Neutron Electric Dipole Moment From QCD Sum Rules

by

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Abstract

Neutron Electric Dipole Moment From QCD Sum Rules

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The electric dipole moments of nucleons (NEDM, d_N) are calculated using the method of QCD sum rules. Our calculations are based on the parity ($\not{P}$) and time reversal ($\not{T}$) violating parameter $\bar{\theta}$ in QCD and establish a functional dependence of the NEDM on $\bar{\theta}$, without assuming a perturbative expansion of this symmetry breaking parameter. The results obtained from the QCD sum rules approach are shown to be consistent with the general symmetry constraints on $\not{P}$ and $\not{T}$ in QCD; including the necessity of quark masses, spontaneous chiral symmetry breaking, and the $U_A(1)$ anomaly. Given the current experimental upper bound on the neutron electric dipole moment (nEDM), $d_n \leq 10^{-26} e \cdot cm$, we find $\bar{\theta} \leq 10^{-10}$. This is compatible with previous calculations of nEDM using different techniques.

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to say thank you all and I promise I will be back soon.

DEDICATION

TO ALL MY TEACHERS IN LIFE

Chapter 1

INTRODUCTION — PHYSICAL ASPECTS OF THE NEUTRON ELECTRIC DIPOLE MOMENT (NEDM)

1.1 Discrete Symmetries P, T and C — Their Realizations and Violations in Nature

· **Symmetries and physics**

Symmetry, or the principle of invariance under symmetry transformations, has been shown to be one of the most important concepts in physics. For example, many conservation laws in physics (e.g. momentum conservation, angular momentum conservation etc.) can be related to the invariance of the underlying dynamical theories under symmetry operations (e.g. translations, rotations etc.) through Noether's theorem [1]; Einstein's theory of relativity is based on the concept of Lorentz invariance, which is a symmetry of space – time coordinate transformations; finally, the concept of phase invariance in quantum mechanics and the principle of locality give rise to gauge symmetries, which are generalized to a gauge principle and generate the interactions of elementary particles in the subnuclear world.

The use of symmetry not only provides a framework for the construction of the physical laws in nature[1] it also deepens our understanding for the implications of a physical theory. The classifications of spectroscopies in the microscopic world, ranging from atoms, molecules, nuclei to hadrons, could not achieve such an impressive success without the study of the symmetry groups of the underlying dynamics. Furthermore, the selection rules for the transtion amplitudes in quantum mechanics generalizes the concept of conservation laws in classical mechanics, and is used as a test of symmetries in a given physical system and the associated theories.

In view of these accomplishments brought out by the symmetry principle, we

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[1] Two promoninent examples are: the construction of a gauge theory of standard model for elementary particles, and the chiral perturbation theory based on the chiral symmetry of QCD.

should also emphasize that symmetry, by itself, may not always be a sacred law of nature. Some symmetries are exact, space – time transformations and gauge symmetries being such examples. Some are approximate, e.g. isospin symmetry (or isosymmetry) in nuclear physics, and the violations of, or departure from a symmetrical theory can be (and/or has been) observed in nature or measured in experiments. On the other hand, while some symmetries are explictly broken in certain theories (e.g. isosymmetry in nuclear physics) , and the effects of symmetry breakings can be modeled by some symmetry breaking parameters (e.g. the mass difference between u and d quarks); other symmetries are broken spontaneously (e.g. chiral symmetry in QCD) and the nature of symmetry breakings often involves nonperturbative dynamics[2]. Moreover, it is possible to have some connections between these two types of symmetry breaking, and therefore, they may not be totally independent and unrelated[3]. Thus, complimentary to the effort of unification, where the hope is that all known fundamental interactions of nature can be described by a single theory via a symmetry principle, the study of symmetry breakings — the patterns, the cause – effect relationship — provides another route to a deeper understanding of the physical laws in nature.

· **The classifications of symmetries**

The symmetries in physics can be classified according to three different schemes:

In the first scheme, the symmetries are classified as (1) **continuous** or (2) **discrete**, depending on the structures of the symmetry groups. Continuous groups have their elements built up from (infinitesmal) generators around the identity, and can be parametrized by a set of continuous variables. A discrete group, on the other hand, contains only finite numbers of group elements.

In the second scheme, the symmetries are classified as (1) **external (or space–time)** or (2) **internal**, depending on the spaces which symmetry operations act on. External symmetries are transformations of space – time coordinates (e.g. translation, rotations, boosts) and field variables defined on the space – time continuum

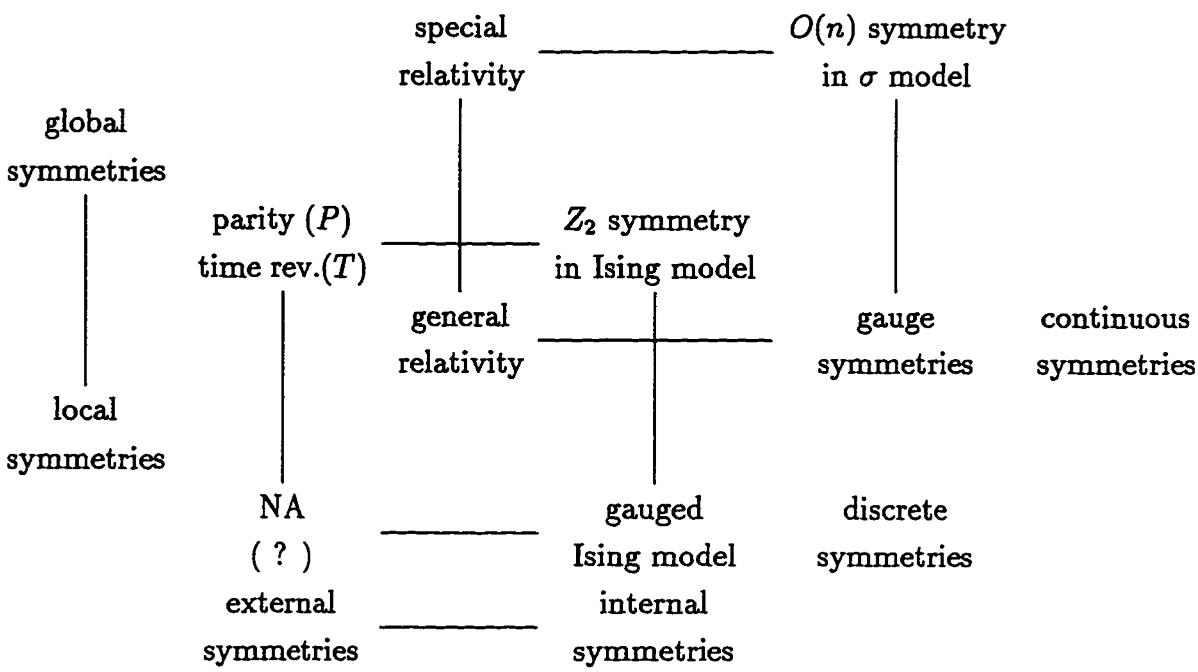
[2] Such a pattern is often required to circument many intrinsic difficulties in a given theory (e.g. The chiral symmetry breaking in QCD is necessary to explain the masses of nucleons consisting of nearly massless quarks and gluons.).

[3] For example, in some model, it is possible to generate effective interactions, which explictly break certain symmetry, from a more "fundamental" theory in which the symmetry is broken spontaneously. See [2].

(e.g. scalars, vectors, tensors). In contrast, internal symmetries consist of transformations on the internal degrees of freedom (e.g. charge, color etc.) which are independent of spce–time coordinates.

Finally, in the third scheme, the symmetries are classified as (1) **global**,when the symmetry transformations act uniformly throught the space – time continuum; or, (2) **local**, where the symmetry operations can have a space – time dependence (it is also called a gauge symmetry)

We can summarize the classifications of symmetries, together with examples, in the following diagram:



We shall focus on the **discrete space–time symmetries** in this work. These include: (1) **parity**, or **space inversion** (P), where the spatial coordinates in a reference frame are transformed into their negatives, (2) **time reversal** (T), where the time axis is changed from $+t$ to $-t$, (3) **charge conjugation** (C), where a particle is changed into its antiparticle[4]. These symmetries were believed to be exact in the fundamental physical theories. However, it turns out that this is not the case.

[4] The reason why we include charge conjugation into the space – time symmetries will be explained in the next chapter.

In particular, we now find that these three symmetries are broken in the subnuclear world[5].

There is no ad hoc reason why the physical laws of nature should respect the discrete symmetries P, T, and C. There is no analogy to the conservation law of momentum, which physicists cherish so much and put to test so severely, that supports our belief in translational invariance. The equivalent conservation laws (in quantum mechanics)— selection rules associated with these discrete symmetries are often broken, as many physical states are not invariant under these discrete symmetry transformations and experimentlly we can detect some rare transitions which are forbidden if these discrete symmetries are exact. However, it is much easier to explain the effects of symmetry breaking for a specific model instead of trying to answer why the symmetry is not there? Consequently, up to present, the research interest in the studies of discrete symmetry mainly lie in two directions: for experimentalists, it is necessary to test the symmetries by measuring various physical observables (e.g. rare or forbidden transition, asymmetry cross sections, some intrinsic quantities related to $\not{P}$, $\not{T}$, like EDMs) and quantify these measurements in a systematic way; for theorists, it is desirable to develop the most economical model (with largest output-input ratio) and give quantitative explinations/predictions to various phenomena consistent with experimental results.

1.2 EDM as a Test of Discrete Symmetry Breaking

In order to test how well the discrete symmetries are realized (or how badly they are broken) in the subnuclear world, we need to look for some observable quantities or physical processes which indicate the violations of or departure from the discrete symmetries. The system we choose to look at has to be simple enough (such that we do not have to deal with many substructures and interactions) and well-understood (such that it is possible to isolate the symmetry breaking physics from other complications). As Purcell and Ramsey [3] pointed out, the possible existence of neutron electric dipole moment (nEDM) would be such an example. The existence of a nEDM implies that both parity (P) and time reversal (T) symmetries are broken.

[5] The statistical mechanics foundations of irreversibility, namely time reversal violation at the macroscopic level, will not be considered in this work.

To see this, we can write down an interaction Hamitonian H_{int} of a nEDM (denoted as $\vec{d}_n$) coupled to an external electric field (denoted by $\vec{E}$) as follows:

$$H_{int} \equiv \vec{d}_n \cdot \vec{E}$$

For an elementary particle like the neutron the only intrinsic vector is spin ($\vec{s}_n$). Therefore, the EDM $\vec{d}_n$ has to be proportional to the spin $\vec{s}_n$ and carries the same transformation properties under P and T. In particular, $\vec{d}_n$ is **even** under the **parity** transformation $\mathcal{P}\vec{d}_n\mathcal{P}^{-1} = \vec{d}_n$ and **odd** under the **time reversal** transformation $\mathcal{T}\vec{d}_N\mathcal{T}^{-1} = -\vec{d}_N$. This, along with the fact that the electric field $\vec{E}$ is odd under P but is even under T, leads to an interaction Hamitonian which breaks both P and T symmetries. Thus, we conclude that a nEDM is an indicator for $\not{P}$ and $\not{T}$ in the subnuclear world.

It is to be emphasised that the electric dipole moments we are discussing are the intrinsic EDMs associated with the elementary particles, which is of purely quantum mechanical nature and should be distingushed from the polarization observables (associated with the distoration of charge distribution) induced by an external field. The former, due to the fact that nucleons are spin 1/2 particles, is the only $\not{P}$ and $\not{T}$ quantum operator we can put in the interaction Hamitonian of nucleons with EM fields[6]. The latter, on the other hand, are much more complicated and of their own interest, both experimentally and theroretically, see [4]. Basically, these effects are due to the modifications of the internal structures of particles under the influence of external fields (so that the combined system; particles plus external field satisfies the Maxwell equations) and no restriction on the pattern of multipole expansions can be made (e.g. an induced quadrupole moment of the nucleon is allowed).

[6] There is no instrinsic quadrupole moment (spin 2) of nucleons, according to the addition rules of angular momentums in quantum mechanics, or in a generalized case, Wigner–Eckart theorem. The generalization to higher spin particles with higher intrinsic $\not{P}$ and $\not{T}$ moments will not be included in the present work.

1.3 Experimental Measurements of nEDM (Review)

At present, the measurement of a neutron[7] electric dipole moment is the most sensitive experimental test of the time reversal violation. However, measurements of atomic EDMs have almost reached the same level of accuracy. The continuing progress in this field has improved the precision by a factor of 10^6 since the early pioneer work by Smith, Purcell, and Ramsey[5]. There are several methods proposed to measure a nEDM, e.g. neutron interferometry, neutron scattering in a strong crystalline electric field. Today the best accuracy is achieved by the combination of nuclear resonace method and the use of ultracold neutron (UCN) technique [6], as done by two major groups; one at the LNPI in Gatchina/Leningrad [7], and the other at ILL in Grenoble [8]. We shall briefly review the basic ideas underlying this experiment.

- **nEDM as measured from nuclear resonace method**

If P and T are broken, the interaction energy of a neutron in the presence of external EM fields $(\vec{E}, \vec{B})$, $\vec{E} \parallel \vec{B}$ is given by

$$H = -(\vec{\mu}_n \cdot \vec{B}) - (\vec{d}_n \cdot \vec{E}) \tag{1.1}$$

In quantum mechanics, there are two energy levels of the interaction Hamitonian (1.1), corresponding to the two possible spin states of the neutron.

To isolate the contribution of a nEDM to the interaction energy, we reverse the direction of the external electric field $\vec{E}$ and keep the direction of the external magnetic field $\vec{B}$ fixed. The energy difference between these two configurations is proportional to the nEDM

$$\Delta H = -2(\vec{d}_n \cdot \vec{E}) \tag{1.2}$$

It is to be noticed that the nEDM does not change sign under the reversal of the external electric field $\vec{E}$. Also, any induced polarization (e.g. quadratic Stark effect) cancels under such a field reversal.

To measure the energy difference between the two spin states of neutrons, we can employ the technique of nuclear magnetic resonance (NMR) [9]. That is, an

[7] Because of the charge neutral property and a relatively long life time, neutron provides a unique chance to the EDM measurements of the elementary particles.

oscillating magnetic field perpendicular to the constant $(\vec{E}, \vec{B})$ fields is introduced, such that a spin-flip transition between two spin eigenstates is possible. the apparatus is shown in Fig. 1.1. A resonance happens when the frequency of the oscillating magnetic field (ω) is equal to the spin precession frequency

$$\omega_0 \equiv \frac{H(\uparrow) - H(\downarrow)}{\hbar} = \frac{2(\mu_n B + d_n E)}{\hbar} \tag{1.3}$$

and an initially spin–up ($\uparrow$) population is converted into a spin–down ($\downarrow$) one. In the final detector of the outgoing neutron beam, which counts the neutron with a prescribed spin state, we would observe a decreased number of the spin–up neutrons in the resonance region, as shown in Fig.1.2.

Taking the difference of the resonance frequency for the two configurations (a constant magnetic field plus parallel or antiparallel constant electric field), we can thus isolate the contribution of the nEDM

$$\Delta\omega_0 \equiv \frac{4 d_n E}{\hbar} \tag{1.4}$$

The difference of resonance frequency $\Delta\omega_0$ is then related to the counting rate of the final neutron detector

$$\Delta N = -\frac{dN}{d\omega}\Delta\omega_0 \tag{1.5}$$

and the nEDM follows from the above equations as

$$\vec{d}_n = \frac{\hbar}{4}\frac{\Delta N}{E}\left(\frac{dN}{d\omega}\right)^{-1} \tag{1.6}$$

With the advent of the UCN (ultra cold neutron) technique, the measurement can achieve a high accuracy due to the ability of storing cold neutrons in a longer period, comparable with the lifetime of the neutron; the current best upper bound on the nEDM is set by this method. The latest publications report the result of the measurement from the LNPI group (1986),

$$d_n = -(14 \pm 6) \times 10^{-26} e - cm$$

, which implies an upper bound of the nEDM

$$|d_n| \leq 2.6 \times 10^{-25} e - cm$$

at the 95% c.l..

The result of the ILL group (1990)

$$d_n = -(3 \pm 5) \times 10^{-26} e - cm$$

, which implies an upper bound of the nEDM

$$|d_n| \leq 1.2 \times 10^{-25} e - cm$$

at the 95% c.l..

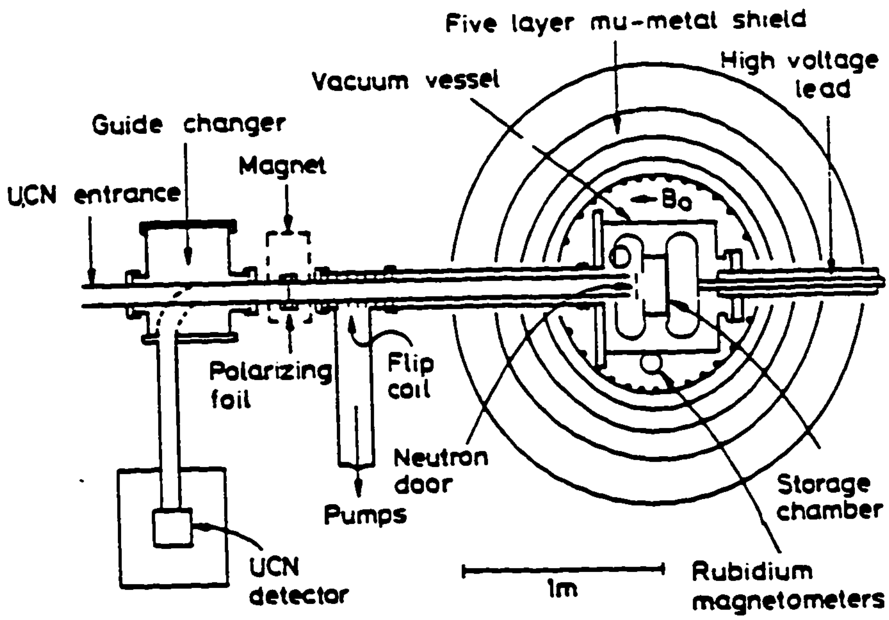


Figure 1.1: The experimental apparatus used at the ILL group to measure the nEDM

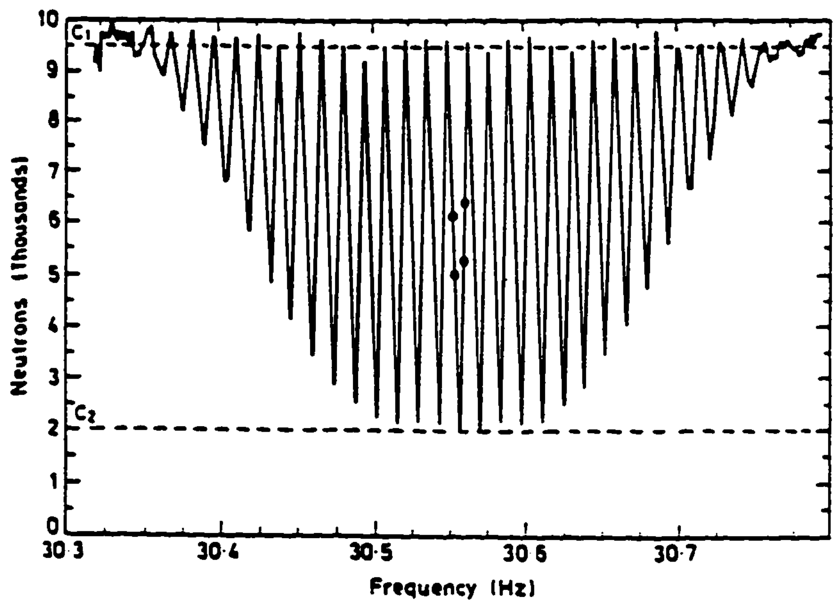


Figure 1.2: The resonance curve in the measurement the nEDM, the counting of neutrons as a function of the oscillating freqency

1.4 Theoretical Study of nEDM (Review)

The theoretical study of the nEDM only began seriously after the breakdown of P and C were discovered in 1956, and more so after the unexpected discovery of CP violation in the neutral kaon system.

The issue has become complicated by the knoeledge of the underlying theory, QCD, and the presence of its nontrival structure of the vacuum state. Baluni [10] first used the chiral rotation technique to transform the strong $\not{P}$ and $\not{T}$ gluon interaction (For more details, see Chap.2.) into a quark pseudo mass term and used the bag model to estimate the relevent matrix element. Crewther et al. [11] improved this approach by employing current algebra techniques and insertions of intermediate nucleon plus pion states to saturate the matrix element. There are also bag model calculation, as done by Morgan and Miller [12]. Generally speaking, these authers obtained similar results and using the current upper bound on the nEDM, the $\bar{\theta}$ parameter (defined in next section) is about 10^{-10} to 10^{-12}. Besides, these calculations are all consistent with the chiral limit constraint, that is the nEDM would vanish as the quark masses go to zero [34][3].

However, in addition to the curious mystery of a small $\bar{\theta}$ parameter in the fundamental theory, many connections of the nEDM problem to the special properties of QCD were discovered. First of all, the nonperturbative nature of the nEDM problem was first pointed out and discussed by Shabalin [14], and based on a σ model, which describes the interactions of nucleons and mesons, he was able to relate the nEDM to spontaneous chiral symmetry breaking. Secondly, Shifman [15] first explained the relation between the strong CP violation and the $U_A(1)$ problem and indicated the dependence of a nEDM to the anomalous gluon condensate $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$, which was later rediscovered by Aoki et al. [16]. These discoveries can be stated as symmetry constraints on the possible $\not{P}$ and $\not{T}$ in the strong interactions, which is the main subject of the next chapter.

Given the quantitative calculations (most of these were done in effective models) and the qualitative symmetry constraints discussed above, it is interesting to see whether these previous calculations are consistent with the symmetry constraints

[3] To be precise, only one massless quark is sufficient to have a zero nEDM. But many of the previous calculations failed to make such limit explictly.

(see sec.2.3). While it is possible to bring in the nonperturbative nature in the parameters of a given model, e.g. the pion mass, most calculations fail to satisfy the anomaly constraints [15] [16]. A recent calculation by H-Y Cheng [18], within the framework of chiral perturbation theory, reexamines these constraints and obtains an improved result which demonstrates all the symmetry constraints explictly. Still, a direct approach based on the QCD Lagrangian does not exist[9]. It is our intention to show that there exists a QCD–based calculation which is consistent with all symmetry constraints.

Because the strong together with the weak CP violation problem could be an indication of physics beyond standard model, there is vigorous research on this subject[19]. Here, our interest will be focused on the low energy realization of QCD. Therefore, our discussion of $\not{P}$ and $\not{T}$ will be limited to the standard model only. That is, without modifying the gauge structures and the particle contents of the theory, we will study the possible consequences of CP violation effect, NEDMs, from the symmetry breaking parameter of the QCD Lagrangian.

1.5 Motivation and Methodology

In this dissertation, we shall study the electric dipole moments of nucleons (NEDM, denoted as d_N), which serves as an indicator of both parity and time reversal symmetry breakings. The main focus is on the possible violations of P and T symmetries in the strong interactions[10], with Quantum Chromodymaics (QCD) as the underlying theory (so called strong CP problem).

In this picture, the nucleons (N) are treated as composite particles consisting of quarks (q) and gluons (G) and the strong P and T violating interaction in the QCD Lagrangian is characterized by a parameter $\bar{\theta}$ (See sec. 2.2 for the definition of this parameter.). Our purpose is to establish a functional dependence of the NEDM on the $\bar{\theta}$ parameter, along with other fundamental parameters of the QCD Lagrangian, e.g. the masses of quarks m_q and the values of quark condensates R_q.

[9] As will be clear in the next chapter, in order to perform a calculation of the nEDM on the lattice, we need to go beyond the quenched approximation.

[10] The possible violations of CP or T symmetries in the electroweak interactions will be briefly reviewed in sec.2.3.

The problem is difficult and interesting, since an analytical calculation of low energy hadronic observables based on QCD Lagrangian[11] is not a trivial task. Furthermore, it turns out that an important property[12] of QCD, namely — chiral symmetry, is closely related to strong CP problem and gives three stringent constraints on the possible breaking of P and T symmetries in QCD (See the discussions in Chap. 2). Thus, we need to face the challenge how to realize these constraints explictly in our calculation[13].

· **The method of QCD sum rules**

Indeed, we find the QCD sum rules [20] approach provides an adequate tool to attack this problem. With some nontrival generalizations of previous studies (e.g. by B.L.Ioffe and A.V.Smilga [21]) on the magnetic moments of nucleons [22] (denoted as μ_N), we are able to apply the same techniques to the calculations of the NEDMs[14] from the QCD Lagrangian.

The method of QCD sum rules, pioneered by Shifman, Vainstein and Sakrarov in 1979, is a semi-empirical approach to low energy hadron physics. The basic idea is a matching[15] between two representations - a hadronic model and a quark - gluon calculation - of some correlation functions at a certain energy scale. The region of matching is chosen such that we can perform reasonably reliable calculations in QCD on the one hand, and so that the correlation function we are interested is dominated by the physical observables of known low lying hadronic states (for details, see chap. 5), on the other. Once we obtain one or more relations (this is what we call **QCD sum rules**) between these two representations, the hadronic variables of interest can then be extracted and expressed as functions of quark and gluon variables.

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[11] That is, we do not rely on any effective theory, e.g. chiral Lagrangian, or numerical calculations, e.g. lattice gauge theory, to extract information of hadronic properties from QCD.

[12] Which is conjectured to be true for QCD but a rigorous proof is still beyond our reach.

[13] A study of NEDM and the strong CP problem has been done within the framework of chiral perturbation theory by Aoki, Hatsuda [17] and H-Y Cheng [18].

[14] As we shall explain later, the quark mass corrections to the magnetic moments of nucleons come out natually in our approach.

[15] The matching is called a dispersion relation in field theory, where we can relate the real part of a correlation function, which can be calculated by using operator product expansion (OPE), to its imaginary part, which is an experimentally measurable function and will be parametrized by an phenomenlogical model in the QCD sum rule approach.

Aside from the virture of maintaining Lorentz covariance and gauge invariance through out the calculations, the QCD sum rule method has the advantage that it deals with quark – gluon degrees of freedom directly without a need to solve hadronic wave functions. Furthermore, we can quantify the nonperturbative nature of the QCD vacuum in terms of various condensates, which are vacuum matrix elements of (color singlet) QCD operators, without a complete solution of confinement. Thus, many physical observables can be related to a set of universal constants and all the symmetries[16] (and/or symmetry breakings) of QCD can be displayed explictly in the sum rule calculations. In this regard, the NEDM problem we study here will serve as an illustrative example how the QCD sum rule method helps us reveal the interesting connections between a global, internal symmetry (chiral symmetry) and the discrete, external P and T symmetries of QCD.

[16] As an example, the current algebra relationships and their corrections due to the presence of gluons can be implimented in the sum rule calculations. These are normally in the form of various low energy theorems. See [24].

Chapter 2

SYMMETRY CONSTRAINTS ON P AND T VIOLATION IN THE STRONG INTERACTION

2.1 Discrete Symmetries P , T and C in Relativistic Quantum Field Theories and the Standard Model of Particle Physics

Parity (or space inversion, P) and time reversal (T) are discrete geometric transformations[1] of the space–time continuum. The basic concepts of these two operations are independent of which theory or what framework is chosen to describe a physical system. Indeed, the concept of P and T symmetries can be applied to both classical and quantum physics, in both non–relativistic and relativistic regimes. Nevertheless, the manifestations and restrictions of these discrete symmetries in different frameworks do have varied implications for the dynamics of a given system (e.g. selection rules in the quantum mechanical transitions). Moreover, as we change the continuous space–time symmetry from Galilean group to Lorentz group and couple the matter fields with the underlying space–time[2](e.g. field theory and gauge symmetry), the realizations and violations of P and T symmetry do receive many constraints from the internal consistency of the underlying framework. The most significant and general example is given by the CPT[3] theorem in the relativistic quantum field theories (RQFT), which states that the combined operations P, T and C (defined in next section) in any local RQFT is equal to identity (up to an abitrary phase). One immediate consequence is that: In local RQFT, the three discrete symmetries P, T and C (and their violations) are not independent![4]

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[1] By geometric we mean there is no dynamics or interactions of the physical systems involved.

[2] The important difference is that, in the non–relativistic region the time coordinate is totally independent from the spatial coordinates. This is no longer true in the relativistic regime, because a Lorentz transformation will mix up space and time coordinates.

[3] C stands for the charge conjugation.

[4] For example, a time reversal violation ($\not{T}$) automatically implies a violation of CP symmetry and vice versa.

Such theorem certainly is not just of pure academic interest, as today we believe that almost all subnuclear phenomena can be well described by a standard model (indeed a local gauged RQFT model), which contains the Glashow-Salam-Weinberg (GSW) model for the electroweak interactions and quantum chromodynamics (QCD) for the strong interaction, and all three discrete symmetries P, T and C are observed to be broken in the weak interactions. In this case, not only is there a need to put in much effort, both experimentally and theoretically, to pin down many unknown parameters in the standard model, hence explain and/or predict many symmetry breaking effects in the real world, but also there is another strong motivation which requires such discrete symmetry breaking in order to explain the formation of our universe. For example, the observed **baryon asymmetry** in our universe is believed to result from CP violations of the particle interactions in the early stage of cosmological evolution [5]. It is important to know whether theoretical models for these symmetry breaking effects, constrained by laboratory experiments, can account for the necessary cosmological phenomena in a quantitative way.

· **Constraints on the discrete symmetry breakings in a RQFT**

Before we dwell on the analysis of the discrete symmetry breakings in strong and electroweak interactions (this is done in the following two subsections), it is useful to see what kind of constraints are put on the discrete symmetry breakings from the internal consistency of the RQFT. First of all, we shall explain what we mean by internal consistency in precise terms; these are basic requirements we impose on all viable RQFTs:

1. **Lorentz invariance** : Since the field theory is based on a Lagrangian formalism, if the theory is independent of the reference frames we choose to describe physics (relativistic invariance), Lorentz invariance requires that every term in the Lagrangian has to be a Lorentz scalar or pseudoscalar (if parity is not conserved). In particular, a fermion operator must accompanied by a anti–fermion operator to make a Lorentz scalar.

2. **Hermiticity** : Any quantum theory describing a closed (non–dissipating)

[5] To generate a nonzero baryon number during the expansion of the universe, three conditions are required: (1) unsuppressed baryon number violating interactions, CP violation, and a departure from thermal equilbrium.

system has to conserve the total probability. This implies that the Hamiltonian (and the Lagrangian), as the generator of an unitary evolution operator, has to be a Hermitian operator. This requirement has a consequence that all coefficients (or coupling constants) in a Lagrangian must be either real or pure imaginary, but not a general complex number. e.g. $3i\bar{q}q$ is not a hermitian operator.

3. **Renormalizability** : In the old days, renormalizability meant that the only allowable operators in the Lagrangian of a RQFT must have dimensions less or equal to the space–time dimension[6] Strictly speaking, this is not an absolute requirement[7]. The basic idea here is power counting, where we can organize the importance of various operators by dimensional analysis, where the lower the dimension, the larger the contribution to a physical observable. Hence, to focus on the simplest case first, we shall restrict our attention to operators of dimension 4 or less in the Lagrangian only.

4. **Gauge invariance** : In the standard model, both electroweak and strong interactions are generated by gauge principles, where we insist that the theory is invariant under local gauge transformations. Thus, any new interactions we add to the standard model in order to describe $\not{P}$ and $\not{T}$ have to maintain the gauge invariance of the standard model Lagrangian.

Let us examine the implications from the internal consistency of the RQFT on the fermion bilinear terms $\bar{\psi}\Gamma\psi$, in a local RQFT Lagrangian, where Γ is a generic Dirac matrix[8]: First of all, we can divide them into 5 types, denoted by S (scalar

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[6] Since we shall choose dimensional regularization (DR) to calculate the loop intergrals and regularize the infinite constants appearing in such calculations, we define the space – time dimension to be 2ω.

[7] As is now realized the proper way of understanding QFT is to use the language of effective field theory, it is no longer correct to treat non–renormalizable interactions as physical pathalogy.

[8] The bilinear terms have dimension 3, which can couple to a boson (dimension 1) to generate a renormalizable interaction, e.g. $i\bar{\psi}\gamma_5\psi\phi$. If we need to generate a fermion–fermion interaction in a RQFT, we need at least dimension 6 operators, e.g. $\bar{\psi}\Gamma_1\psi\bar{\psi}\Gamma_2\psi$ (in 4 dimensional space–time).

$\Gamma = I$), P (pseudoscalar $\Gamma = i\gamma_5$), V (vector $\Gamma = \gamma_\mu$), A (axial vector $\Gamma = \gamma_\mu\gamma_5$) and T (tensor $\Gamma = \sigma_{\mu\nu}$), see [25].

If we consider an interaction between a fermion and a boson through the Yukawa coupling, that is, the interaction Hamiltonian is a product of the fermion bilinear terms and a bosonic field operator (ϕ), $\bar{\psi}\Gamma\psi\phi$. Assuming the boson is a scalar or vector particle (depending on the Γ matrices appearing in the fermion bilinear terms), we can study the transformation properties of these interaction under P, T, and C:

$\bar{\psi}\Gamma\psi$	S	P	V	A	T
P	+	−	+	−	+
T	+	−	+	+	+
C	+	+	+	−	+

As this table shows, the S and V type interactions conserve all three symmetries separately; while P type interaction breaks P and T (which is the case for strong interaction, see the discussions in next section.), and A type interaction breaks C and P (which is the case for weak interaction). There is no interaction which breaks C and T but conserves P. A T odd P even interaction in a local RQFT must come from a higher dimensional (nonrenormalizable) operator.

With these general introductions on the interplay between discrete symmetry breakings and RQFT as a warm-up exercise, we now move on to more specific discussions on the discrete symmetry breakings in the standard model.

2.1.1 P and T Violation in Quantum Chromodynamics (QCD)

We shall be concerned in this work with the possible breaking of parity ($\not{P}$) and time reversal ($\not{T}$) symmetries in the strong interaction. The basis of our study will be Quantum Chromodynamics (QCD), which is believed to be the basic theory of strong interaction. It should be emphasized that, up to this moment, the discrete symmetries P, T and C appear to be exact symmetries in the strong interactions, and there is no experimental evidence that P and T violations are due to strong interactions. However, due to the nontrivial structures of the QCD vacuum and the interconnection between electroweak interaction and QCD within the framework of the standard model, it is not clear why the $\not{P}$ and $\not{T}$ only occurs in the electroweak sector.

Our interest is to relate the possible existence of $\not{P}$ and $\not{T}$ interactions in QCD to a hadronic observable, namely, the NEDM. To this end, we need to know first what kind of $\not{P}$ and $\not{T}$ interactions can we put in the QCD Lagrangian, which is

$$\mathcal{L}_{QCD} \equiv \bar{\psi} i \not{D} \psi + m_q{}' \bar{\psi}\psi + \frac{1}{4} G^2 \tag{2.1}$$

$$\text{where} \qquad \not{D} \equiv (\partial_\mu + i g_s B^a_\mu \frac{\lambda^a}{2}) \cdot \gamma^\mu \tag{2.2}$$

The meanings of various symbols are:

- ψ : quark field
- $\bar{\psi}$: Dirac adjoint of the quark field, $\bar{\psi} \equiv \dagger\psi\gamma_0$
- B^a_μ : gluon field, $a = 1, .., 8$
- $\frac{\lambda^a}{2}$: generators of the color $SU(3)$ gauge group
- $G_{\mu\nu}$: gluonic tensor field, $G_{\mu\nu} \equiv [\partial_\mu + i g_s B_\mu, \partial_\nu + i g_s B_\nu], G^2 \equiv G_{\mu\nu} G^{\mu\nu}$
- g_s : strong coupling constant in QCD

If we now impose all four basic requirements on the constructions of $\not{P}$ and $\not{T}$ interactions in QCD, we find that there are only two possibilities:

$$\mathcal{L}^{\not{P}\not{T}}_{QCD} \equiv i a \bar{\psi} \gamma_5 \psi + b G \tilde{G} \ (a, b \text{ are real numbers}) \tag{2.3}$$

$$\text{where} \qquad \tilde{G}_{\mu\nu} \equiv \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} G^{\alpha\beta} \tag{2.4}$$

the 4 dimensional totally antisymmetric tensor is defined as

$$\varepsilon_{0123} \equiv -\varepsilon^{0123} \equiv 1 \tag{2.5}$$

The $i a \bar{\psi} \gamma_5 \psi$ term will be refered as a pseudo–mass term and the $b G \tilde{G}$ term will be refered as a gluon anomaly term[9]. These operators are odd (they change signs)under the parity and time reversal transformations and give rise to $\not{P}$ and $\not{T}$ hadronic observables, e.g. $\eta \rightarrow \pi\pi$ decay rate and the NEDMs.

A few comments are in order:

[9] We adopt a constructive approach for these $\not{P}$ and $\not{T}$ interactions in QCD in order to avoid the complicated discussions on the nontrivial topological structures associated with a nonabelian gauge theory, where the name of anomaly come from.

- **$\bar{\theta}$ as an angular variable**

  The two terms mentioned above can be rewritten in a different form, (referred as **polar form** or **polar representation** in this work.) by taking the following steps:

  step1 We can combine the pseudo–mass term for quarks (replace ψ by q) $ia\bar{q}\gamma_5 q$ with the usual quark mass term $m_q{}'\bar{q}q$ into a more compact notation , by defining

$$m_q{}^2 \equiv m_q{}'^2 + a^2 \quad (2.6)$$
$$\theta_q \equiv \tan^{-1}\frac{a}{m_q{}'} \quad (2.7)$$

  In so doing, we have replaced two parameters $a, m_q{}'$ by another set m_q, θ_q where θ_q is called a **quark (chiral) phase** and the combination $m_q{}'\bar{q}q + ia\bar{q}\gamma_5 q$ is now replaced by

$$m_q\bar{q}e^{i\theta_q\gamma_5}q \quad (2.8)$$

  step2 The gluon anamoly term $G\tilde{G}$ can be rewritten as a total divergence of a topological current K_μ,

$$K_\mu \equiv \frac{g_s^2}{16\pi^2}\sum \epsilon_{\mu\nu\rho\sigma}B_{a\nu}\{\partial^\rho B_{a\sigma} + \frac{1}{3}f_{abc}B_{b\rho}B_{c\sigma}\} \quad (2.9)$$
$$\partial^\mu K_\mu = \frac{g_s^2}{32\pi^2}G\tilde{G} \quad (2.10)$$

  Then we can rewrite the parameter b as $\frac{g_s^2\theta_G}{32\pi^2}$ The classical instanton configurations have interger values for the topological charge $Q_K \equiv \int d^4x K_0(x)$ [26]. Hence, if we change the value of θ_G by a multiple of 2π, the generating functional of QCD is invariant (See the definition in sec2.2. The integrand of the generating functional is of the form $e^{in\theta_G}$.)

The reason for these changes of variables is twofold:

1. By invoking $U_A(1)$ chiral rotations of the quark fields, (this will be discussed in more details in sec.5.3) we can identify the $U_A(1)$ covariant and invariant parameters in QCD, which hadronic observables depend on.

2. As shown in the equations eq.2.8, eq.2.10, we now have two angular variables θ_q, θ_G in the QCD Lagrangian, which are of period 2π. This implies that all the physical observables calculated from the QCD Lagrangian have to display a similar periodic structure for the dependence on these angular variables.

- **Number of P and T violating parameters**

  From the previous discussion, it seems that we have two symmetry breaking parameters (θ_q and θ_G) which characterize the violation of P and T symmetries in QCD. However, it turns out that this is not correct!! Only one (as we shall show in the next section), namely $\theta_G - \theta_q$ will generate $/P$ and $/T$ observables in QCD; the other parameter (either θ_q or θ_G, or any combination which is independent of $\theta_G - \theta_q$) is just an unobservable phase, which serves as a continuous parameter labeling a family of equivalent representations for the QCD Lagrangian. Furthermore, the physical quantities calculated from any particular representation of the QCD Lagrangian should always give the same result. In other words, physical quantities should be independent of the representation of QCD Lagrangians[10].

With all these prelimilary discussions, we shall define the strong $\not{P}$ and $\not{T}$ interaction in QCD as the term eq.2.8 and eq.2.10 appearing in the QCD Lagrangian, with a single parameter $\bar{\theta}$ characterizes the explicit breaking of P and T symmetries.

2.1.2 P and T Violation in the GSW Model of the Electroweak Theory

It was the study of K_L, K_S mesons decays into pions which first led to the astonishing discovery [28] that CP may not be an exact symmetry of the weak interactions. This type of CP violation (or $\not{T}$ through the CPT theorem) is generally classified as a "weak CP violation", because decay proceeds through the exchange of a weak boson, which changes a s quark in the K meson into a u quark in pion—a flavor changing process.

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[10] This is closely related to the path integral interpretation of the chiral anomaly in QCD, which was first derived by Fujikawa [27].

Within the standard model, it can be shown that, in order to incorporate the CP violation effects without modifying the gauge structure of the theory, it is necessary to have at least 3 generations of quarks or more than one Higgs doublet in the GSW Lagrangian. The former scheme introduces a complex phase in the quark mixing matrix (which is possible only if the generations of quarks is greater than 3) and is called Cabibbo-Kobayashi-Maskawa (CKM) model [29]; the later requires two Higgs doublets with a relative complex phase between their ground state expectation values and was first proposed by S. Weinberg [30]. Beyond the standard model, other models were proposed; most of them require a modification of the gauge groups, hence introduce many new gauge bosons, e.g. the Left–Right symmetric model [31]. The possibility of a spontaneous CP violation in the electroweak interactions was also studied by various authors [32].

Most models mentioned above use the experimental observations in the K meson system to pin down their input parameters, and the contribution of weak CP violations to the NEDMs can be calculated. The severe upper bound set by the experimental constraint $d_N < 10^{-26} e \cdot cm$ rules out many possible candidates. Furthermore, the cosmological observation (e.g. baryon to photon ratio) makes further constraints. In view of these difficulties, it is not clear we have a correct complete explnation of the source of CP violations. Hopefully, future measurements at the B factories [33] will be a vital step toward the unraveling the mystery of CP violations.

2.1.3 Connection and Comparison Between Weak and Strong CP Violations

Since the standard model contains both QCD and GSW models as its ingredients, and for many processes (except for the purely leptonic interactions) we need to take both of these interactions into account; it is important to notice that there are some features which help us understand the natures of weak CP violations and strong ones[11]:

1. **Charge conjugation**

[11] Our comparisons between these two interactions are based on the CKM model and the strong CP violating interaction defined in sec.2.2

There is a very important difference between the strong $\not{P}$ and $\not{T}$ and the CP violations in the electroweak interactions. Namely, the former case, as defined before, still conserves charge conjugation (C) symmetry; the latter, as a combination of the $V-A$ structure (which violates both C and P but conserves T) and a $\not{P}$ and $\not{T}$ (but C conserving) CKM phase, violates C, P and T. The charge conjugation as well as parity are badly broken in the electroweak interactions, their combined effects give rise to a tiny time reversal violating effect through the CPT theorem. In the case of the NEDM, which is both P and T violating but C conserving, the tree level weak interactions will give zero contribution simply because of the charge conjugation symmetry. In order to generate a nonvanishing contribution to the NEDM from the weak interaction, it is necessary to go higher order such that there is a C–even amplitude from the perturbation series.

2. **Fermion generations**

   In the CKM model of weak CP violations, there must be at least three generations of nondegenerate quarks (that is, $m_d \neq m_s \neq m_b$ or $m_u \neq m_c \neq m_t$) to generate a CP violating phase in the mixing matrix [29]. By contrast, in the θ term model of strong CP violations, only one massive quark is sufficient to break CP.

   A consequence is that, in a world of only one (or two) generation(s) of quarks, there would be no CKM type of CP violation. Also, if in some physical processes the contributions of the third generation to a CP violating observable are highly suppressed (possibly through virtual processes only), the weak CP violation should be small compared to the strong one (Assuming both CP violating parameters are of the same order.)

3. **Flavor structure**

   The other feature of the weak mixing matrix is that weak CP violations manifest themselves mainly through flavor changing processes. That is, the leading weak CP violation processes are off–diagonal effects in the flavour space[12].

[12] For flavour conserving processes we need 2nd order weak interactions (CP violating times CP conserving) to have non–zero contributions.

Strong CP violations, on the other hand, add up all the contributions from individual flavours. Therefore, one can describe strong CP violations as diagonal effects in the flavour space. In particular, the NEDM, which probes the nucleon matrix element of the EM current, should be dominated by strong CP violation, unless there are large cancellations or the CP violating parameter $\bar{\theta}$ is anomalously small[13].

4. **Gauge field anomaly**[14]

   Since both QCD and GSW models are based on nonabelian gauge theories, where the nontrival structures (e.g. instantons) of the gauge fields could have some consequences on the CP violations, it is curious to see whether there is a similar anomaly contribution (like the QCD θ term) from the weak gauge fields ($SU_L(2)$ sector, to be specific).

   It turns out that the answer is no, and the reason behind this is closely tied to the V–A structure of weak interactions . If we write down the weak gauge boson anomaly term $W\tilde{W}$ as gluon anomaly term in QCD, one can show that, by performing a $U_V(1)$ rotation on the quark fields, we can rotate the weak θ_W angle away without changing the quark chiral phases θ_q and the gluon angle θ_G[15]. Therefore, like the $\bar{\theta}$ angle in massless QCD, the weak anomaly term is irrelavent for CP violations[16] [17].

We should emphasize that, up to this time, there is no definite theory of CP or T violation for subnuclear interactions. Also, more experimental evidences and observations are needed to help us map out the underlying patterns for these discrete symmetry breakings. Without such further progress, it is not clear whether the

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[13] Unfortunately, this seems to be the case in the real world.

[14] The author would like to thank Prof. Peter Arnold for explaining this point to him.

[15] This certainly has to do with the vector-type coupling nature of QCD and the fact that quark mass terms are invariant under $U_V(1)$ rotations.

[16] The point is, there is no operator in the standard model which violates the $U_V(1)$ symmetry, unlike the QCD case where the quark mass terms break the $U_A(1)$ symmetry explicitly.

[17] However, this weak θ_W term is responsible for the baryon number violations in the weak interactions.

standard model alone is sufficient to provide a complete and satisfactory scenario for the discrete symmetry breakings in the subnuclear world. In view of this, a comprehensive study of the CP violation problem should thus take both weak and strong interactions into account[18]. Only if we can perform reliable theoretical calculations (especially in the strong interaction sector), a comparison with various ongoing measurements should then be able to tell us whether this is the whole story of the CP violation problem, or if there is some new mystery waiting us to explore.

2.2 Functional Integral Formulation of P and T Violation in QCD

2.2.1 The Generating Functional and Correlation Functions in QCD

In order to clarify the points we have made in the previous section, it is convenient to formulate the problem of strong $\not P$ and $\not T$ in QCD in terms of functional integrals. The advantage is that all the correlation functions (or Green's functions) can be obtained from a generating functional through functional derivatives [26], so that many exact identities and theorems can be derived or proven without resorting to a perturbative expansion[19].

The definition of the QCD generating functional (denoted by Z) is:

$$Z[\zeta,\bar{\zeta},J_\mu;\theta_q,\theta_G] \equiv \frac{1}{N}\int [D\psi][D\bar{\psi}][DB_\mu]e^{iS_{QCD_{\theta_q,\theta_G}}+\bar{\zeta}\psi+\bar{\psi}\zeta+JB} \tag{2.11}$$

where

$$S_{QCD_{\theta_q,\theta_G}} \equiv \int d^{2\omega}x\bar{\psi}i\not D\psi + \frac{1}{4}G^2 + m_q\bar{\psi}e^{i\theta_q\gamma_5}\psi + \frac{g_s^2\theta_G}{32\pi^2}G\tilde{G} \tag{2.12}$$

The normalization constant for the generating functional is

$$N \equiv \int [D\psi][D\bar{\psi}][DB_\mu]e^{iS_{QCD_{\theta_q,\theta_G}}} \tag{2.13}$$

such that

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[18] It should be kept in mind that there are only two parameters in the minimal version of the standard model which can generate CP violations, these are the complex phase in the quark mixing matrix and the $\bar{\theta}$ in QCD.

[19] We should also emphasize that the technique of generating functionals not only gives a formal solution to the quantum theory (by generating all the Green's functions), but it is also a useful calculational scheme which is more general than the usual perturbation method.

$$Z[\zeta = 0, \bar{\zeta} = 0, J_\mu = 0; \theta_q, \theta_G] = 1 \qquad (2.14)$$

Notice that we shall restrict our discussion to one quark flavor for simplicity. The generalizations to multiflavour case will be given in sec.2-4.

Since we shall focus only on the quark correlation functions, and in particular, the nucleon correlation function(NCF), which is a time–ordered product of composite quark fields, we can set the source terms for gluon fields J_μ equal to zero and suppress the gluonic source term dependence of the generating functional Z thereafter.

Having defined the generating functional, we can calculate the quark correlation functions by taking functional derivatives of Z with respect to the external sources ζ and $\bar{\zeta}$. For example, the simplest correlation function — quark propagator $S_{F,\theta_q,\theta_G}(p)$, can be written as :

$$S_{F,\theta_q,\theta_G}(p) \equiv \langle T\psi(x)\bar{\psi}(0)\rangle_{\theta_q,\theta_G} = \frac{\delta^2 Z[\zeta,\bar{\zeta};\theta_q,\theta_G]}{\delta\zeta(0)\delta\bar{\zeta}(x)}\Big|_{\zeta=\bar{\zeta}=0} \qquad (2.15)$$

$$= \frac{1}{N}\int [D\psi][D\bar{\psi}][DB_\mu]\psi(x)\bar{\psi}(0)e^{iS_{QCD\theta_q,\theta_G}} \qquad (2.16)$$

In the free theory, that is, we set g_s equal to zero, we have:

$$S_{F,\theta_q}(p) = \frac{1}{\not{p} - m_q e^{i\theta_q\gamma_5}} = \frac{\not{p} + m_q e^{-i\theta_q\gamma_5}}{p^2 - m_q^2} \qquad (2.17)$$

Notice that the new structure associated with the γ_5 matrix, appears in the quark propagator, which is not necessarily tied to the violation of P and T symmetries in QCD, only implies a particular choice of quark chiral basis in the QCD generating functional[20]. In addition, the pole of the quark propagator (which is defined to be the on–shell mass for the particle) is real, as a physical mass of an elementary particle has no imaginary part, this is consistent with the hermiticity requirement we have discussed before (see sec. 2-1). Similarily, the n–point Green's functions and the Green's functions for composite fields can be calculated in the same way. In particular, if we choose a **neutron interpolating field** as [21]

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[20] There is no $\not{p}\gamma_5$ structure because of C–parity. See the discussion in sec.3-3.

[21] Another equally valid choice would be $\eta_n \equiv (d^t C\sigma_{\mu\nu}d)\gamma_5\sigma^{\mu\nu}u$.
For more discussion, see Appendix-D

$$\eta_n \equiv (d^t C\gamma_\mu d)\gamma_5\gamma^\mu u \qquad (2.18)$$

then the **neutron correlation function** $\Pi^n(p)$ can be calculated as follows:

$$\Pi^n(p) = \int d^{2\omega}x e^{ipx}\langle T\eta^n(x)\bar{\eta}^n(0)\rangle_{\theta_q,\theta_G} \qquad (2.19)$$

$$= \int d^{2\omega}x e^{ipx}[((\frac{\delta}{\delta\bar{\zeta}_d(x)})^t C\gamma_\mu \frac{\delta}{\delta\bar{\zeta}_d(x)})\gamma_5\gamma^\mu \frac{\delta}{\delta\bar{\zeta}_u(x)}]\times$$
$$[((\frac{\delta}{\delta\bar{\zeta}_d(0)})^t C\gamma_\nu \frac{\delta}{\delta\bar{\zeta}_d(0)})\gamma_5\gamma^\nu \frac{\delta}{\delta\bar{\zeta}_u(0)}]^\dagger \gamma_0 Z[\zeta,\bar{\zeta};\theta_q,\theta_G]\ |_{\zeta=\bar{\zeta}=0} \qquad (2.20)$$

2.2.2 $U_A(1)$ Chiral Anomaly and the Reparametrization Covariance of the QCD Generating Functional

From the definition of the generating functional of QCD, it is clear that the fermion ψ and gluon B_μ fields appearing in the QCD Lagrangian are just dummy variables and any redefinitions of these variables should not change the generating functional, which is a functional of external sources only. In paricular, we wish to focus on a special class of transformations, which change the fermionic variables in the generating functional of QCD, namely the $U_A(1)$ chiral transformation (or a chiral rotation), which is defined as:

$$q \to q' \equiv e^{i\theta\gamma_5} q \qquad (2.21)$$

$$\text{or } q'_i = [\ \cos\theta I_{ij} + i\sin\theta\gamma_{5ij}\ ]\ q_j \qquad (2.22)$$

One can show that under a chiral rotation, the θ_q variable in the $\not{P}$ and $\not{T}$ QCD Lagrangian changes by 2θ (this will be defined as a chirally covariant quantity) and since the chiral transformation only acts on the fermionic variable, it seems that the gluon field and the θ_G variable should stay the same (or, chirally invariant).

However, it turns out that this is not correct. As shown by Fujikawa [27], in the functional integral formalism, the chiral transformation is not a unitary transformation, and therefore, in the change of variables due to a chiral rotation in the functional integral, we need to include the Jacobian associated with such a transformation, exactly as we do change of variables in ordinary integrations. It is a nontrival task to evalue the Jacobian, we shall not bother the derivations here. The bottom line is, we can rewrite the jacobian in terms of a gluon anamoly term $\frac{g_s^2}{32\pi^2}G\tilde{G}$ times the $U_A(1)$ rotation angle 2θ. Combining this term with the $bG\tilde{G}$ term, we conclude that the θ_q variable also changes by a constant 2θ.

In terms of the formulae we just defined;

$$\begin{aligned} Z[\zeta,\bar{\zeta};\theta_q,\theta_G] &\equiv \frac{1}{N}\int[D\psi][D\bar{\psi}][DB_\mu]e^{iS_{QCD\theta_q,\theta_G}+\bar{\zeta}\psi+\bar{\psi}\zeta} & (2.23)\\ &= \frac{1}{N}\int[D\psi'][D\bar{\psi}'][DB_\mu]e^{iS_{QCD\theta_q-2\theta,\theta_G-2\theta}+\bar{\zeta}e^{-i\theta\gamma_5}\psi'+\bar{\psi}'e^{-i\theta\gamma_5}\zeta} & (2.24)\\ &= Z[e^{-i\theta\gamma_5}\zeta,\bar{\zeta}e^{-i\theta\gamma_5};\theta_q-2\theta,\theta_G-2\theta] & (2.25) \end{aligned}$$

If we redefine the fermionic source term $\zeta,\bar{\zeta}$ by the following:

$$\begin{aligned} \zeta' &\equiv e^{-i\theta\gamma_5}\zeta & (2.26)\\ \bar{\zeta}' &\equiv \bar{\zeta}e^{-i\theta\gamma_5} & (2.27) \end{aligned}$$

which amounts to a redefinition of the chiral angle of the vacuum state. The previous formula simply states that the generating functional with chiral phases θ_q,θ_G is identical (up to the redefinition of the source terms) to the generating functional with chiral phases $\theta_q-2\theta,\theta_G-2\theta$. Therefore, any physical observable must depend on one variable only, which is the difference between θ_q and θ_G. In view of this, we define a chirally invariant angle:

$$\bar{\theta}\equiv\theta_G-\theta_q$$

which is the single $\not{P}$ and $\not{T}$ parameter in the QCD Lagrangian. The apparent redundancy in our construction of a $\not{P}$ and $\not{T}$ QCD Lagrangian simply implies there is a reparametrization invariance in the generating functional of QCD. In particular,

(1) The appearence of a pseudo–mass term and a gluon anomaly term does not necessarily imply a violation of P and T symmetries; A QCD Lagrangian with θ_G equals to θ_q still conserve P and T. Only if they are not equal, we can generate physical observables, e.g. NEDMs from the QCD Lagrangian.

(2) The equivalence relations between different representations of the QCD the generating functionals (i.e. there are two sets of chiral phases (θ_{G1}, θ_{q_1}) and (θ_{G2}, θ_{q_2}) but $\bar{\theta}_1=\bar{\theta}_2$) should be understood in a more careful way, this does not mean all

the correlation functions generated from the two equivalent generating functionals are identical, but rather they differ by a chiral conjugation, for example, if we calculate a quark propagator from Z_1

$$S_{F,\theta_{q1},\theta_{G1}}(p) \equiv \langle T\psi(x)\bar{\psi}(0)\rangle_{\theta_q,\theta_G} = \frac{\delta^2 Z[\zeta,\bar{\zeta};\theta_{q_1},\theta_{G1}]}{\delta\zeta(0)\delta\bar{\zeta}(x)}|_{\zeta=\bar{\zeta}=0} \tag{2.28}$$

According to the identity eq(2-) and using the chain rule, we get,

$$S_{F,\theta_{q1},\theta_{G1}}(p) = \frac{\delta\zeta'(y)}{\delta\zeta(0)}\frac{\delta^2 Z[\zeta',\bar{\zeta}';\theta_{q_2},\theta_{G2}]}{\delta\zeta'(y)\delta\bar{\zeta}'(z)}\frac{\delta\bar{\zeta}'(z)}{\delta\bar{\zeta}(x)}|_{\zeta=\bar{\zeta}=0} \tag{2.29}$$

$$= e^{-i\theta\gamma_5}\frac{\delta^2 Z[\zeta',\bar{\zeta}';\theta_{q_2},\theta_{G2}]}{\delta\zeta'(y)\delta\bar{\zeta}'(z)}|_{\zeta'=\bar{\zeta}'=0}\, e^{-i\theta\gamma_5} \tag{2.30}$$

$$= e^{-i\theta\gamma_5}S_{F,\theta_{q_2},\theta_{G2}}(p)e^{-i\theta\gamma_5} \tag{2.31}$$

[22] Using a similar method, but with more laborious calculus, one can show that the correlation function for a neutron interpolating field Π_n calculated from two equivalent generating functionals are related by

$$\Pi_n = e^{-i\theta\gamma_5}\Pi_n e^{-i\theta\gamma_5} \tag{2.32}$$

This is a general exact result and we derive this without the use of perturbative expansion. The lesson we take from this is that, in any calculation of physical observables from using correlation functions, we need to pay special attention to the representation dependence of the calculations in order to extract a correct (that is, representation independent) answer. Specifically, the chiral conjugation factor which appears in the formula eq.2.29, eq.2.30 should be filtered out before we correctly interpretate a chirally invariant matrix element, We shall see more examples in later sections.

2.2.3 $U_A(1)$ Chiral Anomaly and the Strong $\not{P}$ and $\not{T}$ in QCD

It should be clear by now that, either a pure gluon anamoly term or a single pseudo mass term in the QCD Lagrangian should describe the same physics, as long as

[22] For a free theory, one can obtain the same result by doing a direct differentiation, without using the chain rule relation.

$\theta_q = -\theta_G$. Certainly, we can also work with a more general cases by allowing both θ_q and θ_G to be nonzero, as long as they belong to the same equivalent class. In this section, we shall focus on a special property for the $\not{P}$ and $\not{T}$ QCD generating functionals. Consequently, we shall work with a particular representations where $\theta_G = 0$ and $\theta_q = -\bar{\theta}$. To study this, we first notice that in the functional intergal of the QCD generating functionals, the exponent in the expontential is quadratic in the fermionic variables. Thus, in the Eucledan space calculation, we can integrate out the fermionic variables exactly, which gives rise to a functional determinant with both a gluonic background and a quark pseudo mass term:

$$Z[\zeta,\bar{\zeta};\theta_q,\theta_G] \equiv \frac{1}{N}\int[D\psi][D\bar{\psi}][DB_\mu]e^{iS_{QCD_{\theta_q,\theta_G}}+\bar{\zeta}\psi+\bar{\psi}\zeta} \quad (2.33)$$

$$= \frac{1}{N}\int[DB_\mu]det[i\not{D}+m_q e^{-i\bar{\theta}\gamma_5}]e^{\bar{\zeta}[\not{D}+m_q e^{-i\bar{\theta}\gamma_5}]\zeta+\frac{1}{4}G^2} \quad (2.34)$$

In such a representation, the strong $\not{P}$ and $\not{T}$ effects of the $\bar{\theta}$ come to the calculations in two ways;

(1) The quark propagator, as obtained from the functional derivitives of the generating functional with respect to the fermionic sources ζ and $\bar{\zeta}$, contain a pseudo mass term $m_q e^{-i\bar{\theta}\gamma_5}$. This is normally refered as a connected insertion.

(2) The determinant factor $det\ [i\not{D}+m_q e^{-i\bar{\theta}\gamma_5}]$ in a diagramatical expansion, gives rise to closed disconnected fermion loops, and are normally refered as a disconnected insertion[23].

Not all these insertions really contribute to a $\not{P}$ and $\not{T}$ observable. As we have seen in the last section, the connected insertion can always be eliminated through a redefinition of the fermionic sources. Therefore, they contribute to the representation dependent chiral conjugation phases rather than an intrinsic $\not{P}$ and $\not{T}$ observable.

The second contribution (closed disconnected fermion loops) has to be connected to the connected part (these come from the external lines in the definition of a correlation function) through at least two gluonic propagators (otherwise, it is zero upon taking color trace). In this case, we have a well known $U_A(1)$ chiral anomaly

[23] It should be emphasized that whenever we talk about connected or disconnected insertions we really mean that these are separated fermion lines; however, there are gluonic propagators connect these fermion lines.

triangle, which means that the strong $\not{P}$ and $\not{T}$ effects only contribute through a $U_A(1)$ chiral anomaly.

These mathematical formulations can be rephrased into symmetry constraints on the possible existence of a strong $\not{P}$ and $\not{T}$, which we shall turn to in the next section.

2.3 Symmetry Constraints on P and T Violations in QCD

It is important (and interesting) to realize that there is some connection between chiral symmetry (a continuous symmetry) and strong P and T (two discrete symmetries) violations in QCD. This connection leads to three stringent constraints on these discrete symmetry breakings in QCD. In particular, assuming $\bar{\theta}$ is not equal to zero in the QCD Lagrangian, we shall show that, these constraints imply that as long as chiral symmetry is an exact symmetry of QCD, there will be no strong P and T violations in QCD at all. For this reason, we shall refer to these constraints as **symmetry constraints**, which, along with a nonvanishing $\bar{\theta}$ parameter, provide necessary and sufficient conditions of strong $\not{P}$ and $\not{T}$ in QCD.

2.3.1 Non–Perturbative Nature of P and T Violation in QCD

Before attacking the "difficult" problem of calculating the NEDM, we shall begin with a "simple" case, namely, is there an EDM for a quark (qEDM)? Such a question is motivated by the study of magnetic moments of nucleons in the framework of quark model, in which through the use of nonrelativistic wave functions of nucleons, we can establish a relationship between magnetic moments of constituent quarks and magnetic moments of nucleons. Furthermore, we can deduce the magnetic moments of other hadrons from the flavour symmetry (and their violations). It is therefore of interest to see whether we can establish a similar relationship for qEDMs and NEDMs, and other EDMs of hadrons. On the other hand, it is a well–established result that the radiative corrections in QED gives rise to the anamolous magnetic moment of an electron. Given a $\not{P}$ and $\not{T}$ parameter $\bar{\theta}$ in QCD, we wish to calculate the EDMs of quarks (pertubatively, in the strong coupling constant g_s).

To do the calculations in a diagrammatic way, we need to know how to incoroporate the $\not{P}$ and $\not{T}$ effect into the Feynman rules. At first thought, we need to modify the gluon and quark propagators because of the two terms we mentioned in sec. 2-2;

with the gluon anomaly term gives a correction to the former and the quark pseudo–mass term to the latter. After some algebra, we find that the gluon anamoly term gives no contribution to the gluon propagator. This has to do with the fact that such a term can be written as the total divergence of a topological current, which will vanish upon integration by parts. The inclusion of the quark pseudo–mass term presents no trouble. As we have shown before, the net effect is to introduce an extra chiral phase in the (normal) mass term of the quark propagator.

Such observations immediately lead to an apparent parodox: first of all, in a particular choice of quark chiral basis for the QCD Lagrangian, we can make θ_q vanish and θ_G equal to $\bar{\theta}$. This, together with the fact that the gluon anamoly term gives no contribution to a $\not{P}$ and $\not{T}$ gluonic propagator, implies that the perturbative contributions to a qEDM is zero. But, as we can also perform a chiral rotation such that all the $\not{P}$ and $\not{T}$ effects reside in the quark pseudo–mass term, and a direct calculation (see next section) seems to indicate a nonzero qEDM. Furthermore, such a calculation does not rely on a perturbative expansion in θ_q ($= -\bar{\theta}$). What goes wrong here?

The resolution, not too surprisingly, lies in the very important property of reparameterization invariance of the QCD Lagrangian (under chiral rotations). It is clear that the physical content of the theory should be invariant under any change of variables. In particular, a chiral rotation which shifts the values of θ_G and θ_q should not change the value of a qEDM. If we look back at our calculations, we do find a θ_q (which is not a $U_A(1)$ invariant) dependence of the qEDM. This indicates that we are not calculating a physical quantity and something is missing in our result.

To settle the paradox, we need to go back to the functional integral representation of the QCD Lagrangian, where we can study the transformation property under $U_A(1)$ rotations of Green's functions. If we take the functional derivatives of the QCD generating functional with respect to the external sources, we can show that a two–point Green's function expressed in two chiral bases differ by a chiral conjugation (see sec 2-2-3). Such an unphysical dependence on the chiral basis of Green's functions can be eliminated by including a (representation dependent) chiral phase of the quark wave function, or, equivalently, a redefinition of the chiral phase of the quark source terms in the QCD generating functional . Once we put these ingredients together, we can extract a chirally invariant EDM from the two–point Green's function (that is, a quark propagator in an external EM field), which equals to zero[24]. Consequently, the EDM of quarks in QCD receive no perturbative contribution, and any $\not{P}$ and $\not{T}$ contribution to qEDM has to come from purely nonperturbative effects.

In fact, this conclusion we have derived is more general. It is not too difficult to see that if we use a perturbative approach to calculate the NEDM, we need to include a chiral phase (α_N) for the "nucleon wave function", as parametrized by the matrix element of the interpolating current between the vacuum and a nucleon state. Such chirally covariant phases will compensate the effect of the chiral phase for quark (θ_q) and renders the NEDM to be zero. Furthermore, in addition to the calculations of EDM problems, it is not inconcevible to see this requirement of nonperturbative contributions is actually a necessary constraint on all strong $\not{P}$ and $\not{T}$ observables. Therefore, we can summarize this subsection as follows:

All the strong $\not{P}$ and $\not{T}$ observables in QCD receive only nonperturbative contributions, the perturbative contributions of $\not{P}$ and $\not{T}$ effect give no effect at all.

2.3.2 Chiral limit and P and T Violation in QCD

The second constraint on the strong $\not{P}$ and $\not{T}$ problem has to do with the role played by the quark mass term. Peccei and Quinn [34] were the first to point out that,

If the mass of the quark is zero, then there is no strong $\not{P}$ and $\not{T}$. (*)

We can use the functional integral formalism to understand why this is so. According to the definition of the quark chiral phase θ_q, it is only defined for a nonvanishing quark mass; the chiral phase for a massless quark is undefined. we can take advantage of this fact to choose a quark chiral phase θ_q equal to θ_G such that the $U_A(1)$ invariant phase $\bar{\theta}$ is zero, or put another way, we can always rotate the gluon anomaly phase away such that the resulting QCD Lagrangian is P and T conserving.

This simple argument can be phrased in two different ways: First of all, what we need here is an operator (e.g. the quark mass term) which breaks the chiral symmetry explictly, such that the unphysical change of variable becomes ineffective in reducing the irrelevant operators (remember the discussion on the weak anomaly contribution to the $\not{P}$ and $\not{T}$ observables).

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[24] one can show that this holds true to all orders in the fine coupling constant (α) of QED and also in the strong coupling constant (α_s) of QCD.

Secondly, we can also apply the same argument to the nonrenormalizable higher dimensional operators. Since these operators naturally carry dimensional coupling constants (which will correspond to the mass coefficient for dimension 3 operators), it is possible to define various chiral phases for them, and they will give contributions to the NEDM.

With these two constraints in hand, we conclude that:

It is necessary to have both spontaneously and explictly chiral symmetry breakings to generate strong $\not{P}$ and $\not{T}$ from QCD.

This conclusion not only establish an interesting connection between the discrete space – time symmetries P and T and a continuous chiral symmetry in QCD, but also has an important consequence in the study of early universe, sepecifically, the matter antimatter asymmetry caused by CP violation. In the big bang cosmology, the early universe is in a very high temperature phase, where the chiral symmetry of QCD is supposed to be restored (a phase transtion). Our conclusion here naturally implies that in such case the strong $\not{P}$ and $\not{T}$ does not contribute to the matter antimatter asymmetry. Such asymmetry has to come from other dynamics (e.g. weak interaction) in the early stage of the evolution of our universe[25].

2.3.3 $U_A(1)$ Anomaly Constraint of P and T Violation in QCD

The third constraint was first discussed by Shifman, Vainshtein and Zakharov [15], and then rediscovered recently by S. Aoki et al. [16]; they realized that, in a diagramatical language, the contribution of strong $\not{P}$ and $\not{T}$ only comes in to the physical variables through the internal fermion loops with a pseudo–mass insertion. Such a diagram is exactly identical to the anomalous Ward identity associated with the flavour singlet axial current and has a close relationship with the $U_A(1)$ anomaly in QCD. People had examined such a connection in the context of chiral perturbation theory [18] [17]. Basically, this constraint requires that the chiral anomaly provides a solution to the $U_A(1)$ problem [15]. If this is not the case, then the strong $\not{P}$ and $\not{T}$ contribution to the EDM should be zero.

In our study, which is the first one based on the QCD sum rule approach, the NEDM will be related to various condensates of QCD. We therefore interpret this

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[25] Of course, as the universe cools off in the later stage of its evolution, the strong $\not{P}$ and $\not{T}$ can have some consequence as the chiral symmetry of QCD is broken then.

constraint as that all $\not{P}$ and $\not{T}$ hadronic variables have to be proportional to chially invariant gluonic anomalous condensates, such as $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$ (the lowest dimensional gluonic anomalous condensate).

Given all three general constraints on the strong $\not{P}$ and $\not{T}$ discussed above, we can now study their implications on the $\not{P}$ and $\not{T}$ hadronic variables, based on the operator language. It is natural to expect that all $\not{P}$ and $\not{T}$ observables have to be proprotional to

(1) the quark mass m_q,

(2) Λ_{QCD}, or equivalently, the chiral radius R_q; which is a characterization of nonperturbative effect of QCD,

(3) the gluonic anomalous condensates, e.g. $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$.

The last one is clearly not independent of the second one, there must be some connection relating these two factors. It turn out that there is an exact formula one can use to relate the chiral radius R_q and a gluonic anomalous condensate $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$. Namely, the anomalous Ward identity of QCD. The derivation of this identity and related discussion will be given in sec.2.4.

2.3.4 A Graphic Illustration of the Symmetry Constraints of P and T Violation in QCD

From the previous discussion, it is clear that to generate a $\not{P}$ and $\not{T}$ observable in QCD, we need three nonzero parameters, m_q, R_q and $\bar{\theta}$. These conclusions were derived through the use of a functional integral formalism. While these elegant derivations are exact and nonperturbative in nature, they lack the intuition and simplicity to help us grasp the basic ideas. In view of this, we would like to have a different approach to understand how do these symmetry constraints work in QCD.

In fact, there is one simple way to visualize these symmetry constraints without relying on the functional integral formalism. Specifically, we shall use a graphic illustration to show that there is no strong $\not{P}$ and $\not{T}$ if there is no spontaneous chiral symmetry breaking and/or the quark mass goes to zero[26].

To begin with, the set of $\not{P}$ and $\not{T}$ QCD Lagrangians as defined in sec 2-2 can be represented as a two dimensional plane (phase space), where a given pair of

[26] As we have explained before, the $U_A(1)$ anomaly constraint is not really independent from the first two constraints. Hence, we shall ignore this constraint in this section.

$\not{P}$ and $\not{T}$ parameters corresponds to a point on the plane. It is useful to choose the polar representations for the $\not{P}$ and $\not{T}$ parameters (i.e. θ_q, θ_G) so that the periodic structures of these parameters implies an identification of the boundaries of the squares (e.g. $\theta_q = 0 \equiv \theta_q = 2\pi$) and the phase space of the $\not{P}$ and $\not{T}$ QCD Lagrangians becomes a torus. See Fig. 2.1.

On the two dimensional plane, we can identify the equivalent classes of the $\not{P}$ and $\not{T}$ QCD Lagrangians (those with the same $\bar{\theta}$) with the straight lines $\theta_G - \theta_q = constant$. After the identification of the boundaries of the square, these straight lines map onto a family of nonintersecting closed loops winding over the torus. Any two points on the same curve (which correspond to to equivalent $\not{P}$ and $\not{T}$ QCD Lagrangians of the same $\bar{\theta}$) describe the same physics (reparametrization invariance). See Fig. 2.2.

The connection with the symmetry constraints is established once we specify the length scales of the torus: the large radius, conjugate to the θ_G angular variable, is related to some function[27] of R_q; and the small radius, conjugate to the θ_q angular variable, is some function of m_q. With these specifications, we can study the change of the geometries of the torus in two special limits:

(1) chiral limit ($m_q \to 0$): In this limit, as the small radius shrinks to zero, the torus degenerates into a circle and all equivalent loops collapse onto the equator. If we insist on the single valuedness of the physics as represented in this case, it is natural to conclude that all strong $\not{P}$ and $\not{T}$ vanish. Since all the equivalent classes collapse onto the P and T conserving one. See Fig. 2.3.

(2) no spontaneous chiral symmetry breaking ($R_q \to 0$): In this limit, as the large radius shrinks to zero, the torus degenerates into a sphere. The equivalent loops become eight–shaped curves and they all intersect with the $\not{P}$ and $\not{T}$ conserving loop at two points. Again, using the single–valuedness argument, we conclude that if there is no spontaneous chiral symmetry breaking ($R_q = 0$), the QCD Lagrangians conserve P and T even with a nonzero $\bar{\theta}$. See Fig. 2.4.

It is worth mentioning that such a graphic illustration indicates a dual relationship between m_q and R_q. In addition, we hope that the geometrical pictures can be used quantitatively. For example, by choosing some suitable functions for the m_q and R_q

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[27] Since we are only interested in a qualitative description of the symmetry constraints in this section, the actual form of the function is not important. Except that the function has to vanish when its argument is zero.

dependences of both radii of the torus, we might be able to relate the EM moments of particles to certain geometrical measures (e.g. surface area enclosed by certain contour) or fluxes through the loops.

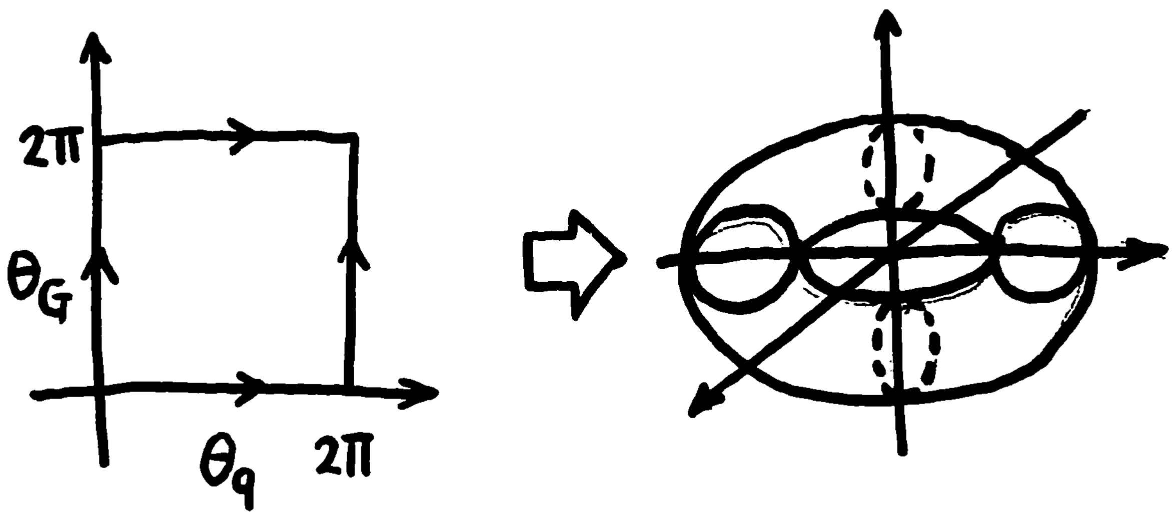

Figure 2.1: The phase plane and CP torus of QCD

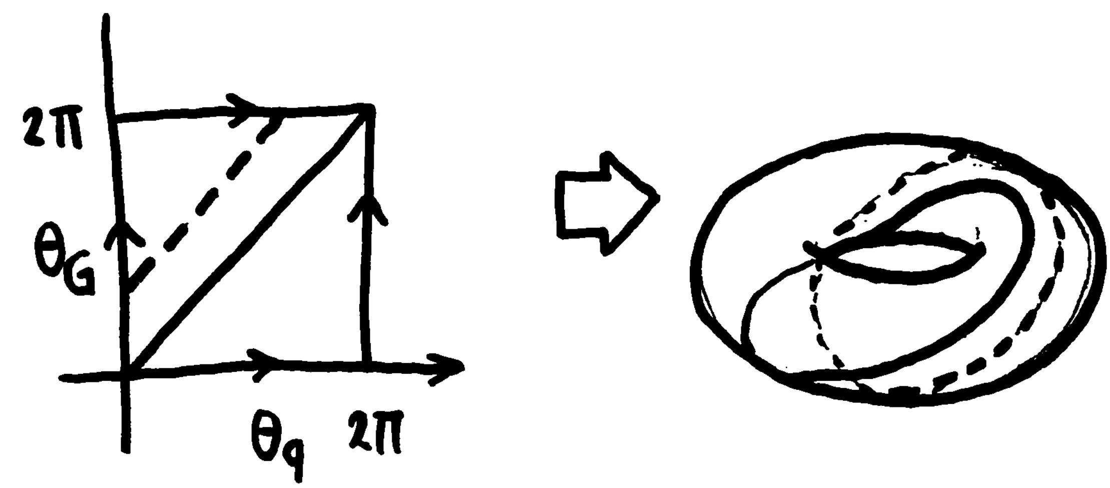

Figure 2.2: The equivalent classes of QCD Lagrangian

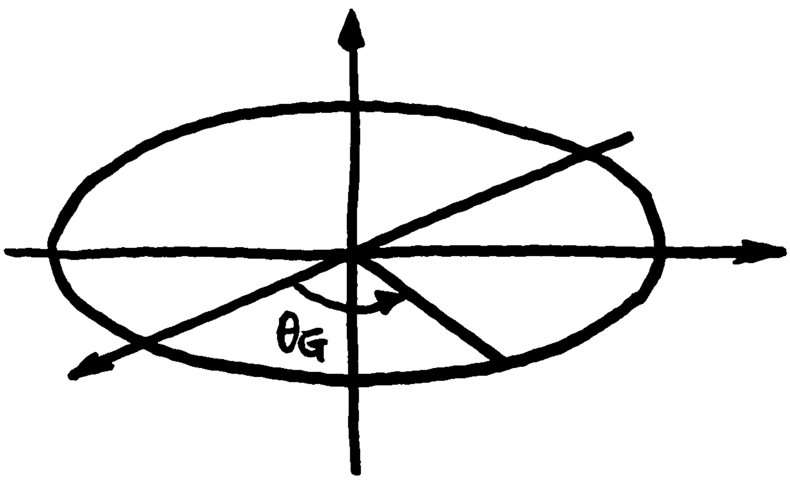

Figure 2.3: The chiral limit of the CP torus of QCD

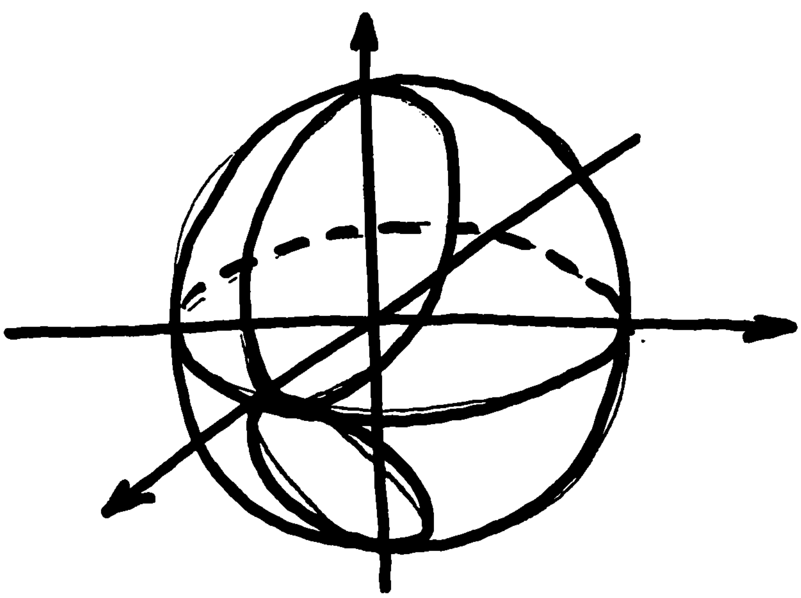

Figure 2.4: The chiral symmetric CP torus of QCD

2.4 QCD with Many Quark Flavors and Crewther's Conditions on Quark Condensates

Up to this point, our discussion of P and T violations in the strong interaction has been based on the simplest picture — QCD with one quark flavour. The reason behind this simplification is clear: the existence of the $U_A(1)$ chiral anomaly and the physics of reparametrization invariance are easily explained in this case. That is, the essential features are flavour independent. The generalization to many quark flavours seems to be straightforward, at least in the case of degeneracy, where all quarks have the same mass.

However, in the real world, not only do we have N_f ($N_f = 6$) quark flavours, but, in addition, they are not degenerate in mass. This flavour asymmetry means that $SU(N_f)$ is not a good symmetry of QCD (a continuous symmetry breaking), which makes a generalization of our previous discussion less trivial:

1. With many quark flavours, we have many quark chiral phases θ_q (associated with the mass terms) in the QCD Lagrangian. At first sight, this seems to increase the number of unknown parameters for strong $\not{P}$ and $\not{T}$ in QCD. However, this is not the case because the reparametrization invariance is enlarged to a greater "symmetry"[28], which helps to reduce the number of physically relevant parameters. In particular, we can apply chiral rotations to each quark flavour independently without affecting the physics. Also, all quarks interact with gluons with an universal constant g_s (as a consqence of the nonabelian gauge symmetry), and the change of gluon phase θ_G under any chiral rotation is additive among quark flavours. These two facts lead to a set of chirally invariant numbers $\bar{\theta}_q$ (flavour - dependent) which we need to determine (together with the chirally covariant phases θ_{G_q}). This is the main subject of this section.

2. On the other hand, since the mass parameters ($m_{q_i}, i = 1, 2, ..., N_f, q_i = u, d, s, ..$) play important roles in P and T violations in QCD (*àla* chiral limit constraint), the naive expectation that all $\not{P}$ and $\not{T}$ observables have to be proportional to mass parameters leads to an apparent contradiction with the

[28] It is inappropreiate to think of this "symmetry" as physical symmetry, as physical states are not invariant under such "symmetry" transformations.

decoupling theorem[29], To resolve this apparent parodox, we would like to see how the contributions of heavy quarks ($M_Q \gg \Lambda_{QCD}$) get suppressed in the hadronic $\not P$ and $\not T$ observables. Indeed, we can limit the quark flavours to the " nuclear domain " (keep only u and d quarks in QCD). Even there, it is still of interest to see how isospin symmetry breaking is coupled with $\not P$ and $\not T$ in QCD.

In view of these subtleties, we need to see how the flavor degrees of freedom affect the previous discussion:

1. **Number of $\not P$ and $\not T$ parameters in QCD**

   Once there are N_f quark flavors in the QCD Lagrangian, the number of $\not P$ and $\not T$ terms consistent with the requirements discussed in sec. 2-1 is $N_f + 1$; while each quark flavor contributes a different pseudo–mass term $m_{q_i}\bar{q}_i e^{i\theta_{q_i}\gamma_5} q_i$ (no summation over i) [30] , the gluon anomaly term $\frac{g_s^2\theta_G}{32\pi^2}G\tilde{G}$ stays the same. Since we can perform a chiral rotation on a single quark flavor q_i, which amounts to redefining the values of θ_{q_i} and θ_G at the same time without changing the physics (reparametrization invariance), the difference between the gluon phase θ_G and the quark phase θ_{q_i} must be a chiral invariant under the chiral rotations of the corresponding flavor, which we define as $\bar{\theta}_{q_i}$. In addition, in the multiflavor case, the reparametrization invariance of the QCD generating functional is enlarged, such that the simltaneous redefinitions of the quark phases $\theta_{q_i}, i = 1, 2, ..., N_f$ leave the physics unchanged, provided the gluon phase θ_G changes accordingly[31].

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[29] That is, the effect of heavy particles on the low–energy physics should vanish as the masses of heavy particles go to infinity. This can be proven in field theory, e.g. the Symanzik – Appelquist – Carrazzone theorem [35].

[30] The possible mixing matrix $m_{q_{ij}}\bar{q}_i e^{i\theta_{q_{ij}}\gamma_5} q_j$ can always be diagonalized by a redefinition of quark flavors (via an unitary matrix in the flavor space).

[31] To be more precise, under a general chiral transformation:

$$q_i \to q_i' \equiv e^{-i\frac{\alpha_i}{2}\gamma_5} q_i i = 1, 2, ..., N_f$$

, the quark phase θ_{q_i} changes by α_i and the gluon phase θ_G changes by $\sum_{i=1}^{N_f} \alpha_i$, the difference

$$\theta_G' - \sum_{i=1}^{N_f} \theta_{q_i}' = (\theta_G + \sum_{i=1}^{N_f} \alpha_i) - \sum_{i=1}^{N_f} (\theta_{q_i} + \alpha_i) = \theta_G - \sum_{i=1}^{N_f} \theta_{q_i}$$

From this, it should be clear that we have only one parameter ($(N_f+1)-N_f = 1$) which characterizes the strength of $\not{P}$ and $\not{T}$ in QCD with N_f quark flavors. Therefore, we define

$$\bar{\theta} \equiv \theta_G - \sum_{i=1}^{N_f} \theta_{q_i} \tag{2.35}$$

which is the single chirally invariant parameter on which all the strong $\not{P}$ and $\not{T}$ observables depend. , we have

$$\bar{\theta} = \sum_{i=1}^{N_f} \bar{\theta}_{q_i} \tag{2.36}$$

2. **The flavor dependence of the quark chiral phases**

As shown in the previous section (See sec.2.2), for any given quark flavor

$$\alpha_q \equiv \tan^{-1}\left(\frac{\langle i\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}}{\langle \bar{q}q\rangle_{\theta_q,\theta_G}}\right) \tag{2.37}$$

transforms (under $U_A(1)$) like θ_q Therefore, $\alpha_q - \theta_q$ is a chirally invariant phase for the i–th quark. We define this chirally invariant difference as $\bar{\theta}_q \equiv \alpha_q - \theta_q$

To actually determine the values of $\bar{\theta}_q$, we need the following information:

3. **The value of $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$ through the anomalous Ward identity**

The anomalous Ward identity for the flavour singlet axial current

$$J_\mu^5 \equiv \sum_{j=1}^{N_f} \bar{q}_j\gamma_\mu\gamma_5 q_j \tag{2.38}$$

reads

$$\partial^\mu J_\mu^5 = 2i\sum_{j=1}^{N_f}(m_{q_j}\bar{q}_j e^{i\theta_{q_j}\gamma_5} q_j) + \frac{g_s^2}{32\pi^2}G\tilde{G} \tag{2.39}$$

Taking the vacuum expectation values on both sides of the equation and using translational invariance, which holds true in the constant background field

is invariant.

method, (so that $\langle \partial^\mu J^5_\mu \rangle_{\theta_q,\theta_G} = 0$), we get

$$2i \sum_{j=1}^{N_f} m_{q_j} \langle \bar{q}_j e^{i\theta_{q_j}\gamma_5} q_j \rangle_{\theta_q,\theta_G} = \frac{-g_s^2}{32\pi^2} \langle G\tilde{G} \rangle_{\theta_q,\theta_G} \tag{2.40}$$

The left hand side of this equation can be further simplified using the polar representation for the quark condensates,

$$2i \sum_{j=1}^{N_f} m_{q_j} R_{q_j} \sin\bar{\theta}_{q_j} = \frac{-g_s^2}{32\pi^2} \langle G\tilde{G} \rangle_{\theta_q,\theta_G} \tag{2.41}$$

and the value of the anomalous gluonic condensate can be determined exactly.

Three comments can be made at this point:

(a) eq.2. 36, being an exact identity of QCD, seems to indicates that $g_s^2 \langle G\tilde{G} \rangle_{\theta_q,\theta_G}$ receives contributions from all quark flavours, which is not true. Because in the definitions of the vacuum condensates, we really need to specify an infrared cutoff, or, a factorization scale between the perturbative Wilson coefficients and the nonperturbative vacuum matrix elements. In the low energy calculation, such cutoff is chosen around the typical hadronic scale ($\approx \Lambda_{QCD}$) and certainly the virtual contributions of heavy quarks should not be included in the vacuum condensates, or equivalently, The chiral radii for heavy quark flavour $R_Q = 0$. Thus, the anomalous gluon condensate only receives contributions from light quarks (For which $N_f = 3$, and $q_j = u, d, s$)

(b) If we postulate a "equipartition rule" for the light quark contributions to the anomalous gluon condensate $\langle G\tilde{G} \rangle_{\theta_q,\theta_G}$, that is,

$$m_{q_i} R_{q_i} \sin\bar{\theta}_{q_i} = m_{q_j} R_{q_j} \sin\bar{\theta}_{q_j} \;\; i,j = 1,2,3 \tag{2.42}$$

We can simplify the Eq.(2.) into the followings,

$$\frac{-g_s^2}{32\pi^2} \langle G\tilde{G} \rangle_{\theta_q,\theta_G} = 2iN_f m_{q_j} R_{q_j} \sin\bar{\theta}_{q_j} \text{No sum over } j \tag{2.43}$$

Furthermore, these conditions, along with the phase addition rule Eq(2.), enable us to slove for the (flavour dependent) invariant chiral angles $\bar{\theta}_q$

in terms of the quark masses m_q and chiral radii R_q. If we take the first order expansion in $\frac{m_q}{\Lambda_{QCD}}$ of the Eq(2.), the chiral radii would be the same $R_{q_i} = R_{q_j}$ and we obtain

$$\bar{\theta} = \sum_{i=1}^{N_f} \bar{\theta}_{q_i} \tag{2.44}$$

$$m_{q_i} \sin \bar{\theta}_{q_i} = m_{q_j} \sin \bar{\theta}_{q_j} i, j = 1, 2, 3 \tag{2.45}$$

Indeed, this conclusion was first derived by R.J. Crewther [36] from a different reasoning. That is, by studying the stability of massless QCD under the perturbation of quark mass terms (which break the $SU_V(N_f) \times SU_A(N_f)$ symmetry explicitly), Crewther was able to derive the same conditions (Eq(2.33) and Eq(2.37)). For this reason, we shall refer to these two sets of formulae as **Crewther's conditions.** Combining these results (N_f equations in total), we can solve for $\bar{\theta}_q$ (as functions of quark masses m_q, chiral radius R_q and $\bar{\theta}$). In particular, in the case of single quark flavour, these conditions reduce to what we have found in the previous section. For two quark flavours, we can write down the analytical solutions to these equations.

$$\tan \bar{\theta}_d = \frac{m_u \sin \bar{\theta}}{m_d + m_u \cos \bar{\theta}} \tag{2.46}$$

$$\tan \bar{\theta}_u = \frac{m_d \sin \bar{\theta}}{m_u + m_d \cos \bar{\theta}} \tag{2.47}$$

For the many flavour case ($N_f \geq 3$), we have no explict solutions. However, the important physics (decoupling of heavy particles[32] from low-energy observables) can be visualized via the following graphical representation: that is, If we plot circles corresponding to various quark flavours, with all the centers located at the origion and radius equal to $m_q R_q$, the solutions to the Crewther's conditions come out as the intersection points

[32] We have to be careful on the definition of heavy particles and the applicability of decoupling theorem in this case. For the following discussion, heavy means $m_u/m_q, m_d/m_q \ll 1$, but $m_q/\Lambda_{QCD} \leq 1$ and we shall apply the decoupling theorem in the limit with m_q/Λ_{QCD} fixed and $m_q \to \infty$.

of a fixed horizontal line (from second condition) with the quark circles, see the following figure:

Thus, as heavier quarks have larger radii $m_q R_q$ and therefore smaller $\bar{\theta}_q$, we see that the $\not{P}$ and $\not{T}$ parameter $\bar{\theta}$ is indeed domainated by light quarks. This fact can also be understood analytically in a special limit (which seems to be the case in practice). Namely, assuming all the phases are very small. In this limit, we can write the Crewther's conditions as:

$$\bar{\theta} = \sum_{i=1}^{N_f} \bar{\theta}_{q_i} \tag{2.48}$$

$$m_{q_i}\bar{\theta}_{q_i} = m_{q_j}\bar{\theta}_{q_j} \;\; i,j = 1,2,3 \tag{2.49}$$

Solving these equations, we have

$$\bar{\theta}_{q_i} = \frac{\bar{\theta}}{(m_{q_i}R_{q_i})\sum_j \left(\frac{1}{m_{q_j}R_{q_j}}\right)} \tag{2.50}$$

In the real world, where $N_f = 3$, we have only one "massive" strange quark s with mass m_s around 150Mev, which is close to the Λ_{QCD} scale (250 $\sim$ 300Mev. As m_s is much larger than m_u and m_d, we get (in the small angle limit)

$$\bar{\theta}_u = \frac{m_d m_s \bar{\theta}}{m_u m_d + m_u m_s + m_d m_s} \rightarrow \frac{m_d \bar{\theta}}{m_u + m_d} \tag{2.51}$$

$$\bar{\theta}_d = \frac{m_u m_s \bar{\theta}}{m_u m_d + m_u m_s + m_d m_s} \rightarrow \frac{m_u \bar{\theta}}{m_u + m_d} \tag{2.52}$$

$$\bar{\theta}_s = \frac{m_u m_d \bar{\theta}}{m_u m_d + m_u m_s + m_d m_s} \rightarrow 0 \tag{2.53}$$

in accord with the small angle solutions to the $N_f = 2$ QCD, (compared Eq(2.) in the all angle limit)

(c) The topological susceptibility constant of QCD, χ is defined as (ref)

$$\chi \equiv \chi(q^2 = 0) \equiv \lim_{q^2 \to 0} \int d^{2\omega}x e^{iqx} \langle T \frac{g_s^2}{32\pi^2} G\tilde{G}(x), \frac{g_s^2}{32\pi^2} G\tilde{G}(0)\rangle_{\bar{\theta}=0} \tag{2.54}$$

which can be shown to be equal to

$$\chi = \frac{d}{d\bar{\theta}} \langle \frac{g_s^2}{32\pi^2} G\tilde{G}\rangle_{\theta_q=0,\theta_G=\bar{\theta}} \tag{2.55}$$

(The easiest way to see this is to use a functional integral representation). Using the result of the anomalous Ward identity Eq(2.) and taking the derivative with respect to $\bar{\theta}$, we have

$$\chi = N_f \left[\frac{\prod_{j=1}^{N_f}(m_{q_j} R_{q_j})}{\sum_{k=1}^{N_f}(\prod_{l \neq k} m_{q_l} R_{q_l})} \right] \tag{2.56}$$

$$= 3 \frac{(m_u R_u)(m_d R_d)(m_s R_s)}{(m_u R_u)(m_d R_d) + (m_u R_u)(m_s R_s) + (m_d R_d)(m_s R_s)} \tag{2.57}$$

This again, was discovered by various authors (E. Witten, G. Veneziano [37]) , using different techniques (large N_c QCD). However, our expression has the advantage that this is an exact identity in terms of the quark - gluon variables.

4. Symmetry constraints

Now we need to turn to the most important part of our discussion — symmetry constraints on $\not{P}$ and $\not{T}$ in multiflavour QCD :

(a) **Chiral limit constraint :**

For many quark flavors, the statement (*) should be generalized to the following:

If QCD contains any massless quark, there is no strong $\not{P}$ and $\not{T}$ even with a non-zero $\bar{\theta}$.

This can be understood in two ways:

First of all, if there is a massless quark in QCD, the chiral angle associated with that particular quark is undefined. We can then take advantage of this fact to choose a new chiral angle for that flavour, such that the $\bar{\theta}$ thus obtained vanishes.

Second, we can see this through Crewther's conditions. If any quark becomes massless, the second condition forces other massive quarks to adjust their $\bar{\theta}_q$ to equal zero. Thus $\bar{\theta}$ receives its entire contribution from the massless quark, and this renders $\bar{\theta}$ to become an irrelevant parameter.

(b) **The necessity of spontaneous chiral symmetry breaking in QCD**

Notice that in Crewther's condition, the mass parameter m_q is always multiplied by the chiral radius R_q. Therefore, the same argument can be made for the requirement of spontaneous chiral symmetry breaking $\lim_{m_q \to 0} R_q \neq 0$ in order to generate $\not{P}$ and $\not{T}$ in QCD.

(c) **Anomaly contribution**

Now it should be clear that the third constraint is not independent from the first two, if we assume the anomalous Ward identity. However, to see the details of how this constraint is satisfied, we need to generate some solutions from the QCD sum rules, which we shall study in Chapter 5.

2.5 Summary and Conclusion

We introduce the strong $\not{P}$ and $\not{T}$ in QCD, using the functional integral formalism (sec.2.2). Where, due to the existence of QCD chiral anomaly, the two possible $\not{P}$ and $\not{T}$ terms, allowed by the general requirements in RQFT (as discussed in sec.2.1) only give rise to one physically relevant (for $\not{P}$ and $\not{T}$) parameter, $\bar{\theta}$. The other degree of freedom implies a reparametrization invariance of the QCD generating functional, which can be used to check the consistency of our calculations.

The connection of chiral symmetry breaking (both explicit and spontaneous) and strong $\not{P}$ and $\not{T}$ in QCD are explained in the context of symmetry constraints (sec.2.3). These constraints, together with a nonvanishing $\bar{\theta}$, consist of sufficient and necessary conditions for strong $\not{P}$ and $\not{T}$ in QCD.

The generalization to multiflavour case is discussed in sec.2.4, where we can determine all the invariant chiral phases $\bar{\theta}_q$ through the Crewther's conditions and the contribution of s quark can be shown to be small. Finally, we examine the previous theoretical studies in strong $\not{P}$ and $\not{T}$ problems form the point of symmetry constraints, and several comments related to the QCD sum rule approach in connection with the symmetry constraints are made in sec.2.5.

Chapter 3

STUDY OF THE NUCLEON CORRELATION FUNCTIONS (NCF) FROM THE HADRON DEGREES OF FREEDOM

3.1 Electromagnetic Properties of Spin 1/2 Particles

Electric dipole moments, along with other observables (e.g. magnetic moment) are basic quantities we use to characterize the electromagnetic properties of elementary particles. These quantities can be measured in the labortory using external electromagnetic probes and reflect the internal structure of these elementary particles. In principle, any theory we use to describe the inner constituents and dynamics of these elementary particles should be able to provide a quantative explanation of these EM observables. Thus, even though our main interest is to study the possible $\not{P}$ and $\not{T}$ in strong interaction, it turns out that the calculation of a NEDM is not totally independent from the other P and T conserving EM observables. Also, to perform such a calculation starting from a RQFT requires some special machinery which relates physical observables to some theoretically calculable objects. It is to these points that we turn next.

If we try to make a list of EM observables for elementary particles, the first and simplest quantity which comes to one's mind would be the charge (denoted as Q). This is a conserved quantum number (due to the symmetry of global phase invariance) and can be related to the EM current through the continuity equation.

$$\partial_\mu j^\mu = 0, \quad Q(t) = \int d^3\vec{x} j_0(x), \quad \frac{dQ(t)}{dt} = 0 \tag{3.1}$$

The second property of an elementary particle with spin is magnetic moment (denoted as μ_N). This is a characterization of the interaction between the spin of the particle and a magnetic field (both are pseudovectors and their interaction conserves both P and T). If we try to generalize this list and remind ourselves of the parallelism between electricity and magnetism, it is tempting to include the magnetic monopole and electric dipole moment, which correspond to charge and magnetic moment, respectively[1]. The latter particpates in a $\not{P}$ and $\not{T}$ interaction with an electric field. As explained in the previous chapter (see sec.1.2), if we ignore polarization variables, these are all the intrinsic EM observables associated with a spin 1/2 particle. The real question is, how do we calculate these numbers from QCD?

To answer such a question and to set up a proper notation for future reference, it is useful to write down the mathematical expressions describing the electromagnetic properties of nucleons. Not too surprisingly, everything we wish to calculate in a quantum theory has to be in the form of a matrix element. To generate these EM observables of nucleons, we need to examine the nucleon (denoted as N) matrix element of an EM current (J_μ). In the momentum space, this can be written as:

$$\begin{aligned} V_\mu(q;p_1,p_2) &\equiv \int d^{2\omega}x e^{iqx}\langle N(p_2)|J_\mu(x)|N(p_1)\rangle \\ &= (2\pi)^{2\omega}\delta^{2\omega}(q-p)\langle N(p_2)|J_\mu(0)|N(p_1)\rangle \\ &\equiv (2\pi)^{2\omega}\delta^{2\omega}(q-p)V_\mu(p_2,p_1) \end{aligned} \tag{3.2}$$

where we have assumed translational invariance (which leads to the momentum conserving delta function) and the momentum transfer is defined as

$$p \equiv p_2 - p_1 = q$$

The EM current $J_\mu(x)$ is defined to be the Noether's current associated with a global $U(1)$ symmetry (charge conservation) of the theory. In QCD, $J_\mu(x)$ can be written as the sum of quark currents over different flavours,

$$J_\mu(x) \equiv \sum_f e_f \bar{q}_f(x)\gamma_\mu q_f(x) \tag{3.3}$$

One can check that, for a free quark, the matrix element of an EM current between two one–particle states is

$$V_q^\mu(p_2,p_1) = \bar{u}(p_2)\gamma^\mu u(p_1) \tag{3.4}$$

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[1] The possible existence of a magnetic monopole is an interesting problem, but we shall not discuss it in this work.

where $u(p_i)$ is a Dirac spinor with 4–momentum p_i satisfying $\hat{p}_i u(p_i) = m_q u(p_i)$

The γ^μ factor appearing between two Dirac spinors is normally called an "interaction vertex"; it also appears in the diagrammatical expansion of field theoretical calculations.

We can further simplify the EM matrix element V^μ by doing a nonrelativistic expansion, assuming $p_1/m_q, p_2/m_q \ll 1$. After that, we obtain an expression for the charge and magnetic moment of a free quark:

$$V_q^\mu(p_2, p_1) \rightarrow e_q \psi^\dagger \{1 + \frac{e_q \hbar}{2m_q} \vec{\sigma}\vec{q}\} \psi \tag{3.5}$$

where ψ is the large component of the Dirac spinor $u(p_i)$, and we obtain a value of the magnetic moment for a free quark, $\frac{e_q \hbar}{2m_q}$. On the other hand, for a strongly interacting theory like QCD, there is no simple way to calculate a quark EM matrix element. Furthermore, since we do not know the solution for a nucleon as a bound state of quarks and gluons, an analytical calculation of nucleon EM matrix elements from the QCD Lagrangian seems to be intractable. However, based on the transformation properties of the EM matrix elements under the Lorentz group (That is, the EM matrix element V_μ transforms like a vector, as the EM current J_μ does.), we can make a general parametrization of these EM matrix elements in terms of some invariant functions, generally referred as form factors; these form factors summarize the detailed information from the complicated dynamics.

To see this, we first use a Dirac spinor (with a mass m_N) to represent a spin 1/2 nucleon and then factorize the nucleon EM matrix element into a product of a Dirac spinor with its conjugate and an interaction vertex. That is,

$$V_N^\mu(p_2, p_1) = \bar{u}(p_2) \Gamma_N^\mu(p_2, p_1) u(p_1) \tag{3.6}$$

Compared with the free quark case, we find that a single constant matrix γ^μ is replaced by a momentum dependent 4×4 matrix. What we need is to decompose the general matrix $\Gamma_N^\mu(p_2, p_1)$ into 16 independent Dirac matricies and extract the momentum dependence in terms of various invariant functions. This is not a trivial task, as we have two independent momenta p_1, p_2; they can give rise to 3 scalars p_1^2, p_2^2 and $p_1 \cdot p_2$. However, the factorization of V_μ into a bilinear form in the Dirac spinors has some important consequence. That is, by using the Dirac equations for the nucleon spinor and its conjugate, we can eliminate mutual dependent combinations. The bottom line is, for the on shell nucleon states, we can decompose the EM matrix elements into 6 terms, and define 6 indepent form factors depend on one scalar variable q^2 only.

$$\begin{aligned}\Gamma^{\mu}_{N}(p_2, p_1) &= \tilde{F}_1(q^2)\cdot p^{+}_{\mu} + \tilde{F}_2(q^2)\cdot p^{-}_{\mu} + \tilde{F}_3(q^2)\cdot \gamma_{\mu} \\ &\quad + \tilde{F}_4(q^2)\cdot p^{+}_{\mu}\gamma_5 + \tilde{F}_5(q^2)\cdot p^{-}_{\mu}\gamma_5 + \tilde{F}_6(q^2)\cdot \gamma_{\mu}\gamma_5 \end{aligned} \tag{3.7}$$

where

$$p^{+} \equiv p_2 + p_1 \text{ and } p^{-} \equiv p_2 - p_1 \tag{3.8}$$

These independent form factors can be further simplified by invoking two conditions: (1) current conservation $\partial_{\mu} j^{\mu} = 0$ and (2) Hermiticity $V^{\mu}_{N}(p_2, p_1) = [V^{\mu}_{N}(p_2, p_1)]^{\dagger}$. These condition implies $\tilde{F}_2 = 0$ and $\frac{q^2}{2m_q}\tilde{F}_5 = \tilde{F}_6$ and we are left with 4 independent form factors. Using some Dirac algebra, we can derive the following

$$V^{\mu}_{N}(p_2, p_1) = F_1(q^2)\gamma^{\mu} + iF_2(q^2)\frac{\sigma^{\mu\nu}q_{\nu}}{2m_q} + F_3(q^2)\frac{\sigma^{\mu\nu}\gamma_5 q_{\nu}}{2m_q} + F_4(q^2)(q^2\gamma^{\mu} - q^{\mu}\hat{q})\gamma_5 \tag{3.9}$$

The decomposition of the interaction vertex into 16 Dirac matrices is not unique because one can freely add terms which vanish upon using the Dirac equation.

Simalarily, by doing a nonrelavistic reduction, one can show that[2]

$$eF_1(q^2 = 0) = Q(\text{ charge }) \tag{3.10}$$

$$\frac{e}{2m}[F_1(q^2 = 0) + F_2(q^2 = 0)] = \mu(\text{ magnetic moment }) \tag{3.11}$$

$$-\frac{e}{2m}[F_3(q^2 = 0)] = d(\text{ electric dipole moment }) \tag{3.12}$$

In addition to these mathematical formulations, we would like to emphasize the physical meaning associated with the nucleon EM matrix element. In view of the lack of a solution for the nucleon wave function, the inner structure of nucleons and their interaction with an EM field are ascribed to a modification of the interaction vertex. In addition to the single vector coupling like a quark–photon–quark vertex, there are other (scalar, tensor and their parity counterparts) types of coupling. Also, the fact that a nucleon has a finite size can be taken into account by generalizing a coupling

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[2] The last form factor $F_4(q^2 = 0)$ is called anapole moment.

constant to a momentum dependent form factor. Our purpose here is to relate this matrix element to a correlation function which we can calculate and use the result to extract the EM moments. This will be the main subject of this chapter.

3.2 Correlation Functions of Interpolating Nucleon Fields in the Presence of External Electromagnetic Fields

- The necessity of introducing an interpolating field in the QCD sum rule calculations

We have described in the previous section that the nucleon matrix element of the EM current operator properly gives rise to 4 independent form factors F_1, F_2, F_3 and F_4. When evaluated at $q^2 = 0$, these are the response coefficients (charge, anomalous magnetic moment, electric dipole moment and anapole moment) which characterize the electromagnetic properties of the spin 1/2 nucleons. In order to compare theoretical predictions with experimental data, one needs to calculate the matrix elements starting from a model for nucleons (which contains the constituents and the interaction among them inside the nucleons) and the interaction of the nucleons with EM fields. The latter is described by the theory of Quantum Electrodynamics (QED), which is a weak coupling theory at low energy and has a classical limit in the long wavelength approximation[3]. Therefore, we can treat the EM fields as classical background and expand the matrix element as power series in the electric charge, assuming higher order terms are negliable. As to the former, it requires a knowledge of the constituents and the interaction which binds them together to form a nucleon state. Now we believe Quantum Chromodynamics (QCD) is such a theory which is responsible for the inner structure of nucleons. Unfortunately, in contrast to the brillant success of QED, the task of describing properties of nucleons from the QCD Lagrangian has proven to be extremly difficult.

- The use of an interpolating field

The composite field operators are usually called "interpolating fields (currents)", in the sense that all the information we need from a nucleon state are summarized in

[3] This is called the Thirring theorem.

the matrix elements of these composite field operators between the physical vacuum and the nucleon state. These matrix elements can be thought as a substitute for the complete wave functions of the nucleon state in the Fock space.

The choice of the nucleon interpolating field is not unique. The number of valid interpolating fields for a given physical state is related to the representation theory of $SU(N_f)$ quark flavour symmetry and Lorentz group, which we shall not discuss in this work[4]. However, the basic idea is to construct composite operators with proper quantum numbers and definite transformation properties under various symmetry group such that the overlap between nucleon states and the interpolating field is maximized[5].

It sould be emphasized that the reason for introducing of an interpolating field is to enables us to perform calculations in terms of microscropic degrees of freedom. The physical obserables of a composite state should not depend on the different choice of the interpolating fields. More discussion will be given on sec.5.3.

3.3 The Invariant Tensor Structures of NCF

- The tensor basis for NCF

As we have emphasized before, the basic idea of the QCD sum rule method is to represent a chosen correlation function in terms of both hadron and quark–gluon pictures. The matching of these two pictures is at the heart of QCD sum rules, from which one can extract useful information of hadron observables in terms of QCD parameters. It is crucial to realize that from a matching of these two pictures of a single correlation function, one can generate several independent sum rules. Because these independent sum rules help us to extract different observables from a single hadronic correlation function. Also, these independent sum rule relations serve as mutual consistency checks for our calculation.

To derive these independent sum rules, we need to decompose the NCFs into a linear combination of various (Lorentz) tensor (Dirac) matrices, with the numerical

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[4] For further details, please refer to App.D.

[5] Ideally, one can choose an interpolating field which couples only to the physical nucleon state and has zero overlap with any higher states. However, this is equivalent to an exact solution for a nucleon wave function.

coefficients as functions of invariant momentum squared p^2. This is a general feature of the NCFs which must be shared by both hadronic models and QCD calculations. Therefore, once we choose a set of independent tensor matrices, which we shall refer to as tensor basis, we should express both representatons of the NCFs in terms of that basis; the QCD sum rules are derived by identifing the invariant coefficient functions associated with each independent tensor matrix from both hadron and quark–gluon representations.

The number of independent basis tensors and sum rules has to do with the spin (and also other internal quantum number) structure of the problem at hand, the choice of currents, the states we are interested, the observables we are after, etc.. In the present case, the NCF $\Pi_N(p)$ consists of a nucleon propagator $\Pi_N^{(0)}(p)$ and a polarization tensor $\Pi_N^{\mu\nu}(p)$, which correspond to a scalar matrix and a tensor matrix, respectivly. Since we have only one momentum variable p_μ, the construction of the tensor basis amounts to finding all possible combinations of the momentum variable p_μ and 16 Dirac matrices that make up the required Lorentz structures. There are many ways to carry out such a construction and it is not a trival task to convince oneself that we have a complete answer. Fortunately, there is a systematic way to generate a basis set which we discuss in detail in App.B . For the moment, we shall simply state the results for the NCFs, without assuming any discrete symmetries P, T and C.

For the nucleon propagator $\Pi_N^{(0)}(p)$, there are 4 independent tensors:

$$I, \quad \gamma_5, \quad \hat{p} \equiv p_\mu \cdot \gamma^\mu, \quad \hat{p}\gamma_5 \tag{3.13}$$

Notice that there is no basis tensor with dimension equal or greater than 2. This is because we have only one independent momentum p_μ, and all higher dimensional scalar matrices ($\propto p^2, p^3$, etc.) can be reduced to a scalar function of p^2 times one of these 4 basis tensors.

For the polarization tensor $\Pi_N^{\mu\nu}(p)$, the tensor basis consists of 8 independent second rank tensor matrices:

$$\sigma^{\mu\nu} \qquad \sigma^{\mu\nu}\cdot\gamma_5 \tag{3.14}$$

$$p^\mu\gamma^\nu - p^\nu\gamma^\mu \qquad (p^\mu\gamma^\nu - p^\nu\gamma^\mu)\cdot\gamma_5 \tag{3.15}$$

$$\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta \qquad \epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta\cdot\gamma_5 \tag{3.16}$$

$$\hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu) \qquad \hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu)\cdot\gamma_5 \tag{3.17}$$

As in the case of the nucleon propagator $\Pi_N^{(0)}(p)$, there is no basis tensor with dimension equal or greater than 3.

- Charge conjugation, chiral rotations and the tensor basis

These basis tensors have definite transformation properties under C, P and T, which add further constraints on the sum rule structures. In the case of strong $\not{P}$ and $\not{T}$, the charge conjugation C is still an exact symmetry. This implies that the coefficient function of $\hat{p}\cdot\gamma_5$ in the nucleon propagator $\Pi_N^{(0)}$ and those associated with $(p^\mu\gamma^\nu - p^\nu\gamma^\mu)$ and $\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta$ in the polarization tensor $\Pi_N^{\mu\nu}$ vanish. Consequently, in the current situation, we have three independent sum rules from the nucleon propagator $\Pi_N^{(0)}$ and six from the polarization tensor $\Pi_N^{\mu\nu}$[6].

Another important feature has to do with the chiral property of these basis tensors. One can show that, under a $U_A(1)$ transformation, those tensors with odd dimensions (also with odd number of gamma matrices) are invariant under a chiral conjugation, while those with even dimensions are covariant (they change by an overall chiral phase). Consequently, it is convenient to combine these even tensors of parity doublets (those differ by a γ_5) in a polar form, such that the $U_A(1)$ chiral covariant form is manifest in our sum rule relations.

Given these classifications of the basis tensors for the NCFs, we can then define various coefficient functions associated with these basis tensors.

$$\Pi_N(p) \equiv \Pi_N^{(0)}(p) + e\Pi_N^{\mu\nu}(p)F_{\mu\nu} + O(e^2) \tag{3.18}$$

$$\Pi_N^{(0)}(p) \equiv f_1(p^2)\cdot\hat{p} + \tilde{f}_2(p^2)\cdot I + i\tilde{f}_3(p^2)\cdot\gamma_5 \tag{3.19}$$

$$\equiv f_1(p^2)\cdot\hat{p} + f_2(p^2)\cdot e^{i\phi p^2\gamma_5} \tag{3.20}$$

with

$$f_2^2(p^2) \equiv \tilde{f}_2^2(p^2) + \tilde{f}_3^2(p^2) \tag{3.21}$$

[6] Not all independent sum rules are equally useful for extracting the hadronic observables, this will depend on their convergence property of the OPE series and the stability against the variations of other adjustable variables (e.g. Borel mass, continuum threshold... etc.). Further complications will be discussed in later sections (see sec. 6-4).

$$\tan\phi(p^2) \equiv \frac{\tilde{f}_3(p^2)}{\tilde{f}_2(p^2)} \tag{3.22}$$

$$\begin{aligned}
\Pi_N^{\mu\nu}(p) &\equiv \tilde{g}_1(p^2) \cdot \sigma^{\mu\nu} && + \tilde{g}_2(p^2) \cdot i\sigma^{\mu\nu}\gamma_5 \\
&+ g_3(p^2) \cdot i\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta\gamma_5 && + g_4(p^2) \cdot i(p^\mu\gamma^\nu - p^\nu\gamma^\mu)\gamma_5 \\
&+ g_5(p^2) \cdot \hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu) && + g_6(p^2) \cdot i\hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu)\gamma_5 \\
&\equiv g_1(p^2) \cdot \sigma^{\mu\nu}e^{i\varphi_1 p^2\gamma_5} && \\
&+ g_3(p^2) \cdot i\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta\gamma_5 && + g_4(p^2) \cdot i(p^\mu\gamma^\nu - p^\nu\gamma^\mu)\gamma_5 \\
&+ g_2(p^2) \cdot \hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu)e^{i\varphi_2 p^2\gamma_5} &&
\end{aligned}$$

with

$$g_1^2(p^2) \equiv \tilde{g}_1^2(p^2) + \tilde{g}_2^2(p^2) \tag{3.23}$$

$$\tan\varphi_1(p^2) \equiv \frac{\tilde{g}_2(p^2)}{\tilde{g}_1(p^2)} \tag{3.24}$$

$$g_2^2(p^2) \equiv g_5^2(p^2) + g_6^2(p^2) \tag{3.25}$$

$$\tan\varphi_2(p^2) \equiv \frac{g_6(p^2)}{g_5(p^2)} \tag{3.26}$$

The discussion in the rest of this chapter will be devoted to the construction of a hadronic parametrization for all these invariant coefficients, including all these nucleon EM moments and some other model parameters.

3.4 The Contribution of Hadronic States to the Invariant Tensor Structures of the Nucleon Propagator

In the previous sections, we have shown that a NEDM d_N can be extracted from the nucleon matrix element of an EM current operator ($d_N = F_3^N(0)$). With the introduction of the interpolating nucleon fields $\eta_N, \bar{\eta}_N$ and their correlation functions, the nucleon matrix element of an EM current operator can be embeded into the nucleon correlation functions in the presence of an external EM field. The purpose of this and the following sections is to show, by inserting complete sets of hadronic states into the NCF, we can decompose the NCF into various contributions of hadronic states and express the NCF in terms of hadronic variables, which include real physical observables, e.g. NEDM, and other model parameters representing the contributions of higher states (resonances + continuum).

Begin with the nucleon propagator $\Pi_N^{(0)}$, wich is a two – point correlation function. We insert a complete set of hadron states between the composite operators η_N, $\bar{\eta}_N$.

$$\Pi_N^{(0)}(p) \equiv \int d^{2\omega}x e^{ipx} \langle \Omega | T(\eta_N(x), \bar{\eta}_N(0)) | \Omega \rangle_{\theta_q,\theta_G} \tag{3.27}$$

$$\equiv \sum_N \int d^{2\omega}x e^{ipx} \theta(x_0) \langle \Omega | \eta_N(x) | N \rangle_{\theta_q,\theta_G} \langle N | \bar{\eta}_N(0) | \Omega \rangle_{\theta_q,\theta_G}$$
$$-\theta(-x_0) \langle \Omega | \bar{\eta}_N(0) | \bar{N} \rangle_{\theta_q,\theta_G} \langle \bar{N} | \eta_N(x) | \Omega \rangle_{\theta_q,\theta_G} \tag{3.28}$$

This amounts to, roughly speaking, a factorization of the nucleon propagator into two wave functions, one for the initial state, the other for the final state. To make the meaning more precise, we need to define the matrix element of the interpolating field between spin 1/2 states and the QCD vacuum Ω.

$$\langle \Omega | \eta_N | N(\vec{p}, s_N) \rangle_{\bar{\theta}} \equiv \lambda_N e^{i\frac{\theta_N}{2}\gamma_5} u(\vec{p}, s_N) \tag{3.29}$$

Here N stands for any physical spin 1/2 state with the same quantum number as the interpolating field η_N and λ_N gives the overlap amplitude of the state N with a 3 quark state. Finally, because of the reparametrization degrees of freedom of quark fields and the presnece of strong $\not{P}$ and $\not{T}$, the quark field, hence the nucleon matrix element may not be an eigenstate of P and T. This effect is taken care of by putting in a nucleon chiral phase $e^{i\frac{\theta_N}{2}\gamma_5}$. Notice that θ_N is not a fixed physical observables; its value will depend on the choice of QCD generating functional. The necessity of a nucleon chiral phase has been emphasized in sec.2.3 from the point of nonperturbative symmetry constraint. Another way to see this is that as we calculate the nucleon propagator to extract the nucleon mass M_N without an inclusion of a nucleon chiral phase, we shall find a nonvanishing contribution to the γ_5 sum rule, which indicates that the nucleon state is not an eigenstate of helicity operator. Therefore we need to perform a chiral rotation to the nucleon state such that an intrinsic EDM with respect to the physical nucleon state is well–defined.

With this new feature, we can proceed to simplify various time orderings and Fourier transforms. The result, for the lowest nucleon state, is

$$e^{i\frac{\alpha_N}{2}\gamma_5} \frac{\lambda_N}{\hat{p} - M_N} e^{i\frac{\alpha_N}{2}\gamma_5} = \lambda_N \frac{(\hat{p} + M_N \cdot e^{i\alpha_N\gamma_5})}{p^2 - M_N^2} \tag{3.30}$$

Any higher state gives a similar result with different numerical values for λ_N, α_N and M_N.

Since we are only interested in the ground state property, we do not want to complicate our calculation by involving the details of the hadronic spectrum. Furthermore, if we include too many unknowns in our calculation ,then this approach has any predictive power. For these reasons, we need to sum over all higher state contributions and make an effective model. This is normally done in the sum rule approach by using a "duality ansatz". Namely, we replace the contribution of higher states by the perturbative quark–gluon contribution in the high energy region. Using the dispersion relation, this translates into a model for the imaginary part of the correlation function at high $Q^2 \equiv -p^2$, which we shall choose to be a step function $\theta(Q^2 - s_N^0)$ times the contributions of leading (in terms of dimension) terms in the OPE calculations. Once we make this choice, the only model parameter in the hadronic representation of the nucleon propagator $\Pi_N^{(0)}$ is the starting point s_N^0 of the higher state, which is normally called "continuum threshold".

It should be emphasized that we do not have any a priori estimate of the importance of the contribution of higher states to the NCF; neither is it clear how we can isolate the ground state matrix element from the contamination of these uncertainty due to higher states[7]. What we can do at best is to choose several sum rules which have similar higher state contributions, and by taking suitable linear combinations of these sum rules to eliminate such dependences.

In short, the success of any QCD sum rule calculations has to rely on the insensetivity of the crude approximations for the hadron model used in the calculations. In particular, the physical observables extracted from the sum rule calculations must have a weak dependences on the model parameters over a reasonable range.

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[7] Unlike the lattice calculations, where the correlation functions can be calculated systematically through the Monte Carlo technique, and the ground state dominance in the correlation functions is achieved if we let the correlation functions evolve in Euclidean time long enough (all higher states are surpressed exponentially). In the QCD sum rule approach, the longest "time" or maximum suppression of higher state contributions is limited by the convergence radius of the OPE series in the left hand side.

3.5 The Contribution of Hadronic States to the Invariant Tensor Structures of the Polarization Tensor

The decomposition of the polarization tensor $\Pi_N^{\mu\nu}(p)$ into various hadronic contributions is more complicated than the previous case. Since the polarization tensor $\Pi_N^{\mu\nu}(p)$ comes from the insertion of an interaction Lagrangian $\int d^4z\mathcal{L}_{int}(z)$ into the 2 – point correlation function of the interpolating fields, i.e. the nucleon propagator $\Pi_N^{(0)}(x)$, we have a 3 – point (x,z,0) product describing a hadron–photon–hadron interacting vertex. In order to rewrite the polarization tensor in terms of hadron variables, we need to insert two hadronic complete sets in between three different space–time locations. Roughly speaking, this implies that for a given pair of inserting states, we can factorize the polarization tensor into a product of 2 hadron propagators and a vertex matrix element coupled to a static external EM field A^μ.

$$
\begin{aligned}
e\Pi_N^{\mu\nu}(p)F_{\mu\nu} &\equiv \int d^{2\omega}x e^{ipx}\int d^{2\omega}z\langle T\mathcal{L}_{int}(z)\eta_N(x)\bar{\eta}_N(0)\rangle_{\theta_q,\theta_G} && (3.31)\\
&= \sum_{N,N'}\int d^{2\omega}x e^{ipx}\int d^{2\omega}z\theta(x-z)\theta(z-0)\times\\
&\times \langle\Omega|\eta_N(x)|N\rangle_{\theta_q,\theta_G}\langle N|J_\mu(z)|N'\rangle_{\theta_q,\theta_G}\langle N'|\bar{\eta}_N(0)|\Omega\rangle_{\theta_q,\theta_G}A^\mu(z)+\\
&+ \text{time ordering} && (3.32)
\end{aligned}
$$

where the vector potential A^μ of a constant EM field $F_{\mu\nu}$ is given by

$$A^\alpha \equiv \frac{1}{2}x_\beta F^{\beta\alpha} \tag{3.33}$$

If we write down the matrix element in the complete Fock space of hadrons, we get a huge matrix which can be divided into 3 regions as follows (See table 3.1):

(1) Region 1 contains only a single matrix element, where N is the lowest–lying nucleon state and this matrix element gives us all the EM response coefficients we have described in detail in sec.3-1

(2) Region 2 contains all the transitions, with N' representing any excited state higher than N. This region is sometimes referred to as off-diagonal transitions for obvious reasons. Notice that such transitions are allowed in a static external background (with no input momentum flow) because the nucleon and excited states we are considering are highly off–shell ($Q^2 \gg M_N^2$) states.

Table 3.1: Nucleon to higher state EM transition matrix element

$\Pi_N^{\mu\nu}$	N	N''
N	$\langle N\vert J_\mu\vert N\rangle_{\theta_q,\theta_G}$ *Region*1	$\langle N\vert J_\mu\vert N''\rangle_{\theta_q,\theta_G}$ *Region*2
N'	$\langle N'\vert J_\mu\vert N\rangle_{\theta_q,\theta_G}$ *Region*2	$\langle N'\vert J_\mu\vert N''\rangle_{\theta_q,\theta_G}$ *Region*3

(3) Region 3 consists of all those contributions from the EM form factors and transition amplitudes of excited states, which we shall treat as an effective resonance without bothering with the fine details of the complete hadronic spectrum.

With all this classification at hand, we proceed in the following three subsections to write down the effective hadronic model for the polarization tensor. All the invariant coefficient functions $g_i, i = 1, ..., 6$, as defined in sec.3.3, will be decomposed in three parts corresponding to the three regions tabulated above. Thus, $g_2 \equiv g_2^{(1)} + g_2^{(2)} + g_2^{(3)}$ and so on.

3.5.1 The Contribution of the Nucleon State to the Invariant Tensor Structures of the Polarization Tnesor

For a given pair of nucleon states N_1, N_2, the contribution to the polarization tensor can be visualized as a nucleon –photon–nucleon vertex. In a constant external EM field, the intial momentum is equal to the final one, and we have only one momentum variable p_μ. It is not too difficult to see, as in the case of the nucleon propagator, that the representation dependence of the QCD Lagrangian can be taken care of by taking the chiral conjugation $e^{i\frac{\theta_N}{2}\gamma_5}$ of an intrinsic $\not{P}$ and $\not{T}$ nucleon –photon–nucleon vertex, which contains a nonzero $F_3(0)$, or equivalently a nonzero α_N (remember that $\tan\alpha_N \equiv \frac{F_3}{F_2}$).

Putting all of the above together, we get the contribution of the nucleon state to the polarization tensor (We factor out an overall constant $\frac{\lambda_N^2 F_{\mu\nu}}{4M_N(p^2-M_N^2)^2}$:

$$g_1^{(1)}(p) = 2i\cos\theta_N M_N^2 F_1 + i\cos\theta_N(p^2 + M_N^2)F_2 - \sin\theta_N(p^2 - M_N^2)F_3 \quad (3.34)$$

$$g_2^{(1)}(p) = 2i\sin\theta_N M_N^2 F_1 i\sin\theta_N(p^2 + M_N^2)F_2 + \cos\theta_N(p^2 - M_N^2)F_3 \quad (3.35)$$

$$g_3^{(1)}(p) = 2M_N(F_1+F_2) \qquad g_4^{(1)}(p) = 2M_N F_3 \tag{3.36}$$
$$g_5^{(1)}(p) = (-2)[\cos\theta_N F_2 + \sin\theta_N F_3] \quad g_6^{(1)}(p) = (-2)[\sin\theta_N F_2 + \cos\theta_N F_3] \tag{3.37}$$

3.5.2 The Contribution of Nucleon–to–Excited State Transitions to the Invariant Tensor Structures of the Polarization Tensor

The second region which contains nucleon to higher states transitions requires some special treatment. Since it is impossible to take care of the complete details of the hadronic spectrum, we need to sum over all contributions from higher states and keep only a few model parameters. For this reason, we define the following matrix element:

$$\begin{aligned}\langle\Omega|\eta_N(0)|N\rangle_{\bar\theta,F_{\mu\nu}\neq 0} &\equiv \int d^{2\omega}z\langle\Omega|T(\mathcal{L}_{int}(z)\eta_N(0))|N(\vec p,s_N)\rangle_{\bar\theta} - \text{pole term}\\ &\equiv [\,\frac{E_B}{M_N}e^{i\frac{\varphi_B}{2}\gamma_5}(p^\mu\gamma^\nu - p^\nu\gamma^\mu) + \frac{E_A}{M_N}e^{i\frac{\varphi_B}{2}\gamma_5}\sigma^{\mu\nu}\,]\,u(\vec p,s_N)\end{aligned} \tag{3.38}$$

Here we introduce 4 unknown model parameters: $E_A, \varphi_A, E_B, \varphi_B$; with E_A, E_B invariant under a $U_A(1)$ rotation, and the phases φ_A, φ_B transform covariantly.

The contribution of the nucleon to higher states transitions to the polarization tensor can be obtained if we substitute eq() to the following:

$$\begin{aligned} e\Pi_N^{\mu\nu}(p)F_{\mu\nu} &\equiv \int d^{2\omega}x e^{ipx}\langle\Omega|T(\eta_N(x),\bar\eta_N(0))|\Omega\rangle_{\bar\theta,F_{\mu\nu}}\\ (N\to N') &\Rightarrow \sum_N\int d^{2\omega}x e^{ipx}\theta(x_0)\langle\Omega|\eta_N(x)|N\rangle_{\bar\theta,F_{\mu\nu}\neq 0}\langle N|\bar\eta_N(0)|\Omega\rangle_{\bar\theta,F_{\mu\nu}=0} +\\ &\quad + \theta(x_0)\langle\Omega|\eta_N(x)|N\rangle_{\bar\theta,F_{\mu\nu}=0}\langle N|\bar\eta_N(0)|\Omega\rangle_{\bar\theta,F_{\mu\nu}\neq 0} +\\ &\quad -\theta(-x_0)\langle\Omega|\bar\eta_N(0)|\bar N\rangle_{\bar\theta,F_{\mu\nu}\neq 0}\langle\bar N|\eta_N(x)|\Omega\rangle_{\bar\theta,F_{\mu\nu}=0} -\\ &\quad -\theta(-x_0)\langle\Omega|\bar\eta_N(0)|\bar N\rangle_{\bar\theta,F_{\mu\nu}=0}\langle\bar N|\eta_N(x)|\Omega\rangle_{\bar\theta,F_{\mu\nu}\neq 0}\end{aligned} \tag{3.39}$$

After collecting terms for those tensor basis of the polarization tensor, we obtain (after factor out the overall constant tensor $\frac{e\lambda_N^2 F_{\mu\nu}}{4(p^2-M_N^2)}$)

$$g_1^{(2)}(p) = 2E_A M_N\cos(\frac{\varphi_A}{2}+\frac{\theta_N}{2}) \tag{3.40}$$
$$g_2^{(2)}(p) = 2E_A M_N\sin(\frac{\varphi_A}{2}+\frac{\theta_N}{2}) \tag{3.41}$$
$$g_3^{(2)}(p) = -2iE_A\cos(\frac{\varphi_A}{2}-\frac{\theta_N}{2}) \tag{3.42}$$

$$g_4^{(2)}(p) = 2i[E_A \sin(\frac{\varphi_A}{2} - \frac{\theta_N}{2}) - E_B \sin(\frac{\varphi_B}{2} - \frac{\theta_N}{2}) \quad (3.43)$$

$$g_5^{(2)}(p) = 2i\frac{E_B}{M_N}\cos(\frac{\varphi_B}{2} + \frac{\theta_N}{2}) \quad (3.44)$$

$$g_6^{(2)}(p) = 2i\frac{E_B}{M_N}\sin(\frac{\varphi_B}{2} + \frac{\theta_N}{2}) \quad (3.45)$$

3.5.3 The Contribution of the Continuum to the Invariant Tensor Structures of the Polarization Tensor

The treatment of the continuum contribution to the polarization tensor is based on the same principle as in the case of the nucleon propagator. Namely, we shall assume the duality ansatz and take the leading terms in the OPE series of the QCD calculation at high energy to model the continuum contribution, using the same threshold parameter s_N^0. Since it is a standard practice to shift the continuum contribution to the left hand side of the QCD sum rules, the detailed calculation from quark–gluon degrees of freedom will be discussed in the next chapter.

3.6 Summary and Conclusion

The nucleon matrix elements of the EM current operator are used to describe their EM properties. In particular, the invariant form factors appearing in the matrix elements, evaluated at $q^2 = 0$, can be identified as EM response coefficients (charge, anamolous magnetic moment, electric dipole moment and anapole moment) for the nucleons.

In order to calculate these matrix elements from a microscopic theory (QCD), we introduce an interpolating field for the nucleon state and embed the EM matrix elements in the correlation function of the interpolating field (Nucleon Correlation Function, NCF) in the presence of an external EM field.

In the context of strong $\not{P}$ and $\not{T}$, the breakdown of P and T symmetries leads to a multitude of tensor structures, which we use to decompose the NCFs. With the help of charge conjugation (C) and the $U_A(1)$ chiral transformation properties, we can organize these covariant tensors in a compact form such that the resulting formulae reflect the reparametrization covariance of the QCD generating functional explictly. See the discussion in sec.3.3.

The NCF, expanded to first order of electric charge, consists of a nucleon propagator and the product of a polarization tensor and the external EM field tensor. Both can be expressed in terms of hadronic variables by inserting complete sets of hadronic states. Dividing the complete spectrum into a nucleon state plus higher states (resonances + continuum) and summing over the total contribution from the higher states in a parametrized form, we obtain an effective hadron model for the NCF. In so doing, not only can we build in the quantities of interest, e.g. NEDMs, to the NCFs, but we also can show that our general parametrizations of higher state contributions to NCFs obey the general requirements that follow from charge conjugation and $U_A(1)$ chiral transformation.

Chapter 4

STUDY OF THE NUCLEON CORRELATION FUNCTIONS (NCF) FROM THE QUARK – GLUON DEGREES OF FREEDOM

4.1 Explanations of the General Scheme of Our Calculations (Method and Approximations)

- The method of operator product expansion

To study the nucleon correlation functionfrom the quark—gluon degrees of freedom in QCD, we use the method of operator product expansion (OPE). This is a genaral method in QFT, and can be used to extract useful information from the corrrelation functions[1]. The basic idea of the OPE is that, given a product of various operators (with different space–time locations, or momentum dependence), we can expand the product into a series of local operators, weighted by numerical functions (called Wilson coefficients). The series is organized in order of increasing dimensions of the local operators, and the space–time or momentum dependence of the original product is lumped into the Wilson coefficients. In the coordinate space representation, we can expand the correlation functions in powers of space–time coordinates x_μ (referred to as a short distance expansion, which is like a Taylor expansion in field theory.). After taking the Fourier transformation of the correlation functions to momentum space, we obtain a seriers in $1/Q^2$ (referred to as a $1/Q^2$ expansion[2], where $Q^2 \equiv -p^2$, with p the momentum variable conjugates to the coordinate variable x).

To really make use of the expansion series, which is an operator identity, we need to take matrix elements of the time ordered product and its operator expansion

[1] The classical example of the OPE method in QCD is the study of deep inelastic scattering (DIS) of leptons off nuclear targets, where the naive parton model interpretation and the prediction of scale invariance violations in the cross sections can be formulated in a rigorous and quantitative manner through the use of the operator formalism.

[2] Using dimensional analysis, one can show that lower dimensional operators carry more singular Wilson coefficients, or less powers in $1/Q^2$, so that all the terminologies are consistent.

series with respect to some physical state. In the current study, we choose this state to be the QCD vacuum (see the discussion in sec.3-3 for the use of the interpolating field). The vacuum expectation value (V.E.V.) of the time ordered product of two interpolating field operators gives the nucleon correlation function, which was studied in the hadronic picture in the last chapter. On the other hand, the vacuum expectation values of these local operators in the expansion series become various condensates which we have discussed extensively in chap. 2. If we need to determine the NCFs as functions of Q^2 from the microscopic theory, to compare with the result from a hadronic model of the NCF, we have to calculate these expansion coefficients in the OPE series, in addition to the input values of various condensates. Henceforth, our task in this chapter is to determine the Wilson coefficients of the nucleon correlation functionin the presence of an external EM field.

- The approximation scheme in our calculation of the OPE series of the NCFs

Given the powerful and general applicability of the OPE method in QFT, we have to confess that in general, the series can not be calculated exactly and certain truncations and approximation have to be made in most realistic applications. Since the OPE series of the NCFs is organized in order of increasing dimensions of the local operators, or, more precisely, as an expansion of Λ^2_{QCD}/Q^2, we shall truncate the series at dimension 6 as an approximation. In addition, the Wilson coefficients, normally calculated by perturbative method, are expanded as a power series in g_s–the strong coupling constant. We only keep terms up to first order in g_s (coming from the mixed condensates $\langle \bar{q}Gq\rangle_{\theta_q,\theta_G}$) in the calculations of various Wilson coefficients[3]. Finally, as we have to include the quark masses in our calculation, the presence of such small parameters, as compared to the intrinsic QCD scale– Λ_{QCD}, will be treat as corrections to the massless theory. For this reason, we shall expand the Wilson coefficients with respect to $m_q\Lambda_{QCD}/Q^2$ and only keep terms up to first order in the quark masses[4].

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[3] Hence the gluonic fluctuation in the physical vacuum, which is at least second order in g_s, will be ignored in our calculation.

[4] There are also nonanalytic terms, which are propotional to $m_q ln\frac{m_q^2}{Q^2}$, contribute to the Wilson coefficients of some infrared singular operators, e.g. $m_q F_{\mu\nu}$ and $F_{\mu\nu}G\tilde{G}$. While these contributions could be numerically important and closely related to the chiral behaviour of physical observables,

- Operator product expansions of the NCFs in the presence of an external field

One feature is specific to our approach: Since we use a classical EM field as an external background, rather than including QED into the full Lagrangian[5], the series of the OPE includes not only the usual scalar (or pseudoscalar) operators but also the (Lorentz) tensor operators (e.g. $\bar{q}\sigma_{\mu\nu}q$) whose expectation values with respect to the polarized vacuum can have non–zero values. These induced condensates are proportional to the external EM field $F_{\mu\nu}$ (and its dual $\tilde{F}_{\mu\nu}$) times some susceptibility constants (except for some dimensional constant, which is normally taken to be the quark condensate $\langle\bar{q}q\rangle_{\theta_q,\theta_G}$.). Thus, the use of the external field method greatly increases the number of of input parameters in our problem. In addition, since our main concern is the study of $\not{P}$ and $\not{T}$ in QCD, many operators are members of a parity doublet in the OPE series of the NCF, e.g. $\bar{q}G_{\mu\nu}q$ and $\bar{q}G_{\mu\nu}\gamma_5 q$. In view of these complications, it is helpful to start with a classification of operators in QCD according to their dimensions. This will serve as a complimentary check with our method of expanding the NCF. Furthermore, as we shall see later, the properties (dimension, chirality ... etc.) of various operators have important consequences for our problem.

4.2 Classification of P and T Violating Operators in QCD (with external EM fields)

We divide the operators appearing in the OPE series of the NCF into two classes[6]:

(1) The (Lorentz) scalar operators: which appear in both the sum rules for the nucleon propagator $\Pi_N^{(0)}(p)$ and the polarization tensor $\Pi_N^{\mu\nu}(p)$.

(2) The (second rank) tensor operators: which appear only in the polarization tensor $\Pi_N^{\mu\nu}(p)$. The V.E.V.s of these operators are induced condensates, which will be discusssed in sec.4.3.

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we manage to avoid these difficulties by taking suiutable combinations of QCD sum rules to eliminate their existence.

[5] Otherwise, we will be calculating a 3-point function with a nucleon–nucleon–photon vertex.

[6] The reason that we have no vector or pseudovector operators is because we only keep gauge invariant operators.

Table 4.1: Basic building blocks for constructing operators in QCD

operators in QCD	scalar	tensor
dim 0	I	NA
dim 1	m_q	$D_\mu \equiv \partial_\mu - ig_s B^a_\mu \frac{\lambda^a}{2}$
dim 3/2	NA	$q, \bar{q}$
dim 2	NA	$F_{\mu\nu}, \tilde{F}_{\mu\nu}; G_{\mu\nu}, \tilde{G}_{\mu\nu}$

The operators will be listed according to their dimensions, and only operators with dimensions less or equal to 6 will be kept.

To begin with, let us write down the basic "building blocks" for composite operators (see Table 4.1):

All composite operators can be built from these elements by (a) doing dimensional analysis, (b) applying Dirac algebra and (c) use equations of motion. After these procedures, we obtain a list in Table 4.2:

We should emphasize that:

(1) Not all of these operators are independent.

Since we shall take expectation values of these operators with respect to the physical vacuum (V.E.V.), one can use equations of motion for quark and gluon fields, valid for physical states, to eliminate redundant operators (in the sense that they give the same V.E.V.). For example, $\langle \bar{q}\gamma_\mu D_\nu q - (\mu \leftrightarrow \nu)\rangle_{\theta_q, \theta_G} = 0$ and $\langle \bar{q}\gamma_5\gamma_\mu D_\nu q - (\mu \leftrightarrow \nu)\rangle_{\theta_q, \theta_G} \propto m_q \bar{q}\sigma_{\mu\nu} q$.

(2) Almost all operators come in pairs (parity doublets, or parity partners), with the partner generated by a multiplicative factor γ_5 or by a dual transformation ($\tilde{A}_{\alpha\beta\mu\nu} \equiv \frac{1}{2}\epsilon_{\alpha\beta\xi\eta}A^{\xi\eta\mu\nu}$).

Again, some of these pairs are not independent; they can be related to other condensates by simple algebraic relations. For example, $\bar{q}\gamma_5\tilde{G}_{\mu\nu}q \propto \bar{q}G_{\mu\nu}q$. Nevertheless, after these manipulations, on account of P and T violations, some of these pairs seem to be new and require extra information (new susceptibility constants, for example.). For example, both $\bar{q}G_{\mu\nu}q$ and $\bar{q}\gamma_5 G_{\mu\nu}q$ can be induced by $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$, and it is not clear how to relate their susceptibility constants.

However, we shall see in the next section that there are generalized chiral circle

Table 4.2: A classification of operators in QCD up to dimension 6

Lorentz structure v.s. dimension	scalar or (pseudoscalar)	antisymmetric tensor or (pseudotensor)
dim 0	I	NA
dim 1	m_q	NA
dim 2	NA	$F_{\mu\nu}, \tilde{F}_{\mu\nu}$
dim 3	$\bar{q}q, \bar{q}\gamma_5 q$	$m_q F_{\mu\nu}, m_q \tilde{F}_{\mu\nu}$ $\bar{q}\sigma_{\mu\nu}q, \bar{q}\sigma_{\mu\nu}\gamma_5 q$
dim 4	$m_q\bar{q}q, m_q\bar{q}\gamma_5 q$ $G^2 = -\tilde{G}^2, G\tilde{G}$	$m_q\bar{q}\sigma_{\mu\nu}q, m_q\bar{q}\sigma_{\mu\nu}\gamma_5 q$ $\bar{q}\gamma_\mu D_\nu q - (\mu \leftrightarrow \nu),$ $\bar{q}\gamma_5\gamma_\mu D_\nu q - (\mu \leftrightarrow \nu)$
dim 5	$\bar{q}\sigma \cdot Gq, \bar{q}\gamma_5\sigma \cdot Gq$ $\bar{q}\sigma \cdot \tilde{G}q, \bar{q}\gamma_5\sigma \cdot \tilde{G}q$ $m_q G^2 = -m_q\tilde{G}^2$ $m_q G\tilde{G}$	$F_{\mu\nu}\bar{q}q, F_{\mu\nu}\bar{q}\gamma_5 q$ $\tilde{F}_{\mu\nu}\bar{q}q, \tilde{F}_{\mu\nu}\bar{q}\gamma_5 q$ $\bar{q}G_{\mu\nu}q, \bar{q}\gamma_5 G_{\mu\nu}q$ $\bar{q}\tilde{G}_{\mu\nu}q, \bar{q}\gamma_5\tilde{G}_{\mu\nu}q$ $\bar{q}\sigma_{\mu\alpha}G^\alpha_\nu q - (\mu \leftrightarrow \nu)$ $\bar{q}\gamma_5\sigma_{\mu\alpha}G^\alpha_\nu q - (\mu \leftrightarrow \nu)$ $\bar{q}\sigma_{\mu\alpha}\tilde{G}^\alpha_\nu q - (\mu \leftrightarrow \nu)$ $\bar{q}\gamma_5\sigma_{\mu\alpha}\tilde{G}^\alpha_\nu q - (\mu \leftrightarrow \nu)$
dim 6	$\bar{q}\Gamma q\bar{q}\Gamma q$ $m_q\bar{q}\sigma \cdot Gq, m_q\bar{q}\gamma_5\sigma \cdot Gq$ $m_q\bar{q}\sigma \cdot \tilde{G}q, m_q\bar{q}\gamma_5\sigma \cdot \tilde{G}q$ $\bar{q}G_{\alpha\beta}\gamma^\alpha D^\beta q, \bar{q}\gamma_5 G_{\alpha\beta}\gamma^\alpha D^\beta q$ $\bar{q}\tilde{G}_{\alpha\beta}\gamma^\alpha D^\beta q, \bar{q}\gamma_5\tilde{G}_{\alpha\beta}\gamma^\alpha D^\beta q$ $G^3, G^2\tilde{G}$	$\bar{q}\Gamma_1 q\bar{q}\Gamma_2 q$ $m_q F_{\mu\nu}\bar{q}q, m_q F_{\mu\nu}\bar{q}\gamma_5 q$ $F_{\mu\nu}G^2 = -F_{\mu\nu}\tilde{G}^2, F_{\mu\nu}G\tilde{G}; F_{\mu\nu} \leftrightarrow \tilde{F}_{\mu\nu}$ $\bar{q}G_{\mu\alpha}\gamma^\alpha D_\nu q - (\mu \leftrightarrow \nu)$ $\bar{q}\gamma_5 G_{\mu\alpha}\gamma^\alpha D_\nu q - (\mu \leftrightarrow \nu)$ $\bar{q}\tilde{G}_{\mu\alpha}\gamma^\alpha D_\nu q - (\mu \leftrightarrow \nu)$ $\bar{q}\gamma_5\tilde{G}_{\mu\alpha}\gamma^\alpha D_\nu q - (\mu \leftrightarrow \nu)$

theorems we can use to reduce the number of these "extra" unknowns, and all these parity doublet condensates are related by a single number, namely, the $\bar{\theta}$ parameter. Thus, the apparent difficulty of too many unknown parameters in the sum rule approach (with external field) is resolved.

4.3 Chiral Rotation, Chiral Phases of Quark Condensates and Susceptibility Constants

In this section, we shall discuss various induced condensates and their susceptibility constants, in order of increasing dimensions.

dimension 3

The simplest induced condensate in the presence of an external EM field $F_{\mu\nu}$ is $\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G}$. To the lowest order in the EM coupling constant, this second rank antisymmetric Lorentz tensor can be written as a linear combination of $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$:

$$\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G} \equiv e_q[u\cdot F_{\mu\nu} + v\cdot\tilde{F}_{\mu\nu}] + O(e_q^2 F_{\mu\alpha}F^{\alpha}{}_{\nu}, e_q^2 F_{\mu\alpha}\tilde{F}^{\alpha}_{\nu}) \tag{4.1}$$

Using the Dirac algebra $\sigma_{\mu\nu}\gamma_5 = \frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$, we have

$$\langle\bar{q}\sigma_{\mu\nu}\gamma_5 q\rangle_{\theta_q,\theta_G} \equiv u\cdot\tilde{F}_{\mu\nu} - v\cdot F_{\mu\nu} + O(e_q^2 F_{\mu\alpha}F^{\alpha}{}_{\nu}, e_q^2 F_{\mu\alpha}\tilde{F}^{\alpha}_{\nu}) \tag{4.2}$$

The linear coefficients u, v in front of $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ characterize the response of the QCD vacuum to the perturbations of an external EM field. Notice here a nonzero v implies that $\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G}$ can be induced by $\tilde{F}_{\mu\nu}$. This is because the quark fields in QCD with nonzero θ_q and θ_G are not eigenstates of P and T. In addition, one should be careful about interpreting u and v as physical observables, since the values of u and v clearly depend on the chiral basis of the quark fields in the QCD generating functional. In order to extract representation independent information, we need to examine the chiral transformation property of $\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G}$:

Under a $U_A(1)$ chiral rotation

$$q \rightarrow q' \equiv e^{i\frac{\theta}{2}\gamma_5}q, \qquad q = e^{-i\frac{\theta}{2}\gamma_5}q' \tag{4.3}$$

$$\begin{aligned}\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G} &= \langle\bar{q}\sigma_{\mu\nu}e^{-i\theta\gamma_5}q\rangle_{\theta_q-\theta,\theta_G-\theta} \\ &= \cos\theta\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q-\theta,\theta_G-\theta} - i\sin\theta\langle\bar{q}\sigma_{\mu\nu}\gamma_5 q\rangle_{\theta_q-\theta,\theta_G-\theta}\end{aligned} \tag{4.4}$$

This shows that, under a $U_A(1)$ chiral rotation, $\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G}$ transforms like $\langle\bar{q}q\rangle_{\theta_q,\theta_G}$. Similarily, one can show that $\langle\bar{q}\sigma_{\mu\nu}\gamma_5 q\rangle_{\theta_q,\theta_G}$ transforms like $\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}$. Therefore, we can generalize the chiral circle theorem to the following:

$$\langle\bar{q}q\rangle_{\theta_q,\theta_G}\cdot\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G}-\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}\cdot\langle\bar{q}\sigma_{\mu\nu}\gamma_5 q\rangle_{\theta_q,\theta_G}$$

is invariant under a $U_A(1)$ chiral rotation.

If we substitute the linear expansion form for $\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G}$, we have:

$$\begin{aligned}
\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G} &\approx e_q u\cdot F_{\mu\nu}+e_q v\cdot\tilde{F}_{\mu\nu}\\
&= \langle\bar{q}\sigma_{\mu\nu}e^{-i\theta\gamma_5}q\rangle_{\theta_q-\theta,\theta_G-\theta}\\
&\approx e_q\cos\theta(u'\cdot F_{\mu\nu}+v'\cdot\tilde{F}_{\mu\nu})-ie_q\sin\theta(\frac{i}{2}\epsilon_{\mu\nu\alpha\beta})(u'\cdot F^{\alpha\beta}+v'\cdot\tilde{F}^{\alpha\beta})\\
&= e_q(\cos\theta u'-\sin\theta v')F_{\mu\nu}+e_q(\sin\theta u'+\cos\theta v')\tilde{F}_{\mu\nu} \qquad (4.5)
\end{aligned}$$

Since the components of $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ are totally independent, by comparing the leading coefficients of these EM tensors in two representations, we get a relation between (u',v') and (u,v):

$$\begin{bmatrix} u \\ v \end{bmatrix}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\cdot\begin{bmatrix} u' \\ v'\end{bmatrix}$$

Such a relation indicates that (u,v) can be viewed as a two dimensional vector, and a $U_A(1)$ chiral rotation on the quark fields acts like a real rotation in this two dimensional vector space. We can then choose a polar representation for these two parameters (u,v):

$$u = \chi\cos\alpha_q \qquad (4.6)$$
$$v = \chi\sin\alpha_q \qquad (4.7)$$

Notice that

(1) The chirally invariant number χ is assumed to be flavour independent.

(2) The chirally covariant phase α_q transforms like θ_{Gq} under a $U_A(1)$ chiral rotation.

To see the last point, we need to combine the chiral tensor theorem with the polar representations for (u, v) and $\langle\bar{q}q\rangle_{\theta_q,\theta_G}$:

$$\begin{aligned}\langle\bar{q}q\rangle_{\theta_q,\theta_G}\cdot\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G}-\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}\cdot\langle\bar{q}\sigma_{\mu\nu}\gamma_5 q\rangle_{\theta_q,\theta_G}\\ = e_q\chi[\ \cos(\alpha_q-\theta_{Gq})F_{\mu\nu}+\sin(\alpha_q-\theta_{Gq})\tilde{F}_{\mu\nu}\ ]\end{aligned} \tag{4.8}$$

Thus, the chiral tensor theorem requires $\alpha_q - \theta_{Gq}$ to be proportional to the chirally invariant phase $\bar{\theta}$. If the proportionality constant is zero, or equivalently, $\alpha_q = \theta_{Gq}$, we have the following simple form for $\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G}$:

$$\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G} = e_q\chi[\langle\bar{q}q\rangle_{\theta_q,\theta_G}F_{\mu\nu}+i\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}\tilde{F}_{\mu\nu}] \tag{4.9}$$

As a side remark, we mention that there is no $m_q\langle G_{\mu\nu}\rangle_{\theta_q,\theta_G}$ induced condensate because gluons carry colors and we only keep color singlet condensates.

dimension 4

There are no dimension 4 induced condensates which require definitions of new susceptibility constants.

- $m_q\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G}$ and $m_q\langle\bar{q}\sigma_{\mu\nu}\gamma_5 q\rangle_{\theta_q,\theta_G}$ are trivial; we simply multiply the previous result by the quark masses.

- Using the equation of motion for quarks, we can show that $\langle\bar{q}\gamma_\mu D_\nu q\rangle_{\theta_q,\theta_G} - (\mu \leftrightarrow \nu) = 0$ and $\langle\bar{q}\gamma_\mu\gamma_5 D_\nu q\rangle_{\theta_q,\theta_G} - (\mu \leftrightarrow \nu) \propto m_q\langle\bar{q}\sigma_{\mu\nu}\gamma_5 q\rangle_{\theta_q,\theta_G}$.

- One can show that $G_{\mu\alpha}G^{\alpha}{}_{\nu}$ and $G_{\mu\alpha}\tilde{G}^{\alpha}_{\nu}$ are symmetrical in $(\mu \leftrightarrow \nu)$; hence they can not be induced by $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$.

dimension 5

At dimension 5, we need to define the susceptibility constants for the following induced mixed condensates:

$$\begin{gathered} g_s\bar{q}G_{\mu\nu}q \quad , \quad g_s\bar{q}\gamma_5 G_{\mu\nu}q; \\ g_s\bar{q}\sigma_{\mu\alpha}G^{\alpha}_{\nu}q-(\mu\leftrightarrow\nu) \quad , \quad g_s\bar{q}\gamma_5\sigma_{\mu\alpha}G^{\alpha}_{\nu}q-(\mu\leftrightarrow\nu)\end{gathered}$$

If we choose to ignore the presence of extra chirally invariant phases and take the following parametrizations

$$g_s\langle\bar{q}G_{\mu\nu}q\rangle_{\theta_q,\theta_G} \equiv \kappa_q F_{\mu\nu}\langle\bar{q}q\rangle_{\theta_q,\theta_G} - i\tilde{\kappa}_q\tilde{F}_{\mu\nu}\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G} \tag{4.10}$$

$$g_s\langle\bar{q}\gamma_5 G_{\mu\nu}q\rangle_{\theta_q,\theta_G} \equiv -i\xi_q\tilde{F}_{\mu\nu}\langle\bar{q}q\rangle_{\theta_q,\theta_G} + \tilde{\xi}_q F_{\mu\nu}\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G} \tag{4.11}$$

$$g_s\bar{q}\sigma_{\mu\alpha}G^{\alpha}_{\nu}q - (\mu \leftrightarrow \nu) \equiv \eta_q F_{\mu\nu}\langle\bar{q}q\rangle_{\theta_q,\theta_G} - i\tilde{\eta}_q\tilde{F}_{\mu\nu}\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G} \tag{4.12}$$

We can show that, by using a similar generalization of the chiral tensor theorem,

$$\tilde{\kappa}_q = \xi_q,\ \tilde{\xi}_q = \kappa_q,\ \tilde{\eta}_q = \eta_q \tag{4.13}$$

Taking the dual transformations for gluon fields and/or γ_5 matrix, we get all the other chiral doublet condensates in terms of the susceptibility constants κ_q, ξ_q, η_q and quark condensates $\langle\bar{q}q\rangle_{\theta_q,\theta_G}, \langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}$. If we assume that all the susceptibility constants are proportional to the quark charge e_q times a flavor independent constant, then we can rewrite the susceptibility constants $\kappa_q = e_q\kappa, \xi_q = e_q\xi, \eta_q = e_q\eta$.

We should emphasize that the susceptibility constants χ, κ, ξ also appear in the QCD sum rule calculations of the magnetic moments of the nucleons, e.g. see [21], therefore they are not new input parameters in the present calculations. However, we do find one dimension 5 induced mixed condensate

$$\langle\bar{q}\sigma_{\mu\alpha}G^{\alpha}_{\nu}q\rangle_{\theta_q,\theta_G} - (\mu \leftrightarrow \nu)$$

which are missing from the previous publications and such a condensate would give a contribution to the magnetic moments of the nucleons.

dimension 6

There are 4 classes of tensor condensates of dimension 6, class 1 and class 2 condensates have explicit EM field dependence. Therefore, no susceptibility constant is needed in these cases.

(1)

$$m_q F_{\mu\nu}\bar{q}q,\ m_q F_{\mu\nu}\bar{q}\gamma_5 q$$

which appear in the third order Taylor expansion of two point quark condensate $\langle\bar{q}(0)q(x)\rangle_{\theta_q,\theta_G}$

(2) For gluonic operators, we have

$$F_{\mu\nu}G^2 = -F_{\mu\nu}\tilde{G}^2, \qquad F_{\mu\nu}G\tilde{G}$$
$$F_{\mu\alpha}G^{\alpha\beta}\tilde{G}_{\beta\nu} - (\mu \to \nu) \;\propto\; F_{\mu\nu}G\tilde{G}$$

(3) There are two four-quark induced condensates:

$$\langle\bar{q}\gamma_\mu q\bar{q}\gamma_\nu q\rangle_{\theta_q,\theta_G} \;-\; (\mu \to \nu)$$
$$\langle\bar{q}\gamma_\mu q\bar{q}\gamma_\nu\gamma_5 q\rangle_{\theta_q,\theta_G} \;-\; (\mu \to \nu)$$

(4) There are also mixed condensates with an inseration of covariant derivatives,

$$\langle\bar{q}G_{\mu\alpha}\gamma^\alpha\vec{D}_\nu q\rangle_{\theta_q,\theta_G} - (\mu \to \nu) \tag{4.14}$$

Class (4) terms are also negelected in the previous sum rule calculations of the nucleon magnetic moments.

The last two classes of condensates are the first (in terms of dimension) chirally invariant induced quark condensates we encounter in the classification. The presence of such induced quark condensates causes trouble since we can no longer rely on the chiral circle theorem, which is very useful for chirally covariant condensates, to reduce unknown susceptibility constants. Thus, the inclusion of these chirally invariant induced condensates implies an increasing number of unknown parameters. What we can do at best, in the case of class 3 induced condensates, is to factorize these four–quark induced condensates into products of normal (but chirally covariant) quark condensates with previously defined susceptibility constants. This is in analogy to the vacuum saturation assumption (VSA) used to factorize a scalar four–quark condensate into products of normal quark condensates. As to the last class of induced condensates, a study show that we need 6 parameters to parametrize them; since this greatly increases the complexity of the problem, we shall choose to ignore these condensates and leave it to future studies.

4.4 Operator Product Expansion for Two–point and Three–point Condensates to Dimension 6

These consecutive two sections (sec.4.4 and sec.4.5) are devoted to some technical points of our calculations. Namely, the operator product expansion (OPE) of the nucleon correlation functions (NCF). As has been explained in sec.4-1, there are two ways to look at the OPE calculations; in the plane wave method, we can think of the expansion series as scattering processes, and interepret the Wilson coefficients as "transition amplitudes", which give the probabilities for a given initial state (as represented by the interpolating fields) to evolve into a particular final state (the chosen operators in the expansion series). This method has the advantage that it is easy to understand pictorially. Nevertheless, it requires considerable effort to carry out actual calculations, together with other technical difficulties mentioned before.

The second method (generalized Wick expansion + background field method), views the NCF as a propagator for the nucleon state, consisting of three valence quarks. the nontrival structure of QCD dynamics is included with the non–perturbative vacuum contributions which modify the quark and gluon propagators and also the quark – gluon vertex.

In practice, one can generate these contributions by inclusion of the normal ordering of operators in the usual Wick expansion of the correlation function. These normal orderings, after taking the expectation value with respect to QCD vacuum, become what we call "n–point condensates". Diagramatically, these n–point condensates can be generated from the usual pertuabative Feynman diagrams by cutting off the particle propagators (in the case of two–point condensate) and/or blurring out the vertices.

Two comments are in order:

1. It is in the second form that people usually present their calculations in QCD sum rules. In so doing, one should be careful not to interepret the NCF as a product of "non-perturbative" propagators of quarks and gluons. Because the non-perturbative contribution of the quark-gluon vertex is generally non-factorizable. Besides, if we need to do a higher order calculation of the Wilson coefficients (radiative correction in QCD), the quantum nature of the vacuum gluon fields should be taken into account explicitly (we ignore such effect in our approximation scheme). There again, the factorization ansatz will not give the correct answers.

2. To compute the Wilson coefficients of the n-point condensates, we need to expand these condensates into a series of local operators. These involve a Taylor

expansion of quark and/or gluon fields in the space-time coordinate x_μ. It is important to replace the normal derivative by a covariant derivative in order to make the whole expression gauge–invariant. Here the combinations of background field method and fixed point gauge make such a procedure simpler, and the addition of an extrnal EM field presents no new difficulty.

3. Every terms generated from the Taylor expansion of n-point condensates can be represented by a diagram. These new diagrams, combined with the normal Feynman rules and Feynman diagrams, can be used to constructed a diagramatical expansion (OPE series) of any correlation functions. We shall discuss this in the next section.

4.5 Operator Product Expansion for Nucleon Correlation Function to Dimension 6

Having studied the OPE for 2–point and 3–point condensates in the previous section, the calculations of the OPE for the NCF become more feasible. The central idea of our approach is to decompose the NCF $\Pi(p)$ into a series of condensates up to dimension 6:

The classification of various operators in sec.4.2 will help here to make sure that we do not lose any possible terms in the operator series. Also, the scheme we explained in sec.4.4 ensures that we have a systematic way of calculating various Wilson coefficients. Combining these ingredients, we shall present our results by the following steps:

step1 Write the generalized Wick expansion for NCF. Keep normal ordered operators with dimension less or equal to 6.

step2 Substitute OPE for 2-point (dimension 3 – in the case of quarks, or dimension 4 – in the case of gluons) and 3-point condensates (dimension 5) for the V.E.V.s of normal–ordered operators; 4-point condensates, normally come from 4-quark condensates, can be approximated by the vacuum saturation assumption, which essentially factorized the 4-quark condensates into product of normal quark condensates.

step3 Draw Feynman diagrams for each contribution and write down the formulae of loop integrals according to the (generalized) Feynman rules.

step4 Do the integral, collecting coefficients for the same operators (many diagrams could correspond to the same operator). and then tabulate the results according to the dimensions of various operators.

step5 Collecting terms for various basis tensors. Because of the relationship between dimension and chirality of condensates (even dimension ⇒ chirally invariant, odd dimension ⇒ chirally covariant.) Odd tensors only receive contributions from even operators and even tensors only receive contributions from odd operators. and we shall use polar form to represent chirally covariant coefficients.

We take the proton correlation as an example to demonstrate the proceses, the neutron correlation function can be easily obtained by changing an u-quark in the proton to a d-quark and all the related parameters (e.g. charges e_q, masses m_q, chiral radii R_q, etc.).

4.6 Summary and Conclusion

In this chapter, using the method of OPE, we study the nucleon correlation functions (NCF) from the quark–gluon degrees of freedom, with QCD as the underlying theory describing the interaction between these constituents of nucleons. The NCF is calculated in the presence of an external EM field, in order to extract the response coefficients of the nucleons (in our case, the quantity of interest is the electric dipole moment of nucleons). We explain the method and our approximation scheme in sec.4.1.

The presence of an external field, toegther with the nontrival structure of QCD vacuum, generate many new induced condensates, which appear in the OPE series of NCF and require definitions of various susceptibility constants. Furthermore, with the parity and time–reversal symmetry breaking effects coming from the $\bar{\theta}$ parameters, we get a doubling of vacuum condensates. A complete classification of scalar and tensor operators, up to dimension 6, is presented in sec.4.2. On the other hand, using a generalized chiral circle theorem, we can relate these chirally covariant operators and reduce the unknown parameters greatly in the calculation. This is discussed in sec.4.3.

Finally, the NCF is expanded into a series of vacuum condensates, weighted by various Wilson coefficients. We explain our method for calculating these coefficients and give results of the NCF for both neutron and proton in sec.4.4, 4.5. The QCD sum rules are obtained by identifying the hadronic model (see Chap.3) and the microscopic calculations (this chapter) of the NCF, which will be analyzed in the next chapter.

Chapter 5

ANALYSIS OF QCD SUM RULES (QSR) FOR NUCLEON CORRELATION FUNCTIONS (NCF) IN THE PRESENCE OF AN EXTERNAL ELECTROMAGNETIC FIELD: PART ONE

5.1 Outline of this Chapter

The QCD sum rules (QSRs) for nucleon correlation functions (NCFs) in the presence of an external electromagnetic field $F_{\mu\nu}$ will be analyzed in this chapter. The main emphasis will be on:

- The consistency of the QCD sum rules for NEDMs with symmetry constraints on strong P and T violations (see Chapter 2).
- An order of magnitude estimate of hadronic variables (e.g. nucleon masses M_N, residues λ_N, EM moments μ_N, d_N) extracted from the QCD sum rules.

This is a primary analysis in the sense that we shall treat the approximate sum rules (due to truncations of the OPE series and the hadronic parametrizations of the spectral functions) as exact identities, and solve for the hadronic observables as unknown variables in the sum rule equations, with the underlying philosophy explained below (see sec.5.3). In view of this, we shall not be concerned with an error analysis at this stage. A more complete study will be presented in next chapter.

A basic outline is as follows: The sum rules derived from the calculations of the nucleon correlation functions (NCFs) are summarized in sec.5.2, where we also introduce the Borel transform to improve the matching between the hadronic parametrizations of the NCFs (see Chap.3) and the QCD side of the sum rule calculation (see Chap.4). Then we proceed in sec.5.3 to examine some special properties of various hadronic variables and QCD parameters. The treatment of sum rule equations will be explained in sec.5.4. In sec.5.5, taking isospin symmetric QCD as a starting point, we solve the sum rule equations and examine all the symmetry constraints. A more comprehensive treatment (without assuming the isospin symmetry in QCD) is given in sec.5.6. We discuss the role of strange quarks in sec.5.7. All the finding in sec.5.5 to sec.5.7 lead to an interesting answer of an important question for parity and time reversal violations in QCD, which is discussed in sec.5.8. Finally, a brief summary of this chapter is given in sec.5.9.

5.2 QCD Sum Rules (QSRs) as extracted from the Invariant Tensor Structures of NCFs and the Use of the Borel Transformation

- **QCD sum rules of the NCFs in the momentum space**

Here we summarize the results of our calculations in Chap.3 and Chap.4 by listing all the invariant coefficient functions of various independent basis tensors[1] for the proton correlation function[2] in the momentum space. The equalities relating hadronic parametrizations and the OPE calculations for various invariant coefficient functions are referred as QCD sum rules. It should be kept in mind that these two representations of NCFs are derived from different expansions (a hadronic complete set in the former and $1/Q^2$ in the latter) of the same correlation functions. Therefore, within our approximations these equalities are only approximate identities.

$$\Pi_0^p(p) = f_1^p(p^2)\hat{p} + f_2^p(p^2)e^{i\phi^p(p^2)\gamma_5} \tag{5.1}$$

(a) $f_1^p(p^2)$ ($\hat{p}$ sum rule)

$$\frac{\lambda_p}{p^2 - M_p^2} + \text{continuum} = \frac{p^4 \ln(-p^2)}{8(2\pi)^3} - \frac{2a_d^2}{3(2\pi)^4 p^2} \tag{5.2}$$

(b) $f_2^p(p^2)e^{i\phi^p(p^2)\gamma_5}$ (I, γ_5 sum rule)

$$\frac{\lambda_p M_p}{p^2 - M_p^2}e^{i\theta_p\gamma_5} + \text{continuum} = \frac{m_d p^4 \ln(-p^2)}{4(2\pi)^3}e^{i\theta_d\gamma_5} + \frac{a_d p^2 \ln(-p^2)}{(2\pi)^4}e^{i\theta_{Gd}\gamma_5} \tag{5.3}$$

$$\Pi_{\mu\nu}^p(p) = \sum_{i=1}^{i=6} g_i^p(p^2) \cdot T^i \tag{5.4}$$

[1] We do not list the $\sigma_{\mu\nu}(I, \gamma_5)$ sum rules, the reasons will be explained in sec.5.4.

[2] For definitions of these invariant coefficient functions, please refer to sec.3.3.

(c) $g_3^p(p^2)$ ($\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta\cdot\gamma_5$ sum rule)

$$\frac{2\lambda_p^2(F_1^p+F_2^p)}{4(p^2-M_p^2)^2}+\frac{2\lambda_p^2E_A^p\cos(\frac{\varphi_A^p}{2}-\frac{\theta_p}{2})}{4(p^2-M_p^2)}+\text{ continuum}$$
$$= e_d\left[\frac{p^2\ln(-p^2)}{2(2\pi)^4}+\frac{m_ua_u\cos\bar{\theta}_u}{p^2}+\frac{a_u^2}{3p^4}\right]+(\propto e_u) \tag{5.5}$$

(d) $g_4^p(p^2)$ ($(p^\mu\gamma^\nu-p^\nu\gamma^\mu)\cdot\gamma_5$ sum rule)

$$\frac{2\lambda_p^2F_3^p}{4(p^2-M_p^2)^2}+\frac{2\lambda_p^2[E_A^p\sin(\frac{\varphi_A^p}{2}-\frac{\theta_p}{2})-E_B^p\sin(\frac{\varphi_B^p}{2}-\frac{\theta_p}{2})]}{4(p^2-M_p^2)}+\text{ continuum}$$
$$=(\propto m_qR_q\sin\bar{\theta}_q)10\text{cm} \tag{5.6}$$

(e) $g_5^p(p^2)+ig_6^p(p^2)\gamma_5$ ($\hat{p}(p^\mu\gamma^\nu-p^\nu\gamma^\mu)$ sum rule)

$$\frac{(-2)\lambda_p^2F_p}{4M_p(p^2-M_p^2)^2}e^{i(\theta_p+\alpha_p)\gamma_5}+\frac{2\lambda_p^2E_B^p}{4M_p(p^2-M_p^2)}e^{i(\frac{\varphi_B^p+\theta_p}{2})\gamma_5}+\text{ continuum}$$
$$=e_u\left[\frac{m_d\ln(-p^2)}{(2\pi)^4}\right]e^{i\theta_d\gamma_5}+\left[e_d\frac{\chi a_d\ln(-p^2)}{3(2\pi)^4}+e_u\frac{a_d}{(2\pi)^4p^2}\right]e^{i\theta_{Gd}\gamma_5} \tag{5.7}$$

The neutron sum rules can be obtained from the proton ones by doing an isospin rotation, namely, replacing the hadronic observables, e.g. M_p by M_n, λ_p by λ_n; and the QCD parameters, e.g. m_d by m_u, and R_d by R_u etc.

- **The Use of the Borel Transformation**

In principle, the QCD sum rules, as summarized above, can be used to extract information on hadronic variables in terms of QCD parameters. However, the validity of a matching between these two representations for the NCFs is severely limited by the convergence property of the OPE series and the uncertainty of the higher state contributions[3]. Fortunately, there are several prescriptions to improve the sum

[3] These limitations can be understood quantitatively in an exactly soluble quantum mechanical model, namely, the simple harmonic oscillator. See [ref.]

rule relations. Specifically, we wish to generate a set of improved sum rules such that the OPE series have better convergence and the contribtuions from a hadronic parametrization is dominated by the ground state observables. For this purpose, we use the Borel transformation, which is defined as [20]:

$$\mathcal{B}_{M_B}[f(Q^2)] \equiv \lim_{\substack{Q^2 \to \infty \\ n \to \infty \\ M_B \equiv \frac{Q^2}{n} fixed}} \left(\frac{1}{n!}\right)(Q^2)^{n+1}\left(-\frac{d}{dQ^2}\right)^{n+1} \tag{5.8}$$

We need to apply this transformation[4] on both sides of the sum rule relations:

On the OPE side of the QCD sum rules, we have perturbative contributions in the form of $(Q^2)^m \ln(Q^2)$ and the nonperturbative contributions represented by various vacuum condensates as power series of $\frac{1}{Q^2}$. The relative importance of the leading contributions in the OPE series are given by the relative powers of Q^2, which may not be useful if we need to restrict ourselves in the low-energy physics. However, if we take the Borel transformation of the OPE series, we find

$$\mathcal{B}_{M_B}[(Q^2)^m \ln(Q^2)] = (-1)^{m+1} m!(M_B)^m \tag{5.9}$$

$$\mathcal{B}_{M_B}\left[\left(\frac{1}{Q^2}\right)^k\right] = \frac{1}{(k-1)!}\left(\frac{1}{M_B^2}\right)^k \tag{5.10}$$

Thus, after the Borel transformation, the contributions of perturbative contributions are enhanced by an additional $m!$ factor and the contributions of higher dimensional operators (accompanied by higher powers $\left(\frac{1}{Q^2}\right)^k$) are suppressed by an additional $\frac{1}{(k-1)!}$ factor; this helps to give the series a better convergence. In addition, any finite polynomials vanish after taking the Borel transformation,

$$\mathcal{B}_{M_B}[(Q^2)^n] = 0 \quad n \geq 0 \tag{5.11}$$

which helps to eliminate the unknown subtraction constants in a dispersion relation.

On the other hand, the hadronic parametrizations generally contain various hadronic propagators, which are relativistic variants of the energy denominators in a non-

[4] Notice that, after the Borel transformation, the virtual 4–momentum variable $Q^2 \equiv -p^2$ appearing in the original sum rules is replaced by the square Borel mass M_B^2.

relativistic perturbative expansion, with the energy eigenvalues replaced by the invariant mass squared M^2. If we choose the virtual 4–momentum variable $Q^2 \equiv -p^2$ close to zero, we find that the ground state (N) observables only dominate the expansion by a ratio M^2/M_N^2, which is not a big number in the case of nearby excited states[5]. Nevertheless, if we apply the Borel transformation to these propagators, we get[6]

$$\mathcal{B}_{M_B}\left[\left(\frac{1}{Q^2+M^2}\right)^k\right]=\frac{1}{(k-1)!}\left(\frac{1}{M_B^2}\right)^k e^{-\frac{M^2}{M_B^2}} \tag{5.12}$$

Thus, after the Borel transformation, the higher states propagator (with larger masses $M_N'^2$) become a $\frac{1}{(k-1)!}e^{-\frac{M_N'^2}{M_B^2}}$ factor and the uncertainty of the higher state contributions can be reduced since the contributions of higher states are relatively supressed by an expontially decaying factor $e^{-\frac{\Delta M_N^2}{M_B^2}}$, with $\Delta M_N^2 \equiv M_N'^2 - M_N^2$[7].

Having established the usefulness of the Borel transformation, we need to specify how the matching of sum rule relations can be realized. As we have mentioned before that the identities we have derived can not be exact over all values of M_B^2. A choice of "matching region" has to be made, and such a choice is a compromise between different convergent properties of the two representations of the NCFs. On the OPE side, we prefer a large value of M_B to surppress power corrections (see Eq.(5.)); on the phenomenological side, we prefer a small value of M_B to enhance ground state

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[5] On the other hand, if we take a large virtuality $Q^2 \geq M^2$, we loose the ground state dominace.

[6] One can check that if we take M^2 to zero, we recover the previous result Eq.(5.8).

[7] In the case of the polarization tensor, we need to compare the contributions of the single nucleon EM matrix elements with those of the $N \to N'$. The latter, after the Borel transformation

$$\begin{aligned}\mathcal{B}_{M_B}\left[\frac{1}{(Q^2+M_N^2)(Q^2+M_N'^2)}\right] &= \left(\frac{1}{M_N'^2-M_N^2}\right)\left(\mathcal{B}_{M_B}\left[\frac{1}{Q^2+M_N^2}\right]-\mathcal{B}_{M_B}\left[\frac{1}{Q^2+M_N'^2}\right]\right)\\ &= \left(\frac{1}{M_N'^2-M_N^2}\right)\frac{e^{-\frac{M_N^2}{M_B^2}}}{M_B^2}\left(1-e^{-\frac{\Delta M_N^2}{M_B^2}}\right),\end{aligned}$$

as compared to the former contribution

$$\mathcal{B}_{M_B}\left[\left(\frac{1}{Q^2+M_N^2}\right)^2\right]=\left(\frac{1}{M_B^2}\right)^2 e^{-\frac{M_N^2}{M_B^2}}$$

, is not neglectible unless $M_N'^2 \gg M_N^2$ and should be taken care of by using some other method.

observables (see Eq.(5.)). In view of this, the matching of the sum rule relations only works for a finite range of the Borel mass M_B, and the region of matching the QCD calculations and the hadronic parametrizations (within certain criterions which will be specified in Chap.6) is generally called a Borel window.

It is a empirical fact that the ground state mass usually lies in the Borel window and within the Borel window, physical quantities should have mild dependence on the Borel mass M_B. Hence, in the primary analysis, we can choosse a value for the Borel mass M_B close to the ground state mass M_N^2 to get a rough estimation for the observables of interest.

Before we write down the full expression of the QCD sum rules, after applying the Borel transformation, we need to discuss the contributions of the continuum in the hadronic parametrizations of various invariant coefficient functions. In the momentum space, these contributions are given by the duality ansatz, where we introduce a continuum threshould s_0^N and then identify the leading term in the OPE series times a step function $\theta(Q^2 - s_0^N)$ as the continuum contribution. It turns out that the Borel transformation of this continuum ansatz can be written in an analytical expression. If we shift the continuum contribution to the sum rule relations from the phenomenological side to the OPE side[8], it is useful to define following functions which will appear in the sum rule identities quite often.

$$E_n(s,w) \equiv 1 - \int_w^\infty dt e^{-st} t^n \quad (5.13)$$

$$E_0(s,w) = 1 - e^{-\frac{w}{s}} \quad (5.14)$$

$$E_1(s,w) = 1 - e^{-\frac{w}{s}}(\frac{w}{s} + 1) \quad (5.15)$$

$$E_2(s,w) = 1 - e^{-\frac{w}{s}}(\frac{w^2}{2s^2} + \frac{w}{s} + 1) \quad (5.16)$$

Applying the Borel transformation to both sides of the QCD sum rules, we get the following improved relationships:

$\Pi_0^p(p)$

(a) $f_1^p(M_B^2)$

[8] Because of the similarity of their mathematical structures, also we want to treat the continuum threshould s_0^N as a parameter, see next section.

$$M_B^6 E_2(M_B^2) + 4m_u a_u \cos\bar{\theta}_d M_B^2 E_0(M_B^2) + \frac{4a_u^2}{3} = \tilde{\lambda}^2 e^{-\frac{M_N^2}{M_B^2}} \tag{5.17}$$

(b) $f_2^p(M_B^2)e^{i\phi^p(p^2)\gamma_5}$

$$2M_B^6 E_2(M_B^2) m_d e^{i\theta_d\gamma_5} + 2M_B^4 E_1(M_B^2) a_u e^{i\theta_{Gd}\gamma_5} = \tilde{\lambda}^2 M_N e^{-\frac{M_N^2}{M_B^2}} e^{i\theta_p\gamma_5} \tag{5.18}$$

$\Pi_{\mu\nu}^p(p)$
(c) $g_3^p(M_B^2)$

$$\begin{aligned} e_u M_B^4 E_1(M_B^2) + e - dm_u a_u \cos\bar{\theta}_d + \frac{a_u^2}{3M_B^2}[-(e_d + 2/3e_u) + (\propto e_u)] \\ = \frac{\tilde{\lambda}^2}{4} e^{-\frac{M_N^2}{M_B^2}} \left(\frac{F_1^p + F_2^p}{M_B^2} + E_A \cos(\frac{\varphi_A}{2} - \frac{\theta_N}{2})\right) \end{aligned} \tag{5.19}$$

(e) $g_5^p(p^2) + ig_6^p(p^2)\gamma_5$

$$\begin{aligned} (4e_u a_d + \propto e_d) e^{i\theta_{Gd}\gamma_5} - (4e_u m_d + \propto e_d) M_B^2 e^{i\theta_d\gamma_5} \\ = \frac{\tilde{\lambda}^2}{M_p} e^{-\frac{M_N^2}{M_B^2}} \left(\frac{F_p}{M_B^2} e^{i(\theta_p + \alpha_p)\gamma_5} + E_B e^{i(\frac{\varphi_B + \theta_p}{2})\gamma_5}\right) \end{aligned} \tag{5.20}$$

5.3 Hadronic Variables and QCD Parameters in the QSR

There are several nucleon observables and model parameters in the phenomenological model for the NCFs we could determine by the sum rule techniques. Before we solve them from the sum rule equations, it is helpful to discuss various important properties of these variables which are closely related to the issue of $U_A(1)$ chiral symmetry of QCD and to the sum rule method itself. An understanding of these properties is essential for a proper usage of the sum rule method. The basic idea is that: (1) Physical observables must be representation independent. Therefore, they can depend on chirally invariant QCD parameters only. (2) Chiral transformation property of the interpolating fields helps us to avoid the scheme dependence (truncations of OPE

Table 5.1: Hadronic variables of the nucleon propagator $\Pi_N^{(0)}$

nucleon state	N	$\lambda_N, \alpha_N; M_N$
higher states	N'	s_0^N

Table 5.2: Hadronic variables of the polarization tensor $\Pi_N^{\mu\nu}$

nucleon state	$N \leftrightarrow N$	$F_1^N(0) = Q^N; F_2^N(0), F_3^N(0)$ or F_N, β_N
nucleon state to higher states	$N \leftrightarrow N'$	$E_A, \varphi_A; E_B, \varphi_B$
higher states	$N_1' \leftrightarrow N_2'$	s_0^N

series + the choice of an interpolating field) in a sum rule calculation of physical observables. See App.D.

In the following, we shall list those nucleon observables and model parameters which appear in the hadronic model for NCFs. Then we proceed to discuss two important properties of these variables. Finally, we explain how to treat these variables as unknowns in the sum rule equations.

A list of the nucleon observables and model parameters in the NCFs

We shall separate the nucleon observables and model parameters into two groups, those that appear in the nucleon propagator $\Pi_N^{(0)}$ and those that appear in the polarization tensor $\Pi_N^{\mu\nu}$:

Dependence on the choice of interpolating fields η_N

As explained in Chap.3, the reason for introducing an interpolating field η_N is purely methodological; it enables us to do calculations in QCD without knowing the solutions for hadron wave functions. Therefore, it should be clear that all the physical observables (e.g. mass M_N, magnetic moment $\mu_N \propto F_1^N(0) + F_2^N(0)$, EDM $d_N \propto F_3^N(0)$) , being inherent properties of a physical state in QCD, should be independent of the choice of interpolating fields. Also, the continuum threshold s_0^N, is a characteric of the QCD spectrum and consequently, should not depend on η_N either.

On the other hand, the dependence of some other varibles (e.g. λ_N, α_N) on the choice of interpolating fields is unavoidable. By definition, they describe the overlap between nucleon states and the interpolating fields, or, the amplitude of the nucleon states being observed in the lowest Fock component of their complete wave functions. Different choices of the interpolating fields may give rise to different overlaps[9]. Similarily, in our treatment for the higher state contributions to the NCF, the dependence of the model parameters $E_A, \varphi_A; E_B, \varphi_B$ on the choice of interpolating fields η_N are built in from the very beginning.

Given such distinct attributes among these hadronic variables, it is clear that for physical observables a reasonable calculation should not generate a dependence on the interpolating fields. Nevertheless, due to the truncations of the OPE series, such natural expectations may not be reflected in a realistic sum rule calculation[10] This is certainly a methodological artifact and should be avoided in any consistent calculation. Besides the quantitative dependence on the choice of nucleon interpolating fields, we have a different concern in the current problem. Since we have put much emphasis on the chiral symmetry constraints for the strong $\not{P}$ and $\not{T}$, such general constraints should hold true no matter what interpolating field operator is chosen to do the calculations. Consequently, it is very crucial to study the chiral property for all possible choices of the interpolating fields and the QCD sum rules derived from them. Such study involve some tedius (but straightforward) algebra and we shall leave all the details to the App.D. In short, our calculations confirm that the chiral symmetry constraints for the strong $\not{P}$ and $\not{T}$ are obeyed for all possible choices of the nucleon interpolating fields.

$U_A(1)$ chiral properties of the hadronic variables and QCD parameters

One important feature of the polar representations of both hadronic variables and QCD parameters is that we can easily identify their transformation properties under a $U_A(1)$ chiral rotation. We can classify these numbers into two classes:

(1) class 1: this class consists of all the chirally covariant numbers, which are changed by a constant phase under a $U_A(1)$ chiral rotation. In the polar representations, these numbers appear as phase variables, e.g. $\alpha_N, \varphi_A, \varphi_B$, and θ_q, θ_G.

---

[9] By taking a nonrelativistic reduction, one can see how the overlap between nucleon states and the interpolating fields converges to a particular choice.

[10] In some cases, by employing a mixture interpolating field operator (with a mixing parameter t), people do find a dependence on t for some generic hadronic observables. See [38].

(2) class 2: this class consists of all the chirally invariant numbers, which stay the same under a $U_A(1)$ chiral rotation. In the polar representations, these numbers appear either as radii variables, e.g. M_N, F_N and m_q, R_q or the difference between two chirally covariant phases, e.g.β_N and $\bar{\theta}$

To summarize, the classification can be tabulated as follows:

nucleon observables and model parameters

chiral property	dependence onη_N	no dependence onη_N
class 1	$\alpha_N, \varphi_A, \varphi_B$	NA
class 2	$\lambda_N, A_N, B_N,$	$M_N, s_0^N, F_1^N(0), F_2^N(0), F_3^N(0), F_N, \beta_N$

QCD parameters

chiral property	phase variables	radii variables
class 1	θ_q, θ_G	NA
class 2	$\bar{\theta}$	m_q, R_q, g_s all s.c. as defined in sec 4-4

Number of unknowns If we can calculate the NCFs exactly in QCD, then according to dispersion relations [25], the imaginary parts of the NCFs are the spectral functions which tell us the masses of physical states (the locations of the peaks) and the strengths of their couplings to the interpolating fields (the heights of the peaks, e.g. λ_N). Unfortunately, this has not been achieved within our current ability. It is for this reason we need to rely on an effective model for the spectral functions to parametrize our ignorance[11]. Clearly, we are mostly interested in the ground state observables (e.g. M_N, F_N) and wish to calculate these numbers from QCD sum rules. However, this does not imply that all other model parameters (e.g. E_A^N, E_B^N, s_0^N) are less important (numerically) and can be ignored. In fact, a QCD sum rule calculation for the nuclear magnetic moments shows that, the nucleon to higher states transitions does give a substantial contribution to the sum rule equations. In view of this, we adopt the following attitude toward the use of these model parameters in the sum rule calculations:

[11] Based on this feature, the QCD sum rule method has to be a semi-empirical approach to hadronic physics and we should remind ourself the inherent limitation of this approach.

(1) Since it is impossible to have an exact formula of the spectral functions for NCFs, we need to rely on experimental inputs to give a realistic description of the numerical importance of higher states. In particular, the continuum threshold s_0^N shall be chosen around $(1.5GeV)^2$ to correspond to the $N^*(1535)$. It is hoped that within a reasonable range of the values for s_0^N, the ground state observables will have negligible dependence on a variation of s_0^N[12]

(2) The nucleon to higher states transitions make comparable contributions as the ground state nucleon EM matrix elements. Therefore, it is not clear that the exponential suppression works well in this case. In addition, we do not have sufficient knowledge to pin down the experimental inputs for these model parameters E_A, E_B. Consequently, we shall eliminate these parameters by applying a differential operator to the sum rule relations (see next section).

The bottom line of our discussion here is that for a given nucleon (proton or neutron) we have 3 unknowns λ_N, θ_N, M_N in the nucleon propagator $\Pi_0^N(p)$ sum rules and 2 unknowns $F_2^N(0), F_3^N(0)$ or F_N, α_N in the polarization tensor $\Pi_{\mu\nu}^N(p)$ sum rules.

5.4 Choice of Sum Rules

Number of equations

The number of sum rules we can derive has to do with the number of independent tensors in the decomposition of a NCF. Furthermore, not all the sum rules thus derived are equally useful. In the current case, due to the neglect of the weak interaction and the conservation of charge conjugation (C) in the strong interaction, we have 3 independent sum rules in the nucleon propagator $\Pi_0^N(p)$ and 6 independent sum rules in the polarization tensor $\Pi_{\mu\nu}^N(p)$[13]. Accroding to our scheme as to what variables should be treated as unknowns in the sum rule equations, it is clear that we can solve

[12] It should be emphasized that the standard " one resonance + a step function " model for the hadronic spectral functions might not be consistent with certain chiral symmetry constraints, as pointed out by T. Cohen et.al [41]. Also, the finite width of ground states may change the values to a certain degree see [39].

[13] This was discussed in sec.3-3 on the tensor structures of the NCFs. Also, the detailed QCD calculations and the hadronic model (including the contributions from the higher states) serve as consistency checks in support of this statement.

for λ_N, θ_N and M_N from the nucleon propagator $\Pi_0^N(p)$ sum rules (as functions of s_0^N, the continuum threshold). In order to slove for the EM response coefficients $F_2^N(0), F_3^N(0)$ or equivalently, F_N and α_N, we need to turn to the polarization tensor $\Pi_{\mu\nu}^N(p)$. It turns out that in this case, there are two major complications in the QCD calculations: (a) The presence of susceptibilitity constants and (b) The problem of nonanalytic dependence of quark mass in the Wilson coefficients (see eq.)[14]. For these reasons, we need to take certain combinations of proton and neutron sum rules in order to avoid these complications.

Given the table in sec.5-2, we need to take the following steps to generate another set of sum rules which allows us to solve for the hadronic variables without involving extra unknown parameters.

step 1: The use of a polar form

step 2: Linear combinations of proton and neutron sum rules

step 3: The eliminations of higher state parameters (transition matrix elements)

Due to the duality ansatz for the continuum, all these contributions (Region 3) to the polarization tensor $\Pi_{\mu\nu}^N(p)$ drop out in step 2 (Again, that is due to the charge dependence). However, the $N \leftrightarrow N'$ contributions(Region 2), as parametrized by the parameters $E_A^N, E_B^N \varphi_A^N$ and φ_B^N, still survive after step 2. Nevertheless, these contributions appear in the sum rules with different powers of Borel mass M_B and do not interfere with the EM moments of nucleons. Consequently, we can eliminate these model parameters by applying a differential operator

$$1 - M_B^2 \cdot \frac{\partial}{\partial M_B^2}$$

to the sum rule equations.

After all these manipulations, we have the following 10 sum rule equations for 10 nucleon variables which we shall solve in next two sections:

---

[14] Both complications actually stem from a common source, namely, the use of a constant external field.

$$
\begin{array}{llcl}
(3) & \Pi_0^p(p) & \Longleftrightarrow & \lambda_p, \alpha_p, M_p \\
 & A_d^p m_d e^{i\theta_d\gamma_5} + B_{G_d}^p R_d e^{i\theta_{Gd}\gamma_5} & = & C^p e^{i\alpha_p\gamma_5} \\
(3) & \Pi_0^n(p) & \Longleftrightarrow & \lambda_n, \alpha_n, M_n \\
 & A_u^n m_u e^{i\theta_u\gamma_5} + B_{G_u}^n R_u e^{i\theta_{Gu}\gamma_5} & = & C^n e^{i\alpha_n\gamma_5} \\
(2) & & & \\
 & & \Longleftrightarrow & F_p, F_n; \beta_p, \beta_n \\
 & A_d^{p'} m_d e^{i\theta_d\gamma_5} + B_{G_d}^{p'} R_d e^{i\theta_{Gd}\gamma_5} & = & C^{p'} e^{i(\alpha_p+\beta_p)\gamma_5} \\
 & A_u^{n'} m_u e^{i\theta_u\gamma_5} + B_{G_u}^{n'} R_u e^{i\theta_{Gu}\gamma_5} & = & C^{n'} e^{i(\alpha_n+\beta_n)\gamma_5}
\end{array}
$$

5.5 Symmetry Constraints: $SU_f(2)$ Isosymmetric QCD

Simplifying QCD sum rules with isosymmetry

To solve the QCD sum rule equations for nucleon EM moments ($F_2^N(0), F_3^N(0)$ or equivalently,F_N and α_N) it is helpful to start with a special case, instead of dealing with the full complexity for the first time. Also, in so doing, it is easy to see how the chiral symmetry constraints on the strong $\not{P}$ and $\not{T}$ are realized in the QCD sum rule approach.

The special case we shall study in this section is the isosymmetric limit of QCD. That is, we set $m_u = m_d$ and $R_u = R_d$ (but $Q_u = -2Q_d$). In this limit, by making a chiral rotation on either flavour to adjust $\theta_u = \theta_d$, we can show that $\theta_{Gu} = \theta_{Gd}$ from Crewther's conditions (see eq.). Thus, there is no need to distinguish u and d quarks and the QCD Lagrangian is invariant under a $SU(2)$ transformation in the flavour space.

On the hadron side, if we ignore the QED corrections to the nucleon masses, which are of second order in charge, we have $M_p = M_n$. Also, the isosymmetry implies that $\lambda_p = \lambda_n$ and $\alpha_p = \alpha_n$. In view of these simplifications, we get a set of reduced sum rules:

$$
A m_q e^{i\theta_q\gamma_5} + B R_q e^{i\theta_{Gq}\gamma_5} = C_N e^{i\theta_N\gamma_5} \tag{5.21}
$$

$$
A' m_q e^{i\theta_q\gamma_5} + B' R_q e^{i\theta_{Gq}\gamma_5} = C_N{}' e^{i(\theta_N+\alpha_N)\gamma_5} \tag{5.22}
$$

where the Wilson coefficients are given by

$$A = \frac{im_q M_N^4}{3 \cdot 2^6 \pi^4} \qquad A' = 0$$
$$B = \frac{-iR_q M_N^2}{3 \cdot 2^4 \pi^2} \qquad B' = \frac{-iR_q M_N^2}{3 \cdot 2^4 \pi^2}(e_d^2 - e_u^2) \tag{5.23}$$

and the unknown hadronic variables

$$C_N = \lambda_N^2 M_N e^{-\frac{M_N}{M_B}} \tag{5.24}$$
$$C_N' = \lambda_N^2 \frac{F_N}{M_N} e^{-\frac{M_N}{M_B}} \tag{5.25}$$

We shall solve for the $\not P$ and $\not T$ chirally invariant EM phase α_N (which is related to the NEDM by $F_3 = F_N \sin \alpha_N$) from these sum rule equations.

Chiral covariance, periodic structures of the QCD sum rules

As emphasized in the previous section, the polar representation helps us to classify the hadronic variables and the QCD parameters according to their transformation properties under the $U_A(1)$ chiral rotations. An important advantage of the use of polar representation in the QCD sum rule approach is that the equations we derive are manifestly $U_A(1)$ chirally covariant (for even tensor sum rules). This chirally covariant form of the sum rule equations is a manifestitation of the reparametrizaion covariance of the QCD generating functional. Thus, in our formulation, either a pure gluonic anomalous interaction $G\tilde{G}$, or a quark pseudo mass term $m_q \bar{q} e^{i\theta_q \gamma_5} q$ (or any mixture between these two interactions, as long as $\bar{\theta} \equiv \theta_G - \theta_q$ is fixed.) gives rise to the same answers for the hadronic variables.

On the other hand, the periodic structures on the $\bar{\theta}$ parameter comes out naturally from the polar representation of the QCD sum rule equations. The explicit examples of the solutions drived from the sum rule equations will be given in next subsections.

The realizations of chiral symmetry constraints on the strong $\not P$ and $\not T$

Another benefit brought about by the polar representation is that, we can examine the chiral symmetry constraints on the strong $\not P$ and $\not T$ by looking at the sum rule equations without actually solving them.

(1) The nonperturbative nature of strong $\not P$ and $\not T$:

If we set $R_u = R_d = 0$ in the sum rule equations, which is equivalent to assuming that there is no SSB of chiral symmetry and we keep only perturbative contributions, we find that $\alpha_N = \theta_q$ from the mass sum rule. If we substitute this into the EM sum rules, we get a zero solution for α_N. Therefore, we have no NEDM and $d_N = 0$.

(2) Chiral limit:

If we set m_q equal to zero, we find that $\alpha_N = \theta_G$ from the mass sum rule. This also leads to a vanishing α_N. Thus, in the massless limit of QCD, we can not generate a strong $\not{P}$ and $\not{T}$ NEDM.

It is interesting to observe that, these two symmetry constraints were originally discovered based on different perspectives of QCD (see the discussion in Chap.2). Furthermore, the relevant parameters have different dimensions (dim $m_q = 1$, dim $R_q = 3$) and seem to be of different physical origins. Neverthless, in the QCD sum rules they appear to be on symmetrical footings and constrain the strong $\not{P}$ and $\not{T}$ observables in exactly the same way. Whether this indicates some profound connections between these parameters (some new duality of QCD ?), which could lead us to a deeper level of understanding QCD, is beyond our present ability.

(3) Anomaly constraint:

From the previous discussion, it is clear that we need both finite m_q and R_q to have a nonzero NEDM. And as $\bar{\theta}$ goes to zero, we have no strong $\not{P}$ and $\not{T}$. Therefore, we can expect that a NEDM should be propotional to the product of these three factors. Again, from the polar form representations of the sum rule equations, we can deduce that a periodic structure has to be there, and give rise to a (continuous) factor $\sin\bar{\theta}$ for the NEDMs. These inferences, together with the anamolous Ward identity we have discussed in Chap.2 (see sec. 2-7), surely establish the relationship between a NEDM with the $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$ which is what we refer to as anomaly constraint on strong $\not{P}$ and $\not{T}$.

One can also see the last point more clearly by solving the sum rule equations. After some algebra, we get

$$\sin\alpha_N = \frac{(AB' - A'B)m_q R_q \sin(\bar{\theta}/2)}{(AA'm_q^2 + BB'R_q^2) + (AB' + A'B)m_q R_q \cos(\bar{\theta}/2)} \tag{5.26}$$

$$= \frac{\langle G\tilde{G}\rangle_{\theta_q,\theta_G}}{(AA'm_q^2 + BB'R_q^2) + (AB' + A'B)m_q R_q \cos(\bar{\theta}/2)} \tag{5.27}$$

$$\cos\alpha_N = \frac{(AA'm_q^2 + BB'R_q^2) + ((AB' + A'B)m_q R_q \cos(\bar{\theta}/2)}{(AA'm_q^2 + BB'R_q^2) + (AB' + A'B)m_q R_q \cos(\bar{\theta}/2)} \tag{5.28}$$

Thus, we do find all the required factors and all the symmetry constraints are satisfied in the QCD sum rule approach in the isosymmetric limit.

A rough estimation of the hadronic variables from the solutions of sum rule equations

With the analytical solutions of the sum rule equations, we can substitute the numerical values of these QCD parameters to get an order of magnetitude estimate of the NEDM. With the following input parameters,

$$m_q = 5 \times 10^{-3}(GeV),\ R_q = 1.4 \times 10^{-3}(GeV)^3,\ M_N = 1(GeV) \tag{5.29}$$

we obtain

$$\tan\alpha_N \leq 10^{-2} \cdot \bar{\theta} \text{ in the small } \bar{\theta} \text{ limit} \tag{5.30}$$

This gives a NEDM

$$d_N = 10^{-16}\bar{\theta}e \cdot cm \tag{5.31}$$

5.6 Why Strong Parity and Time Reversal Violations Are Small Effects

Within our approach, it is possible to factorize a $\not{P}$ and $\not{T}$ hadronic observable like the NEDM into the product two contributions: one is from the (explict) symmetry breaking parameter $\bar{\theta}$, and the other is from the (chiral) symmetry constraints (contains both m_q and R_q). In this form, not only can we show how the symmetry constraints work out explictly, but also it is possible to investigate an interesting question:

> Why strong $\not{P}$ and $\not{T}$ are small effects?

There is certainly an easy answer to this question, that is, because $\bar{\theta}$ is a small number! As long as we have only tiny symmetry breaking terms in the Lagrangian, we should expect all the symmetry breaking effects to be small[15]. However, such an answer against the principle of naturalness[16] , and should be considered undesireable[17]. The

---

[15] After all, $\bar{\theta}$ is an adjustable free parameter in the theory, and it is impossible to calculate a value for it from QCD itself.

[16] The princlple of naturalness (in the context of QFT) was first discussed by t'Hooft [40]; it states that a theory with a small parameter can be consider natural only if the limiting theory (that is, set the parameter equal to zero) posesses a new symmetry.

[17] Remember that in the current case, we have a dimensionless strong coupling constant g_s, which is much greater than $\bar{\theta}$. Such a situation could lead to a hierarchy problem, and it is not clear

next question then is, what other possibility could there be? Foutunately, there exists another alternative. In fact, we can imagine that the (chiral) symmetry constraints work not only in a qualitative manner, but also quantitatively. They generate small coefficients to the strong $\not{P}$ and $\not{T}$ observables. If this is true, then $\bar{\theta}$ needs not to be a small parameter in order to generate a tiny NEDM. Put it in another way,

> the strong $\not{P}$ and $\not{T}$ are small effects because of the internal dynamics of the (P and T conserving) QCD (which gives rise to the symmetry constraints) , rather than a small $\bar{\theta}$!

Unfortunately, such an interesting scenario is not supported by our calculations. In the primary study of the QCD sum rule equations for the NEDM, the experimental measurement of d_n does put a strigent limit ($< 10^{-10}$) on $\bar{\theta}$, (which is compatible with the previous result obtained by R.J. Crewther et al [11]. [18] the numerical coefficients generated by the symmetry constraints contribute a much weaker suppression (compared to the contribution from the $\bar{\theta}$ parameter) of the NEDMs.

What we can conclude from this study is to be able to set an upper bound on the ratio of an NEDM, d_N, to a nucleon magnetic moment μ_N, which is independent of the value for $\bar{\theta}$. Using the analytical formula, we can calculate such an upper bound: The answer is

$$\frac{d_N}{\mu_N} \leq 10^{-2} \tag{5.32}$$

5.7 Summary and Conclusion

By taking suitable combinations of proton and neutron sum rules (in order to eliminate undesireable complications), we can generate useful sum rule equations which relate desired hadronic variables to known QCD parameters.

These equations are manifestly chirally covariant, which reflects the equivalence relation between different rerpresentations (under $U_A(1)$ chiral rotations) of the QCD Lagrangian.

---

that without a fine–tuning, the large ratio of g_s to $\bar{\theta}$ is stable against radiative corrections.

[18] Such a similarity in terms of a numerical factor may not be just a coincidence, since their calculation are based on current algebra relations, which QCD also obeys.

In the primary analysis, we can derive analytic expressions for the hadronic variables (e.g. nucleon masses, EM moments) as solutions to these sum rule equations, with the following special features which we would like to emphasize:

1. Being physical observables, these solutions are invariant under $U_A(1)$ chiral rotations. Indeed, we find that all the hadronic variables (e.g. nucleon masses M_p, M_n, anomalous magnetic moments F_2^p, F_2^n, electric dipole moments F_3^p, F_3^n are functions of m_q, R_q and $\bar{\theta}$ only.

2. The solutions for the EDM of nucleons are consistent with all the symmetry constraints on strong $\not{P}$ and $\not{T}$ discussed in Chap.2.

3. Due to the use of the polar representation, there is no need to do perturbative calculations for $\bar{\theta}$. Therefore, we can drop the unnatural assumption of a small $\bar{\theta}$ in our discussion. Furthermore, the periodic dependence of hadronic observables on $\bar{\theta}$ comes out naturally in the solutions.

4. In the context of OPE, it is possible to factorize the physical observables into two contributions: those that can be characterized as (1) long distance parts or nonperturbative effects, represented by various condensates; (2) short distance parts or perturbative effects, represented by Wilson coefficients. While symmetry constraints are realized in the nonperturbative sector, as represented by the strong $\not{P}$ and $\not{T}$ gluon condensate $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$, we also find the perturbative calculations revealing some intriguing dynamics. In particular, we are able to set an upper bound of the ratio of anamolous magnetic moment to electric dipole moment, irrespective of the value for $\bar{\theta}$. (i.e. no matter how strong the effect of $\not{P}$ and $\not{T}$ in QCD).

With the current experimental result, which set an upper bound for the NEDM at $d_n \leq 10^{-26} e - cm$, together with the analytic solution from the sum rule equations, we find an upper bound for $\bar{\theta} \leq 10^{-10}$.

Chapter 6

ANALYSIS OF QCD SUM RULES (QSR) FOR NUCLEON CORRELATION FUNCTIONS (NCF) IN THE PRESENCE OF AN EXTERNAL ELECTROMAGNETIC FIELD: PART TWO

6.1 Outline of this Chapter

This chapter aims at a more careful study of the QSRs for the NCFs in the presence of an external EM field. We shall extract various nucleon variables, e.g. the nucleon masses M_N, nucleon residues λ_N and their EM moments F_2^N, F_3^N, from the QSRs in a rigorous manner. The emphasis here is to use QCD sum rule method to obtain quantative results for the hadronic variables, including an analysis and/or estimate of the errors and uncertainities in our calculations.

The analysis consists of two steps:

First of all, we need to choose a range of Borel mass such that the matching between the two representations of the NCFs is achieved within the uncertainity of truncations of the OPE series and our modeling of hadronic spectral functions. The details are explained in sec.6.2.

Secondly, in sec.6.3, we use the method of least-squares fits to extract the nucleon variables from the sum rule relations of various invariant coefficient functions. That is, we look for the optimal values of the nucleon variables, which appear in the hadronic model of the NCFs, such that the differences between the OPE series and the hadronic model of the NCFs, as functions of the Borel mass M_B, are minized.

After some comments in sec.6.4, regarding to several intrinsic difficulties in the QCD sum rule calculations, we conclude this chapter with a brief summary in sec.6.5.

6.2 Choice of the Borel Window

As we emphasized before, the extractions of hadronic variables from the QCD sum rules of a given correlation function is achieved through a matching between the OPE series and the hadronic parametrizations. These two representations, being functions of the Borel mass M_B, are derived from different expansions of the same invariant functions. With the inevitable truncations of the OPE series and crude approximation in the hadronic parametrizations, there is no reason to believe that these two functions agree well throughout the entire range of M_B. In particular, while the OPE series, as a series in $\frac{\Lambda^2_{QCD}}{M_B^2}$, acquires better convergence at large M_B, the hadronic models are saturated by the ground state observables, due to the expontential suppression factor $e^{-\frac{M_N^2}{M_B^2}}$, only in the low M_B region. Therefore, it is unlikely that we should trust the sum rules as identities for two functions and a judicious choice of a matching region for these two representations has to be made.

Such a matching region, within which the sum rules are valid, is generally called a **Borel window**. There is no unique rule to find out what region should be chosen and a standardized procedure of its specification does not exist. In this work, we shall adopt the following criteria to choose a Borel window for the QCD sum rules:

(1) On the OPE side of the sum rules, we can obtain a lower bound on the Borel mass M_B, by requiring that the contributions of the highest dimensional operators to the total truncated series is no greater than 20%. That is,

$$\frac{|\text{ highest dimensional operators } C_n(M_B^2)\langle O_n\rangle|}{|\Sigma_{i=0}^{i=n} C_i(M_B^2)\langle O_i\rangle|} \leq 20\%$$

Given the first criterion, we obtain various lower bounds for the Borel mass M_B in different sum rules. Also, because the charge dependences in the Wilson coefficeints of proton and neutron sum rule, their Borel windows could be different. We shall take the largest number among various sum rules as the lower bound for the Borel mass M_B. The net result is

$$M_B \geq 0.9GeV \tag{6.1}$$

(2) On the phenomenological side of the sum rules, in order to achieve ground state dominance of the NCFs, we need to set an upper bound for the Borel mass M_B. To this end, one should require the contributions from the continuum to the NCFs

to be less than 20%[1]. In other words,

$$\frac{|\text{continuum contributions (through duality ansatz)}|}{|\Sigma_{i=0}^{i=n} C_i(M_B^2) O_i|} \leq 20\%$$

From the second criterion, we obtain an upper bounds for the Borel mass M_B

$$M_B \leq 1.2GeV \tag{6.2}$$

As a side remark, we mention that since the OPE series for

$$(p_\mu \gamma_\nu - p_\nu \gamma_\mu)\gamma_5$$

only starts from dimension 6, there is no restriction on the Borel mass M_B from this sum rule based on our two criteria.

The bottom line here is that taking the common intersection of the various restrictions, corresponds to a choice of Borel window

$$0.9GeV \leq M_B \leq 1.2GeV$$

Within this region, the truncated OPE series and the hadronic parametrizations, as functions of the Borel mass M_B for the NCFs are approximately equal, with a 20% uncertainity.

6.3 On the Determination of Hadronic Variables from the QCD Sum Rules

There are two ways to extract hadronic variables from a given set of QCD sum rules:

(1) We can treat the hadronic variables appearing in the RHS of the QSRs as free parameters. The optimal values for these parameters should be determined such

---

[1] Such a requirement can be realized because we adopt the duality ansatz for the continuum contributions, which are equal to a product of a step function and the leading term in the OPE series. In addition, we cannot take a ratio between the continuum contributions and the ground state contributions, since the latter are unknown and have to be determined by the sum rule relations. However, the consistency can be checked once we obtain a solution fo the ground state observables, which we shall discuss in sec.6.5

that the χ^2 for the OPE calculations and the hadronic parametrizations, viewed as functions of the Borel mass M_B, is minized within the prescribed Borel window.

(2) We can solve for the hadronic variables as functions of the Borel mass M_B, along with other input parameters[2]. We determine their values by averaging the solution over the prescribed Borel window (which is equivalent to a least χ^2 fit to the constant functions, because in principle, the hadronic variables should be independent of the Borel mass M_B.)

In general, there is no guarantee that these two methods will give similar answers. However, it is reasonable to expect that the variables we extract from these two methods should not differ by 20uncertainity from the choice of the Borel Window. The success of any sum rule calculation should meet this requirement. In our current case, since we need to rely on the linear combinations and a differential operator $1 - M_B^2 \frac{\partial}{\partial M_B^2}$ to eliminate undesireable parameters, it is more convinent for us to adopt the second approach. Therefore, our discussion below will be based on the solutions we derived in the previous chapter. For simplicity, we restrict the discussion to isospin symmetric $N_f = 2$ QCD only.

6.3.1 On the Determination of Nucleon Observables from the QCD Sum Rules

- On the Determination of Nucleon Masses M_N and the Resdiue $\tilde{\lambda}_N$

On the phenomenalogical side of the sum rules for the nucleon propagator, we get a solution for $\tilde{\lambda}_N e^{-\frac{M_N^2}{M_B^2}}$ from the $\hat{p}$ sum rule and a solution for $\tilde{\lambda}_N M_N e^{-\frac{M_N^2}{M_B^2}}$ from the I, γ_5 sum rule. we can take the ratio to obtain a solution of the nucleon masss M_N, as a function of the Borel mass M_B. The function is plotted in Fig.6-1, within the prescribed Borel window the function dependence on the Borel mass is relatively stable, and we obtain the central value for the nucleon mass

$$M_N = 0.96 GeV \tag{6.3}$$

We can substitute this value to either $\hat{p}$ or I, γ_5 sum rule to get a solution for the resdiue $\tilde{\lambda}_N$, The functions thus obtained are plotted in Fig.6-2(a)(b). While the $\hat{p}$

[2] We choose the continuum threshold s_0^N to be 1.75 GeV.

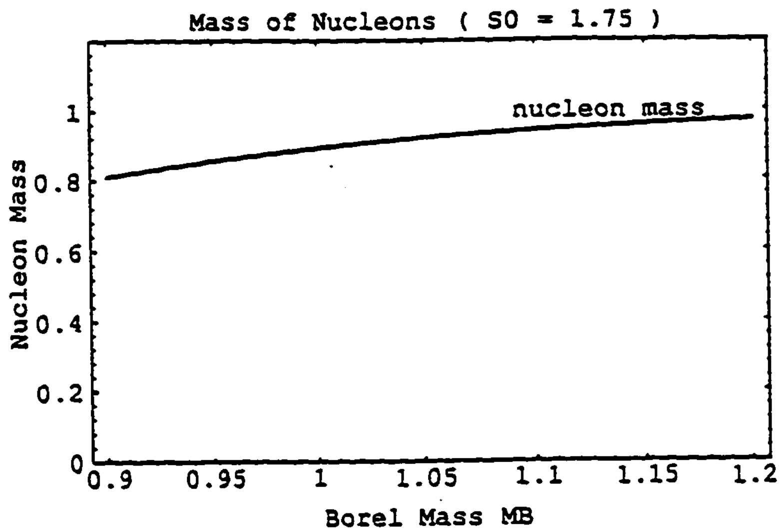

Figure 6.1: Mass of nucleons ($s_0 = 1.75\ GeV$, $\bar{\theta} = 10^{-10}$)

sum rule seems to give a more stable curve, these two solutions does not differ too much and we can conclude the value of the nucleon resdiue $\tilde{\lambda}_N$, as extracted from the $\hat{p}$ sum rule to be

$$\tilde{\lambda}_N = 2.48(GeV)^6 \tag{6.4}$$

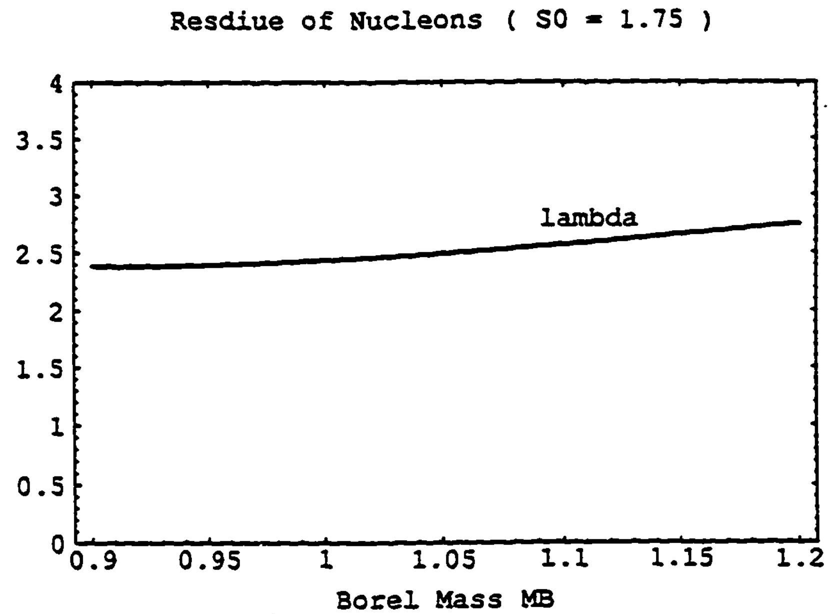

Figure 6.2: Residue of nucleons from the $\hat{p}$ sum rule

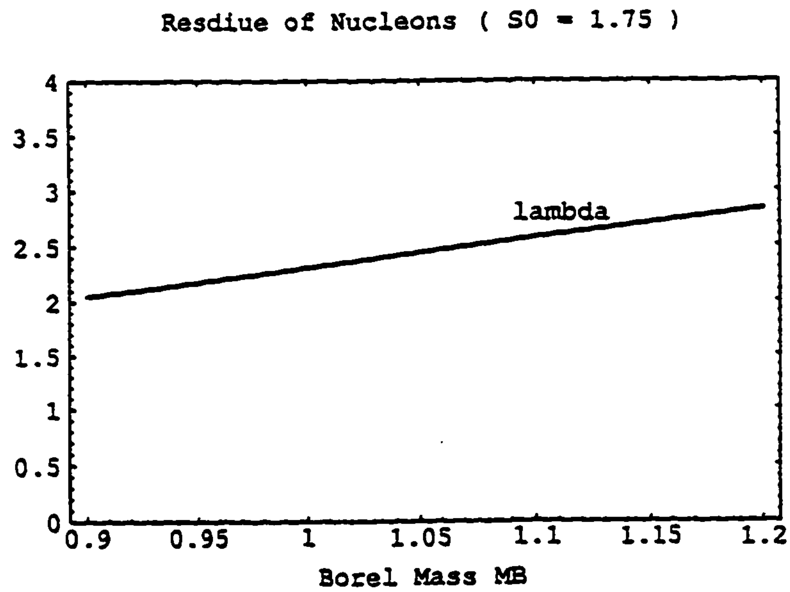

Figure 6.3: Residue of nucleons from the (I, γ_5) sum rule

- On the Determination of the Nucleon EM moments F_2^N, F_3^N

Using the formulae for the nucleon EM moments F_2^N, F_3^N, as given in sec.5.5, we have the following plots Fig.6-3 for their dependence on the Borel mass M_B at $\bar{\theta} = 10^{-10}$:

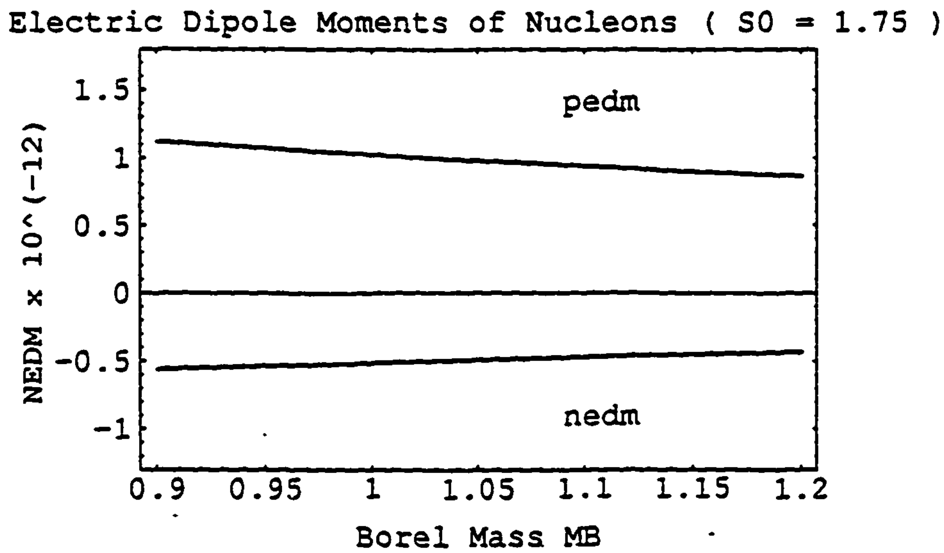

Figure 6.4: Electric dipole moments of nucleons

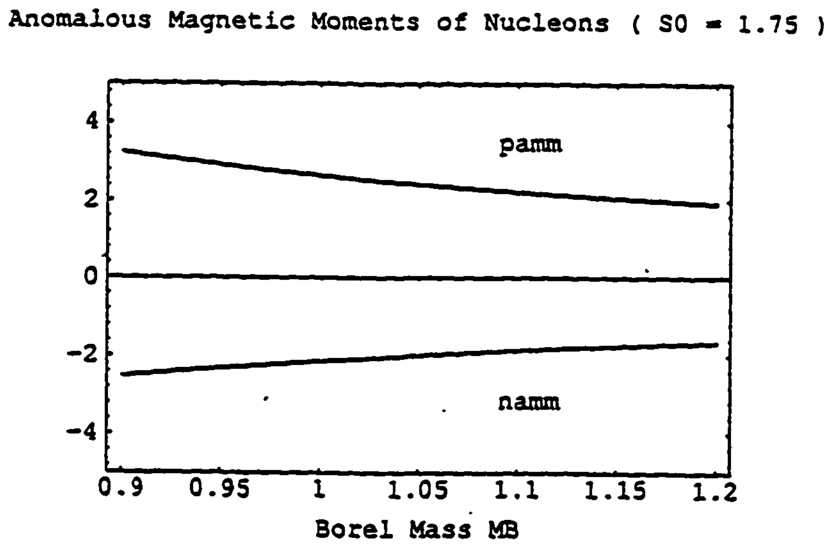

Figure 6.5: Anomalous magnetic moments of nucleons

The numbers we obtain for the nucleon moments are

$$F_2^p = 2.75, \qquad F_2^n = -2 \tag{6.5}$$

$$F_3^p = 1.0 \times 10^{-2}\bar{\theta}, \qquad F_3^n = -0.5 \times 10^{-2}\bar{\theta} \tag{6.6}$$

As we have explained before, the physical observables should have periodic dependence on the $\bar{\theta}$ parameter. In addition, they should have mild dependence on the continuum threshold s_0^N. We examine these properties below by demonstrate the 3 dimensional plots[3] for the EM moments as functions of (s_0^N, M_B) and ($\bar{\theta}$, M_B) .

[3] The apparent discontinuities shown in the F_3^N plot as a periodic function of the $\bar{\theta}$ parameter has to do with the degeneracy of CP violating vacuua at $\bar{\theta} = \pi$, see [36].

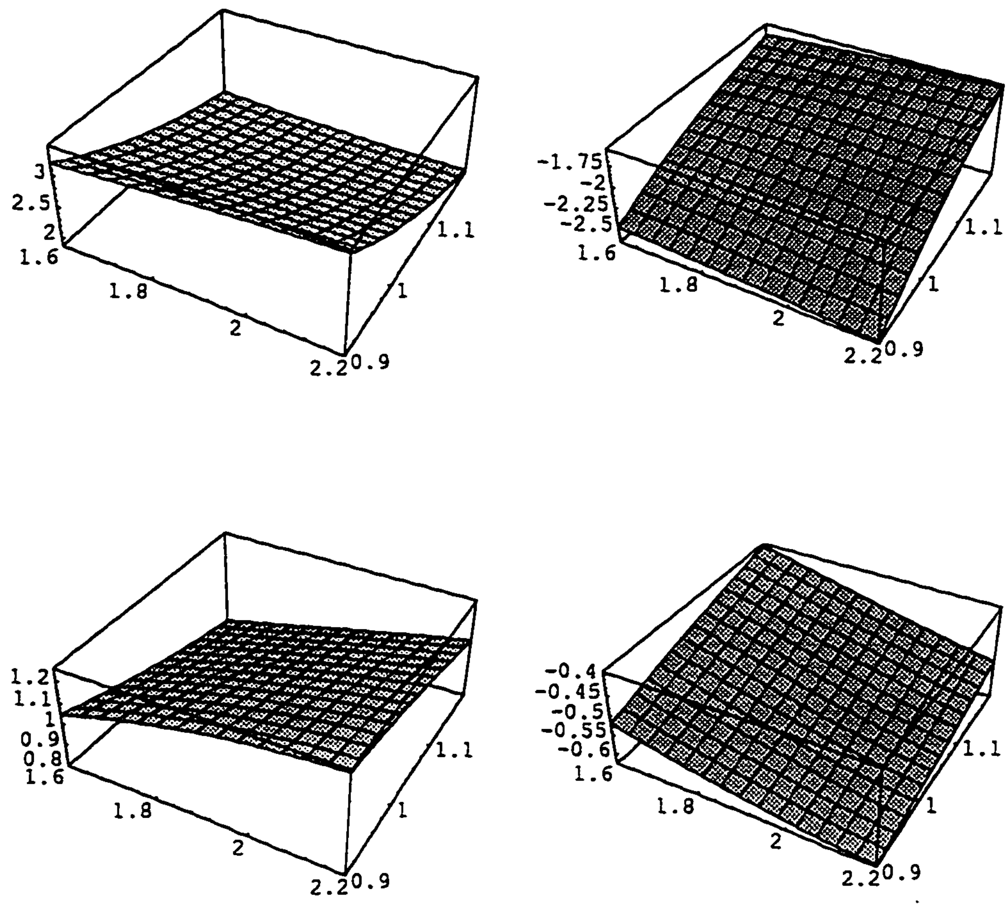

Figure 6.6: EM moments as functions of (s_0^N, M_B)

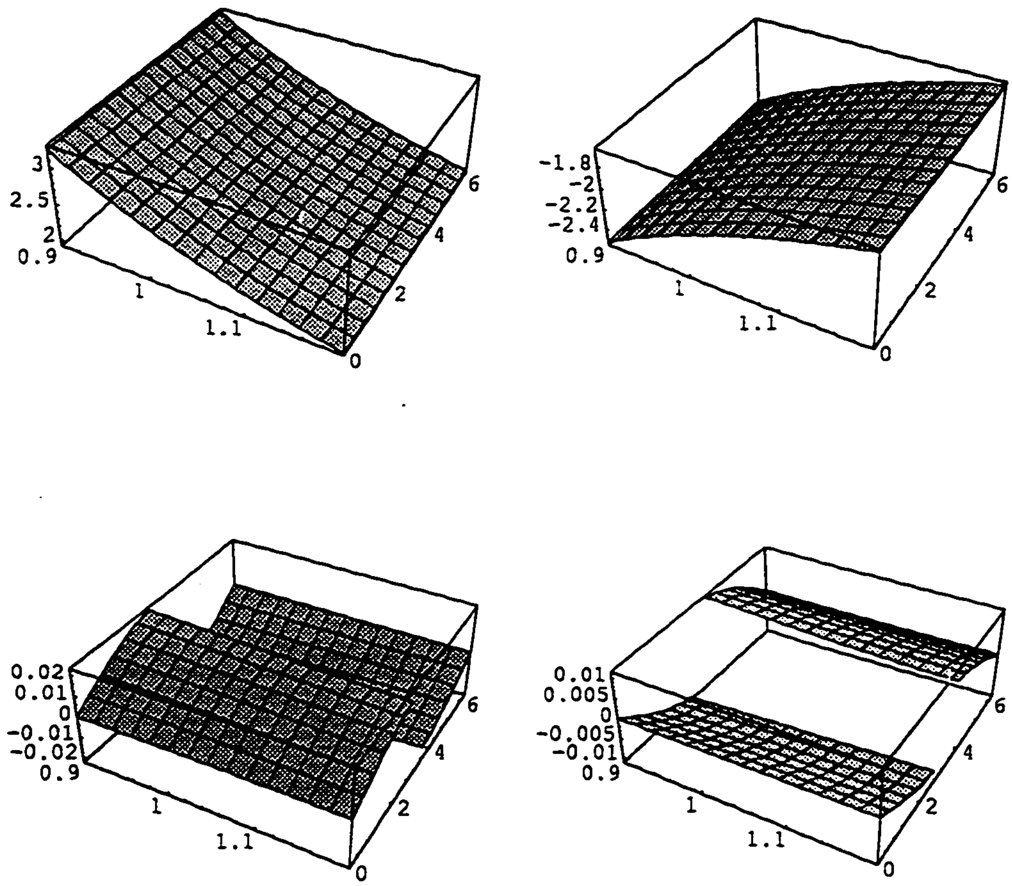

Figure 6.7: EM moments as functions of ($\bar{\theta}$, M_B)

6.4 On the Error Analysis of the QCD Sum Rule Calculations

There are four places in the QCD sum rule calculations which introduce theoretical uncertainities, which should be included as part of the erroer analysis of any QCD sum rule calculations.

(1) The choice of a Borel window:

As discussed before, this uncertainty is related to the truncations of the OPE series and the hadronic parametrizations.

(2) Approximations in the calculations of the Wilson coefficients:

When we calculate the OPE series of a correlation function, the Wilson coefficients are expanded in the powers of "small" parameters, α_s, m_q^2/Q^2 and we make truncations of these series to obtain an approximated results for the Wilson coefficients. Such a truncation procedure adds further uncertainities to our calculations.

(3) The extractions of physical quantities from the matching:

Since physical quantities should be independent of the Borel mass, the departure of such independence of the physical observables in a given Borel window should not be larger than the uncertainty associated with the choice of a Borel window. Nevertheless, the scheme dependent errors (how do we extract physical quantities from the matching) are related to the χ^2 of the matching, and represent the quality of a sum rule calculation.

(4) The dependence of the input parameters:

The QCD sum rule calculations generally rely on input parameters which are derived from other sources, e.g. the quark mass, quark condensates etc.. In many cases, these numbers are not so well determined and their variations from a "standard" values have to be taken into account when we estimate the errors of the physical quantities from a QCD sum rule calculation.

A comprehensive study of all these factors could be very laborious and complicated. We shall present a detailed analysis of our calculations in a future publication.

6.5 Summary and Conclusion

In this chapter, we extract the nucleon observables, including nucleon mass, magnetic moments from the QCD sum rules. The relation between the $\bar{\theta}$ parameter and the nucleon EDM was examined by extracting the linear coefficients in the small $\bar{\theta}$ limit.

The nucleon masses and magnetic moments are close to experimental values and the calculation of the NEDM, combined with the experimental upperbound, give an upper bound for the $\bar{\theta}$, which is less than 10^{-10}.

Chapter 7

FINAL COMMENTS AND CONCLUSIONS

7.1 A Summary of Our Contributions in this Work

Our contributions in this work can be summarized as follows:

(1) We elucidate the connections between three chiral symmetry constraints on the strong $\not{P}$ and $\not{T}$, based on a general $U_A(1)$ covariant representation of the QCD generating functional and the generalized anomalous Ward identity (see sec.2.3). These connections lead to exact identities which relate the anomalous gluonic condensate $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$ and the topological susceptibility constant χ to the masses of quarks m_q and the chiral radii R_q (see sec.2.4). Our graphic illustrations for the chiral symmetry constraints (see sec.2.3.4), together with the sum rule relations, also give a hint to a dual relation of the quark masses m_q and the chiral radius R_q.

(2) We show that the chiral symmetry constraints are explicitly satisfied in the calculation of NEDMs from the QCD sum rule method. In particular, the chiral limit constraint or the existence of a massless quark implies no strong $\not{P}$ and $\not{T}$ can be realized without assuming an isosymmetry in QCD.

(3) Through the use of polar representations of chiral doublet variables, e.g. nucleon EM moments F_2^N and F_3^N; and QCD condensates $\langle\bar{q}q\rangle_{\theta_q,\theta_G}$ and $i\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}$, we can perform a calculation of the NEDMs without assuming a small $\bar{\theta}$ parameter. In addition, the periodic structure of the dependence on the $\bar{\theta}$ parameter for the NEDMs comes out naturally in our derivations.

On the technical side, we have made the following progress:

(1) A systematic way of generating independent tensor basis and their mutal dependence is developed (see App.B). The result is useful for (a) the parametrizations of the nucleon matrix elements $\langle N(p')|J_\mu|N(p)\rangle_{\theta_q,\theta_G}$ in terms of various invariant form factors (see sec.3.1), and (b) the decompositions of nucleon correlation functions $\Pi^N \approx \Pi_0^N + e\Pi_{\mu\nu}^N F^{\mu\nu}$ into various invariant coefficient functions (see sec.3.3).

(2) A complete list of induced condensates (2nd rank tensors) up to dimension 6 is made (see sec.4.2). The relations between susceptibility constants for chiral doublet

condensates, e.g. $\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G}$ and $i\langle\bar{q}\sigma_{\mu\nu}\gamma_5 q\rangle_{\theta_q,\theta_G}$, is clarified by a generalization of the chiral circle theorem (see sec.4.3).

7.2 The Major Results from the QCD Sum Rule Analysis of the NCFs in the Presence of an External EM Field

Our study of the nucleon EM moments, using the method of QCD sum rules, can be summarized as follows:

(1) The electric dipole moments of nucleons d_N, as functions of the $\bar{\theta}$ parameter, are given by:

(a) analytical expressions:

$$F_3^p = \frac{e_u(e_u^2 - e_d^2)(AB_1 - A_1B)m_qR_q\sin\bar{\theta}_q}{e_u^2C_p - e_d^2C_n} \tag{7.1}$$

$$F_3^n = \frac{e_d(e_u^2 - e_d^2)(AB_1 - A_1B)m_qR_q\sin\bar{\theta}_q}{e_u^2C_p - e_d^2C_n} \tag{7.2}$$

where

$$\begin{aligned} A &= 2M_N^6E_2(w, M_N) \\ B &= 2M_N^4E_1(w, M_N) \\ A_1 &= 4M_N^4E_0(w, M_N) \\ B_1 &= -4M_N^2 \\ C_p &= 2\cdot(2\pi)^2R_qM_p^4 \\ C_n &= 2\cdot(2\pi)^2R_qM_n^4 \end{aligned}$$

(b) numerical result:

$$d_p = 1\cdot 10^{-16}\bar{\theta}e - cm \tag{7.3}$$

$$d_n = -0.5\cdot 10^{-16}\bar{\theta}e - cm \tag{7.4}$$

(2) Given the experimental upper bound on the possible existence of a nEDM,

$$d_n \leq 10^{-26}e\cdot cm \text{ (95\%)confid.}$$

We obtain an upper bound for the $\bar{\theta}$ parameter:

$$\bar{\theta} \leq 10^{-10}$$

(3) The upper bound of the ratio $\frac{d_N}{\mu_N}$ with respect to the $\bar{\theta}$ parameter, which is an intrinsic property of the P and T conserving QCD is given by:

$$\frac{d_N}{\mu_N} \leq 10^{-2}$$

7.3 Further Work Related to the Current Study

There are several directions worth exploring which are closely related to the current problem:

(1) Previously the electroweak contributions (via the CKM model) to the NEDMs was either estimated or calculated in an effective approximation, and it was found that these contributions are small. However, in order to set a real upper bound for the $\bar{\theta}$ parameter, it is necessary to do a comprehensive study by including both electroweak and strong CP violations into the calculations.

(2) It is also of interest to see how the presence of nuclear medium effects will change the values of the EM moments of nucleons, in view of the experimental development of RHIC and the cosmological consequences discussed herein. Therefore, a generalization to the finite density and finite temperature case (hot nuclear matter) of the current calculation would be useful.

(3) There are other low energy experiments which look for P and T violations in the subatomic world, for example, the time reversal violations in neutron β decay. The matrix element of interest in that case is $\langle p(p_f)|\bar{d}\gamma_\mu\gamma_5 u|n(p_i)\rangle$, contains $\not{P}$ and $\not{T}$ form factors, and gives rise to a $\not{P}$ and $\not{T}$ observable in the differential cross section. The QCD sum rule method can be applied, with certain generalizations of the current work, to the calculations of $\not{P}$ and $\not{T}$ form factors in these problems.

In addition, there are also some theoretical issues which need to be clarified:

(1) The isospin contents of the nucleon EM moments is an interesting and important subject which we do not discuss in details in the present work[1]. It is not clear whether our analytical results for the nucleon EM moments can be related to

[1] The author would like to thank Prof. Wick Haxton for bring out this issue to his attention.

the chiral model calculation [11] and if not, why the difference? This question is very important phenomenologically in the study of nuclear EDM and the atmoic EDM problem. Also, the general relations between different approaches of low energy QCD are yet to be explored (see the discussion in sec.7.4.3).

(2) It was pointed out by T.D. Cohen and S.H. Lee (see [41]), the use of a simple continuum model (single resonance plus continuum) in the phenomenological parametrization of the sum rule approach can lead to an inconsistent chiral behaviour of the hadronic variables on the QCD parameters (e.g. the dependence on the quark masses). This is often the case, as can be seen if we compare the naive analytic expressions for the hadronic variables (e.g. nucleon mass, magnetic moment, etc) and the similar results from chiral perturbation theory calculations. However, it also has been shown that, by inclusion of pionic tails in the simple continuum model, it is possible to remedy this discrepency between the two approaches [ref]. If we wish to establish a connection between QCD and low energy effective models through the sum rule language, a modified continuum model consistent with the chiral symmetry must be implemented to make the connection and/or comparison meaningful.

(3) As mentioned earlier, the infrared singularity $\frac{\ln Q^2}{Q^2}$, which is closely related to the use of a massless propagator in the presence of an external field, signifies a complicated property of QFT, namely, operator mixing. For example, the lowest dimensional induced operators $m_q F_{\mu\nu}, m_q \tilde{F}_{\mu\nu}$ and $\bar{q}\sigma_{\mu\nu}q$ are mixed under the evolutions of the renormalization group, as expressed by the presence of anomalous dimensions and the dependence of an infrared cutoff in the Wilson coefficients. To settle this superficial dependence on the infrared cut-off in the physical observables requires a careful definition of the infrared cut-off dependence of the vacuum condensates and the definition of the renormalized operators. Such an analysis is necessary in order to make the use of an external field in the QCD sum rule approach self-consistent.

7.4 Prospect and Outlook

7.4.1 The Physics of $\not{P}$ and $\not{T}$ in Low Energy Experiments

The physics of $\not{P}$ and $\not{T}$ is certainly an interesting field and contains a lot of important problems yet to solved. Besides various planned or ongoing high energy programs which search for the CP violations at B factories, there are also many low energy measurements probing similar physics at a different scale.

One important example is given by the atomic EDM measurements (e.g. H_g^{199}), where different components (nuclei, electrons) and interactions (both strong and weak) contribute to the total $\not{P}$ and $\not{T}$ observable – the atomic EDM. To sudy such a compound system requires a combination of different physics at various length scale (from atmoic physics, nuclear physics, particle physics to new physics beyond standard model etc.). These measurements, along with other particle physics experiments and astrophysical observables, surely put many constraints on the possible solutions of the CP violation and guide us toward further understanding of this mystery. In the study of atmoic EDM experiments, we believe that there is a need to reexamine the implication of chiral symmetry constraints on the forms of $\not{P}$ and $\not{T}$ nuclear forces, as these are usually done in the meson exchange model where strong $\not{P}$ and $\not{T}$ contribute[2]. Also, there are related experiments, e.g. $\not{T}$ in neutron β decay, which involve $\not{P}$ and $\not{T}$ hadronic matrix elements and require a detailed analysis. Finally, in the construction of new physics which is used to explain the $\not{P}$ and $\not{T}$ effects (most of these inevitably involve gauge theories), there could be unexpected constraints waiting for us to explore.

7.4.2 The Roles of Gluons in the Low Energy Physics of Baryons and Mesons

In the low energy descriptions of hadrons in terms of QCD, people have to rely on the concept of the constituent quarks[3], and the interactions among these constituent quarks are either given by an effective potential model (mostly derived from a one–gluon–exchange plus confining potential) or the couplings to the Goldstone bosons (e.g. pion, kaon, etc). In any case, the special features of gluons (especially the nonperturbative natures, e.g. $U_A(1)$ anomaly) are either ignored or put in various models by hand. However, we know that there are several low energy hadronic observables which are related the gluons and and can be quantified as matrix elements of gluonic operators. For example:

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[2] For example, the special role played by η' meson, which carries the information of the $U_A(1)$ anomaly, should have some impact on the $\not{P}$ and $\not{T}$ nuclear forces.

[3] These are believed to be the quasi-particles in the full QCD Lagrangian, but a quantitative and microscopic derivation is yet to be found.

(1) $\pi \rightarrow 2\gamma$, which probes the chiral anomaly in QCD [26].

(2) The mass difference between pion and eta particles, $M_{\eta'}^2 - M_\pi^2$, is related to the matrix element $\langle\Omega|G\tilde{G}|\eta'\rangle$.

(3) The mass difference between parity doublet N_+(normal nucleon) and N_- ($N^*(1530)$), $M_{N_+} - M_{N_-}$, is related to the matrix element $\langle N_+|G\tilde{G}|N_-\rangle$.

(4) A complete analysis of $\pi - \eta - \eta'$ mixing , including the effects of the gluon anomaly and the s quark.

These problems address the special roles that the gluons play in the low energy hadronic world and have not been studied directly in terms of the QCD Lagrangian. The QCD sum rule method, as exemplified by this work, would be a useful technique to do a quantitative analysis in this area.

7.4.3 QCD Sum Rules as a Bridge From a Quark—Gluon Picture to the World of Baryons and Mesons

In addition to the lattice gauge theory approach, there are currently three major approaches in the study of low to medium energy physics:

(1) quark models (including bag models),

(2) chiral perturbation theory (and the Skyrme models),

(3) QCD sum rules.

While none of these approaches generate a complete solution of the QCD dynamics, they do provide a rough picture of the low energy world of baryons and mesons and how the symmetries of QCD govern the pattern of these hadronic interactions. For most practical calculations, they often give comparable results (particularly if they agree with experiments). This may not be a surprise, since, they all try to incorporate as many features of QCD as they can. In view of this, it is of interest to ask the following question: Is there any relation or correspondence between these different approaches?

This is certainly not an easy question, as we do not, for instance, really know the actual relations between the constituent quark in the quark model and the current quark in the QCD Lagrangian, nor is it clear how the approximately massless quarks can form a massive nucleon. The only hint seems to root in the symmetries of QCD. The current algebra relations provide one of the earliest examples (e.g. Goldberger-Treiman relation, etc.). It is not too hard to see by following the same spirit, we should be able to establish a connection of the quark—gluon picture (in the operator language, as is normally used in the sum rule approach) and the world of baryons and mesons (which are the effective degrees of freedom in the chiral Lagrangian).

Furthermore, these correspondences can be generalized to the nuclear domain, at least in the few body systems. The various approaches mentioned above have been used extensively to study the properties of single hadrons and the couplings between mesons and baryons as the input parameters used in nuclear physics. Moreover, there has been renewed interest in the study of nuclear forces from the QCD perspective (chiral symmetry, large N_C, etc.). Other issues, like charge symmetry also suggest a request for a deeper understanding of the important fundamental question? How does QCD work in the low energy region?

It is in this area that we may try to apply the QCD sum rule method and establish a connection between various approaches. Specifically, there are three problems we wish to examine in the near future:

(1) Binding energies in the few nucleon systems:

The binding energies of many few nucleon systems is of fundamental importance in nuclear physics, e.g. the binding energy of the deuteron, and the difference between the binding energies of H^3 and He^3. By using a quark interpolating field for nucleon states and an interpolating wave function for nuclei, it is possible to calculate the invariant mass of these few body system, hence the binding energy.

(2) Excited states and transitions:

Normally, the method of QCD sum rules is used to study the physical properties of a ground state for given quantum numbers and the effects of excited states are lumped in the continuum model. In the few body system, it is often of interest to know a particular transition matrix element, e.g. $\Delta \rightarrow N+\gamma$, between a ground state and a nearby excited state. In addition, we have no knowlege how the nonrelativistic description of the internal excitations of hadrons, as given by the quark model, can be derived from the QCD Lagrangian. Therefore, it is of interest to see if there is a generalized scheme in the QCD sum rule approach to allow for a natural inclusion of the excited states as generated by the QCD dynamics.

(3) The nature of nucleon forces:

We can apply the QCD sum rule method to a four point function, which is often related to the scattering amplitude of two particles, where the amputated invariant amplitude could tell us the nature of the exchanged particle, or in the nonrelativistic language, the potential between scattering particles. For example, the Coulomb potential between two charged particles can be related to the exchanged virtual photon in a scattering amplitude. Similarly, the Yukawa potential, responsible for the long range part of the nucleon force, can be related to the exchange of a massive pion between two nucleon, in a hadronic field theory. If we are interested in the realizations of the underlying symmetries of QCD in the nucleon forces, particularly those related to chiral symmetry, it is important that we can quantify these features in terms of QCD parameters. In this regard, QCD sum rule method provides such a tool without involving too much computational complications. Furthermore, this method has the advantages that it is based on a Lorentz covariant formulation and there is no artifical separation between an interaction vertex and a propagator for the interactions between composite fields like hadrons. This could be of interest in the study of charge symmetry breaking in the nuclear forces.

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Appendix A

NOTATION AND CONVENTION

A.1 Metric, Dirac Matrices

For most part of this work, we shall follow the notation and convention of Bjorken and Drell [25].

(1) The space-time metric is defined as $g_{00} = 1, g_{11} = g_{22} = g_{33} = -1$.

(2) The inner product of two 4–vectors is $x \cdot p \equiv x^0 p^0 - \vec{x} \cdot \vec{p}$

(3) The 4–dimensional totally antisymmetric tensor $\epsilon_{0123} \equiv -\epsilon^{0123} = 1$

The Dirac γ matrices satisfy the anticommutation relations:

$$\{\gamma_\mu, \gamma_\nu\} \equiv \gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2 g_{\mu\nu}$$

$$\sigma_{\mu\nu} \equiv \frac{i}{2}[\gamma_\mu, \gamma_\nu] \equiv \gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu, \quad \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \equiv \gamma^5$$

The charge conjugation matrix $C \equiv i\gamma^2\gamma^0$, has the following properties:

$$C = -C^{-1} = -C^\dagger = -C^T, \quad C\gamma_\mu C^{-1} = -\gamma_\mu^T$$

Another useful transformation on the γ matrices:

$$\bar{\Gamma} \equiv \gamma^0 \Gamma^\dagger \gamma^0$$

For $\Gamma = \gamma_\mu, \sigma_{\mu\nu}, i\gamma_5$, one can show that $\bar{\Gamma} = \Gamma$.

For more relations between second rank tensor matrices, please refer to App.B .

A.2 Fourier Transformation

A table of massless Fourier transforms is given in [23]:

$$\mathcal{F}.\mathcal{T}.[f(x)] \equiv [f(p^2)]_{2n} \equiv \int d^4x e^{ipx} \frac{f(x)}{(x^2 - i\epsilon)^{2n}} \tag{A.1}$$

A special case from above is

$$[1]_{2n} = C_n(p^2)^{n-2}ln(-p^2), C_n \equiv \frac{i\pi^2(-1)^n}{(n-2)!(n-1)!4^{(n-2)}} \quad (A.2)$$

A.3 Feynman Propagator

The Feyman propagator for the free spin $\frac{1}{2}$ fermion is

$$S_F(x) \equiv (-i)\langle T\psi(x)\bar{\psi}(0)\rangle \quad (A.3)$$

$$= \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \frac{\hat{p}+m}{p^2-m^2+i\epsilon} \quad (A.4)$$

In momentum space

$$S_F(p) \equiv \frac{1}{\not{p}-m+i\epsilon} = \frac{\hat{p}+m}{p^2-m^2+i\epsilon} \quad (A.5)$$

In coordinate space, the Feyman propagator for the free spin $\frac{1}{2}$ fermion with mass m_q has the leading singular form (in the chiral basis with a θ_q quark phase):

$$S_F(x) \equiv \frac{i\hat{x}}{2\pi^2 x^4} - \frac{m_q e^{-i\theta_q\gamma_5}}{4\pi^2 x^2} \quad (A.6)$$

In the presence of an external EM field $F_{\mu\nu}$, and with the inclusion of nonperturbative effect, parametrized by vacuum condensates, we have the following terms add to the perturbative propagator

$$S_F^{NP}(x) \equiv -\frac{ie_q\tilde{F}^{\xi\eta}x_\xi\gamma_\eta\gamma_5}{8\pi^2x^2} - \frac{R_q e^{-i\theta_{Gq}\gamma_5}}{12} - \frac{\chi R_q}{24}(F\cdot\sigma)e^{-i\theta_{Gq}\gamma_5} + (\geq \text{dim.4}) \quad (A.7)$$

$$\text{where the chiral radius} R_q^2 \equiv \langle\bar{q}q\rangle_{\theta_q,\theta_G}{}^2 + i\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}{}^2 \quad (A.8)$$

$$\text{and the gluon phase} \tan\theta_{Gq} \equiv \frac{i\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}}{\langle\bar{q}q\rangle_{\theta_q,\theta_G}} \quad (A.9)$$

Appendix B

ON THE TENSOR STRUCTURE OF THE NUCLEON CORRELATION FUNCTION (NCF)

In this section, we shall discuss:

- The construction of the complete set of basis tensors for the nucleon correlation function (NCF) in the external field background
- The classifications of the basis tensors (dimensionality v.s. chirality)
- Discrete symmetry transformations (C, P, T) and the basis tensors

B.1 On the construction of the basis tensors for the NCF in the external field background

It is important to know how we can write down a complete set of covariant tensors (these will be refered as **basis tensors**, composed of one Lorentz vector p_μ and 16 Dirac matrices) and decompose the polarization tensor $\Pi^N_{\mu\nu}(p)$ (treated as a 4×4 Dirac matrix and a second rank Lorentz tensor) in terms of these basis tensors. Such structures come out naturally from both QCD calculations (see the discussions in Chap.4) and hadronic representations (see the discussions in Chap.3) for the nucleon correlation function (NCF). Furthermore, the QCD sum rules are extracted from the coefficient functions asociated with these basis tensors. A correct decomposition will help us make sure that there are no omission and redundency in our calculations.

The choice of these basis tensors are not unique; however, any legitimate choice has to meet two criterions:

1. **completeness** Any second rank tensor matrix (as a function of p_μ) must be able to be written as an unique linear combination (with invariant coefficient functions of p^2) of these basis tensors.

2. **independence** None of the basis tensor can be written as a linear combination of the other members in a given representation.

Hence, the basis set we choose is maximal (no omission) in the sense of the first criterion and minimal (no redundency) in the sense of the second.

The answer, as given in the main text (see sec. 3.4) is that, we have 8 independent tensors in the basis set (without imposing any discrete symmetries C, P and T) for $\Pi^N_{\mu\nu}(p)$[1]. Our aim here is to show that there is a systematic way to generate and enumerate a list of basis tensors. Moreover, in so doing, we can assure ourself that both criteria (completeness and independence) are satisfied automatically.

The idea is that, by taking the commutators and the anticommutators of $\not{p}$ and γ_μ in an iterative manner, we can generate all the independent tensors in an ecnomic way:

[1] These can be found by either trial and error or using brute force combinatoric. Nevertheless, these are not desirable approaches.

1. Almost all the independent tensors appear only once in such a procedure.

2. If any term does appear more than once, this simply gives us a relation for mutually dependent tensors from different (but equivalent) basis sets.

3. Such an iterative procedure always ends in finite steps (it either ends with a zero or repeats the pattern over again); hence the algebra of the basis tensors is closed under commutations and anticommutations, so that we are guaranteed that the basis set is complete.

Omitting the detailed calculation, we present the process in the following **"tree graph"** analysis.

$$\sigma_{\mu\nu}$$

$$[\sigma_{\mu\nu}, \hat{p}] = (-2i)(p_\mu\gamma_\nu - p_\nu\gamma_\mu) \qquad \{\sigma_{\mu\nu}, \hat{p}\} = 2\epsilon_{\mu\nu\alpha\beta}p^\alpha\gamma^\beta\gamma_5$$

$$[[\sigma_{\mu\nu}, \hat{p}], \hat{p}] = (-4i)(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\hat{p} \qquad \{[\sigma_{\mu\nu}, \hat{p}], \hat{p}\} = 0 \qquad [\{\sigma_{\mu\nu}, \hat{p}\}, \hat{p}] = 0 \qquad \{\{\sigma_{\mu\nu}, \hat{p}\}, \hat{p}\} = (-4)p^2\sigma_{\mu\nu} + [[\sigma_{\mu\nu}, \hat{p}], \hat{p}]$$

$$[[[\sigma_{\mu\nu}, \hat{p}], \hat{p}], \hat{p}] = (4p^2)[\sigma_{\mu\nu}, \hat{p}] \qquad \{[[\sigma_{\mu\nu}, \hat{p}], \hat{p}], \hat{p}\} = 0$$

$$\sigma_{\mu\nu}\gamma_5$$

$$[\sigma_{\mu\nu}\gamma_5, \hat{p}] = -\{\sigma_{\mu\nu}, \hat{p}\}\gamma_5 = -2\epsilon_{\mu\nu\alpha\beta}p^\alpha\gamma^\beta$$

$$\{\sigma_{\mu\nu}\gamma_5, \hat{p}\} = -[\sigma_{\mu\nu}, \hat{p}]\gamma_5 = (2i)(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\gamma_5$$

$$[[\sigma_{\mu\nu}\gamma_5, \hat{p}], \hat{p}] = (-4)p^2\sigma_{\mu\nu}\gamma_5 + [[\sigma_{\mu\nu}, \hat{p}], \hat{p}]\gamma_5$$

$$\{[\sigma_{\mu\nu}\gamma_5, \hat{p}], \hat{p}\} = 0$$

$$[\{\sigma_{\mu\nu}\gamma_5, \hat{p}\}, \hat{p}] = 0$$

$$\{\{\sigma_{\mu\nu}\gamma_5, \hat{p}\}, \hat{p}\} = (-4i)(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\hat{p}\gamma_5$$

$$[\{\{\sigma_{\mu\nu}\gamma_5, \hat{p}\}, \hat{p}\}, \hat{p}] = 0$$

$$\{\{\{\sigma_{\mu\nu}\gamma_5, \hat{p}\}, \hat{p}\}, \hat{p}\} = (-4p^2)[\sigma_{\mu\nu}, \hat{p}]\gamma_5$$

B.2 The classifications of the basis tensors— dimensionality v.s. chirality and discrete symmetry transformations

There are several comments we would like to add:

1. The number of independent tensors can be obtained from the representation theory of the Lorentz group, which we do not discuss here. However, we believe that this procedure can be generalized to other groups (e.g. internal symmetry groups, $SU(N)$) as well, and give a simple machinery for obtaining the complete basis set in other situations.

2. We can organize these tensors in terms of power counting (in p_μ and chiral pairing (tensor $\times$ I or γ_5). See sec. 3.3.

3. Next, we can examine the (discrete) symmetry properties of these basis tensors. Using the following definitions [25]:

$$P : \bar{\Gamma} \equiv \gamma_0\Gamma^\dagger\gamma_0 \qquad \text{(B.1)}$$

$$C : C \equiv i\gamma_2\gamma_0, \; \Gamma^C \equiv C\Gamma C \qquad \text{(B.2)}$$

$$T : T \equiv CP \qquad \text{(B.3)}$$

The transformation properties of the basis tensors under the discrete symmetries P, T, and C can be obtained by applying these definitions. In particular, for the charge conjugations, we find the tensors $\epsilon_{\mu\nu\alpha\beta}p^{\alpha}\gamma^{\beta}$ and $p_{\mu}\gamma_{\nu} - p_{\nu}\gamma_{\mu}$ have odd charge parity. Therefore, in the sum rule identities, their invariant coefficients vanish as the strong CP violation conserves charge parity.

Appendix C

ON THE SOLUTIONS OF THE NUCLEON EM MOMENTS FROM THE SUM RULE EQUATIONS IN $N_F = 2$ QCD

In this section, we shall discuss:

- The choice of sum rule equations for the EM moments of proton and neutron in $N_f = 2$ QCD.

- The solutions of the EM moments of proton and neutron in $N_f = 2$ QCD.

C.1 The choice of sum rule equations for the EM moments of proton and neutron in $N_f = 2$ QCD

As mentioned in the main text, the symmetry constraints on the $\not{P}$ and $\not{T}$ in QCD should be applied to each flavor independently. Either any quark becomes massless or its chiral radius shrinks to zero, the strong $\not{P}$ and $\not{T}$ should vanish. Therefore, it is important that in the QCD sum rule calculation, we can demonstrate such strigent constraints are indeed satisfied explictly without assuming an isospin symmetry.

Without assuming an isosymmetry in $N_f = 2$ QCD, the QCD sum rules of the NCFs become very complicated. In addition to the doubling of the input parameters, which includes: quark masses m_u, m_d; chiral radii R_u, R_d; quark chiral angles θ_u, θ_d and θ_{Gu}, θ_{Gd}, and many susceptibility constants, the manipulations of the proton and neutron sum rules in order to eliminate these susceptibility constants and infreard cutoff add further difficulties. For these reasons, we shall discuss these subtleties in this appendix.

First of all, the solutions of the nucleon masses, without the inclusion of virtual photon exchange between quarks, should present no trouble if we carefully label the flavour dependence of the input parameters in QCD. Thus, the (I, γ_5) sum rule equations for the nucleon propagator can be written as:

$$Am_q e^{i\theta_q\gamma_5} + BR_q e^{i\theta_{Gq}\gamma_5} = C_N e^{i\theta_N\gamma_5} \tag{C.1}$$

where A and B are proportional to Wilson coefficients (as functions of Borel mass M_B) in the OPE series, and C_N is proportional to the product of nucleon mass and the nucleon propagator. The quark basis dependent nucleon phase θ_N is defined in eq.3.24.

We can find a solution of C_N and θ_N in terms of A, B, m_q, R_q, θ_q and θ_{Gq}:

$$C_N^2 = A^2 m_q^2 + 2ABm_q R_q \cos\tilde{\theta}_q + B^2 R_q^2 \tag{C.2}$$

$$\tan\theta_N = \frac{Am_q \sin\theta_q + BR_q \sin\theta_{Gq}}{Am_q \cos\theta_q + BR_q \cos\theta_{Gq}} \tag{C.3}$$

For future reference, we also need several expressions of these covariant angles. For example, by multiplying the equation (C.1) with a covariant phase $e^{-i\theta_{Gq}\gamma_5}$, we get

$$Am_q e^{-i\tilde{\theta}_q\gamma_5} + BR_q = C_N e^{i(\theta_N - \theta_{Gq})\gamma_5} \tag{C.4}$$

Taking the real and imaginary parts of the equation, we have

$$\cos(\theta_N - \theta_{Gq}) = \frac{Am_q \cos\bar{\theta}_q + BR_q}{C_N} \quad \text{(C.5)}$$

$$\sin(\theta_N - \theta_{Gq}) = \frac{-Am_q \sin\bar{\theta}_q}{C_N} \quad \text{(C.6)}$$

Similarily, by multiplying the equation with a covariant phase $e^{-i\theta_q\gamma_5}$, we get

$$Am_q + BR_q e^{i(\theta_{Gq}-\theta_q)\gamma_5} = C_N e^{i(\theta_N-\theta_q)\gamma_5} \quad \text{(C.7)}$$

Taking the real and imaginary parts of the equation, we have

$$\cos(\theta_N - \theta_q) = \frac{Am_q + BR_q \cos\bar{\theta}_q}{C_N} \quad \text{(C.8)}$$

$$\sin(\theta_N - \theta_q) = \frac{BR_q \sin\bar{\theta}_q}{C_N} \quad \text{(C.9)}$$

These realtions will be used repeatedly when we derive the solutions for the EM moments of nucleons.

To generate useful sum rules for the nucleon EM moments from the polarization tensors, we need to take certain combinations of proton and neutron invariant coefficient functions associated with the following tensors:

$$\begin{array}{rl} \text{odd tensor}: & \hat{p}\sigma_{\mu\nu} + \sigma_{\mu\nu}\hat{p}, \;\; (p_\mu\gamma_\nu - p_\nu\gamma_\mu)\gamma_5 \\ \text{even tensor}: & \hat{p}(p_\mu\gamma_\nu - p_\nu\gamma_\mu)(I, \gamma_5) \end{array}$$

For the $\hat{p}\sigma_{\mu\nu} + \sigma_{\mu\nu}\hat{p}$ sum rules, we have

$$e_d(F_2^p + F_1^p) - e_u(F_2^n + F_1^n) = K_1' \quad \text{(C.10)}$$

where K_1' represents some dimensional 6 condensate. After some algebra, we can rewrite the equation as

$$e_d F_2^p - e_u F_2^n = -e_d + K_1' \equiv K_1 \quad \text{(C.11)}$$

The situation for the other odd tensor sum rule, $(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\gamma_5$ is quite different. The OPE series starts from dimension 6 operator, $\langle G\tilde{G}\rangle_{\theta_q,\theta_G} F_{\mu\nu}$, and there are strong quark mass nonanalyticity assiciated with these diagrams. In view of this, we are unable to calculate the Wilson coefficients at this stage. However, since on the right hand side of the sum rule, we know that the contributions to the invariant coefficient function is from the NEDM only, and it is linear, we can write down a sum rule equation for the tensor $(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\gamma_5$ as follows

$$aF_3^p - bF_3^n = K_2 \propto \langle G\tilde{G}\rangle_{\theta_q,\theta_G} \tag{C.12}$$

where a, b are some linear combinations of e_d and e_u; we shall explore some possible scenario after we solve the sum rule equations.

The real trouble in the $N_f = 2$ QCD is the sum rule associated with the tensor $\hat{p}(p_\mu\gamma_\nu - p_\nu\gamma_\mu)(I, \gamma_5)$, for at least two reasons:

(1) Because of the chiral property (even tensors are chirally covariant) we need to take care of the various chiral phases. The fact that the isosymmetry is broken implies all the chiral phases are flavor dependent.

(2) The eliminations of susceptibility constants and the infrad cutoff dependence cannot be achieved at the same time. Unlike the case of isosymmetric QCD these undesired unknowns have to be handled separately.

To settle these problems, we need to make use of Crewther's conditions and the lemmas derived at the beginning of the section.

To begin with, after the elimination of the single pole contribution through the use of the differential operator $1 - M_B\frac{\partial}{\partial M_B}$, the sum rule associated with the tensor $\hat{p}(p_\mu\gamma_\nu - p_\nu\gamma_\mu)(I, \gamma_5)$ reads:

$$A'_N m_q e^{i\theta_q\gamma_5} + B'_N R_q e^{i\theta_{Gq}\gamma_5} = C'_N e^{i(\theta_N+\alpha_N)\gamma_5} \tag{C.13}$$

where the unknowns C'_N, α_N can be related to the nucleon EM moments

$$C'_N \propto F_N, \;\; \tan\alpha_N \equiv \frac{F_3^N}{F_2^N}$$

To first order in the electric charge e_q, the Wilson coefficients A'_N, B'_N can be written as

$$A'_p \equiv e_u A_1 + e_d A_2 \tag{C.14}$$

$$A'_n \equiv e_d A_1 + e_u A_2 \tag{C.15}$$

$$B'_p \equiv e_u B_1 + e_d B_2 \tag{C.16}$$

$$B'_n \equiv e_d B_1 + e_u B_2 \tag{C.17}$$

where A_2 contains the infrad cutoff dependence (associated with the operator $m_q F_{\mu\nu}$ and B_2 contains many susceptibility constants (associated with the operators $\bar{q}\sigma_{\mu\nu}q$ and $\bar{q}G_{\mu\nu}(1,\gamma_5)q$). As before, we can multiply this equation by a chiral phase $R_q e^{-i\theta_{Gq}\gamma_5}$, and take the imaginary part (which eliminates the contribution of B'_N, hence the dependence of B_2), we get

$$-A'_N\Delta = R_q C'_N \sin(\theta_N - \theta_{Gq} + \alpha_N) \tag{C.18}$$

where $\Delta \equiv m_q R_q \sin\bar{\theta}_q$ is a flavour independent number, which is propotional to the anomalous gluon condensate $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$

It is possible now to eliminate the A_2 contribution by taking a linear combination of proton and neutron sum rules by multiplying the equation for the proton with e_u and that for the neutron with e_d and taking the difference:

$$-(e_u A'_p - e_d A'_n)\Delta = e_u R_d C'_p \sin(\theta_p - \theta_{Gd} + \alpha_p) - e_d R_u C'_n \sin(\theta_n - \theta_{Gu} + \alpha_n) \tag{C.19}$$

After expanding the $\sin(\theta_N - \theta_{Gq} + \alpha_N)$ and using the relation (C.5),(C.6) and (C.8),(C.9), which render the polar form of the sum rule equations into a Cartesian representation, we obtain an equation for $F_2^p, F_3^p, F_2^n, F_3^n$:

$$\begin{aligned} e_u R_d \cos(\theta_p - \theta_{Gd})F_3^p &+ e_u R_d \sin(\theta_p - \theta_{Gd})F_2^p \\ -e_d R_u \cos(\theta_n - \theta_{Gu})F_3^n &- e_d R_u \sin(\theta_n - \theta_{Gu})F_2^n \\ = -(e_u A'_p - e_d A'_n)\Delta & \end{aligned} \tag{C.20}$$

We can replace the proton EM moments F_2^p, F_3^p by the neutron ones, using the sum rule equations associated with the $\hat{p}\sigma_{\mu\nu} + \sigma_{\mu\nu}\hat{p}$ and $(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\gamma_5$ tensors, In so doing, the above equation becomes

$$\begin{aligned} \left[\frac{b}{a} e_u R_d \cos(\theta_p - \theta_{Gd}) - e_d R_u \cos(\theta_n - \theta_{Gu})\right] F_3^n &\quad + \\ \left[\frac{e_u^2}{e_d} R_d \sin(\theta_p - \theta_{Gd}) - e_d R_u \sin(\theta_n - \theta_{Gu})\right] F_2^n &\quad = \\ = -(e_u^2 - e_d^2)A_1\Delta & \\ -\frac{e_u R_d K_1}{e_d}\sin(\theta_p - \theta_{Gd}) - \frac{e_u R_d K_2}{a}\cos(\theta_p - \theta_{Gd}) & \end{aligned} \tag{C.21}$$

With a similar treatment, one can generate another equation by keeping the B_N term and taking another linear combination to eliminate the B_2 contribution. The net result is:

$$\begin{aligned}
\left[\frac{b}{a}e_u m_d \cos(\theta_p-\theta_d) - e_d m_u \cos(\theta_n-\theta_u)\right] F_3^n \quad &+ \\
\left[\frac{e_u^2}{e_d} m_d \sin(\theta_p-\theta_d) - e_d m_u \sin(\theta_n-\theta_u)\right] F_2^n \quad &= \\
(e_u^2-e_d^2)B_1\Delta \\
-\frac{e_u m_d K_1}{e_d}\sin(\theta_p-\theta_d) - \frac{e_u m_d K_2}{a}\cos(\theta_p-\theta_d)
\end{aligned} \tag{C.22}$$

The triangular functions appearing in the coefficients of the equation for the neutron EM moments can be written in terms of chiral invariant phases, $\bar{\theta}_u, \bar{\theta}_d$, using the lemmas we have derived in the begining of this section. After these substitution, we have

$$\begin{aligned}
\left[\left(\frac{be_u}{aC_p}\right)(Am_dR_d\cos\bar{\theta}_d + BR_d^2) - \left(\frac{e_d}{C_n}\right)(Am_uR_u\cos\bar{\theta}_u + BR_u^2)\right] F_3^n \quad &+ \\
+\left[\left(\frac{e_d^2}{C_n}\right) - \left(\frac{e_u^2}{C_p}\right)\right]\frac{A\Delta}{e_d}F_2^n \quad &= \\
-(e_u^2-e_d^2)A_1\Delta + \frac{e_u A\Delta K_1}{e_d C_p} - \frac{e_u K_2}{aC_p}(Am_dR_d\cos\bar{\theta}_d + BR_d^2)
\end{aligned} \tag{C.23}$$

$$\begin{aligned}
\left[\left(\frac{be_u}{aC_p}\right)(Am_d^2 + Bm_dR_d\cos\bar{\theta}_d) - \left(\frac{e_d}{C_n}\right)(Am_u^2 + Bm_uR_u\cos\bar{\theta}_u)\right] F_3^n \quad &+ \\
-\left[\left(\frac{e_d^2}{C_n}\right) - \left(\frac{e_u^2}{C_p}\right)\right]\frac{B\Delta}{e_d}F_2^n \quad &= \\
(e_u^2-e_d^2)B_1\Delta - \frac{e_u B\Delta K_1}{e_d C_p} - \frac{e_u K_2}{aC_p}(Am_d^2 + Bm_dR_d\cos\bar{\theta}_d)
\end{aligned} \tag{C.24}$$

In the end, we have derived two equations for the EM moments of the neutron, which we shall solve in the next section.

C.2 The solutions of the EM moments of proton and neutron in $N_f = 2$ QCD

It is relatively easy to solve for F_3^n, as we can add the previous equations to eliminate F_2^n, and the solution for F_3^p can be obtained by either using the sum rule associated with the $(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\gamma_5$ tensor or by performing an isospin rotation:

$$F_3^n = \frac{a(e_u^2 - e_d^2)(AB_1 - A_1B)\Delta}{be_uC_p - ae_dC_n} - \frac{e_uK_2C_p}{be_uC_p - ae_dC_n} \tag{C.25}$$

$$F_3^p = \frac{b(e_u^2 - e_d^2)(AB_1 - A_1B)\Delta}{be_uC_p - ae_dC_n} - \frac{e_dK_2C_n}{be_uC_p - ae_dC_n} \tag{C.26}$$

It is clear that the symmetry constraints are satisfied for both proton and neutron EDMs, as both are proportional to Δ and K_2, which are proportional to the anomalous gluon condensate $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$.

Appendix D

ON THE CHOICE OF NUCLEON CURRENT η^N

D.1 On the Construction of Nucleon Interpolating Field Operators η^N

We shall not explain how to construct the interpolating field operators for the low lying baryon octet using three quark fields and Dirac matrices. For a detailed explanation, please refer to A. Christo [42]. Instead, we will focus on the interplay between chiral symmetry constraints and the possible choice of nucleon interpolating field operators η^N.

We know that, to the lowest dimension, there are two independent interpolating field operators η_1^N, η_2^N for a given nucleon state N (For definiteness, we shall discuss the case for neutron only):

$$\eta_1^n \equiv ((d^t)^a C\gamma_\mu d^b)\gamma_5\gamma^\mu u^c \epsilon_{abc} \tag{D.1}$$

$$\eta_2^n \equiv ((d^t)^a C\sigma_{\mu\nu} d^b)\gamma_5\sigma^{\mu\nu} u^c \epsilon_{abc} \tag{D.2}$$

Another equivalent choice can be related to the above by the following relations:

$$\eta_1^n = 2[((d^t)^a C u^b)\gamma_5 d^c - ((d^t)^a C\gamma_5 u^b) d^c]\epsilon_{abc} \tag{D.3}$$

$$\eta_2^n = 2[((d^t)^a C u^b)\gamma_5 d^c + ((d^t)^a C\gamma_5 u^b) d^c]\epsilon_{abc} \tag{D.4}$$

One can show that these interpolating field operators have correct quantum numbers (e.g. charge, flavour, etc.) and they transform like spin $\frac{1}{2}$ particles under the Lorentz group.

D.2 $U_A(1)$ Chiral Transformations and the Nucleon Interpolating Field Operators η^N

In our study of the NEDM and strong $\not{P}$ and $\not{T}$ problem, we choose η_1^n to do our calculations. The reason is that η_1^n (and η_2^n) is (are) eigenvector(s) of the $U_A(1)$

chiral transformations: one can show that under a chiral rotation

$$q \to e^{i\theta\gamma_5} q \quad \Rightarrow \quad \eta_1^n \to e^{-i\theta\gamma_5}\eta_1^n, \quad \eta_2^n \to e^{3i\theta\gamma_5}\eta_2^n \tag{D.5}$$

The other set of independent intepolating fields, as mentioned in sec.D.1, will mix under a chiral rotation.

The choice of an eigenvector of the $U_A(1)$ chiral transformations for the nucleon interpolating fields has the advantage that the QCD sum rules derived from the correlation function of these interpolating fields are manifestly $U_A(1)$ covariant. It is important to make sure that all the $U_A(1)$ covariant phases transform in the same way such that the physical observables extracted from the sum rule relations are $U_A(1)$ invariant. This is shown in our calculation where a generic $U_A(1)$ covariant sum rule relation has the form:

$$A m_q e^{i\theta_q\gamma_5} + B R_q e^{i\theta_{Gq}\gamma_5} = C_N e^{i\theta_{N_1}\gamma_5} \tag{D.6}$$

All three $U_A(1)$ covariant phases θ_q, θ_{Gq} and θ_N change by -2θ under the chiral rotation D.5 and $C_N \propto M_N$ or F_N are indeed $U_A(1)$ invariant.

However, an observation of the eigencharge under a $U_A(1)$ chiral rotation D.5 shows that η_1^n transforms differently than η_2^n. While the former carries an eigencharge of -1, the later carries an eigencharge of 3. One important question arises immediately, do we lose the $U_A(1)$ chiral covariance of the sum rule relations, if we choose to work with η_2^n?

The answer is no. As the reparametrization invariance of the QCD generating functional and the chiral symmetry constraints are general exact statements, any correct calculations should not violate these statements. What happens in the case of using η_2^n in the sum rule calculations is that the OPE series for the $U_A(1)$ covariant sum rules have the following structure (The definition of θ_{N_2} is similar to that of θ_{N_1}, see eq.3.26)

$$A_1 m_q^3 e^{-i3\theta_q\gamma_5} + A_2 m_q^2 R_q e^{-i(2\theta_q+\theta_{Gq})\gamma_5} + A_3 m_q R_q^2 e^{-i(\theta_q+2\theta_{Gq})\gamma_5} + A_4 R_q^3 e^{-i3\theta_{Gq}\gamma_5} = C_N e^{i\theta_{N_2}\gamma_5} \tag{D.7}$$

The LHS of the sum rule now consists of 4 terms; all of them carry eigencharges of 3.[1]. Therefore, the $U_A(1)$ chiral covariance of the sum rule relations is maintained

[1] One needs to verify that all terms with eigencharge -1 have zero coefficients. The general cal-

in the case of η_2^n and the symmetry constraints on the strong $\not{P}$ and $\not{T}$ in QCD is also obeyed, as it should.

The reason that we choose to do our calculation in η_1^n instead of η_2^n is because the latter involves either higher dimensional condensates or Wilson coefficients which are higher order in the quark mass (because of the chiral properties), and we either have no knowledge of the values of these higher dimensional condensates or make a truncation of the Wilson coefficients to first order in the quark mass.

culations are straightforward, if tedious. However, the conclusions we derived here is actually a generalization of the massless calculations, e.g. the calculations of nucleon masses using QCD sum rules. From the helicity representations of the quark field operator one can see that the nonvanishing term in the identity sum rule for the nucleon intepolating field η_2^n only starts from dimension 9 $\langle\bar{q}q\rangle^3$.

VITA

Chuan-Tsung Chan was born in Nov. 10, 1965 in Taipei, Taiwan. With the enlightment of his teacher in junier high school, he developed an early interest in mathematics and sciences. This interest was further nurtured when he took the physics and math classes in high school. Unlike other brilliant peers in his age, he always earned low scores in the tests. Still, he kept the prejudice that this is due to bad education and time constraints rather than his own stupidness. Such hopeless optimism was rewarded when he entered the Department of Mathematics of National Taiwan University (NTU) in Oct. 1983, with an **exact** passing score.

While still enjoyed the torturing training in his undergraduate study with insignificant performance, he finally realized that what is his real interest in academics and decided switch over to physics. He earned his master degree at NTU under the supervision of Prof. W-Y Pauchy Hwang, and entered the University of Washington in 1992 as a graduate student of Prof. E.M. Henley.

The search of a life is always a long story. We shall put a small period in the end of 1996, the year that the tired person finally earns his Ph.D..