

Estimation and Identification Issues in Monetary Policy Rules

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A dissertation
submitted in partial of fulfillment of the
requirement for the degree

Doctor of Philosophy

University of Washington

2015

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Program Authorized to Offer Degree:

Economics

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Abstract

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The dissertation explores the links between macroeconomic phenomena and monetary policy and to develop new econometric methods. In the first chapter, “Monetary Policy Rules and Macroeconomic Stability Revisited: Limited Information Approach under Identifying Restrictions, provides a new approach to limited information estimation consistent with the forward-looking monetary policy rule. Recently, the weak identification in the conventional estimation method has drawn attention to the estimation of a forward-looking monetary policy rule. This paper identified a particular range for the value of the concentration parameter, for which the generalized method of moments (GMM) suffers from the weak identification problem, while the proposed method does not. This implies that GMM estimation generates spurious weak identification in the estimation of a forward-looking monetary policy rule.

The proposed approach allows us to provide stronger messages to the estimation of a forward-looking monetary policy rule. The estimation results confirm a change of monetary policy in the U.S. In the 1960-1979 sample, the policy was inactive and it did not react sufficiently to the expected deviation of inflation from its target. In contrast, under the 1979-1997 sample monetary policy actively responds to the inflation with a high degree of interest smoothing.

The second chapter of the dissertation is the extension of the first chapter, “Estimation of a Time-varying Forward-looking Monetary Policy Rule: Limited Information Approach. In this chapter, I estimate a time-varying forward-looking monetary policy rule by considering

a time-varying structural vector auto-regression (VAR) model for the monetary transition mechanism. Assuming that the time variation comes from the coefficients and the variance covariance matrix, I illustrate this via modeling multivariate stochastic volatility. In a foundational paper, Primiceri (2005) estimated a time-varying structural VAR with stochastic volatility after assuming monetary policy shocks to be independent of any other innovations without forward-looking variables. Because agents are assumed to be rational, monetary policy changes can be incorporated into future forecasts. Kim and Nelson (2006) used a single equation to investigate the estimation of a forward-looking monetary policy rule in relation to the forward-looking behavior of agents. To account for the endogeneity, they suggested a two-step estimation technique based on the control function approach. However, as Chon and Kim (2014) argued, the error term in instrumenting equations for forward-looking variables follows moving-average (MA) dynamics, resulting in additional information loss. Consequently, this paper illustrates that one can recover this MA structure after considering the reduced-form of the time-varying VAR; the procedure suggested in this paper resolves the possible weak identification issues.

The third chapter of the dissertation is “Stock Market Reaction to Monetary Policy Changes: Identification through Heteroskedasticity with Markov-switching.” This paper investigates the estimation issues surrounding the response of asset prices to monetary policy changes. Because of the simultaneous relationship between stock prices and policy decisions, and because both react to numerous other variables, estimation of the impact of stock price to monetary policy action is difficult.

In this paper, I use the heteroskedastic structure of monetary policy shocks to identify stock market reactions to monetary policy changes following Rigobon and Sack (2004). Especially, in order to consider all possible sources which affect shifts in monetary policy shocks, such as the alteration of expectations about the future path of the monetary policy and a change in the timing of policy moves, I incorporate the Markov-switching framework to detect different state endogenously. The procedure proposed in this paper can reduce the potential bias caused by mis-specified timings in the shifts of monetary policy shocks and produce more precise estimate of the monetary policy actions on the stock market. Since the stock market is forward-looking, I focus on the surprised part of the policy actions within the

conventional event-study framework. The empirical finding tells us that the heteroskedasticity on event day may well be a consequence of the asymmetric effects on the different types of policy actions: expansionary policy vs. contractionary policy. Also, we found that the unanticipated 25-basis point increase would decrease 1.91 percent in the S&P 500 returns.

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my advisor Professor Chang-Jin Kim for the continuous support of my Ph.D study and research, for his patience, motivation, enthusiasm, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my Ph.D study.

Besides my advisor, I would like to thank the rest of my thesis committee: Professor Stephen Turnovsky, Eric Zivot, and Yu-chin Chen, for their encouragement, insightful comments, and hard questions.

I would also like to thank my husband and my soul mate, Jaeho Kim, supporting me spiritually throughout my research. Completing this work would have been all the more difficult were it not for his help and love.

Last but not the least, I would like to thank my family by giving encouragement and providing the moral and emotional support I needed to complete my thesis.

This dissertation is dedicated to my love, Jaeho Kim.

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Monetary Policy Rules and Macroeconomic Stability Revisited: Limited Information Approach Under Identifying Restrictions

1.1. Introduction

In estimating the forward-looking monetary policy rule, the full information method may not be optimal when we are not interested in estimating other structural equations in the system. For example, Fukač and Pagan (2010), Mavroeidis (2004), and Kurmann (2007) argue that the limited information method may sometimes be preferred, as it is robust with respect to specification errors in other structural equations that one is not estimating. Another advantage of this method is that one does not need to solve for the model prior to estimation. This is why generalized method of moments (GMM) is sometimes employed in estimating the policy reaction function.

Clarida et al. (2000) estimate the forward-looking monetary policy rule by employing GMM. They conclude that the policy reaction did not satisfy the conditions for determinacy prior to the Volcker era, violating the Taylor principle. They also find that the policy over the Volcker-Greenspan era (1979:Q3-1997:Q4 sample) did satisfy the Taylor principle. However, it is well known that GMM estimation of forward-looking models may be subject to weak identifiability of the model's parameters. Mavroeidis (2004) and Nason and Smith (2008) analyze potential sources of weak identifiability in these models, and Stock et al. (2002) provides a survey of issues related weak instruments andn weak identification in GMM. Stock and Wright (2000), Kleibergen (2005, 2007), and Kleibergen and Mavroeidis (2009) discuss identification-robust inferences based on GMM. Especially, Mavroeidis (2010) studied weak identification in the estimation of the Taylor rule by revisiting the original work by Clarida et al. (2000). He asserts that the anaylsis of monetary policy rule based on single-equation approaches may suffer from weak identification, which can lead biased estimators

¹ This chapter is based on the joint work with Chang-Jin Kim.

and inferences. Inoue and Rossi (2011) also confirm the weak identification problem in the Taylor rule based on GMM by proposing a test for weak identification in nonlinear models. However, an identification-robust method based on GMM often produces large confidence sets making inconclusive evidence for the Taylor rule estimation.

As Mavroeidis (2010) mentions, his analysis points out the shortcomings of the limited information method such as GMM. He suggests making use of further identifying assumptions when estimating the monetary policy rule. In this paper, we do this. By assuming that monetary policy shocks are uncorrelated with other macroeconomic shocks and that they do not have contemporaneous effects on output or inflation, we develop a new approach to limited information estimation of the policy rule as an alternative to GMM. Such an identifying assumption, albeit possibly controversial, is standard in the literature and has been extensively adopted.

The limited information method proposed in this paper is more efficient than GMM. The strength of the instruments is characterized in linear instrumental variables regression models by a unitless measure known as concentration parameter. Based on Monte Carlo experiments, we show that there exists a particular range of the concentration parameter such that our method does not suffer from the problem of weak identification, while GMM does. For this range of the concentration parameter, both the Wald test from our approach and the identification-robust test from GMM have correct size. Under the same situation, however, the power of the conventional Wald test from the proposed approach is considerably higher than that of the identification-robust test from GMM.

The proposed approach allows us to provide stronger messages to the estimation of a forward-looking monetary policy rule. The estimation results confirm a change of monetary policy in the U.S. In the 1960-1979 sample, the policy was inactive and it did not react sufficiently to the expected deviation of inflation from its target. In contrast, under the 1979-1997 sample monetary policy actively responds to the inflation with a high degree of interest smoothing. As a diagnostic check, we confirm the validity of the identifying assumptions we employ by showing that the disturbance of the estimated policy rule equation are serially uncorrelated. Furthermore, we show that the proposed approach does not suffer from the weak identification problem, based on a bootstrap-based Monte Carlo experiment.

The plan of this paper is as follow. In Section 2, we present a new approach to limited information estimation of the forward-looking monetary policy with the identifying assumptions. In Section 3, we perform Monte Carlo experiments in order to evaluate the performance of the proposed method in comparison with GMM. Section 4 discusses empirical results, and Section 5 concludes the paper.

1.2. A New Approach to Limited Information Estimation of the Forward-Looking Policy Rule

1.2.1. Underlying Model and Identifying Restrictions

We consider a model that consists of the monetary policy equation and the reduced-form equations for both inflation and the output gap, as given below:

$$i_t = \beta_0 + (1 - \beta_1)(\beta_\pi E_t[\pi_{t+1}] + \beta_x x_t) + \beta_1 i_{t-1} + \epsilon_t, \quad (1.1)$$

$$\pi_t = A_\pi + B'_\pi \tilde{Y}_{t-1} + e_{2,t}, \quad (1.2)$$

$$x_t = A_x + B'_x \tilde{Y}_{t-1} + e_{3,t}, \quad (1.3)$$

where i_t is the interest rate; π_t is inflation; x_t is the output gap. Equation (1.2) and (1.3) form a sub-VAR model for inflation and the output gap with $\tilde{Y}_{t-1} = [Y'_{t-1} \ Y'_{t-2} \ \dots \ Y'_{t-k}]'$, where $Y_t = [i_t \ \pi_t \ x_t]'$.

In deriving a new approach to limited information estimation of the policy rule in equation (1.1), we employ an identifying assumption that monetary policy shocks are uncorrelated with other macroeconomic shocks and that they do not have contemporaneous effects on output and inflation within the above framework. This assumption is standard in the literature and has been extensively adopted. For example, Rotemberg and Woodford (1997), Bernanke and Mihov (1998), Christiano et al. (1999, 2005), and Boivin and Giannoni (2006) employ the assumption in evaluating the effects of a monetary policy shock. More recently, Primiceri (2005) employ the same assumption in identifying the monetary policy rule from a time-varying structural VAR model of output, inflation and the interest rate.

An additional assumption that we employ is that a reduced form for the above structural model is given by a finite-order vector autoregressive (VAR) process, as given below:

$$Y_t = \Phi_0 + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \dots + \Phi_p Y_{t-p} + e_t, \quad e_t \sim i.i.d.N(0, \Omega), \quad (1.4)$$

where the last two rows of equation (1.4) are equations (1.2) and (1.3).

1.2.2. A Limited Information Approach Under Identifying Restrictions

We can rewrite equation (1.1) as:

$$i_t = \beta_0 + (1 - \beta_1)(\beta_\pi \pi_{t+1} + \beta_x x_t) + \beta_1 i_{t-1} + u_{t+1}, \quad (1.5)$$

where $u_{t+1} = \epsilon_t - (1 - \beta_1)(\beta_\pi \pi_{t+1} - E_t[\pi_{t+1}])$ is correlated with π_{t+1} . It is easy to see that, if monetary shocks do not have contemporaneous effects on the output gap, u_{t+1} is serially uncorrelated and it is not correlated with the output gap x_t . The instrumenting equation is given by:

$$\pi_{t+1} = c_1 + c_2' \tilde{Y}_{t-1} + \eta_{t+1}, \quad (1.6)$$

where $\tilde{Y}_{t-1} = [Y'_{t-1} \quad Y'_{t-2} \quad \dots \quad Y'_{t-p}]'$, and η_{t+1} follows an MA(1) process as given below:

$$\eta_{t+1} = g_{t+1} - \theta g_t, \quad g_{t+1} \sim i.i.d.(0, \sigma_g^2), \quad (1.7)$$

where g_{t+1} is serially uncorrelated.

GMM estimation of the equation (1.5) using \tilde{Y}_{t-1} as instruments is equivalent to a two-step estimation procedure. In the first step, we estimate equation (1.6) by OLS and obtain $\hat{\pi}_{t+1} = \hat{c}_1 + \hat{c}_2' \tilde{Y}_{t-1}$. In the second step, we estimate equation (1.5) by replacing π_{t+1} with $\hat{\pi}_{t+1}$. The first-step is inefficient, as autocorrelation in the error terms is ignored. Note that the second-step regression also induces autocorrelation in the error terms, as we have:

$$i_t = \beta_0 + (1 - \beta_1)(\beta_\pi \hat{\pi}_{t+1} + \beta_x x_t) + \beta_1 i_{t-1} + \tilde{u}_{t+1}, \quad (1.8)$$

where $\tilde{u}_{t+1} = u_{t+1} + (1 - \beta_1)\beta_\pi\hat{\eta}_{t+1}$. Here, $\hat{\eta}_{t+1} = \pi_{t+1} - \hat{\pi}_{t+1} = \pi_{t+1} - c_1 - c_2\tilde{Y}_{t-1}$ follows an MA(1) process. Ignoring autocorrelation in the \tilde{u}_{t+1} term in the second-step estimation is an additional source of inefficiency. We argue that the weak identification problem in GMM estimation or two-step estimation of the forward-looking monetary policy rule may be due to inefficient use of information.

In designing a new approach to limited information estimation of the policy rule in the equation (1.5), that u_{t+1} is serially uncorrelated is critical. In this case, it would be correlated with g_{t+1} , but not with g_t in equation (1.7). The identifying assumptions that we employ guarantee this. Thus, by considering an orthogonal projection of u_{t+1} on g_{t+1} , we have:

$$u_{t+1} = \gamma g_{t+1} + \omega_{t+1}, \quad (1.9)$$

where ω_{t+1} is not correlated with g_{t+1} ; and γ is a function of the correlation between u_{t+1} and g_{t+1} as well as the standard deviations of u_{t+1} and g_{t+1} . We can thus rewrite the equation (1.5) as:

$$i_t = \beta_0 + (1 - \beta_1)(\beta_\pi\hat{\pi}_{t+1} + \beta_x x_t) + \beta_1 i_{t-1} + \gamma g_{t+1} + \omega_{t+1}, \quad (1.10)$$

where, conditional on g_{t+1} , the disturbance term ω_{t+1} is not correlated with π_{t+1} or x_t .

We can therefore decompose the log likelihood function for equations (1.6),(1.7) and (1.10), as given below:

$$\begin{aligned} \ln L(\tilde{\Psi}_1, \tilde{\Psi}_2) &= \sum \ln(f(i_t, \pi_{t+1}, x_{t+1} | \tilde{\Psi}_1, \tilde{\Psi}_2, I_{t-1})) \\ &= \sum \ln(f(\pi_{t+1}, x_{t+1} | \tilde{\Psi}_1, I_{t-1}) f(i_t | \tilde{\Psi}_2, \pi_{t+1}, x_{t+1}, I_{t-1})) \\ &= \sum \ln(f(\pi_{t+1}, x_{t+1} | \tilde{\Psi}_1, I_{t-1})) + \sum \ln(f(i_t | \tilde{\Psi}_2, \pi_{t+1}, x_{t+1}, I_{t-1})), \end{aligned} \quad (1.11)$$

where $\tilde{\Psi}_1$ refers to the vector of parameters associated with equation (1.6) and (1.7); $\tilde{\Psi}_2$ refers to the vector of parameters associated with equation (1.10); and I_{t-1} refers to information up to $t - 1$. Then, based on the above decomposition of the likelihood function, we can design the following two-step procedure:

Step 1: Estimate equations (1.6) and (1.7) to get the control functions, g_{t+1} .

Step 2: Estimate equation (1.10) by the nonlinear least squares (NLS) method by replacing the g_{t+1} term with \hat{g}_{t+1} .

Once the control function \hat{g}_{t+1} is obtained from the first step, estimating equation (1.10) is straightforward. Then, one should take into account of the fact that, as the second step regression employs \hat{g}_{t+1} instead of g_{t+1} , there arises the problem of generated regressors when estimating the variance-covariance matrix of the parameter estimates. Taking care of this problem is also straightforward. A major difficulty for the above two-step procedure is in estimating equations (1.6) and (1.7) and in obtaining the control function \hat{g}_{t+1} in the first step.

Estimation of equations (1.6) and (1.7) by maximum likelihood estimation method is sometimes problematic. In finite samples, even when the true value of θ in equation (1.7) is less than 1, there is non-zero probability that it may be estimated to be 1. This is called the pile-up problem, and the probability of the pile-up problem increases as we have more coefficients in equation (1.6).² In fact, when we estimated the first-step regression in equations (1.6) and (1.7) by the maximum likelihood method in our application in Section 4, we obtained an estimate of $\hat{\theta} = 1$. In the next section, we discuss how the pile-up problem can be avoided.

1.2.3. Estimating the Parameters of the First-step Regression and the Control Function

The companion form of the VAR model in equation (1.4) is given by:

$$\tilde{Y}_{t+1} = \tilde{\delta} + F\tilde{Y}_t + \tilde{e}_{t+1}, \quad (1.12)$$

where $\tilde{Y}_t = [Y'_t \ Y'_{t-1} \ \dots \ Y'_{t-p}]'$ is $3p \times 3p$; $\tilde{\delta}$ is a function of Φ_0 ; and F , which is a function of Φ_1, Φ_2, \dots , and Φ_p . If we express \tilde{Y}_{t+1} as a function of \tilde{Y}_{t-1} , we have:

$$\tilde{Y}_{t+1} = \tilde{\delta} + F\tilde{\delta} + F^2\tilde{Y}_{t-1} + F\tilde{e}_t + \tilde{e}_{t+1}, \quad (1.13)$$

² For details, refer to Sargan and Bhargava (1983). Recently, Kim and Kim (2014) investigate the issue within both the classical and Bayesian frameworks.

the second row of which is the same as equation (1.6), and we have:

$$\eta_{j,t+1} = i_2'(F\tilde{e}_t + \tilde{e}_{t+1}), \quad (1.14)$$

where $i_2 = [0 \ 1 \ 0 \ \dots \ 0]'$ is a selection vector. The parameters c_1 and c_2 in equation (1.6) can be expressed as:

$$c_1 = i_2'(\tilde{\delta} + F\tilde{\delta}), \quad (1.15)$$

$$c_2 = i_2'F^2. \quad (1.16)$$

By equating the variances and the first-order autocovariances of η_{t+1} in equation (1.7) and (1.14), we also have:

$$i_2'(F\Omega F' + \Omega) i_2 = (1 + \theta^2)\sigma_g^2, \quad (1.17)$$

$$i_2'(F\Omega) i_2 = -\theta\sigma_g^2. \quad (1.18)$$

Equations (1.15)-(1.18) suggest that the parameters of the first-step regressions in (1.6) and (1.7) can be estimated indirectly by estimating the parameters of the VAR model in equation (1.4). Once all the parameters of the first-step regression are estimated, we can iteratively calculate the control functions as follows:

$$\hat{g}_{t+1} = \hat{\eta}_{t+1} + \hat{\theta}\hat{g}_t, \quad t = 0, 1, 2, \dots, T - 1, \quad (1.19)$$

conditional on the initial value, g_0 . Appendix A describes how g_0 can be estimated.

1.3. Performance of the Proposed Limited Information Approach in Comparison with GMM: Monte Carlo Experiment

1.3.1. Design of the Monte Carlo Experiment

To illustrate the properties of the estimators from the proposed approach in comparison with those from GMM, we conduct a simulation experiment. The size and the power of hypothesis tests are also studied. We consider the following data generating process:

$$y_{1,t} = \alpha_1 E_t y_{2,t+1} + \alpha_2 y_{1,t-1} + \epsilon_{1,t}, \quad (1.20)$$

$$y_{2,t} = \phi_{21} y_{2,t-1} + e_{2,t}, \quad (1.21)$$

$$\begin{bmatrix} \epsilon_{1,t} \\ e_{2,t} \end{bmatrix} \sim i.i.d. \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\epsilon^2 & \rho\sigma_\epsilon\sigma_e \\ \rho\sigma_\epsilon\sigma_e & \sigma_e^2 \end{pmatrix} \right), \quad (1.22)$$

$$\alpha_1 = 1; \alpha_2 = 0.5; \sigma_\epsilon^2 = 1; \sigma_e^2 = 1; \rho = 0, \quad (1.23)$$

where we assume an AR(1) process for $y_{2,t}$ to consider just-identified case, then the concentration parameter ($= \mu^2$) for the identification of the parameters in equation (1.1) can be derived as:³

$$\mu^2 = T \frac{\phi_{21}^4}{(1 - \phi_{21}^4)}, \quad (1.24)$$

which depends only on ϕ_{21} . We consider two alternative values of the concentration parameters per instrument ($= \mu^2/k$, where k is the number of instruments): 1 and 30. When we estimating model by GMM, we use two lags of $y_{1,t}$ and $y_{2,t}$, so that $k = 4$.

For the proposed approach, the control function, \hat{g}_{t+1} is estimated based on a VAR(2) model. The sample size we consider is 1000, and we use 5,000 simulations for our study. Since we conduct our simulation study for fixed values of the concentration parameters, the large sample size is used to reduce the sampling variability.

We investigate the alternative assumptions of the error term distribution by letting the reduced-form shock of the structural equation be $e_t = [e_1 \ e_2]'$, $e_t \sim i.i.d.(0, \Sigma)$ corresponding to equation (1.20)-(1.21), the three experiments can be summarized as follow:

Case A: i.i.d. Normal Distribution.

Case B: i.i.d. Student-t Distribution with degree of freedom 5.

Case C: Conditional Heteroskedasticity of e_t (GARCH(1,1)).

³ The detailed derivation is in Appendix A.

$$\begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} | I_{t-1} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{1,t}^2 & \sigma_{12,t} \\ \sigma_{12,t} & \sigma_{2,t}^2 \end{pmatrix} \right), \quad (1.25)$$

$$\sigma_{1,t-1}^2 = 0.1 + 0.05e_{1,t-1}^2 + 0.9\sigma_{1,t-1}^2, \quad (1.26)$$

$$\sigma_{2,t-1}^2 = 0.1 + 0.05e_{2,t-1}^2 + 0.9\sigma_{2,t-1}^2. \quad (1.27)$$

For each data set generated, the values of coefficients in equation (1.20) are kept fixed for two cases over simulations. We vary the value of concentration parameters to consider different degrees of quality of instruments. The implied values of ϕ_{21} are 0.251 and 0.572 corresponding to $\mu^2/k = 1$ and $\mu^2/k = 30$ for 1,000 observation.

The general pattern in this experiment is when the concentration parameter increases, the performance of GMM and the proposed method is improved. For weak and strong IV, the strength of the identification depends on the value of the concentration parameters. In other words, when the sample sized is large enough, it is the concentration parameters that determines how strong the instruments is for our parameters of interest.

1.3.2. Performance of the the proposed method and GMM Estimation

The upper panel of Table 1.1-1.3 reports simulation results for parameter estimates based on GMM and the proposed method. In Figure 1.1-1.3, we depict the sampling distribution of $\hat{\alpha}_1$ from the proposed method against that from GMM. When $\mu^2/k = 30$, the sampling distribution of $\hat{\alpha}_1$ is centered around the true value of 1, regardless of the estimation procedures employed, even though the sampling variation of $\hat{\alpha}_1$ is smaller the proposed method. However, when $\mu^2/k = 1$, we have very different results for all cases.

While the sampling distribution of $\hat{\alpha}_1$ based on the proposed method is centered around the true value of 1, the sampling distribution of based on GMM is centered around considerable biased values for all cases. The standard deviation of $\hat{\alpha}_1$ from GMM is much larger than that from the proposed method. GMM suffers from the weak identification problem, while the method proposed in this paper does not.

Notice that even if when we have non-Gaussian error distribution such as Student-t distribution and Garch (1,1) process, the proposed method performs better than GMM estimation for each cases. One can achieve reliable estimation results from the proposed method with fat-tailed distributed error terms or in the presence of heteroskedasticity.

We also compute the size of the power of testing $H_0 : \alpha_1 = 1$ against $H_1 : \alpha_1 \neq 1$ at the 5% significance level. We employ three alternative tests: i) a Wald test based on GMM; ii) an identification-robust test based on GMM; and iii) a Wald test based on the proposed method. The lower panel of Table 1.1-1.3 reports the rejection frequencies under the null hypothesis for different values of the concentration parameter. With $\mu^2/k = 30$, none of the three tests appear to be problematic. The rejection frequencies are closed to 5% for all three tests. However, with $\mu^2/k = 1$, unlike the other two tests, the Wald test based on GMM is very problematic, due to the weak identification problem. As neither the identification-robust test from GMM nor the Wald test from the proposed method suffer from size distortions, we compare the power curves for these two tests in Figure 1.4-1.6. The Wald tests from the proposed method for all cases have higher power than the identification-robust test from GMM, regardless of the values of the concentration parameter considered. For $\mu^2/k = 1$, the difference in the power of the two tests is considerably larger than for $\mu^2/k = 30$.

The results from our simulation study can be summarize as follows. If the concentration parameter is high enough, neither GMM nor the proposed method suffer from the weak identification problem. If the concentration parameter is too low, none of the methods may be free from the weak identification problem. An important result is that there exists a particular range for the value of the concentration parameter for which GMM suffered from the weak identification problem, while the proposed method does not. When testing for a hypothesis in this case, the Wald test from the proposed method has much higher power than the identification-robust test from GMM.

1.4. Empirical Results: Monetary Policy Rules for the 1979-1997 Sample and Macroeconomic Stability

1.4.1. Discussion of the Results

We first estimate the monetary policy rule in equation (1.5) using the same quarterly data (the 1960:Q1-1979:Q2 sample and the 1979:Q3-1997:Q4 sample) that Clarida et al. (2000) or Mavroeidis (2010) employ. The interest rate is the average federal funds rate in the first-month of each quarter; inflation is measured by the % change of the GDP deflator; and output gap is the series constructed by CBO. All data are taken from primarily from the St. Louis Federal Reserve Economic Database (FRED). Following Clarida et al. (2000) or Mavroeidis (2010), we use four lags of π_t , x_t , and i_t as instruments. Estimation results based on GMM and the proposed method are reported in Table 1.4, for the 1960:Q1-1979:Q2 sample and the 1979:Q3-1997:Q4 sample.

The weak identification concerns in the 1979:Q3-1997:Q4 sample are confirmed in many empirical literatures (Consolo and Favero, 2009; Inoue and Rossi, 2010; and Mirza and Storjohann, 2014). Especially, Innoe and Rossi (2010) develop a test for a weak identification in nonlinear models and provide empirical results for the US forward-looking Taylor rule. Their test does reject the identification in the 1979:Q3-1997:Q4 sample and emphasizes that there is a problem for the structural parameters in the monetary policy reaction function. Therefore, the estimation results of monetary policy rule based on the GMM estimation should be taken cautiously for the 1979:Q3-1997:Q4 sample.

The point estimates based on the proposed method shows that there is a shift in the conduct of monetary policy from the 1960:Q1-1979:Q2 sample to the 1979:Q3-1997:Q4 sample. The coefficients (β_π, β_x) in the the 1960:Q1-1979:Q2 sample are (0.885, 0.295), their estimates increase to (1.979, 0.647) for the 1979:Q3-1997:Q4 sample. Also, monetary policy under the 1979:Q3-1997:Q4 sample seems to be characterized by a high degree of smoothing as in Mavroeidis (2010) with an even stronger response to the inflation and the output gap. Notice that the standard error for the second period are greater than the 1960:Q1-1979:Q2 sample. Mavroeidis (2010) refer to this high degree of interest of smoothing as excess policy inertia, and it can lead weak identification problems for the 1979:Q3-1997:Q4 sample. These results emphasize that a more aggressive response of monetary policy to inflation and the output gap in the 1979:Q3-1997:Q4 sample. We also report the hypothesis testing results for the

$H_0 : \beta_\pi = 1$ against $H_1 : \beta_\pi > 1$. As Mirza and Storjohann (2014) mentioned, the evaluation using the identification robust method generate large confidence sets to be consistent with both aggressive response and passive response toward inflation. For the 1979:Q3-1997:Q4 sample, the proposed method are able to reject strongly the null hypothesis, and it gives a consistent view of Clarida, Gali and Gertler (2000).⁴

1.4.2. Does the Proposed Method Suffer from the Weak Identification Problem for the 1979:Q3-1997:Q4 Sample? Bootstrapping Analysis

The results for the 1979:Q3-1997:Q4 sample based on the proposed approach critically depend upon i) the validity of the assumption we employ and ii) non-existence of the weak identification problem. Note that, under the assumption that monetary policy shocks do not have contemporaneous effects on output and inflation, we would have serially uncorrelated disturbance terms u_{t+1} in equation (1.5) or ω_{t+1} in equation (1.10). In fact, we cannot reject the null hypothesis that these disturbances are serially uncorrelated.

In order to check whether the proposed method suffer from the problem of weak identification for the 1979:Q3-1997:Q4 sample, we conduct a Bootstrap analysis. We generate 5,000 sets of data by appropriately bootstrapping the residuals obtained from VAR estimation of actual data, and estimate equation (1.5) using the proposed method for each bootstrapped data set. In the absence of the weak identification problem, sampling distributions for the estimates of the β_π and β_x coefficients based on the proposed approach would be centered around $\hat{\beta}_\pi$ and $\hat{\beta}_x$ the estimates based on actual historical data. Also, there would be little size distortion for the conventional Wald tests.

Monetary policy over Volcker’s tenure was designed to control money rather than interest rates, and this resulted in considerable volatility in the interest rate. The inflation rate and the output gap also had higher volatility over Volcker’s tenure than over Greenspan’s tenure. We thus assume that, the reduced-form disturbance terms in equation (1.4) are

⁴ To draw conclusions with respect to Taylor principle, Mavroeidis (2010) considers joint confidence set of the feedback coefficients in the monetary policy equation. In Appendix, we also provide the joint confidence sets based on the proposed method. However, notice that in this paper, we use the reduced-form dynamics for the other equations, it is difficult to interpret the joint confidence sets directly based on the proposed method.

heteroscedastic, so that $e_{it}|S_t \sim i.i.d.(0, \sigma_{i,S_t}^2)$, $i = 1, 2, 3$, where $\sigma_{i,S_t}^2 = (1 - S_t)\sigma_{i,0}^2 + S_t\sigma_{i,1}^2$, with $S_t = 0$ for $t \leq 1987 : Q2$ and $S_t = 1$ for $t > 1987 : Q2$.

In order to preserve the heteroscedastic nature of actual data, we follow the procedure given below, when bootstrapping the residuals:

- i) Estimate the VAR model in equation (1.4) and obtain residuals $\hat{e}_t = [\hat{e}_{1t} \ \hat{e}_{2t} \ \hat{e}_{3t}]$, $t = p + 1, p + 2, \dots, T$, and then estimate σ_{i,S_t}^2 , for $i = 1, 2, 3$, and for $S_t = 0, 1$.
- ii) Calculate standardized residuals, $\hat{e}_{it}^* = \hat{e}_{it}/\hat{\sigma}_{i,S_t}$, for $i = 1, 2, 3$ and for $t = p + 1, p + 2, \dots, T$.
- iii) Draw, with replacement, random samples $\{\tilde{e}_{1t}^* \ \tilde{e}_{2t}^* \ \tilde{e}_{3t}^*\}_{t=p+1}^T$ from standardized residuals.
- iv) Calculate $\tilde{e}_t = [\sigma_{1,S_t}\tilde{e}_{1t}^* \ \sigma_{2,S_t}\tilde{e}_{2t}^* \ \sigma_{3,S_t}\tilde{e}_{3t}^*]'$, for $t = p + 1, p + 2, \dots, T$.
- v) Generate series $\tilde{Y}_t = \hat{\Phi}_0 + \hat{\Phi}_1 Y_{t-1} + \hat{\Phi}_2 Y_{t-2} + \dots + \hat{\Phi}_p Y_{t-p} + \tilde{e}_t$ (for $t = p + 1, p + 2, \dots, T$), where $\hat{\Phi}_j$ refers to the OLS estimate. The initial values Y_1, Y_2, \dots, Y_p are taken from actual data.

Reported in the upper panel of Table 1.5 are the means and the standard deviations for the estimates of β_π and β_x . The bootstrap sampling distributions for these parameter estimates are also depicted in Figure 1.7. They are centered around the true value ($\beta_\pi = 1.979$ and $\beta_x = 1.979$), which are the estimates from actual historical data based on the proposed method, as reported in Table 1.4. The nlower panel of Table 1.5 reports the rejection frequencies of the standard Wald tests for testing $H_0 : \beta_\pi = 1.979$ against $H_1 : \beta_\pi \neq 1.979$, as well as those for testing $H_0 : \beta_x = 0.647$ against $H_1 : \beta_x \neq 0.647$. There does not seem to be large size distortion for the standard Wald tests from the proposed method. All these results suggest that, unlike the GMM approach of Mavroeidis (2010), the proposed approach does not suffer from the weak identification problem for the 1979:Q3-1997:Q4.

1.5. Summary and Conclusions

In the standard simultaneous equations models without forward-looking terms, one can implement limited information methods by jointly estimating a pseudo-system that consists

of the structural equation of interest and the reduced-form equations for the right-hand-side endogenous variables in the structural equations. As Fukač and Pagan (2010) mention, in the presence of a forward-looking term, the limited information approach requires a solution for the expectation of this forward-looking term, which is not readily available from the pseudo system.

The identifying assumption that we employ allows us to derive a limited information approach that does not require a solution for the expectation of the forward-looking term. The approach presented in this paper is more efficient than GMM, as it can effectively handle the moving-average disturbances, a problem that inherently results in inefficiency in GMM estimation of a forward-looking model. The proposed approach provides us the monetary policy reacts strongly to the inflation for the 1979-1997 sample. Furthermore, our bootstrap analysis suggests that the proposed approach does not suffer from the weak IV problem for this sample. The validity of the identifying assumption that we employ is also confirmed by checking the absence of serial correlation in the disturbances of the estimated policy rule.

The proposed approach allows us to provide stronger messages to the estimation of a forward-looking monetary policy rule. The estimation results confirm a change of monetary policy in the U.S. In the 1960-1979 sample, the policy was inactive and it did not react sufficiently to the expected deviation of inflation from its target. In contrast, under the 1979-1997 sample monetary policy rule follows the Taylor principle and actively response to the inflation and the output gap with a high degree of interest smoothing.

**Estimation of a Forward-Looking Time-varying Monetary Policy Rule:
Limited Information Approach**

2.1. Introduction

The monetary policy rule has been investigated for several decades to provide understanding of the outbursts of inflation and unemployment of the 1970s and early 1980s. Many works of literature (e.g., Clarida et al. (2000), Boivin and Giannoni (2003), Cogley and Sargent (2005), Lubik and Schorfheide (2004)) have stressed that the U.S. monetary policy was less active against inflation under the Fed chairmanship of Arthur Burns than under Paul Volcker and Alan Greenspan. However, the empirical conclusion of the significant effect on the macroeconomic variables due to the change in monetary policy is still controversial. The opposite view has been addressed by comparing the relative importance of the non-systematic and systematic aspects of the monetary policy changes (see e.g., Bernanke and Mihov (1998), Primiceri (2005), Sims and Zha (2006)).

Various econometric methodology has been suggested to consider the time variation of a forward-looking Taylor-rule-type monetary policy. Kim and Nelson (2006), for example, propose a two-step procedure to deal with time-varying endogenous regressors with heteroscedasticity. They also account for the changing degrees of uncertainty associated with the Fed's forecasts of future economic conditions. By doing so, they achieve efficiency in estimation by employing the standardized projection errors regarding inflation and output gaps. Their empirical findings support the idea of active response toward inflation during the 1980s. In addition, they questioned the reliability of the lagged inflation rates as IV showing a wide confidence band under the Volcker-Greenspan regime.

To consider the multivariate linear setup, Cogley and Sargent (2005) use time-varying variance in the context of VARs with drifting coefficients. They assume that the simultaneous relations among macroeconomic variables are time invariant. To reveal evidence of the

evolution of monetary policy actions, they estimate a forward-looking Taylor rule with the interest rate smoothing method. They suggest a two-stage least squares analysis on a date-by-date basis. In the first step, they project the Feds forecasts of average inflation and unemployment onto a set of instruments that are a constant and two lags of inflation and unemployment rates. The second step involves projecting the current interest rates onto the fitted values from the first-stage projections along with the lagged nominal interest rates. They found evidence for the monetary policy activism, as in Clarida et al. (2000), that monetary policy was passive in the 1970s and active in the Volcker-Greenspan era. Also, more importantly, they point out that estimates for the Volcker-Greenspan period are less precise because of instrument relevance problems, and the tails of the distributions overlap at various dates.

Recently, Chon and Kim (2014) have stressed the efficiency loss of the conventional IV estimation in simultaneous equation models with forward-looking variables. They employ the identifying assumption that monetary policy shocks do not have any contemporaneous effects on macroeconomic variables to derive a Limited Information procedure for the estimation of a forward-looking monetary policy rule. The procedure they design can effectively handle the moving-average disturbances, a problem that inherently results in inefficiency in the IV estimation of a forward-looking model rendering spurious weak identification issues in the Volcker-Greenspan era. The two-step estimation procedures suggested by Kim and Nelson (2006) and Cogley and Sargent (2005) involve the efficiency loss problems that originated from the moving-average disturbances in the instrumenting equations. These researchers concern the relevance of IV for the Volcker-Greenspan samples but could not handle the additional information in the residuals.

With the same identifying assumption for the monetary policy shocks employed by Chon and Kim (2014), Primiceri (2005) analyzes the systematic and non-systematic parts of monetary policy and their effect on the macroeconomic variables with time-varying parameters demonstrating stochastic volatility. His approaches consider a flexible way to consider changes in policy because it captures both time variation of the simultaneous relations among the variables of the model and the heteroscedasticity of the innovations. The empirical conclusion he made is that the high volatility of the exogenous non-policy shocks seems to

explain a larger fraction of the poor economic performance of the 1970s and early 1980s, which were episodes of high unemployment and inflation. However, the monetary policy rule he considered only responds to the current inflation rates without response to real activity. Furthermore, a Lucas (1976) critique issue arises in the monetary policy rule without consideration for the rational and forward-looking behavior of economic agents. In this paper, we propose a Limited-Information procedure to estimate a forward-looking monetary policy rule by modeling multivariate time varying coefficient with stochastic volatility.

The main contribution of this paper is the assertion that, under the identifying assumption that monetary policy shocks do not have simultaneous effects on macroeconomic variables, we can effectively handle the moving-average dynamics in the instrumenting equations. This would improve the efficiency for the estimation of a time-varying forward-looking monetary policy rule mitigating the issue of relevance to instruments (see Cogley and Sargent, 2005). Also, by incorporating the time-varying coefficients and innovations, we can reasonably model for the effects of the changes in monetary policy on the rest of the economy. Not only the forecasts of future expectation for inflation and output gap but also the moving-average dynamics can be obtained from the posterior mean of reduced-form time-varying VAR. Thus, the estimation results obtained in this paper would provide more reliable evidence to understand whether monetary policy rules have changed and that the persistence of inflation itself has drifted over time.

The rest of the paper is summarized as follows. Section 2 presents the time-varying structural VAR model adopted in this paper for the estimation of a forward-looking monetary policy rule. Section 3 introduces the estimation procedure used in this paper with the key features of the estimation strategy. Section 4 explain the empirical results of the application to a forward-looking monetary policy rule. Concluding remarks are offered in Section 5.

2.2. Model Specification

For an empirical estimation of a forward-looking Taylor rule as in Kim and Nelson (2006), we consider the following linear TVP model of monetary policy transmission mechanism.

With a VAR specification of aggregate supply and demand, our complete model of monetary policy takes a sub-VAR system.

$$i_t = \beta_{0,t} + \beta_{1,t}E_t[\pi_{t+1}] + \beta_{2,t}x_t + \beta_{3,t}i_{t-1} + m_t, \quad (2.1)$$

$$\pi_t = \delta_{\pi 0,t} + \tilde{Y}'_{t-1}\delta_{\pi 1,t} + e_{2,t}, \quad (2.2)$$

$$x_t = \delta_{x 0,t} + \tilde{Y}'_{t-1}\delta_{x 1,t} + e_{3,t}, \quad (2.3)$$

where i_t is the federal funds rate; x_t is a measure of the unemployment at time t ; π_t is the percent change in the price level between time t and $t + 1$; $\tilde{Y}_t = [Y'_t \ Y'_{t-1} \ \dots \ Y'_{t-k+1}]'$, with $Y_t = [i_t \ \pi_t \ x_t]'$; and $E_t(\cdot)$ refers to the expectation formed by the Fed conditional on information available at the beginning of time t , when the federal funds rate is determined. We assume that the monetary policy shocks m_t do not have any contemporaneous effects on macroeconomic variables.

The above forward-looking Taylor rule would have the following reduced form representation, where the coefficients of the following reduced form Taylor rule would be convolutions of the above structural VAR model (2.1)-(2.3).

$$i_t = \delta_{i 0,t} + \tilde{Y}'_{t-1}\delta_{i 1,t} + e_{1,t}, \quad (2.4)$$

Notice that the coefficients $\delta_{i 0,t}$ and $\delta_{i 1,t}$ would be function of the coefficients in equation (2.1). The error terms of the reduced form consists of (2.2),(2.3), and (2.4) are denoted as $e_t = [e_{1,t} \ e_{2,t} \ e_{3,t}]$ are heteroscedastic shocks with variance covariance matrix Ω_t .

To identify the monetary policy shock, a structural VAR model would be recovered via Cholesky decomposition of the variance-covariance matrix of the reduced-form error terms. After we assume that the monetary policy shocks m_t do not have any contemporaneous effects on macroeconomic variables, we place endogenous variables in the order of inflation, output gap, and the interest rate, respectively. Then, we have the following relations of the reduced-form errors.

$$e_t = A^{-1}\epsilon_t, \quad \epsilon_t = \Sigma_t\tilde{\epsilon}_t, \quad \tilde{\epsilon}_t \sim i.i.d.N(0, I_3) \quad (2.5)$$

$$A_t = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{bmatrix} \quad (2.6)$$

and Σ_t is the diagonal matrix:

$$\Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & 0 \\ 0 & \sigma_{2,t} & 0 \\ 0 & 0 & \sigma_{3,t} \end{bmatrix} \quad (2.7)$$

We let B_t to be a vector stacking time-varying reduced-form VAR coefficients including intercepts and lagged variables; σ_t be the vector of the diagonal elements of the matrix Σ_t , and α_t be the vector of non-zero and non-one elements of the matrix A_t , which stacks by rows. Then, we have the following the dynamics of the model's time varying parameters is specified as follows.

$$B_t = B_{t-1} + \nu_t, \quad (2.8)$$

$$\alpha_t = \alpha_{t-1} + \zeta_t, \quad (2.9)$$

$$\log \sigma_t = \log \sigma_{t-1} + v_t, \quad (2.10)$$

where the distributional assumptions as regards $(\tilde{\epsilon}_t, \nu_t, \zeta_t, v_t)$ are state below.

$$V = \text{var} \begin{pmatrix} \tilde{\epsilon}_t \\ \nu_t \\ \zeta_t \\ v_t \end{pmatrix} = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{bmatrix}, \quad (2.11)$$

where I_n is an n -dimensional identity matrix, Q , S , and W are positive definite matrices.

In case $E_t[\pi_{t+1}]$ in (1) is unobservable, we replace them by π_{t+1} , the ex-post measures of inflation. This results in the following equation for monetary policy rule with ex-post data:

$$i_t = \beta_{i0,t} + \beta_{1,t}\pi_{t+1} + \beta_{2,t}x_t + \beta_{3,t}i_{t-1} + u_{t+1}, \quad u_{t+1} \sim i.i.d.N(0, \sigma_{u,t+1}^2), \quad (2.12)$$

where $u_{t+1} = \beta_{1,t}(\pi_{t+1} - E_t[\pi_{t+1}]) + m_t$. Note that the regressors π_{t+1} in equation (2.1) are correlated with the disturbance term u_{t+1} . In order to estimate the equation (2.12), we consider the following instrumenting equation is given by:

$$\pi_{t+1} = c_{1,t} + \tilde{Y}'_{t-1} c_{2,t} + \eta_{t+1}, \quad (2.13)$$

where $\tilde{Y}_t = [Y'_t \ Y'_{t-1} \ \dots \ Y'_{t-k+1}]'$; $c_{1,t}$ and $c_{2,t}$ are functions of the coefficients in the reduced-form (2.2), (2.3), and (2.4); and η_{t+1} follows MA(1) process as given below:

$$\eta_{t+1} = g_{t+1} - \theta_{t+1}g_t, \quad g_{t+1} \sim i.i.d.(0, \sigma_{g,t+1}^2), \quad (2.14)$$

where g_{t+1} is serially uncorrelated. After we assume that the monetary policy shocks do not have any contemporaneous effects on macroeconomic variables, we have u_{t+1} is serially uncorrelated. Then, one is able to consider an orthogonal projection of u_{t+1} on g_{t+1}^* , where $g_{t+1}^* = \frac{g_{t+1}}{\sigma_{g,t+1}}$ is a standardized residuals of g_{t+1} . Then, we have:

$$u_{t+1} = \gamma_{t+1}g_{t+1}^* + \omega_{t+1}, \quad (2.15)$$

where $\omega_{t+1} = \sqrt{1 - \rho_{t+1}^2} \sigma_{u,t+1} \omega_{t+1}^*$; $\omega_{t+1}^* \sim N(0, 1)$; and $\gamma_{t+1} = \rho_{t+1} \sigma_{u,t+1}$, ρ_{t+1} is the correlation between u_{t+1} and g_{t+1}^* . We can thus rewrite the equation (2.12) as:

$$i_t = \beta_{0,t} + \beta_{1,t}\pi_{t+1} + \beta_{2,t}x_t + \beta_{3,t}i_{t-1} + \gamma_{t+1}g_{t+1}^* + \omega_{t+1}, \quad \omega_{t+1} \sim i.i.d.N(0, \sigma_{\omega,t+1}^2), \quad (2.16)$$

where, conditional on g_{t+1}^* , the disturbance term ω_{t+1} is not correlated with any other regressors in the above equation.

2.3. Estimation Procedure

2.3.1. Markov Chain Monte Carlo (MCMC) algorithm

We consider the following two-steps Gibbs sampling algorithm to estimate the time-varying Taylor rule in equation (2.1) along with the reduced-form VAR system in equation (2.2) and (2.3):

Outline for Gibbs Sampling Algorithm

Step 1: Estimate the full reduced-form VAR consisting (2.2)-(2.4) following the Gibbs sampling algorithm by Primiceri (2005). Calculate the posterior means (or medians) of reduced-form parameters for the initial values of $\{\delta_t, \beta_t\}_{t=1}^{T-1}$. And construct $\{g_{t+1}^*\}_{t=1}^{T-1}$.

Step 2: Conditional on $\{g_{t+1}^*\}_{t=1}^{T-1}$, draw $\{\beta_t\}_{t=1}^{T-1}$ in equation (2.1) using the backward simulation algorithm by Carter and Kohn (1994).

In later sections, we discuss the details for the step 1) in order to consider the Limited Information approach.

2.3.2. Construction of the Control Function: Details of Step 1

With VAR specification given by in equation (2.2)-(2.4), we can consider the instrumenting equation for an estimation of equation (2.4). By defining $\tilde{Y}_t = [Y_t' \quad Y_{t-1}' \quad \dots \quad Y_{t-k+1}']'$, we have the following companion form of a time varying VAR:

$$\tilde{Y}_t = \tilde{B}_{0,t} + \Phi_t \tilde{Y}_{t-1} + \tilde{e}_t, \quad e_t \sim i.i.d.N(0, \tilde{\Omega}_t), \quad (2.18)$$

where $\tilde{B}_{0,t}$ is a function of $\delta_{k,t}$ for $k = 1, 2$, and 3; Φ_t is a function of $\delta_{j,t}$ for $j = \pi, x, i$ and the first (3×3) diagonal block of $\tilde{\Omega}_t$ is Ω_t . And, all the parameters are time-varying and follow random walk processes.⁵ The variances are assumed to evolve as geometric random walks, belonging to the class of models known as stochastic volatility. All the innovations in the reduced form VAR model are assumed to be jointly normally distributed and independent with each other.

To consider the proposed estimation for a forward-looking time varying MP rule, Step 1 involves the construction of the control function in equation (14). In the following, we explain how to generate a control function for the second step estimation.

By solving equation (2.18) forward, we have:

⁵ Alternatively, we may use demanded data for the equation (2.18), i.e, $\hat{y}_t = y_t - E(y_t)$.

$$\begin{aligned}
\tilde{Y}_{t+1} &= \tilde{B}_{0,t+1} + \Phi_{t+1}\tilde{Y}_t + \tilde{e}_{t+1} \\
&= \tilde{B}_{0,t+1} + \Phi_{t+1}(\tilde{B}_{0,t} + \Phi_t\tilde{Y}_{t-1} + \tilde{e}_t) + \tilde{e}_{t+1} \\
&= \tilde{B}_{0,t+1} + \Phi_{t+1}\tilde{B}_{0,t} + \Phi_{t+1}\Phi_t\tilde{Y}_{t-1} + \Phi_{t+1}\tilde{e}_t + \tilde{e}_{t+1}
\end{aligned} \tag{2.19}$$

The two step prediction error is obtained as:

$$\eta_{t+1} = i'_\pi(\Phi_{t+1}\tilde{e}_t + \tilde{e}_{t+1}), \tag{2.20}$$

where $i_\pi = [1 \ 0 \ 0 \ 0 \ \dots \ 0]'$ is a selection vector. The parameter $c_{1,t}$ and $c_{2,t}$ in equation (2.13) can be expressed as:

$$c_{1,t} = i'_\pi(\tilde{B}_{0,t+1} + \Phi_{t+1}\tilde{B}_{0,t}), \tag{2.21}$$

$$c_{2,t} = i'_\pi(\Phi_{t+1}\Phi_t). \tag{2.22}$$

By equating the variance and the first-order auto-covariances of η_{t+1} in equation (2.13), we have:

$$i'_\pi(\tilde{\Omega}_{t+1} + \Phi_{t+1}\tilde{\Omega}_t\Phi'_{t+1})i_\pi = (1 + \theta_{t+1}^2)\sigma_{g,t}^2, \tag{2.23}$$

$$i'_\pi(\Phi_{t+1}\tilde{\Omega}_t)i_\pi = -\theta_{t+1}\sigma_{g,t}^2. \tag{2.24}$$

Once all parameters of the VAR model are estimated, we can iteratively calculate the control function as follows:

$$\hat{g}_{t+1} = \hat{\eta}_{t+1} + \hat{\theta}_{t+1}\hat{g}_t, t = 1, 2, \dots, T - 1, \tag{2.25}$$

conditional on the initial value, g_0 .

2.4. Empirical Findings

2.4.1. Priors and Data

The data we employ are quarterly data covering the period 1965:1 to 2011:4. As in Kim and Nelson (2006), the interest rate the average federal funds rate in the first-month of each quarter; inflation is measured by the % change of the GDP deflator; the output gap is the series constructed by CBO. Two lags are used for the estimation.

Priors for state vectors and hyperparameters are obtained from OLS estimates as in Primiceri (2005) with a few modifications. The first 10 years (40 observation, from 1945:1 to 1964:4) are used to calibrate the prior distributions.⁶ For example, we obtain the mean and the variance of B_0 are chosen to be OLS point estimates and four times its variance in a time invariant VAR, estimated on the small initial samples. And, we chose a prior for A_0 in a same way. For $\log\sigma_0$ instead, the mean of the distribution is chosen to be the log of the OLS point estimates of the standard errors of the same invariant VAR, while the variance covariance matrix is assumed to be identity matrix. Also, degrees of freedom and scale matrices are needed for the inverse-Wishart prior distributions of the hyperparameters. The degrees of freedom set to 4 for W and 2 and 3 for the two blocks of S . In sum, the priors take the forms:

$$B_0 \sim N(\hat{B}_{OLS}, 4V(\hat{B}_{OLS})), \quad (2.26)$$

$$A_0 \sim N(\hat{A}_{OLS}, 4V(\hat{A}_{OLS})), \quad (2.27)$$

$$\log\sigma_0 \sim N(\log\hat{\sigma}_0, I_n), \quad (2.28)$$

$$Q \sim IW(0.01^2 40V(\hat{B}_{OLS}), 40), \quad (2.29)$$

$$W \sim IW(0.01^2 4I_n, 40), \quad (2.30)$$

$$S_1 \sim IW(0.1^2 2V(\hat{A}_{1,OLS}), 2), \quad (2.31)$$

$$S_2 \sim IW(0.1^2 3V(\hat{A}_{2,OLS}), 3), \quad (2.32)$$

where S_1 and S_2 denote the two blocks of S , while $\hat{A}_{1,OLS}$ and $\hat{A}_{2,OLS}$ stand for the two corresponding blocks of \hat{A}_{OLS} . The simulation are based on 10,000 iterations of the Gibbs sampler, discarding the first 2000 for convergence.

⁶ Refer to Primiceri (2005) for details.

2.4.2. Estimation Results

Our empirical findings are shown in Figures 2.2-2.6. Results for the forward-looking Taylor rule are quite different from others, which ignored the additional moving average dynamics in the error term. In Figure 2.3, we observe that the Fed's response to inflation during that 1970s was lower than in the 1980s. This tells us that the Fed did not provide sufficient attention to inflation.

However, in the early 1980s, the Fed's response to expected inflation increased sharply and stayed at a high level until the mid-1980s around 2, and it started to decline after the mid-1990s throughout the whole sample. Contrast to Kim and Nelson (2006), which ignores the moving-average dynamics, this paper shows that the response to inflation after the mid-1990s decreased, and it has been lower than 1 from the 2000s until 2010. Especially after 2006, the Fed's response to inflation was the lowest, and it is due to the deviation of the traditional monetary policy tools, which is that the Fed's focused on the long-term interest rate changes rather than the short-term interest rate.

Next, Figure 2.4. depicts the response of the federal funds rate to the expected real GDP gap. In contrast to Kim and Nelson (2006), the response for the whole period from the 1970s to the 2000s is positive and significantly different from zero throughout the whole period. Around the mid-1980s, the response was at the lowest level, but it started to increase again until the 2000s. If one ignores the moving-average dynamics in the error term, it is possible for one to interpret erroneously that the response to the output was insignificant during the 1980s. Keeping the significant response to the real economic condition throughout the whole sample shows that monetary policy became more active after Volcker.

Combining the results of the Fed's response to inflation after the mid-2000s, it is consistent with the fact that the Fed's recent monetary policy toward Quantitative Easing (QE) to adjust the long-term interest rate. This would lead the little responses to the inflation and real economics in the federal fund rate.

The evidence of the aggressive monetary policy toward inflation seems to support the conclusion proposed by Clarida et al. (2000) in contrast to Primiceri (2005). In general, a systematic monetary policy was more responsive to the economic conditions seen in the

early 1980s, and it was passive in the 1970s. Weak Instrument concerns raised in Cogley and Sargent (2005) and Kim and Nelson (2006) after 1979 can be taken into account through the proposed procedure. Finally, in Figure 2.5, the degree of interest rate smoothing is shown. It has been continuously increasing since the mid-1970s. However, it has decreased slightly during the mid-2000s, but it still remained at a high level during the whole sample. Lastly, in Figure 2.6, the time-varying volatility for the federal funds rate is shown. The figure shows the apparent heteroscedasticity in the disturbance of monetary policy rule, as in Kim and Nelson (2006) and Primiceri (2005).

2.5. Conclusion

In this paper, we investigate a Limited-Information procedure to estimate a forward-looking monetary policy rule by modeling the multivariate time varying coefficient with stochastic volatility. By incorporating the time-varying coefficients and innovations, this paper reasonably models the effects of the changes in monetary policy on the rest of the economy. Not only the forecasts of future expectation for inflation and output gap but also the moving average dynamics can be obtained from the posterior mean of reduced-form time-varying VAR. Thus, the estimation results obtained in this paper would provide more reliable evidence to understand whether monetary policy rules have changed and the persistence of inflation itself has drifted over time.

Compared to the existing literature to estimate monetary policy via time-varying VAR specification, this paper takes into account the forward-looking component in the Taylor rule. Having estimated the time-varying VAR in the first step, this paper can effectively recover moving average terms for the estimation of a forward-looking Taylor rule. The main finding of this paper is that, under the identifying assumption that monetary policy shocks do not have simultaneous effects on macroeconomic variables, we can handle the moving-average dynamics in the instrumenting equations. This would improve the efficiency for the estimation of a time-varying forward-looking monetary policy rule mitigating the issue of relevance to instruments (see Cogley and Sargent, 2005; and Kim and Nelson, 2006).

Our empirical findings show that the dynamics of the monetary policy's response to inflation and output gap are quite different from the one which does not consider the additional MA process in the error term. The evidence on aggressive monetary policy toward inflation seems to be supported in this paper, in contrast to Primiceri (2005). In general, a systematic monetary policy was more responsive to the economic conditions in the early 1980s, and it was passive in the 1970s. Also, the recent monetary policy, put in place after the mid-2000s, is far different than the Volcker-Greenspan era. The detailed investigation of the recent monetary policy presents an opportunity for further research.

**Stock Market Reaction to Monetary policy Changes:
Identification through Heteroskedasticity with Markov-switching**

3.1. Introduction

The response of stock prices to monetary policy is a key for analyzing the impact of policy transmission mechanism. Changes in monetary policy are transmitted through the stock market via the wealth effect, the interest effect, and by other mechanisms as well (see, Bernanke and Kuttner, 2005). Understanding the relationship between stock prices and monetary policy is important for several reasons. For policy makers, it would be crucial to have accurate estimates of the reaction of monetary policy in order to form effective policy actions. Also, for market participants, the reliable estimates of the responsiveness of stock prices to monetary policy is important for wise investment and risk decision making.

Estimating the impact of stock prices to monetary policy actions, however, has several difficulties. One of the main issues occurs when estimating the effect of the Fed on stock prices based on typical ordinary-least-squares (OLS). Estimates may be severely biased due to the endogeneity and omitted variable problems. The endogeneity problem⁷ originates from the simultaneous interaction between stock prices and policy decisions, while the omitted variable problem is due to factors that influence both policy rates and stock prices which are commonly excluded from regression analysis. Usually, policy rates and stock prices are determined simultaneously within the data-frequency interval, and it is possible to render the endogeneity problems⁸. In particular, a surprise change in the target rate could reflect the reaction of the Fed to changes in stock prices that occurred earlier in the quarter, month or week, instead of capturing an independent monetary policy shock. Also, changes in policy

⁷ Rigobon and Sack (2003) find that movement in the stock market have a significant impact on the macroeconomy and thus are likely to be an important factor in the determination of monetary policy.

⁸ Bernanke and Kuttner (2005) emphasize the problem of using lower frequency data.

rates and stock prices may be caused by other important news released earlier in the period which are omitted from regression. In either case, the classical regression assumption that the error term is uncorrelated to a surprise changes in the target rate is violated and the estimated can be biased (See e.g., Farka, 2009).

In this paper, in order to address these estimation issues, we extend the work of Rigobon and Sack (2004) by incorporating Markov-switching framework. The estimation procedures employed in this paper captures heteroskedasticity of monetary policy shocks endogenously different to the original work of Rigobon and Sack (2004). Once one estimate the timing of monetary policy changes, the heteroskedasticity structure helps to identify the impact of stock market to policy changes. In a recent example, Gerlach et al. (2006) shows that there are two distinct regime in the response of stock prices to policy surprises. Following Kuttner (2001), we use thirty-day federal funds rate futures to extract unexpected part of policy changes. Since the stock markets are forward-looking, the market is unlikely respond to anticipated policy changes. In this sense, asset prices will response to revisions in future expectations, we thus focus on surprise part of policy changes to figure more clearly the market response to monetary policy.

Traditionally, a popular approach is a event-study framework,⁹ measuring the impact of Fed's policy on asset price is to estimate the reaction to fund rate changes on the day of the change (see, e.g., Bernanke and Kuttner, 2005; Gurkayna et. al., 2005; Davig and Gerlach, 2006; Farka, 2009). A conventional 'event-study' approach, is typically used in estimating the response of stock prices to monetary policy actions using lower frequency data such as monthly or quarterly basis. To account for the endogeneity issues possibly come from the use of lower frequency data, Bernanke and Kuttner (2005) uses daily data to address the problem within a 'event-study' framework. In a seminal paper, they investigate recently the effects of monetary policy on the stock market by using daily CRSP value-weighted returns and a measure of unexpected changes to the target federal funds rate. They conclude that an unexpected 25-basis-point cut in the target federal funds rate is associated with a 1 percent

⁹ It measures the impact of Federal Reserve policy on the stock market is to calculate the market's reaction to funds rate changes on the day of the changes. Because this approach involves looking at the response to specific events, it might be described as an 'event-study' style analysis.

increase in equity prices.

Also, Farka (2009) uses a new data set consisting of high frequency changes in S&P 500 and federal funds futures around the time of monetary policy announcement. He argues that the intra-day data set in his paper reduce the omitted variable bias by decreasing the likelihood that other relevant information is released in the market during the narrow interval around policy announcement. However, as Bernanke and Kuttner (2005) mentioned, if there is contemporaneous response of monetary policy to the stock market or if monetary policy and stock market both responded jointly to new economic information, using high-frequency data still subject to the bias.

More statistical solution is suggested by Rigobon and Sack (2004) within a VAR approach that exploits the heteroskedasticity of monetary policy shocks across event days and non-event days. In order to take care of these possible simultaneous bias, they assume that the variance of monetary policy shocks is higher on days of FOMC meetings and of the Chairman's semi-annual monetary policy testimony to Congress, when a larger portion of the news hitting markets is about monetary policy. They then show that the shift in the variance of the policy shocks on those dates is sufficient to measure the responsiveness of asset price to monetary policy. To employ their method, one need to identify a period of time in which the variance of the policy shocks was higher than at other times, but the other shocks in the system remained unchanged. Thus, if the above assumption for the change of monetary policy shock is violated which means one mis-specify the time in which the variance-covariance matrix shifts, it would generate some bias in the estimates of the parameter.

As Farka (2009) discusses, the monetary policy shocks possibly come from the alteration of expectations about the future path of the monetary policy and a change in the timing of policy moves. He stresses that the importance of these shocks to obtain more precise estimates of the effect of monetary policy on stock returns. Market participants, therefore, would respond to the information content of policy shock which reflect two factors that one is expectational changes regarding the near term path of future policy, and another one is shifts in the timing of an anticipated policy move. In this sense, if we only use the institutional information to capture the change in the policy shock to employ the identification

procedure suggested by Rigobon and Sack (2004), it would not enough to investigate the all possible changes in the policy shocks. In particular, timing of the policy shock shifts caused by revisions in expectations regarding the future path of monetary policy would be very difficult to be found. In this paper, using Markov-switching framework to detect different states, we also account for the uncertainty of timing issue instead of specifying the high and low volatility states exogenously. And, it would reduce the possible bias which may be caused by the uncertainty of the shift of the policy shock. As mentioned earlier, the stock markets are unlikely response to the anticipated changes in policy shocks, we hence use the heteroskedasticity structure focusing on the event days.

Gerlach et al. (2006) present a test of the response of stock prices to Federal Reserve policy shocks using a Markov-switching framework to a conventional event-study approach. They show that in the low volatility regime, the market response to unexpected changes in the target federal funds rates is significantly negative. More recently, Farka (2009) finds that the asymmetric effects in level and volatility of stock returns to policy changes with respect to the type of policy shocks and to the type of policy shocks by specifying an intra-day GARCH model. In this paper, we use the state-dependent structure in the variance to identify the original impact on stock market focusing on the event-days.

The rest of this article is summarized as follows. We first introduce a simultaneous relationship between asset prices and monetary policy, and discuss existing problems in the estimation in Section 2. Section 3 reviews the method proposed by Rigobon and Sack (2004), and we suggest the use of Markov-switching framework in the first step to detect the shift in the variance endogenously. Section 4 explains the data we use for empirical study and show the results for the response of asset prices due to the monetary policy shocks. Concluding remarks are offered in Section 5.

3.2. Model Specification and Estimation Problem

Consider the following simplified systems of equations of monetary policy and asset prices used in Rogobon and Sack (2004):

$$\Delta i_t = \beta \Delta s_t + \gamma z_t + \epsilon_t, \quad (3.1)$$

$$\Delta s_t = \alpha \Delta i_t + z_t + \eta_t, \quad (3.2)$$

where Δi_t is the change in the short-term interest rate and Δs_t is the change in an asset price. Monetary policy reaction can be described in equation (3.1), and it captures the expected response of policy to a set of variables z_t ¹⁰ and to asset price. Equation (3.2) shows the response of the asset price by the interest rate and also z_t . The variable ϵ_t is the monetary policy shock, and η_t is a shock to the asset price. Monetary policy shock¹¹ means deviations from the typical response of the short-term interest rate. We further assume that those disturbances are assumed to be have no serial correlation and to be uncorrelated with each other and with the common shock z_t .¹²

To understand the equation (3.2) more clearly, we use the approximation introduced by Campbell and Shiller (1988) for a dynamic setting, the log level of stock price can be described as follow (see, Rigobon and Sack, 2003):

$$S_t = \frac{k}{1 - \delta} + \sum_{j=0}^{\infty} \delta^j (1 - \delta) E_t(d_{t+1+j}) - \sum_{j=0}^{\infty} \delta^j E_t(h_{t+j}), \quad (3.3)$$

where δ and k are constant; d_t is log dividend; and h_{t+j} is the return holding equities between $t + j$ and $t + j + 1$. The expected holding return for equities can be expressed with the sum of the short-term interest rate and a risk premium, denoted as i_{t+j} and ρ_{t+j} , meaning that $E_t(h_{t+j}) = i_{t+j} + \rho_{t+j}$. To bridge the gap between equation (3.2) and equation (3.3) we further assume that one can able to approximate the expectation of the future dividend and short-term interest rate by current and lagged values of macroeconomic news and the interest rate. By matching the equation (3.2) and (3.3), the shock η_t would be interpreted as stock market shock, which originate the change to risk preference.

In this paper, we are interested in the estimation of equation (3.2) to investigate the response of asset price to the change in the monetary policy. However, the estimation

¹⁰ For notational simplicity, we let z_t is a single variable.

¹¹ The shock could also reflect any factors driving a wedge between the interest rate and the policy expectation.

¹² As mentioned in Rigobon and Sack (2004), it is an oversimplified model of the relationship between movements and in interest rates and asset prices, but they justify this model with the evidence that allowing for a richer lag structure had little effect on the result.

of the response of asset prices to changes in monetary policy is problematic due to the simultaneous relationships and omitted variables (see, Rigobon and Sack, 2004). Because of the simultaneity, the data we observe is the intersection of the interest rates and the asset prices, and it thus may not be useful to investigate the slope of the equation (3.2). Another problem would be the omitted variable bias. The common shock z_t would shift the interest rate and the asset prices at the same time, the realization would be affected by the coefficients γ on those variables. The equation (3.3) also describes the log level of stock prices whereas the equation (3.2) is about the change in stock prices z_t , but nearly identical results hold if the VAR instead uses the log level of stock prices given the lags included in the VAR (see, Rigobon and Sack, 2003).

As an alternative, one can employ VARs to capture the dynamics of asset prices with lagged term in equation (3.1) and (3.2) (see, e.g., Campbell and Ammer, 1993). By employing VARs, the parameters of the structural equation (3.2) can be recovered by imposing restrictions, usually, the identification of VARs often takes the form of exclusion restriction, which is either α or β is zero, which is unrealistic. Holding everything else equal, higher interest rates are associated with lower stock market prices, given the higher discount rate for the expected stream of dividend. Also, the Federal Reserve may respond to higher stock prices by raising interest rate at the same time. Therefore, the exclusion restriction, either α or β is zero, would misinterpret the parameter in the structural equation (See, Rigobon and Sack, 2003).

To resolve this problem, Rigobon and Sack (2003) introduce a novel identification procedure by using the heteroskedasticity, and extended this method for the estimation of the impact of monetary policy on asset price. This intuition was first suggested by Wright (1928). He originally explains how the bias from OLS would disappear if the variance of one of the shocks goes to infinity, in which case one of the equations is identified. For Rigobon and Sack (2003) case, it needs only a shift in the relative magnitudes of the variance of the shocks. In next section, we carefully review the method proposed by Rigobon and Sack (2004), and we raise some issues of potential problems in the estimation of the equation (3.2).

3.3. Identification through heteroskedasticity and Markov-Switching framework

3.3.1. Review of Rigobon and Sack (2004)

In a seminal paper, Rigobon and Sack (2004) develop a technique called identification through heteroskedasticity to estimate equation (3.2). By investigating the changes in the variance of interest rates and asset prices, the response of asset prices to monetary policy shock can be estimated. The idea is based on the assumption that one can identify a period of time in which the variances of the other shocks in the system remained unchanged. One need to identify a period of time of the changes in the variance to use this approach. Suppose one can identify two subsample, denoted s_1 and s_2 , and one need to have the following assumptions for the variance:

$$\sigma_{\epsilon}^{s_1} > \sigma_{\epsilon}^{s_2} \quad (3.4)$$

$$\sigma_{\eta}^{s_1} = \sigma_{\eta}^{s_2} \quad (3.5)$$

$$\sigma_z^{s_1} = \sigma_z^{s_2} \quad (3.6)$$

The assumption (3.4)-(3.6) means that the variance of monetary policy shock would elevate in the subsample s_1 whereas the variance of other shocks remain same. This procedure, thus, relies on the heteroskedasticity of policy shocks that happens on particular dates, including days of FOMC meetings and of the Chairman's semi-annual monetary policy testimony to Congress. Rigobon and Sack (2004) show that the correlation between the policy rate and these other asset prices shifts importantly on those dates, as one would expect given the greater importance of policy shocks justifying the above identification assumption.

In order to understand the identification through heteroskedasticity, consider the following the reduced-form of equation (3.1) and (3.2):

$$\Delta i_t = \frac{1}{1 - \alpha\beta} [(\beta + \gamma)z_t] + \beta\eta_t + \epsilon_t], \quad (3.7)$$

$$\Delta s_t = \frac{1}{1 - \alpha\beta} [(1 + \alpha\gamma)z_t] + \eta_t + \alpha\epsilon_t]. \quad (3.8)$$

Then, the variance-covariance matrix for two subsample, denoted Ω_{s_1} and Ω_{s_2} would be:

$$\Omega_{s_1} = \frac{1}{1 - \alpha\beta} \begin{bmatrix} \sigma_\epsilon^{s_1} + \beta^2\sigma_\eta^{s_1} + (\beta + \gamma)^2\sigma_z^{s_1} & \alpha\sigma_\epsilon^{s_1} + \beta\sigma_\eta^{s_1} + (\beta + \gamma)(1 + \alpha\gamma)\sigma_z^{s_1} \\ \cdot & \alpha^2\sigma_\epsilon^{s_1} + \sigma_\eta^{s_1} + (1 + \alpha\gamma)^2\sigma_z^{s_1} \end{bmatrix}, \quad (3.9)$$

$$\Omega_{s_2} = \frac{1}{1 - \alpha\beta} \begin{bmatrix} \sigma_\epsilon^{s_2} + \beta^2\sigma_\eta^{s_2} + (\beta + \gamma)^2\sigma_z^{s_2} & \alpha\sigma_\epsilon^{s_2} + \beta\sigma_\eta^{s_2} + (\beta + \gamma)(1 + \alpha\gamma)\sigma_z^{s_2} \\ \cdot & \alpha^2\sigma_\epsilon^{s_2} + \sigma_\eta^{s_2} + (1 + \alpha\gamma)^2\sigma_z^{s_2} \end{bmatrix}. \quad (3.10)$$

With the assumption that the parameters α, β , and γ would not vary across the two subsample, the difference in variance-covariance matrices is given as:

$$\Delta\Omega = \Omega_{s_1} - \Omega_{s_2} = \frac{(\sigma_\epsilon^{s_1} - \sigma_\epsilon^{s_2})}{(1 - \alpha\beta)^2} \begin{bmatrix} 1 & \alpha \\ \alpha & \alpha^2 \end{bmatrix}. \quad (3.11)$$

The identification of α is straightforward from equation (3.10). To estimate the response of asset prices to monetary policy shock, captured in α , one can replace the change in the variance-covariance matrices with the sample estimates. By letting $\Delta y_t = [\Delta i_t \quad \Delta s_t]'$ and defining dummy variables, $\delta_t^{s_1}$ and $\delta_t^{s_2}$, that have the value 1 for all days for each subsamples, respectively.

Then, the sample estimates would be:

$$\hat{\Omega}_{s_1} = \frac{1}{T_{s_1}} \sum_{t=1}^T \delta_t^{s_1} \Delta y_t \Delta y_t', \quad (3.12)$$

$$\hat{\Omega}_{s_2} = \frac{1}{T_{s_2}} \sum_{t=1}^T \delta_t^{s_2} \Delta y_t \Delta y_t'. \quad (3.13)$$

Then, the parameter α can be estimated as follows:

$$\hat{\alpha} = \frac{\Delta \hat{\Omega}_{12}}{\Delta \hat{\Omega}_{11}} = \frac{\Delta \hat{\Omega}_{22}}{\Delta \hat{\Omega}_{12}}, \quad (3.14)$$

where $\Delta \hat{\Omega}_{i,j}$ indicates the (i, j) element of the change in the $\hat{\Omega}$ matrix ¹³.

¹³ For details, refer to Rigobon and Sack (2004)

3.3.2. Identification of the change in the variance based on Markov-switching

One of the problems of the implementation of identification through heteroskedasticity is that one need to be aware the time when the variance of monetary policy shock changes. If not, it would generate the bias for the structural parameter that we are interest in. In this sense, timing of the policy moves need to be clear to split samples according to the heteroskedasticity of monetary policy shocks.

Rigobon and Sack (2004) assume that days of FOMC meeting and of the Chairman's semi-annual monetary policy testimony to Congress are likely to have a greater amount of news about monetary policy than other days. Also, they further assume that other types of shocks still take place on these days, but the relative importance of policy shock is likely to increase dramatically. Based on these assumptions, they split two subsamples for the one to indicate that the variance of the policy shock is elevated denoted s_1 , and the other one, denoted s_2 to represent the set of days immediately preceding those included in s_1 to satisfy the identification condition (3.4)-(3.6). However, if the above assumption for the change of monetary policy shock is violated which means one mis-specify the time in which the variance-covariance matrix shifts, it would generate some bias in the estimates of the parameter α . Also, more importantly, because the stock market is unlikely respond to the anticipated monetary policy changes, the regression on event days would help to obtain more precise estimate results of the Fed's impact on the change in stock returns. ¹⁴

Additionally, in a seminal paper, Farka (2009) discusses the type of the policy shocks: 1) those that alter expectations about the future path of the monetary policy, and 2) those that signal a change in the timing of policy moves. He stresses that the importance of these shocks to obtain more precise estimates of the effect of monetary policy on stock returns. As he mentioned, market participants, therefore, would respond to the information content of policy shock which reflect two factors that one is expectational changes regarding the near term path of future policy, and another one is shifts in the timing of an anticipated policy move. In this sense, if we only use the institutional information to capture the change

¹⁴ Rigobon and Sack (2003) shows the bootstrap results to check the robustness in the use of the identification through heteroskedasticity for the exogenously specified the different variance-covariance matrices.

in the policy shock to employ the identification procedure suggested by Rigobon and Sack (2004), it would not enough to investigate the all possible changes in the policy shocks. In particular, timing of the policy shock shifts caused by revisions in expectations regarding the future path of monetary policy would be very difficult to be found.

In this paper, we use the heteroskedasticity of monetary policy shocks that exist in high-frequency data within the event study framework. In order to take care of the uncertainty of timing issue in previous literature, we use Markov-switching framework to detect different states instead of specifying the high and low volatility states exogenously. And, it would reduce the possible bias which may be caused by the uncertainty of the shift of the policy shock, and it accounts for the possible biases in the conventional event study approaches. In what follows, we introduce more details about the identification through heteroskedasticity proposed by Rigobon and Sack (2004) with Markov-switching framework.

From the reduced-form of equation (3.1) and (3.2), the reduced-form can be written with the reduced-form shock as follows.

$$\Delta y_t = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = e_t, \quad e_t \sim N(0, \Omega_{s_t}), \quad (3.15)$$

where s_t is the unobserved state variable, and Ω_{s_t} is given by:

$$\Omega_{s_t} = \begin{bmatrix} \Omega_{11,s_t} & \Omega_{12,s_t} \\ \Omega_{21,s_t} & \Omega_{22,s_t} \end{bmatrix}. \quad (3.16)$$

Also, we assume that two-state Markov-chain with the following transition probability.

$$\Pi = \begin{bmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{bmatrix}, \quad (3.17)$$

where $p_{ij} = Pr[s_t = j | s_{t-1} = i]$ for $i = 0, 1$ and $j = 0, 1$.

Step 1: Estimate equation (3.15) via Maximum-Likelihood Estimation (MLE), and get the reduced-from variance-covariance matrix Ω_{s_t} for each regime.

Step 2: Based on the estimates of Ω_{s_t} , we calculate the change of the variance covariance matrix, denoted as $\hat{\Omega} = \hat{\Omega}_1 - \hat{\Omega}_2$.

Step 3: We apply the procedure developed by Rigobon and Sack (2004) to get an inference of α in equation (3.1).

Note that one can identify two distinct regimes based on Markov-switching framework endogenously in the first step. With taking care of the uncertainty of shifts in monetary policy shock, one can apply the same procedure to Rigobon and Sack (2004). The presence of unconditional heteroskedasticity in the reduce form allows us to identify the structural parameter α , but the limitation of this analysis is that we are only able to obtain the partial identification of the model.

3.4. Estimation Results

3.4.1. Data

Estimating the response of stock prices to monetary policy actions is difficult by the fact that the market is unlikely to respond to policy actions that were already anticipated (see Bernake and Kuttner, 2005). Asset markets are forward-looking and hence tend to incorporate any information about anticipated policy changes. Thus, distinguishing between expected and unexpected policy actions is important for understanding their effect. Kuttner (2001) suggests a way to construct a measure of ‘surprise’ rate changes by using Federal funds futures data ¹⁵. In this paper, we analyze the effects of FOMC target rate decisions between the July of 1989 through the April of 2008. ¹⁶. Following Kuttner (2001) and Bernanke and Kuttner (2005) we use same data form FFR futures contracts in order to use the unexpected part of the FFR change. On the day of the FOMC decision, the FFR shock, is measured by the change in the implied rate of the current-month FFR futures contract, as traded on the CBOT market, relative to the day before the FOMC announcement, scaled by a factor related to the number of days in the month affected by the change. Then, a measure

¹⁵ Rigobon and Sack (2002) utilize the euro-dollar future rate to focus on the surprise component of policy moves.

¹⁶ We use the data provided from <http://econ.williams.edu/people/knk1>.

of the surprise element of any specific change in the Federal funds target can be written as:

$$\Delta i_t = \frac{D}{D-d}(f_{m,d}^0 - f_{m,d-1}^0), \quad (3.17)$$

where Δi_t is the unexpected target rate changes, $f_{m,d}$ is the current-month future rate, and D is the number of days in the month.

Before 1994, changes in the federal funds target rate were not announced by FOMC, in which policy changes were not closed and a significant policy move happened between meetings. After 1994, however, the day of the target rate changes are well known; and the policy action is announced. In this case, the day-to-day change in the future rate on days would reflect changes in the market's expectation of the target funds rate on some future date (Kuttner, 2008). Therefore, in pre-1994, day-to-day changes in future target rate could well be a consequence from either changes in policy preferences or macro news while, in post-1994, policy surprises on event day, are more likely comes from changes in the Fed's preference. The policy of announcing target rate changes, which began in February 1994, eliminates most of the timing uncertainty associated with rate changes in the earlier part of the sample. After 1994 period when all Federal funds rate changes were announced, and most coincided with FOMC meetings. Therefore, to make use of pre-1994 sample, one should be aware of the uncertainty of timing in shifts of monetary policy shocks.

The stock market data we use, which consists of high-frequency observations of the Nasdaq index. The stock market returns are log differences of the Nasdaq index on the close of the day of the FOMC meeting, relative to the previous trading day.

3.4.2. Results

Table 3.2 shows the estimation results. As we can see in Table 3.2, we are able to detect two different regime on event-day. In a seminal paper, Kontonikas et al. (2013) found that the heteroscedastic structure in monetary policy shock and state dependence, with the reaction being stronger during the bad times on the event-days. Therefore, in order to identify the impact of stock price to monetary policy shock one should take care of the asymmetric effects and the heteroscedasticity due to policy changes. Focusing on the event days, we found that

the estimate $\hat{\alpha}$ is -7.859 , implying that the an unanticipated 25-basis point change increase 1.98% decline in the Nasdaq. In Rigobon and Sack (2004), they found that 2.4% decline in stock price, which is larger reaction than our estimates. Within event-study framework, Our results relax the assumption that the larger variance in the monetary policy shocks always happened on the event-days than non event-days. The two distinct regime found in this paper in the reduced form shock tells us the higher volatility state to Fed's expansionary shock, and the low volatility state to Fed's contractionary response to the economy.

In order words, the state dependence heteroscedastic structure on event-day could well be a consequence of the asymmetric effects to the different types of policy actions: expansionary or contractionary policy. It turns out the periods that we have high volatility states coincide the periods the economy in the recession, and this findings are consistent to Kontonikas et al. (2013). Therefore, as Rigobon and Sack (2004), the assumption that the monetary policy shock always would be higher on event-day than on non-event day possibly generate biased estimates due to the uncertainty of timing in the changes of monetary policy shock. If the future policy are already announced, then it is possible that we could have mild reaction in stock price to policy changes. Also, market reaction could have asymmetric effects depending on the type of policy actions: rate cut vs. rate hike. In this sense, we need to employ Markov-switching framework to detect two different regime endogenously in the first step.

3.5. Conclusion

In this paper, we take care of the endogeneity issues in estimating the response of stock prices due to simultaneity and the omitted variable bias. We extend the work of Rigobon and Sack (2004) by incorporating Markov-switching framework. The estimation procedures employed in this paper captures heteroskedasticity of monetary policy shocks endogenously to account for all possible changes in monetary policy shocks. Once one estimate the timing of monetary policy changes, the heteroskedasticity structure helps to identify the impact of stock market to policy changes.

Since the monetary policy shocks possibly come from the alteration of expectations about the future path of the monetary policy and a change in the timing of policy moves. In this sense, if we only use the institutional information to capture the change in the policy shock to employ the identification procedure suggested by Rigobon and Sack (2004), it would not enough to investigate the all possible changes in the policy shocks. In particular, timing of the policy shock shifts caused by revisions in expectations regarding the future path of monetary policy would be very difficult to be found. In this paper, using Markov-switching framework to detect different states, we also account for the uncertainty of timing issue instead of specifying the high and low volatility states exogenously. And, it would reduce the possible bias which may be caused by the uncertainty of the shift of the policy shock.

We found that the two distinct regime in the reduced form shocks, which tells us the higher volatility state to Fed's expansionary shock, and the low volatility state to Fed's contractionary response to the economy. The state dependence heteroscedastic structure on event-day could well be a consequence of the asymmetric effects to the different types of policy actions: expansionary or contractionary policy. Our finding implies that the an unanticipated 25-basis point change increases 1.98% decline in the Nasdaq.

Appendix A. Estimating the Initial Value for the Control Functions, g_0 .

Through recursive substitutions, we rewrite equation (1.18) for $t = 0, 1, 2, \dots, T - 1$ as follows:

$$\begin{aligned}\hat{\eta}_1 &= g_1 - \hat{\theta}g_0, \\ \hat{\eta}_2 &= g_2 - \hat{\theta}\hat{\eta}_1 - \hat{\theta}^2g_0, \\ \hat{\eta}_3 &= g_3 - \hat{\theta}\hat{\eta}_2 - \hat{\theta}^2\hat{\eta}_1 - \hat{\theta}^3g_0, \\ &\vdots \\ \hat{\eta}_T &= g_T - \hat{\theta}\hat{\eta}_{T-1} - \hat{\theta}^2\hat{\eta}_{T-2} - \dots - \hat{\theta}^{T-1}\hat{\eta}_1 - \hat{\theta}^Tg_0.\end{aligned}$$

By rearranging terms in the above equations, we can obtain the following regression equation in which the coefficient is g_0 and the disturbance term is g_t :

$$\tilde{\eta}_t = g_0\tilde{x}_t + g_t, \quad g_t \sim i.i.d.N(0, \sigma^2),$$

where $\tilde{\eta}_t = \hat{\eta}_t + \sum_{i=1}^{t-1} \hat{\theta}^{t-i}\hat{\eta}_i$ and $\tilde{x}_t = -(\hat{\theta})^t$. Then, g_0 , can be estimated by the following OLS estimators:

$$\hat{g}_0 = \frac{\sum \tilde{x}_t\tilde{\eta}_t}{\sum \tilde{x}_t^2}.$$

Appendix B. Macroeconomic Interpretation of Testing Determinacy.

Figure B.1-B.2 depict the 90 percent level confidence sets constructed by inverting three alternative tests on (β_π, β_x) , designed for checking the condition for determinacy given below, and employed by Clarida et al. (2000) and Mavroeidis (2010):

$$\beta_\pi + \frac{1 - \beta}{\lambda}\beta_x \geq 1, \tag{B.1}$$

where the parameter $0 < \psi < 1$ is a discount factor and the λ parameter captures the degree of nominal rigidities in a forward-looking Phillips curve. The three alternative test considered are: i) a Wald test based on GMM, ii) an identification-robust test based on GMM; and iii) a Wald test based on the proposed method. The confidence sets are constructed as in Mavroeidis (2010), by fixing the values of ψ and λ at 0.99 and 0.3, respectively.

The equation (B.1) represent Taylor Principle that the Fed raises real rates in response to inflation to eliminate the possibility of self-fulfilling inflation. The above condition is more general than the original Taylor Principle that simply states that the coefficients on inflation in the Taylor rule is greater than one. With the discount factor close to one, the above condition derived in the New Keynesian framework still means that the coefficient on inflation in the Taylor rule be greater than one. As mentioned in Section 4, even though one cannot interpret the Taylor principle directly based on the proposed method, it would be worthwhile to check the confidence sets based on the proposed method in order to see the advantage of taking care of moving-average term appropriately compared to GMM estimation.

For the 1960:Q1-1979:Q2 sample, we get very robust results regardless of the inference methods employed. The two dimensional 90 % confidence sets on the β_π and the β_x parameters all cover the indeterminacy region. For the 1979:Q3-1997:Q4 sample, the identification-robust inference, we cannot reject the null of indeterminacy. However, the Wald confidence set from the proposed method is much smaller than the identification-robust confidence set from GMM. The proposed approach results in reasonably accurate estimates of the parameters, and the 90 % confidence set for the reaction parameters lies outside the indeterminacy region.

Appendix C: Reduced-form Monetary Policy Rule in Equation (2.4)

Notice that the coefficients of the reduced form Taylor rule given by equation (2.4) would be the function of the semi-structural VAR model consisting of the equation (2.1)-(2.3). In order to understand the relationship between the coefficients in the structural equation and the ones in the reduced form equation, we consider the second order of VAR of the dynamics

of the inflation and the unemployment as follows.

$$i_t = \delta_{i,0t} + \delta_{i,11,t}\pi_{t-1} + \delta_{i,12,t}x_{t-1} + \delta_{i,13,t}i_{t-1} + \delta_{i,14,t}\pi_{t-2} + \delta_{i,15,t}x_{t-2} + \delta_{i,16,t}i_{t-2} + e_{1,t} \quad (C.1)$$

In the following derivation, we assume that i) $E_t\delta_{i,1j,t+1} = \delta_{i,1j,t}$ for $j = 1, 2, \dots, 6$ and ii) $E_t e_{j,t} = 0$; for $k = 1, 2, 3$. After solving equation (2.2) forward, and we combine this with equation (2.1) and (2.3). Then, we would get the following relationship between equation (2.1) and (2.4).

$$\delta_{i,0,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{0,t} + \alpha_{1,t}\{\delta_{\pi,0,t}(1 + \delta_{\pi,11,t})\} + \delta_{x,0,t}(\alpha_{1,t}\delta_{\pi,12,t} + \alpha_{2,t})]$$

$$\delta_{i,11,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}(\delta_{\pi,11,t}^2 + \delta_{\pi,12,t}\delta_{x,11,t} + \delta_{\pi,14,t}) + \alpha_{2,t}\delta_{x,11,t}]$$

$$\delta_{i,12,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}\{\delta_{\pi,12,t}(\delta_{x,11,t} + \delta_{x,12,t}) + \delta_{\pi,13,t}\} + \alpha_{2,t}\delta_{x,12,t}]$$

$$\delta_{i,13,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}\{\delta_{\pi,13,t}(\delta_{x,11,t} + \delta_{x,13,t}) + \delta_{i,16,t}\} + \alpha_{2,t}\delta_{x,13,t} + \alpha_{3,t}]$$

$$\delta_{i,14,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}(\delta_{\pi,11,t}\delta_{\pi,14,t} + \delta_{\pi,12,t}\delta_{x,14,t}) + \alpha_{2,t}\delta_{x,14,t}]$$

$$\delta_{i,15,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}(\delta_{\pi,11,t}\delta_{\pi,14,t} + \delta_{\pi,12,t}\delta_{x,15,t}) + \alpha_{2,t}\delta_{x,15,t}]$$

$$\delta_{i,16,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}(\delta_{\pi,11,t}\delta_{i,16,t} + \delta_{\pi,12,t}\delta_{x,16,t}) + \alpha_{2,t}\delta_{x,16,t}]$$

$$e_{1,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[(\alpha_{1,t}\delta_{\pi,11,t})e_{2,t} + m_t + (\alpha_{1,t}\delta_{\pi,12,t} + \alpha_{2,t}e_{3,t})]$$

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Table 1.1. Simulation Results for α_1 [True $\alpha_1 = 1$] for the Normal distribution of the reduced-form shocks.

$$y_{1t} = \alpha_1 E_t y_{2t+1} + \alpha_2 y_{1t-1} + \varepsilon_{1t}$$

$$y_{2t} = \phi_{21} y_{2t-1} + e_{2t}$$

Parameter estimates				
	GMM		Proposed method	
	Mean	SD	Mean	SD
$\mu^2/k = 1$	0.674	0.784	1.004	0.180
$\mu^2/k = 30$	0.998	0.148	1.002	0.070

Hypothesis testing ($H_0 : \alpha_1 = 1$, 5% level test)			
	Rejection frequencies		
	GMM-Wald	GMM-Robust	Proposed-Wald
$\mu^2/k = 1$	0.182	0.052	0.050
$\mu^2/k = 30$	0.062	0.050	0.051

Note: The sample size that we use is 1000. 5,000 Monte Carlo replications. SD means the Standard deviation of the mean. Threshold values for weak instrument Tests for Two-stage Least Squares (TSLS) based on Concentration parameter that ensure the bias of TSLS is no more than 10% of the inconsistency of OLS is 3.71 for three instruments. And threshold values when weak instruments are defined so that the usual nominal 5% TSLS t-test of the hypothesis testing has size potentially exceeding 15% is 6.36 for three instruments. (Source: Stock and Yogo, 2002). Implied parameters for ϕ_{21} of each case are 0.251 and 0.572, respectively.

Table 1.2. Simulation Results for α_1 [True $\alpha_1 = 1$] for the Student-t distribution of the reduced-form shocks.

$$y_{1t} = \alpha_1 E_t y_{2t+1} + \alpha_2 y_{1t-1} + \varepsilon_{1t}$$

$$y_{2t} = \phi_{21} y_{2t-1} + e_{2t}$$

Parameter estimates				
	GMM		Proposed method	
	Mean	SD	Mean	SD
$\mu^2/k = 1$	0.687	0.634	1.006	0.175
$\mu^2/k = 30$	0.994	0.143	1.001	0.068

Hypothesis testing ($H_0 : \alpha_1 = 1$, 5% level test)			
	Rejection frequencies		
	GMM-Wald	GMM-Robust	Proposed-Wald
$\mu^2/k = 1$	0.190	0.036	0.054
$\mu^2/k = 30$	0.072	0.040	0.058

Note: The sample size that we use is 1000. 5,000 Monte Carlo replications. SD means the Standard deviation of the mean. We assume that the reduced-form errors follow student-t distribution with degree of freedom 5. Threshold values for weak instrument Tests for Two-stage Least Squares (TSLS) based on Concentration parameter that ensure the bias of TSLS is no more than 10% of the inconsistency of OLS is 3.71 for three instruments. And threshold values when weak instruments are defined so that the usual nominal 5% TSLS t-test of the hypothesis testing has size potentially exceeding 15% is 6.36 for three instruments. (Source: Stock and Yogo, 2002). Implied parameters for ϕ_{21} of each case are 0.251 and 0.572, respectively.

Table 1.3. Simulation Results for α_1 [True $\alpha_1 = 1$] for the GARCH (1, 1) distribution of the reduced-form shocks.

$$y_{1t} = \alpha_1 E_t y_{2t+1} + \alpha_2 y_{1t-1} + \varepsilon_{1t}$$

$$y_{2t} = \phi_{21} y_{2t-1} + e_{2t}$$

Parameter estimates				
	GMM		Proposed method	
	Mean	SD	Mean	SD
$\mu^2/k = 1$	0.694	0.894	1.005	0.220
$\mu^2/k = 30$	1.006	0.164	1.004	0.082

Hypothesis testing ($H_0 : \alpha_1 = 1$, 5% level test)			
Rejection frequencies			
	GMM-Wald	GMM-Robust	Proposed-Wald
$\mu^2/k = 1$	0.130	0.046	0.064
$\mu^2/k = 30$	0.052	0.048	0.048

Note 1: The sample size that we use is 1000. 5,000 Monte Carlo replications. SD means the Standard deviation of the mean. Threshold values for weak instrument Tests for Two-stage Least Squares (TSLS) based on Concentration parameter that ensure the bias of TSLS is no more than 10% of the inconsistency of OLS is 3.71 for three instruments. And threshold values when weak instruments are defined so that the usual nominal 5% TSLS t-test of the hypothesis testing has size potentially exceeding 15% is 6.36 for three instruments. (Source: Stock and Yogo, 2002). Implied parameters for ϕ_{21} of each case are 0.251 and 0.572, respectively. We assume that the reduced-form shocks follow GARCH (1, 1) given by:

$$E(e_{1,t}^2 | I_{t-1}) = 0.1 + 0.01e_{1,t-1}^2 + 0.9\sigma_{1,t-1}^2$$

$$E(e_{2,t}^2 | I_{t-1}) = 0.1 + 0.01e_{2,t-1}^2 + 0.9\sigma_{2,t-1}^2$$

Table 1.4. Forward-looking Taylor Rule Equation Estimates

$$i_t = \beta_0 + (1 - \beta_1)(\beta_\pi E_t \pi_{t+1} + \beta_x x_t) + \beta_1 i_{t-1} + \varepsilon_t$$

	1961:1 – 1979:2		1979:3 – 1997:4	
	GMM	Proposed method	GMM	Proposed method
β_π	0.945 (0.167)	0.953 (0.096)	2.244 (0.103)	1.979 (0.317)
β_x	0.658 (0.240)	0.404 (0.114)	0.822 (0.182)	0.647 (0.333)
β_1	0.797 (0.085)	0.574 (0.115)	0.828 (0.215)	0.775 (0.059)
Hypothesis testing ($H_0 : \beta_\pi = 1$, $H_1 : \beta_\pi > 1$)				
	GMM-Robust	Proposed-Wald	GMM-Robust	Proposed-Wald
Test-Statistic (P-value)	0.945 (0.167)	0.235 (0.628)	0.945 (0.167)	9.542 (0.002)

Note: Standard errors are in parenthesis. Data we use as in Mavroeidis. (2010), the interest rate is the average federal funds rate in the first-month of each quarter; inflation is measured by the % change of the GDP deflator; the output gap is the series constructed by CBO. The IVs include intercept, four lags of inflation, output gap, and federal funds rate. The pre-Volcker sample is 1961:2 to 1979:2, and the Volcker-Greenspan sample is the 1979:3–1997:4.

Table 1.5. Monte Carlo Results based on Proposed method (from bootstrapped data)

$$i_t = \beta_0 + (1 - \beta_1)(\beta_\pi E_t \pi_{t+1} + \beta_x x_t) + \beta_1 i_{t-1} + \varepsilon_t$$

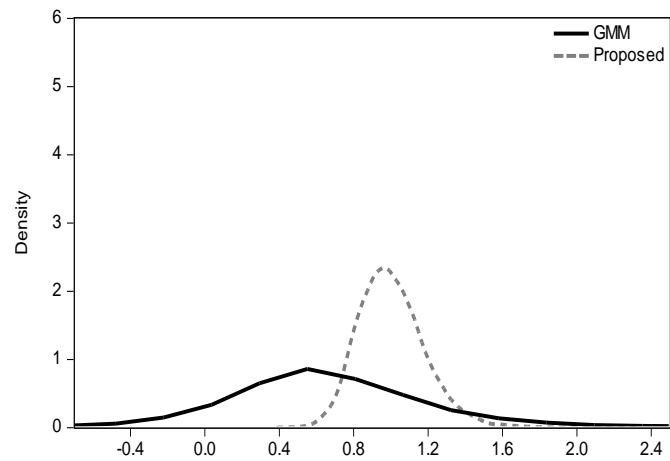
Parameter estimates			
True values	Mean	SD	
$\beta_\pi=1.979$	1.996	0.345	
$\beta_x=0.647$	0.631	0.366	
Hypothesis testing (Size of the test)			
	1%	5%	10%
$H_0 : \beta_\pi=1.979$	0.017	0.046	0.074
$H_0 : \beta_x=0.647$	0.028	0.062	0.097

Note: SD means the Standard deviation of the mean. We generate the same number of bootstrapped samples for Volcker-Greenspan periods in the estimation of the Taylor rule by 5,000 replications. True values are obtained from the estimates based on the Proposed method. We generate artificial series of the variables from the estimated coefficients in the VAR order 4 and the residuals as if they were population values, and estimate new coefficients.

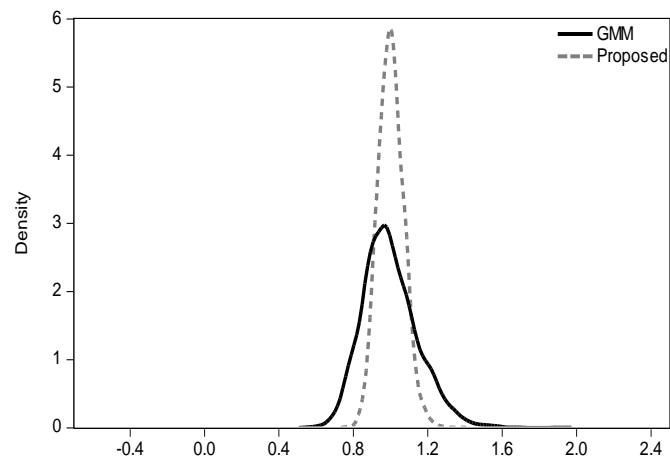
Figure 1.1. Distributions of α_1 [True $\alpha_1 = 1$] for the Normal distribution of the reduced-form shocks.

$$y_{1t} = \alpha_1 E_t y_{2t+1} + \alpha_2 y_{1t-1} + \varepsilon_{1t}$$

$$y_{2t} = \phi_{21} y_{2t-1} + e_{2t}$$



$\mu^2/k = 1$

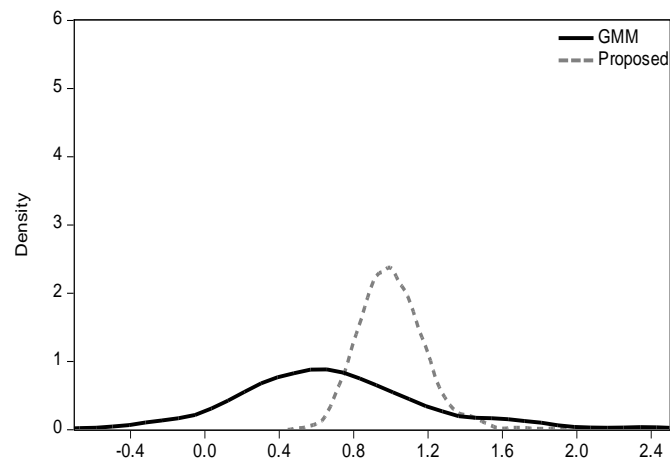


$\mu^2/k = 30$

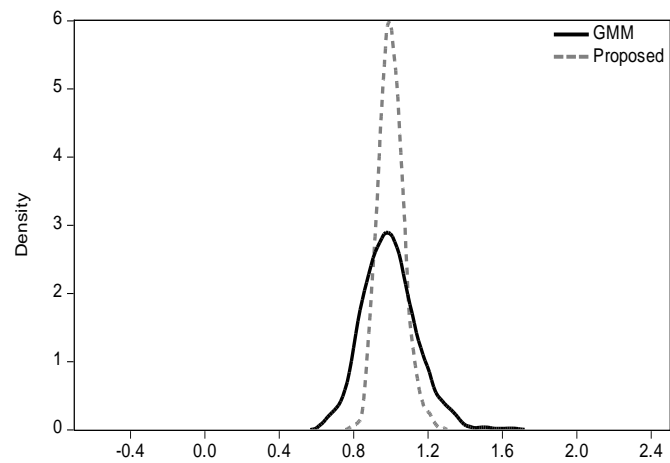
Figure 1.2. Distributions of α_1 [True $\alpha_1 = 1$] for the Student-t distribution of the reduced-form shocks.

$$y_{1t} = \alpha_1 E_t y_{2t+1} + \alpha_2 y_{1t-1} + \varepsilon_{1t}$$

$$y_{2t} = \phi_{21} y_{2t-1} + e_{2t}$$



$\mu^2/k = 1$

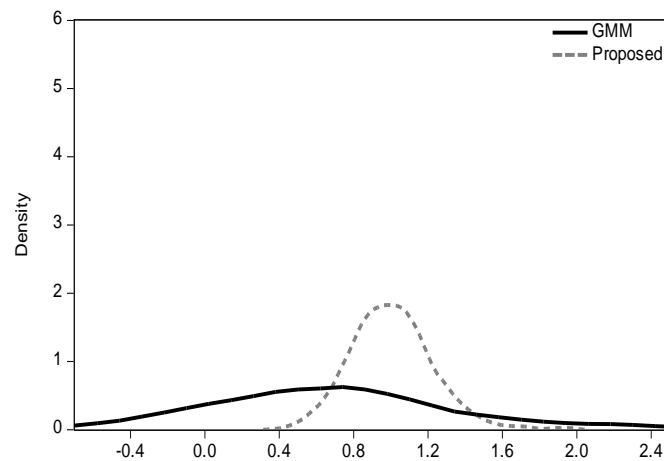


$\mu^2/k = 30$

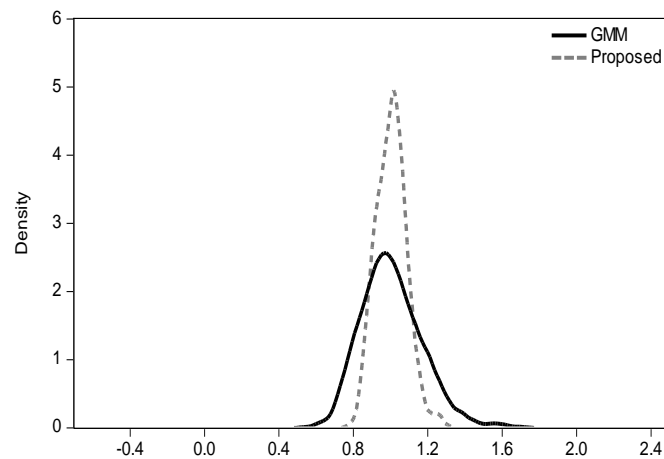
Figure 1.3. Distributions of α_1 [True $\alpha_1 = 1$] for the GARCH (1, 1) distribution of the reduced-form shocks.

$$y_{1t} = \alpha_1 E_t y_{2t+1} + \alpha_2 y_{1t-1} + \varepsilon_{1t}$$

$$y_{2t} = \phi_{21} y_{2t-1} + e_{2t}$$



$\mu^2/k = 1$

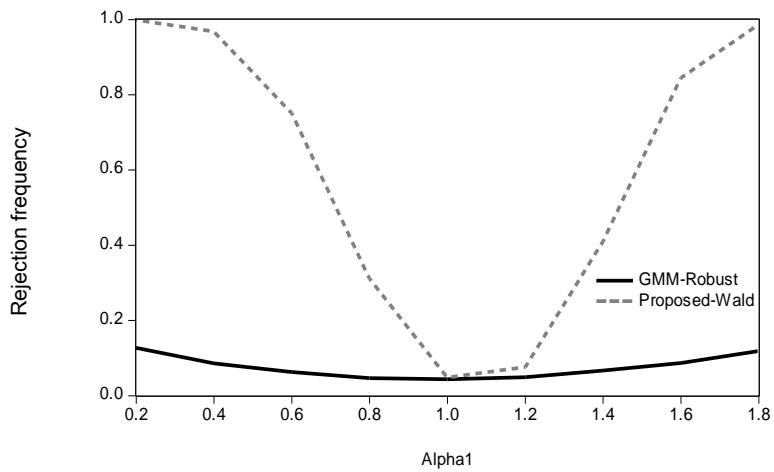


$\mu^2/k = 30$

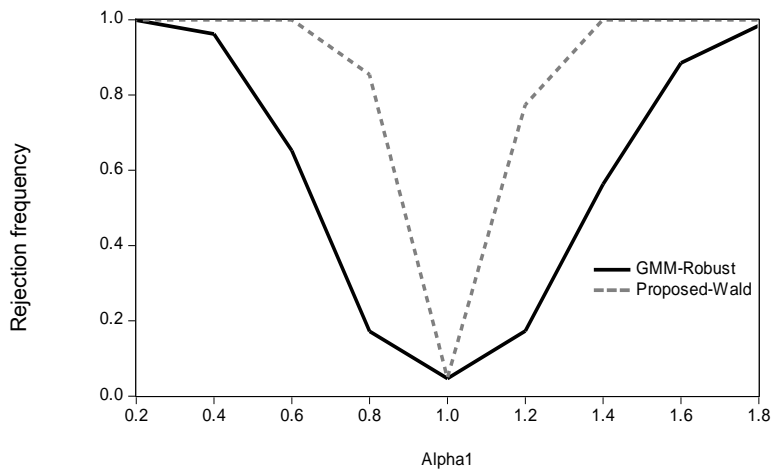
Figure 1.4. Power curves of 5% level tests for $H_0 : \alpha_1 = 1$ against $H_1 : \alpha_1 \neq 1$ for the Normal distribution of the reduced-form shocks.

$$y_{1t} = \alpha_1 E_t y_{2t+1} + \alpha_2 y_{1t-1} + \varepsilon_{1t}$$

$$y_{2t} = \phi_{21} y_{2t-1} + e_{2t}$$



$$\mu^2/k = 1$$

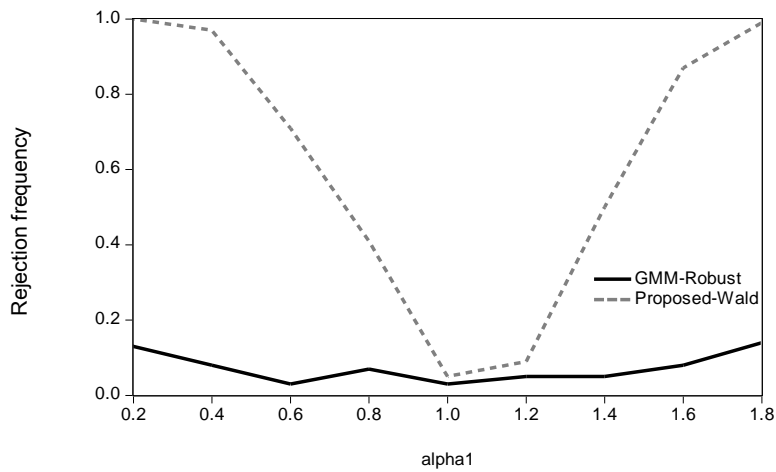


$$\mu^2/k = 30$$

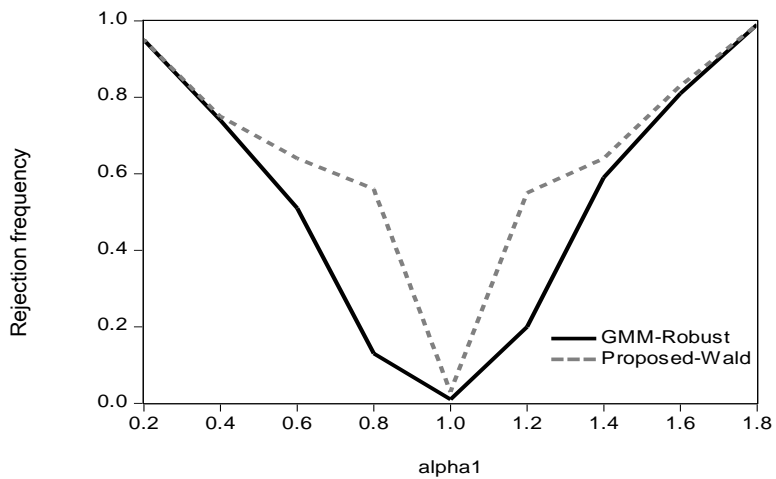
Figure 1.5. Power curves of 5% level tests for $H_0 : \alpha_1 = 1$ against $H_1 : \alpha_1 \neq 1$ $[\alpha_1 = 1]$ for the Student-t distribution of the reduced-form shocks.

$$y_{1t} = \alpha_1 E_t y_{2t+1} + \alpha_2 y_{1t-1} + \varepsilon_{1t}$$

$$y_{2t} = \phi_{21} y_{2t-1} + e_{2t}$$



$$\mu^2/k = 1$$

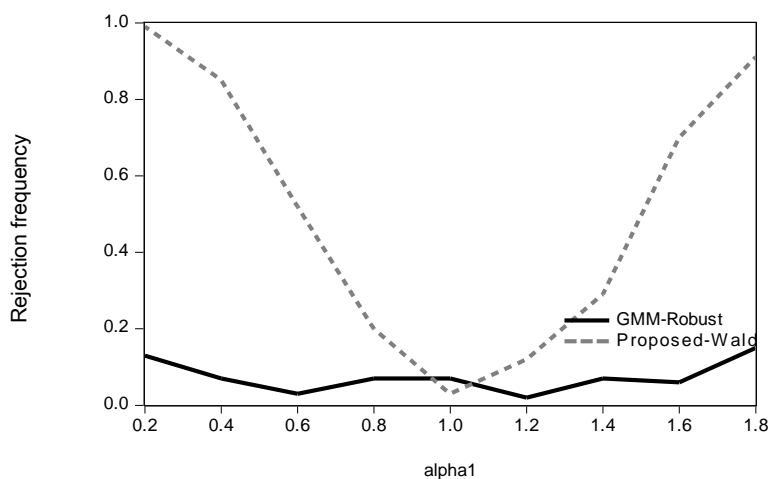


$$\mu^2/k = 30$$

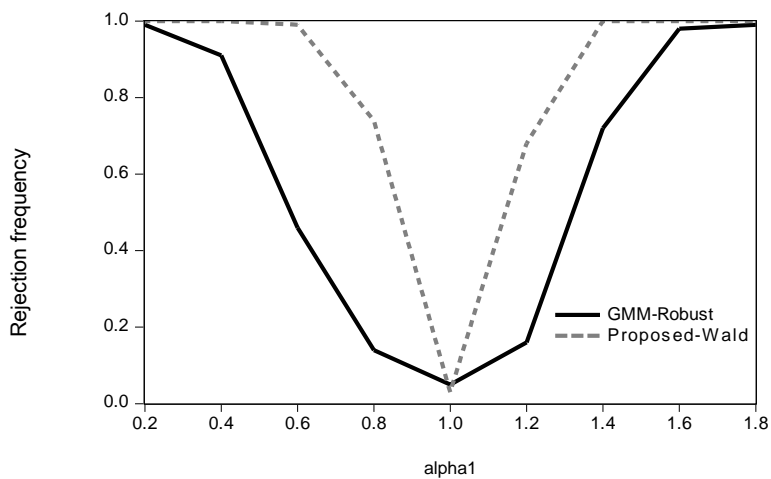
Figure 1.6. Power curves of 5% level tests for $H_0 : \alpha_1 = 1$ against $H_1 : \alpha_1 \neq 1$ for the GARCH (1, 1) distribution of the reduced-form shocks.

$$y_{1t} = \alpha_1 E_t y_{2t+1} + \alpha_2 y_{1t-1} + \varepsilon_{1t}$$

$$y_{2t} = \phi_{21} y_{2t-1} + e_{2t}$$



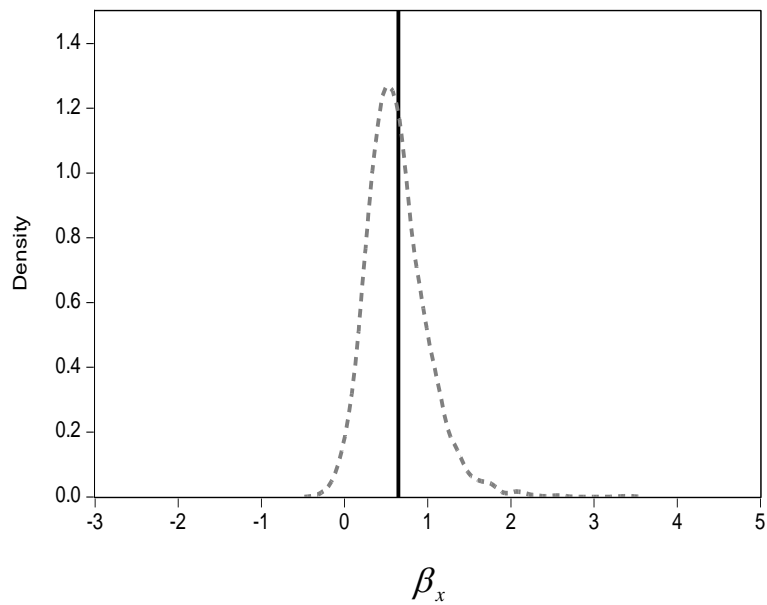
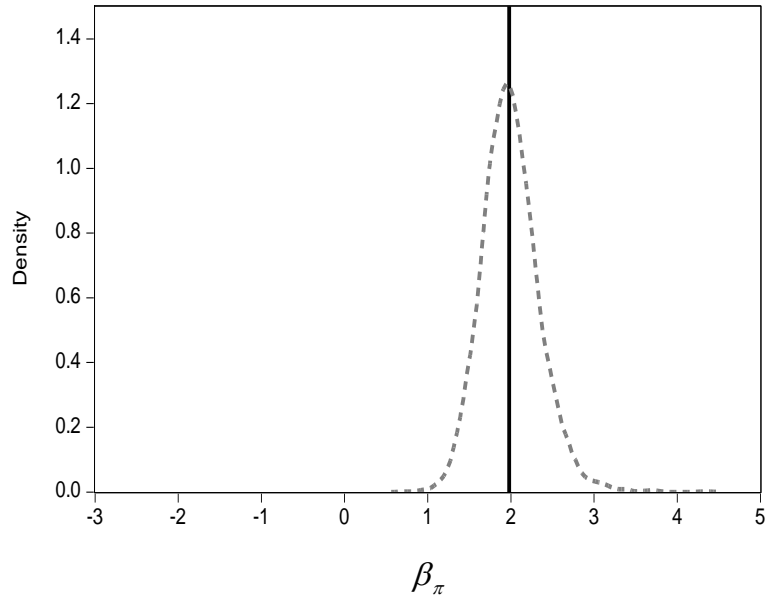
$$\mu^2/k = 1$$



$$\mu^2/k = 30$$

Figure 1.7. Sampling Distribution of the $\hat{\beta}_\pi$ and $\hat{\beta}_x$ obtained from Bootstrapped Samples

$$i_t = \beta_0 + (1 - \beta_1)(\beta_\pi E_t \pi_{t+1} + \beta_x x_t) + \beta_1 i_{t-1} + \varepsilon_t$$



Note: The black vertical line represents true value of the parameters, which are the estimates from actual historical data based on the proposed method.

**Estimation of a Forward-Looking Time-varying Monetary Policy Rule:
Limited Information Approach**

2.1. Introduction

The monetary policy rule has been investigated for several decades to provide understanding of the outbursts of inflation and unemployment of the 1970s and early 1980s. Many works of literature (e.g., Clarida et al. (2000), Boivin and Giannoni (2003), Cogley and Sargent (2005), Lubik and Schorfheide (2004)) have stressed that the U.S. monetary policy was less active against inflation under the Fed chairmanship of Arthur Burns than under Paul Volcker and Alan Greenspan. However, the empirical conclusion of the significant effect on the macroeconomic variables due to the change in monetary policy is still controversial. The opposite view has been addressed by comparing the relative importance of the non-systematic and systematic aspects of the monetary policy changes (see e.g., Bernanke and Mihov (1998), Primiceri (2005), Sims and Zha (2006)).

Various econometric methodology has been suggested to consider the time variation of a forward-looking Taylor-rule-type monetary policy. Kim and Nelson (2006), for example, propose a two-step procedure to deal with time-varying endogenous regressors with heteroscedasticity. They also account for the changing degrees of uncertainty associated with the Fed's forecasts of future economic conditions. By doing so, they achieve efficiency in estimation by employing the standardized projection errors regarding inflation and output gaps. Their empirical findings support the idea of active response toward inflation during the 1980s. In addition, they questioned the reliability of the lagged inflation rates as IV showing a wide confidence band under the Volcker-Greenspan regime.

To consider the multivariate linear setup, Cogley and Sargent (2005) use time-varying variance in the context of VARs with drifting coefficients. They assume that the simultaneous relations among macroeconomic variables are time invariant. To reveal evidence of the

evolution of monetary policy actions, they estimate a forward-looking Taylor rule with the interest rate smoothing method. They suggest a two-stage least squares analysis on a date-by-date basis. In the first step, they project the Feds forecasts of average inflation and unemployment onto a set of instruments that are a constant and two lags of inflation and unemployment rates. The second step involves projecting the current interest rates onto the fitted values from the first-stage projections along with the lagged nominal interest rates. They found evidence for the monetary policy activism, as in Clarida et al. (2000), that monetary policy was passive in the 1970s and active in the Volcker-Greenspan era. Also, more importantly, they point out that estimates for the Volcker-Greenspan period are less precise because of instrument relevance problems, and the tails of the distributions overlap at various dates.

Recently, Chon and Kim (2014) have stressed the efficiency loss of the conventional IV estimation in simultaneous equation models with forward-looking variables. They employ the identifying assumption that monetary policy shocks do not have any contemporaneous effects on macroeconomic variables to derive a Limited Information procedure for the estimation of a forward-looking monetary policy rule. The procedure they design can effectively handle the moving-average disturbances, a problem that inherently results in inefficiency in the IV estimation of a forward-looking model rendering spurious weak identification issues in the Volcker-Greenspan era. The two-step estimation procedures suggested by Kim and Nelson (2006) and Cogley and Sargent (2005) involve the efficiency loss problems that originated from the moving-average disturbances in the instrumenting equations. These researchers concern the relevance of IV for the Volcker-Greenspan samples but could not handle the additional information in the residuals.

With the same identifying assumption for the monetary policy shocks employed by Chon and Kim (2014), Primiceri (2005) analyzes the systematic and non-systematic parts of monetary policy and their effect on the macroeconomic variables with time-varying parameters demonstrating stochastic volatility. His approaches consider a flexible way to consider changes in policy because it captures both time variation of the simultaneous relations among the variables of the model and the heteroscedasticity of the innovations. The empirical conclusion he made is that the high volatility of the exogenous non-policy shocks seems to

explain a larger fraction of the poor economic performance of the 1970s and early 1980s, which were episodes of high unemployment and inflation. However, the monetary policy rule he considered only responds to the current inflation rates without response to real activity. Furthermore, a Lucas (1976) critique issue arises in the monetary policy rule without consideration for the rational and forward-looking behavior of economic agents. In this paper, we propose a Limited-Information procedure to estimate a forward-looking monetary policy rule by modeling multivariate time varying coefficient with stochastic volatility.

The main contribution of this paper is the assertion that, under the identifying assumption that monetary policy shocks do not have simultaneous effects on macroeconomic variables, we can effectively handle the moving-average dynamics in the instrumenting equations. This would improve the efficiency for the estimation of a time-varying forward-looking monetary policy rule mitigating the issue of relevance to instruments (see Cogley and Sargent, 2005). Also, by incorporating the time-varying coefficients and innovations, we can reasonably model for the effects of the changes in monetary policy on the rest of the economy. Not only the forecasts of future expectation for inflation and output gap but also the moving-average dynamics can be obtained from the posterior mean of reduced-form time-varying VAR. Thus, the estimation results obtained in this paper would provide more reliable evidence to understand whether monetary policy rules have changed and that the persistence of inflation itself has drifted over time.

The rest of the paper is summarized as follows. Section 2 presents the time-varying structural VAR model adopted in this paper for the estimation of a forward-looking monetary policy rule. Section 3 introduces the estimation procedure used in this paper with the key features of the estimation strategy. Section 4 explain the empirical results of the application to a forward-looking monetary policy rule. Concluding remarks are offered in Section 5.

2.2. Model Specification

For an empirical estimation of a forward-looking Taylor rule as in Kim and Nelson (2006), we consider the following linear TVP model of monetary policy transmission mechanism.

With a VAR specification of aggregate supply and demand, our complete model of monetary policy takes a sub-VAR system.

$$i_t = \beta_{0,t} + \beta_{1,t}E_t[\pi_{t+1}] + \beta_{2,t}x_t + \beta_{3,t}i_{t-1} + m_t, \quad (2.1)$$

$$\pi_t = \delta_{\pi 0,t} + \tilde{Y}'_{t-1}\delta_{\pi 1,t} + e_{2,t}, \quad (2.2)$$

$$x_t = \delta_{x 0,t} + \tilde{Y}'_{t-1}\delta_{x 1,t} + e_{3,t}, \quad (2.3)$$

where i_t is the federal funds rate; x_t is a measure of the unemployment at time t ; π_t is the percent change in the price level between time t and $t + 1$; $\tilde{Y}_t = [Y'_t \ Y'_{t-1} \ \dots \ Y'_{t-k+1}]'$, with $Y_t = [i_t \ \pi_t \ x_t]'$; and $E_t(\cdot)$ refers to the expectation formed by the Fed conditional on information available at the beginning of time t , when the federal funds rate is determined. We assume that the monetary policy shocks m_t do not have any contemporaneous effects on macroeconomic variables.

The above forward-looking Taylor rule would have the following reduced form representation, where the coefficients of the following reduced form Taylor rule would be convolutions of the above structural VAR model (2.1)-(2.3).

$$i_t = \delta_{i 0,t} + \tilde{Y}'_{t-1}\delta_{i 1,t} + e_{1,t}, \quad (2.4)$$

Notice that the coefficients $\delta_{i 0,t}$ and $\delta_{i 1,t}$ would be function of the coefficients in equation (2.1). The error terms of the reduced form consists of (2.2),(2.3), and (2.4) are denoted as $e_t = [e_{1,t} \ e_{2,t} \ e_{3,t}]$ are heteroscedastic shocks with variance covariance matrix Ω_t .

To identify the monetary policy shock, a structural VAR model would be recovered via Cholesky decomposition of the variance-covariance matrix of the reduced-form error terms. After we assume that the monetary policy shocks m_t do not have any contemporaneous effects on macroeconomic variables, we place endogenous variables in the order of inflation, output gap, and the interest rate, respectively. Then, we have the following relations of the reduced-form errors.

$$e_t = A^{-1}\epsilon_t, \quad \epsilon_t = \Sigma_t\tilde{\epsilon}_t, \quad \tilde{\epsilon}_t \sim i.i.d.N(0, I_3) \quad (2.5)$$

$$A_t = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{bmatrix} \quad (2.6)$$

and Σ_t is the diagonal matrix:

$$\Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & 0 \\ 0 & \sigma_{2,t} & 0 \\ 0 & 0 & \sigma_{3,t} \end{bmatrix} \quad (2.7)$$

We let B_t to be a vector stacking time-varying reduced-form VAR coefficients including intercepts and lagged variables; σ_t be the vector of the diagonal elements of the matrix Σ_t , and α_t be the vector of non-zero and non-one elements of the matrix A_t , which stacks by rows. Then, we have the following the dynamics of the model's time varying parameters is specified as follows.

$$B_t = B_{t-1} + \nu_t, \quad (2.8)$$

$$\alpha_t = \alpha_{t-1} + \zeta_t, \quad (2.9)$$

$$\log \sigma_t = \log \sigma_{t-1} + v_t, \quad (2.10)$$

where the distributional assumptions as regards $(\tilde{\epsilon}_t, \nu_t, \zeta_t, v_t)$ are state below.

$$V = \text{var} \begin{pmatrix} \tilde{\epsilon}_t \\ \nu_t \\ \zeta_t \\ v_t \end{pmatrix} = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{bmatrix}, \quad (2.11)$$

where I_n is an n -dimensional identity matrix, Q , S , and W are positive definite matrices.

In case $E_t[\pi_{t+1}]$ in (1) is unobservable, we replace them by π_{t+1} , the ex-post measures of inflation. This results in the following equation for monetary policy rule with ex-post data:

$$i_t = \beta_{i0,t} + \beta_{1,t}\pi_{t+1} + \beta_{2,t}x_t + \beta_{3,t}i_{t-1} + u_{t+1}, \quad u_{t+1} \sim i.i.d.N(0, \sigma_{u,t+1}^2), \quad (2.12)$$

where $u_{t+1} = \beta_{1,t}(\pi_{t+1} - E_t[\pi_{t+1}]) + m_t$. Note that the regressors π_{t+1} in equation (2.1) are correlated with the disturbance term u_{t+1} . In order to estimate the equation (2.12), we consider the following instrumenting equation is given by:

$$\pi_{t+1} = c_{1,t} + \tilde{Y}'_{t-1} c_{2,t} + \eta_{t+1}, \quad (2.13)$$

where $\tilde{Y}_t = [Y'_t \ Y'_{t-1} \ \dots \ Y'_{t-k+1}]'$; $c_{1,t}$ and $c_{2,t}$ are functions of the coefficients in the reduced-form (2.2), (2.3), and (2.4); and η_{t+1} follows MA(1) process as given below:

$$\eta_{t+1} = g_{t+1} - \theta_{t+1}g_t, \quad g_{t+1} \sim i.i.d.(0, \sigma_{g,t+1}^2), \quad (2.14)$$

where g_{t+1} is serially uncorrelated. After we assume that the monetary policy shocks do not have any contemporaneous effects on macroeconomic variables, we have u_{t+1} is serially uncorrelated. Then, one is able to consider an orthogonal projection of u_{t+1} on g_{t+1}^* , where $g_{t+1}^* = \frac{g_{t+1}}{\sigma_{g,t+1}}$ is a standardized residuals of g_{t+1} . Then, we have:

$$u_{t+1} = \gamma_{t+1}g_{t+1}^* + \omega_{t+1}, \quad (2.15)$$

where $\omega_{t+1} = \sqrt{1 - \rho_{t+1}^2} \sigma_{u,t+1} \omega_{t+1}^*$; $\omega_{t+1}^* \sim N(0, 1)$; and $\gamma_{t+1} = \rho_{t+1} \sigma_{u,t+1}$, ρ_{t+1} is the correlation between u_{t+1} and g_{t+1}^* . We can thus rewrite the equation (2.12) as:

$$i_t = \beta_{0,t} + \beta_{1,t}\pi_{t+1} + \beta_{2,t}x_t + \beta_{3,t}i_{t-1} + \gamma_{t+1}g_{t+1}^* + \omega_{t+1}, \quad \omega_{t+1} \sim i.i.d.N(0, \sigma_{\omega,t+1}^2), \quad (2.16)$$

where, conditional on g_{t+1}^* , the disturbance term ω_{t+1} is not correlated with any other regressors in the above equation.

2.3. Estimation Procedure

2.3.1. Markov Chain Monte Carlo (MCMC) algorithm

We consider the following two-steps Gibbs sampling algorithm to estimate the time-varying Taylor rule in equation (2.1) along with the reduced-form VAR system in equation (2.2) and (2.3):

Outline for Gibbs Sampling Algorithm

Step 1: Estimate the full reduced-form VAR consisting (2.2)-(2.4) following the Gibbs sampling algorithm by Primiceri (2005). Calculate the posterior means (or medians) of reduced-form parameters for the initial values of $\{\delta_t, \beta_t\}_{t=1}^{T-1}$. And construct $\{g_{t+1}^*\}_{t=1}^{T-1}$.

Step 2: Conditional on $\{g_{t+1}^*\}_{t=1}^{T-1}$, draw $\{\beta_t\}_{t=1}^{T-1}$ in equation (2.1) using the backward simulation algorithm by Carter and Kohn (1994).

In later sections, we discuss the details for the step 1) in order to consider the Limited Information approach.

2.3.2. Construction of the Control Function: Details of Step 1

With VAR specification given by in equation (2.2)-(2.4), we can consider the instrumenting equation for an estimation of equation (2.4). By defining $\tilde{Y}_t = [Y_t' \quad Y_{t-1}' \quad \dots \quad Y_{t-k+1}']'$, we have the following companion form of a time varying VAR:

$$\tilde{Y}_t = \tilde{B}_{0,t} + \Phi_t \tilde{Y}_{t-1} + \tilde{e}_t, \quad e_t \sim i.i.d.N(0, \tilde{\Omega}_t), \quad (2.18)$$

where $\tilde{B}_{0,t}$ is a function of $\delta_{k,t}$ for $k = 1, 2$, and 3; Φ_t is a function of $\delta_{j,t}$ for $j = \pi, x, i$ and the first (3×3) diagonal block of $\tilde{\Omega}_t$ is Ω_t . And, all the parameters are time-varying and follow random walk processes.⁵ The variances are assumed to evolve as geometric random walks, belonging to the class of models known as stochastic volatility. All the innovations in the reduced form VAR model are assumed to be jointly normally distributed and independent with each other.

To consider the proposed estimation for a forward-looking time varying MP rule, Step 1 involves the construction of the control function in equation (14). In the following, we explain how to generate a control function for the second step estimation.

By solving equation (2.18) forward, we have:

⁵ Alternatively, we may use demanded data for the equation (2.18), i.e, $\hat{y}_t = y_t - E(y_t)$.

$$\begin{aligned}
\tilde{Y}_{t+1} &= \tilde{B}_{0,t+1} + \Phi_{t+1}\tilde{Y}_t + \tilde{e}_{t+1} \\
&= \tilde{B}_{0,t+1} + \Phi_{t+1}(\tilde{B}_{0,t} + \Phi_t\tilde{Y}_{t-1} + \tilde{e}_t) + \tilde{e}_{t+1} \\
&= \tilde{B}_{0,t+1} + \Phi_{t+1}\tilde{B}_{0,t} + \Phi_{t+1}\Phi_t\tilde{Y}_{t-1} + \Phi_{t+1}\tilde{e}_t + \tilde{e}_{t+1}
\end{aligned} \tag{2.19}$$

The two step prediction error is obtained as:

$$\eta_{t+1} = i'_\pi(\Phi_{t+1}\tilde{e}_t + \tilde{e}_{t+1}), \tag{2.20}$$

where $i_\pi = [1 \ 0 \ 0 \ 0 \ \dots \ 0]'$ is a selection vector. The parameter $c_{1,t}$ and $c_{2,t}$ in equation (2.13) can be expressed as:

$$c_{1,t} = i'_\pi(\tilde{B}_{0,t+1} + \Phi_{t+1}\tilde{B}_{0,t}), \tag{2.21}$$

$$c_{2,t} = i'_\pi(\Phi_{t+1}\Phi_t). \tag{2.22}$$

By equating the variance and the first-order auto-covariances of η_{t+1} in equation (2.13), we have:

$$i'_\pi(\tilde{\Omega}_{t+1} + \Phi_{t+1}\tilde{\Omega}_t\Phi'_{t+1})i_\pi = (1 + \theta_{t+1}^2)\sigma_{g,t}^2, \tag{2.23}$$

$$i'_\pi(\Phi_{t+1}\tilde{\Omega}_t)i_\pi = -\theta_{t+1}\sigma_{g,t}^2. \tag{2.24}$$

Once all parameters of the VAR model are estimated, we can iteratively calculate the control function as follows:

$$\hat{g}_{t+1} = \hat{\eta}_{t+1} + \hat{\theta}_{t+1}\hat{g}_t, t = 1, 2, \dots, T - 1, \tag{2.25}$$

conditional on the initial value, g_0 .

2.4. Empirical Findings

2.4.1. Priors and Data

The data we employ are quarterly data covering the period 1965:1 to 2011:4. As in Kim and Nelson (2006), the interest rate the average federal funds rate in the first-month of each quarter; inflation is measured by the % change of the GDP deflator; the output gap is the series constructed by CBO. Two lags are used for the estimation.

Priors for state vectors and hyperparameters are obtained from OLS estimates as in Primiceri (2005) with a few modifications. The first 10 years (40 observation, from 1945:1 to 1964:4) are used to calibrate the prior distributions.⁶ For example, we obtain the mean and the variance of B_0 are chosen to be OLS point estimates and four times its variance in a time invariant VAR, estimated on the small initial samples. And, we chose a prior for A_0 in a same way. For $\log\sigma_0$ instead, the mean of the distribution is chosen to be the log of the OLS point estimates of the standard errors of the same invariant VAR, while the variance covariance matrix is assumed to be identity matrix. Also, degrees of freedom and scale matrices are needed for the inverse-Wishart prior distributions of the hyperparameters. The degrees of freedom set to 4 for W and 2 and 3 for the two blocks of S . In sum, the priors take the forms:

$$B_0 \sim N(\hat{B}_{OLS}, 4V(\hat{B}_{OLS})), \quad (2.26)$$

$$A_0 \sim N(\hat{A}_{OLS}, 4V(\hat{A}_{OLS})), \quad (2.27)$$

$$\log\sigma_0 \sim N(\log\hat{\sigma}_0, I_n), \quad (2.28)$$

$$Q \sim IW(0.01^2 40V(\hat{B}_{OLS}), 40), \quad (2.29)$$

$$W \sim IW(0.01^2 4I_n, 40), \quad (2.30)$$

$$S_1 \sim IW(0.1^2 2V(\hat{A}_{1,OLS}), 2), \quad (2.31)$$

$$S_2 \sim IW(0.1^2 3V(\hat{A}_{2,OLS}), 3), \quad (2.32)$$

where S_1 and S_2 denote the two blocks of S , while $\hat{A}_{1,OLS}$ and $\hat{A}_{2,OLS}$ stand for the two corresponding blocks of \hat{A}_{OLS} . The simulation are based on 10,000 iterations of the Gibbs sampler, discarding the first 2000 for convergence.

⁶ Refer to Primiceri (2005) for details.

2.4.2. Estimation Results

Our empirical findings are shown in Figures 2.2-2.6. Results for the forward-looking Taylor rule are quite different from others, which ignored the additional moving average dynamics in the error term. In Figure 2.3, we observe that the Fed's response to inflation during that 1970s was lower than in the 1980s. This tells us that the Fed did not provide sufficient attention to inflation.

However, in the early 1980s, the Fed's response to expected inflation increased sharply and stayed at a high level until the mid-1980s around 2, and it started to decline after the mid-1990s throughout the whole sample. Contrast to Kim and Nelson (2006), which ignores the moving-average dynamics, this paper shows that the response to inflation after the mid-1990s decreased, and it has been lower than 1 from the 2000s until 2010. Especially after 2006, the Fed's response to inflation was the lowest, and it is due to the deviation of the traditional monetary policy tools, which is that the Fed's focused on the long-term interest rate changes rather than the short-term interest rate.

Next, Figure 2.4. depicts the response of the federal funds rate to the expected real GDP gap. In contrast to Kim and Nelson (2006), the response for the whole period from the 1970s to the 2000s is positive and significantly different from zero throughout the whole period. Around the mid-1980s, the response was at the lowest level, but it started to increase again until the 2000s. If one ignores the moving-average dynamics in the error term, it is possible for one to interpret erroneously that the response to the output was insignificant during the 1980s. Keeping the significant response to the real economic condition throughout the whole sample shows that monetary policy became more active after Volcker.

Combining the results of the Fed's response to inflation after the mid-2000s, it is consistent with the fact that the Fed's recent monetary policy toward Quantitative Easing (QE) to adjust the long-term interest rate. This would lead the little responses to the inflation and real economics in the federal fund rate.

The evidence of the aggressive monetary policy toward inflation seems to support the conclusion proposed by Clarida et al. (2000) in contrast to Primiceri (2005). In general, a systematic monetary policy was more responsive to the economic conditions seen in the

early 1980s, and it was passive in the 1970s. Weak Instrument concerns raised in Cogley and Sargent (2005) and Kim and Nelson (2006) after 1979 can be taken into account through the proposed procedure. Finally, in Figure 2.5, the degree of interest rate smoothing is shown. It has been continuously increasing since the mid-1970s. However, it has decreased slightly during the mid-2000s, but it still remained at a high level during the whole sample. Lastly, in Figure 2.6, the time-varying volatility for the federal funds rate is shown. The figure shows the apparent heteroscedasticity in the disturbance of monetary policy rule, as in Kim and Nelson (2006) and Primiceri (2005).

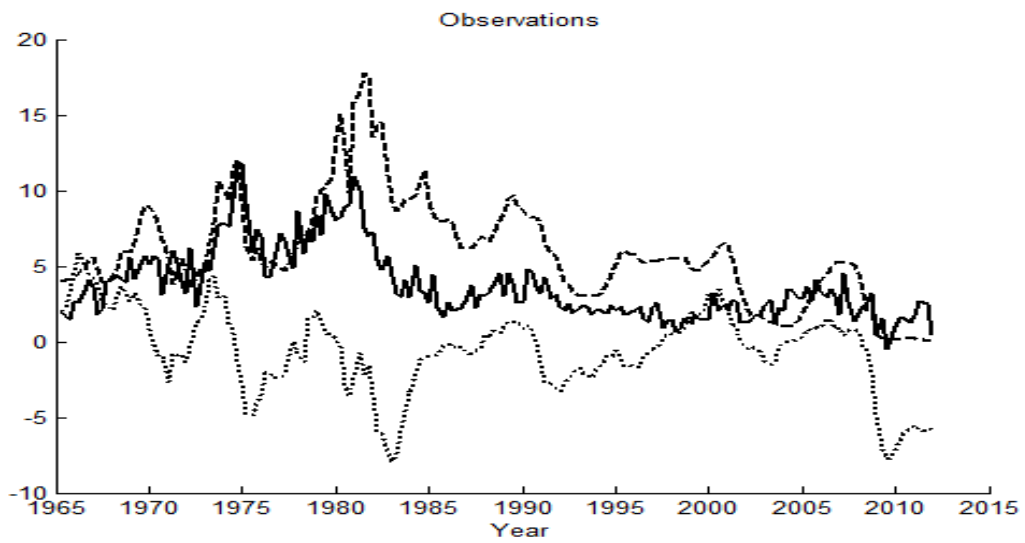
2.5. Conclusion

In this paper, we investigate a Limited-Information procedure to estimate a forward-looking monetary policy rule by modeling the multivariate time varying coefficient with stochastic volatility. By incorporating the time-varying coefficients and innovations, this paper reasonably models the effects of the changes in monetary policy on the rest of the economy. Not only the forecasts of future expectation for inflation and output gap but also the moving average dynamics can be obtained from the posterior mean of reduced-form time-varying VAR. Thus, the estimation results obtained in this paper would provide more reliable evidence to understand whether monetary policy rules have changed and the persistence of inflation itself has drifted over time.

Compared to the existing literature to estimate monetary policy via time-varying VAR specification, this paper takes into account the forward-looking component in the Taylor rule. Having estimated the time-varying VAR in the first step, this paper can effectively recover moving average terms for the estimation of a forward-looking Taylor rule. The main finding of this paper is that, under the identifying assumption that monetary policy shocks do not have simultaneous effects on macroeconomic variables, we can handle the moving-average dynamics in the instrumenting equations. This would improve the efficiency for the estimation of a time-varying forward-looking monetary policy rule mitigating the issue of relevance to instruments (see Cogley and Sargent, 2005; and Kim and Nelson, 2006).

Our empirical findings show that the dynamics of the monetary policy's response to inflation and output gap are quite different from the one which does not consider the additional MA process in the error term. The evidence on aggressive monetary policy toward inflation seems to be supported in this paper, in contrast to Primiceri (2005). In general, a systematic monetary policy was more responsive to the economic conditions in the early 1980s, and it was passive in the 1970s. Also, the recent monetary policy, put in place after the mid-2000s, is far different than the Volcker-Greenspan era. The detailed investigation of the recent monetary policy presents an opportunity for further research.

Figure 2.1. Data (GDP Deflator, CBO Output Gap, Fed Rate): 1965:Q1~2011:Q4



Note: The dotted black line represents GDP Deflator, and the grey dotted line represents CBO Output Gap, and the black line represents Federal Funds Rate. The sample runs 1965:Q1~2011:Q4.

Figure 2.2. 60% HPDI for $\beta_{0,t}$

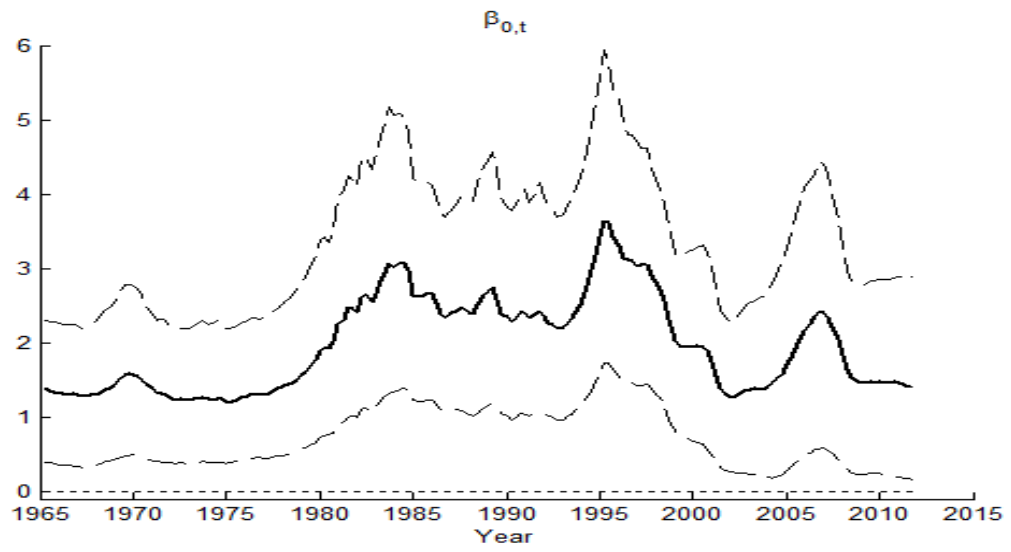


Figure 2.3. 60% HPDI for $\beta_{1,t}$

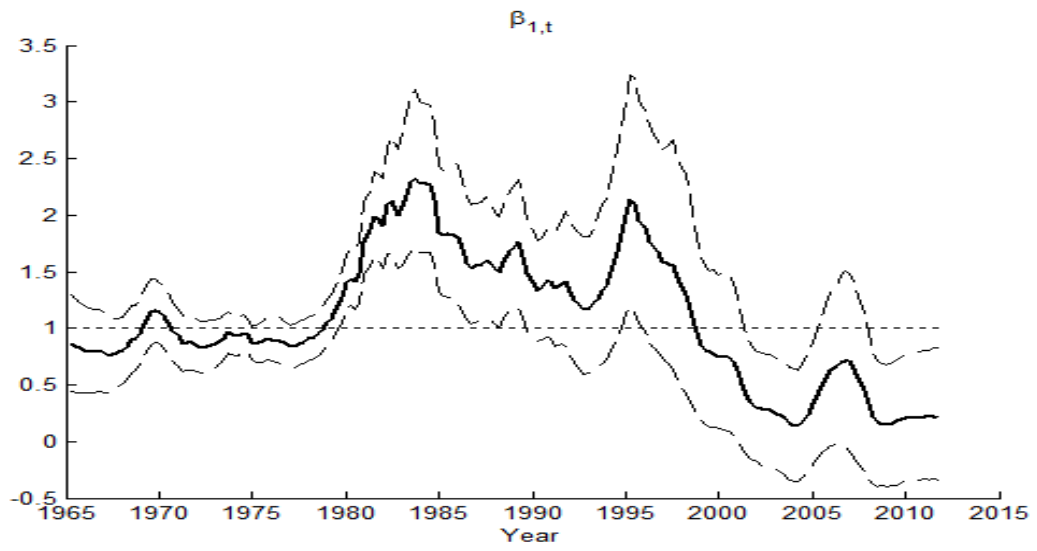


Figure 2.4. 60% HPDI for $\beta_{2,t}$

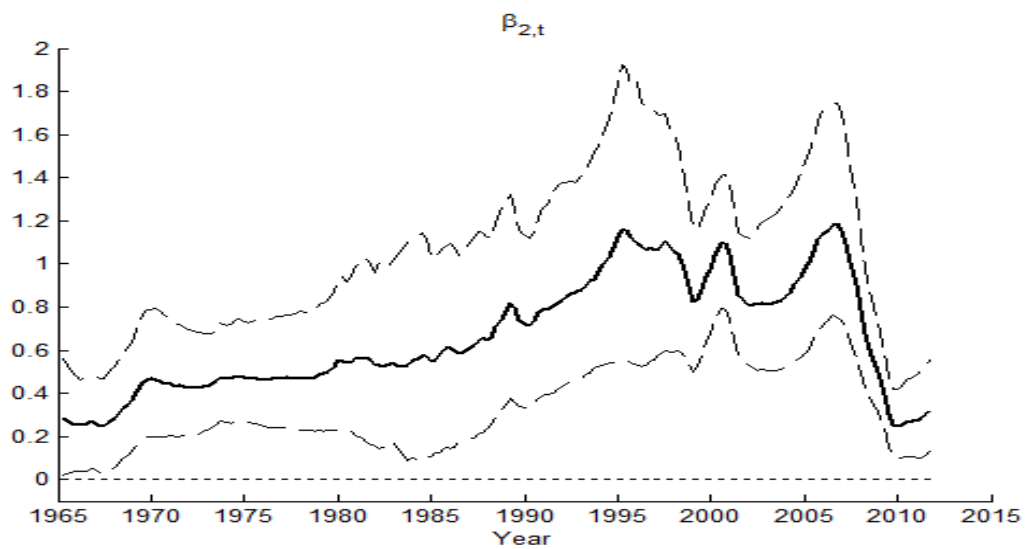


Figure 2.5. 60% HPDI for $\beta_{3,t}$

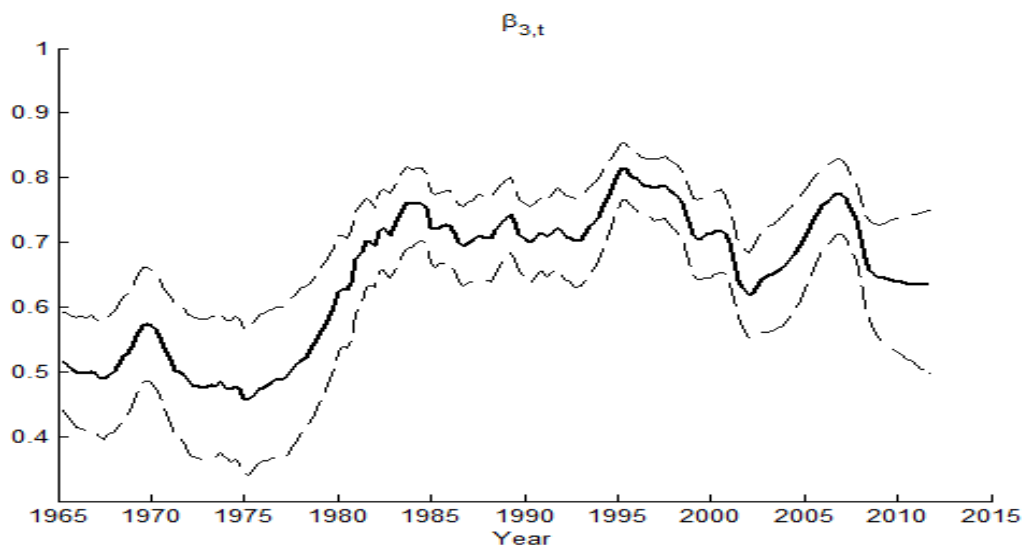
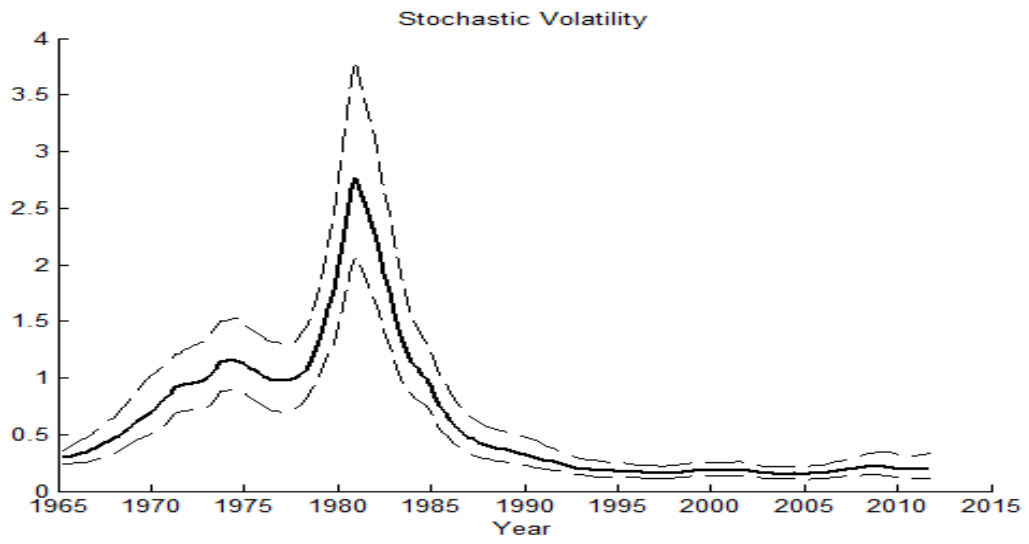


Figure 2.6. 60% HPDI for SV



**Stock Market Reaction to Monetary policy Changes:
Identification through Heteroskedasticity with Markov-switching**

3.1. Introduction

The response of stock prices to monetary policy is a key for analyzing the impact of policy transmission mechanism. Changes in monetary policy are transmitted through the stock market via the wealth effect, the interest effect, and by other mechanisms as well (see, Bernanke and Kuttner, 2005). Understanding the relationship between stock prices and monetary policy is important for several reasons. For policy makers, it would be crucial to have accurate estimates of the reaction of monetary policy in order to form effective policy actions. Also, for market participants, the reliable estimates of the responsiveness of stock prices to monetary policy is important for wise investment and risk decision making.

Estimating the impact of stock prices to monetary policy actions, however, has several difficulties. One of the main issues occurs when estimating the effect of the Fed on stock prices based on typical ordinary-least-squares (OLS). Estimates may be severely biased due to the endogeneity and omitted variable problems. The endogeneity problem⁷ originates from the simultaneous interaction between stock prices and policy decisions, while the omitted variable problem is due to factors that influence both policy rates and stock prices which are commonly excluded from regression analysis. Usually, policy rates and stock prices are determined simultaneously within the data-frequency interval, and it is possible to render the endogeneity problems⁸. In particular, a surprise change in the target rate could reflect the reaction of the Fed to changes in stock prices that occurred earlier in the quarter, month or week, instead of capturing an independent monetary policy shock. Also, changes in policy

⁷ Rigobon and Sack (2003) find that movement in the stock market have a significant impact on the macroeconomy and thus are likely to be an important factor in the determination of monetary policy.

⁸ Bernanke and Kuttner (2005) emphasize the problem of using lower frequency data.

rates and stock prices may be caused by other important news released earlier in the period which are omitted from regression. In either case, the classical regression assumption that the error term is uncorrelated to a surprise changes in the target rate is violated and the estimated can be biased (See e.g., Farka, 2009).

In this paper, in order to address these estimation issues, we extend the work of Rigobon and Sack (2004) by incorporating Markov-switching framework. The estimation procedures employed in this paper captures heteroskedasticity of monetary policy shocks endogenously different to the original work of Rigobon and Sack (2004). Once one estimate the timing of monetary policy changes, the heteroskedasticity structure helps to identify the impact of stock market to policy changes. In a recent example, Gerlach et al. (2006) shows that there are two distinct regime in the response of stock prices to policy surprises. Following Kuttner (2001), we use thirty-day federal funds rate futures to extract unexpected part of policy changes. Since the stock markets are forward-looking, the market is unlikely respond to anticipated policy changes. In this sense, asset prices will response to revisions in future expectations, we thus focus on surprise part of policy changes to figure more clearly the market response to monetary policy.

Traditionally, a popular approach is a event-study framework,⁹ measuring the impact of Fed's policy on asset price is to estimate the reaction to fund rate changes on the day of the change (see, e.g., Bernanke and Kuttner, 2005; Gurkayna et. al., 2005; Davig and Gerlach, 2006; Farka, 2009). A conventional 'event-study' approach, is typically used in estimating the response of stock prices to monetary policy actions using lower frequency data such as monthly or quarterly basis. To account for the endogeneity issues possibly come from the use of lower frequency data, Bernanke and Kuttner (2005) uses daily data to address the problem within a 'event-study' framework. In a seminal paper, they investigate recently the effects of monetary policy on the stock market by using daily CRSP value-weighted returns and a measure of unexpected changes to the target federal funds rate. They conclude that an unexpected 25-basis-point cut in the target federal funds rate is associated with a 1 percent

⁹ It measures the impact of Federal Reserve policy on the stock market is to calculate the market's reaction to funds rate changes on the day of the changes. Because this approach involves looking at the response to specific events, it might be described as an 'event-study' style analysis.

increase in equity prices.

Also, Farka (2009) uses a new data set consisting of high frequency changes in S&P 500 and federal funds futures around the time of monetary policy announcement. He argues that the intra-day data set in his paper reduce the omitted variable bias by decreasing the likelihood that other relevant information is released in the market during the narrow interval around policy announcement. However, as Bernanke and Kuttner (2005) mentioned, if there is contemporaneous response of monetary policy to the stock market or if monetary policy and stock market both responded jointly to new economic information, using high-frequency data still subject to the bias.

More statistical solution is suggested by Rigobon and Sack (2004) within a VAR approach that exploits the heteroskedasticity of monetary policy shocks across event days and non-event days. In order to take care of these possible simultaneous bias, they assume that the variance of monetary policy shocks is higher on days of FOMC meetings and of the Chairman's semi-annual monetary policy testimony to Congress, when a larger portion of the news hitting markets is about monetary policy. They then show that the shift in the variance of the policy shocks on those dates is sufficient to measure the responsiveness of asset price to monetary policy. To employ their method, one need to identify a period of time in which the variance of the policy shocks was higher than at other times, but the other shocks in the system remained unchanged. Thus, if the above assumption for the change of monetary policy shock is violated which means one mis-specify the time in which the variance-covariance matrix shifts, it would generate some bias in the estimates of the parameter.

As Farka (2009) discusses, the monetary policy shocks possibly come from the alteration of expectations about the future path of the monetary policy and a change in the timing of policy moves. He stresses that the importance of these shocks to obtain more precise estimates of the effect of monetary policy on stock returns. Market participants, therefore, would respond to the information content of policy shock which reflect two factors that one is expectational changes regarding the near term path of future policy, and another one is shifts in the timing of an anticipated policy move. In this sense, if we only use the institutional information to capture the change in the policy shock to employ the identification

procedure suggested by Rigobon and Sack (2004), it would not enough to investigate the all possible changes in the policy shocks. In particular, timing of the policy shock shifts caused by revisions in expectations regarding the future path of monetary policy would be very difficult to be found. In this paper, using Markov-switching framework to detect different states, we also account for the uncertainty of timing issue instead of specifying the high and low volatility states exogenously. And, it would reduce the possible bias which may be caused by the uncertainty of the shift of the policy shock. As mentioned earlier, the stock markets are unlikely response to the anticipated changes in policy shocks, we hence use the heteroskedasticity structure focusing on the event days.

Gerlach et al. (2006) present a test of the response of stock prices to Federal Reserve policy shocks using a Markov-switching framework to a conventional event-study approach. They show that in the low volatility regime, the market response to unexpected changes in the target federal funds rates is significantly negative. More recently, Farka (2009) finds that the asymmetric effects in level and volatility of stock returns to policy changes with respect to the type of policy shocks and to the type of policy shocks by specifying an intra-day GARCH model. In this paper, we use the state-dependent structure in the variance to identify the original impact on stock market focusing on the event-days.

The rest of this article is summarized as follows. We first introduce a simultaneous relationship between asset prices and monetary policy, and discuss existing problems in the estimation in Section 2. Section 3 reviews the method proposed by Rigobon and Sack (2004), and we suggest the use of Markov-switching framework in the first step to detect the shift in the variance endogenously. Section 4 explains the data we use for empirical study and show the results for the response of asset prices due to the monetary policy shocks. Concluding remarks are offered in Section 5.

3.2. Model Specification and Estimation Problem

Consider the following simplified systems of equations of monetary policy and asset prices used in Rogobon and Sack (2004):

$$\Delta i_t = \beta \Delta s_t + \gamma z_t + \epsilon_t, \quad (3.1)$$

$$\Delta s_t = \alpha \Delta i_t + z_t + \eta_t, \quad (3.2)$$

where Δi_t is the change in the short-term interest rate and Δs_t is the change in an asset price. Monetary policy reaction can be described in equation (3.1), and it captures the expected response of policy to a set of variables z_t ¹⁰ and to asset price. Equation (3.2) shows the response of the asset price by the interest rate and also z_t . The variable ϵ_t is the monetary policy shock, and η_t is a shock to the asset price. Monetary policy shock¹¹ means deviations from the typical response of the short-term interest rate. We further assume that those disturbances are assumed to be have no serial correlation and to be uncorrelated with each other and with the common shock z_t .¹²

To understand the equation (3.2) more clearly, we use the approximation introduced by Campbell and Shiller (1988) for a dynamic setting, the log level of stock price can be described as follow (see, Rigobon and Sack, 2003):

$$S_t = \frac{k}{1 - \delta} + \sum_{j=0}^{\infty} \delta^j (1 - \delta) E_t(d_{t+1+j}) - \sum_{j=0}^{\infty} \delta^j E_t(h_{t+j}), \quad (3.3)$$

where δ and k are constant; d_t is log dividend; and h_{t+j} is the return holding equities between $t + j$ and $t + j + 1$. The expected holding return for equities can be expressed with the sum of the short-term interest rate and a risk premium, denoted as i_{t+j} and ρ_{t+j} , meaning that $E_t(h_{t+j}) = i_{t+j} + \rho_{t+j}$. To bridge the gap between equation (3.2) and equation (3.3) we further assume that one can able to approximate the expectation of the future dividend and short-term interest rate by current and lagged values of macroeconomic news and the interest rate. By matching the equation (3.2) and (3.3), the shock η_t would be interpreted as stock market shock, which originate the change to risk preference.

In this paper, we are interested in the estimation of equation (3.2) to investigate the response of asset price to the change in the monetary policy. However, the estimation

¹⁰ For notational simplicity, we let z_t is a single variable.

¹¹ The shock could also reflect any factors driving a wedge between the interest rate and the policy expectation.

¹² As mentioned in Rigobon and Sack (2004), it is an oversimplified model of the relationship between movements and in interest rates and asset prices, but they justify this model with the evidence that allowing for a richer lag structure had little effect on the result.

of the response of asset prices to changes in monetary policy is problematic due to the simultaneous relationships and omitted variables (see, Rigobon and Sack, 2004). Because of the simultaneity, the data we observe is the intersection of the interest rates and the asset prices, and it thus may not be useful to investigate the slope of the equation (3.2). Another problem would be the omitted variable bias. The common shock z_t would shift the interest rate and the asset prices at the same time, the realization would be affected by the coefficients γ on those variables. The equation (3.3) also describes the log level of stock prices whereas the equation (3.2) is about the change in stock prices z_t , but nearly identical results hold if the VAR instead uses the log level of stock prices given the lags included in the VAR (see, Rigobon and Sack, 2003).

As an alternative, one can employ VARs to capture the dynamics of asset prices with lagged term in equation (3.1) and (3.2) (see, e.g., Campbell and Ammer, 1993). By employing VARs, the parameters of the structural equation (3.2) can be recovered by imposing restrictions, usually, the identification of VARs often takes the form of exclusion restriction, which is either α or β is zero, which is unrealistic. Holding everything else equal, higher interest rate are associated with lower stock market prices, given the higher discount rate for the expected stream of dividend. Also, the Federal Reserve may respond to higher stock prices by raising interest rate at the same time. Therefore, the exclusion restriction, either α or β is zero, would misinterpret the parameter in the structural equation (See, Rigobon and Sack, 2003).

To resolve this problem, Rigobon and Sack (2003) introduce a novel identification procedure by using the heteroskedasticity, and extended this method for the estimation of the impact of monetary policy on asset price. This intuition was first suggested by Wright (1928). He originally explains how the bias from OLS would disappear if the variance of one of the shocks goes to infinity, in which case one of the equation is identified. For Rigobon and Sack (2003) case, it needs only a shift in the relative magnitudes of the variance of the shocks. In next section, we carefully review the method proposed by Rigobon and Sack (2004), and we raise some issues of potential problems in the estimation of the equation (3.2).

3.3. Identification through heteroskedasticity and Markov-Switching framework

3.3.1. Review of Rigobon and Sack (2004)

In a seminal paper, Rigobon and Sack (2004) develop a technique called identification through heteroskedasticity to estimate equation (3.2). By investigating the changes in the variance of interest rates and asset prices, the response of asset prices to monetary policy shock can be estimated. The idea is based on the assumption that one can identify a period of time in which the variances of the other shocks in the system remained unchanged. One need to identify a period of time of the changes in the variance to use this approach. Suppose one can identify two subsample, denoted s_1 and s_2 , and one need to have the following assumptions for the variance:

$$\sigma_{\epsilon}^{s_1} > \sigma_{\epsilon}^{s_2} \quad (3.4)$$

$$\sigma_{\eta}^{s_1} = \sigma_{\eta}^{s_2} \quad (3.5)$$

$$\sigma_z^{s_1} = \sigma_z^{s_2} \quad (3.6)$$

The assumption (3.4)-(3.6) means that the variance of monetary policy shock would elevate in the subsample s_1 whereas the variance of other shocks remain same. This procedure, thus, relies on the heteroskedasticity of policy shocks that happens on particular dates, including days of FOMC meetings and of the Chairman's semi-annual monetary policy testimony to Congress. Rigobon and Sack (2004) show that the correlation between the policy rate and these other asset prices shifts importantly on those dates, as one would expect given the greater importance of policy shocks justifying the above identification assumption.

In order to understand the identification through heteroskedasticity, consider the following the reduced-form of equation (3.1) and (3.2):

$$\Delta i_t = \frac{1}{1 - \alpha\beta} [(\beta + \gamma)z_t] + \beta\eta_t + \epsilon_t], \quad (3.7)$$

$$\Delta s_t = \frac{1}{1 - \alpha\beta} [(1 + \alpha\gamma)z_t] + \eta_t + \alpha\epsilon_t]. \quad (3.8)$$

Then, the variance-covariance matrix for two subsample, denoted Ω_{s_1} and Ω_{s_2} would be:

$$\Omega_{s_1} = \frac{1}{1 - \alpha\beta} \begin{bmatrix} \sigma_\epsilon^{s_1} + \beta^2\sigma_\eta^{s_1} + (\beta + \gamma)^2\sigma_z^{s_1} & \alpha\sigma_\epsilon^{s_1} + \beta\sigma_\eta^{s_1} + (\beta + \gamma)(1 + \alpha\gamma)\sigma_z^{s_1} \\ \cdot & \alpha^2\sigma_\epsilon^{s_1} + \sigma_\eta^{s_1} + (1 + \alpha\gamma)^2\sigma_z^{s_1} \end{bmatrix}, \quad (3.9)$$

$$\Omega_{s_2} = \frac{1}{1 - \alpha\beta} \begin{bmatrix} \sigma_\epsilon^{s_2} + \beta^2\sigma_\eta^{s_2} + (\beta + \gamma)^2\sigma_z^{s_2} & \alpha\sigma_\epsilon^{s_2} + \beta\sigma_\eta^{s_2} + (\beta + \gamma)(1 + \alpha\gamma)\sigma_z^{s_2} \\ \cdot & \alpha^2\sigma_\epsilon^{s_2} + \sigma_\eta^{s_2} + (1 + \alpha\gamma)^2\sigma_z^{s_2} \end{bmatrix}. \quad (3.10)$$

With the assumption that the parameters α, β , and γ would not vary across the two subsample, the difference in variance-covariance matrices is given as:

$$\Delta\Omega = \Omega_{s_1} - \Omega_{s_2} = \frac{(\sigma_\epsilon^{s_1} - \sigma_\epsilon^{s_2})}{(1 - \alpha\beta)^2} \begin{bmatrix} 1 & \alpha \\ \alpha & \alpha^2 \end{bmatrix}. \quad (3.11)$$

The identification of α is straightforward from equation (3.10). To estimate the response of asset prices to monetary policy shock, captured in α , one can replace the change in the variance-covariance matrices with the sample estimates. By letting $\Delta y_t = [\Delta i_t \quad \Delta s_t]'$ and defining dummy variables, $\delta_t^{s_1}$ and $\delta_t^{s_2}$, that have the value 1 for all days for each subsamples, respectively.

Then, the sample estimates would be:

$$\hat{\Omega}_{s_1} = \frac{1}{T_{s_1}} \sum_{t=1}^T \delta_t^{s_1} \Delta y_t \Delta y_t', \quad (3.12)$$

$$\hat{\Omega}_{s_2} = \frac{1}{T_{s_2}} \sum_{t=1}^T \delta_t^{s_2} \Delta y_t \Delta y_t'. \quad (3.13)$$

Then, the parameter α can be estimated as follows:

$$\hat{\alpha} = \frac{\Delta \hat{\Omega}_{12}}{\Delta \hat{\Omega}_{11}} = \frac{\Delta \hat{\Omega}_{22}}{\Delta \hat{\Omega}_{12}}, \quad (3.14)$$

where $\Delta \hat{\Omega}_{i,j}$ indicates the (i, j) element of the change in the $\hat{\Omega}$ matrix ¹³.

¹³ For details, refer to Rigobon and Sack (2004)

3.3.2. Identification of the change in the variance based on Markov-switching

One of the problems of the implementation of identification through heteroskedasticity is that one need to be aware the time when the variance of monetary policy shock changes. If not, it would generate the bias for the structural parameter that we are interest in. In this sense, timing of the policy moves need to be clear to split samples according to the heteroskedasticity of monetary policy shocks.

Rigobon and Sack (2004) assume that days of FOMC meeting and of the Chairman's semi-annual monetary policy testimony to Congress are likely to have a greater amount of news about monetary policy than other days. Also, they further assume that other types of shocks still take place on these days, but the relative importance of policy shock is likely to increase dramatically. Based on these assumptions, they split two subsamples for the one to indicate that the variance of the policy shock is elevated denoted s_1 , and the other one, denoted s_2 to represent the set of days immediately preceding those included in s_1 to satisfy the identification condition (3.4)-(3.6). However, if the above assumption for the change of monetary policy shock is violated which means one mis-specify the time in which the variance-covariance matrix shifts, it would generate some bias in the estimates of the parameter α . Also, more importantly, because the stock market is unlikely respond to the anticipated monetary policy changes, the regression on event days would help to obtain more precise estimate results of the Fed's impact on the change in stock returns. ¹⁴

Additionally, in a seminal paper, Farka (2009) discusses the type of the policy shocks: 1) those that alter expectations about the future path of the monetary policy, and 2) those that signal a change in the timing of policy moves. He stresses that the importance of these shocks to obtain more precise estimates of the effect of monetary policy on stock returns. As he mentioned, market participants, therefore, would respond to the information content of policy shock which reflect two factors that one is expectational changes regarding the near term path of future policy, and another one is shifts in the timing of an anticipated policy move. In this sense, if we only use the institutional information to capture the change

¹⁴ Rigobon and Sack (2003) shows the bootstrap results to check the robustness in the use of the identification through heteroskedasticity for the exogenously specified the different variance-covariance matrices.

in the policy shock to employ the identification procedure suggested by Rigobon and Sack (2004), it would not enough to investigate the all possible changes in the policy shocks. In particular, timing of the policy shock shifts caused by revisions in expectations regarding the future path of monetary policy would be very difficult to be found.

In this paper, we use the heteroskedasticity of monetary policy shocks that exist in high-frequency data within the event study framework. In order to take care of the uncertainty of timing issue in previous literature, we use Markov-switching framework to detect different states instead of specifying the high and low volatility states exogenously. And, it would reduce the possible bias which may be caused by the uncertainty of the shift of the policy shock, and it accounts for the possible biases in the conventional event study approaches. In what follows, we introduce more details about the identification through heteroskedasticity proposed by Rigobon and Sack (2004) with Markov-switching framework.

From the reduced-form of equation (3.1) and (3.2), the reduced-form can be written with the reduced-form shock as follows.

$$\Delta y_t = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = e_t, \quad e_t \sim N(0, \Omega_{s_t}), \quad (3.15)$$

where s_t is the unobserved state variable, and Ω_{s_t} is given by:

$$\Omega_{s_t} = \begin{bmatrix} \Omega_{11,s_t} & \Omega_{12,s_t} \\ \Omega_{21,s_t} & \Omega_{22,s_t} \end{bmatrix}. \quad (3.16)$$

Also, we assume that two-state Markov-chain with the following transition probability.

$$\Pi = \begin{bmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{bmatrix}, \quad (3.17)$$

where $p_{ij} = Pr[s_t = j | s_{t-1} = i]$ for $i = 0, 1$ and $j = 0, 1$.

Step 1: Estimate equation (3.15) via Maximum-Likelihood Estimation (MLE), and get the reduced-from variance-covariance matrix Ω_{s_t} for each regime.

Step 2: Based on the estimates of Ω_{s_t} , we calculate the change of the variance covariance matrix, denoted as $\hat{\Omega} = \hat{\Omega}_1 - \hat{\Omega}_2$.

Step 3: We apply the procedure developed by Rigobon and Sack (2004) to get an inference of α in equation (3.1).

Note that one can identify two distinct regimes based on Markov-switching framework endogenously in the first step. With taking care of the uncertainty of shifts in monetary policy shock, one can apply the same procedure to Rigobon and Sack (2004). The presence of unconditional heteroskedasticity in the reduce form allows us to identify the structural parameter α , but the limitation of this analysis is that we are only able to obtain the partial identification of the model.

3.4. Estimation Results

3.4.1. Data

Estimating the response of stock prices to monetary policy actions is difficult by the fact that the market is unlikely to respond to policy actions that were already anticipated (see Bernake and Kuttner, 2005). Asset markets are forward-looking and hence tend to incorporate any information about anticipated policy changes. Thus, distinguishing between expected and unexpected policy actions is important for understanding their effect. Kuttner (2001) suggests a way to construct a measure of ‘surprise’ rate changes by using Federal funds futures data ¹⁵. In this paper, we analyze the effects of FOMC target rate decisions between the July of 1989 through the April of 2008. ¹⁶. Following Kuttner (2001) and Bernanke and Kuttner (2005) we use same data form FFR futures contracts in order to use the unexpected part of the FFR change. On the day of the FOMC decision, the FFR shock, is measured by the change in the implied rate of the current-month FFR futures contract, as traded on the CBOT market, relative to the day before the FOMC announcement, scaled by a factor related to the number of days in the month affected by the change. Then, a measure

¹⁵ Rigobon and Sack (2002) utilize the euro-dollar future rate to focus on the surprise component of policy moves.

¹⁶ We use the data provided from <http://econ.williams.edu/people/knk1>.

of the surprise element of any specific change in the Federal funds target can be written as:

$$\Delta i_t = \frac{D}{D-d}(f_{m,d}^0 - f_{m,d-1}^0), \quad (3.17)$$

where Δi_t is the unexpected target rate changes, $f_{m,d}$ is the current-month future rate, and D is the number of days in the month.

Before 1994, changes in the federal funds target rate were not announced by FOMC, in which policy changes were not closed and a significant policy move happened between meetings. After 1994, however, the day of the target rate changes are well known; and the policy action is announced. In this case, the day-to-day change in the future rate on days would reflect changes in the market's expectation of the target funds rate on some future date (Kuttner, 2008). Therefore, in pre-1994, day-to-day changes in future target rate could well be a consequence from either changes in policy preferences or macro news while, in post-1994, policy surprises on event day, are more likely comes from changes in the Fed's preference. The policy of announcing target rate changes, which began in February 1994, eliminates most of the timing uncertainty associated with rate changes in the earlier part of the sample. After 1994 period when all Federal funds rate changes were announced, and most coincided with FOMC meetings. Therefore, to make use of pre-1994 sample, one should be aware of the uncertainty of timing in shifts of monetary policy shocks.

The stock market data we use, which consists of high-frequency observations of the Nasdaq index. The stock market returns are log differences of the Nasdaq index on the close of the day of the FOMC meeting, relative to the previous trading day.

3.4.2. Results

Table 3.2 shows the estimation results. As we can see in Table 3.2, we are able to detect two different regime on event-day. In a seminal paper, Kontonikas et al. (2013) found that the heteroscedastic structure in monetary policy shock and state dependence, with the reaction being stronger during the bad times on the event-days. Therefore, in order to identify the impact of stock price to monetary policy shock one should take care of the asymmetric effects and the heteroscedasticity due to policy changes. Focusing on the event days, we found that

the estimate $\hat{\alpha}$ is -7.859 , implying that the an unanticipated 25-basis point change increase 1.98% decline in the Nasdaq. In Rigobon and Sack (2004), they found that 2.4% decline in stock price, which is larger reaction than our estimates. Within event-study framework, Our results relax the assumption that the larger variance in the monetary policy shocks always happened on the event-days than non event-days. The two distinct regime found in this paper in the reduced form shock tells us the higher volatility state to Fed's expansionary shock, and the low volatility state to Fed's contractionary response to the economy.

In order words, the state dependence heteroscedastic structure on event-day could well be a consequence of the asymmetric effects to the different types of policy actions: expansionary or contractionary policy. It turns out the periods that we have high volatility states coincide the periods the economy in the recession, and this findings are consistent to Kontonikas et al. (2013). Therefore, as Rigobon and Sack (2004), the assumption that the monetary policy shock always would be higher on event-day than on non-event day possibly generate biased estimates due to the uncertainty of timing in the changes of monetary policy shock. If the future policy are already announced, then it is possible that we could have mild reaction in stock price to policy changes. Also, market reaction could have asymmetric effects depending on the type of policy actions: rate cut vs. rate hike. In this sense, we need to employ Markov-switching framework to detect two different regime endogenously in the first step.

3.5. Conclusion

In this paper, we take care of the endogeneity issues in estimating the response of stock prices due to simultaneity and the omitted variable bias. We extend the work of Rigobon and Sack (2004) by incorporating Markov-switching framework. The estimation procedures employed in this paper captures heteroskedasticity of monetary policy shocks endogenously to account for all possible changes in monetary policy shocks. Once one estimate the timing of monetary policy changes, the heteroskedasticity structure helps to identify the impact of stock market to policy changes.

Since the monetary policy shocks possibly come from the alteration of expectations about the future path of the monetary policy and a change in the timing of policy moves. In this sense, if we only use the institutional information to capture the change in the policy shock to employ the identification procedure suggested by Rigobon and Sack (2004), it would not enough to investigate the all possible changes in the policy shocks. In particular, timing of the policy shock shifts caused by revisions in expectations regarding the future path of monetary policy would be very difficult to be found. In this paper, using Markov-switching framework to detect different states, we also account for the uncertainty of timing issue instead of specifying the high and low volatility states exogenously. And, it would reduce the possible bias which may be caused by the uncertainty of the shift of the policy shock.

We found that the two distinct regime in the reduced form shocks, which tells us the higher volatility state to Fed's expansionary shock, and the low volatility state to Fed's contractionary response to the economy. The state dependence heteroscedastic structure on event-day could well be a consequence of the asymmetric effects to the different types of policy actions: expansionary or contractionary policy. Our finding implies that the an unanticipated 25-basis point change increases 1.98% decline in the Nasdaq.

Table 3.1. Descriptive statistics for FFR unexpected changes and S&P500 index

	Obs.	Min	Max	Mean	St. Dev.
Unexpected FFR Change	131	-42.000	15.000	-3.023	9.745
S&P500	131	311.400	1498.130	843.738	371.696

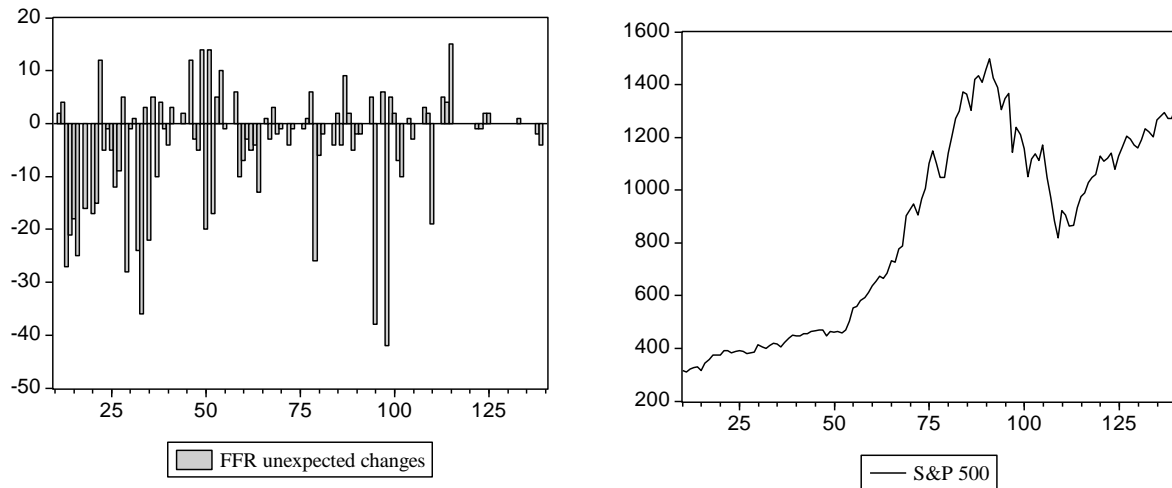
Note: Unexpected FFR Change, is measured by the change in the implied rate of the current-month FFR futures contract, as traded on the CBOT market, relative to the day before the FOMC announcement, scaled by a factor related to the number of days in the month affected by the change. We use the same data with Bernanke and Kuttner (2005). S&P500 for stock prices changes on the day of FOMC meetings. . The sample runs from the March of 1990 to the December of 2006.

Table 3.2. Maximum Likelihood Estimates and the response of stock price to monetary policy

Parameter	Estimates
$\sigma_{e1,H}$	14.027 (1.373)
$\sigma_{e2,H}$	1.311 (0.117)
$\sigma_{e1,L}$	1.419 (0.378)
$\sigma_{e2,L}$	0.814 (0.088)
P_{00}	0.821 (0.066)
P_{11}	0.820 (0.080)
Log-likelihood	-491.09667
$\hat{\alpha}$	-7.654 (2.579)

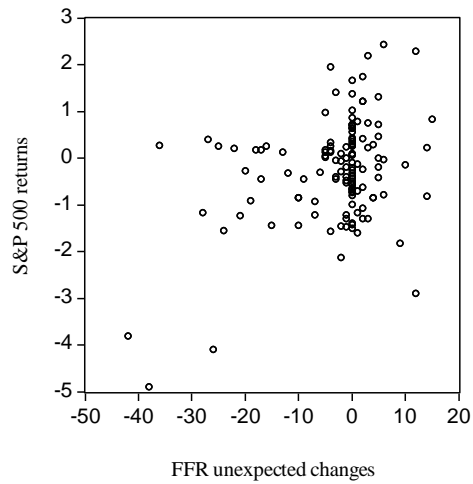
Note: The sample runs from the March of 1990 to the December of 2006. H means the high volatility states, and L means the low volatility states of the reduced-form shocks. The e_1 represents the reduced-form shocks in the reduced-form of stock prices changes, and the e_2 represents the reduced-form shocks in the reduced-form of unexpected FFR changes. P_{00} is the probability for the low volatility states, and P_{11} means the probability for the high volatility states. Alpha is obtained indirectly by employing the method of Rigobon and Sack (2004) after we get the estimates from MLE. Standard errors are reported in parenthesis. We use the S&P500 for stock prices changes, and these are defined as 100 times the first difference of the log of the S&P 500 on close of the day of the FOMC meeting, relative to the previous trading day.

Figure 3.1. Federal funds rate surprises and S&P 500 stock price index



Note: The figures are the unexpected FFR changes and 1-day S&P 500 index, for the 131 event-days of the sample. The sample runs from the March of 1990 to the December of 2006.

Figure 3.2. S&P 500 returns and FFR unexpected changes



Note: The figure is a scatterplot of 1-day S&P 500 returns against the unexpected FFR changes, for the 131 events of the sample. The sample runs from the March of 1990 to the December of 2006. The S&P500 returns for stock prices changes, and these are defined as 100 times the first difference of the log of the S&P 500 on close of the day of the FOMC meeting, relative to the previous trading day.

Appendix A. Estimating the Initial Value for the Control Functions, g_0 .

Through recursive substitutions, we rewrite equation (1.18) for $t = 0, 1, 2, \dots, T - 1$ as follows:

$$\begin{aligned}\hat{\eta}_1 &= g_1 - \hat{\theta}g_0, \\ \hat{\eta}_2 &= g_2 - \hat{\theta}\hat{\eta}_1 - \hat{\theta}^2g_0, \\ \hat{\eta}_3 &= g_3 - \hat{\theta}\hat{\eta}_2 - \hat{\theta}^2\hat{\eta}_1 - \hat{\theta}^3g_0, \\ &\vdots \\ \hat{\eta}_T &= g_T - \hat{\theta}\hat{\eta}_{T-1} - \hat{\theta}^2\hat{\eta}_{T-2} - \dots - \hat{\theta}^{T-1}\hat{\eta}_1 - \hat{\theta}^Tg_0.\end{aligned}$$

By rearranging terms in the above equations, we can obtain the following regression equation in which the coefficient is g_0 and the disturbance term is g_t :

$$\tilde{\eta}_t = g_0\tilde{x}_t + g_t, \quad g_t \sim i.i.d.N(0, \sigma^2),$$

where $\tilde{\eta}_t = \hat{\eta}_t + \sum_{i=1}^{t-1} \hat{\theta}^{t-i}\hat{\eta}_i$ and $\tilde{x}_t = -(\hat{\theta})^t$. Then, g_0 , can be estimated by the following OLS estimators:

$$\hat{g}_0 = \frac{\sum \tilde{x}_t\tilde{\eta}_t}{\sum \tilde{x}_t^2}.$$

Appendix B. Macroeconomic Interpretation of Testing Determinacy.

Figure B.1-B.2 depict the 90 percent level confidence sets constructed by inverting three alternative tests on (β_π, β_x) , designed for checking the condition for determinacy given below, and employed by Clarida et al. (2000) and Mavroeidis (2010):

$$\beta_\pi + \frac{1 - \beta}{\lambda}\beta_x \geq 1, \tag{B.1}$$

where the parameter $0 < \psi < 1$ is a discount factor and the λ parameter captures the degree of nominal rigidities in a forward-looking Phillips curve. The three alternative test considered are: i) a Wald test based on GMM, ii) an identification-robust test based on GMM; and iii) a Wald test based on the proposed method. The confidence sets are constructed as in Mavroeidis (2010), by fixing the values of ψ and λ at 0.99 and 0.3, respectively.

The equation (B.1) represent Taylor Principle that the Fed raises real rates in response to inflation to eliminate the possibility of self-fulfilling inflation. The above condition is more general than the original Taylor Principle that simply states that the coefficients on inflation in the Taylor rule is greater than one. With the discount factor close to one, the above condition derived in the New Keynesian framework still means that the coefficient on inflation in the Taylor rule be greater than one. As mentioned in Section 4, even though one cannot interpret the Taylor principle directly based on the proposed method, it would be worthwhile to check the confidence sets based on the proposed method in order to see the advantage of taking care of moving-average term appropriately compared to GMM estimation.

For the 1960:Q1-1979:Q2 sample, we get very robust results regardless of the inference methods employed. The two dimensional 90 % confidence sets on the β_π and the β_x parameters all cover the indeterminacy region. For the 1979:Q3-1997:Q4 sample, the identification-robust inference, we cannot reject the null of indeterminacy. However, the Wald confidence set from the proposed method is much smaller than the identification-robust confidence set from GMM. The proposed approach results in reasonably accurate estimates of the parameters, and the 90 % confidence set for the reaction parameters lies outside the indeterminacy region.

Appendix C: Reduced-form Monetary Policy Rule in Equation (2.4)

Notice that the coefficients of the reduced form Taylor rule given by equation (2.4) would be the function of the semi-structural VAR model consisting of the equation (2.1)-(2.3). In order to understand the relationship between the coefficients in the structural equation and the ones in the reduced form equation, we consider the second order of VAR of the dynamics

of the inflation and the unemployment as follows.

$$i_t = \delta_{i,0t} + \delta_{i,11,t}\pi_{t-1} + \delta_{i,12,t}x_{t-1} + \delta_{i,13,t}i_{t-1} + \delta_{i,14,t}\pi_{t-2} + \delta_{i,15,t}x_{t-2} + \delta_{i,16,t}i_{t-2} + e_{1,t} \quad (C.1)$$

In the following derivation, we assume that i) $E_t\delta_{i,1j,t+1} = \delta_{i,1j,t}$ for $j = 1, 2, \dots, 6$ and ii) $E_t e_{j,t} = 0$; for $k = 1, 2, 3$. After solving equation (2.2) forward, and we combine this with equation (2.1) and (2.3). Then, we would get the following relationship between equation (2.1) and (2.4).

$$\delta_{i,0,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{0,t} + \alpha_{1,t}\{\delta_{\pi,0,t}(1 + \delta_{\pi,11,t})\} + \delta_{x,0,t}(\alpha_{1,t}\delta_{\pi,12,t} + \alpha_{2,t})]$$

$$\delta_{i,11,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}(\delta_{\pi,11,t}^2 + \delta_{\pi,12,t}\delta_{x,11,t} + \delta_{\pi,14,t}) + \alpha_{2,t}\delta_{x,11,t}]$$

$$\delta_{i,12,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}\{\delta_{\pi,12,t}(\delta_{x,11,t} + \delta_{x,12,t}) + \delta_{\pi,13,t}\} + \alpha_{2,t}\delta_{x,12,t}]$$

$$\delta_{i,13,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}\{\delta_{\pi,13,t}(\delta_{x,11,t} + \delta_{x,13,t}) + \delta_{i,16,t}\} + \alpha_{2,t}\delta_{x,13,t} + \alpha_{3,t}]$$

$$\delta_{i,14,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}(\delta_{\pi,11,t}\delta_{\pi,14,t} + \delta_{\pi,12,t}\delta_{x,14,t}) + \alpha_{2,t}\delta_{x,14,t}]$$

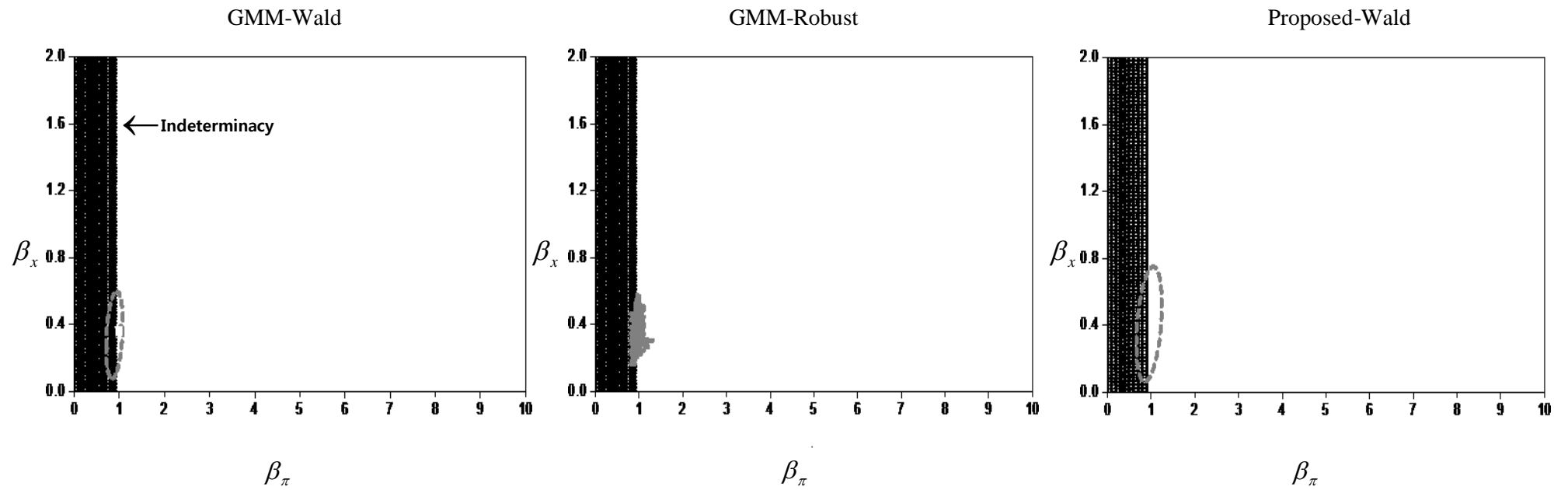
$$\delta_{i,15,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}(\delta_{\pi,11,t}\delta_{\pi,14,t} + \delta_{\pi,12,t}\delta_{x,15,t}) + \alpha_{2,t}\delta_{x,15,t}]$$

$$\delta_{i,16,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[\alpha_{1,t}(\delta_{\pi,11,t}\delta_{i,16,t} + \delta_{\pi,12,t}\delta_{x,16,t}) + \alpha_{2,t}\delta_{x,16,t}]$$

$$e_{1,t} = \left(\frac{1}{1 - \alpha_{1,t}\delta_{\pi,13,t}}\right)[(\alpha_{1,t}\delta_{\pi,11,t})e_{2,t} + m_t + (\alpha_{1,t}\delta_{\pi,12,t} + \alpha_{2,t}e_{3,t})]$$

Figure B.1. 90 percent level of confidence sets for (β_π, β_x) in the Taylor Rule [1960:1 – 1979:2]

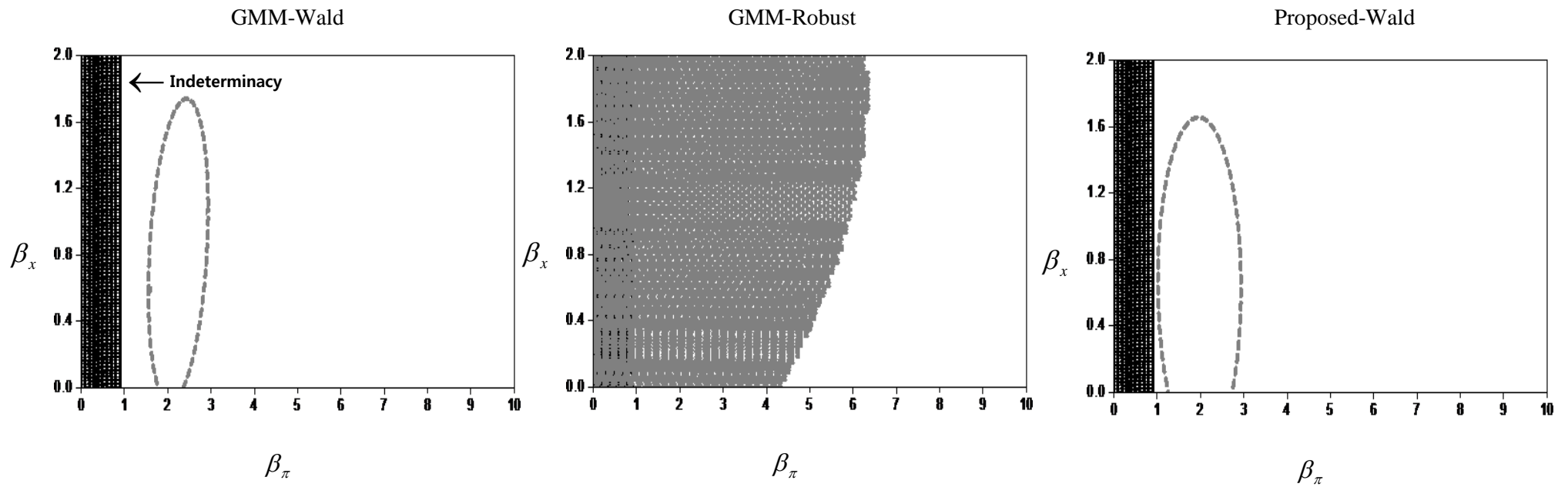
$$i_t = \beta_0 + (1 - \beta_1)(\beta_\pi E_t \pi_{t+1} + \beta_x x_t) + \beta_1 i_{t-1} + \varepsilon_t$$



Note: GMM-Wald refers to Wald test based on GMM. GMM-Robust refers to identification-robust test based on GMM. Proposed-Wald refers to Wald-test based on the proposed method.

Figure B.2. 90 percent level of confidence sets for (β_π, β_x) in the Taylor Rule [1979:3 – 1997:4]

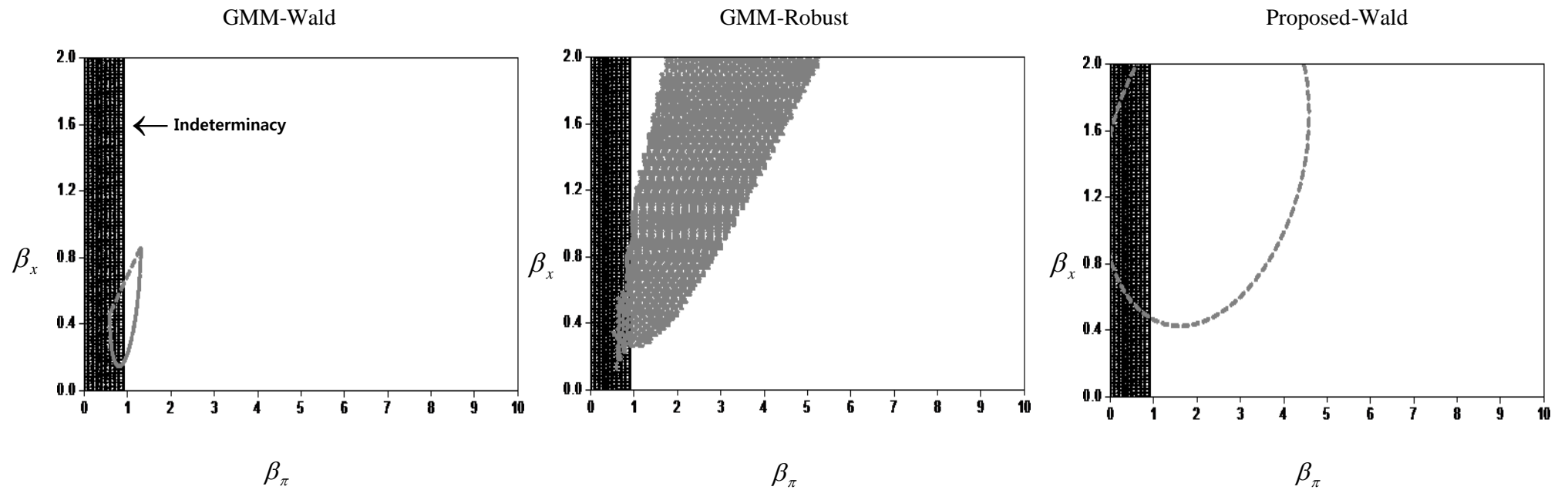
$$i_t = \beta_0 + (1 - \beta_1)(\beta_\pi E_t \pi_{t+1} + \beta_x x_t) + \beta_1 i_{t-1} + \varepsilon_t$$



Note: GMM-Wald refers to Wald test based on GMM. GMM-Robust refers to identification-robust test based on GMM. Proposed-Wald refers to Wald-test based on the proposed method.

Figure B.3. 90 percent level of confidence sets for (β_π, β_x) in the Taylor Rule [1987:3 – 2006:1]

$$i_t = \beta_0 + (1 - \beta_1)(\beta_\pi E_t \pi_{t+1} + \beta_x x_t) + \beta_1 i_{t-1} + \varepsilon_t$$



Note: GMM-Wald refers to Wald test based on GMM. GMM-Robust refers to identification-robust test based on GMM. Proposed-Wald refers to Wald-test based on the proposed method.

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