Essays in Financial Economics

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Abstract

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This dissertation consists of two chapters. The first chapter studies the spillover effect of fire sales transmitted by dealers in the corporate bond market. I use the monthly exclusion events from the Bloomberg Barclays US Corporate Index to identify dealers stressed from taking an unusually large amount of excluded bonds. Stressed dealers actively offload non-excluded bonds. Due to the fact that a stressed dealer is more effective at trading bonds in which she has a higher market share of volume, she sells more of those bonds. On the bond level, non-excluded bonds that are collectively traded more by stressed dealers experience lower returns and worse liquidity even 6 months after the event. Finally, I examine the impact of the Volcker Rule on the fire sale spillover and find that the spillover effect is stronger under the Volcker Rule than under the preceding Dodd-Frank Act. The results are consistent with the view that the Volcker Rule hinders liquidity provision of bank-affiliated dealers.

The second chapter is joint work with Lucas Rooney and Mark Westerfield. We present a model in which an agent exerts hidden effort to create unobservable and durable expertise (human capital) that generates noisy cash flows. The impact of the agent's effort is long-lasting, so the agent responds to the entire stock of future cash-flow rights (pay-for-performance). The principal manages this promise of future cash flow rights, which must

be consistent across time. The optimal contract features two regions. When the stock of future cash flow rights is low, the contract resembles training in which the principal builds up a stock of future pay-for-performance sensitivities by offering none today – pushing cash flow sharing into the future. Once the stock future cash flow rights reaches a threshold, the contract enters the active region in which the principal uses short-term cash flow rights to control the growth of the package of incentives and the path of the agent's effort. In the active region, the correlation between cash flows and future cash flow rights is optimally negative, meaning good performance results in higher future consumption and lower future expertise.

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DEDICATION

To my wife, Ying-Chin,

and my son, Peien.

Chapter 1

FIRE SALE SPILLOVERS IN THE CORPORATE BOND MARKET

1.1 Introduction

Dealers play a vital role in facilitating transactions in over-the-counter markets. Since customer orders arrive sporadically, the dealers commit capital to absorb incoming orders and subsequently look for buyers for the securities. In the meantime, the dealers park the securities on their own balance sheets. Therefore, the functioning of the market crucially depends on the capacity of dealers' balance sheets.

The market is subject to distress when a fire sale happens and the dealers do not have immediate buyers lined up. During a fire sale, a large amount of securities flows into the dealers sector, their prices are heavily discounted and liquidity suffer. However, the costs imposed by the fire sale can even go beyond the securities that directly experience the fire sale. A dealer usually makes a market for multiple securities simultaneously. Before she can find buyers for the fire sale securities, they stay on the dealer's balance sheet and take up a substantial amount of space. This in turn impedes the dealer's market making capacity overall. To alleviate the balance sheet stress, the dealer actively offloads other securities in her inventory that are not directly experiencing the fire sale, thus transmitting the selling pressure from one group of securities to another, and resulting in depressed prices and deteriorated liquidity for other securities.

In this paper, I study the aforementioned fire sale spillover in the corporate bond market. To test the mechanism, I examine the fire sales induced by bond exclusions from the

Bloomberg Barclays US Corporate Index. This index tracks the investment grade corporate bonds and covers a substantial fraction of the corporate bond market. The index is rebalanced at the end of every month and the index trackers, in order to minimize their tracking errors, adjust their portfolios accordingly by selling the bonds that exit the index. [21] show that the index trackers demand immediacy exactly when the index is rebalanced from exclusion events and large selling pressure emerges. The rules for entering and exiting the index are mechanical and transparent, making the sales of excluded bonds free of information about the fundamentals. Hence, the index exclusion events are ideal experiments to test my hypotheses.

Using the academic version of Trade Reporting and Compliance Engine (TRACE) maintained by the Financial Industry Regulatory Authority (FINRA), I calculate the cumulative order flows of excluded bonds for each dealer during the exclusion events. I classify dealers as stressed if they have taken an unusually large position of the excluded bonds. An average stressed dealer takes in roughly $25 million worth of excluded bonds over an 5-day event window prior to the exclusion day, while her inventory for non-excluded bonds barely changes over the same period.

After identifying each dealer's stress status, I study the stressed dealers' trading behavior in non-excluded bonds. My hypothesis is that in response to a large selling pressure from excluded bonds, the stressed dealers would offload non-excluded bonds to alleviate inventory constraint. Nevertheless, the stressed dealers would not sell bonds randomly. Instead, they would sell more of the bonds that are easier for them to sell. I characterize this cross-sectional difference among non-excluded bonds for each stressed dealer by calculating the market share of trading volume by the stressed dealer, which I call the *idiosyncratic dealer stress exposure* (I - $Expo$). I find that non-excluded bonds within a stressed dealer's portfolio follow similar inventory dynamics prior to the exclusion event, but their inventory levels start to diverge post-exclusion. Stressed dealers reduce their holdings more in bonds with higher *I - Expo*. To examine whether bonds with higher *I - Expo* are actually easier to sell for the stressed dealer, I calculate the marginal trading costs for non-excluded bonds and find that the marginal trading costs are equalized across non-excluded bonds. This result suggests that stressed dealers behave optimally when it comes to offloading and the *I - Expo* measure captures the easiness to trade feature in the cross section relatively well.

Next, I examine the implications of stressed dealers trading behavior for bond pricing and liquidity. For each non-excluded bond, I calculate the *systematic dealer stress exposure* (*S - Expo*) by aggregating its *I - Expo* from the stressed dealers. I find that bonds with higher *S - Expo* experience lower returns even 6 months after the exclusion event. Specifically, an inter-quartile move in *S - Expo* from 25th to 75th percentile would result in a decrease in the 6-month cumulative return by 80 basis points. This change represents 50% of the average 6-month cumulative return. I also find that bonds with high dealer stress exposure experience liquidity deterioration, as measured by the change in Amihud price impact, over the same 6 months period. This difference comes mainly from the stressed dealers.

In light of the regulation changes after the 2008 Financial Crisis, I investigate how the spillover effect varies under different policy regimes, especially under the Volcker Rule. The Volcker Rule is one of the most controversial topics among the many regulatory changes since the Great Financial Crisis. As pointed out by [5], the Volcker Rule is intended to limit bank risk taking by restricting or prohibiting certain speculative activities. However, critics argue that it blurs the line between speculating and market making, and therefore makes banks reluctant to commit capital for liquidity provision. I find that the average magnitude of the fire sale spillover is stronger under the Volcker Rule than under the preceding Dodd-Frank Act. The evidence suggests that the Volcker Rule limits liquidity provision of bank-affiliated dealers. It is possible that the composition of dealers has changed since the Volcker

Rule and the non-bank-affiliated dealers has stepped in to provide liquidity. However, the capital committed by non-bank-affiliated dealers does not seem to be enough to replenish the deficiency caused by the Volcker Rule.

One might be concerned about the validity of the exclusion events due to the index tracker's objectives and possibly superior information. It is possible that minimizing tracking errors might not be the only objective of index trackers. They might want to deliver excess returns against their benchmarks had they been able to do so. They could sell more excluded bonds that have lower returns going forward, thus creating heterogeneous selling pressure. The dealers, upon observing the heterogeneous selling pressure, would update their beliefs and adjust their portfolios accordingly. In order to assess the plausibility of this argument, I calculate the selling pressure of the excluded bonds surrounding the exclusion events and find that the selling pressure is fairly homogeneous across the excluded bonds. Therefore, it is unlikely that the heterogeneous selling pressure drives my results.

Another concern is that the stressed dealers offload the non-excluded bonds simply because of the concentration in characteristics brought by the excluded bonds. When the stressed dealers receive a large volume of excluded bonds, their portfolio-level characteristics tilt towards the ones of excluded bonds. To manage the sudden increase in certain characteristics, they would sell off non-excluded bonds that are similar to the excluded ones. In order to see if this channel is at work, I calculate the average characteristics of the excluded bonds and non-excluded bonds with high and low $S\text{-}Expo$. I find that high exposure bonds are not more similar than low exposure bonds to the excluded bonds.

One might also argue that the return and liquidity differentials along the $S\text{-}Expo$ dimension are driven by differences that already exist before the exclusion event. I find no evidence to support this argument. For non-excluded bonds, their returns and changes in liquidity do not vary with $S\text{-}Expo$ in months leading up to the exclusion event. So it is unlikely that pre-existing differences would drive my results.

This study contributes to three strands of literature. First, it documents a novel channel through which fire sales can create negative externalities. Previous literature has mainly focused on the interaction between mutual fund holdings and fire sales. Faced with a large redemption, a mutual fund is forced to liquidate its existing positions, thus generating negative price impact. Other funds that hold the same securities will suffer from performance deterioration due to the price impact, which in turn triggers capital outflow from and forced liquidation by the other funds. [23] document that in the corporate bond market, redemption induced fire sales have negative effects on peer funds who hold the same assets, thereby amplifying the negative price impact. However, [15] find little evidence for asset fire sales because of corporate bond mutual funds liquidity management strategies. [14] find evidence of fire sale externalities in the equity mutual fund industry.

Second, this paper provides evidence to the literature that links dealers inventory to bond return and liquidity dynamics, and more broadly, the intermediary asset pricing literature. Recent advances in this literature are made by [29] and [31]. More specifically, both [29] and [31] construct time series of aggregate inventory of investment grade bonds and find that it has predictive power for the aggregate credit spread and return of high-yield bonds. And similarly, the aggregate inventory of high yield bonds can predict credit spread and return of investment grade bonds. My study differs from the extant literature in that instead of looking at aggregate, time series variations between inventory and bond returns, I look at the granular, cross sectional relationship.

Lastly, this paper contributes to the debate on the impact of the Volcker Rule on market quality. Prior literature has found mixed evidence in this regard. [46] find no evidence that post-crisis regulations hurt bond market liquidity. [8] find that while liquidity in the corporate bond market has not worsen markedly after the financial crisis, capital commitment by bank-affiliated dealers has decreased ever since. Their results support the interpretation that the Volcker Rule has changed the market for the worse. [6] use credit rating downgrades as stress events and show that downgraded bonds experience worse liquidity after the Volcker Rule. Similar to [6], this paper suggests that the Volcker Rule makes the market more vulnerable to stress and the scope of the effect is even wider than previously thought.

The remainder of the paper is organized as follows. I describe the mechanics of index exclusion in Section 1.2. In Section 1.3, I describe the data used in this paper and how I construct the sample. In Section 1.4, I study the stressed dealers trading behavior in non-excluded bonds. I investigate the implications of stressed dealers trading for bond-level pricing and liquidity in Section 1.5. Section 1.6 presents findings on the impact of the Volcker Rule on the spillover effect. In Section 1.7, additional tests are performed to rule out alternative explanations. Section 1.8 offers concluding remarks.

1.2 Index Exclusion Events

I focus on the exclusion events from a major market index in the corporate bond space, the Bloomberg Barclays US Corporate Index. This index covers a large fraction of corporate bonds in the US and plays an important role for index mutual funds and ETFs. As of July 31, 2021, for Vanguard, Charles Schwab, and BlackRock, the total AUM of mutual funds and ETFs that track the index and their variations equals to $114 billion.[1]

In order to identify index eligible and excluded bonds, I follow the methodologies described by [12]. The index includes USD-denominated, fixed-rate securities publicly issued

[1]Information obtained from their respective websites, Vanguard: https://institutional.vanguard.com/VGApp/iip/institutional/csa/investments/benchmarks/barclays/funds, Charles Schwab: https://www.schwabassetmanagement.com/insights/insights/resource-center/insights/prospectus?combine=&field_product_solution_target_id%5B%5D=291&field_product_solution_target_id%5B%5D=296&field_asset_class_target_id%5B%5D=271, and BlackRock: https://www.blackrock.com/us/individual/products/investment-funds#!type=all&style=44342&fsac=43549%7C43774%7C43566%7C43567%7C43588&view=perfNav. Table A.1 in the Appendix provides a detailed list of mutual funds and ETFs with their AUMs.

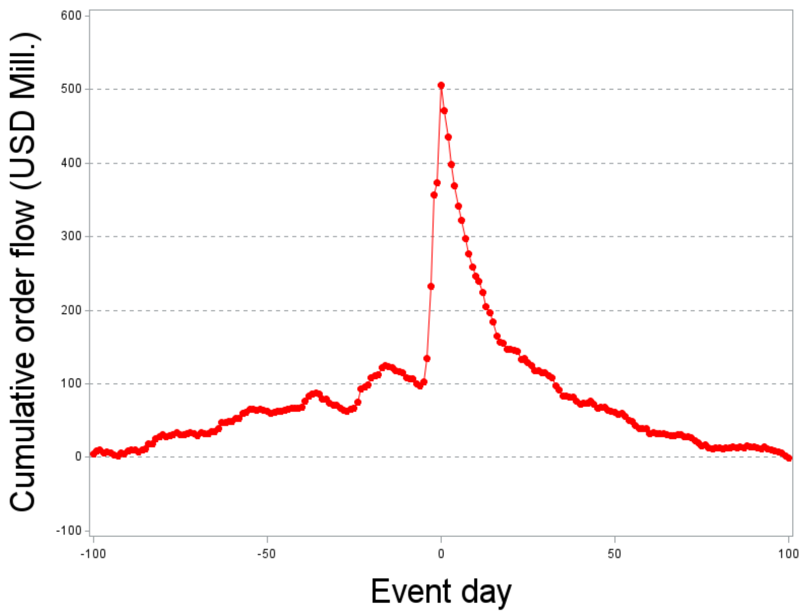
by US and non-US industrial, utility, and financial issuers, with at least one year time to maturity. It rebalances on the last trading day of each month. Furthermore, the securities must be investment grade using the middle rating of S&P, Moody's, and Fitch. If there are only two ratings available, the lower rating is used. If there is only one rating, that rating is used. Before March 31, 2017, the minimum requirement for amount outstanding is $250 million. After April 1, 2017, the requirement was raised to $300 million. Table A.2 in the Appendix describes the requirements for the types of bonds to be included in the indexes and their corresponding variables used in Mergent Fixed Income Securities Database (FISD).

Figure 1.1 plots the aggregate inventory dynamics of the excluded bonds. For each exclusion event, I calculate the cumulative order flow of all excluded bonds received by all dealers over the 200-day event window, setting the initial level at 0. Then I calculate the average inventory on each event day. The graph shows that, from day -100 to -95, the aggregate inventory level stays relatively flat, going from 0 to about $100 million. The aggregate selling pressure starts to intensify on day *-4* and the aggregate inventory peaks at roughly $500 million on day 0. From day 1 on, the inventory series gradually drifts down. This picture is consistent with the findings by [21] and that index trackers minimize tracking errors by delaying their portfolios rebalancing until the index is actually rebalanced.

To gauge the magnitude of the shock, I collect data on dealer balance sheet space from two sources: the Federal Reserve Bank of New York Primary Dealer Statistics and the aggregate dealer gross position calculated by [29].[2] Both data sources aim to capture the aggregate holdings of corporate bonds by dealers. During my sample period, the dealers collectively hold about $20 billion worth of corporate bonds and the aggregate inventory shock of excluded bonds takes up around 2.5% of the aggregate balance sheet space. However,

[2]Data on FRBNY Primary Dealer Statistics can be downloaded here: https://www.newyorkfed.org/markets/counterparties/primary-dealers-statistics. Data from [29] can be found in the Supporting Information section on the webpage: https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.12991

Figure 1.1: Aggregate dealer inventories (in USD mill.) of excluded bonds over time. This figure plots the cumulative aggregate order flow (inventory) of excluded bonds over the [-100,100] event window. At the end of each month (exclusion day), I calculate the cumulative aggregate order flow of excluded bonds from 100 trading days prior to 100 trading days after, setting the initial value at 0. Finally, I take the average on each event day.



this number is an underestimate of the true magnitude of the shock since I do not observe the initial holding level of the excluded bonds and set it to 0.

1.3 Data and Sample Construction

I use the academic version of Trade Reporting and Compliance Engine (TRACE) provided by the Financial Industry Regulatory Authority (FINRA). In addition to the transaction level information such as trade date, time, price, quantity, buy/sell indicator, etc., this data set contains masked dealer identifiers, which enable me to calculate each dealer's inventory position over time. The sample period is from January 2006 to June 2018.

I first apply the cleaning procedures according to [20] to account for cancellations, corrections, and reversals. I then duplicate locked-in trades, remove wash trades, and delete trades with anomalous prices.[3] Next, the cleaned TRACE dataset is merged with FISD by CUSIPs. Bonds must be dollar-denominated with non-missing offering dates and maturity dates, and the issuers of the bonds must be in one of the three industry groups: industrial, financial, and utility. Trades that are larger than the issue size and trades that happen less than a month after the issuance are also excluded. Following [16], I delete trades with non-FINRA member affiliates.[4] Finally, I follow the definitions of [12] to select relevant bonds. Table A.2 in the Appendix summarizes the filtering procedures and the number of bonds and trades remaining after each step. The final sample includes around 30 thousand bonds and 124 million trades.

1.4 Stressed Dealers Trading Behavior

1.4.1 Identifying Stressed Dealers

As discussed earlier, a dealer becomes stressed when she receives a large volume of excluded bonds and is unable to locate immediate buyers. Therefore, identifying stressed dealers involves time-series comparisons of each dealer's inventory levels surrounding the exclusion event. Figure 1.1 indicates that the aggregate selling pressure starts to intensify 5 days prior to the rebalancing date (day 0). In order to identify stressed dealers, I calculate the cumulative order flow of excluded bonds over 5-day intervals using data from the past 100

[3]Locked-in trades are interdealer trades reported only once in the dataset. Wash trades are trades in which a dealer transacts with itself. Trades with reported prices less than 10 and greater than 300 are considered anomalous. Trades with anomalous prices are likely due to reporting errors and bonds in default.

[4]For trades reported before November 2, 2015, I match trades that are of the same bond, dealer, volume, price, and day, of opposite sides, and within 60 seconds of each other. If the matched trades are both with customers, I delete them both. If one of the matched trades is with a customer and the other one is with a dealer, I delete the customer trade. For trades reported on and after November 2, 2015, I delete trades that have their counter-party identifiers as 'A'.

trading days for each dealer d. Specifically,

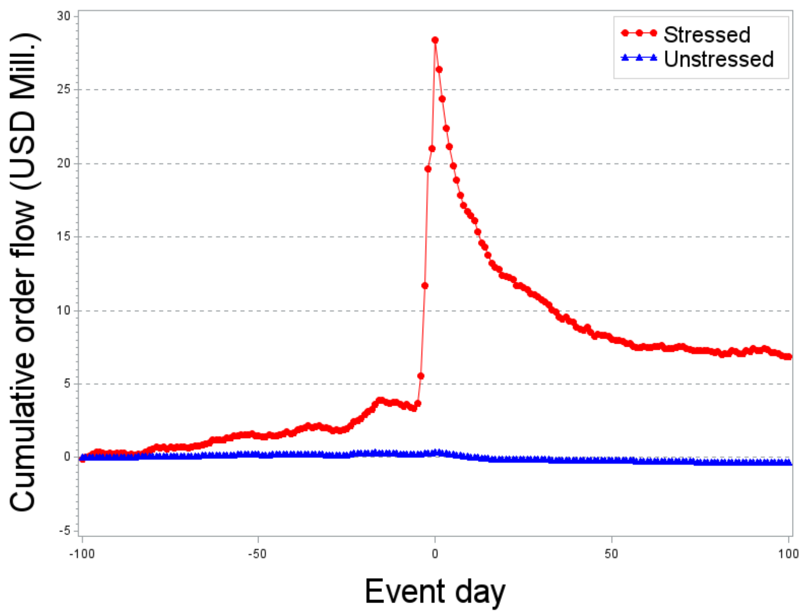
$$I_{e,d,t} = \sum_{i \in e} I_{i,d,t} = \sum_{i \in e} \sum_{s=4}^{s=0} q_{i,d,t-s},$$

where e is the set of excluded bonds, and $q_{i,d,t-s}$ is the order flow of bond i handled by dealer d on date $t-s$. $q_{i,d,t-s}$ is positive if the dealer is a net buyer and negative if a net seller. For each 100 day pre-exclusion event window, I have 20 inventory observations for each dealer. I calculate the average and standard deviation for the first 19 observations and a dealer is classified as stressed if its 20th observation is 2 standard deviations above the average.

This classification method aims to avoid the confounding effect of dealers' sheer sizes while still retaining the 'surprise' element. Suppose that I simply rank dealers based on their day 0 inventory levels and consider the top ones stressed. I might end up picking the wrong dealers who are large and slowly building up their holdings over time. Figure A.1 provides an example of this simpler method. I sort dealers into terciles (three equal-sized groups) based on their inventory levels on day 0 and consider the top group stressed. The graph shows that the group of dealers that are considered stressed start accumulating their inventories of excluded bonds long before the exclusion day. This unnormalized classification scheme undermines my ability to use the exclusion events as inventory shocks.

Figure 1.2 plots the results of the original classification method proposed above. The red line with dots represents the average inventory of excluded bonds by stressed dealers and the blue line with triangles corresponds to the unstressed dealers. Similar to the aggregate inventory dynamics, the stressed dealers inventory level stays relatively flat up until day -5 and spikes up from day -4 to 0, reaching its peak at roughly \$28 million. Starting from day 1, the inventory drifts back down. On the other hand, the inventory level of unstressed dealers remains stable throughout the event window. This figure provides evidence that my

Figure 1.2: Average inventories of excluded bonds by dealer stress status over time. This figure plots the cumulative order flow (inventory) of excluded bonds over the [-100,100] event window. The red line with dots represents the average inventory of stressed dealers. The blue line with triangles represents the average inventory of unstressed dealers. At the end of each month (exclusion day), for each dealer, I calculate the cumulative order flow of excluded bonds from 100 trading days prior to 100 trading days after, setting the initial value at 0. On each event day, the averages are taken within each dealer group and across exclusion events.



classification scheme picks up the "right" dealers in the cross section.

1.4.2 Idiosyncratic Dealer Stress Exposure

After classifying which dealers are stressed at the end of each month, I proceed to study how stressed dealers trade their portfolios of non-excluded bonds. If she wishes to offload non-excluded bonds to alleviate balance sheet constraint, a rational dealer would sell more of the bonds that are easier for her to sell until the marginal transaction costs for all bonds are equalized. In order to capture such "easiness" on the dealer-bond level, I calculate the Idiosyncratic Dealer Stress Exposure, which I call $I\text{-}Expo$, as the market share of the stressed dealer during the same month of the exclusion event, i.e.,

$$I\text{-}Expo_{i,d,m} = \frac{Vol_{i,d,m}}{\sum_{d \in D} Vol_{i,d,m}},$$

where $Vol^S_{i,d,m}$ denotes the par volume of bond i traded by stressed dealer d in month m and $\sum_{d \in D} Vol_{i,m}$ is the total volume of bond i traded by all dealers in month m.

This measure aims to capture the expertise a dealer has trading a bond. If *I - Expo* is high, it suggests that the dealer has ample experience trading and deep knowledge about the bond. Therefore, the dealer should have a easier time selling the bond.

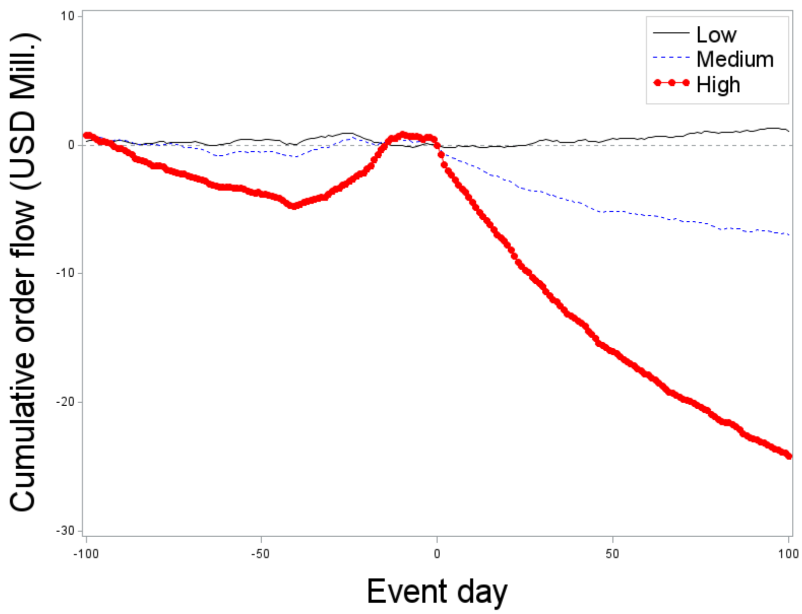
1.4.3 Stressed Dealers Offloading Non-excluded Bonds

In this subsection, I look at the stressed dealers trading behavior around the exclusion events. If the fire sales spillover mechanism is at work and *I - Expo* truly captures how easily a stressed dealer can sell certain bonds, the stressed dealers are likely to sell more bonds with higher *I - Expo* to manage their overall balance sheet space.

To test this idea, I look at the stressed dealers inventory in non-excluded bonds surrounding exclusion events. For each stressed dealer, the non-excluded bonds are sorted into terciles (3 equal-sized groups) at the end of every month based on their idiosyncratic dealer stress exposures, *I - Expo*. Within each tercile, the cumulative order flow is calculated during the [-100,100] event window. I set the day 0 inventory level to be 0 for each tercile for ease of comparison. For each exclusion event, I take the average of all tercile-event day observations across stressed dealers. Finally, the tercile-event day observations are averaged over exclusion events. Figure 1.3 presents the findings.

Figure 1.3 shows the inventory dynamics for bonds in different *I - Expo* terciles. From event day -100 to 0, all bonds follow similar trajectories. Starting on day 1, their paths start

Figure 1.3: Average stressed dealers inventory in non-excluded bonds with different levels of *I - Expo*. At the end of each month, all non-excluded bonds within a stressed dealer's portfolio are sorted into terciles based on *I - Expo*. I calculate the aggregate cumulative order flow for each tercile from 100 tradings days prior to the exclusion event to 100 days after, setting the inventory level on day 0 to be 0. Finally, the tercile-event day observations are averaged across stressed dealers and exclusion events.



to diverge. Bonds with higher *I - Expo* are being offloaded more heavily by stressed dealers after the exclusion event. This monotonic relationship between *I - Expo* and inventory level is consistent with the spillover effect.

Furthermore, I investigate the transaction costs the stressed dealers face when trading the non-excluded bonds. If the stressed dealers behave optimally, they would equalize the *marginal* transaction costs across bonds. I use the Amihud price impact of the each dealer's last sale on each day to proxy for the marginal transaction cost. Specifically,

$$MPI_{i,d,t} = \frac{|P_{i,d,t,n} - P_{i,d,t,n-1}|}{Vol_{i,d,t,n}},$$

where $P_{i,d,t,n}$ is the clean price of the n-th trade in bond i by dealer d on date t, and $Vol_{i,n,t}$ is the par volume in millions of dollars. When calculating this measure, I keep only principal dealer sales that are larger than $100,000. The marginal price impact numbers are averaged within each *I - Expo* tercile, across stressed dealers, and over exclusion events. Figure 1.4a and 1.4b show the results.

Figure 1.4a displays the tercile level marginal price impact over time. The differences in marginal price impact between high and low *I - Expo* terciles and their 95% confidence intervals are presented in Figure 1.4b. The three groups have very similar levels of marginal price impact. For example, bonds in the low *I - Expo* tercile have an average marginal price impact of 31.49 while those in the high group have an average of 28.70. The differences between the top and bottom groups are statistically indistinguishable from 0 on most days. The results support the interpretation that the stressed dealers behave optimally when selling non-excluded bonds. The findings also confirm that the *I - Expo* measure successfully captures the bond-level 'easiness to trade' feature ex-ante.

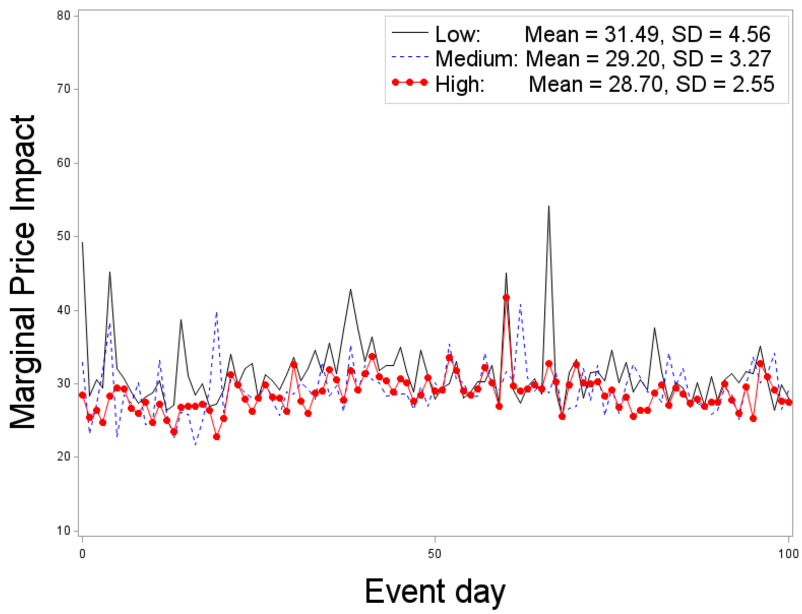
1.5 Implications for Bond Pricing and Liquidity

1.5.1 Systematic Dealer Stress Exposure

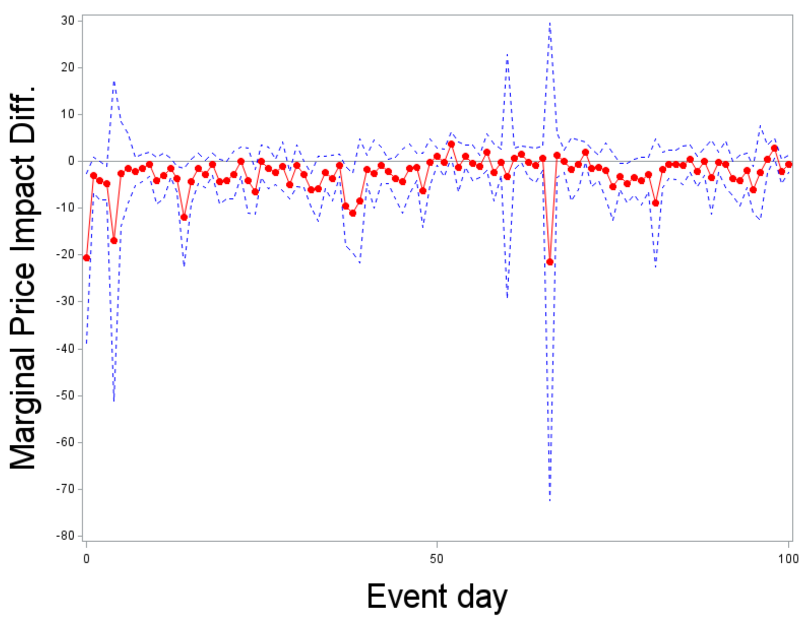
Having established that stressed dealers actively offload non-excluded bonds to manage their balance sheet spaces, I now examine the implications of the spillover effect for bond pricing and liquidity. The stressed dealers sell more heavily bonds in which they have higher market shares, i.e., *I - Expo*. This trading behavior will result in price drop and liquidity deterioration on the bond level. If a bond is collectively traded more by the stressed dealers, the impact on pricing and liquidity will be amplified. To capture such variation in the cross section of non-excluded bonds, I calculate the Systematic Dealer Stress Exposure, or *S - Expo*,

Figure 1.4: Marginal price impact by *I - Expo* tercile. Panel A exhibits the average marginal price impact by *I - Expo* tercile. The bond-level marginal price impact numbers are averaged within each *I - Expo* tercile, across stressed dealers, and over exclusion events. Panel B displays the differences between the high and low *I - Expo* terciles and their confidence intervals.

(a) Panel A: Average marginal price impact by *I - Expo* tercile.



(b) Panel B: Average differences in marginal price impact between high and low *I - Expo* terciles (solid, red line with dots) and their 95% confidence intervals (blue, dashed lines).



as the aggregate stressed dealers market share of trading volume. Specifically,

$$S\text{-}Expo_{i,m} = \sum_{d \in D} I\text{-}Expo_{i,d,m} \times \mathbf{1}\{Stress\}_{d,m},$$

where D is the set of all dealers trading bond i in month m, and $\mathbf{1}\{Stress\}_{d,m}$ is an indicator variable that takes the value of 1 if dealer d is classified as stressed in month m.

1.5.2 Corporate Bond Return

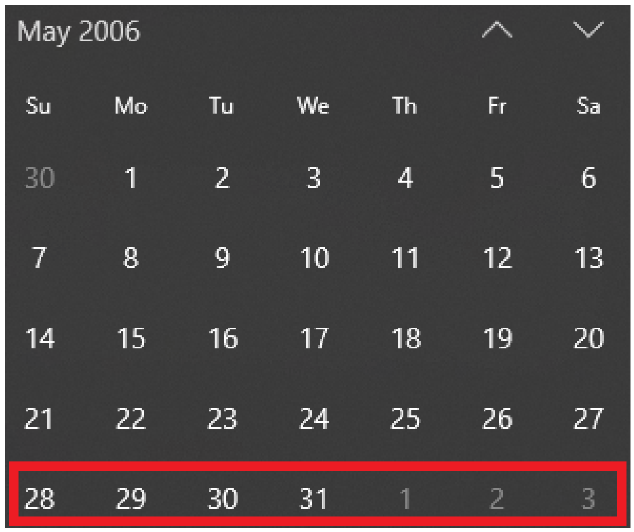
The rate of return of bond i from month $m+h-1$ to $m+h$ is

$$R_{i,m+h-1\to m+h} = \frac{P_{i,m+h} + AI_{i,m+h} + Coupon_{i,m\to m+h}}{P_{i,m+h-1} + AI_{i,m+h-1}} - 1,$$

where h is the monthly horizon that ranges from 1 to 12, $P_{i,m+h}$ is the volume-weighted clean price of bond i at the end of month $m+h$, $AI_{i,m+h}$ is the accrued interest, and $Coupon_{i,m+h-1\to m+h}$ is the coupon payment in month $m+h$. Following [9], I keep only trades greater than \$100,000 when calculating the weighted average price. I follow [3] to restrict price observations to be in the last week of the month or the first week in the following month. For example, the last week of May 2006 starts from May 28 and the first week of June 2006 ends on June 3. If there are multiple prices in that week, I keep the one that has the shortest distance to the last day of the month, in this case May 31, 2006. If there are two days that are equally far from the last day of the month, I keep the date that is in the month instead of the following month. Figure 1.5 illustrates the eligible dates for month end price calculation in May 2006.

There is a concern that the number of days between the two price observations might vary with dealer stress exposure in some way, thereby biasing my empirical results. Table A.3 in the Appendix shows that there is no meaningful variation along the dimension of

Figure 1.5: An example of eligible dates for month end price calculation in May 2006. The dates highlighted by the red rectangle are the ones that fit my selection criterion.



dealer stress exposure.

1.5.3 Corporate Bond Liquidity Change

My measure for bond liquidity is the price impact in the spirit of [1]. It is calculated as the price difference between two consecutive transactions divided by the volume of the newer trade, i.e.,

$$PI_{i,n,t} = \frac{P_{i,n,t} - P_{i,n-1,t}}{Vol_{i,n,t}} \times Trade\ Direction_{i,n,t},$$

where $P_{i,n,t}$ is the clean price of n-th trade in bond i on date t, $Vol_{i,n,t}$ is the par volume in millions of dollars, and $Trade\ Direction_{i,n,t}$ equals to 1 if the trade is a buy from customers and -1 if a sell. When calculating this measure, I keep only principal trades with customers that are larger than \$100,000. I set the value to missing for the first trade of the day to avoid new information revelation over night.

At the end of each month, I calculate the monthly average price impact, $PI_{i,m}$, by taking the average of all the values in month m and giving all the values an equal weight. Similarly, I calculate the monthly price impact of stressed dealers and unstressed dealers, $PI^S_{i,m}$ and $PI^U_{i,m}$, using transactions with respective dealers only.

Finally, I calculate the change in liquidity over time as

$$\Delta PI_{i,m+h} = PI_{i,m+h} - PI_{i,m}$$

where h is an integer that ranges from 1 to 12 and represents the number of months after the exclusion event. $\Delta PI^S_{i,m+h}$ and $\Delta PI^U_{i,m+h}$ are calculated similarly.

1.5.4 Summary statistics

Table 1.1 presents summary statistics and time series averages cross-sectional correlations of non-excluded bonds characteristics. Panel A displays the time series average of cross-sectional moments. On average, I have around 8186 unique bonds for each cross section. The dealer stress exposure has an average of 0.27 and a median of 0.20 in the cross section. *Return*, PI, PI^S, and PI^U are all on the monthly frequency. The credit ratings are in numeric scale, with 1 corresponding to an S& P's AAA rating and 21 to a S& P's C. Table A.4 in the Appendix shows the letter to numeric ratings conversion scheme for the three rating agencies. Ratings from 1 to 10 are considered investment grade, and ratings of 11 or higher are considered high yield. For each individual bond, the rating is calculated as the median of S&P, Moody's, and Fitch ratings. If two ratings are available, the lower rating is used. If only one rating is available, that rating is used. The average rating for the bonds is 9.12 (BBB for S& P), and the majority of bonds are investment grade. Size refers to the bond's amount outstanding. The average bond has a size of $583 million. The average bond

Table 1.1: Summary statistics and time-series averages of cross-sectional correlations. Panel A presents the summary statistics of bond characteristics, including systematic dealer stress exposure, monthly return, monthly average price impact, stressed dealers price impact, unstressed dealers price impact, credit rating, size, age, and maturity. Each moment is calculated from the monthly cross section and then averaged over time. N is the average number of bonds in each cross section. Ratings are the median of S&P, Moody's, and Fitch ratings. If two ratings are available, the lower rating is used. If only one rating is available, that rating is used. Ratings are in conventional numerical scales, where 1 corresponds to an AAA rating and 21 to a C rating. Ratings from 1 to 10 are considered investment grade and ratings of 11 or higher are considered high yield. Size is the bond's amount outstanding in millions of dollars. Age is the number of years from offering date to the end of the month. Maturity is the number of years to maturity at the end of each month. Panel B displays the time series averages of cross-sectional correlations between the characteristics. The sample period is from January 2006 to June 2018.

Panel A: Summary statistics

					Percentiles			
	N	Mean	Median	SD	5th	25th	75th	95th
$S\text{-}Expo$	8186	0.27	0.20	0.28	0.00	0.04	0.43	0.84
R	8186	0.27	0.24	3.66	-3.93	-0.85	1.35	4.25
PI	8186	19.76	8.40	72.45	0.50	3.00	21.01	67.69
PI^S	8186	14.98	4.90	48.40	0.22	1.51	14.45	57.86
PI^U	8186	20.18	6.76	84.68	0.30	2.04	20.15	74.63
Rating	8186	9.12	8.58	3.98	3.76	6.42	10.92	16.83
Size	8186	583.38	413.57	584.87	67.63	246.55	709.48	1710.95
Age	8186	5.13	3.63	5.05	0.37	1.66	7.01	16.14
Maturity	8186	8.87	5.91	9.36	0.71	3.08	9.66	27.54

Panel B: Time series averages of cross-sectional correlations

	$Return$	PI	PI^S	PI^U	Rating	Size	Age	Maturity
$S\text{-}Expo$	-0.01	-0.02	-0.12	0.04	-0.01	0.08	-0.09	-0.01
$Return$		0.01	0.01	0.01	0.02	-0.01	0.00	0.02
PI			0.69	0.96	-0.01	-0.01	0.11	0.07
PI^S				0.41	0.00	0.00	0.06	0.08
PI^U					-0.02	0.00	0.09	0.07
Rating						-0.18	-0.08	-0.11
Size							-0.23	-0.02
Age								0.09

also has roughly 9 years to maturity and has existed for 5 years at the time of the exclusion event.

Panel B presents the time-series averages of cross-sectional correlations between the variables. It can be seen that $S\text{-}Expo$ does not high correlation with any of the bond level characteristics. Though not a sufficient condition, the low correlations between $S\text{-}Expo$ and other variables are a necessary condition for dealer stress exposure being randomly assigned across bonds.

1.5.5 Fama-MacBeth Regressions

For the bond level analyses, my main specification is the following:

$$y_{i,m+h} = \alpha_h + \beta_h \cdot S\text{-}Expo^{SD}_{i,m} + Controls_{i,m} + \varepsilon_{i,m+h}, \tag{1.1}$$

where $y_{i,m+h}$ is the outcome variable such as return and changes in liquidity from month m to $m + h$, h is the horizon in months, ranging from -4 to 12, $S\text{-}Expo^{SD}$ is the standardized $S\text{-}Expo$ in each month m with an average of 0 and standard deviation of 1, and $Controls_{i,m}$ include bond level characteristics such as PI, rating, size, age, and industry dummies, all of which are standardized in the cross section to have mean 0 and standard deviation 1. The coefficient of interest here is β_h. It represents the cross-sectional difference in the outcome variable associated with 1 standard deviation increase in $S\text{-}Expo$.

I standardize the right hand side variables in the cross section for two reasons. On the one hand, it helps with the interpretation of the regression coefficients. On the other hand, as pointed out by [24], in the context of Fama-MacBeth regression ([25]), the β coefficient represents the return on a zero-investment portfolio that sets the portfolio's value of $S\text{-}Expo$ to one and the values of other standardized characteristics to zero.

Since my focus is on the cross section, it is natural to use Fama-MacBeth regression as in [25] to estimate the coefficients. Given that the exclusion events happen at the end of every month and the outcome variables can span over multiple months, the outcome variables will inevitably overlap. This overlapping nature might create complicated correlation structures in the time series. In order to correct for such correlation, I use the [40] standard error with automatic lag selection. In this case, 13 lags are selected.

Another concern that comes with studying long run outcomes is the omitted variable bias. If the exclusion events have persistent effects over the next several months, $\hat{\beta}_h$ would be biased due to the omission of lagged $S\text{-}Expo^{SD}$'s. In order to see how important this omitted variable bias is for my main specification, I run the following panel regression:

$$S\text{-}Expo^{SD}_{i,m} = \lambda_i + \sum_{l=1}^{L} \phi_l \cdot S\text{-}Expo^{SD}_{i,m-l} + \epsilon_{i,m}, \tag{1.2}$$

where λ_i is the bond fixed effect, and L represents the number of lags. This regression exploits the time series variation within each bond. Table 1.2 presents the results with L being 1, 3, 6, and 12.

As is shown in Table 1.2, there is barely any persistence in the time series. The lack of persistence ensures that my specification is safe from the omitted variable bias discussed above while having the advantage of being parsimonious and easy to interpret.

Table 1.3 presents the main results of this section. Column 1 shows that, after the exclusion event, bonds with higher $S\text{-}Expo$ experience lower returns. This negative effect on return can last as long as 6 months. A 1 standard deviation increase in $S\text{-}Expo$ can predict 40 basis points decrease in return 6 months post exclusion. The positive, albeit insignificant, coefficient in month 1 seems to be inconsistent with the fire sale spillover mechanism. However, it is possible that, shortly after the exclusion event, the market understands that

Table 1.2: Time series dynamics of dealer stress exposure. This table shows the persistence of $S\text{-}Expo^{SD}$ over time. Each column presents the point estimates and standard errors (in parenthesis) of the regression (1.2). Standard errors are clusterd by both bond and month. *, **, *** indicate statistical significance at 10%, 5%, and 1%, respectively.

Lags	$L = 1$	$L = 3$	$L = 6$	$L = 12$
1	0.02	0.01	0.00	-0.00
	(0.02)	(0.02)	(0.03)	(0.03)
2		0.04	0.03	0.03
		(0.02)	(0.03)	(0.03)
3		0.01	-0.00	-0.03
		(0.03)	(0.03)	(0.03)
4			0.07***	0.07**
			(0.03)	(0.03)
5			-0.02	-0.04
			(0.03)	(0.03)
6			-0.02	0.00
			(0.03)	(0.03)
7				-0.02
				(0.03)
8				-0.02
				(0.03)
9				0.02
				(0.03)
10				-0.07**
				(0.03)
11				0.07**
				(0.03)
12				0.05
				(0.04)

Table 1.3: Fama-MacBeth cross-sectional regressions of return and changes in liquidity. This table presents the estimation results of equation (1.1) with left hand side variables being R, the monthly return (column 1), ΔPI, the change in overall price impact (column 2), ΔPI^S, the change in stressed dealers price impact (column 3), or ΔPI^U, the change in unstressed dealers price impact (column 4). The standard errors (in parentheses) are calculated according to [40] with 13 lags. *, **, and *** indicate 10%, 5%, and 1% level of statistical significance, respectively.

Horizon	R	ΔPI	ΔPI^S	ΔPI^U
1	0.14	0.40***	1.58***	-0.04**
	(0.11)	(0.14)	(0.09)	(0.02)
2	-0.19**	0.29*	1.63***	-0.03*
	(0.09)	(0.17)	(0.07)	(0.02)
3	-0.07	0.17	1.65***	-0.04*
	(0.09)	(0.18)	(0.08)	(0.02)
4	-0.18*	0.49**	1.55***	-0.02
	(0.10)	(0.21)	(0.15)	(0.02)
5	0.03	0.58***	1.70***	0.01
	(0.09)	(0.22)	(0.08)	(0.02)
6	-0.16**	0.37*	1.70***	-0.01
	(0.08)	(0.22)	(0.08)	(0.02)
7	-0.04	0.36*	1.58***	0.02
	(0.08)	(0.22)	(0.11)	(0.02)
8	-0.10	0.50**	1.71***	0.01
	(0.09)	(0.24)	(0.08)	(0.02)
9	-0.09	0.46*	1.79***	0.02
	(0.09)	(0.26)	(0.08)	(0.03)
10	-0.01	0.39	1.76***	0.01
	(0.10)	(0.28)	(0.08)	(0.03)
11	0.04	0.43	1.78***	0.01
	(0.11)	(0.27)	(0.08)	(0.03)
12	0.05	0.54*	1.71***	0.03
	(0.10)	(0.28)	(0.09)	(0.03)

the stressed dealers need to offload non-excluded bonds and has the capacity to absorb the sales. As the sell-off continues, the market capacity gets depleted and the negative effect on return emerges. This is also indicated by the coefficients in Column 3, where the effect on the change in stressed dealers price impact becomes stronger after month 1. In column 2, almost all point estimates are positive and statistically significant, suggesting that bonds with higher $S\text{-}Expo$ have become more illiquid after the exclusion event and stayed illiquid ever since. Columns 3 tells a similar story to the one in columns 2, except that the focus is on transactions with stressed dealers. The coefficients in columns 4 are mostly statistically insignificant. Taken together, columns 2 to 4 suggest that the spillover effect on liquidity stems from the stressed dealers while the unstressed dealers are not affected by the exclusion events and do not contribute to change in liquidity on the bond level.

The point estimates in column 1, 2, and 3 are not only statistically significant, but also economically meaningful. For example, the average 6-month cumulative return is around 1.6%. An inter-quartile move in $S\text{-}Expo$ from 25th to 75th percentile would result in a bond earning 0.8% lower return 6 months after. This change represents 50% of the average. Moreover, the average changes in price impact, ΔPI and ΔPI^S, are -0.28 and 0.88 over the 6 months period. A 1 standard deviation increase in $S\text{-}Expo$ results in 0.37 increase in ΔPI and 1.70 in the same time frame.

1.6 The Impact of the Volcker Rule on the Spillover Effect

Since the 2008 Financial Crisis, a series of regulatory changes regarding bank capital has been introduced. One of the most recent and controversial changes is the Volcker Rule. The Volcker Rule intends to curb risky proprietary trading by banks that have access to FDIC insurance or to the Federal Reserve's discount window. However, the Volcker Rule might have unintended consequences because it blurs the line between normal market making and

Table 1.4: Sizes and differences (in $100 million) of exclusion induced inventory shocks in subperiods. The subperiods are defined as: Pre-Crisis (January 1, 2006 to June 30, 2007), Crisis (July 1, 2007 to April 30, 2009), Post-Crisis (May 1, 2009 to July 20, 2010), Post-Dodd-Frank (July 21, 2010 to March 31, 2014), and Post-Volcker (April 1, 2014 to June 30, 2018). The first row displays the average size of the inventory shocks in each subperiod. The second row shows the differences between the respective subperiod and Post-Volcker. The standard errors (in parentheses) are in the third row.

	Pre-Crisis	Crisis	Post-Crisis	Post-Dodd-Frank	Post-Volcker
Shock Size	3.79	2.99	3.38	3.43	3.16
Difference	0.63	-0.17	0.22	0.26	
	(0.79)	(0.65)	(0.77)	(0.53)	

proprietary trading. In the face of the new regulation, bank-affiliated dealers might be more reluctant to commit capital for market making due to higher risk of regulatory sanction. Meanwhile, non-bank-affiliated dealers might not be big enough to step up and provide liquidity for the market, especially during times of stress.

To add to the debate on the effect of the Volcker Rule, I study how the spillover effect changes under different policy regimes, especially under the Volcker Rule. Following prior literature such as [5] and [8], I divide the full sample into five non-overlapping subperiods: Pre-Crisis (January 1, 2006 to June 30, 2007), Crisis (July 1, 2007 to April 30, 2009), Post-Crisis (May 1, 2009 to July 20, 2010), Post-Dodd-Frank (July 21, 2010 to March 31, 2014), and Post-Volcker (April 1, 2014 to June 30, 2018).

First, I look at how the sizes of the exclusion induced inventory shocks vary over time. I calculate the variable Shock Size as the cumulative order flow of excluded bonds handled by stressed dealers collectively from day -4 to day 0 of the exclusion event. Table 1.4 presents the average Shock Size (in $100 million) in the subperiods and the differences between the first four subperiods and Post-Volcker. Table 1.4 shows the results.

Table 1.4 indicates that the size of the inventory shock stemming from the exclusion events does not change significantly over the years. In the Post-Volcker period, the average size of the shock is \$316 million. In other subperiods, the sizes range from \$299 million to \$379 million. However, none of the differences between the Post-Volcker period and all the other subperiods is statistically different from 0.

Next, I look at how the average magnitudes of the spillover effect change over time. I use the following regression:

$$\begin{aligned} y_m = \alpha &+ \gamma_1 \cdot Pre\text{-}Crisis_m + \gamma_2 \cdot Crisis_m + \gamma_3 \cdot Post\text{-}Crisis_m \\ &+ \gamma_4 \cdot Post\text{-}Dodd\text{-}Frank_m + \delta_0 \cdot ShockSize_m + \varepsilon_m, \end{aligned} \tag{1.3}$$

where y_m is one of the three outcome variables: $\hat{\beta}_6^R$, which is the coefficient obtained from regression (1.1) with monthly bond return 6 months post-exclusion as the left hand side variable; $\hat{\beta}_6^{\Delta PI}$ and $\hat{\beta}_6^{\Delta PI^S}$, which are similarly defined as $\hat{\beta}_6^R$ but from regressions with ΔPI, the change in overall price impact, and ΔPI^S, the change in price impact of stressed dealers, as left hand side variables respectively. I focus on the spillover effect 6 months post-exclusion because the effect can last as long as 6 months, as indicated by Table 1.3. $Pre\text{-}Crisis_m$, $Crisis_m$, $Post\text{-}Crisis_m$, and $Post\text{-}Dodd\text{-}Frank_m$ are dummy variables corresponding to the subperiods defined above. The coefficients of interest are γ_1 to γ_4, which represent the differences in y_m between the respective subperiods and the Post-Volcker period, while the size of the inventory shock is held constant. Table 1.5 presents the results.

Two noticeable patterns emerge from Table 1.5. First, the coefficients in front of the dummy variable $Crisis$ are statistically significant, indicating that the magnitudes of spillover effect are substantially larger in the Crisis period than in the Post-Volcker period. This result makes sense because during the financial crisis, liquidity in the financial markets dried up and any type of offloading by the stressed dealers would have much larger impact on

Table 1.5: Subperiod analysis of the spillover effect. This table presents the estimation results of regression (1.3) with left hand side variables being $\hat{\beta}_6^R$, the coefficient from regression (1.1) with monthly return 6 months post-exclusion as the left hand side variable (column 1), $\hat{\beta}_6^{\Delta PI}$, the coefficient from regression (1.1) with change in overall price impact 6 months post-exclusion as the left hand side variable (column 2), and $\hat{\beta}_6^{\Delta PI^S}$, the coefficient from regression (1.1) with change in stressed dealers price impact 6 months post-exclusion as the left hand side variable (column 3). The standard errors (in parentheses) are calculated according to [40] with 13 lags. *, **, and *** indicate 10%, 5%, and 1% level of statistical significance, respectively.

	β_6^R	$\beta_6^{\Delta PI}$	$\beta_6^{\Delta PI^S}$
Panel A: Spillover Effect in the Post-Volcker period			
Post-Volcker	-0.04	0.50**	1.67***
	(0.13)	(0.23)	(0.03)
Panel B: Differences from the Post-Volcker period			
Pre-Crisis	-0.10	-0.15	-0.07***
	(0.38)	(0.18)	(0.02)
Crisis	-0.16*	0.24***	0.61***
	(0.09)	(0.06)	-0.1
Post-Crisis	0.03	-0.25	0.03**
	(0.08)	(0.36)	(0.01)
Post-Dodd-Frank	0.03	-0.39**	-0.26***
	(0.08)	(0.18)	(0.01)

pricing and liquidity than during normal times. Second, two out of three point estimates for $Post\text{-}Dodd\text{-}Frank$ are statistically significant, meaning that the spillover effect is stronger in the Post-Volcker than in the Post-Dodd-Frank period. This finding is consistent with the interpretation that the Volcker Rule hinders liquidity provision of bank-affiliated dealers.

Prior literature argues that while the Volcker Rule targets bank-affiliated dealers, non-bank-affiliated dealers are not affected. Instead, they step in and provide extra liquidity. If this argument is true, then the impact of the Volcker Rule is twofold. On the one hand, the Volcker Rule lowers the aggregate amount of capital committed to market making, as shown in Table 1.5. On the other hand, the composition of capital changes. A larger fraction of the capital comes from non-bank-affiliated dealers under the Volcker Rule than under previous policy regimes. Due to the limitations of the data, I am unable to identify which dealers are bank-affiliated. Therefore, I cannot formally test how much of the effect brought by the Volcker Rule is due to the dampening of banks market making versus the change in dealer composition.

1.7 Alternative Explanations

1.7.1 Private Information and Heterogeneous Selling Pressure among Excluded Bonds

One might be concerned about the validity of the exclusion events. Instead of minimizing tracking errors, the index funds might be interested in delivering excess returns against their benchmarks had they been able to do so. Suppose the index trackers have superior, private information about which excluded bonds will perform well going forward. They would sell less winners and more losers, thereby creating heterogeneous selling pressure across the excluded bonds. Observing the patterns from the index trackers, the stressed dealers would update their beliefs and adjust their portfolios of non-excluded bonds accordingly.

To assess whether this argument is plausible, I calculate the selling pressure of the excluded bonds surrounding the exclusion events. I use the cumulative order flow during an event window scaled by the bond's amount outstanding to proxy for its selling pressure. The reason for scaling is to adjust the size difference between the excluded bonds. Since the index is value weighted, when a large bond is excluded, a large sell volume of the bond is expected even the sale is entirely mechanical and information free. If the index funds do not possess private information and rebalance their portfolios purely for tracking errors, the selling pressure across excluded bonds would be relatively homogeneous.

Table 1.6 presents the findings. The results support the argument that the index trackers do not have superior, private information. First, in each panel, the standard deviations are much smaller than the means. This suggests that the selling pressure of the excluded bonds

Table 1.6: Order flow dynamics of excluded bonds. This table shows the average cumulative order flows of excluded bonds scaled by their amount outstanding during different event windows and by reasons of exclusion. Columns 1 and 2 display the means and standard deviations of net order flow over amount outstanding. Columns 3 and 4 exhibit those of inflow to the stressed dealers over amount outstanding. Columns 5 and 6 present outflow from the stressed dealers over amount outstanding.

	Net		Inflow		Outflow	
	Mean	SD	Mean	SD	Mean	SD
Panel A: Event day window [-100,0]						
All	1.25	0.15	6.73	0.35	5.47	0.31
Size	1.26	0.13	6.73	0.31	5.47	0.30
Maturity	1.33	0.16	6.77	0.34	5.44	0.30
Rating	1.21	0.16	6.69	0.37	5.48	0.34
Panel B: Event day window [1,100]						
All	-1.25	0.13	5.45	0.32	6.70	0.39
Size	-1.25	0.13	5.45	0.40	6.70	0.22
Maturity	-1.21	0.14	5.46	0.30	6.68	0.36
Rating	-1.24	0.14	5.44	0.33	6.69	0.41
Panel C: Event day window [-4,0]						
All	1.00	0.23	2.08	0.39	1.07	0.37
Size	1.00	0.26	2.06	0.45	1.06	0.25
Maturity	1.02	0.30	2.11	0.33	1.10	0.38
Rating	0.94	0.23	2.02	0.37	1.08	0.38
Panel D: Event day window [1,5]						
All	-0.26	0.15	1.23	0.16	1.48	0.28
Size	-0.25	0.18	1.24	0.18	1.49	0.28
Maturity	-0.23	0.19	1.23	0.11	1.46	0.28
Rating	-0.22	0.12	1.23	0.16	1.45	0.28

is tightly clustered around its mean. Second, the means across exclusion reasons are also very close to each other. This indicates that the index trackers do not sell any group of bonds particularly heavily.

1.7.2 Concentration in Certain Characteristics among Excluded Bonds

Another alternative explanation for my results is the similarity in characteristics between excluded and non-excluded bonds. When the stressed dealers receive a large volume of excluded bonds, their portfolio-level characteristics would heavily tilt towards the characteristics of the excluded bonds. In response to this sudden increase in concentration of certain characteristics, the stressed dealers would sell non-excluded bonds with similar characteristics, thus generating negative price pressure. If the non-excluded bonds with similar characteristics happen to have high $S\text{-}Expo$, my results would overestimate the impact stemming from inventory stress.

In order to see if this mechanism is at work, I first divide the excluded bonds into three groups based on their reasons for exclusion, downgrade, change in amount outstanding, and change in time to maturity. Next, I calculate the absolute difference in average characteristics between the excluded bonds and non-excluded bonds in the bottom $S\text{-}Expo$ tercile, ΔLow, and that between the excluded bonds and non-excluded bonds in the top $S\text{-}Expo$ tercile, $\Delta High$. Finally, I calculate the difference between ΔLow and $\Delta High$, $\Delta Low - \Delta High$. If $\Delta Low - \Delta High$ is negative, it means that low exposure bonds are closer to the excluded bonds, which is against the characteristics management argument. Conversely, if $\Delta Low - \Delta High$ is positive, then it supports the argument. Table 1.7 presents the results.

In Panel A, for bonds excluded due to rating downgrade, high S- bonds have ratings that are closer to the excluded bonds. However, the difference is small and statistically insignificant. For amount outstanding related exclusions in panel B, low S- bonds actually have more similar sizes. This is against the characteristics management motive. In panel C, for exclusions due to time to maturity, it is true that high exposure bonds are closer in time to maturity to the excluded bonds. However, the average time to maturity of the excluded bonds are less than 1 year, and that of the non-excluded bonds are around 9 years. It is

Table 1.7: Characteristic differences between excluded and non-excluded bonds with varying levels of S-$Expo$. This table presents the average characteristics of the excluded bonds (column 1), the absolute difference in average characteristics between the excluded bonds and low exposure bonds in the bottom S-$Expo$ tercile (column 2), the absolute difference between the excluded bonds and bonds in the top S-$Expo$ tercile (column 3), and the difference in differences (column 4). The standard errors are calculated according to [40] with 13 lags. *, **, and *** indicate 10%, 5%, and 1% level of statistical significance, respectively.

	Excluded	ΔLow	$\Delta High$	$\Delta Low - \Delta High$
Panel A: Downgrade				
Rating	11.68	2.57	2.50	0.07
		(0.20)	(0.19)	(0.11)
Size	622	313	36	277***
		(30.77)	(29.00)	(18.83)
Maturity	8.20	1.70	1.04	0.66
		(0.39)	(0.40)	(0.72)
Panel B: Amount Outstanding				
Rating	8.32	0.79	0.86	-0.05
		(0.14)	(0.13)	(0.08)
Size	112	209	474	-265***
		(10.30)	(13.12)	(9.13)
Maturity	11.24	1.33	2.00	-0.66
		(0.72)	(0.64)	(0.61)
Panel C: Time to Maturity				
Rating	6.37	2.74	2.81	-0.07
		(0.10)	(0.09)	(0.10)
Size	762	449	182	267***
		(18.06)	(21.42)	(8.73)
Maturity	0.95	8.95	8.30	0.45***
		(0.07)	(0.08)	(0.10)

unlikely that the stressed dealers would sell the high S- bonds just to drive up the portfolio-level maturity. Taken together, the evidence for the characteristics management motives is rather weak.

1.7.3 Pre-existing Trends in Return and Liquidity

One concern for the results in Table 1.3 is that the outcome variables are already trending in that direction before the exclusion event. If that's the case, then it is difficult to attribute the difference in pricing and liquidity in the cross section to the inventory shock. In order to address this concern, I run the same regression as in Table 1.3 but use returns and changes in price impact before the exclusion event. For h from -4 to -1, I measure *Return* as

$$R_{i,m+h \to m+h+1} = \frac{P_{i,m+h+1} + AI_{i,m+h+1} + Coupon_{i,m+h+1 \to m+h}}{P_{i,m+h} + AI_{i,m+h}} - 1,$$

ΔPI is calculated as

$$\Delta PI = PI_{i,m+h} - PI_{i,m}, \tag{1.4}$$

and ΔPI^S and ΔPI^U are calculated the same way as ΔPI. Table 1.8 presents the regression results.

Overall, there is little evidence suggesting that pre-trend exists. In column 1, all the point estimates are very close to 0 and statistically insignificant. In column 2 and 3, the negative coefficients are indeed in line with the existence of a pre-trend. However, as time goes on, the coefficients become positive and insignificant. In column 4, all the point estimates are very close to 0. Therefore, there is not enough support for the existence of a pre-trend.

1.8 Conclusion

This study explores the possibility of dealer transmitted fire sale spillovers in the corporate bond market. I use the index exclusion events as plausibly exogenous shocks to dealer inventory and classify dealers in the cross section as stressed if they have taken an usually large amount of excluded bonds into their balance sheets. I construct a dealer-bond level

Table 1.8: Fama-MacBeth cross-sectional regression of pre-event return and changes in liquidity. This table presents the estimation results of equation (1.1) with the left hand side variable being R (column 1), ΔPI (column 2), ΔPI^S (column 3), or ΔPI^U (column 4), all of which are calculated in the pre-event period. The standard errors (in parentheses) are calculated according to [40] with 13 lags. *, **, and *** indicate 10%, 5%, and 1% level of statistical significance, respectively.

Horizon	R	ΔPI	ΔPI^S	ΔPI^U
-4	-0.00	-0.24*	-0.48*	-0.01
	(0.01)	(0.14)	(0.25)	(0.02)
-3	0.02	-0.13	-0.01	-0.01
	(0.02)	(0.16)	(0.32)	(0.02)
-2	0.02	-0.04	0.43	-0.04**
	(0.03)	(0.14)	(0.56)	(0.02)
-1	0.01	0.09	1.47	-0.04**
	(0.02)	(0.13)	(0.97)	(0.02)

variable to capture how likely a dealer would sell certain non-excluded bonds, which I call *idiosyncratic dealer stress exposure*. I find that in response to the inventory shock, the stressed dealers would sell more of the non-excluded bonds with higher exposure. Moreover, the stressed dealers equalize the marginal transaction costs when trading the non-excluded bonds. The evidence suggests that the stressed dealers behave optimally when managing balance sheet spaces.

Next, I investigate the implications of the spillover effect for pricing and liquidity of non-excluded bonds. I construct a bond level variable called *systematic dealer stress exposure* by aggregating the market shares of stressed dealers within each non-excluded bonds. I find that bonds with higher exposure experience more severe price drop and liquidity deterioration post-exclusion. The effect can last as long as 6 months.

Finally, I examine the effect of the heavily debated Volcker Rule on the spillover effect. I find that the average magnitude of the fire sale spillover is higher under the Volcker Rule than under the preceding Dodd-Frank Act. This finding supports the interpretation that the

Volcker Rule negatively affects the willingness of bank-affiliated dealers to commit capital for market making.

Chapter 2

DYNAMIC CONTRACTS WITH HUMAN CAPITAL ACCUMULATION

2.1 Introduction

In principal-agent models, the principal hires an agent to do a job that the principal does not wish to do themself. Often this is motivated by an appeal to expertise: the agent has specialized skills or knowledge that the principal lacks. This motivation has the additional benefit of generating hidden information; the principal cannot observe the agent's actions because the principal does not share the agent's expertise and so cannot untangle the exact relationship between expertise and output. For example, [17] and related work consider a setting where the agent can take an instantaneous hidden action that leads to private benefits and reduces the mean of the cash flow of the period. While the probability distribution of the cash flows is publicly known, the principal is unable to tell whether a low realization of the cash flow is due to the agent's action or bad luck. Thus, contracts are commonly designed to elicit short-term effort: the agent is granted a portion of cash flow rights in order to induce them to use their expertise to increase output. In this paper, we push one step back and ask what cash flow rights are required to generate and maintain the initial expertise.

We model cash flows as being the result of durable human capital (expertise) that the agent must work to accumulate. The agent has two degrees of hidden information, hidden effort to accumulate durable expertise and persistent private information in the level of expertise. The agent can accumulate expertise but at a cost, and the effort or resources spent on learning cannot be observed by the principal. Similarly, the agent's level of expertise is not observable by the principal. However, unlike effort, expertise is durable: expertise depreciates over time, but not quickly, and it can be re-accumulated. Expertise is used to generate noisy output, which the principal can observe. Thus our model can accommodate education, training, or learning-by-doing.

Aside from persistent private information over expertise, the model is standard. The agent has CARA preferences over consumption and a quadratic cost of effort. The principal is risk-neutral and the principal and agent share a discount rate. The agent maintains hidden savings, which create the standard limitation that the agent's marginal utility of consumption must be a martingale. Cash flows are modelled as human capital plus a brownian motion shock.

This model can be applied to any setting in which the primary determinant of output is not today's effort but instead the slower accumulation of expertise or knowledge. This includes any version of the "knowledge economy", such as the research and development departments in technology firms. For example, Microsoft Research does not have customer-facing products but produces research papers and files patents related to computer science. However, their accumulated knowledge is infused in other Microsoft products, such as Bing, LinkedIn, etc, and contributes to earnings growth in the long run. The model can also be interpreted to cover relationships like networks or client lists. Specifically, it can be used to analyze the compensation structure of employees in the professional service industries, such as investment banking, consulting, and accounting. In these industries, the employees develop a network of clients over time and their clients are likely to use their services repeatedly. Therefore, their effort to acquire an customer today generates profits in the future.

Because the agent's expertise is durable, their incentive to accumulate it derives from future cash flow sharing. Incentives are durable as well. In the solution to the optimal contract model, we show that the principal manages a stock of incentives – the average value of all future cash flow rights granted to the agent – and doles them out over time. The stock of incentives is what induces effort by the agent to accumulate expertise. At the same time, the agent is risk-averse, so they require compensation for volatility in their consumption – compensation for being exposed to cash flow volatility. However, like in standard moral hazard models, today's compensation is driven by the today's volatility. This means that there is a mismatch in duration – incentives are the stock of long duration average cash flow rights, matching the impact of long duration expertise, but risk compensation is a flow of consumption based on the flow of volatility.

We show that the principal engages in a dynamic incentives management problem. The contract has two regions. In the first, the agent begins the contract with a low level of incentives – the stock of future cash flow rights granted to the agent is small. The principal responds by *training* the agent: the principal pushes all the agent's cash flow rights into the future so that the stock of incentives grows, and the agent accumulates human capital. Once the stock of incentives passes a threshold, the contract enters a second region, and the principal begins to grant cash flow rights and to actively manage the agent's expertise. Once incentives have reached an upper threshold, the amount and cost of future cash flow rights becomes too high and the principal holds the stock of incentives constant. If given a choice, the principal will optimally choose to hire an agent just out of training.

A key feature of managing the agent's incentives is that it is done stochastically. The stock of future incentives optimally has a negative volatility loading on the cash flow brownian motion. There are three effects as time passes. The first is from discounting: any cash flow rights not granted today must be larger if granted in the future, so incentives drift up. The second is from the negative loading on volatility: when the cash flow shock is unexpectedly positive, the agent's utility increases but their stock of future incentives declines. The third is from correlation and complementarity: CARA utility means that higher continuation value implies a lower marginal cost of effort, so future incentives (the marginal benefit from effort) optimally decline after a positive cash flow shock.

This paper builds on the literature of optimal dynamic contracting in continuous time, including [35], [11], [17], [41], [48], among others. A common theme in these models is the immediate impact of the agent's action. Whenever the agent takes an action, it affects the expected output in that instance but not thereafter. Our model differs from the previous ones by considering the long-term impact of the agent's action.

Our paper is also part of a burgeoning literature on persistent private information. [50] analyzes the optimal contracting problem with persistent private information using a system of ordinary differential equations. [47] develops the change of variable method to account for the persistence and provides a general characterization of the optimal contract. Subsequently, a number of papers apply these techniques to study a variety of economic situations. [44] studies the effect of renegotiation under persistent private information. [42] studies the role of termination plays in dynamic contracting. [18] and [33] introduce learning into the canonical moral hazard model in continuous time and study its implications. [39] analyze the optimal contract when the agent can boost his current output at the expense of the firm's long term value. [26] embeds an agency friction in the investment model where the agent observes a private signal about productivity and can misreport for his personal gain. [45] introduces ambiguity aversion in a principal-agent framework. Contrary to the extant literature, our model investigates the optimal design of incentive provision when the agent builds up productive capital over time.

This paper is organized as follows. Section 2.2 describes the model. Section 2.3 presents the optimal contract for the benchmark model with observable human capital. We characterize and analyze the optimal contract in Section 2.4. Section 2.5 offers concluding remarks.

2.2 *The Model*

2.2.1 Agents and Production

In this section, we set up a principal-agent problem in continuous time with both hidden action and hidden information. The principal has fixed assets that, when managed by an agent with human capital, produce cumulative cash flow

$$dX_t = H_t dt + \sigma dZ_t, \tag{2.1}$$

where H is the agent's stock of human capital, $\sigma > 0$ is the constant volatility, and Z is a standard Brownian motion. While the realization of the cash flow process X is publicly observable, the process Z and the level of human capital H are only observable to the agent.

The agent can make a costly investment $h \geq 0$ in human capital and the overall level of human capital H depreciates at rate $\rho > 0$. Thus, human capital evolves as

$$dH_t = (h_t - \rho H_t)dt. \tag{2.2}$$

Our model is consistent with standard intuitions about human capital: it is unobservable, durable, and renewal is costly. This requires two levels of hidden information. First, human capital impacts cash flows, but is not observable. Second, the agent privately builds up human capital over time, and that capital depreciates. These two levels imply that the effect of the any deviation by the agent from the desired investment has a persistent effect on cash flows. Conversely, the principal will need to provide long-term incentives that are consistent across time to induce the desired action by the agent.

We similarly allow for both hidden action and hidden information for the agent's consumption and savings. The principal awards the agent some level of payment m based on observed cash flows, which the agent deposits into a savings account that is privately observed and grows at rate r. The agent then privately consumes out of savings. Thus, the agent's savings evolve as

$$dS_t = (rS_t + m_t - c_t)dt, \tag{2.3}$$

where c is the agent's consumption.

The agent has exponential utility over consumption flow and human capital investment:

$$u(c, h) = -\frac{1}{\gamma} \exp\left[-\gamma\left(c - \frac{\theta}{2}h^2\right)\right], \tag{2.4}$$

where $\gamma > 0$ is the constant coefficient of absolute risk aversion. The agent faces a quadratic financial adjustment costs for improving human capital, with cost parameter $\theta > 0$. This can be viewed as a financial cost (e.g. the agent is buying education) or a non-pecuniary cost (e.g. learning requires painful effort).

The principal is risk neutral over cash flows and receives utility $dX_t - m_t dt$. The principal and the agent share the common discount rate r.

2.2.2 Formulation of the Optimal Contracting Problem

We use "hats" to denote the principal's desired actions and plain letters to denote the agent's actual decisions. So, for example, $\hat{c}$ is the amount of consumption the principal desires for the agent and c is the amount of consumption actually undertaken. Similarly, $\hat{S}$ is the level of hidden savings consistent with the principal's desires and S_t is the true level of hidden savings.

At time 0, the principal hires the agent and observes the agent's human capital and private savings. The initial levels are irrelevant and we can set them equal to zero, i.e.,

$\hat{H}_0 = H_0 = \hat{S}_0 = S_t = 0$. The recommended levels of human capital and savings evolve according to

$$d\hat{H}_t = (\hat{h}_t - \rho \hat{H}_t)dt \tag{2.5}$$

$$d\hat{S}_t = (r\hat{S}_t + m_t - \hat{c}_t)dt. \tag{2.6}$$

Starting at time 0, a contract $\mathcal{C}$ specifies a series of payments m_t conditional on the principal's information set up to time t. The contract also specifies a recommended path of actions $\{\hat{c}, \hat{h}\}$ to the agent with an implied path for $\hat{H}$ and $\hat{S}$. The principal and agent share the same discount rate r, so it is without loss of generality to assume that the principal does all the savings for the agent, meaning $\hat{S}_t = 0$ and $\hat{c} = m$ throughout (see e.g. [48]).

The agent acts to maximize expected discounted utility:

$$W(\mathcal{C}) = \max_{c,h} \; E_0^{c,h} \left[\int_0^\infty e^{-rt} u(c_t, h_t) dt \right]. \tag{2.7}$$

where $E_0^{c,h}$ is the expectation operator under the measure induced by the action (c, h). The contract is incentive compatible if the agent selects the contract that induces the agent to choose the course of action $c = \hat{c}$ and $h = \hat{h}$ recommended by the contract. The agent has an outside option $\underline{W}$.

The principal's objective is to maximize the expected value of net cash flows from the project:

$$V_0 = E_0^{c,h} \left[\int_0^\infty e^{-rt} (dX_t - m_t dt) \right]. \tag{2.8}$$

Consequently we have the following definition:

Definition 1 (Contracts) *A contract $\mathcal{C}$ offers a recommended path of actions and states*

$\{\hat{c}, \hat{h}, \hat{S}, \hat{H}\}$ and a series of payments $\{m\}$ to the agent based on the path of X.

An incentive compatible contract induces the agent to choose $\{c, h, S, H\} = \{\hat{c}, \hat{h}, \hat{S}, \hat{H}\}$.

An optimal contract is incentive compatible, grants the agent at least their outside option $W(\mathcal{C}) \geq \underline{W}$ at $t = 0$, and gives the principal the highest level of value given the first two conditions.

2.3 The Model with Observable Human Capital

In this section, we solve for the benchmark contract with observable human capital. Because cash flow noise is now observable and irrelevant, and there is no other noise in the model, the principal can offer the agent a contract with full insurance that grants constant consumption and utility:

$$W_t = \underline{W}. \tag{2.9}$$

Because the principal's opportunity set is constant and without noise, the desired level of investment in human capital will also be constant. This result gives the agent constant consumption whether or not the agent has hidden savings. Hidden savings might allow the agent to smooth consumption across time, but there are no shocks to smooth and the level of human capital investment does not vary:

Proposition 1 *With observable human capital, the contract specifies a constant level of consumption and effort by the agent*

$$c = \frac{\theta}{2}h^2 - \frac{1}{\gamma}\ln(-\gamma r \underline{W}) \tag{2.10}$$

$$h = \frac{1}{\theta(r + \rho)}. \tag{2.11}$$

The long run human capital is

$$H^{FB} \equiv \lim_{t\to\infty} H_t = \frac{1}{\rho\theta(r+\rho)}. \tag{2.12}$$

Equation (2.10) states that the agent's consumption net of the cost of human capital growth must be high enough to match his outside opportunity. Equation (2.11) sets h so that the marginal increase in expected discounted cash flows ($\frac{1}{r+\rho}$) equals the marginal cost of investment (θh). This generates a constant investment in human capital which brings H_t to its long-run stable level H^{FB}.

2.4 The Optimal Contract

2.4.1 The Agent's Problem

We first analyze the agent's decision rule under an arbitrary contract and then characterize the incentive compatibility conditions using the decision rule. Continuing the "hat" notation, Z is the true Brownian motion under the measure induced by (c, h), namely $\mathcal{P}$, while $\hat{Z}$ is the implied Brownian motion under the measure induced by $(\hat{c}, \hat{h})$, namely $\hat{\mathcal{P}}$. We will proceed by using the Stochastic Maximum Principal to obtain the necessary incentive compatibility conditions. We later verify that the IC conditions are indeed sufficient.

The agent solves the following problem: maximize (2.7) subject to (2.1), (2.2), (2.3), and (2.4). Two relevant state variables emerge from the agent's problem. The first state variable is the agent's promised continuation utility $\hat{W}$,

$$\hat{W}_t = E_t^{\hat{\mathcal{P}}}\left[\int_t^{\infty} e^{-r(s-t)} u(\hat{c}_s, \hat{h}_s) ds\right]. \tag{2.13}$$

It is now well known in the dynamic contracting literature that the continuation utility $\hat{W}$ summarizes all future promises from the principal to the agent.

Because total utility (past utility plus continuation utility) is a martingale, we can use the martingale representation theorem to show that there exists a predictable, measurable β such that

$$d\hat{W}_t = r\hat{W}_t dt - u(\hat{c}_s, \hat{h}_s)dt - \gamma r \hat{W}_t \beta_t \sigma d\hat{Z}_t. \tag{2.14}$$

The principal controls β_t indirectly by awarding the agent payments based on cash flows. It will be shown to be the case that $\beta \geq 0$, meaning it is never optimal to engage in value destruction. β is interpreted as the sensitivity of the agent's continuation utility to cash flow shocks: if cash flows are higher than expected by some amount δ, the agent captures $-\gamma r \beta_t W_t \delta > 0$ as additional utility. One can think of $\frac{d\hat{W}}{dX}$ as $-\gamma r \beta W$.

The second important state variable is the stock of future incentives. Because the agent's choice of h affects all future cash flows, it must also be true that all future cash flow sensitivities affect the agent's choice of h. We can characterize the sum of all future incentives as

$$P_t^{\Delta H} = E_t^{\hat{\mathcal{P}}} \left[\int_t^{\infty} e^{-(r+\rho)(s-t)} \left[-\gamma r \beta_s \hat{W}_s \right] ds \right]. \tag{2.15}$$

The benefit of investing in human capital today is that all future cash flows are higher, and the agent will receive these increases discounted by the interest rate r plus the depreciation rate of human capital ρ.

Because total incentives (past incentives plus future incentives) is a martingale, we can use the martingale representation theorem to show that there exists a predictable, measurable η such that

$$dP_t^{\Delta H} = \left((r+\rho) P_t^{\Delta H} + \gamma r \hat{W}_t \beta_t \right) dt + \sigma P_t^{\Delta H} \eta_t d\hat{Z}_t. \tag{2.16}$$

The principal controls η_t indirectly by controlling all future β through cash payments to the agent. In this way, the principal can respond to cash flow shocks by increasing or decreasing future incentives.

We now characterize incentive compatibility:

Proposition 2 *Necessary and sufficient conditions for the contract to be incentive compatible are that the agent's policies are given by*

$$\hat{c}_t = m_t \tag{2.17}$$

$$r\hat{W}_t = u(m_t, \hat{h}_t) \tag{2.18}$$

$$\hat{h}_t = -\frac{P_t^{\Delta H}}{\theta\gamma r\hat{W}_t} \tag{2.19}$$

and the state variables $\hat{W}$ and $P^{\Delta H}$ evolve according to (2.14) and (2.16).

Appendix B.1 proves the necessary conditions. Appendix B.2 proves the necessary conditions are sufficient.

The first condition (2.17) means that the agent does not save privately – the principal and the agent share a discount rate, and the principal can commit to the contract, so the principal can save on behalf of the agent. The second condition (2.18) provides the required incentives for the agent to maintain zero savings. The agent does not need to smooth consumption because the agent's marginal utility of consumption, $u(\hat{c}_t, \hat{h}_t)$, is a martingale. This also implies that the agent's utility is a martingale, which simplifies the analysis. The third condition (2.19) sets the agent's marginal cost of effort equal to the marginal benefit. The marginal benefit is $P_t^{\Delta H}$, the discounted value of future cash flow surprises. The marginal cost is the derivative of utility, (2.4), with respect to h, and plugging in (2.18).

2.4.2 The Principal's Problem

The previous subsection characterizes the incentive compatibility conditions for the agent. We now move on to the principal's problem. The principal solves the following problem

$$V(\hat{H}_t, \hat{W}_t, P_t^{\Delta H}) = \max_{\beta,\eta} E_t \left[\int_t^\infty e^{-r(s-t)} \left(dX_s - \hat{c}_s ds \right) \right], \tag{2.20}$$

subject to the evolution of state variables (2.5), (2.14), and (2.16), and the IC conditions (2.18) and (2.19).

The problem is simplified by characterizing the solution using $z = -\frac{P^{\Delta H}}{\gamma r \hat{W}}$ and $\hat{W}$ as the state variables, rather than $P^{\Delta H}$ and $\hat{W}$. z represents the agent's future incentives scaled by current continuation utility and is more directly linked to the agent's choices, (2.19). Using Ito's lemma, we can derive the evolution of z:

$$dz_t = \left((r + \rho + \sigma^2 \gamma r \beta_t \phi_t) z_t - \beta_t \right) dt + \sigma \phi_t z_t d\hat{Z}_t, \tag{2.21}$$

where $\phi_t = \gamma r \beta_t + \eta_t$. With the introduction of z, (2.18) and (2.19) imply

$$\hat{h}_t = \frac{1}{\theta} z_t \tag{2.22}$$

$$\hat{c}_t = -\frac{1}{\gamma} \ln(-\gamma r \hat{W}_t) + \frac{1}{2\theta} z_t^2. \tag{2.23}$$

This means that the principal's net cash flows can be written as

$$dX_t - \hat{c}_t dt = \left(\hat{H}_t + \frac{1}{\gamma} \ln(-\gamma r \hat{W}_t) - \frac{1}{2\theta} z_t^2 \right) dt \tag{2.24}$$

Integrating by parts and using the fact that W_t is a martingale yields

$$E_t\left[\int_t^\infty e^{-r(s-t)}\hat{H}_s ds\right] = \frac{1}{r+\rho}\hat{H}_t + E_t\left[\int_t^\infty e^{-r(s-t)}\frac{1}{\theta(r+\rho)}z_s ds\right]$$

$$E_t\left[\int_t^\infty e^{-r(s-t)}\frac{1}{\gamma}\ln(-\gamma r\hat{W}_s)ds\right] = \frac{1}{r\gamma}\ln(-\gamma r\hat{W}_t) + E_t\left[\int_t^\infty e^{-r(s-t)}\left(-\frac{\gamma r\sigma^2}{2}\beta_s^2\right)ds\right]$$

This allows us to write the principal's problem as a simple dynamic programming problem with a single state variable:

$$V(z_t,\hat{H}_t,\hat{W}_t) = v(z_t) + \frac{1}{r+\rho}\hat{H}_t + \frac{1}{r\gamma}\ln(-\gamma r\hat{W}_t), \tag{2.25}$$

$$v(z_t) = \max_{\beta,\phi} E_t\left[\int_t^\infty e^{-r(s-t)}\left(\frac{1}{\theta(r+\rho)}z_s - \frac{1}{2\theta}z_s^2 - \frac{\gamma r\sigma^2}{2}\beta_s^2\right)ds\right] \tag{2.26}$$

We have decomposed the principal's value function into three parts. The second term is the perpetuity value of today's human capital. The third term is the perpetuity value of payments to the agent based on today's continuation utility. The first term captures all the future variation from incentives, including the benefits of human capital accumulation ($\frac{1}{\theta(r+\rho)}z_s$), costs of additional human capital accumulation ($-\frac{1}{2\theta}z_s^2$), and the risk sharing cost of giving the agent volatile consumption ($-\frac{\gamma r\sigma^2}{2}\beta_s^2$).

To complete our characterization of the principal's problem, we will need to see how the economy evolves when the principal stops dynamically managing incentives. Stopping in this economy means $dz_t = 0$ and reverting to a steady state. This is accomplished with $\phi_{stop} = 0$ and $\beta_{stop} = (r+\rho)z_t$. Substituting these values into (2.26) yields

$$v_{stop}(z) = \frac{1}{r\theta(r+\rho)}z - \frac{1+\gamma r\theta\sigma^2(r+\rho)^2}{2r\theta}z^2. \tag{2.27}$$

We can now solve the principal's problem as a dynamic programming problem with one state variable:

Proposition 3 *Define* $z^* = \arg\max_z v(z)$*. Then,* $v(z)$ *exists, is unique, concave and twice continuously differentiable on* $z \in (0, z^*)$*.* $v(z)$ *exists on* $z \in (z^*, z_R)$*, and in both regions it must solve the HJB equation*

$$\max_{\beta \geq 0, \phi} \Bigg\{ -rv(z) + \frac{1}{\theta(r+\rho)} z - \frac{1}{2\theta} z^2 - \frac{\gamma r \sigma^2}{2} \beta^2 + v'(z) \left((r + \rho + \sigma^2 \gamma r \beta \phi) z - \beta \right) + \frac{1}{2} v''(z) \sigma^2 \phi^2 z^2 \Bigg\} = 0. \tag{2.28}$$

In addition, we have boundary conditions

$$v(0) = 0 \tag{2.29}$$

$$v(z_R) = v_{stop}(z_R) \tag{2.30}$$

$$v'(z_R) = v'_{stop}(z_R) \tag{2.31}$$

The proof of this proposition is in Appendix B.3. Figure 2.1 shows a numerical solution to the principal's problem.

2.4.3 Two Regions

The principal faces a truly dynamic incentives smoothing problem. z_t is the stock of future cash-flow sensitivities, and it determines the agent's choice of human capital accumulation through the incentive compatibility condition. To be consistent, the principal must give these incentives out over time. However, those future incentives must be given out slowly because they have a quadratic cost: the agent must be compensated for the variance of his consumption and utility flows, so the cost of current incentives is proportional to β^2. Because incentives are discounted, they – and their costs – are larger if paid out later. This

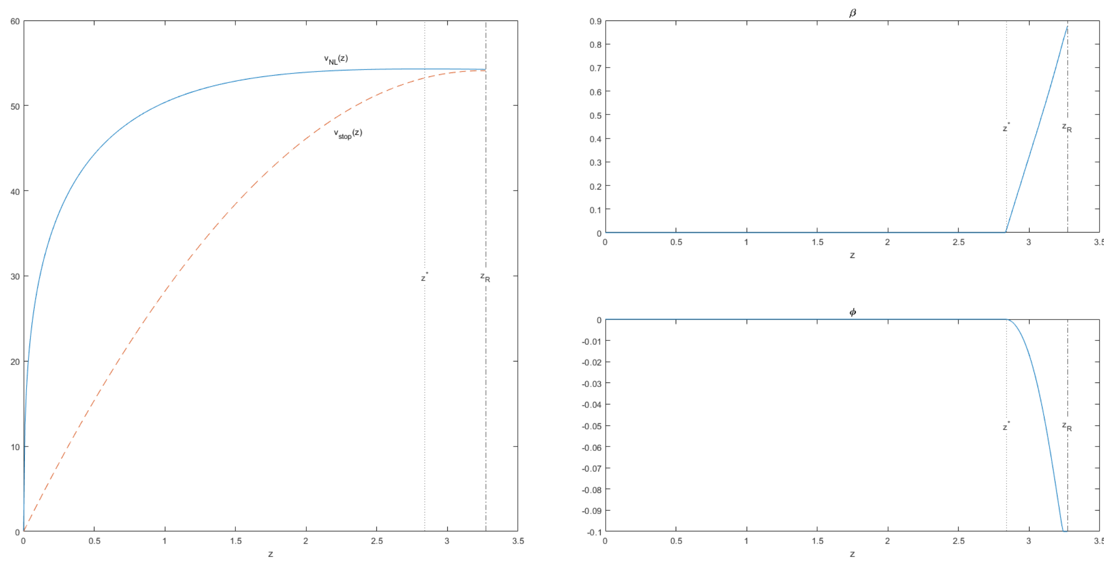


Figure 2.1: Numerical solution to principal's problem. Parameters: $r = 0.1$, $\rho = 0.2$, $\gamma = 3$, $\sigma = 1$, and $\theta = 1$. z^* is the point where $v'(z^*) = 0$.

creates two regions, one in which the principal accumulates this stock of future cash flow sensitivities, and one in which the principal spends that stock:

Corollary 4 *For* $0 \leq z_t \leq z^*$ *the principal chooses* $\beta = \phi = 0$. *For* $z^* < z_t < z_R$, *the principal chooses* $\beta > 0$ *and* $\phi < 0$. *For* $z = z_R$, *the principal chooses* $\phi = 0$ *and* $\beta = (r + \rho)z_R$.

The contract has two regions, an accumulation or training region with $z \leq z^*$ and an active incentives region with $z > z^*$. In the training region, the principal does not give the agent any cash flow sensitivity ($\beta = \phi = 0$). However, that does not mean the agent has no incentive to accumulate human capital. Instead, by pushing cash flow sensitivity into the future, the principal accumulates incentives for the agent to invest in human capital: $dz_t = (r + \rho)z_t dt$. We call this training because the agent is accumulating human capital

($h > 0$), accumulating incentives to invest in human capital ($dz > 0$) and yet has no cash flow sensitivity ($\beta = \phi = 0$). Of course this training is costly – the principal has to re-imburse the agent for the effort spend on human capital accumulation. If the principal chooses the initial level of incentives, he will choose $z_0 = z^*$ – meaning the principal hires an agent exactly at the end of their training.

The second region is the active incentives region: $z > z^*$. Here the accumulated future cash flow sensitivity is high enough to incentivize the agent to invest in human capital, but the principal begins to pay out incentives – to give the agent cash flow sensitivity today – so that z does not become excessively high. Interestingly, the principal sets $\phi < 0$, which also implies $\eta < 0$. This means that the principal optimally chooses to make the stock of future incentives stochastic, and negatively correlated with cash flows. Thus, when there is a surprise positive cash flow, the agent's continuation utility goes up while the agent's accumulated future incentives go down from two sources. The first source is direct use of cash flow incentives, and the second is a negative stochastic shock to future incentives.

This feature is driven by the fact that the marginal cost of effort is positive and decreasing with existing utility: $\frac{\partial}{\partial h}u(c,h) = -\theta\gamma r W h > 0$. This means that β plays a dual role: it is the sensitivity of utility to cash flows and also the negative sensitivity to the marginal cost of effort to cash flows. By making the volatility of future incentives (η) sufficiently negative that the volatility of scaled incentives ($\phi = \beta\gamma r + \eta$) is also negative, the principal incudes a correlation based drift in z_t. From (2.21), z drifts downward. Since z is already too high – $z > z^*$ is what defines the active management region – the principal induces a negative drift.

Numerically, it is always the case that $z_R < \frac{1}{r+\rho}$, which implies that h_t is less than the level it would be with observable human capital. However, we have been unable to prove the result. Intuitively, it is clear: $z = \frac{1}{r+\rho}$ maximizes the principal's value (2.26) if we ignore the quadratic risk-sharing cost to using cash flow sensitivity. This cost can only induce the principal to use less cash flow sensitivity overall, which means that he will stop the evolution of z before reaching $\frac{1}{r+\rho}$.

2.4.4 *Interpreting the HJB Equation*

We can discuss the optimal β and ϕ using equation (2.28). The optimal level of β is determined by maximizing

$$-\frac{\gamma r\sigma^2}{2}\beta^2 + \gamma r\sigma^2\phi v'(z)z\beta - v'(z)\beta. \tag{2.32}$$

$-\frac{\gamma r\sigma^2}{2}\beta^2$ is the cost for exposing the agent to uncertainty. $\gamma r\sigma^2\phi v'(z)z\beta$ represents the marginal effect of β on $v(z)$ through the growth rate of z. This effect comes from Ito's lemma. $-v'(z)\beta$ represents the marginal effect on $v(z)$ by directly extracting β from z. This effect is straightforward since z serves as the storage account for β. Hence, the optimal β depends on the slope of $v(z)$. When $v'(z)$ is positive, z is relatively low. Given that the agent faces a quadratic cost of labor, incentivizing him to work harder is beneficial to the principal. At this point, invoking β is very costly since all three terms are negative. Therefore, setting $\beta = 0$ when $v'(z)$ is positive is the optimal choice since it allows z to grow at the maximum speed. On the other hand, when $v'(z)$ is negative, z is relatively high. The cost for compensating the agent for exerting effort starts to out weight the benefit. This is again due to the quadratic nature of the cost function of labor. At this point, using a positive amount of β is optimal. Since the level of z is too high, using β can slow the growth of z through the two forces: discount rate and direct accumulation. By trading off the cost $-\frac{\gamma r\sigma^2}{2}\beta^2$ and benefit $\gamma r\sigma^2\phi v'(z)z\beta - v'(z)\beta$, the principal determines the optimal level of β.

The optimal ϕ maximizes

$$\gamma r\sigma^2\beta v'(z)z\phi + \frac{\sigma^2}{2}v''(z)z^2\phi^2. \tag{2.33}$$

When $v'(z)$ is positive, and β takes the value of 0, it is optimal to choose $\phi = 0$ because it helps z grow at the maximum speed. When $v'(z)$ is negative, invoking z creates two effects. First, it reduces the growth rate of z, and its marginal benefit is indicated by $\gamma r\sigma^2\beta v'(z)z$. Second, subjecting z to the Brownian shock is costly to the principal. Therefore, the principal picks ϕ by equating the marginal benefit and marginal cost.

In summary, the principal manages incentive provision over time. When z is low, the marginal benefit of exerting effort is low. Since z grows over time, the principal expects the agent to work harder in the future. In order to accelerate the process of inducing higher future effort, the principal mutes the pay performance sensitivity now, i.e., $\beta = 0$. ϕ affects the accumulation of z through the discount rate and the Brownian shock. When $\beta = 0$, the discount rate effect is absent. Therefore, implementing $\phi \neq 0$ is costly but does not have the benefit of influencing the growth of z.

When z is high, despite the large benefit of putting in effort, the cost of effort is also very high. Without any intervention from the principal, z will continue to grow, making the future situation for the principal even worse. To limit the growth of z, the principal would start using pay-performance sensitivity. And a higher level of z would require a higher level of β to deter future growth. Another way to help the principal slow down the growth of z is by setting $\phi < 0$ since a negative ϕ decreases the discount rate of z. Again, a higher level of z would require a more negative ϕ. By implementing $\phi < 0$, the principal makes $P^{\Delta H}$ and W negatively correlated. When a positive Brownian shock happens, $P^{\Delta H}$ decreases while W increases. Therefore, when z becomes higher, good performance reduces future incentives.

2.5 Conclusion

This paper studies the optimal contract when the agent exerts hidden effort to build productive, durable human capital. Since the agent's action has long-lasting impact on future cash flows, the principal uses the stock of short-term incentives as a state variable, in addition to the agent's continuation utility and the perceived level of human capital.

The optimal contract features two regions. When the stock of short-term incentives is low, the contract is deterministic. In this region, the principal grants no cash flow rights and lets the stock to accumulate at the fastest speed. This is because when the stock is low, using short-term incentives is particularly expensive because it reduces the accumulation of expertise in the future. However, once the stock reaches certain threshold, the principal starts to use short-term incentives and lets the stock evolve stochastically. The optimal level of short-term incentives is determined by the agent's risk aversion, cost of effort, and the current level of the stock. These forces create a non-linear profile of the short-term incentives. Moreover, the stock of future incentives has a negative loading on the cash flow shock. The negative loading curbs the growth of the stock when it gets too high. In short, good performance accelerates vesting of cash flow rights.

Appendix A

APPENDIX FOR FIRE SALE SPILLOVERS IN THE CORPORATE BOND MARKET

Figure A.1: Classfication method based on simple cross-sectional sorts on day 0 inventory levels. This figure plots the cumulative order flow (inventory) of excluded bonds over the [-100,100] event window. The red line with dots represents the average inventory of dealers in the top quintile of inventory. The blue line with triangles represents the average inventory of dealers in the bottom four quintiles. At the end of each month (exclusion day), for each dealer, I calculate the cumulative order flow of excluded bonds from 100 trading days prior to 100 trading days after, setting the initial value at 0. On each event day, the averages are taken within each group and across exclusion events.

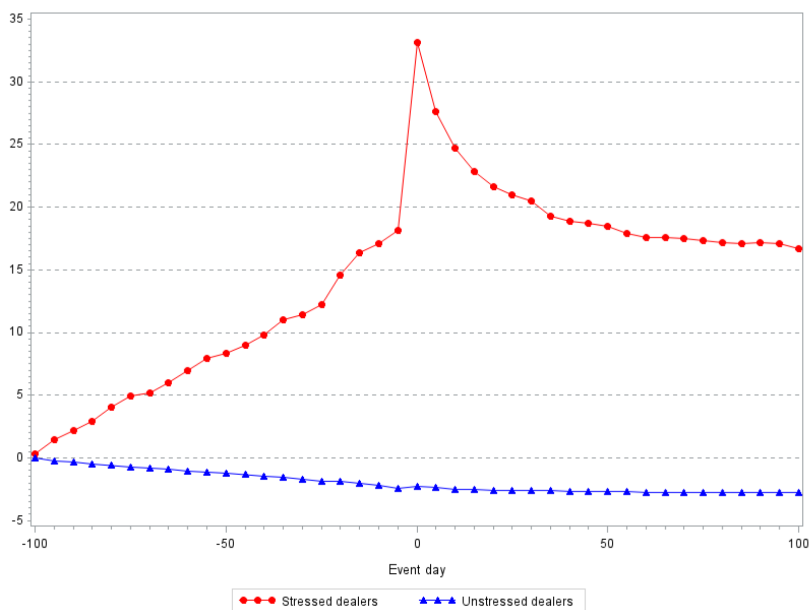


Table A.1: Asset under management (in $Billions) of mutual funds and ETFs that track the Bloomberg Barclays Indexes.

Fund family	Fund name	AUM	Benchmark
Vanguard	Short-Term Corporate Bond Index Fund Institutional Shares	47.9	Bloomberg Barclays U.S. 1-5 Year Corporate Bond Index
Vanguard	Intermediate-Term Corporate Bond Index Fund Institutional Shares	48.3	Bloomberg Barclays U.S. 5-10 Year Corporate Bond Index
Vanguard	Total Corporate Bond ETF	0.72	Bloomberg Barclays U.S. Corporate Bond Index
Vanguard	Long-Term Corporate Bond Index Fund Institutional Shares	6	Bloomberg Barclays U.S. 10+ Year Corporate Bond Index
Charles Schwab	Schwab 1-5 Year Corporate Bond ETF	0.63	Bloomberg Barclays US 1-5 YearCorporate Bond Index
Charles Schwab	Schwab 5-10 Year Corporate Bond ETF	0.35	Bloomberg Barclays US 510 YearCorporate Bond Index
BlackRock	iShares iBonds Dec 2023 Term Corporate ETF	1.59	Bloomberg Barclays December 2023 Maturity Corporate Index
BlackRock	iShares iBonds Dec 2022 Term Corporate ETF	1.57	Bloomberg Barclays December 2022 Maturity Corporate Index
BlackRock	iShares iBonds Dec 2024 Term Corporate ETF	1.43	Bloomberg Barclays December 2024 Maturity Corporate Index
BlackRock	iShares iBonds Dec 2021 Term Corporate ETF	1.39	Bloomberg Barclays December 2021 Maturity Corporate Index
BlackRock	iShares iBonds Dec 2025 Term Corporate ETF	1.13	Bloomberg Barclays December 2025 Maturity Corporate Index
BlackRock	iShares Aaa - A Rated Corporate Bond ETF	1.19	Bloomberg Barclays U.S. Corporate Aaa-A Capped Index
BlackRock	iShares iBonds Dec 2026 Term Corporate ETF	0.74	Bloomberg Barclays December 2026 Maturity Corporate Index
BlackRock	iShares iBonds Dec 2027 Term Corporate ETF	0.47	Bloomberg Barclays December 2027 Maturity Corporate Index
BlackRock	iShares iBonds Dec 2028 Term Corporate ETF	0.27	Bloomberg Barclays December 2028 Maturity Corporate Inde8
BlackRock	iShares iBonds Dec 2029 Term Corporate ETF	0.12	Bloomberg Barclays December 2029 Maturity Corporate Index
BlackRock	iShares iBonds Dec 2030 Term Corporate ETF	0.09	Bloomberg Barclays December 2030 Maturity Corporate Index
Total		113.89	

Table A.2: Filtering procedures and number of bonds and trades remaining after each step.

Filter	Cusips	Trades
Apply Dick-Nielsen and Poulsen (2019) filter to account for cancellations, corrections, and reversals	115585	171424364
Duplicate locked-in trades	115585	175490503
Delete wash trades	115585	175490134
Delete trades with anomalous price (<10 or >300)	114892	175186454
Merge with FISD, keep USD denominated bonds, and keep industrial, financial, and utility issuers	90172	162060925
Exclude trades with size greater than amount outstanding	89181	161819716
Exclude trades after maturity date	89100	161671993
Remove trades less than a month after issuance	84606	154962199
Delete trades with non-FINRA member affiliates	84054	146233204
Select bonds accroding to Bloomberg (2017, 2020): (corresponding FISD variables in parathesis)		
Fixed-rate coupon only (coupon type = F)	51067	139138300
Exclude perpetual bonds (perpetual = N)	51065	139136302
Exclude bonds with equity type features: unit deals, convertibles, exchangeables, and preferreds	47632	133977490
(unit deal = N, convertible = N, bond type $\neq$ CCOV, exhangeable = N, preferred security = N)		
Exclude private placements (private placement = N)	47597	133924800
Exclude retail bonds (bond type $\neq$ RNT)	30533	124536286
Exclude bonds with missing, 0, 25, or 50 par amounts (principal amt $\neq$ missing, 0, 25, or 50)	30477	124518178
Exclude pass-through certificates (bond type $\neq$ CPAS)	29947	124408743

Table A.3: Fama-MacBeth cross-sectional regressions of number of days apart between two price observations for calculating returns.

	1	2	3	4	5	6	7	8	9	10	11	12
$S\text{-}Expo^{SD}$	-0.001	0.001	0	0.001	0.005*	0.002	0.002	0.003	0.003	0	0	-0.029
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.038)
Rating	-0.001	0.001	0	0.001	0	0	0	0	0	0.001	0.002	0.012***
	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)
Maturity	0	0	0	0	0	0	0	0	0	0	0	-0.001*
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
ln(Size)	-0.006	0.004	-0.004	-0.003	-0.004	-0.003	-0.005	-0.001	-0.007	-0.009	0.005	0.339***
	(0.059)	(0.055)	(0.036)	(0.059)	(0.049)	(0.042)	(0.061)	(0.049)	(0.04)	(0.065)	(0.048)	(0.028)
Age	0	0.001	0	0	0	0.001	0.001	0.003	0.001	0	0.001	-0.006***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.002)
$\overline{PI}$	0	0	0	0	0	0	0	0	0	0	0	0.001***
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
Finance	0.001	-0.002	0.003	0	0	0.005	-0.003	0.002	0	0.006	0.004	-0.019**
	(0.007)	(0.008)	(0.008)	(0.008)	(0.008)	(0.007)	(0.008)	(0.008)	(0.007)	(0.009)	(0.009)	(0.008)
Utility	0.005	-0.012	0.003	-0.005	-0.004	0.002	-0.007	-0.004	0.005	0.004	-0.001	-0.07***
	(0.016)	(0.016)	(0.013)	(0.017)	(0.016)	(0.014)	(0.017)	(0.015)	(0.015)	(0.018)	(0.016)	(0.014)

Table A.4: Letter to numeric rating conversion.

S & P		Moody's		Fitch	
Letter	Numeric	Letter	Numeric	Letter	Numeric
AAA	1	Aaa	1	AAA	1
AA+	2	Aa1	2	AA+	2
AA	3	Aa	3	AA	3
AA-	4	Aa2	3	AA-	4
A+	5	Aa3	4	A+	5
A	6	A1	5	A	6
A-	7	A	6	A-	7
BBB+	8	A2	6	BBB+	8
BBB	9	A3	7	BBB	9
BBB-	10	Baa1	8	BBB-	10
BB+	11	Baa	9	BB+	11
BB	12	Baa2	9	BB	12
BB-	13	Baa3	10	BB-	13
B+	14	Ba1	11	B+	14
B	15	Ba	12	B	15
B-	16	Ba2	12	B-	16
CCC+	17	Ba3	13	CCC+	17
CCC	18	B1	14	CCC	18
CCC-	19	B	15	CCC-	19
CC	20	B2	15	CC	20
C	21	B3	16	C	21
		Caa1	17		
		Caa	18		
		Caa2	18		
		Caa3	19		
		Ca	20		
		C	21		

Appendix B

APPENDIX FOR DYNAMIC CONTRACTS WITH HUMAN CAPITAL ACCUMULATION

B.1 *Proof of Proposition 2, Necessity*

We characterize the agent's choices as deviations from the principal's recommended values: $\Delta H = H - \hat{H}$, $\Delta S = S - \hat{S}$, $\Delta c = c - \hat{c}$, and $\Delta h = h - \hat{h}$. Using (2.2), (2.3), (2.5), and (2.6) we have

$$d\Delta H_t = (\Delta h_t - \rho \Delta H_t)dt \tag{B.1}$$

$$d\Delta S_t = (r\Delta S_t - \Delta c_t)dt, \tag{B.2}$$

Then, the cash flow process evolves according to

$$dX_t = \hat{H}_t dt + \sigma d\hat{Z}_t = H_t dt + \sigma \underbrace{(d\hat{Z}_t - \frac{\Delta H_t}{\sigma} dt)}_{dZ_t}. \tag{B.3}$$

We define the Radon-Nikodym derivative Γ between $\mathcal{P}$ and $\hat{\mathcal{P}}$:

$$\Gamma_t = \exp\left(-\frac{1}{2}\int_0^t (\frac{\Delta H_s}{\sigma})^2 ds + \int_0^t \frac{\Delta H_s}{\sigma} d\hat{Z}_s\right), \tag{B.4}$$

$$d\Gamma_t = \frac{\Delta H_t}{\sigma} \Gamma_t d\hat{Z}_t. \tag{B.5}$$

This allows us to write the agent's objective function (2.7) as

$$W_t(\mathcal{C}) = \max_{c,h} \; E_t^{c,h}\left[\int_t^\infty e^{-r(s-t)}u(c_s,h_s)ds\right] \tag{B.6}$$

$$= \max_{\Delta c,\Delta h} \; E_t^{\hat{c},\hat{h}}\left[\int_t^\infty e^{-r(s-t)}\Gamma_s u(\hat{c}_s+\Delta c_s,\hat{h}_s+\Delta h_s)dt\right] \tag{B.7}$$

subject to the state variables (B.1), (B.2), and (B.5).

We set up the Stochastic Hamiltonian (see, e.g., [49]) as

$$\begin{aligned}\mathcal{H}(\Delta c,\Delta h,\Gamma,\Delta H,\Delta S,P,Q) =& \Gamma u(\hat{c}+\Delta c,\hat{h}+\Delta h) + Q^\Gamma\frac{\Delta H}{\sigma}\Gamma \\ &+ P^{\Delta H}(\Delta h-\rho\Delta H) + P^{\Delta S}(r\Delta S-\Delta c).\end{aligned} \tag{B.8}$$

Taking first order conditions and evaluating along the optimal path ($\Delta c^* = \Delta h^* = 0$, $\Gamma^* = 1$, and $\Delta H^* = \Delta S^* = 0$) gives

$$[\Delta c] \qquad -\gamma u(\hat{c},\hat{h}) = P^{\Delta S} \tag{B.9}$$

$$[\Delta h] \qquad \theta\gamma\hat{h}u(\hat{c},\hat{h}) = -P^{\Delta H}, \tag{B.10}$$

while the adjoint processes evolve as

$$dP_t^\Gamma = \left(rP_t^\Gamma - u(\hat{c}_t,\hat{h}_t)\right)dt + Q_t^\Gamma d\hat{Z}_t \tag{B.11}$$

$$dP_t^{\Delta H} = \left((r+\rho)P_t^{\Delta H} - \frac{q_t^\Gamma}{\sigma}\right)dt + Q_t^{\Delta H}d\hat{Z}_t \tag{B.12}$$

$$dP_t^{\Delta S} = Q_t^{\Delta S}d\hat{Z}_t. \tag{B.13}$$

Then, integrating (B.11) recovers the promised continuation utility:

$$\hat{W}_t = P_t^{\Gamma} = E_t\left[\int_t^{\infty} e^{-r(s-t)} u(\hat{c}_s, \hat{h}_s) ds\right], \tag{B.14}$$

Similarly, integrating (B.12), we obtain the stock of future cash flow sensitivities,

$$P_t^{\Delta H} = E_t\left[\int_t^{\infty} e^{-(r+\rho)(s-t)} \frac{q_s^{\Gamma}}{\sigma} ds\right]. \tag{B.15}$$

Combining (B.13) and (B.9) shows that flow utility $u(\hat{c}, \hat{h})$ is a martingale. (B.14) shows that promised continuation utility $\hat{W}$ must also be a martingale, and (B.11) implies $u(\hat{c}_t, \hat{h}_t) = r\hat{W}_t$, (2.18). Similarly, (B.10) implies $\theta\gamma r\hat{h}\hat{W}_t = -P^{\Delta H}$, (2.19). By redefining $Q^{\Gamma} = -\sigma\gamma r\hat{W}\beta$ and $Q^{\Delta H} = \sigma P^{\Delta H}\eta$, we have (2.14) and (2.16).

B.2 Proof of Proposition 2, Sufficiency

To prove sufficiency we follow [39] in constructing an upper bound for the payoff after deviation $(\Delta S, \Delta H)$. If such an upper bound exists, then the Stochastic Hamiltonian demonstrates a global optimum rather than a local optimum. We construct an upper bound of the form

$$F(W, \Delta S, \Delta H, z) = W \exp\left(-\gamma r\Delta S - \gamma r z\Delta H - L_1\Delta H^2\right), \tag{B.1}$$

where L_1 is a constant and W is the agent's true (as opposed to promised) continuation utility. Remembering the change of measure Γ, (B.5), the agent faces the state variables:

$$d\Delta S_t = (r\Delta S_t - \Delta c_t)dt \tag{B.2}$$

$$d\Delta H_t = (\Delta h_t - \rho\Delta H_t)dt \tag{B.3}$$

$$dW_t = -\gamma r W_t \beta_t \Delta H_t dt - \sigma\gamma r W_t \beta_t dZ_t \tag{B.4}$$

$$dz_t = \left(\left(r + \rho + \sigma^2\gamma r\beta_t\phi_t + \phi_t\Delta H_t\right) z_t - \beta_t\right) dt + \sigma\phi_t z_t dZ_t. \tag{B.5}$$

We define

$$G_{t_0,t} = \int_{t_0}^{t} e^{-r(s-t_0)} u(\hat{c}_s + \Delta c_s, \hat{h}_s + \Delta h_s) ds + e^{-r(t-t_0)} F(W_t, \Delta S_t, \Delta H_t, z_t). \tag{B.6}$$

Moreover,

$$u(\hat{c} + \Delta c, \hat{h} + \Delta h) = rW \exp\left(-\gamma\Delta c + \frac{\gamma\theta}{2}\Delta h^2 + \gamma z\Delta h\right). \tag{B.7}$$

We differentiate $G_{t_0,t}$ to obtain the drift term

$$e^{r(t-t_0)} E[dG_{t_0,t}] = u(\hat{c}_t + \Delta c_t, \hat{h}_t + \Delta h_t) + \mu_t^F F(W_t, \Delta S_t, \Delta H_t, z_t) - rF(W_t, \Delta S_t, \Delta H_t, z_t), \tag{B.8}$$

where

$$\begin{aligned}\mu^F = & -\gamma r\beta\Delta H - r\gamma(r\Delta S - \Delta c) - (\gamma r z + 2L_1\Delta H)(\Delta h - \rho\Delta H) \\ & -\gamma r\Delta H\left(\left(r + \rho + \sigma^2\gamma r\beta\phi + \phi\Delta H\right)z - \beta\right)\right) + \frac{\sigma^2\gamma^2 r^2}{2}\Delta H^2\phi^2 z^2 + \sigma^2\gamma^2 r^2\Delta H\beta\phi z.\end{aligned}$$

We want to show that the drift of dG is negative, which is sufficient to show that F is an upper bound. To do so, we'll first pull out an F term to simplify. $e^{r(t-t_0)}E[dG_{t_0,t}]$ can be written as

$$F\left[r\exp\left\{\gamma r\Delta S+\gamma rz\Delta H+L_1\Delta H^2-\gamma\Delta c+\frac{\gamma\theta}{2}\Delta h^2+\gamma z\Delta h\right\}+\mu^F-r\right] \tag{B.9}$$

Because $F<0$, we need to show that

$$r\exp\left\{r\gamma\Delta S+\gamma rz\Delta H+L_1\Delta H^2-\gamma\Delta c+\frac{\gamma\theta}{2}\Delta h^2+\gamma z\Delta h\right\}+\mu^F-r\geq 0. \tag{B.10}$$

Substituting in for μ^F, we want

$$\begin{aligned}&r\exp\left\{r\gamma\Delta S+\gamma rz\Delta H+L_1\Delta H^2-\gamma\Delta c+\frac{\gamma\theta}{2}\Delta h^2+\gamma z\Delta h\right\}-r-\gamma r\beta\Delta H\\&-r\gamma(r\Delta S-\Delta c)-(\gamma rz+2L_1\Delta H)(\Delta h-\rho\Delta H)-\gamma r\Delta H\left((r+\rho+\sigma^2\gamma r\beta\phi+\phi\Delta H)z-\beta\right)\\&+\frac{\sigma^2\gamma^2r^2}{2}\Delta H^2\phi^2z^2+\sigma^2\gamma^2r^2\Delta H\beta\phi z\geq 0.\end{aligned} \tag{B.11}$$

Using the inequality $e^x-1\geq x$, it is sufficient to show that

$$\begin{aligned}&\gamma r^2\Delta S+\gamma r^2z\Delta H+rL_1\Delta H^2-\gamma r\Delta c+\frac{\gamma r\theta}{2}\Delta h^2+\gamma rz\Delta h-\gamma r\beta\Delta H\\&-r\gamma(r\Delta S-\Delta c)-(\gamma rz+2L_1\Delta H)(\Delta h-\rho\Delta H)-\gamma r\Delta H\left((r+\rho+\sigma^2\gamma r\beta\phi+\phi\Delta H)z-\beta\right)\\&+\frac{\sigma^2\gamma^2r^2}{2}\Delta H^2\phi^2z^2+\sigma^2\gamma^2r^2\Delta H\beta\phi z\geq 0.\end{aligned} \tag{B.12}$$

which can be simplified to

$$rL_1\Delta H^2+\frac{\gamma r\theta}{2}\Delta h^2-2L_1\Delta H\Delta h+2\rho L_1\Delta H^2-\gamma r\phi z\Delta H^2+\frac{\sigma^2\gamma^2r^2}{2}\phi^2z^2\Delta H^2\geq 0. \tag{B.13}$$

Taking the minimum with respect to Δh yields $\Delta h = \frac{2L_1}{\gamma r\theta}\Delta H$. Substituting that in yields

$$rL_1\Delta H^2 - \frac{2L_1^2}{\gamma r\theta}\Delta H^2 + 2\rho L_1\Delta H^2 - \gamma r\phi z\Delta H^2 + \frac{\sigma^2\gamma^2 r^2}{2}\phi^2 z^2\Delta H^2 \geq 0 \tag{B.14}$$

ΔH^2 is comon to all terms; cancelling, we obtain a quadratic function of L_1:

$$-\frac{2}{\gamma r\theta}L_1^2 + (r+2\rho)L_1 + \frac{\sigma^2\gamma^2 r^2}{2}z^2\phi^2 - \gamma r z\phi \geq 0. \tag{B.15}$$

From the principal's problem, we have $\phi \leq 0$, which implies there is a value of L_1 that obtains the desired result.

B.3 *Proof of Proposition 3*

The necessary conditions for Proposition 3 leading to the HJB equation are standard from dynamic programming. We include here additional necessary details and a sketch of what would be required for sufficiency.

Because $v_{stop}(z)$ uses fixed policies, it must be the case that $v(z) \geq v_{stop}(z)$. The two functions are equal (to 0) at $z = 0$, which is also the minimum value of z, so $v'(0) \geq v'_{stop}(0) > 0$. Inspection of the HJB equation yields $\beta = \phi = 0$ if $v'(z) \geq 0$, and the HJB equation becomes

$$-rv(z) + \frac{1}{\theta(r+\rho)}z - \frac{1}{2\theta}z^2 + (r+\rho)v'(z)z = 0. \tag{B.1}$$

Thus, $v'(z) > (=)0$ implies $rv(z) > (=)\frac{1}{\theta(r+\rho)}z - \frac{1}{2\theta}z^2$. This implies $\frac{1}{\theta(r+\rho)}z - \frac{1}{2\theta}z^2$ is strictly increasing and passes through $v(z)$ from below at z^*. Taking the derivative of (B.1) yields

$$(r+\rho)v'(z)z = rv(z) - \left[\frac{1}{\theta(r+\rho)}z - \frac{1}{2\theta}z^2\right] \tag{B.2}$$

$$(r+\rho)v''(z)z = -\rho v'(z) - \left[\frac{1}{\theta(r+\rho)}z - \frac{1}{2\theta}z^2\right]' \tag{B.3}$$

Since the term in square brackets is strictly increasing, $v'(z) \geq 0$ implies $v''(z) < 0$. Thus $v(z)$ is increasing and strictly concave for $z \leq z*$. Moreover, The ODE in (B.1) is first-order and can be solved analytically up to a constant.

We now consider the $z \geq z^*$ region. A solution to $v(z)$ exists everywhere by the Peano existence theorem. We have existence and uniqueness everywhere except in a right neighborhood around z^* where $v'(z^*) = 0$, from Lipschitz continuity and the Picard-Lindelof Theorem.

For sufficiency we would require

- A proof that $v(z)$ as described is concave on $z >\geq z^*$.
- A verification theorem that the agent cannot do better that $v_{stop}(z_R)$ at z_R.
- A proof that the solution to the HJB equation is unique at $v(z)$.

The final point can be dealt with through a limit. The problem for ODE uniqueness is that in the neighborhood around z^*, we have $\frac{1}{\phi^2(z)} \approx \frac{1}{(v'(z))^4}$ is not integrable because $\phi = 0$ at z^*. We can make an economically innocuous and mathematically simplifying assumption that ϕ cannot take values in $(0, \epsilon)$ for some $\epsilon > 0$. We are asserting that the principal cannot give the agent an incentive volatility that is between some arbitrarily small number and zero. So, for example, if $\epsilon = 10^{-45}$, then incentive volatility cannot be less than 10^{-45} without being zero. This is an economically meaningless restriction. Yet, it allows us to use standard theorems to prove the classical uniqueness of the ODE instead of resorting to viscosity solutions or other methods. In particular, this gives us Lipschitz continuity everywhere, and therefore existence and uniqueness.

With concavity, uniqueness, and the welfare condition, we would have sufficiency as well as necessity for the conditions of the proposition.

B.4 Proof of Proposition 1

This is a corollary of the Proof of Proposition 2 in Sections B.1 and B.2. Because H is observable, any h can be implemented with $\beta = 0$. Thus $r\hat{W} = u(\hat{c}, \hat{h})$ and $\hat{W}$ is constant. With $\beta = 0$, (2.25) and (2.26) can be maximized point-wise to obtain (2.11). Using $r\hat{W} = u(\hat{c}, \hat{h})$ yields (2.10). Because c is constant, the same value would be chosen if S were observable as well.

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