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Development of a Generalizable Understanding of Cross-Flow Turbine  
Design: Intracycle Velocity Control and Parametric Changes in Rotor  
Geometry

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## **Abstract**

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Cross-flow turbines, often referred to as vertical-axis wind turbines (VAWTs), offer potential for wind and hydrokinetic energy generation. However, their complex, fluctuating angle-of-attack, dynamic stall, and flow recovery cause periodic and unsteady dynamics that are difficult to model, and an optimal design process remains undetermined. This dissertation investigates two independent pathways to improve turbine performance by regulating blade-flow interactions: (1) intracycle variable-velocity control to adjust the relative flow experienced by the blades, and (2) modifying rotor geometry—including camber, blade pitch, and chord-to-radius ratio ( $c/R$ )—to influence the blade aerodynamics. These approaches are explored through a combination of laboratory performance measurements and theoretical analyses rooted in classical steady-state aerodynamic principles to inform generalizable design rules. By focusing on optimizing both the control scheme and blade geometry, significant gains can be achieved without adding substantial increases in complexity. All experiments were conducted in the Alice C. Tyler flume at the University of Washington, where temperature and flow conditions were held constant, to enable broader non-dimensional interpretation of results.

To modify and control cyclic fluctuations in angle-of-attack, several works have evaluated active blade pitch control and found benefits; however, this introduces design complexity and additional failure pathways. A simpler fixed-geometry approach is sinusoidal variable-

velocity (intracycle) control, where velocity varies with blade position to regulate relative velocity and improve efficiency. A preliminary study by [Strom et al. \(2017\)](#) demonstrated a 53% increase in efficiency over constant-speed control, but the broader implications remain poorly understood. For example, while increased performance has been generally shown to increase loading, the extent of this effect has not previously been studied for this type of control. This work experimentally explores a systematic sweep of intracycle kinematics by varying the phase and amplitude of sinusoidal changes in rotation rate, and examining their impact on efficiency and loading. Performance was found to be most sensitive to changes in the sinusoid's phase shift, which regulates synchronization between angular velocity, torque production, and near-blade hydrodynamics. Optimal kinematics increased efficiency by up to 13.2% over constant-speed control or reduced maximum forces by 20%. Performance gains depended strongly on the Reynolds number, which explains the smaller improvements observed here (on a percentage basis) relative to prior work, despite higher peak efficiency. It was found that triggering an earlier stall, followed by blade acceleration, can reduce maximum loading by 12% while simultaneously increasing efficiency by 3% by broadening the performance peak. Particle image velocimetry connected near-blade flow structures with key intracycle kinematics and identified beneficial hydrodynamic structures for future control strategies.

To understand how fixed rotor geometry can impact turbine performance, loading, and near-blade hydrodynamics of cross-flow turbines, the influences of blade camber, pitch, and  $c/R$  were examined simultaneously. These geometric parameters have a coupled effect on performance, adding to design complexity. The circular path of cross-flow turbine blades causes the incident flow to appear curved, inducing a virtual camber and pitch, which alter the lift, drag, and pitching moment of the blades. While virtual camber and pitch are largely determined by  $c/R$ , they are expected to interact directly with geometric pitch and camber, complicating turbine performance optimization. While cambered blades may be beneficial, there is little consensus in the literature on which camber direction might be most favorable. By considering both virtual and geometric effects, this work clarifies how

pitch, camber, and  $c/R$  are coupled, enabling a better understanding that can help reduce design complexity.

Optimum geometric camber is seen to be somewhat rotation-rate-dependent, with concave-in (negative) camber showing potential to reduce loading while enhancing performance at higher rotation rates. Peak efficiency was not found to be particularly sensitive to percent camber, though a  $-2\%$  (NACA-2418) cambered blade shows relatively consistent benefits across the tested rotors. The overall optimum net camber, defined as the combination of geometric and virtual components, was found to vary somewhat across  $c/R$ . For the higher tested  $c/R$  values,  $0.49 \leq c/R \leq 0.74$ , performance is quite similar, and the optimal net camber appears to lie between 6% and 8%. This range of optimum net camber is likely due to the non-linearity of geometric camber's influence on dynamic stall and recovery. Departures in performance at low  $c/R$  deserve further exploration, as they may be related to either Reynolds number effects or a change in the sign of geometric camber to achieve the same net camber. Importantly, by utilizing net pitch, defined similarly to net camber, optimal performance collapses to a greater extent across  $c/R$ . The optimum net pitch for all foils (independent of camber) is approximately zero, explaining previous observations for symmetric foils that demonstrated the optimal geometric blade pitch to be related to  $c/R$ . Finally, thin airfoil theory is used to draw parallels between pitch and camber using the zero-lift angle-of-attack, and to define a new “total” pitch parameter that encapsulates the effects of camber, pitch,  $c/R$ , and their flow-curvature effects into a single variable that is correlated with the range of performance and loading within a single rotation. This new parameter provides a physical representation of the lift generation at infinite tip-speed ratio and demonstrates a linear collapse of both the maximum force and the variation in efficiency over a cycle.

Together, the research contained in the chapters of this dissertation establishes experimental links between turbine kinematics, blade geometry, and near-blade hydrodynamics, demonstrating how control and aerodynamic design can enhance efficiency while mitigating structural loads. The results provide new physical insight and practical frameworks for the

future optimization of cross-flow turbines for marine and wind energy applications.

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## NOMENCLATURE

### Rotor Design Variables

|                |                                                                              |
|----------------|------------------------------------------------------------------------------|
| $\alpha_g$     | Geometric pitch [ $^{\circ}$ ]                                               |
| $\Delta\theta$ | Angular difference between quarter-chord and mid-chord radius [ $^{\circ}$ ] |
| $\dot{\omega}$ | Angular acceleration [ $\text{rad/s}^2$ ]                                    |
| $\gamma$       | Camber [%]                                                                   |
| $\gamma_g$     | Geometric camber [%]                                                         |
| $\lambda_0$    | Intracycle phase mean tip-speed Ratio                                        |
| $\omega$       | Angular velocity [ $\text{rad/s}$ ]                                          |
| $\phi$         | Intracycle sinusoidal phase shift [ $^{\circ}$ ]                             |
| $\sigma$       | Solidity                                                                     |
| $\theta$       | Azimuthal position [ $^{\circ}$ ]                                            |
| $A_{\lambda}$  | Non-dimensional amplitude of intracycle velocity change                      |
| $c$            | Chord length [m]                                                             |
| $c/R$          | Chord-to-radius ratio [m]                                                    |
| $D$            | Diameter the quarter chord [m]                                               |
| $D_o$          | Outermost diameter [m]                                                       |

|       |                                 |
|-------|---------------------------------|
| $h$   | blade height/span [m]           |
| $R$   | Radius to the quarter-chord [m] |
| $R_o$ | Outermost radius [m]            |
| $A$   | Area [m <sup>2</sup> ]          |
| $N$   | Number of blades                |

### **Flow Parameters**

|            |                                         |
|------------|-----------------------------------------|
| $\alpha$   | Angle-of-attack [°]                     |
| $\alpha_n$ | Nominal angle-of-attack [°]             |
| $\nu$      | Kinematic viscosity [m <sup>3</sup> /s] |
| $\rho$     | Fluid density [kg/m <sup>3</sup> ]      |
| $Fr_h$     | Depth based Froude number               |
| $Fr_s$     | Submergence based Froude number         |
| $H$        | Dynamic depth [m]                       |
| $Re_b$     | Blade-relative Reynolds number          |
| $Re_c$     | Chord-based Reynolds number             |
| $Re_D$     | Diameter-based Reynolds number          |
| $s$        | Submergence depth [m]                   |
| $TI$       | Turbulence intensity                    |



$U_\infty$  Inflow velocity [m/s]

$U_{rel,n}$  Nominal relative velocity[m/s]

$W$  Channel width [m]

### **Performance Metrics**

$\lambda$  tip speed ratio

$\lambda_{Opt}$  Optimal tip speed ratio

$\tau$  Torque

$C_P$  Power coefficient

$C_Q$  Torque coefficient

$C_{F_X}$  Thrust force coefficient

$C_{F_Y}$  Lateral force coefficient

$C_F$  Force coefficient

$C_{P,Opt}$  Optimal power coefficient

### **Flow Curvature Variables**

$\alpha_v$  Virtual pitch [°]

$\alpha_{L0}$  Zero lift angle-of-attack [°]

$\alpha_{net}$  Net pitch [°]

$\alpha_{tot}$  Total pitch [°]

$\gamma_v$       Virtual camber [%]

$\gamma_{net}$     Net camber [%]

### **Inertial Parameters**

$\delta\theta$       Phase offset of the center of mass from the quarter chord [°]

$I$         Moment of inertia [kg m<sup>2</sup>]

$mr_g$     Mass times the radius to the center of mass [kg m]

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## Chapter 1

# INTRODUCTION

With the expanding population and increasing electrification of the globe, there is an ever-growing demand for electrical power. Renewable methods for generating electricity are critical to addressing this need and reducing the detrimental impacts of climate change and pollution on the planet. Marine hydrokinetic energy, such as tidal or river current generators, is an area of renewable energy production that has seen minimal implementation but maintains significant potential [8]. Marine energy would be ideal to assist in the electricity demands of many, as over one-third of the global population lives within 100 km of oceans or rivers [9]. Additionally, harnessing local marine energy in remote communities like Alaska would replace the need for diesel generators and fuel transportation.

The two main methods of extracting marine hydrokinetic energy are axial flow turbines, where the flow is parallel to the axis of rotation, and cross-flow turbines (sometimes called vertical-axis in wind applications), where the flow is perpendicular to the rotation direction. Although axial flow turbines have traditionally dominated the wind energy sector due to their high individual efficiency, straightforward dynamics, and thus easier optimization and modeling, recent research in cross-flow turbines has shown they may be superior in several ways. Cross-flow turbines show promise for various power applications due to the potential for constructive interference when used in arrays, low rotation rates leading to decreased environmental impacts and acoustic pollution, direction-agnostic operation, simplified maintenance, and rectangular profile useful for confinement exploitation [10–12]. Despite these advantages, cross-flow turbines are still not fully understood. Cross-flow turbines exhibit complex, unsteady flow separation due to large periodic variations in the blade angle-of-attack, making modeling, simulation, and design of these turbines challenging [12, 13]. Consequently, while some general geometric testing and ideal control schemes have been explored, definitive design guidelines for the practical deployment of cross-flow turbines have

yet to be established. The work presented here aims to contribute to the ongoing pursuit of such guidelines by experimentally examining the effects of variable-velocity control and the codependent interaction of rotor variables with blade camber on their performance and dynamics.

Current methods to enhance cross-flow turbine performance are limited. Several approaches, such as active blade pitching, plasma actuators, jet injection, and profile cutouts along the blades' edges, have been tested to control the flow over turbine blades and improve their power output [11, 12, 14–16]. However, these tactics increase the complexity of the turbine and often require power investment to see benefits. Despite analysis of these complex optimization methods, a fundamental understanding of the impacts of scale, chord-to-radius ratio, blade geometry, and other design parameters remains murky. Axial-flow turbines have seen converged designs across scales of application, resulting in complex blade geometries. However, the tools and engineering design knowledge to make informed decisions and designs for a particular application do not exist for cross-flow turbines. Further, robust and relatively simplistic designs are crucial for the initial implementation of this emerging technology, where catastrophic failure can lead to lasting setbacks in broader cross-flow turbine applications.

To expand knowledge of the cross-flow turbine design space, this work experimentally investigates the performance and flow-field effects of two low-complexity performance-improvement methods that show significant potential: variable-velocity control and leveraging interactions among rotor design variables. Prior works studying sinusoidal intracycle velocity control have shown substantial efficiency improvements of up to 53% over a turbine operated at a constant rotation rate [1, 17–19]. These previous studies employed optimization schemes to find ideal control kinematics. However, the sensitivity of the intracycle control parameter space and its potential to control turbine loading alongside power output have remained unexplored. Additionally, while symmetrical blades have traditionally been utilized in cross-flow turbines, some numerical works investigating wind turbine designs have explored cambered blades and have shown they enhance performance, facilitate wake recovery (which would allow more compact array deployment), and have the potential for improved self-starting [11, 20–22]. However, the literature contains conflicting evidence



regarding the appropriate direction and magnitude of camber, and to date, no detailed experimental work has examined its effects. It is thought that the optimum camber must be a function of the relative curvature of the blade's path, parameterized by the chord-to-radius ratio. This curvature leads the blade to behave as if it has a virtual camber, and a change in pitch, and so any generalized guidance for blade camber and pitch must be explored as a function of this curvature.

### **1.1 Objectives**

This work aims to determine the fundamental impacts of variable-velocity control, cambered blades, and the interactions of the rotor design variables in cross-flow turbines. This research seeks to establish connections between these effects and a deeper understanding of hydrodynamic phenomena and theory, while also evaluating their utility for future application and deployment of the technology. The design parameters and overarching objectives of this work are as follows:

1. Identify the sensitivity of cross-flow turbine performance and loading to the intracycle control parameter space and to explore other potential utilities for future implementation beyond performance optimization (Chapter 4).
2. Determine the impacts of cambered blades on experimental performance and flow fields of cross-flow turbines while connecting results to the estimated virtual camber due to rotational geometry and flow curvature (Chapter 5).
3. Ascertain the interactions between design changes in chord-to-radius ratio, blade pitch, camber, and their coupled impacts on the performance and loading of cross-flow turbines (Chapter 6).

## Chapter 2

### BACKGROUND

To understand cross-flow turbine performance and dynamics. The following section summarizes key performance metrics, geometric parameters, and non-dimensional effects demonstrated in prior work to provide context necessary for this work.

#### 2.1 *Cross-flow Theory*

This study aims to streamline complexity while retaining the capacity for fundamental insights and generalizability to more complex designs by focusing on straight-bladed, fixed-pitch cross-flow turbines. At a constant rotation rate, turbine performance depends on the tip-speed ratio,  $\lambda$ , and blade position. The tip-speed ratio provides a non-dimensional measure of the turbine's rotation rate relative to the incoming flow, and is defined as  $\lambda = \omega R / U_\infty$ . Here,  $\omega$  is the angular velocity of the turbine,  $R$  is the turbine radius measured to the quarter-chord mounting point, and  $U_\infty$  is the free stream velocity far upstream of the turbine. For non-dimensional parameters normalized by the turbine projected area, such as turbine efficiency, the radius to the outermost swept point of the blade,  $R_o$  was used and multiplied by two to represent the maximum diameter,  $D_o$ . A schematic of turbine blade coordinates and nominal relative velocity used in this work is shown in Fig. 2.1. The blade angular position represented as  $\theta$  is defined to be  $0^\circ$  when the blade tangential velocity vector is pointed directly upstream.

For a given  $\lambda$  and blade azimuthal blade position,  $\theta$ , in the absence of rotor influence on the inflow, the nominal angle-of-attack is defined as

$$\alpha_n(\theta) = \tan^{-1} \left[ \frac{\sin(\theta)}{\lambda + \cos(\theta)} \right] + \alpha_g, \quad (2.1)$$

where  $\alpha_g$  is the geometric (preset) pitch of the blade. Figure 2.2(a) shows how the  $\alpha_n$  varies as a function of position and rotation rate.

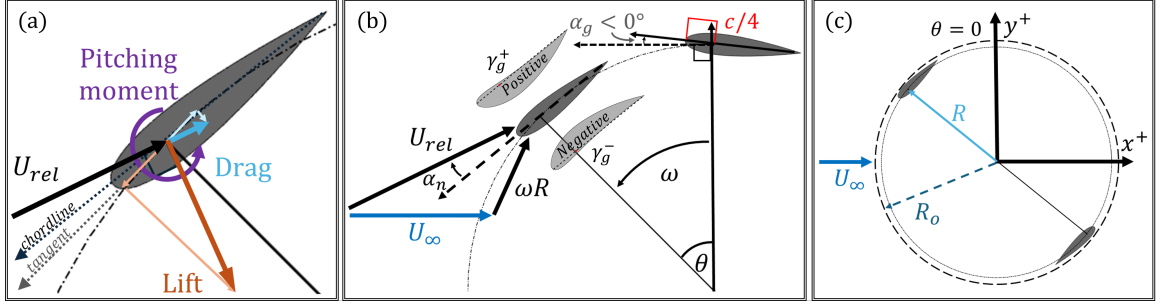


Figure 2.1: Cross-flow turbine geometry and kinematics: (a) Near blade decomposition of hydrodynamic lift, drag, and pitching moment along the blade. (b) Nominal blade-level coordinates and key variables used in this investigation. (c) The turbine-level turbine-level cartesian coordinate system is defined relative to the inflow velocity.

For low  $\lambda$ , it is clear that the nominal angles-of-attack of a cross-flow turbine far exceed the standard operating envelope in traditional aerodynamic applications, exceeding the static stall angle of the airfoil,  $\pm(12^\circ$  to  $15^\circ)$  for the NACA 0018 [2, 3]. Additionally, the  $\alpha_n$  switches from large positive to negative values over a single cycle. Beyond the large fluctuations in  $\alpha_n$ , the nominal relative velocity,  $U_{rel,n}$ , of the blade fluctuates greatly through the cycle and can be observed as,

$$U_{rel,n}(\theta) = \sqrt{\lambda^2 + 2\lambda \cos(\theta) + 1} \quad (2.2)$$

with nominal profiles shown in Fig. 2.2(b). We see that the relative velocity on the blade reaches a maximum when the blade is traveling upstream ( $\theta = 0^\circ$ ) and a minimum at about  $180^\circ$  while moving with the flow towards the downstream.

The relative velocity and angle-of-attack experienced by the blade generate lift, drag, and a parasitic pitching moment on the blade (shown in Fig. 2.1a); these forces and moments are combined to produce power and fluctuate throughout the cycle.

## 2.2 Key Performance Metrics

To interpret the performance of cross-flow turbines, defining some key metrics used in the analysis is necessary. The torque produced by the fluid-blade interaction of the turbine

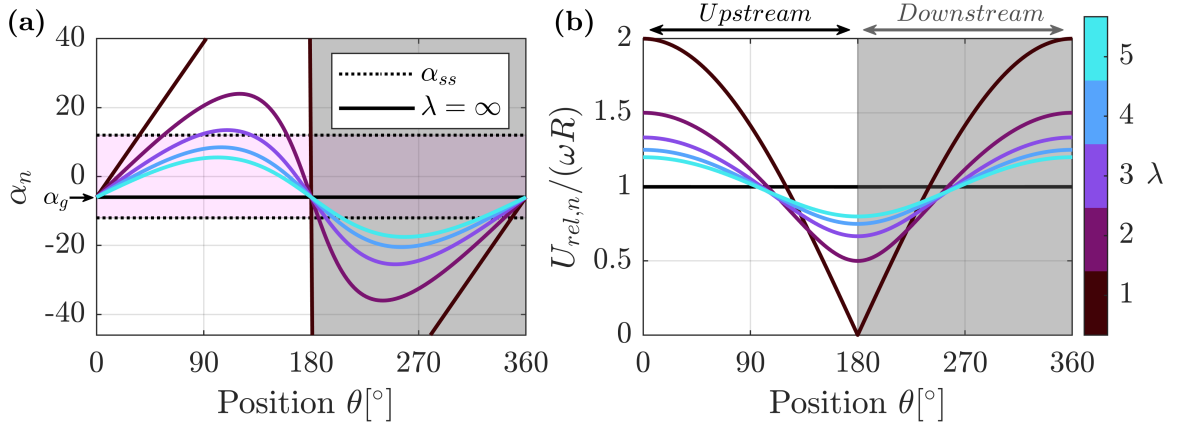


Figure 2.2: (a) Nominal angle-of-attack ( $\alpha_n$ ) experienced by the blade throughout the cycle as a function of  $\lambda$ . The shaded pink denotes conditions bounded by the static stall angle  $\alpha_{ss} = \pm 12^\circ$  for the blades and Reynolds numbers used in this work [2, 3]. (b) Nominal relative velocity as a function of  $\lambda$ . The solid black lines represents the infinite  $\lambda$  condition where  $\alpha_n = \alpha_g$ , the gray shaded rectangle indicates the downstream region ( $\theta = 180^\circ - 360^\circ$ ).

generates power and is non-dimensionalized as

$$C_Q(\theta) = \frac{\tau(\theta)}{\frac{1}{2}\rho AU_\infty^2 R} \quad (2.3)$$

where  $\rho$  is the fluid density,  $A = hD_o$  ( $h$  is the blade height/span) is turbine projected area, and  $\tau$  is the torque. Turbine performance or efficiency is defined in terms of the coefficient of power, or the ratio of extracted mechanical power,  $\tau\omega$ , to the kinetic power present in the water flowing through the cross-sectional area of the turbine as,

$$C_P(\theta) = \frac{\tau(\theta)\omega}{\frac{1}{2}\rho AU_\infty^3} = C_Q(\theta)\lambda \quad (2.4)$$

where  $\omega$  is the angular velocity. Similarly, the horizontal force coefficient is defined as a normalized vector sum of the lateral force,  $F_Y$  (perpendicular to the flow,  $y$  direction), and the thrust force,  $F_X$  (in line with the free stream,  $x$  direction), both orthogonal to the turbine's central shaft,

$$C_F(\theta) = \frac{\sqrt{F_X^2(\theta) + F_Y^2(\theta)}}{\frac{1}{2}\rho AU_\infty^2}. \quad (2.5)$$

Time-averaged versions of all metrics are chosen to encompass an integer number of rotations and denoted by an overbar, e.g.  $\overline{C_F}$ .

### 2.2.1 non-dimensional Parameters

Some key non-dimensional parameters have been shown to impact turbine performance substantially and have received ongoing research. The Reynolds number of the turbine can be defined in several ways, but is most commonly expressed as diameter-based, chord-based, blade-relative, or .

$$Re_D \frac{DU_\infty}{\nu} \quad \text{or} \quad Re_c = \frac{cU_\infty}{\nu} \quad \text{or} \quad Re_b = \frac{cU_{rel}(\theta)}{\nu} \quad (2.6)$$

respectively, which are the ratios of kinetic to viscous effects.  $c$  is the chord length and  $\nu$  is the kinematic viscosity. Turbine performance increases with  $Re$  until an independent regime is reached and performance plateaus [23, 24]. Typically, chord- and diameter-based  $Re$  values are used to define experimental conditions, since  $U_\infty$  can be directly measured. Yet, from the perspective of the blade, the true blade-relative  $Re$  is explanatory. However, no general model exists for the fluctuations in  $U_{rel}$ , and so nominal estimates ( $U_{rel,n}$ ) are typically averaged over a rotation as a function of  $\lambda$ , to approximate it, ignoring induction.

Another parameter particularly important in experimental work is the blockage,

$$\beta = \frac{A_{turbine}}{A_{channel}} = \frac{hD_o}{HW}, \quad (2.7)$$

the ratio of turbine projected area to fluid cross-sectional area. As  $\beta$  increases, the turbine occupies a larger portion of the channel, constraining the flow and forcing more of the freestream to pass through the turbine, resulting in higher power output.  $\beta$  can be accounted for by applying blockage-correction methods to approximate unconfined systems when reporting experimental results to allow more generalizability [25].

The channel Froude number,

$$Fr_h = U_\infty / \sqrt{gH}, \quad (2.8)$$

governs the ratio of kinetic energy to potential energy in the system, where  $g$  is the acceleration of gravity and  $H$  is the dynamic depth of the water channel. A submergence-based

Froude number,

$$Fr_s = U_\infty / \sqrt{gs}, \quad (2.9)$$

can also be defined, where  $s$  is the distance between the free surface and the turbine. While changes in  $Fr$  are observed to have a small effect on performance, the effects of Froude number are significantly smaller than the effects of  $\beta$  or  $Re$  and can be neglected, particularly for sufficiently small Froude numbers (i.e.,  $Fr_s \leq 0.44$ ) [24].

### 2.3 Geometric Parameters

This work investigates fixed/straight-blade cross-flow turbines. Although these turbines have relatively few design parameters, the complex interactions among them make rotor optimization challenging. The three parameters in particular govern the rotor geometry, and aerodynamic performance of the blades are introduced here and explored in detail in Chapters 5 and 6.

1. The blade *geometric camber*,  $\gamma_g$ , defines curvature of the blade as a percentage calculated as the ratio between the maximum displacement of the camber line relative to the chord line (shown in Fig. 2.1b). This curvature strongly influences the lift generation and pitching moment, through the cycle, affecting both the timing and characteristics of stall and recovery [26–28]. The optimal mounting direction “concave in” (negative) or “concave out” (positive) is an area of contention in the literature, which has often lacked consideration [11, 29, 30]
2. The *geometric pitch* of the blades (often referred to as the preset pitch),  $\alpha_g$ , is the angle the blade is rotated about the quarter chord (shown in Fig. 2.1b), measured between the chord line and tangent line of the swept circle. It determines the baseline angle-of-attack the turbine blades experience through their rotation, which in turn affects torque production, stall timing, and flow recovery [7, 31, 32].
3. The *chord-to-radius ratio*,  $c/R$ , where  $c$  is typically a fraction of the length of  $R$  and governs the curvature of the flow experienced by the blade, creating virtual camber

and pitch effects that act similarly to combining the effects of both  $\alpha_g$  and  $\gamma_g$  changes (see Section 2.5) [33, 34].

Another parameter of note is the number of blades  $N$ , which reduces variation in power production and loading over the cycle, and impacts blade wake interactions [7]. Other parameters, not evaluated in detail in this work, include the blade surface roughness, which affects skin friction/drag and boundary layer separation; blade cant or helix angle, which arises from angled or curved blades that sweep about the circle to smooth power production; the aspect ratio of the rotor; and the foil thickness. While each of these parameters can significantly influence aerodynamic performance, they are not the focus of the present study.

Another non-dimensional geometric parameter often introduced in the analysis of cross-flow turbines is the turbine solidity,  $\sigma$ , which combines  $N$  and  $c/R$ . The solidity represents the fraction of the turbine perimeter occupied by the blades and is defined as

$$\sigma = \frac{Nc}{2\pi R}. \quad (2.10)$$

This parameter has been shown to most strongly correlate with the  $\lambda$  of optimal efficiency,  $\lambda_{opt}$ , while only weakly explaining other performance trends [7, 35]. Solidity alone does not reliably capture turbine performance, since the same value can be achieved through multiple designs (e.g. few large blades, or many small blades), and does not account for blade-wake interactions or changes in flow curvature [7].

## 2.4 Hydrodynamic behavior

As the turbine blades actively exceed the static stall angle, the rapid increase in angle-of-attack through the upstream sweep leads to dynamic stall before switching from positive to negative. Dynamic stall involves the generation, growth, and shedding of vorticity from the blade [36]. During the generation phase, this can produce a temporary lift overshoot that can exceed static limits while attachment is maintained due to the actively changing  $\alpha_n$ . During the upstream sweep, a vortex forms along the suction side of the blade (inside), increasing lift; however, as this vortex is shed and advected off the foil near the end of the upstream sweep, a sudden loss of lift and an increase in drag occur, which is detrimental to

performance. The passage of the vortex also induces nonlinear disturbances in the pressure field, affecting lift, drag, pitching moment, and force hysteresis, meaning that loads and moments differ when pitching up versus down. The passage of this vortex induces nonlinear disturbances in the pressure field, which can significantly impact lift, drag, pitching moment, and force hysteresis (i.e., the loads and moments can differ when pitching up versus down). Dynamic stall tends to be much more severe and persistent than static stall and remains the subject of many fundamental studies, though largely for rectilinear pitching foils [28, 37, 38]. Dynamic stall in cross-flow turbines experiences additional complexities due to the rotating reference frame discussed in Section 2.5. In cross-flow turbines, this phenomenon is harnessed for power generation; during the initial dynamic stall vortex formation, the lift can be increased incredibly, which can drive power production [39]; however, considerable drag can reduce the benefits post-stall, so this process must be carefully investigated to allow stall to be harnessed optimally [36].

Another aspect of cross-flow turbine hydrodynamics that is key to understanding their performance is flow induction. Flow induction in cross-flow turbines refers to the turbine altering the fluid flow in its immediate vicinity. As turbine rotation increases, the higher thrust loading further diverts the flow around the turbine. Additionally, because the upstream blade passage extracts power from the free stream, the inflow experienced by the downstream blades includes the wake of the upstream blades and a greatly reduced inflow velocity. This feature of cross-flow turbines remains difficult to model because induction depends on accurately predicting blade-level forces, which are themselves hard to resolve due to induction effects, dynamic stall, and reattachment. As a result of induction, the nominal predictions of  $U_{rel,n}$ , and  $\alpha_n$  are flawed, particularly in the downstream sweep, though they can remain an estimate for a general understanding of flow. Furthermore, because the downstream blade experiences a much-reduced inflow, the relative velocity is dominated by its tangential component. This reduction in free-stream flow makes the reattachment of flow more difficult post-stall in the downstream blade path. After complete flow separation in the upstream occurs with dynamic stall, reattachment of the flow is needed downstream.

While power is primarily produced during the upstream sweep, maximizing upstream power is not always optimal, as shown by Snortland et al. [40]. As  $\lambda$  increases beyond the



optimal performance point, the turbine’s performance diminishes; however, the upstream blade’s performance continues to increase. At these higher  $\lambda$ , the downstream  $\alpha_n$  approaches the geometric pitch angle, and performance is dominated by a combination of drag on the blade and a parasitic pitching moment. Therefore, reducing downstream drag at above-optimal rotation rates and or enhancing lift production in the upstream at low  $\lambda$  are vital to the optimization of cross-flow turbines.

## 2.5 Flow Curvature Effects and Virtual Forces

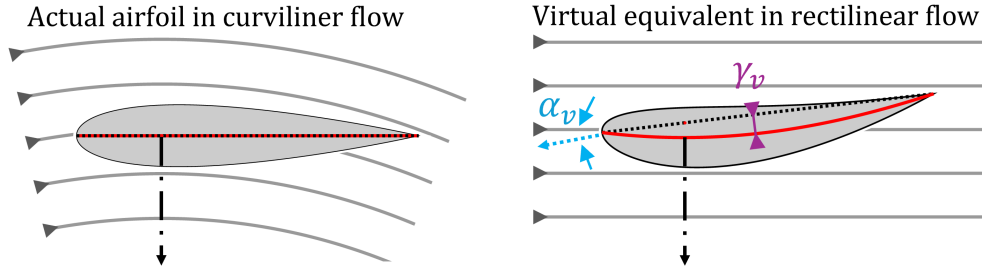


Figure 2.3: Schematic of flow-curvature in the blade reference frame of a cross-flow turbine with  $0^\circ$  geometric pitch, resulting in a virtual camber ( $\gamma_v$ ) and pitch ( $\alpha_v$ ) after being transformed to the rectilinear axis.

The blade of a cross-flow turbine experience curvilinear streamlines exhibits a similar behavior to a cambered foil in rectilinear flow. This phenomenon, which leads to higher lift generation than expected for a symmetrical foil, was first noted by Muraca et al. [41] and later examined and labeled “virtual camber” by Migliore et al. [33]. In their initial work, Migliore et al. [33] developed a geometric conformal mapping transformation, which preserves the local  $\alpha_n$  along the chord and transforms the foil into a virtual equivalent as if it were in a rectilinear flow. This transformation assumes an airfoil can be reduced to its chord line (a thin airfoil) and that nominal flow conditions apply, neglecting blade wake interactions and induction. Subsequently, a number of other researchers have developed alternative approaches. Extensions to include the effects of active pitching-induced velocity variation were developed by two groups [42, 43]. Others used alternative mathematical approaches to the transformation that have been both more straightforward, such as using

a virtual camber line of a circular arc [44, 45], and more complex, utilizing exponentials and conformal transformations [46]. Despite the range of methods, all approaches yield similar predictions for  $c/R < 0.4$  [34]. For larger  $c/R$ , the results of these methods diverge slightly.

Migliore’s original technique is used throughout this dissertation due to its simplicity and greater prevalence in the literature. For the turbine configuration tested in this work, the virtual equivalent airfoil at infinite  $\lambda$  has a virtual camber of close to 6%. The phenomenon of virtual camber and pitch has an observed impact on the pressure profiles of the blades and effects on overall performance, leading to significant errors if not considered in numerical estimates [47, 48]. While induction and blockage effects are neglected in the calculation of  $\alpha_n$ , and flow curvature effects, making phase-resolved behavior a significant approximation, the transformation remain helpful for rough interpretation. And as the rotation rate increases, the true downstream  $\alpha$  and flow curvature are expected to approach infinite  $\lambda$  behavior increasing the expected accuracy.

Additionally, similar methods have been developed for analysis of submarines making constant radius turns and specialized wind tunnels to evaluate these curved flows for automobile cornering, illustrating the broader reliance and impacts of curved flows [49, 50].

Beyond the effects of flow curvature on aerodynamic phenomena, the blade’s rotating reference frame also introduces three additional virtual forces not present in traditional aerodynamics. These virtual forces arise in the rotating reference frame version of the Navier-Stokes equation

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\nabla p + \frac{1}{Re}\nabla^2 u - \frac{d\Omega}{dt} \times R - 2\Omega \times u - \Omega \times (\Omega \times u).$$

The Coriolis force  $= -2\Omega \times u$ , which acts perpendicular to the relative velocity vector at every point in the flow (Fig. 2.4). This impacts local flow structures and has been shown to result in the vortex capture phenomena by Tsai and Colonius [51] and Bridges et al. [52], where the upstream dynamic stall vortex remains attached and follows the blade path longer than in a rectilinear pitching foil.

The centrifugal force  $= -\Omega \times (\Omega \times u)$ , acts radially on the flow traveling with the blade. This force has been shown to increase the radial pressure gradient within the boundary

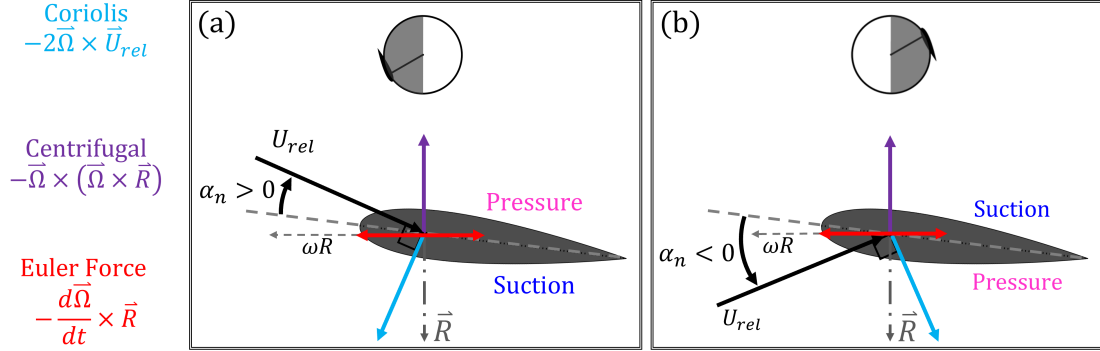


Figure 2.4: Schematic of nominal directions of virtual forces for (a) the upstream sweep and (b) downstream sweep. The direction of the Euler force is dependent on the sign of acceleration.

layer by at least an order of magnitude compared to what is experienced in non-rotating curved flows [53, 54]. The change in pressure gradient is expected to help maintain flow attachment on the blade's inner surface at high  $\alpha_n$  through the upstream sweep [33].

And the last fictitious force is the Euler force  $= -\frac{d\vec{\Omega}}{dt} \times \vec{R}$ , which only arises when the blade angular velocity varies through the cycle. This force is expected to have two primary effects. First, slightly delaying or promoting flow separation due to the change in tangential velocity, which has received limited study, and primarily in helicopter rotor applications [55]. Second, it more directly modifies the relative velocity and angle-of-attack thereby influencing the blade aerodynamics more explicitly, observed in prior studies using variable-velocity control [1, 19], though not explicitly attributed to the Euler force. However, for small velocity fluctuations (e.g., under torque control), this force is several orders of magnitude smaller than other inertial terms and is likely overshadowed by the dominant periodic aerodynamic forces that cross-flow turbines inherently experience.

## Chapter 3

### COMMON METHODS

#### 3.1 *Experimental Facilities*

The following work was completed at the University of Washington in the Alice C. Tyler recirculating water flume, which has a 4.6 m long and 0.76 m wide  $W$  test section, and can operate at a dynamic depth of up to,  $H$ , of 0.60 m. The temperature, depth, and inflow velocity were controlled to maintain consistent non-dimensional parameters such as the blockage, Reynolds, and Froude numbers across a set of experiments.

Temperature was manually monitored (Omega Ultra-Precise RTD temperature sensor) and kept within  $\pm 0.1^\circ\text{C}$  of the target value throughout the tests using an external chiller and an inline pool heater. The dynamic depth of the flume was recorded at a 0.5 Hz sample rate by a sonic altimeter (Omega LVU30 Series) upstream of the turbine. The inflow velocity was measured five turbine diameters upstream of the turbine using an acoustic Doppler velocimeter (ADV), Nortek Vectrino Profiler. The ADV data were collected at a vertical point centered on the turbine position at a sample rate of 16 Hz, and recorded low turbulence intensities between 2 – 4% for the conditions in this work.

#### 3.2 *Cross-flow Turbine Test Rig*

Two turbine configurations were employed: one for performance and loading measurement and another for flow-field particle image velocimetry (PIV) acquisition (Fig. 3.1). Both turbines could be operated in single- or multi-bladed modes. The standard blades investigated in this work were NACA 0018 airfoils with a  $c/R$  of 0.49 and outward geometric pitch of  $-6^\circ$  measured about a quarter-chord mounting point (Fig. 2.1), a configuration found to be optimal by prior work at the University of Washington [7, 32]. All blades were machined from aluminum, so span-wise bending was considered negligible. After machining, all blades were sanded smooth to a dull finish, anodized, and painted to reduce corrosion and provide

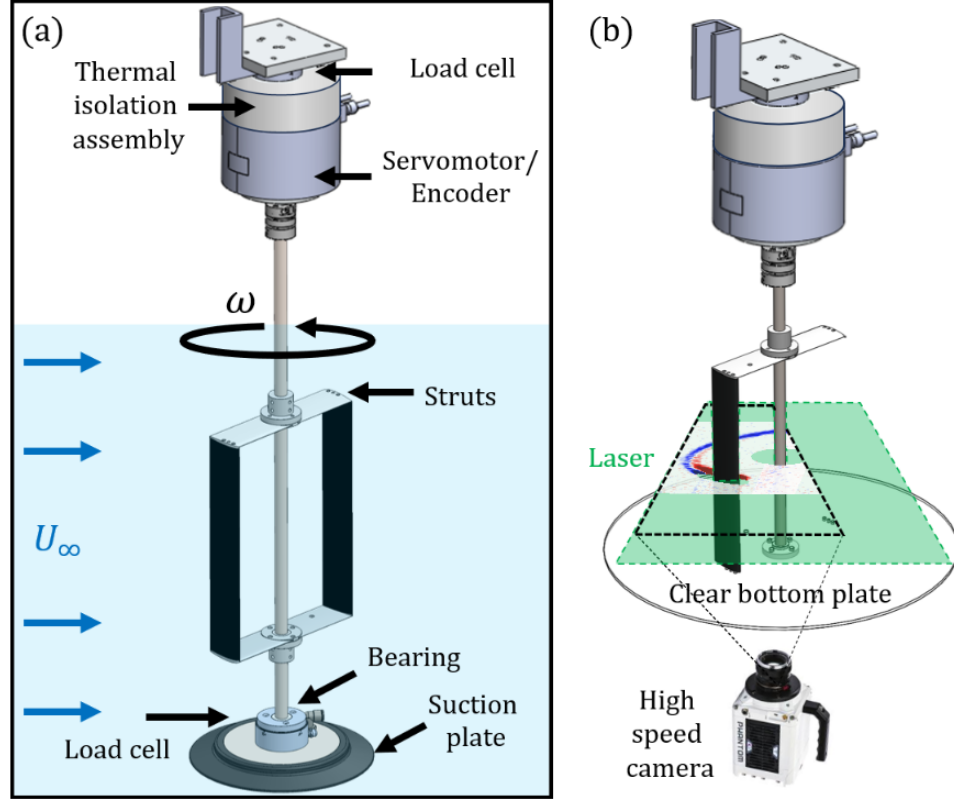


Figure 3.1: (a) Experimental configuration used to collect force and performance data. (b) Cantilevered experimental setup used to capture in rotor mid-span planar PIV with representative vorticity measurement.

a consistent surface finish.

### 3.2.1 Performance Measurements

The performance turbine was a pinned configuration (shown in Fig. 3.1a), employing a suction plate on the bottom of the flume. The blades were fastened to the central shaft by two thin NACA 0008 profile strut-type end plates to reduce drag. Two ATI 6-axis load cells, Mini45's, were mounted at the top and bottom of the turbine central shaft, respectively. These load cells measured the turbine's moments and forces at 1 kHz and were recorded in MATLAB using National Instruments PCIe-6353 DAQs. A 5 Nm servomotor (Yaskawa SGMCS-05B3C41), equipped with an internal encoder with resolution  $2^{18}$  counts-

per-revolution, rotated the turbine to the desired control kinematics and measured the position throughout the cycle. All turbine performance information was recorded using *Matlab Simulink Desktop Real-time*. During operation, data were collected for 45 s for each control set point, which was sufficient to achieve convergence of cycle-averaged performance. A low-pass Butterworth filter with a pass frequency of 10 Hz and a cutoff of 110 Hz was applied to the extracted data to remove high-frequency motor noise. This filter had no impact on time-averaged performance, but improved the phase-average representation [56].

Prior studies using this experimental setup at the University of Washington have quantified sensor uncertainty in detail [7, 57]. Those analyses showed that although instrument specifications imply relatively large total system uncertainties, the measured variability is much smaller. In particular, the mean standard deviation of cycle-averaged  $C_P$  at each  $\lambda$  is approximately 0.015, near the optimal operating point. The discrepancy between observed repeatability and that predicted from a traditional uncertainty analysis arises primarily from the ATI load cells, whose practical repeatability is substantially better than their quoted specifications. Prior work also documented cycle-to-cycle fluctuations in the inflow and ADV-measured  $U_\infty$  [57]; however, with the extended 45-s test durations, the influence of this variation becomes negligible.

### 3.2.2 Estimation of Blade-level Forces and Torques

To isolate the hydrodynamic performance and loading on the blades, it is necessary to estimate and remove the effects of support structure losses and inertial loading recorded by the load cells. The following outlines the approach used throughout this dissertation to resolve the blade-level performance so that performance changes are not obscured by secondary losses.

#### *Compensation for Inertial Loading*

First, inertial effects are corrected. For a multi-bladed turbine with equal blade spacing, the center of mass lies on the axis of rotation, and inertial effects do not affect turbine phase-resolved forces. In contrast, single-bladed turbines have an offset between the center

of mass and the axis of rotation. Therefore, the inertial loads caused by this offset must be corrected for when analyzing phase-averaged blade-level forces for a one-bladed turbine. Inertial contributions are corrected for using the method proposed by Snortland et al. [56]. The correction is applied to each planar force component independently,  $F_{X,m}$  and  $F_{Y,m}$ , where the subscript  $m$  denotes load cell measured quantities. Under constant speed control, inertial loads measured by the load cells are removed by employing

$$\begin{aligned} F_X &= F_{X,m} + mr_g \omega^2 \sin(\theta - \delta\theta) \\ F_Y &= F_{Y,m} - mr_g \omega^2 \cos(\theta - \delta\theta), \end{aligned} \quad (3.1)$$

where  $mr_g$  represents the inertial effect through the product of the rotating mass and radius to the center of mass, and  $\delta\theta$  term corrects for the azimuthal offset of the blade center of mass relative to the quarter-chord. Supplemental experiments conducted in air are used to back out the  $mr_g$  and  $\delta\theta$  terms of our turbine. In these tests, the turbine velocity was sinusoidally varied as a function of position. By assuming drag to be negligible in air, Newton's second law is applied, and inertial constants are solved for as detailed in [56]. This inertial correction was not applied to axisymmetric configurations (including support structure cases), for which  $mr_g$  is assumed negligible. Furthermore, although single-bladed turbines experience inertial loading, these contributions integrate to zero over a full revolution and therefore do not affect time-averaged performance. Additional terms arise under variable-speed operation due to angular acceleration and are discussed in Chapter 4.2.

#### *Isolation of Blade-level Performance*

Second, to analyze blade-level hydrodynamic performance, the influence of the supports and any bearing losses must be estimated and subtracted from the measured full-turbine performance. A superposition method is utilized, following the analysis of Strom *et al.* [58] and Snortland *et al.* [56]. Auxiliary experiments were conducted under matching conditions to measure the influence of the blade support structure without the blades present. To isolate blade-level power and torque coefficients, a simple superposition,

$$C_{[P,Q],blade}(\theta) = C_{[P,Q],turb}(\theta) - C_{[P,Q],sup}(\theta) \quad (3.2)$$

was used, where subscripts  $turb$  and  $sup$  correspond to full turbine experiments, and supports with blades removed (i.e., struts + center shaft), respectively. This approach has been shown to effectively collapse performance for different support structures [56, 59].

To isolate blade-level force coefficients  $C_{F_X}$  and  $C_{F_Y}$

$$C_{[F_X, F_Y], blade}(\theta) = C_{[F_X, F_Y], turb}(\theta) - [C_{[F_X, F_Y], sup}(\theta) - C_{[F_X, F_Y], shaft}(\theta)] - C_{[F_X, F_Y], shaft}(\theta) \frac{H-h}{H}, \quad (3.3)$$

where  $shaft$  corresponds to an additional shaft only case. This approach only considers the shaft influence outside of the turbine area ( $\frac{H-h}{H}$ ), because due to induction, the force is negligible on the shaft inside of the rotor [56].

The in-plane force magnitude is found by combining the component blade-level forces using,

$$C_{F, blade}(\theta) = \sqrt{C_{F_X, blade}^2(\theta) + C_{F_Y, blade}^2(\theta)}. \quad (3.4)$$

While the above equations are presented for performance as a function of  $\theta$ , averaging the results over an integer number of rotations provides the time-averaged performance. Alternatively, this is equivalent to applying the same equations but simply substituting the functions of  $\theta$  for their time-averaged equivalent (i.e.  $C_{F, turb}(\theta) \rightarrow \overline{C_{F, turb}}$ ). As all turbine data presented in this dissertation is estimated blade-level performance, the subscripts are omitted and performance metrics simplified, e.g.,  $C_{F, blade} \rightarrow C_F$ .

Although the measurements of all forces and torques are synchronous, velocity measurements obtained with the ADV are asynchronous, because they are collected upstream (introducing an advection delay) and are sampled at a lower rate. Since values of  $U_\infty^2$  and  $U_\infty^3$  are required to non-dimensionalize performance coefficients, it is important to clarify how they are computed from the ADV data. Two main approaches can be used. First, it is possible to calculate cycle-specific performance with instantaneous velocity measurements by using an empirical correction for an advection time based on a cross correlation of  $U_\infty$  with cycle-averaged power [60]. However, this approach relies on Taylor's frozen-flow hypothesis, which has been found to amplify the relative variation in  $C_P$  compared to dimensional power when the advection time is uncertain. Instead, in this work, we use the second approach, taking the average of squared or cubed ADV velocity measurements



at a given tip-speed ratio set-point for normalization, following prior work in this facility [7, 57, 60]. As variation in inflow and turbulence intensity is relatively small (2-4%) across all the tests, this approximation is reasonable and reduces variability in non-dimensional performance relative to a dimensional representation.

### *3.2.3 Flow-Field Measurements*

The second turbine configuration was a cantilevered assembly (shown in Fig. 3.1b) rigidly mounted to the same motor with one strut-type endplate and acrylic disk support at the bottom of the blades. This was chosen to enable near-blade acquisition of in-rotor two-component planar PIV to be collected at the midspan of the blades, where the flow is assumed to be approximately two-dimensional. This configuration enabled the acquisition of in-rotor phase-locked PIV using a high-speed camera beneath the flume; more details can be found in [61], and are discussed in further detail in Chapter 4.2.

## Chapter 4

### THE SENSITIVITY OF INTRACYCLE VELOCITY CONTROL AND ITS IMPACT ON TURBINE PERFORMANCE AND LOADING

The work presented in this chapter includes material published in [62] and presented at the associated conference, and coauthored by Abigale Snortland, Isabel Scherl, Brian Polagye, and Owen Williams. All data collection was conducted by the author, with initial support from Isabel Scherl during control commissioning and Abigale Snortland during PIV acquisition. An expanded draft has been prepared for publication. Data analysis and figure generation was conducted by the author, who has also led drafting and editing, with contributions from Owen Williams.

#### 4.1 *Introduction*

The cyclic, complex hydrodynamics experienced by cross-flow turbines have made modeling difficult, and an optimal design has yet to see convergence. While cyclic variations in  $\alpha_n$  and relative velocity lead to dynamic stall, loss of lift, and blade wake interactions, it is possible, through modifications in design or control, to harness this complex fluid structure interaction and magnify power production [16, 63]. Intracycle velocity control employs sinusoidal perturbations in rotational speed as a function of position to affect turbine power output and loads. The variation in angular velocity allows control of the angle-of-attack and relative velocity experienced by the blades, similar to actively pitching configurations but without the additional complexity and introduction of failure mechanisms [1, 16, 63]. Initial work with intracycle control has shown the potential to improve efficiency by up to 53% over a turbine operated at a constant rotation rate [1, 17, 19]. These previous studies employed an optimization scheme to find power-optimal control kinematics. However, intracycle control may introduce trade-offs, such as increased drivetrain stress and higher structural loads, which remain unexplored. In this work, we explore the sensitivity of the intracycle control parameter space and its potential to control turbine loading in addition to power output

through an exploration of the broader parameter space.

Traditionally, cross-flow turbines operate under constant speed or torque control, and performance is often characterized as a function of  $\lambda$  [60]. Under the simplest form of intracycle control, the velocity is changed as a function of blade position in a sinusoidal manner. The variable-velocity controller explored in this work takes the form,

$$\lambda_{control} = \lambda_0 + A_\lambda \cos(N(\theta - \phi)) \quad (4.1)$$

where  $\lambda_0$  is the baseline tip-speed ratio,  $A_\lambda$  is the amplitude, and  $\phi$  is the phase shift corresponding to the azimuthal position of the blade where the maximum velocity is seen. The azimuthal blade position,  $\theta$  is defined according to the coordinate system shown in Fig. 2.1b, and  $N$  is the number of turbine blades. We note that the time-averaged  $\lambda_{control}$  decreases with increasing  $A_\lambda$  as less time is spent at phases of high velocity, and so  $\overline{\lambda_{control}} = \lambda_0$  is only true for constant speed control<sup>1</sup>.

Increased efficiency resulting from intracycle control was empirically found to occur when the peak cycle torque is aligned with the maximum rotation rate, leading to increased power production [1]. These results are corroborated by the computational studies of Dave *et al.*[17, 19], which demonstrated that intracycle control flow fields see higher relative velocity and delayed leading-edge vortex formation through the power-producing regions of the cycle when compared to constant speed control, prolonging torque generation [19]. In the previous experimental work, a Nelder-Mead search algorithm was utilized to converge to kinematics with optimal efficiency. Since the full range of potential kinematics has not been thoroughly explored, a study of the amplitude-phase parameter space would yield significant new information concerning the sensitivity of kinematic optima. In addition, enhancements to power output are often associated with increases in loading, accentuating structural challenges. If other optimization objectives beyond power could be used, and kinematics could be uncovered that manage loading while enhancing or maintaining performance, such schemes could significantly impact the cost and maintenance of deployed systems. This information would be invaluable when designing control schemes for operational deployment.

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<sup>1</sup>Averaging relations for intracycle control, where  $\langle \rangle$  represents an average, and the subscript denotes the basis to average with respect too:  $\lambda_0 = \overline{\lambda_{control}}(A_\lambda = 0) = \langle \lambda_{control} \rangle|_\theta \neq \overline{\lambda_{control}}(A_\lambda > 0)$

Some prior works have explored the utility of sinusoidally pitching turbine blades as a function of position through the cycle, showing significant benefits to performance and further shedding light on beneficial flow phenomena [20, 63]. Notably among these studies, recent work by Le Fouest and Mulleners [16] utilized a genetic algorithm to obtain an optimal pitching profile for a single blade. They connected this to the timing of dynamic stall and nominal  $\alpha_n$ , noting that the optimal pitching motion drives a rapid initial increase in  $\alpha_n$  to a value slightly above the static stall angle before plateauing through the majority of the power-producing upstream sweep. This resulting  $\alpha_n$  profile allowed maximum power production by maintaining attached flow, under maximum lift conditions, to generate an increase in torque and therefore power [16]. Despite these advances, a key limitation remains: most prior studies have focused primarily on power maximization, with comparatively little attention given to the coupled impact of structural loading, particularly for intracycle speed control.

This investigation is further motivated by the potential application of this control strategy to a turbine deployment by the Applied Physics Laboratory (APL) at the Pacific Northwest National Laboratory test facility in Sequim Bay, WA [4]. The turbine’s anchoring strategy is a gravity foundation from self-weight with four legs contacting the channel bottom. Figure 4.1 shows a schematic of the system deployed in 2023. Intracycle control results from this work were used to determine whether power could be optimized while mitigating lateral loading and reducing the risk of overturning.

This chapter experimentally explores the sensitivity of turbine performance to sinusoidal intracycle control parameters and uncovers the hydrodynamic sources of these sensitivities. We investigate the impact of intracycle kinematics on power, torque, and loading throughout a large parameter space of intracycle amplitudes and phase-shifts for single and double-bladed turbines. The objective is to investigate the utility of intracycle control to reduce turbine forces while mitigating the impact on power production and tie changes in performance to variations in hydrodynamic features.

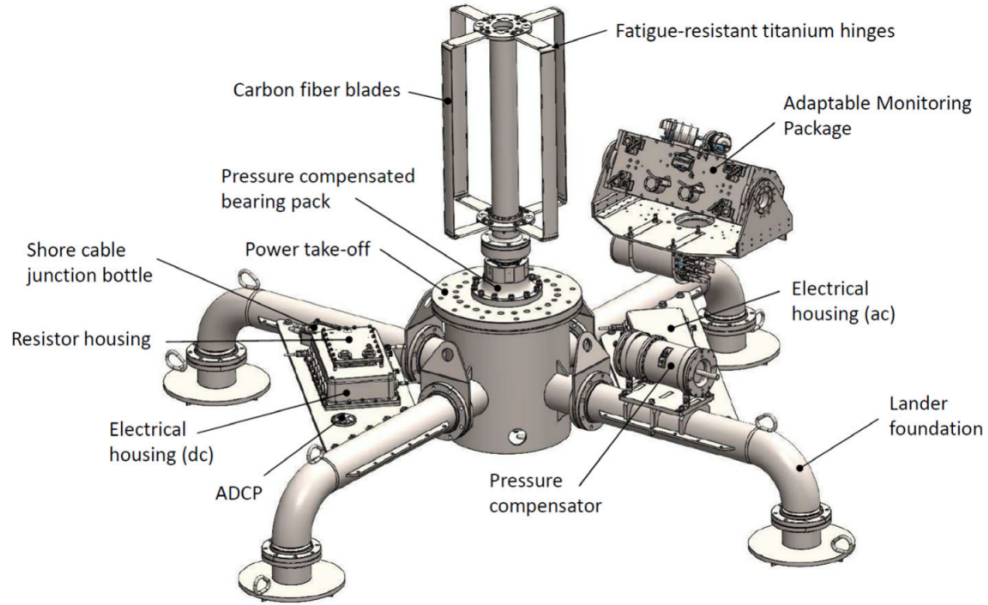


Figure 4.1: Schematic of APL seafloor-mounted lander turbine project from 2023. Reproduced from [4].

## 4.2 Methods

This section summarizes the experimental setup of this work and specific details associated with intracycle tests; additional details are presented in Chapter 3.1.

### 4.2.1 Experimental Conditions

This work was conducted at the University of Washington, Alice C. Tyler Flume, with a constant water temperature of 29 °C, inflow velocity of 0.885 m/s, and dynamic depth of 57.2 cm. The inflow turbulence intensity was between 2 and 3%, measured  $5D_o$  upstream of the turbine position using the ADV. The dynamic depth of the flume,  $H$ , was measured at a 0.5 Hz sample rate by a sonic altimeter, and the turbine was centered in the water column sufficiently downstream of the inlet to experience uniform flow. All instrumentation specifics and sample rates are discussed in detail in Chapter 3.1.

Two turbine configurations were employed: one for performance measurement and another for flow-field particle image velocimetry (PIV) acquisition (Section 3.1). Both tur-

bines could be operated in single- or double-bladed configurations. Symmetrical NACA 0018 blades were used for all tests, and the two configurations matched the standard setup discussed in the common methods discussed in detail in Section 3.1. The blades had a chord,  $c$ , of 4.06 cm and a span,  $h$ , of 23.4 cm, and were mounted at a geometric pitch,  $\alpha_g$ , of  $-6^\circ$ , rotated outwards about the quarter chord. A 1.27 cm diameter central shaft supported both configurations. The outermost turbine diameter,  $D_o$ , was 17.2 cm and quarter-chord radius 8.23 cm, resulting in a chord-to-radius ratio of 0.49 and a chord-based Reynolds number,  $Re_c$  of  $44 \times 10^3$ . The depth based Froude number,  $Fr_h$ , was 0.38 and the turbine blockage ratio,  $\beta = hD_o/HW$ , was 9.3%. The turbine solidity,  $\sigma = Nc/(\pi D)$ , was 0.15 for the two-bladed turbine and  $\sigma$  of 0.08 for the single-bladed turbine.

The performance turbine was a pinned configuration (shown in Fig. 3.1a), employing a suction plate on the bottom of the flume. The blades were fastened to the central shaft by two thin NACA 0008 profile strut-type endplates to reduce drag. The motor with internal encoder rotated the turbine to the desired control kinematics and measured the position and velocity throughout the cycle. All turbine performance information was acquired using Matlab Simulink Desktop in Real-time including forces and moments at 1 kHz. To obtain a better understanding of the intracycle performance space, a sweep of turbine performance was conducted across a wide range of intracycle phases and amplitudes. The position of max velocity,  $\phi$ , defined as the phase shift, was varied between 0 and  $180^\circ$  (limited due to the symmetry of a two-bladed turbine). The normalized amplitude of velocity change,  $A_\lambda/\lambda_0$ , was varied between 0 (constant speed control) and 0.64 (i.e. 64 percent of the phase mean tip-speed ratio). These two variables were discretized in a  $21 \times 22$  grid, resulting in the testing of 462 individual control-kinematic profiles. The single-bladed configuration was tested using key two-bladed kinematics to assess blade-level performance changes under intracycle control. While this approach provides useful insight, it is not a direct one-to-one comparison, as induction differs for a single-bladed turbine. For each kinematic set point, the hydrodynamics were allowed to stabilize before collecting data for 45 s, the time required to converge cycle-to-cycle fluctuations across all configurations.

Torque, forces, and accelerations increase greatly with intracycle control amplitude, leading to potential degradation of the motor controller's ability to track prescribed motion.

Controller efficacy was explored by comparing the R-squared metric,  $R^2 = 1 - \sum_{i=1}^n (y_i - f(x_i))^2 / \sum_{i=1}^n (y_i - \bar{y})^2$ , where  $y_i$  is the commanded rotation rate,  $\bar{y}$  is the mean value, and  $f(x_i)$  is the actual measured value of  $y_i$  calculated using the commanded turbine position and velocity, and those recorded by the motor encoder. The  $R^2$  values were found to be greater than 0.9997 until  $A_\lambda = 0.56\lambda_0$ , after which values gradually dropped to 0.995. This slight error is a result of the servomotor struggling to match the commanded values, leading to a maximum undershooting of the phase shift by  $2.5^\circ$ , an increase in  $\lambda_0$  by a maximum 0.02 and an increase in  $A_\lambda$  of 1.5% relative to the commanded values. As these errors were relatively small, the turbine can be assumed to have followed the specified parameters sufficiently closely across the full range of explored values. However, the results of this analysis informed the maximum amplitudes tested in subsequent experiments.

#### 4.2.2 Removal of Support Losses and Inertial Effects

While the basic approach to account for support losses and inertial loading under constant speed control has been discussed in Section 3.2.2, additional terms must be introduced to account for inertial influences of angular acceleration present in intracycle variable-velocity control. Unlike constant speed control where inertial effects only arise in the forces of imbalanced turbine configurations (e.g., single-bladed), under intracycle control, there is also an impact on the torque of axisymmetric turbines. The updated subtraction equations are shown as follows.

To extract blade-level phase-averaged torque, the method of Strom *et al.*[1] is used to account for inertial influences and combined with the hydrodynamic superposition shown in equation 3.2. The resulting, the hydrodynamic (*blade*) torque, or efficiency, is calculated using,

$$\tau_{blade}(\theta) = \tau_{turb,m}(\theta) + I_{turb}\dot{\omega}(\theta) - (\tau_{sup,m}(\theta) + I_{sup}\dot{\omega}(\theta)) \quad (4.2)$$

where  $I_{turb}$  is the moment of inertia of the full turbine, and  $I_{sup}$  is the moment of inertia of the supports. The moments of inertia are calculated using the same supplementary experiments conducted in air discussed in Section 3.2.2 to reveal  $mr_g$  and  $\delta\theta$  in the identification of fluid forces measured by the load cells. By assigning an arbitrary acceleration scheme and

rotating the turbine in air, the moment of inertia could be estimated, assuming negligible air drag. The moment of inertia for the one- and two-bladed turbines tested here was found to be  $I_{turb} = 7.3 \times 10^{-3} \text{ kgm}^2$  and  $6.4 \times 10^{-3} \text{ kgm}^2$ , respectively. The moment of inertia of the turbine with no blades,  $I_{sup} = 5.5 \times 10^{-3} \text{ kgm}^2$ , was also needed in the

To analyze the blade-level phase average one-bladed loading, an additional term is introduced to equation 3.1 (Section 3.2.2), to account for angular acceleration present during intracycle control. The modified equations for fluid forcing applicable for intracycle subtraction are,

$$F_X(\theta) = F_{X,m}(\theta) + mr_g[\omega(\theta)^2 \sin(\theta - \delta\theta) - \dot{\omega}(\theta) \cos(\theta - \delta\theta)] \text{ and} \quad (4.3)$$

$$F_Y(\theta) = F_{Y,m}(\theta) - mr_g[\omega(\theta)^2 \cos(\theta - \delta\theta) + \dot{\omega}(\theta) \sin(\theta - \delta\theta)]. \quad (4.4)$$

After the inertial effects are accounted for, blade-level coefficients are calculated by non-dimensionalizing the results and removing support losses (equations 3.2 and 3.3, from section 3.2.2). For a support-only and balanced two-bladed turbine,  $mr_g$  is assumed to be negligible, causing the inertial terms to drop out. To calculate blade-level time-averaged loading or performance (designated by an overbar, e.g.  $\bar{X}$ ), it is not necessary to subtract the inertial effects of intracycle control as they integrate to zero across a full cycle; however, removing the time-averaged influence of the support structure is still necessary.

#### 4.2.3 PIV Turbine Configuration

The second turbine configuration was a cantilevered assembly (shown in Fig. 3.1b) rigidly mounted to the same motor with one strut-type blade support at the top, and an acrylic disk support at the bottom of the blades. This configuration allowed the acquisition of in-rotor phase-locked PIV using a high-speed camera beneath the flume; more details can be found in [61]. Controller accuracy for this configuration was checked and found to be comparable to that of the pinned performance configuration, indicating that both performance and flow fields should be consistent between the two systems.



#### 4.2.4 Flow-Field Measurements

A single Phantom V641 high-speed camera with 2560 x 1600 resolution and a 55 mm focal length lens captured images within a plane normal to the blades at their center span, discussed in Section 3.1 and shown in Fig. 3.1b. The flow was illuminated by a 30 mJ per pulse Continuum Terra PIV Nd:YLF laser, producing a light sheet approximately 2 mm thick. PIV was acquired at 30 phases around the cycle, with timing controlled by the *MATLAB Simulink* turbine controller to ensure phase consistency of the acquisition. For some control kinematics, phases were captured with an equal spacing of  $12^\circ$ . In other cases, data were captured with constant time separation, resulting in slight changes in phase separation across the cycle to ease acquisition. The exact phases were recorded in the Simulink model and accounted for in analysis and visualization. At each phase, 85 image pairs were captured. These replicates enabled compilation and analysis of low-noise phase-averaged flow fields. The inter-frame time was chosen to be 350-370  $\mu\text{s}$ , so maximum in-plane displacements were limited to approximately 9 pixels such that vortex cores could be sufficiently resolved. The image calibration was 10.16 pixels/mm. Auxiliary experiments, run under matching conditions and with ambient illumination, allowed the turbine central shaft to be tracked as a function of phase and small precession to be eliminated prior to processing of the PIV images. This approach follows the method developed by Snortland et al. [57].

Two mirrors positioned on the opposite side of the flume were used to reflect the laser sheet into the rotor to reduce the extent of shadowed regions and improve data yield. Fields of view (FoV) of  $15.7 \times 25.1 \text{ cm}^2$  were collected by maintaining the same laser position and changing the relative turbine position to collect data in multiple regions relative to the turbine after measuring to a datum. Two FoVs were collected, one capturing the upstream blade path and the other the downstream. FoVs were overlapped by approximately 3 cm to create a larger, synthesized phase-averaged domain.

PIV data were analyzed using *LaVision DaVis 10.2* software employing a multi-grid, multi-pass approach with iterative image deformation. Butterworth high-pass background subtraction was applied to a 49-frame moving block of images as a preprocessing step to

remove background illumination. Hand-drawn polygon masks were created for each phase to remove non-illuminated regions, such as the central shaft, blade shadows, and any remaining poorly illuminated regions. The interrogation window size of the final pass was  $32 \times 32$  pixels with 75% overlap. A vector post-processing step was applied three times to remove erroneous vectors. It involved a universal outlier detection filter on a  $5 \times 5$  vector domain with a threshold of three standard deviations. The resulting processed data was exported and phase-averaged for further plotting and analysis. The uncertainty of the flow fields is quantified using the correlation method employed in the DaVis (LaVision) processing software, which assesses the pixel-wise contribution to intensity peaks between each PIV image pair. Further details and validation can be found at [64]. An uncertainty field is then output for each PIV frame for a particular field of view (e.g., 30 phases  $\times$  85-cycles at each phase). Averaging across all frames resulted in a mean uncertainty in velocity-magnitude of  $\pm 0.038$  m/s, corresponding to roughly 4% of the free-stream velocity. Most uncertainty occurred in partially shadowed regions or within the blade wakes; when these areas were excluded, the background uncertainty was reduced.

#### 4.2.5 Intracycle Estimates of $\alpha_n$ and $U_{rel,n}$

Nominal calculations of  $\alpha_n$  and  $U_{rel,n}$  were conducted for intracycle control cases by including the fluctuations of  $\lambda$  as a function of  $\theta$  in the equations outlined in Section 2.1. These were used to provide an approximate understanding of near-blade hydrodynamics and to qualitatively connect measured results to theory.

### 4.3 Results

This section describes the sensitivity of control outcomes to the structure (amplitude and phase) of the intracycle waveform and the range of potential trade-offs between increasing performance and reducing loading as visualized through Pareto fronts. The observed turbine-level performance trends are then related to blade-level hydrodynamics using performance and flow-field measurements.

#### 4.3.1 Sensitivity of Intracycle Control Performance to Changes in Kinematics

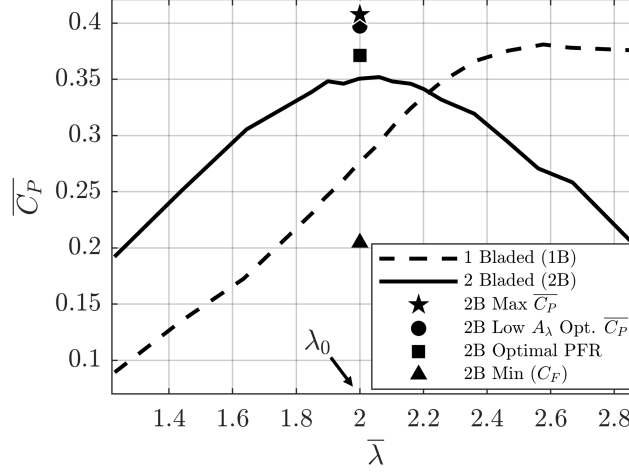


Figure 4.2: Support subtracted turbine efficiency under constant speed control of both one and two-bladed turbines. The black symbols represent performance under intracycle kinematics of interest for a two-bladed turbine plotted as a function of  $\lambda_0$ . These kinematics are selected based on the data shown in Fig. 4.3.

The optimal tip-speed ratio, for a two-bladed turbine under constant speed control is found to be  $\lambda = 2$  as shown in Fig. 4.2. This corresponds closely to that of prior work for a similar turbine at similar conditions, which found peaks at  $\lambda = 1.9$  [1, 19], and approximately 2 [65]. A sweep of single-bladed constant-speed performance is also shown, revealing an optimum performance at a higher rotation rate of  $\lambda = 2.6$ , consistent with the difference in solidity and induction. Performance of the one-bladed turbine is lower at the two-bladed optimal rotation rate. Throughout this study and the subsequent intracycle control parameter sweep, the baseline tip-speed ratio  $\lambda_0 = 2$  is fixed to assess the effects of  $A_\lambda$  and  $\phi$ .

The two-bladed turbine power coefficient (support influences subtracted), as a function of intracycle phase,  $\phi$ , and amplitude,  $A_\lambda/\lambda_0$ , is visualized in Fig. 4.3a. Consistent with previous studies, significant increases in time-averaged performance are possible under intracycle control. In this case, under optimal kinematics, the percent change in  $\overline{C_P}$  increases

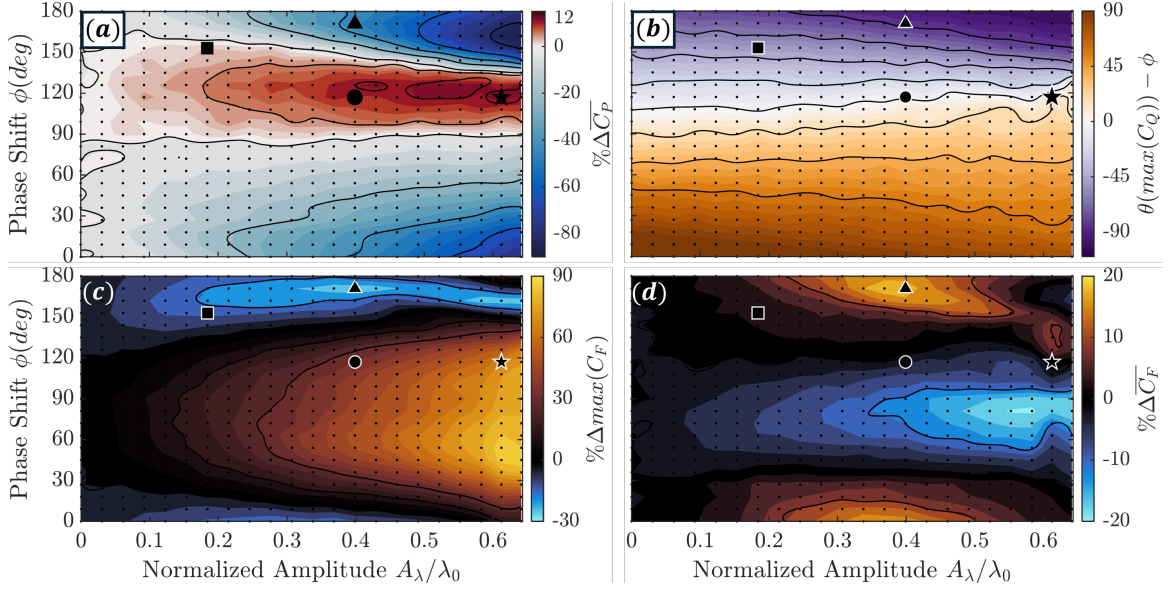


Figure 4.3: Heatmaps of select support-subtracted metrics as a function of intracycle control parameters for a two-bladed turbine. Results are shown relative to constant-speed control ( $\lambda = 2$ ). (a) Percent change in blade-level efficiency,  $(\overline{C_{P, intracycle}} - \overline{C_{P_0}})/\overline{C_{P_0}}$ . (b) Angular difference between the location of the torque peak,  $\text{Max}(C_Q(\theta))$ , and max velocity. (c) Percent change in maximum  $C_F$ . (d) Percent change in  $\overline{C_F}$ . Small black dots correspond to experimental kinematics tested. The black symbols represent kinematics of interest discussed in Table 4.2.

by up to 13.2%. A comparison of optimal kinematics and turbine output is made with previous studies in Table 4.1. Interestingly, the percentage increase in  $\overline{C_P}$  observed in the current study is lower than observed in prior works, both experimentally by Strom et al. [1](53%) and computationally by Dave and Franck [19](40%) using Large Eddy Simulation (LES). Other computational studies using a Reynolds-averaged Navier-Stokes (RANS) method show closer agreement with the current experiments, with reported improvements of 5% [18, 66] and 12.2% [65] over constant-speed control. We note that the measured maximum efficiency in the current study ( $C_P = 0.408$ ) is the highest reported to date, necessitating a close examination of the sources of these discrepancies.

A number of sources for the difference in performance augmentation between this and

earlier studies are identified (see Table 4.1). First, this current work operates at a slightly lower confinement compared to Strom *et al.*[1] and Dave *et al.*[19], which may slightly reduce performance, but is unlikely to be explanatory. The primary differences are more directly attributable to two sources, and are accentuated by the sensitivity of percentage change metrics to the baseline value of  $\overline{C_P}$  under constant speed control. First, considering the experimental work, Strom *et al.* [1] reported performance changes at the turbine level while using disk end-plates. These end-plates have subsequently been found to incur relatively high parasitic losses [58]. In contrast, this work utilized aerodynamic strut end plates and subtracted losses to isolate blade-level performance. Second, the Reynolds number is appreciably lower in Strom *et al.* [1] ( $Re_c = 31k$ ) than the present work ( $Re_c = 44k$ ). Under intracycle control, the turbine blades experience sequential acceleration and deceleration, leading to changes in blade-level Reynolds numbers that would amplify performance during the portions of the cycle with the highest rotation rates. Because maximum rotation rates correspond to the region of maximum torque, the relative increase in Reynolds number (compared to constant speed control) would have been more significant for the conditions reported in Strom *et al.* [1]. Due to both of these differences, the baseline efficiency at constant speed was substantially lower at  $\overline{C_{P,turb}} = 0.203$  than the  $\overline{C_{P,blade}} = 0.35\%$  reported here, inflating the percentage increase with intracycle control. Further analysis and proof of Reynolds number and support structure experimental effects are shown in appendix A.2. While the Reynolds number effect or losses due to end plates would not apply to the computational studies Dave and Franck [67], those LES simulations predicted substantially greater performance gains than both corrected prior RANS [18, 66] and subsequent RANS results by [65]. This discrepancy, under matching geometric and control conditions, suggests the deviation likely arises from the simulation methodology. Overall, the closer correspondence between the performance of this study and that of Hartke *et al.* [65], supports this hypothesis.

Figure 4.3a provides an opportunity to explore turbine power sensitivities in greater detail than earlier studies that sought a single optimal set of kinematics for maximum power output through downhill simplex optimization. The hydrodynamic efficiency is referenced as the percent change relative to the performance at constant  $\lambda = 2$ , highlighting the kinematics

Table 4.1: Experimental conditions and performance comparison between this work and previous intracycle experiments

|                                | This work       | Strom 2017      | Dave 2023       | Hartke 2025     |
|--------------------------------|-----------------|-----------------|-----------------|-----------------|
| Approach                       | Exp.            | Exp.            | 2D-LES          | 2D-RANS         |
| Blade Supports                 | Struts          | Disks           | None            | None            |
| $Re_c = \frac{cU_\infty}{\nu}$ | 44k             | 31k             | 45k             | 45k             |
| Blockage $\beta$               | 9.3%            | 11%             | 10.6%           | 10.6%           |
| Optimal $C_P \phi$             | $117^\circ$     | $112^\circ$     | $110^\circ$     | $117^\circ*$    |
| Optimal $C_P A_\lambda$        | $0.61\lambda_0$ | $0.68\lambda_0$ | $0.68\lambda_0$ | $0.64\lambda_0$ |
| Optimal $C_P \lambda_0$        | 2               | 1.9             | 1.9             | 2               |
| Optimal $C_P$                  | 0.408           | 0.311           | 0.396           | 0.381           |
| $C_P$ Gain                     | 13.2%           | 53%             | 40%             | 13.4%           |

that produce performance benefits (red) and reductions (blue). Within an approximate  $50^\circ$  range of phase offsets ( $90 - 140^\circ$ ), performance is not particularly sensitive to kinematics, and we see a consistent performance improvement with increasing intracycle control amplitude. Not all intracycle kinematics are beneficial, however, and large losses are present for some  $\phi$  values outside this range. The importance of  $\phi$  is apparent from the vertical stratification observed in Fig. 4.3a. While  $\phi$  appears to have a larger role in controlling the sign of relative performance, changes in amplitude appear to primarily amplify the performance (positive or detrimental) at fixed values of  $\phi$ .

Due to our definition of intracycle control velocity as a function of blade position, it should be noted that the time-averaged  $\lambda$  is affected by changes in  $A_\lambda$ . Although the baseline tip-speed ratio is fixed at ( $\lambda_0 = 2$ ) across all cases, the time-averaged tip-speed ratio, ( $\bar{\lambda}$ ), decreases with increasing amplitude due to more time being spent at low angular velocities. For example at  $A_\lambda = 0.64$ ,  $\bar{\lambda} = 1.54$ . This effect, noted by Athair et al. [62], Hunt et al. [68] and further explored by Hartke et al. [65], may contribute to the observed performance decline at the highest ( $A_\lambda$ ). While one could compare intracycle control performance to

constant-speed cases at the same  $\bar{\lambda}$ , this would overestimate its benefit. A more appropriate baseline for comparison is the peak efficiency at  $\bar{\lambda} = 2$ , the global optimum for constant-speed operation, and is the approach utilized throughout this work.

Figure 4.3b illustrates the synchronization of performance with  $\phi$  supporting the hypothesis proposed by Strom et al. [1] that optimal efficiency occurs when the phase of peak torque aligns with the peak velocity. With this dataset, this alignment can be seen to hold across the kinematic parameter space as the angular difference between the torque peak and max angular velocity coincides with  $\phi$  values of peak performance in (a). Where the difference is near zero (alignment, represented by white in (b)) optimal efficiency is observed. Further, when the torque peak leads or lags  $\phi$  by  $\sim 90^\circ$  (i.e., torque and velocity are out of phase), the poorest performance is observed.

Increases in power output are often associated with increases in average turbine loading; however, the impact of intracycle control kinematics on turbine loading has received only limited discussion in previous work. We examine turbine loading in Fig. 4.3(c-d), through the percent change in maximum in-plane phase- and time-averaged mean forces, respectively. Interestingly, these two parameters are shown to be largely in opposition, where the maximum force increases at high  $A_\lambda$  and  $\phi \approx 70^\circ$  (c), while the mean force experiences a minimum for the same kinematics (d). Efficiency contours (a) are not seen to perfectly align with loading metrics in (c) or (d), showing regions where increased efficiency does not correspond to high loading. This visual misalignment highlights the inconsistent correlation between these metrics and suggests the exciting potential of multi-objective optimization tuned to a specific application. As a result, it should be possible to augment power without greatly impacting max force by finely tuning  $\phi$  and  $A_\lambda$ . We explore these tradeoffs throughout the remainder of this paper.

To better understand the turbine dynamics responsible for the variation in performance and loading behaviors observed in Fig. 4.3, four reference kinematics were selected (Table 4.2). These include: (1) the maximum efficiency (star), (2) lower amplitude enhanced efficiency with matching  $\phi$  (circle), (3) the highest ratio of efficiency enhancement to maximum force reduction (referred to as Power to Force ratio or PFR) (square) and (4) the reduced efficiency kinematic which experiences minimum max-force (triangle). The optimal efficiency

case showed a  $C_P$  value of 0.408 corresponding to a 13.17% enhancement over the constant-speed case, and also an almost 70% increase in maximum force. The lower-amplitude case showed almost the same efficiency improvement ( $\sim 10\%$ ) while only increasing the maximum force by 37%. The optimal PFR case produced a moderate, 3%, improvement in efficiency while reducing the maximum force by 12%. The minimum max-force case reduces peak force by 23.7% but at a cost of an efficiency reduction of 43.2%.

These four cases are chosen to highlight control kinematics that exhibit key trade-offs between efficiency gains and loading changes, ranging from high-performance/high-loading conditions to more balanced or unfavorable behavior. In Section 4.3.4, these kinematics will be revisited to explore local flow fields, once their performance is put into the broader context of all the potential kinematic choices.

Table 4.2: Key intracycle kinematics and associated blade-level performance metrics for a two-bladed turbine

| Control scheme                            | $\phi$      | $A_\lambda$      | $\overline{C_P}$ | $\overline{C_P}$ Gain | $\text{Max}(C_F)$ | $\text{Max}(C_F)$ Gain | $\overline{C_F}$ | $\overline{C_F}$ Gain |
|-------------------------------------------|-------------|------------------|------------------|-----------------------|-------------------|------------------------|------------------|-----------------------|
| Constant $\lambda$                        | —           | 0                | 0.360            | —                     | 2.23              | —                      | 1.25             | —                     |
| $\text{Max}(\overline{C_P})$ (★)          | $117^\circ$ | $0.612\lambda_0$ | 0.408            | 13.17%                | 3.79              | 69.68%                 | 1.28             | 2.28%                 |
| Low $A_\lambda$ Opt. $\overline{C_P}$ (●) | $117^\circ$ | $0.399\lambda_0$ | 0.398            | 10.32%                | 3.06              | 37.07%                 | 1.23             | -1.49%                |
| Optimal PFR (■)                           | $153^\circ$ | $0.184\lambda_0$ | 0.371            | 3.02%                 | 1.96              | -11.95%                | 1.29             | 3.64%                 |
| min $\text{Max}(C_F(\theta))$ (▲)         | $171^\circ$ | $0.399\lambda_0$ | 0.205            | -43.18%               | 1.70              | -23.68%                | 1.49             | 19.39%                |

#### 4.3.2 Tradeoffs Between Performance and Loading

To better understand the trade-offs between loading and power output in the intracycle performance space, we employ a two-dimensional (power, maximum, or mean loading) Pareto front analysis[69] of all 462 two-bladed kinematics tested. An intracycle kinematic trajectory is considered to be part of the Pareto front if no other kinematics showed better performance over both objectives: a more reduced loading (either maximum or mean loading depending on the case), and/or more improved efficiency, similar to Le Fouest and Mulleners [16].



Bounding conditions are also set so that kinematics that reduce efficiency by more than 40% or increase loading by more than 50% compared to the constant-speed case are excluded from the Pareto front. The resulting points highlight the combined potential of intracycle control to trade loading and efficiency, should multi-objective optimization be desired. To examine loading trade-offs, we evaluated both maximum force, which typically determines structural design, and mean force to better understand differences in the underlying mechanisms; this resulted in two distinct Pareto fronts, each defined with respect to turbine efficiency. Our approach follows similar analysis by Le Fouest and Mulleners Le Fouest and Mulleners [16] who examined optimized pitching trajectories for a single-bladed turbine and evaluated performance compared to the mean standard deviation of the pitching moment.

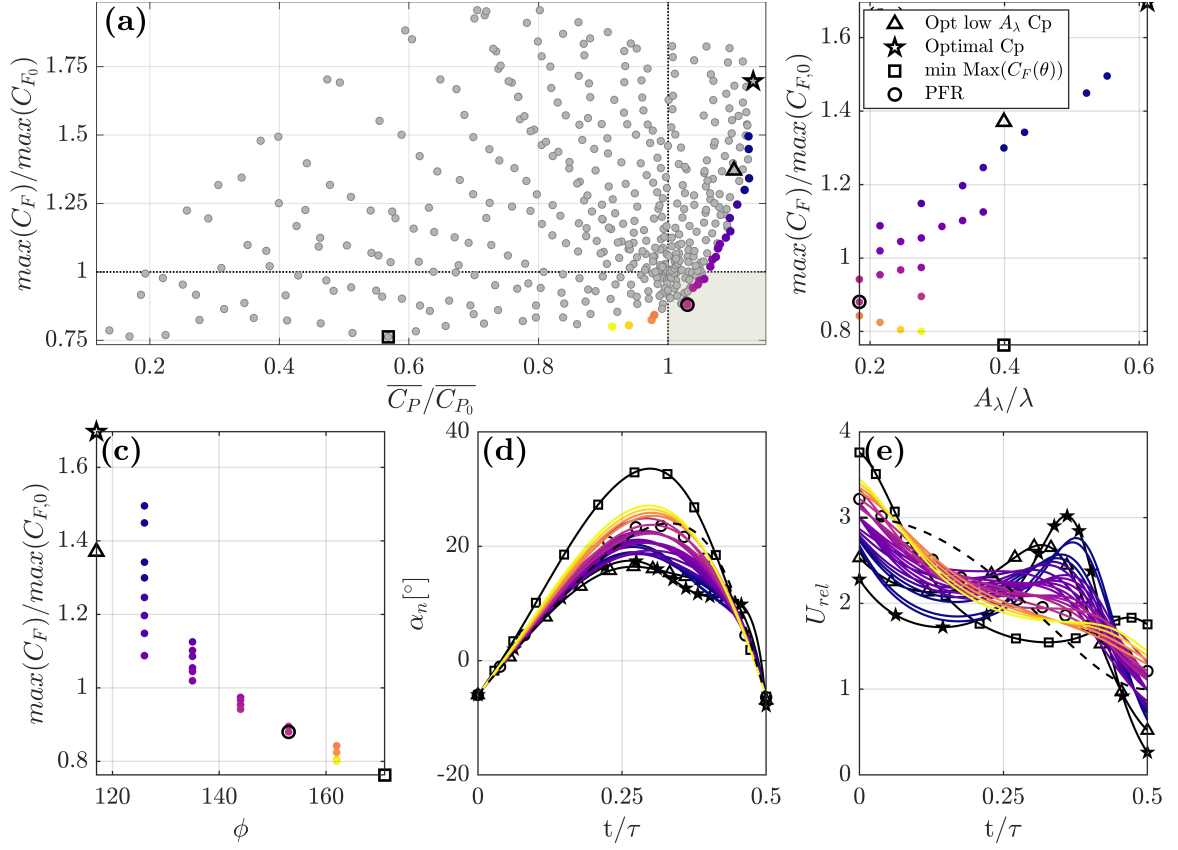


Figure 4.4: (a) Exploration of the tradeoffs between maximum  $C_F(\theta)$  and  $\overline{C_P}$ , relative to constant speed control kinematics (denoted by  ${}_0$ ). Pareto front cases are colored according to their respective efficiency. All other tested kinematics in grey. Black symbols are the highlighted kinematics of Table 4.2. Other sub-figures highlight the kinematics and blade-relative flow conditions of Pareto-optimal conditions. (b) Maximum force as a function of intracycle amplitude  $A_\lambda$ , and (c) the position of maximum velocity,  $\phi$ . (d) Nominal angle-of-attack, and (e) relative velocity variation through the upstream as a function of normalized time. For (d,e), the dashed line represents the  $\lambda = 2$ , constant rotation rate case.

Figure 4.4a shows the mean efficiency, and maximum force for all intracycle kinematics normalized by their respective constant speed performance (denoted by,  ${}_0$ ), Pareto front points are indicated by coloring according to their  $\overline{C_P}$ . Note that all of the highlighted kinematics of Table 4.2 closely correspond to this front (designated black symbols). The

Pareto front spans up to a 13.2% increase in efficiency, corresponding to a loading increase of 50%, and lower performing cases with a 10% reduction in efficiency and a 20% reduction in maximum force. The yellow points show some cases where a small sacrifice to efficiency can yield a beneficial further reduction in loading. A few cases lie in the gray quadrant of greatest interest, where both efficiency and maximum force are reduced. While benefits are available, only a limited number of phase and amplitude combinations are effective, and if kinematics are not carefully selected, detrimental increases in maximum loading and/or efficiency reductions are likely. In deployments with nonuniform or turbulent flow, where variability in the flow direction would affect the phase, reliably targeting these limited control combinations will be particularly difficult.

To take a deeper look at the trade-offs between the maximum  $C_F$  and  $\overline{C_P}$ , the associated kinematic parameters are explored in (b) and (c). Maximum force is shown to generally increase with increasing  $A_\lambda$ , however, there is still a  $\phi$  dependency. The exception occurs for the lowest performing cases on the Pareto front, where increasing amplitudes result in lower maximum forces. For the Pareto-optimal points,  $\overline{C_P}$  represented by the point colors, shows a similar trend with increasing  $A_\lambda$ . In (c), it is observed that loading increases as  $\phi$  decreases, with the largest increases below  $135^\circ$ .

To identify the potential mechanisms that underpin these trends, Fig. 4.4(d-e) examines the variation in nominal angle-of-attack and relative velocity as a function of the Pareto optimal kinematics. These metrics are plotted as a function of time, normalized by the period ( $\tau$ ) of rotation, as time spent in a given state is more clearly linked to  $\overline{C_P}$ , rather than blade position,  $\theta$ . Because these nominal calculations neglect induction, any apparent downstream trends are not physically meaningful and were excluded. Variations in the upstream portion of the rotation ( $t/T < 0.5$ ) are less influenced by induction, and have the largest impact on power production, making them the most relevant for this analysis. With this in mind, reducing the upstream nominal angle-of-attack peak ( $t/T \approx 0.3$ ) leads to an increase in  $\overline{C_P}$  and maximum  $C_F$ . Maximum-power cases (more purple) exhibit a more constant angle-of-attack and relative-velocity through the upstream sweep. Maintaining  $\alpha_n$  close to an optimal value for longer, while  $U_{rel,n}$  is also increasing, extends the period of high lift power generation, similar to results observed under optimized blade pitch control

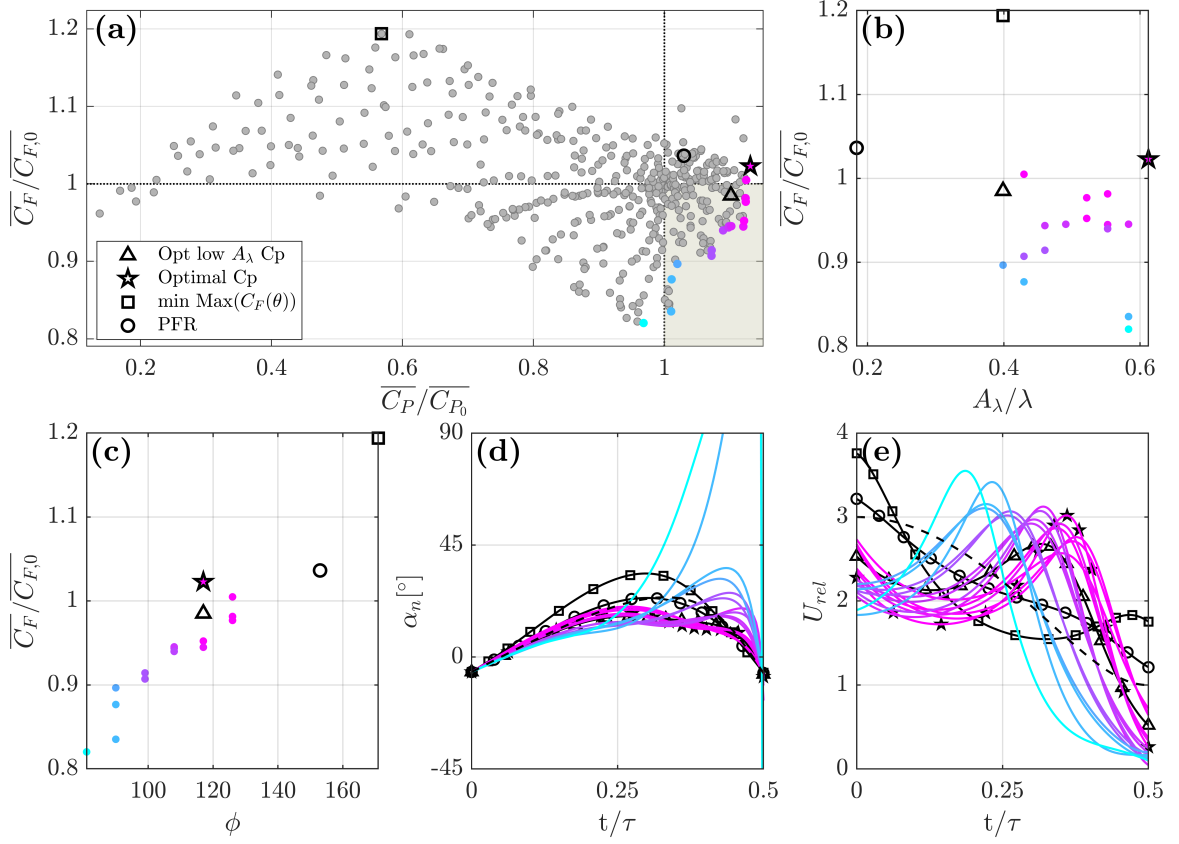


Figure 4.5: (a) Exploration of the tradeoffs between  $\overline{C_F}$  and  $\overline{C_P}$ , relative to constant speed control kinematics (denoted by  $o$ ). Pareto front cases are colored according to their respective efficiency. All other sub-figures and labels as in Fig. 4.4.

Le Fouest and Mulleners [16]. A smoother and lower magnitude  $U_{rel,n}$  curve also appears necessary to decrease maximum loading. The Pareto optimal cases with the lowest loading occur when  $\alpha_n$  peaks earlier and at a greater magnitude. It is hypothesized that this may trigger earlier stall such that overall forces are reduced.

The Pareto front generated when comparing the normalized time-averaged force  $\overline{C_F}$  to  $\overline{C_P}$  (shown in Fig. 4.5a), exhibits different trends from the maximum force cases. The range of mean forces is smaller (only  $\pm 20\%$ ); however, a greater proportion of the front lies in the quadrant where the mean force can be reduced while enhancing efficiency. In addition, we note that only the optimal and low-amplitude optimal cases highlighted in Table 4.2 lie

close to this new Pareto front. Once again, across the entire range of kinematics tested, the number of effective phase and amplitude combinations is relatively few to yield a benefit. In (b), it is noted that the intracycle amplitude does not significantly influence Pareto-optimal conditions; however, generally higher amplitudes than seen under maximum force cases in Fig. 4.4b, are necessary here. Considering both Pareto fronts, we observe that the maximum force-reducing Pareto cases span a broader range of normalized amplitudes,  $A_\lambda/\lambda_0 \approx 0.2\text{--}0.55$ , with higher  $\phi = 125\text{--}162^\circ$ . In contrast, the mean force-reducing Pareto cases are concentrated at higher amplitudes  $A_\lambda/\lambda_0 = 0.4\text{--}0.6$  and lower  $\phi = 80\text{--}125^\circ$ .

Similar to the maximum force Pareto front, Fig. 4.5c shows  $\phi$  correlates closely with turbine efficiency and mean force for the mean force Pareto front. However, in this case, both  $\overline{C_F}$  and  $\overline{C_P}$  increase with increases in  $\phi$  from  $80^\circ$  to  $125^\circ$ . As previously, optimal efficiency once again occurs when the upstream  $\alpha_n$  is held near constant for a wide portion of the upstream half of the rotation, allowing for extended maximal lift generation. As the maximum  $C_P$  cases are the same for both Pareto fronts, once again it is observed that higher  $C_P$  cases (pink) exhibit a spike in  $U_{rel,n}$  (e) near  $t/T = 0.35$ . Reductions in mean loads are attributed to an earlier  $U_{rel,n}$  spike, a later, more extreme spike in angle-of-attack, in contrast to what is observed for the maximal force Pareto kinematics.

Figure 4.6 illustrates phase-averaged turbine performance for Pareto-optimal cases, with colors matching the original Pareto front kinematics. The cases that reduce the maximum forces (Fig. 4.6a–b), show that this reduction is produced by broadening and shifting the force and power peaks earlier in the cycle. A closer examination of the  $\phi$  (Fig. 4.4c) suggests that this is achieved through a slight misalignment of velocity and torque peaks, incentivizing earlier power generation and promoting early stall, thus limiting peak loading and then extending the peak width to compensate over time. Conversely, the cases that reduce the mean loading (Fig. 4.6c–d) produce narrower, more distinct power peaks due to higher amplitude motion. The close alignment of high velocity and torque results in sharp, impulsive power and loading spikes. This alignment yields high instantaneous power, and lowers the mean force by compressing the area under the curve in (d), but it also produces high peak forces as a detrimental side effect. And although these kinematics can effectively reduce mean forces, this approach may negatively impact generator efficiency and structural

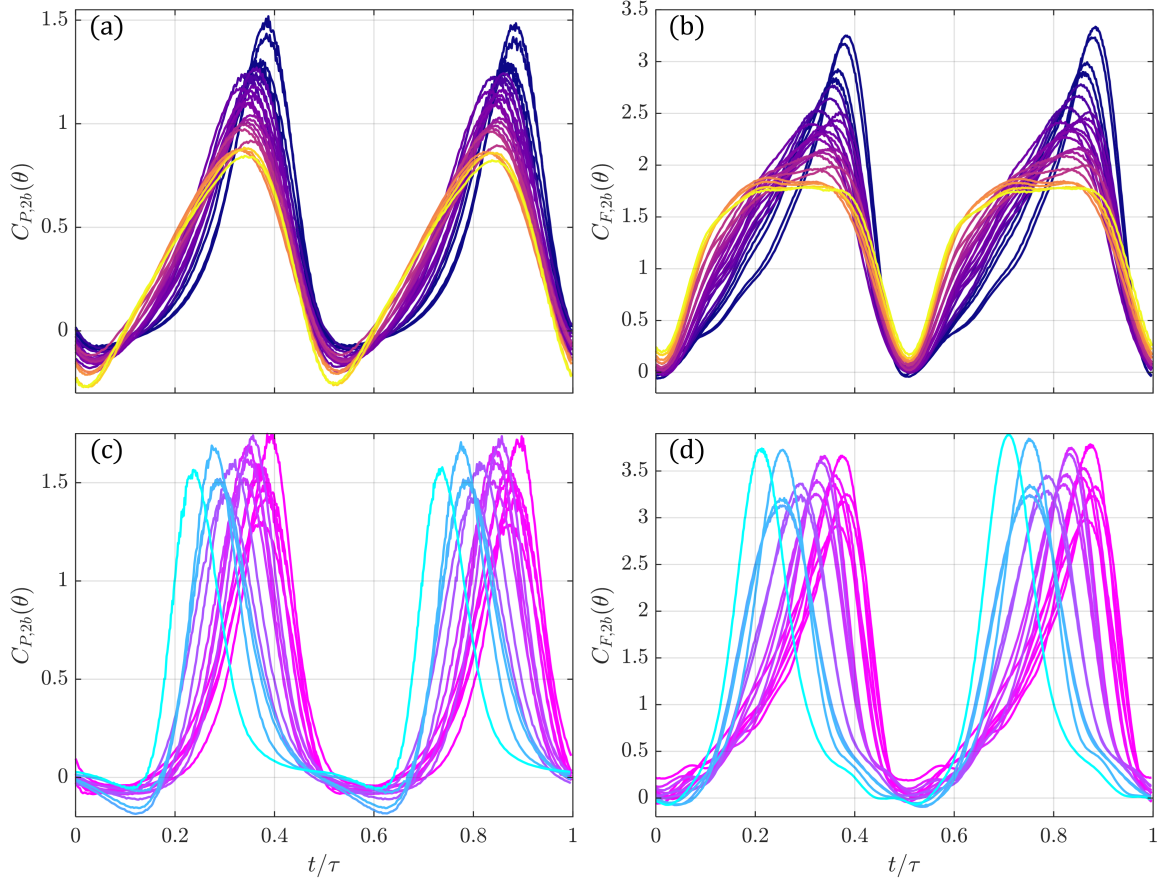


Figure 4.6: Two-bladed Pareto front phase averages, as a function of normalized time. (a)  $C_P(\theta)$ , (b)  $C_{Force}(\theta)$  selected from maximum force front, and (c)  $C_P(\theta)$ , and (d)  $C_{Force}(\theta)$  selected from mean force front. All results have been supported and inertial subtracted to isolate the hydrodynamic behavior only.

performance due to the impulsive power and loading incident on the turbines.

Though these two-bladed phase-average figures provide broad insight into the underlying changes in performance across the cycle of Pareto-optimal points, the nuanced changes in blade-level performance are obscured. As our experimental setup utilizes load cells mounted at the central shaft, phase-averaged measurements of a two-bladed turbine inherently convolve the contributions of both the upstream and downstream blades. As a result, blade-specific features are obscured, and interpretation is complicated. By operating the turbine in a single-bladed configuration and matching kinematics to the two-bladed case, a qualitatively representative blade-level performance analysis is possible despite the difference in solidity. As a result, one-bladed data helps isolate hydrodynamic differences and identify blade-resolved loading.

#### *4.3.3 Explaining Performance Trends Using Single-Bladed Data*

Overall, high-efficiency cases rely on strong, short-duration loading and power generation, aided by tight torque-velocity alignment. In contrast, low-force cases benefit from broader power and load profiles driven by early stall and a deliberate torque-velocity misalignment to broaden the loading and power production. To further understand the blade-level mechanisms underlying intracycle control enhancement, a selected range of two-bladed kinematics was reproduced with a single-bladed turbine, enabling direct analysis of blade-level hydrodynamics. It is acknowledged that single and double-bladed turbines are not equivalent due to differences in solidity, blade-wake interactions, and turbine induction. These influences result in a higher optimum  $\lambda$  for a single turbine under constant speed control (see Fig. 4.2). However, we argue that the relative trends in efficiency and maximum loading should remain consistent between the single and double-bladed turbines under intracycle control at the same mean  $\lambda$ .

To validate the use of a one-bladed turbine performance operating under two-bladed kinematics to interpret blade resolved intracycle control effects, one-bladed heatmaps and Pareto fronts were compared with two-bladed results (Appendix A.3). One-bladed efficiency and maximum-loading closely matching two-bladed turbine performance, with the two-

bladed Pareto front cases lying along the same boundaries of the one-bladed Pareto front. However, the single-bladed mean  $C_F$  heatmaps differ from the two-bladed configuration (Fig. 4.3d). As a result, one-bladed maximum force trends are considered representative of the two-bladed turbine, and are used for the remainder of this work to examine blade-level behavior influencing turbine maximum loads and efficiency.

Figure 4.7 examines the phase-averaged performance of a one-bladed turbine under matching kinematics to the maximum force Pareto-optimal cases identified in Fig. 4.4. Broadly, the tradeoffs between maximum loading and power output are seen to be consistent with two-bladed results from Fig. 4.6(a-b), suggesting that the downstream blade does not dominate the dynamics of the two-bladed turbine at these conditions. Panel (a and c) once again shows that higher-performing cases (in purple) have a narrower, later peak in force and efficiency, while lower peak force cases show a broader, earlier peak. As expected, the highest-performing cases arise when velocity and torque peaks are most closely aligned. In contrast, cases that optimized for lower maximum force (in yellow) show an earlier and smaller torque peak, which leads the maximum velocity peak by up to  $\approx 35^\circ$ , likely due to earlier flow separation and loss of lift. However, because of the slightly misaligned velocity peak, the lower power generation post-stall is still amplified, causing the peak in power to be wider than the peak in torque for these cases. Loading exhibits quite similar behavior with low peak force cases seeing more significant lateral and thrust forces earlier in the cycle, indicating enhanced early lift and thus power generation, before flattening out earlier, resulting in more uniform upstream loading. Downstream, power and loading variations are minimal across all kinematics, suggesting that there is little influence of the downstream blade on kinematic choices. Overall, high-efficiency cases rely on strong, short-duration loading and power generation, aided by tight torque-velocity alignment, while low-peak-force kinematics benefit from broader power and load profiles driven by early stall and a deliberate torque-velocity misalignment to extend power generation.



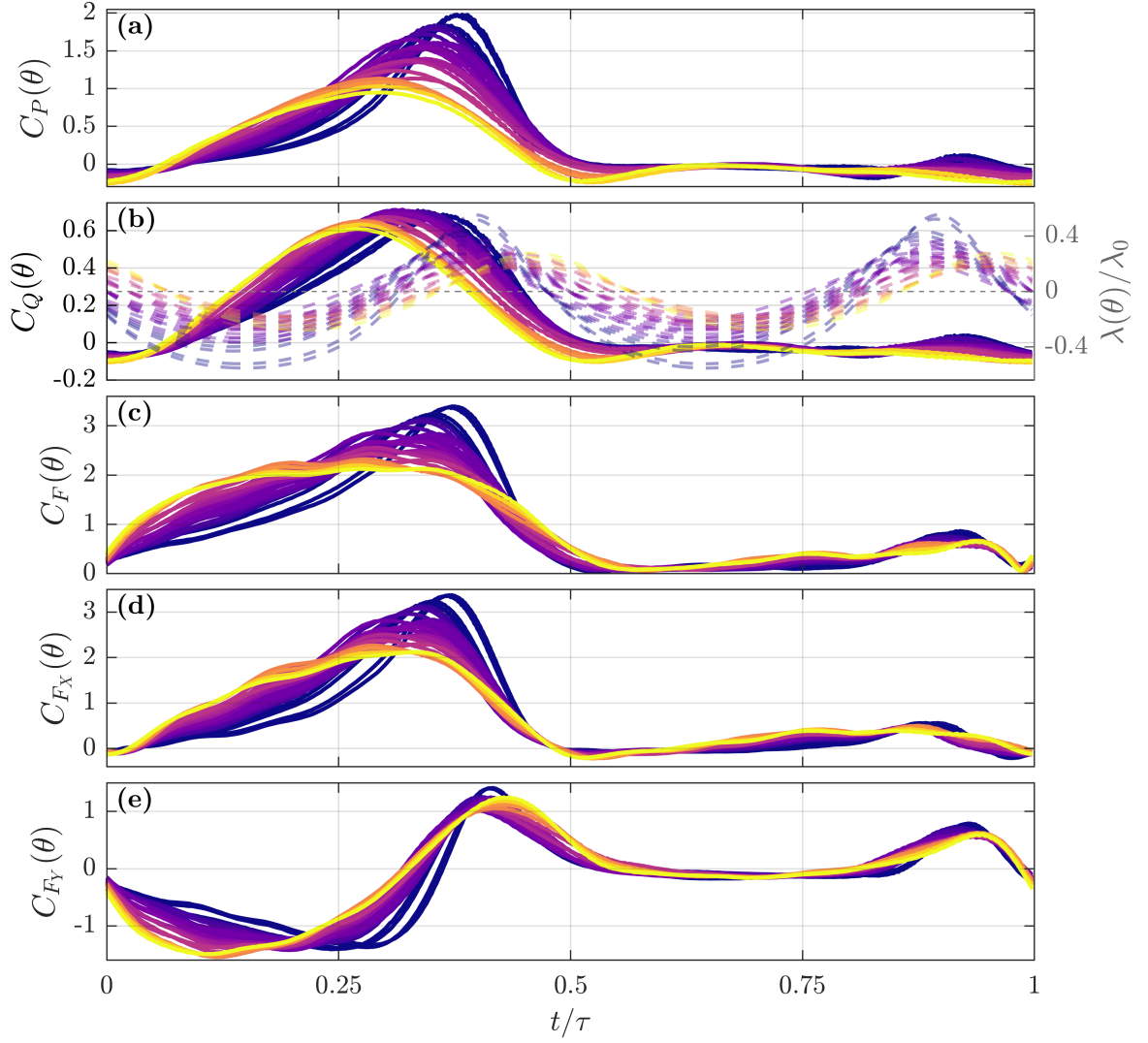


Figure 4.7: One-bladed phase-averaged turbine performance metrics for kinematic cases selected from the two-bladed maximum force Pareto front, as a function of normalized time. In (b)  $C_Q$  (solid) and  $\lambda(\theta)/\lambda_0$  (dashed).

Broadly, to reduce peak force, higher control velocities (b) are observed at the beginning of the cycle, enhancing earlier flow recovery and the transition from downstream dynamics to power production, as observed in earlier power production. This is followed by an early angle-of-attack peak, followed by a gentle drop resulting in earlier stall, and prompts an earlier and lower peak  $C_P$  while cutting off the continued increase in force, effectively ampu-

tating the curve. The turbine control velocity at this point accelerates, lagging behind the diminishing torque production, and causes the power generation and loading to extend later in the cycle. In contrast, the case sees a maximum  $C_P$  and  $C_F$  peak, considering the alignment of torque and velocity, resulting in the compounded production of power and paired force, like the behavior observed by the prior work of Strom et al. [1], Dave and Franck [19] who found optimal performance to occur when velocity and torque were aligned.

#### 4.3.4 *Near-blade Hydrodynamics*

To corroborate above hypotheses derived from phase-averaged performance related to the timing and magnitude of dynamic stall, PIV flow-fields were acquired for the one-bladed turbine under three intracycle conditions and one constant speed case. These were the conditions of Table 4.2 but without the case corresponding to the true maximum  $\overline{C_P}$  of the two-bladed turbine. This case was excluded due to the loading limitations of the cantilevered PIV rotor assembly. The time average performance of the one-bladed turbine under these chosen kinematics is highlighted in Figs. A.3 and A.4. As discussed in Section 4.3.3, the hydrodynamics in the power-producing portion of the cycle should be similar between the one and two-bladed turbines.

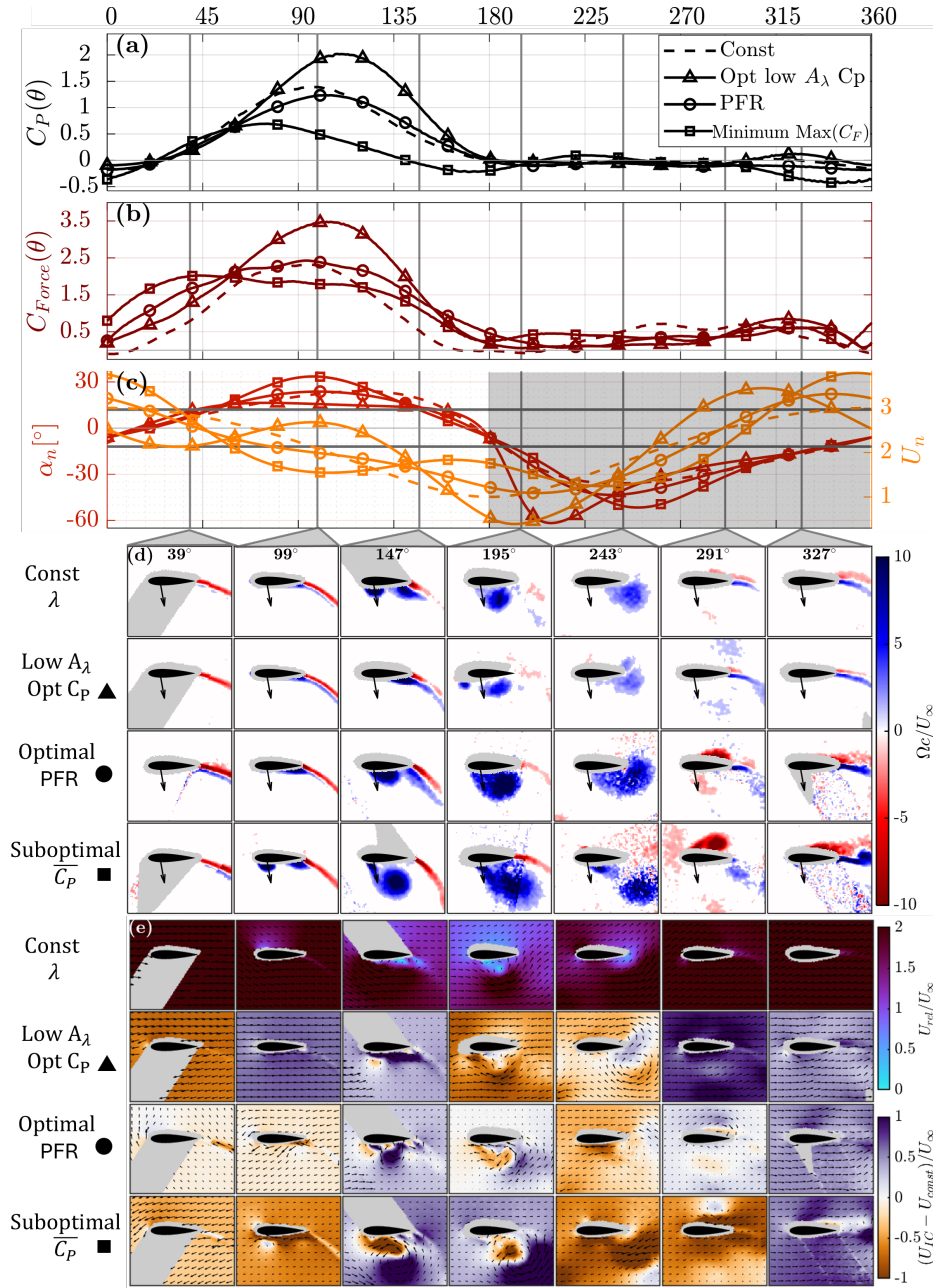


Figure 4.8: Comparison of one-bladed phase-averaged performance and loading with changes in near-blade hydrodynamics. Support subtracted (a)  $C_P(\theta)$ , (b)  $C_F(\theta)$ , (c)  $\alpha_n, U_{rel,n}$ , (d) normalized vorticity fields in the blade reference frame, with black arrows indicating radial direction to the axis of the turbine. Red and blue indicate clockwise and counterclockwise vortex rotation, respectively. (e) Relative velocity fields normalized by the freestream in proximity of the blade, intracycle cases are shown relative to the constant-speed case. Masked regions (shadows or occlusions) are shown in gray.

Near-blade vorticity flow-fields rotated into a blade relative frame are shown in Fig. 4.8d, highlighting key phases of the cycle in the power-producing portion of the rotation and downstream flow recovery. Red and blue vorticity represent clockwise and counterclockwise rotations, respectively. Under constant speed control,  $C_P$  and  $C_F$  see a maximum that peaks early in the cycle (around  $100^\circ$ ), see Fig. 4.8a. After the performance peak, a leading edge vortex forms along the inside of the blade and is shed well before the blade enters the downstream sweep, resulting in the loss of power production within  $100\text{-}180^\circ$ . The shed vortex (blue) then advects with the blade as it moves downstream, as can be seen at phases  $195^\circ$  and  $243^\circ$ , before the switch of the suction side leads to gradual flow recovery through the rest of the downstream sweep. In contrast, in the upstream, kinematics for optimal intracycle power generation hold the vorticity closer to the blade for longer, with the vortex shedding much closer to the upstream/downstream transition. This would appear to be the benefit of higher relative velocity and more constant  $\alpha_n$  after exceeding the static stall angle. The result appears to be prolonged lift and power generation. The shed vortex is also smaller, potentially leading to faster flow recovery.

The minimized max-force kinematics produce almost opposite effects, with a much larger and earlier formation of the dynamic stall vortex, due to higher angle-of-attack and reduced relative velocity. This vortex fully separates from the blade, causing an early loss of lift, and is sufficiently far from the blade, so no benefit is gained. Interestingly, a secondary dynamic stall vortex appears to shed from the blade around  $270^\circ$ , consistent with observed lift generation in phases just before this (Fig. 4.7). Later in the cycle,  $291\text{ - }327^\circ$ , discrete clockwise and counterclockwise vortices are seen along the blade, which reduces lift and increases drag, resulting in the notable negative performance shown in Fig. 4.8a at the end of the rotation that was not observed in the other cases.

Optimal PFR kinematics produce only subtly different flow-fields from the constant rotation rate baseline case, which is consistent with the modest relative increases in power and reductions in force. The stall vortex is slightly larger and appears to develop and shed somewhat earlier and more diffusely than in the constant-speed and optimal-efficiency cases. The blade motion also closely tracks the advecting stall vortex, maintaining its proximity closer than the minimized max-force case between  $147\text{-}243^\circ$ . A more significant red vortex

is seen downstream near  $291^\circ$ , indicating a relative delay in flow recovery. These changes in hydrodynamic behavior are connected to their effect on power in Fig. 4.8a, where the PFR case shows a decrease in peak efficiency, but the width of the power-producing region is slightly larger. This is further magnified when considered as a function of time, such as Fig. 4.7. Comparing optimal PRF to constant-speed kinematics, the relative velocity near  $\theta = 99^\circ$  is lower (Fig. 4.8e), reflecting the lack of alignment between the torque peak and the rotation rate. After the upstream vortex begins to shed, the increased tangential velocity causes the relative velocity to accelerate at  $\theta = 147^\circ$ . Although earlier stall reduces lift at this point, the higher rotation rate still extends power production upstream. Additionally, at the end of the cycle, the blade accelerates again, promoting flow recovery and restoring power and load production to levels observed at the beginning of the cycle, despite the lower tangential velocities at  $\theta = 39^\circ$ .

#### 4.4 Conclusions

The sensitivity of sinusoidal intracycle control of cross-flow turbines to changes in amplitude and phase was experimentally investigated to broaden understanding of the optimization opportunities for power/efficiency and loading. Interactions between performance and changes to in-rotor hydrodynamics were studied for both single and double-bladed turbines at a tip-speed ratio of two, corresponding to the optimum time-averaged performance of the two-bladed design under constant speed control. One-bladed data were acquired to explore variations in the phase-averaged performance and demonstrated to be informative of the two-bladed turbine despite differences in solidity, induction, and optimum mean  $\lambda$ .

The two-bladed performance could be increased by up to 13.2% compared to the same turbine under constant speed control at the same baseline tip-speed ratio. Performance increases could be consistently obtained within a narrow band of phase offsets ( $\phi = 90 - 140^\circ$ ) and were seen to continuously increase with intracycle amplitude,  $A_\lambda$ , if  $A_\lambda < 0.61\lambda_0$  for the two-bladed turbine. At higher amplitudes,  $\bar{\lambda}$  decreased significantly, and the controller's ability to track the specified intracycle control profile degraded, potentially explaining the diminishing returns observed at even larger amplitudes.

Optimum control inputs for the intracycle turbine were  $\phi = 117^\circ$  and  $A_\lambda = 0.61\lambda_0$ ,

corresponding closely to the kinematics found by Strom *et al.*[1], Dave *et al.*[19], and Hartke *et al.*[65] for a similar turbine and comparable Reynolds numbers [1, 19, 65]. The maximum-performing kinematics corresponded closely to cases in which the velocity and torque peaks aligned, confirming prior observations and extending them to the full parameter space. While the maximum  $C_P = 0.408$  found in this work was found to be higher, a much lower percent improvement in power production relative to constant speed control was observed than in [1, 19]. This is primarily due to lower blade-relative Reynolds number in the prior experimental work, leading to a perceived amplification of power output when intracycle control operates in a region of high Reynolds-number sensitivity (Appendix A.2). The amplified Reynolds number variation throughout the cycle under intracycle control is also expected to affect simulation accuracy. The power augmentation percentages presented in this study more closely match those of Hartke *et al.* [65], and are thought to be more in line with what might be seen in deployed systems due to the higher Reynolds numbers of the test and subtraction of support parasitic drag from the resulting power estimate.

Explorations of in-rotor flow fields and nominal blade-relative angles of attack and velocities ( $\alpha_n$  and  $U_{rel,n}$ , respectively) highlight sources of power augmentation through intracycle control. Analysis of the  $\alpha_n$  profiles indicates that maintaining  $\alpha_n$  just above the expected static stall angle in the upstream region is seen to delay stall while enhancing the power-producing region overall, in a manner similar to that observed by Le Fouest and Mulleners [16] with intracycle blade pitch control. At the same time, for optimal power output under intracycle control, blade-relative velocities remain higher for longer in this part of the rotation, further enhancing power output. Both of these trends should increase lift and prolong power extraction. These findings are supported by in-rotor flow field observations, indicating a delayed and smaller stall vortex in cases with optimal power coefficient. Optimal efficiency kinematics also appear to help manage the post-stall flow recovery with an increased relative velocity along the blade between  $270^\circ$  -  $360^\circ$ , as shown in the flow-fields. Surprisingly, this trend is also captured by  $U_{rel,n}$ , despite its higher uncertainty downstream due to the neglect of induction effects. In contrast, minimum max-force kinematics appear to promote increased angles-of-attack early in the cycle, lower blade-relative velocities at the phases of peak lift, and the formation of a strong dynamic stall vortex that sheds early

in the cycle. When the intracycle phase shift was chosen to improve force reduction over efficiency augmentation, secondary stall events are also prominent in the downstream blade sweep near  $290^\circ$ .

Two Pareto front analyses reveal interesting tradeoffs between turbine power output and loading. These extreme points highlight potential strategies for balancing power generation and force reduction, offering insights for future implementation and control considerations. The Pareto fronts for exploring a combination of (1) power output and maximum turbine loading and (2) power output and mean loading reveal that only two of the three parameters (power, average loading, and peak loading) can be optimized simultaneously. While a couple of points exist that allow maximum and mean force to both be reduced, this is only achieved when nearly all power is shed, and mean and maximum blade forces are found to approximately be inversely proportional. This is intuitively plausible: to generate power from the flow, an opposing thrust is required to convert kinetic energy into useful torque. Both Pareto fronts highlight clear opportunities for power augmentation while reducing loading, with careful selection of intracycle phase and amplitude. Mean loading can be minimized through careful selection of the intracycle phase, such that the power output is primarily controlled by the amplitude. In contrast, if a management of peak forces and power output is the goal, the peak intracycle amplitude must be moderated, and the range of useful intracycle phase offsets narrows as a slight offset appears to be desired between the phases of peak torque and rotation rate. It is revealed that the ratio of power and peak loading can be improved by up to 17.3% through a 12% reduction of the peak loading while still slightly increasing power output in a two-bladed turbine ( $\phi = 153^\circ$  and  $A_\lambda = 0.18\lambda_0$ , denoted as the PFR case above). This performance is achieved by inducing a slightly earlier stall seen in the flow fields, to limit the maximum force generation when compared with the optimal  $C_P$  case. At the same time, a slightly delayed velocity peak causes the blade to track the stall vortex and amplifies the diminishing torque, thereby extending power output later in the cycle. The result is a low maximum but wider phase-averaged performance curve. Note that the range of intracycle phase offsets for both Pareto fronts is relatively narrow ( $\sim 40$  degrees), highlighting the need for careful selection based on the overall control objective.

Reductions in peak loading are seen to be associated with a broadening of the loading curve brought about by smoothing of the nominal angle-of-attack and blade-relative velocity profiles relative to the case that solely optimized power output. In-rotor flow-fields representative of cases that reduce peak loading (the PFR and minimum max-force cases) show dynamic stall vortex formation slightly earlier than in the constant speed case, but a more diffuse vortex is shed later in the cycle. However, these reductions in peak loading result in a moderately high and broad loading profile, which increases average loading (area under the curve). In contrast, cases that generate lower mean forces produce power over a very narrow range resulting in highly impulsive power and force generation. Close alignment between peak velocity and torque results in taller and narrower power and force production due along with a flattened angle-of-attack upstream and a high relative velocity during the power stroke. Although we did not observe the flow field of the minimum mean loading case, the magnification of the maximum loading and more impulsive power generation indicates that this would likely be a poor kinematic choice for field deployment.

While prior work has highlighted the potential for intracycle control to be utilized for power enhancement, this work shows the potential for its use to be tailored to a specific site and project. Depending on a particular application and materials available, it may be more economical to optimize power at the expense of force or reduce loading at the detriment of power at extreme ends of the Pareto optimal points. There may also be applications where transitions to intracycle control may be useful for power or load shedding above a turbine's rated speed, though a high degree of certainty of the flow direction as well as torque rating of the control motor would be necessary to provide the necessary input and ensure control is achieving the desired dynamics.

Additional results from this work have been disseminated through the coauthor publication [65], which used the results presented here to validate LES and RANS models at the University of Wisconsin, with data openly available in the MHK-DR repository, linked [here](#).



## Chapter 5

### THE IMPACTS OF GEOMETRIC CAMBER ON CROSS-FLOW TURBINE PERFORMANCE AND HYDRODYNAMICS

Instead of employing variable-velocity control to regulate the relative flow along the foil, this chapter explores the connections between blade camber and turbine performance and hydrodynamics under constant-speed control.

The work in this chapter includes content from Athair et al. (2026) [70] recently accepted for publication in *Wind Energy*. This work is coauthored by Caelan Consing, Jennifer A. Franck, and Owen Williams. All experimental data collection and processing were conducted by the author. Numerical simulations were conducted by Caelan Consing, building upon the model and work of Dave and Franck [19] and Bridges [71]. All data analysis and figure generation were conducted by the author, who also led drafting and editing of the manuscript, to which all coauthors contributed.

#### 5.1 Introduction

Traditionally, symmetrical foils have been favored for cross-flow turbines. It has been argued that this approach accommodates the positive and negative  $\alpha_n$  experienced by cross-flow turbines. However, introducing the concept of virtual camber and its implications, it becomes evident that the traditional rationale for using symmetric blades overlooks the curvature of the flow [33]. The impact of this virtual geometry and its effect on turbine performance remains unclear, making it essential to ascertain whether such changes are beneficial, detrimental, or exploitable for enhancing turbine efficiency. This understanding is crucial to future turbine optimization, underscoring the importance of studying camber to elucidate cross-flow turbine dynamics and identify optimal blade geometries. This chapter, explores the influence of cambered foils on turbine performance and loading, highlighting the utility of these geometries in cross-flow turbines relative to the estimated variation in virtual camber.

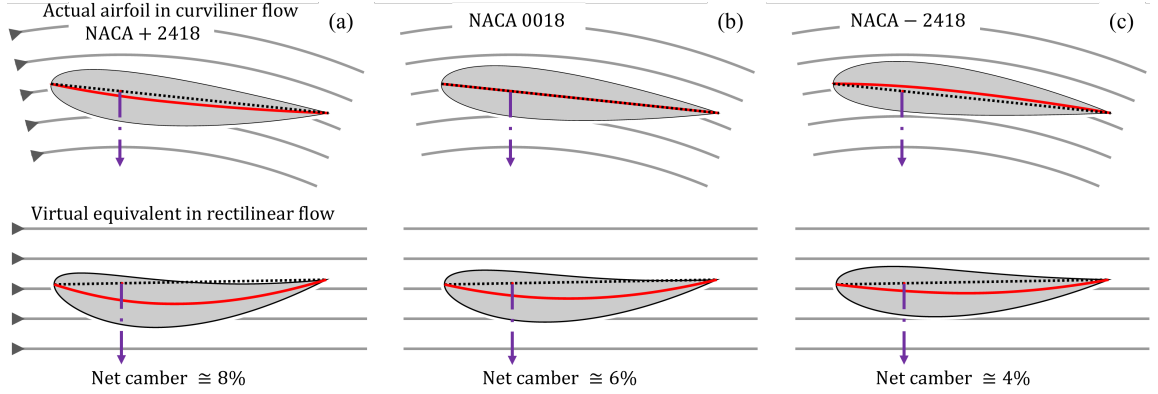


Figure 5.1: Schematic equivalent blades of cross-flow turbines result in virtual camber and virtual pitch behavior for blades tested in this paper. Dotted black lines correspond to the chord line of the foils, (red) corresponds to the camber line, and purple represents the radial direction to the center of the turbine. Transformed using Migliore’s method,  $c/R = 0.49$ , and  $-6^\circ$  geometric pitch, under conditions of  $\lambda = \infty$ . (a),(b) and (c) show the effects of geometric camber on the virtual equivalent foils and the resulting net camber. The virtual pitch for all three foils was  $\sim 7^\circ$  which negates the geometric pitch, resulting in a  $\sim 1^\circ$  inward pitch shown in the rectilinear equivalents.

It is critical to clarify the sign convention used to discuss camber effectively. We adopt a sign convention consistent with  $\alpha_n$  and geometric pitch. In this convention,  $\alpha_n$  is positive in the upstream where the productive lift is generated (power generating) (See equation 2.1 and Fig. 2.2). Conversely, a negative geometric pitch ( $\alpha_g$ ), as used in this work, reduces  $\alpha_n$  and lift in the upstream power-producing region (see Fig. 2.1). Virtual camber is consistently concave out through the cycle, which enhances the upstream lift and is defined as positive. Geometric camber mirrors this, with positive camber (concave out) accentuating the virtual and enhancing upstream lift. In contrast, negative camber (concave in) reduces the virtual and degrading upstream lift production (see Fig. 5.1).

In steady-state aerodynamics, camber typically increases lift for a given  $\alpha_n$  but also increases drag and pitching moment. The influence of virtual camber and cambered foils on turbine performance may vary depending on the flow region’s sensitivity to lift, drag,

and pitching moment. It might be seen as advantageous to camber the foil to increase lift in the power-producing region, as other optimization techniques have focused on, using a positive cambered blade and accentuating the virtual [11]. However, it remains challenging to transfer steady-state aerodynamic results to cross-flow turbines due to the added effects of virtual camber, virtual forces from the rotating blade reference frame, and dynamic stall present from extreme cyclic angles-of-attack experienced. Recent work by Snortland et al. [40] has shown that the upstream region is primarily influenced by lift, while the downstream is more influenced by drag and pitching moment. Thus, choosing a blade profile that improves upstream lift (positive camber) or reduces drag downstream (negative camber) may be optimal, but there is no current theory that can determine which influence will win out.

As discussed in Sec. 2.5, virtual camber and pitch are important to cross-flow turbines. This phenomenon stems from variations in the  $\alpha_n$  along the chord, resulting in behavior akin to the aerodynamic performance of a cambered blade with a certain  $\alpha_n$ . When examined considering only the curvilinear flow ( $\lambda = \infty$ ), the virtual profile becomes independent of position. The design parameters that determine the virtual camber and pitch of a given turbine are  $c/R$ , mounting point, and geometric pitch, with  $c/R$  having the dominant effect [33]. During actual turbine operation, where rectilinear inflow is superimposed, the virtual profile becomes a function of  $\lambda$  and blade position through the cycle. As a result, the influence of curvature is most often quoted in the simplest form of  $\lambda = \infty$  (as if there were no inflow). The conformal transformation method detailed in Migliore and Wolfe [5], while insightful, relies on several key assumptions, including negligible induction effects and chord-wise relative velocity gradient, ignoring virtual forces, absence of blade wake interactions, and a thin airfoil assumption reducing the foil to its chord line during transformation. Consequently, this model for virtual camber provides valuable insights into turbine dynamics but should be viewed as an approximate tool rather than an exact predictive method.

After Migliore et al. described virtual camber in cross-flow wind turbines in 1980 [33], other researchers have developed alternative transformation methods to estimate the equivalent airfoil in a curvilinear flow. Extensions that include the effects of active pitching-induced velocity variation were developed by two groups [42, 43]. Further alternative mathematical

approaches to transform to a virtual equivalent have been developed, both more straightforward, using a virtual camber line of a circular arc, and more complex, utilizing exponentials [44–46, 49]. However, these all result in similar predictions when compared at  $\lambda = \infty$  until  $c/R$  exceeds 0.4, where slight diverging occurs [34]. There is a degree of uncertainty stemming from induction and underlying assumptions, likely surpassing the variations among methods. Therefore, we will rely on the method proposed by Migliore et al. [33] to assess magnitudes and trends throughout this, acknowledging this inherent uncertainty. The importance of considering virtual profiles was underscored in 2015 by Bianchini et al. [47], by comparing surface pressure profiles of transformed foils in a wind tunnel, with those observed in the rotating reference frame using CFD [47]. Their work highlighted the criticality of including both the transformed airfoil and adjusting to include the pitch predicted using Migliore’s method before applying blade element momentum theory to produce a rough estimate of performance [3, 47].

Additionally, while virtual camber transformations have primarily been applied to symmetrical blades, the mathematical framework for extending these transformations to geometrically cambered blades is well-defined. Since Migliore’s method assumes a thin airfoil for the transformation, the camber line is the input, as a result, for foils that have a camber centered between 0.4-0.6  $x/c$  the transformation is essentially a superposition of the virtual camber and geometric camber of the foil. This effective superposition can be seen in Fig. 5.1, for the cambers explored in this work, where the positive camber increases the equivalent virtual cambered foil and the negative cambered foil decreases it compared to the symmetrical foil.

Understanding the effects of virtual or geometric camber begins with exploring the impact of camber on dynamic stall using pitching foils, which avoid rotational-path influences. However, the implications of camber on dynamic stall have not been exhaustively evaluated [28]. Limited experimental findings suggest that dynamic stall vortex formation occurs closer to the blade’s trailing edge during upward pitching for more cambered geometries [37]. However, this observation was made with reflexive blades, raising uncertainty about its general applicability to camber. Numerical simulations using Large Eddy Simulation (LES) showed delayed stall and improved flow reattachment with a higher camber

for pitching foils NACA 4412 vs NACA 0012 [27]. Another recent numerical work explored six airfoil design parameters on dynamic stall behavior of a sinusoidally pitching foil and found camber to the strongest impact on both lift and moment coefficient characteristics and camber and thickness had the most significant impacts on stall angles but were not clear on the direction of these effects [28]. However, that study was only validated for a baseline NACA0012, and results may deviate for other baselines. Moreover, the impact of the rotating frame of a cross-flow turbine on dynamic stall in combination with cambered foils remain uninvestigated.

The effects of geometric camber on cross-flow turbine performance have been sporadically explored. An early investigation by Healy [72] suggested that close-to-symmetrical blades performed optimally. In contrast, Baker [73] suggested there might be potential power output benefits of camber without specifying the direction relative to turbine geometry. Subsequent research by Kirke and Lazauskas [20] demonstrated higher torque coefficients at low rotation rates, demonstrating the potential for applications of camber for improved self-starting behavior but lower peak power when using positively cambered blades [20]. Kirke also touched on the potential for approximate cancellation of virtual camber using geometric camber and tested positive and negatively cambered blades and proposed that a positive camber performed best due to upstream positive incidence and increased lift on the blade at low  $c/R = 0.1$  [74]. More recently, some work has directly compared inward and outward cambering performance [21, 22, 29, 75]. The conclusions from these works showed contradictory trends; Danao et al. [21] proposed that a negative camber performed best while Kirke and Lazauskas [20] and Abotaleb et al. [75] indicated that positively cambered blades improve performance across  $\lambda$  for a single  $c/R$ . Alternatively, Abotaleb et al. [75] suggested that utilizing adaptive (actively morphing) camber would be ideal, and [22] concluded positive camber promotes wake flow recovery without touching on the impacts on power. These explorations were all conducted using double multiple stream tube (DMST) analysis or CFD models. Given the complex dynamics and flow separation inherent in these turbines, such approaches would benefit from experimental validation. However, experimental assessments of camber effects have been limited, leaving a gap in our understanding the optimal camber geometry and sign, relative to turbine rotation [12, 29].

In addition, many of these prior tests were conducted at either a blade-geometric pitch angle of zero degrees, which can be sub-optimal even for symmetrical blades [32], depending on the mounting point. Further, these works were performed at smaller  $c/R$  where virtual camber is less impactful. As a result, many questions remain, including the sources of these conflicting conclusions, the influence of  $c/R$ , and the generality of these conclusions across the broader turbine design space. In short, it is still unclear what direction of camber may benefit overall turbine performance and under what conditions.

This study aims to address the gap in understanding by experimentally evaluating the effects of camber on cross-flow turbines. Specifically, it investigates whether virtual camber enhances or diminishes performance by comparing blades that reduce the virtual, and others that enhance it. PIV is employed to explore the near-blade hydrodynamics and its correlation to upstream-downstream performance, shedding light on the flow field impacts of camber. The study establishes a sign convention for camber relative to turbine geometry, crucial for clarifying camber direction—an often overlooked detail. This work also contributes to understanding optimal blade geometry and informs future decisions regarding camber positioning for enhanced turbine performance. Furthermore, experimentally validated computational fluid dynamics (CFD) simulations conducted by collaborators at the University of Wisconsin-Madison are used to explore a broader range of cambers and assess performance sensitivity across a wider parameter space. A theoretical analysis of flow curvature was conducted using the method detailed by Migliore and Wolfe [5], which compares curvature-related changes in virtual camber and  $\alpha_n$  with geometric camber.

## 5.2 Methods

This work was completed at the same test facilities, under the same flow conditions and instrumentation discussed in detail in sections 3.1 and 4.2. The same two-turbine configurations were also employed: one for performance measurement and another for PIV acquisition, Fig. 3.1. However, to assess the effects of camber, this work explored three different blade profiles under constant-speed control: a symmetrical NACA 0018 and a NACA 2418 with mounting pins along the chordline, allowing it to be mounted positively or negatively (Fig. 5.1).

### 5.2.1 Turbine Configurations

Both one- and two-bladed turbines were tested with three different blade cambers: a symmetrical NACA 0018, and a NACA 2418 that could be mounted in either the positive or negative camber directions. The mounting and dimensions of the blades remained uniform across all three configurations; with a chord length ( $c$ ) of 4.06 cm, a span ( $h$ ) of 23.4 cm, and geometric pitch angle ( $\alpha_g$ ) of  $-6^\circ$ , rotated leading edge outward about the quarter-chord, a configuration previously found to be optimal for a symmetrical blade [7]. The experimental flow condition, turbine position in the water column, and non-dimensional parameters matched Chapter 4.2. These non-dimensional numbers were held constant across all tests to alleviate any secondary changes to performance across experiments [24]. The turbine radius to the quarter-chord, was held constant at 8.23 cm, resulting in  $c/R = 0.49$ , matching Chapter 4.2; however, the outermost swept radius of the three blades,  $R_o$ , had slightly different values of 8.55, 8.60, and 8.66 cm for the positive, symmetrical, and negatively cambered blades, respectively. These different radii were incorporated into the area calculation used to non-dimensionalize power and force coefficients, consistent with previous work [7].

Although, the Reynolds number in this study ( $Re_c = 4.4 \times 10^4$  or  $Re_D = 1.9 \times 10^5$ ) lies in a transitional regime and are, therefore, not representative of full-scale performance [76], prior studies indicate that increasing Reynolds number consistently delays flow separation and reattachment, leading to higher performance and loads [24, 76–78]. Consequently, increasing Reynolds number is expected to improve efficiency and delay stall for all blade geometries tested here. The effect of Reynolds number on the relative performance between geometries, through potential changes in upstream and downstream contributions, remains an area for future investigation when extending these results to different flow conditions.

### 5.2.2 PIV Methods

Near-blade mid-span flow-fields were captured using the same single-camera planar PIV system shown in Fig. 3.1b, discussed in detail in Section 4.2. Two FoV were captured one upstream and one downstream to fully resolve the inrotor flow. PIV was captured at 30 equally spaced phases with a  $12^\circ$  separation, with timing controlled by the MATLAB

Simulink turbine controller to ensure phase locking of data. At each phase, 85 image pairs were captured. These replicates were captured for each experiment and FoV so that low-noise phase-averaged flow fields could be reconstructed and analyzed. The inter-frame time was chosen to be 350-370  $\mu\text{s}$ , so maximum in-plane displacements were limited to approximately 9 pixels, and vortex cores could be resolved. The image calibration was 10.16 pixels/mm. Again, mild shaft procession was tracked during auxiliary experiments under matching conditions, so it could be accounted for in post, following the method developed by Snortland et al. [57]. The resulting processed data was exported and phase-averaged for further plotting and analysis in MATLAB.

### 5.2.3 Numerical Setup

Computational fluid dynamic (CFD) simulations were conducted by collaborators at the University of Wisconsin-Madison to explore a broader range of camber geometries. The governing equations are the incompressible unsteady Reynolds-averaged Navier-Stokes (URANS) equations, solved in two-dimensions. The solver is implemented using the second-order finite-volume spatial discretization in OpenFOAM. The Reynolds stress tensor is modeled with  $k-\omega$  shear stress transport (SST) equations, which is a closure model demonstrated to handle separated flows [79], and used in prior URANS simulations of cross-flow turbines [80–83].

The meshing, numerical solver, and turbine set-up for the computations very closely follows previous turbine simulations validated with experimental force and PIV data [66, 84]. The only deviations from these prior works is that the current simulations are single-bladed turbines, and each blade has camber. Single-bladed turbines allow for more in-depth examination of the blade-level forces and their impact on power, and enable one-to-one comparison with the single-bladed turbine experiments.

The computational turbine has  $c/R = 0.49$  with an outwards pitch of  $-6^\circ$ , matching the experiments, but does not consider the struts or center shaft. The two-dimensional domain has a blockage of 12.8%, in the transverse direction, closely resembling experiments, and has slip walls along the top and bottom. The computational domain extends 300 chord lengths



in the streamwise direction with the turbine placed in the center, and has incompressible inlet and outlet conditions. The Reynolds number based on chord and freestream is  $4.5 \times 10^4$ .

The baseline blade shape is the same as the experiments, a NACA 0018, a symmetric blade with 18% maximum thickness. The mesh surrounding the airfoil is structured and body-fitted, surrounded by an unstructured mesh in the far-field, as noted in Fig. 5.2a. There are 50 layers and 105,465 total cells in the body-fitted region, with a  $\Delta y/c = 1 \times 10^{-4}$ . Rotation is accomplished with a sliding and rotating arbitrary mesh interface (AMI), inside of which constant mesh resolution is maintained to extract the flow structures close to the turbine blade. Outside the AMI inner region mesh resolution drops since wake structure and dynamics are not investigated in this study. All URANS models are sensitive to mesh resolution, especially around the surface of an airfoil with flow separation. Thus, camber was generated via a mesh-motion algorithm that deforms the baseline mesh to the desired camber, demonstrated in Fig. 5.2b. The mesh morphing is done prior to the start of the simulation. This approach eliminates the need for remeshing and ensures that the near-wall resolution remains constant for an equal comparison across different cambers. The mesh motion algorithm is fully documented in prior work [71, 85]. Mesh resolution studies on the uncambered turbine can be found in Dave et al. [66], whereas a mesh resolution study on the structured layers of the cambered blade are presented in the supplemental material of [70].

#### 5.2.4 Estimation of Virtual Camber and Pitch

All virtual camber methods require an estimate of the blade-relative inflow velocity. As this is not known analytically because of turbine induction (which cannot be predicted accurately *a priori*, at present), the estimation of virtual camber has a degree of uncertainty, particularly downstream. Acknowledging this fact, we rely on the method developed by Migliore and Wolfe [5], to assess magnitudes and trends of virtual geometry, as deviations due to induction surpass those due to the assumptions inherent to each alternative model. At high  $\lambda$ , the relative velocity of the blade rotation converges to the tangential velocity. Although induction has been neglected, its effects can be qualitatively observed, reducing

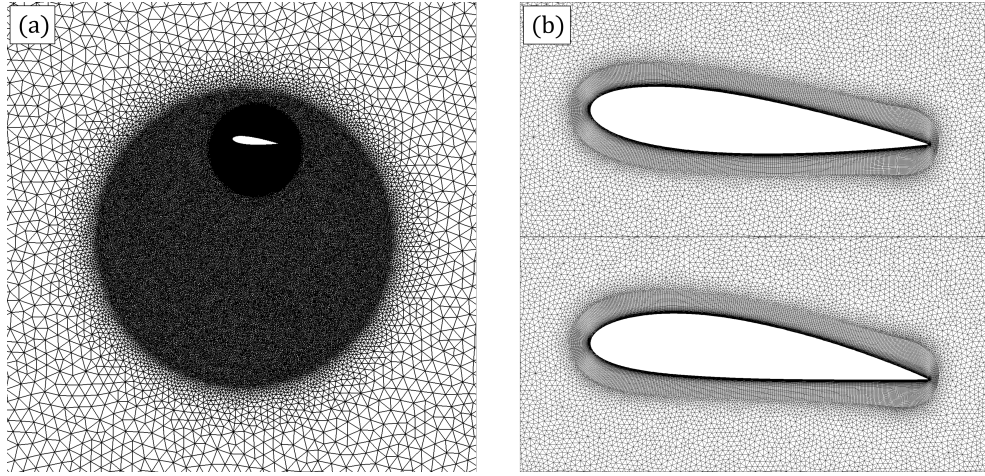


Figure 5.2: (a) AMI region (b) Symmetric NACA 0018 (top) and negative camber NACA 3418 (bottom). Mesh images demonstrating the meshing strategy for the single-bladed turbine comprised of structured and unstructured cells: a) shows the rotating and sliding AMI region; b) demonstrates the mesh motion that cambers the mesh while maintaining the same mesh structure and resolution.

the effective inflow velocity both upstream and, more significantly, downstream due to momentum extraction [86, 87]. This reduction in incident velocity increases the blade local tip-speed ratio, accelerating the convergence towards the  $\lambda = \infty$  case faster than predicted by the nominally defined  $\lambda$  [88]. Since downstream induction further reduces the flow impinging on the blade, the downstream portion of the rotation is expected to approach the virtual camber and pitch of the steady  $\lambda = \infty$  prediction, at lower rotation rates than the upstream portion.

While virtual camber transformations have primarily been applied to symmetrical blades in the literature, the mathematical framework for extending these transformations to geometrically cambered blades is well-defined. Since the method of Migliore and Wolfe [5] assumes a thin airfoil for the transformation, the camber line is the primary input. The resulting virtual camber is primarily independent of the input geometric camber, and so the transformation can be thought of as a superposition of the foil's virtual and geometric camber (particularly for small cambers  $< \pm 4\%$ ). For foils with geometric camber centered

between 0.4 and 0.6  $x/c$ , the virtual camber line is smooth, and the net camber magnitude is approximately the sum of the geometric camber and the virtual camber of a symmetrical foil. This is illustrated in Fig. 5.1 for the blade profiles studied in this work at the limit of  $\lambda = \infty$ . The virtual camber of the symmetrical foil, with  $c/R = 0.49$  and quarter-chord pitch  $\alpha_g = -6^\circ$  (Fig. 5.1b), is seen to be 6% while the positive and negative 2% foils are shown to increase and decrease the net camber to 8% and 4% respectively. For foils with more extreme positions of maximum camber,  $x/c < 0.4$  or  $x/c > 0.6$ , the net camber can exhibit multiple maxima or a reflexive profile depending on the sign of the geometric camber, increasing the final profile complexity. We intentionally selected profiles with a position of max camber at  $x/c = 0.4$  to avoid exploring such foils in the current study for this reason.

Although changes to blade mounting location and geometric pitch are not explicitly explored here, their effects merit discussion. Changing the geometric pitch directly shifts the nominal angle-of-attack throughout the cycle. However, it can be shown that changing the mounting point along the chord is geometrically equivalent to introducing a change in blade pitch and a small adjustment in mounting point radius. As a result, blade pitch and mounting point can be treated as interchangeable in their influence, provided they are properly documented [89, 90]. Virtual camber and pitch are also largely insensitive to either pitch or mounting point parameters, though a small dependence remains for large  $c/R$ ,  $> 0.8$  (detailed in Chapter 6 and Appendix B.1 and recently noted by Reingewirtz and Greenblatt [48] for pitch as well). As a result, the influences of camber explored in this study are thought to be broadly applicable to a wide range of turbines. The broader interaction between geometric pitch and camber, and their virtual counterparts, is explored in Chapter 6.

### 5.3 Results

#### 5.3.1 Exploration of Virtual Camber and Pitch Dependencies

We explore the variation of estimated curvature-induced virtual camber and pitch so that they can be compared to the geometric camber of the chosen airfoils. Throughout this analysis, we assume the blade mounting point to be at the quarter-chord. Figure 5.3 shows

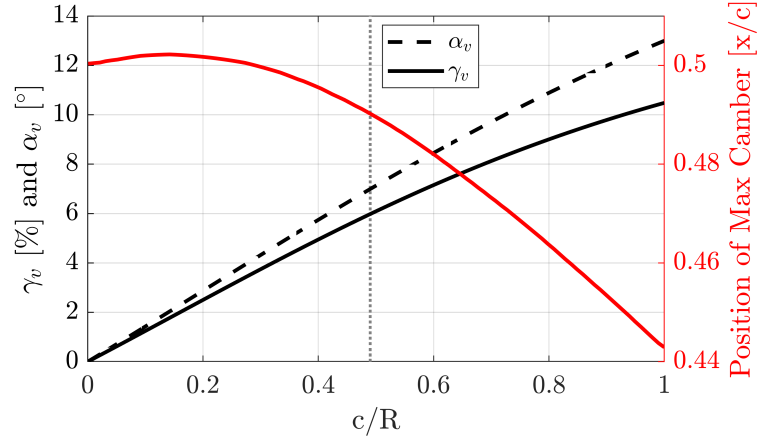


Figure 5.3: Impact of  $c/R$  on virtual camber ( $\gamma_g$ ) and  $\alpha_v$ , as well as the location of maximum virtual camber for  $\lambda = \infty$ . The vertical dotted line corresponds to the blade geometry explored in this work,  $c/R = 0.49$ ,  $\alpha_g = -6^\circ$  and  $c/4$  mounting point.

the effects of  $c/R$  on the virtual equivalent airfoil geometry in a purely rotational flow (i.e., at infinite  $\lambda$ , no inflow). The virtual pitch and camber are seen to vary in an approximately linear fashion with  $c/R$ , up to a maximum of  $13^\circ$  and 10%, respectively. The position of maximum virtual camber is non-linear and varies between  $0.44c$  and  $0.50c$ . For  $c/R = 0.49$ , used by the turbines in the current study, a virtual camber of 6% is observed, three times the maximum geometric camber that was tested. Similarly, the virtual pitch ( $\alpha_v$ ) is  $7^\circ$  and opposes the geometric pitch of  $-6^\circ$ , resulting in a  $\sim 1^\circ$  inward net pitch. The position of the maximum virtual camber is  $\sim 0.49$  for the current study, close to the location of the maximum camber of the NACA-2418 foils. This geometry was explicitly chosen to be close to matching the virtual profile while also being a foil whose aerodynamic performance has been well documented.

For a finite and constant  $\lambda$ , the virtual geometry varies as a function of blade position  $\theta$ , as explored in Fig. 5.4 for a  $c/R$  of 0.49. While the virtual camber is consistently positive through the cycle, the greatest variations are seen to occur at low  $\lambda$ , where the inflow to the turbine has a greater influence. As such, expectations from steady-state aerodynamics suggest virtual camber would enhance upstream lift compared to a symmetrical blade at

the same  $\alpha$ . The virtual pitch profiles (Fig. 5.4b) follow those of virtual camber very closely, maintaining a positive value through the cycle, unlike the nominal  $\alpha_n$  observed in 2.2a, which changes sign. This positive virtual pitch corresponds to a nose-in pitch of the blade, increasing the upstream angle-of-attack and lift. For  $\lambda > 2$ , the virtual camber and pitch increase throughout the cycle until just after  $\theta = 180^\circ$  before diminishing, while lower  $\lambda$  shows a brief and moderate reduction of virtual pitch and camber upstream. The position of the maximum virtual camber, Fig. 5.4c, moves from the mid-chord towards the blade's leading edge before dramatically shifting to the aft half of the foil for  $\theta > 180^\circ$ . For the majority of the cycle, the resulting virtual airfoil (see examples in Figs. 5.4(d,e)) appears similar to traditional NACA airfoils except under low  $\lambda$  conditions and for phases near  $\theta = 180^\circ$  where we see more extreme shapes due to a high rate of change in  $\alpha_n(\theta)$ . With increasing  $\lambda$ , virtual pitch and virtual camber increase upstream and asymptotically approach the steady-state  $\lambda = \infty$  case (represented by the dashed horizontal lines in Fig. 5.4). As induction in the downstream is neglected in the transformation method [5], the virtual geometry predictions have higher uncertainty for  $\theta > 180^\circ$  and are expected to deviate toward the infinite result  $\lambda$ , due to lower inflow velocities at the blade, as noted previously.

### 5.3.2 Influence of Geometric Camber on Turbine Performance

The time-averaged performance of one- and two-bladed configurations are shown in Fig. 5.5a for the different blade cambers tested (symmetric and  $\pm 2\%$  camber). It is revealed that the optimal geometric camber is dependent on the operating condition of the turbine for both configurations. For one- or two-blades, the symmetric or negatively cambered foils produce very similar time-averaged power output for most rotation rates, whereas the performance of the positively cambered foil produces less power at all but the lowest  $\lambda$ , contradicting earlier expectations that the positively cambered foil would give the best overall performance due to expected enhancements to upstream lift. Overall, the negatively cambered foil performs best at the highest  $\lambda$ , and symmetric foils provide the best time-averaged performance at more moderate-to-low  $\lambda$ .

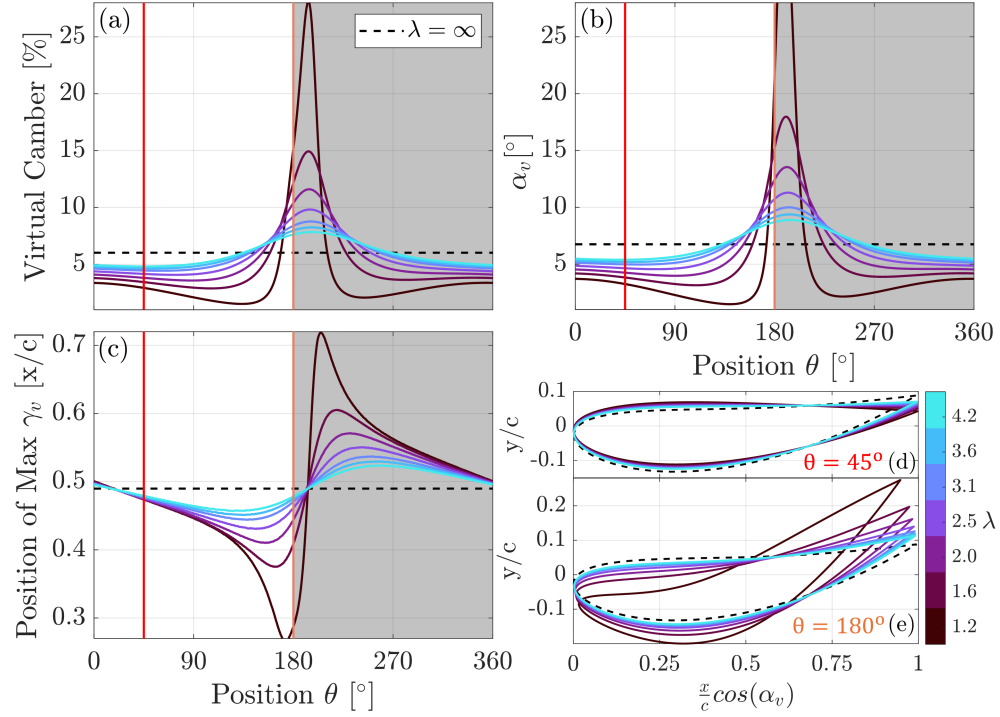


Figure 5.4: Trends of virtual camber (a), virtual pitch (b), and the position of max camber (c) for a range of  $\lambda$ , as a function of blade position,  $\theta$ . Calculated using the method of Migliore and Wolfe [5] for  $c/R = 0.49$  and geometric pitch of  $-6^\circ$  measured from a quarter-chord mounting point. Cases with a purely rotational flow ( $\lambda = \infty$ ) are shown with the dashed black lines. The downstream region is shown by a gray background, where the true behavior is expected to more closely match the  $\lambda = \infty$  case. The associated virtual equivalent profiles for a NACA 0018 foil are shown for azimuthal positions of  $45^\circ$  (d) and  $180^\circ$  (e).

Peak performance for one- and two-bladed turbines with symmetrical foils occurs at  $\lambda$  values of 2.8 and 2, respectively and we see minimal variation due to geometric camber. For two-bladed turbines, the optimal  $\lambda$  slightly decreases with increased camber, showing values of 2.15, 2.05, and 1.98 for negative, symmetrical, and positive cambered blades. While the one-bladed turbine shows similar trends overall, the difference in optimal tip-speed of symmetric and negatively cambered blades lies within experimental accuracy.

The one-bladed data is used to isolate the upstream and downstream contributions to overall turbine performance for each camber (see Fig. 5.5b,c). Following the analysis of Snortland et al. [88], these results further highlight the linkages between the downstream portion of the rotation and the optimal  $\lambda$ , with the optimal rotation rate governed by degradation in downstream performance observed at higher rotation rates. The upstream performance continues to increase past the cycle-average optimum, and the downstream is approximately neutral (or slightly negative) until a critical  $\lambda$  at which the performance decreases quickly and in an approximately linear fashion.

While the positively cambered foil has better performance than the negatively cambered foil in the upstream portion of rotation for lower  $\lambda$ , as expected, this is more than offset by diminished performance in the downstream sweep in almost all cases. Negative camber is seen to reduce upstream performance, but the performance is much improved in the downstream region and even generates a small amount of positive power near optimal conditions. In other words, performance benefits in one region are generally offset to some extent within the other, with improvements in flow recovery in the downstream portion of the rotation appearing to be a key factor for this turbine. The upstream performance is also seen to peak at a much lower  $\lambda$  for the positively cambered foil, significantly contributing to its poor overall performance. To provide a bound for potential improvements in turbine performance due to camber, we consider a hypothetical blade that changes camber through the cycle to match the best performing profile. The resulting time-averaged performance would be as shown in by the red profile in 5.5a. Neglecting unsteady effects from switching camber, such a blade would have between 10% and 61% improvement in efficiency compared to the symmetrical blade, with a higher potential for improvement by percentage at extreme  $\lambda$  (high or low). Beyond the rated speed where power shedding is necessary, it may be bene-

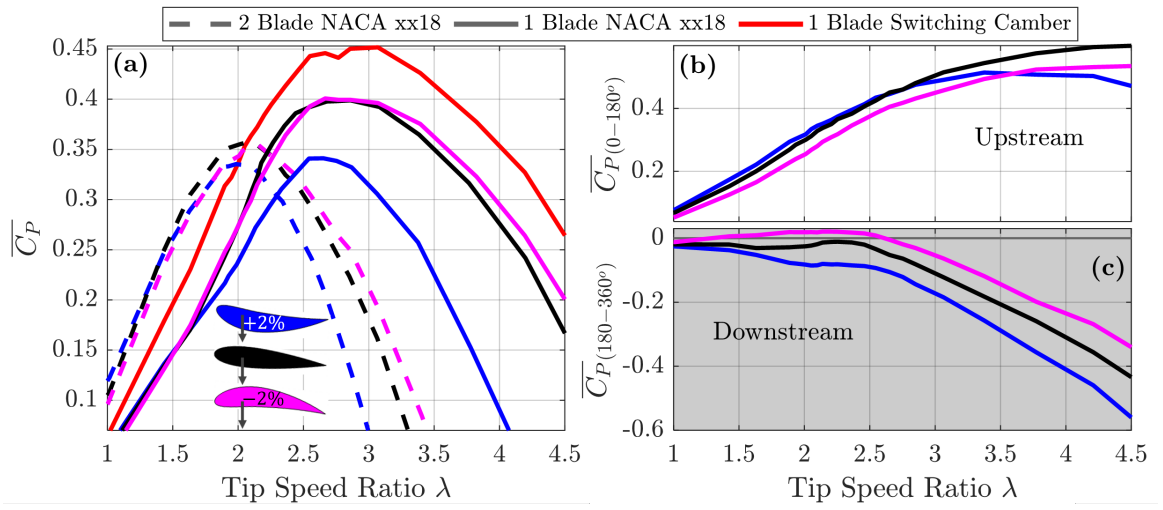


Figure 5.5: (a) Time-averaged performance of a one- (solid) and two-bladed (dashed) turbine as a function of rotation rate. (b) Time averaged upstream and (c) downstream performance of a single blade such that the sum is equal to performance of the whole turbine in (a). Positively cambered foils are shown in blue, symmetrical in black, and negative in pink. Arrows attached to the blade in the legend of (a) point toward the center of rotation. Red corresponds to the average performance of a hypothetical one-bladed turbine with the best performing profiles in the upstream portions of the rotation.



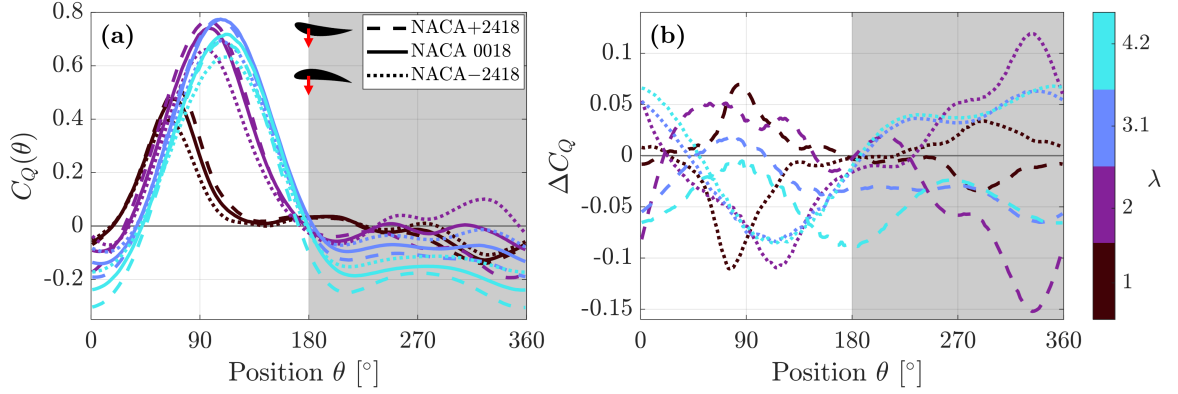


Figure 5.6: (a) Phase-averaged  $C_Q$  of the single-bladed turbine and (b) the difference in phase-averaged  $C_Q$  between cambered cases and the symmetrical-bladed turbine. Arrows attached to the blade in the legend of (a) point toward the center of rotation. The colors correspond to  $\lambda$  and line styles to blade geometry.

ficial to flip the dynamic cambering behavior to achieve the worst attributes in each region. Preliminary work has already designed airfoils with variable camber [91], which could be tested in cross-flow turbine applications to take full advantage of upstream/downstream preferences.

Further exploration of phase-averaged  $C_Q$  of the one-bladed turbine (see Fig. 5.6a) illustrates a more detailed comparison of performance throughout the cycle as a function of  $\lambda$ . By subtracting the symmetrical-bladed performance at equivalent  $\lambda$ 's the phase-dependent changes in torque due to different cambered geometries are highlighted in Fig. 5.6b. As might be expected, it is seen that more positive geometrically cambered blades have a higher and slightly later upstream torque peak for  $\lambda = 1 - 3$  (indicating stall delay), however for  $\lambda > 3.4$  it is seen to fall off and become less than that of the symmetrically bladed turbine. The azimuthal location of the peak torque is shifted later in the rotation with increasing camber for below-optimal  $\lambda$ . In contrast, the phase of peak torque appears to be approximately independent of geometric camber above the optimal tips-speed. We see that previously observed decreases in the integrated upstream torque ( $0 < \theta < 180$ ) at high  $\lambda$  for positive camber (seen in as  $C_P$  in Fig 5.5b) is likely due a continuation of

poor flow recovery as the blade passes  $\theta = 0^\circ$ , delaying the start of the lift-generating process to later in the cycle. Downstream, torque becomes progressively more negative at the highest rotation rates. This effect is amplified for the more positively cambered foils. In contrast, the negative foil shows a positive secondary downstream peak around  $\theta = 315^\circ$  at moderate  $\lambda$  (i.e.,  $\lambda = 2$ ), suggesting a secondary stall event that produces lift. Furthermore, as the geometric camber increases, more detrimental pitching moment values are expected, which lead to performance degradation at high  $\lambda$ . This observation builds on the findings of Snortland et al.[56] and provides further insight into how camber and  $\lambda$  impact performance.

#### *Influence of Geometric Camber on Turbine Loading*

The time-averaged in-plane, support-subtracted loading on one- and two-bladed configurations are shown in Fig. 5.7a. While the optimal geometric camber for performance is shown to be dependent on the rotation rate, loading is much less nuanced. The mean and peak turbine loading are seen to be larger for increasingly positive camber across all tip-speed ratios, and the difference in average loading between the tested blade geometries grows with rotation rate. Conversely, turbine loading is also seen to be reduced with a negative geometric camber. The loading on a single blade is seen to be notably higher with increasing  $\lambda$  because no oppositional second blade is present.

Fig. 5.7(b-c), reveals that the loading in the upstream half of the rotation increases directly with increasingly more positive camber, while the camber that produces the greatest average downstream loading shows a small  $\lambda$  dependence. For  $\lambda < 2$ , downstream loading is seen to be relatively independent of camber profile. Near the  $\lambda$  of optimal performance ( $\lambda = 2 - 2.5$  for the two-bladed turbine), the negative camber sees the highest mean downstream loading. And as  $\lambda$  increases above a value of 2.5, the average downstream loading starts to increase, with higher camber seeing consistently higher mean loads once again. This is expected to occur due to the higher drag and radial lift generation that is generated for the positive camber as the blades approach steady-state conditions. These results suggest that, at the optimal one and two-bladed  $\lambda$ , the average in-plane blade loading can be reduced by 18% and 0.5%, respectively, by changing from a symmetrical blade to the negatively

cambered one.

We can similarly examine the maximum in-plane blade loading by examining the phase-averaged variation of the one-bladed turbine in Fig. 5.7d. We see that a more positively cambered blade increases the maximum loading (which occurs in the upstream portion of the rotation). Conversely, at the highest  $\lambda$  tested ( $\lambda=4.5$ ), the negatively cambered blade produced a peak blade-level loading 20% less than the positively cambered foil and 12% less than the symmetrical. At the optimum  $\lambda$ , the maximum loading on the negatively cambered blades was 18% less than the positively cambered turbine and 10% less than the symmetrical. In addition, the position of maximum force shifts later in the cycle for lower geometric camber, opposing what is observed for  $C_Q$ , where a higher camber shows the most delayed peak.

Section 5.3.2 discussed the performance of a hypothetical blade that can switch camber direction to obtain beneficial characteristics of both the upstream and downstream regions. The influence of such a variable foil on average blade loading is modeled in Fig. 5.7a (redline), showing up to a 13% increase below the optimal  $\lambda$  but a 6.5-8.2% reduction at or beyond peak  $\lambda$  compared to a traditional symmetrical foil, with the largest reduction at high  $\lambda$ . Overall, the loading on such an actively cambering foil is only slightly higher than a fixed negative camber foil while further enhancing performance.

### 5.3.3 Flow Field Analysis for $\lambda = 2$

Exploring the phase-averaged flow fields allows direct linkages to be made with the performance and loading trends observed above. The interconnections between shed vorticity and the phases and types of dynamic stall and reattachment are explored. In Fig. 5.8, we compare phase-averaged  $C_P$ , the in-plane force coefficient,  $C_F$ , and phase-averaged near-blade vorticity and blade-relative velocity fields for each airfoil at  $\lambda = 2$ , which is close to optimal for the two-bladed turbine. The PIV results are presented in a blade-centric coordinate system. Single-bladed results are shown here to allow direct connection between blade-level performance and observed hydrodynamics. The time-averaged performance of the symmetric and negatively cambered one-bladed turbine at  $\lambda = 2$  is almost equivalent, while the

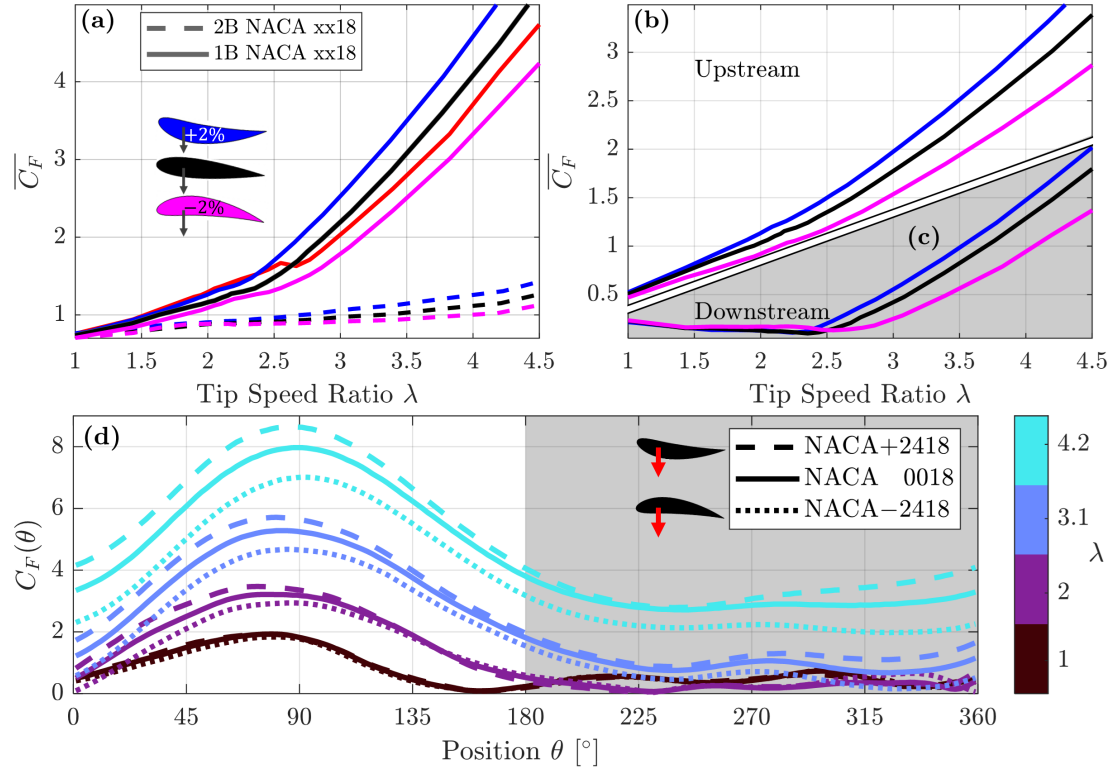


Figure 5.7: (a) Time-averaged one- and two-bladed in-plane loading. (b) Upstream and (c) downstream average force on a one-bladed turbine. (d) Single-blade, phase-averaged horizontal force, with colors corresponding to  $\lambda$  and line style representing blade geometry. Arrows attached to the exaggerated blade in the legend of (a) and (d) point toward the center of rotation.

positively cambered blade performs significantly worse (Fig. 5.5a). The two-bladed data shows similar trends in both efficiency and force at this rotation rate, so we expect the one-bladed hydrodynamic results to also be representative of the two-bladed configuration at these conditions.

The vorticity fields for the symmetric blade (black foils in Fig. 5.8(c)) are typical of documented trends of flow separation throughout the cycle [36]. Early in the cycle  $\theta = 0 - 90^\circ$ , attached flow is observed along the blade, where angle-of-attack increases and thus power production and forces. As the blade progresses to  $\theta \approx 120^\circ$ , visible dynamic stall features are observed, with growing development of leading- and trailing-edge vortices (blue) near the blade, which induces an unfavorable change in the direction of lift relative to torque generation, resulting in a loss of power production. The dynamic stall vortex (blue) originating at the blade tip grows through the cycle, and advects along the blade before shedding off,  $\theta = [120 - 225^\circ]$ . As the blade enters the downstream sweep ( $\theta = [180 - 225^\circ]$ ), the loss of lift, increased drag, sign change of the angle-of-attack, and reduced relative velocity lead to a dip in power production. As the blade progresses, its relative velocity increases, and flow reattachment occurs as a uniform shear wake is observed near  $\theta = 270^\circ$ , restarting the process.

In contrast, the positively cambered blade sees both a different stall and recovery process compared to the symmetric blade. Early in the cycle at the vorticity along the blade appears almost identical to the symmetric blade at  $\theta = 90^\circ$ : however, differences in  $C_P$  are still observed; likely linked to the ability of camber to enhance lift. This difference in lift can be inferred by the velocities being lower on the outer surface of the foil and accelerated along the inner in Fig. 5.8d. The positively cambered foil is seen to have a slightly delayed stall compared to the symmetric foil, and it starts more prominently at the trailing edge. This delay in stall, along with accelerated relative velocity on the inside of the blade, allows for higher power and force generation upstream, matching the performance measurements  $\theta = 40 - 120^\circ$ . The positively cambered blade also exhibits different dynamic stall behavior, with both significant leading and trailing edge vorticity observed ( $\theta = 170^\circ$ ). A larger stall vortex is seen to form near the trailing edge, along with a very small counter-clockwise vortex at the blade trailing edge (red). This small vortex is less noticeable in the presence

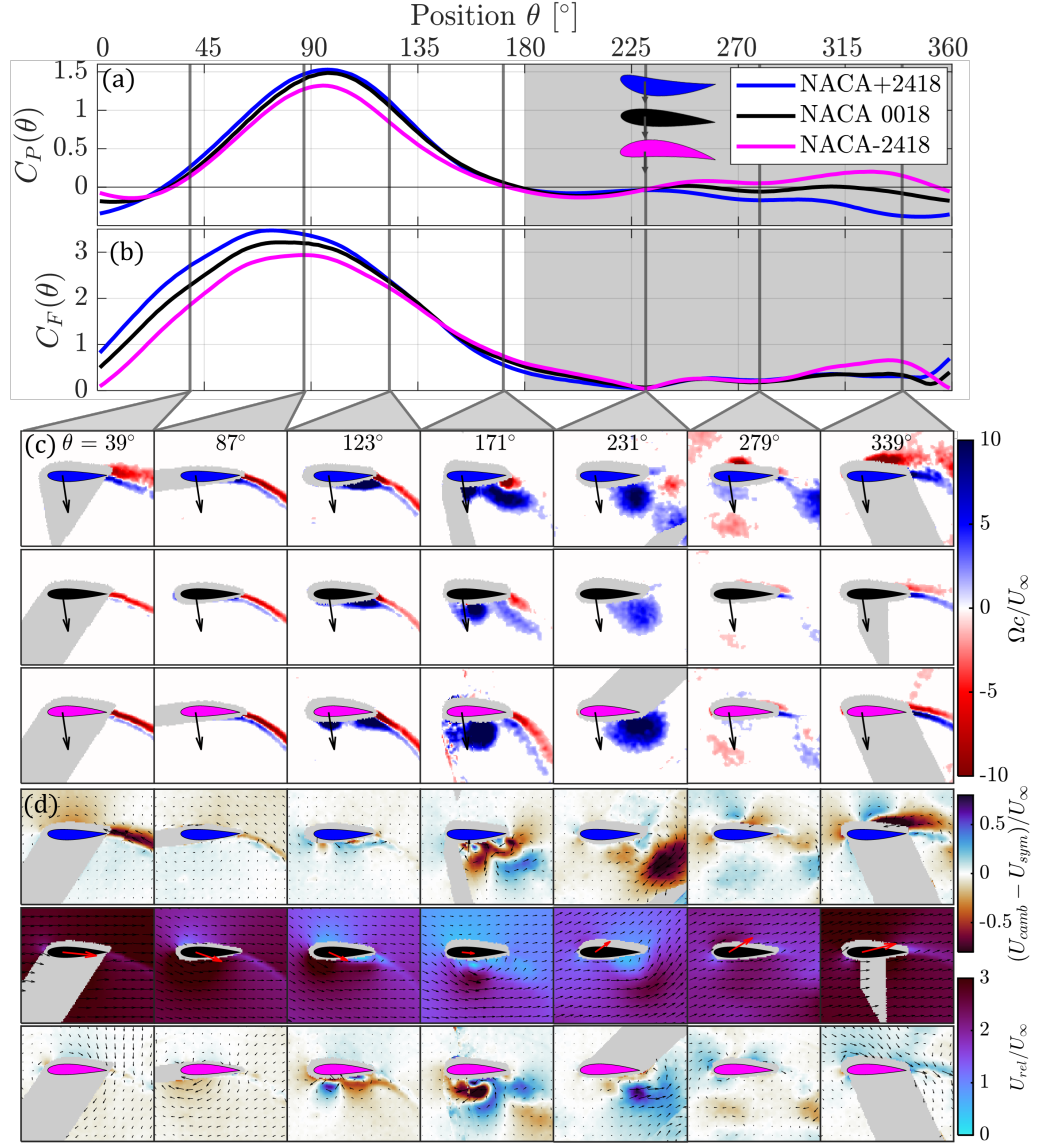


Figure 5.8: Comparison of one-bladed phase-averaged performance and near-blade hydrodynamic structures across blade profiles at  $\lambda = 2$ . Support subtracted phase-averaged (a)  $C_P$ , (b)  $C_F$ . (c) Near blade vorticity fields with black arrow indicating radial direction, and (d) shows both blade relative velocity fields and the difference in relative velocity between the symmetrical case and cambered at matching conditions. In (d), the red arrows attached to the blades indicate the nominal blade-relative velocity independent of geometric camber. Arrows attached to the blades in the legend of (a) point toward the center of rotation.

of the stronger vortices of the opposite sign close downstream, resulting in a less severe shedding of what appears to be two clockwise (blue) vortices rather than the single shed vortex observed for the symmetric foil. The later shedding of vorticity by the more positively cambered foil leads to increased lift later in the cycle, as resulting blade-relative velocities are lower on the underside of the blade for  $\theta = 170 - 220^\circ$ . The stronger trailing edge vortex formation during dynamic stall due to positive camber follows the limited observations in prior work with pitching foils [27, 28, 92], but this is the first time this has been observed experimentally while holding other airfoil design parameters constant in the rotating cross-flow turbine frame. In addition, in the the downstream portion of the rotation the positively cambered blade shows poor reattachment and sheds a secondary red vortex on the outside of the blade between  $\theta = 280 - 340$  that was not present in the symmetrical case. The poor downstream performance of the positive camber prevents flow reattachment till  $\theta = 40^\circ$ , leading to a broader wake and power degradation between  $\theta = 270 - 40^\circ$ .

The negatively cambered blade sees an opposite trend in both recovery speed and stall strength, relative to the positively cambered blade. While at  $90^\circ$  the vorticity along the blade is broadly similar to the other blades, the relative velocity on the inside of the blade is diminished in comparison to the symmetrical cases, resulting in the reduction of power production and loading. During the power stroke, the negatively cambered blade shows a notably larger clockwise leading edge stall vortex ( $\theta = 120^\circ$ ) compared to the symmetrical blade, matching observations by McCroskey [92] where less cambered foils (net camber in this application) tend to exhibit more of a leading edge stall. This more dramatic stall leads to a loss of lift and force along the blade between  $180$  and  $225^\circ$  and is reflected in the force and performance data here. However, once this vortex is convected out of frame, the flow is largely attached on both sides of the foil, leading to lift generation and positive performance ( $300^\circ < \theta < 360^\circ$ ) and a coupled rise in forces relative to the more positively cambered blades in the latter portions of the cycle.

Across the three blade profiles, we observe nonlinear changes in the stall and recovery processes. While the strength and phases of vortex separation and reattachment are altered, greater changes are also present. More negatively cambered blades promote stall closer to the leading edge, while the opposite is true for more positively cambered blades. Secondary

downstream stall events are also observed for the positive camber. Although the observed changes to stall dynamics with camber have seen some exploration in pitching foils, there remains no general model for the effect of camber on dynamic stall. Moreover, in the rotating frame of these turbines, the influence of increasing virtual and geometric camber may influence stall differently. As a result, the use of net camber as a design parameter may be overly simplistic, despite its utility.

#### 5.3.4 *Exploring a Wider Range of Geometric Camber Through Simulation*

In an effort to estimate the optimal camber for this foil, we now explore the sensitivity of performance and loading to a wider range of geometric camber ( $-5\%$  to  $+3\%$ ) at a constant  $\lambda = 2$  through validated 2D-URANS simulations of a one-bladed configuration.

Computational time-averaged performance and loading, as well as the relative contributions from the upstream and downstream portions of the rotation, are shown in Fig. 5.9. While the proportion of the performance derived from the upstream and downstream parts of the rotation varies slightly between the CFD and experimental results, both show similar trade-offs with camber, and computational trends broadly align with experimental findings. Negative geometric camber is seen to increase downstream performance across all tested foils, and positive camber shows improved upstream performance for camber of less than  $+2\%$ , at which point the upstream performance peaks and begins to reduce. Once again, improvements to upstream performance are observed to come at the cost of downstream performance. The loading trends are also similar to experiments, despite overall forces being slightly higher in the simulations, potentially due to their two-dimensional nature. Overall, the in-plane  $\overline{C_F}$  shows a consistent increase with increasing camber, primarily from the upstream contribution, with a slight, unexplained dip at  $+2\%$  before continuing to increase. The rate at which loading improves with negative camber is seen to reduce at approximately  $-4\%$  camber, but is still decreasing at the minimum camber tested of  $-5\%$ .

The simulations suggest that the camber of optimal performance for a one-bladed turbine with  $c/R = 0.49$ , at  $\lambda = 2$ , is approximately  $-3\%$ , and that the loading is also significantly reduced for this profile. This amount of geometric camber corresponds to  $\sim 50\%$  of the



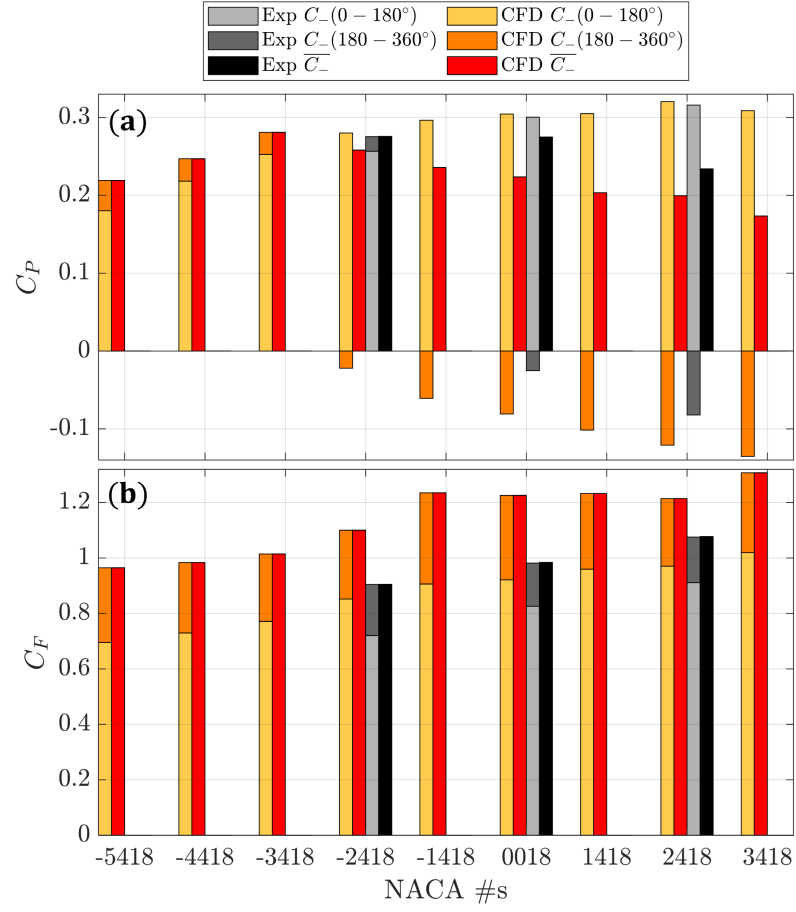


Figure 5.9: Blade-level performance of one-bladed turbines simulated using 2D-URANS, at  $\lambda = 2$ ,  $c/R = 0.49$  and  $\alpha_g = -6^\circ$  about the quarter-chord, across an expanded range of cambers with constant thickness and position of max camber (NACA X418). (a) Time-averaged  $\overline{C_P}$  and, (b)  $\overline{C_F}$  and their contributions from the upstream and downstream portions of the rotation over an expanded range of blade camber.

virtual camber, leading to a net camber of +3%, assuming the limiting case of  $\lambda = \infty$  is representative of the virtual camber effect. Comparing the loading and power generation of this case to the simulated symmetric NACA 0018 performance shows a 25.6% increase in  $\overline{C_P}$  and 17.2% decrease in  $\overline{C_F}$  at the same  $\lambda$ .

#### 5.4 Discussion

Results from this study indicate that, at or above the optimal tip-speed, performance can be increased by reducing the magnitude of net camber. This is consistent with reductions in both detrimental drag and pitching moment caused by camber. Recent work by Snortland et al.[88] showed that, at high tip-speed, performance is limited by downstream drag and the pitching moment, as the blade-relative velocity tends toward the tangential velocity, and the dynamics become more steady-state. Similarly, loading is dominated by lift production normal to the blade in this regime. The speed of downstream flow recovery, seen to be vastly different between different cambered blades, would further enhance the disparity. The hypothesis that enhancements to blade lift in the upstream portions of the rotation would enhance overall performance turns out only to be valid when the downstream flow recovery has a reduced influence (and only to a limited extent), such as seen at lower tip-speeds.

While tip-speed dependent, the results of the current study suggest that a positive net camber is likely desired as, at the  $c/R = 0.49$  explored in this work, the magnitude of virtual camber (+6%), is greater than the -2% to -3% geometric camber that was found to be optimal at a tip-speed ratio of two. Previous studies have found negative or positive geometric camber to be optimal. While variations in tip-speed, approach, or objectives may partially explain this, we hypothesize that differences in the magnitude of the virtual camber contribute significantly to this reported variation. If differences in flow curvature ( $c/R$ ) and resulting virtual camber, calculated for an infinite tip speed, are taken into account (and terminology and sign conventions modified to match the current paper), results from earlier cambered studies appear more consistent. The optimal foil of Kirke [20], from a self-starting perspective, was a NACA 4418, resulting in a net camber of +5.6% ( $c/R=0.123$ , virtual camber = 1.6%). The optimal profile from Bausas and Danao [26] ( $c/R = 0.1$ ,

virtual camber = 1.3%) had a net camber of  $-0.2\%$ . Zanforlin [22] found an optimal profile ( $c/R = 0.33$ , virtual camber = 2.1%) had a net camber of 4.1%. Differences in the optimal value likely still stem from variations in the objective, such as power optimization [26], self-starting [20], or wake recovery [22], as well as the tip-speeds examined, or the overall blade profile (such as thickness). Direct comparison with prior work, however, remains challenging due to differences in rotation rates, as well as validation or modeling approaches. Simplified models, such as DMST, blade element momentum, and vortex panel, can fail to accurately capture the complex, unsteady separation present in cross-flow turbines [11]. This limitation is exacerbated by the the limited availability of experimental data for validation, as well as the common practice of neglecting the impact the pitching moment [56]. Despite this, the relatively small differences in optimal camber magnitude, when combined with our results, suggest that a net camber of close to zero or slightly positive may be beneficial across the design space. Furthermore, our work highlights the utility of a negative geometric camber to reduce excessive virtual camber and enhance efficiency.

Although this work fixed the mounting point and geometric pitch (matching the optimum for the same turbine with symmetrical foils), it is perhaps unsurprising that more consistent trends are revealed when re-examining these previous studies using net camber. Because virtual camber and virtual pitch are primarily governed by  $c/R$  and largely insensitive to blade pitch and mounting point (see appendix B.1), the use of net camber provides a more consistent basis for comparison across designs. While net camber seems to reveal broader trends, our conclusions are restricted to the turbine design studied here. And though pitch and mounting point do not alter virtual camber, their combined effects with virtual pitch to produce net pitch, and their interactions with net camber, merit further investigation, which is the topic of Chapter 6.

## 5.5 Conclusions

This work has shown that ideal blade camber depends on  $\lambda$ , with a positive geometric camber favored (if only slightly) at low rotation rates, where upstream performance is critical to overall power production, and a lower geometric camber favored at higher rotation rates, where the downstream region and blade stall recovery dominate the overall perfor-

mance [88]. This is in contrast to the assumption, based on linear airfoil behavior, that positive camber should enhance total performance due to improved lift in the upstream sweep, independent of rotation rate. Explorations of phase-averaged performance indicate that changing camber exhibits trade-offs in performance between the upstream and downstream regions. Increasing camber, which has been shown to benefit the upstream power production, is coupled with a drop in performance in the downstream portion of the rotation at high tip-speed-ratios. Overall performance is a subtle balance between upstream power production and downstream flow recovery, and the optimum camber will depend on final turbine operational conditions.

This work, therefore, highlights the significant potential of negative geometric camber to enhance turbine efficiency and loading. For the tested turbine with a  $c/R$  of 0.49, power output improvements of up to 20% were observed compared to the same turbine with symmetric blades beyond optimal  $\lambda$ . At the same time, peak and average loading were reduced by up to 8% and 13%, respectively. If the blade could dynamically switch between positive and negative camber throughout the cycle, the performance enhancement is estimated to be even greater, up to 61% of that of the symmetric foil, if operated at higher tip-speed ratios, while reducing loading by 6.5-8.2%. More negatively cambered blades show broad benefits that may prove crucial to practical turbine deployments. Reduced maximum loading and torque, achieved through a weaker power stroke, combined with faster flow recovery during the downstream rotation, will produce smoother power delivery and structural advantages. In addition, the performance curves of the negatively cambered blades showed an expanded range of  $\lambda$ 's which produced positive power, increasing the area under the  $C_P - \lambda$  curve. This, broadening of the domain of power-producing  $\lambda$ , is critical in applications with large-scale turbulence and significantly fluctuating inflow velocities, suggesting that negatively cambered blades may have additional value in certain field deployments. Furthermore, as more negatively cambered blades are utilized, power generation shifts from the upstream peak to the downstream region, reducing the peak-to-average ratio and the variability of power generation, which has beneficial implications for generator efficiency and grid connection.

In this study, we have interpreted blade camber as the net camber, the sum of geometric

and virtual camber. When doing so, a net camber of approximately +3% (or −3% geometric camber) is found to be optimal for power generation at this  $c/R$  at a tip-speed ratio close to optimal for the two-bladed turbine ( $\lambda = 2$ ). This corresponds to a geometric camber of around 50% of the virtual component. Furthermore, reanalysis of datasets from the literature suggests that a net camber of close to zero or slightly positive may be optimal more generally, despite the many techniques and objectives used in these previous studies.

The net camber thus appears to be a useful tool for comparing turbines at different  $c/R$  and reconciling past studies. However, we must be cautious. PIV results from the current study show that the shedding and reattachment processes change nonlinearly with variations in geometric camber (promoting either leading- or trailing-edge stall, for example). It is thought that virtual camber is unlikely to have precisely the same influence as geometric camber on local pressure fields, flow separation, and reattachment processes. As a result, the net camber, while a promising metric, should be used only for guidance at present until more conclusive results about its generality can be obtained.

Any global parameters that alter the phase of upstream blade stall or flow induction, such as blade pitch, Reynolds number, confinement, or turbine solidity, might be expected to shift this balance between upstream and downstream, and therefore the optimum net camber for a given tip-speed ratio. Since the relative performance of the cambered blades has been found to be tip-speed ratio dependent, the relative performance of more positive or negative cambered blades shown in Fig. 5.9 may change at more extreme values of  $\lambda$  or Reynolds number. Variations in  $c/R$  will also affect the optimum turbine geometry by altering the total net pitch and camber, and so blade pitch and shape must be tuned simultaneously. Although virtual camber is relatively insensitive to changes in pitch and mounting location, the coupled effects of  $c/R$ , blade pitch, and airfoil profile require additional investigation (Chapter 6). Finally, while this work has focused on a single family of NACA airfoils with a fixed maximum camber location at 40% chord, future studies should explore other airfoil families and camber locations to verify the generality of our observations.

Finally, blade geometric camber is seen to have a significant impact on cross-flow turbine performance, loading, and hydrodynamics, providing a basis for tailoring cross-flow turbine blades to specific needs. Collectively, our work highlights the complex interplay among blade

camber, overall turbine geometry, and operation, with net camber appearing to capture the combined influences of virtual and geometric camber, despite nonlinearities in the flow-field response to changes in geometric camber. It is demonstrated that the operating range of tip speeds and loading should also be considered, with multifaceted design implications extending beyond turbine efficiency to include power quality and structural integrity.

## Chapter 6

# HARNESSING THE INTERACTIONS OF PITCH, CAMBER, AND THE CHORD-TO-RADIUS RATIO OF CROSS-FLOW TURBINE ROTORS AND TO OPTIMIZE PERFORMANCE

This work contains material expanding upon the EWTEC conference paper [93], and is presently being drafted for publication with coauthors Aidan Hunt, Brian Polagye, and Owen Williams.

To further generalize the implications of net camber for turbine design explored in Chapter 5, this chapter investigates the interrelated effects of parametric variations in geometric pitch and  $c/R$  along with a wider range of geometric cambers. Chapters 4 and 5 have demonstrated that cross-flow turbine performance depends on tightly coupled geometric and hydrodynamic interactions near the blade, and existing semi-analytical models (e.g., double multiple streamtube theory) lack the fidelity required for reliable optimization [94].

### 6.1 Introduction

Cross-flow turbine blades operate in inherently unsteady, cyclically varying flow, creating strong coupling between blade geometry and local flow curvature (see Section 5.3) and complicating both modeling and design decisions. While key geometric parameters,  $c/R$ , blade geometric pitch ( $\alpha_g$ ), and geometric blade camber ( $\gamma$ ), are known to influence performance, their coupled effects have not seen comprehensive consideration. Building on Chapter 5, which examined geometric camber effects at fixed  $c/R = 0.49$ , changes in  $c/R$  will impact virtual pitch and camber such that blade geometric pitch and camber may need to be tuned in unison to compensate. This chapter, therefore, investigates the individual and coupled influences of  $c/R$ ,  $\alpha_g$ , and  $\gamma$  on turbine performance through the lens of flow curvature transformations. By clarifying the connections between these geometric parameters and the analogous virtual effects, we aim to advance a more unified understanding of cross-flow turbine optimization and provide practical insights for future designs.

Chapter 5 showed that geometric camber is a key but nontrivial design parameter in cross-flow turbines affecting lift generation through the cycle. Its optimal value arises from a balance between upstream power production and downstream flow recovery, which shifts with the direction of geometric camber and the tip-speed ratio. It was demonstrated that many of the conflicting results in the literature about the effects of cambered blades likely stem from differing  $c/R$  which are rarely isolated from their effects on virtual camber, compounded by differing experimental objectives. While Hunt et al. [7] qualitatively discusses the increase in flow curvature with increasing  $c/R$  for symmetrical blades. Recent work by Reingewirtz and Greenblatt [48] evaluated how virtual camber affects performance and stall behavior of a symmetric blade of a two- and three-bladed turbine with equal solidity,  $c/R = [0.75, 0.5]$ . Pressure profiles confirm virtual camber significantly alters their aerodynamics; however, differences in induction and blade-wake interactions between the two configurations introduce additional performance variations, complicating attempts to isolate virtual camber. However, the quantitative examination of how geometric camber interacts with flow curvature effects and virtual pitch remains unclear. While Migliore and Wolfe [5] and Rainbird et al. [95] have shown that virtual camber has a noticeable effect on performance and should be accounted for, which has been incorporated with mixed success into some reduced order modeling [95, 96], flow curvature analysis has not been systematically used to evaluate experimental results and predict cross-flow turbine performance outcomes. These gaps, together with the results of Chapter 5, highlight an opportunity to expand the quantitative analysis of  $c/R$  and camber, linking performance more directly to both geometric and virtual geometries through conformal transformation.

While the prior chapter maintained constant blade pitch (optimal for the  $c/R$ ), changes in  $c/R$  alter virtual pitch (Fig. 5.3 above) and are therefore expected to interact with  $\alpha_g$ . Recent experimental work by Hunt et al. [7] showed that optimal geometric pitch is primarily dependent on the  $c/R$ , with weaker dependence on the number of blades or Reynolds number. Specifically, as  $c/R$  increased (leading to a more positive virtual pitch), a more negative geometric pitch is needed to be optimal. This indicates that, as with camber above, geometric and virtual pitch may interact directly.

To develop a broader understanding of how rotor design variables affect turbine dynam-



ics, it is informative to examine the behavior of well-understood, synonymous variables from steady, rectilinear aerodynamics. These relationships are visualized in Fig. 6.1. Changing geometric pitch can be equated to a shift in angle-of-attack, resulting in movement of the nominal operating point of an airfoil diagonally along a lift coefficient-versus-angle-of-attack ( $C_L - \alpha$ ) curve (Fig. 6.1 blue) [97]. For a cross-flow turbine, if hysteresis and induction changes are neglected, changing geometric pitch affects lift at the start of the rotational cycle ( $\theta = 0^\circ$ ). Conversely, airfoil camber produces a shift of the whole lift curve to the left, by some zero-lift angle-of-attack ( $\alpha_{L0}$ ) (yellow). This change typically increases the maximum lift prior to stall and is accompanied by an increasingly parasitic pitching moment (red). Each change in these variables has a secondary effect on the range of angles-of-attack at which the flow remains attached to the foil. This range is critical for cross-flow turbines due to their cyclic and periodic variation in angle-of-attack (Fig. 2.2). Furthermore, since changes in  $c/R$  affect the optimal tip-speed ratio (via solidity), virtual pitch, and virtual camber, it will compound changes in the  $C_L - \alpha$  curve behavior of the blades. Even from this simplified model, it is clear that pitch and camber are linked by their effect on lift production and should be explored in conjunction to achieve a better understanding of cross-flow turbine behavior. While drag and pitching moment both have implications for the downstream sweep and overall turbine performance [88, 98], will also be affected, the following discussion will largely focus on lift for the sake of brevity.

Making direct comparisons to traditional rectilinear airfoil theory and thin airfoil theory is a considerable oversimplification of the complexity present in cross-flow turbines. Dynamic stall and transients from cyclic fluctuations in angle-of-attack (Fig. 2.2) make direct application of these theories questionable. However, the understanding of general airfoil behavior gained from rectilinear flow may be used to provide a pathway toward greater understanding of cross-flow turbine design variables. Further, this analysis is shown to align with the results of Chapter 5. In the upstream region where the majority of power is produced, positive  $\alpha_n$  is observed, lift to dominate over drag, and more positive camber is seen to typically augment performance. Conversely, in the downstream region, the nominal model for angle-of-attack provides a questionable (lack of induction) negative value, and performance depends on the ability to minimize power losses through improved flow

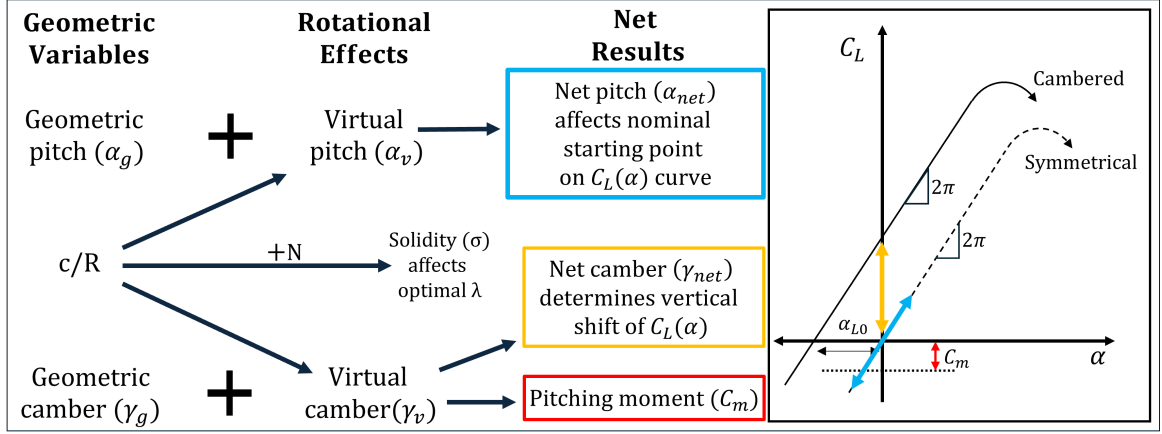


Figure 6.1: Theoretical impacts of pitch, camber, and  $c/R$ , and their expected influence on blade lift and pitching moment developed from a simplified steady aerodynamic perspective.

recovery, aided by negative camber [88]. In addition, Snortland et al. [56] showed that the pitching moment about the quarter-chord of the blade that arises from virtual camber increasingly opposes power production as  $\lambda$  increases, especially in the downstream sweep. The alignment of these turbine design trends with rectilinear theory becomes more evident as  $\lambda$  increases toward infinity (the limiting case of no inflow or high induction), where the nominal angle-of-attack approaches a steady-state condition that approximates the classical aerodynamic behavior above.

Considering the rectilinear aerodynamic expectations presented (Fig. 6.1), and observed in prior work both virtual and geometric parameters appear to influence performance in similar ways. Section 5.3.1 showed that virtual geometry (camber and pitch) increases with larger  $c/R$  and is largely independent of the corresponding geometric values. Because of their independence and similar behavior, we expect their behavior to be additive and can be evaluated in isolation or combined into net parameters. While the concepts of net camber and net pitch were introduced in Chapter 5, this work will directly investigate change and evaluate the utility of these parameters, and so explicitly defining these net variables in equation form is beneficial. We define net pitch as

$$\alpha_{net} = \alpha_v + \alpha_g$$

which represents the contributions from virtual pitch (or incidence) and geometric pitch. Similarly, net camber is defined as

$$\gamma_{net} = \gamma_v + \gamma_g$$

which combines virtual,  $\gamma_v$ , and geometric,  $\gamma_g$ , camber and is approximately equal to the flow-curvature transformation applied to a cambered foil. The definition of these net variables implicitly assumes that the effects of virtual and real geometry on the flow and turbine performance can be superimposed linearly. Although likely a simplification, it is the simplest conceptual approach and a logical starting point. By assuming these parameters are valid descriptions of the local blade aerodynamics, they are expected to exhibit a net behavior akin to changes in angle-of-attack or camber of a foil in rectilinear flow, shown in Fig. 6.1.

The rectilinear approximation of variable interactions and the results of Chapter 5 highlight an opportunity to apply a conformal transformation (e.g., Migliore and Wolfe [5], Akimoto et al. [46], etc.) to expand quantitative analysis of  $\alpha_g$  with  $c/R$  and camber and link performance more directly between geometric and virtual variables. While prior work has explored some isolated geometric effects on performance, coupled changes in virtual camber and pitch, producing multiple effects and rendering true isolation of parametric influences uncertain. This chapter explores the interplay between the  $c/R$ , camber, and geometric pitch angle on cross-flow turbine performance through experimental testing of three different  $c/R$ , between three and seven geometric cambers at each  $c/R$ , and five different geometric pitches at each  $c/R$ -camber combination. Overall, this results in 75 unique combinations of pitch, camber, and  $c/R$ , that are tested across a range of tip-speed ratios. The selected geometries were chosen to maintain equal net camber and isolate individual virtual or geometric parameters. Critically, the flow curvature (calculated using Migliore and Wolfe's [5] conformal transformation method) is accounted for when considering blade relative virtual pitch, and camber. This serves to quantify the extent to which camber,  $c/R$ , and  $\alpha_g$  are linked through flow curvature and whether they have a consistent impact on performance. Additionally, parallels between pitch and camber effects are explored utilizing the zero-lift angle-of-attack to draw broader design implications. Through simultaneous

consideration of pitch, camber, and  $c/R$ , and by comparing resulting virtual camber and pitch, our aim is to isolate individual effects and demonstrate their distinct influence on cross-flow turbine performance.

## 6.2 Methods

This work was conducted utilizing the same test facilities and experimental apparatus as those shown in Section 3.1 and discussed in detail in Section 4.2. Different flow conditions were employed and were chosen to match the non-dimensional conditions of Hunt et al. [7]. Water temperature was increased relative to Chapter 4 and 5, and held at  $39^{\circ}\text{C} \pm 0.1^{\circ}\text{C}$  and maintained by a pool heater to keep non-dimensional parameters constant across all tests. The freestream velocity was held constant at 1.1 m/s with a TI of 3-4%. The dynamic water depth was held at 0.48 m, resulting in a constant nominal blockage of  $\approx 11\%$ . These different conditions maximize the achievable Reynolds number in the facility, thereby reducing transition region artifacts.

### 6.2.1 Experimental Apparatus

The freestream velocity and dynamic depth were recorded using the ADV and ultrasonic transducer at the same positions and sample rate relative to the turbine noted in Section 4.2. The motor and load cells all remained the same as specified in Section 3.1 and 4.2. The experimental apparatus is similar to that used in prior work [7, 25, 60, 99] and is shown in Fig. 3.1a. The turbine position in the water column was fixed at 17 cm below the free surface and resulted in a constant submergence Froude number,  $Fr_s = 0.85$ , depth-based Froude number  $Fr_h = 0.51$ , and diameter-based Reynolds number  $Re_D = 2.7 \times 10^5$  across all tests.

To assess the interactions of the chord-to-radius ratio with virtual camber, a range of blade profiles were explored. All blades have a span of 23.4 cm and an approximate diameter of 17.2 cm. Three chord lengths were tested: (2.03 cm, 4.06 cm, and 6.12 cm), corresponding to nominal  $c/R = 0.25, 0.49$  and  $0.74$ , and from 50% to 150%  $\times$  the baseline utilized in Chapter 5. To allow the smallest chord length blades (2.03 cm) to interface with the same hole mounting pattern, the final 4% of each end was gradually lofted to a

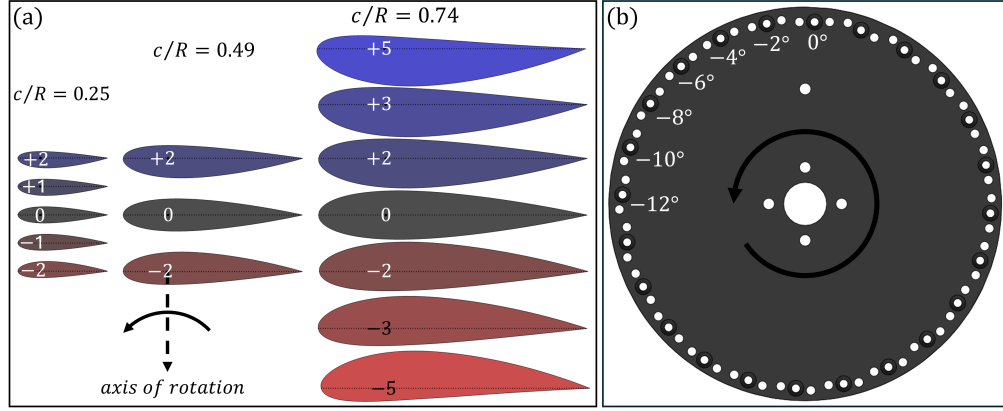


Figure 6.2: (a) NACA x418 profiles of all tested blades with camber percentages indicated on the blades and corresponding chord-to-radius ratios indicated by column header. “Blue” blades are the mirror image of the “red” with the sign of camber designated with respect to the axis of rotation. (b) Representative schematic of the turbine end plate used to systematically vary geometric pitch.

NACA 0018 profile with 4.06 cm chord, following the same approach as Hunt et al. [7]. This maintained the inner 92% of the span at the target chord and profile. Five different NACA airfoil profiles were used (Fig. 6.2a): a symmetrical NACA 0018 and four cambered blades with the same thickness. The cambered blades fixed the position of maximum camber at 40% of the chord, following a NACA x418 profile, where “x” was the percent camber,  $\gamma_g$ . The mounting structure on the blades was aligned with the chord line such that a single cambered blade could be used to test the effects of positive and negative blade camber. The blade profiles were selected to highlight cases with comparable net camber, geometric camber, and ratio of geometric-to-virtual camber across the three  $c/R$ . Experiments were conducted with  $\gamma_g = -2\%$  to  $+2\%$  camber for all  $c/R$ , with a wider range of  $\gamma_g$  tested at the highest  $c/R$  where a larger geometric camber was necessary to counteract the elevated virtual camber and achieve a similar net camber.

These profiles were selected to maximize the number of parametric comparisons while minimizing the number of blades fabricated. Additionally, three  $c/R = 0.49$  blades (NACA 0018 and NACA 2418 profiles) were fabricated to examine how single-bladed trends extended

to two- and three-bladed turbines. An overview of test cases is shown in Table 6.1. Five geometric pitch values with a  $2^\circ$  discretization were tested for each combination of  $c/R$  and camber. To minimize the size of the test matrix, only pitches near the optimal, along with adjacent points on either side, were captured for each configuration, rather than testing the full range for each camber and  $c/R$ .

Table 6.1: Overview of the 105 explored turbine geometric configurations tested in this work.

|                       | $N$  | $c/R$ | $\alpha_g[^\circ]$ | $\gamma_g[\%]$         | Total Cases |
|-----------------------|------|-------|--------------------|------------------------|-------------|
| Single Bladed Tests   | 1    | 0.23  | 0:-2:-8            | -2, -1, 0, 1, 2        | 25          |
|                       |      | 0.49  | -2:-2:-10          | -2, 0, 2               | 15          |
|                       |      | 0.74  | -4:-2:-12          | -5, -3, -2, 0, 2, 3, 5 | 35          |
| Increased Blade Count | 2, 3 | 0.49  | -4:-2:-10          | -2, 0, 2               | 30          |

The geometric pitch angle was varied using disk end plates with different blade mounting hole patterns as in Hunt et al. [7] (Fig. 6.2b), in contrast to the aerodynamic struts used in Chapters 4 and 5. All unused holes were covered with electrical tape to reduce drag and increase the signal-to-noise ratio for blade-level contributions. The hole pattern was arranged to pitch the blade about its quarter-chord location and configured to allow mounting different numbers of equally spaced blades. The radial mounting distance was arranged such that the outermost portion of the blade, measured from the axis of rotation, was held constant for the symmetrical blade across all pitch angles for the middle chord-to-radius ratio ( $c/R = 0.49$ ).

Each geometric combination of pitch, camber, and  $c/R$  was tested using constant speed control and swept through a range of  $\lambda$ . The minimum  $\lambda$  step size was 0.05 and utilized about the peak of the performance curve. The spacing between successive  $\lambda$  set points was progressively expanded to 0.1, 0.2, and 0.5 at higher and lower  $\lambda$  values to uncover a broader proportion of the performance curve and ensure optimal performance was captured while maintaining feasible experimental duration. The minimum  $\lambda$  tested was 0.5, while the

maximum achievable  $\lambda$  was constrained by the DAQ motor command voltage to  $\sim 52$  rad/s, corresponding to  $\lambda \approx 4$ .  $\lambda$  was intentionally limited below this maximum to avoid excessive loading on the test rig for single-bladed tests of  $c/R = 0.74$ .

### 6.2.2 Performance Evaluation

All post-processing and filtering were the same as in Chapter 3.1 and were conducted within MATLAB. The use of the disk end-plate mounting structure, along with a wider range of geometric variables, introduces additional considerations into the analysis. While the outermost diameter was held constant for a symmetrical blade with a 4.06 cm chord length, the radius to the quarter chord,  $R$ , changed up to 1.9% across the range of  $\alpha_g$ . Further, as  $c/R$  and/or camber changed, the outermost diameter also saw  $\pm 5\%$  variation relative to the baseline condition. Because of this, the true radius to the quarter chord,  $R$ , and outermost diameter,  $D_o$ , of each configuration were incorporated into the non-dimensionalization of performance (noted in Chapter 3.1 and Section 5.2) and virtual camber calculations.

Since forces and moments are measured at the turbine center shaft, it is necessary to subtract support losses and account for loading to resolve blade-level dynamics. In addition to the turbine tests (Table 6.1), supplemental experiments were conducted with the blades removed under matching kinematic and flow conditions to measure the effects of the support structure on overall performance. Further supplementary experiments were conducted in air for each single-bladed turbine to characterize the inertial parameters  $mr_g$ ,  $\delta\theta$ , and  $I$  necessary to isolate the fluid forcing on the blade. Using the resulting support losses and inertial parameters for each geometric configuration, blade-level loading is estimated using the methods developed by Snortland et al. [56], Strom et al. [59], as discussed in Section 3.2.2.

Although the channel blockage ratio was held approximately constant at  $11\% \pm 0.6\%$  across experiments, the degree to which blockage augments turbine performance depends on the thrust coefficient [100], which is influenced by rotor geometry. For example, at a given  $\lambda$ , turbines with more positive  $\alpha_g$ , geometric cambers, and/or larger  $c/R$  have higher thrust coefficients, and thus experience greater performance gains from blockage. However, prior

work [101] has shown that single-bladed turbines—the primary focus of this work—are less affected by blockage than multi-bladed rotors due to their lower solidity, which generally results in lower thrust at a given  $\lambda$ . Similarly, while analytical blockage corrections could be applied to the measured performance data [102, 103], these methods have not been validated for blade-level forces or support structure correction, and cannot be used in a phase-average manner. Consequently, no corrections have been applied to these data.

### 6.2.3 Virtual Camber Calculation

Following the analysis in Section 5.3.1, virtual camber and pitch were estimated using Migliore and Wolfe’s [5] method. While other authors have developed similar virtual transformations, each produces similar results for  $c/R \lesssim 0.6$  and their gradual spread at higher  $c/R$  is less significant than the uncertainty in the underlying assumptions [34, 45, 46, 49]. Although Migliore and Wolfe’s [5] method assumes a thin airfoil, neglects induction, and does not model stall, which are all present in a realistic cross-flow turbine, it remains a valuable tool for examining the effects of virtual geometry, and we adopt it in this work to explore relationships between blade camber, pitch, and  $c/R$ . Applying this transformation to the three  $c/R$  explored in this chapter, their nominal virtual cambers, in order of increasing  $c/R$ , were 3%, 6%, and 9%, respectively.

### 6.2.4 Estimation of Zero-lift Angle-of-Attack

To evaluate equivalencies between pitch and camber, we utilize  $\alpha_{L0}$  to draw parallels between their effects on lift (shown in Fig. 6.1). Thin airfoil theory provides a direct way to calculate an estimate zero-lift angle-of-attack, for a given camber line. While typically not applicable to cross-flow turbines, it can be used here by applying the same assumptions and justifications adopted for virtual camber and pitch calculations (e.g.  $\lambda \rightarrow \infty$ , induction free, and thin blades defined by their camber lines). To estimate  $\alpha_{L0}$  geometric, virtual, or net profile, the process is straightforward: normalize the corresponding camber line to a unit chord and apply the equations outlined in Appendix C.2. Values of  $\alpha_{L0}$  were calculated for all tested rotor geometry.



### 6.3 Results

This section presents performance changes across the range of tested geometries and examines the influence of each geometric parameter. The observed behavior is then connected to flow-curvature calculations of virtual camber and pitch, as well as the zero-lift angle-of-attack, to unify their effects.

#### 6.3.1 Time-average Optimal Efficiency

The performance of all tested one-bladed rotor geometries at their respective optimal  $C_P$  conditions is extracted, and metrics of interest are shown in Figs. 6.3 and 6.4.

Comparing the changes in performance between the rows of Fig. 6.3 shows how performance changes with  $c/R$ . Two main trends can be seen. First, as  $c/R$  increases, the maximum  $C_P$  also increases. Second, with increasing  $c/R$ , the optimal blade pitch shifts to more negative values consistent with the findings of Hunt et al. [7] for symmetrical blades. These trends hold across the tested range of geometric camber. The increase in peak  $C_P$  with  $c/R$  aligns with prior observations for cross-flow turbines at similar Reynolds numbers [7]. In the middle and right columns of Fig. 6.3, the single-bladed turbine  $C_P$  is shown for the upstream half of the cycle ( $\theta = 0 - 180^\circ$ , middle column), and the downstream sweep ( $\theta = 180 - 360^\circ$ , right column), allowing us to identify where performance differences originate. It is observed that increasing  $c/R$  leads to greater power production upstream, accompanied by downstream degradation.

To evaluate the effects of changing geometric pitch on optimal performance, we examine changes in performance moving vertically within each tile, thereby holding geometric camber and  $c/R$  constant. It is clear that there is an optimum blade pitch for each camber, even if the breadth of this optimum increases with larger  $c/R$ . For a fixed  $c/R$ , the optimal pitch is largely constant (i.e., general vertical contours for  $C_P$ ). As  $c/R$  increases, the optimal pitch decreases by several degrees, but the optimum pitch is seen to be largely insensitive to geometric camber. In retrospect, since  $c/R$  determines  $\alpha_v$ , expected to directly interact with  $\alpha_g$ , it is reasonable that the optimum  $\alpha_g$  is almost exclusively a function of  $c/R$ . This optimum appears to be governed by a balance between upstream and downstream

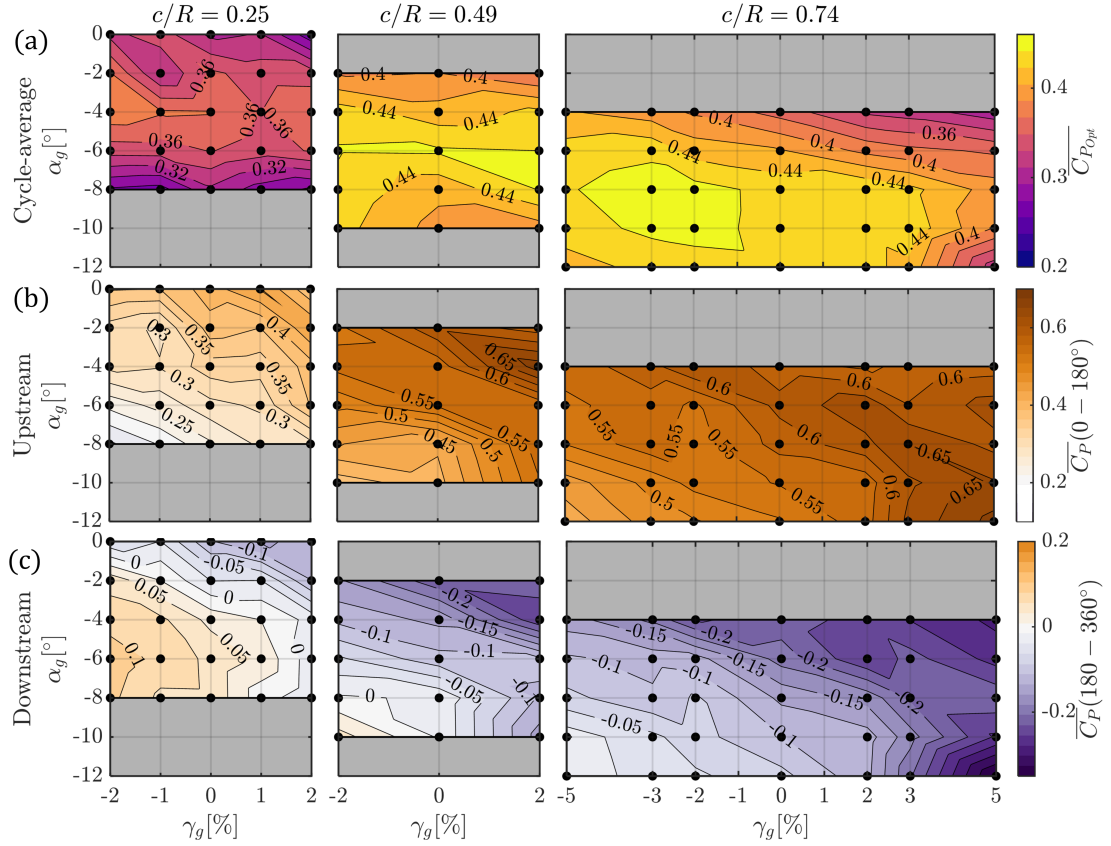


Figure 6.3: Performance at optimal tip-speed ratio for one-bladed turbines as a function of  $c/R$  (columns), geometric camber,  $\gamma_g$ , and geometric blade pitch,  $\alpha_g$ . The rows indicate (a) the maximum  $C_P$ , (b) time-averaged upstream, and (b) time-averaged downstream regions under equivalent conditions. Black points indicate tested configurations.

influences, with more positive  $\alpha_g$  producing higher upstream power output, and vice versa for downstream.

Similar to what was observed in Chapter 5 at a lower Reynolds number, changing camber, while holding pitch and  $c/R$  constant (moving vertically within each tile) shows that more negative camber seems to slightly increase performance. At the highest  $c/R$ , we observed that the most positive camber values, +3% and +5%, have a significant reduction in performance. However, compared to pitch or  $c/R$ , changes in geometric camber appear to have a smaller and less directly proportional impact on optimal  $C_P$ . Examining the change

in performance between the upstream and downstream (Fig. 6.3b-c), shows that geometric camber amplifies the difference between the upstream and downstream, with more positive camber increasing upstream power production and downstream degradation. We also note that for the most negative cambers tested at the two smallest  $c/R \leq 0.49$ , downstream power production occurs, a behavior that is rarely observed for symmetric blades (see Chapter 5).

Broad trends in performance with geometric camber and pitch match expectations from steady-state aerodynamics. In the upstream portion of the rotation, more power is produced as geometric camber becomes more positive (increasing lift) and  $\alpha_g$  becomes more positive, increasing the angle-of-attack. Meanwhile, in the downstream portion of the rotation, the trends reverse, with more negative camber and pitch benefiting performance.

To understand how the optimal operating point changes with parameters, we are examining the optimal tip-speed ratio for each geometric configuration (Fig. 6.4a). As  $c/R$  increases, optimal  $\lambda$  decreases, consistent with the expected inverse relationship between optimal  $\lambda$  and solidity [7, 35, 104]. At a fixed  $c/R$ , the optimal  $\lambda$  weakly depends on  $\alpha_g$ , and  $\gamma_g$ . The optimal  $\lambda$  slightly increases when camber magnitude is increased (either positive or negative) to the most extreme values tested, or  $\alpha_g$  is at the edges of the tested domain. Interestingly, the differences at the extremes of the configuration envelope for a fixed  $c/R$  are relatively small in comparison to  $C_P$  and approach the limit of our  $\lambda$  discretizations.

Evaluating the effects of pitch and camber on turbine forces (Fig. 6.4b,c) shows similar trends to the changes in upstream and downstream power generation. For the higher two  $c/R = [0.49, 0.74]$ , as  $\gamma_g$  and  $\alpha_g$  become more positive, the thrust coefficient largely follows suit, and the magnitude of the lateral force (negative in our coordinate system) grows as well. It is perhaps unsurprising that the conditions that lead to the largest loads occur when camber is expected to generate the largest lift and drag during the power stroke, and a more positive  $\alpha_g$  leads to stall earlier in the rotation [31]. While the  $c/R = 0.25$  results exhibit similar lateral force behavior, the thrust force sees less consistent trends with more negative  $\alpha_g$  increasing thrust for some cambers. The reasons for this difference are unclear.

Following Chapter 5, negatively cambered blades are once again seen to exhibit significant loading benefits. Negative camber can mildly improve or maintain efficiency while reducing both the variation between upstream and downstream and the time-averaged struc-

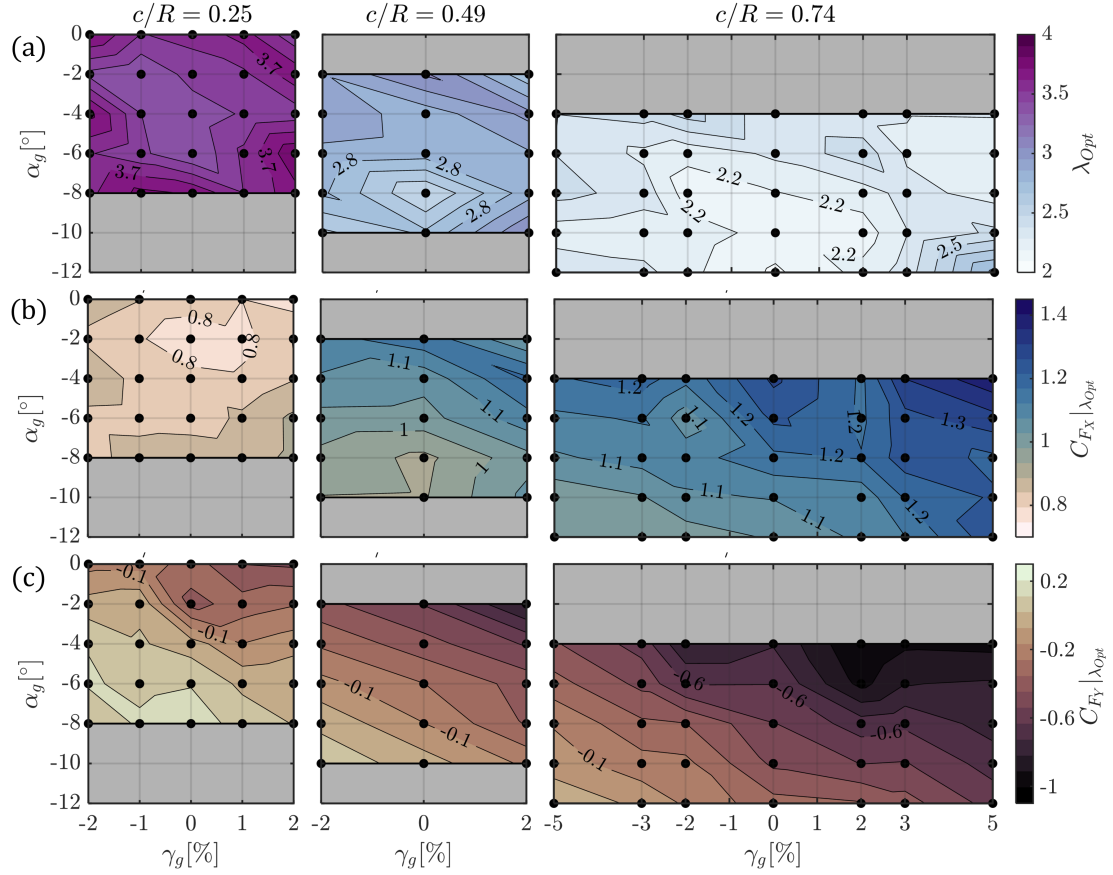


Figure 6.4: Performance metrics at optimal efficiency (matching Fig. 6.3) for a one-bladed turbine as a function of  $c/R$  (columns),  $\gamma_g$ , and  $\alpha_g$ . (a) Variation in optimal tip-speed ratio ( $\lambda_{Opt}$ ) (top row), (b) time-average  $C_{F_x}$  at  $\lambda_{Opt}$ , and (c) time-average  $C_{F_y}$  at  $\lambda_{Opt}$ . Black points indicate tested configurations.

tural loads. These benefits match observations in Chapter 5 at a lower Reynolds number and  $c/R = 0.49$ , showing that those results extend to a wider range of  $c/R$  and  $\alpha_g$ .

### 6.3.2 Generalization across $\lambda$

To this point, we have considered performance only at the tip-speed ratio corresponding to maximum blade-level performance. This operating condition is desirable up to the rated inflow velocity for a given turbine, which is driven by economic considerations. As inflow

velocity increases beyond that point, the rotation rate is adjusted to reduce efficiency and maintain time-averaged power. Additionally, results from Chapter 5 indicate that optimal camber is  $\lambda$  dependent. We therefore examine the influences of these geometric parameters across a broader range of tip-speed ratios.

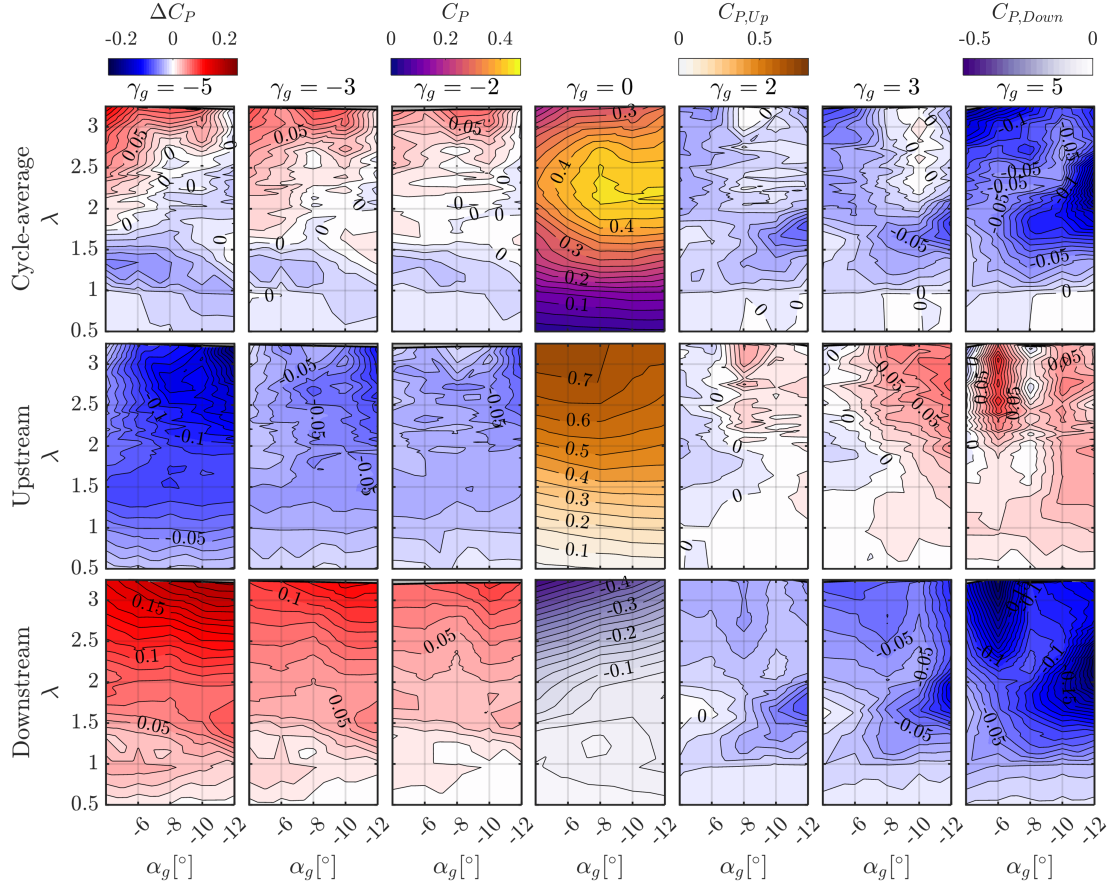


Figure 6.5: Performance as a function of geometric configuration and tip-speed ratio. (top row)  $C_P$  for  $c/R = 0.74$  for a one-bladed turbine as a function of  $\lambda$  and blade pitch  $\alpha_g$ . The geometric camber  $\gamma_g$  for each column is constant and noted. (Middle row) Upstream averaged performance,  $C_{P,Up}$ . (Bottom row) Downstream averaged performance,  $C_{P,Down}$ . Absolute performance is presented for the symmetrical-blade ( $\gamma_g = 0$ ), and the performance of cambered-blades is presented as the difference referenced to this baseline for visualization.

Figure 6.5 highlights the changes in time-averaged performances as a function of  $\lambda$ ,  $\alpha_g$ , and  $\gamma_g$  for  $c/R = 0.74$ , as well as the proportion of that performance derived from the upstream,

and downstream regions. The contours are an aggregation of all  $C_P(\lambda)$  information collected for these configurations. For the symmetrical blade ( $\gamma_g = 0\%$ ) (middle column), at relatively low ( $\lambda < 1.1$ ) or high ( $\lambda > 3$ ) tip-speed ratios, cycle-averaged performance is largely insensitive to blade pitch. Optimal performance corresponds to a relatively broad range of  $\lambda$ , but has a stronger sensitivity to blade pitch. As camber is introduced, the difference in performance with respect to the symmetrical blade at equal conditions varies with both geometric pitch and  $\lambda$ . Broadly, a more positive camber reduces efficiency (blue shading) across almost all  $\lambda$ . Two regions in particular stand out for the positively cambered foils. For  $\lambda=2$ , just below the optimal tip-speed ratio, more negative pitch is found to be detrimental, while this reverses for  $\lambda \geq 3$ , where more positive pitch is found to decrease performance. The magnitude of this detrimental change in performance increases with more positive camber. In contrast, for negative camber, more positive pitch slightly reduces performance at lower  $\lambda \leq 1.5$  but improves performance at higher  $\lambda$ , with a moderate dependence on the camber magnitude. This broadens the overall operating window, and is consistent with what was observed in Chapter 5, but is now found to be a general result across a range of geometric pitch values and  $c/R$ . While the negatively cambered blades significantly outperform the symmetric blades at tip-speed ratios  $\geq 3$ , the peak performance across cambers corresponds to tip-speed ratios of  $\approx 2.2$  (noted in Fig. 6.4a).

These trends in cycle-averaged performance can be explained by considering performance in the upstream and downstream portions of the rotation (Fig. 6.5, middle and bottom row). Positive camber leads to broad, mild improvements upstream (though interestingly, not for all values of geometric pitch), but causes more significant performance degradation downstream. Conversely, negative camber produces moderate degradation upstream for  $\gamma_g = -2\%$  and  $-3\%$ , while the  $-5\%$  results in a more significant decrease, particularly at higher  $\lambda$  and more negative blade pitch. Downstream, the negative camber greatly improves the performance, with benefits increasing with tip-speed ratio and camber, but only weakly influenced by  $\alpha_g$ . So, while the optimal  $\lambda$  is relatively consistent across geometries (Fig. 6.4), downstream performance degradation can be reduced by extending the performance curves through the use of negatively cambered blades.

### 6.3.3 Effects of Blade Count

To verify that single-bladed behavior extends to higher blade counts, all camber and pitch combinations for the  $c/R = 0.49$  cases were replicated for two- and three-bladed turbines.

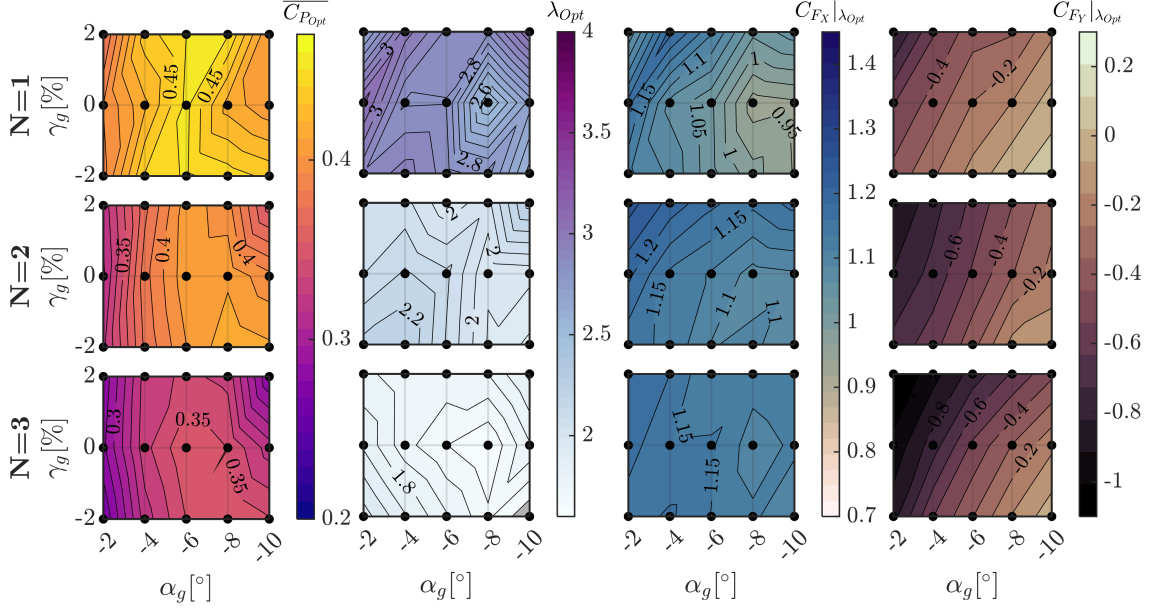


Figure 6.6: Effect of blade count (rows) on the optimal  $C_P$ ,  $\lambda_{opt}$ ,  $C_{F_X}$  and  $C_{F_Y}$  at  $\lambda_{opt}$  for  $c/R = 0.49$  as a function of geometric camber and pitch. Black points indicate configurations at which data were collected.

While the maximum  $C_P$  decreases with increasing  $N$  (Fig. 6.6), the relative trends in performance metrics are consistent with the single-bladed results. As  $N$  increases, peak performance occurs at slightly lower camber, but the optimal  $\alpha_g$  remains nearly constant. The trends with  $N$  and  $\alpha_g$  agree with prior results for symmetrical blades [7], suggesting that these behaviors likely extend with geometrically cambered blades.

Although optimal  $\lambda$  decreases with increasing blade count, the relative trends remain similar as  $N$  increases. Across all three blade counts, the lowest optimal  $\lambda$  corresponds to a symmetrical blade at  $\alpha_g = -8$ , with a higher optimal  $\lambda$  for more positive  $\alpha_g$  and any geometric camber, positive or negative. However, consistent with the single-bladed results, the variation in optimal  $\lambda$  is not strongly affected by pitch or camber for cases close to



optimal performance.

The thrust force is less affected by changes in pitch for higher blade counts, consistent with Hunt et al. [7]. This trend also holds for changes in camber. However, lateral force is sensitive to pitch and camber, particularly as blade count increases. The overall contours for  $C_{F_X}$  and  $C_{F_Y}$  are consistent with the one-bladed results (Fig. 6.3), with more negative pitch and camber both reducing  $C_{F_X}$  and the magnitude of  $C_{F_Y}$ . Consequently, with increasing blade count, more negative camber (moving vertically down the tiles of Fig. 6.6) can still improve power production and reduce time-average loads.

#### 6.3.4 Comparison of Virtual and Geometric Parameters with Phase-averaged Performance

By examining the phase-averaged performance of key geometries at peak performance, we can assess the relative effects of changing each parameter of interest through the cycle. Isolating each geometric and virtual effect allows us to verify the expectations gained from theoretical rectilinear aerodynamics discussed in the introduction and shown in Fig. 6.1. To evaluate the effects of  $c/R$  on symmetric blades and the resulting virtual behavior at finer discretization, we incorporate data from Hunt et al. [6]. These data were obtained under identical experimental conditions and shown to be repeatable across equivalent cases.

Fig. 6.7 demonstrates the manner in which blade pitch, virtual camber, and geometric camber cause similar trends in performance. The comparison is made at the optimal tip-speed ratio for each configuration. The  $c/R$  has the greatest influence on optimum tip speed, varying from 2.3–3.6 between the highest and lowest  $c/R$ , respectively. In contrast, a minimal difference in optimal tip-speed-ratio is seen across the geometric cambered cases (supported by Fig. 6.4a). Increasing toe-out blade pitch (more negative  $\alpha_g$ ) for constant  $c/R$  and camber (Fig. 6.7a) reduces performance in the upstream and increases it downstream. This is attributed to a reduction in the nominal angle-of-attack during the upstream sweep, delaying stall, but an increase in the nominal angle-of-attack in the downstream sweep, delaying reattachment. Increasing net camber while keeping net pitch constant (Fig. 6.7b,c) has a similar phase-average effect whether achieved through virtual (b) or geometric camber (c), with higher camber increasing upstream power production for



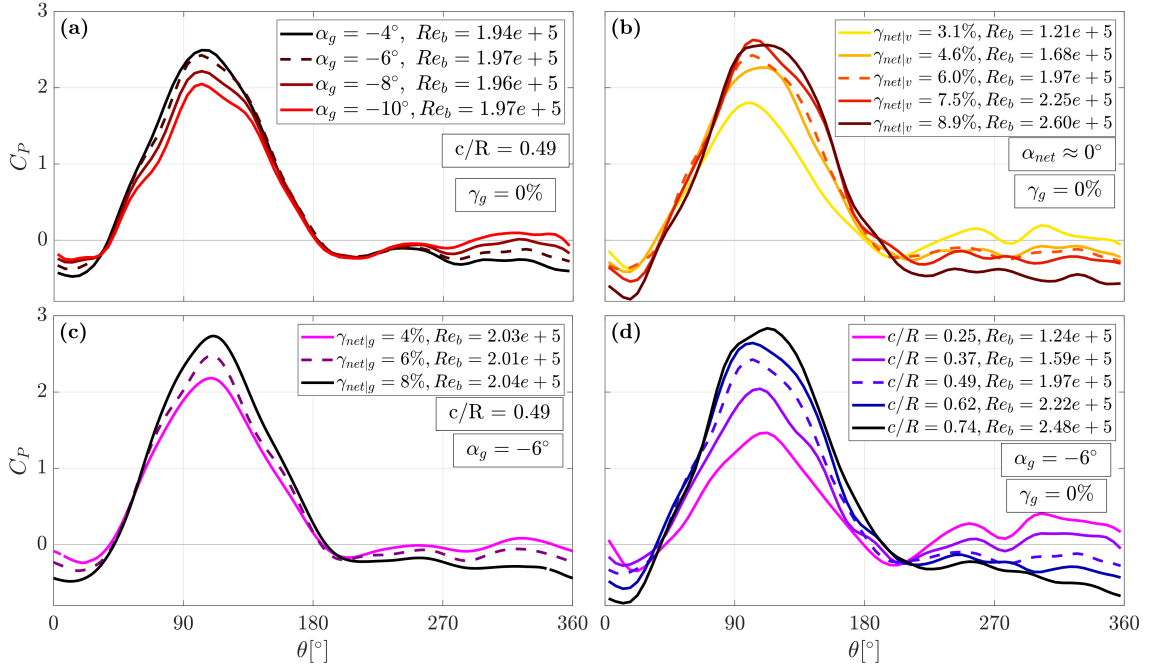


Figure 6.7: One-bladed phase-averaged performance for various turbine geometries at optimal tip-speed ratio as a function of pitch and camber. (a) Changes in blade pitch for  $c/R = 0.49$  and  $\gamma_g = 0\%$ . (b) Changes in net camber through changes in  $c/R$  ( $\gamma_v$ ) for  $\gamma_g = 0\%$ , with  $\alpha_g$  varied to maintain approximately constant ( $\pm 1^\circ$ ) net pitch. (c) Changes in net camber solely through geometric camber ( $\gamma_g$ ) for  $c/R = 0.49$  and  $\alpha_g = -6^\circ$  (constant  $\gamma_v$ ). (d) Changes in  $c/R$  at  $\alpha_g = -6^\circ$  and  $\gamma_g = 0\%$ , resulting in changes to both net pitch and virtual camber. Each line corresponds to the optimal  $\lambda$  for the given configuration, and the dashed line denotes the consistent geometry across all four cases. Data for (a-b & d) are from Hunt et al. [6] while (c) was collected here.

positive angles-of-attack and reducing it downstream. Solely changing  $c/R$  while keeping  $\alpha_g$  fixed (Fig. 6.7d) has more pronounced effect which indicates that the combined increases in virtual camber and pitch have a cumulative influence (i.e., as if the influences of Fig. 6.7a and 6.7c were combined, though perhaps not in a linear manner). This compounded influence of changes in  $c/R$  aligns with expectations from steady aerodynamic analysis in Section 6.1. The key takeaways are that all of these parameters produce similar effects on

phase-average performance (both upstream and downstream trends), and the magnitude of change produced by each of these variables is comparable, and they need to be considered in conjunction during the design and optimization of turbine geometry.

## 6.4 Discussion

Having considered the effects of each geometric parameter, we now turn to the question of integrating these parameters into a comprehensive framework.

### 6.4.1 Factors Contributing to Uncertainty: Reynolds and Blockage Affects

While the diameter-based Reynolds number and blockage are held approximately constant across these cases,  $Re_D = 2.65 \times 10^5 \pm 0.06 \times 10^5$ ) and  $\beta = 11\% \pm 0.6\%$  respectively, the influences of local Reynolds number and blockage should be considered. The blade local Reynolds number changes with chord length and tip-speed ratio and turbine thrust changes the induction, which in turn affects blockage augmentation [7]. As a result, we explore whether the changes observed in Fig. 6.7 and the prior heatmaps might be the result of Reynolds number or blockage effects, which could not be wholly controlled for.

While the true blade-level relative Reynolds number is indeterminate, due to induction-modified infow, the nominal relative velocity (Equation 2.2) is used to approximate the Reynolds number experienced by the blade as a function of  $\lambda$ . The resulting blade-relative Reynolds numbers,  $Re_b$ , are included in Fig. 6.7 and are not in a Reynolds independent regime, which occurs at chord-based Reynolds number  $Re_c \gtrsim 1.5 - 2 \times 10^5$  or diameter-based Reynolds number  $Re_D \gtrsim \times 10^6$  [24, 104–106]. Within this range, a higher  $Re_b$  is expected to delay flow separation and improve performance. This trend may partially explain the observations from Fig. 6.3, where higher  $c/R$  cases generally exhibit improved performance. However, the significant performance changes with  $\alpha_g$  (where Reynolds number is largely invariant) and similar  $C_P$  magnitude between the two highest  $c/R$  shown in Fig. 6.3, indicate that performance changes are largely attributable to geometric variables rather than  $Re_b$  alone. The notably lower performance in the  $c/R = 0.25$  case suggests that a threshold may have been crossed, at which the effects of the Reynolds number appear to be more pronounced.

Similarly, while Reynolds-number effects may also be present in phase-averaged performance trends, there are multiple reasons to conclude that this is of secondary importance. First, when comparing the geometric camber cases (Fig. 6.7c) where  $Re_b$  is approximately constant, we still see strong correspondence in performance to similar values of virtual equivalence from changes in  $c/R$  (Fig. 6.7b). This suggests that changes in  $Re_b$  have limited impact relative to flow curvature effects for  $Re_b > 1 \times 10^5$ . In addition, examination of the broader dataset from Hunt et al. [6] suggests that the magnitude of change in  $Re_b$  between different  $c/R$  cases is too small to explain the change in performance for  $Re_b = 1.6 - 2.7 \times 10^5$ . Specifically, for equivalent physical geometries tested at  $Re_b = 1.5 \times 10^5$  and  $2.6 \times 10^5$ , the difference in peak phase-averaged  $C_P(\theta)$  at optimal  $\lambda$  is 0.26 (shown in Appendix C.3) [6]. In contrast, in this work, when varying virtual camber for  $Re_b$  between  $1.2 \times 10^5$  and  $2.6 \times 10^5$ , the change in peak  $C_P(\theta)$  is 0.88 (Fig. 6.7b). This suggests that changes in virtual camber produce a more substantial change in peak performance than can be explained solely by Reynolds number.

While the blockage ratio remained approximately constant, the extent to which blockage augments turbine performance depends on the thrust coefficient, which in turn depends on rotor geometry. Thrust is most strongly affected by increases in  $c/R$ , followed by more positive  $\gamma_g$  and lastly more positive  $\alpha_g$  (Fig. 6.6b). Consequently, observed trends with constant geometry but varying  $\alpha_g$  are likely the most robust to blockage effects, whereas increasing  $c/R$  has a larger effect. This introduces a secondary coupling between blockage and design variables. Nevertheless, the focus on single-bladed turbines, which have lower solidity and thus thrust at a given  $\lambda$ , helps reduce the overall sensitivity to blockage augmentation in our results [103]. While analytical blockage corrections exist to scale performance data [25, 101, 107], their applicability to phase-resolved measurements and blade-level loading remains uncertain, and the effects of rotor geometry and confinement on thrust cannot be easily separated. As a result, the observed trends are interpreted as primarily geometry-driven, with blockage acting as a modest but non-negligible bias on absolute performance levels.

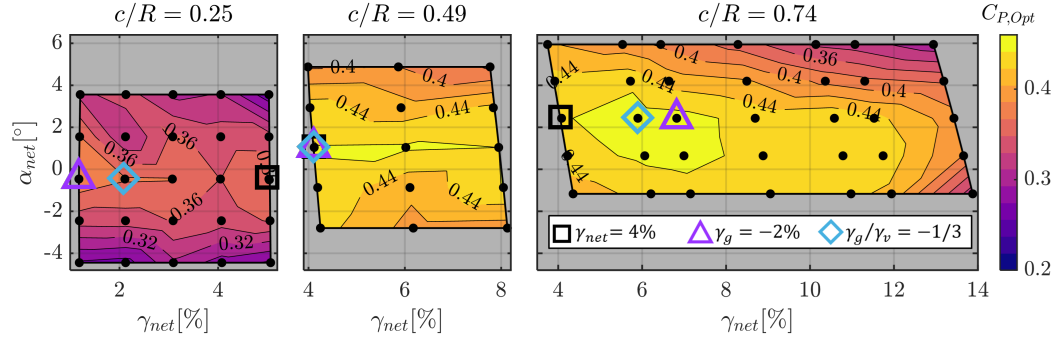


Figure 6.8: Optimal  $C_P$  as a function of net camber ( $\gamma_{net}$ ) and net pitch ( $\alpha_{net}$ ), representing the data from Fig. 6.3a against combined parameters. Symbols denote geometric configurations with parallel camber behavior across  $c/R$ .

#### 6.4.2 Evaluating the Effectiveness of “Net” Parameters

By converting the geometric axes of Fig. 6.3 into net camber and net pitch, it is possible to examine how well these parameters determine turbine performance. Because virtual camber and pitch are most dependent on  $c/R$ , the representation with net parameters (Fig. 6.8) bears strong similarities at first glance to the representation involving only geometric variables for pitch and camber (Fig. 6.3a). Critically, however, net pitch reduces the optimal pitch range from  $\alpha_{g,Opt} = [-11^\circ, -4^\circ]$  to  $\alpha_{net,Opt} = [-1^\circ, +3^\circ]$ , largely offsetting the  $c/R$  dependence of optimal  $\alpha_g$ . This suggests that optimal  $\alpha_g$  effectively compensates for the virtual pitch at a given  $c/R$ , and is therefore largely determined by flow-curvature effects.

The collapse of optimal performance as a function of  $\alpha_{net}$  is demonstrated more directly by examining line plots of performance as a function of pitch. Vertical slices Fig. 6.3a (constant geometric camber and  $c/R$ ) are extracted and normalized by the best performing  $C_P(\alpha_g)$ , to compensate for differences in performance. Figure 6.9 shows the resulting scaled performance as a function of  $\alpha_g$  (grey) and  $\alpha_{net}$  (colored). In both cases, performance decreases on either side of the optimal pitch, where normalized performance is unity. For  $\alpha_{net}$  the optimal range is noticeably narrower, reinforcing the reduced dependence on  $c/R$  when accounting for the associated change in  $\alpha_v$ .

A slight residual dependence of power-optimal net pitch on  $c/R$  remains (Fig. 6.9),

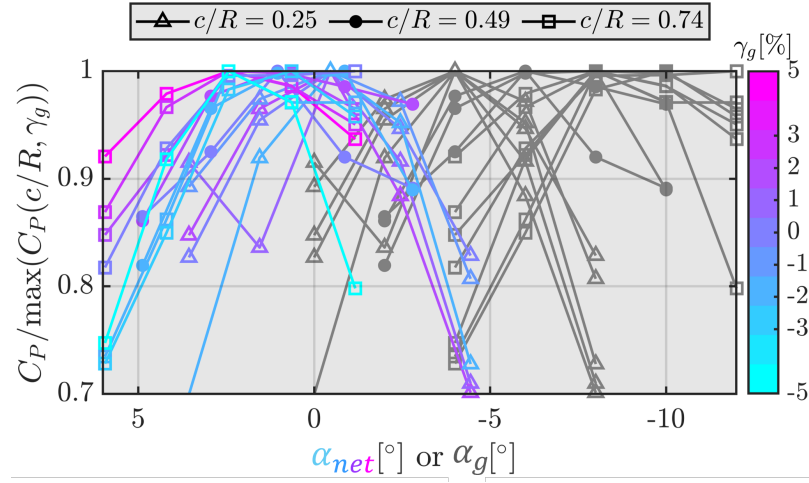


Figure 6.9: Normalized efficiency curves for constant  $\gamma_g$  and  $c/R$  as a function of net and geometric pitch angles.  $C_P$  is normalized by the maximum value for each combination of  $\gamma$  and  $c/R$ . Colored curves, show performance as a function of  $\alpha_{net}$ , with the colors indicating the geometric camber. Gray curves are the same as a function of  $\alpha_g$ , corresponding to vertical slices of Fig. 6.3a. Marker shapes denote the  $c/R$ .

with the optimal value increasing slightly with larger  $c/R$ . There are two potential causes. First, the calculation of virtual pitch employed here assumes induction is negligible and  $\lambda = \infty$ , which likely overestimates the true average virtual pitch experienced by the blade, such that a more accurate estimate of virtual pitch might result in an optimal  $\alpha_{net}$  entirely independent of  $c/R$ . Second, other features of changing  $c/R$  (e.g., solidity and pitching moment) likely impact performance in a way that is not fully described by  $\alpha_{net}$ . However, net pitch calculated for  $\lambda = \infty$ , despite the slight remaining dependence on  $c/R$ , provides an effective means of estimating the optimal pitch for a cross-flow turbine during initial design iterations.

Net camber also provides a useful normalization by combining geometric camber and  $c/R$  induced virtual camber into a single parameter. Broadly, the power-optimum net camber is fairly wide for each  $c/R$ , although trends vary slightly across different  $c/R$  values. For the two higher  $c/R$  values, 0.49 and 0.74, introducing net camber causes the optimal geometric cambers to converge. At  $c/R = 0.49$ , the optimal geometric camber ranged from  $-2\%$  to

+2%, while at  $c/R = 0.74$ , it ranged from  $-5\%$  to  $\sim -1\%$ . These optimal geometric camber ranges, which initially only slightly overlapped, collapse into a fully overlapping net camber of 4–8%, slightly higher than the 3% suggested by RANS at a lower Reynolds number in Chapter 5. At lower  $c/R$ , there seems to be a slight bias toward lower camber, although limited sampling at intermediate  $c/R$  precludes identification of a distinct optimum. For  $c/R = 0.25$ , we observe a broad plateau of performance that is slightly reduced in the overlapping range around  $\gamma_{net} = 4\%$ , and slightly better performing at lower  $\gamma_{net} = 1\%$  to  $3\%$ , though higher values were not tested.

To further consider the slight difference in behavior at low  $c/R$ , it is important to consider how net camber is achieved. At  $c/R = 0.25$ , to reach comparable net camber values (6–8%) requires substantially larger positive geometric camber due to the reduced virtual camber, compared to the higher  $c/R$  cases where optimal performance is obtained with negative or small positive geometric cambers. This distinction suggests that performance may depend not only on the magnitude of net camber but also on how it is partitioned between geometric and virtual components. At lower  $c/R$ , the increased magnitude and sign change of geometric camber to achieve the same net geometry likely introduces an excess of surface curvature, potentially altering the unsteady dynamics, contributing to the observed difference in contours.

As previously mentioned, in addition to rotor geometries with constant net camber across  $c/R$ , we also captured configurations with equal geometric cambers of  $\pm 2\%$  and  $0\%$  as well as cases where the ratio of geometric camber to virtual camber was held constant at  $\pm 1/3$  for all  $c/R$ . The icons shown in Fig. 6.8 highlight a subset of these cases at their respective optimal  $\alpha_{net}$ . This subset includes configurations with a geometric camber  $-2\%$ , a geometric-to-virtual camber ratio of  $-1/3$ , and an equivalent net camber of approximately  $+4\%$ . For the cases with equal net camber of  $+4\%$ , the position relative to the optimal contour shifts as  $c/R$  changes: for smaller  $c/R$ , the  $\gamma_{net} = 4\%$  seems to lie above the peak, while for larger  $c/R$ , it falls below. Examining the  $\gamma_g = -2\%$  and  $\gamma_g/\gamma_v = -1/3$  cases, both consistently fall within the highest performance contours across all tested  $c/R$ . This result indicates that either the geometric camber or the geometric-to-virtual camber ratio may serve as a more reliable design metric for identifying the maximum  $C_P$ . The partial

collapse with  $\gamma_{net}$ , and potential improvement by these other metrics, supports the view that camber effects are somewhat nonlinear, and that geometric and virtual camber—while similar in phase-averaged analyses—may not be equivalent. Suggesting that more nuance may be needed than just examining net camber, which was proposed at the end of Chapter 5 before virtual camber was simultaneously varied. However, further work is needed to confirm whether this holds across a wider range of geometry.

#### *Comparison of $C_P$ across $\lambda$ for Equivalent Net Geometry*

Since it is possible to achieve approximately the same net camber and net pitch with different combinations of geometric camber, blade pitch, and  $c/R$ , we can assess whether holding net camber and net pitch constant yields similar performance curves across different physical rotor configurations. Figure 6.10 shows performance for combinations of camber,  $c/R$ , and  $\alpha_g$  that all have  $\gamma_{net} = +4\%$  and equal net pitch ( $\alpha_{net} \approx 2^\circ$  and  $\alpha_{net} \approx 0^\circ$ ). Given the pronounced differences, it appears that net camber and net pitch are not explanatory for optimal performance, as foreshadowed by Fig. 6.8, or general performance curves. Specifically, the difference in chord length, which directly affects the local Reynolds number, appears to dominate. The lack of explanatory power for optimal  $\lambda$  is unsurprising, given that this parameter is well-described by solidity, which varies across the cases considered.

Although net camber and pitch may determine the lift coefficient and the angle-of-attack experienced by the blades throughout the cycle, the dimensional lift force produced by the blade depends on blade area, which increases with chord length, and the incident flow speed. If the blade area were the only difference in performance, a simple scaling by chord length could collapse the performance curves. However, as discussed previously, the change in chord length also affects the solidity and true blade-level Reynolds number, known to affect performance. To understand how these parameters impact performance, the anticipated effects of each are previewed. Increasing  $c/R$  (and thus solidity) is expected to primarily shift the performance curve left by reducing the optimal  $\lambda$ , while narrowing the performance peak. As  $c/R$  increases, the chord-based Reynolds number is also expected to increase, potentially explaining the notably lower peak  $C_P$  for  $c/R = 0.25$ . Comparison

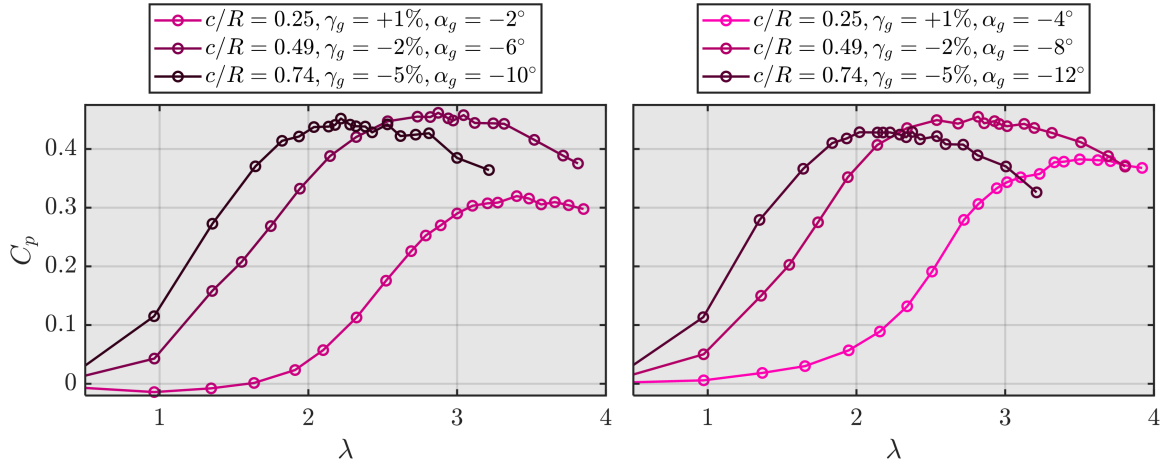


Figure 6.10: Comparison of cases with equivalent net geometry across  $c/R$ . For all cases,  $\gamma_{net} \approx +4\%$ . (right)  $\alpha_{net} \approx 2^\circ$  and (left)  $\alpha_{net} \approx 0^\circ$ .

with the results of Hunt et al. [7], who also explored Reynolds number influences, indicated peak performance for each  $c/R$  of symmetrical blades to be similar to those seen in Fig. 6.10, while at higher  $Re_D = 8.3 \times 10^5$ , optimal performance flips and occurs for lower  $c/R$ . Consequently, the differences in peak  $C_P$  for increasing  $c/R$  is expected to be due to a combination of Reynolds number effects and larger blade area, while the shifts in optimal operating point governed by solidity.

#### 6.4.3 The Equivalency of Pitch and Camber Using Thin Airfoil Theory

Virtual pitch appears to determine the optimal blade pitch through net pitch (Section 6.4.2) while net camber provides insight into the optimal geometric camber of turbine blades (Section 6.3.4). As their phase-averaged behavior has demonstrated similar tradeoffs between the upstream and downstream portions of the rotation, this opens the question of whether these can be combined into a single metric. The zero lift angle-of-attack (Fig. 6.1),  $\alpha_{L0}$ , arises when camber is introduced. This means that camber can be treated as a lateral shift in the lift curve for an equivalent symmetrical blade [97] or a  $\Delta\alpha_g$ . In practice, camber and pitch are found to be approximately proportional for small values, with 1% camber corresponding to about  $1^\circ$  pitch [108]. The validity of this assumption depends on the slope of



the lift curve being the same between symmetrical and cambered foils, which varies only to a limited extent [97]. Thin airfoil theory provides a means of directly estimating  $\alpha_{L0}$  using only the net camber line produced using flow curvature transformation as input (Appendix C.2 contains the necessary thin airfoil equations).

Although cross-flow turbines violate key thin airfoil theory assumptions, we apply the same simplifications used for virtual camber calculation (neglecting induction and  $\lambda \rightarrow \infty$ ) to apply thin airfoil theory and translate net camber into an equivalent pitch. In doing so, net camber can be converted to an angular value and combined with net pitch to produce a “total” pitch as

$$\alpha_{tot} = \alpha_{net} - \alpha_{L0}.$$

The subtraction is needed to convert the negative  $\alpha_{L0}$  of a positive camber into positive pitch, so the pitch effect of camber follows the same sign convention as all others in this work.

This new parameter notionally combines the effects of  $\alpha_g$ ,  $\gamma$ , and  $c/R$ , through flow curvature effects, into a single value. Physically, this pitch would define the angular displacement along a symmetric lift curve, with the turbine starting the cycle at  $\theta = 0$ . Looking back at the upstream and downstream performance contours in Fig. 6.3b-c, the diagonal slopes are approximately 1% camber per  $1^\circ$  pitch. This indicates that performance changes between the upstream and downstream vary closely with the approximate pitch-camber equivalence ( $\alpha_{L0}$ ), suggesting that the weighting of power production over the cycle can be described by considering both with  $\alpha_{tot}$ .

While this new parameter combines the effects of  $\alpha_g$ ,  $\gamma$ , and  $c/R$ , through flow curvature effects, into a single value, it is illuminating to explore the contribution of each element to  $\alpha_{tot}$ . By breaking all virtual and geometric parameters into their contributions to total pitch, the expanded form is

$$\alpha_{tot} = \alpha_g + \alpha_v - \alpha_{L0,g} - \alpha_{L0,v},$$

where  $\alpha_{L0,g}$  and  $\alpha_{L0,v}$  are the zero-lift angle of attack contributions due to the geometric camber and virtual camber via  $c/R$ , respectively.

While cycle-averaged performance is not explained by this parameter (nor would we expect it to be, based on the results presented previously in this chapter),  $\alpha_{tot}$  is found to be a good predictor of the maximum power variation throughout the cycle,  $\text{Max}(\Delta C_P(\theta)) = \text{max}(C_P(\theta) - \text{min}(C_P(\theta)))$  (Fig. 6.11 bottom row), which is the difference between the highest and lowest points on the phase-averaged performance curve, often located in the upstream and downstream portions of the rotation, respectively. This metric is useful for design purposes, as high variability will reduce water-to-wire efficiency and can lead to structural issues.

Examining the top row of Fig. 6.11 shows  $\text{Max}(\Delta C_P(\theta))$  plotted against the individual parameters contributing to total pitch: geometric pitch, virtual pitch, geometric camber, and virtual camber. Considered in isolation, none of these parameters exhibits a strong correlation with power variability. The partially combined parameters (net pitch and net camber) shown in the middle row also fail to fully elucidate trends in variability across the design space. In contrast, the bottom row shows that when all design-parameter contributions are synthesized into a single parameter, a clear linear relationship emerges. More negative  $\alpha_{tot}$  is observed to correlate linearly with reduced  $\text{Max}(\Delta C_P(\theta))$ . Observations from Chapter 5, and Fig. 6.3b-c, suggest that optimal camber and pitch are driven by a balance of upstream and downstream influences to get optimal performance. The clustering of optimal performance cases (more yellow) in the middle of the range of  $\alpha_{tot}$  supports the presence of an optimum balance.

Examining the change in behavior as the blade count increases (black filled markers) shows that total turbine variability (evaluated with  $\text{Max}(\Delta C_P(\theta))$ ) decreases significantly. The linear relationship with  $\alpha_{tot}$  is maintained, but the slope is reduced as additional blades smooth the overall profile. Interpreting the linear correlation across blade counts and considering how parameters are integrated into  $\alpha_{tot}$  highlights how geometric changes can reduce variability. More negative  $\alpha_{tot}$ , can be achieved by using more negative  $\alpha_g$ , more negative  $\gamma_g$  (which reduces  $\alpha_{L0}(\gamma_g)$ ) and smaller  $c/R$  which diminishes the virtual contributions of both  $\alpha_{L0}(\gamma_v)$  and  $\alpha_v$ .

Additionally, as high upstream generation is typically seen to coincide with higher maximum horizontal force,  $\alpha_{tot}$  exhibits a similar explanatory value for the analysis of peak

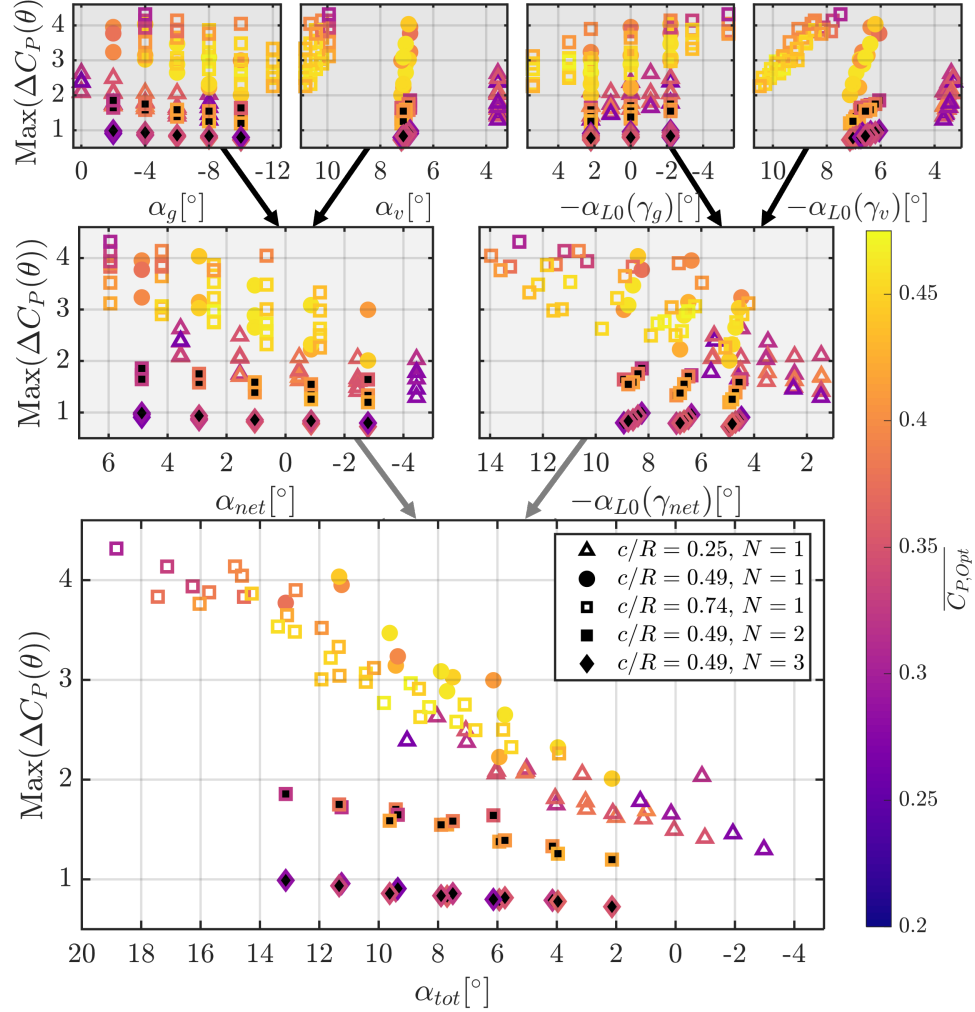


Figure 6.11: Turbine performance at  $\lambda_{opt}$ , maximum difference in phase-average  $C_P(\theta)$  over one cycle, (top row) as a function of component effects converted into respective pitch values, from left to right geometric pitch, virtual pitch, zero-lift angle due to geometric camber and the same for virtual camber, (middle row) as a function of net parameter pitches: (left) net pitch and (left)  $\alpha_{L0}$  due to net camber, and (bottom row) as a function of "total pitch", combining all contributions. Symbols are colored according to their  $\overline{C_{P,Opt}}$  and their shapes designate their respective blade geometry, with black-filled shapes denoting  $c/R = 0.49$  with  $N > 1$ .

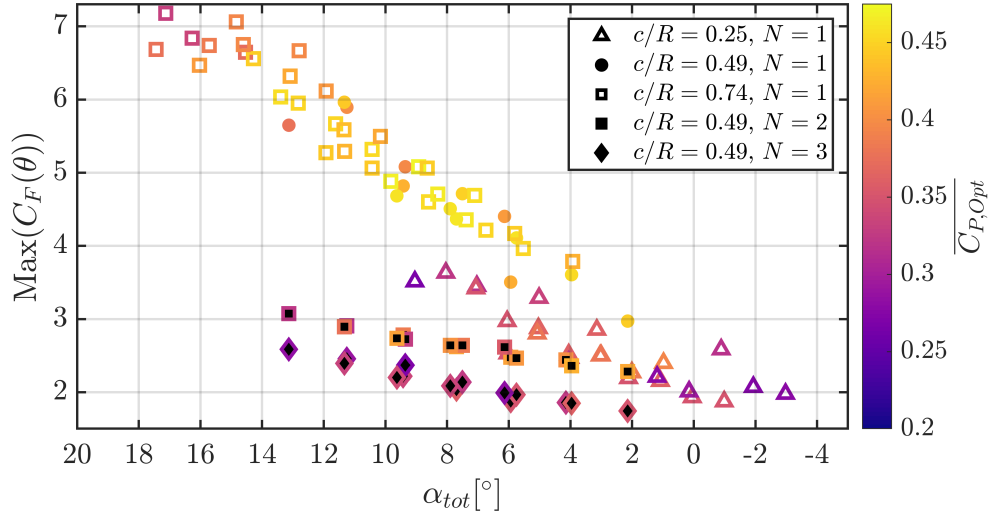


Figure 6.12: Turbine maximum instantaneous horizontal force coefficient ( $\text{Max}(C_F(\theta))$ ) as a function of  $\alpha_{tot}$  at  $\lambda_{opt}$ . Symbols are colored according to their  $\overline{C_{P,Opt}}$ , and their shapes designate their respective blade geometry, with black-filled shapes denoting  $c/R = 0.49$  with  $N > 1$ .

loading (Fig. 6.12). The maximum horizontal force, which combines  $C_{F_X}$  and  $C_{F_Y}$ , is a key parameter driving structural design constraints. Again, a direct linear correlation is observed with  $\alpha_{tot}$ , and increasing blade count reduces maximum loads while preserving the linear trend.

Overall, the linear relationships suggest that while  $\alpha_{tot}$  does not effectively capture the time-averaged performance, it does approximate the maximum upstream lift generation, which in turn drives the power variability and maximal force throughout the cycle. These are both important design constraints for cross-flow turbines. Additionally, the linear relationship between these performance parameters and  $\alpha_{tot}$  suggests that this relationship can be used to clarify trade-offs aimed at reducing power variability and peak force within a cycle. Through this lens, the reductions in the range of power and force within a cycle through reducing net camber (more negatively cambered blades), smaller  $c/R$ , and more negative pitch, as well as increasing the number of blades, are all consistent. Specifically, while each of these changes has been shown to reduce upstream lift in earlier results in this

chapter,  $\alpha_{tot}$  enables the combined effects to be evaluated with a single parameter.

## 6.5 Conclusions

This work experimentally investigates the interplay between the chord-to-radius ratio, geometric camber, blade pitch, and the effects of virtual camber and pitch, which have interrelated effects on cross-flow turbines. Theoretical analysis of rectilinear equivalent variables and their steady-state aerodynamic behavior was used to identify expected implications of geometric changes, while providing a pathway to combine parameters into net camber and net pitch, or total pitch, thus reducing the dimensionality of the geometric design space. This work highlights the multifaceted interactions of design parameters in cross-flow turbines and the importance of considering the effects of flow curvature in conjunction with geometric parameters.

Considering all three parameters at once ( $c/R$ ,  $\alpha_g$ , and  $\gamma_g$ ), the sensitivity and relative strength of their effect on performance are evaluated. Higher  $c/R$  consistently produces the largest shift in optimal  $C_P$ , and alters the optimal  $\alpha_g$ . Blade pitch primarily controls performance at a given  $c/R$ , strongly influencing the balance between upstream and downstream power production. Camber has a secondary effect on  $C_P$ , with its influence increasing at higher  $c/R$ , but generally remaining weaker than pitch effects. When  $\alpha_g$  and  $\gamma_g$  are both more positive, the performance difference between upstream and downstream is magnified, whereas moving both more negatively tends to smooth power production over the cycle, reducing peak-to-average performance variations. Overall,  $c/R$  is the strongest driver of the performance envelope,  $\alpha_g$  provides the primary tuning of performance, and camber offers a more subtle effect on the weighting of performance between the upstream and downstream.

Net pitch is found to closely predict the blade pitch that yields optimal efficiency, with values of net pitch between  $-1^\circ$  and  $+3^\circ$  resulting in maximum performance across different combinations of cambers and  $c/R$ . This behavior can be explained by the role of  $\alpha_{net}$  to account for variations in  $\alpha_v$  that arise with changing  $c/R$ . In effect, the optimal geometric blade pitch is that which offsets the virtual pitch experienced by the foil. The interpretation of virtual pitch depends on the choice of blade mounting point, and therefore on the reference coordinate system. However, expressing results in terms of net pitch removes

this dependence, and these choices do not affect the underlying aerodynamic performance (see Appendix B.1). While the quarter-chord mounting point in this work is typical and originates from classical aerodynamics, we recommend using a coordinate system about the mid-chord for cross-flow turbines, which minimizes the impact of virtual pitch, which has not been regularly considered in the design and study of cross-flow turbines. The point at which the blade is mounted does not affect fluid forcing, so using the mid-chord as the datum for defining blade pitch and  $c/R$  relative to it simply improves the clarity and generalizability of the results. Further, while the mount point does not impact performance for fixed geometry, it does impact how loading is translated through support structures, and should be selected to reduce moments about the strut blade connection point to reduce fatigue on the support structure.

Though net pitch illustrated an approach to defining optimal blade pitch, net camber did not yield a similarly clear result for optimal geometric camber. As  $c/R$  increases, the camber associated with the highest efficiency decreases, indicating that using camber to counteract the virtual camber appears beneficial. The optimal net camber does not converge to a single value but rather appears to increase with  $c/R$ . There is some ambiguity in this result, as the two lower  $c/R$  cases did not have a clearly optimal geometric camber. Similarly, while the same net cambers of  $\sim 6\%$  to  $8\%$  performed best for both  $c/R = 0.49$  and  $0.74$ , the smallest  $c/R$  case saw slightly better performance at a net cambers of  $\sim 1\%$  to  $3\%$ . This may be a consequence of the larger ratio of geometric camber to virtual camber or lower chord-based Reynolds numbers for this configuration. These trends, together with the strong performance of cases defined by constant  $\gamma = -2\%$  or a fixed geometric-to-virtual camber ratio of  $-1/3$ , suggest that net camber alone may be insufficient to collapse the data across  $c/R$ . Instead, a more nuanced treatment that distinguishes between geometric and virtual camber may be required to reliably identify optimal configurations.

This work presented the first comparison of equivalent net geometry across three  $c/R$  by careful selection of  $c/R$ , blade pitch, and camber. While the resulting performance of each was near the optimal of their respective  $c/R$ , the performance curves as a function of  $\lambda$  did not collapse, indicating that the additional effects of Reynolds number and solidity remain important. Though the smallest  $c/R$  exhibited notably lower performance, the two higher

$c/R$  showed similar performance as a function of  $\lambda$ , with only a small increase in  $\lambda_{opt}$  with increasing  $\lambda$ . The deviations in performance with net geometry highlight the limitations of these parameters, given the remaining performance differences with chord length.

The optimal tip-speed ratio was largely found to be independent of pitch and camber, but was inversely proportional to and strongly dependent on  $c/R$  or  $N$ , consistent with previous work on solidity. Examining off-optimal  $\lambda$  revealed connections between pitch and camber, highlighting areas for improving turbine performance relative to traditional symmetrical blades. For  $\lambda \geq$ , the optimal value for a symmetrical blade, negative cambers, and more positive blade pitch together show an avenue to enhance performance by reducing virtual camber and downstream losses, which is magnified for high  $c/R$ , and extends the performance at higher rotation rates, broadening the range of viable  $\lambda$  that can produce power. Additionally, negative cambers consistently provide a means of reducing loads simultaneously, extending the results of Chapter 5.

By combining the zero-lift angle-of-attack of net camber blades with net pitch, it is possible to relate maximal loading and peak-to-average power generation with a single parameter, total pitch. This provides a way to reduce blade pitch, geometric camber,  $c/R$ , and resulting virtual camber and pitch into a single metric. While not predictive of maximum  $C_P$ ,  $\alpha_{tot}$  linearly correlates with the maximum power fluctuation over a cycle and maximal instantaneous loading.  $\alpha_{tot}$  provides a novel way to consolidate blade camber and pitch parameters into a single metric that can inform pathways to reduce peak loads and power variability.

### ***Data Availability***

Data from this work will be made available on Dryad and linked to the Hunt et al. [6] dataset, as it effectively extends that work to new geometries under matching conditions.

## Chapter 7

**CONCLUSIONS**

Cross-flow turbines are a promising technology for diversifying renewable energy generation in both wind applications and the currently largely underutilized marine current resource. However, their complex cyclic hydrodynamics, unsteady loading, unique rotational reference frame, and coupled design parameters keep their design and optimization an open area of research. Empirical models still often fall short, and CFD remains computationally expensive and, in many cases, unvalidated by experiments, making experimental research on cross-flow turbines essential. While some studies have explored complex approaches to enhance performance, these designs typically offer only modest performance benefits while introducing new pathways for structural failure. Moreover, the fundamental relationships between performance and control, or geometric design changes, have not been fully established. The objective of this research was to investigate previously overlooked, relatively simple kinematic and geometric pathways to enhance the performance of cross-flow turbines, linking changes to blade-level hydrodynamics and fundamental aerodynamic theory to fill gaps in the understanding of cross-flow turbine performance. In this work, exploring a much broader intracycle velocity control parameter space demonstrated the use of control to alter turbine hydrodynamics and optimize performance and loading trade-offs for a single turbine geometry. Explorations of geometry (geometric pitch, blade camber, and chord-to-radius ratio) provide an alternative approach to enhancing performance by altering the rotor's aerodynamic response under constant-speed control. While each of these approaches to enhance performance affects blade-flow interactions, they do so through different pathways: one controls the relative velocity on a fixed blade, and the other alters aerodynamic performance by changing the geometry. Beyond the published results of this work, openly accessible datasets for numerical validation of cross-flow turbine simulations (CFD or semi-analytical) have been generated for both performance and flow-



field comparison, which may be used for the development and benchmarking of numerical models.

This dissertation summarizes the motivations for this research and relevant background in Chapter 1 and 2. Chapter 3.1 provides an overview of the experimental apparatus and data processing methods used in this work, and outlines key performance metrics. Exploring the full intracycle control parameter space in Chapter 4 revealed the utility of intracycle control for loading reduction, in addition to the already-known potential for power augmentation. While peak performance of only 13.2% was achieved over constant-speed control, reducing the apparent benefit of intracycle control compared to prior works, we were able to identify the significant impact of Reynolds number on performance enhancement, that biased previous results. Optimal power-producing kinematics were shown to delay stall and enhance flow recovery, matching the historically expected desirable flow characteristics of cross-flow turbines. Pareto front, multi-objective analysis showed the potential to balance maximum or mean loading and power production; however, all three could not be optimized simultaneously. By triggering slightly earlier stall, and ensuring the peak blade velocity occurred just after the maximum torque, it was possible to halt the growth of loading, reducing the peak forces by 12%, and extend power output later in the cycle, achieving a simultaneous 3% efficiency improvement. The corresponding phase-average performance curve was seen to have a reduced maximum, but a broader range of power production through the cycle. This suggests that alternative hydrodynamic mechanisms can be targeted for optimization, and benefits beyond simply maximizing power should also be considered.

Chapter 5 focused on the effects of positive and negative cambered blades under constant speed control, providing clearer insights into their utility in cross-flow turbines. While prior literature has yielded conflicting results, two main reasons were identified for this discrepancy while exploring the underlying hydrodynamics and future utility of blade camber. First, in prior studies, different metrics were investigated and selected as optimal: starting torque, maximum efficiency, or wake recovery, for example. Each of these is strongly dependent on the blade camber. Second, the combined consideration of virtual camber in conjunction with geometric camber has largely been neglected. As a result, the selection of cambered blades and the magnitude of camber relative to the chord-to-radius ratio (which

drives virtual camber effects) have not been considered. And so, while prior work may have explored a range of blade cambers, the net camber effect, accounting for differences in their virtual cambers, can also help resolve some conflicts. In addition, the lack of validation of numerical explorations has led to much uncertainty in prior simulations. This work experimentally tested both positive and negative 2% cambered blades at a moderate chord-to-radius ratio,  $c/R = 0.49$  where the virtual camber is  $\approx 6\%$ . Results demonstrated that positive camber increases the cycle maximum torque generation at low speeds, indicating the potential benefit to starting torque. However, the largest benefits were found with negatively cambered blades, which showed the potential to improve efficiency, broadening the range of power producing  $\lambda$  beyond the point at which symmetrical blade performance drops off. Negative camber was also seen to simultaneously decrease both mean and maximum loading, an additional benefit. The broader range of power-producing  $\lambda$  is an exciting discovery, as it is expected to benefit field-scale deployments by providing greater resistance to turbulent fluctuations, thereby significantly improving annual power production. Additionally, by considering the  $c/R$ -induced virtual camber in conjunction with the blade geometric camber using the concept of net camber, it was possible to partially collapse and further rationalize the discrepancies in literature observations. However, flow fields showed a clear change in stall behavior with negatively cambered blades, resulting in earlier leading edge stall upstream but improved downstream reattachment, while positive camber caused delayed upstream stall paired with poor downstream flow recovery. This non-linear change in stall dynamics suggests that net camber, with its linear addition of geometric and virtual influences, might be an oversimplification. The best performing configuration in Chapter 4 showed the potential to both delay stall and improve flow re-attachment relative to symmetrical blades.

To both explore broader geometric interactions and evaluate the generality of Chapter 5 when virtual geometry is varied, Chapter 6 explored how the performance of cross-flow turbines changes with blade pitch, camber, and chord-to-radius ratio. This chapter showed how geometric design variables can impact turbine performance and demonstrated the interdependence of these variables through the phenomena of virtual camber and pitch. A direct connection was established between fundamental aerodynamics in rectilinear flow

and changes in cross-flow turbine design parameters to compare the effects of lift versus angle-of-attack and highlight similarities and differences in cross-flow turbine performance. Surprisingly, the optimal blade pitch was largely independent of changes in blade camber. Combining virtual pitch and present pitch into net pitch was shown to provide a good estimate of optimal pitch across  $c/R$  and camber. Consistently more positive cambers and more positive pitch configurations exhibited higher upstream performance. More negative cambers and pitches resulted in more balanced power production during blade passage, both upstream and downstream, potentially benefiting generator performance and causing a reduction in maximum loading. By incorporating the zero-lift angle-of-attack, maximal loading and power fluctuations were found to follow an approximately linear relationship across all configurations of pitch, camber, and  $c/R$ . This chapter also introduced a new approach to decoupling rotor design parameters, enabling the effects of solidity to be assessed independently from the paired influence of virtual camber for the first time.

On top of the direct results of this research, this dissertation highlights the importance of investigating fundamental parameters before adding complexity to a poorly understood system. While camber is one of the most basic airfoil parameters to vary, it has received little direct exploration, due to the persistence of initial assumptions that have endured for decades and continue to inform current research. While some parametric investigations of design parameters have provided deeper insight into the effects of individual variables on turbine performance, the inherent interconnectedness of parameter adjustments has been largely overlooked. While simple at first glance, the underlying dynamics of cross-flow turbine designs are highly complex, and few studies consider the multiple pathways by which a single perturbation in a variable can cascade to affect performance. While the cross-flow turbines explored here have a simple geometry, the nonlinear interactions among their design parameters create complexity that must be better understood to enable a generalizable optimization approach. In future work, care must be taken to consider the implications of design changes and expand the more holistic analysis presented here.

## 7.1 Future Work

While intracycle control has been explored using a sinusoidal function in terms of blade position, this results in a decrease in mean  $\lambda$  with increasing amplitude. Though [1, 65] also tested modifying  $\lambda_0$  to compensate for this effect, biasing the time blades spend at lower rotation rates still remains, as does aggressive acceleration and deceleration through the cycle. Future work should investigate sinusoidal intracycle time control to address both of these features. By controlling rotation rate as a function of time, synchronized to a single cycle, the amount of time spent below and above the mean would be equal, and so no change in time-average  $\lambda$  would occur for increasing  $A_\lambda$ . This may provide additional benefit because velocity and acceleration are derivatives of time, and so the motor acceleration requirements would be smaller, as well. The differential equation required to solve for sinusoidal variation in time, synchronized to the turbine period and position, does not yield a closed-form solution and must therefore be solved numerically before being prescribed.

Chapter 5 approximated the potential for turbine enhancement by use of an actively cambering foil through the cycle, with positive camber upstream to maximize power extraction and negative camber downstream to reduce drag and enhance flow recovery. In this work, it is shown that it may be able to significantly enhance performance and control loading, and even alter the range of power-producing  $\lambda$ . Although some initial work has investigated passively flexing or articulating trailing edges to vary camber throughout the cycle, these approaches also modify blade pitch, thereby precluding the isolation of the effects of dynamic (intracycle) cambering. Although integrating both strategies (active pitching and cambering) may ultimately be beneficial, establishing a fundamental understanding of active camber control is a necessary step before combining multiple approaches.

A currently overlooked aspect of flow curvature transformations is that while the rectilinear equivalent enforces the angle-of-attack along the blade to be constant a variation in chord local relative velocity remains. In the limit of  $\lambda \rightarrow \infty$ , the relative velocity at any point along the blade becomes proportional to the radial distance to it, equivalent to a blade experiencing linear shear inflow [34]. While this analysis still assumes purely rotational flow and ignores induction, which distorts the true flow around the blades, this velocity gradi-

ent would likely have an additional unconsidered impact on the aerodynamics. Although this effect would likely be negligible at small  $c/R$ , larger  $c/R$  would produce more extreme changes in relative velocity along the chord and may compromise the validity of assuming equivalent behavior of a virtual foil in uniform flow. Future work should explore approaches to compensate for this radial velocity gradient, which could clarify its impact and improve the accuracy of virtual camber transformation models.

Much remains to be learned to fully optimize the hydrodynamic performance of cross-flow turbines. There will always be more questions to ask and answer. Considering the current state of turbine performance, many other system-wide aspects are likely to be more critical to investigate and solve than small percentage changes in blade performance. Generator efficiency, robust control for start-up and above-rated speed conditions, debris diversion, corrosion mitigation for marine applications, permitting, and environmental considerations. In parallel, responsible deployment requires careful consideration of energy policy, environmental tradeoffs, resource availability, and future demand. Valuable lessons can be drawn from established marine and energy industries operating in prospective deployment regions. Ultimately, analytical and experimental advances must transition to real-world implementation: devices must be deployed, tested, and refined in operational environments. At some point, devices must be immersed in water, and the resulting challenges must be addressed if cross-flow turbines are ever to become a viable technology for marine deployment.

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## APPENDICES

### A Intracycle Control Appendices

#### A.1 The Impact of Increasing $A_\lambda$ on $\bar{\lambda}$

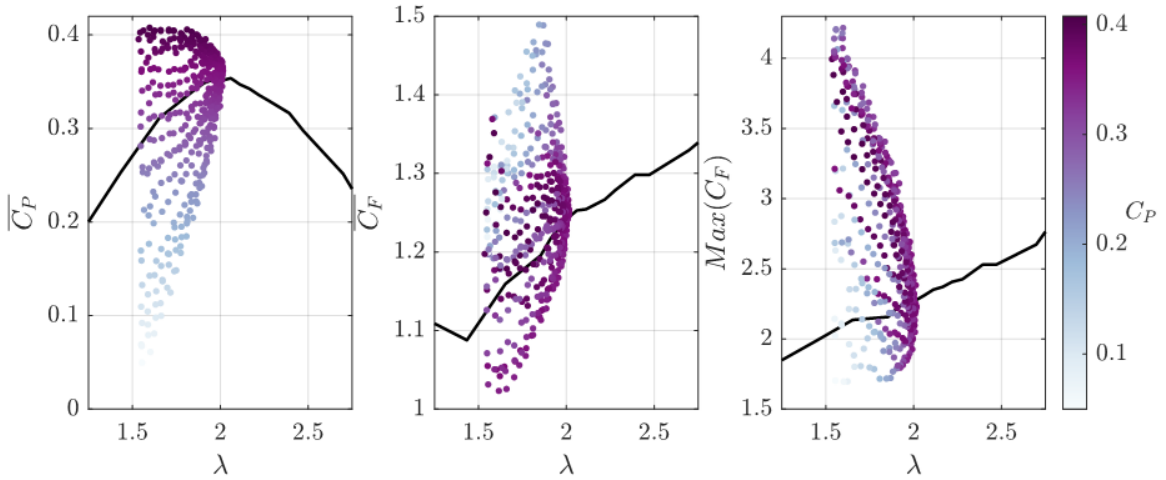


Figure A.1: Effect of changing amplitude on time average  $\lambda$  across  $\overline{C_P}$ ,  $\overline{C_F}$  and  $Max(C_F)$ , for all intracycle control kinematics compared to the constant speed case shown in black for a two-bladed turbine.

Figure A.1 presents all the data points from the two-bladed heatmaps (Fig. 4.3), plotted in comparisons their individual  $\bar{\lambda}$  and the constant-speed equivalent curves in black. These figures highlight that at the higher amplitudes, lower  $\bar{\lambda}$ , higher potential gains can be observed.

#### A.2 Reynolds Number and Support Impacts on Intracycle Control

To better visualize the contributing effects on the difference in the percent change in  $C_P$  with intracycle performance, Fig. A.2 shows performance at two chord-based Reynolds numbers and with different support structures present. Intracycle performance and the

peak constant speed case they are compared against are represented by the diagonal lines. The higher Reynolds number case,  $Re_c = 44k$  with struts, comes from the experiments from Chapter 4, while all those with disk-type supports and  $Re_c = 31k$  were conducted by Greg Talpey using the same experimental apparatus in 2023, and matching the conditions of Strom et al. [1]. Intracycle cases are plotted against their respective  $\overline{\lambda}_{control}$  values, unlike in Fig. 4.2, so lines do not all overlap.

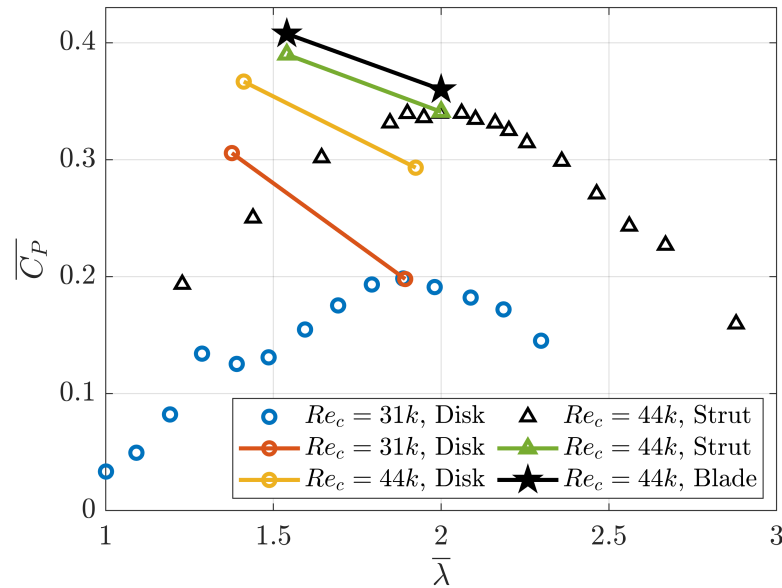


Figure A.2: Comparison of intracycle performance across Reynolds numbers and support structures. Curves without lines show the performance curves as a function of constant-speed control, for  $Re_c = 44k$  explored in Chapter 4 (triangles) and  $Re_c = 31k$  conducted by Greg Talpey in 2023 under matching control and conditions of Strom et al. [1] (circles). Intracycle cases and their compared peak constant speed are shown by diagonal lines. Marker type denotes support type, “Disks” represented by circles, “Struts” as triangles, and loss subtracted results (“Blades”) as stars.

The recorded percent change in efficiency for the raw disk turbine at  $Re_c = 31k$  was 54% (orange) within  $\sim 1\%$  of Strom et al. [1]’s results. Utilizing the same kinematics but conducted at  $Re_c = 44k$  more than halved the efficiency gain to 25% (yellow), indicating

the magnitude that Reynolds number affects these cases. Utilizing struts at  $Re_c = 44k$ , and the slightly different optimal kinematics identified in 4.2 (green) produced a 14% efficiency improvement, while fully subtracting the strut losses slightly reduced the benefit further to 13.2% (the value reported in this work in Section 4.3). These additional reductions in the percent change in efficiency with intracycle control demonstrate the secondary contribution of support losses, in addition to Reynolds number effects.

### A.3 Examination of One-Bladed Intracycle for Analysis of Two-Bladed Trends

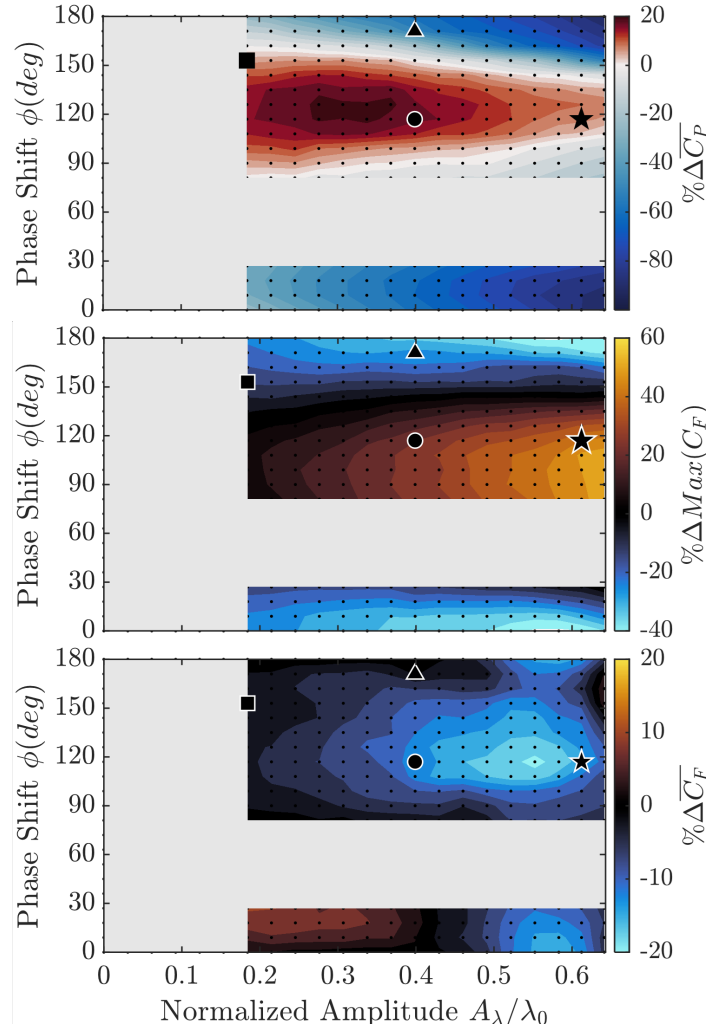


Figure A.3: one-bladed heatmaps of two-bladed kinematics as a function of intracycle control parameters  $A_\lambda/\lambda_0$  and  $\phi$ , for  $\lambda_0 = 2$  compared to constant speed equivalent performance. (a) Percent change in blade-level  $\overline{C_P}$ , (b) percent change in maximum  $C_F$ , (c) percent change in  $\overline{C_F}$ .

Efficiency contours are found to be consistent (Fig. A.3a) with that of the 2-bladed heatmaps shown in Fig. 4.3. The only difference is the percentages and the slight reduction in optimal  $A_\lambda/\lambda_0$ , which is expected to be due to the reduced induction present in the one-bladed

turbine. Maximum  $C_F$  in Fig. A.3b also shows quite similar topography to the two-bladed turbine, with a more significantly reduced force region at low and high  $\phi$ . Examining the mean force, a more notable difference is observed, with a vertical shift in  $\phi$  where the peak mean loading is observed. As  $\overline{C_F}$  is dependent on the addition of each blade out of phase with additional induction effects, the difference shown in the one-bladed heatmap is perhaps not unsurprising. However, we expect the dynamics of the one-bladed turbine to remain representative and insightful for improving our understanding of the two-bladed turbine and how intracycle control affects performance and loading across the design space, so we will continue to use the one-bladed data for analysis of key cases moving forward. Use of the one-bladed data allows interpretation of phase-resolved performance changes and directly ties them to the flow fields and the hydrodynamic mechanisms causing the improvement.

To further verify the validity of the single bladed Pareto optimal for analysis of the two bladed data, Fig. A.4(a-b) shows the one-bladed data from the heatmaps above presented as a function of Pareto highlighted parameters shown for the two bladed data in Fig. 4.4a and 4.5a. The colored points correspond to matching kinematics and coloring identified by the two-bladed Pareto fronts above. While the single-bladed heatmaps shown in Fig. A.3(a-b) exhibit behavior broadly similar to the two-bladed data Fig. A.4, a confirms the Pareto optimal points of the two-bladed turbine do align with the optimal position of the one-bladed data. This suggests the single-bladed performance at these points is representative and can allow blade-level analysis of performance to understand the root cause of performance changes of the two-bladed intracycle data. In contrast, the  $\overline{C_F}$  behavior in Fig. A.4b shows that many of the two-bladed Pareto optimal do not correspond to similar single-bladed behavior. Therefore, these points may not represent the same underlying dynamics and will not be explored using phase-averaged analysis. However, because maximum force is generally more deterministic of structural design, we believe it remains the more critical parameter for investigation moving forward.

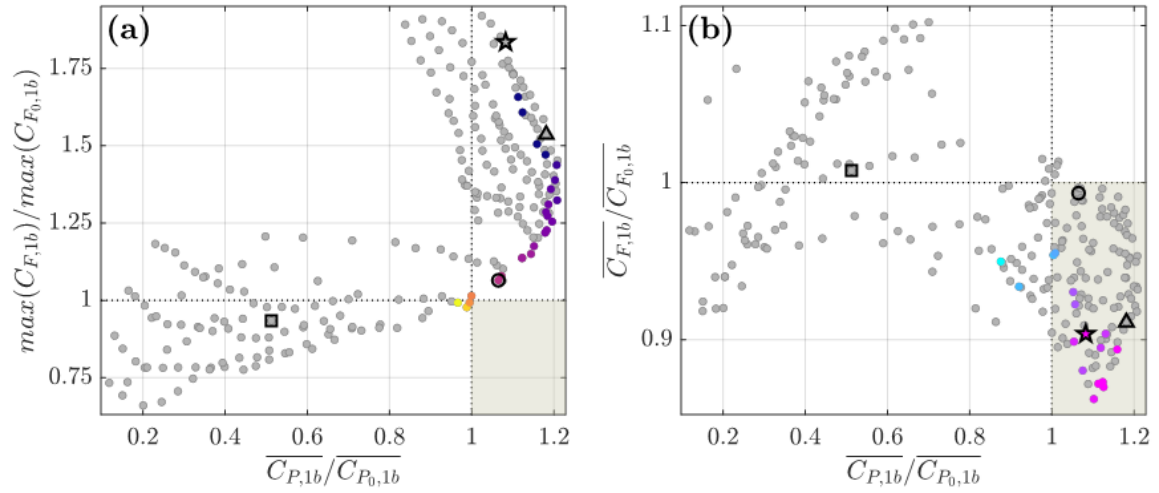


Figure A.4: One-bladed performance cases with lighted kinematics, shown with colors, from the two-bladed Pareto fronts in Fig. 4.4a and 4.5a. Intracycle performance is normalized by the constant speed one-bladed performance for  $\lambda = 2$ . (a) Normalized  $\max(C_F(\theta))$  as a function of  $\overline{C_P}$  for all one-bladed performance under intracycle kinematics. (b) Normalized  $\overline{C_F}$  as a function of  $\overline{C_P}$ . Coloring of Pareto front points is held constant according to the respective kinematics and performance of the two-bladed turbine. The prior noted kinematics of interest are again shown with the matching black symbols; all other tested kinematics are presented as gray points.



## B Camber Appendices

### B.1 Impacts of Pitch and Mounting Point on Virtual Camber and $\alpha_v$

Changing the mounting point is geometrically equivalent to changing the pitch and radius. For a constant geometry turbine with blades fixed relative to the central axis, two mounting point radii can be drawn to any two different points along the chord, and turbines with each mounting solution would perform identically [89, 90]. The primary difference is a matter of a shifted coordinate system origin; the local blade pitch at each “mounting point” will differ by the angle between the two radii,  $\Delta\theta$ , and there would be a slight change in radius to the mounting location [89]. The ratio of pitching moment-to-tangential force, responsible for the power generation, will also differ due to the mounting point, but it is again due solely to a change in the coordinate system and results in the same net performance [90]. As a result, geometric pitch and mounting point can be treated interchangeably in practice, provided the coordinate systems and parameters are clearly defined.

Considering the infinite tip-speed ratio case, the conformal mapping can be visualized as “unwrapping” the circular streamlines and bending the foil about the point where the blade chord is tangent to a streamline; virtual camber emerges from the deformation of the chord line away from that point. Both pitch and mounting point shift this tangent location somewhat, but since the circular streamlines determine the curvature, the virtual camber is largely independent of pitch/mount and primarily depends on  $c/R$ . See Fig. B.1a, which shows the change in virtual camber (black) and virtual pitch (red), for different mounting points, as a function of  $c/R$ . Shaded lines represent changes in geometric pitch, measured at the quarter-chord. While the variation in virtual camber and pitch is quite small, increases in nose-out (negative) pitch slightly increase the virtual camber (though this difference is small at the  $c/R$ ’s explored in this work).

Redrawing a new radius to the mid-chord, while keeping the same geometry, and then utilizing the resulting radius and the equivalent pitch measured at  $c/2$  into Migliore and Wolfe’s [5] transformation method, results in Fig. B.1b. Once again, the virtual camber, now calculated about the mid-chord, is largely unchanged by the geometric pitch (as observed by [109]). The virtual camber is likewise little changed by the mounting position (see the

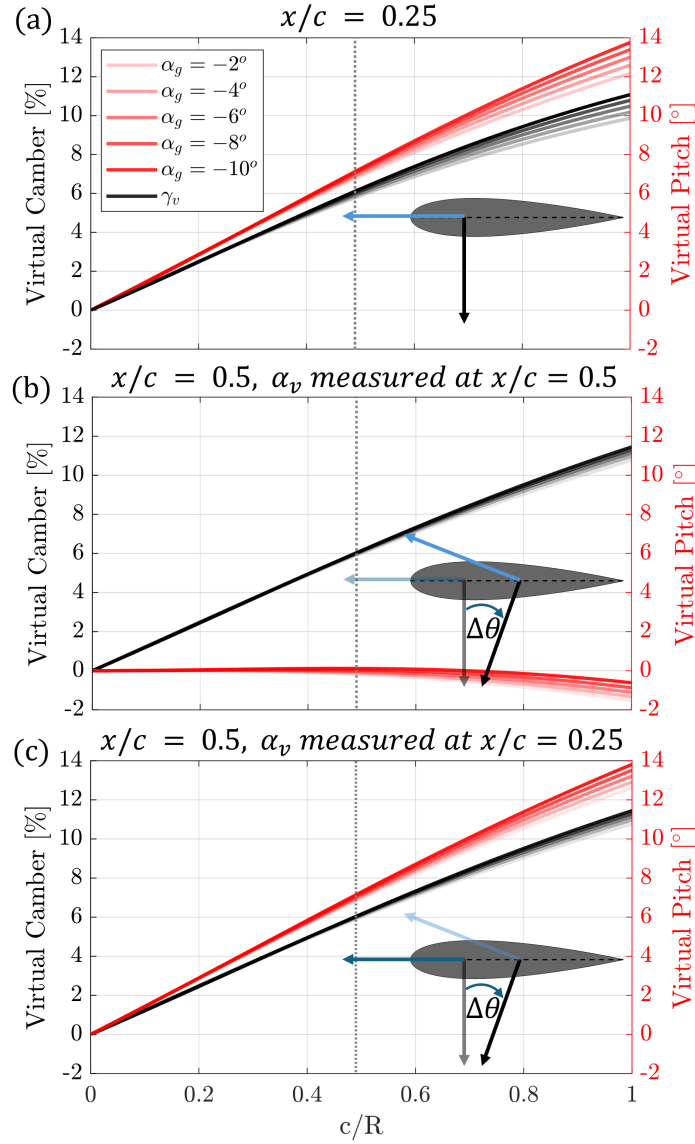


Figure B.1: Virtual camber and pitch as a function of  $c/R$  assuming  $\lambda = \infty$  (a) for a quarter-chord mounting point, (b) for a mid-chord mounting point, (c) for a mid-chord mounting point with virtual pitch corrected by the rotation of the reference frame. Color gradient lines correspond to the blade geometric pitch,  $\alpha_g$ . The vertical dashed line corresponds to  $c/R = 0.49$ , the same geometry examined in this work. Example foil and coordinate systems are shown for a  $0^\circ$  quarter-chord pitch for each configuration, with the radial direction (black) and local angle-of-attack (blue).

difference between Figs. B.1a,b).

At first glance, virtual pitch is significantly altered by the chosen mounting point, becoming almost negligible and independent of  $c/R$  for a midpoint-mounted blade. However, the change in virtual pitch between Figs. B.1 a and b arise from a change in the reference location (the mounting point) where the geometric angle-of-attack is measured. Changing the mounting point introduces a rotation of the coordinate system by an angle  $\Delta\theta$ , equal to the angular separation between the two mounting-point radii. This rotation is neglected in (b). Therefore, to enable direct comparison, the virtual pitch computed at the mid-chord can be corrected to the quarter-chord reference by adding the rotation angle  $\Delta\theta$  (also equal to the change in local geometric pitch between the two mounting points), which increases with  $c/R$ . The resulting comparable virtual pitch is given by

$$\alpha_v(x/c = 0.25) = \Delta\theta + \alpha_v(x/c = 0.50).$$

Figure B.1c shows the result of this correction: though virtual pitch and camber are calculated at the mid-chord, the virtual pitch is referenced to the quarter-chord frame, allowing for direct comparison. With this adjustment, the virtual pitch closely matches that in Fig. B.1a, highlighting that the underlying virtual behavior is largely unaffected by the mounting point.

By utilizing the virtual pitch to calculate the net pitch (discussed in detail in Chapter 6), geometry can be placed in a common coordinate system to facilitate direct comparison to other work. Virtual pitch effectively acts as a correction factor for blade pitch when the mounting point is offset from the blade's mid-chord. Moving forward, care should be taken in the documentation of the reference mounting point, particularly for blade-level analysis. While the mounting point does not affect aerodynamics and performance for fixed geometry, it will impact moments and how loads are transmitted through support structures. Consequently, this should be selected to reduce moments about the strut-blade connection point [89, 90].

## C Camber, Pitch and $c/R$ Interaction Appendices

### C.1 $\lambda$ Dependence of $C_P$ at Lower $c/R$

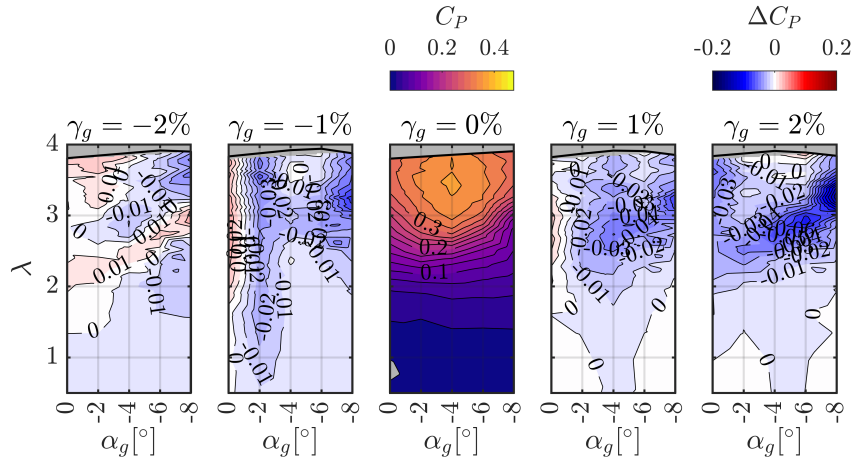


Figure C.1:  $C_P$  for  $c/R = 0.25$  for a one-bladed turbine as a function of  $\lambda$  and blade pitch  $\alpha_g$ , with geometric camber  $\gamma_g$  constant for each individual tile.  $C_P$  presented for the symmetrical (*NACA0018*) bladed turbines, while the performance of cambered blades is presented as the difference in  $C_P$  with respect to the symmetrical blade under matching  $\lambda$  and  $\alpha_g$ .

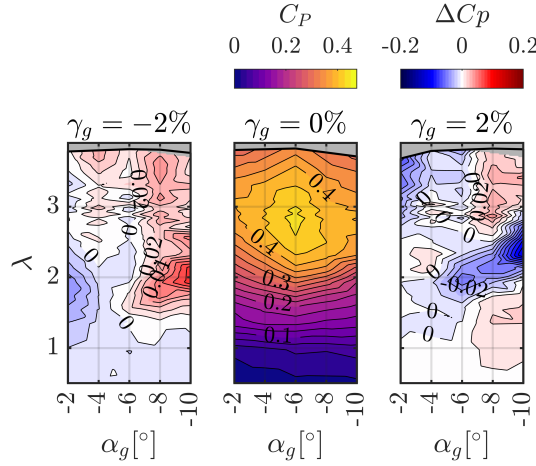


Figure C.2:  $C_P$  for  $c/R = 0.49$  for a one-bladed turbine as a function of  $\lambda$  and blade pitch  $\alpha_g$ , with geometric camber  $\gamma_g$  constant for each individual tile.  $C_P$  presented for the symmetrical (NACA 0018) bladed turbines, while the performance of cambered blades is presented as the difference in  $C_P$  with respect to the symmetrical blade under matching  $\lambda$  and  $\alpha_g$ .

### C.2 Thin Airfoil Estimation of $\alpha_{L0}$

To estimate the zero-lift angle-of-attack ( $\alpha_{L0}$ ), thin airfoil theory provides a direct analytical method that uses the blade's camber line as input. This can be done for the geometric, virtual, or combined virtual camber line. The equations utilized here are adapted from Anderson [97].

The slope of the camber line is defined by,

$$\frac{dz}{dx}$$

while utilizing a standard NACA coordinate system, with the chord line normalized to equal one, and aligned such that the leading edge is positioned at the origin and the chord line is oriented along the positive x-axis. The “x” coordinate, therefore, represents the position along the chord line relative to the leading edge, and “z” corresponds to the perpendicular distance from the chord line to the camber line.  $\alpha_{L0}$  can then be calculated using,

$$\alpha_{L0} = \alpha_{[C_L=0]} = -\frac{1}{\pi} \int_0^\pi \frac{dz}{dx} (\cos \theta_0 - 1) d\theta_0$$

where  $\theta_0$  is defined by the relation,

$$\theta_0 = \cos^{-1}(1 - 2x/c).$$

These equations are combined and solved numerically using a second-order central-difference scheme to estimate the derivative and a trapezoidal approximation for integration. The resulting  $\alpha_{L0}$  of a net camber profile is used to provide equivalence between camber and pitch and define the total pitch  $\alpha_{tot}$  (Section 6.4).

### C.3 Assessment of Reynolds Number across $c/R$

To evaluate the potential impacts of the blade relative Reynolds number across geometry in Chapter 6, we examine data from Hunt et al. [6] with a matching experimental set-up, symmetrical blades, across a finer discretization of  $c/R$ , and varied Reynolds numbers compared to Chapter 6, all other non-dimensional parameters were held constant.

Although the tests in Chapter 6 were conducted below the Reynolds number independence threshold, analysis of lower  $Re_D$  data from Hunt et al. indicates that, despite being in a transitional flow regime, both phase-averaged and time-averaged results exhibit largely uniform trends up to a critical blade-relative Reynolds number of approximately  $Re_b < 5 \times 10^4$ . This is reached by examining data across the full range of lower Reynolds numbers tested by Hunt et al. [6], shown in Fig. C.3. The magnitude change in  $Re_b$  between different  $c/R$  cases is determined to have a smaller impact than would explain the amount of change in performance for  $Re_D = 1.6 - 2.7 \times 10^5$  (the lower tested Reynolds number  $Re_D = 0.75 \times 10^5$  exhibited different trends and is not applicable to this work), indicating virtual geometry is still playing a significant roll in Fig. 6.7(b). For equivalent physical geometries run at two different blade-relative Reynolds numbers,  $Re_b = 2.60 \times 10^5$  and  $Re_b = 1.45 \times 10^5$ , the difference in peak phase-average  $C_P(\theta)$  at optimal  $\lambda$  is 0.26. In contrast, for a fixed diameter-based Reynolds number, but varying virtual geometries (determined by  $c/R$ ), the change in peak  $C_P(\theta)$  is 0.88. The range of tested  $c/R$  corresponded to associated blade-relative Reynolds numbers of  $Re_b = 2.60 \times 10^5$  and  $Re_b = 1.21 \times 10^5$ . This result indicates that changes in virtual geometry lead to approximately a 3-fold greater change in peak performance than changes in the Reynolds number alone. Below this Reynolds num-

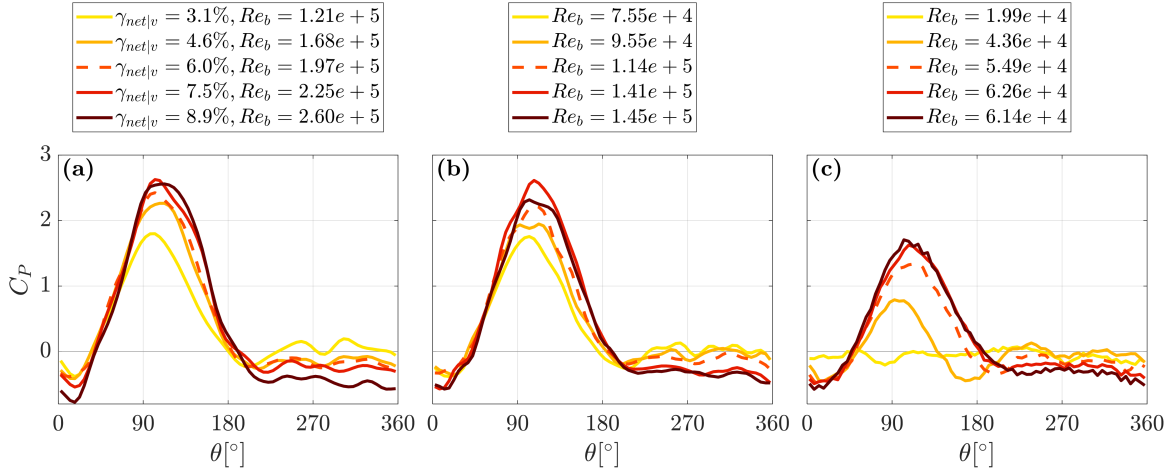


Figure C.3: One-bladed phase-averaged performance as a function  $c/R$  at  $\lambda_{Opt}$  across the lower Reynolds numbers explored by Hunt et al. [7]. All blade profiles are symmetrical, NACA 0018. The virtual camber noted in the legend is due to the range of tested chord-to-radius ratios,  $c/R = [0.25, 0.37, 0.49, 0.62, 0.74]$ , while  $\alpha_g$  is varied to maintain approximately constant ( $\pm 1^\circ$ ) net pitch. (a)  $Re_D = 2.7 \times 10^5$  matched in Chapter 6, (b)  $Re_D = 1.6 \times 10^5$ , and (c)  $Re_D = 0.75 \times 10^5$ . Nominal blade-relative Reynolds numbers for each case are in the legend. Data are from Hunt et al. [6].

ber, Reynolds number effects are expected to play a more dominant role in performance. All chord relative Reynolds numbers explored in Chapter 6 were greater than this threshold of  $Re_b < 5 \times 10^4$ .