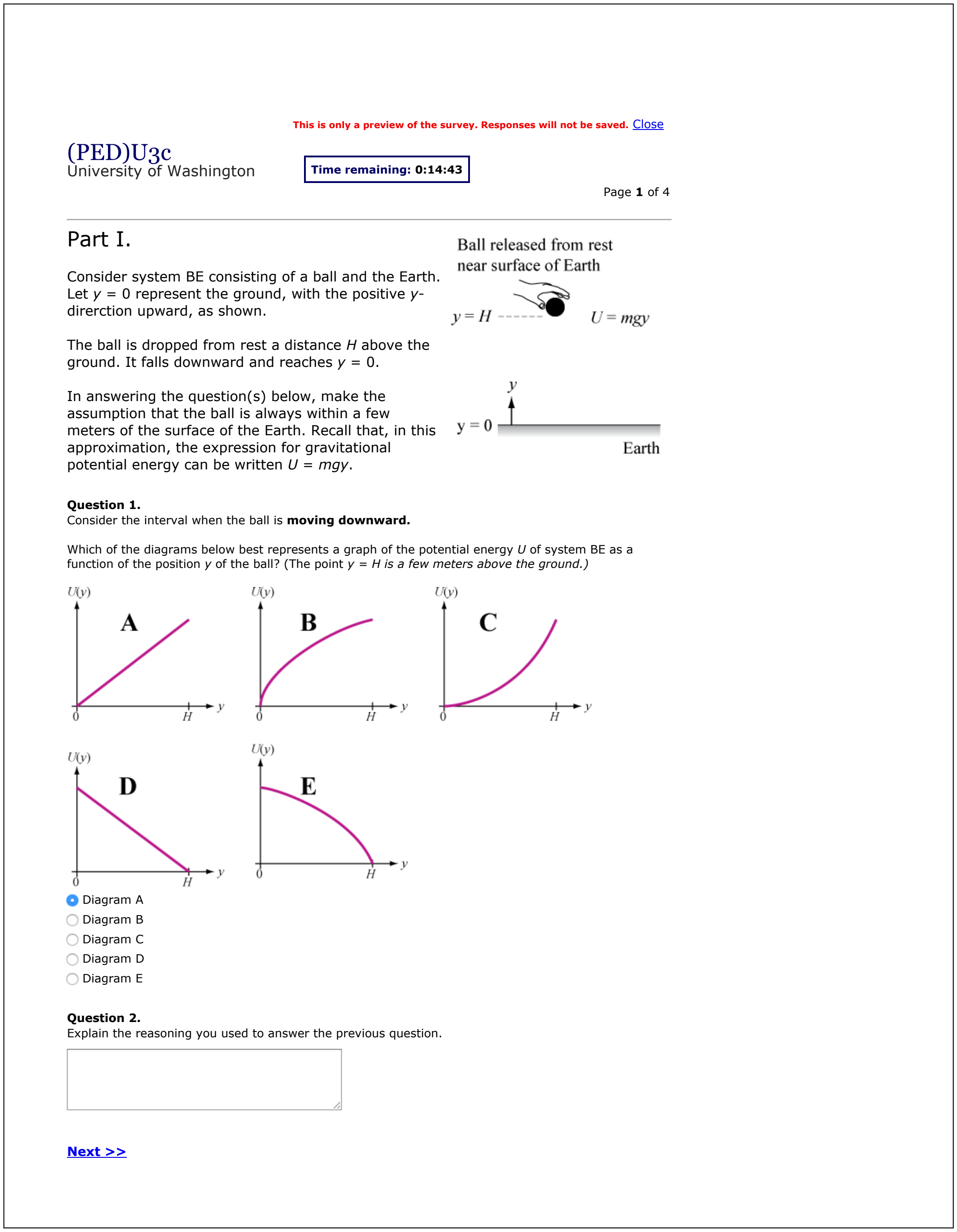
Question 7.
For which of the graphs, if either, **could** the particle be **speeding up** when it is located at ***x* = 40 cm?**

- ○ Neither graph
- ○ Only graph A
- ○ Only graph B
- ● Both graphs A and B

Question 8.
Explain the reasoning you used to answer the previous question.

Submit responses

Questions or Comments?



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Time remaining: 0:14:43

Page **1** of 4

Part I.

Consider system BE consisting of a ball and the Earth. Let $y = 0$ represent the ground, with the positive y-direrction upward, as shown.

The ball is dropped from rest a distance H above the ground. It falls downward and reaches $y = 0$.

In answering the question(s) below, make the assumption that the ball is always within a few meters of the surface of the Earth. Recall that, in this approximation, the expression for gravitational potential energy can be written $U = mgy$.

Question 1.
Consider the interval when the ball is **moving downward.**

Which of the diagrams below best represents a graph of the potential energy U of system BE as a function of the position y of the ball? (The point *y = H is a few meters above the ground.)*

- (●) Diagram A
- ( ) Diagram B
- ( ) Diagram C
- ( ) Diagram D
- ( ) Diagram E

Question 2.
Explain the reasoning you used to answer the previous question.

Next >>

Figure A.21: Catalyst pretest version (PED)U3c administered in autumn 2014 to Phys 121C. (Four pages.)

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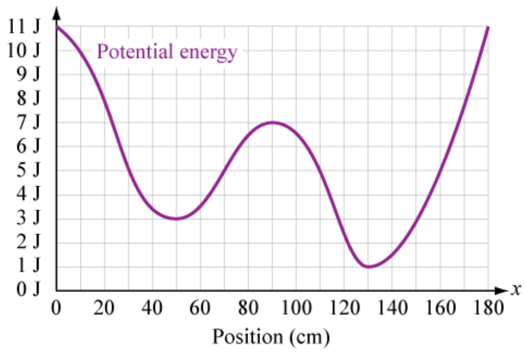
Part II.

A particle is confined to a one-dimensional system. The solid curve below shows the potential energy *U* of the system as a function of the position *x* of the particle.

The only forms of energy in the system are potential energy and translational kinetic energy of the particle. Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.



Question 3.
Suppose the particle were released from rest at *x* = 30 cm.

What is the maximum *x*-coordinate that the particle reaches (i.e., how far does the particle move in the rightward direction on the graph)?

70 cm

Question 4.
Explain the reasoning you used to answer the previous question.

Next >>

Questions or Comments?

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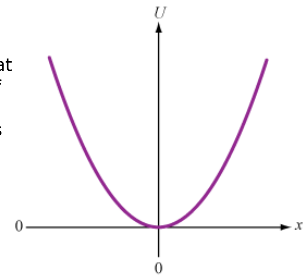
Part III.

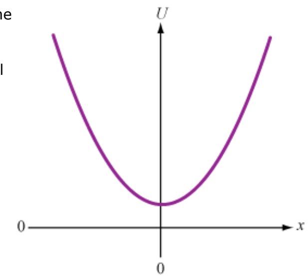
Consider a spring that obeys Hooke's law. (Recall that Hooke's law is *F=-kx,* where *x* is the displacement of the spring from equilibrium.) The diagram at right represents elastic potential energy *U* of the spring as a function of *x*.



Question 5.

Consider now the diagram at right. This graph is identical to the one above except that it has been vertically "shifted."

Could this new diagram also be used to represent the potential energy of the same spring?



- (selected) Yes, this new graph could be used to represent the potential energy of the spring instead of the original graph above.
- No, this new graph could NOT be used to represent the potential energy of the spring instead of the original graph above.

Question 6.
Explain the reasoning you used to answer the previous question.

Next >>

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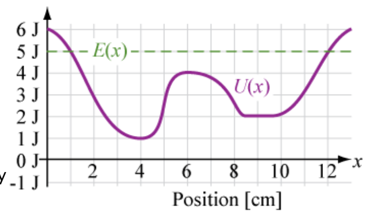
Part IV.

A particle is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle.



The only forms of energy in the system are potential energy and translational kinetic energy of the particle. Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.

Question 7.
Suppose it is known that the particle is located at *x* = 2 cm.

Is it possible that the particle is moving in the **positive *x*-direction** at *x* = 2 cm?

- (•) Yes, it is possible.
- ( ) No, it is not possible.

Question 8.
Explain the reasoning you used to answer the previous question.

Question 9.
Suppose again it is known that the particle is located at *x* = 2 cm.

Is it possible that the particle is moving in the **negative *x*-direction** at *x* = 2 cm?

- (•) Yes, it is possible.
- ( ) No, it is not possible.

Question 10.
Explain the reasoning you used to answer the previous question.

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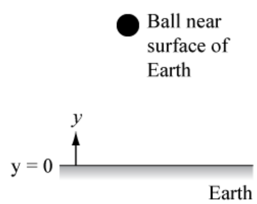
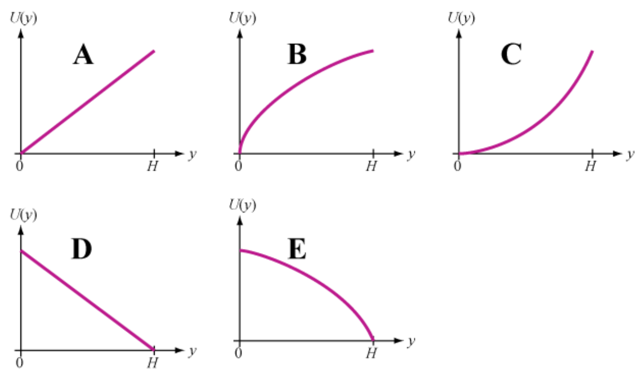
Part I.

Consider system BE consisting of a ball and the Earth. Let $y = 0$ represent the ground, with the positive y-direrction upward, as shown.

In answering the question(s) below, make the assumption that the ball is always within a few meters of the surface of the Earth.



Question 1.
Which of the diagrams below best represents a graph of the potential energy U of system BE as a function of the position y of the ball? (The point $y = H$ *is a few meters above the ground.)*



- (•) Diagram A
- ( ) Diagram B
- ( ) Diagram C
- ( ) Diagram D
- ( ) Diagram E

Question 2.
Explain the reasoning you used to answer the previous question.

Next >>

Questions or Comments?

Figure A.22: Catalyst pretest version (PED)U3d administered in autumn 2014 to Phys 121B (honors) and Phys 121D. (Three pages.)

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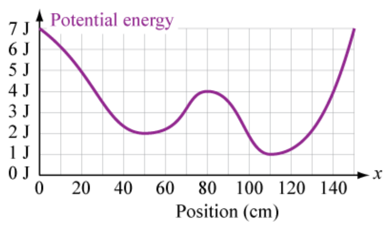
Page **2** of 3

Part II.

A particle is confined to a one-dimensional system. The solid curve below shows the potential energy *U* of the system as a function of the position *x* of the particle.

The only forms of energy in the system are potential energy and translational kinetic energy of the particle. Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.



Question 3.
Suppose the particle were located at *x* = 70 cm and moving.

What is the direction of the acceleration of the particle?

- ○ The acceleration is in the positive direction.
- ● The acceleration is in the negative direction.
- ○ The acceleration is zero.
- ○ There is not enough information to answer.

Question 4.
Explain the reasoning you used to answer the previous question.

Question 5.
Suppose the particle were released from rest at *x* = 20 cm.

What is the *maximum kinetic energy* in joules that the particle attains after being released?

4 J

Question 6.
Explain the reasoning you used to answer the previous question.

Next >>

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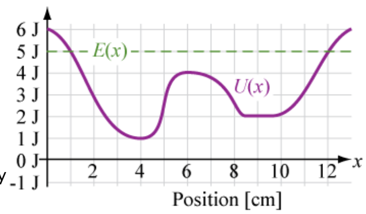
Part III.

A particle is confined to a one-dimensional system. The solid curve at right shows the potential energy *U* of the system as a function of the position *x* of the particle.



The only forms of energy in the system are potential energy and translational kinetic energy of the particle. Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.

Question 7.
Suppose that the particle were located at ***x* = 2 cm.**

Which of the following best describes the **direction of motion** of the particle?

- ○ Positive *x*-direction
- ○ Negative *x*-direction
- ● Either positive or negative *x*-direction
- ○ The particle is not moving

Question 8.
Explain the reasoning you used to answer the previous question.

Question 9.
Suppose again that the particle were located at ***x* = 2 cm.**

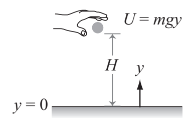
Which of the following best describes how the **speed of the particle** is changing at this position?

- ○ The speed is increasing.
- ○ The speed is decreasing.
- ● The speed is either increasing or decreasing (can't tell which).
- ○ The speed is not changing.

Question 10.
Explain the reasoning you used to answer the previous question.

Name ________________________________ **Student ID** ______________ **Score** __________
last *first*

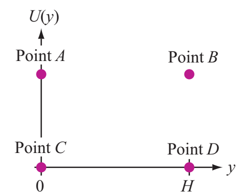
22. A ball is dropped from rest at $y = +H$ (a few meters above the ground). It falls downward and reaches the ground at $y = 0$. Take the positive y-direction to be upward.

    Ball released from rest from $y = H$

    Recall that when the ball is always close to the surface of the Earth, the expression for gravitational potential energy can be written as $U = mgy$, defining the gravitational potential energy of the system to be zero when the ball is at $y = 0$.

    Which of the four points at right best represents the potential energy of the ball-Earth system <u>at the instant the ball is released?</u> (Note that the horizontal axis represents position y.)

    A. Point A
    B. Point B
    C. Point C
    D. Point D

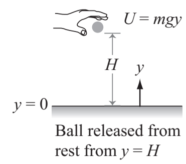
Physics 114A, Winter 2015 Final Exam ME-UWA114A151L-EF(PED).doc

Figure A.23: Multiple-choice question administered in winter 2015 to PHYS 114A.

Name ______________________________ **Student ID** _____________ **Score** __________

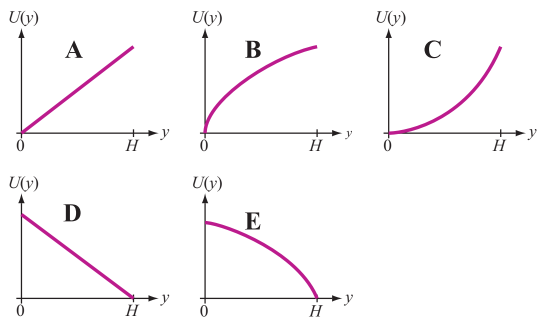
last *first*

22. A ball is dropped from rest at $y = +H$ (a few meters above the ground). It falls downward and reaches the ground at $y = 0$. Take the positive y-direction to be upward.



Recall that when the ball is always close to the surface of the Earth, the expression for gravitational potential energy can be written as $U = mgy$, defining the gravitational potential energy of the system to be zero when the ball is at $y = 0$.

Which of the five graphs below best represents a plot of the potential energy U of the ball-Earth system as a function of the height y of the ball? (Note that the horizontal axis represents position y.)



Physics 114B, Winter 2015 Final Exam ME-UWA114B151L-EF(PED).doc

Figure A.24: Multiple-choice question administered in winter 2015 to Phys 114B.

3. [3 pts] Suppose that an object is moving in one dimension and that its acceleration is in the positive direction. Which of the following best describes how the speed of the particle is changing?
   A. the speed is increasing
   B. the speed is decreasing
   C. the speed is not changing
   D. there is not enough information to answer

Physics 121 A,B Winter 2015 Exam 3 pg. 1

Figure A.25: Multiple-choice question administered in winter 2015 to Phys 121A and 121B on the third midterm exam.

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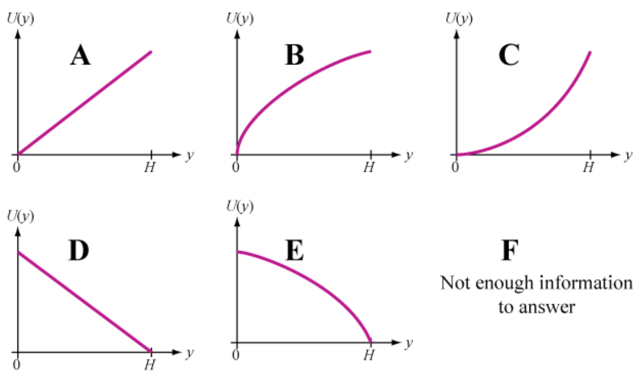
Part I-Prelim.

Consider a system consisting of a ball and the Earth. Let $y = 0$ represent the ground, with the positive y-direction upward.

In answering the question below, assume that the ball is always within a few meters of the surface of the Earth. Recall that, in this approximation, the expression for gravitational potential energy can be written as $U = mgy$.

Question 2.
Which of the graphs below best represents a plot of the potential energy U of the ball-Earth system as a function of the height y of the ball?



Required.
- (•) Graph A
- ( ) Graph B
- ( ) Graph C
- ( ) Graph D
- ( ) Graph E

- ( ) Option F: Not enough information

Question 3.
Explain the reasoning you used to answer the previous question.

Next >>

Figure A.26: Catalyst pretest U4A_VIDEO administered in winter 2015 to PHYS 121A&B. This pretest incorporated logic, so there was not a single path through it that students proceeded. Included are the main questions and some hints that accompanied incorrect answers. (Seven pages.)

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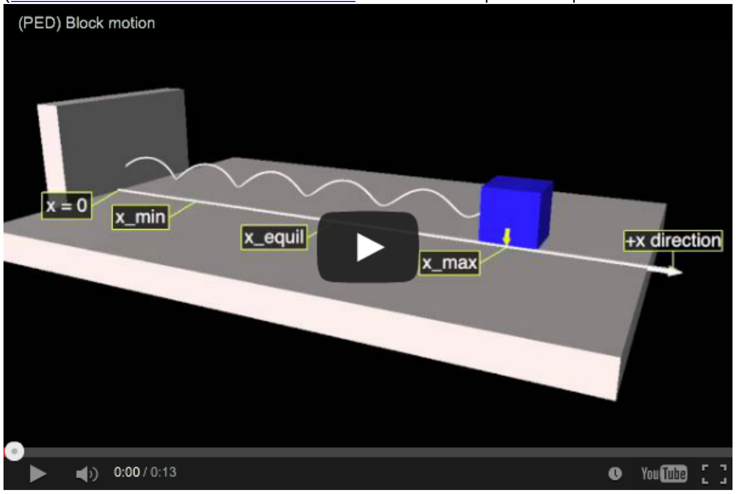
Part I.

A wall is located at $x = 0$. Let the positive x-direction be to the right. One end of a spring is attached to the wall and the other is attached to a block. The spring is initially stretched and the block is held in place at $x = +x_{max}$. All surfaces are frictionless.

The block is then released and undergoes the motion shown in the video below. Click the play button to show the motion.

(Please click here if the video does not load. The video will open in a separate tab or window.)



A summary of the video follows:

- The block begins moving in the negative x-direction and speeds up.
- It attains its maximum speed at the equilibrium position x_{equil}, and begins to slow down.
- The block continues to slow down until it momentarily stops at x_{min}, and then begins moving back in the positive x-direction.
- The block reaches the same maximum speed at x_{equil} as it did before, and eventually returns to

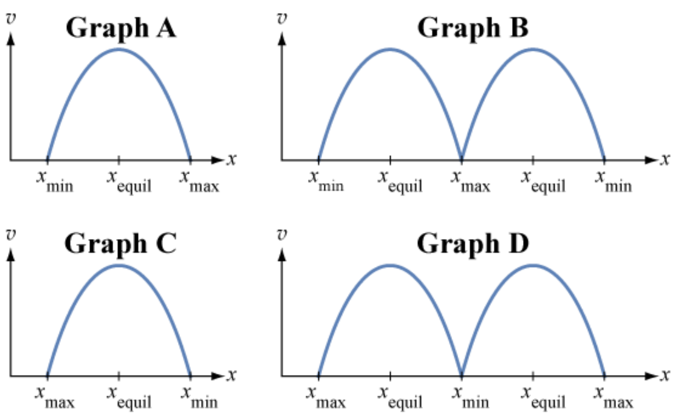
  x_{max} and stops.

For the following questions, consider the **entire interval** that begins when the block is **released from rest at x_{max}** and ends when it **returns to x_{max}**.

Question 4.

Which of the graphs below best represents a plot of **speed** of the block as a function of **position** of the block? (*Note:* The vertical axis represents speed—not velocity—and the horizontal axis represents position—not time.)

Graphs A and C differ only by the locations of the labels for x_{min} and x_{max}, as do graphs B and D.



Required.

- (•) Graph A best represents the speed vs position of the block
- ( ) Graph B best represents the speed vs position of the block
- ( ) Graph C best represents the speed vs position of the block
- ( ) Graph D best represents the speed vs position of the block

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Part II.

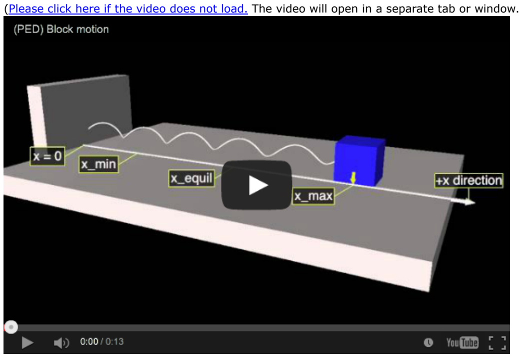
In this part of the pretest, you will be asked about certain features of the graph of speed vs position for the block. You will return to the original question in part III.

For your convenience, an abbreviated description of the block's motion, as well as the video, are given below.

- The block is released from rest at x_{max} and begins moving in the negative x-direction.
- It turns around at x_{min} and eventually returns to x_{max}.
- The block's speed is greatest at x_{equil}, and is the same when the block moves in both directions.

You may replay the video at any time.

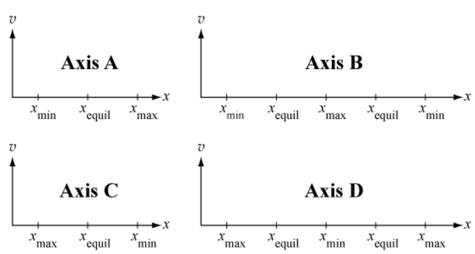
(Please click here if the video does not load. The video will open in a separate tab or window.)



Question 5.

Suppose you were to graph the speed of the block vs its position.

Which of the choices below best represents the correct labels to use on the horizontal axis?



Required.

- (•) Axis A best represents the correct horizontal axis
- ( ) Axis B best represents the correct horizontal axis
- ( ) Axis C best represents the correct horizontal axis
- ( ) Axis D best represents the correct horizontal axis

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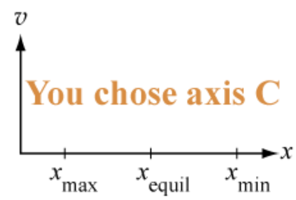
Part II (cont.)

You believe axis C represents the horizontal axis for a plot of speed vs position.



This is incorrect.

Recall that values on horizontal axes typically increase to the *right* of the graph, and values on vertical axes typically increase toward the *top* of the graph.

Consider how x_{min} compares to x_{max}, then decide how these two coordinates should be plotted relative to each other on the horizontal axis.

Click the Next >> button below to reattempt this question.

Next >>

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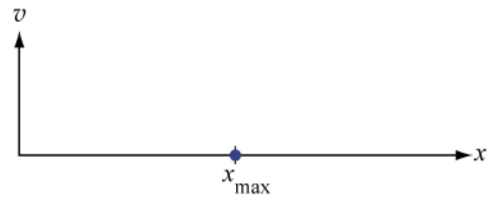
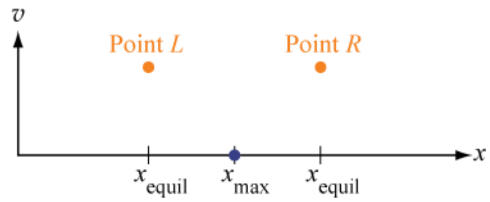
Recall that at the moment of release, the block is located at x_{max} and has a speed of zero. This point in the motion is plotted on the axes at right.



Question 3.

Now consider the first instant when the block is located at x_{equil} and moving in the negative x-direction.

Which choice at right best represents where this point should be plotted on the axes? (The point you have already considered is shown in dark blue.)



Required.

(•) Point *L*

( ) Point *R*

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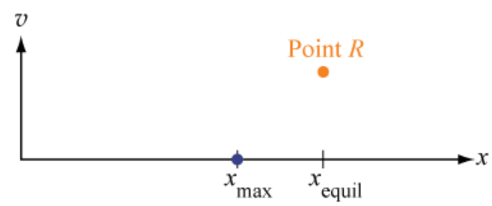
Part II.A (cont.)

You believe point R represents the instant when the block is located at x_{equil}.

This is incorrect.

Recall that values on horizontal axes typically increase to the right of the graph, and values on vertical axes typically increase toward the top of the graph.



Consider how x_{equil} compares to x_{max}, then decide how these two coordinates should be plotted relative to each other on the horizontal axis.

Click the Next >> button below to reattempt this question.

Next >>

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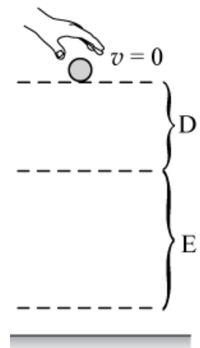
Page **1** of 3

Part I.

A ball is dropped from rest a short distance above the ground. The ball rebounds from the ground and returns to the same height after each bounce. Assume that the ball bounces forever.

The two regions shown, D and E, are of unequal size. Region D is smaller than region E.

After the ball has been bouncing for a very long time, its position is measured at a random time by taking a photograph. Ignore effects due to the size of the ball (*i.e.,* treat the ball as a point mass.)



Ball rebounds and returns to same height

Question 1.

Rank, from greatest to least, the **probabilities** of finding the ball in the two regions D and E. Use the symbols **D** and **E** only once each, along with **>** or **=** as appropriate. If there is not enough information, enter "not enough information."

Not enough information

Question 2.
Explain the reasoning you used to answer the previous question.

Next >>

Figure A.27: Catalyst pretest version U4B_PCQ administered in spring 2015 to PHYS 121A. Part I covers basic probability ideas that are relevant to the tutorial *Probability in classical and quantum mechanics*. (Six pages.)

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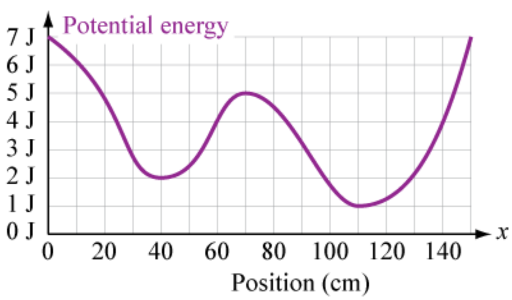
Part II.

A particle is confined to a one-dimensional system. The solid curve shows the potential energy *U* of the system as a function of the position *x* of the particle.

The only forms of energy in the system are potential energy and translational kinetic energy of the particle. Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.



Question 3.
Suppose it is known that the particle is located at ***x* = 70 cm.**

Which of the following best describes the **direction of motion** of the particle?

- ○ The particle is moving in the positive *x*-direction
- ○ The particle is moving in the negative *x*-direction
- ○ The particle is not moving.
- ⦿ There is not enough information to answer.

Question 4.
Explain the reasoning you used to answer the previous question.

Question 5.
Suppose **instead** it is known that the particle is located at ***x* = 60 cm.**

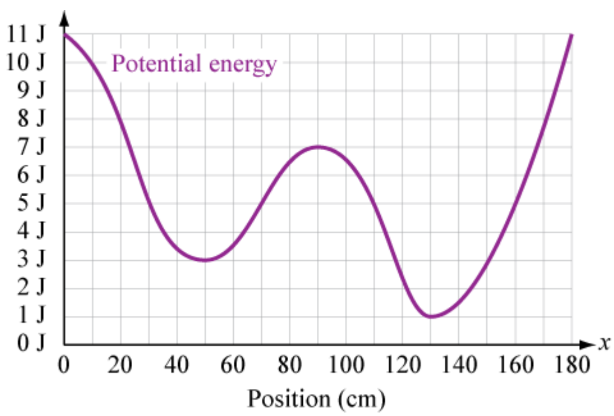
Which of the following best describes the **direction of the force** of the particle?

- ◯ The force is in the *x*-direction
- ◉ The force is in the negative *x*-direction
- ◯ The force is zero.
- ◯ There is not enough information to answer.

Question 6.
Explain the reasoning you used to answer the previous question.

Consider the different graph of potential energy vs. position shown below.

As before, the only forms of energy in the system are potential energy and translational kinetic energy of the particle. Assume that the only force on the particle is that associated with the potential energy $U(x)$ shown.



Suppose that the particle is released **from rest at x = 30 cm.** Its subsequent motion is observed for a very long time.

Question 7.
After being released from rest at x = 30 cm, what is the **smallest *x*-position** (*i.e.,* farthest left on the graph) that the particle reaches?

	0 cm	30 cm	50 cm	70 cm	90 cm
$x_{smallest}$ =	○	⦿	○	○	○

Question 8.
After being released from rest at x = 30 cm, what is the **largest *x*-position** (*i.e.,* farthest right on the graph) that the particle reaches?

	50 cm	70 cm	90 cm	160 cm	180 cm
$x_{largest}$ =	○	⦿	○	○	○

Question 9.

Explain the reasoning you used to answer the previous **two** questions.

Next >>

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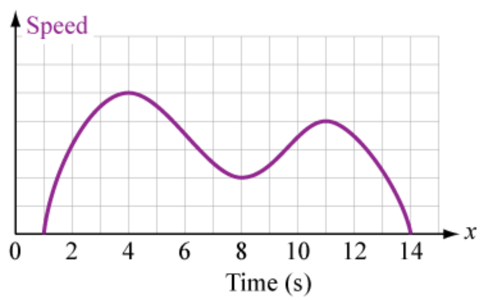
Page **3** of 3

Part III.

A particle is confined to a one-dimensional system. The solid curve below shows the **speed *v*** of the particle as a function of **time *t*.**



Question 10.

Describe in words how the **speed** of the particle is changing at time ***t* = 8 s.** If there is not enough information to answer, state so explicitly.

Question 11.
Explain the reasoning you used to answer the previous question.

A particle moves back and forth in a one-dimensional system with potential energy. The only forms of energy are potential energy and the translational kinetic energy of the particle. Assume that the total energy in the system is constant.

The system is composed of two regions, A and B (not shown).

Suppose it is known that the kinetic energy of the particle in region A is twice that in region B.

Question 12.
Which of the following is a true statement regarding the potential energy of the system in regions A and B?

- ◯ The potential energies in regions A and B are **the same.**
- ◯ The potential energy in region A is **half that** in region B.
- ⊙ The potential energy in region A is **less than** that in region B, but not necessarily half.
- ◯ The potential energy in region A is **twice that** in region B.
- ◯ The potential energy in region A is **more than** that in region B, but not necessarily twice.

Question 13.
Explain the reasoning you used to answer the previous question.

Submit responses

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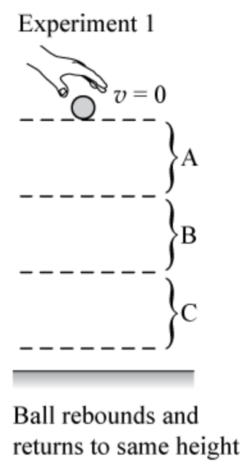
Page **1** of 4

Part I.



In experiment 1, a ball is dropped from rest a short distance above the ground. The ball rebounds from the ground and returns to the same height after each bounce. Assume that the ball bounces forever. The three regions shown, A, B, and C, are of equal size.

After the ball has been bouncing for a very long time, its position is measured at a random time by taking a photograph. Ignore effects due to the size of the ball (*i.e.,* treat the ball as a point mass.)

Question 1.

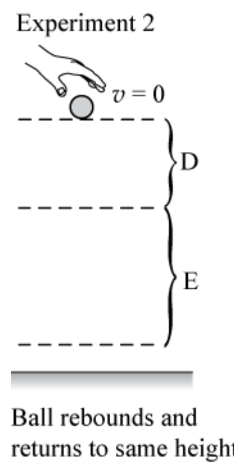
Rank, from greatest to least, the **probabilities** of finding the ball in the three regions A, B, and C. Use the symbols **A**, **B**, and **C** only once each, along with **>** and/or **=** as appropriate. If there is not enough information, enter "not enough information."

Question 2.

Explain the reasoning you used to answer the previous question.

Figure A.28: Catalyst pretest version U4c_PCQ administered in spring 2015 to Phys 121B&C. Part I covers basic probability ideas that are relevant to the tutorial *Probability in classical and quantum mechanics.* (Seven pages.)

Experiment 2 is identical to experiment 1, except that there are now two regions, D and E, of unequal size. Region D is smaller than region E.



Question 3.

Rank, from greatest to least, the **probabilities** of finding the ball in the two regions D and E. Use the symbols **D** and **E** only once each, along with **>** or **=** as appropriate. If there is not enough information, enter "not enough information."

Question 4.
Explain the reasoning you used to answer the previous question.

Next >>

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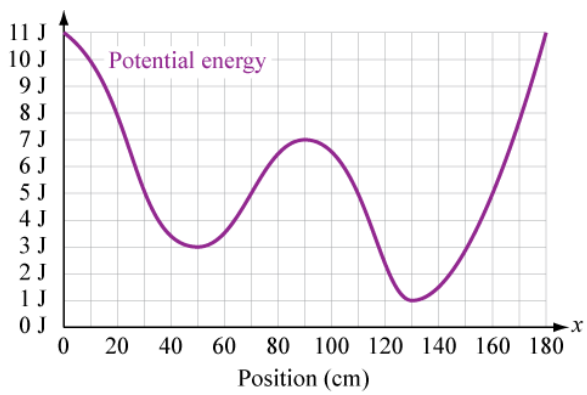
Part II.

A particle is confined to a one-dimensional system. The solid curve shows the potential energy *U* of the system as a function of the position *x* of the particle.

The only forms of energy in the system are potential energy and translational kinetic energy of the particle. Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.



Question 5.
Suppose that the particle is **released from rest** at ***x* = 70 cm.**

Which of the following best describes the **initial motion** of the particle?

- ◯ The particle will initially move in the positive *x*-direction.
- ◯ The particle will initially move in the negative *x*-direction.
- ◯ The particle will remain stationary.
- ◯ There is not enough information to answer.

Question 6.
Explain the reasoning you used to answer the previous question.

Question 7.
Suppose again that the particle is **released from rest** at ***x* = 70 cm.**

What is the **total energy** E_{tot} of the system?

Select one...

Question 8.
Explain the reasoning you used to answer the previous question.

Next >>

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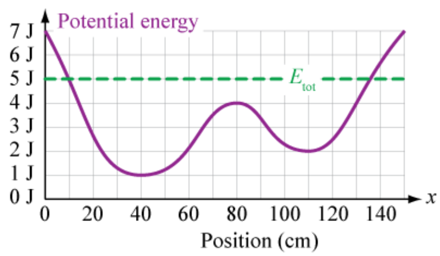
Part III.

A particle is confined to a one-dimensional system. The solid curve below shows the potential energy *U* of the system as a function of the position *x* of the particle. The dashed line represents the total energy E_{tot} of the system.

As before, the only forms of energy in the system are potential energy and translational kinetic energy of the particle. Assume that the only force on the particle is that associated with the potential energy *U*(*x*) shown.



Question 9.

Suppose it is known that the particle is located at ***x* = 80 cm.**

Describe in words how the **speed** of the particle is changing at that location. If there is not enough information to answer, state so explicitly.

Question 10.
Explain the reasoning you used to answer the previous question.

Next >>

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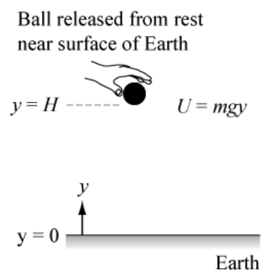
Part IV.



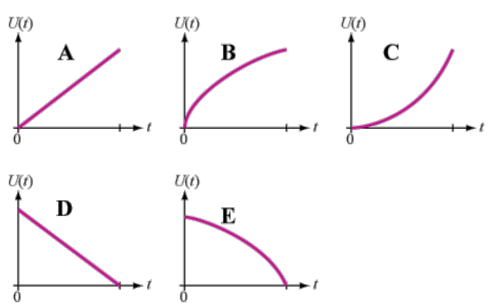
Consider system BE consisting of a ball and the Earth. Let $y = 0$ represent the ground, with the positive y-direction upward, as shown.

The ball is dropped from rest a distance H above the ground. It falls downward and reaches $y = 0$.

In answering the question(s) below, make the assumption that the ball is always within a few meters of the surface of the Earth. Recall that, in this approximation, the expression for gravitational potential energy can be written as $U = mgy$.

Question 11.
Consider the interval when the ball is **moving downward.**

Which of the diagrams below best represents a graph of the **potential energy *U*** of system BE as a function of **time *t*?**

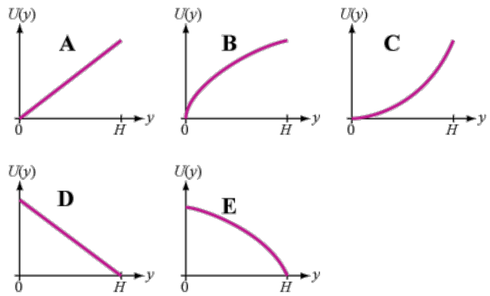


- ◯ Diagram A
- ◯ Diagram B
- ◯ Diagram C
- ◯ Diagram D
- ◯ Diagram E

Question 12.
Explain the reasoning you used to answer the previous question.

Question 13.
Consider again the interval when the ball is **moving downward.**

Which of the diagrams below best represents a graph of the **potential energy *U*** of system BE as a function of the **position *y*** of the ball?



- ◯ Diagram A
- ◯ Diagram B
- ◯ Diagram C
- ◯ Diagram D
- ◯ Diagram E

Question 14.
Explain the reasoning you used to answer the previous question.

Submit responses

A.2 Curriculum related to Potential energy diagrams

In this section we provide curricular materials related to the tutorial *Potential energy diagrams.*

Contents of section

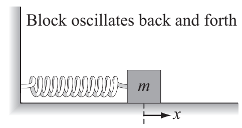
A.2.1 In-class worksheets for Potential energy diagrams

In this section we provide the in-class worksheets for the tutorial *Potential energy diagrams.*

POTENTIAL ENERGY DIAGRAMS

Mech 1

I. Graphical representations of potential energy



A block of mass m is attached to an ideal spring, as shown. It oscillates back and forth on a level, frictionless surface. The x-coordinate denotes the location of the center of mass of the block. The spring is at its equilibrium position when $x = 0$. The block can oscillate between $x = -x_{max}$ and $x = +x_{max}$.

A. Consider a single period of the block's motion (*i.e.*, from $-x_{max}$ to x_{max} and back again to $-x_{max}$).

Suppose you know only the position of the block. Based only on that information:

- can you determine its direction of motion? Explain or give an example.

- can you determine whether it is speeding up or slowing down? Explain or give an example.

B. Consider the system consisting of the block alone. Is the total energy for this system constant? If so, explain why. If not, which other object(s) must be included in the system in order for the energy of the new system to be constant? Explain.

C. Let S represent the system for which the energy is conserved.

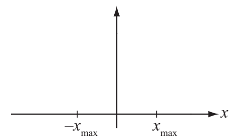
Write an expression for the potential energy U of system S as a function of x.

Figure A.29: In-class tutorial worksheet administered in spring 2013 to all sections of PHYS 121. (Five pages.)

Mech 2 ***Potential energy diagrams***

D. On the axes at right, sketch a curve of the potential energy U for system S as a function of x between $-x_{max}$ and x_{max}. Label the curve $U(x)$.

1. On the same axes, draw a dashed curve or line to represent the total energy. Label the curve $E(x)$.
2. Based on your graph, how can you tell that the energy in the system is constant? Explain.



E. Recall the definition $E = K + U$, which relates the two forms of mechanical energy to the total energy in a simple system.

1. Based on this definition, what *geometric* feature of your graph above represents the kinetic energy at a particular position? Explain.
2. Indicate on your graph the kinetic energy when the particle is at $x = x_{max}/2$.
3. Where is the block moving (i) the fastest and (ii) the slowest? Answer based solely on your graph. Explain.

F. Two students attempt to use their graph of potential energy versus position to describe the motion of the block at $x = x_{max}/2$.

Student 1: "At $x = x_{max}/2$ the graph of potential energy is increasing. This means the kinetic energy is decreasing since the energy is constant. A decrease in kinetic energy means the block is slowing down at this position."

Student 2: "I was looking at the slope of the graph. Slope corresponds to velocity. Since the slope of the graph is getting steeper near $x = x_{max}/2$, the block must be speeding up."

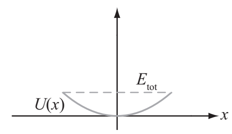
With which student, if either, do you agree? For the student(s) with whom you disagree, explain the error in reasoning.

Is your response to the student statements above consistent with your answers to part A? If not, resolve any inconsistencies.

➪ Discuss your answers with a tutorial instructor.

II. Allowed and forbidden regions

A. The solid curve at right represents the potential energy for the spring-block system of the previous section. The dashed line represents the total energy.



Suppose that the spring-block system were given more energy so that the block is able to reach positions $-2x_{max} < x < 2x_{max}$.

1. On the graph above, sketch the new curves representing (i) potential energy and (ii) total energy.

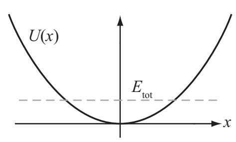
2. Describe in words how the curve representing potential energy changed, if at all. Explain.

   Did the portion of the curve between $-x_{max}$ and x_{max} change?

3. Describe in words how the line representing total energy changed, if at all.

It is commonplace on graphs of potential energy versus position to draw the curve for all positions, regardless of whether or not the object of interest can actually reach those positions. In addition, the line representing total energy is also extended to include all positions.

B. The graph at right represents the potential and total energy of the original block-spring system using the conventions described above.



On the graph, (i) label the positions $-x_{max}$ and x_{max}, and (ii) indicate the region in which the block can be located.

C. Generalize your results thus far. What mathematical relation must exist between total energy E_{tot} and potential energy $U(x)$ for regions in which objects can actually be located?

Graphs such as the one in part B above are called *potential energy diagrams*. Regions in which the object of interest can be located are called *allowed regions*. Regions in which the object cannot be located are called *disallowed* or *forbidden regions*. Locations at which objects stop and change directions are called *turn-around points*.

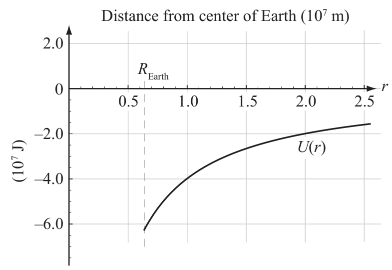
Mech 4 *Potential energy diagrams*

III. Planetary gravitational potential energy

Recall that the gravitational potential energy for a system of two objects with masses m_A and m_B, with their centers of mass separated by a distance r, is $U(r) = -Gm_Am_B/r$. This expression is valid when the potential energy at infinite distance is defined to be zero.

A. The graph at right represents the gravitational potential energy for a system consisting of a 1 kg projectile and the Earth. The position R_{Earth} indicates the surface of the Earth.

Suppose that the projectile is launched from the surface of the Earth with kinetic energy of 2.0×10^7 joules. (Assume that the projectile only moves in the radial direction r.)



Draw a line segment at $x = R_{\text{Earth}}$ indicating the kinetic energy at this position.

1. Draw and label a line indicating the total energy of the Earth-projectile system.

2. Approximately how far from the center of the Earth does the projectile reach before turning around? Explain.

3. Indicate the allowable and the forbidden region(s) for positions $r > R_{\text{Earth}}$. Explain.

4. Is the total energy of the system *positive*, *negative*, or *zero?*

   Recall the relation between E_{tot} and $U(x)$ for allowable regions from part II.C of section II.

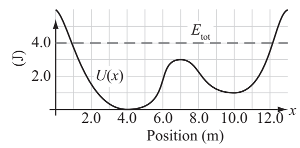
   Does the allowable region in this situation also satisfy the relation? If not, resolve the inconsistency.

B. Suppose instead that the projectile were given a kinetic energy of 7.0×10^7 joules.

Discuss with your partners (i) the location of the allowable region and (ii) the location of the turn-around point, if any. Write down your thoughts after your discussion.

➪ Discuss your answers with a tutorial instructor.

IV. Relating force to potential energy diagrams



The potential energy diagram at right represents an unknown one-dimensional system. The coordinate x represents the position of a particle within that system.

A. Suppose that the particle were at $x = 8.0$ m and moving in the positive x–direction.

1. At this location, is the particle *speeding up*, *slowing down*, or *moving at constant speed?* Explain.

2. At this location, in what direction is the acceleration of the particle? Explain.

3. At this location, in what direction is the net force on the particle? Explain.

B. Repeat part A above assuming instead that the particle were at $x = 8.0$ m moving in the negative x–direction.

Did the direction of the acceleration and force depend on the direction of motion?

C. What is the direction of (i) the acceleration and (ii) the force at $x = 6.0$ m? Explain.

D. Generalize your results thus far. What geometric feature of potential energy diagrams can be used to determine the direction of the force on a particle? Be sure to include any relevant signs.

Recall from lecture that when a conservative force $\vec{F}$ acts on an object, one can define a change in potential energy $\Delta U_{a \to b} = -\int_a^b \vec{F} \cdot d\vec{s}$. In one dimension (*e.g.*, the x-direction), the relation between force and potential energy can also be described by $F = -dU/dx$.

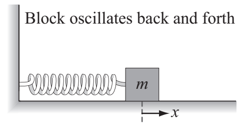
E. Is your response to part D consistent with the relation $F = -dU/dx$? If not, resolve any inconsistencies.

F. Determine the magnitude and direction of the force on the particle at $x = 8.0$ m.

POTENTIAL ENERGY DIAGRAMS

Mech
1

I. Graphical representations of potential energy



A block of mass *m* is attached to an ideal spring of spring constant *k*, as shown. It oscillates back and forth on a level, frictionless surface. The *x*-coordinate denotes the location of the center of mass of the block. The spring is at its equilibrium position when $x = 0$. The block oscillates between $x = -x_{max}$ and $x = +x_{max}$.

A. Consider one period of the block's motion (*i.e.*, from $-x_{max}$ to x_{max} and back again to $-x_{max}$).

Suppose you know only the position of the block (*e.g.*, $x = x_{max}/2$). Based only on that information:

- can you determine its direction of motion? Explain or give an example.

- can you determine whether it is speeding up or slowing down? Explain or give an example.

B. Consider the system consisting of the block alone.

Is the total energy for this system constant? If so, explain why. If not, which other object(s) must be included in the system in order for the energy of the new system to be constant? Explain.

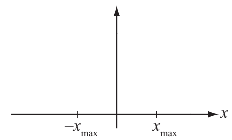
C. Let *S* represent the system for which the energy is constant.

Write an expression for the potential energy *U* of system *S* as a function of *x*.

Figure A.30: In-class tutorial worksheet administered in summer 2013 to Phys 121A. (Five pages.)

Mech 2 ***Potential energy diagrams***

D. On the axes at right, sketch a curve of the potential energy U for system S as a function of x between $-x_{max}$ and x_{max}. Label the curve $U(x)$.



1. On the same axes, draw a dashed curve or line to represent the total energy. Label the curve $E(x)$.
2. Based on your graph, how can you tell that the energy in the system is constant? Explain.

E. Recall the definition $E = K + U$, which relates the two forms of mechanical energy to the total energy in a simple system. Note that this relation can also be written $K = E - U$.

1. Based on this definition, what *graphical feature* of your graph above represents the kinetic energy at a particular position? Explain.
2. Indicate on your graph the kinetic energy when the particle is at $x = x_{max}/2$.
3. Where is the block moving (i) the fastest and (ii) the slowest? Answer based solely on your graph. Explain.

F. Two students attempt to use their graph of potential energy versus position to describe the motion of the block at $x = x_{max}/2$.

Student 1: "At $x = x_{max}/2$ the graph of potential energy is increasing. This means the kinetic energy is decreasing since the energy is constant. A decrease in kinetic energy means the block is slowing down at this position."

Student 2: "I was looking at the slope of the graph. Slope corresponds to velocity. Since the slope of the graph is getting steeper near $x = x_{max}/2$, the block must be speeding up."

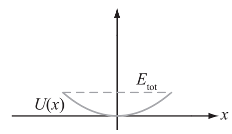
With which student, if either, do you agree? For the student(s) with whom you disagree, explain the error in reasoning.

Is your response to the student statements above consistent with your answers to part A? If not, resolve any inconsistencies.

➪ Discuss your answers with a tutorial instructor.

II. Allowed and forbidden regions

A. The solid curve at right represents the potential energy for the spring-block system of the previous section. The dashed line represents the total energy.



Suppose that the spring-block system were given more energy so that the block is able to reach positions $-2x_{max} < x < 2x_{max}$.

1. On the graph above, sketch the new curves representing (i) potential energy and (ii) total energy.

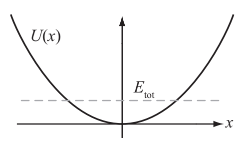
2. Describe in words how the curve representing potential energy changed, if at all.

   Did the portion of the curve between $-x_{max}$ and x_{max} change?

3. Describe in words how the line representing total energy changed, if at all.

It is commonplace on graphs of potential energy versus position to draw the curve for all positions, regardless of whether or not the object of interest can actually reach those positions. In addition, the line representing total energy is also extended to include all positions.

B. The graph at right represents the potential and total energy of the *original* block-spring system using the conventions described above.



On the graph, (i) label the positions $-x_{max}$ and x_{max}, and (ii) indicate the region in which the block can be located.

C. Generalize your results thus far. What mathematical relation must exist between total energy E_{tot} and potential energy $U(x)$ for regions in which objects can actually be located?

Graphs such as the one in part B above are called *potential energy diagrams*. Regions in which the object of interest can be located are called *allowed regions*. Regions in which the object cannot be located are called *disallowed* or *forbidden regions*. Locations at which objects stop and change directions are called *turn-around points*.

D. Label the turn-around points on the graph from part B.

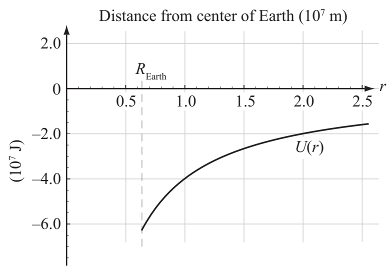
Mech 4 *Potential energy diagrams*

III. Planetary gravitational potential energy

Recall that the gravitational potential energy for a system of two objects with masses m_A and m_B, with their centers of mass separated by a distance r, is $U(r) = -Gm_Am_B/r$. This expression is valid when the potential energy at infinite distance is defined to be zero.

A. The graph at right represents the gravitational potential energy for a system consisting of a 1 kg projectile and the Earth. The position R_{Earth} indicates the surface of the Earth.

Suppose that the projectile is launched from the surface of the Earth with kinetic energy of 2.0×10^7 joules. (Assume that the projectile only moves in the radial direction r.)



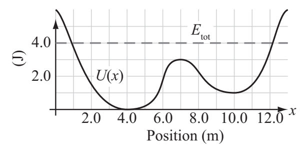
1. Draw a line segment at $x = R_{Earth}$ indicating the kinetic energy at this position. (*Hint:* Recall your responses in parts E.1 and E.2 of section I.
2. Draw and label a line indicating the total energy of the Earth-projectile system.
3. Approximately how far from the center of the Earth does the projectile reach before turning around? Explain.
4. Indicate the allowed and the forbidden region(s) for positions $r > R_{Earth}$. Explain.
5. Is the total energy of the system *positive*, *negative*, or *zero?*

B. Suppose instead that the projectile were given a kinetic energy of 7.0×10^7 joules.

Discuss with your partners (i) the location of the allowable region and (ii) the location of the turn-around point, if any. Write down your thoughts after your discussion.

➪ Discuss your answers with a tutorial instructor.

IV. Relating force to potential energy diagrams



The potential energy diagram at right represents an unknown one-dimensional system. The coordinate x represents the position of a particle within that system.

A. Suppose that the particle were at $x = 8.0$ m and moving in the positive x–direction.

1. At this location, is the particle *speeding up, slowing down,* or *moving at constant speed?* Explain.

2. At this location, in what direction is the acceleration of the particle? Explain.

3. At this location, in what direction is the net force on the particle? Explain.

B. Repeat part A above assuming instead that the particle were at $x = 8.0$ m but moving in the negative x–direction.

Did the direction of the acceleration and force depend on the direction of motion?

C. What is the direction of the force at $x = 6.0$ m? Explain.

D. Generalize your results thus far. What graphical feature of potential energy diagrams can be used to determine the direction of the force on a particle? Be sure to include any relevant signs.

Recall from lecture that when a conservative force $\vec{F}$ acts on an object, one can define a change in potential energy $\Delta U_{a \to b} = -\int_a^b \vec{F} \cdot d\vec{s}$. In one dimension (*e.g.*, the x-direction), the relation between force and potential energy can also be described by $\vec{F} = -(dU/dx)\hat{x}$.

E. Is your response to part D consistent with the relation above? If not, resolve any inconsistencies.

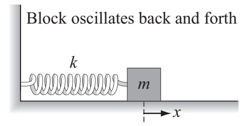
F. Determine the magnitude and direction of the force on the particle at $x = 8.0$ m.

POTENTIAL ENERGY DIAGRAMS

Mech 1

I. Graphical representations of potential energy

A block of mass m is attached to an ideal spring of spring constant k, as shown. It oscillates back and forth on a level, frictionless surface. The x-coordinate denotes the location of the center of mass of the block. The spring is at its equilibrium position when $x = 0$. The block oscillates between $x = -x_{max}$ and $x = +x_{max}$.



A. Consider one period of the block's motion (*i.e.*, from $-x_{max}$ to x_{max} and back again to $-x_{max}$).

Suppose you know only the position of the block (*e.g.*, $x = x_{max}/2$). Based only on that information:

- can you determine its direction of motion? Explain or give an example.

- can you determine whether it is speeding up or slowing down? Explain or give an example.

B. Consider the system consisting of the block alone.

Is the total energy for this system constant? If so, explain why. If not, which other object(s) must be included in the system in order for the energy of the new system to be constant? Explain.

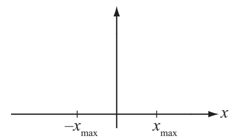
C. Let S represent the system for which the energy is constant.

Write an expression for the potential energy U of system S as a function of the position x of the block. Your expression should involve the spring constant k.

Figure A.31: In-class tutorial worksheet administered in autumn 2013 and winter 2014 to all sections of Phys 121, including honors. (Five pages.)

Mech 2 ***Potential energy diagrams***

D. On the axes at right, sketch a curve of the potential energy U for system S as a function of x between $-x_{max}$ and x_{max}. Label the curve $U(x)$. Such graphs are called *potential energy diagrams*.



1. On the same axes, draw a dashed curve or line to represent the total energy. Label the curve $E(x)$.
2. What feature of your diagram indicates that the total energy in the system is constant?

E. Write down the relationship between total, kinetic, and potential energy for a system.

1. Based on this relationship, draw on your diagram a *single vertical line segment* whose length represents the kinetic energy at $x = x_{max}/2$. (*Hint:* The bottom of your line segment should coincide with the potential energy curve at $x = x_{max}/2$.)
2. Where is the block moving (i) the fastest and (ii) the slowest? Draw and label line segments on your graph to indicate the kinetic energies at these positions.

F. Two students attempt to use their potential energy diagram to describe the motion of the block at $x = x_{max}/2$.

Student 1: "At $x = x_{max}/2$ the graph of potential energy is increasing. This means the kinetic energy is decreasing, so the block must be slowing down at this position."

Student 2: "I was looking at the slope of the graph. Slope corresponds to velocity. Since the slope of the graph is getting steeper near $x = x_{max}/2$, the block must be speeding up."

Neither student is correct. Explain the errors in each of their reasoning.

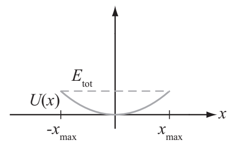
Is your response to student 1 consistent with your answers to part A?

G. Generalize your results from this section: Is it possible to determine (i) the direction of motion of an object or (ii) whether the object is speeding up or slowing down, based only on the object's location on a graph of potential energy *vs*. position?

➪ Discuss your answers with a tutorial instructor.

II. Allowed and forbidden regions

A. The potential energy diagram and total energy from the previous section are reproduced at right.



Suppose that the same spring-block system were given more energy so that the block oscillates between $-2x_{max} < x < 2x_{max}$.

1. On the graph above, sketch new curves that would represent (i) potential energy and (ii) total energy.

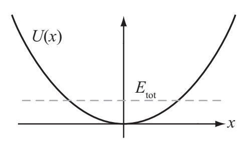
2. Describe in words how the curve representing potential energy would change, if at all.

   Would the portion of the curve between $-x_{max}$ and x_{max} change?

3. Describe in words how the line representing total energy would change, if at all.

It is common for potential energy diagrams to show the curves for both $U(x)$ and total energy for the entire range of x-values on the diagram.

B. The potential energy diagram at right corresponds to the *original* block-spring system using these conventions.



Does the block reach all of the positions x for which U is drawn?

Regions in which the object of interest can be located are called *allowed regions*. Regions in which the object cannot be located are called *disallowed* or *forbidden regions*.

C. Label the allowed and forbidden regions on the potential energy diagram above.

What mathematical relation must exist between total energy E_{tot} and potential energy $U(x)$ for the *allowed regions?* Express your answer as an inequality.

Locations at which objects stop and reverse direction of motion are called *turn-around points*.

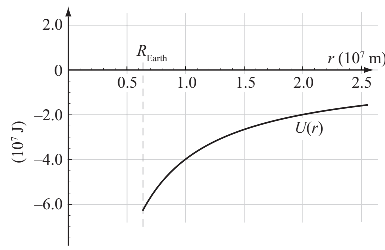
D. Explain how to locate turn-around points on a potential energy diagram.

Label the turn-around points on the potential energy diagram in part B.

Mech 4 ***Potential energy diagrams***

III. Planetary gravitational potential energy

Recall that the gravitational potential energy for a system of two objects with masses m_A and m_B, with their centers of mass separated by a distance r, is $U(r) = -Gm_Am_B/r$. This expression is valid when the potential energy at infinite distance is defined to be zero.



A. The graph at right represents the gravitational potential energy for a system consisting of a 1.0 kg projectile and the Earth. Consider only $r > R_{\text{Earth}}$, where r represents the distance from the center of the Earth and R_{Earth} is the radius of the Earth.

Suppose that the projectile is launched from the surface of the Earth with kinetic energy of 2.0×10^7 joules. (Assume that the projectile only moves in the radial direction r.)

1. Draw a line segment at $r = R_{\text{Earth}}$ that represents the *kinetic energy* at this position. (*Hint:* Recall your work in part I.E. How long should the line segment be?)
2. Draw and label a line indicating the *total energy* of the Earth-projectile system.
3. Use the potential energy diagram to determine how far from the center of the Earth the projectile reaches before turning around. Explain.
4. Indicate the allowed and the forbidden region(s) for positions $r > R_{\text{Earth}}$.
5. Is the total energy of the system *positive*, *negative*, or *zero?*

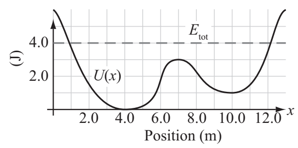
B. Suppose instead that the projectile were given a kinetic energy of 7.0×10^7 joules at the surface of the Earth.

Discuss with your partners (i) the location of the allowable region and (ii) the location of the turn-around point, if any. Write down your thoughts after your discussion.

➪ Discuss your answers with a tutorial instructor.

IV. Relating force to potential energy diagrams

The potential energy diagram at right represents an unknown one-dimensional system. The coordinate x represents the position of a particle within that system. The only force on the particle is that associated with the potential energy.



A. Fill in the table below for an instant when the particle is located at $x = 6.0$ cm. Do not fill in the last column.

Did the directions of the force or acceleration depend on the direction of motion?

B. Complete the table for an instant when the particle is located at $x = 8.0$ cm.

	$x = 6.0$ cm		$x = 8.0$ cm
Direction of motion	Positive x-direction	Negative x-direction	
Speeding up or *slowing down*			
Direction of acceleration (+ or –)			
Direction of force (+ or –)			
Sign of dU/dx			

C. Generalize your results above. What graphical feature of potential energy diagrams can be used to determine the *direction of the force* on a particle *(e.g.,* sign of potential, slope of potential, *etc.)?* Include any relevant signs.

In one dimension (*e.g.,* the x-direction), a conservative force $\vec{F}$ is related to the associated potential energy U by $\vec{F} = -(dU/dx)\hat{x}$. This relation can be used to determine the change in potential energy: $\Delta U_{a \to b} = -\int_a^b \vec{F} \cdot d\vec{x}$.

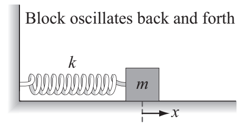
D. Is your response to part C consistent with the relation above? If not, resolve any inconsistencies.

E. Determine the force on the particle at $x = 7.0$ m. Explain.

POTENTIAL ENERGY DIAGRAMS

Mech 1

I. Graphical representations of potential energy



A block of mass m is attached to an ideal spring of spring constant k, as shown. It oscillates back and forth on a level, frictionless surface. The x-coordinate denotes the location of the center of mass of the block. The spring is at its equilibrium position when $x = 0$. The block oscillates between $x = -x_{max}$ and $x = +x_{max}$.

A. Consider one period of the block's motion (*i.e.*, from $-x_{max}$ to x_{max} and back again to $-x_{max}$).

Suppose you know only the position of the block (*e.g.*, $x = x_{max}/2$). Based only on that information:

- can you determine its direction of motion? Explain or give an example.

- can you determine whether it is speeding up or slowing down? Explain or give an example.

B. Consider the system consisting of the block alone.

Is the total energy for this system constant? If so, explain why. If not, which other object(s) must be included in the system in order for the energy of the new system to be constant? Explain.

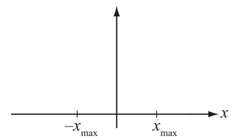
C. Let S represent the system for which the energy is constant.

Write an expression for the potential energy U of system S as a function of the position x of the block. Your expression should involve the spring constant k.

Figure A.32: In-class tutorial worksheet administered in spring 2014 to all sections of PHYS 121. (Five pages.)

Mech 2 ***Potential energy diagrams***

D. On the axes at right, sketch a curve of the potential energy U for system S as a function of x between $-x_{max}$ and x_{max}. Label the curve $U(x)$. Such graphs are called *potential energy diagrams*.



1. On the same axes, draw a dashed curve or line to represent the total energy. Label the curve $E(x)$.
2. What feature of your diagram indicates that the total energy in the system is constant?

E. Write down the relationship between total, kinetic, and potential energy for a system.

1. Based on this relationship, draw on your diagram a *single vertical line segment* whose length represents the kinetic energy at $x = x_{max}/2$. (*Hint:* The bottom of your line segment should coincide with the potential energy curve at $x = x_{max}/2$.)
2. Where is the block moving (i) the fastest and (ii) the slowest? Draw and label line segments on your graph to indicate the kinetic energies at these positions.

F. Two students attempt to use their potential energy diagram to describe the motion of the block at $x = x_{max}/2$.

Student 1: "At $x = x_{max}/2$ the graph of potential energy is increasing. This means the kinetic energy is decreasing, so the block must be slowing down at this position."

Student 2: "I was looking at the slope of the graph. Slope corresponds to velocity. Since the slope of the graph is getting steeper near $x = x_{max}/2$, the block must be speeding up."

Neither student is correct. Explain the errors in each of their reasoning.

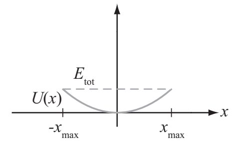
Is your response to student 1 consistent with your answers to part A?

G. Generalize your results from this section: Is it possible to determine (i) the direction of motion of an object or (ii) whether the object is speeding up or slowing down, based only on the object's location on a graph of potential energy *vs*. position?

➪ Discuss your answers with a tutorial instructor.

II. Allowed and forbidden regions

A. The potential energy diagram and total energy from the previous section are reproduced at right.



Suppose that the same spring-block system were given more energy so that the block oscillates between $-2x_{max} < x < 2x_{max}$.

1. On the graph above, sketch new curves that would represent (i) potential energy and (ii) total energy.

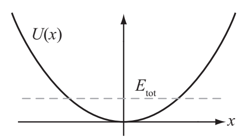
2. Describe in words how the curve representing potential energy would change, if at all.

   Would the portion of the curve between $-x_{max}$ and x_{max} change?

3. Describe in words how the line representing total energy would change, if at all.

It is common for potential energy diagrams to show the curves for both $U(x)$ and total energy for the entire range of x-values on the diagram.

B. The potential energy diagram at right corresponds to the *original* block-spring system using these conventions.



Does the block reach all of the positions x for which U is drawn?

Regions in which the object of interest can be located are called *allowed regions*. Regions in which the object cannot be located are called *disallowed* or *forbidden regions*.

C. Label the allowed and forbidden regions on the potential energy diagram above.

What mathematical relation must exist between total energy E_{tot} and potential energy $U(x)$ for the *allowed regions?* Express your answer as an inequality.

Locations at which objects stop and reverse direction of motion are called *turn-around points*.

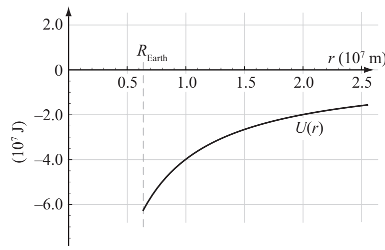
D. Explain how to locate turn-around points on a potential energy diagram.

Label the turn-around points on the potential energy diagram in part B.

Mech 4 ***Potential energy diagrams***

III. Planetary gravitational potential energy

Recall that the gravitational potential energy for a system of two objects with masses m_A and m_B, with their centers of mass separated by a distance r, is $U(r) = -Gm_Am_B/r$. This expression is valid when the potential energy at infinite distance is defined to be zero.



A. The graph at right represents the gravitational potential energy for a system consisting of a 1.0 kg projectile and the Earth. Consider only $r > R_{Earth}$, where r represents the distance from the center of the Earth and R_{Earth} is the radius of the Earth.

   Suppose that the projectile is launched from the surface of the Earth with kinetic energy of 2.0×10^7 joules. (Assume that the projectile only moves in the radial direction r.)

   1. Draw a line segment at $r = R_{Earth}$ that represents the *kinetic energy* at this position. (*Hint:* Recall your work in part I.E. How long should the line segment be?)

   2. Draw and label a line indicating the *total energy* of the Earth-projectile system.

   3. Use the potential energy diagram to determine how far from the center of the Earth the projectile reaches before turning around. Explain.

   4. Indicate the allowed and the forbidden region(s) for positions $r > R_{Earth}$.

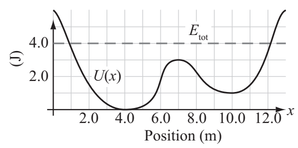
   5. Is the total energy of the system *positive*, *negative*, or *zero?*

B. Suppose instead that the projectile were given a kinetic energy of 7.0×10^7 joules at the surface of the Earth.

   Discuss with your partners (i) the location of the allowable region and (ii) the location of the turn-around point, if any. Write down your thoughts after your discussion.

➪ Discuss your answers with a tutorial instructor.

IV. Relating force to potential energy diagrams



The potential energy diagram at right represents an unknown one-dimensional system. The coordinate x represents the position of a particle within that system. The only force on the particle is that associated with the potential energy.

A. Fill in the table below for an instant when the particle is located at $x = 6.0$ m. Do not fill in the last column.

Do we need to know the direction of motion in order to determine the direction of force or acceleration? Explain.

B. Complete the table for an instant when the particle is located at $x = 8.0$ m.

	$x = 6.0$ m		$x = 8.0$ m
Direction of motion	Positive x-direction	Negative x-direction	
Speeding up or *slowing down*			
Direction of acceleration (+ or –)			
Direction of force (+ or –)			
Sign of dU/dx			

C. Generalize your results above. What graphical feature of potential energy diagrams can be used to determine the *direction of the force* on a particle *(e.g.,* sign of potential, slope of potential, *etc.)?* Include any relevant signs.

In one dimension (*e.g.*, the x-direction), a conservative force $\vec{F}$ is related to the associated potential energy U by $\vec{F} = -(dU/dx)\hat{x}$. This relation can be used to determine the change in potential energy: $\Delta U_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{x}$.

D. Is your response to part C consistent with the relation above? If not, resolve any inconsistencies.

E. Determine the force on the particle at $x = 7.0$ m. Explain.

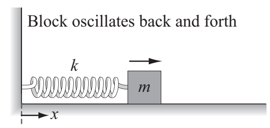
➪ Discuss your answers with a tutorial instructor.

POTENTIAL ENERGY DIAGRAMS

Mech
1

I. Graphical representations of potential energy

A block is attached to an ideal spring of spring constant k, as shown. It oscillates back and forth on a level, frictionless surface. The x-coordinate denotes the location of the center of the block. The spring is at its equilibrium position when $x = x_o$. Let system SB be composed of the spring and block.



A. During one period of its motion, the block starts at $x_{min} > 0$, moves right toward x_o, reaches the turn-around point at x_{max}, moves left toward x_o, and reaches x_{min} again.

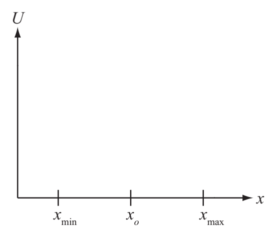
In the space provided, sketch the shape of the graph of potential energy U of system SB *vs*. position x for the entire period. Use the convention that $U = 0$ at the equilibrium position. Explain.

B. Recall from lecture that the potential energy U for a spring can be written $U(x) = \frac{1}{2}k(x - x_o)^2$, where k is the spring constant and x_o is the equilibrium position.

Sketch a curve of $U(x)$ based on the expression above.

On the diagram draw and label dots to indicate an instant when:

1. The block is at $x = x_o$ and moving to the right.
2. The block is at $x = x_o$ and moving to the left.



Graphs of $U(x)$ are commonly called *potential energy diagrams*.

C. Based on your work above, can an object be located at a particular position on a potential energy diagram more than once?

Can an object's direction of motion be determined using only a potential energy diagram?

D. Check that your sketches in parts A and B are consistent. If not, resolve any inconsistencies.

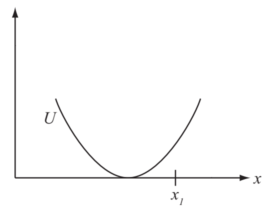
➪ Discuss your answers with a tutorial instructor.

Figure A.33: In-class tutorial worksheet administered in summer 2014 to Phys 121A. (Six pages.)

Mech 2 ***Potential energy diagrams***

II. Using potential energy diagrams

Consider again system SB from the previous section. An arbitrary position x_1 is labeled on the diagram at right.



A. Is the total energy of system SB constant? Base your answer on the work-energy theorem.

On the graph above, draw a line to represent the total energy and label it E_{tot}.

What feature of your diagram indicates that the total energy in the system is constant?

B. For a system that has only kinetic and potential energy, write down an equation relating total energy E_{tot}, kinetic energy K, and potential energy U.

Based on this relationship, draw on the diagram above a single *vertical line segment* whose length represents the kinetic energy K at $x = x_1$. *(Hint:* The bottom of your line segment should coincide with the potential energy curve at $x = x_1$.*)*

Where is the block moving (i) the fastest and (ii) the slowest? Draw and label line segments on your graph to indicate the kinetic energies at these positions.

C. Consider again the arbitrary position labeled x_1.

1. Can you determine whether the particle is *speeding up* or *slowing down* at x_1? If so, state which is true and explain. If not, explain why not.

   Is your answer consistent with your responses to part C in the previous section? If not, resolve the inconsistency.

2. Can you determine the direction of the acceleration of the particle at x_1? If so, describe the direction and explain. If not, explain why not.

3. Suppose that the particle were moving in the positive x-direction (to the right) at x_1.

   - Would the particle be *speeding up* or *slowing down?*
   - What does your answer indicate about the direction of the acceleration?

Suppose instead that the particle were moving in the negative x-direction (to the left).

- Would the particle be *speeding up* or *slowing down?*
- What does your answer indicate about the direction of the acceleration?

Does the direction of acceleration appear to depend on the direction of motion?

4. Are your answers to parts 2 and 3 consistent? If not, resolve the inconsistency.

D. Consider the following statement regarding the potential energy diagram.

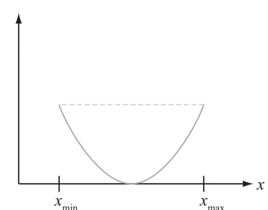
"At $x = x_1$, potential energy U is increasing. Since total energy is constant, the kinetic energy must be decreasing."

Do you agree with this statement? If not, explain what mistake the student is making.

➪ Discuss your answers with a tutorial instructor.

III. Allowed and forbidden regions

A. The potential and total energies of system SB are reproduced at right.



Suppose that the system were given more energy so that the amplitude of oscillation were larger. The block now reaches $x_{new\ min} < x_{min}$ and $x_{new\ max} > x_{max}$.

Label locations for $x_{new\ min}$ and $x_{new\ max}$ on the diagram.

Sketch new curves on the diagram that could represent (i) the potential energy and (ii) the total energy for this modified experiment.

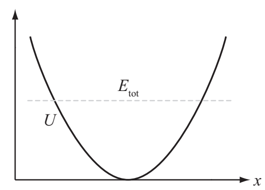
Describe in words how the curve of $U(x)$ between x_{min} and x_{max} changed, if at all. Explain.

Describe in words how the line representing the total energy changed, if at all.

Mech 4 ***Potential energy diagrams***

Potential energy diagrams often show the curves for both U and E_{tot} for the *entire range* of x-values on the diagram, whether or not the object of interest actually reaches those positions. That is, the diagrams show what U would be *if* the particle were located at those positions.

B. The potential energy diagram at right corresponds to the *original* block-spring system, in which the block oscillates only between x_{min} and x_{max}, using the convention discussed above.



Indicate the region(s) in which the block can be located.

Indicate the region(s) in which the block cannot be located.

Regions in which the object of interest can be located are called *allowed regions*. Regions in which the object cannot be located are called *disallowed* or *forbidden regions*.

C. What mathematical relationship must exist between the total energy E_{tot} and the potential energy $U(x)$ for the *allowed regions?* Express your answer as an inequality.

D. Indicate on the diagram the location(s) at which the block stops and turns around.

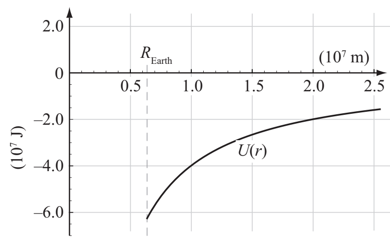
Locations at which objects stop and reverse their direction of motion are called *turn-around points*.

E. What mathematical relationship exists between the total energy E_{tot} and the potential energy $U(x)$ for the *turn-around points?*

IV. Planetary gravitational potential energy

Recall that the gravitational potential energy for a system of two objects with masses m_A and m_B, with their centers of mass separated by a distance r, is $U(r) = -Gm_Am_B/r$. This expression is valid when the potential energy at infinite distance is defined to be zero.

A. The graph at right shows the potential energy for the system of a 1.0 kg projectile and the Earth. Consider only $r > R_{\text{Earth}}$, where r is the distance from the center of the Earth and R_{Earth} is the Earth's radius.



Now suppose that the projectile is launched in the radial direction from the surface of the Earth with $K = 4.0 \times 10^7$ J.

1. Draw a line segment at $r = R_{\text{Earth}}$ that represents the *kinetic energy* at this position. (*Hint:* Recall your work in part II.B.)

2. Draw and label a line indicating the *total energy* of the Earth-projectile system.

3. Use your diagram to determine the location r where the projectile turns around. Explain.

4. Indicate the allowed and the forbidden region(s) for positions $r > R_{\text{Earth}}$.

5. Does the relationship $E_{\text{tot}} = K + U$ still hold?

   Can the total energy of a system be negative?

   Can the potential energy be negative?

   Can the kinetic energy be negative?

Mech 6 ***Potential energy diagrams***

B. Determine the direction of the acceleration of the projectile at $x = 1.0 \times 10^7$ m. Explain.

Use your answer to determine the direction of the *force* on the projectile. Assume that the only force on the particle is that associated with the potential energy shown.

➪ Discuss your answers with a tutorial instructor.

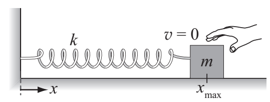
Potential energy diagrams can simplify certain problems, such as finding turn-around points graphically rather than by solving kinematic equations. These simplifications can be done without knowing whether the potential energy is due to a spring, gravitational interactions, or some other unknown interactions. In the homework, you will consider potential energy diagrams of arbitrary shape and unknown source.

POTENTIAL ENERGY DIAGRAMS

Mech 1

I. Graphical representations of potential energy

A block is attached to an ideal spring of spring constant k on a level, frictionless surface, as shown. Let system SB be composed of the spring and block.

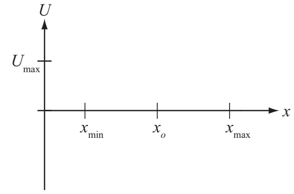


A. The block is initially held in place at $x = x_{max}$. It is then released. It starts to move to the left toward the equilibrium position x_o, reaches the turn-around point x_{min}, moves right toward x_o, and finally returns to x_{max}.

In the space provided, sketch a graph of the potential energy U *vs*. position x for system SB. Use the convention that $U = 0$ at the equilibrium position x_o. Briefly explain.

B. In this part, you will be guided in constructing the graph of potential energy to check your answer above.

The positions x_{min}, x_o, and x_{max} are labeled on the horizontal axis at right. The maximum potential energy U_{max} is labeled on the vertical axis.



1. On the diagram, draw and label *five separate dots* to indicate when the particle: (i) is initially released from x_{max}; (ii) passes through x_o for the first time; (iii) reaches x_{min}; (iv) passes through x_o again; and (v) returns to x_{max}.
2. Connect your five dots by a smooth curve.
3. Recall from lecture that the potential energy $U(x)$ for a spring above is $\frac{1}{2}k(x-x_o)^2$. Are your curves consistent with this expression? If not, resolve any inconsistencies.

Graphs of $U(x)$, such as the one you drew above, are commonly called *potential energy diagrams*.

C. Answer the following questions based on your work above.

1. Can an object/system pass through a particular point on a potential energy diagram more than once?
2. Can the direction of motion of a particle be inferred from a potential energy diagram?
3. Can the shape of a potential energy diagrams be determined using only the functional form of $U(x)$ (*i.e.*, can you determine the shape without information about the motion)?

➪ Discuss your answers with a tutorial instructor.

Figure A.34: In-class tutorial worksheet administered in autumn 2014 to Phys 121X. (Five pages.)

Mech 2 ***Potential energy diagrams***

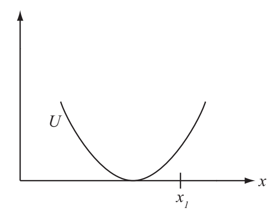
II. Using potential energy diagrams

Consider again system SB from the previous section.

A. Is the total energy of system SB constant? Base your answer on the work-energy theorem.

On the graph at right, draw a line to represent the total energy. Label it E_{tot}.

What feature of your diagram indicates that the total energy of the system is constant?



B. For a system that has only kinetic and potential energy, write down an equation relating the total energy E_{tot}, kinetic energy K, and potential energy U.

Based on this relationship, draw on the diagram above at $x = x_1$ a single *vertical line segment,* the length of which represents the kinetic energy K at this position. *(Hint:* The bottom of your line segment should coincide with the potential energy curve at $x = x_1$.*)*

Where is the block moving (i) the fastest and (ii) the slowest? If possible, draw and label line segments on your graph to indicate the kinetic energies at these positions.

C. Consider again the position labeled x_1.

1. Can you determine whether the particle is *speeding up* or *slowing down* at x_1*?* If so, state how the speed is changing and explain. If not, explain why not.

   Is your answer consistent with your responses to part C of section I? If so, explain how it is consistent. If not, resolve any inconsistencies.

2. Can you determine the direction of the acceleration of the particle at x_1*?* If so, describe the direction at that point and explain. If not, explain why not.

D. The following questions can be used as a guide to check your answers to part C.

1. Suppose that the particle were moving in the positive x-direction at $x = x_1$ (see the top diagram at right).

   Dir of motion →

   - Would the particle be *speeding up* or *slowing down?* Circle your answer in the box at right. Explain briefly.

     Speeding up?
     Slowing down?

   - Based on your answer, what is the direction of the acceleration? Draw an arrow to indicate the direction at right. Explain briefly.

     Dir of acceleration

   Suppose instead that the particle were moving in the negative x-direction at $x = x_1$ (see the top diagram at right).

   Dir of motion ←

   - Would the particle be *speeding up* or *slowing down?* Circle your answer in the box at right. Explain briefly.

     Speeding up?
     Slowing down?

   - Based on your answer, what is the direction of the *acceleration?* Draw an arrow to indicate the direction at right. Explain briefly.

     Dir of acceleration

2. Answer the following questions based on your work above.

   - Does a positive acceleration imply that an object is speeding up?

   - Can the direction of the acceleration at a point on a potential energy diagram be determined without knowing the direction of motion?

3. Based on your work above, do you still agree with your answers to part C? If not, resolve any inconsistencies.

E. Consider the following student statement regarding the potential energy diagram.

"Just based on the graph we can tell that at $x = x_1$ the potential energy of the system is increasing. Since total energy is constant, the particle must be slowing down."

Do you agree with this statement? If so, explain why. If not, explain what mistake or assumption the student is making.

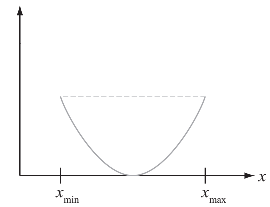
➪ Discuss your answers with a tutorial instructor.

Mech 4 ***Potential energy diagrams***

III. Allowed and forbidden regions

A. The potential and total energies of system SB are reproduced at right.

Suppose that the same system were given more energy so that the amplitude of oscillation were larger. The block now reaches $x_{new\ min} < x_{min}$ and $x_{new\ max} > x_{max}$.



1. Label locations for $x_{new\ min}$ and $x_{new\ max}$ on the diagram.
2. Sketch new curves on the diagram that could represent (i) the potential energy and (ii) the total energy for this modified experiment.
3. How is your new curve of $U(x)$ different? How is it the same? Explain.

   Describe in words how the line representing the total energy changed, if at all.

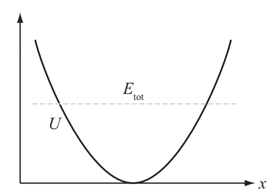
   Does the functional form of $U(x)$ appear to depend on the total energy?

Potential energy diagrams often show the curves for both U and E_{tot} for the *entire range* of x-values on the diagram, whether or not the object of interest actually reaches those positions. That is, the diagrams show what U would be *if* the particle were located at those positions, even though the object may never reach all of the positions shown.

B. The potential energy diagram at right corresponds to a block-spring system that uses the convention above.

On the potential energy diagram:

- indicate the positions where the block can be located.
- indicate the positions where the block cannot be located.



Positions where an object *can* be located are called *allowed regions*. Positions where the object cannot be located are called *disallowed* or *forbidden regions*.

C. For each of the following regions, write the mathematical relationship that exists between the total energy E_{tot} and the potential energy $U(x)$.

- The allowed region

- The forbidden region

D. Indicate on the diagram the location(s) at which the block stops and turns around.

Locations at which objects stop and reverse their direction of motion are called *turn-around points* or *turning points*.

E. What mathematical relationship exists between the total energy E_{tot} and the potential energy $U(x)$ for *turn-around points?*

Potential energy diagrams can simplify certain problems, such as finding turn-around points graphically rather than by solving kinematic equations. These simplifications can be done without knowing whether the potential energy is due to a spring, gravitational interactions, or some other unknown interactions. In the homework, you will consider potential energy diagrams of arbitrary shape and unknown source.

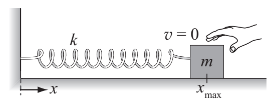
➪ Discuss your answers with a tutorial instructor.

POTENTIAL ENERGY DIAGRAMS

Mech 1

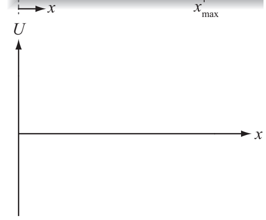
I. Graphical representations of potential energy

A block is attached to an ideal spring of spring constant k on a level, frictionless surface, as shown. Let system SB be composed of the spring and block.

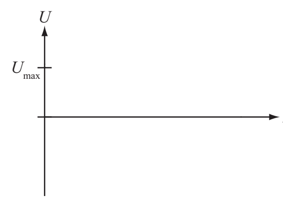


A. The block is initially held in place at $x = x_{max}$. It is then released. It starts to move to the left toward the equilibrium position x_o, reaches the turn-around point x_{min}, moves right toward x_o, and finally returns to x_{max}.

In the space provided, *predict* the shape of the graph of the potential energy U *vs*. position x for system SB. Let $U = 0$ at $x = x_o$. Briefly explain.



B. In this part, you will check your answer above.



1. On the horizontal axis at right, label the positions x_{min}, x_o, and x_{max}.
2. Are your labels consistent with the relation $x_{min} < x_o < x_{max}$? If not, resolve any inconsistencies.
3. On the diagram, draw and label *five dots* to indicate when the block: (i) is initially released from x_{max}; (ii) passes through x_o for the first time; (iii) reaches x_{min}; (iv) passes through x_o again; and (v) returns to x_{max}.
4. Connect your five dots by a smooth curve.
5. Recall from lecture that the potential energy $U(x)$ for a spring above is $\frac{1}{2}k(x-x_o)^2$. Is your curve consistent with this expression? If not, resolve the inconsistency.

Graphs of $U(x)$, such as the one you drew above, are commonly called *potential energy diagrams*.

C. Answer the following questions based on your work above.

1. Can an object/system pass through a point on a potential energy diagram more than once?
2. Can the direction of motion of an object be inferred from a potential energy diagram?
3. Can the shape of a potential energy diagram between x_{min} and x_{max} be determined using *only* the expression for $U(x)$ (*i.e.*, without knowing the motion of the block)?

➪ Discuss your answers with a tutorial instructor.

Figure A.35: In-class tutorial worksheet administered in winter 2015 to Phys 121X. (Six pages.)

Mech 2 ***Potential energy diagrams***

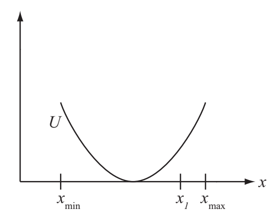
II. Kinematic quantities from potential energy diagrams

Consider again system SB from the previous section.

A. Is the total energy of system SB constant? Base your answer on the work-energy theorem.

On the graph at right, draw and label a line E_{tot} that represents the total energy.

What feature of your graph indicates that the total energy of the system is constant?



B. For a system that has only kinetic and potential energy, write down an equation relating the total energy E_{tot}, kinetic energy K, and potential energy U.

Based on this relationship, draw on the diagram above a single *vertical line segment* at $x = x_1$, the length of which represents the kinetic energy K at this position. *(Hint:* The bottom of your line segment should coincide with the potential energy curve at $x = x_1$.*)* Do not draw the graph of $K(x)$.

Where is the block moving (i) the fastest and (ii) the slowest? If possible, draw and label vertical line segments on your graph to indicate the kinetic energies at these positions.

C. Consider again the position labeled x_1. Base your answers to the following questions on the potential energy diagram above.

1. Can you determine whether the block is *speeding up* or *slowing down* at x_1? If so, state how the speed is changing and explain. If not, explain why not.

   Is your answer consistent with your responses to part C of section I? If so, explain how it is consistent. If not, resolve any inconsistencies.

2. Can you determine the direction of the acceleration of the block at x_1? If so, describe the direction at that point and explain. If not, explain why not.

D. The following questions can be used as a guide to check your answers to part C.

1. Suppose that the block were moving in the positive *x*-direction at $x = x_1$ (see the top diagram at right).

Dir of motion →

- Based on the potential energy diagram, would the block be *speeding up* or *slowing down?* Circle your answer at right.

Speeding up? *Slowing down?*

- Use your answer to determine the direction of the acceleration. Draw an arrow to indicate the direction at right. Explain briefly.

Dir of acceleration

Suppose instead that the block were moving in the negative *x*-direction at $x = x_1$ (see the top diagram at right).

Dir of motion ←

- Based on the potential energy diagram, would the block be *speeding up* or *slowing down?* Circle your answer at right.

Speeding up? *Slowing down?*

- Use your answer to determine the direction of the acceleration. Draw an arrow to indicate the direction at right. Explain briefly.

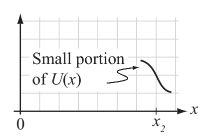
Dir of acceleration

2. Based on your work above, does a positive acceleration imply an increase in speed?

   Based on your work above, can the direction of the acceleration be determined *without* knowing the direction of motion?

3. Do you still agree with your answers to part C? If not, resolve any inconsistencies.

E. A portion of a *different* potential energy diagram is shown at right. Assume the particle is at $x = x_2$ and is moving.



1. Consider the following student statement.

   "Based on the graph we can tell that at $x = x_2$ the potential energy of the system is decreasing. This means the kinetic energy is increasing, so the particle is speeding up."

   Do you agree with this statement? Explain.

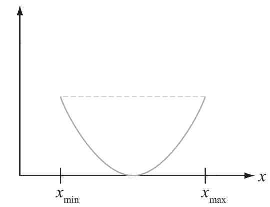
2. Determine the direction of the *acceleration* of the particle at $x = x_2$. If it is not possible to answer, state so explicitly. Explain.

➪ Discuss your answers with a tutorial instructor.

III. Allowed and forbidden regions

A. The potential and total energies of system SB are reproduced at right.

Suppose that the same system were given more energy so that the amplitude of oscillation were larger. The block now reaches $x_{\text{new min}} < x_{\text{min}}$ and $x_{\text{new max}} > x_{\text{max}}$.



1. Label locations for $x_{\text{new min}}$ and $x_{\text{new max}}$ on the diagram.
2. Sketch new curves on the diagram that could represent (i) the potential energy and (ii) the total energy for this modified experiment.
3. Did your curve of $U(x)$ between x_{min} and x_{max} change? Explain why or why not.

   Describe in words how the line representing the total energy changed, if at all.

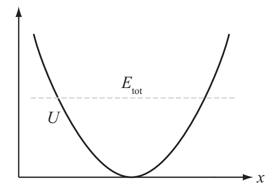
B. Based on your work above, does the shape of a potential energy diagram appear to depend on the total energy?

Potential energy diagrams often show the curves for both U and E_{tot} for the *entire range* of x-values on the diagram, whether or not the object of interest actually reaches those positions. That is, the diagrams show what U would be *if* the object were located at those positions, even though the object may never reach all of the positions shown.

C. The potential energy diagram at right corresponds to a block-spring system that uses the convention above.

On the potential energy diagram:

- indicate the positions where the block can be located.
- indicate the positions where the block cannot be located.



Positions where an object *can* be located are called *allowed regions*. Positions where the object cannot be located are called *disallowed* or *forbidden regions*.

D. For each of the following regions, write the mathematical relationship that exists between the total energy E_{tot} and the potential energy $U(x)$.

- The allowed region

- The forbidden region

E. Indicate on the diagram the location(s) at which the block stops and turns around.

Locations at which objects stop and reverse their direction of motion are called *turn-around points* or *turning points*.

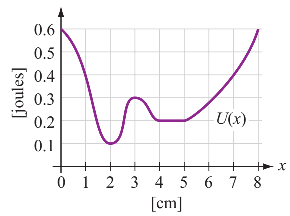
F. What mathematical relationship exists between the total energy E_{tot} and the potential energy $U(x)$ for *turn-around points?*

➪ Discuss your answers with a tutorial instructor.

Mech 6 ***Potential energy diagrams***

IV. Applications of potential energy diagrams

The potential energy diagram for some system is shown at right. The variable *x* represents the position of a particle within that system.



A. Is the particle ever located at $x = 7$ cm? If there is not enough information to answer, state so explicitly. Explain.

Suppose that the particle *were* at $x = 7$ cm. Determine the direction of the acceleration of the particle. If there is not enough information, state so explicitly. Explain.

B. At which positions would the acceleration of the particle be zero? Explain.

At which positions could the particle be released from rest and remain at rest? Explain.

C. The particle is now released from rest at $x = 7.0$ cm.

What is the *maximum kinetic energy* attained by the particle? Express your answer in joules (J). (*Hint:* You may find your work in part II.B helpful.) Explain and show your work.

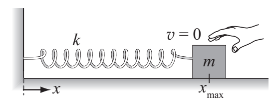
➪ Discuss your answers with a tutorial instructor.

POTENTIAL ENERGY DIAGRAMS

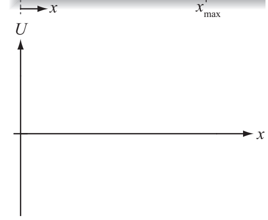
Mech 1

I. Graphical representations of potential energy

A block is attached to an ideal spring on a level, frictionless surface. It is initially held in place at $x = x_{max}$ and then released. It starts to move to the left toward the equilibrium position x_o, reaches the turn-around point x_{min}, moves right toward x_o, and finally returns to x_{max}. Consider the system of both the block and spring.

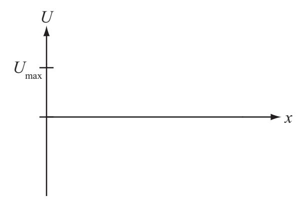


A. In the space provided, *predict* the shape of the graph of the potential energy U *vs*. position x. Let $U = 0$ at $x = x_o$. Briefly explain.



B. In this part, you will check your answer above.

1. On the horizontal axis at right, label the positions x_{min}, x_o, and x_{max}.
2. Are your labels consistent with the relation $x_{min} < x_o < x_{max}$? Resolve any inconsistencies.
3. On the diagram, draw and label *five dots* to indicate when the block: (i) is initially released from x_{max}; (ii) passes through x_o for the first time; (iii) reaches x_{min}; (iv) passes through x_o again; and (v) returns to x_{max}.
4. Connect your five dots by a smooth curve.
5. Recall from lecture that the potential energy $U(x)$ for the spring above is $\frac{1}{2}k(x-x_o)^2$. Is your curve consistent with this expression? If not, resolve the inconsistency.



Graphs of $U(x)$, such as the one you drew above, are commonly called *potential energy diagrams*.

C. Answer the following questions based on your work above.

1. Can an object/system pass through a point on a potential energy diagram more than once?
2. Can the direction of motion of an object be inferred from a potential energy diagram?
3. Does the leftmost position on a potential energy diagram always correspond to where the particle "begins" its motion?
4. If the potential energy of a system is *increasing* with respect to time at some position, must the slope of the graph of $U(x)$ be *positive* at that position?

➪ Discuss your answers with a tutorial instructor.

Figure A.36: In-class tutorial worksheet administered in spring 2015 to Phys 121X. (Six pages.)

Mech 2 *Potential energy diagrams*

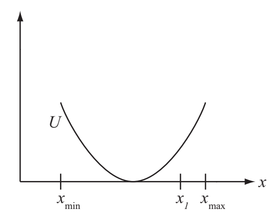
II. Kinematic quantities from potential energy diagrams

Consider again system SB from the previous section.

A. Is the total energy of system SB constant? Base your answer on the work-energy theorem.

On the graph at right, draw and label a line E_{tot} that represents the total energy.

What feature of your graph indicates that the total energy of the system is constant?



B. For a system that has only kinetic and potential energy, write down an equation relating the total energy E_{tot}, kinetic energy K, and potential energy U.

Based on this relationship, draw on the diagram above a single *vertical line segment* at $x = x_1$, the length of which represents the kinetic energy K at this position. *(Hint:* The bottom of your line segment should coincide with the potential energy curve at $x = x_1$.*)* Do not draw the graph of $K(x)$.

Where is the block moving (i) the fastest and (ii) the slowest? If possible, draw and label vertical line segments on your graph to indicate the kinetic energies at these positions.

C. Consider again the position labeled x_1. Base your answers to the following questions on the potential energy diagram above.

1. Can you determine whether the block is *speeding up* or *slowing down* at x_1? If so, state how the speed is changing and explain. If not, explain why not.

   Is your answer consistent with your responses to part C of section I? If so, explain how it is consistent. If not, resolve any inconsistencies.

2. Can you determine the direction of the acceleration of the block at x_1? If so, describe the direction at that point and explain. If not, explain why not.

D. The following questions can be used as a guide to check your answers to part C.

1. Suppose that the block were moving in the positive x-direction at $x = x_1$ (see the top diagram at right).

Dir of motion →
Speeding up? / *Slowing down?*
Dir of acceleration

- Based on the potential energy diagram, would the block be *speeding up* or *slowing down?* Circle your answer at right.
- Use your answer to determine the direction of the acceleration. Draw an arrow to indicate the direction at right. Explain briefly.

Suppose instead that the block were moving in the negative x-direction at $x = x_1$ (see the top diagram at right).

Dir of motion ←
Speeding up? / *Slowing down?*
Dir of acceleration

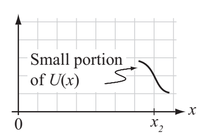
- Based on the potential energy diagram, would the block be *speeding up* or *slowing down?* Circle your answer at right.
- Use your answer to determine the direction of the acceleration. Draw an arrow to indicate the direction at right. Explain briefly.

2. Based on your work above, does a *positive acceleration* imply an *increase in speed?*

Based on your work above, does the direction of the acceleration depend on the direction of motion (*i.e.*, do you need to know the actual direction of motion to determine the direction of the acceleration)?

3. Do you still agree with your answers to part C? If not, resolve any inconsistencies.

E. A portion of a *different* potential energy diagram is shown at right. Assume the particle is moving at $x = x_2$ and that the total mechanical energy in the system of interest is constant.



1. Consider the following student statement.

"Based on the graph we can tell that at $x = x_2$ the potential energy of the system is decreasing. This means the kinetic energy is increasing, so the particle is speeding up."

Do you agree with this statement? Explain.

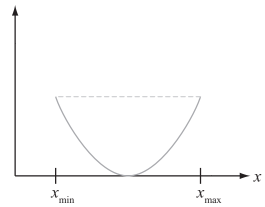
2. Determine the direction of the *acceleration* of the particle at $x = x_2$. If it is not possible to answer, state so explicitly. Explain.

➪ Discuss your answers with a tutorial instructor.

Mech 4 ***Potential energy diagrams***

III. Allowed and forbidden regions

A. The potential and total energies of system SB are reproduced at right.



Suppose that the same system were given more energy so that the amplitude of oscillation were larger. The block now reaches $x_{new\ min} < x_{min}$ and $x_{new\ max} > x_{max}$.

1. Label locations for $x_{new\ min}$ and $x_{new\ max}$ on the diagram.
2. Sketch new curves on the diagram that could represent (i) the potential energy and (ii) the total energy for this modified experiment.
3. Did your curve of $U(x)$ between x_{min} and x_{max} change? Explain why or why not.

Describe in words how the line representing the total energy changed, if at all.

B. Base your answers to the following questions on your work above.

Does the value of the potential energy $U(x)$ at a particular position x (*e.g.*, $x = x_{max}/2$) depend on the total energy?

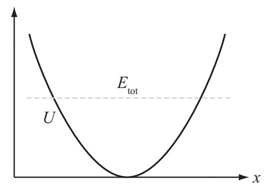
Does the value of potential energy $U(x)$ at a particular position x depend on either the speed or the direction of velocity of the particle at that position?

Potential energy diagrams often show the curves for both U and E_{tot} for the *entire range* of x-values on the diagram, whether or not the object of interest actually reaches those positions. That is, the diagrams show what U would be *if* the object were located at those positions, even though the object may never reach all of the positions shown.

C. The potential energy diagram at right corresponds to a block-spring system that uses the convention above.

On the potential energy diagram:

- indicate the *positions* where the block can be located.
- indicate the *positions* where the block cannot be located.



Positions where an object *can* be located are called *allowed regions*. Positions where the object cannot be located are called *disallowed* or *forbidden regions*.

D. For each of the following regions, write the mathematical relationship that exists between the total energy E_{tot} and the potential energy $U(x)$.

- The allowed region

- The forbidden region

E. Indicate on the diagram the location(s) at which the block stops and turns around.

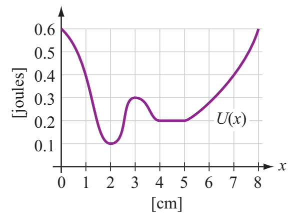
Locations at which objects stop and reverse their direction of motion are called *turn-around points* or *turning points*.

F. What mathematical relationship exists between the total energy E_{tot} and the potential energy $U(x)$ for *turn-around points?*

➪ Discuss your answers with a tutorial instructor.

Mech 6 ***Potential energy diagrams***

IV. Applications of potential energy diagrams



The potential energy diagram for some system is shown at right. The variable x represents the position of a particle within that system. Assume the total mechanical energy of the system is constant.

A. Is the particle ever located at $x = 7$ cm? Explain how you can tell. If there is not enough information to answer, describe what additional information is needed.

Suppose that the particle *were* at $x = 7.0$ cm. Determine the direction of the acceleration of the particle. If there is not enough information, state so explicitly. Explain.

B. At which positions would the acceleration of the particle be zero? Base your answer on your work in this tutorial. Explain.

At which positions could the particle be released from rest and remain at rest? Explain.

C. The particle is now released from rest at $x = 7.0$ cm.

What is the *maximum kinetic energy* attained by the particle? Express your answer in joules (J). (*Hint:* You may find your work in part II.B helpful.) Explain.

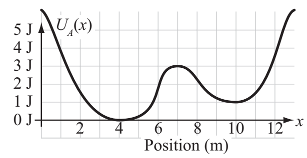
➪ Discuss your answers with a tutorial instructor.

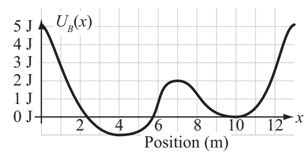
A.2.2 Written homework for Potential energy diagrams

In this section we provide the written homework for the tutorial *Potential energy diagrams*.

POTENTIAL ENERGY DIAGRAMS Name ______________________ Mech HW–1

1. Student 1 studies a one-dimensional system (system A) that contains potential energy. The student correctly constructs the potential energy diagram $U_A(x)$ at right. Student 2 also studies a one-dimensional system (system B) that contains potential energy. The student correctly constructs the potential energy diagram $U_B(x)$ at right. The particles in both systems are of the same mass.





a. Describe in words how the potential energy diagrams $U_A(x)$ and $U_B(x)$ differ.

In the following questions, suppose that the particles of interest are released from rest at $x = x_{released} = 1.0$ m.

b. Fill out the table at right as you work through the following questions.

	System A	System B
$U(x_{released})$		
E_{tot}		
$x_{turn\ around}$		
$K(x = 7\text{ m})$		
$x_{max\ speed}$		

i. What is the potential energy $U(x_{released})$ of system A when the particle is released?

How, if at all, does your answer differ for system B?

ii. What is the total energy E_{tot} for system A? Explain.

How, if at all, does your answer differ for system B? Explain.

Draw a dashed line on each of the potential energy diagrams above to represent the total energy of each system.

iii. At what position $x_{turn\ around}$ does the particle of system A first turn around, if at all? Explain using your potential energy diagram.

How, if at all, does your answer differ for system B? Explain.

Figure A.37: Written homework administered from spring 2013 until spring 2014 to all sections of Phys 121. (Five pages.)

Mech HW–2 ***Potential energy diagrams***

iv. What is the value of kinetic energy $K_{7\text{ m}}$ of the particle in system A at $x = 7$ m? Express your answer in joules. Explain and show your work.

How, if at all, does your answer differ for system B? Explain.

v. At what position $x_{\text{max speed}}$ does the particle of system A obtain its maximum speed? Explain.

How, if at all, does your answer differ for system B? Explain.

Do the particles of systems A and B obtain the same maximum v_{max}? If not, which has a larger maximum speed? Explain.

If you have not done so already, complete the table on the previous page.

c. Is it possible that both students are studying the same system (*i.e.*, that these two potential energy diagrams represent the same physical system)? If so, how can you account for the different potential energy diagrams? If not, state so explicitly. In either case, explain.

d. Is your answer to part c consistent with the idea that only *changes* in potential energy ($\Delta U_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{s}$) are physically significant? If so, explain how it is consistent. If not, resolve the inconsistency.

e. Consider the following *incorrect* student statement.

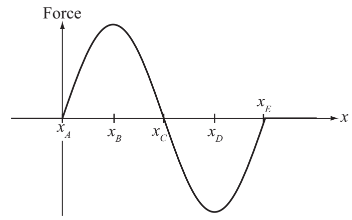
"Systems A and B have different total energies. Energy is conserved, meaning it cannot be created from nothing. Therefore, systems A and B cannot be the same system since they have different total energies. Energy is not conserved here."

Explain the error in reasoning made by this student.

Potential energy diagrams Name ____________________________ Mech HW–3

2. The diagram at right shows the direction and magnitude of a conservative *force* exerted on a particle when the particle is located at position x. The force is zero for $x < x_A$ and zero for $x > x_E$.

   a. Identify the regions(s) in which the force is in the positive x-direction. Explain.

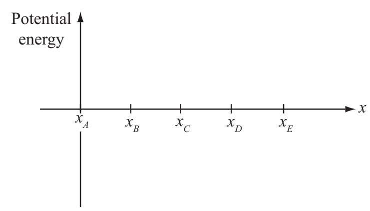


   How is a positive force represented geometrically on a potential energy diagram (*e.g.*, positive slope, positive value, *etc.*)? Explain.

   How is a zero force at a particular position represented on a potential energy diagram?

   b. In the space at right, sketch a potential energy diagram that is consistent with the given conservative force. Choose the value of potential energy at infinity to be zero.

   Explain your reasoning.



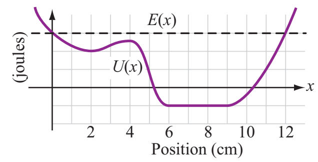
   Describe how choosing a different value for potential energy at infinity would affect your potential energy diagram.

   c. For what values, if any, of total energy $E = K + U$ would the allowed region be restricted to between x_A and x_E? Explain.

   d. Suppose it is known that the allowed region includes positions $x < x_A$. What, if anything, can be said about the value of total energy E? Explain.

Mech HW–4 ***Potential energy diagrams***

3. A particle of mass m_o is part of a one-dimensional system. The potential energy U and total energy E of the system as a function of the position x of the particle are shown at right. The horizontal lines are separated by 1 joule.



a. At what locations or regions, if any, does the particle have a speed of zero? Explain.

b. Label the position x_o on the potential energy diagram where the magnitude of the force on the particle is the strongest. Explain.

What is the direction of the force at $x = x_o$? Explain.

At $x = x_o$, is the particle *speeding up, slowing down,* or *neither?* If there is not enough information to answer, state so explicitly. In any case, explain.

c. Describe the motion of the particle for the region 6.0 cm < x < 9.0 cm *(e.g.,* speeding up, not moving, *etc.).* Explain.

Consider the following *incorrect* student statement regarding the region 6.0 cm < x < 9.0 cm.

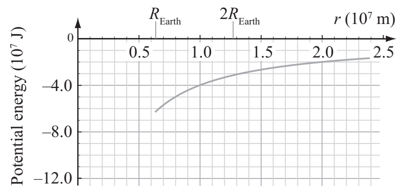
"The value of potential energy is the lowest in this region. This means all of the energy is in the form of only kinetic energy. Therefore, the kinetic energy in this region is equal to the total energy of the system."

Describe the error in reasoning made by the student.

Potential energy diagrams Name ____________________ Mech HW–5

4. This question examines the effect of changing the mass of the particle for two different systems.

   a. The potential energy diagram at right corresponds to a system composed of the Earth and a projectile. Assume the projectile is only able to move in the radial direction r.



   Suppose that the original projectile were replaced with a new projectile with twice the mass.

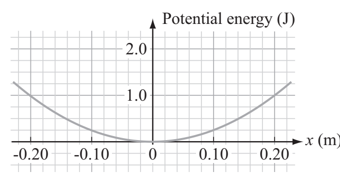
   i. On the diagram, draw the potential energy diagram for the new system. Explain.

   ii. After this change, would the amount of time it takes for the projectile to travel from $r = 2R_{Earth}$ to $r = R_{Earth}$ *increase*, *decrease*, or *remain the same?* Assume both projectiles are released from rest at $r = 2R_{Earth}$. Explain. (*Hint:* Consider how the acceleration at each position r changes, if at all.)

   b. The potential energy diagram at right corresponds to a spring-block system on a frictionless, horizontal surface.

   Suppose that the original block were replaced by a block with twice the mass.

   i. On the diagram, draw the potential energy diagram for the new system. Explain.

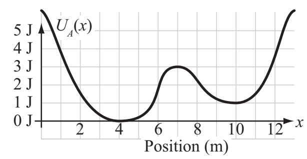


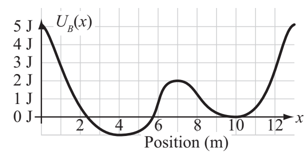
   ii. After this change, would the amount of time it takes for the block to travel from $x = 0.20$ m to $x = 0$ cm *increase, decrease,* or *remain the same?* Assume both blocks are released from rest at $x = 0.20$ m. Explain.

   c. Generalize your results thus far. Does changing the mass of the moveable particle within a one-dimensional system always affect the potential energy diagram and the motion of the particle in the same way?

POTENTIAL ENERGY DIAGRAMS Name ______________________________ Mech HW–1

1. Student 1 studies a one-dimensional system (system A) that contains potential energy. The student correctly constructs the potential energy diagram $U_A(x)$ at right. Student 2 also studies a one-dimensional system (system B) that contains potential energy. The student correctly constructs the potential energy diagram $U_B(x)$ at right. The particles in both systems are of the same mass.





a. Describe in words how the potential energy diagrams $U_A(x)$ and $U_B(x)$ differ.

In the following questions, suppose that the particles of interest are released from rest at $x = x_{\text{released}} = 1.0$ m.

b. Fill out the table at right as you work through the following questions.

	System A	System B
$U(x_{\text{released}})$		
E_{tot}		
$x_{\text{turn around}}$		
$K(x = 7\text{ m})$		
$x_{\text{max speed}}$		

i. What is the potential energy $U(x_{\text{released}})$ of system A when the particle is released?

How, if at all, does your answer differ for system B?

ii. What is the total energy E_{tot} for system A? Explain.

How, if at all, does your answer differ for system B? Explain.

Draw a dashed line on each of the potential energy diagrams above to represent the total energy of each system.

iii. At what position $x_{\text{turn around}}$ does the particle of system A first turn around, if at all? Explain using your potential energy diagram.

How, if at all, does your answer differ for system B? Explain.

Figure A.38: Written homework administered in summer 2014 to Phys 121A. (Six pages.)

iv. What is the value of kinetic energy $K_{7\ \mathrm{m}}$ of the particle in system A at $x = 7$ m? Express your answer in joules. Explain and show your work.

How, if at all, does your answer differ for system B? Explain.

v. At what position $x_{\text{max speed}}$ does the particle of system A obtain its maximum speed? Explain.

How, if at all, does your answer differ for system B? Explain.

Do the particles of systems A and B obtain the same maximum v_{max}? If not, which has a larger maximum speed? Explain.

If you have not done so already, complete the table on the previous page.

c. Is it possible that both students are studying the same system (*i.e.*, that these two potential energy diagrams represent the same physical system)? If so, how can you account for the different potential energy diagrams? If not, state so explicitly. In either case, explain.

d. Is your answer to part c consistent with the idea that only *changes* in potential energy ($\Delta U_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{s}$) are physically significant? If so, explain how it is consistent. If not, resolve the inconsistency.

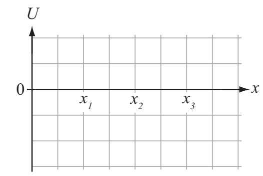
e. Consider the following *incorrect* student statement.

"Systems A and B have different total energies. Energy is conserved, meaning it cannot be created from nothing. Therefore, systems A and B cannot be the same system since they have different total energies. Energy is not conserved here."

Explain the error in reasoning made by this student.

Potential energy diagrams Name ______________________ Mech HW–3

2. A particle is part of a one-dimensional system. The only forms of energy are the potential energy U of the system and the translational kinetic energy K of the particle.



Assume that the only force acting on the particle is that associated with the potential energy.

a. Suppose that the following information about the acceleration of the particle and potential energy U are known.

- At $x = x_1$, the acceleration is in the positive x-direction and $U > 0$.
- At $x = x_2$, the acceleration is in the positive x-direction and $U < 0$.
- At $x = x_3$, the acceleration is in the negative x-direction and $U < 0$.

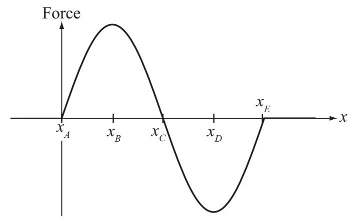
On the diagram above, sketch a graph of $U(x)$ that is consistent with the information given above. Explain your reasoning. (*Hint:* You may find part II.C of the tutorial *Potential energy diagrams* helpful.)

Part d of homework problem 1 gives the relationship between a conservative force $\vec{F}$ and the associated potential energy U. Another way to express the same relationship in one dimension is $\vec{F} = -(dU/dx)\hat{x}$.

b. Is your sketch from part 0 consistent with the relationship $\vec{F} = -(dU/dx)\hat{x}$? If so, explain how it is consistent. If not, resolve the inconsistency.

Mech HW–4 ***Potential energy diagrams***

3. The diagram at right shows the direction and magnitude of a conservative *force* exerted on a particle when the particle is located at position x. The force is zero for $x < x_A$ and zero for $x > x_E$.

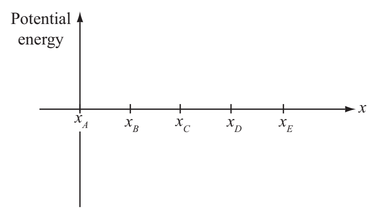


a. Identify the regions(s) in which the force is in the positive x-direction. Explain.

How is a positive force represented geometrically on a potential energy diagram (*e.g.*, positive slope, negative value, *etc.*)? Explain.

b. In the space at right, sketch a potential energy diagram that is consistent with the given conservative force. Choose the value of potential energy at infinity to be zero.

Explain your reasoning.



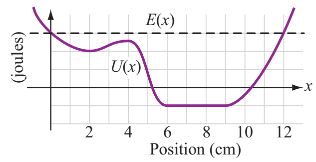
Describe how choosing a different value for potential energy at infinity would affect your potential energy diagram.

c. For what values, if any, of total energy $E = K + U$ would the allowed region be restricted to between x_A and x_E? Explain.

d. Suppose it is known that the allowed region includes positions $x < x_A$. What, if anything, can be said about the value of total energy E? Explain.

Potential energy diagrams Name ______________________________ Mech HW–5

4. A particle of mass m_o is part of a one-dimensional system. The potential energy U and total energy E of the system as a function of the position x of the particle are shown at right. The horizontal lines are separated by 1 joule.



   a. At what locations or regions, if any, does the particle have a speed of zero? Explain.

   b. Label the position x_o on the potential energy diagram where the magnitude of the force on the particle is the strongest. Explain.

   What is the direction of the force at $x = x_o$? Explain.

   At $x = x_o$, is the particle *speeding up*, *slowing down*, or *neither?* If there is not enough information to answer, state so explicitly. In any case, explain.

   c. Describe the motion of the particle for the region 6.0 cm < x < 9.0 cm (*e.g.*, speeding up, not moving, *etc.*). Explain.

   Consider the following *incorrect* student statement regarding the region 6.0 cm < x < 9.0 cm.

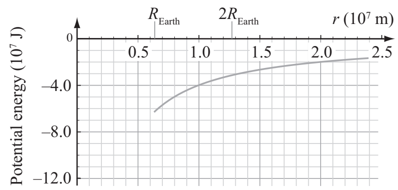
   "The value of potential energy is the lowest in this region. This means all of the energy is in the form of only kinetic energy."

   Describe the error in reasoning made by the student.

Mech HW–6 ***Potential energy diagrams***

5. This question examines the effect of changing the mass of the particle for two different systems.

   a. The potential energy diagram at right corresponds to a system composed of the Earth and a projectile. Assume the projectile is only able to move in the radial direction r.

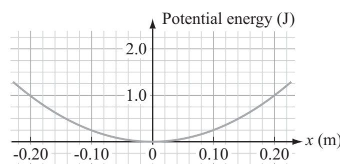
   

   Suppose that the original projectile were replaced with a new projectile with twice the mass.

      i. On the diagram, draw the potential energy diagram for the new system. Explain.

      ii. After this change, would the amount of time it takes for the projectile to travel from $r = 2R_{Earth}$ to $r = R_{Earth}$ *increase, decrease*, or *remain the same?* Assume both projectiles are released from rest at $r = 2R_{Earth}$. Explain. (*Hint:* Consider how the acceleration at each position r changes, if at all.)

   b. The potential energy diagram at right corresponds to a spring-block system on a frictionless, horizontal surface.

   Suppose that the original block were replaced by a block with twice the mass.

      i. On the diagram, draw the potential energy diagram for the new system. Explain.

      ii. After this change, would the amount of time it takes for the block to travel from $x = 0.20$ m to $x = 0$ cm *increase, decrease*, or *remain the same?* Assume both blocks are released from rest at $x = 0.20$ m. Explain.

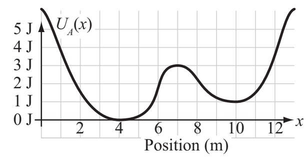
   c. Generalize your results thus far. Does changing the mass of the moveable particle within a one-dimensional system always affect the potential energy diagram and the motion of the particle in the same way?

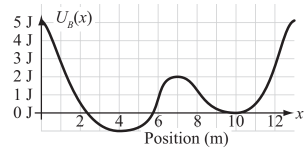
POTENTIAL ENERGY DIAGRAMS Name ____________________________ Mech HW–1

1. Student 1 studies a one-dimensional system (system A) that contains potential energy. The student correctly constructs the potential energy diagram $U_A(x)$ at right.

   Student 2 also studies a one-dimensional system (system B) that contains potential energy. The student correctly constructs the potential energy diagram $U_B(x)$ at right below.

   The particles in both systems are of the same mass.





   a. Describe in words how the potential energy diagrams $U_A(x)$ and $U_B(x)$ differ.

   In the following questions, suppose that the particles of interest are released from rest at $x = x_{\text{released}} = 1.0$ m.

   b. Fill out the table at right as you work through the following questions.

	System A	System B
$U(x_{\text{released}})$		
E_{tot}		
$x_{\text{turn around}}$		
$K(x = 7\text{ m})$		
$x_{\text{max speed}}$		

      i. What is the potential energy $U(x_{\text{released}})$ of system A when the particle is released?

         How, if at all, does your answer differ for system B?

      ii. What is the total energy E_{tot} for system A? Explain.

         How, if at all, does your answer differ for system B? Explain.

         Draw a dashed line on each of the potential energy diagrams above to represent the total energy of each system.

      iii. At what position $x_{\text{turn around}}$ does the particle of system A first turn around, if at all? Explain using your potential energy diagram.

         How, if at all, does your answer differ for system B? Explain.

Figure A.39: Written homework administered in autumn 2014 to Phys 121X. (Six pages.)

Mech HW–2 ***Potential energy diagrams***

iv. What is the value of kinetic energy $K_{7\,\mathrm{m}}$ of the particle in system A at $x = 7$ m? Express your answer in joules. Explain and show your work.

How, if at all, does your answer differ for system B? Explain.

v. At what position $x_{\text{max speed}}$ does the particle of system A obtain its maximum speed? Explain.

How, if at all, does your answer differ for system B? Explain.

Do the particles of systems A and B obtain the same maximum v_{max}? If not, which has a larger maximum speed? Explain.

If you have not done so already, complete the table on the previous page.

c. Is it possible that both students are studying the same system (*i.e.*, that these two potential energy diagrams represent the same physical system)? If so, how can you account for the different potential energy diagrams? If not, state so explicitly. In either case, explain.

d. Is your answer to part c consistent with the idea that only *changes* in potential energy ($\Delta U_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{s}$) are physically significant? If so, explain how it is consistent. If not, resolve the inconsistency.

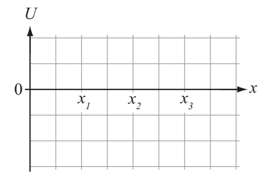
e. Consider the following *incorrect* student statement.

"Systems A and B have different total energies. Energy is conserved, meaning it cannot be created from nothing. Therefore, systems A and B cannot be the same system since they have different total energies. Energy is not conserved here."

Explain the error in reasoning made by this student.

Potential energy diagrams Name ______________________ Mech HW–3

2. A particle is part of a one-dimensional system. The only forms of energy are the potential energy U of the system and the translational kinetic energy K of the particle.



Assume that the only force acting on the particle is that associated with the potential energy.

a. Suppose that the following information about the acceleration of the particle and potential energy U are known.

   - At $x = x_1$, the acceleration is in the positive x-direction and $U > 0$.
   - At $x = x_2$, the acceleration is in the positive x-direction and $U < 0$.
   - At $x = x_3$, the acceleration is in the negative x-direction and $U < 0$.

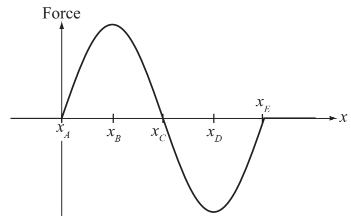
   On the diagram above, sketch a graph of $U(x)$ that is consistent with the information given above. Explain your reasoning. (*Hint:* You may find part II.C of the tutorial *Potential energy diagrams* helpful.)

Part d of homework problem 1 gives the relationship between a conservative force $\vec{F}$ and the associated potential energy U. Another way to express the same relationship in one dimension is $\vec{F} = -(dU/dx)\hat{x}$.

b. Is your sketch from part 0 consistent with the relationship $\vec{F} = -(dU/dx)\hat{x}$? If so, explain how it is consistent. If not, resolve the inconsistency.

Mech HW–4 ***Potential energy diagrams***

3. The diagram at right shows the direction and magnitude of a conservative *force* exerted on a particle when the particle is located at position x. The force is zero for $x < x_A$ and zero for $x > x_E$.

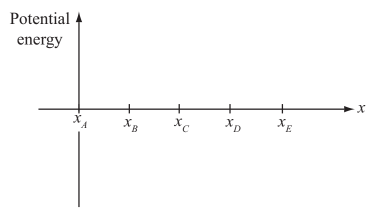


a. Identify the regions(s) in which the force is in the positive x-direction. Explain.

How is a positive force represented geometrically on a potential energy diagram (*e.g.*, positive slope, negative value, *etc.*)? Explain.

b. In the space at right, sketch a potential energy diagram that is consistent with the given conservative force. Choose the value of potential energy at infinity to be zero.

Explain your reasoning.



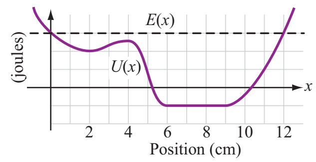
Describe how choosing a different value for potential energy at infinity would affect your potential energy diagram.

c. For what values, if any, of total energy $E = K + U$ would the allowed region be restricted to between x_A and x_E? Explain.

d. Suppose it is known that the allowed region includes positions $x < x_A$. What, if anything, can be said about the value of total energy E? Explain.

Potential energy diagrams Name ______________________________ Mech HW–5

4. A particle of mass m_o is part of a one-dimensional system. The potential energy U and total energy E of the system as a function of the position x of the particle are shown at right. The horizontal lines are separated by 1 joule.



a. At what locations or regions, if any, does the particle have a speed of zero? Explain.

b. Label the position x_o on the potential energy diagram where the magnitude of the force on the particle is the strongest. Explain.

What is the direction of the force at $x = x_o$? Explain.

At $x = x_o$, is the particle *speeding up*, *slowing down*, or *neither?* If there is not enough information to answer, state so explicitly. In any case, explain.

c. Describe the motion of the particle for the region 6.0 cm < x < 9.0 cm (*e.g.*, speeding up, not moving, *etc.*). Explain.

Consider the following *incorrect* student statement regarding the region 6.0 cm < x < 9.0 cm.

"The value of potential energy is the lowest in this region. This means all of the energy is in the form of only kinetic energy."

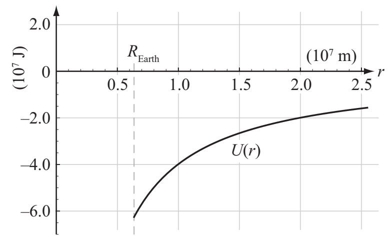
Describe the error in reasoning made by the student.

Mech HW–6

Potential energy diagrams

5. Recall from lecture that $U(r) = -Gm_A m_B/r$ is the universal gravitational potential energy for a system of two objects of masses m_A and m_B, with their centers separated by a distance r. This expression is valid when the potential energy at infinite distance is defined to be zero.

The graph at right shows the potential energy for system S consisting of a 1.0 kg projectile and the Earth. Consider positions outside the radius of the Earth ($r > R_{Earth}$).



a. Suppose that a projectile is launched from the surface of the Earth. At $r = 1.0\times10^7$ m it is moving away from the surface with kinetic energy 1.0×10^7 J.

   i. Is the total energy of system S constant? Explain.

   Draw and label a line on the indicating the *total energy* of the Earth-projectile system.

   ii. Use your diagram to determine the kinetic energy of the projectile just after it was launched from the surface of the Earth. Explain.

   iii. Use your diagram to determine the projectile's turn-around point. Explain.

   Indicate on the diagram the allowed and the forbidden region(s) for positions $r > R_{Earth}$.

   iv. Base your answers to the following questions on your work above.

   - Can the total energy of a system be negative?
   - Can the potential energy be negative?

b. Determine the *direction* of the acceleration of the projectile at $r = 1.0\times10^7$ m. Use the methods you developed in part II.D of the tutorial to explain your reasoning.

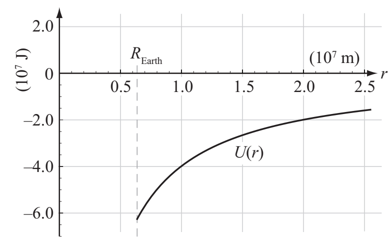
   Use your answer and Newton's second law to determine the *direction* of the net force on the projectile. Assume the only force on the particle is that associated with the diagram shown.

   Is your answer consistent with your knowledge of gravitational forces? Explain.

POTENTIAL ENERGY DIAGRAMS Name ____________________ Mech HW–1

1. Recall from lecture that $U(r) = -Gm_Am_B/r$ is the universal gravitational potential energy for a system of two objects of masses m_A and m_B, with their centers separated by a distance r. This expression is valid when the potential energy at infinite distance is defined to be zero.

The graph at right shows the potential energy for system S consisting of a 1.0 kg projectile and the Earth. Consider positions outside the radius of the Earth ($r > R_{Earth}$).



a. Suppose that the projectile is launched from Earth. At $r = 1.0\times10^7$ m it is moving away from the surface with kinetic energy 1.0×10^7 J.

i. Is the total energy of system S constant? Briefly explain.

Draw and label a line on the diagram to indicate the *total energy* of system S.

ii. Use your diagram to determine the kinetic energy of the projectile just after it was launched from the surface of the Earth. Explain.

iii. Use your diagram to determine the projectile's turn-around point. Explain.

iv. Indicate on the diagram the allowed and the forbidden region(s) for positions $r > R_{Earth}$.

v. Base your answers to the following questions on your work above.

- Can the total energy of a given system be negative?
- Can the potential energy of a given system be negative?

b. Determine the *direction* of the acceleration of the projectile at $r = 1.0\times10^7$ m. Use the method you developed in part II.D of the tutorial to explain your reasoning.

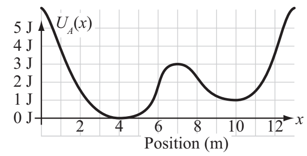
Use your answer and Newton's second law to determine the *direction* of the net force on the projectile. Explain. (Assume the only force on the particle is that associated with $U(x)$).

Is your answer consistent with your knowledge of gravitational forces? Explain.

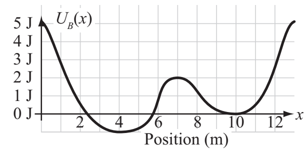
Figure A.40: Written homework administered in winter and spring 2015 to Phys 121X. (Six pages.)

Mech HW–2 ***Potential energy diagrams***

2. Student 1 studies a one-dimensional system (system A) and correctly constructs the potential energy diagram $U_A(x)$ at right.



Student 2 also studies a one-dimensional system (system B) and correctly constructs the potential energy diagram $U_B(x)$ below.



The particles in both systems have the same mass.

a. Briefly describe how the potential energy diagrams differ.

b. *Predict* whether it is possible that students 1 and 2 are studying the same system (*i.e.*, is it possible that the two potential energy diagrams represent the same system)? Explain why or why not.

In the following questions you will be guided in checking your answer to part b.

c. Suppose that the particles of interest are released from rest at $x = 1.0$ m.

i. What is the kinetic energy of the particle in system A at $x = 1.0$ m? Explain.

How, if at all, would your answer differ for system B? Briefly explain.

ii. At what location x does the particle in system A attain its *maximum* kinetic energy? Explain.

How, if at all, would your answer differ for system B? Briefly explain.

iii. Determine the maximum kinetic energy attained by the particle in system A. Express your answer in joules (J). Explain and show your work.

How, if at all, would your answer differ for system B? Explain.

Potential energy diagrams Name ______________________________ Mech HW–3

iv. Do the particles in the two systems attain the same maximum *speed?* Explain.

v. Determine the locations of the turn-around point(s) for system A. Explain.

How, if at all, would your answer differ for system B? Briefly explain.

d. Consider the following *correct* student statement.

"If a conservative force $\vec{F}$ acts on a particle, a potential energy U can be associated with the system of the particle and the object exerting the force. The relation between the force and potential energy can be written, in one dimension, as $\vec{F} = -(dU/dx)\hat{x}$."

Based on this student statement, are the two potential energy diagrams, $U_A(x)$ and $U_B(x)$, associated with the *same* force $\vec{F}(x)$? Explain.

e. Based on your answers to parts c and d, could the two potential energy diagrams describe the same system? Explain.

Do you still agree with your prediction to part b? Explain.

f. Consider the following student statement about systems A and B.

"If the particles are released from rest at x = 1.0 m, system A will have 4 J of total energy and system B will have 3 J of total energy. Energy is conserved; it cannot change. Therefore, systems A and B cannot represent the same system because they have different amounts of total energy."

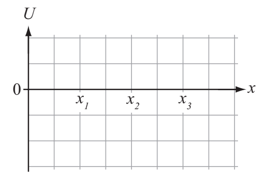
Does this student come to a correct conclusion about systems A and B? If so, state so explicitly. If not, describe the error in his or her reasoning.

Mech
HW–4

Potential energy diagrams

3. A particle is part of a one-dimensional system. The only forms of energy are the potential energy U of the system and the translational kinetic energy K of the particle.

Assume that the only force acting on the particle is that associated with the potential energy.



a. Suppose that the following information about the acceleration of the particle and potential energy U are known.

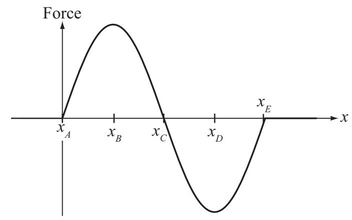
- At $x = x_1$, the acceleration is in the positive x-direction and $U > 0$.
- At $x = x_2$, the acceleration is in the positive x-direction and $U < 0$.
- At $x = x_3$, the acceleration is in the negative x-direction and $U < 0$.

On the diagram above, sketch a graph of $U(x)$ that is consistent with the information given above. Explain your reasoning. (*Hint:* You may find part II.C of the tutorial *Potential energy diagrams* helpful.)

b. Is your sketch above consistent with the relationship between force and potential energy given in problem 2 of this homework? If so, carefully explain how it is consistent. If not, resolve the inconsistency.

Potential energy diagrams Name ____________________ Mech HW–5

4. The diagram at right shows the direction and magnitude of a conservative *force* exerted on a particle when the particle is located at position x. The force is zero for $x < x_A$ and zero for $x > x_E$.

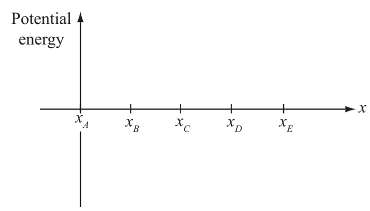


a. Identify the regions(s) in which the force is in the positive x-direction. Explain.

How is a force in the positive direction represented geometrically on a potential energy diagram *(e.g.*, positive slope, negative value, *etc.)?* Explain.

b. In the space at right, sketch a potential energy diagram that is consistent with the given conservative force. Choose the value of potential energy at infinity to be zero.

Explain your reasoning.



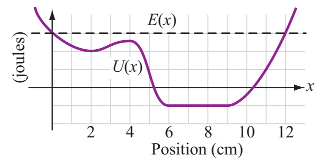
Describe how choosing a different value for potential energy at infinity would affect your potential energy diagram.

c. For what values, if any, of total energy $E = K + U$ would the allowed region be restricted to between x_A and x_E? Explain.

d. Suppose it is known that the allowed region includes positions $x < x_A$. What, if anything, can be said about the value of total energy E? Explain.

Mech HW–6 ***Potential energy diagrams***

5. A particle of mass m_o is part of a one-dimensional system. The potential energy U and total energy E of the system as a function of the position x of the particle are shown at right. The horizontal lines are separated by 1 joule.



a. Identify the turn-around point(s). Explain.

b. Label the position x_o on the potential energy diagram where the magnitude of the force on the particle is the strongest. Explain.

What is the direction of the force at $x = x_o$? Explain.

At $x = x_o$, is the particle *speeding up*, *slowing down*, or *neither?* If there is not enough information to answer, state so explicitly. In any case, explain.

c. Describe the motion of the particle for the region 6.0 cm < x < 9.0 cm *(e.g.,* speeding up, not moving, *etc.)*. Explain.

Consider the following student statement regarding the region 6.0 cm < x < 9.0 cm.

"The value of potential energy is the lowest in this region. This means there is no potential energy and all of the energy is in the form of kinetic. So, since the total energy is 3 J, the maximum kinetic energy of the particle must also be 3 J."

Does this student correctly determine the maximum kinetic energy of the particle? If so, state so explicitly. If not, describe the error in his or her reasoning and determine the correct maximum kinetic energy.

A.2.3 Online interactive practice homework

As discussed in §7.1, the online interactive practice homework (OIPH) consists of over 100 Catalyst separate pages.[1] The order in which most of these pages are seen by students varies depending on their answers throughout the OIPH. Thus, it is not possible to present the Catalyst pages in chronological order. Instead, they are grouped by *function,* as indicated in the list of contents below.[2] The possible destinations are always shown in the caption below each page, as well as other relevant pages.

Readers interested in getting a sense of how the OIPH would be presented to students may start on page 480 (the first one presented to students) and use the information in the caption.

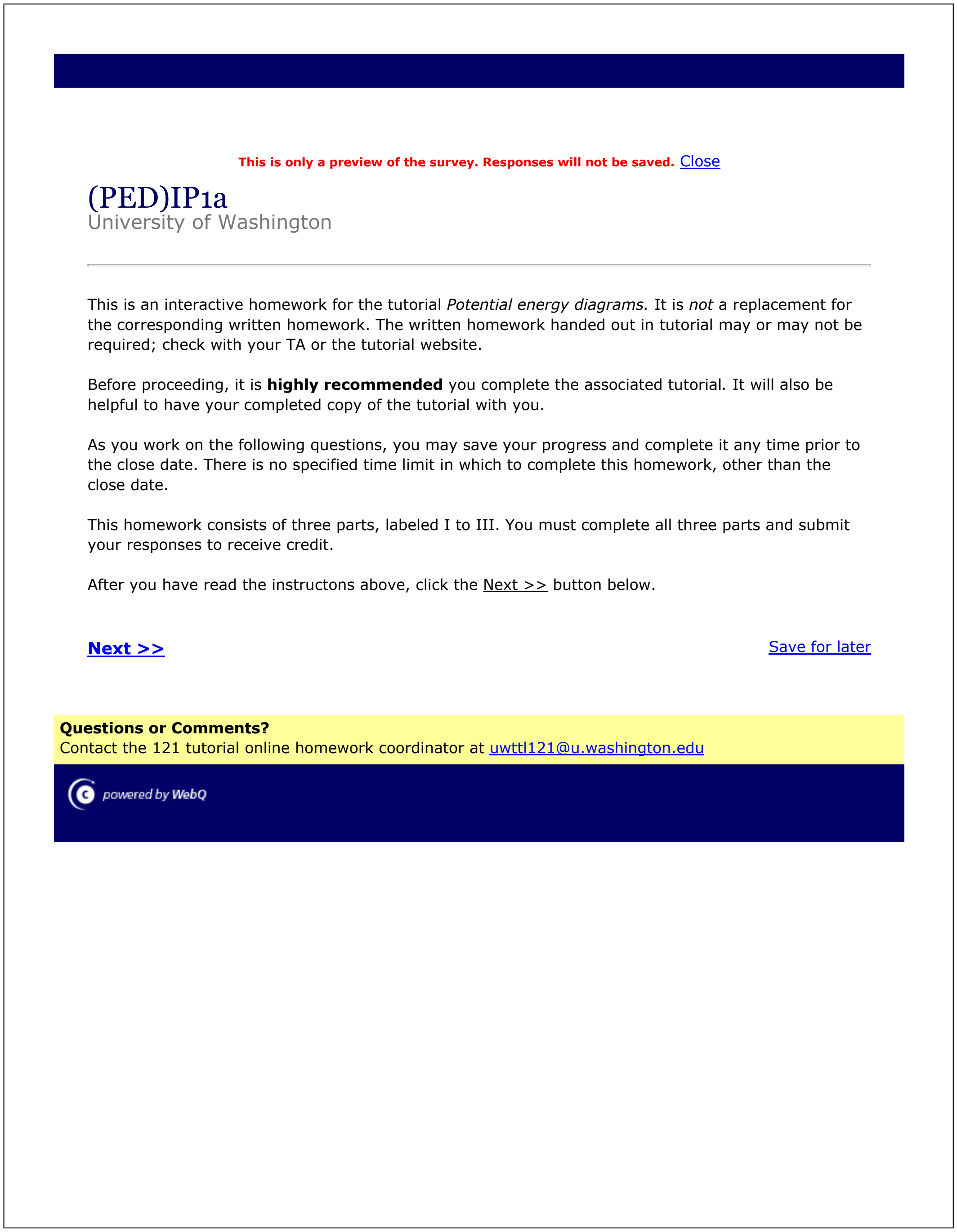
Contents of subsection

A.2.3-a Preliminary and commentary pages

Below are the preliminary and commentary pages of the OIPH.

[1]Most of these pages are identical to other pages, which allowed a maximum number of attempts to be set.

[2] A flowchart showing some of these question functions is shown in Figure 7.1 on page 186.



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(PED)IP1a
University of Washington

This is an interactive homework for the tutorial *Potential energy diagrams.* It is *not* a replacement for the corresponding written homework. The written homework handed out in tutorial may or may not be required; check with your TA or the tutorial website.

Before proceeding, it is **highly recommended** you complete the associated tutorial. It will also be helpful to have your completed copy of the tutorial with you.

As you work on the following questions, you may save your progress and complete it any time prior to the close date. There is no specified time limit in which to complete this homework, other than the close date.

This homework consists of three parts, labeled I to III. You must complete all three parts and submit your responses to receive credit.

After you have read the instructons above, click the Next >> button below.

Next >> Save for later

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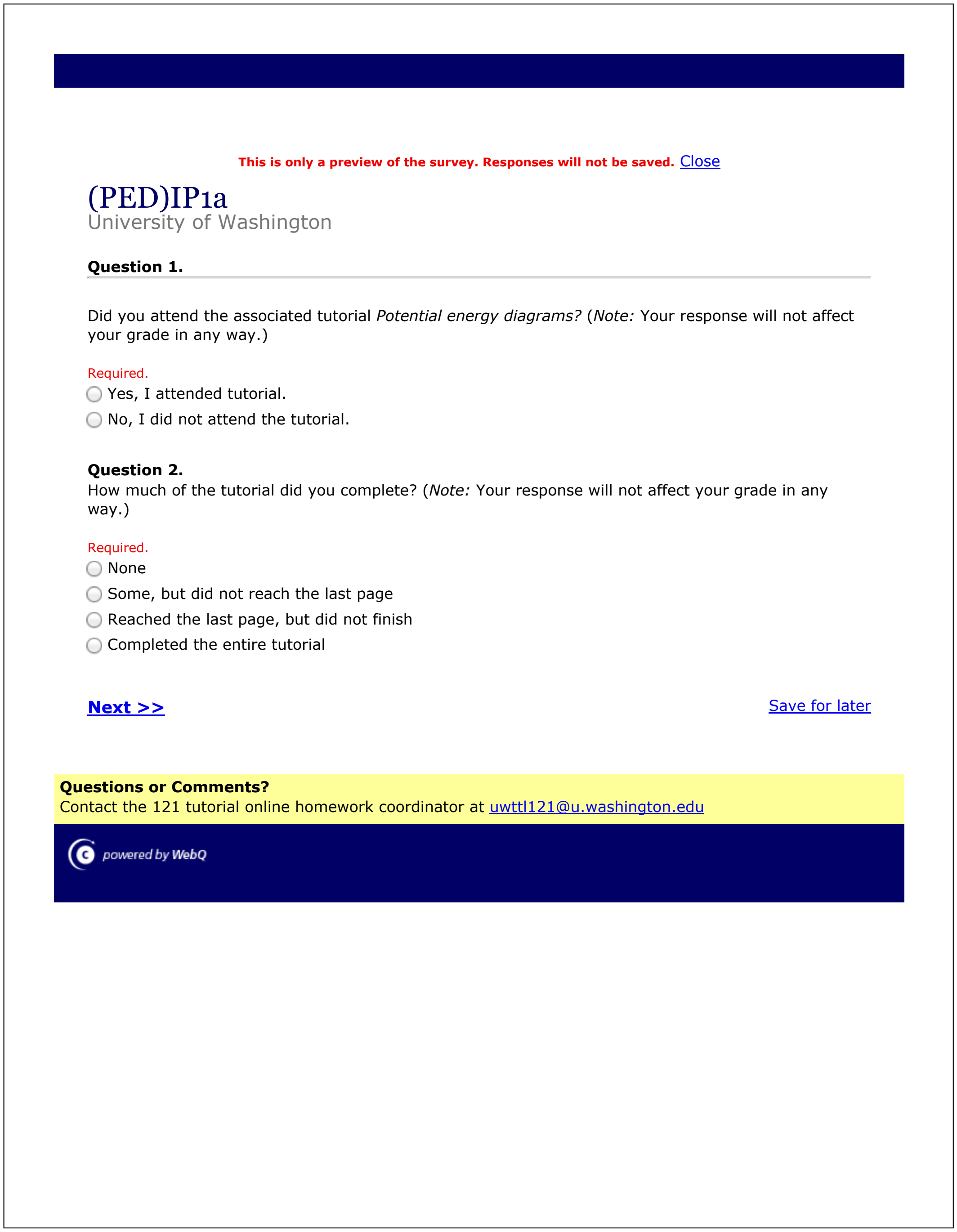
Figure A.41: Introductory page of OIPH administered in spring 2014 to Phys 121X. This is the first page students see when beginning the OIPH. The next page students see is shown on page 481.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

Question 1.

Did you attend the associated tutorial *Potential energy diagrams?* (*Note:* Your response will not affect your grade in any way.)

Required.

- Yes, I attended tutorial.
- No, I did not attend the tutorial.

Question 2.

How much of the tutorial did you complete? (*Note:* Your response will not affect your grade in any way.)

Required.

- None
- Some, but did not reach the last page
- Reached the last page, but did not finish
- Completed the entire tutorial

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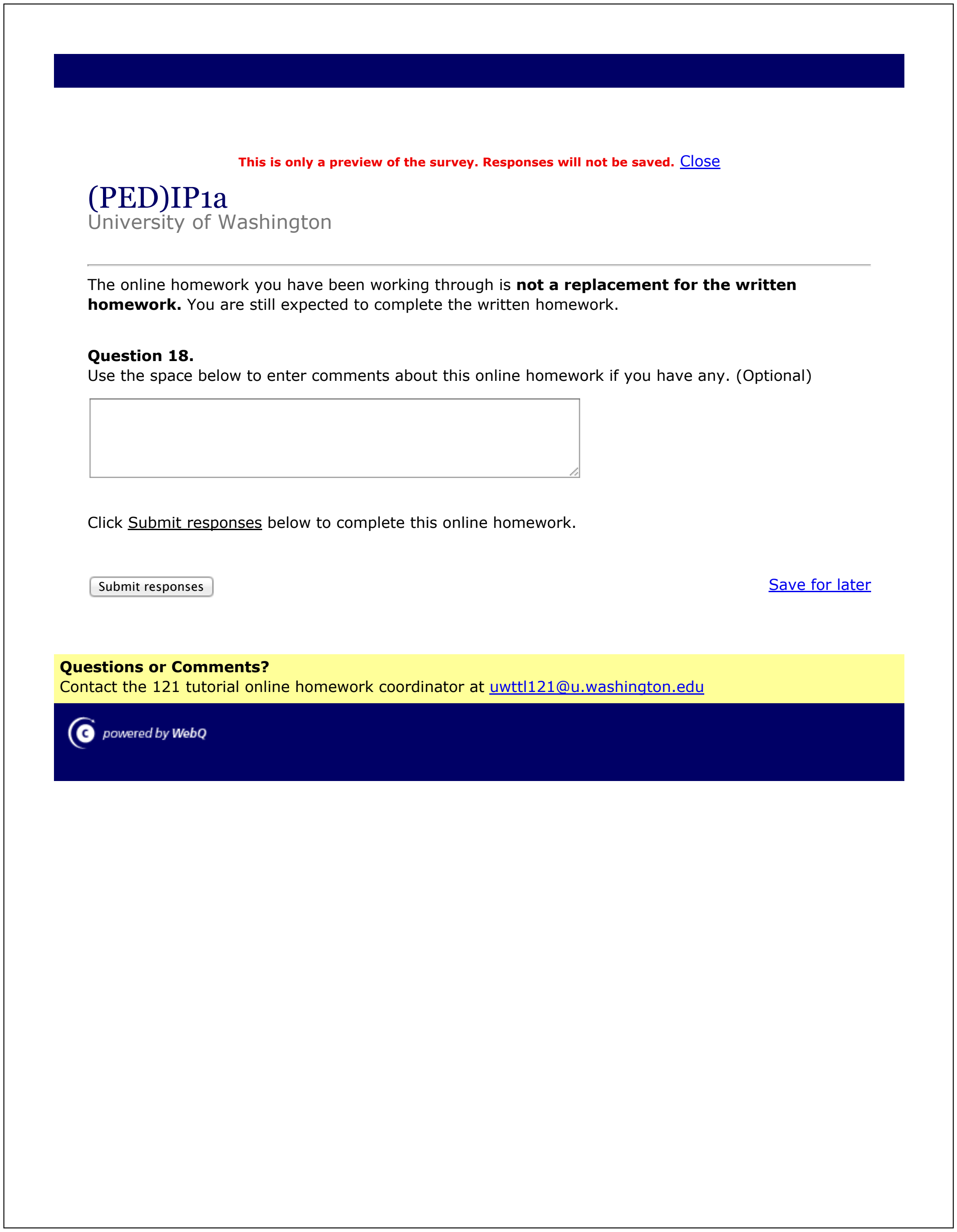
Figure A.42: Preliminary questions of OIPH administered in spring 2014 to Phys 121X. The next page students see is the first PED-related question, which is shown on page 484.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

The online homework you have been working through is **not a replacement for the written homework.** You are still expected to complete the written homework.

Question 18.
Use the space below to enter comments about this online homework if you have any. (Optional)

Click Submit responses below to complete this online homework.

Submit responses

Save for later

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Figure A.43: Final page of the OIPH administered in spring 2014 to Phys 121X. Student answers to the entire OIPH are submitted after pressing "Submit," after which the OIPH ends.

A.2.3-b The six PED-related questions

Below are the six PED-related questions that comprise most of the content of the OIPH.

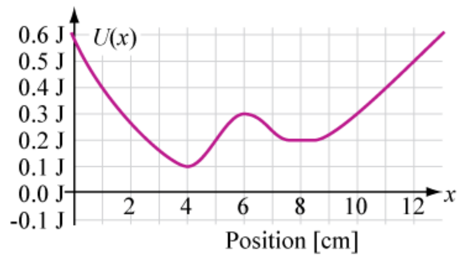
This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

Part I.

For all problems in this online homework, a particle of mass *m* is confined to a one-dimensional system. The solid curve shows the potential energy *U* of the system as a function of the position *x* of the particle.



The only forms of energy in the system are potential energy *U* (shown above) and translational kinetic energy *K* of the particle (not shown).

Assume that the total energy *E* is constant and that the only force on the particle is that associated with the potential energy.

Question 3.

Suppose that the particle is located **at *x* = 2 cm** and is **moving.**

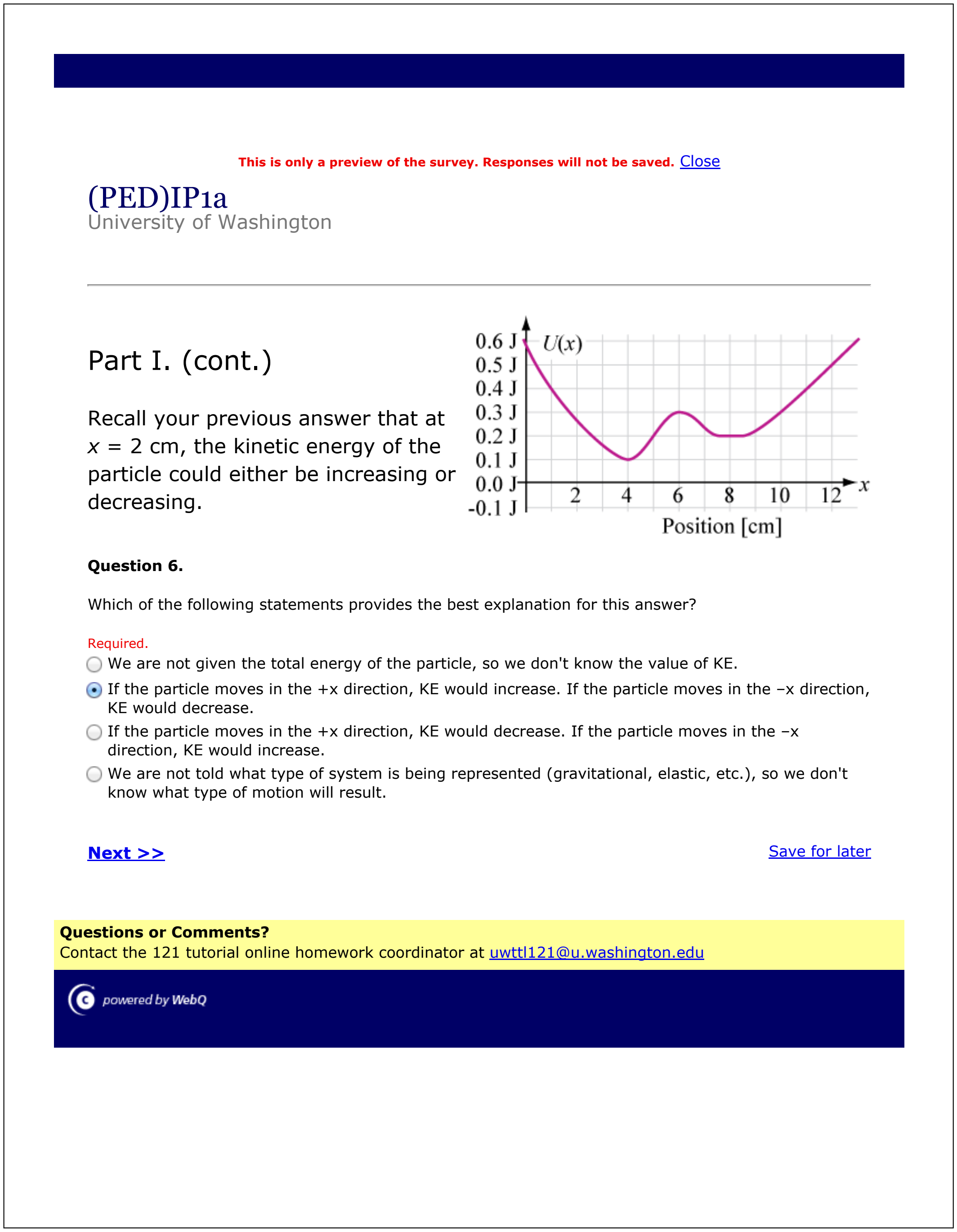
Which of the following best describes how the kinetic energy of the particle is changing?

Required.

- ○ The kinetic energy is increasing.
- ○ The kinetic energy is decreasing.
- ◉ The kinetic energy could be either increasing or decreasing.
- ○ The kinetic energy is not changing.
- ○ The kinetic energy could be increasing, decreasing, or it could not be changing.

Next >> Save for later

Figure A.44: **First** PED-related question of OIPH administered in spring 2014 to PHYS 121X. The correct answer is selected. The next page students see depends on which of the five answers they choose; these pages are, from the top answer downward: 491, 492, 519, 493, 494. After three incorrect attempts students see a guide that is shown on page 512. The second PED-related question is shown on page 485.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

Part I. (cont.)

Recall your previous answer that at x = 2 cm, the kinetic energy of the particle could either be increasing or decreasing.

Question 6.

Which of the following statements provides the best explanation for this answer?

Required.

- ○ We are not given the total energy of the particle, so we don't know the value of KE.
- ◉ If the particle moves in the +x direction, KE would increase. If the particle moves in the −x direction, KE would decrease.
- ○ If the particle moves in the +x direction, KE would decrease. If the particle moves in the −x direction, KE would increase.
- ○ We are not told what type of system is being represented (gravitational, elastic, etc.), so we don't know what type of motion will result.

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Figure A.45: **Second** PED-related question of OIPH administered in spring 2014 to Phys 121X. The correct answer is selected. The next page students see depends on which of the four answers they choose; these pages are, from top answer downward: 495, 520, 496, 497. After three incorrect attempts students see a guide that is shown on page 513. The third PED-related question is shown on page 486.

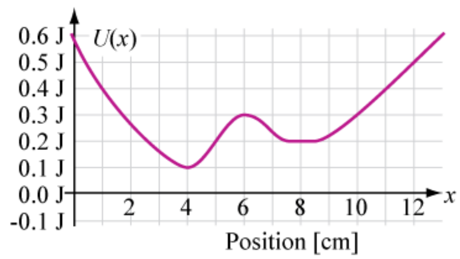
This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

Part II.

Consider again the potential energy diagram shown at right.



Question 8.

Suppose now that the particle were located at ***x* = 5 cm** and is **moving.** (Note that this is a different location than the previous questions.)

Which of the following statements best describes the **direction of the acceleration** of the particle?

Required.

- ○ The acceleration is in the +x-direction.
- ◉ The acceleration is in the −x-direction.
- ○ The acceleration could be in either the +x-direction or −x-direction.
- ○ The acceleration is zero.

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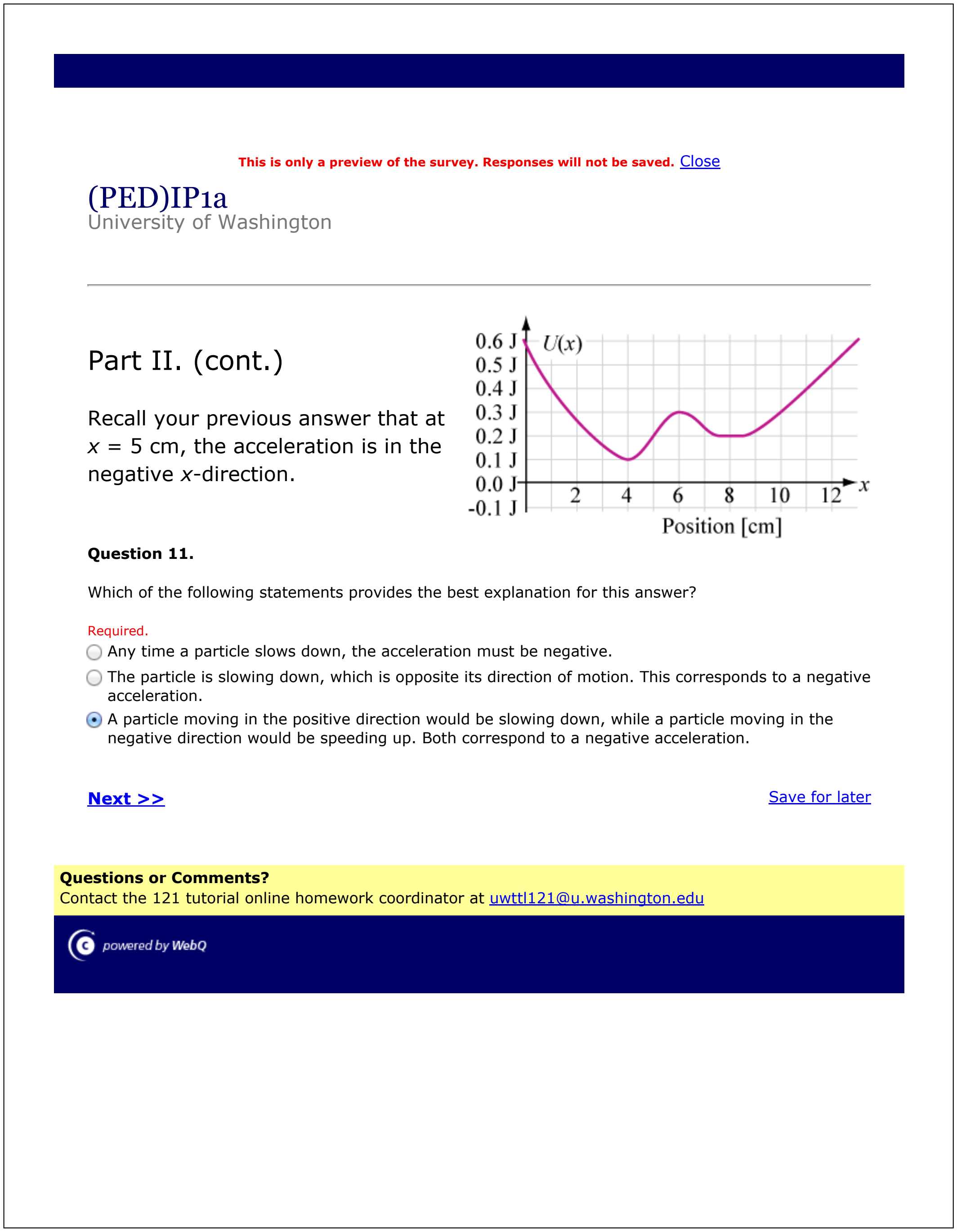
Figure A.46: **Third** PED-related question of OIPH administered in spring 2014 to Phys 121X. The correct answer is selected. The next page students see depends on which of the four answers they choose; these pages are, from top answer downward: 498, 521, 499, 500. After three incorrect attempts students see a guide that is shown on page 514. The fourth PED-related question is shown on page 487.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

Part II. (cont.)

Recall your previous answer that at x = 5 cm, the acceleration is in the negative x-direction.

Question 11.

Which of the following statements provides the best explanation for this answer?

Required.

- Any time a particle slows down, the acceleration must be negative.
- The particle is slowing down, which is opposite its direction of motion. This corresponds to a negative acceleration.
- A particle moving in the positive direction would be slowing down, while a particle moving in the negative direction would be speeding up. Both correspond to a negative acceleration.

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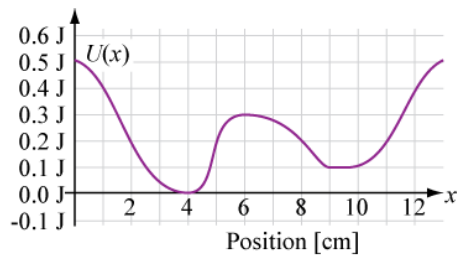
Figure A.47: **Fourth** PED-related question of OIPH administered in spring 2014 to Phys 121X. The correct answer is selected. The next page students see depends on which of the three answers they choose; these pages are, from top answer downward: 501, 502, 522. After three incorrect attempts students see a guide that is shown on page 515. The fifth PED-related question is shown on page 488.

This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

Part III.

Consider now the different potential energy diagram shown at right.



Question 14.

Suppose that the particle were **released from rest at x = 2 cm.**

After being released, at what location will the particle **first** attain a speed of zero?

Required.

- ○ x = 4 cm
- ◉ x = 5 cm
- ○ x = 6 cm
- ○ x = 8 cm
- ○ x = 11 cm

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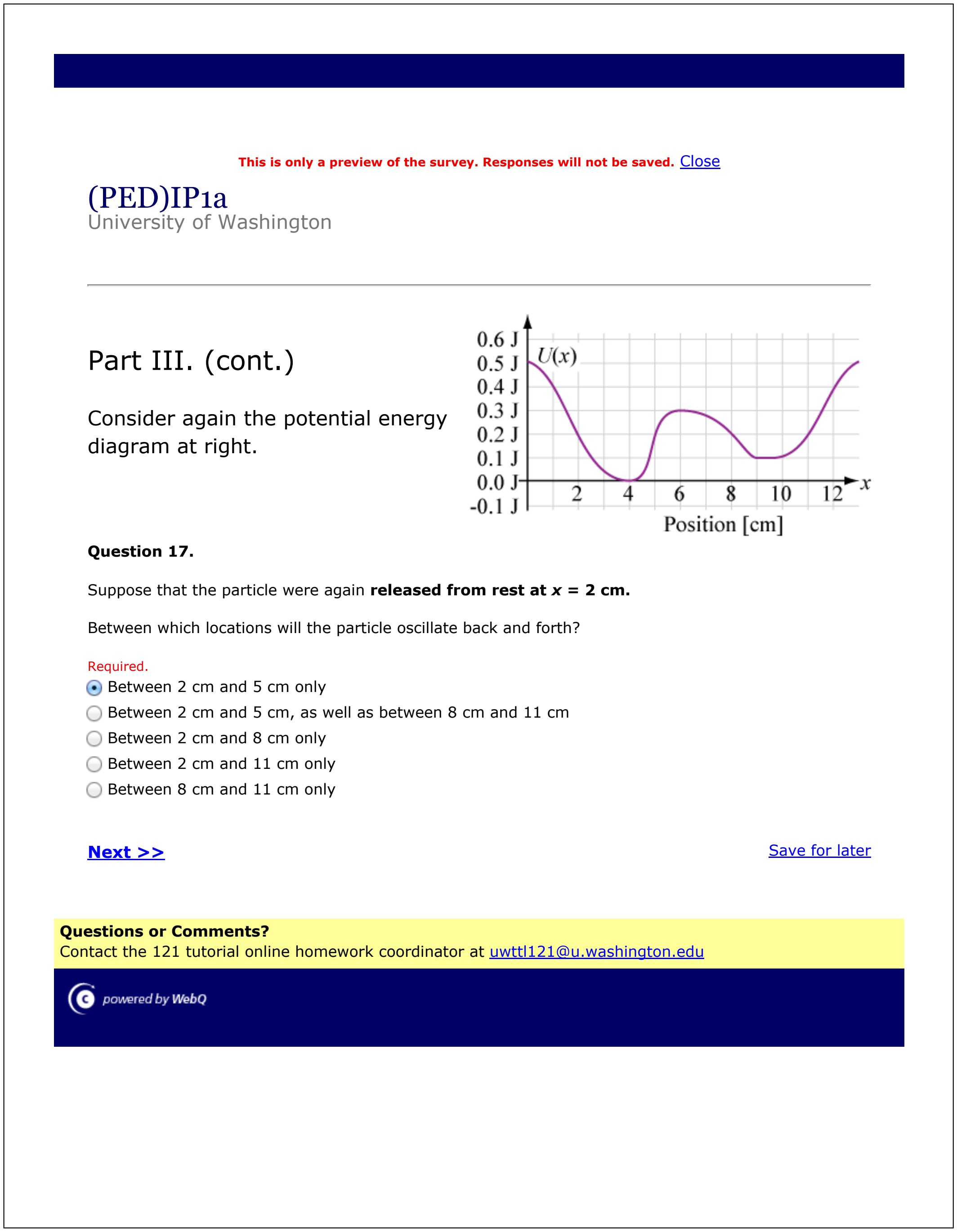
Figure A.48: **Fifth** PED-related question of OIPH administered in spring 2014 to PHYS 121X. The correct answer is selected. The next page students see depends on which of the five answers they choose; these pages are, from top answer downward: 503, 523, 504, 505, 506. After three incorrect attempts students see a guide that is shown on page 516. The sixth and final PED-related question is shown on page 489.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

Part III. (cont.)

Consider again the potential energy diagram at right.

Question 17.

Suppose that the particle were again **released from rest at *x* = 2 cm.**

Between which locations will the particle oscillate back and forth?

Required.
- (selected) Between 2 cm and 5 cm only
- Between 2 cm and 5 cm, as well as between 8 cm and 11 cm
- Between 2 cm and 8 cm only
- Between 2 cm and 11 cm only
- Between 8 cm and 11 cm only

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Figure A.49: **Sixth** and final PED-related question of OIPH administered in spring 2014 to Phys 121X. The correct answer is selected. The next page students see depends on which of the five answers they choose; these pages are, from top answer downward: 524, 507, 508, 509, 510. After three incorrect attempts students see a guide that is shown on page 517. The next page of the OIPH students see is shown on page 482.

A.2.3-c ‘Hint’ pages shown after incorrect attempts

Below are the ‘hint’ pages that are shown to students after selecting an incorrect answer.

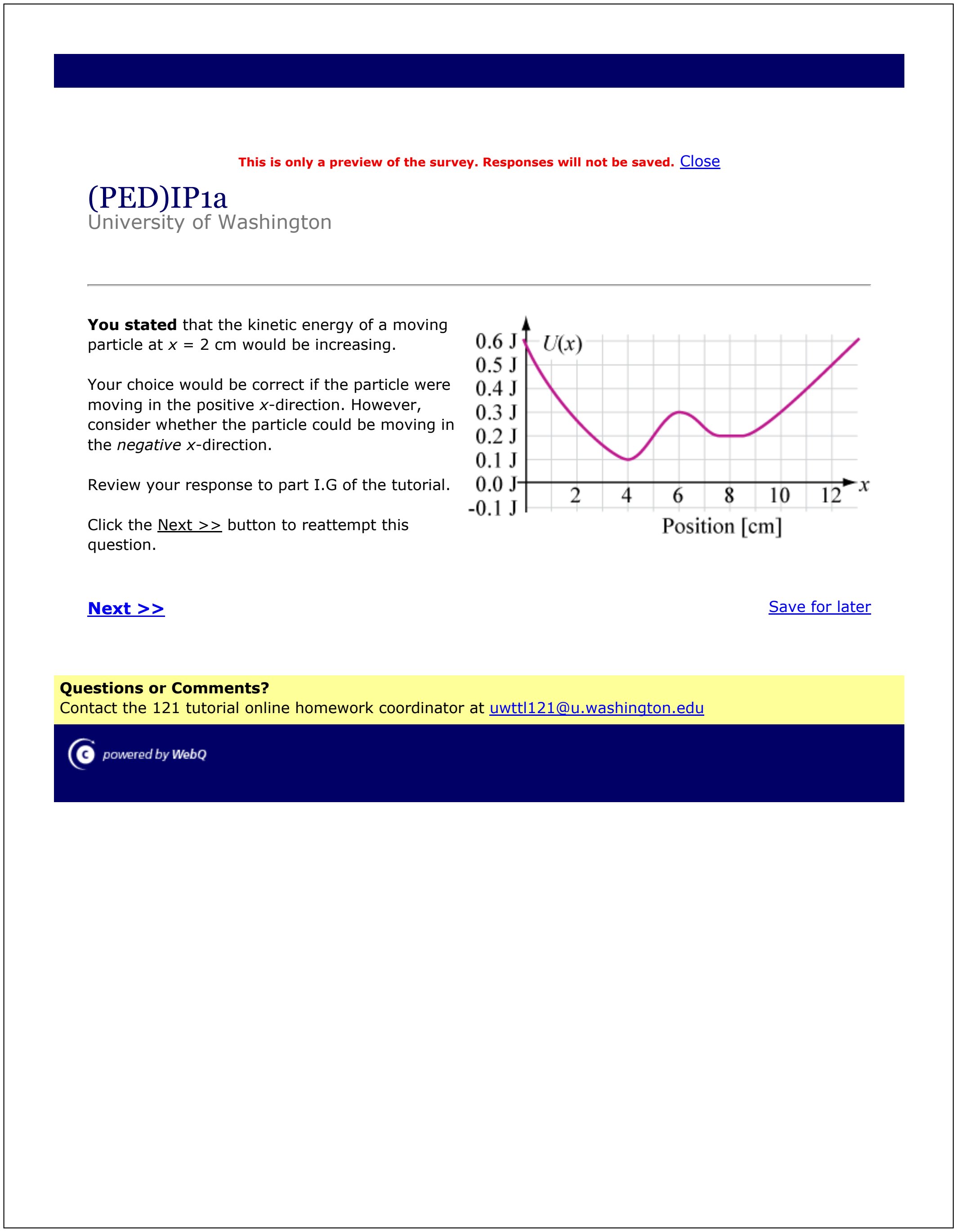


Figure A.50: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the first answer on the first PED-related question, which was shown on page 484. If this is the students' first or second incorrect attempt the next page is the first question again. If this is their third incorrect attempt the next page is the guide shown on page 512.

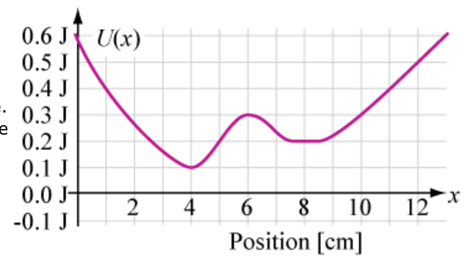
This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

You stated that the kinetic energy of a moving particle at x = 2 cm would be decreasing.

You may have chosen this answer because the slope of the potential energy diagram is negative. However, this only implies that near x = 2 cm the potential energy is becoming smaller for increasing values of x, and larger for decreasing values of x.



Additionally, note that the horizontal axis represents position, not time.

Review parts I.E through I.G of the tutorial.

Click the Next >> button to reattempt this question.

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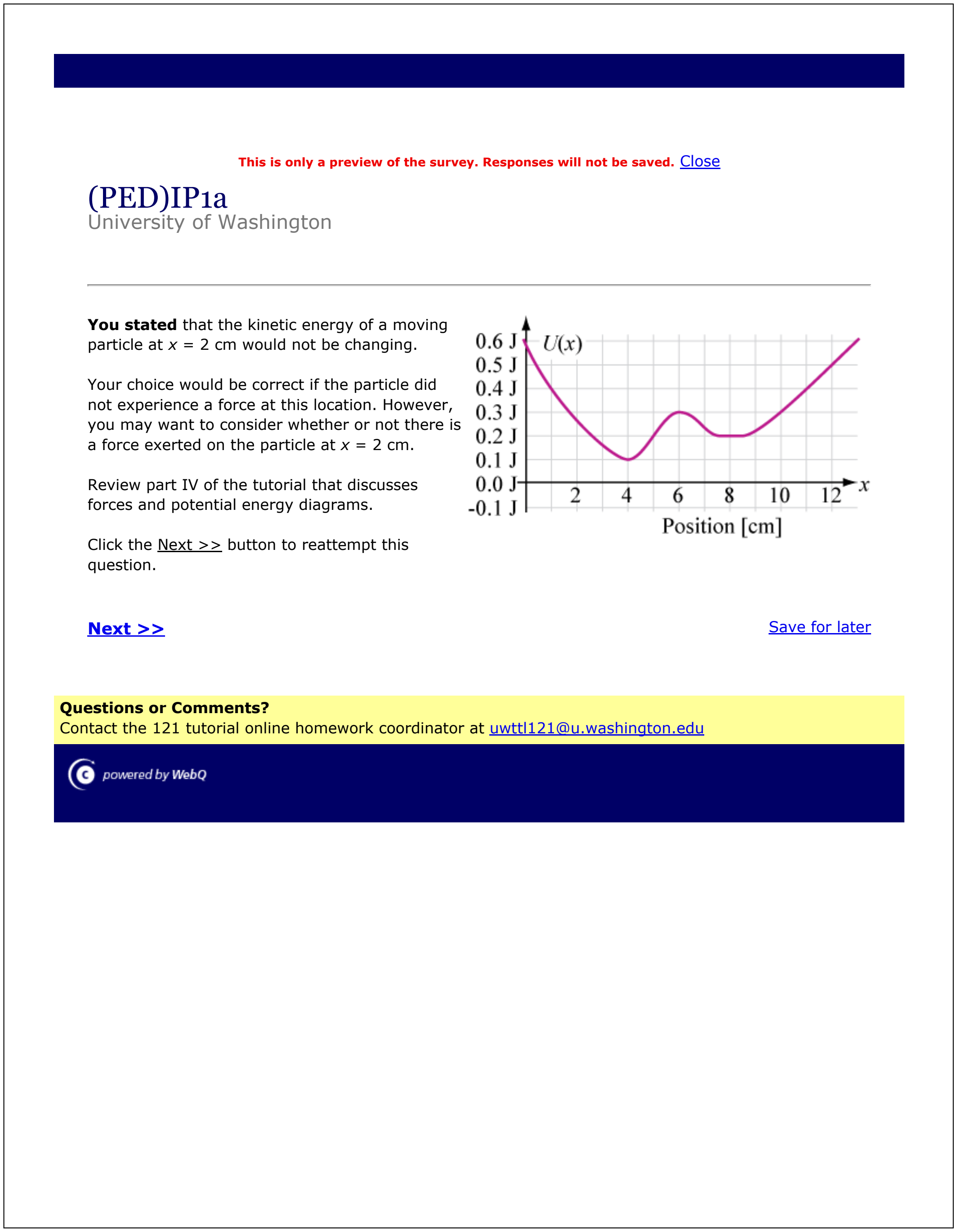
Figure A.51: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the second answer on the first PED-related question, which was shown on page 484. If this is the students' first or second incorrect attempt the next page is the first question again. If this is their third incorrect attempt the next page is the guide shown on page 512.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

You stated that the kinetic energy of a moving particle at x = 2 cm would not be changing.

Your choice would be correct if the particle did not experience a force at this location. However, you may want to consider whether or not there is a force exerted on the particle at x = 2 cm.

Review part IV of the tutorial that discusses forces and potential energy diagrams.

Click the Next >> button to reattempt this question.

Next >>

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Figure A.52: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the fourth answer on the first PED-related question, which was shown on page 484. If this is the students' first or second incorrect attempt the next page is the first question again. If this is their third incorrect attempt the next page is the guide shown on page 512.

This is only a preview of the survey. Responses will not be saved. Close

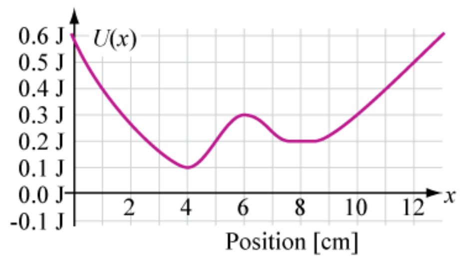
(PED)IP1a
University of Washington

You stated that the kinetic energy of a moving particle at *x* = 2 cm could be increasing, decreasing, or not changing.

Your answer includes the possibility that the kinetic energy is not changing. However, for a particle moving in one dimension, this is only possible if it does not experience a force. You may want to consider whether or not there is a force exerted on the particle at *x* = 2 cm.

Review part IV of the tutorial that discusses forces and potential energy diagrams.

Click the Next >> button to reattempt this question.



Next >>

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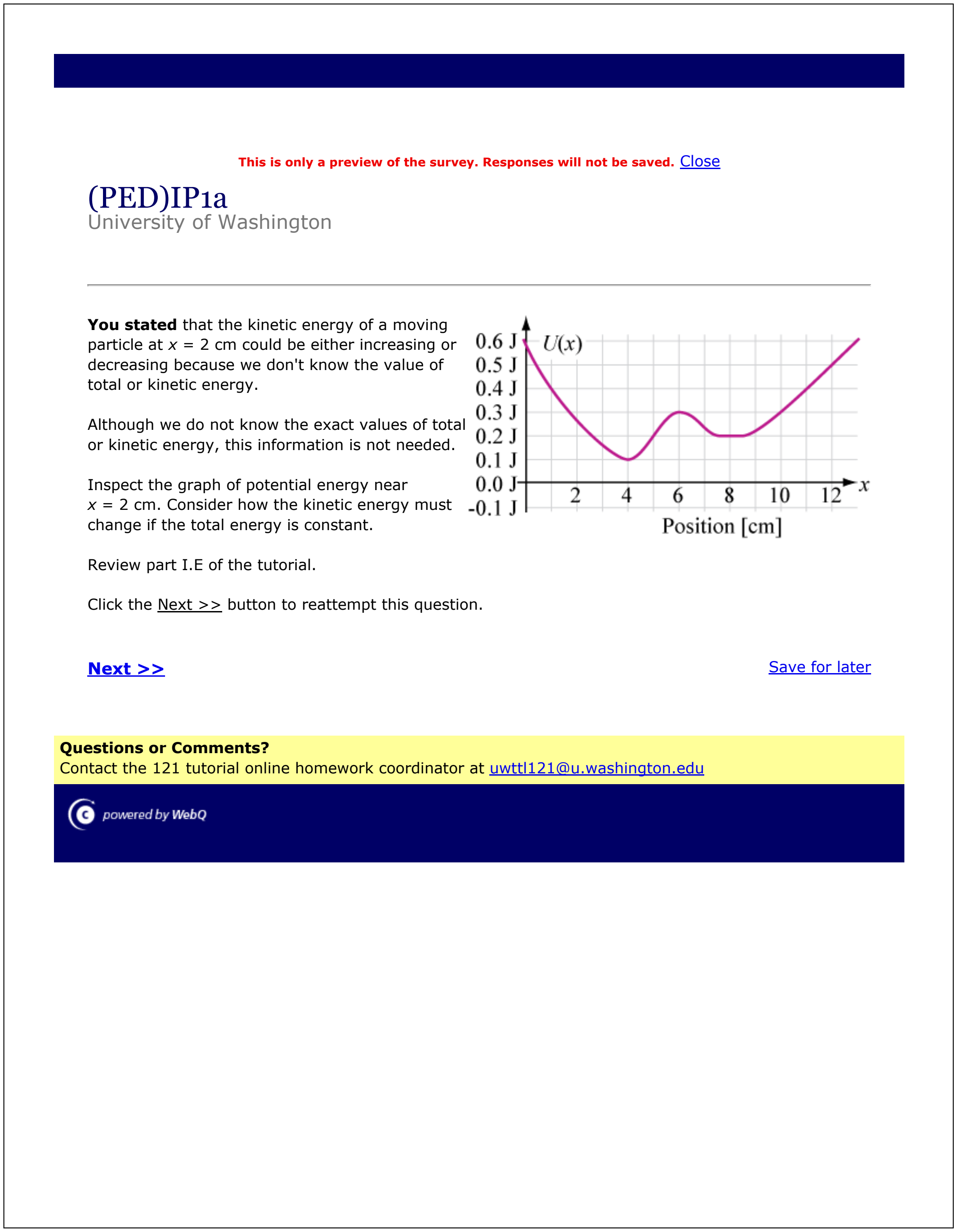
Figure A.53: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the fifth answer on the first PED-related question, which was shown on page 484. If this is the students' first or second incorrect attempt the next page is the first question again. If this is their third incorrect attempt the next page is the guide shown on page 512.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

You stated that the kinetic energy of a moving particle at *x* = 2 cm could be either increasing or decreasing because we don't know the value of total or kinetic energy.

Although we do not know the exact values of total or kinetic energy, this information is not needed.

Inspect the graph of potential energy near *x* = 2 cm. Consider how the kinetic energy must change if the total energy is constant.

Review part I.E of the tutorial.

Click the Next >> button to reattempt this question.

Next >>

Save for later

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Figure A.54: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the first answer on the second PED-related question, which was shown on page 485. If this is the students' first or second incorrect attempt the next page is the second question again. If this is their third incorrect attempt the next page is the guide shown on page 513.

This is only a preview of the survey. Responses will not be saved. Close

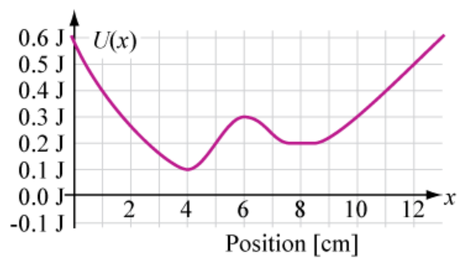
(PED)IP1a

University of Washington

You stated that the kinetic energy of a moving particle at *x* = 2 cm could be either increasing or decreasing because:

- The kinetic energy would decrease if the particle were moving in the +x-direction
- The kinetic energy would increase if the particle were moving in the −x-direction.

Recall, however, that the graph shows *potential* energy, not kinetic energy.



Review part I.E of the tutorial.

Click the Next >> button to reattempt this question.

Next >>

Save for later

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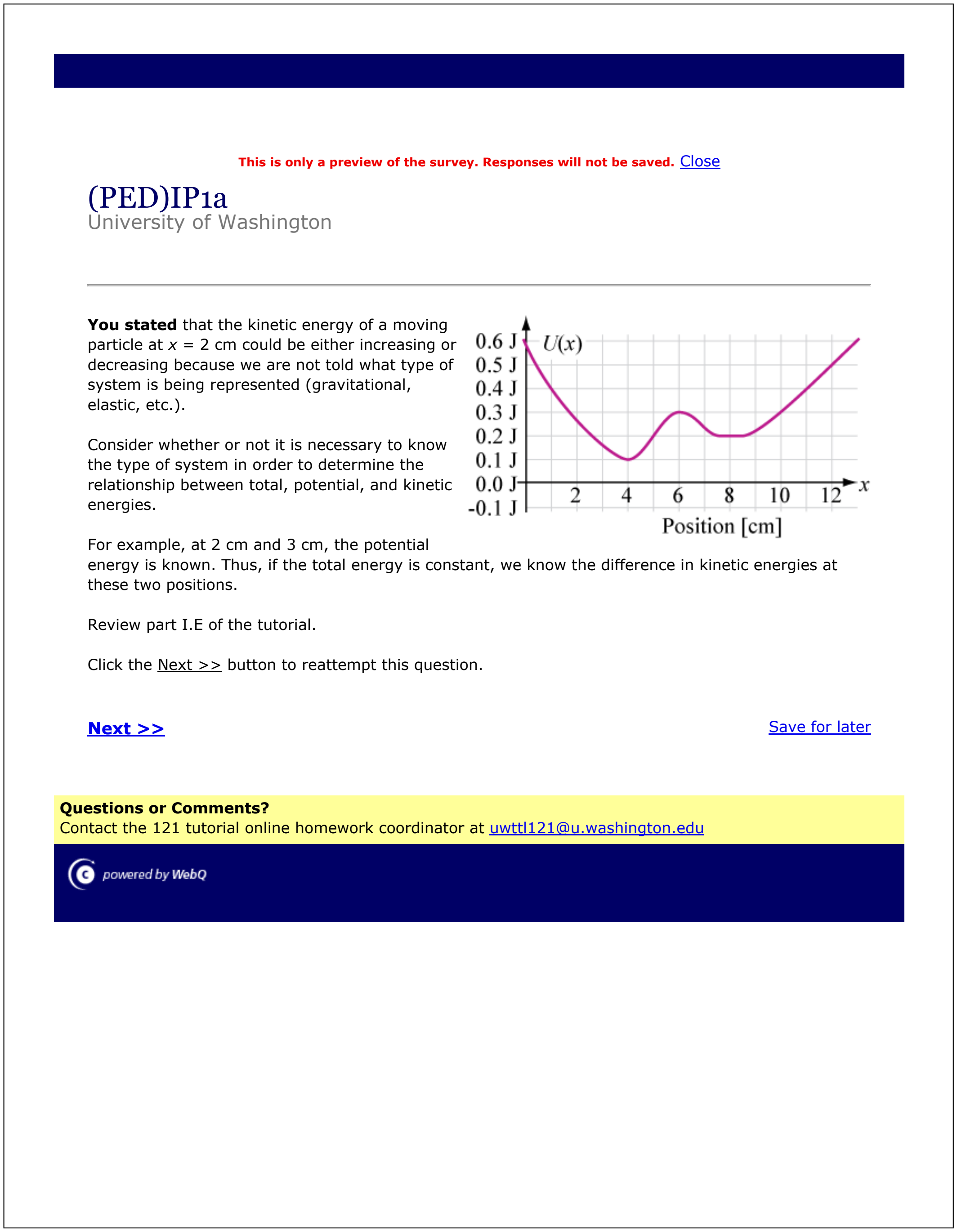
Figure A.55: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the third answer on the second PED-related question, which was shown on page 485. If this is the students' first or second incorrect attempt the next page is the second question again. If this is their third incorrect attempt the next page is the guide shown on page 513.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

You stated that the kinetic energy of a moving particle at *x* = 2 cm could be either increasing or decreasing because we are not told what type of system is being represented (gravitational, elastic, etc.).

Consider whether or not it is necessary to know the type of system in order to determine the relationship between total, potential, and kinetic energies.

For example, at 2 cm and 3 cm, the potential energy is known. Thus, if the total energy is constant, we know the difference in kinetic energies at these two positions.

Review part I.E of the tutorial.

Click the Next >> button to reattempt this question.

Next >> Save for later

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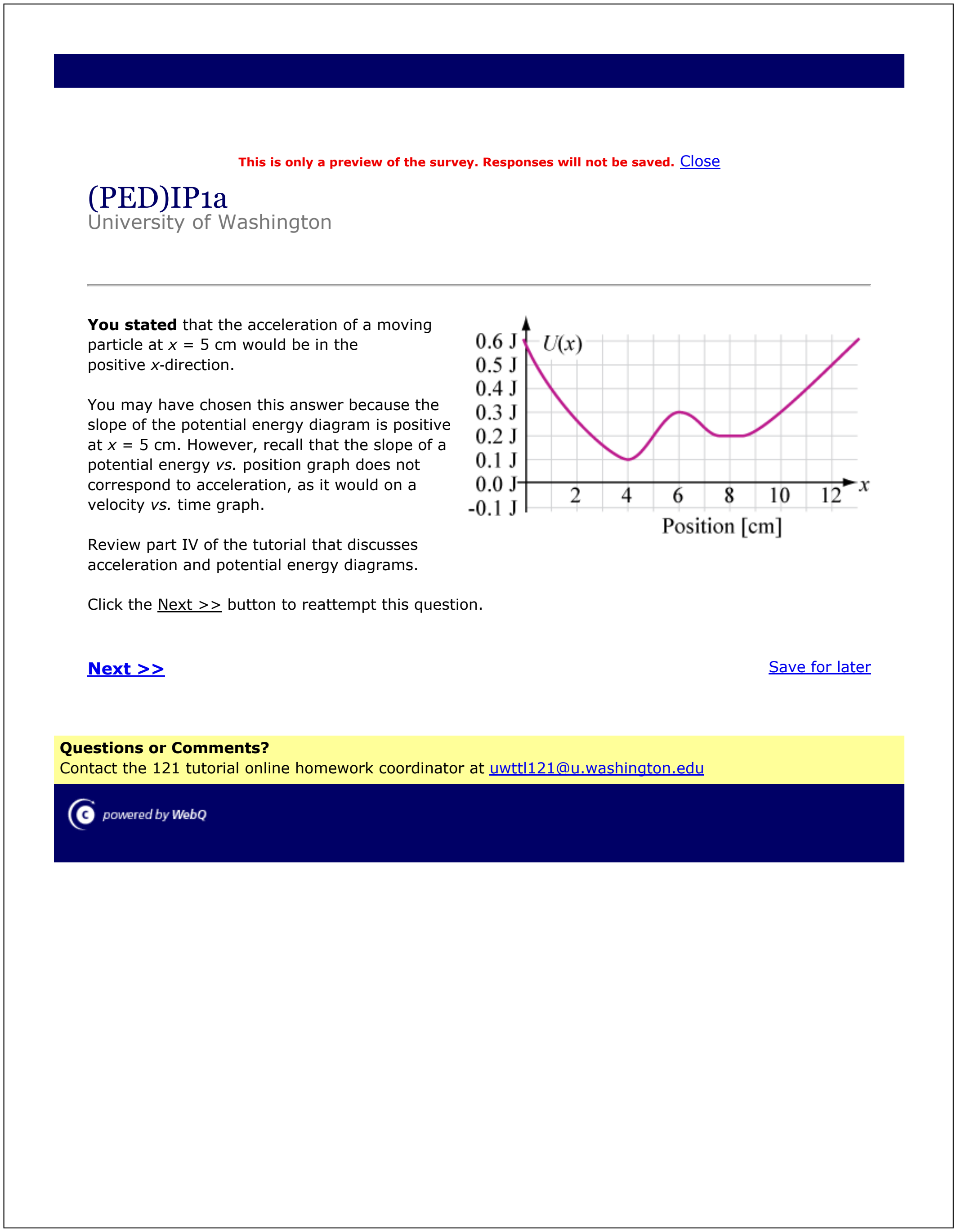
Figure A.56: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the fourth answer on the second PED-related question, which was shown on page 485. If this is the students' first or second incorrect attempt the next page is the second question again. If this is their third incorrect attempt the next page is the guide shown on page 513.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

You stated that the acceleration of a moving particle at *x* = 5 cm would be in the positive *x*-direction.

You may have chosen this answer because the slope of the potential energy diagram is positive at *x* = 5 cm. However, recall that the slope of a potential energy *vs.* position graph does not correspond to acceleration, as it would on a velocity *vs.* time graph.

Review part IV of the tutorial that discusses acceleration and potential energy diagrams.

Click the Next >> button to reattempt this question.

Next >> Save for later

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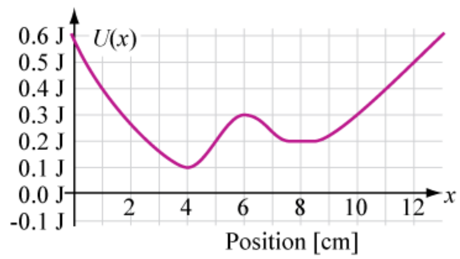
Figure A.57: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the first answer on the third PED-related question, which was shown on page 486. If this is the students' first or second incorrect attempt the next page is the third question again. If this is their third incorrect attempt the next page is the guide shown on page 514.

This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

You stated that the acceleration of a moving particle at $x = 5$ cm could be in either the +x-direction or the −x-direction.



You may have chosen this answer because the direction of motion of the particle cannot be determined. (Recall Part I of this online homework.) However, it may still be possible to know the direction of acceleration even without the direction of motion.

Consider what the direction of the acceleration *would* be if:

- the particle were moving in the +x-direction
- the particle were moving in the −x-direction

It may also help to review part IV of the tutorial that discusses acceleration and potential energy diagrams.

Click the Next >> button to reattempt this question.

Next >> Save for later

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Figure A.58: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the third answer on the third PED-related question, which was shown on page 486. If this is the students' first or second incorrect attempt the next page is the third question again. If this is their third incorrect attempt the next page is the guide shown on page 514.

This is only a preview of the survey. Responses will not be saved. Close

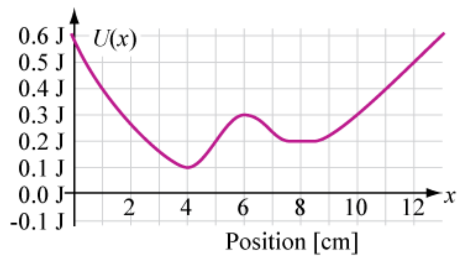
(PED)IP1a

University of Washington

You stated that the acceleration of a moving particle at x = 5 cm would be zero.

You may have chosen this answer because the second derivative of the graph at x = 5 cm is zero. However, the second derivative of a potential energy *vs.* position graph does not correspond to acceleration, as it would on a position *vs.* time graph.

Review part IV of the tutorial that discusses acceleration and potential energy diagrams.



Click the Next >> button to reattempt this question.

Next >> Save for later

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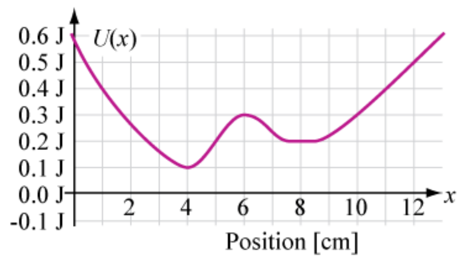
Figure A.59: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the fourth answer on the third PED-related question, which was shown on page 486. If this is the students' first or second incorrect attempt the next page is the third question again. If this is their third incorrect attempt the next page is the guide shown on page 514.

This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

You stated that the acceleration of a moving particle at *x* = 5 cm would be in the negative *x*-direction because any time a particle slows down the acceleration must be negative.



Recall that the *sign* one attributes to acceleration corresponds to a *direction.*

It is possible, for example, for an object to be speeding up but have negative acceleration. This is the case for a ball being dropped from rest (assuming upward is the positive direction) since the direction of motion (negative/downward) is in the same direction of acceleration (negative/downward).

Review part IV of the tutorial that discusses acceleration and potential energy diagrams.

Click the Next >> button to reattempt this question.

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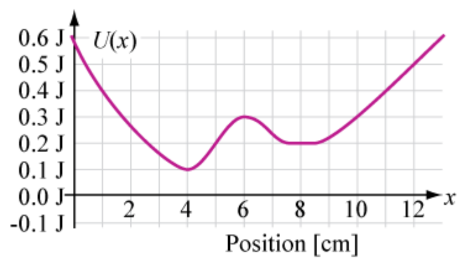
Figure A.60: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the first answer on the fourth PED-related question, which was shown on page 487. If this is the students' first or second incorrect attempt the next page is the fourth question again. If this is their third incorrect attempt the next page is the guide shown on page 515.

This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

You stated that the acceleration of a moving particle at x = 5 cm would be in the negative x-direction because the particle is slowing down, which is opposite its direction of motion.



You may have chosen this answer because you assumed that the particle is moving in the positive x-direction. However, the direction of motion is not given.

Review part IV of the tutorial that discusses acceleration and potential energy diagrams.

Click the Next >> button to reattempt this question.

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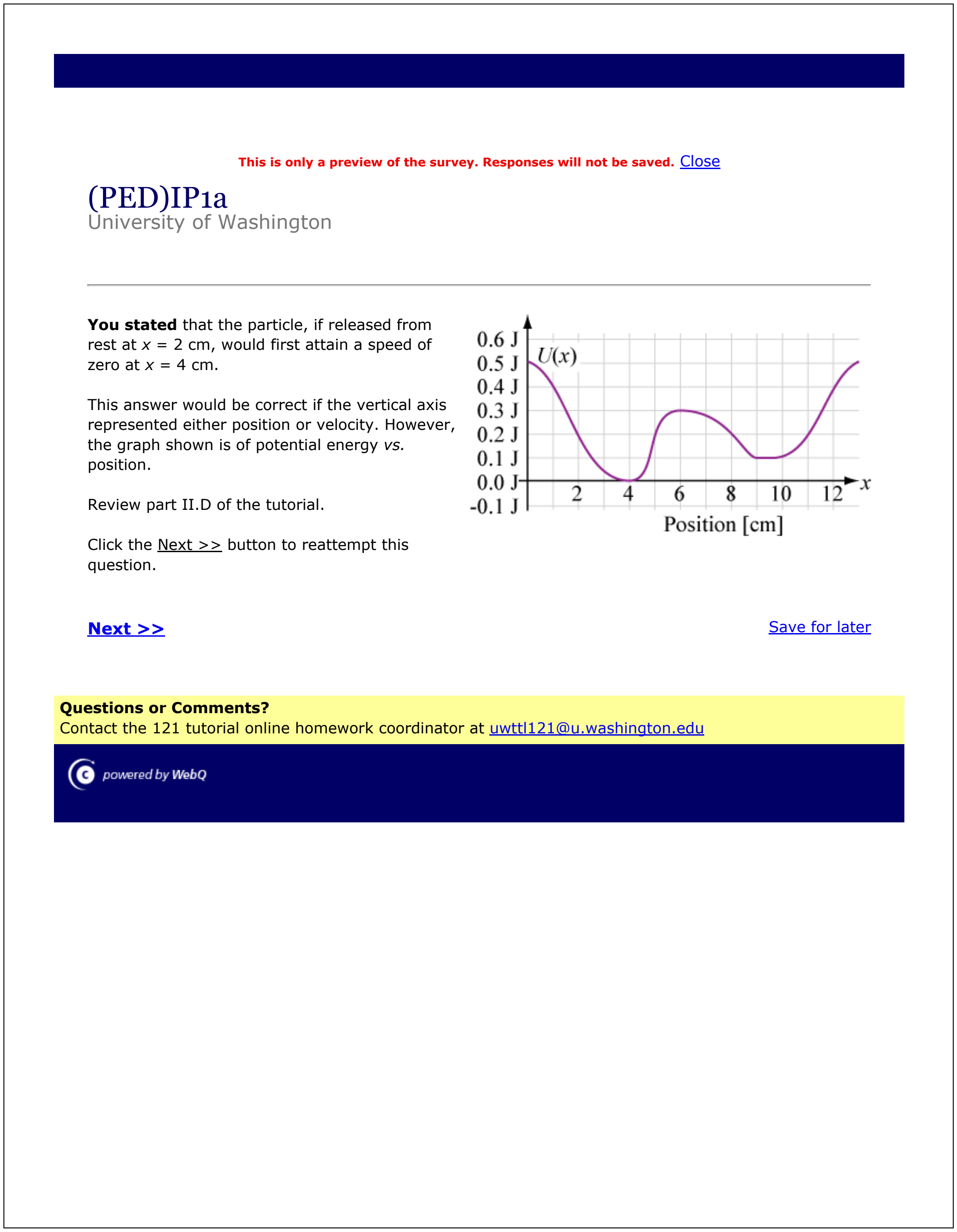
Figure A.61: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the second answer on the fourth PED-related question, which was shown on page 487. If this is the students' first or second incorrect attempt the next page is the fourth question again. If this is their third incorrect attempt the next page is the guide shown on page 515.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

You stated that the particle, if released from rest at x = 2 cm, would first attain a speed of zero at x = 4 cm.

This answer would be correct if the vertical axis represented either position or velocity. However, the graph shown is of potential energy *vs.* position.

Review part II.D of the tutorial.

Click the Next >> button to reattempt this question.

Next >> Save for later

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Figure A.62: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the first answer on the fifth PED-related question, which was shown on page 488. If this is the students' first or second incorrect attempt the next page is the fifth question again. If this is their third incorrect attempt the next page is the guide shown on page 516.

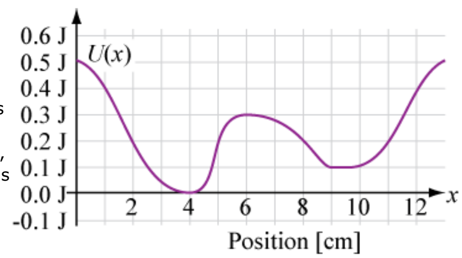
This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

You stated that the particle, if released from rest at x = 2 cm, would first attain a speed of zero at x = 6 cm.

You may have chosen this answer because at this location the graph of $U(x)$ is at a local maximum. Consider instead the relationship between kinetic, potential, and total energies. (*Hint:* If the speed is zero, what is the value of kinetic energy? What does this imply about the relationship between total and potential energies?)



Review part II.D of the tutorial.

Click the Next >> button to reattempt this question.

Next >>

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Figure A.63: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the third answer on the fifth PED-related question, which was shown on page 488. If this is the students' first or second incorrect attempt the next page is the fifth question again. If this is their third incorrect attempt the next page is the guide shown on page 516.

This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

You stated that the particle, if released from rest at *x* = 2 cm, would first attain a speed of zero at *x* = 8 cm.

You may have chosen this answer because at this location the kinetic energy is zero. However, this is not the *first* location where this occurs.

Review part II.D of the tutorial. Then consider what the value of kinetic energy is at several locations along the path of the particle, starting from where the particle is released from rest.

Click the Next >> button to reattempt this question.

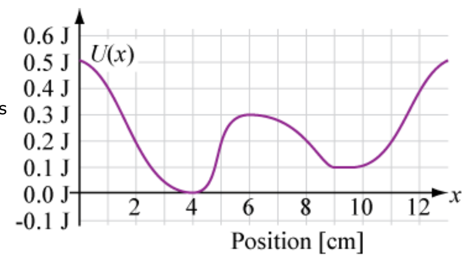

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Figure A.64: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the fourth answer on the fifth PED-related question, which was shown on page 488. If this is the students' first or second incorrect attempt the next page is the fifth question again. If this is their third incorrect attempt the next page is the guide shown on page 516.

This is only a preview of the survey. Responses will not be saved. Close

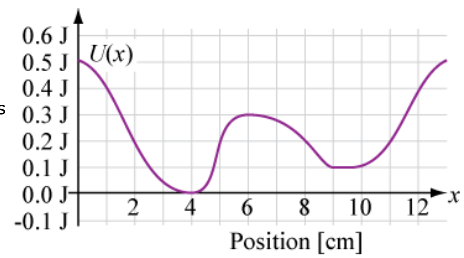
(PED)IP1a

University of Washington

You stated that the particle, if released from rest at *x* = 2 cm, would first attain a speed of zero at *x* = 11 cm.

You may have chosen this answer because at this location the kinetic energy is zero. However, this is not the *first* location where this occurs.

Review part II.D of the tutorial. Then consider what the value of kinetic energy is at several locations along the path of the particle, starting from where the particle is released from rest.



Click the Next >> button to reattempt this question.

Next >>

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Figure A.65: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the fifth answer on the fifth PED-related question, which was shown on page 488. If this is the students' first or second incorrect attempt the next page is the fifth question again. If this is their third incorrect attempt the next page is the guide shown on page 516.

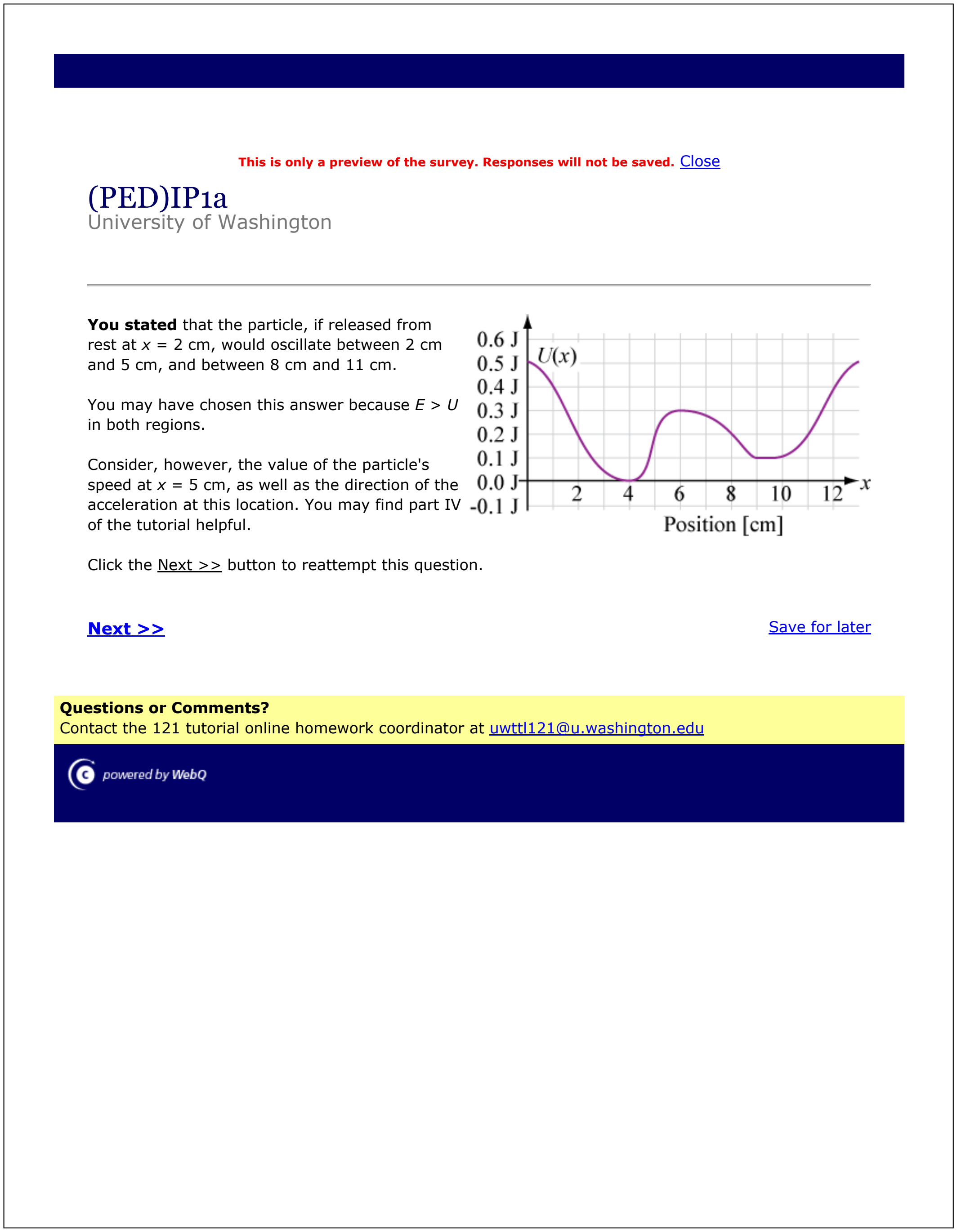


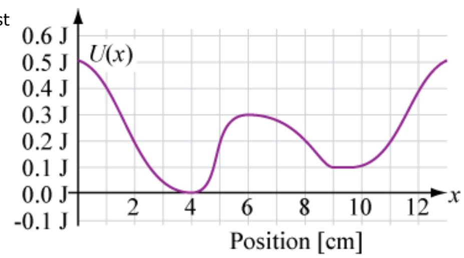
Figure A.66: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the second answer on the sixth PED-related question, which was shown on page 489. If this is the students' first or second incorrect attempt the next page is the sixth question again. If this is their third incorrect attempt the next page is the guide shown on page 517.

This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

You stated that the particle, if released from rest at x = 2 cm, would oscillate between 2 cm and 8 cm only.



You may have chosen this answer because the total energy E is equal to the potential energy U at both of these locations. However, consider the direction of the force on the particle at both of these locations. The directions are not consistent with oscillation between these two locations.

Review part II.D and part IV of the tutorial.

Click the Next >> button to reattempt this question.

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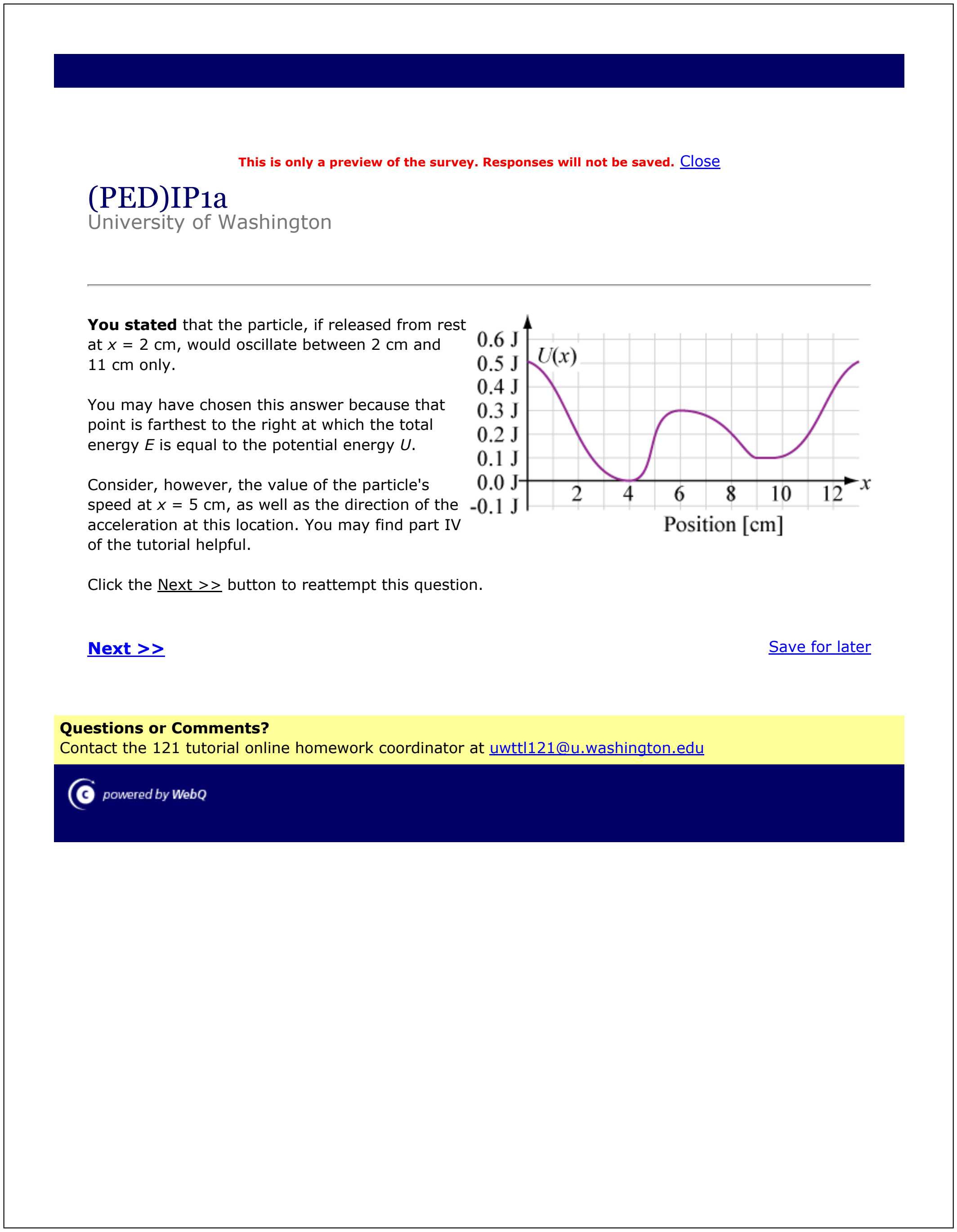
Figure A.67: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the third answer on the sixth PED-related question, which was shown on page 489. If this is the students' first or second incorrect attempt the next page is the sixth question again. If this is their third incorrect attempt the next page is the guide shown on page 517.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

You stated that the particle, if released from rest at x = 2 cm, would oscillate between 2 cm and 11 cm only.

You may have chosen this answer because that point is farthest to the right at which the total energy E is equal to the potential energy U.

Consider, however, the value of the particle's speed at x = 5 cm, as well as the direction of the acceleration at this location. You may find part IV of the tutorial helpful.

Click the Next >> button to reattempt this question.

Next >> Save for later

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Figure A.68: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the fourth answer on the sixth PED-related question, which was shown on page 489. If this is the students' first or second incorrect attempt the next page is the sixth question again. If this is their third incorrect attempt the next page is the guide shown on page 517.

This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

You stated that the particle, if released from rest at x = 2 cm, would oscillate between 8 cm and 11 cm only.

You may have chosen this answer because the total energy E is equal to the potential energy U at both of these locations. However, recall that the particle is released from x = 2 cm.

Consider whether or not the location where the particle is released from rest should be included in the region where it oscillates back and forth.

Click the Next >> button to reattempt this question.

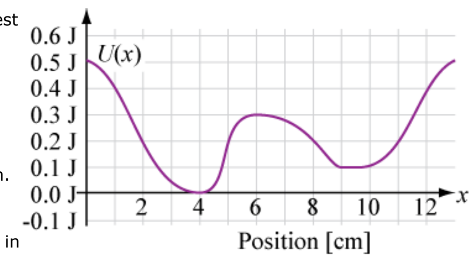

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Figure A.69: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *incorrectly* select the fifth answer on the sixth PED-related question, which was shown on page 489. If this is the students' first or second incorrect attempt the next page is the sixth question again. If this is their third incorrect attempt the next page is the guide shown on page 517.

A.2.3-d Answer ‘guides’ shown after three incorrect attempts

Below are the ‘guide’ pages shown to students after they answer incorrectly three times. There is one guide page for each of the six questions.

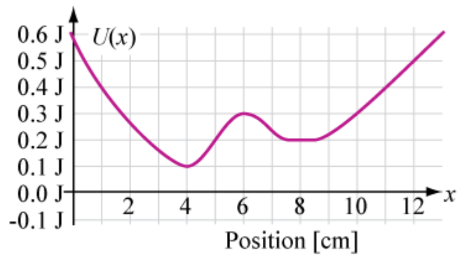
This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

The following description is a guide to help you understand the reasoning required to answer this question:

> *Suppose that the particle is located at x = 2 cm and is moving. How is the kinetic energy of the particle is changing?*



At x = 2 cm, there is a force on the moving particle. (See part IV of the tutorial.) Therefore, the kinetic energy of the particle must be changing.

The particle could be moving either in the positive or negative x-directions.

- If it were moving in the positive direction, the potential energy is decreasing, so the kinetic energy is increasing.
- However, if it were moving in the negative direction, the potential energy is increasing, so the kinetic energy is decreasing.

Therefore, either of the two possibilities above could be true.

On the next page, you will be asked to summarize the reasoning used to arrive at the correct answer.

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Figure A.70: Part of the OIPH administered in spring 2014 to Phys 121X. This "guide" is shown when students make three incorrect attempts on the first PED-related question, shown on page 484. The next page students see is the second question, shown on page 485.

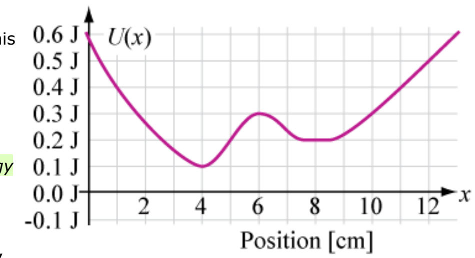
This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

The following description is a guide to help you understand the reasoning required to answer this prompt:

> *Suppose that the particle is located at x = 2 cm and is moving. Explain the reasoning used to arrive at the following conclusion: At x = 2 cm, the kinetic energy could either be increasing or decreasing.*



At x = 2 cm, there is a force on the moving particle. (See part IV of the tutorial.) Therefore, the kinetic energy of the particle must be changing.

The particle could be moving either in the positive or negative x-directions.

- If it were moving in the positive direction, the potential energy is decreasing, so the kinetic energy is increasing.
- However, if it were moving in the negative direction, the potential energy is increasing, so the kinetic energy is decreasing.

Therefore, either of the two possibilities above could be true.

Click the Next >> button to proceed.

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Figure A.71: Part of the OIPH administered in spring 2014 to Phys 121X. This "guide" is shown when students make three incorrect attempts on the second PED-related question, shown on page 485. The next page students see is the third question, shown on page 486.

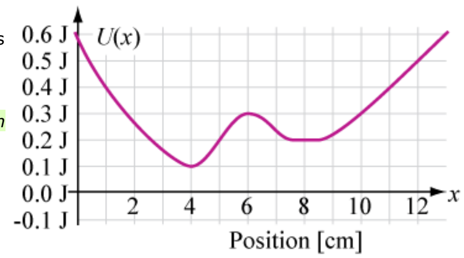
This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

The following description is a guide to help you understand the reasoning required to answer this question:

> *Suppose that the particle is located at x = 5 cm and moving. What is the direction of the acceleration of the particle?*



There are at least two approaches for this problem. One approach is to use the direction of the force on the particle, along with Newton's second law, to deduce the direction of the acceleration. (See part IV of the tutorial.)

The other approach is determine whether the particle is slowing down or speeding up based on its direction of motion:

- If the particle were moving in the positive direction at x = 5 cm, it would be slowing down. Thus, the acceleration is opposite its direction of motion (negative x-direction).
- If the particle were moving in the negative direction at x = 5 cm, it would be speeding up. Thus, the acceleration is parallel to its direction of motion (negative x-direction).

In both cases, we arrive at the same answer.

On the next page, you will be asked to summarize the reasoning used to arrive at the correct answer.

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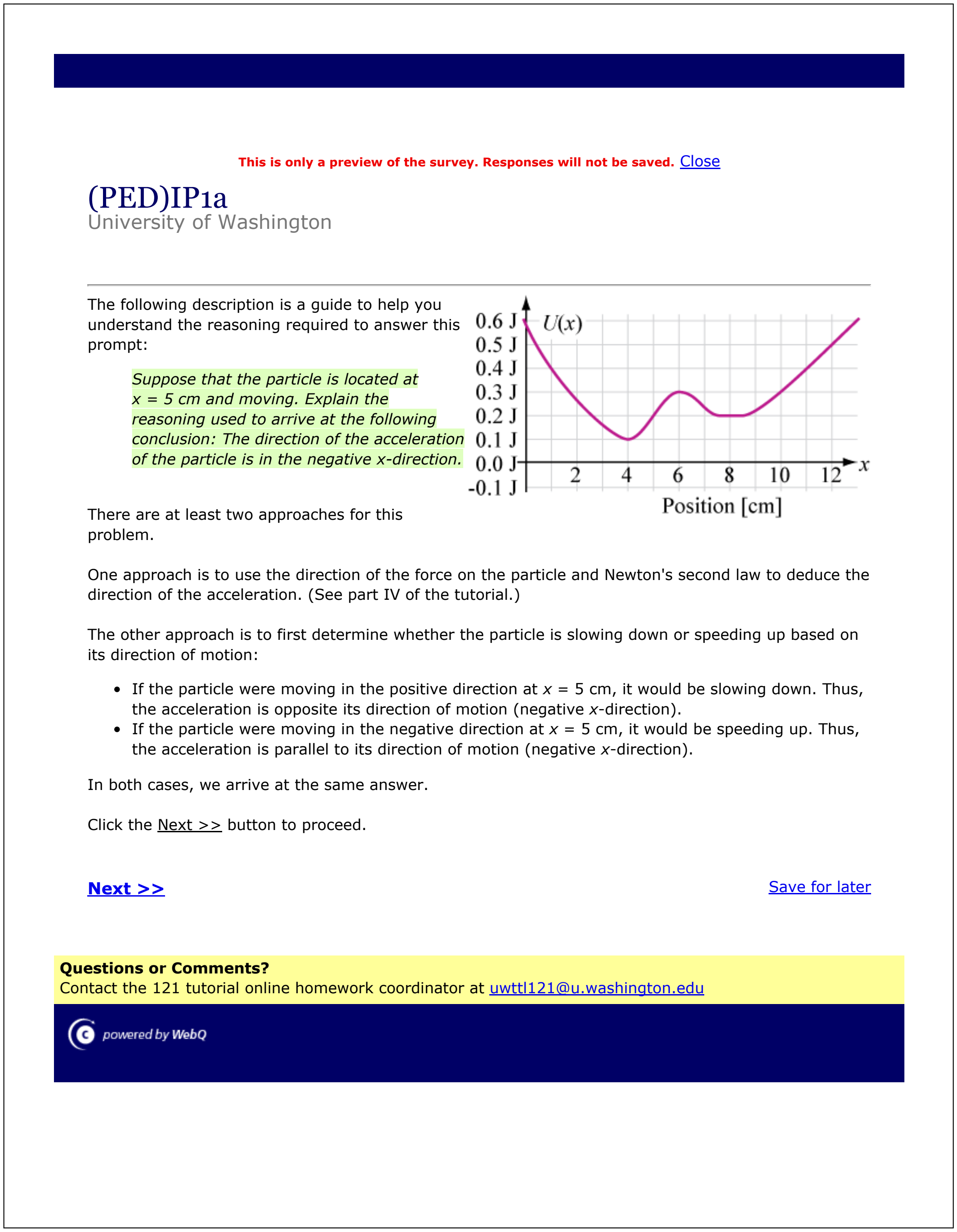
Figure A.72: Part of the OIPH administered in spring 2014 to Phys 121X. This "guide" is shown when students make three incorrect attempts on the third PED-related question, shown on page 486. The next page students see is the fourth question, shown on page 487.



This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

The following description is a guide to help you understand the reasoning required to answer this prompt:

> *Suppose that the particle is located at x = 5 cm and moving. Explain the reasoning used to arrive at the following conclusion: The direction of the acceleration of the particle is in the negative x-direction.*

There are at least two approaches for this problem.

One approach is to use the direction of the force on the particle and Newton's second law to deduce the direction of the acceleration. (See part IV of the tutorial.)

The other approach is to first determine whether the particle is slowing down or speeding up based on its direction of motion:

- If the particle were moving in the positive direction at x = 5 cm, it would be slowing down. Thus, the acceleration is opposite its direction of motion (negative x-direction).
- If the particle were moving in the negative direction at x = 5 cm, it would be speeding up. Thus, the acceleration is parallel to its direction of motion (negative x-direction).

In both cases, we arrive at the same answer.

Click the Next >> button to proceed.

Next >> Save for later

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Figure A.73: Part of the OIPH administered in spring 2014 to Phys 121X. This "guide" is shown when students make three incorrect attempts on the fourth PED-related question, shown on page 487. The next page students see is the fifth question, shown on page 488.

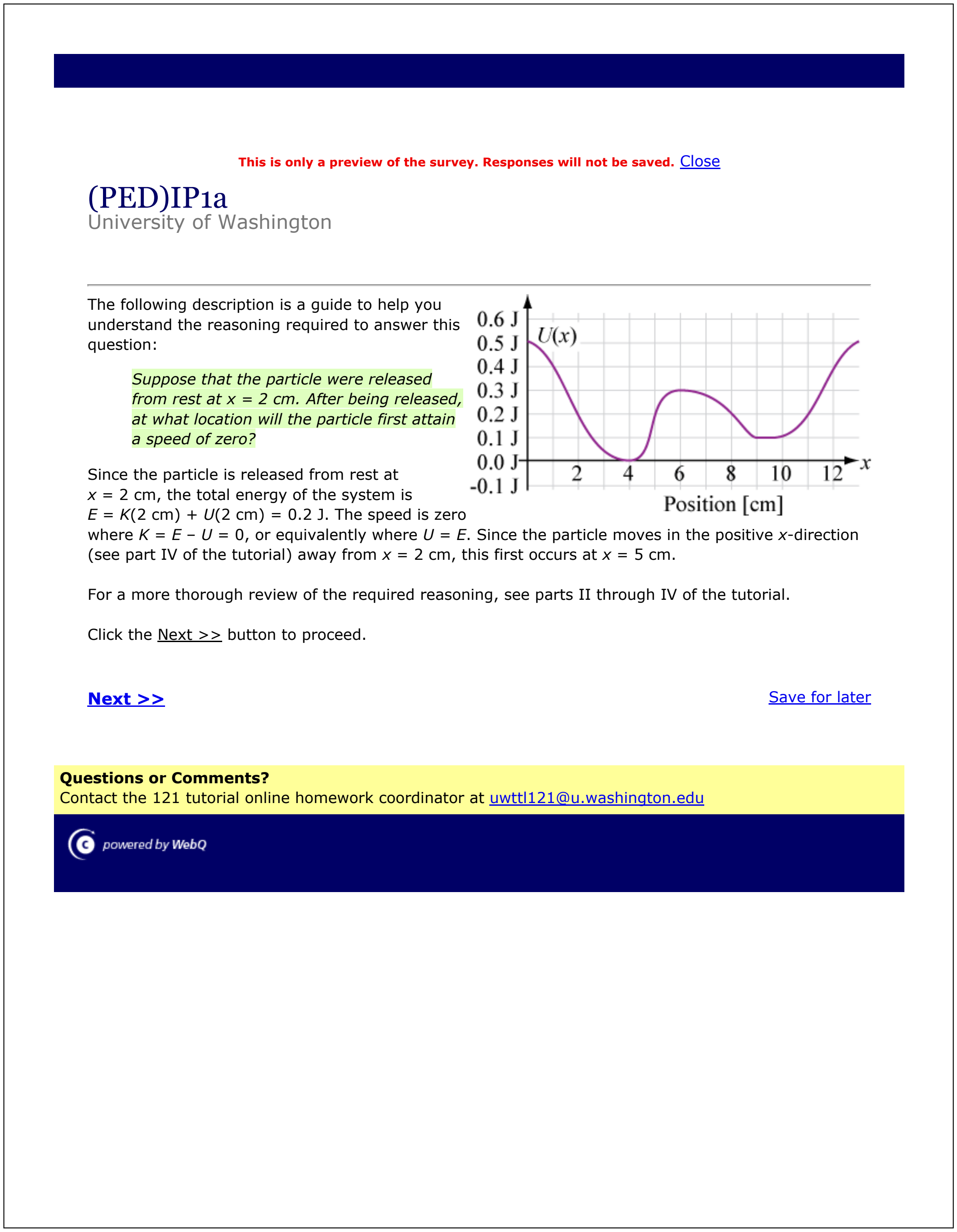


Figure A.74: Part of the OIPH administered in spring 2014 to Phys 121X. This "guide" is shown when students make three incorrect attempts on the fifth PED-related question, shown on page 488. The next page students see is the sixth and final question, shown on page 489.

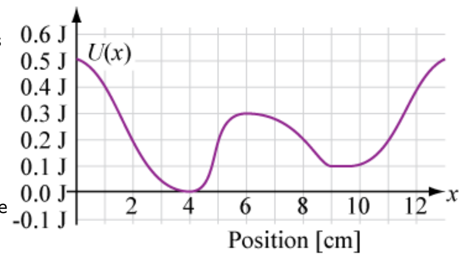
This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

The following description is a guide to help you understand the reasoning required to answer this question:

> *Suppose that the particle were released from rest at x = 2 cm. Between which locations will the particle oscillate back and forth?*



Recall from the previous question that the particle will first attain a speed of zero at x = 5 cm. At this position, the force is in the negative x-direction (see part IV of the tutorial for an explanation). Thus, just after reaching 5 cm, the particle will move in the negative x-direction. For this reason, the particle is not able to move past x = 5 cm. Therefore, the particle can only move between x = 2 cm and x = 5 cm.

Click the Next >> button to proceed.

Next >> Save for later

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Figure A.75: Part of the OIPH administered in spring 2014 to Phys 121X. This "guide" is shown when students make three incorrect attempts on the sixth and final PED-related question, shown on page 489. The next page students see is the commentary page, shown on page 482.

A.2.3-e ‘Confirmation’ pages shown after correct attempts

Below are the ‘confirmation’ pages shown to students after answering *correctly*. There is one confirmation page for each of the six questions.

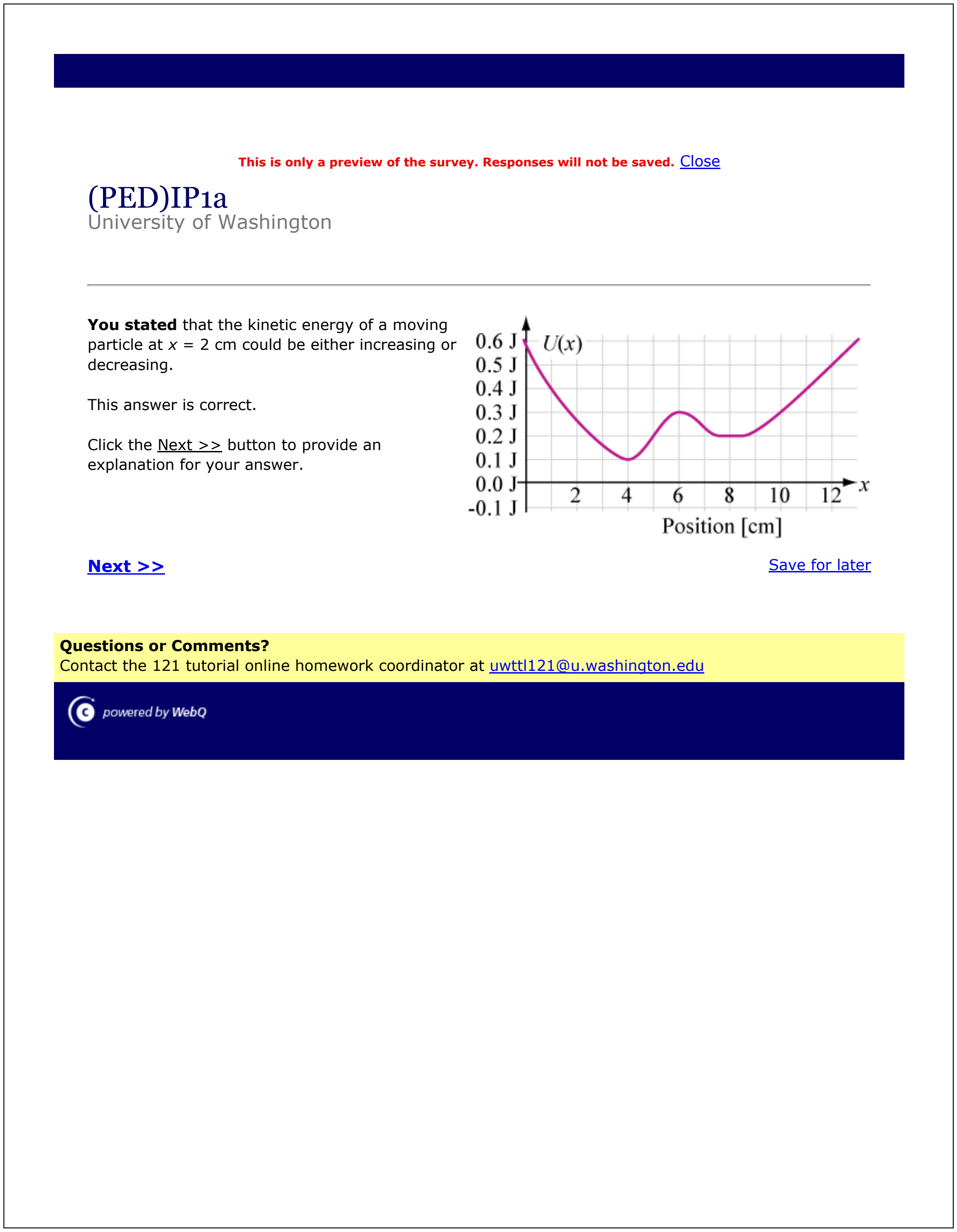


Figure A.76: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *correctly* select the third answer on the first PED-related question, which was shown on page 484. The next page is the second question shown on page 485.

This is only a preview of the survey. Responses will not be saved. Close

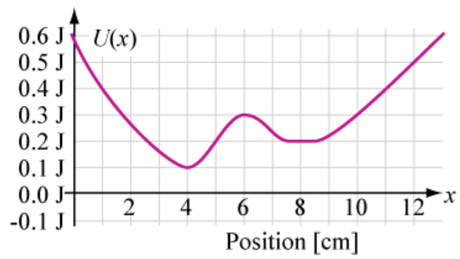
(PED)IP1a

University of Washington

You stated that the kinetic energy of a moving particle at *x* = 2 cm could be either increasing or decreasing because:

- The kinetic energy would increase if the particle were moving in the +x-direction
- The kinetic energy would decrease if the particle were moving in the -x-direction.

This answer is correct.



Since the particle could be moving in either direction at *x* = 2 cm, we do not know whether the *potential* energy increases or decreases (in time). Thus, we do not know how the *kinetic* energy changes.

Click the Next >> button to proceed.

Next >> Save for later

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Figure A.77: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *correctly* select the second answer on the second PED-related question, which was shown on page 485. The next page is the third question shown on page 486.

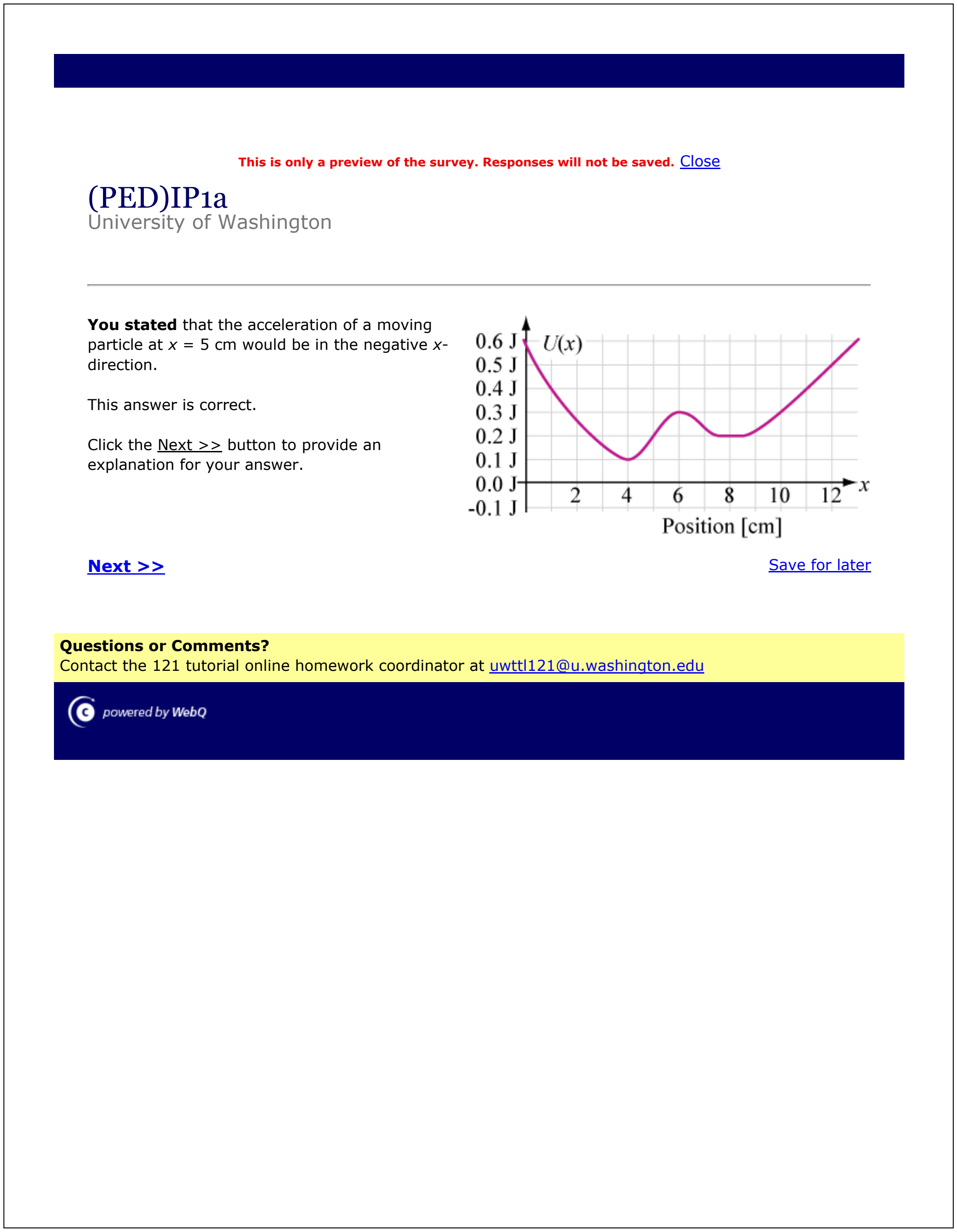
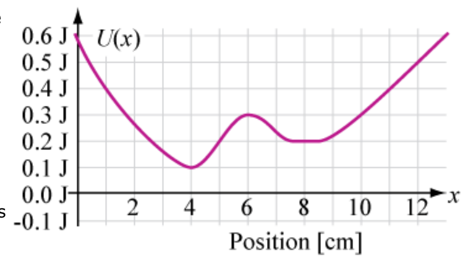


Figure A.78: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *correctly* select the second answer on the third PED-related question, which was shown on page 486. The next page is the fourth question shown on page 487.

This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a
University of Washington

You stated the acceleration of a moving particle at x = 5 cm would be in the negative x-direction because either direction of motion of the particle would correspond to a negative acceleration at this position.



This answer is correct.

- If the particle were moving in the positive x-direction, it would be slowing down, so its acceleration would be opposite in direction to its velocity.
- If instead the particle were moving in the negative x-direction, it would be speeding up, so its acceleration would be in the same direction as velocity.

Both of these correspond to an acceleration in the negative direction.

Click the Next >> button to proceed.

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Figure A.79: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *correctly* select the third answer on the fourth PED-related question, which was shown on page 487. The next page is the fifth question shown on page 488.

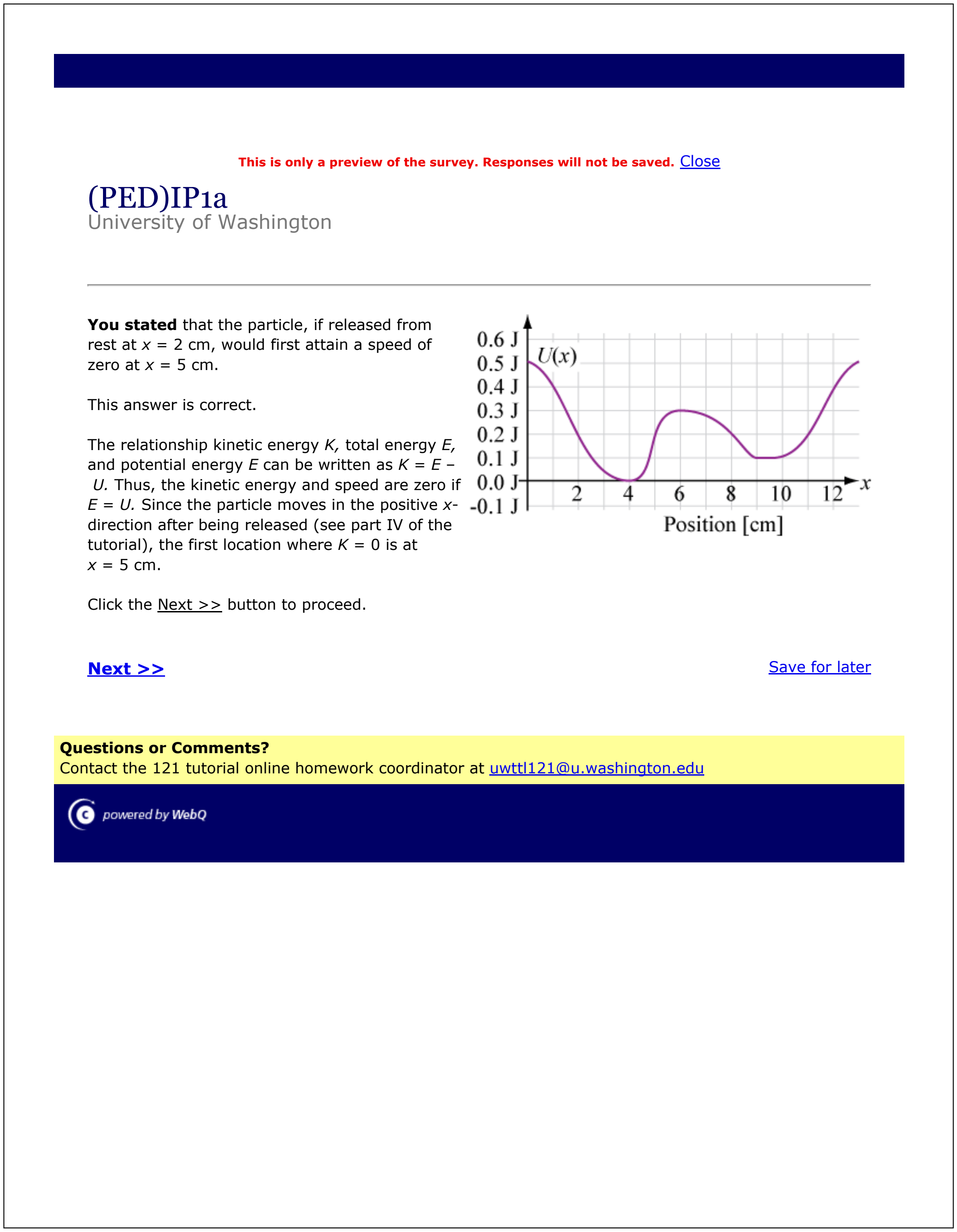


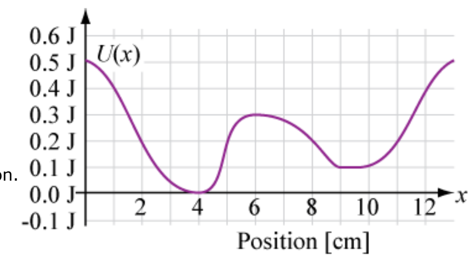
Figure A.80: Part of the OIPH administered in spring 2014 to PHYS 121X. This page is shown after students *correctly* select the second answer on the fifth PED-related question, which was shown on page 488. The next page is the sixth question shown on page 489.

This is only a preview of the survey. Responses will not be saved. Close

(PED)IP1a

University of Washington

You stated that the particle, if released from rest at x = 2 cm, would oscillate between 2 cm and 5 cm only.



This answer is correct.

When the particle reaches x = 5 cm, it has zero velocity and the acceleration is in the $-x$ direction. Hence, it moves back toward x = 2 cm. It never reaches the region between x = 8 cm and x = 11 cm that also satisfies the relationship $E > U$.

Click the Next >> button to proceed.

Next >>

Save for later

Questions or Comments?
Contact the 121 tutorial online homework coordinator at uwttl121@u.washington.edu

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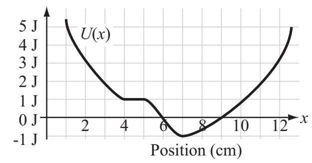
Figure A.81: Part of the OIPH administered in spring 2014 to Phys 121X. This page is shown after students *correctly* select the first answer on the sixth PED-related question, which was shown on page 489. The next page students see is the commentary page, shown on page 482.

A.3 Posttests related to Potential energy diagrams

In this section we provide post-tutorial assessments (posttests) for *Potential energy diagrams*.

Name ____ **Student ID** _______________ **Score**________
last *first*

IV. [15 points total] A particle of mass m is part of a one-dimensional system. A graph of potential energy *vs.* position of the particle is shown at right. The only forms of energy are the potential energy of the system and the kinetic energy of the particle. Consider parts A, B, and C below separately.



A. [5 pts] Suppose it is only known that the maximum kinetic energy obtained by the particle is 3 joules.

On the diagram, clearly indicate the turn-around point(s) if any. Explain.

B. [4 pts] Suppose it is only known that the particle is located at $x = 9.0$ cm and is moving.

In this case, would the particle be *speeding up*, *slowing down*, or *neither?* If there is not enough information to answer, state so explicitly. Explain.

C. [5 pts] Suppose it is only known that the particle has zero speed for all time (*i.e.*, it never moves).

In this case, what are the possible values of total energy E_{tot} of the system? If there is not enough information to answer, or if this is not a possible situation, state so explicitly. Explain.

D. [1 pts] Answer the following questions to help us improve the tutorial and homework on potential energy diagrams. You will receive credit for answering, regardless of your particular answers.

i. In your tutorial section, how far did you and your group get into the last tutorial of the quarter, *Potential energy diagrams?* (Circle one.)

- None/Did not attend
- Did not make it to the last page
- Made it to the last page, but did not finish
- Finished

ii. How much of the **homework** associated with the tutorial *Potential energy diagrams* did you complete? (*Note:* This homework was not collected.)

- None
- Some, but not all
- All

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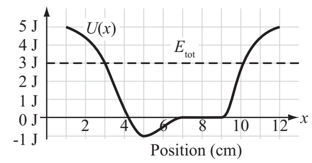
Figure A.82: Written final exam administered in spring 2013 to Phys 121A.

Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

IV. [15 points total] A particle of mass *m* is part of a one-dimensional system. Graphs of potential energy *U* and E_{tot} *vs*. position of the particle are shown at right. The only forms of energy are the potential energy of the system and the kinetic energy of the particle.



A. [4 pts] At what location(s) could the particle have been released from rest? If there is not enough information to answer, state so explicitly. Explain.

B. [5 pts] What is the maximum kinetic energy obtained by the particle? Express your answer in joules. If there is not enough information to answer, state so explicitly. Explain.

C. [5 pts] At what location(s) or region(s) is the magnitude of the force on the particle the smallest? If there is not enough information to answer, state so explicitly. Explain.

D. [1 pts] Answer the following questions to help us improve the tutorial and homework on potential energy diagrams. You will receive credit for answering, regardless of your particular answers.

i. In your tutorial section, how far did you and your group get into the last tutorial of the quarter, *Potential energy diagrams?* (Circle one.)

- None/Did not attend
- Did not make it to the last page
- Made it to the last page, but did not finish
- Finished

ii. How much of the **homework** associated with the tutorial *Potential energy diagrams* did you complete? (*Note:* This homework was not collected.)

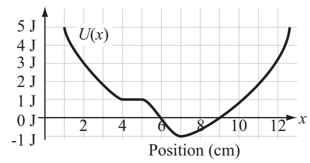
- None
- Some, but not all
- All

Physics 121B, Spring 2013 Final Exam ME-UWA121B132T-EF(PED)

Figure A.83: Written final exam administered in spring 2013 to Phys 121B.

Name ___________________________________ **Student ID** _______________ **Score** ___________
last *first*

III. [20 points total] A particle of mass *m* is part of a one-dimensional system. A graph of potential energy *vs*. position of the particle is shown at right. The only forms of energy are the potential energy of the system and the translational kinetic energy of the particle.



Consider the four questions below separately.

18. [5 pts] Suppose it is only known that the maximum kinetic energy obtained by the particle is 3 joules.

    In this case, which two points below best describe the turn-around points, if any?

    A. 1.0 cm & 12.5 cm
    B. 2.0 cm & 12.0 cm
    C. 3.0 cm & 11.0 cm
    D. 5.3 cm & 10.2 cm
    E. 6.0 cm & 9.0 cm

19. [5 pts] Suppose it is only known that the particle is located at $x = 9.0$ cm and is moving.

    Which of the following best describe the motion of the particle at this location?

    A. The particle is speeding up
    B. The particle is slowing down
    C. The particle could be either speeding up or slowing down
    D. The particle is at a turn-around point
    E. None of the above

20. [5 pts] Suppose it is only known that the particle has zero speed for all time (*i.e*., it never moves).

    In this case, what are the possible values of total energy E_{tot} of the system?

    A. $E_{tot} = -1$ J only
    B. $E_{tot} = -1$ J or $E_{tot} = 1$ J
    C. $E_{tot} = 0$ J only
    D. $E_{tot} = 1$ J only
    E. $E_{tot} = 5$ J only

21. [5 pts] Suppose it is only known that the system has total energy 3 J and that it takes 1.0 second for the particle to move between its turn-around points. If the mass of the particle were **increased,** but the potential energy and total energy of the system were unchanged, which of the following best describes the amount of time it would take the new particle to move between its turn-around points?

    A. Less than 1.0 second.
    B. Equal to 1.0 second
    C. Greater than 1.0 second
    D. Not enough information to answer

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Figure A.84: Multiple-choice final exam administered in summer 2013 to Phys 121A.

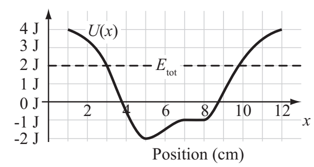
Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

XI. [10 points total] For each of the following two problems, a point particle is part of a system. The potential energy U of the system is shown as a function of the position x of the particle. Assume that the only forms of energy are potential energy of the system and translational kinetic energy of the particle.

Parts A and B below are independent. Answer each question separately.

A. [5 pts] In the potential energy diagram at right, the horizontal line E_{tot} represents the total energy of the system.

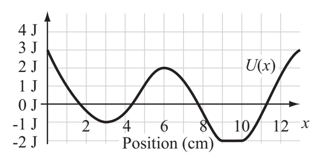
When the particle is at $x = 6$ cm, is it moving in the *positive x-direction,* the *negative x-direction,* or is the particle *not moving?* If there is not enough information to answer, state so explicitly. Explain.



B. [5 pts] Consider the graph of potential energy *vs*. position shown at right.

Suppose that the particle were released from rest at $x = 1$ cm.

Determine the maximum kinetic energy obtained by the particle. Express your answer in joules. Explain.

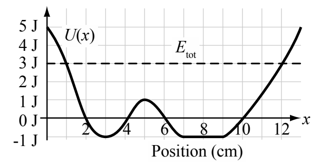


Physics 121A, Autumn 2013 Final Exam ME-UWA121A134T-EF(PED).doc

Figure A.85: Written final exam administered in autumn 2013 to Phys 121A.

Name ___________________________________ **Student ID** ______________ **Score**________
last *first*

X. [10 points total] For the following two problems, a point particle is part of a system. The graph at right shows the potential energy U of the system as a function of the position x of the particle. Assume that the only forms of energy are potential energy of the system and translational kinetic energy of the particle. The horizontal line E_{tot} represents the total energy of the system.



A. [5 pts] At what location(s) is the magnitude of the force on the particle the smallest? Explain.

B. [5 pts] At what location(s) could the particle have been released from rest? Explain.

Physics 121C, Autumn 2013 Final Exam ME-UWA121C134T-EF(PED)

Figure A.86: Written final exam administered in autumn 2013 to Phys 121C.

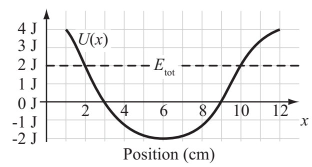
Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

XI. [10 points total] For each of the following two problems, a point particle is part of a system. The potential energy U of the system is shown as a function of the position x of the particle. Assume that the only forms of energy are potential energy of the system and translational kinetic energy of the particle.

Parts A and B below are independent. Answer each question separately.

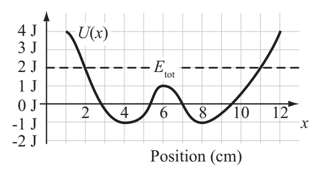
A. [5 pts] In the potential energy diagram at right, the horizontal line E_{tot} represents the total energy of the system.

Determine the location(s) of the turn-around point(s) for the particle. If there are no turn-around points, state so explicitly. Explain.



B. [5 pts] Consider the potential energy diagram at right. The horizontal line E_{tot} represents the total energy of the system.

When the particle is at x = 7 cm, is the particle *speeding up*, *slowing down*, or is the particle *not moving?* If there is not enough information to answer, state so explicitly. Explain.



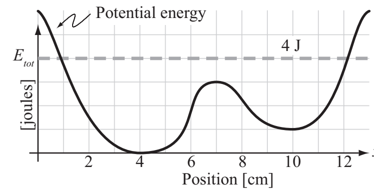
Physics 121D, Autumn 2013 Final Exam ME-UWA121D134T-EF(PED).doc

Figure A.87: Written final exam administered in autumn 2013 to Phys 121D.

Name ___________________________________ **Student ID** _______________ **Score** ___________
last *first*

Questions 30 and 31 involve a point particle that is part of a one-dimensional system. The only forms of energy of the system are potential energy of the system and translational kinetic energy of the particle. The total energy of the system is conserved. The *type* of interaction(s) between the particle and the rest of the system (*e.g*., gravitational, magnetic, other, etc.) is not given, nor is it necessary for these questions.

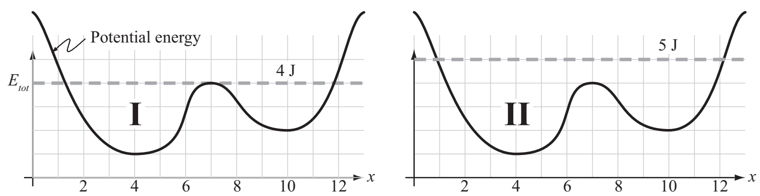
30. [5 pts.] The graph of potential energy of the system versus position of the particle is shown at right. The total energy of the system is 4 joules, as indicated.



Is it possible to determine the direction of the **velocity** of the particle when it is located at $x = 8$ cm?

A. Yes. It is moving in the $+x$ direction.
B. Yes. It is moving in the $-x$ direction.
C. Yes. The velocity is zero.
D. No, it is not possible. More information is needed.

31. [5 pts.] The graphs below differ from the graph above in that the potential energy and/or the total energy curves have been shifted up or down by some amount.



Which of the graphs above could represent **the same physical system** as the original system from question 16 **and** have the particle undergo **the same motion?** [*Note:* Having the *same motion* means the particle has the same speed at every position that it had in the original system.]

A. None of the graphs
B. Only I
C. Only II
D. Both I and II

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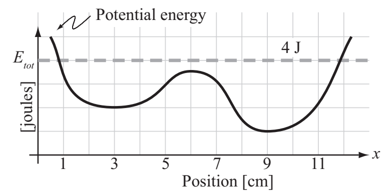
Figure A.88: Multiple-choice final exam administered in winter 2014 to Phys 121A. Identical to a pretest administered in spring 2011 to Phys 121A (see page 322).

Name ___________________________________ **Student ID** _______________ **Score** ___________
last *first*

Questions 30 and 31 involve a point particle that is part of a one-dimensional system. The only forms of energy of the system are potential energy of the system and translational kinetic energy of the particle. The total energy of the system is conserved. The *type* of interaction(s) between the particle and the rest of the system (*e.g.*, gravitational, magnetic, other, etc.) is not given, nor is it necessary for these questions.

The graph of potential energy of the system versus position of the particle is shown at right. The total energy of the system is 4 joules, as indicated.



30. Is it possible to determine the direction of the **velocity** of the particle when it is located at $x = 5$ cm?
    A. Yes. It is moving in the $+x$ direction.
    B. Yes. It is moving in the $-x$ direction.
    C. Yes. The velocity is zero.
    D. No, it is not possible. More information is needed.

31. Is it possible to determine the direction of the **acceleration** of the particle when it is located at $x = 5$ cm?
    A. Yes. The acceleration is in the $+x$ direction.
    B. Yes. The acceleration is in the $-x$ direction.
    C. Yes. The acceleration is zero.
    D. No, it is not possible. More information is needed.

Physics 121B, Winter 2014 Final Exam ME-UWA121B141T-EF(PED)mc.doc

Figure A.89: Multiple-choice final exam administered in winter 2014 to Phys 121B. Identical to a pretest administered in spring 2011 to Phys 121B (see page 323).

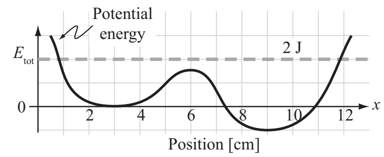
Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

IV. [15 points total] In the following questions, a particle is confined to a one-dimensional system. The only forms of energy are translational kinetic energy of the particle and potential energy of the system. All graphs of $U(x)$ represent the potential energy of the system as a function of the position of the particle. Assume the only force present is that corresponding to the potential energy diagram shown.

Parts A, B, and C below are independent.

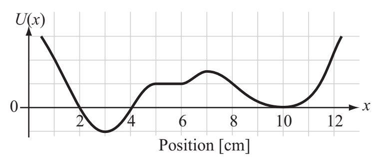
A. [4 pts] Consider the graph of $U(x)$ at right. The dashed horizontal line represents the total energy of the system.

Determine the direction of the acceleration of the particle when it is at $x = 5$ cm. If there is not enough information to answer, state so explicitly. Explain.



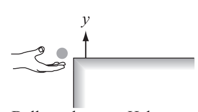
B. [5 pts] Consider the graph of $U(x)$ at right.

List all of the positions or regions where the particle could be released from rest and remain stationary. Explain.

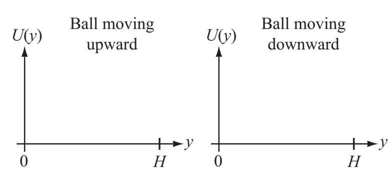


C. [6 pts] A ball is thrown upward from $y = 0$, as shown. It moves in the positive y-direction until $y = H$ (a few meters), before falling back down to $y = 0$. Let upward be the positive y-direction.

On the graphs below, sketch for the ball-Earth system a graph of potential energy U of the system *vs*. position y of the ball when the ball is (1) moving upward, and another when it is (2) moving downward. Explain.



Ball reaches $y = H$ then falls back to $y = 0$



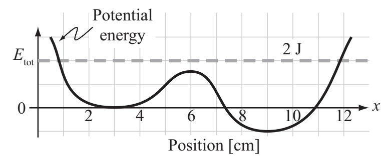
Physics 121A, Spring 2014 Final Exam ME-UWA121A142T-EF(PED).doc

Figure A.90: Written final exam administered in spring 2014 to Phys 121A. Students worked through the online interactive practice homework this quarter.

Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

IV. [15 points total] A particle is confined to a one-dimensional system. The only forms of energy are the potential energy of the system and the translational kinetic energy of the particle. Assume the only force present is that corresponding to the potential energy diagram shown.

A. For parts i and ii below, use the potential energy diagram shown at right. The horizontal dashed line represents total energy.



i. [5 pts] Rank, from greatest to least, the **magnitudes of the force** on the particle when the particle is at the following four positions: x = **2 cm,** x = **3 cm,** x = **6 cm,** and x = **11 cm**. If any of the forces are equal to zero, state so explicitly. If there is not enough information to answer, state so explicitly. Explain.

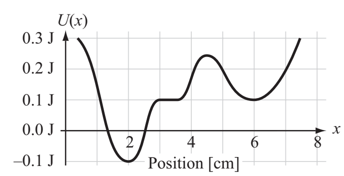
ii. [5 pts] Suppose that it takes the particle 2.0 seconds to travel from one turn-around point to the other.

Now suppose that the particle were replaced with a less massive particle ($m_{new} < m_{old}$) while the total and potential energies remained the same. Would the **time it takes** the new particle to traverse the same path be *greater than, less than,* or *equal to* 2.0 seconds? If there is not enough information, state so explicitly. Explain.

B. [5 pts] Now consider the potential energy diagram shown at right.

Suppose that the particle is released from rest at x = 7 cm.

After being released, what is the **maximum kinetic energy** attained by the particle? Express your answer in joules (J). Explain.



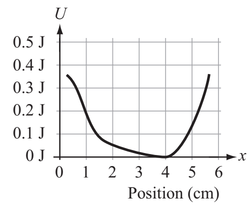
Physics 121B, Spring 2014 Final Exam ME-UWA121B142T-EF(PED).doc

Figure A.91: Written final exam administered in spring 2014 to Phys 121B. Students worked through the online interactive practice homework this quarter.

Name ______________________________ **Student ID** _____________ **Score** __________
last *first*

IV. [20 points total] For all questions on this page, an object is part of a one-dimensional system. The only forms of energy are the potential energy of the system and the translational kinetic energy of the object.

For questions 18 and 19 consider the potential energy diagram shown at right. Suppose that the particle is located at x = 1.0 cm and moving.

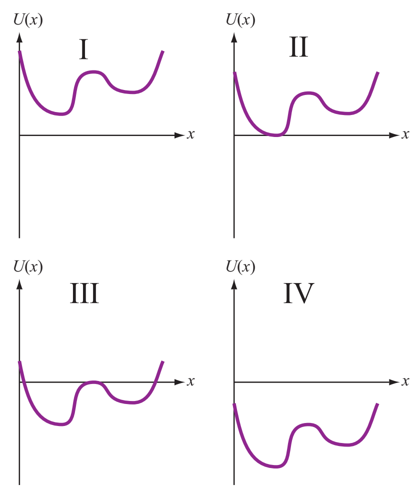


18. [4 pts] Which of the following best describes how the speed of the particle is changing at x = 1.0 cm?
    - A. The particle is speeding up.
    - B. The particle is slowing down.
    - C. The particle's speed is not changing.
    - D. There is not enough information to answer.

19. [4 pts] Which of the following best describes the direction of the acceleration at x = 1.0 cm?
    - A. The acceleration is in the positive x-direction.
    - B. The acceleration is in the negative x-direction.
    - C. The acceleration is zero.
    - D. Not enough information to answer.

20. [4 pts] Consider the four graphs of $U(x)$ shown at right.

    Which of the diagrams shown could correspond to a possible physical system?
    - A. Only I and II
    - B. Only I and IV
    - C. Only II and III
    - D. Only I, II, and III
    - E. All of them are possible (I, II, III, and IV)



Physics 121A, Summer 2014 Final Exam ME-UWA121A143T-EF(PED).doc

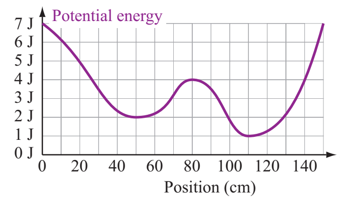
Figure A.92: Multiple-choice final exam administered in summer 2014 to Phys 121A.

Name ______________________________________ **Student ID** _______________ **Score**________

last *first*

X. [15 points total] For all problems on this page, a point particle is part of a one-dimensional system. The only forms of energy in the system are translational kinetic energy of the particle and potential energy of the system. Assume the only force on the particle is that associated with the potential energy. All potential energy diagrams are functions of the position of the particle within the system.

A. Use the potential energy diagram at right for the following two questions.



i. [5 pts] For this question only, suppose that the particle is located at x = 100 cm and is moving.

Determine the *direction of the force* on the particle at this position. If there is not enough information to answer, state so explicitly. Explain.

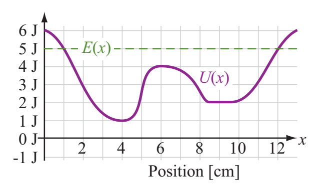
ii. [5 pts] Suppose that, as the particle moves in the system with potential energy $U(x)$ shown above, it is observed that its maximum kinetic energy is 4 J.

Determine the *total energy* of the system. Express your answer in joules (J). If there is not enough information to answer, state so explicitly. Explain.

B. [5 pts] Consider now the potential energy diagram at right.

Suppose it is known that the particle is located at x = 5 cm.

Determine the *direction of motion* of the particle. If there is not enough information to answer, state so explicitly. Explain.



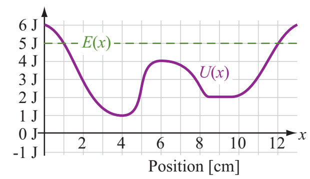
Physics 121A, Autumn 2014 Final Exam ME-UWA121A144T-EF(PED).doc

Figure A.93: Written final exam administered in autumn 2014 to Phys 121A.

Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

X. [15 points total] For all problems on this page, a point particle is part of a one-dimensional system. The only forms of energy in the system are translational kinetic energy of the particle and potential energy of the system. Assume the only force on the particle is that associated with the potential energy. All potential energy diagrams are functions of the position of the particle within the system.

A. Use the potential energy diagram at right for the following two questions.



i. [5 pts] Suppose the particle is located at $x = 5$ cm.

Determine the *direction of motion* of the particle. If there is not enough information to answer, state so explicitly. Explain.

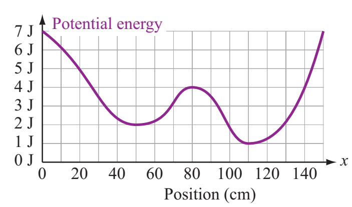
ii. [5 pts] Suppose again the particle is located at $x = 5$ cm.

Is the speed of the particle *increasing*, *decreasing*, *not changing*, or is there *not enough information* to answer? Explain.

B. [5 pts] Consider now the potential energy diagram at right.

Suppose the particle were released from rest at $x = 20$ cm.

Determine the *maximum kinetic energy* attained by the particle. Express your answer in joules (J). If there is not enough information to answer, state so explicitly. Explain.



Physics 121C, Autumn 2014 Final Exam ME-UWA121C144T-EF(PED).doc

Figure A.94: Written final exam administered in autumn 2014 to Phys 121C.

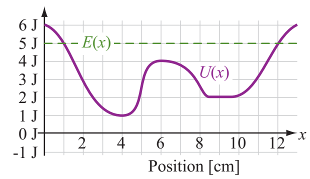
Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

X. [15 points total] For all problems on this page, a point particle is part of a one-dimensional system. The only forms of energy in the system are translational kinetic energy of the particle and potential energy of the system. Assume the only force on the particle is that associated with the potential energy. All potential energy diagrams are functions of the position of the particle within the system.

A. [5 pts] Consider the potential energy diagram at right.

Suppose the particle is located at $x = 5$ cm.

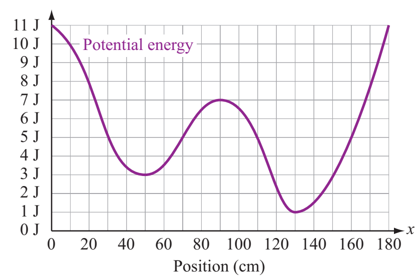
Determine the *direction of the acceleration* of the particle. If there is not enough information to answer, state so explicitly. Explain.



B. [5 pts] Consider the potential energy diagram at right.

Suppose the particle is released from rest at $x = 30$ cm.

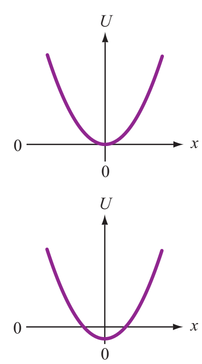
Determine the *maximum kinetic energy* attained by the particle. Express your answer in joules (J). If there is not enough information to answer, state so explicitly. Explain.



C. [5 pts] Consider a spring that obeys Hooke's law. (Recall that Hooke's law is $F = -kx$, where x is the displacement of the spring from equilibrium.) The diagram at right represents elastic potential energy U of the spring as a function of x.

Consider now the bottom diagram. This graph is identical to the first except that it has been vertically shifted.

Could this new diagram also be used to represent the potential energy of the same spring? Explain.



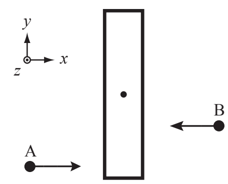
Physics 121D, Autumn 2014 Final Exam ME-UWA121D144T-EF(PED).doc

Figure A.95: Written and multiple-choice final exam administered in autumn 2014 to Phys 121D. (Two pages.)

Name ____________________________________ **Student ID** _______________ **Score** ___________
last *first*

IX. [15 points total]

A large uniform, rectangular board is at rest on a horizontal, frictionless table. Two identical small pucks labeled A and B approach the board with equal speeds as shown at right. Both pucks collide with the board simultaneously and stick to it.



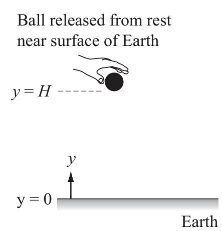
28. [5 pts] After the collision: Which statement best describes the **linear momentum** of the system composed of puck A, puck B, and the board?
   - A. Zero
   - B. To the right (+*x* direction)
   - C. To the left (+*x* direction)
   - D. Out of the page (+*z* direction)
   - E. None of the above

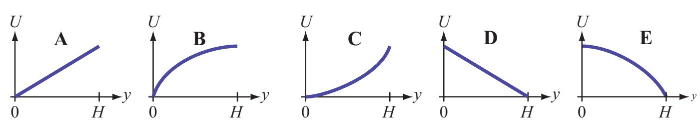
29. [5 pts] Which of the following must be true when the **angular momentum** of any system is constant?
   - A. The net force on the system by external objects is zero.
   - B. The net torque on the system by external forces is zero.
   - C. The net work done on the system by external forces is zero.
   - D. The sum of the angular velocities of the objects in the system is constant.
   - E. More than one of the choices A–D above must be true when the angular momentum is constant.

30. [5 pts] Consider system BE consisting of a ball and the Earth. Let $y = 0$ represent the ground, with the positive *y*-direction upward, as shown. The ball is dropped from rest a distance H above the ground. It falls downward and reaches $y = 0$. Make the assumption that the ball is always within a few meters of the surface of the Earth.

   Consider the interval when the ball is moving downward.

   Which of the five diagrams below best represents a graph of the potential energy U of system BE as a function of the position y of the ball? (The point $y = H$ is a few meters above the ground.)

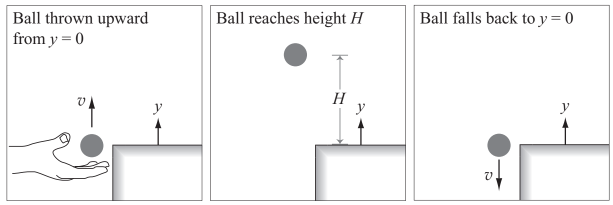




Name ______________________ **Student ID** ____________ **Score** ________
last *first*

IX. [20 points total] For this part of the exam, assume all systems are one-dimensional (*i.e.*, only a single object or particle within the system of interest can move in one dimension).

For questions 1 and 2, a ball is thrown upward from $y = 0$ (see first diagram). It slows down, turns around at $y = H$ (less than a meter above $y = 0$), speeds up, and reaches $y = 0$ again.

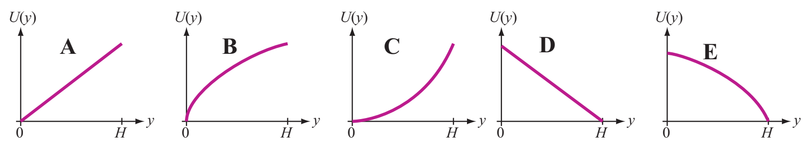


Take the positive y-direction to be upward.

Recall that near the surface of the Earth, the gravitational potential energy can be written as $U = mgy$.

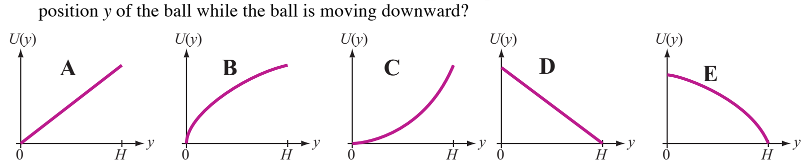
28. [5 pts] Consider the interval while the ball is **moving upward.**

Which of the diagrams A–E below best represents a plot of potential energy U, as a function of the position y of the ball while the ball is moving upward?



29. [5 pts] Consider the interval while the ball is **moving downward.**

Which of the diagrams A–E below best represents a plot of potential energy U, as a function of the position y of the ball while the ball is moving downward?

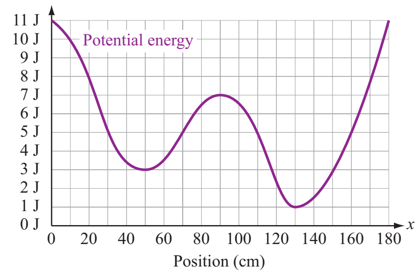


Physics 121A, Winter 2015 Final Exam ME-UWA121A151T-EF(PED).doc

Figure A.96: Multiple-choice final exam administered in winter 2015 to Phys 121A. (Two pages.)

Name ___________________________________ **Student ID** _______________ **Score** ___________
last *first*

For the following two questions, consider the potential energy diagram at right. Assume that the total mechanical energy is constant.



Suppose the particle were released from rest at $x = 30$ cm. Its subsequent motion is observed for a very long time.

30. [5 pts] What is the **smallest *x*-position** (*i.e.*, farthest left on the graph) that the particle reaches?

    A. 0 cm

    B. 30 cm

    C. 50 cm

    D. 70 cm

    E. 90 cm

31. [5 pts] What is the **largest *x*-position** (*i.e.*, farthest right on the graph) that the particle reaches?

    A. 50 cm

    B. 70 cm

    C. 90 cm

    D. 160 cm

    E. 180 cm

Physics 121A, Winter 2015 Final Exam ME-UWA121A151T-EF(PED).doc

Name ______________________________ **Student ID** _____________ **Score** __________
last *first*

III. [20 points total] For this part of the exam, assume all systems are one-dimensional (*i.e.*, only a single object or particle within the system of interest can move in one dimension).

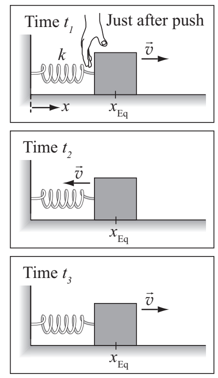
28. [5 pts] A block is attached to an ideal massless spring of spring constant k on a level, frictionless surface. Just before time t_1, a hand gives the block a quick push to the right.

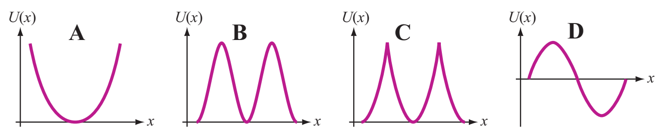
At time t_1, the block is at the equilibrium position x_{Eq} moving to the right (see top diagram). Later, at time t_2, the ball is again at x_{Eq}, but moving to the left. Finally, at time t_3, the ball is at the equilibrium position x_{Eq} moving to the right again.

Assume that the potential energy of the block-spring system is zero at the equilibrium position. Let rightward be the positive x-direction.

Consider the entire interval shown, from t_1 to t_3.

Which of the choices A–E below best represents a plot of potential energy U of the block-spring system as a function of the position x of the block for the *entire* motion described above? (If none of the graphs correctly show the plot, choose E.)





E
None of these/Other

29. [5 pts] For this question, consider the potential energy diagram at right. Assume that the total mechanical energy is constant.

Suppose the particle were released from rest at x = 30 cm. What is the maximum kinetic energy that the particle attains?

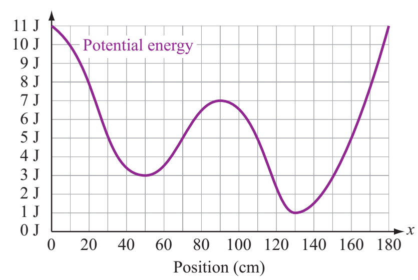
A. 2 J

B. 4 J

C. 6 J

D. 10 J

E. None of the above

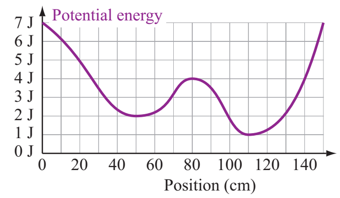


Physics 121B, Winter 2015 Final Exam ME-UWA121B151T-EF(PED).doc

Figure A.97: Multiple-choice final exam administered in winter 2015 to Phys 121B. (Two pages.)

Name ___________________________________ **Student ID** _______________ **Score** ___________
last *first*

For the next two questions, consider the potential energy diagram at right. Assume that the total mechanical energy is constant.



30. [5 pts] Suppose it is known that the particle is at $x = 20$ cm and moving. Which of the following best describes how the speed of the particle is changing at this position?

   A. The speed is increasing

   B. The speed is decreasing

   C. The speed is not changing

   D. Not enough information to answer

31. [5 pts] Suppose instead it is known that the particle is at $x = 70$ cm and moving. Which of the following best describes the direction of the particle's acceleration at this position?

   A. Positive x-direction

   B. Negative x-direction

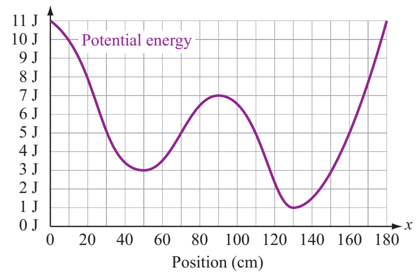
   C. Zero

   D. Not enough information to answer

Name ______________________________ **Student ID** ______________ **Score** __________
last *first*

III. [18 points total] On this page, treat all systems as one-dimensional and ignore rotational kinetic energy. Assume that the total energy (potential and translational kinetic) is constant. Assume also that the only force on the particle is that associated with potential energy.

For the following **two questions,** consider the potential energy diagram shown at right.



27. [4 pts] Suppose that the particle is released **from rest** at ***x* = 70 cm.** Which of the following statement best describes the **initial motion** of the particle?

    A. The particle will initially move in the positive x-direction.

    B. The particle will initially move in the negative x-direction.

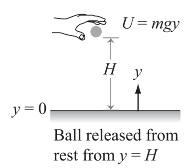
    C. The particle will remain stationary.

    D. There is not enough information to answer.

28. [5 pts] Suppose again that the particle is released **from rest** at ***x* = 70 cm.** What is the **total energy** of the system?

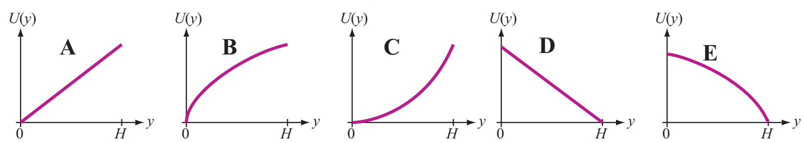
    A. 5 J B. 6 J C. 7 J D. 11 J E. None of these

29. [5 pts] Consider system BE consisting of a ball and the Earth. Let $y = 0$ represent the ground, with the positive y-direction upward, as shown. The ball is dropped from rest a distance H (a few meters) above the ground. The ball is always near the surface of the Earth so the gravitational potential energy can be written as $U = mgy$.



Consider the interval when the ball is moving downward.

Which of the diagrams below best represents a graph of the **potential energy *U*** of system BE as a function of the **position *y*** of the ball?



30. [4 pts] Consider an arbitrary system. Assume the only forms of energy are kinetic energy and potential energy. Which of the following is a **true statement** regarding kinetic energy and total energy at a given instant in time?

    A. The value of the kinetic energy of a system must be <u>less than</u> the value of total energy.

    B. The value of the kinetic energy of a system must be <u>less than</u> or <u>equal to</u> the value of total energy.

    C. The value of the kinetic energy can be <u>greater than,</u> <u>less than,</u> or <u>equal to</u> the value of total energy.

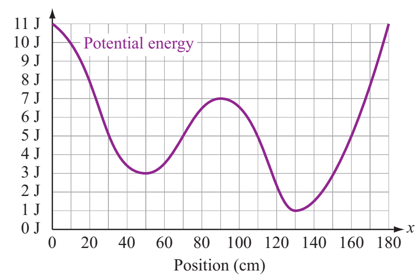
Physics 121A, Spring 2015 Final Exam ME-UWA121A152T-EF(PED).doc

Figure A.98: Multiple-choice final exam administered in spring 2015 to Phys 121A.

Name ___________________________________ **Student ID** _______________ **Score** ___________
last *first*

III. [18 points total] On this page, treat all systems as one-dimensional and ignore rotational kinetic energy. Assume that the total energy (potential and translational kinetic) is constant. Assume also that the only force on the particle is that associated with potential energy.

For the following **three questions,** consider the potential energy diagram shown at right.



29. [5 pts] Suppose that the particle is located at ***x* = 120 cm** and is moving**.** What is the **direction of the force** on the particle?
    A. The force is in the positive *x*-direction.
    B. The force is in the negative *x*-direction.
    C. The force is zero.
    D. There is not enough information to answer.

30. [4 pts] Suppose again that the particle is located at ***x* = 120 cm** and is moving**.** Which of the following statement best describes how the **speed** of the particle is changing?
    A. The particle is speeding up.
    B. The particle is slowing down.
    C. The speed of the particle is not changing.
    D. There is not enough information to answer.

31. [5 pts] Suppose instead that the particle is released **from rest** at ***x* = 30 cm.** What is the **maximum kinetic energy** that the particle attains?
    A. 2 J
    B. 4 J
    C. 5 J
    D. 6 J
    E. 10 J

32. [4 pts] Consider a system (not shown) composed of two regions, A and B. It is known that the kinetic energy of the particle in region A is twice that in region B. Which of the following is a **true statement** regarding the potential energy of the system in regions A and B?
    A. The potential energies in regions A and B are the same.
    B. The potential energy in region A is **half that** in region B.
    C. The potential energy in region A is **less than** that in region B, but not necessarily half.
    D. The potential energy in region A is **twice that** in region B.
    E. The potential energy in region A is **more than** that in region B, but not necessarily twice.

Physics 121B, Spring 2015 Final Exam ME-UWA121B152T-EF(PED).doc

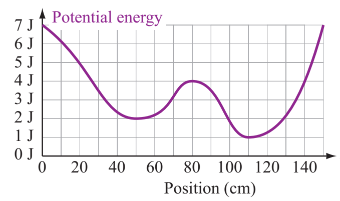
Figure A.99: Multiple-choice final exam administered in spring 2015 to Phys 121B.

Name ______________________________ **Student ID** ______________ **Score** __________

last *first*

III. [18 points total] On this page, treat all systems as one-dimensional and ignore rotational kinetic energy. Assume that the total energy (potential and translational kinetic) is constant. Assume also that the only force on the particle is that associated with potential energy.

For the following **two questions,** consider the potential energy diagram shown at right.



27. [5 pts] Suppose that the particle is located at **x = 100 cm** and is moving**.** What is the **direction of the force** on the particle?

    A. The force is in the positive x-direction.

    B. The force is in the negative x-direction.

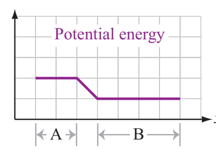
    C. The force is zero.

    D. There is not enough information to answer.

28. [5 pts] Suppose instead that the particle is released **from rest** at **x = 20 cm.** What is the **maximum kinetic energy** that the particle attains?

    A. 2 J B. 4 J C. 5 J D. 6 J E. 7 J

29. [4 pts] Consider the potential energy diagram at right. The potential energy in region A is twice that in region B. (The potential energy to the left of region A and to the right of region B is not shown.) Assume that the particle is able to reach both regions.



    Which of the following is a **true statement** regarding the kinetic energy in regions A and B?

    A. The kinetic energies in regions A and B are the same.

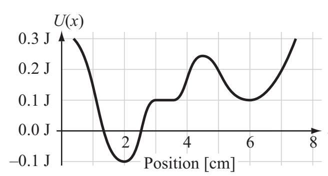
    B. The kinetic energy in region A is **half that** in region B.

    C. The kinetic energy in region A is **less than** that in region B, but not necessarily half.

    D. The kinetic energy in region A is **twice that** in region B.

    E. The kinetic energy in region A is **more than** that in region B, but not necessarily twice.

30. [4 pts] Which of the following statements **must be true** regarding the potential energy diagram at right?



    A. This potential energy diagram is not possible because $U < 0$ in some regions.

    B. There is an instant when the particle is located at $x = 7$ cm.

    C. The particle will reach $x = 2$ cm and immediately stop.

    D. None of these statements must be true.

Physics 121C, Spring 2015 Final Exam ME-UWA121C152T-EF(PED).doc

Figure A.100: Multiple-choice final exam administered in spring 2015 to Phys 121C.

Appendix B

QUESTIONS AND CURRICULAR MATERIAL FOR THE NEW TUTORIAL ON SPATIAL PROBABILITY DENSITY

In this portion of the Appendix we provide all pretests, in-class worksheets, homework, posttests, and other curricular material for the tutorial *Probability in classical and quantum mechanics*.

Contents of appendix

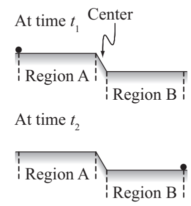
B.1 Pretests related to Probability in classical and quantum mechanics

In this section we provide written and online pretests related to the tutorial *Probability in classical and quantum mechanics*.

PRETEST: Classical Probability **Name** ________________________________

□ I have studied probability before □ I have NOT studied probability before Intended Major_______________

I. A track is composed of two frictionless levels joined in the center by a steep frictionless ramp, as shown at right. At time t_1, a ball is at the left end of the track moving to the right at constant speed. At time t_2, the ball reaches the right end of the track. The ball never leaves the surface of the track.



A. Is the speed of the ball in Region A *greater than, less than,* or *equal to* the speed of the ball in Region B?

B. During the interval from t_1 to t_2, a large number of photographs of the ball are taken at random times.

One particular photograph is selected at random. Is the likelihood of that photograph showing the ball in Region A *greater than, less than,* or *equal to* the likelihood of that photograph showing the ball in Region B? Explain.

How confident are you of your answer & explanation? Very Confident □ □ □ □ □ Not confident

C. Walls are now placed at either end of the track, allowing the ball to bounce back and forth between the ends of the track. Assume there is no friction or loss of energy in the system such that the ball can bounce back and forth forever. The ball never leaves the surface of the track.

A large number of photographs are taken at random times while the ball bounces back and forth.

i. One particular photograph is selected at random. Is the likelihood of that photograph showing the ball in Region A *greater than, less than,* or *equal to* the likelihood of that photograph showing the ball in Region B? Explain.

How confident are you of your answer & explanation? Very Confident □ □ □ □ □ Not confident

ii. The position of the ball in each photograph is recorded and the average position is calculated. Does the average position of the ball lie to the *left of,* to the *right of,* or *exactly at* the center of the track? Explain.

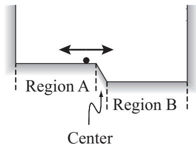
How confident are you of your answer & explanation? Very Confident □ □ □ □ □ Not confident

Physics 123A, Autumn 2007 CPR Pretest WO-UWA123A074P-P1(CPR)

Figure B.1: Written pretest version (CPR)W1A administered in autumn 2007 in Phys 123A. This is one of three versions of pretests administered this quarter.

PRETEST: Classical Probability **Name** ______________________________

☐ I have studied probability before ☐ I have NOT studied probability before Intended Major_______________



. A frictionless track is composed of two levels joined in the center by a steep ramp, as shown at right. A ball is given an initial push and then allowed to bounce between the ends of the track. There are no frictional forces and no energy loss in the system, such that the ball can bounce back and forth forever. The ball never leaves the surface of the track.

A. Is the speed of the ball in Region A *greater than, less than,* or *equal to* the speed of the ball in Region B?

B. Is the time spent in Region A *greater than, less than,* or *equal to* the time spent in Region B? Briefly explain.

C. While the ball is bouncing back and forth, a large number of photographs of the ball are taken at random times.

i. Is the number of photographs showing the ball in Region A *greater than, less than,* or *equal to* the number of photographs showing the ball in Region B? Explain

How confident are you of your answer & explanation? Very Confident ☐ ☐ ☐ ☐ ☐ Not confident

ii. One particular photograph is selected at random. Is the likelihood of that photograph showing the ball in Region A *greater than, less than,* or *equal to* the likelihood of that photograph showing the ball in Region B? Explain.

How confident are you of your answer & explanation? Very Confident ☐ ☐ ☐ ☐ ☐ Not confident

iii. The position of the ball in each photograph is recorded and the average position is calculated. Does the average position of the ball lie to the *left of,* to the *right of,* or *exactly at* the center of the ramp? Explain.

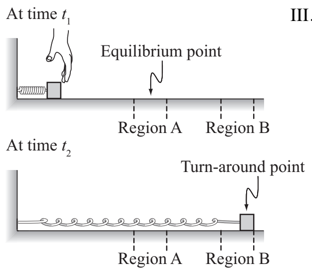
How confident are you of your answer & explanation? Very Confident ☐ ☐ ☐ ☐ ☐ Not confident

Physics 123A, Autumn 2007 CPR Pretest WO-UWA123A074P-P1(CPR)

Figure B.2: Written pretest version (CPR)W2A administered in autumn 2007 in PHYS 123A. This is one of three versions of pretests administered this quarter.

PRETEST: Classical Probability **Name** ______________________________

☐ I have studied probability before ☐ I have NOT studied probability before Intended Major______________



III. At time t_1, a block is released from rest after being compressed, as shown at right. At time t_2, it reaches the turn-around point. The track is frictionless.

Region A is located near the equilibrium point. Region B is of equal length and located near the turn-around point.

A. Is the average speed of the block in Region A *greater than, less than,* or *equal to* the average speed of the ball in Region B?

B. Is the time spent in Region A *greater than, less than,* or *equal to* the time spent in Region B? Briefly explain.

C. During the interval from t_1 to t_2, a large number of photographs of the block are taken at random times.

One particular photograph is selected at random. Is the likelihood of that photograph showing the ball in Region A *greater than, less than,* or *equal to* the likelihood of that photograph showing the ball in Region B? Explain.

How confident are you of your answer & explanation? Very Confident ☐ ☐ ☐ ☐ ☐ Not confident

D. The block is now allowed to oscillate back and forth. Assume there are no frictional forces or energy loss in the system, such that the block can oscillate back and forth forever.

A large number of photographs of the block are taken at random times while the block is oscillating back and forth.

i. One particular photograph is selected at random. Is the likelihood of that photograph showing the ball in Region A *greater than, less than,* or *equal to* the likelihood of that photograph showing the ball in Region B? Explain.

How confident are you of your answer & explanation? Very Confident ☐ ☐ ☐ ☐ ☐ Not confident

Physics 123A, Autumn 2007 CPR Pretest WO-UWA123A074P-P1(CPR)

Figure B.3: Written pretest version (CPR)W3A administered in autumn 2007 in Phys 123A. This is one of three versions of pretests administered this quarter.

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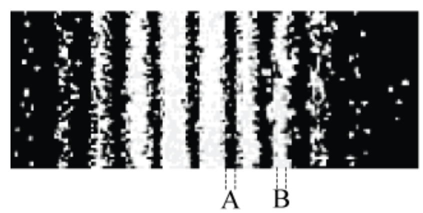
This is only a preview of the survey. Responses will not be saved. Close

(WPD,CPR)U1c
University of Washington

Page **1** of 2

Part I.

A large number of particles are incident on a mask containing narrow slits. A screen is placed far from the mask. It records the locations of the particles striking it as white dots. The photograph below shows the screen at the end of the experiment. Regions A and B are of equal width.



Question 1.
After the experiment, suppose you were to randomly choose one of the white dots that corresponds to a particle striking the screen.

How does the likelihood that the particle struck within region A compare to the likelihood that the particle struck within region B?

- ○ unanswered
- ○ A > B > 0
- ○ A > B = 0
- ○ A = B > 0
- ○ A = B = 0
- ◉ B > A > 0
- ○ B > A = 0
- ○ not enough information given

Question 2.
Explain your reasoning.

Figure B.4: Catalyst pretest version (WPD,CPR)U1c administered in winter 2008 to Phys 123B. (Four pages.)

Question 3.
Suppose the experiment described above is repeated. This time, however, only one particle is incident on the mask and reaches the screen.

How does the likelihood that the particle will strike within region A compare to the likelihood that it will strike within region B?

- ◯ unanswered
- ◯ A > B > 0
- ◯ A > B = 0
- ◯ A = B > 0
- ◯ A = B = 0
- ◉ B > A > 0
- ◯ B > A = 0
- ◯ not enough information

Question 4.
Explain your reasoning.

Next >>

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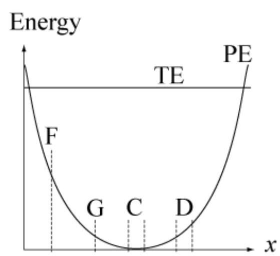
(WPD,CPR)U1c
University of Washington

Page **2** of 2

Part II.

Consider a system containing an object constrained to move in one dimension. The potential energy of the system is given by the curve PE in the graph below. The total energy of the system is given by the line TE. Regions C and D are of equal width, as shown.



Question 5.

Is it possible to describe the motion of the object? If so, do so in the space below. If it is not possible to describe the motion of the object, state what additional information is needed.

Question 6.
Suppose the position of the center of the object were measured at a random time.

How does the likelihood of finding the center of the object within region C compare to the likelihood of finding the center of the object within region D?

- ○ unanswered
- ○ C > D > 0
- ○ C > D = 0
- ○ C = D > 0
- ○ C = D = 0
- ⦿ D > C > 0
- ○ D > C = 0
- ○ not enough information

Question 7.
Explain your reasoning.

Question 8.
Suppose the position of the center of the object were measured at a random time.

How does the likelihood of finding the center of the object exactly at point F compare to the likelihood of finding it exactly at point G?

Time remaining:
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- ○ unanswered
- ○ F > G > 0
- ○ F > G = 0
- ○ F = G > 0
- ⦿ F = G = 0
- ○ G > F > 0
- ○ G > F = 0
- ○ not enough information

Question 9.
Explain your reasoning.

Submit responses

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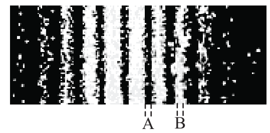
Name ____**UW Net ID** ____________________
last *first*

Note: If any of the following quantities are equal to zero, or if it is not possible to rank the quantities, state so explicitly.



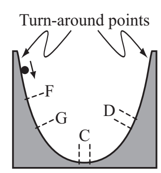
I. A large number of particles are incident on a mask containing narrow slits. A screen is placed far from the mask. It records the locations of the particles striking it as white dots. The photograph at right shows the screen at the end of the experiment. Regions A and B are of equal width, as shown.

A. After the experiment, suppose you were to randomly choose one of the white dots that corresponds to a particle striking the screen.

Is the likelihood that the particle struck within region A *greater than*, *less than*, or *equal to* the likelihood that it struck within region B? Explain.

B. Suppose the experiment described above is repeated. This time, however, only one particle is incident on the mask and reaches the screen.

Is the likelihood that the particle will strike within region A *greater than*, *less than*, or *equal to* the likelihood that it will strike within region B? Explain.



II. A ball oscillates back and forth on a frictionless ramp, as shown. Regions C and D are of equal width.

Suppose the position of the *center* of the ball were measured at a random time.

i. Is the likelihood of finding the center of the ball within region C *greater than*, *less than*, or *equal to* the likelihood of finding it within region D? Explain.

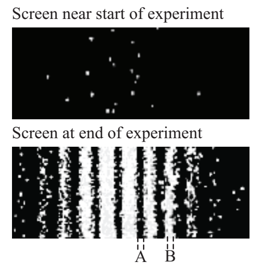
ii. Is the likelihood of finding the center of the ball exactly at point F *greater than*, *less than*, or *equal to* the likelihood of finding it exactly at point G? Explain

Physics 225/315A, Winter 2008 QM-UWA225A081T-O1(WPD,CPR)U1a

Figure B.5: Written pretest version 225A_U1A administered in winter 2008 to PHYS 225A.

Name ____**UW Net ID** ________________
last *first*

Note: If any of the following quantities are equal to zero, or if it is not possible to rank the quantities, state so explicitly.



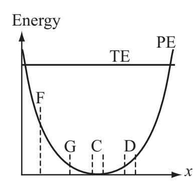
I. Particles are incident one at a time on a mask containing narrow slits. A screen is placed far from the mask. It records the locations of particles striking it as white dots. Pictures of the screen near the start and the end of the experiment are shown at right. Regions A and B are of equal width, as shown.

A. After the experiment, suppose you were to randomly choose one of white dots corresponding to a particle that struck the screen.

Is the likelihood that the particle struck region A *greater than, less than,* or *equal to* the likelihood that it struck within region B? Explain.

B. Suppose the experiment described above is repeated. This time, however, only *one* particle is incident on the mask and reaches the screen.

Is the likelihood that the particle will strike within region A *greater than, less than,* or *equal to* the likelihood that it will strike within region B? Explain.



II. Consider a system containing an object constrained to move in one dimension. The potential energy of the system is given by the curve PE in the graph at right. The total energy of the system is given by the line TE. Regions C and D are of equal width, as shown.

Suppose the position of the center of the object were measured at a random time.

A. Is the likelihood of finding the center of the object within region C *greater than, less than,* or *equal to* the likelihood of finding it within region D? Explain.

B. Is the likelihood of finding the center of the object exactly at point F *greater than, less than,* or *equal to* the likelihood of finding it exactly at point G? Explain.

Physics 225A/315A, Winter 2008 WO-UWA225A081T-O1(WPD,CPR)U1b

Figure B.6: Written pretest version 225A_U1B administered in winter 2008 to PHYS 225A.

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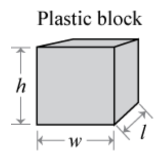
(PCQ)U1a&U1b

University of Washington

Part I(1a).

A solid block of plastic of height ***h***, width ***w***, and length ***l*** has a total mass of ***M*** uniformly distributed throughout its volume, as shown.



Question 2.

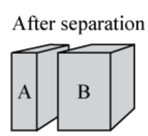
What is the volume mass density ρ_o of the plastic block? Express your answer in terms of the given variables.

M/(hwl)

Question 3.

Explain your reasoning.

The block is now broken into two pieces, labeled A and B, as shown.



Question 4.

Rank, from **greatest to least,** the volume mass densities of the **o**riginal block, piece **A**, and piece **B**. (Use only the symbols **O** (for original), **A**, **B**, **>**, and **=**. Do not use spaces or the < symbol.)

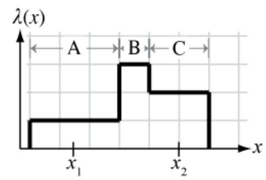
O=A=B

Question 5.

Explain your reasoning.

Figure B.7: Catalyst pretest version U1A administered in spring 2008 to PHYS 123A&C. Students in these two courses received either this version or version U1B (see Figure B.8 on page 564) based on the last digit of their student ID number. (Four pages.)

A one-dimensional rod contains a positive charge. A graph of the amount of charge per unit length λ versus position x is shown at right.



Question 6.

Rank, from **greatest to least,** the amount of charge in regions A, B, and C. (Use only the symbols **A**, **B**, **C**, **>**, and **=**. Do not use spaces or the < symbol.)

C>A=B

Question 7.

Explain your reasoning.

Next >>

Questions or Comments?
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0:11:34

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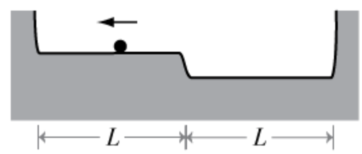
(PCQ)U1a&U1b

University of Washington

Part II(1a).

A pebble of mass m = 5 g moves back and forth forever on a frictionless track with very steep sides, as shown. The two levels shown are each of length L and are joined by a steep ramp. Ignore the intervals in which the pebble is on the steep portions of the track and assume it never leaves the surface of the track.



Suppose a photograph of the pebble were taken at a random time.

Question 8.
Is the likelihood that the photograph shows the pebble on the upper level greater than, less than, or equal to the likelihood that it shows the pebble on the lower level?

likelihood on upper level is greater than likelihood on lower level

Question 9.

Explain your reasoning.

Next >>

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0:09:32

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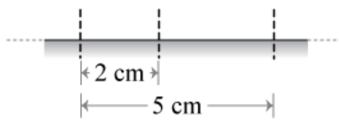
(PCQ)U1a&U1b

University of Washington

Part III(1a&1b).

A pebble of mass m = 5 g moves back and forth in a frictionless track of unknown shape. A short, horizontal segment of the track is shown at right. It is known that the probability of finding the pebble within a particular 2 cm region on the track is 0.08 (i.e., 8%), as shown.



Question 10.

What is the probability of finding the pebble within the indicated 5 cm region? Express your answer in decimal format. (If there is not enough information to answer, enter "not enough information.")

0.20

Question 11.

Explain your reasoning.

Submit responses

Questions or Comments?
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Time remaining:
0:07:02

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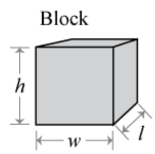
(PCQ)U1a&U1b

University of Washington

Part I(1b).

A block of height ***h***, width ***w***, and length ***l*** has a total positive charge of ***Q*** uniformly distributed throughout its volume.



Question 2.

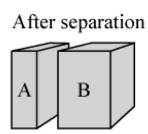
What is the volume charge density ρ_0 of the block? Express your answer in terms of the given variables.

Q/(hwl)

Question 3.

Explain your reasoning.

The block is now broken into two pieces, labeled A and B, as shown.



Question 4.

Rank, from greatest to least, the volume charge densities of the **o**riginal block, piece **A**, and piece **B**. (Use only the symbols **O** (for original), **A**, **B**, **>**, and/or **=**. Do not use spaces or the < symbol.)

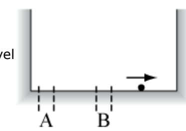
O=A=B

Question 5.

Explain your reasoning.

Figure B.8: Catalyst pretest version U1B administered in spring 2008 to PHYS 123A&C. Students in these two courses received either this version or version U1A (see Figure B.7 on page 560) based on the last digit of their student ID number. (Four pages.)

A pebble of mass m = 5 g moves back and forth in a one-dimensional region with rigid walls, as shown. Assume that the pebble rebounds from the walls elastically (*i.e.,* without loss of energy) and that the surface of the region is level and frictionless. Regions A and B are of equal width, as shown.



Suppose a photograph of the pebble were taken at a random time.

Question 6.
Is the likelihood that the photograph shows the pebble in region A *greater than, less than,* or *equal to* the likelihood that it shows the pebble in region B?

likelihood in region A is equal to likelihood in region B

Question 7.

Explain your reasoning.

Next >>

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0:06:31

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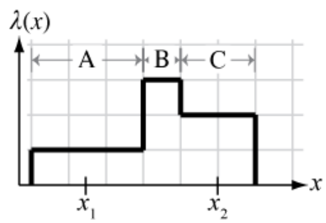
(PCQ)U1a&U1b

University of Washington

Part II(1b).

Many photographs are taken of a pebble confined to some unknown frictionless track. A graph of the probability of a photograph showing the pebble per unit length λ versus position x is shown at right.



Question 8.

Rank, from largest to smallest, the probability of a photograph showing the pebble in regions A, B, and C. (Use only the symbols **A**, **B**, **C**, **>** and/or **=**. Do not use spaces or the < symbol.)

C>A=B

Question 9.

Explain your reasoning.

Next >>

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0:05:56

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(PCQ)U1a&U1b

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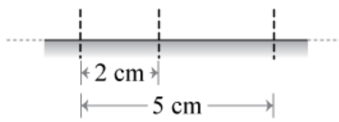
Part III(1a&1b).

A pebble of mass m = 5 g moves back and forth in a frictionless track of unknown shape. A short, horizontal segment of the track is shown at right. It is known that the probability of finding the pebble within a particular 2 cm region on the track is 0.08 (i.e., 8%), as shown.



Question 10.

What is the probability of finding the pebble within the indicated 5 cm region? Express your answer in decimal format. (If there is not enough information to answer, enter "not enough information.")

0.20

Question 11.

Explain your reasoning.

Submit responses

Questions or Comments?
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0:13:09

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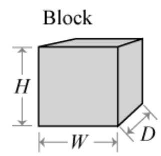
(PCQ)U1c

University of Washington

Page **1** of 3

Part I.

A block of material of height ***H,*** width ***W,*** and depth ***D*** contains a single marble that is located in a randomly chosen region.



Question 1.

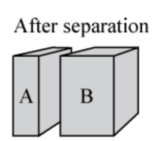
Write an expression for the *probability density* of finding the marble in the block of material in terms of the given variables *(i.e.,* in terms of *H, W, and D).*

1/(HWD)

Question 2.

Explain your reasoning.

The block of material, with the marble still inside, is now broken into two unequal-sized pieces, labeled A and B, as shown.



Rank, from **greatest to least,** the probability densities of finding the marble in the original block, piece A, and piece B. (Be sure to select a different object in each of the drop-down for questions 5, 7, and 9.) If there is not enough information to answer, leave 5-9 unanswered and explain your reasoning.

Figure B.9: Catalyst pretest version U1c administered in summer 2008 to Phys 123A. (Four pages.)

Question 3.

The probability density in the original block

Question 4.

is equal to

Question 5.

that in piece A

Question 6.

is equal to

Question 7.

that in piece B.

Question 8.

Explain your reasoning.

Next >>

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(PCQ)U1c

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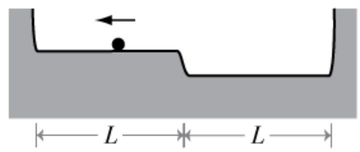
Page **2** of 3

Part I.

A pebble of mass *m* = 5 g moves back and forth forever on a frictionless track with very steep sides, as shown. The two levels shown are each of length *L* and are joined by a steep ramp. Assume the pebble never leaves the surface of the track.



Suppose a photograph of the pebble were taken at a random time.

Question 9.
Is the likelihood that the photograph shows the pebble on the upper level *greater than, less than,* or *equal to* the likelihood that it shows the pebble on the lower level?

likelihood on upper level is greater than likelihood on lower level

Question 10.

Explain your reasoning.

Next >>

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(PCQ)U1c

University of Washington

Page **3** of 3

Part III.

A ball is confined to a frictionless track that is composed of two regions, A and B. It is known that the probability of finding the ball in region A is *greater than* the probability of finding the ball in region B.

Question 11.
Is the *probability density* in region A *greater than, less than,* or *equal to* the probability density in region B? Assume that the probability density in each region is constant.

not enough information

Question 12.

Explain your reasoning.

Submit responses

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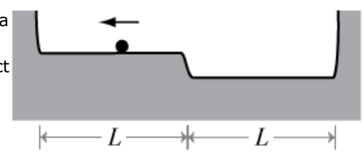
(PCQ)U1d

University of Washington

Part I. 0-4

A classical object (*e.g.,* a ball) moves back and forth forever on a frictionless track with very steep sides, as shown. The two levels are of equal length *L* and are joined by a ramp. The object is able to make it up and down the ramp, and so it never "gets stuck" in the lower level.



Suppose a single photograph of the pebble is taken at a random time.

Question 2.
Is the probability that the photograph shows the object on the upper level *greater than, less than,* or *equal to* the probability that it shows the object on the lower level?

P(upper) is greater than P(lower)

Question 3.

Explain your reasoning.

Next >>

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Figure B.10: Catalyst pretest version U1d administered in autumn 2008 to Phys 123A. About half of the students in this class had the first two pages switched; the other half had the pretest presented as it is here. (Four pages.)

Time remaining:
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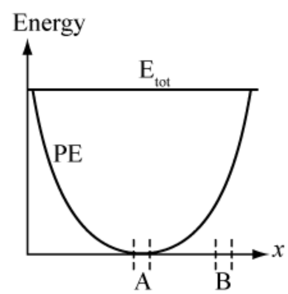
(PCQ)U1d

University of Washington

Part II. 0-4

A classical object (*e.g.,* a ball) is part of a system that includes potential energy and is confined to move in one dimension on the *x*-axis. The potential energy of the system as a function of position of the object is labeled PE. The total energy of the system (potential and translational kinetic) is labeled E_{tot}. Assume there are no dissipative forces (e.g., friction). Regions A and B are of equal width, as shown.

Suppose a single photograph of the object is taken at a random time.



Question 4.
Is the probability that the photograph shows the object in region A *greater than, less than,* or *equal to* the probability that it shows the object in region B?

P(A) is less than P(B)

Question 5.

Explain your reasoning.

Next >>

Questions or Comments?
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Time remaining:
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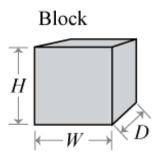
(PCQ)U1d

University of Washington

Part III.

A block of material of height ***H***, width ***W***, and depth ***D*** contains ***N*** sand particles that are evenly distributed throughout its volume.



Question 6.

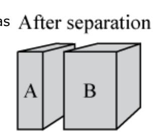
Write an expression for the **number density** of sand particles in the block of material (i.e., number of particles per unit volume) in terms of the given variables *(i.e.,* in terms of *H, W, D,* and *N).*

N/(HWD)

Question 7.

Explain your reasoning.

The block of material is now broken into two unequal-sized pieces, labeled A and B, as shown.



Question 8.
Is the **number density** of sand particles in piece A *greater than, less than,* or *equal to* the number density of sand particles in piece B?

number density in A is equal to that in B

Question 9.

Explain your reasoning.

Time remaining:
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(PCQ)U1d

University of Washington

Part IV.

An object is known to either be located in one of two regions, region A or B. It is known that the probability of finding the object in region A is greater than that in region B. The sizes of regions A and B are unknown.

Assume the probability density in each region is constant, but possibly different.

Question 10.
Is the *probability density* in region A *greater than, less than,* or *equal to* the probability density in region B? If there is not enough information, state so explicitly.

not enough information

Question 11.

Explain your reasoning.

Submit responses

Questions or Comments?
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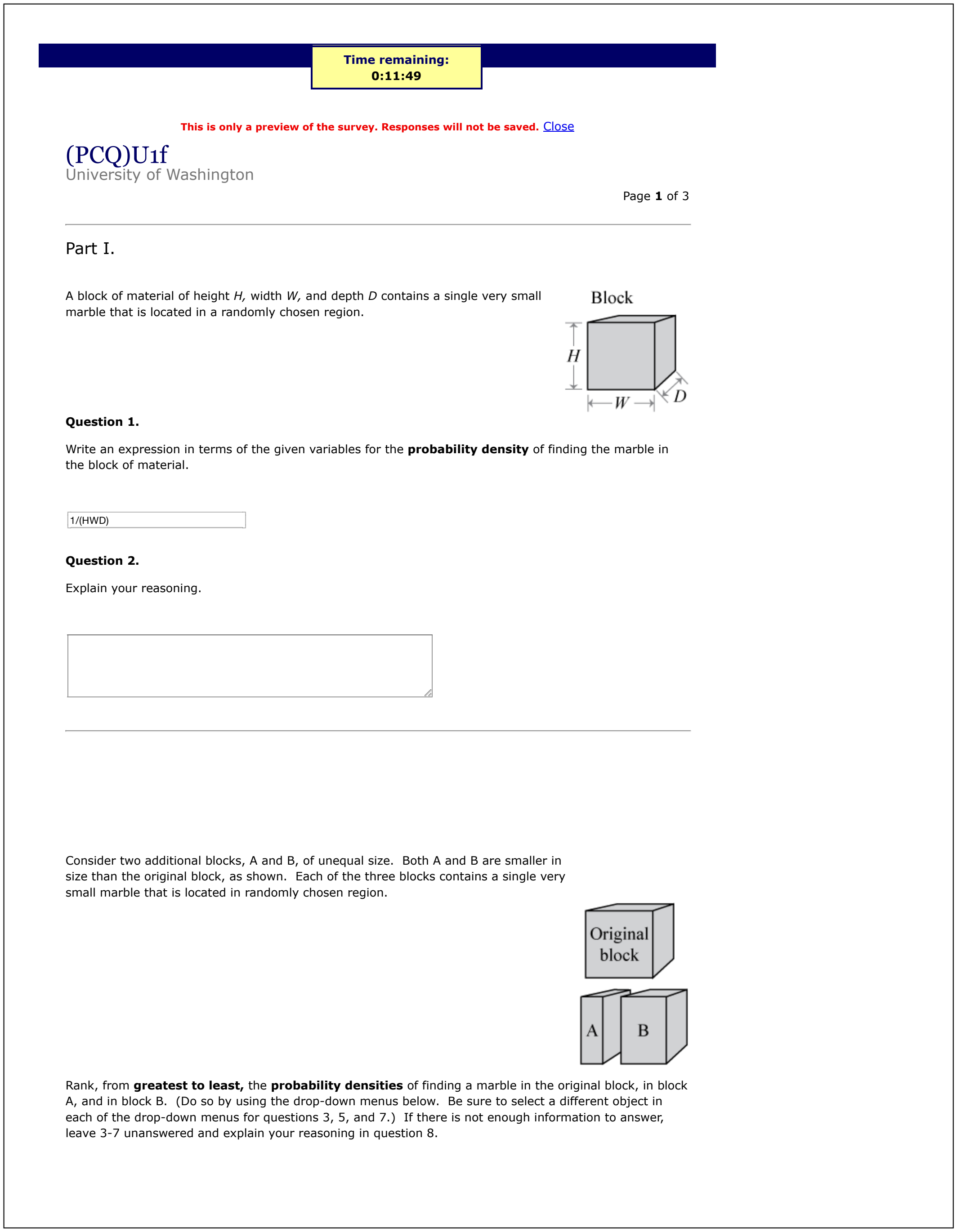

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Page **1** of 3

Part I.

A block of material of height *H,* width *W,* and depth *D* contains a single very small marble that is located in a randomly chosen region.

Block

H

W

D

Question 1.

Write an expression in terms of the given variables for the **probability density** of finding the marble in the block of material.

1/(HWD)

Question 2.

Explain your reasoning.

Consider two additional blocks, A and B, of unequal size. Both A and B are smaller in size than the original block, as shown. Each of the three blocks contains a single very small marble that is located in randomly chosen region.

Original block

A

B

Rank, from **greatest to least,** the **probability densities** of finding a marble in the original block, in block A, and in block B. (Do so by using the drop-down menus below. Be sure to select a different object in each of the drop-down menus for questions 3, 5, and 7.) If there is not enough information to answer, leave 3-7 unanswered and explain your reasoning in question 8.

Figure B.11: Catalyst pretest version U1f administered in winter 2009 to Phys 123A. (Four pages.)

Question 3.

The probability density in block A

Question 4.

is greater than

Question 5.

that in block B

Question 6.

is greater than

Question 7.

that in the original block.

Question 8.

Explain your reasoning.

Next >>

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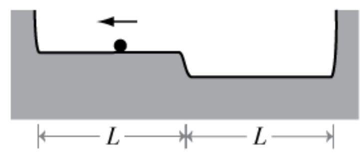
Page **2** of 3

Part II.

A ball moves back and forth forever on a frictionless track with very steep sides, as shown. The two levels of the track are of equal length and are joined by a ramp. Assume that the ball is able to make it up and down the ramp and it never leaves the surface of the track.



Suppose a photograph of the ball were taken at a random time.

Question 9.
Is the likelihood that the photograph shows the ball on the upper level *greater than, less than,* or *equal to* the likelihood that it shows the ball on the lower level?

likelihood on upper level is greater than likelihood on lower level

Question 10.

Explain your reasoning.

Next >>

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Page **3** of 3

Part III.

A ball moves back and forth on an arrangement of two horizontal tracks (A and B) that are connected by a short inclined segment. The tracks are at different heights and have different lengths.

It is known that the probability of finding the object on track A is greater than that of finding it on track B.

Question 11.
Is it possible to compare the probability densities on the two tracks? If so, how do they compare? (You may assume the probability density on each track is constant.)

No, the probability densities cannot be compared

Question 12.

Explain your reasoning.

Submit responses

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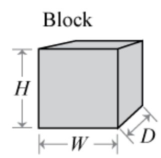
Page **1** of 3

Part I.

A block of material of height *H,* width *W,* and depth *D* contains a single very small marble that is located in a randomly chosen region.



Question 1.

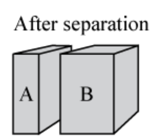
Write an expression in terms of the given variables for the **probability density** of finding the marble in the block of material.

1/(HWD)

Question 2.

Explain your reasoning.

The block of material, with the marble still inside, is now broken into two unequal-sized pieces, labeled A and B, as shown.



Rank, from **greatest to least,** the **probability densities** of finding the marble in the original block, in piece A, and in piece B. (Do so by using the drop-down menus below. Be sure to select a different object in each of the drop-down menus for questions 3, 5, and 7.) If there is not enough information to answer, leave 3-7 unanswered and explain your reasoning in question 8.

Figure B.12: Catalyst pretest version U1e administered in winter 2009 to Phys 123B. (Four pages.)

Question 3.

The probability density in the original block

Question 4.

is equal to

Question 5.

that in piece A

Question 6.

is equal to

Question 7.

that in piece B.

Question 8.

Explain your reasoning.

Next >>

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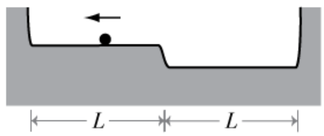
Page **2** of 3

Part II.

A ball moves back and forth forever on a frictionless track with very steep sides, as shown. The two levels of the track are of equal length and are joined by a ramp. Assume that the ball is able to make it up and down the ramp and it never leaves the surface of the track.



Question 9.
Is the speed of the ball in the upper level *greater than, less than,* or *equal to* that in the lower level?

speed in upper level < speed in lower level

Question 10.

Explain your reasoning.

Suppose a photograph of the ball were taken at a random time.

Question 11.
Is the likelihood that the photograph shows the ball on the upper level *greater than, less than,* or *equal to* the likelihood that it shows the ball on the lower level?

likelihood on upper level is greater than likelihood on lower level

Question 12.

Explain your reasoning.

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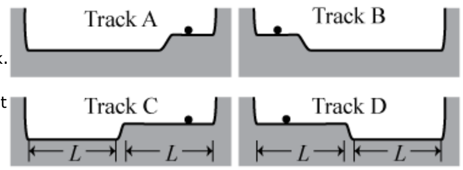
Page **3** of 3

Part III.

Four balls are confined to move back and forth in the four frictionless tracks shown. The balls are able to make it up to the upper level in each track.

Suppose a photograph of each track were taken at a random time.



Question 13.
For which of the tracks shown could the probability of the photograph showing the ball in the left region be greater than that in the right region (*i.e.,* for which track could Prob(left) > Prob(right) be true)? Select all that apply.

- [x] Prob(left) > Prob(right) could be true for track A
- [x] Prob(left) > Prob(right) could be true for track B
- [ ] Prob(left) > Prob(right) could be true for track C
- [x] Prob(left) > Prob(right) could be true for track D
- [ ] P(left)>P(right) could not be true for any of the shown tracks

Question 14.

Explain your reasoning.

Submit responses

Questions or Comments?

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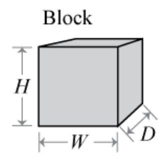
Page **1** of 3

Part I.

A block of material of height *H,* width *W,* and depth *D* contains a single very small marble that is located in a randomly chosen region.



Question 1.

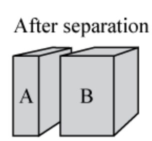
Write an expression in terms of the given variables for the **probability density** of finding the marble in the block of material.

1/(HWD)

Question 2.

Explain your reasoning.

The block of material, with the marble still inside, is now broken into two unequal-sized pieces, labeled A and B, as shown.



Rank, from **greatest to least,** the **probability densities** of finding the marble in the original block, in piece A, and in piece B. (Do so by using the drop-down menus below. Be sure to select a different object in each of the drop-down menus for questions 3, 5, and 7.) If there is not enough information to answer, leave 3-7 unanswered and explain your reasoning in question 8.

Figure B.13: Catalyst pretest version U1h administered in spring 2009 to Phys 123A&C. (Four pages.)

Question 3.

The probability density in the original block

Question 4.

is equal to

Question 5.

that in piece A

Question 6.

is equal to

Question 7.

that in piece B.

Question 8.

Explain your reasoning.

Next >>

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(PCQ)U1h
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Page **2** of 3

Part II.

A ball moves back and forth forever on a frictionless track with very steep sides, as shown. The two levels of the track are horizontal and are joined by a ramp. Assume that the ball is able to make it up and down the ramp and it never leaves the surface of the track.

Suppose the position of the ball were measured at a random time.

Question 9.
Is the **probability** of finding the ball on the upper level *greater than, less than,* or *equal to* the **probability** of finding the ball on the lower level?

Not enough information to answer

Question 10.

Explain your reasoning.

Question 11.
Is the **probability density** of finding the ball on the upper level *greater than, less than,* or *equal to* the **probability density** of finding the ball on the lower level?

density on upper level is greater than density on lower level

Question 12.

Explain your reasoning.

Next >>

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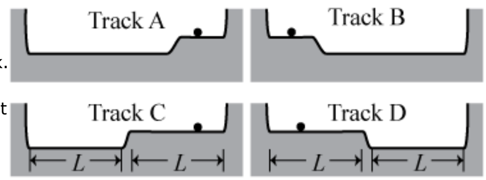
Page **3** of 3

Part III.

Four balls are confined to move back and forth in the four frictionless tracks shown. The balls are able to make it up to the upper level in each track.

Suppose a photograph of each track were taken at a random time.



Question 13.
For which of the tracks shown could the probability of the photograph showing the ball in the left region be greater than that in the right region (*i.e.,* for which track could Prob(left) > Prob(right) be true)? Select all that apply.

- [x] Prob(left) > Prob(right) could be true for track A
- [x] Prob(left) > Prob(right) could be true for track B
- [ ] Prob(left) > Prob(right) could be true for track C
- [x] Prob(left) > Prob(right) could be true for track D
- [ ] P(left)>P(right) could not be true for any of the shown tracks

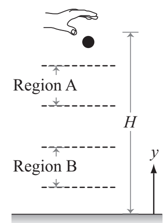
Question 14.

Explain your reasoning.

Submit responses

Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

[12 points total] A ball is dropped from rest at a height H (a couple of meters) above the surface of the earth, as shown. While the ball is falling, its position is measured by taking a photograph at a random time. Regions A and B are of equal size.

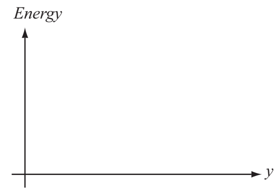


Regions A and B are of equal size

Treat the ball classically (*i.e.,* do not use quantum mechanics in answering the following questions). The following questions are intended to be answered qualitatively; no involved calculations are expected.

A. [4 pts] On the axes below at right, sketch for the Earth-ball system both (i) a qualitative graph of the *potential energy* vs *height* of the ball and (ii) a qualitative graph of the total energy E_{tot} vs *height* of the same system. Take $y = 0$ to be the ground. Clearly label each graph.

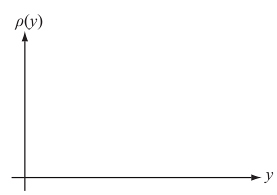
i. Explanation:



ii. Explanation:

B. [4 pts] Is the likelihood that the photograph shows the ball in region A *greater than, less than,* or *equal to* the likelihood that the photograph shows the ball region B? Explain.

C. [4 pts] On the axes at right, sketch a qualitative graph of the probability density $\rho(y)$ associated with measuring the position y of the ball at a random time. Take $y = 0$ to be the ground. Explain.



Physics 225A, Winter 2010 Final Exam QM-UWA225A101L-EF(PCQ).doc

Figure B.14: Written final exam administered in winter 2010 to Phys 225A. Since this question includes ideas related to potential energy diagrams, it is also shown in the potential energy diagram section in Appendix A.1.

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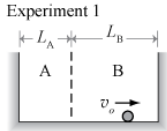
Page **1** of 3

Part I.

In experiment 1, a classical object moves with speed v_o in the flat track shown at right. The surface is frictionless and the object bounces off of the walls elastically. Assume the object bounces forever and always moves with the same speed.

An imaginary line divides the track into unequal-sized sections A and B of length L_A and L_B, with $L_A < L_B$.

Experiment 1



Classical object moves without loss of speed

After the object has been moving for a very long time, its position is measured at a random time by taking a photograph.

Question 1.
Is the **probability** of finding the object in section A *greater than, less than,* or *equal to* the **probability** of finding the object in section B?

- (•) Greater than
- ( ) Less than
- ( ) Equal to
- ( ) Not enough information

Question 2.

Explain your reasoning.

Question 3.
Is the **probability density** associated with section A *greater than, less than,* or *equal to* the **probability density** associated with section B?

- ( ) Greater than
- ( ) Less than
- (•) Equal to
- ( ) Not enough information

Figure B.15: Catalyst pretest version U2b administered in spring 2010 to Phys 123A. (Five pages.)

Question 4.

Explain your reasoning.

Question 5.
Write an expression for the **probability** of finding the object in section A in terms of the given variables.
If there is not enough information, state so explicitly.

L_A/(L_A+L_B)

Question 6.

Explain your reasoning.

Question 7.
Write an expression for the **probability density** associated with section A in terms of the given variables.
If there is not enough information, state so explicitly.

1/(L_A+L_B)

Question 8.

Explain your reasoning.

Next >>

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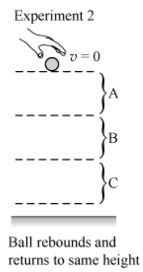
Page **2** of 3

Part II.

In experiment 2, a ball is dropped from rest a short distance above the ground. The ball rebounds from the ground and returns to the same height after each bounce. Assume that the ball bounces forever. The three regions shown, A, B, and C, are of equal size.

After the ball has been bouncing for a very long time, its position is measured at a random time by taking a photograph. Ignore effects due to the size of the ball (*i.e.,* treat the ball as a point mass).



Question 9.
Rank, from greatest to least, the **probabilities** of finding the ball in the three regions A, B, and C. Use the symbols **A**, **B**, and **C** only once each, along with **>** and/or **=** as appropriate. If there is not enough information, enter "not enough information."

A>B>C

Question 10.

Explain your reasoning.

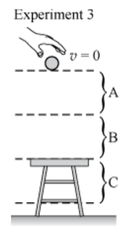
Question 11.
Rank, from greatest to least, the **probability densities** at the centers of regions A, B, and C. Use the symbols **A**, **B**, and **C** only once each, along with **>** and/or **=** as appropriate. If there is not enough information, enter "not enough information."

A>B>C

Question 12.

Explain your reasoning.

Experiment 3 is identical to experiment 2, except that a bench is placed in region C, as shown. The ball is released from rest from the same location as in experiment 2, bounces elastically off of the bench, and returns to the same height after each bounce. Assume that the ball bounces forever.



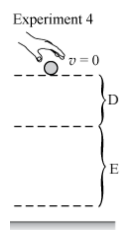
Question 13.
Is the **probability** of finding the ball in region A in experiment 3 (with the bench) *greater than, less than,* or *equal to* the **probability** of finding the ball in region A in experiment 2 (without the bench)?

- (•) Greater than
- ( ) Less than
- ( ) Equal to
- ( ) Not enough information

Question 14.

Explain your reasoning.

Experiment 4 is identical to experiment 2, except that there are now two regions D and E of unequal size. Region D is smaller than region E.



Question 15.
Is the **probability** of finding the ball in region D *greater than, less than,* or *equal to* the **probability** of finding the ball in region E?

- ( ) Greater than
- ( ) Less than
- ( ) Equal to
- (•) Not enough information

Question 16.

Explain your reasoning.

Question 17.
Is the **probability density** at the center of region D *greater than, less than,* or *equal to* the **probability density** at the center of region E?

- (•) Greater than
- ( ) Less than
- ( ) Equal to
- ( ) Not enough information

Question 18.

Explain your reasoning.

Next >>

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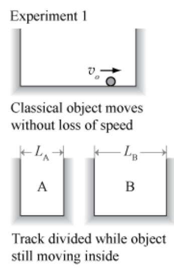
Page **1** of 3

Part I.

In experiment 1, a classical object moves with speed v_o in the flat track shown at right. The surface is frictionless and the object bounces off of the walls elastically. Assume the ball bounces forever and always moves with the same speed.



After the object has been moving for a very long time, a partition is inserted at a random time that divides the track into unequal-length sections A and B ($L_A < L_B$). The two sections are then spatially separated, as shown, and the position of the object is measured by taking a photograph.

Question 1.
Is the **probability** of finding the object in section A *greater than, less than,* or *equal to* the **probability** of finding the object in section B?

- ○ Greater than
- ◉ Less than
- ○ Equal to
- ○ Not enough information

Question 2.

Explain your reasoning.

Question 3.
Is the **probability density** associated with section A *greater than, less than,* or *equal to* the **probability density** associated with section B?

- ○ Greater than
- ○ Less than
- ◉ Equal to
- ○ Not enough information

Question 4.

Explain your reasoning.

Figure B.16: Catalyst pretest version U2c administered in spring 2010 to Phys 123C and in autumn 2011 to Phys 123A. (Four pages.)

Question 5.
Write an expression for the **probability** of finding the object in section A in terms of the given variables. If there is not enough information, state so explicitly.

L_A/(L_A+L_B)

Question 6.

Explain your reasoning.

Question 7.
Write an expression for the **probability density** associated with section A in terms of the given variables. If there is not enough information, state so explicitly.

1/(L_A+L_B)

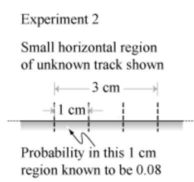
Question 8.

Explain your reasoning.

Part II.

In experiment 2, an object moves back and forth on a frictionless track. The shape of the track is unknown (*i.e.,* it may have multiple ramps and levels).

A short, horizontal segment of the track is shown at right. It is known that when the position of the object is measured at a random time, the probability of finding the object within the indicated 1 cm region is 0.08 (*i.e.,* 8%).



Question 9.
What is the probability of finding the object within the indicated 3 cm region? Express your answer in decimal format. If there is not enough information, enter "not enough information."

0.24

Question 10.

Explain your reasoning.

Next >>

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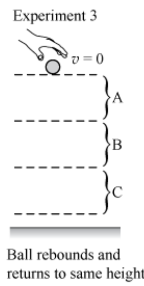
Part III.

In experiment 3, a ball is dropped from rest a short distance above the ground. The ball rebounds from the ground and returns to the same height after each bounce. Assume that the ball bounces forever. The three regions shown, A, B, and C, are of equal size.

After the ball has been bouncing for a very long time, its position is measured at a random time by taking a photograph. Ignore effects due to the size of the ball (*i.e.,* treat the ball as a point mass).



Question 11.
Rank, from greatest to least, the **probabilities** of finding the ball in the three regions A, B, and C. Use the symbols **A**, **B**, and **C** only once each, along with **>** and/or **=** as appropriate. If there is not enough information, enter "not enough information."

A>B>C

Question 12.

Explain your reasoning.

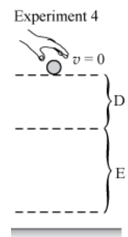
Question 13.
Rank, from greatest to least, the **probability densities** at the centers of regions A, B, and C. Use the symbols **A**, **B**, and **C** only once each, along with **>** and/or **=** as appropriate. If there is not enough information, enter "not enough information."

A>B>C

Question 14.

Explain your reasoning.

Experiment 4 is identical to experiment 3, except that there are now two regions D and E of unequal size. Region D is smaller than region E.



Question 15.
Is the **probability** of finding the ball in region D *greater than, less than,* or *equal to* the **probability** of finding the ball in region E?

- Greater than
- Less than
- Equal to
- Not enough information

Question 16.

Explain your reasoning.

Question 17.
Is the **probability density** at the center of region D *greater than, less than,* or *equal to* the **probability density** at the center of region E?

- Greater than
- Less than
- Equal to
- Not enough information

Question 18.

Explain your reasoning.

Next >>

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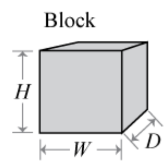
Page **2** of 2

Part IV. A block of material has height *H*, width *W*, and depth *D*. One of your friends has selected a random point P inside the block. You do not know which point was chosen.

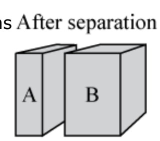


Question 7.
Write an expression in terms of the given variables for the **probability density** (not probability) associated with point *P* being located in a region of the block of material. If there is not enough information to answer, state so explicitly.

1/(HWD)

Question 8.
Explain the reasoning you used to answer the question above. If your answer is "not enough information," please discuss what additional information is necessary in order to answer the question.

Part V. The original block is separated into two unequal-sized portions A and B. Portion A is smaller than portion B.



Question 9.
Rank, from greatest to least, the **probability densities** associated with point *P* being located in the original block, portion A, and portion B. Use the symbols **O**, **A**, **B**, **>**, and **=**. If there is not enough information to answer, state so explicitly.

O=A=B

Question 10.
Explain the reasoning you used to answer the question above. If your answer is "not enough information," please discuss what additional information is necessary in order to answer the question.

Figure B.17: Catalyst pretest version U2A_SSD administered in autumn 2010 to PHYS 123A. This is part of a larger pretest; only the relevant portion is shown. An interactive lecture version of *Probability in classical and quantum mechanics* rather than the tutorial section version was used this quarter. (One page.)

Time remaining:
0:13:53

This is only a preview of the survey. Responses will not be saved. Close

(PCQ)U3b

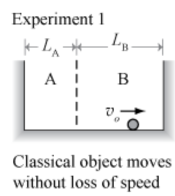
University of Washington

Part I.

A classical object moves with speed v_o in the flat track shown at right. The surface is frictionless and the object bounces off of the walls elastically. Assume the object bounces forever and always moves with the same speed.

An imaginary line divides the track into unequal-sized sections A and B of lengths L_A and L_B, with $L_A < L_B$.

After the object has been moving for a very long time, its position is measured at a random time by taking a photograph.



Question 2.
Is the **probability** of finding the object in section A *greater than, less than,* or *equal to* the probability of finding the object in section B?

- ◯ Greater than
- ◯ Less than
- ◯ Equal to
- ◯ Not enough information

Question 3.

Explain your reasoning.

Question 4.
Is the **probability density** associated with section A *greater than, less than,* or *equal to* the **probability density** associated with section B?

- ◯ Greater than
- ◯ Less than
- ◯ Equal to
- ◯ Not enough information

Figure B.18: Catalyst pretest version U3b administered in spring 2015 to Phys 123A&C. There were three parts to this pretest, and each part had two variants. Students saw only one of the variants on their pretest. For each part that is listed on the subsequent pages, variant 2 follows variant 1. (Eight pages.)

Question 5.

Explain your reasoning.

Question 6.

Write an expression for the **probability** of finding the object in section A in terms of the given variables. If there is not enough information, state so explicitly.

Question 7.

Explain your reasoning.

Question 8.

Write an expression for the **probability density** associated with section A in terms of the given variables. If there is not enough information, state so explicitly.

Question 9.

Explain your reasoning.

Next >>

Questions or Comments?
Contact the 123 tutorial pretest coordinator at uwttl123@u.washington.edu

Time remaining:
0:13:43

This is only a preview of the survey. Responses will not be saved. Close

(PCQ)U3b

University of Washington

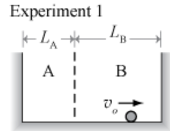
Part I.

A classical object moves with speed v_o in the flat track shown at right. The surface is frictionless and the object bounces off of the walls elastically. Assume the object bounces forever and always moves with the same speed.

An imaginary line divides the track into unequal-sized sections A and B of lengths L_A and L_B, with $L_A < L_B$.

After the object has been moving for a very long time, its position is measured at a random time by taking a photograph.

Experiment 1



Classical object moves without loss of speed

Question 2.
Is the **probability** of finding the object in section A *greater than, less than,* or *equal to* the **probability** of finding the object in section B?

- ◯ Greater than
- ◯ Less than
- ◯ Equal to
- ◯ Not enough information

Question 3.

Explain your reasoning.

Question 4.
Is the **probability per unit length** associated with section A *greater than, less than,* or *equal to* the **probability per unit length** associated with section B?

- ◯ Greater than
- ◯ Less than
- ◯ Equal to
- ◯ Not enough information

Question 5.

Explain your reasoning.

Question 6.

Write an expression for the **probability** of finding the object in section A in terms of the given variables. If there is not enough information, state so explicitly.

Question 7.

Explain your reasoning.

Question 8.

Write an expression for the **probability per unit length** associated with section A in terms of the given variables. If there is not enough information, state so explicitly.

Question 9.

Explain your reasoning.

Next >>

Questions or Comments?
Contact the 123 tutorial pretest coordinator at uwttl123@u.washington.edu

Time remaining:
0:13:01

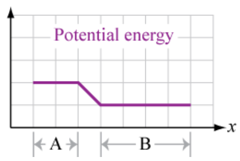
This is only a preview of the survey. Responses will not be saved. Close

(PCQ)U3b
University of Washington

Part II.

A classical object (e.g., a ball) moves back and forth in a one-dimensional system. The only forms of energy are potential energy of the system and translational kinetic energy of the particle. The total energy of the system is constant.

The graph at right shows the potential energy of the system as a function of the position of the object. (The potential energy to the left of region A and to the right of region B is not shown.) Regions A and B are of different sizes and have different potential energies. It is known that the object reaches both regions.



While the object is moving back and forth, a photograph of the object is taken at a random time.

Question 10.
Is the **probability** that the photograph shows the ball in region A *greater than, less than,* or *equal to* the **probability** that it shows the ball in region B?

- ◯ Greater than
- ◯ Less than
- ◯ Equal to
- ◯ Not enough information

Question 11.

Explain your reasoning.

Question 12.
Is the **probability density** in region A *greater than, less than,* or *equal to* the **probability density** in region B?

- ◯ Greater than
- ◯ Less than
- ◯ Equal to
- ◯ Not enough information

Question 13.

Explain your reasoning.

Next >>

Time remaining:
0:12:59

This is only a preview of the survey. Responses will not be saved. Close

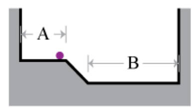
(PCQ)U3b
University of Washington

Part II.

A ball moves back and forth on a track composed of two horizontal levels. The ball is able to make it up the ramp to the upper level. The track is frictionless so that the ball moves back and forth forever.

Regions A and B are of different sizes and have different potential energies.

While the ball is moving back and forth, a photograph of the ball is taken at a random time.



Ball moves back and forth on a frictionless track

Question 10.
Is the **probability** that the photograph shows the ball in region A *greater than, less than,* or *equal to* the **probability** that it shows the ball in region B?

- Greater than
- Less than
- Equal to
- Not enough information

Question 11.

Explain your reasoning.

Question 12.
Is the **probability density** in region A *greater than, less than,* or *equal to* the **probability density** in region B?

- Greater than
- Less than
- Equal to
- Not enough information

Question 13.

Explain your reasoning.

Next >>

Time remaining:
0:09:06

This is only a preview of the survey. Responses will not be saved. Close

(PCQ)U3b

University of Washington

Part III.

It is known that a particular object is located in one of several one-dimensional regions. Two of these regions are labeled Region 1 and Region 2.

Region 1 has length ***A***. The probability that the object is located in region 1 is ***P***.

Region 2 has length ***B***.

There are several other regions of unknown length and probability.

Note: In this problem, assume that equal-sized regions have equal probabilities (i.e., the probability is uniformly distributed throughout all regions).

Question 14.

What is the **probability** of the object being located in **region 2?** Express your answer in terms of the given variables and/or numerical constants.

Question 15.

Explain your reasoning.

Submit responses

This is only a preview of the survey. Responses will not be saved. Close

(PCQ)U3b

University of Washington

Part III.

It is known that a particular object is located in one of several one-dimensional regions. Two of these regions are labeled Region 1 and Region 2.

Region 1 has length ***5 cm***. The probability that the object is located in region 1 is ***0.10*** (i.e., 10%).

Region 2 has length ***B***.

There are several other regions of unknown length and probability.

Note: In this problem, assume that equal-sized regions have equal probabilities (i.e., the probability is uniformly distributed throughout all regions).

Time remaining:
0:11:58

Question 14.

What is the **probability** of the object being located in **region 2?** Express your answer in terms of the given variables and/or numerical constants.

Question 15.

Explain your reasoning.

Submit responses

B.2 Curriculum related to Probability in classical and quantum mechanics

In this section we provide curricular materials related to the tutorial *Probability in classical and quantum mechanics.*

Contents of section

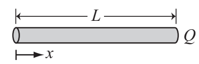
B.2.1 In-class worksheets for Probability in classical and quantum mechanics

In this section we provide the in-class worksheets for the tutorial *Probability in classical and quantum mechanics.*

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS

ST 1

I. Review of linear density



A. A rod of length L has a total positive charge of Q evenly distributed over its length, as shown.

1. Suppose you wanted to calculate the amount of charge between $x = 0.4\,L$ and $x = 0.6\,L$. Describe the steps you would need to take.

2. Can the quantity Q/L aid in the calculation of the amount of charge in some given region of length Δx? If so, demonstrate how.

B. Two students are trying to interpret the quantity Q/L:

Student 1: "It describes how much charge each point contains. So if the charge density is 5 μC/cm, each point on the rod contains 5 μC of charge. The cm tells us what units we are working in."

Student 2: "It describes the amount of charge in a region. If this quantity were 5μC/cm, then the amount of charge in a 1 cm region of the rod would be 5 μC. So the amount of charge in a 3 cm region would be three of these, or 15 μC."

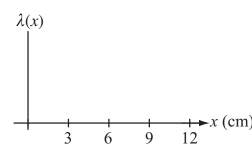
1. With which student, if either, do you agree? Explain.

2. Discuss how the interpretation with which you agree is consistent with your answers to part A.

The quantity Q/L is called the *linear charge density.*

C. Suppose the rod had length L = 12 cm and charge Q = 60 μC. Draw a quantitatively correct graph of the linear charge density λ versus position x. Label relevant values on the vertical axis.

D. What geometric feature of your graph represents the amount of charge between x = 3 cm and x = 6 cm? Indicate this feature on your graph.

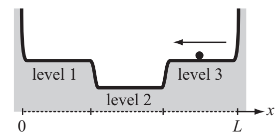


➪ Discuss your answers to this section with a tutorial instructor.

Figure B.19: In-class tutorial worksheet administered in spring 2008 to Phys 123A&B. (Five pages.)

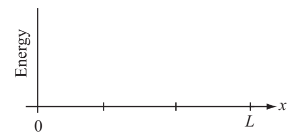
ST 2 ***Probability in classical and quantum mechanics***

II. Qualitative arguments of classical probability



A ball moves back and forth forever on a frictionless track with very steep sides. The three levels of the track are of equal length and are joined by steep ramps. Levels 1 and 3 are of equal height. Assume that the time spent on the steep portions is negligible and that the ball never leaves the surface of the track.

In the following questions, consider the system consisting of the ball, track, and Earth.



A. On the axes at right, sketch a qualitatively correct graph of potential energy versus position of the ball.

   On the same axes, draw a line that could represent the *total* energy of the system. How is the fact that the total energy of the system is conserved reflected in your graph?

B. Suppose the position of the ball were measured by taking a photograph of the system at a random time. Rank, from greatest to least, the likelihoods of the photograph showing the ball on levels 1, 2, and 3. Explain.

C. Consider the following three statements made by students:

   Student 1: "All three levels are the same length, so the ball is equally likely to be found on any level."

   Student 2: "The ball is most likely to be found on level 2 since it has the lowest potential energy. Levels 1 and 3 are equally likely but less likely than level 2."

   Student 3: "The ball will go through the middle twice as often each round trip, so it is most likely to be found on level 2. Levels 1 and 3 are equally likely but less likely than level 2."

   With which student(s), if any, do you agree? Explain.

D. Rank the speed of the ball when it is on each level. Is your answer consistent with your graphs of total energy and potential energy?

Rank the amount of time that the ball spends on each level while it moves once from the left side to the right side. Explain.

Would this ranking of amount of time on each level change if instead you were to consider the ball as it makes a full oscillation (*i.e.*, from left to right, then back to left)? Explain.

E. Review your answers to parts B and C above. On the basis of your results thus far, do you still agree with your answers? Resolve any inconsistencies.

➪ Discuss your answers to this section with a tutorial instructor.

III. Probability density

Consider again the ball and track situation from section II. Assume each level is of length 10 cm.

A. Suppose that, out of a large number of photographs, four out of every 100 (*i.e.*, 0.04) show the ball in the *leftmost* 2 cm region of level 2. Is the fraction of the photographs showing the ball in the *center* 2 cm region of level 2 *greater than*, *less than*, or *equal to* 0.04? Explain.

B. Calculate the probability of finding the ball within a region of length 1 cm on level 2. Explain how you determined your answer.

ST 4 ***Probability in classical and quantum mechanics***

C. Use your answer to part B to calculate the probability of finding the ball anywhere on level 2 (*i.e.*, the entire 10 cm region of level 2). Explain.

D. In section I you considered a charge density. Explain how the number you calculated in part B can be considered a density. (Review section I if necessary.)

The quantity you calculated in part B is called the *probability density* of finding the ball.

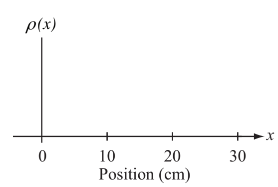
E. Give an interpretation of probability density. (*Hint:* Review part B of section I.)

F. What is the probability of finding the ball anywhere on level 1? Explain.

What is the probability density on level 1? Show any work and include units in your answer.

G. Draw a quantitatively correct graph of probability density ρ versus position x for the system. Label relevant values on the vertical axis.

H. What geometric feature of your graph represents the probability of finding the ball between $x = 5$ cm and $x = 15$ cm? Indicate this feature on your graph and use it to calculate the specified probability.



I. What would you expect for the probability of finding the ball *anywhere* in the system? Explain.

Show that your graph above is consistent with what you expect. Resolve any inconsistencies.

➪ Discuss your answers to this section with a tutorial instructor.

IV. Comparing classical and quantum physics

A. A ball is confined to a one-dimensional region of length L. The system has total energy E and the potential energy is constant in the region. (This situation is often called a particle in a box.)

Use the ideas developed in this tutorial to predict a quantitatively correct graph of probability density ρ versus position x.

Label relevant values on the vertical axis. (*Hint:* Review your answer to part I above.)

B. The experiment above is performed with an *electron* (instead of a ball) and a graph of probability density versus position is determined. Ask a tutorial instructor for the results of this experiment.

C. Two students discuss the results of the experiment.

Student 1: "No, the electron passes through any given region equally as much, but the average of many position measurements will be in the middle. The results match my prediction."

Student 2: "This does not match my prediction. There must be something different about an electron when compared to a ball."

With which student, if either, do you agree? Describe the error in the reasoning of the other student(s).

➪ Discuss your answers to this section with a tutorial instructor.

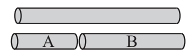
You will reconsider the above situation in the homework.

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS

ST 1

I. Review of linear density

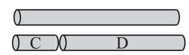
A. An insulating rod has a positive charge evenly distributed over its length. The rod is then broken into two pieces (A and B), as shown.



1. Is the charge on piece A *greater than*, *less than*, or *equal to* that on piece B? Explain.

2. Is the amount of charge in a region of unit length located on piece A *greater than*, *less than*, or *equal to* that in a region of unit length located on piece B? Explain.

B. A different insulating rod is broken into two pieces (C and D) that contain the *same* amount of charge. Piece D is three times the size of piece C. The charge on each piece is uniformly distributed.



Is the amount of charge in a region of unit length located on piece C *greater than*, *less than*, or *equal to* that in a region of unit length located on piece D? Explain.

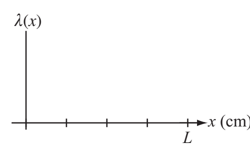
In parts A and B above, you considered the amount of charge in a region of unit length. We call this quantity *linear charge density,* or just *charge density.*

C. Calculate the amount of charge contained in a region that has a length of 5 units if the charge density in that region were 3 μC/unit length. Explain in words why your method works.

Give an expression for the charge density in a uniformly charged region of length d containing charge q. Is your expression consistent with your answers to A.2 and B? Explain.

D. Suppose the rod from part B above has *total* positive charge Q and length L. Draw a graph of the linear charge density λ vs. position x. (Take $x = 0$ to be the left edge of the rod.) Label relevant values on the vertical axis.

What geometric feature of your graph represents the amount of charge on the right half of the rod? Indicate this feature on your graph and use it to calculate this charge.



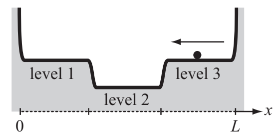
➪ Discuss your answers to this section with a tutorial instructor.

Figure B.20: In-class tutorial worksheet administered in summer 2008 to Phys 123A. (Five pages.)

ST 2 ***Probability in classical and quantum mechanics***

II. Qualitative reasoning about classical probability

A ball moves back and forth forever on a frictionless track with very steep sides. The track has three horizontal levels that are of equal length and are joined by steep ramps. Levels 1 and 3 are of equal height. Assume that the time spent on the steep portions is negligible and that the ball never leaves the surface of the track.

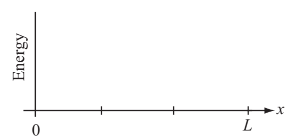


A. Suppose the position of the ball were measured by taking a photograph of the system at a random time. Rank, from greatest to least, the likelihoods of the photograph showing the ball on levels 1, 2, and 3. Explain your reasoning.

B. Consider the system consisting of the ball, track, and Earth.

1. On the axes below, sketch a qualitatively correct graph of potential energy vs. position of the ball. Draw a dashed line to represent the *total* energy of the system.

   How are the following reflected in your graph:
   (i) The total energy of the system is constant.
   (ii) The ball makes it up the ramps from level 2.
   (iii) The ball remains between $x = 0$ and $x = L$.



2. Rank the speed of the ball when it is on each level. Is your ranking consistent with your graphs of total energy and potential energy? Explain.

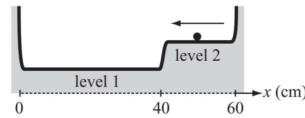
   What does your ranking imply about the relative amounts of time the ball spends on each level during one complete oscillation (*i.e.*, from $x = 0$ to $x = L$ and back)? A large number of oscillations? Explain.

C. Suppose that a complete oscillation takes 20 s and that the ball spends eight of those seconds on level 3. What is the probability of finding the ball on *each* level? Explain.

D. Do you still agree with your answer to part A? Resolve any inconsistencies.

➪ Discuss your answers to this section with a tutorial instructor.

III. Probability density



A ball moves back and forth on the frictionless track shown at right. Level 1 is of length 40 cm and level 2 is of length 20 cm. It is known that, out of a large number of photographs, 10 out of every 100 (*i.e.*, a fraction equal to 0.10) show the ball in the leftmost 4 cm region on level 2.

A. Is the fraction of photographs showing the ball in another 4 cm region on level 2 *different than* or *equal to* 0.10? Explain.

B. What is the probability of finding the ball within a region of length 1 cm on level 2? Explain.

Use your answer to calculate the probability of finding the ball on level 2 (*i.e.*, the entire 20 cm region of level 2). Explain.

C. In what way(s) can the first number you calculated in part B be considered a type of density?

The quantity that you calculated in the first part of B is called *probability density.*

D. In the context of section I, charge density could be interpreted as "the amount of charge in a region of unit length." Give an analogous interpretation for probability density.

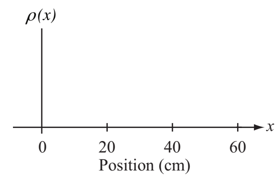
E. What is the probability of finding the ball on level 1? Explain.

What is the *probability density* on level 1? Show any work and include units in your answer.

ST 4 ***Probability in classical and quantum mechanics***

F. Draw a graph of probability density ρ vs. position x for the system. Label relevant values (including units) on the vertical axis.

What geometric feature of your graph represents the probability of finding the ball in the right half of the track? Indicate this feature on your graph and use it to calculate the specified probability.



What would you *expect* for the probability of finding the ball between $x = 0$ and $x = 60$ cm? How would this be reflected geometrically in your graph?

G. Suppose that at the start of this section you were given only the shape of the frictionless track and told the ball is able to make it up the ramp to level 2. Would it be possible to compare: (i) The time spent on each level? (i) The probabilities of finding the ball on each level? (ii) The probability densities on each level? Explain.

H. Three students are discussing the difference between probability and probability density:

Student 1: "Whenever the probabilities in two regions are the same, the probability densities should be the same as well."

Student 2: "No, we have to consider the sizes of the regions as well. The smallest region of a track will have the largest probability density since you divide by the length."

Student 3: "Objects are more likely to be found on horizontal levels that have higher probability densities."

None of the students are correct. Describe the flaw(s) in the reasoning of each student.

➪ Discuss your answers to this section with a tutorial instructor.

IV. Comparing classical and quantum mechanics

A. A ball is confined by rigid walls to move back and forth in a horizontal frictionless region of length L. A graph of potential energy versus position of the ball is shown at right. (This situation is often called an *infinite square well* or a *particle in a box*.)

Use the ideas developed thus far to draw a graph of probability density ρ vs. position x. Label relevant values on the vertical axis. Explain.

B. Ask a tutorial instructor for a graph of probability density that corresponds to a *neutron* with kinetic energy 8.2×10^{-4} eV confined to box of size L = 1.0 nm.

Suppose many measurements of position were made in the case of a ball and the case of the neutron described above. Describe how the distributions of position measurements vary in the two cases.

C. In part A above, you used the ideas developed in sections I – III of this tutorial to construct a graph of probability density for a ball. Are those methods appropriate to use in the case of a neutron confined to a one-dimensional box?

The methods you have developed to predict the likelihoods of finding objects in certain regions are consistent with the ideas of *classical mechanics.* In some cases (*e.g.,* small masses and small sizes), the behavior of particles may not match these predictions. This observation provides motivation for different description or model of the world in this regime. This model is called *quantum mechanics.*

D. In the homework, you will reconsider the situation involving the neutron in an infinite square well by using the ideas of quantum mechanics.

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS

ST 1

I. Review of linear density

A. An insulating rod has a positive charge evenly distributed over its length. A line divides the rod into two pieces (A and B), as shown.

1. Is the charge on piece A *greater than*, *less than*, or *equal to* that on piece B? Explain.

2. Is the charge density of piece A *greater than*, *less than*, or *equal to* that of piece B? Explain.

B. A line divides a different insulating rod into two pieces (C and D) that contain the *same* amount of charge. Piece D is three times the size of piece C. The charge on each piece is uniformly distributed.

Is the charge density of piece C *greater than*, *less than*, or *equal to* that of piece D? Explain.

C. Generalize your answers thus far. Is there a one-to-one relationship between: (i) charge density and the amount of charge? (ii) charge density and the size of a region? Explain.

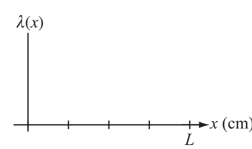
D. Do you agree or disagree with the following interpretation of charge density for a region with constant charge density: "The amount of charge in a region of unit length." Explain.

E. Suppose the rod from part B above has *total* positive charge Q and *total* length L.

1. Calculate the charge density of the two pieces (C and D). Show your work.

2. Draw a graph of the linear charge density λ vs. position x. (Take $x = 0$ to be the left edge of the rod.)

3. What geometric feature of your graph represents the amount of charge on the *left half* of the rod? Indicate this feature on your graph and use it to calculate this charge.



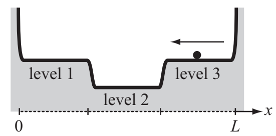
➪ Discuss your answers to this section with a tutorial instructor.

Figure B.21: In-class tutorial worksheet administered in autumn 2008 to Phys 123A. (Five pages.)

ST 2 ***Probability in classical and quantum mechanics***

II. Qualitative reasoning about classical probability

A ball moves back and forth forever on a frictionless track with very steep sides. The track has three horizontal levels that are of equal length. Levels 1 and 3 are of equal height. Assume that the ball is able to make it up and down the ramps and never leaves the surface of the track.

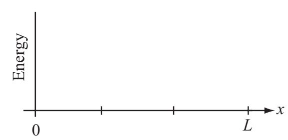


A. Suppose the position of the ball were measured by taking a photograph of the system at a random time. *Predict* the ranking, from greatest to least, of the likelihoods of the photograph showing the ball on levels 1, 2, and 3. Explain your reasoning.

B. Consider the system consisting of the ball, track, and Earth.

1. On the axes below, sketch a qualitatively correct graph of potential energy vs. position of the ball. Draw a dashed line to represent the *total* energy of the system.

   How are the following reflected in your graph:
   (i) The total energy of the system is constant.
   (ii) The ball makes it up the ramps from level 2.
   (iii) The ball remains between $x = 0$ and $x = L$.



2. Rank the speed of the ball when it is on each level. Is your ranking consistent with your graphs of total energy and potential energy? Explain.

   What does your ranking imply about the relative amounts of time the ball spends on each level during one complete oscillation (*i.e.*, from $x = 0$ to $x = L$ and back)? A large number of oscillations? Explain.

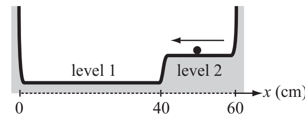
C. Suppose that a complete oscillation takes 20 s and that the ball spends eight of those seconds on level 3. What is the probability of finding the ball on *each* level? Explain.

D. Do you still agree with your answer to part A? Resolve any inconsistencies.

➪ Discuss your answers to this section with a tutorial instructor.

III. Probability density

A ball moves back and forth on the frictionless track shown at right. Level 1 is of length 40 cm and level 2 is of length 20 cm. It is known that, out of a large number of photographs, 8 out of every 100 (*i.e.*, a fraction equal to 0.08) show the ball in the centermost 4 cm region on level 2.



A. Is the fraction of photographs showing the ball in other 4 cm regions on level 2 *different than* or *equal to* 0.08? Explain.

B. What is the probability of finding the ball within a region of length 1 cm on level 2? Explain.

Use your answer to calculate the probability of finding the ball on level 2 (*i.e.*, the entire 20 cm region of level 2). Explain.

C. In what way(s) can the first number you calculated in part B be considered a type of density?

The quantity that you calculated in the first part of B is called *probability density*.

D. In the context of section I, charge density could be interpreted as "the amount of charge in a region of unit length." Give an analogous written interpretation for probability density.

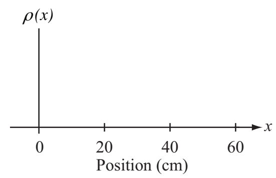
E. What is the probability of finding the ball on level 1? Explain.

What is the *probability density* on level 1? Show any work and include units in your answer.

ST 4 ***Probability in classical and quantum mechanics***

F. Draw a graph of probability density ρ vs. position x for the system. Label relevant values (including units) on the vertical axis.

What geometric feature of your graph represents the probability of finding the ball in the *right half* of the track? Indicate this feature on your graph and use it to calculate the specified probability.



What would you *expect* for the probability of finding the ball between $x = 0$ and $x = 60$ cm? How is this reflected geometrically in your graph?

G. Suppose that at the start of this section you were given the exact shape (*i.e.*, lengths and heights) of the frictionless track and told the ball is able to make it up the ramp to level 2. Would it be possible to compare: (i) the probabilities of finding the ball on each level? (ii) the probability densities on each level? (You may find it helpful to refer to your interpretation of probability density.) Explain.

H. Three students are discussing the difference between probability and probability density:

Student 1: "Whenever the probabilities in two regions are the same, the probability densities should be the same as well."

Student 2: "The smallest region of any track will have the largest probability density since you divide by the length."

Student 3: "Objects are more likely to be found on horizontal levels that have higher probability densities."

None of the students are correct. Describe the flaw(s) in the reasoning of each student.

➪ Discuss your answers to this section with a tutorial instructor.

IV. Comparing classical and quantum mechanics

A. A ball is confined by rigid walls to move back and forth in a horizontal frictionless region of length L. A graph of potential energy versus position of the ball is shown at right. (This situation is often called an *infinite square well* or a *particle in a box*.)

Use the ideas developed thus far to draw a graph of probability density ρ vs. position x. Label relevant values on the vertical axis. Explain.

B. A *neutron* with kinetic energy 8.2×10^{-4} eV is confined to box of size $L = 1.0$ nm. Ask a tutorial instructor for a graph of probability density that corresponds to this situation.

Suppose many measurements of position were made in the case of a ball and the case of the neutron described above. Describe how the distributions of position measurements would vary in the two cases.

C. In part A above, you used the ideas developed in sections I – III of this tutorial to construct a graph of probability density for a ball. Are those methods appropriate to use in the case of a neutron confined to a one-dimensional box?

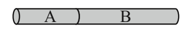
The methods you have developed to predict the likelihoods of finding objects in certain regions are consistent with the ideas of *classical mechanics.* In some cases (*e.g.,* small masses and small sizes), the behavior of particles may not match these predictions. This observation provides motivation for different description or model of the world in this regime. This model is called *quantum mechanics.*

D. In the homework, you will reconsider the situation involving the neutron in an infinite square well by using the ideas of quantum mechanics.

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS — ST 1

I. Review of linear density

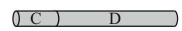
A. An insulating rod has positive charge evenly distributed over its length. A line divides the rod into two pieces (A and B), as shown.



1. Is the charge on piece A *greater than*, *less than*, or *equal to* that on piece B? Explain.

2. Do the charge densities of the two pieces have the same ranking as above? Explain.

B. A line divides a different insulating rod into two pieces (C and D) that contain the *same* amount of charge. The charge on each piece is uniformly distributed.



Is the charge density of piece C *greater than*, *less than*, or *equal to* that of piece D? Explain.

C. Generalize your answers thus far. Is it possible to rank the charge densities in different regions based solely on the ranking of (i) the amount of charge? (ii) the size? Explain.

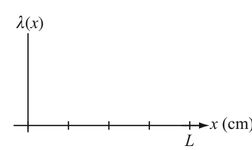
D. Do you agree or disagree with the following interpretation of charge density for a region with constant charge density: "The amount of charge in a region of unit length." Explain.

E. Suppose the rod from part B above has *total* positive charge Q and *total* length L. Piece D is three times the size of piece C.

1. Determine the charge density of the two pieces (C and D). Show any work.

2. Draw a graph of the linear charge density λ vs. position x. (Take $x = 0$ to be the left edge of the rod.)

3. How is the fact that the amounts of charge in the two parts of the rod are the same reflected in your graph? Explain.

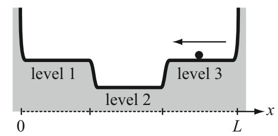


➪ Discuss your answers to this section with a tutorial instructor.

Figure B.22: In-class tutorial worksheet administered in winter 2009 to all sections of Phys 123. (Five pages.)

II. Qualitative reasoning about classical probability

A ball moves back and forth forever on a frictionless track with very steep sides. The track has three horizontal levels that are of equal length. Levels 1 and 3 are of equal height. Assume that the ball is able to make it up and down the ramps and never leaves the surface of the track.

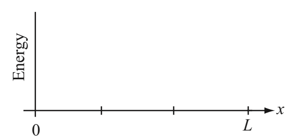


A. Suppose the position of the ball were measured by taking a photograph of the system at a random time. *Predict* the ranking, from greatest to least, of the likelihoods of the photograph showing the ball on levels 1, 2, and 3. Explain your reasoning.

B. Consider the system consisting of the ball, track, and Earth.

1. On the axes below, sketch a qualitatively correct graph of potential energy vs. position of the ball. Draw a dashed line to represent the *total* energy of the system.

   How are the following reflected in your graph?
   (i) The total energy of the system is constant.
   (ii) The ball makes it up the ramps from level 2.



2. Rank the speed of the ball when it is on each level. Is your ranking consistent with your graphs of total energy and potential energy? Explain.

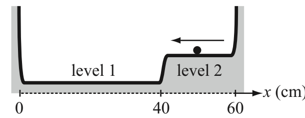
   What does your ranking imply about the relative amounts of time the ball spends on each level during one complete oscillation (*i.e.*, from $x = 0$ to $x = L$ and back)? A large number of oscillations? Explain.

C. Suppose that a complete oscillation takes 20 s and that the ball spends eight of those seconds on level 3. What is the probability of the photograph showing the ball on *each* level? Explain.

D. Do you still agree with your answer to part A? Resolve any inconsistencies.

III. Probability density

A ball moves back and forth on the frictionless track shown at right. Level 1 is of length 40 cm and level 2 is of length 20 cm. The ball is able to make it up to level 2.



A. Is it possible to predict on which level the ball is most likely to be found? Explain briefly.

Suppose now it is known that there is an 8% chance of a photograph showing the ball in the rightmost 4 cm region on level 2.

B. Is the probability of a photograph showing the ball in other 4 cm regions on level 2 *different than* or *equal to* 0.08? Explain.

C. What is the probability of finding the ball within a region of length 1 cm on level 2? Explain.

Use your answer to calculate the probability of finding the ball on level 2 (*i.e.*, the entire 20 cm region of level 2). Explain.

D. In what way(s) can the first number you calculated in part C be considered a type of density?

The quantity that you calculated in the first part of C is called *probability density*.

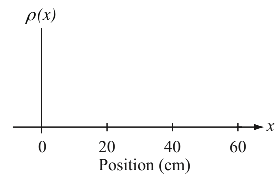
E. In the context of section I, charge density could be interpreted as "the amount of charge in a region of unit length." Give an analogous written interpretation for probability density.

F. What is the probability *density* on level 1? Show any work and include units in your answer.

ST 4 ***Probability in classical and quantum mechanics***

G. Draw a graph of probability density ρ vs. position x for the system. Label relevant values (including units) on the vertical axis.

What geometric feature of your graph represents the probability of finding the ball between $x = 30$ and $x = 60$ cm? Indicate this feature on your graph.



What would you *expect* for the probability of finding the ball between $x = 0$ and $x = 60$ cm? How is this reflected geometrically in your graph?

H. Suppose that at the start of this section you were only given the shape of the track and told the ball is able to make it up the ramp to level 2. Given only this information, is it possible to

(i) rank the probabilities of finding the ball on each level?
(ii) rank the probabilities in two small regions of the *same width* on each level?
(iii) rank the probability *densities* on each level? (*Hint:* Use your answer to question ii above.)

In each case, explain.

I. Do you agree or disagree with any of the following student ideas about probability and probability density as they relate to objects moving on tracks? Explain.

Student 1: "The ball is most likely to be found on the horizontal section that is highest."

Student 2: "You're most likely to find the ball on the horizontal section with the greatest probability density."

Student 3: "You cannot rank the probability densities on horizontal sections without knowing both the probabilities and the lengths of the sections."

Is your answer to H.iii consistent with your response to student 3? If not, resolve the inconsistency.

➪ Discuss your answers to this section with a tutorial instructor.

IV. Comparing classical and quantum mechanics

A. A ball is confined by rigid walls to move back and forth in a horizontal frictionless region of length L. A graph of potential energy versus position of the ball is shown at right. (This situation is often called an *infinite square well* or a *particle in a box*.)

Use the ideas developed thus far to draw a graph of probability density ρ vs. position x. Label relevant values on the vertical axis. Explain.

B. A *neutron* with constant kinetic energy 8.2×10^{-4} eV is confined to a box of size $L = 1.0$ nm. Ask a tutorial instructor for a graph of probability density that corresponds to this situation.

Suppose many measurements of position were made in the case of a ball and the case of the neutron described above. Describe how the distributions of position measurements would vary in the two cases.

C. In part A above, you used the ideas developed in sections I – III of this tutorial to construct a graph of probability density for a ball. Are those methods appropriate to use in the case of a neutron confined to a one-dimensional box?

The methods you have developed to predict the likelihoods of finding objects in certain regions are consistent with the ideas of *classical mechanics.* In some cases (*e.g.,* small masses and small sizes), the behavior of particles may not match these predictions. This observation provides motivation for a different description or model of the world in this regime. This model is called *quantum mechanics.*

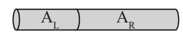
In the homework, you will reconsider the situation involving the neutron in an infinite square well by using the ideas of quantum mechanics.

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS

ST 1

I. Review of linear density

A. An insulating rod has positive charge evenly distributed over its length. A line divides the rod into two pieces (A_L and A_R), as shown.



1. Is the charge on piece A_L *greater than*, *less than*, or *equal to* that on piece A_R? Explain.

2. Do the charge densities of the two pieces have the same ranking as above? Explain.

B. A line divides a different insulating rod into two pieces that contain the *same* amount of charge. The charge on each piece is uniformly distributed.

Do the charge densities of the two pieces have the same ranking as the amount of charge? Explain.

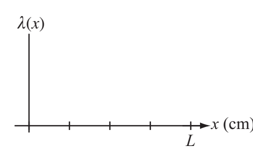
C. Is it possible to rank the charge densities in different regions based solely on the ranking of (i) the amount of charge? (ii) the size? Explain.

D. Suppose the rod from part B above has *total* positive charge Q and *total* length L. Piece B_L has length $L/4$.

1. Determine the linear charge density of the two pieces (B_L and B_R). Show any work.

2. Draw a graph of the linear charge density λ vs. position x. (Take $x = 0$ to be the left edge of the rod.)

3. How is the fact that the amounts of charge in the two parts of the rod are the same reflected geometrically in your graph? Explain briefly.



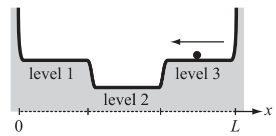
➪ Discuss your answers to this section with a tutorial instructor.

Figure B.23: In-class tutorial worksheet administered in spring 2009 to all sections of Phys 123. (Five pages.)

ST 2 ***Probability in classical and quantum mechanics***

II. Qualitative reasoning about classical probability

A ball moves back and forth forever on a frictionless track with very steep sides. The track has three horizontal levels that are of equal length. Levels 1 and 3 are of equal height. Assume that the ball is able to make it up and down the ramps and never leaves the surface of the track.

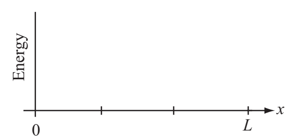


A. Suppose the position of the ball were measured by taking a photograph of the system at a random time. *Predict* the ranking, from greatest to least, of the likelihoods of the photograph showing the ball on levels 1, 2, and 3. Explain your reasoning.

B. Consider the system consisting of the ball, track, and Earth.

1. On the axes below, sketch a qualitatively correct graph of potential energy vs. position of the ball. Draw a dashed line to represent the *total* energy of the system.

   How are the following reflected in your graph?
   (i) The total energy of the system is constant.
   (ii) The ball makes it up the ramps from level 2.



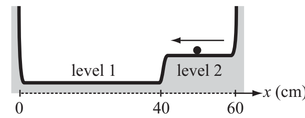
2. Rank the speed of the ball when it is on each level. How is your ranking consistent with your graphs of total energy and potential energy? Explain.

   What does your ranking imply about the relative amounts of time the ball spends on each level during (i) one complete oscillation (*i.e.*, from $x = 0$ to $x = L$ and back)? (ii) a large number of oscillations? Explain.

C. Do you still agree with your prediction in part A? Resolve any inconsistencies.

III. Probability density

A ball moves back and forth on the frictionless track shown at right. Level 1 is of length 40 cm and level 2 is of length 20 cm. The ball is able to make it up to level 2.



A. Is it possible to predict on which level the ball is most likely to be found? (It may be helpful to consider extreme cases of the ball's speed.) Explain.

Suppose now it is known that there is an 8% chance of a photograph showing the ball in the rightmost 4 cm region on level 2.

B. Is the probability of a photograph showing the ball in other 4 cm regions on level 2 *different than* or *equal to* 0.08? Explain.

C. What is the probability of finding the ball within a region of length 1 cm on level 2? Explain.

Use your answer to calculate the probability of finding the ball on level 2 (*i.e.*, the entire 20 cm region of level 2). Explain.

D. In what way(s) can the first number you calculated in part C be considered a type of density?

The quantity that you calculated in the first part of C is called *probability density*.

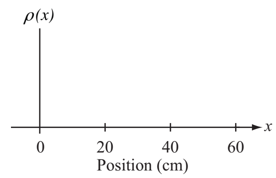
E. In the context of section I, charge density could be interpreted as "the amount of charge in a region of unit length." Give an analogous written interpretation for probability density.

F. What is the probability *density* on level 1? Show any work and include units in your answer.

ST 4 ***Probability in classical and quantum mechanics***

G. Draw a graph of probability density ρ vs. position x for the system. Label relevant values (including units) on the vertical axis.

What geometric feature of your graph represents the probability of finding the ball between $x = 30$ and $x = 60$ cm? Indicate this feature on your graph.



What would you *expect* for the probability of finding the ball between $x = 0$ and $x = 60$ cm? How is this reflected geometrically in your graph?

H. A ball moves back and forth on the frictionless track at right and is able to make it up the ramp. Given only this information, is it possible

(i) to rank the probabilities of finding the ball on each level? (*Hint:* Review your answer to part A on the previous page.)
(ii) to rank the probabilities in small regions of the *same width* on each level? (See figure for locations of regions.)
(iii) to rank the probability *densities* on each level? (*Hint:* Use your answer to question ii.)

In each case, explain.

I. Do you agree or disagree with the following student ideas about probability and probability density as they relate to objects moving on tracks? Explain.

Student 1: "The ball is most likely to be found on the horizontal level that is highest."

Student 2: "You're most likely to find the ball on the horizontal level with the greatest probability density."

Student 3: "You must know the probabilities and length of the levels in order to rank the probability densities on the levels."

Is your answer to H.iii consistent with your response to student 3? If not, resolve the inconsistency.

➪ Discuss your answers to this section with a tutorial instructor.

IV. Comparing classical and quantum mechanics

A. A ball is confined by rigid walls to move back and forth in a horizontal frictionless region of length L. A graph of potential energy versus position of the ball is shown at right. (This situation is often called an *infinite square well* or a *particle in a box*.)

Use the ideas developed thus far to draw a graph of probability density ρ vs. position x. Label relevant values on the vertical axis. Explain.

B. A *neutron* with constant kinetic energy 8.2×10^{-4} eV is confined to a box of size $L = 1.0$ nm. Ask a tutorial instructor for a graph of probability density that corresponds to this situation.

Suppose many measurements of position were made in the case of a ball and the case of the neutron described above. Describe how the distributions of position measurements would vary in the two cases.

C. In part A above, you used the ideas developed in sections I – III of this tutorial to construct a graph of probability density for a ball. Are those methods appropriate to use in the case of a neutron confined to a one-dimensional box?

The methods you have developed to predict the likelihoods of finding objects in certain regions are consistent with the ideas of *classical mechanics.* In some cases (*e.g.,* small masses and small sizes), the behavior of particles may not match these predictions. This observation provides motivation for a different description or model of the world in this regime. This model is called *quantum mechanics.*

In the homework, you will reconsider the situation involving the neutron in an infinite square well by using the ideas of quantum mechanics.

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS

ST
1

I. Review of linear density

A. A thin insulating rod has positive charge evenly distributed over its length. A line divides the rod into two pieces (A_L and A_R), as shown. Treat the rod as one-dimensional.

Rank the (i) charge and (ii) charge densities associated with pieces A_L and A_R. Explain.

B. A different rod is composed of two pieces that have *equal* amounts of charge. The charge is uniformly distributed on each piece.

Rank the charge densities associated with pieces B_L and B_R. Explain.

C. Consider the following student statements.

Student 1: "The charge density at a point is the amount of charge at that point."

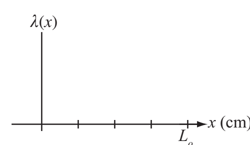
Student 2: "The charge density at a point is the charge per unit length near that point. You calculate it with Q_{total} / L_{total}."

Do you agree with either of the students? Are there situations in which the student(s) with whom you agreed might not be correct?

D. The rod from part B has *total* positive charge Q_o and *total* length L_o. Piece B_L has length $L_o/4$.

Determine the charge densities of the two pieces, λ_L and λ_R. Show your work.

Draw a graph of the linear charge density λ *vs*. position x. Take $x = 0$ to be the left edge of the rod.



What geometric feature of your graph indicates that the charges on the two pieces are the same?

What would you expect for the quantity $\int_{\text{all } x} \lambda(x)\, dx$,

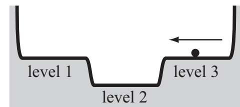
where $\lambda(x)$ is the charge density λ at position x?

➪ Discuss your answers to this section with a tutorial instructor.

Figure B.24: In-class tutorial worksheet administered in spring 2010 to all sections of Phys 123. (Five pages.)

ST 2 ***Probability in classical and quantum mechanics***

II. Qualitative reasoning about classical probability



A. A ball moves back and forth forever on a frictionless track with very steep sides. The track has three horizontal levels that are of equal length. Levels 1 and 3 are of equal height. Assume that the ball is able to make it up and down the ramps and never leaves the surface of the track.

After a very long time, the position of the ball is measured by taking a photograph of the system at a random time. *Predict* the ranking, from greatest to least, of the probability of the photograph showing the ball on levels 1, 2, and 3. Explain.

B. The following questions can be used as a guide to check your prediction above.

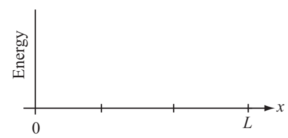
Consider the system consisting of the ball, track, and Earth.

On the axes below, sketch (i) a qualitatively correct graph of potential energy *vs*. position of the ball and (ii) a dashed line to represent the *total* energy of the system.

How are the following reflected in your graph?

The total energy of the system is constant.

The ball has enough kinetic energy to make it up the ramps from level 2.



Suppose you were only given the graph above. Describe how you could use it to (i) rank the speed of the ball on each level and (ii) rank the amount of time the ball spends on each level during a very large number of oscillations.

Do you still agree with your prediction in part A? Resolve any inconsistencies.

C. Write an expression for the probability, P_1, of the photograph showing the ball on level 1 in terms of Δt_1, Δt_2, and Δt_3 (the amounts of time the ball spends on levels 1, 2, and 3).

Suppose you wanted to decrease the probability of finding the ball on level 1. What physical change could you make (i) to level 1 to achieve this? (ii) to level 3?

III. Probability density

If the position of a classical or quantum object is not known exactly, it is possible to define a quantity called *probability density for position*, or just *probability density*.

A. In the context of section I, an interpretation for charge density is the *amount of charge in a particular region of unit length*. Give an analogous written interpretation for probability density.

What are the units of probability density?

How could you calculate a numerical value for probability density ρ in a region where probability density is constant?

How could you use probability density ρ to calculate probability?

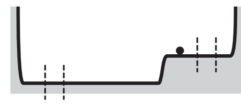
B. A ball moves back and forth on the frictionless track at right. The ball is able to make it up to level 2. Its position is measured at a random time while the ball is moving.

Is the probability density the same at all points in level 1, or does it vary? Explain.

Rank both (i) the *probabilities* and (ii) probability *densities* on the two levels using only the given information. If either is not possible, state so explicitly. Explain.

ST 4 ***Probability in classical and quantum mechanics***

The following questions can be used as a guide to check your answer to (ii) above.



How do the amounts of time that the ball spends in regions of unit length on the two levels (shown at right) compare?

Use your ranking above and your *interpretation* of probability density to rank the probability densities on the two levels. Explain.

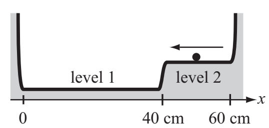
Do you still agree with your response to (ii) above? Resolve any inconsistencies.

C. What would you expect for the quantity $\int_{\text{all } x} \rho(x)\,dx$, where $\rho(x)$ is the probability density ρ at position x? Briefly explain.

The process of ensuring that $\int_{\text{all } x} \rho(x)\,dx$ has the value you expect is referred to as *normalization.*

D. Suppose that the lengths of levels 1 and 2 are 40 and 20 cm, respectively, and that the probability of finding the ball in the rightmost 4 cm region on level 2 is 8%.

Calculate the probability density on the two levels. Show work and include units



➪ Discuss your answers to this section with a tutorial instructor.

IV. Comparing classical and quantum mechanics

A system consisting of a particle with the potential energy shown at right is often called an *infinite square well* or *a particle in a box*. (The potential energy is zero between $x = 0$ and L and infinite elsewhere.)

A. Use the ideas developed thus far to draw a graph of $\rho(x)$ for the case of *a ball* confined to an infinite square well. Briefly explain.

Label relevant value(s) on the vertical axis.

B. An *electron* with a kinetic energy of 1.49 eV is confined to a box of width $L = 1.00$ nm.

Ask a tutorial instructor for a graph of the corresponding probability density versus position. Copy the graph on the axes at right.

C. Does using the classical ideas of speed and/or time spent in regions lead to a correct prediction for the probability distribution of the electron confined to an infinite square well?

Could a *single* measurement of position distinguish between a ball and an electron confined to an infinite square well? If so, explain why. If not, explain how you *could* distinguish between the two cases using position measurements.

The methods you have developed to predict the likelihoods of finding objects in certain regions are consistent with the ideas of *classical mechanics.* In some cases (*e.g.,* small masses and small sizes), the behavior of particles may not match these predictions. This observation provides motivation for a different description or model of the world in this regime. This model is called *quantum mechanics.*

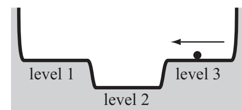
D. Recall from lecture or consult your textbook for the relation between the *wave function* Ψ in quantum mechanics and the probability density ρ.

Obtain a handout from a tutorial instructor. Do the graphs support this relation?

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS

ST 1

I. Qualitative reasoning about classical probability



A. A ball moves back and forth forever on a frictionless track with very steep sides. The track has three horizontal levels that are of equal length. Levels 1 and 3 are of equal height. Assume that the ball is able to make it up and down the ramps and never leaves the surface of the track.

After a very long time, the position of the ball is measured by taking a photograph of the system at a random time. *Predict* the ranking, from greatest to least, of the probability of the photograph showing the ball on levels 1, 2, and 3. Explain.

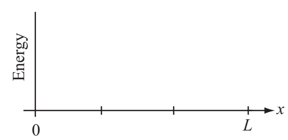
B. The following questions can be used as a guide to check your prediction of part A above.

Consider the system consisting of the ball, track, and Earth.

On the axes below, sketch both (i) a qualitatively correct graph of potential energy *vs*. position of the ball and (ii) a dashed line to represent the *total* energy of the system.

How are the following reflected in your graph?

- The total energy of the system is constant.
- The system has enough energy so that the ball makes it up the ramps to levels 1 and 3.



Suppose you were *only* given the graph above.
Describe how you could use it to (i) rank the speed of the ball on each level and (ii) rank the amount of time the ball spends on each level during a very large number of oscillations.

Do you still agree with your prediction in part A? Resolve any inconsistencies.

C. Write an expression for the probability, P_1, of finding the ball on level 1 in terms of Δt_1, Δt_2, and Δt_3 (the time the ball spends on levels 1, 2, and 3 during a complete oscillation).

Suppose you wanted to decrease the probability of finding the ball on level 1. What physical change could you make (i) to level 1 to achieve this? (ii) to level 3?

Figure B.25: In-class tutorial worksheet administered in autumn 2011 to Phys 123A. (Four pages.)

ST 2 ***Probability in classical and quantum mechanics***

II. Probability density

If the position of a classical or quantum object is not known exactly, it is possible to define a quantity called *probability density for position*, or just *probability density*.

A. Recall from electromagnetism that an *interpretation* for linear charge density is the amount of charge in a region of unit length (*i.e.,* charge per unit length).

Give an analogous *written* interpretation for probability density.

What are the units of probability density?

Suppose you knew the probability P of finding an object in a region of length L. How could you calculate a numerical value for probability density ρ? Are there any assumptions that you are making about the region?

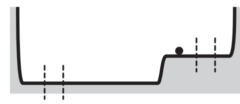
How could you use the probability density ρ and the length L of a region to calculate the probability P of finding an object in that region? Are there any assumptions that you are making about that region?

B. A ball moves back and forth on the frictionless track at right. The ball is able to make it up to level 2. Its position is measured at a random time while the ball is moving.

Is the probability density the same at all points in level 1, or does it vary? Explain.

Rank both (i) the *probabilities* and (ii) probability *densities* on the two levels using only the given information. If either is not possible, state so explicitly. Explain.

The following questions can be used as a guide to check your answer to (ii) above.



How do the amounts of time that the ball spends in regions of unit length on the two levels (shown at right) compare?

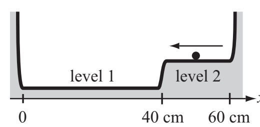
Use your ranking above and your *interpretation* of probability density from part II.A to rank the probability densities on the two levels. Explain.

Do you still agree with your response to (ii) above? Resolve any inconsistencies.

C. What would you expect for the quantity $\int_{\text{all } x} \rho(x)\, dx$, where $\rho(x)$ is the probability density ρ at position x? Briefly explain.

The process of ensuring that $\int_{\text{all } x} \rho(x)\, dx$ has the value you expect is referred to as *normalization.*

D. Suppose that the lengths of levels 1 and 2 are 40 and 20 cm, respectively, and that the probability of finding the ball in the rightmost 4 cm region on level 2 is 8%.



Calculate the probability density on the two levels. Show your work and include units.

➪ Discuss your answers to this section with a tutorial instructor.

ST 4 ***Probability in classical and quantum mechanics***

III. Comparing classical and quantum mechanics

A system consisting of a particle with the potential energy shown at right is often called an *infinite square well* or *a particle in a box*. (The potential energy is zero between $x = 0$ and L and infinite elsewhere.)

A. Use the ideas developed thus far to draw a graph of $\rho(x)$ for the case of *a ball* confined to an infinite square well. Briefly explain.

Label relevant value(s) on the vertical axis.

B. An *electron* with a kinetic energy of 1.49 eV is confined to a box of width $L = 1.00$ nm.

Ask a tutorial instructor for a graph of the corresponding probability density versus position. Copy the graph on the axes at right.

C. Does using the classical ideas of speed and/or time spent in regions lead to a correct prediction for the probability distribution of the electron confined to an infinite square well?

Could a *single* measurement of position distinguish between a ball and an electron confined to an infinite square well? If so, explain why. If not, explain how you *could* distinguish between the two cases using position measurements.

The methods you have developed in this tutorial to predict the likelihoods of finding objects in certain regions are consistent with the ideas of *classical mechanics.* In some cases (*e.g.,* small masses and small sizes), the behavior of particles may not match these predictions. This observation provides motivation for a different description or model of the world in this regime. This model is called *quantum mechanics.*

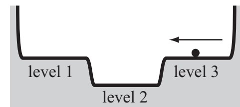
D. Recall from lecture or consult your textbook for the relation between the *wave function* Ψ in quantum mechanics and the probability density ρ.

Obtain a handout from a tutorial instructor. Do the graphs on the handout support this relation?

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS

ST 1

I. Qualitative reasoning about classical probability



A. A ball moves back and forth forever on a frictionless track with very steep sides. The track has three horizontal levels that are of equal length. Levels 1 and 3 are of equal height. Assume that the ball is able to make it up and down the ramps and never leaves the surface of the track.

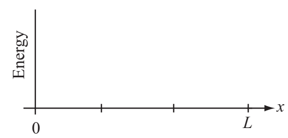
After a very long time, the position of the ball is measured by taking a photograph of the system at a random time. *Predict* the ranking, from greatest to least, of the probability of the photograph showing the ball on levels 1, 2, and 3. Explain.

B. The following questions can be used as a guide to check your prediction in part A above.

Consider the system consisting of the ball, track, and Earth.

1. On the axes below, sketch both (i) a qualitatively correct graph of potential energy *vs*. position of the ball and (ii) a dashed line to represent the *total* energy of the system.

2. Base your answers to the following questions on the diagram you drew.



- How can you tell the total energy is constant?
- What geometric feature represents the kinetic energy of the ball at a particular position *x?*
- How can you tell the ball makes it up the ramps to levels 1 and 3?

3. Describe how you can use your diagram to (i) rank the speed of the ball on each level and (ii) rank the amount of time it spends on each level during a large number of oscillations.

4. Do you still agree with your prediction in part A? Resolve any inconsistencies.

C. Write an expression for the probability, P_1, of finding the ball on level 1 in terms of Δt_1, Δt_2, and Δt_3 (the time the ball spends on levels 1, 2, and 3 during a complete oscillation).

Suppose you wanted to decrease the probability of finding the ball on level 1. What physical change could you make (i) to level 1 to achieve this? (ii) to level 3?

Figure B.26: In-class tutorial worksheet administered in spring 2015 to Phys 123A&C. (Four pages.)

ST 2 ***Probability in classical and quantum mechanics***

II. Probability density

If the position of a classical or quantum object is not known exactly, it is possible to define a quantity called *probability density for position*, or just *probability density*.

A. Recall from electromagnetism that an *interpretation* for linear charge density is the amount of charge in a region of unit length (*i.e.*, charge per unit length).

Give an analogous *written* interpretation for probability density. (*Note:* An equation is not an interpretation.)

What are the units of probability density?

Suppose you knew the probability P of finding an object in a region of length L. How could you calculate a numerical value for probability density ρ? Are there any assumptions that you are making about the region?

How could you use the probability density ρ and the length L of a region to calculate the probability P of finding an object in that region? Are there any assumptions that you are making about that region?

B. A ball moves back and forth on the frictionless track at right. The ball is able to make it up to level 2. Its position is measured at a random time while the ball is moving.

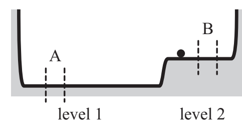
1. Is the probability density the same at all points in level 1, or does it vary? Explain.

2. Rank the probabilities P_1 and P_2 on the two levels. If there is not enough information to answer, state so explicitly. Explain.

3. Rank the probability densities ρ_1 and ρ_2 on the two levels. If there is not enough information to answer, state so explicitly. Explain. (You will check your answer on the next page.)

It is sometimes difficult to rank probability densities in two regions when the ranking of the probabilities cannot be determined. An alternative method to ranking the probability densities is to consider two small, equal-sized portions within the regions of interest. You will use this method in part C below.

C. The following questions can be used as a guide to check your answer to part B.3 above.



How do the amounts of time that the ball spends in regions of unit length on the two levels (shown at right) compare?

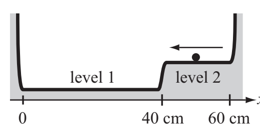
Use your ranking above and your *interpretation* of probability density from part II.A to rank the probability densities on the two levels. Explain.

Do you still agree with your response to part B.3 above? Resolve any inconsistencies.

D. What would you expect for the quantity $\int_{\text{all } x} \rho(x)\, dx$, where $\rho(x)$ is the probability density ρ at position x? Briefly explain.

The process of ensuring that $\int_{\text{all } x} \rho(x)\, dx$ has the value you expect is referred to as *normalization.*

E. Suppose that the lengths of levels 1 and 2 are 40 and 20 cm, respectively, and that the probability of finding the ball in the rightmost 4 cm region on level 2 is 8%.



Calculate the probability density on the two levels. Show your work and include units.

➪ Discuss your answers to this section with a tutorial instructor.

ST 4 ***Probability in classical and quantum mechanics***

III. Comparing classical and quantum mechanics

A system consisting of a particle with the potential energy shown at right is often called an *infinite square well* or *a particle in a box*. (The potential energy is zero between $x = 0$ and L and infinite elsewhere.)

A. Use the ideas developed thus far to draw a graph of $\rho(x)$ for the case of *a ball* confined to an infinite square well. Briefly explain.

Label relevant value(s) on the vertical axis.

B. An *electron* with a kinetic energy of 1.49 eV is confined to a box of width $L = 1.00$ nm.

Ask a tutorial instructor for a graph of the corresponding probability density versus position. Copy the graph on the axes at right.

C. Does using the classical ideas of speed and/or time spent in regions lead to a correct prediction for the probability distribution of the electron confined to an infinite square well?

Could a *single* measurement of position distinguish between a ball and an electron confined to an infinite square well? If so, explain why. If not, explain how you *could* distinguish between the two cases using position measurements.

The methods you have developed in this tutorial to predict the likelihoods of finding objects in certain regions are consistent with the ideas of *classical mechanics.* In some cases (*e.g.,* small masses and small sizes), the behavior of particles may not match these predictions. This observation provides motivation for a different description or model of the world in this regime. This model is called *quantum mechanics.*

D. Recall from lecture or consult your textbook for the relation between the *wave function* Ψ in quantum mechanics and the probability density ρ.

Obtain a handout from a tutorial instructor. Do the graphs on the handout support this relation?

B.2.2 Interactive lecture for Probability in classical and quantum mechanics

In this section we provide the materials (worksheet and clicker questions) for the interactive lecture for *Probability in classical and quantum mechanics.* Information about the interactive lecture can be found in §10.4 on page 303.

INTERACTIVE LECTURE: PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS

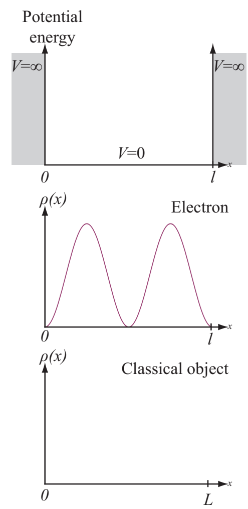
ST 1

I. Comparing classical and quantum mechanics

CLICKER: Use your clicker to register your answer to the following question individually:

An electron is confined to an infinite square well of width *l*. The graph at right shows the probability density ρ as a function of position *x* for a particular value of the total energy.

Now a classical object (*e.g.*, a ball) with non-zero kinetic energy is confined to an infinite square well of width *L*. Suppose the position of the object were going to be measured at a random, unknown time. What would the graph of probability density versus position look like for this situation? Explain.



Continue to the next part. You will return to the above situation at the end of the interactive lecture.

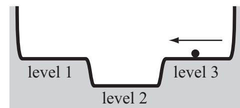
Physics Education Group, Department of Physics
University of Washington (Autumn 2010)

Figure B.27: Accompanying student worksheet for the interactive lecture administered in autumn 2010 to Phys 123A. (Four pages.)

ST 2 ***Interactive Lecture: Probability in classical and*** *quantum mechanics*

II. Qualitative reasoning about classical probability

CLICKER: Use your clicker to register your answer to part A below:



A. A classical object moves back and forth on a frictionless track composed of three flat levels joined by ramps. All three levels are of equal width and levels 1 and 3 are at equal heights. Assume the ball is able to make it up and down the ramps and never leaves the surface of the track.

Suppose a photograph were to be taken at a random time. *Predict* the ranking, from greatest to least, of the probability of the photograph showing the ball on levels 1, 2, and 3. Explain.

B. The following sequence of questions can be used as a guide to check your response above.

How do the speeds compare on the three levels? (What role do the ramps play?)

How do the relative amounts of time that the ball spends on the three levels compare? Explain.

Write an expression for the probability, P_1, of the photograph showing the ball on level 1 in terms of Δt_1, Δt_2, and Δt_3 (the amounts of time the ball spends on levels 1, 2, and 3).

C. Do you still agree with your answer to the clicker question in part II.A? If so, has your explanation changed?

CLASS DISCUSSION: Please wait before proceeding.

III. Probability density

CLICKER: Use your clicker to register your answer to part A below:

A. A classical object moves back and forth on the frictionless track shown at right. Suppose the position of the object were going to be measured at a random, unknown time.

How do the *probability densities* on the two levels compare? Explain.

B. Three students are discussing their ideas of probability density.

Student 1: "Probability density at a point is the probability of finding the object at that point."

Student 2: "Probability density at a point tells you the probability of finding the object per unit length near that point. This is just like charge density, but with probability."

Student 3: "You can't define probability density at a point. Probability density is only defined for regions. Probability density is probability divided by length, similar to charge density."

Do you agree with any of these students? Explain.

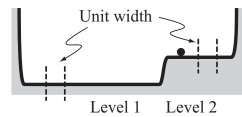
CLASS DISCUSSION: Please wait before proceeding.

C. Before continuing, write a verbal interpretation of the term *probability density*.

D. The following questions can be used as a guide to check your response to the clicker question of part III.A.

Is the probability density the same at all points in level 1, or does it vary?

Rank the probabilities of the ball being found in the regions of unit length on the two levels (shown at right).



Use your ranking above and your interpretation of probability density to rank the probability densities on the two levels.

Do you still agree with your response to the clicker question of part A? If so, has your explanation changed?

CLICKER and DISCUSSION: Re-answer the clicker question from part III.A.

E. Having worked through this interactive lecture, how would your answer and explanation to the first clicker question (part I.A.) change, if at all?

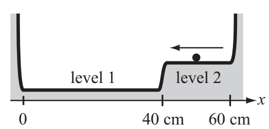
IV. If there is time, discuss the following questions with your neighbor

A. What would you expect for the quantity $\int_{\text{all } x} \rho(x)\, dx$, where $\rho(x)$ is the probability density ρ at position *x?* Briefly explain.

The process of ensuring that $\int_{\text{all } x} \rho(x)\, dx$ has the value you expect is called *normalization.*

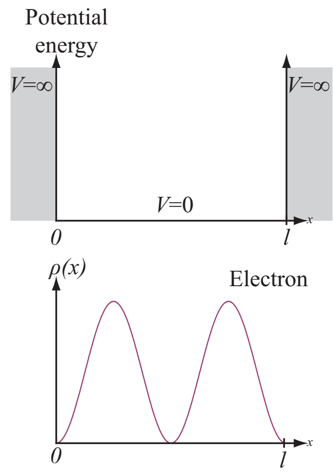
B. Consider again the track from section III. Suppose that the lengths of levels 1 and 2 are 40 and 20 cm, respectively, and that the probability of finding the ball in the rightmost 4 cm region on level 2 is 8%.

Calculate the probability density on the two levels. Show work and include units.



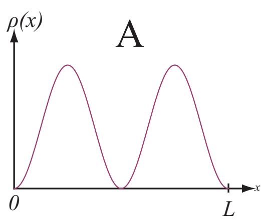
An electron is confined to an infinite square well of width l. The potential energy *vs.* position graph is at right.

The probability density for the electron as a function of position is shown at right for a particular value of the total energy.

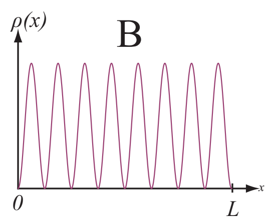


Now a classical object (*e.g.,* a ball) is confined to an infinite square well of width L. Its position is going to be measured at a random, unknown time. What would the graph of probability density vs. position look like?

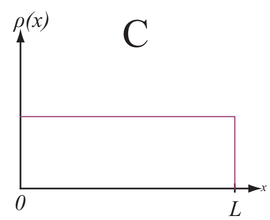
A



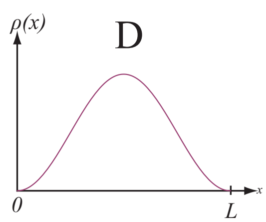
B



C



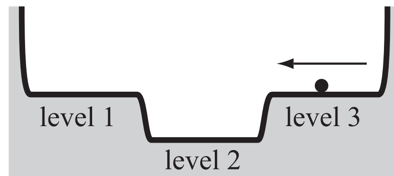
D



E

Similar to choices A or B, but depends on total energy

Figure B.28: Accompanying clicker questions for the interactive lecture administered in autumn 2010 to Phys 123A. The correct answers for these four clicker questions are: C, B, C, C. The third and fourth questions are identical. (Four pages.)



A classical object moves back and forth forever on a frictionless track made of three flat levels joined by ramps. All three levels have equal widths. Levels 1 and 3 are at equal heights. The ball is able to make it up each ramp and stays on the surface of the track.

A photograph will be taken at a random time. *Predict* the ranking of the probability that the photograph shows the ball on levels 1, 2, and 3.

A. P(1) = P(2) = P(3)
B. P(1) = P(3) > P(2)
C. P(2) > P(1) = P(3)
D. P(3) > P(2) > P(1)
E. Not enough information

A classical object moves back and forth on the frictionless track at right. Suppose the position of the object were going to be measured at a random, unknown time.

How do the *probability densities* on the two levels compare? Explain.

A. $\rho(1) > \rho(2)$
B. $\rho(1) > \rho(2) = 0$
C. $\rho(1) < \rho(2)$
D. $\rho(1) = \rho(2)$
E. Not enough information

A classical object moves back and forth on the frictionless track at right. Suppose the position of the object were going to be measured at a random, unknown time.

How do the *probability densities* on the two levels compare? Explain.

A. $\rho(1) > \rho(2)$
B. $\rho(1) > \rho(2) = 0$
C. $\rho(1) < \rho(2)$
D. $\rho(1) = \rho(2)$
E. Not enough information

B.2.3 Written homework for Probability in classical and quantum mechanics

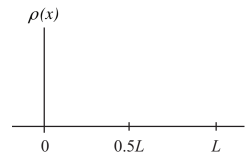
In this section we provide the written homework for the tutorial *Probability in classical and quantum mechanics*.

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS

Name ______________________

ST HW–1

1. In part A of section IV of the tutorial you considered a system in which a ball is confined to a one-dimensional region of length L with constant potential energy. On the axes at right, redraw your *predicted* graph of probability density versus position. Label relevant values on the vertical axis.

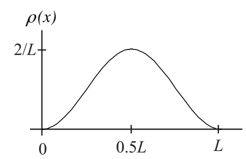


   a. Explain how you determined your predicted graph.

   b. Based on your predicted graph above, how does the probability of finding the ball in a given region of specified width vary along the track? Explain.

   c. An experiment is performed to determine the probability density of an *electron* confined to an infinite well of width L. The resulting graph is shown at right.

   Using the experimentally determined graph, in what region(s) are you most likely to find the electron? Explain.



   d. In the case above, classical mechanics does not correctly predict the probability density for an electron in an infinite well. Describe how the classical result differs from the result for an electron.

The method you used to predict the probability density in part a above is consistent with the ideas of *classical mechanics*. In some cases (e.g., small masses and small sizes), the behavior of particles may not match these predictions. This observation provides motivation for another description of the world in this regime. This model is called *quantum mechanics*.

Figure B.29: Written homework administered in spring 2008 to Phys 123A&B. (Four pages.)

ST
HW–2

Probability in classical and quantum mechanics

2. A *classical* object is confined to move on a frictionless track. The track is composed of multiple regions of various lengths and at various heights.

   Consider a particular region, region C. Assume this region is horizontal.

   a. Is the speed of the particle in the left half of region C the *same as* or *different from* the speed of the particle in the right half of region C? Explain.

   b. Suppose you knew the following three quantities:

   - L_C, the length of region C
   - v_C, the speed of the object in region C
   - T, the amount of time it takes to traverse from the left to right side of the entire track

   i. In terms of the given quantities, determine the probability of the object being found in region C. Explain.

   ii. In terms of the given quantities, determine the probability density in region C. Explain.

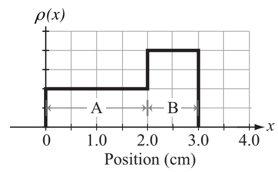
   c. Is the probability density *constant throughout* region C, or does it *vary* in region C? Explain.

   d. Suppose it is known that the probability density in a different region is constant throughout.

   What must be true of the shape of the track in that region? Explain.

Probability in classical and quantum mechanics Name ______________________ ST HW–3

3. The graph at right shows the probability density ρ versus position x for some unknown situation. Region A lies between $x = 0$ cm and $x = 2.0$ cm. Region B lies between $x = 2.0$ cm and $x = 3.0$ cm, as shown.



a. Label relevant values (including units) on the vertical axis. Explain.

b. In which region, A or B, is the object more likely to be found? If the likelihoods are equal, state so explicitly. Explain.

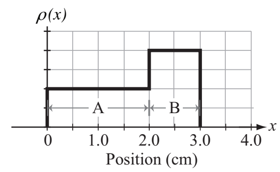
What is the probability of the object being found between $x = 1.5$ cm and $x = 2.5$ cm? Explain.

What is the probability of the object being found exactly at $x = 1.0$ cm? Explain.

ST
HW–4

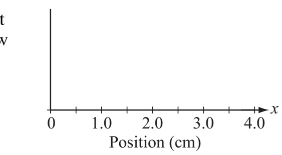
Probability in classical and quantum mechanics

4. The graph of probability density from problem 3 is reproduced at right. Assume the graph corresponds to a *classical* object moving back and forth in a frictionless track.



   a. Is the amount of time spent by the object in region A *greater than, less than,* or *equal to* the time it spends in region B? Explain.

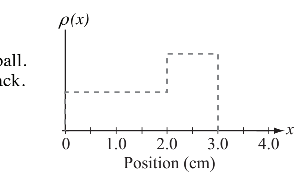
   b. Sketch the shape of a track that is qualitatively consistent with the graph of probability density above. Explain how you determined the shape of the track.



   c. In each case below, suppose a single change is made to the system.

      i. The total energy of the ball-track-Earth system is decreased by decreasing the kinetic energy of the ball. The object is still able to reach both sides of the track.

      Draw a new graph of probability density *versus* position. (Your graph need only be qualitatively correct.) Explain.



      ii. The length of region B is increased while keeping the length of region A the same.

      Will the probability density ρ in region A *increase, decrease,* or *remain the same?* Explain.

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS — Name ____________________ — ST HW–1

1. If you have not done so already, complete section IV of the tutorial. The handout described in the tutorial is shown below.

In classical mechanics (CM), probability densities can be constructed can be constructed using Newton's second law ($\vec{F}_{net} = m\vec{a}$) since the motion of an object depends on its acceleration. In quantum mechanics (QM), probability densities are constructed using Schrödinger's equation ($-(\hbar^2/2m)\partial^2\Psi/\partial x^2 + U\Psi = i\hbar\partial\Psi/\partial t$). As shown in the case of the infinite square well, the predictions of the models do not agree.

Complete the second and fourth columns of the table below for an electron confined to an infinite square well of width L = 10 cm. In each case, the width of region a is 2 cm.

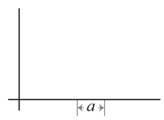
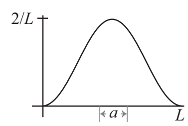
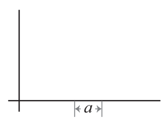
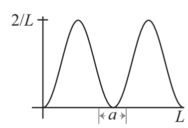
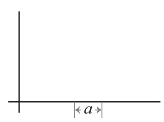
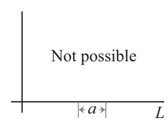
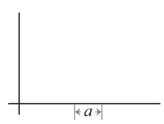
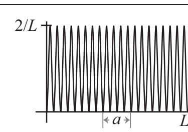
Kinetic energy of electron	Prediction of probability density by CM	Prediction of probability density by QM	Appoximate probability for object to be found in region a
3.8×10^{-17} eV			For CM: For QM:
1.5×10^{-16} eV			For CM: For QM:
4.0×10^{-16} eV			For CM: For QM: N/A
3.0×10^{-15} eV			For CM: For QM:

Figure B.30: Written homework administered in summer 2008 and autumn 2008 to PHYS 123A. (Four pages.)

ST HW–2 ***Probability in classical and quantum mechanics***

2. An object is confined to move on a frictionless track. The track is composed of multiple regions of various lengths and various heights. Treat the object classically.

   Consider a particular region, region C. Assume this region is horizontal.

   a. Suppose you knew the following three quantities:

      - L_C, the length of region C
      - v_C, the speed of the object in region C
      - T, the amount of time it takes to travel from the left to the right end of the track.

      i. In terms of the given quantities (L_C, v_C, and T), determine the probability of the object being found in region C. Explain.

      ii. In terms of the given quantities (L_C, v_C, and T), determine the probability density in region C. Simplify your answer as much as possible. Explain.

   b. Is the probability density *constant throughout* region C, or does it *vary* in region C? Explain.

   c. Suppose it is known that the probability density in a different region of the track is constant throughout that region.

      What must be true of the *shape* of the track in that region? Explain briefly.

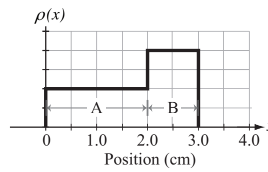
   d. Consider the following *incorrect* student statement:

      "My expression for the probability density in region C did not depend on L_C, so the probability density in region C should have the same value regardless of how long it is."

      Describe the error(s) in this student's reasoning.

Probability in classical and quantum mechanics Name ______________________ ST HW–3

3. The graph at right shows the probability density ρ versus position x for some unknown situation. Region A lies between $x = 0$ cm and $x = 2.0$ cm. Region B lies between $x = 2.0$ cm and $x = 3.0$ cm, as shown.



a. Is the probability of finding the object in region A *greater than*, *less than*, or *equal to* that in region B? Explain.

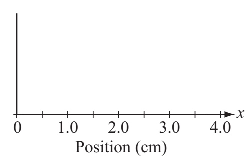
b. Label relevant values (including units) on the vertical axis. Explain.

What is the probability of the object being found in the right half of the track (*i.e.*, from $x = 1.5$ cm to $x = 3.0$ cm)? Explain and show any work.

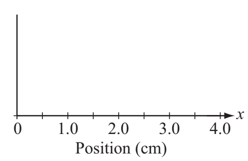
What is the probability of the object being found *exactly* in the middle of the track (*i.e.*, at $x = 1.5$ cm)? Explain.

c. Assume the graph of probability density above corresponds to a classical object moving back and forth in a frictionless track.

i. Sketch the shape of a track that would produce the graph of probability density. Explain briefly. (Your sketch need only be qualitatively correct.)

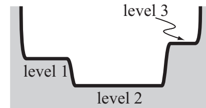


ii. Suppose the object's speed was reduced such that the speed of the object on the upper level were *very* small. Sketch a qualitatively correct graph of probability density in this case. Explain briefly.



ST
HW–4

Probability in classical and quantum mechanics



4. A classical object moves back and forth in the frictionless track show at right. The three horizontal levels are joined by steep ramps. Level 2 is the lowest and longest, while level 3 is the highest and shortest. Assume the ball is able to make it up each ramp.

   a. Rank, from greatest to least, the probability *densities* on the three levels. If there is not enough information to answer, state so explicitly. Explain. (*Hint:* Consider the interpretation of probability density described in the tutorial.)

   b. Rank, from greatest to least, the *probabilities* of the object being found on the three levels. If there is not enough information to answer, state so explicitly. Explain.

   c. Suppose the *height* of level 3 were increased but the total energy of the ball-Earth system remains the same. The ball is still able to make it up to level 3. In each of the following questions, explain your reasoning.

      i. Does the total time spent in region 3 *increase, decrease,* or *remain the same?*

      ii. Would the probability of the object being found in region 1 *increase, decrease,* or *remain the same?*

      iii. Would the *probability density* in region 1, *increase, decrease,* or *remain the same?*

      iv. Would the *probability density* in region 3, *increase, decrease,* or *remain the same?*

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS Name ____________________ ST HW–1

If you have not done so already, complete section IV of the tutorial.

1. In classical mechanics (CM), probability densities can be constructed using Newton's second law ($\vec{F}_{\text{net}} = m\vec{a}$) since the motion of an object depends on its acceleration. In quantum mechanics (QM), probability densities are constructed using Schrödinger's equation [$-(\hbar^2/2m)\partial^2\Psi/\partial x^2 + U\Psi = i\hbar\partial\Psi/\partial t$]. As shown in tutorial, the predictions of the two models do not agree even in the simple case of a particle in an infinite square well. In addition, only certain energies of the particle in an infinite square well are possible in quantum mechanics.

   In this problem, we explore the differences between the two models for the case of an electron in an infinite square well of width L = 1 nm. Complete the last two columns of the table below. (Note that you only need to estimate the probabilities for the last column using the graph. For example, you may use the value $2/L = 2\ \text{nm}^{-1}$ for the probability density in the QM case for the first row, and the width of region a is approximately 0.2 nm.)

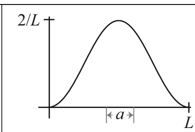
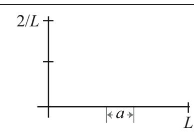
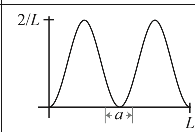
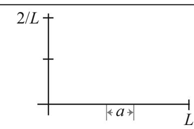
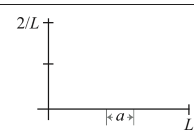
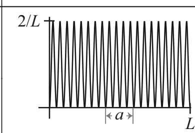
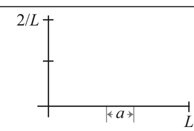
Kinetic energy of electron	Prediction of probability density by QM	Prediction of probability density by CM	Appoximate probability for object to be found in region a
E_1 = 0.38 eV			For QM: For CM:
$E = 4\ E_1$			For QM: For CM:
$E = 5\ E_1$	Energy not allowed		For QM: N/A For CM:
$E = 400\ E_1$			For QM: For CM:

Figure B.31: Written homework administered in winter 2009 and spring 2009 to all sections of Phys 123. (Four pages.)

ST HW–2 ***Probability in classical and quantum mechanics***

2. An object is confined to move on a frictionless track. The track is composed of multiple regions of various lengths and various heights. Treat the object classically.

   Consider a particular region, region C. Assume this region is horizontal.

   a. Suppose you knew the following three quantities:

      - L_C, the length of region C
      - v_C, the speed of the object in region C
      - T, the amount of time it takes to travel from the left to the right end of the entire track.

      i. In terms of the given quantities (L_C, v_C, and T), determine the probability of the object being found in region C. Explain.

      ii. In terms of the given quantities (L_C, v_C, and T), determine the probability density in region C. Simplify your answer as much as possible. Explain.

   b. Is the probability density *constant throughout* region C, or does it *vary* in region C? Explain.

   c. Suppose it is known that the probability density in a different region of the track is constant throughout that region.

      What must be true of the *shape* of the track in that region? Explain briefly.

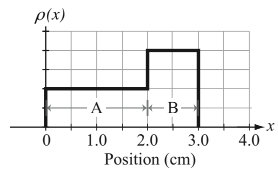
   d. Consider the following *incorrect* student statement:

      "My expression for the probability density in region C did not depend on L_C, so even if the length of region C doubled and everything else stayed the same, the density won't change."

      Describe the error(s) in this student's reasoning.

Probability in classical and quantum mechanics Name ______________________ ST HW–3

3. The graph at right shows the probability density ρ versus position x for some unknown situation. Region A lies between $x = 0$ cm and $x = 2.0$ cm. Region B lies between $x = 2.0$ cm and $x = 3.0$ cm, as shown.



   a. Is the probability of finding the object in region A *greater than*, *less than*, or *equal to* that in region B? Explain.

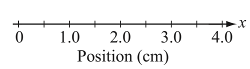
   b. Label relevant values (including units) on the vertical axis. Explain.

   What is the probability of the object being found in the right half of the track (*i.e.*, from $x = 1.5$ cm to $x = 3.0$ cm)? Explain and show any work.

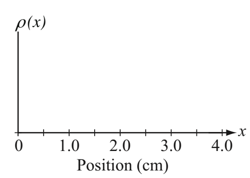
   What is the probability of the object being found *exactly* in the middle of the track (*i.e.*, at $x = 1.5$ cm)? Explain.

   c. Assume the graph of probability density above corresponds to a ball moving back and forth in a frictionless track.

      i. Sketch the shape of a track that would produce the graph of probability density. (Your sketch need only be qualitatively correct.) Explain briefly.



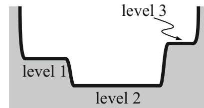
      ii. Suppose that the total energy were reduced so that the speed of the ball on the upper level were *extremely* small. Sketch a qualitatively correct graph of probability density in this case. Explain briefly.



ST
HW–4

Probability in classical and quantum mechanics

4. A ball moves back and forth in the frictionless track shown at right. The three horizontal levels are joined by steep ramps. Level 2 is the lowest and longest, while level 3 is the highest and shortest. Assume the object is able to make it up each ramp.



   a. Rank, from greatest to least, the probability *densities* on the three levels. If there is not enough information to answer, state so explicitly. Explain. (*Hint:* Consider the interpretation of probability density described in the tutorial.)

   b. Rank, from greatest to least, the *probabilities* of the ball being found on the three levels. If there is not enough information to answer, state so explicitly. Explain.

   c. Suppose that the *height* of level 3 were increased but the total energy of the ball-Earth system remained the same. The ball is still able to make it up to level 3. In each of the following questions, explain your reasoning.

      i. Would the total time spent in region 3 *increase, decrease,* or *remain the same?*

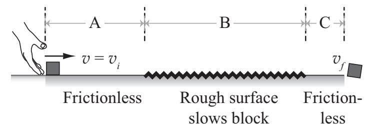
      ii. Would the probability of the object being found in region 1 *increase, decrease,* or *remain the same?*

      iii. Would the *probability density* in region 1, *increase, decrease,* or *remain the same?*

      iv. Would the *probability density* in region 3, *increase, decrease,* or *remain the same?*

PROBABILITY IN CLASSICAL AND QUANTUM MECHANICS — Name ____________ — ST HW–1

1. A series of small blocks enter the track shown from the left side with speed v_i. Region A is frictionless. Region B is rough and causes the blocks to eventually slow to speed $v_f < v_i$. Region C is frictionless. As soon as a block falls off of the right edge of the track (with speed v_f), a new one enters from the left (with speed v_i) such that there is always exactly one block on the track.

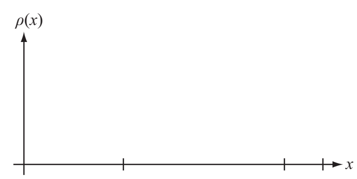


Region B is larger than region A, which is larger than region C, as shown. A photograph of the track is taken at a random time.

a. Is the probability of finding a block in region A *greater than, less than,* or *equal to* that in region C? If there is not enough information to answer, state so explicitly. Explain.

Is the probability density at the center of region A *greater than, less than,* or *equal to* that at the center of region C? If there is not enough information, state so explicitly. Explain.

b. Sketch a qualitative graph of the probability density ρ versus position x. The shape of your sketch in region B need only be approximate. Explain.



Indicate the geometric feature of your graph that corresponds to the probability of finding a block in the left half of region B. Explain.

Is the probability of finding a block in the left half of region B *greater than, less than,* or *equal to* that in the right half of region B? Explain.

Figure B.32: Written homework administered in spring 2010, autumn 2011, and spring 2015 to all sections of PHYS 123. (Three pages.)

ST
HW–2

***Probability in classical and quantum** mechanics*

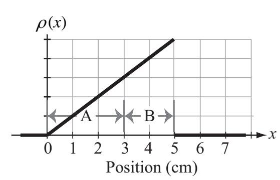
c. Suppose that the length of region C were increased. Regions A and B are unchanged.

Would the total amount of time it takes a block to traverse the entire track *increase, decrease,* or *remain the same?* If there is not enough information, state so explicitly. Briefly explain.

Would the probability density at the center of region A *increase, decrease,* or *remain the same?* If there is not enough information, state so explicitly. Explain.

Would the probability density at the center of region C *increase, decrease,* or *remain the same?* If there is not enough information, state so explicitly. (*Hint:* Consider a small region of unit length near the center of region C.) Explain.

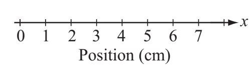
2. The graph at right shows ρ versus x for a classical particle confined to move in some unknown potential. Region A lies between $x = 0$ cm and $x = 3$ cm. Region B lies between $x = 3$ cm and $x = 5$ cm, as shown.



a. What is the *probability* of the particle being found between $x = 0$ and $x = 2.5$ cm? Write your answer in decimal format. Explain and show any work.

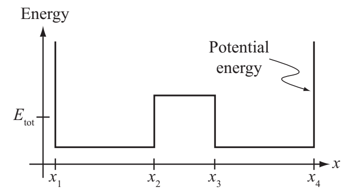
b. Assume that the graph of probability density above corresponds to a ball moving back and forth on a frictionless track.

Sketch the shape of a track that would produce the given graph of probability density. Your sketch need only be qualitatively correct. Explain.

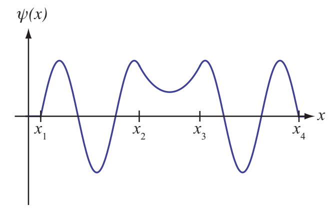


***Probability in classical and quantum** mechanics* Name ______________________ ST HW–3

3. The energy diagram at right shows the potential energy versus position of a particle in a certain potential well.

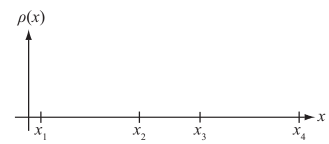


a. Suppose that the particle were quantum mechanical and the system has total energy E_{tot}, as indicated in the diagram. The graph below at right shows a wave function $\psi(x)$ corresponding to energy E_{tot}. Assume $\psi(x)$ is entirely real.

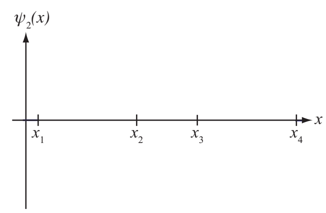


On the axes below at right, sketch a qualitative graph of the corresponding probability density $\rho(x)$. Explain how you determined the shape of the graph.

If the position of the particle is measured, could it be found between x_2 and x_3? Explain.



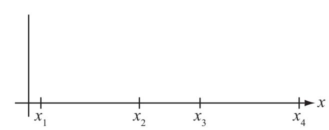
Is there another wave function that could produce the same probability density $\rho(x)$? (Assume the wave function is entirely real.) If so, sketch the wave function $\psi_2(x)$ in the space at right. If not, state so explicitly. Explain.



b. Suppose instead that the particle were classical and the system had the same total energy E_{tot}.

If the position of the particle is measured, could it be found between x_2 and x_3? Explain.

In the space at right, sketch the shape of a frictionless track to which the classical particle could be confined that would produce the energy diagram above. Assume any two adjacent levels are joined by a ramp. Explain.

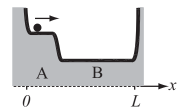


B.3 Posttests related to Probability in classical and quantum mechanics

In this section we provide post-tutorial assessments (posttests) for *Probability in classical and quantum mechanics.*

Name ____________________________________ **Student ID** ________________ **Score** ____________
last *first*

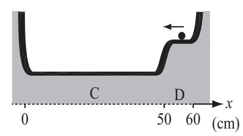
V. [20 points total] A ball moves back and forth forever on a frictionless track consisting of two horizontal sections, shown at right (not to scale). Assume that the ball has enough kinetic energy to get from region B to region A, and that the time the ball spends on the steep sections of the track is negligible. The exact heights and lengths of the two regions are unknown, except that region B is lower and longer than region A.



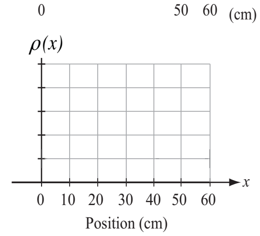
A. [5 pts] Is the probability density in region A, ρ_A, *greater than, less than,* or *equal to* the probability density in region B, ρ_B? Explain. If not enough information is given, explain what additional information is needed.

B. [5 pts] Is the probability of finding the ball in region A, P_A, *greater than, less than,* or *equal to* the probability of finding the ball in region B, P_B? Explain. If not enough information is given, explain what additional information is needed.

For the next two questions, consider a ball moving back and forth forever on a different track, shown at right. Region C is 50 cm long and region D is 10 cm long. Both regions are horizontal, and their exact heights are unknown. As before, assume that the time spent on the steep portions of the track is negligible.



C. [5 pts] It is observed that the ball spends the same amount of time in region C as in region D. On the grid at right, draw a *quantitatively correct* graph of the probability density as a function of position on the track. Label relevant values on the vertical axis, with correct units. Explain how you determined the shape of the graph, and the values on the vertical axis.



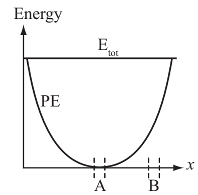
D. [5 pts] Suppose that the height of region D were increased, while region C remained unchanged. The total energy of the system is unchanged, and the ball still has sufficient kinetic energy to get from region C to region D. Does the probability density in region D *increase, decrease,* or *remain the same?* Explain. If not enough information is given, explain what additional information is needed.

Physics 123A, Spring 2008 Exam 3 WO-UWA123A082T-E3(PCQ).doc

Figure B.33: Written midterm exam administered in spring 2008 to Phys 123A.

Name ________________________________ **Student ID** ______________ **Score** __________
last *first*

VIII. [20 points total] This problem is composed of two unrelated parts.



A. [7 pts] A classical object is confined to move along the *x*-axis and is part of a system that includes potential energy. The graph at right shows the potential energy of the system as a function of position of the object and is labeled PE. The total energy of the system is labeled E_{tot}. Assume that there are no dissipative forces (*e.g.*, friction) and that the system has no rotational energy. Regions A and B are of *equal width* and lie along the *x*-axis, as shown.

Suppose that the position of the object were measured at a random time.

Would the probability of finding the object in region A be *greater than, less than,* or *equal to* the probability of finding it in region B? Explain.

B. A classical object is confined to a frictionless track. The track is composed of two horizontal levels, levels C and D, joined by a frictionless ramp. The heights and lengths of the two levels are unknown.

For each of the following cases, use the given information to determine if the *probability density* on level C is *greater than, less than,* or *equal to* the probability density on level D. If there is not enough information to answer, state what additional information is needed. *Consider each case separately and explain your reasoning.*

i. [7 pts] Out of a *very* large number of photographs, more two-thirds (2/3) show the object on level C.

ii. [6 pts] The speed of the object on level C is greater than the speed of the object on level D.

Figure B.34: Written midterm exam administered in spring 2008 to Phys 123C.

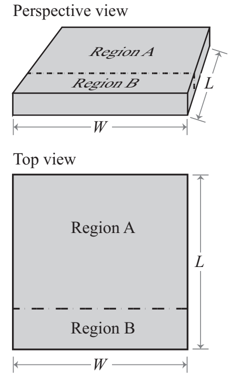
Name ______________________________ **Student ID** _____________ **Score** _________
last *first*

III. [25 points total] This page is composed of two unrelated problems.

A. A rock is placed at a random location inside of a two-dimensional box of length *L* and width *W*.

Perspective view

Top view



i. [6 pts] What is the probability **density** for the rock inside the box? Answer in terms of the given variables *(L* and *W)*. Explain.

ii. [6 pts] A dashed line divides the box into two unequal-sized regions (A and B).

Is the probability **density** in region A *greater than, less than*, or *equal to* the probability density in region B? Explain.

B. A ball moves back and forth forever on the frictionless track shown at right. The ball is able to make it up to the upper level and never leaves the surface of the track.

i. [6 pts] Is the probability **density** of finding the ball on the upper level *greater than, less than*, or *equal to* that on the lower level? If there is not enough information, state so explicitly. Explain.

ii. [7 pts] Suppose the length of the lower level is *increased*. The length of the upper level stays the same and the total energy does not change.

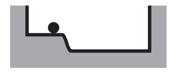
Would the probability **density** in the lower level *increase, decrease*, or *remain the same?* Explain.

Physics 123A, Summer 2008 Final Exam WO-UWA123A083T-EF(PCQ)
pg5

Figure B.35: Written final exam administered in summer 2008 to Phys 123A.

Name ___________________________________ **Student ID** _______________ **Score** ___________
last *first*

IV. [25 points total] This page is composed of two unrelated parts, A and B.



A. [15 pts] A classical object moves back and forth forever on the frictionless track shown at right. The object is able to make it up the ramp.

i. Suppose that the height of the **upper** level were increased. Assume that the object is still able to make it up the ramp and that the total energy of the object-Earth system is unchanged.

Would the probability **density** on the **lower level** *increase, decrease,* or *remain the same?* Explain.

ii. Suppose instead that the length of the **lower** level of the original track is increased. The length of the upper level and the total energy of the object-Earth system are unchanged.

Would the probability **density** on the **lower** level *increase, decrease,* or *remain the same,* as compared to the original probability density on the lower level? Explain.

B. [10 pts] A classical object is confined to a frictionless track composed of two horizontal regions joined by a ramp. The object is able to make it up and down the ramp. The two regions are of unequal heights and unequal lengths. Consider the two questions below **separately.**

i. Suppose that the **probability** of finding the object in the left region were greater than that in the right region. Would the probability **density** in the left region be *greater than, less than,* or *equal to* that in the right region? If there is not enough information, state so explicitly. Explain.

ii. Suppose that the probability **density** in the left region were greater than that in the right region. Would the **probability** of finding the object in the left region be *greater than, less than,* or *equal to* that in the right region? If there is not enough information, state so explicitly. Explain.

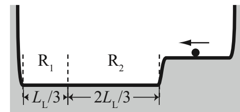
Physics 123A, Autumn 2008 Final Exam WO-UWA123A084T-EF(PCQ)

Figure B.36: Written final exam administered in autumn 2008 to Phys 123A.

Name ________________________________ **Student ID** ______________ **Score** __________
last *first*

IX. [25 point total] This page consists of two unrelated parts, A and B.

A. A classical (*i.e.*, non-quantum mechanical) object moves back and forth forever on the frictionless track at right. The two levels are horizontal and the object is able to make it up the ramp to the upper level. An imaginary line divides the lower level into two regions, R_1 and R_2, as shown.



i. [6 pts] Is the probability density in region R_1 *greater than, less than,* or *equal to* that in region R_2? If there is not enough information, state so explicitly. Explain.

ii. [6 pts] The probabilities of the ball being found in the lower and upper levels are P_L and P_U, respectively. The length of the entire lower level is L_L.

Write an expression for the probability density of the object being found in region R_1. Express your answer in terms of the given variables. If there is not enough information, state so explicitly. Explain.

B. A classical object is confined to a frictionless track that is composed of two horizontal levels (C and D) joined by a short ramp. The heights and lengths of the two levels are unknown.

For each of the following cases, use the given information to determine if the probability density on level C is *greater than, less than,* or *equal to* the probability density on level D. If there is not enough information, state so explicitly. *Consider each case separately and explain your reasoning.*

i. [6 pts] Out of a very large number of photographs taken at random times, more than half show the object on level C.

ii. [7 pts] The speed of the object on level C is greater than the speed on level D.

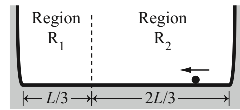
Physics 123A, Winter 2009 Final Exam WO-UWA123A091T-EF(PCQ)fr.doc

Figure B.37: Written final exam administered in winter 2009 to Phys 123A.

Name ______________________________ **Student ID** ______________ **Score** __________
last *first*

IX. [25 points total] This page consists of two unrelated parts, A and B.

A. A classical (*i.e.*, non-quantum mechanical) object moves back and forth forever on the horizontal frictionless track at right. An imaginary line divides the track into two regions, R_1 and R_2.



The length of the entire track is *L*.

i. [6 pts] Is the probability density of finding the object in region R_1 *greater than, less than,* or *equal to* that of finding the object in region R_2? If there is not enough information, state so explicitly. Explain.

ii. [6 pts] Write an expression for the probability density in region R_1. Write your expression in terms of the given variables. If there is not enough information, state so explicitly. Explain.

B. A classical object moves back and forth forever on the frictionless track at right. The two levels are horizontal and the object is able to make it up the ramp to the upper level.

i. [6 pts] Is the probability of finding the object in the lower level *greater than, less than,* or *equal to* that in the upper level? If there is not enough information, state so explicitly. Explain.

ii. [7 pts] Is the probability density of finding the object in the lower level *greater than, less than,* or *equal to* that in the upper level? If there is not enough information, state so explicitly. Explain.

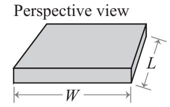
Physics 123B, Winter 2009 Final Exam WO-UWA123B091T-EF(PCQ)fr.doc

Figure B.38: Written final exam administered in winter 2009 to Phys 123B.

Name ___________________________________ **Student ID** _______________ **Score** ___________
last *first*

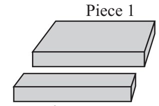
IX. [20 points total] Parts A and B below are independent.

A. A very small pebble is embedded at a random location inside of a two-dimensional piece of clay of length *L* and width *W*.



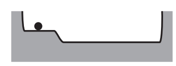
i. [5 pts] What is the probability **density** of finding the pebble inside the box? Answer in terms of the given variables *(L* and *W)*. Explain.

ii. [5 pts] The piece of clay, with the pebble still inside, is separated into two unequal sized pieces, labeled piece 1 and piece 2. Piece 1 is larger than piece 2, as shown.



Is the probability **density** of finding the pebble in piece 1 *greater than, less than,* or *equal to* the probability density of finding the pebble in piece 2? Explain.

B. A ball moves back and forth forever on the frictionless track composed of two horizontal levels joined by a ramp shown at right. The ball is able to make it up to the upper level and never leaves the surface of the track.



Suppose the position of the ball were measured at a random time.

i. [5 pts] Is the probability **density** of finding the ball on the upper level *greater than, less than,* or *equal to* that on the lower level? If there is not enough information, state so explicitly. Explain.

ii. [5 pts] Suppose the length of the lower level is increased. The length of the upper level stays the same and the total energy does not change.

Would the probability **density** of finding the ball on the **upper level** *increase, decrease,* or *remain the same?* If there is not enough information, state so explicitly. Explain.

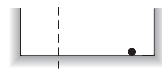
Physics 123A, Spring 2009 Final Exam WO-UWA123A092T-EF(PCQ)fr

Figure B.39: Written final exam administered in spring 2009 to Phys 123A.

Name ______________________________ **Student ID** _____________ **Score** __________
last *first*

IX. [20 points total] Parts A and B below are independent.

A. A ball moves back and forth forever on a frictionless track composed of a single horizontal section, as shown. (Ignore any loss in energy from collisions with the walls.) An imaginary line divides the track into two regions. The position of the ball is measured at a random time.



28. [5 pts] Is the probability **density** of finding the ball in the left region *greater than, less than,* or *equal to* that in the right region? Explain.

29. [5 pts] Suppose the ball is replaced by a ball moving with half the speed of the original.

Is the probability **density** in the left region *greater than, less than,* or *equal to* the probability density in the left region of the original case above? Explain.

B. The track above is replaced by a different frictionless track composed of two horizontal levels (1 and 2) that are joined by a ramp. The relative heights and lengths of the two levels are unknown. A ball moves back and forth forever on the track and has enough energy to move on both levels of the track.

For each of the following cases, use the given information to determine if the probability **density** on level 1 is *greater than, less than,* or *equal to* the probability **density** on level 2. If there is not enough information, state so explicitly. Consider each case separately and explain your reasoning.

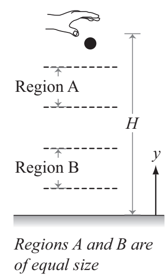
30. [5 pts] It is only known that out of a very large number of photographs taken at random times, more than half show the object on level 1.

31. [5 pts] It is only known that the speed of the object on level 1 is greater than that on level 2.

Figure B.40: Written final exam administered in spring 2009 to Phys 123C.

Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

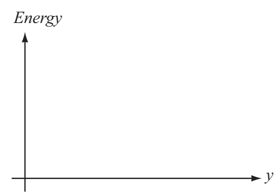
[12 points total] A ball is dropped from rest at a height H (a couple of meters) above the surface of the earth, as shown. While the ball is falling, its position is measured by taking a photograph at a random time. Regions A and B are of equal size.



Treat the ball classically (*i.e.,* do not use quantum mechanics in answering the following questions). The following questions are intended to be answered qualitatively; no involved calculations are expected.

A. [4 pts] On the axes below at right, sketch for the Earth-ball system both (i) a qualitative graph of the *potential energy* vs *height* of the ball and (ii) a qualitative graph of the total energy E_{tot} vs *height* of the same system. Take $y = 0$ to be the ground. Clearly label each graph.

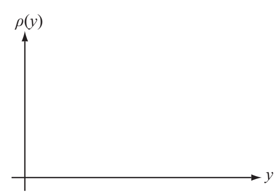
i. Explanation:



ii. Explanation:

B. [4 pts] Is the likelihood that the photograph shows the ball in region A *greater than, less than,* or *equal to* the likelihood that the photograph shows the ball region B? Explain.

C. [4 pts] On the axes at right, sketch a qualitative graph of the <u>probability density</u> $\rho(y)$ associated with measuring the position y of the ball at a random time. Take $y = 0$ to be the ground. Explain.

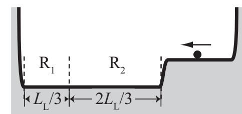


Physics 225A, Winter 2010 Final Exam QM-UWA225A101L-EF(PCQ).doc

Figure B.41: Written final exam administered in winter 2010 to Phys 225A. This is categorized as a posttest since most students would have taken the tutorial *Probability in classical and quantum mechanics* in spring 2009. Performance on this exam partially motivated the research into potential energy diagrams; thus, this exam is also included in the probability density section on page 321.

Name ________________________________ **Student ID** ______________ **Score** __________
last *first*

V. [20 point total] This page consists of two unrelated parts, A and B.



A. A classical object moves back and forth forever on the frictionless track at right composed of two horizontal levels. The object is able to make it up to the upper level and never leaves the surface of the track. While the ball is moving, its position is measured at a random time.

An imaginary line divides the lower level into two regions, R_1 and R_2, as shown.

i. [5 pts] Is the **probability density** in region R_1 *greater than, less than*, or *equal to* that in region R_2? If there is not enough information, state so explicitly. Explain.

ii. [5 pts] The probabilities of the object being found in the lower and upper levels are P_L and P_U, respectively. The lengths of the lower and upper levels are L_L and L_U, respectively.

Write an expression for the **probability density** in region R_1. Express your answer in terms of the given variables. If there is not enough information, state so explicitly. Explain.

B. A classical object is confined to a *different* frictionless track (not shown) composed of two horizontal levels (C and D) joined by a short ramp. The heights and lengths of the two levels are unknown.

For each of the following cases, use the given information to determine if the **probability density** on level C is *greater than, less than*, or *equal to* the **probability density** on level D. If there is not enough information, state so explicitly. *Consider each case separately and explain your reasoning*.

i. [5 pts] Suppose that, out of a very large number of photographs taken at random times, more than half show the object on level C.

ii. [5 pts] Suppose instead that the speed of the object on level C is greater than that on level D.

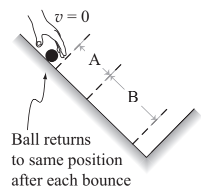
Physics 123A, Spring 2010 Final Exam WO-UWA123A102T-EF(PCQ)fr.doc

Figure B.42: Written final exam administered in spring 2010 to Phys 123A.

Name ________________________________ **Student ID** ______________ **Score** __________
last *first*

V. [20 points total] This page is composed of two unrelated problems.

A. A ball is released from rest near the top of a slanted track, as shown. The ball rebounds from the bottom edge and returns to the starting position after each bounce. Assume the ball bounces forever. The size of region A is smaller than the size of region B, as shown.



After the ball has been bouncing for a very long time, its position is measured at a random time.

i. [5 pts] Is the **probability** of finding the ball in region A *greater than, less than,* or *equal to* that in region B? If there is not enough information, state so explicitly. Explain.

ii. [5 pts] Is the **probability density** at the center of region A *greater than, less than,* or *equal to* the probability density at the center of region B? If there is not enough information, state so explicitly. Explain.

B. A ball moves back and forth forever on the frictionless track shown at right. The ball is able to make it up to the upper level and never leaves the surface of the track.

i. [5 pts] Is the **probability density** on the upper level *greater than, less than,* or *equal to* that on the lower level? If there is not enough information, state so explicitly. Explain.

ii. [5 pts] Suppose the length of the **lower** level is **increased**. The length of the upper level stays the same and the total energy does not change.

Would the **probability density** in the **lower** level *increase, decrease,* or *remain the same?* If there is not enough information, state so explicitly. Explain.

Physics 123C, Spring 2010 Final Exam WO-UWA123C102T-EF(PCQ)

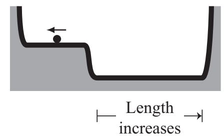
Figure B.43: Written final exam administered in spring 2010 to Phys 123C.

Name ________________________________ **Student ID** ______________ **Score** __________
last *first*

10. A ball moves back and forth forever on the frictionless track at right. The ball is able to make it up to the upper level and never leaves the surface of the track.

    Suppose the length of the lower level is increased. The length of the upper level stays the same and the total energy does not change.

    What happens to the probability density in the lower level after the length of the lower level is increased?

    A. Probability density in lower level *increases*.

    B. Probability density in lower level *decreases*.

    C. Probability density in lower level *remains the same*.

    D. There is not enough information.

11. A ball moves back and forth forever in a *different* frictionless track composed of two levels (A and B) joined by a ramp. The heights and the lengths of the two levels are unknown.

    Suppose that, out of a very large number of photographs taken at random times, more than half show the object on level A.

    How does the probability density in level A compare to the probability density in level B?

    A. Probability density in level A is *greater than* that in level B.

    B. Probability density in level A is *less than* that in level B.

    C. Probability density in level A is *equal to* that in level B.

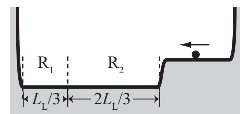
    D. There is not enough information.

Physics 123A, Autumn 2010 Exam 3 page 3 WO-UWA123A104L-E3(PCQ)mc.doc

Figure B.44: Multiple-choice midterm exam administered in autumn 2010 to Phys 123A. *Probability in classical and quantum mechanics* was administered as an interactive lecture this quarter (rather than a tutorial).

Name ________________________________ **Student ID** ______________ **Score** __________
last *first*

IV. [20 points total] This page consists of two unrelated parts, A and B.



A. A classical object moves back and forth forever on the frictionless track at right composed of two horizontal levels. The object is able to make it up to the upper level and never leaves the surface of the track. While the ball is moving, its position is measured at a random time.

An imaginary line divides the lower level into two regions, R_1 and R_2, as shown.

i. [5 pts] Is the **probability density** in region R_1 *greater than, less than,* or *equal to* that in region R_2? If there is not enough information to answer, state so explicitly. Explain.

ii. [5 pts] The probabilities of the object being found in the lower and upper levels are P_L and P_U, respectively. The lengths of the lower and upper levels are L_L and L_U, respectively.

Write an expression for the **probability density** in region R_1. Express your answer in terms of the given variables. If there is not enough information to answer, state so explicitly. Explain.

B. A classical object is confined to a *different* frictionless track (not shown) composed of two horizontal levels (C and D) joined by a short ramp. The heights and lengths of the two levels are unknown.

For each of the following cases, use the given information to determine if the **probability density** on level C is *greater than, less than,* or *equal to* the **probability density** on level D. If there is not enough information, state so explicitly. *Consider each case separately and explain your reasoning.*

i. [5 pts] Suppose that, out of a very large number of photographs taken at random times, more than half show the object on level C.

ii. [5 pts] Suppose instead that the speed of the object on level C is greater than that on level D.

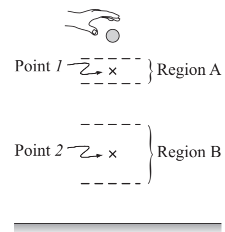
Physics 123A, Autumn 2011 Exam 3 WO-UWA123A114T-E3(PCQ).doc

Figure B.45: Written midterm exam administered in autumn 2011 to Phys 123A.

Name ______________________________________ **Student ID** _______________ **Score**________
last *first*

VI. This page is composed of two unrelated parts, A and B. Treat each part as a classical one-dimensional problem (*i.e.*, do not use quantum mechanics).

A. [13 pts] A ball is dropped a few meters above the ground. It bounces elastically from the ground and comes back up to the same height before moving back downward. Ignore air resistance and assume the ball is able to bounce and up and down in this fashion forever, always reaching the same height.



Ball bounces elastically and returns to same height

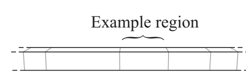
Two regions (A and B) and two points *(1* and *2)* are shown at right. The size of region B is larger than that of region A.

After the ball has been bouncing for a long time, its precise position is measured at a random time.

i. Is the **probability** P_A of the center of the ball being measured in region A *greater than, less than,* or *equal to* the **probability** P_B of its center being measured in region B? If there is not enough information, state so explicitly. Explain.

ii. Is the **probability density** ρ_1 at *point 1 greater than, less than,* or *equal to* the **probability density** ρ_1 at point *2?* If there is not enough information, state so explicitly. Explain.

B. [7 pts] A small grain of sand is placed in a random location in a long box. The box is composed of multiple regions of different lengths. Some of these regions are shown at right. Consider regions K and U (not shown):



Grain of sand located somewhere in long box

- Region K has length L_K. It is known that the probability of the grain of sand being located in region K is P_K.
- Region U has length L_U.

What is the probability P_U that the grain of sand is located in region U? Write your answer only in terms of the given variables (*i.e*, only in terms of L_K, P_K, and/or L_U). Assume the probability density is constant throughout the box. Explain.

Physics 123X, Spring 2015 Exam 3 WO-UWA123X152T-E3(PCQ).doc

Figure B.46: Written midterm exam adminisitered in spring 2015 to Phys 123A&C.

VITA

Brian Stephanik grew up in what is now Lakewood, Washington. He enrolled as an undergraduate at the University of Washington in 2003 intending to major in computer science. He subsequently switched to astronomy after many nights on his parents' roof with a pair of binoculars and a telescope. An academic advisor convinced him to major in physics. He completed his Bachelor of Science degree in Physics & Astronomy in 2007. Shortly after he married his wife Anya. He joined the Evening Master of Science program in physics at the University of Washington in 2007. While partially through the master's program, he applied and was accepted to the University's Ph.D. physics program in 2008. In early 2013 Anya gave birth to their son, Jamie. In 2015 he earned a Doctor of Philosophy in physics.

Index for potential energy diagrams

Entries indicate page number of full text of pretest, posttest, or curriculum used in this investigation.

Index for probability density

Entries indicate page number of full text of pretest, posttest, or curriculum used in this investigation.