COMMUTING SERVICE PLATFORM: CONCEPT, METHOD, AND ANALYSIS

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Abstract

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Commuting congestion increases alongside the prosperity of urban cities. With the rapid development of ridesourcing services and the advances of connected and automated vehicles (CAV), researchers are seeking innovative approaches to alleviate commuting congestion. This research aims to build a commuting service platform (CSP) that alleviates congestion and increases mobility during peak hours. The pricing strategies and the route choices of the proposed CSP are studied. On the planning level, how CSP leverages emerging mobility services for commuting and connects directly commuters (employees) and their worksites (employers) is investigated. The planning level model shows how the price allocation, i.e., the prices charged to commuters and worksites, can impact the participation and profit of the CSPs, which is extended to consider locations of homes and worksites using a general modeling framework. On the operational level, a route choice model for an integrated, multimodal CAV ridesourcing and transit system is proposed. The route choice model captures the economic behaviors and interactions of the major players (i.e.. the ridesourcing company and customers) in the commuting scenario by optimizing the profit of the ridesourcing company and the utility of customers, as well as the optimal route choice to minimize congestion. The proposed CSP is prominent in building the next-generation travel demand management (TDM) strategies.

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GLOSSARY

BRT	Bus Rapid Transit
AV	Auronomous Vehicles
CAV	Connected and Automated Vehicles
CSP	Commuting Service Platform
CTR	Commut Trip Reduction
ESTP	Employer Sponsored Transportation Program
MaaS	Mobilibty as a Service
MCP	Mixed Complimentarity Problem
NMS	New Mobility Services
NWF	non work flex
SAV	Shared Autonomous Vehicles
SOV	Single Occupancy Vehicle
TDM	Transportation Demand Management
WF	work flex
WFH	work from home

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DEDICATION

to my dear grandmother, Yindi, who showed me how to be strong and have inner peace in difficult times

Chapter 1
INTRODUCTION

1.1 Research Motivation

The first two decades of the 21st Century have witnessed growing commuting congestion, various proposed solutions to mitigate the congestion, and emerging technologies and systems such as new mobility services (e.g., Uber/Lyft) and connected and automated vehicles (CAVs). All these will have a profound impact to shape future commuting especially in congested urban areas.

1.1.1 Commuting congestion

Accounting for a substantial portion of the total daily trips (e.g., commuting trips are about 28% of the total daily trips in the US (Federal Highway Administration, 2020)), commuting trips are important to businesses (employers), local economy, and people's daily life, which also experience the most congestion and related problems, especially in fast-growing urban areas. Among all trips, they are probably the easiest to describe: Businesses (employers) lease/purchase space for their activities (often in urban areas), and abide by city ordinance. Employees work at the businesses, meeting the work and arrival times set by employers. Normally, employers make decisions that largely determine employees' work schedule. In response to such schedule, employees are commuters who create the commuting traffic (i.e., travel demand) as they decide on mode/vehicle of travel, time of travel, route, and the like. At the same time, public agencies manage/maintain the transportation infrastructure, providing proper capacity (and policies) to serve the demand. Congestion happens when demand exceeds capacity, often at specific periods of time, e.g., the peak periods that are in most cases the commuting periods. This research focuses on commuting to work, while the above can also apply to “commuting to school.”

In the existing urban transportation system, commuting related decisions by the major players (i.e., commuters, employers, and transportation agencies) are only loosely connected and largely isolated. Businesses (employers) are the major attractors of commuting trips, but have no or little responsibility of managing traffic or congestion; commuters (employees) form the commuting traffic, and to a great extent, have to follow the work schedule established by their employers and thus do not have much flexibility in their commuting schedule (Holguín-Veras *et al.*, 2011); transportation agencies, who provide transportation infrastructure and system capacity, do not have any direct control on travel schedule, demands, etc. Some TDM strategies did recognize this issue and developed policies and programs to keep employers in the loop (such as employer-based transit passes, vanpooling, telecommuting, parking management, etc.) to manage commuting demands by encouraging their employees to switch from single occupancy vehicle (SOV) travels to more efficient modes or avoid travel at all (Georgia Institute of Technology, 1994; Young, 1992; WSDOT, 2019). Furthermore, many technology companies and government agencies have been implementing flexible work hours or even telecommuting policies so that their employees do not need to follow strict work schedule or go to work every day. However, such strategies are for individual employers/employees and mostly on a voluntary basis, lacking coordinations among different employers and even employees within the same employer. This results in much less significant impact to reduce commuting problems than what they could have achieved, which can be shown clearly by the steady increase in commuting related congestion, e.g., the number of hours wasted in traffic by an average US commuter in urban areas has grown for about 40% in 6 years, from 36 hours in 2009 to 50 hours in 2015 (Inrix, 2015). Therefore, to make TDM strategies more effective, urban cities need mechanisms that can more closely connect/coordinate employers and employees (and also agencies) so that the effect of commuting trips can be directly reflected in their decision makings. Current Covid-19 pandemic has shown this more vividly: widely adopted work from home (WFH) policies (mainly by tech companies) have had a huge impact on commuting (Gao *et al.*, 2021). Although many uncertainties still remain, recognizing and leveraging the role of employers and their interactions with employees will continue to be critical for commuting during and after the Covid-19 pandemic.

1.1.2 Commuting related TDM strategies

Reducing commuting congestion and related issues have been a long-lasting challenge in transportation. In addition to infrastructure expansion (rare nowadays) and efficient traffic control schemes (e.g., traffic signal control, routing, etc.), adequate TDM methods are crucial. TDM focuses on developing relatively longer term planning and coordination strategies to help manage people's time and modes of travel (FHWA, 2012), with the purposes of eliminating certain trips, switching them to more efficient modes (such as transit), or changing trip starting times (e.g., peak spreading). TDM has been studied very extensively in the last several decades (Ferguson, 1990) especially for commuting traffic. There are TDM programs in some cities in the US and around the world that developed TDM strategies to reduce congestion by promoting non SOV travels, among others. One example is the commute trip reduction (CTR) program of the State of Washington (WSDOT, 2019). As discussed above, commuters, their employers, and transportation management agencies are the major players for commuting related decisions. Effective TDM methods should recognize and take advantage of the behavior and interactions of these major players.

1.1.3 Emerging mobility services

Recent technology advances have produced novel mobility modes that have transformed (and will continue to transform) urban transportation. For example, mobile-app based new mobility services have led to the paradigm of mobility as a service (MaaS), the "integration of various forms of transport services into a single mobility service accessible on demand" (ERTICO, 2016). MaaS connects transportation service providers and travelers directly, which includes various forms (Shaheen & Ismail, 2016) such as ridesourcing (also called ride-hailing or e-hailing), ridesharing, carpooling, carsharing, bikesharing, on-demand shuttle services, among others.

Over the last decade or so, MaaS or ridesourcing (e.g., Uber/Lyft/Didi) have made remarkable progress on serving urban travel demands through real-time matching of drivers and customers (Ke *et al.*, 2019). In New York City, e.g., ridesourcing served 4.2 billion trips in 2018, up from about 2 billion trips in 2016, which was more than eight times the number of trips served by taxis in 2018 (Schaller Consulting, 2018). Platforms that offer ridesourcing services may also provide shared-ride (i.e. "ridesplitting") services for separate customers traveling from nearby locations, such as Uber Pool. This dissertation uses "single rides" or "shared rides" to distinguish ridesourcing services for single customer pickups or multiple customer pickups, respectively (Li *et al.*, 2019).

By focusing on all types of travels (commuting, entertainment, shopping, etc.), current MaaS only connects travelers with service providers, thus largely excluding key players of important trips (e.g., employers in commuting). As a result, the current form of MaaS may not be effective in solving commuting problems. After all, we have been trying to "nudge" travelers by technologies, incentives, etc., to the point that we probably need other innovative ways as well to collectively solve commuting problems. Meanwhile, there are employer-sponsored transportation programs (ESTP), in which employers are directly involved with providing commuting services to employees (Apple Inc. *et al.*, 2012). This is mostly in the form of providing carpool or shuttle services to employees from home to work and vice versa, by either operating the services directly (e.g., Amazon) or by outsourcing the services to a third party (e.g., Microsoft). More recently, industry innovators are tapping into ESTP by helping design TDM strategies (Luum, 2019) and provide carpool services to co-workers (Scoop, 2019).

1.1.4 Integrated transit system

On-demand transit systems, which integrate demand responsive services and transit, have been studied and implemented long before the era of ridesourcing. For example, Stein studied the integration of Dial-a-Ride services with fixed transit routes to solve the first/last mile problem (Stein, 1978), which has been the main challenge for many potential transit users. Later on, studies on similar problems showed that the integration of on-demand or flexible route services with fixed-route transit could reduce the operation cost of transit agencies (Black, 1995; Malucelli *et al.*, 1999).

The wide deployment of MaaS has inspired research on the integration of ridesourcing and transit (Feigon & Murphy, 2016). Chen & Nie (2017) compared the line-based design and the zone-based design for the integration of ridesourcing and transit. Their analytical and simulation results both showed that line-based services can achieve higher efficiency and allow ridesharing. Ma *et al.* (2019) proposed a ridesharing scheme with integrated transit in which ridesourcing serves either the whole trip or the first/last mile of a transit route. Their objective was to optimize vehicle dispatch and idle vehicles relocation for an integrated, multimodal transit-rideshare system. The numerical experiments indicated that the transit-rideshare system outperforms the rideshare-only system by 32% reduction in user travel time and 64% reduction in vehicle travel time. Compared with the rideshare-only system, the transit-ridesharing system also reduces the operation cost and customer waiting times, and achieves better performance in regions where passenger demands are heterogeneous. Pinto *et al.* (2020) proposed a bi-level mathematical programming formulation for the joint transit network redesign and mobility service fleet size determination. The integration in their study was to replace inefficient transit routes/patterns with shared autonomous mobility services, while the on-demand first/last mile for transit is not considered. In addition, their model only had one objective function to minimize the disutility of customers, with no platform profit maximization or congestion consideration. Ban *et al.* (2019) proposed a general equilibrium model that consists of the optimization of three main players: service providers, passengers and the network congestion. Their study provided insights for the transportation planning agencies on the congestion impact of ridesourcing. The deadhead miles of either taxi or ridesourcing services generally exacerbate the network congestion, but when the demand pattern has high level of symmetry, the deadhead miles can be significantly reduced.

In addition to studies on the operation of the integrated system of transit and ridesourcing, researchers also investigated the user experience and preferences of passengers. Yan *et al.* (2019) evaluated travelers' responses to the integrated transit pilot MTransit and showed that ridesourcing could complement transit by serving the first/last mile and/or replacing fixed transit routes with low usage, leading to reduced passenger waiting times and lower operation costs of transit agencies.

It suffices to say that most studies so far have focused on the integrated ridesourcing and transit mode without considering other related modes that customers may choose, e.g., ridesourcing services alone (eithr single rides or shared rides) for an entire trip. Pinto *et al.* (2020) did consider multimodal whereas platform profit maximization or congestion effect was not considered.

1.1.5 Connected and Automated vehicles (CAV)

CAVs are currently under rapid development, and are expected to have profound impacts to transportation and the daily life of everyone. CAVs especially automated vehicles (AVs) may dramatically change car ownership and travel behavior, which can significantly change travel demands and the resulting congestion, energy use, and emissions in the transportation system. First, the vehicle ownership may become slimmer if people have convenient access to CAV services, especially in cases when shared automated vehicles (SAVs) and dynamic ride-sharing can operate efficiently (Fagnant, 2014). By modeling autonomous vehicles based dynamic ride-sharing for Austin, Texas, it was found that each shared vehicle is promising to have the same function as ten traditional vehicles under the model setting (Fagnant & Kockelman, 2015). Thus, when CAVs are widely deployed, instead of owning private car, most people may choose to rent a CAV or switch to on-demand mobility services (e.g., taxis and ridesourcing/ridesharing) and mass transit services. Secondly, CAVs may change people's travel behavior in many aspects. For example, AVs can free up the occupant's in-car time for a range of leisurely and economically-productive activities that are either not possible at all while one is driving, or are not as productive while driving because the driver must continuously devote a share of his/her cognitive resources to driving-related tasks (Smith, 2012;

Spieser *et al.*, 2014; Anderson *et al.*, 2014; Priemus & van Wee, 2013). It was also pointed out that AV can reduce travelers' time costs, as drivers will not need to pay attention to traffic anymore, and the overall driving experience will be altered considerably. Drivers, who in effect may be considered passengers for most of the journey, will be able to pursue activities such as reading, working or sleeping ,while traveling in their cars (Le Vine *et al.*, 2015). Furthermore, the advent of the AV may allow for the emergence of novel business models such as SAVs, which could provide inexpensive mobility on-demand services and could play a vital role in sustainable transportation systems, by providing convenient first-mile/last-mile solutions, which could facilitate multimodality. System-wide coordination of SAVs could mitigate congestion and could facilitate the integration of advanced propulsion systems (Burns, 2013). As a result of the car ownership and travel behavior changes, CAVs may lead to profound changes in terms of travel demands, congestion, emissions, and fuel consumption at the transportation network system level. Such impact, i.e., how CAVs will affect traffic congestion and related energy/emission issues at the system level, remains largely unclear and is currently an active research area.

1.2 Commuting Service Platform (CSP)

With the fast development of MaaS, it is imperative to understand how to best leverage MaaS to more effectively engage employers to resolve the commuting related issues of their employees (commuters). At the same time, while MaaS/ridesourcing, ESTP, and CAVs are rapidly evolving, there is a growing trend of the integration of the three. It is expected that integrating them to focus on commuting trips, facilitated by proper TDM policies, may provide the needed mechanism to better connect employers and commuters, and as a result providing new ways to solve commuting challenges. In particular, it is envisioned that such integration may produce the so-called employer-based *commuting service platform* (CSP) to serve future urban commuting. Figure 1.1 shows the conceptual structure of a CSP. It first encapsulates and provides multimodal commuting services. Different from existing MaaS that treat commuters as individual travel decision makers and only connect them with

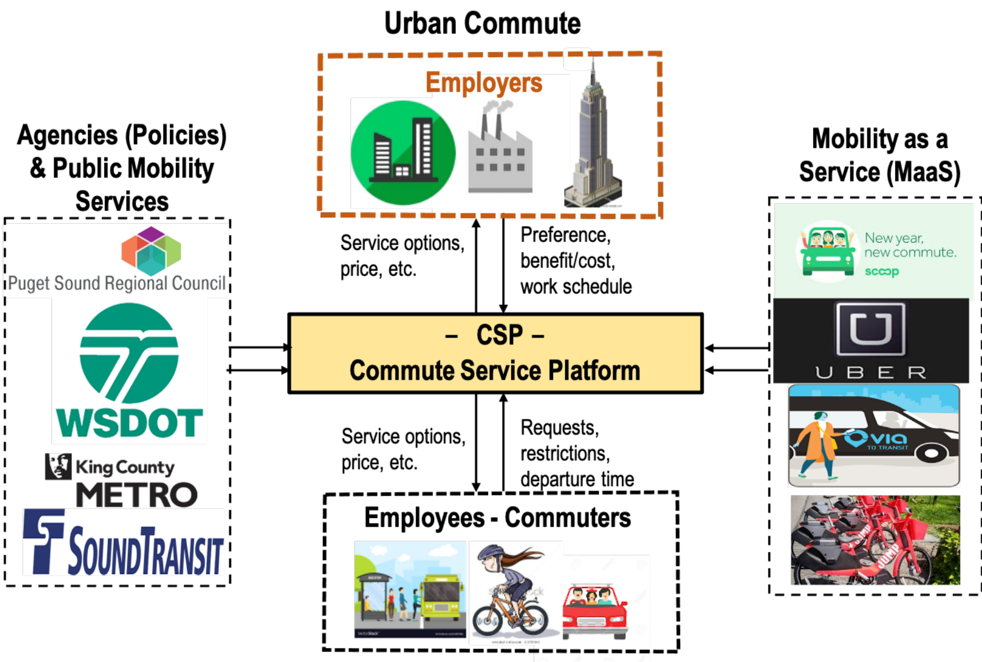


Figure 1.1: Conceptual Structure of the CSP

service providers (e.g. Uber services), CSP integrates employers to leverage their important role in influencing the employees (i.e. commuters) to use more efficient commute options.

CSP can help match an employer with its potential employees in the long term (called the *planning* level) and provide commuting services on a daily basis (called the *operational* level). Employers are motivated to join CSP because (i) they are increasingly aware of the commuting issues and have started to help directly or indirectly their employees' daily commuting (e.g., the above-discussed ESTPs), which has become one of the important strategies for them to recruit/retain the needed talents (Commute Seattle, 2016, 2017; Harrington, 2019); (ii) more companies are supporting sustainability and are becoming more socially responsible (including how to deal with congestion and related issues caused by commuting); (iii) there are pressures from local communities, cities, and even states for companies to take more actions to help resolve commuting issues, e.g., the CTR program in the State of Washington. For employees, it is always in their best interest to find the optimal commuting options that can better balance work and family. Therefore, when selecting employers, possible commuting options and the commuting packages an employer can provide will have an important impact on their decisions. CSP can thus be considered as a platform to connect employers and their potential employees when commuting is concerned, similar to how Amazon connects sellers and potential buyers via its online platform. Currently, a real-world, full-scale CSP does not exist yet. However, there are early prototypes of CSP. For example, for one of its business models, Scoop charges employers to provide carpool services to their employees. It is expected that such early platforms will rapidly grow and evolve to the future CSP to provide a wider spectrum of commuting services.

Joining the CSP is beneficial to employers in the long term. Currently, the ESTP include a variety of subsidies, such as parking subsidies, free transit passes, shuttles, commuting-related reimbursement (mostly fixed upper limit per month), etc. However, it's difficult to measure whether these subsidies help employees to get more convenient commuting services. CSP can help with ESTP, so that ESTP can evolve from its current role of monetary subsidy to a program that benefits morning commutes and provides employees with more mobility choices. CSP can analyze the commuting pattern, such as the average commuting time of a company, the percentage of transit trips or shared ride trips of a company, or the average commute delay caused by traffic. These data can help the employer to understand its role in the commuting scenario and make changes. For example, the employer can reduce the parking subsidy and subscribe for more integrated transit services or shared ride services. This can help the employer in the long term if less commuters choose single-ride services and there is less commuting congestion.

1.3 Research Objective

Building the envisioned CSP needs to solve various planning- and operational-level challenges, including understanding the behaviors of its key players (employers, employees (commuters), service providers, transportation agencies) and their interactions, pricing mechanisms of CSP when considering the interactions of the two sides (employers and commuters) on the platform, operational issues of CSP (such as matching, pricing, routing especially on multimodal and intermodal services), data collection/storage and privacy related issues, among others. This dissertation focuses on some key aspects of the planning level and the operational level analyses of CSP. On the *planning level*, (i) Short/long term management for market expansion. CSP needs to adjust its business strategies when the coverage of its services increases. For example, the business model will be different when CSP is subscribed by only a few employers (and so do their employees) in one city versus when CSP services cover hundreds of employers (and their employees) in multiple cities; (ii) The optimal pricing strategy towards the two groups (i.e., commuters and worksites) where CSP can be modeled as a two-sided market (see Chapter 3). On the *operational level*, (a) The matching of passengers and vehicles for single rides and shared rides. Matching has always been an interesting topic either for traditional taxi or the emerging ridesourcing services. Since CSP has access to more information, such as the location of worksites, commuters and the pick-up time, it's prominent that an optimal matching strategy can be developed to further reduce the waiting time of commuters. (b) The dispatching strategy (route choices). CSP can develop a dispatching strategy that minimize the total travel time of commuters and network congestion simultaneously. (c) The integration with transit, CSP can cooperate with the transit system to further relieve peak hour congestion by providing first-mile services, i.e., sending commuters to transit stations. Noteworthy is that the operational analysis is conducted from the perspective of the transportation network system, not from the individual providers' perspectives.

More detailed discussions are provided below on the planning level analysis of CSP (the static pricing strategy of the CSP) and the operational level analysis of the CSP (integrated transit and ridesourcing services).

1.3.1 Planning level: pricing strategies

With a CSP, an employer needs to subscribe for the platform (by paying an annual fee or per "transaction" fee; here a transaction means a commuting service for one of its employees)

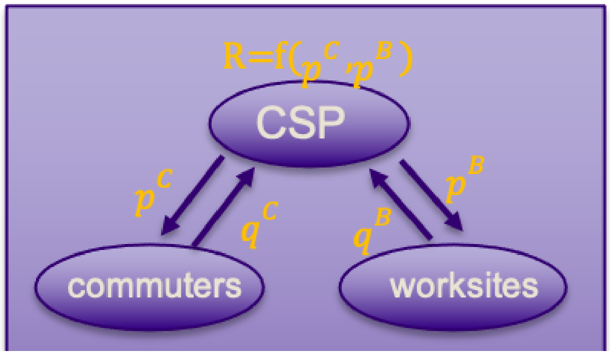


Figure 1.2: Pricing strategy of CSP

so that its employees can use the service. Employees will also be charged each time the CSP service is used. For the platform (i.e., CSP), the cost of service will still be the cost of labor and vehicle depreciation, fuel, maybe also the cost of negotiation with employers, etc. However, the source of revenue has new components. Instead of charging every commuter (employee) a fare as traditionally done, CSP has potential revenue sources from both the commuter side and the employer side. That is, CSP can closely connect employers and employees, with proper policies / management strategies from the agencies, which is similar to a two-sided market. Two-sided market is characterized by two distinct sides (e.g., employers and employees on the CSP) who get ultimate benefit by interacting through a common platform (e.g., the CSP in our study) (Rochet & Tirole, 2003). A platform is said to be two-sided if the price allocation but not only the aggregated price of the two sides affects the profit (or participation) (Rochet & Tirole, 2006), shown in Figure 1.2. Notice here that two-sided market methods have been applied to analyze MaaS where the two sides are service providers and travelers (Zha *et al.*, 2016; Djavadian & Chow, 2017). In a CSP, however, the two sides are employers and employees (commuters), which is markedly different from the analysis for MaaS.

The planning level study of CSP focuses on understanding the interactions of employers and employees on CSP, under what conditions a CSP will be a two-sided market, and if so how to apply the two-sided market analysis framework to study the basic interactions of employers and employees on the CSP in a systematic manner. Such analyses can help understand the interactions of key players and how different policies (such as prices charged by CSP for employers and employees) by the platform may lead to different behaviors of employers/employees and the resulting system effects, based on which to develop operational level methods of CSP and related TDM strategies in the future. To begin with, a particular TDM strategy called proximate commute (Mullins, 1999) is related to the envisioned CSP. Proximate commute allows employees who work for a multi-worksites company/employer (e.g., Starbucks, Key Bank, etc.) to be assigned to worksites closer to their homes, which is beneficial to employees, employers, community, and the environment. Allowing qualified employees voluntarily swapping worksites is one of the ways that a multi-worksites employer could help resolve commuting issues (Mullins, 1995). This study assumes that one or two CSPs are providing the commuting services to all the employees of the employer. Two-sided market analysis is conducted to understand the interactions of worksites and employees when proximate commute is implemented, and the effect and implications of such interactions/behavior.

1.3.2 Operation level: integrated transit and ridesourcing services

On the operational level, the integrated transit and ridesourcing services for the morning commuting problem is studied. All vehicles are CAVs operated by a single CSP, providing on-demand ridesourcing services to commuters. CAVs provide single-ride or shared-ride services, and can drop off passengers at the transit stations or at their destinations (Figure 1.3). For the transit part, fixed-route mass transit are considered, which operates on dedicated right-of-way according to a fixed schedule, such as light rail, subways, or bus rapid transit (BRT). With this approach, there are three major players when integrating ridesourcing and transit services with CAVs: i) Customers who choose mobility services based on their values of time and other personal/social characteristics; ii) a ridesourcing platform providing both

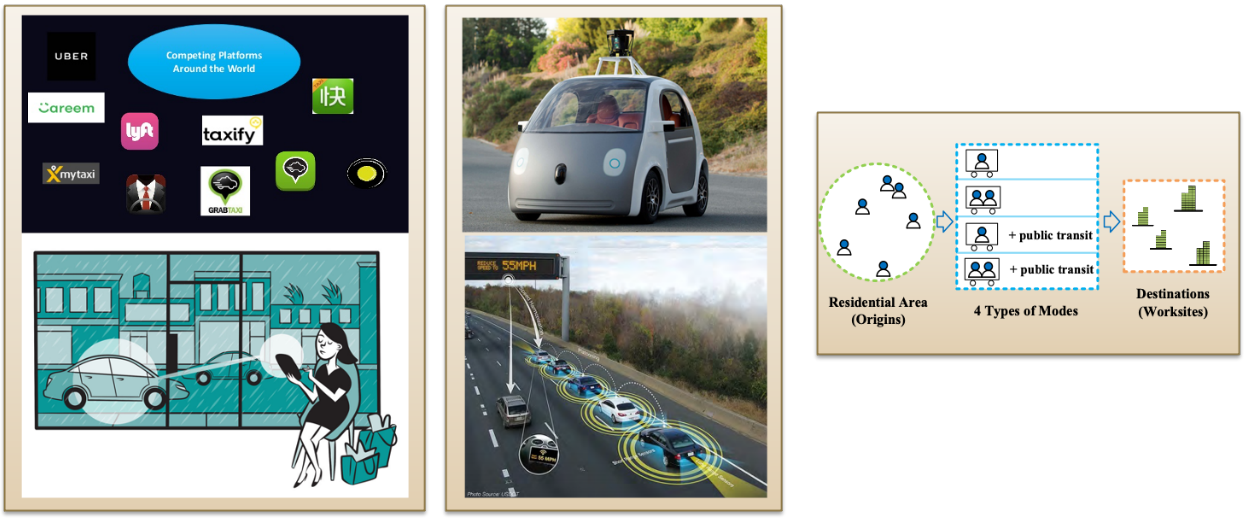


Figure 1.3: Background for the route choice model

single ride and shared ride services, which serves customers and dispatches vehicles (CAVs) to maximize its profit; and iii) the route choices of CAVs that impact the overall network congestion. Clearly, the three players have distinct objectives; however, they interact with each other on the multimodal transportation network of an urban area, resulting in the use pattern of each travel mode and the overall network congestion. Therefore, understanding and modeling the behavior and the interactions of these three players is crucial to better integrate ridesourcing and transit and to evaluate their collective effect on the urban transportation system. This research aim to develop a modeling framework that is not only mathematically rigorous, but also captures the key behaviors and interactions of the major players when integrating ridesourcing with transit.

1.4 Research Contributions

The planning level model investigates the optimal pricing strategies of the CSPs and extends the two-sided market analysis to consider multiple locations for the two groups; the main contributions are:

(1) Propose the concept of CSP to connect worksites and commuters (as the two sides) for commuting, and apply the two-sided market analysis methods to analyze the participation and prices of the two sides, and the profit of the CSP. The analysis is done for both the monopoly platform and the duopoly platform;

(2) Extend the analysis to consider the locations of homes and worksites, and develop a modeling framework to model/analyze the general case with commuters and worksites from multiple locations on multiple CSPs;

(3) The proposed CSP and the associated two-sided analysis methods provide useful tools and insights to build CSPs to connect employers and their employees to better solving future commuting issues.

The operation level model studies the integrated transit and ridesourcing services; the main contributions are:

(i) Methods to model the behavior of the three major players (customers, ridesourcing providers, CAV route choices) on a multimodal network, including the development of an extended network structure;

(ii) Formulation of a general equilibrium model with single rides, shared rides, and integrated CAV ridesourcing and transit services, for which formal analysis can also be conducted including the existence and uniqueness of its solution, and solution methods;

(iii) Evaluation of network effects as a result of different CAV dispatching strategies, different values of time of customers, and different usage rates of transit. This may provide useful insights on developing policies to manage ridesourcing and to better integrate it with transit services.

Collectively, this dissertation research investigates some of the basic questions for building and operating the proposed CSP, which are expected to shed insights on building practice CSPs to serve commuting trips in future urban areas.

Chapter 2

LITERATURE REVIEW

2.1 Proximate Commute

Originated from 1970s, TDM has been extensively studied (Ferguson, 1990). The detailed review of TDM in this dissertation are omitted; readers may refer to Andrea Broaddus (2009) for a comprehensive review. This section highlights some TDM programs implemented by government agencies. In the early 1990s, Southern California launched an employer-based TDM program to improve air quality by trip reduction, which required employers with over 100 employees to conduct trip reduction plans during peak hours (Georgia Institute of Technology, 1994). The program was shown to reduce 8.4 % daily vehicle trips in an accounting firm (Young, 1992). The Washington State Legislature passed the CTR Law in 1991 to encourage travelers to drive alone less, reduce carbon emissions and keep the busiest commute routes flowing (Kadesh & Roach, 1997). By 2018, over 1000 worksites and over 550,000 commuters have joined the program. The wide adoption of CTR program reduced "almost 13% daily vehicle miles traveled by employees between 2007 and 2018" (WSDOT, 2019).

Proximate commute is a CTR strategy that allows employees of a multi-worksite organization to be assigned to the worksites close to their homes (Mullins, 1995). It aims to reduce commute distances by swapping workers in different worksites or taking commute distances/times into consideration when building a new worksite or recruiting new workers. Key Bank of Washington conducted a demonstration project of proximate commute in 1995. The project lasted for 15 month, during which nearly 500 employees at 30 Key Bank branches in Washington were given the opportunity to voluntarily switch to the branches closer to their homes. 17% of eligible employees enrolled in this program, for whom commute miles reduced by 65%. There was a 33% reduction in the longest commute per Key Bank worksite. The results showed that proximate commute is a low-cost method for reducing employee commute time, distance, expense, which can help increase work force productivity. Employers were also willing to implement proximate commute because of reduced absenteeism, higher morale and productivity, and other improvements.

Existing TDM strategies and CTR programs have correctly recognized the importance of involving employers in commuting. Evaluations have also been done on the impact of involving employers in TDM to reduce traffic congestion and related issues (Yushimito *et al.*, 2014, 2015). However, those programs and evaluations were often done for individual employers/employees, and lacked coordinations among different employers and employees. More critically, they have not taken the full advantage of emerging mobility options (such as ridesourcing) into consideration. This dissertation investigates a specific TDM strategy, i.e., the next-generation proximate commute strategies that are implemented via the CSP.

2.2 Two-sided market

Two-sided markets are defined as markets where one or several platforms enable interactions between the the two sides and get the two sides “on board" by appropriately charging each side (Rochet & Tirole, 2006). For example, the Uber platform matches drivers and riders, and charges riders while pays wages to drivers (wages can be considered as negative prices charged to drivers). The two sides choose to join the platform (i.e., consume the services provided by the platform) that makes them better-off. The platform bears the cost of services and charges the two sides to obtain profits. Because of the same-side negative and cross-side positive network effects, (see definitions in Chapter 3.1), the price allocation but not only the total price of services will affect the participation of both sides and the profit of the platform (Rochet & Tirole, 2006). Same-side effects capture the consumer behavior that an agent will usually be worse-off if more agents from the same side join the same platform, whereas cross-side effects exist if an agent from one side benefits from the increasing participation from the other side on a common platform. Depending on the number of platforms and the relations among them, a two-sided market may consist of a single monopoly platform or multiple competitive platforms. When there are competitive platforms, competition among platforms affects the participation and profits on each platform (Armstrong & Wright, 2007). Users from either side could choose to join a single platform, referred to as "singlehome", or choose to use multiple platforms, which is called "multihome".

Rochet & Tirole (2003) provided the first comprehensive investigation of the theory of the two-sided market based on a single platform. With an analytical solution of the price allocation for different governance structures, their study unveiled how a platform makes profit by courting the two-sides. While illustrated in the context of credit cards, their study provided a benchmark model that is applicable to a wide range of applications of two-sided markets. Armstrong (2006), Armstrong & Wright (2007) analyzed two-sided markets under different degrees of product differentiation on each side of the market. They analyzed the conditions of strong product differentiation on both sides, in which case agents from both sides singlehome. The conditions when sellers view the platforms as homogeneous while buyers view them as heterogeneous are also discussed.

Two-sided market provides a method to analyze how price structure affects profits and economic efficiency. Take credit cards for an example. Different credit card issuers are platforms; buyers choose to own one (singlehome) or multiple (multihome) types of credit cards for purchase; sellers choose to accept one (singlehome) or multiple (multihome) types of credit cards. A transaction happens on a platform if a buyer purchases from a seller using the credit card issued by the platform. To optimize its profit, the platform needs to decide which side to bear the price burden. This usually leads the platform to make less money on one side, or even subsidize this side, and recoup its cost from the other side. The platform loses profit when subsidizing one side, and this side is regarded as a "loss leader". In the credit card example, buyers are usually the loss leaders and the many promotion programs by credit card issuers (such as points or rebates) are the subsidies.

There are many examples of real-world markets involving two groups of agents interacting via common platforms, which may be characterized as two-sided markets. Examples include:

Table 2.1: Applications of Two-sided Market

Field	Platform(s)	Two sides	Findings	Authors
Academic Journals	Academic journals	authors ; readers	Open access policy makes publications free to readers and charges high publication fees to authors. This policy is good when considering maximizing social welfare, but may harm readers utility, the impact or profit of the journal.	Jeon & Rochet (2010)
Payment card	credit card, debit card	merchants ;customers	Benchmark model shows that HAC rule not only benefits the multi-card platform but also raises social welfare. However, in the extended model HAC rule may no longer raise social welfare under all parameter settings.	Rochet & Tirole (2008)
Magazine	Magazine companies	readers; advertisers	Higher demand on the reader side increases advertising rates. Higher demand on the advertiser side reduces the price of magazine to readers.	Kaiser & Wright (2006)
Internet	Internet Service Provider	content providers; users	Network neutrality regulation increases the total surplus under certain parameter ranges for both monopoly and duopoly platforms.	Economides & Tåg (2012)
Flexible mobility services	The built environment	operators ; travelers	The differences between one-sided and two-sided market. The threshold when the network externalities lead to two-sidedness.	Djavadian & Chow (2017)
Ride-sourcing	Ride-sourcing services	drivers; passengers	The matching condition/regulation policy of the first/ second best solution in monopoly case is found. The study of competing platforms suggests merging of platforms as competition won't lower the price level or improve social welfare	Zha *et al.* (2016)

1) academic publishing; 2) advertising media market; 3) payment systems, such as credit cards; 4) Internet service providers. There are a handful applications of the two-sided market theory in transportation. One example is the matching of drivers and customers in taxi or ridesourcing. By treating the ridesourcing platform as a two-sided market with customers and drivers as the two sides, Zha *et al.* (2016) and Wang *et al.* (2016) studied the matching process with negative same-side externality and the positive cross-side externality. Djavadian & Chow (2017) evaluated an agent-based stochastic day-to-day adjustment process in a two-sided market. A collection of publications in these markets is summarized in Table 2.1. Despite these efforts of two-sided market applications, no study so far has attempted to connect employers and employees by a common platform (e.g., CSP, as proposed in this dissertation) when commuting trips are considered. Consequently, no study has applied the two-sided market framework to analyze the behavior and interactions of commuters and employers on CSP.

2.3 Integrated transit system

2.3.1 Demand responsive transportation system and paratransit

As early as 1970s, the U.S. Department of Transportation had been researching "demand responsive" transportation system and "paratransit" (Orski, 1975). The purpose is to have on-demand operation systems to provide transit services to people who live far away from the transit stations, elder people or the handicapped people, so as to improve the equity of transit service and increase the usage rate of transit.

One of the outcomes of this research is an integration of Dial-a-Ride and transit. Stein (1978) proposed an analytical investigation for a Dial-a-Ride transportation system where the first/last mile of public transit are served by Dial-a-Ride. The static case and the dynamic case were both studied under the assumption of uniformly distributed demand. The proposed transportation system design is computational efficient and easy to implement. Malucelli *et al.* (1999) designed a demand adaptive system where the buses can pick up passengers either at compulsory stops or optional stops. Compared with the demand responsive system such as Dial-a-Ride, the demand adaptive system still operates within a conventional line transportation framework and is considered as a more cost effective design.

2.3.2 The integration of ridesourcing with transit

The wide deployment of ridesourcing services has inspired research on the integration of ridesourcing and transit. Feigon & Murphy (2016) examined the relationship between public transportation and ridesourcing. With the survey data collected from transportation officials and shared mobility users, the authors analyzed capacity, demand, and comparative travel times of ridesourcing and transit. An assessment of the integration of ridesourcing with transit was presented and a complication of current related business models was evaluated. Their study showed that the increased usage of ridesourcing is associated with higher transit usage rate, lower private car ownership and reduced transit operation cost. Ridesourcing, especially shared ridesourcing can be a complement to public transit and enhances urban mobility. Based on these findings, public transportation agencies are encouraged to engage with ridesourcing platforms. According to the interviews and survey, the public transit agencies and the ridesourcing platforms agree on such collaborations, some of them have already launched related projects.

Chen & Nie (2017) explored design options for the integration of e-hailing services into public transportation networks. Relative spatial position (RSP) of e-haling services were designed, which disaggregates e-hailing services in space and match them with transit. Two types of RSPs are analyzed and compared: a zone-based design that assigns a set of e-haling vehicles to serve first/last mile within a relatively small zone surrounding a transit stop; and a line-based design that operates e-hailing vehicles along a fixed-route transit line to serve passengers who need a ride between their origin/destination and the closest transit stop. Their study contributes to the research field by proposing an unifying analysis framework based on continuous approximation approach and offering the first comparative study of RSP design. Their analytical and simulation results suggest that line-based services can achieve higher efficiency, since the e-haling vehicles in the zone-based design incur more deadhead miles.

Ma *et al.* (2019) proposed a ridesharing scheme with integrated transit in which ridesourcing serves either the whole trip or the first/last mile of a transit route. Their objective is to optimize vehicle dispatch and idle vehicles relocation for an integrated, multimodal transit-rideshare system. The model has three types of modes: ridesharing only, rideshare-transit-rideshare, and rideshare-transit-walk (and vice versa). Dynamic queueing-theoretic vehicle dispatch and idle vehicle relocation algorithms are customized to solve the model. The numerical experiments indicate that the transit-rideshare system outperforms the rideshare-only system by 32% reduction in user travel time and 64% reduction in vehicle travel time. Compared with rideshare-only system, the transit-ridesharing system also reduces the operation cost and customer waiting times, and achieves better performance in regions where passenger demands are heterogeneous.

Pinto *et al.* (2020) proposed a bi-level mathematical programming formulation for the joint transit network redesign and mobility service fleet size determination. The authors tested their model framework using real world travel demand data. Their study showed that the integrated system reduces average customer waiting time compared with the transit-only system. The integration in their study is to replace inefficient transit routes/patterns with shared connected autonomous mobility services, while the on-demand first/last mile for transit is not considered. In addition, their model only has one objective function to minimize the disutility of customers, with no ridesourcing platform profit maximization or congestion consideration.

In addition to studies on the operation of the integrated system of transit and ridesourcing, researchers also investigated the user experience and preferences of passengers. Yan *et al.* (2019) evaluated travelers' responses to the integrated transit pilot MTransit. MTtransit is a pilot of on-demand transit services in the north campus of the University of Michigan, where twelve fixed bus routes are replaced with four fixed high-frequency bus routes in the central high-volume corridors, together with on-demand shared shuttle services in the outer area.

Both revealed preference and stated preference data are collected from students, faculties and staffs from the campus. Their study showed that ridesourcing could complement transit by serving the first/last mile and/or replacing low usage transit routes, leading to reduced passenger waiting times and lower operation costs of transit agencies.

While the other literatures tell us how the integrated transit system is designed and optimized, it is also important to evaluate how the actual pilot program bring changes to the transit system. Terry & Bachmann (2020) evaluated the types of trips passengers are taking through a transit-integrated ridesourcing pilot, and compared them with transit and walking alternatives. Results suggested that most rides of the pilot have a great travel time saving compared to the alternative modes. Ridesourcing has the potential to complement or supplement the existing transit services. When ridesourcing only serves the first/last miles, passengers choose ridesourcing over walking and are more likely to take transit, thus transit usage rate increases. However, there are cases (18% trips) when ridesourcing trips are designed alongside transit, which often leads to the decrease of transit usage rate. There is a great chance (65% of the trips) that the ridesourcing does not send passengers to their closest transit stop.

2.4 Multi-modal network models

Di & Ban (2019) proposed a theoretical framework of generic traffic network equilibria to model a multimodal network that consists of solo driving, ridesharing and e-hailing service. It is challenging to fit ridesharing in an equilibrium model. Thus the author proposed an extended network representation, including two layers, vehicular flow layer and person flow layer. It is critical to split the network because the vehicular flow is different from the person flow for ridesharing mode. In the operational model in chapter 4, the ridesharing mode are formulated to be based on ridesharing pairs instead of origin destination pairs. Two layers of network are designed for ridesourcing (single ride and shared ride) and transit, respectively. In this way, the different properties of transit network and ridesourcing network, such as capacity and routes, can be captured.

Ban *et al.* (2019) proposed a general equilibrium model that consists of the optimization of three main players: profit optimization of service providers including traditional taxi and e-hailing services; utility maximization of the customers who can choose among solo driving, taxi or e-hailing; network congestion minimization, following the Wardrop's principle so that vehicles always choose the route with minimum travel time (Wardrop and Whitehead, 1952). Their numerical experiments show the customer choices under different value of time or different prices of the mobility services and the resulting congestion pattern of the network. Their study provides insights for the transportation planning agencies to leverage the mode split of multimodal network in order to reduce congestion. The deadhead miles of either taxi or e-haling services generally exacerbate the network congestion, but when the demand pattern has high level of symmetry, the model can optimize the route choice to reduce deadhead miles. In this dissertation, the proposed scheme to optimize ridesourcing services extends the model in Ban et al. (2019) significantly to encompass shared rides and transit under the CAV environment.

2.5 Post-pandemic commute

Coronavirus pandemic, one of the deadliest pandemic along the history, hit the world in 2019 and is still ongoing. The pandemic caused major cultural, economical, and technological disruptions. People across the globe has witnessed/experienced a shift to remote work. Here is a brief overview of the change in commuting demand during the pandemic and its affect on people's commuting behavior and preference.

Quarantine was tested to be an effective measure to fight against Covid. Cities all over the world have different levels and duration of quarantine strategies from 2019 to 2022. Based on the county-level daily mobility dataset provided by Descartes Labs (Warren and Skillman, 2020), Kim & Kwan (2021) analyzed the mobility of 2639 counties in the US from March to September 2020. Their study shows that in the 1st wave (March to June), the mobility across US first declined abruptly and then slowly recovered back to the pre-pandemic level around April to June. In the 2nd wave (July to September), despite the severity of Covid and the stay-at-home regulation, mobility only decreased a little. The level of the mobility restriction policy is only one of the factors that affect mobility, political partisanship, economical status also play roles in affecting mobility.

Pandemic-related mobility restrictions have long-term effects on people's commuting preferences. Alexander *et al.* (2021) conducted a survey of 5,043 full-time employees who work in corporate or government settings. 52% employees prefer hybrid work post-pandemic while the percentage is only 30% pre-pandemic. More than 50% of employees prefer to work from home for at least 3 days each week in the future. Such a global-wide shift to hybrid/remote work will have an unprecedented impact on work life. In order to aid companies, employees, and society to conduct efficient work practices, Teevan *et al.* (2021) from Microsoft studied how remote work affects working efficiency and employees' well-being, as well as social implications. Their study shows that the average working hours got longer during COVID due to the interweaving of life and work tasks and the elimination of commutes. The Microsoft survey shows that respondents need to commute for 1 hour per day on average before the pandemic. Half of the respondents think no commutes is an important benefit of remote work. 59% of the respondents continue to work remotely after the office reopened to reduce commute time.

In the planning level analysis, flexible work hours are considered in the duopoly model, where the employer side and the employee side can choose from the work-flexible (WF) CSP and the none work-flexible (NWF) CSP. With the trend of the interweaving between life and work since the pandemic, flexible working hours will continue to be a popular choice. Compared to the shuttles, CSP can set optimal prices for both WF and NWF commute trips to improve the efficiency of the morning commute.

Chapter 3

PLANNING LEVEL ANALYSIS

3.1 Problem statement

The planning model studies the proximate commute problem of an employer with multiple worksites when there is a CSP to provide commuting services for its employees. There are many examples of such employers in urban areas. For example, as shown in Figure 3.1, there are 17 Starbucks stores (worksites) in the Seattle downtown area. The planning model will investigate when proximate commute with CSP will present two-sidedness, and when this happens how the pricing schemes and other mechanisms may impact the participation of the two sides (i.e., worksites and employees) and the profit of the CSP. Both the monopoly platform and the duopoly platforms in the two-sided markets are analyzed. For the monopoly platform, there will be only one type of CSP service in the market. For the duopoly platforms, I assume there are two types of CSPs to provide NWF services and WF services respectively. Then the analysis is extended to consider the locations of commuters' homes and worksites using a general framework. Through the investigations, this research hopes to gain deeper understanding of CSPs, how the price allocation of a CSP may impact its scale (i.e., the participation of the two sides) and profit, which can provide useful insight on how to design CSP and next-generation TDM strategies based on emerging technologies and mobility options.

The followings are several key definitions that will be used hereafter in this chapter.

• **Cross-side benefit network effect** refers to the benefit (or utility) an agent would obtain when joining a CSP because of the existence of agents on the *other side* of the CSP. For example, the benefit for a commuter to join a CSP is to receive commuting service to be able to arrive at the worksite; the benefit for a worksite is to attract and transport its workers

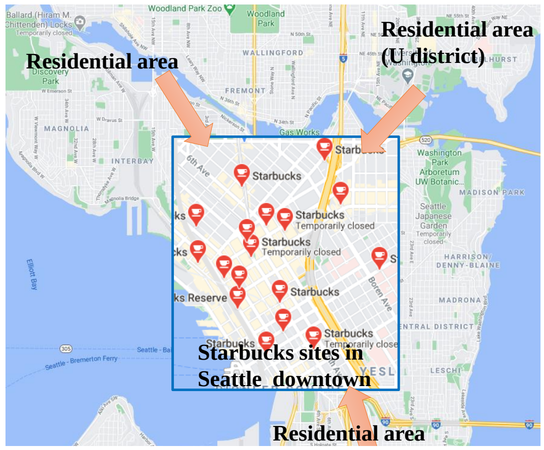


Figure 3.1: The distribution of Starbucks in downtown Seattle and the commuting trends

(i.e. commuters). Clearly the existence of worksites (or commuters) on the other side is the primary reason for a commuter (or worksite) to join a CSP.

• **Same-side negative network effect** refers to the negative benefit (or disutility) an agent would bear when joining a CSP due to the existence of other agents on the *same side* of the CSP. It is also called the same-side "**congestion**" effect in the literature. This leads to the "**inconvenience cost**" when an agent decides to join a CSP. For example, a commuter may have a harder time to use transit services if there are many other commuters going to the same worksite. Similarly, more worksites in a certain area also imply more commuters in the area, creating more congestion and unreliable travel times.

• **Multihome** means an agent (a commuter or a worksite) can subscribe for multiple CSPs when they exist.

• **Singlehome** means an agent can subscribe for only one CSP when multiple CSP services exist.

The real world proximate commute problem is simplified by the following assumptions:

(a) One or multiple CSPs exist to provide commuting services. CSPs charge both sides for using the service.

(b) The total number of commuters or worksites (either choose to join a CSP or not) is assumed to be fixed (and normalized to 1 as shown later in this dissertation). In reality, this may change, e.g., more worksites in an area may attract potential workers from other cities to relocate to that particular area. This research does not consider this case.

(c) A commuter can choose which worksite to work for based on his/her own preference.

(d) The participation of worksites and employees is not pre-defined, which is determined by market equilibria.

(e) For an agent (commuter or worksite), the cross-side benefit is positive and increases with the increased number of agents on the other side. This is generally understandable; in particular, with more worksites on the other side, the commuter usually has more options to choose from (including quicker and more convenient commuting choices), leading to larger cross-side benefits. Similar logic also applies to worksites.

(f) For an agent (commuter or worksite), this research assumes the "same-side" negative effect exists. Further, this same-side negative effect increases with the increased number of agents on the same side. This could be understand as: more commuters mean more competition, more congested traffic on the road, and more crowded buses, leading to more negative effects.

Assumption (a) ensures that worksites and employees interact on the CSP, and the CSP's price strategy may impact their behavior of using the platform. Assumption (b) is applied to simplify the discussions and may be relaxed in future research. Under assumption (c), employees are exchangeable among different worksites, which is the key concept of proximate commute. From the two-sided market analysis, one would be able to obtain the proportion of commuters/worksites participating in a platform and the resulting profits. Assumption (d) indicates the behavior principle of commuters and worksites so that the participation rates of employees and worksites are not pre-specified, which can be determined by profit maximization (or similar objectives) of the CSP. Assumptions (e) and (f) guarantee that the proposed CSP is a two-sided market.

One should notice here that while the cross-side benefits should be generally positive, in practice, there can be multiple (and potentially conflicting) factors that may influence the same side negative effect (that is related to the inconvenience cost) of either worksites or commuters. This research simply assumes the same side "negative" effect is larger than the "positive" effect for an agent of either side of a CSP. It is worth emphasizing that how to determine the actual values of the cross-side benefit and same-side negative effects, as well as other parameters of the proposed CSP (see the notation list next and the models later), is not trivial. Rigorous investigations need to be conducted on the behavior, interaction, and attitude of commuters and their employers (worksites) regarding the proposed CSP and various commuting services that may be provided by the CSP. See Section 3.6 for more discussions on this.

3.1.1 Notation

The following is a list of notations used in the planning model.

Sets:

- i labels of platforms, $i \in \{N, W\}$; N, W denote NWF CSP, WF CSP, respectively.
- k labels of different groups, $k \in \{B, C\}$; B, C denote worksites, commuters (employees), respectively.

Variables:

- q^k (monopoly model) the fraction of group k agents (i.e., demand) that join the CSP.
- p^k (monopoly model) subscription price of a group k agent charged by the CSP, $p^k \geq 0$.
- q_i^k (duopoly model) the fraction of group k agents singlehoming on CSP i.
- Q^k (duopoly model) the fraction of group k agents multihoming on both CSPs.
- p_i^k (duopoly model) subscription price of a group k agent charged by CSP i, $p_i^k \geq 0$.
- x^k range from $[0, 1]$, the location of a group k agent at the unit interval in Hotelling Model.
- U_i^k the utility of a group k agent on CSP i.
- R_i the profit of CSP i.

Parameters:

U_0^k the initial benefit of a group k agent when joining a CSP.

t^k the rate of same-side negative effect.

β_i the rate of cross-side benefit of commuters on CSP i. A commuter obtains benefit $\beta_i q_i^B$ by joining CSP i, as s/he has the potential to choose from q_i^B worksites (singlehoming).

α_i the rate of cross-side benefit of worksites on CSP i. A worksite obtains benefit $\alpha_i q_i^C$ by joining CSP i, as it has the potential to choose from q_i^C commuters (singlehoming).

b^k the rate of cross-side benefit of group k agents in monopoly model

f_i^k the cost of CSP i to serve group k agents in duopoly model.

f^k the cost of the CSP to serve group k agents in monopoly model.

Auxiliary parameters:

α^+, α^- the sum/difference of the cross-side benefit rate of *worksites* on the two CSPs, respectively, defined for analytical purpose, $\alpha^+ = \alpha_N + \alpha_W$, $\alpha^- = \alpha_N - \alpha_W$

β^+, β^- the sum/difference of the cross-side benefit rate of *commuters* on the two CSPs, respectively, defined for analytical purpose, $\beta^+ = \beta_N + \beta_W$, $\beta^- = \beta_N - \beta_W$

3.2 Hotelling model and linear demand specification

In this chapter, the inconvenience cost of worksites and employees joining the CSP is captured by Hotelling model. Hotelling model (Hotelling, 1929) has been applied to many studies in two-sided markets. Economides & Tåg (2012) used Hotelling model for monopoly and duopoly two-sided markets. Rochet & Tirole (2003), Armstrong (2006) and Kaiser & Wright (2006) applied Hotelling model for duopoly two-sided market models. This study applies Hotelling model in both the monopoly platform and duopoly platforms. A classic Hotelling model depicts that customers are uniformly distributed on a unit length street, and two stores locate at the two ends ($x = 0$ and $x = 1$) of the street. Hotelling competition assumes that consumers purchase at those stores if and only if the minimum utility of consuming at the two stores is larger than some constant $\bar{U}$ (Fudenberg & Tirole, 1991). Under this assumption, the Nash equilibrium is achieved when the utility of purchasing at the two stores are the same. Denote the Nash equilibrium as x^*, $x^* \in [0, 1]$. Under equilibrium, consumers located to the left of x^* choose the store at $x = 0$, the rest of consumers choose the store at $x = 1$. Therefore, x also indicates the proportion of customers, i.e., the participation rate, who choose the store at $x = 0$ ($1 - x$ is the proportion of customers who choose the store at $x = 1$). The role of Hotelling model in monopoly platform and duopoly platforms are similar. Figure 3.2 uses duopoly platforms as an example to illustrate how the Hotelling model is applied to represent inconvenience cost. Notice that this chapter focuses on the scenario where all agents from either side will join one of the two CSPs; see the discussion in Section 3.4.1 and the proof in Appendix A.2.

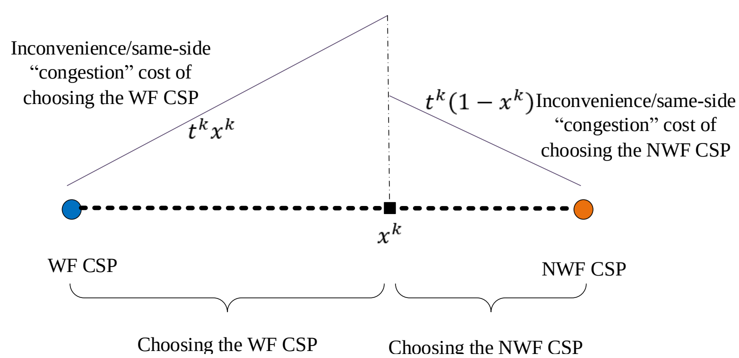


Figure 3.2: An illustration of Hotelling Model ($k \in \{B, C\}$)

In duopoly platforms, the NWF CSP ($x = 1$) and the WF CSP ($x = 0$) are located at the two ends of a unit interval, as shown in Figure 3.2. The demands of worksites and commuters are both specified by Hotelling models. Take worksites as an example, worksites distribute uniformly along the unit interval. x^B denotes the location of a worksite, which also indicates preference of the worksite over the WF CSP (or the NWF CSP): smaller x^B implies the worksite prefers more of the WF CSP (and less of the NWF CSP) . t^B denotes the rate of inconvenience cost (i.e., the same-side negative effect) of worksites. The worksite located at

x^B experiences a inconvenience cost of $t^B x^B$ when joining the WF CSP, or a inconvenience cost of $t^B(1-x^B)$ when joining the NWF CSP (shown in equation (3.15) and (3.16)). Under such setting, the cost term induced by Hotelling model ($t^B x^B$ or $t^B(1-x^B)$) reflects the same-side negative effects. It means that a worksite will be worse-off if more other worksites join the same CSP.At the equilibrium, the optimal x^{B*} represents the participation rate of businesses in the WF CSP, and $1-x^{B*}$ is the participation rate of businesses in the NWF CSP.

Same analysis can be applied to the commuter side, which is omitted here. Hotelling model can also be applied to the monopoly platform, with a similar interpretation as shown in Figure 3.2 except that one end is the CSP and the other end is "not joining the CSP".

3.3 Monopoly platform

In a monopoly model, agents from the two sides choose to join the CSP or be off the market. The utilities of agents are affected by the same-side and cross-side effects. This section first discusses two-sidedness of the CSP, then analyzes how network effects impact the participation, price strategies, and profits of the CSP.

3.3.1 Two-sidedness in commuting problems

Before introducing the model, one wants to answer why the envisioned CSP is a two-sided market, and why the two-side market theory is important in studying the CSP. To do so, a benchmark model of the monopoly platform similar to Armstrong (2006) is presented to illustrate the two-sidedness of a market with CSP. Note that in this benchmark model, the same-side negative effect will not be considered, which will be added later in Section 3.3.2. As stated above, worksites and commuters are the two sides, denoted as $k \in \{B, C\}$. This research assumes unit mass for both sides, i.e., $q^B, q^C \in [0,1]$. q^B and q^C are then also the participation rates of the two sides. The CSP platform charge worksites and commuters with prices p^B and p^C. By joining the CSP, a group k agent incurs a fixed benefit U_0^k. In the monopoly model, there is only one CSP sending commuters to worksites. The costs for the

CSP to serve the two groups are f^B and f^C, respectively. f^C represents the per commuter transportation cost, while f^B represents the negotiation cost of attracting a worksite to join the CSP. The rate of the cross-side network benefits are measured by b^k. By joining the CSP, each agent from group k experiences a benefit of $b^k q^l$ under the assumption that s/he values the participation of the other group l. This is intuitively understandable. A commuter on the CSP will be better-off if more worksites join the CSP since s/he will have more choices of worksites. similar logic applies to worksites. Then the utility of a group k agent is determined by

$$U^k = U_0^k + b^k q^l - p^k \quad \forall k, l \in \{B, C\} \text{ and } k \neq l \tag{3.1}$$

Supposing that the participation of group k agents on the CSP can be measured by an increasing function of utility, the participation of group k agents is:

$$q^k = \phi^k(U^k) \tag{3.2}$$

From equation (3.1), the price of group k can be expressed as $p^k = U_0^k + b^k q^l - U^k$. Profit of the CSP is:

$$\begin{aligned} R &= (p^B - f^B)\phi^B(U^B) + (p^C - f^C)\phi^C(U^C) \\ &= \Big[(U_0^B + b^B\phi^C(U^C) - U^B) - f^B\Big]\phi^B(U^B) + \Big[(U_0^C + b^C\phi^B(U^B) - U^C) - f^C\Big]\phi^C(U^C) \end{aligned} \tag{3.3}$$

Here the cost of the commuter side (f^B) or the worksite side (f^C) is modeled as a one-time fixed cost per agent. In practice, the cost for providing commuting services for a commuter may also be modeled as per-transaction cost (i.e. the cost per commuting trip). This per-transaction cost can be converted to the one-time fixed cost by assuming a fixed cost per trip (transaction) and the estimated number of trips per year or per month. For more general cases, e.g., the per transaction cost may change as the number of commuter changes, more advanced modeling approaches will be needed. This is particularly true when one studies the operation of the CSP.

Theorem 1. *The equilibrium price can be obtained by maximizing the profit of the platform:*

$$p^k = f^k - U_0^k - b^l \phi^l(U^l) + \frac{\phi^k(U^k)}{[\phi^k(U^k)]'} \quad \forall k, l \in \{B, C\} \text{ and } k \neq l \tag{3.4}$$

The proof of **Theorem 1** can be found in Appendix A.1. In order to study the elasticity of demand, the Lerner index (Lerner, 1934) is introduced. The elasticity can be written as,

$$\eta^B(p^B \mid q^C) = \frac{p^B[\phi^B(U^B)]'}{\phi^B(U^B)}; \quad \eta^B(p^C \mid q^B) = \frac{p^C[\phi^C(U^C)]'}{\phi^C(U^C)} \tag{3.5}$$

Substituting Equation (3.4) into Equation (3.5) yields the expression of Lerner indices at the equilibrium:

$$\frac{1}{\eta^B(p^B \mid q^C)} = \frac{p^B - (f^B - U_0^B - b^C q^C)}{p^B}; \quad \frac{1}{\eta^C(p^C \mid q^B)} = \frac{p^C - (f^C - U_0^C - b^B q^B)}{p^C} \tag{3.6}$$

Based on this model (Equation 3.1-3.6), two important concepts are illustrated:

(i) Loss leader: under the optimal price structure, it is possible for group k agents to act as the loss leader (to be subsidized), that is, when $p^k < f^k$. From equation (3.6), this occurs if the group's elasticity of demand $\eta^k(p^k \mid q^l)$ is large and/or the cross-side benefit (b^l) enjoyed by group l ($k, l \in \{B, C\}, k \neq l$) is large. On the other hand, when b^l is small and/or $\eta^k(p^k \mid q^l)$ is small, the CSP charges higher price to group k agents. This implies that, if worksites value the number of commuters that join the CSP, and/or commuters' elasticity of demand is high, the CSP will subsidize the commuters to attract more commuters, thus more worksites to join the platform.

(ii) Two-sidedness: two-sidedness is conceptually defined from either of the two aspects: 1) "the volume of transaction on the platform depends on the allocation of price between the two sides but not only on the aggregated price level" (Rochet & Tirole, 2006); 2) the decision of one group affects the outcomes of the other group, typically through an externality, i.e. cross-side positive network effects and same-side negative network effects (Rysman, 2009; Caillaud & Jullien, 2003; Armstrong, 2006). Because CSP services transport employees to their worksites, no agent from one group would be willing to join the CSP unless agents from the other group also join the CSP (if no commuter joins the CSP, it's irrational for

worksites to join the CSP and vice-versa). As a result, the price of one group is affected by the participation and network effects of both groups. In the benchmark model, if there is no network effects (i.e., $b^k = 0$) and only the aggregated price level affects the transaction, the participation of the two sides will be the same, $q^B = q^C = \phi(p^B + p^C)$. The model will be reduced to $R = (p^B + p^C - f^B - f^C)\phi(p^B + p^C)$. In the reduced model, if the CSP holds the aggregated price (i.e., $p^B + p^C$) to be constant, its profit will not change even when the price allocation differs. In this case, the model will reduce to one-sided. The models hereafter in this chapter assumes that there are network effects in the market. Therefore, participation always changes with price allocation and network effects from both groups, indicating that the CSP is indeed a two-sided market. Two-sidedness is important for us to unveil how to incentivize participation on the CSP under different level of network effects.

Based on our discussion of two-sideness, the condition of two-sidedness of the monopoly platform is:

(A0) $b^k > 0, t^k > 0$, ensures that network effects exist.

When the CSP extracts profits from both sides in a market with network effects, the participation of one side will be affected by the decisions of both sides, thus two-sidedness holds. Condition **(A0)** is the basic feature of a two-sided market, which is applied to all of the models in this dissertation.

The discussion so far assumes that the CSP's objective is to maximize profit, which is reasonable. Similar analysis can be done for other objectives of the CSP. For example, if the CSP aims to maximize social welfare, one can replace the profit function in Equation (3.3) by the following social welfare function (Armstrong, 2006):

$$\begin{aligned} w =& R + v^B(U^B) + v^C(U^C) \\ =& \Big[(U_0^B + b^B\phi^C(U^C) - U^B) - f^B\Big]\phi^B(U^B) + \Big[(U_0^C + b^C\phi^B(U^B) - U^C) - f^C\Big]\phi^C(U^C) \\ &+ v^B(U^B) + v^C(U^C). \end{aligned} \tag{3.7}$$

Here w is the welfare, and $v^B(U^B)$ and $v^C(U^C)$ are the aggregate consumer surplus of worksites and commuters respectively. Notice that $[v^B(U^B)]' = \phi^B(U^B)$ and $[v^C(U^C)]' = \phi^C(U^C)$.

Maximizing w can obtain the equilibrium price of the platform $p^k = f^k - U_0^k - b^l\phi^l(U^l)$. Comparing with Equation (3.4), one can easily see that the elasticity related term (the last term) in (3.4) disappears in the CSP's price structure if the social welfare objective is considered. Since it is arguably true that most CSPs will focus on profit maximization, this chapter will focus on this objective for the analyses. When designing specific TDM strategies associated with CSP, in particular, objectives of social welfare should be investigated. The analysis methods presented here can be applied (with proper modifications) if other CSP objectives are considered.

3.3.2 Monopoly platform with linear demand specification

In this subsection, the inconvenience cost is added to the benchmark model mentioned in section 3.3.1. Hotelling Model is applied in the utility function to represent the horizontal differentiation between joining vs. not joining the CSP. Group k agents are uniformly distributed on a unit interval $[0, 1]$ with the CSP at $x = 0$. t^k is the rate of inconvenience cost (same-side negative effect) when an agent from group k joins the CSP. A group k agent located at x^k experiences an inconvenience cost of $t^k x^k$ to join the CSP, or a cost of $t^k(1-x^k)$ if s/he does not join the CSP. Adding inconvenience cost $-t^k x^k$ to equation (3.1) yields the following utility functions:

$$U^C = U_0^C + b^C q^B - t^C x^C - p^C; \qquad U^B = U_0^B + b^B q^C - t^B x^B - p^B \tag{3.8}$$

Note that Equation (3.8) actually specifies that the demand function (i.e., $\Phi^k(U^k)$ in Equation (3.2)) is linear with respect to the utility. The conditions that ensure the equilibrium prices of the monopoly model are (Economides & Tåg, 2012) :

(A1) Cross-side positive effects are not strong. When same-side negative effects and cross-side positive effects follow the condition $4t^B t^C > (b^B + b^C)^2$, the profit function is concave and the equilibrium prices are feasible;

(A2) The parameter setting satisfies $4t^B t^C - (b^B + b^C)^2 \geq \max\{(U_0^C - f^C)(b^B + b^C) + 2t^C(U_0^B - f^B), (b^B + b^C)(U_0^B - f^B) + 2t^B(U_0^C - f^C)\}$, which ensures $q^B, q^C \leq 1$.

Equilibrium exists when worksites or commuters are indifferent between choosing and not choosing the CSP. In the Hotelling model, agents from both sides are uniformly distributed on a unit interval. According to the assumption of unit mass for both sides, $x^k = q^k$. Substitute $x^k = q^k$ to Equation 3.8 and solve for the demand of two groups:

$$q^B = \frac{b^B(U_0^C - p^C) + t^C(U_0^B - p^B)}{t^B t^C - b^B b^C} \qquad q^C = \frac{b^C(U_0^B - p^B) + t^B(U_0^C - p^C)}{t^B t^C - b^B b^C} \tag{3.9}$$

Notice that demand function (3.9) is different from the classic demand theory (Mas-Colell *et al.*, 1995). In the classic demand theory, the demand of a person who can choose to buy n goods is obtained by maximizing the sum of utility from n goods under a budget constraint. In comparison, demand function (3.9) is calculated under the equilibrium condition of the Hotelling model, which represents the number of agents on the CSP when all agents have the highest utility. Demand function (3.9) is derived similarly as the demand function in Armstrong (2006); Economides & Tåg (2012). The same method is used to derive the demand function in section 3.4.

Substituting Equation (3.9) to profit function, CSP profit can be written as a function of the prices:

$$R = \frac{b^B(U_0^C - p^C) + t^C(U_0^B - p^B)}{t^B t^C - b^B b^C}(p^B - f^B) + \frac{b^C(U_0^B - p^B) + t^B(U_0^C - p^C)}{t^B t^C - b^B b^C}(p^C - f^C) \tag{3.10}$$

Maximize CSP's profit using the first order condition of equation (3.10). The equilibrium prices are

$$p^B = \frac{-b^{B2} f^B - b^{C2} U_0^B + (2t^B t^C - b^B b^C)(f^B + U_0^B) + t^B(b^B - b^C)(U_0^C - f^C)}{4t^B t^C - (b^B + b^C)^2} \tag{3.11}$$

$$p^C = \frac{-b^{C2} f^C - b^{B2} U_0^C + (2t^B t^C - b^B b^C)(f^C + U_0^C) + t^C(b^B - b^C)(f^B - U_0^B)}{4t^B t^C - (b^B + b^C)^2} \tag{3.12}$$

Substitute equation (3.11) and (3.12) into equation (3.9). Demands on the CSP platform are

$$q^B = \frac{(U_0^C - f^C)(b^B + b^C) + 2t^C(U_0^B - f^B)}{4t^B t^C - (b^B + b^C)^2} \quad q^C = \frac{(b^B + b^C)(U_0^B - f^B) + 2t^B(U_0^C - f^C)}{4t^B t^C - (b^B + b^C)^2} \tag{3.13}$$

Substituting equation (3.11) and (3.12) into equation (3.10) yields the profit at equilibrium price

$$R = \frac{(b^B + b^C)(f^B - U_0^B)(f^C - U_0^C) + t^C(f^B - U_0^B)^2 + t^B(f^C - U_0^C)^2}{4t^B t^C - (b^B + b^C)^2} \tag{3.14}$$

Equation (3.9) shows the relation between price and participation. When model parameters are given, demand quantity can be expressed as a linear combination of prices, $q^k = f(p^B, p^C)$. This also indicates the key characteristic of a two-sided market, i.e., the participation of one side of the market is affected by the allocation of prices between the two sides. It is hard to draw conclusions on the distribution of price allocations based on the equilibrium expressions shown in equation (3.11) and (3.12). A comprehensive analysis on how network effects impact the distribution of prices, quantities and profits are presented in the numerical experiments next.

Remark: From equation (3.13), to ensure that some agents from both sides will join the CSP (i.e. $q^B \geq 0, q^C \geq 0$), the initial benefits of the two sides need to be large enough, i.e., $U_0^C \geq f^C$ and $U_0^B \geq f^B$. Provided all other parameters given, further increasing say U_0^B will increase q^B, until $q^B = 1$, i.e., all agents from the worksite side will join the CSP. This threshold value of U_0^B can be derived from (3.13). Same applies to the commuter side.

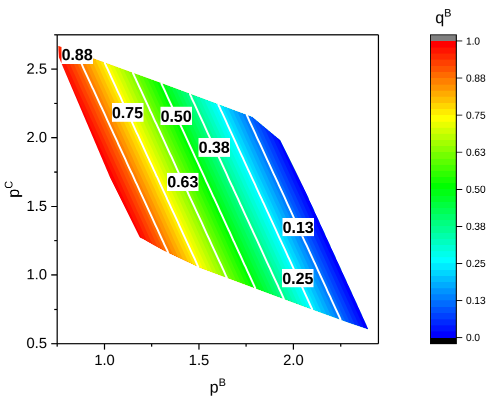
3.3.3 Numerical experiments of monopoly platform

This numerical experiment uses the Starbucks stores in downtown Seattle as an example (Figure 3.1) to explain the participation of the two sides. For this, Starbucks commuters from the U-District (by the University of Washington campus) and the downtown stores (worksites) are regarded as the two sides of the CSP, and maximize the profit of the CSP by selecting the optimal price strategies under different network effects. The baseline parameters for this case study are $U_0^B = 1.9$, $U_0^C = 2.1$, $b^B = 0.5$, $b^C = 0.7$, $t^B = 1.1$, $t^C = 1.5$, $f^B = 0.73$, $f^C = 0.75$. The following part briefly explains how these parameters are selected and how they may relate to this case study. Transit (buses and light rail) plays a key role in commuting from the U-District to Downtown Seattle. The (single ride) bus fare is \$2.25 and the light rail fare is \$2.5. The operational costs of CSP are related to (but not the same as) these fares. It is assumed that in normal cases, the cost of transporting a commuter by the CSP is \$1.5, which can be roughly evenly split into the costs to the commuter side and the worksite side as $f^B = 0.73$, $f^C = 0.75$. This numerical experiment assumes that each Starbucks store needs five workers, resulting in 85 workers (commuters) for the 17 stores. For illustration purposes, this chapter assumes the cross-side benefits (b^B, b^C) and the same-side negative effects (t^B, t^C) are linearly related to these numbers as $b^k = \gamma_k N^l (k \neq l), t^k = \theta_k N^k$. N^k is the total number of agents on side k (i.e., 17 for worksites and 85 for commuters). γ_k, θ_k are the coefficients for the cross-side benefits and same-side effects respectively. The coefficients are set as $b^B = 0.5$, $b^C = 0.7$, $t^B = 1.1$, and $t^C = 1.5$. The relative values between b^B and b^C, and between t^B and t^C are more important than their absolute values. Under the proximate commute setting studied here, commuters value the number of worksites more than worksites value commuters (i.e., $b^C > b^B$), while commuters dislike the participation of other commuters more than that of worksites (i.e., $t^C > t^B$). In addition, a commuter incurs higher benefit when joining the CSP than a worksite (i.e., $U_0^C > U_0^B$) under proximate commute, and thus it is reasonable to set $U_0^C = 2.1 > U_0^B = 1.9$. These parameters are also selected to satisfy conditions **(A0)** - **(A2)** presented above. Note that, in practice, these parameters need to be determined via rigorous investigations about the behavior, attitude, and preference of an agent (commuters or worksites) regarding a CSP, as well as the specific commuting options provided by the CSP. This would help determine, e.g., the actual function forms and the coefficients of the cross-side benefits and same-side negative effects parameters.

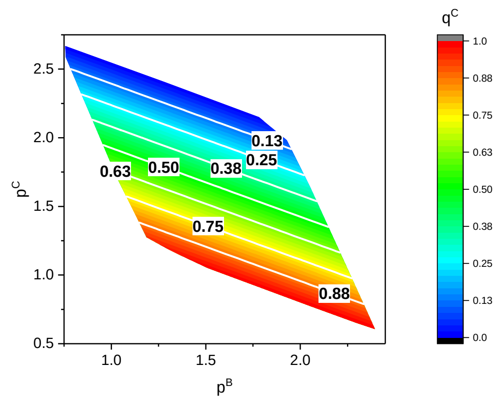
Demand-price relation

According to equation (3.9), demand can be written as a linear function of prices. An example of the demand-price relation is shown in Figure 3.3 using the baseline parameters. Because of the cross-side positive effects, the choices of Starbucks worksites affect the choices of commuters (and vice versa). Therefore, if the CSP increases the price of commuters (p^C), less commuters will choose the CSP, the participation of worksites will decrease as well.

Under the current parameter setting, the price of one side (e.g., p^C) has dominant impact on the participation of the same side (e.g., q^C), and has minor impact on the participation of the other side (e.g., q^B).



(a) q^B



(b) q^C

Figure 3.3: The change of participation as a function of (p^B, p^C)

Cross-side positive effects

In order to explore the cross-side positive effects, b^B and b^C are tested while fixing other parameters to the baseline values. Results in Figure 3.4 show that when the overall cross-side effect $(b^B + b^C)$ increases, the participation (q^B and q^C) from both sides increases, the aggregated price $(p^B + p^C)$ keeps steady, and the CSP's profit (R) increases. Notice that the aggregated price does not change, implying that CSP profit increases as a result of the increased participation from both sides. This tells us that the price allocation, not only the aggregated price, is effective to change the participation and profit of the CSP, indicating the two-sidedness of the CSP. The CSP can achieve higher profit when the two sides value the choices of each other (i.e., larger cross-side effects). It is common for big companies like Starbucks to provide commuting subsidies to improve employees' satisfaction at work. This means that worksites are willing to pay subscription fees to the CSP that can then charge worksites with higher prices and take the commuters side as a loss leader.

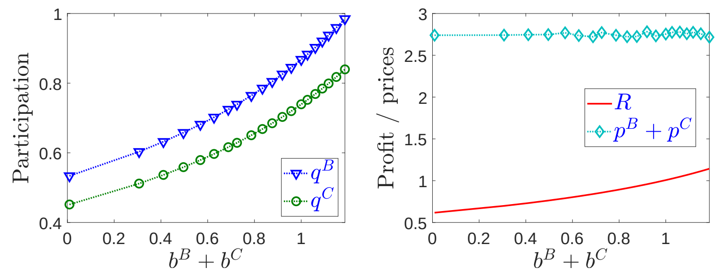
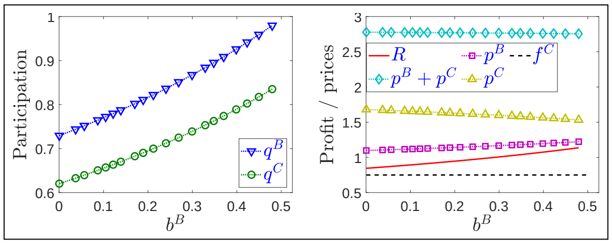


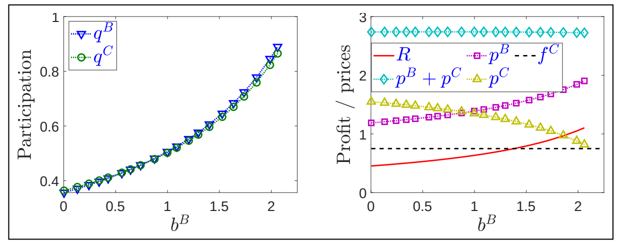
Figure 3.4: Sensitivity analysis of $b^B + b^C$

Since the equilibrium price structure is affected by both b^B and b^C, b^B is tested unilaterally in order to capture the change of the price structure (same can be done if one changes b^C only). Results are shown in Figure 3.5a. When worksites value the number of commuters more (b^B increases), the CSP can make more profit by reducing the price of commuters (thus attracting more participation from both sides) and recouping profit from worksites. By setting lower prices to commuters, more commuters will be willing to join the CSP. Worksites highly value the number of commuters, and as a result they will join CSP even if their price is high.

In Figure 3.5a, increasing cross-side positive effect of worksites reduces the price of commuters. In the monopoly model, all parameters follow the conditions described in **(A1)** and **(A2)**. So under the baseline parameter setting, b^B cannot exceed 0.8. With an attempt to show the "loss leader", t^B, t^C is changed to $t^B = 2, t^C = 2.2$, while keeping the other parameters the same as the baseline values. Figure 3.5b show the results when unilaterally changing b^B with adjusted t^B, t^C. Results show that CSP will subsidize commuters when b^B exceeds 2.2: the price of commuters p^C falls below cost $f^C = 0.75$, indicating the commuter side is the "loss leader". At the same time, the CSP recoups profit by charging worksites much higher prices.



(a) baseline parameters



(b) adjusted parameters, $t^B = 2, t^C = 2.2$

Figure 3.5: Sensitivity analysis of b^B

Same-side negative effects

I use the baseline parameters and only change the value of t^C and t^B to investigate the same-side negative effects. The resulting patterns of participation, prices and profits are presented in Figure 3.6. Intuitively, when t^B becomes smaller, a worksite will experience less disutility if s/he joins the CSP given existing worksites on the CSP, meaning the participation of worksites are less discouraged. The results shown in Figure 3.6a agree with our intuition. When each worksite is less discouraged by fellow worksites on CSP (t^B is small), the participation of worksites increases. Also notice that, as t^B gets smaller, t^C will have weaker impact on q^B. This means that when t^B is small, the number of worksites on the CSP (q^B) is mainly affected by the same-side negative effect of worksites (t^B). On the other hand, if the same-side negative effect of worksites (t^B) is larger, the same-side negative effect of the commuter side (t^C) will have a greater impact on the participation of worksites (q^B). Thus. when t^B is large, the same-side negative effect of both sides will discourage the participation of worksites. Commuters have similar behaviors, as shown in Figure 3.6b.

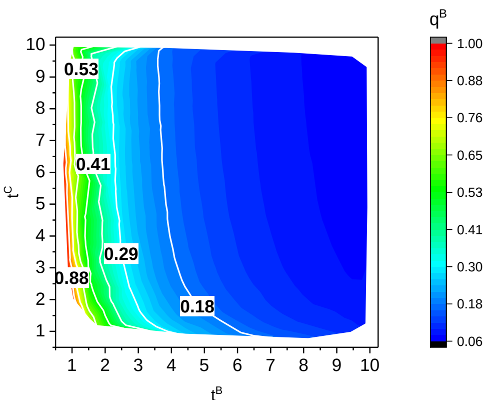
Figure 3.6c and 3.6d show that the price allocation changes very slowly with the same-side negative effects, with a price variation below 0.08. But it is interesting that the price of worksites is generally more sensitive to the same-side negative effect of commuters. When t^B becomes larger given small t^C, the CSP will lose many worksites and a few commuters, as shown in Figure 3.6a and 3.6b. The CSP fails to maintain profits in such case. Therefore, when the same-side negative effect gets too large, more agents from both sides will leave the CSP. The same holds when t^C becomes larger given small t^B. Ultimately, t^B, t^C will affect the CSP profits. Increasing same-side negative effects of either side reduces CSP profits mainly because of the loss of participantion, which follows from the discussion of participation and prices. The contour lines in Figure 3.6e show that for a range of combination of t^B, t^C, i.e., t^B, t^C fall onto the same contour line, the CSP can manage to maintain the same profit level (e.g., with a profit of 0.54). In practice, when the participation of both sides is large for the CSP, it could happen that the commuting services of the CSP may degrade if the number of commuters exceeds the capacity of the service. In such scenario, the potential CSP commuters or worksites will experience higher same-side negative effects, which could lead to dramatic decrease of participation. This implies that when there are increasing participation of the CSP, the CSP needs to control the same-side effects in order to maintain the profit and service quality.

3.4 Duopoly platforms

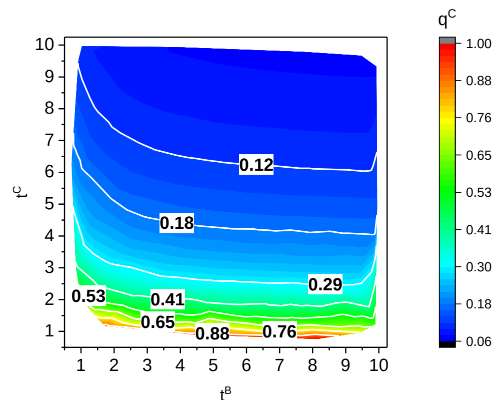
3.4.1 General Model for duopoly platforms

In practice, there are often multiple options for commuting, which indicates there should be multiple CSPs in real life commuting services. Thus, a two-sided market with more than one platform is also valuable for understanding the competition among CSPs. Here the competition between two CSPs is investigated. Worksites and commuters are still regarded as the two sides. The two CSPs are specified as the WF CSP and the NWF CSP, denoted as $i \in \{W, N\}$. Here I first apply the model similar as Armstrong (2006) to the general scenario (i.e. both commuters and worksites multihome), and then discuss two specific scenarios: (i) worksites and commuters singlehome (i.e., a worksite or a commuter can only join one of the two CSPs); (ii) worksites multihome (i.e., a worksite can join both CSPs), commuters singlehome.

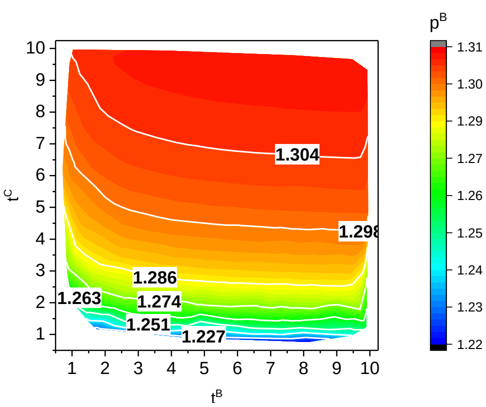
For the general scenario, it is assumed that an agent from group k has an incurred utility



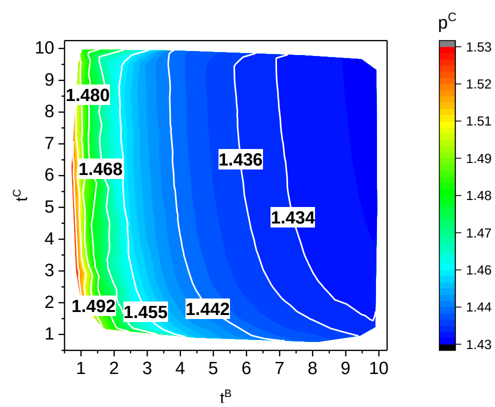
(a) q^B



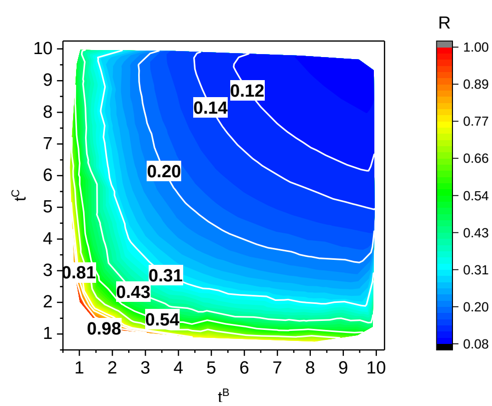
(b) q^C



(c) p^B



(d) p^C



(e) R

Figure 3.6: Sensitivity analysis of t^B, t^C

U_0^k by joining the market. The cost for platform i to serve a group k agent is f_i^k. Capital notation Q^k denotes the number of group k agents joining both CSPs, i.e., multihoming. An agent from group C obtains benefit $\beta_i(q_i^B + Q^B)$ by joining CSP i with β_i the cross-side benefit rate of commuters joining CSP i, as s/he will have the potential to interact with $(q_i^B + Q^B)$ agents from group B. Similarly, an agent from group B obtains cross-side benefit $\alpha_i(q_i^C + Q^C)$ by joining CSP i, with α_i the cross-side benefit rate of worksites on CSP i. CSP i charges a subscription price p_i^k to agents from group k. According to assumption **(f)**, an agent will be worse off when more agents from the same group join the same platform (i.e., the same-side negative effect). Here motivated by the Hotelling model in Section 3.2, I model the same-side negative effect (inconvenience cost) as $t^k(q_i^k + Q^k)$ for agent k choosing platform i. The utility of a worksite can then be expressed as:

$$U_W^B = \underbrace{U_0^B}_{\text{fixed benefit}} - \underbrace{p_W^B}_{\text{subscription price}} - \underbrace{t^B(q_W^B + Q^B)}_{\substack{\text{same-side/inconvenience}\\ \text{costs}}} + \underbrace{\alpha_W(q_W^C + Q^C)}_{\text{cross-side benefits}} \tag{3.15}$$

$$U_N^B = \underbrace{U_0^B}_{\text{fixed benefit}} - \underbrace{p_N^B}_{\text{subscription price}} - \underbrace{t^B(q_N^B + Q^B)}_{\substack{\text{same-side/inconvenience}\\ \text{costs}}} + \underbrace{\alpha_N(q_N^C + Q^C)}_{\text{cross-side benefits}} \tag{3.16}$$

When a worksite multihomes, the utility is:

$$U_{NW}^B = \underbrace{U_0^B}_{\text{fixed benefit}} - \underbrace{(p_W^B + p_N^B)}_{\text{subscription price}} - \underbrace{t^B(q_W^B + q_N^B + Q^B)}_{\substack{\text{same-side/inconvenience}\\ \text{costs}}} + \underbrace{[\alpha_W(q_W^C + Q^C) + \alpha_N(q_N^C + Q^C)]}_{\text{cross-side benefits}} \tag{3.17}$$

Here notice that a multihoming worksite needs to pay both prices of joining the two platforms and has to interact with all agents from the same side (i.e., $q_W^B + q_N^B + Q^B$ for the same-side negative effect). At the same time, it also enjoys interacting with the agents of the other side on both platforms (for the cross-side benefits).

Similarly, the utility of a commuter is given by:

$$U_W^C = \underbrace{U_0^C}_{\text{fixed benefit}} - \underbrace{p_W^C}_{\text{subscription price}} - \underbrace{t^C(q_W^C + Q^C)}_{\substack{\text{same-side/inconvenience}\\ \text{costs}}} + \underbrace{\beta_W(q_W^B + Q^B)}_{\text{cross-side benefits}} \tag{3.18}$$

$$U_N^C = \underbrace{U_0^C}_{\text{fixed benefit}} - \underbrace{p_N^C}_{\text{subscription price}} - \underbrace{t^C(q_N^C + Q^C)}_{\substack{\text{same-side/inconvenience} \\ \text{costs}}} + \underbrace{\beta_N(q_N^B + Q^B)}_{\text{cross-side benefits}} \tag{3.19}$$

$$U_{NW}^C = \underbrace{U_0^C}_{\text{fixed benefit}} - \underbrace{(p_W^C + p_N^C)}_{\text{subscription price}} - \underbrace{t^C(q_W^C + q_N^C + Q^C)}_{\substack{\text{same-side/inconvenience} \\ \text{costs}}} + \underbrace{[\beta_W(q_W^B + Q^B) + \beta_N(q_N^B + Q^B)]}_{\text{cross-side benefits}} \tag{3.20}$$

The profit of platform i is:

$$R_i = \underbrace{(q_i^B + Q^B)(p_i^B - f_i^B)}_{\text{profit collected from worksites}} + \underbrace{(q_i^C + Q^C)(p_i^C - f_i^C)}_{\text{profit collected from commuters}} \quad \forall i \in \{W, N\} \tag{3.21}$$

For this general scenario, the following lemma is introduced:

Lemma 1. *For the general multihome model of the duopoly CSPs, i.e. Equations* (3.15) - (3.21)*, when there exists non-participating agents on both sides, i.e.,* $q_W^B + q_N^B + Q^B < 1$ *and* $q_W^C + q_N^C + Q^C < 1$*, the two platforms are independent or weakly dependent.*

The definitions of “independence” and “weak dependence" and the detailed proof of the lemma can be found in Appendix A.2. With a focus on the interactions and competitions of the two CSPs in the duopoly model, this section focuses on the cases when all agents from either side will join one of the CSPs or both CSPs, i.e., I assume $q_W^B + q_N^B + Q^B = 1$ and $q_W^C + q_N^C + Q^C = 1$ and derive conditions that can lead to those cases. This is also the scenario that was focused in most previous studies; see Armstrong & Wright (2007). This section analyzes duopoly platforms when both sides singlehome. The scenario of worksites multihome and commuters singlehome is presented in Appendix A.4. Also noteworthy is that the results in Lemma 1 only applies to the scenarios when the owners of the platforms aims to maximize their profits. When other objectives are used, the analysis methods in Appendix A.2 may still apply but the results may not be as clean as those in the lemma.

3.4.2 Duopoly platforms when worksites and commuters singlehome

The following conditions ensure that agents from both sides are singlehoming and the equilibrium price is feasible, as formally shown in Theorem 2:

(B1) U_0^B and U_0^C are sufficiently high such that all agents wish to subscribe to at least one CSP;

(B2) $t^B > \alpha_W q_W^C + \alpha_N q_N^C$ and $t^C > \beta_W q_W^B + \beta_N q_N^B$: ensures that the incremental utility from singlehome to multihome is always negative, so that no agent multihomes at any non-negative prices set by the two CSPs;

(B3) $4t^B t^C > (\alpha^+ + \beta^+)^2$: ensures that the profits of the CSPs are positive. $\alpha^+ = \alpha_N + \alpha_W$, $\beta^+ = \beta_N + \beta_W$.

Condition (**B1**) is intuitively understandable, which may be obtained in a similar way as in the Remark at the end of Section 3.3.2. In particular, the equilibrium solution for each scenario of the duopoly model derived in Appendix A.2 may be used for this. The actual derivation can be cumbersome and is omitted here.

Theorem 2. *(i) Under condition (B1)-(B3), an equilibrium exists and all agents singlehome. In other words, (B1)-(B3) are the sufficient conditions for equilibrium existence.*

(ii) For simplicity, this proof only shows the formulation of equilibrium when the two CSPs have the same pricing strategy, $p_W^B = p_N^B$ *and* $p_W^C = p_N^C$. *Half of the agents from each group will join each platform, i.e.,* $q_W^B = q_W^C = q_N^B = q_N^C = 0.5$. *If* $f_W^B + t^B + \psi_W^B > \frac{\beta^+}{2}$ *and* $f_W^C + t^C + \psi_W^C > \frac{\alpha^+}{2}$, *the equilibrium prices are:*

$$p_W^B = f_W^B + t^B - \frac{\beta^+}{2} + \psi_W^B \geq 0 \qquad p_W^C = f_W^C + t^C - \frac{\alpha^+}{2} + \psi_W^C \geq 0 \tag{3.22}$$

where

$$\psi_W^B = \frac{2(\beta^+ - \alpha^+)\beta^- t^B + (\beta^{+2} - 4t^B t^C)\alpha^-}{8t^B t^C - 2\alpha^+\beta^+} \qquad \psi_W^C = \frac{2(\alpha^+ - \beta^+)\alpha^- t^C + (\alpha^{+2} - 4t^B t^C)\beta^-}{8t^B t^C - 2\alpha^+\beta^+} \tag{3.23}$$

Each CSP makes profit:

$$R_i = \frac{t^B + t^C - \frac{\beta^+ + \alpha^+}{2} + \psi_W^B + \psi_W^C}{2} > 0 \quad \forall i \in \{W, N\} \tag{3.24}$$

Proof. The proof of (i), i.e., all agents singlehome and the existence of equilibrium solutions, follows similar logic as lemma 1 in Armstrong & Wright (2007); see **Appendix A.3** for details. Compared with Armstrong & Wright (2007), our model differentiates cross-side benefit on different CSPs, i.e., α_N, α_W for worksites and β_N, β_W for commuters, which is a better description of reality since different CSPs bring about different level of “attraction" between the two groups.

To prove (ii), first, under condition (**B2**), all agents single home as shown in part (i). Notice that $q_N^k + q_W^k = 1$. According to Nash equilibrium, an agent experiences the same utility from either CSP, i.e., $U_W^B = U_N^B$ and $U_W^C = U_N^C$. From equation (3.15)-(3.19) one gets the following relations:

$$U_0^B - p_W^B - t^B x^B + \alpha_W(q_W^C + Q^C) = U_0^B - p_N^B - t^B(1 - x^B) + \alpha_N(q_N^C + Q^C) \tag{3.25}$$

$$U_0^C - p_W^C - t^C x^C + \beta_W(q_W^B + Q^B) = U_0^C - p_N^C - t^C(1 - x^C) + \beta_N(q_N^B + Q^B) \tag{3.26}$$

Substituting $x^k = q_W^k$ in Equation 3.25 and 3.26 yields the participation of each group on a CSP. Set $\alpha^+ = \alpha_N + \alpha_W$, $\beta^+ = \beta_N + \beta_W$, $\alpha^- = \alpha_N - \alpha_W$, $\beta^- = \beta_N - \beta_W$, the number of group k agents joining the WF CSP can be written as:

$$q_W^B = \frac{1}{2} + \frac{\alpha^+(p_N^C - p_W^C) + 2t^C(p_N^B - p_W^B)}{4t^B t^C - \alpha^+\beta^+} - \frac{t^C\alpha^- + \frac{\alpha^+\beta^-}{2}}{4t^B t^C - \alpha^+\beta^+} \tag{3.27}$$

$$q_W^C = \frac{1}{2} + \frac{\beta^+(p_N^B - p_W^B) + 2t^B(p_N^C - p_W^C)}{4t^B t^C - \alpha^+\beta^+} - \frac{t^B\beta^- + \frac{\alpha^-\beta^+}{2}}{4t^B t^C - \alpha^+\beta^+} \tag{3.28}$$

Condition **(B3)** ensures that $4t^B t^C - \alpha^+\beta^+ > 0$ and the profit function is concave with respect to prices. Substitute the demand functions, equation (3.27) and (3.28), into the profit function (3.21). Assume $p_W^B = p_N^B$ and $p_W^C = p_N^C$. Based on the first order condition of profit function (3.21) over prices, one can obtain the optimal prices.

$$\frac{\partial R_i}{\partial p_i^k} = 0 \qquad \forall i \in \{W, N\}, k \in \{B, C\} \tag{3.29}$$

$$s.t. \qquad p_W^B = p_N^B, p_W^C = p_N^C$$

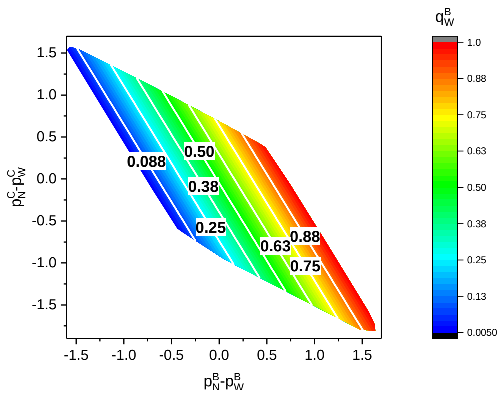
■

Note that I assumed $p_W^B = p_N^B$ and $p_W^C = p_N^C$ in Theorem 2 for more concise expressions of the equilibrium prices. Such assumptions are removed when generating numerical results in section 3.4.2.

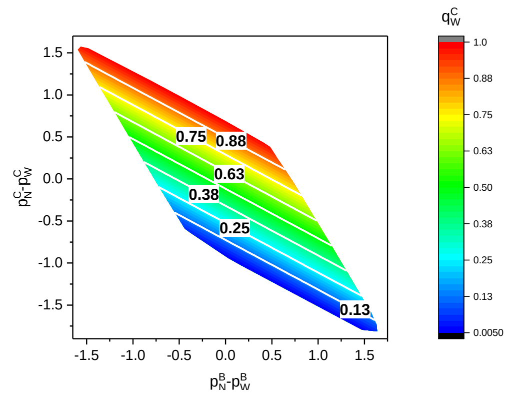
Numerical experiments for duopoly model (singlehoming)

To avoid redundancy, the results for the duopoly model are presented selectively. The same-side effects of the duopoly model is similar to those of the monopoly model, which are omitted here. Also, the analysis for the cross-side effects of the two sides are similar, thus only the results of the worksites are presented. The same Starbucks example from section 3.1 is tested in this subsection. The baseline parameters are set similarly as those of the monopoly model with minor modifications: $U_0^B = 2$, $U_0^C = 2$, $\alpha_N = 0.7$, $\alpha_W = 0.6$, $\beta_N = 0.5$, $\beta_W = 0.8$, $t^B = 1.1$, $t^C = 1.2$, $f_W^B = 0.7$, $f_N^B = 0.73$, $f_W^C = 0.73$, $f_N^C = 0.75$. This parameter setting ensures that worksites experience more cross-side benefits when joining the NWF CSP than the WF CSP ($\alpha_N > \alpha_W$). In practice, this may be due to the reduced cost when a worksite sets fixed working hours for their employees. Commuters obtain more cross-side benefits from the number of worksites on the WF platform ($\beta_W > \beta_N$). This is because commuters tend to value more flexible working hours. Commuters dislike the participation of other commuters more than that of worksites ($t^C > t^B$). For CSPs, the service cost of each commuter (f_i^C) is higher than the per-worksite cost (f_i^B). The WF CSP spends less than the NWF CSP to serve customers from the same group ($f_W^k < f_N^k$). Also the values of the parameters are set to satisfy conditions **(B1)** - **(B3)**, which implies that the conditions of Theorem 2 hold, i.e., all agents singlehome and an equilibrium solution holds for the duopoly model.

According to equations (3.27) and (3.28) , participation (demand) can be expressed as a linear function of $(p_N^C - p_W^C)$ and $(p_N^B - p_W^B)$ when the other parameters are constant. Using the baseline parameters, the demand-price relation is obtained, shown in Figure 3.7. The number of agents from either group on the WF CSP increases when the WF CSP sets lower prices than the NWF CSP. With the current parameters, $p_N^B - p_W^B$ has larger impact than $p_N^C - p_W^C$ on the participation of worksites on the WF CSP (q_W^B). Similarly, $p_N^C - p_W^C$ has larger impact than $p_N^B - p_W^B$ on the participation of commuters on the WF CSP (q_W^C). This indicates that the participation of group k on CSP i is affected, but not at the same level, by the prices of both groups on the two CSPs. Therefore, CSPs should be more strategic to allocate the prices between the two groups in order to be more competitive.



(a) q_W^B

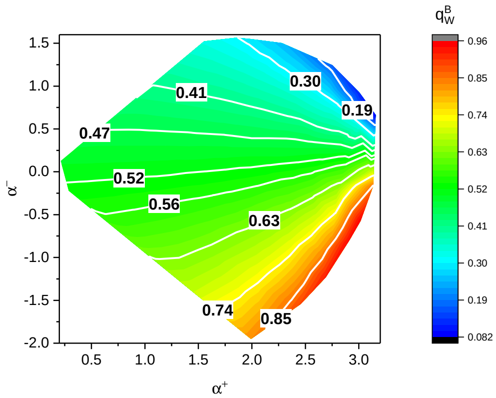


(b) q_W^C

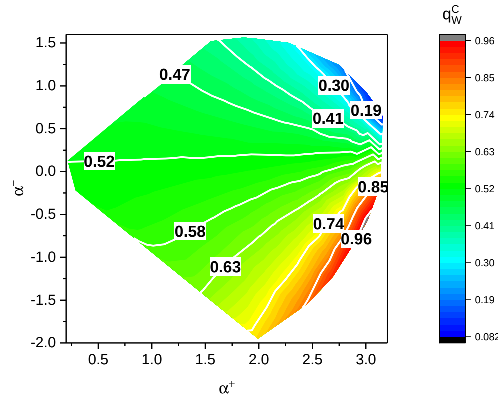
Figure 3.7: The change of participation as a function of $(p_N^B - p_W^B, p_N^C - p_W^C)$

α_N and α_W are the cross-side benefit rates of worksites on the two CSPs. I define $\alpha^+ = \alpha_N + \alpha_W$ as the overall cross-side benefit rate of worksites from the two CSPs, and $\alpha^- = \alpha_N - \alpha_W$ as the difference of cross-side benefit rate of worksites between the NWF CSP and the WF CSP. α^+, α^- are linear combinations of α_N and α_W. When $\alpha^- > 0$, worksites value higher of the commuters on the NWF CSP (the baseline case); when $\alpha^- < 0$, worksites value higher of the commuters on the WF CSP. Here the sensitivity of α^+, α^- are tested in order to see how the cross-side benefits of the two CSPs affect participation, prices and CSP profit. Notice that the impact of β^+ and β^- can be analyzed in a similar way, which is omitted here.

Fixing other parameters as the baseline values, the values of α^+, α^- are changed unilaterally. Figure 3.8a and 3.8b show that when $\alpha^- \approx 0$, the participation of worksites / commuters does not change much with α^+. Under this scenario, worksites/commuters are indifferent



(a) q_W^B



(b) q_W^C

Figure 3.8: The change of participation with α^+, α^-

toward the two CSPs and the overall level of cross-side benefit rate (α^+) marginally affects the participation of worksites/commuters on the WF CSP (or the NWF CSP). When the cross-side benefit rate on the WF CSP exceeds that of the NWF CSP ($\alpha^- < 0$), the WF CSP becomes more attractive to both worksites and commuters. When α^- is negative and fixed, if α^+ increases, the WF CSP becomes more attractive to both sides. Note that in this case, $\alpha_N < \alpha_W$ and α_N increases together with α_W. But even α_N increases, the participation on the NWF CSP is still decreasing (since the participation on the WF CSP keeps increasing), showing that α_W becomes the dominant factor to the change of participation (q_W^B and q_W^C). Similar results can be obtained when α^N is larger (α^- is positive). Therefore, the larger cross-side benefit is dominant in deciding the participation of the two CSPs from both groups.

When α^+, α^- changes, each CSP adjusts its price structure to maximize its profit. The price structure patterns are shown in Figure 3.9. It is shown that the prices of worksites are mainly affected by α^-, while the prices of commuters are mainly affected by α^+. Comparing Figure 3.8a and Figure 3.9a, one notices that for worksites, prices have the similar pattern as that of participation. The reason is that α_W and α_N are the cross-side benefit of worksites, and the relative value of these two parameters (α^-) can be understood as the cross-side benefit discrepancy of the two CSPs from the perspective of worksites. When $\alpha^- < 0$, worksites get higher cross-side benefits on the WF CSP. Knowing that worksites care less about price and care more about the number of commuters on the WF CSP, the WF CSP sets higher price to worksites (bottom-right part of Figure 3.9a). In the meantime, worksites are less attracted to the number of commuters on the NWF CSP. The NWF CSP tries to set lower price to worksites to encourage participation (bottom-right part of Figure 3.9b), but the participation is still low (bottom-right part of Figure 3.8a and 3.8b, the high participation on the WF CSP also means that the participation on the NWF CSP is low). The case when $\alpha^- > 0$ can be understood in a similar way. From commuters' perspective, the overall cross-side effect of worksites (α^+) matters more because α^+ measures how much the worksites value commuters as a group. If the worksites value commuters more, i.e. $\alpha^+ > \beta^+ = 1.3$, CSPs will set lower prices to commuters to attract both groups on board. When α^+ is much larger than β^+, the CSP will even take the commuters group as a loss leader. Remember that in section 3.3.1, the threshold of subsidization/loss leader is defined as $p_i^k < f_i^k$. In Figure 3.9c, $p_W^C < f_W^C = 0.73$ when α^+ exceeds 2.4, in which case the commuter side is the loss leader to the WF CSP. Otherwise, if the worksites value commuters less, i.e. $\alpha^+ < \beta^+ = 1.3$, CSPs will increase the prices of commuters and set lower prices to worksites. This shows that the aggregated cross-side benefit of worksites (α^+) in the duopoly model is similar to that of worksites (b^B) in the monopoly model (also β^+ is similar to b^C). Hence, the price patterns in Figure 3.9 are consistent with the findings in the monopoly model.

Figure 3.10 shows how profits on the two CSPs change with α^+, α^-. When $\alpha^- < 0$ ($\alpha_W > \alpha_N$), the WF CSP can make higher profit. Similarly, the NWF CSP tends to make higher profit when $\alpha^- > 0$ ($\alpha_N > \alpha_W$). In contrast, large overall cross-side benefit rate (α^+) is not always desired by CSPs. When α^- is fixed, profits on both CSPs decrease with α^+. Referring to Figure 3.8 and 3.9, the following part explains why profits decrease with α^+. Take the WF CSP as an example, when $\alpha^- > 0$ and α^+ is large (the top-right part of Figure 3.10a and 3.10b), the WF CSP charges low prices for both groups (the top-right part of

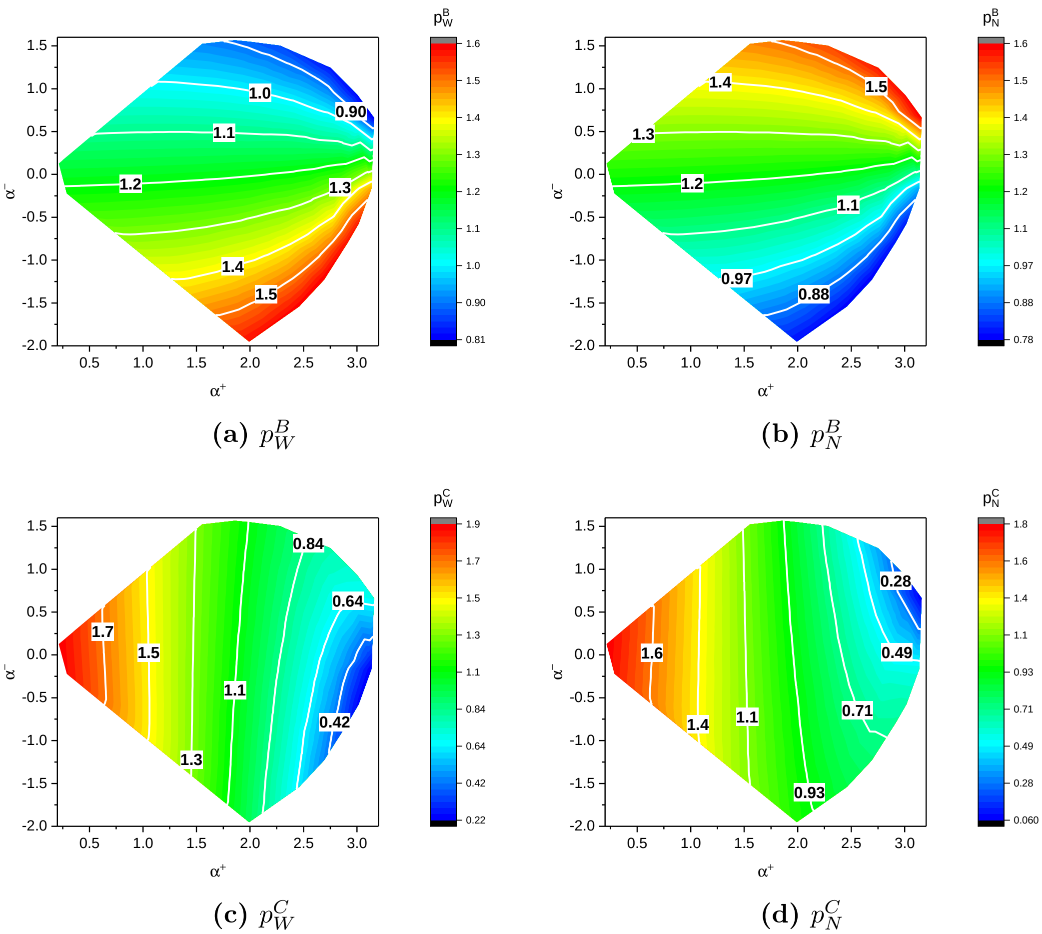
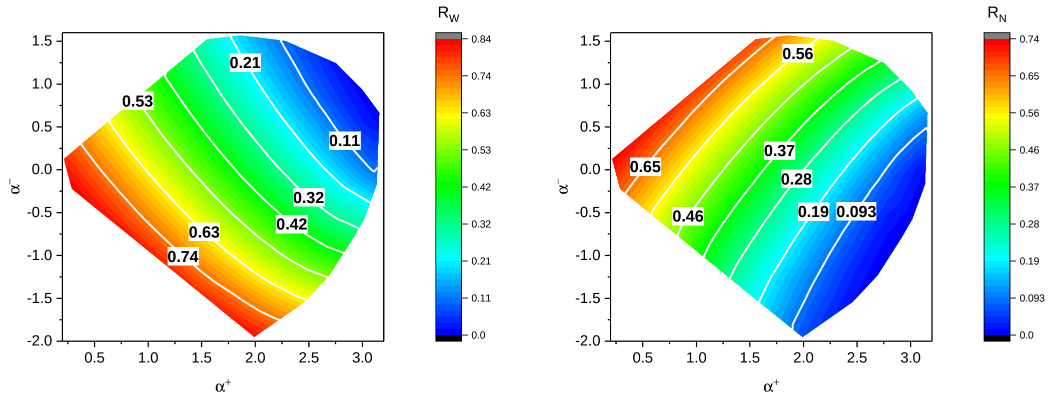


Figure 3.9: The change of prices with α^+, α^-

Figure 3.9a and 3.9c), but few agents from either group join WF CSP (the top-right part of Figure 3.8a and 3.8b). Thus, the WF CSP gains low profit because of the low participation and low prices. Consider another situation, when $\alpha^- < 0$ and α^+ is large (the bottom-right part of Figure 3.10a and 3.10b). The WF CSP can attract lots of participants from both groups (the bottom-right part of Figure 3.8a and 3.8b), and even takes advantage of high α_W to charge worksites high price (the bottom-right part of Figure 3.9a), but it still makes low profit. The reason is that, the WF CSP subsidizes commuters too much and fails to recoup enough profits from worksites. From Figure 3.9c, one can see that in this case the price of commuters decreases with the overall cross-side benefit (α^+). The commuters are charged with only 0.42 ($0.42 = p_W^C < f_W^C = 0.73$) when $\alpha^+ \approx 2.7$, which implies more commuters on WF CSP means more profit loss. Similar analysis can also applies to the NWF CSP. The bottom-right part of Figure 3.10a and 3.10b shows that the profit of NWF CSP is lower than that of WF CSP. This means that large α^+ intensifies the competition between the two CSPs. One of the CSP attracts lots of participants by making use of cross-side network effects and over subsidizing, while the other platform fails to attract consumers even if it sets very low prices. Neither of the CSPs makes good profits in this case. The profit patterns also distinguish the CSPs from the one-sided market. Even the highest participation from both sides (the bottom-right part of Figure 3.8a, 3.8b and 3.10a) cannot ensure high profit on the WF CSP, although its profit is higher than that of the NWF CSP. The most desirable cases for the WF CSP are on the bottom-left part of Figure 3.10a, when the CSP charges medium prices to worksites and commuters without taking any of them as loss leaders (the bottom-left part of Figure 3.9a, 3.9c), and is also able to have more than 50% of the customer share from the both sides (the bottom-left part of Figure 3.8a, 3.8b).

3.5 Modeling Commuters' Home and Worksite Locations

So far the actual locations of commuters' homes and worksites have not been considered. In other words, all commuters' homes are aggregated into one location and all worksites into one location. In this section, I extend the above two-sided market analysis method to



(a) R_W **(b)** R_N

Figure 3.10: The change of CSP profit with α^+, α^-

consider the locations of commuters' homes and worksites. "Location" here does not mean the exact address of a single home or worksite; rather it is a geographic area that contains multiple homes of commuters (e.g., a neighborhood or community) or multiple employers (e.g., a few blocks in the downtown area). The ability of modeling home/worksite locations can enable us to analyze how specific location characteristics (e.g. distance or commuting services) may impact the choices of CSPs and/or worksites. This section assumes there are multiple CSPs; for simplicity, the case when both commuters and worksites singlehome is studied. That is, a commuter can choose one CSP for his/her commuting, or do not choose any CSP. Similarly, a worksite can make the choice of joining a CSP or not.

Additional notations are defined to denote home and worksite locations:

m label of home locations of commuters, $m \in \{1, 2, \ldots, M\}$.

n label of worksites, $n \in \{1, 2, \ldots, N\}$

Also notice that, as in previous sections, i denotes the label of CSPs, $i \in \{1, 2, \ldots, I\}$. Some of the notation in previous sections are redefined to capture different home/worksite locations, and different CSPs. Capital letters U, U_0, B, T, P, Q, F now denote the utility, initial utility, cross-side benefit effect, same-side negative effect, price, demand, and cost for the commuter side, respectively. Superscript (m, n) indicates the home and worksite pair (m, n), while subscript i indicates CSP i. For example, $B_i^{m,n}$ denotes the cross-side benefit of commuters from home location m who choose the commuting service provided by CSP i to go to worksite n.

Similarly, the lower case variables u, u_0, b, t, p, q, f denote the utility, initial utility, cross-side benefit effect, same-side negative effect, price, demand, and cost for the worksite side, respectively. Notice that worksites do not distinguish employees from different home locations - this is the essential idea of Proximate Commute. Hence I only use i to indicate CSP i and n to indicate worksite n for worksite related parameters and variables. For example, q_i^n denotes the demand quantify of worksites at location n that join CSP i. Note also that here P, Q, U, p, q, u are variables of the model while the others are parameters of the model.

The following sections, two cases are present: the basic case and the general case. The basic case is the simplest case with one home location, two worksite locations, and one CSP. For this case, analytical results are derived to help illustrate how considering the locations of worksites may impact the choices of commuters, worksites, and the CSP. The general case considers multiple home and worksite locations, and multiple CSPs. For this, due to the complexity of the model, it is difficult to derive any analytical results. Instead a general modeling framework is proposed, based on the generalized Nash game (Facchinei & Pang, 2009), to help conduct analysis and compute numerical results to assess the choices of commuters, worksites, and CSPs in general.

3.5.1 Basic case

Since the basic case only considers one home location and one CSP, indices i and m are omitted from the notation. For example, B^n denotes the cross-side benefit effect when commuters choose the CSP to go to worksites at location n. In this basic case, commuters or worksites can choose to join or not join the CSP, as shown in Figure 3.11. The two worksite locations are often distinguished (when commuting is concerned) by distances from the home location, available commuting modes (e.g., if worksite location 2 is closer to the home location, a commuter may choose to bike, while for a worksite that is farther away, the commuter has to consider driving, carpooling, or taking the transit), etc. Therefore, commuters going to different worksite locations may experience different same-side effects, denoted as T^n. Similarly, the costs for the CSP to transport a commuter also depends on the actual worksite locations, implying that the cost of joining the CSP for either the commuter side or the worksite side also depends on the actual location. This is represented by F^n and f^n for commuters and worksites respectively. For other parameters, they may not be significantly impacted by the actual commuting experiences of commuters. To present clean results, for this basic case, I assume $t^1 = t^2 = t, B^1 = B^2 = B, b^1 = b^2 = b$. Under the similar assumption, $U_0^1 = U_0^2 = U_0, u_0^1 = u_0^2 = u_0$. Figure 3.11 also shows the values of these parameters used in the numerical experiments later in this subsection. These parameters are based on those in the previous sections (e.g., Section 3.3.3), which are adjusted to show that worksite location 2 is closer to the home location (and thus $F^2 < F^1, T^2 < T^1, f^2 < f^1$). Notice that these assumptions are relaxed in the modeling framework for the general case in the next subsection.

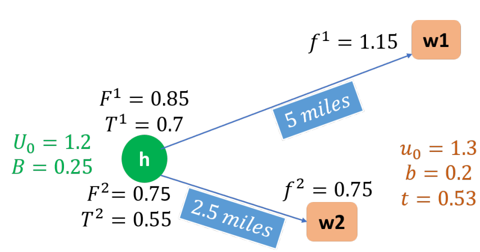


Figure 3.11: Network for the basic case: one home location and two worksite locations

Similar to those discussed in the previous sections, the utility functions for commuters and worksites are:

$$U^n = U_0 - P^n + Bq^n - T^nQ^n \quad \forall n \in 1,2 \tag{3.30}$$

$$u^n = u_0 - p^n + bQ^n - tq^n \quad \forall n \in 1,2 \tag{3.31}$$

$$\tag{3.32}$$

The profit can be expressed as:

$$R = \sum_n [(P^n - F^n)Q^n + (p^n - f^n)q^n] \tag{3.33}$$

At equilibrium, commuters (worksites) are indifferent of joining or not joining the CSP at equilibrium, indicating the utility of two choices are the same. Hence $U^1 = U^2 = \tilde{U}$, $u^1 = u^2 = \tilde{u}$, where $\tilde{U}$ ($\tilde{u}$) is a constant representing the utility of commuters (worksites) not joining the CSP. Since an initial utility (U_0 or u_0) is already included in the utility function, one can treat $\tilde{U} = 0, \tilde{u} = 0$ for simplicity. In summary:

$$\begin{cases} U_0 - P^1 + Bq^1 - T^1Q^1 & = 0 \\ U_0 - P^2 + Bq^2 - T^2Q^2 & = 0 \\ u_0 - p^1 + bQ^1 - tq^1 & = 0 \\ u_0 - p^2 + bQ^2 - tq^2 & = 0 \end{cases} \tag{3.34}$$

In the system of linear equations in (3.34), the expression of P^1, P^2, p^1, p^2 are written as:

$$P^n = U_0 + Bq^n - T^nQ^n \quad \text{and} \quad p^n = u_0 + bQ^n - tq^n \tag{3.35}$$

The profit function can be rewritten as a function of demand quantities, $R(Q^1, Q^2, q^1, q^2)$, by substituting P^n and p^n in (3.33) with (3.35). The profit maximization problem can then be formulated as an NLP.

$$\max_{Q^1,Q^1,q^1,q^2} R = \sum_n [(U_0 + Bq^n - T^nQ^n - F^n)Q^n + (u_0 + bQ^n - tq^n - f^n)q^n] \tag{3.36}$$

$$\text{s.t.} \quad Q^1 + Q^2 \leq 1 \tag{3.37}$$

$$0 \leq Q^1, Q^2, q^1, q^2 \leq 1 \tag{3.38}$$

The following theorem summarizes the proprieties and quantifies the solutions of this base case.

Theorem 3. *The basic case has the following properties:*

(a) To ensure the profit is a concave function with respect to Q^1, Q^2, q^1, q^2*, one needs* $4tT^n \geq (b+B)^2, \quad \forall n \in \{1,2\}$

(b) When $Q^1 + Q^2 < 1$*, i.e., there are commuters who do not join the CSP to go to any of the two worksites, the demand quantities can be derived as:*

$$Q^n = \frac{2t(U_0 - F^n) + (B+b)(u_0 - f^n)}{4tT^n - (b+B)^2} \qquad \forall n \in \{1,2\} \tag{3.39}$$

$$q^n = \frac{(B+b)(U_0 - F^n) + 2T^n(u_0 - f^n)}{4tT^n - (b+B)^2} \qquad \forall n \in \{1,2\} \tag{3.40}$$

(c) When $Q^1 + Q^2 = 1$*, i.e., all commuters join either of the two worksites, the demand quantities are:*

$$Q^n = \frac{-(b+B)^2 + (-f^n + f^\ell) + 2t(-F^n + F^\ell + 2T^\ell)}{4t(T^1 + T^2) - 2(b+B)^2} \qquad \forall n, \ell \in \{1,2\}, n \neq \ell \tag{3.41}$$

$$q^n = \frac{-(b+B)^3 + (f^1 + f^2 - 2u_0)(b+B)^2 + 2t(-F^n + F^\ell + 2T^\ell)(b+B) + 4t(u_0 - f^n)(T^1 + T^2)}{8t^2(T^1 + T^2) - 4t(b+B)^2}$$
$$\forall n, \ell \in \{1,2\}, n \neq \ell \tag{3.42}$$

The proof of **Theorem 3** can be done straightforwardly: for (a), the Hessian matrix of the profit function (3.35) can be analyzed to derive the condition in (a); for (b) and (c), the KKT conditions of the NLP can be analyzed, and the above solutions can be derived for $Q^1 + Q^2 < 1$ and $Q^1 + Q^2 = 1$ respectively. Details are omitted here.

For the above theorem, (b) and (c) merit further discussions. First, the corresponding prices for both cases can be readily derived based on (3.35). For (b), it is easily seen that (Q^1, q^1, P^1, p^1) are independent from (Q^2, q^2, P^2, p^2): the former are only determined by the common parameters and parameters related to worksite location 1, while the latter are determined by common parameters and those related to location 2. That is, when there are commuters who do not join the CSP to commute to the worksites, the decisions of all agents (commuters, worksites, the CSP), regarding which worksite location to select (for commuters), how to participate (for commuters and worksites), or how to price the two sides (for the CSP), are independent. This is similar to the independence discussion for selecting CSPs in the duopoly case in **Appendix A.2**, with the distinct difference that this subsection deals with commuters selecting one of the two worksite locations (not CSPs). In (c), however, when all commuters join the CSP to commute to their worksites, agents' decisions are dependent on each other as reflected by the fact that the decisions related to one worksite location are also impacted by the parameters of the other location.

Numerical results

The baseline parameters for the network are: $F^1 = 0.85, F^2 = 0.75, T^1 = 0.7, T^2 = 0.55, f^1 = 1.15, f^2 = 0.75, B = 0.25, b = 0.2, U_0 = 1.2, u_0 = 1.3, t = 0.53$. There are two commuting trips in the network, denoted as $h \to w1$ and $h \to w2$. Worksite location 2 is closer to the home than location 1. The cost parameters F^n, f^n are the operation costs, which are related to the distance of the commuting trip. Trip $h \to w1$ is 5 miles, longer then trip $h \to w2$. Thus the transportation cost of $h \to w1$ is set as \$2, which is split into the commuter group and the worksite group so that $F^1 = 0.95, f^1 = 1.15$. The transporting cost of $h \to w2$ is set as \$1.5, which is split into the two groups so that $F^2 = 0.75, f^2 = 0.75$. Under the baseline parameter setting, the same-side negative effect on trip $h \to w1$ is larger than $h \to w2$ ($T^1 > T^2$). For shorter trips, there may be more alternative mode to choose from. Thus the same-side negative effect is lower for trip $h \to w2$. Since commuters prefer more the proximate commute policy, the cross-side benefit of commuters is set to be higher than that of the worksites ($B > b$). Again, it's nontrivial to know the exact values of these network effect parameters, and the numerical experiments emphasis more on their relative values for illustration purposes.

Under the above parameter setting, sensitivity analyses are conducted to study how the

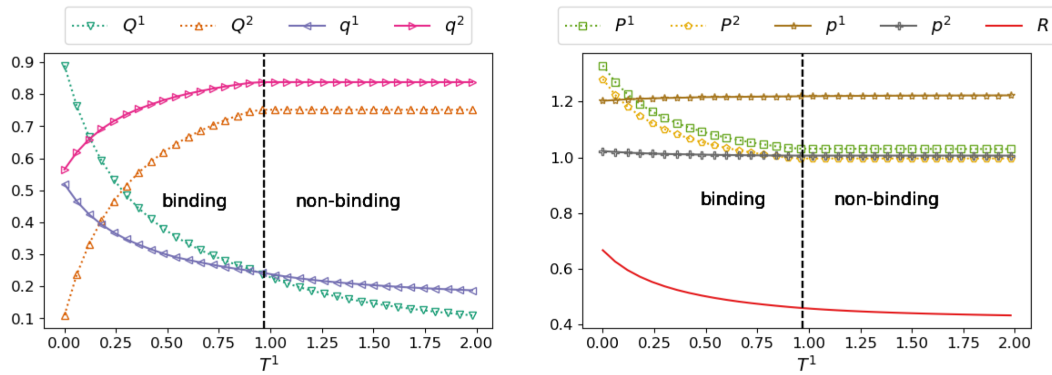


Figure 3.12: Sensitivity analysis of T^1

equilibrium solution responds to different parameters by unilaterally changing one parameter at a time. The sensitivity analysis of the same-side negative effect T^1 is presented in Figure 3.12; similar analyses can be done for other parameters and are omitted here. First $T^1 = 0.97$ (shown as the vertical dashed line in Figure 3.12) delineates whether the constraint $Q^1 + Q^2 = 1$ is active: it is active (binding) on the left side (i.e., $T^1 < 0.97$) and not active (not binding) on the right hand side (i.e., $T^1 > 0.97$). This makes sense as when T^1 is relatively small (the left side of the vertical dashed line), both worksite locations are somewhat attractive and all agents will join the CSP to commute to worksites in one of the locations. In this case, as T^1 increases, worksite location 1 becomes less attractive (i.e. commuter utility decreases). This leads to reduced commuter choices of location 1 (Q^1) which subsequently reduces the participation of worksites in location 1 (q^1) as well due to the two-sidedness of the CSP. As Q^1 decreases, Q^2 increases accordingly and $Q^1 + Q^2 = 1$ always holds. The increase of Q^2 also motivates more worksite at location 2 to join the CSP, i.e., q^2 also increases. This verifies that when $Q^1 + Q^2 = 1$ holds, i.e., all commuters join the CSP, agents' decisions at the two locations depend on each other. This dependence breaks on the right side of the vertical dashed line. In this case, $Q^1 + Q^2 < 1$ and agents' decisions regarding the two locations are independent: while larger T^1 will further reduce the participation of commuters and worksites with respect to worksite location 1 (i.e., both

Q^1 and q^1 decrease), their participation with respect to worksite location 2 does not change at all. The change patterns of the prices are similar, while the profit of the CSP reduces continuously when T^1 increases due to the reduced participation. Also noteworthy is that T^1 has more significant impacts to the prices of the commuter side (P^1, P^2) than to the prices of the worksite side (p^1, p^2). This is similar to the findings of the monopoly and duopoly cases.

3.5.2 General case

The basic case is extended to the general case with multiple home locations, multiple worksite locations, and multiple CSPs. Such a general situation has three types of players: commuters, worksites, and CSPs. The utility of each player can be modeled as shown previously in this dissertation. And the players' decisions can be modeled by equilibrium of the utility (for commuters and worksties) or maximizing the profit (for CSPs). This research proposes to formulate the decisions of all players simultaneously using the generalized Nash game (Facchinei & Pang, 2009), for which a general equilibrium may be reached (Ban *et al.*, 2019; Fan *et al.*, 2021). The optimization of all players is interconnected by endogenous and exogenous variables. Next the utility and decision (optimization) model of the three players are presented.

Commuters decision model: As shown previously, the utility of commuters from home location m to worksite n via CSP i can be modeled as:

$$U_i^{m,n} = U_{0,i}^{m,n} + B_i^{m,n} q_i^n - T_i^{m,n} Q_i^{m,n} - P_i^{m,n}. \tag{3.43}$$

The demand quantify (participation) of commuters who reside at home location m choose to go to worksite n via CSP i can be determined via the following complementarity condition:

$$0 \le U^{*,m} - U_i^{m,n} \perp Q_i^{m,n} \ge 0, \tag{3.44}$$

$$0 \le (1 - \sum_{i,n} Q_i^{m,n}) \perp U^{*m} \ge 0. \tag{3.45}$$

Here $\perp$ denotes "perpendicular", i.e., for two vectors x, y, $x \perp y \Leftrightarrow x^T y = 0$. $U^{*,m}$ denotes the maximum utility commuters from home location m may obtain by choosing any worksite and any CSP. The first complementarity condition states that if $Q_i^{m,n} > 0$, i.e., worksite n and CSP i are selected, the utility of making such a choice must be the maximum utility, i.e., $U_i^{m,n} = U^{*,m}$. On the other hand, if the utility of choosing worksite n via CSP i is strictly less than the maximum utility, i.e., $U_i^{m,n} < U^{*,m}$, which yields $Q_i^{m,n} = 0$, i.e., no commuter will go to worksite n via CSP i. This condition guarantees that if any commuter from home location m chooses a worksite n on any CSP, the utlity of that choice has to be the maximum utility. The second complementarity condition states that if there are commuters from home location m who do not choose to go to any worksite via any CSP, i.e., $\sum_{i,n} Q_i^{m,n} < 1$, which yields $U^{*,m} = 0$, i.e., the maximum utility of commuters from home location m is zero. This makes sense according to the assumption that those who do not use any CSP have zero utility. On the other hand, if all commuters decide to get to their worksites via the CSPs, one has $\sum_{i,n} Q_i^{m,n} = 1$, which implies that $U^{*,m} \geq 0$. That is, the maximum utility could be larger than zero in this case. Also noteworthy is that in the above complementarity formulation for commuters' decisions, $Q_i^{m,n}$ and $U^{*,m}$ are the endogeneous decision variables, while q_i^n and $P_i^{m,n}$ are considered as exogeneous variables to commuters' decisions.

Worksites decision model: First the utility of worksites at location n that joins CSP i can be modeled as (notice that a worksite does not distinguish employees based on their home locations):

$$u_i^n = u_{0,i}^n + b_i^n \sum_m Q_i^{m,n} - t_i^n q_i^n - p_i^n. \tag{3.46}$$

Similar to the decision model of commuters, the worksites' decision model can be expressed as the following complementarity conditions:

$$0 \leq u^{*,n} - u_i^n \perp q_i^n \geq 0, \tag{3.47}$$

$$0 \leq 1 - \sum_i q_i^n \perp u^{*,n} \geq 0. \tag{3.48}$$

Here $u^{*,n}$ denotes the maximum utility of worksites at location n, similar to $U^{*,m}$ defined for commuters. The rational of the above complementarity conditions as the decision model for worksties are also similar to that for the commuters; detailed explanations are omitted here. Note that for the worksite decision model above, $u^{*,n}$ and q_i^n are the endogenous decision variables, while $Q_i^{m,n}$ and p_i^n are exogenous variables.

CSP decision model: For CSPs, their decisions can be modeled by profit maximization. Assuming one entity owns all the CSPs, the profit maximization model for all the CSPs of the entity can be expressed as

$$\max_{P_i^{m,n}, p_i^n} R = \sum_{i,m,n} (P_i^{m,n} - F_i^{m,n}) Q_i^{m,n} + \sum_{i,n} (p_i^n - f_i^n) q_i^n \tag{3.49}$$

$$s.t. \qquad P_i^{m,n} \geq 0, p_i^n \geq 0 \tag{3.50}$$

For the above CSP decision model, the prices $P_i^{m,n}$ and p_i^n are the endogenous decision variables, while $Q_i^{m,n}$ and q_i^n are exogenous variables. I simply assume the only constraint for prices are the nonnegativity constraints. The actual constraints may be more complex for real application, for which the modeling method proposed here can still apply. Using (3.43) and (3.46), one can derive $Q_i^{m,n}$ and q_i^n using $P_i^{m,n}$ and p_i^n. Then the above maximization problem can be reformulated using its KKT conditions as:

$$0 \leq 2P_i^{m,n} + U_i^{m,n} - F_i^{m,n} - U_{0,i}^{m,n} - B_i^{m,n} q_i^n \perp P_i^{m,n} \geq 0 \tag{3.51}$$

$$0 \leq 2p_i^n + u_i^n - u_{0,i}^n - f_i^n - b_i^n (\sum_m Q_i^{m,n}) \perp p_i^n \geq 0 \tag{3.52}$$

These three decision models are all expressed by complementarity conditions. They depend on each other as shown by the exogenous variables identified above. They collectively form a GNEP. Since this paper focuses mainly on presenting the concept of the many-to-many CSP, the formulation especially the constraints of the three decision models are relatively simple. It is easy to see (e.g., using the results in Chapter 1 of Facchinei & Pang (2003)) that each of the three decision models is equivalent to a variational inequality (VI) problem. We omit the details of showing this equivalence here but explicitly give the constraint sets of the three decision models: for commuters at home location m: $K^m = \{Q_i^{m,n} | 1 - \sum_{i,n} Q_i^{m,n} \geq 0, Q_i^{m,n} \geq 0\}$, for worksties at location n: $K^n = \{q_i^n | 1 - \sum_i q_i^n \geq 0, q_i^n \geq 0\}$, and for the entity that owns the CSPs, $K^{CSP} = \{P_i^{m,n}, p_i^n | P_i^{m,n} \geq 0, p_i^n \geq 0\}$. Clearly the constraint set of the decision model of each player only depends on the variables of the player, i.e., the above GNEP model has disjoint constraint sets among all players. This is a special case of the jointly convex GNEP, originally studied in Rosen (1965) and later formally defined and discussed more comprehensively in Facchinei and Kanzow (2007). A jointly convex GNEP has an important mathematical property, i.e., it has an equivalent VI reformulation (Facchinei & Kanzow, 2007). This applies to the three decision models presented here. For brevity, we omit the details of such an equivalent VI. It suffices to say that the mathematical property (especially the existence and uniqueness of the solutions) and solution method of the proposed problem can then be studied by following the VI literature (e.g., Facchinei & Pang (2003)), which has been extensively applied and analyzed in the transportation literature. In particular, the model can be solved as a Mixed Complementarity Problem (MCP) using the PATH solver in GAMS. We omit the details here with a note: since the function of the VI is linear and thus monotone but not strongly monotone, the proposed GNEP model does not have a unique solution, for which certain conditions for local uniqueness may be derived.

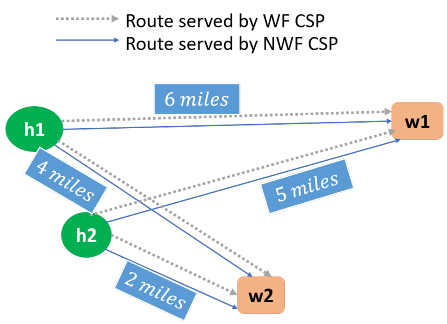
However, if more real-world constraints and conditions are considered (especially those between the commuters and worksites, such as shared resources, budget/incentive limits, etc.), common constraints may exist among commuters, worksites, and the platforms. When this happens, one needs to further investigate if these common constraints are jointly convex. If so, the equivalent VI reformulation still exists and the above analysis and solution approaches also apply. Yang & Ban (2017) provides more easy-to-check conditions for verifying the jointly convex property of a GNEP for transportation network modeling problems with common constraints. However, if the model turns out to be not jointly convex, such an equivalent VI reformulation may not exist. Instead a general (i.e., not jointly convex) GNEP is equivalent to a Quasi-VI (QVI, see Facchinei & Pang (2003)) for which the research on its mathematical properties and solution method is still at its beginning stage. For such general GNEPs, solution existence and uniqueness conditions have to be established by exploring the special structure of the GNEP. In particular, there is no general uniqueness theory for such general GNEPs (or QVIs).

We also note that for simplicity, we only consider both commuters and worksites single-home. This can be readily extended to model the multihoming case by properly revising the utility functions of commuters and worksites and the profit function of the CSPs, similar to those in Section 3.4.1. We also assume an entity owns all CSPs. If there are multiple entities, each of which owns different CSPs, we could model each entity as a separate player and model its decisions as the same profit maximization model as we have shown here. In those extensions, it is more likely that the resulting model will be a general GNEP, which calls for further investigations.

Numerical results

The above modeling framework can be applied to model and solve the general case with multiple home/worksite locations and multiple CSPs. Here for illustration purposes, the results of 2 home locations, 2 worksite locations, and 2 CSPs are presented (Figure 3.13). To be consistent with the new notation, I use $i = 1$ to denote the WF CSP, and $i = 2$ to denote the NWF CSP. The parameters of this case study are set similarly as those for the simple case above, which also follow the same setting between the WF CSP and the NWF CSP.

The general equilibrium model for this case study can be formulated and solved very easily. The sensitivity analysis are conducted on how parameter changes impact the choices of commuters of the worksite locations, participation of the two sides on the CSP, prices, and the profit. There are much more parameters even for this case study with 2 home locations, 2 worksite locations, and 2 CSPs (see Table 3.1), compared with the case studies covered so far. Therefore, for brevity, this section only focuses on illustrating the impacts of the cross-side benefit effects and the same-side negative effects. In particular, the analyses show how the participation (demand) changes with respect to $B_1^{1,1}, B_1^{1,2}$ and $T_1^{1,1}, T_1^{1,2}$, i.e., the cross-side benefits effects and the same-side negative effects, respectively, of commuters from home location 1 to choose worksite location 1 and 2 on the WF CSP. Prices and the platform profit can be analyzed in a similar way, which are omitted here.



	WF CSP (i=1)	NWF CSP (i=2)
fixed cost	$F_1^{1,1} = 0.96, F_1^{1,2} = 0.65$ $F_1^{2,1} = 0.86, F_1^{2,2} = 0.45$	$F_2^{1,1} = 1.05, F_2^{1,2} = 0.78$ $F_2^{2,1} = 0.95, F_2^{2,2} = 0.58$
	$f_1^1 = 1.14, f_1^2 = 1.05$	$f_2^1 = 1.35, f_2^2 = 1.22$
same-side effect	$T_1^{1,1} = 0.7, T_1^{1,2} = 0.6$ $T_1^{2,1} = 0.65, T_1^{2,2} = 0.5$	$T_2^{1,1} = 0.72, T_2^{1,2} = 0.63$ $T_2^{2,1} = 0.67, T_2^{2,2} = 0.53$
	$t_1^1 = 0.7, t_1^2 = 0.6$	$t_2^1 = 0.72, t_2^2 = 0.63$
cross-side effect	$B_1^{1,1} = 0.35, B_1^{1,2} = 0.33$ $B_1^{2,1} = 0.37, B_1^{2,2} = 0.38$	$B_2^{1,1} = 0.33, B_2^{1,2} = 0.3$ $B_2^{2,1} = 0.35, B_2^{2,2} = 0.34$
	$b_1^1 = 0.3, b_1^2 = 0.28$	$b_2^1 = 0.33, b_2^2 = 0.3$
initial utility	$U_{0,1}^{1,1} = 1.75, U_{0,1}^{1,2} = 1.75$ $U_{0,1}^{2,1} = 1.7, U_{0,1}^{2,2} = 1.7$	$U_{0,2}^{1,1} = 1.7, U_{0,2}^{1,2} = 1.7$ $U_{0,2}^{2,1} = 1.75, U_{0,2}^{2,2} = 1.75$
	$u_{0,1}^1 = 1.65, u_{0,1}^2 = 1.65$	$u_{0,2}^1 = 1.7, u_{0,2}^2 = 1.7$

Figure 3.13 & Table 3.1: The general case: network and baseline parameters

Figure 3.14 shows, on the WF CSP (i.e., the same CSP as the four parameters studied here), how the participation of the commuter side and the worksite side changes with $B_1^{1,1}$ and $B_1^{1,2}$. One can see that, when $B_1^{1,1}$ increases, the choice of commuters from home location 1 to worksite location 1 using the WF CSP (i.e., $Q_1^{1,1}$) increases as well. This is reasonable as increased cross-side benefit motivates more commuters to choose the worksite location using the CSP. As a result, worksite location 2 appears less attractive to commuters with the increase of $B_1^{1,1}$, leading to reduced $Q_1^{1,2}$, albeit at a much less significant level. For the worksite side, as $B_1^{1,1}$ increases, participation of worksites at location 1 using the WF CSP (i.e., q_1^1) also increases due to the increase of $Q_1^{1,1}$, while at worksite location 2, worksite participation (i.e., q_1^2) has little or no change. The increase of q_1^1 and no change of q_1^2 also impact the choices of commuters from home location 2 in terms of worksite locations: $Q_1^{2,1}$ increases (at a much less significant level compared with $Q_1^{1,1}$), while $Q_1^{2,2}$ does not change much. The impacts of $B_1^{1,2}$ to all the demand variables in the figure are just opposite to those of $B_1^{1,1}$, which are not repeated here.

On the alternative NWF CSP, the impacts of $B_1^{1,1}, B_1^{1,2}$ to participation are almost opposite to those of the WF CSP, as shown in Figure 3.15, with some noticeable differences. Again this paragraph explains the impacts of $B_1^{1,1}$, while the impacts of $B_1^{1,2}$ can be done in a similar way. First due to the competition of the two CSPs, the increase of $B_1^{1,1}$, i.e., the cross-side benefit of commuters from home location 1 to select worksite 1 using the WF CSP, will make it less attractive for commuters from the same home location to select the same worksite using the alternative NWF CSP. This leads to reduced $Q_2^{1,1}$. Different from Figure 3.14, $Q_2^{1,1}$ also decreases with the increase of $B_1^{1,2}$. The impacts of the two parameters on $Q_2^{1,2}$ are the same as those for $Q_2^{1,1}$. On the worksite side, due to the decreases of the commuters' participation, the worksite demands (both q_2^1 and q_2^2) also decrease on the NWF CSP, but with a much less significant level. The demand of a particular worksite (e.g., q_2^1) does not change much with the parameter for the other worksite (e.g., $B_1^{1,2}$), which is similar to the patterns in Figure 3.14. For commuters from the other home location (i.e., location 2), the change patterns of the demands to both worksite locations ($Q_2^{2,1}, Q_2^{2,2}$) using the NWF CSP are the same as the change patterns of the worksite demands (q_2^1, q_2^2).

To better illustrate and compare how B_1^{11}, B_1^{12} may change the participation of both sides on both CSPs, the results are summarized in Table 3.2. In the table, the directions of changes are marked using the directions of the arrows or no change using the solid box; the significance of the changes are highlighted using the boldness of the arrows. It can be seen that typically a parameter associated with a particular side (commuters or worksites), a location (home or worksite), and a CSP (WF or NWF) impacts more the participation of the same side, same location, and same CSP than the other side, the other location, and/or the other CSP. The interactions (competition) of the two sides, different home/worksite locations, and the two CSPs are also clearly shown in the table.

Table 3.2: Results of sensitivity analysis for the general case

	Change pattern with respect to B_1^{11}, B_1^{12}		Change pattern with respect to T_1^{11}, T_1^{12}	
	commuter side	worksite side	commuter side	worksite side
Same CSP, same home loc.	Q_1^{11} **↑** ↓ Q_1^{12} ↓ **↑**	q_1^1 **↑** ■ q_1^2 ■ **↑**	Q_1^{11} **↓** ↑ Q_1^{12} ↑ **↓**	q_1^1 **↓** ■ q_1^2 ■ **↓**
Same CSP, diff. home loc.	Q_1^{21} ↑ ■ Q_1^{22} ■ ↑		Q_1^{21} ↓ ■ Q_1^{22} ■ ↓	
Diff. CSP, same home loc.	Q_2^{11} **↓** **↓** Q_2^{12} **↓** **↓**	q_2^1 ↓ ■ q_2^2 ■ ↓	Q_2^{11} **↑** **↑** Q_2^{12} **↑** **↑**	q_2^1 ↑ ■ q_2^2 ■ ↑
Diff. CSP, diff. home loc.	Q_2^{21} ↓ ■ Q_2^{22} ■ ↓		Q_2^{21} ↑ ■ Q_2^{22} ■ ↑	

Remark: (1) **↑** indicates increasing as the parameter increases; **↓** indicates decreasing as the parameter increases; bold arrows indicate major impacts, while thin arrows indicate minor impacts; ■ indicates no obvious changes

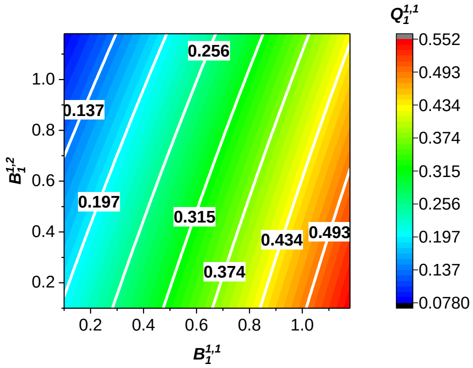
(2) Arrows and boxes to only represent the direction and extend of the sensitivity effect (minor, major, still), they do not indicate the real "slop", many results show obvious nonlinear trend;

(3) The first arrow (or solid box) is for B_1^{11} (or T_1^{11}), and the second arrow (or solid box) is for B_1^{12} (or T_1^{12})

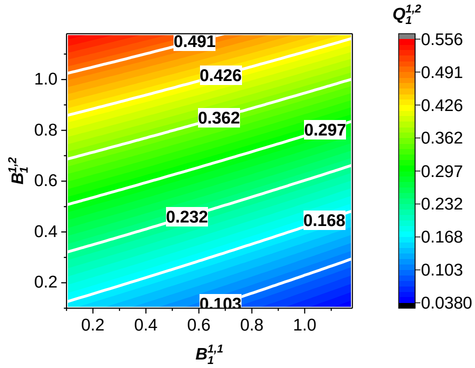
Table 3.2 also summarizes the analysis results of the same-side negative effects T_1^{11}, T_1^{12} on the participation (demands) of the two sides, which are almost opposite to the results of B_1^{11}, B_1^{12}. Detailed discussions are omitted here for brevity. In a similar way, the modeling framework proposed here also allows us to analyze how these parameters may impact other variables (e.g. prices and profit), as well as how other parameters may impact the participation, prices, and profit. Again those details are omitted here.

3.6 Discussion

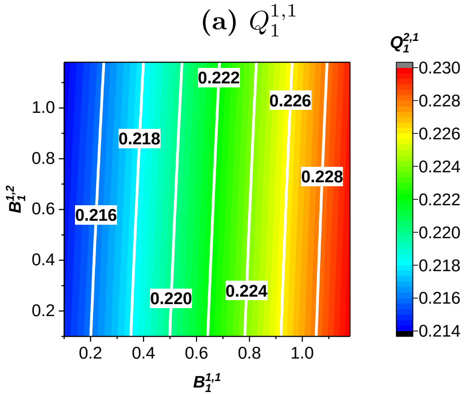
The above analysis of CSP using the two-sided market theory leads to some interesting findings. In **the monopoly model**, when a CSP changes the price allocation between the two sides, the participation from both sides will be affected. Even if the CSP only changes the price of one side, e.g., worksites, the participation of both sides will change (Figure 3.3). Given the specific parameter settings in this dissertation, the participation of one side (e.g., worksites) is more sensitive to the price of the same side (worksites), and less sensitive to the price of the other side (commuters); see Figure 3.3. Since the cross-side positive network effects bring more benefits to one side if the participation of the other side increases (see equation (3.1)), increasing cross-side network effects from one side or both sides raise the



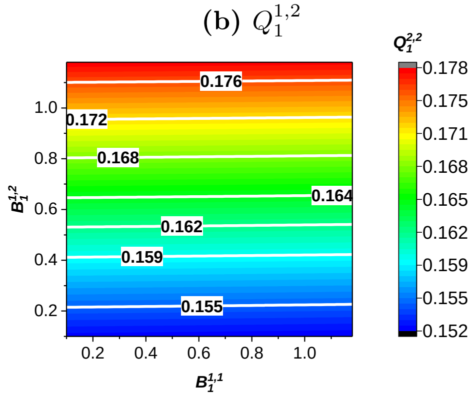
(a) $Q_1^{1,1}$



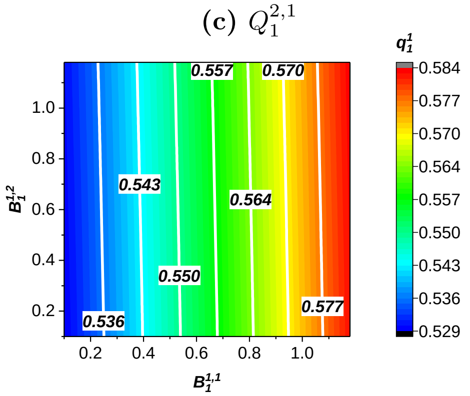
(b) $Q_1^{1,2}$



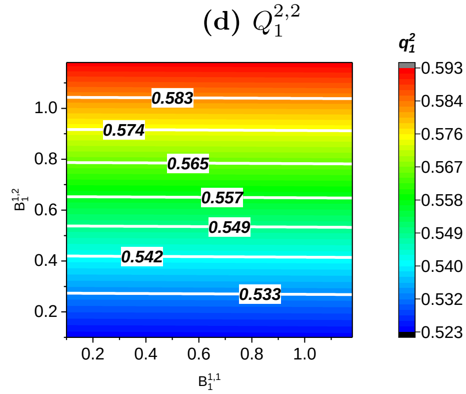
(c) $Q_1^{2,1}$



(d) $Q_1^{2,2}$

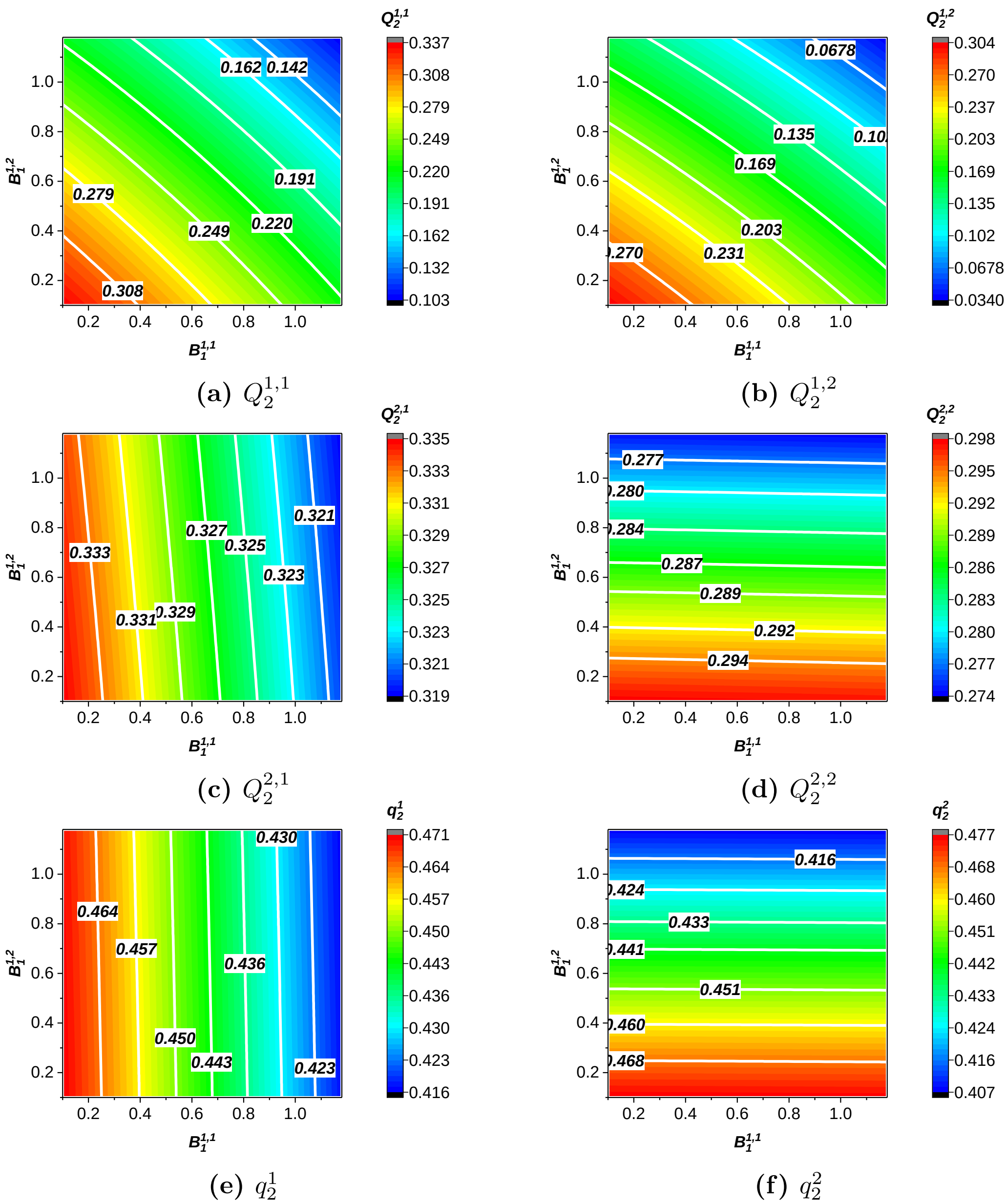


(e) q_1^1



(f) q_1^2

Figure 3.14: Participation on WF CSP vs. $B_1^{1,1}, B_1^{1,2}$



(a) $Q_2^{1,1}$ **(b)** $Q_2^{1,2}$

(c) $Q_2^{2,1}$ **(d)** $Q_2^{2,2}$

(e) q_2^1 **(f)** q_2^2

Figure 3.15: Participation on NWF CSP vs. $B_1^{1,1}, B_1^{1,2}$

participation of both sides (Figure 3.4, 3.5a, 3.5b). Furthermore, when worksites highly value the number of commuters on a CSP, the CSP may reduce the price charged on commuters and recoup its profit from the worksites (Figure 3.5b). Worksites are attracted by the commuters on the CSP and will not be discouraged by the high price. The reverse will be also true if commuters value highly the number of worksites on the CSP. Under some specific parameter settings, the CSP is willing to subsidize the commuters to maintain or increase the overall profit (Figure 3.5b). The same-side negative effects discourage an agent to join the CSP when many agents from the same side have already chosen the same CSP (Figure 3.6). Thus, high level of same-side effects reduce participation and profit on a CSP. However, the CSP can maintain its profit under certain combinations of the same-side effects, shown by the contour line in Figure 3.6e.

The duopoly model inherits the main characteristics of the monopoly model, and helps us understand the competition between the two CSPs. The participation of one side is still affected by the price allocation of both sides (Figure 3.7), but with more complex patterns. For example, the participation of worksites is more sensitive to the relative price of worksites ($p_N^B - p_W^B$), and less sensitive to the relative price of commuters ($p_N^C - p_W^C$). The same also applies to commuters. Generally, the cross-side benefits of worksites on the two CSPs (α_W and α_N) encourage participation (Figure 3.8), which is consistent with the monopoly model. However, the actual participation pattern is more complex in that the higher cross-side benefit has the dominant effects on participation. The price of worksites is mainly affected by the relative value of the cross-side benefits (α^-; Figure 3.9a, 3.9b), while the price of commuters is mainly affected by the aggregated cross-side benefits (α^+; Figure 3.9c, 3.9d). Therefore, the CSP with higher cross-side benefits can attract more worksites even if it sets high prices on worksites; see the right-bottom part of Figure 3.9a. A CSP makes more profit when the participation is high and the subsidization level is relatively low. However, there exist "bad" competitions where neither of the CSPs makes high profit. For example, in the right-bottom part of Figure 3.10, the NWF CSP has low profit because of low participation from both sides. Although the WF CSP successfully attracts participation from both sides, its profit is still low (slightly higher than that of the NWF CSP) because of the subsidization to the commuters side.

When specific **locations of homes and worksites** are considered, more competitions/interactions are introduced to the choices of commuters, worksites, and CSPs. For the basic case (1 home location, 2 worksite locations, and 1 CSP), when there are non-participating commuters on the CSP, commuters' choices of the two worksite locations, as well as worksites' choices and prices, are independent. Participation and prices of the two sides with respect to the two locations become dependent when all commuters join the CSP. This is shown clearly in Figure 3.12. When multiple home and worksite locations and multiple CSPs are considered (i.e. the general case), the interactions and choices become more complex. A modeling framework, based on the GNEP, can be applied to model this general case, which helps analyze and solve the problem. Numerical experiments in Figure 3.14 clearly show the two-sidedness of the model and the competitions among worksite locations and the CSPs. They also show that a parameter associated with a particular side (commuters or worksites), a location (home or worksite), and a CSP (WF or NWF) impacts more the participation of the same side, same location, or same CSP than the other side, the other location, or the other CSP.

The above analysis results help draw some initial insights on how to build CSPs in practice. First, the envisioned CSP can help enhance the collaboration among CSPs, employers, and commuters, which is beneficial to all players in the market: (i) CSP can manage to obtain profits from both sides; (ii) employers can out-source the commuting subsidization to their employees to a third party (in our case, the CSPs) conveniently which helps recruit/retain needed talents; and (iii) commuters can have more affordable/accessible commuting choices. The profit of a CSP is affected by cross-side positive effects as well as same-side negative effects. In practice, cross-side benefits exist because employers and employees value each other's participation. More importantly, employers usually play an important role in the commuting decisions of their employees. For example, the work schedule is often set by employers which determines the departure time of employees, and employers' commuting-related programs (e.g., those related to transit passes and parking) also impact their employers' commuting decisions. Therefore, understanding and leveraging the strong interactions of employees and employers to adjust price strategies according to the cross-side effects is crucial for a CSP to attract participation. For instance, if worksites value commuters more, a CSP can set a lower price to the commuters and set a higher price to the worksites to attract more commuters, thus encouraging more worksites to participate. Meanwhile, high same-side effects may exist when the participation on a CSP exceeds the maximum number of services the CSP can provide, or the CSP becomes less efficient when the amount of customers increases. Imagine when a CSP hires big vans to pick up employees at their homes and then send them to a worksite. If the number of commuters taking the van increases, the pickup time will increase. Knowing this, some commuters may choose more time-efficient ways of commuting and choose not to join the CSP. Thus, when a CSP tries to attract more customers, it is important to develop strategies that can serve the increasing demand properly.

The proposed CSP may be built by extending existing ridesourcing (MaaS) platforms, such as Uber or Scoop platforms. For example, a ridesourcing company may set up contracts with business owners and assign vehicles to transport their employees to worksites (each vehicle picks up multiple commuters from the same or different employers). Such vehicles can either send commuters from their homes to the worksites directly, or send them from homes to hubs of public transit. In fact, industry pioneers such as Scoop is doing this by providing carpool services to co-workers of the same company or people living in the same neighborhood. To be more effective, the envisioned CSP platform needs to directly and actively engage employers and enable (real-time) communications between commuters and their employers (e.g., managers) in order to resolve commuting-related (and work schedule-related) issues promptly, which is currently lacking and should be the key focus of building practical CSPs. This will hopefully help promote more efficient ridesourcing modes (such as ridesplitting, carpooling, vanpooling) that are currently underutilized (Li *et al.*, 2019). The integration of employers into the CSP platform will also lead to a win-win-win situation: commuters can choose convenient and less expensive commuting services, CSP has access to larger demands (by working with employers directly) that may result in larger profit, and business owners can also benefit from CSPs because their employees have more convenient ways to get to work and the total commuting trips of their employees are causing less congestion (and thus better meet regulations on commute trip reductions).

Our analysis may also help may help develop the next-generation, CSP-based and *employer-centered* **TDM strategies** to better leverage the emerging MaaS technologies and to actively engage employers. First, employer-centered TDM strategies can be developed for CSPs to enable and facilitate the communications between employers and employees regarding commuting decisions. For example, a CSP can negotiate with different worksites on the flexible working hours for commuters who live close to each other but work for different worksites. Assume Commuter A and Commuter B live in the same neighborhood but work at different worksites (which are also close to each other): A leaves for work at 7:30am and B leaves for work at 8:00am based on their work schedules. If both worksites require fixed working hours, A and B need to go to work separately, probably by driving alone (i.e., two cars). If the CSP can negotiate with worksites and adjust the working hours, A and B may carpool to work together at 7:45am, reducing SOV trips in the road network. Currently, a carpool happens in most cases only if two commuters have the same working hours and close-by worksites. With CSP, there will be more chances for carpool or ridesharing to occur by actively involving employers in commuting related decisions. As a result, SOV travels can be further reduced due to CSPs. Second, by understanding/analyzing the interactions/participation of employers and employees to different CSP services (via the two-sided market analysis method discussed above), TDM strategies can be developed to help increase the usage of certain types of commuting services of the CSP that are more beneficial to the urban transportation network. For example, agencies can provide incentives to encourage employers to subscribe to the WF CSP since the platform encourages peak spreading (and thus helps reduce peak hour congestion). This will motivate more commuters (employees) to choose the WF platform. As a result, the actual use of the WF CSP will increase, which could help ease the peak hour congestion. Thus, instead of putting forward regulations that directly guide individual commuters/companies to adopting TDM strategies as traditionally done, transportation agencies can implement and increase the impact of TDM strategies by working with CSPs who may then have more (positive) influences on the commuting related decisions of employers and commuters.

Lastly, in the numerical experiments in this dissertation, I provided some preliminary discussions on how to set the parameters (i.e., the initial utility, the cost of CSP to provide services for each side, the cross-side benefit effect, and the same side negative effect) and how they may be related to the transportation system. These are much simplified and the parameters in this dissertation are selected mainly for illustration purposes. More realistic values of these parameters should be determined via rigorous investigations on the characteristics, behavior, attitude, and preference of a player (commuter or worksite) regarding the commuting services provided by the CSPs, as well as the actual or perceived pros and cons of using certain services. Here I offer some initial thoughts in Table 3.3 on the major factors/considerations that may impact the values of the parameters. Formal studies on this are beyond the scope of the current dissertation and should be pursued in future research.

Table 3.3: Major factors/considerations for model parameters

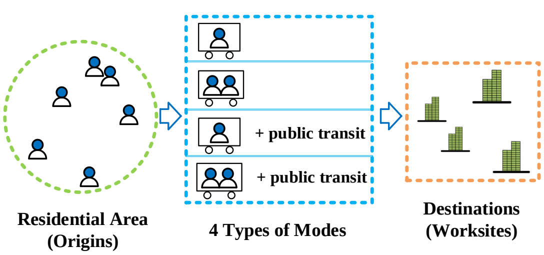
	Commuter Side	**Worksite Side**
CSP Cost	Transportation cost (fuel, labor, etc.) to send commuters from homes to work, and vice versa	Negotiation and related costs for worksites to join the CSP
Initial utility	• Reduced car ownership • More affordable/convenient commuting options	• Better ways to implement commuting programs and meet commuting related regulations • Retain / recruit needed talents
Cross-side benefit effects	• More worksites imply more choices to select the employers, potentially those with commuting options that fit better with a commuter's needs • Commuter's sociodemographic variables matter	• More commuters imply more choices to recruit workers, potentially those who can fit better with a worksite's preferred commuting options • The types and locations of worksites also matter
Same-side negative effects	Depending on the types of commuting services and the characteristics of the commuter: • Negative (e.g., for transit/taxi/ridesourcing services): more commuters lead to more congestion (or more crowded buses); • Negative (e.g., for carpool/vanpool): more commuters may lead to degraded services (e.g. longer waiting times or detours) if CSP vehicles could not serve all requests efficiently due to unbalanced temporal/spatial distributions of commuters • Positive (e.g., for carpool/vanpool): more commuters may lead to more efficient matching/routing, and shorter waiting times for detours	Depending on the types of commuting services and the types/characteristics of worksites: • Negative: more worksites mean it is more competitive to recruit workers and may require a worksite to subscribe commuting services that are attractive to workers but not to the worksite • Negative: more worksites may create more congestion (and thus longer/unreliable commuting times of their workers) during peak hours • Positive: more worksites may enable CSP to more efficiently match commuters or provide more efficient/inexpensive services (such as shuttle buses instead of carpools)

Chapter 4

OPERATIONAL LEVEL ANALYSIS

4.1 Problem statement

Figure 4.1a illustrates the morning commute scenario. There are four types of modes, $m \in \{1, 2, 3, 4\}$, which can serve commuters from their residential areas to the worksites: i) single rides from origin to destination, m=1; ii) shared rides that take passengers from two separate nearby locations from origin to destination, m=2. iii) single rides as the first mile then transfer to transit, m=3; iv) shared rides that take passengers from two separate locations as the first mile then transfer to transit, m=4. Single-ride services and shared-ride services are operated by the same ridesourcing platform. For simplicity, the four types of modes are referred to as taxi (m=1), ridesharing (m=2), Ttransit (m=3), RStransit (m=4); see Figure 4.1b. A customer chooses a particular mode based on individual value of time and other characteristics.



(a) Morning commute scenario

	Single ride	Shared ride
Ridesourcing mode	Taxi, m= 1	Ridesharing, m=2
Integrated mode	Ttransit, m=3	RStransit, m=4

(b) Four types of modes

Figure 4.1: Integrated multimodal network

This study does not consider the mode where a commuter owns a CAV and drives alone to the worksite (i.e. the "solo driving" mode as defined in (Ban *et al.*, 2019), in order to better investigate the interaction between CAV ridesourcing and transit. The transit services considered are mass transit with fixed routes/schedules (i.e. on-demand transit services are not considered), such as light rail, subway, or BRT, which usually run on separate right-of-way and thus have little or no interaction with vehicular traffic congestion. Additionally, for the current study, this dissertation only considers shared rides in which a CAV picks up customers from a maximum of two separate nearby locations. This is consistent with the ridesplitting services in most ridesourcing platforms (e.g. Uber Pool) and research suggests that in practice, more than 90% of shared rides include two pickup locations (Li *et al.*, 2019). The proposed modeling framework may be further extended to include solo driving, on-demand transit, and street bus services, which can be studied in future research.

4.1.1 General equilibrium overview

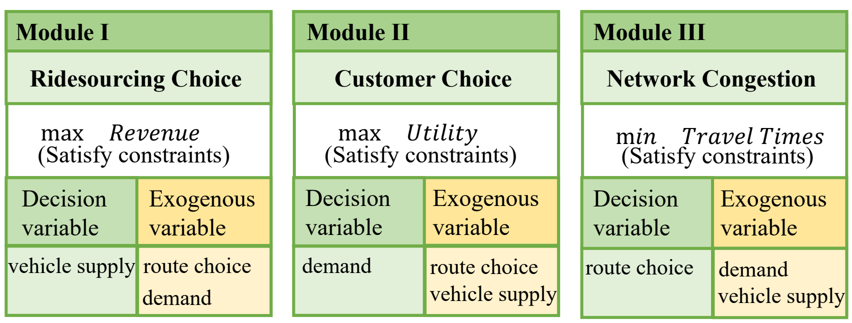
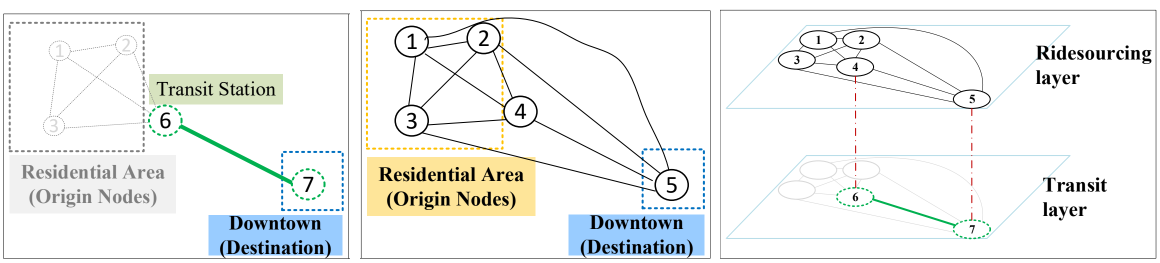


Figure 4.2: A summary of the general equilibrium model

A general equilibrium model with three modules is proposed (Figure 4.2). The ridesourcing choice module maximizes the revenue of the provider. The endogenous variable is the CAV dispatch (i.e. vehicle supply), while customer demand and route choice are exogenous variables. The customer choice module minimizes the disutility of customers. The decision variables are the demand of each mode with vehicle supply and route choices as the exogeneous variables. The network congestion module captures the flow interaction and congestion effect due to the choices and interaction of customers and service providers. The behavior of vehicles' route choices are modeled according to Wardrop's first principle, i.e., the CAVs always choose the route with the minimum travel time. The route choice is the decision variable, while demand and vehicle supply are exogenous variables. The three modules combine to form a general equilibrium model.

4.1.2 Extended network structure



(a) Transit layer **(b)** Ridesourcing layer **(c)** Two-layer network

Figure 4.3: Extended network structure of the small network

An extended network of two layers is constructed to model the integrated multimodal network, similar to the method in Di & Ban (2019) but more concise. Figure 3 shows the extended structure for a small network. Figure 4.3b is the ridesourcing layer, where node 5 is the vehicle destination for ridesourcing (m=1,2), and transit station node 4 is the vehicle destination for the integrated modes (m=3,4). In the transit layer (Figure 4.3a), transit nodes 6 and 7 are defined, where node 6 has the same location with node 4 in the ridesourcing layer, and node 7 has the same location as node 5 (Figure 4.3c). Vehicle flow is based on the ridesourcing layer, while passenger flow is based on both layers. Recall that our model assumes that transit runs on separate right-of-way and therefore does not interact with congestion on the ridesourcing layer. To simplify the discussion, this dissertation only considers CAV ridesourcing serving the first mile of transit and thus assume customers' destinations coincide with transit stations. A similar method can be applied to model the scenario where CAV ridesourcing serves the last mile of transit when customers' destinations are different from transit stations. The proposed model can also deal with multiple transit stations at the origins and/or at the destinations, although only one such station is shown (at the origins or at the destinations) in Figure 4.3 for illustration purposes.

4.2 Methodology

In our model, the following sets are defined : $m \in \{1, 2, 3, 4\}$, four types of modes, introduced in section 2.1; N , the set of nodes; O , the set of origins, where vehicles pick up passengers; D^v , the set of destinations of ridesourcing vehicles, where vehicles drop off all passengers and then head to the next pick-up location; D^c , the set of destinations of customers, which are the worksites; $K \in O \times D^c$, the set of origin-destination (OD) pairs for customers; T , the set of transit stations, where customers of mode 3 or 4 transfer to transit. In the model formulation, O_k denotes the origin of an OD pair k, D_k^v denotes the vehicle destination of the OD pair k, D_k^c denotes the customer destination of the OD pair k, T_k denotes the transit station of the OD pair k for mode 3 or 4. That is, vehicles and customers have different destinations for the integrated transit modes (m=3,4), but have the same destination if the whole trip is served by the ridesourcing modes (m=1,2). Here a mapping set is defined $MAP \coloneqq \{(m, D^v, D^c), k \in K | (1, D_k^c, D_k^c), (2, , D_k^c, D_k^c), (3, D_k^v, D_k^c), (4, D_k^v, D_k^c)\}$ to connect the sets of customer destinations and CAV destinations of each mode. To illustrate these sets using the network in Figure 4.3, one obtains $N \coloneqq \{1, 2, 3, 4, 5\}$, $O \coloneqq \{1, 2, 3\}$, $D^v \coloneqq \{4, 5\}$, $D^c \coloneqq \{5\}$, $T_k \coloneqq \{6\}$, $MAP \coloneqq \{(m, D^v, D^c) | (1, 5, 5), (2, 5, 5), (3, 4, 5), (4, 4, 5)\}$.

4.2.1 Ridesourcing choice module (Module I)

The revenue of each shared-ride vehicle consists of a fixed fare, time-based fare, and distance-based fare for each passenger. The fixed fares $F_{O_k}^m$ and $F_{O_{k'}}^m$ for two shared-ride passengers with OD pairs k and k' are based on the locations of the origin nodes, O_k and $O_{k'}$. Notice that although the vehicle takes detours to pick up passengers, the fare that passengers need to pay is based on the distance between their origins and the destination. Thus the time- and distance-based fares are formulated as $\alpha_1^m \left(t_{O_k D_k} + t_{O_{k'} D_k}\right)$ and $\alpha_2^m \left(d_{O_k D_k} + d_{O_{k'} D_k}\right)$, where α_1^m is the time-based fare rate and α_2^m is the distance-based fare rate. Notice here that for simplicity, this chapter assumes the two groups of passengers on a shared ride go to the same destination (i.e. the same transit stop or the same downtown worksite). The modeling method proposed here can be properly revised to incorporate the case when the two destinations are separate and close to each other.

The cost of each shared-ride vehicle also includes time- and distance-based costs of the actual route taken, represented by $\beta_1^m \left(t_{jO_k} + t_{O_k O_{k'}} + t_{O_{k'} D_k}\right)$ and $\beta_2^m \left(d_{jO_k} + d_{O_k O_{k'}} + d_{O_{k'} D_k}\right)$, where β_1^m is the time-based cost rate and β_2^m is the distance-based cost rate. Since this chapter specifically models CAVs, the time-based cost will be lower than it is for traditional taxis. However, if a vehicle takes longer than usual to finish a trip, it indicates less efficiency and lower customer satisfaction, which cause extra cost to the platform. In summary, the profit function of a shared-ride vehicle that drops off the previous passenger(s) at j and then picks up passengers first at O_k and then at $O_{k'}$ can be formulated as:

$$\begin{aligned} R_{jkk'}^m = F_k^m + F_{k'}^m + \underbrace{\alpha_1^m (t_{O_k D_k} + t_{O_{k'} D_k})}_{\substack{\text{time based} \\ \text{revenue}}} + \underbrace{\alpha_2^m (d_{O_k D_k} + d_{O_{k'} D_k})}_{\substack{\text{distance based} \\ \text{revenue}}} - \underbrace{\beta_1^m (t_{jO_k} + t_{O_k O_{k'}} + t_{O_{k'} D_k})}_{\substack{\text{travel time} \\ \text{based cost}}} \\ - \underbrace{\beta_2^m (d_{jO_k} + d_{O_k O_{k'}} + d_{O_{k'} D_k})}_{\substack{\text{travel distance} \\ \text{based cost}}} \quad \forall m \in \{2, 4\} \end{aligned} \tag{4.1}$$

Similarly, the profit of a single-ride vehicle that drops off the previous passenger(s) at j and then picks up passengers at O_k and drop off passengers at destination D_k can be formulated as:

$$R_{jk}^{m} = F_{O_k}^{m} + \underbrace{\alpha_1^m t_{O_k D_k}}_{\substack{\text{time based}\\ \text{revenue}}} + \underbrace{\alpha_2^m d_{O_k D_k}}_{\substack{\text{distance based}\\ \text{revenue}}} - \underbrace{\beta_1^m (t_{jO_k} + t_{O_k D_k})}_{\substack{\text{travel time}\\ \text{based cost}}} - \underbrace{\beta_2^m (d_{jO_k} + d_{O_k D_k})}_{\substack{\text{travel distance}\\ \text{based cost}}} \quad \forall m \in \{1,3\} \tag{4.2}$$

For m= 1 or 2, commuters take single rides or shared rides to their actual destinations (e.g., in Figure 4.3, the drop-off location is node j=5); for m= 2 or 4, commuters take single rides or shared rides to the transit stations (not their actual destinations), e.g., in Figure 4.3, the drop-off location is node j=4 (the same as node 6 in the transit layer). This needs to be properly modeled.

The decision variables of the ridesourcing choice module are $z_{jkk'}^{m}$ (where O_k , $O_{k'} \in O$) for shared-ride vehicles and z_{jk}^{m} (where $O_k \in O$) for single-ride vehicles. $z_{jkk'}^{m}$ denotes the number of shared-ride vehicles currently at destination j that will next pick up customers first at O_k and then at $O_{k'}$. z_{jk}^{m} denotes the number of single-ride vehicles currently at destination j that will next pick up customers at O_k. Although there are four different types of modes, the ridesourcing platform is only providing two types of services, either single rides or shared rides. This chapter assumes that for the same service, the platform has the same price strategies for the first-mile trips and whole trips. The objective function of *Module I* is to maximize the total profit from single-ride and shared-ride services:

$$\max_{z_{jk}^m \geq 0, z_{jkk'}^m \geq 0, q_{kk'}^m, b_{kk'}^m} \underbrace{\sum_{m \in \{1,3\}} (R_{jk}^m \cdot z_{jk}^m) + \sum_{m \in \{2,4\}} (R_{jkk'}^m \cdot z_{j\tilde{k}\tilde{k}'}^m - w^m \cdot |q_{\tilde{k}\tilde{k}'}^m| - w^m \cdot |b_{\tilde{k}\tilde{k}'}^m|)}_{\text{average trip profit}} \tag{4.3}$$

Subject to:

Constraints for single rides, $m = 1, 3$:

$\sum_{j \in D} z_{jk}^m \geq Q_k^m \ \forall \ k \in K$ *vehicle supply satisfies customer demand*

$\sum_{k \in K} z_{jk}^m = \sum_{k' : j = D_k} Q_{k'}^m \ \forall \ j \in D$ *vacant single-ride CAVs are avaible again to serve next* *trip*

Constraints for shared rides, $m = 2, 4$:

$\sum_{j\in D} z^m_{jkk'} - Q^m_{k,kk'} = b^{+m}_{kk'} - b^{+m}_{kk'}\ \forall k \in K$ *shared vehicles pick up 2 customers in each trip*

$\sum_{j\in D} z^m_{jkk'} - Q^m_{k',kk'} = q^{+m}_{kk'} - q^{+m}_{kk'}\ \forall k \in K$ *shared vehicles pick up 2 customers in each trip*

$\sum_{k,k'\in K} z^m_{jkk'} = \sum_{k\in K}\sum_{k':j=D_{k'}} Q^m_{k',kk'}\ \forall j \in D$ *vacant shraed-ride CAVs are avaible again*

Constraint for the total number of vehicles:

$$\underbrace{\sum_{m=1,3}(\sum_{k\in K}\sum_{j\in D} z^m_{jk} t_{jO_k} + \sum_{k\in K} Q^m_k t_{O_k D_k})}_{number\ of\ vehicles\ for\ sigle\ ride} + \underbrace{\sum_{m=2,4}(\sum_{k,k'\in K}\sum_{j\in D} z^m_{jkk'} t_{jO_k} + \sum_{k,k'\in K} Q^m_{k,kk'} t_{O_k O_{k'}} + \sum_{k,k'\in K} Q^m_{k',kk'} t_{O_{k'} D_{k'}})}_{number\ of\ vehicles\ for\ shared\ ride} \leq N \tag{4.4}$$

Here Q^m_k is the demand of mode m along OD pair k. $Q^m_{kk'}$ and $Q^m_{k'k}$ are the demand of the fist pickups and second pickups of the shared ride services (m=2,4). As illustrated in Figure 4.2, customer demand variables are exogoneous to *Module I*.

A mismatch occurs when shared rides fail to pick up 2 passengers from 2 different origin nodes. $q^m_{kk'}$ and $b^m_{kk'}$ are defined to capture the mismatch of shared rides. There is no mismatch when both $q^m_{kk'}$ and $b^m_{kk'}$ equals 0. ω^m is the rate of penalty, with a higher value indicating a greater cost per mismatch. Constraints for *Module I* ensure that: i) the number of vehicles dropping off customers at destination j is equal to the number of vehicles departing from j; ii) each customer request is served ; iii) shared-ride vehicles can only pick up at most two passengers from up to two different origins, and penalization will be imposed if they pick up passengers from only one location; iv) the number of vehicles in operation is no larger than the total number of vehicle operated by the platform.

4.2.2 Customer choice module (Module II)

Customers can choose to take ridesourcing to a transit station or directly to worksites. Thus, the customer flow is present on both the ridesourcing layer and the transit layer.

When calculating the disutility of the shared-ride customers, either for the first mile or for the whole trip, there is a trade-off between precisely capturing the behavior of the customers and the feasibility of the model. For shared-ride trips, the first customer tends to have a longer ride time while the second customer tends to have a longer waiting time. But if the shared-ride trips benefit the first customer more than the second customer, it will be hard for the model to find a second customer so that the model is feasible. Thus, travel/waiting time-based disutility are simplified so that two customers in the same shared-ride trip expect the same (worst-case) disutility caused by the detour. The disutility of a shared-ride customer that is picked up first can be formulated as,

$$V^m_{kk'} = \underbrace{F^m_{O^k}}_{\substack{\text{fixed}\\ \text{fare}}} + \underbrace{\alpha^m_1 t_{O_k D_k}}_{\substack{\text{travel time}\\ \text{based fare}}} + \underbrace{\alpha^m_2 d_{O_k D_k}}_{\substack{\text{distance}\\ \text{based fare}}} + \underbrace{\gamma^m_1 (t_{O_k O_{k'}} + t_{O_k D_k})}_{\substack{\text{travel time}\\ \text{based disutility}}} + \underbrace{\gamma^m_2 (t_{O_k O_{k'}} + w^m_{kk'})}_{\substack{\text{disutility}\\ \text{due to waiting}}} \quad (4.5)$$

$$+ \underbrace{\gamma^m_3 \lambda^m_{kk'}}_{\substack{\text{disutility}\\ \text{due to matching}}} + Tr^m \quad \forall m \in \{2, 4\}$$

Where O_k is the first pick-up location, $O_{k'}$ is the second pick-up location. This model have disutility due to the fare of the shared rides. $F^m_{O_k}$ is the location-based fixed fare. The time-based fare $\alpha^m_1 t_{O_k D_k}$ and the distance-based fare $\alpha^m_2 d_{O_k D_k}$ depend only on the OD pair $O_k \to D_k$, which is reasonable because the additional disutility of shared rides is already considered in other terms. γ^m_1 is the disutility rate for travel time. The travel time disutility depends on the travel time of OD pair $O_k \to D_k$ and the travel time of detour $O_k \to O_{k'}$. γ^m_2 is defined as the waiting time disutility rate and $w^m_{kk'}$ as the waiting time disutility from the previous drop off location to the first pick-up location. Thus the summation of $\gamma^m_2 t_{O_k O_{k'}}$ and $w^m_{kk'}$ captures the worst-case waiting time from the vehicle's previous drop-off location j to the second shared-ride customer. The matching of the two customers in a shared ride is also regarded as the cost of the platform, which is modelled as the Lagrange multiplier of the shared ride demand constraints in *Module I*, denoted as $\gamma^m_3 \lambda^m_{kk'}$, where γ^m_3 is the rate of matching cost and $\lambda_{kk'}^m$ is the multiplier. See Ban *et al.* (2019) for more discussions of why this multiplier could be regarded as (proportional to) the matching cost. The disutility of the second picked-up customer, $V_{k'k}^m$, can be derived similarly, which is omitted here.

This chapter assumes high capacity for the transit route $T_k \to D_k$, so the travel time and distance are constant for the transit route. Consequently, travel time and travel distance can be represented by the same parameter, $t_{T_kD_k} \cdot const = d_{T_kD_k}$. The disutility of taking transit consists of the time/distance-based transit fare $\alpha_3^m d_{T_kD_k}$, time/distance-based disutility $\gamma_4^m d_{T_kD_k}$ and the transfer cost γ_5^m . When $m \in \{1,2\}$, ridesourcing vehicles serve the whole trip, so $Tr_k^m = 0$. Therefore the the disutility of transit is

$$Tr^m = 0 \quad \forall m \in \{1,2\} \tag{4.6}$$

$$Tr^m = \underbrace{\alpha_3 d_{T_kD_k}}_{\substack{\text{travel distance} \\ \text{based fare}}} + \underbrace{\gamma_4 d_{T_kD_k}}_{\substack{\text{travel distance} \\ \text{based disutility}}} + \underbrace{\gamma_5}_{\substack{\text{disutility} \\ \text{of transfer}}} \quad \forall m \in \{3,4\} \tag{4.7}$$

Similar to shared rides, the disutility of a customer of a single-ride trip can be formulated as,

$$V_k^m = \underbrace{F_{O^k}^m}_{\substack{\text{fixed} \\ \text{fare}}} + \underbrace{\alpha_1^m t_{O_kD_k}}_{\substack{\text{travel time} \\ \text{based fare}}} + \underbrace{\alpha_2^m d_{O_kD_k}}_{\substack{\text{distance} \\ \text{based fare}}} + \underbrace{\gamma_1^m t_{O_kD_k}}_{\substack{\text{travel time} \\ \text{based disutility}}} + \underbrace{w_k^m}_{\substack{\text{disutility due} \\ \text{to waiting}}} + Tr^m \tag{4.8}$$

$$\forall m \in \{1,3\}$$

Customer waiting time is calculated as the average deadhead miles from the previous destinations to the next origin (first origin for shared ride trips); see Ban *et al.* (2019),

$$w_k^m = \gamma_2^m \frac{\sum_{j \in D} z_{jk}^m t_{jO_k}}{\sum_{j \in D} z_{jk}^m} \quad \forall k \in K, m \in \{1,3\} \tag{4.9}$$

$$w_{kk'}^m = \gamma_2^m \frac{\sum_{j \in D} z_{jkk'}^m t_{jO_k}}{\sum_{j \in D} z_{jkk'}^m} \quad \forall k, k' \in K, m \in \{2,4\} \tag{4.10}$$

The decision variable for *Module II* is the customer demand. The customer demand for single rides is Q_k^m . The customer demand for shared rides are $Q_{kk'}^m$ and $Q_{k'k}^m$ for the first pick-up and second pick-up, respectively. The objective function minimizes the disutility of customers:

$$\min_{Q_{k'k}^m, Q_{kk'}^m, Q_k^m} \sum_{m=1,3} V_k^m Q_k^m + \sum_{m=2,4} V_{kk'}^m Q_{kk'}^m + \sum_{m=2,4} V_{k'k}^m Q_{k'k}^m \tag{4.11}$$

$$\text{s. t.} \sum_{m=1}^{4} Q_k^m = Q_k \quad \forall k \in K \tag{4.12}$$

$$\sum_{k' \in K} (Q_{k'k}^m + Q_{kk'}^m) = Q_k^m \quad \forall m \in \{2, 4\}, k \in K \tag{4.13}$$

The constraints for *Module II* ensure that: i) the customers requesting different types of modes sum up to the total travel demand; ii) the number of customers taking shared rides along OD pair $O_k \to D_k$ is equal to the summation of O_k as the first and second pick up location from all shared-ride trips. The above module minimizes the total disutility of all customers. Since V_k^m , $V_{kk'}^m$, $V_{k'k}^m$ are all exogeneous to *Module II* (as they are independent of Q_k^m , $Q_{k'k}^m$, $Q_{kk'}^m$), our formulation ensures that each customer chooses the mode with the least disutility. An analogy to this is that the shortest path search problem on a transportation network can be reformulated to a linear program to minimize the total cost of all users of the network; more discussions on this can be found in Ban *et al.* (2019).

4.2.3 Network congestion module (Module III)

Because this research only considers the congestion effect of ridesourcing services, *Module III* is based on the ridesourcing layer. There are three types of network flow: i) deadhead miles, i.e., the distance CAVs travel from drop-off locations to the next pickup locations, which apply to both single rides and shared rides; ii) detours, when CAVs are occupied by only the first group of shared ride customers to pick up the second group of customers, which only apply to shared rides; iii) occupied trips, when a CAV travels from the final pick-up location to the destination, which apply to both single rides and shared rides. The following equations show how the three types of flow can be calculated.

(i) Deadhead miles:

$$\sum_{m=1,3} z_{jk}^{m} t_{jO_k} + \sum_{m=2,4} \sum_{k' \in K} z_{jkk'}^{m} t_{jO_k} \tag{4.14}$$

(ii) Detours:

$$\sum_{m=2,4} \sum_{k' \in K} Q_{kk'}^{m} t_{O_k O_{k'}} \tag{4.15}$$

(iii) Ocuppied trips:

$$\sum_{m=1,3} Q_{k}^{m} t_{O_k D_k} + \sum_{m=2,4} \sum_{k' \in K} Q_{k'k}^{m} t_{O_k D_k} \tag{4.16}$$

Each type of flow can be formulated as complimentary conditions to ensure that the route choice follows Wardrop's principle. Now that the full formulation of the general equilibrium model is obtained, consisting of the three optimization problems from *Module I-III*, one can derive the KKT conditions of the three optimization problems and reformulate the general equilibrim model to a mixed complimentarity problem (MCP). The solution existence and uniqueness conditions to such a formulation are studied and established, using similar methods as reported in Ban *et al.* (2019), which is omitted here.

4.3 Numerical Experiments of small network

The general equilibrium model (an MCP) is solved using the PATH solver in GAMS. Sensitivity analysis is conducted on a small network and the Sioux Falls network. Due to space limitations, the results of the Sioux Falls network is omitted. When conducting the sensitivity analysis, I first set the baseline values for all parameters; then either unilaterally change one parameter at a time or change two parameters at the same time to see how customers' mode choice and the overall network congestion react to the change of the parameters.

The link properties of the small network (Figure 4.3) are shown in Table 4.1. The demand from nodes 1-3 to destination node 5 is 40 for all three origins. Table 4.2 lists the baseline parameters. Essentially, parameters for all single ride vehicles are the same, either for whole trips or first mile trips. The single-ride parameters for $m = 1, 3$ is set as the same values, e.g., $\alpha_1^1 = \alpha_1^3$. Similarly, the shared-ride parameters for mode $m = 2, 4$ is set as the same values, e.g., $\alpha_1^2 = \alpha_1^4$. The baseline parameter settings in Table 2 reflect the following relationship between the fares of different modes: single ride $>$ shared ride $>$ transit, while customer disutility rates follow the relation: shared ride $>$ single ride. When conducting the sensitivity analysis later in this section, some of these baseline parameters are modified.

Link	**From**	**To**	**Length (mile)**	**FFT (h)**	**Capacity**
1	1	2	0.4	0.02	40
2	1	3	0.5	0.025	40
3	1	4	2.1	0.07	60
4	2	1	0.4	0.02	40
5	2	3	0.6	0.03	40
6	2	4	1.8	0.06	50
7	3	1	0.5	0.025	40
8	3	2	0.6	0.03	40
9	3	4	1.8	0.06	50
10	4	1	2.1	0.07	60
11	4	2	1.8	0.06	60
12	4	3	1.8	0.06	60
13	2	5	10	0.5	100
14	3	5	11	0.55	100
15	4	5	9	0.45	120
16	5	2	10	0.5	100
17	5	3	11.5	0.55	100
18	5	4	9	0.45	120
19	6	7	9	—	—

Table 4.1: Parameters of the small network

4.3.1 Results of unilaterally changing one parameter

Table 4.3 shows the results when unilaterally changing the travel distance-based fare rate of single rides or shared rides. Under the current baseline parameter setting (Table 4.2), the demand for serving the whole trip using ridesourcing ($m = 1, 2$) is always zero, indicating that customers prefer the integrated modes that contain both ridesourcing and transit. Compared with Ttransit, RStransit saves more VMT, thus higher demand of RStransit corresponds to lower VMT. When unilaterally increasing the distance-based fare rate of single rides ($\alpha_2^1 = \alpha_2^3$), the cost of requesting Ttransit increases, thus the demand for Ttransit decreases while the demand for RStransit increases. When unilaterally increasing the distance-based

Illustration of parameters	**Notation** (m =1,2,3,4)	**Values**
The fixed fare for different modes ($)	F^m	5, 2.9, 5, 2.9
Time-based fare rate ($/h)	α_1^m	4.1, 1.2, 4.1, 1.2
Distance-based fare rate ($/mile)	α_2^m	1.5, 1.7, 1.5, 1.7
Conversion factor from time to cost ($/h)	β_1^m	7, 2.6, 7, 2.6
Conversion factor from distance to cost ($/mile)	β_2^m	1, 1.1, 1, 1.1
Value of time of customers, while traveling ($/h)	γ_1^m	2, 2.7, 2, 2.7
Value of time of customers, while waiting ($/h)	γ_2^m	3, 4.2, 3, 4.2
Value of time of customers, whilce mathing in shared rides ($)	$\gamma_3^m (m = 2, 4)$	NA, 2.5, NA, 2.5
Travel distance-based fare rate of transit ($/h)	α_3	0.37
Conversion factor from distance to cost for transit ($/mile)	γ_4	0.22
Transfer cost of transit ($/transfer)	γ_5	1.1

Table 4.2: Baseline parameters

fare of shared ride ($\alpha_2^2 = \alpha_2^4$), the demand of RStransit decreases while the demand of Ttransit increases. This example shows that when the cost parameter related to a particular mode increases (which represents certain cost/disutility of selecting the mode), customer choice of that mode will decrease, and the VMT of the entire network will also change accordingly.

When unilaterally changing the waiting time cost parameter for single rides (γ_2^1), modes 1,3,4 have non-zero demand (Table 4.4). When γ_2^1 is increased, the cost of both taxi and Ttransit increases. As a result, the demand of taxi and Ttransit decreases while the demand of RStransit increases.

4.3.2 Results of changing two parameters at the same time

To better show how different mode choices may impact VMT, I compare the VMT of a certain mode split scenario to the case when all customers drive alone in a personal vehicle. When all customers choose solo driving (no ridesourcing or transit involved), the VMT is equal to 1266.11 vehicle miles based on the UE solution. Thus, the VMT change can be

Table 4.3: Unilaterally change α_2^1 or α_2^2

$\alpha_2^1 = \alpha_2^3$	$\alpha_2^2 = \alpha_2^4$	**Taxi, m=1**	**Ridesharing, m=2**	**Ttransit, m=3**	**RStransit, m=4**	**VMT (miles)**
1.45	1.73	0%	0%	100%	0%	455.90
1.47		0%	0%	70%	30%	393.14
1.53		0%	0%	33%	67%	316.00
1.85		0%	0%	21%	79%	294.69
1.93		0%	0%	0%	100%	258.00
1.52	1.19	0%	0%	23%	77%	298.36
	2.12	0%	0%	46%	54%	342.05
	2.13	0%	0%	73%	27%	399.96
	2.18	0%	0%	90%	10%	434.37
	2.20	0%	0%	100%	0%	456.00

Table 4.4: Unilaterally change γ_2^1

$\gamma_2^1 = \gamma_2^3$	**Taxi, m=1**	**Ridesharing, m=2**	**Ttransit, m=3**	**RStransit, m=4**	**VMT (miles)**
1.75	33%	0%	67%	0%	1149.57
1.79	24%	0%	58%	18%	927.45
1.81	18%	0%	51%	31%	763.75
1.83	8%	0%	42%	50%	521.50
1.85	2%	0%	35%	63%	359.27

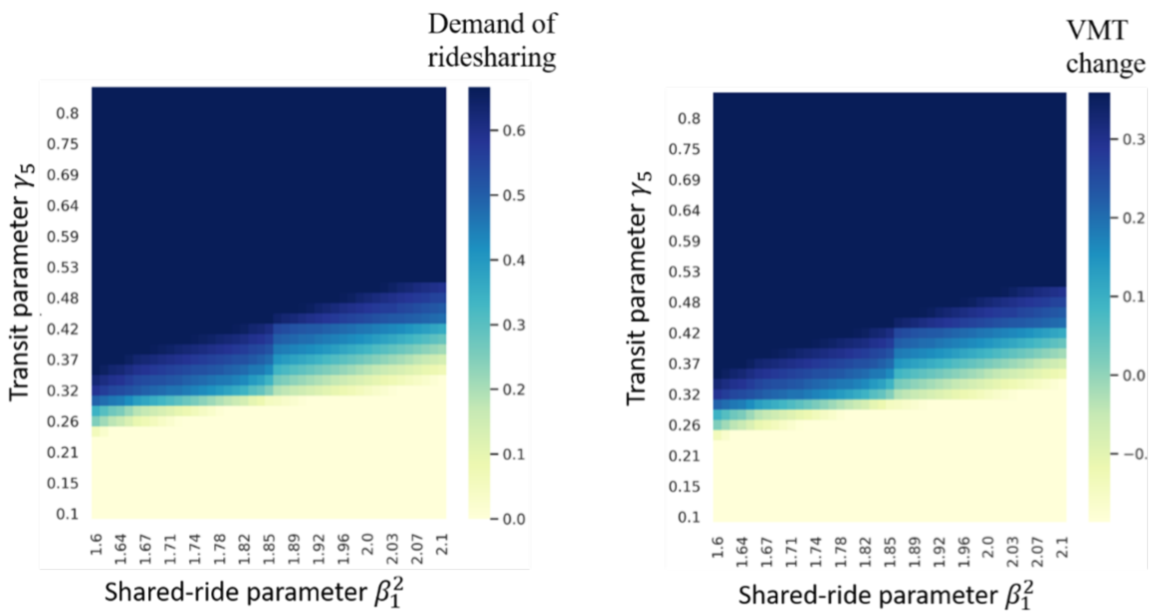
calculated as:

VMT change= (VMT of a particular scenario-1266.11)/1266.11

VMT change is a better indicator of congestion level compared with the absolute value of VMT. When VMT change is positive, it indicates that there are deadhead miles traveled, which make the congestion level higher than the solo driving scenario. When VMT change is negative, it implies that many customers choose transit or shared rides, so that the network is less congested than the solo driving scenario.

Figure 4.4 shows the results when one changes the time-based disutility rate for shared ride ($\beta_1^2 = \beta_1^4$) and the transfer cost of transit (γ_5) at the same time. The demand of single rides is zero; customers choose between ridesharing and RStransit. Since the demand of ridesharing and RStransit sum up to 1, Figure 4.4 only shows the demand pattern of ridesharing (Figure 4 (a)). Figure 4 is divided into 2 regions, with the top part as the high ridesharing demand region, and the bottom part as the high RStransit region. When γ_5 increases, the cost of taking transit increases, thus the demand of RStransit decreases and the demand of ridesharing increases. When β_1^2 increases and the transit parameter $\gamma_5 \approx 0.3$, the cost of ridesharing increases, so the demand of RStransit increases while the demand of ridesharing decreases. When customers only choose between ridesharing and RStransit, the demand pattern is mainly affected by transit parameter γ_5. When $\gamma_5 \in [0.25, 0.48]$, the ridesharing parameter also affects the demand pattern. Figure 4 (b) indicates that VMT change is consistent with the mode split pattern: VMT increases when more customers switch from RStransit to ridesharing. When γ_5 is small and most or all demand is served by RStransit, the VMT may be lower than solo driving; however, when more than about 20% of trips choose ridesharing, the VMT is greater than for solo driving.

When more than 2 types of modes have non-zero demand, the mode split pattern can be more complex. Figure 5 shows the results of changing the distance-based fare rate of shared rides ($\alpha_2^2 = \alpha_2^4$) and the travel distance-based cost of transit (α_3). The demand of Ttransit is always zero; customers choose among the other three modes: taxi, ridesharing and RStransit. There are five mode split patterns, marked in Figure 4.5a & 4.5b. The demand



(a) Percentage of Demand of ridesharing (m=2) **(b)** VMT change

Figure 4.4: Sensitivity analysis of changing $\beta_1^2(\beta_1^2 = \beta_1^4)$ and γ_5

and VMT change of these five mode split patterns are summarized in Table 4.5. Along the boundary of different mode split patterns, the demands of different modes change gradually, shown by the zoom-in version of the boundaries (Figure 4.5a & 4.5b). Mode split pattern ① lies in the region where the transit parameter is low, $\alpha_3 < 0.1$. The disutility of taking transit is low and all customers choose RStransit. Mode split patterns ②-⑤ roughly divide the space of transit parameter and ridesharing parameter into four quadrants. Mode split pattern ② takes place in the right bottom quadrant, where transit parameter $\alpha_3 < 0.64$ and shared-ride parameter α_2^2 is large. Compared with pattern ①, transit cost and shared-ride cost are higher in ②. Thus 33% of customers switch from RStransit to taxi. Mode split pattern ③ lies next to ②, with smaller shared-ride cost parameter. As a result of the decreased shared-ride cost, ridesharing demand increases to 67%, taxi demand drop to zero. Compared with ②, mode split ③ also has higher transit cost parameters, thus the transit use decreases to 33%. Mode split ③ has lower VMT compared to ②, since no travelers

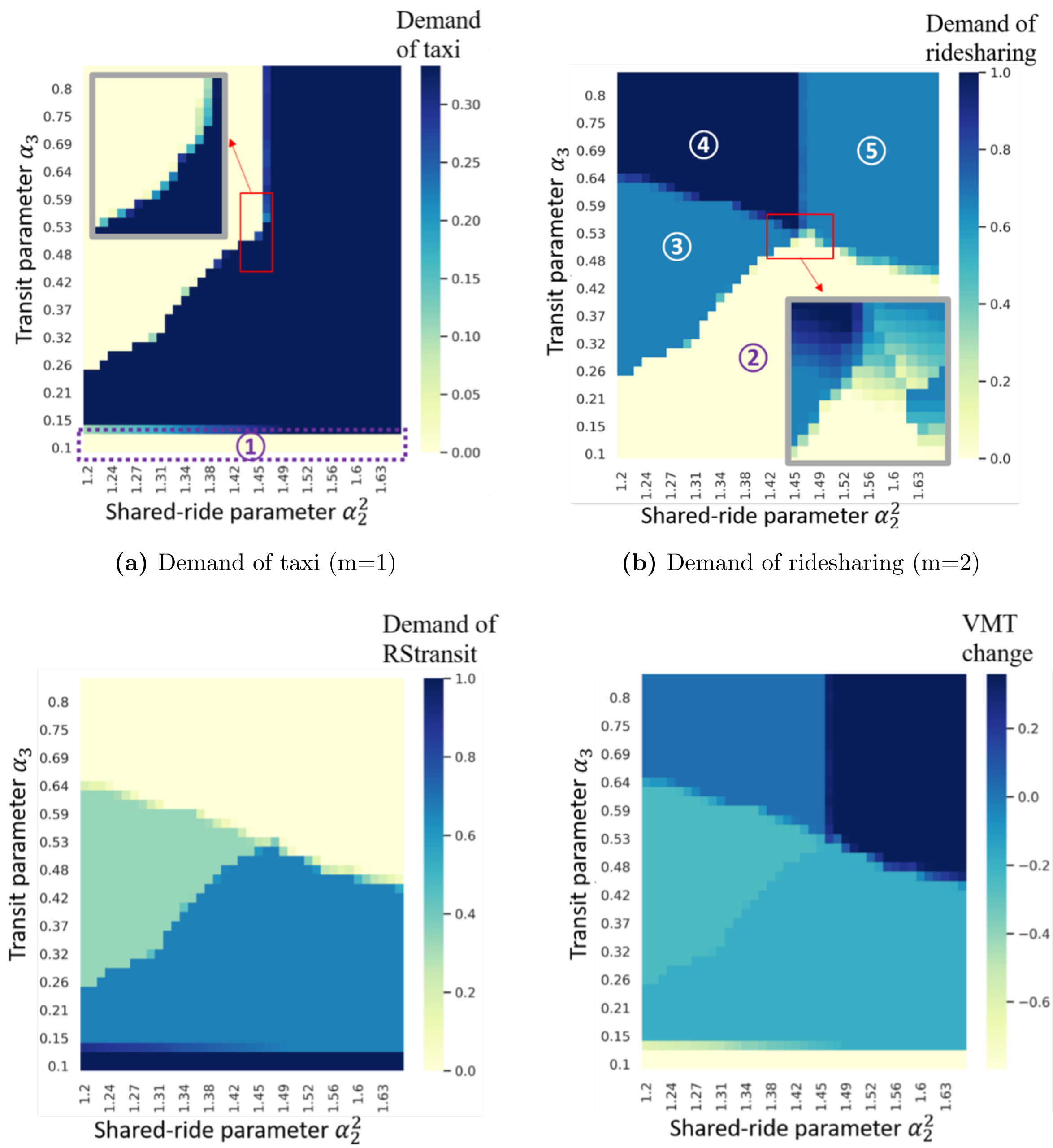
Mode split pattern	Taxi	Ridesharing	Ttransit	RStransit	VMT change
①	0%	0%	0%	100%	-80%
②	33%	0%	0%	67%	-25%
③	0%	67%	0%	33%	-30%
④	0%	100%	0%	0%	1%
⑤	33%	67%	0%	0%	33%

Table 4.5: Summary of the mode split patterns in Figure 4.5

request taxi rides in ③. The mode split patterns ④ and ⑤ can be explained in a similar manner. This example shows that the mode split in the integrated multimodal network has complex patterns. Especially in the era of CAVs, when the operations of vehicles are highly coordinated, it will be even more important to study the customers' mode choice systematically. The VMT change in Figure 4.5d are consistent with the mode split pattern in Figure 4.5a, 4.5b & 4.5c. The relation between mode split and VMT change is further disccussed in the following section.

4.3.3 Model choices versus VMT change

Figure 4.6a – 4.6d shows the relation between the demand (expressed in terms of the percentage of the total demand) of a particular mode $m \in \{1, 2, 3, 4\}$ and the VMT change. The plots are generated by 1) randomly changing all the model parameters (and thus the mode choices of customers and the resulting VMT of the network); and 2) showing the percentage of the demand m and the VMT change. For a given demand percentage of a mode, for example ridesharing (Figure 4.6b), customers may choose among the other three modes depending on the actual parameter combinations, resulting in different mode choices and different levels of VMT changes. Therefore, the relation between ridesharing demand and VMT change is scattered and forms a triangular region – the variations of mode splits



(a) Demand of taxi (m=1)

(b) Demand of ridesharing (m=2)

(c) Demand of RStransit (m=4)

(d) VMT change

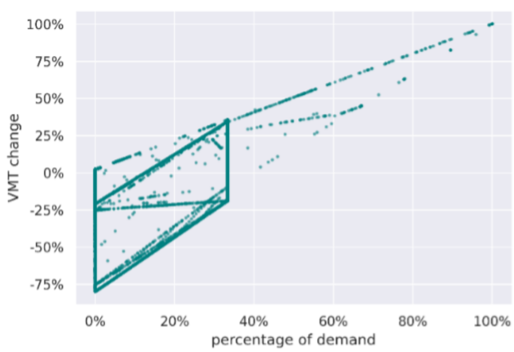
Figure 4.5: Sensitivity analysis of changing $\alpha_2^2(\alpha_2^2 = \alpha_2^4)$ and α_3

and VMT changes reduce as the demand percentage of ridesharing increases which diminish when the demand percentage is 100%. Similar patterns can be found for the other three modes. Further examination of the plots and results reveal that the upper boundary and lower boundary of each plot in Figure 4.6a – 4.6d captures the cases when exactly 2 modes are selected. For example, in Figure 4.6d, along the upper boundary, customers choose between taxi and RStransit, while the demand for ridesharing or Ttransit is zero. Thus, the VMT change starts at the corner case of all customers choosing taxi (VMT change = 100%), ends at the corner case of all customers choosing RStransit (VMT change ≈ -80%). Along the lower boundary in Figure 4.6d, customers choose between Ttransit and RStransit while the demand for taxi or ridesharing is zero. The VMT change starts at the corner case of all customers choosing Ttransit (VMT change ≈ -64%) and ends at the corner case of all customers choosing RStransit (VMT change ≈ -80%). The data points within the triangular region are cases when 3 or more types of modes are selected.

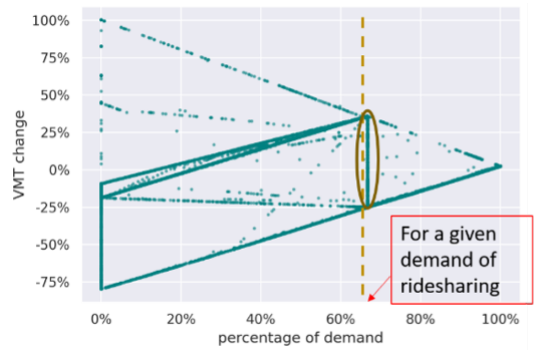
The corner cases are shown (i.e. when customers only choose one of the four modes) in Figure 4.6e. The plot shows that RStransit saves the most VMT: VMT decreases by 80% compared with solo driving when all customers choose RStransit. One the other hand, Ttransit reduces VMT by 64%, ridesharing has similar VMT with solo driving, and taxi causes the most congestion with 100% VMT increase due to deadhead miles.

4.4 Numerical Experiments of Seattle U-district network

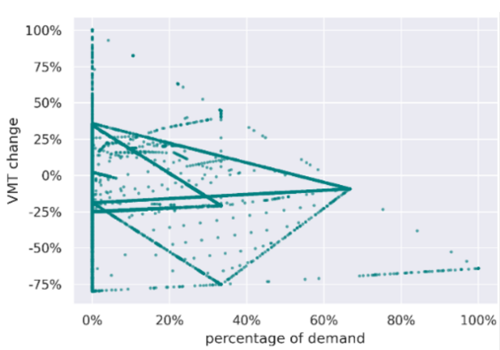
U-district Seattle is the home to the University of Washington Seattle campus and is also a major residential area in Seattle. In the following numerical experiment, I treat U-district in Seattle as the residential area that generates the commute demand. Commuters from U-district can commute to downtown Seattle for work by four modes, single/shared rides for the whole trip or the first mile (then transfer to transit). The following numerical experiment will analyze how commuters' mode choices change with their value-of-time, and the resulting congestion pattern. The numerical experiments on the Seattle network will bring us more insights into the operation of the CAV integrated transit system, as well as help with building



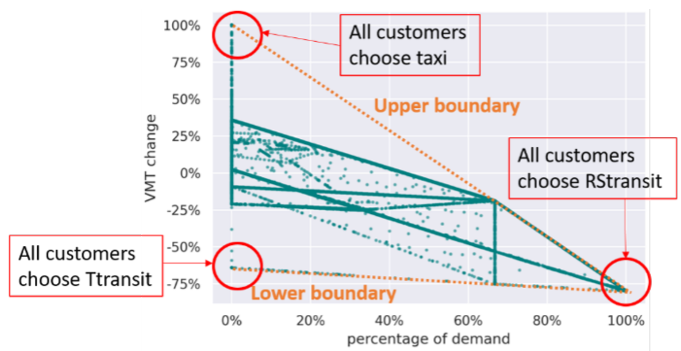
(a) Demand of taxi (m=1)



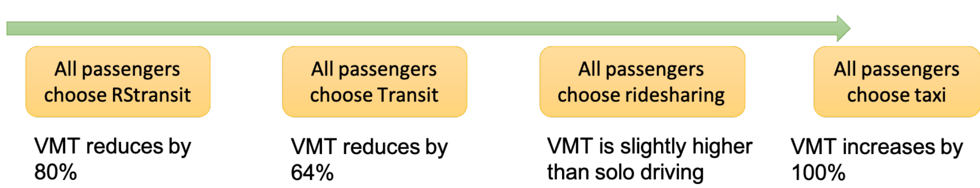
(b) Demand of ridesharing (m=2)



(c) Demand of Ttransit (m=3)



(d) Demand of RStransit (m=4)



(e) Summary of corner cases

Figure 4.6: Demand versus VMT change

the next-generation TDM strategies that increase the mobility of the urban city and relieve congestion simultaneously.

4.4.1 Data processing: simplification of the Seattle U district network

The network provided by PSRC (2021) includes 439 nodes and 1241 links (Figure 4.7). I simplify this network (combine nodes that are too close to each other, combine or eliminate small links, etc.) so that it only includes 151 nodes and 491 links, as shown in Figure 4.8. The simplified network consists of two types of links: (i) the original links, i.e., blue links in Figure 4.8; (ii) new links, i.e., red links in Figure 4.8. The links that are cut into smaller sections are replaced with new links that directly connect the start node and the end node. For the morning commuting scenario, I assume that all passengers (customers for the integrated transit system) travel from U-district to Seattle downtown area. The location of the Westlake link station is the destination node, as shown in Figure 4.9. I also assume that each passenger chooses one of the three routes from U-district to downtown area, and the three routes are simplified as three hyperlinks, see the three links that connect U-district with downtown in Figure 4.9. Link properties, such as length, free flow time, and capacity are modified for the new links.

The demand from U district is aggregated at the Traffic Analysis Zone (TAZ) level. Table 4.6 summarize the label of the TAZ selected from U district and downtown area. U district has 50 TAZs, downtown has 137 TAZs. Figure 4.10 shows these TAZs in a map. In U district, one node is selected within each TAZ and set as an origin. The origin nodes are marked as the yellow dots in Figure 4.8. The demand from the origin nodes to destination node 151 is shown in Table 4.7. I aggregated the demand of the 137 TAZs in downtown area, i.e., in Table 4.7, the demand from node 25 to node 151 is 218, meaning that the demand from TAZ 221 to the whole downtown area is 218 people per hour.

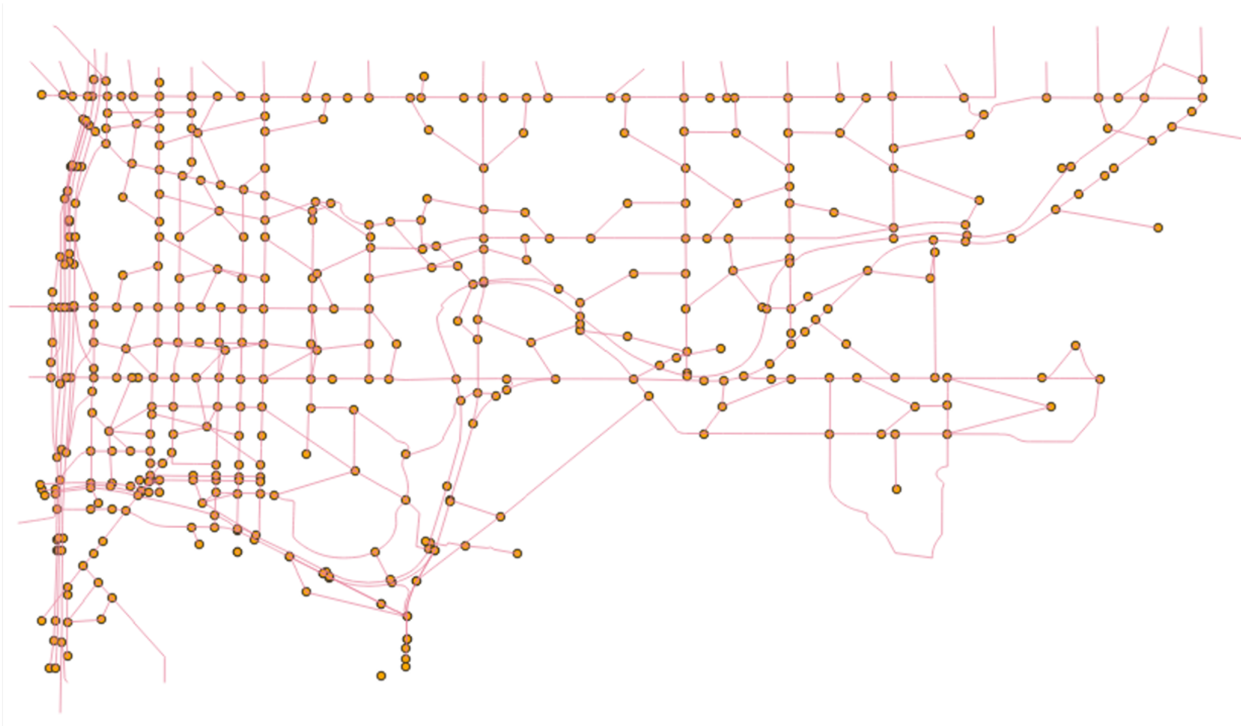


Figure 4.7: U-district: the original network (439 nodes, 1241 links)

Table 4.6: TAZs in U district and Downtown

	The list of TAZs
U-district	201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308
Downtown	441, 438, 439, 437, 435, 433, 431, 429, 607, 616, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 609, 610, 611, 612, 613

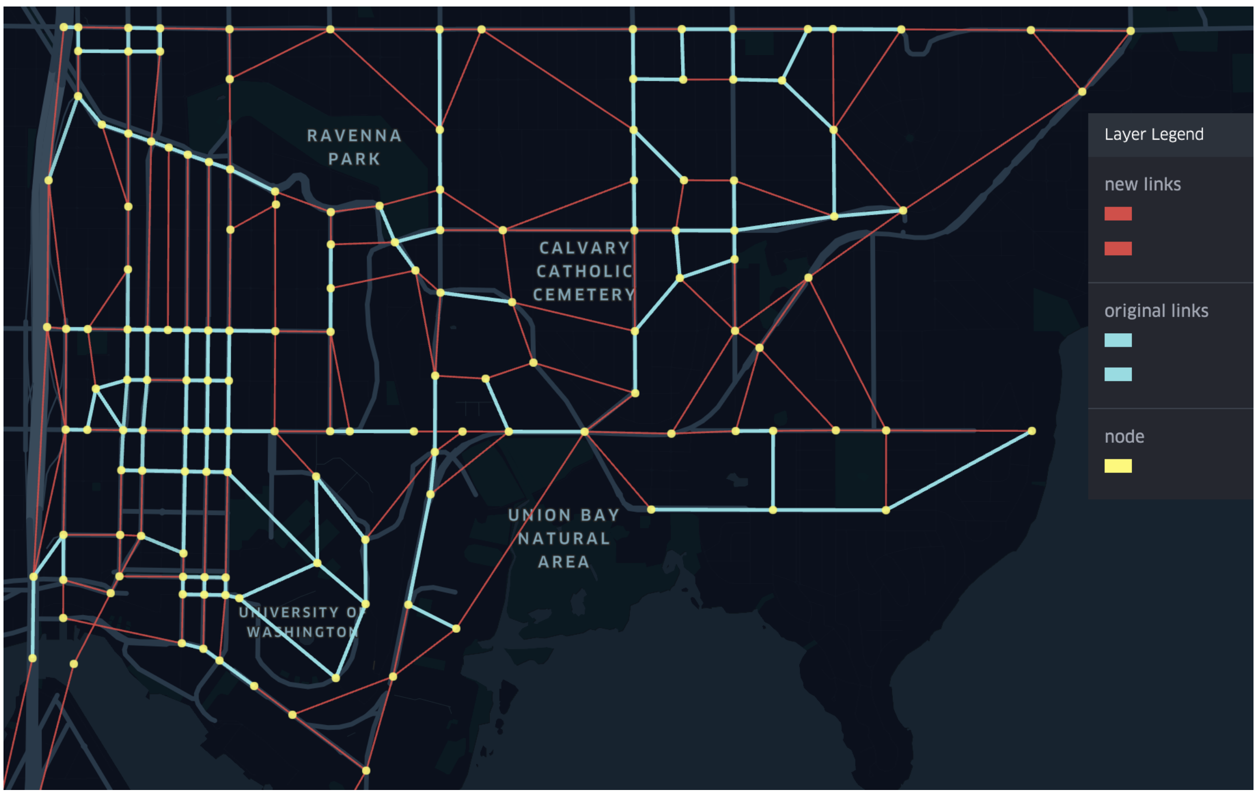


Figure 4.8: U-district: simplified network (151 nodes, 491 links)

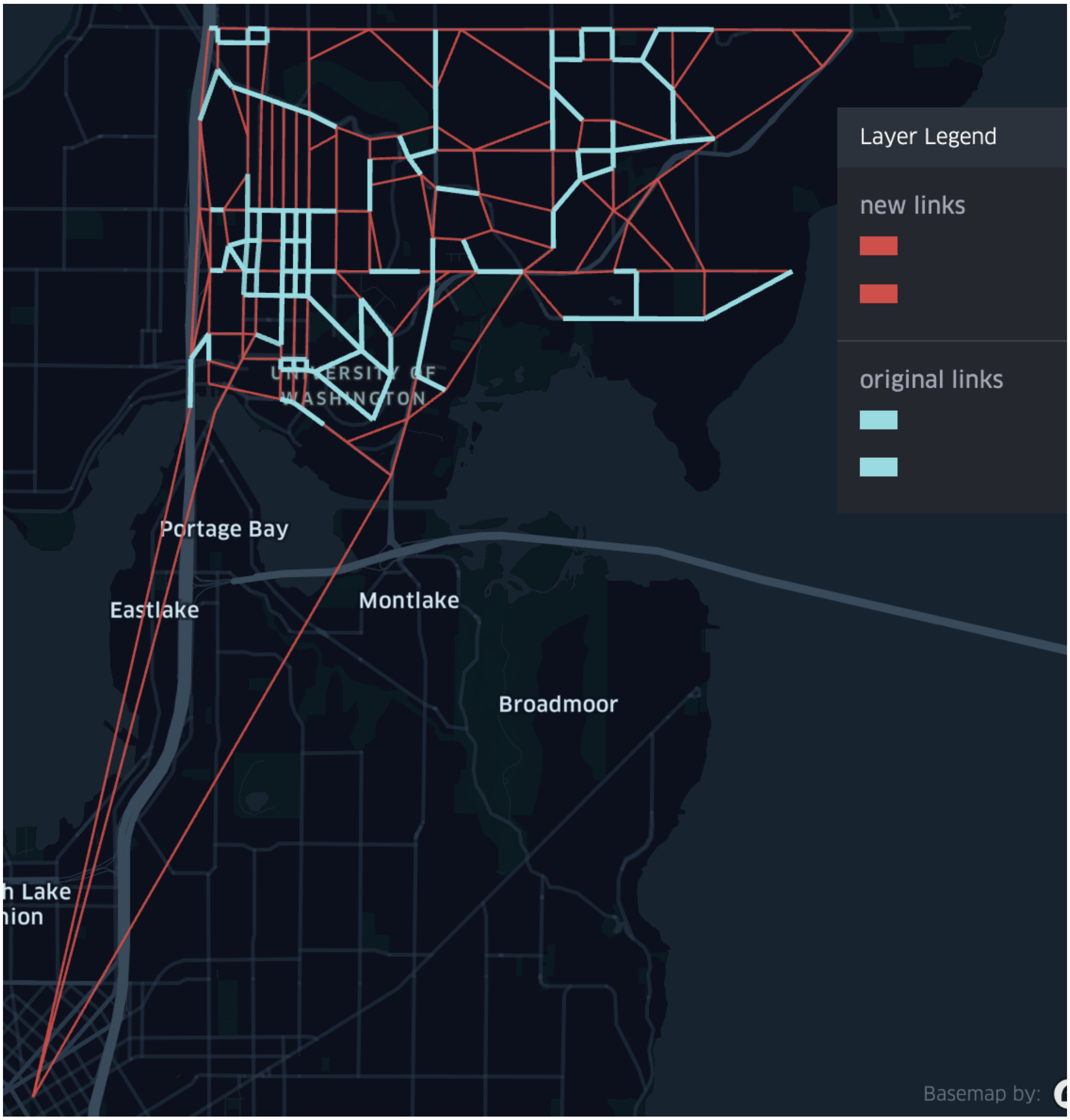


Figure 4.9: Hyperlinks to Seattle downtown area

Figure 4.10: TAZs of the Seattle area (Seattle downtown is marked using the red box)

Table 4.7: Demand Table of the Seattle Network

origin - destination	demand		origin - destination	demand		origin - destination	demand
25 - 151	218		33 - 151	195		83 - 151	66
17 - 151	224		18 - 151	124		74 - 151	124
15 - 151	67		26 - 151	125		58 - 151	122
45 - 151	89		55 - 151	125		75 - 151	279
13 - 151	81		147 - 151	235		103 - 151	151
38 - 151	90		47 - 151	344		114 - 151	76
11 - 151	95		79 - 151	475		113 - 151	269
146 - 151	63		59 - 151	205		112 - 151	95
10 - 151	90		91 - 151	152		150 - 151	808
37 - 151	125		57 - 151	185		1 - 151	0
9 - 151	78		93 - 151	100		142 - 151	1546
40 - 151	89		80 - 151	68		107 - 151	539
8 - 151	103		49 - 151	56		127 - 151	36
43 - 151	23		148 - 151	252		134 - 151	15
7 - 151	69		50 - 151	7		105 - 151	508
39 - 151	122		76 - 151	63		133 - 151	9
20 - 151	206		149 - 151	101			

4.4.2 Ridesharing pairs (RS pairs)

In the small network, because there are only 3 origin nodes, shared rides can happen between each pair of origins. For a large network, such as the Seattle network, it will be computationally expansive and practically uneconomical to allow shared rides between each pair of origin nodes. For example, if a shared ride vehicle first picks up a passenger at node 132 (node 132 is at the left-down area in Figure 4.11), it does not make sense for the platform to allow this vehicle to pick up the second passenger at node 16 (node 16 is at the right-up area in Figure 4.11) and then go to the destination in the downtown area. Note that our model chooses from the top k shortest paths between each OD pair (origin-destination pair) and RS pair. If one have one path between each RS pair, there will be 1*50*(50-1)=2450 paths for RS pairs. With a large number of paths, the running time of the MCP solver can be several hours, and it will be time-consuming for us to conduct the sensitivity analysis. To reduce the computation time, I assume that the distance between the two nodes in an RS pair is less or equal to 1 mile, shown in Figure 4.11. Under this assumption, there are 1164 RS pairs in total. The average number of RS pairs from a node is 23.28, which means after picking up the first passenger from an origin node, there are 23 among the other 49 nodes that a shared-ride vehicle can choose from to pick up the second passenger. Table 4.8 summarizes the statistics of the number of RS pairs from a node. The 25% percentile of the number of RS pairs a node belongs to is 18, meaning that 3/4 of nodes are the origins of more than 18 RS pairs.

Table 4.8: Statistics of the number of RS pairs from a node

mean	**std**	**min**	**25%**	**50%**	**75%**	**max**
23.28	7.761864	4	18	24	27.75	37

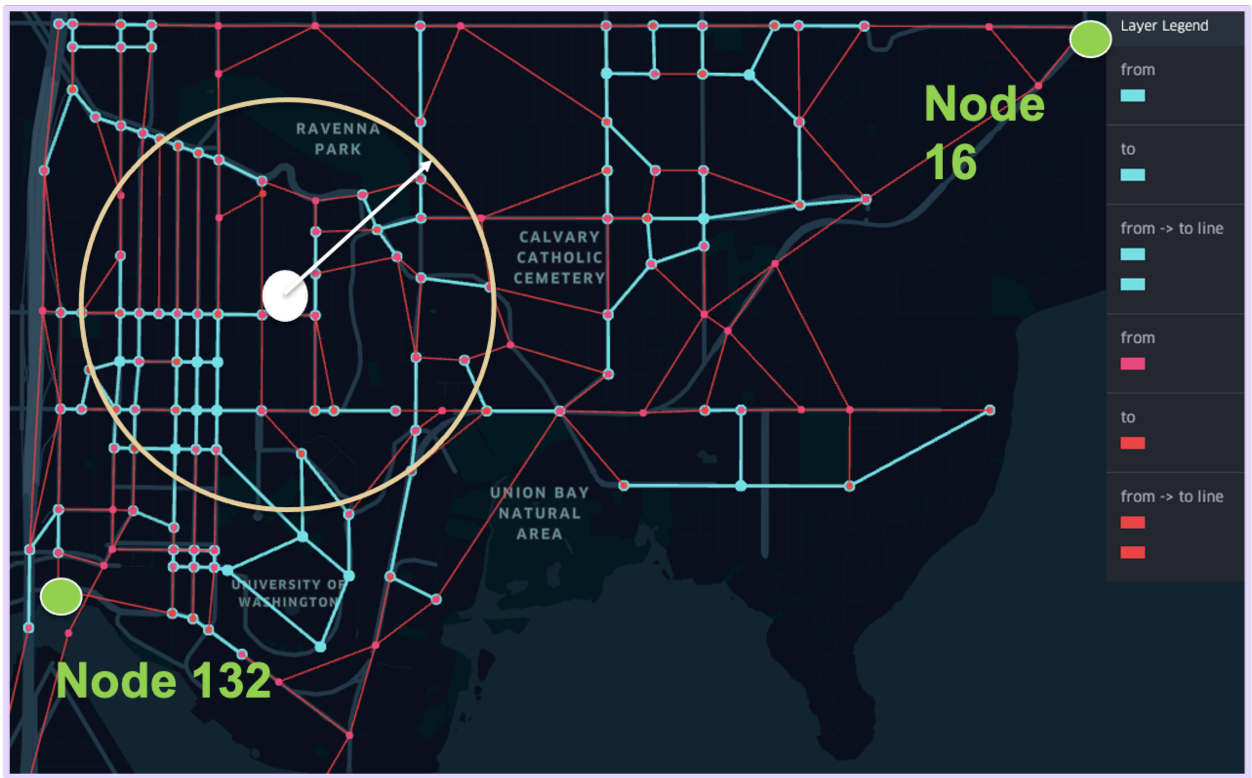


Figure 4.11: Select two nodes within 1 mile distance as RS pairs

4.4.3 Cost parameters

Table 4.9 lists the parameters of the U-District network. Table 4.9 reflects the following relationship between the fares of different modes: single ride > shared ride > transit, while customer disutility rates follow the relation: single ride > shared ride. When conducting the sensitivity analysis later in this section, I modify some of these baseline parameters.

Table 4.9: Baseline parameters (Seattle network)

Illustration of parameters	**Notation (m =1,2,3,4)**	**Value**
The fixed fare for different modes ($)	F^m	6.8, 5.8, 6.8, 5.8
Time-based fare rate ($/h)	α_1^m	1.1, 0.5, 1.1, 0.5
Distance-based fare rate ($/mile)	α_2^m	1.5, 1.2, 1.5, 1.2
Conversion factor from time to cost ($/h)	β_1^m	3, 2.6, 3, 2.6
Conversion factor from distance to cost ($/mile)	β_2^m	1.2, 1.1, 1.2, 1.1
Value of time of customers, while traveling ($/h)	γ_1^m	3, 2.8, 3, 2.8
Value of time of customers, while waiting ($/h)	γ_2^m	3, 2.8, 3, 2.8
Value of time of customers, while mathing in shared rides ($)	$\gamma_3^m, (m = 2, 4)$	NA, 1, NA, 1
Travel distance-based fare rate of transit ($/h)	α_3	0.5
Conversion factor from distance to cost for transit ($/mile)	γ_4	0.5
Transfer cost of transit ($/transfer)	γ_5	0.1

4.4.4 Results of unilaterally changing one parameter

Table 4.10 shows the results when I unilaterally change the travel distance-based fare rate of single rides. Under the current baseline parameter setting (Table 8), the demand for serving the whole trip using ridesharing (m=2) is always zero, indicating that customers prefer the integrated modes or taxi over ridesharing. When taxi has low travel distance-based fare rate ($\alpha_2^1 = 0.8$), the majority of customers (98.89% customers) choose taxi. When I unilaterally increase the distance-based fare rate of single rides ($\alpha_2^1 = \alpha_2^3$), the cost of requesting taxi increases, more customers switch to request Ttransit, thus the demand for taxi mode decreases while the demand for Ttransit increases. When the distance-based fare rate of single rides ($\alpha_2^1 = \alpha_2^3$) continues to increase to 1.4, around 29% customer switch from Ttransit to RStransit, since for some routes, taking RStransit cost less than Ttransit. This example shows when I increase the cost parameter of taxi, customer choice of taxi will decrease, and the VMT of the entire network decreases. The VMT decreases because customer switch from high VMT mode taxi to low VMT mode Ttransit and RStransit.

Table 4.10: Unilaterally change α_2^1

$\alpha_2^1 = \alpha_2^3$	**Taxi, m=1**	**Ridesharing, m=2**	**Ttransit, m=3**	**RStransit, m=4**	**VMT (miles)**
0.8	98.89%	0.00%	1.11%	0.00%	97402.20
0.9	32.36%	0.00%	67.64%	0.00%	46892.46
1.4	27.75%	0.00%	72.25%	0.00%	44019.52
1.5	27.36%	0.00%	43.29%	29.35%	38618.89
2	27.36%	0.00%	25.77%	46.87%	36500.68

4.4.5 Mode choices vs. VMT change

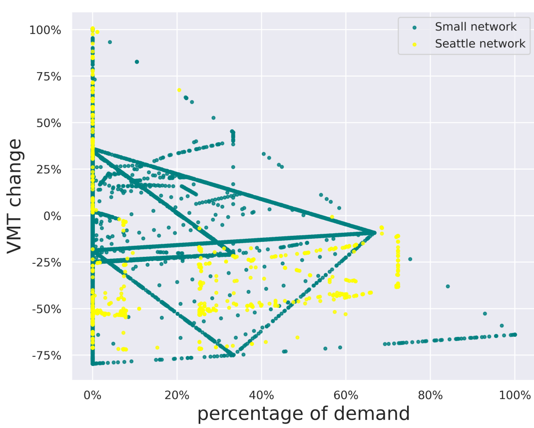
Similar to section 4.3.3, I compare the VMT of a certain mode split scenario to the case when all customers drive alone in a personal vehicle. When all customers choose solo driving (no ridesourcing or transit involved), the VMT is equal to 49414.34 vehicle miles based on the UE solution. Thus, the VMT change can be calculated as:

VMT change= (VMT of a particular scenario-49414.34)/49414.34

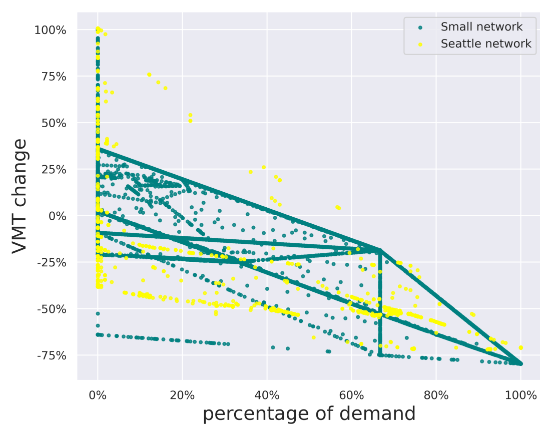
Figure 4.12a – 4.12d shows the relation between the demand (expressed in terms of the percentage of the total demand) of a particular mode $m \in 1, 2, 3, 4$ and the VMT change. I plot the results from the small network and the results from the Seattle network in the same graph. Figure 4.12, shows that the results from the Seattle network lies in the same triangular region as the small network, which indicates that our findings in the small network can be generalized to large-scale real networks.

(a) Demand of taxi (m=1)

(b) Demand of ridesharing (m=2)



(c) Demand of Ttransit (m=3)



(d) Demand of RStransit (m=4)

Figure 4.12: Demand versus VMT change (Seattle network))

4.5 Discussion

In the numerical experiments in section 4.3 and 4.4, I change the parameters of the four modes, including the fixed fare, time-/distance-based fare, time-/distance-based disutility, customer waiting time cost, transfer cost, and matching cost. In reality, customers choose one mode over another based on various factors. By setting different parameters, one can better reveal customers' mode choices. It is intuitive that when I increase the disutility parameters for some mode, fewer customers will choose that mode. The results can capture this properly (Table 4.3, 4.4 & 4.10). The proposed model can also capture more complex scenarios due to the interactions of different modes. For example, when I change the parameters of transit, the disutility of both Ttransit and RStransit changes. When I change the parameters of shared rides, the disutility of both ridesharing and RStransit changes. This resembles the interactions of ridesourcing companies and transit agencies in practice. When a ridesourcing company changes the price of its shared-ride service, it may affect ridesharing and RStransit differently. Similarly, the disutility due to transit transfer may change when the transit company make changes to its routes, stops, schedules, and fare. The increased transfer cost of taking transit may affect Ttransit and RStransit differently. Therefore, when there are changes to the parameters of transit and shared rides, how customers exactly react to such changes can be quite complex, which can be captured by the model proposed in this dissertation as shown in Figures 4.4 & 4.5.

On the other hand, different mode splits (resulting from customers' mode choices) may lead to varied VMT changes compared to all customers driving alone. Numerical experiments (Figure 4.6e in particular) show that higher usage of transit or shared rides reduces the VMT. Shared rides alone without transit (i.e. the ridesharing mode (m=2)) may still lead to slightly higher VMT compared with solo driving due to the deadhead miles. The results thus clearly illustrate the important role of transit in serving commuters in urban areas. Although this study only considers fixed route, fixed schedule mass transit (sometimes less efficient compared to on-demand transit) in this dissertation, the numerical results show that if all customers choose to use transit, the VMT reduction can be tremendous: 64% if the first mile is served by single rides (for Ttransit) or 80% if the first mile is served by shared-rides (for RStransit), as shown in Figure 4.6e. Further, the numerical results in Section 4 are for a morning commute scenario where travel demands are extremely asymmetric, i.e., all demands are from a common residential area to downtown worksites. This also explains why the VMT changes are large in general, from 80% reduction (RStransit only) to 100% increase (Taxi only). For more symmetric demand patterns, I expect the VMT changes are milder; see Ban *et al.* (2019). However, I believe that the general trend of the changes should be the same.

The operational level model can thus help provide insights to transit agencies and ridesourcing companies for making sensible policies related to their operations and for the potential collaboration on integrated ridesourcing and transit systems. The ability to capture the network-wide congestion effect is also a highlight of the model since the evaluation of the congestion level in a multimodal network is important, especially if CAVs are widely deployed.

Chapter 5

CONCLUSION REMARKS AND FUTURE WORK

5.1 Planning level analysis

The planning level model proposed the concept of *commuting service platforms* (CSP) and applied the two-sided market analysis methods to study the demand-price relation and network effects in a market where employers and employees are directly connected by the envisioned CSP. A benchmark model was proposed to clarify the definition of the two-sidedness and the threshold of subsidization. Models for both the monopoly and the duopoly platforms were constructed and analyzed. The models were further extended to consider locations of homes and worksites. The analyses presented in this dissertation allow us to obtain a basic understanding of CSP at the planning level and the interactions of its major players (employers, employees, and the platform), as well as how such interactions may impact the participation of the platform and its prices and profit. Such analyses and findings can help gain useful insights on how to build CSPs and how to develop associated TDM strategies in practice.

In the monopoly case, more agents from the two sides will join the CSP when either of their prices decreases. Under the given parameter settings, the CSP can use commuters as a loss leader and attract participation from both sides; the opposite may occur if different parameters are chosen. The increase of the same-side "congestion" effects from either side will reduce CSP's profit. However, the CSP is able to maintain its profit for a range of the same-side "congestion" effects. In the duopoly model, the participation from the two sides is affected by the relative price of the two CSPs ($p_N^B - p_W^B$ and $p_N^C - p_W^C$). Relatively lower prices on a CSP increase its participation. Consistent with the monopoly model, the cross-side positive effects encourage participation on a CSP. However, high overall cross-side effects intensify the competition between the two CSPs and no CSP can achieve high profits. Demand constraints are effective in controlling the number of employees on each worksite. Under the demand constraints, both CSPs make good profits under most parameter settings (α^- and α^+), which indicates that both CSPs will sustain in this two-sided market.

The proposed CSP and the two-sided market based analysis methods and results presented in this dissertation are just an initial step toward building practical CSPs and TDM strategies. The proximate commute scenario studied here is also a very simplified version that a general CSP can help serve. However, I believe that the envisioned CSP and the two-sided market analysis methods are the critical building block to develop future mobility systems (i.e. CSP) to solve commuting challenges. Future research will extend the above analyses in several important ways.

- I will investigate how to properly capture and model key components of the CSP, such as the key parameters (see Table 3.3), utility functions, and demand functions of employees and worksites/employers in their commuting decisions. For this, understanding their behaviors in terms of commuting decisions is crucial and should be investigated.

- I will relax some of the assumptions imposed in this dissertation, e.g., Assumption (b), to consider how the numbers of worksites and commuters may change with respect to CSP and related policies.

- The general case presented in Section 3.5.2 merits further extensions (e.g., by considering more practical constraints/interactions among the players) and investigations (e.g., the proprieties of the resulting GNEP model).

- I need to extend proximate commute to more general commuting scenarios. For this, I will model how employees are matched with employers, with commuting services as one option provided by employers.

- I only focus on the planning-level analysis of CSP in this dissertation. To be more practical, one will need to consider the myriad existing and future mobility services and think about innovative ways to combine them into an integrated CSP to serve practical commuting needs. To do so, one need to study the *operational level* challenges of CSP. Our operational level analysis is an initial endeavor in this direction, but there are more aspects to consider. For example, designing the optimal number of commuters that each vehicle picks up, the matching of multiple commuters to one serving vehicle, the optimal pick-up locations, the optimal routes for the CSP vehicles in real-time, how to model the transfers between different modes, how employers' decisions may impact commuting service operations, etc. Such CSP operational issues are challenging and merits further investigations.

- Understanding the behaviors and interactions of the major players (employers, employees, and platforms) with respect to commuting options (e.g., WF and NWF) and the operations of CSPs can be leveraged to develop the next-generation TDM strategies for commuting that take advantage of the proposed CSP and emerging mobility options. For this, different objectives of the CSP (such as profit maximization, welfare maximization, and others) should be investigated and compared to derive useful insights. I will pursue these topics in future research and results may be reported in future papers.

- Future studies can expand the general model in section 3.5.2 to include the dimension of mode choice, i.e., investigate how the preferences and demands of employees and employers change across different types of modes. Single-ride service is the most expensive type of mode. The employer side is likely to have few or no subscriptions for the single-ride service. Thus, employees need to pay at the full price most of the time if they choose single-ride. In comparison, RStransit is the least expensive type of mode. The employer side is likely to subscribe for RStransit and subsidize most of the cost. The employee can get to work by RStransit at a low price or for free. In addition to the four types of modes studied in the operational level analysis, the CSP can incorporate a variety of services. When transit is not available, the CSP can send one group of vehicles to pick up commuters and send them to the closest station, where a minivan can pick up all commuters from the station and send them to their worksite. Future studies can investigate the optimal location of the station and how to minimize waiting time and transfer time.

- Future research can study how CSP can benefit the hybrid working scenario. One can expand the general model described in section 3.5.2 so that there can be multiple employers joining the CSP. The CSP can predict the commuting trip demand of each company on each day and assign vehicles to serve their commuting trips. Different companies tend to have different anchor days, so that the CSP can have a stable supply of vehicles to provide commuting services to all companies on each day of the week. The CSP can also leverage the demand by setting lower prices for commuting trips on the day of the week that has the lowest demand.

5.2 Operational level analysis

The era of CAVs may bring about a significant reduction in car ownership. This dissertation envisions a morning commuting scenario when there is one ridesourcing platform operating all CAVs, providing single- or shared-ride services for commuters. CAVs can either send customers to their worksites or send them to transit hubs (major stations) to take transit to the worksites. This resulted in four modes for a customer to choose: taxi, ridesharing, Transit, and RStransit. I modeled the value of time/inconvenience of customers as an overall disutility by choosing a mode, encompassing various factors such as distance, waiting, matching, transfer, and the fare of the mode. At equilibrium, the ridesourcing company maximizes its profit, each customer chooses the mode with the lowest disutility, and the vehicular flow is assigned to the network routes based on Wardrop's first principle, revealing the network congestion. The proposed model captures the behavior and interactions of the ridesourcing platform and the customers and can assess the congestion level of the network. I also discussed the potential insights of the modeling methods and results for integrating CAV-based ridesourcing services with the fixed-route mass transit. Future studies are recommended in the following areas:

- Test the proposed model on large, real-world networks. I tested the validity of the operational level model using the small network and then applied the model to the Seattle U district network. Future studies can be conducted to apply the proposed model to the entire Seattle City. The computation can be expensive for large real-world networks. The scalability of the model will be the main challenge of such studies.

- Extend the current model to include other ridesourcing modes such as on-demand transit (i.e., micro-transit) and bikeshare services. When adding on-demand transit to the current model, the profit/efficiency of transit needs to be considered. When adding bikeshare services to the current model, the demand-supply balance of bikes in each zone needs to be considered. One way to do this is to use the pricing strategy, i.e., set low prices for bike trips to zones that have low bike count and high demand for bikes; and set high prices for bike trips to zones that have high bike count and low demand for bikes

- Collaborate with ridesourcing companies to test the model using large-scale real-world trip data. I can use the trip price data from ridesourcing companies as the trip price parameters in our model. Human behavior data will also be useful. Currently, the customers' disutility function consists of major factors such as waiting/matching related costs or travel time/distance-based costs. Alternatively, I can estimate customer disutility using passenger behavior data. I can compare the two methods to discuss the pros and cons of using data to estimate customer disutility.

- Test the model for more general scenarios. Currently, I only consider the morning commuting scenarios. The operational model can also test the network where the demand is symmetric between each OD pair, i.e., for the Seattle network, there will be demand from U district to downtown area, as well as demand from downtown area to U district.

5.3 *At the end*

This dissertation covers the basic planning level and operation level analysis of CSP. The planning level model focuses on the consumer incentives on the commuter side and the employer side, i.e., incentives to attract companies and their employees to join the platform, elastic pricing for flexible working start time, and providing convenient company-wise commuting services. The operational level model focuses on providing alternative commuting modes, the integration of ridesourcing and transit, and a sustainable commuting operation strategy that considers network congestion.

Remote work has become more common since Covid. Less commuting hours is the top reason for more employees to work from home (Teevan *et al.*, 2021). Other reasons include the convenience of finishing family-related tasks such as taking care of their kids. On the other hand, employers still want employees to work at the office so that some meetings can be held in person to enhance communication. In order to balance the preference of employers and employees, hybrid work has become widely adopted by companies, which allows employees to work from home 2-3 days each week and work at the office on anchor days. Working hours are more flexible in the case of hybrid work. Employees can choose to join the meetings in person but go back home to finish individual tasks. Section 3.4 analyzed the pricing of WF and NWF CSP. The CSP can set lower prices for the agents on the WF CSP so that more employees will choose WF CSP, which will reduce the peak hour congestion. Some existing commuting services such as shuttle is not efficient in the hybrid working scenario because of the oscillation in commuting demand. The demand of commuting trips is high on anchor days and low on remote work days. The employers however usually have a fixed fleet for the shuttle service, which is a waste of fleet resources on remote working days. CSP however, has the potential to gather data from both the employer side (location, anchor day, flexible work hours, etc) and the employee side, which helps to balance vehicle supply and commuter demand.

In recent decades, we have witnessed the landscape change in cities motivated by big corporations. A relaxed urban transportation network and efficient commute services are the common wishes of companies and residents of the city. With the development of the smart, connected city and the wide acceptance of hybrid work since COVID, I look forward to future studies and practices on the systematic improvement of urban mobility with elasticity on time/location of travel and road network-level applications of two-sided market theory.

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Appendices

Appendix 1

A.1 Proof of Theorem 1

This proof mainly refers to the utility function in Equation (3.1), and the profit function in Equation (3.3). First, rewrite the profit function as: (3.3) as

$$R = \sum_{(k,\ell)\in\{(B,C),(C,B)\}} \Big[(U_0^k + b^k\phi^\ell(U^\ell) - U^k) - f^k\Big]\phi^k(U^k) \quad (3)$$

Notice that here the behavior of commuters/worksites is modeled using the demand function $\phi^k(U^k)$ for side k in order to derive the Lerner Index. This allows us to derive the equilibrium prices of the CSP using the demand functions of both sides. In particular, CSP can optimize its profit by using the first order conditions of R w.r.t. U^B and U^C:

$$\frac{\partial R}{\partial U^k} = -\phi^k(U^k) + \Big[(U_0^k + b^k\phi^\ell(U^\ell) - U^k) - f^k\Big]\frac{\partial\phi^k(U^k)}{\partial U^k} + b^\ell\phi^\ell(U^\ell)\frac{\partial\phi^k(U^k)}{\partial U^k} = 0 \tag{A.1}$$

$$0 = -\phi^k(U^k) + \Big[U_0^k + (b^k + b^\ell)\phi^\ell(U^\ell) - U^k - f^k\Big]\frac{\partial\phi^k(U^k)}{\partial U^k} \tag{A.2}$$

$$\frac{\phi^k(U^k)}{[\phi^k(U^k)]'} = U_0^k + (b^k + b^\ell)\phi^\ell(U^\ell) - U^k - f^k \tag{A.3}$$

$$U^k = -\frac{\phi^k(U^k)}{[\phi^k(U^k)]'} + U_0^k + (b^k + b^\ell)\phi^\ell(U^\ell) - f^k \tag{A.4}$$

Comparing (A.4) with $U^k = U_0^k + b^k q^\ell - p^k$ yields the optimal prices in equation (3.4),

$$p^k = f^k - U_0^k - b^\ell\phi^\ell(U^\ell) + \frac{\phi^k(U^k)}{[\phi^k(U^k)]'} \quad \forall k,\ell \in \{B,C\} \text{ and } k \neq \ell$$

A.2 Proof of Lemma 1

This proof first defines **platform independence** and **weak dependence**. The two CSPs (the WF CSP and the NWF CSP) in the duopoly model are: 1) *independent* if the equilibrium solution (participation, price, etc.) of one CSP only depends on the parameters of that particular CSP (e.g., f_W^B as the cost for the worksite side if using the WF CSP) and the parameters that are common to both CSPs (e.g. the initial utility of joining the platforms for worksite, U_0^B); and 2) "weakly dependent" if the equilibrium solution of one CSP mainly depends on the parameters of that CSP and the common parameters with one more parameter or variable that introduces specific interactions between the two CSPs (as reflected in their solutions). Recognize that this weak dependence definition is a bit vague, which will become clearer when explained using some examples below.

The proof assumes that there are no participating agents on both sides, i.e., $q_W^B + q_N^B + Q^B < 1$ and $q_W^C + q_N^C + Q^C < 1$. The proof starts by enumerating all possible corner cases, based on the possible values of demands (positive or zero). Consider one side, say the worksites, based on the demand variables, q_W^B, q_N^B, Q^B, there are eight possible cases: 1) $q_W^B > 0, q_N^B > 0, Q^B > 0$; 2) $q_W^B > 0, q_N^B > 0, Q^B = 0$; 3) $q_W^B > 0, q_N^B = 0, Q^B > 0$; 4) $q_W^B = 0, q_N^B > 0, Q^B > 0$; 5) $q_W^B = 0, q_N^B = 0, Q^B > 0$; 6) $q_W^B > 0, q_N^B = 0, Q^B = 0$; 7) $q_W^B = 0, q_N^B > 0, Q^B = 0$; and 8) $q_W^B = 0, q_N^B = 0, Q^B = 0$. We may call cases 1), 3), 4), and 5) as multihoming cases, and 2), 6), and 7) single homing cases, while case 8) has no agent joining any platform.

The proof can be conducted by enumerating all possible scenarios (when considering both sides): there are $8 \times 8 = 64$ scenarios in theory considering the possibilities on the two sides. This number can be reduced since 8) above is not interesting to us, while 6) and 7) lead to a single platform, which all lead to independent platform (as there is no or only a single platform). So these three cases can be dropped and the corresponding scenarios. The next theorem shows that Q^B can be readily calculated for case 1).

Theorem 4. *In case 1) above,* $Q^B = U_0^B / t^B$ *when* $q_W^B + q_N^B + Q^B < 1$.

Proof. At the equilibrium, since $q_W^B + q_N^B + Q^B < 1$, so that $U_W^B = U_N^B = U_{NW}^B = 0$ as according to the assumption that the utility of not joining any CSP is zero. Then by comparing equations (3.15) - (3.17), one can derive that $Q^B = U_0^B / t^B$.

As shown in the theorem above, since Q^B in case 1) can be expressed by common parameters

(U_0^B, t^B), the analysis of case 1) is very similar to that of case 2) with $Q^B = U_0^B/t^B$. Thus we can focus on case 2) only. Furthermore, case 5) is a special case to case 3) and case 4) is symmetric to case 3). This proof thus focuses on case 2) and case 3) in the following proof. Here case 2) is the singlehoming case and case 3) is the multihoming case. This will result in three scenarios: both sides singlehome, both sides multihome, and one side singlehome and the other side multhome. It is shown next that for all three scenarios, if there are no participating agents on both sides ($q_W^B + q_N^B + Q^B < 1$ and $q_W^C + q_N^C + Q^C < 1$), the two CSPs are independent or weakly dependent.

Scenario 1: both sides singlehome. Under this scenario, $q_W^B > 0, q_N^B > 0, Q^B = 0$ and $q_W^C > 0, q_N^C > 0, Q^C = 0$. The utilities of worksites and commuters can be expressed as:

$$U_W^B = U_0^B - p_W^B - t^B q_W^B + \alpha_W q_W^C; \qquad U_N^B = U_0^B - p_N^B - t^B q_N^B + \alpha_N q_N^C \tag{A.5}$$

$$U_W^C = U_0^C - p_W^C - t^C q_W^C + \beta_W q_W^B; \qquad U_N^C = U_0^C - p_N^C - t^C q_N^C + \beta_N q_N^B \tag{A.6}$$

Since there are agents who do not join any CSP, the optimal utility is zero. The optimal prices can be derived as:

$$p_W^B = U_0^B - t^B q_W^B + \alpha_W q_W^C; \qquad p_N^B = U_0^B - t^B q_N^B + \alpha_N q_N^C \tag{A.7}$$

$$p_W^C = U_0^C - t^C q_W^C + \beta_W q_W^B; \qquad p_N^C = U_0^C - t^C q_N^C + \beta_N q_N^B \tag{A.8}$$

Substituting the optimal prices into the profit of CSPs yields:

$$\begin{aligned} R =& q_W^B(p_W^B - f_W^B) + q_N^B(p_N^B - f_N^B) + q_W^C(p_W^C - f_W^C) + q_N^C(p_N^C - f_N^C) \\ =& - t^B(q_W^B)^2 - t^B(q_N^B)^2 - t^C(q_W^C)^2 - t^C(q_N^C)^2 + (\alpha_W + \beta_W)q_W^B q_W^C + (\alpha_N + \beta_N)q_N^B q_N^C \\ &+ q_W^B(U_0^B - f_W^B) + q_N^B(U_0^B - f_N^B) + q_W^C(U_0^C - f_W^C) + q_N^C(U_0^C - f_N^C) \end{aligned} \tag{A.9}$$

The optimal values of the demands (Q's and q's) can be obtained by setting the partial derivative of R to each demand variable to zero (notice that the constraints $q_W^B + q_N^B < 1$

and $q_W^C + q_N^C < 1$ are not binding), i.e.,

$$-2t^B q_W^B + (\alpha_W + \beta_W) q_W^C + U_0^B - f_W^B = 0; \quad -2t^C q_W^C + (\alpha_W + \beta_W) q_W^B + U_0^C - f_W^C = 0 \tag{A.10}$$

$$-2t^B q_N^B + (\alpha_N + \beta_N) q_N^C + U_0^B - f_N^B = 0; \quad -2t^C q_N^C + (\alpha_N + \beta_N) q_N^B + U_0^C - f_N^C = 0 \tag{A.11}$$

Clearly, the worksite and commuter participation (demand) of the WF CSP (q_W^B, q_W^C) is only determined by Equation (A.10), while the participation of the NWF CSP (q_N^B, q_N^C) are only determined by Equation (A.11). Equation (A.10) only involves parameters of the WF CSP and common parameters, while Equation (A.11) only involves parameters of the NWF CSP and common parameters. This implies that the equilibrium participation of two CSPs is independent. It is easy to actually derive the equilibrium demands based on these two equations to verify this observation, which is omitted here. Similar results can also be found for the equilibrium prices.

Scenario 2: commuters singlehome and worksites multihome Without loss of generality, we have $q_W^B > 0, q_N^B = 0, Q^B > 0$ (i.e. worksites multihome) and $q_W^C > 0, q_N^C > 0, Q^C = 0$ (i.e. commuters singlehome). This leads to the following utilities for commuters and worksites:

$$U_W^B = U_0^B - p_W^B - t^B(q_W^B + Q^B) + \alpha_W q_W^C; \qquad U_N^B = U_0^B - p_N^B - t^B Q^B + \alpha_N q_N^C \tag{A.12}$$

$$U_{NW}^B = U_0^B - (p_W^B + p_N^B) - t^B(q_W^B + Q^B) + \alpha_W q_W^C + \alpha_N q_N^C; \tag{A.13}$$

$$U_W^C = U_0^C - p_W^C - t^C q_W^C + \beta_W q_W^B; \qquad U_N^C = U_0^C - p_N^C - t^C q_N^C + \beta_N q_N^B \tag{A.14}$$

Similarly to what is done for Scenario 2 above, at optimality (i.e. equilibrium), the utilities $U_W^B = U_{NW}^B = 0, U_W^C = U_N^C = 0$ (and $U_N^B <= 0$). The optimal prices and the corresponding profit function can be derived as:

$$
\begin{aligned}
R =& (q_W^B + Q^B)(p_W^B - f_W^B) + Q^B(p_N^B - f_N^B) + q_W^C(p_W^C - f_W^C) + q_N^C(p_N^C - f_N^C) \\
=& - t^B(q_W^B + Q^B)^2 - t^C(q_W^C)^2 - t^C(q_N^C)^2 + (\alpha_W + \beta_W)(q_W^B + Q^B)q_W^C + (\alpha_N + \beta_N)Q^B q_N^C \\
& + (q_W^B + Q^B)(U_0^B - f_W^B) - f_N^B Q^B + q_W^C(U_0^C - f_W^C) + q_N^C(U_0^C - f_N^C)
\end{aligned} \tag{A.15}
$$

Set the derivative of R to each demand variable to zero to obtain the optimal demands:

$$q_N^C = \frac{f_N^B}{(\alpha_N + \beta_N)} \tag{A.16}$$

$$Q^B = \frac{2t^C q_N^C}{\alpha_N + \beta_N} - \frac{U_0^C - f_N^C}{\alpha_N + \beta_N} \tag{A.17}$$

$$q_W^B = \frac{2t^C(U_0^B - f_W^B) + (\alpha_W + \beta_W)(U_0^C - f_W^C)}{4t^B t^C - (\alpha_W + \beta_W)^2} - Q^B \tag{A.18}$$

$$q_W^C = \frac{2t^B(U_0^C - f_W^C) + (\alpha_W + \beta_W)(U_0^B - f_W^B)}{4t^B t^C - (\alpha_W + \beta_W)^2} \tag{A.19}$$

In this scenario, the demand of multihome worksites and the demand of commuters on the NWF CSP only depend on the parameters of the NWF CSP or the common parameters. On the other hand, the worksite and commuter demands on the WF CSP (q_W^B, q_W^C are almost only determined by the parameters of the NW CSP or common parameters, except Q^B as in the expression for q_W^B. This shows that the WF and NWF CSPs are weakly dependent. Similar results can also be found for the optimal prices. It interesting to see that in this scenario, Q^B is only dependent on the NWF platform, which brings interactions to the WF platform via q_W^B. We call the two CSPs weakly dependent.

Scenario 3: both sides multihome. There are four possible cases in this scenario depending on the sign of q_W^B, q_N^B (one of them is zero and the other is positive) and q_W^C, q_N^C (one of them is zero and the other is positive). Next we show the case of $q_W^B > 0, q_N^B = 0, Q^B > 0$ and $q_W^C > 0, q_N^C = 0, Q^C > 0$. The other three cases can be analyzed in a similar way. First the utilities of worksites and commuters are:

$$U_W^B = U_0^B - p_W^B - t^B(q_W^B + Q^B) + \alpha_W(q_W^C + Q^C) \tag{A.20}$$

$$U_N^B = U_0^B - p_N^B - t^B Q^B + \alpha_N Q^C \tag{A.21}$$

$$U_{NW}^B = U_0^B - (p_W^B + p_N^B) - t^B(q_W^B + Q^B) + \alpha_W(q_W^C + Q^C) + \alpha_N Q^C; \tag{A.22}$$

$$U_W^C = U_0^C - p_W^C - t^C(q_W^C + Q^C) + \beta_W(q_W^B + Q^B) \tag{A.23}$$

$$U_N^C = U_0^C - p_N^C - t^C Q^C + \beta_N Q^B \tag{A.24}$$

$$U_{NW}^C = U_0^C - (p_W^C + p_N^C) - t^C(q_W^C + Q^C) + \beta_W(q_W^B + Q^B) + \beta_N Q^B; \tag{A.25}$$

The profit function is:

$$R = (q_W^B + Q^B)(p_W^B - f_W^B) + Q^B(p_N^B - f_N^B) + (q_W^C + Q^C)(p_W^C - f_W^C) + Q^C(p_N^C - f_N^C) \tag{A.26}$$

Similar to the analyses for Scenario 1 and Scenario 2, at the optimality, $U_W^B = U_{NW}^B = 0$ ($U_N^B <= 0$), and $U_W^C = U_{NW}^C = 0$ ($U_N^C <= 0$). This can help derive the optimal prices, which can be substituted into (A.26) to obtain the optimal demands. The details are omitted. The demand functions are:

$$Q^B = \frac{f_N^C}{(\alpha_N + \beta_N)} \tag{A.27}$$

$$Q^C = \frac{f_N^B}{(\alpha_N + \beta_N)} \tag{A.28}$$

$$q_W^B = \frac{2t^C(U_0^B - f_W^B) + (\alpha_W + \beta_W)(U_0^C - f_W^C)}{4t^B t^C - (\alpha_W + \beta_W)^2} - Q^B \tag{A.29}$$

$$q_W^C = \frac{2t^B(U_0^C - f_W^C) + (\alpha_W + \beta_W)(U_0^B - f_W^B)}{4t^B t^C - (\alpha_W + \beta_W)^2} - Q^C \tag{A.30}$$

Clearly Q^B and Q^C are determined only by the NWF CSP, while q_W^B and q^{WC} depend almost solely on the NW CSP except the involvement of Q^B in q_W^B and Q^C in q_W^C. These are also the only places where the interactions of the two CSPs occur. We thus call the two CSPs weakly dependent.

The above discussions of the three scenarios complete the proof of Lemma 1.

A.3 Proof of Part (i) of Theorem 2

Proof. singlehome case can be further divided into sub-cases: (1) all agents from group k prefer one CSP to another, $q^k_W q^k_N = 0, q^k_W + q^k_N = 1$; (2) agents from group k choose either CSPs, $q^k_W q^k_N > 0, q^k_W + q^k_N = 1$. To guarantee all agents singlehome, one needs to show that group B agents with the lowest utility does not want to multihome. (The proof of group C singlehoming can be derived similarly)

Case (1): all group B agents prefer the WF CSP to the NWF CSP, so that $U^B_W > U^B_N$. Here, the agents with the lowest utility is located at $x^k = 1$, and they are most likely to multihome. Evaluated at $x^B = 1$, the incremental utility of multihoming with respect to singlehoming is $U^B_{WN} - U^B_W|_{x^B=1} = U^B_0 - (p^B_W + p^C_W) - t^B + (\alpha_W q^C_W + \alpha_N q^C_N) - \big[U^B_0 - p^B_W - t^B + \alpha_W q^C_W\big] = -p^B_N + \alpha_N q^C_N$. We assume $U^k_W > U^k_N$, thus $U^B_W|_{x^B=1} > U^B_N|_{x^B=1}$ holds, from which yields $-p^B_W - t^B + \alpha_W q^C_W > -p^B_N + \alpha_N q^C_N$. Condition (A2) states that $t^B > \max\{\alpha_W q^C_W, \alpha_N q^C_N\}$, so $-p^B_W - t^B + \alpha_W q^C_W$ is negative. Hence the incremental utility of multihoming for group B is also negative. Similarly, one can prove that the incremental utility of multihoming for group C is also negative under (A2).

Case (2): agents from group k are most likely to multihome when they are indifferent between the two CSPs, namely, $U^B_W|_{x^B=x^*} = U^B_N|_{x^B=x^*}$. This is the lowest utility an agent experiences by choosing singlehome. Evaluate at location x^*, the incremental utility from multihoming with respect to singlehoming is $U^B_{WN} - \frac{1}{2}(U^B_W|_{x^B=x^*} + U^B_W|_{x^B=x^*}) = U^B_0 - (p^B_W + p^C_W) - t^B + (\alpha_W q^C_W + \alpha_N q^C_N) - \frac{1}{2}\big[2U^B_0 - p^B_W - p^B_N - t^B + \alpha_W q^C_W + \alpha_N q^C_N\big] = \frac{1}{2}\big[- p^B_W - p^B_N - t^B + \alpha_W q^C_W + \alpha_N q^C_N\big]$, given condition (A2), $t^B > \alpha_W q^C_W + \alpha_N q^C_N$, so the incremental utility is negative. ■

A.4 Duopoly model when workistes multihome

Worksites tend to view the CSPs as homogenous. In contrast, commuters tend to view the CSPs as heterogeneous because they often evaluate the level of service of a CSP based on various factors: (i) how convenient it is to choose a CSP based on a commuter's schedule;

(ii) total vehicle travel time on a CSP, which is likely to be different on the WF CSP and the NWF CSP; (iii) the comfort level of the CSP services, etc. In section 3.4.2, we assume that the same-side "congestion" effects are high enough (condition (B2)) so that no agents will multihome. Here we relax this constraint and allow worksites to multihome. Here are the conditions for the duopoly model where the worksites multihome and the commuters singlehome:

(C1) $U_0^B = 0$ and U_0^C is high enough so that commuters wish to join at least one of the CSPs

(C2) (i) $t^B = 0$, the cost of joining a CSP for worksites is low, so that it is possible for worksites to choose both CSPs; (ii) $t^C > \beta_N q_N^B + \beta_W q_W^B$, the same-side "congestion" effects of commuters are high, ensuring that commuters singlehome

(C3) $f_W^B < \min\{\frac{\alpha_W}{3}\}$, $f_i^C < \min\{\frac{3\alpha_W}{4}\}$: ensures that both CSPs are willing to serve worksites.

To find the equilibrium in this setting, the consistent demand configurations need to be characterized. We first list all the possible configurations of worksites in a duopoly model, and then show that worksites will always choose Configuration 1 under conditions (C1) $\sim$ (C3). Commuters singlehome under condition **(C2)** since Part (i) of **Theorem 2** still applies.

Configuration 1: worksites multihome

Given that $Q^B = 1, q_W^B = q_N^B = 0$, the fraction of commuters joining the WF CSP is determined by the Hotelling Model,

$$q_W^C = \frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W - \beta_N}{2t^C} \tag{A.31}$$

The number of commuters joining the NWF CSP is $1 - q_W^C$. We assume that when a worksite is indifferent between join and not join a CSP, it will join the platform. It is optimal for worksites to mutlti-home when $U_{NW}^B \geq \max\{U_N^B, U_W^B, 0\}$, i.e., $-(p_W^B + p_N^B) - t^B + \alpha_W q_W^C + \alpha_N q_N^C \geq \max\{-p_W^B - t^B x^B + \alpha_W q_W^C, -p_N^B - t^B(1 - x^B) + \alpha_N q_N^C, 0\}$, which implies that multihoming is preferred over singlehoming or not joining any of the CSPs. The first two inequalities can be written as,

$$p_W^B \leq (\frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W - \beta_N}{2t^C})\alpha_W \tag{A.32}$$

$$p_N^B \leq (\frac{1}{2} + \frac{p_W^C - p_N^C + \beta_N - \beta_W}{2t^C})\alpha_N \tag{A.33}$$

Profits of the CSPs are,

$$R_W = p_W^B - f_W^B + (p_W^C - f_W^C)(\frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W - \beta_N}{2t^C}) \tag{A.34}$$

$$R_N = p_N^B - f_N^B + (p_N^C - f_N^C)(\frac{1}{2} + \frac{p_W^C - p_N^C + \beta_N - \beta_W}{2t^C}) \tag{A.35}$$

Configuration 2: worksites singlehome on the WF CSP

The fraction of commuters joining the WF CSP is,

$$q_W^C = \frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W}{2t^C} \tag{A.36}$$

Worksites choose to singlehome on the WF CSP when $U_W^B \geq \max\{U_N^B, U_{NW}^B, 0\}$, i.e., $-p_W^B - t^B x^B + \alpha_W q_W^C \geq \max\{-p_N^B - t^B(1-x^B) + \alpha_N q_N^C, -(p_W^B + p_N^B) - t^B + \alpha_W q_W^C + \alpha_N q_N^C, 0\}$. These inequalities can be written as,

$$p_W^B \leq (\frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W}{2t^C})\alpha_W \tag{A.37}$$

$$p_N^B \geq (\frac{1}{2} + \frac{p_W^C - p_N^C - \beta_W}{2t^C})\alpha_N \tag{A.38}$$

Profits of the CSPs are,

$$R_W = p_W^B - f_W^B + (p_W^C - f_W^C)(\frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W}{2t^C}) \tag{A.39}$$

$$R_N = (p_N^C - f_N^C)(\frac{1}{2} + \frac{p_W^C - p_N^C - \beta_W}{2t^C}) \tag{A.40}$$

Configuration 3: worksites singlehome on the NWF CSP

The fraction of commuters on the WF CSP is,

$$q_W^C = \frac{1}{2} + \frac{p_N^C - p_W^C - \beta_N}{2t^C} \tag{A.41}$$

Worksites choose to singlehome on the WF CSP when $U_N^B \geq \max\{U_W^B, U_{NW}^B, 0\}$, i.e., $-p_N^B - t^B(1-x^B) + \alpha_N q_N^C \geq \max\{-p_W^B - t^B x^B + \alpha_W q_W^C, -(p_W^B + p_N^B) - t^B + \alpha_W q_W^C + \alpha_N q_N^C, 0\}$. These inequalities can be written as,

$$p_W^B \geq (\frac{1}{2} + \frac{p_N^C - p_W^C - \beta_N}{2t^C})\alpha_W \tag{A.42}$$

$$p_N^B \leq (\frac{1}{2} + \frac{p_W^C - p_N^C + \beta_N}{2t^C})\alpha_N \tag{A.43}$$

Profits of the CSPs are,

$$R_W = (p_W^C - f_W^C)(\frac{1}{2} + \frac{p_N^C - p_W^C - \beta_N}{2t^C}) \tag{A.44}$$

$$R_N = p_N^B - f_N^B + (p_N^C - f_N^C)(\frac{1}{2} + \frac{p_W^C - p_N^C + \beta_N}{2t^C}) \tag{A.45}$$

Configuration 4: worksites join neither of the CSPs

The proportion of commuters joining the WF CSP is the same as that in Configuration 1. Worksites do not want to join any CSP when $0 \geq \max\{-p_W^B - t^B x^B + \alpha_W q_W^C, -p_N^B - t^B(1-x^B) + \alpha_N q_N^C, -(p_W^B + p_N^B) - t^B + \alpha_W q_W^C + \alpha_N q_N^C, 0\}$. This requires each inequality (A.32) and (A.33) be reversed. The CSPs only make profits from the commuter side. There exists price range when some of the configurations overlap. To make the explanation more concise, we assume that the two CSPs set the same prices, i.e., $p_W^B = p_N^B = p^B, p_W^C = p_N^C = p^C$. Configuration 1, 2 and 3 are all consistent when $\max\{(\frac{1}{2} - \frac{\beta_N}{2t^C})\alpha_N, (\frac{1}{2} - \frac{\beta_N}{2t^C})\alpha_W\} \leq p^B \leq \min\{(\frac{1}{2} + \frac{\beta_W - \beta_N}{2t^C})\alpha_W, (\frac{1}{2} + \frac{\beta_N - \beta_W}{2t^C})\alpha_N\}$. Configuration 4 is the reverse of configuration 1, so it is impossible for them to overlap. Configuration 2, 3 and 4 are consistent at the same time when $\max\{(\frac{1}{2} + \frac{\beta_W - \beta_N}{2t^C})\alpha_W, (\frac{1}{2} + \frac{\beta_N - \beta_W}{2t^C})\alpha_N\} \leq p^B \leq \min\{(\frac{1}{2} + \frac{\beta_W}{2t^C})\alpha_W, (\frac{1}{2} + \frac{\beta_N}{2t^C})\alpha_N\}$.

One-sided cross-side network effects

The analysis is straightforward when the cross-side benefits are one-sided. In this section we will discuss about such cases. The simplified network effects will still unveil important insights from the duopoly model. If we do not consider the cross-side benefits of commuters, then $\beta_W = \beta_N = 0$. In this case, the equilibrium is described in the following theorem,

Theorem 5. *Let condition **(C1)-(C3)** hold and assume* $\beta_W = \beta_N = 0$*. Then the equilibrium is unique, CSPs will serve both sides of the market, with worksites multihome and commuters singlehome. The optimal prices for worksites are* $p_W^B = (\frac{1}{2} + \frac{f_N^C - f_W^C - \alpha^-}{6t^C})\alpha_W, p_N^B = (\frac{1}{2} + \frac{f_W^C - f_N^C + \alpha^-}{6t^C})\alpha_N$*. The equilibrium prices of commuters are depend on the parameter settings of the model. If* $\frac{f_N^C}{3} + \frac{2(f_W^C)}{3} + t^C \geq \frac{\alpha_N}{3} + \frac{2\alpha_W}{3}$ *and* $\frac{2(f_N^C)}{3} + \frac{f_W^C}{3} + t^C \geq \frac{2\alpha_N}{3} + \frac{\alpha_W}{3}$*, the optimal prices for commuters are,*

$$p_W^C = \frac{f_N^C - \alpha_N}{3} + \frac{2(f_W^C - \alpha_W)}{3} + t^C \tag{A.46}$$

$$p_N^C = \frac{2(f_N^C - \alpha_N)}{3} + \frac{f_W^C - \alpha_W}{3} + t^C \tag{A.47}$$

CSPs make profits,

$$R_W = -f_W^B + (\frac{f_N^C - f_W^C - \alpha^-}{6t^C} + \frac{1}{2})(\frac{f_N^C - f_W^C - \alpha^-}{3} + t^C) \tag{A.48}$$

$$R_N = -f_N^B + (\frac{f_W^C - f_N^C + \alpha^-}{6t^C} + \frac{1}{2})(\frac{f_W^C - f_N^C + \alpha^-}{3} + t^C) \tag{A.49}$$

If $\frac{f_N^C}{3} + \frac{2(f_W^C)}{3} + t^C < \frac{\alpha_N}{3} + \frac{2\alpha_W}{3}$ *and* $\frac{2(f_N^C)}{3} + \frac{f_W^C}{3} + t^C < \frac{2\alpha_N}{3} + \frac{\alpha_W}{3}$ *, the equilibrium prices for commuters are* $p_W^C = 0, p_N^C = 0$*. The profits of the CSPs are,*

$$R_W = -f_W^B + (\frac{f_N^C - f_W^C - \alpha^-}{6t^C} + \frac{1}{2})(\alpha_W - f_W^C) \tag{A.50}$$

$$R_N = -f_N^B + (\frac{f_W^C - f_N^C + \alpha^-}{6t^C} + \frac{1}{2})(\alpha_N - f_N^C) \tag{A.51}$$

Proof. It takes 2 steps to proof **Theorem 5**. First we show how we derive the equilibrium prices when both CSPs are willing to serve worksites. In the second step, we will explain why each CSP is better off by serving worksites.

Step (i): Suppose both CSPs are willing to serve the worksites, then the equilibrium prices follow the expressions presented in **Theorem 5**.

Since the decisions of commuters are not affected by worksites ($\beta_W = 0, \beta_N = 0$), the participation of commuters takes the form of equation (A.31). Worksites, knowing the decisions of commuters, choose the WF CSP if $p_W^B \leq q_W^C \alpha_W$, or choose the NWF CSP if $p_N^B \leq q_N^C \alpha_N$, as characterized in inequalities (A.37) and (A.43). Worksites' decisions of joining one CSP is independent of their decisions of joining the other CSP. Thus, CSPs will fully extract the surplus from worksites. In other words, each CSP will set the prices to worksites as high as possible. Set $p_W^B = q_W^C \alpha_W$, yields the profit function of the WF CSP. Similarly, the profit function of the NWF CSP can be obtained when setting $p_N^B = q_N^C \alpha_N$.

$$R_W = - f_W^B + (p_W^C + \alpha_W - f_W^C)(\frac{1}{2} + \frac{p_N^C - p_W^C}{2t^C}) \tag{A.52}$$

$$R_N = - f_N^B + (p_N^C + \alpha_N - f_N^C)(\frac{1}{2} + \frac{p_W^C - p_N^C}{2t^C}) \tag{A.53}$$

From the perspective of CSPs, the revenue from worksites can be regarded as a reduction to the marginal cost of commuters, from f_i^C to $f_i^C - \alpha_i$. Given that both CSPs serve worksites, the equilibrium is unique. Under condition **(C3)**, the profits of CSPs are non-negative. The profit maximization problems of CSPs are,

$$\frac{\partial R_i}{p_i^C} = 0 \quad \forall i \in \{W, N\}$$

which yield the price structure in **Theorem 5**.

Step (ii): Each CSP is better off when serving worksites.

In this part, we are going to prove that the WF CSP is better off when serving worksites. Similar proof applies to the NWF CSP. First, we study the cases when the equilibrium prices of commuters are positive, i.e., $\frac{f_N^C}{3} + \frac{2(f_W^C)}{3} + t^C \geq \frac{\alpha_N}{3} + \frac{2\alpha_W}{3}$. Profit functions are given in equation (A.48) and (A.49). Suppose on the contrary, the WF CSP stops serving worksites, its profit is,

$$R_W = (p_W^C - f_W^C)(\frac{1}{2} + \frac{\frac{2(f_N^C - \alpha_N)}{3} + \frac{f_W^C - \alpha_W}{3} + t^C - p_W^C}{2t^C}) \tag{A.54}$$

when worksite set price p_W^C to commuters. The profit function is maximized at $p_W^C = \frac{f_N^C}{3} + \frac{f_W^C}{3} - \frac{\alpha_W}{6} - \frac{\alpha_N}{3} + t^C$. The WF CSP makes positive profits only if the price is larger than cost, i.e., $t^C > \frac{f_W^C - f_N^C}{3} + \frac{\alpha_W}{6} + \frac{\alpha_N}{3}$, in which case it obtains profit,

$$\widetilde{R}_W = \frac{1}{72t^C}(2\alpha_N + \alpha_W - 2f_N^C + 2f_W^C - 6t^C)^2$$

Under condition **(C3)**, $\widetilde{R}_W$ is less than the profit obtained in Theorem **5** equation (A.48). The proof is as follows.

Condition **(C3)** states that $f_i^C < \frac{3}{4}\alpha_W$, thus, $-\alpha_W^2 + \frac{4}{3}\alpha_W(f_W^C - f_N^C) < 0$, so that,

$$
\begin{aligned}
& \widetilde{R}_W < R_W \\
\iff\ & \frac{1}{72t^C}(2\alpha_N + \alpha_W - 2f_N^C + 2f_W^C - 6t^C)^2 < -f_W^B + (\frac{f_N^C - f_W^C - \alpha^-}{6t^C} + \frac{1}{2})(\frac{f_N^C - f_W^C - \alpha^-}{3} + t^C) \\
\iff\ & 4\alpha_N\alpha_W + \alpha_W^2 - 4\alpha_W f_W^C - 12\alpha_W t^C < -8\alpha_N\alpha_W + 4\alpha_W^2 + 8\alpha_W f_N^C - 8\alpha_W f_W^C + 24\alpha_W t^C - 72 f_W^B t^C \\
\iff\ & 12\alpha_W(\alpha_N - 3t^C) < 3\alpha_W - 72f_W^B t^C + 4\alpha_W(f_W^C - f_N^C) \\
\iff\ & 4\alpha_W(3t^C - \alpha_N) > -\alpha_W^2 + 24 f_W^B t^C + \frac{4}{3}\alpha_W(f_W^C - f_N^C) \\
\iff\ & 8\alpha_W t^C > -\alpha_W^2 + 24 f_W^B t^C + \frac{4}{3}\alpha_W(f_W^C - f_N^C) \quad (-\alpha_W^2 + \frac{4}{3}\alpha_W(f_W^C - f_N^C) < 0) \\
\iff\ & \alpha_W t^C > 3 f_W^B t^C \\
\iff\ & \frac{1}{3}\alpha_W > f_W^B
\end{aligned}
$$

Condition **(C3)** states that $\frac{1}{3}\alpha_W > f_W^B$. So the inequality $\frac{1}{72t^C}(2\alpha_N + \alpha_W - 2f_N^C + 2f_W^C - 6t^C)^2 < -f_W^B + (\frac{f_N^C - f_W^C - \alpha^-}{6t^C} + \frac{1}{2})(\frac{f_N^C - f_W^C - \alpha^-}{3} + t^C)$ holds, which proves that $\widetilde{R}_W < -f_W^B + (\frac{f_N^C - f_W^C - \alpha^-}{6t^C} + \frac{1}{2})(\frac{f_N^C - f_W^C - \alpha^-}{3} + t^C)$. In the same way, one can show that when $\frac{f_N^C}{3} + \frac{2(f_W^C)}{3} + t^C < \frac{\alpha_N}{3} + \frac{2\alpha_W}{3}$, the WF CSP will lose profit if not serving worksites. ■

Theorem 5 finds the conditions under which an equilibrium exists where worksites multihome and have their surplus fully extracted.