

Identifying and addressing student difficulties with electrostatics and magnetostatics in
introductory and advanced courses

Ryan Lowell Crites Hazelton

A dissertation
submitted in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy

University of Washington

2015

Reading Committee:

Paula Heron, Chair

Peter Shaffer

Leslie Rosenberg

Program Authorized to Offer Degree:

Physics

©Copyright 2015

Ryan Lowell Crites Hazelton

University of Washington

Abstract

Identifying and addressing student difficulties with electrostatics and magnetostatics in introductory and advanced courses

Ryan Lowell Crites Hazelton

Chair of the Supervisory Committee:
Professor Paula R. L. Heron
Department of Physics

This dissertation describes an investigation of student conceptual understanding of statics concepts in E&M, at the introductory and junior level. At the introductory level, this research led to the development of three new research-based and research-validated worksheets for *Tutorials in Introductory Physics*. The topics are electric potential difference, electric properties of conductors, and Ampère's law. The research at the junior level provides the initial research base for the creation of a set of junior-level worksheets, *Tutorials in Physics: Electrodynamics*. This research primarily focuses on identifying advanced students' difficulties with various topics across the E&M course, as well as comparing the difficulties encountered by introductory and advanced students. We also describe some initial attempts to address these conceptual difficulties at the junior level in electrostatics and magnetostatics.

TABLE OF CONTENTS

List of Figures	vii
List of Tables.....	x
Chapter 1: Introduction	1
A. Background and context for research.....	1
1. Main results from physics education research.....	1
2. Curriculum development at the University of Washington	2
B. Topic coverage in the introductory and junior courses.....	3
1. Introductory course.....	3
2. Junior course.....	5
C. Motivation for research.....	6
1. Introductory course.....	6
2. Junior course.....	7
D. Methods of investigation	8
1. Data sources.....	8
2. Description of data.....	9
E. Organization of dissertation.....	10
F. References for chapter 1	11
Chapter 2: Student difficulties with electric potential difference.....	12
A. Conceptually understanding potential difference	13
1. Incorrect choice of systems.....	13

2. Negative work by the field.....	17
3. Internal vs. external work.....	19
B. Conflating force and potential energy.....	20
C. Physical meaning of potential difference	24
D. Potential difference in charge distributions.....	25
E. Discussion	28
F. References for chapter 2.....	29
Chapter 3: Curriculum development to address difficulties with electric potential	
difference	30
A. Initial version of the electric potential difference tutorial.....	30
1. Student performance on path-independence, before tutorial	31
2. Student performance of path-indendence, after tutorial	33
3. Discussion of the initial version	33
B. Developing a new electric potential difference tutorial.....	34
1. Version 1.....	34
2. Version 2.....	35
3. Version 3.....	36
4. Version 4.....	37
5. Version 5.....	38
6. Version 6.....	39
C. Comparison of student post-test performance after each version.....	40

D. Addressing student difficulties with a pre-tutorial and homework	44
1. Developing and assessing a pre-tutorial for potential difference	44
2. Modifying the tutorial homework to address test-charge independence.....	49
E. Discussion	51
F. References for chapter 3.....	51
Chapter 4: Student difficulties with conductors.....	52
A. The net electric field inside a conductor.....	52
B. Blocking of electric fields by conductors	58
C. Student misconceptions about ground.....	60
D. Conclusions.....	62
E. References for chapter 4.....	63
Chapter 5: Curriculum development to address student difficulties with conductors.....	65
A. Developing the electric properties of conductors tutorial.....	65
1. Version 1.....	66
2. Version 2.....	67
3. Version 3.....	68
4. Version 4.....	68
5. Version 5.....	69
B. Evaluating each version through post-test performance	71
1. Control post-test	71
2. Version 1.....	73

3. Version 2.....	74	
4. Version 3.....	76	
5. Version 4.....	76	
6. Version 5.....	78	
C. Discussion	80	
Chapter 6: Comparing introductory and junior-level conceptual difficulties in		
electrostatics	81	
A. Difficulties with physics concepts.....	84	
1. Potential difference.....	84	
2. Conductors.....	86	
B. Relating mathematical and physical concepts	88	
1. Introductory students.....	88	
2. Junior-level students	90	
a. Mathematics as a descriptive language for physics.....	91	
b. Integration of Coulomb's law	93	
c. Using symmetry arguments	96	
d. Vector derivatives	100	
C. Conclusions and implications for junior-level instruction.....	103	
D. References for chapter 6.....	104	
Chapter 7: Refining a tutorial on Ampère's law		105
A. Initial version of the Ampère's law tutorial.....	106	

B. Relating line integrals to the presence of current.....	109
1. Student difficulties	109
2. Modifying curriculum to address difficulties	112
3. Evaluating curriculum modifications.....	112
C. Relating the line element to the area vector	113
1. Student difficulties	113
2. Modifying curriculum to address difficulties	114
3. Evaluating curriculum modifications.....	115
D. Conflating line integrals with flux.....	116
1. Student difficulties	116
2. Modifying curriculum to address difficulties	118
3. Evaluating curriculum modifications.....	119
E. Inappropriate use of a dot product.....	120
1. Student difficulties	120
2. Modifying curriculum to address difficulties	121
3. Evaluating curriculum modifications.....	122
F. Utility of Ampère’s law	122
G. Discussion	125
Chapter 8: Comparing introductory and junior-level conceptual difficulties with Ampère’s law.....	128
A. Junior-level difficulties with the integral form of Ampère’s law	130

1. Choosing the correct loop	130
2. Use of symmetry arguments	134
3. Discussion	135
B. Junior-level difficulties with the differential form of Ampère's law	136
1. Zero curl implies zero field.....	137
2. Curl is a property of systems as a whole	138
3. Discussion	140
C. Utility of Ampère's law	140
D. Conclusions.....	143
E. References for Chapter 8	143
Guide to appendices	144
Appendices	148

LIST OF FIGURES

<i>Number</i>	<i>Page</i>
Figure 1.1 Coverage of a physics topic in the introductory course	3
Figure 1.2 Coverage of a physics topic in the junior course	5
Figure 2.1 The two connected conductors problem	14
Figure 2.2 Potential difference question from midterm examination, Fall 2013.....	19
Figure 2.3 The like-charged and opposite-charged block problems.....	21
Figure 2.4 Conducting sphere grounded through an ammeter.....	24
Figure 2.5 Two sets of questions probing student ability to reason about distributed charges	26
Figure 3.1 Pretest question probing student understanding of path independence.....	32
Figure 3.2 A variant of the two-charged block problem used in version 2 of the potential difference tutorial	35
Figure 3.3 Two experiments with charged blocks, to help students relate external work to changes in potential energy.....	38
Figure 3.4 Post-test question used to compare versions of the potential difference tutorial.....	40
Figure 3.5 Comparison of three potential difference post-test questions across five tutorial versions.....	41
Figure 3.6 Like-charged blocks question for the pre-tutorial.....	45
Figure 3.7 Opposite-charged blocks question for the pre-tutorial.....	46
Figure 3.8 Effect of the pre-tutorial on completion time for the first page of the potential difference tutorial.....	47
Figure 3.9 Post-test question to assess student understanding of the effect of reference point on potential difference.....	48
Figure 3.10 Homework question guiding students to derive test-charge independence ...	49

Figure 4.1	Pretest question about the field and potential differences in a conductor	53
Figure 4.2	Example of a shielding question	59
Figure 4.3	Post-test question about field and potential difference in a conductor	61
Figure 5.1	Diagram from the initial version of the properties of conductors tutorial	66
Figure 5.2	Diagram from the second version of the properties of conductors tutorial.....	67
Figure 5.3	Modified diagram for the two-connected conductors problem.....	68
Figure 5.4	Equipment for the foil-covered-cup electric field experiments.....	70
Figure 5.5	Control post-test question, run in a quarter when the properties of conductors tutorial was not run	72
Figure 5.6	Post-test question investigating time-dependence in conductors	73
Figure 5.7	Student response to the “charged-plane” question	75
Figure 5.8	Post-test question about grounding and shielding.....	76
Figure 5.9	Post-test question about a neutral sphere centered on a dipole	77
Figure 5.10	Two post-test questions to evaluate student use of model-based reasoning...	79
Figure 6.1	Diagram from the junior tutorial on potential.....	85
Figure 6.2	Homework question probing students’ understanding of shielding	86
Figure 6.3	Post-test question probing student understanding of charge density and divergence	92
Figure 6.4	Post-test question probing student ability to integrate charge distributions ..	93
Figure 6.5	A spherical Gaussian surface around a uniformly charged sphere	97
Figure 6.6	Three charge distributions to test the use of Gauss’s law	98
Figure 6.7	Student response to a question on the junior-level Gauss’s law tutorial homework.....	101
Figure 7.1	Pretest question about magnetic flux and Gauss’s law	106

Figure 7.2	Four Ampèrian loops used in the first part of the Ampère’s law tutorial.....	107
Figure 7.3	Post-test question about using Ampère’s law ‘in reverse’	109
Figure 7.4	Tutorial homework question about a nonuniform field	112
Figure 7.5	Circular Ampèrian loop around a wire	114
Figure 7.6	Tutorial homework question about a semicircular loop	117
Figure 7.7	Uniform-field question added to the tutorial to help students practice line integrals.....	118
Figure 7.8	Question about two rotated loops, to test for student confusion about the correct dot product in Ampère’s law	120
Figure 7.9	Lecture exam question about the utility of Ampère’s law	123
Figure 7.10	Pretest question about five Ampèrian loops around a wire	124
Figure 8.1	Junior post-test question about a sheet and slab of current.....	131
Figure 8.2	Incorrect answer for the question about Figure 8.1	132
Figure 8.3	A hypothetical students’ proposed loop for a straight wire	132
Figure 8.4	A portion of a long solenoid.....	133
Figure 8.5	Tutorial question involving five points around a wire, to introduce symmetry ideas	134
Figure 8.6	Post-test question about a coaxial cable with time-varying current	137
Figure 8.7	The magnetic field of a long wire	138
Figure 8.8	Probing curl with a ‘pinwheel’ analogy	139
Figure 8.9	The electric field of a point charge.....	139
Figure 8.10	Pretest question probing student understanding of the utility of Ampère’s law.....	141
Figure 8.11	Tutorial question to demonstrate that Ampère’s law is not always useful	142

LIST OF TABLES

<i>Number</i>		<i>Page</i>
Table 1.1	List of introductory E&M tutorials.....	6
Table 1.2	List of junior-level E&M tutorials.....	8
Table 2.1	Results from two versions of the two-connected-conductors problem, with either a positive or negative rod.....	15
Table 2.2	Results from two versions of the two-disconnected-conductors problem, with either a positive or negative rod.....	17
Table 2.3	Comparison of student responses to the two-block problems.....	22
Table 4.1	Results from two questions about the electric field and the potential difference within a conductor.....	53
Table 4.2	Comparison of results about the electric field in a conductor to common misconceptions about ground	62
Table 7.1	Results for questions about each side of Ampère's law for a line current	110

CHAPTER 1: INTRODUCTION

This dissertation describes an investigation into student reasoning difficulties with electro- and magnetostatics, at the introductory and junior level. The research at the introductory level described here deals with a set of difficulties relating to electric potential difference in electrostatics, and with Ampère's law in magnetostatics. The research into junior-level difficulties is much broader, though is less in-depth and forms the smaller piece of this dissertation.

A. Background and context for research

1. Main results from physics education research

Research into physics education, i.e. how students learn physics topics and the best practices to adopt in physics instruction, began over thirty years ago. A very large body of research has since been produced that identifies common student difficulties and misconceptions, provides analytical tools to identify and quantify these difficulties, and both identifies teaching methods that do not address these difficulties as well as provides suggestions about methods that do.

One of the major results from the field is that after traditional lecture-based instruction, students may have learned quantitative skills but do not tend to learn qualitative reasoning skills¹. Thus while many students can answer typical end-of-chapter problems on midterm or final examinations, and hence appear to have mastered the

material, the same students cannot answer simple qualitative questions about the same topics.

Another major result is that instruction that focuses on getting students to actively engage with each other, the instructors, and the curriculum can be very effective at teaching students qualitative reasoning skills². It has been suggested² that part of the reason that these strategies are successful when standard lectures are not is that students are passive observers in standard lectures. They receive very little explicit instruction or practice with individual skills, and are instead expected to absorb them by watching the lecturer demonstrate them. Conversely, in active-engagement exercises students must practice skills and create knowledge structures on their own, leading to greater mastery and understanding of both individual skills and physics as a whole.

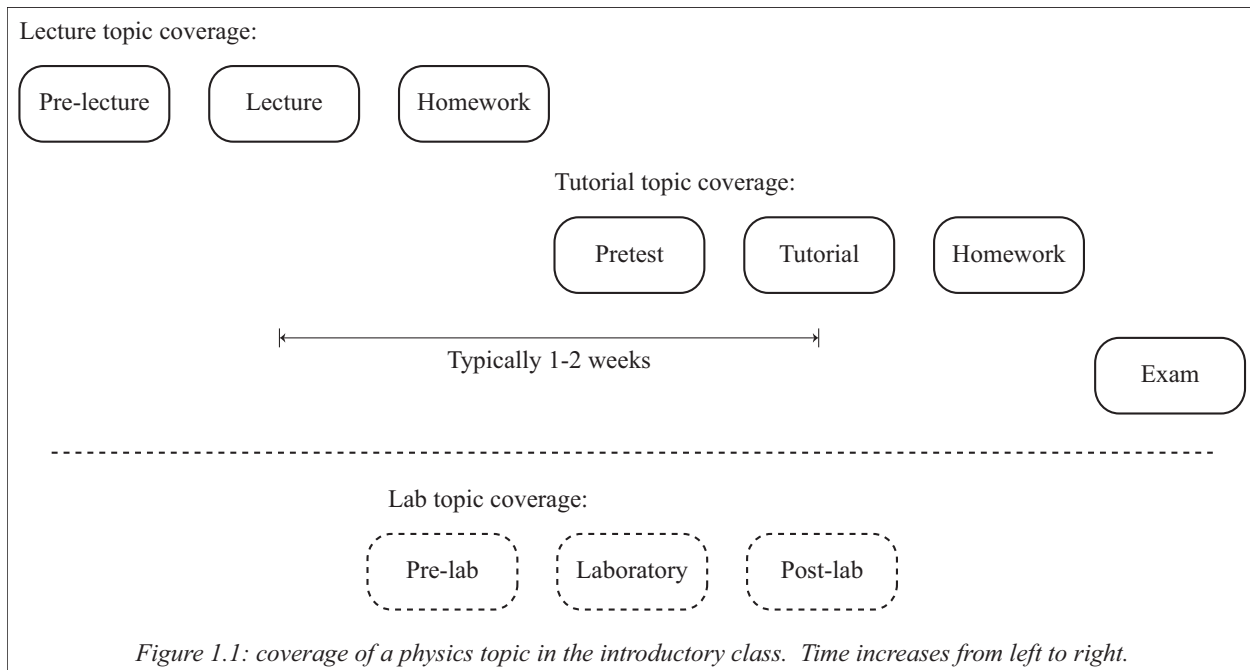
2. Curriculum development at the University of Washington

At the University of Washington, as at many colleges and universities, the introductory calculus-based physics course has an associated discussion section. The Physics Education Group at the University of Washington has developed a model for active-engagement instruction in this discussion section, involving a set of worksheets called *Tutorials in Introductory Physics*³ that are designed to address known conceptual difficulties that students struggle with when learning introductory physics. These worksheets are extensively research-based and research-validated, and are designed to help students learn the qualitative reasoning skills that most students do not learn from standard lecture instruction.

B. Topic coverage in the introductory and junior courses

1. Introductory course

Figure 1.1 shows how a given topic is typically covered in the introductory calculus-based course. The introductory course has three main parts; lecture, tutorial, and instructional laboratory. Each of which in turn comprise a pre-instructional component, an in-class component, and a homework or otherwise post-instructional component. The introductory sequence is three quarters long: Physics 121 covers mechanics; Physics 122 covers electricity and magnetism; and Physics 123 covers waves, optics, modern physics, and quantum mechanics. Unless otherwise indicated, all introductory data described in this dissertation is drawn from the second quarter.



The pre-instructional component in the lecture introduces students to a given topic, as an alternative to having students read the textbook before coming to class. Students are

introduced to the theory and mathematical expressions associated with the topic during the in-class lecture, and then practice using the mathematical expressions and laws by doing quantitative problems on the homework.

The tutorial portion as a whole is designed to help students learn to reason qualitatively about the topic, to complement the quantitative skills that students primarily learn from lecture. The pretest for most of the tutorials in the introductory sequence is administered after all relevant lecture instruction; all pretests discussed in this dissertation were administered after lecture instruction. The pretest serves as a measurement of what students know after lecture, before qualitative instruction in the tutorial. The in-class tutorial portion focuses on qualitative reasoning, as does the tutorial homework. Between half and two-thirds of the homework grade is awarded for correct reasoning, and less than half for the correct answer, to promote students' use of qualitative reasoning.

The laboratory section is organized similarly to the tutorial section; a pre-lab investigates students' initial understanding, students work through an in-class lab with a standard lab manual, and then answer post-lab questions.

The course also includes three midterm examinations and one final examination. The midterm examinations include questions from all three components of the course, most of which are multiple-choice. There are also two free-response pages, one associated with the lecture portion of the course and one associated with the tutorial portion. The tutorial exam questions emphasize qualitative reasoning and are graded very similarly to how tutorial homework is graded.

Around 1500 students per year take the introductory calculus-based course at the University of Washington, of which roughly 5% are physics majors, 70% engineering majors, and the rest a mix of other physical-science and life-science majors. The course textbook is Tipler's *Physics for Scientists and Engineers*, special UW edition¹⁰.

2. Junior course

The coverage of a given topic in the junior level course is fairly similar to that in the introductory course, shown in Figure 1.2. Notable differences are that there is no associated laboratory section at the junior level, and students are expected to read the textbook on their own so there is no pre-lecture portion. There are typically two midterms and a final, all of which are free-response. At the junior level, instruction on electricity and magnetism covers three quarters: Physics 321 covers electrostatics; Physics 322 covers magnetostatics and the beginning of EM waves; and Physics 323 covers electrodynamics, waves, optics, time-dependence, and relativistic electrodynamics.

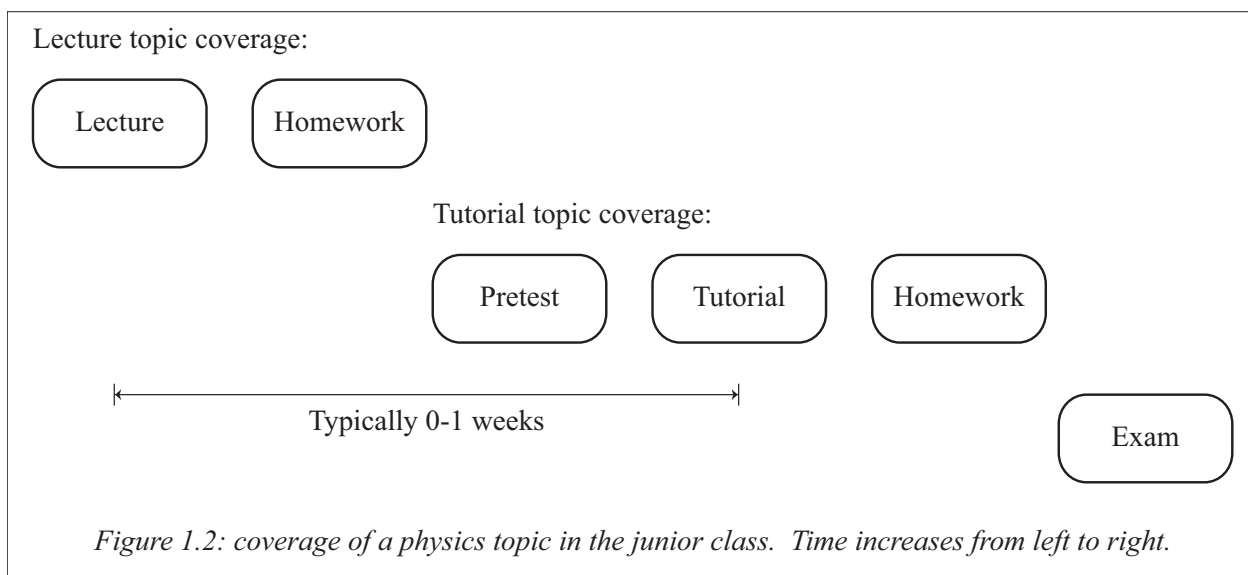


Figure 1.2: coverage of a physics topic in the junior class. Time increases from left to right.

Around 150 students take the junior-level courses each year, all of which are declared physics majors. The textbook for the class is Griffith's *Introduction to Electrodynamics*¹¹.

C. Motivation for research

1. Introductory course

The initial motivation for this research is that there is a known gap in student understanding between electrostatics and circuits⁴. Many circuits ideas such as voltage are the same concepts introduced in electrostatics, though presented in a different way with a different name. Table 1.1 shows the current list of introductory E&M tutorials; bolded entries are tutorials developed over the course of this investigation, three of which are described in this dissertation.

Electrostatics	Circuits	Magnetostatics
Charge	Capacitance	Magnets and Magnetic Fields
Electric Field and Flux	Model for Circuits Part 1	Magnetic Interactions
Gauss's Law	Model for Circuits Part 2	Ampère's Law
Electric Potential	Model for Circuits Part 3	Lenz's Law
Difference	RC Circuits	Faraday's Law
Electric Properties of Conductors		

Table 1.1: List of introductory E&M tutorials. New tutorials developed in the course of this investigation are bolded.

This investigation began with an evaluation of student difficulties on the Capacitance pretest, post-electrostatics instruction. We found that most students had

difficulties with capacitance ideas because they struggled with earlier electrostatics concepts that they needed to apply to conductors. This ultimately led to a complete re-design and re-write of the *Electric Potential Difference* tutorial, as well as the creation of a new tutorial, *Electric Properties of Conductors*.

2. Junior course

The junior-level E&M course at the University of Washington originally had a discussion section that was typically used by the instructor as a problem-solving review session, wherein a graduate student TA would work through examples from the text or from homework. In contrast, the junior-level quantum mechanics course used a set of tutorials originally designed by A. Crouse⁵, and later revised by several members of our group⁶ into *Tutorials in Physics: Quantum Mechanics*⁷. Nearly all students that were exposed to these tutorials in quantum mechanics asked for tutorials to be implemented in the electromagnetism courses as well.

We initially implemented 16 of the E&M tutorials written at CU Boulder⁸, but quickly discovered that they were not well matched to our students in terms of difficulty level, required familiarity with math, or length. We then completely rewrote these materials to better fit the University of Washington courses and students, as well as writing 13 new tutorials to fill gaps that the CU Boulder tutorials did not. All these tutorials have been re-written twice based on initial research and observations during the tutorial sessions. A full list of the worksheets in *Tutorials in Physics: Electrodynamics*⁹ are shown in Table 1.2.

Tutorials are now implemented in the same manner in both the junior-level E&M and quantum mechanics courses, though there are some differences in how the junior-level and introductory courses are designed, as described in the next section.

Electrostatics	Magnetostatics	Electrodynamics
Vector Derivatives	Curling Fields	EM Plane Waves
Integrals with Charge	<i>Ampère's Law</i>	<i>Waves: Normal Incidence</i>
<i>Gauss's Law</i>	<i>Vector Potential</i>	<i>Waves: Oblique Incidence</i>
<i>Potential</i>	Magnetization	<i>Gauge Invariance</i>
Conductors	Auxiliary Field	<i>Retarded Potentials</i>
<i>Separation of Variables</i>	<i>Electromotive Force</i>	Radiation
<i>Fourier's Trick</i>	<i>Maxwell-Ampère's Law</i>	Relativity: Space & Time
<i>Multipole Expansion</i>	<i>Energy Flow</i>	Relativity: Paradoxes
<i>Polarization</i>		Relativity: Intervals
<i>Displacement Field</i>		Relativity: Energy
		Relativistic E&M

Table 1.2: List of junior-level E&M tutorials. Tutorials initially based on CU Boulder versions are italicized.

D. Methods of investigation

1. Data sources

The quantitative data discussed comes primarily from tutorial pretests and tutorial exam questions in both courses. In all cases, data were coded using an emergent coding scheme. The set of codes was then recursively cross-compared to eliminate code over-lap, typically resulting in a basis of up to 7 distinct codes per data set. Often multiple different data sets could share the same coding scheme, if the questions were similar and/or required similar reasoning. In these cases the initial data set was blinded and re-analyzed

with the minimal basis set of codes, to ensure consistency in later applications of the code set. If there was disagreement between the two initial and blinded coding instances, the code set was modified and re-run until consistency between two parallel coding instances was reached.

Tutorial homework provided an additional source of quantitative data, as did the multiple-choice and free-response lecture exam questions. Data from these sources are rarely used in this dissertation, and are explicitly noted when they are discussed.

Qualitative data are also included, based on informal observations of students as they work through the tutorial worksheet or in office hours.

2. Description of data

All tutorials are given a three-letter code, and all quarters are given a three-number code; the first two numbers are the last two digits of the year, and the last number refers to specific quarter in that year. Data sets are named to refer to the tutorial and quarter from which they came. For instance, data from the *Capacitance* pretest taken in the Autumn quarter of 2013 (the 4th quarter of the year) would be labeled as CAP_134. The same naming scheme is used for exam data. Full copies of all questions asked and a guide to these questions can be found in the Appendices.

Tutorial curriculum is labeled by three-letter code and version number; tutorial homework is labeled with the version number of the tutorial it accompanies. Copies of all curricula and a guide to these materials are also in the Appendices.

E. Organization of dissertation

The body of this dissertation consists of seven chapters. Chapter 2 describes our research into identifying introductory students' difficulties with the concepts of electric potential energy and electric potential difference. The preliminary 2nd edition of the introductory tutorials already contained a tutorial to address difficulties with potential difference. However, we found that most students did not struggle with some of the material in the tutorial, and did have significant issues with concepts not covered in the tutorial. Chapter 3 describes the modifications we made to this tutorial to better match it to our students' needs.

We found that difficulties with potential difference could be split into two categories: difficulties with the base concept, which could be addressed by modifications to the potential difference tutorial, and difficulties reasoning about potential difference in conductors. While investigating the latter set, we found that many students could not reason correctly about multiple electrostatics concepts in the context of conductors, not just potential difference. This research is described in chapter 4. Chapter 5 details the steps we took to create a new tutorial on the electric properties of conductors.

Chapter 6 presents a categorization of student difficulties in physics, and uses this categorization to compare the sets of difficulties we've encountered at the introductory and junior level, in the context of electrostatics. This investigation reveals that the junior-level students have serious difficulty relating mathematics to physics. Preliminary efforts to adapt the advanced tutorials to support student learning in this area are also included.

The preliminary 2nd edition of the tutorials did not include a tutorial on Ampère's law. Paul van Kampen from Dublin City University provided us with such a tutorial. However, we discovered that it was not well-suited to the introductory students in our course. Chapter 7 discusses research into student difficulties with Ampère's law, and how we refined the initial version of the tutorial to address the difficulties our students had.

Chapter 8 again compares introductory and junior-level students, in this case with Ampère's law, and extends the discussion of difficulties with math-physics connections.

F. References for chapter 1

- [1] L. C. McDermott, "Research on conceptual understanding in mechanics," *Phys. Today* 37, p. 24 (1984)
- [2] A. V. Heuvelen, "Learning to think like a physicist: a review of research-based instructional strategies," *Am. J. Phys.* 59, p. 891 (1991)
- [3] L. C. McDermott, P. S. Shaffer, and the Physics Education Group at the University of Washington, *Tutorials in Introductory Physics*. (Prentice Hall, New Jersey, 2002).
- [4] L. Borghi, A. D. Ambrosio, and P. Mascheretti, "Microscopic models for bridging electrostatics and currents," *Phys. Educ.* 42, p. 146 (2007)
- [5] A. Crouse, "Student understanding of quantum mechanics," Ph.D. Dissertation, University of Washington (2007)
- [6] G. Passante, P. J. Emigh, and P. S. Shaffer, "Testing tutorials in upper-division: an example from quantum mechanics," PERC proceedings (2014)
- [7] L. C. McDermott, P. S. Shaffer, and the Physics Education Group at the University of Washington, *Tutorials in Physics: Quantum Mechanics*, Preliminary edition (2014)
- [8] colorado.edu/physics/EducationIssues/cts/index.htm
- [9] L. C. McDermott, P. S. Shaffer, and the Physics Education Group at the University of Washington, *Tutorials in Physics: Electrodynamics*, Preliminary edition (2015)
- [10] P. A. Tipler and G. Mosca, *Physics for Scientists and Engineers*, special UW edition, volume 2.
- [11] D. J. Griffiths, *Introduction to Electrodynamics* 4th edition. (Addison-Wesley, 2012).

CHAPTER 2: STUDENT DIFFICULTIES WITH ELECTRIC POTENTIAL DIFFERENCE

“I forgot the voltage formula, but I don’t think distance was in it.”
– Response on an introductory pretest

This chapter focuses on identifying introductory students’ difficulties with electric potential and electric potential difference (which we will refer to henceforth as “potential” and “potential difference”), and on characterizing the reasoning students typically use when thinking about these concepts. Unless otherwise specified, all data in this chapter is from introductory pretests.

As mentioned in the introduction, we have investigated student understanding of several different electrostatic concepts over a five-year period. We wrote tutorial materials to address the initial difficulties we found, and then adapted and modified these materials as we identified new ideas that students struggled with. We also continuously modified the tutorial materials to better address topics that the tutorials were not adequately helping students with. For clarity and ease of comprehension, however, we are presenting all data on student difficulties together in this chapter. Chapter 3 describes our efforts to modify tutorial materials to address these difficulties.

The data in this chapter comes both from the *Electric potential difference* pretest, as well as from the *Capacitance* pretest the following week. Both pretests were administered after all lecture instruction on potential and potential difference, as well as all the electrostatics labs. Both the lecture and the textbook introduce electric potential difference

as $\Delta V_{i \rightarrow f} = V_f - V_i \equiv -\frac{W_{\text{electric field}}}{q_{\text{test}}}$, where $W_{\text{electric field}}$ is the work done on a test charge by the electric field. Students have also seen SmartPhysics coverage of potential difference before lecture instruction, which also defines it in terms $-W_{\text{electric field}}$.

A. Conceptually understanding potential difference

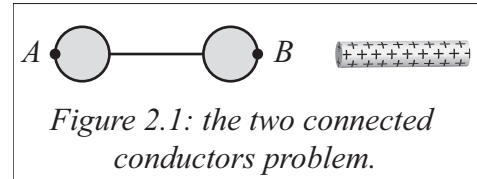
The data shows that students have three main issues with comprehending the definition. First, more than half the students don't realize that to find the work done by the field, they need to consider the entire system of all charges, not just a subsystem. Second, many students don't remember to include the negative in their reasoning, which could be because they don't understand why the work by the field should be negative. Third, many students do not comprehend that there is a relative sign difference between internal work done by the field and external work done on the system, so they find the wrong sign when thinking about potential difference from an energy perspective. These issues are discussed in turn in the following sub-sections.

1. *Incorrect choice of systems*

Most students tend not to struggle with questions that involve simple systems (i.e. a small number of ideal point charges). This could be because point charges are not complicated: students can simply use the memorized formula $V = kQ/r$ with the principle of superposition to answer these questions. It could also be because this is the type of question that is often asked in end-of-chapter problems, so students are familiar with this type of problem when they appear on exams. Questions that

involve more complicated systems or that involve distributed charges tend to be much harder for students.

For example, the two-connected-conductors problem (2CC) is shown in Figure 2.1 at right. Two conducting spheres are attached via a thin conducting



wire. A positively charged object is brought near the right sphere, and remains in place long enough for the charges to reach equilibrium. The question first asks students to compare the charge on each sphere; the second question asks if the potential at the right sphere is greater than, less than, or equal to that at the left sphere. This question appeared on the CAP_104 pretest, shown in full in Appendix A.

The correct answer to the first question is that negative charges will be attracted to the right sphere by the positively charged object, and an equal number of positive charges will be repelled to the left sphere since the two spheres are net neutral. In equilibrium no charges are moving, so the net electric field inside the conductors and the wire is zero. Thus there will be no change in potential energy if a test charge is moved from one sphere to the other, and hence the answer to the second question is that the potential difference between the spheres is zero.

Student responses to two variants of 2CC are shown in Table 2.1. The first column involves a positive charge as in Figure 1, and the second column has the same setup but with a negatively charged rod.

For both variants, about half the students gave the correct answer, but most did not use correct reasoning. Over half the students with correct answers argued that the

potential of each sphere was equal because the amount of charge on each sphere was equal. These students made two mistakes: first, incorrectly using the point-charge formula $V = kQ/r$ with the *magnitude* of the charge on the spheres, ignoring the fact that the sign of the charge on each sphere was opposite. Second, these students ignored the presence of the charged rod, only considering the sub-system consisting of the two connected spheres.

	Positive rod (N=135)	Negative rod (N=155)
Equal to (correct) <i>Entire system</i>	52% <i>17%</i>	54% <i>13%</i>
Greater than <i>External object only</i> <i>Internal charge distribution only</i>	18% <i>12%</i> <i>6%</i>	30% <i>8%</i> <i>17%</i>
Less than <i>External object only</i> <i>Internal charge distribution only</i>	16% <i>2%</i> <i>15%</i>	8% <i>3%</i> <i>5%</i>

Table 2.1: Two versions of the two-connected-conductors problem, with either a positive or negative rod. Correct answers are bolded, reasoning is italicized.

All the students who gave an incorrect answer did not consider the entire system of charges. Most of the students who chose “greater than” discussed either the electric field of the rod or the relative positions of the rod to each sphere, completely ignoring the electric field of the induced charges on the spheres, as in these statements:

“The potential of B is greater than that of A as the opposing charged rod is closest to the sphere B.”

“Point B is closer than point A to the charged rod and distance in the equation for electric potential is in the denominator.”

Students who chose “less than,” on the other hand, tended to discuss either the charge distribution on the spheres or the electric field this distribution creates, ignoring the presence of the external charged object:

“Since the electric potential is directly related to the charge, the overall electric potential of the more positive ball is greater than the negative ball.”

“Electric potential is greater at a negatively charged point.”

A second, paired question on the same pretest involved removing the wire between the two conducting objects, creating the two-disconnected-conductors problem (2DC). The question text specified that no objects moved, no charge distributions changed as a result of removing the wire, and that the charge on the wire was negligible. The question is the same: is the potential at the right sphere greater than, less than, or the same as that at the left sphere?

Correct answer: Since no charge distributions changed as a result, no electric fields changed either, so the potential at each sphere and the potential difference between the spheres must be the same as in 2CC. Therefore the potential at each sphere is still the same.

2DC is a harder question than 2CC, and fewer students answered correctly, as shown in Table 2.2. Again we see a similar pattern of incorrect reasoning, with most students not considering the entire charged system, but only a piece of it. What is particularly interesting is that only 39% of students gave a consistent answer between 2CC and 2DC, even with the prompt that no charges moved. Two-thirds of the students who correctly answered “equal to” in 2CC chose “greater than” in 2DC.

	Positive rod (N=135)	Negative rod (N=155)
Equal to (correct)	30%	18%
<i>Entire system</i>	2%	3%
Greater than	46%	62%
<i>External object only</i>	21%	28%
<i>Internal charge distribution only</i>	7%	13%
Less than	16%	15%
<i>External object only</i>	5%	7%
<i>Internal charge distribution only</i>	5%	4%

Table 2.2: Two versions of the two-disconnected-conductors problem, with either a positive or negative rod. Correct answers are bolded, reasoning is italicized.

However, all the students who correctly answered 2CC with correct reasoning (including the effects of all charges in the system) correctly answered 2DC as well. These findings suggest that misusing the idea of a system underlies some of the difficulty students have learning about potential difference, and that guiding students to think about the correct choice of system could help them understand potential differences.

2. Negative work by the field

Electric potential difference was historically defined in terms of the change in potential energy of a system: $\Delta V = \Delta U / q_{test}$. One way to apply this is to consider a system that contains a set of source charges and a test charge, and let an external force move the test charge from an initial point to a final point at constant speed. By the work-energy theorem the work done by the external force is equal to the change in total mechanical energy of the entire system, kinetic plus potential. If a test charge is moved at constant speed, then the change in kinetic energy is zero, so the work is equal to the potential difference. The net force on the charge is zero since it is not accelerating, so the external

force must be equal and opposite to the internal electric force of the source charges on the test charge. Thus the work done by the external force is equal and opposite to that done by the internal force. Mathematically, $\Delta U = W_{ext} (if \Delta K = 0) = -W_{by\ field}$.

This approach relies on students imagining that there is an external force that moves the charge at constant speed, even when such a force is not present. Since the electrostatic force is conservative, the work done by the field only depends on the start and end points. Therefore textbooks often simply define the change in potential energy as $\Delta U = -W_{by\ field}$ to avoid having to discuss this hypothetical external force, so thus $\Delta V = \Delta U / q_{test} = -W_{by\ field} / q_{test}$.

However, this minus sign in the standard definition of potential difference is confusing to students. In qualitative observations during tutorial sessions, most groups of students either asked a TA directly about the origin of the minus sign, or mentioned it during a TA checkout. Many students were unsure about the sign of the potential difference in some cases because of the minus sign, or forgot it entirely and gave an answer with an incorrect sign. Other than these qualitative observations, we have relatively little data. Many post-test exam questions on potential difference shared a similar quantitative question, where a given test charge gained a given amount of kinetic energy, and students were asked to calculate the associated potential difference. Some students calculated an answer of the correct magnitude and opposite sign, but since the question asked students to show their work instead of explaining their reasoning, it's difficult to state that the incorrect sign is due to misremembering or misapplying the definition.

Student difficulty with negatives is documented in other contexts, particularly energy¹⁻³. Most textbooks do not explain the minus sign in potential difference, or reference it very briefly (e.g. Knight⁴; Halliday, Resnick and Walker⁵; Mazur⁶). It is often left as a footnote; and many lecturers do not go through the derivation in class, leaving the negative as a definition, which students then are less likely to remember.

3. Internal vs. external work

We have asked a variety of pretest questions about work, changes in potential energy, and potential difference. On many of these, between 15 and 20% of students incorrectly answer potential difference questions because they conflate work by an external agent with internal work done by the electric field. Most of the post-test exam questions on potential difference included a version of the question shown in Figure 2.2 (exam question EPD_134, in Appendix B). A hand moves a positive test charge at constant speed from point 1 to point 2, away from a positively charged rod. The question asks whether the potential difference from point 1 to point 2 is positive, negative, or zero.

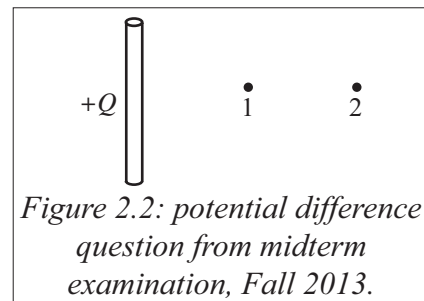


Figure 2.2: potential difference question from midterm examination, Fall 2013.

This question could be answered by using the work done by the field, as follows: the electrostatic force points to the right, and the charge moves to the right, so the work done is positive. Due to the negative in the definition, the potential difference is negative.

This question could also be answered by considering the work done by an external force: to keep the test charge moving at constant speed, the hand must exert a force to the left (opposite the electrostatic force). The hand does negative external work, and by the

work-energy theorem the change in total energy is also negative. There is no change in kinetic energy, so the change in potential energy is negative, and the potential difference is again negative.

Most students answered this question, and variants of it, correctly. However between 16%-25% of students (N=404) used one of the methods above with the incorrect work, resulting in an overall incorrect sign. The most likely explanation for this is that students are first taught the work-energy theorem in mechanics, so they are used to associating energy with external work. In E&M electric potential energy is typically introduced first, and then potential difference. Students are primed to think about *external* work and energy being the same sign, so when they see potential energy and potential difference defined in terms of *internal* work, they tend to forget the sign.

B. Conflating force and potential energy

Though most textbooks define potential difference in terms of the internal work done by the field, some use the definition involving changes in potential energy. This avoids the conceptual difficulties discussed in Section A, but it can elicit conceptual problems with the definition of potential energy instead. In particular, many students tend to conflate force and potential energy, as found by B. Lindsey³.

Lindsey gave students two different scenarios involving two charged blocks: the like-charged blocks (LCB) problem and the opposite-charge blocks (OCB) problem, shown in Figure 2.3. In each case, two hands move the charged blocks as shown. The blocks begin

and end at rest. In both cases, the question asks if the electric potential energy increases, decreases, or remain the same as a result of the blocks' movement.

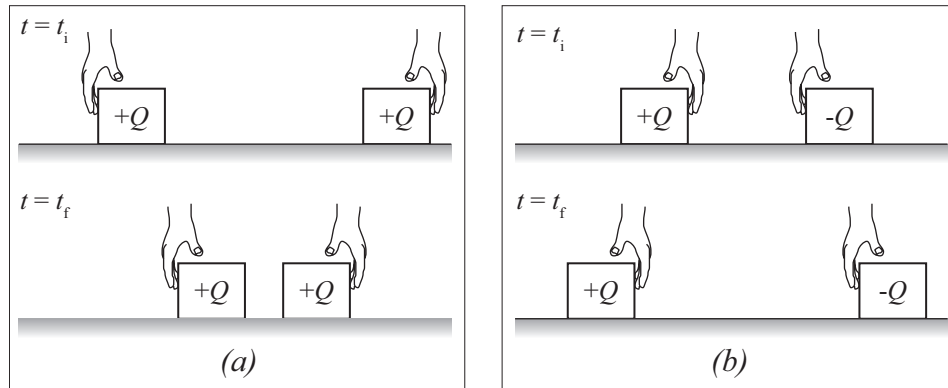


Figure 2.3: (a) the like-charged block problem, (b) the opposite-charged block problem.

Lindsey found that the majority of students correctly answered LCB, but less than half correctly answered OCB. Also, half the students who incorrectly answered OCB reasoned based on distance, and some students used force arguments on both questions, consistent with conflating force and energy. Lindsey did not analyze the reasoning that students used to support their answers, so she could not conclude that students were definitely conflating force with energy.

To answer this question, we conducted a duplication study using the same figure and questions with introductory students at UW. To see if this force-energy conflation was specific to the electrostatic force or was more general, we also asked analogous questions in the context of springs. We found that many students simply stated formulae for the electrostatic force or potential energy, without providing any deeper explanation. To try to elicit explanations that did not depend on equations we created two new sets of questions that are homomorphic to either the electrostatics or springs questions, in terms of a

general conservative force. For the cases homomorphic to the charged blocks, the question states that the blocks attract or repel each other with an unknown force, which is weaker when the blocks are farther apart. The cases homomorphic to the spring systems are identical, except that the unknown force is stronger when the blocks are farther apart.

Table 2.3 shows the data from each experiment. Lindsey's results are shown in the first column, and the rest of the columns show UW pretest data. Results from the replication study are in column 2, springs in column 3, and the unknown-force cases are columns 4 and 5.

	Lindsey (N=70)	Charged blocks (N=134)	Springs (N=373)	Homo- morphic (charge) (N=133)	Homo- morphic (springs) (N=135)
Like-charged blocks					
Increases (correct) <i>Force</i>	84% –	84% 48%	89% 11%	81% 47%	47% 5%
Decreases <i>Force</i>	10% –	9% –	6% –	5% –	47% 39%
Does not change	5%	7%	4%	12%	6%
Opposite-charged blocks					
Increases (correct) <i>Force</i>	36% –	49% –	85% 33%	42% –	73% 33%
Decreases <i>Force</i>	55% –	39% 32%	8% –	44% 31%	22% –
Does not change	8%	10%	6%	14%	5%

Table 2.3: Comparison of student responses to the two-block problems. In the upper table the blocks move together, and in the lower they move apart.

The replication study agrees with Lindsey's data – almost half the students who correctly answer LCB do not answer OCB correctly. This is supported by the percent of

students who are using force reasoning, as half the students who gave the correct answer to LCB are not using work or energy reasoning, but are basing their answers on force:

“ r is decreasing, so force between blocks increases, increasing potential.”

“The same sign charged blocks will have a greater electric force to repel them as they get closer, thus the electric potential energy increases.”

Almost all students who incorrectly answered OCB use similar reasoning:

“The pull of the blocks is less when they are farther apart. This [sic] the potential is less.”

“As the field becomes weaker the potential energy [...] becomes weaker”

In the context of springs, we found no real difference between the two questions: nearly all students answered the compression and extension problems similarly. This is most likely due to the fact that the spring force is symmetric (force increases with distance from equilibrium in both directions) whereas the electrostatic force is asymmetric. Thus, both spring questions could be viewed as being analogous to LCB, as the pattern of answers and explanations is very similar for both questions.

The pattern of responses for the unknown-force questions is nearly identical, with the LCB and OCB homomorphs switched, and are also very similar to the original charged-blocks problems. This is very strong evidence for a force-energy conflation, since 30 to 50% of students choose answers for potential difference consistent with their knowledge of how the magnitude of the force varies with distance, independent of the specific details of the force.

C. Physical meaning of potential difference

Potential difference is one of the most abstract and mathematical concepts that introductory students are introduced to in their study of E&M. Students often have difficulty relating potential difference to a physical process, or understanding its significance in a given situation. This could be in part due to the fact that students struggle to conceptually understand electric fields, at both the introductory and advanced levels⁷⁻¹⁰, and the standard definition of potential involves the electric field.

A particularly illustrative example of this tendency not to relate potential to physical behavior comes from the CAP_104 post-test (Appendix B), shown in Figure 2.4 at right. A positively charged rod is brought near a conducting sphere that is connected to ground through an ammeter, which briefly records a positive value. The

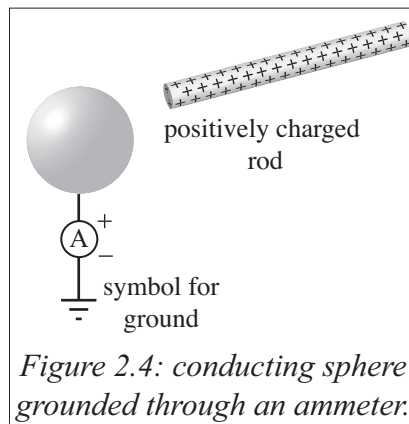


Figure 2.4: conducting sphere grounded through an ammeter.

question asks whether the potential difference from the sphere to the ground is positive, negative, or zero while the ammeter records a non-zero reading.

One way to answer this question is that while the rod moves toward the sphere, the rod will exert a downward force on positive charges in the sphere. A hand would have to do negative external work to move a positive test charge at constant speed from the sphere to ground, so ΔU is negative and hence ΔV is as well. Alternately conventional current flows to ground, and current flows from higher to lower potential, so the potential difference is negative.

Only a quarter of students (N=135) correctly answered the question, and all of these students used the direction of current and the relation between current and voltage. Nearly half the students only considered the negative charges in the sphere, and gave answers with an incorrect sign. All students who used the definition of potential in terms of work by the electric field arrived at the wrong answer, and all students who considered the direction of current found the correct answer. No students used reasoning based on physical phenomena such as energy.

We hypothesized that one reason that students did not view potential differences as being physically meaningful was that students were introduced to potential difference by a definition involving the electric field. If so, conceptually motivating potential difference from changes in potential energy might help students relate potential differences to physical processes. This hypothesis is validated in chapter 3, though the change in definition needs to be supported with other changes to the tutorial to be fully effective.

D. Potential difference in charge distributions

Overall, students did more poorly on the connected-conductors problems in section A.1 than on the charged-block systems in section B. There are several possible explanations for this disparity. It could be that the systems in section A.1 were more complicated, with more component pieces to analyze; that these questions required students to visualize charges on the spheres that were not shown; or simply that questions involving distributed charges are more difficult than questions involving point charges. Student performance on standard lecture questions on post-tests can rule out the first

possibility; more than 85% of students correctly answer systems involving multiple point charges, regardless of the complexity of the system. This is likely because typical end-of-chapter problems focus on systems of point charges, so students are used to thinking about multiple-charge systems. It seemed likely that students were performing poorly on questions involving non-point objects because they didn't understand how to find charge distributions on these objects, or the electric fields these charge distributions create.

To test this idea we designed a series of questions involving various charge distributions on isolated spheres, so that the system was as simple as possible. Figure 2.5 shows two sets of questions that were administered to the same population of 207 students. Question 1 was administered on the EFF_111 pretest (Appendix A), and questions 2a and 2b were administered as part of the lecture multiple-choice post-test.

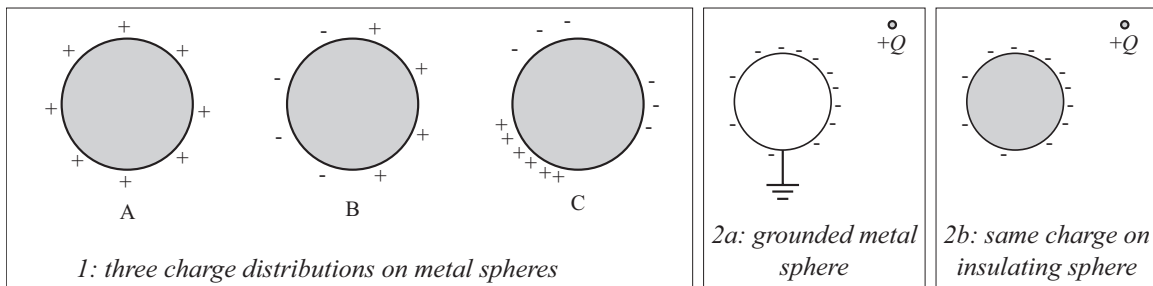


Figure 2.5: two sets of questions probing student ability to reason about distributed charges.

Question 1 gives three stable charge distributions on three metal spheres. With the given the charge distribution, the question asks whether each sphere could be alone in the system, or if there must be other charges in the system. In case A, charges are uniformly distributed and the metal is an equipotential, so no other charges are necessary. In cases B and C, the spheres are not equipotentials on their own; there must be other charges in the system to make the charge distribution stable.

In question 2a, a positive charge is placed next to a grounded metal sphere, which gains the stable charge distribution shown. The question asks if the surface of the sphere is an equipotential or not. The correct answer is yes, because the net electric field inside a conductor is zero so there is no potential difference across the conductor. Question 2b replaces the metal sphere with an insulating sphere that has the same charge distribution as the metal sphere. The question is otherwise identical. The correct answer is again yes; the charge distribution is identical, so the fields and potential differences in this system are the same as in the original system.

Between 80% and 88% of students correctly answered cases A and B in question 1, but only 60% correctly answered case C. In case C, 35% of students said that it was impossible for this charge distribution to be stable, even with other charges in the system. This could be due to the fact that in the textbook and in homework students see problems involving uniformly charged conductors, or dipole distributions, but no “strange” distributions. Similar results appear in analogous questions; students are comfortable with simple, highly symmetric charge distributions, but struggle much more with asymmetric conductors or asymmetric charge distributions.

Question 2 was designed to test this hypothesis. Only 30% of students correctly answered 2a, with nearly 40% stating that the sphere couldn't be an equipotential because charge could flow to or from ground.

The second part of the question is more difficult, since the correct answer requires a correct answer on the first part. Thus we focused on consistency instead of correctness. Replacing the material of the sphere should not change anything, since all charges in both

cases are identical. Slightly less than half the students were consistent with their answer to the previous question. Between 70% and 80% of students stated either that insulators could never be equipotentials, or could only be equipotentials if they had a uniform charge distribution.

E. Discussion

Taken together, the results of this investigation show that students perform fairly well on quantitative problems involving electric fields, potential energy, and potential differences in the context of point charges. However, many students struggle with conceptual questions about potential energy and potential difference even in point-charge contexts, and have significant difficulty with systems involving extended objects or conductors.

These results led to two hypotheses that drove the curriculum development phase of this investigation. First, that guiding students to think about potential difference in terms of potential energy could address their conceptual difficulties with potential difference. The disadvantage of this approach is that it could lead to conflation between force and energy, but that could be addressed by guiding students to use the work-energy theorem. The goal then is to build a strong conceptual understanding of potential difference in terms of potential energy per test charge, and then tie that to the more practically useful line integral of the electric field.

The second is that distributed charges and conductors pose a different set of difficulties that are somewhat separate from the concept of potential difference. Thus after

students work through a tutorial on potential difference, a second tutorial could address their conceptual difficulties with extended objects since those issues would be addressed separately from conceptual difficulties directly relating to potential difference.

These hypotheses will be discussed and tested in the following chapters: Chapter 3 will deal with the first by further developing the existing *Electric Potential Difference* tutorial, and chapter 5 will address the second by developing a new tutorial, *Electric Properties of Conductors*.

F. References for chapter 2

- [1] B. Stephanik and P. S. Shaffer, "Examining student ability to interpret and use potential energy diagrams for classical systems," PERC Proceedings 2011
- [2] G. Lee and J. Yi, "Addressing Students' difficulties in understanding two different expressions of gravitational potential energy (I): mgh & $-GMm/r$," PERC Proceedings 2006
- [3] B. A. Lindsey, "Student reasoning about electrostatic and gravitational potential energy: an exploratory study with interdisciplinary consequences," PRST-PER 10, p. 013101 (2014)
- [4] R. D. Knight, *Physics for Scientists and Engineers: A Strategic Approach*, 3rd ed. (Addison-Wesley, 2012), pp. 900-911
- [5] D. Halliday, R. Resnick, K. Krane *Fundamentals of Physics*, 4th ed. (Wiley, 1992), pp. 651-654
- [6] E. Mazur, *Principles & Practice of Physics*, 1st edition (Addison-Wesley, 2014), pp. 664-667
- [7] S. J. Pollock and S. V. Chasteen, "Cognitive issues in upper-division electricity & magnetism," PERC Proceedings 2009
- [8] S. E. Kanim, "An investigation of student difficulties in qualitative and quantitative problem solving: examples from electric circuits and electrostatics," PhD thesis, University of Washington (1999)
- [9] R. R. Harrington, "An investigation of student understanding of electric concepts in the introductory university physics course," PhD thesis, University of Washington (1995)
- [10] J. Li, "Improving students' understanding of electricity and magnetism," PhD thesis, University of Pittsburgh (2012)

CHAPTER 3: CURRICULUM DEVELOPMENT TO ADDRESS DIFFICULTIES WITH ELECTRIC POTENTIAL DIFFERENCE

“I have no idea how to answer these questions. I have spent 15 minutes trying to figure these out plus another 15 trying to answer them. I am confused, so I will now spend the next 30 minutes reading the textbook.”

– Response on an introductory pretest

This chapter details the process of developing curriculum to address the student difficulties with electric potential difference discussed in chapter 2. I will present a brief description of the initial version of the *Electric Potential Difference* tutorial (EPD), a description of each subsequent version, as well as the research motivation that drove each version. A quantitative comparison of post-test data for each version is shown in section C. Finally, section D details the development of a teaching pretest and homework designed to support and extend the ideas developed in the tutorial.

A. Initial version of the EPD tutorial

The initial version of the EPD tutorial (published in the first edition of *Tutorials in Introductory Physics*) was developed at a time when there was not a strong research base on this topic. It has comparatively little discussion of the concept of electric potential difference itself. The first page guides students through a brief review of work in the context of a general force, and then a review of kinetic energy and the work-energy theorem. The next two pages are devoted to motivating the fact that the work done by the electric field between two points is the same, no matter what path is taken between the

points (henceforth referred to as path-independence). There is a half-page discussion of potential difference, defined as $\Delta V = -\frac{W_{\text{electric field}}}{q_{\text{test}}}$, and then the last section is a mathematical exercise in computing potential differences based on changes in kinetic energy.

As discussed in the previous chapter, students struggle significantly with several aspects of potential difference, and the low fraction of the tutorial devoted to directly discussing potential difference seemed like it could be improved. We decided to assess student understanding of path-independence before students worked through the tutorial, to determine whether we could replace those 2 pages with more material on potential difference.

1. Student performance on path-independence, before tutorial

Throughout the duration of this research, the electric potential difference pre-test was always run after students had seen all relevant lecture coverage on the topic. We asked several different pre-test questions about path-independence, about both the work done by the electric field, and potential difference directly.

Figure 3.1 shows a typical example of one of these questions, from the EPD_114 pretest (shown in full in appendix A). The question shows four points around a dipole, and first asks about the sign of the work done by the electric field between various pair of points. The next questions ask about moving a test charge from point C to point D along three different paths (directly from C to D, from C to B to D, and from C to B to A to D). Between 60-75% of students correctly found the sign of the work for each movement; there were between 10-20% of students who had an overall sign error, but were otherwise self-consistent. 86% of the students stated that the work was the same for all paths in this question.

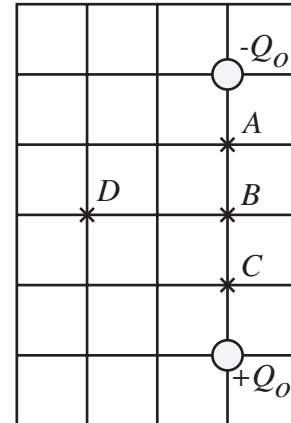


Figure 3.1: pretest question probing student understanding of path independence

Over this entire set of questions, between 85-93% of students answered that work and potential difference were the same for all paths. Of particular interest was the fact that nearly every student who stated that the work was the same for all paths had correct reasoning. In addition, the reasoning most students gave was fairly detailed. For instance:

“This is a trick question. Work is independent of the path taken it only depends on the start point + end point. It doesn’t matter if you do the work required to move the distance vertically and horizontally at the same time or at different times.”

“Potential difference, which is $\Delta U/q_{test}$, is actually a state function; it doesn’t matter which path is taken.”

“The electric force is a conservative force meaning the amount of work done is independent of the path taken.”

Thus only 10-15% of the students did not seem to recognize that work and potential difference are path-independent, before they work through the tutorial.

2. Student performance on path-independence, after tutorial

A question similar to Figure 3.1 was used on several old midterm post-test questions, using horizontal sheets of charge instead of point charges. In each case, between 85%-90% of students correctly used path-independence on the post-test. This suggests that the 2 pages on path-independence on the tutorial were not significantly impacting students. To test this hypothesis, we removed those pages from the next version of the tutorial, and asked another path-independence question on the post-test. 86% of students answered correctly, statistically indistinguishable from the pre-test results on a chi-squared test.

3. Discussion of the initial version

These results suggested that before tutorial instruction, most students are able to apply the path-independence concepts that were covered in the tutorial. We decided to remove the two pages that covered this topic from the potential difference tutorial, and expand the explicit coverage of potential difference.

In addition, we had observed that students often worked through the last page of the tutorial very slowly, and typically did not complete it. We thought that this could be due to the fact that the last page required students to use the work-energy theorem. This is a topic that students had last seen in mechanics, three months or more before they were

taught potential difference. Although, we also noted that the last page was much more mathematical than the typical introductory tutorials are, so students may simply have been slow to work through the required algebra.

Thus, our long-term strategy for developing the next versions of this tutorial was to emphasize energy and the work-energy theorem, and define potential difference in terms of changes in potential energy. Our goal was to help students conceptually understand potential difference in terms of physical quantities and the physical behavior of systems, instead of as a memorized formula.

B. Developing a new EPD tutorial

In order to ensure that the final tutorial was both research-motivated and research-validated, the tutorial evolved in fairly small increments. There are six distinct versions of the tutorial, presented in full in Appendix C as EPD_Version1, etc. In the followings subsections I will describe the major changes between each version, as well as the research-based motivation for each change. A plot comparing post-test performance after various versions of the tutorial had been used is presented in section C, along with a discussion of the effect of each version.

1. Version 1

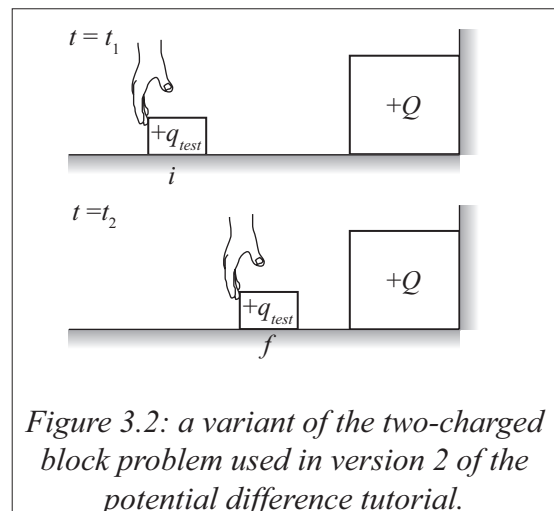
In version 1, we dropped the path-independence pages, which had the side-effect of introducing potential difference earlier in the tutorial so students had more time to think about the concept. We kept the definition of potential difference as work done by the field per test charge. We also added a page where students worked through the two-connected-

conductors problem of chapter 2 (described in Figure 2.1, page 14). The last section, which involved a lot of calculation, was also replaced with two pages condensed out of the *Capacitance* tutorial, to give students an example of using potential difference in a physical context.

This version was intended to both test the effect of removing the pages on path-independence, and to have students think about potential in a physical context. As previously discussed, removing the pages on path-independence didn't negatively impact students. However, the two-connected conductors problem was surprisingly hard for nearly all students, and almost no students got past the third page of the tutorial. In addition, one of the questions in the tutorial was designed to show students that potential difference does not change if the test charge was changed, but many TAs noted that very few students correctly answered these questions without TA prompting.

2. Version 2

The second version of the tutorial cast the first page on work and energy in terms of an electrostatic interaction between two like-charged blocks as shown in Figure 3.2, where a hand pushes a small mobile block towards a large fixed block. Students were asked about the external forces on each block, and which forces did work on the two-block system. They then



were asked to determine if there is a change in the total energy of the system, the kinetic energy of the system, or the potential energy of the system.

The definition of potential difference was also switched to $\Delta V = \Delta U/q_{test}$ instead of $\Delta V = -W_{electric\ field}/q_{test}$. This modification was in response to the earliest findings described in chapter 2, in which we noted that students struggled both with the negative sign and with using the correct work in the definition. The section on test-charge independence was expanded to address the issues the TAs had noted, and several ‘helper’ questions were added to the two-connected-conductors page to assist students with it.

In using this version we identified a major flaw: the work-energy theorem was not mentioned, as we assumed that students would remember it from their mechanics course. We found that this was not true for most students, and TAs often had to remind students of the theorem and how to use it. Additionally, the lecture and textbook defined changes in potential energy as the negative work done by the field, whereas the work-energy theorem relates *external* work to changes in energy, so many students had the incorrect sign for most answers. As a result, many students took an inordinate amount of time on the first page, and often needed a significant amount of TA help and involvement before they could finish this page. Also, even with the ‘helper’ questions we added to the two-connected-conductors page, students still struggled with this context.

3. Version 3

Due to the problems students had with work in the first sections of version 2, we tried starting version 3 with the definition of potential, and then motivating the change in potential energy both in terms of the internal work (as in lecture coverage), as well as with

the work-energy theorem. We also expanded the discussion of test-charge independence. These changes were partially motivated by poor post-test performance both on changes in energy, and in test-charge independence questions. TA observations consistently suggested that students struggled with potential difference in the two-connected-conductors page, so we added some exercises before this section to help students think about charge distributions, electric field, and potential differences in conductors.

During observations in tutorial sections, we found that many students were confused by the definition of potential difference, and were not sure what formulas to use to find ΔU from internal or external work. At the end of the tutorial section most students were in the middle of the new page on conductors, which was difficult. Nonetheless, as discussed in the next section, student performance increased on test-charge independence questions on the post-test so expanding the section on test-charge independence was beneficial.

4. Version 4

Since the first page in the previous versions was difficult for many students, in the fourth version we decided to present students with the work-energy theorem and the relation between change in potential energy and internal work initially, and have them show that the two definitions were consistent. The two-connected-conductors page had been hard for students in every version, so we moved it from this tutorial into a new tutorial, *Electric Properties of Conductors*, which will be discussed in chapter 5.

Ultimately we found that the fourth version was too difficult for many students; almost none worked through more than the first 2 pages. The post-test question for this

version only included a few questions about potential difference since the rest were assessing the new tutorial on conductors. In addition, when analyzing student responses we found that many students failed to interpret the problem correctly. Thus, this version of the post-test is not included in the comparison of post-tests in section C.

5. Version 5

In all of the previous versions of the tutorial, we have found that students had difficulty with finding changes in potential energy on the first page. For the fifth version, we used the two-block systems in Figure 3.3 to help students relate the external work done by the hands to the change in potential energy of the system. The experiments, as well as the sequence of questions, are very similar to those in the *Conservation of Energy* tutorial that students had worked through in mechanics (Physics 121). We removed any mention of internal work by the field, and were very explicit in guiding students to use the work-energy theorem with the external work done by the hands.

In previous versions TAs had noted that students didn't seem to have a firm grasp of potential energy before they were introduced to potential difference. We therefore added a

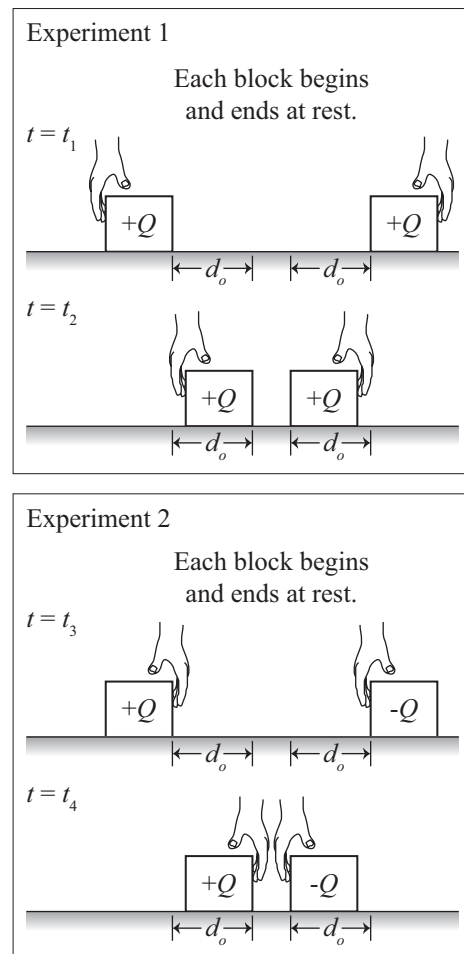


Figure 3.3: two experiments with charged blocks, to help students relate external work to changes in potential energy

second page of simple examples to give students practice with reasoning about changes in potential energy, before they needed to use it in the definition of potential difference.

Post-test performance increased by a small but statistically significant amount from the previous version. Students didn't tend to get stuck on any section of the tutorial, and a much higher fraction completed the tutorial by the end of the period. However, we observed that in many cases students were not using the work-energy theorem given on the first page, but were instead using a memorized formula for electrostatic potential

energy, $\Delta U = \frac{kQq}{r}$.

6. Version 6

In designing the sixth (and final) version of the EPD tutorial, we wanted to present electric potential energy in such a way that students could not use a memorized formula. We therefore recast the first page in terms of a general, unknown conservative force, and then shifted to using electric forces on the second page. The section on test-charge independence was slightly expanded again, by adding a questions in which students were explicitly asked whether or not potential difference depended on the test charge. We had anticipated that these changes would decrease the average completion time, so we also added a page on the reference point of potential and on relating absolute potential to potential differences. We also added a page on how charged systems react to potential differences, to help relate electric potential back to the physical behavior of objects.

Our observations in tutorial suggested that these changes had the desired effect. By the end of the tutorial most students had completed the tutorial or were working on the

last page. Student performance on the post-test was also very good for this version, around 85% for all questions (except test-charge independence). It is interesting to note, however, that students took a significant amount of time to complete the first page of this version. In most cases this seemed to be because they did not recall the work-energy theorem from mechanics.

C. Comparison of student post-test performance after each version

The post-tests for each version except version 4 shared three similar questions, allowing for straightforward comparison of student performance after each version. A typical example is shown in Figure 3.4. Students were initially told that a positive test charge of a specific

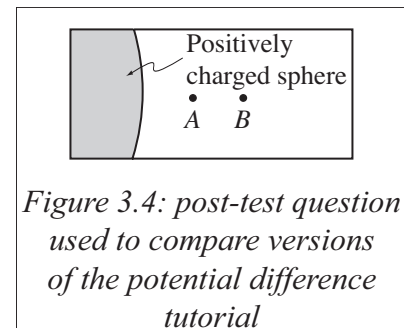
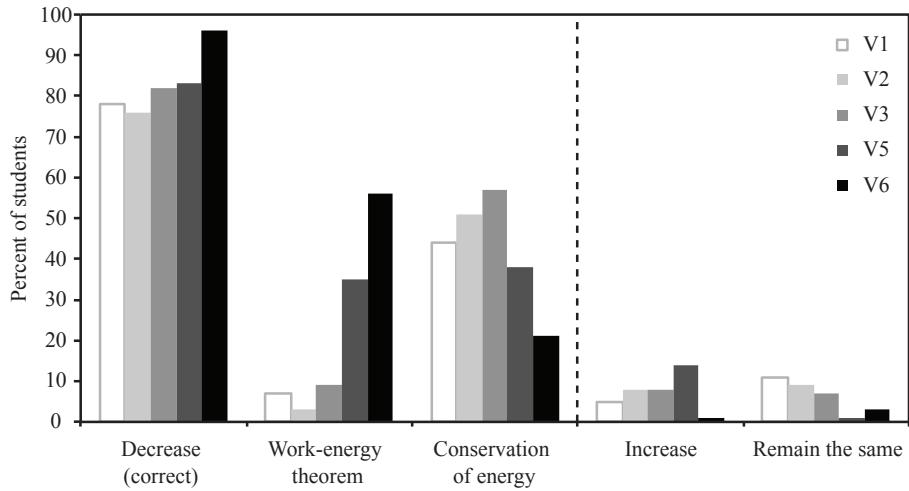
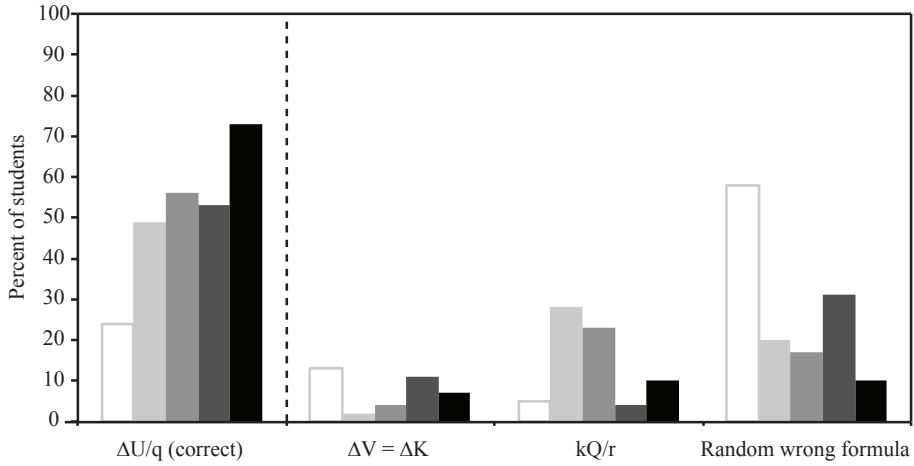


Figure 3.4: post-test question used to compare versions of the potential difference tutorial

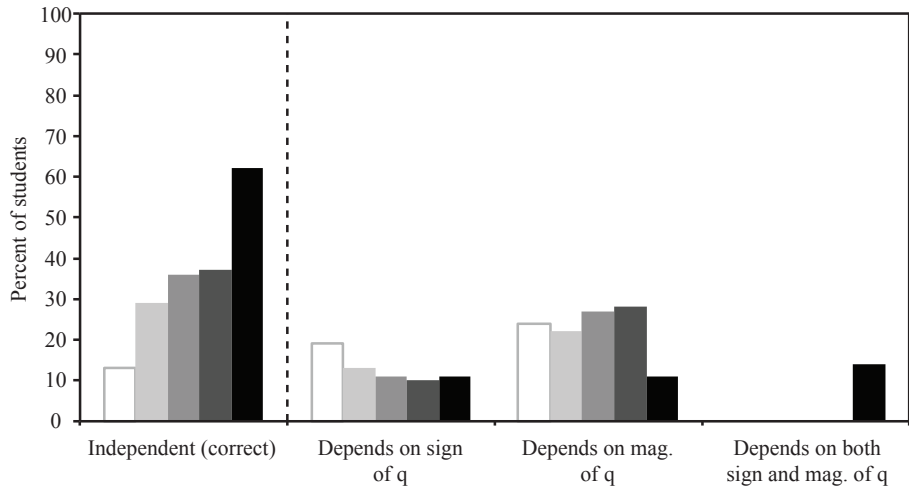
magnitude was released from rest at point *A*, and it then crossed point *B* with a specified kinetic energy. The first question (part a) asks if the change in potential energy of the system is positive, negative or zero. The most complete correct answer is that in the system of both charges, there are no external forces, so there is no external work done. Thus the total energy is a constant, so if the test charge gains kinetic energy it must have lost potential energy. This question evaluates the effect of the tutorial on students' understanding of potential energy, as the definition of potential difference relies on changes in potential energy. Results for this question from each administration are shown in Figure 3.5 (a). All results are discussed together, after describing each question.



(a): change in potential energy due to the movement of a test charge



(b): calculating ΔV from the movement of the test charge



(c): dependence of ΔV on the test charge

Figure 3.5: comparison of three EPD post-test questions across five tutorial versions. Correct answers are to the left of the dashed line; figure (a) also includes two types of correct reasoning. Incorrect answers are to the right of the dashed line.

The second question (part b) asks students to find the potential difference between the two points. The definition of potential difference is $\Delta V = \frac{\Delta U}{q_{\text{test}}}$, and since there is no change in total energy, $\Delta E = 0 = \Delta K + \Delta U$ and thus $\Delta V = -\frac{\Delta K}{q_{\text{test}}}$. This question is related to the first, as it also probes students' ability to reason about potential difference, in this case quantitatively. Results from this question are shown in Figure 3.5 (b).

In the third question (part c) the original test charge is replaced by a different test charge, with different magnitude and the opposite sign, which is moved by a hand between the same two points. The question asks how the potential difference between the two points when the second test charge is moved compares to the potential difference when the first charge is moved. The correct answer is that potential difference is independent of test charge, so the potential difference is the same for both. An alternate explanation is that the change in the test charge causes an identical change in ΔU , so ΔV is unchanged. Figure 3.5 (c) shows results from this question for each version.

To control for possible variations between each class, we compared student performance on the first three pretests of each quarter, and on the lecture multiple-choice portion of the midterm that included the EPD post-test questions. χ^2 tests showed no significant difference in pretest performance between the five classes, which is consistent with Heron's findings¹ that variation in lecture instruction statistically has little to no effect on students' pretest performance. There were small differences in the total multiple-choice exam scores, but this is most likely due to variance in overall difficulty of the exams,

particularly since these differences were not correlated with student performance on the qualitative tutorial questions. Since there was no significant difference between each class, we concluded that the post-test performance on each question could be used as a measure of the effect of each tutorial version.

Student performance on part a increases with version number, reaching 96% correct on version 6, which is unusually high for post-test questions. The second group of columns in Figure 3.5 (a) shows the students who gave the correct answer with correct reasoning, e.g. that the total energy is conserved since there is no external work on the system. The third group of columns shows students who gave the correct answer and an incomplete explanation: these students stated that the total energy is conserved without referencing the work-energy theorem to explain why.

The prevalence of this incomplete explanation diminishes somewhat with version, and the fraction of students who are correctly using the work-energy theorem on versions 5 and 6 is much higher than those in any other version. These were the versions with a first page of examples in using the work-energy theorem, which the plot shows is an effective strategy compared to the approaches taken in earlier versions.

Across each version, part a was easier than parts b and c. The change in student performance across versions is more striking in parts b and c than in part a, which could be due to a ceiling effect. If more students already understand the concepts necessary to answer part a than parts b or c, and some fraction of students learn these concepts from the tutorial, then the absolute change in an easier question will be smaller than on a more difficult question.

The graphs in Figure 3.5 show that student performance as a whole increased with each successive version of the tutorial, and also supports the program of small, research-motivated and research-validated curriculum changes.

D. Addressing student difficulties with a pre-tutorial and homework

Student performance after version 6 is much better than after any other version, but there were two persistent issues that remained. First, no matter how carefully the first page of the tutorial was worded, many students did not remember the work-energy theorem, and so worked through the first page fairly slowly. This seemed to be compounded by the force-energy conflation described in chapter 2 (pages 20-23). Second, although performance on test-charge independence questions increased with each version, it still lags behind performance on other questions for all versions.

Given that extensive tutorial development had not satisfactorily addressed these issues, we decided to see if modifications to the tutorial pretest or homework could address these issues. Specifically, we hoped that adding “teaching” material to the pretest could decrease the time necessary for students to complete the first page, and that adding quantitative questions on test-charge independence to the homework could increase post-test performance.

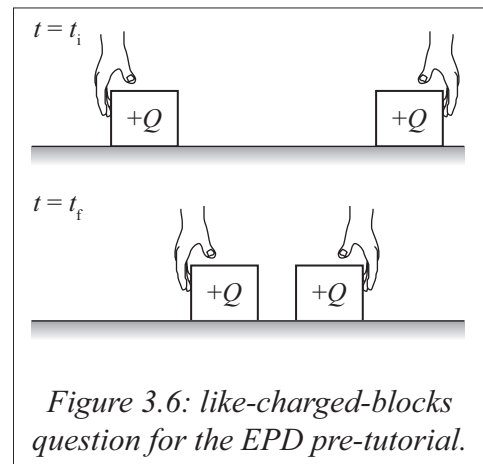
1. Developing and assessing a pre-tutorial for potential difference

As mentioned above, the issues students seemed to have with the first page were related to a difficulty remembering the work-energy theorem, and to a conflation of force with potential energy. To address this, we added an elicit-confront-resolve cycle of

questions to the *like-charged-blocks* and *opposite-charged-blocks* pretest (described in chapter 2 on page 2.8 and 2.9, and shown in full as the EPD_152 pretest in Appendix A).

Though this has a similar form compared to a standard tutorial pretest, this “pre-tutorial” is intended to both remind students about the work-energy theorem, as well as to point out the associated error in conflating force and potential energy.

This pre-tutorial first asks students about the change in potential energy as two like-charged blocks were pushed together (Figure 3.6). This is the elicit phase, in which many students exhibit force-energy conflation. The pre-tutorial then shows students a discussion between two students about their answers, where a hypothetical Student 1 conflates force and

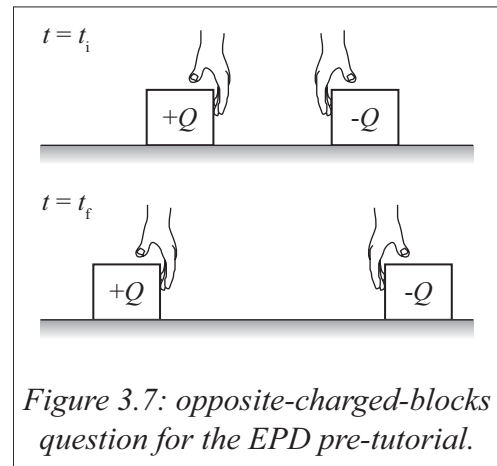


energy and a hypothetical Student 2 uses the work-energy theorem:

Student 1: *“I think the system gains potential energy as the blocks are pushed together. The force between the blocks is greater when they’re closer together, so the energy must be greater as well.”*

Student 2: *“I agree that the system gains potential energy, but I thought about what the hands were doing. Both hands push the blocks inward, so they both do positive work. The net external work is positive, and there’s no change in kinetic energy since the blocks start and end at rest, so the potential energy increases.”*

The pre-tutorial then shows students the opposite-charged blocks case (Figure 3.7), and asks what Student 1 and Student 2 would answer if they used the same reasoning as they had in the previous case. In the confront phase, students are shown a graph of potential energy as a function of distance, which supports the reasoning used by Student 2, and asked whether the line of reasoning presented by Student 1 or Student 2 was correct. The teaching aspect of this pre-tutorial occurs as a result of the resolve step, where students compare the two lines of reasoning against the correct graph.



The pre-tutorial was run in the Spring quarter of 2015, when there were two lecture sections of Physics 122, A and B ($N_A = 176$, $N_B = 171$). Both sections had the same lecture and lab instructor, and the same pool of TA. Section A was given the pre-tutorial, and section B was given a standard pretest that had the same questions about the two-block systems. Other than on EPD-related materials, the two sections were statistically indistinguishable on all pretests, post-tests, and overall course grade.

The first-page completion times of the EPD tutorial for both sections are shown in Figure 3.8. The average completion time for section A was 11 minutes, compared to 14 minutes for section B. This result is statistically significant at the $p < 0.05$ level, according to a t-test between the two classes. The standard deviation of section A is also significantly less than of section B, as the plot shows. Students in the test group were also much less likely to make mistakes on the first page of the tutorial, which reduced the amount of TA

intervention necessary. To contrast, at least one table of students in each section of the control group needed a significant amount of TA help (5-10 minutes) to complete the first page of the tutorial.

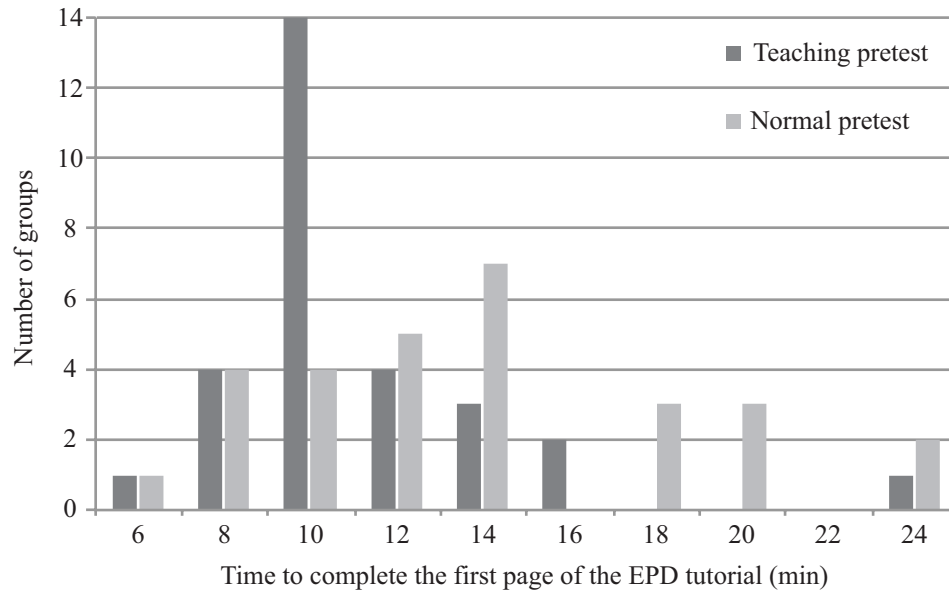


Figure 3.8: effect of the pre-tutorial on first-page completion time of EPD

It seems that the pre-tutorial is achieving its designed purpose, which was encouraging, but did speeding up the first page of the tutorial have any effect on student learning? To answer this question, we designed a 2-part post-test, administered on the second midterm examination two weeks later (EPD_152 in Appendix B). The first part involved the same context that was used for the post-test comparison (Figure 3.4), and asked questions a and c (described in section C). The pre-tutorial had no significant effect on these questions: 69% of students in section A had correct answers with correct reasoning, and 61% of students in section B had correct answers with correct reasoning ($\chi^2 = 1.2$, $p = 0.27$).

In the second part of this post-test, students were shown Figure 3.9, and told that the potential at point C was positive. The question asks which of the other marked points could be the reference point for potential. To determine the correct answer, students had to use the fact that the potential at the reference point is defined to be zero. For C to have positive potential, the potential difference from the reference point to C must be

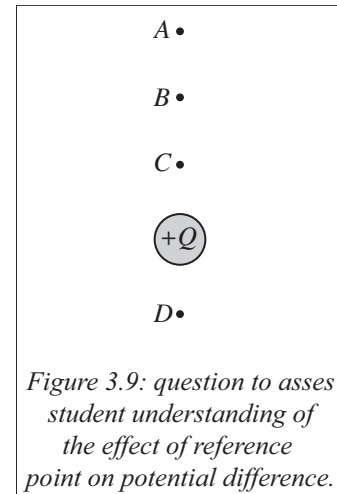


Figure 3.9: question to assess student understanding of the effect of reference point on potential difference.

positive. Since C and D are equidistant from the charge, the potential difference between them is zero. The potential difference between both points A and B to point C is positive, so thus either A or B could be the reference point of potential. This line of reasoning was covered in the fourth page of the tutorial version given to these students. Most students in both classes got to this page, but students in section A usually had more time to work on the page, as well as to get help from the TAs on the material, as a result of working through the first page more quickly.

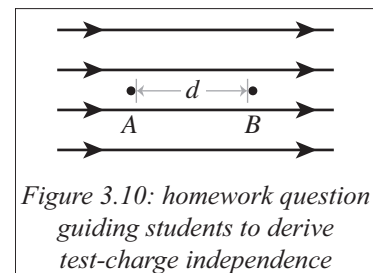
On the post-test, 75% of students in section A chose a set of points that included A and B , though only 69% correctly stated that it could only be A and B . 64% of students in section B chose a set of points that included A and B , and 59% correctly chose only those two points. These results are significant at the $p = 0.05$ level with a χ^2 test. Many students in section B also did not answer the question at all, or chose C or D exclusively, showing that they did not understand the meaning of the reference point. In contrast, two thirds of students in section A who answered incorrectly reasoned based on the definition of reference point, but usually made a mistake or applied the definition incorrectly.

These results suggest that the pretest decreased the time it took students to work through the first page of the tutorial, which had the side-effect of increasing student learning, as measured by post-test performance.

2. *Modifying the EPD homework to address test-charge independence*

We extended the section involving test-charge independence several times as we developed successive versions of the tutorial, but post-test performance on this topic was lower than for other topics related to potential difference. We thought that giving students a numerical exercise, guiding them to prove to themselves that potential difference was test-charge independent, could help cement their understanding.

To that end, we added a homework question based on Figure 3.10 to EPD_HW_Version6 (Appendix D). Students were given the magnitude of the electric field, the distance



between the points, and the magnitudes of three different test charges (two positive, one negative). The question asks students to find the work needed to move each of the test charges from point *A* to point *B*, the change in potential energy due to this movement, and the potential difference for each case (which is the same for each test charge). The question then asks if changing the magnitude of the field or the distance between the points would change the potential difference (both of which would). Finally, there is a synthesis question that asks which of the variables discussed (test charge, field strength, and distance between points) affects the potential difference.

Students' submitted homework was analyzed the same way the post-tests were, with an emergent coding scheme (described in chapter 1). This homework was given in

the Spring quarter of 2015, with both the test and control populations from the pre-tutorial comparison seeing the same homework. 82% of students found that the potential difference between the two points was the same for all three test charges, and stated that potential was independent of test charge, which is higher than student performance on any post-test before this homework; there was no variation between groups that had or had not seen the pre-tutorial. However, only 60% of students stated that potential was independent of test charge on the post-test described in the previous section; this is statistically indistinguishable from the post-test results from students who did not see these new homework questions.

This 22% drop between homework and post-test performance is surprising, and suggests that the homework modifications were not effective. We added an additional set of questions to EPD_HW_Version6b about Figure 3.10, which guide students to derive the definition of potential difference in terms of the line integral of the electric field. The equation they derive does not include q_{test} , which we hoped would further help students realize that potential is test-charge independent.

Though nearly every student correctly answered the new homework questions, post-test performance on the same test-charge independence post-test was unchanged. This is strikingly similar to the results found by Sayre and Heckler², who found that student performance typically peaked while students were learning, and then either spontaneously decayed or suffered interference from learning related topics. Our results could be explained by this phenomenon; student performance on the second iteration of EPD_HW_Version6 improved, but then decayed to the same level as before by the time of

the post-test. It is surprising, however, that we have only observed this disparity with test-charge independence and not with the other related concepts that we investigated. This implies that Sayre and Heckler's model may not be completely applicable in this case, or that some other mechanism entirely is responsible for this. However we have not identified any other plausible explanation that accounts for both the variance in homework performance and the similarity in post-test performance.

E. Discussion

As we have shown, a program of research and curriculum development have significantly impacted student learning of electric potential difference, and related concepts. However, we found limited success by only doing research to improve in-class curriculum, and decided to increase the impact by developing other elements. This has implications for instruction: including instruction on course elements outside of the classroom can have a greater effect than only focusing on the in-class elements.

F. References for chapter 3

- [1] P. R. L. Heron, "Student performance on conceptual questions: does instruction matter?" PERC Proceedings 2012
- [2] E. C. Sayre and A. F. Heckler, "Peaks and decays of student knowledge in an introductory E&M course," PRST-PER 5, p. 013101 (2009)

CHAPTER 4: STUDENT DIFFICULTIES WITH CONDUCTORS

“The net electric field inside a conductor is zero, but it takes positive work to move a test charge which is why electric potential difference is negative.”

– Response on an introductory pretest

There have been many studies investigating student ability to reason about voltage, current, and other concepts in the context of electric circuits, as well as on basic electrostatics concepts such as Coulomb’s and Gauss’s laws¹. However, there are very few publications dealing with electrostatics concepts such as charge, fields, and potential differences in conductors. Because the literature is so sparse in this area, this research is groundbreaking, particularly dealing with potential differences in conductors. Original research described in this chapter can therefore only be loosely compared with prior research.

The next few sections detail broad categories of issues students have with applying electrostatics concepts to conductors. Most of the original research detailed here comes from pretest data, as well as some post-test data from the *Electric Potential Difference* tutorial.

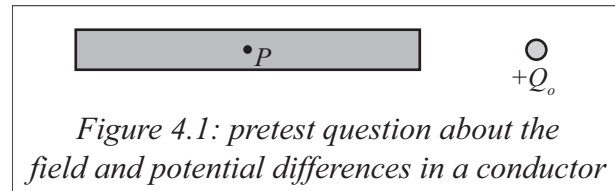
A. The net electric field inside a conductor

The fact the net electric field inside a conductor in equilibrium is zero is standard to include in introductory textbooks. Many texts state it as a given or give a brief description, but some texts such as Chabay and Sherwood² motivate this fact in terms of a mechanistic model of forces on charges. If the conductor is in static equilibrium, then no charges are

moving; each charge has zero acceleration, zero force, and therefore experiences no net electric field.

Most students tend to take this statement about the net electric field as a definition, and do not understand the fact that that it is a logical consequence of the fact that charges can freely move in conductors. If students don't understand *why* the electric field inside a conductor is zero, they are very unlikely to be able to apply further electrostatics concepts that depend on the electric field, to conductors.

Figure 4.1 shows a pretest question from the EPC_152 pretest (Appendix A) which exemplifies this lack of conceptual



understanding. Students are told that the point charge had been in place for a long time, and the question asks about the direction of the net electric field at point P . A second question then asks whether the potential difference from the left side of the rod to the right side of the rod is positive, negative, or zero. The results from this question are shown in Table 4.1.

Sign of the potential difference across the rod:		Direction of the electric field at point P :		
		Zero (correct)	To the left	To the right
Positive	34%	47%	33%	15%
Negative	29%	9%	18%	5%
Zero (correct)	35%	12%	9%	6%
		26%	5%	3%

Table 4.1: Results from 331 students on the direction of the electric field at point P , and the sign of the potential difference across the rod (while the charge was in place). The lower-right corner shows the breakdown of student responses on both questions. Correct percentages are in bold.

Half the students chose the correct direction of the electric field. A third of students stated the field at P was to the left, most of which only referenced the external field in their explanation. 15% thought the field was to the right, and only discussed the induced field of the charge distribution on the rod.

Only a quarter of students stated both that the electric field was zero and that the potential difference across the rod was zero. About a third stated the potential difference was positive, consistent with only considering the induced field, and another third chose negative, consistent with only considering the external field. This is further evidence of the inconsistent choice of systems that was described in chapter 2 (pages 13-17).

Interestingly, the proportion of self-consistent answers was less than half, across all students. For instance, 6% of students stated that the field was to the right (considering the field of the induced charge distribution) but that the potential difference was negative (basing their answers on proximity of each end of the rod to the external charge).

Many students explicitly stated both that the field was zero and that potential was non-zero:

“Electric field in a conductor is zero. The left side is positive and the right side is negative so the potential from left to right is negative.”

“The net electric field is zero in all conductors but the induced charge will create a gradient.”

“The net electric field is zero, because in the center of a conductor the E field is always zero. The potential difference is positive because the V_{right} is larger than

V_{left}, due to the E field being larger on the right side, creating a positive outcome.”

Students showed similar behavior while working through the two-connected-conductors page of the *Electric Potential Difference* tutorial, in those versions that included it (shown in Figure 2.1, on page 14). Most groups of students ended up incorrectly deciding that the potential difference between the two conductors was non-zero; initially, some students tentatively thought that the potential difference was zero but tended to be convinced otherwise by students who incorrectly based their answer on the net charge on either conductor.

We made observations of all tutorial sections during one of the quarters when the version of the *Electric Potential Difference* tutorial that was run included the two-connected conductors contexts. All the tutorial TAs were instructed to ask students who decided the potential difference was non-zero, what the net electric field inside the conductors was. All students correctly answered that it was zero, but could not spontaneously use that fact to determine the correct answer, and needed further guiding questions from the TA.

Christian and Talanquer³ characterized reasoning used in small study groups in chemistry classes, and found that students' reasoning could be characterized as rule-based, case-based, or model-based. Rule-based reasoning tends to be based on generalized patterns; people can deduce patterns from observations of the world or from mental models, and can cast these patterns as empirical generalizations or symbolic rules. This is often an advantage because a rule requires less cognitive processing to apply to new

situations, since most of the cognitive processing necessary has already occurred to generate the rule. However, “novices often ... tend to overextend the scope of [the rule].” In the context of conductors, rules are often provided by the instructor or textbook instead of from observed patterns, which perhaps makes students even more likely to overgeneralize them. For instance, “the electric field in a conductor is equal to zero” is a rule – students can apply this statement to a situation without needing to spend time thinking about why the statement is true or how it applies.

Case-based reasoning relies on recalling old experiences (cases) to apply to new, similar situations. This is often what lies behind a physics expert’s “intuition” – if an expert has solved dozens of similar problems in the past, they are very likely to intuit the answer to a new problem because it matches an old case they are familiar with. However, this can be a danger if students recall a case that is not appropriate or related to the current task. The work on expert-novice characterization of physics problems by Chi et al⁴ can be viewed as an example of this – students build cases based on surface features, whereas experts build cases based on physics content. This type of reasoning typically occurs less often in the introductory E&M course than other types of reasoning, as students solve a given type of problems only a few times, not enough to create a substantial base of cases to reason from.

Model-based reasoning is the most powerful, and least common, type of reasoning found by Christian and Talanquer. Most if not all of physics can be described by models of systems or processes, that associate a descriptive mechanism to a physical concept. Reasoning with models could be more powerful because it is more flexible: students using

model-based reasoning only need to recall one idea (the model) and apply it, whereas students using rule-based reasoning could have many rules and might need to spend time thinking about which rule to use.

Christian and Talanquer analyzed cognitive processing associated with these modes of reasoning using the revised Bloom's taxonomy^{5,6}. They found that rule- and case-based reasoning was frequently based on remembering (the lowest scale of cognitive processing in the taxonomy) whereas model-based reasoning more frequently led to higher levels of analytical thinking ("analyzing" is the second-highest scale of the taxonomy).

When applying this framework to the pretest question about Figure 4.1, we see evidence of widespread use of rule-based reasoning among students, and comparatively little model-based reasoning. The large fraction of students who correctly answered that the electric field in the rod from Figure 4.1 is zero but the potential difference is nonzero seem to be using rule-based reasoning when dealing with conductors, in particular the rule "the electric field inside a conductor is zero." These students do not seem able to apply their rule to thinking about potential differences; instead they activate a different rule, that $V=kq/r$. This rule is not incorrect as long as students consider all charges in the system and use the principle of superposition. Most students that used this rule did not consider all charges in the system, and so this reasoning almost exclusively led students to the incorrect answer.

Some students (around 15%) do use model-based reasoning, however:

"The net electric field inside a conductor is zero, so the electric potential difference is the definite integral of zero, which is zero."

“Since the electric field in a conductor is zero, it doesn’t take any work to move a test charge around inside it, so the potential difference between any points inside are zero.”

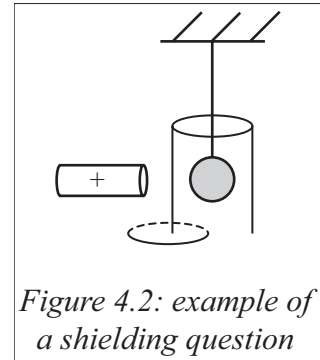
Electric potential depends on the electric field, so if students do not conceptually understand the electric field they are unlikely to use correct reasoning about potential differences. This research suggests that in contexts such as conductors, rule-based reasoning about the net field in a conductor does not support students’ ability to reason about potential differences based on these fields. In these contexts, it seems that students are more likely to be successful if they use model-based reasoning. If curriculum could be designed to explicitly help students build a model for how conductors behave, it could help them replace rule-based reasoning with model-based reasoning, which is more likely to help them understand related concepts. A version of the *Electric Properties of Conductors* tutorial based on this hypothesis will be discussed in chapter 6, as well as the effect it has on students’ reasoning level and ability.

B. Blocking of electric fields by conductors

The previous section discussed students who seemed to treat the field in a conductor in terms of rules, and could not correctly reason using these rules with more complex ideas. There is another subset of students between these students and students who were using model-based reasoning: these students understood that *an* electric field inside a conductor was zero, but not that it was the *net* electric field. Many students believe that only the *external* field is zero inside a conductor, and they often use language such as

“the conductor blocks the field,” instead of realizing that the conductor creates an internal electric field that cancels the external field.

This has been seen in research on students’ understanding of charge in conductors. Figure 4.2 shows a question that Bilak and Singh⁷ showed to introductory students and first-year graduate students. The question stated that a small aluminum ball was hung at the center of a tall metal cylinder, and a positively charged plastic rod was brought near the cylinder. The question asked if there was an induced charge distribution on the ball.



They found that only a third of introductory students, and a quarter of first-year graduate students, correctly answered that there was no induced charge on the ball. Most students stated that the cylinder shielded the ball from the plastic rod, but that the charge distribution on the cylinder then induced a charge distribution on the ball.

Bilak and Singh did not include data about student reasoning, so we conducted a replication study with the same questions that also asked students to explain their reasoning. Unsurprisingly, we found similar patterns to those described by Bilak and Singh. Of the 15% of students who incorrectly answered that the electric field inside the rod was to the right, around $\frac{3}{4}$ explicitly stated that the external electric field was zero inside the rod, and many said the rod “blocked” or “excluded” the external field, leaving only the induced field.

Nearly all students who incorrectly believed that the external field was blocked then used correct reasoning to find the potential difference across the rod, due to the induced

field alone. Although these students were not using completely correct model-based reasoning, the fact that they could correctly reason about potential differences based on their electric field suggests that their reasoning was more flexible than the quarter of students using rule-based reasoning to conclude that E was zero but potential difference was not.

In contrast, the third of students who stated that the electric field was to the left tended to assume that the *induced* electric field was zero, but that the external electric field was not. This is consistent with Guruswamy's work⁸, who found that when asked about the transfer of charge between two conductors, students tended to ignore the forces between charges on the same conductor. These students who think that the induced electric field is zero could be thinking of the induced charge distribution as a single object, instead of being comprised of many individual charges, and thus having no effect on the charges in the metal.

C. Student misconceptions about ground

Leinweber⁹ found a small set of common misconceptions about ground: the most prevalent of which is that ground neutralizes the charge on an object, so that a grounded object has zero net charge, as in the quote below.

“[The object] is grounded which neutralizes its charge (by the definition of grounding).”

The basic reasoning behind these answers is similar to students who only consider the effect of external electric fields. In this case students were not including the effect of

local charge distributions, only that of the ground (which is external to the entire charged system).

The other common misconception is that ground “takes away charge.” On some questions (i.e. ones in which there is no external field) this idea can lead to the correct answer, but with incomplete reasoning. It can also lead to the incorrect answer in a small fraction of students who fixate on the fact that charge carriers are negative. For example, Figure 4.3 shows a question from the CAP_104

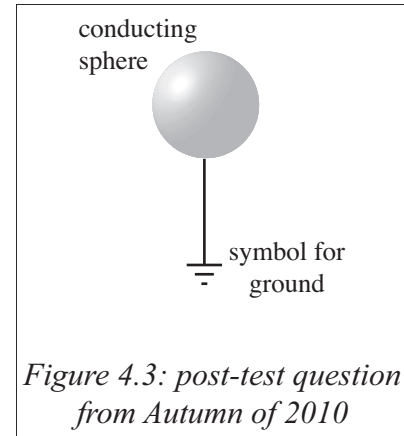


Figure 4.3: post-test question from Autumn of 2010

post-test (Appendix B). The first question asks about the net charge on the sphere and the potential difference between the sphere and ground, both of which are zero. This question was intended to be a simple ‘warm-up’ question before a charged object was added to the system. Almost all students correctly answered that the sphere was neutral, but 7% of students stated that the sphere was positive, because ground takes all the (negative) mobile charge carriers.

Table 4.2 compares the prevalence of these misconceptions about ground to those described about electric fields in conductors. Students who used more-sophisticated cognitive processing primarily viewed ground as a large conductor, so grounding an object simply allows charge to distribute over a much larger object. The proportion of students using correct reasoning is fairly similar in both cases (first column).

Students in the second column tend to not think about the induced charge distributions; for the field question students are discounting the induced field, and in the

grounding case students don't believe that there is an induced charge distribution at all. On the other hand, students in the last column tend to ignore external effects entirely, and only focus on the motion of charges inside the conductor. These percentages are not equal and should not be compared against each other; instead, this table shows that the patterns of answers and reasoning that appear in questions about the field in conductors are similar to the patterns in other questions about conductors, such as grounding questions.

	Zero	To the left	To the right
Direction of the electric field:	47%	33%	15%
	Large conductor	Neutralizes	Takes charge
Effect of ground:	50%	42%	7%

Table 4.2: Comparison of results about the electric field in a conductor (top row, taken from Table 4.1) to common misconceptions about ground (bottom row). Correct answers are bolded.

D. Conclusions

This research into student difficulties suggests that designing a tutorial to help students construct and then engage with a simple mechanistic model of charges in conductors could help them reason about the behavior of charges in conductors. Such model-building could not only help students understand the behavior of charge distributions in conductors, but could also help students with further down-stream concepts that depend on an understanding of charge distributions and fields in conductors.

The impact of instruction based on conceptual models of charge distributions has been tested by several other researchers in the context of electric circuits¹⁰⁻¹². Thacker et

al showed that in a traditional lecture-based engineering course, the use of a textbook that emphasized microscopic models of charge distributions had a significant impact on student ability to reason about macroscopic behavior, often about behavior that students were completely unable to explain without considering the microscopic charge distributions. Relating to the hierarchy of cognitive reasoning proposed by Christian and Talanquer, Hirvonen showed that an instructional approach based on a microscopic model of forces on charges and charge movement helped almost all students to abandon naïve microscopic models of charge in circuits in favor of more correct, complete models. However, all of these studies have used these models in the context of circuits, as opposed to starting with the conceptual models to deal with conductors in the context of electrostatics. Chapter 5 discusses the development of curriculum to aid students in constructing a model for the electrostatic behavior of conductors.

E. References for chapter 4

- [1] See, for example, L. C. McDermott and E. F. Redish, "Resource letter: PER-1: physics education research," *AJP* 67, p. 755 (1999)
- [2] R. W. Chabay and B. A. Sherwood, *Matter and Interactions*, 3rd edition (Wiley, 2010), pp. 597-603
- [3] K. Christian and V. Talanquer, "Modes of reasoning in self-initiated study groups in chemistry," *Chem. Educ. Res. Pract.* 13, p. 286 (2012)
- [4] M. T. H. Chi, P. J. Feltovich, and R. Glaser, "Categorization and representation of physics problems by experts and novices," *Cog. Sci.* 5, p. 121 (1981)
- [5] L. W. Anderson and D. R. Krathwohl, *A taxonomy for learning, teaching, and assessing: a revision of Bloom's taxonomy of educational objectives*, (New York: Longman, 2001)
- [6] D. R. Krathwohl, "A revision of Bloom's taxonomy: a review," *Theor. Pract.* 42, p. 212 (2002)

- [7] J. Bilak and C. Singh, "Improving students' conceptual understanding of conductors and insulators," PERC Proceedings 2007
- [8] C. Guruswamy, "Students' understanding of the transfer of charge between conductors," *Phys. Educ.* 32, p. 91 (1997)
- [9] I. Leinweber, "An investigation into student reasoning patterns of electrostatic phenomena," general exam presentation at UW (2010) (unpublished)
- [10] B. A. Thacker, D. Boys, and U. Ganiel, "Macroscopic phenomena and microscopic processes: student understanding of transients in direct current electric circuits," *Am. J. Phys.* 67, p. s25 (1999)
- [11] B. S. Eylon and U. Ganiel, "Macro-micro relationships: the missing link between electrostatics and electrodynamics in students' reasoning," *Int. J. Sci. Educ.* 12, p. 79 (1990)
- [12] P. E. Hirvonen, "Surface-charge-based micro-models – a solid foundation for learning about direct current circuits," *Eur. J. Phys.* 28, p. 581 (2007)

CHAPTER 5: CURRICULUM DEVELOPMENT TO ADDRESS DIFFICULTIES WITH CONDUCTORS

“The electric field [inside the conductor] is zero because ... the E-field inside is equal to the E-field outside and by superposition they will cancel. Thus the E-field inside is zero.”
– Introductory response on a homework question

This chapter details the development of a new tutorial, *Electric Properties of Conductors* (abbreviated EPC), to address student difficulties with applying electrostatics concepts to conductors discussed in chapter 4. In section A I will describe each version of each version of the EPC tutorial and homework, as well as the research motivation for each successive change. A quantitative comparison of student post-test performance after each version is presented in section C.

A. Developing the EPC tutorial

As discussed in chapter 3, the first few versions of the *Electric Properties of Conductors* tutorial included a page dealing with the two-connected-conductors problem (shown in Figure 2.1, on page 14). The first three versions of EPD attempted to cover both the basic concept of electric potential difference, as well as applying it to conductors. Based on the research described in chapter 4, and the persistent difficulties students had with this section of the tutorial, we decided to break the EPD tutorial into two pieces: the new EPD tutorial would cover changes in potential energy, potential difference, and reference points, and EPC would deal with charge distributions, fields, and potential differences inside conductors. All the versions described in this section are shown in full in Appendix C.

1. Version 1

When this new tutorial was created, the two-connected-conductors page was to be the the body of EPC. In previous versions of EPD that included this context, students had significantly struggled with it, so we added two pages of preliminary material before the connected-conductors questions. These initial questions were designed to guide students to conclude that the net electric field in a conductor is zero, and then to use that fact to find potential differences between the two conductors in the third page.

Students were shown the neutral conducting sphere in Figure 5.1, and told that a net positive charge was placed at point A. The first page walked students through how the charges would move, and that when they stopped moving the

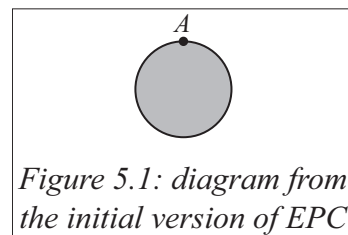


Figure 5.1: diagram from the initial version of EPC

net electric field inside was zero. In page 2, the neutral sphere was placed into a uniform external electric field, and students were expected to use their findings about the net field to conclude the direction of the induced field inside the sphere. At the end of the first section students determined the charge distribution that would be required to create this electric field.

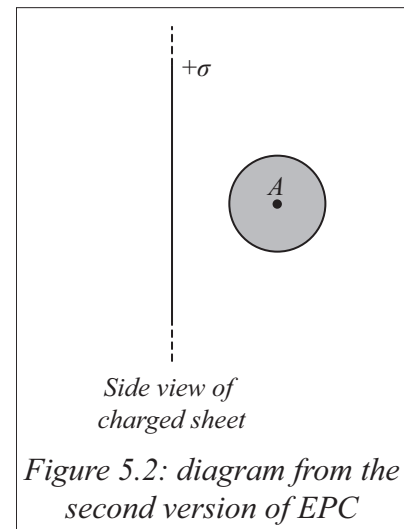
The third page was identical to that from EPD_Version3, asking students about the field inside two connected conductors and the potential difference between them. The last page had students think about Figure 5.1 again *before* equilibrium, to determine that the net field inside a conductor is only zero at equilibrium.

Many students found this version fairly difficult. In many cases the step size between questions was too large, so earlier questions did not adequately scaffold students'

understanding to tackle harder questions. The second page was intended to guide students from net external field, to induced field, to induced charge distributions, but most students did not use this logical structure, often thinking about the induced charge first. Many students stated that the induced charge blocked the external field, and that the induced field inside the conductor was zero. Students did fairly well on the post-test question about potential difference between conductors, though most students had issues with the connected conductors in tutorial.

2. Version 2

The second version streamlined the first two pages to address the difficulties students had with the first version, and more strongly emphasized the induced field on page 2. Students were shown Figure 5.2, and were asked about the net field at point A. They were then given a student statement that the net field inside the conductor was zero, but that the external field was to the right, so the induced field must be to the left. Finally, they were asked about the induced charge distribution and asked about how it affected the net field inside and outside the sphere.



Side view of charged sheet
Figure 5.2: diagram from the second version of EPC

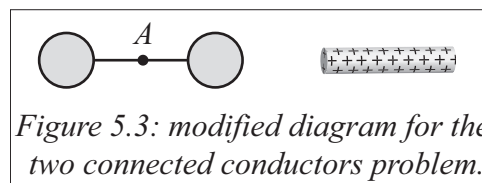
Students responded much more positively to this page than in the previous version. During tutorial observations, very few students used “blocking” language, and focused more on determining the induced field. Post-test performance also increased (see section

B). However, the logical order (finding induced charge distributions last) still confused many students, and many students misinterpreted the post-test question.

3. Version 3

We made significant changes to version 3 compared to the previous iterations. Since so many students were struggling with the order of questions on page 2, in this version students are first asked to draw the stable charge distribution on the sphere in Figure 5.2, and then to draw the field inside the sphere due to those charges. We also removed the explicit questions about the induced field, which seemed to lead to student confusion on the post-test.

The figure for the two-connected conductors page was also changed, explicitly marking point A as shown in Figure 5.3. Students were asked about the



net field at point A , and then extrapolated what the field elsewhere in the wire would be, before they were asked about potential difference. Though this page was still difficult for students, these changes resulted in fewer questions for the TAs during the tutorial section.

We also added a discussion of grounding, using the same sequence of questions we asked about the two-connected conductors.

4. Version 4

There were no major changes in terms of overall structure or content between versions 3 and 4, but since there was persistent student confusion about what some questions were asking, or the purpose of some sections, each page of version 3 was carefully rewritten. In particular, on page 2 students were explicitly asked to draw the

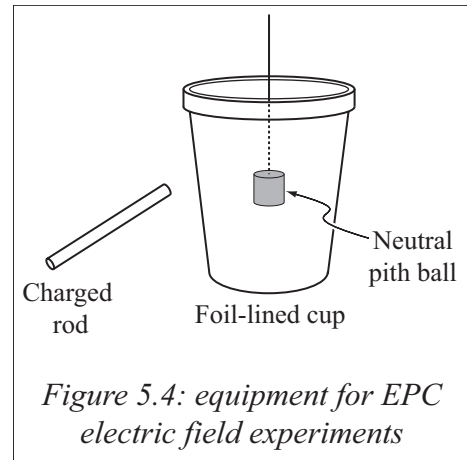
external, induced, and net fields inside the neutral metal sphere, and to explain their reasoning. The discussion about ground was also extended, to explicitly confront the common misconceptions that (1) grounding neutralizes an object, and that (2) potential is always kQ/r so connected conductors must have the same sign of charge.

5. Version 5

Student performance on electric-field questions had significantly risen throughout the development of this tutorial, to 80-90% depending on the question (described further in section B). However, student performance on potential difference questions seemed to plateau at about 40%. The prior versions all had two similarities: first, all the examples were thought experiments. Second, though some of the prior versions attempted to motivate or explain why the net field is zero, they all to some extent relied on students recalling the fact that the net field is zero from lecture or the textbook.

From the perspective of cognitive complexity and reasoning skills described in chapter 4, these versions all focused to some extent on rule-based reasoning, and did not encourage students to use model-based reasoning. To address this we re-wrote the tutorial to (a) depend on physical observations instead of abstract thought experiments, and (b) focus on helping students to build a mechanistic model for charge movement in conductors, which they could then use to explain other conductor-based phenomena such as grounding.

Thus for version 5, we replaced the first three pages of thought experiments with a physical experiment, shown in Figure 5.4. Students observe that the neutral pith ball is attracted to the charged rod if the ball is outside the cup, but not if the ball is inside the cup. They find the direction of the induced field from their results, and then the induced charge distribution on the cup.



The connected-conductors page was consistently the hardest page for students in any version, and since the reasoning students needed to use on that page was identical to what they would use in the page on grounding, we replaced it with a more simple page about finding potential differences between various points in the cup.

Student engagement during tutorials was much higher with the hands-on equipment than in any previous version, and nearly all students completed the tutorial by the end of the period, compared to around 50% in previous versions. Students also seemed to have fewer questions for the TAs, typically being able to work through the material correctly on their own without requiring as much assistance.

Finally, since the cup is shielding the pith ball from the charged rod, students were asked to extend the ideas of shielding on the homework, to consider how charges outside the cup react to charges inside the cup.

B. Evaluating each version through post-test performance

Unlike in the case of the EPD tutorial, the post-tests for EPC were not very similar. Since tutorial coverage and post-test question both differed, it is difficult to evaluate the relative difficulty of each question. Simple comparisons of correct and incorrect percentages are therefore unhelpful, so in this section I will instead focus more on the relative prevalence of correct reasoning. The relevant questions on each post-test (shown in full in Appendix B) are described in each section, as well as patterns in student reasoning for each question. The first sub-section will describe the control question, given in a quarter when the EPC tutorial was not run.

The pretests for each quarter were the same, and student performance on the pretest was statistically indistinguishable for each quarter, based on two-tailed t-tests between each pair of quarters. Each pretest was run between one and two weeks after lecture instruction, and after students had completed the same set of labs in each quarter. Since all the initial variables are comparable, differences in reasoning patterns can be attributed to the effect of each version of the tutorial.

1. Control post-test

The control post-test used Figure 5.5 below, in post-test EPC_141. This question asked students to relate potential difference in two conductors to the net charge on each conductor, and was run as a lecture free-response question during a quarter in which EPC was not run. The question stated that the switches were simultaneously closed, and asked students to state whether or not the spheres became charged, and if so to find the charge on each. Students were told that the spheres were far apart, that the wire was thin enough

that the net charge on it was negligible, and that the reference point for potential was at infinity. The correct answer is that the left sphere acquires enough positive charge to give it 6 V of potential, and the right sphere acquires enough negative charge to give it -6 V of potential. The charges on each sphere can then be calculated using the formula $V = kQ/r$.

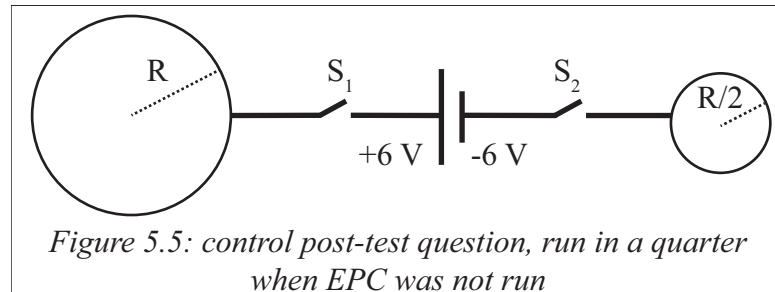


Figure 5.5: control post-test question, run in a quarter when EPC was not run

55% of students stated that both spheres would become charged, though only 27% of students found a positive charge for the left sphere and a negative charge for the right sphere ($N = 212$). 32% of students stated that the spheres would not become charged, despite the fact that there is a potential difference between the spheres, and 11% of students left the question blank (typically only a handful of students leave a post-test question blank).

The 27% with the correct answer all reasoned based on the rule " $V = kQ/r$." This reasoning ranks low in cognitive complexity on the modified Bloom scale described on page 57, but the question did not require complex reasoning to solve. 28% of students stated that the spheres would be charged but did not arrive at the correct answer; the majority of these neglected to include the sign of potential when applying their rule. The 32% of students who stated that the spheres do not become charged all reasoned with a variant of "this is not a complete circuit, so current does not flow." This is true in an

equilibrium case, but not for transients; this misconception is related to those found by Thacker et al when they investigated student understanding of charging capacitors. These students' reasoning could also be interpreted as an over-generalization of the rule that current only flows in complete circuits.

2. Version 1

The first version of EPC focused on the time-behavior of conductors and on potential difference between charged conductors, so the post-test (EPC_132) asked one question on each topic, using Figure 5.6. In the first

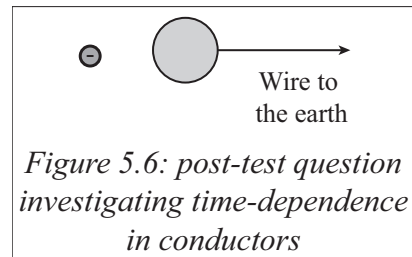


Figure 5.6: post-test question investigating time-dependence in conductors

question, students were told the negative point charge was suddenly brought near the grounded sphere, and asked about the potential difference from the sphere to the ground before the system reached equilibrium. The second question asked about the potential difference after the system reached equilibrium.

Students seemed to struggle with the first question, and only 29% correctly answered that the potential difference was positive ($N = 182$). 20% of students naïvely used kQ/r to conclude that since the sphere is positively charged, points with greater values of r would have less potential, so the potential difference from the sphere to ground is negative. A further 10% stated that the electric field in a conductor was zero, so the potential difference was zero. In this latter group of students, it seems that the rule “the electric field in a conductor is zero” is very strongly held. Even though the tutorial emphasized that potential difference was not zero if the conductor was not in equilibrium, it was not enough to overcome these students' tendency to over-generalize this rule.

Three quarters of the students correctly answered the second question, though only 27% used model-based reasoning based on the net field in the wire. Most of the rest of the students who answered correctly used a rule: 20% stated that potential differences in a conductor are zero in equilibrium, with no justification. A further 11% stated that potential differences were zero because charges were not moving, which is closely related to the previous rule.

3. *Version 2*

The post-test for this version (EPC_142) looked at simpler ideas about charge movement; an example of a correct student response is shown in Figure 5.7. The question asks about a passenger inside a charged plane. The first question asks if the passenger would get shocked if they touched the inside surface of the plane; this question tests students' understanding of charge movement in the context of shielding. The correct answer is that since the net field inside a conductor is zero, no charges will experience a force and thus the passenger will not be shocked. The second question is more general, asking if the charge on the plane could be determined by measuring the electric field inside the plane. The correct answer is that the net field within a shielded region is zero, independent of the charge on the conductor, so the net charge could not be determined by measuring the field inside.

[3 pts] Suppose you were flying in a small plane through a cloud, and the plane picked up a large amount of charge from the clouds. If you touch the aluminum wall next to your seat, would you be shocked? Explain why or why not.

No because even though the plane is charged, aluminum is a conductor so all the charge picked up will be on the outside surface of the plane. You are hopefully inside the plane where there is no charge.

[3 pts] Describe how you could determine how much charge was on the plane by measuring the electric field inside. Explain your reasoning. If it is not possible to do so, explain why not.

It is not possible because the electric field within the plane will be zero. You can pretend the plane is a sphere and given $E_r = 0$ if $r < R$, meaning if we are measuring inside, it will be impossible.

Figure 5.7: student response to the “charged-plane” post-test question

Overall, two-thirds of students correctly answered both questions ($N = 154$). All students who did not answer correctly used reasoning consistent with ignoring part of the charged system – they only took into account the charge on the nearest wall of the plane, not the entire plane.

Reasoning about shielding is slightly more challenging than reasoning about fields in conductors, since the shielded region is not actually inside a conductor. An expert’s explanation would involve a generalization about the behavior of conductors; charges move to create zero net field within the conductor, and as long as there is no charge in the shielded region the conductor will create zero net field in the shielded region as well. However, most students still answered this question by simply stating the rule that the electric field in a conductor is zero, without mentioning this added complexity.

4. Version 3

In this post-test (EPC_144) students were presented with Figure 5.8, which shows a neutral metal ball inside a neutral hollow metal cube; the entire system is placed in a uniform external electric field. Students were first asked if

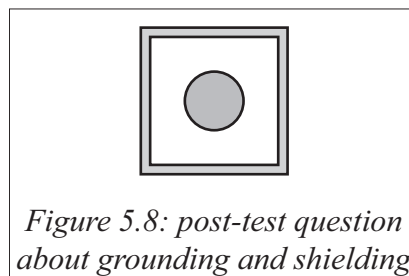


Figure 5.8: post-test question about grounding and shielding

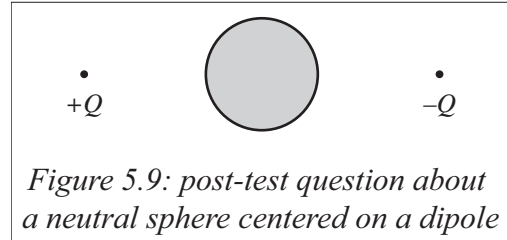
there was a net force on the neutral ball. Then the cube was then grounded, and students were asked to sketch the charge distribution on the ball and the cube after the system came to equilibrium. The correct answer to the first question is that since the net electric field inside the hollow cube is zero, there will be no force on the ball. In the second question, grounding the cube allows positive charges to flow to ground, so the cube becomes negatively charged.

Student performance on the first question dropped compared to the analogous post-test question on electric fields after version 3; 54% of students stated that the electric field was zero so the force was zero ($N = 234$). Overall 88% of students got the correct answer, but about 30% incorrectly stated that both the cube and ball would polarize, and due to the symmetry of the situation the forces on each side of the ball would cancel.

72% of students correctly answered the grounding question, with correct reasoning, which is significantly higher than control data (e.g. the CAP_104 data presented in chapter 2, page 24). Only 13% stated that ground neutralizes the cube, compared to the 45% that Leinweber found, discussed in chapter 4 (pages 60-62). Although this version was fairly long and not all students got through the page on grounding, it appears that the tutorial positively impacts students' reasoning about ground.

5. Version 4

Figure 5.9 shows the post-test for this version (EPC_151). A neutral metal sphere is placed at the center of a physical dipole. Students are first asked to draw the charge distribution on the sphere. The sphere is then grounded, and students are asked if charges flowed to ground. The wire is removed, and the last question asks about the potential difference between the sphere and ground.



The sphere will polarize with negative charges on the left surface and positives on the right; however, due to the symmetry of the charge distribution, no charge will flow to ground. The potential difference between the sphere and ground is zero while the sphere is grounded, and removing the wire changes no charge distributions, so the potential difference is still zero.

The changes between version 3 and version 4 are smaller than between earlier versions, so we expected that student performance would not significantly differ between the two post-tests. Unfortunately, the last question could be answered in two ways: either because grounding sets the potential of the sphere to be equal to ground (which is discussed in tutorial), or because the sphere is neutral and the charge distribution is symmetric; thus comparison of students who correctly model grounding on this post-test with that of version 3 is impossible. On this post-test 74% of students gave a correct answer, though only 30% used model-based reasoning similar to what students were prompted to use in tutorial; a χ^2 test between this post-test and the post-test of version 1

shows no difference in the fraction of students using model-based reasoning about potential differences.

More than half the students who correctly found that the potential difference is zero based their answer on the fact that no charges flow to or from ground before equilibrium is reached. This does not obviously fit into either rule- or case-based reasoning, but it is not model-based reasoning either.

6. Version 5

The post-tests described above show improvement in the fraction of students who correctly answer questions about the field in conductors, but there is much less improvement in student reasoning about potential differences in conductors. To address this, we made large changes to version 5, to have students construct a model for conductors based off of physical experiments and observations. We also hoped this would lead to a larger fraction of students using model-based reasoning after working through the tutorial.

The post-test for this version (EPC_153) is similar to that for version 4, to make the comparison of student reasoning easier. As shown in Figure 5.10 a, the symmetry of the charge distribution was broken, to avoid the ambiguity in solution methods. The sphere was then hollowed out (Figure 5.10 b) to create a shielding question, to test the effect of the shielding questions added to the homework for this version.

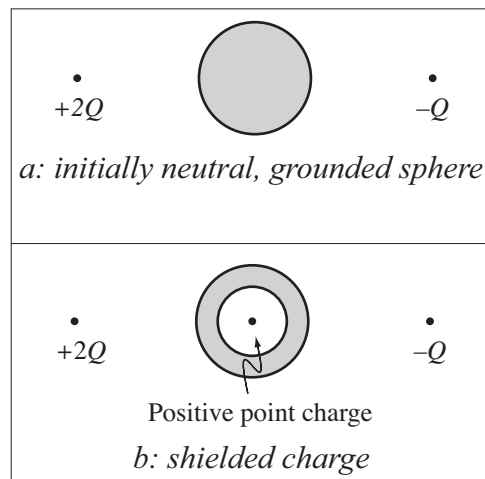


Figure 5.10: two post-test questions to evaluate student use of model-based reasoning

Students are first asked to draw the charge distribution on the sphere, and then are asked what would happen when the sphere was grounded. In this case, the net electric field of the two point charges has a downward component all along the wire, so positive charges will flow to ground. The third question asks about the potential difference between the sphere and ground after the wire is removed, which is again zero. In the last question, the sphere is hollowed out and a positive charge is placed inside, and students are asked if the charge experiences a force. Since the charge is inside a conductor, the electric field is zero and the net force on the charge is also zero.

Two thirds of the 67 students who took this exam correctly answered that the potential difference between the sphere and ground is zero, and over 80% of these students used model-based reasoning in their explanations. 80% correctly answered that the shielded charge would experience no net force, and essentially all these students all used model-based reasoning:

“The hollow conducting shell will redistribute [the charges in the metal] to balance the outer electric field, so inside the hollow there is no net E .”

“The outside charges will rearrange the charge on the outer surface of the shell, but since E net must be zero inside of a conductor the $(-)$ charge that accumulates on the inner surface must cancel the E of the $(+)$ middle charge. Thus the outer charges don’t affect the middle one at all.”

As well as the greater prevalence of model-based reasoning, the explanations provided by most students on this post-test also tended to be longer, more complex, and more detailed than the average explanation on earlier post-tests.

C. Discussion

For complex ideas, particularly those that depend on other concepts, curriculum development aimed at addressing misconceptions is not always sufficient. The first several versions of the *Electric Properties of Conductors* tutorial that we developed incrementally improved student understanding of electric fields in conductors, but did not improve student understanding of potential difference in conductors to the same degree. These versions also did not improve the cognitive level of students’ reasoning at all. However, combining these reforms with an overall structure that promoted model-building had a large impact on students’ reasoning ability, particularly about potential difference.

CHAPTER 6: COMPARING INTRODUCTORY AND JUNIOR-LEVEL CONCEPTUAL DIFFICULTIES IN ELECTROSTATICS

“Tutorials are really helpful in not only the tutorial part of the class but also the lecture part of the class. I learned the most from going to tutorial.”
– Junior-level student on a course evaluation

Our research into student difficulties, some of which is discussed in previous chapters, suggests that introductory students typically face three major sets of challenges when learning and assimilating physics knowledge into coherent models and frameworks. The first involves understanding physical concepts. As the previous chapters have shown, particularly chapters 2 and 4, students typically do not learn to qualitatively reason about many concepts covered in lecture or the textbook. Students often memorize physics “sound bites,” or specific rules to be applied to physical situations, without understanding the conceptual reason for the rule or considering the limitations of the rule. However, this pitfall can be overcome with curriculum based on appropriate research-based techniques such as peer instruction, guided inquiry, active engagement, etc. The tutorials discussed in chapters 3 and 5 are examples of this.

The second challenge to students comes from understanding mathematical concepts. Most physics questions involve mathematics to some degree, either explicitly or implicitly. Algebra, proportional reasoning, calculus, etc. are all important tools for understanding physics. While determining and addressing student difficulties with mathematics is an active area for mathematics education researchers, many students leave

their required math classes able to perform mathematical calculations and operations without understanding the underlying concepts.

The third major issue for students deals with the connections between mathematics and physics. In some sense mathematics is the language that physicists use to talk about physics; one of the key skills necessary to understanding physics is the ability to use mathematical tools in the context of physics. This type of conceptual difficulty is somewhat different than the previous two categories, in that conceptual difficulties in this area are not due to misconceptions or misunderstanding of concepts or ideas, but instead stem from an inability to transfer ideas from one domain to another¹.

This chapter compares the difficulties that introductory students have to those faced by junior-level students, primarily with physics concepts or with math-physics connections. Perhaps unsurprisingly, the overall description of the conceptual challenges to each group is largely identical, except that the concepts the upper-division students face are more complex than those presented in the introductory course.

Two differences in the data sets make direct quantitative comparisons difficult. First, the number of students the upper-division classes ranges from 25 to 120, versus 300 to more than 1000 in the introductory classes. The much smaller number of students means that an individual measurement in the upper division only allows for a general sense for the prevalence of a conceptual difficulty. Second, though the introductory tutorials discussed in this dissertation were developed over the course of several years, the upper-division tutorials have been in development for a much shorter period of time. Given the limited number of exams each quarter, the few implementations of each tutorial,

and the large number of advanced tutorials, many of the advanced tutorials have yet to be carefully post-tested.

This chapter serves mostly as a description of the types of difficulties that junior-level students encounter, as well as to give a broad sense of how these difficulties compare to those faced by introductory students. Statements such as “some students struggle with...” should be read “some students incorrectly answer questions about this concept, and these students’ reasoning is similar enough to hint that there is an underlying difficulty leading to these incorrect answers.” Most of the evidence for these difficulties comes from observations during the tutorial section, as well as estimates of how long students took on particular sections of the tutorial or how often TAs had to explain concepts to students. Even where quantitative data exists, such as post-tests or homework, concrete conclusions are hard to draw because these questions were not designed to elicit specific student difficulties. For these reasons, we cannot conclude how representative a particular set of students’ answers to a question are of the junior-level student population as a whole.

Based on these findings, we are currently performing more research to identify the scope, nature, and prevalence of the ideas that students struggle with. This includes designing new pretest, homework, and exam questions to draw out particular difficulties, as well as designing tutorials to address the underlying issues that we hypothesize are the cause of these observed issues. Descriptions of this process and the resultant data will be published in upcoming papers. The resulting material, *Tutorials in Physics: Electrodynamics* will also be published when a strong research base has been developed, and when the tutorials have been thoroughly research-validated.

A. Difficulties with physics concepts

1. Potential difference

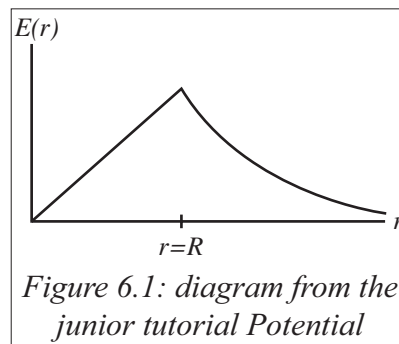
We have found that many of the post-lecture instruction difficulties that introductory students have with potential difference either appear rarely or do not appear at all in post-lecture instruction responses for junior-level students. This could be interpreted as a success of the introductory tutorial on potential difference, as the two measurements occur before and after the introductory tutorial; however there is also a substantial selection bias between the large introductory service course and the much smaller junior course for physics majors. Also, though many of the juniors took the introductory courses with tutorials, transfer students from other colleges may not have.

As an example of this difference, all junior-level students correctly answered the pretest questions we asked about test-charge dependence. Also, on both pretests and lecture homework, nearly all junior-level students correctly used the definitions of potential difference in terms of a line integral of the electric field or as the ratio of change in energy per test charge. This was a difficult task for introductory students; between one-third and one-half of students either did not know the correct formula or did not know how to use it on most pretests (described in chapter 2).

However, our research suggests that many students at both levels do not fully grasp the physical interpretation of potential differences. This is addressed in the introductory *Electric Potential Difference* tutorial, which emphasizes that potential difference is a change in potential energy per test charge (in the limit that the magnitude of the test charge goes to zero). Students are helped to reason that to find this potential difference a test charge

must be moved from one point to another, and the resulting change in energy divided by the test charge. After the final version of the *Electric Potential Difference* tutorial, most introductory students used correct reasoning about moving a test charge between two points (see Figure 3.5 on page 41).

At the junior-level, however, we have found that students again struggle with the physical nature of potential differences, particularly when dealing with reference points. For example, during the upper-division tutorial *Potential* (shown as POT_Version1 in Appendix F),



students are given the electric field of a uniformly charged sphere (Figure 6.1). Students were first asked to plot the potential difference with the reference point at the center of the sphere, and then with the reference point at infinity. In each case, students needed to integrate from the reference point to the point at which they were finding the potential.

During tutorial nearly all students could find the potential for points in the same region as the reference point: when the reference point is at the center students could find the potential for points inside the sphere, and when the reference point was at infinity students could find the potential for points outside the sphere. However, less than a quarter of students correctly integrated over both regions to find the potential in the different region from the reference point. Some students integrated the exterior field from infinity to points inside the sphere, despite the fact that the field inside the sphere had a different functional dependence than the field outside. Some students only integrated one region or the other, or integrated over bounds that made no physical sense. Nearly all

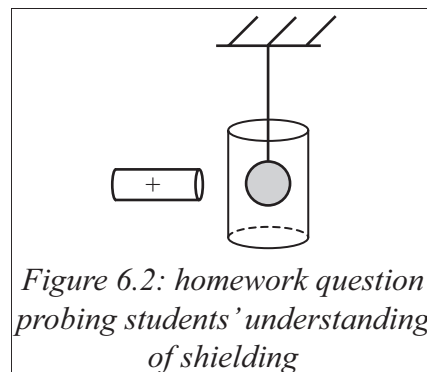
students recognized that they needed to integrate between two different points, but they did not connect potential to the physical motion of some test charge, even after the tutorial prompted them.

This may appear to be a math-physics difficulty since it has to do with setting up an integral, but we argue that the difficulty is primarily with physics, not math. When TAs helped students with this part of the tutorial, they found that most students did not understand that to measure potential difference between two points (or the potential at a point relative to a reference point) that they must imagine moving a charge from one point to the other. This issue is explicitly addressed in both the introductory and junior-level tutorials on potential, but it seems to be a difficult enough concept that students in both courses still struggle with it post-tutorial.

2. Conductors

Although about half of both sets of students incorrectly answered pretest questions about the net field inside a conductor, nearly 80% of junior-level students correctly answered similar questions post-tutorial. Figure 6.2 shows a question from the junior-level *Conductors* homework. All students stated that the net field inside the

conducting shell was zero, compared to 54% of introductory students (see Figure 5.8 on page 76 and accompanying discussion). However, about a third of students in each class stated that the object inside the conducting shell would become polarized, as shown by quotes from the junior-level homework:



“The cylinder wants an $E = 0$ inside, [... so] there is an induced E inside. There will be an induced field on the ball which polarizes the ball.”

“We have no E field inside the conductor. The ball will be polarized but there won't be a force on the ball.”

Though these students are correctly reasoning about the net electric field inside a conductor, they still do not have a complete conceptual model for the behavior of conductors. These students seem to correctly find the induced charge distribution that will create zero net field inside, but when they turn their attention to an object inside the conductor they use the nearest charge distribution instead of the net field to determine the effect on the object

There are also differences between the two sets of students in terms of the behavior of fields outside conductors. Many introductory students assume during tutorial that the induced charge distribution on a conductor only affects the net field inside the conductor, and that the net field outside the conductor is unchanged. The static boundary conditions at the surface of a conductor state that the field must be normal to the surface, for if there were some tangential component then charges would be moving. Thus to satisfy these boundary conditions the net field around the conductor must change.

Junior-level students are introduced to boundary conditions before they work through the *Conductors* tutorial, and on the *Conductors* homework, students are asked about the net field just outside the surface of a conductor. All but 2 of the 72 students who turned in this homework stated that the field must be normal to the surface. However,

about half the students stated that it could only point away from the surface, instead of either toward or away. One student said:

“The field must be normal to the surface or else charges at the surface would move. It can’t point into the surface because $E = 0$ inside, so it must point out.”

Thus though nearly all students know that the field outside must be perpendicular to the surface, half of them did not understand that boundary conditions allow fields directed toward or away from the surface (corresponding to negative and positive surface charge densities).

Though the junior-level students do not share some of the same difficulties that the introductory students have, they have new difficulties with the more advanced concepts they had not seen in the introductory class. Other than this, the overall pattern of student difficulties is fairly similar between both classes.

B. Relating mathematical and physical concepts

1. Introductory students

Yeatts and Hundhausen¹ showed that a significant challenge to introductory students was what they termed a “compartmentalization of knowledge,” or an inability to transfer concepts learned in a mathematical context to a physical context. For instance, students had practiced finding the mean value of a function over a closed interval in a calculus class, and could correctly solve related homework and post-test questions in a purely mathematical context. However, when students were given an analogous question

involving a varying velocity, many students simply averaged the initial and final velocities instead of integrating to find the mean value.

Many students also learn mathematics as a set of computational tools, instead of as conceptual ideas. Grundmeier et al³ found that most introductory students could procedurally evaluate an integral, independent of their conceptual understanding of an integral. Most of the students they interviewed had very limited understanding that an integral was a limit of estimation for the area under a curve.

Nguyen and Rebello⁴ viewed integration in physics as a four-step process: (1) recognizing the need for an integral, (2) expressing the infinitesimal quantity, (3) determining how to accumulate the infinitesimal quantities, and (4) computing the integral. They asked introductory students about integration in electrostatics, and found that students generally did not have significant difficulty with steps (1) and (4), but did have significant difficulty with the middle steps.

We can interpret Nguyen and Rebello's results in terms of the broad categories of difficulties mentioned at the beginning of this section. Students did not seem to have significant difficulty with simple physics concepts (in the case of this study, resistors and capacitors in series and electric fields of point charges) or with recognizing the need for mathematics. However, most of the conceptual difficulties students had dealt with the connection between mathematics and physics. Their study did not investigate whether or not students understood integration at a conceptual level, but they found that most students didn't struggle with evaluating the integrals.

Much of the work to develop research-based curriculum has focused on increasing students' conceptual understanding of physics, the first group of difficulties mentioned. However, several authors have found that qualitative, conceptual instruction can also impact students' ability to solve quantitative questions^{5,6}. In particular, Kanim found that students who had worked through conceptual-based tutorials as well as quantitative homework designed to help students apply mathematical models performed significantly better than students who had only seen traditional lecture instruction. However, he also found that students who had only seen the concept-based tutorials performed better than students who had only seen lecture instruction, though less well than students who were exposed to both sets of curriculum.

2. Junior-level students

Physics majors in upper-division classes face the same categories of challenges as students in the introductory classes, though typically with higher-level math and more challenging concepts. As discussed in previous sections, junior-level students typically struggle with some of the same concepts that introductory students as well as new conceptual difficulties. Just as with introductory students, conceptual-based instruction can successfully address these difficulties for most students.

The mathematics that juniors need is significantly more complex than what students are exposed to at the introductory level. Students are expected to be able to fully understand vector calculus, ordinary and partial differential equations, and special techniques used to solve various types of vector and differential equations. Students are typically taught vector calculus and differential equations in the mathematics department.

However, the special topics necessary to solve the type of differential equations that appear in physical systems are usually taught in a course in the physics department.

Since both physical and mathematical concepts introduced in the junior-level courses are more complex, the connection between them is even more important. A successful strategy that some introductory students employ is “equation hunting” – searching for an equation or sequence of equations that contains all the variables they have been given, and manipulating them until they get an answer⁷. This strategy cannot work for most upper-division problems, because the math is typically advanced enough that long calculations that require a physical understanding of the system are necessary. In this section I will present several categories of difficulties junior-level students have with the math-physics connection. I will also briefly discuss preliminary results from curriculum designed to help students address some of these difficulties.

a. Mathematics as a descriptive language for physics

Many students do not seem to realize that mathematical expressions can be *descriptions* of some aspect of a physical system. These students tend to view mathematical expressions as tools or facts, not descriptors. This viewpoint can allow students to appear deceptively competent with some math-heavy concepts that require mathematical manipulation instead of understanding, and yet lead to very poor performance in other cases.

For example, many students in Physics 321 (electrostatics) struggle with the meaning of charge density. On the first part of a lecture homework question, students were asked to find the charge density everywhere

in space for a uniformly charged sphere with charge Q and volume V , shown in Figure 6.3.

The correct answer is that the charge density is a function of position (a scalar field); if the function is evaluated at a point inside the sphere the charge density is Q/V , and if the function is evaluated at a point outside the sphere the charge density is zero. Only about half of the students ($N = 100$) included the fact that the charge density outside the sphere is zero in their answers.

A later part of the question asked about the divergence of the electric field. Gauss's law in differential form relates the divergence to the charge distribution: $\nabla \cdot \vec{E} = \rho / \epsilon_0$. Thus the correct answer is that the divergence is positive inside the sphere and zero outside. Most students did not reference location at all, simply stating that the divergence of the field was constant. These students seemed to think that charge density is a property of an object, like mass or shape, instead of as a descriptive definition of the amount of charge at each point.

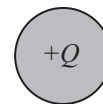


Figure 6.3: question probing student understanding of charge density and divergence

To probe the extent to which students held this idea, students in two classes were given a post-test question based on Figure 6.4 (COU_144 in Appendix E), about a charged cube with charge distribution $\rho = bz$. The question simply asks what the net charge in the cube is. Students took this midterm after all lecture and tutorial

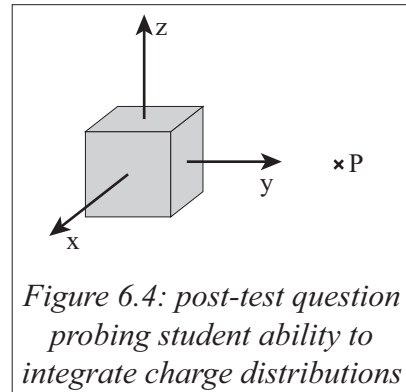


Figure 6.4: post-test question probing student ability to integrate charge distributions

coverage of Coulomb's law, superposition, and integration. The correct answer is that the net charge is zero, since the charge distribution is symmetric above and below the x - y plane, so there is an equal amount of positive charge in the upper half of the cube (where z , and thus ρ , is positive) and negative charge in the lower half of the cube (where z is negative).

Only a third of students found that the total charge was zero. A third of students correctly set up the integral, but evaluated the z -integral from 0 to a instead of from $-a/2$ to $+a/2$, and found the total charge was positive. The last third of students assumed that charge density is $\rho = Q/V$, so the total charge is $Q = \rho V = bza^3$. Thus, even though these students were explicitly shown a non-uniform charge density, they could not interpret the mathematical statement as a description of the charge distribution.

b. Integration of Coulomb's law

Students were also asked to set up (but not solve) an integral for the electric field at point $P = (0, 7, 0)$ in Figure 6.4. The integral expression can be found by superposition and

from Coulomb's law: the electric field from a point-charge element is $d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\rho}{(\vec{P} - \vec{r}')^2} \hat{\eta}$,

where $\hat{\eta} = \frac{(\vec{P} - \vec{r}')}{|\vec{P} - \vec{r}'|}$ is the unit vector from a point in the cube to point P . The final integral

expression is given by
$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0} \int_{-a/2}^{a/2} dx' \int_{-a/2}^{a/2} dy' \int_{-a/2}^{a/2} dz' \frac{bz'(-x', 7-y', -z')}{[x'^2 + (7-y')^2 + z'^2]^{3/2}}.$$

There are six main parts to this integral: (1) the correct form of the differential electric field element, (2) expressing the charge density in terms of the dummy variable r' , (3) expressing the integral in terms of r' , (4) evaluating the separation vector $r - r'$, (5) including the correct unit vector, and (6) correct bounds.

Almost no students had a completely correct integral. Many students did not express the integral in terms of the dummy variable r' , which in this case was less important because r was specified to be $(0, 7, 0)$. About half the students either did not include the unit vector, or did not express the unit vector in terms of r and r' . About half the students did not include bounds, or had incorrect bounds. Most worrying, 10% of students did not have the correct overall form of the integral, for instance not including the $1/r^2$ dependence of the electric field.

This question was administered after all relevant instruction and homework, in both lecture and tutorial. The first tutorial students worked through was designed to address difficulties with the separation vector $r - r'$, and how to use it to set up Coulomb's law integrals. Even after targeted instruction, less than half the students knew how to set up the general structure of such an integral, much less include all the details such as correct bounds and integration variables. However, over 90% of students correctly answered all

conceptual pretest questions about Coulomb's law, so clearly this is an issue with math-physics connections. Similar results were published by Wilcox et al⁸, who found that upper-division students have serious issues with coordinating mathematical and physical representations of systems; particularly with expressing the differential charge element, the limits of integration, and the difference vector.

To address the issues of charge distributions and integration, we wrote the junior-level tutorial *Coulomb's Law* to focus more on the mathematical process of integration and less on the physical concepts, since students generally were not struggling with applying the law on the pretest and on lecture homework. The tutorial (COU_Version1 in Appendix F) first has students think about integrating charge density to address some of the conceptual difficulties with integrals in a simpler context (a scalar field) first. Students are then given a differential expression of Coulomb's law for a differential charge element, and practice with integrating a vector field, including the unit vector.

This tutorial is new enough that we have no quantitative data on its effectiveness. However, students tend to complete this tutorial fairly quickly, with few instances of needing TA intervention to interpret a question compared to the other junior-level tutorials. Students also seem to perform better on lecture homework after working through this tutorial than in previous classes; most still have some errors with their integrals, but they tend to be about small details as opposed to errors in the overall form of the integral. This preliminary data suggests that developing curriculum to explicitly address conceptual difficulties in the math-physics connection can increase students'

conceptual understanding of integration, as well as their ability to carry out the process of setting up an integral for a physical quantity.

c. Using symmetry arguments

Exploiting the symmetry of a system to obtain information about physical quantities associated with that system can be an important tool for physicists. However, properly constructing a symmetry argument seems to be a difficult task for most students. This may be because many students do not understand what symmetry means, or because students do not understand the value of using symmetry to find information about physical phenomena, as reported by Pepper et al⁹.

For example, most textbooks and lecturers introduce Gauss's law as a tool to find the electric field of select charge distributions, for which it is much easier to use than Coulomb's law. Gauss's law in integral form states that the surface integral of the electric field over a closed volume is proportional to the net charge enclosed in the volume:

$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$. This equation is usually used to find the electric field of a given charge

distribution, but it is generally not possible to evaluate a closed-form integral of an unknown function. Thus, more information about the electric field is necessary to partially evaluate the integral, so that we may solve for the electric field.

A typical case in which Gauss's law would be used is shown in Figure 6.5. The sphere is rotationally symmetric about any axis through its center, and since the Gaussian surface is centered on the sphere it is also rotationally symmetric about the same axes. We can

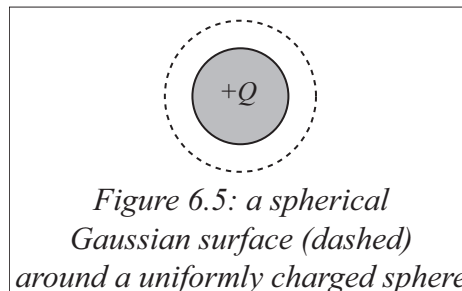


Figure 6.5: a spherical Gaussian surface (dashed) around a uniformly charged sphere

then conclude that the magnitude of the electric field at every point on the Gaussian surface is constant. In addition, because the sphere is also reflectionally symmetric about any plane through its center, we can conclude that the electric field at any point on the Gaussian surface must point radially outward; if it had any $\hat{\phi}$ or $\hat{\theta}$ components they would not be symmetric under reflections. Thus since $d\vec{A}$ always points outward from the surface by convention, $\vec{E} \cdot d\vec{A}$ is constant everywhere on the surface, and the left-hand side of the integral can be simplified to $E(4\pi r^2)$. The electric field can then be solved for in terms of the net charge on the sphere.

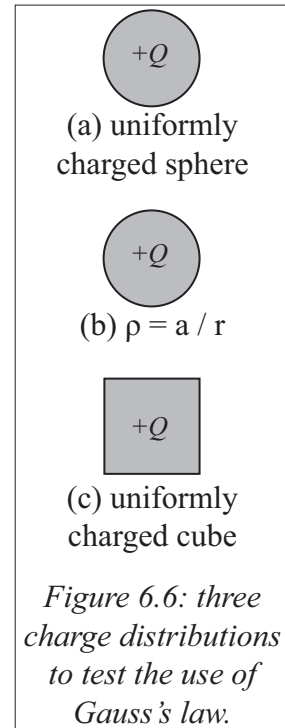
The initial version of the junior-level tutorial *Gauss's Law* (GSL_Version1 in Appendix F) focused on the mathematical aspects of Gauss's law; relating the divergence of E to the charge density, using the divergence theorem to relate the differential and integral forms of Gauss's law, and expressing the differential area $d\vec{A}$ for various Gaussian surfaces. The initial version was motivated by the student difficulties with integration of Coulomb's law discussed above, to help students with the mechanics of integrating Gauss's law. However, post-test data shows that students do not typically struggle with carrying out the

integral, but instead struggle with when Gauss's law can be used to find the electric field, and how to choose a Gaussian surface.

For example, three of the questions given on the junior-level *Gauss's Law* tutorial homework are summarized in Figure 6.6. The questions involved a uniformly charge sphere, a non-uniform but spherically symmetrically charged sphere, and a uniformly charged cube. In each case, students were asked whether or not they could use Gauss's law to find the electric field outside the charged object. The answer is yes for both spheres; since the charge distributions are spherically symmetric, the quantity $\vec{E} \cdot d\vec{A}$ is constant over a spherical Gaussian surface, so the integral can be solved for the electric field. However, since the cube does not possess any

continuous rotational symmetries, no Gaussian surface can be chosen such that $\vec{E} \cdot d\vec{A}$ is a constant, and the integral cannot be solved for the electric field.

Of the 48 students who turned in this homework, all correctly answered that Gauss's law can be used with the uniform sphere. An eighth of the students incorrectly stated that Gauss's law could not be used with the non-uniform sphere because it was not symmetric. Half the students incorrectly stated that Gauss's law could be used with the cube, most of whom merely stated that the Gaussian surface had to be the same shape as the charge distribution. These students seem to be using a memorized rule instead of thinking about what symmetry tells them about the electric field, and whether or not that knowledge is



sufficient to solve the integral. Many students who stated that a cubical Gaussian surface could allow them to find the electric field included statements such as:

“You can use Gauss’s law for any enclosed surface. A cube is very symmetrical and has a uniform charge which makes this a great shape for Gauss’s law.”

“[Gauss’s law] works with any closed surface and the flux is easily calculated which is Q/ϵ_0 . From here the surface area can be found and thus E can easily be found.”

These students seem to be over-generalizing the utility of Gauss’s law. While Gauss’s law is always *true*, it is not always *useful*; it can only be used to find the electric field in situations with extremely high symmetry.

The second version of the *Gauss’s Law* tutorial (GSL_Version2 in Appendix F) was designed to address the problems students had in understanding and applying symmetry to Gauss’s law. The first page focused on the symmetries of charge distributions, and what those symmetries imply about the electric field. The second page applied that knowledge to choose a Gaussian surface such that $\vec{E} \cdot d\vec{A}$ is easy to evaluate, and thus allowing the integral to be solved for the electric field.

Students worked through the first page fairly quickly, but still significantly struggled with applying symmetry to the field, and to finding an appropriate Gaussian surface. The question about the charged cube (Figure 6.6 c) was included in the tutorial as part of a TA checkout; many students incorrectly answered the question, and often it took a significant amount of TA time to lead students to see why Gauss’s law could not be used with the cube.

The homework associated with this version was the same as that for the first version. Given that symmetry had been emphasized and that every group discussed the charged cube with the TAs, we expected the sequence of three questions described in Figure 6.6 would be much easier for students. The fraction of students who correctly used symmetry arguments increased compared to after the previous version. In addition, fewer students stated that Gauss's law could be used with the charged cube compared to after the previous version. However, more than a quarter of students still believed a cubical Gaussian surface could be used to find the electric field of the cube, and misapplied symmetry arguments to support their answers.

Clearly symmetry arguments are difficult for students, and even a tutorial specifically aimed at understanding symmetry and constructing symmetry arguments is not sufficient to address all students' difficulties with them. Symmetry is a broadly-applicable idea, not tied to specific physical concepts, so we are in the process of incorporating small sections on symmetry arguments in many different tutorials, homework, and exams, since a single instance of instruction is insufficient to address all students' difficulties with them. This will be explored in upcoming publications, as no data has yet been collected about the effectiveness of this broad approach.

d. Vector derivatives

One of the most important new pieces of mathematics that junior-level students are expected to understand is the set of vector derivatives: gradient, divergence, and curl. Introductory students typically see E&M laws in their integral forms (for instance, Gauss's

law, Coulomb's law, and Faraday's law), while junior level students are expected to be able to relate the integral forms of these laws to their differential form, all of which involve vector derivatives.

However, our observations suggest that students tend to understand these derivatives in terms of a procedural formalism, and do not typically understand them on a conceptual basis. This tends to lead to logical inconsistencies in students' answers between their conceptual understanding and their mathematical answers.

On the *Gauss's Law* homework, students were asked to find the divergence of the electric field of a line charge, and relate it to the integral form of Gauss's law. A correct student response is given in Figure 6.7. The divergence of the electric field is zero everywhere except at $s = 0$, so the divergence is proportional to a delta function at $s = 0$.

1. The electric field of an infinite line charge that runs along the z-axis is equal to $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s} \hat{s}$ in cylindrical coordinates (this should be the electric field you found in the tutorial).

a. Find the divergence of the E-field for $s \neq 0$. *Hint:* Divergence in cylindrical coordinates is given by: $\vec{\nabla} \cdot \vec{V} = \frac{1}{s} \frac{\partial}{\partial s}(sV_s) + \frac{1}{s} \frac{\partial}{\partial \phi} V_\phi + \frac{\partial}{\partial z} V_z$

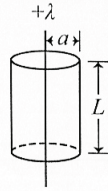
$$\vec{\nabla} \cdot \vec{V} = \frac{1}{s} \frac{\partial}{\partial s} \left(\frac{s \lambda}{2\pi\epsilon_0 s} \right)$$

$$= 0$$

E doesn't have ϕ & z components

b. Calculate the electric flux out of the imaginary Gaussian cylinder shown, which has a radius a and a length L .

$$\Phi = \int \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}$$

$$= \frac{\lambda L}{\epsilon_0}$$


c. Given parts a and b, where is the divergence of this \vec{E} field? *Hint:* What corresponds to a source of \vec{E} field?

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

use Dirac delta.

$$= \frac{\lambda \delta^2(s)}{\epsilon_0}$$

$$\rho = \frac{\lambda \delta^2(s)}{\frac{s^2}{r}}$$

$$\int_s \vec{E} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{E}) d\tau$$

Figure 6.7: correct student response to a question on the upper-division Gauss's law homework

Nearly every student ($N = 82$) correctly calculated that the divergence was zero for $s > 0$, but nearly half the students did not find the correct divergence in the last question. Most students stated that the divergence was a constant, not including the delta function, and almost a quarter of students had no idea how to do that part of the problem and left it blank.

Similar difficulties arise when students consider the curl of a vector field, typically when considering Ampere's law or the vector potential, which is discussed in chapter 8. We have found that essentially all students can compute vector derivatives mathematically, usually as part of a lecture homework question, but many students cannot answer conceptual questions about vector derivatives.

To help students connect the procedural and conceptual aspects of vector derivatives, we developed physical analogies for each derivative. Students are introduced to gradients in terms of slope and height on a topographic map; divergence is treated in terms of water flow, with sources and sinks; and curl is introduced with a small pinwheel, such that the pinwheel rotates at points where the vector field has curl, and does not when the curl is zero. One page on divergence was added to the *Coulomb's Law* tutorial, one page on divergence was added to the *Gauss's Law* tutorial, and one page on curl was added to the *Potential* tutorial. Full versions of each can be found in Appendix F; COU_Version2, GSL_Version2, and POT_Version2 respectively.

As before, we have very little quantitative post-test data on the effect of this effort to address student understanding of vector derivatives. However, after the change over three

quarters of students found the correct divergence on the *Gauss's Law* homework question shown in Figure 6.7, compared to about half of students before the page on divergence was added to the tutorial. Students who have worked through these additional pages seem to have less difficulty with later tutorials that rely on a conceptual understanding of vector derivatives.

C. Conclusions and implications for junior-level instruction

When we began developing curriculum for the junior level, we assumed that upper-division students are more capable than introductory students. Upper-division students work with more complex mathematics, have seen physical concepts at a deeper level than introductory students, and are all physics majors. However they face exactly the same set of barriers to learning physics that introductory students do: they do not have some of the concept-specific difficulties that introductory students have, but as they are exposed to more advanced physics concepts they also have more advanced difficulties. We have seen that both introductory and upper-division students struggle to understand mathematics at a conceptual level, and both have issues connecting physics ideas to mathematical descriptions.

Over the course of developing tutorial curriculum for the junior-level E&M courses, we have observed similarities in terms of overall ability as well. The initial tutorial versions we developed assumed that junior-level students could cover more topics in a 50-minute tutorial section, with larger step size between questions, than introductory students could. We have found that this is not the case; the upper-division tutorials have

evolved to cover less material than they did initially, with step sizes comparable to that in introductory tutorials. The upper-division tutorials start with more complex ideas than the introductory tutorials do, but are otherwise very similar.

The only significant difference between the two sets of E&M tutorials is that since most junior-level students seem to have learned the basic physics concepts in the introductory course, the advanced tutorials focus more on the mathematical concepts and the math-physics connections.

D. References for chapter 6

- [1] F. R. Yeatts and J. R. Hundhausen, "Calculus and physics: challenges at the interface," *Am. J. Phys.* 60, p. 716 (1992)
- [2] C. Baily, M. Dubson and S. J. Pollock, "Developing tutorials for advanced physics students: processes and lessons learned," PERC Proceedings 2013
- [3] T. A. Grundmeier, J. Hansen, and E. Sousa, "An exploration of definition and procedural fluency in integral calculus," *PRIMUS* 16, p. 178 (2006)
- [4] D. H. Nguyen and N. S. Rebello, "Students' difficulties with integration in electricity," *PRST-PER* 7, p. 010113 (2011)
- [5] S. E. Kanim, "An investigation of student difficulties in qualitative and quantitative problem solving: examples from electric circuits and electrostatics," PhD thesis, University of Washington (1999)
- [6] B. Thacker, E. Kim, K Trefz, and S. Lea, "Comparing problem solving performance of physics students in inquiry-based and traditional introductory physics courses," *Am. J. Phys.* 62, p. 627 (1991)
- [7] E. Kuo, "More than just 'plug-and-chug': exploring how physics students make sense with equations," PhD thesis, University of Maryland (2013)
- [8] B. R. Wilcox, M. D. Caballero, R. E. Pepper, and S. J. Pollock, "Upper-division student understanding of Coulomb's law: difficulties with continuous charge distributions," PERC Proceedings 2012
- [9] R. E. Pepper, S. V. Chasteen, S. J. Pollock, and K. K. Perkins, "Our best juniors still struggle with Gauss's law: characterizing their difficulties," PERC Proceedings 2010

CHAPTER 7: REFINING A TUTORIAL ON AMPÈRE'S LAW

“The dot product of I and B are different for both by an angle of 45 degrees. This will make the [rotated loop] smaller than the horizontal loop by $\sqrt{2}$.”
– Introductory pretest response about the tilted loops in Figure 7.8

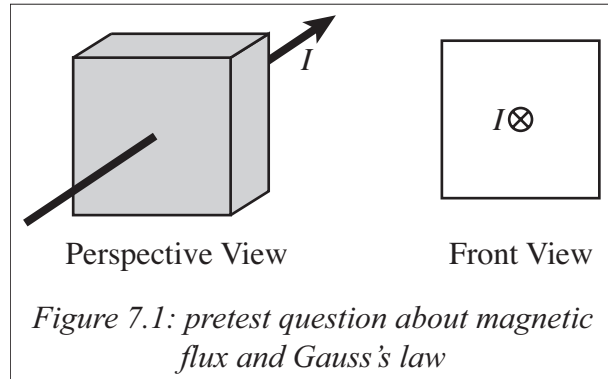
Paul van Kampen (Dublin City College) wrote an introductory-level tutorial that covered Gauss's law for magnetic fields and Ampère's law, and in early 2011 asked us to test it in the introductory class at the University of Washington. The latter part of his tutorial was heavily based on the curricular sequence in the *Gauss's Law* tutorial developed at UW.

The tutorial was extremely dense, with very little scaffolding or helper questions. We found that our students had significant difficulties with the material, so we decided to adapt and refine the tutorial to address those issues. Section A provides a brief overview of the tutorial and each category of difficulties students encountered working through the tutorial, and the subsequent sections describe the curricular changes we made to address each category.

Unlike the *Electric Potential Difference* and *Electric Properties of Conductors* tutorials, this tutorial did not significantly change form over the course of its development at UW. Instead, the changes version-to-version tended to be refinements in wording, question order, and question content. Thus the sections describing our modifications are grouped in terms of addressing each area of conceptual difficulties, instead of by version. Including van Kampen's initial version there are three distinct versions, which are shown in full in Appendix C.

A. Initial version of the Ampère's law tutorial

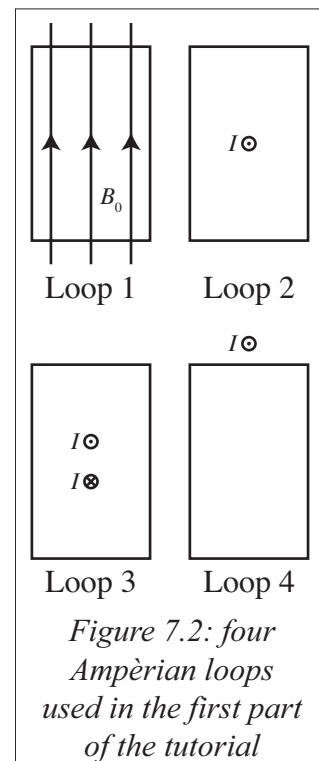
The version we received from van Kampen is shown as AMP_Version0 in Appendix C. The first page has students apply Gauss's law for magnetism to the cubical surface in Figure 7.1. The question asks about the flux through each face and the



net flux through the cube. The correct answer is that the flux through the front and back faces are zero since the magnetic field is parallel to the surface, and the flux through the other four faces is also zero since the wire is symmetrically placed so each side has an equal amount of positive and negative flux. The net flux is zero.

The surface is then lowered so the line current is off-center, and students are again asked about the flux through the right and top face; in this case the flux through the top face is still zero due to symmetry, but the flux through the right face is now non-zero. However the flux through the left face is equal and opposite to the flux through the right face, again by symmetry, so the net flux is still zero. Students are then expected to synthesize their answers to conclude that Gauss's law for magnetism cannot give them useful information about the magnetic field.

The second and third pages deal with Ampère's law. The second page asks students to determine the sign of the line integral around each of the four loops shown in Figure 7.2, and the third page gives a statement of Ampère's law and asks students to apply it to a few simple examples. For loop 1, the line integrals along the top and bottom are zero and equal but opposite on the left and right sides, so the loop integral is zero. For loop 2, the path element and magnetic field have the same sign of dot product on all four sides, so the loop integral is positive (taken counterclockwise) or negative (taken clockwise). The loop integral for loop 3 is zero since by symmetry each point on the loop with a positive dot product is balanced by a point with the opposite dot product. Loop 4 is impossible to determine without using Ampère's law.



We found this initial version very hard for nearly all students. As mentioned above, the tutorial is very dense, with almost no scaffolding to help students answer the harder questions. The tutorial also assumes that students already understand line integrals, and know how to use them; however as described in Chapter 6, many introductory students have difficulty conceptually understanding scalar integrals, much less vector integrals.

The fact that this version begins with Gauss's law and flux integrals further exacerbates this issue. We have seen with the tutorial *Gauss's Law* that many students tend to think about flux in terms of field lines; for instance if a charge is outside a Gaussian surface, students often reason that the net flux is zero since every field line that enters the

surface also leaves the surface. This is correct reasoning for a flux integral, but many students tend to use the same reasoning for line integrals, i.e. loop 2 in Figure 7.2. Many students incorrectly extend the fact that the net magnetic flux through any Gaussian surface is zero to conclude that the line integral around any Ampèrian loop is also zero.

Another issue students face is the vector nature of the integral. Gauss's law also involves a vector integral, but by convention the area vector is chosen to point out of the enclosed surface. In Ampère's law there are two choices of area vector for any loop, and two directions that the path element $d\vec{l}$ can point, each of which are equally valid. The two choices cannot be made independently or else the signs of the left-hand and right-hand sides of Ampère's law would be inconsistent. This is a point that most students miss; the initial version states that the direction of circulation is taken to be consistent with the right-hand rule, but many students do not understand why this is so, or how to apply it to choose the direction of the line integral.

A third issue is that many students interpret Ampère's law as a mathematical definition, instead of a physical relation. That is, nearly every student can find the net current crossing a loop and equate that to the line integral around the loop, but many students do not recognize the inverse statement: if the line integral around a loop is non-zero, there must be net current crossing the loop.

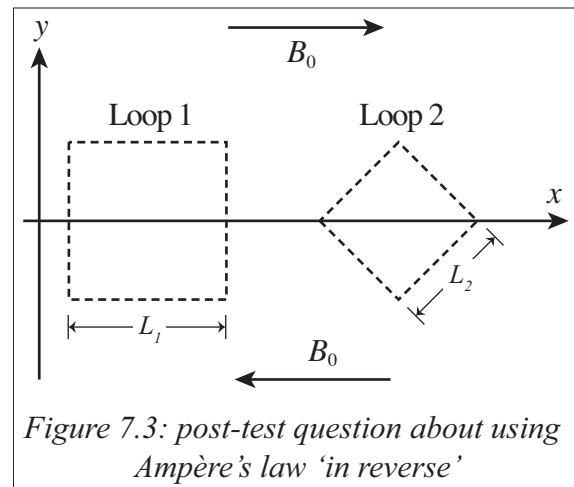
In the next sections I will discuss specific issues that our introductory students struggled with. Each section has three sub-sections: (1) discussion of an observed difficulty or set of related difficulties; (2) a description of the curricular changes we made to address

these difficulties; and (3) post-tutorial analysis to determine effectiveness of the curricular changes.

B. Relating line integrals to the presence of current

1. Student difficulties

Many students struggle with using Ampère's law "in reverse" – that is, most students are comfortable relating the net current crossing through a loop to the line integral, but have difficulty when given a magnetic field and asked to find the current. Figure 7.3 shows an example of a post-test



question on this topic. The magnetic field uniformly points right above the x-axis, and uniformly points left below the x-axis. The question first asks students to find the line integral around loop 1, and then to find the magnitude and direction of the net current through it. One way to answer is to observe that the line integral taken clockwise is positive, so the current is in the same direction as the area vector. A clockwise line integral means an area vector into the page, so the current through the loop is into the page.

This exam (AMP_124 in Appendix B) was administered to 197 students, two weeks after students had worked through version 2 of the tutorial and version 1 of the homework. This exam tested the transfer of Ampère's law ideas into other contexts; the tutorial and homework versions the students had seen focused on using Ampère's law to find the

magnetic field from a given current distribution, and although this exam required students to use similar reasoning, it was in an unfamiliar context. Transfer problems have been shown to be challenging in a wide variety of contexts, and this exam was similarly difficult.

	$2B_0L$	Zero	Other
$\oint \vec{B} \cdot d\vec{l} :$	70%	26%	4%

	Correct magnitude			Zero
	Into the page	Clockwise	No direction	
Encircled current:	18%	14%	24%	44%

Table 7.1: Responses to two questions about loop 1 in Figure 7.3: the line integral around the loop, and the encircled current. Correct answers are bolded.

Most students correctly found the line integral around the loop. A quarter of students believed that the line integral was zero, most of whom stated that since the magnetic field switched directions above and below the x-axis, that the line integral contributions would be opposite. 10% of students stated that since no current was shown on the diagram, that no current passed through the loop and thus the line integral was zero.

However, very few students could correctly relate the line integral to the direction of current through the loop. Overall, 56% found the correct magnitude of the current, but only a third of these found that it pointed into the page. Many of the 24% who found the correct magnitude but did not specify a direction stated that the direction could be found by the right-hand-rule, but did not know how to do so.

The disparity between the students who found a zero line integral and those who found zero encircled current is particularly interesting. All of the students who stated that the line integral was zero also stated that there was no encircled current, consistent with

Ampère's law. A further 18% found a non-zero line integral but still stated that the current was zero. Most of these students fell into three categories, represented by the three statements below:

“There is no current since there is no change in flux so no EMF.”

“There is no current passing through the loop, because there is no velocity and therefore no magnetic force on the charges in the loop.”

“The current is zero, because \vec{B} and \vec{A} are perpendicular and

$$\oint \vec{B} \cdot d\vec{A} = \mu_0 I_{enc}.”$$

Lenz's law accounted for slightly more than half these explanations (e.g., the first statement), which is likely due to the fact that Lenz's law and motional EMF were covered immediately before the exam, and also that this question is very similar to the type of questions asked about Lenz's law. A few students tried to use the magnetic force (e.g., the second statement), and nearly half used some sort of flux argument without mentioning Lenz's law or changing flux. These students seem to be conflating the line integral with magnetic flux, which will be discussed further in section D.

2. Modifying curriculum to address difficulties

To address this idea, we added a homework question based on Figure 7.4 to AMP_HW_Version1, where an unknown current distribution creates a spatially non-uniform magnetic field (henceforth referred to as the non-uniform field problem). Students are told that that the field in region I increases in magnitude towards the bottom of the page, the field in region II is uniform, and the field in region III is zero. Students are asked to choose

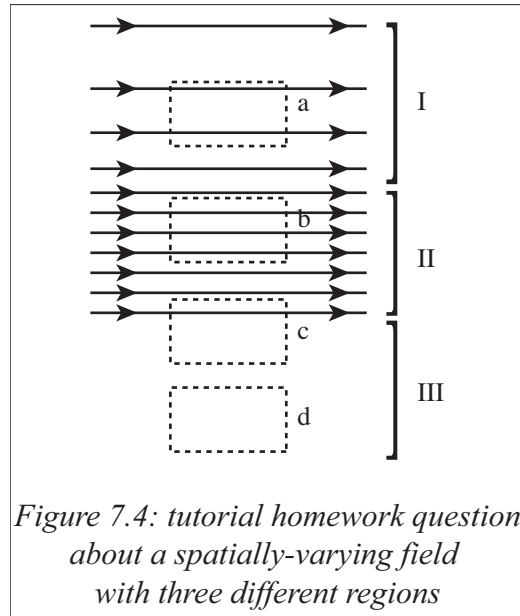


Figure 7.4: tutorial homework question about a spatially-varying field with three different regions

an area vector and find the sign of the line integral, and then explicitly told to use Ampère's law to determine the direction of current flowing through each loop.

3. Evaluating curriculum modifications

The non-uniform field problem on version 1 of the homework was very difficult for students because it involved multiple different conceptual difficulties. These difficulties, and further modifications to this question, are described in the next three sections. Student performance after version 1 was fairly low due to the effect of these different conceptual difficulties, and it was not possible to accurately differentiate between students who struggled with relating line integrals to current versus the other difficulties about the line integral. However, very few students had inconsistent answers about the line integral and the current, and including the statement that the field was created by an unknown current

distribution significantly decreased the number of students who stated that all line integrals were zero because no current was shown.

Version 2 of the homework incorporates several modifications to address the difficulties with line integrals, described below. Altogether, 85% of students correctly answer this question and almost no students are inconsistent in their answers or use alternate explanations such as Lenz's law. Thus, prompting students to use Ampère's law in the problem statement appears to successfully address these difficulties.

C. Relating $d\vec{l}$ and \vec{A}

1. Student difficulties

There are two choices for the direction to integrate around any line integral, just as there are two choices for the area vector describing the surface bounded by the line integral. These choices are not independent, since for Ampère's law to be true the sign of each side must be consistent. However, there is still an ambiguity left in the choices of $d\vec{l}$ and \vec{A} , which is resolved by the right-hand rule. AMP_Version1 merely stated that the two vectors were related by the right-hand-rule, without motivation or explanation. Very few students understood why the vectors were related, much less how to relate them when using Ampère's law.

This is not usually an issue when students are given the current and are asked to find the magnetic field; in this case students typically use Ampère's law only to find the magnitude of the magnetic field, and to determine the direction by using the right-hand-rule between current and magnetic field. However, this is important when students are

given an inverted problem, where they know the magnetic field and are asked to find the associated current distribution. In this case, particularly if the field is not trivial, students need to choose a direction in which to integrate and then relate the sign of the line integral and the direction of the area vector to determine the direction of current.

2. *Modifying curriculum to address difficulties*

In AMP_Version1 we added a table to have students compare the effects of each choice. They are shown Figure 7.5, and asked to find the sign of the encircled current with the area vector either into or out of the page, and the sign of the line

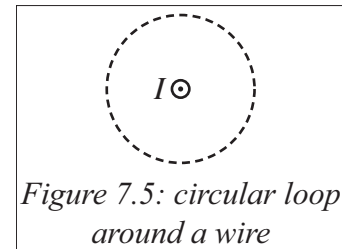


Figure 7.5: circular loop around a wire

integral with the path either clockwise or counterclockwise. Finally, they are asked to find a right-hand-rule that relates the choices, so that the signs of each side of Ampère's law are consistent.

This set of questions was still somewhat abstract to students, as this was presented after Ampère's law was provided, and many students didn't seem to understand that the sign of the line integral and the direction of the area vector both affect Ampère's law. Thus in AMP_Version2 we had students complete a similar table about the sign of the line integral and encircled current for each case in Figure 7.2, and then explicitly asked about how the choices of area vector or path element affect the signs of each side of Ampère's law. Last, they are asked about how to relate the two choices to make the signs of each side consistent.

This idea is also reinforced by the non-uniform field problem, which was added to the second version of the homework. This question proved fairly challenging to students,

so in AMP_HW_Version2 we added a helper question using Figure 7.5, which includes a student dialogue about the sign of the line integral around the loop. The two hypothetical students disagree about the sign of the line integral because they chose different directions of the area vector or line integral, and the question asks whether or not both students could be correct. Students see this question immediately before the non-uniform field problem, so that they are reminded about the relation between the area vector and the line integral before approaching the next question.

We also added an intermediate step to the non-uniform field problem. After students are asked to choose an area vector, they are asked which direction to integrate around the loops given their choice of area vector. This is also intended to prompt recall of the right-hand-rule between area and line integral, before asking them about the sign of the line integral.

3. Evaluating curriculum modifications

As mentioned in the previous section, the table in AMP_Version1 was somewhat confusing to students. About one-quarter of students filled out the table with the correct signs, but did not know where to go from there. They did not seem to understand that the sign of each side of the line integral was determined by their choices, and that the signs of each side should be consistent. This set could generally have their confusion addressed during TA checkouts, but often required a fairly long intervention.

A similar fraction of students had difficulty with the non-uniform field problem. Of 164 students who saw the second version of the homework, 30% found a sign for the line integral around loop (a) that was inconsistent with the direction of area vector they had

chosen for the loop. Even among students that arrived at the correct answer, very few explicitly mentioned the right-hand-rule. Some students instead stated that if the area vector is out of the page then the line integral is counterclockwise. This is a true statement, but it does not offer any evidence that students understand the relation, as opposed to having simply memorized a rule.

The changes to the tutorial and homework in version 2 effectively addressed this difficulty. In tutorial, adding questions about how the choice of area vector or path direction affected either side of Ampère's law helped students see that the sign of each side was a consequence of their choice, and thus the fact that each side had to be consistent did not cause most groups any difficulty. On the homework, essentially all students correctly related the direction of integration to their choice of area vector.

D. Conflating line integrals with flux

1. Student difficulties

In informal observations made during tutorial sessions with the initial version, as many as three-quarters of student groups incorrectly believed that the line integral along loop 2 in Figure 7.2 is zero. These students typically needed a fairly involved TA interaction to see that the line integral is not zero; typically the TA needed to ask the group to consider the sign of the dot product between \vec{B} and $d\vec{l}$ at several points around the loop before students grasped that the dot product does not switch sign when a field line crosses out of or into the area bounded by the loop.

On the initial version of the homework, students were asked to find the line integral from Y to X along the flat side of the loop in Figure 7.6; in later questions, the curved side of the loop is extended to the right until it encircles the wire. One-third of students stated that the line integral is zero; this is a much smaller fraction than

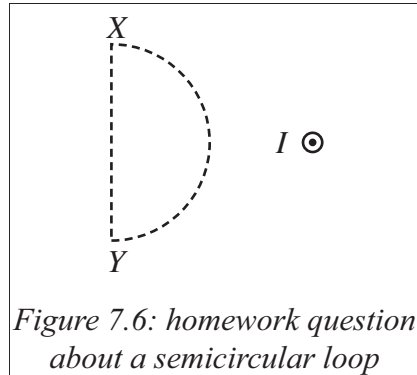


Figure 7.6: homework question about a semicircular loop

during tutorial, but a sizeable number of students still struggle with this idea after tutorial instruction. Some of these students did not provide any explanation of their homework answers, but the majority used language consistent with flux, e.g. “field lines in equals field lines out,” or “no net field crosses through the loop.”

We suspected that a partial explanation for the wide-spread prevalence of this conflation between flux and line-integrals was due to including Gauss's law for magnetic fields on the first page of the tutorial. Also, given that we had not observed many students struggling with understanding Gauss's law in the introductory labs, it might not be necessary to include coverage of Gauss's law at all. Students are introduced to Gauss's law in lecture first, followed by lab (typically the next week), and then the Ampère's law tutorial (typically the week after that). Thus, if we could demonstrate that most students did not struggle with Gauss's law pre-tutorial, we could remove coverage of this idea from the tutorial.

To test this, we duplicated the Gauss's law treatment on the AMP_113 pretest. Students are shown Figure 7.1, and are asked about the flux through various sides of the cube. The current flowing into the front face confused some students, who thought that

this meant the flux was positive, and only half the students correctly answered that the flux through the front face was zero. Three quarters of students correctly answered all other questions with correct reasoning, however, indicating that most students do not have significant difficulty with magnetic flux.

2. Modifying curriculum to address difficulties

Since we believed that including Gauss's law for magnetic fields led to significant confusion with Ampère's law, we removed the first page on Gauss's law in AMP_Version1. We replaced this section with a brief overview of line integrals, using Figure 7.7 as an example: students find the line integral along each side in turn, and

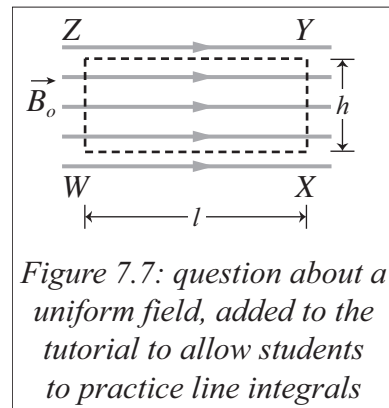


Figure 7.7: question about a uniform field, added to the tutorial to allow students to practice line integrals

then the line integral around the entire loop. This was added because integral calculus is a co-requisite of Physics 122, not a pre-requisite, so many students have not been exposed to line integrals.

This addition decreased, but did not eliminate, the prevalence of this conflation. We then added an incorrect student statement in AMP_Version2:

“When field lines go out of the loop the dot product is positive, and when they go in the dot product is negative. Every field line that leaves the loop also re-enters the loop, so the line integral around the loop is zero.”

Students are told that the statement is incorrect, and asked to explain why. This statement cues students to think about what quantity is dotted with the magnetic field.

3. *Evaluating curriculum modifications*

Adding a section at the beginning of the tutorial about reviewing line integrals had a significant impact, as the number of students who conflated line integrals and flux in tutorial section dramatically dropped. However, between 20-25% of students who saw version 1 of the tutorial incorrectly believed that the line integral for a closed loop was zero even if non-zero net current crossed through the loop, and needed TA input to realize their mistake.

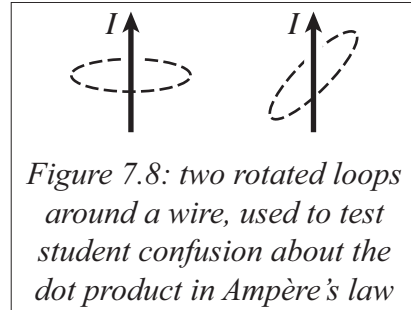
This conflation also appears on the non-uniform field problem. As mentioned in previous sections this question was difficult for students as it invoked multiple different conceptual difficulties. In the quarters where version 1 of the homework was administered, between 20-30% of students used flux reasoning to conclude that the line integral around all four loops was zero.

After adding the incorrect statement to AMP_Version2, the conflation essentially disappeared in tutorial sections, as well as homework; less than 5% of students incorrectly used flux reasoning for any loop in the homework.

E. Inappropriate use of a dot product

1. Student difficulties

There are two distinct ways in which students inappropriately use a dot product in Ampère's law. For the first, students tend to think that the current encircled involves a dot product. For example, Figure 7.8 shows a pretest question about two Ampèrian loops around two



identical wires, where the right loop is tilted 45° relative to the left loop. Students are asked about the line integral around each loop; the correct answer is that the loops both encircle the same amount of current, so the line integrals are equal. When this was asked on the AMP_121 pretest (Appendix A, N=2800), 63% believed that the line integral for the right loop was different than the left loop, and 36% stated that the line integrals were equal. These students typically state that the line integrals around each loop differ by a factor of $\sqrt{2}$, consistent with taking a dot product between the direction of current and the area vector.

The second case involves a mistake in relating the line integral to the direction of current. We have only seen this difficulty manifest in contexts where the magnetic field is given and the current is unknown, as in the non-uniform field problem. If the clockwise line integral is positive, some students state that current flows clockwise around the loop. These students could either be simply mapping the direction of the magnetic field onto the direction of current, or trying to relate the sign of the current to the relative directions of

current and the line element. We have not seen any evidence for the former misconception, however, so the second explanation is more likely.

2. *Modifying curriculum to address difficulties*

To address the first case, we added a student discussion about Figure 7.8 to AMP_Version1. Student 1 states that the line integral is less in the tilted loop because the current crosses the loop at an angle; student 2 states that the line integral is greater in the tilted loop because portions of the loop are closer to the wire than for the horizontal loop and the magnetic field is greater close to the wire; and student 3 correctly applies Ampère's law to conclude that the line integrals are equal. Students are asked which statement they agree with, and to explain the errors in the other statements. In AMP_Version2 this question was moved to the homework, to make room for questions aimed at addressing the second difficulty with dot products.

When this discussion was moved to the homework, a different student discussion was added to the tutorial to replace it. Three students know that the clockwise line integral around a loop is $2\mu_0 I$, and discuss what this implies about the current through the loop:

“The line integral is positive, so current is in the same direction as the area vector. There must be two wires crossing through the loop, each carrying a current I into the page.”

“I disagree. I chose the area vector out of the page, so current is flowing out of the page.”

“The line integral clockwise around the loop is positive, so current should be flowing clockwise around the loop.”

Students are told that all three statements are incorrect, and asked to explain why. The first student has the correct direction of current, but incorrectly assumes that there must be two wires. The second student does not correctly relate the line integral direction with the area vector. The third student conflates the direction of integration with the direction of current. All three statements are adapted from student responses to the non-uniform field problem in version 1 of the homework.

3. *Evaluating curriculum modifications*

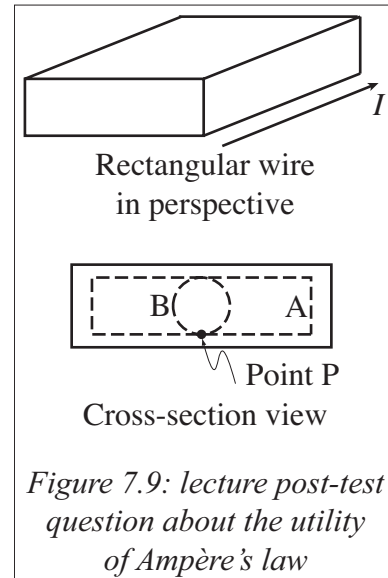
In tutorial, students sometimes struggle with identifying the incorrect piece of the first two statements, but not typically the third. We only have data from one quarter for version 2 of the homework, but overall no students out of 90 stated that the current flowed around the loop.

F. Utility of Ampère's law

Many students struggle to understand when Ampère's law can be used to find the magnetic field of some current distribution. A physics expert would say that Ampère's law is always *true*, but it is not always *useful* in that it cannot always be used to find the magnetic field. The magnetic field appears in Ampère's law inside an integral and a dot product, and thus to be able to extract useful information about the magnetic field, some information must already be known about the magnetic field. For instance, the magnetic field of a wire could be found by considering a circular loop centered on the wire. Since a long straight wire has rotational and reflectional symmetries, the magnetic field must be purely azimuthal, so the dot product is one. Every point on the loop is the same distance

from the wire, so by symmetry arguments the magnitude of the magnetic field is constant on the loop. This combination of arguments means that the magnetic field can be treated as a constant in this integral, pulled out of the integral, and then solved for in terms of the current in the wire.

Physics novices on the other hand, such as introductory physics students, typically do not make this distinction between truth and usefulness. For example, one of the physics faculty teaching the 122 course one quarter placed Figure 7.9 on the final exam as a multiple-choice question. Two possible loops are shown inside a rectangular wire carrying uniform current density. The question is

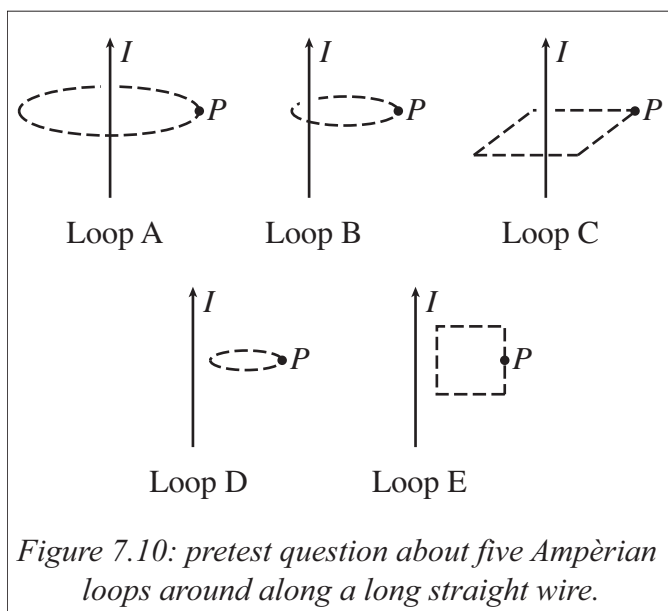


whether either, both, or neither of the loops can be used to find the magnetic field at point P. The correct answer is that since the rectangular wire does not possess continuous rotational symmetry, no loop can be found such that the magnitude of the magnetic field is constant and the dot product with the path element is also constant. Thus neither loop can be used to find the magnetic field at point P.

The students that quarter did very poorly on this exam, much to the surprise of the instructor. The instructor asked us why so many students answered incorrectly, particularly since the instructor had discussed in lecture similar problems where Ampère's law could not be used. We put this question on the AMP_151 pretest, and asked students to explain their reasoning.

Of 174 students who took the pretest, most chose both loops. Only 20% stated that neither loop could be used. However, the vast majority of the explanations stated that the loops could not be used because they did not enclose all of the current in the wire. Only 3% (6 students) stated that Ampère's law required symmetry, so neither loop would work.

To further explore this idea we also added a new pretest question based on Figure 7.10. Students were asked whether or not they could use each loop to find the magnetic field at point P . The correct answer is loop A only, as described above. However, though loop E cannot be used to find the field, it does offer useful information about the vertical components of the field.



Over 300 students took a version of the pretest that contained this question. Only 7% of students correctly answered that loop A alone would allow them to find the magnetic field. The most common incorrect pattern (60%) was yes to loops A-C, and no to loops D and E; these students seem to think that as long as the loop encloses the wire, that it can be used to find the field. This is supported by their explanations:

“For Ampère's law, we need to have the current going through the enclosed surface chosen.”

“Current must be enclosed in the shape, no matter the shape, in order for us to use Ampère's law. The last two do not work because there is no current enclosed.”

Even among the 7% that found the correct answer, very few students could correctly articulate the symmetry arguments needed to prove that Ampère's law would work with this loop. Some students simply stated that the loop was symmetric, without further explanation. Many students recalled that this shape was what had been used in lecture or in the textbook, and did not explain why the other loops were incorrect other than that they did not match what the students had been shown.

G. Discussion

These results imply that even with an intervention, Ampère's law remains an extremely difficult tool for students to understand. The first four conceptual difficulties, relating to the mechanics of using the tool, can apparently be successfully addressed through curriculum development, as we have shown. However, helping students *use* the tool can never be very successful if students do not *understand* the tool or how to apply it. Though not discussed here, the same argument holds for Gauss's law as well.

Another important related point to this is that at the introductory level, the integral form of Ampère's law does not teach students new physics, as students already know how to find the magnetic field by using the Biot-Savart law. It is a very elegant tool to find the magnetic field, but it must be used in physical situations with very high symmetry. In these situations it offers an alternative to brute-force integration of the Biot-Savart law, but it is

not an inherently new physical concept in its own right. Thus the question must be raised: why teach either Gauss's or Ampère's laws at the introductory level at all?

At the junior level, Ampère's law is fundamental to understanding electromagnetism, especially in differential form, as it constitutes a quarter of Maxwell's equations. As we will discuss in chapter 8, junior-level students have more tools at hand to understand and use Ampère's law, though even they can significantly struggle with it. One possible reason to teach the integral form of both laws at the introductory level is for physics majors who will see it in their advanced courses; for these students, being exposed to the tools early could in principle help students master the tools in later courses. However, in a broad introductory service course such as the one at the University of Washington where physics majors constitute roughly 5% of the class, this argument loses much of its weight.

Another possible argument is that often the introductory physics course is a required survey course not because other majors need to understand basic physics, but because there is an expectation that a physics course teaches students to think critically and lean problem-solving skills. By this argument, physics *content* is not very important for non-majors, as the emphasis is on critical reasoning proficiency. However, our results suggest that Gauss's and Ampère's laws are contexts that the majority of introductory students struggle to understand, and it is questionable how much critical reasoning students can use about concepts they do not grasp. If physics instructors primarily cared about teaching students to reason, it would be logical to do so with a concept that is not so complex and poorly-understood to most students.

Thus, if it is necessary to include coverage of Ampère's law in a broad introductory physics course, this research shows that curriculum can be designed to help students with the main sets of conceptual difficulties associated with using this tool. However, the fact that so many students struggle to understand it, even at the junior level, suggests that it would be better to leave Ampère's law to either physics-major specific introductory courses, or to junior-level courses.

CHAPTER 8: COMPARING INTRODUCTORY AND JUNIOR-LEVEL CONCEPTUAL DIFFICULTIES WITH AMPÈRE'S LAW

“Loops that have no enclosed current will not have magnetic field. If a loop does enclose current we can learn about the magnetic field.”
– Junior-level pretest response

As described in chapter 6, we have found that physics students face challenges with physics concepts, mathematical concepts, and with connecting mathematical language to physical situations. In our experience, introductory students tend to struggle more with the physics concepts, probably because they are encountering these ideas for the first time. Junior-level students tend not to struggle with physical concepts as much as introductory students, perhaps because they have already been exposed to them, or because there is a significant selection bias between a large introductory service course and a junior-level course for physics majors. In either case, the junior-level class treats these concepts in greater mathematical depth and complexity than in the introductory class, so the struggle that advanced students have with mathematical concepts and with connecting math and physics is more evident.

These trends hold true in the context of Ampère's law as well. Other than recognizing when Ampère's law will and will not be useful (described in section F of chapter 7), we have not found any evidence that junior-level students exhibit the difficulties described in the chapter 7 about the introductory course. Instead, nearly all of the difficulties we find in the junior-level class have to do with connecting math to physics. This is supported by the ideas of Manogue et al¹, who state that students need “a

foundation for their learning in physics and their future ability to solve new problems. This foundation consists not so much of learning how to solve new kinds of problems as of connecting the knowledge they already have into a coherent understanding of what it means to solve problems.” Similarly, Wallace and Chasteen² categorized all the difficulties with Ampère’s law that they observed with upper-division students into two groups: either related to not thinking of the integral as a sum, or not using information about the magnetic field to solve the line integral.

The data in this chapter comes from pretests, in-class observations, and post-tests of the junior-level class at the University of Washington. The class sizes are much smaller than the introductory class, typically between 25 and 40 in the summer, and between 90 and 100 in the winter; this is about a tenth the size of the introductory class.

The research described in this chapter serves primarily to describe the types of difficulties encountered by juniors, as well as a rough idea of the prevalence of these difficulties. Statements such as “some students struggle with...” should be read as “some students incorrectly answer questions about this concept, and these students’ reasoning is similar enough to hint that there is an underlying difficulty leading to these incorrect answers.” The pretests, tutorials, and tutorial homeworks were being actively revised during this process, usually in response to some of the difficulties described here. Many of the research questions were not designed to elicit particular difficulties, and thus any individual measurement described in this chapter can only give a general sense for the prevalence of a particular conceptual difficulty.

We are continuing to research student difficulties, and to further develop the tutorials to address these difficulties. This work will be published in future papers.

A. Junior-level difficulties with the integral form of Ampère's law

Chapter 7 described several different difficulties that introductory students have with the integral form of Ampère's law, one dealing with the applicability of Ampère's law, and the rest had to do with evaluating the integral. We have found that juniors also have difficulty with when to use Ampère's law and with what loop to choose, but have not seen evidence at the junior level of any of the difficulties related to evaluating the integral.

As illustrated below, we found that few students considered the dot product in the line integral when choosing their loop, consistent with Wallace and Chasteen's results. Most students do not choose the correct loop to calculate the magnetic field at a point, and often cannot explain why they chose the loop they did. In addition, many students only understand how the magnetic field is related to the line integral at a very superficial level.

1. *Choosing the correct loop*

We have found that most junior-level students do not understand that additional information is necessary to solve Ampère's law for the magnetic field in addition to the current encircled by a loop, and that by extension the loop must be chosen carefully to allow this calculation to be carried out.

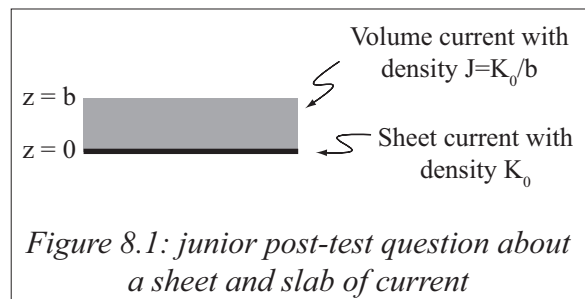
In the initial version of the junior-level *Ampère's Law* tutorial, students were shown a cylindrical wire carrying uniform current, asked to choose an appropriate loop that would allow them to find the field both inside and outside the wire, and then to calculate

the field. Nearly every student immediately chose circular loops, and those students who were initially hesitant were quickly convinced by the rest of their group that circular loops were correct.

However, during TA checkouts following this task, only one or two students per section could explain why this was the correct loop, and typically those students only mentioned that the magnitude of the magnetic field was constant on these circles, without mentioning that the dot product with the path element was also constant. It was almost impossible for the TAs to lead these students to explain why this loop was correct with Socratic reasoning and leading questions; the TA was usually forced to explicitly explain why the loop had to be circular and centered on the wire.

Based on these observations, we designed a post-test question about Figure 8.1, administered two weeks after tutorial coverage (AML_143 in Appendix E). The slab carries volume current into the page, and the

sheet carries equal current out of the page, so the net current of the system is zero. The first question asked students to draw an Ampèrian loop that would allow them to find the field at the center of the slab, $z = b/2$. Later questions then asked students to find the magnetic field at various locations, and to consider locations where the net magnetic field was zero.



The first question was difficult for students, with less than 10% of students giving both correct answers and somewhat correct reasoning. 32

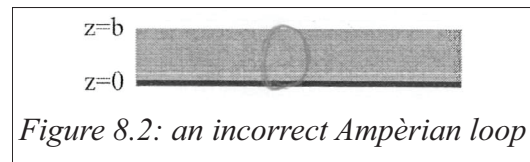


Figure 8.2: an incorrect Ampèrian loop

students took the exam, of which 34% drew a square or rectangular loop through the center of the slab. A further 47% of students also drew rectangular loops, though they did not draw them through the center of the slab; most students drew a loop with one side above $z = b$ and one side below $z = 0$. 16% of students incorrectly drew a circular loop, one of which is shown in Figure 8.2.

Most students stated that the loop had to be square due to the shape of the current distribution but did not elaborate further. Only one student explicitly stated that the line integral along the left and right sides of their rectangle was zero due to the dot product, so the loop integral would simplify enough to allow them to solve for the magnetic field at the center.

We had written the initial version of the tutorial to focus on the curl and the differential form of Ampère's law, with little motivation of how to use the line integral to find the magnetic field. After working through this version of the tutorial students were clearly not learning how to choose a loop or how to apply the law to find the magnetic field, so we rewrote the tutorial to focus on these issues.

The new section of the second version, dealing with the choice of Ampèrian loop, starts with an incorrect statement about Figure 8.3:

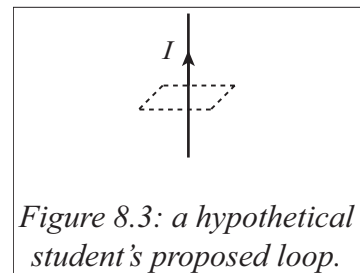


Figure 8.3: a hypothetical student's proposed loop.

“I think the magnetic field points around the wire. I can use a square as my Ampèrian loop because the path vector at the center of each side has the same direction as the magnetic field. This makes finding the total line integral easy.”

Students are asked why this statement is incorrect, and then asked to choose a more appropriate shape (in tutorial section, all students choose circular loops at this point).

Next, students are explicitly asked if the dot product $\vec{B} \cdot d\vec{l}$ is easy to evaluate everywhere on the path. Finally, for the portions of the loop where the dot product is non-zero, students are asked if the magnetic field on each section is constant in magnitude. If neither of the above are true, the tutorial suggests that students should consider a different path.

The post-test for version two of the tutorial included Figure 8.4 (AML_153 in Appendix E). In one of the questions students were asked to draw a loop they could use to find the magnetic field a distance $R/2$ from the center of the solenoid. Of the 25 students who took this post-test, three quarters correctly

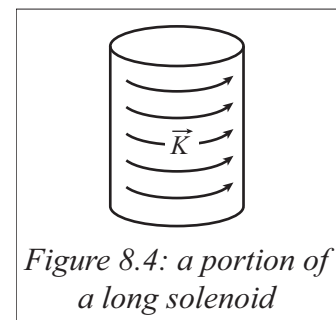


Figure 8.4: a portion of a long solenoid

chose a rectangular loop with one side at $R/2$ and one side outside the solenoid. This is surprisingly high, since students typically choose loops with similar symmetry to the current distribution, which in this case has cylindrical symmetry. Only 16% of students chose a circular loop (centered on the solenoid, with radius $R/2$). Because of the small number of students who took this post-test, statistical comparisons with the previous version are not appropriate, but this is a large shift from the previous post-test. In particular, all students in this post-test chose an Ampèrian loop that passed through the point of interest, compared to only a third in the previous case.

2. Use of symmetry arguments

One of the easiest ways to obtain information about the magnetic field, such that a loop can be chosen to make solving the integral easy, is to use a symmetry argument. If the current distribution is highly symmetric, then a symmetry argument can prove that the magnetic field has similar symmetry, which indicates the loop that should be used. For example, the wire in Figure 8.5 is rotationally symmetric about an axis along the wire, so the magnetic field at points D

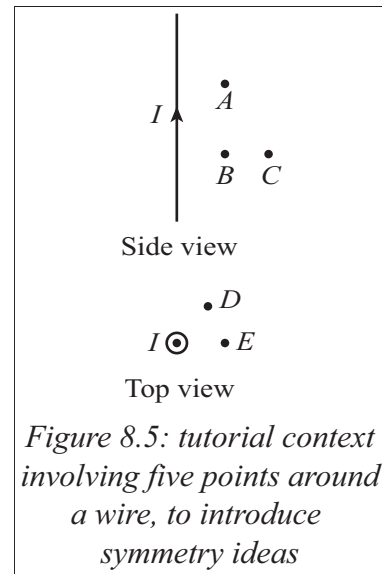


Figure 8.5: tutorial context involving five points around a wire, to introduce symmetry ideas

and E are the same. The wire is also translationally symmetric, so the magnetic field at points A and B are the same. In addition the magnetic field cannot have a radial component without violating Gauss's law for magnetism, and it cannot have a vertical component without violating the Biot-Savart law. Thus the field must point azimuthally, and must have constant magnitude on a path of constant radius from the wire, and the line integral is trivial to evaluate for a circular loop.

This type of argument is very hard for students to construct without guidance, as discussed in chapter 6 and by Pepper et al³ in the context of Gauss's law for electric fields. An intermediate version of the *Ampère's Law* tutorial between versions 1 and 2 asked students to use a symmetry argument to show that the field at a given radius was constant, but very few students during tutorial could manage to do so. This part was significantly expanded in version 2, based on Figure 8.5, on the first page of the tutorial. Students are asked to use symmetry arguments and other pieces of knowledge (such as Gauss's law for

magnetic fields) to determine the behavior of the magnetic field around the wire, and then they see the section on appropriate choice of loops (as described in the previous section). This is also reinforced in the homework; students are given several different current distributions, and asked to determine the directions the magnetic field can point and the variables the magnetic field can depend on for each case.

Two related questions were also asked on the post-test with Figure 8.4. The questions ask which direction(s) it is impossible for the magnetic field of the solenoid to point, and what spatial variable(s) it is impossible for the magnetic field to depend on. This is still a fairly difficult topic; 36% of students correctly stated and explained that the field cannot point in the azimuthal or radial direction, and 40% correctly stated and explained that the field cannot depend on z or ϕ . A further third of students correctly answered that one or both of the directions or variables were impossible, usually with incorrect, incomplete, or no reasoning. Nearly a quarter of students left these questions blank, while answering the rest of the questions on the page.

3. Discussion

We found that an effective strategy for helping students develop a functional understanding of Ampère's law was to emphasize the necessity of having prior knowledge about the magnetic field (such as from symmetry) before choosing an Ampèrian loop. However, more work still needs to be done to fully address this set of difficulties students have with connecting the mathematical description of the line integral to the physical behavior of magnetic fields.

In particular, understanding symmetry and using symmetry arguments seems to be challenging for most junior-level students. This idea is not tied to specific physics concepts, so we have begun to introduce the idea at several different points in the E&M curriculum to help students better understand the utility of symmetry arguments as a reasoning tool. For example, the *Gauss's Law* tutorial has been rewritten in an analogous manner to the first two parts of the *Ampère's Law* tutorial, focusing on symmetry to gain knowledge of the electric field and then using that knowledge to pick the correct Gaussian surface. However, these modifications are new enough that no data has yet been taken to gauge their effectiveness.

B. Junior-level difficulties with the differential form of Ampère's law

Ampère's law in integral form is a tool that allows for easy calculation of the magnetic field provided the current distribution has high enough symmetry. In practice, only infinite sheets of current, infinite cylinders of current, and solenoids have sufficient symmetry to be easily solved via Ampère's law (and of course superpositions of these cases). The differential form of Ampère's law, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, is much more useful. It describes the general relationship between magnetic fields and current distributions, allows the direct calculation of currents from a given magnetic field (e.g. finding bound currents of a permanent magnet), and with the rest of Maxwell's equations, admits wave solutions for electromagnetic radiation.

The differential form seems to be harder for students to grasp than the integral form, as we will discuss in the following sections.

1. Zero curl implies zero field

Many students believe that if the curl of a field is zero at a point, then the magnetic field itself is also zero at that point. We have observed this belief in several different contexts: on the *Ampère's Law* tutorial and homework; on the *Vector Potential* pretest and homework; and on the *Electromotive Force* tutorial and post-tests. We initially observed this belief in the *Ampère's Law* pretest, and included an explicit question about whether zero curl implied zero field in that tutorial. The same students struggled with it again in the context of Faraday's law, so we included the question on all the tutorials that directly involved curl.

We post-tested this misconception in the context of Faraday's law with a question about Figure 8.6 (EMF_153 in Appendix E). The current in the coaxial cable is increasing linearly with time, so the time derivative of the magnetic field is azimuthal. This is analogous to a solenoid, so the electric field inside the cable points upward. On one of the questions,

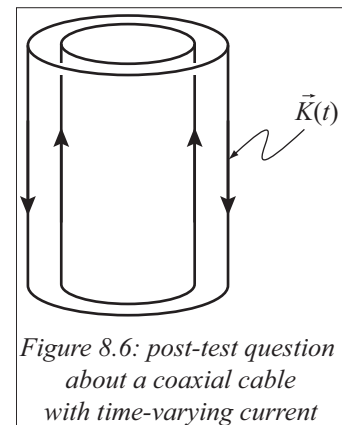


Figure 8.6: post-test question about a coaxial cable with time-varying current

students were told that an electron was released from rest at the center of the solenoid, and then asked to describe the electron's motion. Of the 26 students who took the exam, 42% stated that the charge would not move since $\vec{\nabla} \times \vec{E} = -\partial \vec{B} / \partial t$ and $\vec{B} = 0$ at the center of the cable, so $\vec{E} = 0$ as well. Only 8% had the correct answer with correct reasoning. Clearly, including single questions on each tutorial that involves curl is not enough to address the issue.

2. Curl is a property of systems as a whole

The opposite difficulty is also common; assuming that if curl exists somewhere in space, that it will exist everywhere. For instance, the *Vector Potential* tutorial begins with a review of curl and Ampère's law from the previous tutorial. Students are reminded of the differential form of Ampère's law, and asked where in space the magnetic field of a long wire has curl. The correct answer is that the curl of \vec{B} is specified by the current density, which is only non-zero at the wire, so \vec{B} only has curl at the wire. However, in tutorial section nearly every student stated that the magnetic field had curl everywhere in space. The TAs had to discuss this with every group, often for several minutes, before they changed their answers.

This could be because vector derivatives are difficult to understand without analogies, and analogies are often inexact. In this case, the difficulty likely has two sources: the name itself, and the common "pinwheel" analogy. In terms of the name, "curl" evokes a sense of "things rotating around."

The field of a long straight wire is shown in Figure 8.7, and the magnetic field everywhere in space is rotating around the wire. The issue is that curl at a point measures rotation *about that point*: so while there is obviously curl at the wire, there is not curl anywhere else because the field is not rotating about any other point. Students tend not to realize that curl is defined with respect to the point in question, and is not a generalized property of the system.

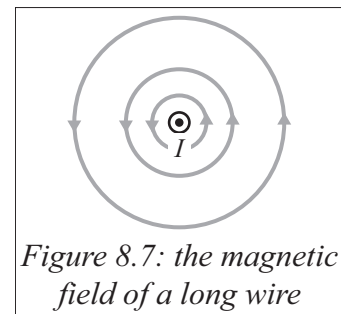
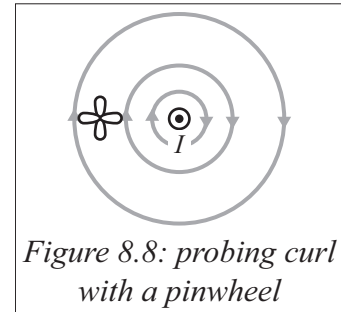


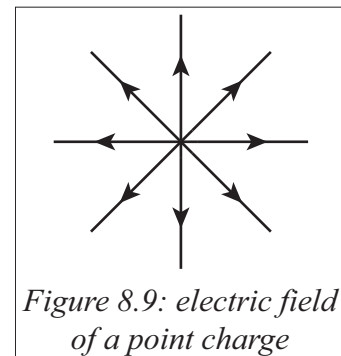
Figure 8.7: the magnetic field of a long wire

The pinwheel analogy could also be to blame for this particular misunderstanding. The curl of a vector field is usually visualized in terms of small pinwheels: imagine that the vector field is a pattern of wind, place a pinwheel at some point in the field, and if the pinwheel rotates then the field has curl. However



this simple analogy can be difficult to apply, for example in Figure 8.8 about the long straight wire again. The field at the right side of the pinwheel is stronger than the field at the left side, so it is natural to assume that the pinwheel there will rotate. However the field at the left side is curving *with* the curvature of the pinwheel but the field at the right side is curving *away from* the curvature of the pinwheel, so the field at the left edge has a greater area of influence on the pinwheel than the field at the right edge. These two effects exactly cancel out, but students typically do not consider the second effect unless prompted to.

This difficulty is not specific to the curl. We see the same type of difficulties with divergence, in the differential form of Gauss's law. In the *Gauss's Law* tutorial students are shown Figure 8.9 and asked where there is non-zero divergence. As



before the answer is that the divergence is only non-zero at the point charge. However, the name could be misleading to students; we observed that around three quarters of students stated that the field in Figure 8.9 had divergence everywhere. When we probed students' reasoning, many students did not have clear explanations, but about a quarter stated that divergence was "something spreading apart,"

and the field looks like it is spreading apart everywhere in space. The analogy that is typically used for divergence is water flow: points of positive divergence are “sources” where water is being added to a pool, and points of negative divergence are “sinks” where water is being drained from a pool. Unlike the pinwheel analogy for curl this analogy does not support this misconception, since field lines are only created at the point charge (water is spreading out from a single point source, with no other sources around). However, in this case students typically default to reasoning based on the name, instead of the analogy, which suggests that students have not sufficiently internalized the analogy and struggle to apply it to physical situations.

3. Discussion

Attempting to address these difficulties individually, in each tutorial that triggers them, has not worked. We are designing a new tutorial on general fields that have curl but not divergence, to address the misconceptions above in more detail.

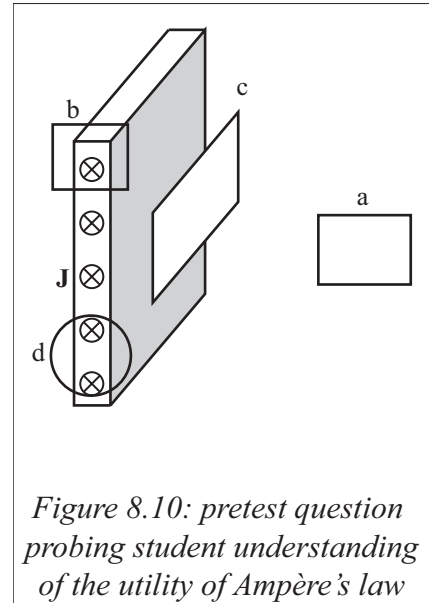
Nearly all the fields in magnetostatics have curl but not divergence: the magnetic field, the vector potential, the dynamically-created electric field in Faraday's law, etc. We hope that starting the magnetostatics series with this new tutorial on general, divergenceless, curly fields will provide a common base that students can refer back to as they are introduced to each new case.

C. Utility of Ampère's law

As described in section F of chapter 7, many introductory students think that Ampère's law is always useful as long as it encloses current, without considering whether

or not the integral is solvable. This is true of junior-level students as well, though we have found that the curriculum changes described above could impact this.

A good example of this difficulty is a question from the *Ampère's Law* pretest. This question incorporates Figure 8.10, where several possible Ampèrian loops are shown near a portion of a two-dimensionally infinite slab of current. The question asks which of the loops could prove useful in learning something about the magnetic field at some location. A physics expert would likely say that loop b would allow the field of the slab to be calculated, and loop a would show that the field cannot



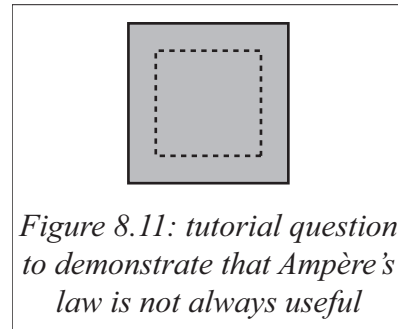
depend on the distance from the sheet. Loop c would show that the field cannot depend on the location next to the sheet, and only loop d would not be directly useful since the dot product is not easily evaluated at every point on the loop.

On this question many students tend to think that loops b and d are the most useful; of 89 students, 56% stated that b and d would allow them to calculate the magnetic field directly, with a few more stating that various other loops could also give some information. Only 8% stated loop b only, all of which had good reasoning:

"[Loops] a and c are both zero because they enclose no current. [Loops] b and d will tell you about current density, but b is probably the easier integral"

"D is a circle which would be an awful integral."

At the end of the tutorial, after students work through the part about choosing an appropriate loop to make evaluating the dot product and the integral easy, they are shown a student dialogue about Figure 8.11. One hypothetical student maintains that there is not enough information about the magnetic field to do the path integral, while the second states that since the symmetry of the loop is the same as that of the wire, the magnetic field is constant along the loop so Ampère's law is simple to evaluate.



During the tutorial section, we find that nearly all students correctly disagree with the second student, but many struggle to explain their reasoning for why the second student is incorrect. The second student espouses an idea that we found to be fairly common when first developing the symmetry section, which is that the Ampèrian loop should match the symmetry of the current distribution, not the symmetry of the field. Of course, in the instances where Ampère's law can be used, the correct loops do reflect both symmetries. Since the line integral involves the loop and the magnetic field, it is those symmetries that must be similar, and we have seen that many students find it difficult to grasp this distinction.

The curriculum development on symmetry and choosing an appropriate loop had a positive impact on students' understanding about when Ampère's law was useful, but it did not address some of the deeper issues surrounding symmetry arguments and their application to Ampère's law. Those will need to be addressed with more specific, targeted curriculum development.

D. Conclusions

Ampère's law is a fundamental part of classical electromagnetic theory, and as we have shown, even junior-level students struggle with some aspects of it. The tutorial development we have described seems somewhat promising for improving students' understanding. However, the problems seem not to be trivial, and more work is necessary to fully address them. In particular, the fact that the integral and differential forms are so different and raise such distinct conceptual issues seems to make this a difficult topic. We have found that junior-level students typically do not have most of the difficulties identified in the lower-division population, described in chapter 7, but the same pattern of challenges in making connections between math and physics appears in both classes. The major difference between the classes is that introductory students typically operate only at the level of understanding the process of applying Ampère's law; whereas the upper-division students have enough mathematical and physical background to attempt to understand Ampère's law in both forms at a fundamental level, though not all succeed.

E. References for chapter 8

- [1] C. A. Manogue, K. Browne, T. Dray, and B. Edwards, "Why is Ampère's law so hard? A look at middle-division physics," *Am. J. Phys.* 74, p. 344 (2006)
- [2] C. S. Wallace and S. V. Chasteen, "Upper-division students' difficulties with Ampère's law," *PRST-PER* 6, p. 020115 (2010)
- [3] R. E. Pepper, S. V. Chasteen, S. J. Pollock, and K. K. Perkins, "Our best juniors still struggle with Gauss's law: characterizing their difficulties," *PERC Proceedings* 2010

GUIDE TO APPENDICES

All appendix materials are labeled by a three letter code assigned to the associated tutorial. Pretests and post-tests are also labeled with a three number code that describes the quarter in which the questions were administered: the first two numbers are the last two digits of the year, and the last number refers to which quarter in the year. For instance, Summer of 2013 (the 3rd quarter) would be labeled as 133, and Autumn of 2010 as 104. Tutorials are labeled by three-letter code and version number, and homework is labeled by the version number of the tutorial with which it is associated.

The table below shows the associated three-letter codes for the tutorials at each level.

Introductory tutorial	Code	Junior tutorial	Code
Ampère's Law	AML	Ampère's Law	AML
Capacitance	CAP	Conductors	CON
Electric Field and Flux	EFF	Coulomb's Law	COU
Electric Potential Difference	EPD	Electromotive Force	EMF
Electric Properties of Conductors	EPC	Gauss's Law	GSL
		Potential	POT

LIST OF APPENDICES

Appendix A: Introductory pretests

AMP_113.....	148
AMP_151.....	151
CAP_104.....	154
EFF_111.....	158

EPC_152	160
EPD_152.....	164

Appendix B: Introductory post-test questions

AMP_124.....	166
CAP_104.....	167
EPC_132	168
EPC_141	169
EPC_142	170
EPC_144	171
EPC_151	172
EPC_153	173
EPD_134.....	174
EPD_152.....	175

Appendix C: Introductory tutorial versions

AMP_Version0	176
AMP_Version1	179
AMP_Version2	184
EPC_Version1.....	189
EPC_Version2.....	193
EPC_Version3.....	197
EPC_Version4.....	201

EPC_Version5.....	206
EPD_Version0	211
EPD_Version1	217
EPD_Version2	222
EPD_Version3	227
EPD_Version4	231
EPD_Version5	235
EPD_Version6	239

Appendix D: Introductory tutorial homework versions

AMP_HW_Version0	244
AMP_HW_Version1	246
AMP_HW_Version2	249
EPC_HW_Version1.....	252
EPC_HW_Version2.....	254
EPC_HW_Version3.....	256
EPC_HW_Version5.....	259
EPD_HW_Version0	262
EPD_HW_Version4	266
EPD_HW_Version5	270
EPD_HW_Version6	274
EPD_HW_Version6b.....	276

Appendix E: Junior post-test questions

AML_143	279
AML_153	280
COU_144	281
EMF_153	282

Appendix F: Junior tutorial versions

CON_Version1	283
COU_Version1	287
COU_Version2	292
GSL_Version1	296
GSL_Version2	300
POT_Version1	304
POT_Version2	309

8/18/2015

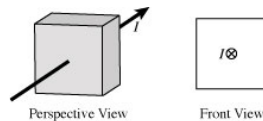
Catalyst WebQ

Print view of '(AMP)U1a'[Print this page](#)

Earlier in the course, you learned about electric flux. We can define magnetic flux through a surface similarly:

$$\vec{\Phi}_B = \vec{B}_1 \cdot d\vec{A}_1 + \vec{B}_2 \cdot d\vec{A}_2 + \vec{B}_3 \cdot d\vec{A}_3 \dots$$

For the following questions, consider a cubical surface. A wire carrying a current I goes through the center of the front and back faces of the cube as shown. While responding to these questions, base your answers on your knowledge of the magnetic field around a current-carrying wire.

**Question 1.**

The magnetic flux through the front surface is:

- Positive.
 Negative.
 Zero.

Question 2.

Explain your reasoning.

Question 3.

The magnetic flux through the right surface is:

- Positive.
 Negative.
 Zero.

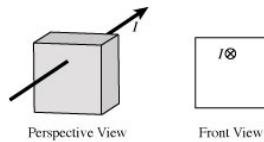
Question 4.

Explain your reasoning.

The cubical surface is shifted downward, as shown below.

8/18/2015

Catalyst WebQ

**Question 5.**

The magnetic flux through the front surface is:

- Positive.
 Negative.
 Zero.

Question 6.

Explain your reasoning.

Question 7.

The magnetic flux through the right surface is:

- Positive.
 Negative.
 Zero.

Question 8.

Explain your reasoning.

As shown below, an Amperian loop has a wire passing through it, carrying a current I .

**Question 9.**

The wire is bent into a semicircle as shown below. How does the magnitude of the magnetic field along the loop compare to that for the straight wire?



- The magnitude of the magnetic field along the loop is less than that for the straight wire on the right side, and greater than on the left side.
 The magnitude of the magnetic field along the loop is greater than that for the straight wire on the right side, and less than on the left side.
 The magnitude of the magnetic field along the loop is the same as that for the straight wire all around the loop.
 The magnitude of the magnetic field is greater than that for the straight wire all around the loop.

Question 10.

Explain your reasoning.

Question 11.

How does the line integral around the loop compare to that for the straight wire?

<https://catalyst.uw.edu/webq/build/tipem/133569>

2/3

8/18/2015

Catalyst WebQ

- The line integral is greater than that for the straight wire.
- The line integral is the same as that for the straight wire.
- The line integral is less than that for the straight wire.
- Ampere's law doesn't apply here since the wire is not straight.

Question 12.

Explain your reasoning.

Questions or comments?
[Contact us or email catalysthelp@uw.edu](mailto:catalysthelp@uw.edu)

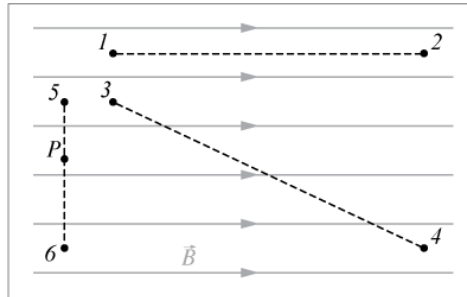
8/18/2015

Catalyst WebQ

Print view of '(AMP)U1d1'

[Print this page](#)

Shown at right are three paths in a uniform magnetic field. One path is from point 1 to point 2, one path is from point 3 to point 4, and one path is from point 5 to point 6.



Question 1.

At point P, what is the direction of $d\vec{l}$ along the path from point 5 to 6?

I - zero

⊗ J - into page

⊙ K - out of page

- Arrow A
- Arrow B
- Arrow C
- Arrow D
- Arrow E
- Arrow F
- Arrow G
- Arrow H
- I - Zero
- J - Into the page
- K - Out of the page
- None of these options accurately specifies the direction of $d\vec{l}$ at point P.

Question 2.

Rank the absolute value of the line integral of the magnetic field along the given paths from greatest to smallest. If any line integral is zero, state so explicitly. Use the following variables to represent these line integrals:

$$X = \left| \int_1^2 \vec{B} \cdot d\vec{l} \right| \quad (\text{Note: } X \text{ is the absolute value of the line integral of the magnetic field from point 1 to point 2.})$$

$$Y = \left| \int_3^4 \vec{B} \cdot d\vec{l} \right|$$

$$Z = \left| \int_5^6 \vec{B} \cdot d\vec{l} \right|$$

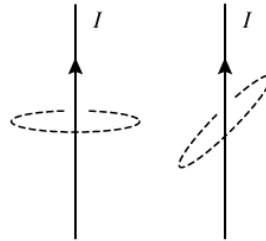
8/18/2015

Catalyst WebQ

Example: $Z > X = Y = 0$ **Question 3.**

Explain your reasoning for the previous question.

Two wires, each with current I , pass through the two Amperian loops shown at right. The loops are identical, except the loop on the right has been rotated 45° from horizontal.

**Question 4.**

Is the absolute value of the **current enclosed** by the rotated loop *greater than*, *less than*, or *equal to* that of the horizontal loop?

- Greater than that of the horizontal loop, by a factor of 2.
 Greater than that of the horizontal loop, by a factor of $\sqrt{2}$.
 Equal to that of the horizontal loop.
 Less than that of the horizontal loop, by a factor of $\sqrt{2}$.
 Less than that of the horizontal loop, by a factor of 2.

Question 5.

Explain your reasoning for the previous question.

Question 6.

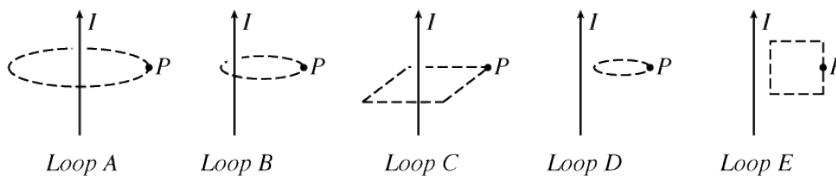
Is the absolute value of the **line integral of the magnetic field** around the rotated loop *greater than*, *less than*, or *equal to* that around the horizontal loop?

- Greater than that around the horizontal loop, by a factor of 2.
 Greater than that around the horizontal loop, by a factor of $\sqrt{2}$.
 Equal to that around the horizontal loop.
 Less than that around the horizontal loop, by a factor of $\sqrt{2}$.
 Less than that around the horizontal loop, by a factor of 2.

Question 7.

Explain your reasoning for the previous question.

Five Amperian loops are shown through point P next to a wire carrying current I .



<https://catalyst.uw.edu/webq/build/tipem/262137>

2/3

8/18/2015

Catalyst WebQ

Question 8.

For each of the Amperian loops above, indicate whether or not you could use Ampere's law with that Amperian loop to calculate the magnetic field at point P .

Rows

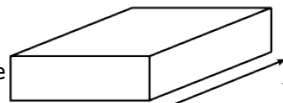
A
B
C
D
E

- Yes.
 No.

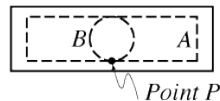
Question 9.

Explain your reasoning for the previous question.

A rectangular wire carries a current I into the page, uniformly distributed through the wire. Two Amperian loops are shown inside the wire: loop A (rectangular), and loop B (circular).



*Rectangular wire
in perspective*



*Cross-section with two
Amperian loops (A and B)*

Question 10.

Could you use either of the Amperian loops shown to find the magnetic field at point P inside the wire?

- Yes, loop A only.
 Yes, loop B only.
 Yes, both loops A and B.
 No, neither loop would help me find the magnetic field.

Question 11.

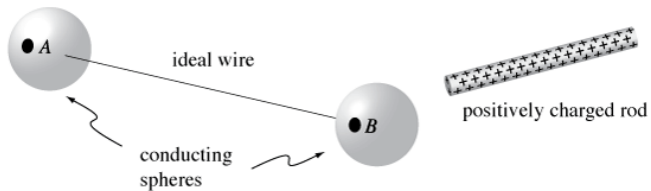
Explain your reasoning for the previous question.

© Lillian C. McDermott, Peter S. Shaffer and the Physics Education Group
University of Washington, 2015

Questions or comments?
Contact us or email catalysthelp@uw.edu

8/18/2015

Catalyst WebQ

Print view of '(CAP)U2a'[Print this page](#)

Two conducting spheres, initially uncharged and held in place on insulating stands (not shown), are connected by an ideal conducting wire as shown. A positively charged rod is brought close to the right sphere. Let Q_L be the absolute value of the net charge on the left sphere and Q_R be the absolute value of the net charge on the right sphere.

Question 1.

At the instant shown in the diagram, the absolute value of the net charge on the right sphere, Q_R , is:

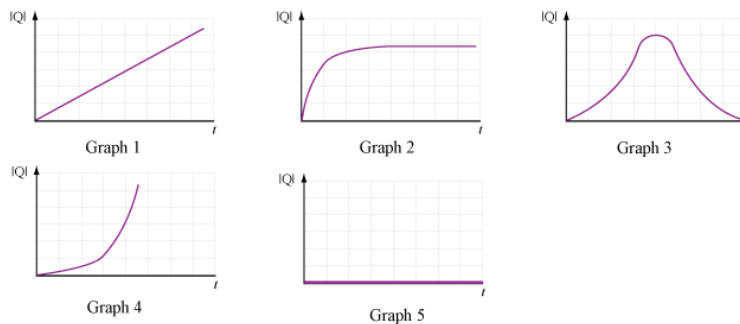
- greater than* the absolute value of the net charge on the left sphere, $Q > Q_L$.
- less than* the absolute value of the net charge on the left sphere, $Q < Q_L$.
- equal to* the absolute value of the net charge on the left sphere, $Q = Q_L$.
- There is not enough information provided.

Question 2.

Explain the reasoning you used to answer the question above.

Question 3.

Which of the graphs below best represents the absolute value of the net charge Q_R on the right sphere as a function of time, once the rod is close to the right sphere and held in place as shown above?



<https://catalyst.uw.edu/webq/build/tipem/114826>

1/4

8/18/2015

Catalyst WebQ

- Graph 1
 Graph 2
 Graph 3
 Graph 4
 Graph 5
 Other: Please describe in your explanation below.

Question 4.

Explain the reasoning you used to answer the question above.

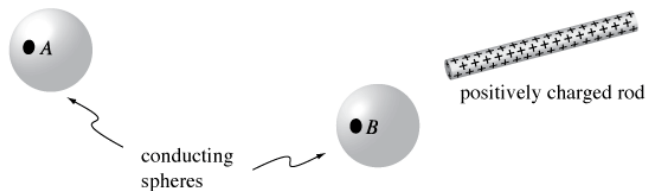
Question 5.

After the charged rod has been in place for a long time, the electric potential V_B at point B on the right sphere is:

- greater than* the electric potential V_A at point A on the left sphere.
 less than the electric potential V_A at point A on the left sphere.
 equal to the electric potential V_A at point A on the left sphere.
 There is not enough information provided.

Question 6.

Explain the reasoning you used to answer the question above.



The wire connecting the two spheres is now removed, as shown.

Question 7.

The electric potential V_B at point B on the right sphere is:

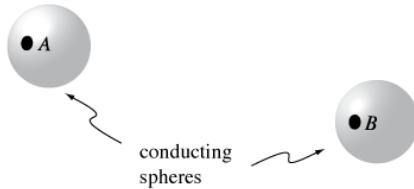
- greater than* the electric potential V_A at point A on the left sphere.
 less than the electric potential V_A at point A on the left sphere.
 equal to the electric potential V_A at point A on the left sphere.
 There is not enough information provided.

Question 8.

Explain the reasoning you used to answer the question above.

8/18/2015

Catalyst WebQ



The charged rod is now removed from the system, as shown.

Question 9.

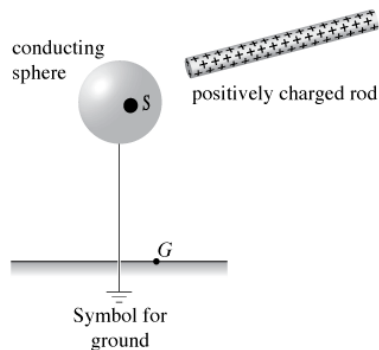
The electric potential V_B at point B on the right sphere is:

- greater than* the electric potential V at point A on the left sphere.
- less than* the electric potential V at point A on the left sphere.
- equal to* the electric potential V at point A on the left sphere.
- There is not enough information provided.

Question 10.

Explain the reasoning you used to answer the question above.

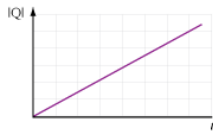
A positively charged rod is brought close to a grounded conducting sphere as shown.

**Question 11.**

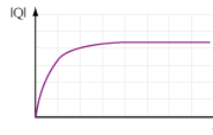
Which of the graphs below best represents the absolute value of the net charge Q on the sphere as a function of time, once the rod is close to the sphere and held in place as shown above?

8/18/2015

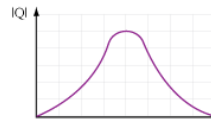
Catalyst WebQ



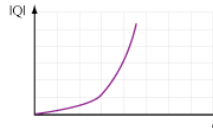
Graph 1



Graph 2



Graph 3



Graph 4



Graph 5

- Graph 1
 Graph 2
 Graph 3
 Graph 4
 Graph 5
 Other: Please describe in your explanation below.

Question 12.

Explain the reasoning you used to answer the question above.

Question 13.

After the charged rod has been in place for a long time, the electric potential at point S on the sphere, V_S , is:

- greater than* the electric potential V_G at point G.
 less than the electric potential V_G at point G.
 equal to the electric potential V_G at point G.
 There is not enough information provided.

Question 14.

Explain the reasoning you used to answer the question above.

© Lillian C. McDermott, Peter S. Shaffer and the Physics Education Group
University of Washington, 2012

Questions or comments?
Contact us or email catalysthelp@uw.edu

Time remaining: 0:14:56

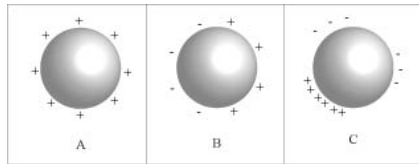
This is only a preview of the survey. Responses will not be saved. [Close](#)

(EFF)U4a
University of Washington

Page 1 of 2

Part I

Consider the diagrams below. In each diagram, a conducting metal sphere has a static charge distribution on its surface as shown.



Question 1.

Under what conditions, if any, can the sphere in diagram A have the charge distribution shown?

- The sphere can be alone in the system; there is no need for additional charges that are not shown.
- There must be additional charges (not shown) in the system.
- It is impossible for the sphere to have this charge distribution.

Question 2.

Explain your reasoning. If you believe that there must be additional charges or charge distributions in the system, describe their approximate locations as well as the size and shape of any charge distributions.

Question 3.

Under what conditions, if any, can the sphere in diagram B have the charge distribution shown?

- The sphere can be alone in the system; there is no need for additional charges that are not shown.
- There must be additional charges (not shown) in the system.
- It is impossible for the sphere to have this charge distribution.

Question 4.

Catalyst WebQ - Preview

https://catalyst.uw.edu/webq/preview/?solstice_selected_button...

Explain your reasoning. If you believe that there must be additional charges or charge distributions in the system, describe their approximate locations as well as the size and shape of any charge distributions.

Time remaining: 0:14:56**Question 5.**

Under what conditions, if any, can the sphere in diagram C have the charge distribution shown?

- The sphere can be alone in the system; there is no need for additional charges that are not shown.
- There must be additional charges (not shown) in the system.
- It is impossible for the sphere to have this charge distribution.

Question 6.

Explain your reasoning. If you believe that there must be additional charges or charge distributions in the system, describe their approximate locations as well as the size and shape of any charge distributions.

[Next >>](#)**Questions or Comments?**

Contact the 122 tutorial pretest coordinator at uwttl122@u.washington.edu



8/18/2015

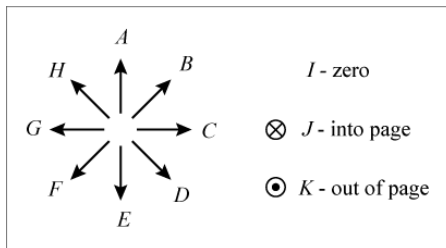
Catalyst WebQ

Print view of '(EPC)U4a'[Print this page](#)

A positive charge, $+Q_o$, is held to the right of a neutral conducting rod as shown. Point P is located at the center of the rod. The charge has been held in place for a long time.

**Question 1.**

Select the arrow from the figure below that most accurately indicates the direction of the electric field at point P . If the net electric field is zero at point P , select that option.



- Arrow A
 Arrow B
 Arrow C
 Arrow D
 Arrow E
 Arrow F
 Arrow G
 Arrow H
 The net electric field is zero at point P .
 None of these options accurately specifies the direction of the net electric field at point P .

Question 2.

Is the electric potential difference from the left end of the rod to the right end of the rod ($\Delta V_{\text{left} \rightarrow \text{right}} = V_{\text{right}} - V_{\text{left}}$) *positive, negative, or zero?*

- positive.
 negative.
 zero.
 There is not enough information provided.

Question 3.

Explain the reasoning you used to answer the questions above.

An additional charge, $-Q_o$, is now held in place to the left of the rod.

8/18/2015

Catalyst WebQ

$-Q_0$

$+Q_0$

Question 4.

After both charges have been held for a long time, is the electric potential difference from the left end of the rod to the right end of the rod ($\Delta V_{\text{left} \rightarrow \text{right}} = V_{\text{right}} - V_{\text{left}}$) *positive, negative, or zero*?

- positive.
 negative.
 zero.
 There is not enough information provided.

Question 5.

Explain the reasoning you used to answer the question above.

Question 6.

At some time $t = t_0$, the two charges are suddenly removed.

Immediately after t_0 , is the electric potential difference from the left end of the rod to the right end of the rod ($\Delta V_{\text{left} \rightarrow \text{right}} = V_{\text{right}} - V_{\text{left}}$) *positive, negative, or zero*?

- positive.
 negative.
 zero.
 There is not enough information provided.

Question 7.

Explain the reasoning you used to answer the question above.

Question 8.

A long time after t_0 , is the electric potential difference from the left end of the rod to the right end of the rod ($\Delta V_{\text{left} \rightarrow \text{right}} = V_{\text{right}} - V_{\text{left}}$) *positive, negative, or zero*?

- positive.
 negative.
 zero.
 There is not enough information provided.

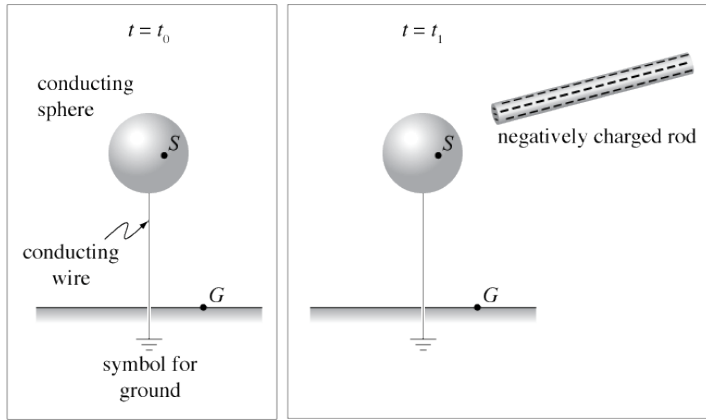
Question 9.

Explain the reasoning you used to answer the question above.

At t_0 , a negatively charged rod is far away from a grounded conducting sphere. From t_0 to t_1 , the rod is gradually brought close to the sphere. At t_1 , the rod is held in place near the sphere as shown.

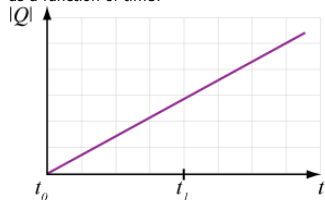
8/18/2015

Catalyst WebQ

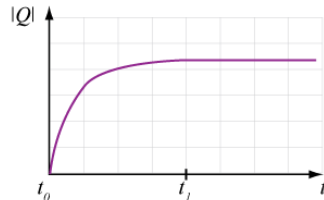


Question 10.

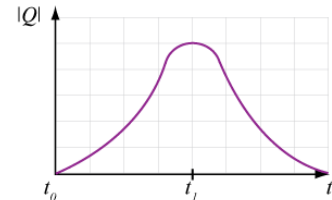
Which of the graphs below best represents the absolute value of the net charge Q on the sphere as a function of time?



Graph 1



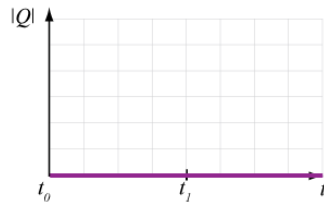
Graph 2



Graph 3



Graph 4



Graph 5

- Graph 1
- Graph 2
- Graph 3
- Graph 4
- Graph 5
- Other: Please describe in your explanation below.

Question 11.

Explain the reasoning you used to answer the question above.

Question 12.

After the charged rod has been in place for a long time, is the electric potential difference from point S on the sphere to point G on the ground ($\Delta V_{S \rightarrow G} = V_G - V_S$) positive, negative, or zero?

- positive.
- negative.
- zero.
- There is not enough information provided.

<https://catalyst.uw.edu/webq/build/tipem/258765>

3/4

8/18/2015

Catalyst WebQ

Question 13.

Explain the reasoning you used to answer the question above.

© Lillian C. McDermott, Peter S. Shaffer and the Physics Education Group
University of Washington, 2015

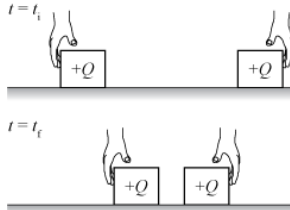
Questions or comments?
Contact us or email catalysthelp@uw.edu

8/18/2015

Catalyst WebQ

Print view of '(EPD)U3c_Research5a'[Print this page](#)

A. Two identical positively charged blocks are at rest on a frictionless surface. Two hands push the blocks toward each other from $t = t_i$ to $t = t_f$. The blocks begin and end at rest.

**Question 1.**

Does the *electric potential energy* of the two-block system above *increase, decrease, or remain the same* as a result of the movement?

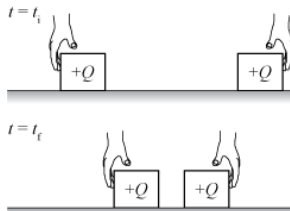
- Increase.
 Decrease.
 Remain the same.

Question 2.

Explain your reasoning for the previous question.

Question 3.

B. Two students discuss their answers about the two identical blocks from the previous page, shown again at right.



Student 1: "I think the system gains potential energy as the blocks are pushed together. The force between the blocks is greater when they're closer together, so the energy must be greater as well."

Student 2: "I agree that the system gains potential energy, but I thought about what the hands were doing. Both hands push the blocks inward, so they both do positive work. The net external work is positive, and there's no change in kinetic energy since the blocks start and end at rest, so the potential energy increases."

With which student's reasoning, if either, do you agree?

- Student 1.
 Student 2.
 Neither student is using correct reasoning.

Question 4.

Explain your reasoning for the previous question.

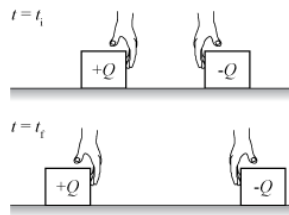
C. The sign of one of the blocks is switched from positive to negative. Two hands push the blocks away from each other from $t = t_i$ to $t = t_f$. The blocks begin and end at rest.

<https://catalyst.uw.edu/webq/build/tipem/268366>

1/2

8/18/2015

Catalyst WebQ

**Question 5.**

If Student 1 used the same reasoning for this case as they did for the first case, would they think that the electric potential energy of this system *increases*, *decreases*, or *remains the same* as a result of this movement?

- Increase.
 Decrease.
 Remain the same.

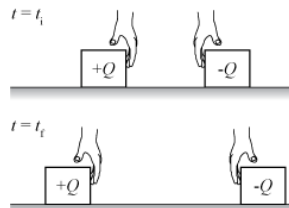
Question 6.

If Student 2 used the same reasoning for this case as they did for the first case, would they think that the electric potential energy of this system *increases*, *decreases*, or *remains the same* as a result of this movement?

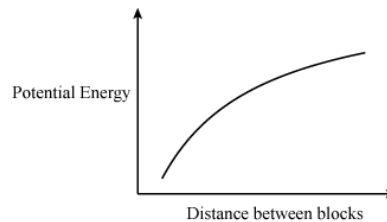
- Increase.
 Decrease.
 Remain the same.

Student 1 reasoned that force was related to changes in potential energy. In this case the force between the blocks is decreasing as the blocks are moving apart, so Student 1 would conclude that the potential energy is also decreasing.

Student 2 reasoned that external work was related to changes in potential energy. In this case the hands are both doing positive work, so the net external work is positive, and Student 2 would conclude that the potential energy is increasing.



A graph of the potential energy of the two-block system as a function of the distance between the blocks is shown at right.

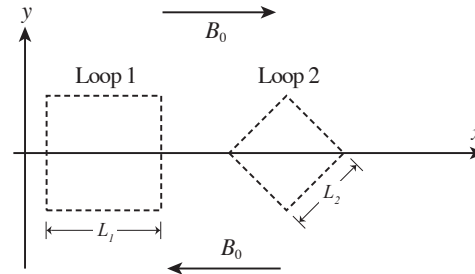
**Question 7.**

Which student was correct? What line of reasoning should you use to think about electric potential energy? Explain.

Name _____ Student ID _____ Score _____
 last first

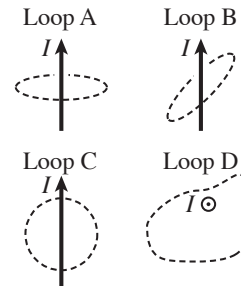
III. [20 points total] This question consists of two independent parts, A and B.

- A. A constant magnetic field with magnitude B_0 points to the right when $y > 0$ and to the left when $y < 0$. Two Amperian loops in the x - y plane are shown: loop 1 is a square with side length L_1 centered on the x -axis, while loop 2 is a square with side length L_2 , also centered on the x -axis, but rotated 45 degrees as indicated.



- i. [3 pts] Determine $\oint \vec{B} \cdot d\vec{l}_1$ (the line integral of the magnetic field around loop 1) in terms of the variables given.
- ii. [3 pts] Find the magnitude and direction of the current passing through loop 1. If the current is zero, state so explicitly. Explain.
- iii. [4 pts] Determine $\oint \vec{B} \cdot d\vec{l}_2$ (the line integral of the magnetic field around loop 2) in terms of the variables given.
- iv. [4 pts] It is observed that the current passing through each loop is the same. Determine L_2 (the side length of loop 2) in terms of L_1 and the other variables given.

- B. Four Amperian loops are shown at right around very long current-carrying wires. Loops C and D lie in the plane of the page, while loops A and B do not. The currents for loops A, B, and C point toward the top of the page; the current for loop D points out of the page.

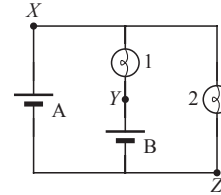


- i. [3 pts] Rank the loops in terms of the absolute value of the line integral of the magnetic field ($\int \vec{B} \cdot d\vec{l}$) around each loop. If the line integral around any loop is zero, state so explicitly. Explain.
- ii. [3 pts] Which of the loops would be easiest to use to calculate the magnetic field near a wire? Explain.

Name _____ Student ID _____ Score _____
last first

IV. [20 points total] This question consists of two separate parts, A and B.

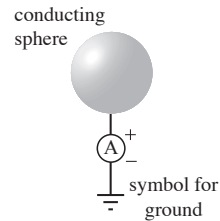
A. The circuit shown at right consists of two identical bulbs, and two ideal batteries. The voltage of battery A is greater than the voltage of battery B.



i. [5 pts] Rank points X, Y, and Z according to electric potential. Explain.

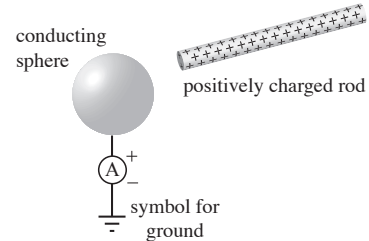
ii. [5 pts] On the diagram above, indicate the direction of the current through all four elements (A, B, 1, and 2). Explain.

B. A solid conducting sphere, of radius R , has been connected to ground through an ideal ammeter for a very long time.



i. [5 pts] Is the potential difference, ΔV_{SG} , between the sphere and ground *positive, negative, or zero*? (Note: $\Delta V_{SG} = V_S - V_G$.) Explain.

ii. [5 pts] A positively charged rod is now brought close to the sphere as shown, but held far enough away so that no spark jumps. A negative reading on the ammeter reading is briefly observed during this process.

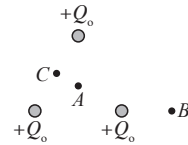


While the ammeter reading is negative, is the potential difference, ΔV_{SG} , between the sphere and ground *positive, negative, or zero*? Explain.

Name _____ Student ID _____ Score _____
last first

IV. [20 points total] This question consists of two independent parts, A and B.

A. Three identical positive point charges are arranged in an equilateral triangle. Point A is at the center of the triangle and B is outside the triangle. A and B are equally far from the bottom-right charge. Point C is halfway between the left and top charges.

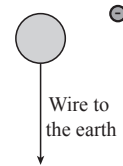


i. [6 pts] Determine whether the electric potential difference between each of the following pairs of points is *positive*, *negative*, or *zero*. Explain your reasoning in each case.

- A to B :

- A to C :

B. A neutral conducting metal sphere is connected to the earth with a wire. At time t_0 a negative point charge is brought close to the sphere as shown.



i. [5 pts] Describe what happens to the system after t_0 . Explain.

ii. [5 pts] Before the system reaches equilibrium, is the electric potential difference between the sphere and the earth *positive*, *negative* or *zero*? Explain.

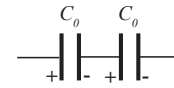
iii. [4 pts] After a long time, is the electric potential difference between the sphere and the earth *positive*, *negative*, or *zero*? Explain.

Name _____ Student ID _____
 last *first*

III.! [25 pts] Parts A and B are independent.

A.! A student charges two identical parallel-plate capacitors by connecting them one at a time across a 9-volt battery. Each capacitor is carefully disconnected from the battery so that it is not discharged. The student then connects the charged capacitors in series as shown at right, but the capacitors are *not* connected to a battery and the circuit is open.

Two original capacitors
connected in series



Suppose now that the **left** capacitor is modified such that the distance between the plates is increased, as shown below right.

After the left capacitor
is modified



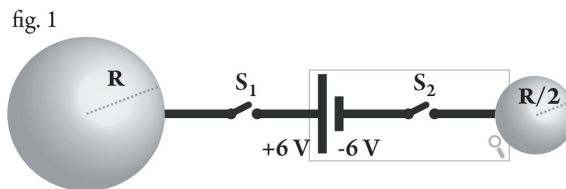
1.! [5 pts] After the change, does the charge on the **left** capacitor *increase*, *decrease*, or *remain the same*? Explain.

2.! [6 pts] After the change, does the potential difference across the **left** capacitor *increase*, *decrease*, or *remain the same*? Explain.

3.! [6 pts] After the change, does the potential difference across the **right** capacitor *increase*, *decrease*, or *remain the same*? Explain.

B.! [8 pts] Two conducting spheres of different radii are connected to a battery as shown in the figure below.

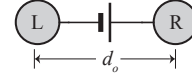
If both switches are closed simultaneously, will the spheres become charged? If so, calculate the charge on each. If not, explain why not. Assume that potential is zero at infinity.



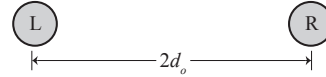
Name _____ Student ID _____ Score _____
last first

IV. [20 points total] This question consists of two independent parts, A and B.

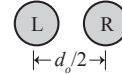
- A. Two identical neutral metal spheres on insulating stands (not shown) are placed a distance d_o apart. A battery is briefly connected between the two spheres and then removed. The potential difference from the left sphere to the right sphere is $\Delta V_{L \rightarrow R} = +\Delta V_o$.



- i. [3 pts] Find the work it would take to move a test charge q_{test} at constant speed from the right sphere to the left sphere. Express your answer in terms of the given variables.
- ii. [5 pts] The stands are now moved apart so that the two spheres are a distance $2d_o$ apart. Does $\Delta V_{L \rightarrow R}$ change? If so, how? Explain your reasoning.



The stands are moved closer together so that the spheres are a distance $d_o/2$ apart. The battery is reconnected and then removed, so $\Delta V_{L \rightarrow R} = +\Delta V_o$ again.



- iii. [6 pts] Does the total amount of charge on either sphere *increase*, *decrease*, or *remain the same* when the battery is reconnected? Explain your reasoning.

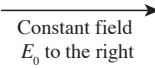
B. Storm clouds often become charged.

- i. [3 pts] Suppose you were flying in a small plane through a cloud, and the plane picked up a large amount of charge from the clouds. If you touch the aluminum wall next to your seat, would you be shocked? Explain why or why not.
- ii. [3 pts] Describe how you could determine how much charge was on the plane by measuring the electric field inside. Explain your reasoning. If it is not possible to do so, explain why not.

Name _____ Student ID _____ Score _____
last first

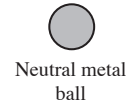
- V. [20 points total] Four experiments are conducted with a small neutral metal ball. In each case, there is a uniform electric field \vec{E}_0 pointing to the right as shown.

Constant field \vec{E}_0 to the right



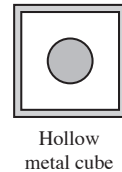
- A. In experiment 1, the ball is placed in the uniform field.

- [1 pt] On the diagram, sketch the stable charge distribution on the ball.
- [3 pts] Is the net force on the ball *to the right*, *to the left*, or *equal to zero*? Explain.



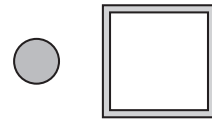
- B. [4 pts] In experiment 2 a neutral, hollow metal cube is placed around the ball as shown. The external electric field is unchanged.

- Is the net force on the ball *to the right*, *to the left*, or *equal to zero*? Explain.



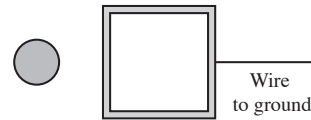
- C. In experiment 3, the cube is moved to the right of the ball. The external electric field is unchanged.

- [1 pt] Sketch the stable charge distribution on the cube.
- [3 pts] Is the net force on the ball *to the right*, *to the left*, or *equal to zero*? Explain.



- D. In experiment 4, the cube is grounded as shown. Neither the ball nor the cube are moved, and the external electric field is unchanged.

- [1 pt] Sketch the stable charge distribution on the cube.
- [3 pts] Is the net force on the ball *to the right*, *to the left*, or *equal to zero*? Explain.

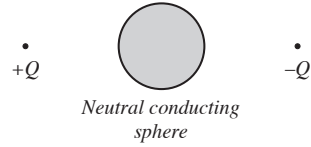


- [4 pts] How does the magnitude of the net force on the ball in experiment 4 compare to that in experiment 3? Explain.

Name _____ Student ID _____ Score _____
last first

V. [20 points total] Parts A and B of this problem are independent.

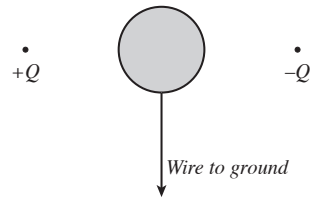
A. A neutral conducting sphere is placed halfway between two point charges, with charge $+Q$ and $-Q$ respectively.



i. [2 pts] On the diagram, sketch the induced charge distribution on the sphere.

ii. [4 pts] The sphere is now connected to ground through a wire, as shown.

When the sphere is grounded, do any charges flow to or from ground? If no charges flow or if there is not enough information, state so explicitly. Explain.



iii. [4 pts] The wire connecting the sphere to ground is now removed.

Is the electric potential difference from the sphere to ground *positive, negative, or zero*? Explain.

B. The magnetic field lines due to two wires with equal currents into the page are shown. Point 1 is equidistant from the wires. Which vector best represents the magnetic field at point 1 and at point 2?

i. [5 pts] Point 1:
 (Circle the correct answer below.)

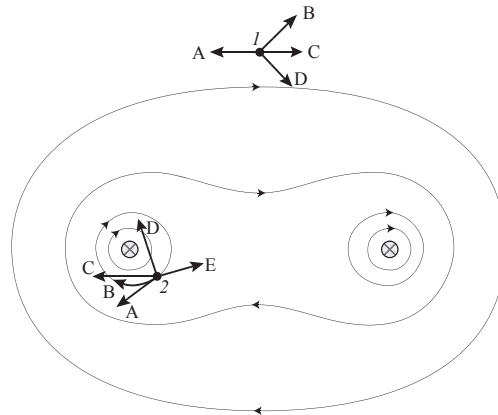
A B C D Zero

Explain.

ii. [5 pts] Point 2:
 (Circle the correct answer below.)

A B C D E

Explain.

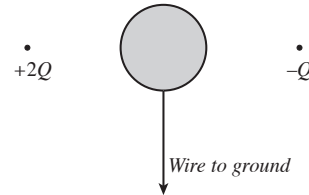


Name _____ Student ID _____ Score _____
last first

IV. [20 points total] Parts A and B of this problem are independent.

A. Two point charges are suddenly placed on either side of a grounded conducting sphere as shown.

- i. [4 pts] Just after the point charges are added to the system, do any charges flow between the sphere and ground? Explain.

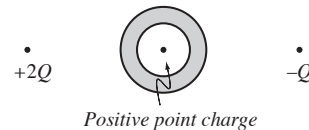


- ii. [8 pts] Is the electric potential difference from the sphere to ground *positive, negative, or zero* at the following times? Explain your reasoning for each case.

- Just after the point charges are placed near the sphere.

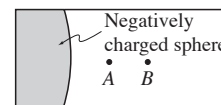
- After the charge distribution on the sphere stabilizes.

- iii. [4 pts] The grounded sphere is replaced with a neutral, hollow conducting shell as shown. A positive point charge is placed at the center of the shell.



Is there a net force on the point charge, and if so, in what direction? Explain.

- B. [4 pts] Two experiments are conducted near a large, negatively charged insulating sphere as shown at right. In experiment 1, a test particle with charge -1 nC is moved from point A to point B . In experiment 2, a test particle with charge $+2.5\text{ nC}$ is moved from point A to point B . Let ΔV be measured from the starting point to the ending point in both experiments.



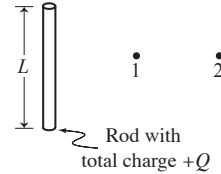
How does ΔV in experiment 1 compare to ΔV in experiment 2? Compare both magnitude and sign. Explain your reasoning.

Name _____ Student ID _____ Score _____
last first

V. [20 points total] The following questions involve a charged plastic rod. It has length L , diameter d , and a total charge $+Q$ that is uniformly distributed over its surface.

A. [4 pts] Test charge A (+1 nC) is moved by a hand from point 1 to point 2, at constant speed.

As a result of this motion, is the change in electrostatic potential energy of the system ΔU_E *positive, negative, or zero*? Explain.



B. [4 pts] Test charge B (-1.5 nC) is also moved by a hand from point 1 to point 2 at constant speed.

As a result of this motion, is ΔU_E *positive, negative, or zero*? Explain.

C. [4 pts] Suppose the electric potential difference for charge A's movement is ΔV_A , and for charge B's movement it is ΔV_B .

Is ΔV_B *greater than, less than, or equal to* ΔV_A ? Explain.

D. [4 pts] Test charge B is launched from point 1 with an initial kinetic energy of 6 J, and briefly comes to rest at point 2.

Find the electric potential difference ΔV_B for this motion.

E. [4 pts] Suppose you want to increase the magnitude of ΔV_B by moving the charged rod.

Should you move the rod to the right, closer to the points, or to the left, farther from the points? If neither of these movements will increase ΔV_B state so explicitly. Explain your reasoning.

Name _____ Student ID _____ Score _____
last first

III. [20 points total] This question consists of two independent parts, A and B.

A. Four points are shown next to a positive charge $+Q$. Points C and D are the same distance from the charge. A•

i. [7 pts] A hand moves a positive test charge $+q_{\text{test}}$ from rest at point B to rest at each of the other points. For each movement, does the hand do *positive*, *negative*, or *zero* work on the test charge? Explain your reasoning for each case. B•

• W_{BA} :



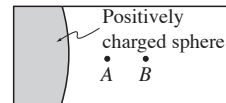
C•
D•

• W_{BC} :

• W_{BD} :

ii. [5 pts] Suppose the potential at point C was positive. Which labeled point could be the reference point for potential in this system? If more than one point is possible, state so explicitly. Explain.

B. A portion of a large, positively charged insulating sphere is shown at right. Point A is 10 cm from the surface of the sphere, and point B is 20 cm from the surface of the sphere.



i. [4 pts] A positive test charge with charge $+1 C$ is released from rest at point A . It is observed to cross point B with 3 J of kinetic energy.

What is the potential difference ΔV from point A to point B in this case? Show your work.

ii. [4 pts] The positive test charge is replaced with a negative test charge with charge -2.5 nC . A hand moves the test charge from point A to point B .

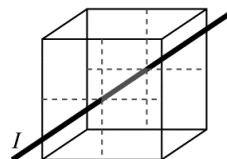
How does the magnitude and sign of the ΔV when the negative test charge moves compare to the magnitude and sign of ΔV when the positive test charge moves? Explain your reasoning.

AMPÈRE'S LAWEM
57**I. Magnetic flux**

The magnetic flux through a set of imaginary surfaces dA_i with normal vectors \hat{n}_i is given by

$$\Phi_B = (\vec{B}_1 \cdot \hat{n}_1)dA_1 + (\vec{B}_2 \cdot \hat{n}_2)dA_2 + (\vec{B}_3 \cdot \hat{n}_3)dA_3 + \dots$$

- A. Consider a cubical Gaussian surface around a current-carrying wire. The wire goes through the centre of the front and back faces, as shown at right. The direction of the current is from the back of the cube to the front.



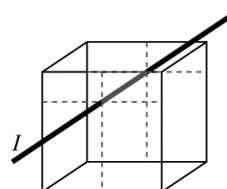
What is the value of the magnetic flux through each of the faces of the cube? Explain in each case.

Face	Φ_B (N·m/A)	Explanation
front		
back		
left		
right		
top		
bottom		

- B. The Gaussian surface is lowered as shown at right. The magnitude of the magnetic flux through the left face of the cube is $2 \text{ N}\cdot\text{m}/\text{A}$.

What is the sign of the magnetic flux through the left face? Explain.

What is the sign and magnitude of the magnetic flux through the right face? Explain.



- C. In the *Gauss' Law* tutorial, you saw that Gauss' Law can give useful information about the *electric* field for a given charge distribution.

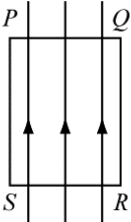
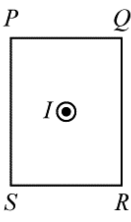
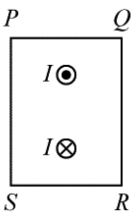
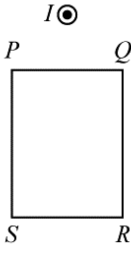
Can you use Gauss' Law to get information about the *magnetic* field for a given current distribution? Explain.

EM
58 **AMPÈRE'S LAW**

II. The line integral of the magnetic field

Consider the four cases, A–D, below. In case A, the centre of a hypothetical rectangular loop $PQRS$ is located such that the magnetic field lines are parallel to two sides of the rectangle. In cases B–D, the currents I in and out of the page are perpendicular to the area bounded by the loop.

- A. What is the magnetic flux through each loop? Explain.
- B. Are the following line integrals *positive, negative, or zero*? Base your answers for the circulation of the magnetic field, taken in the clockwise direction in each case, on the line integrals you considered. If it is not possible to find an answer for the circulation in this way, state so explicitly.

	 <p>Case A</p>	 <p>Case B</p>	 <p>Case C</p>	 <p>Case D</p>
$\int_P^Q \vec{B} \cdot d\vec{l}$				
$\int_Q^R \vec{B} \cdot d\vec{l}$				
$\int_R^S \vec{B} \cdot d\vec{l}$				
$\int_S^P \vec{B} \cdot d\vec{l}$				
$\oint \vec{B} \cdot d\vec{l}$				

- C. Consider the following statement about the circulation of the magnetic field:
- “The work done by the magnetic field when I move a test charge around the loop is zero, so the circulation of the magnetic field must be zero in all four cases.”*
- Do you agree with this statement? Explain.

AMPÈRE'S LAWEM
59**III. Ampère's Law**

Ampère's Law states that the circulation (*i.e.*, the line integral around a *closed* loop) of the magnetic field around a loop is directly proportional to the net current piercing the surface enclosed by the loop, called the enclosed current:

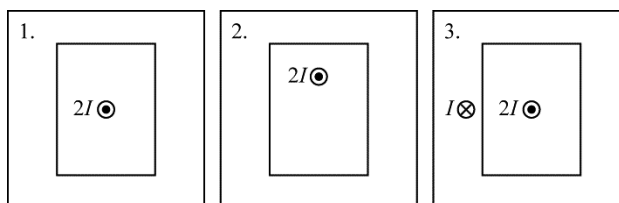
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}.$$

Unless stated otherwise, we will choose the direction of the circulation such that it is in agreement with the right-hand rule.

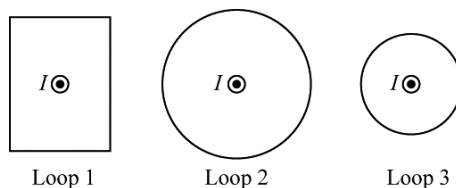
- A. Check that your answers to Part B of Section II are consistent with Ampère's Law. If you were unable to complete the table, does Ampère's Law allow you to do this now? Explain.

What role did the right-hand rule play in your answer?

- B. What is the value of the circulation of the magnetic field for each of the loops below?



- C. Three students discuss three possible loops around a current carrying wire shown below. Loops 1 and 2 have the same length.



- Student 1: "Loop 2 has the same length as Loop 1. In Loop 2, the magnetic field always points along the path, but in Loop 1, only a component of the magnetic field is along the path. So the circulation around Loop 2 is greater."
- Student 2: "The circulation around Loop 3 is greater than around Loop 2, because the magnetic field is stronger."
- Student 3: "The circulation around Loop 3 is less than around Loop 2, because the path is shorter."

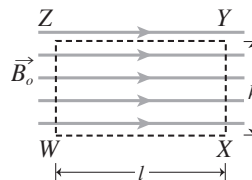
Comment on each statement.

AMPERE'S LAW

EM
1**I. Line integrals around closed loops**

A *line integral* is an integral of a function along a defined path, in a specified direction. The line integral of a vector \vec{V} along a path from point 1 to point 2 is written as $\int_1^2 \vec{V} \cdot d\vec{l}$. The placement of the 1 and 2 signifies that the line element $d\vec{l}$ is directed along the path from point 1 to point 2.

A. As shown at right, a magnetic field \vec{B}_o is pointing to the right through a rectangular surface with length l and height h .



1. Find $\int_W^X \vec{B} \cdot d\vec{l}$, the line integral of the magnetic field from point W to point X, in terms of the given variables.

2. Find the line integral along the paths shown connecting the following points.

X to Y:

Y to Z:

Z to W:

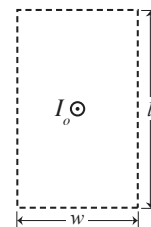
3. Find $\oint_{\text{loop WXYZ}} \vec{B} \cdot d\vec{l}$, the line integral around the entire loop.

II. The line integral of the magnetic field

In the following questions, consider an imaginary rectangular loop. The loop borders a surface with length l and width w and is placed near one or more wires all with the same current I_o . Consider each line integral of the magnetic field using a *counterclockwise* path.

- A. A single wire with current out of the page passes through the loop.

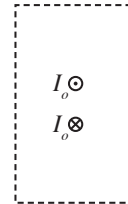
Is the sign of the line integral of the magnetic field around the entire loop *positive*, *negative*, or *zero*? (*Hint*: Consider the sign of the line integral around each side, one at a time). Explain.



EM *Ampere's Law*
2

- B. Two symmetrically placed wires, one with current into the page and one with current out of the page, pass through the loop.

Is the sign of the line integral of the magnetic field around the entire loop *positive*, *negative*, or *zero*? (*Hint*: Consider the effect of both wires on each side, one at a time). Explain.



- C. A single wire with current out of the page is above the loop.

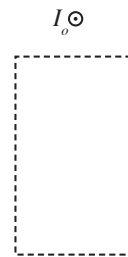
For each side of the loop, state whether the line integral is *positive*, *negative*, or *zero*. Explain.

The top side:

The bottom side:

The left side:

The right side:



Do you have enough information to determine the sign of the line integral around the entire loop? Explain why or why not.

- D. Suppose you evaluated the line integral in a *clockwise* direction instead. How, if at all, would your answers to parts A through C above change? Explain.

⇒ Check your results for part II with a tutorial instructor before continuing.

©Tutorials in Introductory Physics, Physics Education Group, Department of Physics
University of Washington (Spring 2015)

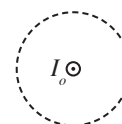
III. Ampère's law

Ampère's law states that the line integral of the magnetic field around a closed loop (often called an Amperian loop) is directly proportional to the current encircled by the loop:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}, \text{ where } \mu_0 \text{ is a constant.}$$

The *encircled current* is the *net* current passing through the surface bounded by the loop (*i.e.*, the amount of charge per unit time passing from one side of the loop to the other). The sign of the encircled current is defined by convention to be positive if the current passes through the loop in the same direction as the area vector.

- A. Consider the Ampèrian loop around the current-carrying wire shown at right. In the table below, determine the sign of the encircled current for each direction of area vector and the sign of the line integral of the magnetic field for each direction around the loop.



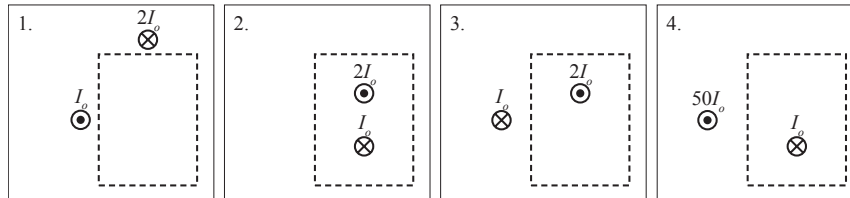
What is the sign of the encircled current if the area vector is:	Into the page?	Out of the page?
What is the sign of the line integral if the path is taken to be:	Clockwise?	Counterclockwise?

The table above shows that the direction of the area vector affects the sign of the right-hand side of Ampère's law, and the direction used for the line integral affects the sign of the left-hand side.

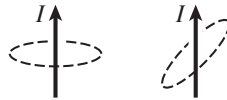
- To make Ampère's law consistent on each side, which direction should be used for the line integral if the area vector is out of the page?
 - Devise a rule by which you can use your right hand to identify the direction that should be used for the line integral around a loop based on your choice of area vector for the loop.
- B. If you have not done so already, use Ampère's Law to find the line integral of the magnetic field in part C of section II.

EM
4 *Ampere's Law*

- C. Use Ampère's law to find the line integral of the magnetic field around each of the loops below.



- D. Two identical Ampèrian loops around a wire with current I are shown below. Consider the following student discussion.



- Student 1: "I think the line integral of the magnetic field should be smaller in the right loop since the current is crossing the loop at an angle compared to the left loop."
- Student 2: "No, it should be larger. Since the right loop is tilted, the top and bottom of the loop are closer to the wire than the left loop, and the magnetic field is stronger close to the wire. That means the line integral is greater in the right loop."
- Student 3: "The amount of charge passing through the right loop per second is the same as the left one, even though they're crossing the loops in different directions. The encircled current is the same, so the line integral will be the same."

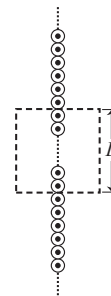
Do you agree with any of the students? Explain how the reasoning of the other students is incorrect.

⇨ Check your results for part III with a tutorial instructor before continuing.

IV. Applications of Ampère's law

A. A square Ampèrian loop with length L is centered on a large conducting sheet, which carries current out of the page. We can model this sheet as a row of parallel wires, each with a current I , shown below. The density of wires (number of wires per unit length) is n .

- In terms of n and other relevant quantities, how much current is encircled by the Ampèrian loop?
- Sketch magnetic field lines on both sides of the sheet. Use the principle of superposition to determine your answer.
- Let B_L and B_R represent the magnitudes of the magnetic field at the left and right sides of the square.



How do B_L and B_R compare? Explain.

- Along which of the sides of the Ampèrian loop is there a non-zero line integral? Explain using a sketch showing the orientation of the magnetic field vector and the line element vector.

Write an expression for the line integral around the entire loop in terms of L , B_L , and B_R .

Use the relationship between B_L and B_R to simplify your expression for the line integral around the entire loop.

- Ampère's law relates the line integral around an Ampèrian loop (which you found in part 4) to the net current encircled by the loop (which you found in part 1). Use this relationship to find the magnitude of the magnetic field at the left side of the square in terms of I .

What is the magnetic field at the right side of the square?

Does the magnetic field near a large sheet of current depend on the distance from the sheet? Use your results above to justify your answer.

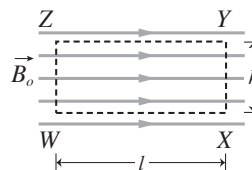
↔ Check your results for this page with a tutorial instructor.

AMPERE'S LAW

EM
1**I. Line integrals around closed loops**

A *line integral* is an integral of a function along a defined path, in a specified direction. The line integral of a vector \vec{V} along a path from point 1 to point 2 is written as $\int_1^2 \vec{V} \cdot d\vec{l}$. The placement of the 1 and 2 signifies that the line element $d\vec{l}$ is directed along the path from point 1 to point 2.

A. A uniform magnetic field \vec{B}_0 is pointing to the right. The rectangular loop shown at right has length l and height h .



- Find $\int_W^X \vec{B} \cdot d\vec{l}$, the line integral of the magnetic field from point W to point X , in terms of the given variables.

- Find the line integral along the paths shown connecting the following points.

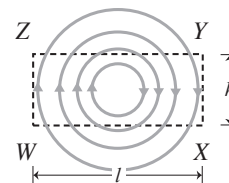
X to Y :

Y to Z :

Z to W :

- Find $\oint_{\text{loop } WXYZ} \vec{B} \cdot d\vec{l}$, the line integral around the entire loop.

B. The rectangular loop is now placed in a circular field as shown.



- Consider the following *incorrect* statement:

"When field lines go out of the loop the dot product is positive, and when they go in the dot product is negative. Every field line that leaves the loop also re-enters the loop, so the line integral around the loop is zero."

Explain the error(s) in this statement.

- Is the line integral around the entire loop *positive*, *negative*, or *zero*? Explain.

EM *Ampere's Law*

2

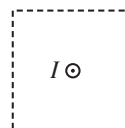
II. The line integral of the magnetic field

In each case below, a square loop is placed near one or more wires, all carrying the same current I . Consider each line integral of the magnetic field using a *counterclockwise* path.

- A. Case 1: A single wire with current out of the page passes through the loop.

Find the sign of the line integral along each side:

Top: Bottom: Left: Right:

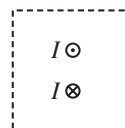


In the first column of the table below, indicate whether the line integral around the entire loop is *positive*, *negative*, or *zero*.

- B. Case 2: Two symmetrically placed wires, one with current into the page and one with current out of the page, pass through the loop.

Find the sign of the line integral along each side:

Top: Bottom: Left: Right:

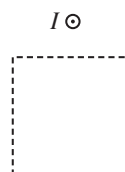


In the first column of the table below, indicate whether the line integral around the entire loop is *positive*, *negative*, or *zero*.

- C. Case 3: A single wire with current out of the page is above the loop.

Find the sign of the line integral along each side:

Top: Bottom: Left: Right:



Do you have enough information to determine the sign of the line integral around the entire loop? Explain why or why not.

- D. Suppose you evaluated the line integral in a *clockwise* direction instead. In the second column of the table below, indicate the sign of the line integral around each loop in this case. Do not fill in the last two columns now; you will be asked to do so on the next page.

	$\oint \vec{B} \cdot d\vec{l}$, taken:		Encircled current, with \vec{A} :	
	Counterclockwise	Clockwise	Into the page	Out of the page
Case 1:				
Case 2:				
Case 3:				

Ampère's law states that the line integral of the magnetic field around a closed loop (often called an Ampèrian loop) is directly proportional to the net current encircled by the loop:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}, \text{ where } \mu_0 \text{ is a constant.}$$

The sign of the encircled current is defined to be positive if the current passes through the loop in the same direction as the area vector.

- E. If you have not done so already, use Ampère's law to determine the line integral around the loop in case 3 on the previous page, and write your answers in the table.
- F. In the last two columns of the table on the previous page, indicate the value and sign of the encircled current through each loop, for each direction of the area vector.
1. How does changing the choice of area vector affect the sign of the enclosed current?
 2. How does changing the choice of a clockwise or counterclockwise path affect the sign of the line integral?
 3. To make the sign of Ampère's law consistent on both sides of the equation, which direction should be used for the line integral if the area vector is out of the page?
 4. Devise a rule by which you can use your right hand to identify the direction that should be used for the line integral around a loop based on your choice of area vector for the loop.

↔ Check the results in your table with a tutorial instructor before continuing.

EM *Ampere's Law*

4

- G. Some number of current-carrying wires (not shown) creates a magnetic field around the Amperian loop shown. The clockwise line integral of the magnetic field around the loop is $2\mu_0 I$.



1. Consider the following student dialogue:

Student 1: "The line integral is positive, so current is in the same direction as the area vector. There must be two wires crossing through the loop, each carrying a current I into the page."

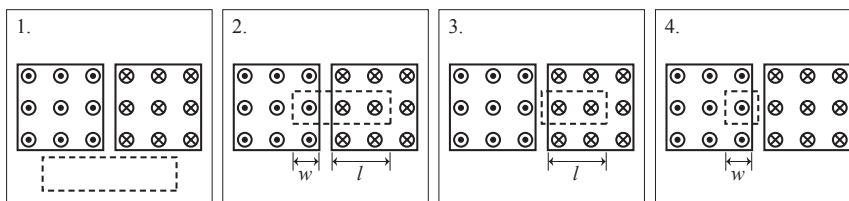
Student 2: "I disagree. The area vector should be out of the page, so current is flowing out of the page"

Student 3: "The line integral clockwise around the loop is positive, so current should be flowing clockwise around the loop."

All three students are incorrect. Describe the errors in their reasoning.

2. What is the correct conclusion that can be drawn about this system? Explain.

- H. Each square wire below has a constant current density of 2 A/m^2 . Use Ampère's law to find the line integral of the magnetic field around each of the loops below, each with height h .

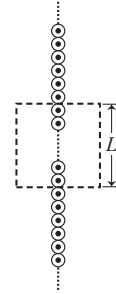


⇨ Check your results for part II with a tutorial instructor before continuing.

III. Applications of Ampère's law

A. A square Ampèrian loop with length L is centered on a large conducting sheet, which carries current out of the page. We can model this sheet as a row of parallel wires, each with a current I , shown below. The density of wires (number of wires per unit length) is n ; many such wires pass through the square loop.

- In terms of n and other relevant quantities, how much current is encircled by the Ampèrian loop?
- Sketch magnetic field lines on both sides of the sheet. Use the principle of superposition to determine your answer.
- Let B_L and B_R represent the magnitudes of the magnetic field at the left and right sides of the square.



How do B_L and B_R compare? Explain.

- Along which of the sides of the Ampèrian loop is there a non-zero line integral? Explain using a sketch showing the orientation of the magnetic field vector and the line element vector.

Write an expression for the line integral around the entire loop in terms of L , B_L , and B_R .

Use the relationship between B_L and B_R to simplify your expression for the line integral around the entire loop.

- Ampère's law relates the line integral around an Ampèrian loop (which you found in part 4) to the net current encircled by the loop (which you found in part 1). Use this relationship to find the magnitude of the magnetic field at the left side of the square in terms of I .

What is the magnetic field at the right side of the square?

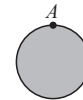
Does the magnetic field near a large sheet of current depend on the distance from the sheet? Use your results above to justify your answer.

ELECTRIC PROPERTIES OF CONDUCTORS

EM
1**I. Charged conductors**

A *conductor* is a material in which positive and negative charges are free to move.

A. Shown at right is a neutral conducting sphere. The sphere has been undisturbed for a long time, and is in electrostatic equilibrium. Many positive charges, with total charge $+Q_0$, are placed at point A on the sphere.



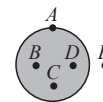
1. Will the charges remain at point A? Explain your reasoning.

2. A long time later, the sphere is in equilibrium and we can assume that no charges are moving. What does this imply about the magnitude of the electric force on any charge in the sphere? Explain.

3. What is the net electric field inside the sphere?

4. How are the positive charges distributed on the sphere? Explain.

5. Suppose you move a test charge from B to C. What is the electric potential difference from B to C? Explain. (Recall that $\Delta V_{B \rightarrow C} = \Delta U_E / q_{\text{test}}$, where ΔU_E is the change in electrostatic potential energy when the test charge moves from B to C).



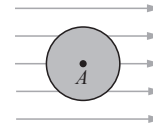
An *equipotential* is a set of points which have no electric potential difference between each other.

(i.e. $\Delta V_{i \rightarrow j} = 0$ for all points i and j in the equipotential).

6. Which of the points A-E are part of the same equipotential? Explain.

EM
2 *Electric properties of conductors*

B. An identical neutral conducting sphere is placed in an external electric field as shown.



1. Draw a vector at point *A* representing the external electric field. Explain.

2. At equilibrium, what is the net electric field at point *A*? Explain.

3. Two students discuss their answers to questions 1 and 2 above.

Student 1: "The electric field inside a conductor is zero, so the external electric field at *A* is zero."

Student 2: "No, we know the *net* electric field is zero. The external field points to the right, so there must be another field to the left that cancels it."

Student 1: "No, conductors block electric fields. The external field can't go through the metal."

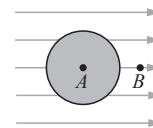
With which student, if either, do you agree? Explain.

Is your answer consistent with your answers to questions 1 and 2? If not, resolve any inconsistencies.

4. Draw a vector at point *A* representing the *induced electric field* of the sphere. How will the direction and magnitude of this field compare to that of the external field? Explain.

5. How do the positive and negative charges in the sphere have to be arranged to produce the induced electric field? Draw the induced charge distribution on the sphere.

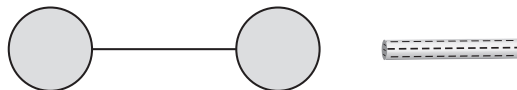
6. Point *B* is outside the sphere, but is still within the external electric field. Will the magnitude of the net electric field at point *B* *increase*, *decrease*, or *remain the same* due to the induced charge distribution? Explain.



⇨ Discuss your responses to this page with a tutorial instructor.

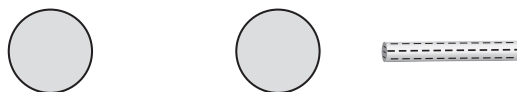
C. Two neutral conducting spheres are connected by a conducting wire. Then a negatively charged rod is brought close to the right sphere (but no sparks jump from the rod). The wire is small enough that the amount of charge on it is negligible.

1. Sketch the final charge distribution on each sphere below.



2. Is the electric potential difference $\Delta V_{L \rightarrow R}$ between the left sphere and the right sphere positive, negative, or zero? Explain. (*Hint*: How is this situation similar to part B on the previous page?)

The wire is now removed from the spheres. The rod remains in place and no charges move.



3. Predict whether $\Delta V_{L \rightarrow R}$ is positive, negative, or zero after the wire is removed. (*Hint*: Consider moving a test charge along the path where the wire used to be.)

4. Two students discuss their predictions to question 3 above.

Student 1: "The left sphere is negatively charged, and the right sphere is positive. If we move a positive test charge from the left sphere to the right the potential energy will increase, so the potential difference is positive."

Student 2: "Before the wire was removed, the potential difference between the two spheres was zero since they were part of the same conductor. No charges moved when the wire was removed, so the potential difference is still zero."

Discuss these statements with your partners. With which student do you agree?

5. Check that your prediction in question 3 is consistent with your answer to question 4. Resolve any inconsistencies.

⇒ Discuss your responses to part C with a tutorial instructor before you continue.

EM *Electric properties of conductors*
4

D. In this tutorial we've seen that the electric field inside a conductor is zero at equilibrium, after the conductor is undisturbed for a long time. However that is not necessarily true for some time intervals.

1. Suppose we consider the situation from part A again. A large amount of positive charges are placed at A at time t_0 . At that instant, before the charges have begun to move, what is the direction of the electric field at point B ? Explain.

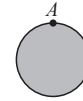


2. Draw the electric field inside the conductor at t_0 before charges have moved.
3. Which direction will a positive test charge at point B experience a force? Explain.
4. Describe the motion of the positive charges after t_0 . When will charges stop moving?
5. Under what conditions can we assume that the electric field inside a conductor is zero? Explain.

ELECTRIC PROPERTIES OF CONDUCTORS

EM
1**I. Charged conductors**

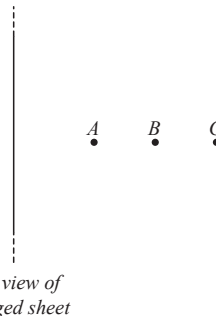
A. Shown at right is a neutral conducting sphere. The sphere has been undisturbed for a long time, and is in electrostatic equilibrium. Many positive charges, with total charge $+Q_0$, are placed at point A on the sphere.



1. Will the charges stay there? Explain why or why not.
2. After the sphere comes to equilibrium, how will the charges be arranged on the sphere? Explain your reasoning.
3. What is the net electric field inside the sphere?
(*Hint: Consider a Gaussian surface inside the sphere. How much charge is enclosed by it?*)
4. Is the potential difference between the left side of the sphere and the right side *positive*, *negative*, or *zero*? Explain. (Recall from last tutorial that electric potential difference is the ratio $\Delta U_E / q_{\text{test}}$.)

B. An infinite sheet has uniform charge density $+\sigma_0$.

1. On the diagram, draw electric field vectors at each of the marked points. How does the magnitude of the field at each of the points compare?
2. Is the potential difference between B and C *positive*, *negative*, or *zero*?



EM
2 *Electric properties of conductors*

A neutral metal sphere is placed at point A.

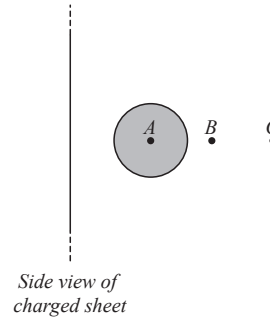
3. What is the *net* electric field at point A after the sphere has been placed?

4. Two students discuss their answers above.

Student 1: "The field due to the sheet was to the right. The sphere is neutral, so it doesn't affect the electric field at all."

Student 2: "But we know that the net electric field in a conductor is zero. There must be another electric field to the left that cancels the external field."

With which student, if either, do you agree?

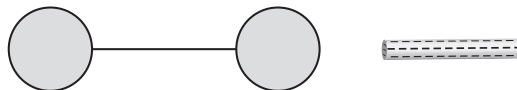


5. Draw a vector at point A representing the *induced electric field* of the sphere. How will the magnitude and direction of this field compare to that of the charged sheet?
6. How do the positive and negative charges in the sphere have to be arranged to produce the induced electric field? Draw the induced charge distribution on the sphere.
7. At point B, outside the sphere, will the magnitude of the net electric field be *larger*, *smaller*, or *the same as* what it was before the sphere was added? Explain.
8. Is the magnitude of the potential difference between B and C *larger*, *smaller*, or *the same as* what it was before the sphere was added? Explain.

⇒ Discuss your responses to this page with a tutorial instructor.

- C. Two neutral conducting spheres are connected by a conducting wire. Then a negatively charged rod is brought close to the right sphere (but no sparks jump from the rod). The wire is small enough that the amount of charge on it is negligible.

1. Sketch the final charge distribution on each sphere below.



2. Is the electric potential difference $\Delta V_{L \rightarrow R}$ between the left sphere and the right sphere positive, negative, or zero? Explain. (*Hint: What is the electric field inside the wire?*)

The wire is now removed from the spheres. The rod remains in place and no charges move.



3. Predict whether $\Delta V_{L \rightarrow R}$ is positive, negative, or zero after the wire is removed. (*Hint: Consider moving a test charge along the path where the wire used to be.*)

4. Two students discuss their predictions to question 3 above.

Student 1: "The left sphere is negatively charged, and the right sphere is positive. If we move a positive test charge from the left sphere to the right the potential energy will increase, so the potential difference is positive."

Student 2: "Before the wire was removed, the potential difference between the two spheres was zero since they were part of the same conductor. No charges moved when the wire was removed, so the potential difference is still zero."

Discuss these statements with your partners. With which student do you agree?

5. Check that your prediction in question 3 is consistent with your answer to question 4. Resolve any inconsistencies.

⇔ Discuss your responses to part C with a tutorial instructor before you continue.

EM *Electric properties of conductors*
4

D. In this tutorial we've seen that the electric field inside a conductor is zero at equilibrium, after the conductor is undisturbed for a long time. However that is not necessarily true for some time intervals.

1. Suppose we consider the situation from part A again. A large amount of positive charges are placed at A at time t_0 . At that instant, before the charges have begun to move, what is the direction of the electric field at point B ? Explain.



2. If you placed a test charge at point B , would it experience a force?
3. Is the potential difference between A and B *positive*, *negative*, or *zero*? Explain.
4. Describe the motion of the positive charges after t_0 . When will charges stop moving?
5. Under what conditions can we assume that the electric field inside a conductor is zero? Explain.

ELECTRIC PROPERTIES OF CONDUCTORS

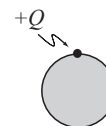
EM
1**I. Charged conductors**

A. Shown at right is a solid metal sphere with positive charge $+Q$. A positive test charge $+q$ is moved from point A to point B . Consider the system of the metal sphere and test charge.



1. When the test charge moves from point A to point B , is the change in electric potential energy, ΔU_E , *positive*, *negative*, or *zero*? Explain.
2. Is the electric potential difference $\Delta V_{A \rightarrow B}$ from point A to point B *positive*, *negative*, or *zero*?
3. Suppose instead that a *negative* test charge were moved from point A to point B . Would the electric potential difference be *positive*, *negative*, or *zero*? Explain.

B. A second metal sphere is initially uncharged. At time t_0 many positive charges with total charge $+Q$ are placed at the top as shown at right.



1. At the instant the charges are placed (before any charges have had time to move), is the potential difference $\Delta V_{T \rightarrow B}$ from the top to the bottom of the sphere *positive*, *negative*, or *zero*?
2. Is this charge distribution stable, or will it change in time? Explain.
3. If it is not stable, sketch the stable charge distribution on the diagram above.
4. Two students discuss the electric field of the stable distribution they found:

Student 1: "In a stable distribution no charges are moving. This means that inside the sphere the net force on each charge must be zero so the electric field inside must also be zero."

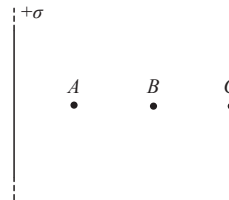
Student 2: "I was thinking about a point near the left side of the sphere. The electric field from the charges on the left will be much stronger than from the charges on the right, so the net field inside points towards the center."

With which student, if either, do you agree? Explain.

EM
2 *Electric properties of conductors*

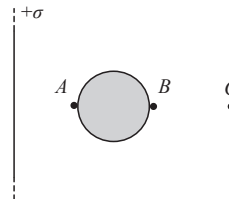
C. An infinite sheet has uniform charge density $+\sigma$ as shown at right.

1. A positive test charge is moved from point A to point B . Does the electric potential energy *increase*, *decrease*, or *remain constant*?



The test charge is removed and a neutral metal sphere is placed near the sheet as shown.

2. On the diagram, draw the stable charge distribution.
3. Sketch the electric field inside the sphere due to the charges on the sphere.
4. What is the *net* electric field inside the sphere? Explain.

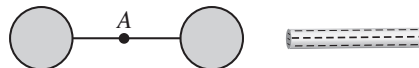


5. Is the magnitude of the electric potential difference from point A to point B *greater than*, *less than*, or *equal to* what it was before the sphere was added? Explain.
6. Does the magnitude of the net electric field at point C *increase*, *decrease*, or *remain the same* when the sphere is added? Explain.
7. Is the magnitude of the electric potential difference from point B to point C *greater than*, *less than*, or *equal to* what it was before the sphere was added? Explain.

⇔ Discuss your responses to part I with a tutorial instructor before you continue.

II. Connected conductors

A. Two neutral conducting spheres are connected by a conducting wire. At time t_0 a negatively charged rod is placed to the right of the spheres as shown.



1. At time t_0 , before any charges can move, is the electric potential difference $\Delta V_{L \rightarrow R}$ from the left sphere to the right sphere *positive*, *negative*, or *zero*? Explain.
2. Draw the stable charge distribution. Once this distribution is established, is $\Delta V_{L \rightarrow R}$ *positive*, *negative*, or *zero*?

The wire is now removed from the spheres. No other changes are made to the system.



3. Is the magnitude of the electric field at point A *greater than*, *less than*, or *equal to* what it was before the wire was removed? Explain.
 4. Is $\Delta V_{L \rightarrow R}$ positive, negative, or zero? Explain.
- The negative rod is now removed, but the spheres are left undisturbed. The charges on the spheres are allowed to reach a stable distribution.
5. Is the magnitude of the electric field at point A *greater than*, *less than*, or *equal to* what it was before the rod was removed?
 6. Is $\Delta V_{L \rightarrow R}$ positive, negative, or zero? Explain.
 7. If the wire were replaced would any charges move? Describe the final charge distribution.

⇔ Discuss your responses to part II with a tutorial instructor before you continue.

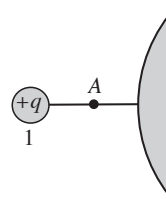
EM *Electric properties of conductors*

4

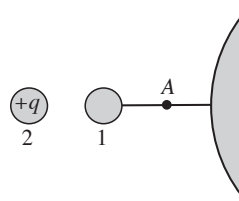
III. Conductors and ground

A *ground* is a conducting object that can donate or accept any amount of charge to another conductor. No matter how many charges flow to or from ground, the ground has no electric field anywhere in space.

- A. A small, metal sphere (sphere 1) with positive charge $+q$ is connected to a very large, neutral conducting sphere (a ground) as shown.



1. Describe what happens when the two spheres are connected.
 2. Once a stable distribution is established, what is the net charge on sphere 1? Explain.
 3. Is the electric potential difference $\Delta V_{1 \rightarrow L}$ between sphere 1 and the large sphere *positive*, *negative*, or *zero*? Explain.
- B. Another positively charged sphere, sphere 2, is brought near the grounded sphere as shown below.



1. Immediately after sphere 2 is placed, what is the net electric field at point A?
2. Once a stable distribution is established, what is the net charge on sphere 1? Explain.

3. Consider the following student dialogue:

Student 1: "When conductors are grounded they lose their charge. Sphere 1 is grounded both before and after sphere 2 is added, so sphere 1 remains neutral."

Student 2: "However the second sphere is positively charged, so negative charges will be attracted to it. Thus sphere 1 will become negatively charged."

With which student, if either, do you agree? Explain.

Check that your answer is consistent with your responses to question 2 above.

©Tutorials in Introductory Physics, Physics Education Group, Department of Physics
University of Washington (Fall 2014)

ELECTRIC PROPERTIES OF CONDUCTORS

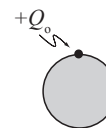
EM
1**I. Charged conductors**

A. Shown at right is a solid metal sphere with positive charge $+Q_0$. A positive test charge $+q_0$ is moved from point A to point B. Consider the system of the metal sphere and the test charge.



1. When the test charge moves from point A to point B, is the change in electric potential energy ΔU_E positive, negative, or zero? Explain.
2. Recall from the tutorial *Electric potential difference* that electric potential difference is defined as $\Delta U_E/q_{\text{test}}$. Is $\Delta V_{A \rightarrow B}$ positive, negative, or zero?
3. Would your answer to question 2 above change if a *negative* test charge were moved from point A to point B? Explain.

B. A second metal sphere is initially uncharged. At time t_0 many positive charges with total charge $+Q_0$ are suddenly placed on the sphere as shown at right.

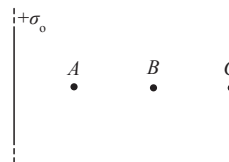


1. Sketch the electric field in the sphere at time t_0 (before the positive charges or any charges in the sphere have had time to move).
2. At t_0 is the potential difference from the top of the sphere to the bottom of the sphere $\Delta V_{T \rightarrow B}$ positive, negative, or zero?
3. Is this charge distribution stable, or will it change in time? If it is not stable, explain why not and describe a stable distribution.
4. Consider the following student statement about the stable distribution on the sphere.
 "A stable charge distribution isn't changing in time. This means that inside the sphere the net force on each charge must be zero. Thus the electric field inside must also be zero."
 Do you agree with this student's statement? Explain why or why not.
5. Is your answer consistent with the charge distribution you described in question 3 above? If not, resolve any inconsistencies.

EM
2 *Electric properties of conductors*

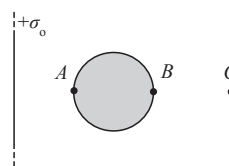
C. An infinite sheet has uniform charge density $+\sigma_0$ as shown.

1. A positive test charge is moved from point A to point B . Is $\Delta V_{A \rightarrow B}$ *positive, negative, or zero*? Explain.



The test charge is removed and a neutral metal sphere is placed near the sheet as shown.

2. On the diagram, draw a possible stable charge distribution.
3. Draw an arrow to indicate the direction of the following electric fields at the center of the sphere. If any of the electric fields are zero, state so explicitly.

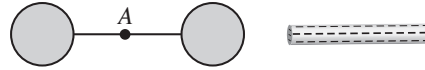


- The electric field due *only* to charges on the sheet.
 - The electric field due *only* to the charges on the sphere.
 - The *net* electric field.
4. Consider the following student statement about the metal sphere.
 "The electric field inside a conductor is zero. The metal blocks the external electric field from penetrating into the sphere."
 Do you agree with this student's statement? Explain why or why not.
 5. Is the absolute value of $\Delta V_{A \rightarrow B}$ *greater than, less than, or equal to* what it was before the sphere was added? Explain.
 6. Does the magnitude of the net electric field at point C *increase, decrease, or remain the same* when the sphere is added? Explain.
 7. Based on your answers above, does the induced charge distribution on the metal sphere affect the electric field and ΔV inside the sphere, outside the sphere, or both?

⇔ Discuss your responses to part I with a tutorial instructor before you continue.

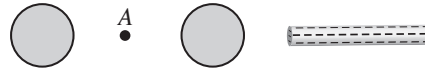
II. Application: connected conductors

A. Two neutral conducting spheres are connected by a conducting wire. At time t_0 , a negatively charged rod is placed to the right of the spheres as shown.



1. At time t_0 , before any charges move, what is the direction of the electric field at point A?
2. At time t_0 is the electric potential difference from the left sphere to the right sphere $\Delta V_{L \rightarrow R}$ positive, negative, or zero? Explain.
3. Draw the stable charge distribution. Once this distribution is established, is $\Delta V_{L \rightarrow R}$ positive, negative, or zero?

B. The wire is now removed from the spheres, and no other changes are made to the system. Assume the net charge on the wire is small enough to be ignored.



1. Is the magnitude of the electric field at point A zero or non-zero after the wire is removed? Explain.
 2. Is $\Delta V_{L \rightarrow R}$ zero or non-zero? Explain.
- C. The negatively charged rod is now removed, but the spheres are left undisturbed. The charges on the spheres are allowed to reach a stable distribution.
1. In the space below, sketch the stable charge distribution on the two spheres. Explain your reasoning.
 2. After the rod is removed, does the magnitude of the electric field at point A increase, decrease, or remain the same? Explain.

⇔ Discuss your responses to part II with a tutorial instructor before you continue.

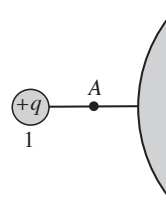
EM *Electric properties of conductors*

4

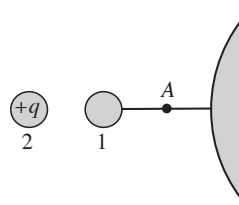
III. Application: ground

A *ground* is a conducting object that we assume can give or take any amount of charge to or from another conductor. As part of this assumption we assume the electric field due to the ground is negligible.

- A. A small metal sphere, sphere 1, with positive charge $+q$ is connected to a very large, neutral conducting sphere as shown. Assume the sphere can be treated as a ground.



1. The instant that the spheres are connected, before any charges have moved, what is the direction of the net electric field at point A ? Explain.
 2. Describe what happens when the two spheres are connected. Explain.
 3. After a stable charge distribution is reached, what is the net charge on sphere 1? Explain.
 4. After the charge distribution is stable, is the electric potential difference from sphere 1 to the large sphere *positive*, *negative*, or *zero*? Explain.
- B. Another positively charged sphere, sphere 2, is placed near the grounded sphere as shown below.



1. The instant that sphere 2 is placed, before any charges have moved, what is the direction of the net electric field at point A ? Explain.
2. Once a stable distribution is established, is the net charge on sphere 1 *positive*, *negative*, or *zero*? Explain.

-
3. Two students discuss their answers to question 2 above.

Student 1: "When conductors are grounded they lose their charge. Sphere 1 is grounded both before and after sphere 2 is added, so sphere 1 remains neutral."

Student 2: "I disagree. The second sphere is positively charged, so negative charges will be attracted to it. Thus sphere 1 will become negatively charged."

With which student, if either, do you agree? Explain.

Check that your answer is consistent with your response to question 2 above.

4. A third student joins the discussion.

Student 3: "After the charge distribution is stable, the potential difference between sphere 1 and the large sphere is zero. How can ΔV be zero if sphere 1 is charged?"

How would you address student 3's concerns? Explain how you could help this student resolve their confusion.

5. Based on your answers above, what can you say about the net charge on two connected conductors if you only know that the potential difference between them is zero? Explain.

ELECTRIC PROPERTIES OF CONDUCTORS

EM
1**I. Electric field in conductors**

Obtain the following equipment:

- rod (which could be made of PVC, acrylic, glass, *etc.*)
- piece of fleece, wool, or other material
- small pith ball attached to an insulating thread
- small cup lined with aluminum foil

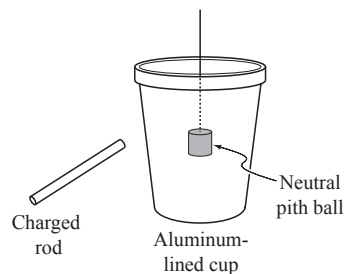
A. Charge the rod by rubbing it with wool or some other material.

1. Hold the pith ball by the string and bring the charged rod toward the pith ball, but don't let it touch the pith ball.

Describe what you observe.

2. Touch the pith ball to make sure that it is neutral, and hold it inside the foil-lined cup as shown. Bring the charged rod towards the cup, but don't let it touch the cup.

Repeat your experiment for several other points inside the cup, and describe what you observe.



3. What do your results suggest about the net electric field inside the cup? Explain.

4. Two students discuss their answers to question 3 above:

Student 1: "The electric field inside the cup can't be zero. The rod is charged, so it will create an electric field inside the cup."

Student 2: "I agree that the rod creates an electric field inside the cup, but I think the charges in the cup will create an induced electric field that opposes it. Thus the net electric field inside the cup would be zero."

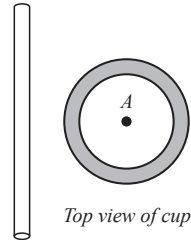
With which student, if either, do you agree? Explain your reasoning.

5. Why would it be *incorrect* to say that the cup 'blocks' the electric field of the rod?

EM *Electric properties of conductors*
2

B. Ask a tutorial instructor what the sign of the charge on the rod is.

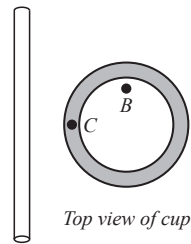
1. On the top-view diagram at right, draw an arrow to indicate the electric field due to the charged rod at point A.
2. Based on your observations, in which direction must the induced electric field point? How does the magnitude of the induced field compare to the magnitude of the field due to the rod? Explain.



3. What charge distribution on the cup would create this induced field? Explain, and draw the charge distribution on the diagram.

C. Consider points B and C, shown at right.

1. On the diagram, draw vectors to represent the electric fields of the rod and the induced charge distribution at point B.
2. What is the net electric field at point B? Is your answer consistent with your observations? Explain.



3. Sketch vectors at point C representing the electric field of the rod and the electric field of the induced charge distribution.
4. Are your vectors consistent with the fact that the net electric field within a conductor is zero? Explain.

D. Summarize your results thus far to describe the electric field at any point inside the cup.

⇨ Discuss your responses to this section with a tutorial instructor before you continue.

II. Potential difference in conductors

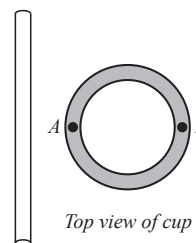
A. A positive test charge $+q$ is moved from point A to point B , near a point charge $+Q$. Consider the system consisting of both charges.



1. When the test charge moves from point A to point B , is the change in electric potential energy ΔU_E *positive*, *negative*, or *zero*? Explain.
2. Recall from the tutorial *Electric potential difference* that electric potential difference is defined as $\Delta U_E/q_{\text{test}}$. Is $\Delta V_{A \rightarrow B}$ *positive*, *negative*, or *zero*?
3. Would your answer to question 2 above change if a *negative* test charge were moved from point A to point B ? Explain.

B. The cup from section I is shown at right. Consider moving a positive test charge from point A to point B .

1. Is $\Delta V_{A \rightarrow B}$ *positive*, *negative*, or *zero*? Explain.



What would the potential difference be between two other points in the cup? Explain.

Suppose that the rod is very quickly moved from the left side of the cup to the right side of the cup.

2. The instant the rod is moved, before any charges on the cup have moved, in which direction is the electric field at points A and B ? Explain.
3. At this instant, will $\Delta V_{A \rightarrow B}$ be *positive*, *negative*, or *zero*? Explain.

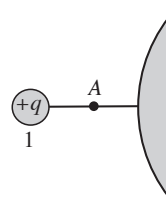
EM *Electric properties of conductors*

4

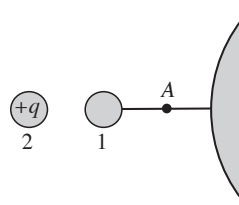
III. Application: ground

A *ground* is a conducting object that we assume can give or take any amount of charge to or from another conductor. As part of this assumption we assume the electric field due to the ground is negligible.

- A. A small metal sphere, sphere 1, with positive charge $+q$ is connected to a very large, neutral conducting sphere as shown. Assume the large sphere can be treated as a ground.



1. The instant that the spheres are connected, before any charges have moved, what is the direction of the net electric field at point A ? Explain.
 2. Describe what happens when the two spheres are connected. Explain.
 3. After a stable charge distribution is reached, what is the net charge on sphere 1? Explain.
 4. After the charge distribution is stable, is the electric potential difference from sphere 1 to the large sphere *positive*, *negative*, or *zero*? Explain.
- B. Another positively charged sphere, sphere 2, is placed near the grounded sphere as shown below.



1. The instant that sphere 2 is placed, before any charges have moved, what is the direction of the net electric field at point A ? Explain.
2. Once a stable distribution is established, is the net charge on sphere 1 *positive*, *negative*, or *zero*? Explain.

-
3. Two students discuss their answers to question 2 above.

Student 1: "When conductors are grounded they lose their charge. Sphere 1 is grounded both before and after sphere 2 is added, so sphere 1 remains neutral."

Student 2: "I disagree. The second sphere is positively charged, so negative charges will be attracted to it. Thus sphere 1 will become negatively charged."

With which student, if either, do you agree? Explain.

Check that your answer is consistent with your response to question 2 above.

4. A third student joins the discussion.

Student 3: "After the charge distribution is stable, the potential difference between sphere 1 and the large sphere is zero. How can ΔV be zero if sphere 1 is charged?"

How would you address student 3's concerns? Explain how you could help this student resolve their confusion.

5. Based on your answers above, what can you say about the net charge on two connected conductors if you only know that the potential difference between them is zero? Explain.

ELECTRIC POTENTIAL DIFFERENCE

EM
1**I. Review of work**

A. Suppose an object moves under the influence of a force. Sketch arrows showing the relative directions of the force and displacement when the work done by the force is:

Positive	Negative	Zero
----------	----------	------

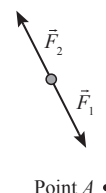
B. An object travels from point A to point B while two constant forces, \vec{F}_1 and \vec{F}_2 , of equal magnitude are exerted on it.

1. Is the total work done on the object by \vec{F}_1 *positive, negative, or zero*? • Point B

2. Is the total work done on the object by \vec{F}_2 *positive, negative, or zero*?

3. Is the net work done on the object *positive, negative, or zero*? Explain.

4. Is the speed of the object at point B *greater than, less than, or equal to* the speed of the object at point A ? Explain how you can tell.



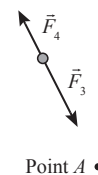
C. An object travels from point A to point B while two constant forces, \vec{F}_3 and \vec{F}_4 , of *unequal* magnitude are exerted on it as shown.

1. Is the total work done on the object by \vec{F}_3 *positive, negative, or zero*? • Point B

2. Is the total work done on the object by \vec{F}_4 *positive, negative, or zero*?

3. Is the net work done on the object *positive, negative, or zero*? Explain.

4. Is the speed of the object at point B *greater than, less than, or equal to* the speed of the object at point A ? Explain how you can tell.



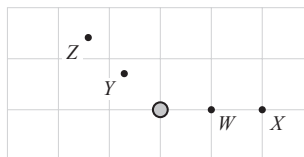
EM
2 **Electric potential difference**

- D. State the work-energy theorem in your own words. Are your answers in part B consistent with this theorem? Explain.

Are your answers in part C consistent with the work-energy theorem? Explain.

II. Work and electric fields

The diagram at right shows a top view of a positively charged rod. Points W , X , Y , and Z lie in a plane near the center of the rod. Points W and Y are equidistant from the rod, as are points X and Z .

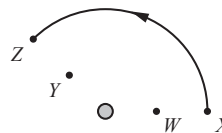


- A. Draw electric field vectors at points W , X , Y , and Z .
- B. A particle with charge $+q_0$ travels along a straight line path from point W to point X .

Is the work done *by the electric field* on the particle *positive, negative, or zero*? Explain using a sketch that shows the electric force on the particle and the displacement of the particle.

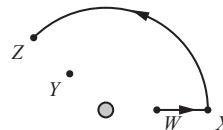
Compare the work done by the electric field when the particle travels from point W to point X to that done when the particle travels from point X to point W .

- C. The particle travels from point X to point Z along the circular arc shown.
- Is the work done *by the electric field* on the particle *positive, negative, or zero*? Explain. (*Hint*: Sketch the direction of the force on the particle and the direction of the displacement for several short intervals during the motion.)



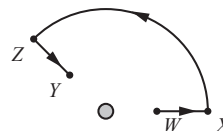
Electric potential difference EM
3

2. Compare the work done by the electric field when the particle travels from point W to point X to that done when the particle travels from point W to point Z along the path shown. Explain.



- D. Suppose the particle travels from point W to point Y along the path $WXZY$ as shown.

1. Compare the work done by the electric field when the particle travels from point W to point X to that done when the particle travels from point Z to point Y . Explain.



What is the total work done by the electric field on the particle as it moves along the path $WXZY$?

2. Suppose the particle travels from W to Y along the arc shown. Is the work done by the electric field on the particle *positive*, *negative*, or *zero*? Explain using force and displacement vectors.



3. Suppose the particle travels along the straight path WY . Is the work done by the electric field on the particle *positive*, *negative*, or *zero*? Explain using force and displacement vectors. (*Hint*: Compare the work done along the first half of the path to the work done along the second half.)



EM *Electric potential difference*

4

-
- E. Compare the work done as the particle travels from point W to point Y along the three different paths in part D.

It is often said that the work done by a static electric field is *path independent*. Explain how your results in part D are consistent with this statement.

III. Electric potential difference

- A. Suppose the charge of the particle in section II is increased from $+q_0$ to $+1.7q_0$.
1. Is the work done by the electric field as the particle travels from W to X *greater than*, *less than*, or *equal to* the work done by the electric field on the original particle? Explain.
 2. How is the quantity *the work divided by the charge* affected by this change?

The *electric potential difference* $\Delta V_{W \rightarrow X}$ in going from point W to point X is defined as:

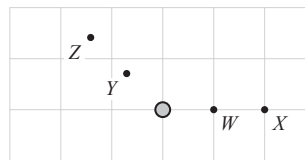
$$\Delta V_{W \rightarrow X} = V_X - V_W \equiv -\frac{W_{\text{elec}}}{q}$$

where W_{elec} is the work done by the electric field on a charge q as it travels from point W to point X . [Note that some textbooks use different notation, (e.g., ΔV_{WX} rather than $\Delta V_{W \rightarrow X}$) or a different convention (e.g., $\Delta V_{W \rightarrow X} = V_W - V_X$). Check how your textbook expresses this relationship.]

3. Does this quantity depend on the *magnitude* of the charge of the particle that is used to measure it? Explain.
4. Does this quantity depend on the *sign* of the charge of the particle that is used to measure it? Explain.

Electric potential difference EM
5

B. Shown at right are four points near a positively charged rod. Points W and Y are equidistant from the rod, as are points X and Z . A charged particle with mass $m_o = 3 \times 10^{-3}$ kg is released from rest at point W and later is observed to pass point X .



1. Is the particle positively or negatively charged? Explain.

2. Suppose that the magnitude of the charge on the particle is 2×10^{-6} C and that the speed of the particle is 40 m/s as it passes point X .
 - a. Find the change in kinetic energy of the particle as it travels from point W to point X .

 - b. Find the work done by the electric field on the particle between point W and point X . (*Hint*: See part D of section I.)

 - c. Find the electric potential difference between point W and point X , $\Delta V_{W \rightarrow X}$ (*i.e.*, $V_X - V_W$).

 - d. If the same particle were released from point Y , would its speed as it passes point Z be *greater than*, *less than*, or *equal to* 40 m/s? Explain.

3. Suppose that a second particle with the same mass as the first but nine times the charge (*i.e.*, 18×10^{-6} C) were released from rest at point W .
 - a. Would the electric potential difference between points W and X change? If so, how, if not, why not?

 - b. Would the speed of the second particle as it passes point X be *greater than*, *less than*, or *equal to* the speed of the first particle as it passed point X ? Explain.

EM
6 *Electric potential difference*

4. A particle with mass $m_o = 3 \times 10^{-8}$ kg is launched toward the rod from point Z and turns around at point Y .
- a. If the particle has charge $q_o = 2 \times 10^{-6}$ C, with what speed should it be launched? Explain.
- b. If instead the particle has charge $9q_o$ (*i.e.*, 18×10^{-6} C) with what speed should it be launched? Explain.

ELECTRIC POTENTIAL DIFFERENCE

EM
1**I. Review of work**

A. Suppose an object moves under the influence of a force. Sketch arrows showing the relative directions of the force and displacement when the work done by the force is:

Positive	Negative	Zero
----------	----------	------

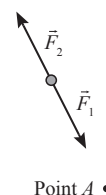
B. An object travels from point A to point B while two constant forces, \vec{F}_1 and \vec{F}_2 , of equal magnitude are exerted on it.

1. Is the total work done on the object by \vec{F}_1 *positive, negative, or zero*? • Point B

2. Is the total work done on the object by \vec{F}_2 *positive, negative, or zero*?

3. Is the net work done on the object *positive, negative, or zero*? Explain.

4. Is the speed of the object at point B *greater than, less than, or equal to* the speed of the object at point A? Explain how you can tell.



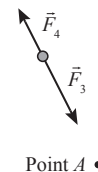
C. An object travels from point A to point B while two constant forces, \vec{F}_3 and \vec{F}_4 , of *unequal* magnitude are exerted on it as shown.

1. Is the total work done on the object by \vec{F}_3 *positive, negative, or zero*? • Point B

2. Is the total work done on the object by \vec{F}_4 *positive, negative, or zero*?

3. Is the net work done on the object *positive, negative, or zero*? Explain.


4. Is the speed of the object at point B *greater than, less than, or equal to* the speed of the object at point A? Explain how you can tell.





EM *Electric potential difference*

2

II. Electric potential difference

- A. The diagram at right shows a top view of a positively charged rod that is fixed in place. Points R and S lie in a plane near the center of the rod. 

A particle with positive charge $+q_{test}$ travels along a straight path from point R to point S only under the influence of the charged rod.  

1. Is the work $W_{R \rightarrow S}$ done by the electric field as the particle moves from R to S positive, negative, or zero?

2. Suppose that the charge on the particle were halved.

Is the work done by the electric field as the particle moves from R to S greater than, less than, or equal to the work done by the electric field on the original particle?

Would the ratio $W_{R \rightarrow S}/q_{test}$ change? If so, how? If not, explain why not.

The *electric potential difference* in going from point R to point S is defined as:

$$\Delta V_{R \rightarrow S} = V_S - V_R \equiv -\frac{W_{R \rightarrow S}}{q_{test}}$$

where $W_{R \rightarrow S}$ is the work done by the electric field on a charge q as it travels from point R to point S . [Note: some textbooks use different notation, (e.g., ΔV_{RS} rather than $\Delta V_{R \rightarrow S}$) or a different convention. Check how your textbook expresses this relationship.]

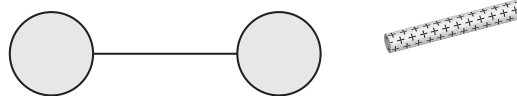
3. Would the electric potential difference $\Delta V_{R \rightarrow S}$ change if:
 - the magnitude of the charge on the rod were increased? Explain.
 - the sign of the test charge were reversed? Explain.
4. Based on your answers, does the electric potential difference between points R and S depend on the test charge, the rod, or both? Explain.

⇨ Check your results for part II with a tutorial instructor.

III. Applications of Electric Potential Difference

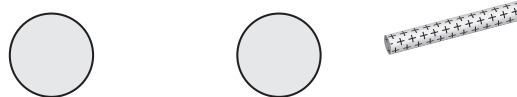
A. Two neutral conducting spheres are connected by a conducting wire. Then a positively charged rod is brought close to the right sphere (but no sparks jump from the rod). The magnitude of the charge on the wire is small and can be ignored.

1. Draw the final charge distribution on each sphere below.



2. Suppose a positive test charge were to be moved through the wire from the left sphere to the right sphere. Would the work done by the electric field on the test charge be *positive*, *negative*, or *zero*? Explain. (*Hint*: What is the magnitude of the electric field inside a conductor in static equilibrium?)
3. Is the electric potential difference between the left sphere and the right sphere *positive*, *negative*, or *zero*?

The wire is now removed from the spheres, but no other changes are made to the system and no charges move.



4. Does the electric potential difference between the left sphere and the right sphere change, and if so, how?
5. Two students discuss their answers to question 4 above.

Student 1: "The left sphere is positively charged so the potential is positive there, and it's negative at the right sphere. We're going from positive to negative potential, so the potential difference is negative."

Student 2: "Before the wire was removed, the two spheres were at the same potential since they were connected by a conductor. Nothing changed when the wire was removed, so the potential difference is still zero."

Discuss these statements with your partners. With which student do you agree? Would this change your answer to part A.4 above?

EM *Electric potential difference*

4

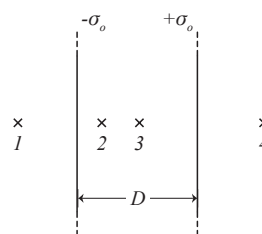
B. A small portion near the center of a large conducting sheet of area A_o is shown magnified at right. The sheet is positively charged, with a uniform charge density $+\sigma_o$.

- Write an expression for the magnitude of the electric field to the left and to the right of the charged sheet.



A sheet with uniform charge density $-\sigma_o$, is now placed a distance D away from the first. The sheets are large enough and close enough together that fringing effects near the edges can be ignored.

- At each labeled point, draw two vectors, one to represent the electric field at that point due to the left sheet and one for the electric field due to the right sheet.
- Write an expression for the net electric field at points 1, 2, 3, and 4 in terms of the given variables.



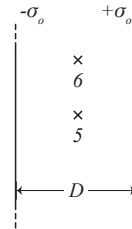
- Find the work done by the electric field on a positively charged particle with charge $+q_{\text{test}}$ as it moves from the left sheet to the right sheet.
- Find the electric potential difference $\Delta V_{L \rightarrow R}$ between the left sheet and the right sheet.
- Find $\left| \frac{Q}{\Delta V} \right|$ (the absolute value of the ratio of the net charge on one sheet to the electric potential difference between the sheets).

The electric potential difference between two isolated conductors depends on their net charges and their physical arrangement. If the conductors have charge $+Q$ and $-Q$, the quantity $\left| \frac{Q}{\Delta V} \right|$ is called the *capacitance* (C) of the particular arrangement of conductors.

Electric potential difference EM
5

C. Consider again the sheets from part B above.

1. Find the work done on the test charge as it moves from point 5 to point 6. Explain.



2. Find the potential difference between points 5 and 6.

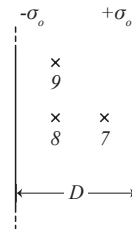
If the electric field would do no work on a test charge as it moves between two points, those points are said to have the same electric potential.

3. Identify other points that have the same electric potential as point 5. Indicate these points on your diagram above.

A set of points that all have the same electric potential is called an *equipotential*.

4. Sketch a few different equipotential lines on your diagram above.

5. Suppose that the electric potential difference between point 7 and point 8 is ΔV . Find the potential difference between point 7 and point 9. Explain.

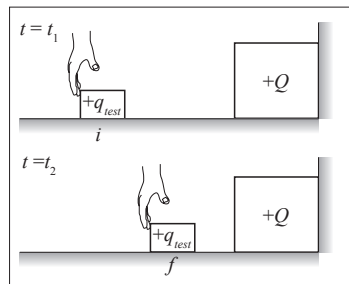


⇔ Check your results for part III with a tutorial instructor.

ELECTRIC POTENTIAL DIFFERENCE

EM
1**I. Work and electric potential energy**

A. A positively charged block with charge $+q_{test}$ is at rest at point i on a level frictionless surface, near another charged block with charge $+Q$ that is attached to a wall. A hand pushes the $+q$ block to point f , where it ends at rest.



1. In the space below, draw a free-body diagram for the left block at an instant during the motion. (Recall that a free-body diagram shows all the external forces acting on an object).

For each of the forces on your free-body diagram, determine whether the work done by them on the block is *positive*, *negative*, or *zero*.

2. Is the net work done on the left block from t_1 to t_2 *positive*, *negative*, or *zero*? Explain.

Free-body diagram
for the left block

B. Now consider the system consisting of both blocks.

1. In the space at right, draw a free-body diagram showing the forces acting on the system.
2. For each of the forces on your free-body diagram, determine whether the work done on the system is *positive*, *negative*, or *zero*.

Free-body diagram
for the system

3. Is the net work done on the system from t_1 to t_2 *positive*, *negative*, or *zero*? Explain.
4. Is ΔK , the change in kinetic energy of the system, *positive*, *negative*, or *zero*? Explain.
5. Is ΔU_E , the change in electrostatic potential energy of the system, *positive*, *negative*, or *zero*? Explain.

EM *Electric potential difference*

2

C. Suppose the left block had a $-q_{test}$ charge, and the hand moved it from point i to point f . The test charge again begins and ends at rest.

1. Is the work done by the hand on the system *positive*, *negative*, or *zero* in this case? Explain.
2. Is ΔU_E *positive*, *negative*, or *zero*? Explain.
3. Would the ratio $\Delta U_E/q_{test}$ change if the sign of the test charge were changed? Explain.

The *electric potential difference* between point i and point f is defined as:

$$\Delta V_{i \rightarrow f} = V_f - V_i \equiv \frac{\Delta U_E}{q_{test}}$$

where ΔU_E is the change of electric potential energy of a system as a charge q_{test} travels from point i to point f . [Note: some textbooks use different notation, (e.g., ΔV_{if} rather than $\Delta V_{i \rightarrow f}$) or a different convention. Check how your textbook expresses this relationship.]

4. In the table below, indicate whether each of the quantities is *positive*, *negative*, or *zero* for the motion of a test charge from point i to point f near the large block.

Sign of test charge	ΔK_{system}	$W_{net ext}$	ΔU_E	$\Delta V_{i \rightarrow f}$
Positive				
Negative				

5. Would the electric potential difference between points i and f change if:
 - the magnitude of the test charge were decreased? Explain.
 - the magnitude of the charge on the large block was increased? Explain.
6. Based on your answers, does the electric potential difference between points i and f depend on the test charge, the large block, or both?

⇒ Check your results for part II with a tutorial instructor.

II. Applications of electric potential difference

A. Two neutral conducting spheres are connected by a conducting wire. Then a positively charged rod is brought close to the right sphere (but no sparks jump from the rod). The magnitude of the charge on the wire is small and can be ignored.

1. Draw the final charge distribution on each sphere below.



2. On your diagram above, draw and label the electric field vector at point A due to:

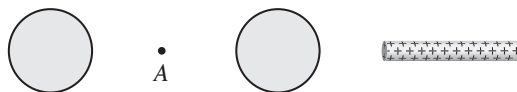
- the charged rod.
- the charges induced on the spheres. (*Hint: What is the net electric field at a point inside a conductor at equilibrium?*)

3. Suppose a hand moves a positive test charge through the wire from the left sphere to the right sphere. Is the work required by the hand to move the charge *positive*, *negative*, or *zero*? Explain.

Would ΔU_E be *positive*, *negative*, or *zero*? Explain.

4. Is the electric potential difference between the left sphere and the right sphere *positive*, *negative*, or *zero*? Explain.

The wire is now removed from the spheres, but no other changes are made to the system and no charges move.



5. On your diagram above, draw the electric field vector at point A due to:

- the charged rod.
- the charges induced on the spheres.

6. Suppose a hand moves a positive test charge from the left sphere to the right sphere. Is the work required to move the charge *positive*, *negative*, or *zero*? Explain.

7. Is the electric potential difference between the left sphere and the right sphere *positive*, *negative*, or *zero*? Explain.

EM *Electric potential difference*

4

8. Two students discuss their answers to question 7 above.

Student 1: "The left sphere is positively charged so the potential is positive there, and it's negative at the right sphere. We're going from positive to negative potential, so the potential difference is negative."

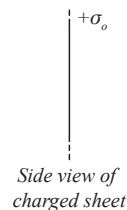
Student 2: "Before the wire was removed, the two spheres were at the same potential since it didn't take any work to move a charge between them. Nothing changed when the wire was removed, so the potential difference is still zero."

Discuss these statements with your partners. With which student do you agree? Would this change your answer to part 7 above?

III. Supplement: Distributed charges

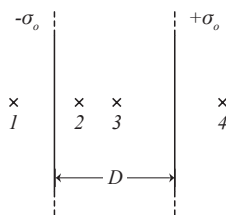
- A. A small portion near the center of a large conducting sheet of area A_o is shown magnified at right. The sheet is positively charged, with a uniform charge density $+\sigma_o$.

1. Write an expression for the magnitude of the electric field to the left and to the right of the charged sheet.



A sheet with uniform charge density $-\sigma_o$ is now placed a distance D away from the first. The sheets are large enough and close enough together that fringing effects near the edges can be ignored.

2. At each labeled point, draw two vectors, one to represent the electric field at that point due to the left sheet and one for the electric field due to the right sheet.
3. Write expressions for the following quantities in terms of the given variables:

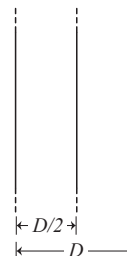


- the electric field at points 1, 2, 3, and 4
- the work needed to move a positively charged particle with charge $+q_{\text{test}}$ from the left sheet to the right sheet
- the electric potential difference $\Delta V_{L \rightarrow R}$ between the left sheet and the right sheet

Electric potential difference EM
5

4. The right sheet is moved halfway toward the left sheet as shown. Describe how the magnitude of each of the following quantities will change (if at all). Explain.

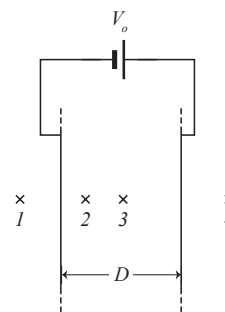
- the charge density on each sheet
- the electric field both outside and between the sheets
- the electric potential difference $\Delta V_{L \rightarrow R}$ between the left sheet and the right sheet



B. Suppose the sheets are discharged, then held a distance D apart and connected to a battery.

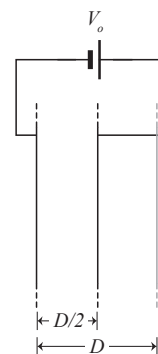
1. Write expressions for the following quantities in terms of the given variables. Explain your reasoning in each case.

- the electric potential difference $\Delta V_{L \rightarrow R}$ between the left sheet and the right sheet
- the electric field at points 1, 2, 3, and 4
- the charge density on each sheet.



2. The right sheet is moved halfway toward the left sheet. Describe how the magnitude of each of the following quantities will change (if at all). Explain.

- the electric potential difference $\Delta V_{L \rightarrow R}$ between the left sheet and the right sheet
- the electric field both outside and between the sheets
- the charge density on each sheet.



ELECTRIC POTENTIAL DIFFERENCE

I. Work and electric potential energy

Recall from lecture or your textbook that the electric potential difference between two points i and f is defined as $\Delta V_{i \rightarrow f} = V_f - V_i \equiv \frac{\Delta U_E}{q_{\text{test}}}$, where ΔU_E is the change in electrostatic potential energy of the system when a charge q_{test} moves from point i to point f . (The system consists of q_{test} and all charges that interact with it).

A. A positively charged sphere with charge $+Q$ is shown at right. A test charge $+q_{\text{test}}$ is released from rest at point A , and it passes point B at a later time.



1. Is the work done by the electric field on the test charge *positive*, *negative*, or *zero* as the test charge moves from A to B ?
 2. Is ΔU_E *positive*, *negative*, or *zero* when the charge moves from A to B ? Explain.
- B. Consider the system of the sphere and the test charge.
1. Is the net external work done on the system *positive*, *negative*, or *zero*? Explain.
 2. During this movement, is the change in kinetic energy *positive*, *negative*, or *zero*?
 3. Recall that the work-energy theorem states that $W_{\text{net ext}} = \Delta K + \Delta U$. Is electrostatic potential energy *increasing*, *decreasing*, or *remaining the same* during this motion?
 4. Is your answer consistent with what you found in part A? If not, resolve any inconsistencies.
 5. Is $\Delta V_{A \rightarrow B}$ *positive*, *negative*, or *zero*? Explain.
- C. Suppose that q_{test} , the charge of the test particle, were halved.
- Would ΔU_E , the change in electrostatic energy as the particle moves from A to B , change? If so, how? If not, explain why not.
- Would $\Delta V_{A \rightarrow B}$ change? If so, how? If not, explain why not.

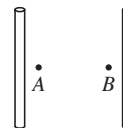
- D. In the table below, indicate if each single change to the system will change ΔU_E or $\Delta V_{A \rightarrow B}$. In each case, no other change is made to the original system.

Change to the system:	ΔU_E	$\Delta V_{A \rightarrow B}$
Magnitude of the test charge increases.		
Sign of the test charge changes.		
Magnitude of the charge on the sphere increases.		
Sign of the charge on the sphere changes.		

Is there a difference between the change in electrostatic potential energy ΔU_E and electric potential difference $\Delta V_{A \rightarrow B}$?

Based on your answers above, does $\Delta V_{A \rightarrow B}$ depend on the test charge, the sphere, or both?

- E. Points A and B are equally close to two identical positively charged rods. A positive test charge is moved from point A to point B .

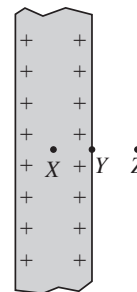


- Briefly ignore the right rod, and only consider the system consisting of the test charge and the left rod. Is ΔU_E *positive*, *negative*, or *zero* when the particle moves from A to B ? Explain.
- Now consider the system consisting of the test charge and the right rod. Is ΔU_E *positive*, *negative*, or *zero*? Explain.
- In the entire system of both rods and the test charge, is $\Delta V_{A \rightarrow B}$ *positive*, *negative*, or *zero*? Explain.

⇨ Discuss your responses with a tutorial instructor before you continue.

II. Applications of electric potential difference

A. Shown at right is a portion of a large metal slab, with a net positive charge. Suppose a test charge $+q_{\text{test}}$ is moved from point X in the center of the slab to point Y at the right surface.



Side view of metal slab

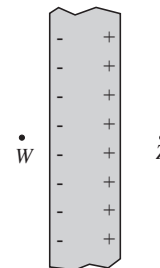
1. Predict if $\Delta V_{X \rightarrow Y}$ is *positive*, *negative*, or *zero* as a result of this movement.
2. Two students discuss their predictions to part 1:
 Student 1: "There is no net electric field inside a conductor, so there's no change in energy from X to Y , and so ΔV is zero."
 Student 2: "No, electrostatic potential energy should be increasing, because the test charge is getting closer to the positive charges on the side of the slab. ΔV is positive."

Do you agree with either student? Explain. Check your prediction in part 1.

The test charge is now moved from point Y to point Z , outside the slab.

3. Is $\Delta V_{Y \rightarrow Z}$ *positive*, *negative*, or *zero*? Explain.
4. Suppose the test charge moves from X to Z . Would $\Delta V_{X \rightarrow Z}$ be *greater than*, *less than*, or *equal to* $\Delta V_{Y \rightarrow Z}$? Explain.

B. An identical neutral slab is placed in a uniform electric field to the right (not shown), and then becomes polarized.

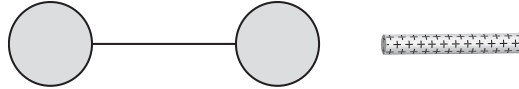


Side view of metal slab

1. Is $\Delta V_{L \rightarrow R}$ from the left edge of the slab to the right edge *positive*, *negative*, or *zero*? Explain.
2. Is $\Delta V_{W \rightarrow Z}$ *positive*, *negative*, or *zero*? Explain.
3. Suppose the metal slab were removed (the uniform electric field is still there). Would the magnitude of $\Delta V_{W \rightarrow Z}$ be *greater than*, *less than*, or *equal to* what it was before the slab was removed? Explain.

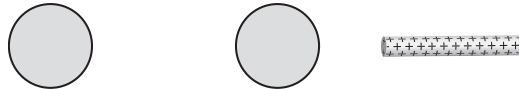
- C. Two neutral conducting spheres are connected by a conducting wire. Then a positively charged rod is brought close to the right sphere (but no sparks jump from the rod). The magnitude of the charge on the wire is small and can be ignored.

1. Draw the final charge distribution on each sphere below.



2. Is the electric potential difference $\Delta V_{L \rightarrow R}$ between the left sphere and the right sphere *positive, negative, or zero*? Explain. (*Hint*: How is this situation similar to part B on the previous page?)

The wire is now removed from the spheres, but no other changes are made to the system and no charges move.



3. Suppose a positive test charge moves from the left sphere to the right sphere. Predict whether $\Delta V_{L \rightarrow R}$ is *positive, negative, or zero*.
4. Two students discuss their predictions to question 3 above.
- Student 1: "The left sphere is positively charged, and the right sphere is negative. We're going from positive to negative, so the potential difference is negative."
- Student 2: "Before the wire was removed, the two spheres were at the same potential since they part of the same conductor. No charges moved when the wire was removed, so the potential difference is still zero."
- Discuss these statements with your partners. With which student do you agree?
5. Check that your prediction in question 3 is consistent with your answer to question 4. Resolve any inconsistencies.

⇨ Discuss your responses to part B with a tutorial instructor.

ELECTRIC POTENTIAL DIFFERENCE

EM
1**I. Work and electric potential energy**

- A. A positively charged sphere with charge $+Q$ is shown at right. A test charge $+q_{\text{test}}$ is released from rest at point A , and it passes point B with speed v_B .



Determine if ΔU_E , the change in electrostatic potential energy of the system is *positive*, *negative*, or *zero*:

1. Using the work done by the electric field, $W_{\text{elec}} = -\Delta U_E$.
 2. Using the work-energy theorem, $W_{\text{net ext}} = \Delta E_{\text{total}}$.
 3. Are your answers consistent with each other? If not, resolve any inconsistencies.
- B. The same test charge is now moved from point A to point B at constant speed.
1. How does ΔU_E in this case compare to the ΔU_E you found in part A? Explain.
 2. Would your answer change if the charge were accelerated towards point B ? Explain.
 3. If one of the charges in a system of charges is moved, does the change in electrostatic potential energy depend on how that charge is moved? Explain.
- C. Suppose that q_{test} , the charge of the test particle, were doubled.
- Would ΔU_E , the change in electrostatic energy of the system as the particle moves from A to B , change? If so, how? If not, explain why not.

EM *Electric potential difference*

2

II. Electric potential difference

Recall from lecture or your textbook that the electric potential difference between two points i and f is defined as $\Delta V_{i \rightarrow f} = V_f - V_i \equiv \frac{\Delta U_E}{q_{\text{test}}}$, where ΔU_E is the change in electrostatic potential energy of the system when a charge q_{test} moves from point i to point f . (The system consists of q_{test} and all charges that interact with it.)

- A. Shown at right is the system from part A of section I.

Is $\Delta V_{A \rightarrow B}$, the electric potential difference between A and B, *positive, negative, or zero*?



- B. Three new systems are made by making one change to the figure above. In the table below, indicate if the magnitude or sign of ΔU_E or $\Delta V_{A \rightarrow B}$ would change if:

	ΔU_E	$\Delta V_{A \rightarrow B}$
the magnitude of the test charge were decreased.		
the sign of the test charge were changed.		
the magnitude of the charge on the <i>sphere</i> were increased.		

Based on your answers above, does $\Delta V_{A \rightarrow B}$ depend on the test charge, the sphere, or both?

- C. The experiment from part A is repeated, with an identical charged sphere symmetrically placed as shown. (*i.e.* the distance between the left sphere and point A and between the right sphere and B are equal).



- Briefly ignore the right sphere, and only consider the system consisting of the test charge and the left sphere. Is ΔU_E for this system *positive, negative, or zero* when the particle moves from A to B? Explain.
- Now consider the system consisting of the test charge and the right sphere. Is ΔU_E for this system *positive, negative, or zero*? Explain.
- In the entire system consisting of all charges, is $\Delta V_{A \rightarrow B}$ *positive, negative, or zero*? Explain.

⇔ Discuss your responses with a tutorial instructor before you continue.

Electric potential difference EM
3

D. An infinite sheet has charge density $-\sigma_o$. A side view of a small portion of the sheet is shown at right.

1. Draw electric field vectors at points A, B, and C. How does the magnitude of the field at these three points compare?

2. Is the electric potential difference between points A and B positive, negative, or zero?

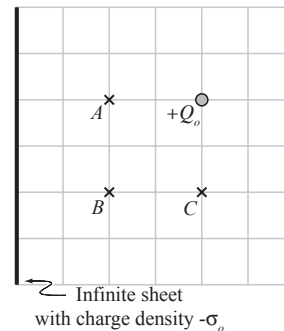
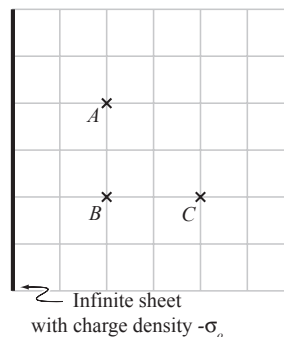
3. Is the electric potential difference between A and C positive, negative, or zero?

4. There is more than one way to get from A to C; for instance, a point charge could move on a straight line from A to C, or it could move straight down to B and then right to C.

Would the value of $\Delta V_{A \rightarrow C}$ change if you used a different path from A to C? Explain.

5. A positive point charge $+Q_o$ is placed to the right of the sheet as shown.

In the table below, indicate whether the electric potential difference between each pair of points is positive, negative, or zero for the sheet alone, for the point charge alone, and for the entire system. Explain your reasoning in each case.



	Sheet	Point charge	Entire system
$\Delta V_{A \rightarrow B}$			
$\Delta V_{A \rightarrow C}$			

EM
4 *Electric potential difference***III. Applications of electric potential difference**

A. A portion of a positively charged infinite conducting slab is shown at right (no other charges are nearby). Suppose a test charge $+q_{\text{test}}$ is moved from point X in the center of the slab to point Y at the right surface.

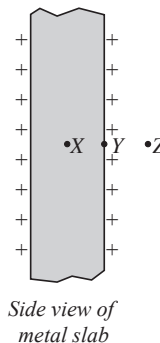
1. Would $\Delta V_{X \rightarrow Y}$ be *positive*, *negative*, or *zero* as a result of this movement?

2. Two students discuss their answers to part 1:

Student 1: "There is no net electric field inside a conductor, so there is no change in energy from X to Y , and so ΔV is zero."

Student 2: "I disagree, the electrostatic potential energy should be increasing. The test charge is getting closer to the positive charges on the side of the slab, so ΔV is positive."

Do you agree with either student? Explain. If necessary, revise your answer to part 1.



The test charge is now moved from point Y to point Z , outside the slab.

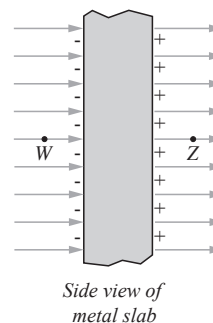
3. Is $\Delta V_{Y \rightarrow Z}$ *positive*, *negative*, or *zero*? Explain.

4. Suppose the test charge moves from X to Z . Would $\Delta V_{X \rightarrow Z}$ be *greater than*, *less than*, or *equal to* $\Delta V_{Y \rightarrow Z}$? Explain.

B. A portion of a neutral infinite conducting slab is shown at right. The slab is placed in a constant electric field to the right, and becomes polarized as shown.

1. Is $\Delta V_{L \rightarrow R}$ from the left edge of the slab to the right edge *positive*, *negative*, or *zero*? Explain.

2. Is $\Delta V_{W \rightarrow Z}$ *positive*, *negative*, or *zero*? Explain.



⇒ Discuss your responses with a tutorial instructor.

ELECTRIC POTENTIAL DIFFERENCE

EM
1**I. Work and electric potential energy**

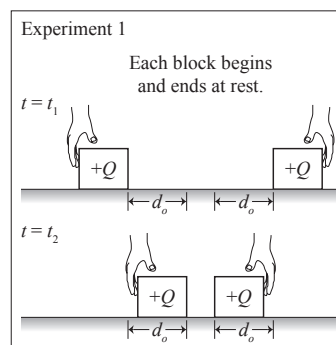
In the tutorial *Conservation of energy* or in lecture, we found that the total energy of a system can only be changed by interactions with objects outside the system. The change in total energy is related to the net external work done on the system by the following equation:

$$W_{\text{net ext}} = \Delta E_{\text{tot}}$$

- A. In experiment 1 shown at right, two charged objects are on a level, frictionless surface. Each block starts from rest, is pushed by a hand through a distance d_0 , and ends at rest.

In the table below, indicate if each quantity is *positive*, *negative*, or *zero* for system 1, consisting of the two blocks. (*Note*: ΔU_E represents the change in electrostatic potential energy of the system.)

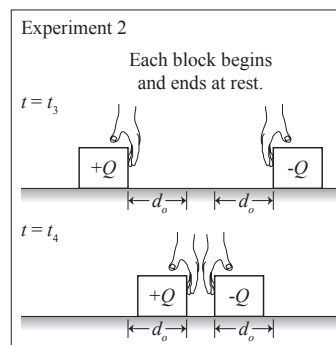
	$W_{\text{net ext}}$	ΔE_{tot}	ΔK	ΔU_E
System 1				



- B. In experiment 2, the sign of the charge on one block is reversed. The hands now exert a force to keep each block moving at constant speed. Each block still moves the same distance, and begins and ends at rest.

In the first row of the table below, indicate if each quantity is *positive*, *negative*, or *zero* for the system of the two blocks in experiment 2.

	$W_{\text{net ext}}$	ΔE_{tot}	ΔK	ΔU_E
System 2				
System 3				



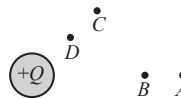
- C. A third experiment is set up the same as experiment 2, but the blocks are released from rest at their initial positions. After they each move a distance d_0 , they each have a velocity v_0 .

1. Fill out the second row of the table above, for the system of the blocks in experiment 3.
2. Is the magnitude of the change in electrostatic potential energy, ΔU_E , in experiment 3 *greater than*, *less than*, or *equal to* what it was in experiment 2? Explain.
3. Suppose that in experiment 3, that the blocks were launched towards each other with some initial velocity v_0 , but still move a distance d_0 . Would this change ΔU_E ? Explain.

EM *Electric potential difference*

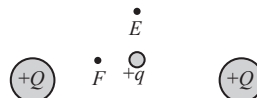
2

D. Two pairs of points, A - B and C - D , are shown next to a $+Q$ charge at right. A second point charge $+q$ can either be moved from A to B , or from C to D . The distance between each pair of points is the same.



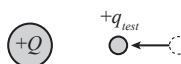
1. Would you need to exert more force on the point charge to move it from A to B , or from C to D ? Explain.
2. Which movement would give a larger change in electrostatic potential energy of the system? Explain.
3. Suppose both charges were negative instead of positive. Would your answer above change? Explain why or why not.

E. Another charge is added to the right of the $+Q$ charge. A smaller charge $+q$ is placed between them as shown, slightly above the axis on which the two $+Q$ charges lie.



1. Suppose the $+q$ charge is moved upwards to point E . Would ΔU_E be *positive*, *negative*, or *zero*? Explain.
2. Suppose instead the charge is moved left to point F . Would ΔU_E be *positive*, *negative*, or *zero* in this case? Explain.

F. You move a small test charge towards a $+Q$ charge. As a result of your movement, the change in electrostatic energy of the system is ΔU_E .



1. Suppose the magnitude of the test charge q_{test} was halved but you still moved it the same distance.
Would ΔU_E change? If so, how? If not, explain why not.
(*Hint: Use superposition to calculate the change in energy.*)
2. Would the ratio $\Delta U_E/q_{test}$ change? If so, how? If not, explain why not.

⇔ Discuss your responses to section I with a tutorial instructor before you continue.

II. Electric potential difference

Recall from lecture or your textbook that the electric potential difference between two points i and f is defined as $\Delta V_{i \rightarrow f} = V_f - V_i \equiv \frac{\Delta U_E}{q_{\text{test}}}$, where ΔU_E is the change in electrostatic potential energy of the system when a charge q_{test} moves from point i to point f . (The system consists of q_{test} and all charges that interact with it.)

- A. A large charged ball is shown at right. A small test charge with charge $+q_{\text{test}}$ is moved from point A to point B .



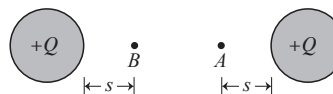
1. Is ΔU_E *positive*, *negative*, or *zero*? Explain.
2. Is $\Delta V_{A \rightarrow B}$, the electric potential difference between A and B , *positive*, *negative*, or *zero*?

- B. In the table below, indicate how ΔU_E or $\Delta V_{A \rightarrow B}$ from part A would change if:

	ΔU_E	$\Delta V_{A \rightarrow B}$
the magnitude of the test charge were increased.		
the sign of the test charge were changed.		
the magnitude of the charge on the <i>sphere</i> were decreased.		

Based on your answers above, does $\Delta V_{A \rightarrow B}$ depend on the test charge, the sphere, or both?

- C. The experiment from part A is repeated, with an identical charged sphere symmetrically placed as shown.



- Is $\Delta V_{A \rightarrow B}$ *positive*, *negative*, or *zero*? Explain.

- ⇔ Discuss your responses to section II with a tutorial instructor before you continue.

EM 4 *Electric potential difference*

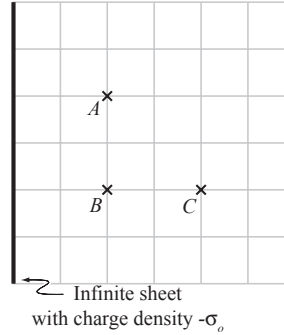
III. Applications of electric potential difference

A. An infinite sheet has charge density $-\sigma_o$. A side view of a small portion of the sheet is shown at right.

1. Draw electric field vectors at points A , B , and C . How does the magnitude of the field at these three points compare?
2. Suppose you move a test charge $+q_{test}$ at constant speed from A to B .

Is the electric potential difference between points A and B *positive, negative, or zero*?

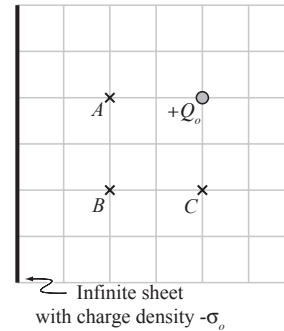
3. Is the electric potential difference between A and C *positive, negative, or zero*?
4. There is more than one way to get from A to C ; for instance, a point charge could move on a straight line from A to C , or it could move straight down to B and then right to C .
Would the value of $\Delta V_{A \rightarrow C}$ change if you used a different path from A to C ? Explain.



5. A positive point charge $+Q_o$ is placed to the right of the sheet as shown.

In the table below, indicate whether the electric potential difference between each pair of points is *positive, negative, or zero* for the sheet alone, for the point charge alone, and for the entire system. Explain your reasoning in each case.

	Sheet	Point charge	Entire system
$\Delta V_{A \rightarrow B}$			
$\Delta V_{A \rightarrow C}$			



ELECTRIC POTENTIAL DIFFERENCE

EM
1**I. Work and electric potential energy**

In the tutorial *Conservation of energy* and in lecture, you found that if a system interacts with an external object or system then the total energy of the system can change. The change in energy is related to the work done on the system according to the following equation:

$$W_{\text{net ext}} = \Delta E_{\text{tot}} = \Delta K + \Delta U$$

Two experiments are conducted with identical blocks on a level, frictionless surface. The blocks experience an attractive conservative force toward each other (not shown). Initially, hands keep each block at rest.

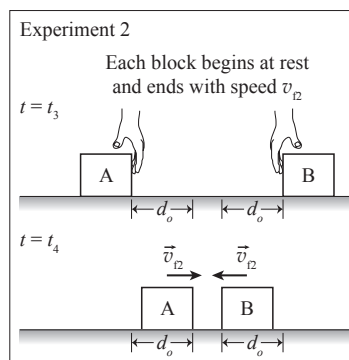
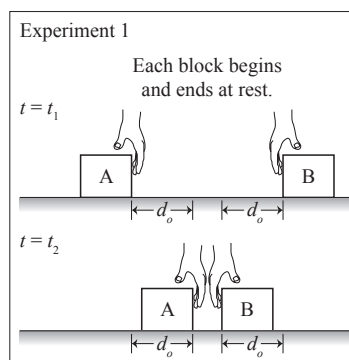
In experiment 1, the hands allow each block to move a distance d_o toward the other. The blocks start and end at rest.

In experiment 2, the hands release the blocks from rest. Each block has a speed of v_{12} after moving a distance d_o .

- A. Consider the system consisting of the two blocks. In the table below, indicate if each quantity is *positive*, *negative*, or *zero* for each experiment.

	$W_{\text{net ext}}$	ΔE_{tot}	ΔK	ΔU
Experiment 1				
Experiment 2				

How did you use the fact that the blocks are attracted to each other to fill out the table above? Explain.



Consider the following student statement:

"For a conservative force, potential energy only depends on where the blocks start and end. The blocks move the same distance in both experiments, so ΔU for both experiments is the same."

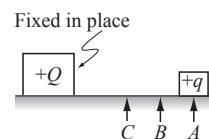
Do you agree with this statement? Explain why or why not.

EM *Electric potential difference*

2

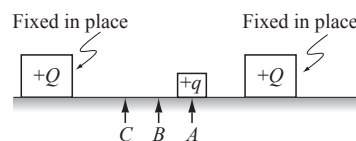
The experiments on the previous page involve a change in potential energy due to the change in position of a system with an unknown interaction. If the interaction is due to an electrostatic force, the associated energy change is called a change in *electrostatic potential energy*, or ΔU_E .

- B. Three equally-spaced points (A , B , and C) are shown next to a positively charged block, fixed in place on a frictionless plane. A smaller positively charged block is initially placed at point A .



- The small $+q$ block is moved from point A to point B . The block begins and ends at rest. For the system of the two blocks, is ΔU_E *positive*, *negative*, or *zero*? Explain in terms of the work-energy equation given on page 1.
- Is ΔU_E due to moving $+q$ from point A to point B *greater than*, *less than*, or *equal to* ΔU_E due to moving $+q$ from point B to point C ? Explain.
- The $+q$ block is moved from point C to point B . In this case, is ΔU_E *positive*, *negative*, or *zero*? Explain.

- A second large block is added, also fixed in place. Point B is halfway between the two fixed blocks. Consider the system consisting of all three blocks.



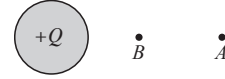
Suppose the $+q$ charge again moves from point A to point B . Would ΔU_E be *positive*, *negative*, or *zero*? Explain.

Would your answer change if the $+q$ block were now replaced by a block with charge $-q$? Explain why or why not.

⇔ Discuss your responses with a tutorial instructor before you continue.

II. Electric potential difference

A. A small positive test charge $+q$ is moved from point A to point B, toward a sphere with charge $+Q$.



1. Is ΔU_E positive, negative, or zero? Explain.

 2. Suppose the magnitude of the test charge q_{test} were halved. Would ΔU_E change? If so, how? If not, explain why not.

 3. Would the ratio $\Delta U_E/q_{\text{test}}$ change? If so, how? If not, explain why not.
- B. The quantity $\Delta U_E/q_{\text{test}}$ is called the *electric potential difference* between the test charge's initial and final positions. It is represented as $\Delta V_{i \rightarrow f}$.
- Is $\Delta V_{A \rightarrow B}$, the electric potential difference from point A to point B, positive, negative, or zero? Explain.
- C. Would the magnitude or sign of $\Delta V_{A \rightarrow B}$ from part A change if:
- the magnitude of the test charge were increased? Explain.

 - the sign of the test charge were changed? Explain.

 - the magnitude of the charge on the *sphere* were decreased? Explain.

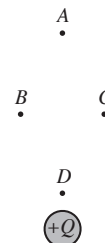
Based on your answers above, does $\Delta V_{A \rightarrow B}$ depend on the test charge, the sphere, or both?

⇨ Discuss your responses to section II with a tutorial instructor before you continue.

EM
4 *Electric potential difference*

III. Potential and reference points

A. Four points $A - D$ are shown near a positively charged sphere.



- Suppose you move a test charge $+q$ from point B to each of the other points in turn. For each movement, indicate whether ΔV is *positive*, *negative*, or *zero*.

B to A :

B to C :

B to D :

- Suppose the electric potential at point B were zero. Indicate whether the electric potential at each point is *positive*, *negative*, or *zero*.

Point A :

Point C :

Point D :

Point B is an example of a *reference point* – a point where the electric potential is defined to be zero. The electric potential of any other point can be defined with respect to this point.

- Suppose we had chosen point A as a reference point instead. What would the sign of the electric potential at points $B - D$ be in this case? Explain.

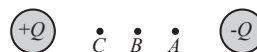
- The reference point is now moved toward the top of the page, infinitely far away. Does the electric potential at point B *increase*, *decrease*, or *remain the same* compared to when the reference was at point A ? Explain.

Does setting the reference point at infinity change the electric potential differences between the other points, for instance $\Delta V_{B \rightarrow D}$? Explain your reasoning.

IV. Applications of electric potential difference

In the previous sections, you investigated the change in electrostatic potential when a charged system was altered in some way. In this section we will explore how a charged object reacts to potential differences.

- A. Three points are shown between a positively charged and a negatively charged sphere. Consider a positive point charge $+q$ placed at point B .

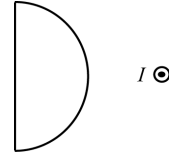


1. If you released the point charge from rest at point B , toward which of the other two points would it move?
 2. As a result of this movement, is ΔU_E *positive*, *negative*, or *zero*?
 3. Is the electric potential difference between the starting and ending positions of the test charge *positive*, *negative*, or *zero*? Explain.
- B. The positive point charge is replaced with a negative point charge $-q$, which is again placed at point B .
1. If you released the point charge from rest at point B , toward which of the other two points would it move?
 2. As a result of this movement, is ΔU_E *positive*, *negative*, or *zero*?
 3. Is the electric potential difference between the starting and ending positions of the test charge *positive*, *negative*, or *zero*? Explain.
- C. When a *positive* charge is free to move, will it move toward a region with higher or lower electric potential?

When a *negative* charge is free to move, will it move toward a region with higher or lower electric potential?

ED
HW-51**AMPÈRE'S LAW**ED
HW-51

1. The closed loop shown at right consists of a straight side and a curved side. To the right of the loop there is a current I out of the page perpendicular to the area bounded by the loop.

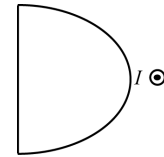


- a. What is the circulation around the entire closed loop? Explain.

Let $\int_L \vec{B} \cdot d\vec{l}$ represent the line integral of the straight left-hand part of the loop. Write an expression in terms of $\int_L \vec{B} \cdot d\vec{l}$ for the line integral of the curved part of the loop, $\int_C \vec{B} \cdot d\vec{l}$.

- b. Suppose that the curved part of the loop is made larger as shown. The straight part is unchanged.

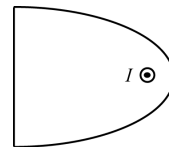
- i. Does the value of $\int_L \vec{B} \cdot d\vec{l}$ change? Explain.



- ii. How does the line integral of the new curved part of the loop compare to the line integral of the original curved portion of the loop? Explain.

- c. Suppose that the curved part of the loop is made even larger so that it now encloses the current as shown. The straight part is still unchanged.

- i. Does the value of $\int_L \vec{B} \cdot d\vec{l}$ change? Explain briefly.



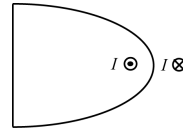
- ii. How does the line integral of the new curved part of the loop compare to the line integral of the original curved portion of the loop? Explain.

ED
HW-52 *Ampère's Law*

iii. Use Ampère's Law to write an expression in terms of $\int_L \vec{B} \cdot d\vec{l}$ and I for the line integral of the curved part of the loop.

d. A second current is introduced to the right of the loop as shown.

i. Is the value of $\int_L \vec{B} \cdot d\vec{l}$ greater than, less than, or equal to the value of $\int_L \vec{B} \cdot d\vec{l}$ in part c? Explain.



ii. Is the value of the circulation around the entire loop *greater than, less than, or equal to* the value of the circulation around the entire loop in part c? Explain.

2. In the *Gauss' Law* tutorial and homework, you learnt how you could compare the electric flux under certain circumstances by judiciously choosing closed surfaces. In this tutorial and homework, you have compared the line integral of the magnetic field by judiciously choosing closed loops.

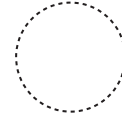
Choose one tutorial or homework question from the *Ampère's Law* tutorial or homework and compare it in detail with its counterpart in the *Gauss' Law* tutorial and homework.

AMPERE'S LAW

Name _____

EM
HW-1

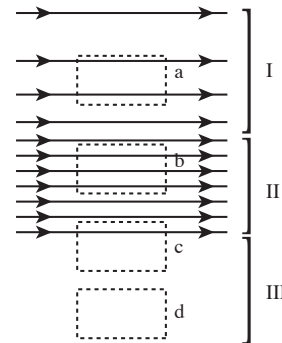
1. Some number of current-carrying wires (not shown) creates a magnetic field around the Amperian loop shown. The clockwise line integral of the magnetic field around the loop is $2\mu_0 I$. Consider the following student statement:



"There must be 2 wires that cross perpendicularly through the loop, each carrying a current I into the page. That's the only possible arrangement that works."

Is the student's conclusion correct? If the student is incorrect, what is the correct conclusion that can be drawn? Explain your reasoning.

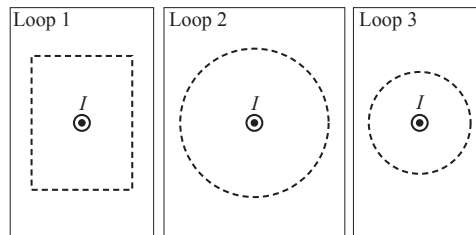
2. A current distribution (not shown) creates the spatially varying magnetic field shown at right. In region I the magnetic field increases in magnitude toward the bottom of the page. In region II the magnetic field is uniform, and in region III the magnetic field is zero.



- a. Consider loop a (in region I).
- Draw and label an area vector for loop a on the diagram.
 - Is the line integral of the magnetic field around loop a *positive*, *negative*, or *zero*? Explain.
 - Which direction is net current passing through the loop? Use Ampere's law to explain your answer. If no current passes through the loop, state so explicitly.
- b. Use the reasoning you developed in part a to find the direction that current is passing through loop b (in region II). Explain your reasoning.

EM *Ampere's law*
HW-2

- c. Use the reasoning you developed in part a to find the direction that current is passing through loop c (the top part of which is in region II, and the bottom part of which is in region III).
- d. Use the reasoning you developed in part a to find the direction that current is passing through loop d (in region III):
3. The line integral is the same for all three loops shown below. Loops 1 and 2 have the same perimeter. Consider the following discussion about the *incorrect* predictions made by three students:



- Student 1: "It seems like the integral around Loop 2 should be greater. They're the same length, but in Loop 2 the magnetic field always points along the path, and in Loop 1 only a component of the magnetic field points along the path."
- Student 2: "I thought that Loop 3 should be the largest integral, since the loop is closest to the wire so the magnetic field is strongest."
- Student 3: "I predicted the opposite. The path is shorter around Loop 3, so the integral around Loop 3 should be less than that around Loop 2."

Explain why each student's prediction is incorrect.

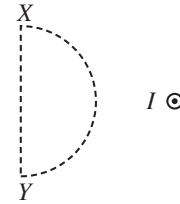
Ampere's law

Name _____

EM
HW-3

4. An Amperian loop is shown at right near a wire with current I out of the page.

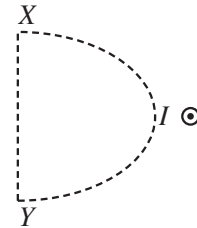
- a. What is the line integral of the magnetic field around the *entire* closed loop? Explain.



Let $\int_s \vec{B} \cdot d\vec{l}$ be the line integral from point X to point Y along the straight left-hand segment of the loop. Write an expression for $\int_c \vec{B} \cdot d\vec{l}$, the line integral from point Y to point X along the curved segment of the loop, in terms of $\int_s \vec{B} \cdot d\vec{l}$.

- b. Suppose that the curved side of the Amperian loop in part a is expanded as shown. The straight edge is unchanged.

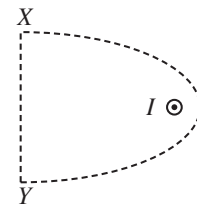
- i. Does the value of $\int_s \vec{B} \cdot d\vec{l}$ change? Explain.



- ii. Does the value of $\int_c \vec{B} \cdot d\vec{l}$ change? Explain.

- c. Suppose that the curved part is made even larger so that it now encloses the current-carrying wire, as shown. The straight side is still unchanged.

- i. Does the value of $\int_s \vec{B} \cdot d\vec{l}$ change? Explain.



- ii. Does the value of $\int_c \vec{B} \cdot d\vec{l}$ change? Explain.

AMPERE'S LAW

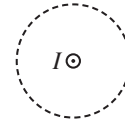
Name _____

EM
HW-1

1. Current is flowing out of the page, through the Amperian loop shown. Consider the following student dialogue:

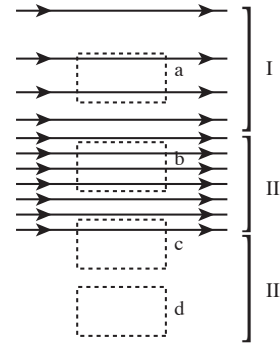
Student 1: "By the right hand rule the magnetic field will point counter-clockwise, so the line integral will be positive."

Student 2: "I disagree. The area vector should be into the page which makes the encircled current negative, so the line integral is also negative."



Is either student correct? Could both be correct? Explain.

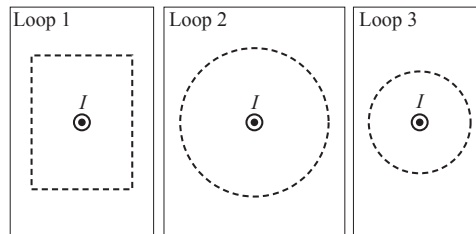
2. A current distribution (not shown) creates the spatially varying magnetic field shown at right. In region I the magnetic field increases in magnitude toward the bottom of the page. In region II the magnetic field is uniform, and in region III the magnetic field is zero.



- a. Consider loop a (in region I).
- Draw and label an area vector for loop a on the diagram.
 - Which direction should you integrate around the loop? Explain.
 - Is the line integral of the magnetic field around loop a *positive, negative, or zero*? Explain.
 - Which direction is net current passing through the loop? Use Ampere's law to explain your answer. If no current passes through the loop, state so explicitly.
- b. Which direction is current passing through loop b (in region II)? Explain, using the reasoning you developed in part a.

EM *Ampere's law*
HW-2

- c. What direction is current passing through loop c (the top part of which is in region II, and the bottom part of which is in region III)? Explain, using the reasoning you developed in part a.
- d. What direction is current passing through loop d (in region III)? Explain, using the reasoning you developed in part a.
3. The line integral is the same for all three loops shown below. Loops 1 and 2 have the same perimeter. Consider the following discussion about the *incorrect* predictions made by three students:



- Student 1: "It seems like the integral around Loop 2 should be greater. They're the same length, but in Loop 2 the magnetic field always points along the path, and in Loop 1 only a component of the magnetic field points along the path."
- Student 2: "I thought that Loop 3 should be the largest integral, since the loop is closest to the wire so the magnetic field is strongest."
- Student 3: "I predicted the opposite. The path is shorter around Loop 3, so the integral around Loop 3 should be less than that around Loop 2."

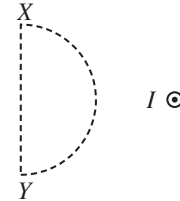
Explain why each student's prediction is incorrect.

Ampere's law

Name _____

EM
HW-3

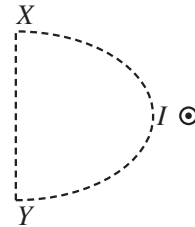
4. An Amperian loop is shown at right near a wire with current I out of the page.
- a. What is the line integral of the magnetic field around the *entire* closed loop? Explain.



Let $\int_s \vec{B} \cdot d\vec{l}$ be the line integral from point X to point Y along the straight left-hand segment of the loop. Write an expression for $\int_c \vec{B} \cdot d\vec{l}$, the line integral from point Y to point X along the curved segment of the loop, in terms of $\int_s \vec{B} \cdot d\vec{l}$.

- b. Suppose that the curved side of the Amperian loop in part a is expanded as shown. The straight edge is unchanged.

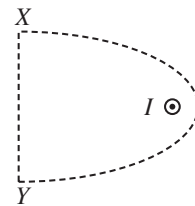
i. Does the value of $\int_s \vec{B} \cdot d\vec{l}$ change? Explain.



ii. Does the value of $\int_c \vec{B} \cdot d\vec{l}$ change? Explain.

- c. Suppose that the curved part is made even larger so that it now encloses the current-carrying wire, as shown. The straight side is still unchanged.

i. Does the value of $\int_s \vec{B} \cdot d\vec{l}$ change? Explain.



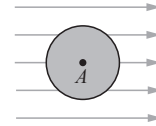
ii. Does the value of $\int_c \vec{B} \cdot d\vec{l}$ change? Explain.

ELECTRIC PROPERTIES OF CONDUCTORS Name _____

Mech
HW-1

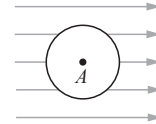
1. In the tutorial you considered a neutral metal sphere in an external electric field.

- a. On the diagram, draw the induced charge distribution you found in the tutorial.



- b. The conducting sphere is replaced with a plastic sphere, with the exact same charge distribution that you drew for the metal sphere.

- i. Is the net electric field at point *A* in the new sphere *greater than, less than, or the same as* what it was in the metal sphere? Explain your reasoning.

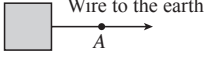


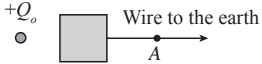
- ii. Is the plastic sphere an equipotential? Explain why or why not.

- iii. If the external electric field were removed, would the plastic sphere be an equipotential? Explain.

- c. Suppose the neutral sphere was placed in the external field at time t_0 . The instant the sphere is placed in the field, is the sphere an equipotential? Explain.

Mech *Electric Properties of Conductors*
HW-2

2. A small metal cube is grounded by connecting it to the earth with a wire.  Wire to the earth
A
- a. Is the electric potential difference between the cube and the earth *positive, negative, or zero*? Explain.

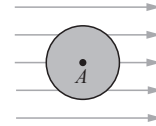
- b. A positive point charge is placed next to the cube, and the system reaches equilibrium.  $+Q_o$
Wire to the earth
A
- Indicate the direction of the electric field at point A, between the cube and the earth. If the electric field is zero, state so explicitly. Explain.

- c. The wire is removed without disturbing any charges, and then the point charge is removed.
- i. Indicate the direction of the electric field at point A. Explain.
- ii. Is the electric potential difference between the cube and the earth *positive, negative, or zero*? Explain.

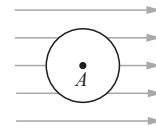
ELECTRIC PROPERTIES OF CONDUCTORS Name _____
EM
HW-1

1. In the tutorial you considered a neutral metal sphere in an external electric field of a charged sheet (the sheet is not shown, but the field it produces is indicated).

- a. On the diagram, draw the induced charge distribution you found in the tutorial.



- b. The conducting sphere is replaced with a plastic sphere, with the exact same charge distribution that you drew for the metal sphere.



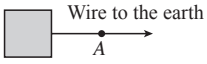
- i. Is the net electric field at point A in the new sphere *greater than, less than, or the same as* what it was in the metal sphere? Explain your reasoning.


- ii. Is the plastic sphere an equipotential? Explain why or why not.

- iii. If the external electric field were removed, would the plastic sphere be an equipotential? Explain.

- c. Suppose the neutral conducting sphere was placed in the external field at time t_0 . The instant the sphere is placed in the field, is the sphere an equipotential? Explain.

EM *Electric Properties of Conductors*
 HW-2

2. A small metal cube is grounded by connecting it to the earth with a wire. 
- a. Is the electric potential difference between the cube and the earth *positive, negative, or zero*? Explain.

- b. A positive point charge is placed next to the cube, and the system reaches equilibrium. 
- Indicate the direction of the electric field at point A, between the cube and the earth. If the electric field is zero, state so explicitly. Explain.

- c. The wire is removed without disturbing any charges, and then the point charge is removed.
- i. Indicate the direction of the electric field at point A. Explain.
- ii. Is the electric potential difference between the cube and the earth *positive, negative, or zero*? Explain.

ELECTRIC PROPERTIES OF CONDUCTORS Name _____

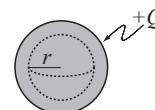
EM
HW-1

1. In tutorial you discussed the stable charge distribution of a charged, conducting sphere. Consider the following student statement:

"I think the charges will be uniformly distributed through the sphere in a stable distribution."

Answer the following questions as if this student's charge distribution is correct, *i.e.* $+Q$ is uniform throughout the volume of the sphere.

- a. Is the net flux through the Gaussian surface shown *positive, negative, or zero*?



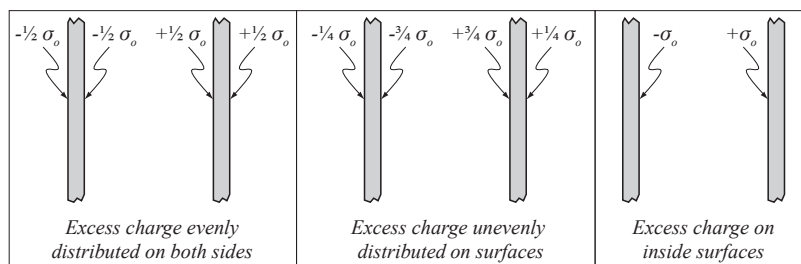
- b. What is the direction of the electric field at radius r ? If the electric field is zero state so explicitly. Explain.

- c. How, if at all, would charges at radius r move?

- d. Is your answer to part c consistent with a stable distribution? Explain.

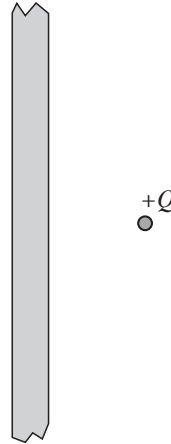
2. Two conducting plates are placed close together. The left plate is given a negative charge, and the right an equal amount of positive charge.

The diagrams below show various distributions of charge on the two plates. Decide which arrangement is physically correct. Explain.

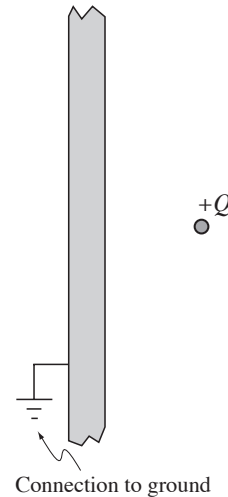


EM *Electric Properties of Conductors*
 HW-2

3. A portion of a large, conducting slab is shown at right.
- A point charge $+Q$ is placed near the right side of the slab.
 - On the diagram, draw the electric field inside the slab generated only by the $+Q$ charge.
 - Use another color to sketch the electric field that must be generated by the slab.
 - Sketch a charge distribution that could make this electric field.
 - Is the magnitude of the surface charge density on the slab uniform? Explain.



- The conducting slab is now connected to ground.
 - When a stable distribution is reached, what is the net charge on the slab? Explain.
 - The connection to ground is removed. Is the electric potential difference between the slab and ground *positive*, *negative*, or *zero*? Explain.



- The $+Q$ charge is also removed. Sketch the stable charge distribution on the slab.
- Describe the electric field generated by this charge distribution.

Electric Properties of Conductors

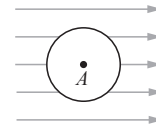
Name _____

EM
HW-3

4. This question has three independent parts. In each case the insulators are perfect.

a. A neutral, insulating sphere is placed in a uniform external field \mathbf{E}_0 .

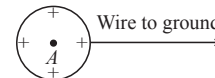
i. Describe how, if at all, the charges on the sphere move.



ii. What is the magnitude and direction of the electric field at point A?

b. An isolated, insulating sphere has a positive charge $+Q$ uniformly distributed on its surface. At time t_0 the sphere is connected to ground.

i. Describe how, if at all, the charges on the sphere move.

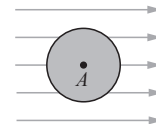


ii. Is the electric potential difference between point A and ground *positive*, *negative*, or *zero*? Explain.

c. A neutral, conducting sphere is placed in a uniform external field \mathbf{E}_0 .

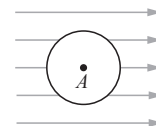
i. Sketch the stable charge distribution.

ii. What is the magnitude and direction of the electric field at point A?



The conducting sphere is replaced with an insulating sphere that has exactly the same charge distribution drawn above.

iii. What is the magnitude and direction of the electric field at point A? Explain.



The direction of the external field is reversed so that it points to the left.

iv. What is the magnitude and direction of the electric field at point A? Explain.

ELECTRIC PROPERTIES OF CONDUCTORS

EM
HW-1

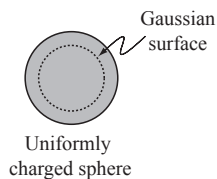
1. At t_0 , a uniform volume charge density is placed on a metal sphere, with total charge $+Q$. Consider the following student statement:

"The charges are uniformly distributed throughout the sphere, so the distribution should be stable."

Use the following questions as a guide to determine whether or not this statement is correct.

- a. A spherical Gaussian surface is centered inside the sphere.

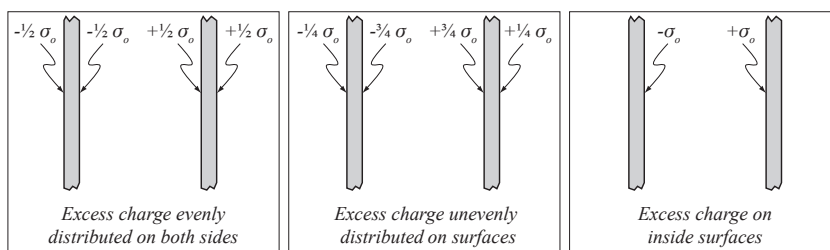
At t_0 , is the net flux through the Gaussian surface *positive, negative, or zero*?



- b. What is the direction of the electric field at a point on the Gaussian surface? If the electric field is zero, state so explicitly. Explain.
- c. Would a positive charge at this point experience a force? If so, in which direction?
- d. Is this student's statement correct? Explain why or why not.

2. Two conducting plates are placed close together. The left plate is given a negative charge, and the right an equal amount of positive charge.

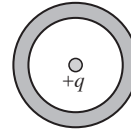
The diagrams below show various distributions of charge on the two plates. Decide which arrangement is physically correct. Explain why each case is or is not physically correct.



EM *Electric Properties of Conductors*
 HW-2

3. A point charge $+Q$ is placed next to a hollow, spherical conducting shell. A second point charge $+q$ is placed at the center of the cavity.

$+Q$



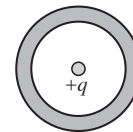
Spherical conducting shell

- a. On the diagram, sketch the net electric field inside the shell. Explain.
- b. Would the $+q$ charge experience a force? If so, in which direction? Explain why or why not.

The $+Q$ charge is now moved closer to the shell as shown.

- c. After a new stable distribution is reached, sketch the net electric field inside the shell. Explain.

$+Q$

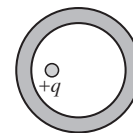


- d. Suppose you were inside the shell. Could you tell that the $+Q$ charge had changed positions? Explain why or why not.

The $+q$ charge is now moved off-center, toward the $+Q$ charge as shown.

- e. Would this movement affect the $+Q$ charge? Explain why or why not.

$+Q$



- f. Based on your answers above, does any change to the charge distributions either inside or outside of the sphere affect the electric field in the other region? Explain.

Electric Properties of Conductors

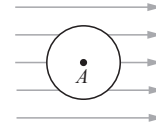
Name _____

EM
HW-3

4. The following questions involve three identical, perfectly insulating spheres.

a. A neutral, insulating sphere is placed in a uniform external field \vec{E} .

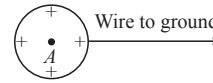
i. Describe how, if at all, the charges on the sphere move.



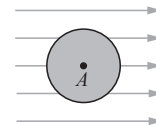
ii. What is the magnitude and direction of the electric field at point A?

b. An isolated, insulating sphere has a positive charge $+Q$ uniformly distributed throughout its volume. At time t_0 the sphere is connected to ground.

i. Describe how, if at all, the charges on the sphere move.

ii. Is the electric potential difference from point A to the ground *positive*, *negative*, or *zero*? Explain.c. A neutral, conducting sphere is placed in a uniform external field \vec{E} .

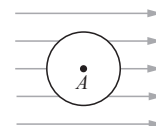
i. Sketch the stable charge distribution.



ii. What is the magnitude and direction of the electric field at point A?

The conducting sphere is replaced with an insulating sphere that has exactly the same charge distribution that you drew in part i above.

iii. What is the magnitude and direction of the electric field at point A? Explain.



The direction of the external field is reversed so that it points to the left.

iv. What is the magnitude and direction of the electric field at point A? Explain.

ELECTRIC POTENTIAL DIFFERENCE

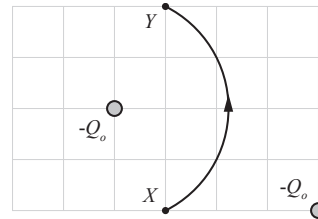
Name _____

EM
HW-1

1. Two charged rods, each with net charge $-Q_o$, are held in place as shown in the top-view diagram below.

a. A small test charge $-q_o$ travels from point X to point Y along the circular arc shown.

- i. Draw an arrow on the diagram at each point (X and Y) to show the direction of the electric force on the test charge at that point. Explain why you drew the arrows as you did.



- ii. Is the work done on the charge by the electric field *positive*, *negative*, or *zero*? Explain.

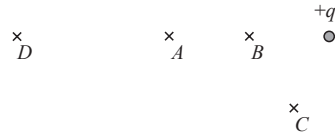
- iii. Is the electric potential difference $\Delta V_{X \rightarrow Y}$ (i.e., $V_Y - V_X$) *positive*, *negative*, or *zero*? Explain.

- b. The test charge is launched from point X with an initial speed v_o and is observed to pass through point Y . Is the speed of the test charge at point Y *greater than*, *less than*, or *equal to* v_o ? Explain your reasoning.

EM *Electric potential difference*
HW-2

2. A positive charge of magnitude q_o is shown in the diagram below.

a. Points B and C are a distance r_o away from the charge, point A is a distance $2r_o$ from it, and point D is even farther away.



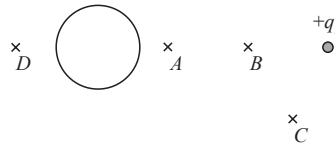
i. On the diagram, indicate the direction of the electric field at points A , B , C , and D .

ii. Let W_{AB} represent the absolute value of the work done by an external agent in moving a small test charge from rest at point A to rest at point B .

- Would the absolute value of the work done by an external agent in moving the same test charge from point B to point C be *greater than*, *less than*, or *equal to* W_{AB} ? Assume the particle is at rest at both points. Explain.

- Would the absolute value of the work done by an external agent in moving the same test charge from point A to point C be *greater than*, *less than*, or *equal to* W_{AB} ? Assume the particle is at rest at both points. Explain.

b. A large metal sphere with zero net charge is now placed to the left of point A as shown.



i. Sketch the charge distribution on the metal sphere in the diagram at right.

ii. Has the *magnitude* of the electric field at the following points *increased*, *decreased*, or *remained the same*? Explain.

- point B

- point D

Electric potential difference

Name _____

EM
HW-3iii. Has the *direction* of the electric field at the following points changed? Explain.

- point *B*

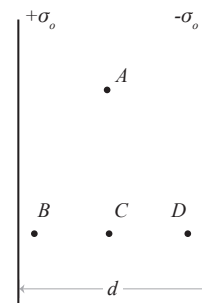
- point *C*

iv. Has the absolute value of the electric potential difference $\Delta V_{A \rightarrow B}$ (i.e., $|V_B - V_A|$) *increased*, *decreased*, or *remained the same*? Explain your reasoning.

3. Two very large sheets of charge are separated by a distance d . One sheet has a surface charge density $+\sigma_o$; the other, a surface charge density $-\sigma_o$. A small region near the center of the sheets is shown.

a. Draw arrows on the diagram to indicate the direction of the electric field at points *A*, *B*, *C*, and *D*.

i. Compare the magnitudes of the electric fields at points *A*, *B*, *C*, and *D*. Explain.



ii. How would the electric force exerted on a charged particle at point *A* compare to the electric force exerted on the same particle at:

- point *B*?

- point *C*?

- point *D*?

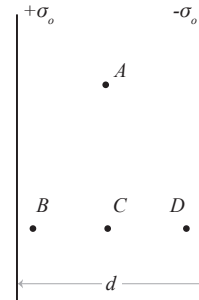
EM HW-4 **Electric potential difference**

The diagram from the previous page is reproduced at right for your convenience.

b. A positively charged test particle moves from point *A* to point *C*.

i. Is the work done on the particle by the electric field *positive*, *negative*, or *zero*? Explain.

ii. Find $\Delta V_{A \rightarrow C}$ (i.e., $V_C - V_A$). Explain how you determined your answer.



c. A positively charged test particle moves from point *A* to point *D*.

i. Is the work done on the particle by the electric field *positive*, *negative*, or *zero*? Explain.

ii. Is $\Delta V_{A \rightarrow D}$ (i.e., $V_D - V_A$) *positive*, *negative*, or *zero*? Explain how you can tell.

d. Determine the magnitude and direction of the electric field at points *A*, *B*, *C*, and *D*. (*Hint*: Use superposition and your results for the electric field of a large sheet from the tutorial *Gauss' law*.)

e. A particle of mass m_0 and charge $-q_0$ is released from rest at a point just to the left of the right sheet. Find the speed of the particle as it reaches the left sheet. Express your answer in terms of given variables.

ELECTRIC POTENTIAL DIFFERENCE

Name _____

Mech
HW-1

1. A student makes some measurements of electric potential difference near two charged systems.

a. The student moves a positive test charge from point S to point T as shown, and finds that $\Delta V_{S \rightarrow T} = 0$. \times
 S

i. Indicate on the diagram the position of the charge(s) that would give this measurement, and state whether each charge is positive or negative. Is there more than one possible arrangement? Explain.

 \times
 T ii. The test charge is launched from point S with an initial speed v_o and is observed to pass through point T . Is the speed of the test charge at point T *greater than*, *less than*, or *equal to* v_o ? Explain your reasoning.b. Four points W , X , Y , and Z are near a different charged system. The student observes that $\Delta V_{W \rightarrow Y} = 0$, $\Delta V_{W \rightarrow Z} = -V_o$, and $\Delta V_{W \rightarrow X} = -2V_o$. \times \times \times
 Z Y X

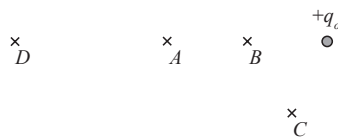
i. Indicate on the diagram the position of the charge(s) that would give these readings, and state whether each charge is positive or negative. Explain.

 \times
 W ii. A small negative test charge is placed at point W . On the diagram, draw an arrow showing which direction the test charge would move. Explain.

Mech HW-2 **Electric potential difference**

2. A positive charge of magnitude q_o is shown in the diagram below.

a. Points B and C are a distance r_o away from the charge, point A is a distance $2r_o$ from it, and point D is even farther away.



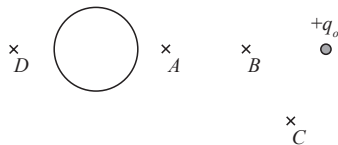
i. On the diagram, indicate the direction of the electric field at points A , B , C , and D .

ii. Let W_{AB} represent the absolute value of the work done by an external agent in moving a small test charge from rest at point A to rest at point B .

- Would the absolute value of the work done by an external agent in moving the same test charge from point B to point C be *greater than*, *less than*, or *equal to* W_{AB} ? Assume the particle is at rest at both points. Explain.

- Would the absolute value of the work done by an external agent in moving the same test charge from point A to point C be *greater than*, *less than*, or *equal to* W_{AB} ? Assume the particle is at rest at both points. Explain.

b. A large metal sphere with zero net charge is now placed to the left of point A as shown.



i. Sketch the charge distribution on the metal sphere in the diagram at right.

ii. Has the *magnitude* of the electric field at the following points *increased*, *decreased*, or *remained the same*? Explain.

- point B

- point D

Electric potential difference

Name _____

Mech
HW-3iii. Has the *direction* of the electric field at the following points changed? Explain.

- point *B*

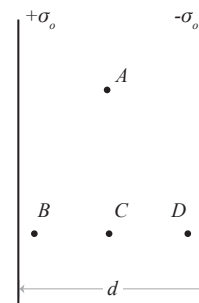
- point *C*

iv. Has the absolute value of the electric potential difference $\Delta V_{A \rightarrow B}$ (i.e., $|V_B - V_A|$) *increased*, *decreased*, or *remained the same*? Explain your reasoning.

3. Two very large sheets of charge are separated by a distance d . One sheet has a surface charge density $+\sigma_o$; the other, a surface charge density $-\sigma_o$. A small region near the center of the sheets is shown.

a. Draw arrows on the diagram to indicate the direction of the electric field at points *A*, *B*, *C*, and *D*.

i. Compare the magnitudes of the electric fields at points *A*, *B*, *C*, and *D*. Explain.



ii. How would the electric force exerted on a charged particle at point *A* compare to the electric force exerted on the same particle at:

- point *B*?

- point *C*?

- point *D*?

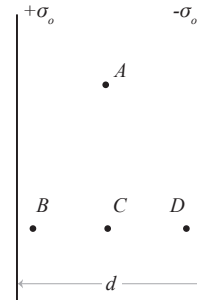
Mech HW-4 **Electric potential difference**

The diagram from the previous page is reproduced at right for your convenience.

b. A positively charged test particle moves from point *A* to point *C*.

i. Is the work done on the particle by the electric field *positive*, *negative*, or *zero*? Explain.

ii. Find $\Delta V_{A \rightarrow C}$ (i.e., $V_C - V_A$). Explain how you determined your answer.



c. A positively charged test particle moves from point *A* to point *D*.

i. Is the work done on the particle by the electric field *positive*, *negative*, or *zero*? Explain.

ii. Is $\Delta V_{A \rightarrow D}$ (i.e., $V_D - V_A$) *positive*, *negative*, or *zero*? Explain how you can tell.

d. Determine the magnitude and direction of the electric field at points *A*, *B*, *C*, and *D*. (*Hint*: Use superposition and your results for the electric field of a large sheet from the tutorial *Gauss' law*.)

e. A particle of mass m_0 and charge $-q_0$ is released from rest at a point just to the left of the right sheet. Find the speed of the particle as it reaches the left sheet. Express your answer in terms of given variables.

ELECTRIC POTENTIAL DIFFERENCE

Name _____

EM
HW-1

1. A test charge is placed at point A, next to a positively charged sphere as shown.



- a. In this problem, consider the system consisting of both charges. In each of the following cases, if it is not possible to answer the question, state so explicitly. Explain your reasoning in each case.

A small black dot with the letter "A" below it.

- i. The test charge has charge $+q$. How could you move the test charge so that the potential energy of the system *increases*?
- ii. The test charge is replaced with a negative test charge, with charge $-q$. How could you move the test charge so that the potential energy of the system *increases*?

- The charge on the sphere is made negative as shown. A test charge is still placed at point A.



- b. In this problem, consider the system consisting of both charges. In each of the following cases, if it is not possible to answer the question, state so explicitly. Explain your reasoning in each case.

A small black dot with the letter "A" below it.

- i. The test charge has charge $+q$. How could you move the test charge so that the potential energy of the system *decreases*?
- ii. The test charge is replaced with a negative test charge, with charge $-q$. How could you move the test charge so that the potential energy of the system *decreases*?

2. Suppose you move a test charge from point S to point T, and find that $\Delta V_{S \rightarrow T}$ is positive.

A small black dot with the letter "S" below it.

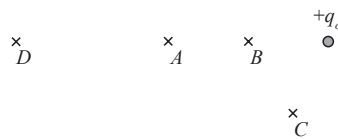
- a. Suppose that the system that produces this potential difference consists of a single positively charged ball. Where could the positive charge be located? Is there more than one possible answer? Explain.

A small black dot with the letter "T" below it.

- b. Suppose instead that the system producing the electric potential difference was a negatively charged ball. Where could it be located?

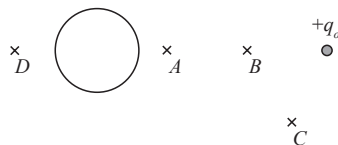
EM *Electric potential difference*
HW-2

3. A positive charge of magnitude q_o is shown in the diagram at right.



- a. Points B and C are a distance r_o away from the charge, point A is a distance $2r_o$ from it, and point D is even farther away.
- On the diagram, indicate the direction of the electric field at points A , B , C , and D .
 - Let W_{AB} represent the absolute value of the work done by an external agent in moving a small test charge from rest at point A to rest at point B .
 - Would the absolute value of the work done by an external agent in moving the same test charge from point B to point C be *greater than*, *less than*, or *equal to* W_{AB} ? Assume the particle is at rest at both points. Explain.
 - Would the absolute value of the work done in moving the same test charge from point A to point C be *greater than*, *less than*, or *equal to* W_{AB} ? Assume the particle is at rest at both points. Explain.

- b. A large metal sphere with zero net charge is now placed to the left of point A as shown.



- Sketch the charge distribution on the metal sphere in the diagram at right.
- Has the *magnitude* of the electric field at the following points *increased*, *decreased*, or *remained the same*? Explain.
 - point B
 - point D

Electric potential difference

Name _____

EM
HW-3

iii. Has the *direction* of the electric field at the following points changed? Explain.

- point *B*

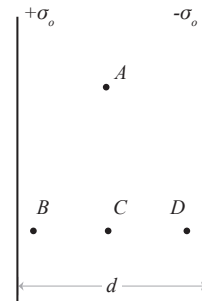
- point *C*

iv. Has the absolute value of the electric potential difference $\Delta V_{A \rightarrow B}$ (i.e., $|V_B - V_A|$) *increased*, *decreased*, or *remained the same*? Explain your reasoning.

4. Two very large sheets of charge are separated by a distance d . One sheet has a surface charge density $+\sigma_o$; the other, a surface charge density $-\sigma_o$. A small region near the center of the sheets is shown.

a. Draw arrows on the diagram to indicate the direction of the electric field at points *A*, *B*, *C*, and *D*.

i. Compare the magnitudes of the electric fields at points *A*, *B*, *C*, and *D*. Explain.



ii. How would the electric force exerted on a charged particle at point *A* compare to the electric force exerted on the same particle at each of the following points. Explain your reasoning in each case.

- point *B*

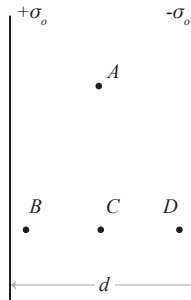
- point *C*

- point *D*

EM *Electric potential difference*
HW-4

The diagram from the previous page is reproduced at right for your convenience.

- b. You move a positively charged test particle from point *A* to point *C*.
- i. During the movement, do you do *positive*, *negative*, or *zero* work on the particle? Explain.



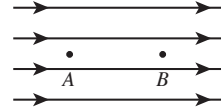
- ii. Find $\Delta V_{A \rightarrow C}$ (i.e., $V_C - V_A$). Explain how you determined your answer.
- c. You move a positively charged test particle from point *A* to point *D*.
- i. During the movement, do you do *positive*, *negative*, or *zero* work on the particle? Explain.
- ii. Is $\Delta V_{A \rightarrow D}$ (i.e., $V_D - V_A$) *positive*, *negative*, or *zero*? Explain how you can tell.
- d. Determine the magnitude and direction of the electric field at points *A*, *B*, *C*, and *D*. (*Hint*: Use superposition and your results for the electric field of a large sheet from the tutorial *Gauss' law*.)
- e. A particle of mass m_0 and charge $-q_0$ is released from rest at a point just to the left of the right sheet. Find the speed of the particle as it reaches the left sheet. Express your answer in terms of the given variables.

ELECTRIC POTENTIAL DIFFERENCE

Name _____

EM
HW-1

1. Points *A* and *B* are 1 m apart, in a region of uniform electric field with magnitude $E_0 = 2 \text{ N/C}$. Three test charges (with magnitudes $q_1 = 1 \text{ nC}$, $q_2 = 2 \text{ nC}$, and $q_3 = -0.5 \text{ nC}$) are moved from point *A* to point *B*.

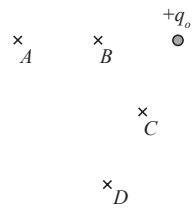


- a. Complete the table below with the answers to each of the following questions. For each case, consider the system consisting of the test charge and the uniform field.
- How much force would a hand need to exert on each charge to hold it at rest at point *A*? Specify both magnitude and direction.
 - How much work would it take for a hand to move each charge from point *A* to point *B* at constant speed?
 - What would be the change in potential energy of each system as a result of each movement?
 - What would be the potential difference from point *A* to point *B* in each case?

	Force by hand	Work by hand	ΔU	ΔV
q_1				
q_2				
q_3				

- b. Now points *A* and *B* are moved farther apart, so the distance between them is 2 m. Would any of the values in the table above change? If so, state how these values would change. Explain.
- c. Now the magnitude of the electric field is increased to 3 N/C. Would any of the values in the table above change? If so, state how these values would change. Explain.
- d. In the experiments above, indicate which of the following variables affected ΔV . Explain.
- The magnitude of q_{test}
 - The sign of q_{test}
 - The external field
 - The distance between the points

EM *Electric potential difference*
HW-2

2. A positive charge of magnitude q_o is shown in the diagram at right. Points B and C are a distance r_o away from the charge, and points A and D are a distance $2r_o$ from it.
- 
- a. On the diagram, indicate the direction of the electric field at each point.
- b. Suppose a hand moves a positive test charge from point A . To which of the other points could the charge be moved without doing any net work on the charge? Explain.

What is the potential difference from point A to any of the points you identified above? Explain.

A set of points that all have the same value of potential are called an *equipotential* or *equipotential surface*.

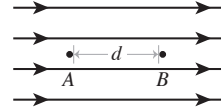
- c. What shape are the equipotential surfaces around q_o ? Explain, and draw a few equipotentials on the diagram above. (*Hint*: The test charge can move in any direction, including into and out of the page.)
- d. Let W_{AB} represent the work done by a hand to move a positive test charge from point A to point B , and W_{AC} represent the work done by a hand to move a positive test charge from point A to point C .
- Is W_{AB} *positive*, *negative*, or *zero*?
 - Would the absolute value of W_{AB} be *greater than*, *less than*, or *equal to* the absolute value of W_{AC} ? Explain.

ELECTRIC POTENTIAL DIFFERENCE

Name _____

EM
HW-1

1. Points *A* and *B* are a distance $d = 1$ m apart, in a uniform electric field with magnitude $E_o = 2$ N/C. A hand moves three test charges (with magnitudes $q_1 = 1$ nC, $q_2 = 2$ nC, and $q_3 = -0.5$ nC) in turn from point *A* to point *B*.



- a. Consider the system consisting of each test charge and the uniform field. Fill out the table below as you answer the four questions below. For each quantity, specify the numerical value, including sign or direction where appropriate.
- i. How much force would a hand need to exert on each charge to hold it at rest at point *A*? Draw an arrow to indicate the direction of the force.
 - ii. How much work would it take for a hand to move each charge from point *A* to point *B* at constant speed?
 - iii. What would be the change in potential energy of each system as a result of each movement?
 - iv. What would be the potential difference from point *A* to point *B* in each case?

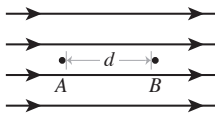
	Force by hand	Work by hand	ΔU	ΔV
q_1				
q_2				
q_3				

- b. Now points *A* and *B* are moved farther apart, so $d = 2$ m. Would any of the quantities in the table above change? If so, state how these values would change. Explain your reasoning for each quantity.
- c. Now the magnitude of the electric field is increased to 3 N/C. Would any of the quantities in the table above change? If so, state how these values would change. Explain your reasoning for each quantity.
- d. Circle each of the variables in these experiments that affect ΔV .

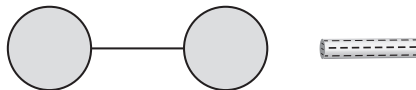
E_o d The sign of q_{test} The magnitude of q_{test}

Explain your reasoning.

EM *Electric potential difference*
HW-2

2. The electric field from problem 1 is shown at right, with magnitude E_0 . A hand moves a positive test charge $+q_{\text{test}}$ from point A to point B at constant speed.
- 
- During the motion, what is the magnitude and direction of the force that the hand must exert on the charge to keep it moving at constant speed? Write your answer in terms of the
 - Write an integral expression for the net external work the hand does on the charge, as the charge moves from point A to point B .
 - Use the definition $\Delta V = \Delta U / q_{\text{test}}$ and your answer above to write an integral expression for potential difference, in terms of the electric field vector.
 - Based on your answer above, does ΔV depend on the test charge? Explain why or why not.

3. Two small metal spheres are connected by a wire, and a negatively charged rod is placed next to the right sphere.



Consider the following student discussion about the potential difference from the left sphere to the right sphere:

Student 1: "Since the right sphere is positively charged, the potential is higher there than at the left sphere. Thus the potential difference from left to right is positive."

Student 2: "I disagree. The right sphere is closer to the negative rod, so the potential at the right sphere is lower. The potential difference from left to right is negative."

Student 3: "Positive charges want to move to regions of lower potential, but no charges are moving since the system is in equilibrium. Thus the potential difference is zero."

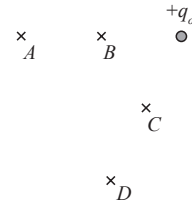
With which student, if any, do you agree? Explain.

Electric potential difference

Name _____

EM
HW-3

4. A positive charge of magnitude q_o is shown in the diagram at right. Points B and C are a distance r_o away from the charge, and points A and D are a distance $2r_o$ from it.



- a. On the diagram, indicate the direction of the electric field at each point.
- b. Suppose a hand moves a positive test charge from point A . To which of the other points could the charge be moved without doing any net work on the charge? Explain.

What is the potential difference from point A to any of the points you identified above? Explain.

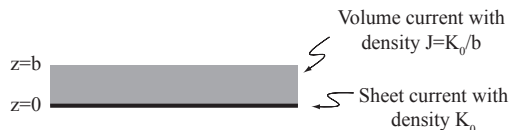
A set of points that all have the same value of potential are called an *equipotential* or *equipotential surface*.

- c. What shape are the equipotential surfaces around q_o ? Explain, and draw a few equipotentials on the diagram above. (*Hint*: The test charge can move in any direction, including into and out of the page.)
- d. Let W_{AB} represent the work done by a hand to move a positive test charge from point A to point B , and W_{AC} represent the work done by a hand to move a positive test charge from point A to point C .
- Is W_{AB} *positive*, *negative*, or *zero*?
 - Would the absolute value of W_{AB} be *greater than*, *less than*, or *equal to* the absolute value of W_{AC} ? Explain.

Name _____ Student ID _____ Score _____
 last *first*

IV. [33 points total] Tutorial question.

The diagram at right shows an infinite sheet of current at the surface of an infinite slab of current. At $z=0$ there is a uniform surface current K_0 into the page, and above it the slab has uniform current density out of the page with magnitude K_0/b , from $z=0$ to $z=+b$.

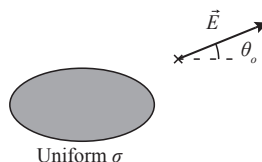


- A. Suppose you wanted to know the magnetic field at the center of the slab, at $z=b/2$. Sketch an Amperian loop that would help you find \mathbf{B} at the center. Explain your reasoning, and any assumptions you are making.
- B. Use superposition to find the magnetic field at any point inside the slab, from $z=0$ to $z=+b$. Show your reasoning.
- C. If you inspect your answer to part (ii), you should find that $\mathbf{B}=0$ for one value of z inside the slab. If the current density in the slab is doubled, but the sheet current is unchanged, would there still be a value of z with $\mathbf{B}=0$ inside the slab? If so, find it. If not, explain why not.
- D. In this case, use superposition to find \mathbf{B} for $z<0$. Explain your reasoning, and any assumptions you are making.

Name _____ Student ID _____ Score _____
last first

III. [33 points total] Tutorial question.

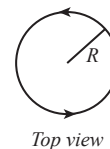
- A. [12 pts] A uniformly charged ellipse in the plane of the page is shown below. At the point shown, the electric field is measured to have a magnitude E_o at an angle θ_o above the horizontal.



At what other point(s) is behavior of the electric field similar to the electric field of the point shown? For each point, label and sketch the electric field at that point on the diagram above. Explain your reasoning for each point in the space below.

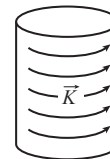
- B. A portion of a long solenoid is shown at right, in perspective and top view. The solenoid has uniform surface current density K .

- i. [7 pts] Based on your knowledge of symmetry and the behavior of magnetic fields, which direction(s) is it *impossible* for the magnetic field of the solenoid to point? Explain your reasoning in each case.



Top view

- ii. [7 pts] Which spatial variable(s) is it *impossible* for the magnitude of \vec{B} to depend on? Explain your reasoning in each case.



Portion of a very long solenoid

- iii. [3 pts] Suppose you wanted to find \vec{B} a distance $s = R/2$ from the center of the solenoid. Clearly draw an Ampèrian loop on the relevant diagram that would allow you to find the field at this distance.
- iv. [4 pts] Inside the solenoid, how does the magnitude of the magnetic field depend on the distance s from the center of the solenoid? Explain *without* explicitly finding a formula for \vec{B} .

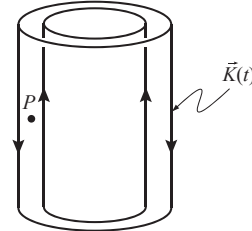
Name _____ Student ID _____ Score _____
last first

III. [33 points total] Tutorial question.

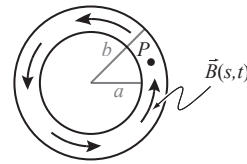
A. [9 pts] A long coaxial cable has a time-dependent surface current at radius a and at radius $b > a$.

The magnetic field is given as $\vec{B} = \frac{I_0}{2\pi s} t \hat{\phi}$ for $a < s < b$, and zero everywhere else.

Describe a method to find the vector potential \vec{A} at point P within the cable at a radius R where $a < R < b$. Explain in detail. (You may assume that the vector potential at $s \rightarrow \infty$ is zero.)

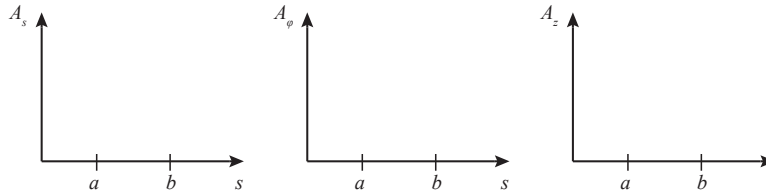


Coaxial cable with surface currents



Cross sectional view with magnetic field

B. [16 pts] Graph the vector potential as a function of radius at a time $t \neq 0$. If a particular segment or graph is zero, state so explicitly.



For the segments that are non-zero, what is the dependence on s ? (E.g., s , s^2 , $1/s$, e^s , $\ln(s)$, constant.) Explain.

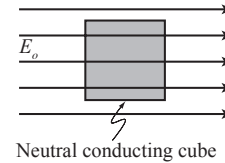
C. [8 pts] Suppose a positive charge were placed at the center of the cable. Would the charge move? If so, in what direction? Explain.

CONDUCTORS

ED
1

I. Conductors in electric fields

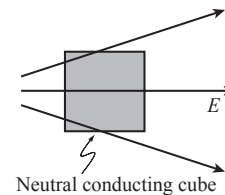
A. A conducting cube is placed in a uniform electric field with magnitude E_o pointing to the right, as shown at right.



1. After a long time, what is the net electric field inside the cube?
2. Given the direction of the external field, what does your answer above imply about the direction and magnitude of the induced electric field inside the cube? Explain.
3. How would charges have to be arranged on the surface of the cube to generate this induced electric field? Explain.
4. As a result of this induced charge distribution, does the electric field to the right of the cube *increase*, *decrease*, or *remain the same*? Explain.

B. The cube is now placed in a non-uniform field as shown.

1. What will the induced electric field inside the cube be in this case? Explain.



2. On the diagram, sketch the charge distribution that would create this new induced field.
3. Are positive and negative charges evenly distributed on the sides of the cube, or is the density in some regions greater than in other regions? Explain.

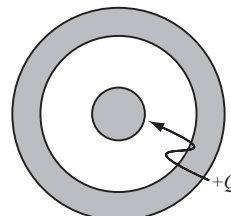
C. In general, when a conductor is placed in an external electric field, what will be the induced electric field inside the conductor?

ED *Conductors*

2

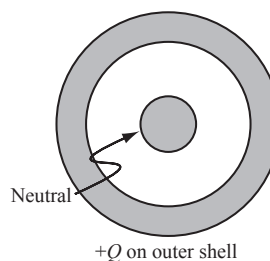
II. Charge distributions on conductors

A. A conducting sphere is placed at the center of a neutral conducting shell as shown. A total charge of $+Q$ is placed on the inner sphere.



1. How will the charge on the surface of the inner sphere be distributed? Explain how you know.
2. On the diagram, sketch the electric field produced *only* by the inner sphere everywhere in space.
3. What direction does the field of the $+Q$ point within the metal of the spherical shell?
4. Within the metal of the spherical shell, what direction must the *induced* electric field of the shell point? Explain.
5. On the diagram, sketch the charge distribution on the shell that would produce this induced electric field. Explain.

B. The $+Q$ is removed from the inner sphere and put on the shell instead (the inner sphere is now neutral).

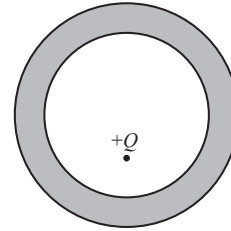


1. How are the $+Q$ charges distributed on the shell? Explain how you know.
2. How will the charge on the inner sphere be arranged in response to the field of the shell? Explain.

Conductors ED
3

III. Shielding

A. A point charge with charge $+Q$ is placed off-center inside a *neutral* spherical shell as shown.



Neutral conducting shell

1. Sketch the electric field of the point charge everywhere in space.
2. How much charge is on the inner surface of the shell? How is it distributed? Explain.
3. Three students discuss the charge distribution on the outer surface of the shell.

Student 1: "The point charge makes a stronger electric field at the bottom of the shell than at the top, so the bottom of the outer surface should have more positive charge to cancel that out."

Student 2: "I disagree. What if the inner surface completely cancels out the electric field in the metal caused by the point charge? Then there can't be any induced charge on the outer surface."

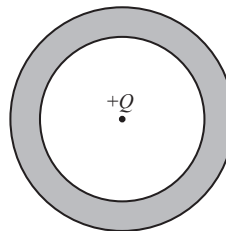
Student 3: "That can't be right. The shell is net neutral, so there has to be a way to have positive charge on the outer surface without affecting the field inside."

Do you agree with any of the students above? Explain your reasoning.
- ⇒ Ask a tutorial instructor for a handout showing the electric field of this charge distribution.
4. Is the electric field inside the cavity of the shell affected by the charge distribution of the shell, or is it only due to the point charge? Explain your reasoning.
5. Will the point charge experience a force? If so, in which direction? Explain.

ED
4 **Conductors**

B. Suppose the $+Q$ from part A was moved to the center of the shell.

1. Will the charge distribution on the **inner** surface of the shell change? Explain.

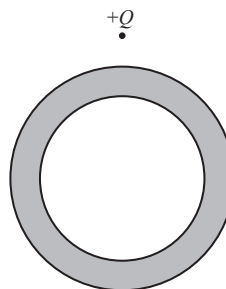


Neutral conducting shell

2. Will the charge distribution on the **outer** surface of the shell change? Explain.

C. Suppose the point charge was placed outside of the spherical shell.

1. Will there be a non-zero charge distribution on the **outer** surface of the shell? Explain.



Neutral conducting shell

2. Will there be a non-zero charge distribution on the **inner** surface of the shell? Explain.

3. Would the charge distribution on either surface of the shell change if you moved the point charge closer to the shell? Explain why or why not.

D. Does moving a charge on one side the shell affect the electric field on the other side?

This phenomenon is known as *shielding* – the charges on a conductor rearrange to cancel any change to the field inside a conductor. If a conductor separates two regions of space, changes to the electric field on either side of the conductor do not affect the field on the other side.

©Tutorials in Introductory Physics, Physics Education Group, Department of Physics
University of Washington (Spring 2015)

COULOMB'S LAW

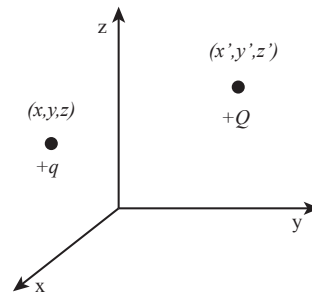
ED
1**I. The separation vector in multiple coordinate systems**

Coulomb's Law describes the effect that a point charge (often called the "source" charge in textbooks) has on another charge (often called the "test" charge). You should be familiar with Coulomb's law from your introductory physics classes, but in this tutorial we will look at Coulomb's law in a more formal way.

A. Shown at right is a source charge $+Q$ at a point (x', y', z') , and a test charge $+q$ at a point (x, y, z) .

1. On the graph, draw the vectors \vec{r} and \vec{r}' .

2. Your textbook defines a third vector, the separation vector \vec{r}_{sep} . In terms of the locations of the source and test charge, which direction should the separation vector point? Explain.



3. Draw this vector on the graph, and express it in terms of the vectors \vec{r} and \vec{r}' .

4. What are the Cartesian components of \vec{r}_{sep} ? Express them in terms of the Cartesian components of \vec{r} and \vec{r}' .

5. What are the Cartesian components of \hat{r}_{sep} , the unit vector in the direction of \vec{r}_{sep} ?
(Hint: You can make any vector into a unit vector by normalizing it to have length 1.)

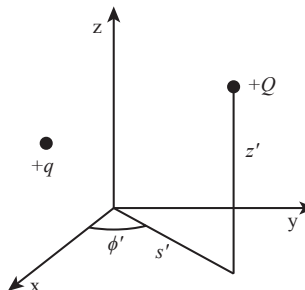
6. Write an expression for the force the source charge exerts on the test charge, including the direction in which the force acts.

ED *Coulomb's Law*

2

B. The same two charges from part A are shown at right, in cylindrical coordinates this time.

- How can you write the coordinates of the source charge (x', y', z') in terms of the cylindrical coordinates s', ϕ' , and z' ?



- Using your answer to part 1, write the Cartesian components of \vec{r}_{sep} in cylindrical coordinates.

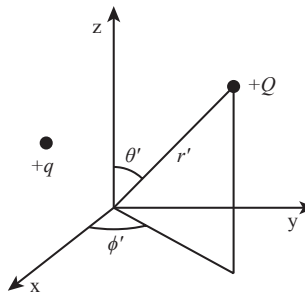
$$r_{\text{sep } x} =$$

$$r_{\text{sep } y} =$$

$$r_{\text{sep } z} =$$

C. The charges are now put into spherical coordinates as shown.

- How can you write the coordinates of the source charge (x', y', z') in terms of the spherical coordinates r', θ' , and ϕ' ?



- Using your answer to part 1, write the Cartesian components of \vec{r}_{sep} in cylindrical coordinates.

$$r_{\text{sep } x} =$$

$$r_{\text{sep } y} =$$

$$r_{\text{sep } z} =$$

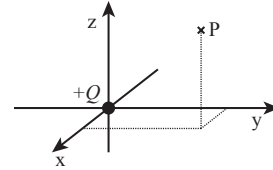
⇒ Check your answers with a tutorial instructor before continuing.

II. The separation vector and electrostatic potential

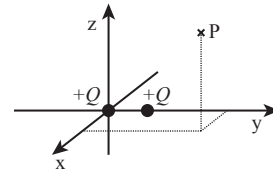
Recall from freshman physics that the potential at a test point P due to a point charge $+Q$ is given by $V = kQ/r_{\text{sep}}$.

- A. Shown at right is a charge $+Q$ at the origin, and a test point P at an arbitrary point.

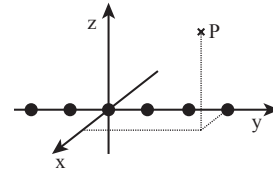
What are the vectors \vec{r} and \vec{r}' in this case? Use this to write an expression for the potential at point P.



- B. A second $+Q$ charge is added, located at $(0, y_2, 0)$. What is the potential at point P due to both charges?

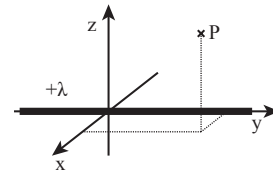


- C. Many more charges are added, forming a string of N identical point charges along the y -axis, with y_N the y -coordinate of the N th charge. Write an expression for the potential at point P due to the string of charges.



- D. The string of charges is replaced by an infinite line of charge on the y -axis with uniform linear charge density $+\lambda$.

1. What is the potential at point P due to this line?
(Hint: A good way to check to see if you've converted from a sum to an integral correctly is to verify that both expressions have the same units.)



2. In the context of this integral, what does the vector \vec{r} correspond to?

What does the vector \vec{r}' correspond to?

ED *Coulomb's Law*

4

III. Charged bike tires

A hundred years in the future, a bicyclist named Thomas gets lost in the Cascades east of Seattle and gets a flat tire. Thomas pulls off the tire and consults his Tricorder to find out if there are nearby life forms that could help him. However, the flat tire has somehow become uniformly positively charged. The Tricorder complains that the electric field from the tire is very annoying.

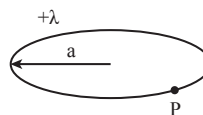
Your goal is to calculate the electric field produced by the charged tire (a ring with linear charge density $+\lambda$).

- A. The origin is at the center of the tire, and the Tricorder is off to the side as shown. An arbitrary point P is shown on the tire.

Tricorder



1. Label the diagram with points (x, y, z) and (x', y', z') .



2. Draw and label the three vectors \vec{r} , \vec{r}' , and \vec{r}_{sep} .

3. Given the shape of the charge distribution, what choice of coordinates would be most convenient?

4. Using these coordinates, write an expression for the electric field at the Tricorder from a small piece of charge dq at point P. Be explicit – you should not leave your answer in terms of \vec{r}_{sep} , you should substitute it for the coordinates you know.

⇒ Check your expression with a tutorial instructor before continuing.

- B. Use this answer to write a full integral expression for the electric field at the Tricorder. Simplify your expression as much as possible.

C. Thomas notices that if he puts his Tricorder on the z -axis (straight above the center of the ring) it complains much less about the electric field.

1. What are the coordinates of the Tricorder (in the coordinate system you chose) at this position?

2. Plug these coordinates into your expression from part B, and evaluate the integral.

(Hint: Most of the terms in your integral are either constant, zero, or integrate to zero. It's not a hard integral.)

3. Check the limits of your expression both for very large z , and for $z = 0$. Do these answers make sense? Sketch the E field along the z axis, including both positive and negative values of z .

4. Where should the Tricorder be placed to avoid interference from the electric field?

COULOMB'S LAW

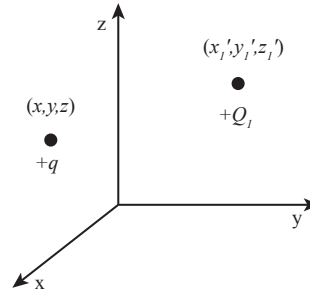
ED
1**I. The separation vector**

Coulomb's Law describes the effect that a point charge (often called the "source" charge in textbooks) has on another charge (often called the "test" charge). You should be familiar with Coulomb's law from your introductory physics classes, but in this tutorial we will look at Coulomb's law in a more formal way.

A. Shown at right is a source charge $+Q_1$ at a point (x_1, y_1, z_1) , and a test charge $+q$ at a point (x, y, z) .

1. On the graph, draw the vectors \vec{r} and \vec{r}' .

2. In what direction would you expect the force on the test charge to point?



This direction is related to a third vector, known as the separation vector \vec{r}'' . The separation vector is defined as a displacement from the source charge to the test charge.

3. Draw the separation vector on the graph, and express it in terms of the vectors \vec{r} and \vec{r}' .

(Hint: Which two vectors add to make the third?)

4. What are the Cartesian components of \vec{r}'' ? Express them in terms of the Cartesian components of \vec{r} and \vec{r}' .

5. How would you express the unit vector \hat{r}'' , in terms of variables associated with \vec{r}'' ?

(Hint: You can make any vector into a unit vector by normalizing it to have length 1.)

6. Write an expression for the force the source charge exerts on the test charge, including the direction in which the force acts.

◇ Check your answers with a tutorial instructor before continuing.

ED
2 Coulomb's law

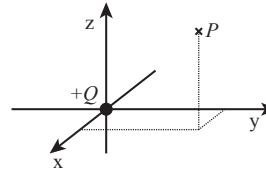
II. Superposition in distributed charge

The expression to the force you wrote in the previous section should be linear in the source charges. Due to linear algebra, this means that the effect on a test charge by multiple point charges can be described as the linear sum of individual effects by each source charge.

Recall from freshman physics that the electric field at a test point P due to a point charge $+Q$ is given by $\vec{E}(\vec{r}) = \vec{F}/q_{\text{test}}$.

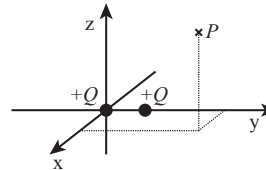
A. Shown at right is a charge $+Q$ at the origin, and a test point P with coordinates (x, y, z) .

1. What are the vectors \vec{r} and \vec{r}' in this case?



2. Use this to write an expression for the y-component of the electric field at point P .

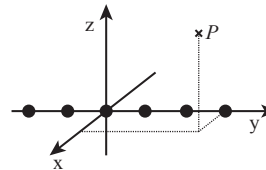
B. A second $+Q_2$ charge is added, located at $(0, y_2', 0)$. What is the electric field at point P due to both charges, in terms of the separation vectors \vec{r}_1 and \vec{r}_2 ?



In order to add vectors, we need to use a Cartesian coordinate system.

C. Rewrite the y-component of the net electric field in terms of Cartesian coordinates.

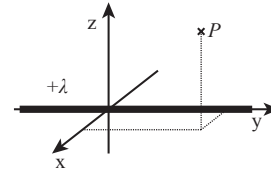
D. Many more charges are added, forming a string of identical point charges along the y-axis, with y_N' the y-coordinate of the N th charge. Write an expression for the y-component of the electric field at point P due to the string of charges as a sum.



Coulomb's law ED
3

- E. The string of charges is replaced by an infinite line of charge on the y -axis with uniform linear charge density $+\lambda$.

Suppose we take a small segment centered on $(0, y', 0)$ with length dy' .



1. How much charge is contained in that segment? Call that charge dq' .

2. What is the contribution to the y -component of the electric field by that dq' ? Call that contribution dE_y .

3. Compare your answer above to the answer you found in part C. Write an integral for the total y -component of the electric field in terms of the dummy variable dy' . You do not need to solve this integral.

⇒ Check your answers with a tutorial instructor before continuing.

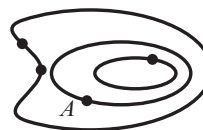
ED
4**III. Supplemental: Gradient**

The *gradient* is a vector derivative that acts on a scalar field ϕ , returning a vector field \vec{V} , written as $\vec{\nabla}\phi = \vec{V}$.

- A. Suppose you had some scalar field ϕ . What does the gradient of ϕ physically tell you about this field?

What are some physical systems where the gradient would be a useful thing to use? List as many as you can think of.

- B. One way the gradient is often visualized is with topographic maps. A map of a hill is shown at right; each closed loop on the map shown represents a specific elevation.



The gradient at a point shows the direction of the steepest ascent at that point.

1. Sketch vectors showing the gradient at each of the marked points.

What do the magnitude of your vectors represent?

2. Suppose you were at point A. Which direction relative to the gradient would you have to move in order to travel:

Downhill?

Uphill?

Stay at the same height?

What do your answers above imply about the direction of the gradient compared to lines of constant height?

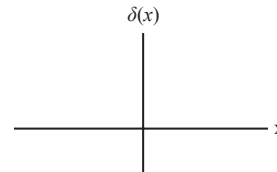
3. Is the gradient at a point the same thing as the slope at that point? Explain why or why not.
4. Does the gradient at all points give you as much information as the map does? Explain why or why not.

GAUSS'S LAW

ED
1**I. Delta functions**

A. A delta function is defined to be zero everywhere except for when the argument is zero, where its value is such that if you integrate across the delta function you get 1.

1. On the plot at right, sketch $\delta(x)$. How would your plot change if you instead sketched $\delta(x-3)$?



2. What is $\int_{-1}^1 \delta(x) dx$? If position is measured in meters, what are the units of $\delta(x)$?

Would the value of the integral change if you integrated from -100 to 100 instead? Explain.

3. What happens if you integrate a delta function with another function, *i.e.* $\int_{-1}^1 f(x)\delta(x) dx$?
(*Hint:* Think about where a delta function is zero and non-zero.)

What is $\int_{-10}^{10} a \delta(x-3) dx$, where a is a constant?

What is $\int_{-10}^{10} x \delta(x-3) dx$?

4. Suppose you are given a volume charge density $\rho(x,y,z) = b \delta(x-3)$.

What physical situation does this charge density describe? What are the units of b ? Explain.

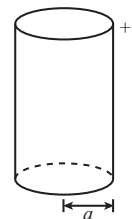
⇒ Check your answers with a tutorial instructor before continuing.

ED Gauss's Law
2

II. Gauss's law

Gauss's law in integral form states that the electric flux through a Gaussian surface is proportional to the net charge enclosed in the surface: $\oint \vec{E} \cdot d\vec{A} = Q_{enc} / \epsilon_0$.

A. Shown at right is a very long cylinder with uniform charge density σ on its surface.



1. What direction does the electric field point inside and outside of the cylinder? Explain.
2. Suppose you wanted to find the electric field outside the cylinder. What Gaussian surface would you use with this charge distribution (*i.e.* a sphere, a box, a cylinder, etc)? Sketch it on the figure above.
3. On your Gaussian surface, draw a small square representing an infinitesimal area $d\vec{A}$.
4. What is the height of the square, in terms of the coordinate system you are using? What is the area of the square?
5. What direction does $d\vec{A}$ point?
6. Using your answers above and Gauss's law, find the electric field inside and outside the cylinder.

III. Divergence of the electric field

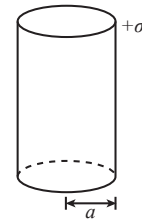
A. Each of the following charge distributions has a total charge $+Q$. For each case, find the volume charge density ρ . Make sure that your answers have the correct units.

1. A uniformly charged sphere centered on the origin with radius a .
2. A uniformly charged spherical shell centered on the origin with radius a . (Your answers to page 1 will likely help here).
3. An infinite line charge with density λ on the z -axis.
4. A point charge at the origin.

⇒ Check your answers with a tutorial instructor before continuing.

B. The charged cylinder from the previous page is shown at right.

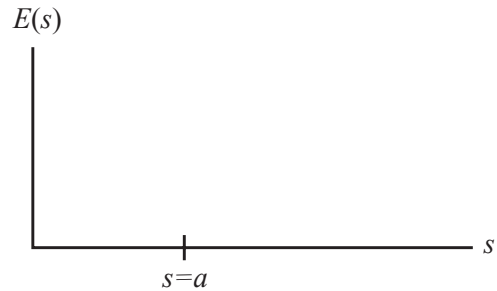
1. How much charge is there on a section of the cylinder with length L ?



2. What is the volume charge density ρ for this distribution?
3. Integrate your charge density over the section of the cylinder with length L . Does your answer agree with your answer to part 1?

ED
4 **Gauss's Law**

4. In part II you found the electric field of the cylinder everywhere in space. In the plot below, sketch the electric field as a function of the cylindrical coordinate s .



5. In cylindrical coordinates, the divergence of a field that only depends on s is given by

$$\nabla \cdot \vec{v} = \frac{1}{s} \frac{\partial}{\partial s} (s v_s).$$

Is the divergence of the electric field *zero* or *nonzero* inside the cylinder?

Is the divergence of the electric field *zero* or *nonzero* outside the cylinder?

Is the divergence of the electric field *zero* or *nonzero* at the surface of the cylinder, *i.e.* at $s=a$?

6. In differential form, Gauss's law relates the divergence of the electric field at a point in space to the volume charge density at that point: $\nabla \cdot \vec{E} = \rho / \epsilon_0$.

Based on your answers above, where in space is there a charge density? Explain.

Is your answer consistent with your answer to B.2 on the previous page? If not, resolve any inconsistencies.

GAUSS'S LAW

ED
1**I. Symmetry and electric fields**

Several points are shown next to an infinite straight wire with a linear charge density $+\lambda$. Points A and B have different z -values, points B and C have different s -values, and points D and E have different ϕ -values.

A. For the following questions, *do not use Coulomb's or Gauss's Law*. Use only symmetry arguments to answer each question.

1. What types of symmetry does this charge distribution have? (e.g. translational, rotational, inversional)

2. Is the magnitude of the electric field at point A greater than, less than, or equal to that at point B ? Explain.

3. Is the magnitude of the electric field at point D greater than, less than, or equal to that at point E ? Explain.

4. Do symmetry arguments alone allow you to compare the magnitude of the electric field at point B to that at point C ? Explain why or why not.

B. Consider the electric field at point A . State whether symmetry forbids the electric field to point in the following directions. Explain *without using Coulomb's or Gauss's Law*.

\hat{s} -direction

$\hat{\phi}$ -direction

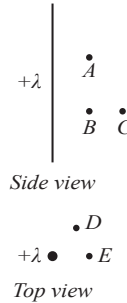
\hat{z} -direction

C. Summarize your conclusion about the electric field of the line charge thus far:

On what variable(s) can the electric field depend?

In what direction(s) can the electric field point?

⇨ Check your answers to this page with a tutorial instructor before continuing.



ED
2 *Gauss's law*

II. Choosing a Gaussian surface

Gauss's Law in differential form states that the divergence of electric field is proportional to the volume charge density: $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$. In integral form it states that the electric flux through a Gaussian surface is proportional to the net charge enclosed in the surface:

$$\oiint \vec{E} \cdot d\vec{A} = \iiint \rho dV / \epsilon_0 = Q_{enc} / \epsilon_0.$$

Note: A Gaussian surface is an imaginary surface that fully encloses a volume, such that the area vector $d\vec{A}$ points outwards by convention.

A. A student proposes the Gaussian surface shown at right:

"I think the electric field points away from the wire. I can use a cube as my Gaussian surface because the top and bottom surfaces have no flux, and side surfaces have electric field in the same direction as the area. This makes finding the total flux easy."



The student is incorrect. Identify the flaw(s) in the student's reasoning. Explain.

B. What is a more appropriate shape for the Gaussian surface in this situation? Explain based on the properties of the charge distribution and your chosen shape.

C. On the whiteboard, sketch the charge distribution, your Gaussian surface, and the electric field at multiple points on the Gaussian surface. Label the dimensions of your Gaussian surface.

1. How much charge is enclosed in your Gaussian surface? Express your answer in terms of λ and the dimensions of your Gaussian surface.

Gauss's law ED
3

-
2. Consider the flux integral $\oiint \vec{E} \cdot d\vec{A}$.
- For your Gaussian surface, is the dot product $\vec{E} \cdot d\vec{A}$ easy to determine for all points on the surface? Explain.
 - For the sections of the surface where the dot product is non-zero, is the electric field on each section constant in magnitude? If not, you may need to try a different surface. If so, how does this simplify the flux integral?
3. Using Gauss's Law, find the electric field of the line charge at the surface of your Gaussian surface. Express your answer in terms of the dimensions of your Gaussian surface.
4. Suppose you changed the size of your Gaussian surface. How does the magnitude of the electric field depend on each dimension of your Gaussian surface? Explain.
- D. The symmetric properties of the line charge allows us to choose an appropriate Gaussian surface such that Gauss's Law can solve for the electric field. For the following charge distributions, what shape (if any) would be an appropriate Gaussian surface? Explain.
- Uniformly charged infinite sheet
- Uniformly charged spherical shell
- Uniformly charged cube
- ◇ Check your answers to this page with a tutorial instructor before continuing.

ED Gauss's law

4

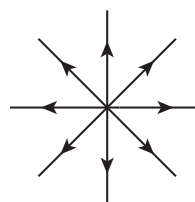
III. Supplement: Divergence

The *divergence* is a vector derivative that acts on a vector field \vec{V} , returning a scalar field ϕ , written as $\vec{\nabla} \cdot \vec{V} = \phi$.

- A. We can conceptually think about the divergence by thinking about a tank of water. If water is being added to the tank at a point then the divergence is positive there. If water is flowing out of the tank through a drain then the divergence is negative there.

Points of positive divergence are called *sources*, and points of negative divergence are *sinks*.

1. The field of a single source at the origin is shown at right. Where does this field have positive divergence? Explain.



2. Does this field have negative or zero divergence anywhere?

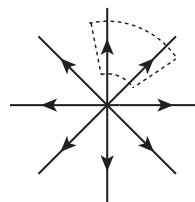
- B. The divergence theorem can be used to express a volume integral of the divergence of a vector field inside a region of space in terms of the net flux of the field through that region:

$$\iiint \vec{\nabla} \cdot \vec{V} \, d\tau = \iint \vec{V} \cdot d\vec{A}$$

1. Based on the equation above, if the net flux through the region is positive, what do you know about the divergence inside?

If the net flux is zero, does this tell you the field has no divergence? Think carefully.

2. Consider the dashed region of the field from part B shown. What's the flux through this region? Does this agree with your answers to part B above?

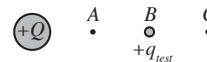


- C. In what physical systems could you use the divergence? List as many as you can think of.

POTENTIAL

ED
1**I. Electric potential**

- A. A small positive test charge is placed at point B near a uniformly charged positive sphere with total charge $+Q$.



1. Suppose a hand move the test charge to point A . As a result of this movement is ΔU_E , the electric potential energy of the two-charge system, *positive*, *negative*, or *zero*? Explain. (It may be helpful to recall the work-energy theorem from classical mechanics, and to think about the work the hand does.)
2. Suppose instead you move the test charge to point C . What is the sign of ΔU_E in this case? Explain.
3. How could you move the test charge such that ΔU_E is zero? Explain.

The electric potential difference between two points in a system of charges is defined as:

$$\Delta V_{A \rightarrow B} = V_B - V_A \equiv \frac{\Delta U_E}{q_{test}}$$

where ΔU_E is the change in potential energy when q_{test} moves between the points.

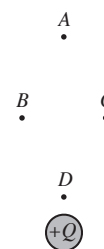
- B. Four points are shown near another positively charged sphere.

1. Suppose you move a test charge from point B to each of the other points in turn. For each movement, indicate whether ΔV is *positive*, *negative*, or *zero*.

B to A :

B to C :

B to D :

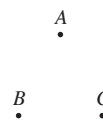


2. Suppose the electric potential at point B were zero. In this case, which points would have positive potential? Negative potential? Zero potential? Explain.

Point B is an example of a *reference point* – a point where the potential is defined to be zero. We can then define the potential of any other point with respect to this reference point.

ED
2 **Potential**

3. Suppose we had chosen point A as a reference point instead. Would your answers to question 2 change? If so, what would the sign of the potential at points B - D be in this case?



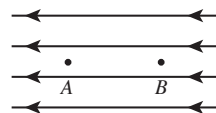
How could you use the definition of electric potential difference to measure the electric potential at point D ? Explain your method.



4. Point A (the reference point) is now moved infinitely far away from the sphere. What affect does this have on the potential at point B ? Explain.

Does setting the reference point at infinity affect the potential differences between the other points, for example $\Delta V_{B \rightarrow D}$? Why or why not?

- C. A hand moves a positive test charge $+q$ from point A to point B at constant speed against an electric field with magnitude E as shown.



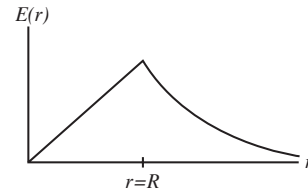
1. If the charge is moved at constant speed, how much force does the hand need to exert? Express your answer in terms of \vec{E} .
2. If points A and B are a distance d apart, how much work does the hand do moving the charge?
3. What is the electric potential difference from A to B ?
4. Rewrite your answer as an integral of \vec{E} . What are the bounds on the integral?

This is a general result: the electric potential difference between two points can be written as a line integral of the electric field between the two points.

Potential ED
3

D. The electric field of a uniformly charged sphere with radius R is shown at right.

1. In the space below, sketch the electric potential as a function of r , with the reference point of potential at the origin.



2. Now sketch the potential with the reference point at infinity.

3. What does a different choice of reference point do to your graph of potential? Does it change the overall shape of the graph? Explain.

II. Curl of a vector field

The curl operator measures the rotation of a vector field at a point. The direction of $\vec{\nabla} \times \vec{H}$ is given by the right-hand rule: if the vector field \vec{H} curls in a counterclockwise direction around the point at which you are evaluating the curl, the curl points out of the page.

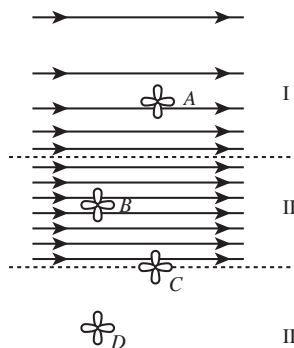
- A. In the space below, sketch a vector field that has a curl that points into the page.

ED *Potential*

4

- B. One way to conceptually view the curl is to imagine that the vector field represents a pattern of wind. Then imagine placing a small pinwheel at some point in the vector field. If the pinwheel will rotate, the curl of the field is nonzero.

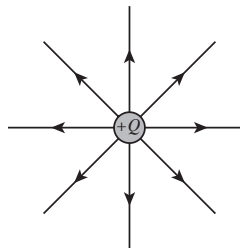
A spacially varying vector field is shown at right. In region I the field increases in magnitude towards the bottom of the page. In region II the field is uniform, and in region III the field is zero. The dashed lines show the edge of each region.



- Four pinwheels are placed in the field as shown. Which of the pinwheels would rotate? Explain.
- At each point A - D , which direction does the curl of the vector field point? If the curl is zero at any point, state so explicitly.
- Considering the entire field, where in space is there curl? Explain.
- If the curl of a vector field is zero at some point, can you conclude that the field is zero at that point? If not, what can you conclude about the vector field there?

- C. The electric field of a point charge is shown at right.

- What is the curl of this field everywhere in space? Explain.



Potential ED
5

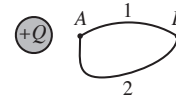
2. Stokes' theorem relates the curl of a vector field through a closed loop to the line integral of the field around the loop:

$$\iint (\nabla \times \vec{H}) \cdot d\vec{A} = \oint \vec{H} \cdot d\vec{l}$$

Using Stokes' theorem and your answer to question 1, what is the line integral of the electric field of a point charge around a closed loop?

3. Any charge distribution can be viewed as a superposition of point charges. Given your answers above, what is the line integral of the electric field of any charge distribution around a closed loop?

- D. A closed loop is shown at right, consisting of path 1 from point A to point B , and then continuing through path 2 back to point A .



- Given your answer above, how does the integral of E along path 1 from A to B compare to the integral along path 2? Explain.
- If we reverse the direction of path 2, then both paths go from point A to point B . How does the line integral from A to B along each path compare?
- Does the potential difference from A and B depend on the path you take between the points?
- If you were finding the potential difference between A and B , what path would you use? Why would you pick that path? Explain your reasoning.

POTENTIAL

ED
1**I. Gradient revisited**

A topographical map represents a two-dimensional scalar field where the x - and y - coordinates represent the position of a point and the height $h(x,y)$ is a scalar function of the position.

Consider the topographical map of an island hill shown at right. Lines of constant height are labeled h_1 , h_2 , and h_3 , where h_1 is sea level.



A. Recall that the gradient is a directional derivative of a scalar field.

1. In what direction does the gradient of the height point?

2. What does the magnitude of the gradient tell you?

B. Suppose you were at point A. What range of angles relative to the gradient would you move in order to travel:

Uphill?

Downhill?

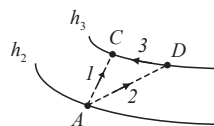
Stay at the same height?

1. Suppose you were to take a step along a path $d\vec{l}$. What is your change in height dh in terms of the vectors $\vec{\nabla}h$ and $d\vec{l}$? This relation should turn two vectors into a scalar.

2. Suppose you were to walk to the top of the hill at point B. How would you find the height of the hill if you knew the value of h_2 ? Write your answer as an integral of the gradient.

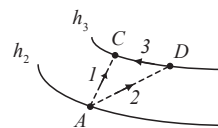
C. Several paths are drawn on a section of the topographical map.

1. Suppose you go from point A to point C via path 1. What is the change in height? Express your answer in terms of h_2 and h_3 .



ED
2 **Potential**

2. Suppose you go from point A to point D via path 2. What is the change in height?



3. Suppose you go from point A to point C through point D via path 2 and 3. What is the change in height?

Does the change in height from point A to point C depend on which path you take?

- D. Suppose the entire hill was raised such that h_1 is at an elevation of 150m above sea level. Does this change the height difference between point A and point C ? Explain.

In this example, sea level is the *reference point* where the absolute height is defined as zero. The reference point only changes the absolute height at all points, but does not change the physically observable features of the hill.

II. Electric field as a gradient

Recall from introductory mechanics that a conservative force is the negative spatial derivative of a potential energy. The vector form of this relation uses the gradient: $\vec{F} = -\vec{\nabla}U$.

- A. Electrostatic forces are conservative, so there is an associated electrostatic potential energy.
1. How is the electrostatic force on a test charge related to the electric field at a point?
 2. How is the electric potential energy of a test charge related to the electric potential at a point?
 3. How is the electric field related to the electric potential?

⇒ Check your answers to this page with a tutorial instructor before continuing.

B. The relationship between electric field and potential is analogous to the topographical map.

1. What electrostatic quantity is related to the following:

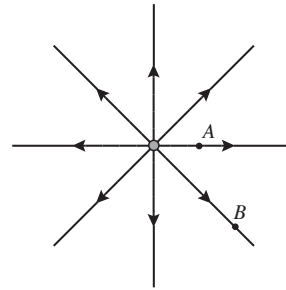
Elevation at a point?

Direction of steepest ascent?

Rate of change in height?

2. The electric field lines of a point charge are drawn at right. Sketch two equipotential lines that correspond to positions where the value of the potential is the same. Include points A and B on your equipotential lines.

Does point A or point B have a higher electric potential? Explain.



3. Suppose you were to take a step along a path $d\vec{l}$. What is the change in potential dV in terms of the vectors \vec{E} and $d\vec{l}$?

Using your expression above, construct an integral for the potential difference from point A to point B .

4. Does the potential difference from point A to point B depend on the path? Explain.

Describe a path for which potential difference is easy to evaluate. Explain.

5. If you add a constant to the potential at all points by changing the reference point of the potential, does this affect physically observable quantities like the electric field? Explain.

⇨ Check your answers to this page with a tutorial instructor before continuing.

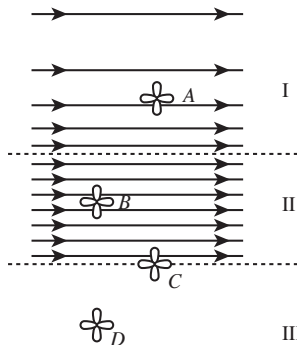
ED
4 *Potential*

III. Supplement: Curl

The *curl* is a vector derivative that acts on a vector field \vec{V} , returning another vector field \vec{W} , written as $\vec{\nabla} \times \vec{V} = \vec{W}$.

One way to conceptually view the curl is to imagine that the vector field represents a pattern of wind. Then imagine placing a small pinwheel at some point in the vector field. If the pinwheel will rotate, the curl of the field is nonzero.

- A. A spatially varying vector field is shown at right. In region I the field increases in magnitude towards the bottom of the page. In region II the field is uniform, and in region III the field is zero. The dashed lines show the edge of each region.



- Four pinwheels are placed in the field as shown. Which of the pinwheels would rotate? Explain.
- At each point A-D, which direction does the curl of the vector field point? If the curl is zero at any point, state so explicitly.
- Considering the entire field, where in space is there curl? Explain.

Stokes' theorem relates the curl of a vector field through a closed loop to the line integral of the field around the loop:

$$\iint (\vec{\nabla} \times \vec{V}) \cdot d\vec{A} = \oint \vec{V} \cdot d\vec{l}$$

- Suppose the vector field is a gradient of a scalar function. If you integrate the line integral of the gradient along any closed path, does the value of the scalar function change at the start-and-end point? Explain.

What does this imply about the curl of a gradient?