# Learning to Facilitate Productive Mathematical Discussion:

## Teacher candidate participation in a practice-based elementary mathematics methods course

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## Abstract

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A prevailing challenge in teacher education is the "problem of enactment," a discrepancy in what teacher candidates (TCs) know or say about practice and what they can actually do or enact (Kennedy, 1999). Current practice-based teacher education research focuses on articulating pedagogies of practice and core practices to integrate theory and practice, attend to the sociocultural nature of TC learning, and support TCs to *enact* practices (Grossman, Compton, et al., 2009; Grossman et al., 2019; McDonald et al., 2013). While there is research on practice-based learning cycles, core practices, and pedagogies of practice, there is limited research documenting how TCs participate in these types of contexts and how they enact core practices.

**Purpose and Research Question.** The purpose of this study was to understand TC participation: How does TC participation manifest and change in their practice-based mathematics

methods course? I sought to characterize TCs' participation around the practice of facilitating a productive mathematical discussion.

**Data and Methodology.** Relevant data sources for this study included videotaped observations of TCs' participation in the course, TC assessments and lesson plans, and field notes. Data was analyzed for broad themes and categories related to TC participation around facilitating discussion. Coding was guided by my conceptual framework, which focused on TC learning as participation in a community of practice. Iterative passes of the data looked for patterns and reoccurring themes related to development of a shared repertoire and joint enterprise.

**Findings.** Findings documented changes in TCs' talk about and enactment of facilitating productive mathematical discussion. TCs' *specificity* in articulating the instructional goal, *intentionality* in facilitating discussions to reach these goals, and *ownership* over articulating and working on goals increased over the 10-week course. TCs were able to use tools (particularly lesson plans, board work, and talk moves) more intentionally in response to student thinking and in service of their instructional goal as they became familiar with these tools and routinized their use. As part of the shared repertoire, these tools reified practice as representations, decompositions, and approximations of practice and provided routines and opportunities for deliberate practice.

**Implications.** Understanding what TCs learn in practice-based courses, particularly how they enact practice, contributes to understanding the efficacy of current practice-based approaches in teacher education. This study is one of a limited number of studies that examines TCs' participation and what they learn and enact. My dissertation makes two contributions that address gaps in the literature:

1. Existence proof that there are shifts in TC beliefs and talk about teaching *and* enactment of the practice of facilitating a productive mathematical discussion in a 10-week practice-based methods course.
2. An image of TC participation to contribute to trajectories of TC learning and enactment of core practices in teacher education.

My study considered the sociocultural approach common in teacher education and operationalized learning from that perspective, laying the groundwork for research on what particular pedagogies offer for TC learning. Understanding the changes in how TC participate and enact practice is useful to support teacher educators to continually refine and improve their practices and coursework.

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## Acknowledgements

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## Dedication

To Dad (my first role model of coaching, expertise, and innovation) and his unwavering and enthusiastic belief in me.

# Chapter 1: Introduction

*"What must be the aim and spirit of theory in order that practice work may really serve the purpose of an educational laboratory? We are met here with the belief that instruction in theory is merely theoretical, abstruse, remote, and therefore relatively useless to the other teacher as a teacher, unless the student is at once set upon the work of teaching; that only 'practice' can give a motive to professional learning, and supply material for educational courses" (Dewey, 1904).*

This study seeks to understand teacher candidate (TC) learning of high-quality mathematics teaching practice. The purpose of this qualitative case study is to analyze their learning as it manifested in participation, particularly how a small group of elementary TCs worked on mathematics teaching practice. To better understand the nature of TC learning, I examined the relationship and patterns in TC participation as characterized by their talk about and enactment of practice. To study TC learning as participation, my qualitative approach draws from sociocultural theory to examine their experiences through observations, video recordings, field notes, and course documents. Participants of this study were graduate students in a teacher credential program at a large public institution.

This chapter begins with a brief overview of the background and context of practice-based teacher education. Following this context, I provide my statement of purpose, research questions, and theoretically framing. I outline my research methodology and approach (covered in greater detail in Chapter 4). I include my position as a researcher and the assumptions carried into this study. This chapter concludes with a short list of definitions of key terminology used in this study and an outline of the dissertation.

## Background and Context of Practice-Based Teacher Education

Dewey (1904) highlights a perennial issue plaguing teacher education: the divide between

theory and practice. Teacher education has oscillated between practice-centric approaches and theoretical, knowledge-based approaches in a way that perpetuates this perceived gap and unproductively dichotomizes theory and practice (Forzani, 2014). Given the discrepancy in what teacher candidates (TCs) know or say about practice and what they can actually do or enact (what Kennedy, 1999, calls a "problem of enactment"), a group of teacher educators have attended to supporting TCs to *enact* practices central to the work of teaching (Forzani, 2014; McDonald et al., 2013). These teacher educators focus on addressing the problem of enactment by thoughtfully integrating theory and practice through practice-based structures, pedagogies, and curriculum (Grossman & Pupik Dean, 2019; Kennedy, 1999; Lampert et al., 2013; McDonald et al., 2013). They aim to integrate knowledge and doing by situating learning in practice and focusing on enactment of practice.

The current practice-based movement focuses on specifying core practices of high-quality instruction and pedagogies of practice that provide opportunities for deliberate practice and enactment of these practices (Forzani, 2014; Grossman & Pupik Dean, 2019; Lampert et al., 2013; McDonald et al., 2013; Zeichner, 2012). This approach to teacher education is intentional in organizing learning for deliberate practice of the interactive and complex work of teaching and enactment of core practices (Forzani, 2014; Grossman & Pupik Dean, 2019). For mathematics education, the practice of facilitating a productive mathematical discussion is vital to enacting high-quality instruction where students are pressed to collectively reason about key mathematical ideas (Franke et al., 2007; Shaughnessy et al., 2019).

**Problem Statement**

As teacher education makes progress towards specifying core practices and associated pedagogies of practice, it is equally important to document what TCs are learning. There is a

limited but growing body of research documenting how TCs learn in these types of contexts, particularly how they investigate and reflect on practice *and* enact core practices (Grossman, 2018; Hauser & Kavanagh, 2019; Kazemi, 2017; Lampert, 2010; Shaughnessy et al., 2019). For this dissertation, core practices are teaching practices essential to the work of teaching and that occur often in teaching, address common problems TCs will face, are approachable, preserve the complexity of teaching, allow novices to learn about students and teaching, and are researched and show potential to improve student learning (Grossman, Hammerness, et al., 2009; Grossman, 2018; Lampert, 2010).[1] Literature documenting the nature of TC learning and enactment of practice, as well as research on how to examine and assess that learning, is limited.

**Research Purpose and Questions**

This study seeks to understand TCs' learning as it changes over the course of a practice-based elementary mathematics methods class. The methods course aimed to develop TCs' pedagogical content knowledge (particularly about children's learning trajectories in number and operations and content knowledge in number, operations, and computational algorithms), develop competence in a set of instructional activities to enact several core practices of ambitious teaching, learn to support students participation through discussion, and create a classroom culture where all students are positioned competently.[2] Course goals were centered around a particular view of ambitious teaching.[3]

[1] For this dissertation, I focus on the following core practices identified in syllabus of the course studied: Teach towards an instructional goal; Elicit and respond to student reasoning; Orient students to one another's ideas and to the mathematical goal; Set and maintain expectations for student participation; Position students as competent; Assess students' understanding; and Represent mathematical ideas visually, numerically, and/or verbally.
[2] See Appendix B for the course goals and Appendix D for the set of instructional activities.
[3] See Appendix C for the course's named Principles for Teaching and Chapter 2 for a review of literature on high-quality mathematics instruction and ambitious mathematics teaching practices.

I adopt a sociocultural perspective where learning involves a transformation of participation. This focus on TC participation, particularly around how they talk about and enact practice, provides one way of understanding TC learning. The following research question guided my study: *How did TC participation manifest and change in their practice-based methods course?* I am interested in TC participation as they investigated and enacted the practice of facilitating productive mathematical discussions. I characterize their investigation and enactment of practice by looking at two sub-questions:

1. How did TCs facilitate discussions to reach particular instructional goals?
2. What shared repertoire did TCs develop, and how did they use these tools?

## Conceptual Framing: A Sociocultural Approach to Teacher Candidate Learning

My dissertation takes a sociocultural approach to TC learning as participation and negotiation of meaning (Lave, 1996; Lave & Wenger, 1991; Tharp & Gallimore, 1988). Sociocultural theory purports that interactions are socially mediated, and meaning is negotiated and constructed socially (Vygotsky & Cole, 1978). Drawing from this theoretical framing, I define the process of learning as becoming a fuller participant in a community of practice and negotiating and renegotiating one's identity through participation in a community. Below, I expand on how these ideas frame my research questions and guide my analytical approach.

### *Learning as Changing Participation*

I define learning as changing participation in a social activity and in the practices of a community (Lave, 1996; Lave & Wenger, 1991; Rogoff, 2003; Tharp & Gallimore, 1988). For this study, the community includes the TCs and TEs in the mathematics methods course. TC learning is fuller participation in the community of teaching practice, including development of a vision of high-quality mathematics instruction, enactment of core practices of that vision of

instruction, and engagement in the discourse of teaching. TC learning is evidenced by their increased participation in teaching practice as “guided by a shared sense of value and meaning" (Stein & Brown, 1997, p. 163; Lave & Wenger, 1991). Thus, analysis of TC learning needs to "involve the identification of the goals, values, and work practices of the community, followed by the tracing of trajectories of newcomers as they move from peripheral to fuller and fuller forms of participation in the work practices of the community" (p. 164).

### *Using A Sociocultural Unit of Analysis*

Learning is a social process and negotiation of meaning, something that happens between people. Thus, my sociocultural-framed study does not use the individual TC as the unit of analysis. Rather, the activity and social practice they engage in is the unit of analysis. With a sociocultural framing, the "focus of analysis shifts away from pedagogical activity towards an analysis of the community's activities and learning resources" (Stein & Brown, 1997, p. 162). In this study, I examine interactions between TCs and the teacher educator, the purpose of their work, and the tools that they use to better understand TC learning as "changes over time in social participation patterns in the work practices of the community" (p. 162). That participation is in a community of practice which includes three aspects (see Figure 1): mutual engagement, joint enterprise, and a shared repertoire (Wenger, 1998). These aspects frame how I examined TC participation.

**Mutual Engagement.** Mutual engagement refers to the idea that people in a community of practice are doing something in relation to one another. How TCs engage in a community, what they do, what roles they play, and how they navigate their engagement in the work all matter to understanding their participation.

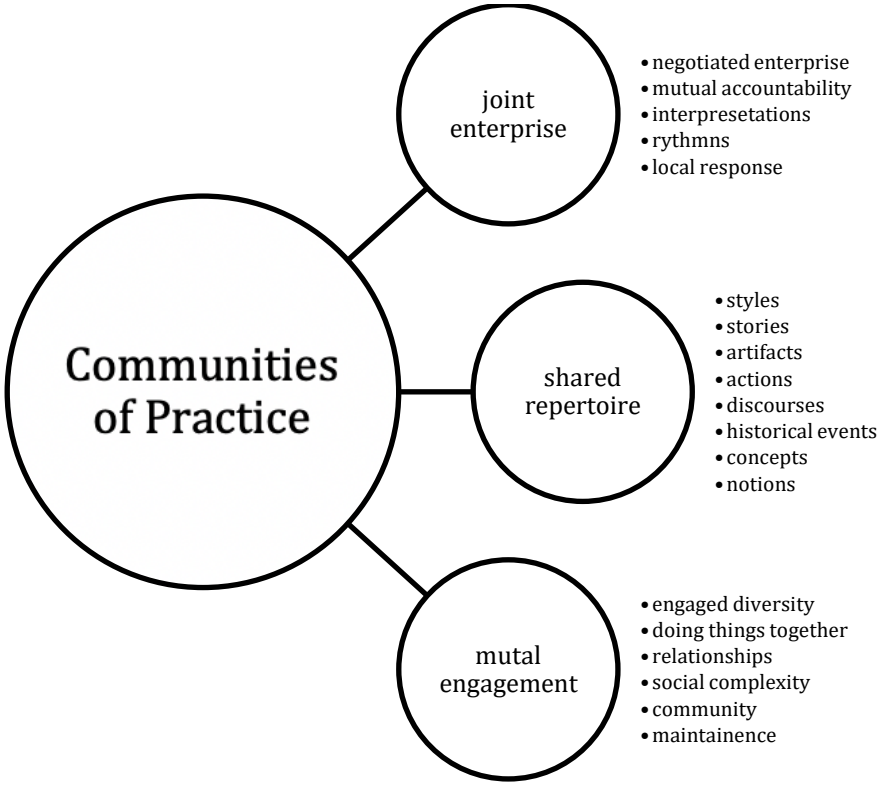
**Joint Enterprise.** As TCs are mutually engaged in the community, they are negotiating a joint enterprise, sometimes referred to as a joint productive activity. The term “joint productive

activity" describes how "individuals come together with a shared goal and work toward a joint product that is meaningful to all participants" (Stein & Brown, 1997, p. 162). This joint enterprise is the purpose of their work.

**Shared Repertoire.** As TCs engage in joint productive activity, they develop a shared repertoire that serves as a location for negotiating meaning. This repertoire includes "routines, words, tools, ways of doings things, stories, gestures, symbols, genres, actions, or concepts that the community has produced or adopted in the course of its existence, and which have become a part of its practice" (Wenger, 1998, p. 83). In understanding how TC participation becomes reified in the practice, part of my analysis involved looking at the tools, language, and other aspects of their shared repertoire.

In summary, through mutual engagement, TCs engage in joint enterprise that produces a shared repertoire.

***Figure 1.*** *Three Aspects of Communities of Practice adapted from Wenger (1998).*



Analytically, my sociocultural framing allows me to understand TC learning by examining the joint enterprise, mutual engagement, and shared repertoire of the community of practice (Wenger, 1998). Drawing from these constructs, I am interested in analyzing the shared purpose of what they are working on, the interactions and conversational discourse among TCs, and the tools and expertise within the group.

**Research Approach**

Qualitative case study methodology was used to characterize the nature of TC participation and how it changed over time within one elementary mathematics methods course. A qualitative approach is useful for understanding TC participation by allowing for rich description of how TCs talked about and enacted practice. The setting and methods course sampled was chosen based on the program's implementation of practice-based methods coursework and use of pedagogies of enactment. The participant group included seven focal TCs that were purposefully sampled as part of a larger research study.

Data was collected over the ten-week course through observations, video recordings, field notes, and documents. Data was analyzed in an iterative process aiming to uncover how TCs facilitated discussions to reach particular instructional and practice goals and how TC participation was mediated through tools. Methods of sampling, data collection, and analysis included strategies to address issues of trustworthiness to support the validity and reliability of findings. The purpose of this study and analytical approach were to characterize the nature and changes of TC participation in a practice-based mathematics methods course as they learned to facilitate mathematical discussions.

## Researcher Positionality

At the time of conducting this study, my role as a researcher was one of observer as participant, where my observer role was known and prioritized over participation in the course. As an observer as participant, I could provide insight into the social dynamics of the group and interpretations of TC participation. This role of data collection was part of a larger research project. Through this role, I collected and organized all data and have examined this data with the teacher educator of the course and with other educational researchers around a different set of research questions. This allowed me to understand the data deeply and from multiple perspectives.

It is also important that in prior years, my role in the course was as a teaching assistant. This prior experience provided an insider understanding and insight into the goals and structure of the course but could also provide bias in how I interpreted and understood events in the course. To address issues of bias and subjectivity, I employed strategies for increasing trustworthiness of my analyses and findings, such as triangulation with multiple sources of data, researcher reflectivity and critical self-reflection, member checks with study participants (particularly the teacher educator), and peer review of my analytic methods. This experience and positionality are valuable to my understanding of how learning is organized in the coursework, but also must be acknowledged and considered in how I analyzed the data.

## Assumptions

As articulated above, my position as a researcher is not neutral. I approach this work with several assumptions about learning and research. These assumptions come from my interpretation of teacher education literature, my experience researching and coaching TCs, and my perspective as a classroom teacher. While not an exhaustive list, the following assumptions underscored my work and approach to analysis:

- Theory and practice are not distinct entities. Practices of teaching, principles of ambitious teaching, and knowledge are intertwined.
- Teaching is a complex practice. This requires professional judgment and professional expertise including, but not limited to, relation expertise, pedagogical content knowledge, and mathematical knowledge for teaching.
- Teaching is something that can be learned. Through deliberate practice, teachers can improve their practice and enact high-quality teaching practices.
- People learn in practice. Learning to teach is situated in particular contexts.
- TCs need opportunities in methods courses to enact and try out practices with intentional coaching and feedback.
- Universities can play a vital role in the preparation of teachers and the research of teacher education.
- High-quality teaching practice is something that can be articulated and decomposed for TCs to learn.
- High-quality mathematics teaching involves orchestrating opportunities for all students to reason and think deeply, to engage in mathematical practices beyond simply memorization, and to develop understanding of important mathematical ideas.

**Rationale and Significance**

This study has implications for improving instruction by understanding TC learning and enactment of teaching practices. As a field, we have made substantial progress in articulating pedagogies of teacher education, particularly around pedagogies of enactment, and specifying core practices and related decompositions of practice (Grossman, 2018; Grossman & Pupik, 2019; Kavanagh, Conrad, et al., 2020; McDonald et al., 2013; Shaughnessy et al., 2019). There is also

articulation of tools and scaffolds to support learning in practice and TE moves to facilitate opportunities to learn (OTL) (e.g., Boerst et al., 2011; Ghousseini et al, 2015; Lampert et al., 2013). However, research documenting what TCs *enact* and how they are taking up practices is limited. Understanding how TC participation changes in methods courses and how they take on core practices complements the work around specifying core practices and pedagogies of enactment.

The purpose of teacher education is to prepare TCs for the profession of teaching. Thus, understanding what TCs learn in practice-based courses, particularly how they enact practice, would contribute to understanding the efficacy of a core practices approach. My dissertation makes two contributions that address gaps in the literature:

1. Existence proof that there are shifts in TC beliefs and talk about teaching *and* enactment of the practice of facilitating a productive mathematical discussion in a 10-week practice-based methods course.
2. An image of TC participation to contribute to trajectories of TC learning and enactment of core practices in teacher education.

This study is one of a limited number of studies that looks at TCs' participation and how they work on and enact core practices. I provide an initial attempt at illustrating trajectories of TC learning as participation in current practice-based approaches to teacher education. My study considered the sociocultural approach common in teacher education and operationalized learning from that perspective, laying the groundwork for research on what particular pedagogies offer for TC learning. Understanding the changes in how TCs participate and enact practice is useful to support teacher educators to continually refine and improve their practices and coursework.

Teacher education that is situated in practice can help TCs develop the skills, knowledge, dispositions, and identities necessary for enacting ambitious teaching practices. Continued

research is needed to develop practice-based teacher education that better understands what TCs are learning and how they enact practices. Practice-based teacher education that draws from both university and school expertise has the potential to help TCs enact high leverage core practices and to integrate theory and practice. There is much work to be done to transform teacher education. This type of study is essential to understand the possibilities for supporting TC enactment of high-quality practice.

**Definitions of Key Terminology Used in This Study**

While key terminology used in this study are noted below, these terms are contextualized and discussed in more detail in following chapters.

- **Ambitious teaching –** Teaching that is responsive to students and student thinking and engages students in meaningful mathematical activity in a way that positions all students competently (Kazemi et al., 2009; Kelley-Petersen, 2010). Ambitious teaching is high-quality teaching (see below) with explicit attention to equity: treating students as sense-makers (and mathematics and sense-making activity), building on funds of knowledge, and enacting culturally and linguistically responsive practices.
- **Community of practice –** A community where participants mutually engage in joint enterprise that produces a shared repertoire. The boundary of this community is determined by the practices of the group, not necessarily and organizational structure or design (Wenger, 1998).
    - **Mutual engagement –** How participants engage in a community of practice and do work in relation to one another.

    - **Joint enterprise –** The shared goal and joint product that participants negotiate and work towards. This is what the participants are working on as understood and continually renegotiated by participants in the community of practice.
    - **Shared repertoire –** The tools, discourse, norms, stories, histories, ideas, and ways of participating that the community of practice has negotiated as part of the practice.
- **Core practice -** A teaching practice that is essential to the work of teaching. These practices are "identifiable components of teaching that teachers enact to support learning" (Grossman 2018, p. 184). Core practices occur often in teaching, address common problems TCs will face, are approachable, preserve the complexity of teaching, allow novices to learn about students and teaching, and are researched and show potential to improve student learning (Grossman, Hammerness, et al., 2009; Grossman, 2018; Lampert, 2010).
- **The Core Practices Consortium (CPC) –** A cross-institutional and cross-discipline group of teacher educators focused practice-based approaches to improving preparation of TCs. Work has focused on creating a common language around teaching and teacher education through specification and study of pedagogies of practice and core practices (Core Practices Consortium, 2020; Grossman, 2018).
- **Enactment (of practice) -** An instance where TCs are facilitating instruction and interacting with elementary students. For the course under study, an important learning goal is for TCs to be able to enact several core practices of ambitious teaching, as outlined in the course syllabus.

- **High-quality mathematics instruction –** Instruction that advances student thinking and reasoning and develops students' mathematical proficiency (e.g., facilitating a productive mathematical discussion).
    - **Productive mathematical discussion –** Discussion that is responsive to student thinking, attends to mathematical goal(s), and supports meaningful student participation and learning (Chapin et al., 2013; Kazemi & Hintz, 2014; Smith & Stein, 2011).
- **Participation –** How TCs engage and interact in a setting. For this study, I focused on aspects of their practice as characterized by how they talked about practice and how they enacted practice during their mathematics methods course.
- **Pedagogies of practice –** Pedagogies in professional education, including approximations, decompositions, and representations of practice. I use the definitions outlined by Grossman and colleagues (2009, 2018).
    - **Approximation** (e.g., rehearsal, microteaching, role plays) **–** "Opportunities to rehearse and enact discrete components of complex practice in settings of reduced complexity" (Grossman, Hammerness, et al., p. 283).
    - **Decomposition** (e.g., following up on student ideas, eliciting student ideas, decomposed components lesson protocol) **–** "Breaking down practice into its constituent parts for the purposes of teaching and learning" (Grossman, Compton, et al., 2009, p. 2059).
    - **Representation** (e.g., video analysis, narratives and case studies, modeling, lesson plans) **–** "The different ways that practice is represented in professional education and what these various representations make visible to novices" (p. 2059).

- **Pedagogy of enactment –** Pedagogies that support TCs to learn to enact practices. This includes approximations of practice and pedagogies that provide opportunities for deliberate practice (with coaching) of core practices (Grossman, Hammerness, et al., 2009).
- **Practice-based teacher education -** Teacher education that integrates knowledge and doing by situating learning in practice and enactment of core practices. While there have been many historical approaches to being "practice-based," I refer to teacher education that is intentional in organizing learning for deliberate practice of the interactive work of teaching and skillful, knowledgeable enactment of core practices in response to student thinking (Forzani, 2014; McDonald et al., 2013; Hauser & Kavanagh, 2020).
- **Teacher candidate (TC) -** A student enrolled in the elementary teacher education program. For this study, TCs were full-time graduate students.
- **Teacher educator (TE) -** A person involved in facilitating TC learning. For this teacher education program, teacher educators included coursework instructors, classroom teachers, and university-based coaches. For the course studies, teacher educators included a university professor, two teaching assistants (a doctoral student and post-doctoral associate), and a classroom teacher.

**Organization of the Dissertation**

Chapter 2 is a critical review of current literature on TC learning in mathematics education and practice-based teacher education. The purpose of this literature review was to synthesize and characterize existing literature on TC learning, particularly supporting enactment of high-quality mathematics instruction. Considering the study purpose, three major areas of literature were reviewed: 1) high-quality mathematics instruction, 2) teacher learning, and 3) practice-based teacher education.

In Chapter 3, I describe context of my research: the mathematics methods course and study participants. The mathematics methods course was designed in light of the practice-based shift to articulate pedagogies of practice and specify high-quality core practices for teacher education (articulated in the previous chapter). The practice-based design of the course situated TC learning in cycles of enactment and investigation around a set of ambitious practices. In this chapter I discuss: 1) a learning cycle of enactment and investigation to organize learning around core practices; 2) the methods course; and 3) the study participants.

In Chapter 4, I describe my analytic methods. I discuss my decision to use a qualitative approach examine TC learning. I outline my sampling methods, choice of case study methodology, and role as a researcher. I discuss my data collection and analytic methods and conclude with a brief discussion on the issues of reliability, validity, and limitations to my study.

In Chapter 5, I focus on my sub-question around goals: How did TCs facilitate discussions to reach particular instructional goals? I examine how TCs worked on goals and what aspects of practice they attended to as they worked on facilitating discussions. I describe how TCs made shifts in their talk *and* enactment of facilitating discussions towards instructional goals. This analysis documented changes in TCs' specificity of goals, intentionality in working towards these goals, and ownership of goals.

In Chapter 6, I discuss the shared repertoire TCs developed and how they used tools within this repertoire. I discuss my selection of the three tools: lesson plans, board work and materials, and talk moves. This is followed by a three-part discussion of my findings for each tool, including how they were used in the course and how they facilitated OTL. I conclude with a summary of these findings across the three tools.

In Chapter 7, I discuss the implications and contributions of this study.

## Chapter 2: Literature Review

The purpose of my case study is to understand how TCs investigate and enact the practice of facilitating a productive mathematical discussion in a practice-based methods course. To frame my analysis of TC participation, this chapter presents a critical review of current literature on TC learning in mathematics education and practice-based teacher education. This review was ongoing throughout the proposal, data collection and analysis, and synthesis of this study.

This review focuses on what teacher education researchers know about TC learning, particularly supporting enactment of high-quality mathematics instruction. The purpose of this literature review is to synthesize and characterize existing literature on TC learning, particularly supporting enactment of high-quality mathematics instruction. Considering the study purpose, three major areas of literature were reviewed: 1) high-quality mathematics instruction, 2) teacher learning, and 3) practice-based teacher education.

A review of research on high-quality mathematics instruction provides an image of the type of teaching practice math teacher educators hope to support TCs in enacting. Research emphasizes the importance of responsive, discourse-based practices (e.g., facilitating a productive mathematical discussion) and highlights the complex and relational nature of teaching mathematics. This literature also justifies my choice of focusing on the practice of facilitating productive mathematical discussions.

Considering this research on high-quality mathematics instruction, I review literature on teacher learning, with a focus on mathematics education and practice-based teacher education research. I review this body of literature to understand teacher education efforts to support TC learning and to characterize how this learning has been studied. Research on teacher learning, particularly on deliberate practice and adaptive expertise, provides a framework for understanding

how TCs learn to enact complex and relational practices in high-quality mathematics instruction articulated in this review.

To situate my study within the broader teacher education landscape, I review the literature on practice-based teacher education, particularly an approach centered around pedagogies of investigation and enactment (McDonald et al., 2013). I discuss the issues that have been studied regarding this approach, including studies on practice-based efforts to support TC enactment of high-quality mathematics instruction. A more detailed description of the actual design of the practice-based methods course being studied can be found in the following chapter.

To conduct this literature review, I used multiple information sources, including books, journal articles, dissertations, and internet resources. These sources were accessed through ERIC, ProQuest, and the UW Libraries Search Engine. I sought out literature that helped answer my review question: What do we know about TC learning of discussion facilitation (particularly in practice-based mathematics methods courses)? I searched for peer-reviewed, sociocultural-framed literature on teacher learning, mathematics education, and practice-based teacher education. After finding articles that met my search criteria, I used a snowball strategy to find other seminal pieces and related articles. In the case of practice-based teacher education literature, I drew on research from the Core Practices Consortium (CPC). The CPC represents a group of teacher educators across the United States who aim to specify core practices and pedagogies for teacher education. Their work is central to articulating the emerging core practices – focused approach and opportunities for supporting TC learning in this type of teacher education.

**Defining High-Quality Mathematics Instruction**

Research on high-quality mathematics instruction (reviewed in this section) has indicated the importance of discussion in supporting student learning. Thus, my dissertation analysis focuses

on the practice of facilitating a productive mathematical discussion (I discuss core practices later in the review). The research reviewed highlights the importance of responsive, discourse-based practices and the challenge in supporting TCs to enact these types of complex and relational practices. Below I discuss the mathematics education literature and vision of high-quality mathematics instruction which I draw on for my study.

### *A Vision of High-Quality Mathematics Instruction*

Over the past few decades, mathematics educators have built a robust literature base on high-quality mathematics instruction (Franke et al., 2007; Hiebert et al., 1997; Kelley-Petersen, 2010; Munter, 2014; National Council of Teachers of Mathematics, 2014; Smith & Stein, 2011). The National Council of Teachers of Mathematics (2014) names eight research-based mathematics teaching practices:

> *Establish mathematics goals to focus learning.*
>
> *Implement tasks that promote reasoning and problem solving.*
>
> *Use and connect mathematical representations.*
>
> *Facilitate meaningful mathematical discourse.*
>
> *Pose purposeful questions.*
>
> *Build conceptual fluency from conceptual understanding.*
>
> *Support productive struggle in learning mathematics.*
>
> *Elicit and use evidence of student thinking. (p. 10)*

These practices reflect a vision of mathematics instruction as complex and interactive work, requiring teachers to enact practices based on instructional goals and student thinking. Research has examined how particular discussion-based instructional practices support students to develop

mathematical proficiency (Franke et al., 2007; Munter, 2014; National Council of Teachers of Mathematics, 2014; Smith & Stein, 2011).

### *The Role of Discussion in High-Quality Mathematics Instruction*

Discussion-based practices can support students in developing mathematical proficiency, particularly when students are pushed to reason mathematically, to communicate their ideas and listen to other students' thinking, and to develop conceptual understanding (Franke et al., 2007; Kazemi & Stipek, 2001; Munter, 2014). This research documents the importance of eliciting student thinking and building on what students already know, including the informal and intuitive mathematical ideas brought into the classroom (Carpenter et al., 2015; Franke et al., 2007; Munter, 2014; Smith & Stein, 2011). Teachers who can adjust both mathematics content and methods to what they observe in students are more likely to enable all learners to succeed. Facilitating discussions that attend to student thinking and that reach mathematical goals is necessary to fostering more equitable, high-quality experiences for students. This type of teaching is a move away from more traditional, teacher-directed practices to instruction centered on productive mathematical discussions.

**Classroom discourse communities.** This vision of high-quality mathematics instruction, specifically the focus on the practice of leading a productive mathematical discussion, requires a particular classroom discourse community. In synthesizing much of the research on this type of mathematics instruction, Munter (2014) describes these classroom discourse communities as including the following:

> "(a) Talk that is of a conceptual orientation is likely composed of mathematical arguments, relations among multiple strategies, and explorations of errors and contradictions (Kazemi & Stipek, 2001; Simon & Schifter, 1991); (b) teachers' questions drive investigations, help

> students explain their problem-solving strategies, and help the teacher understand students' thinking (Borko, 2004); (c) student discussions spawn new questions and help students recognize confusions (Engle & Conant, 2002); and (d) students hold others accountable for producing sufficient mathematical arguments by repeating their own or others' questions until satisfied with answers (Hufferd-Ackles et al., 2004)" (p. 591).

Orchestrating these types of discourse communities requires a particular role of the teacher that contrasts traditional, lecture-based teaching roles.

**The teacher's role in the discourse community.** Increasing student talk and facilitating productive discussions requires intentional facilitation and work on the part of the teacher. In a discourse community, the teacher is an active facilitator and a *more knowledgeable other*: "one who proactively supports students' learning by intentionally structuring classroom activities to afford genuine mathematical work and then coparticipating in that work" (Munter, 2014, p. 590; Rogoff et al., 1996). Teachers who play an active role in classroom discussion and student learning "continually upgrade their understanding of how each child thinks, select activities that will engage all the children in problem solving and enable their mathematical knowledge to grow, and create a learning environment where all children are able to communicate about their thinking and feel good about themselves in relation to mathematics" (Carpenter et al., 1999, p. 101). Teachers who facilitate productive mathematics discussions have a purpose in mind, elicit and respond to student thinking in ways that move the discussion towards that purpose, make mathematical ideas explicit through appropriate representations, talk moves, and modeling, and manage the social aspects of discussion including establishing discussions norms and positioning students competently.

***Facilitating discussions around mathematical goals.*** Productive mathematical discussions work towards a clear, mathematical goal (Chapin et al., 2013; National Council of

Teachers of Mathematics, 2014; Sleep, 2012; Smith & Stein, 2011). Productive discussion facilitation is not just about facilitating more *quantity* of student talk but more *quality* of student talk around mathematics and reasoning (Chapin et al., 2013). Being clear about the mathematical goal from the beginning of planning and during discussion facilitation guides teachers in making instructional decisions that increase the quality of student talk. These goals guide planning and tasks selection as well as in-the-moment teacher moves that orient students towards key mathematics (National Council of Teachers of Mathematics, 2014; Smith & Stein, 2011; Sleep, 2012). Kazemi and Hintz (2014) argue that the goal of the discussion guides in-the-moment instructional decisions and the structure of discussion: "The goal helps you decide what to listen for, which ideas to pursue, and which to highlight….[and] the mathematical ideas at the heart of a lesson will help you distinguish between different types of classroom discussions you can have with your students" (p. 2). In productive discussion facilitation, teachers' choices of mathematical representations and talk moves and the focus of argumentation and explanation are all in service of the mathematical and social goals.

***Supporting learning with mathematical representations.*** Careful use of mathematical representations is crucial to supporting learning (Kazemi & Hintz, 2014; National Council of Teachers of Mathematics, 2014). This includes mathematical models as tools and use of a board to represent students' ideas (Fosnot & Dolk, 2001; National Council of Teachers of Mathematics, 2014). Mathematical models serve as representations of our world and mathematical ideas:

> "Models are representations of relationships that mathematicians have constructed over time as they have reflected on how one thing can be changed into another and as they have generalized ideas, strategies, and representations across contexts. Although models are used as a lens when new mathematical questions are being explored, they are themselves constructed in the development of our mathematical awareness…Models are mental maps used by mathematicians to organize their activity, solve problems, or explore relationships" (Fosnot & Dolk, 2001, p. 77).

Teachers work with students to help them develop ideas around modeling and generalization. Representation on a class board serve as a public record of ideas, facilitates communication, and provide space to shape student thinking (Hiebert et al., 1997). The National Council of Teachers of Mathematics (2014) names "use and connect mathematical representations" as one of the eight practices of mathematics teaching. High-quality mathematics instruction supports student "in making connections among mathematical representations to deepen understanding of mathematical concepts and procedures and as tools for problem solving" (p. 24). Using mathematical representation is a practice of both the discipline of teaching and the discipline of mathematics. Using representations orients students to key mathematics and engages both students and teachers in mathematical practices such as modeling with mathematics and using tools strategically.

***Supporting talk though scaffolding.*** There are many strategies for supporting productive math talk and discourse in the classroom. Sherin (2002) suggests that teachers need to provide social scaffolding that "helps establish and support classroom norms for how students should talk about mathematics" as well as analytic scaffolding that "[structures] how and what mathematical ideas are discussed in class" (p. 209). For example, the filtering technique happens in classroom discussions after idea generation and comparison and evaluation of these ideas. When filtering, the "class [narrows] the space of consideration" and examines and investigates a few ideas in more depth. To scaffold the content of mathematical discourse, "each time that the teacher narrows the scope of ideas that are considered during filtering, he takes control of some of the mathematics that is discussed" (p. 226). The study suggests that new structures for classroom discourse, such as filtering, are needed to support both process and content of mathematics discourse (p. 227). Filtering is one possibility to understanding "how teachers respond to the ideas that students raise

in class – how teachers help other members of the class interpret and respond to these ideas, how teachers coordinate discussion of multiple ideas, and how teachers begin to pursue particular ideas in more details" (p. 228). Other strategies can provide alternative moves and talk scaffolds for teachers to elicit and respond to student thinking and ideas.

***Supporting talk through talk moves.*** In addition to the overall social interaction and discourse scaffolding strategies, the individual talk moves used by the teacher can support mathematical learning (Chapin et al., 2013). Chapin and colleagues articulate four steps toward productive talk (see **Table 1**). They name supporting talk moves for helping students clarify or share their thoughts, orient to other's ideas, deepen their own thinking, and engaging with other's reasoning.

***Table 1*** *Four Steps to Support Productive Talk and Talk Moves to Support These Goals*

| Steps Toward Productive Talk | Supporting Talk Moves |
|---|---|
| Helping individual students clarify and share their own thoughts | Wait time<br>Turn and talk<br>Stop and jot<br>Will you share that with the class?<br>Say more…<br>So are you saying…? (revoicing) |
| Helping students orient to the thinking of others | Who can repeat…? |
| Helping students deepen their own reasoning | Press for reasoning: Why do you think that? |
| Helping students engage with the reasoning of others | What do you think about that?<br>Do you agree or disagree…and why?<br>Who can add on? |

*Note: Adapted from Chapin et al., 2013.*

Each move can serve a different purpose (see **Table 1** above) to help increase math learning and to center the discussion on the mathematical goal. The purpose of using talk moves is to increase the amount of productive math talk to support students' conceptual learning of mathematics.

Talk moves, including revoicing, repeating, reasoning, adding on, and waiting, can help individuals clarify and share, deepen reasoning, orient to the thinking of others, and engage with the reasoning of others. Revoicing clarifies student thinking and allows others to hear again and understand. Similarly, the repeating move gives another version of the idea, provides evidence that other students could and did hear what was said, clarifies the original idea, and allows student thinking to be taken up seriously. The reasoning move elicits respectful discussion of ideas and supports mathematical learning by asking students to explain their reasoning and an adding on move can increase participation and allow students to dive deeper into the mathematical idea. Teachers can also use wait time before calling on students (to set up the expectation that students need to participate and to allow students enough thinking time to participate) or after calling on students (to allow student time to organize thoughts). TCs not only need to learn about these talk moves but also to understand when to use them and have opportunities to practice and reflect with mentors or teacher educators.

***Supporting talk around argumentation.*** Increasing productive math talk not only requires intentional use of talk scaffolds and talk moves but also attentiveness to support argumentation norms and establish what counts as mathematics. Yackel (2002) proposes collective argumentation, establishing argument from the collective rather than individual notion, as one avenue for increasing productive math talk. Sociomathematical norms in collective argumentation establish the base of mathematical knowledge: "what might be taken as backing for a collective argumentation by a mathematics class" and "the mathematical knowledge that is available for public use by a particular community in constructing mathematical claims and to justify them to others" (p. 426). Argumentation is used as a means of opening the discussion to new mathematical concepts and tools and to further the mathematical agenda. Additionally, the teacher should

provide discussion norms to help students develop support and evidence for arguments, provide reasons to justify claims, attempt to make sense of others' ideas, and understand there may be multiple arguments for a claim. In a mathematical discourse community, productive math talk through collective argumentation requires teacher to know their students as well as the content and underlying mathematics. For teacher education programs, this means supporting TCs in learning how to use collective argumentation to facilitate productive math talk (including the classroom norms, justification, notation, diagrams, and tools to help facilitate).

***Planning for discussion facilitation.*** Planning for responsive discussion facilitation requires attention to mathematics, students, and responsive teacher moves. Facilitating discussions that respond to student thinking and move towards a mathematical goal is a challenging and complex practice. This type of practice requires careful planning that anticipates student thinking and possible responsive teacher moves. Chapin et al. (2013) name four steps for planning talk-based lessons: (1) Identifying the mathematical goals; (2) Anticipating confusion; (3) Asking questions; and (4) Planning the implementation. Teachers must identify the mathematical goal (which drives planning and in-the moment instructional decisions) and anticipate student thinking and confusion around this topic. Based on the goal and anticipated student thinking, teachers can plan questions to promote talk around reasoning and conceptual thinking. As teachers plan for implementation, they consider talk moves to support productive math talk, talk formats and structure of the discussion, and a lesson closing that summarizes the big ideas. Planning the implementation involves linking talk moves to the four steps of productive math talk and choosing a talk format and summarizing the discussion in a way that highlights the goal. The purpose of this planning is not to script teacher moves, but to be strategic in how to facilitate the discussion and when to use particular talk moves:

> "Although it is not necessary to plan, in advance, every talk move you will ask, thinking ahead about how to incorporate these moves in you instruction can increase the productivity of your class discussions….make a note in your lesson plan of when you might use specific talk moves to support students' learning" (p. 268)

Other mathematics educators provide possible templates for thinking about these planning practices. For example, in *Intentional Talk,* Kazemi and Hintz (2014) include different planning templates based on the goal and structure of the discussion. Although the templates vary in content, they all include a place to anticipate student ideas and possible moves and representations to support student thinking. Smith and Stein (2011) provide a lesson plan template based on the Thinking Through a Lesson Protocol that centers around students in a manner similar to lesson planning during Japanese lesson study:

> "What is noteworthy about the planning in which teachers engage as part of the lesson study process is the attention that they pay to anticipating what students are likely to do during the lesson, considering the kinds of guidance that they could give to students who show misconceptions in their thinking, and determining how to end the lesson in a way that will advance students' understanding. In this planning process, the attention shifts from the teacher as the key actor in the classroom to the students: What are they thinking? How are they making sense of the content? How can their mathematical understanding be advanced during the lesson" (Smith & Stein, 2011).

From this perspective, lesson planning is both a place to plan for discussion facilitation and an opportunity for improving teaching and learning (Lewis, 2002; Smith & Stein, 2011; Stigler and Hibert 1999).

While there is not commonly accepted lesson plan format, lesson plans for facilitating productive mathematical discussion center around student thinking and mathematics (Chapin et al., 2013; Kazemi & Hintz, 2014; Smith & Stein, 2011; Van de Walle et al., 2019). High-quality mathematics instruction requires planning that attends to mathematical goals, anticipates student ideas and thinking, and considers teacher moves to respond to student thinking and attend to the goal.

**Discourse practices as part of the teacher education curriculum.** With this vision of high-quality mathematics instruction, the curriculum of teacher education programs should help TCs develop discourse and teaching practices necessary to facilitate productive mathematical discussions. These discussion practices require TCs to attend to social management, balance procedures and conceptual understanding, understand overarching goals, and listen to students, among many other things (Kazemi et al., 2009; Lampert, 2010; Lampert & Graziani, 2009; Leinhardt & Steele, 2005). Facilitating discussions that press on student thinking and highlight mathematical ideas is complex work, and many researchers have thought carefully about how to decompose this practice for the purpose of learning to teach (discussed later in the review).

### *Dissertation Contribution to Research on High-Quality Mathematics Instruction*

Mathematics educators have made great progress in articulating a vision of high-quality mathematics instruction, particularly around facilitating productive mathematical discussions. Much of this work has been done in classrooms with practicing classroom (and often veteran) teachers, not novices. In contrast, my study examines TCs efforts to facilitate productive discussions. I examine how they facilitate discussions to reach particular goals. My study also contributes to understanding how TCs begin using tools (including lesson plans, representations, and talk moves) to support discussion facilitation. Findings provide an image of beginning practices along a teacher learning trajectory of high-quality mathematics practice.

**The need to support learning and enactment.** Despite research supporting discussion-based practices, mathematics classrooms remain overwhelmingly teacher-directed and traditional in format. This indicates the need for support for teachers, particularly novices, to engage in this complex, interactive work and to enact practices based on student thinking. Because these discourse-based, high-quality practices are complex and often not what districts emphasize in their

math curricula and/or what TCs experienced themselves as students, teacher education programs will need to provide images of this type of high-quality teaching, appropriate support in developing necessary content and pedagogical knowledge, and opportunities for TCs to try out these practices with deliberate coaching and feedback. In the following section, I review the literature on TC learning, particularly around the high-quality mathematics instruction discussed above.

## Understanding Teacher Candidate Learning

In this section, I examine how TC learning has been characterized and studied. I discuss the shift to sociocultural approaches to TC learning and how this approach shapes what teacher educators attend to. I then expand on literature that examines TC learning through deliberate practice aimed at developing adaptive expertise and opportunities to enact high-quality mathematics instruction discussed above.

### *Learning as Changing Participation in a Community of Practice*

Contemporary research on TC learning attends to the situatedness and sociocultural nature of learning. From a sociocultural perspective, learning is changing participation in a social activity and in the practices of a community (Lave, 1996; Lave & Wenger, 1991; Rogoff, 2003; Tharp & Gallimore, 1988). A focus on communities of practice reframes learning as happening in practice via socialization and membership into a group, in this case the profession of teaching (Wenger, 1998). Teacher education plays a role in shaping learning communities and supporting the socialization of newcomers (TCs) into the practice of teaching (Lampert, 2010; Lave & Wenger, 1991; Wenger, 1998). Through communities of practice, TCs and TEs can analyze teaching problems, evaluate effective teaching strategies, develop a knowledge base for teaching, and refine practices that address the improvisational nature of teaching. In methods coursework, initial participation such as observing instructional activities or co-planning lessons acts as an

approximation of practice. TCs gradually enact practices that move them from peripheral to full participation as teachers with their own students.

Learning how to teach simultaneously shapes TCs' identities and develops their understanding of common values, language, and tools of teaching (Lampert, 2010; Ball & Forzani, 2009). As TCs participate in learning communities, they "develop a *vision* for their practice; a set of *understandings* about teaching, learning and children; *dispositions* about how to use this knowledge; *practices* that allow them to act on their intentions and beliefs; and *tools* that support their efforts" (Darling-Hammond & Bransford, 2005, p. 385). From this perspective, teacher education should support TCs to develop a vision of high-quality teaching, such as the vision discussed above, and should provide practice-based opportunities to enact practices that draw on knowledge and professional judgment in response to student thinking.

**Professionalism and adaptive expertise.** Literature on high-quality mathematics instruction highlights the improvisational, relational, and interactional nature of teaching. Teaching is an improvisational and complex profession that requires a strong knowledge base (something the field has struggled to codify) as well as professional judgment to make instructional decisions in relation to content goals and student thinking. Loughran (2019) describes this professional judgement and "the thinking that underpins professional practice" as *pedagogical reasoning* (p. 526). Ambitious teaching includes "invisible cognitive work," or pedagogical reasoning, that positions teachers as "decision-makers who consistently face pedagogical dilemmas and employ professional judgment" (Kavanagh, Conrad, et al., 2020, p. 3). In other words, in response to pedagogical dilemmas and problems of practice, teachers draw on professional judgement to make instructional decisions. The instructional moves, or observable teacher behaviors, reflect the unseen pedagogical reasoning employed by expert teachers. This

unseen pedagogical reasoning is similar to "disciplined improvisation" (Sawyer, 2004) and adaptive expertise (Hatano and Inagaki, 1986; Darling-Hammond & Bransford, 2005; Timperley, 2013). Teaching is complex and requires adaptive expertise and pedagogical reasoning as teachers navigate nonroutine and dynamic pedagogical dilemmas.

Hatano and Inagaki (1986) describe two kinds of professional expertise: routine expertise and adaptive expertise. Routine experts focus on the dimension of efficiency and developing "a core set of competencies that they apply throughout their lives with greater and greater efficiency" (Darling-Hammond & Bransford, 2005, p. 48-49). Adaptive expertise involves innovating and restructuring beliefs, practices, and identity in response to new information and nonroutine inputs (Anthony et al., 2015; Darling-Hammond & Bransford, 2005; Hatano & Inagaki, 1986). Teachers who have adaptive expertise attend to their practice in relation to students:

> "While it is important that PTs master routines, what distinguishes adaptive teachers is their constant attention to the impact of teaching and learning routines on students' engagement, learning, and wellbeing" (Anthony et al., 2015, p. 109).

The high-quality mathematics instruction outlined in the previous section is an ambitious vision associated with this conception of adaptive expertise.

As a profession, teaching requires expertise that balances the two dimensions of *efficiency*, performing tasks in a routinized or automatized manner and without much attentional resources, and *innovation*, "[going] beyond one's existing efficiency-oriented skills and strategies to adapt to new situations" to "rethink key ideas, practices, and even values in order to change what they are doing" (Darling-Hammond & Bransford, 2005, p. 77, 361; Hatano & Inagaki, 1986). Building the ideas of routine expertise and adaptive expertise, Timperley (2013) positions teacher learning as situated in practice and teachers as "responsive and adaptive experts for whom the engagement, learning, and well-being of all students is the basis of their professional identity" (p. 4). In reviewing literature on practice-based experiences that support novices to meet standards for

graduation, Timperley provides a framework for understanding shifts that novices make as they learn to teach. The review asserts that "routine expertise and adaptive expertise are not mutually exclusive" and expertise does not develop on a linear progression from novice to routine expert to adaptive expert (p. 9). Professionalism requires both routine expertise and adaptive expertise. As novices develop routine and adaptive expertise, they make two shifts: 1) Focus from self to students; and, 2) Increasingly complex understandings about teaching and learning. These shifts can be seeing in **Table 2**.

***Table 2*** *Markers of Routine and Adaptive Expertise*

| Focus of shift | From | To |
|---|---|---|
| | Self | Students |
| Identity | Self as a student learning how to teach. | Routine: Job is to construct an effective learning environment.<br>Adaptive: Professional identity is focused on promoting valued outcomes for each learner. |
| Agency | Self-preservation: Surviving the reality shock. | Routine: Strong sense of self-efficacy that provides the confidence necessary for teaching.<br>Adaptive: Agency depends on developing relationships with learners that promotes their learning – particularly priority learners. |
| Normality | Self as the norm: All learners are like them and learn like they do. | Routine: Realizes that some learners are different from them<br>Adaptive: Learn to identify and use cultural and linguistic resources of diverse learners |
| | Simplicity | Complexity |
| Complexity and knowledge | Teaching is about transmitting knowledge. | Routine: Teaching needs to flexibly provide opportunities to learn accepted knowledge.<br>Adaptive: Teaching is the co-construction of knowledge. |
| Interactions, relationships, and responsibility | If taught in ways familiar to the teacher, students should learn. Marks reflect student ability (absolute view). | Routine: Students' responsibility to learn, teacher to present as well as possible.<br>Adaptive: Teaching and learning is a function of complex relationships between teachers and students, whanau [families] and communities. |
| Location of learning | Learning happens primarily in the classrooms. Other environments such as the home get learners ready for schooling. | Routine: The school needs to build on this learning.<br>Adaptive: Learning draws on resources from multiple environments. Teaching develops powerful educational connections between these environments |

*Note: Adapted from Timperley, 2013 and Anthony et al., 2015.*

Anthony et al. (2015) use Timperley's framework to study teacher learning as the development of adaptive expertise. This case study of two TCs used interview and journal transcripts to analyze shifts in beliefs and values of teaching and learning. They found both TCs made shifts from self to students and towards more complex understanding about teaching. These shifts reflected markers of routine expertise and beginning markers of adaptive expertise. These shifts were supported by repeated opportunities to experiment with instructional practices and to work with students over time, attention to student learning outcomes, and individual and group planning and reflections on enactments:

> "Overall, the act of engagement in an inquiry and knowledge building cycle…appears to have enhanced PTs' valuing of teacher inquiry as a way of learning – the essence of adaptive expertise" (p. 115)

Articulating the shifts from self to students and from a simple to increasingly complex understanding of teaching and learning is a useful framework for thinking about how TCs learn, particularly how they develop routine expertise and adaptive expertise. The shifts articulated by Timperley, Anthony, and colleagues focus on TC beliefs, identity, and knowledge about teaching and learning. My dissertation builds on this idea of shifts that teachers make by analyzing the shifts in *enactment* of practice. I extend this framework to consider shifts of enacted practice, not just as stated in interviews and journal reflections of practice. In other words, I examine the development of routine and adaptive expertise reflected in their *enacted* practices.

**Deliberate practice to develop professional judgment and adaptive expertise.** Given the complexity of teaching, deliberate practice is needed for TCs to develop the professional judgment and expertise required of teachers. Deliberate practice is "purposefully and critically rehearsing certain kinds of performances…[and] is particularly important to the development of expertise" (Darling-Hammond & Bransford, 2005, p. 401). The assumption here is that high-quality teaching practices do not develop hap hazardously or just by simply being in a classroom.

Rather, through deliberate practice, prior knowledge is considered, tasks are crafted explicitly to improve specific areas of teaching, and feedback and self-assessment is incorporated (Ericsson et al., 1993). A major assumption here is that TCs learn through practice and need opportunities to practice teaching with feedback and coaching.

The challenge for teacher education is to create opportunities for deliberate practice of the unseen pedagogical reasoning and adaptive expertise TCs need to teach ambitiously (in addition to observable instructional moves and behaviors of ambitious teaching). Thoughtful practice-based experienced and pedagogical dilemmas can be used to create opportunities to learn (OTL) for TCs to engage in pedagogical reasoning and to develop their adaptive expertise (Kavanagh, Conrad, et al., 2020). Recent approaches to practice-based teacher education focus on specifying pedagogies of practice and core practices to engage TCs in pedagogical reasoning and deliberate practice (Forzani, 2014; Grossman & Pupik Dean, 2019; Kavanagh, Conrad, et al., 2020; Timperley, 2013). I discuss the literature on this practice-based approach in the last part of this review.

### *Efforts to Support TC Learning in Methods Courses*

Mathematics educators have made efforts to study TC learning of high-quality mathematics instruction and through deliberate practice focused on developing adaptive expertise. In thinking about the knowledge and practice needed for adaptive expertise, literature has focused on TCs development of content and pedagogical knowledge, beliefs about teaching and learning, and opportunities to learn.

**Characterizing learning as the development of pedagogical content knowledge.** Beginning in the 1980s, teacher education shifted to approach teacher learning of professional knowledge and from a cognitive approach (Forzani, 2014). As a result, the field has a robust literature base on teacher development of content knowledge and pedagogical knowledge. An

important contribution of this work was Shulman's concept of pedagogical content knowledge (1986). More recent work in mathematics education has focused on TC learning of pedagogical content knowledge, reasoning about student thinking, and mathematical knowledge for teaching (e.g., Ball et al., 2009; Silver et al., 2007). While mathematical knowledge and pedagogical content knowledge are important curricular goals of teacher education, examining how TCs develop professional knowledge does not provide a comprehensive understanding of TCs' development of adaptive expertise and the types of practice they learn to enact.

**Characterizing learning as changing participation.** Research from the last two decades has focused on studying TC learning as changing participation in a community of practice. Studies that focus on participation and development of adaptive expertise have examined TCs' opportunities to learn (OTL), often by examining participation of how TCs talk about practice. Data sources include interviews, written reflections, self-reports about practice, and videos of TC discussing practice. These studies are useful for understanding changes in how TCs talk about teaching and problems of practice and how investigation and reflection on practice supports their development of professional knowledge and a vision of practice. However, participation includes more than beliefs and understanding about teaching and learning. Participation also includes the enactment of instruction and the practices TCs appropriate. Thus, I separate my review of teacher learning studies into two categories: studies that characterize talk and OTL and studies that characterize participation (including talk) and enactment.

***Characterizing talk and opportunities to learn (OTL).*** With the increased attention to communities of practice and framing of learning as a social activity, teacher educators have characterized learning by examining OTL. OTLs in teacher learning communities are "affordances for changing participation and practice….[and] understanding a learner's trajectory involves

hypotheses about affordances that are available to the learner to participate in particular ways" (Horn & Kane, 2015, p. 172; Greeno & Gresalfi, 2008). Although my dissertation focuses on TC learning, there are studies on inservice teacher learning that are helpful for characterizing the research on OTL. The following three studies focus on opportunities for inservice teacher learning.

Horn and Little (2010) analyzed in-service teacher work group interactions and how teachers' talk provided OTL. They used videos of teacher work group interactions, interviews, and material artifacts to understanding how teacher work groups used conversational routines to open opportunities to talk about problems of practice:

> "By conversational routines, we refer to patterned and recurrent ways that conversations unfold within a social group. Routines are constituted by moves, turns of talk that shape the interaction's progress by setting up and constraining the response of the subsequent speakers." (p. 184)

One conversational routine (normalizing, specifying, revising, and generalizing problems of practice) allowed teacher talk to move between specific and general accounts of practice, and the practice of "normalizing" fostered a sense of membership and legitimized teachers' experiences as worthwhile problems of practices. Visible teacher leadership also created OTL by "posing questions, eliciting specific accounts of classroom practice, preserving a focus on both student and teacher learning, and encouraging initiative of these sorts by others" (p. 210-211). Furthermore, the group's shared curriculum enabled them "to situate problems of practice in relation to students' observed or anticipated encounter with a common set of mathematics problems, activities, and instructional materials" (p. 210). This contrasted the talk-through routine where participants walked through a lesson. In the walk-through routine, the conversation became about showing and telling what happened or deliberating and evaluating teacher decisions and curriculum. Methodologically, this study contributes to understanding of how OTL and problems of practice can be examined through teacher's talk at the level of conversational routines.

Subsequent research on conversational OTLs in secondary math teacher's work groups linked teacher talk and professional learning. Horn & Kane (2015) studied talk as collective ZPD, Zone of Proximal Development (Engstrom, 1987), in beginning, emergent, and sophisticated workgroups (based on teacher experience). Findings indicated accomplished teachers are positioned to learn more from talking with colleagues because of the available conceptual resources:

> "By relying primarily on language as a medium to interact with artifacts, constitute representations, and convey questions about teaching, teachers' talk about instruction also conveys problem frames and epistemic stances about the nature of teaching. These different conceptual resources—problem framings, representations, and epistemic stances—provide substantively different traction on instructional problem solving and, we claim, teachers' OTLs" (p. 380).

In addition to conceptual resources within the group, the emergent group benefitted from coach (not present in the other groups) in the role of a more knowledgeable other. The important role of the coach as a more knowledgeable other is similar to the importance of a teacher educator working with novice TCs.

Sherin and Han (2004) analyzed teacher talk in the context of Video Clubs. In these Video Clubs, teachers examined videos of their practice, providing representations of practice and access for the group to make sense of classroom interactions. They documented two changes as teachers participated in Video Clubs. First, there was a shift in *what* teachers discussed: from raising pedagogical issues (teacher-focused) to attending to student thinking. Second, there was a shift in *how* they discussed practice: "Over the course of the video clubs, the teachers selected more complex issues related to student conceptions to examine" (p. 180). Earlier discussions focused on evaluating and suggesting alternative teacher moves or strategies, while later discussions attended to more complex pedagogical issues and connecting pedagogical reasoning with student thinking: "Specifically, once the teachers began to focus on student conceptions, they developed

sophisticated techniques for analyzing the student ideas that stood out to them" (p. 180). This shift in *how* teachers discussed practice reflected development of teachers' professional vision of high-quality instruction.

These studies contribute to understanding OTL created by the pedagogies of Video Club (Sherin & Han, 2004) and replays and rehearsals in teacher workgroups (Horn & Kane, 2015; Horn & Little, 2010). Their findings are similar to the shifts in focus from self to student and in understanding of teaching and learning in Timperley's framework (2013). These studies serve as a starting point for conversation around the OTL that teacher talk can create. The authors articulate the need for further research on how this talk relates to enactment and changes in practices. The literature presented thus far provides useful ways for thinking about OTL in the context of *practicing* teachers. Below I highlight how OTL have been studied in the context of TCs (preservice teachers).

Ghousseini and Herbst (2016) examined OTL in a practice-based, secondary mathematics methods course. They examined OTLs by analyzing video of methods class sessions on leading discussions, an end-of-course TC portfolio, and interviews with a curricular framework (Darling-Hammond & Bransford, 2005) and pedagogical framework (Grossman, Compton, et al., 2009). Their curricular framework for learning to teach (FLT) focused on the goals of teacher learning and included "a vision of practice, knowledge of students and content, dispositions for using this knowledge, and a repertoire of practices and tools" (p. 79). Findings documented how representations of practice provided opportunities for TCs to develop content knowledge and vision of discussion practices. Decomposition of practice afforded OTL simultaneously about moves and routines to facilitate discussion as well as dispositions for teaching and vision of classroom discussions. Approximations afforded different OTL around all four components of the

FLT. Findings around OTLs facilitated by approximations highlighted to role of the TE in thoughtfully sequencing experiences and approximations, using pedagogies deliberately, and calling TC attention to practices and curricular goals. Their findings align with practice-based literature advocating for decompositions and approximations in addition to representations of practice and suggest that coordinating *all* three (rather than focus on just one) better supports OTL around curricular goals:

> "Opportunities to learn about classroom mathematics discussions as viewed from the FLT (professional vision, dispositions, teacher moves and tools, and knowledge of content and students) were distributed across the three pedagogies of practice, with each pedagogy making additional aspects of the practice available for the PSTs' understanding and investigation" (p. 100).

While their study only examines two 3-hours class meetings, the authors suggest TCs need many opportunities to do this work and to deliberate practice enactment of facilitating productive mathematical discussions.

Lampert and colleagues' study (2013) on the pedagogy of rehearsal examined what TEs and TCs do during rehearsals and the OTL the knowledge, practices, and dispositions of "ambitious teaching" (aligned with the high-quality mathematics instruction articulated earlier). Studio Code was used to examine the substance and structure of TE/TC exchanges in 30 rehearsals. They found that rehearsals, within a cycle of enactment and investigation, facilitated deliberate practice of ambitious teaching as the TE and TCs worked on multiple practices and variations of those practices. The split time between actively teaching the activity and exchanges discussing aspects of practices facilitated more authentic approximation to ambitious teaching than simply enacting a lesson without deliberate feedback and discussion through the exchanges. Eliciting and responding were the most frequent substance codes within exchanges (followed by representations then student engagement) and were worked on both in isolation (i.e. it was the only focus of the

exchange) and simultaneously with other codes. They assert that rehearsals provide opportunities for deliberate practice to develop adaptive expertise:

> "Rehearsals created rich opportunities for novice and TEs to learn to navigate the social and intellectually complex demands of teaching. Rehearsals are designed both for novices to try out enactments of IAs and interact with TEs to develop the judgment need to respond appropriately to student performance" (p. 238).

My dissertation builds on this analysis of OTL in TE/TC exchanges in rehearsals to begin documenting how TCs appropriate ambitious practices across the cycle of enactment and investigation in their methods course.

The literature reviewed highlight a couple trends in research on teacher learning. First, aligned with the vision of high-quality mathematics instruction noted in the previous section, nearly all the studies talk about the importance of putting students and student thinking at the center of teacher learning. This literature represents a growing focus on trying to understand how TCs learn the complex and relational practice of facilitating productive mathematical discussions.

Another commonality was the characterization of teacher learning as opportunities for teacher learning (OTL). Many of these studies provide images of the types of learning communities and pedagogies that can support a range of important aspects of learning to teach (e.g., development of a vision of high-quality practice, pedagogical content knowledge, a focus on student thinking, etc.). However, understanding what TCs believe about teaching and learning and OTL do not capture what TCs are able to *enact* and how they appropriate these ideas *in* their instruction. Disappointingly, research indicates a gap between TC beliefs about high-quality instruction and enacted practice (Clift & Brady, 2005; Kennedy, 1999; van Es & Conroy, 2009). For example, van Es and Conroy (2009) found that even if TCs could talk about classroom discourse and articulate the vision of high-quality instruction proposed by their teacher education program, they weren't able to ask questions to students to promote sense making, reasoning, and

conceptual thinking. This is not to say that TCs cannot be supported to make changes in their beliefs *and* practice. Changes in beliefs about teaching (including developing a vision of high-quality teaching practice) is not equivalent to changes in ability to *enact* high-quality teaching practice.

Further research is needed to examine the gap between talk and practice. Developing TC vision of high-quality instruction and being able to talk about this kind of practice are necessary but not sufficient outcomes of teacher education. Thus, the field needs literature characterizing TC enactment in addition to these robust studies discussed above. In the following section, I discuss the limited but growing body of literature that seeks to characterize participation and enactment of practice.

***Studies that characterize participation and enactment.*** To understand how practice-based methods courses support TC learning around facilitating discussion, it is useful to have an image of their participation within that course. In this section, I review several studies to discuss how the field has examined TC enactments of practice (see **Table 3** below). There are a limited number of studies that characterize participation and enactment, particularly when refining the search criteria to studies of mathematics methods courses. The eight studies reviewed provide methodological examples of how to analyze participation and findings illustrate the field's limited by growing knowledge base of TC participation and enactments.

***Table 3*** *Studies that characterize participation and enactment around discussions*

| Study | Discipline and Context | Research Question(s) | Aspects of Practice Examined |
|---|---|---|---|
| van Es & Sherin. (2010) The influence of video clubs on teachers' thinking and practice. | Elementary Mathematics<br><br>Video club of inservice teacher enactments of whole group and small group discussions | How does participation in a video club focused on examining student mathematical thinking influence teachers' thinking and practice? | Shifts in teaching practices:<br>• Making space for student thinking<br>• Publicly recognizing students have ideas to contribute<br>• Providing extended opportunities for student thinking<br>• Eliciting multiple methods or solutions from students<br>• Probing student thinking: Asking for explanations<br>• Probing student thinking: Probing for explanations<br>• Teacher as a learner: Learner stance |
| Ghousseini et al. (2015) Investigating the potential of guided practice with an enactment tool for supporting adaptive performance. | Elementary Mathematics<br><br>TC rehearsals of instructional activities within the methods course (activities were taught to elementary students immediately following rehearsals) | What problem space arises for novices when they us an enactment tool in the context of learning a complex interactive practice like teaching?<br>How can this problem space afford opportunities for novices and instructors, as pore competent others, to connect tools to goals in ways that guide novices' adaptive practices? | Problems of practice in question sequences and connections to professional commitment of ambitious teaching Commitment categories included:<br>• Students as sensemakers.<br>• Participation and equitable access.<br>• Clear instructional goals.<br>• Knowledge of students as learners. |
| Sleep & Boerst. (2013) Preparing beginning teachers to elicit and interpret students' mathematical thinking. | Elementary Mathematics<br><br>Audio recording of TCs conducing a Student Thinking Interview in their field placement but as part of a methods course assignment. | How can teacher education assignments be designed to support interns in learning to do the work of teaching? | Types of teacher questions/prompts<br>• Initial posing<br>• Eliciting student thinking<br>• Neutral<br>• Leading |
| Sleep. (2012) The work of steering instruction toward the mathematical point: A decomposition of teaching practice. | Elementary Mathematics<br><br>Enactments of lessons in field placements. | What are the central tasks of steering instruction toward the mathematical point during a lesson's enactment?<br>What teaching strategies can be used to steer instruction toward the mathematical point?<br>What problematic issues arise for beginning teachers when trying to steer instruction toward the mathematical point? | Seven central tasks of steering instruction towards the mathematical point:<br>• Attending to and managing multiple purposes.<br>• Spending instructional time on mathematical work.<br>• Making sure students are doing the mathematical work.<br>• Developing and maintaining a mathematical storyline.<br>• Opening up and emphasizing key mathematical ideas.<br>• Keeping a focus on meaning. |
| Kazemi & Waege. (2015) Learning to teach within practice-based methods courses. | Elementary Mathematics<br><br>TC rehearsals of instructional activities within the methods course followed by enactments of these activities in a partner elementary classroom | How did PTs develop in their teaching of ambitious practices as envisioned by the mathematics methods course? | Ambitious teaching practices:<br>• Aiming toward a mathematical goal.<br>• Eliciting and responding to students' mathematical ideas.<br>• Orienting students to each others' ideas, setting and maintaining expectations for student performance.<br>• Positioning students competently.<br>• Assessing students' understanding.<br>• Using mathematical representations. |

| Study | Discipline and Context | Research Question(s) | Aspects of Practice Examined |
|---|---|---|---|
| Reisman et al. (2019) Evidence of emergent practice: Teacher candidates facilitating historical discussions in their field placements. | Elementary and Secondary Social Studies<br><br>Discussion assignment: Lesson (enactment) of TCs' facilitating discussion around historical texts with students in their field placements. | How did teacher candidates enact discussion in their field placements after being introduced to discussion through a practice-based approach in their methods course?<br>How and to what extent did teacher candidates' discussion facilitation vary depending on the scaffolds (e.g., lesson structure, instruction materials) they received in methods instruction? | Four-Pronged Framework for Historical Discussions:<br>• Engage students as sensemakers.<br>• Orient students to each other.<br>• Orient student to the text.<br>• Orient students to the discipline. |
| Kloser et al. (2019) Connecting the dots: Secondary science teacher candidates' uptake of the core practice of facilitating sensemaking discussions from teacher education experiences. | Secondary Science<br><br>TC rehearsals of sensemaking discussions within the methods course and enactments of discussion facilitation with students in their field placements. | Within and across each problem, what elements of the practice do (facilitating sensemaking discussions in science) do teacher candidates take up when:<br>a. Facilitating a discussion with peers during rehearsal, and<br>b. Facilitating a discussion with students in an authentic classroom environment?<br>How do teacher candidates' uptake of the practice in rehearsals and classroom discussions vary based on whether elements of the practice are presented implicitly or explicitly in the methods course? | Coding Framework for Facilitating Sensemaking Discussions:<br>• Goals for sensemaking discussions<br>• Framing the discussion<br>• Facilitating the discussion<br>• Closing the discussion<br>• Equity considerations<br>• Pre-requisites for quality discussion<br>• Centrality of models to discussion<br>• Centrality of tools to discussion<br>• Coordinated set of practices<br>• Pacing<br>(child codes were specified under these parent codes) |
| Thompson et al. (2013) Developing a Theory of Ambitious Early-Career Teacher Practice. | Secondary Science<br><br>TC enactments in their methods course, field placement, and first year of teaching. | How are novices' trajectory of practice influenced by their own developing pedagogical reasoning about teaching and learning, as well as their participation in multiple professional communities that may have contrasting visions and messages about good teaching?<br>How do tools and professional routines, used in different learning-to-teach settings, creating opportunities and tensions for the development of practice? | Ambitious teaching practices of science:<br>• Selecting big ideas/models.<br>• Working on students' ideas.<br>• Working with science ideas.<br>• Pressing for explanation.<br>(practices were coded with levels of sophistication and focus on students) |

These studies focus on practices used within the larger practice of facilitating productive mathematical discussions (Ghousseini et al., 2015; Kazemi & Waege, 2015; Sleep, 2012; Sleep & Boerst, 2012; van Es & Sherin, 2010) or are studies of the practice of discussion facilitation in other disciplines (Kloser et al., 2019; Reisman et al., 2019; Thompson et al., 2013). I discuss these studies to illustrate what the field knows about TCs' enactments of practices central to facilitating productive mathematical discussions and cross-discipline examples of how to examine discussion facilitation.

The first study is a continuation of the Video Club research discussed in the previous section: van Es and Sherin (2010) provide a methodological example of how to characterize teacher talk *and* instruction by examining inservice teachers' talk during Video Clubs, teachers' self-reports of the effects of the Video Club, and instruction over the year. The following three mathematics education studies examined TCs' development of practices within discussion facilitation: eliciting and responding to student thinking (Ghousseini et al., 2015), eliciting and interpreting student thinking (Sleep & Boerst, 2012), and steering instruction towards a mathematical point (Sleep, 2012). In particular, Sleep (2012) provides a useful framework for how TCs learn to facilitate discussions towards particular mathematical goals, and I use this framework in my analysis of goals in Chapter Five. Three additional studies outside of mathematics education (Reisman et al., 2019; Kloser et al., 2019; Thompson et al., 2013) characterize TC enactments and their attempts to facilitate discussion. Although these studies are in science and social studies, the methodology and characterization of how TCs facilitated discussion parallels much of my methodological approach, desire to understand how practice-based methods courses support TC learning and aim to characterize TCs' discussion facilitation. I summarize their findings and methodologies and connect these to this dissertation study. The final study, examining TC

participation in an earlier iteration of the course studied in this dissertation, used the course's named instructional practices as a lens for characterizing TCs' enactment so practice (Kazemi & Waege, 2015). My dissertation builds on their findings on how TCs worked on managing interactions with students, setting up tasks to facility equitable access to the task, and using representations to work on particular mathematical ideas

Although in the context of inservice teachers, van Es and Sherin (2010) provide an example of how to characterize teacher talk *and* instruction by examining teacher talk during Video Clubs, teachers' self-reports of the effects of the Video Club, and instruction over the year. The goal of the Video Clubs was to support teachers to notice particular ideas around student thinking. Videos of discussions were coded for participation (who commented, the topic (classroom management, climate, mathematical thinking, pedagogy, other), stance (describe, evaluate, interpret), specificity (specific, general), and source of evidence (video or not). Analysis of interviews focused on teacher's beliefs about the importance of student thinking, role of student thinking in instruction, and the curriculum. To analyze for changes in practices related to student thinking, observations of lessons were coded for three related practices: publicly recognizing students have ideas to contribute, providing extended opportunities for student thinking, and eliciting multiple methods or solutions from students. From analyzing the interview, they found teachers believed they were paying closer attention to student thinking and believed they learned about curriculum. Findings noted changes in how they attended to student thinking during discussion facilitation:

> "Teachers increasingly made space for students' thinking to become public in the classroom, they more frequently probed students' thinking, and they took on the stance of learner in the context of teaching" (p. 169).

This study offers an image of how researchers might connect analysis of teacher talk (and subsequent OTL and development of professional vision) with changes in instructional practice.

Methodologically, they analyze teacher learning as changing in participation and use data sources that capture both teacher *talk* about and *enactment* of practice.

Ghousseini et al. (2015) analyzed video rehearsals to understand how tools (specifically a question sequence) could be used adaptively within a community of practice. They examined problem spaces, opportunities to develop TCs' adaptive expertise, and how enactment tools mediated TC learning in a community of practice. They found one enactment tool (a routine question sequence) created opportunities for deliberate practice around eliciting and responding to student thinking in rehearsals. By stabilizing and routinizing some aspects of practice, TCs were able to focus on this eliciting and responding practice:

> "The use of enactment tools, like the question sequence at the center of our investigation, can function as a stable procedure that can reduce some of the initial complexity of relational practice. At the same time, however, enacting the procedure opens up a rich problem space that scaffold performance. When novices have opportunities to navigate these problem spaces in the company of more experienced others, enactment tools can mediate learning about goals of profession practice, including the commitments that enable practitioners' judgment in situations of uncertainty" (p. 494-495).

Findings also indicated the importance of the TE in connecting tool use with a vision of high-quality mathematics teaching.

Sleep and Boerst (2013) examined the practices of eliciting and interpreting students thinking. Their study examined how scaffolds supported TCs in developing these assessment practices as evidenced in the Student Thinking Interview assignment. For this elementary mathematics methods assignment, TCs interviewed a student in their field placement and made evidence-based claims for formative assessment. Researchers coded the types of questions TCs asked to elicit student thinking and their assertion sketch (their evidence-based claim about student thinking) before feedback, in relation to student thinking, and after instructor feedback. They found TC eliciting practice was scaffolded by provided prompts (they almost always used questions from the pool of possible prompts). They also tracked discrepancies between assertions and evidence

and between evidence and interviews with students. Instructor feedback was tailored to these discrepancies, and TCs overwhelmingly took up this feedback. The assertion sketch provided scaffolding of the format of claims and evidence but did not seem to impact the substance and specific language modeled in the sample. Findings indicate the importance of attending to the purpose and level of the scaffold. Although they only examine a small aspect of practice (and not in the context of facilitating discussions), this study provides one way to characterize TC practice (in this case formative assessment practice) and how to scaffold that practice.

Sleep (2012) examined enactments of discussion facilitation from mathematics lessons taught in teaching practicums to decompose the practice of "steering instruction towards the mathematical point." She outlined seven central tasks to this work of steering instruction:

1. attending to and managing multiple purposes,
2. spending instructional time on mathematical work,
3. spending instructional time on the intended mathematics,
4. making sure students are doing the mathematical work,
5. developing and maintaining a mathematical storyline,
6. opening up and emphasizing key mathematical ideas, and
7. keeping a focus on meaning.

For each of these seven tasks, she identified teaching strategies and problematic issues arising for beginners as they worked on steering discussions. She argues her tasks and associated teaching strategies decompose the work of steering discussions to a useable grain size that helps beginnings learn what to *do* to steer discussions. The positioning of strategies for each task and nesting of these tasks within the larger practice of steering discussions towards a mathematical point maintains the how and why of practice, as Boerst and colleagues (2011) discussed in their study.

Sleep also suggests developing tools (e.g., lesson plans, observation/debriefing protocols, etc.) based on these tasks could help focus TC attention on the how to attend to instructional goal and steer the discussion to reach that goal. This study provides a useful lens for examining practice and decomposing the work of facilitating discussion towards an instructional goal.

Certain tasks, scaffolding, tools, and pedagogies can help TCs enact practices in relation to mathematical goals and student thinking. Sleep and Boerst (2013) and Ghousseini et al. (2015) examined practices that could be nested in the practice of facilitating discussion: the practices of eliciting, interpreting, and responding to student thinking. Sleep's (2009) articulation of seven tasks within "steering discussion towards the mathematical point" is another perspective on working on discussion facilitation. Other studies examine the practices of orienting students to each other's ideas (Ghousseini, 2015) and planning practices for selecting, sequencing, and connecting student strategies (Tyminski et al., 2014). While these studies document enactment of practices needed to facilitate discussions, it would be useful to have a trajectory for understanding TCs' development of the larger practice of facilitating productive mathematical discussions in relation to connected practices and nested smaller practices. Four studies examined this kind of trajectory and documented TCs' attempts to facilitate discussion in elementary and secondary social studies practicums (Reisman et al., 2019), in secondary science methods courses and practice (Kloser et al., 2019), in the practicum and first year of secondary science teaching (Thompson et al., 2013), and in an elementary mathematics methods course (Kazemi & Wæge 2015).

Kloser et al (2019) and Reisman et al. (2019) examined TC enactments in relations to tools, similar to Ghousseini et al. (2015). Reisman and colleagues (2019) used a four-pronged framework of facilitating historical discussions (see **Table 3** above) to document how TCs facilitated

discussion, where they seemed to struggle, and how tools supported their facilitation of discussions. They noted that TCs used prescribed questions sequences and instructional materials from methods courses and “oriented students to the text, to each other, and to the disciplines as demonstrated by their instructors” in the methods courses (p. 153). They also found that although TCs "made efforts to orient students to each other, they struggled to engage students in collective knowledge construction" (p. 154). Similarly, Kloser and colleagues (2019) found that although TCs used discursive tools from the methods courses and were able to take up particular talk moves, they “did not always invite or result in students taking up other students' ideas and moving collective understanding forward" (Kloser et al., 2019, p. 126). Research questions for this study focused on what aspects of the discussion framework TEs made explicit or working on implicitly, what aspects of discussion TCs took up (and how), and the connection between the TE pedagogies and TC practice. Both studies documented trajectories of how TCs facilitated discussions (rather than assessing quality of practice, e.g., Thompson et al., 2013) and found TCs were able to use discursive tools provided from methods courses. Both studies documented the challenge for TCs to facilitate productive discussions that supported students’ collective knowledge construction and noted the importance of identifying instructional goals to facilitate productive sensemaking discussions. Similarly, I examined how TCs worked on and facilitated discussions in relation to instructional goals and tools (including discursive tools). However, I do not examine practice at the level of utterance (Reisman et al., 2019) nor do I explicitly connect TE pedagogies to TCs trajectories of practice from coursework to the field (Kloser et al., 2019). I document the trajectory of participation (using the interactions of TCs as the unit of analysis) and TCs’ facilitation of discussions within their mathematics methods course.

Thompson and colleagues (2013) also examined trajectories of science teaching practice in the context of TCs' practicum and first year of teaching (rather than the methods course). They used both observations of teaching practice and interviews to understand how and why novice teachers appropriated ambitious science teaching practices. Instruction was coded for instances of ambitious practices and rated on a scale of sophistication. Interviews and talk about practice were coded for how and why novice teachers use ambitious practices. They examined the relationship between tools and appropriation of ambitious teaching practices, finding that "well-designed tools supported modifications to discourses and practices and helped novice teachers address vision-to-practice gaps" (p. 605). This study provides one way of documenting TC appropriation of ambitious teaching practices and tools and of connecting visions of teaching and talk about practice to what teachers enacted. While the context is science, methods and findings provide a useful example of how teacher educators could begin documenting trajectories of teachers' *enactments* of practice and the teacher education tools that supported productive appropriation of ambitious practice. Using data of appropriation over two contexts and multiple years provided a rich description of trajectories of practice and how and why teachers' enactment and vision of practice changed. Researchers could adopt a similar kind of methodological approach to understanding participation in an earlier and shorter time frame: methods courses.

In their case study of three TCs' trajectories of learning, Kazemi and Wæge (2015) characterize learning as participation and experience in their methods course. Their research focused on development of ambitious practices as envisioned by the course. They used StudioCode video analysis to code participation in "1) how they appeared to try out the instructional practices the course aimed to help them learning and (2) how they identified with the principles of the course" (p. 130). Findings noted how TCs worked on three dimensions of practice during the

methods course: managing interactions with students (with a focus on eliciting and responding to student thinking), setting up tasks to facility equitable access to the task, and using representations to work on particular mathematical ideas. They noted that the learning cycle used in the course appeared to support TCs in enacting the practice of eliciting student ideas. However, only one of the three TCs seemed able to support student to use questioning practices and talk moves that developed mathematical justifications.

The challenge for novices to facilitate productive mathematical discussions (e.g., collective work to articulate mathematical justifications) and not just increased talk is similar to findings from Kloser et al. (2019) and Reisman et al. (2019) regarding the difficulty for novices to "invite or result in students taking up other students' ideas and moving collective understanding forward" (Kloser et al., 2019, p. 126) and "engage students in collective knowledge construction" in text-based social studies discussions (Reisman et al., 2019, p. 154). Reisman et al. (2019) suggest the need to be explicit about the purpose of text-based discussions (e.g., the purpose of explaining a scientific phenomenon for sensemaking discussion in science). Similarly, Kazemi and Wæge suggest explicit work prior to enactment (e.g., during representations of practice or when being introduced to instructional activities) on identifying the goal to help TCs "reflect on talk moves not just to elicit students' initial ideas but to build explanations and develop reasoning" (p. 142). They also found the representing student thinking and use of the board was an important starting point:

> "Beginning with representing students' thinking on the board was a common starting point for the three PTs. The use of representations to advance students' ideas is a complex task and our hope is that PTs would also learn to relate different representations to one another and to the mathematical story that unfolds during a lesson" (p. 142).

This study contributes to a trajectory of how TCs might develop practice during their methods course. It is one of a handful of studies documenting how TCs learn and what kinds of practice they enact in practice-based methods courses.

***Dissertation Contribution to Research on TC Learning***

Despite the plethora of studies on teacher learning (and OTL), there are limited studies that define learning as participation then analyze that participation *and* enactment of practice. Characterizing TC learning in this way contrasts approaches that examine talk or reflection about practice (e.g., Anthony et al., 2015 or Horn & Little, 2010) or learning spaces created by pedagogies or TEs (e.g., Ghousseini & Herbst, 2016 or Lampert et al., 2013). Studies that examine teacher talk through interviews, self-reports, and discussion about practice are helpful for examining what TCs attend to, including changing beliefs and understandings about teaching and learning. However, these studies do not examine *enactment* of practice. Some studies examine enactments of practice with a focus on a smaller practice or technique nested within the practice of facilitating discussion (e.g., Sleep, 2012 or Ghousseini, 2015), enactments of practices outside of discussion facilitation with students (e.g., Sleep & Boerst, 2012), approximations without students present (e.g., Tyminski et al., 2014), or enactments outside of methods coursework (e.g., Sleep, 2012 or Thompson et al., 2013).

There is a growing body of empirical studies on TCs' uptake of practice and what they take from their practice-based methods courses for social studies and science (Reisman et al., 2019; Kloser et al, 2019; Windschitl et al, 2011). Mathematics educators pioneered foundational research on the pedagogy of rehearsal (e.g., Lampert et al., 2013), specified a mathematics version of the CPC's specification of discussion (e.g., Shaughnessey et al., 2019), and documented how teachers enacted core practices with students (e.g., Franke et al., 2007); However, there is limited research

on TCs' uptake of practices from mathematics methods courses (in comparison to the research from social studies and science colleagues). My choice to examine enactment within the course's rehearsals and classroom visits provides an image of the changes in practice and trajectory of TC learning to facilitate a productive discussion within the course. In other words, I provide evidence that TCs can learn to facilitate discussion (to some extent) within the ten-weeks of a methods course, and I document these changes their instructional practice.

My literature builds on these existing studies that document shifts in TC talk about facilitating discussions to document shifts in how they *enact* facilitating discussions. I examined their enactment and characterize what the enacted practice communicates about their vision of high-quality instruction (not vice versa). I am contributing existence proof that there are shifts in TC beliefs and talk about teaching (evidenced by the literature above) *and* enactment of teaching practice in a 10-week practice-based mathematics methods course. Research documents the challenge in changing practice and how TC practice often remains unchanged even if beliefs about high-quality mathematics instruction change; thus, my documentation that beliefs *and* enactment of teaching practice can change in a methods course is important. My dissertation provides and image of how TCs work on the practice of facilitating discussion: the changes in their talk, changes in their enactment, and use of tools in that work.

As evidenced and summarized above, there have been few studies that document TC learning in relation to their enactment of practice. In response to the problem of enactment (Kennedy, 1999), there is a recent focus on practice-based teacher education that incorporates pedagogies of enactment centered around core practices (Forzani, 2014; McDonald et al., 2013). This line of research discusses how certain tasks and pedagogies can help TCs enact principles, practices, and knowledge in relation to mathematical goals and student thinking.

## Practice-Based Teacher Education

In this section, I review literature on practice-based teacher education. I describe a current practice-based approach, including the problems and issues it is responding to, how it differs from previous approaches, the focus on core practices and pedagogies of enactment, and the existing research around this approach. I also discuss recent research that has worked to decompose high-quality mathematics instruction (specifically the practice of leading a productive mathematical discussion) into core practices for practice-based learning.

### *Problematizing Teacher Education: The Theory – Practice Gap*

Teacher education has long been plagued with critiques of being disconnected from practice (Fraser, 2007; Feiman-Nemser & Buchmann, 1985; Grossman, 2005; Zeichner, 2012). Historically, there has been a structural divide between university coursework and clinical practice (or fieldwork) (Grossman, Hammerness, et al., 2009). Critics of teacher education have argued that teacher educators emphasize the theoretical and view the school as places TCs can enact these theories. This problematically assumes TCs will make the transfer into their practice, and the onus is on TCs to connect what is "learned" at the university to classroom practice (Feiman-Nemser & Buchmann, 1985; McDonald et al., 2013). This assumption has resulted in a perceived disconnect between theory and practice and a critique that teacher education programs inadequately prepare teachers to enact effective teaching practices (Darling-Hammond et al., 2005; Feiman-Nemser & Buchmann, 1985). In response to this theory – practice gap, teacher education has historically oscillated between practice-centric approaches and theoretical, knowledge-based approaches in a way that perpetuates this perceived gap and unproductively dichotomize theory and practice (Forzani, 2014).

**A history of "practice-based" teacher education reform.** To address the perceived theory – practice gap, there has been a plethora of "practice-based" approaches to teacher education and attempts to name aspects of teaching practice. Teacher education has a long history of attempting to specify the skills, components, and knowledge of teaching to improve instruction (Forzani, 2014). Teacher education reform in the 1960's and 1970's focused on competency-based teacher education (CBTE), naming discrete skills that TCs could practice in low-stakes settings. Micro-teaching, often structured within learning cycles where TCs tried out these practices in peer small groups and used video recordings of teaching, was a signature pedagogy of CBTE. This skills-based approach lacked attention to student thinking, a central component of high-quality teaching. CBTE lacked attention to establishing a larger vision of high-quality practice, content knowledge, and development of TCs' professional judgment of when to use skills. In reaction to these shortcomings, a cognitive approach in the 1980s focused on teacher knowledge and professional judgment. This shift was marked by a focus on case-based methods that aimed to develop knowledge and moment-to-moment instructional decisions based on professional judgment. Cognitive approaches still lacked a codified knowledge base for the profession as well as pedagogies to ensure TCs would be able to enact practices based on this knowledge. While these approaches have attended, in various ways, to teaching skills and knowledge, neither approach addressed the problem of enactment (Grossman & Pupik Dean, 2019; Kennedy, 1999). Furthermore, these approaches lacked attention to the sociocultural and situated nature of TC learning.

### *Current Research on Practice-Based Structure, Pedagogy, and Curriculum*

A group of practice-based educators have worked to address the problem of enactment and thoughtfully integrate theory and practice through practice-based structures, pedagogies, and

curriculum. (Forzani, 2014; Grossman & Pupik Dean, 2019; Kennedy, 1999; McDonald et al., 2013). The assumption is that practice and theory are connected and intertwined "as teachers exercise professional judgment about how to contextualize their practice for individual students or groups of students, they need to be conversant with theories about knowledge, identity, and learning" (Grossman et al., 2019, p. 97). Rather than view theory and practice as dichotomous entities on opposite ends of a spectrum, teacher educators are thinking about how to integrate knowledge and practice with a focus on the situated nature of TC learning. Integrating knowledge and doing involves framing, more specifically, the "disconnectedness" of theory and practice as a lack of specification of how to integrate knowledge and doing in ways that provide opportunities to investigate *and* enact practice (Kennedy, 1999; Forzani, 2014).

Ball and Cohen (1999) laid the groundwork for understanding this integration of theory and practice in their chapter on "a practice-based theory of professional education." They argued that dramatically changing schooling and instructional opportunities for students involves significant changes in how professional learning is supported in practice. Professional learning must support teachers' knowledge of their subject matter, students as individual learners and as members of diverse communities, learning theory, and pedagogy, as well as their ability to use this knowledge in "complex interactions in the unpredictable situations that we call classrooms" (p. 10). This assumes that teaching is a complex and relational practice that requires opportunities for sustained professional inquiry and practice-based learning in communities of practice. At the core of their argument is the idea that professional education must be grounded in practice by centering professional learning around the work and activities of the profession, in this case, teachers. Being "practice-based" and situating learning "in practice" does not simply mean putting teacher education and professional development in schools and classrooms. Practice-based learning

involves grounding opportunities to learn both in teachers' individual classroom experiences and practice as well as in records and artifacts of practice that open up spaces for inquiry and reflection on comparative perspectives of practice.

Ball and Cohen assert that professional learning that is situated in practice develops teachers' abilities to improvise and experiment in response to students and problems of practice and to use these experiences as opportunities to inquire about and improve practice. Teacher education (and teaching) needs to be professionalized, overcome teachers' "apprenticeship of observation," and provide professional counter-socialization. From this perspective, radically transforming teaching and teacher education involves challenging the existing and "ordinary cultural, knowledge, and professional norms of teaching and teacher education" (p. 30). This type of practice-based learning requires particular kinds of opportunities to learn, examples of practice-based materials, artifacts from practice, and pedagogies to support learning in practice, and in the past two decades, teacher educators have worked to build a repertoire of these kinds of practice-based teacher education materials, curricula, and pedagogies. Building on Ball and Cohen's arguments and conceptualization of practice-based teacher education, teacher educators have worked to integrate theory and practice through approaches that are structural, pedagogical, curricular, or some combination (Hauser & Kavanagh, 2019).

**Structurally mediating university and school spaces.** One way to integrate theory and practice is attend to structures in teacher education that mediate university and school spaces. Researchers using this approach focus on structural changes to intentionally design methods courses and other teacher education experiences that create third spaces between the academic knowledge and theory generated from the university with practitioner knowledge and practices

gleaned from the field (Zeichner et al., 2015). The context of this study, a practice-based mathematics methods course, is an example of this kind of design (Kazemi & Wæge, 2015).

**Pedagogically connecting spaces.** In thinking about context and mediating university and school spaces, teacher educators are also thinking about what is happening in these spaces including the pedagogies to support TC learning (Forzani, 2014; Grossman, Hammerness, et al., 2009; Grossman et al., 2019). Building on Ball and Cohen's (1999) theory of practice-based learning (particularly the emphasis on pedagogies of investigation and use of artifacts from practice to center learning in practice), Grossman, Compton, et al. (2009) developed a framework of pedagogies of practice. Their pedagogies of practice framework was developed through a comparative study of professional education for three professions involving relational practices: clergy, teaching, and clinical psychology. Their framework named "three key concepts for understanding the pedagogies of practice in professional education: representation, decompositions, and approximations of practice" (p. 2055). Representations include the ways practice is represented to novices. Decompositions of practice involve "breaking down practice into its constituent parts for the purposes of teaching and learning" (p. 2058). Approximations of practice are "opportunities for novices to engage in practices that are more or less proximal to the practices of a profession" (p. 2058).

For teacher education, representations of practices such as lesson plans, student work, and video make different aspects of teaching practice visible to TCs. Approximations of practice simulate the real practice of teachers and "require students to engage in practice that is related but not identical, to the work of practicing professional…and can allow students to experiment with new practices under easier conditions" (Grossman, 2011, p. 2840). These approximations of practice can be useful pedagogies of enactment. Teaching can be broken down into components

that are more easily learned by TCs, the process of "decomposing" teaching. Core practices (discussed below) are a particular kind of decomposition of practice. Decomposing teaching allows TCs to target specific components of practice and should be situated in a context that approximates teaching. Decompositions, representations, and approximations of practice can be used strategically to develop TCs' adaptive expertise (Ghousseini & Herbst, 2016; Grossman, Compton, et al., 2009).

An important finding from Grossman, Compton, and colleagues' (2009) comparative study was for the programs studied, "teachers [had] fewer opportunities to engage in approximations that focus on contingent, interactive practice than do novices in the other two professions" (p. 2056). This lack of opportunities of approximation in teacher education reflect lack of pedagogies of enactment. Historically, much of teacher education has focused on providing pedagogies of reflection and investigation, where TCs talk practice and rarely move out of their role as a student. There has been a lack of pedagogies of enactment, where TCs enact practice and engage in activities that position them in the role of the teacher (Ball & Cohen, 1999; Grossman, Hammerness, et al., 2009; Kennedy, 1999). These pedagogical issues in supporting TCs to enact effective teaching practices are often attributed to the structural separation between university coursework and practical experiences. Integrating theory and practice requires both attention to structural changes *and* pedagogical changes in teacher education.

**Curricular approaches to practice-based learning.** Adding to structural and pedagogical dilemmas, attempts (such as in CBTE) to name aspects of practice for the sake of learning to teach historically decomposed practice into lists of discrete techniques or skills without enough attention to developing TCs' adaptive expertise, pedagogical content knowledge, and overarching vision of high-quality instruction (Forzani, 2014). Without a codified knowledge base, the teaching

profession (and teacher education) has been left vulnerable to critique. However, decomposing practice in a way that maintains the complexity of teaching but also simplifies practice enough to be learned by a novice has proved challenging. Earlier "practice-based" approaches (e.g., the 1929 Common Wealth Teacher Training Study) have overly discretized practice, creating long lists or practices that are unvetted by research, too overwhelming and lengthy to learn, or too technical in ways that reduce practice to simple moves, skills, or techniques (Forzani, 2014). Thus, researchers (e.g., Kennedy, 2016; Zeichner, 2012) caution against focusing on just "core practices" and observed behaviors. The challenge for teacher education is how to support TCs in developing both a vision of high-quality teaching (the *why* of teaching) and naming practice to do or enact (the *how* of teaching). Teacher education has struggled to find appropriate grain sizes and ways of identifying practice to balance complexity and learnability. As a result, specifying practice, particularly facilitating discussion, has been a major aim of the Core Practices Consortium, discussed below.

Current research on practice-based teacher education assumes that high-quality teaching practice is not only complex, but also teachable and learnable through pedagogies of practice (Ball & Forzani, 2009; Grossman, Compton, et al., 2009; Lampert & Graziani, 2009; Lampert, 2010; Zeichner, 2012). This renewed practice-based movement attempts to identify pedagogies that "weave together novices' development of meaningful knowledge for teaching with their capacity to actually enact ambitious teaching in particular disciplines in the classroom" (McDonald et al., 2013, p. 379). The practice-based teacher approach articulated in this review is based on a particular vision of teaching (e.g., the high-quality instruction articulated in the beginning of this review) and identifies core practices within that vision of teaching as one aspect of the work within practice-based teacher education.

### *Defining Practice-Based Teacher Education*

The current practice-based movement focuses on specifying core practices of high-quality instruction and pedagogies of practice that provide opportunities for deliberate practice and enactment of these practices to develop TCs' pedagogical reasoning and adaptive expertise (Forzani, 2014; Grossman & Pupik Dean, 2019; Lampert et al., 2013; McDonald et al., 2013; Zeichner, 2012). From this perspective, "practice-based" does not simply mean extending time in field placements and school settings, focusing on decontextualized lists of moves or techniques, or ignoring the role of theory, professional judgment, and content knowledge. Without attention to developing TCs' vision of high-quality teaching and pedagogical reasoning, practice-based teacher education runs the risk of providing a reductionist approach to teacher education. In light of these challenges, Hauser and Kavanagh (2019) propose a definition of "practice-based teacher education" based on the growing body of research:

> "Practice-based teacher education is an approach to preparing novice teachers that focuses on the importance of developing novices' ability to enact teaching practices. Ambitious approaches to PBTE attend to the development of teacher belief, knowledge, and judgment but do so through work on practicing instructional routines that occur with frequency in the work of teaching (e.g., facilitating discussion, modeling) (Zeichner, 2012)." (p. 2)

This approach to teacher education is intentional in organizing learning for deliberate practice of the interactive and complex work of teaching, enactment of core practices, and development of adaptive expertise (Forzani, 2014; Grossman & Pupik Dean, 2019).

**The Core Practices Consortium (CPC): An agenda for developing a common language and vision for teacher education.** In 2012, a cross-institutional and cross-discipline group of teacher educators formed the Core Practices Consortium (CPC) to collaborate on practice-based teacher education. The CPC has focused on creating a common language around teaching and teacher education and to attend to student thinking and the situatedness of learning. The CPC's research agenda focuses on two areas: 1) Pedagogy: Articulating and investigating *pedagogies* of

teacher education, and, 2) Core practices: Decomposing and naming *practices* central to high-quality instruction (Grossman et al., 2018).

**Core practices: Decomposing and naming practices central to high-quality instruction.** As teacher education makes a shift back to practice, a group of educators, including members of the CPC, have proposed positioning core practices as part of the curriculum of teacher education (Ball et al., 2009; Grossman, 2018; Grossman, Hammerness, et al., 2009; Grossman & Pupik Dean, 2019; McDonald et al., 2013; Zeichner, 2012). These core practices are complex, relational practices that are situated in a vision of high-quality instruction. Focusing on core practices is an attempt to use these practices as "contexts for novices to learn teaching moves they can adapt to particular communities and students, engage with the complexities of teaching, and hone identities as teachers committed to equity, opportunity, and advocacy for children and youth" (Grossman et al., 2019, p. 97). This assumes that teachers make decisions based on particular students and in particular contexts. Core practices are thoughtful decompositions of practice for the sake of learning to teach. They help make teaching practice teachable and learnable to novices:

> "***Core practices in teaching*** are identifiable components fundamental to teaching that teachers enact to support learning. Core practices include both general and content-specific practices and consist of strategies, routines, and moves that can be unpacked and learned by teachers. Their enactment occurs within the complexities of teaching and thus cannot be decontextualized from the histories and policies of schooling, where teaching occurs, or who students are. Core practices must be grounded in principles for high-quality, equity-centered instruction" (CPC, 2020).

When looking across various definitions of core practices, Grossman, Hammerness, et al. (2009) noted several characteristics that all definitions shared. Core practices are:

- Practices that occur with high frequency in teaching;
- Practices that novices can enact in classrooms across different curricula or instructional approaches;
- Practices that novices can actually begin to master;

- Practices that allow novices to learn more about students and about teaching;
- Practices that preserve the integrity and complexity of teaching; and
- Practices that are research-based and have the potential to improve student achievement.

Core (or high leverage) practices are essential teaching practices that occur often in teaching, address common problems TCs will face, are approachable, preserve the complexity of teaching, allow novices to learn about students and teaching, and are researched and show potential to improve student learning (Ball & Forzani, 2009; Ball et al., 2009; Grossman, Hammerness, et al., 2009; Lampert, 2010; McDonald et al., 2013).

***Grain size of decompositions: The relationship between a core practice and smaller techniques or skills.*** In trying to name and decompose practice, attention must be given to the grain size of how practice is decomposed. When advocating that teacher education could include a set of core practices, teacher educators are not imagining a discrete list of skills, such in the case of CBTE or the Commonwealth Teacher Training Study (Forzani, 2014). They are not generic practices, such as a "best practices" list (e.g., Doug Lemov's 2012 Teach Like a Champion Field Guide) that adopt what Kavanagh and colleagues (2020) critique as a "Pavlovian perspective on teacher learning" and focus on visible behaviors of teachers (p. 2). Core practices decompose practice to make teaching practice learnable, but must share the characteristics that Grossman, Hammerness, et al. (2009) articulate. Larger practices and pedagogies (e.g., facilitating a productive mathematical discussion), core practices (e.g., eliciting student thinking), and smaller, discrete moves "entail the invisible, cognitive part of teaching (e.g., professional judgment, instructional decisions, knowledge for teaching)" (Kavanagh, Conrad, et al., 2020). Core practices can then be decomposed into smaller techniques and skills that constitute a teacher's repertoire of moves and are nested in larger practices and pedagogies.

Boerst et al. (2011) note that traditional approaches to decomposing teaching either focus on large "domains" of practice (e.g., creating a classroom community) or smaller, particular techniques (e.g., questioning techniques or Lemov's techniques). Focusing only on large domains leaves TCs without moves to use. In contrast, focusing on discrete techniques does not support TCs to develop a vision of instruction. Furthermore, discrete techniques are not applicable in all contexts and covering all possible scenarios in teaching would require TCs learn an overwhelming (and arguably endless) number of techniques. Boerst and colleagues propose an alternative approach: decomposing practice is into nested practices of varying grain size to connect larger domains with particular techniques. Connecting larger domains of practice with smaller techniques or routines supports TCs to attend to the *how* and *why* of practice and to develop a vision of high-quality mathematics instruction. This attention to grain size and nesting practices within core practices also "[provides] a map of teaching that teacher educators can use to support the learning of teaching" (p. 2845). Boerst and colleagues document the ways their methods course nested approximations and increased the complexity and authenticity of approximations to support recomposition. Thus, in addition to nested practices, nested approximations with varying levels of complexity could also be used strategically to support learning.

Janssen et al. (2015) also note the importance of attending to the grain size of practice to both articulate learnable practices and maintain the integrity of high-quality teaching practice. They present a theoretical argument for hierarchical modularity to manage complexity by decomposing into learnable parts or modules and recomposing practices to innovate and understand teaching. Modularity provides a way of understanding how to balance effective and meaningful practices that novices can enact with generative properties and generalizability of larger practices to new settings. From this perspective, core practices are generative of teacher

learning and adaptive expertise.

Janssen and colleagues argue that practice-based research has focused on routinization and decomposition, lacking attention to recomposing practice to develop TCs' adaptive expertise and abilities to innovate in new teaching scenarios. They propose the use of instructional activities one method to recompose practice:

> "In the context of deliberate practice of core practices, it is important for student teachers to place core practices within instructional activities to understand the meaning and function of practice" (p. 141).

In this model, practices are enacted in service of the goal of the instructional activity[4]. Core practices are situated and given meaning (in contrast to less authentic approximations such as microteaching).

The above arguments focus on connecting enactments of practice with a larger vision of practice. This attention to the how *and* why of practice is crucial for development of TC adaptive expertise. Focusing on core practices, rather than just a list of discrete practices, aims to develop knowledge and professional judgment in practice. Unlike a list of moves or techniques, core practices serve a purpose. They are situated in a vision of high-quality practice and enacted in response to and in service of student thinking. In contrast to practice discrete techniques, core practices should be practiced in relation to students and student thinking to develop adaptive expertise:

> "Attending to practice links the focus on skill with the focus on knowledge required for the complexity of teaching, leading to a view of teaching practice as the application of knowledge in action. Core practices in teaching, then, are identifiable components fundamental to teaching and grounded in disciplinary goals that teachers enact to support learning. Core practices consist of the enactment of knowledge, beliefs, and dispositions through strategies, routines, and moves that

[4] In the following chapter, I discuss use of instructional activities in methods courses to support TC learning in cycles of enactment and investigation.

> can be unpacked and learned by teachers. They are distinguished from other efforts to focus on teaching practice (e.g., Lemov & Atkins, 2014) in that core practices recognize the highly intersubjective and complex nature of teaching and are deeply connected to the goals of disciplinary learning (McDonald et al., 2013).” (Grossman & Pupik Dean, 2019)

From this perspective, core practices make up the “building blocks” of high-quality mathematics instruction. Teaching practice (including core practices and smaller practices and techniques) must be learned and practiced in context, not as isolated pieces of information to be learned outside of instruction.

***Core practices as a common language not prescription.*** While proponents of core practices-focused teacher education do not advocate for establishing one set or cannon of core practices, they do advocate for development of a common language around practice and the pedagogies to support TC learning. For example, “facilitating discussion” has widely been identified as a central practice across disciplines and in multiple settings and approaches to teacher education. What does *facilitating discussion* mean? What do TCs need to know and be able to do as they facilitate discussions? If multiple teacher educators talk about facilitating discussion, do they share the same vision of what that practice is? To improve instruction, it is necessary to have some common language to talk about, work on, and learn about practice.

The CPC collaborated on developing a common language and created specifications that steered away from standardization and prescription (Grossman & Pupik Dean, 2019). There was value in having both a common understanding of core practices and room for adaptation to content areas, teacher education contexts, schools, and students. In generating common language, teacher educators created space to learn from one another:

> “Developing and revising specifications of both the practices of teaching such as facilitating discussion and the pedagogies of teacher education both surfaced disagreements and built common ground among group members. The specificity around the definitions of discussion pushed forward different conceptions around the practice, in ways that might not occur when simply agreeing about the importance of student discourse. Practice lives in the details, and only by surfacing these details, can we begin to see where we agree and disagree. This focus on

> specification of practice is perhaps a distinctive element of our approach to reform in teacher education" (p. 165).

Specifications were intended to be generative and flexible rather than static and served as boundary objects as teacher educators questioned, shared, and learned from one another.

***Specification of cross-discipline core practices and math-specific core practices.*** There have been efforts to specify core practices across disciplines as well as core practices specific to mathematics (Grossman, Hammerness, et al., 2009; Grossman & Pupik Dean, 2019; Shaughnessy et al., 2019; TeachingWorks, 2013). Grossman and Pupik Dean (2019) examined how teacher educators from multiple institutions and disciplines collaborated to develop a shared decomposition of the practice of facilitating discussion. This collaborative work allowed teacher educators space to learn from each other about the work in preparing TCs to facilitate discussion. Specifying this practice also made space for negotiating shared language and understanding and addressing tensions. One important tension was subject area differences. Grossman and Pupik Dean (2019) documented this tension and the move to have both generic (cross-discipline) and subject-specific specifications of practice:

> "The CPC's initial effort was to see if the group could agree on some broad specifications of discussion that cut across subject matter and grade levels. However, because researchers within the CPC share a concern for the importance of subject matter in instruction, they decided that subject area teams would adapt the common specifications of core practice to their specific subject area. This led to a practice of generating a specification of the practice across the whole group and then giving the subject area teams discretion to change the specification to fit their subject area. This allowed the differences across subject matter communities to be maintained, even as the group agreed to a more generic description" (p. 162).

Importantly, Grossman and Pupik Dean (2019) assert this shared language and understanding does not require standardization of this core practice. Specifying a core practice still leaves room for adaptation based on context and discipline.

***Is* discussion facilitation a *core practice*?** Thus far, I have named characteristics of core practices and the collaborations and work to specify core practices. Development of a common

language around teaching is necessary, and this work has been instrumental in moving the field forward. Despite the promising collaborations through the CPC, it would be misleading to claim that teacher education has, as a whole, agreed on a set of particular core practices or even what practices can be considered "core." Much of the CPC work focused on specifying the practice of facilitating discussion. This practice is an essential component of ambitious and high-quality teaching across disciplines; however, educators are not in agreement around if it constitutes a core practice, a domain of practice (as defined by Boerst et al., 2011), a participation structure (e.g., like an instructional activity), or some other grain size of practice. Although TeachingWorks and some members of the CPC consider facilitating discussion as a core practice (e.g., Reisman et al., 2019), my dissertation looks at core practices within this larger domain of facilitating discussion.

Facilitating productive mathematical discussions represents much of the work involved in teaching ambitiously but is not comprehensive of the practice of teaching. For example, formal assessment practices or building meaningful home – school connections are also part of the work of high-quality teaching but are not in the scope of this study. Furthermore, a "discussion" can be specified as a temporal participation structure. Shaughnessey et al., (2019) do just this in specifying the practice of leading a discussion with four parts: Planning, launching a discussion, orchestrating interaction, and closing the discussion. With this specification, the practice of discussion is a kind of temporal participation structure. Although some teacher educators use terms such as instructional activities and core practices interchangeably (e.g., Reisman et al, 2019), I make a distinction between instructional activities (discussed in Chapter 3), temporal participation structures, and core practices.

For this study, I view *facilitating a productive mathematical discussion* as a pedagogy (or larger "practice") of high-quality mathematics instruction. My choice to focus on the pedagogy

or "practice" of *facilitating productive mathematical discussion* is an attempt to: 1) name the pedagogical approach and type of ambitious teaching practice aimed for (i.e. not the traditional IRE format); and 2) to bind the scope of this study by examining one area of practice related to student – teacher interactions (i.e. leading discussions). Framing the work of *facilitating a productive mathematical discussion* as a pedagogy of high-quality teaching positions this work as different from a core practice and more akin to Boerst and colleagues (2011) notions of larger domains of teaching. I view the practice of facilitating a productive mathematical discussion as pedagogy and larger "practice" of ambitious teaching in contrast to the course's named practices of ambitious teaching as core practice (e.g., *eliciting and responding to student thinking* or *using mathematical representations*). Since this dissertation also uses the term "pedagogy" to refer to teacher educator pedagogies (e.g., rehearsals), I refer to *facilitating a productive mathematical discussion* simply as a "practice" of high-quality instruction. Within the practice of leading discussions and within core practices, there are smaller instructional moves or techniques (e.g., a *revoicing talk move*) that serve different purposes and are enacted in response to pedagogical dilemmas and based on professional judgment (Boerst et al., 2011; Kavanagh, Conrad, et al., 2020).

While there is not consensus around which core practices should be worked on in teacher education, it is evident that high-quality teaching practice can be represented and decomposed for TCs to learn. The CPC research and my study focus on practices that are important to the interactions between teachers, content, and students within the classroom. This is not to say other important core practices do not exist outside of these interactions. The purpose of identifying core practices is to select and make visible to TCs what goes into some part of the work of teaching. In mathematics teacher education, facilitating a productive mathematical discussion is central to

high-quality instruction. Below I highlight some of the key ideas and research on decomposing this practice.

***Decomposing the practice of facilitating productive mathematical discussions for teacher education.*** Though much of the literature agrees on the importance of leading discussions, there are a vast number of efforts to decompose this practice. In some ways, the research on discussion-based mathematics instruction (discussed in the first part of the review) decomposes the practice of facilitating discussion and names components of discussions (e.g., identifying a mathematical goal, talk moves, using representations, etc.). However, it is unreasonable to expect TCs to have *mastered* their teaching practice or for methods courses to support TC development of *all* practices associated with the vision of high-quality mathematics instruction outlined in the first section of this review. This does not mean that teacher education should not focus on high-quality practice. Rather, it indicates the importance in understanding what is "core" in high-quality instruction and what components are necessary to provide a foundation for TCs to begin enacting high-quality mathematics instruction.

While there is not consensus on the way to decompose the practice of facilitating productive mathematical discussions, looking at teacher educator specification efforts provides a curricular framework for how teacher educators are attending to specifying and learning of this practice in math methods courses. Teacher educators share commonalities and are developing a common language that leaves space for adaptation and variation, indicating the goal of specifying practice is not standardization or prescription. In this section, I highlight several specifications and work around decomposing practice of facilitating productive mathematical discussion. The purpose of this section is to define this practice for the sake of my dissertation analysis.

Smith and Stein (2011) decompose and describe the practice of facilitating productive mathematical discussions into five practices: anticipate, monitor, select, sequence, and connect. They describe this type of teaching as "skillful improvisation" and highlight how particular questions and talk moves can ensure active thinking, increase participation, and hold students accountable to the discussion and the mathematics. Related to this work on practices to orchestrate discussions, Chapin et al., (2013) name specific talk moves to elicit and orient students to different mathematical ideas and to press them to reason and develop mathematical arguments (see earlier section on talk moves).

In another variation of decomposing this practice, Kazemi and Hintz (2014) elaborate on different discussion structures that can be implemented based on facilitating particular goals. They give five different targeted discussion structures based on the goal of the discussion such as the "Why? Let's Justify" structure to "generate justifications for why a particular mathematical strategy works" or the "Define and Clarify" structure to "define and discuss appropriate ways to use mathematical models, tools, vocabulary, or notation" (p. 3). They articulate four principles that guide discussions to deepen student thinking:

- "Discussions should achieve a mathematical goal;
- Students need to know what and how to share;
- Teachers need to orient students to one another and the mathematical ideas; and,
- Teachers must communicate that all students are sense makers and that their ideas are valued" (p. 2).

Their work further decomposes the work of leading discussions in the context of addressing goals, content, and practices specific to the discipline of mathematics.

Within the CPC, mathematics educators worked to decompose the practice of facilitating discussion specifically for learning to teach, in relation to approximations of practice, and across two different teacher education contexts (Shaughnessy et al., 2019). The articulated four

components of leading a mathematical discussion: "(1) *Planning*, including identifying a clear instructional goal, (2) *Launching* a discussion, (3) *Orchestrating* interaction around the mathematics, and (4) *Closing* the discussion" (p. 169). Findings suggested that teacher educators did not work on these four components in isolation, suggesting the cyclical nature of learning and the necessity for multiple opportunities to discuss and practice facilitating discussions. They investigated the OTL facilitated by use of this decomposition:

> "A common decomposition can be enacted with attention to the social and intellectual dimensions of teaching, where TCs have opportunities to problem solve together around questions of what counts as participation, whose voices are heard in discussions, who is positioned competently and who defines competence" (p. 177).

In addition to supporting OTL for TCs, the specification became a tool "to talk about teacher education practice across contexts" (p. 177).

Decomposition of discussion facilitation is dependent on a vision of high-quality mathematics instruction. Teachers who facilitate productive mathematics discussions have a purpose in mind, elicit and respond to student thinking in ways that move the discussion towards that purpose, make mathematical ideas explicit through appropriate representations, talk moves, and modeling, and manage the social aspects of discussion. When looking at research on decomposing this complex practice, two big ideas emerge. First, the practice of facilitating productive mathematical discussions is composed of nested practices (e.g., eliciting) and smaller moves (e.g., talk moves). Facilitating discussions requires anticipating, eliciting, responding, and orienting moves enacted in relation to student thinking. Second, facilitating discussions that are productive and mathematical require attention to student thinking in relation to mathematical goals. Thus, methods courses should focus TC attention on goals and opportunities to work on these goals with students.

**Summarizing the practice-based teacher education research landscape.** In the last decade, teacher educators have worked hard to reimagine practice-based teacher education by reorganizing curriculum around a vision of high-quality instruction (and core practices within that vision) and using pedagogies to support TC enactment of principles, practices, and knowledge in relation to mathematical goals and student thinking. Research on this work focuses on developing a common language around teacher education pedagogies and core practices. Hauser and Kavanagh (2019) note that practice-based teacher education approaches can be categorized as structural, pedagogical, and curricular changes (as I discussed above). The issues *researched* on practice-based teacher education can be categorized similarly: 1) Structural: Documenting structural and logistical changes in reorganizing courses in practice and around core practices; 2) Pedagogical: Articulating pedagogies of practice (including articulation of TE moves); and, 3) Curricular: Naming core practices (and decompositions of these practices).

***Structural issues: Documenting structural and logistical changes in reorganizing courses in practice and around core practices.*** A growing body of research documents teacher educators' efforts to structure courses to be practice-based and the affordances and challenges they experienced in their courses (e.g., Davis & Boerst, 2014; Virmani et al., 2018). The research provides images of different models of practice-based teacher education and adaptations to different content areas and institutional contexts. One approach to structuring methods courses around core practices is a learning cycle with embedded instructional activities (Lampert et al., 2010; Lampert et al., 2013; McDonald et al., 2013). I expand on this learning cycle in the following chapter on the study context.

***Pedagogical issues: Articulating pedagogies of practice.*** In response to the critiques that teacher education has lacked pedagogies of enactment, many teacher educators have worked to

specify teacher education pedagogies (Grossman, 2018; McDonald et al., 2013). Rehearsals emerged as an early pedagogy for approximation (Lampert et al., 2013), and other teacher educators have documented how they adapted this pedagogy for their context (Arbaugh et al., 2018; Averill et al., 2016; Davis et al, 2017; Ghousseini, 2017; Wæge & Fauskanger, 2020). Teacher educators have specified and studied other pedagogies included clinical interviews (Sleep & Boerst, 2012; Chao et al., 2018), analysis of transcripts (Singletary et al., 2018), analysis of video (Casey et al., 2018; Charalambous et al., 2018; Schutz et al, 2018; Sherin & Han, 2004; van Es & Sherin, 2010), coteachers huddle (Soslau et al., 2018), modeling (McGrew et al., 2018; DeLuca et al, 2013), simulations and role plays (Schutz et al., 2018), and Teacher Time Out (Gibbons et al, 2017; Wæge & Fauskanger, 2020). The studies are contributing to a repertoire of teacher education pedagogies focusing on supporting TC enactment and document the various problems of practice that be surfaced and worked on. Other studies focus on the scaffolds (e.g., Sleep & Boerst, 2012), tools (e.g., Ghousseini et al., 2015), and assignments (e.g., Sleep & Boerst, 2012; Tyminski et al., 2014) within methods courses to support learning in practice. Additional studies examine the use of instructional activities as a routine for trying out practices and structure for recomposing practice (Hunter et al., 2013; Lampert & Graziani, 2009; Lampert et al., 2010)

There is a growing body of studies examining OTL afforded by different pedagogies and tools (Boerst et al., 2011; Ghousseini & Sleep, 2013; Jenset et al., 2018; Lampert et al., 2013). Other studies analyze the interplay of representations, approximations, and decompositions and suggest the importance of using pedagogies in purposeful sequences and cycles (Boerst et al., 2011; Ghousseini & Herbst, 2014; Lampert et al., 2013). A growing body of literature documents teacher educator moves within these pedagogies (Averill et al., 2016; Lampert et al., 2013; Shaughnessy et al., 2019). These studies found the role of the educator as a more knowledgeable

other and the moves she made were consequential for OTL. The plethora of studies documents various aspects of pedagogical issues demonstrates the field's efforts over the last decade to answer the call for specification of pedagogies of practice.

***Curricular issues: Naming core practices.*** In addition to investigating pedagogies of practice, teacher educators have also worked to decompose practice and specify core practices (see earlier section for more on core practices). Four important ideas have emerged from this body of research: 1) Grain size of practice; 2) The need to recompose practice; 3) Common language is being develop through specification of practices; and, 4) Core practices are embedded in particular visions of high-quality instruction. First, researchers have noted that there are various grain sizes and nesting of practices within larger core practices (Boerst et al., 2011; Shaughnessy et al., 2019). Some studies examine enactment of particular parts of practice (e.g., smaller practices nested within the practice of facilitating a discussion) such as orienting students to each other's ideas (Ghousseini, 2015), planning practices (including selecting and sequencing strategies) and selecting types of questions (Tyminski et al., 2014), or eliciting and responding to student thinking (Sleep & Boerst, 2012; Kavanagh, Metz, et al., 2020). Second, in addition to decompositions of practice, TCs need opportunities to recompose practice (Boerst et al., 2011; Janssen et al., 2015). Furthermore, collaborations made possible through the CPC have focused on specifying *core* practices. In particular, they created common language around the practice of facilitating discussion both across disciplines (Grossman & Pupik Dean, 2019) and specific to mathematics (Shaughnessy et al., 2019). Finally, core practices do not represent the entire curriculum of teacher education. Core practices are embedded in (and reflect) particular visions of high-quality teacher education. Kennedy (2016) cautions against using only observed behaviors for understand TCs' development of practice. While core practices (and pedagogies of enactment to try out these

practices) are necessary and can be used as foundational building blocks of ambitious teaching practice, teacher education must continue to attend to both the *how* and *why* of teaching, utilizing pedagogies of investigation and reflection. This includes developing TCs' pedagogical content knowledge, professional judgment, pedagogical reasoning, vision of high-quality instruction, etc. *and* their repertoire of teaching moves and practices.

Teacher educators must think carefully about how to organize learning around their selected core practices. To develop TCs' vision of high-quality instruction and knowledge and skills to enact this vision, teacher education must attend to both larger core practices and smaller nested practices and techniques and must consider how to use approximations to provide opportunities to enact these practices (Boerst et al., 2011; Grossman, Hammerness, et al., 2009; Shaughnessy et al., 2019). TEs can make decisions about which practices to highlight and how to work on these practices for the sake of TC learning, and, at the same time, must balance the reality and complexity that multiple practices are being worked on simultaneously (Shaughnessy et al., 2019).

***Approaches to studying TC learning in practice-based teacher education.*** Teacher education has made great progress in creating a repertoire of practice-based pedagogies of teacher education and specifying core practices. Within practice-based teacher education literature, studies examine TC learning based on the following:

- OTL facilitated by particular pedagogies, teacher moves, or course activities (e.g., Boerst et al., 2011; Ghousseini & Sleep, 2013; Jenset et al., 2018; Lampert et al., 2013)
- Interviews, self-reports, and reflections on practice (e.g., Auhl & Daniel, 2014; Davis, 2006; Lai et al., 2015; Mathewson et al 2017; Solslau et al., 2018).

- Enactment of (core) practice(s) nested within the practice of facilitating discussion (e.g., Ghousseini, 2015; Sleep, 2009).
- Enactments in practicum (e.g., Charalambous et al., 2018; Sleep, 2012; Thompson et al., 2013).
- Enactments of discussions in other disciplines outside of mathematics (e.g., Kloser et al., 2019; Reisman et al., 2019).

Literature on OTL within methods courses highlights the problems of practice that are surfaced and the role of the teacher educator in facilitating these OTL. Existing literature on OTL is framed from the perspective of what the teacher educator did or how the course was centered in practice and less about how TC participation and enactment changed. This literature documents the teacher educator moves and pedagogies can support TC learning but does not provide a complete picture of how TCs enact practice. Research that focuses on TCs has used interviews, self-reports, and reflection of practice. While it is important for TC to develop new identities and beliefs about teaching, documenting these changes also does not provide an image of what TCs are able to enact.

Research on practices that are nested within the practice of facilitating discussion are helpful for our understanding of how TC enact these practices, and additional research is needed to build an image of how TCs enact this practice in service of the larger practice of facilitating productive mathematical discussions. Research on enactments in practicum are methodologically useful for understanding how to examine TC participation and enactment of practice but are not helpful in understanding changes within methods courses. A few mathematics educators have begun documenting TC enactment of the practice of facilitating productive mathematical discussions (Kazemi & Wæge, 2015; Ghousseini, 2015; Ghousseini et al., 2015), and additional research in needed to build on this emerging literature base.

### *Dissertation Contribution to Practice-Based Teacher Education Literature*

This study seeks to contribute to research on pedagogies of practice by providing an initial attempt to understand TC enactment of and participation around the practice of facilitating productive mathematical discussion. I examine TC experience in a practice-based course that utilizes a cycle of investigation and enactment to engage TCs in core practices. Findings provide an image of how TCs work on core practices, including the changes in their talk, enactment, and use of tools in practice. This complements the work of the Core Practice Consortium around decomposing and specifying the practice of facilitating mathematical discussion (Shaughnessy et al., 2019). Documenting changes in how investigate and enact the practice of facilitating a productive mathematical discussion contributes to understanding and beginning to articulate a trajectory of TC learning in practice-based teacher education.

## Chapter Summary

This chapter presented a critical review of literature on TC learning, particularly around supporting enactment of high-quality mathematics instruction. The purpose of this literature review was to examine how the field of teacher education has studied teacher learning in mathematics education and practice-based teacher education. For this purpose, my review answers three questions:

1. What high-quality mathematics instruction are we trying to prepare TCs for?
2. What do we know about TC learning around this vision of high-quality mathematics instruction?
3. How can practice-based teacher education support this learning and what issues have been studied in practice-based teacher education?

First, I discussed literature around high-quality mathematics instruction to understand the vision of practice that teacher education is trying to prepare TCs to enact. High-quality mathematics instruction is complex and interactive work, requiring teachers to enact practices based on instructional goals, content knowledge, and student thinking. Research has examined how particular discussion-based instructional practices support students to develop mathematical proficiency and the importance of facilitating productive mathematical discussions. While literature documents the importance of this practice, additional research is needed to provide images of beginning practices along a teacher learning trajectory of high-quality mathematics practice. My study examines how TCs facilitate discussions to reach particular goals and begin using tools (including lesson plans, representations, and talk moves) to support discussion facilitation.

Mathematics educators have made efforts to study TC learning of high-quality mathematics instruction and through deliberate practice focused on developing adaptive expertise. In thinking about the knowledge and practice needed for adaptive expertise, literature has focused on TCs development of content and pedagogical knowledge, beliefs about teaching and learning, and opportunities to learn.

Research documents how TCs make shifts in their focus (from self to students) and understanding of teaching and learning (as more complex) during methods coursework. Research has also studied TC learning of pedagogical content knowledge. During methods courses, TCs can make shifts in how they attend to goals in planning and in how they talk about practice. There is also research documenting how TCs make shifts in enacting individual, smaller, discrete practices (e.g., responding to student work, questioning practices, orienting students to one another).

While there are existing studies that document shifts in TC talk about facilitating discussions, there are limited studies documenting shifts in how they *enact* facilitating discussions. I examine their enactment and characterize what the enacted practice communicates about their vision of high-quality instruction (not vice versa). I am contributing an existence proof that there are shifts in TC beliefs and talk about teaching (evidenced by the literature above) *and* enactment of teaching practice in a 10-week practice-based methods course.

Practice-based teacher education (e.g., the context of my study on TC learning) attempts to address the problem of enactment by specifying the core practices and the pedagogies in ways that integrate teacher knowledge and enactment of practice. Rather than narrowly focus on either skills or knowledge, teacher educators have begun work on supporting TCs to learn and develop a principled vision of high-quality teaching, practices to enact this vision, judgment in executing these practices, and knowledge about math and students to base the decisions on student thinking. Because specification of core practices and pedagogies of practice are recent and ongoing work in practice-based teacher education, there is limited research documenting TCs' experiences and what they are actually learning.

Over a decade ago, Lampert (2009) critiqued that research has not vetted and provided evidence of the value of this kind of practice-based approach: "The efficacy of such approaches…has yet to be explored" (p. 31). As a field, we have made substantial progress in articulating pedagogies of teacher education, particularly around pedagogies of enactment, and specifying core practices and related decompositions of practice. There is also articulation of tools and scaffolds to support learning in practice and TE moves to facilitate OTL. The purpose of teacher education is to prepare TCs for the profession of teaching. Thus, understanding what TCs learn in practice-based courses, particularly how they enact practice, would contribute to

understanding the efficacy of a core practices approach. My dissertation makes two contributions that address gaps in the literature:

1. Existence proof that there are shifts in TC beliefs and talk about teaching *and* enactment of the practice of facilitating a productive mathematical discussion in a 10-week practice-based methods course.
2. An image of TC participation to contribute to a trajectory of TC learning and enactment of core practices in teacher education.

Much like the argument that teachers need to focus on students, *teacher educators* need to focus on *TCs*. As a field, we have limited images of changes in TC practice and the trajectory of learning that we hope to cultivate in our methods courses. The field has made progress articulating a vision of high-quality mathematics instruction and pedagogies of practice to support TC learning in practice. It is the ability to *enact* high-quality teaching practice that has been problematic for teacher education. The purpose of my case study is to understand how TCs investigate and enact the practice of facilitating a productive mathematical discussion in a practice-based methods course. Building on the literature documenting pedagogies of practice, I document TC enactment of practice and characterize their participation in a practice-based setting.

## Chapter 3: Context – The Mathematics Methods Course and Participants

In this chapter, I describe the context of my research: the mathematics methods course and study participants. The mathematics methods course was designed in light of the practice-based shift to articulate pedagogies of practice and specify high-quality core practices for teacher education (articulated in the previous chapter). The practice-based design of the course situated TC learning in cycles of enactment and investigation around a set of ambitious practices. In this chapter I discuss: 1) a learning cycle of enactment and investigation to organize learning around core practices, 2) the methods course (including the classroom context), and 3) the study participants. My study examined TC participation in the context of a practice-based mathematics course; thus, it is useful to understand the design and structure of the course, the elementary classroom context, and the background of the participants.

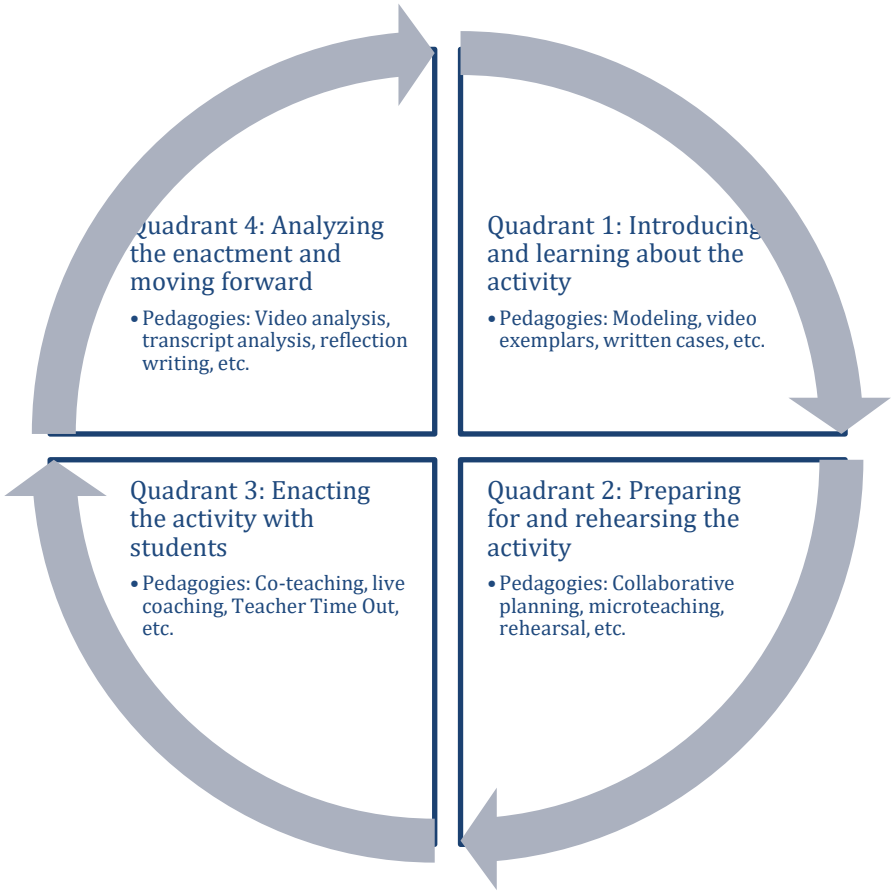
### Organizing Learning Around Core Practices: Cycles of Enactment and Investigation

Mathematics teacher educators (along with other practice-based scholars) have begun to specify what it means for teacher education to center around a set of core teaching practices (Ball & Cohen, 1999; Forzani, 2014; Grossman, Hammerness, et al., 2009; Grossman & Pupik Dean, 2019; Lampert et al., 2013; McDonald et al., 2013). One way to frame TCs learning in practice is to organize learning cyclically (see **Figure 2** below). TEs support TCs to learn core practices and associated knowledge, skills, and professional judgment through "recurrent cycles during which teacher educators support novice teachers to analyze and observe, to plan and rehearse, and to experiment with the instructional activities" (Lampert et al., 2010, p. 137). Building on Hatano and Inagaki's (1986) observations about expertise, Lampert and colleagues (2013) note the importance of repeated cycles for deliberate practice:

> "Improvement of adaptive performance requires a balance between repeated practice to develop the efficiency of routines and the development of conceptual understanding to be able to innovate and adapt to new situations" (p. 228).

Using this approach, methods courses can be structured to provide opportunities in a hybrid context (often in the setting of an elementary school) that rejects dichotomizing theory and practice and instead strategically integrates expertise, practice, and knowledge from both university TEs and classroom teachers (Kazemi, 2017; Lampert et al., 2013; Zeichner, 2010).

***Figure 2.*** *Cycles of Investigation and Enactment.*




*Note: A representation of learning organized in a cycle of investigation and enactment adapted from McDonald et al., 2013.*

Structuring learning in a learning cycle of investigation and enactment is one attempt to move to situate learning in practice and to provide TCs with multiple opportunities to enact core practices (Kazemi, 2017; McDonald et al., 2013). This framework for organizing learning assumes that "learning is a process that occurs over time in interaction with the particular settings in which

and students with whom teachers learn to teach" (McDonald et al., 2013; p. 381). This cycle involves collective work in learning about an instructional activity (discussed in the following section), preparing for and rehearsing the activity, enacting the activity with students, and analyzing the enactment and looking forward (see **Figure 2**). Through these four quadrants, TEs work to specify the core practice(s) being worked on as well as the pedagogies to support TCs' learning and enactment of practice.

Within the learning cycle, the expertise and role of TE is critical to providing skilled coaching and feedback to facilitate TC learning and enactment of high-quality practice (Kazemi et al., 2009). Through repeated cycles of deliberate practice, TCs learn core practices while simultaneously developing their vision of high-quality teaching and content knowledge. TEs use pedagogies of practice including decomposing, representations, and approximations of practice (discussed in the previous chapter) to support TC learning of core practices (Grossman, Compton, et al., 2009; McDonald et al., 2013). Within this cycle, TCs need an activity structure to recompose practice in meaningful, yet doable, level of complexity (Lampert et al., 2013). In the following section, I discuss the use of instructional activities as *structure* for managing and recomposing practice within cycles of enactment and investigation.

### *Instructional Activities as "Structures" for Managing and Recomposing Practice*

Instructional activities are routine participation structures for managing the interactions of teacher, students, and content. Instructional activities make high leverage teaching moves explicit, structure teacher – student interactions, enable novices to enact principles of ambitious teaching, and are generative of other activities and high leverage practices (Kazemi et al., 2009). In mathematics teaching, common instructional activities include choral counting, computational

strings, and solving word problems (Lampert et al., 2010). The following instructional activities[5] were used in the methods course selected for this study.

***Choral Counting.*** The teacher leads students in group skip counting, recording the numbers on the board. The teacher stops at particular points to ask students how they are finding the next number, to discuss what patterns students notice, and to explain and use patterns. The teacher annotates the count as students share. Discussions can focus on mathematical explanation, pattern and structure in number system, place value, or predictions.

***Estimation 180.*** The teacher provides an image and poses a mathematical question related to estimation. Students make predictions, use a strategy to estimate, then discuss estimation strategies. Estimations and actual answers can be compared (more at estimation180.com).

***Quick images.*** The teacher flashes an image of a collection of objects such as dots. Students must quickly discern the number of dots, encouraging them to subitize and use structure. Discussions focus on composing numbers and developing number sense.

***Computational problem or string of problems.*** The teacher poses a computational problem (Mental Math) or sequence of computational problems (String). Students analyze computational strategies. Sequenced problems for the String are designed for students to make explicit connections and relationships between the problems.

***Solving word problems.*** The teacher launches a word problem, monitors as students solve, then selects and sequences student strategies to discuss as a group.

These instructional activities exist in various curricula as smaller activities such as warm-ups (e.g., computational strings) and larger activities that serve as the core of a lesson (e.g., solving word

[5] A detailed description (articulated in the course syllabus) of the activities can be found in Appendix D.

problems). The instructional activities support students to develop number sense and work on important domains of elementary mathematics including the structure of number and number system, meaning of operations, and strategies for computation. The structure of these activities also provides opportunities for students to engage in mathematical practices.

In methods courses, instructional activities provide a routine participation structure and recompose practice into short 10-15-minute lessons (Janssen et al., 2015; Lampert et al., 2010). Instructional activities are tools for structuring interaction in a manageable format and as *structures* for TCs to develop ambitious teaching practices:

> "Preparing novices for ambitious mathematics teaching would mean findings structured ways to enable them to get deep enough into authentic interactions with specific learners to practice inventing educative responses while not being overwhelmed with the unpredictability and complexity of creating improvised interaction" (Lampert et al., 2010, p. 135).

They enable broad student participation and create opportunities for TCs to elicit and respond to student thinking. In addition to structuring participation to facilitate opportunities for productive mathematical discussion, instructional activities also provide a routine structure to "allow the practitioner temporarily to hold some things constant while working on others" (p. 130). Instructional activities support both student and teacher learning and are designed to be *structures* for (and vehicles to develop) ambitious teaching, the curriculum of methods courses.

To introduce teachers to instructional activities, TEs used Planning Protocols and Planning Templates for each activity (see TEDD.org). Planning Protocols[6] are intended to support teachers to understand the structure of the instructional activity and outline the "steps" for the activity as well as possible teacher moves and questions. These are scaffolds TEs use often in quadrant 1 of the learning cycle (Introducing and Learning about the Activity). Planning Templates[7] focused on

[6] See an example of a Choral Counting Planning Protocol from the course in Appendix E.
[7] See an example of a Choral Counting Planning Template from the course in Appendix F.

the specific task and include places to anticipate student thinking (and possible teacher responses), plan for how to use representations (including board work), and ways to highlight key mathematics.

***Towards a Curriculum for Methods Courses: Ambitious Practices, Principles, and Knowledge***

A group of mathematics educators have envisioned curriculum of their methods courses as consisting of practices, principles, and knowledge of ambitious teaching (Kazemi et al., 2009; Lampert et al., 2013). The instructional activities discussed above are "intended to enable novices to engage in learning the work of teaching and TEs to engage in teaching the work of teaching along three dimensions simultaneously: principles of high-quality teaching, essential teaching practices which embody those principles, and mathematical concepts and processes to be learned that provide direction for teaching practices" (Kazemi et al., 2009, p.15). An example of the practices and principles can be seen in **Table 4** below. This table includes the seven ambitious teaching practices and five principles for teaching that were articulated in the methods course studied. The course curriculum also included pedagogical content knowledge, with a particular focus on "specialized mathematical knowledge about children's learning trajectories in number and operations and…understanding of number, operations, and computational algorithms" (see Appendix B on Course Goals).[8]

Different approaches can be taken to name practices that embody and reflect principles of the high-quality mathematics instruction. In the previous chapter, I argued that facilitating a productive mathematical discussion is a practice of high-quality mathematics instruction.

[8] Balancing the short duration of methods courses and coverage of practices, principles and knowledge for high-quality mathematics instruction is challenging. To address this challenge, the teacher education program required TCs take an undergraduate-level Math for Elementary Teachers prerequisite course prior to the two-quarter mathematics methods course sequence. Thus, the course studied placed emphasis on the practices and principles of ambitious teaching.

Facilitating a productive mathematical discussion requires enactment of related practices such as *eliciting and responding to student reasoning* and *orienting students to one another's ideas and to the mathematical goal.* The methods course identified these types of practices as *ambitious practices*.

Ambitious teaching is responsive to students and student thinking and engages students in meaningful mathematical activity in a way that positions all students competently (Kazemi et al., 2009; Kelley-Petersen, 2010). While *ambitious teaching* is sometimes used interchangeably with *high-quality mathematics instruction*, mathematics educators have used the term *ambitious teaching* with a particular focus on equity: treating students as sense-makers (and mathematics and sense-making activity), building on funds of knowledge, and enacting culturally and linguistically responsive practices (Kazemi et al., 2009; Kelley-Petersen, 2010). In my analysis, I use the term *ambitious practices* to refer to the ambitious practices articulated in the course (see **Table 4**). The methods course studied worked to support TCs in developing an ambitious vision and practice of high-quality mathematics instruction.

***Table 4*** *Ambitious Practices and Principles for Teaching*

| Ambitious Practices | Principles for Teaching |
|---|---|
| • Teach towards an instructional goal<br>• Elicit and responding to student reasoning<br>• Orient students to one another's ideas and to the mathematical goal<br>• Set and maintain expectations for student participation<br>• Position students as competent,<br>• Assess students' understanding<br>• Represent mathematical ideas visually, numerically, and/or verbally | 1. Teachers must position students as sense-makers and knowledge-generators, who desire to invest and succeed in school. This involves noticing children, building relationships with them, their families and communities, valuing their perspectives, and attending to their thinking, curiosities and capabilities<br>2. Teaching is both intellectual work and a craft. Deep knowledge of content and pedagogy, creativity, and passion fuel both learning and teaching.<br>3. Teachers must design equitable learning environments in which all children are engaged in robust and consequential learning.<br>4. Teachers' instruction and student learning is always conducted within the context of larger social systems, structures, and hierarchies.<br>5. What we do and say matters and must be analyzed. Our language and action constructs or constrains opportunities for children to build meaningful, positive, and sustained relationships to learning and one another. |

*Note: Excerpt from the course syllabus.*

Above, I described a framework for organizing learning in a cycle of enactment and investigation of core practices. Mathematics educators have worked to identify and create protocols for a set of instructional activities (to be used within these cycles) that function as *structures* for working simultaneously on practices, principles and knowledge needed for high-quality mathematics instruction (see tedd.org). The methods course selected for my study used this learning cycle of enactment and investigation and instructional activities. Below I describe the mathematics methods course.

**A Description of the Mathematics Methods Course**

The mathematics methods course[9] was the first of a two-quarter course sequence in a graduate-level, elementary teacher credentialing program. The course was chosen based on its use of pedagogies of enactment, the focus on core practices, and my experience as a TE in the course sequence. In the first quarter (discussed below), TCs met at a local elementary school, participating in a cycle of enactment and investigation that involved pedagogies such as rehearsals and opportunities for enactment of instructional activities with elementary students (McDonald et al., 2012; Kazemi et al., 2009; Lampert et al., 2013). Building on the experiences from the first course, the second course (sequenced in the following quarter) was located at the university and included work on unit and lesson planning, assignments on mathematical content, and explorations of curriculum and Common Core Standards for elementary mathematics.

While it would be productive to study TCs' practice during the entire course sequence and into the field, I examined TCs' experiences and OTL within the fall course. My purpose was to understand the trajectory of TC learning within a quarter-long practice-based methods course. The practice-based course in the fall quarter was designed with opportunities for TCs plan and enact

[9] Kazemi (2017) provides a description of this same course from the perspective of the lead teacher educator.

lessons with the TE present. Because of the designed hybrid setting in the elementary school, the use of a cycle of enactment and investigation around a set of core practices, and the opportunities for enactment with students, this timeframe and setting were particularly relevant for better understanding how TC participation changes, particularly in their talk and enactment of the practice of facilitating a productive mathematical discussion.

***The Fall Quarter Course and Classroom Context***

The fall course[10] aimed to develop TCs' specialized mathematical knowledge for teaching, their enactment of several core teaching practices through a set of instructional activities, and their abilities to create a classroom learning environment and support student participation. The course was designed to support TC enactment of several core practices of *ambitious teaching* based on underlying principles for teaching and principles for learning to teach (see Table 3 above). This vision of ambitious teaching and assumptions about *teaching* and *learning to teach* guided the design of the course, use of activities, and choices of TE moves and pedagogies.

The course included 31 TCs and four TEs. The TCs were divided into four groups, each led by one of the TEs and assigned a group of 7-8 elementary students. Within these groups, each TC had their own individual "Husky Buddy" who they were responsible for following during the ten-week course. The TEs for the course included the course instructor (a mathematics education faculty member), two teaching assistants (one graduate student and one post-doctoral associate), and the classroom teacher. The course instructor was assigned to the TC teaching group selected for this study and served as their coach during small group planning time, rehearsals, classroom

[10] See Appendix B for the Course Goals as articulated in the syllabus.

enactments, and debriefing discussions for the duration of the ten-week course. Thus, for the remainder of the dissertation I refer to the course instructor as the TE (of the small group).

**A Description of the Teacher Educator**

The course instructor was the lead teacher educator and facilitator for the small group selected for this study. She is a mathematics education faculty member in the College of Education. She had taught in elementary classrooms for two years before studying mathematics teaching and learning at a prestigious doctoral program and researching and teaching as a faculty member in mathematics teacher education. In the fall of 2014, when the data was collected for this study, the TE had been working with TCs using this practice-based design for more than eight years. During this iteration of the methods course, the TE was collaborating with mathematics teacher educators across multiple institutions as part of the Core Practice Consortium's project analyzing teacher educator pedagogy.

As one of the mathematics educators that pioneered research on rehearsals, the learning cycle of investigation and enactment, and instructional activities, the TE is well-versed in the practice-based teacher education discussed in the previous chapter. The TE is an experienced coach committed to regularly learning and teaching alongside classroom teachers and continually revising and improving the practice-based design of the methods course based on experiences in classrooms and new teacher education research. She regularly facilitated professional development in mathematics education for practicing elementary teachers, in addition to teaching, researching, and designing the mathematics methods course.

**A Description of the Classroom Teacher**

The classroom teacher was intentionally selected because of his commitment to social justice, his student-centered pedagogy, and his effort to cultivate a classroom culture that valued

student voices, ideas, and experiences. He had been a cooperating teacher in the teacher educator program for several years and was both interested in teacher education and open to inviting others into his classroom and practice. He was familiar with CGI (Cognitively Guided Instruction) and the instructional activities covered in the course and committed to a vision of mathematics instruction that aligned to the course goals. Additionally, he was passionate and articulate in talking about his identity as a white male teacher in a diverse school and how he navigated social justice and equity issues that emerged in his class and the broader school environment.

**A Description of Oak Park Elementary and the Classroom Context**

Oak Park is a K-6 Title 1 school in the Northwest and a partner school with the university teacher education program. They offer Spanish and Vietnamese dual language programs, and eighty percent of the students speak a language in addition to English. Oak Park Elementary is partnered with an early learning center in an adjacent building. The mathematics methods course's four-hour class sessions met in one of the rooms in this adjacent building for planning and debriefing activities, and TCs walked over to the classroom teacher's room for the classroom visit.

**A Description of the Course Activities and Schedule**

The course began with a two day "Math Institute" at the university for TCs to build on their knowledge from the prerequisite Math for Elementary Teachers content course, to introduce the instructional activities, and to introduce course principles, ambitious practices, and content of Cognitively Guided Instruction (CGI) (Carpenter et al, 2015). After the Math Institute, the TCs met weekly for nine 4-hour class sessions at a local elementary school, Oak Park Elementary School.

On the first day at Oak Park, TCs met and talked with their Husky Buddies and watched the classroom teacher and TE model a lesson with the whole class. TCs did not teach a lesson

during this session. TCs also did not teach during the eighth session (11-25) because it was a school holiday for the elementary students. Instead, they worked with the classroom teacher and TEs to think about parent conferences, working with families, and how to launch problem solving tasks. In all other sessions, each small group of TCs taught one or two lessons (in the structure of an instructional activity). TCs taught these lessons in pairs. This enabled each TC to teach a lesson approximately every other week. **Table 5** (below) shows the activities that TEs modeled (both with students and out of the classroom with TCs in the role of students) and the instructional activities led by TCs. To introduce the instructional activities, TCs were provided with a Planning Protocol and a Planning Template.[11]

***Table 5*** *Instructional Activities Led by the TEs and TCs*

| **Date** | **9-30** | **10-07** | **10-14** | **10-21** | **10-28** | **11-04** | **11-18** | **12-02** |
|---|---|---|---|---|---|---|---|---|
| **TE Modeled Lesson (with students)** | Choral Counting | Estimation 180 | none | none | Launch of a Word Problem | String | Discussion Comparing Student Strategies | none |
| **TE Modeled Lesson (without students)** | none | Estimation 180 | none | none | Mental Math | none | none | none |
| **Instructional Activities led by TCs** | none | Choral Counting | Estimation 180 | Choral Counting and Estimation 180 | Choral Counting and Solving a Word Problem | Mental Math and Solving a Word Problem | Mental Math and String | Solving a Word Problem and String |

Sessions began with an hour of structured planning time in the morning, as well as small group time to rehearse lessons that TCs prepared in advance. After the planning session, the TCs and TEs walked over the third-grade classroom for the classroom visit. After the classroom teacher and/or a TE modeled a short lesson with the entire class, the students would break up into their small groups and TCs would teach the instructional activities. There were typically two lessons

[11] See Appendix E for an example of the Choral Counting Planning Protocol and Appendix F for the Choral Counting Planning Template.

taught in each small group during each session, with each lesson co-taught by two TCs. After the classroom visit, TCs debriefed the classroom visit first with their smaller teaching group then with the entire class. The sessions concluded with time for additional math content work, introductions and explorations of instructional activities (including TE modeling of activities), and planning time for the following week.

**Study Participants**

The focal group of TCs were selected as part of a larger research study documenting the practice of the TE (Shaughnessy et al., 2019). As part of that study, the TE intentionally created a group of TCs that represented a variety of grade levels and schools (for their practicum placements) as well as mathematical and personal backgrounds.

In terms of mathematical experiences, all but one of the TCs (Danielle), described their formal experiences with mathematics as procedural and involving activities like timed multiplication tests. Only Sam and Danielle described their mathematical experiences as being consistently positive. Four of the TCs described a positive turning point in their mathematics experiences, and three TCs (Katelina, Orianne, Tina) explicitly referenced the university's Mathematics for Elementary Teachers course, a prerequisite course for the program, as facilitating that turning point. Below is a brief description of each teacher candidate based on their mathematics autobiography assignment[12] and informal conversations captured through field notes.

Katelina was placed in a fifth-grade classroom at a local elementary school with a strong professional development partnership with the university facilitated by the instructor of the course. Katelina speaks Russian, and she and her family moved from Belarus when she was 8 years old to an area just south of Seattle. She described her experiences in ELL classes, with cultural

[12] See Appendix G for The Mathematics Autobiography assignment details.

adjustments in school. She noted her experiences with mathematics were always about memorization and computing formulas. Although she stayed after school for mathematics support in elementary school, she always enjoyed mathematics until reaching more advanced courses such as pre-algebra. She said that the university's Mathematics for Elementary Teachers course "helped me be more confident in my own abilities of understanding and teaching mathematics since I am actually able to understand the content through a variety of strategies rather than just the algorithms" (Mathematics Autobiography). She described how mathematics is "about developing understanding and mathematical relationships and structures" and "bringing meaning and sensemaking into math is extremely important" (Mathematics Autobiography). While she noted the importance of students' confidence in mathematics, she also felt nervous about teaching.

Flora was placed in a first and second split grade classroom at a local elementary school in its second year as a university field placement school. She is conversational in Spanish, and before coming to graduate school, she had travelled overseas to visit family in Brazil, to study abroad, and to teach English. These travelling experiences piqued her interest in global education, children's experiences both in and out of the classroom, and culturally responsive teaching. For her own elementary schooling, she was always in the highest level of mathematics. Flora enjoyed mathematics until middle school, where her school implemented a self-leveled program that she describes was based on many worksheets, handouts, and tests. She described herself as a slow thinker who needs time to process. Additionally, she was reserved and negative about mathematics and felt the "least prepared to teach and least excited to teach." She stated, "I always assume that kids will think math is boring" (Mathematics Autobiography).

Melissa was placed at a local elementary school in 4th and 5th split grade Spanish-English dual immersion classroom. Born in California to Mexican immigrant parents, Melissa grew up in

Oregon. In describing her "love-hate relationship" with mathematics, she talked about "fear of failure" and "fretting." Memories from elementary school brought back experiences with multiplication timed tests, and high school experiences were negative and felt competitive. In her college years, she avoided taking math classes and took advantage of the tutoring services offered to student athletes such as herself. After graduating from a state university in Oregon, she taught English in Chile and worked in a Montessori classroom. She was surprised at how much her experiences in school different from the way mathematics was taught in the Montessori classroom. From her experiences, she felt that students' experiences with mathematics can be positive or negative and believed "I need to make math fun and let the students know that it is not a competition rather than a personal journey of understanding the procedure rather than focusing on the solutions" (Mathematics Autobiography).

Orianne was placed in a kindergarten classroom a local elementary school that has hosted the mathematics methods coursework in previous years and that has a strong relationship with the university. Orianne is Japanese American and moved from Tokyo to study at the university as an undergraduate student. She entered the graduate program immediately following her graduation. Orianne noted that she did not like math in elementary school and remembered "silently copying methods and memorizing formulas that [she] wouldn't be able to recall just hours after taking the test" (Mathematics Autobiography). Her view of mathematics changed positively during a calculus class, when she was pressed to explain her strategies and to work in groups. She attributed her positive experience in the course to her teacher's facilitation of student-centered lessons and determination to help her succeed. Like other TCs, Orianne took the Mathematics for Elementary Teachers course at the university. This was another positive experience with mathematics, though she noted that she was still intimidated by the student-centered approach. For her, good

mathematics teaching stresses the "importance of identifying each students' strengths, making each lesson fun and applicable, incorporating partner and group work so that students can bounce ideas and learn from one another, and establishing strong relationships with each student and family so that appropriate goals and plans can be developed" (Mathematics Autobiography).

Sam was placed in a kindergarten classroom at a long-time partner and field placement school for the university. Sam is a white female from a predominantly white, rural community in Eastern Washington. She speaks only a minimal amount of Spanish. Like other TCs, her elementary memories of mathematics included timed multiplication tests. She struggled with mathematics beginning in geometry, and after barely passing algebra two, decided to retake the course to have a better understanding of the content. She felt that this was a good decision, and despite her negative experiences, stated that she now had a strong, positive relationship with mathematics. Like Orianne, she entered the program immediately following her undergraduate studies.

Danielle was placed in a second-grade classroom at a local elementary school. Danielle is a white female who returned to school for a career change after working in another profession for several years. While the school is in a more affluent district that the other TCs, the school has a more diverse student population in comparison to the district. In addition, her mentor teacher was a graduate of university teacher education program and active in working with the teacher education program. Danielle noted that she always liked math. Before entering the program, Danielle studied public relations and sociology. After working in public relations for five years, she returned to graduate school to pursue a more “meaningful” career.

Tina was placed in a fifth-grade classroom at Oak Park Elementary, where the mathematics methods course took place. Tina is Vietnamese and grew up in California. She described her

experiences with mathematics as including timed multiplication tests, a lot of computations, and memorization. Tina felt mathematics was an important part of her life and her family stressed the importance of mathematics for future career opportunities such as medicine or engineering. Mathematics and science were emphasized in her household, and her family members supported her learning and helped her with homework. She noted that after taking the Mathematics for Elementary Teachers Course, she moved away from thinking that mathematics was just about memorization and computation. Instead, in mathematics classrooms: "[Students should] understand the process and be able to apply these concepts to real life. They should also be able to apply in different formats, not just in forms of word problems or numerical sentences. This can open up many more opportunities for students to enjoy math and feel competent" (Mathematics Autobiography).

## Chapter 4: Research Strategy and Design

The purpose of this study was to characterize teacher candidate (TC) participation as they learned to facilitate productive mathematical discussions in a practice-based mathematics methods course. To characterize TC participation around facilitating discussion, my analytic approach examined changes in participation during the small group time of planning, rehearsing, enacting, and debriefing of lessons, a common activity cycle in this practice-based methods course (see Chapter 3 for a description of this cycle). I examined the following research question: How does TC participation manifest and change in their practice-based mathematics methods course? I characterized TC investigation and enactment of practice by dividing my larger research question into two major analytic questions:

- How did TCs facilitate discussions to reach particular instructional goals?
- What shared repertoire did TCs develop, and how did they use these tools?

Attending to how discussion-based lessons were planned, talked about, and enacted was appropriate for understanding how a practice-based approach focused on core practices could address the problem of enactment (Grossman, 2018; Kennedy, 1999; Shaughnessy et al., 2019). In the following section, I discuss my decision to use a qualitative approach to examine TC learning. I outline my sampling methods, choice of case study methodology, and role as a researcher. Following this section, I discuss my data collection and analytic methods and conclude with a brief discussion on the issues of reliability, validity, and limitations to my study.

### A Qualitative Approach to Understanding Teacher Candidate Learning

A qualitative analysis is particularly useful to understand how TC participation changed over time. Qualitative research focuses on process, understanding, and meaning making (Merriam, 2009). Qualitative research study is useful for understanding how people interpret their

experiences, how they construct their worlds, and what meaning they attribute to their experiences. Through this inductive process from particular experiences to more general findings, analysis provided a rich description of TC experiences. Through qualitative analysis, I interpreted TCs' participation and how practice was worked on, learned, and enacted. Examining qualitative data sources such as observations, videotape recordings of sessions, and relevant artifacts allowed for analysis of both the substance of what was talked about and enacted and how TCs engaged in these discussions and lessons.

This study is case study of TC participation in methods coursework. These case (the group of TCs) was selected as an instance of TC learning in a community of practice. The choice of what data to collect was determined by the desire "to understand teaching as participation in ongoing practice." (Lave, 1996, p. 158). Relevant data sources included observations of practice and participation in the course, videos of the course sessions, documents from the course, and field notes. For my case study analysis, I identified recurring patterns, which became my findings (Merriam, 2009). The purpose of my analysis was to develop a "thick description" of TC learning through "some sort of organization of the data…to convey to the reader the sociocultural patterns characteristic of the group under study" (p. 29).

Case study investigates a phenomenon in a real-life context by examining one or more cases that are instances of a process, issue, or concern (Merriam, 2009). Here, the group of TCs represented an "instance" or example of TC learning and enactment of practices in a practice-based mathematics methods course. Focusing on TC learning was helpful for analyzing particular, practical problems and provided a heuristic to "illuminate the reader's understanding of the phenomenon under study" by discovering new meaning, extending the reader's experience, or confirming what is known (p. 44). Case study methodology is anchored to real life, provides a rich

description, and offers reader interpretation (Merriam, 2009). This case study was observational, using participant observation as major data source. Limitations of case study include issues of generalizability and whether the colorful, rich description provided in my analysis are transferrable findings and prototypes for TC learning.

## Sampling: Settings, Timeframes, Events, and Participants

While the research question posed could be asked in the setting of any teacher education program, my specific interest was TC participation in *practice-based* methods courses focused on core practices. Sampling for this study was purposeful: "based on the assumption that the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned" (Merriam, 2009, p. 77). There were two levels of sampling: selection of the methods course and sampling of the TC group within the course. The TCs in the methods course sampled were divided into four teaching groups with several TCs in each group. There were seven TCs in the teaching group I sampled.

### *The Mathematics Methods Course*

The setting was chosen based on the program's implementation of practice-based methods coursework and use of pedagogies of enactment. The course (described in Chapter 3) was selected because of its location in a traditional, university-based teacher education program and its design as a practice-based space focusing on core practices. Additionally, it was chosen based on my experience as a teacher educator in the coursework.

### *Selection of the Teacher Candidate Group*

This group was purposefully sampled as part of a larger, cross-institutional research project with the Core Practices Consortium (Grossman, 2018; Shaughnessy et al., 2019). The larger research project focused on teacher educator pedagogies and specifications of decompositions of

practice. For data collection, the research team selected the TC group coached by the lead teacher educator, a collaborating researcher of the Core Practices Consortium project and the university faculty member leading the course. I purposefully selected the same TC group. All seven TCs in the group were included as participants (see previous chapter for descriptions of the TCs).

### *Researcher Role*

This study was conducted while I was in the role of a research assistant collecting data for one methods course site and collaborating on cross-institutional analysis for the Core Practices Consortium. I observed and followed one of the teaching groups for the entire quarter. My role was one of observer as participant, where my observer role in known and participation is subordinate to my observer activities (Merriam, 2009). This role allowed me to "have access to many people and a wide range of information" without the distraction and limitations of self-study and simultaneously fulfilling duties as the teacher educator (p. 124).

## Data Sources and Collection

This data was collected as part of the Core Practices Consortium's mathematics education research that examined teacher educator pedagogies and specifications of core practices from the perspective of the teacher educator. Examining a subset of this data in terms of TC participation provides a different lens and complementary findings to the larger project. My data is a subset of the larger project.

### *Selected Course Sessions*

I examined seven course sessions that included all lessons taught by the focal group (12 TC-led instructional activities). This included 7 out of 9 sessions at the school and did not include the two-day Math Institute, which was held at the university and which did not include any TC-led lessons or pedagogies of enactment. I focused on the small group portion of the session. This

included TC participation and documents for small group planning, rehearsals, enactment, and debriefing of the lesson. I chose not to examine portions of the sessions that involved whole groups activities: activities around content/teaching standards and rehearsals and enactments of other TCs, TEs, and the classroom teacher. While this work was certainly relevant and impacted the work done in the small group, I chose this boundary to focus on the activities and interactions of the selected group. This decision was based on my purpose to examine TC participation around enactment of a core practice and within the sampled TC group. Further research could build on this study and illuminate how different aspects within the course, between courses, and across the entire program interact to support TC development of their practice and knowledge.

***Observations and Video Recordings***

For the selected teaching group, I observed all meetings for their mathematics methods course, including their common planning time, rehearsals, enactment, and reflection and debriefing time. I video recorded and took field notes during observations of the meetings. Observations expose things that have become routine to participants and to use the researcher's own knowledge and expertise to interpret what is observed firsthand (Merriam, 2009). Observations take place in the setting where the phenomenon of interest (TC participation) naturally occurred and provide a firsthand encounter with the phenomenon of interest. Thus, observations were particularly useful to document how TCs participated and what they talked about and enacted.

During observations, I focused on TCs' actions, interactions, and content of conversations, recorded descriptions of the instruction, and wrote anecdotal notes about how the practice was worked on throughout the cycle of enactment and investigation. Observation field notes included descriptions of the setting, activities, people, and their interactions as well as direct quotes capturing the substance of what was said. For each observation, I wrote up analytical memos with

preliminary analysis and interpretation for initial coding. Video recordings were analyzed in StudioCode© (as described later in the chapter).

### *Document Review*

Collected documents from the course included TCs' lesson plans, course assignments, and written reflections, as well as teacher educator lesson plans and materials for the sessions. Reviewing documents provided descriptive information that was both stable and nonreactive (Merriam, 2009). Document review was useful to triangulate findings about participation and to uncover any discrepancies or questions from observations. These documents were used to better understand individual TCs' planning and reflection of practice and what resources were provided to TCs. Personal documents, especially daily reflections, provided a first-person source of "an individual's actions, experiences, and beliefs" and were a "reliable source of data concerning a person's attitudes, beliefs, and view of the world" (p. 142-143). Documents reflected the participants' perspectives, and during analysis I considered the highly subjective nature of these documents.

## Analytic Methods

Data was analyzed for broad themes and categories related to TC participation and enactment of the practice of facilitating a productive mathematical discussion. The purpose of this research was to better understand how TC participation changed in a quarter-long methods course. Analysis was an iterative process examining regularities in the data (Merriam, 2009). Coding organized data and involved "generating concepts from and with our data, using coding as a means of achieving this" (Coffey & Atkinson, 1996, p. 26). I began with open coding to assign codes to pieces of data (e.g., transcripts of sessions, documents, observation notes), investigating the ways TCs talked about and enacted practice.

The first cycle of coding these chunks of data involved descriptive and in vivo coding, based on my conceptual framing and research questions. Subsequent coding focused on identifying patterns and subsuming these particulars into broader, more general emergent categories: "trying to categorize a particular action, event, participant, state, and so on, into a more abstractly defined class" (Miles et al., 2014, p. 285). My analysis (see **Table 8**), related discrete findings to each other, named patterns, and identified corresponding constructs.

***Analysis of TC Participation***

I began analysis by selecting a subset of data (videos from the first and last course sessions and written documents, assessments, and lesson plans for one TC). I descriptively coded TC participation around facilitating productive mathematical discussion. This was an iterative process to figure out the appropriate unit of analysis and chunking. I referred to my conceptual framework and research questions to refine my method of analyzing video and my initial coding scheme based on early reviews of this subset of data (see Figure 3). This included refining initial descriptive codes as well as articulating sub-codes. These sub-codes came from in vivo coding to capture TCs participation and voice around the practice of facilitating a productive mathematical discussion (Miles, et al, 2014). Coding decisions were recorded in analytic memos. After testing and revising this coding scheme by coding two videos and two documents, I coded the remaining data set.

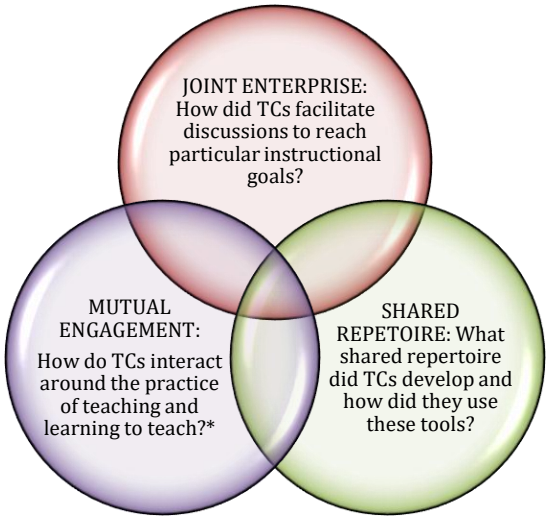
To answer my research question, I examined the joint enterprise (goals analysis) and the shared repertoire (tools analysis) of the group. Two main analytic questions guided my analysis:

1. How did TCs facilitate discussions to reach particular instructional goals?
2. What shared repertoire did TCs develop and how did they use these tools?

Answering these questions involved coding for instances when participants were working on facilitating discussion in videos of the course and in the TC lesson plans, assessments, and

assignments. I analyzed changes in what they attend to, how they talked about practice, and their enactment of facilitating mathematical discussion. In my analysis of goals, I talk about their joint enterprise: how they work on instructional and practice goals. In my analysis on tools, I examined the shared repertoire of the group: how TCs used talk moves, lesson plans, and course and teaching materials (focusing on board work).

***Figure 3:*** *Analytic Questions Based on Joint Enterprise, Mutual Engagement, and Shared Repertoire.*




*Note: These questions and figure draw from Wenger's (1998) Communities of Practice framework.**[13]

**Analyzing the joint enterprise: A goals analysis.** In my analysis of the joint enterprise of the group, I examined goals. This involved analysis of their instructional goal (mathematical learning target for students) and practice goal (what aspects of teaching practice were attend to). The purpose of this analysis was to understand how TCs worked on instructional goals and teaching practice.

[13] *My framing of mutual engagement as "how TCs interacted" differs from the original definition that Wenger (1998) provides. I also embed my findings to the analytic question of how TCs interact (mutual engagement) in my analysis on goals (joint enterprise) and my analysis on tools (shared repertoire).

***Descriptive coding and analytic summaries of instructional goals and practice goals.*** I reviewed the lesson plans, field notes, and videos to track how instructional goals and practice goals were worked on over the duration of the ten-week course. While reading field notes and reviewing documents (particularly lesson plans), I used descriptive coding to note when they were attending to goals and process coding to tag how they worked on components of discussion facilitation (practice goal).

For video recordings, I used StudioCode© software to code in a similar but digital approach, using the video timeline, a "chronologically organized multilayered graphical representation of all codes, descriptors, and narrative comments attached to a particular video" (Lampert et al., 2013, p. 230). I created a "goals" line, where I tagged instances of TCs working on goals. I examined how TCs worked on instructional goals and practice goals during each part of the course session (planning time, rehearsals, the classroom visit, and the debrief). Each instance had a comment box where I recorded a descriptive summary of the state and observed goal(s) (including level of specificity) and how TCs worked on discussion to facilitate this goal (including their intentionality in working towards the goal and what aspect of discussion they were working on). This detailed coding and commentary were used to create summary tables and analytic memos recording instructional and practice goals for each session.

***Table 6:*** *Definitions and examples for specificity of stated and written instructional goals*

| Specificity | Description | Example |
|---|---|---|
| None | No goal is written in their lesson plan or said out loud. | NA |
| Vague | A vague goal is written in the lesson plan and/or stated out loud (or goal is determined by TE but not expanded on by TCs) and/or multiple generic goals are listed. | Danielle's goals for 10-7 Choral Count: Use patterns to make predictions (articulated by TE). Explain patterns.<br>Orianne's written goal for 11-04 Choral Count: See what strategies students use when mentally subtracting. See if students can take on the perspectives of peers.<br>Sam's written goal for 11-4 String (not taught): To elicit student thinking and see how they mentally solve a subtraction problem. |
| Approaching | A goal is articulated that is clear but not specific to the task or that has a mathematical idea. | Sam and Flora's goal for 11-4 Solving a Word Problem: Students could solve the problem using an array.<br>Written goal for Katelina's 11-18 String: Use the first problem to help you solve the second problem. |
| Clear | A clear and mathematical goal is articulated and is appropriate for the task. | Katelina's stated goal for 12-2 String: Break apart the second number in a subtraction problem by place value. |

***Table 7:*** *Definitions and examples for intentionality in orchestrating discussions towards instructional goals*

| Intentionality | Description | Example |
|---|---|---|
| Not intentional | Moves are not intentionally connected to the goal; teacher moves may try to elicit ideas, but TCs don't build on these ideas or respond in relation to a goal without TE prompting | In the first choral count on 10-7, the rehearsal focused on teacher moves and logistics (e.g., "Quick question, is it awkward for me to stand here?"). During enactment, TCs used a repeating move for all patterns (not strategically to orient to particular mathematics). |
| Loose | TCs are aware of the goal, but don't know what moves to choose or choose moves that don't necessarily steer the conversation towards that goal. Moves that are connected to the goal are enacted or prompted by the TE. | In the first choral count, Danielle asked student to make a prediction by using a story context: "How many minutes do you think the total would be after...45 days, right?" This context was not framed in a way that was helpful for the goal of using patterns to make predictions and they didn't discuss the story context in a way that might help students make sense of the mathematical task. |
| Approaching | TCs begin to make decisions based on the goal and/or they take up suggestions by TE that are connected to the goal | On 10-21, the TE made suggestions to visually show multiplicative relationships and for questions based on the idea of "groups of three." Orianne took up these representations and questions about "groups of three" the enactment. |
| Intentional | TCs make clear efforts to teach towards this goal. Talk moves and representations are selected and used in relation to the goal. | During the 11-18 Mental Math, Danielle selected a strategy to discuss based on the goal (breaking apart numbers). As the student shared, she drew the incremental strategy on a number line. She asked others to repeat. |

***Data display: Narrative storylines and goal specificity and intentionality matrices.*** From my coding, summary tables, and analytic memos, I created a narrative storyline of each day, providing a rich description of how TCs worked on instructional goals and discussion facilitation. Using the coding, summary tables, analytic memos, and storylines above, I created a detailed goals matrix, outlining how instructional goals and practice goals were worked on during each course session and each part of the course sessions. From this matrix, I created matrices of the level of specificity of the instructional goal(s) and intentionality in working towards these goals. I linked these to each IA and when each TC taught. These time-ordered matrices allowed me to see the trajectory of goal specificity over entire course, within particular IAs, and for particular TCs (See Appendix I).

**Analyzing the shared repertoire: A tools analysis.** In the second part of my analysis, I examined the shared repertoire TCs developed and how they used tools in the group. The following tools were coded and analyzed to examine the shared repertoire: lesson plans, the board (including posters and markers), and talk moves. By examining how TCs used tools, I could see what aspects of practice TCs attended to through each group of tools. I wanted to understand how these tools were used to mediate participation and to support opportunities for TC learning.

***First cycle of coding: Descriptive and in vivo codes of tools.*** StudioCode© software was used to tag instances when TCs were engaging with lesson plans, the board, and talk moves (i.e. the "shared repertoire") (Wenger, 1998). I created a "tools" timeline and tagged instances when each of these tools were used or talked about. I attached short descriptions and in vivo coding to these instances, describing how the tool was being used.

***Second cycle of coding and memos: Patterns and themes in tools.*** A second cycle of coding (analyzing within tagged instances of "tools") helped me examine themes and changes related to

the shared repertoire. Using the "transcription" feature in StudioCode©, I created a chronological matrix of tagged instances that included my first cycle of coding and descriptive summaries of each instance. This allowed me to examine each of the three categories of tools and the patterns and themes in how they were used and talked about. Analytic memos captured these patterns as well as how the lesson plans, board, and talk moves facilitated opportunities for TC learning.

In addition to video analysis, analytic memos were used to capture emerging patterns in written lesson plans and course materials. I reviewed lesson plans before and after watching videos to connect how lesson plans were used in the rehearsal, classroom visit, and debriefing with the substance of what TCs wrote in the plans. I noted how this connected to the instructional goal and practice goal for that day and how these lesson plans facilitated opportunities for TC learning (see Appendix J: Findings Chart for Tool Use and OTL).

**Iterative examination of patterns and themes across the goals and tools coding and analyses.** Although rounds of the two analyses (and coding cycles within these analyses) were done in successive passes (e.g., coding the "goals" timeline was followed by coding for "tools"), I continually revisited different parts of my analyses in an iterative analytical process. In other words, after each successive stage of coding, I revisited earlier coding and analyses, refining or creating additional analytic memos, summary tables, and storylines to reflect new layers of analysis. This allowed me to understand the interrelatedness of the joint enterprise (goals analysis) and shared repertoire (tools analysis) of the community of practice and to document mutual engagement of participants.

Through this iterative process, I could understand how TCs interacted around the practice of facilitating productive mathematical goals (mutual engagement). An important theme of "ownership" emerged from these successive passes and iterative examination. To capture this idea

of ownership, I coded where the ownership resided (TE ownership, joint ownership, TC ownership) in instances in the timelines and reviewed summary tables analytic memos to summarize how this ownership changed over the ten-week course.

***Table 8:*** *Overview of Process for Analyzing TC Participation*

Research Question: How does TC participation manifest and change in their practice-based mathematics methods course?

| Question | Analyzed for… | Analytic Methods |
|---|---|---|
| JOINT ENTERPRISE: How did TCs facilitate discussions to reach particular instructional goals? | **What was the instructional goal?**<br>• What did they say was their goal? What was their instructional goal, as indicated by their rehearsal and enactment?<br>• How do they work on these goals and how does this change over time?<br>• How did TCs intentionally facilitate discussions to reach their goals?<br>**What seemed to be their practice goal?**<br>• What aspects of practice did they work on?<br>• What did they talk about?<br>• What were they trying to do/enact?<br>• How was this connected to the TE goal?<br>**How did TC interact with students and student ideas?** | GOALS CODING:<br>Hand coded lesson plans, field notes, and course materials:<br>• Descriptive coding to note when they were attending to goals.<br>• Process coding to tag how they worked on components of discussion facilitation (practice goal).<br>Coded video on "goals" line in StudioCode©:<br>• Segmented video into instances of TCs working on instructional goals and components of discussion facilitation.<br>• Instructional Goal: Tagged instances when TCs were talking about or enacting instructional goals and wrote descriptive summary of the state and observed goal(s) (including level of specificity) in the comment box for the instance.<br>• Practice Goal: Tagged instances when TCs were talking about or enacting different components of discussion and wrote descriptive summary of how TCs worked on discussion to facilitate this goal (including their intentionality level) in the comment box for the instance.<br><br>ANALYTIC SUMMARIES AND DATA DISPLAYS:<br>Summary tables and analytic memos:<br>• Recorded instructional and practice goals for each session.<br>Narrative storylines:<br>• Described how TCs worked on instructional goals and discussion facilitation in each session.<br>Time-ordered goal specificity and intentionality matrices:<br>• Level of specificity and intentionality.<br>• Articulated by Date, Instructional Activity, and TC. |
| SHARED REPETOIRE: What shared repertoire did TCs develop and how did they use these tools? | • How did TCs use their lesson plans?<br>• How did TCs use the board?<br>• What talk moves were used and how?<br>**How were OTL mediated by tools?** | TOOLS CODING:<br>Coded video on "tools" line in StudioCode©:<br>• Tagged instances of TCs engaging with lesson plans, board, and talk moves.<br>• Attached in vivo codes and descriptive summaries of how the tool was used in comment box to instances.<br>Hand coding of lesson plans and course materials:<br>• Coded for the content TCs included and how they used planning templates.<br><br>CHRONOLOGICAL MATRIX AND ANALYTIC MEMOS<br>Patterns and Themes in Tools:<br>• Used the "transcription" feature in StudioCode© to create chronological matrix of instances of tools (with attached codes and descriptive summaries).<br>• Wrote analytic memos for each tool and how it was used and talked about. |
| MUTUAL ENGAGEMENT: How did TCs interact around the practice of teaching and learning to teach?* | • How did TCs interact with one another and the TE?<br>• How did TCs interact with practice and tools?<br>• Where did ownership of the work on instructional goals reside?<br>• Where did ownership of the work on practice goals reside? | ITERATIVE EXAMINATION<br>Iterative examination of patterns and themes across the goals and tools coding and analyses:<br>• Revisited earlier coding and analyses after each successive stage of coding.<br>• Refined or created additional analytic memos, summary tables, and storylines to reflect new layers of analysis.<br>Coded for where the ownership resided (TE ownership, joint ownership, TC ownership):<br>• Coded instances in the timelines.<br>• Coded and reviewed summary tables analytic memos.<br>• Wrote analytic memo summarizing how this ownership changed. |

**Data organization: Structural codes.** The substance of my analysis came from the coding outlined above. However, for data organization, substance codes were useful for tagging segments of the learning cycle and TCs associated with particular video segments and documents. These structural codes were used for data management as I moved from coding to displaying my analysis into different matrices.

### *Ethical Considerations*

This study analyzed a subset of data from a larger, multi-institutional research project. Through the larger research project, IRB approval was granted, and IRB procedures were carefully followed throughout the research process. TCs in the focal group were consented in person prior to any data collection at their field placement or at the university. TCs were fully informed about consent process and their obligations as a research participant, as well as the protections in place for them. They could ask questions before they made their final decision to proceed and sign the consent forms.

Participants had the right to withdraw from the study at any time. Participant names and workplaces were not named in any accounts since pseudonyms have been used. TC were informed that their supervisors, mentor teachers, and program instructors would not have access to data for evaluation purposes. Names and contact information were recorded in order to connect data with information source, a feature that is important for scheduling future data collection, thanking participants, or clarifying information already collected. A pseudonym was assigned each school and participant to prevent against disclosure, and a reference document linking pseudonyms with identifying information was kept in a separate, secure location. Pseudonyms were used on revised field notes, artifacts, analytic memos, and writing. Study data is kept on a password-protected laptop and a hard drive in a locked facility.

### *Issues of Trustworthiness: Validity, Reliability, Credibility, Dependability, Confirmability, Transferability*

During design, data collection, and analysis, I was careful to attend to issues of validity and reliability using several strategies that Merriam (2009) described:

- triangulation – complementary methods and data sources and use of multiple sources of data (e.g., video recordings, field notes, and documents) to confirm emerging findings and produce converging conclusions.
- member checks – testing plausibility by taking tentative interpretations back to TEs of the course.
- researcher reflexivity – critical self-reflection of my assumptions, biases, theoretical orientation, and interpretations.
- peer review – discussions with colleagues about methodology, emerging findings/interpretations through mathematics education research group (MERG), the Core Practices Consortium collaborators, and university-based qualitative research groups.
- audit trail – detailed account of design, framing, and interpretive decisions through analytic memos and use of Microsoft Word, Excel, and OneNote applications.
- rich, thick descriptions – contextualizing descriptions through narrative descriptions in the first round of data analysis and as written in the context and findings chapters.

### *Limitations*

In addition to attending to issues of validity and reliability, I acknowledge the limitations of my study. Teacher education research has been plagued by the problem of ambiguity in comparisons that rely on teacher educators' self-descriptions and generalization about programs that exist in multiple sites or contexts (Zeichner & Conklin, 2008). The role of practitioner research

can be problematic as the dual role of researcher and teacher complicates research methods. Although this study is not technically practitioner research (I was not the teacher educator in the course being studied), I have worked in this course in previous years as a teaching assistant and coach, and I was aware of my positionality to the program studied. Triangulating my findings, having a heightened sense of researcher reflexivity, using peer review with colleagues outside of the course and affiliated with other institutions, and establishing a clear audit trail backed with data (rather than just insider insights) was important to establish reliability and validity.

Existing teacher education studies also raise issues about how particular pedagogies connect to and are situated in courses and the teacher education program as a whole (Grossman, 2005). I examined an individual course at one institution. Within this course, I only examined TC participation in the context of their small group time. It would be fruitful for additional studies to examine TC enactments over a longer trajectory across the whole teacher education program (including field placements) and beyond the program as well as TC enactments across sites and programs (areas of research the Core Practices Consortium has started).

In addition, it is difficult to untangle effectiveness of particular pedagogies (Grossman, 2005). My findings center on TC participation and the OTL that emerge in pedagogies of enactment. I do not make claims about the effects of pedagogies on TC learning. Instead, I characterize TC learning as participation *in the context of* (not *as an evaluation of*) practice-based pedagogies and core practices. It would be productive for research to continue examine what TCs can learn from representations of others' practice and how particular pedagogies help TCs learn from their own practice (another line of research the Core Practices Consortium is pursuing).

The goal of my study was not to provide generalizable findings about TC participation in *all* practice-based methods courses. Nor was it to say that this course is the *right* way to structure

a methods course. Rather, the goal of this qualitative research is to provide a rich, thick description of TC learning within a practice-based setting focused on core practices. Findings from this research contribute images for understanding a trajectory of TC learning and enactment of core practices and existence proof that TCs can make shifts in both talk and enactment of practice within a methods course.

**Chapter Summary**

This chapter provided a detailed description of my methodology. Qualitative case study methodology was used to characterize the nature of TC participation and how it changed over time. The setting and methods course sampled was chosen based on the program's implementation of practice-based methods coursework and use of pedagogies of enactment. The participant sample included seven focal TCs that were purposefully sampled as part of a larger research study. Data was collected over the ten-week course through observations, video recordings, field notes, and documents.

A review of the literature provided the background for my conceptual framework, which framed TC learning as participation in joint enterprise and with a shared repertoire. Data was analyzed in an iterative process aiming to uncover how TCs facilitated discussions to reach particular instructional and practice goals and how TC participation was mediated through tools. Methods of sampling, data collection, and analysis included strategy to address issues of trustworthiness to support the validity and reliability of findings. The purpose of this study and analytical approach were to characterize the nature and changes of TC participation in a practice-based mathematics methods course as they learned to facilitate mathematical discussions.

## Chapter 5: Teacher Candidate Participation Around Instructional and Practice Goals

This chapter examines how TCs facilitated discussions to reach instructional goals. There is considerable research asserting that productive mathematical discussions work towards a clear, mathematical goal (Chapin et al., 2013; National Council of Teachers of Mathematics, 2014; Smith & Stein, 2011), and a growing body of research documents pedagogies of practice and OTL about teaching towards instructional goals (Ghousseini & Herbst, 2016; Grossman, 2018; Tyminski et al., 2014; Sleep, 2012). However, there is limited research that captures both the planning and talk around goals with enacted practice working towards these goals (Kazemi & Wæge, 2015; Sleep, 2012). I focus on my sub-question around goals: How did TCs facilitate discussions to reach particular instructional goals? To determine their instructional goal, I examined what they said was their goal and what their instructional goal was as indicated by their participation in rehearsal and enactment. I examined how TCs worked on goals and what aspects of practice they attended to as they worked on facilitating discussions (their practice goal). Findings suggest TCs' *specificity* in articulating the instructional goal, *intentionality* in facilitating discussions to reach these goals, and *ownership* over articulating and working on goals increased over the 10-week course.

This analysis builds on the work of several important studies. First, it builds on Tyminski and colleagues' (2014) study that documented how TCs made pedagogical decisions in an approximation of practice for facilitating discussions (extended planning task and rehearsal with peers acting as students). Using similar methods, I examined TCs' pedagogical decisions in enactments with *students*, not just the planning and approximation of practice. This analysis also builds on Sleep's (2012) study of TC enactments of facilitating discussions towards a mathematical instructional goal. I used her seven tasks of "steering instruction towards the mathematical point" as a lens to understand the work of facilitating discussions to reach instructional goals as I analyzed

how TCs worked on facilitating discussions (including what counts as a "productive" or "mathematical" discussion).

Findings also build on research documenting shifts in TC efficacy and agency (e.g., Anthony et al., 2015) in engaging in facilitating discussions towards an instructional goal and the important role of the TE as a more knowledgeable other, particularly in earlier approximations and enactments (Drake, 2016; Ghousseini & Sleep, 2011; Grossman, Hammerness, et al., 2009; Kazemi et al., 2009; Lampert et al., 2013; Shaughnessy et al., 2019). I examined TC participation in the context of the TE's scaffolding and moves. Much like Kazemi and Wæge's (2015) suggested (in a study of a previous iteration of this course), the TE can attend to identifying the goal to help TCs "reflect on talk moves not just to elicit students' initial ideas but to build explanations and develop reasoning" (p. 142).

This analysis examined the iterative trajectory of TC participation, particularly the development of TCs' talk and enactment of practices and techniques nested facilitating discussions towards mathematical goals. This iterative trajectory builds on Boerst and colleagues' (2011) attention to grain size of practice (including larger domains, nested in smaller techniques, and connected intermediate practices) and to intentional sequencing and nesting of approximations of practice.

My findings provide an image of the iterative and cyclical trajectory of TC participation in a methods course. The methods course focused on seven practices of ambitious teaching including *teaching towards an instructional goal* and *orienting students to one another's ideas and to the mathematical goal* (course syllabus). I focused on the shifts related to working on goals in their enacted practice.

## Changes in Specificity, Intentionality, and Ownership of Goals

My analysis found three changes in how TCs worked on facilitating discussion to reach instructional goals during the ten-week methods course:

1. SPECIFICITY: TCs were more specific in articulating the instructional goal for their lesson.
2. INTENTIONALITY: TCs were more intentional in facilitating discussions to reach these goals.
3. OWNERSHIP: TCs took on more ownership over articulating and working on the instructional goal and aspects of facilitating productive mathematical discussion (the practice goal).

### *Specificity in Articulating Instructional Goals*

To understand how TCs articulated goals, I examined the stated and observed instructional goal for each lesson. Stated goals included the instructional goal written in their lesson plans and any instructional goal explicitly articulated during planning and rehearsal discussions. In the beginning of the course, goals were either absent or vaguely[14] stated. As the course progressed, TCs began articulating goals that addressed particular mathematical ideas or practices (see **Table 9** below). They wrote more specific goals, explicitly talked about these goals, and stated them during their rehearsals of facilitating discussions and actual facilitation of discussions with students.

[14] See **Table 6** and **Table 7** in Chapter 4 for definitions and examples of specificity and intentionality levels.

### *Intentionality in Facilitating Discussions to Reach the Instructional Goals*

In addition to noting the specificity of how TCs articulated the instructional goal, I also examined how TCs worked on this goal, analyzing their intentionality in facilitating discussions towards an instructional goal. In early facilitation of discussions, TC moves and instructional decisions were less intentional in working towards the instructional goal (see **Table 9** below). In later sessions, TCs were more intentional in their decision making to actively steer the mathematical discussion towards these goals.

***Table 9:*** *Specificity and Intentionality Chart*

| Session | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Date | 7-Oct | 14-Oct | 10-21a | 10-21b | 10-28a | 10-28b | 11-4a | 11-4b | 11-18a | 11-18b | 12-2a | 12-2b |
| Instructional Activity | Choral Count | Estimation 180 | Choral Count | Estimation 180 | Word Problem | Choral Count | Word Problem | Mental Math | String | Mental Math | Word Problem | String |
| Specificity of goal | None/ vague | vague | clear | approaching | clear | clear | approaching | vague | approaching | clear | clear | clear |
| Intentionality in facilitating discussions | Not intent | loose | loose | loose | loose | approaching | loose | loose | approaching | approaching | intentional | intentional |

My analysis found the level of goal specificity and intentionality varied based on when the lesson happened in the course (later lessons were more specific and articulated), the TC (whether it was their first time teaching or a later lesson), and the instructional activity (e.g., goals for Estimation 180 activities were less specific). Over the ten-week course, five out of the seven TCs moved from no goal or vague goal to being able to clearly state an instructional goal and intentionally making teaching decisions based on this goal. A sixth TC moved from vague goals to approaching more intentionality. Only one TC stayed in the middle level of specificity.

The level of goal specificity and intentionality also varied by instructional activity for problem solving, mental math, and strings. This could be attributed to the different purposes and affordances of different instructional activities (Cunard, 2014; Kazemi & Wæge, 2015). For example, Estimation 180 stayed at the vague/approaching level, perhaps because the activity was only used twice (and both times at the beginning of the course) or because the activity itself focuses on more open strategy sharing, not necessarily a specific mathematical idea. Choral counting was done three times and during the beginning of the course. Thus, there was only a small observed increase in goal specificity and intentionality. In contrast, during strings, TCs were specific in articulating goals and more intentional in working towards goals during strings activities. This could be attributed to 1) strings being introduced and enacted later in the course (when TCs had already been refining and enacting more complex practices); and 2) the structure of the strings, which "places particular emphases on the use of representation as well as questioning practices that steer students towards mathematical goals in particular way" (Kazemi & Wæge, 2015, p. 142). During strings, TCs worked on *eliciting and responding to student ideas*, *orienting student to key mathematics*, and *representing mathematical ideas* in service of the instructional goal. In earlier enactments (e.g., choral counting and Estimation 180), TCs did not show the same level of intentionality in *eliciting and responding to student thinking in service* of instructional goals and did not attend as explicitly to *orienting* and *representing* practices.

While evaluating the OTL and effects of using particular instructional activities is not in the scope of this study, other studies have attended to the affordances and constraints of particular activities, and TEs will need to think intentionally about how to use and sequence these activities as well as approximations of practice (Cunard, 2014; Kazemi & Wæge, 2015). The types of tools (including scaffolding tools and instructional activities as structures for recomposing practice) and

both increasingly complex and nested approximations of practice can be used intentionally to support the iterative development of TC practice and provide multiple opportunities to try out and refine practices (Boerst et al., 2011; Janssen et al., 2015; McDonald et al., 2013; Shaughnessy et al., 2019).

### *Ownership Over Articulating and Working on Instructional Goals and Practice Goals*

Over the ten-course, TCs took more ownership over instructional goals. Early in the course, TCs did not write or say goals (or they were extremely vague) unless prompted by the TE. Towards the end of the course, they began writing and stating goals that addressed particular mathematical ideas and practices. Additionally, these goals were written or stated by the TCs without (or with less) prompting from the TE. This change indicated a shift in ownership as TCs moved from "practice-shock" to increased efficacy in working collaboratively on practice, confidence in their contributions to the group and willingness to try practices, and agency in building relationships with students to facilitate meaningful discussions (Timperley, 2013).

## Zooming in to Three Class Sessions

In this section, I use three class sessions to illustrate the changes in specificity, intentionality, and ownership of goals. I describe three lessons led by TCs: a choral count on the first day, a choral count from the middle of the course, and a string activity on the last day of the course. These episodes show the development in TCs' abilities to identify instructional goals and enact them. They illustrate the changes in 1) TCs' *specificity and identification* of instructional goals during the course; 2) how TCs *intentionally* facilitated discussions to reach these particular goals; and 3) *ownership* of how goals were articulated and worked on.

The first episode is a case of working to "get through" the lesson. TCs did not specify a goal and were not intentional in how they facilitated the discussion in relation a goal. They

conveyed nervousness and uncertainty about leading the activity both during rehearsal and with the students. This was one of their first attempts to teach a lesson, and TCs were in survival mode, focusing on themselves as they experienced a kind of "practise-shock" (Timperley, 2013). Much of their attention and instructional time was focused on nonmathematical work (Sleep, 2009). Their enactment of the lesson, both in rehearsal and in the classroom with students, can be characterized as working just to "get through" the activity. They looked to the TE as the authority during both the rehearsal and enactment.

The second episode is a case of TCs specifying goals but looking to the TE for support on how to reach those goals. The TCs identified a goal on their lesson plans and during rehearsal and enactment. However, they continued to depend on the TE for guidance in how to reach these goals. In the beginning and middle of the course, the TE (in the role of a more knowledgeable other) reminded TCs of their goal(s) and suggested moves to work towards that goal (Ghousseini & Sleep, 2011; Grossman, Hammerness, et al., 2009; Kazemi et al., 2009; Lampert et al., 2013). Towards the middle of the course (e.g., this episode), TCs were intentional in thinking about how to facilitate discussions to reach the goal but did not quite know *how* to do that nor did they remember to do that throughout the lesson.

The final episode is a case of intentionally facilitating discussion towards particular mathematical ideas. TCs worked on tasks that Sleep (2009) outlines as central to steering the discussion towards a goal including spending time on the intended mathematics and emphasizing key mathematical ideas. TCs articulated a goal with specific mathematical ideas, intentionally discussed how to reach that goal, and made teaching decisions based on that goal. These discussions and intentional decisions were led by the TCs rather than the TE. Thus, their

participation in this session (and other later sessions) was characterized by more confidence and ownership in facilitating discussions to reach the goals they identified.

***First Lesson: Working to "Get Through" the Lesson***

This first episode is a case of TCs working to "get through" the lesson. TCs vaguely identified a goal: Danielle mentioned asking students to explain why a pattern worked (a possible goal), but Katelina did not talk about this idea or specify a goal. TCs were not intentional in facilitating discussion towards a particular goal and looked to the TE as authority on how to lead an instructional activity. This was the second day of the course, and a pair of TCs came prepared to teach a choral count. They had seen the TE model a choral count, and this was the first activity TCs led with students. At this early point in the course, the practice goal for TC learning was *eliciting student thinking*, particularly eliciting patterns in the case of this choral count. As indicated on the TE lesson plans, earlier cycles focused on getting to know students, *eliciting student thinking,* and understanding the instructional activity, while later cycles focused on explicit work around goals (e.g., scaffolding TCs to support students in explaining why a pattern happens) and other practices, such as *using representations* or *orienting students to each other or to mathematical ideas*. TCs' work on the practice of facilitating discussion happened in the context of the TEs moves and goals for TC learning. Thus, we see a focus on eliciting ideas in this first cycle.

**First lesson: Description of the lesson plan.** A planning template and the choral counting task (choral counting by twenty) were provided to the TCs in the previous week. On their planning template, both TCs left the "goal" line blank. They wrote potential questions to ask students including repeating questions and why a pattern works (*explaining a pattern*), but they did not offer any anticipated student explanations for these questions. Danielle tried to explain why a

pattern works in her own words on her lesson plan, but it is unclear how she planned to use this information. This goal of explaining a pattern was also written in the TE lesson plan as a focus for course session (*to use or explain a pattern*).

**First lesson: Description of the rehearsal.** In the twenty-minute rehearsal, TCs seemed anxious: standing in front with excited but nervous smiles, tightly clutching their lesson plans, and talking quickly and with choppy sentences. As they rehearsed, the TE and other TCs played the part of elementary students[15]. TC began by posing a story context:

Katelina: So I'm reading at home 20 minutes every day, right? Cause I love to read all the time. So I'm trying to keep track of how many minutes I read in a month or even more. So, um, right now we're gonna – do you guys remember we did choral counting last time? Yeah? So right now we're going to be starting by twenties. So we're going to start with twenties, and we're going to start by counting by twenties. And so, I want you to – I'll be writing it, but don't talk faster than I can write, because then I'm gonna just lost it, and I'm gonna – cause you guys are just so smart. I won't be able to keep up. So, keep – um, uh, so write – talk – write as I talk, no you talk and I'll, I'll record what you're saying, okay? And then, so, you two are going to talk to each other. You two are going to talk to each other. And you two are going to talk to each other. Okay, so, and before, before we start, let's just remember some of our discussion stems. Can, can someone suggest some of the sentence stems that we can use when we're talking to other people? Yeah, Melissa?

Melissa: I agree with.

Katelina: I agree with. Something else?

TE: That feels a little too general to me. I'm afraid that they are going to say, "What do you mean 'discussion stems?'" I think it would be better if you gave them a suggestion.

Katelina: Okay, okay.

TE: Today when we are listening, um, we might, we want to listen to and try to repeat each other's ideas. I heard so and so say.

Katelina: Okay, okay, good. Okay.

Danielle: Quick logistics question, is it awkward for me to stay standing here? Like, it's not, I shouldn't sit?

TE: No.

Danielle: Okay.

In lines 1-3, Katelina tried to provide a meaningful story context to engage "students" in the count. Before this class meeting, the TCs watched a video where a teacher began with a story context.

[15] I refer to elementary students as *students* and the graduate students in the teacher education program as *teacher candidates* (TCs). During rehearsals, the TE and TCs who were not teaching acted as "students." Thus, in rehearsals, the word "student" refers to TCs and the TE playing the part of an elementary student (i.e. during rehearsal). In other parts of the cycle of enactment and investigation, my use of the word "student" means elementary students (not TCs role playing elementary students).

They also read how story contexts can support students to make sense of the problem and mathematics. Katelina knew that providing a context *could* be useful, but she was less clear about how to carry her context through this choral count to elicit patterns and support students to *explain patterns* (a possible goal for the lesson). In the enactment, Danielle attempted to use this context when asking students to *use a pattern to make a prediction* (another possible goal, see description of the enactment below). Katelina reflected in the debrief that the context was not very useful and that she included it simply because she had seen the teacher in the video use one. Situating problems in "nonmathematical contexts that take time to explain and establish," as evidenced in this excerpt is an issue that can interfere with time spent on mathematical work (Sleep, 2012, p. 948). Using an example about herself (loving to read) functioned to build rapport with students. However, Katelina did not connect this story to the students nor did she return to this original example. Because of this, the story context became a possible idea for why the number twenty was picked rather than a meaningful tool to support student thinking.

In lines 3-13, Katelina attempted to follow several of the elements of the models she experienced in the course: reminding students of previous activities/content, stating the count (by twenties), telling the students to keep pace, indicating who should be partners, providing talk moves, and positioning students competently. Through several starts and revisions, her directions did not come out smoothly. Katelina tried to implement ideas from the course; however, the repetitive and choppy phrasing of her sentences was difficult to follow. Although the wording of the launch of the count and introduction of talk moves may not seem like mathematical work, concise and intentional wording can communicate the goal to students, support student participation, and prevent nonmathematical contexts and ideas from derailing the conversation (Sleep, 2012). The TE directed Katelina to think about this wording (lines 16-18). Katelina was

trying to remember and use moves from class discussions with the TE and from course materials such as readings and TE-provided planning templates. Both TCs in this rehearsal were conscientiously trying to incorporate many different elements of what they had seen or experienced in the models of choral counts and lessons experienced prior to this day.

This excerpt highlights the complexity and difficulty involved when trying out different moves and making in-the-moment decisions about how to use these moves to facilitate discussions. This was their first rehearsal, and TCs worked hard to incorporate the various ideas that they had been discussing. They were working to understand and "get through" the activity. As they grappled with the complexity of how to enact multiple ideas and how to facilitate the instructional activity, ideas from the course (e.g., talk moves) functioned as a checklist of teacher moves to implement rather than moves teachers can choose to use intentionally to work towards particular goals. From this checklist approach, their goal was to use these moves to get through the lesson. Thus, Katelina attempted to incorporate them *all* into her lesson introduction. She, along with her group members, was making sense of how and when to *do* or *enact* practices and ideas that they had talked about and investigated.

After the introduction, the TCs worked on getting the count out. The choral counting was slow, as Katelina struggled to simultaneously hold her lesson plan and write the numbers. Her numbers were not lined up, and she was at the edge of the chart paper when the count reached 200, leaving no space for annotations of patterns. She stopped at 200, asking what students thought the next number would be. This general prompt came from the choral counting planning protocol. When Katelina asked this, Melissa interjected and suggested lining up numbers so the students could see the patterns. As a "student," Melissa was looking at the board for patterns, which were

difficult to see with the unevenly sized and spaced numbers. Being in the student role allowed her to notice and attend the importance of careful representations.

In the following excerpt, the TE added to Melissa's suggestion by offering a specific strategy for spacing and lining up the numbers and emphasized the importance doing this to the support students to see patterns.

TE: Sometimes what people do, to make sure you leave room, you could very faintly put a little dot along the side, then you know you have room. It looks like you will, but you know how you want to line up –

Katelina: Yeah, I was worried, that, yeah. [lifts marker up, then pauses] So like, even before the lesson, but dots where each number is going to go?

TE: Yeah, if you want to. Because that's the thing about counts. Making sure you line up numbers makes all the difference so you can see the patterns.

In lines 30-31, Katelina tried to make sense of the TE's suggestion and think through how she might more neatly record the count. By emphasizing the lining up, the TE was focusing Katelina's attention on the purpose of the recoded count in a choral counting activity (lines 32-33). After the TE's comment, Danielle suggested that Katelina start writing the numbers before and in anticipation of students saying the number to speed up the pace, noting that the TE did that in the previous week. The TE noted that that was unintentional and that she should have waiting for students to say the number.

As Katelina continued the count, they stopped again (when students faltered at 300, mumbling and saying different numbers), and four suggestions were made. First, the TE made suggestions for what to do if students falter or are unsure of a number by modeling possible teacher language:

TE: "If we're adding 20, let's think about how we're figuring that out. What's a way that we can figure out what 20 more is? Check with a partner." Then, "Oh, so I heard Hailey said she's going to count by tens, so let's try that, 280, 290, 300." So you can always pick something out from the turn and talk and know that you're going to be successful with that child if you are worried about putting that kid on the spot.

Sam suggested "rewinding" if they get stuck at 300, going back a few numbers to get back into the rhythm. Katelina tried this suggestion out then paused before asking about patterns. The TE

suggested where to stand and reminded Katelina to position students so everyone could see. Katelina mentioned again that she needed to redo the spacing to leave room for the rest of the count. As they talked about the placement of the numbers on the board and where to stop, Flora suggested to be clear when Katelina wanted to stop so students don't keep counting.

While Melissa's first suggestion (lining up numbers) was consequential to the success of a choral counting activity, the other TC suggestions indicated TCs were attending to issues that were not necessarily central to the task (or ambitious teaching practices) and to naming teacher moves and simplistic understanding of teaching. Danielle's suggestion to write before the students speak was an attempt to address Katelina's slow pacing but would have taken ownership of the count away from student. Sam's suggestion to rewind is useful when students get out of sync or the pacing is off, but it isn't a move to address why students might be faltering at 300 (nor does it support student attention to mathematical ideas). Flora's suggestion of being clear about when to stop was more of an opinion of how to transition from counting to discussion of patterns. Despite their eagerness to contribute, collaborate, and work on practice together, TCs didn't have a big repertoire of classroom experiences and teacher moves to draw from. Thus, their suggestions are more evaluative statements and opinions about teaching decisions, rather than professional judgments based on a vision of high-quality teaching.

After listening to these suggestions and talking about how to get the count out and recorded, Danielle asked the group to look for patterns and to turn and talk about these patterns. She elicited two patterns (*all numbers end in zero* and 111112222233333 in the hundreds place) and underlined them in the count. Thinking out loud, Danielle noted this would be a good time to say, "Why? Why does it go 1111?" Sam added it would have been good to ask this same why question for the first pattern as well. The TE interjected, suggesting they get more patterns out and get students

comfortable with the TCs and with talking in the group. She noted this question may be too early, given it was early in the count and that this lesson was the first lesson with their students.

Danielle elicited and annotated one more pattern (2, 4, 6, 8, 0 in the tens place), then the rehearsal ended in a short two-minute discussion about what questions to ask, how to ask the questions, and when to use repeating and turn and talk moves. The TE noted that if they want to ask a why question, they need to think about what pattern to choose in the moment (i.e. choose based on what patterns have been shared). The group talked about how they could use a Teacher Time Out to pause and collectively make an instructional decision of which pattern to explain (Danielle used a Teacher Time Out in the enactment).

Throughout the rehearsal, the group worked on how to support students to talk and how to make patterns visible to students. Danielle's move to ask "why" certain patterns work indicated emerging attention to an instructional goal, but they did not expand on this in the rehearsal or enactment. TCs were very focused on their moves: how they should stand, what they should say, when they should stop the count, and how they should annotate patterns on the board. Rather than working towards an instructional goal for students, the goal of the lesson became how to complete the choral count.

**First lesson: Description of enactment.** During the twenty-minute enactment of the choral count, TCs focused on how to represent patterns on the board and teacher moves to get students to share ideas. The took up some of the suggestions from the rehearsal but led the choral count much like their rehearsal. They repeatedly used the same teacher questions and talk moves, indicating they might not have been attending to when or how to use these talk moves to intentionally work towards a goal. Their use of a story context (particularly at the end of the activity) was not clearly connected to their effort to have students *use a pattern to make a*

*prediction*. Their choice of moves and questions suggest they were working on understanding the instructional activity and their students.

Katelina began the lesson with a revised version of her introduction. In the following excerpt, we see this introduction and how she took up ideas from the rehearsal.

Katelina: I'm going to sit right here so you can see (directly related to the rehearsal). So I love reading, and I read 20 minutes each day. And so we're going to do, we're going to count by 20s. Do you hear how everybody is in the class today? So we're going to have to whisper count. Whisper count the number, then the next number. I wanna hear all your voices, but we're just whispering. I wanna see all your mouths moving. So we're going to count by 20s, because I read every day 20 minutes. And then, when we're talking with each other, so, let's see. [looks at the group of students] Maybe you two can talk to each other, and then you two, Thahn and Arianna, you guys are going to talk to each other. And when you're talking, remember to listen, right? And you're going to repeat your partner's ideas, and if you agree, say I agree, and if you disagree say why. Okay. You guys are going to do great, because you're all geniuses! I'll write, and as you speak, I'll write, so don't speak too quickly. So we're going to start at 20 and count by twenties. [Writes "+20" in the corner at "20" in the first spot of the count]

In this minute-long introduction, Katelina was working on introducing the count (lines 40-41 and 51) and setting up expectations for how to talk and participate (lines 41-50). She took up suggestions from the rehearsal, including checking in to make sure students could see (line 39), providing sentence stems (line 48-49), and recording the count (+20) on the board (line 51-52). In an attempt to set up participation and talk expectations, Katelina gave many directions and explanations of talk moves. Just like in the rehearsal Katelina was working hard to incorporate what she had been learning in class and make sense of how to take up these ideas in her practice.

After this introduction, Katelina spent another minute asking students about the next three numbers. In the next excerpt, we see a problem of practice (responding to student error) arise when Katelina asks for the third number.

Katelina: Can you think, what the first number? And give me a thumbs up when you have it. So what's the next number? [Students: 40!] What's the next number?
Students: (loudly): 80!
Other students (softly): 60.
Katelina (noticing two students said 80): Okay, so let's think about it [points to 20] so the next number is going to be 40 [points to the second spot] then the next number is going to be…?
One student: 80!

| | |
|---|---|
| Other students: 60! | 61 |
| Katelina (to one of the students that said 60): And why is that? | 62 |
| Student: Because we're counting by twenties. | 63 |
| Katelina: Yeah! We're counting by twenties. | 64 |

By having students think about the first three numbers, Katelina used a common strategy for starting choral counts. This move is intended to prepare students to participate in the count and think about the first few numbers. The moment when two students said "80" (line 55) was not inconsequential (later in the activity students struggled to count by 20s). This exchange showed how a problem of practice (responding to student error) was opened up in one of the first interactions with students. Katelina hadn't anticipated how to respond to this kind of error and prompted students to "think about it" (lines 57). She made the decision to call on a student who said the correct answer and asked that student why it was 60 (line 62), but she did not check in with the students who said 80.

After this exchange, the group started counting, and some of the students started counting by tens. The TE paused the count and asked students what they were counting by (Students: twenty) and what numbers get skipped (every other ten). In the protocol, one of the suggested moves to support student to learn from peers and hear different strategies for counting was, "How are you getting the next number?" The TCs did not ask this prompt, but the TE suggested this move in a later rehearsal of choral counting. Katelina was able to start the count and record the number more neatly, but the TE had to interject to support students to figure out what number comes next. In that sense, Katelina did not support the goal of the first part of choral counting, which was to provide a way for students to figure out what comes next.

The count continued, and Katelina took up the suggestion for spacing and lining up numbers and the TE's suggestions of how to address when students falter and going further into the count before eliciting patterns. Five minutes into the count, Danielle gave directions to think

about what patterns students noticed. In the next ten minutes, they elicited four patterns (2, 4, 6, 8, 0 pattern in tens place, +200 across columns, always having zero in the ones place, and adding 20 going down). The TE made three suggestions during this time: 1) Using a repeating move after a student shared a pattern (this resulted in TCs asking students to use a repeating move after every pattern); 2) asking students to turn to their peers (not the board) when sharing their pattern (something they talked about in the debriefing discussion); and 3) asking students if the second pattern shared (+200 across columns) happened in other places (this resulted in a student making a prediction for 600 in the last row of the third column). With each pattern, TCs asked students to come to the board to show their pattern, underlined the pattern they saw, asked other students to repeat this pattern, and then repeated what the students said.

TCs used the repeating talk move and "what (other) patterns do you notice?" multiple times during the ten minutes of eliciting strategies. This indicated they were working to use talk moves to increase student talk. It also wasn't clear if they had a stopping point for when to close the lesson or transition beyond sharing patterns. Intentional discussion facilitation includes knowing what to listen for (related to the goal) and responding appropriately when this happens. The formulaic sharing structure and repeatedly asking for additional patterns indicated TCs were focused on increasing talk and number of shared patterns, not necessarily listening for particular patterns to use or explain (the possible goals for the activity).

In the last four minutes, the TE reminded them that they wanted to ask a *why* question. In the following exchange, they talk about what question to ask but ultimately decide to have students use patterns to make a prediction based on time and what a student did earlier.

TE: Do you guys want to ask a why question? Do you feel like it? Or where do you– which one?

Danielle: Teacher time out, I'm thinking…across the columns? [motioning along the bottom row]

TE: That's pretty– that takes some time. I'm wondering if they'll get, why are they all ending in zero? Should we try that one? 70 71
Danielle: Yeah, that sounds good, because we still need to predict. 72
TE: Oh, well you still have that do. [Danielle: Yeah, the predictions.] Well, do you want to do that instead? Use one of the patterns? [Danielle nods] Why don't you do that? Use the patterns to make a prediction. 73 74

In line 67 Danielle suggested asking students to explain why there is always a zero in the ones place. Thinking about the limited time remaining, the TE suggested a different pattern that would take less time to explain. When Danielle said that they still need to predict (line 71), the TE (realizing that they would not have time to explain a pattern *and* use patterns to make predictions) suggested using the remaining time to make a prediction. The TCs were simultaneously trying to address multiple goals within the choral count (noticing, explaining, *and* using patterns) while not being clear about mathematically what they were trying to accomplish (e.g., specifying the goal of explaining why there are zeros with the idea of multiples of 20). In the following excerpt, Danielle asked students to use a pattern to make a prediction. She framed this question in Katelina's "reading 20 minutes per day" context.

Danielle: Now, we want to see. Katelina reads 20 minutes a day, right? [she draws a box in the fifth row of the fourth column, the 35th spot] How many minutes do you think the total would be after [she pauses, pointing to and counting the columns by 10 to herself] 35, no 45 days? 45 days, right? 75 76 77 78
Katelina: So right here, what would be the total in this box right here? And use your patterns that you've already said. 79 80

In lines 75-78, Danielle made a connection back to the reading context. This context wasn't framed in a way that was helpful for the goal of using patterns to make predictions and didn't help students make sense of the mathematical task. It could have been useful for a different goal[16] around multiplicative relationships and making sense of groups of twenty (a later rehearsal worked on a goal with this type of multiplicative reasoning). Additionally, Danielle miscounted the groups of

[16] E.g. "I love to read 20 minutes each day, and I was thinking, how could I keep track of the number of minutes each day? Let's count by 20s so we can add up the number of minutes each day!"

20, and her box was predicting the number of minutes for the 35th day (not the 45th spot). In lines 79-80, Katelina reminded students of the goal: *to use patterns*. This was the only move she used that seemed linked to a possible goal.

In the last two and a half minutes, TCs asked students to predict how many minutes they would have cumulatively read by day 45 and drew a box for that place on the choral count chart. One student shared 700, and the other students agreed. When Danielle asked students why they thought it was 700, the TE reminded them of the limited time remaining (TE: And this is the last thing we will say. That's all we have time for today!). In the final two minutes, three students shared how they used different patterns to predict the 700, and Danielle pointed to numbers and spaces on the board as students shared. In the following excerpt, we see the final remarks made after these students shared how they used patterns.

| | |
|---|---|
| TE: That's all we have time for. Look at all the things you noticed about those numbers. | 81 |
| Katelina: Thank you so much, you guys. | 82 |
| Danielle: You did a great job! | 83 |
| TE: You found patterns and you used them to make predictions. That's good mathematical work that you did today. | 84<br>85 |

In lines 81-82, the TE interjected to remind TCs of the time and prompt them to close the activity. The TCs' moves to close were positive compliments (lines 83-84), rather than moves to facilitate student voice and highlight key mathematical ideas based on the goal. In productive mathematical discussions, closures are integral to explicate a mathematical claim, generalization, or big idea (connected to the goal) and to support student voice and shared meaning around this idea (Chapin et al., 2013; Kazemi & Hintz, 2014; Shaughnessy et al., 2019; Smith & Stein, 2011, etc.). The TE modeled a phrasing for an abbreviated closure based on the goal of using patterns to make predictions (lines 85-86). The lack of time management and lack of meaningful closure are common novice challenges. This reflected a simplistic view of teaching and indicated their attention was on getting through the activity and using moves to increase student talk and sharing.

During this first enactment of a choral counting activity, TCs asked students to share patterns and asked other students to repeat these patterns, something that they rehearsed and discussed. One of the students started to make a prediction using one of the stated patterns. Using a pattern was one possible focus according to the TE lesson plan (not the TC lesson plans), but TCs had not planned for or rehearsed this. The TE suggested they use one of the patterns to predict a number. This is an example of how the TE had a possible instructional goal in mind, but the TCs may have been more focused on getting through the activity. The TCs asked students to use the patterns to make predictions, but they did not ask students any "why" questions nor did they ask students to explain any patterns (the possible goal they discussed in the rehearsal). They used moves from their lesson plans and rehearsals in service of completing the instructional activity. Without a clearly articulated instructional goal, their goal became completion of the steps of the instructional activity, rather than using the instructional activity as a structure for facilitating discussions that intentionally moved towards an instructional goal.

**First lesson: Description of the debriefing conversation.** In the debrief, TCs talked about how to pay attention to and understand what students say. However, the TE explicitly focused on attending to and making sense of student thinking, highlighting this idea in her framing for the day and discussion during the rehearsals and debriefing conversation. In early cycles, the TE deliberately focused on shifting TC attention to students and focusing on eliciting student thinking. Understanding and attending to student thinking is different than how to respond to these ideas and what to do. Trying to make sense of student thinking is an important foundational step in learning to use a goal to guide what to do with student responses (Sleep & Boerst, 2012). In terms of the instructional goal, they talked about students' abilities to find and explain patterns. TCs talked about how they were nervous but glad to have the rehearsal to figure out things like lining

up place value on the board. They talked about paying attention to what students say and the differing purposes of using a story context to support thinking around what the numbers mean (versus using it as just an attention grabber). The group debriefed finding patterns, what each student noticed, and how to make sure that you understand what students are sharing.

**Analysis of the first lesson: Working to "get through" the lesson.** In this first cycle of enactment and investigation, the TCs were actively making sense of the instructional activity of choral counting and working to "get through" their lesson. In the rehearsal, they worked on launching their choral count and making sense of the instructional activity's structure. Problems of practice focus on the structure of the activity and teacher moves: *What needs to be said to get started? Where should TCs stand? How can the numbers be lined up in columns? What questions should be asked and when? What should be done next in the count?* At this early point of the quarter, TCs were not quite facilitating discussion[17]. Their first enactments reflected efforts to understand the instructional activities, their students, and moves to increase talk. Thus, they were learning to facilitate student talk and supporting students to show-and-tell different ideas[18]. As these more basic components of discussion-based practices became routinized (e.g., later lessons), TCs attended to more complex aspects of practice, including using moves intentionally to facilitate discussion towards a goal (Janssen et al., 2015; Lampert et al., 2010). The group built shared understanding around the instructional activities, which served as structures for recomposing practice and focusing on more ambitious aspects of practice (Lampert et al., 2010).

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[17] Per the literature review, productive mathematical discussion includes discourse-practices that are responsive to student thinking in relation to mathematical goals and position all students competently to engage in mathematical practices and to develop identities as mathematicians (Chapin et al., 2013; Kazemi & Hintz, 2014; Shaughnessy et al., 2019; Smith & Stein, 2011, etc.).

[18] This type of show-and-tell talk does not support mathematical proficiency and reasoning (Chapin et al., 2013; Franke et al., 2007; National Council of Teachers of Mathematics, 2014).

The routine of choral counting provides opportunities for different instructional goals: "examine number relationships, build place-value understanding, develop computational skills and fluency, and make mathematical justifications and generalizations" (Chan Turrou et al., 2017). While Danielle loosely named the goal of *explaining why a pattern works* (written in her lesson plan), she did not follow through with this goal during the rehearsal or enactment, and her teaching partner, Katelina, did not explicitly share this goal in her lesson plan, rehearsal, and the way she facilitated discussion. The goal of explaining why a pattern works was related to the focus outlined in the TE's lesson plan for that course session (*to use or explain a pattern*).

When a student predicted a number during the enactment, their goal switched to the *using a pattern to predict a number*. While shifting the goal can be a responsive move based on student thinking and the intended mathematics (Sleep, 2009), this shift in focus indicates they may not have had a clear idea of which goal they were working towards, given the various possible goals that could be facilitated through choral counting. Additionally, the content of their lesson plans and discussion about goals indicated they were not clear about the kinds of explanations or predictions they wanted students to do (or what counted as an explanation, e.g., Yackel, 2002).

In the lesson plan and rehearsal, Katelina and Danielle focused on teacher moves and logistical issues. Discrete moves and logistical issues by themselves are not ambitious teaching practices; however, they are part of the work of teaching. More experienced teachers routinize these discrete moves and attention to logistical issues, leaving space to attend to student thinking and more complex practices responding to student thinking and the instructional goal. In contrast, novices need opportunities to begin routinizing these moves and to work through these issues (Lampert & Grazaini, 2009; Lampert et al., 2013). Preparing for and practicing these smaller moves and logistical issues (e.g., trying out the wording of instructions and questions, managing

materials and board space, considering where to stand so that students can see, etc.) can increase instructional time spent on mathematical work and create a learning space for discussion (Sleep, 2012). It would be concerning if this were the *end* of the trajectory of TC learning in a methods course. However, as a first enactment in teaching a lesson, it is reasonable that TCs were attending to teacher moves and logistical issues arising from leading the choral count.

Based on previous studies of teacher learning, the focus on self and teacher actions is not surprising. In this first enactment, they were experiencing a kind of "practise-shock" and were making sense of what they (as the teachers) needed to do to lead the choral count (Timperley, 2013). The TCs *wanted* to facilitate a productive mathematical discussion. They read and talked about using talk moves and "discussion stems" to facilitate discussions (Chapin et al., 2013). However, they were unsure of how to facilitate discussion and use these moves, a problem of enactment and professional judgment and knowledge in practice (Kennedy, 1999; Kazemi & Wage, 2015). In this first cycle, they attended to how to "get through" the lesson rather than how to facilitate discussion towards an instructional goal.

At this early stage in the course, the TCs were trying on the activity and including a range of ideas in their lesson. The focus on their own moves, lack of follow through for the story context, and forgotten goal of explaining why a pattern works is not surprising. In this early point of their program (and without a deep base of teaching experience to draw from), TCs were uncertain about what students might say, how to articulate instructional goals, and how they should steer the conversation towards these goals. It is reasonable that their focus was figuring out how to get through the activity and how to elicit student ideas. Both are necessary for learning to facilitate discussions productively. *Eliciting student thinking* was one of the practices of ambitious teaching

outlined in the syllabus and was an aspect of discussion facilitation they worked on in increasing complexity throughout the course.

Trying out new practices while simultaneously making this practice public is challenging work. Teaching has long been a profession of isolation and making one's practice public, while necessary for teacher learning, can feel intimidating and vulnerable (Bryk et al., 2010; Timperley, 2013). Learning to teach, especially when they are shifting discourses, dispositions, and long held beliefs about teaching, is an emotional process for TCs (Timperley, 2013). It is not surprising that TCs seemed nervous about teaching the lesson and doing this work in front of their peers. The group was establishing their norms for working on practice together and creating shared frameworks for understanding the practice of facilitating discussion and choral counting as an activity. As they became more comfortable teaching in front of their peers, they made a shift from focusing on themselves and understanding teaching as a checklist of moves/steps to focusing on student thinking and facilitating discussions intentionally to reach instructional goals. We see the beginning of this shift in the following episode.

***Mid-course Lesson: TCs Specifying Goals and Needing Support***

In the first few class sessions, TCs led activities that were intended to be open strategy sharing discussions. As TCs became more familiar with the instructional activities, their students, and how to teach and elicit ideas in their small groups, the TE prompted them to think about how their facilitation could target particular mathematical ideas. In contrast to discussing a broad range of strategies or ideas (e.g., open strategy sharing), during the fourth class session (October 21) TCs began working on more "targeted discussions," where the purpose of the discussion was to "target" (or highlight) a particular mathematical idea (Kazemi & Hintz, 2014). The class session below (October 28) was the TCs' second attempt to facilitate discussions to highlight particular

mathematical content. Unlike the first class session, where only one lesson was taught, there was a second activity during this day, which I have excluded to keep this section concise.

The instructional activity was a choral count by 2 starting at two. The focus of the lesson was supporting students to construct mathematical explanations of why particular patterns existed. TCs had read an excerpt from Kazemi and Hintz's (2014) *Intentional Talk*, which discussed different ways to structure productive discussions based on the purpose of the discussion. This choral count was intended to have a more focused goal (*constructing mathematical explanations*) than earlier open strategy sharing discussions. In addition to reading the book excerpt, TCs also watched videos on choral counts. Throughout the rehearsal, Sam continued to reference the teacher moves from this video.

Compared to the first enactment, TCs seemed more confident and relaxed teaching in front of a group and eliciting student ideas. They specified their goal and the mathematical ideas they wanted to highlight during the rehearsal. There were several instances where they talked about their goal and possible moves based on this goal. However, they looked to the TE for support on how to reach this goal. While they seemed more confident facilitating discussion and stating the lesson's goal, they were less sure of how to address the goal through this discussion facilitation. They highlighted two patterns in the rehearsal that they wanted to address but were unsure how to respond to student ideas to support students to explain one of these patterns during the enactment. The TE provided modeling of how to elicit student thinking around these ideas.

**Mid-course lesson: Description of lesson plans.** In the lesson plan, the stated mathematical goal was still vague: "Students identify patterns and explain their thinking." For the closure, Melissa wrote that she would state how students discovered different patterns and explained their thinking. She used the lesson planning template and wrote a script to follow that

included repeating and add on moves, when to stop, and teacher questions with anticipated student responses. Throughout the rehearsal, she wrote revisions to this template and her script.

**Mid-course lesson: Description of the rehearsal.** Before the rehearsal started, the TE asked if there were particular patterns they wanted to students to notice. This move pressed TCs to not just acknowledge that they want students to explain (something they wrote in their lesson plans) but also to consider *what* they want students to explain. Sam shared several ideas she hoped students would notice and explain. Later in the rehearsal and in the enactment, she tried to highlight these ideas, particularly the idea that there were five groups of two in each column.

TE: Remember how we, we're trying to work on getting kids to think about why those patterns are occurring, right? So this'll be–
Sam: And getting them to explain. [inaudible]
TE: Getting them to explain. Is there a particular pattern you think you want them to explain?
Sam: Well I'm hoping – yes. That they, um, notice that the twos, er, the ones, are the same digits are across columns.
TE: Okay.
Melissa: And to see that it's a reoccurring pattern.
Sam: And also – that's one of them, and – the explanation could be, because there's a distance of ten, and if you add ten, then you're only adding one ten and zero ones.
TE: And that's why the ones – [Sam inaudible comment]. Something like that. That's why the ones digit, okay.
Sam: And then, uh, the distance of ten between the column is, um, between the columns, because you can use that to figure out other numbers in the count, and we can also get a little bit at multiplication with that. So if we have, um, five rows, um, of fifty, how many more rows to get a hundred? Or something like that? [shrugs her shoulders and looks at the TE] I dunno.
TE: Okay. Alright.
Sam: Something like that. So those are the two patterns that I think they'll say, and that I think that we're able to push at and that they might be able to tell us a little more about.
TE: Alright.
Sam: Oh! Or, and the reason for ten, also, five times two. [points to the columns] Five times two.
TE: So there's five counts of two.
Sam: "How many twos are in one ten?" would be another question we could push at.
TE: Which I think we should also be able to do together if they look at you. I think you're right. I think this one we could probably provide a little support if we need to. Alright, this is great! I'm excited!

The TE prompted TCs to state *what* patterns they hoped students would share and to begin thinking about their goal of explaining patterns (lines 4-5). In the excerpt, Sam named two related ideas: the ones digit stays the same across columns (lines 6-7) and the distance of ten (or group of ten)

between columns (lines 10-11 and lines 14-15). Although the patterns of "plus ten" and the repeating digits across columns in the ones place were both identified in her lesson plan, there wasn't an *explanation* for these patterns. This lack of planning for an explanation could have contributed to the clunky wording of her explanation as she thought out loud about explaining why the pattern worked. In their lesson plans, both TCs wrote that they wanted students to explain patterns, but the TE's question during rehearsal (lines 4-5) prompted them to identify which patterns to explain and what the explanation might be. In the rehearsal, Sam was attending to mathematics of the repeating digits across columns in the ones place and tried to connect how the distance of ten provided an explanation for this pattern (lines 10-11, 14-17, 24-25, and 27).

After this discussion of patterns and explanations, the rehearsal began. In the excerpt below, Melissa provided an introduction about how they (the students) had been counting bigger numbers and were going to "step back" and count by smaller numbers. She read their goal from her lesson plan and asked students to repeat. The TE paused her, noting their lengthy introduction might be a lot for students to repeat (lines 48-51).

Melissa: The past few weeks we've been choral counting with big numbers. [reading her lesson plan] I want to step back and do one we are all familiar with. Our goal is to see how many patterns we can find and also use your teacher voices with the patterns that you do find.
[looks down at her lesson plan, then motions to Sam to start]
Sam: And then can someone repeat –
Melissa: Yeah, I was going to say –
Sam: What Melissa just said? What are we counting by today?
Danielle: She didn't say.
Sam: She didn't say? (looking back at Melissa) Oh, hah! [the group laughs] Well, (looking playfully at the group) can you guess? [they laugh again]
Melissa: Our goal is to see –
TE: She said what our goal is.
Sam: I see. Okay.
Melissa: I can do that part. Can anyone repeat what I just said? Our goal today? It's to explain our thinking and use teacher voice.
Sam: Okay.
TE: I think that, um, you said a lot. So if all of a sudden you asked me to repeat that, I might be feeling like, "What is she asking me to do?" So I'm going to share something about what we're doing today and I'm going to check to see if that makes sense to you. So give them a heads up.
Sam (nodding in agreement): Yeah yeah.

TE: So give them a heads up if you want them to repeat the goal. 53

TCs worked on launching the choral count and attending to the goal. Melissa stated the goal (lines 32-34), and Sam asked students to repeat both the goal and what they are counting by (lines 36 and 38). There was a moment of confusion as Sam realized they hadn't introduced what they were counting by (counting by 2s starting at 2). After the group talked about the wording of introduction and repeating the goal, the TE asked the TCs why they want students to repeat the goal (line 54). In the excerpt below, Sam explains how she saw the repeating move in a video and wanted to try it.

TE: Why do you want them to repeat the goal? 54
Sam: Um, I saw it in a video I watched (raises her eyebrows playfully). And it was just 55
good to see and it was good to, uh, get – she had them repeat – well, they were counting 56
by two hundreds, so it's a little bit bigger, [TE: Yeah.] but she had them repeat back, 57
um, the number, and then like a separate – er, um, she had like, "We're counting by 58
two hundreds today, what are we counting by? [TE: Yeah.] Two hundreds. Today 59
we're going to be looking for patterns in numbers. [TE: Yeah.] Can somebody repeat, 60
what are we looking for? 61
TE: Yes, I know which video you saw. That's why you want to do – Ms. Todds. 62
Sam: Yeah! That girl! And it was awesome. So a lot of what's in this, I just pulled. 63
TE: That's great. I'm glad you're going to try it. You see how she doesn't say very much? 64
[Sam: Yeah, yeah.] Cause you said a lot. Right? So repeating that might be quite 65
demanding on me. So, break it up a little bit more like she did if you want to do that. 66
Alright? 67
Melissa: Yeah. [writes notes on her lesson plan] 68
TE: So, "We're going to look for patterns, and we're going to try to explain them today." 69
Sam: Yeah, and she breaks it into two, so like, "We're counting by twos, can somebody 70
tell me what we are counting by?" And then come back. 71
TE: Okay, do you want to try that again? Just that, just that piece of it? 72
Melissa: Yeah. [continues to write notes on her lesson plan for a minute] Today we are 73
going to look for patterns and explain them. [Looks at TE and nods for approval] 74
Yeah? Then ask…? 75
TE: And you can say, "What will we do with our patterns today?" You know? And you 76
can like, target the repeating. [Melissa writes more notes on her lesson plan] The 77
follow up can be – [pauses]. 78
Melissa (turns to the students): So what will we do with our patterns? 79
TCs (acting as students): Explain them! 80

By asking students to repeat, the TCs were thinking about how to get students to attend to the goal and the number they were skip counting. They had seen a teacher use a repeating move in the video but weren't exactly sure how to use this repeating move in a way that students can easily

understand. The repeating moves were written in their lesson plans, but their introduction was very verbose and difficult to follow in the rehearsal.

When asked why she was having students repeat (line 54), Sam wasn't sure how to respond and gave her reasoning: she saw it in a video, and it was "good to see" (line 55-56). The TE helped them think about the language and phrasing they could use to be clear to students. In lines 64-66, the TE pointed out how the teacher in the video did not speak as much as they did. She suggested breaking up their introduction and gave a sample prompt to students in lines 69.

When they tried the introduction again, Melissa repeated the TE's suggested prompt (line 73-75) and looked to the TE for approval. The TE responded (lines 76-77) by giving another possible question. Melissa wrote this wording down verbatim on her lesson plan, suggesting her view of the TE as the expert. Throughout this rehearsal, Melissa was clutching her lesson plan and reading her pre-thought-out prompts to ask students. She was thinking about their goal of asking students to explain; however, she continued to look at the TE to seek approval of her moves and teaching decisions. The excerpt is an example of the TCs, particularly Melissa at the end of the excerpt, looking to the TE for suggestions of what to do and confirmation of new moves she was trying out.

After Melissa finished her revised version of the introduction, Sam told students they were going to count by 2s and asked a student to repeat. They continued with the count until they reached eighteen. Melissa asked students to think of patterns. Sam talked about how the teacher in the video asked for patterns early on and how they thought it would be "good to do" because of their goal of explaining patterns. In the excerpt below, the TE provided background about what kinds of questions to ask depending on where they were in the count.

Sam: And here, that video, she also stopped, like really early on, and had them identify 81
patterns. And I guess we thought that would be good to do since our objective is to 82
get them to explain their patterns. So maybe stopping early on? 83

TE: It's really – um, for finding patterns you actually want more of the count out. So I would revise that question, "Does it look like any patterns are starting?" [Sam: Okay] Right? Because you, uh, kinda sometimes need more of the count out to say more. 84 85 86 87
Sam: Uh huh, yeah, so like, why do you think in that video she stopped early? 88

Sam was explicitly thinking about the goal (lines 82-83) and mimicking moves she seen either in the video or modeled by the TE. She was still unfamiliar with the purpose of the different components and moves in the activity structure of choral counts. For example, she knew that identifying patterns is a component of choral counts (lines 81-82), but she didn't realize they had not counted enough to support students to identify patterns in this count. Thus, the TE provided feedback to revise the question, given that it may be too early to identify patterns (lines 84-87). This provided more explanation about the choral counting activity and when and how to use particular questions or teacher moves.

After Sam's question (line 88), the TE talked about the video and the groups discussed possible phrasings of questions. Before moving on, the TE checked in with the TCs by asking them what question they would pose. In the excerpt below, the TCs were still unsure about what question to use and looked to the TE for more direction. Sam then repeated the TE's suggestion.

TE: So how are you going to phrase that question to us if you stop there? 89
Melissa: Does it look like any patterns are starting? [looks to the TE for approval] 90
Katelina: Maybe – [trails off] 91
Sam: Maybe can you predict the next number, uh, using a pattern, er? 92
TE: Or, how are you figuring out the next number? 93
Sam (takes up the TE's suggested question): How are you figuring out the next number? 94
TE: Okay, that might say, "I'm noticing a pattern" or "I'm going 19, 20." [holds up fingers to count two] Or, "I know my twos." 95 96
Sam: How are you figuring out the next number. 97
TE: Yeah. 98

Melissa was still unsure and looked to the TE for approval (line 90). Sam followed up by asking a question (line 92) that didn't quite capture the feedback the TE gave. The TE gave another possible question (line 93) and explained what students might do (lines 95-96). Unlike the first day, where TCs depended completely on the TE for suggestions, in this mid-course session, TCs thought aloud

about their ideas, the goal, and what they could ask. However, they ultimately looked to the TE for affirmation or for an alternative move.

They continued counting from 18 to 34. Sam noticed and commented that it was difficult to write on the board. They stopped at fifty, and Sam asked students to look at the count and turn and talk. She did not prompt students to explain the patterns but did listen in to select a student who noticed one of the patterns she was hoping to discuss. She purposefully called on this student and, after checking in with the TE, annotated the board to show the increase by tens.

The TE mentioned that students sometimes only see patterns in one spot and suggested asking, "Is that the only place that's happening?"[19] After annotating the pattern, they elicited and talked about a few other patterns. Melissa pointed out a diagonal pattern, and Sam asked how they could explain it. In the excerpt below, the TE did a think aloud about why the pattern worked to support TCs in explaining the pattern.

| | |
|---|---|
| TE: Let me just think about this out loud with you for just a second. So it's ten when you go across, right? So, you know that it's, from two to twelve it's ten, so that means from four to fourteen it's ten, but from two to fourteen, how many counts is that going to be? | 99<br>100<br>101<br>102 |
| Sam: Twelve, right? [TCs talk simultaneously about how it is twelve more] | 103 |
| TE: Right? It's, its twelve, so that means that – what's happening to the ones digit? [TC inaudible] Five? And is that what's happening in the pattern? Yeah. | 104<br>105 |
| Sam: So if they were to figure it out, would you just annotate it plus twelve? | 106 |
| TE: Yeah. Plus twelve. Yeah. I mean that you need a follow up question. How much bigger is the number getting. | 107<br>108 |

Although this pattern did not get elicited in the classroom visit, the TE was thinking aloud and modeled an explanation about why the pattern worked, which was the goal of their lesson. Although Melissa elicited this pattern, it was the TE who made the move to explain why it worked and to think aloud with the TCs. Even though she had her own idea, Sam checked in with the TE about how to annotate it (line 106).

[19] In the enactment, this is exactly what happened. Students only saw the "plus ten" happening on the bottom row, and although Sam took up the TE's suggestion, students still struggled to see it happening in other rows.

After eliciting a couple more patterns, Sam returned to the original pattern of 10, 20, 30, 40, 50 that Katelina (acting the part of her buddy, Arianna) shared and asked her why her pattern was happening. This pattern was a pattern Sam hoped students would see, and asking "why" aligned with their goal of having students explain patterns. Sam referenced the teacher from the video again and asked the group what they thought about the possible question of "What do we think about Arianna's pattern?"

The TCs were considering how to support student participation and to position them competently. They were also thinking about when to use talk moves (e.g., turn and talk) purposefully, unlike the first lesson where the purposes of using moves were less clear. The TE acknowledged their efforts to do this, then commented on how to support student thinking around this particular mathematical idea. The TE modeled ways to pose questions to students around this pattern and suggested, "Why don't you think about why that might be happening?" Sam took up her suggestion and, jumping back into the rehearsal, used the same phrasing. The group continued to discuss how they might support students to explain the pattern. Sam asked how she should rephrase, and the TE suggested following up with the open ended "why questions" by asking more specific questions such as "Would it help us to think about how many twos are in each column?"

During this discussion, the TE pointed out that they want to work on mathematical explanation not just noticing, and TCs asked the TE about these explanations. The TCs offered suggestions for how to phrase the teacher questions, but ultimately looked to the TE for her suggestion. They ended the rehearsal by talking about what their particular students might say. Tina asked if students should have something to write on, and the TE suggested that they could write out the count on a whiteboard or separate sheet of paper. The rehearsal ended with TCs saying that having students explain is difficult.

During the rehearsal, the TE supported the TCs to anticipate patterns and to work on mathematical explanations. When moving to the enactment, TCs tried to take up some of these ideas. However, the TE needed to remind them to ask students to *explain* the pattern. Lessons in the middle of the course were characterized in this way: TCs were beginning to identify instructional goals and attempted to facilitate discussions to reach these goals but were still dependent on the TE.

**Mid-course lesson: Description of the enactment.** In this enactment, the TE was particularly active in guiding the conversation and co-facilitating the discussion, modeling teacher moves and questions to use, and thinking aloud about how to press students to explain a pattern. The TE intervened multiple times, especially when the TCs looked perplexed at how to support students to provide an explanation. Throughout the lesson, the TE directly asked questions to the students and thought aloud to TCs about how to do this. In other lessons, the TE played a less intervening role, making suggestions or providing guidance when elicited by the TCs.

The enactment began with Melissa stating the goal: "Today we're going to look for patterns and explain our thinking. What are we going to do today?" They led students in counting by twos, and after stopping at 50, they passed out papers with the count so students could point as they talked. The introduction and counting from 2 to 50 took 3 minutes. This was an improvement in TCs' efficiency and use of time to introduce and get the count started. After the paper sheets of the count were passed out, Sam asked Sahra to share her pattern. In the excerpt below, the TCs elicit the idea that each column increases by ten, something they discussed in the rehearsal.

Sam: Sahra noticed a pattern, and I've asked her to share it for us. So Sahra, what was the pattern that you noticed? 109 110
Sahra: [inaudible] 111
Sam: So what – so each column is increasing by how many? 112
Sahra: Ten. 113
Sam: By ten. So like this. (writes "+10" on the board) Does that happen anywhere else or just the bottom row? 114 115
Sahra: (inaudible) 116

Sam: Just the bottom row? (draws arcs and "+10") connecting the numbers in the bottom row) 117 118
Melissa: Let's try with this row. (Points to the second from the bottom) Does the same thing happen? 119 120
Students: No. 121
Student: Going up by eights. 122
Sam: By eights? (starts to underline the "8" in the ones digits in the row) 123
Melissa: So you're saying it's going by eights? (Points to the "8" in the ones digit in the row) So how about this? (Points to the eight and the 18) Eight, then 18. Does it also go up by ten? 124 125 126
Thahn: Yeah. 127
Sam: Yeah? So it's happening here too? (draws the arcs and "+10" on this row as well) 128
TE: Let's count that! 129
Everyone: Eight, 18, 28, 38, 48. 130
TE/Sam(simultaneously): What would be the next one? 131
Students: 58! 132

In lines 109-110, Sam called on a student whose pattern connected to what the TCs had discussed in the rehearsal: each column increased by ten. To highlight the idea of increasing by ten, Sam took up a question modeled earlier by the TE: "Does that happen anywhere else?" (lines 114-115). She was trying to support students to identify that the pattern was in other rows (an idea they mentioned in the rehearsal that they wanted to have students explain). However, the TCs struggled to support students to identify that the pattern happened in other places. When Melissa asked students if the "same thing" from the bottom row happened in other rows (lines 119-120), students said no (line 121). It is unclear if the students were noticing that the bottom row is skip counting by tens or if they were noticing each column increases by ten.

After eliciting Sahra's strategy and writing "58," Sam elicited two more ideas (with the TE providing help interpreting the student's patterns), then asked if there were any more patterns (line 133, below). In the excerpt below, the TE suggested they try explaining the pattern (their instructional goal).

Sam: Any other patterns that we don't have up there? 133
TE: Should we try to give it a shot to explain one of these patterns? 134
Sam: Yeah! Let's give it a shot. I was thinking plus ten? What do you think? (looking at the TE) 135 136
TE: The plus ten? 137
Sam: Or the pattern, 2, 4, 6, 8, zero? 138
TE: I know. Why does it go to zero after eight? That could be something else too. I don't know do you want to try that one first? 139 140

Sam: Yeah! So, (to the kids) this was a little bit of Arianna and Inga's pattern. What do you guys think about Inga and Arianna's pattern? And her pattern was that in each column, the ones digit goes 2, 4, 6, 8, 0. What do you think about that? Do you want a second to turn and talk? 141 142 143 144
TE: Yeah, let's think about that. 145
Sam: Let's go back to our buddies and turn and talk. 146

In the rehearsal, TCs asked students to explain after sharing just one pattern, but the TE suggested that TCs get a few patterns out before having students try to explain. This suggestion was an attempt to help TCs elicit more ideas and patterns students could choose from and explain. TCs took up this suggestion. In this part of the enactment, they had elicited a few patterns, including the one that they were hoping to have students explain (plus ten). In line 134, the TE suggested asking students to explain. Sam quickly identified the "plus ten" idea and turned to the TE for confirmation of her choice (lines 135-136). Sam continued to consider other options and again looked to the TE for confirmation for the second pattern of 2, 4, 6, 8, 0 (line 138).

As they thought aloud together, the TE put the choice back on Sam (lines 139-140). She provided a possible question for students ("Why does it go to zero after eight?" in line 139) that was connected to the second pattern Sam has suggested (line 138). This question could have supported students to think about why each column increases by ten, the idea discussed in the rehearsal and the first idea Sam suggested (line 135). The TE was both validating Sam as she thought aloud and providing a question that pressed students to consider ideas relevant to their goal and ideas discussed during the rehearsal. In lines 141-144, Sam skillfully took up the TE's suggestion, posing the question to students and asking them to turn and talk about the pattern.

As students did a turn and talk, Sam, Melissa, and the TE listened in and asked students questions including, "Why does it end in zero?" and "Why does it happen in every single column?" Their questions indicate that they were working to listen to students and facilitating the discussion towards their goal. Melissa listened to Thahn and then asked him to share his idea to the whole group. In the excerpt below, she asked him to share, and the TE signaled to students that they were

working on "explaining," the focus for the lesson. Students were reluctant to share, and the TCs and TE tried different questions to elicit ideas about the increase of tens in each column.

Melissa: Thahn, do you want to share something? 147
TE: This is really important what we are doing now, because we're trying to explain a pattern. Mathematicians like to really explain why when they notice something. So good thinking. 148 149 150
Sam: Uh, so what do you guys think about the pattern we were talking about? (students are silent) What do you think? Why is it happening? 152 153
TE: It's not magic? Why is it happening? There's a reason for it. How come? (students are quiet) 154 155
Inga: Zero might not be an odd number. 156
Sam: Zero might not be an odd number? 157
Melissa: Does anyone want to add on to what Inga is saying? 158
TE: How come it's doesn't go 2, 4, 6, 8, 9 or 2, 4 ,6 8, 2? How come it goes to zero? Can you think about that? 159 160
Thahn: Because it's a ten. 161
Sam: Because it's a ten? 162
Melissa: What makes it into a ten? What did we do? 163
Thahn: Because we added two. 164
Melissa: We added two to the eight. 165
Sam: How many twos are there in a ten? 166
Thahn: Twenty? 167
Other students: Five! 168
Sam: Can we count them? 169
Everyone: One, two, three, four five. (Sam points to each number as they count) 170
Sam: So, can we also say… (looks over at the TE) 171
TE: Can that help us? Can that idea? 172
Sam: There are five groups of two in ten, right? 173
Thahn: Five times two. 174
Sam: (nods her head in agreement) So five times two. (writes "5 x 2 = 10") So five times two equals ten. So there are five groups of two in one ten…. (pauses to think while looking at the poster) 175 176 177

In lines 147-155, Melissa, Sam, and the TE asked students to explain, but students were reluctant to share. Finally, Inga shared "zero might not be an odd number" (line 156), which was not what TCs were expecting. Unsure of what to do, Melissa asked if others wanted to elaborate on this idea (line 158). In lines 159-160, the TE intervened and redirected students to the original question by providing two nonexamples and asking why the pattern goes to zero. At this point, Thahn finally shared that it was a ten (line 161). This was the idea Melissa had heard in the turn and talk and was hoping to highlight when she selected Thahn to share (line 147).

Melissa thoughtfully selected Thahn, based on his comments and their goal, but needed some guidance from the TE when this idea was not immediately shared. In line 163, Melissa asked

questions to support students to think about why there was an increase of ten: "What makes it into a ten? What did we do?" Thahn answered her second question literally when he responds that they added a two (line 164). Sam responded (line 166) by asking students how many twos there are in a ten, a move that connected to their goal of explaining why there is an increase of ten between the columns. They counted the twos (line 170), and Sam wrote "5 x 2 = 10" to emphasize the fives groups of two for each column (lines 175-177). This "connection to multiplication" and idea that ten is made of five groups of two was something Sam talked about during rehearsal.

Although Sam wrote "5 x 2 = 10" to emphasize the five groups of two, the group hadn't articulated *why* there was a zero. In the following excerpt, the TE stepped in and asked questions to support explanation around this pattern.

| | |
|---|---|
| TE: So how come when we get to 28, why does the next number have zero in the ones? | 178 |
| Like at 38, it goes to zero again. | 179 |
| Raul: Because it's the third column. | 180 |
| TE: Why is that? Why does it go back to zero again in the ones place? | 181 |
| Arianna: (inaudible) | 182 |
| Melissa: Think about this, how many twos are in a twenty? | 183 |
| Students: Ten! | 184 |
| Melissa: How many twos are in the thirty? | 185 |
| Students: Fifteen. | 186 |
| Melissa: Fifteen? Okay, do you see a pattern going on there too? How many twos are in | 187 |
| forty? | 188 |
| Students: Twenty. | 189 |
| Sam: What about 50? | 190 |
| Students: Twenty five! | 191 |
| Sam: Wow! Okay, counting by fives. (still looking at the poster, thinking) | 192 |
| TE: Yes, the twos are counting by fives. Hmm, I don't know, what do we think? Do we | 193 |
| have the beginnings of an explanation? | 194 |
| Sam: I think so. (looks at students) We're working it out with you. | 195 |

The TE directed students back to the original question of what it goes back to zero in the ones place (lines 178-179 and 181). Melissa and Sam then asked about how many twos are in 10, 20, 30, 40, and 50. Although students said five, ten, fifteen, twenty, and twenty-five twos (respectively), it is unclear if students understood the increase of ten between each column and why that ten was happening. The TE and Sam both expressed this uncertainty (lines 193-195). After this exchange, the TE thought aloud to the TCs about why she gave two nonexamples and

how she hoped that would support students to think about why the pattern was happening. She directly talked to students, asking them questions and pointing out to TCs when the students were beginning to provide an explanation, not just a noticing. In the following excerpt, we see Sam return to the original idea, increase of ten between columns, as she ends the lesson.

| | |
|---|---|
| Sam: Why is this happening? (points to the 10, 20, 30, etc. in the bottom row) Why there is a distance of 10 in each column? | 196<br>197 |
| Arianna: I think because the columns are five times two. | 198 |
| Sam: Two columns make five times two? So this and this make 5 times two? (points to the first two columns) | 199<br>200 |
| Arianna: (inaudible, repeats strategy) | 201 |
| Sam: If we kept going, would we get to 157? | 202 |

Sam returned to the goal but wasn't sure how to conclude the lesson. After asking why there was a distance of 10 in each column (lines 196-197), Sam asked if they would get to 157 in their count (line 202). While this question seems random, it is something she mentioned in the rehearsal. When the TE (in the rehearsal) asked if there was a particular pattern they wanted students to explain, Sam mentioned using the distance of ten between the column to figure out other numbers in the count. This is not something they discussed or took up in rehearsal since it was not part of the goal of explaining a pattern. Using the pattern to figure out other numbers is a different goal than explaining why a pattern happens, and they worked on this goal in other choral counts. Sam was attending to how to facilitate discussion and elicit student ideas but was still learning what moves steer the discussion towards their goal and strategies for focusing on the intended mathematics (Sleep, 2009). In this case, Sam was sidetracked with an idea that was interesting and mathematical but not related to this particular task nor their chosen goal. Sam also asked if they would land on two hundred but ran out of time. The TE had them stop the lesson but commented that students could continue to think about and try to explain why they would or would not land on 157 or 200. Although the TCs returned to the idea of "plus ten" between columns, the final questions showed

they still needed support in focusing on the intended mathematics and concluding in a way that highlights key mathematics related to the goal.

**Analysis of the mid-course lesson: Specifying goals but looking to the TE for support on how to reach those goals.** This class session shows the TCs in transition during the middle of the course. TCs were becoming more specific in articulating their instructional goal. They wrote the goal (*to explain patterns*) in their lesson plans and noted several possible patterns for this count. They did not include possible explanations in their lesson plan but started to think through these explanations during the rehearsal. This indicates they were still making sense of high-quality mathematics instruction, particularly the role of *planning* in facilitating a discussion that is responsive to students. They were developing their understanding of the purpose of lesson planning and value in using lesson plans[20] as a space to anticipate student thinking, think through mathematical explanations, and consider closures that highlight the intended mathematics.

Melissa and Sam explicitly stated the goal to students in both the rehearsal and enactment with students. In the enactment, students shared patterns, and the TCs asked students to *explain patterns* (their goal). Students were reluctant to explain patterns and TCs struggled to use moves to support talk through scaffolding (Sherin, 2002) and establish sociomathematical norms supporting student talk for argumentation and justification, including establishing what counts as an argument or any explanation (Yackel, 2002). Moves to scaffold talk and establish sociomathematical norms could have facilitated discussion towards their goal of explaining patterns. This indicates the need to support TCs to: 1) Understand what counts as argumentation and explanation; 2) Facilitate sociomathematical norms in discussion; and 3) Be clear about the possible mathematical explanations connected to their goal (Yackel, 2002, Sleep, 2012). The lack

[20] I discuss the purpose and use of lesson plans in more depth in the following chapter.

of a written explanation in their lesson plans indicates they may not have thought about a possible explanation for different patterns and/or they may have needed support in developing their content knowledge about what constitutes a mathematical explanation. Kazemi and Wæge (2015) also found the TCs needed deliberate feedback and support not just in eliciting student ideas but also using talk moves to explain and reason.

At this point in the course, TCs knew moves could be used intentionally to serve a purpose, and they talked about steering instruction towards goals. They also understood and used many of the moves and ideas that had been introduced in readings and course resources and covered in the class sessions. However, it was challenging for them to use these to steer instruction towards their goal. Although they talked about intentionally using moves, they were less sure in the moment of which moves or ideas to use for the purpose of steering the discussion towards their goal. They did not quite know *how* to steer the conversation towards their instructional goal nor did they remember to do that throughout the lesson.

Although TCs were more comfortable teaching their small group, they still needed guidance from the TE about the instructional activity and how to teach towards their goal. The TE provided guidance, such as how much of the count to get out before students could identify patterns, how many patterns to elicit, and when to have students explain a pattern. In this episode, the TCs worked in response to the TE's guidance and prompting to specify their goal and work towards it in the lesson. While not evident in the transcript, the video data illustrated that with each statement, Sam ended with a rising inflection, indicating that this was something she is still thinking about. Many of her statements were spoken as if she was checking in and asking for confirmation from others about her ideas. Melissa showed even more hesitancy in her body language by turning to the TE for confirmation after she made a revision or tried out a new move.

In this lesson, the TE intervened by modeling how to teach towards this goal and providing suggestions for teacher moves and phrasing. Although the TE was quite active in this lesson, her questions and interventions were not evaluative or judgmental. She was empathic and authentic in co-thinking and co-teaching with the TCs in reaching their goal. She guided TCs to understand how to facilitate a discussion that pressed students to explain and to understand the rationale behind particular moves and how moves can work towards particular goals.

The TCs were gaining confidence in their teaching, but the TE still had ownership over the goal, particularly how to facilitate the discussion to reach this goal. This is not surprising, given that the TCs were just halfway through the first of two methods courses and in the second quarter of their teacher education program. The TCs knew what their goal was and potential moves to use, but the TE helped them think about when to use these moves, how to reach the goal, and how to phrase questions to support student thinking. As a more knowledgeable other, the TE provided coaching and deliberate feedback that scaffolded TC enactment, highlighted particular aspects of ambitious practice, and centered attention on student thinking and mathematics (Ericsson, 2002; Ghousseini & Sleep, 2011; Grossman, Hammerness, et al., 2009; Lampert et al., 2013). Through purposeful coaching in practice and sequencing of approximations of practice, the TE supported TCs to develop a vision for ambitious teaching and to acquire knowledge and skills in practice and in relation to this vision (Ghousseini & Herbst, 2016; Grossman, Hammerness, et al., 2009; Kazemi et al., 2009).

The TCs' moves during enactment reflected an effort to facilitate discussion based on the goal, but they were not quite able to do this work on their own. Although I do not provide an analysis of the TE moves, other studies document the critical role of the TE and OTL facilitated by intentional moves and coaching (Ghousseini et al., 2015; Grossman, Hammerness, et al., 2009;

Kazemi et al., 2009; Lampert et al., 2013; Shaughnessy et al., 2019). The TE facilitates opportunities for deliberate practice of ambitious teaching practices and opportunities for TCs to develop adaptive expertise. The changes in TC participation and shifts in talk and enactment of practices (particularly teaching towards an instructional goal) happened in the context of the TE's careful coaching and deliberate feedback.

This was an important moment in the trajectory of TC learning. Many methods courses and studies on TC learning stop at this critical point between developing new frames of reference and developing practices and adaptive expertise to enact high-quality mathematics instruction (Grossman, Hammerness, et al., 2009; Kennedy, 1999; McDonald et al., 2013). TCs were in the process of shifting beyond simply *talking* about facilitating a productive mathematical discussion to being able to *enact* this ambitious practice. Development of ambitious teaching practice was an iterative process as TCs revisited practices in increasing complexity. In this mid-course episode and the first episode, the TCs were developing and refining their shared understanding of teaching and shifting focus from just "getting through" the lesson to talking about how they could facilitate discussions in response to students and the mathematical goal. In tandem with the TE's scaffolding, deliberate coaching, and intentionally sequenced approximations, TCs began to develop and refine *enactments* of practice in increasingly ambitious and complex ways. We see this shift in enactment, not just talk, in the final episode.

### *Final Day of the Course: Intentionally Facilitating Discussions Around Particular Mathematical Ideas*

On the final day of the course, the TCs led a problem-solving task and two computational string activities (or what I call a "string"). In this section, I describe the TCs' participation and work on goals during the first string lesson. The string lesson provides a clear image of the changes

observed during the 10-week course in the TCs' *specificity* of instructional goals and *intentionality* in facilitating discussions to reach these goals.

In a string, two or more computational problems are posed, one at a time. The problems are sequenced intentionally to highlight a particular mathematical relationship or idea. For this string activity, Katelina and Orianne planned to pose 24 – 4 followed by 24 – 6. The goal was to understand that when subtracting, the second number (in this case the six) can be decomposed and subtracted from the first number incrementally, in the decomposed chunks. By posing the problem 24 – 4, students were primed to subtract an additional two more by breaking the six from 24 – 6 into four and two. First, they subtracted 24 – 4 = 20, then 20 – 2 = 18. Thus, students could use what they knew about an easier problem (24 – 4) to solve a more difficult problem (24 – 6).

**Final day of the course: Description of the rehearsal.** Katelina began the rehearsal by stating the goal: "Our goal for today is to use our strategy that we used to solve the first problem to help us solve the next one." She asked the group if she should have students repeat. Another TC said no, so she shrugged and kept going. This confidence in thinking out loud and casually asking the group for suggestions was a change in her participation from the first lesson, where she was nervous in front of her peers, and the second rehearsal, where she continued to look to the TE for direction. After introducing the goal, Orianne reminded students to keep the answer in their heads. Katelina also reminded them not to shout out, instructing them to hold a quiet thumb, then posed the first problem.

The flow of this introduction is markedly different from the first lesson. She began by explicitly stating a goal. Her directions were clear, concise, and easy for students to follow. The TCs seemed more comfortable working with one another, rehearsing, and thinking through their

lesson collectively. The entire introduction took 45 seconds, even with their short dialogue around if they should ask students to repeat.

Katelina wrote the first problem (24 - 4) on the board, moving where she sat, adjusting the level of the poster, and writing big numbers to ensure students could see the problem. She gave wait time and prompted students to show a thumb when they had an idea. After students signaled with their thumb, Katelina asked them to shout out the answer together, a move they used throughout the course to support student participation and to hear if students had the same ideas. They all said twenty. Katelina wrote "20" and asked if someone could share how they solved the problem. "Raul"[21] (Melissa playing the part of her buddy Raul) explained that he did four minus four, then twenty minus zero.

Katelina raised her hand to write on the board but stopped herself and asked the group how to represent this. In the following excerpt, the group discussed how to both validate the student's idea and use representations to highlight particular mathematics.

Melissa (as Raul): I just did four minus four.
Katelina: [pointing to the 4 in 20 and the second 4] Four minus four.
Melissa (as Raul): Which is zero, then add the two for twenty. [inaudible]
Katelina: [tilting her head in a thinking pose] Okay...how would I…should I just say... [trails off]
Melissa (as Raul): I did the twenty and the four, then four minus four, then put it back together.
TE (to Katelina): What are you thinking?
Katelina (stepping out of teaching mode to ask for help): Like, how do I record that? I don't want to do a number line, because that's not what he did. So he did… [trails off again]
Danielle: He broke the 24 into twenty and four.
Orianne: 24 into twenty and four. [motioning to two imaginary parts from coming from the 24 written on the board]

[21] As TCs rehearsed, they realized it would be helpful to pretend to be their particular third grade buddy, anticipating what they might say or do. Playing the part of students indicates a focus on learning about students' thinking, experiences, and cultural and linguistic resources and understanding teaching as knowing particular students and responding to their ideas (Anthony et al., 2015). This focus on students is a marker of emerging adaptive expertise. They started pretending to be their particular buddy in the 5th class meeting (October 28), and by the end of the quarter had become quite adept predicting what their buddy might say or do.

Katelina (back in the role of the teacher, writing down what Orianne was motioning): So you broke this into a 20 and a 4 [draws a caret-shaped number bond attaching 20 and 4 to the 24] 17

As "Raul" shared his strategy (lines 1 and 3), Katelina was hesitant and unsure of what to write on the board (lines 4-5). Sensing this uncertainty, the TE invited her to step out of teaching and think aloud with the group (line 8). Katelina had prepared a number line representation in her lesson plan (to highlight subtraction of four and then two more) but did not want to use a number line because she didn't think it represented what the student said (lines 9-10). Danielle and Orianne thought out loud with her about breaking the 24 into 20 and 4 (lines 12-14), and Katelina used the caret-shaped number bond model to show this decomposition. In contrast to the first day (when the TCs faced the TE the whole time), Katelina looked to the other TCs (in addition to the TE) for support and drew on the group's collective expertise to think about how to use representations.

In response to this number bond model (which usefully showed the decomposition of 24), the TE prompted the group to think about how they might represent the student's strategy and attend to mathematical precision. The TE was supporting TCs to show the students' strategy in the model by asking them what they could do (lines 18) and suggesting they draw an X to show the subtracted part (line 21). Danielle suggested crossing out both the four under the 24 and the subtrahend four, which created an invalid equation. Katelina took up the TE and Danielle's suggestions, and when she drew an X through the 4 under the 24 and through the subtrahend 4 (lines 23-26), her symbolic notation became invalid. The TE directed TCs to think about their representation and noted this change in making the number sentence invalid (lines 26-31). This move directed their attention to mathematical precision and addressed pedagogical content knowledge in practice.

The entire exchange (beginning with Raul's sharing and ending with the use of the number line) took just under two minutes. This exchange, made possible by the pedagogy of rehearsal and

the TE's deliberate feedback, opened space for TCs to work on mathematical precision and representations in practice, thus being a closer approximation of ambitious practice than simply enacted a lesson with feedback or doing an uncoached run through of the activity (Lampert et al., 2013). In talking about the students' strategy and representations, the group was working on the goal of this string: To understand that if you subtract a two-digit number, you can decompose the second number and subtract it in chunks. They worked on representations to support students to use this strategy and moves to highlight the idea of decomposition. Even though the TE provided prompting and guidance around precision and representations, the TCs were actively collaborating and co-thinking about how to intentionally use representations and facilitate the discussion to reach their particular goal. This contrasts with the mid-course lesson (October 28) where the TE constantly reminded the TCs of their goal and modeled moves to work towards this goal.

After the discussion of number lines and representing student ideas, Katelina provided the following prompt before posing the second problem: "We'll solve another problem and again, try think how to use the same strategies to solve the next one." This prompt is a generic prompt for transitioning to the next computational problem in a string. The TE suggested being explicit in orienting students to using this previous problem: "If your goal is to connect this one, 'We know that 24-4 equals 20, so let's remember that,' be very explicit about it. 'Let's use that to solve this next one.'" Katelina repeated the directions with the same explicitness the TE suggested, then posed (24 – 6). She used the same wait time, thumb strategy, and collective shouting out of the answer as with the first problem, then checked in with her peers again to ask if group counting was an appropriate next move. The TE made a quick suggestion, "Now this feels like a turn and talk." The group had developed a shared understanding that a turn and talk move can be used to create opportunity for the teacher to listen in and select student ideas to share and can be used to support

student participation by allowing students to try saying their thinking out loud in pair setting prior to talking with the whole group. Thus, the TE could use a quick prompt to signal this moment as an opportunity to use a turn and talk for that purpose.

During the turn and talk, "Arianna" (Tina role playing her buddy) shared a counting by one strategy. The group discussed how there might be kids still counting by ones, but the purpose of the string was to encourage students to use the previous problem. The TE suggested listening for students who were using the strategy of breaking apart, then making the strategy explicit for other students. Katelina elicited a student's strategy that uses this idea of breaking apart numbers. She used the same number bond model to show "Inga's" strategy (Danielle role playing her buddy). Before writing her number line, she asked students to repeat. Using this repeating move showed Katelina was thinking about her goal and orienting students to a key mathematical idea. "Raul" repeated that the six was broken into four and two, and Katelina asked a follow up question, "And what does she do with that four and two?" Her repeating move and follow up question pressed students to think about the idea of decomposition in relation to their goal. This shows intentional use of talk moves, enacting them in greater complexity compared to use of these same moves in earlier lessons. In the remainder of this string, Katelina closed by asking students to think about how their ideas connect to the first problem.

After this first string, Orianne led a second string. To keep this description concise, I illustrate just the first string, even though a second string activity was also taught. The group worked on attending to similar issues around representations, intentional use of talk moves, and moves to orient students to the mathematical goal.

**Final day of the course: Description of the enactment.** In the enactment, Katelina again stated the goal, but this time with a cue connected to the idea of decomposition: "So our goal today

is to use an easier problem to help us solve the second problem **by breaking down the numbers**." She used the same directions (not shouting out, thumb signal, wait time, then collectively shouting out the answer) as in the rehearsal. Raul shared that he "took the two off" to do four minus four. This was similar to what Melissa predicted Raul would do, but Katelina didn't use the number bond model. She wasn't sure how to represent this, and Orianne suggested she write 4 – 4 = 0. Katelina connected the student's strategy to a number line, just like the rehearsal. Very similarly to the changes made during the rehearsal, she gave the same explicit prompt, posed the second problem, and asked students to turn and talk about how they used the first problem to help them solve the second one. After listening to students during turn and talk, Orianne purposefully selected a pair of students to share who talked about subtracting incrementally. In the following excerpt, Orianne asked these students to share. It is evident that their moves were intentional in supporting student to think about this goal.

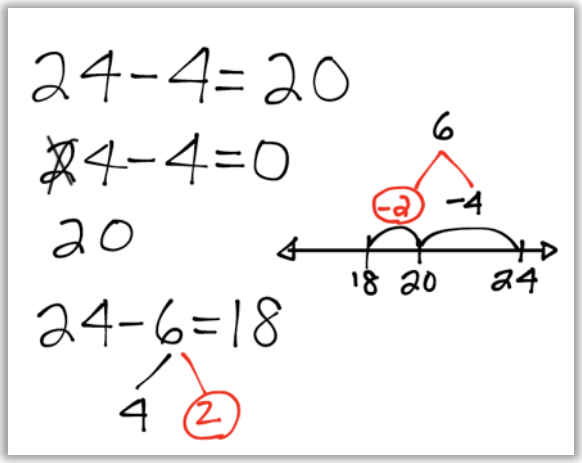
Orianne: So yeah, so do you two want to share what you two came up with? They had two different strategies.
Inga: Eighteen is two away from the twenty.
Katelina: Two away from the twenty? Okay. [holds her pen up to the equation 24 – 6 = 18]
Inga: [inaudible] [Orianne points to the number line]
Katelina: Oh from here? [moves her hand and points to the twenty on the number line]
TE: Can you add that to the number line?
Katelina: Yeah, so you took away, minus two. [writes "-2" with and arrow and "18"]
Kristina: That's what I did!
Katelina: Oh really? But how did you, where did – how did you know to take away this two? [motions at the "-2" she just wrote on the board] Where did it come from?
Inga: Because two plus four equals six.
Katelina: Okay. [puts hand up to write but pauses] Can someone repeat what Inga just said?
Kristina: Can I say it in a different way?
Katelina: Of course!
Kristina: Um, I minused four and then got twenty, then I minused two more because two plus four is six.
Katelina: Did everyone hear that? Yeah? [students nod their heads] Sahra, can you repeat what Inga just said? [student talks quietly; inaudible] Wait! Raul, can you hear Sahra? [he nods his head no] You've got to speak way louder!
Sahra: Twenty-four minus for is twenty, minus two is eighteen.
Katelina: So, we know that this six is four plus two. [draws two lines under the six and writes "2" and "4"] And we already know that twenty-four minus four is twenty, so this two is right here. [circles the two in both places in red] So what is this, where does

this, um, where's the six in here? If we are going from twenty-four to eighteen, where is the twenty-four? 45 46

Kristina: So the six is – [points to the two and four on the number line] 47

Katelina: Right here! So these two numbers are six. [draws two lines from the "-2" and "-4" above the number line and writes "6" in blue] Nice job! 48

***Figure 4:*** *Final Representation on the Board.*




In this excerpt, TCs were asking questions and using moves to press students to reason and think about decomposing the six. When Inga said, "Eighteen is two away from twenty" (line 20), Katelina pointed to the number line to confirm the student's idea and to make it visible for the rest of the students (line 21). Katelina asked Inga how she knew to take away the two (lines 28-29). This was a critical part of using the decomposition strategy. Katelina actively worked to highlight the mathematical ideas central to their goal of decomposing the second number (the six) to subtract in chunks (first the four from twenty-four and then the two from eighteen). Because this was a key idea, she asked other students to repeat multiple times (lines 31-32 and again in lines 37-38). She revoiced the idea (lines 41-43), highlighted the decomposed numbers in her representations, and continued to press students to reason about where the decomposed numbers were in the original equations (lines 43-45). This excerpt is an example of TCs using talk moves and discussion stems (e.g., "Can anyone repeat…" "Who would like to add on...") to steer the conversations towards the instructional goal and key mathematical ideas. Their teacher moves were deliberate and

reflected their goal of understanding how to decompose two-digit numbers to solve subtraction problems.

After Orianne led a second string with the same goal, Katelina closed by asking about the goal, then revoiced the breaking apart strategy.

| | |
|---|---|
| Katelina: So today, what goal – what strategy did we focus on? Who can recap what strategy we worked on? | 49<br>50 |
| Orianne: How did we subtract? What strategy did we use? | 51 |
| Student: Number line. | 52 |
| Katelina: A number line! [points to the number line on the board]. And we worked on breaking apart the number to make it easier for us! | 53<br>54 |
| Kristina: We used our first strategy to help us solve the second problem. | 55 |

In lines 49-50, Katelina tried to ask a question to have students repeat the goal. Orianne added on with her prompt of what strategy they used for subtraction (line 51). These questions showed they were trying to close the activity in a way that highlighted the goal through student voice. When the student said number line, they were referring to the tool that was used to model the strategy. During earlier enactments, both the TCs and students confounded the idea of mathematical *tools* for modeling (e.g., number line, base-10 blocks) with a mathematical *strategy* (e.g., breaking apart numbers). They didn't have time to clarify and expand on this as a group, but Katelina repeated and connected their mathematical goal around decomposition (*breaking apart the numbers*) to the tool used for modeling. Although there wasn't time to support more student voice, Katelina and Orianne were intentional in highlighting the goal in the closure. Supporting student voice is another aspect of the complex work of teaching towards instructional goals. Although this was the last day of the methods course, TCs seemed primed to continue building on this complex practice of facilitating productive mathematical discussions by attending to student voice.

**Analysis of the final day of the course: Intentionally facilitating discussions around particular mathematical ideas.** From rehearsal to enactment, we see important shifts in TC participation. During the rehearsal, Katelina told students the goal was to "use the strategy for the

first problem to help solve the second one." This goal was a more general goal for the activity of strings, which sequence problems to highlight particular mathematics. Through their conversations and enactment, the TCs realized this general goal for strings (using the first problem to help solve the second) and their choice of problems supported student thinking around a more specific goal. Katelina revised her statement for the enactment with students, stating the goal was "to use easy problems to solve harder problems ***by breaking down the numbers.***" This subtle revision demonstrates that she not only understood the general purpose of the activity of strings, but also worked towards facilitating the discussion in a way that highlighted a particular mathematical idea. The TCs' choice of problems and articulated goal of *breaking down the numbers* focused instructional time on intended mathematics and created and maintained a coherent mathematical storyline in their lesson (Sleep, 2009).

The idea of decomposition was carried throughout the day as TCs thought about how to support student thinking around this mathematical idea. They worked on *eliciting and responding to student thinking* and using representations and steered the discussion by making sure students were doing the mathematical work, opening up and emphasizing key ideas, and keeping a focus on meaning (Sleep, 2009). In the rehearsal and debriefing, the TCs talked about how to represent student ideas to highlight how the numbers were decomposed. They focused on student thinking and teaching moves that highlighted the key mathematical idea of decomposition. TCs actively worked to *orient students to the mathematical goal* and *represent mathematical ideas visually, numerically, and in words*, two common practices of ambitious teaching articulated in the course. TCs were revisiting the practice of *eliciting and responding the student thinking* in increased complexity (c.f. previous episodes) and in relation to additional practices around *orienting students*,

*representing mathematical ideas*, and *teaching towards a mathematical goal*. Thus, TCs were facilitating discussion in more ambitious and complex ways.

The fact that the TE was still providing support around what to expect from students and how to highlight the key mathematical idea during the discussion is not surprising. While the TCs' specification of instructional goals and intentionality in facilitating discussion changed dramatically, there was still much room for the TCs to improve their practice. The ten weeks of practice-based methods provided opportunities to develop routine and beginning adaptive expertise by shifting focus from self to students and building a shared framework around the complexity of facilitating productive mathematical discussions (Timperley, 2013). The TE continued to provide deliberate feedback in coaching that further developed TCs' routine expertise and challenged them to develop adaptive expertise. In this final episode, there were substantial differences in how TCs thought about and worked on instructional goals in their practice (understanding teaching as complex). They had more ownership (agency and efficacy) working with their peers and in instructional decision-making.

In contrast to early rehearsals, TCs had increased ownership about why they would use particular moves and how to use representations more effectively as a public record of ideas being discussed. This shift in ownership contained markers of emerging adaptive expertise. This ownership showed development of their identity as professionals working on ambitious teaching practices, agency in building relationships with elementary students and contributing to discussion in how to support those students' learning, and efficacy in believing that they could facilitate productive mathematical discussions. It is encouraging that as the course progressed, TCs became more adept at incorporating ideas from prior course sessions and the preceding rehearsal into their enactment.

## Conclusion

This chapter examined how TCs facilitated discussion to reach instructional goals. Findings showed how TCs made shifts in their talk *and* enactment of facilitating discussions towards instructional goals. This analysis documented changes in TCs' specificity of goals, intentionality in working towards these goals, and ownership of goals.

Discussions that are productive and mathematical focus on *quality* (over *quantity*) of talk and work towards a mathematical goal (Chapin et al., 2013). Arguably, goals guide all instructional decisions: which mathematical ideas are pursued and worked on, what questions to ask students, how to respond to and represent student ideas, etc. (Kazemi & Hintz, 2014; Sleep, 2012). *Eliciting and responding to student thinking*, *orienting students to each other's ideas and key mathematics*, *representing mathematical ideas*, and doing these practices in relation to the goal and student thinking is ambitious and challenging practice (Kelley-Peterson, 2010; Kazemi & Wæge, 2015; Lampert et al., 2013). My decision to focus on instructional goals within discussions was motivated by the need for teachers to develop these kinds of ambitious practices to support students' mathematical proficiency.

As the course progressed, TCs were more specific in articulating the instructional goal for their lesson. They wrote more specific goals, explicitly talked about these goals, and stated them during their rehearsals of discussions and facilitation of discussions with students. They began articulating goals that addressed particular mathematical ideas or practices and were more intentional in facilitating discussions to reach these goals. Early in the course, TCs often did not write or talk about goals unless prompted and guided by the TE. Towards the end of the course, TCs identified and worked on goals that addressed particular mathematical ideas without (or with less) prompting from the TE. TCs took on more ownership over articulating and working on the

instructional goal and enacting ambitious practices (e.g., *eliciting and responding to student thinking* or *representing mathematical ideas*) and moves (e.g., talk moves) needed to facilitate productive mathematical discussion.

### *An Iterative Trajectory of TC Participation in Facilitating Productive Mathematical Discussions*

Zooming into three episodes provided an image of the trajectory of development of ambitious practice around working towards instructional goals. In the first few cycles, TCs worked on understanding instructional activities, teaching a lesson with kids, and eliciting ideas. Once the initial "practise-shock" dissipated, logistical issues and less ambitious moves (e.g., *how to pass out materials* or *how to record numbers without running out of space*) became routinized, and TCs could attend to more ambitious aspects of practice (e.g., *how to use representations to orient students to key mathematics*) (Ericsson et al., 1993; Darling-Hammond & Bransford, 2005; Hatano & Inagaki, 1986; Lampert et al., 2013; Timperley, 2013). Their focus shifted from self to students and making sense of their thinking in service of *positioning students competently*, an ambitious practice. They worked hard to set up a talk community (a marker of routine expertise) and support students to talk and participate in discussions (Chapin et al., 2013; Timperley, 2013).

In the middle of the course, TCs began actively working on goals, shifting from how to set up the learning environment (routine expertise) to talking about intentionally supporting individual student learning (adaptive expertise). Through learning cycles in practice, TCs developed a shared understanding of facilitating productive mathematical discussion and could *talk* about this practice. They were intentional in thinking about how to facilitate discussions to reach the goal but did not quite know *how* to do that. This was an important pivot point: ending the methods course here would perpetuate the problem of enactment, the gap between how TCs *talk* about ambitious

practice versus what they are able to *enact* (Kennedy, 1999). As a more knowledgeable other, the TE was critical in addressing this gap and supporting TCs to contextualize and develop knowledge, skill, and professional judgment *in practice* (Drake, 2016; Ghousseini & Sleep, 2011; Grossman, Hammerness, et al., 2009; Kazemi et al., 2009; Lampert et al., 2013; Shaughnessy et al., 2019). The TE supported them to move beyond *talking* about facilitating a productive mathematical discussion to being able to *enact* this ambitious practice. The TE provided deliberate feedback and coaching to support TCs working towards their instructional goal.

In the final episode, TCs were intentionally facilitating discussions around particular mathematical ideas. TCs focused on student thinking and teaching moves that could highlight key mathematical ideas. TCs worked on multiple practices of ambitious teaching in relation to one another, in service of the goal, and in greater complexity than previous enactments. TCs believed they had ideas to contribute to the group (efficacy) and agency over enacting practices in response to student thinking and in support of their learning.

### *The Relationship of Iterative Trajectories and Decomposition: Domains, Intermediate Practices, and Techniques*

By analyzing how TCs worked on instructional goals, my findings provide an image of the iterative nature and trajectory of how TCs learned to facilitate productive mathematical discussions. This iterative trajectory of TC participation, as they revisited and refined practices, builds on Boerst and colleagues' (2011) use of "continuum of grain sizes of practice" to frame the role of specificity in addressing both the *why* of teaching by naming larger core domains of practice (e.g., *mathematical discussion* or *learning community*) and the *how* of teaching by giving "practical" guidance through specific and high-leverage techniques and moves (e.g., a *revoicing* move) that could serve multiple purposes and that are nested within larger domains. Considering this

continuum of specificity, supporting TC learning also involves identifying intermediate practices (e.g., *eliciting student thinking*) and making explicit connections between nested and related practices of varying grain size.

This chapter analyzed participation around the domain of leading a mathematical discussion. TCs began their work at a fine grain level, examining teacher moves and logistical issues need to lead an instructional activity. For example, they used the talk move of *repeating*, even in early enactments. As their focus shifted to students, they worked on intermediate practices and began talking about how these intermediate practices, such as *eliciting student thinking*, could be used in service of the instructional goal. With TE scaffolding and repeated approximations of practice, TCs began *enacting* intermediate practices and revisited techniques and moves in service of the goal. TCs continually revisited and refined the larger domain (discussion facilitation), intermediate practices, and techniques in increasing complexity as cycled between specific interactions with students and generalized ideas about ambitious practice.

In working towards instructional goals, TC participation and learning was not a linear process nor was it a process of *applying* learned theories and pedagogical knowledge. They did not progress from mastering small, discrete moves (e.g., talk moves) to then working on intermediate practices (e.g., *eliciting*) to finally mastering larger core practices (e.g., facilitating a discussion). Likewise, they did not master a vision for ambitious teaching outside of practice and then *apply* this vision in their instructional practice with students (this approach exacerbates the problem of enactment). In repeated cycles of enactment and investigation, TCs revisited and refined their understanding and use of discrete techniques (e.g., talk moves), larger domains of practice (e.g., facilitating productive mathematical discussions), and related intermediate practices

(e.g., *working towards an instructional goal* or *eliciting student ideas*). In this iterative process, they cycled between decomposing and recomposing practice in varying grain sizes.

Shaughnessy et al. (2019) provide a framework for decomposing the domain of leading mathematical discussion (including articulating the centrality of instructional goals in this work): "(1) *Planning*, including identifying a clear instructional goal, (2) *Launching* a discussion, (3) *Orchestrating* interaction around the mathematics, and (4) *Closing* the discussion" (p. 169). Like other research on high-quality mathematics instruction, the specification of "including identifying a clear instructional goal" in planning signals the importance of goals within productive mathematical discussions and provided the underlying motivation this chapter's analysis and focuses on goals. In contrast to these four large components of discussion facilitation, Sleep (2012) articulates smaller techniques with a "useful" grain size of strategies specifically for steering discussions towards the mathematical point. Her framework of strategies within the work of steering discussions towards mathematical goals and related issues that beginners often face provided a useful lens for understanding TC participation at a small grain size. The ambitious practices named in the course (e.g., *eliciting and responding to student thinking* or *orienting students towards one another's ideas and to the mathematical goal*) functioned as intermediate practices.

This analysis of TC participation complements existing research documenting pedagogies of enactment, core practices of ambitious teaching, and TE moves to facilitate OTL (e.g., Ghousseini & Sleep, 2011; Grossman, Hammerness, et al., 2009; Grossman, 2018; Lampert et al., 2013; Shaughnessy et al., 2019). The changes in TC participation around facilitating discussions towards an instructional goal happened in the context of the TE's purposeful sequencing and use of approximations, decompositions, and representations of practice. This chapter contributes to

understanding how TCs develop understanding of and intentionality in facilitating productive mathematical discussions.

## Chapter 6: Teacher Candidate Participation Around a Shared Repertoire

In this chapter, I discuss the shared repertoire TCs developed and how they used tools within this repertoire. Wenger (1998) describes shared repertoire as the tools, discourse, norms, stories, histories, ideas, and ways of participating in a community of practice that members have negotiated as part of the practice. The shared repertoire I examined included the ways TCs used lesson plans, teaching materials (focusing on use of the board), and talk moves. In the previous chapter, I examined how TCs worked on instructional goals and teaching practice, framing my analytical approach with Wenger's (1998) notions of joint enterprise and mutual engagement. As they interacted (mutual engagement) and negotiated meaning around instructional goals and how to facilitate productive mathematical discussions (joint enterprise), they also developed a shared repertoire. Shared repertoire is a "resource for negotiating meaning" in a community of practice and includes "routines, words, tools, ways of doings things, stories, gestures, symbols, genres, actions, or concepts that the community has produced or adopted in the course of its existence, and which have become a part of its practice" (Wenger, 1998, p. 83). I analyzed a selected set of tools within the shared repertoire and the cycle between TC learning of practice and how they made sense of and used these tools in practice.

This analysis builds on an emerging body of literature focused on enactment tools within practice-based teacher education (Chapin et al., 2013; Ghousseini et al., 2015; Lampert et al., 2010; Thompson et al., 2013). Building off Wertsch's (1998) theory that tools mediate activity and goals, Ghousseini and colleagues (2015) discussed the use of tools to open up problem spaces and possibilities for OTL:

> "A tool mediates action by translating what may stand as an abstract conceptual problem for a beginner into a series of concrete operations at which one can become proficient. Thus, learning to use a tool entails developing technical skills. Wertsch also argued that a tool can support enactment thorough affordances (and sometimes constraints) it contributes to the development of goal-directed activity… the use of a tool towards particular ends opens up *problem spaces*,

> problem-solving situations… These problem spaces, emerging as a result of the practitioner acting on the world using particular mediational means, provide the user with an opportunity to engage in a form of inquiry" (p. 467).

They found one enactment tool, a specific question sequence, created opportunities for deliberate practice of eliciting student thinking. In another study on tool use, Thompson et al. (2013) examined how teachers appropriated ambitious science teaching practices in their practicum and first year of teaching. They framed tools and routines as reifications of ambitious teaching and found "taking up well-designed tools supported modifications to discourses and practices and helped novice teachers address vision-to-practice gaps" (p. 605). Both studies document how tools can be used to support enactment and manage complexity of teaching while "maintaining its improvisational nature" (Ghousseini et al., 2015, p. 492). Additional studies, such as my analysis of tools presented in this chapter, are needed to continue this work studying how tools can guide TC participation and learning in practice-based coursework.

In this chapter I discuss my selection of the three tools: lesson plans, board work and materials, and talk moves. This is followed by a three-part discussion of my findings for each tool, including how they were used in the course and how they facilitated OTL. I conclude with a summary of these findings across the three tools.

**Selection of Analyzed Tools**

The selected set of tools (lesson plans, teaching materials, and talk moves) was chosen based on the ways that practice was reified in the context of the course and current research and theories about tools, routines, and scaffolds for TC learning. Like Thompson and colleagues' (2013) choice to examine tool and practices that supported ambitious science teaching, I narrowed my scope to examine tools that represented the work to facilitating productive mathematical discussion. In the discussion below, I highlight this process and justification for selecting these three tools.

### *Lesson Plans*

A central component of facilitating productive mathematical discussions is planning, including identifying a mathematical goal (Chapin et al., 2013; Smith & Stein, 2011; Shaughnessy et al., 2019). Lesson plans have been used in teacher education as tools for investigating and representing practice (Grossman, Hammerness, et al., 2009). Despite the focus on planning practices in teacher education, less attention has been placed on how TCs use these lesson plans in enactments of practice. For the course studied, TCs were provided with a Planning Protocol and a Planning Template[22] for each instructional activity. By examining how they used these protocols and planning templates throughout the cycle of enactment and investigation, I could understand TCs intentions in leading the instructional activities, how they were interpreting the work of planning, what aspects of facilitating productive mathematical discussion they were attending to, and how lesson plans scaffolded enactment and provided OTL.

### *Board Work and Materials*

The second set of tools I examined were teaching materials. I focused on examining use of the board and representations on this board. Mathematics education research highlights the importance of using representations and "engaging students in making connections among mathematics representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving" (National Council of Teachers of Mathematics, 2014, p. 24). Additionally, *represent mathematical ideas visually, numerically, and/or verbally* was named as one of the seven ambitious practices worked on in the course and is an important part of facilitating productive mathematical discussion (see Appendix B; Kelley-Peterson, 2010). Analyzing use of

[22] See Appendix E for an example of the Choral Counting Planning Protocol and Appendix F for the Choral Counting Planning Template.

the board offered a perspective of how mathematics was worked on and revealed how TCs attended to *positioning students competently* (by valuing and taking up their ideas on the board) and *orienting students to each other's ideas and the mathematical goal* (two additional ambitious practices named in the course).

***Discourse Tools and Talk Moves***

Given my dissertation's focus on facilitating productive mathematical discussions, I wanted to include a discourse tool (or routine) in my selection of tools. Mathematics educators have articulated many routines and techniques to support productive math talk including talk moves (Chapin et al. 2013), exchanges routines (Leinhardt & Steele, 2005), funneling and focusing techniques (Sherin, 2002), question types (Boerst et al., 2011, Tyminski et al., 2014) and sociomathemaitcal norms for argumentation (Yackel, 2002). My choice to examine talk moves was guided by the literature documenting the productive ways talk moves can support discourse (and the different types of talk moves) and the context of the course. As part of the course, TCs read chapters from Chapin et al. (2013) *Classroom Discussions in Math* and explicitly talked about and worked on using these moves in lessons. Talk moves were both a named part of the shared repertoire of the group and discrete techniques within high-quality mathematics instruction that are particularly useful for facilitating discussion.

For all three groups of tools, I examined how they were used in practice and how they facilitated OTL for TCs. Findings documented the changes in the content and use of lesson plans, ownership and use of the board, and use of talk moves to facilitate discussion. Across all three tools, there was a shift from a focus on self (teacher) to students, particularly in using these tools to anticipate student thinking and to consider possible responses and representations that moved towards a mathematical goal. Lesson plans scaffolded learning about the structure of the

instructional activities, directed TCs' attention to students, representations, and mathematics, and provided a space for the TE to provide individual feedback. The board served as a collective space to practice representations and mathematical ideas, an opportunity to think through the lesson (including the mathematics, representations, teacher moves, and possible student ideas), and a record of public thought and artifact for discussion. Talk moves became a common language to name discrete moves and collectively negotiate how to use these moves intentionally in discussion facilitation.

**Lesson Plans**

In this section I highlight changes in 1) the content of the lesson plans, 2) the use and purpose of the lesson plans, and 3) how lesson plans were tools to facilitate opportunities for TC learning. TCs were provided with a Planning Protocol and a Planning Template[23] for each instructional activity. The Planning Protocols and Planning Templates were created by the TE and colleagues[24] and had slightly different formats depending on the structure and purpose of each instructional activity. Planning Templates focused on the specific task and included places to anticipate student thinking (and possible teacher responses), plan for how to use representations (including board work) and note ways to highlight key mathematics. They were designed to help TCs specify the task, articulate their goal, anticipate student thinking based on the task, and ask questions and respond to student thinking. Although all TCs used the TE-provided protocols and planning templates, the content of what TCs wrote in them varied among TCs and over time. The

[23] See Appendix E for an example of the Choral Counting Planning Protocol and Appendix F for the Choral Counting Planning Template.

[24] Creation of instructional activity Planning Protocols and Planning Templates is an ongoing collaborative effort with TEs from the course and colleagues from Teacher Education by Design (TEDD) and LTP: Learning in, from, and for teaching practice. For more examples of these protocols, templates, the collaborative work, see TEDD.org, *Intentional Talk* (Kazemi & Hintz, 2014), and *Choral Counting and Counting Collections* (Franke et al., 2018).

level of detail and mathematical content in these plans improved over the ten weeks. Additionally, the ways these lesson plans were used and the purpose they served also varied throughout the quarter. In the following sections, I discuss these changes.

### *Changes in the Content of the Lesson Plans*

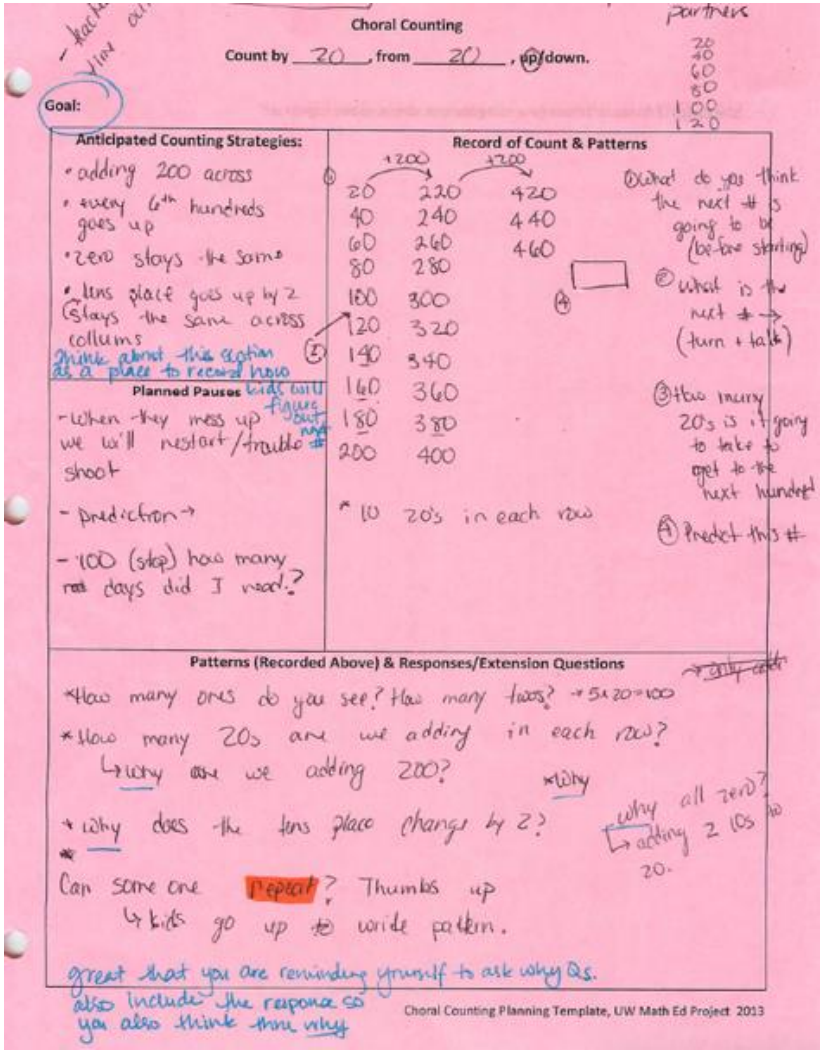
As the course progressed, the content of the lesson plans changed. The content of early lesson plans focused on the teacher moves and questions. These early lesson plans were either very teacher-centered scripts or (in contrast) filled-in planning templates with many blank spaces and incomplete areas. Anticipated patterns or strategies were either not specific to the task, very general ideas, or not included. The goal and closure were not included or did not connect to a clear mathematical goal.

In later sessions, TCs included a mathematical goal, anticipated student strategies and representations for these ideas, and outlined teacher moves and questions connected to the goal. The representations in the lesson plans improved over the ten weeks by becoming more mathematically accurate, more clearly representing student thinking and mathematical ideas, and specific to the particular mathematics and task. Changes in the content of the lesson plans can be characterized in the following ways:

1. More complete plans and planning around a particular task.
2. Inclusion and increased specificity of a mathematical goal.
3. Less teacher-focused scripting or listing of teacher moves and more focus on moves connected to student strategies and mathematical ideas.
4. Inclusion and anticipation of student strategies and representations.
5. Increased accuracy and anticipation of representations.

I included several lesson plans (below) that characterize the changes in the lesson plan content over time. Lesson Plan 1 is Katelina's choral counting lesson plan from the first lesson (**Figure 5**). This example is typical of an earlier lesson plan that is vague and incomplete in places. Lesson Plan 2 (**Figure 6**) is Orianne's typed lesson plan for Estimation 180 on the second day of teaching (the third course meeting). This is an example of a very detailed, teacher-focused lesson plan. Lesson Checklist 1 (**Figure 7**) is Melissa's handwritten lesson checklist for the same Estimation 180 activity. Lesson Plan 3 (**Figure 8**) is Melissa's lesson plan for a String activity in the second to last day (including a handwritten checklist). Lesson Plan 4 (**Figure 9**) is Katelina's lesson plan for a Mental Math activity on the same day. Lesson Plan 3 and 4 are examples of later lesson plans that included a mathematical goal and focused on student strategies and representations. In the following section, I use these four example lesson plans to elaborate on the changes in the lesson plan content over time. I detail what kinds of scripts and moves were included in their lesson plans and the content and ideas TCs included.

**Figure 5** Lesson Plan 1: Katelina's lesson plan for the choral counting activity on the first day of teaching.



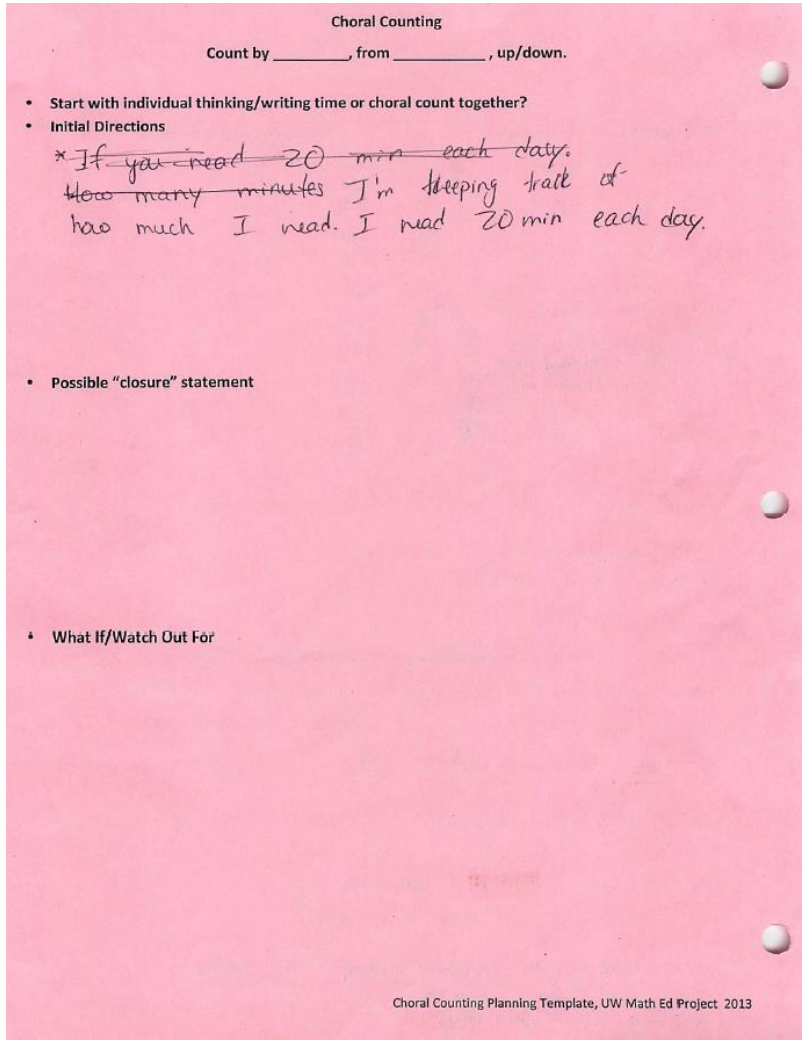
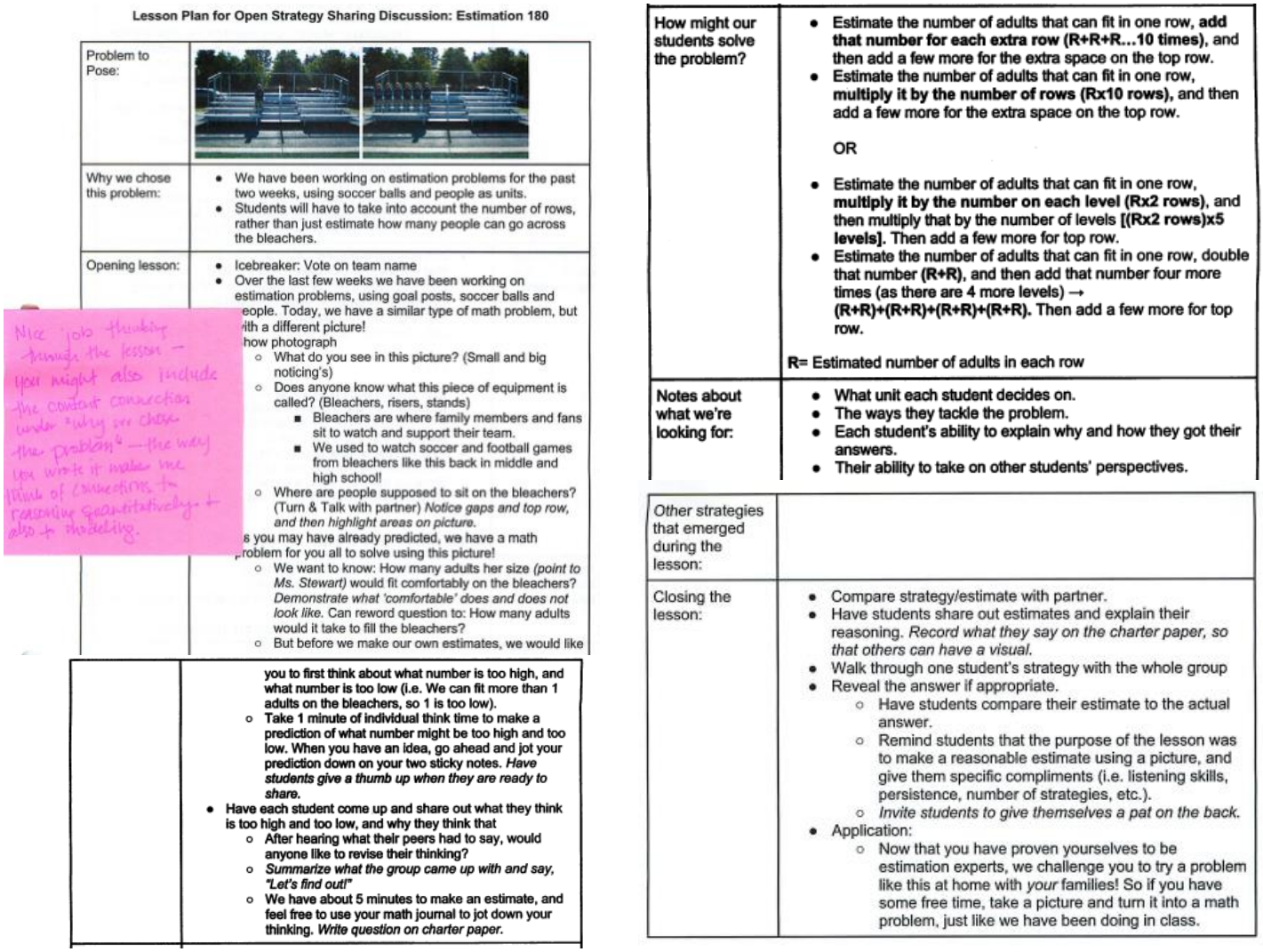



**Figure 6.** Lesson Plan 2: Melissa and Orianne's typed lesson plan for Estimation 180 on the second day of teaching.



**Lesson Plan for Open Strategy Sharing Discussion: Estimation 180**

| | |
|---|---|
| Problem to Pose: |  |
| Why we chose this problem: | • We have been working on estimation problems for the past two weeks, using soccer balls and people as units.<br>• Students will have to take into account the number of rows, rather than just estimate how many people can go across the bleachers. |
| Opening lesson: | • Icebreaker: Vote on team name<br>• Over the last few weeks we have been working on estimation problems, using goal posts, soccer balls and eople. Today, we have a similar type of math problem, but ith a different picture!<br>how photograph<br>○ What do you see in this picture? (Small and big noticing's)<br>○ Does anyone know what this piece of equipment is called? (Bleachers, risers, stands)<br>▪ Bleachers are where family members and fans sit to watch and support their team.<br>▪ We used to watch soccer and football games from bleachers like this back in middle and high school!<br>○ Where are people supposed to sit on the bleachers? (Turn & Talk with partner) *Notice gaps and top row, and then highlight areas on picture.*<br>s you may have already predicted, we have a math problem for you all to solve using this picture!<br>○ We want to know: How many adults her size *(point to Ms. Stewart)* would fit comfortably on the bleachers? *Demonstrate what 'comfortable' does and does not look like.* Can reword question to: How many adults would it take to fill the bleachers?<br>○ But before we make our own estimates, we would like |
| | **you to first think about what number is too high, and what number is too low (i.e. We can fit more than 1 adults on the bleachers, so 1 is too low).**<br>**○ Take 1 minute of individual think time to make a prediction of what number might be too high and too low. When you have an idea, go ahead and jot your prediction down on your two sticky notes. *Have students give a thumb up when they are ready to share.***<br>**• Have each student come up and share out what they think is too high and too low, and why they think that**<br>**○ After hearing what their peers had to say, would anyone like to revise their thinking?**<br>**○ *Summarize what the group came up with and say, "Let's find out!"***<br>**○ We have about 5 minutes to make an estimate, and feel free to use your math journal to jot down your thinking. *Write question on charter paper.*** |
| **How might our students solve the problem?** | • **Estimate the number of adults that can fit in one row, add that number for each extra row (R+R+R...10 times), and then add a few more for the extra space on the top row.**<br>• **Estimate the number of adults that can fit in one row, multiply it by the number of rows (Rx10 rows), and then add a few more for the extra space on the top row.**<br>**OR**<br>• **Estimate the number of adults that can fit in one row, multiply it by the number on each level (Rx2 rows), and then multiply that by the number of levels [(Rx2 rows)x5 levels]. Then add a few more for top row.**<br>• **Estimate the number of adults that can fit in one row, double that number (R+R), and then add that number four more times (as there are 4 more levels) → (R+R)+(R+R)+(R+R)+(R+R). Then add a few more for top row.**<br>**R= Estimated number of adults in each row** |
| **Notes about what we're looking for:** | • **What unit each student decides on.**<br>• **The ways they tackle the problem.**<br>• **Each student's ability to explain why and how they got their answers.**<br>• **Their ability to take on other students' perspectives.** |
| *Other strategies that emerged during the lesson:* | |
| Closing the lesson: | • Compare strategy/estimate with partner.<br>• Have students share out estimates and explain their reasoning. *Record what they say on the charter paper, so that others can have a visual.*<br>• Walk through one student's strategy with the whole group<br>• Reveal the answer if appropriate.<br>○ Have students compare their estimate to the actual answer.<br>○ Remind students that the purpose of the lesson was to make a reasonable estimate using a picture, and give them specific compliments (i.e. listening skills, persistence, number of strategies, etc.).<br>○ *Invite students to give themselves a pat on the back.*<br>• Application:<br>○ Now that you have proven yourselves to be estimation experts, we challenge you to try a problem like this at home with *your* families! So if you have some free time, take a picture and turn it into a math problem, just like we have been doing in class. |

Nice job thinking through the lesson — you might also include the content connection under "why we chose the problem" — the way you wrote it makes me think of connecting to reasoning quantitatively + also to modeling.

**Figure 7.** Lesson Checklist 1: Melissa's handwritten checklist lesson "plan" for Estimation 180 activity on the second day of teaching.

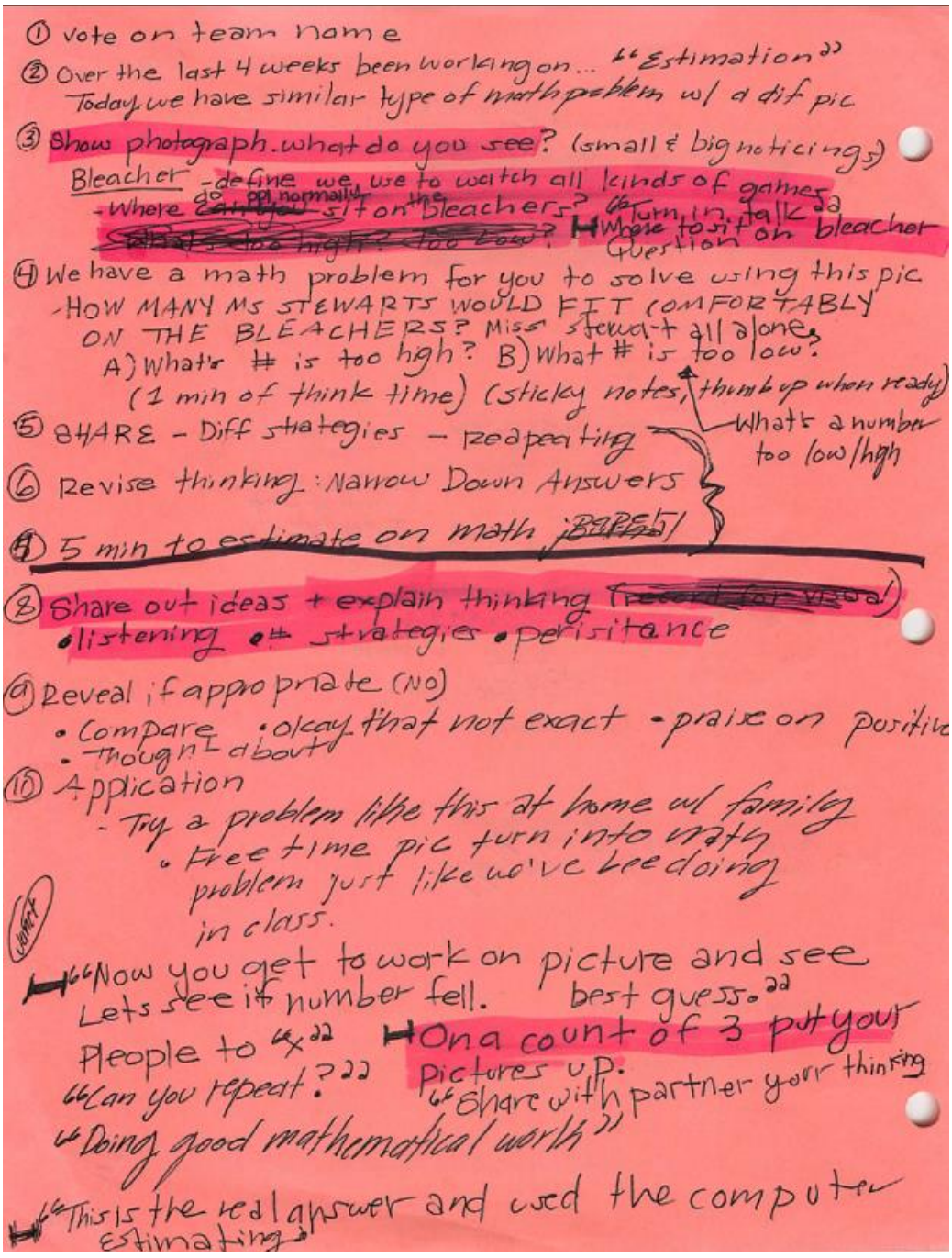
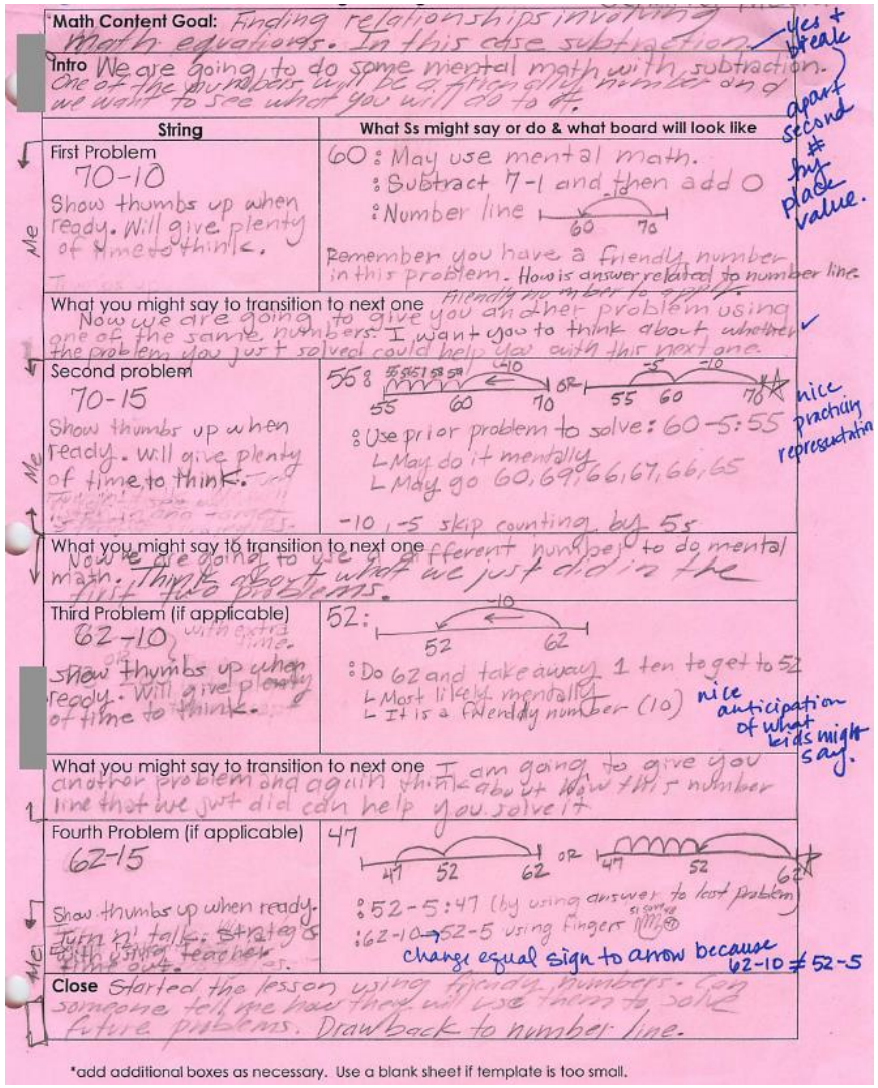


**Figure 8** Lesson Plan 3: Melissa's lesson plan and written script for a String activity on the second to last day.



**Math Content Goal:** Finding relationships involving math equations. In this case subtraction.

**Intro** We are going to do some mental math with subtraction. One of the numbers will be a friendly number and we want to see what you will do to it.

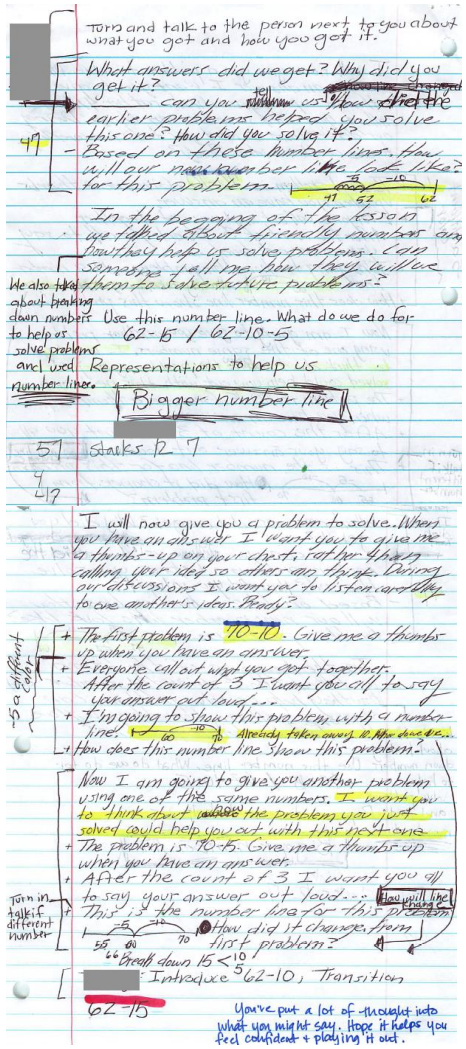
| String | What Ss might say or do & what board will look like |
|---|---|
| First Problem<br>70-10<br>Show thumbs up when ready. Will give plenty of time to think. | 60 : May use mental math.<br>: Subtract 7-1 and then add 0<br>: Number line<br>Remember you have a friendly number in this problem. How is answer related to number line. |
| What you might say to transition to next one<br>Now we are going to give you another problem using one of the same numbers. I want you to think about whether the problem you just solved could help you with this next one. | |
| Second problem<br>70-15<br>Show thumbs up when ready. Will give plenty of time to think. | 55 : 55 60 70 OR 55 60 70<br>: Use prior problem to solve: 60-5: 55<br>└ May do it mentally<br>└ Mdy go 60, 69, 66, 67, 66, 65<br>-10, -5 skip counting by 5s |
| What you might say to transition to next one<br>Now we are going to use a different number to do mental math. Think about what we just did in the first two problems. | |
| Third Problem (if applicable)<br>62-10<br>Show thumbs up when ready. Will give plenty of time to think. | 52 : 52 62<br>: Do 62 and take away 1 ten to get to 52<br>└ Most likely mentally<br>└ It is a friendly number (10) |
| What you might say to transition to next one<br>I am going to give you another problem and again think about how this number line that we just did can help you solve it. | |
| Fourth Problem (if applicable)<br>62-15<br>Show thumbs up when ready. Turn n' talk: Strategies with partner teacher time out. | 47 : 47 52 62 OR 47 52 62<br>: 52-5: 47 (by using answer to last problem)<br>: 62-10→52-5 using fingers<br>change equal sign to arrow because 62-10 ≠ 52-5 |
| **Close** Started the lesson using friendly numbers. Can someone tell me how they will use them to solve future problems. Draw back to number line. | |

*add additional boxes as necessary. Use a blank sheet if template is too small.

yes + breaks apart second # by place value.

nice practicing representation

nice anticipation of what kids might say.



Turn and talk to the person next to you about what you got and how you got it.

What answers did we get? Why did you get it?

- can you [illegible] use your [illegible] earlier problems helped you solve this one? How did you solve it?
- Based on these number lines, how will our mental number line look like for this problem.

47 52 62

In the beggining of the lesson we talked about friendly numbers and how they help us solve problems. Can someone tell me how they will use them to solve future problems?

We also talked about breaking down numbers to help us solve problems and used number lines.

Use this number line. What do we do for 62-15 / 62-10-5

Representations to help us

Bigger number line

57 Starks 12 7

I will now give you a problem to solve. When you have an answer I want you to give me a thumbs-up on your chest, not for you calling your idea so others can think. During our discussions I want you to listen carefully to one another's ideas. Ready?

- The first problem is 70-10. Give me a thumbs up when you have an answer.
- Everyone call out what you got together. After the count of 3 I want you all to say your answer out loud...
- I'm going to show this problem with a number line. Already taken away 10 from 70
- How does this number line show this problem.

-5 a different color

Now I am going to give you another problem using one of the same numbers. I want you to think about whether the problem you just solved could help you out with this next one.

- The problem is 70-15. Give me a thumbs up when you have an answer.
- After the count of 3 I want you all to say your answer out loud...
- This is the number line for this problem. How will the thing

Turn in talk if different number

How did it change from first problem?

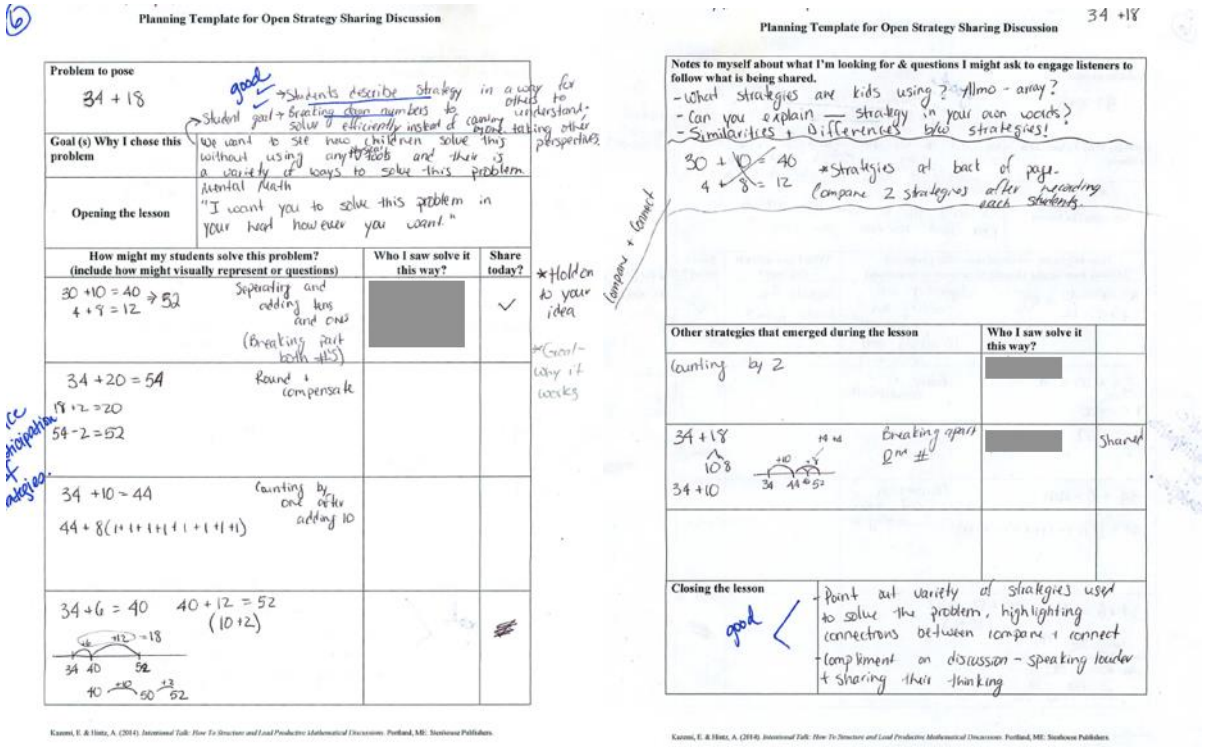
Break down 15 < 10

Introduce 62-10, Transition

62-15

You've put a lot of thought into what you might say. Hope it helps you feel confident + playing it out.

**Figure 9.** Lesson Plan 4: Katelina's lesson plan for a Mental Math activity on the second to last day.




**More complete plans and planning around a particular task.** In earlier sessions, TCs left some planning spaces blank and (when using Protocols) relied on the general guidelines in the Protocol for what to do next in the instructional activity, despite there being space next to the general description to elaborate and plan for the particular task. This suggests TCs may have seen the instructional activity guideline and template as enough or may not have known how to plan around a task. Teacher moves were not specific to the task or were very general ideas. They either had very teacher-focused scripts or templates with many blank spaces and incomplete areas.

Lesson Plan 1, Katelina's choral counting lesson plan from the first lesson, is an example of a generic and incomplete lesson plan (**Figure 5**). There were several places left blank, including the goal, the "closure" statement, and "What if/Watch Out For." These three sections are important for thinking about how to move beyond simply facilitating interaction and eliciting student talk to

facilitating a productive mathematical discussion. It is not clear why she left these blank. The blank spaces suggest she may not have been planning the choral counting activity with a particular goal or mathematical idea in mind. As a novice, she may have been unsure about what to write or what would be useful in planning. She included initial directions, but they were written as a hook to interest students and did not necessarily set up how students should participate or direct students to think about a mathematical idea. Like most of the lesson plans, she listed teacher moves, but the teacher moves as well as anticipated strategies and board work were generic.

The contrast in completeness and specificity around a particular task between this first lesson plan and the third and four lesson plans (both from the second to last day) is striking. Lesson Plan 3, Melissa's lesson plan for a String activity (**Figure 8**), and Lesson Plan 4, Katelina's lesson plan for a Mental Math activity (**Figure 9**), are examples of lesson plans that are mostly complete and specific to a task. They included specific instructional goals, how to open and close the lesson, anticipated representations and student responses, and teacher moves that are specific to the task. For example, in Lesson Plan 3 (**Figure 8**), Melissa included a content goal ("Finding relationships involving math equations. In this case subtraction.") and anticipated representations for each problem and possible student responses. In her lesson plan, it is clear how she planned to progress through the activity. She included an introduction, teacher problems for each problem and for transitions, and a closure that is specific to this number string. The TE comments (written on the post it) were also more about the mathematics and clarify the goal as opposed to clarifying what the template means and reminding TCs to fill in the different spaces (e.g., "Nice anticipation of what kids might say" and "Change equal sign to arrow because 62 – 1- does not equal 52 – 5").

**Inclusion and increased specificity of the goal.** As discussed in the previous analyses, the goal in earlier lesson plans was either unwritten, not a mathematical goal, or vague. Rather than

articulate a specific goal, the TCs would list their task (e.g., count by 20s from 20) and leave vague and generic descriptions of what to do. For example, in Lesson Plan 1, Katelina left the "Goal" line blank (**Figure 5**). In the "Why we chose this problem" section for Lesson Plan 2, Orianne wrote, "Students will have to take in to account the number of rows, rather than just estimate how many people go across the bleachers" (**Figure 6**).

This could have connected to a mathematical goal around multiplicative relationships. However, her discussion facilitation didn't indicate this was the focus of her lesson. In her note, the TE pressed on this by asking about how this connects to a content goal and student thinking (reasoning quantitatively, modeling, etc.).

Although her goal was not clear, Orianne included detailed options for a closure. These included comparing a strategy with a partner, having students share out estimates, walking through a student's strategy with the whole group, reveal the answer, and applying what they learned to a new problem (though a new problem is not offered). She also stated the purpose in this closure, "make a reasonable estimate using a picture, and give them specific compliments," which seemed include both a student goal (former half) and teacher move (latter half).

In the last two sessions, TCs specified instructional goals that were tied to particular mathematical ideas or practices. For example, in Lesson Plan 3 (**Figure 8**), Melissa wrote, "Finding relationships involving math equations. In this case subtraction," as her goal. The TE left her a note to be more specific: "Yes + break apparent second # by place value." Although her goal was still a broad mathematical content goal, having this starting point allowed the TE to provide feedback on her stated goal and to press on what specific mathematical idea was being addressed. In Lesson Plan 4 (**Figure 9**), Katelina included this more mathematically specific type of goal: "Students describe strategy in a way for others to understand; Student goal → Breaking down numbers to solve

efficiently instead of counting by ones." This goal is mathematical (focusing on the idea of decomposition of numbers), specific to the task posed (mental math for "34 + 18"), and student-centered (specifies what students will do). Later in the lesson plan, she included places to compare and connect strategies that highlight her math content goal.

Related to the goal, their closure section also improved. In the beginning, the closure was left blank or was very generic (e.g., "Good job mathematicians!"). In Lesson Plan 3 (**Figure 8**), Melissa made notes to connect back to the strategies being used and the number line representations that she anticipated: "Started the lesson using friendly numbers. Can someone tell me how they will use them to solve future problems. Draw back to number line." This closure opened space for student voice and circled back to the math content goal and representations. Similarly, in Lesson Plan 4 (**Figure 9**), Katelina planned on highlighting connections between strategies that they compared and connected: "Point out a variety of strategies used to solve the problem, highlighting connections between compare and connect." The inclusion and increased specificity indicate TCs were able to include a more specific math content goal and were improving in being able to anticipate representations and student strategies and teacher moves to support learning around this goal. I elaborated on these findings in the previous chapter.

**Scripting or listing teacher moves to anticipating strategies and mathematics.** In early sessions, lesson plans were very teacher-focused and usually included a teacher script and/or a list of teacher moves and questions (often generic and not specific to the mathematical goal). In later sessions, TCs included a mathematical goal, anticipated student strategies and representations for these ideas, and outlined specific teacher moves and questions that were connected to the goal. The focus shifted to anticipating student strategies and mathematical ideas.

In Lesson Plan 1, Katelina listed four questions which functioned as a checklist of teacher questions. This may indicate Katelina was still making sense of the activity and possible teacher moves and/or that she had a limited repertoire for more task-specific and student-focused planning. It's not clear if these were written before the rehearsal or as a result of the rehearsal, but they did serve as a list and order of questions to ask during the enactment. This first lesson plan is an example of a beginning and partially filled-in lesson plan that focused on generic teacher moves. In contrast, Lesson Plan 2 was very detailed and scripted and is an example of the type of scripting TCs did in earlier lessons. The typed lesson plan was a joint lesson plan written by Orianne and Melissa. They used the Estimation 180 template to write down the teacher moves, including what to say and what to do. Though this lesson plan was more detailed than Katelina's, it still did not include a specific mathematical goal, anticipate a wide range of strategies, or explicitly include a plan for how to facilitate discussion after students have estimated (like Katelina).

Orianne's lesson plan (**Figure 6**) was very detailed and included what they would say and ask students to do. While she was considering what questions to ask and how to support student thinking (particularly in the launch and set up of the task), she needed support and practice with anticipating student strategies, articulating a mathematical goal, and being explicit about how she would facilitate discussion after students estimated (e.g., she said "walk through one student's strategy" rather than planning for how she would facilitate discussion). Though she didn't explicitly outline how she would facilitate a discussion, Orianne included talk moves, such as a turn and talk about "where people are supposed to sit on the bleachers" and a revision move: "After hearing what their peers had to say, would anyone like to revise their thinking?"

For a novice's first lesson plan, her planning was impressive. Her lesson plan included many of the elements that had been discussed in the class: talk moves, launching the task to help

students make sense of the problem, etc. This attention to planning was reflected in her enactment, where she successfully launched the activity, used talk moves, and elicited student ideas. She was the only TC during the beginning of the quarter who used her observational notes about students to decide who should share. While her later lesson plans seemed less formal (many are handwritten, not typed), she maintained this level of detail while also increasing her ability to anticipate student thinking and articulate how she would facilitate the discussion. While her lesson plan was one of the more detailed lesson plans, the content indicates she was still very self-focused and needed to know what to do and say next. Like all earlier lesson plans, TC attention was on themselves and teacher moves, not the mathematical goal or student thinking. This focus on self may indicate that she was still making sense of her role as the teacher and making sense of the instructional activity.

In Lesson Checklist 1, we see Melissa's "plan" for the same lesson (**Figure 7**). Although the longer typed up lesson plan (**Figure 6**) was created together, Melissa had her own handwritten to-do list scripting teacher move and questions. She used this handwritten "plan" rather than the shared, detailed lesson plan that Orianne used, perhaps because the detailed lesson plan was overwhelming to use in the moment or for some other reason. She took notes throughout the rehearsal, so it's not clear how much of this checklist was written during the rehearsal and if any of it was written in advance before class or in the morning planning time. This handwritten plan is very much a to-do checklist. She wrote down what she needs to do (e.g., "vote on team name" or "Reveal if appropriate") and possible wording and phrasing for what she could say (e.g., "Now you get to work on picture and see if number fell…" or "This is the real answer and used the computer estimating"). While later lesson plans included some phrasing and teacher moves, in contrast to

these plans, there was also inclusion and anticipation of student strategies. Below, I discuss this change in lesson plan content.

**Inclusion and anticipation of student strategies.** All the templates had space to anticipate student strategies. In the beginning, TCs wrote narratively and wrote a general strategy that students might do. As they had more experiences rehearsing and teaching students, they anticipated a wider range of strategies based on what they knew about their group of students.

In the choral counting template, there was space, "Record of Count and Patterns," to anticipate student thinking and board work. Katelina filled in these spaces in Lesson Plan 1 (**Figure 5**); however, the content was not detailed. She did not use color in the "Record of Count and Patterns" and only highlighted two patterns. In the space to record counting strategies, she noted different patterns, but not student strategies (the TE made a comment about this in blue). In Lesson Plan 2 (**Figure 6**), Orianne only had two anticipated strategies, both which were symbolically focused (estimating the number in each row and then using repeated addition or multiplication to calculate). She didn't note how students might estimate nor did she anticipate any drawing-based strategies, which were the strategies students used during enactment. Her limited anticipated strategies and "what to look for" are not surprising, given that she was a novice teacher in the beginning of the course.

With more classroom visits, TCs got to know their particular students much better and improved their abilities to anticipate what students might say or do socially and mathematically. In Lesson Plans 3 and 4, Melissa and Katelina came up with several strategies to solve their respective tasks and possible student responses. In Lesson Plan 4, Katelina anticipated a *tens and ones* strategy that involved breaking apart numbers (connected to her mathematical goal). Four of the six students used this strategy. Because she anticipated the most commonly used strategy, when

a student used a different decomposition strategy that she did not anticipate, she was able to have students compare and connect these similar but slightly different decomposition strategies.

**Accuracy and anticipation of representations.** The representations in the lesson plans improved over the ten weeks by becoming more mathematically accurate, clearer in representing student thinking and mathematical ideas, and specific to the mathematics and task at hand. However, they never used color on their lesson plans to practice what the board might look like, which was highlighted and discussed in rehearsals and taken up in enactments. In some planning templates used in this course sequence, there was space for anticipating board work. Whether they anticipated board work or not and how mathematically accurate it was varied throughout the quarter. I expand on their use of the board and representations later in this chapter.

In the section above, I illustrated how the content of the lesson plans changed. Earlier lesson plans focused on the teacher moves/questions and could be quite vague or incomplete. In contrast, later lesson plans were more mathematically accurate, included specific goals and anticipated strategies, and focused on moves connected to student strategies and mathematical ideas. As the content of the lesson plans changed, the TCs changed how they used their lesson plans. In the following section, I document the changes in the use of the lesson plans.

### Changes in the Use and Purpose of the Lesson Plans

In video analysis of the sessions, TCs moved from clutching the lesson plan as a "script" or to-do list during their rehearsal and enactment to (in later sessions) using the lesson plans as a place for anticipatory work that can be referenced during the discussion if needed.

In the beginning of the quarter, TCs used their lesson plans either as 1) rigid scripts of teacher talk, clutching them as they rehearsed and taught the lesson, or 2) as a list of general teacher moves for the instructional activity (not for a particular mathematical goal). Because the lesson

plans included scripts and/or teacher moves, the TCs used the lesson plans as a checklist of what to do or say and kept their lesson plans in hand or next to them to reference in both rehearsals and enactments. When suggestions were made during the rehearsal, TCs wrote these teacher moves or phrasing of questions down in their lesson plan. This may indicate that while the instructional activities were intended to provide a "stable and rehearsable backdrop for the dynamic work of responding to student thinking," TCs were working to understand and make sense of these routines, especially in the beginning of the course (Lampert et al., 2010, p. 133).

Their rehearsal and enactment closely followed what they wrote. For the first few sessions, most TCs held on to the lesson plans and used them like scripts to open the lesson and when the ask a question or go to a new part of the lesson. In the rehearsals, they held their lesson plans or set them next to where they were teaching. They did the same thing in the enactment, looking at their lesson plan for how to phrase questions and the next move (indicating that they had not yet routinized talk moves and the steps of the instructional activities). TCs took notes on their lesson plans, especially when the TE made comments, asked questions, or gave suggestions. These notes included directly transcribing the TE phrasing of questions and teacher talk or adding notes on what to do next in different parts of the instructional activity.

Halfway through the quarter, TCs were still rehearsing with the lesson plans sitting next to them and adding notes on their plans throughout the rehearsal. However, they seemed more relaxed in adding notes, unlike the first lessons where they were quickly scribbling in their lesson plans when the TE talked. For the enactment, they had their lesson plans next to them or in hand, and reference as necessary (not for every move).

In later lessons, TCs looked at the lesson plans to remember specifics about the problem(s) or task and occasionally what question to ask, rather than using the lesson plan as a script. They

still took notes during rehearsals when TE or other TCs made suggestions. In these later sessions, lesson plans became opportunities to anticipate student strategies and to plan moves and questions that connected to a specific mathematical goal. This marked a shift in the purpose of the lesson plans in relation to facilitating productive mathematical discussion and their vision of teaching (Chapin et al., 2013; Thompson et al., 2013; Timperley, 2013). Instead of writing down a script of teacher questions (as in the first lessons), they anticipated student thinking and wrote possible questions to elicit this thinking and press on student reasoning. The possible moves they listed were specific to the mathematical goal and what they anticipated their particular students might do.

All TCs used the given format provided in the lesson planning templates, and a couple TCs use their own format. There was variation in how the TCs initially used lesson plans. Some lesson plans were extremely scripted, and others were quite vague. For example, Melissa really needed a script and was rarely able to deviate from this script, even if they made changes during rehearsal. Katelina's lesson plans indicated she wanted to make sure that she had thought about the activity structure. Initially, she did not include a mathematical goal or closure, and her anticipated strategies, representations, and teacher questions were generic. Orianne wrote the most thorough lesson plans, thinking through the activity and carefully attending to student thinking. Despite these differences in how TCs initially approached lesson plans, all TCs moved towards using the lesson plans as an opportunity to anticipate student thinking, representations, and teacher moves to plan for their facilitation of a mathematical discussion. Lesson plans became a place to practice representations, to note specific teacher prompts and phrasing based on the rehearsal, and to take notes about student thinking and strategies during enactment.

Though the content of the lesson plans changed and there was variation in how TCs initially approached lesson plans, they consistently used lesson plans in every session during planning time, rehearsal, enactment, and debriefs. Throughout the quarter, they used lesson plans as references for phrasing and what move to use. Thus, what they included in the lesson plan was also seen in how they rehearsed and enacted the lesson. They were consistent in taking notes in their lesson plans during rehearsals, and the non-teaching TCs used the observation templates, a blank sheet of paper, or lesson plans (if they wrote one) to take notes about students. Observation notes during lesson helped them record student thinking, strategies, and participation, which they used in the debriefs.

In the beginning, lesson plans were like to-do lists: do this, ask this, say this, etc. Later in the quarter, TCs used the lesson plans less as a way to know what to do next and more as a way to think through how to facilitate a discussion (thinking both before the rehearsal, together during the rehearsal to learn from everyone, and during the debrief as a record of what happened). Thus, the use and purpose of the lesson plans changed from being a script and/or list of teacher moves, to being a place to anticipate and record student strategies and use of representations on the board. In later sessions, lesson plans were used as opportunities to anticipate student strategies and to plan moves and questions that connected to a specific mathematical goal (Chapin et al., 2013; Smith & Stein, 2011).

***Lesson Plans as Tools to Facilitate Opportunities for TC Learning***

As a tool, lesson plans facilitated opportunities for TC learning by 1) providing scaffolding to the structure/purpose of the instructional activity (e.g., start with an intro, hold up the image, etc.), 2) focusing TC attention (e.g., there was a space to anticipate strategies, for closure, to anticipate representations on the board, etc.), and 3) becoming a boundary object mediating the

thinking of TCs and feedback and comments from the TE. Lesson plans became places to work on practice outside of the lesson enactment and are a tool for learning before, during, and after teaching.

**Provided scaffolding to the structure and purpose of the instructional activity.** In all four lesson plans, TCs worked to make sense of the instructional activity and instructional sequence (what to ask, what to do next, the order of the activity, etc.). The lesson plans provided scaffolding to the structure and purpose of the instructional activity. Although they initially left many spaces blank or vague, having possible teacher prompts, places to fill in the problem and anticipate board work and student thinking, and a space for closure in the lesson plan provided scaffolding for understanding and decomposing the activities (Boerst et al., 2011; Grossman, Hammerness, et al. 2009). As a representation of practice, the format of the lesson plan template guided how TCs went thought the instructional activity, prompting them to consider different aspects of practice and mathematics.

The format and content of the lesson plan seems important when thinking about how to support TC participation. Lesson plans that are too lengthy and detailed may be too overwhelming, perhaps contributing to the blank spaces left by TCs and their initial use of their own simplified teacher scripts and lists of teacher moves. However, as representations of practice, lesson plans make practice studyable and should include decompositions of elements essential to high-quality teaching practice to focus TC attention (Grossman, Hammerness, et al., 2009; Thompson et al. 2013). Lessons plans that include places to anticipate and think about student thinking can support TCs to develop pedagogical content knowledge and to enact student-centered teaching practices.

Examining the intentionality and goal specific by instructional activity (see Appendix I) revealed that early enactments of activities were less specific than later enactments and certain

instructional activities had increased levels of specificity and intentionality. This indicates the need to purposefully sequence instructional activities and approximations of practice (Boerst et al., 2011) and to better understand the affordances and constraints of particular activities (Cunard, 2014). This also indicates the potential usefulness of using the same instructional activity more than once (Boerst et al., 2011; Lampert et al., 2010). As the instructional activity becomes routinized (TC become more familiar with the activity structure and what to do next), the instructional activity becomes a vehicle for practicing more ambitious and complex practices and to learn about student thinking (Lampert et al., 2010; McDonald et al., 2013). Even in the lesson plans, we see that in the beginning of the quarter, TCs are just trying to figure out what to do next. Once they make sense of the structure and purpose of the instructional routine, they are better able to focus on student thinking, the mathematical goal, and how they will facilitate discussion around that goal.

**Focused TC attention.** While lesson plans started as scripts that TCs clutched in their hands through the lesson, they eventually used lesson plans as a more anticipatory space. The lesson plans decomposed and represented practice in particular ways, focusing TC attention by having blank spaces for their goal, opening and closing, and anticipated representations and student strategies. As they improved their planning and more completely filled in the lesson plans, they also became more specific about their math content goals, and they connected representations, anticipated student strategies/responses, and teacher moves to these goals. Having space to fill these things in focused the TC attention. TCs didn't initially fill out these spaces, perhaps because of their limited experience with students or because they were not sure what the spaces meant. Regardless of the reason, actually teaching the lesson supported TCs in understanding what components in the lesson plan meant and what needed to be planned. They moved from using the

lesson plans as a script and a list of teacher moves to attending to anticipation of student thinking and ways to represent ideas.

**Lesson plans as a boundary object mediating the thinking of TCs and feedback and comments from the TE.** Lesson plans allowed the TE to see what TCs were focused on during planning (and notetaking during lesson) to provide feedback. The lesson plans became an object to interact with: a way to document practice and a way to talk about practice before (lesson planning, rehearsal) and after (debriefing) the actual teaching. Lesson plans were reifications of practice, reflecting the TCs' understanding of practice (Thompson et al., 2013; Wenger, 1998). Their lesson plans made TC thinking more visible to the TE and other TCs in the groups. This indicates the importance of lesson plans as tools to mediate interactions and opportunities for TC learning when the TE provides feedback (Wenger, 1998).

## Board Work and Materials

In the previous section, I examined how TC participation and opportunities to learn were facilitated through lesson planning. In this section, I focus on board work and materials, particularly how these were used in rehearsals and the enactment. Board work included the board that was used with the small group, in most cases, chart paper hung up in front of the group. Materials included objects such as pencils, individual student boards, chart paper markers, math manipulatives, and chart paper. I highlight how TCs used the board and materials and how board work and materials became tools to facilitate opportunities for TC learning.

### *Changes in the Use of the Board and Materials*

In analyzing how TCs used the board and materials, five ideas emerged around ownership of the materials and board use, logistical management, active use of the board during rehearsal time, representing mathematical ideas, and quality of representations. On the first day, much of

the discussion was about logistics, and TCs needed assistance managing their materials. While they took up TE suggestions, the TE had much of the ownership over how to use materials and board. Several times in the rehearsal and enactment, the TCs pointed to the board but didn't write anything. They also used a limited selection of ways to annotate patterns and represent ideas. In contrast, on the last day, TCs had ownership of the board use. They were comfortable using a variety of representations (e.g., number line, number tree/bond) to highlight the mathematics that were central to their goal. In the rehearsal, they talked about how to represent student thinking and how to highlight key ideas. They took up ideas from rehearsal into the enactment. In the section below, I expand on these five findings:

1. Ownership over the use of the materials and discussion about materials and board work moved from being TE-initiated to residing in the group.
2. Focus in rehearsals moved from logistical management of materials to focusing on how to use the board.
3. TCs used the board more actively and intentionally during later rehearsals.
4. In later sessions, TCs used the board based on their mathematical purpose.
5. TC repertoire and quality of mathematical representations increased.

**Ownership of the materials and board use.** Ownership over the use of the materials and discussion about materials and board work moved from being TE-initiated to residing in the group. In the beginning of the quarter, the TE directed the rehearsal discussion around materials and board work. Later in the quarter, TCs initiated questions, contributed suggestions, and discussed not just logistical issues but also representations of mathematics.

To examine the change in ownership over the materials and board use, I noted instances where the group talked about materials or board use. These instances were coded as either TE

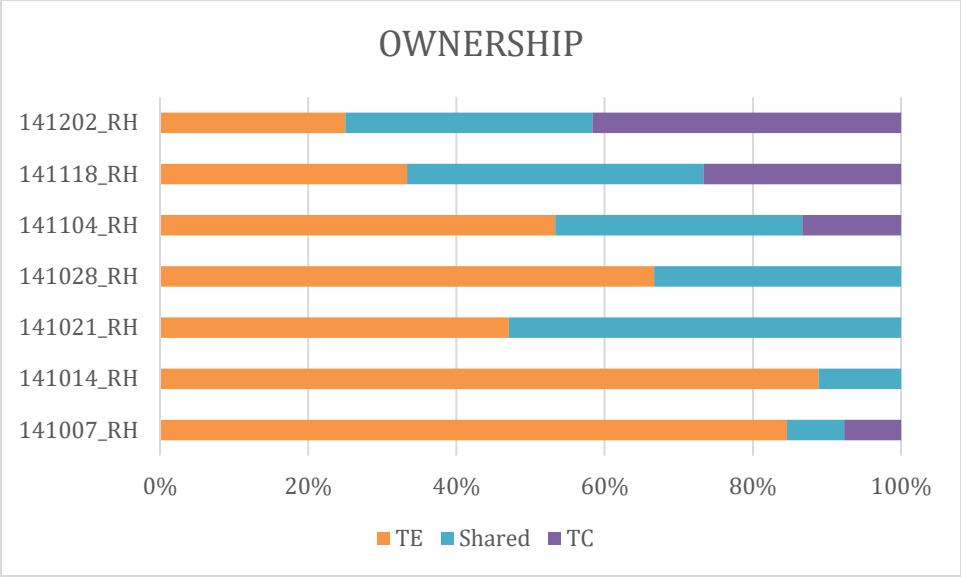
ownership, shared ownership, or TC ownership. TE ownership included instances where the TE made a suggestion or comment. These instances were TE initiated, usually included some type of suggestion or comment about mathematics or student thinking, and typically did not have much discussion. For example, in almost all choral counting rehearsals, the TCs forgot to leave space to annotate the count. The TE suggested leaving space for annotations and suggested strategies for ensuring that there was space and that numbers lined up (e.g., putting light dots on the paper).

Shared ownership included instances where the group engaged in a discussion with TCs and the TE actively offering questions and comments. Ownership over the materials and board use was shared between TCs and the TE. For example, during a choral counting rehearsal in the fourth session, Sam asked the group how she should annotate a diagonal pattern. Both the TE and Danielle offered ideas as they discussed as a group how to annotate the pattern.

TC ownership instances included times where TCs offered a suggestion or comment that was not initiated by the TE and where TCs directed and managed the conversation. For example, during the last rehearsal, Flora asked if they should write the numbers in for the word problem or leave the spaces blank, and Sam suggested putting them in to support students to understand the problem.

In this chart (**Figure 10**), we see the shift of ownership from the TE to the TCs (like the shift in ownership over goals in the previous chapter). Nearly all the instances coded in the first two sessions were TE directed. Much of the discussion was about logistics, and TCs needed prompting and assistance around managing their materials. While they took up some TE suggestions, the TE had most of the ownership over how to use materials and board. Most of the TE suggestions were logistical such as ensuring that students could see, reminding TCs to write down ideas, or making sure materials were brought and passed out. Below I provide examples of TE ownership, shared ownership, and TC ownership.

***Figure 10.*** *Shift in ownership of the materials and board.*




On the first day, Katelina struggled with spacing as she recorded her choral count. The TE provided suggestions for spacing and lining up the numbers:

TE: Sometimes what people do, to make sure you leave room, you could very faintly put a little dot along the side, then you know you have room. It looks like you will, but you know how you want to line up –

Katelina: Yeah, I was worried, that, yeah. [lifts marker up, then pauses] So like, even before the lesson, but dots where each number is going to go?

TE: Yeah, if you want to. Because that's the thing about counts. Making sure you line up numbers makes all the difference so you can see the patterns.

In lines 27-29, the TE provided a specific strategy for spacing and lining up the numbers. Katelina asked a follow up question, as she tried to make sense of the TE's suggestion (lines 30-31). The TE also brought the poster paper and markers to the classroom visit and reminded TCs to straighten the poster paper during the enactment. These suggestions indicate ownership of the board and materials still resided the TE.

In contrast to the beginning session, the last two days had less TE ownership and more shared or TC ownership around materials and board work. The excerpt below is an example of TC ownership, with TCs problem solving around what to do next and how to represent the student's

idea. Ownership became shared with the TE, as she offered a small suggestion just at the end of the conversation. A student had just showed a counting strategy to 17 – 9 using her fingers.

| | |
|---|---|
| Orianne (to the group): Should I write that down or have everyone try it? With her? Because I can draw her fingers. | 1<br>2 |
| Tina: Yeah, you can draw. | 3 |
| (Orianne starts drawing) | 4 |
| Orianne: So this is Aiah's. (Writes "Aiah" on the board). | 5 |
| Flora: I feel like it might be more engaging to have everyone do it. 'Cause they might be a little like, (pauses) they've been sitting for a long time. Oh! They just had their break though. | 6<br>7<br>8 |
| Orianne: So maybe while I have everyone do it, I can write it? | 9 |
| Katelina: Yeah! | 10 |
| TE: Well you (points to Tina) can help us while she's drawing it. | 11 |
| Tina (to the group; steps back in to teaching mode): So as Orianne is drawing it, let's practice Aiah's way together. So…(holds up her fingers as she counts with the group) | 12<br>13 |
| Whole group: Nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen. | 14 |
| (Orianne writes each number "9, 10 11, 12, 13, 14, 15, 16, 17" over the fingers as students count) | 15<br>16 |
| Tina: How many fingers do you have? | 17 |
| Whole group: Eight. | 18 |
| (Orianne counts the eight fingers, then pauses and looks perplexed. She laughs, and writes "8" next to the picture of the hands) | 19<br>20 |
| Tina (to Orianne): The way that you showed it before was…(she writes "1, 2, 3, 4, 5, 6, 7, 8" over each finger) | 21<br>22 |
| Orianne: Oh, yeah. | 23 |
| TE: And two different colors might be nice too. | 24 |

In lines 1-2, Orianne posed a question to the group about whether she should record the student's strategy on the board. Her teaching partner, Tina, suggested yes, she should record it. In lines 6-8, Flora suggested having students do the finger strategy to keep them engaged, and Orianne took up the suggestion. The TE added on (line 11) that they could share the writing (Orianne) and counting (Tina), presumably to allow Orianne to focus on the representation. Orianne wrote the numbers that the students were saying out loud above the fingers (lines 16-17) but then realized she had not recorded how a student would know that they have eight fingers up. In lines 20-21, Orianne was genuinely perplexed about how to record this counting. Her teaching partner Tina reminded her of a representation they had used previously (for a counting on strategy). In the last line, the TE added the suggestion of two colors to keep track of what each of the numbers are counting.

In this excerpt, we see collective TC ownership over the use of the board, as TCs jumped in with suggestions and ideas. Orianne wasn't showing stress or anxiety in being put on the spot, as this was not a showcase. She was collectively thinking through her teaching practice with the support of her peers and the TE. When a genuine problem of practice arose (how do I represent this idea?), TCs and the TE actively contributed to the discussion, offering ideas and suggestions as co-learners. This type of shared ownership became increasingly common as the quarter progressed. Additionally, as the ownership shifted from the TE to being shared in the group, what the TE highlighted and what the TCs attended to shifted. In a later finding, I discuss how the content of these instances related to the TE and TC contributions.

In addition to coding ownership over the discussion and interactions, I also coded for who was literally managing the materials. In the first few lessons, the TE not only made all suggestions about space and materials, but also physically brought them or provided them for TCs during the rehearsals. Midway through the quarter, the TCs even made a joke about how they never remembered to ask students to have pencils to write with:

| | |
|---|---|
| Flora (in a sarcastic and playful tone): So everyone has your pencils, right? | 1 |
| (The group laughs) | 2 |
| TE: Oh, we need pencils? | 3 |
| Katelina: We never have them. | 4 |
| Orianne: It's very important. | 5 |
| Sam: You're right, we're always like, 'Go do it!' and they're like, 'Uh…' | 6 |
| (The group laughs) | 7 |
| (TE passes out boards to write on) | 8 |

In this lighthearted exchanged, the ownership of managing the materials started to shift to the TCs. Although the TE still passed out the boards to write on, the TCs remembered that they would need to think about materials and reflected (jokingly) about how they had not done this in the past.

As the TE moved away from literally managing materials (e.g., passing out the papers/where kids sit) to managing the materials/board space for mathematics (e.g., when to write, what to write), and (eventually) to directing the conversation towards the goal (which TCs only

get to in the final days), TCs took more ownership over managing the materials. In the following sections, I expand on how TCs managed the materials, board, and space.

The above findings focused on rehearsals. Classroom visits were coded for ownership as well; however, each classroom visit had only 1-4 instances that were coded, and these were almost always a TE suggestion. This could be because there was less discussion and reflection in the lesson (as compared with rehearsal, with no students present) and because other TCs tried to not "intervene" during lessons to keep it as authentic as possible. During the classroom visits, the TE suggestions were often brief and served to address logistical issues or to elicit or clarify student ideas and representations.

Ownership over the use of the materials and discussion about materials and board work moved from being TE-initiated to residing in the group. In this section, I discussed the shift of ownership of managing materials from the TE to the TCs. Below, I focus on the TCs and how their focus shifted from logistical management of materials to how to use the board.

**Logistical management versus using the board.** Focus in rehearsals moved from logistical management of materials to how to use the board and represent mathematical ideas. In the beginning of the quarter, TCs focused on logistical management of students, materials, and board work. In early rehearsals, TCs did not explicitly think about the board and materials, and (as discussed above) the TE initiates discussion around materials and board work. When TCs started to explicitly think about materials, it was very logistical (e.g., where do we put and when do we use stickies; using colors for the sake of having a variety of color; how to hold the markers; when to pass out materials). When TCs co-planned and prepared for rehearsal, they discussed when they would trade off teaching, who would hold the markers, when to pass the papers out, and other

logistical issues. In later lessons, their questions and discussion focused on how to use the board and represent particular mathematical ideas.

For example, in the first rehearsal, the first question Danielle asked was about where to stand: “Quick logistics question, is it awkward for me to stay standing here? Like, it’s not, I shouldn’t sit?” In this same rehearsal, Katelina struggled to concurrently hold her lesson plan and record the count on the board, and Danielle didn’t have markers nearby when she went to annotate patterns (Katelina brought them over to her). The TE’s early comments were likewise focused on logistics as she reminded TCs to make sure students could see, brought materials, reminded them leave space on the board to annotate, and prompted them to think about how they would pass out materials. In the first few activities, the TE provided and managed the materials during most of the activity. During the fourth course meeting, the TE’s coaching shifted to focusing on how to use stickies, how to display work side-by-side to compare, and how to make an array with tiles. While still logistical, these coaching moves were moving towards discussion of mathematical ideas (e.g., using an array or positioning strategies side by side for a compare and connect discussions). In the last few weeks, they worked on particular representations on the board.

Over the course of the ten weeks, the TE’s coaching shifted from managing materials and space to discussing board work in relation to students thinking and mathematical ideas. The TCs made a similar shift in their focus. The TC shift seemed to shadow the TE focus. This analysis does not establish a causal relationship or correlation, but further research around this interaction of TE focus and TC focus would be beneficial to better understanding how to support TC learning and attention on key aspects of practice.

**Active use of the board during rehearsal time.** As the quarter progressed, TCs used the board more actively and intentionally during rehearsals. In the beginning of the quarter, the board

was not used actively nor intentionally to rehearse how they would record student strategies and mathematical ideas. In beginning rehearsals, they started talking about what they would do rather than doing it, often pointing to the board instead of recording. When first rehearsing, teaching in front of peers can feel awkward and unnatural, prompting TCs to talk about what they would do, rather than do it (Kazemi et al., 2015). Several times in the first rehearsal and enactment, the TCs pointed to the board and needed prompting from the TE to move from talking about practice to recording the strategy or idea. Other times, TCs recorded things on the board that didn't seem relevant to supporting students thinking around the mathematical goal. In later rehearsals, TCs thought about why they were writing things (as evidenced by their exchanges and discussions) and the usefulness of "practicing." As they became more comfortable with the logistics of using the board and recording ideas, they talked more about how to represent student thinking and how to highlight key ideas (as opposed to the logistics of the board and materials). They also took up ideas from rehearsal into the enactment.

**Representation of mathematical ideas.** In later sessions, TCs used the board based on their mathematical purpose. The goal(s) that was included on their lesson plans and/or that they discussed during the rehearsal was reflected in their board work. This relates to findings from my previous analyses on goals: as the quarter progressed, TCs included more specific instructional goals on their lesson plans and worked towards highlighting more specific mathematical ideas. Similar to my findings of how TCs worked on goals, the representation of mathematical ideas increased over the quarter as well as representations related to the goal. Below I look at two examples to show the relationship between board work and mathematics and instructional goals.

In the first lesson, there was no goal written on their lesson plans or specified during the rehearsal. Similarly, there was not a clear mathematical idea highlighted in their board work.

During the rehearsal, Katelina did not line up the place value of the numbers and did not record what they are counting by (a suggestion made by the TE during the rehearsal and taken up during the classroom enactment). In the enactment, Katelina tried to line the numbers up and wrote the "+20" when prompted by the TE. In the board work from their enactment (**Figure 11**), we can see they recorded the count (counting by twenty starting at twenty) and indicated some student patterns. There is some variation in color based on the patterns, but the misalignment of the numbers and the lack of spacing make it difficult for TCs to clearly annotate ideas and for students to clearly see patterns (an instructional goal in future choral counting lessons).

***Figure 11***. *Katelina and Danielle's board from the enactment on first day.*

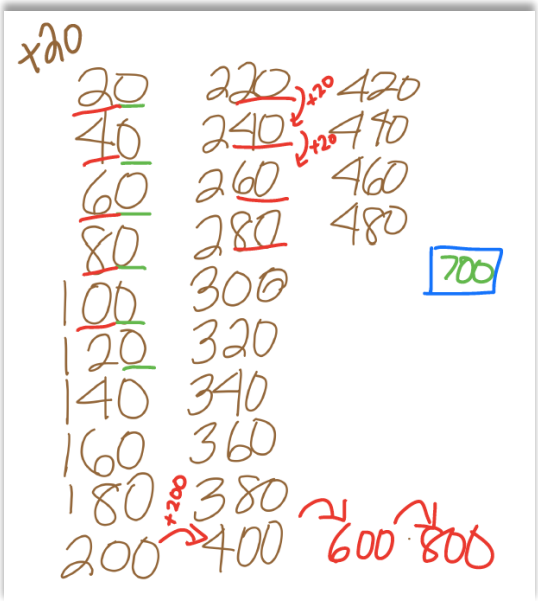
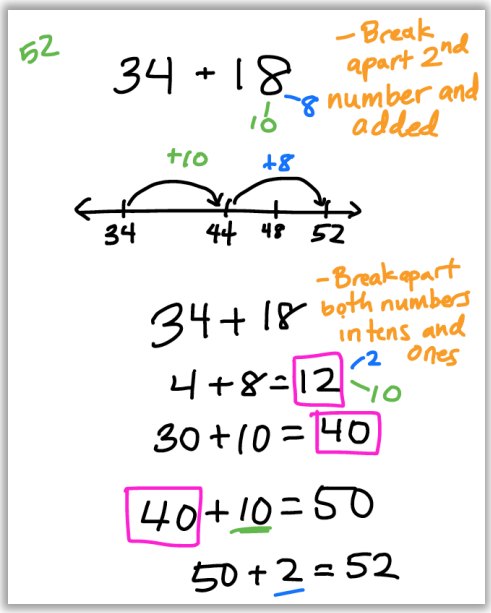



***Figure 12.*** *Katelina's board from the enactment on second to last day.*




The second image (**Figure 12**) is Katelina and Danielle's board work from the mental math enactment on the second to last day. On her lesson plan, Katelina wrote, "Student Goal: Students describe strategy in a way for others to understand; Breaking down numbers to solve efficiently instead of counting by one." Her board work reflected this goal. The mental math problem posed was 34 + 18. One student broke 18 into ten and eight, then added up from 34 in chunks. Katelina

showed this strategy on a number line with "+10" in green and "+8" in blue on the number line and below the 18 in the original problem.

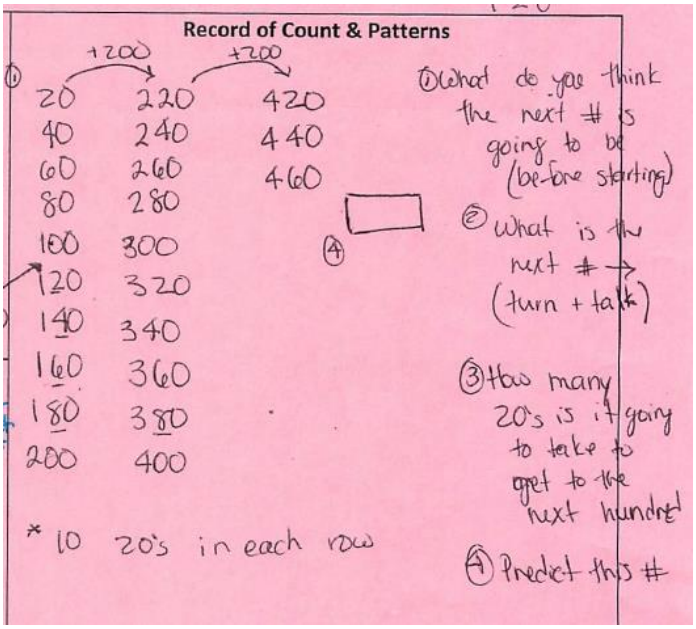
A second student broke the numbers apart based on place value, and she showed this strategy using number sentences. In both the number line and number sentences, she used color to correspond to different parts of the number. They also, after prompting from the TE, named the strategies as a group and recorded these on the poster: "Break apart 2nd number and added" and "Break apart both numbers." Katelina's board work used both number lines, number sentences, words, and caret-shaped number bonds to highlight their goal around how the numbers were broken apart. In this example, Katelina incorporated a specific mathematical idea in her board work. Board work more clearly represented particular mathematical ideas and instructional goals as the quarter progressed. In addition to an increase in TCs' inclusion of particular mathematical ideas, there was also improvement in the quality of their representations, which I discuss next.

**Quality of mathematical representations.** The TCs' repertoire and quality of mathematical representations increased over the course. Like earlier lesson plans, earlier board work included vague representations or representations that were either inaccurate or did not clearly highlight particular mathematical ideas. They also used a limited selection of representing patterns and ideas. TE had to prompt them to write, add color strategically, put in arrows, line up numbers, write words that match their goal, and use a number line or array accurately. Later board work included representations that were mathematically clear and accurate and that highlighted particular mathematical ideas. Their board work showed that TCs were comfortable using a variety of representations (e.g., number line, number tree, etc.) to highlight mathematics that is central to their goal. To illustrate these findings, I contrast representations from earlier board work and

lessons plans (**Figure 11** above and **Figure 13** below) with later representations (**Figure 14** and **Figure 15**).

The first image (**Figure 13** below) is Katelina and Danielle's board work from the first day. This is an example of the TCs' first attempt to records ideas. As they wrote out ideas and the count during the rehearsal and enactment, issues emerged around spacing, color, and ways to show student ideas. In their representation of a choral count by 20, it was difficult to see mathematical patterns because the numbers were not lined up by place value. In both the rehearsal and enactment, Katelina noted that it was hard to line up the numbers. Because of this, it was difficult for students to see the patterns based on place value. Katelina and Danielle did attempt to annotate student patterns, but they only used a few colors, and used the same method (underlining) to show two different patterns (zeros going down in the ones place and counting by twenties in the tens place in the second column). They also only showed patterns in one column (or row respectively), so the idea that the patterns are present in other places was not highlighted.

**Figure 13.** Representations from Katelina's lesson plan on the first day.



For example, one student noted that they were counting by twenties in the second column. She underlined 20, 40, 60 and so on in the second column and wrote +20 with an arrow but did not show or ask students about if this happens in the other columns. The board work from the enactment was similar to their lesson plan (image on the right). In the excerpt from Katelina's lesson plan, we see even less annotation of patterns, the same difficulty lining up digits, and no color for highlighting patterns.

The TE collected lesson plans and gave feedback on their representations. Additionally, she prompted them to look at their board work, including how they were spacing our representations and annotating student ideas. During the fourth course session, the TE prompted TCs to think about how they could intentionally use color to highlight particular ideas. After this rehearsal, TCs used color more intentionally during the rehearsal and enactments, and they had more discussions on how to represent mathematical ideas. Despite the change in their practice during rehearsals and enactments, they never practiced this color with their representations on their lesson plans.

The above images (**Figure 11** and **Figure 13**) show early attempts to use the board and represent ideas. The following images show improvement in the quality of representations. **Figure 14** (below) is an excerpt of the anticipated student strategies and representations from Orianne's mental math (17 – 9) lesson plan from the middle of the quarter. Although she does not use color, she uses a wide array of representations (number sentences, words, number lines, representations of fingers for direct modeling, and base-ten blocks) that are clear and mathematically accurate. Her annotations of these representations show that she understood the mathematics in each strategy. She had mathematically accurate representations for each strategy. For example, she included two different strategies that involve fingers. The first strategy involved counting up from

nine to find the different between seventeen and nine. She wrote the numbers students would say as they count up (10, 11, 12, 13, 14, 15, 16, 17) as well as the count for how many fingers it took to get to seventeen (1, 2, 3, 4, 5, 6, 7, 8, "8 fingers to get to 17"). Similarly, she showed a counting backwards strategy and labels the numbers students would count (16, 15, 14, 13, 12, 11, 10, 9) and the numbers that represent how many fingers were needed to count back to nine. There were also number lines and representations of base-ten blocks that clearly show the different strategies for solving this problem.

**Figure 14.** Representations from Orianne's lesson plan from the middle of the quarter.

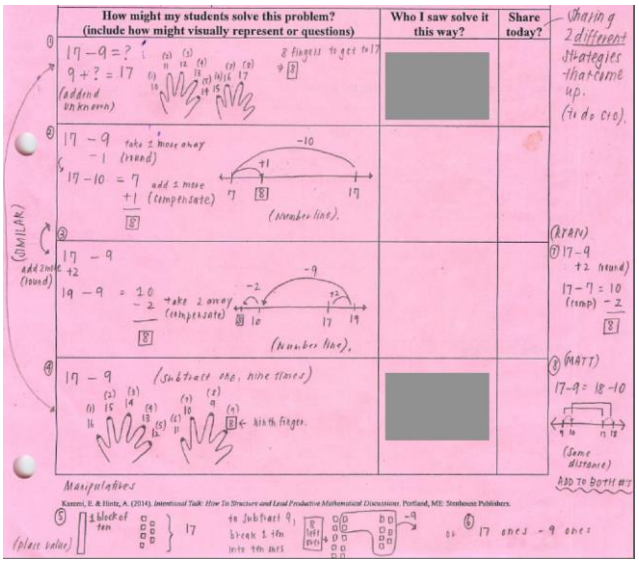
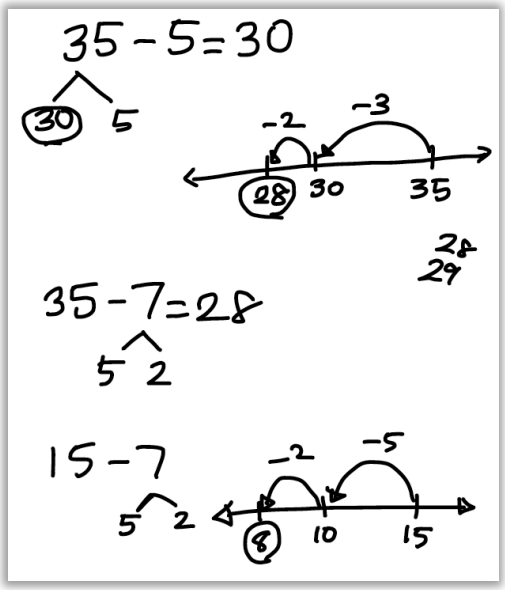



**Figure 15.** Orianne's board work from the last day.




Like Orianne's lesson plan from the middle of the quarter (**Figure 14**), we can also see the increased repertoire and quality of mathematical representations in the TCs' board work. **Figure 12**, Katelina's board from her enactment of the mental math activity (34 + 18) on the second to last day, is another example of increased quality of mathematical representations. This image shows her using color to show the corresponding parts of the numbers on from the caret-shaped number bond, number line, and number sentence. For example, one of the students said that they

added ten then eight by breaking apart the eighteen. Under "18" she wrote the 10 in green and the 8 in blue. She used these same colors on the number line to show the incremental strategy. Katelina also used words to verbally represent what the strategies were. Using and connecting contextual, visual, verbal, physical, and symbolic representational forms are important parts of supporting students to deepen understanding of mathematical ideas and make connections across representations (National Council of Teachers of Mathematics, 2014). Katelina used multiple representations, including verbal, symbolic and visual representations and strategic use of color and a number line model to support students in developing ideas around decomposition of number.

Similarly, we see a wider and more mathematically accurate array of representations in the second image, Orianne's board work from the number string on the last day (**Figure 15**). Orianne also coordinates colors and uses multiple representations to show how students were breaking apart numbers. **Figure 12**, **Figure 14**, and **Figure *15*** show improvement in TCs' abilities to clearly represent mathematical ideas. Their representations became more mathematically accurate, they used color and space intentionally, and they used a variety of representations.

***Board Work and Materials as Opportunities to Facilitate TC Learning***

The board and materials facilitated opportunities for TC learning in three ways. First, the board was a space to practice representations and mathematical ideas. TCs also practiced the logistics of spacing and how they would manage materials in rehearsals and enactments. Second, the board became a way to think through the lesson, including the mathematics, representations, teacher moves, and possible student ideas. Finally, the board served as a record of public thought and artifact for discussion. TCs could build on this artifact to discuss ideas around mathematical content and teaching practice.

**The board as a space to practice.** Through the rehearsal and enactment, TCs were pressed to move beyond *talking* about how they would manage materials and use the board to actually *practicing* and *doing* that work. Rehearsals were opportunities to practice managing materials, and the board served as a space to practice representing ideas. When TCs first rehearsed, they struggled logistically with the board and materials management. Having to rehearse and enact lessons immediately surfaced logistically challenges (e.g., how to pass out materials, how to hold the markers) as well as problems of practice connected to how to facilitate productive mathematical discussions (e.g., how to represent ideas, what representations were useful based on the math content goal).

Teaching requires logistical management of materials and board space as well as using the board to represent ideas and facilitate discussion. With the guidance of the TE and peers, TCs moved from focusing on managing materials and board work to representing mathematical ideas and using the board to navigate the discussion. In the rehearsals and enactment, TCs practiced representing student thinking and using space wisely.

**The board as a collective space to think through the lesson.** The board serves as a collective space to think through the activity, mathematics, representations, and possible student strategies. Once representations and ideas were on the board, TCs and the TE could interact with what was written. They looked at the board to think about potential teacher moves, questions to ask, when to stop in a choral count, and what to highlight. The TE provided feedback and referenced what was written to highlight ideas about mathematics, student thinking, instructional activities, and teaching practice. In other words, TCs were able to learn about mathematical content for teaching and teaching practice through the ideas shared on the board. The materials and board become tools to think about mathematics, students, and practice.

**The board as a record of public thought and artifact for discussion.** The board and student work became artifacts for discussion in the debriefing. The student work was used in debriefing as evidence of student learning and what happened in the lesson and as a starting point for discussions. Similarly, the board became the record of the rehearsal, lesson, and student thinking. Most importantly, the board served as an integral record of public thought both mathematically for students and later in the debriefing discussion with the TCs. Early in the quarter, the TE said that the board was their public representation that the group would interact with. In productive mathematical discussions, this is, essentially, the purpose of the board. After the TE explicitly highlighted this idea, the TCs used the board for this purpose. The board served as a space to create a coherent mathematical storyline based on the mathematical goal (Sleep, 2012). The board became an integral part of their lessons and public record of their discussions, not an afterthought.

## Talk Moves

In the previous sections, I discussed lesson plans, board work, and materials. In this section, I focus on talk moves as tools for mediating TC participation. I analyzed how talk moves were used as tools for TCs to facilitate discussion and found that the tools provided both a common language around techniques to facilitate talk as well as particular teacher moves that could be revisited in increasing complexity.

Chapin et al. (2013) define talk moves as "strategic ways of asking questions and inviting participation in classroom conversations" (p. 11). To facilitate productive mathematical talk, they propose four "steps" or "goals" (what I call "talk goals") to supporting meaningful discussion, and

they name talk moves as possible tools to accomplish these four talk goals[25]. This idea of using talk moves intentionally to accomplish talk goals in facilitating productive mathematical discussions was part of my analysis of instructional and practice goals. In the previous chapter, I looked at how TCs facilitated discussions to reach instructional goals, and I documented the ways the used moves (including talk moves) intentionally (or not) towards instructional goals. For this part of my analysis, I focus specifically on talk moves to understand how this tool mediated TC participation around the practice of facilitating productive mathematical discussions. To illustrate how use of talk moves changed, I compare how talk moves were used in rehearsals and enactments during the first and last TC led activities. These two course sessions were described in more detail in the previous chapter.

***Changes in the Use of Talk Moves: Illustrations of the First to Last Instructional Activities***

From the first rehearsal, TCs talked about and used talk moves, but the way they talked about and used talk moves changed over the ten-weeks. On the first day, the introduction of talk moves and discussion norms was very clunky. In beginning activities, the TCs used talk moves with vague prompts (e.g., "Turn and talk with your partner about this.") and listed generic versions of talk moves. The TE was very active in modeling times to use particular moves, suggesting talk move prompts and opportunities for talk moves, and highlighting the purpose of different talk. As the quarter progressed, TCs were more active and took the lead in suggesting talk moves and discussing when to use them. In response to prompting by the TE, they also started strategically anticipating talk moves in the lesson plans and used them deliberating for specific tasks and

[25] The four steps for facilitating productive talk are 1) Helping individual students clarify and share their own thoughts; 2) Helping students orient to the thinking of others; 3) Helping students deepen their own reasoning; and 4) Helping students engage with the reasoning of others (Chapin et al., 2013). See **Table 1** in Chapter 2 for a list of the supporting talk moves.

mathematical goals. As the course progressed, the TCs' repertoire of talk moves increased, and they began using talk moves intentionally towards productive talk. In the last few lessons, they used the talk moves purposefully and at times where students could discuss key or puzzling mathematics. Below, I expand on these ideas using excerpts from the first and last lessons of the course.

**Talk moves in the first rehearsal and enactment.** In the first TC-led instructional activity, Katelina and Danielle worked to include a variety of talk moves[26] in their choral count by twenties. In both the rehearsal and enactment, Katelina asked students about "discussion stems" (i.e. talk moves) in her opening of the activity. The following excerpt is Katelina's revised opening, which was still quite long as she listed many talk moves and directions about talk moves all at the beginning.

Katelina: I'm going to sit right here so you can see (directly related to the rehearsal). So I love reading, and I read 20 minutes each day. And so we're going to do, we're going to count by 20s. Do you hear how everybody is in the class today? So we're going to have to whisper count. Whisper count the number, then the next number. I wanna hear all your voices, but we're just whispering. I wanna see all your mouths moving. So we're going to count by 20s, because I read every day 20 minutes. And then, when we're talking with each other, so, let's see. [looks at the group of students] Maybe you two can to talk to each other, and then you two, Thahn and Arianna, you guys are going to talk to each other. And when you're talking, remember to listen, right? And you're going to repeat your partner's ideas, and if you agree, say I agree, and if you disagree say why. Okay. You guys are going to do great, because you're all geniuses! I'll write, and as you speak, I'll write, so don't speak too quickly. So we're going to start at 20 and count by twenties. [Writes "+20" in the corner at "20" in the first spot of the count]

In trying to set up talk expectations, Katelina gave many directions and explanations of talk moves. Although Katelina was attending to the importance of introducing new forms of talk and participation to students (as Chapin and colleagues suggest), asking students to think of

[26] Although they used talk moves, they didn't use them deliberately for accomplishing the four "talk goals" and steps towards productive talk that Chapin et al. (2013) articulate. Because this analysis examined how participation around this tool changed over time, I coded all instances of talk moves (and emerging attempts at using talk moves) even if the implementation of the move didn't serve purpose of facilitating productive talk.

"discussion stems" could be its own lesson and was both vague (i.e. third grade students might not know the pedagogical term "discussion stems") and not appropriate for the short timing and structure of the instructional activity. It is possible that Katelina hadn't considered students might not know the term "discussion stem" or might talk about and name talk moves differently in their class. The TE suggested that Katelina provide the discussion stems (which she did in the enactment). Danielle used a wait time move followed by a turn and talk when she asked what patterns students noticed, a couple of reasoning moves ("Why does it go 1111?" and "Is that a pattern or does it just happen that one time?"), and a repeating move of one of the patterns (+200 across columns). The turn and talk served the purpose of *helping individual students clarify and share their own thoughts*.

Towards the end of the rehearsal, the TE suggested using a repeating move after a student shared a pattern. This exchange happened at the end of the count when the group was discussing how they could support students to explain or extend a pattern. In attempt to use the +200 pattern across columns, Danielle used multiple prompts to elicit ideas. In the following exchange, we see these prompts and the group's discussion about repeating moves, what question to ask, and how to call on students.

Danielle: Can anyone repeat what Orianne said? Or come up to the board and show if it happens anywhere else in the chart?

TE: That's too many questions. So ask us, what is she noticing is happening as we go from 20 to 220 to 420?

Danielle: So what is Orianne noticing when we go from 20 to 220 to 420?

Sam: She's noticing a jump of 200. There's going to be 200 between the columns.

Danielle: Does anyone see that anywhere else?

TE: Or ask somebody else to repeat. 'Cause sometimes you wanna make sure that –

Katelina: Can we call on a person? Like, Flora, can you repeat what Orianne said? Or what Sam said? Or do you kinda don't want to call out students?

TE: Uh, it kinda depends. You can always say, "Try repeating to your partner" or "If you have a question or you're not sure what she said you can ask her to explain it again."

Danielle: Can everyone turn and talk to your partner about what you were hearing Orianne and Sam saying about that pattern?

Danielle used a repeating move and asked students if they wanted to show if the patterns happened in other places (lines 1-2). These questions were two different prompts, and the TE proposed an alternate, clearer question (lines 3-4) which Danielle took up (line 5). After the student answered this first question (line 6), Danielle followed up with one of her original questions about if the pattern happened in other places (line 7). The TE suggested a repeating move (perhaps about checking in with other students) and Katelina asks about putting students on the spot with a repeating move (lines 9-10). Katelina's question indicated she was making sense of the purpose of repeating moves and trying to understand how to support students to feel comfortable sharing (and not put on the spot). The TE's alternate suggestions (lines 11-12) addressed this issue of supporting students to feel comfortable participating, and Danielle took up this move as a turn and talk (lines 13-14). In this exchange, TCs were developing shared understanding of the purpose and use of talk moves.

In the enactment, the TCs used talk moves in similar ways. During the count, the TCs used multiple repeating moves, two reasoning moves (Do you agree with her?), and two turn and talks. After they counted to 480, Danielle (exactly like in the rehearsal) asked students what patterns they noticed, gave "private think time" (wait time), and asked them to turn and talk about their patterns. After the turn and talk, Kristina shared the 2, 4, 6, 8, 0 pattern in the tens place, and Katelina repeats this pattern. By repeating Kristina's pattern, Katelina was valuing her contribution but not holding other students accountable for listening to their peers or checking in to see if other students understood this pattern. Before the TCs elicited another pattern, and the TE suggested a repeating move. For the three other elicited patterns, the TCs followed a repetitive format: Students shared a pattern, TCs asked someone to repeat, and then the TC repeated the pattern.

After the second pattern was shared (+200 across columns), the TE asked if this happened in other places. Inga said yes and then used the pattern to predict the bottom number of the third column (600) and fourth column (800). Danielle repeated that Inga made a prediction using the pattern (an emergent form of the revoicing move) and asked other students to give a thumbs up if they agreed with the prediction (she did not ask students why they agreed, making this a simplified version of a reasoning talk move). As students gave a thumbs up, she asked if someone could repeat: Can someone else repeat what Inga is saying about what happens in each column? This was the first time TCs attempted to use talk moves in relation to an instructional goal (*using a pattern to make a prediction*). Although Danielle was not always clear about which goal she was working on during this activity, asking students if they agreed and to repeat this idea indicated she was beginning to think about how talk moves could be used intentionally to *orient students to the thinking of others*. After the third pattern is shared (always having zero in the ones place), Katelina asked other students if they agreed (Students: Yeah.). Unlike Danielle's reasoning move (if students agreed with Inga's prediction), this reasoning talk move seemed random, and the TCs moved on to the last pattern.

In this first TC-led instructional activity, TCs worked on how to introduce talk moves to students and how to use them in a choral counting activity. Ownership over the talk moves resided with the TE as she scaffolded their use of moves, modeled language around using talk moves, and made suggestions for which talk moves and why. With the exception of Danielle's agree/repeating move at the end of the count, most of the moves were not used strategically to highlight key mathematics and/or towards the talk goals that Chapin and colleagues (2013) articulated. They used talk moves before group sharing and multiple repeating moves after students shared patterns. They also repeated student ideas. Their repeating of student ideas could be interpreted as an

emerging form of a revoicing talk move as TCs worked to make sense of what students shared. Teacher "revoicing" can help *students clarify and share their own thoughts*, particularly when an idea is unclear, when students share something that is long and complicated or when the teacher wants to repeat an idea with precise mathematical language (Chapin et al. 2013). Kazemi and Hintz (2014) summarize this move:

> "Repeat some or all of what the student has said, then ask the student to respond and verify whether or not the revoicing is correct. Revoicing can be used to clarify, amplify, or highlight an idea" (p. 21).

The TCs repeating did not match this format (verifying with the student that the revoicing accurately captured their idea) and purposes (clarifying, amplifying, and highlighting ideas) for revoicing (although Danielle used an emergent form of revoicing when Inga made a prediction). In this early stage of the course, TCs are developing a shared language around talk moves and negotiating meaning around how these moves could support productive mathematical discussion.

**Talk moves in the last rehearsal and enactment.** To illustrate the change in intentionality and how talk moves were used in later lessons, I discuss talk moves in the two strings activities[27] led by Katelina and Orianne on the last day of the course. Although the TCs used the same two talk moves as the first day (repeating move and turn and talks), the use of these moves and how they talked about the moves changed. The mathematical goal for both strings was to use an easier problem to solve a second problem by breaking down number (an incremental strategy), and they deliberately used talk moves to orient students to mathematical ideas around decomposition of

[27] String 1: 24 – 4 and 24 – 6; String 2: 35 – 5, 35 – 7, and 15 – 7; There was also a Solving a Word Problem activity that I have omitted for brevity.

numbers and models to represent this. In both the rehearsal and enactment, turn and talks were used to listen to and select a student who used a break apart strategy (their goal).

Katelina began the rehearsal with clear directions for how to participate (e.g., not shout out the answer). She posed the first problem (24 – 4), used wait time, reminded students to signal they were ready with their thumb, and prompted students to hold up multiple fingers if they thought of additional ways to solve the problem. This format of wait time and thumb signal (used for both problems) was consistent from the first activity and throughout the quarter, and the added prompt to hold up fingers (for additional strategies) directed students who solved quickly to consider other strategies and stay engaged. After a student shared how they solved it, the group talked about how to represent the students' strategy and connect it to a number line. Katelina posed the second problem (24 – 6) with the same wait time/thumb prompt. Before she asked for a student strategy, the TE suggested doing a turn and talk and the group discussed the use of AB partners, which we see in the excerpt below.

| | |
|---|---|
| TE: Now this feels like a turn and talk, so you can listen to how they figured it out. | 29 |
| Katelina: (to TE) Yeah. (stepping back into teacher mode) Let's turn and talk and tell your | 30 |
| partner how you used the second problem, using the strategy of this one. [motions to | 31 |
| the equation and representations for the first problem] | 32 |
| Orianne (pausing the activity): Should we do AB partners? Like, As first share, then we'll | 33 |
| tell you when to switch. [Other TCs: Yeah.] | 34 |
| Katelina (back in teacher mode): So As will share first. | 35 |

In line 29, the TE reminded them that turn and talks could be used to listen in and select students. Katelina took up this suggestion with the added prompt of using the first strategy (lines 30-31), a prompt that connected to her goal. Orianne asks about using the AB partner strategy (lines 33-34), which the TCs use in all three activities during this day. Unlike the first choral count, where Katelina assigned AB partners and then never revisited the idea during the activity, the TCs on this last day successfully (and quickly) introduced the AB partner talk structure (done during the

Solving a Word Problem activity) and used it to support participation and make space for all students to share and listen.

After this exchange, Katelina listened as the students shared, and Orianne prompted partners to switch to the B partner. This was followed by a one-minute exchange discussing how to use the turn and talk to select a student who used a break apart strategy. The TE noted that there would be different strategies, and Katelina repeated that they would listen during the turn and talk to select a student who used the first problem to help them solve the second one. Katelina asked what to do if a student shared a strategy that wasn't related to breaking apart of numbers. The TE reiterated and suggested that they select and start with a student who used a break apart strategy (decomposing numbers). She also emphasized the need to make the strategy explicit and visible (the excerpt below).

TE: And then you want to maybe make explicit what the strategy is. So what the strategy is, is that a 6 is a 4 and a 2. So just like you took away that 4, and then you only have 2 left. So you really actually want to make visible, what is the strategy that we're really working on. By the end of the conversation, you want to make sure that comes out. 36 37 38 39

This explanation of what the strategy is (lines 36-38) and how to make the strategy visible (lines 38-39) framed the TCs' decisions of when to use repeating moves and follow up questions. In the next excerpt, Inga (Danielle role playing her buddy, Inga) shared a break apart strategy, and Katelina asked another student to repeat.

Inga (Danielle role playing her buddy): I broke the six into a four and two [Katelina uses the caret shaped number bond and writes 4 and 2 under the 6] because that's what we did there [Danielle points to the first problem] and 24 minus four is twenty, so then I just took away two more. 40 41 42 43
Katelina: So let's see if we can show that on a number line. [starts to draw a number line] So we start with – oh! [pausing her number line] Can somebody repeat Inga's strategy for me? 44 45 46
Raul (Melissa role playing her buddy): She broke down the six into a four and two. 47
Katelina: And then what did she do with that four and two? 48

In lines 40-43, Inga (strategically selected during the turn and talk by Katelina) shared the incremental break apart strategy. Katelina was about to ask about number lines (lines 44-45) when

she realized that this was the important mathematics of the lesson. Thus, she asked another student to repeat. Using the repeating move at this moment oriented students to each other's ideas and, importantly, the key mathematical idea of string. When Raul wasn't explicit about how the six and two were used (line 47), Katelina pressed for further explanation (line 48). Raul elaborated, and Katelina followed up with the question, "And how does that connect to our first problem?" again, addressing the TE's comment to be make the strategy and decomposition explicit.

The second string (35 – 5, 35 – 7) had the same goal, and Orianne used the turn and talk in the same manner, listening to students and strategically selecting a student to share based on the goal. In the same way that she pressed Raul to elaborate on how decomposing six was helpful, when a student shared they subtracted two more from 35 – 5 (to solve 35 – 7), Katelina used follow up questions and talk moves to deepen students' reasoning and make the mathematical idea of decomposition and subtracting incrementally explicit. She tried out several variations of this question and repeating talk move: "Can someone repeat, why did we subtract 2? Why is this 7? Can someone repeat TE's strategy? Or, why can she subtract 2 from this number line?" Her think aloud of what wording she wanted to use showed that she was attending to the key mathematics and how a talk move or follow up question could be used intentionally to support productive math talk.

In the enactment, TCs used talk moves with similar intentionality as the rehearsal. They used consistent directions (not shouting out, thumb signal, wait time, then collectively shouting out the answer), turn and talks to listen in and select students, and repeating moves to orient students to the mathematical goal. After listening to students during turn and talk, Orianne purposefully selected Inga to share her incremental strategy. Inga said, "Eighteen is two away from the twenty." Like the rehearsal, this explanation needed elaboration. In the following excerpt, we

see them asking these students to share. It is evident that their moves were intentional in supporting students to think about this goal.

Katelina: But how did you, where did – how did you know to take away this two? [motions at the "-2" she just wrote on the board] Where did it come from? 49 50
Inga: Because two plus four equals six. 51
Katelina: Okay. [puts hand up to write but pauses] Can someone repeat what Inga just said? 52 53
Kristina: Can I say it in a different way? 54
Katelina: Of course! 55
Kristina: Um, I minused four and then got twenty, then I minused two more because two plus four is six. 56 57
Katelina: Did everyone hear that? Yeah? [students nod their heads] Sahra, can you repeat what Inga just said? [student talks quietly; inaudible] Wait! Raul, can you hear Sahra? [he nods his head no] You've got to speak way louder! 58 59 60
Sahra: Twenty-four minus for is twenty, minus two is eighteen. 61
Katelina: So, we know that this six is four plus two. [draws two lines under the six and writes "2" and "4"] And we already know that twenty-four minus four is twenty, so this two is right here. [circles the two in both places in red] So what is this, where does this, um, where's the six in here? If we are going from twenty-four to eighteen, where is the twenty-four? 62 63 64 65 66
Kristina: So the six is – [points to the two and four on the number line] 67
Katelina: Right here! So these two numbers are six. [draws two lines from the "-2" and "-4" above the number line and writes "6" in blue] Nice job! 68 69

Katelina prompted the student to say more (a variation of a talk move) about how she knew to take away the two (lines 49-50). Katelina actively worked to highlight how decomposing the six could be useful for an incremental break apart strategy. Because this was the key mathematical idea of the string, she asked multiple students to repeat (lines 52-53 and 58-59) and revoiced the idea (lines 62-64). This revoicing was more deliberate and intentional than "revoicing" moves in earlier activities, where TCs simply repeated every shared student idea. She also pressed students to reason (reasoning talk move) about where the decomposed numbers and the original equations (lines 64-66), taking up the TEs comment about making the strategy explicit. In the second string, Orianne used a turn and talk, repeating, and reasoning moves with similar purpose. The rehearsal and enactment on the last day illustrated how TCs used the talk moves strategically to orient students to each other and the mathematical goal and to highlight key mathematics and

representations. In both strings, they used a turn and talk to select students and repeating and reasoning moves when students shared the break apart strategy.

### *Talk Moves as Facilitating (Iterative) Opportunities for TC Learning*

Talk moves facilitated iterative OTL by providing a common language (to talk about productive math talk), routine and tangible moves to repeatedly try out (and enact in increasing complexity), and representations and reifications of practice that mediated TCs' understanding of ambitious practice. Talk moves gave something specific for TCs to do and, at the same time, were generative, high-leverage techniques that could be used for a number of purposes and that shaped TCs' understanding of facilitating productive mathematical discussion.

**Opportunity to work on practice with a common language.** Talk moves provided a common language to understand teacher moves within discussion facilitation. By looking at a specific, named techniques, the group had a starting point for talking about more complex aspects of practice and facilitation of productive mathematical discussion. This builds on Boerst and colleagues' (2011) idea of supporting TC learning by connecting and nesting practices of varying size (domains, intermediate practices, and techniques) to attend to both the how (enacting practices and moves) and why (principled vision of high-quality teaching). Through this common language, TCs and TEs collectively worked on practice and developed shared meaning of discussion facilitation.

**Opportunity for deliberate practice.** Talk moves provided "tools" (adding to TCs' repertoire of teacher moves) that TCs could try out and talk about. As routine structures, talk moves could be repeatedly and deliberately practiced in increasing complexity (Boerst et al., 2011). Having a repertoire of discourse-based techniques and moves, particularly talk moves, gave TCs tools to respond to student thinking and participation within the larger practice of discussion

facilitation. Talk moves served as routine structures to enact more complex intermediate practices such as eliciting student thinking or orienting students to key mathematical ideas. In the same way that instructional activities routinize teacher – student – content interactions (and thus creates a "stable and rehearsable backdrop for the dynamic work of responding to student thinking," Lampert et al., 2010, p. 133), talk moves were used as routinized teacher moves that TCs could use in increasingly complex ways.

**Opportunity for generative learning.** Talk moves are high-leverage techniques: moves that can be used for a number of purposes and that are generative of learning. For the same reasons that teacher educators are working on naming core practices, use of talk moves creates OTL by providing a core set of "cross-cutting" moves (Boerst et al., 2011). As high-leverage techniques, talk moves can be nested within more complex core practices and domains. Thus, teacher educators don't have to name all moves within teaching but can focus on a manageable number of high-leverage and generative techniques and moves that support enactment of core practices. In my analysis, talk moves opened up problems of practice and OTL about intentionally choosing moves within intermediate practices (e.g., eliciting student thinking) to respond to student thinking and attend to mathematical goals.

**Opportunity to develop a shared vision of practice.** Talk moves provided opportunities to develop a shared vision of ambitious practice. As Thompson et al. (2013) and Ghousseini et al. (2015) found, tools (in this case talk moves) reify practice. Talk moves facilitated iterative OTL as the group cycled between using talk moves and negotiating meaning around the practice of facilitating productive mathematical discussion. Using talk moves established that teaching is deliberate: moves are chosen based on purpose (Boerst et al., 2011). As TCs collectively worked on using talk moves, they developed shared meaning that productive mathematical discussions

require attending and responding to student thinking, using talk moves purposefully, and working toward mathematical ideas. It is possible to use talk moves like a checklist and in non-ambitious ways (e.g., the first day that they used talk moves). Thus, talk moves need to be embedded and made sense in a vision of ambitious teaching and with deliberate coaching and feedback from the TE:

> "Enactment tools can be more than scripts or processes that scaffold performance. When novices have opportunities to navigate these problem spaces in the company of more experienced others, enactment tools can mediate learning about goals of professional practices, including the commitments that enable practitioners' judgment in situations of uncertainty" (Ghousseini et al., 2015, p. 495).

My findings, like Ghousseini and colleagues (2015) found with their specified question sequence, articulate how talk moves mediated TC participation and "illustrate the way in which an enactment tool and its discursive nature mediates learning of professional goals and in a community of practice" (p. 492). Talk moves facilitated iterative OTL by providing a common language, providing routine, high-leverage moves that were generative of TC learning, and reifying practice and mediating TCs' understanding of facilitating productive mathematical discussion.

**Conclusion**

The purpose of this chapter was to analyze how TC participation was mediated through a shared repertoire. The shared repertoire examined included the ways TCs used lesson plans, teaching materials (focusing on use of the board), and talk moves. My choice in selecting these tools was based in mathematics education research on facilitating productive mathematical discussions and a desire to study enactment tools that were used explicitly and throughout the ten-week course. I documented the ways these tools were used in practice and how they facilitated OTL for TCs. All three groups of tools created opportunities to work collectively on practice in increasingly complex ways. Both enactment of discussion facilitation and use of tools were

iterative processes. Tools reified practice as representations, decompositions, and approximations of practice and provided routines and opportunities for deliberate practice.

### *Summarizing of Findings on Lesson Plans*

There were changes in both the *content* and *use* of lesson plans. A major change in content was the shift in focus on student thinking. Over the ten-week course, content of lesson plans became completer and more specific to the task, identified a mathematical goal, included and anticipated student strategies and representations, and increased in accuracy and anticipation of representations. Lesson plans changed from listing teacher-focused actions and scripts to being a space to anticipate student strategies and possible representations for the board. This shift to using lesson plans as opportunities to plan possible moves in response to student thinking and towards a mathematical goal reflected a change in TCs' understanding of the purpose of lesson planning and discussion facilitation. Lesson plans provided scaffolding to the structure and purpose of the instructional activity, directed TC attention to student thinking, representations, and mathematical goals, and served a boundary object mediating TC thinking and TE coaching (via written feedback and comments).

### *Summarizing Findings on Board Work and Materials*

There were also changes in the use of the board and materials. In rehearsals, participation shifted from working on logistical management of materials to focusing on how to use the representations on the board. In later rehearsals, TCs used the board more actively and intentionally. The variety and quality of TCs' mathematical representations increased, and in later sessions, TCs used boards based on their mathematical purpose. Ownership over the use of the materials and discussion about materials and board work moved from being TE-initiated to residing in the group. The board and materials facilitated opportunities for TC learning as a space

to practice representations and mathematical ideas, opportunity to think through the lesson (including the mathematics, representations, teacher moves, and possible student ideas), and a record of public thought and artifact for discussion.

### *Summarizing Findings on Talk Moves*

Facilitating discussion can feel daunting and overwhelming, but having talk moves allowed TCs to consider teacher moves in relation to student thinking and steer the conversation towards the instructional goal (Boerst et al., 2011; Sleep, 2012). Throughout the quarter, TCs used talk moves to listen carefully to student thinking. They were consistent in how they used wait time, the thumb signal, and participation expectations. They used repeating moves, revoicing, and turn and talks throughout the quarter, and additional talk moves in later lessons as they increased their repertoire of talk moves. Over time, they developed a shared (and more nuanced) understanding of the moves and intentionality in using them to facilitate productive talk. Talk moves facilitated iterative OTL by providing a common language, providing routine, high-leverage moves that were generative of TC learning, and reifying practice and mediating TCs' understanding of facilitating productive mathematical discussion.

In this chapter, I discussed how TCs used tools and how these tools mediated OTL. My findings do not assert that these are the *right* tools. My intention was to characterize their use as embedded in a practice-based methods course. Facilitating productive mathematical discussions is an interactive and complex practice, and carefully selected tools can help support enactment, managing complexity, while "maintaining its improvisational nature" (Ghousseini et al., 2015, p. 492). These tools decomposed, represented, and approximated the practice of facilitating productive mathematical discussion and created a circular relationship between tool use and understanding of ambitious practice.

# Chapter 7: Discussion and Implications for Teacher Education

Practice-based teacher education attempts to address the problem of enactment by supporting TCs to develop a principled vision of high-quality teaching, practices and professional judgment to enact this vision, and knowledge about math and students to base the decisions on student thinking (Forzani, 2014; Grossman & Pupik Dean, 2019; Lampert et al., 2013; McDonald et al., 2013; Zeichner, 2012). As a field, we have made substantial progress specifying core practices of high-quality instruction and pedagogies of practice that provide opportunities for deliberate practice and enactment of these practices (Grossman, 2018; Kazemi, 2017; Lampert et al., 2013; Shaugnessy et al., 2019). However, we have limited images of changes in TC practice and trajectories of learning that we hope to cultivate in our methods courses (Boerst et al., 2011; Kazemi & Wæge, 2015; Shaughnessy et al., 2019).

The purpose of this study was to understand how TCs investigated and enacted the practice of facilitating productive mathematical discussions. My analysis and findings answered the following: How does TC participation manifest and change in their practice-based mathematics methods course? I examined TC learning as evolving participation in the course. I characterized their investigation and enactment of this practice by looking at two sub-questions:

1. How did TCs facilitate discussions to reach particular instructional goals?
2. What shared repertoire did TCs develop, and how did they use these tools?

Building on work of the Core Practice Consortium and other practice-based scholars, I documented TCs' enactment of practice and characterized their participation in a practice-based setting. My dissertation made two contributions to the literature:

1. Existence proof that there are shifts in TC beliefs and talk about teaching *and* enactment of the practice of facilitating a productive mathematical discussion in a 10-week practice-based methods course.
2. An image of TC participation to contribute to a trajectory of TC learning and enactment of core practices in teacher education.

The purpose of teacher education is to prepare TCs for the profession of teaching. Understanding what TCs learn in practice-based courses, particularly how they enact practice, informs and refines teacher educator practice and contributes to understanding the efficacy of a core practices approach.

In Chapter 5, I discussed how TCs worked on the practice of facilitating productive mathematical discussions using the lens of instructional goals. This chapter answered my first sub-question: How did TCs facilitate discussions to reach particular instructional goals? I examined how TCs worked on goals and what aspects of practice they attended to as they worked on facilitating discussions. I described how TCs made shifts in their talk *and* enactment of facilitating discussions towards instructional goals. Findings documented changes in participation around instructional and practice goals. As the course progressed, TCs were more specific in articulating the instructional goal and more intentional in orchestrating discussions to reach these goals. Ownership over this work on goals shifted from residing with the TE to residing in the group. Findings indicated that through repeated cycles of enactment and investigation, TCs revisited and refined their understanding and use of discrete techniques, intermediate practices, and larger domains of practice in increasing complexity.

In the second part of my analysis (Chapter 6), I discussed the shared repertoire TCs developed and how they used tools within this repertoire (particularly lesson plans, board work,

and talk moves). As TCs became more familiar with the tools and routinized their use, they were able to use them more intentionally in response to student thinking and in service of their instructional goal. My findings articulated how these tools opened up OTL around different problems of practice. Across all three tools, findings indicated both enactment of discussion facilitation and use of tools were iterative processes. All three groups of tools created opportunities to work collectively on practice in increasingly complex ways. Tools reified practice as representations, decompositions, and approximations of practice and provided routines and opportunities for deliberate practice. In the following sections, I discuss implications for teacher education in relation to these findings.

## Iterative Trajectories of TCs' Work on Facilitating Discussions Towards Instructional Goals: Discussion and Implications for Teacher Education

My findings illustrated the iterative nature of TCs' development of practice and trajectories of participation in relation to core practices and decompositions of practice. In repeated cycles of enactment and investigation, TCs revisited and refined their understanding and use of discrete, high-leverage techniques (e.g., talk moves), larger domains of practice (e.g., facilitating productive mathematical discussions), and related intermediate practices (e.g., *orienting students to the mathematical goal*). Through this iterative process (revisiting and refining nested and connected practices of varying grain sizes) TCs attended to both the *why* of teaching (developing their practice of facilitating productive mathematical discussion and their vision of ambitious teaching) and the *how* of teaching (appropriating and using high-leverage techniques and moves that served multiple purposes and were nested within larger domains and intermediate practices) (Boerst et al., 2011; Janssen et al., 2015). The iterative nature of TCs' development of practice, particularly how they worked on facilitating discussions towards instructional goals with increasing intentionality

and complexity, has implications for how teacher education can frame what TCs are working on (and towards) and how this practice can be decomposed and developed iteratively.

### *Facilitating Discussion versus Facilitating Interaction: What Counts as a Productive and Mathematical Discussion?*

The changes in how TCs worked on discussion and instructional goals highlights the relationship between facilitating *interaction* and facilitating *discussion*. I framed my analysis as how TCs worked on discussion practices. However, in the beginning of the course, it is not accurate to say TCs were facilitating productive mathematical discussion. Instead, they were working to facilitate interaction, and facilitating interaction and talk with real students necessitated attention to what to do in response. Before TCs could attend to more ambitious aspects of discussion practices (and how to facilitate *productive* and *mathematical* discussions), they needed support to facilitate talk and interaction. As they developed shared meaning and routinized practices and moves to facilitate talk and interaction, the TE then directed their attention to goals, reflecting a shift from working on facilitating interaction to facilitating discussion. She prompted them to think about their moves in relation to student thinking and goals and provided tools to scaffold their facilitation of talk *and* discussion (e.g., modeling of specific talk moves to orient students to goals; providing the compare and connect discussion structure; choosing instructional activities whose structure lent itself to more targeted mathematical goals). The TE was providing opportunities for deliberate practice with feedback, first attending to more general practices of facilitating talk, then increasing complexity to address the demands of facilitating discussions in response to student thinking and towards mathematical goals.

The distinction between facilitating talk and facilitating discussion matters for teacher education because it indicates the need for multiple opportunities to enact and refine practice.

While facilitating talk and interaction by itself is not ambitious practice, it is part of the work of facilitating discussion. Teachers who facilitate productive mathematical discussions have practiced and routinized more logistical, less ambitious moves of facilitating interaction, allowing them to enact complex practices that are responsive to students and mathematical goals. In contrast, TCs needed time to practice and routinize these moves and practices for facilitating interaction. TCs' early focus on teacher moves and logistical issues reflected this work (e.g., how to stand in a way so that students could see, remembering to bring and pass out materials, remembering to use talk moves and figure out how to word questions to increase student talk). Until these less complex and less ambitious moves were practiced and routinized to some degree, there was less capacity and attention to work on responsive and ambitious discussion practices. While it isn't productive for teacher education to only focus on logistical aspects of practice or small-sized discrete moves, practicing and routinizing moves to facilitate interaction is necessary work. Learning in practice takes time. If methods courses only have one or two cycles of enactment and investigation (or a few opportunities for enactment and to be in classrooms), not a lot of progress will be made.

This distinction between working to facilitate *talk* and working to facilitate *discussion* also prompts an important question: What counts as *discussion*? More specifically, what counts as *productive mathematical* discussion? *Facilitating discussion towards instructional goals* is a practice of high-quality teaching across content areas (Grossman, 2018; TeachingWorks, 2020). The Core Practices Consortium worked to specify what constituted discussion facilitation versus interaction facilitation and defined the practice of facilitating discussion in the following way:

> "In a whole class-discussion, the teacher and all of the students work on specific content together, using one another's ideas as resources. The purposes of a discussion are to build collective knowledge and capability in relation to specific instructional goals and to allow students to practice learning, speaking, and interpreting" (Grossman, 2018, p. 185).

To count as discussion, there has to be some kind of goal, a "joint enterprise," that people are contributing to and working towards (this conception of discussion is also true for TC learning and their work and joint enterprise around practice).

My choice to specify this practice as facilitating *productive mathematical* discussion was an attempt to distinguish facilitating interaction versus facilitating discussion towards a goal *and* to understand discussion facilitation in the context of the discipline of mathematics. Productive discussions support meaningful student participation and talk towards an instructional goal, as "the teacher and a wide range of students contribute orally, listen actively and respond to and learn from others' contributions" (Grossman, 2018, pl. 185; Chapin et al., 2013). A *mathematical* discussion includes discipline specific practices and facilitates collective meaning and learning around mathematics (e.g., a mathematical concept, practice, or model). This kind of discussion helps students figure out a mathematical idea, build on members' contributions, and collectively make sense of this idea.

By analyzing how TCs worked on instructional goals, I provided an image of this trajectory from working on facilitating interaction to facilitating productive mathematical discussion. My analysis of TCs' level of specificity of instructional goals and intentionality in facilitating productive mathematical discussions was not an assessment of the sophistication of their discussion facilitation (e.g., Thompson et al., 2013). Rather, I sought to illustrate the change in their practice around goals and to distinguish their work on facilitating interaction and facilitating discussion. Though there is some degree of facilitating interaction that needs to be practiced and routinized before discussion facilitation can happen, I am not proposing that TCs' trajectories of discussion facilitation are linear learning progressions moving from working on facilitating interaction to facilitating discussion. Similarly, TCs do not "master" small techniques, then work

only on intermediate practices, and finally "graduate" to trying these out in larger domains. My findings illustrated how this trajectory was iterative as TCs enacted intermediate practices and techniques in more complex ways and continually revisited them with more nuanced and complex understanding of ambitious teaching and the work of facilitating productive mathematical discussions.

**Decomposing and Working on Practice: The Relationship Between Intermediate Practices and the Practice of Facilitating a Productive Mathematical Discussion**

In the previous section, I focused the relationship and difference between facilitating interaction and facilitating productive mathematical discussion. We can see how smaller techniques (e.g., talk moves) and related intermediate practices (e.g., *eliciting student thinking* and *orienting students to the mathematical goal*) are important components of discussion facilitation. Particular to the methods course studied, seven ambitious practices (e.g., *eliciting and responding to student thinking* or *orienting students towards one another's ideas and to the mathematical goal*) were articulated and explicitly worked on (see Appendix B). My findings around the iterative trajectories of developing practice illustrated how these ambitious practices functioned as *intermediate practices* in service of the larger practice of facilitating productive mathematical discussion.

In addition to *facilitating a productive mathematical discussion*, the ambitious practices outlined in the course functioned as core practices. They are nested and related to discussion facilitation and serve as building blocks of ambitious teaching to "innovate and improvise" (Janssen et al., 2015, p. 144). In the course, supporting TC development of these ambitious practices involved sequencing approximations of practice and working on practices in increasing complexity. Nested within these ambitious practices, smaller high-leverage techniques and moves

(which serve a number of purposes) were used to support TC practice (e.g., talk moves). The attention to varying grain sizes of practice (and how they are nested and connected) and the iterative process in revisiting and refining these practices supported development of TC practice and purpose of teaching:

> "By moving upwards in the system through recomposing core practices into instructional activities and lessons, teachers can obtain a deeper understanding of the connection between core practices and instructional activities and the goals of teaching. By moving downwards in the system by decomposing practice, teachers can develop a better grasp of how these broad goals can be enacted in a classroom" (Janssen et al., 2020, p. 144).

Facilitating productive mathematical discussions *is* ambitious teaching, and thus requires attention to ambitious core practices such as *eliciting and responding to student thinking*, *positioning students competently*, and *orienting students to one another's ideas and the mathematical goal*. The ambitious practices provided a useful lens for understanding nested and connected *intermediate practices* central to facilitating productive mathematical discussions. For teacher education, this implies that core practices should not be worked on in isolation and outside of a vision of ambitious teaching.

## The Iterative Process of Using Tools and Negotiating Meaning of Practice: Discussion and Implications for Teacher Education

Both enactment of discussion facilitation and use of tools were iterative processes. Tools reified practice as representations, decompositions, and approximations of practice and provided routines and opportunities for deliberate practice. This implies that teacher education needs a repertoire of core enactment tools and validates the importance of using tools in enactment and learning cycles.

### *The Need for a Repertoire of Core Enactment Tools*

Given the iterative process of tool use and negotiating meaning of practice, practice-based teacher education needs for a repertoire of high-leverage, core enactment tools (in addition to high-

leverage core practices and pedagogies, and instructional activities). Having a set of high-leverage, core enactment tools that are generative of learning and able to serve a number of purposes would allow TEs to be flexible and responsive to TCs' needs while at the same time working with a manageable set of tools. In the course studied, the TE identified smaller high-leverage techniques and moves (which serve a number of purposes) to support TC practice (e.g., talk moves). Other tools to scaffold discussion facilitations included: 1) providing discussion structures from *Intentional Talk,* 2) identifying different types of discussions based on the discussion goal, and 3) use of instructional activities to provide a routine for structure interaction and recompose practice. These tools (varying in grain size) provided common language and routines to work on practice. Building a shared repertoire of high-leverage, core enactment tools is ongoing work, and mathematics educators continue to specify enactment tools, scaffolds, and course assignments for supporting TC learning (e.g., Boerst & Sleep, 2012; Chapin et al., 2013; Ghousseini et al., 2015; Tyminski et al, 2014).

***The Importance of Using Tools in Enactment (and Learning Cycles)***

The iterative trajectory of revisiting and refining both practice and tool use validates the usefulness of a learning cycle of enactment and investigation and opportunities to enact practice and work in classrooms within a methods course. Using tools in practice and with students shaped how TCs appropriated and made sense of tools and the practice of facilitating productive mathematical discussion. After careful planning, TCs experienced their plan in action and with students. Seeing what students did and experiencing teaching and decision making in the moment (both in rehearsal and in enactments) informed the next cycle of planning, tool use, and enactments. TCs' collective understanding of and beliefs about what it meant to facilitate discussions shaped how they used tools. At the same time, the ways that they used tools and the problems of practice

that were opened during enactments shaped how they understood practice. Having approximations of practice (e.g., rehearsals), enactments with students, and feedback through this work (beyond pretending to teach or talking about teaching) provided different lenses to problematize practice and tool use. Enacting lessons necessitated that TCs work through the logistics of using tools, leading instructional activities, and facilitating interaction and talk. In each repeated cycle and revisiting of tools (including instructional activities) and practice, TCs reified practice and "theory" of how students learn (Thompson et al., 2013; Ghousseini et al., 2015). Using tools in enactments and learning cycles prompted TCs to revisit and refine their practice and use of tools.

**Attending to the Sequence of TC Learning Experiences**

In the discussion above, I have alluded to the need to attend to how TC learning experiences are sequenced. A learning cycle model is a useful framework for considering how to: 1) situate TC learning in practice, 2) provide TCs with multiple opportunities to enact core practices, and 3) introduce, prepare, enact, and reflect on activities and core practices (McDonald et al., 2013). It is also important that teacher educators build on this framework to look beyond experiences in one cycle and to examine the trajectory of TC experiences through multiple cycles during the course and their time in the teacher education program. For example, if a course goes through ten rounds of the instructional cycle, what tools would be useful for scaffolding TC practice, and what aspects of practice would be useful to focus on in the first round? How is this different from the fifth or tenth rounds? The iterative nature of TC learning suggests TC learning experiences could be thoughtfully sequenced through particularly instructional activities, use of various enactment tools, the level of teacher educator scaffolding, and what aspects of practice are worked on. For example, it seems useful to begin with a small set of moves (e.g., a few selected talk moves) and instructional activities and to provide the instructional goal. It also seems useful to start with an instructional

activity that scaffolds TCs' facilitation of interaction and talk and is structured to elicit student ideas. Further research is needed to understand how focusing on particular aspects of practice and using particular tools and instructional activities can scaffold TC learning and how they could be sequenced to support TC development of practice.

**The Role of the Teacher Educator in Scaffolding and Coaching for Deliberate Practice**

Numerous studies document the critical role of the TE in coaching TCs and scaffolding deliberate practice to develop adaptive expertise (e.g., Anthony et al., 2015; Ghousseini et al., 2015; Kavanaugh, 2020; Lampert et al., 2013). Although I examined TC participation, this participation was situated in the context of the methods course and in relation to the TE and her feedback and coaching. Deliberate practice is not just experience in a domain: It requires scaffolding and approximations "as a way to create an environment within which teachers could specifically and repeatedly focus on one goal and receive feedback on their progress towards that goal" (Kavanaugh et al., 2020, p. 104). The TE's role as a more knowledgeable other and her feedback and coaching on enactments of practice created opportunities for deliberate practice. This facilitated TC learning beyond understanding practice in discrete, non-ambitious ways (e.g., practicing discrete moves like in microteaching or the Commonwealth Study's attempt to outline all the moves of teaching) and developed TCs' practice, vision of teaching, and professional expertise. The TE is an adaptive expert, responding to the needs of her TCs. This involves sequencing approximations and representations of practice to give TCs ways of seeing and understanding practice and connecting moves and practices with principles teaching.

Given this critical role, the expertise and experience of the TE in this study, particularly in designing and teaching this practice-based course, coaching TCs, and working in classrooms, were important for supporting TC learning. Her experience in classrooms enabled her to anticipate what

students might do, to provide possible moves in relation to student thinking, and to scaffold TCs' understanding and use of the instructional activities. My findings demonstrated how in early sessions, TCs often did not anticipate student thinking (or were vague in how they planned to represent and address student ideas) and needed support in understanding the structure and purpose of the instructional activities. This highlights for TEs facilitating practice-based methods courses to have classroom experiences and strong coaching practices.

Through the course's planning and debriefing meetings and conversations with the TE, it is evident that the TE was continually revisiting her vision of ambitious teaching and stance on learning to teach and thought intentionally about how to support TC learning. She worked to scaffold this learning and position TCs competently (and for success) based on her vision of ambitious teaching and what she knew about TCs and students. Just as elementary teachers have a vision of mathematics teaching, the TE also has a vision of what TCs should learn, including the learning goals and anticipation of what TCs might do. Based on her expertise, the TE in this study could provide relevant and timely coaching in rehearsals and scaffold experiences to support TCs to successfully facilitate instructional activities and facilitate interaction. As TCs became more comfortable facilitating the instructional activities and interaction, the TE coached them to refine their practice and begin facilitating discussions in ways that were productive and mathematical. Thus, a TE's vision of teaching and learning to teach plays a large role in shaping practice-based instruction and sequencing experiences to support TC learning.

My study documents the TC participation in the context of the TE's deliberate coaching and feedback. Continued research is needed to explicitly connect this participation, learning, and OTL with particular TE pedagogies, tools and moves. Additionally, my findings indicate the need to attend to the preparation of TEs and how they are supported to coach within practice-based

settings. This practice-based design necessitated multiple coaches for to provide the coaching and guidance needed for deliberate practice within the small group time (particularly during rehearsals). The TE did not facilitate the methods course alone and worked with a team of teacher educators, including the classroom teacher and a graduate assistant and postdoctoral researcher. The other three TEs were well-versed in CGI, the instructional activities, and the ambitious practices discussed in the course. Additionally, they met weekly to debrief the session, discuss observations about TC practice and learning needs, and plan next steps for supporting TC learning. In later iterations of the course, teaching assistants met weekly to rehearse rehearsals and to discuss and work on coaching practices, instructional activities, the session's focus and learning goal for TCs. The course instructor often met separately with the classroom teacher to accommodate their teaching schedule. In the following section, I zoom in on the importance of this particular teacher educator: the classroom teacher.

**The Role of the Classroom Teacher and School Context in Developing TC Practice**

In addition to being situated in the TE practice and coaching, TC learning and participation was situated in the partner elementary teacher's classroom. Changes in TC participation (documented in my findings) signal the important role of this partner teacher and the school context in supporting TCs' discussion facilitation. If teacher education hopes to support TCs to take up ambitious practices and to facilitate productive mathematical discussions, it seems useful to select classrooms where these types of practices are already happening. By selecting a classroom where this type of discussion and ambitious teaching practices take place, TCs can: 1) Observe the classroom teacher (and/or teacher educator) modeling these ambitious practices; and, 2) Try out these practices in a classroom where productive mathematical discussions are already happening.

With these two purposes in mind, the partner teacher and classroom can be strategically selected to connect course goals and opportunities for TC learning.

One purpose of having a partner classroom (and having this field experience embedded in the methods course) is to provide a space where TCs can observe the instructional activities and ambitious practices in action. The classroom teacher (along with the teacher educator) modeled discussion practices and instructional activities. Because the classroom teacher participated in part of the sessions' planning and debriefing conversations, TCs could also investigate and ask questions about his particular elementary students and what they observed, as well as how he addressed issues of social justice through his practice and negotiated the realities of the school context with his vision of ambitious teaching. Through his particular experiences and practices, TCs could investigate larger course goals around teaching equitably and addressing social justice issues through mathematics teaching. This type of active role in the methods course, particularly in modeling ambitious practices and discussing the relationship between social justice and mathematics teaching, suggests that partner teachers should have a vision of teaching that aligns with course goals and be able to facilitate productive mathematical discussions (or at the least be committed to working on this practice). While it is possible to partner with a teacher who is not an expert in facilitating productive mathematical discussion, this type of partnership would require the TE to balance actively supporting the partner teacher's practice while simultaneously being respectful of and valuing the partner teacher's practice and willingness to invite others' into the classroom. Although always important, it would be especially important for the TE to carefully frame what TCs are observing and working on to both position the teacher competently and ensure TCs do not focus on unintended practices (e.g., practices that are contrary to facilitating productive mathematical discussions).

The classroom teacher's role was not limited to demonstration and providing perspective and background information on the students and class community. In the course studied, the TE framed the classroom teacher's role as a co-teacher educator, and he (like the TE) positioned himself as a learner alongside the TCs: openly sharing and reflecting on his practice, rehearsing and receiving feedback from the TE, sharing his classroom experiences and knowledge during whole group discussions, and mentoring and coaching a small group of TCs. The classroom teacher's role as an additional teacher educator has important implications. The classroom teacher's role in the course was intentional and strategic for connecting coursework and in-classroom experiences: his vision of mathematics instruction aligned with course goals and his practice was grounded in principles from CGI, the instructional activities, and discussion practices. The productive partnership in the course studied suggests practice-based methods courses could support TC learning by partnering with teachers whose vision and practice align with course goals, who are open and willing to share their practice, who are willing to learn alongside the TE and TCs, and who are committed to student-centered practices that are ambitious and equity-focused.

In addition to providing a space for observation and investigation, the classroom also provided a space for TCs to try out the instructional activities and ambitious practices. Time and intentional culture building are needed to create a classroom culture and community where productive mathematical discussion takes place and where students feel valued and safe to share. By selecting a classroom where this type of discussion already happens, TC learning can be more "scaffolded" in the sense that students are used to sharing their ideas and using talk moves. TCs can build on the class culture that the teacher has already fostered and learn to enact practices in a space where elementary students are accustomed to sharing their ideas and participating in mathematical discussions. Although TCs still have to establish discussion norms and facilitate

student talk, this context allows TCs to learn to facilitate discussion in a setting that is already supportive of discussion practices (as seen in the course studied).

In the same way that instructional activities provide a backdrop that allows the TE to hold certain elements of teaching constant and to focus on particular aspects of practice, strategic selection of a classroom can provide a productive learning space for TCs, who are just starting to try out discussion practices (Lampert et al., 2010). The purpose of the partner classroom is to provide a space for TCs to learn to teach ambitiously (for the course studied, a large part of this work was learning to *facilitate productive mathematical discussions*). Thus, the classroom context for practice-based methods courses should scaffold learning for novices and provide opportunity for them to build their repertoire of discourse tools and discussion practices in increasing complexity and increasing authenticity to the challenge and ambitious nature of teaching. With limited time and opportunities to teach in the ten-week course, the existing culture of the elementary classroom (where students already participate in productive mathematical discussion) can be leveraged to support TC learning. In other words, its seems useful for teacher educators to partner with classrooms where students are already participating in productive mathematical discussions.

For various logistical and programmatic reasons, teacher education has, historically, haphazardly created field experiences or left them largely unguided (Zeichner, 2010). Despite the challenges of supporting field experiences, existing research documents that thoughtfully connecting coursework to field experiences, including deliberate coaching and mentoring, can support TCs "to successfully enact complex teaching practices" (Zeichner, 2010, p. 95). The course studied is one of many efforts to bridge the gap between coursework and field experiences and to provide deliberate coaching and practice. In my discussion above, I described the important

role of the classroom teacher and the intentionality of the TE in carefully connecting her methods course with the classroom experience. While my dissertation focused on TC participation and changes in how they facilitated discussion, it is clear the TE, classroom teacher, and school context all played important roles in shaping this participation. Creating third spaces that mediate campus courses and field experiences, such as in the course studied, can support TCs to enact complex practices. This dissertation documented the changes in TC participation and enactment within one such "third space."

The TE's expertise and coaching was consequential for connecting "theory" and "practice" and supporting TC to facilitate productive mathematical discussions. In addition, having the classroom teacher as a partner teacher educator leverages the expertise from the school, builds capacity within the school and teacher education program, and positions the teacher as an instructional leader and coach. As a teacher educator, the classroom teacher can also mediate the course experiences and co-creates the third space as someone who engages in this ambitious work and who knows the school, students, and realities of teaching. Their classroom serves not only as a place where TCs can see the instructional activities and ambitious practices in action, but also a space to try out and learn these ambitious practices. Thus, the classroom context is a central aspect of creating this third space in practiced-based methods courses. My findings demonstrate how TC practice developed and how their participation changed in the context of this careful coordination, deliberate coaching, and practice-based design.

**Limitations of this Study**

There are several limitations to my study. First, this study is not an *assessment* of TCs' practice of facilitating productive mathematical discussions nor is it an assessment of their development of pedagogical content knowledge. When analyzing their intentionality in working

towards an instructional goal or the changes in how they used tool, I was not assessing their sophistication of practice in the same way as Thompson et al. (2013) or IQA Mathematics Rubrics (Boston, 2012). I examined how TCs worked on mathematics through articulation of mathematical goals and their mathematical representations and board work. However, this analysis was not an assessment of pedagogical content knowledge. Other studies have worked on assessing pedagogical content knowledge (e.g., Hill et al., 2008; Hill, 2010) and more work is needed to understand how we might assess enactments of practice within methods courses.

My findings center on TC participation and OTL facilitated through pedagogies of enactment. I do not make claims about the effects of pedagogies on TC learning, nor do I make causal claims about the TE's effect on TC learning. I discussed the TE pedagogies and moves in relation to TC participation and to situate their investigation and enactment of practice. I characterize TC learning as participation *in the context of* (not *as an evaluation of*) practice-based pedagogies, core practices, and the TEs of the course.

I examined a limited number of tools (lesson plans, board work, and talk moves), but other tools were used in the course and certainly influenced and supported TC participation and enactment of practice. In particular, the use of video was an important tool in this course and has been widely researched as a representation of practice (e.g., van Es & Sherin, 2010; Grossman, 2018). In the methods course, TCs watched videos of experienced teachers, watched videos of their own practice immediately after enactment, and used on online video platform to reflect on their practice through course assignments (this online platform was mostly used in the second course in the methods sequence). It would be useful to understand how these videos influenced practice and facilitated OTL.

In addition, this study raises an important question about the nature of discipline-specific practice: To what degree are these findings generalizable to other disciplines? The CPC work highlights both the tension between discipline-specific practices (and methods courses) and general teaching practices as well as the opportunities and potential of this cross-discipline collaboration and learning (Grossman & Pupik Dean, 2019). While aspects of my findings around TCs' development of the practice of facilitating productive *mathematical* discussions (and related intermediate practices and techniques) are specific to the discipline of mathematics teaching, there are contributions that are not discipline specific (e.g., the iterative nature of TC learning and development of practice, the need to use tools in enactments, etc.). Further research and cross-discipline collaborations are needed and could support teacher education programs in building coherent and thoughtfully sequenced learning experiences for TCs.

The goal of my study was not to provide generalizable findings about TC participation in *all* practice-based methods courses. Nor was it to say that this course is the *right* way to structure a methods course or that the tools that I examined are the *right* tools for supporting enactment. Rather, the goal of this qualitative research is to provide a rich, thick description of TC learning within a practice-based setting focused on facilitating productive mathematical discussion. Findings from this research contribute images for understanding a trajectory of TC learning and enactment of core practices and existence proof that TCs can make shifts in both talk and enactment of practice within a methods course.

**Further Research**

This study warrants further research on several topics. First, given the importance of the TE, it would be useful to connect the iterative trajectory of TCs' development of practice with research on the effectiveness particular TE moves and pedagogies. Given the complexity of

facilitating deliberate practice and supporting TCs' iterative development of practice, continued research on teacher educator pedagogies and pedagogies to support new teacher educators is also needed.

As discussed above, continued research is needed to develop a repertoire of high-leverage, core enactment tools within the pedagogies of practice and supporting particular core practice. This includes continued research on instructional activities as routines, the different OTL and problems of practice each routine offers, and how instructional activities might be sequenced to support TCs in their iterative development of practice.

Existing teacher education studies also raise issues about how particular pedagogies connect to and are situated in courses and the teacher education program as a whole (Grossman, 2005). I examined an individual course at one institution. Within this course, I only examined TC participation in the context of their small group time. It would be fruitful for additional studies to examine TC enactments over a longer trajectory across the whole teacher education program (including field placements) and beyond the program as well as TC enactments across sites and programs (areas of research the Core Practices Consortium has started).

I provided images contributing to possible trajectories of TC learning and enactment of practices. Further research is needed to articulate the type of enactment we hope to see at the end of methods courses and in field placements. As Boerst et al. (2011) assert, teacher education needs tools for assessing enactment, not just TCs' reflections on their practice, and opportunities to provide feedback on this enactment: "Because practice is a goal of beginning teacher learning, providing feedback on the enactment of practice and gauging the skill with which beginning teachers engage in practice over time are essential" (p. 2859). Mathematics educators have begun this important work of specifying practice for the sake of assessment and developing tools to assess

TC enactment (Boerst & Sleep, 2012; Boerst et al., 2011; Tyminski et al., 2014). My dissertation provides images of changes in enactments of facilitating productive mathematical discussions that could support articulation of the how we might assess and examine enactment.

We also do not know how TCs enact and continue to develop their practice beyond their teacher education program (particularly facilitating productive mathematical discussion). This question would be a fruitful continuation of my study (much like Thompson et al., 2013, documented for ambitious science teaching). As the literature has suggested, current iterations of practice-based teacher education attempt to prepare TCs to enact high-quality teaching practice that currently does not consistently exist in schools (Grossman, 2018; Lampert et al., 2010). If teacher education hopes to prepare TCs to enact high-quality teaching practice, it will be critical to understand the support and constraints they have in their schools to continue developing their practice and the ways they appropriate core practices. If teacher education has prepared them to enact ambitious practices that are currently not seen in schools, it would be helpful to understand what their participation looks like in their school placement. For teacher education to be effective in preparing teachers, it must consider what TCs experience beyond their programs.

While my findings around the shift in talk and enactment of practice within a ten-week course are encouraging, ambitious and high-quality mathematics instruction (particularly facilitating productive mathematical discussion) is not a practice that can be mastered in one year. Thus, it will be critical for future research to connect the work being done in practice-based teacher education programs with practice-based professional development. It seems that there are possibilities for creating continuity of professional learning between pre-service teacher education and placement as a classroom teacher. Current practice-based literature provides opportunities for TCs to participate in a promising learning cycle of both investigation and enactment of practice.

Long-term improvement of instructional practice and enactment of high leverage core practices could be facilitated through a continuation of this type of hybrid learning for TCs in and beyond their teacher education program. This type of hybrid learning values both the university-based and school-based expertise. TCs who have begun to develop their practice could continue learning in a similar format if it was present in their schools. Math Labs or Lesson Study are examples of such learning cycles for teachers (e.g., Kazemi, Hintz, & Gibbons, 2014 and Fernandez & Yoshida, 2004). What is imagined here is an agenda for teacher education research that strives to provide continuity in continued practice-based professional learning.

**Conclusion**

The field has made progress articulating a vision of high-quality mathematics instruction and pedagogies of practice to support TC learning in practice (Franke et al, 2007; Grossman, 2018; Munter, 2014; Shaughnessy et al., 2019). It is the ability to *enact* high-quality teaching practice that has been problematic for teacher education (Kennedy, 1999; Grossman, Hammerness, et al., 2009). As a field, we have limited images of changes in TC practice and trajectories of learning that we hope to cultivate in our methods courses. The purpose of this study was to understand how TCs investigated and enacted the practice of facilitating a productive mathematical discussion. Building on the work of the Core Practices Consortium and literature documenting pedagogies of practice, this dissertation documented TC enactment of practice and characterized their participation in their practice-based mathematics methods course.

This study is one of a limited number of studies examining TC participation and how they work on and enact core practices. I provided an initial attempt at illustrating trajectories of TC learning as participation in current practice-based approaches to teacher education. My study considers the sociocultural approach common in teacher education and operationalizes learning

from that perspective to lay the groundwork for research on what particular pedagogies offer for TC learning. Understanding the changes in how TC participate and enact practice informs how teacher educators can refine and improve their practices and coursework.

The shifts in TCs' beliefs and talk about teaching *and* enactment of the practice of facilitating a productive mathematical discussion in their methods course are encouraging. Focusing on *enactment* of practice and integrating knowledge and doing by situating learning in practice can support TCs in developing the skills, knowledge, dispositions, and identities necessary for enacting ambitious teaching practices (Forzani, 2014; Grossman, Hammerness, et al., 2009; McDonald et al., 2013). There is much work to be done to transform teacher education. The collective effort to create a common language around pedagogies of practice and core practices is promising and invites continued research around supporting and preparing TCs for the professional work of ambitious teaching.

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# Appendix A: Analytic Methods

## Overview of Process for Analyzing TC Participation

Research Question: How does TC participation manifest and change in their practice-based mathematics methods course?

| Question | Analyzed for… | Analytic Methods |
|---|---|---|
| JOINT ENTERPRISE: How did TCs facilitate discussions to reach particular instructional goals? | **What was the instructional goal?**<br>• What did they say was their goal? What was their instructional goal, as indicated by their rehearsal and enactment?<br>• How do they work on these goals and how does this change over time?<br>• How did TCs intentionally facilitate discussions to reach their goals?<br>**What seemed to be their practice goal?**<br>• What aspects of practice did they work on?<br>• What did they talk about?<br>• What were they trying to do/enact?<br>• How was this connected to the TE goal?<br>**How did TC interact with students and student ideas?** | GOALS CODING:<br>Hand coded lesson plans, field notes, and course materials:<br>• Descriptive coding to note when they were attending to goals.<br>• Process coding to tag how they worked on components of discussion facilitation (practice goal).<br>Coded video on "goals" line in StudioCode©:<br>• Segmented video into instances of TCs working on instructional goals and components of discussion facilitation.<br>• Instructional Goal: Tagged instances when TCs were talking about or enacting instructional goals and wrote descriptive summary of the state and observed goal(s) (including level of specificity) in the comment box for the instance.<br>• Practice Goal: Tagged instances when TCs were talking about or enacting different components of discussion and wrote descriptive summary of how TCs worked on discussion to facilitate this goal (including their intentionality level) in the comment box for the instance.<br>ANALYTIC SUMMARIES AND DATA DISPLAYS:<br>Summary tables and analytic memos:<br>• Recorded instructional and practice goals for each session.<br>Narrative storylines:<br>• Described how TCs worked on instructional goals and discussion facilitation in each session.<br>Time-ordered goal specificity and intentionality matrices:<br>• Level of specificity and intentionality.<br>• Articulated by Date, Instructional Activity, and TC. |
| SHARED REPETOIRE: What shared repertoire did TCs develop and how did they use these tools? | • How did TCs use their lesson plans?<br>• How did TCs use the board?<br>• What talk moves were used and how?<br>**How were OTL mediated by tools?** | TOOLS CODING:<br>Coded video on "tools" line in StudioCode©:<br>• Tagged instances of TCs engaging with lesson plans, board, and talk moves.<br>• Attached in vivo codes and descriptive summaries of how the tool was used in comment box to instances.<br>Hand coding of lesson plans and course materials:<br>• Coded for the content TCs included and how they used planning templates.<br>CHRONOLOGICAL MATRIX AND ANALYTIC MEMOS<br>Patterns and Themes in Tools:<br>• Used the "transcription" feature in StudioCode© to create chronological matrix of instances of tools (with attached codes and descriptive summaries).<br>• Wrote analytic memos for each tool and how it was used and talked about. |
| MUTUAL ENGAGEMENT: How did TCs interact around the practice of teaching and learning to teach?* | • How did TCs interact with one another and the TE?<br>• How did TCs interact with practice and tools?<br>• Where did ownership of the work on instructional goals reside?<br>• Where did ownership of the work on practice goals reside? | ITERATIVE EXAMINATION<br>Iterative examination of patterns and themes across the goals and tools coding and analyses:<br>• Revisited earlier coding and analyses after each successive stage of coding.<br>• Refined or created additional analytic memos, summary tables, and storylines to reflect new layers of analysis.<br>Coded for where the ownership resided (TE ownership, joint ownership, TC ownership):<br>• Coded instances in the timelines.<br>• Coded and reviewed summary tables analytic memos.<br>• Wrote analytic memo summarizing how this ownership changed. |

*My framing of mutual engagement as "how TCs interacted" differs from the original definition that Wenger (1998) provides. I also embed my findings to the analytic question of how TCs interact (mutual engagement) in my analysis on goals (joint enterprise) and my analysis on tools (shared repertoire).

## Appendix B: Syllabus Content: Course Goals

### Course Goals

1. To develop specialized mathematical knowledge about children's learning trajectories in number and operations and to strengthen your own understanding of number, operations, and computational algorithms in order to be able to interpret the Common Core Standards;
2. To develop competence in teaching a set of instructional activities in order to learn to:*
    a. *teach towards an instructional goal*,
    b. *elicit and responding to student reasoning,*
    c. *orient students to one another's ideas and to the mathematical goal,*
    d. *set and maintain expectations for student participation*,
    e. *position students as competent,*
    f. *assess students' understanding, and*
    g. *represent mathematical ideas visually, numerically, and/or verbally*
3. To learn to support broad participation by students in your mathematics classroom through classroom discussions;
4. To learn to create a classroom culture in which all children attain important mathematics and develop productive mathematics identities

**While not articulated in the syllabus, the instructor framed the features of a discussion on a PowerPoint slide (in multiple course meetings) in the following manner:**

### Feature of a Discussion

To have a discussion, we want to have:

- a purpose in mind
- open and support students' entry into discussion
- elicit student thinking and orient students to one another and the math
- close the discussion

## Appendix C: Syllabus Content: Principles for Teaching and Principles for Learning to Teach

We are guided by a set of **principles for teaching**:

1. Teachers must position students as sense-makers and knowledge-generators, who desire to invest and succeed in school. This involves noticing children, building relationships with them, their families and communities, valuing their perspectives, and attending to their thinking, curiosities and capabilities
2. Teaching is both intellectual work and a craft. Deep knowledge of content and pedagogy, creativity, and passion fuel both learning and teaching.
3. Teachers must design equitable learning environments in which all children are engaged in robust and consequential learning.
4. Teachers' instruction and student learning is always conducted within the context of larger social systems, structures, and hierarchies.
5. What we do and say matters and must be analyzed. Our language and action constructs or constrains opportunities for children to build meaningful, positive, and sustained relationships to learning and one another.

The design of the activities and the coursework in general is guided by the following **principles for learning to teach**, which we expect you to endorse in this course and beyond:

1. Teaching is intellectual work and requires specialized knowledge.
2. Teaching is something that can be learned.
3. Learning to do something requires repeated opportunities to practice.
4. There is value in making teaching public.
5. We all bring our histories forward. Our own learning experiences and identities shape what we know and do. Our developing identities as math teachers matter to our work with children.

## Appendix D: Syllabus Content: Instructional Activities

We will focus on a set of instructional activities. We have selected these because: 1) They are *types* of activities that are central to the work of elementary teaching; 2) They contain important mathematics and have the potential to improve student achievement; 3) You will be able to use these activities across elementary grade levels and mathematical content; 4) They will enable you to attend to student thinking and engage in ambitious teaching practices in ways that should transfer into and support your daily work as a teacher; and 5) You may find them useful to regularly use in your classroom. The activities are:

Activity 1: Quick Images

In this activity, you will flash an image of a collection of objects or symbols such as dots. You will engage students in quickly discerning the number of dots in the image and will then engage them in a discussion of how they organized and subitized items to count them. This activity supports children in moving beyond counting by ones, helps them think about how numbers are composed, and supports the development of number sense. Teaching the activity requires that you deliberately select images based on your mathematical goals and productively manage student participation to help them develop number sense.

Activity 2: Estimation 180

This activity is inspired by middle school teacher Andrew Stadel (see estimation180.com). You will present students with an image about which a question related to estimation is posed. You will first support students to understand what the image is as a context for a problem situation. Then they will go about figuring out and sharing their ideas about the mathematical question related to the image. You can ask your students to compare their estimates with the actual answer.

Activity 3: Choral Counting

In this activity, you will lead students in skip counting together and you will record the count on chart paper. You will engage children in noticing patterns that emerge in the count and in speculating about mathematical explanations for those patterns. Counting is something that any child can do at any grade level as a warm-up. It is an activity that allows opportunities for students to understand pattern and structure in the number system, develop place value understanding and investigate the four operations. It allows you to engage the whole class at once and open up multiple access points for participation.

Activity 4: Posing one or more computational problems in order to elicit a range of strategies or work on a particular strategy

In this instructional activity, you will lead students through a mini-lesson in a focused, analytic exploration of computational strategies. Each problem in the string is designed to build on the thinking used to solve previous problems: there are explicit connections and relationships among the problems. To enact well, you must understand the rationale underlying the sequence of problems so that your instructional purpose guides your decision-making. You will need to anticipate student solution strategies and plan instructional moves accordingly. Although this activity begins with students sharing multiple strategies, you play an active role

in guiding them toward particular strategies and increased understanding of operations by drawing on models of operational concepts. The task can be used to introduce key mathematical concepts in a day's lesson.

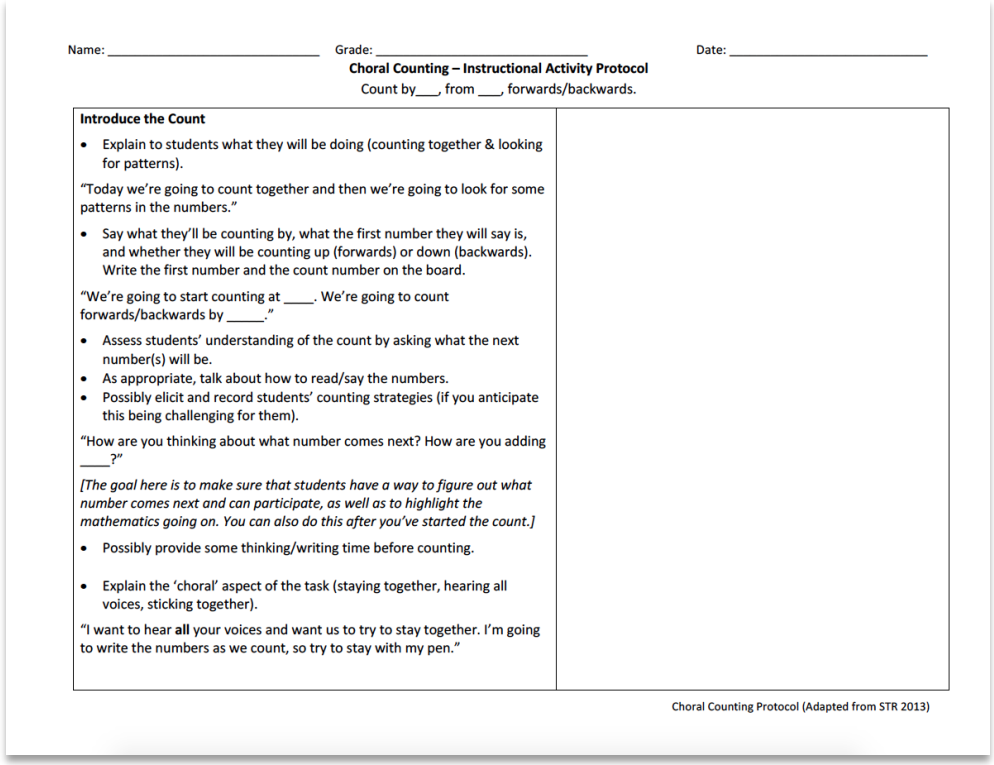
Activity 5: Posing true-false equations

Students are asked to determine whether an equation is true or false. They reason about the veracity of equations using relational thinking about quantities and the meaning and behavior operations. For example, True or False: $75 + 50 = 100 + 25$ or $9 \times 12 = 10 \times 12 - 1$. Students could determine the truth of these statement by calculating. But, the goal for leading discussions is to support students to think about how the quantities are related. Problem contexts and visual representations can help students develop an argument for whether the statements are true or false and why.

Activity 6: Posing and sharing solutions to word problems

Problem solving lessons can be utilized in any classroom setting, across a wide variety of mathematical topics, and with most curriculum materials. You will work on launching problems to make them accessible to all students. You will also attend to monitoring students as they work, listening to their ideas, and interpreting how their ideas relate to the mathematical goal for your lesson. In that process, you will pose questions to elicit and support students' reasoning, and help them draw on mathematical tools and models. During the discussion phase of problem solving, you will help children communicate their thinking, encourage them to engage with one another's contributions, and make decisions about when and how to respond to student contributions in order to meet your mathematical goal.

# Appendix E: Course Content: Planning Protocol for Choral Counting Instructional Activity



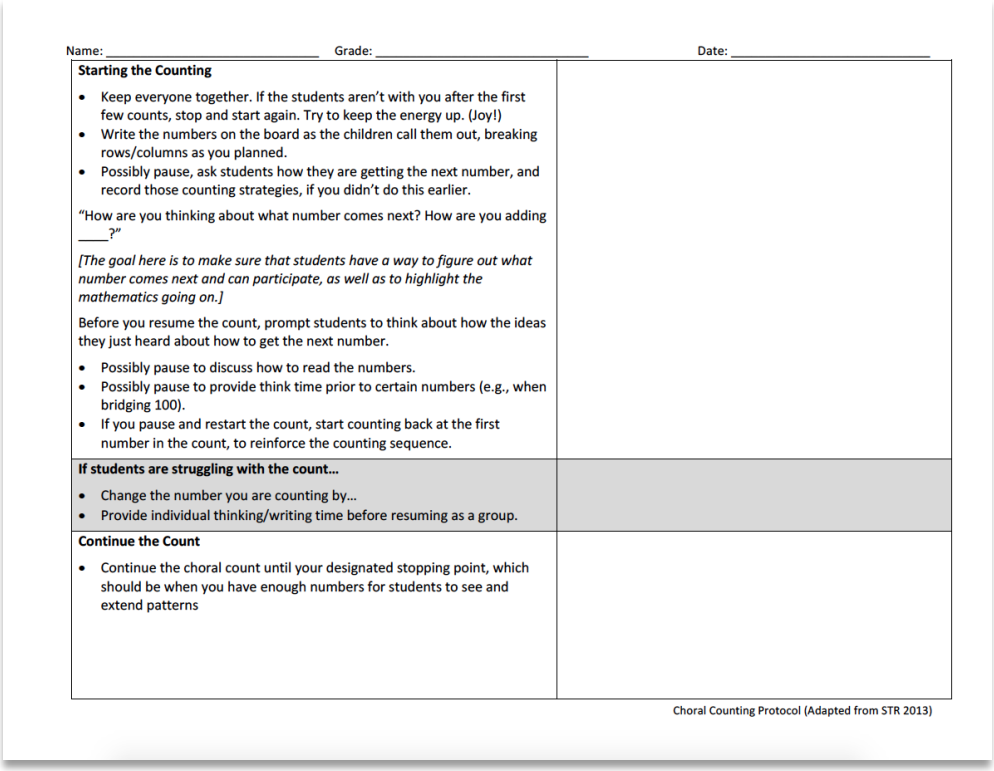
Name: _____ Grade: _____ Date: _____

**Choral Counting – Instructional Activity Protocol**

Count by___, from ___, forwards/backwards.

| | |
|---|---|
| **Introduce the Count**<br>• Explain to students what they will be doing (counting together & looking for patterns).<br>"Today we're going to count together and then we're going to look for some patterns in the numbers."<br>• Say what they'll be counting by, what the first number they will say is, and whether they will be counting up (forwards) or down (backwards). Write the first number and the count number on the board.<br>"We're going to start counting at ____. We're going to count forwards/backwards by _____."<br>• Assess students' understanding of the count by asking what the next number(s) will be.<br>• As appropriate, talk about how to read/say the numbers.<br>• Possibly elicit and record students' counting strategies (if you anticipate this being challenging for them).<br>"How are you thinking about what number comes next? How are you adding ____?"<br>*[The goal here is to make sure that students have a way to figure out what number comes next and can participate, as well as to highlight the mathematics going on. You can also do this after you've started the count.]*<br>• Possibly provide some thinking/writing time before counting.<br>• Explain the 'choral' aspect of the task (staying together, hearing all voices, sticking together).<br>"I want to hear **all** your voices and want us to try to stay together. I'm going to write the numbers as we count, so try to stay with my pen." | |

Choral Counting Protocol (Adapted from STR 2013)



Name: _____ Grade: _____ Date: _____

| | |
|---|---|
| **Starting the Counting**<br>• Keep everyone together. If the students aren't with you after the first few counts, stop and start again. Try to keep the energy up. (Joy!)<br>• Write the numbers on the board as the children call them out, breaking rows/columns as you planned.<br>• Possibly pause, ask students how they are getting the next number, and record those counting strategies, if you didn't do this earlier.<br>"How are you thinking about what number comes next? How are you adding ____?"<br>*[The goal here is to make sure that students have a way to figure out what number comes next and can participate, as well as to highlight the mathematics going on.]*<br>Before you resume the count, prompt students to think about how the ideas they just heard about how to get the next number.<br>• Possibly pause to discuss how to read the numbers.<br>• Possibly pause to provide think time prior to certain numbers (e.g., when bridging 100).<br>• If you pause and restart the count, start counting back at the first number in the count, to reinforce the counting sequence. | |
| **If students are struggling with the count…**<br>• Change the number you are counting by…<br>• Provide individual thinking/writing time before resuming as a group. | |
| **Continue the Count**<br>• Continue the choral count until your designated stopping point, which should be when you have enough numbers for students to see and extend patterns | |

Choral Counting Protocol (Adapted from STR 2013)

Name: _____ Grade: _____ Date: _____

| | |
|---|---|
| **Discuss Patterns in the Numbers, Guided by Goal**<br>• Elicit the patterns that students notice in the numbers. Possibly have them look quietly and hold up a thumb when they notice something about the numbers. Possibly ask them to 'turn and talk' about the patterns they see.<br><br>"What patterns do you notice in the numbers?" or "What do you notice about the numbers?"<br>• Record students' ideas so that the patterns they notice are made visible to the class (different colors, different marks).<br>• Follow-up on patterns by asking students to extend them or asking students why they hold.<br>"If this pattern continues, what do we know about this number (draw box)? And what about this one (draw box)?"<br>"Why do you think __? What about how we counted made it so that__?"<br>• Possibly focus on a particular mathematical idea by asking a more pointed question.<br>"Who can show us which of these numbers are whole numbers?"<br>"Who can circle all of the even numbers on board?"<br>• Possibly ask students to apply patterns by predicting what will happen if we extend the count.<br>"Based on the patterns we've noticed, if we keep counting, will we land on _____?" "Why not?" or "How do you know?"<br>"If we are going to land on ____, which row/column will that be in? How do you know?"<br><br>**Note:** You need to be strategic here. And focus on one thing you want to follow up on. This discussion shouldn't go on and on. | |



Name: _____ Grade: _____ Date: _____

| | |
|---|---|
| **Closure**<br>• Bring the counting task to a close by repeating patterns students shared or a response they gave to a question. **In doing so, your goal is to repeat and highlight an important mathematical idea for your students.**<br>• The written count should still be on the board so you can use this to point out what you are talking about. You may want to leave this counting sequence written for a while. | |

**Anticipated White Board**

# Appendix F: Course Content: Planning Template for Choral Counting Instructional Activity

Name: _____ Grade: _____Date: _____

**Choral Counting**

**Count by _____, from _____ , up/down.**

| **Anticipated Counting Strategies:** | **Record of Count & Patterns** |
|---|---|
| **Planned Pauses** | |
| **Patterns (Recorded Above) & Responses/Extension Questions** | |

Choral Counting Planning Template, UW Math Edu Project 2013

Name: _____ Grade: _____Date: _____

- **Mathematical goal/focus** _____
- **Back-up plan if too challenging: writing/thinking time or change count # to _____.**
- **Start with individual thinking/writing time or choral count together?**
- **Initial Directions**

- **Possible "closure" statement**

- **What If/Watch Out For**

Choral Counting Planning Template, UW Math Edu Project 2013

## Appendix G: Math Autobiography Assignment

**Introducing yourself**: Please answer the following questions in 2-3 pages, single-spaced

1. Tell us about your placement: grade level, school, name of teacher, math curriculum in use.
2. Write a brief Math Autobiography so that we can get to know you and a bit about your relationship to mathematics:

   Describe your experiences with mathematics over the course of your life, both in and out of school. How have these experiences shaped your stance towards becoming a teacher of mathematics?
3. What languages other than English do you speak fluently enough to have instructional conversations with children?
4. What are the top five things you would like us to know about you?
5. Is there anything else in particular you'd like us to know about you that would affect your participation in class?

## Appendix H: Template for Storylines

MM-DD-14 Goals Storyline

Instructional Activity 1

| | Instructional Goal | Practice Goal |
|---|---|---|
| TE LP | | |
| TC LP | | |
| RH | | |
| CV | | |
| DB | | |

Instructional Activity 2

| | Instructional Goal | Practice Goal |
|---|---|---|
| TE LP | | |
| TC LP | | |
| RH | | |
| CV | | |
| DB | | |

On this day, the focus was…

The instructional goal(s) …

Their practice goals …

**Narrative Storyline:**

[Narrative description of the lesson plans, rehearsals, enactments, and debriefing]

**Other notes:**

- 

| | |
|---|---|
| TE LP | Teacher Educator Lesson Plan |
| TC LP | Teacher Candidate Lesson Plan |
| RH | Rehearsal |
| CV | Classroom Visit |
| DB | Debrief Session |

# Appendix I: Goal Intentionality and Specificity Matrices

| | Specificity Description |
|---|---|
| none | No goal is written in their lesson plan or said out loud. |
| vague | A vague goal is written in the lesson plan and/or stated out loud (or goal is determined by TE but not expanded on by TCs) and/or multiple generic goals are |
| developing | A goal is articulated that is clear but not specific to the task or that has a mathematical idea. |
| clear | A clear and mathematical goal is articulated and is appropriate for the task. |

| | Intentionality |
|---|---|
| Not intent | Moves are not intentionally connected to the goal; teacher moves may try to elicit ideas, but TCs don't build on these ideas or respond in relation to a goal without TE prompting. |
| loose | TCs are aware of the goal, but don't know what moves to choose or choose moves that don't necessarily steer the conversation towards that goal. Moves that are connected to the goal are enacted or prompted by the TE. |
| developing | TCs begin to make decisions based on the goal and/or take up suggestions by TE. |
| intentional | TCs begin to make decisions based on the goal and/or they take up suggestions by TE that are connected to the goal. |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 7-Oct | 14-Oct | 10-21a | 10-21b | 10-28a | 10-28b | 11-4a | 11-4b | 11-18a | 11-18b | 12-2a | 12-2b |
| Specificity of the goal | vague | vague | clear | approaching | clear | clear | approaching | vague | approaching | clear | clear | clear |
| Intentionality in facilitation | Not intent | loose | loose | loose | loose | | loose | loose | | intentional | intentional | intentional |
| IA/Task | CC | EST | CC | EST | PS | CC | PS | MM | STR | MM | PS | STR |
| EST | | vague | | vague | | | | | | | | |
| CC | none | | g | | | g | | | | | | |
| PS | | | | | g | | vague | | | | clear | |
| MM | | | | | | | | vague | | clear | | |
| STR | | | | | | | | | g | | | clear |
| Katelina | none | | | | g | | | | | clear | | clear |
| Flora | | | | vague | | | vague | | | | clear | |
| Isabella | | vague | | | | g | | | g | | | |
| Orianne | | vague | g | | | | | vague | | | | clear |
| Sam | | | | vague | | g | vague | | | | clear | |
| Danielle | vague | | | | g | | | | | clear | | |
| Tina | | | g | | | | | vague | g | | | |

| | |
|---|---|
| CC | Choral Counting |
| EST | Estimation |
| MM | Mental Math |
| PS | Solving a Word Problem |
| STR | Computational Strings |

# Appendix J: Findings Chart for Tool Use and OTL

| | Tool Use | OTL |
|---|---|---|
| Lesson Plans | Changes in the content:<br>1. More complete plans and planning around a particular task.<br>2. Inclusion and increased specificity of a mathematical goal.<br>3. Less teacher-focused scripting or listing teacher moves and more focus on moves connected to student strategies and mathematical ideas.<br>4. Inclusion and anticipation of student strategies and representations<br>5. Increased accuracy and anticipation of representations.<br>Changes in the use/purpose:<br>• From being a script and/or list of teacher moves, to being a place to anticipate and record student strategies, use of representations on the board, moves and questions that connected to a specific mathematical goal. | Lesson plans facilitated OTL by:<br>1. Providing scaffolding to the structure/purpose of the instructional activity (e.g., start with an intro, hold up the image, etc.).<br>2. Focusing TC attention (e.g., there was a space to anticipate strategies, for closure, to anticipate representations on the board, etc.).<br>3. Becoming a boundary object mediating the thinking of TCs and feedback and comments from the TE. |
| Board Work | Changes in use of the board:<br>1. Ownership over the use of the materials and discussion about materials and board work moved from being TE-initiated to residing in the group.<br>2. Focus in rehearsals moved from logistical management of materials to focusing on how to use the board.<br>3. TCs used the board more actively and intentionally during later rehearsals.<br>4. In later sessions, TCs used to board based on their mathematical purpose.<br>5. TC repertoire and quality of mathematical representations increased. | The board and materials facilitated OTL as:<br>1. Space to practice representations and mathematical ideas.<br>2. A way to think through the lesson, including the mathematics, representations, teacher moves, and possible student ideas.<br>3. A record of public thought and artifact for discussion. |
| Talk Moves | Changes in use of talk moves:<br>• Talk moves were revisited with increasingly complex and nuanced understanding.<br>• Over time, TCs developed a shared (and more nuanced) understanding of the moves and intentionality in using them to facilitate productive talk. | Talk moves facilitated:<br>1. Opportunity to work on practice with a common language.<br>2. Opportunity for deliberate practice.<br>3. Opportunity for generative learning.<br>4. Opportunity to develop a shared vision of practice |

## Vita

Elizabeth Sugino David grew up in California in the rural town of Fallbrook. She received her Bachelor of Arts in East Asian Languages and Cultures from the University of Southern California and her Master of Arts in Teaching, including mathematics and elementary teaching certification, from Pepperdine University Graduate School of Education and Psychology. After teaching high school mathematics in Los Angeles, she moved to Seattle to pursue graduate studies in mathematics education at the University of Washington (UW). In addition to working as a teaching assistant, research assistant, university coach, and instructor in UW's teacher education programs, Elizabeth also researched, led professional development, and taught courses in comparative education as a faculty member at the University of Fukui. Elizabeth currently teaches elementary school as a fourth-grade teacher in Seattle Public Schools and mathematics education courses as teaching faculty at Western Washington University.