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Rachel St John

# Explicitly Controlling Geometric Characteristics of Corridors in Spatial Optimization

Rachel St John

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Reading Committee:

Sándor F. Tóth, Chair

Zelda B. Zabinsky

W. Art Chaovalitwongse

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University of Washington

**Abstract**

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Rachel St John

Chair of the Supervisory Committee:  
Professor Sándor Tóth  
School of Environmental and Forest Sciences

Spatially-explicit mixed-integer programming models (MIPs) allow decision makers to explore a variety of complex scenarios and determine optimal sets of actions across a landscape. In reserve selection problems, the landscape is partitioned into units, and the decision maker must select which units to include in a wildlife reserve. As areas of habitat on the landscape are often scarce, connectivity of these regions through wildlife corridors is critical for species protection. Mixed integer programming models have been used in the past to create wildlife corridors, but they lack the capacity to control corridor geometry.

In this dissertation, I propose an approach, called the Optimal Corridor Construction Approach (OCCA), that employs path planning techniques from artificial intelligence to account for and control corridor geometry, such as width and length. By combining path planning with

network optimization, the OCCA allows the user to control and optimize the geometric characteristics of corridors. The OCCA may be used in other applications involving route construction (e.g., vehicle routing) or barrier construction (e.g., fire break design).

I illustrate the use of the OCCA on the 1,363 unit El Dorado forest in California. I find that the OCCA is extremely effective in selecting maximal width corridors, both with and without corridor length restrictions.

In many spatial optimization approaches, computational performance issues lead to intractable problems. I explore the computational performance of the OCCA by considering a variety of landscape factors to determine which may affect formulation and solution times, as well as problem size. I determine that the number of units, degree of unit adjacency and variation in unit size all affect problem size and performance. I also find that problem size, specifically the number of gate pairs, is linearly correlated with computational performance, specifically run time.

Lastly, I demonstrate how the OCCA can be used in a complex, real world scenario through a case study in Northern Sweden, where I include reindeer corridors in a forest harvest scheduling model. Current commercial forest practices reduce the amount of reindeer habitat and have made it difficult for reindeer to move through the forests. I combine the OCCA with a harvest scheduling model to explore the relationship between timber revenues and the selection and maintenance of reindeer corridors. Since harvest scheduling occurs over a planning horizon, the spatial configuration of corridors can change from one time period to the next in order to accommodate harvesting activities. If no corridors are included in the harvest scheduling model, the optimal harvest schedule results in a forest that is impassable for reindeer. When corridors are included, the combined model produces a harvest schedule that supports reindeer passage on the landscape

throughout the planning horizon. Results from this case study indicate that collaborative management is highly beneficial to reindeer herders, with a minimal cost to foresters.

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## Chapter 1. INTRODUCTION

### 1.1 MOTIVATION

As human activity expands across the landscape, natural areas dwindle. From 1963 to 1997, six million hectares of forest land in the United States were lost to development, and an additional 9.4 million hectares are projected to disappear by 2050 (Alig et al. 2003). As human activity encroaches on previously undisturbed natural areas, large, contiguous patches of habitat areas shrink and fragment. These newly formed “islands” are often inadequate for sustaining existing wildlife, which, in many cases, results in the loss of entire populations.

Habitat fragmentation separates wildlife from critical resources and reduces the amount of available space for populations to grow, move and disperse. One method to reduce fragmentation and increase connectivity is to create wildlife corridors (Beier and Noss 1998). Wildlife corridors provide paths for species in need of protection to move freely between conserved areas that are otherwise separated by human activity. Wildlife corridors can facilitate access to resources, migration, dispersal and population mixing. Major wildlife corridors include the Paséo del Jaguar, a corridor for jaguars stretching from the southern United States to Argentina; the European Green Belt, a corridor for a variety of species running from Norway to Turkey; and the Siju-Rewak Corridor in India, which protects over 139 species of mammals, including Asian elephants, Bengal tigers, clouded leopards and Himalayan black bears (Panthera.org, European Green Belt, World Land Trust).

Wildlife corridors are often designed on a two-dimensional landscape. Property lines and natural boundaries (such as rivers or cliffs) typically partition the landscape into polygon-shaped parcels (see Figure 1.1). The core reserves (dark parcels in Figure 1.1) may be connected

via a collection of other parcels (lightly shaded parcels in Figure 1.1). A wildlife corridor comprises parcels in the landscape that connect these areas. To be successful, wildlife corridors must be conducive to travel. In particular, corridors cannot be too narrow or too long (Soule and Gilpin 1991). Narrow corridors do not provide adequate buffer from the surrounding unsuitable habitat, and long, winding corridors are difficult for some species to travel. Given the spatial configuration of the landscape and corridor width and length requirements, it is not obvious which set of parcels will create an ideal corridor. The combinatorial nature of the problem, as well as ownership structures, land markets and pricing make corridor selection difficult for even small landscapes. Mixed integer programming (MIP) models have been used for corridor design in the past, but lack the capacity to explicitly control corridor geometry (e.g., width or length) without making assumptions about parcel shape.

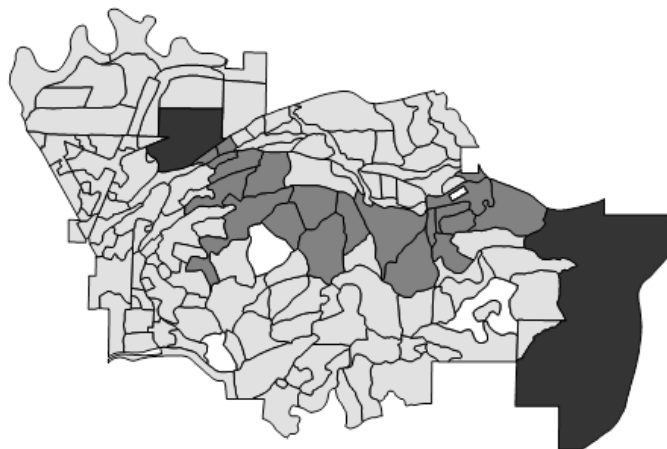


Figure 1.1. A landscape with a corridor.

## 1.2 CONTRIBUTION

I introduce a new approach called the Optimal Corridor Construction Approach (OCCA) that combines techniques from path planning (a field of artificial intelligence) with network optimization models in order to calculate and control geometric characteristics of corridors, such as width and length. This approach allows the user to construct optimal corridors by selecting parcels in realistic landscapes (landscapes of any spatial configuration). Borrowing tools from path planning, I am able to explicitly control geometric characteristics that were previously outside the scope of spatial optimization. The OCCA is demonstrated to be effective on a realistic landscape.

Computational tractability is often a concern when using spatially-explicit MIP methods with adjacency based constraints, such as the OCCA. While techniques for improving computational performance on a case-by-case basis have been developed, the landscape and problem characteristics that can cause computational issues have not been explicitly explored. In this dissertation, I test the effects of various landscape characteristics on problem size and the computational performance of the OCCA. To do so, I generate synthetic but realistic landscape maps using a software package called rlandscape (Passolt et al. 2012). I create several landscapes with specific spatial characteristics and use the OCCA to determine the widest corridor across each. Computational performance is measured by the total run time for each instance. I then use multifactor analysis of variance (ANOVA) to determine which landscape factors affect performance. The results show that the number of units, average degree of adjacency and variation in unit area all significantly impact run time. In addition, the number of gate pairs in the model is linearly correlated with run time.

Lastly, I demonstrate the use of the OCCA in a real world scenario. In Northern Sweden, there are two groups that use the forest as a resource: reindeer herders and the commercial forestry sector. As interwoven as the two industries are, conflicts have arisen. Forest practices have reduced reindeer habitat and increased forest density, compromising the animals' ability to move through forestland on their migration routes. In an attempt to reduce impacts on reindeer husbandry, I embed the OCCA in a harvest scheduling model for a commercial forest in Northern Sweden. This approach allows for timber revenue maximization while simultaneously selecting and maintaining reindeer corridors throughout the planning horizon. I determine that maintaining reindeer corridors in harvest scheduling can be done at minimal cost, and that including corridor constraints in the harvest scheduling model is critical to guarantee connectivity of reindeer habitat.

### 1.3 DISSERTATION ORGANIZATION

This dissertation is organized as follows. Chapter 2 provides the necessary background for introducing my approach, as well as background on computational performance and the Swedish case study. In Chapter 3, I present the Optimal Corridor Construction Approach, and demonstrate its use in an illustrative example. Chapter 4 illustrates the use of the OCCA using two real world landscapes, and explores the effects of different landscape factors on the computational performance of the OCCA. In Chapter 5, I implement the OCCA in a real world case study in Northern Sweden. Lastly, in Chapter 6, I summarize and discuss the dissertation, as well as remark on future research directions.

## Chapter 2. BACKGROUND

### 2.1 WILDLIFE RESERVES AND CORRIDORS

In land management, decision makers can create wildlife reserves by preserving or restoring parcels on the landscape that contain critical habitat. Along with habitat conservation, habitat connectivity is crucial to the vitality of some wildlife populations. One method for increasing the connectivity of wildlife reserves is to create spatial linkages called corridors (Beier and Noss 1998). A corridor should have certain spatial attributes to be successful. Some of these attributes are specific to the target population (such as width), while others are universally important, such as habitat area.

To be utilized, a corridor must contain suitable habitat. Many species have specific needs as to what type of terrain and vegetation they need to survive and disperse. For prey species and birds that prefer interior forest habitat, a patchy corridor may not offer sufficient protection against predation (Soule and Gilpin 1991). Grizzly bears in British Columbia show preference for high elevation, Douglas fir forests, and abundant vegetation (Proctor et al. 2008). Bennett et al. (1994) found that transient chipmunks preferred traveling along fence rows through farmland.

Corridors also cannot be too narrow. Animals that require wildlife corridors prefer to avoid human interaction and areas of exposure. Thus, if a corridor is too narrow, it might not be used (Beier and Loe 1992, Williams et al 2005). Anderson et al. (1977) studied birds in corridors of various widths, and found strong positive correlation between corridor width and the abundance of several species. In addition to birds, many species of mammals are sensitive to corridor width. Simulation studies conducted by Soule and Gilpin (1991) show that the probability of corridor success increases with width until it reaches an asymptote. Harrison

(1992) derived minimum corridor widths for seven different mammal populations based on various studies. The author argued that determining “the minimum width of effective corridors” was a critical research need.

Lastly, corridors cannot be too long. If the distance between two resources is too great, or the corridor itself winds around the landscape, making an unnecessarily long path, animals will be less likely to use it. For slow moving prey species, corridors that are too long can actually be detrimental, as long corridors between shelter areas can give fast-moving predators an advantage in hunting (Soule and Gilpin 1991).

## 2.2 MATHEMATICAL PROGRAMMING FOR RESERVE SELECTION

Reserve selection models choose a set of parcels (called a reserve) on the landscape that will best meet conservation goals. The first reserve selection models employed heuristics to help managers select reserves. As computational power increased, exact optimization techniques became more attractive (Csuti et al. 1997, Rodrigues and Gaston 2002). Set covering models have been used to ensure that all target species are protected (Csuti et al. 1997), while maximal covering models focused on maximizing the number of species protected subject to some form of a budget or area constraints (Church et al. 1996, Csuti et al. 1997). ReVelle et al. (2002) provided a comparison of such reserve selection models to facility location problems. Most reserve selection models are deterministic. Haight et al. (2000) is a notable early exception who introduced an approach for incorporating probabilistic presence-absence data for a species at the parcel level, thereby creating a more reliable reserve selection strategy. For an excellent summary of reserve selection models, see Rodrigues and Gaston (2002).

A common shortcoming of these models is that they ignore the location of the parcels they select, and can create reserves that are highly fragmented. In order to improve upon these aspatial models, another generation of reserve selection models were developed to include spatial control of the reserve selected. These models controlled reserve aspects such as density of reserve units, the ratio of reserve edge to area and connectivity. For literature reviews of reserve selection models that address such spatial characteristics of reserves, see Williams et al. (2005) and, more recently, Billionnet (2013).

Many approaches have been introduced in order to minimize fragmentation without explicitly enforcing connectivity. Some models reduce spatial discontinuities within the reserve by minimizing pairwise distances between selected sites (Önal and Briers 2002), or by minimizing the gaps between reserve fragments (Önal and Briers 2005, Önal and Wang 2008). Others increase contiguity by maximizing the number of pairs of adjacent sites (Nalle et al. 2002) or by maximizing the contiguous size of the reserve fragments (Tóth et al. 2009).

Often, in instances such as wildlife corridor construction, the reserve must be completely connected in order to be successful. Sessions (1992) was the first to model landscapes as graphs, where each parcel is represented by a node, and adjacency among the parcels is represented by edges (see Figure 2.1 with four parcels labeled  $p_h, p_i, p_j$  and  $p_k$ ). Sessions posed the connected reserve selection problem as a Steiner network problem- a network problem in which one must find the minimum cost connected network that contains a given set of nodes.

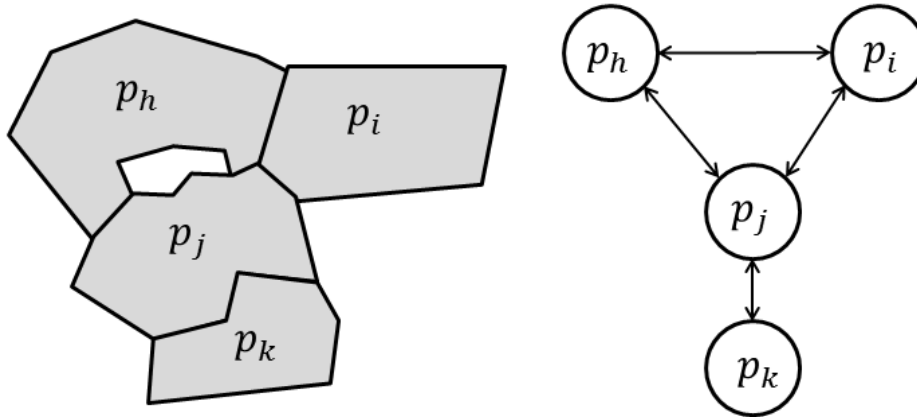


Figure 2.1. A landscape and its graph-theoretical representation.

Others have used network optimization in reserve selection models cast as integer programs. Cerdeira et al. (2005) used a heuristic approach to find a solution to a connected set covering problem. Connectivity of reserves can also be modelled using network flow concepts. Shirabe (2005) used a network flow model in which a single parcel “sink” is preselected to be in the reserve, and all other parcels in the reserve must be spatially linked to the sink. Jafari and Hearne (2013) modelled the problem as a transshipment problem, where the connected reserve is unrooted, that is, no individual parcel is required to be in the reserve.

Graph theory techniques are commonly used in integer programming-based reserve selection models to ensure connectivity. Williams (1998) proposed integer programming (IP) to solve the Steiner network problem in order to find optimal wildlife corridors, and in Williams (2002), he used the same technique to find minimum spanning trees of a given size using primal/dual graphs. Önal and Briers (2006) used integer programming to create a species covering connected subgraph that minimizes the number of selected parcels. Their model uses linear inequalities and a novel, monotonously increasing so-called “tail” function as a “cycle

breaking” feature in order to ensure connectivity. In order to improve computational performance, the authors introduce a two-step preprocessing technique to reduce problem size.

Conrad et al. (2012) also considered the problem as a Steiner network problem, and used a network flow model to find the optimal connected network. They introduced a two-phase method for improving solution times where minimum cost Steiner trees are to be found first (if one exists) and then used as initial solutions to MIP-based (mixed integer programming) optimization models. Dilkina and Gomes (2010) compared a Steiner network formulation to single and multi-commodity network flow formulations. Using their proposed algorithm, the Steiner network model solved two orders of magnitude faster than the network flow formulations.

The need for continuous canopy corridors, or corridors of forest stands above a certain minimum age often arises in managed forest ecosystems. Carvajal et al. (2013) introduced a model that used only node variables and implemented a cutting-plane approach to achieve computational tractability on large-scale (more than 1000 parcels) harvest scheduling problems with connected reserves.

While the problem of connectivity has been addressed with integer programming, little has been done to explicitly incorporate width and length of wildlife corridors into the models. The issue has either not been addressed in documented models, or it was assumed that a one parcel wide corridor was sufficient (Williams 1998) and length could be measured in terms of the number of units (Conrad et al. 2012). The approach used by Conrad et al. (2012) allowed corridors to be multiple parcels wide, but in their application, the authors assumed the landscape was partitioned into a square grid.

In many real world problems, the landscape cannot easily be partitioned into a grid. Boundaries such as mountains, coastlines and property lines result in parcels that can be highly irregular in shape and may even have holes due to lakes or developed areas (for example, see Figure 1.1). On landscapes such as these, it is not clear how to create optimal corridors of certain geometric features. When the landscape is translated to a network graph, as in Figure 2.1, much of the information on parcel geometry is lost. Thus, analysts must either ignore such characteristics as width and length, or superimpose a regular grid on an irregularly partitioned landscape, leading to suboptimal solutions.

Current tools in mathematical programming are insufficient to control geometric characteristics on an irregularly partitioned landscape. Without constraints on spatial configuration, characteristics such as width and length cannot be measured. Moreover, the graph-theoretical representation of the landscape that is currently being used doesn't allow for incorporation of such characteristics into an integer program. A means to calculate geometric characteristics of landscapes is required, as well as a new graph representation of the landscape. For measuring geometric characteristics, I adopt techniques from an area in artificial intelligence called path planning. Such path planning methods can measure the width and length of corridors on landscapes of any spatial configuration in a mathematical programming framework.

## 2.3 PATH PLANNING

Path planning is an area of study within the field of artificial intelligence. Path planning has applications in areas such as robotic surgery (Kiraly et al. 2004), unmanned bomb disposal (Jian-Jun et al. 2007), video game artificial intelligence (Demyen 2007) and molecular motion (Cortés et al. 2005). The classic problem of path planning is called the “piano mover’s problem”

(LaValle 2006). In the piano mover's problem, an agent (a piano) is placed in an enclosed area (a room) filled with obstacles (such as furniture). One must determine an optimal path that will successfully move the piano without colliding with the walls or furniture. The optimal path may be the shortest path, the widest path or the path with fewest turns. Geometric path characteristics such as width, length (Demyen 2007), turn angle (Pinter 2001) and steepness (Roles and ElAarag 2013) are explicitly controlled by partitioning the area into a set of convex polygons called a *navigation mesh*, then using search algorithms to determine the optimal path.

In this dissertation, I adapt techniques from path planning, from Demyen (2007) in particular, to calculate corridor width and length and then use that information in optimal reserve selection.

## 2.4 COMPUTATIONAL PERFORMANCE

Spatially-explicit MIPs are powerful tools that allow users to consider complex scenarios and determine optimal sets of actions for a landscape. In scenarios with adjacency constraints, such as area restriction models for harvest scheduling (Goycoolea et al. 2009, McDill et al 2002, Murray 1999) and contiguous reserve selection models (Conrad et al. 2012, Jafari and Hearne 2013, Önal and Briers 2006), it is difficult to maintain computational tractability for realistic scenarios and large landscapes. In Önal and Briers (2006), a feasible solution to their reserve selection model could not be found within two hours for a landscape of 391 units. In Könnyű and Tóth (2013), area restriction models on many realistic landscapes failed to produce a feasible solution within six hours.

Many authors have developed procedures for dealing with computational tractability. Önal and Briers (2006) used a two-stage procedure in which an optimal solution was found for the landscape with aggregated units first. Then, they sought optimal solutions for the original landscape by considering only units selected by the aggregated solution. A two-stage method taking advantage of Steiner tree structure was also introduced in Conrad et al (2012) to handle the scalability of their reserve selection model. Cutting planes have been successfully implemented for both area restriction models (Könnyű and Tóth 2013) and contiguous reserve selection (Carvajal et al 2013).

Landscape characteristics are thought to affect formulation and solution times. McDill and Braze (2000) ran experiments using hypothetical forests and found the number of units to be a factor in problem solution time. In Tóth et al (2012), the authors conclude that the degree of adjacency affect computational performance. Variation in unit size is also known to impact solution times (Constantin et al. 2008). In this dissertation, I investigate the effects of these landscape characteristics on the computational performance of the Optimal Corridor Construction Approach.

## 2.5 MULTI-USE LANDSCAPE PLANNING IN NORTHERN SWEDEN

In Northern Sweden, two groups use the same forestland as a resource: the forest owners and the reindeer herders. Roughly 50% of the forest in Northern Sweden is commercially owned and managed, and approximately 75% of the forest is used for reindeer husbandry (Eriksson, Sandewall, and Wilhelmsson 1987).

Reindeer (*Rangifer tarandus*) are central to the livelihood and cultural identity of the Sami, the indigenous people of Sweden. The Sami have practiced reindeer husbandry for over 400 years (Lundmark 2007). As stated by Sandström et al. (2003), “The importance of reindeer husbandry for the Sami cannot be overemphasized”. An integral part of Sami culture, reindeer traditionally provided transportation, food, fur and a source of income. Today, 20% of Sami are actively involved in reindeer husbandry. There are 51 herding communities with an estimated total of 240,000 reindeer in Sweden (Rural Development Programme for Sweden 2008).

Reindeer are highly migratory, and require vast tracts of land for habitat. The Sami have legal rights to graze their herds on any land regardless of ownership (RDPS). During the summer months, reindeer use the mountainous western region of Sweden. However, as temperatures drop and snow begins to fall, lichen and other vegetation tied to reindeer habitat gradually become inaccessible. As a result, Sami herders move their herds east towards the coast, where the climate is milder and snow conditions are more suitable. After winter, the herders move the reindeer back towards the mountains to their calving grounds in the foothills. This annual migration crosses up to 500 km each way mostly through forestland (Sandström et al. 2003). During winter and early spring, almost 80% of the reindeer’s diet consists of lichen (Heggeberget et al. 2002). Ground lichen (mostly *Cladina* spp.) is the primary food source for reindeer in winter, but if snow layers on the ground harden, they rely heavily on arboreal lichen (mainly *Bryoria* spp.) for food (Bostedt et al. 2003, Kumpula 2001).

Forestry practices have drastically reduced the amount and accessibility of lichen in northern forests, and many studies have concluded that current forest practices have been detrimental to reindeer husbandry (Berg et al. 2008, Kivinen et al. 2010, Roturier & Bergsten 2006, Roturier & Roué 2009, Widmark 2006). Specifically, short rotations, soil treatments and

dense stands negatively impact lichen presence and accessibility in the forest. Shortened rotations have reduced the amount of old growth forest in which arboreal lichen grows (Esseen et al. 1996). Soil scarification destroys the ground lichen layer, which can take up to 50 years to grow back (Sundén 2003). Dense forests are less suitable to ground lichen growth and difficult for reindeer to move through (Kivinen et al. 2012). Moreover, a North American species *Contorta* (*Pinus contorta*), also known as lodgepole pine, has become a common commercial tree in Sweden due to its short rotations, but it forms stands too dense for reindeer (SSR 2008). In sum, forests abundant in ground lichen have decreased with 71% since the introduction of modern forest practices at around 1955 (Sandström et al. 2016). In some areas, this has forced herders to move reindeer via trucks, which is expensive and stressful for the animals. In order for reindeer husbandry to remain economically viable, more active steps must be taken to create accessible winter habitat.

The 1979 Swedish Forestry Act dictates that forest owners must account for reindeer husbandry in their timber harvesting proposals. However, it is often unclear as to how much effort foresters are supposed to make. This has been subject to debates for many years. Forest management plans that can satisfy the needs of the herders and prove profitable for forest companies are critical for the welfare of both industries, but are difficult to create.

Unfortunately, there has been little research in multi-use planning despite this need. Bostedt et al. (2003) considered a scenario in which a herder and forester co-manage a forest in order to maximize revenue for both parties. The authors found that joint management greatly improved the revenue for the herders with minimal negative effects on the forester. Korosuo et al. (2014) used long term forest simulations to explore the trade-offs and possible synergies in harvest revenue versus lichen habitat for two forests. They found that current practices were not

only suboptimal for harvest revenue, but that they were also devastating for lichen availability. One forestry scenario yielded 2% higher net present value (NPV) than the “business-as-usual” scenario and more than doubled the amount of lichen for reindeer. The scenario that explicitly considered reindeer husbandry needs decreased NPV by only 5%. The results from both of these studies suggest that collaborative management can result in high gains for reindeer herders at a low cost to foresters.

In commercial forestry, harvest schedules are tools used to plan for management actions across a planning horizon. The forest is partitioned into management units or stands, and in a harvesting schedule, every stand is assigned a treatment schedule, which dictates the set of actions that will be applied to that stand for the planning horizon. For instance, a stand may be fertilized in 5 years, thinned in 15 years, then harvested and replanted with lodgepole pine in 35 years.

Similar to optimal reserve selection, mathematical programming is also an effective tool for creating spatially explicit harvest schedules. Given a forest landbase, MIPs find optimal management plans with regards to management goals and restrictions. In a typical harvest scheduling MIP, the model finds the plan that maximizes net present value (NPV) subject to various restrictions, such as maximum clear-cut size (Murray 1999, McDill et al. 2002, Goycoolea et al. 2005, Constantino et al. 2008) and road construction costs (Richards & Gunn 2000).

Carvajal et al. (2013) proposed a model that integrates wildlife corridors with harvest scheduling. Their cutting plane approach to connectivity proved computationally tractable for forests up to 1,363 units when planning for 3 time periods. However, their model did not account for, or control, corridor geometry.

To date, no approach for collaborative forest and reindeer planning has incorporated accessibility of lichen resources. Not only does lichen need to be present on the landscape, but it must be accessible for reindeer. Reindeer corridors through managed forests can facilitate movement and provide sufficient amounts of lichen for reindeer migrating between summer and winter pastures as well as between grazing areas within a seasonal range. Thus, the challenge is to create harvest schedules that are economically attractive for foresters but also maintain reindeer corridors at the same time.

## Chapter 3. THE OPTIMAL CORRIDOR CONSTRUCTION APPROACH

### 3.1 OVERVIEW

The Optimal Corridor Construction Approach is an approach that allows analysts to create optimal connected reserves by explicitly controlling geometric characteristics on a landscape of any spatial configuration. I demonstrate the OCCA by controlling the width and length of wildlife corridors, but the concepts and framework of the approach can be applied to more general landscape management problems.

To incorporate geometric characteristics such as width and length in a corridor selection model, I use a new graphical interpretation of the landscape. Rather than defining each parcel on the landscape as a node and each adjacency between parcels as an edge (as in Figure 2.1), define the nodes of the graph as transitions between polygons called *gates*, and edges as the optimal routes through polygons from one gate to the next (see Figure 3.1 for an example).

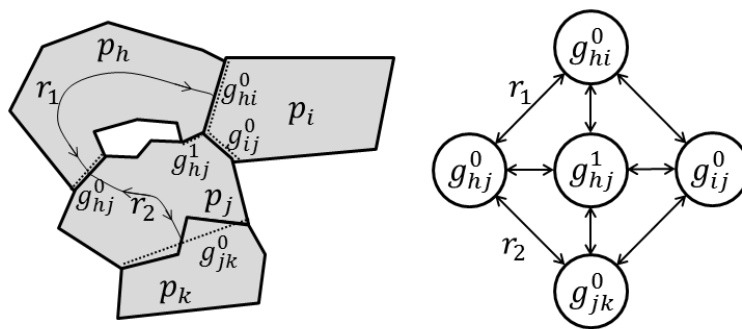


Figure 3.1. New graph-theoretical representation.

I then use pathfinding and MIP-based network optimization models to calculate the geometric characteristics (such as width and length) of optimal routes through each polygon.

The geometric characteristics are used as weights of their corresponding edges on the graph. Once the graph is formed and the edge weights calculated, a MIP-based reserve selection model is used to select the optimal corridor.

The OCCA involves five steps, as outlined in Table 3.1. First, the corridor objectives and constraints are specified in Step 1. For example, analysts may wish to find the minimum length corridor that satisfies a minimum width requirement. In Step 2, the corridor building blocks (sets of one or more land parcels) are defined. Next, the building blocks are used to create the graph-theoretical representation of the landscape (such as in Figure 3.1) in Step 3. In Step 4 path planning techniques are implemented to calculate geometric characteristics that will be used as edge weights for the graph. Finally, the graph and its edge weights are used to formulate a mixed-integer program. The solution to the MIP defines the optimal corridor on the landscape (Step 5). Each step of the Optimal Corridor Construction Approach (OCCA) is described in detail in Section 3.3 after introducing terminology, notation and path planning.

Table 3.1. The Optimal Corridor Construction Approach

<b>The Optimal Corridor Construction Approach</b>
1. Specify corridor objectives and constraints
2. Select eligible polygons for corridor construction
3. Find gate pairs for each triplet of polygons
4. For each gate pair, find the optimal route and associated width and length
5. Create and solve mixed integer program for optimal corridor construction

The OCCA allows analysts to obtain a set of parcels from the landscape that form a corridor of maximum width subject to length restrictions, or minimum length subject to

minimum allowable width constraints. This novel integration of path planning techniques and mathematical programming allows for control over geometric aspects (such as path width, length, steepness or angle) that were previously beyond the capacity of spatial optimization.

### 3.2 TERMINOLOGY

Consider a landscape  $\Omega$  that is composed of a set of polygons. Let  $V = \{v_i\}$  be the set of *vertices* on the Cartesian plane, and an *edge*  $e = (v_i, v_j)$  be a straight, closed line segment connecting two vertices. A *polygon*  $p$  is a continuous region enclosed by a set of edges  $\varepsilon(p) = \{e\}$  which is the *boundary* of  $p$ . Every edge  $e \in \varepsilon(p)$  shares each endpoint with exactly one other edge in  $\varepsilon(p)$  and does not intersect any other edge in  $\varepsilon(p)$ . In this study, *polygons with holes* are also considered. Let  $p_s$  be a polygon, and let  $p_0, \dots, p_k$  be a set of non-intersecting polygons within the boundary of  $p_s$ . Then, the polygon  $p_* = p_s \setminus p_0 \setminus \dots \setminus p_k$  is a polygon with holes, with boundary  $\varepsilon(p_*) = \varepsilon(p_s) \cup \varepsilon(p_0) \cup \dots \cup \varepsilon(p_k)$ . Two polygons  $p_i$  and  $p_j$  are *non-overlapping* if  $(p_i \setminus \varepsilon(p_i)) \cap (p_j \setminus \varepsilon(p_j)) = \emptyset$ . Two polygons  $p_i$  and  $p_j$  are *adjacent* if they are non-overlapping and  $\varepsilon(p_i) \cap \varepsilon(p_j) \neq \emptyset$ . A vertex, edge or polygon  $a$  is *contained* in a polygon  $p$  if  $a \cap p = a$ . An edge or polygon  $b$  is *partially contained* in a polygon  $p$  if  $b \cap p \neq b$  and  $b \cap p \neq \emptyset$ .

In this dissertation landscapes are *contiguous*, that is,  $\Omega = \bigcup_{i=0}^n p_i$  is a polygon.

Given a landscape  $\Omega$ , a *corridor*  $C = \{p_0, p_1, \dots, p_\omega\}$  is a sequence of non-overlapping polygons in  $\Omega$  where  $p_i, p_{i+1}$  are adjacent for all  $i < \omega$ , and polygons  $p_0, p_\omega$  represent the areas to connect.

In order to formally define corridor width and length, I first introduce the concept of agent. An *agent* is a circular region of a given diameter whose location is defined by its

centerpoint. An agent moves through a corridor  $C$  by beginning in  $p_0$ , travelling through  $p_1, p_2, \dots, p_{\omega-1}$  without intersecting  $\varepsilon(\cup_{p_i \in C} p_i) \setminus \{ \varepsilon(p_0) \cap \varepsilon(p_1) \} \setminus \{ \varepsilon(p_{\omega-1}) \cap \varepsilon(p_\omega) \}$ , and ending in  $p_\omega$ . The collection of points on which the agent travels is called a *path*. Note that the number of paths through a corridor is infinite. The *path width* is the maximum diameter an agent can have and still follow the path. The *path length* is the distance the agent travels following the path.

In this dissertation, I consider centered paths called *routes*, since populations of interest are typically edge adverse, in that they typically travel through the interior of the corridor (Soule and Gilpin 1991). The route is a proxy for the average path that the population is likely to follow. Note that the route is not necessarily the shortest path (Figure 3.2a).

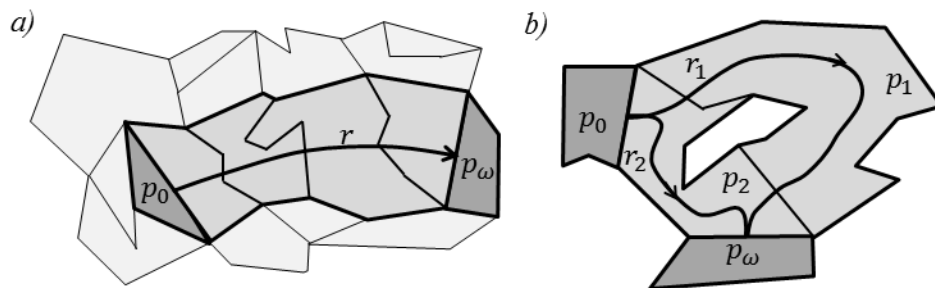


Figure 3.2. a) a landscape, a corridor and a route; b) a corridor with two routes

Given a corridor, the width and length of each route in a corridor is calculated to determine whether it is usable, and if it is, whether it is optimal for wildlife. A corridor with holes has many potential routes. For example, in Figure 3.2b, the corridor has two routes. Depending on the needs of the populations of interest, the best route through a corridor may be different. In Figure 3.2b,  $r_1$  is the optimal route if a route of maximum width is desired, but if

the shortest route is preferred, the optimal route is  $r_2$ . For a given landscape, a corridor that contains the optimal route is an *optimal corridor*.

Given a landscape, and two reserve polygons, the objective of the OCCA is to create a wildlife corridor to connect the reserve polygons with the maximal width route or the shortest length route subject to geometric, logistic and economic constraints.

### 3.3 METHODOLOGY OF THE OPTIMAL CORRIDOR CONSTRUCTION APPROACH

#### 3.3.1 *Specifying Corridor Objectives and Constraints*

The first step of the Optimal Corridor Construction Approach is to specify the corridor objectives and constraints (see Table 1). Given a landscape partitioned into parcels available for corridor use, a wildlife corridor is to be created by selecting parcels that will connect two areas of habitat (e.g., Figure 1.1). Not only must the corridor be connected, but it must also be comprised of suitable habitat, and contain a route that is neither too narrow nor too long. An optimal corridor may be a corridor with the widest route or a corridor with the shortest route. If a corridor of maximal width is desired, a constraint with an upper bound on length may be added.

Alternatively, if the objective is a corridor of minimal length, a minimum width threshold can be specified.

#### 3.3.2 *Selecting Eligible Polygons for Corridor Construction*

Once the characteristics of an optimal corridor are determined, the second step of the OCCA is to create a set of eligible polygons. Typically in corridor selection MIPs, the corridor is one parcel wide. These corridors may be suboptimal if width is of concern. I allow the corridor to be

multiple parcels wide by defining *polygons* as sets of one or more contiguous parcels. These potentially overlapping polygons are analogous to clusters in area restriction models, introduced by McDill et al. (2002), and also used by Goycoolea et al. (2005), Könnnyű & Tóth S. (2013) and Tóth et al. (2013). In these models, constraints prevent overlapping and adjacent clusters from being selected simultaneously, thus ensuring that contiguous areas selected for harvest do not exceed a predefined threshold. For wildlife corridors, using polygons rather than single parcels allows for the geometric control of corridors that are several parcels wide.

Theoretically, it is possible to define every combination of parcels as a polygon. However, even for landscapes with just a few management units, the number of potential polygons can be large, and calculating the widths and lengths of each potential corridor unwieldy. For large, realistic landscapes with hundreds or thousands of parcels, there would be a combinatorial explosion in the number of potential polygons. The problem would be computationally expensive, or even intractable. Thus, I create rules for *eligible polygons* based on the landscape and computational capabilities. For example, in Figure 3.3, define the set of eligible polygons as all single parcels, and all contiguous sets of parcels with area strictly less than 20 hectares. Then, the set of eligible polygons is  $\{p_1, p_2, p_3, p_4, p_5, p_6, p_1 \cup p_2, p_2 \cup p_3, p_3 \cup p_4, p_5 \cup p_6\}$ .

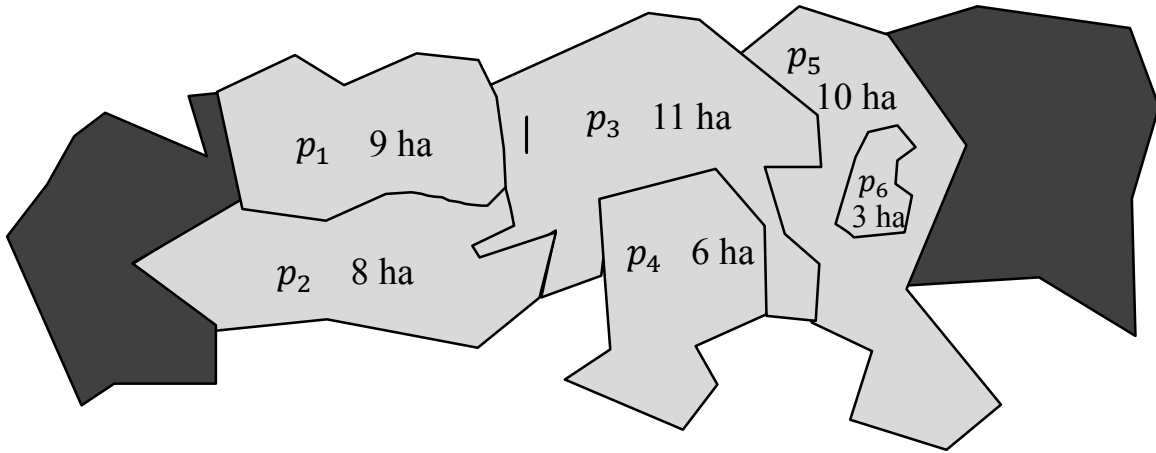


Figure 3.3 Example landscape for defining polygons.

### 3.3.3 Find Gate Pairs for Each Triplet of Polygons

The width and length of a corridor depends on the width and length of the route through each polygon in the corridor. How a route travels through its polygon determines its width and length. *Gate pairs* specify where a route enters and exits the polygon. Gate pairs also depend on the previous and subsequent polygons in the corridor. For example, in Figure 3.4a, the width and length of an optimal route through corridor  $(p_2, p_3, p_4)$  is different than that of the optimal route through corridor  $(p_1, p_3, p_4)$ . Define a *triplet* as a set of three non-overlapping polygons  $(p_i, p_j, p_k)$  such that  $p_i$  and  $p_j$  are adjacent and  $p_j$  and  $p_k$  are adjacent. Note that  $p_i$  and  $p_k$  may be the same polygon (for example, in Figure 3.2b, route  $r_1$  crosses triplet  $(p_2, p_1, p_2)$ ). In Figure 3.4a, there are nine triplets associated with polygon  $p_3$ :  $(p_1, p_3, p_1)$ ,  $(p_1, p_3, p_2)$ ,  $(p_1, p_3, p_4)$ , ...,  $(p_4, p_3, p_4)$ . Note the width and length of corridor  $(p_i, p_j, p_k)$  is equal to the width and length of its reversed corridor  $(p_k, p_j, p_i)$ .

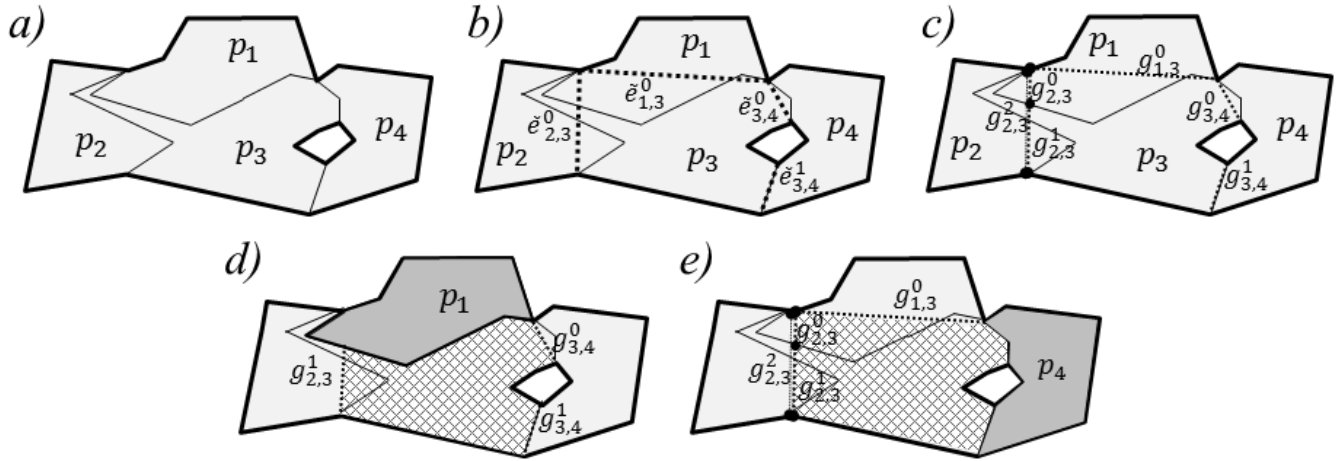


Figure 3.4. a) A set of polygons, b) their associated new edges, c) gates, d) gate pairs for  $(p_2, p_3, p_4)$  and e) gate pairs for  $(p_1, p_3, p_2)$

Given a triplet  $(p_i, p_j, p_k)$ , a *gate pair* consists of two *gates*, an entering gate representing the transition from  $p_i$  to  $p_j$  and an exiting gate, representing the transition from  $p_j$  to  $p_k$ . Define  $G_{ij}$  as the set of gates between  $p_i$  and  $p_j$ , where  $p_i$  and  $p_j$  are adjacent. Gates between  $p_i$  and  $p_j$  are generated by constructing *pseudo-edges* that represent where a route may enter  $p_j$  from  $p_i$ . The midpoint of a gate serves as a transition point for a route moving from polygon  $p_i$  to polygon  $p_j$ . Finding  $G_{ij}$  is done through a gate finding procedure (see Table 3.2).

Table 3.2. Procedure for finding gates given adjacent polygons.

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Gate Finding Procedure.

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Let  $p_i, p_j$  be adjacent polygons.

1. Let  $E_c$  be a set of contiguous edges in  $\varepsilon(p_i) \cap \varepsilon(p_j)$  such that no other edge in  $\varepsilon(p_i) \cap \varepsilon(p_j)$  is contiguous to the edges in  $E_c$ .
-

2. For a given  $E_c$ , let  $v_1, v_2$  be the endpoints of the contiguous edges in  $E_c$ , and construct an edge  $\tilde{e}_{ij}^c = (v_1, v_2)$ .
    - a. If  $\tilde{e}_{ij}^c$  is contained in  $p_i \cup p_j$ ,  $\tilde{e}_{ij}^c \in G_{ij}$
    - b. else, if  $\tilde{e}_{ij}^c$  is partially contained in  $p_i \cup p_j$ , define  $\{\tilde{e}_{ij}^{c0}, \tilde{e}_{ij}^{c1}, \dots, \tilde{e}_{ij}^{ct}\}$  as the segments of  $\tilde{e}_{ij}^c$  partitioned by  $\varepsilon(p_i \cup p_j)$ . Then  $\tilde{e}_{ij}^{c0}, \tilde{e}_{ij}^{c1}, \dots, \tilde{e}_{ij}^{ct} \in G_{ij}$  and all contiguous combinations of  $\{\tilde{e}_{ij}^{c0}, \tilde{e}_{ij}^{c1}, \dots, \tilde{e}_{ij}^{ct}\}$  are in  $G_{ij}$ ,
    - c. else, if  $\tilde{e}_{ij}^c \cap (p_i \cup p_j) = \emptyset$ , then  $\tilde{e}_{ij}^c$  is not a gate.
  3. Repeat steps 1 and 2 for all  $E_c$ .
- 

The gate finding procedure starts by identifying contiguous sets of shared edges between adjacent polygons  $p_i$  and  $p_j$ . For example, in Figure 3.4a,  $p_2$  and  $p_3$  share one set of contiguous edges and  $p_3$  and  $p_4$  share two sets of contiguous edges. For each set of contiguous edges, construct a new edge,  $\tilde{e}_{ij}^c$  that connects the endpoints of the contiguous edge set, where  $c$  indexes the contiguous edge sets. In Figure 3.4b, the new edge between  $p_1$  and  $p_3$  is  $\tilde{e}_{1,3}^0$ , the new edge between  $p_2$  and  $p_3$  is  $\tilde{e}_{2,3}^0$  and the new edges between  $p_3$  and  $p_4$  are  $\tilde{e}_{3,4}^0$  and  $\tilde{e}_{3,4}^1$ .

If  $\tilde{e}_{ij}^c$  is contained in  $p_i \cup p_j$ , then it is a gate. Since  $\tilde{e}_{1,3}^0$  is contained in  $p_1 \cup p_3$ , it is a gate denoted by  $g_{1,3}^0$  and shown in Figure 3.4c. Similarly,  $\tilde{e}_{3,4}^0$  and  $\tilde{e}_{3,4}^1$  are gates, denoted by  $g_{3,4}^0$  and  $g_{3,4}^1$ , respectively. If no part of  $\tilde{e}_{ij}^c$  is contained in  $p_i \cup p_j$ , then it is not a gate. A route using such an edge as a gate may not be contained in the corridor.

If  $\tilde{e}_{ij}^c$  is partially contained in  $p_i \cup p_j$ , partition  $\tilde{e}_{ij}^c$  at each point it crosses  $\varepsilon(p_i \cup p_j)$ . Each partition of  $\tilde{e}_{ij}^c$  is a gate, as well as any contiguous combination of partitions. In Figure

3.4b,  $\tilde{e}_{2,3}^0$  is partially contained in  $p_2 \cup p_3$ , so the procedure creates three gates: two gates via the partitioning,  $g_{2,3}^0$  and  $g_{2,3}^1$ , and the contiguous combination gate  $g_{2,3}^2 = g_{2,3}^0 \cup g_{2,3}^1$ . In Figure 3.4c, there are gate sets  $G_{1,3} = \{g_{1,3}^0\}$ ,  $G_{2,3} = \{g_{2,3}^0, g_{2,3}^1, g_{2,3}^2\}$  and  $G_{3,4} = \{g_{3,4}^0, g_{3,4}^1\}$ . Notice that  $g_{2,3}^0$  is not contained in  $p_2 \cup p_3$ , although it is a possible gate for triplet  $(p_1, p_3, p_2)$ .

For each triplet  $(p_i, p_j, p_k)$ , I use its corresponding gate sets,  $G_{ij}, G_{jk}$  to create the set of gate pairs,  $\Phi_{ijk}$ . Each gate pair  $(g_{ij}^m, g_{jk}^n)$  is comprised of one gate from  $G_{ij}$  that is contained in  $p_i \cup p_j \cup p_k$  and one gate from  $G_{jk}$  that is also contained in  $p_i \cup p_j \cup p_k$ . In Figure 3.4d, the gate pairs for triplet  $(p_2, p_3, p_4)$  are  $\Phi_{2,3,4} = \{(g_{2,3}^1, g_{3,4}^0), (g_{2,3}^1, g_{3,4}^1)\}$  and in Figure 3.4e, the gate pairs for triplet  $(p_1, p_3, p_2)$  are  $\Phi_{1,3,2} = \{(g_{1,3}^0, g_{3,2}^0), (g_{1,3}^0, g_{3,2}^1), (g_{1,3}^0, g_{3,2}^2)\}$ .

Once all gate pairs are found, define a graph-theoretical representation for the landscape where each node is a gate and each edge connects a gate pair (for example, Figure 3.1). Next, for every gate pair, I determine the optimal route through its core polygon by embedding pathfinding techniques into a network optimization model.

### 3.3.4 For Each Triplet and Each of its Gate Pairs, Find the Optimal Route and Associated Width and Length

Given a triplet  $(p_i, p_j, p_k)$  and one of its gate pairs,  $(g_{ij}^m, g_{jk}^n)$ , Step 4 of the OCCA is to find the optimal route from  $g_{ij}^m$  to  $g_{jk}^n$ . The optimal route from  $g_{ij}^m$  to  $g_{jk}^n$  must remain within  $p_i \cup p_j \cup p_k$ . Define the *core polygon*,  $\tilde{p}_{ijk}$ , as the polygon contained in  $p_i \cup p_j \cup p_k$  where  $\varepsilon(\tilde{p}_{ijk})$  includes all gates from all gate pairs in  $\Phi_{ijk}$ . In Figure 3.4d,  $\tilde{p}_{2,3,4}$  is indicated with crosshatching, as is  $\tilde{p}_{1,3,2}$  in Figure 3.4e.

For each gate pair  $(g_{ij}^m, g_{jk}^n)$  and its core polygon  $\tilde{p}_{ijk}$ , I find the optimal route from the midpoint of gate  $g_{ij}^m$ , denoted by  $mid(g_{ij}^m)$ , to the midpoint of gate  $g_{jk}^n$ ,  $mid(g_{jk}^n)$ , through  $\tilde{p}_{ijk}$ . Path planning techniques (Demyen 2007) determine the widths and lengths of route segments, which are then used to construct the optimal route via a network optimization model. Demyen (2007) describes a scenario in which a circular agent must travel from a starting point to an ending point in a room filled with obstacles. He partitions the room with a triangular navigational mesh, and introduces a method to find the width of the maximum-sized agent that can travel collision-free from the starting point to the ending point.

#### 3.3.4.1 Triangulating $\tilde{p}_{ijk}$

Following Demyen (2007), Constrained Delaunay Triangulation (Kallman et al. 2003) is used to create a navigational mesh by decomposing  $\tilde{p}_{ijk}$  into triangles. For any finite set of vertices  $V = \{v_i\}$  on a plane, a triangulation is a navigational mesh of triangles of the convex hull of  $V$ . A Delaunay Triangulation, described in detail in de Berg et al. (2008), is a triangulation in which no vertex in  $V$  lies inside the circumcircle of any other triangle. For example, Figure 3.5a shows a set of vertices, and Figure 3.5b shows its Delaunay Triangulation. A Constrained Delaunay Triangulation (CDT) is a Delaunay Triangulation that is formed with preexisting edges. Given the vertices and edges in Figure 3.5c, the corresponding CDT is shown in Figure 3.5d. There are several algorithms for finding CDTs, such as those in Chew (1989) and Sloan (1993).

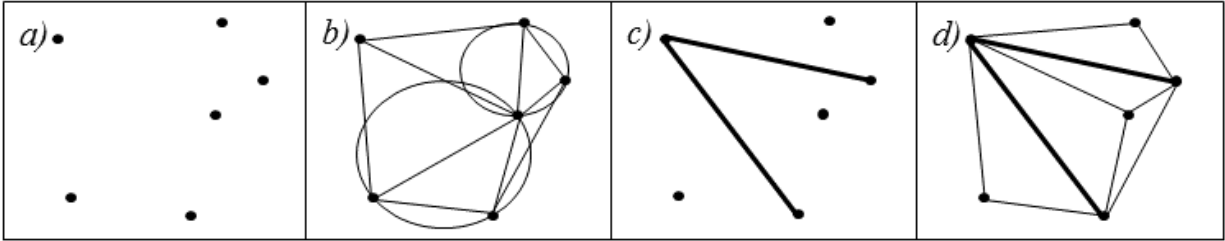


Figure 3.5. a) A set of vertices  $V$  and b) its Delaunay Triangulation. c) A set of vertices  $V$  and edges  $E$  and d) its Constrained Delaunay Triangulation.

Given  $\tilde{p}_{ijk}$  (for example, Figure 3.6a), consider the corresponding CDT (Figure 3.6b). Notice some edges of the CDT may not be contained in  $\tilde{p}_{ijk}$ : they either lay outside the outer boundary of  $\tilde{p}_{ijk}$  or they are in a hole. Let  $CDT'(\tilde{p}_{ijk})$  be the set of all edges of the CDT of  $\tilde{p}_{ijk}$  that are internal to  $\tilde{p}_{ijk}$  (Figure 3.6c). Thus,  $CDT'(\tilde{p}_{ijk}) \cup \varepsilon(\tilde{p}_{ijk})$  is the triangle decomposition of  $\tilde{p}_{ijk}$ .

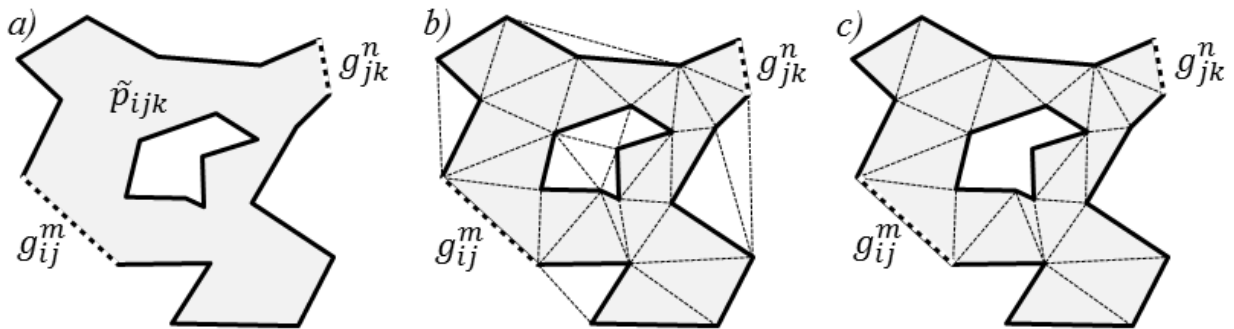


Figure 3.6. a) Core polygon  $\tilde{p}_{ijk}$ , b) CDT of  $\tilde{p}_{ijk}$  and (c)  $CDT'(\tilde{p}_{ijk})$ .

### 3.3.4.2 Calculating the Width and Length of Triangle Edge Pairs

Define a *triangle edge pair*  $(t_a, t_b)$  as an ordered pair of edges from  $CDT'(\tilde{p}_{ijk}) \cup \{g_{ij}^m, g_{jk}^n\}$  that share a vertex and  $(mid(t_a), mid(t_b))$  does not intersect any other edge in  $CDT'(\tilde{p}_{ijk}) \cup \{g_{ij}^m, g_{jk}^n\}$ . The set of all triangle edge pairs associated with gate pair  $(g_{ij}^m, g_{jk}^n)$  is denoted as  $T_{mn}(\tilde{p}_{ijk})$ .

Given a gate pair  $(g_{ij}^m, g_{jk}^n)$  and its core polygon  $\tilde{p}_{ijk}$ , a route from  $mid(g_{ij}^m)$  through  $\tilde{p}_{ijk}$  to  $mid(g_{jk}^n)$  can be represented by the sequence of triangle edge pairs it crosses:

$(g_{ij}^m, t_1), (t_1, t_2), \dots, (t_s, g_{jk}^n)$  (see Figure 3.7 for example). The width and length of the route is determined by the width and length of each triangle it crosses, entering the triangle at  $mid(t_a)$  and exiting the triangle at  $mid(t_b)$ .

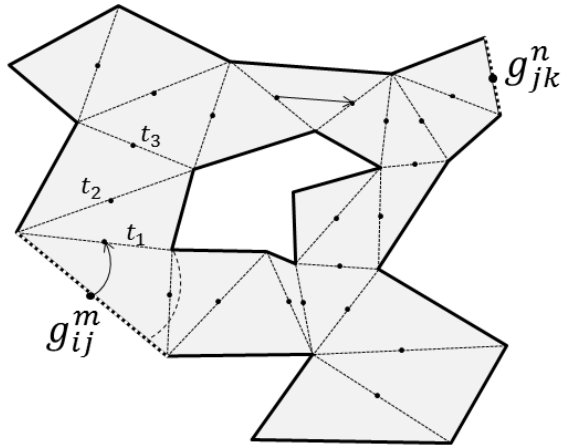


Figure 3.7. Triangle edge pair routes.

The width of the route from  $t_a$  to  $t_b$ , denoted by  $\psi_{ab}$  is calculated by following Demyen (2007). This algorithm determines the width using 1) the angle created by the triangle edge pair, 2) the lengths of  $t_a$  and  $t_b$ , and 3) whether the third edge of the triangle is in the

boundary of  $\tilde{p}_{ijk}$ . Based on these triangle edge pair attributes, the algorithm determines whether the maximal width route from  $t_a$  to  $t_b$  is curved (such as the route from  $g_{ij}^m$  to  $t_1$  in Figure 3.7), then calculates the narrowest width of the route.

For wildlife corridors, knowing the exact route length through a triangle edge pair is not necessary. The route represents the estimated preferred path, and there is no guarantee that wildlife will follow it exactly. When the route through the triangle edge pairs curves, calculating route length requires integration and may be computationally expensive. Since an exact length is not a priority, a quickly calculated proxy will suffice. The distance between the midpoints of  $t_a$  and  $t_b$ ,  $\delta_{ab} = |mid(t_a) - mid(t_b)|$ , serves as a proxy measure of route length, since minimizing  $\delta_{ab}$  will also minimize exact length.

### 3.3.4.3 Network MIP for Optimal Route.

Given a gate pair  $(g_{ij}^m, g_{jk}^n)$ , there may exist several routes through  $\tilde{p}_{ijk}$ . To determine the optimal route, I formulate and solve a network optimization model using the widths and lengths of the triangle edge pairs.

Let  $y_{ab}$  be the binary decision variable that indicates if  $(t_a, t_b)$  is included in the optimal route through  $\tilde{p}_{ijk}$ . Also, let  $M = \max_{a,b} \psi_{ab}$ . If the goal is to find the widest route from  $mid(g_{ij}^m)$  to  $mid(g_{jk}^n)$  through  $\tilde{p}_{ijk}$ , with a maximum length threshold,  $L_{max}$ , then let  $Z$  be the variable that accounts for route width.

$$\max Z \quad (3.1)$$

subject to:

$$Z - M \leq (\psi_{ab} - M)y_{ab} \quad \forall a, b: (t_a, t_b) \in T_{mn}(\tilde{p}_{ijk}) \quad (3.2)$$

$$Z \geq 0 \quad (3.3)$$

The objective (3.1) is to maximize  $Z$ , a non-negative number which cannot exceed the minimum edge pair width in the route, as enforced by constraints (3.2) and (3.3). Constraint (3.4) ensures the length of the optimal route does not exceed a limit,  $L_{max}$ .

$$\sum_{a,b:(t_a,t_b) \in T_{mn}(\tilde{p}_{ijk})} \delta_{ab} y_{ab} \leq L_{max} \quad (3.4)$$

Constraints (3.5) through (3.7) are the network flow constraints, which ensure a connected route.

$$\sum_{b:(g_{ij}^m, t_b) \in T_{mn}(\tilde{p}_{ijk})} y_{mb} = 1 \quad (3.5)$$

$$\sum_{a:(t_a, g_{jk}^n) \in T_{mn}(\tilde{p}_{ijk})} y_{an} = 1 \quad (3.6)$$

$$\sum_{a:(t_a, t_b) \in T_{mn}(\tilde{p}_{ijk})} y_{ab} - \sum_{c:(t_b, t_c) \in T_{mn}(\tilde{p}_{ijk})} y_{bc} = 0 \quad \forall b: t_b \in CDT'(\tilde{p}_{ijk}) \quad (3.7)$$

Constraint (3.5) requires the route to start at  $g_{ij}^m$ , constraint (3.6) requires the route to end at  $g_{jk}^n$  and constraint (3.7) is the network flow constraint. Lastly, constraint (3.8) defines  $y_{ab}$  as binary.

$$y_{ab} \in \{0,1\} \quad \forall a, b : (t_a, t_b) \in T_{mn}(\tilde{p}_{ijk}) \quad (3.8)$$

Alternatively, if the shortest route with a minimum width threshold is desired, the network optimization problem is a shortest path problem, and its formulation is:

$$\min \sum_{a,b:(t_a,t_b) \in T_{mn}(\tilde{p}_{ijk})} \delta_{ab} y_{ab} \quad (3.9)$$

subject to:

$$\psi_{ab} y_{ab} \geq W_{min} \quad \forall a, b: (t_a, t_b) \in T_{mn}(\tilde{p}_{ijk}) \quad (3.10)$$

Constraints (3.5) to (3.8)

The objective (3.9) is to minimize the length of the chosen route. Since corridors with nonzero widths are desired, it is reasonable to assume that there is a minimum width requirement,  $W_{min}$ , which is enforced in constraint (3.10). Alternatively, one can define  $y_{ab}$  only for  $a, b: \psi_{ab} \geq W_{min}$ . Constraints (3.5) through (3.8) are used to ensure a connected route and that  $y_{ab}$  is binary.

After solving the appropriate MIP, the optimal solution  $y_{ab}^*$  is used to calculate the width for the optimal route between the gate pairs,  $w_{ijk}^{mn} = \min_{a,b:y_{ab}^*=1} \psi_{ab}$  and the length  $\ell_{ijk}^{mn} = \sum_{a,b:y_{ab}^*=1} \delta_{ab}$ .

Given a landscape  $\Omega$ , I formulate a graphical representation in which each node is a gate  $g_{ij}^m$  and each edge connects a gate pair,  $(g_{ij}^m, g_{jk}^n)$ . For each gate pair, the width and length of the optimal route from  $g_{ij}^m$  to  $g_{jk}^n$  through  $\tilde{p}_{ijk}$  is determined by finding triangle edge pairs  $T_{mn}(\tilde{p}_{ijk})$ , calculating their widths and lengths, and finding the optimal route using a network optimization model. The widths and lengths of gate pairs are edge weights on the landscape graph. In the final step of the OCCA, this graph is used in a mixed-integer program to construct the optimal corridor.

### 3.3.5 Create and Solve Mixed-Integer Program for Optimal Corridor Construction

The final step in the Optimal Corridor Construction Approach (Step 5 in Table 3.1) is to use the gate pair route widths  $w_{ijk}^{mn}$  and lengths  $\ell_{ijk}^{mn}$  to create and solve an MIP for optimal corridor construction. A variety of MIP models exist for corridor construction (see Section 2.2). For ease of presentation, I demonstrate the OCCA using a network flow model that connects two core reserve polygons. However, other corridor selection MIPs may be implemented using the graph created in Steps 2-4.

The corridor construction MIPs used are analogous to the network optimization models for finding an optimal route between gate pairs. Rather than selecting triangles to traverse a polygon, I select polygons to traverse a landscape. The optimal corridor MIP finds the corridor that contains the optimal route, thus it is the optimal corridor.

Let  $x_{ijk}^{mn} \in \{0,1\}$  be the binary decision variable that indicates whether the route from gate  $g_{ij}^m$  to gate  $g_{jk}^n$  through polygon  $\tilde{p}_{ijk}$  is included in the corridor and let the set of all gate pairs on the landscape  $\Omega$  be  $\Phi_\Omega$ . Also let  $D$  be the upper bound on  $w_{ijk}^{mn}$ . If the objective is to create a maximal width corridor with an upper bound on corridor length  $L_{max}$ , then let  $V$  be the variable associated with corridor width. As in the model for finding the maximum width route between gate pairs, the objective (3.11) is to maximize the corridor width,  $V$ , which is defined in constraints (3.12) and (3.13). Corridor length is controlled in constraint (3.14).

$$\max V \quad (3.11)$$

subject to:

$$V - D \leq (w_{ijk}^{mn} - D)x_{ijk}^{mn} \quad \forall m, n: (g_{ij}^m, g_{jk}^n) \in \Phi_\Omega \quad (3.12)$$

$$V \geq 0 \quad (3.13)$$

$$\sum_{m,n:(g_{ij}^m, g_{jk}^n) \in \Phi_\Omega} \ell_{ijk}^{mn} x_{ijk}^{mn} \leq L_{max} \quad (3.14)$$

Let  $p_0, p_\omega$  denote the polygons to connect. Corridor connectivity is enforced through constraints (3.15) through (3.17).

$$\sum_{m,n:(g_{0j}^m, g_{jk}^n) \in \Phi_\Omega} x_{0jk}^{mn} = 1 \quad (3.15)$$

$$\sum_{m,n:(g_{ij}^m, g_{j\omega}^n) \in \Phi_\Omega} x_{ij\omega}^{mn} = 1 \quad (3.16)$$

$$\sum_{n:(g_{ij}^m, g_{jk}^n) \in \Phi_\Omega} x_{ijk}^{mn} - \sum_{r:(g_{\{hi\}}, g_{\{ij\}}^m) \in \Phi_\Omega} x_{hij}^{rm} = 0 \quad \forall m: g_{ij}^m \in \bigcup_{i \neq 0, j \neq \omega} G_{ij} \quad (3.17)$$

Recall that the landscape was originally partitioned into parcels  $Q$ , which were used to create polygons. For each parcel  $q_a$ , let  $\lambda_a$  be the set of polygons that contain  $q_a$ . Constraints (3.18) through (3.20) ensure that each parcel is in at most one polygon selected for the corridor.

$$\sum_{(g_{ij}^m, g_{jk}^n): p_i \in \lambda_a} x_{ijk}^{mn} \leq 1 \quad \forall q_a \in Q \quad (3.18)$$

$$\sum_{(g_{ij}^m, g_{jk}^n): p_j \in \lambda_a} x_{ijk}^{mn} \leq 1 \quad \forall q_a \in Q \quad (3.19)$$

$$\sum_{\{(g_{ij}^m, g_{jk}^n): p_k \in \lambda_a\}} x_{ijk}^{mn} \leq 1 \quad \forall q_a \in Q \quad (3.20)$$

Lastly, constraint (3.21) defines  $x_{ijk}^{mn}$  as binary.

$$x_{ijk}^{mn} \in \{0,1\} \quad \forall m,n: (g_{ij}^m, g_{jk}^n) \in \Phi_\Omega \quad (3.21)$$

Alternatively, if the shortest route with a minimum width threshold is wanted, the MIP is:

$$\min \sum_{m,n:(g_{ij}^m, g_{jk}^n) \in \Phi_\Omega} \rho_{ijk}^{mn} x_{ijk}^{mn} \quad (3.22)$$

subject to:

$$w_{ijk}^{mn} x_{ijk}^{mn} \geq W_{min} \quad \forall m,n: (g_{ij}^m, g_{jk}^n) \in \Phi_\Omega \quad (3.23)$$

Constraints (3.15) to (3.21)

When selecting the minimum length corridor, the objective (3.22) is to minimize the total length of the route. This problem is analogous to finding the minimum length route between gate pairs. Constraint (3.23) ensures the corridor satisfies the minimum width requirement, and the remaining constraints ensure the corridor is connected.

### 3.4 DISCUSSION

In this chapter I introduced the Optimal Corridor Construction Approach. Given a landscape and reserve polygons to connect via a corridor, the OCCA allows users to select optimal corridors given geometric requirements and goals. A new graph representation of the landscape is created by defining gates and selecting gate pairs, which can incorporate geometric information about the polygons on the landscape. These geometric characteristics are then calculated using techniques from path planning. Lastly, a corridor selection MIP is constructed to determine the optimal corridor on the landscape. Pre-existing connectivity models can be implemented for corridor

selection through the new graph representation of the landscape, allowing for a variety of corridor scenarios to be considered.

## Chapter 4. EMPIRICAL STUDIES OF THE OCCA

### 4.1 OVERVIEW

In this chapter, I apply the OCCA to one real and several simulated landscapes to illustrate its use and to test landscape characteristics that may affect problem formation and solution times. I use the maximal width gate pair and corridor selection models from Chapter 3. All MIPs were solved using CPLEX (2011) version 12.4 (64-bit) on a Dell Precision T7500 machine with Intel Xeon CPU, E5630@2.53Ghz (2 processors) with 4 GB of RAM and 64-bit Windows.

### 4.2 EL DORADO EXAMPLE

To demonstrate the use of the OCCA, consider the 1,363 unit El Dorado National Forest in California (shown in Figure 4.1a), obtained from the Forest Management Optimization Site, a landscape data repository (FMOS 2014). Suppose this is a landscape of parcels for sale, in which all parcels are comprised of suitable habitat and are eligible to be in the corridor. The objective is to purchase land for a corridor of maximal width that connects the dark polygons. While maximal width is the objective, a long winding corridor is also not desirable. Let the price of each parcel be proportional to its size, and suppose the budget allows the purchase of up to 15% of the total land area.

I first solved the corridor construction MIP with no maximum length threshold (see Figure 4.1b). I then included a maximum corridor length restriction of 40 km, roughly 1.2 times the distance between the polygons (Figure 4.1c). The first corridor found maximized width, but was long and winding. When I included a restriction on corridor length, the resulting corridor

was only slightly (3.2%) narrower, but was 8.02 km (16.7%) shorter than the corridor with no length restriction. Even though the length-restricted corridor is narrower, it may be a more appealing corridor because of the significant improvement in length.

For this example, the set of polygons (step 2 of the OCCA) includes 1) all 1,363 individual parcels, 2) 446 multiple parcel polygons that did not exceed an area of 20 hectares, and 3) 5 sets of parcels in which one parcel completely surrounds the other parcel (the smaller parcel is the hole in the larger parcel), resulting in a total of 1,814 polygons. This set of polygons results in approximately 112,000 gate pairs.

The gate pair MIPs (Step 4 of the OCCA) solved to optimality, taking less than 48 minutes for all 112,000 problems. With no maximum length requirement, the maximal width corridor MIP (Step 5) solved to optimality within 137 minutes and found the corridor shown in Figure 4.1b, which is approximately 309.1 meters wide and 48.0 km long. With the maximum length restriction, the problem solved to optimality in 12 minutes. The optimal corridor shown in Figure 4.1c, is approximately 299.1 m wide and 39.98 km long.

Although there were over 100,000 gate pair MIPs, they solved to optimality very quickly due to their network model structure. The corridor MIPs also proved to be computationally tractable. The length restricted corridor solved within minutes and the MIP without a length restriction solved in under 3 hours. Based on this example, it appears that restricting the solution space results in faster solve times for the corridor selection MIP.

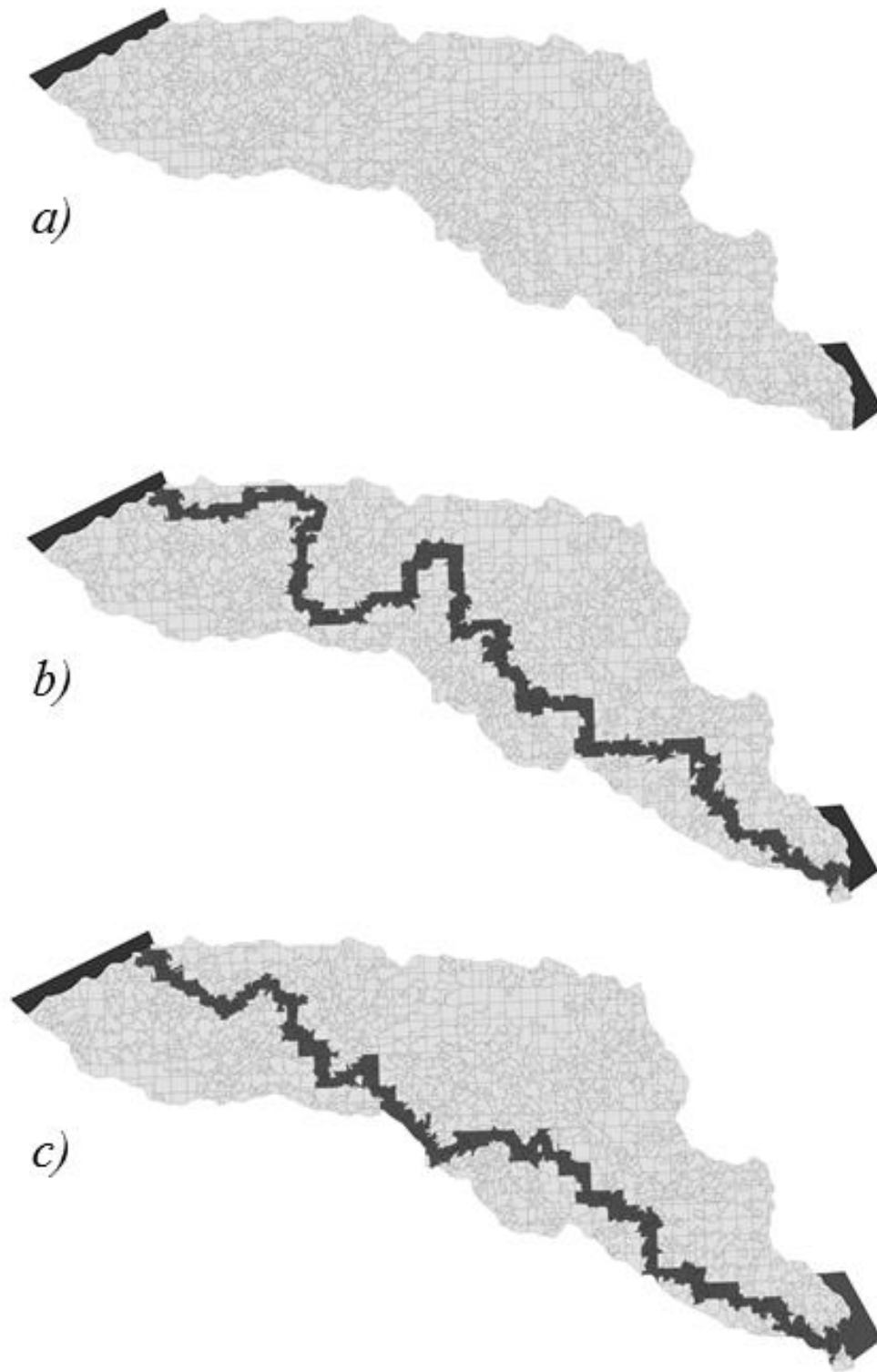


Figure 4.1. a) El Dorado landscape, b) maximal width corridor with no max length threshold and c) maximal width corridor with maximum length threshold of 40 km.

### 4.3 COMPUTATIONAL EXPERIMENTS WITH SIMULATED LANDSCAPES

#### 4.3.1 *Motivation*

Application of spatially explicit MIPs to realistic landscape often results in computational performance issues. Specifically, the number of units, degree of unit adjacency and variation in unit size are known to affect performance on spatially explicit MIPs with adjacency constraints. Values for each of these landscape characteristics for seven real landscapes are shown in Table 4.1. In this section, I explore the effects of these three different landscape factors on problem size and computational performance of the OCCA.

Table 4.1 Landscape characteristics of several real landscapes

Landscape	Number of Units	Degree of Adjacency	Area Variation
Bear Town	71	4.1	126
Buttercreek	351	3.8	40
El Dorado	1,363	5.3	62
Holmen Forest*	1,996	4.3	139
NBCL5	5,881	4.0	123
Pack Forest	195	4.9	72
Shulkell	1,039	4.0	138
<b>Average</b>	<b>1,557</b>	<b>4.3</b>	<b>100</b>

#### 4.3.2 *Experimental Design*

To explore the effects of landscape characteristics on computational performance, I generate realistic landscapes using a software package called rlandscape (Passolt et al. 2012).

Rlandscape is an R package that implements a Voronoi tessellation-based approach to create landscape maps based on user defined characteristics. Users can specify the number of units, area coefficient of variation, distribution of the degree of adjacency, frequency of holes in the landscape, landscape aspect ratio and more. The resulting landscapes have holes and units in a variety of shapes and sizes (see Figure 4.2 for examples). These landscape features make them realistic enough to represent real world forests for the purposes of computational experiments.

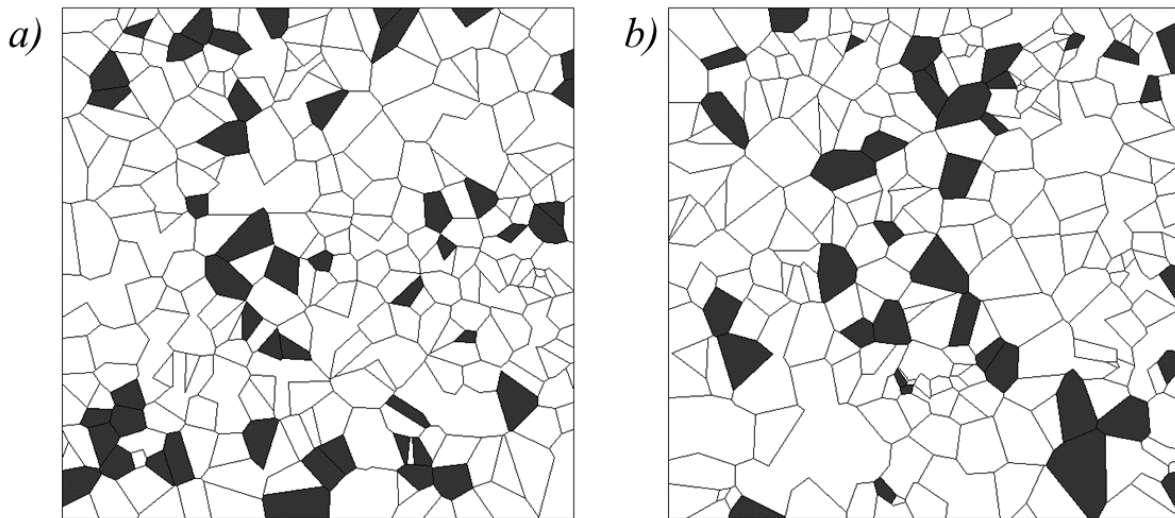


Figure 4.2. Landscapes generated by rlandscape with a) low and b) high variation in unit area.

In this experiment, I explicitly control the number of units, average degree of unit adjacency and the variation in unit size of each landscape. I use the OCCA to determine the widest corridor connecting the bottom-left most unit to the top-right most unit for each landscape, and measure problem size (number of gate pairs) and computational performance (total run time). From the results, I identify the landscape factors that are significant in

computational performance and conclude that run time is linearly related to the number of gate pairs.

To test the effects of the three factors on computational performance, I perform a full factorial experiment with three levels for each factor (a low value, a high value and their midpoint). The values for all factors and levels are shown in Table 4.2. Each value has a small margin around it for ease of landscape generation. For number of units, the values (200, 400 and 600 units) are smaller than the average realistic landscape from Table 4.1 (1,508 units) to ensure problem tractability, but still within the range (71 to 5,881 units). The values for average degree of adjacency and variation in unit size represent the range of values for realistic landscapes.

Table 4.2. Factors and levels for computational experiment.

Factor	Level		
	1	2	3
Number of Units	200±1	400±1	600±1
Average Degree of Adjacency	3.9 ±0.1	4.8±0.1	5.7±0.1
Area Variation	56.0±1.0	98.0±1.0	140.0±1.0

I generated five landscapes for each combination of factor levels, resulting in  $3^3 * 5 = 135$  landscapes total. All experiments are run on a Dell Precision T7500 machine with Intel® Xeon® CPU E5630 @2.53 GHz (2 processors), 4 GB RAM, 64-bit OS, Windows 7. I used CPLEX 12.6.1 (CPLEX 2011) for all optimization and Python 2.7 to generate gate pair MIPs (Section 3.3.4) and corridor MIPs (Section 3.3.5) for all 135 landscapes.

For each instance, the total run time is recorded. This includes the time to determine multiunit polygons and gate pairs, to formulate and solve gate pair MIPs and to formulate and solve the final corridor MIP. The number of gate pairs is also recorded.

### 4.3.3 Results

A summary of the data involving total run time and the number of gate pairs is given in Table 4.3, where the average values for the five landscapes at each factor level combination is reported. The distribution of time for each factor individually is shown in Figure 4.3. Each of the factors appear to be positively correlated with total run time. The p-value for the full factorial ANOVA are also shown in Table 4.4.

Table 4.3 Summary of results of computational experiments.

Number of Units	Degree of Adjacency	Area Variation	Gate Pairs (average)	Total Run Time (average)
200	3.9	56	2752	59.7
200	3.9	98	4928	86.9
200	3.9	140	21421	334.3
200	4.7	56	4835	88.3
200	4.7	98	6502	130.7
200	4.7	140	14992	355.6
200	5.5	56	5371	111.1
200	5.5	98	10491	213.4
200	5.5	140	43781	1112.9
400	3.9	56	6695	96.5
400	3.9	98	27736	436.9
400	3.9	140	19435	411.1
400	4.7	56	7964	159.2
400	4.7	98	20557	454.7
400	4.7	140	43511	1171.1
400	5.5	56	11381	220.5
400	5.5	98	31537	702.1
400	5.5	140	114489	4204.3
600	3.9	56	9973	137.6
600	3.9	98	65495	2039.5
600	3.9	140	10229	202.5
600	4.7	56	11864	225.4
600	4.7	98	71994	2771.7
600	4.7	140	15419	301.9

600	5.5	56	16929	313.4
600	5.5	98	60746	1969.5
600	5.5	140	29925	622.4

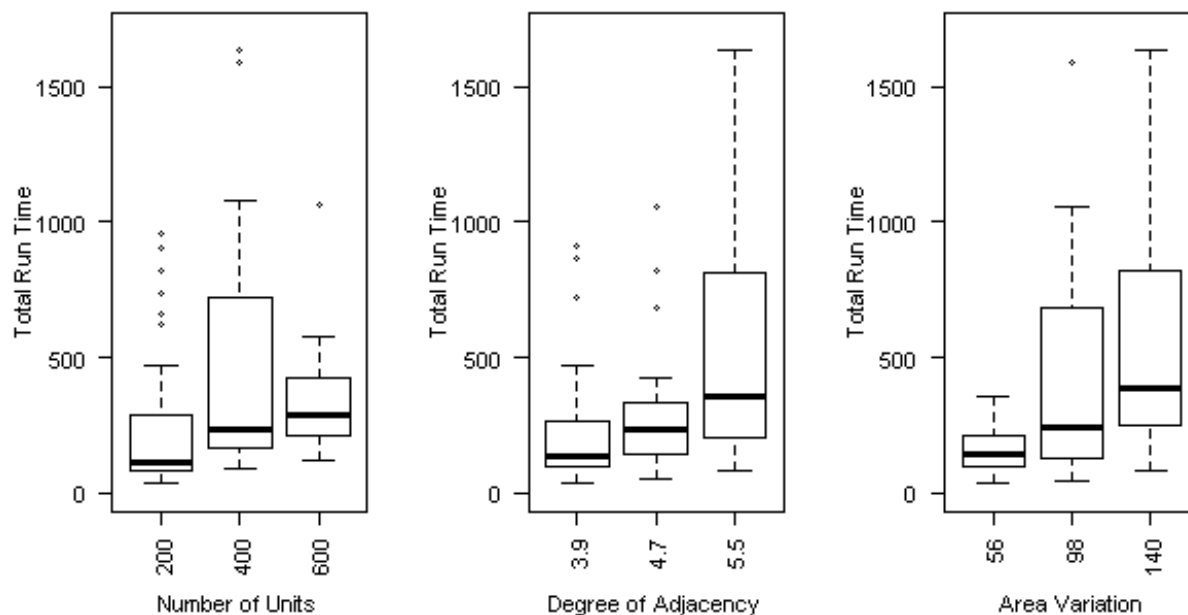


Figure 4.3 Distribution of total time for each factor

Table 4.4 ANOVA p-values

Factor(s)	P Value
num_units	0.0512
deg_adj	0.0522
area_var	0.0136
num_units:deg_adj	0.9731
num_units:area_var	0.6771
deg_adj:area_var	0.0383
num_units:deg_adj:area_var	0.9316

For each landscape factor, the distribution of the number of gate pairs at each level is shown in Figure 4.4. Figure 4.5 illustrates the relationship between the number of gate pairs and total problem time across all factors and factor levels. Since the number of gate pairs appears to

have a positive linear correlation with computational time, I fit a linear model to the data. Figure 4.5 shows the fitted line,  $\hat{y} = -237.4 + 0.03789 * \hat{x}$ , with adjusted  $R^2 = 0.9677$ , and Figure 4.6 shows the resulting residual and Q-Q plots. Based on ANOVA, the number of gate pairs is strongly related to computational time with a p-value of  $< 2.0 * 10^{-16}$ .

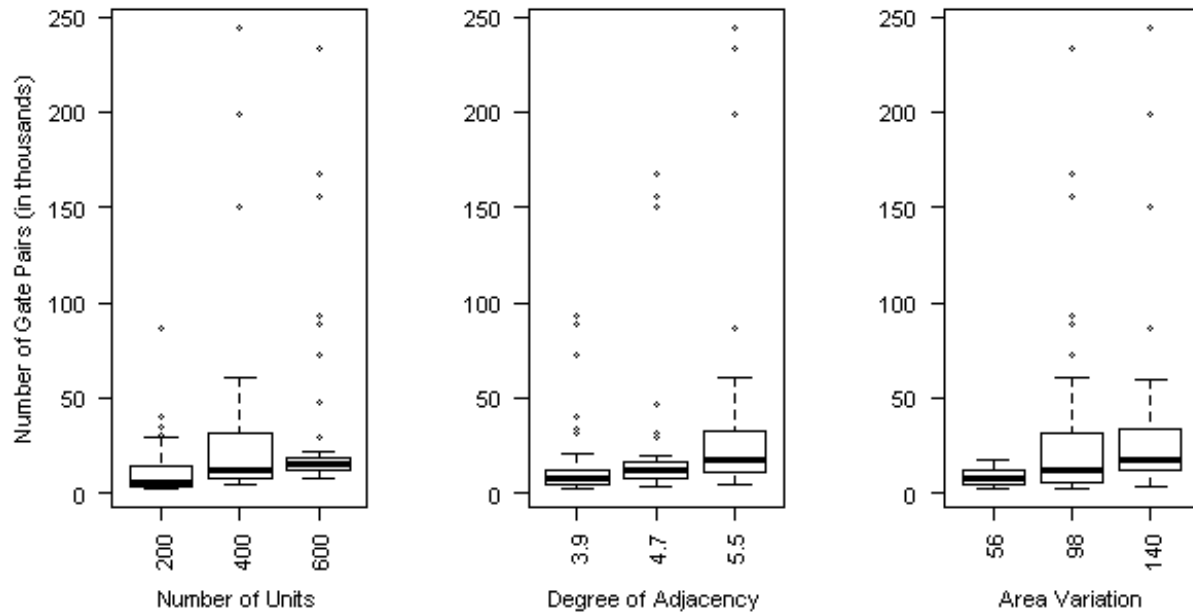


Figure 4.4. Distribution of gate pairs for individual factors.

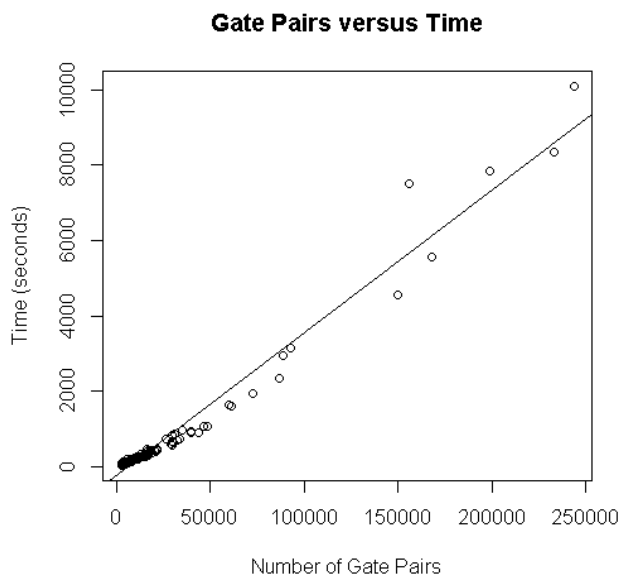


Figure 4.5. Number of gate pairs versus total time

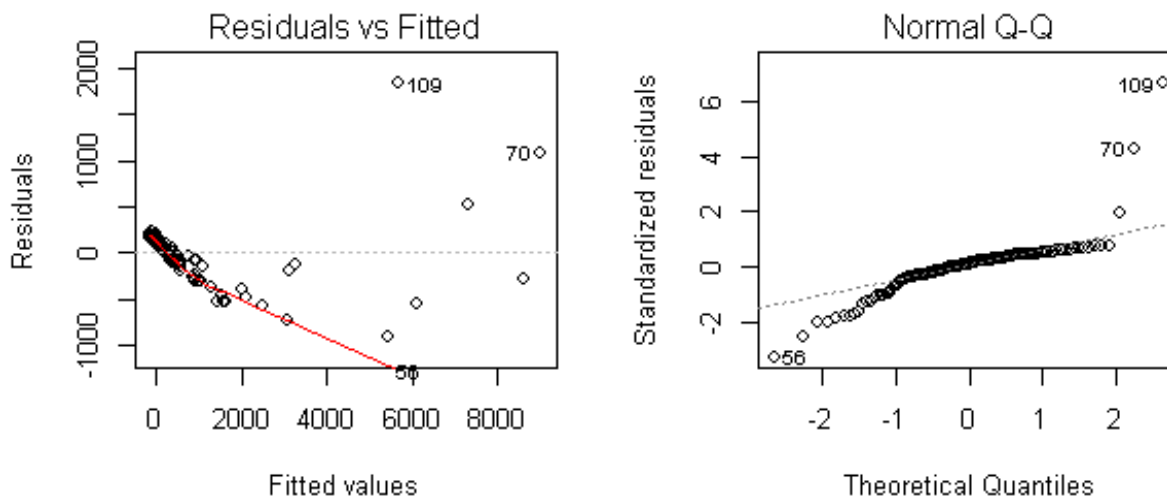


Figure 4.6 Residual and Q-Q plots for the linear fit of gate pairs versus time.

#### 4.3.4 Discussion

The ANOVA results indicate that number of units, average degree of adjacency and variation of area all affect total run time (Table 4.4). The interaction of average degree of adjacency and variation in area also affects problem time, whereas the other pairwise interactions (number of

units with average degree of adjacency and number of units with variation in unit area) and the three-way interaction of all factors are not significant.

The number of gate pairs in a problem tends to increase as each landscape factor increases (see Figure 4.4). When I combined the datasets to explore the effect of problem size on computational performance, I found the number of gate pairs to also be correlated with computational performance. The relationship between the number of gate pairs and time appears to be roughly linear (Figure 4.5), particularly for smaller problems (less than 100,000 gate pairs). Knowledge of this relationship can help users predict whether a problem may take a long time to solve, or even be intractable, based on the number of gate pairs.

## Chapter 5. MAINTAINING REINDEER CORRIDORS IN COMMERCIAL FORESTS IN NORTHERN SWEDEN

### 5.1 OVERVIEW

In northern Sweden, foresters and reindeer herders seek forest management plans that allow both groups to use the forest simultaneously. In this chapter, I present a case study in which the effects of collaborative management are explored. I combine the Optimal Corridor Construction Approach with harvest scheduling to create a harvest schedule for commercial forests that maintains corridors for reindeer herds.

The case study uses a commercial forest (Figure 5.1) covering approximately 14,000 hectares located in the county of Västernorrland, Sweden, an area actively used by reindeer. I investigate whether it is possible to maintain reindeer corridors, and if it is, at what cost to the forest owner in terms of forgone timber revenues.

### 5.2 CORRIDOR HABITAT

In Kivinen et al. (2010), the authors state “Ideal winter grazing areas have a high abundance of both ground growing and arboreal lichens that are easily accessible to reindeer”. This is also true of the routes the reindeer travel in between their winter and summer grazing areas. To be successful, the reindeer corridors need to contain sufficient amounts of both ground and arboreal lichen habitat, and must allow unobstructed travel. Arboreal lichen is typically present in old growth stands (Esseen et al. 1996), so I consider arboreal lichen present in all stands that are older than 120 years old.

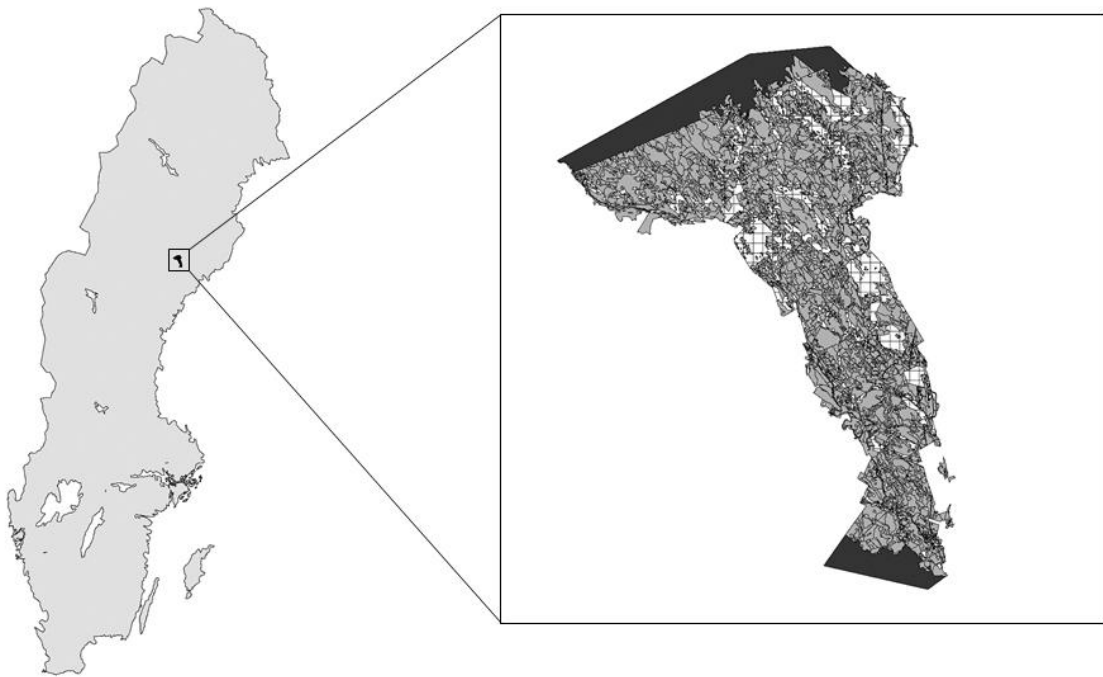


Figure 5.1. Location and landscape map of commercially managed forest for case study. The darker polygons represent the areas to connect via corridors.

Ground lichen grows in managed pine stands with basal area less than 20 m<sup>2</sup>/ha (Dettki and Esseen 1998, Sandström et al 2016). Reindeer avoid grazing in early successional stage forests (Horskotte et al. 2014), thus I assume ground lichen is not available for 10 years after a clearcut. If soil treatment occurred, ground lichen has been destroyed and re-establishment can take up to 50 years (Sundén 2003).

Reindeer corridors must have a certain width to facilitate the travel of animals. If the corridor becomes too narrow, it can be difficult to move reindeer herds along. There is no information available on reindeer corridor width requirements, so using expert opinion, I set the

minimum corridor width at 50 meters in the model. Corridors also must allow for movement. Some units such as Contorta stands (SSR 2008) are impenetrable for reindeer, thus are excluded from the corridor. Similarly, stands with trees taller than 3 meters with more than 1500 stems/ha and any stand with more than 2000 stems/ha are also considered too dense for reindeer movement (SSR 2008).

### 5.3 COMBINING HARVEST SCHEDULING AND REINDEER CORRIDORS

A traditional harvest scheduling MIP, as discussed in Section 2.5, maximizes net present value with various logistic, economic and environmental constraints by selecting treatment schedules for each unit. I formulate and solve a combined harvest scheduling and corridor selection MIP. I include several constraints that will ensure reindeer corridors are maintained by the harvest schedule that meet minimum width requirements, minimum ground and arboreal lichen requirements, and do not contain impenetrable units.

To ensure connectivity of the corridors, I implement the transshipment-based model from Jafari and Hearne (2013). Given a landscape of units, Jafari and Hearne's model selects a set of management units for a connected reserve that maximizes the utility of the reserve, subject to budget constraints. Their model considers flow across an unrooted fully connected network to ensure connectivity. Each unit selected in the reserve has flow entering it, either from another unit in the reserve or by being the initial unit, and "consumes" an amount of flow equal to the cost of including it in the reserve. The total amount of flow allowed is less than or equal to the budget for reserve selection.

I use this model with my graph-theoretical landscape representation. I first create pseudo-polygons that represent the areas adjacent to the landscape that I wish to connect. In

Figure 5.1, the pseudo-polygons are the shaded polygons. These pseudo-polygons are forced to be the first and last units selected for the reserve. Following the OCCA, define the polygons of the landscape as 1) individual units (forested and otherwise), and 2) contiguous sets of units whose combined area does not exceed 2 ha.

The variables, coefficients, indices and sets used in the combined MIP are defined in Table 5.1.

Table 5.1 Variables, coefficients and sets for model.

<b>Decision Variables</b>	
$x_{ij}$	binary decision variable that equals 1 when unit $i$ follows treatment schedule $j$
$z_{mn}^t$	binary decision variable that equals 1 if gate pair $(m, n)$ is in the corridor at time $t$
<b>Intermediate Variables</b>	
$y_{mn}^t$	amount of flow crossing gate pair $(m, n)$ at time $t$
$g_p^t, a_p^t$	amount of ground and arboreal lichen present in polygon $p$ in hectares if $p$ is in the corridor at time $t$ , equals zero otherwise
<b>Coefficients</b>	
$\beta_{ij}$	NPV associated with unit $i$ following treatment schedule $j$
$h_{ij}^t$	volume harvested from unit $i$ at time $t$ when following treatment schedule $j$
$f$	allowable fluctuation in harvest volume from one time period to the next
$\lambda_i$	area of unit $i$
$\gamma_{ij}^t$	binary coefficient that equals one if unit $i$ is impenetrable at time $t$ when following treatment schedule $j$
$\theta_{ij}^t, \phi_{ij}^t$	binary coefficients indicating if ground lichen and arboreal lichen are present in unit $i$ in time $t$ if treatment schedule $j$ is followed
$G_{min}, A_{min}$	minimum amount of ground and arboreal lichen in hectares required in the corridors
<b>Sets and Indices</b>	
$i, U$	forest management unit $i$ and the set of units $U$
$j, J_i$	treatment schedule $j$ and the set of all possible treatment schedules for unit $i$
$t, T$	time period $t$ and the set of all time periods $T$
$(m, n), G$	gate pair $(m, n)$ and the set of all gate pairs $G$
$p, P$	polygon (potentially multiunit) $p$ and the set of all polygons $P$
$S(G), E(G)$	sets of gate pairs adjacent to the starting and ending pseudo-polygons
$mid(m, n)$	polygon associated with gate pair $(m, n)$
$Q$	set of all polygons on the landscape

The full combined model is as follows:

$$\max \sum_{i,j} \beta_{ij} x_{ij} \quad (5.1)$$

subject to:

$$\sum_j x_{ij} = 1 \quad \forall i \in U \quad (5.2)$$

$$\sum_{i,j} h_{ij}^t x_{ij} \leq (1+f) \sum_{i,j} h_{ij}^{t-1} x_{ij} \quad \forall t \in \{1, 2, \dots, T\} \quad (5.3)$$

$$\sum_{i,j} h_{ij}^t x_{ij} \geq (1-f) \sum_{i,j} h_{ij}^{t-1} x_{ij} \quad \forall t \in \{1, 2, \dots, T\} \quad (5.4)$$

$$\sum_{(m,n) \in S(G)} z_{mn}^t = 1 \quad \forall t \in \{0, 1, 2, \dots, T\} \quad (5.5)$$

$$\sum_{(m,n) \in E(G)} z_{mn}^t = 1 \quad \forall t \in \{0, 1, 2, \dots, T\} \quad (5.6)$$

$$\sum_{(m,n) \in S(G)} y_{mn}^t \leq |G| \quad \forall t \in \{0, 1, 2, \dots, T\} \quad (5.7)$$

$$\sum_{\substack{n: \\ (m,n) \in G}} y_{mn}^t - \sum_{\substack{\ell: \\ (\ell,m) \in G}} y_{\ell m}^t = \sum_{\substack{n: \\ (m,n) \in G}} z_{mn}^t \quad \forall m, t \in \{1, 2, \dots, T\} \quad (5.8)$$

$$\sum_{m: (m,n) \in G} z_{mn}^t \leq 1 \quad \forall n, t \in \{0, 1, 2, \dots, T\} \quad (5.9)$$

$$y_{mn}^t \geq z_{mn}^t \quad \forall (m, n) \in G, t \in \{0, 1, 2, \dots, T\} \quad (5.10)$$

$$y_{mn}^t \leq |G| z_{mn}^t \quad \forall (m, n) \in G, t \in \{0, 1, 2, \dots, T\} \quad (5.11)$$

$$\sum_{mid(m,n) \cup p \neq \emptyset} z_{mn}^t + \sum_{i \in p, j} \gamma_{ij}^t x_{ij} \leq 1 \quad \forall p \in P, t \in \{0, 1, 2, \dots, T\} \quad (5.12)$$

$$g_p^t \leq \sum_{i \in p, j \in J_i} \lambda_i \theta_{ij}^t x_{ij} \quad \forall p \in P, t \in \{0, 1, 2, \dots, T\} \quad (5.13)$$

$$a_p^t \leq \sum_{i \in p, j \in J_i} \lambda_i \phi_{ij}^t x_{ij} \quad \forall p \in P, t \in \{0, 1, 2, \dots, T\} \quad (5.14)$$

$$g_p^t \leq \left( \sum_{i \in p} \lambda_i \right) \sum_{mid(m,n)=p} z_{mn}^t \quad \forall p \in P, t \in \{0, 1, 2, \dots, T\} \quad (5.15)$$

$$a_p^t \leq \left( \sum_{i \in p} \lambda_i \right) \sum_{mid(m,n)=p} z_{mn}^t \quad \forall p \in P, t \in \{0, 1, 2, \dots, T\} \quad (5.16)$$

$$g_p^t, a_p^t \geq 0 \quad \forall p \in P, t \in \{0, 1, 2, \dots, T\} \quad (5.17)$$

$$\sum_{p \in P} g_p^t \geq G_{min} \quad \forall t \in \{0, 1, 2, \dots, T\} \quad (5.18)$$

$$\sum_{p \in P} a_p^t \geq A_{min} \quad \forall t \in \{0, 1, 2, \dots, T\} \quad (5.19)$$

$$x_{ij} \in \{0,1\} \quad \forall i \in U, j \in J_i \quad (5.20)$$

$$z_{mn}^t \in \{0,1\} \quad \forall (m,n) \in G, t \in \{0,1,2, \dots, T\} \quad (5.21)$$

The objective (5.1) is to maximize net present value of the forest. This is a function of the timber revenue generated and costs incurred by each unit  $i$  following a treatment schedule  $j$  throughout the planning horizon, as well as the state of the unit at the end of the planning horizon. The first set of constraints address harvest scheduling logistics. Constraint (5.2) requires that only one treatment can be applied to each unit. Constraints (5.3)-(5.4) ensure that the fluctuation of volume harvested in adjacent time periods is less than a predefined bound,  $f$ .

The next set of constraints, (5.5)-(5.11), enforces corridor connectivity. I ensure that the corridors begin and end at the pseudo-polygons by requiring one gate pair from  $S(G)$  and one gate pair from  $E(G)$  to be in the corridor in constraints (5.5) and (5.6). Flow is injected into the

network in constraint (5.7). Constraint (5.8) requires that if a gate pair is selected to be in a corridor, there must be flow entering the gate pair, and it consumes 1 unit of flow. Next, constraint (5.9) ensures that every gate  $n$  is selected to be an exit gate of a gate pair  $(m, n)$  at most once. Constraints (5.10) and (5.11) ensure that there is no flow if the gate pair is not selected, and define the range of values the flow variable  $y_{mn}^t$  can take, i.e.,  $0 \leq y_{mn}^t \leq |G|$ .

Since multiunit polygons are used, constraint (5.12) ensures the gate pair polygons do not overlap. Constraint (5.12) also does not allow impenetrable units in the corridor.

Next in the model, ground and arboreal lichen within the corridor is accounted for and controlled through constraints (5.13)-(5.17). Constraints (5.13) and (5.14) define the upper bound on the amount of ground and arboreal lichen available in a polygon in a time period. The right side of the inequalities in constraints (5.13) and (5.14) calculate the amount of ground and arboreal lichen available in each polygon based on the treatment schedules selected for each unit in the polygon. In constraints (5.15), (5.16) and (5.17), I ensure that the amount of ground and arboreal lichen,  $g_p^t$  and  $a_p^t$ , are nonzero only if polygon  $p$  is selected to be in the corridor in time  $t$ . For each time period, the corridor must contain at least  $G_{min}$  hectares of ground lichen and  $A_{min}$  hectares of arboreal lichen, enforced by constraints (5.18) and (5.19).

Lastly, constraints (5.20) and (5.21) define  $x_{ij}$  and  $z_{mn}^t$  as binary.

#### 5.4 COMPUTATIONAL EXPERIMENT

The case study is based on a 1,996 unit forest owned and managed by Holmen, a Swedish forestry company and used by the Vilhelmina Norra reindeer herding community during the prewinter and winter grazing season. The forest is located in an area that is used by reindeer herders every winter as they move their herds between their summer and winter pastures as well as for winter grazing. I consider the northern and southern boundaries of the forest (shaded

regions in Figure 5.1) as the regions to be connected with corridors. Along with timberland, areas without productive forest are contained within the forest boundary. I partition these non-forested areas (mostly comprised of frozen lakes, mires, streams, etc.) using ESRI ArcGIS's fishnet tool with a 500 x 500 m grid. The resulting non-forested units are considered as possible parts of the corridor system (light grey units in Figure 5.1), but they cannot contain lichen. The final landscape includes 1,996 forested units and 1,827 non-forested units for a total of 3,823 units.

The overall planning horizon is set to 50 years and divided into 10 periods of 5 years (i.e.,  $T$  was set to 10). Then, for each stand, a set of possible treatment schedules for the 50 year planning horizon based on its associated management category was generated by Dr. Karin Öhman (Swedish University of Agricultural Sciences) based on Holmen's 2007 forest inventory data. This resulted in an average of 28 treatment schedule alternatives per stand. Economic data (timber, regeneration, and harvesting costs) used for calculating the NPV for each schedule is based on a timber price list from the current price list for the forest owner's organization in northern Sweden. The discount rate is set to 2.5%. In the model, harvested volume in each time period is required to be within 15% of the volume of the previous time period (e.g.,  $f = 0.15$ ) to ensure that harvesting is relatively consistent across the planning horizon.

I used IBM ILOG's CPLEX 12.6 on a Windows Server 2012 R2 Standard with Intel® Xeon® CPU E5-2680 v2 @ 2.80 GHz (2 processors) with 128 GB RAM and a 64 bit OS to solve all MIPs.

I compared a harvest schedule without corridors to a harvest schedule with reindeer corridors to investigate the potential of collaborative management. The model without reindeer

corridors (using only constraints (5.2)-(5.4) and constraint (5.20)) solved quickly to near optimality (<0.01% gap).

The full combined model for harvest scheduling with reindeer corridors proved too large to solve in a reasonable amount of time. After running for over a week it had not found a feasible solution. Therefore, I created an initial feasible (but suboptimal) solution to use as a warm start for a reasonable solve time (see Figure 5.2 for flowchart of procedure).

The initial solution was found by first finding a harvest schedule with no corridors that limited the number of impenetrable units (Step 1 in Figure 5.2). For this, I maximized NPV subject to constraints (5.2)-(5.4) and constraint (5.20). For many units, there exist treatment schedules in which the unit remains penetrable for all time periods. In such cases, I include a constraint that restricts the possible treatment schedules for the unit to include only these schedules. When no such penetrable treatment schedule exists for a unit, its set of possible treatment schedules are unchanged. The solution was a harvest schedule that had as few impenetrable units as possible.

Next, I fixed the harvest schedule to the solution from Step 1 and, for each time period, solved an MIP to find a corridor on the landscape with no lichen requirements (Step 2 in Figure 5.2). Following steps 2-3 of the OCCA (Table 3.1), the 3,823-unit landscape resulted in 4,461 polygons and 120,572 gate pairs. All gate pair MIPs solved to optimality (0% gap) within 124 minutes total. The eleven corridor MIPs (one for each  $t \in \{0,1, \dots, 10\}$ ) included constraints (5.5)-(5.12) and (5.21) for their respective time period, and a constraint that fixed  $x_{ij}$  to the solution from Step 1.

The output of Step 2 provided a corridor for each time period ( $z_{mn}^t$ ) associated with the harvest schedule from Step 1 ( $x_{ij}$ ). This solution is infeasible for the full model due to ground

and arboreal lichen constraints. Thus, I created and solved an MIP that maximized the minimum ground lichen in the corridor over all time periods (Step 3 in Figure 5.2). The solutions from Step 2 are combined to create an initial feasible solution for the following Step 3 model:

$$\max V \quad (5.22)$$

subject to:

$$V \leq \sum_p g_p^t \quad \forall t \in \{0, 1, 2, \dots, T\} \quad (5.23)$$

Constraints (5.2)-(5.13), (5.15), (5.17), (5.20) and (5.21)

The Step 3 model was run until the objective value exceeded the minimum threshold for ground lichen,  $G_{min}$ . The result of Step 3 is a new harvest schedule and set of corridors for all time periods that meets ground lichen requirements.

Step 4 model maximizes the minimum arboreal lichen, analogous to the Step 3 model. The output from Step 3 was used as an initial feasible solution to the Step 4 model. The Step 4 model was run until the objective value exceeded the threshold for arboreal lichen,  $A_{min}$ .

The resulting solution from Step 4 is used as an initial feasible solution to the full combined model, which I ran for 30 hours.

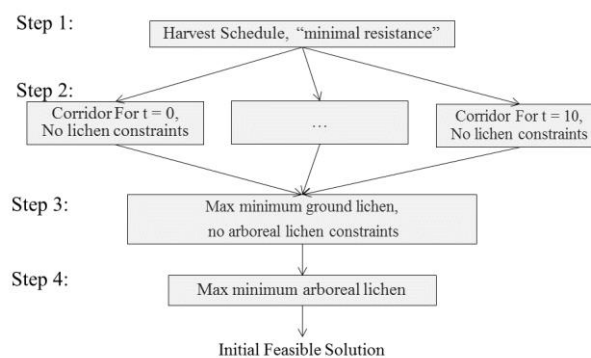


Figure 5.2. Procedure for finding an initial feasible solution to the full combined model.

## 5.5 RESULTS

Combining the OCCA with a harvest scheduling model allowed me to explore the trade-offs between an optimal harvest schedule ignoring reindeer corridors with a harvest schedule that includes viable reindeer corridors.

When no corridors were included in harvest scheduling, the optimal schedule had an NPV of approximately 7,839,000.00 SEK (<0.01% gap). Figure 5.3 and Figure 5.4 illustrate the amount of arboreal lichen, ground lichen and impenetrable stands at the initial forest state and for time periods 3, 7 and 10 if the optimal harvest schedule was followed. Over time, the amount of impenetrable forest increases, until it would be impossible for reindeer to move from the northern border to the southern, regardless of lichen requirements.

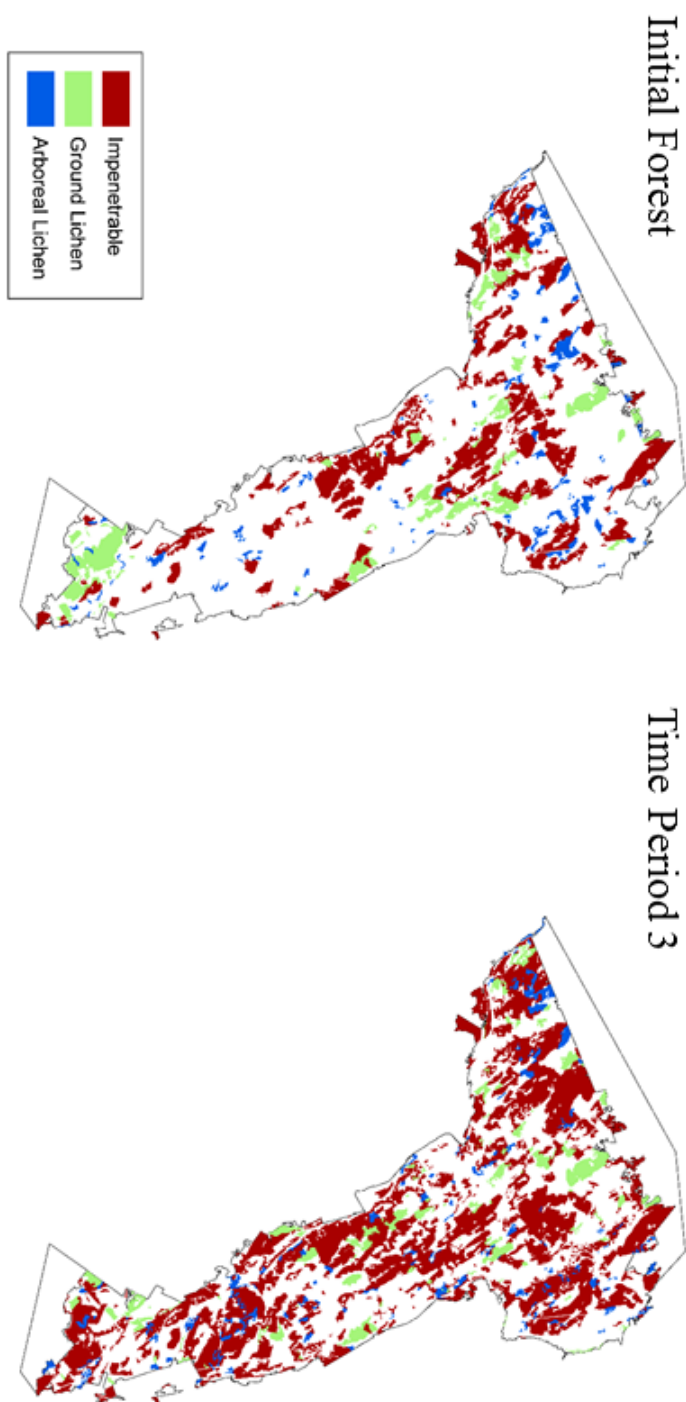


Figure 5.3. Forest conditions resulting from harvest scheduling without corridors for initial forest and time period 3.

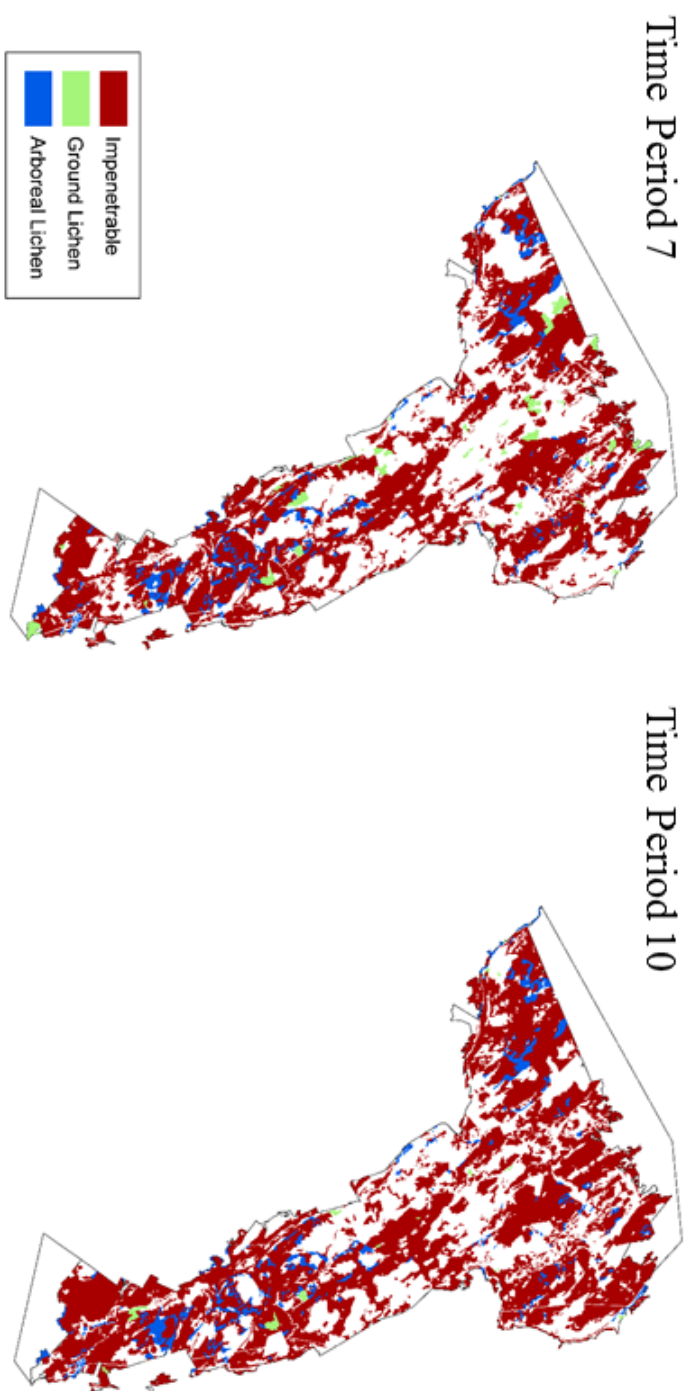


Figure 5.4. Forest conditions resulting from harvest scheduling without corridors for time periods 7 and 10.

Before solving the full combined model, I first found an initial feasible solution using the procedure outlined in Figure 5.2. The solution (after Step 4) was evaluated in the full combined model, and the NPV was approximately 7,539,000, with an optimality gap of 3.82% (upper bound  $\approx 7,838,000$  SEK). After running CPLEX for 30 hours, the solution found yielded a harvest schedule with NPV of approximately 7,579,000 SEK, with an optimality gap of 3.42% (upper bound  $\approx 7,838,000$  SEK). Thus, to include corridors, the amount of NPV forgone from is approximately 260,000 SEK, or 3.32% (compared to the harvesting only model's objective value of  $\approx 7,839,000$  SEK). Table 5.2 shows the volumes harvested in each time period for the harvest schedules with and without corridors. Although harvest volumes at each time period differ between the two harvest schedules, there is little difference in the overall harvested volumes.

Table 5.2. Harvest volumes in thousand  $m^3$  across planning horizon for each harvest schedule for 5 year long time periods.

Time period	Schedule with no corridors	Schedule with corridors
1	66.019	58.789
2	56.117	49.996
3	47.700	42.504
4	40.548	36.164
5	34.471	30.760
6	29.305	26.150
7	24.917	22.228
8	21.178	23.959
9	18.005	27.076
10	15.307	31.102
Total	353.571	348.728

The corridor conditions for each time period are described in Table 5.3. Maps showing the corridors, lichen and penetrability of the forest for the initial landscape and in time periods 3, 7 and 10 are shown in Figure 5.5 and Figure 5.6. The solution found different corridors for each

time period, but all corridors satisfy the requirements related to corridor quality. The amount of ground lichen within the corridor decreased over time, with the exception of time periods 2 to 3. Arboreal lichen increased over time, starting at 53 ha and ending with over 320 ha within the corridor, with one exception. Corridor length was not restricted, so in some cases corridors were long and winding in order to meet lichen requirements. For example, the corridor for time period 9 is nearly double the length of the corridor in time period 1. Also, the corridor for time period 10 touches the northern forest boundary at the beginning of the corridor and in the middle (see Figure 5.6). The amount of impenetrable forest across the landscape increases over time (see Figure 5.5 and Figure 5.6), but the corridors guarantee reindeer can cross the forest from the northern boundary to the southern throughout the 50 year planning horizon.

Table 5.3. Information on corridors selected by model.

Time Period	Ground Lichen (ha)	Arboreal Lichen (ha)	Minimum Width (m)	Length (km)
0	248.81	53.45	51.93	30.91
1	147.30	69.04	50.41	26.10
2	167.53	75.00	50.92	35.04
3	251.83	50.66	50.92	43.34
4	148.18	124.35	50.64	35.00
5	144.52	238.99	51.42	38.49
6	111.11	289.05	51.42	41.76
7	118.23	361.03	50.58	40.94
8	105.53	346.55	50.00	43.38
9	103.09	415.29	50.90	51.49
10	102.43	324.77	51.34	46.56

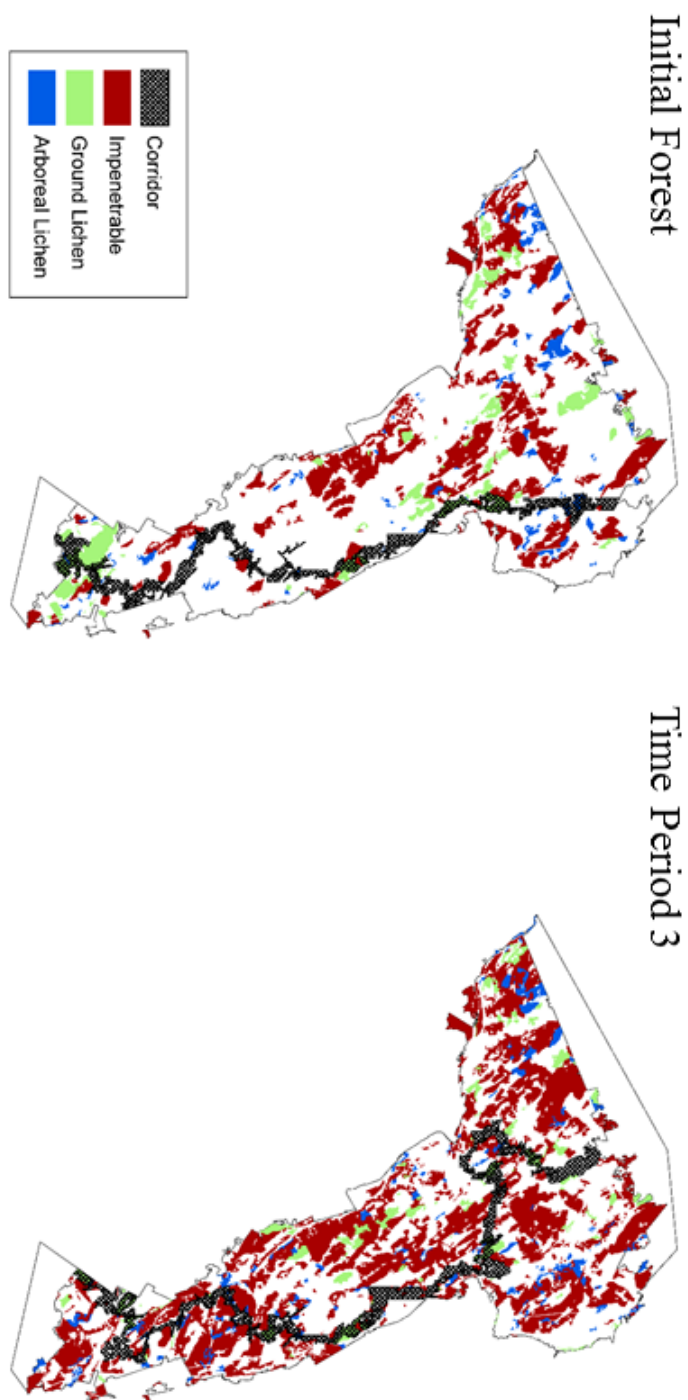


Figure 5.5. Forest conditions and corridors resulting from harvest scheduling with corridors for initial forest and time period 3.

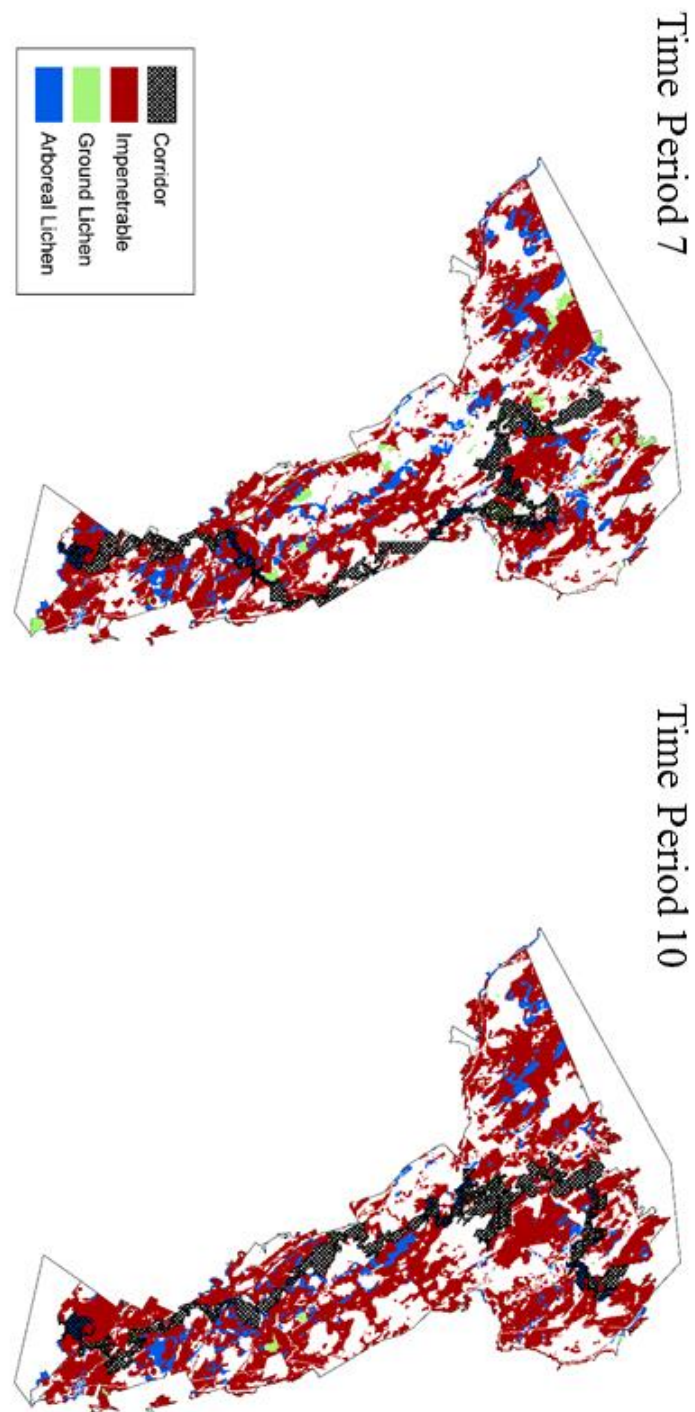


Figure 5.6. Forest conditions and corridors resulting from harvest scheduling with corridors for time periods 8 and 10.

## 5.6 DISCUSSION

In this chapter, I have shown how to incorporate high quality reindeer corridors into a harvest schedule with a small impact on the commercial production of the forest. Since the corridors only directly affect a small portion of the forest, the cost of their inclusion is also relatively low. For instance, in Korosuo et al. (2014), the NPV forgone in order to maximize lichen retention on the landscape was 5%, compared to the 3.42% foregone to include corridors. Meanwhile, I ensure connectivity of the landscape for the reindeer over the planning horizon. These findings offer a possibility for a more comprehensive way to ensure viable conditions for reindeer husbandry when planning forest management.

In both harvest schedule scenarios, there is a dramatic increase in the amount of impenetrable forest by the end of the planning horizon (Figure 5.3-Figure 5.6). By time period 10, both forests are almost completely impenetrable. However, in the harvest schedule with corridors, there exists at least one path through the forest that meets the needs of the reindeer. In the harvest schedule that did not consider corridors, it is impossible to traverse the forest from the northern boundary to the southern. Thus, including corridor constraints in the harvest scheduling model is critical to guarantee reindeer passage.

When finding a harvest schedule with corridors, computational tractability proved to be difficult to achieve. Only by incrementally building up a feasible solution, and using a powerful server, I was able to solve the full problem to within a reasonable optimality gap. Even so, after running the full problem for 30 hours, the gap only improved by 0.40%. Running CPLEX with a “good” initial feasible solution seems critical to finding a near optimal solution.

## Chapter 6. SUMMARY

Wildlife corridors are a critical tool for wildlife conservation. Many MIPs for corridor selection exist, however, they lack the capacity to control geometric characteristics of corridors on realistic landscapes. I have developed a method for explicitly controlling geometric characteristics of corridors in spatial optimization. My approach, the Optimal Corridor Construction Approach, uses methods from path planning in combination with a new graph-theoretical representation of landscapes to give users the ability to optimize or restrict corridor aspects such as width and length. I introduce the OCCA by controlling width and length of wildlife corridors. However, the OCCA has the potential to control other geometric characteristics of a corridor. For instance, if analysts want explicit control over how straight the corridor is, a metric for angle for each gate pairs can be calculated via the gate pair MIP and included in the corridor construction MIP. If a narrow path or long path are desired, rather than a short or wide path, the objectives can be redefined accordingly.

The OCCA in Chapter 3 constructs a corridor that connects two predetermined polygons, however this approach can be extended to address a variety of connected subgraph problems, by modifying previously published models. Rather than using the traditional graph interpretation of the landscape (see Figure 2.1), my new graph (Figure 3.1) can be implemented with other connectivity models. For example, in Chapter 5, I use a model from Jafari and Hearne (2013), which was originally intended to create “unrooted” reserves. For the purposes of the case study, I created a rooted model by adding constraints.

Also in this dissertation, the OCCA is used to construct corridors on a variety of landscapes, both real and simulated. In the case of a real landscape, namely the El Dorado region, the OCCA is an effective method in creating corridors with explicit control of corridor

width and length. Using simulated landscapes, I find that number of units, degree of unit adjacency, and variation in unit area all play a role in computational performance of the OCCA. The number of gate pairs is found to be linearly related to the run time of the problem. With this relationship established, it is possible to determine how computationally tractable a problem is, and adjust the multiunit polygon size accordingly. Thus, the best solution given computational limitations can be achieved.

Lastly, I demonstrated how to use the OCCA to incorporate corridors in a harvest scheduling model using a case study. Specifically, I show that it is possible to create spatially explicit forest management plans in which the needs of reindeer are satisfied, while maximizing net present value. Historically, finding solutions that both herders and foresters are satisfied with has been extremely challenging. However, with the combined OCCA and harvest scheduling model, solutions can be found that satisfy both parties with minimal compromise. In situations such as that of the conflict between the Sami reindeer herders and the commercial forest owners, it is possible to find real and practical solutions in which both parties are satisfied.

Using the OCCA, geometric characteristics (e.g., length, width, orientation, turn angle) as well as ecological characteristics (e.g. food, shelter, accessibility) of corridors can be controlled across time. For the reindeer case study, I found corridors that satisfy a list of requirements, but it would be straightforward to find corridors that maximize or minimize some characteristic, and also to find corridors that improve over time. For example, I can maximize the amount of ground lichen in the corridor, or require that the width of the corridor increases over time.

The OCCA can also be extended beyond wildlife corridors to other scenarios in which “paths” are selected on a landscape. In landscape management problems such as creating

firebreaks for wildfires (Davis 1965) and emergency evacuation routes (Cova and Johnson 2003) connectivity of selected areas is required and controlling geometric aspects of the connected area may be important.

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## VITA

Rachel St John was born in Fairbanks, Alaska. She completed a B.S. in Mathematics and Statistics at the University of Alaska Fairbanks, and a M.S. in Quantitative Ecology and Resource Management at the University of Washington. She has worked as a teaching assistant for courses in probability and statistics, and taught a course on statistical quality control. She has presented at several local and international conferences including the INFORMS Annual Meeting and the Symposium for Systems Analysis on Forest Resources.