Essays on Procurement, Scoring Auction, and Quality Manipulation Corruption

Yangguang Huang

A dissertation

submitted in partial fulfillment of the

requirements for the degree of

Doctor of Philosophy

University of Washington

2016

Reading Committee:

Quan Wen, Chair

Yanqin Fan, Chair

Xu Tan

Program Authorized to Offer Degree:

Economics

University of Washington

Abstract

Essays on Procurement, Scoring Auction, and Quality Manipulation Corruption

Yangguang Huang

Chair of the Supervisory Committee:

Professor Quan Wen

Department of Economics

Professor Yanqin Fan

Department of Economics

This dissertation studies the procurement problem with focus on the issue of quality. Most target items of procurement are not standardized goods, but are some customized goods with quality measured by non-monetary attributes. Scoring auction is one of the most popular procurement schemes used in practice. In a scoring auction, each supplying firm chooses its bid as a combination of price and quality attributes according to a pre-announced scoring rule. The scoring rule ranks all submitted multi-dimensional bids and award the contract to the firm with highest score. To implement a scoring auction, quality assessment is necessary, but the buyer usually does not possess the relevant industrial expertise. So the buyer has to hire an intermediary agent and the problem of quality manipulation arises when the quality reports of bids are distorted by the agent. In particular, the agent may exaggerate the corrupted firm's quality score in exchange for bribe. Chapter 1 provides an theoretical analysis on the optimal procurement scheme design problem under quality manipulation. Chapter 2 is an empirical study on scoring auctions. Chapter 3 shows how we can statistically test quality manipulation from scoring auction data.

Acknowledgments

For all the guidance, supporting, and mentoring, I want to express my deepest gratitude to my main adviser, professor Quan Wen. Throughout the last three years at UW, I get the opportunity to spend time with doctoral supervisory committee professor Wen, who is the pattern for me for excellent researcher, insightful teacher, deep-minded scholar, and noble man. I would never forget the days he spends time on revising my paper word by word. His wisdom and patience always support on the path of pursing PhD during these years.

I also would like to present my sincere thankfulness to lots of people. I am deeply indebted with professor Yanqin Fan. I still remember how professor Fan provide help and encouragement when I brought initial research proposals to her. I thank professor Xu Tan for the time, effort, and contribution on my research projects and supporting me. I am grateful to professor Jijun Xia as both my co-author and mentor. I thank professor Christopher Anderson for his advise and time.

I would like to extend profound thanks to professor Fahad Khalil, Jacques Lawarrée, and Patrick Bajari. I receive valuable training and priceless inspiration from them. I want to truly thank my fellow students: Ming He, Xuetao Shi, Jenny Ho, Ziran Ding, Ken Ho, An-tsu Chen. Thanks for answering my questions and supports my as companions. I also would like to thank Simon Reeve-Parker for all the practical supports.

Lastly, I would like to say thank you to my wife Emma, other family members, friends, and saints from the church. Thanks for all the life and love you supply to me. Because all of you, I will "go out with rejoicing, and be led forth in peace".

For just as the rain comes down And the snow from heaven, And does not return there, Until it waters the earth And makes it bear and sprout forth, That it may give seed to the sower and bread to the eater; So will My word be which goes forth from My mouth; It will not return to Me vainly, But it will accomplish what I delight in, And it will prosper in the matter to which I have sent it. For you will go out with rejoicing,

And you will be led forth in peace; The mountains and the hills Will break forth before you with a ringing shout, And all the trees of the field will clap their hands. In place of the thornbush, the fir tree will come up; In place of the brier, the myrtle will come up; And it will be to Jehovah as a name, As an eternal sign that will not be cut off.

- Isaiah 55:10-13

Contents

Chapter 1

Procurement Auctions under Quality Manipulation Corruption[1]

Abstract

In the procurement of a project with differential quality, quality manipulation arises when the quality-evaluating agent is bribed to exaggerate the quality score of a corrupted firm. We study how the buyer shall adjust the procurement scheme under the threat of quality manipulation. Besides the classical tradeoff between efficiency and information rent, deterrence of corruption by the inefficient firm plays a key role in the optimal mechanism design. We show that, instead of shading as in a second-best mechanism without corruption, the buyer may overstate her preference on quality. Moreover, the buyer may be better off because the efficient firm's rent is eroded by corruption. In comparison of the two popular procurement schemes, the dominance of scoring auctions over minimum-quality auctions in the literature does not hold under quality manipulation.

[1]This Chapter is a joint work with Jijun Xia from Shanghai University of Finance and Economics.

1.1 Introduction

We study the optimal procurement mechanism design problem under quality manipulation corruption in this Chapter. Tadelis and Bajari (2006) define *procurement problem* as “the procurer (buyer) hires a contractor who supplies some customized goods according to a set of desired specifications”. By nature, the target items in most procurements are not standardized goods, but are of differential *quality* measured by multiple non-monetary attributes. For example, in procurement of a construction project, at the moment of transaction, the supplying firm needs to specify the design, materials, equipments, delivery date, safety, service, and maintenance details. Therefore, if contract is awarded through a competitive tendering auction process, each *bid* is multi-dimensional object that consists of a listed price and other non-price quality attributes written as a proposal. Depending on the rules of the auction, the winning bid either obligates the bidder contractually or serves as a guideline for writing a detailed contract.

To implement a procurement, quality assessment is necessary in the auction process, but the complexity and subjectivity of quality evaluation make it particularly susceptible for corruption. This is primarily because, in a typical procurement, the buyer (she) is not an expert in the industry and lacks the specific industrial expertise expertise to evaluate quality of bids submitted by supplying firms (they/it). In some cases, at the time of determining the auction winner, the quality is unobservable to the buyer. In other cases, even after the item or project is delivered, the quality is very costly to verify and may not reveal immediately. Then the buyer is forced to hire mediators who have ability to evaluate the quality of offered items. In real practice, these mediators may include a street-level bureaucrat with industrial experiences, a professional procurement agency company, an auctioneer, and a committee of industrial experts. In this thesis, we abstract multiple layers of mediators into one layer, called the *agent* (he). Corruption would not be an issue if the buyer could observe quality directly without leaving any discretion to the agent. But because the agent is given some discretion in evaluating quality, he can exert it to seek bribes from firms. In particular, the agent may exaggerate the quality assessment of the corrupted firm. Because the process of quality evaluation is both complex and subjective, to some extent, such manipulation may not trigger investigation. If this kind of corruption indeed occurs, then the quality evaluation report is manipulated. Hence, we call this problem *quality manipulation*.

Quality manipulation is a prominent issue in procurements. International (2014) reports that the member states of the European Commission lost around €120 billion in corruption of procurements each year.[2] It points out that "The cost of corruption in public contracting is not only measured by money lost. Corruption distorts competition, can **reduce the quality**, sustainability and safety of public projects and purchases, and reduce the likelihood that the goods and services purchased really meet the public's needs."

In this Chapter, we focus on the two most widely used procurement schemes in practice: scoring auctions and price-only auctions with minimum-quality standards.[3] In a *scoring auction*, firms are asked to submit bids as price-quality combinations. A pre-announced scoring rule ranks these bids and the contract is awarded to the firm with the highest score. In a *price-only auction with minimum-quality standards* (minimum-quality auctions), the buyer publishes a minimum quality requirement and all bids satisfying the requirement are evaluated on a price basis. According to Che (1993), Asker and Cantillon (2008), and Nishimura (2015), in a corruption-free environment, the optimal procurement scheme has two main features. First, the buyer under-reports (shades) the her true preference on quality because of the tradeoff between efficiency (of quality production) and extraction of information rent.[4] Second, scoring auctions dominate minimum-quality auctions with respect to the buyer's payoff.[5] Intuitively, for any given actual procurement quality from the

[2] Data and facts of corruption can be found from Trace International Compendium, OECD Anti-bribery Convention, and Transparency International Corruption Report.

[3] Lots of procurements are not done in auctions, but the process of searching for goods with the desired price-quality combination has many aspects similar to auctions. Bajari et al. (2009) study the difference between several procurement schemes: (bilateral) negotiation, invited bidders, prequalified bidders, and open competitive bidding.

[4] See Proposition 4 in Che (1993) and Proposition 3 in Burguet and Che (2004). This result can be considered as a generalization of the optimal auction in Myerson (1981)

[5] See Theorem 6 in Asker and Cantillon (2008). They show that scoring auctions also dominate beauty contests, and menu auctions. In a beauty contest, or design-build auction, there is no pre-announced scoring rule. Buyers submit bids as price-quality combination and the winner is determined by the buyer's preference over submitted bids. In a menu auction, bidders are allow to submit multiple price-quality combination bids, instead of only one in scoring auctions. The buyer will then determine the winner and the item on its menu.

winner, compared to the fixed quality choice in minimum-quality auctions, the opponents' choice of quality are more flexible in scoring auctions. Therefore, the winner's information rent is lower in scoring auctions and the buyer is better off.

When quality manipulation is introduced to this environment, the auction outcome is critically determined by the relative magnitudes of the *technological rent*[6] of the efficient firm and the *corruption rent* of the corrupted firm. The buyer can influence the sizes of these two rents by adjusting the procurement scheme. She will prefer to deter corruption by the inefficient firm under certain conditions. The optimal mechanism in this case has some properties of the different from the prediction of existing literature. These new properties form the three main results of our study:

Result 1: In selecting the scoring rule or minimum-quality requirement, the buyer *over-report* her preference on quality in order to deter corruption. According to single-crossing property, the efficient firm has a stronger technological advantage in producing high quality. If the auction rule induces firms to produce at a low quality (distorted downward from the first-best quality), the efficient firm can be defeated by the less-efficient-but-corrupted firm with manipulated quality. To deter corruption, the buyer must provide incentive to induce a high quality, where the efficient firm can beat the corrupted firm. In this case, the optimal scoring rule may over-report her preference on quality and the actual procurement quality may even be above the first-best level.

Result 2: The superiority of scoring auctions over minimum-quality auctions in corruption-free environment no longer holds when the buyer chooses to deter corruption by over-reporting quality preference. In this case, minimum-quality auctions result in higher payoff to the buyer because the actual procurement quality is less distorted than scoring auctions. Our model provides answer to the paradox that minimum-quality auctions are consider sub-optimal in the literature, but are widely used in real world practice (see Lundberg et al., 2006). It supports the empirical findings in Tran (2011), which shows that buyers benefit from switching from scoring auctions to minimum-quality auctions.

[6]Because the buyer don't know the efficiency (type) of the firm, the most efficient firm receives a rent due to incomplete information. It is usually called the information rent in the literature. In this thesis, because there is also incomplete information on corruption, we use technological rent and corruption rent to distinct them.

Result 3: Compared to the corruption-free environment, the buyer may be better off from corruption deterrence due to rent saving and efficiency improvement. It happens when the most efficient firm is unlikely to be corrupted. Intuitively, when some inefficient firm is corrupted, its corruption rent will erodes the technological rent of the efficient firm. In this case, the buyer only needs to pay the difference between these two rents. Essentially, the efficient firm is facing stronger competition than the case without corruption and the buyer reaps the benefit. This is against the conventional wisdom[7] that the presence of corruption hurts the buyer.

1.2 Literature Review

When the procurement target items are of differential quality, scoring auctions are commonly used. The advantage of scoring auction is proven both by theory and its popularity. In practice, each bidder is asked to submit one bid as a price-quality combination. The contract is awarded to the bidder that receives the highest score based on a pre-announced scoring rule. By specifying a transparent scoring rule, firms are able to compute the monetary value of supplying at each quality level and submit proposals desirable to the buyer. In the seminal paper by Che (1993), he derives the equilibrium of scoring auctions under quasilinear scoring rules and shows that firm's quality and price choice can be separated. He shows that both first-score auctions (FSA) and second-score auctions (SSA) implement the optimal mechanism and yield the same expected utility to the buyer.

Asker and Cantillon (2008) introduce multi-dimensionality of private information and quality attributes to Che's model. They characterize the equilibrium and expected score equivalence of FSA and SSA. In addition, they show that a scoring auction with a quasilinear scoring rule dominates other alternative procurement schemes including price-only auctions with minimum quality standards, beauty contests, and menu auctions.[8] David et al. (2006) and Chen-Ritzo et al. (2005) provide experimental evidence indicating that scoring auctions dominate traditional price-

[7]The buyer (uninformed party) is hurt by corruption in the literature of bidding rings (e.g. Graham et al. (1990), McAfee and McMillan (1992), and Hendricks et al. (2008)), auctioneer-bidder cheatings (e.g. Compte et al. (2005), Burguet and Perry (2009), and Burguet and Perry (2014)), and quality manipulation (e.g. Celentani and Ganuza (2002) and Burguet and Che (2004)).

[8]In a menu auction, bidders are allow to submit multiple price-quality combination bids, instead of only one in scoring auctions. The buyer will then determine the winner and the item on its menu.

only ones. Wang and Liu (2014), Dastidar (2014), and Hanazono et al. (2015) extend the model to non-quasilinear scoring rule environments. Among these papers, Hanazono et al. (2015) consider the most general setting that covers price-quality ratio, fixed price best proposal, and convex scoring rules. They characterize the equilibrium of FSA and SSA and show that their expected score rankings depends on the curvature of the induced utility of firms.

In general, it is difficult to characterize the optimal mechanism and its implementation by a scoring auction when the environment is complicated. In Branco (1997), costs of different firms have a common component and thus are correlated. In this case, an optimal contract cannot be implemented by first or second-score auctions, but instead requires a two stage mechanism: first select a firm through an auction, then readjust the level of quality via bilateral bargaining. David et al. (2006) characterize an optimal scoring rule within the class of weighted criteria rules with restriction of additively separability of attributes on both preference and cost. Asker and Cantillon (2010) find the optimal mechanism in a specific environment where firm's types are two binary distributed random variables. They show that a scoring auction yields a performance closed to that optimal mechanism numerically. Nishimura (2015) show that implementation of the optimal mechanism via a scoring rule requires substantial cost complementarity among quality attributes. In other words, the widely used linear weighted scoring rule is sub-optimal because it does not exhibit enough complementarity among attributes to provide the correct incentive.

Concerning quality manipulation, it receives less attention from economists than collusion problems, such as bidding rings.[9] Celentani and Ganuza (2002) introduce an endogenous corruption relation forming process to the scoring auction model of Che (1993). Their model focuses on the formation of the corruption side-contract and allow the corrupted firm to win for sure once the agent accept a bribe. They show that increasing the competitiveness of the environment (more firms) may not reduce corruption. Burguet and Che (2004) consider a Bertrand-style environment

[9] Concerning collusion in procurement, there is a large body of literature on bidding rings. In a *bidding ring* or a *cartel*, a group of bidders coordinate their bids to suppress rivalry and capture some of the rents that otherwise would be transferred to the buyer. In an efficient cartel, the cartel leader is the only serious bidder, while the other cartel members submit high phony bids. The literature on bidding rings are rich both theoretically (e.g. Graham et al. (1990), McAfee and McMillan (1992), and Hendricks et al. (2008)), empirically (e.g. Porter and Zona (1993), Pesendorfer (2000), Bajari and Ye (2003), and Asker (2010)).

of two firms with complete information. The corruption relation depends on the outcome of a bribery competition, where two firms competes on bribing the agent and the winner's quality score is exaggerated. Burguet and Che (2004) show that when the scope of quality manipulation is large, the existence of both bribery competition and market competition brings in mixed strategy equilibrium and causes efficiency loss. The efficient firm cannot guarantee to win the contract because the inefficient firm may spend all its resource on either paying a high bribe or submitting a low bid.

1.3 Model and Equilibrium

1.3.1 Environment

The buyer (she) seeks procurement of a project with differential quality $q \in \mathbb{R}_+$ from firms. If the project is delivered at quality q and the compensation is $p \in \mathbb{R}_+$, the buyer's payoff is $U(q,p) = q - p$. The buyer may choose from two procurement schemes: scoring auction with linear scoring rule and minimum quality auction. If the buyer uses a scoring auction, she chooses a quality weight α. The scoring rule $S(q,p) = \alpha q - p$ is a function mapping quality and price into a score. The firm whose bid receives the highest score wins the contract. If the buyer uses a minimum-quality auction, she specifies a minimum-quality standard $\underline{q}$. The contract is awarded to the firm who satisfies the requirement and asks for the least compensation.

After the procurement scheme is announced, each firm is asked to submit a sealed-bid as price-quality combination. Firm i is characterized by the cost function $C(q, \theta_i)$ with one dimensional *type* θ_i, which can be considered as an efficiency parameter or marginal cost. Assume that $C(q, \theta)$ satisfies the following assumption:

Assumption CF: (i) $C(q, \theta)$ is strictly increasing in q, decreasing in θ, and twice continuously differentiable with respect to both parameters. (ii) $C(0, \theta) = 0$, $C_q > 0$, $C_{qq} > 0$, $C_{q\theta} > 0$, $C_{qq\theta} > 0$, $\lim_{q \to \infty} C_q = \infty$, and $\lim_{q \to 0} C_q = 0$. (iii)[10] $C_{qqq} > -C_{qq}^2$.

If the firm wins the contract with bid (p,q), its payoff is $\pi(p,q;\theta) = p - C(q,\theta)$. Firm's payoff

[10]This is a sufficient condition for uniqueness of the solution for analytical tractability. It can be relaxed without affecting qualitative prediction of the model.

is normalized to zero if it does not win the contract. In the benchmark model, we assume there are two firms with type θ_1 and θ_2 respectively, with $\theta_1 < \theta_2$. We refer firm 1 as the *efficient firm* and firm 2 as the *inefficient firm*.

Because the buyer does not possess the expertise to evaluate quality of submitted bids, she hires an *agent* (he). Following Burguet and Che (2004), we assume that the agent can manipulate the evaluation by raising the corrupted firm's quality score by a parameter $m \geq 0$, which is called *the scope of quality manipulation*.[11] If the corrupted firm submits a bid (p,q), its score is exaggerated from $S(p,q)$ to $S(p,q+m)$. When $m = 0$, there is no quality manipulation and we say the agent is honest.

The corruption relation is formed exogenously: the agent is matched with the efficient firm with probability x and the inefficient firm with probability $1-x$, $x \in [0,1]$. The buyer knows this probability x, but does not know which firm is corrupted. Similarly, the buyer knows one firm is of type θ_1 and the other θ_2, but does not knows which one is efficient.[12] In the benchmark model, we assume complete information among firms on both the cost and the corruption relation. Each firm knows its type and its opponent's type. If a firm is corrupted, it knows its opponent is not corrupted, vice versa.

The procurement auction game follows the timeline:

$t = 1$, types and the corruption relation realize. Both firms learn these information.

$t = 2$, the buyer chooses and announces a scoring rule $S(q,p)$ (a minimum-quality standard $\underline{q}$).

$t = 3$, two firms submit their sealed bids (p,q) simultaneously.

$t = 4$, the agent evaluates quality score of the corrupted firm. In a scoring auction, the corrupted firm's quality score is exaggerated by m. The contract is awarded to the firm with the highest score according to the scoring rule. In a minimum-quality auction, the corrupted firm's quality requirement reduced to $\underline{q} - m$. The contract is awarded to the firm bids at lower price.

[11] In Burguet and Che (2004), this parameter m is called the agent's *manipulation power*. They show that if there is no monitoring or the monitoring intensity is not correlated with m, the agent will always exert his full manipulation power. We assume that preventing quality manipulation is impossible or too costly because the buyer lacks industrial expertise.

[12] In other words, the buyer knows the probability distributions of corruption relation and type, but does not know their realization.

We consider a relative simple environment in the benchmark model. In Section 1.5, we provide extension on having more than two bidders, incomplete information on cost, and incomplete information on corruption relation. There are three remarks on the model setup:

Remark 1: We consider a one dimensional quality measure and let it enters both the buyer's payoff function and scoring rule linearly. It is less restrictive than it appeared to be. First, if the cost function is convex and the scoring rule is quasilinear, there is no loss of generality to consider a one dimensional quality measure (Lemma 5). Suppose the cost function is $C(\mathbf{q},\theta)$ and the scoring rule is $S(\mathbf{q},p) = V(\mathbf{q}) - p$, where $\mathbf{q} \in \mathbb{R}^L_+$, then one can consider the firm is producing a quality score $v = V(\mathbf{q})$ with cost function $C(v,\theta)$. Second, as long as the form of cost function is flexible, one can rescale the quality measure so that it enters the payoff function linearly. The linearity of scoring rule does impose some restriction on the optimal scoring rule design. We adapt the linearity setting due to both analytical simplicity and the widely use of weighted linear scoring rule used in procurement practice.

Remark 2: Concerning the formation of corruption relation, Burguet and Che (2004) consider it as the result of a bribery competition. So the efficient firm has advantages in both the auction competition and bribery competition. In our model, the corruption relation is exogenously determined,[13] which allows us to disentangle rents from technological advantage and corruption. It paves the way to characterize the optimal procurement scheme and makes the model easy to generalized. It is also a realistic assumption: explicit bribery competition is nearly impossible under a well-functioned legal system. The bribery competition loser has incentive to report the corruption activity to the antitrust authority because it hurts its interest directly. Therefore the corruption relation must be based on trust and repeated interaction, which can be taken as exogenous for a particular procurement.

Remark 3: We assume that each firm has complete information on all firm's costs and corruption relations (the buyer has incomplete information on them). This assumption is widely used in studies of corruption in auction (e.g. Bajari and Ye (2003), Burguet and Perry (2009) and Athey

[13] In the literature of bidding rings, it is common to assume the collusion relation is exogenously given, for example Porter and Zona (1993), Porter and Zona (1999), Bajari and Ye (2003), and Athey et al. (2011).

et al. (2011)). It circumvents the difficulty of having two layers of incomplete information on both cost and corruption relation. In Section 1.5.3, we analyze a model with incomplete information on corruption relations, which is another contribution to the literature. In Chapter 3, we use an alternative assumption on knowledge of corruption relation: we assume other bidders are completely unaware of the corruption so they follow the original equilibrium strategy (e.g. Porter and Zona (1993), Aryal and Gabrielli (2013) and Tian and Liu (2008).

1.3.2 Scoring Auctions with Linear Scoring Rule (L)

Suppose the buyer uses a *scoring auction with linear scoring rule* (L, hereafter): $S(q,p) = \alpha q - p,\ \alpha \geq 0$, where she selects a *quality weight* to represents the *monetary equivalent for quality*.[14] Using one parameter α to characterize the optimal scoring rule allows for best analytical tractability when quality manipulation is introduced. Once the scoring rule is announced, firms will select their quality according to the following lemma:

Lemma 1:[15] *Given $S(q,p) = \alpha q - p$, for both the honest firm and the corrupted firm, it is a weakly dominant strategy for firm with type θ to choose quality*

$$q^* \equiv \arg\max_q \alpha q - C(q,\theta). \tag{1.1}$$

With convex cost function, the solution q^* is unique. Given α, denote $q_i(\alpha) \equiv \arg\max_q \alpha q - C(q,\theta_i)$ and $C_i(\alpha) \equiv C(q_i(\alpha),\theta_i)$, $i = 1,2$. They have several properties useful for further analysis.

Lemma 2: *For any $\alpha > 0$,* (i) a higher quality weight α induces higher quality, i.e. $q_i'(\alpha) > 0$*;* (ii) the efficient firm picks higher quality under the same α and is more responsive to change in α, i.e. $q_1(\alpha) > q_2(\alpha)$ *and* $q_1'(\alpha) > q_2'(\alpha)$*;* (iii) $C_1(\alpha) > C_2(\alpha)$.

After pinning down quality choice, firms choose their prices in the way of Bertrand competition

[14] The definition can be found in Dini et al. (2006).

[15] It is an extension to Lemma 1 in Che (1993). The same argument holds for all scoring rule with when price and quality are additively separable.

due to complete information. With a honest agent ($m = 0$), firm 1 always wins by slightly out-bidding firm 2 in score. It earns a profit as its technological (information) rent, which corresponds to the outcome of a second-best mechanism.

Proposition 1: *When $m = 0$, the (unique) equilibrium prices are*

$$\begin{cases} p_1(\alpha) & = \alpha q_1(\alpha) - \alpha q_2(\alpha) + C_2(\alpha) = C_1(\alpha) + R_T^L, \\ p_2(\alpha) & = C_2(\alpha), \end{cases}$$

where $R_T^L \equiv \alpha q_1(\alpha) - C_1(\alpha) - \alpha q_2(\alpha) + C_2(\alpha)$ is the efficient firm's technological rent *under L. In the equilibrium, for all $\alpha \geq 0$, $R_T^L \geq 0$, $dR_T^L/d\alpha > 0$, and $d^2R_T^L/d\alpha^2 > 0$.*

Proposition 2: *When $m = 0$, at the equilibrium$\{q_1(\alpha), p_1(\alpha), q_2(\alpha), p_2(\alpha)\}$, firm 1 wins the contract and the buyer's payoff is*

$$U_{SB}^L(\alpha) \quad = \quad q_1(\alpha) - C_1(\alpha) - R_T^L. \tag{1.2}$$

There is a unique quality weight α_{SB} characterizes the optimal linear scoring rule which maximizes $U_{SB}(\alpha)$ and $0 < \alpha_{SB} < 1$.

Figure 1.1: Illustration of Proposition 1

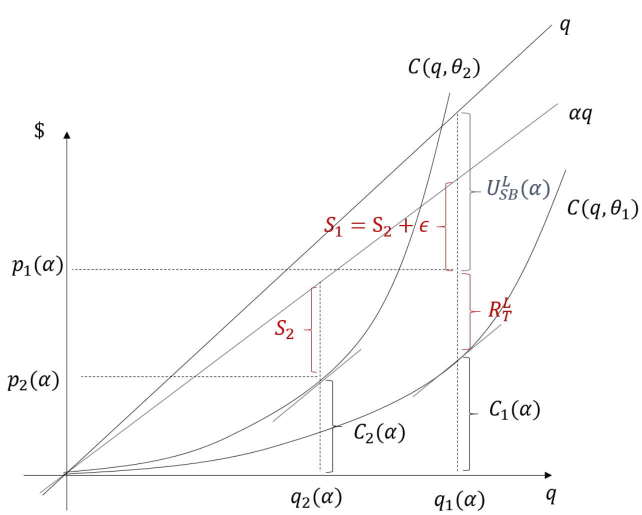


Figure 1.1 illustrate Proposition 1 and 2. Firms choose quality according to on the slope of scoring rule and its cost function. The maximum score firm 2 can get is $S_2 = \alpha q_2(\alpha) - C_2(\alpha)$. Competing under complete information, firm 1 matches firm 2's score and beat firm 2 by score S_1. Hence, in the equilibrium, firm 1's payoff is its technological rent R_T^L and buyer's payoff is $U_{SB}^L(\alpha)$. In the optimal linear scoring rule, the buyer chooses α by the classical trade-off between efficiency of quality and extracting firm 1's rent. As shown in Che (1993) and Asker and Cantillon (2008), the buyer under-reports (shades) his preference on quality ($\alpha_{SB} < 1$) in a second-best mechanism.

When there is quality manipulation ($m > 0$), under linear scoring rule, the corrupted firm receives a *corruption rent* $R_C^L \equiv \alpha m$ because its score is exaggerated by αm. Because technological rent R_T^L is increasing and convex in α, we have the following lemma,

Lemma 3: *For any* $0 < m < \sup_{\alpha \in [0,\infty)} R_T^L(\alpha)/\alpha$, (i) *there exists a unique positive solution* $\tilde{\alpha} > 0$ *to equation* $\tilde{\alpha} m = R_T^L(\tilde{\alpha})$, (ii) *for* $\alpha < \tilde{\alpha}$, $\alpha m > R_T^L(\alpha)$; *for* $\alpha \geq \tilde{\alpha}$, $\alpha m \leq R_T^L(\alpha)$; *and* (iii) $\tilde{\alpha}$ *increases in m.*

Therefore, there exists a cutoff quality weight $\tilde{\alpha}$ of the scoring rule that determines the relative magnitude of R_T^L and R_C^L. When the efficient firm is corrupted, it wins the contract for sure. When the inefficient firm is corrupted, the outcome depends on how the buyer chooses α. Proposition 3 presents the firms' equilibrium bidding strategies:

Proposition 3: *The equilibrium prices, outcome, and the buyer's payoffs are listed below.*

Corrupted Firm	Outcome	Price	Buyer's Payoff
Firm 1 with prob. x	Firm 1 wins	$\begin{cases} p_1(\alpha) = C_1(\alpha) + R_T^L + R_C^L \\ p_2(\alpha) = C_2(\alpha) \end{cases}$	$q_1(\alpha) - p_1(\alpha)$
Firm 2 with prob. $1-x$	Firm 1 wins when $\alpha \geq \tilde{\alpha} \Leftrightarrow R_T^L \geq R_C^L$	$\begin{cases} p_1(\alpha) = C_1(\alpha) + R_T^L - R_C^L \\ p_2(\alpha) = C_2(\alpha) \end{cases}$	$q_1(\alpha) - p_1(\alpha)$
	Firm 2 wins when $\alpha < \tilde{\alpha} \Leftrightarrow R_T^L < R_C^L$	$\begin{cases} p_1(\alpha) = C_1(\alpha) \\ p_2(\alpha) = C_2(\alpha) - R_T^L + R_C^L \end{cases}$	$q_2(\alpha) - p_2(\alpha)$

Given the firms' equilibrium, the buyer's payoff[16] is a function with a discontinuity at $\tilde{\alpha}$.

$$U^L(\alpha) = \begin{cases} U_A^L(\alpha), & \text{if } \alpha \geq \tilde{\alpha}, \\ U_B^L(\alpha), & \text{if } \alpha < \tilde{\alpha}, \end{cases} \tag{1.3}$$

where

$$U_A^L(\alpha) = x\left[q_1(\alpha) - C_1(\alpha) - R_T^L - R_C^L\right] + (1-x)\left[q_1(\alpha) - C_1(\alpha) - R_T^L + R_C^L\right], \tag{1.4}$$

$$U_B^L(\alpha) = x\left[q_1(\alpha) - C_1(\alpha) - R_T^L - R_C^L\right] + (1-x)\left[q_2(\alpha) - C_2(\alpha) + R_T^L - R_C^L\right]. \tag{1.5}$$

The two terms in both U_A^L and U_B^L demonstrate the effect of corruption is opposite depending on the realized corruption relation. If the efficient firm is corrupted, the buyer is worse-off by losing both the technological rent and the corruption rent. But if the inefficient firm is corrupted, the technological rent and the corruption rent will offset each other. Therefore, the buyer need to consider both possibilities in choosing the optimal quality weight α^*.

Theorem 1: *The optimal quality weight α^* can be characterized by three cases shown in Figure 1.2. Define $\alpha_A \equiv \arg\max_{\alpha\in[0,\infty)} U_A^L(\alpha)$, $\alpha_B \equiv \arg\max_{\alpha\in[0,\infty)} U_B^L(\alpha)$, and $\bar{m}^L$ as the solution of $\tilde{\alpha}(m) = \alpha_A(m)$. Given that $m > \bar{m}^L$, there exists a unique $\hat{\alpha} \in (\alpha_A, \infty)$ such that $U_A^L(\hat{\alpha}) = U_B^L(\alpha_B)$. Then*

(i) *If $m \leq \bar{m}^L$, $\alpha^* = \alpha_A$, illustrated in diagram* (A).

(ii) *If $m > \bar{m}^L$ and $\tilde{\alpha} \leq \hat{\alpha}$, $\alpha^* = \tilde{\alpha}$, illustrated in diagram* (B).

(iii) *If $m > \bar{m}^L$ and $\tilde{\alpha} \leq \hat{\alpha}$, $\alpha^* = \alpha_B$, illustrated in diagram* (C).

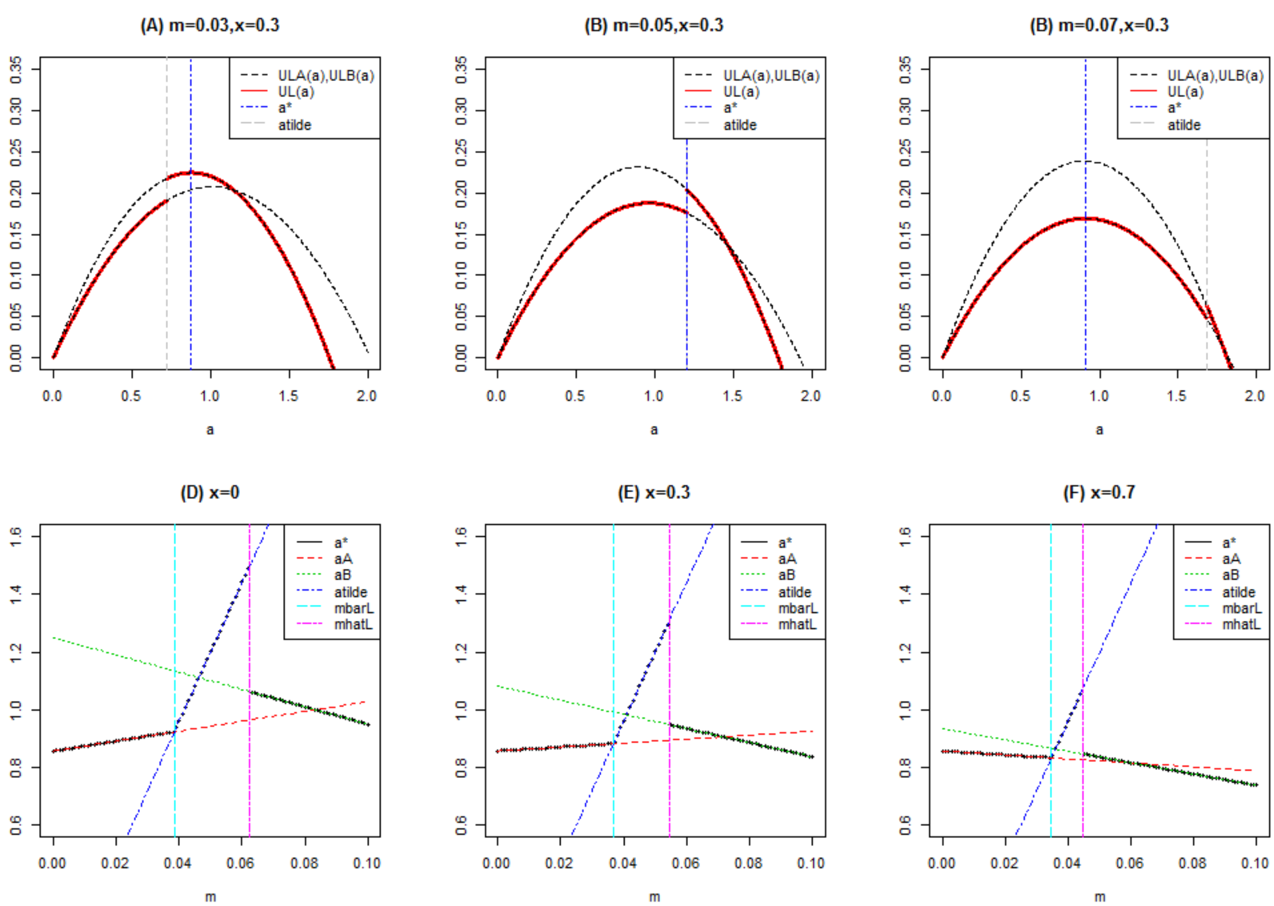
Corollary 1: *The buyer may over-report its preference on quality, i.e. $\alpha^* > 1$.*

Intuitively, Theorem 1 states how the buyer chooses an optimal quality weight at different scopes of quality manipulation. When *m* is relatively small, the buyer still want to procure from

[16] When $m \to 0$, the buyer's choice of α and her payoff converge to those in the case with a honest agent.

firm 1. She is worse-off when firm 1 is corrupted by giving out two rents, but better-off when firm 2 is corrupted as two rents offset each other. When m gets large, a larger quality weight is needed to keep firm 1 winning and deter corruption of firm 2. Raising quality weight increases the size of both rents and changes the induced procurement quality. The buyer may choose a large quality weight to keep procuring from the efficient firm; or she may give up deterring corruption and choose a small quality weight that grands the contract whichever firm is corrupted.

Figure 1.2: Payoff and Optimal α under L



1.3.3 Price-only Auction with Minimum-quality Standards (M)

Now suppose the buyer uses a price-only auction with minimum-quality standard (M, hereafter)‘[17], which requires each bid with quality at least $\underline{q}$. Because firms have no incentive to pro-

[17] The scoring rule of M can be considered as $S(q,p) = \underline{q} - p$ if $q \geq \underline{q}$; $S(q,p) = -\infty$ otherwise.

duce a quality higher than $\underline{q}$, they will set quality $q = \underline{q}$. The buyer's strategy space is choosing *minimum quality q*. Denote $C_1(q) \equiv C(q, \theta_1)$ and $C_2(q) \equiv C(q, \theta_2)$. When the agent is honest ($m = 0$), because $C_1(q) < C_2(q)$, firm 1 wins the contract and the equilibrium prices are $p_1(q) = C_2(q)$, $p_2(q) = C_2(q)$.[18] Firm 1 earns a rent $R_T^M \equiv C_2(q) - C_1(q)$, which is its *technological rent* under M. The buyer's payoff is $U_H^M(q) = q - C_2(q)$, which has a unique maximum q_H.

When there is quality manipulation ($m > 0$), the corrupted firm is allowed to produce at quality $q - m$ and still meets the minimum-quality requirement. When firm 1 is corrupted, it wins the contract for sure. When firm 2 is corrupted, it receives a *corruption rent* $R_C^M \equiv C_2(q) - C_2(q - m)$. The outcome depends on the relative magnitude of technological rent and corruption rent. Firm 2 wins when $R_T^M < R_C^M$, or equivalently, $C_1(q) > C_2(q - m)$. The following lemma shows that we can find a cutoff quality $\tilde{q}$ that determines the relative magnitude of R_T^M and R_C^M.

Lemma 4: *Define function f mapping cost back to quality, which means that for* $c = C(q, \theta)$, $q = f(c, \theta)$. *For any* $0 < m < \sup_{c \in [0, \infty)} [f(c, \theta_1) - f(c, \theta_2)]$, (i) *there exists a unique solution* $\tilde{q} > 0$ *to equation* $C_1(\tilde{q}) = C_2(\tilde{q} - m)$; (ii) *for* $q \geq \tilde{q}$, $C_1(q) \leq C_2(q - m)$; *for* $q < \tilde{q}$, $C_1(q) > C_2(q - m)$; *and* (iii) $\tilde{q}$ *increases in m.*

The equilibrium of the auction competition is described below:

Proposition 4: *When* $m > 0$, *the uncorrupted firm chooses quality q and corrupted firm chooses quality* $q - m$. *The equilibrium prices, outcome and buyer's payoffs are listed below.*

[18] The proof is trivial by Bertrand competition model.

Corrupted Firm	Outcome	Price	Buyer's Payoff
Firm 1 with prob. x	Firm 1 wins	$\begin{cases} p_1(q) = C_2(q) \\ p_2(q) = C_2(q) \end{cases}$	$q - m - p_1(q)$
Firm 2 with prob. $1-x$	Firm 1 wins when $q \geq \tilde{q} \Leftrightarrow R_T^M \geq R_C^M$	$\begin{cases} p_1(q) = C_2(q-m) \\ p_2(q) = C_2(q-m) \end{cases}$	$q - p_1(q)$
	Firm 2 wins when $q < \tilde{q} \Leftrightarrow R_T^M < R_C^M$	$\begin{cases} p_1(q) = C_1(q) \\ p_2(q) = C_1(q) \end{cases}$	$q - m - p_2(q)$

Similar to Proposition 3, the buyer's payoff is a function with a discontinuity at $\tilde{q}$.

$$U^M(q) = \begin{cases} U_A^M(q), & \text{if } q \geq \tilde{q}, \\ \\ U_B^M(q), & \text{if } q < \tilde{q}, \end{cases} \tag{1.6}$$

where

$$U_A^M(q) = x\left[q - m - C_2(q)\right] + (1-x)\left[q - C_2(q-m)\right], \tag{1.7}$$

$$U_B^M(q) = x\left[q - m - C_2(q)\right] + (1-x)\left[q - m - C_1(q)\right]. \tag{1.8}$$

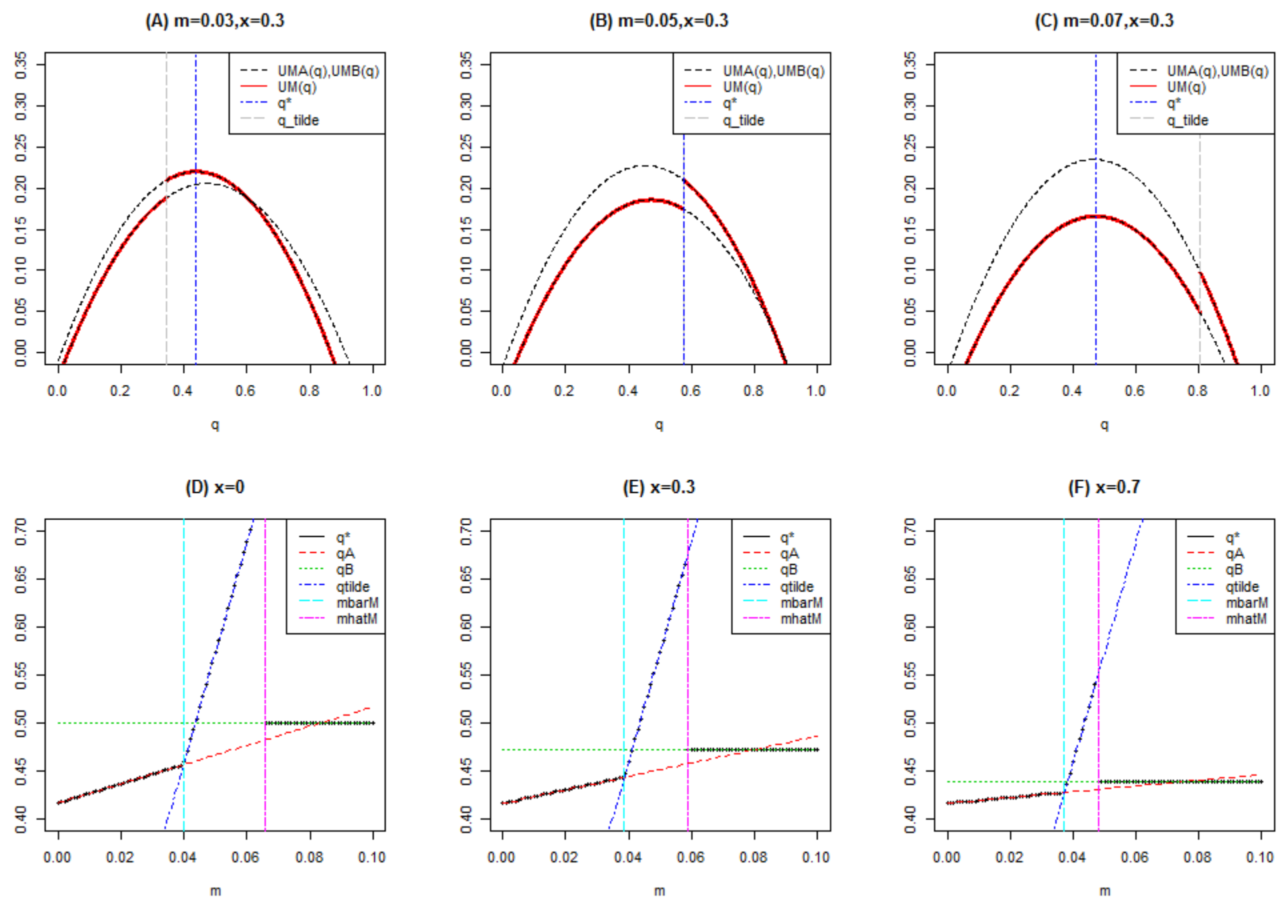
Theorem 2: *The optimal minimum-quality q^* can be characterized by three cases shown in Figure 1.3. Define $q_A \equiv \arg\max_{q\in[0,\infty)} U_A^M(q)$, $q_B \equiv \arg\max_{q\in[0,\infty)} U_B^M(q)$, and $\bar{m}^M$ as the solution of $\tilde{q}(m) = q_A(m)$. Given that $m > \bar{m}_M$, there exists a unique $\hat{q} \in (q_A, \infty)$ such that $U_A^M(\hat{q}) = U_B^M(q_B)$. Then*

(i) *If $m \leq \bar{m}^M$, $q^* = q_A$, illustrated in diagram* (A).

(ii) *If $m > \bar{m}^M$ and $\tilde{q} \leq \hat{q}$, $q^* = \tilde{q}$, illustrated in diagram* (B).

(iii) *If $m > \bar{m}^M$ and $\tilde{q} > \hat{q}$, $q^* = q_B$, illustrated in diagram* (C).

Figure 1.3: Payoff and Optimal q under M



Corollary 2: *The buyer may over-report its preference on quality. Specifically,.* $q^* > q_{FB}$*, where* q_{FB} *is the first-best minimum quality choice when there is complete information and no corruption.* The structure and intuition of Theorem 2 are similar to Theorem 1. If m is relatively small, the buyer may raise the minimum quality to keep firm 1 winning and deter corruption of firm 2. However, when m is large, the buyer gives up large rents in order to allow an uncorrupted firm 1 to win. She may rather pick a low q.

1.4 Buyer's Payoff

We have characterized the buyer's optimal scoring rules and buyer's payoffs under linear scoring auctions and minimum-quality auctions separately. In this section, we compared buyer's payoffs under L and M given the same set of parameters m and x. Given a set of parameters m and x, the buyer chooses $\alpha^*(m,x)$ and $q^*(m,x)$ according to Theorem 1 and 2. There are two main results concerning the buyer's payoff.

Firstly, we can show that the buyer is better off with positive amount of corruption, provided that the inefficient firm is more likely to be corrupted.[19]

Theorem 3: *When $x < 0.5$, the buyer's "optimal" scope of quality manipulation is strictly positive, both in scoring auction and price-only auction, i.e.* $m^{L*} = \arg\max_{m\geq 0} U^L(\alpha^*(m,x)) > 0$ *and* $m^{M*} = \arg\max_{m\geq 0} U^M(q^*(m,x)) > 0$.

Figure 1.4: Comparison of U^L and U^M at $x = 0$

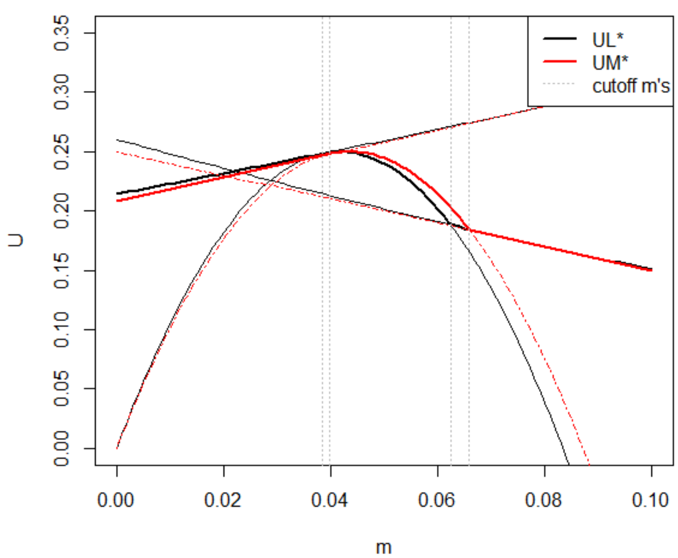


Figure 1.4 illustrate the theorem. Intuitively, without corruption, firm 1 can reap all its technological rent. But with corruption, firm 1's rent is eroded by firm 2's corruption rent. Therefore, the existence of corruption may increase the buyer's expected payoffs.

[19] One can easily show that when $x > \frac{1}{2}$, $U_L(\alpha^*(m,x)) < U_L(\alpha^*(0,x))$ and $U_M(q^*(m,x)) < U_M(q^*(0,x))$, illustrated in Figure 1.5, diagram (B). If the efficient firm is more likely to be corrupted, the buyer is worse-off by giving both technological rent and corruption rent with a large probability.

Secondly, given the same environment, the buyer may prefer using M to L. Asker and Cantillon (2008) show that L dominates M when there is no corruption ($m = 0$).[20] But this dominance relation does not hold when quality manipulation is present.

Theorem 4: *When $x < 0.5$, there exists a set of m such that $U^M(q^*(m,x)) > U^L(\alpha^*(m,x))$, i.e. minimum-quality auction is optimal for the buyer.*

Since Theorem 4 is about existence, we show the theorem by the following parametric example. Assume the cost function takes the form $C(q,\theta) = \theta q^2$, satisfying assumption CF. Set firm 1's efficiency parameter as $\theta_1 = 1$ and firm 2's efficiency parameter as $\theta_2 = \theta > 1$, i.e. $C_1(q) = q^2$, $C_2(q) = \theta q^2$.

Under L, by Lemma 1, $q_1(\alpha) = \frac{\alpha}{2}$, $q_2(\alpha) = \frac{\alpha}{2\theta}$. Equilibrium costs are $C_1(\alpha) = \frac{\alpha^2}{4}$ and $C_2(\alpha) = \frac{\alpha^2}{4\theta}$. Firm 1's technological rent is $R_T^L \equiv \alpha q_1(\alpha) - C_1(\alpha) - \alpha q_2(\alpha) + C_2(\alpha) = \frac{\alpha^2}{4}\left(\frac{\theta-1}{\theta}\right)$. When $m = 0$, the buyer's payoff is $U_H^L(\alpha) = q_1(\alpha) - C_1(\alpha) - R_T^L = \frac{\alpha}{2} - \frac{\alpha^2}{4}\left(\frac{2\theta-1}{\theta}\right)$. The optimal quality weight is $\alpha_H = \frac{\theta}{2\theta-1} < 1$ and the maximized payoff is $U_H^L(\alpha_H) = \frac{1}{4}\left(\frac{\theta}{2\theta-1}\right)$.

When $m > 0$, the corruption rent is $R_C^L = \alpha m$. By lemma 3, the threshold quality weight $\tilde{\alpha}$ that determined the relative magnitude of two rents can be found:

$$R_C^L \leq R_T^L \Leftrightarrow \alpha m \leq \frac{\alpha^2}{4}\left(\frac{\theta-1}{\theta}\right) \Leftrightarrow \alpha \geq \frac{4\theta m}{\theta-1} \equiv \tilde{\alpha}(m).$$

The buyer's payoff function

$$U^L(\alpha) = \begin{cases} U_A^L(\alpha) = \frac{\alpha}{2} - \frac{\alpha^2}{4}\left(\frac{2\theta-1}{\theta}\right) + (1-2x)\alpha m & , \text{ if } \alpha \geq \frac{4\theta m}{\theta-1}, \\ U_B^L(\alpha) = x\left[\frac{\alpha}{2} - \frac{\alpha^2}{4}\left(\frac{2\theta-1}{\theta}\right)\right] + (1-x)\left[\frac{\alpha}{2\theta} - \frac{\alpha^2}{4}\left(\frac{2-\theta}{\theta}\right)\right] - \alpha m & , \text{ if } \alpha < \frac{4\theta m}{\theta-1}. \end{cases}$$

[20] The proof is straight-forward. Consider $\alpha = 1$, $q_2(\alpha = 1) = \arg\max_q q - C_2(q) = q_{SB}$, then $U^L(\alpha = 1) = q_2(\alpha = 1) - C_2(q_2(\alpha = 1)) = U^M(q_{SB})$. By Proposition 2, U^L is maximized at $\alpha_{SB} < 1$, therefore $U^L(\alpha_{SB}) > U_L(\alpha = 1) = U^{SB}(q_{SB})$.

The peak of U_A^L and U_B^L are reached at $\alpha_A = \frac{\theta(1+2m-4xm)}{2\theta-1}$ and $\alpha_B = \frac{x\theta-x-2\theta m+1}{3x\theta-3x-\theta+2}$ respectively.

Using M, when $m = 0$, the buyer's payoff is $U_H^M(q) = q - C_2(q) = q - \theta q^2$. The optimal minimum-quality is $q_H = \frac{1}{2\theta}$ and the maximum payoff is $U_H^M(q_H) = \frac{1}{4\theta}$. Compared to L, one can easily show that $U_H^M(q_H) < U_H^L(\alpha_H)$ for $\theta > 1$.

When $m > 0$, firm 1's technological rent is $R_T^M = C_2(q) - C_1(q) = \theta q^2 - q^2 = (\theta - 1)q^2$ and firm 2's corruption rent (given it is corrupted) is $R_C^M = \theta q^2 - \theta(q-m)^2 = \theta(2qm - m^2)$. By Lemma 4, the threshold quality $\tilde{q}$ that determined the relative magnitude of two rents can be found:

$$R_C^M \leq R_T^M \Leftrightarrow \theta(2qm - m^2) \leq (\theta - 1)q^2 \Leftrightarrow q \geq \frac{m(\theta + \sqrt{\theta})}{\theta - 1} \equiv \tilde{q}(m).$$

The buyer's payoff function

$$U^M(q) = \begin{cases} U_A^M(q) = q - m - \theta q^2 + 2(1-x)\theta mq - (1-x)\theta m^2 & , \text{ if } q \geq \frac{m(\theta+\sqrt{\theta})}{\theta-1}, \\ U_B^M(q) = q - m - x\theta q - (1-x)q^2 & , \text{ if } q < \frac{m(\theta+\sqrt{\theta})}{\theta-1}. \end{cases}$$

The peak of U_A^M and U_B^M are reached at $q_A = \frac{1+2(1-x)\theta m}{2\theta}$ and $q_B = \frac{(1-x\theta)}{2(1-x)}$ respectively.

It is easy to verify that this parametric example provides the three main results of this paper, illustrated in Figure 1.5 and 1.6, with detail in the appendix.

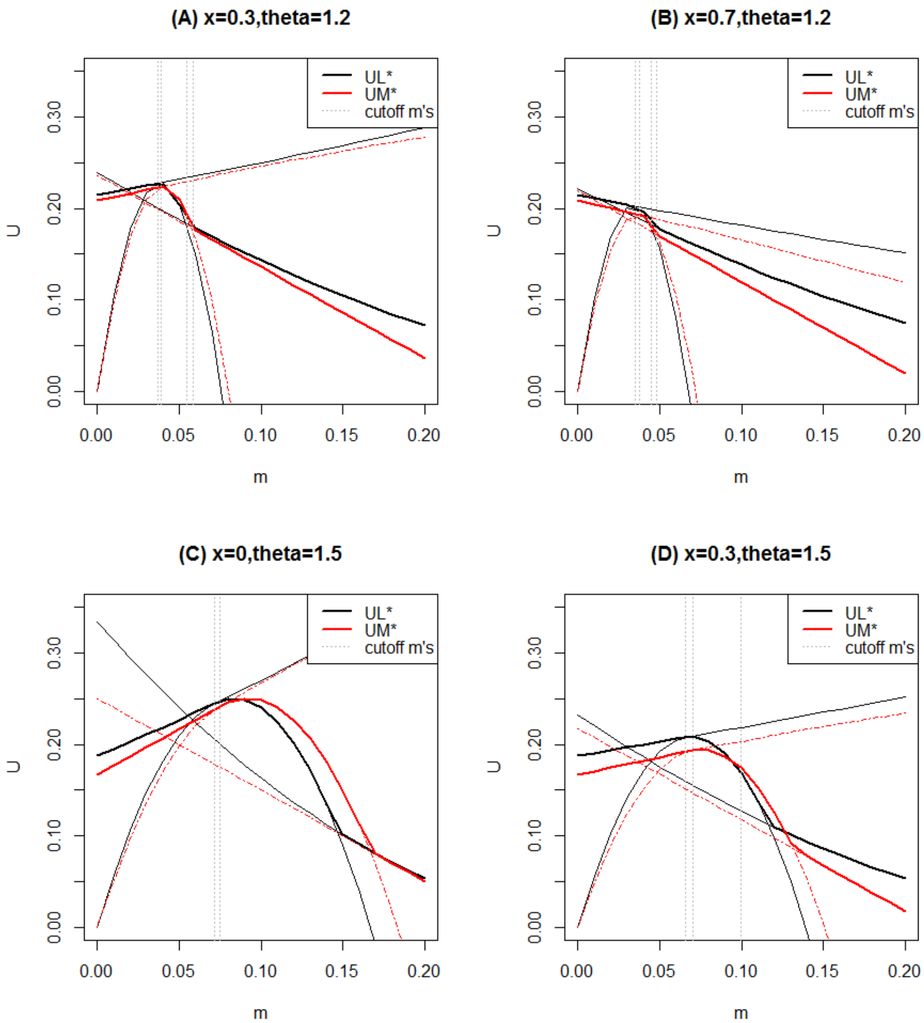
Discussion of the intuition

To understand the intuition of why L may not dominate M, we need to consider the actual *procurement quality* of L. Setting α^* induces firm 1 and firm 2 choose $q_1(\alpha^*)$ and $q_2(\alpha^*)$ respectively under L. Both of them are strictly increasing functions by Lemma 2, therefore we can define

$$\alpha(q) \equiv \begin{cases} q_1^{-1}(q), & \text{if firm 1 wins,} \\ q_2^{-1}(q), & \text{if firm 2 wins.} \end{cases} \tag{1.9}$$

Figure 1.6 shows the procurement quality of L and M at various scopes of quality manipulation.

Figure 1.5: Comparison of U^L and U^M



q_{FB} and q_{FB} are defined as the first-best (the buyer has complete information on costs) and second-best (the buyer has incomplete information on on costs) quality level when there is no corruption. In the parametric example, $q_{FB} = \frac{1}{2}$ and $q_{SB} = \frac{1}{2\theta}$.

When there is no corruption, the buyer faces the tradeoff of distortion of procurement quality and the size of technological rent under incomplete information on costs. By introducing corruption, the buyer needs to consider the relative magnitudes of **two rents** and whether to deter corruption or not. Depending on the scope of quality manipulation, the optimal procurement scheme corresponds to three intervals, as shown in Figure 1.6 (Theorem 1 and 2).

In the first interval, when m is small, the buyer can simply use α_A (q_A) at the peak of U_A^L (U_A^M), which is enough to deter corruption of firm 2. In the second interval, as m goes up, using α_A (q_A) cannot deter corruption. As deterrence is still desirable to the buyer, she has to use a higher quality weight $\tilde{\alpha}$ (minimum-quality standard $\tilde{q}$). In the third interval, m is large, the upward quality distortion is so huge that deterring corruption is no longer desirable. It is optimal for the buyer to choose α_B (q_B) that allows the corrupted-and-inefficient firm wins. It induces a low procurement quality, which reduces the size of both technological rents and corruption rent.

The three main results of this paper are rooted in the corruption deterrence behavior. In the second interval, corruption deterrence by using a high $\tilde{\alpha}$ ($\tilde{q}$) may lead to a upward distortion of the procurement quality above the first-best level.[21] Therefore, the quality weight in the scoring rule (the minimum-quality standard) over-reports the buyer's preference on quality, as shown in Result 1 (Corollary 1 and 2).[22]

From the parametric example, we see that the actual procurement quality is higher in L than M in the second interval. When over-reporting happens, quality may be distorted less in M than L compared to the first-best level.[23] This is why the buyer can be better off by using minimum-quality auctions, as described in Result 2 (Theorem 4).

When the buyer chooses to deter corruption, the efficient firm will win the contract. If the efficient firm is corrupted, the buyer is worst off by paying the summation of two rents, compared to the case without corruption. If the inefficient firm is corrupted, its corruption rent erodes the efficient firm's technological rent, so the buyer saves rent. The buyer may be better off when the latter happens with a higher probability ($x < 0.5$). Specifically, as m goes up initially in the first interval, the buyer will benefit from higher rent saving[24] and efficiency improvement because the procurement quality rises towards the first-best level. In the second interval, efficiency improve-

[21] In this case, two rents offset each other exactly and the buyer is paying zero rent if the inefficient firm is corrupted,

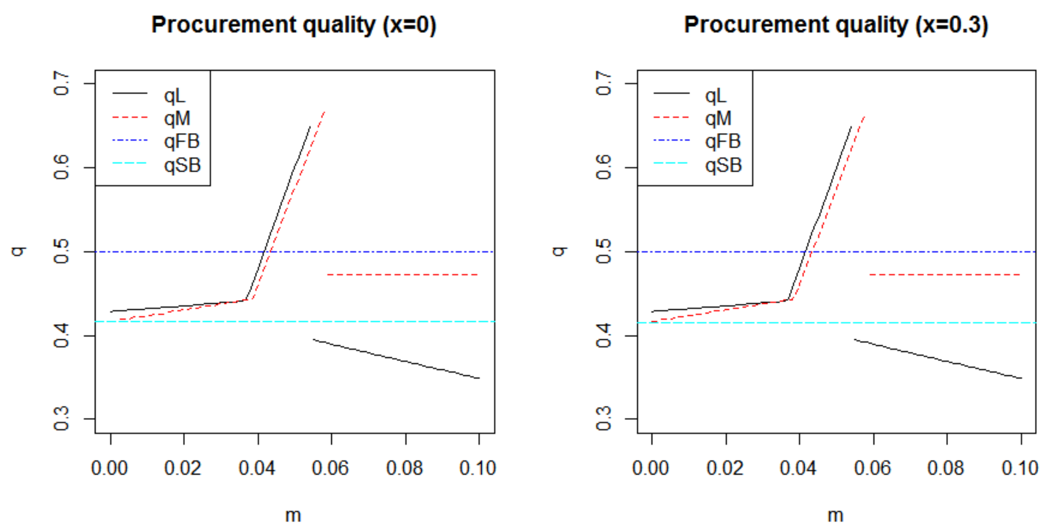
[22] The procurement quality in L $q_1(\tilde{\alpha}) = \frac{2\theta m}{\theta-1} > q_{FB}$ ($\tilde{\alpha} > \alpha_{FB}$), when $m > \frac{\theta-1}{4\theta}$. In M $\tilde{q} = \frac{m(\theta+\sqrt{\theta})}{\theta-1} > q_{FB}$ when $m > \frac{\theta-1}{2(\theta+\sqrt{\theta})}$.

[23] The procurement quality is $q_1(\tilde{\alpha}) = \frac{2\theta m}{\theta-1}$ in L and $\tilde{q} = \frac{m(\theta+\sqrt{\theta})}{\theta-1}$ in M. Provided $\theta > 1$, $2\theta > (\theta+\sqrt{\theta})$, so $q_1(\tilde{\alpha}) > \tilde{q}$. Concerning the size of distortion, when $m > \frac{\theta-1}{3\theta+\sqrt{\theta}}$, $|q_1(\tilde{\alpha}) - q_{FB}| > |\tilde{q} - q_{FB}|$.

[24] The rent paid by the buyer is $R_T^L - R_C^L$ ($R_T^M - R_C^M$). Because R_T^L (R_T^M) does not depend on m and R_C^L (R_C^M) is rising in m, a larger m makes the difference smaller.

ment disappears when the buyer over-reports quality preference. The benefit from rent saving cannot cover the loss from upward quality distortion when m becomes large. Therefore, the buyer is better off under corruption with small m and $x < 0.5$, as shown in Result 3 (Theorem 3).

Figure 1.6: Procurement Quality Comparison



1.5 Extensions

1.5.1 More Than Two Firms

Suppose there are total n firms in the auction and each firm's type θ is drawn independently from $F(\theta)$. Label firms' identities by their the ranking of their types: $\theta_1 < \theta_2 < \cdots < \theta_n$, where a smaller θ indicates higher efficiency according to Assumption CF. The probability of firm i being matched to the agent and forming a corruption relation is denoted as x_i. Denote $\overrightarrow{x} = (x_1, x_2, ..., x_n)$ and $\sum_{i=1}^{n} x_i = 1$.

When firm 1 is corrupted, it wins for sure and the buyer pays the summation of technological rent and corruption rent. When firm $j \neq 1$ is corrupted, the winner depends on the relative size of corruption rent of firm j and technological rent of firm 1. Therefore, by treating firm j as the inefficient firm in the benchmark model, the equilibrium of the benchmark model can be directly applied. The winner will still be either the most efficient firm or the corrupted firm.

When there are more firms, competition reduces the size of technological rent of firm 1. The efficient firm's technological rent decreases in n because the difference of θ_1 and θ_2 decreases in n.[25] On the other hand, the magnitude of corruption rent is not sensitive to the change of n.[26] Therefore, it is more difficult to deter corruption by adjusting the procurement scheme when there are more firms. The buyer is more likely to procure from a corrupted firm and received a project with manipulated quality. As a result, encouraging competition may not reduce corruption or increase the buyer's payoff. The optimal procurement scheme and prediction on the buyer's payoff will depends on $\overrightarrow{x}$ and the scope of corruption. In general, the effect of increasing n on the buyer's payoff is ambiguous.

There is a body of literature studying the relation between competition and corruption in procurement auction. It is commonly believed that increasing competition is a way to reduce corruption, for example Rose-Ackerman (1996), Ades and Tella (1997), and Ades and Di Tella (1999). In contrast, Celentani and Ganuza (2002) show the opposite that competition may not reduce corruption. In the model of Li and Xu (2016), the agent control the invitation of firms. Depending on the specific form of bribery, Li and Xu (2016) show that corruption may or may not result in inviting less firms into the auction. Whether competition reduces corruption is an important policy question. In practice, there are lots of regulation aiming for encouraging competition and reducing entry barrier. For example, it is common to see centralized procurement market, public announcement of procurements information, requirement of minimum number of firms, compensation for entry, and so on. However, as shown in this paper, competition reduces the efficient firm's technological rent and may allow the corrupted firms win more contracts.

[25]The technological rent depends on the difference between the highest order statistic $\theta_{(1:n)}$ and the second highest order statistic $\theta_{(2:n)}$, which is decreasing in n.

[26]The corruption rent is αm under L and $C(q,\theta_j) - C(q-m,\theta_j)$ under M.

1.5.2 Incomplete Information on Costs

The benchmark model assumes that firm can observe the opponent's cost (type θ). If θ becomes private information to each firm, the model turns into a standard auction model. When there is no corruption, the most efficient firm wins the contract for sure at Bayesian Nash equilibrium and receives a technological rent. When there is corruption, the auction outcome again depends on the relative magnitude of the technological rent of the efficient firm and the corruption rent of the corrupted firm. Therefore, the main intuition of the complete information (among firms) can be applied to the case with incomplete information. In Chapter 2, we show that the corruption rent can be reflected by increment of the firm's *pseudotypes* (defined in Asker and Cantillon (2008)), which is the total social surplus of the firm at its equilibrium quality. Asker and Cantillon (2008) show that with a quasilinear scoring rule, there is no loss of generality to consider each firm bidding according to its pseudotype and the equilibrium has a closed form. However, expressing the buyer's expected payoff and deriving the optimal scoring rule are difficult and beyond the scope of this paper.

1.5.3 Incomplete Information on the Corruption Relation

Introducing incomplete information on the corruption relation is not trivial because we need to spell out each agent's belief on all other agents' likelihoods of being corrupted. Given the uncertainty about its opponent, the efficient firm may choose to beat its opponent no matter the opponent is corrupted or not, or it may choose to only beat an uncorrupted opponent. The equilibrium strategy is usually a mixed strategy of these two choices.

To see this, consider the same procurement auction game in Section 1.3. Assume that there is at most one corrupted firm. The agent and the corrupted firm knows their corruption relation, but the buyer and the uncorrupted firm do not know the corruption relation for sure. Let $(1,0)$, $(0,1)$ and $(0,0)$ denote three state in state space of corruption relation, the information structure is described in the following table

	State Indicator	Probability of being corrupted		
		Buyer's belief	Firm 1's belief	Firm 2's belief
Firm 1 is corrupted	$(1,0)$	x_1	1	$\frac{x_1}{1-x_2}$
Firm 2 is corrupted	$(0,1)$	x_2	$\frac{x_2}{1-x_1}$	1
No corruption	$(0,0)$	$1-x_1-x_2$	$\phi \equiv \frac{1-x_1-x_2}{1-x_1}$	$\frac{1-x_1-x_2}{1-x_2}$

The buyer believes that firm 1 and firm 2 are corrupted with probability x_1 and x_2 respectively. When $x_1+x_2=0$ or $x_1+x_2=1$, the model reduced to the one analyzed in Section 1.3. When $x_1+x_2 \in (0,1)$, the buyer believes that there is no corruption with probability $1-x_1-x_2$. If firm 1 knows he is not corrupted, it believes that firm 2 is corrupted with probability $\frac{x_2}{1-x_1}$ and firm 2 is not corrupted with probability $\phi \equiv \frac{1-x_1-x_2}{1-x_1}$. Denote the technological rent of firm 1 as $R_T^L = \alpha(q_1-q_2)-c_1+c_2$ and $R_T^M = C_1(q)-C_2(q)$. The equilibrium with incomplete information on corruption relation is described as follows.

Theorem 5: *Under linear scoring rule $S(q,p)=\alpha q-p$, the equilibrium quality follows $q_i = \arg\max_q \alpha q - C(q,\theta_i)$, $i=1,2$. The equilibrium outcomes and prices are listed in the following table:*

Parameter Values	State	Price	Outcome
$\alpha m \le (1-\phi)R_T^L$	Firm 1 is corrupted	$p_1(\alpha)=C_1(\alpha)+R_T^L-\alpha m$	Firm 1 wins
	Firm 1 is not corrupted	$p_1(\alpha)=C_1(\alpha)+R_T^L+\alpha m$	
	Firm 2 is corrupted	$p_2(\alpha)=C_2(\alpha)$	
	Firm 2 is not corrupted	$p_2(\alpha)=C_2(\alpha)$	
$\alpha m > (1-\phi)R_T^L$	Firm 1 is corrupted	$p_1(\alpha)=C_1(\alpha)+R_T^L+\alpha m$	Firm 1 wins
	Firm 1 is not corrupted	$p_1(\alpha)\sim f_L(p_1)$	Uncertain
	Firm 2 is corrupted	$p_2(\alpha)\sim g_L(p_2)$	Uncertain
	Firm 2 is not corrupted	$p_2(\alpha)=C_2(\alpha)$	Firm 1 wins

$f_L(p_1)$ is some density function with support $[C_1(\alpha)+\phi R_T^L, C_1(\alpha)+R_T^L)]$ and $g_L(p_2)$ is some density function with support $[C_2(\alpha)-(1-\phi)R_T^L+\alpha m, C_2(\alpha)+\alpha m]$.

Theorem 6: *Under minimum-quality q, the equilibrium outcomes and prices are listed in the following table:*

Parameter Values	State	Price	Outcome
$C_2(q-m) \geq C_1(q) + \phi R_T^M$	Firm 1 is corrupted	$p_1(q) = C_2(q)$	Firm 1 wins
	Firm 1 is not corrupted	$p_1(q) = C_2(q-m)$	
	Firm 2 is corrupted	$p_2(q) = C_2(q-m)$	
	Firm 2 is not corrupted	$p_2(q) = C_2(q)$	
$C_2(q-m) < C_1(q) + \phi R_T^M$	Firm 1 is corrupted	$p_1(q) = C_2(q)$	Firm 1 wins
	Firm 1 is not corrupted	$p_1(q) \sim f_M(p_1)$	Uncertain
	Firm 2 is corrupted	$p_2(q) \sim g_M(p_2)$	Uncertain
	Firm 2 is not corrupted	$p_2(q) = C_2(q)$	Firm 1 wins

$f_M(p_1)$ and $g_M(p_2)$ are some density functions with property that

$$f_M(p_1) = \begin{cases} 1 - \frac{1}{p_1 - C_2(q-m)}, & \text{if } (1-\phi)C_1(q) + \phi C_2(q) \leq p_1 < C_2(q), \\ 1, & \text{if } p_1 = C_2(q), \end{cases}$$

$$g_M(p_2) = \begin{cases} 1 + h - \frac{1}{p_2 - C_1(q)}, & \text{if } (1-\phi)C_1(q) + \phi C_2(q) \leq p_1 < C_2(q), \\ 1, & \text{if } p_1 = C_2(q). \end{cases}$$

In summary, under some combination of parameter values, the outcome of the procurement become uncertain. In particular, when a uncorrupted firm 1 and a corrupted firm 2 cannot beat the other for sure, there exists no pure strategy equilibrium. In Burguet and Che (2004), a similar mixed strategy equilibrium is shown when efficient firm cannot beat a corrupted-and-inefficient firm for sure.

1.6 Chapter Conclusion

In a price-only procurement auction, the efficient firm's (technological) rent is the gap of between its cost and the strongest opponent's cost. In a multi-attribute auction, the model primitive is no longer a private cost distribution, but a cost function define on the domain of quality. The technological rent depends on equilibrium quality choice, which in turn depends on the procurement scheme. Therefore, the buyer can affect the efficient firm's technological rent by the scoring rule or the minimum quality requirement. Che (1993) show that, under proper design of scoring rules, both first-score auctions and second-score auctions implement the optimal mechanism. However, when the environment gets complicated, in general, the optimal mechanism is hard to characterize (e.g. Asker and Cantillon (2010)). In this paper, we introduce quality manipulation corruption which grands a corruption rent to the corrupted firm. The adjustment of procurement scheme now affects both the technological rent and technological rent, so the buyer's optimal mechanism design problem reflects consideration on the sizes of two rents and distortion of quality. The optimal procurement yields three results that are distinct from existing literature. First, the buyer may over-report her preference on quality. Second, a minimum-quality auction may perform better than a scoring auction in certain situation. Third, the buyer may be better off under corruption compared to a corruption-free environment.

According to these findings, there are several policy implications. Firstly, to design a better procurement scheme under corruption requires research and good record of data. Consider the case that the buyer needs to select the best quality weight in the scoring rule. In Lengwiler and Wolfstetter (2006), they suggest reducing the weight of quality when the quality evaluation is subject to corruption. However, using a sub-optimal scoring rule with low quality weight does not provide proper incentive to firms and causes direct efficiency loss. Moreover, a low quality weight reduces the technological rent of the efficient firm, which may increase the winning probability of the inefficient-but-corrupted firms. Therefore, understand the nature of corruption and the scope of quality manipulation are important in design of procurement scheme. The relevant research will require data directly related to corruption, such as observation of side payments and identities of corrupted firms. These data may come from convicted corruption cases. There are significant works on bidding rings using data from legal records, such as Porter and Zona (1993), Porter and Zona (1993), and Asker (2010). But studies on agent-bidder corruption with direct corruption related data are still rare.

Secondly, in almost all countries, public procurements are regulated and selection of procurement schemes is restricted. For example, the Chinese Law of Tender[27] requires that all high-valued

[27] Law of the People's Republic of China on Tenders and Bids (click link for full article in English).

government related projects must go through the open bidding process and evaluated by a three-factor weighted linear scoring rule. Giving more flexibility to the buyers can be beneficial, for example allowing for a large quality weight (Result 1) and minimum-quality auctions (Result 2). The regulation policies shall not take away useful tools from buyers' hands, especially in complicated environments. Bajari et al. (2009) and Tadelis (2012) show a similar result that suggests the buyer can be better off by using some seemingly less competitive procurement schemes, like auctions with pre-qualified bidders and negotiations. By considering *ex post* renegotiation and adaption costs, forgoing the advantage of auctions and using bilateral negotiations may be better for the buyers.

Lastly, in industries with high risk of quality manipulation, eradicating all corruption cases is costly and may even be impossible. In this case, it is more important to investigate large and experienced firms than fringe firms. Large firms are typically more efficient and win more contracts. According to Result 3, allowing some less efficient firm to be corrupted may make it a stronger competitor and reduce the large firm's rent. So the antitrust authority shall spend more resource on large firms and harshly punish them if corruption is found. One practical policy is linking the amount of punishment to the value of contracts the convicted firm has won historically.

Chapter 2

An Empirical Study of Scoring Auctions

Abstract

We propose a structural estimation method of scoring auction data, where bids are multi-dimensional, including a price and several quality attributes. We show that by imposing reasonable restriction on the scoring rule and cost function, each bidder's "pseudotype", cost, and rent can be nonparametrically identified and estimated. The method is applied to a data set of server room construction project procurement auctions. The estimation results provide empirical evidence for the three primary implications of the theoretical scoring auction model: (i) bidder's quality choice shall not depends on the number of bidders; (ii) higher quality weight of the scoring rule induces bids with higher quality; and (iii) most winning bids are high-quality-and-expensive ones. These results can be used to predict the effect of scoring rule adjustment and guide the design of optimal procurement scheme.

2.1 Introduction

This Chapter is an empirical study of scoring auctions. Compared to a standard price-only auction data set, the key feature of scoring auction data is that each bid consist of multiple elements. We study a data set from records of server room construction project procurement auctions and provide key evidences for the theoretical prediction of scoring auctions. We propose a structural estimation method and show how it may improve the procurement scheme based on historical data.

Our theoretical model is constructed based on Che (1993), Asker and Cantillon (2008), and Hanazono et al. (2015). The equilibrium presented in Che (1993) is not directly applicable because it predicts that the auction winner's always submits the highest quality. In reality, the winning bid could be high-quality and expensive, or low-quality and cheap. To accommodate the model prediction and the data, We allow each firm's type to be multi-dimensional. On the other hand, the result in Asker and Cantillon (2008) allows us to transform the problem into bidding according to a one-dimensional pseudotype, which avoids the complexity of multi-dimensional private information. We show that equilibrium cost and total social surplus of each firm can be nonparametrically identified and structurally estimated. Our identification and estimation do not require a parametric cost function and hold under environments with multiple quality attributes.

We apply the method to a series of procurement auctions of server room construction projects in China.[1] The data and estimation results provide empirical evidence for these three key implications of the theoretical model. Firstly, Che (1993) predicts that each firm choose their quality maximizing the total social surplus given its cost function and the scoring rule. We show that the choice of quality is indeed separable from the choice of score because quality choice is uncorrelated to the number of bidders. Secondly, firms have incentive to submit bids with higher quality score if the quality weight (slope) of the scoring rule increases. We show this incentive provision effect by the data and quantify it. We find that the project procured by a scoring rule with high quality weight tends to result in higher payoffs for both buyer and firms. Lastly, in scoring auction, people compete more on offering high quality instead of undercutting others by price. We find that

[1] Server room is an indoor place designed to contain machines of data storage, servers, and large computers.

74.24% of winning bids have the highest quality in that auction, while only 4.01% of winning bids have the lowest price.

2.2 Related Literature

We have reviewed the theoretical side literature of scoring auctions in Section 1.2. Let us now turn to the empirical sides. There is a growing literature on the empirical analysis of scoring auctions and other multi-dimensional auctions. In a scoring auction data set, each bid consists of a price and a number of quality attributes. It can potentially answer richer questions than price-only auction data. Lewis and Bajari (2011) explore a highway procurement data set from California generated from "A+B auctions", where bids are evaluated on both price and time of delivery. They show that by introducing time incentive, the overall gain in social welfare is significant. Bajari et al. (2014) analyze another highway procurement data set where each bid consists of a complete list of unit prices. These unit prices are multiplied by quantities estimated by engineers to determine which bid has the lowest cost. Their analysis focuses on the *ex post* adjustment of final payments and how firms strategically reflect potential adaption costs in their bids. Koning and Van de Meerendonk (2014) study data from welfare service provider procurement auctions under weighted scoring rule. They explore how variation of weights on different components affect bids and procurement outcomes.

Nakabayashi and Hirose (2015) study a Japanese scoring auction data set similar to the one in this paper. They provide identification and structural estimation results based on a parametric cost function. This cost function is common knowledge to all except for several parameters as bidder's private information. The identification is based on invertibility conditions of the system of equations obtained by best responses in choosing the multi-dimensional bid. In our analysis, we consider only the class of quasilinear scoring rules, but our cost function is nonparametric.

The study of beauty contests and design-built auctions are also closely related scoring auctions. Krasnokutskaya et al. (2014) study data from online programming service market. They provide an identification and estimation strategy for data that features both auction and discrete choice.

Yoganarasimhan (2013) and Yoganarasimhan (2015) study the online freelancing market where the contract award rule is not pre-announced. They supplying firms offer their price and proposal, while the buyer make decision by considering price, quality, and firms' reputation. She explicitly estimate the value of reputation by the review system of the freelancing platform. Takahashi (2015) study the evaluation uncertainty problem in design-built auctions. The uncertainty dampen difference in firms' chance of winning and he propose a mechanism that asks each bidder to submit a price per unit of design score. He show the new mechanism improve the auction outcome substantially based on structural estimation results.

2.3 Model of Scoring Auctions

A *buyer* (she) seeks procurement of a project that can be delivered at various level of quality $\mathbf{q} \in \mathbb{R}_+^L$. The buyer faces a price-quality tradeoff between cheap-low-quality and expensive-high-quality projects. Setting up a scoring rule $S(p, \mathbf{q}) : \mathbb{R}_+^{L+1} \to \mathbb{R}_+$ reflects the buyer's willingness-to-pay for procuring the project at a higher quality.[2] The scoring rule ranks different price-quality combinations and provide supplying firms incentives to submit desirable project proposals. Che (1993) shows that the buyer will under-report her preference on quality in the optimal scoring rule, but if she lacks commitment power, the only feasible scoring rule is the one that reflects her true preference. In this paper, because researchers usually only observe the score but not buyer's "payoff" empirically, we put aside the buyer's optimal scoring rule design problem and simply treat $S(p, \mathbf{q})$ as her objective function. We focus on the firm's equilibrium bidding behavior and implication of quality manipulation with the goal of conducting an empirical study.

After a scoring rule $S(p, \mathbf{q})$ is announced, suppose there are n symmetric risk neutral supplying *firms* (they/it) enter the auction exogenously, indexed by $i = 1, 2, ..., n$. A generic firm i's type (private information) is a vector of efficiency parameter $\theta \in \mathbb{R}^M$, drawn independently from an identical distribution F. F is absolutely continuous and has density $f = F'$ with support $[\underline{\theta}, \overline{\theta}] \subset \mathbb{R}^M$. Firm i with type θ_i pays a cost $C(\mathbf{q}, \theta_i)$ if it delivers the project with quality $\mathbf{q}$. Provided the

[2] Dini et al. (2006) provides a practical survey on selecting scoring rules.

scoring rule, each firm submits its sealed bid as a price-quality combination. If the firm wins the contract with bid $(p, \mathbf{q})$, its payoff is $\pi(p, \mathbf{q}; \theta_i) = p - C(\mathbf{q}, \theta_i)$. Firm's payoff is normalized to zero if it does not win the contract. Submitted bids are evaluated according to $S(p, \mathbf{q})$ and the firm with the highest score wins the contract. We only consider first-score auctions (FSA) and independent private information framework in this thesis.

When the buyer does not possess the expertise to evaluate quality of submitted bids, she hires an *agent* (he) to evaluate them. In Chapter 2, we consider the competitive model as benchmark. The agent is honest and report the true quality of proposals of submitted bids. Because both the firms and the agent are experts in the industry, they agree on how to evaluate quality. Therefore, except for some tiny uncertainty, firms are effectively choosing quality evaluation scores of their proposals and the agent verifies these quality scores for the buyer.

We restrict our attention to cost functions and scoring rules satisfying the following two assumptions throughout Chapter 2 and 3.

Assumption CF': For all $\mathbf{q} \in \mathbb{R}^L$, $C(\mathbf{q}, \theta)$ is continuous, $C(\mathbf{q}, \theta) \geq 0$, $\frac{\partial C(\mathbf{q}, \theta)}{\partial \mathbf{q}} \gg 0$, and $\frac{\partial^2 C(\mathbf{q}, \theta)}{\partial \mathbf{q}' \partial \mathbf{q}}$ is positive definite.

Assumption QL: The scoring auction uses a quasilinear scoring rule $S(p, \mathbf{q}) = V(\mathbf{q}) - p$, where $V(\mathbf{q})$ is increasing, continuously differentiable, and weakly concave.[3]

With these two assumption, we have the following lemma:

Lemma 5: *Consider a scoring auction with scoring rule $S(p, \mathbf{q}) = V(\mathbf{q}) - p$, where $\mathbf{q} \in \mathbb{R}_+^L$ and $L > 1$. For a firm with cost function $C(\mathbf{q}, \theta)$, it is equivalent to consider it bids according to a transformed cost function $\tilde{C}(v, \theta) \equiv \min_{\text{s.t. } V(\mathbf{q}) = v} C(\mathbf{q}, \theta)$, $v \in \mathbb{R}_+$. $\tilde{C}(v, \theta)$ is single-valued, continuous, increasing, and strictly convex in v.*

Given Lemma 5, there is no loss of generality to reduce the dimensionality of quality attributes to one. For notational convenience, we will consider the cost function and the scoring rule as

[3] Any scoring rule that is additively seprarable in price can be transformed into a quasilinear one without affecting the equilibrium bidding strategy.

$C(q,\theta)$ and $S(p,q)$ respectively, where q is one-dimensional. The firm's profit maximization problem is

$$\max_{p,q}\left[p-C(q,\theta)\right]\Pr\left(\text{win}|S(p,q)\right). \tag{2.1}$$

As shown in Che (1993), the quality choice can be separated from score or price choice. Each firm will choose its quality according to

$$q(\theta)=\arg\max_{q}\left\{V(q)-C(q,\theta)\right\}. \tag{2.2}$$

The proof of (2.2) will be show under a more general setting at Theorem 8. Assumption CF and QL ensure $q(\theta)$ to be a single-valued continuous function by the Maximum Theorem (Berge, 1963). The firm's problem (2.1) is equivalent to a two-step optimization problem where the firm first chooses its score s, then choose a (p,q) combination to fulfill that score.

$$\begin{aligned}(2.1) \quad &\Leftrightarrow \quad \max_{s}\left\{\max_{(p,q)\text{ s.t. }S(p,q)=s}\left[p-C(q,\theta)\right]\Pr\left(\text{win}|s\right)\right\}\\ &\Leftrightarrow \quad \max_{s}\left\{\max_{q}\left[V(q)-s-C(q,\theta)\right]\Pr\left(\text{win}|s\right)\right\}\\ &\Leftrightarrow \quad \max_{s}\left\{\left[V(q(\theta))-C(q(\theta),\theta)-s\right]\Pr\left(\text{win}|s\right)\right\}. \end{aligned} \tag{2.3}$$

At the second and third step, we plug in $p=V(q)-S(p,q)$ and (2.2) respectively. Following Asker and Cantillon (2008), we define the *pseudotype*[4] of a firm as the value function

$$K(\theta) \equiv \max_{q}\left\{V(q)-C(q,\theta)\right\}=V(q(\theta))-C(q(\theta),\theta), \tag{2.4}$$

which is the *total social surplus* of a firm (if $S(p,q)$ represents the true payoff of the buyer). Again by the Maximum Theorem, $K(\theta)$ is a single-valued and continuous function. The distribution of pseudotype K can be obtained from the (joint) distribution of θ by probability transformation

[4] It is called effective cost in Hanazono et al. (2015) and productive potential in Che (1993).

formula:

$$F_K(k) = \Pr(K(\theta) \leq k) = \Pr(\theta \in D_{\{\theta : K(\theta) \leq k\}}) = \int_{\theta \in D} f(\theta) d\theta. \tag{2.5}$$

Denote $\underline{k} = \min_{\theta \in [\underline{\theta}, \overline{\theta}]} K(\theta)$ and $\overline{k} = \max_{\theta \in [\underline{\theta}, \overline{\theta}]} K(\theta)$. We assume $\underline{k} \geq 0$ so that the least efficient firm participates. The support of pseudotype is $[\underline{k}, \overline{k}] \subset \mathbb{R}_+$. Asker and Cantillon (2008) show that pseudotypes are sufficient statistics to describe the equilibrium of scoring auctions under quasilinear scoring rules. Therefore, instead of dealing with the multi-dimensional type θ, it is equivalent to consider that firms draw their one-dimensional pseudotype k from distribution F_K. Problem (2.3) can then be further rewritten as if the firm is selecting its score based on its pseudotype:

$$\max_s (k - s) \Pr(\text{win}|s). \tag{2.6}$$

Directly from Asker and Cantillon (2008), we have the following theorem and two corollaries:

Theorem 7: *Every equilibrium in a scoring auction where firms bid on the basis of their types θ, is type-wise outcome equivalent to an equilibrium in the scoring auction where firms are constrained to bid only on the basis of their pseudotypes $k = K(\theta)$. Firm with type θ and pseudotype $k = K(\theta)$ bids its quality according to* (2.2) *and score according to*

$$s(k) = k - \frac{\int_{\underline{k}}^{k} [F_K(t)]^{n-1} dt}{[F_K(k)]^{n-1}}. \tag{2.7}$$

The corresponding price is $p(\theta) = V(q(\theta)) - s(K(\theta))$.

Corollary 7.1: *The conditional expectation of the winner's score equals to the strongest rival's pseudotype, i.e., $E\left[s(k_{(1:n)})\right] = E\left[k_{(2:n)}\right]$.*

Corollary 7.2: *The buyer receives a higher expected utility in a scoring auction than a price-only auction with minimum quality standards.*

Corollary 7.1 is the expected utility equivalence of FSA and SSA, which is similar to the revenue equivalence principle in Vickrey (1961). Corollary 7.2 is describing the superiority of scoring auctions. Throughout the paper, $X_{(j:n)}$ denotes the jth highest order statistic from an i.i.d.

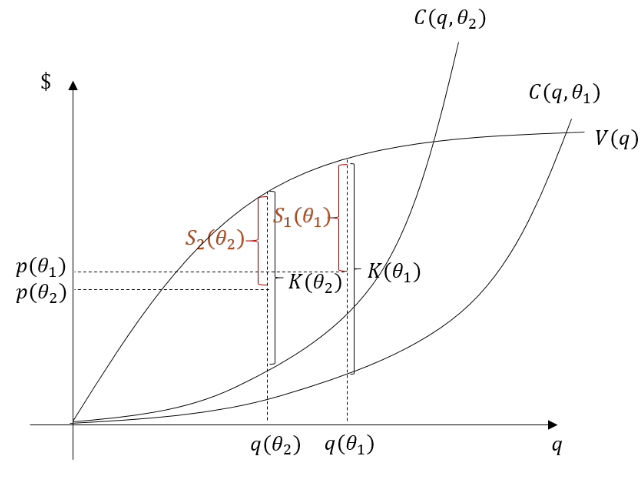
sample of size n from distribution F_X. The distribution function of order statistic $X_{(j:n)}$ is denoted as $F_X^{(j:n)}$. If θ is one-dimensional, the equilibrium in Theorem 7 reduced to the one in Che (1993):

Lemma 6: *When θ is one-dimensional and $C_\theta > 0$, $C_{q\theta} > 0$, there is a symmetric Bayesian Nash equilibrium of a first-score auction where each firm with type θ submits its bid as $q(\theta) = \arg\max_q V(q) - C(q,\theta)$ and $p(\theta) = C(q(\theta),\theta) + \int_\theta^{\overline{\theta}} C_\theta(q(t),t)[1-F(t)]^{n-1}dt/[1-F(\theta)]^{n-1}$.*

However, using a one dimensional θ implies the monotonicity property of $q(\theta)$. The existence of the equilibrium requires the assumption $C_{q\theta} > 0$ and Topkis Theorem (Topkis, 1978) Theorem immediately implies that $q'(\theta) < 0$. To see this, because $q(\theta)$ satisfies first-order condition $V_q(q) - C_q(q,\theta) = 0$, by implicit function theorem, $q'(\theta) = C_{q\theta}/(V_{qq} - C_{qq}) > 0$ under assumption CF', QL, and $C_{q\theta} > 0$. This monotonicity feature implies that the most efficient firm with lowest θ always wins by submitting the highest quality. It does not fit real world data because some contracts are awarded to firms offering cheap-low-quality bids. Therefore, we drop the assumption $C_{q\theta} > 0$ and assume θ to be at least two dimensional. In this way, we relaxes the monotonicity property of quality, which is shown later in the Monte Carlo Example.

In summary, the equilibrium of a competitive scoring auction has three implications that can be tested empirically. First, in choosing the quality, firm only consider the scoring rule and its cost function, but not how many rivals are competing with him. So the number of bidders in the auction shall not affect the choice of quality, but affect the choice of score or price. Second, according to (2.2), a higher slope of $V(\cdot)$ induces firms to bid higher quality. Lastly, because $V(\cdot)$ is the upper bound of pseudotype (total social surplus), firms endogenously choosing high quality in general have higher pseudotypes and win the contract with larger probability. Therefore, in a scoring auction, the competition of firms is mainly reflected on the quality dimension instead of undercutting each other by price. Although the quality choice is not monotone in pseudotype, we expect to see a majority of winning bids are of the high quality and relative high price, instead of cheap-low-quality ones. In the empirical study, we provide evidence for each of these three model implications.

Figure 2.1: Illustration of the Equilibrium of Scoring Auction



2.4 Econometrics of Scoring Auctions

Consider a sample of T independent and repeated scoring auctions of the same industry with the same scoring rule.[5] For scoring auction t, assume researchers observe the number of firms n_t, some auction-specific covariates z_t (with dimension d), bids of each firm $\{p_{it}, q_{it1}, q_{it2}, ..., q_{itL}\}_{i=1}^{n_t}$ (with dimension $L+1$) and scores $s_{it} = V(q_{it1}, q_{it2}, ..., q_{itL}) - p_{it}$. We set aside endogenous entry and reserve price issues in this paper. By result in Theorem 7, the identification result can be established by the method in Guerre et al. (2000).

Theorem 8: *Under assumption CF' and QL, pseudotypes and equilibrium costs of firms are non-parametrically identified.*

Proof: Because the equilibrium bidding strategy in Theorem 7 is monotone, $G_S(s) = \Pr(S \leq s) = \Pr(K \leq k) = F_K(k)$ and $g_S(s) = f_K(k)/s'(k)$. By (3.6), pseudotype k is identified from the observation of scores via

[5] We will discuss variations of scoring rules later in the empirical application section.

$$k \quad = \quad s(k)+s'(k)\frac{F_K(k)}{(n-1)f_K(k)} = s+\frac{G_S(s)}{(n-1)g_S(s)}, \tag{2.8}$$

The equilibrium cost is then identified by the definition of pseudotype,

$$C(q(\theta),\theta) \quad = \quad V(q(\theta))-k = p(\theta)-\frac{G_S(s)}{(n-1)g_S(s)}. \tag{2.9}$$

Q.E.D.

Given observations of n_t, z_t, and s_{it}, the conditional distribution function and density of score can be estimated by kernel estimators,

$$\begin{aligned} \hat{G}_S(s|n_t,z_t) &= \frac{1}{Th_1h_2^d}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}\mathbb{I}(s\leq s_{it})\kappa_G\left(\frac{n-n_t}{h_1},\frac{z-z_t}{h_2}\right), \\ \hat{g}_S(s|n_t,z_t) &= \frac{1}{Th_3h_4h_5^d}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}\kappa_g\left(\frac{s-s_{it}}{h_3},\frac{n-n_t}{h_4},\frac{z-z_t}{h_5}\right). \end{aligned}$$

We use Gaussian kernels and select bandwidths $h_1,...,h_5$ by least-square cross validation. Pseudotypes and equilibrium costs at corresponding quality are estimated by

$$\hat{k}_{it} \quad = \quad s_{it}+\frac{\hat{G}_S(s_{it}|n_t,z_t)}{(n_t-1)\hat{g}_S(s_{it}|n_t,z_t)}, \tag{2.10}$$

$$\hat{c}_{it} \quad = \quad V(q_{it})-\hat{k}_{it}. \tag{2.11}$$

Equation (2.10) shows that the firm chooses a score s as the portion delivered to the buyer from total social surplus k. The second term is the firm's *rent*, reflecting its competitive advantage and information rent.

Notice that in a price-only auction, quality of the target item is fixed, so the model primitive is a cost distribution, which can be identified by the method in Guerre et al. (2000). But in a scoring auction, the model primitive is the cost function defined on the domain of quality attributes. Costs estimated via (2.11) are not randomly drawn from a fixed cost distribution, but rather chosen by firms. Therefore, to fully identified the model, one must make a parametric assumption on cost function. Nakabayashi and Hirose (2015) show the condition for parametric identification. In our study, testing corruption does not require fully identifying the cost function, so we take the nonparametric approach.

Monte Carlo Example

Suppose the scoring rule is $S(q,p) = 2q - p$ and cost function is $C(q,\theta) = \theta_0 + q^2/\theta_1$. Each firm draws its two-dimensional type $(\theta_0,\theta_1) \equiv \theta$ independently from Uniform[0,1] and Uniform[1,2], respectively. Assume that θ_0 and θ_1 are independent, so their joint density equals to one on the support $[0,1]\times[1,2]$. By Theorem 7, the optimal quality choice of a firm with type θ is $q(\theta) = \theta_1$ and its pseudotype is

$$K(\theta) = V(q(\theta)) - C(q(\theta),\theta)) = 2\theta_1 - \theta_0 - \frac{\theta_1^2}{\theta_1} = \theta_1 - \theta_0.$$

The support of k is $[0,2]$. Because θ_0,θ_1 are jointly uniformly distributed with density equals 1 at the area $[0,1]\times[1,2]$., the distribution of pseudotype can be derived by (2.12) . We obtain Figure 2.2 by $F_K(k) = \Pr(K(\theta_0,\theta_1) < k) = \Pr(\theta_1 - \theta_0 < k) = \Pr(\theta_1 < k + \theta_0)$. When $k \in [0,1]$,

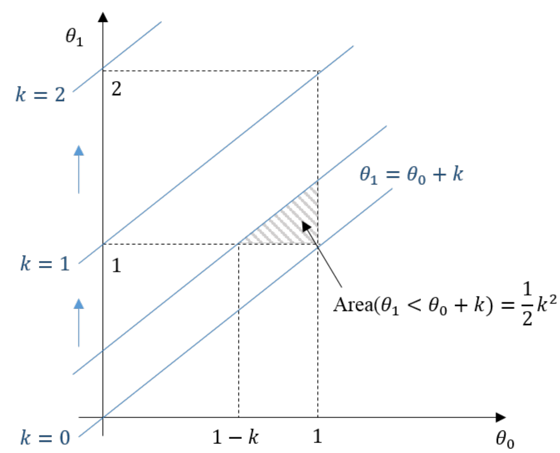
$$\begin{aligned} F_K(k) &= \int_{1-k}^{1}\int_{1}^{\theta_0+k} 1 d\theta_1 d\theta_0 = \int_{1-k}^{1} (\theta_0 + k - 1)\, d\theta_0 \\ &= \left[\frac{1}{2}\theta_0^2 + (k-1)\theta_0\right]_{1-k}^{1} = \frac{1}{2} + (k-1) - \frac{1}{2}(1-k)^2 - (k-1)(1-k) = \frac{k^2}{2}. \end{aligned}$$

When $k \in (1,2]$,

$$\begin{aligned} F_K(k) &= 1 - \int_{0}^{2-k}\int_{\theta_0+k}^{2} 1 d\theta_1 d\theta_0 = \int_{0}^{2-k} (2 - \theta_0 - k)\, d\theta_0 \\ &= \left[(2-k)\theta_0 - \frac{1}{2}\theta_0^2\right]_{0}^{2-k} = 1 - (2-k)^2 + \frac{1}{2}(2-k)^2 = 1 - \frac{(2-k)^2}{2}. \end{aligned}$$

$$F_K(k) = \Pr(\theta_1 - \theta_0 < k) = \begin{cases} \frac{k^2}{2}, & \text{for } k \in [0,1], \\ 1 - \frac{(2-k)^2}{2}, & \text{for } k \in (1,2]. \end{cases} \tag{2.12}$$

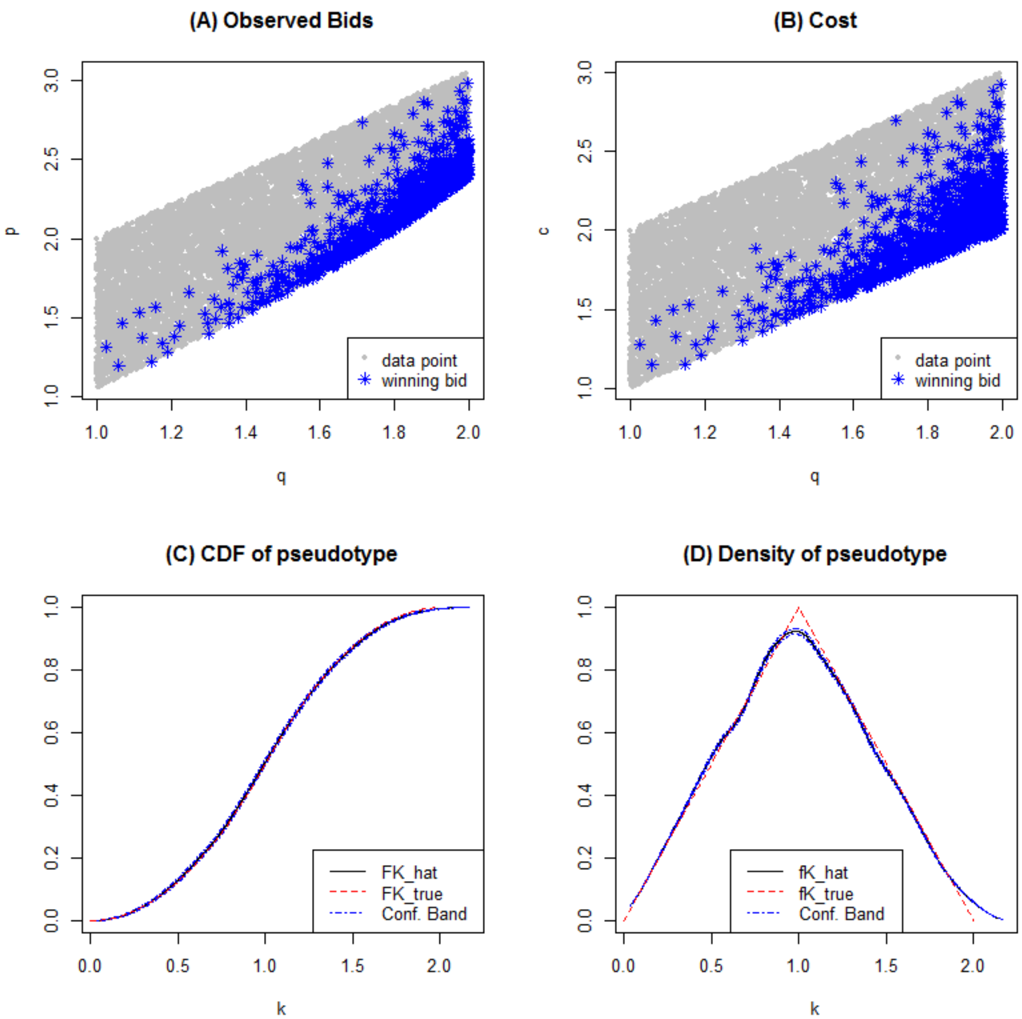
Figure 2.2: Derivation of $F_K(\cdot)$



Notice that, by allowing types to be two-dimensional, firm who submits a high equilibrium quality does not necessarily has a high pseudotype. For example, when firm 1 is type $(0.5, 1.5)$ and firm 2 is type $(0.1, 1.2)$, firm 1 will produce at $q = 1.5$ and have pseudotype $k = 1$; firm 2 will produce at a lower level $q = 1.2$ but have a higher pseudotype $k = 1.1$. The numbers of firms n are randomly draws from 3 to 20 with equal probability. Using (2.7), we can generate a simulated data set and apply our estimator (2.10) and (2.11), as illustrated in Figure 2.3. The estimation is based on 1000 auctions.

In this example, if researchers know the parametric form of the cost function, they can identify two structural parameters by conditions of optimal quality and score choice: $\theta_1 = q$ and $\theta_0 = q - s - G(s|n)/[(n-1)g(s|n)]$. In general, as long as $K(\theta)$ is monotone in θ under the parametric assumption, θ is identified. In application to an actual data set, determining the parametric family of cost function is usually difficult.

Figure 2.3: Illustration of Data and Estimation



2.5 Data and Server Room Construction Industry

Our data set comes from two major procurement platforms: Guangzhou Public Resource Trading Center and Public Resources Trading Center in Guangdong Province.[6] Nearly all procurements conducted in these two trading centers are sealed-bid scoring auctions due to both legal requirements and their economic advantages. The Chinese Law of Tender[7] requires government related

[6]Website: http://gzggzy.cn/, http://www.bcmegp.com/. Starting November 2009, these two major procurement platforms publicly announced auction results of all government related projects.

[7]Law of the People's Republic of China on Tenders and Bids (click this link for its full article in English).

projects with values over certain thresholds to go through the open tender process coordinated by these trading centers. The law also provides guidelines to forming evaluation committees, selecting industrial experts, designing of scoring rules, and the detailed process of auction. Besides public sector, private sector buyers also use these two trading centers frequently because trading centers have connections to a large pool of industrial experts that perform bid evaluations.

The data set covers a series of procurement auctions of server room construction projects. Server room (or data center) is an indoor place designed for containing machines like data storage, servers, and large computers. To ensure reliability and safety, the construction of server rooms have detailed technical requirements on various aspects like temperature, humidity, electricity supply, fire control, etc.. Evaluating quality of a server room construction proposal needs specific expertise. Each bid contains a full construction proposal and a itemized price list. Firm's reputation, experience, certificate, and financial status need to be considered in the bids evaluation. Therefore, compared to lands or cargo, server room construction procurements are subject to higher risks of quality manipulation.

During the two year period (01/01/2012 to 12/31/2013) of our data set, there are total 2147 observed auctions. On average, 8.8 bidders enter and submit valid bids for each auction. The summation of engineer's estimated costs of all observed projects is over 10 billion CNY (1.6 billion USD). Hence, the industry is both large and has enough observations for structural estimations. For each project, our information includes its engineer's estimated cost, number of bidders, factor weights, city, the buyer's name, and the winning firm's identity. On bid level, we observe factor scores of each bid. Table 2.1 and Figure 2.4 summarize the data set. All price data are in units of 1,000 CNY. Transformed quality and transformed score are defined later in this section.

Several remarks about the data set:

(1) Our data set contains much less firm-specific covariates than those in Porter and Zona (1993) and Bajari and Ye (2003). Among the 1046 winning firms, 451 firms win only one contract. Therefore, tracking firm's bidding history to construct variables like "backlog", "capacity", or

Table 2.1: Descriptive Statistics of the Data

Variables	Obs.	Mean	SD	Min.	Max
Project/Auction-specific					
Engineer's estimated cost, p_0	2,147	5,049.90	1,478.49	835	13,239
Weight on tech. factor, w_p	2,147	0.4958	0.0627	0.4	0.55
Weight on price factor, w_q	2,147	0.4042	0.0627	0.35	0.5
Weight on business factor, w_r	2,147	0.1	0	0.1	0.1
Number of firms, n	2,147	8.832	3.857	3	36
Winning score, s	2,147	78.550	6.068	52.49	95.13
Project city	2,147	(21 cities in Guangdong province)			
Bid-specific					
Price factor score, s_p	18,963	69.80	10.43	1.0881	100
Tech. factor score, s_q	18,963	60.47	28.55	0	100
Business factor score, s_r	18,963	72.30	10.15	29	100
Price, p	18,963	4,162.96	1,843.70	363.3	18,417.38
Savings rate, $\rho = \frac{p_0 - p}{p_0}$	18,963	0.1980	0.1044	-0.4891	0.50
Weighted score, s	18,963	66.50	11.10	26.98	95.1337
Transformed quality, $\tilde{q}$	18,963	8,260.86	2,915.77	1,140.24	29,135.76
Transformed score, $\tilde{s}$	18,963	4,097.89	1,481.16	712.09	11,559.27

"utilization rate" is not practical.[8] In addition, we do not observe the identities of all losing firms, so we cannot construct explanatory variables like rival firm's distance or rival capacity.

(2) The market structure of this industry is relative simple. There is a large number of supplying firms and no buyers or firms dominates the industry. Table 2.3 show that the largest firm only takes a 1% market share. Also, because server room project design and construction costs are not much affected by their geographic location, combining data from different cities is reasonable. Moreover, subcontracting is common in this industry, a firm's distance to the project is less important when most components of the project are carried out by subcontractors. These features support the independent private information setting of our model.

(3) The scoring rule of this data set is relative easy to analyze. The business factor weights is constant at 0.1 across all projects and (w_p, w_q) combination takes only five pairs of values.[9] Hence, the variation of the scoring rule can be controlled by w_q. The price factor evaluation rule

[8] See Porter and Zona (1993), Section IV for definitions of backlog, capacity, and utilization rate.

[9] The law of tender requires all construction projects shall economic factor weight $w_p \geq 0.4$.

Table 2.2: Summary of Market Structure

	Market Share		Number of Projects			
	Mean	H.H. index	Mean	SD	Min	Max
Firms	0.0956%	0.0015	2.0525	1.6044	1	22
Total= 1046						
No. of firm wins one project= 451						
Buyers	1.1236%	0.0659	24.1236	53.4929	1	292
Total= 89						
No. of buyer procures one project= 10						
No. of buyer procures less than 10 project= 50						

is consistent and not interdependent. In our sample, the engineer's estimated costs and prices of the bid are transferred into a 100 point price factor score by formula (2.13) below. We will now discuss the scoring rule in details.

All projects in our data set use the most popular "comprehensive evaluation method" in China. It is simply a linear weighted scoring rule consisting of three components: an economic factor, a technical factor, and a business factor. The e*conomic factor* (s_p) evaluates the price of the bid.[10] Denote the *engineer's estimated cost* as p_0, if a company submits price p in his bid, his economic factor score is computed by

$$s_p = \begin{cases} 0, & p > \frac{3}{2}p_0, \\ 100\left(\frac{1}{2}+\rho\right) = 100\left(\frac{1}{2}+\frac{p_0-p}{p_0}\right), & \frac{1}{2}p_0 \le p \le \frac{3}{2}p_0\,, \\ 100, & p < \frac{1}{2}p_0. \end{cases} \tag{2.13}$$

$\rho = \frac{p_0-p}{p_0}$ is called the *savings rate* of that bid. The *technical factor* (s_q) evaluates quality of the construction proposal including the design, building standard, equipment, server machines, follow-up service, warranty, delivery date, payment condition, insurance, etc.. The *business factor* (s_r) evaluates the firm's reputation, experience, risk of default, risk of bankruptcy etc.. Technical

[10] As mentioned in Bajari et al. (2014), in reality price evaluation is not just "the low the better". A highly unbalanced bid (extreme itemized price) or a bid lower than cost could be penalized or rejected.

Table 2.3: Summary of Market Structure

Index	No. of Project	Market Share	Total Value	Share of Value
Top five firms:				
1	22	0.0102	94,641	0.0087
2	15	0.0070	69,990	0.0065
3	14	0.0065	63,264	0.0058
4	14	0.0065	72,496	0.0067
5	12	0.0056	64,504	0.0059
Top five buyers:				
1	292	0.1360	1,463,947	0.1350
2	273	0.1272	1,370,988	0.1264
3	224	0.1043	1,154,855	0.1065
4	220	0.1025	1,110,387	0.1024
5	117	0.05449	579,093	0.05341
Procurement platforms (agents):				
1	1,466	0.6828	7,374,657	0.6802
2	681	0.3172	3,467,485	0.3198

and business factors are evaluated by a committee of experts.[11] Each bid receives three 100-scale scores on three factors, and then a grand score is computed via

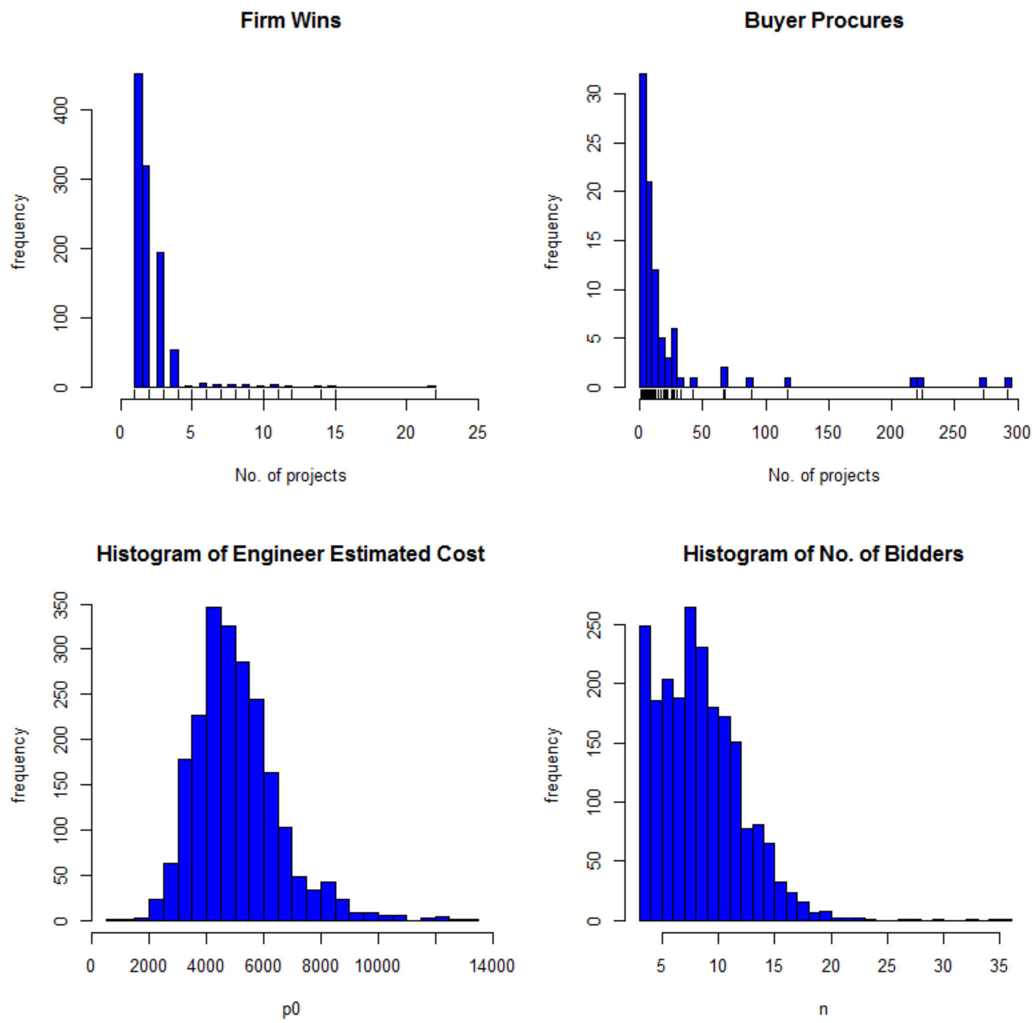
$$S(s_p, s_q, s_r) = w_p s_p + w_q s_q + w_r s_r, \tag{2.14}$$

where weights, w_p, w_q, and w_r, add up to one. The firm who receives the highest 100-scale grand score wins the contract. Because firms are experts of the industry and may have repeated interactions with the agents, they understand the score evaluation process. Therefore, they are effectively selecting technical factor scores by submitting corresponding construction proposals. Business factor scores are also endogenously chosen by firms because firms can hire experienced engineers, acquire relevant certificates, form bidding consortia, and allocate more financial resources to raise s_r.

The linear weighted scoring rule can be transformed into a quasilinear one that reflects the

[11] The law of tender require the committee shall contains five or more members (odd number). There is one representative from the buyer. All the other members are either randomly selected from the pool of experts connected to the procurement agency company.

Figure 2.4: Visualization of Some Variables



same preference of the buyer. Define the *transformed score*, $\tilde{s} \equiv \frac{p_0}{100w_p}\left(S(s_p, s_q, s_r) - 50w_p\right)$ and the *transformed quality*, $\tilde{q} \equiv \frac{p_0}{100w_p}\left(100w_p + w_q s_q + w_r s_r\right)$, then (2.14) can be transformed into

$$\tilde{S}(\tilde{q}, p) = \tilde{q} - p. \tag{2.15}$$

Because the transformation is monotone, firm's winning probability and bidding strategy are not affected. We assume the cost of supplying the transformed quality $\tilde{q}$ satisfies assumption CF. Then the environment satisfies the condition of Lemma 5, which allows us to use the single dimensional quality index $\tilde{q}$ for structural estimation.

Both the original scoring rule (2.14) and the transformed one (2.15) reflect the buyer's willingness-to-pay for a higher quality. The scoring rule determines the *monetary equivalent for quality*.[12] If the firm raises the price by 1 CNY, then the 100-scale grand score will reduce by $100\frac{w_p}{p_0}$. To remain at the same payoff, the buyer needs to be compensated by a higher quality, which requires $w_q \Delta s_q + w_r \Delta s_r = 100\frac{w_p}{p_0}$. We add a boundary condition that the buyer receives zero payoff from a contract with $s_q = 0$, $s_r = 0$, and $p = p_0$. Therefore, both scoring rules (2.14) and (2.15) have are the same monetary equivalent for quality and the same boundary condition. Given (2.15), $\tilde{q}$ represents the benefit of the buyer from a project delivered at (s_q, s_r) , and $\tilde{s}$ as the buyer's *payoff* after compensating the winning firm p. Because the transformed quality and score are anchored on price that has real monetary interpretation, they can be compared across auctions.

2.6 Reduced-form Estimation

We have three main findings in the following reduced-form empirical study:

(1) We test two implications of the theoretical model. First, a higher quality weight (lower price weight) shall induce firms to submit bids with higher quality and higher grand score. Second, firm's choice of quality and price are separated under additively separable scoring rule. By using the *original strategy space* (s_p, s_q, s_r) as the dependent variable, we do not find robust evidence. But we find the evidence supporting both model implications by using the *transformed strategy space* ($\tilde{q}$,$\tilde{s}$), which in turn justifies our use of the transformed strategy space for structural estimations and corruption detection tests.

(2) We tests for unobserved heterogeneity of projects with respect to fringe/non-fringe firms, fringe/non-fringe buyers, and two agents. We do not find strong evidence of unobserved heterogeneity among these projects.

(3) Based on the transformed strategy space, we find that projects with high engineer's estimated costs end up with winning contracts of both high quality scores and high prices. Projects with low engineer's estimated costs induce more competition on price and end up with higher

[12]The definition of monetary equivalent for quality reflected by a scoring rule can be found in Dini et al. (2006).

savings rates (lower markups).

Consider the reduced-form regression model:

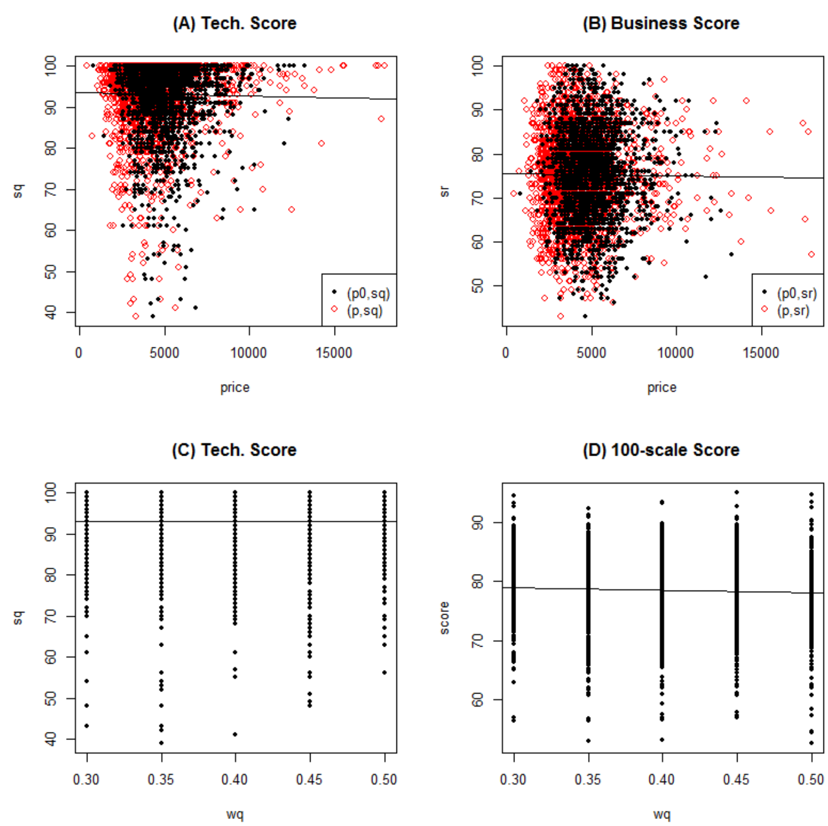
$$Y_t = \alpha_0 + \alpha_1 p_{0,t} + \alpha_2 n_t + \alpha_3 w_{q,t} + \alpha_4 D_{\text{fringe.firm},t} + \alpha_5 D_{\text{fringe.buyer},t} + D_{\text{agent2},t} + \varepsilon_t,$$

where Y_t stands for the dependent variable. The main independent project-specific covariates are engineer's estimated cost and the number of bidders. On bids level, we only observed three factor scores that are endogenously chosen by firms. Price, grand score, transformed quality, and transformed score are all functionally correlated with factor scores. The data also lacks the losing firm's identities, so there is no explanatory variable on bid level. Therefore, we estimate reduced-form models on project level with only winning bids. Because there are lots of firms or buyers that only appear in one project, we do not include firm or project fixed effects in the model. Instead, we add two indicators for fringe firms and fringe buyers: $D_{\text{fringe.firm}} = 1$ if the winning firm is fringe (wins only one project) and $D_{\text{fringe.buyer}} = 1$ if buyer is fringe (procures less than 10 projects). D_{agency2} is the indicator for if the project is process by agent 2.

Figure (2.5) and Table 2.4 summarize the estimation results. Based on the coefficients of technical factor weight, a higher w_q results in a higher technical score, a higher price, and a lower savings rate. Therefore, if a buyer wants to procure the project at a higher quality, the cost will also increase significantly. In other word, firms ask for higher markups in high technical weight procurements. More entry of the procurement auction increases competition and results in positive effects on all five dependent variables. In addition, diagram (A), (B) and regression (I), (II) show that variations of engineer's estimated costs does have significant effects on technical scores and business scores of the winning bids. None of these regressions shows significant differences between the large and fringe firms (buyers). The two agents also appear to be similar.

These regressions with the dependent variable within the observed strategy space have one major drawback: scores on the 100-scale are intangible concepts and hard to compare across auctions. Receiving the same 100 point technical scores may mean completely different things for two projects. We also find that increasing w_q does not significantly increase grand scores in regression (IV), which is not consistent with theoretical model prediction. In addition, the goodness-of-fit, measured by R^2, are relatively low except for regression (III).

Figure 2.5: Illustration of Winning Bids in Observed Strategy Space



Nevertheless, we can consider the transformed strategy space with $\tilde{q}$ and $\tilde{s}$. These two variables are directly related to price and thus can be compared across auctions. Table 2.5 displays estimation results using $\tilde{q}$ and $\tilde{s}$ as dependent variables. For all six regressions, their R^2s improve and show significantly positive coefficient estimates of w_q. We can interpret that by increasing the technical factor weight for 5%, it induces the project to be delivered at a higher quality and higher score to the buyer. The estimated average buyer's payoff increment ranges from 800,858 to 1,044,340 CNY (129,171 to 168,442 USD).

Concerning the coefficient estimates of regressor n, increasing the number of bidders has no significant effect on $\tilde{q}$, but has significant positive effects on $\tilde{s}$. It provides the evidence supporting one important theoretical model implication: firms choose their quality level based on their own social surplus maximization problem (equation (2.2) or (3.1)), hence n does not affect their choice of $\tilde{q}$. Because this property of independent quality choices relies on fairly weak assumption, confirming it in empirical study also supports the validity of our strategy space transformation.

In addition, we also observe some meaningful patterns across the winning bids. In Figure 2.6 diagram (A) and (B), we plot the density of savings rate and transformed quality respectively. The black curve represents the density of all observed bids while the red dashed curve represent the density of only winning bids. These two density diagrams show that winning bids have consistent pattern of higher quality and higher price, compared to other bids. Table 2.6 shows that 74.24% of winning bids have the highest transformed quality in that auction. On the other hand, only 4.01% of winning bids have the highest savings rate in that auction. On average, winning bids ask for higher prices (a lower savings rate) than losing bids. Among the 2147 auction we observed, 145 projects ended up with negative savings rates, meaning that the winning contracts have prices higher than the engineer's estimated costs. All these 145 negative savings rates occur at projects with engineer's estimated costs higher than 5,565 thousand CNY. Hence, a high p_0 project is more likely to be awarded to a high quality and high price bidder, illustrated in Figure 2.7. So we see high quality and high price contracts concentrating at high p_0 projects, where there are more room for quality manipulation. These susceptible signs of corruption motivate the corruption detection tests in Chapter 3.

Table 2.4: Reduced-form Regressions in Observed Strategy Space

	(I)	(II)	(III)	(IV)	(V)
Dep.Var.	s_q	s_r	p	s	ρ
p_0	0.0000	0.0000	1.288**		
	(0.0001)	(0.0001)	(0.0050)		
n	0.7447**	0.1236*	4.341*	0.3809**	0.0013*
	(0.048)	(0.057)	(1.99)	(0.034)	(0.0006)
w_q	11.17**	-4.95	3294.64**	1.53	-0.5758**
	(2.92)	(3.51)	(122.03)	(2.09)	(0.0377)
$D_{\text{fringe.firm}}$	0.1008	-0.5357	-28.67	-0.5463	-0.0093
	(0.4378)	(0.5265)	(18.30)	(0.3127)	(0.0056)
$D_{\text{fringe.buyer}}$	1.0405	-0.5909	-4.416	0.7195	0.0064
	(0.5998)	(0.7212)	(25.07)	(0.4285)	(0.0077)
D_{agent2}	-0.0694	0.3887	-30.78	-0.1587	-0.0034
	(0.3826)	(0.4601)	(16.0)	(0.2734)	(0.0049)
Constant	81.83**	76.48**	-3511.04**	74.66**	0.3927**
	(1.50)	(1.81)	(62.87)	(0.98)	(0.0176)
R^2	0.1050	0.0050	0.9687	0.0595	0.1123
Obs	2147	2147	2147	2147	2147

Note: Significance levels are denoted by asterisks (* $p < 0.05$, ** $p < 0.01$).

Table 2.5: Regression in Transformed Strategy Space

	(I)	(II)	(III)	(IV)	(V)	(VI)
Dep.Var.	Transformed Quality $\tilde{q}$			Transformed Score $\tilde{s}$		
Data	All	$p_0 < 5,565$	$p_0 \geq 5,565$	All	$p_0 < 5,565$	$p_0 \geq 5,565$
n	3.05	-0.85	47.56	20.15**	16.38**	39.08**
	(16.90)	(11.44)	(25.71)	(6.38)	(5.71)	(7.01)
w_q	19,007.23**	16,876.99**	26,367.82**	16,017.17**	14,326.23**	20,886.79**
	(1,038.50)	(690.20)	(1,652.03)	(391.94)	(344.24)	(450.61)
$D_{\text{fringe.firm}}$	200.15	186.62	-69.02	69.71	80.97	-41.14
	(155.65)	(103.21)	(248.59)	(58.75)	(51.48)	(67.81)
$D_{\text{fringe.buyer}}$	-158.57	-80.29	-560.93	-24.58	-21.19	-131.02
	(213.30)	(141.10)	(342.64)	(80.50)	(70.37)	(93.46)
D_{agent2}	121.59	124.31	-242.29	70.75	61.07	-28.83
	(136.08)	(89.99)	(218.49)	(51.36)	(44.88)	(59.60)
Constant	2,021.96**	1,552.37**	2,431.78**	-1,234.99**	-987.43**	-2,068.26**
	(485.21)	(325.33)	(756.52)	(183.13)	(162.26)	(206.35)
R^2	0.1421	0.2943	0.3066	0.4446	0.5378	0.7863
Obs	2147	1551	596	2147	1551	596

Note: Significance levels are denoted by asterisks (* $p < 0.05$, ** $p < 0.01$).

Table 2.6: Pattern of Winning Bids

	Mean of All Bids	Mean of Winning Bids	Highest in the Auction		Lowest in the Auction	
			Number	Percentage	Number	Percentage
s_q	60.48	93.05	1582	73.68%	4	0.19%
s_r	72.30	75.32	455	21.19%	193	8.99%
ρ	0.1980	0.1691	86	4.01%	530	24.69%
$\tilde{q}$	8260.86	9796.17	1594	74.24%	3	0.14%

Figure 2.6: Illustration of Winning Bids in Transformed Strategy Space

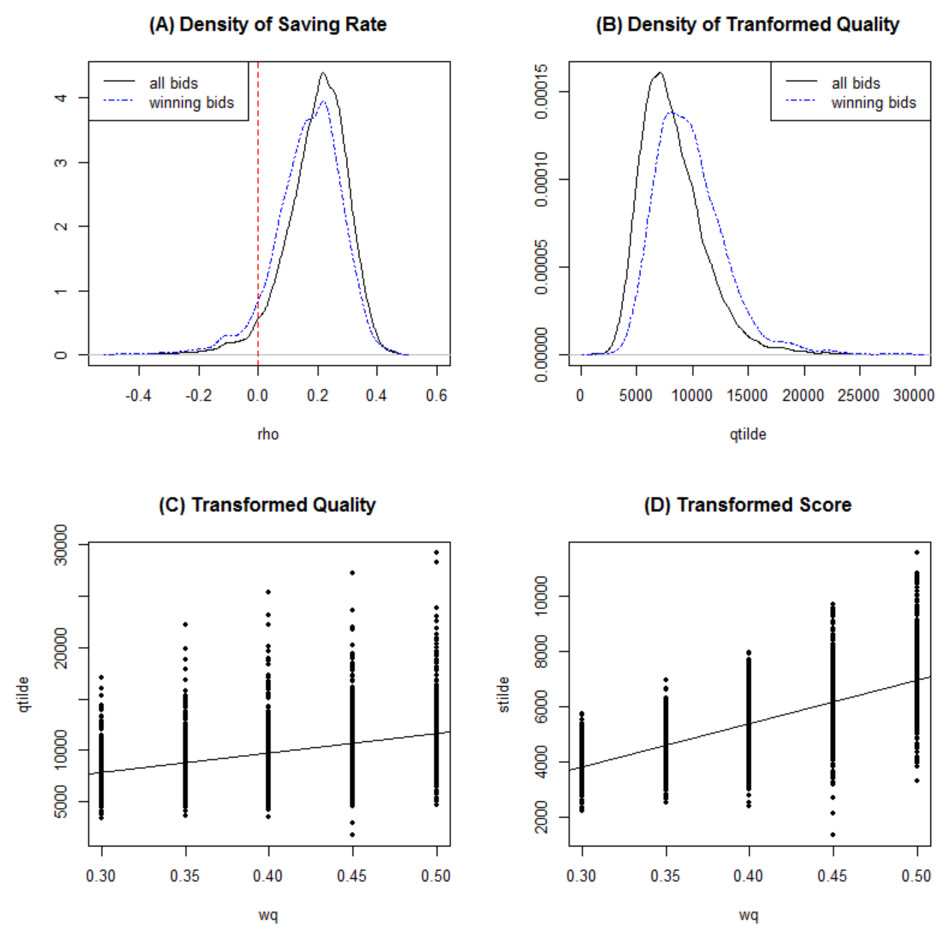
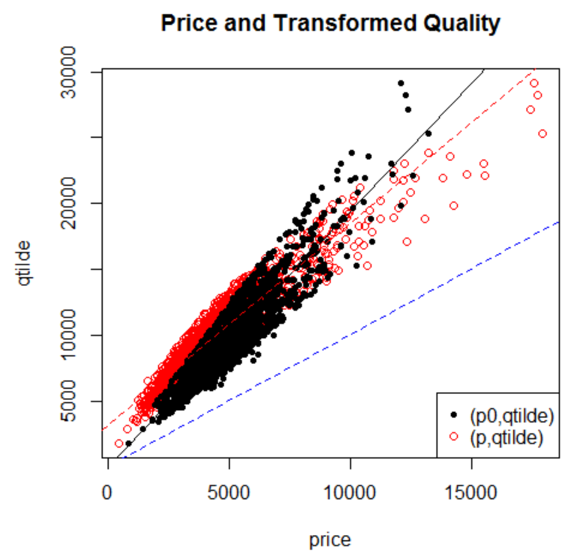


Figure 2.7: Illustration of Winning Bids in Transformed Strategy Space (Continue)



2.7 Structural Estimation

Lacking bid-specific covariates, project level regressions put aside information in all losing bids because they are endogenous, but structural estimation can draw information from all bids. Varying the scoring rule affects the distribution of pseudotype, therefore we consider sub-samples according to technical weights and agents. For each sub-sample, we apply formula (2.10) to structurally estimate pseudotypes. The estimation results are reported in Table 2.7 and Figure 2.8.

Recall that $\hat{k}$ represents the total social surplus of each firm producing at its efficient level, and $\tilde{s}$ represents how much of the social surplus is retained by the buyer. The difference, $\hat{k}-\tilde{s}$, is the *estimated rent* of the firm. Table 2.8 compares performance of two agents. The projects processed by two agents are similar in their observed characteristics, but we find that in general, firms bid in agent 1 gets higher rent than agent 2. Specifically, overall, firms at agent 1 ask for 63,030 CNY (10,166 USD) more rent compared to agent 2. But if we consider only winning bids, at each sub-sample, winning firms at agent 1 do not earns significantly more rent than those at agent 2.

Consider comparative statics on technical factor (quality) weights: a higher w_q in general leads to both higher transformed scores and higher rents, benefiting both parties. Quality weights reflect the buyers' willingness-to-pay for high quality projects, while the supplying firms only care monetary compensations. Serving buyers with higher willingness-to-pay naturally lead to higher payoffs for both sides. The theoretical model (Corollary 7.2), reduced-form, and structural estimation results are consistent in this prediction.

At the end, we want to point out that our nonparametric approach is based on pseudotypes and the distribution of pseudotype changes as scoring rule varies. A fully nonparametric method is may not be applicable when there is a great deal of variation in scoring rules across auctions and the sample size is relatively small. In this case, a parametric or semi-parametric approach shall be adopted. Once a parametric cost function is specified, the optimal quality and scores can be expressed as a system of equation of parameters. Nakabayashi and Hirose (2015) show the exact conditions that ensure the data on quality and score can reverse-engineer the parameters.

Table 2.7: Structural Estimation Results

Sub-sample w_q	Agent	No. of Projects	No. of Bids		Mean	SD	Min	Max
0.3	1	223	2384	$\hat{k}$:	3,358.97	1,292.52	1,156.09	27,485.10
				$\tilde{s}$:	3,126.94	797.31	1,135.51	5,747.27
0.3	2	87	913	$\hat{k}$:	3,389.06	1,088.43	1,308.24	10,132.73
				$\tilde{s}$:	3,173.24	811.30	1,264.24	5,407.96
0.35	1	244	2266	$\hat{k}$:	3,913.30	1,750.54	891.22	35,700.95
				$\tilde{s}$:	3,546.35	1,032.03	821.27	6,945.76
0.35	2	134	1228	$\hat{k}$:	3,942.98	1,473.08	1,540.13	19,494.30
				$\tilde{s}$:	3,617.00	1,014.27	1,494.09	6,303.84
0.4	1	384	3437	$\hat{k}$:	4,451.63	2,250.47	1,253.65	56,690.62
				$\tilde{s}$:	3,980.14	1,242.36	1,175.52	7,971.81
0.4	2	195	1687	$\hat{k}$:	4,448.67	1,805.02	1,052.06	22,355.00
				$\tilde{s}$:	4,053.84	1,227.13	1,015.22	7,568.83
0.45	1	407	3335	$\hat{k}$:	5,342.17	3,181.92	773.96	68,737.29
				$\tilde{s}$:	4,691.41	1,515.09	712.09	9,711.02
0.45	2	176	1510	$\hat{k}$:	5,131.95	2,360.78	1,603.51	28,208.93
				$\tilde{s}$:	4,585.34	1,505.13	1,431.86	9,324.88
0.5	1	208	1567	$\hat{k}$:	6,054.45	3,463.00	1,724.88	58,419.10
				$\tilde{s}$:	5,224.02	1,870.11	1,620.04	11,559.27
0.5	2	89	636	$\hat{k}$:	6,474.78	3,054.39	1,699.51	28,500.12
				$\tilde{s}$:	5,667.42	1,894.45	1,539.81	10,811.87

Table 2.8: Comparison of Two Agents

	Agent 1		Agent 2		t-test of Equal Mean	
	Mean	SD	Mean	SD	Statistic	p-value
n	8.860	3.937	8.772	3.681	0.5029	0.6151
p_0	5030.46	1515.45	5091.75	1395.77	-0.9212	0.3571
$\tilde{s}$	5435.89	1489.60	5484.52	1470.62	-0.7101	0.4778
	Estimated Rent ($\hat{k}-\tilde{s}$) of All Bids					
$w_q = 0.3$	232.03	749.33	215.82	380.90	0.8162	0.4144
$w_q = 0.35$	366.95	1,014.52	325.98	664.14	1.4365	0.1509
$w_q = 0.4$	471.49	1,396.39	394.82	873.68	2.4007	**0.0164**
$w_q = 0.45$	650.76	2,261.30	546.60	1,191.91	2.0940	**0.0363**
$w_q = 0.5$	830.42	2,089.09	807.36	1,551.44	0.2846	0.7760
Overall	498.63	1,634.84	435.60	985.48	3.2844	**0.0010**
	Estimated Rent ($\hat{k}-\tilde{s}$) of Winning Bids					
$w_q = 0.3$	816.54	2,212.67	646.67	821.04	0.9857	0.3251
$w_q = 0.35$	1,239.56	2,667.93	967.06	1,640.22	1.2279	0.2203
$w_q = 0.4$	1,534.06	3,529.86	1,208.71	2,164.06	1.3692	0.1715
$w_q = 0.45$	2,006.02	5,095.38	1,632.98	2,743.71	1.1428	0.2536
$w_q = 0.5$	2,228.15	4,393.84	2,183.71	3,214.99	0.0972	0.9226
Overall	1,605.41	3,914.21	1,326.43	2,330.57	2.0551	**0.0400**

Note: Bold numbers indicate rejection of the null at 0.05 significance level.

2.8 Chapter Conclusion

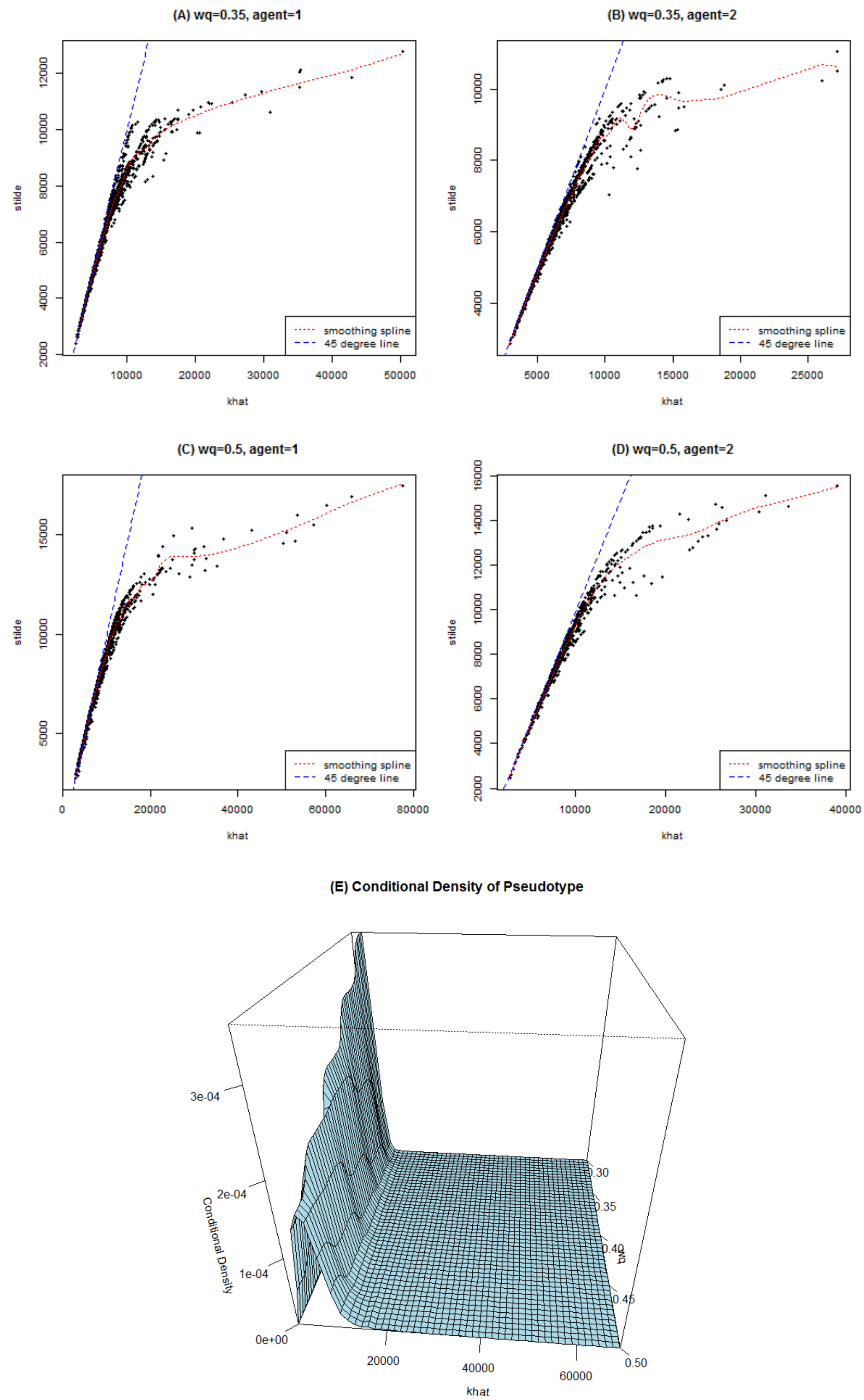
In this Chapter, we develop a structural estimation method and three corruption detection tests of scoring auctions. They are built upon fairly standard data of procurement auctions and can be applied to data from a wide range of industry with enough observations. The estimation method complements the theoretical study of the optimal scoring rule problem. By using historical data, they provide quantitative prediction of the effect of varying the scoring rule. These predictions are particularly useful for designing desirable procurement schemes.

We applied the method to a data set of server room construction project procurement auctions. The data patterns and estimation results provide key evidence for the theoretical scoring auction model. First, under additively separable scoring rules, the choice of quality can be separated from the choice of price and score. The reduced-form estimation shows that transformed quality scores are not affected by the number of competing bidders, but scores are. Second, with competition on both price and quality, firms mainly compete on offering high quality and expensive contracts. In the data set, over 70% of winning bids have the highest quality, but only about 4% of winning bids have the lowest price. Therefore, a reliable quality evaluation procedure is very important in keeping the auction efficient.

We also explore the effect of varying quality weight. The theoretical model predicts that a higher weight on quality induces firms to submit bids at higher quality and score, which is confirmed by estimation results in the transformed strategy space. The structural estimation results show that projects procured with higher quality weights result in both higher payoffs for the buyers and the winning firms. However, the buyer is restricted in picking the quality weight because the scoring rule must reflect her willingness-to-pay of higher quality. The theoretical model of scoring auctions shows that the buyer will not over-state its preferences on quality,[13] instead, the optimal scoring rule "shade" buyer's preference on quality to avoid giving up too much rent to the efficient firm.

[13] However, in Chapter 1, we show that the buyer may over-state it when there is quality manipulation corruption.

Figure 2.8: Illustration of Structural Estimation Result



Chapter 3

Detecting Quality Manipulation Corruption in Scoring Auctions

Abstract

Scoring auctions are particularly susceptible for corruption because the quality assessment usually requires special expertise that the buyer does not possess, which necessitates the participation of a skilled intermediary agent to evaluate quality. Corruption via quality manipulation causes a systematic distortion of bids and such distortion is testable. Based on the structural estimation method of scoring auction data, I construct three tests for detecting quality manipulation. These tests are based on standard auction record data and can be used to reduce anti-trust investigation cost. Applying these tests on the data set studied in Chapter 2, I find some signs of corruption in subsamples with high quality weight scoring rules and large engineer's estimated costs. Therefore, in designing the scoring rule, the buyers need to balance the efficiency gain from incentive provision and the risk of quality manipulation.

3.1 Introduction

Aforementioned, to implement a scoring auction, quality assessment is necessary in the auction process, but having an agent to do evaluation brings in the problem of corruption. Corruption is a prominent issue in procurement both in the public sector and private sectors, especially in developing countries. According to OECD (2013), money drained through corruption amounts to between 20% and 25% of the procurement budget, that is around US$2 trillion annually. Existing studies of corruption in auctions focus on either bidding rings among bidders, or bid revision cheatings between the auctioneer and a bidder (defined later in Section 3.2). Bidding rings and bid revision cheatings suppress competition and causes monetary loss for the buyers, but they ignore another perceivable consequence of corruption: inferiority of quality.

Take the bridge construction industry as example. Ji and Fu (2010) found that there were a total of 85 major bridge collapse accidents in China between 2000 and 2009. Forty cases among them were later convicted to be caused by corruption during the procurements. In a 2012 media report titled "Chinese-style of bridge collapse",[1] government officials and industry experts concluded three main frauds causing bridge collapse accidents: (i) the construction design proposal failed to meet industrial regulation, (ii) the construction was carried out at low quality, and (iii) the finished bridge lacks necessary maintenance. No buyer will purchase a bridge if she knows it will collapse in the near future. Quality manipulation results in the discrepancy between the quality written on the winning bid and the quality actually delivered, which may lead to deadly tragedies.

Current solutions for detecting corruption in auctions focus on auctions where price is the only corruptible outcome variable, and these tests are not applicable to auction settings where quality evaluation is involved in the auction process and can be manipulated. We will continue with the scoring auction model we built up in earlier chapter based on Che (1993), Asker and Cantillon (2008), and Hanazono et al. (2015). We introduce quality manipulation into the scoring auction model and characterize the systematic distortion of corrupted firm's bidding behaviors. The corruption model implication and structural estimation method are put together to construct

[1] http://club.kdnet.net/dispbbs.asp?page=1&boardid=89&id=8581905

three tests of detecting corruption via quality manipulation. A corrupted firm with exaggerated quality will bid more aggressively compared to a competitive counterpart. Based on one auction outcome, one cannot tell whether the aggressive bidding is due to corruption or the efficiency of the firm. But with a large sample of auctions, the abnormally aggressive bids[2] by corrupted firms will reject the competitive bidding model null hypotheses of our tests. The novel feature of these tests is that they are based on data of standard scoring auction records. Performing them require neither prior knowledge of identities of the corrupted firms, nor repeated observation of bids from the same set of firms, nor rich firm-specific covariates.

Third, we apply the structural estimation method and corruption detection tests to a series of procurement auctions of server room construction projects in China.[3] The data and estimation results provide empirical evidence for the three key implications of the theoretical model. Che (1993) predicts that each firm choose their quality maximizing the total social surplus given its cost function and the scoring rule. We show that the choice of quality is indeed separable from the choice of score because quality choice is uncorrelated to the number of bidders. We find that the project procured by a scoring rule with high quality weight tends to result in higher payoffs for both buyer and firms, but it is also subject to a higher risk of corruption. Corruption is also more likely to happen at projects with high engineer's estimated costs.

3.2 Literature Review of Detecting Corruption in Auction

In the *Handbook of Procurement* (edited by Dimitri et al. 2006), Lengwiler and Wolfstetter (2006) point out procurement auction participants may suppress competition by four major forms of collusion or corruption. In the literature, *collusion* usually refers to a *bidding ring* or a *cartel*, where a group of bidders coordinate their bids to suppress rivalry and capture some of the rents

[2]Our tests on abnormally aggressive bidding behavior are closely related to the literature of abnormally low tenders/bids. Putting aside the issue of quality, our tests can also be applied to detecting abnormally low bids. According to Dini et al. (2006) and Eun (2016), the problem of abnormally low bids in procurement can be of particular concerns for two main reasons. Firstly, firms are protected by limited liability and tend to bid aggressively. Once contract is awarded, replacing the incumbent firm is very costly and the selected firm is usually compensated with all cost overrun. Secondly, awarding contracts to "too-good-to-be-true" bids typically lead to reduction of quality or defaults.

[3]Server room is an indoor place designed to contain machines of data storage, servers, and large computers.

that otherwise would be transferred to the buyer. In an efficient cartel, the *cartel leader* (the one with the lowest cost or the winner of an internal pre-auction knockout) is the only serious bidder, while the other cartel members submit high *phony bids*. There is a body of literature on bidding rings both theoretically (e.g. Graham et al. (1990), McAfee and McMillan (1992), and Hendricks et al. (2008)) and empirically (e.g. Pesendorfer (2000), Bajari and Ye (2003), and Asker (2010)). *Corruption* usually refers to the auctioneer (who runs the auction) twisting the auction rule in exchange for bribes. It can take three major forms: (i) *bid revision* (*bid rigging* or "*magic number" cheating*), meaning that the auctioneer allows a favored bidder to adjust his bid after receiving information about rival bids (e.g. Compte et al. (2005) and Burguet and Perry (2009)). (ii) *Bid orchestration*, meaning that the auctioneer serves as the ring manager of a collusive cartel and coordinates their bids.[4] (iii) *Quality manipulation* (or *distortion of quality ranking*), meaning that the agent of bid evaluation is bribed to submit biased quality scores (e.g. Celentani and Ganuza (2002) and Burguet and Che (2004)).

In this section, we briefly review existing empirical works on corruption in auctions and its detection. We focus on a relative small number of papers and only sketch their key insights. For a more comprehensive reviews including the theoretical side literature, readers can consult other surveys like Hendricks and Porter (2007), Harrington (2008), and John Asker's note.

Porter and Zona (1993) is one of the earliest works on collusion detection. They study bidding rings in procurement auctions of Long Island highway construction contracts. Because some bidders are of relative large size and interact with each other in a sequence of auctions, they are able to coordinate as a cartel. They estimate parameters of a linear bid function and a logistic bid ranking model. Because the model can be estimated from using either the whole sample or only winning bids, two sets of parameter estimate shall be equal in a competitive environment. But when there is a bidding ring, the ranking of bids will not fully reflect the economic factors of bidders, leading to different estimates.

Colluding bidders' behaviors can be studied and tested by reduced-form models when detailed

[4] Just how rotten? (http://www.economist.com/node/3308447), *The Economists*, October 21, 2004

data of cartel members identities and characteristics are available from records of antitrust investigation. Porter and Zona (1999) analyze data from school milk contract auctions in Ohio, where a group of firms in Cincinnati were convicted for colluding. The bidding behavior of cartel members is compared to a controlled group. They show that collusion raised market prices by 6.5% on average. Pesendorfer (2000) also analyzes data from school milk contract auctions, where some firms in Florida and Texas were found colluding. He considers the effect of both bid rigging and market splitting. He estimates the coefficients of reduced-form bid function regressions using three subsamples: low cartel bids plus all non-cartel bids, low cartel bids, and all non-cartel bids. A Chow test for equality of coefficients shows that the cartel firms bid less aggressively than non-cartel firms. Feinstein et al. (1985) point out that a cartel may seek not only a higher winning bid, but also collectively use bids to pass false information to the buyer to avoid a "ratchet effect" (Freixas et al. 1985). It happens when the buyer uses past information to form expectations of future auctions. Feinstein et al. (1985) found empirical evidence from data of convicted collusion cases in North Carolina highways procurements.

However, if the data does not provide exact identities of the cartel and non-cartel bidders, the methods above cannot be implemented (unless one runs regression on all possible partitions of the cartel and non-cartel bidders). In addition, the data may not be rich in bidder's characteristics. Harrington (2008) points out that an abnormally high profit margin is not the evidence of collusion, but the evidence of market power. According to Baldwin et al. (1997), there are three (non-mutually exclusive) ways to explain a high profit margin: collusion, demand side factors, and supply side factors. The supply side can be captured by auction-specific covariates describing the object. To identify collusion, researchers need to control demand side factors by enough bidder-specific covariates. To encounter these data limitations, researchers start using structural model to detect collusion.

Bajari and Ye (2003) construct their test based on two distinct model implications of the competition and the collusion model: the competitive bids data shall satisfy conditional independence and exchangeability features. If bidders are competitive, bids must be independent controlling for all publicly observable information on costs under IPV framework. But if there is a cartel, their bids may be correlated and such correlation can be detected. Moreover, a competitive bidder's bid shall not depend on other bidder's identities, so exchanging other bidders' characteristics shall not change the distribution of competitive bidder's bid. In a regression specification, if one regresses bidder i's bid on the covariates of bidder j and k (with other controls), then these two coefficients should be equal. An F-test can be used to check this exchangeability restriction. Identities of potential cartel members can be found by testing each pair of bidders. In addition, Bayesian estimation of the structural model provides the likelihood of the data coming from the collusion model. Aryal and Gabrielli (2013) take a full structural approach to test collusion based on the estimation method of auction data in Guerre et al. (2000). For the same set of bids data, two sets of costs are structurally estimated by assuming the competitive model and the collusion model, denoted as $\left\{\hat{c}^A\right\}$ and $\left\{\hat{c}^B\right\}$ respectively. Because collusion lowers competition, for the same bid b, it implies $\hat{c}^A(b) \geq \hat{c}^B(b)$. Detecting collusion boils down to testing for first-order stochastic dominance of two cost distributions recovered from two models.

Besides bidding rings, Ingraham (2005) studies the corruption between the auctioneer and a bidder. His model is based on the bid revision model in Compte et al. (2005). The auctioneer let the corrupted firm observe others' bids before submitted its. When the corrupted firm's cost is lower than the lowest bid of other firms, it will submit a bid that barely wins the contract. As a result, the difference between the lowest and second lowest bid is smaller than a usual competitive sample. This is a testable model implication.

All works mentioned above are based on first-price sealed-bid auction. Collusion can be a more prominent problem in open auctions where tacit collusion is easier. Athey et al. (2011) study a timber auction data set with two auction formats (sealed-bid and open) and two sets of bidders (mills and loggers). They assume mills are potential cartel and use the sealed-bid auction as benchmark to evaluate whether bids in open auctions satisfy the competitive hypothesis. Bajari and Yeo (2009) studies collusion in FCC spectrum auction and Klemperer (2002) in telecoms license auction. Some other empirical works are based on the unique features of their data set.

Asker and Cantillon (2010) study internal knockout auction from side-transfer data of a stamp dealers cartel. They test the theory of internal organization of bidding rings and measure ring members' benefit from colluding. Tran (2011) uses internal bribery data of a company to compare corruption under two different auction formats. Kawai and Nakabayashi (2014) study an auction data set from Japanese government procurements. Because the reserve price is secret, observation of bids may consist of multiple rounds and the ranking of bidders across rounds can be used to detect collusion. Kaplan et al. (2016) propose a method to identify and test collusion in English auctions.

In summary, to detect collusion, researchers need to derive some key model implications distinguishing the competition model and the collusion model, and then test which model the data support. Hence, all these collusion detection methods suffer from some common problems: (i) When the null hypothesis of the competitive model is rejected, it is hard to tell whether the reason is collusion or model mis-specification (See Figure 3.4). For example, Hendricks and Porter (2007) point out that bid rotation[5] and incumbency pattern[6] are sometimes observational equivalent to collusion. (ii) If corrupted bidders coordinate their bids in a sophisticated way, the recorded bids can pass nearly all these tests. It is called "beating a test of collusion" , discussed in Harrington (2008). (iii) Nearly all these tests rely on repeated observations of bids from the same set of potential corrupted bidders. Dynamic interaction between bidders are very informative of whether they are competing or colluding. But one implicit assumption made here is that the identities of cartel and non-cartel members do not change across auctions.

Our tests are subject to problem (i) as others, but suffer less from problem (ii) and (iii). The quality manipulation problem usually only happens to one bidder. If the agent and corrupted firm wants to avoid being detected, they must reduce the manipulation power. Therefore, beating our tests will directly restrict corruption. Besides that, our tests are also useful for antitrust authorities because they requires only standard auction data. In particular, we do not need a prespecified set of suspicious corrupted bidders, identities of bidders, or repeated bidding behaviors of bidders across

[5]Firms with idle capacity are more likely to win the contract because they are less constraint in their capacity.
[6]Firms who have won a lot in the past may have a unobserved cost advantage.

auctions. Our tests can be perform with very little or even no bidder-specific covariates.

3.3 Model of Quality Manipulation

Assume that the agent randomly matches with one firm and forms a *corruption relation*. This relation can be the result of a bribery side-contract, a long term relationship, favoritism, or other reasons. According to Lengwiler and Wolfstetter (2006), by approaching only one bidder, the auctioneer minimize the number of side-contracts and thus the risk of detection. Large coalitions are of course possible, but the detection risk obviously increases with the number of people who know about the corruption. For simplicity, we assume that the agent matches with each firm with equal probability.[7]

Following Burguet and Che (2004), we assume that the agent can manipulate the evaluation of quality by raising the corrupted firm's quality score by $m > 0$. It means that if the corrupted firm submits a bid (p,q), the score is exaggerated from $S(p,q)$ to $S(p,q+m)$. This parameter m is called the agent's *manipulation power*. The interpretation of m can be (i) the quality score of the corrupted firm is exaggerated; (ii) the actual delivered quality is lower than the one written on the proposal; or (iii) the evaluation method is twisted to give an advantage to the corrupted firm. The magnitude of manipulation power is determined by the discretion given to the agent and the nature of the industry. It is restricted by the extent of not being suspicious and not triggering investigation. For example, in the procurement of a bridge, the agent may claim that the corrupted firm's bridge can serve 30 years while the actual building code is designed for only 25 years. However, he will not say the bridge will last 100 years because it would be very suspicious. The manipulation power is assumed to be an constant number known by the agent and the corrupted firm, but not others. Empirically, researchers don't observe how much quality is manipulated and the magnitude of manipulation can vary across auctions.

The timeline of the auction with corruption is as follows. The buyer announces a scoring

[7] In Huang and Xia (2016), the probability of each firm being corrupted is explicitly modeled. We can relax this equal probability assumption. However, if inefficient firms are being corrupted with significant probabilities and thus the winning bids are mostly not corrupted, then power of our tests will be very weak.

rule and hires the agent. A number of firms enter the auction exogenously and draw their private information θ from F. The agent then randomly matches with one firm and offers him a side contract that exaggerates the firm's quality score by m in exchange for a bribe. The firm decides whether to accept this side contract or not. Then every firm submits a sealed-bid simultaneously as a price-quality combination. If the matched firm accepts the side contract, his quality will be exaggerated by m. The auction outcome is then revealed and the firm with highest score wins the contract.

We skip a detailed model of the endogenous formation process of the corruption relation. We assume the agent is an expert in this industry and is able to design a bribery side contract that the matched firm will accept. For example, if the agent learns θ of the matched firm, he can make a take-it-or-leave-it offer, asking for a bribe slightly less than the difference between the expected payoff of being corrupted and being honest. Our simple model is enough from an empirical point of view, because variables directly related to corruption are usually unobservable (e.g. side payments, identities of corrupted firms, amounts of quality distortion). Writing a complicated model of quality manipulation usually ends up with the same qualitative prediction. We further impose an assumption on other firms' knowledge about the existence of the corruption relation.

Assumption UA: The buyer and other uncorrupted firms are unaware of the existence of the corruption relation.

This assumption is different from the complete information assumption we used in Chapter 1. There are both realistic and technical reasons for this assumption. In reality, if either the buyer or some other firms notice the existence of corruption, they will report it to the antitrust authority because corruption directly hurts their interests. The agent and the corrupted firm will control the scope of quality manipulation so that it does not trigger investigation. Moreover, for technical reason, with incomplete information on costs, adding another layer of incomplete information brings in mixed strategies and the equilibrium becomes both complicated and uninformative as shown in Theorem 5 and 6.

Given assumption UA, all uncorrupted firms follow the same strategy as in Theorem 7. The corrupted firm, once matched with the agent, solves a modified problem:

$$\max_{p,q}\left[p-C(q-m,\theta)\right]\Pr\left(\mathrm{win}|S(q,p)\right).$$

The equilibrium bidding strategy is summarized as the following theorem.

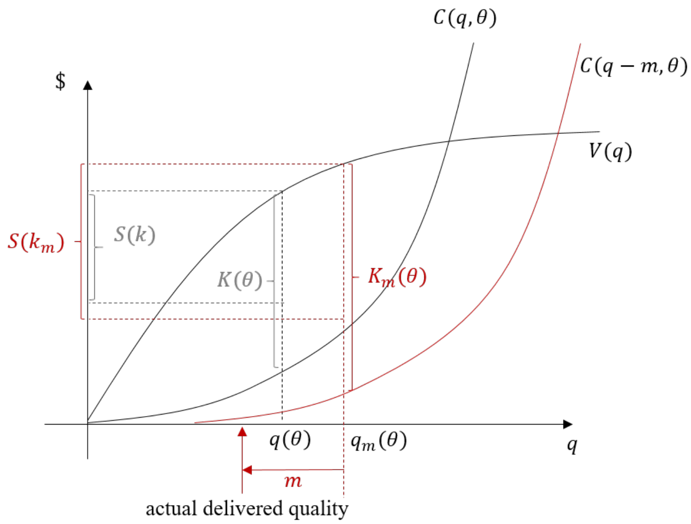
Theorem 9: *Under assumption CF, QL, and UA, the corrupted firm bids*

$$\begin{aligned} q_m(\theta) &= \arg\max_q V(q)-C(q-m,\theta), && (3.1)\\ p_m(\theta) &= V(q_m(\theta))-s(K_m(\theta)), && (3.2)\\ s(k_m) &= \frac{k_m-\int_{\underline{k}}^{k_m}[F_K(t)]^{n-1}dt}{[F_K(k_m)]^{n-1}} \end{aligned}$$

where $k_m=K_m(\theta)\equiv\max_q\{V(q)-C(q-m,\theta)\}$ is the corrupted firm's pseudotype. Compared to an uncorrupted firm with the same type, the corrupted firm has a higher pseudotype, bids a higher quality, and reaches a higher score. All three effects magnify as m increases.

Therefore, the corrupted firm will bid *more aggressively* compared to a competitive firm of the same type. Because the corrupted firm has a large winning probability, it causes a systematic distributional shift of the winning bid. It is the key factor that allows us to construct corruption detection tests. Note that the "more aggressive" prediction is different from the implication of bidding ring models. When an auction involves a bidding ring, both the ring leader and other members bid less aggressively to suppress competition. But with quality manipulation, the corrupted firm pays a lower cost with help from the agent. As a result, the corrupted firm will bid more aggressively to increase its chance of winning the contract.

Figure 3.1: Illustration of the Equilibrium with Corruption



3.4 Detecting Quality Manipulation

The key of detecting quality manipulation corruption lies in checking abnormally aggressive bidding behaviors of corrupted firms. It is impossible to distinguish normally competitive bidding behavior and abnormally predatory bidding behavior by a single observation because the manipulation power (m) in unknown. But when the sample size gets large, the consistent pattern of aggressive winning bids can be captured by statistical tests. In this section, we propose three tests and show them in a Monte Carlo example.

The basic intuition of our corruption detection tests is capturing the persistent abnormally aggressive bidding behaviors associated with the winning bids. Corruption distorts only the corrupted firm's bid and all other bids remain competitive. The distorted bid is the winning bid with a large probability. Hence, even we don't observe the identity of the corrupted bidder, we can test for the existence of systematic deviations from competitive bidding behaviors by comparing the winning bids and other bids. For all these tests, the null hypothesis is that the data are generated from the competitive model, i.e. H_0: $m=0$. It is tested against the alternative hypothesis that the data are generated from the corruption model, i.e. H_1: $m>0$. Test I and II can be performed on one sample of auctions processed by the same agent. Test III can only be performed on two or more sub-samples that are different in their agents or other aspects. These tests are illustrated in a Monte Carlo example and later applied to a real procurement data set. We denote the observed highest score or pseudotype of each auction by subscript "win". The observed second highest and third highest ones are denoted by subscript "rival" and "third", respectively.

Test I

Under the competitive model, the observed scores are associated with underlying order statistics of pseudotypes. In a scoring auction with n firms, the winner and the *strongest rival* are of pseudotypes $k_{(1:n)}$ and $k_{(2:n)}$. By Corollary 7.1, the competitive model implies $E\left[s_{win}\right]=E\left[s(k_{(1:n)})\right]=E\left[k_{(2:n)}\right]=E\left[k_{rival}\right]$. By Theorem 9, for any $m>0$, $E\left[s_{win}\right]>E\left[s(k_{(1:n)})\right]$ and $E\left[s(k_{(2:n)})\right]>E\left[s_{rival}\right]$. Hence, the corruption model implies $E\left[s_{win}\right]>E\left[s(k_{(1:n)})\right]=E\left[k_{(2:n)}\right]>E\left[k_{rival}\right]$. The corruption detection problem becomes testing

$$H_0: E\left[s_{win}\right]=E\left[k_{rival}\right] \quad \text{vs.} \quad H_1: E\left[s_{win}\right]>E\left[k_{rival}\right].$$

We use the Welch's t-test with test statistic

$$\mathscr{T}^I=\frac{T^{-1}\sum_{t=1}^{T}s_{win,t}-T^{-1}\sum_{t=1}^{T}\hat{k}_{rival,t}}{\sqrt{T^{-1}var(s_{win,t})+T^{-1}var(\hat{k}_{rival,t})}}, \tag{3.3}$$

where $\hat{k}_{rival,t}$ are estimated from (2.10). Lucking-Reiley (1999) also uses t-test for revenue equivalence but their samples are generated from different auction formats. For our application, we are studying one sample of the same auction format, therefore we use a bootstrap critical value to account for the correlation between scores and estimated pseudotypes.

Test II

For auctions with symmetric independent private value bidders, Athey and Haile (2002) show that the underlying value distribution is nonparametrically identified even when only one bid of each auction (an order statistic) is observed. When there is no corruption, the estimates of pseudotype distribution from all bids and from only the winning bids should be the same except for some statistical errors. When there is corruption, the winning bids are distorted and the two methods will result in statistically different estimates.

Practically, we construct the test by comparing two empirical CDFs of pseudotypes of winners from two estimation methods. By using all bids, pseudotypes of all firms, $\{\hat{k}_{1t},\cdots,\hat{k}_{n_t t}\}$, can be estimated via (2.10). Denote the pseudotype corresponding to the winning bid as $\hat{k}_{win,t}$ and its empirical CDF $\hat{F}_K^{win}(k)=\frac{1}{T}\sum_{t=1}^{T}\mathbb{I}\left(\hat{k}_{win,t}\leq k\right)$. By using only winning bids, these winning scores have distribution function $G_W(s_{win}|n)=G_S^{(1:n)}(s_{win})=[G_S(s_{win}|n)]^n$ and density $g_W(s_{win}|n)=n\left[G_S(s_{win}|n)\right]^{n-1}g_S(s_{win}|n)$. By replacing relevant terms in (2.8), the winners' pseudotypes are identified by $k_{win}=s_{win}+nG_W(s_{win}|n)/(n-1)g_W(s_{win}|n)$. The underlying pseudotype of each winning bid can then be estimated, denoted as $\check{k}_{win}$. The empirical CDF of $\check{k}_{win}$ is $\check{F}_K^{win}(k)=\frac{1}{T}\sum_{t=1}^{T}\mathbb{I}\left(\check{k}_{win,t}\leq k\right)$. The corruption detection problem becomes testing

$$H_0:\forall k\in[\underline{k},\overline{k}],\ \hat{F}_K^{win}(k)=\check{F}_K^{win}(k),\quad \text{vs.}\quad H_1:\exists k\in[\underline{k},\overline{k}],\ \hat{F}_K^{win}(k)\leq\check{F}_K^{win}(k).$$

The natural option is Kolmogorov–Smirnov (KS) test. The test is one-sided because the aggressive scores in the corruption model results in higher estimate of k. The KS test statistic is

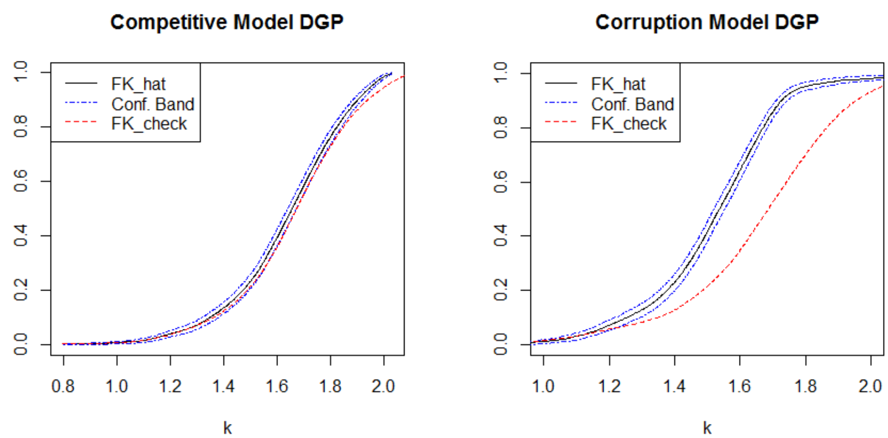
$$\begin{aligned}\mathscr{T}^{II} &= \sup_{k\in[\underline{k},\overline{k}]}\left|\hat{F}_K^{win}(k)-\check{F}_K^{win}(k)\right| \\ &= \sup_{k\in[\underline{k},\overline{k}]}\left|\frac{1}{T}\sum_{t=1}^{T}\mathbb{I}\left(\hat{k}_{win,t}\leq k\right)-\frac{1}{T}\sum_{t=1}^{T}\mathbb{I}\left(\check{k}_{win,t}\leq k\right)\right|.\end{aligned}$$

Similar to test I, there is dependence between the two sets of estimated pseudotypes of the winners, so we use bootstrap critical values. Test II is illustrated in Figure 3.2.

Test III

Inspired by Ingraham (2005), test III is based on the following Markovian property of the conditional distribution of order statistics (see the proof in Arnold et al. (1992)):

Figure 3.2: Illustration of Test II



Lemma 7: *Denote the first spacing of the highest two order statistics as* $X_{12} = X_{(1:n)} - X_{(2:n)}$. *The conditional distribution of* X_{12} *only depends on the third order statistic, that is*

$$f_{X_{12}}(x_{12}|X_{(3:n)} = x_3) = f_{X_{12}}(x_{12}|X_{(3:n)} = x_3, X_{(4:n)} = x_4, \cdots, X_{(n:n)} = x_n).$$

Test III is easy to implement but needs at least two sub-samples. Suppose the observed auctions can be divided into two (or several) sub-samples that are different in their procurement agencies or other aspects. Let D_τ be the dummy variable indicating the observation is from sub-sample τ. We can set up the regression

$$(\hat{k}_{win,t} - \hat{k}_{rival,t}) = \beta_0 + \beta_1 \hat{k}_{third,t} + \beta_2 D_{\tau,t} + \beta_3 z_t + \varepsilon_t,$$

where z_t controls for other auction-specific covariates. In a competitive auction, $\hat{k}_{win}$, $\hat{k}_{rival}$, and $\hat{k}_{third}$ coincide with $k_{(1:n)}$, $k_{(2:n)}$, and $k_{(3:n)}$. According to Lemma 7, the conditional distribution of the first spacing of pseudotypes, $k_{12} = k_{(1:n)} - k_{(2:n)}$, is the same across auctions if we control the third highest pseudotype $k_{(3:n)}$. Therefore the conditional means of two sub-samples are equal if $m = 0$. We can apply a standard t-test for $H_0 : \beta_2 = 0$ versus $H_1 : \beta_2 \neq 0$ with test statistic $\mathscr{T}^{III} = \hat{\beta}_2 / SE(\hat{\beta}_2)$. We can also skip the first stage estimation of pseudotypes and directly use score data to perform the test by the regression

$$(s_{win,t} - s_{rival,t}) = \beta_0 + \beta_1 s_{third,t} + \beta_2 D_{\tau,t} + \beta_3 z_t + \beta_4 n_t + \varepsilon_t,$$

If $\hat{\beta}_2$ is significantly greater than 0, the gap between the winner and the strongest rival is larger in the sub-sample with , which implies a higher likelihood of corruption.

Monte Carlo Example (Continue)

The identification result in Theorem 9 is established in a competitive bidding environment. If there is corruption, the manipulation power is unobservable and may vary across auctions. From a single observation, a researcher cannot conclude whether a high pseudotype is due to a real competitive advantage or a manipulated quality. In the example, for some $m > 0$,

$$q_m(\theta) = \arg\max_q \left\{ 2q - \theta_0 - \frac{(q-m)^2}{\theta_1} \right\} = \theta_1 + m = q(\theta) + m, \tag{3.4}$$

$$K_m(\theta) = 2(\theta_1 + m) - \theta_0 - \theta_1 = \theta_1 - \theta_0 + 2m = K(\theta) + 2m. \tag{3.5}$$

Therefore one cannot separately identify k and m. Although it is not an identified model, the systematic distortion of submitted bids can be captured with a large sample.

We continue with the previous example to illustrate the corruption detection tests. To find the distribution of test statistics under the null, m can be an unknown positive number. But to study the powers of these tests, we let m to be a known fixed number across observations. We generate $B = 199$ samples under the null hypothesis ($m = 0$) and compute test statistics for each sample, $\{\mathcal{T}_b^j\}_{b=1}^B$, $j = I$ and II.[8] Setting the significance level at 5%, the relevant *bootstrap critical value* of the test, $CV(\mathcal{T}^j)$, is the 190th highest among these test statistics (since $(B+1) \times (1 - 0.05) = 190$). In both diagrams of Figure 3.3, the blue line and the black dashed line denote the test statistic and the bootstrap critical value respectively, while the black curve represents the density of 199

[8]To check the validity of bootstrap, we use the data generating process to repeatedly generate data sets and construct the compare the distribution of test statistic. It is use to compare with the bootstrap distribution of the test statistics. They are similar.

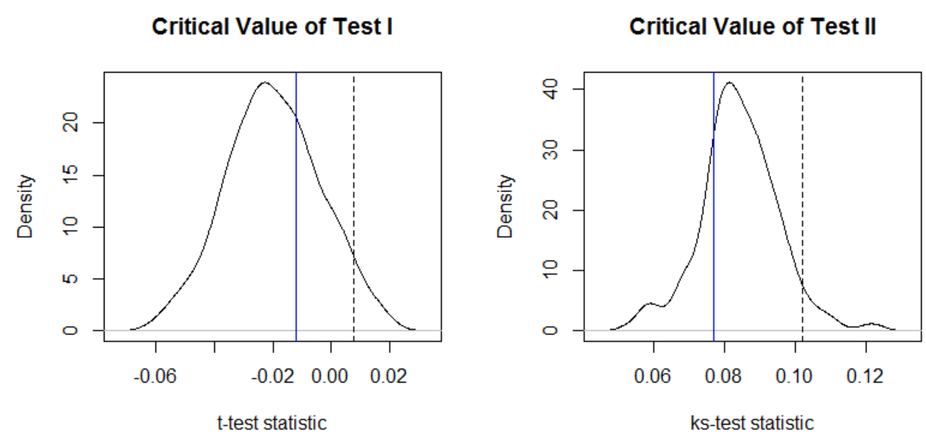
bootstrap test statistics.

We explore the powers of these tests under three alternative hypotheses by taking m equals 0.2, 1, and 2, shown in Table 3.1. A randomly selected corrupted firm will choose a higher quality and have a higher pseudotype according to (3.4) and (3.5) respectively. The *bootstrap power of the test* is defined and computed by

$$\text{power} \quad = \quad 1 - \Pr(\text{accept } H_0 | H_1 \text{ is true}) = 1 - \frac{1}{B} \sum_{b=1}^{B} \mathbb{I}\left(\mathscr{T}_b^j \leq CV(\mathscr{T}^j)\right).$$

The Monte Carlo results show that as the manipulation power m and the sample size T increase, the powers of all three tests improve. The power of test I is relatively weak compared to test II and III, especially in the case of small m.[9]

Figure 3.3: Distribution of Test Statistics Under the Null and Bootstrap Critical Values



[9] Additional Monte Carlo study on the sizes and powers can be found as a supplemental material on the author's website: https://sites.google.com/site/sunnyelan/.

Table 3.1: Power of the Tests

Manipulation Power	Test	Sample Size 200	500	1000
	I	0.2462	0.2362	0.2613
$m = 0.2$	II	0.9246	0.9347	0.9497
	III	0.8241	0.7889	0.8291
The corrupted firm wins with probability 0.2348.				
	I	0.3869	0.4925	0.5528
$m = 1$	II	0.9749	0.9648	0.9849
	III	0.9246	0.9397	0.9598
The corrupted firm wins with probability 0.4596.				
	I	0.9347	0.9648	0.9749
$m = 2$	II	0.9899	0.9950	0.9950
	III	0.9950	0.9950	1.0000
The corrupted firm wins with probability 0.9618.				

Note: For test III, one half of the sample is generated under the null, the other half under the alternative.

Discussion

Compared to most existing collusion detection tests, our tests require less data, so it can be performed on a lot of procurement auction data sets. Existing tests generally require identities of bidders, (rich) bidder-specific covariates, repeated observation from the same set of bidders in several auctions. Some of these tests requires exact identities of (suspected) colluding bidders, for example Porter and Zona (1993), Pesendorfer (2000), and Athey et al. (2011). Some tests, like Bajari and Ye (2003), can be conducted without identities of the corrupted firms, but need to be run on each combination of bidder pairs. Some tests are constructed upon repeated observations from the same set of bidders, which reveal the systematic difference between colluding bidders and competing bidders. Our tests do not require any of these data and hence can be performed before the case-by-case antitrust investigation.

Moreover, with different sub-samples, our tests do not need to specify a prior on which sub-sample is more likely to be corrupted. (For example, Athey et al. (2011) assumes that the sample from open auctions are collusive and sealed-bid auctions are competitive.) Test I and II can be performed on each of the sub-sample and compare their likelihoods of corruption by p-values. Test III estimates a "fixed effect" for each sub-sample by regression and can rank their likelihoods of being corrupted. However, because these tests are constructed on fairly limited sample information, there are several shortcomings:

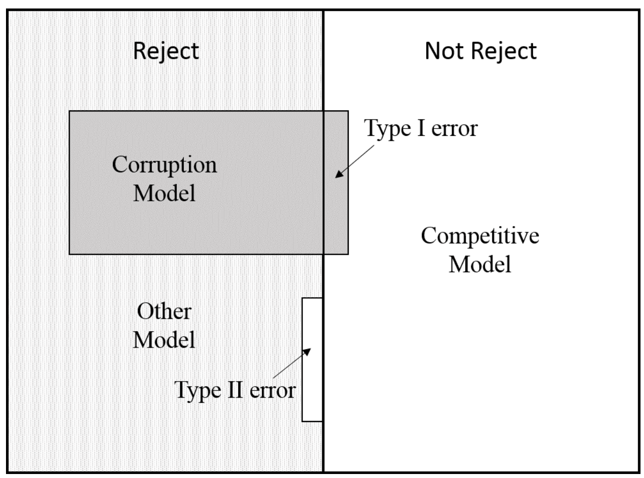
(1) Our test statistics involve two-step estimation based on pseudotypes. Because estimated pseudotypes are correlated, asymptotic distribution under the null is hard to derive analytically. We therefore use bootstrap critical values to make rejection decisions. The analytical study of the asymptotic property may be derived based on the recent findings in Ma et al. (2016). We will leave it for future research. Also, researchers start developing inference methods and tests based on one-step estimation of auction data. For example, Liu and Luo (2014) propose a test of exogenous entry based on empirical quantile of bids, which circumvents the correlation issue of estimated pseudo-values. However, because bids depend on the number of bidders (n), one cannot pool data from auctions with different n together, but needs to conduct the test separately on sub-samples according to n. In our application, there is a great deal of variation of n (see Figure 2.4), so we choose to take the two-step approach.

(2) The powers of our tests are also very difficult to be studied analytically. First, they depend on the manipulation power m, which is unobservable and may vary across auctions. Second, m cannot be estimated even if we assume it follows a parametric distribution. The model is not identified under the alternative hypothesis because the corrupted firm is not always the winner. So the manipulation power cannot be recovered without knowing the exact identities of the corrupted firms. In other words, the corrupted firm's bid and other bids are not generated from the same data generation process, and we don't know which bid comes from the corrupted firm. These complications restrict us from studying the powers of the tests rigorously. A desirable data set to study corruption should include some *ex post* information of convicted corruption records. With identities of corrupted firms, then it is possible to identify the entire corruption model. Researchers can then study the powers and "in-sample" prediction correctness of these tests. We don't have such a data set for now and the main contribution of these tests are their *ex ante* applicable feature

in corruption detection.

(3) Figure 3.4 illustrates a common problem of our tests and most collusion detection tests in the literature. When the data does not reject the null, it supports that the data rationalizes the competitive model. But when the data rejects the competitive model, it cannot distinguish whether the reason is corruption or model mis-specification. For example, rejecting test I can be due to any reason related to failures of expected score equivalence, like bidder's risk aversion. The one-sided tests in test I and test II alleviate this problem: if we find that the winning bid is not aggressive but conservative, we do not reject the null.

Figure 3.4: Interpretation of Test Results



3.5 Empirical Application

The pattern of winning bids and losing bids together reveal whether the bidding behaviors are competitive. Continue using the data set of Chapter 2, the tests proposed above can be directly run based on the structural estimation results in Chapter 2. Table 3.2 and Figure 3.5 show results of test I and II. For test I, there are a total five sub-samples rejecting the competitive model. In general, they happen at auctions with high w_q. For test II, none of sub-samples rejects the competitive model. For test III, we consider six regression models shown in Table 3.3 and find only one coefficient of D_{agency2} being significant. Regression (VI) is run on the sub-sample with high engineer's estimated costs. It implies the first spacing of transformed scores is larger at agent 2, which is the sign of aggressive bidding behavior. Since we also find that rent at agent 2 is generally lower, the reality could be that firms are earning their rent under the table by delivering low quality projects.

In summary, a majority of the data set passes our corruption detection tests. Recall Figure 3.4 these failures of rejection support the theoretical prediction of the competitive model, which makes the structural estimation trustworthy for this data set. For some sub-samples of the data set, we find signs of quality manipulation. The data patterns shown in Figure 2.7 and results of corruption detection tests suggest that antitrust authorities should spend more investigation resources on projects with high technical weights and high engineer's estimated costs, especially on those processed by agent 2.

It is worth mentioning that high technical weight and estimated cost are proxy for complexity of the project. Bajari and Tadelis (2001) and Tadelis (2012) compare auction and negotiation at different levels of project complexity. Complexity may potentially jeopardize the advantage of competitive tendering because the uncertainty in project design stage may lead to costly renegotiation of *ex post* adjustment. The buyer may choose a bilateral negotiation with a reputable supplier, because in the negotiation, the reputable firm can help design the complex project and save the *ex post* adaption cost. In a scoring auction, quality and design of the project are chosen by firms, so it reaps benefit from both price-only auctions and negotiation. However, all these cross-procurement-scheme comparisons are not robust if quality is not perfectly observable and/or verifiable at the moment of transaction. The corruption problem analyzed in this paper not only affects the optimal scoring rule and auction format, but also optimal procurement scheme.

Table 3.2: Results of Test I and II

Sub-sample		Test I			Test II		
w_q	Agent	Test Stat.	BT c.v.	BT p.v	Test Stat.	BT c.v.	BT p.v
0.3	1	-6.7284	-4.0347	0.5400	0.2422	0.2870	0.6600
	2	-3.3526	-5.0863	**0.0000**	0.2644	0.3563	0.8350
0.35	1	-6.2109	-4.9973	0.2800	0.2131	0.2623	0.7000
	2	-5.0055	-4.6895	0.1050	0.2836	0.3209	0.3800
0.4	1	-7.3851	-5.8942	0.2600	0.2396	0.2656	0.4750
	2	-4.7708	-5.6530	**0.0000**	0.2513	0.2923	0.5850
0.45	1	-5.3859	-5.8718	**0.0000**	0.2260	0.2604	0.6500
	2	-5.5837	-5.8815	**0.0150**	0.2330	0.2784	0.6700
0.5	1	-5.3204	-5.2688	0.0700	0.2067	0.2500	0.5900
	2	-4.4933	-4.8648	**0.0100**	0.2584	0.3258	0.5300

Note: BT c.v. and BT p.v stand for "bootstrap critical value " at 0.05 significance level and "bootstrap p-value" respectively. They are computed based on 199 bootstrap samples at project level. Bold numbers indicate rejection of the null.

Figure 3.5: Result of Test I and Test II

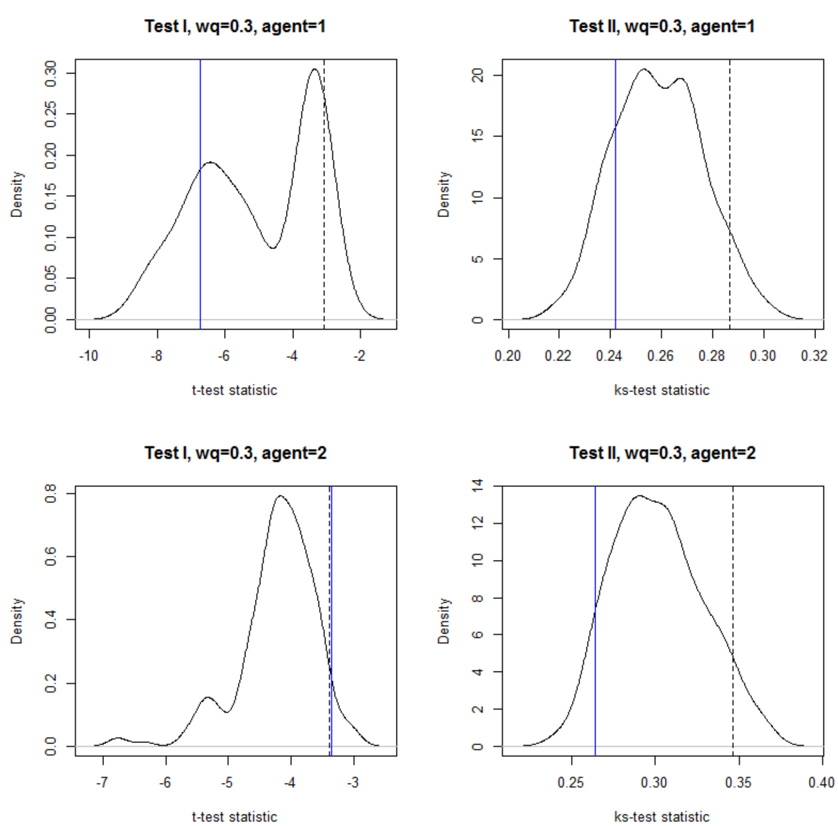


Table 3.3: Results of Test III

	(I)	(II)	(III)	(IV)	(V)	(VI)
Dep.Var		First Spacing of $\hat{k}$			First Spacing of $\tilde{s}$	
Data	All	$p_0 < 5565$	$p_0 \geq 5565$	All	$p_0 < 5565$	$p_0 \geq 5565$
D_{agency2}	-39.78	3.372	-245.88	20.45	4.750	103.25*
	(110.36)	(29.02)	(591.77)	(16.84)	(18.05)	(46.37)
3rd order	0.3082**	-0.0863**	0.0568	-0.0506**	-0.0922**	-0.2180**
statistic	(0.0296)	(0.0129)	(0.1194)	(0.0072)	(0.0098)	(0.0246)
w_q	-1,120.81	3,472.47**	5,910.49	1,813.97**	2,298.35**	4,947.44**
	(929.72)	(268.23)	(5274.83)	(160.63)	(180.07)	(578.31)
n				-24.62**	-17.96**	-9.485
				(2.199)	(2.432)	(6.388)
Constant	-236.29	-547.17**	187.46	64.50	-56.40	-207.70
	(337.84)	(88.85)	(1857.27)	(60.86)	(66.11)	(175.94)
R^2	0.0556	0.0986	0.0089	0.1588	0.1875	0.2875
Obs	2147	1551	596	2147	1551	596

Note: Significance levels are denoted by asterisks (* $p < 0.05$, ** $p < 0.01$).

3.6 Chapter Conclusion

We propose three corruption detection tests of scoring auctions based on the structural estimation method in Chapter 2. These tests are based on standard record of auctions and are easy to implement. They quantify the risk of quality manipulation of sub-samples and guide the antitrust authority to suspicious cases for further investigation. This saves antitrust cost, deter large scale quality manipulation, and may potentially improve performance of procurements that involve quality.

Our corruption detection tests are *ex ante* in the sense that they can label identities of corrupted firms, projects, and agents without specifying a prior of suspects. For future research, one important complement is an *ex post* study of corruption behaviors by data from convicted corruption cases, for example, from investigation reports of collapsed bridges. Then researchers can study the "in-sample" property of these corruption detection tests (e.g. Bajari and Ye (2003)) and even the internal organization of corrupted agents (e.g. Asker (2010)). In this way, historical auction data, antitrust records, and economic analysis can together construct stronger tools for antitrust purposes.

Last but not the least, in selecting the scoring rule, besides the shading for optimal screening, a higher quality weight gives more room for quality manipulation. As Lengwiler and Wolfstetter (2006) suggested, when quality scores are problematic due to the possibility of corruption, the quality weight shall be reduced. We run the three corruption detection tests proposed in this paper and find that, in general, the data set passes our tests. But there are some signs of corruption in sub-samples with higher quality weights and higher engineer's estimated costs. Therefore, in designing the scoring rule, the buyers need to balance the efficiency gain from incentive provision and the risk of quality manipulation.

Appendix

Proof of Lemma 1: Consider a firm with quality manipulation $m \geq 0$. Suppose it submits some (q,p) such that $q \neq q^*$, then there always exist another bid (q^*,p') such that the score is the same but the payoff upon winning is strictly higher. Specifically, by taking $p' = \alpha(q^* - q) + p$, then $\alpha(q^* + m) - p' = \alpha(q+m) - p$, $S(q^*,p') = S(q,p)$. Because $q \notin \arg\max_q \alpha q - C(q,\theta)$, (q^*,p') yields a higher payoff,

$$\begin{aligned}\pi(q^*,p') - \pi(q,p) &= p' - C(q^*,\theta) - p + C(q,\theta) \\ &= \alpha q^* - \alpha q + p - C(q^*,\theta) - p + C(q,\theta) \\ &= \alpha q^* - C(q^*,\theta) - [\alpha q - C(q,\theta)] > 0.\end{aligned}$$

The honest firm will also choose q^*. *Q.E.D.*

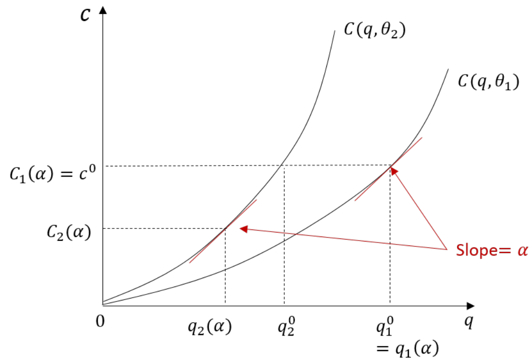
Proof of Lemma 2: (i) $q_i(\alpha)$ satisfies first order condition of (1.1): $\alpha - C_q(q,\theta_i) = 0$. Because $C_{qq} > 0$, $q_i'(\alpha) = \frac{1}{C_{qq}(q,\theta_i)} > 0$.

(ii) Because $C_{q\theta} > 0$, $\frac{\partial q^*}{\partial \theta} = -\frac{C_{q\theta}}{C_{qq}} < 0$. Because $\theta_1 < \theta_2$, $q_1(\alpha) > q_2(\alpha)$.

Because $C_{qq\theta} > 0$ and $C_{qq}(q,\theta_1) < C_{qq}(q,\theta_2)$,$q_1'(\alpha) - q_2'(\alpha) = \frac{1}{C_{qq}(q,\theta_1)} - \frac{1}{C_{qq}(q,\theta_2)} > 0$.

(iii) For any cost level c^0, suppose firm 1 and 2 can produce quality q_1^0, q_2^0, $C(q_1^0,\theta_1) = C(q_2^0,\theta_2) = c^0$. Because $C_{q\theta} > 0$ and $\theta_2 > \theta_1$, $C_q(q_2^0,\theta_2) > C_q(q_1^0,\theta_1)$ (at the same level of cost c, the cost function of firm 1 is steeper). When $q_1^0 = q_1(\alpha)$ is the solution to FOC $C_q(q,\theta_1) = \alpha$, then $\alpha < C_q(q_2^0,\theta_2)$. Firm 2's equilibrium quality $q_2(\alpha)$ must satisfy $\alpha = C_q(q,\theta_2)$ and because $C_{qq} > 0$, hence $q_2(\alpha) < q_2^0$, $C_2(\alpha) = C(q_2(\alpha),\theta_2) < C(q_2^0,\theta_2) = C(q_1^0,\theta_1) = C(q_1(\alpha),\theta_1) = C_1(\alpha)$.*Q.E.D.*

Figure 3.6: Illustration of Lemma 2



Proof of Proposition 1: Let ε be some small positive number. Under Bertrand competition, firm 2 chooses price at its cost $p_2 = c_2(\alpha)$ in equilibrium. Firm 1 chooses $p_1(\alpha) = \alpha q_1(\alpha) - \alpha q_2(\alpha) + C_2(\alpha) - \varepsilon$ to match firm 2's score and slightly over-bids it

$$\begin{aligned} S_1 &= \alpha q_1(\alpha) - p_1(\alpha) \\ &= \alpha q_1(\alpha) - [\alpha q_1(\alpha) - \alpha q_2(\alpha) + C_2(\alpha) - \varepsilon] \\ &= \alpha q_2(\alpha) + C_2(\alpha) + \varepsilon \\ &= \alpha q_2(\alpha) + p_2(\alpha) + \varepsilon > S_2 \end{aligned}$$

Given $p_1(\alpha) = \alpha q_1(\alpha) - \alpha q_2(\alpha) + C_2(\alpha) - \varepsilon$, firm 2 cannot decrease its price below its marginal cost and don't have incentive to increase its price. Ignoring ε for conciseness, $p_2(\alpha) = C_2(\alpha)$ and $p_1(\alpha) = \alpha q_1(\alpha) - \alpha q_2(\alpha) + C_2(\alpha)$ are equilibrium prices.

Because in the equilibrium, firm 1 earns a positive rent, $R_T^L = p_1(\alpha) - C_1(\alpha) > 0$. We can show that R_T^L is increasing and convex in α:

$$
\begin{aligned}
R_T^{L'}(\alpha) &= q_1(\alpha)+\alpha q_1'(\alpha)-C_q(q,\theta_1)q_1'(\alpha)-q_2(\alpha)-\alpha q_2'(\alpha)+C_q(q,\theta_2)q_2'(\alpha) \\
&= q_1(\alpha)-q_2(\alpha)+q_1'(\alpha)\underbrace{\left[\alpha-C_q(q,\theta_1)\right]}_{=0}-q_2'(\alpha)\underbrace{\left[\alpha-C_q(q,\theta_2)\right]}_{=0} \\
&= q_1(\alpha)-q_2(\alpha)>0.
\end{aligned}
$$

$$
R_T^{L''}(\alpha)=q_1'(\alpha)-q_2'(\alpha)>0.
$$

These signs are obtained by results in Lemma 2. *Q.E.D.*

Proof of Proposition 2: α_H satisfies the following first-order condition:

$$
\begin{aligned}
U_H^{L'}(\alpha) &= (1-\alpha)q_1'(\alpha)-q_1(\alpha)+\alpha q_2'(\alpha)+q_2(\alpha)-C_q(q,\theta_2)q_2'(\alpha) \\
&= (1-\alpha)q_1'(\alpha)-q_1(\alpha)+q_2(\alpha)+q_2'(\alpha)\underbrace{\left[\alpha-C_q(q,\theta_2)\right]}_{=0} \\
&= (1-\alpha)q_1'(\alpha)-q_1(\alpha)+q_2(\alpha)=0,
\end{aligned}
$$

Assumption CF (iii) is a sufficient condition for the second-order condition holds

$$
\begin{aligned}
U_H^{L''}(\alpha) &= (1-\alpha)q_1''(\alpha)-q_1'(\alpha)-q_1'(\alpha)-q_2'(\alpha) \\
&< (1-\alpha)q_1''(\alpha)-q_1'(\alpha) \\
&= -(1-\alpha)\frac{C_{qqq}}{C_{qq}^3}-\frac{1}{c_{qq}}<0 \\
&\Leftrightarrow\ C_{qqq}>-\frac{C_{qq}^2}{1-\alpha}
\end{aligned}
$$

Therefore, a sufficient condition for the second-order condition to hold on $\alpha\in[0,1]$ is $C_{qqq}>-C_{qq}^2$.

By Lemma 2 and $C_{qq} > 0$,

$$U_H^{L'}(0) = \frac{1}{C_{qq}(q,\theta_1)} - \underbrace{q_1(0)}_{=0} + \underbrace{q_2(0)}_{=0} > 0,$$

$$U_H^{L'}(1) = 0 - q_1(1) + q_2(1) < 0.$$

U_H^L is continuous in α, therefore, there exists at lease one $\alpha_H \in (0,1)$ satisfies $U_H^{L'}(\alpha) = 0$. So the solution to $U_H^{L'}(\alpha) = 0$ lies within $(0,1)$. *Q.E.D.*

Proof of Lemma 3: (i) Define $k(\alpha) \equiv \frac{R_T^L(\alpha)}{\alpha}$. This function is increasing because

$$k'(\alpha) = \frac{1}{\alpha^2}\left[\alpha R_T^{L'}(\alpha) - R_T^L(\alpha)\right] = \frac{1}{\alpha^2}\left[C_1(\alpha) - C_2(\alpha)\right] > 0.$$

If $m < \sup_{\alpha\in[0,\infty)} k(\alpha)$, by continuity of $k(\alpha)$, there exists a unique $\tilde{\alpha} > 0$ such that $m = k(\tilde{\alpha}) \Leftrightarrow \tilde{\alpha}m = R_T^L(\tilde{\alpha})$.

If $m > \sup_{\alpha\in[0,\infty)} k(\alpha)$, the scope of quality manipulation is so large that the corruption rent always dominates. In this case, all the following result holds by treating $\tilde{\alpha} = \infty$.

Note, $\alpha = 0$ always makes $\alpha m = R_T^L(\alpha)$, but the buyer does not want to induce zero quality.

(ii) Following monotonicity of $k(\alpha)$, obviously, for $\alpha < \tilde{\alpha}$, $k(\alpha) < m \Leftrightarrow \alpha m > R_T^L(\alpha)$; for $\alpha \geq \tilde{\alpha}$, $k(\alpha) \geq m \Leftrightarrow \alpha m \leq R_T^L(\alpha)$.

(iii) By identity $k(\tilde{\alpha}) = m$, $\tilde{\alpha}'(m) = \frac{1}{k'(\alpha)} > 0$. *Q.E.D.*

Proof of Proposition 3: The proof is straightforward by similar argument of Bertrand competition model. If firm 1 is corrupted, under Bertrand competition, firm 2 bidding above its cost cannot be an equilibrium. To see this, consider some $p_2 > C_2(\alpha)$, firm 1 can choose a price $p_1 \in (C_2(\alpha), p_2)$, but then firm 2 can still undercut, so $p_2 > C_2(\alpha)$ is not an equilibrium. Therefore, in equilibrium, firm 2 chooses price equals its cost $p_2 = C_2(\alpha)$ in equilibrium. Firm 1 chooses $p_1(\alpha) = C_1(\alpha) + R_T^L + R_C^L - \varepsilon$ (ε is some small positive number) to match firm 2's score and slightly over-bids it:

$$
\begin{aligned}
S_1 &= \alpha q_1(\alpha) + \alpha m - p_1(\alpha) \\
&= \alpha q_1(\alpha) + \alpha m - [C_1(\alpha) + R_T^L + R_C^L] \\
&= \alpha q_1(\alpha) + \alpha m - [\alpha q_1(\alpha) - \alpha q_2(\alpha) + C_2(\alpha) + \alpha m - \varepsilon] \\
&= \alpha q_2(\alpha) + C_2(\alpha) + \varepsilon \\
&= \alpha q_2(\alpha) + p_2(\alpha) + \varepsilon > S_2.
\end{aligned}
$$

The buyer's payoff in this case is $q_1(\alpha) - p_1(\alpha) = q_1(\alpha) - C_1(\alpha) - R_T^L - R_C^L$.

If firm 2 is corrupted, consider two sub-cases: $\alpha \geq \tilde{\alpha}$ and $\alpha < \tilde{\alpha}$.

When $\alpha \geq \tilde{\alpha}$, technological rent dominates corruption rent. Firm 2 chooses $p_2 = C_2(\alpha)$ and received exaggerated score $S_2 = \alpha q_2(\alpha) - p_2(\alpha) + \alpha m$. Firm 1 chooses $p_1(\alpha) = C_1(\alpha) + R_T^L - R_C^L - \varepsilon$ to match firm 2's score and slightly over-bids it:

$$
\begin{aligned}
S_1 - S_2 &= \alpha q_1(\alpha) - p_1(\alpha) - [\alpha q_2(\alpha) - p_2(\alpha) + \alpha m] \\
&= \alpha q_1(\alpha) - C_1(\alpha) - R_T^L + \alpha m + \varepsilon - \alpha q_2(\alpha) + C_2(\alpha) - \alpha m \\
&= \alpha q_1(\alpha) - C_1(\alpha) - \alpha q_2(\alpha) + C_2(\alpha) - R_T^L + \varepsilon \\
&= \varepsilon > 0.
\end{aligned}
$$

The buyer's payoff is $q_1(\alpha) - p_1(\alpha) = q_1(\alpha) - C_1(\alpha) - R_T^L + R_C^L$.

When $\alpha < \tilde{\alpha}$, corruption rent dominates technological rent and firm 2 wins at the equilibrium. Firm 1 chooses $p_1 = C_1(\alpha)$ and received score $S_1 = \alpha q_1(\alpha) - C_1(\alpha)$. Firm 2 chooses $p_2(\alpha) =$

$C_2(\alpha) - R_T^L + R_C^L - \varepsilon$ to match firm 1's score and slightly over-bids it:

$$\begin{aligned} S_2 - S_1 &= \alpha q_2(\alpha) - p_2(\alpha) + \alpha m - [\alpha q_1(\alpha) - C_1(\alpha)] \\ &= \alpha q_2(\alpha) - C_2(\alpha) + R_T^L - \alpha m + \varepsilon + \alpha m - \alpha q_1(\alpha) + C_1(\alpha) \\ &= \alpha q_2(\alpha) - \alpha q_1(\alpha) + C_1(\alpha) - C_2(\alpha) + R_T^L + \varepsilon \\ &= \varepsilon > 0 \end{aligned}$$

The buyer's payoff is $q_2(\alpha) - p_2(\alpha) = q_2(\alpha) - C_2(\alpha) + R_T^L - R_C^L$. *Q.E.D.*

Proof of Theorem 1: Because both the shape of payoff function and its jump point both depend on the parameter m, it is not trivial to fully express α^* as function of parameter x and m. But we can characterize it by the following analysis:

(1) We first discuss some properties of U_A^L and U_B^L. Recall

$$\begin{aligned} U_A^L(\alpha) &= x\left[q_1(\alpha) - C_1(\alpha) - R_T^L - R_C^L\right] + (1-x)\left[q_1(\alpha) - C_1(\alpha) - R_T^L + R_C^L\right], \\ &= q_1(\alpha) - C_1(\alpha) - R_T^L + (1-2x)R_C^L \\ &= U_H^L(\alpha) + (1-2x)R_C^L. \end{aligned}$$

It has first and second order derivative $U_A^{L'}(\alpha) = U_H^{L'}(\alpha) - (2x-1)m$ and $U_A^{L''}(\alpha) = U_H^{L''}(\alpha) < 0$ (Proposition 2). Therefore, U_A^L has a unique maximum at $\alpha_A(m) \equiv \arg\max_{\alpha \in [0,\infty)} U_A^L(\alpha)$ because it is strictly concave. Because α_H is determined by FOC $U_H^{L'}(\alpha) = 0$ and α_A is determined by $U_H^{L'}(\alpha) - (2x-1)m = 0$, we know that for $x > \frac{1}{2}$, $\alpha_A < \alpha_H$ and α_A decreases in m; for $x < \frac{1}{2}$, $\alpha_A > \alpha_H$ and increases in m.

U_B^L is not generally concave. But U_B^L is bounded above, continuous, and $\lim_{\alpha \to \infty} U_B^L(\alpha) = -\infty$ Therefore, U_B^L reaches its maximum (may not be unique) at a finite α, we define the smallest among these maximum points as $\alpha_B \equiv \arg\max_{\alpha \in [0,\infty]} U_B^L(\alpha)$.

(2) $\forall\alpha \in [0,\tilde{\alpha}]$, $U_A^L(\alpha) > U_B^L(\alpha)$.

$$\begin{aligned} & U_A^L(\alpha) - U_B^L(\alpha) \\ = \; & (1-x)\left[q_1(\alpha) - C_1(\alpha) - R_T^L + R_C^L - q_2(\alpha) - C_2(\alpha) - R_T^L + R_C^L\right], \\ = \; & (1-x)\left\{\underbrace{q_1(\alpha) - C_1(\alpha) - [q_2(\alpha) - C_2(\alpha)]}_{>0} + 2\underbrace{\left(R_C^L - R_T^L\right)}_{\geq 0 \text{ for } \alpha\leq\tilde{\alpha}}\right\} > 0. \end{aligned}$$

(3) When the scope of quality manipulation is relatively small.

Define $\bar{m}^L$ as the solution of $\tilde{\alpha}(m) = \alpha_A(m)$, then if $m \leq \bar{m}^L$, $\tilde{\alpha} \leq \alpha_A$; and if $m > \bar{m}^L$, $\tilde{\alpha} > \alpha_A$. Because U_A^L is defined on $[\tilde{\alpha},\infty)$, when $m \leq \bar{m}^L$, the maximum of $U_A^L(\alpha)$ can be achieved, $\alpha^* = \alpha_A$.

As an extreme case, if $m \to 0$, $\tilde{\alpha} \to 0$, $\alpha_A \to \alpha_H$, $U_A^L(\alpha_A) \to U_H^L(\alpha_H)$, which is the payoff with honest agent.

(4) When the scope of quality manipulation gets large.

If $m > \bar{m}^L$, $\tilde{\alpha} > \alpha_A$, the maximum of U_A^L cannot be achieved. By property shown in (1) and (2), $U_B^L(\alpha_B) < U_A^L(\alpha_B) \leq U_A^L(\alpha_A)$ and U_A^L is decreasing on $[\alpha_A,\infty)$, there exists a unique $\hat{\alpha} > \alpha_A$ such that $U_A^L(\hat{\alpha}) = U_B^L(\alpha_B)$.

The optimal α^* now depends on the relative magnitude of $\tilde{\alpha}$ and $\hat{\alpha}$.

If $\tilde{\alpha} \leq \hat{\alpha}$, $U_A^L(\tilde{\alpha}) \geq U_A^L(\hat{\alpha}) = U_B^L(\alpha_B)$, then $\alpha^* = \tilde{\alpha}$. This α^* is now greater than α_A, which means that the buyer still induce the efficient firm to win the contract but need to use a larger quality weight.

If $\tilde{\alpha} > \hat{\alpha}$, $\forall\alpha \in [\tilde{\alpha},\infty)$, $U_A^L(\alpha) < U_A^L(\tilde{\alpha}) < U_A^L(\hat{\alpha}) = U_B^L(\alpha_B)$, the optimal solution is $\alpha^* = \alpha_B$. It means that the buyer allows whichever the firm be corrupted to win the contract. *Q.E.D.*

Proof of Corollary 1: The parametric example in Section 3, illustrated in Figure 1.2, is an example.

Proof of Proposition 4: The proof is straight forward. When firm 1 is corrupted, firm 2 bid at its cost $C_2(q)$, firm 1 can match and slightly over-bid it by $p_1 = C_2(q) - \varepsilon$. The buyer thus receives a distorted quality compare to no corruption case: $q - m - p_2(q) = q - m - C_2(q)$.

When firm 2 is corrupted, the outcome is determined by relative magnitude of $C_1(q)$ and $C_2(q-m)$. When $C_1(q) \le C_2(q-m)$, technological rent still dominates, firm 1 wins by slightly over-bidding firm 2 at its cost $C_2(q-m)-\varepsilon$. The buyer receives the non-distorted quality q produced by firm 1 at a lower cost, $q - C_2(q-m)$. On the other hand, when $C_1(q) > C_2(q-m)$, corruption rent now dominates technological rent, firm 2 wins for sure by slightly over-bidding firm 2 at its cost $C_1(q)-\varepsilon$. The buyer receives the project from firm 2 with distorted quality. The payoff is then $q-m-C_1(q)$. *Q.E.D.*

Proof of Lemma 4: (i) We first show that as the cost increases, the difference of corresponding qualities also increases. To see this, by assumption $C_{q\theta} > 0$, then

$$\frac{\partial^2 f}{\partial c \partial \theta} = \frac{\partial}{\partial \theta}\left(\frac{1}{\frac{\partial C}{\partial q}}\right) = -\frac{\frac{\partial^2 C}{\partial q \partial \theta}}{(\frac{\partial C}{\partial q})^2} = -\frac{C_{q\theta}}{(\frac{\partial C}{\partial q})^2} < 0.$$

Because $\theta_2 > \theta_1$, for any $c' > c > 0$, $\frac{\partial^2 f}{\partial c \partial \theta} < 0$ implies

$$\frac{\partial\left[f(c,\theta_2)-f(c,\theta_1)\right]}{\partial c} < 0 \Rightarrow f(c',\theta_2)-f(c,\theta_2) < f(c',\theta_1)-f(c,\theta_1),$$

$$\Rightarrow f(c,\theta_1)-f(c,\theta_2) < f(c',\theta_1)-f(c',\theta_2) \Rightarrow q_1-q_2 < q_1'-q_2'.$$

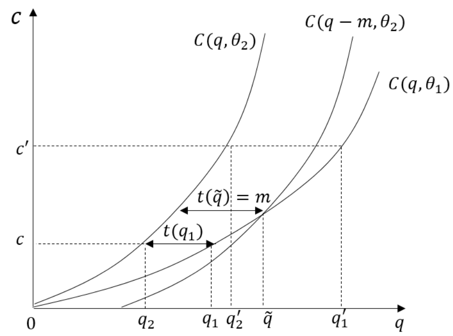
Therefore, quality difference $f(\cdot,\theta_1)-f(\cdot,\theta_2)$ is increasing. Suppose firm 1 is producing at quality $0 < q < \sup_{c\in[0,\infty)}\left[f(c,\theta_1)-f(c,\theta_2)\right]$, then there exists a unique *quality difference* t such that $C_1(q) = C_2(q-t)$. Define this quality difference t as a function q, it follows $\frac{\partial^2 f}{\partial c \partial \theta} < 0$ that $t(\cdot)$ increases in q. By continuity of $t(\cdot)$, for $0 < m < \sup_{c\in[0,\infty)} t(q)$, there exists a unique $\tilde{q} > 0$ such that $t(\tilde{q}) = m$, then $C_1(\tilde{q}) = C_2(\tilde{q}-t(\tilde{q})) = C_2(\tilde{q}-m)$.

(ii) Because t is increasing in q, for $q \ge \tilde{q}$, $t(q) \ge t(\tilde{q}) = m$, $C_2(q-m) = C_2(q-t(\tilde{q})) \ge C_2(q-t(q)) = C_1(q)$; for $q < \tilde{q}$, $t(q) < t(\tilde{q})$, $C_2(q-m) = C_2(q-t(\tilde{q})) < C_2(q-t(q)) = C_1(q)$.

(iii) Straightforward by monotonicity of $t(\cdot)$. *Q.E.D.*

Proof of Theorem 2: We characterize q^* by the following steps:

Figure 3.7: Illustration of Lemma 4



(1) Payoff functions U_A^M and U_B^M are strictly concave.

$$\begin{aligned} U_A^{M'}(q) &= 1 - xC_2'(q) - (1-x)C_2'(q-m) \\ U_A^{M''}(q) &= -x\underbrace{C_2''(q)}_{>0} - (1-x)\underbrace{C_2''(q-m)}_{>0} < 0. \end{aligned}$$

$$\begin{aligned} U_B^{M'}(q) &= 1 - xC_2'(q) - (1-x)C_1'(q) \\ U_B^{M''}(q) &= -x\underbrace{C_2''(q)}_{>0} - (1-x)\underbrace{C_1''(q)}_{>0} < 0. \end{aligned}$$

Therefore, we can define $q_A \equiv \arg\max_{q\in[0,\infty)} U_A^M(q)$ and $q_B \equiv \arg\max_{q\in[0,\infty)} U_B^M(q)$.

(2) For any $q \in [0, \tilde{q}]$, $U_A^M(q) > U_B^M(q)$.

$$\begin{aligned} & U_A^M(q) - U_B^M(q) \\ =\ & (1-x)\left[C_2(q) - C_2(q-m)\right] - xm - (1-x)\left[C_2(q) - C_1(q)\right] + m \\ =\ & (1-x)\left[\underbrace{C_1(q) - C_2(q-m)}_{\geq 0\ for\ q\leq\tilde{q}} + m\right] > 0 \end{aligned}$$

(3) When the scope of quality manipulation is relatively small.

Define $\bar{m}^M$ as the solution of $\tilde{q}(m) = q_A(m)$, then if $m \leq \bar{m}^M$, $\tilde{q} \leq q_A$; and if $m > \bar{m}^M$, $\tilde{q} > q_A$.

When $m \leq \bar{m}^L$, the maximum $U_A^M(q_A)$ can be achieved and by (2) $U_A^M(q_A) > U_B^M(q_A)$, therefore $q^* = q_A$.

(4) When the scope of quality manipulation gets large.

If $m > \bar{m}^M$, $\tilde{q} > q_A$, the maximum of U_A^M cannot be achieved. By property shown in (1) and (2), $U_B^M(q_B) < U_A^M(q_B) \leq U_A^M(q_A)$ and U_A^M is decreasing on $[\alpha_A, \infty)$, there exists a unique $\hat{q} > q_A$ such that $U_A^M(\hat{q}) = U_B^M(q_B)$.

The optimal q^* now depends on the relative magnitude of $\tilde{q}$ and $\hat{q}$.

If $\tilde{q} \leq \hat{q}$, $U_A^M(\tilde{q}) \geq U_A^M(\hat{q}) = U_B^M(q_B)$, then $q^* = \tilde{q}$.

If $\tilde{q} > \hat{q}$, $\forall q \in [\tilde{q}, \infty)$, $U_A^M(\alpha) < U_A^M(\tilde{\alpha}) < U_A^M(\hat{\alpha}) = U_B^M(\alpha_B)$, then $q^* = q_B$. *Q.E.D.*

Proof of Corollary 2: The parametric example in Section 3, illustrated in Figure 1.3, is sufficient to guarantee the Corollary holds.

Proof of Theorem 3: Recall payoff functions (1.3) and (1.6). Both of them are continuous functions of m. We prove the Theorem by showing that $\partial U^L/\partial m\big|_{m=0} > 0$ and $\partial U^M/\partial m\big|_{m=0} > 0$ for any $x < \frac{1}{2}$. When m is small, optimal scoring rule induce outcome A under both L and M.

$$\begin{aligned}
\frac{\partial U^L(\alpha^*(x,m))}{\partial m} &= \frac{\partial U_A^L(\alpha^*(x,m))}{\partial m} \\
&= \frac{\partial}{\partial m}\left\{U_H^L(\alpha^*) + (1-2x)\alpha^* m\right\} \\
&= \frac{dU_H^L}{d\alpha}\frac{\partial \alpha^*}{\partial m} + (1-2x)\alpha^* + (1-2x)m\frac{\partial \alpha^*}{\partial m}.
\end{aligned}$$

As $m \to 0$, because $dU_H^L/d\alpha \to 0$, $1-2x > 0$, and $\partial \alpha^*/\partial m > 0$, we have $\partial U^L/\partial m\big|_{m=0} > 0$.

$$\begin{aligned}
\frac{\partial U^M(q^*(x,m))}{\partial m} &= \frac{\partial U_A^M(q^*(x,m))}{\partial m} \\
&= \frac{\partial}{\partial m}\left\{x\left[q^* - m - C_2(q^*)\right] + (1-x)\left[q^* - C_2(q^* - m)\right]\right\} \\
&= (1-x)C_2'(q^* - m) - x
\end{aligned}$$

Let $m \to 0$, then $q^*(x,m) \to q_H$, which is determined by first-order condition $U'_{M,H}(q) = 0$, so $C'_2(q_H) = 1$. Hence, we have $\partial U^M(q^*(x,m))/\partial m\big|_{m=0} = (1-x)C'_2(q_H) - x = 1 - x - x > 0$, for any $x < \frac{1}{2}$. *Q.E.D.*

Verification of Three Results of the Parametric Example: For clarity of expression, we show the analytical result when $x = 0$.[10] The optimal quality weight[11]

$$\alpha^* = \begin{cases} \alpha_A = \frac{\theta(1+2m)}{2\theta-1}, & \text{for } 0 \le m \le \frac{\theta-1}{6\theta-2}, \\ \tilde{\alpha} = \frac{4\theta m}{\theta-1}, & \text{for } \frac{\theta-1}{6\theta-2} < m \le \hat{m}^L, \\ \alpha_B = \frac{1-2\theta m}{2-\theta}, & \text{for } \hat{m}^L < m \le \frac{1}{2\theta}. \end{cases}$$

The buyer's payoff of L

$$U^L(\alpha^*(m,0)) = \begin{cases} U_A^L(\alpha_A) = \frac{\theta(1+2m)^2}{4(2\theta-1)}, & \text{for } 0 \le m \le \frac{\theta-1}{2\theta(\sqrt{\theta}+1)}, \\ U_A^L(\tilde{\alpha}) = \frac{2\theta(\theta-1)m-4\theta^2m^2}{(\theta-1)^2}, & \text{for } \frac{\theta-1}{6\theta-2} < m \le \hat{m}^L, \\ U_B^L(\alpha_B) = \frac{(1-2\theta m)^2}{4\theta(2-\theta)}, & \text{for } \hat{m}^L < m \le \frac{1}{2\theta}, \\ 0 & \text{for } m > \frac{1}{2\theta}. \end{cases}$$

[10]Consider again the extreme case that the inefficient firm is always corrupted, i.e. $x = 0$. From the buyer's point of view, it is the most desirable situation because the efficient firm's technological rent is being eroded to the largest extent.

[11] The second cutoff value $\hat{m}^L$ can be found by solving $U_B^L(\alpha_B) = U_A^L(\tilde{\alpha})$. Here $\hat{m}^L = \frac{-b-\sqrt{b^2-4ac}}{2a}$, where $a = 12\theta^4 - 24\theta^3 - 4\theta^2$, $b = -8\theta^4 + 28\theta^3 - 24\theta^2 + 4\theta$, $c = -(\theta-1)^2$.

The optimal minimum-quality[12]

$$q^* = \begin{cases} q_A = \frac{1+2\theta m}{2\theta} & \text{for } 0 \le m \le \frac{\theta-1}{2(\theta+\sqrt{\theta})}, \\ \tilde{q} = \frac{m(\theta+\sqrt{\theta})}{\theta-1}, & \text{for } \frac{\theta-1}{2(\theta+\sqrt{\theta})} < m \le \hat{m}^M, \\ q_B = \frac{1}{2}, & \text{for } \hat{m}^M < m \le \frac{1}{4}. \end{cases}$$

$$U^M(q^*(m,0)) = \begin{cases} U_A^M(q_A) = \frac{1}{4\theta} + m, & \text{for } 0 \le m \le \frac{\theta-1}{2(\theta+\sqrt{\theta})}, \\ U_A^M(\tilde{q}) = \frac{(\theta+\sqrt{\theta})m}{\theta-1} - \frac{\theta(\sqrt{\theta}+1)^2 m^2}{(\theta-1)^2}, & \text{for } \frac{\theta-1}{2(\theta+\sqrt{\theta})} < m \le \hat{m}^M, \\ U_B^M(q_B) = \frac{1}{4} - m, & \text{for } \hat{m}^M < m \le \frac{1}{4}, \\ 0 & \text{for } m > \frac{1}{4}. \end{cases}$$

Figure 1.4 illustrate these two payoff functions. When $m \in (\underline{m}, \overline{m})$, $U^L(\alpha^*(m,0)) < U^M(q^*(m,0))$. Specifically, $\underline{m} = \frac{(\theta-1)(\theta-\sqrt{\theta})}{3\theta^2 - 2\theta\sqrt{\theta} - \theta}$, $\overline{m} = \frac{-b-\sqrt{b^2-4ac}}{2a}$, where $a = -4\theta^2\left[(\sqrt{\theta}+1)^2(2-\theta) + (\theta-1)^2\right]$, $b = \left[4\theta(\theta+\sqrt{\theta})(\theta-1)(2-\theta) + 4\theta(\theta-1)^2\right]$, and $c = -(\theta-1)$. Figure 1.5 depict the payoffs under L and M when x and θ various values, which shows that the interval of m that makes M dominating L is larger for smaller x and greater θ.

Proof of Theorem 5: Given Lemma 1, firm's choice of its quality can be separated from its price. Therefore we can consider best responses and equilibrium pricing strategies as functions of other firm's prices along.

Denote firm i's choice of quality as $q_i = \arg\max \alpha q - C(q, \theta_i)$ and its equilibrium costs as $c_i = C(q_i, \theta_i)$. We use subscripts $i = 1, 2$ indicate firms; subscript 0 or m indicates firm i is corrupted or not corrupted. Score of firm i is a function of its price. If firm i is not corrupted, $s_{i0}(p_{i0}) = \alpha q_i^* - p_{i0}$; if firm i is corrupted, $s_{im}(p_{im}) = \alpha(q_i^* + m) - p_{im}$. The equilibrium prices contain each firm's pricing strategy when it is corrupted or not under two sets of parameter values.

[12] The second cutoff value $\hat{m}^M$ can be found by solving $U_B^M(\alpha_B) = U_A^M(\tilde{\alpha})$. Here $\hat{m}^M = \frac{-b-\sqrt{b^2-4ac}}{2a}$, where $a = -\frac{\theta(\sqrt{\theta}+1)^2}{(\theta-1)^2}$, $b = \frac{2\theta+\sqrt{\theta}-1}{\theta-1}$, $c = -\frac{1}{4}$.

(1) Uncorrupted firm 2

It has no opportunity to win. At the equilibrium, $p_{20}(p_{10}, p_{1m}) = c_2$ by regular Bertrand equilibrium derivation.

(2) Corrupted firm 2

It has the opportunity to win a uncorrupted firm 1 with positive profit when $p_{10} > c_2 + \alpha(q_1 - q_2) - \alpha m$. Therefore the corrupted firm 2's best response is

$$p_{2m}(p_{10}) = \begin{cases} c_2, & \text{if } p_{10} \leq c_2 + \alpha(q_1 - q_2) - \alpha m \\ p_{10} - \varepsilon, & \text{if } p_{10} > c_2 + \alpha(q_1 - q_2) - \alpha m \end{cases}$$

(3) Corrupted firm 1

A corrupted firm 1 has both technological and corruption rent and therefore will surely wins the contract. Its best response is

$$p_{2m}(p_{20}) = \begin{cases} c_1, & \text{if } p_{20} \leq c_1 - \alpha(q_1 - q_2) - \alpha m \\ p_{20} - \varepsilon, & \text{if } p_{20} > c_1 - \alpha(q_1 - q_2) - \alpha m \end{cases}$$

(4) Uncorrupted firm 1

This case is not trivial. With probability ϕ, its opponent is not corrupted. Then firm 1 can surely win by its technological rent. With probability $1 - \phi$, his opponent is corrupted, winning or not depends on value of parameters. Therefore firm 1's payoff consists of three cases depending on relative magnitude of its score and firm 2's score

$$E\pi_{10}(p_{10}, p_{20}, p_{2m}) = \begin{cases} 0, & \text{if } s_{10} < s_{20}, \\ \phi(p_{10} - c_1), & \text{if } s_{20} \leq s_{10} < s_{2m}, \\ p_{10} - c_1, & \text{if } s_{10} \geq s_{2m}, \end{cases}$$

where $\phi \equiv \frac{1-x_1-x_2}{1-x_1}$ denotes firm 1's belief of the probability that firm 2 is not corrupted.

The basic choice of firm 1 is whether it choose a *conservative pricing strategy* that only beats uncorrupted firm 2, or an *aggressive pricing strategy* that beats both a corrupted firm 2 or a uncorrupted firm 2. When $s_{2m} > \alpha p_1 - c_1$, firm 1 cannot beat a corrupted firm 2, therefore picks the highest price beating a uncorrupted firm 2. When $s_{2m} \leq \alpha p_1 - c_1$, firm 1 has the opportunity to beat a corrupted firm 2. Whether using conservative or aggressive strategy depends on the profit upon winning. Only when p_{2m} is high enough, firm 1 will choose aggressive price, The threshold condition is given by $p_{2m} - \phi p_{20} > (1-\phi)\left[c_1 - \alpha(q_1 - q_2)\right] + \alpha m$. The best response of uncorrupted firm 1 is

$$p_{10}(p_{20}, p_{2m}) = \begin{cases} p_{20} + \alpha(q_1 - q_2) - \varepsilon, & \text{if } p_{2m} \leq c_1 - \alpha(q_1 - q_2) + \alpha m, \\ p_{20} + \alpha(q_1 - q_2) - \varepsilon, & \text{if } p_{2m} > c_1 - \alpha(q_1 - q_2) + \alpha m, \\ & \text{and } p_{2m} - \phi p_{20} \leq (1-\phi)\left[c_1 - \alpha(q_1 - q_2)\right] + \alpha m \\ p_{2m} + \alpha(q_1 - q_2) - \alpha m - \varepsilon, & \text{if } p_{2m} > c_1 - \alpha(q_1 - q_2) + \alpha m, \\ & \text{and } p_{2m} - \phi p_{20} > (1-\phi)\left[c_1 - \alpha(q_1 - q_2)\right] + \alpha m \end{cases}$$

(5) Given parameter m and α, the equilibrium is determined by the solution of the system of four best responses. We can show that when corruption rent is small ($\alpha m \leq (1-\phi)R_T^L$), there exists a pure strategy Bayesian Nash equilibrium, where $p_{10} = c_1 + R_T^L - \alpha m$, $p_{1m} = c_1 + R_T^L(+\alpha m$, $p_{20} = c_2$, and $p_{2m} = c_2$.

When corruption rent is large ($\alpha m > (1-\phi)R_T^L$), there is no pure strategy Bayesian Nash equilibrium. We can show that $p_{1m} = c_1 + R_T^L + \alpha m$ and $p_{20} = c_2$, while p_{10} and p_{2m} are given by some distribution function defined on the continuous strategy spaces, shown in the Theorem. *Q.E.D.*

We skip the proof of Theorem 6 because it is quite similar to the proof of Theorem 6.

Proof of Lemma 5: For the minimization problem $\min_{\mathbf{q}} C(\mathbf{q}, \theta)$ such that $V(\mathbf{q}) = v$, the Lagrangian expression is

$$\mathscr{L} = C(\mathbf{q},\theta) - \lambda(V(\mathbf{q}) - v).$$

The first-order condition yields a system of these $L+1$ equations of λ and $\mathbf{q}$

$$\begin{cases} \frac{\partial}{\partial \mathbf{q}} C(\mathbf{q},\theta) - \lambda \frac{\partial}{\partial \mathbf{q}} V(\mathbf{q}) = \mathbf{0}_{L\times 1}, \\ V(\mathbf{q}) - v = 0 \end{cases}.$$

By assumption CF, $C(\cdot,\theta)$ is strictly convex in $\mathbf{q}$, matrix $\frac{\partial^2 C(\mathbf{q},\theta)}{\partial \mathbf{q}' \partial \mathbf{q}}$ is positive definite. By assumption QL, $V(\mathbf{q})$ is weakly concave, $\frac{\partial^2 V(\mathbf{q})}{\partial \mathbf{q}' \partial \mathbf{q}}$ is negative semi-definite, there is a unique solution to this system equations, denoted as $\mathbf{q}(v|\theta)$ and $\lambda(v|\theta)$. The value function of the minimization problem is $\tilde{C}(v,\theta) = C(\mathbf{q}(v|\theta),\theta)$. By the Maximum Theorem (Berge, 1963), it is single-valued and continuous in v.

By envelop theorem, the value function satisfies $\tilde{C}_v = \lambda$. Plug $\mathbf{q}(v|\theta)$ into the constraint $V(\mathbf{q}(v|\theta)) = v$. Differentiate with respect to v implies $\frac{\partial}{\partial \mathbf{q}'} V(\mathbf{q}(v|\theta))\mathbf{q}_v = 1$. Therefore, $\frac{\partial}{\partial \mathbf{q}} C(\mathbf{q}(v|\theta),\theta) - \lambda(v|\theta)\frac{\partial}{\partial \mathbf{q}} V(\mathbf{q}(v|\theta)) = \mathbf{0}$ implies $\lambda(v|\theta) = \frac{\partial}{\partial \mathbf{q}} C(\mathbf{q}(v|\theta),\theta) / \frac{\partial}{\partial \mathbf{q}} V(\mathbf{q}(v|\theta)) = \frac{\partial}{\partial \mathbf{q}} C(\mathbf{q}(v|\theta),\theta)\mathbf{q}_v = \tilde{C}_v > 0$.

To show $\tilde{C}_{vv} > 0$ is equivalent to show $\lambda_v(v|\theta) > 0$. Differentiate the first-order condition above with respect to v:

$$\begin{cases} \frac{\partial}{\partial v}(\frac{\partial \mathscr{L}}{\partial \mathbf{q}}) = \frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} C(\mathbf{q},\theta)\mathbf{q}_v - \lambda \frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} V(\mathbf{q})\mathbf{q}_v - \frac{\partial}{\partial \mathbf{q}} V(\mathbf{q})\lambda_v(v|\theta) = 0 \\ \frac{\partial}{\partial v}(\frac{\partial \mathscr{L}}{\partial \lambda}) = \frac{\partial}{\partial \mathbf{q}} V(\mathbf{q})\mathbf{q}_v - 1 = 0 \end{cases}$$

The first L equation yields

$$\frac{\partial}{\partial \mathbf{q}} V(\mathbf{q})\lambda_v(v|\theta) = \left[\frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} C(\mathbf{q},\theta) - \lambda \frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} V(\mathbf{q})\right]\mathbf{q}_v.$$

Premultiply by $\mathbf{q}_v^T$,

$$\mathbf{q}_v^T \frac{\partial}{\partial \mathbf{q}} V(\mathbf{q}) \lambda_v(v|\theta) = \mathbf{q}_v^T \left[\frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} C(\mathbf{q},\theta) - \lambda \frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} V(\mathbf{q}) \right] \mathbf{q}_v,$$

because $\frac{\partial}{\partial \mathbf{q}} V(\mathbf{q})\mathbf{q}_v = 1$ and $\frac{\partial^2 \mathscr{L}}{\partial \mathbf{q}' \partial \mathbf{q}} = \frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} C(\mathbf{q},\theta) - \lambda \frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} V(\mathbf{q})$ is positive definite (PD), we have

$$\lambda_v(v|\theta) = \mathbf{q}_v^T \underbrace{\left[\frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} C(\mathbf{q},\theta) - \lambda \frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} V(\mathbf{q}) \right]}_{PD} \mathbf{q}_v > 0$$

Therefore, $\tilde{C}_v = \lambda(v|\theta) > 0$ and $\tilde{C}_{vv} = \lambda_v(v|\theta) > 0$. *Q.E.D.*

Proof of Theorem 7: (2.2) holds as as a special case by taking $m = 0$ in the proof below. Problem (2.6) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following $s(\cdot)$, a generic firm solves $\max_s (k-s)\Pr(\text{win}|s) = (k-s)[F_K(s^{-1}(s))]^{n-1}$. The first-order condition yields $(k-s)(n-1)[F_K(s^{-1}(s))]^{n-2} f_K(s^{-1}(s)) \frac{ds^{-1}(s)}{ds} - [F_K(s^{-1}(s))]^{n-1} = 0$. At the symmetric equilibrium, we have differential equation

$$s(k)(n-1)[F_K(k)]^{n-2} f_K(k) + s'(k)[F_K(k)]^{n-1} = k(n-1)[F_K(k)]^{n-2} f_K(k). \tag{3.6}$$

$$\Leftrightarrow \frac{d\left(s(k)[F_K(k)]^{n-1}\right)}{dk} = k(n-1)[F_K(k)]^{n-2} f_K(k).$$

Integrate on both side with boundary condition $s(\underline{k}) = 0$,

$$s(k) \quad = \quad \frac{\int_{\underline{k}}^{k} t(n-1)[F_K(t)]^{n-2} f_K(t)dt}{[F_K(k)]^{n-1}} = k - \frac{\int_{\underline{k}}^{k} [F_K(t)]^{n-1} dt}{[F_K(k)]^{n-1}}.$$

The last equality is obtained via integration by parts. The equilibrium price can be computed by $p(\theta) = V(q(\theta)) - s(K(\theta))$. *Q.E.D.*

Note that when θ is one-dimensional and $C_\theta < 0$, it reduces to (**??**) in Che (1993). By envelop theorem, from the value function $K(\theta) = \max V(q) - C(q,\theta)$, we have $K'(\theta) = C_\theta(q(\theta),\theta) < 0$. The lowest type $K(\overline{\theta}) = \min K(\theta) = \underline{k}$. $[1-F(\theta)]^{n-1} = [\Pr(\Theta > \theta)]^{n-1} = [\Pr(K(\Theta) < K(\theta))]^{n-1} = [F_K(k)]^{n-1}$. Let $k = K(\theta)$, $dk = K'(\theta)d\theta = C_\theta(q(\theta),\theta)d\theta$,

$$\begin{aligned} s(K(\theta)) &= K(\theta) - \frac{\int_{\underline{k}}^{K(\theta)} [F_K(t)]^{n-1} dt}{[F_K(k)]^{n-1}} \\ &= K(\theta) - \frac{\int_\theta^{\overline{\theta}} [1-F(\tau)]^{n-1} K'(\tau) d\tau}{[1-F(\theta)]^{n-1}} \\ &= K(\theta) - \frac{\int_\theta^{\overline{\theta}} [1-F(\tau)]^{n-1} C_\theta(q(\tau),\tau) d\tau}{[1-F(\theta)]^{n-1}}. \end{aligned}$$

Hence, $p(\theta) = V(q(\theta)) - s(K(\theta)) = C(q(\theta),\theta) + \int_\theta^{\overline{\theta}} C_\theta(q(\tau),\tau) \frac{[1-F(\tau)]^{n-1}}{} d\tau / [1-F(\theta)]^{n-1}$.

Proof of Corollary 7.1: $k_{(1:n-1)}$ has distribution function $F_K^{(1:n-1)}(t) = [F_K(t)]^{n-1}$, and density $f_K^{(1:n-1)}(t) = (n-1)[F_K(t)]^{n-2} f_K(t)$. If the winner has pseudotype k, the conditional expectation of the highest rival's pseudotype is

$$E[k_{(1:n-1)} | k_{(1:n-1)} < k] = \frac{\int_{\underline{k}}^{k} t(n-1)[F_K(t)]^{n-2} f_K(t) dt}{[F_K(k)]^{n-1}} = s(k),$$

which is equal to the score the winner will bid. At the equilibrium, the winner has pseudotype being the highest order statistic $k_{(1:n)}$, while the second highest bidder has pseudotype $k_{(2:n)}$, hence $E\left[s(k_{(1:n)})\right] = E\left[k_{(2:n)}\right]$. *Q.E.D.*

Proof of Corollary 7.2: Asker and Cantillon (2008) shows a straightforward proof. Suppose the minimum quality standard is set at q and the scoring rule represents the buyer's true preference. By Corollary 7.1, Expected utility of price-only auction is

$$\begin{aligned} V(q) - E\left[C(q,\theta_{(n-1:n)})\right] &= E\left[(V(q) - C(q,\theta))_{(2:n)}\right] \\ &\leq E\left[\max_q (V(q) - C(q,\theta))_{(2:n)}\right] = E\left[k_{(2:n)}\right], \end{aligned}$$

which is the expected utility in scoring auction. *Q.E.D.*

Proof of Theorem 9: (1) Quality

Suppose the corrupted firm with type θ bids (p', q') at some $q' \neq q_m$, we can show that by choosing q_m, the corrupted firm can always find a price p_m that yields a higher payoff upon winning. Let $p_m = V(q_m) - V(q') + p'$, then (p', q') and (p_m, q_m) have the same score because $S(q', p') = V(q') - p' = V(q_m) - p_m = S(q_m, p_m) = s$. These two bids has the same expected payoff $\Pr(\text{win}|s)$. Their expected payoffs satisfies

$$\begin{aligned}
\pi(p_m, q_m) - \pi(p', q') &= \left[p_m - C(q_m - m, \theta) - p' + C(q' - m, \theta)\right] \Pr(\text{win}|s) \\
&= \left[V(q_m) - V(q') + p' - C(q_m - m, \theta) - p' + C(q' - m, \theta)\right] \Pr(\text{win}|s) \\
&= \left[V(q_m) + C(q_m - m, \theta) - \left(V(q') - C(q' - m, \theta)\right)\right] \Pr(\text{win}|s) > 0,
\end{aligned}$$

because q_m is chosen by (3.1). The scoring rule being quasilinearity (additively separable) is essential for this result to hold.

(2) Score and price

Under assumption UA, all other firms pick their score according to (2.7), so the corrupted firm's pick its core according to

$$\max_{s_m} (k_m - s_m) \Pr(\text{win}|s_m) = (k_m - s_m) \left[F_K(s^{-1}(s_m))\right]^{n-1}.$$

Following the same step in getting (2.7), the corrupted firm choose its score according to $s(k_m) = k_m - \int_{\underline{k}}^{k_m} [F_K(t)]^{n-1} dt / [F_K(k_m)]^{n-1}$. The corresponding price is $p_m(\theta) = V(q_m(\theta)) - s(K_m(\theta)) = C(q_m(\theta) - m, \theta) + \int_{\underline{k}}^{K_m(\theta)} [F_K(t)]^{n-1} dt / [F_K(K_m(\theta))]^{n-1}$.

(3) For any $m > 0$, at the equilibrium, $q_m(\theta) > q(\theta)$, $K_m(\theta) > K(\theta)$, and $s(K_m(\theta)) > s(K(\theta))$.

The unique solution of quality choice of (2.2) and (3.1) are both determined by their first-order conditions. Suppose $\tilde{q}$ solves $V_q(q) = C_q(q, \theta)$. Because $C_{qq} > 0$, the cost function has increasing slope, $V_q(\tilde{q}) = C_q(\tilde{q}, \theta) > C_q(\tilde{q} - m, \theta)$. By assumption QL, $V_{qq} \leq 0$, the solution to $V_q(q) = C_q(q - m, \theta)$ must be strictly larger than $\tilde{q}$, therefore $q_m(\theta) > q(\theta)$.

The other two are straight-forward. Because $C_q > 0$, $C(q - m, \theta) < C(q, \theta)$ for all q and θ, $K_m(\theta) = \max_q V(q) - C(q - m, \theta) > \max_q V(q) - C(q, \theta) = K(\theta)$. The equilibrium score bidding function $s(\cdot)$ is increasing, hence $s(K_m(\theta)) > s(K(\theta))$. It is obvious that all three effects magnify as m increases. *Q.E.D.*

A sample bid[13]

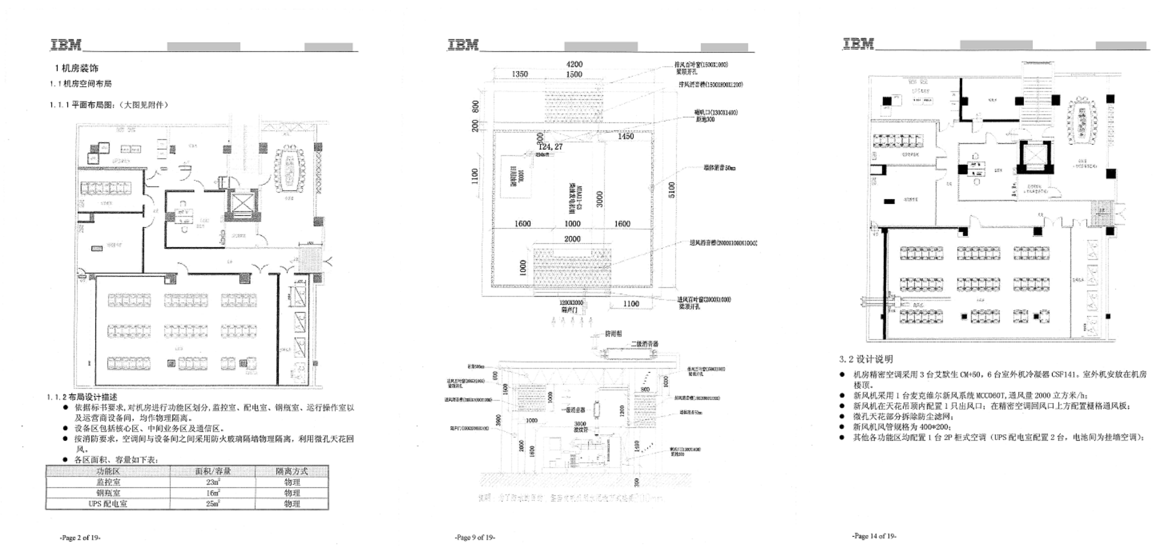
This is a bid of a server room construction project. The buyer is Bank of Dongguan, a regional bank centered at Dongguan, Guangdong province, China. The firm is IBM Engineering Technology (Shanghai) Co., Ltd.. The bid consists of a construction proposal and a detail list of items and their costs. The construction proposal is a 19-page document including standard of construction, condition of delivery, delivery date, equipment purchase plan, payment plan etc. Some selected pages are shown in Figure 3.8. The itemized price list is a 11-page spreadsheet. Table 3.4 shows its major categories, categorical prices, and total price (3,630,000 CNY).

[13]The author receives authorization to disclose the document for non-profit academic research purpose. The original document is in Chinese. All technical details are remain confidential and the relevant copyrights are owned by Bank of Dongguan and IBM Engineering Technology (Shanghai) Co., Ltd. The author declares that he has no relevant or material financial interests that relate to the research described in this paper.

Table 3.4: Summary of the Itemized Price List

Category	Price (CNY)	No. of Items
Data center room renovation	924,295	17
Main power distribution system	108,185	11
Auxiliary power distribution system	176,830	14
Uninterrupted power supply (UPS) system	913,680	13
Generators and environmental engineering	413,050	14
Air conditioning	99,170	11
Precision air conditioning	528,570	2
Cabinets and cabling system	242,230	9
Lightning protection	23,820	3
Room monitoring	185,120	43
Room bridging	15,050	4
Total	3,630,000	141

Figure 3.8: Selected Pages of the Construction Proposal



Bibliography

Ades, A. and R. Di Tella (1999). Rents, competition, and corruption. *The american economic review 89*(4), 982–993.

Ades, A. and R. D. Tella (1997). National champions and corruption: Some unpleasant interventionist arithmetic*. *The Economic Journal 107*(443), 1023–1042.

Arnold, B. C., N. Balakrishnan, and H. N. Nagaraja (1992). *A first course in order statistics*, Volume 54. Siam.

Aryal, G. and M. F. Gabrielli (2013). Testing for collusion in asymmetric first-price auctions. *International Journal of Industrial Organization 31*(1), 26–35.

Asker, J. (2010). A study of the internal organization of a bidding cartel. *The American Economic Review 100*(3), 724–762.

Asker, J. and E. Cantillon (2008). Properties of scoring auctions. *The RAND Journal of Economics 39*(1), 69–85.

Asker, J. and E. Cantillon (2010). Procurement when price and quality matter. *The Rand journal of economics 41*(1), 1–34.

Athey, S. and P. A. Haile (2002). Identification of standard auction models. *Econometrica 70*(6), 2107–2140.

Athey, S., J. Levin, and E. Seira (2011). Comparing open and sealed bid auctions: Evidence from timber auctions*. *The Quarterly Journal of Economics 126*(1), 207–257.

Bajari, P., S. Houghton, and S. Tadelis (2014). Bidding for incomplete contracts: an empirical analysis of adaptation costs. *The American Economic Review 104*(4), 1288–1319.

Bajari, P., R. McMillan, and S. Tadelis (2009). Auctions versus negotiations in procurement: an empirical analysis. *Journal of Law, Economics, and Organization 25*(2), 372–399.

Bajari, P. and S. Tadelis (2001). Incentives versus transaction costs: A theory of procurement contracts. *RAND Journal of Economics*, 387–407.

Bajari, P. and L. Ye (2003). Deciding between competition and collusion. *Review of Economics and Statistics 85*(4), 971–989.

Bajari, P. and J. Yeo (2009). Auction design and tacit collusion in fcc spectrum auctions. *Information Economics and Policy 21*(2), 90–100.

Baldwin, L. H., R. C. Marshall, and J.-F. Richard (1997). Bidder collusion at forest service timber sales. *Journal of Political Economy 105*(4), 657–699.

Berge, C. (1963). Topological spaces (oliver and boyd, edinburgh and london). *1st English edition*.

Branco, F. (1997). The design of multidimensional auctions. *The RAND Journal of Economics*, 63–81.

Burguet, R. and Y.-K. Che (2004). Competitive procurement with corruption. *RAND Journal of Economics*, 50–68.

Burguet, R. and M. K. Perry (2009). Preferred suppliers in auction markets. *The RAND Journal of Economics 40*(2), 283–295.

Burguet, R. and M. K. Perry (2014). Preferred suppliers in asymmetric auction markets. Technical report.

Celentani, M. and J.-J. Ganuza (2002). Corruption and competition in procurement. *European Economic Review 46*(7), 1273–1303.

Che, Y.-K. (1993). Design competition through multidimensional auctions. *The RAND Journal of Economics*, 668–680.

Chen-Ritzo, C.-H., T. P. Harrison, A. M. Kwasnica, and D. J. Thomas (2005). Better, faster, cheaper: An experimental analysis of a multiattribute reverse auction mechanism with restricted information feedback. *Management Science 51*(12), 1753–1762.

Compte, O., A. Lambert-Mogiliansky, and T. Verdier (2005). Corruption and competition in procurement auctions. *Rand Journal of Economics*, 1–15.

Dastidar, K. G. (2014). Scoring auctions with non-quasilinear scoring rules.

David, E., R. Azoulay-Schwartz, and S. Kraus (2006). Bidding in sealed-bid and english multi-attribute auctions. *Decision Support Systems 42*(2), 527–556.

Dimitri, N., G. Piga, and G. Spagnolo (2006). *Handbook of procurement*. Cambridge University Press.

Dini, F., R. Pacini, and T. Valletti (2006). Scoring rules. *Handbook of procurement*, 293–321.

Eun, D. J. (2016). Procurement with a no loss constraint: Evidence and a new mechanism from korea. *working paper*.

Feinstein, J. S., M. K. Block, and F. C. Nold (1985). Asymmetric information and collusive behavior in auction markets. *The American Economic Review*, 441–460.

Freixas, X., R. Guesnerie, and J. Tirole (1985). Planning under incomplete information and the ratchet effect. *The review of economic studies*, 173–191.

Graham, D. A., R. C. Marshall, and J.-F. Richard (1990). Differential payments within a bidder coalition and the shapley value. *The American Economic Review*, 493–510.

Guerre, E., I. Perrigne, and Q. Vuong (2000). Optimal nonparametric estimation of first-price auctions. *Econometrica 68*(3), 525–574.

Hanazono, M., J. Nakabayashi, and M. Tsuruoka (2015). Procurement auctions with general price-quality evaluation.

Harrington, J. E. (2008). Detecting cartels. *Handbook of antitrust economics 213*, 245.

Hendricks, K., R. Porter, and G. Tan (2008). Bidding rings and the winner's curse. *The RAND Journal of Economics 39*(4), 1018–1041.

Hendricks, K. and R. H. Porter (2007). An empirical perspective on auctions. *Handbook of Industrial Organization 3*, 2073–2143.

Huang, Y. and J. Xia (2016). Procurement auction under quality manipulation corruption. *working paper*.

Ingraham, A. T. (2005). A test for collusion between a bidder and an auctioneer in sealed-bid auctions. *Contributions in Economic Analysis & Policy 4*(1).

International, T. (2014). Curbing corruption in public procurement: a practical guide. *http://www.transparency.org/whatwedo/publication/*.

Ji, B. and Z. Fu (2010). Analysis of chinese bridge collapse accident cause in recent years. *China Civil Engineering Journal 43*, 495–498.

Kaplan, U., V. Marmer, A. Shneyerov, et al. (2016). Identifying collusion in english auctions. Technical report, Vancouver School of Economics.

Kawai, K. and J. Nakabayashi (2014). Detecting large-scale collusion in procurement auctions. *Available at SSRN 2467175*.

Klemperer, P. (2002). How (not) to run auctions: The european 3g telecom auctions. *European Economic Review 46*(4), 829–845.

Koning, P. and A. Van de Meerendonk (2014). The impact of scoring weights on price and quality outcomes: An application to the procurement of welfare-to-work contracts. *European Economic Review 71*, 1–14.

Krasnokutskaya, E., K. Song, and X. Tang (2014). The role of quality in on-line service markets.

Lengwiler, Y. and E. Wolfstetter (2006). Corruption in procurement auctions. *Available at SSRN 874705*.

Lewis, G. and P. Bajari (2011). Procurement contracting with time incentives: Theory and evidence. *The Quarterly Journal of Economics 126*(3), 1173–1211.

Li, D. Z. and M. Xu (2016). Competition in procurement auctions with corruption. *Working paper*.

Liu, N. and Y. Luo (2014). A nonparametric test of exogenous participation in first-price auctions. Technical report.

Lucking-Reiley, D. (1999). Using field experiments to test equivalence between auction formats: Magic on the internet. *American Economic Review*, 1063–1080.

Lundberg, S. et al. (2006). Auction formats and award rules in swedish procurement auctions. *Rivista di Politica Economica 96*(1), 91–114.

Ma, J., V. Marmer, and A. A. Shneyerov (2016). Inference for first-price auctions with guerre, perrigne, and vuong's estimator. *working paper*.

Maskin, E. S. and J. G. Riley (1985). Auction theory with private values. *The American Economic Review*, 150–155.

McAfee, R. P. and J. McMillan (1992). Bidding rings. *The American Economic Review*, 579–599.

Myerson, R. B. (1981). Optimal auction design. *Mathematics of operations research 6*(1), 58–73.

Nakabayashi, J. and Y. Hirose (2015). Structural estimation of the scoring auction model. *Unpublished paper. ISER, Osaka University, Japan*.

Nishimura, T. (2015). Optimal design of scoring auctions with multidimensional quality. *Review of Economic Design 19*(2), 117–143.

OECD (2013). Implementing the oecd principles for integrity in public procurement. *http://www.oecd-ilibrary.org/*.

Pesendorfer, M. (2000). A study of collusion in first-price auctions. *The Review of Economic Studies 67*(3), 381–411.

Porter, R. H. and J. D. Zona (1993). Detection of bid rigging in procurement auctions. *Journal of political economy*, 518–538.

Porter, R. H. and J. D. Zona (1999). Ohio school milk markets: An analysis of bidding. *The Rand Journal of Economics*, 263–288.

Rose-Ackerman, S. (1996). Democracy and corruption. *International Social Science Journal 48*(149), 365–380.

Tadelis, S. (2012). Public procurement design: Lessons from the private sector. *International Journal of Industrial Organization 30*(3), 297–302.

Tadelis, S. and P. Bajari (2006). Incentives and award procedures: competitive tendering vs. negotiations in procurement. *Handbook of Procurement, Dimitri, Pigo and Spagolo, Eds.*

Takahashi, H. (2015). Strategic design under uncertain evaluations: Structural analysis of design-build auctions. Technical report.

Tian, G. and C. Liu (2008). Limited corruption in sealed-bid auctions. *Economic Research Journal 3*, 130–141.

Topkis, D. M. (1978). Minimizing a submodular function on a lattice. *Operations research 26*(2), 305–321.

Tran, A. (2011). Which regulations reduce corruption? evidence from the internal records of a bribe-paying firm. Technical report, working paper.

Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *The Journal of finance 16*(1), 8–37.

Wang, M. and S. Liu (2014). Equilibrium bids in practical multi-attribute auctions. *Economics Letters 123*(3), 352–355.

Yoganarasimhan, H. (2013). The value of reputation in an online freelance marketplace. *Marketing Science 32*(6), 860–891.

Yoganarasimhan, H. (2015). Estimation of beauty contest auctions. *Marketing Science*.